

Title: Graviton propagator from EPRL spinfoam model

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Abstract: We derive geometric correlation functions in the new spinfoam model with coherent states techniques, making connection with quantum Regge calculus and perturbative quantum gravity. In particular we recover the expected scaling with distance for all components of the propagator. We expect the same technique to be well-suited for other spinfoam models.

Graviton propagator from new spinfoam models

Claudio Perini

Centre de Physique Théorique de Luminy, Marseille
Università degli studi Roma Tre

Perimeter Institute 2009

Harmonic-radial gauge in classical EM and GR

CP, Magliaro, Rovelli

Wave-packet propagation in QG

CP, Magliaro, Rovelli - CP, Alesci, Bianchi, Magliaro

Asymptotics of fusion coefficients

CP, Alesci, Bianchi, Magliaro

Divergencies (?) of new spinfoam models

CP, Rovelli, Speziale

Graviton propagator in the new models

CP, Magliaro

Outline of the talk

- Graviton propagator - the physical picture

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- Spinfoams
- New models

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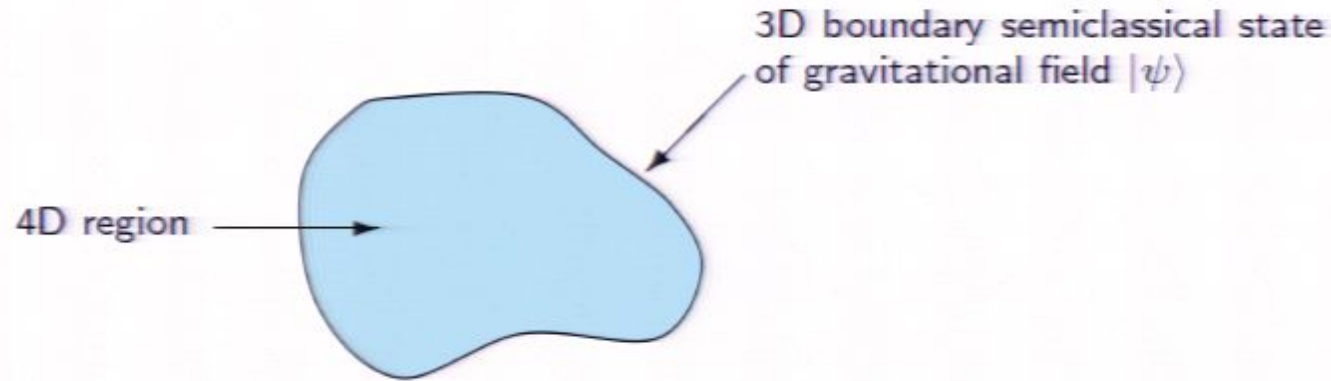
- Graviton propagator - the physical picture
- Spinfoams
- New models
- The new propagator

Outline of the talk

- Graviton propagator - the physical picture
- Spinfoams
- New models
- The new propagator
- Conclusions, work in progress and outlook

Graviton propagator - the physical picture

Colosi, Conrady, Doplicher
Modesto, Oeckl, Rovelli, Testa



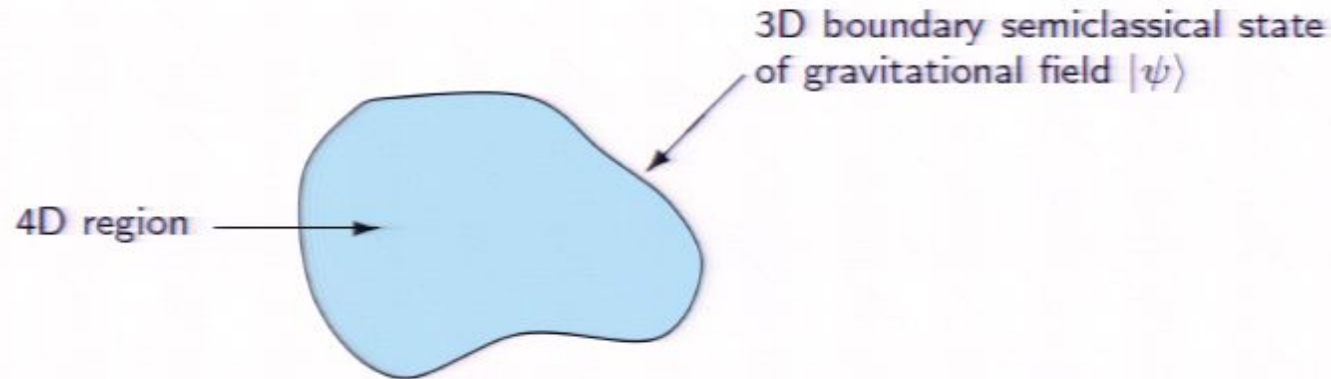
Spinfoams assign an amplitude to this boundary state: $\langle W|\psi\rangle$

$$\text{If } \psi = \psi_{\text{in}}^* \otimes \psi_{\text{out}} \rightarrow \langle W|\psi\rangle \sim \langle \psi_{\text{out}}|e^{iHt}|\psi_{\text{in}}\rangle$$

$\langle W|$ codes the dynamics of quantum general relativity.

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$\langle W|$ codes the dynamics of quantum general relativity. Define

$$\langle \cdot \rangle = \frac{\langle W| \cdot |\psi\rangle}{\langle W|\psi\rangle}$$

that we call physical expectation value, in contrast to the kinematical expectation value

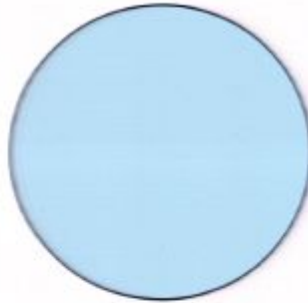
$$\langle \cdot \rangle_{\text{kin}} = \frac{\langle \psi| \cdot |\psi\rangle_{\text{kin}}}{\langle \psi|\psi\rangle_{\text{kin}}}$$

Graviton propagator - the physical picture



boundary semiclassical state coding
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Graviton propagator - the physical picture



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Then construct an excited boundary state by acting with metric field operators at two points:

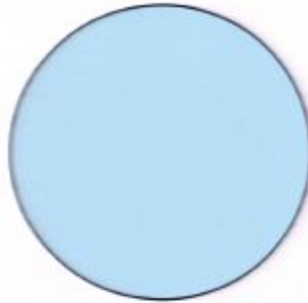
$$(g(x) - \langle g(x) \rangle)(g(y) - \langle g(y) \rangle) |\psi\rangle$$

We can construct the analog of the linearized gravity propagator $\langle 0 | h_{\mu\nu} h_{\rho\sigma} | 0 \rangle$:

$$\langle (g(x) - \langle g(x) \rangle)(g(y) - \langle g(y) \rangle) \rangle = \langle g(x)g(y) \rangle - \langle g(x) \rangle \langle g(y) \rangle$$

It is called **2-point function**, or **graviton propagator**, in (nonperturbative) quantum gravity. *What is it?*

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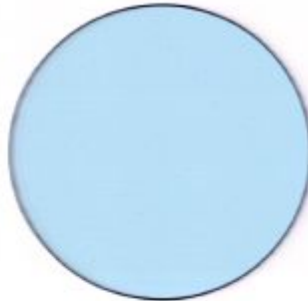
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It is called **2-point function**, or **graviton propagator**, in (nonperturbative) quantum gravity. *What is it?* It represents the probability of detecting two excitations at the spacetime points x and y over flat space-time (here we consider only the Euclidean signature). More correctly, by flat space-time we mean a semiclassical state peaked over this classical geometry.

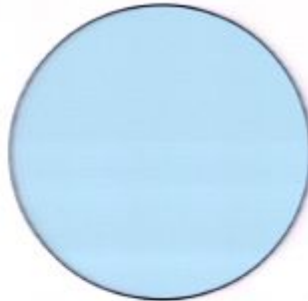
Graviton propagator - the physical picture



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OBS: how can a 3D boundary state code the flat 4D geometry? the boundary data are intrinsic and extrinsic curvature, which classically determine, via Einstein equations, the geometry in the interior.

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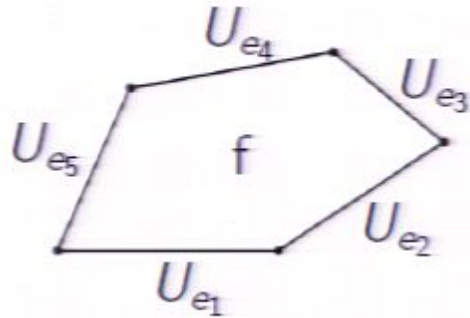
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Perturbativity inside non-perturbativity!

Spinfoams - discretization of classical

Start with a triangulated manifold (faces/triangles, edges/tetrahedra, vertices/4-simplices) and discretize the Holst-Plebanski action for GR:

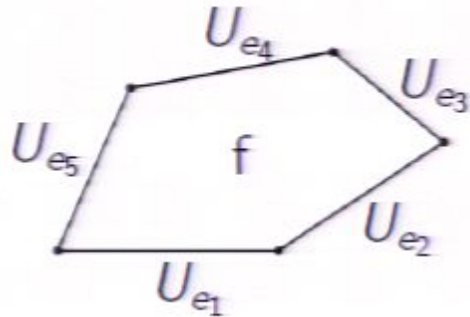


$$S_{\text{discr}} = \sum_f \text{tr}(*B_f U_f + \frac{1}{\gamma} B_f U_f)$$

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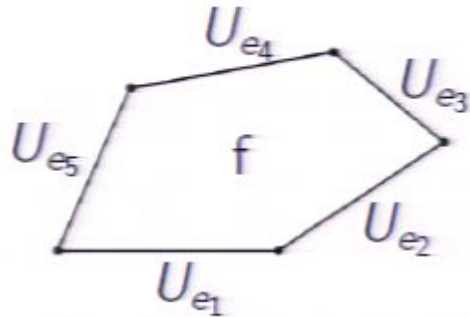
- **Simplicity constraint** $*B_f \cdot B_f = 0$
- **Cross-simplicity constraint** $*B_f \cdot B_{f'} = 0 \quad f, f' \subset t$
- **Closure constraint** $\sum_{f \subset t} B_f = 0$

Replace cross-simplicity by:

$$\exists n_t \text{ s.t. } n_t \perp B_f \quad f \subset t$$

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Boundary variables: $B_f(t_1), B_f(t_2), U_f(t_1, t_2)$

The variable conjugate to U_f is $J_f = *B_f + \frac{1}{\gamma} B_f$

Spinfoams - quantization

Quantize the theory choosing an appropriate Hilbert space. Similarly to lattice Yang-Mills theory, define the kinematical Hilbert space as ($G = \text{Spin}(4)$)

$$L^2(G^L) \quad L = \# \text{ of boundary faces (links)}$$

and quantize

$$J_f(t_1) \longrightarrow \text{l.i. vector field}$$

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$$f(g) = f(hgh^{-1})$$

Then an orthonormal basis of this subspace is given by spin-networks where each link is labeled by irreps of $\text{Spin}(4) = \text{SU}(2) \otimes \text{SU}(2)$, i.e. couples of spins (j^+, j^-) , and each node is labeled by intertwiners between irreps

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- **Simplicity constraint** restricts these spin labels to be of the form

$$j^+ = \left\lfloor \frac{1 + \gamma}{1 - \gamma} \right\rfloor j^-$$

Spinfoams - the model of Barrett and Crane

- **Strong cross-simplicity constraint** In the Barrett-Crane model the cross-simplicity constraint is imposed (too) strongly and restricts the intertwiner space to the 1-dimensional space spanned by the BC intertwiner. The vertex amplitude (4-simplex amplitude) depends *only* on the spins. Two problems arise:

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the boundary state space **doesn't match** with the one of LQG
and

despite some components of the graviton propagator work well (Bianchi, Modesto, Rovelli, Speziale), the other components have the **wrong scaling** (Alesci, Rovelli)

These problems could trace back to the fact that we have overconstrained the system; the intertwiner space is too small!

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Each (j^+, j^-) component in the Peter-Weyl decomposition of $L^2(\text{Spin}(4))$ can be further decomposed:

$$(j^+, j^-) = j^+ \otimes j^- = |j^+ - j^-| \oplus \dots \oplus (j^+ + j^-)$$

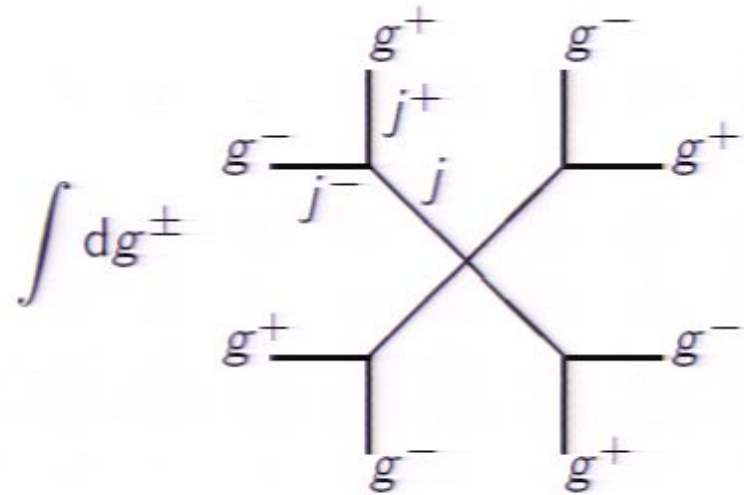
The cross-simplicity constraint selects the component j satisfying the following relation

$$j^\pm = \frac{|1 \pm \gamma|}{2} j$$

i.e. the highest or the lowest weight.

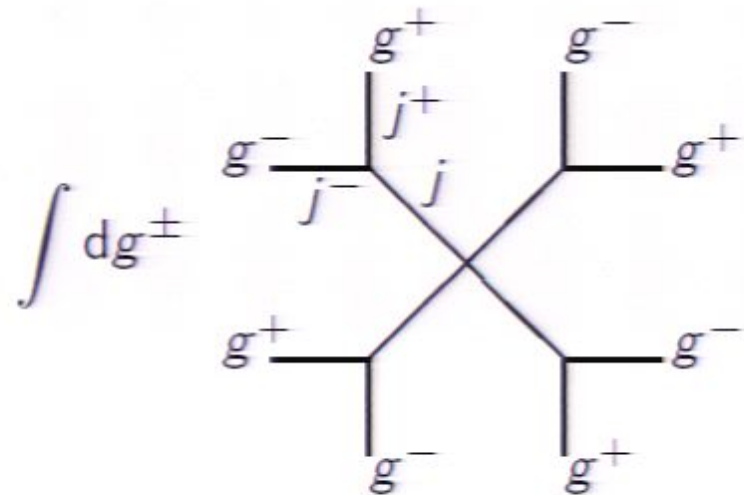
Spinfoams - new models (2)

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⇒ intertwiners are of the form (1 ↔ 1 with LQG intertwiner space):



The 4-simplex amplitude is the contraction of 5 of them. In the recoupling basis:

$$W_{\text{EPRL}}(j, i) = \sum_{i_a^+, i_a^-} 15j(j^+, i^+) 15j(j^-, i^-) f(i_1, i_1^+, i_1^-) \dots f(i_5, i_5^+, i_5^-)$$

Where

$$f(i, i^+, i^-) = i^{abcd} C_a^{a^+ a^-} \dots C_d^{d^+ d^-} i_{a^+ b^+ c^+ d^+}^+ i_{a^- b^- c^- d^-}^-$$

Coherent intertwiners

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$$\mathbb{1}_j = \sum_m |j, m\rangle \langle j, m| = \int dg |j, g\rangle \langle j, g| = \int dn |j, n\rangle \langle j, n|$$

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so that coherent states form an overcomplete basis of H_j . We define a coherent (4-valent) intertwiner by taking the tensor product of four coherent states and then projecting onto the invariant subspace:

$$|j_1 \dots j_4, n_1 \dots n_4\rangle = \int dg g \triangleright |j_1, n_1\rangle \otimes \dots \otimes |j_4, n_4\rangle$$

Spinfoams - EPRL $_{\gamma}$ model (3)

Consider the EPRL $_{\gamma < 1}$ or FK $_{\gamma < 1}$ vertex in the coherent intertwiner basis. Using the following decomposition property of coherent states

$$\begin{array}{c}
 j_1 \quad j_2 \\
 \diagdown \quad \diagup \\
 \text{---} \\
 j_1 + j_2 \\
 \text{---} \\
 n
 \end{array}
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 j_1 \\
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we can untie the nodes (see 2) and rewrite the vertex amplitude as a product of contractions of coherent states:

$$W(j, n) = \int d^5 g^{\pm} \prod_{a < b} \langle -n_{ab} | (g_a^+)^{-1} g_b^+ | n_{ba} \rangle^{2j_{ab}^+} \prod_{a < b} \langle -n_{ab} | (g_a^-)^{-1} g_b^- | n_{ba} \rangle^{2j_{ab}^-}$$

We have used also the tensoring property of coherent states

$$|1/2, n\rangle \underbrace{\otimes \dots \otimes}_{2j \text{ times}} |1/2, n\rangle = |j, n\rangle$$

to write in powers of the 1/2 representation.

$$W(j, n) = \int dg^\pm e^{S(j, n, g)} \quad S = \sum_{a < b} 2j_{ab}^+ \log \langle -n_{ab} | (g_a^+)^{-1} g_b^+ | n_{ba} \rangle + (+ \leftrightarrow -)$$

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$$\max \operatorname{Re} S \implies g_a^\pm n_{ab} = -g_b^\pm n_{ba}$$

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Solutions exist if the normals (boundary data) are s.t. they come from a geometrical 4-simplex defined by the ten areas (boundary spins): $n = n(j)$.

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It follows:

$$(g_a^\pm)^{-1} g_b^\pm | n_{ba} \rangle = e^{i\phi_{ab}^\pm} | -n_{ab} \rangle \quad \phi_{ab}^+ - \phi_{ab}^- = \Phi_{ab} \text{ dihedral angle between } t_a \text{ and } t_b$$

Then

$$W(j, n) \sim \begin{cases} A(j)(e^{iS_{\text{Regge}}} + \text{c.c}) & n = n(j) \\ \text{suppressed} & \text{otherwise} \end{cases} \quad S_{\text{Regge}} = \sum \gamma j_{ab} \Phi_{ab}(j)$$

The new propagator

$$G_{nm}^{abcd} = \langle \mathbf{E}_n^a \cdot \mathbf{E}_n^b \mathbf{E}_m^c \cdot \mathbf{E}_m^d \rangle - \langle \mathbf{E}_n^a \cdot \mathbf{E}_n^b \rangle \langle \mathbf{E}_m^c \cdot \mathbf{E}_m^d \rangle$$

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- **New boundary state** superposition of semiclassical coherent 4-simplices, peaked around $j_{ab} = j_0$, and $\delta j / j_0 \rightarrow 0$

$$|\Psi\rangle = \sum_j \psi(\mathbf{j}) |\mathbf{j}\rangle_0 \quad \psi(\mathbf{j}) \equiv e^{-\frac{1}{j_0} \delta \mathbf{j}^T \alpha \delta \mathbf{j} + i\gamma \phi_0 \sum_{a < b} \delta j_{ab}}$$

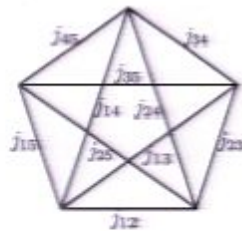
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Key assumption: $|\mathbf{j}\rangle_0$ is the pentagonal state with coherent intertwiners at nodes.



$$|\mathbf{j}\rangle_0 = |j_{ab}, n_{1a}(j), \dots, n_{5a}(j)\rangle$$

It is well-defined as a fluctuation around the equilateral configuration.

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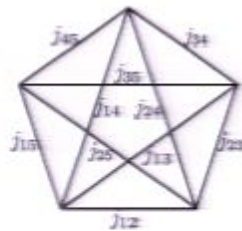
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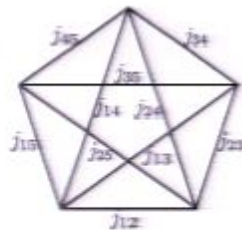
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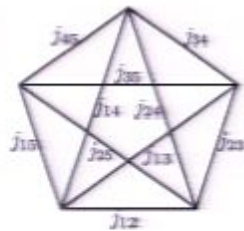
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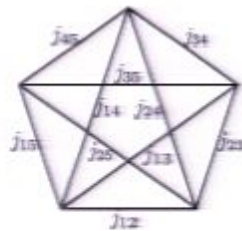
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The new propagator (2)

- Grasping operators** The grasping operator \mathbf{E}_n^a acts at node n creating a 3-valent node “near” the original node along the link na . Double-grasping creates two nodes and joins the free hands.

$$\mathbf{E}_n^a \cdot \mathbf{E}_n^b \begin{array}{c} \text{---} j_{na} \text{---} \\ \diagdown \quad \diagup \\ \text{---} j_{nb} \text{---} \end{array} = \begin{array}{c} \text{---} j_{na} \text{---} \\ \diagdown \quad \diagup \\ \text{---} j_{nb} \text{---} \end{array} \quad (\text{double-grasping action})$$

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The propagator is a simplicial non-perturbative version of the standard propagator of perturbative theory:

$$G_{nm}^{abcd} \longrightarrow G^{\mu\nu\rho\sigma}(x, y) = \langle 0 | h^{\mu\nu}(x) h^{\rho\sigma}(y) | 0 \rangle$$

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Consider the action on a coherent 4-simplex,

$$\mathbf{E}_n^a \cdot \mathbf{E}_n^b |j\rangle_0 \quad \text{and} \quad \mathbf{E}_n^a \cdot \mathbf{E}_n^b \mathbf{E}_m^c \cdot \mathbf{E}_m^d |j\rangle_0$$

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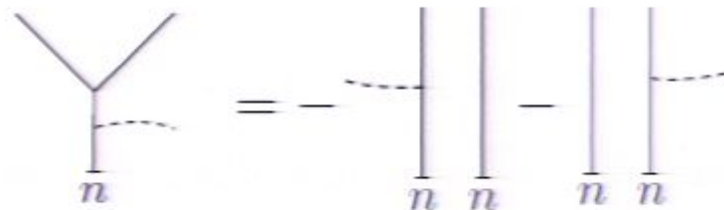
The diagram shows a Y-junction with a vertical line at the bottom and two diagonal lines at the top. A dashed line (grasp) is attached to the bottom vertical line. This is equal to the difference of two configurations of two vertical lines. In the first configuration, the grasp is on the left vertical line. In the second configuration, the grasp is on the right vertical line.

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$$\int d^5 g^\pm \langle j_{na}, -n_{na} | \sigma(g_n^+)^{-1} g_a^+ | j_{na}, n_{an} \rangle \langle j_{nb}, -n_{nb} | \sigma(g_n^+)^{-1} g_b^+ | j_{nb}, n_{bn} \rangle$$

\times (non-grasped factors) $+$...

It is just the new vertex evaluated on a grasped-intertwiner configuration. This expression seems quite complicated *but...*

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... we have the identity:

$$\langle j, n_1 | \sigma | j, n_2 \rangle = j \frac{\langle 1/2, n_1 | \sigma | 1/2, n_2 \rangle}{\langle 1/2, n_1 | 1/2, n_2 \rangle} \langle j, n_1 | j, n_2 \rangle \equiv j \frac{\langle n_1 | \sigma | n_2 \rangle}{\langle n_1 | n_2 \rangle} \langle j, n_1 | j, n_2 \rangle$$

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Since we are interested in the large distance (large j_0) limit, we may regard \mathbf{A} as an insertion and evaluate the integral for large spins with the saddle point method. Remarkably,

$$\mathbf{A}_n^a |_{\text{saddle}} = j_{na} n_{na}$$

namely, on the saddle point \mathbf{A} (that we recall we obtained recasting the action of the grasping operator as an insertion in the group integral) is the classical simplicial quantity measured by the grasping operator.

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All ingredients are ready. The propagator in terms of \mathbf{A} is:

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- **Nondiagonal components:** $a \neq b, c \neq d$ Here we have one contribution from the Regge sector and one contribution from the non-Regge sector:

$$G_{nm}^{abcd} \sim \partial_j(j_{na} j_{nb} n_{na} \cdot n_{nb}) \partial_j(j_{mc} j_{md} n_{mc} \cdot n_{md}) (iS_R'' - \frac{\alpha}{j_0})^{-1} + \text{non-Regge term}$$

The new propagator (7)

Final remarks

The new propagator has the right scaling properties. When all the normalizations are correctly implemented, it reproduces the Newtonian law

$$\frac{1}{L^2}$$

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The new propagator (4)

... we have the identity:

$$\langle j, n_1 | \sigma | j, n_2 \rangle = j \frac{\langle 1/2, n_1 | \sigma | 1/2, n_2 \rangle}{\langle 1/2, n_1 | 1/2, n_2 \rangle} \langle j, n_1 | j, n_2 \rangle \equiv j \frac{\langle n_1 | \sigma | n_2 \rangle}{\langle n_1 | n_2 \rangle} \langle j, n_1 | j, n_2 \rangle$$

Using this, the (grasped) 4-simplex evaluation becomes simpler...

$$\langle W | \mathbf{E}_n^a \cdot \mathbf{E}_n^b | \mathbf{j} \rangle_0 = \int d^5 g^\pm \mathbf{A}_n^a \cdot \mathbf{A}_n^b e^{S(g)}$$

where

$$\mathbf{A}_n^a = A_n^{a+} + A_n^{a-} \quad A_n^{a\pm} = j_{na}^\pm \frac{\langle -n_{na} | \sigma (g_n^\pm)^{-1} g_a^\pm | n_{an} \rangle}{\langle -n_{na} | (g_n^\pm)^{-1} g_a^\pm | n_{an} \rangle}$$

Since we are interested in the large distance (large j_0) limit, we may regard \mathbf{A} as an insertion and evaluate the integral for large spins with the saddle point method. Remarkably,

$$\mathbf{A}_n^a |_{\text{saddle}} = j_{na} n_{na}$$

namely, on the saddle point \mathbf{A} (that we recall we obtained recasting the action of the grasping operator as an insertion in the group integral) is the classical simplicial quantity measured by the grasping operator.

The new propagator (5)

All ingredients are ready. The propagator in terms of \mathbf{A} is:

$$\frac{\sum_j \psi(j) \int dg^\pm \mathbf{A} \cdot \mathbf{A} \mathbf{A} \cdot \mathbf{A} e^S}{\sum_j \psi(j) \int dg^\pm e^S} = \frac{\sum_j \psi(j) \int dg^\pm \mathbf{A} \cdot \mathbf{A} e^S}{\sum_j \psi(j) \int dg^\pm e^S} \frac{\sum_j \psi(j) \int dg^\pm \mathbf{A} \cdot \mathbf{A} e^S}{\sum_j \psi(j) \int dg^\pm e^S}$$

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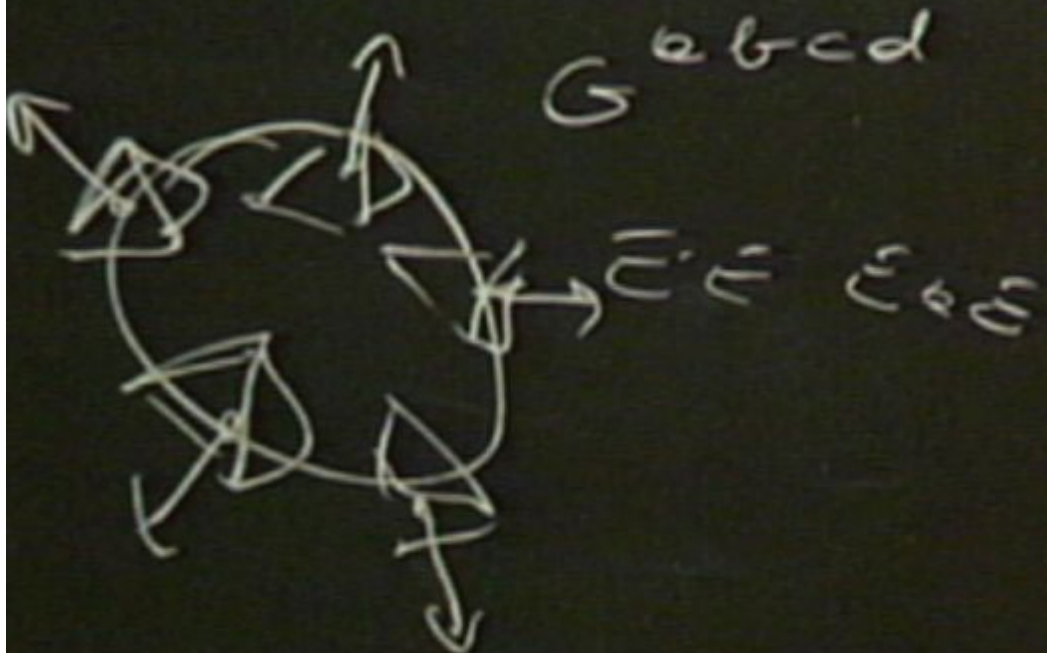
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- We should check what happens when going beyond the single 4-simplex level and/or switching to the Lorentzian signature.

We could...we could...we could...

**We could...we could...we could...
That's why I love physics!**

thank you for the attention

**special thanks to E. Alesci, E. Bianchi, R. Pereira, C. Rovelli
for long beautiful discussions together**



$\partial_m h^{mn} = 0$



$$g_{mn} = \Lambda$$

$$\rightarrow h_{mn} = 0$$

$$h_{mp} = 0$$