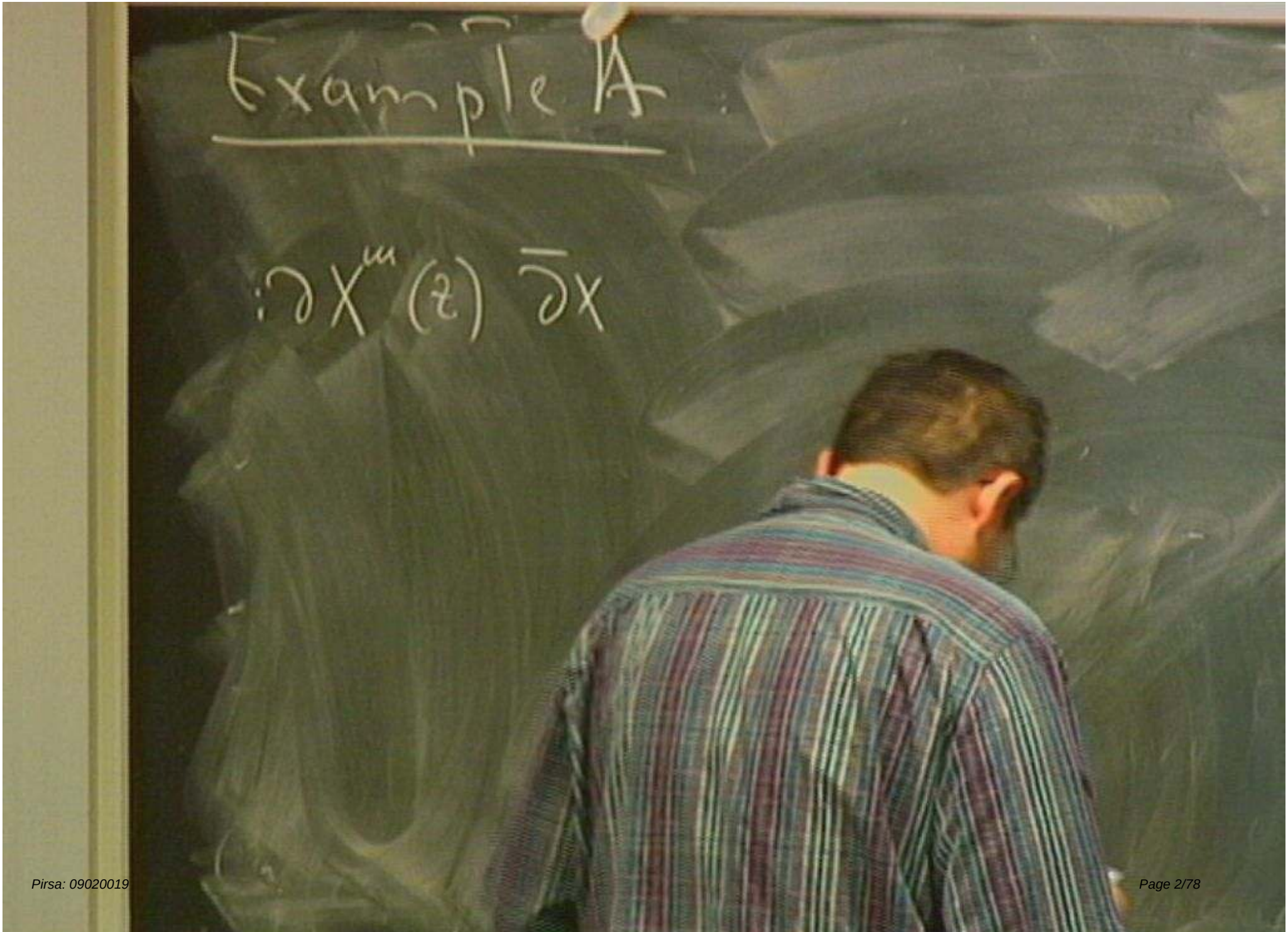
Title: Introduction to the Bosonic String Part B

Date: Feb 13, 2009 12:00 PM

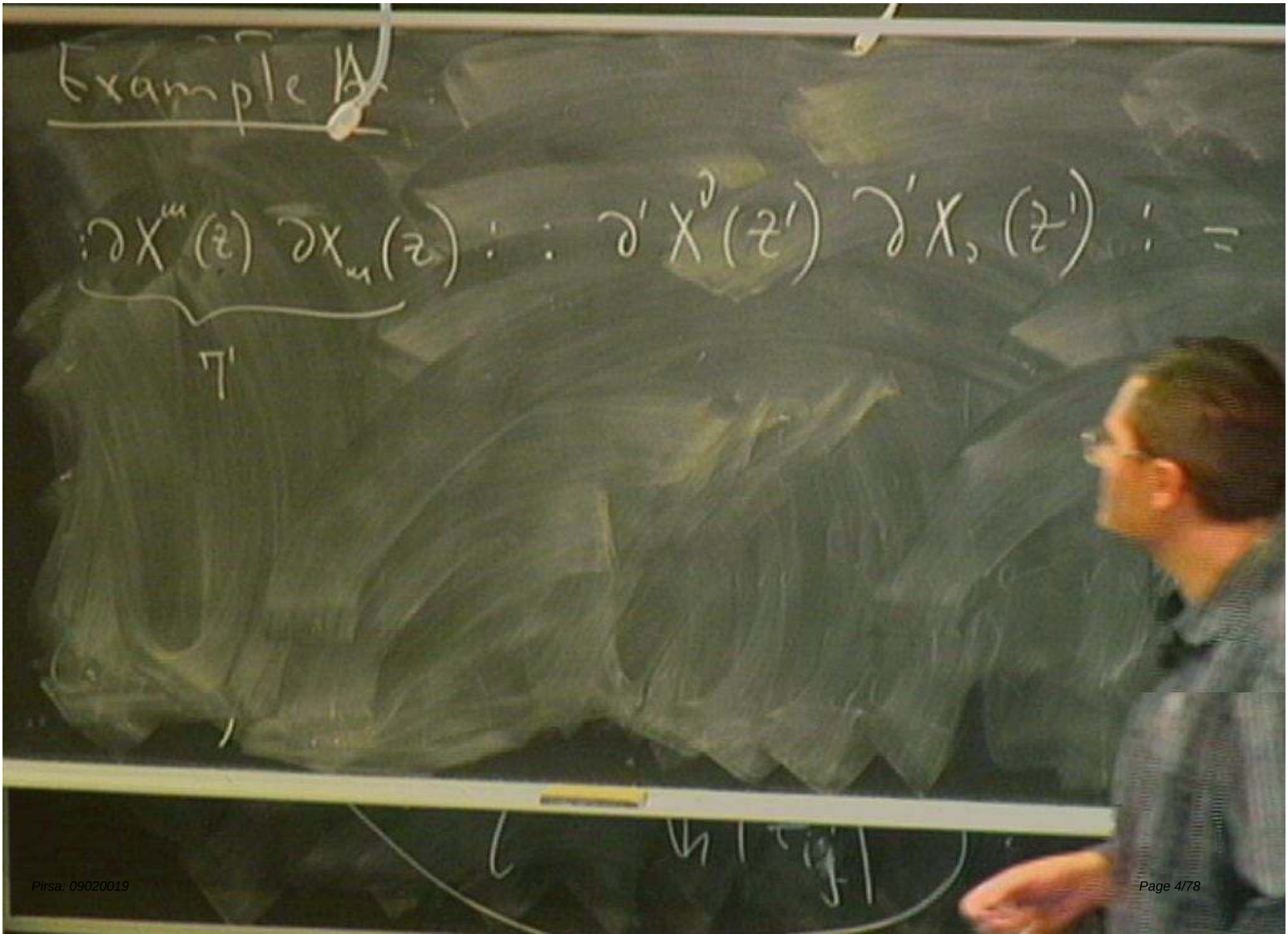
URL: http://pirsa.org/09020019

Abstract: This course provides a thorough introduction to the bosonic string based on the Polyakov path integral and conformal field theory. We introduce central ideas of string theory, the tools of conformal field theory, the Polyakov path integral, and the covariant quantization of the string. We discuss string interactions and cover the tree-level and one loop amplitudes. More advanced topics such as T-duality and D-branes will be taught as part of the course. The course is geared for M.Sc. and Ph.D. students enrolled in Collaborative Ph.D. Program in Theoretical Physics. Required previous course work: Quantum Field Theory (AM516 or equivalent). The course evaluation will be based on regular problem sets that will be handed in during the term. The primary text is the book: 'String theory. Vol. 1: An introduction to the bosonic string. J. Polchinski (Santa Barbara, KITP) . 1998. 402pp. Cambridge, UK: Univ. Pr. (1998) 402 p.' All interested students should contact Alex Buchel at abuchel@uwo.ca as soon as possible.


Example A
:∂X^μ(z) ∂̄x

Example A

$$:\partial X^m(z)\,\partial X_m(z): \;:\; \partial' X^0(z')\,\partial' X_0(z'):$$


Example
$:\partial X^m(z)\,\partial X_m(z):\;:\partial' X^0(z')\,\partial' X_0(z'):\;=$
η'

Example 2:

$$:\partial X^m(z)\,\partial X_m(z):\;:\partial' X^0(z')\,\partial' X_0(z'):\;=$$

related to stress energy tensor

$=$

Example II

$:\partial X^\mu(z)\,\partial X_\mu(z):\;:\partial' X^\nu(z')\,\partial' X_\nu(z'):\;=$

related to stress energy tensor

$=:\partial x^\mu \partial X_\mu\;\partial x^\nu \partial X_\nu:$

z $\quad$ z'

Example A

$:\partial X^\mu(z)\,\partial X_\mu(z):\;:\partial' X^\nu(z')\,\partial' X_\nu(z'):\;=$

related to stress energy tensor

$=:\bar\partial x^\mu \partial X_\mu\;\partial x^\nu \partial X_\nu:$

z $\quad$ z'

no singularities

...traction = replace a pair of fields

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Example A

$:\partial X^\mu(z)\,\partial X_\mu(z):\;:\partial' X^\nu(z')\,\partial' X_\nu(z'):=$

related to stress energy tensor

$=:\partial X^\mu \partial X_\mu\;\partial X^\nu \partial X_\nu: -\frac{d'}{2}\cdot$

z z'

no singularities

subtraction = replace a pair of fields

$\frac{1}{2}d'\eta^{\mu_i\mu_j}\ln|z_{ij}|^2$

Example A

$$:\underbrace{\partial X^m(z)\,\partial X_m(z)}_{F}: \;:\partial' X^\nu(z')\,\partial' X_\nu(z'): \;=$$

related to stress energy tensor

$$=: \partial x^m \partial X_m \; \partial x^\nu \partial X_\nu : \; - \frac{d'}{2} \cdot 4$$

z $\quad$ z'

no singularities

subtraction = replace a pair of fields

$$\frac{1}{2} d' \eta^{\mu;\mu} \ln|z_{ij}|^2$$

Example A

$$:\underbrace{\partial X^\mu(z)\,\partial X_\mu(z)}_{F}: \;:\partial' X^\nu(z')\,\partial' X_\nu(z'):\;=$$

related to stress energy tensor

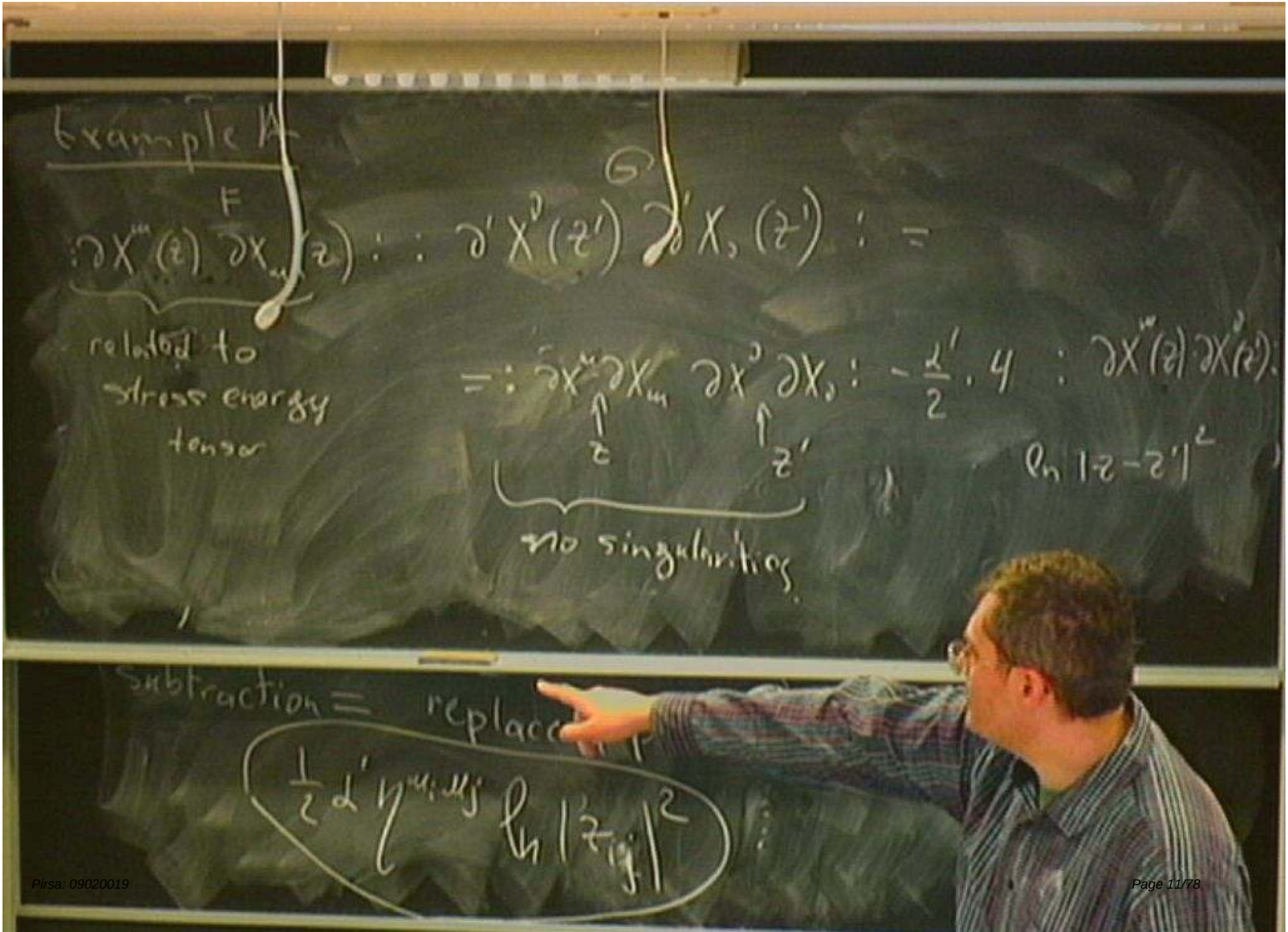
$$=:\partial X^\mu \partial X_\mu\;\partial X^\nu \partial X_\nu: \;-\frac{d'}{2}\cdot 4\;:\partial X^\mu(z)\,\partial X^\nu(z):$$

z z'

no singularities

Subtraction = replace a pair of fields

$$\frac{1}{2}d'\eta^{\mu_i\mu_j}\ln|z_{ij}|^2 \;;$$


Example A
related to stress energy tensor
no singularities
Subtraction = replace
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Example A

$$:\partial X^m(z)\,\partial X_m(z): \;:\; \partial' X^\nu(z')\,\partial' X_\nu(z') \;:\; =$$

related to stress energy tensor

$$=: \partial X^m \partial X_m\, \partial X^\nu \partial X_\nu : \; - \frac{d'}{2} \cdot 4 \; : \partial X^m(z)\,\partial X^\nu(z): \; \lim_{z' \to z} \left[\partial\partial' \ln |z-z'|^2 \right]$$

z z'

no singularities

Subtraction = replace a pair of fields

$$\frac{1}{2} d' \eta^{\mu_i \mu_j} \ln |z_{ij}|^2 \; ;$$

related to stress energy tensor

$=: \partial X^\mu \partial X_\mu \; \partial X^\nu \partial X_\nu : - \frac{1}{2} \cdot 4 \; : \partial X^\mu(z) \partial X^\nu(z) :$

z z'

$\int dz' \ln |z - z'|^2$

no singularities

$+ \frac{1}{2!} \left(- \frac{1}{2} \right)^2$

related to stress energy tensor

$=: \partial X^\mu \partial X_\mu \; \partial X^\nu \partial X_\nu : \; - \frac{1}{2} \cdot 4 \; : \partial X^\mu(z) \partial X_\mu(z)$

z z'

no singularities

$\int w \, d[\partial z' \ln |z - z'|^2]$

$+ \frac{1}{2!} \left(- \frac{1}{2} \right)^2 4$

related to stress energy tensor

$$=: \partial X^\mu \partial X_\mu \; \partial X^\nu \partial X_\nu : \; - \frac{d'}{2} \cdot 4 \; : \partial X_F^\mu(z) \cdot \partial X_G^\nu(z') :$$

z z'

no singularities

$$\cdot \eta_{\mu\nu} \ln\left[\partial\partial' \ln|z-z'|^2\right]$$

$$+ \frac{1}{2!}\left(-\frac{d'}{2}\right)^2 4\, \eta^{\mu\nu}\eta_{\mu\nu}\left(\partial\partial' \ln|z-z'|\right)^2 \cdot (\;\;)$$

related to stress energy tensor

$$=: \partial X^\mu \partial X_\mu \; \partial X^\nu \partial X_\nu : - \frac{1}{2} \cdot 4 \; : \partial X^\mu_F(z) \partial X^\nu_G(z) :$$

z z'

no singularities

$$\eta_{\mu\nu} \left[\partial \partial' \ln |z - z'|^2 \right]$$

$$+ \frac{1}{2!} \left(-\frac{1}{2} \right)^2 4 \, \eta^{\mu\nu} \eta_{\mu\nu} \left(\partial \partial' \ln |z - z'| \right)^2 + 0$$

$$=: \partial^\mu X \partial X_\mu \, \partial' X^\nu \partial' X_\nu : + \int (z')^2$$

related to stress energy tensor

$$=: \partial X^\mu \partial X_\mu \; \partial' X^\nu \partial' X_\nu : \; - \frac{1}{2} \cdot 4 \; : \partial X^\mu(z) \partial X_\mu(z') :$$

z z'

no singularities

$$\eta^{\mu\nu} \partial_z \partial_{z'} \left[\ln |z - z'|^2 \right]$$

$$+ \frac{1}{2!} \left(- \frac{1}{2} \right)^2 4 \, \eta^{\mu\nu} \eta_{\mu\nu} \left(\partial \partial' \ln |z - z'| \right)^2 + \cdots$$

$$=: \partial^\mu X \partial X_\mu \, \partial' X^\nu \partial' X_\nu : + \int (z')^2 \cdot D$$

related to stress energy tensor

$$=: \partial X^\mu \partial X_\mu \; \partial X^\nu \partial X_\nu :\; -\frac{d'}{2}\cdot 4 \;:\; \partial X^\mu(z)\,\partial X_\mu(z')\,:$$

z z'

no singularities

$$\text{Wick}\left[\partial\partial' \ln|z-z'|^2\right]$$

$$+\frac{1}{2!}\left(-\frac{d'}{2}\right)^2 4\,\eta^{\mu\nu}\eta_{\mu\nu}\left(\partial\partial' \ln|z-z'|\right)^2 + 0$$

$$=: \partial^\mu X \partial X_\mu \; \partial' X^\nu \partial' X_\nu :\; + \left\{ \frac{(d')^2}{2}\,\frac{D}{(z-z')^4} \right.$$

related to stress energy tensor

$$=: \partial X^\mu \partial X_\mu \; \partial' X^\nu \partial' X_\nu : - \frac{1}{2} \cdot 4 \; : \partial X^\mu(z) \partial X^\nu(z') : \underbrace{\eta_{\mu\nu} \left[\partial\partial' \ln |z-z'|^2 \right]}_{\text{no singularities}}$$

z z'

$$+ \frac{1}{2!} \left(-\frac{1}{2} \right)^2 4 \, \eta^{\mu\nu} \eta_{\mu\nu} \left(\partial\partial' \ln |z-z'| \right)^2 + 0$$

$$=: \partial^\mu X \partial X_\mu \, \partial' X^\nu \partial' X_\nu : + \left\{ \frac{(2')^2}{2} \frac{D}{(z-z')^4} - \frac{2'}{(z-z')^2} : \partial X^\mu \partial X_\mu : \right.$$

related to stress energy tensor

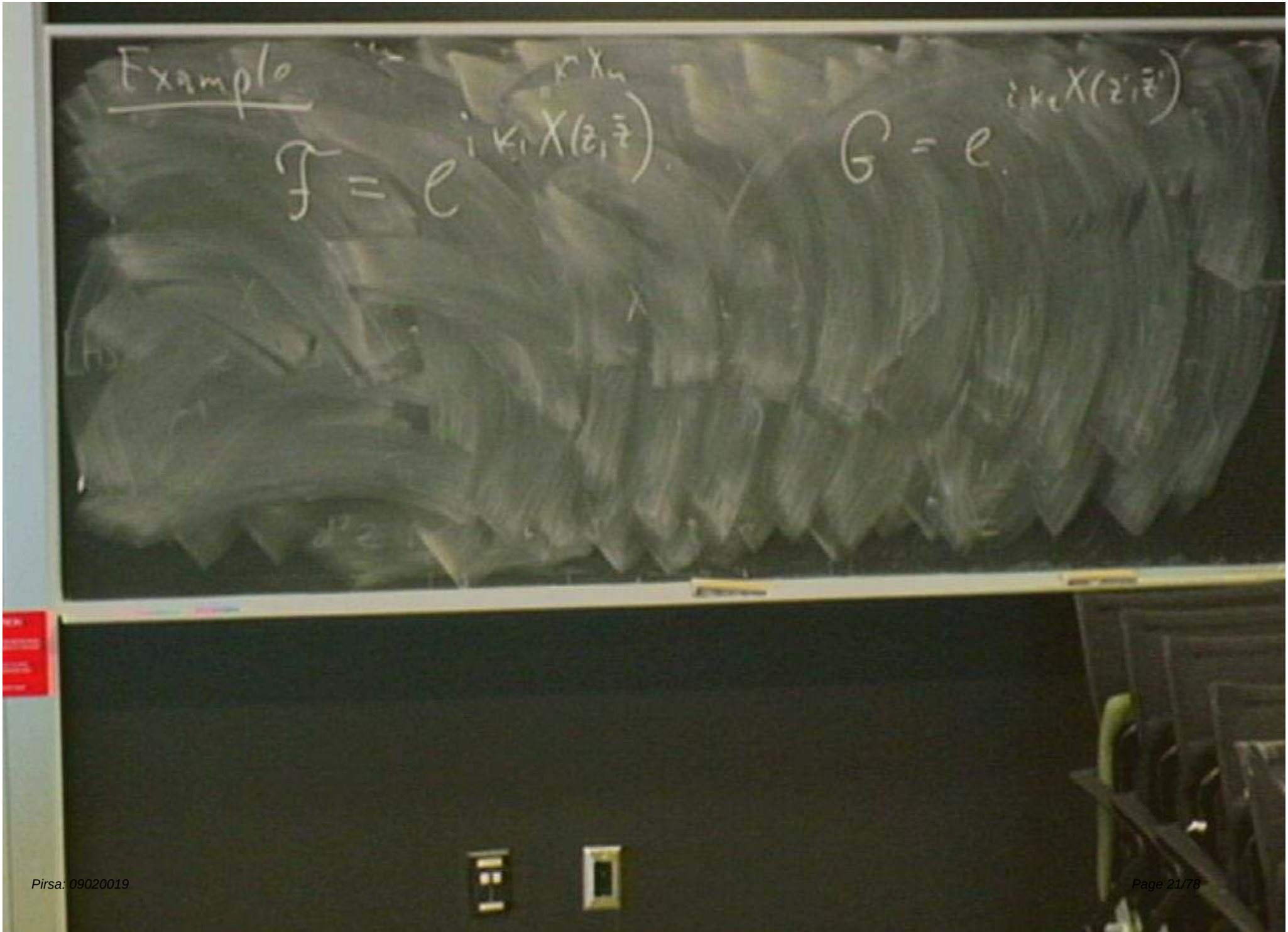
$$=: \partial X^\mu \partial X_\mu \; \partial' X^\nu \partial' X_\nu : \; - \frac{1}{2} \cdot 4 \; : \partial X^\mu(z) \partial X^\nu(z') : \eta_{\mu\nu} \left[\partial\partial' \ln |z-z'|^2 \right]$$

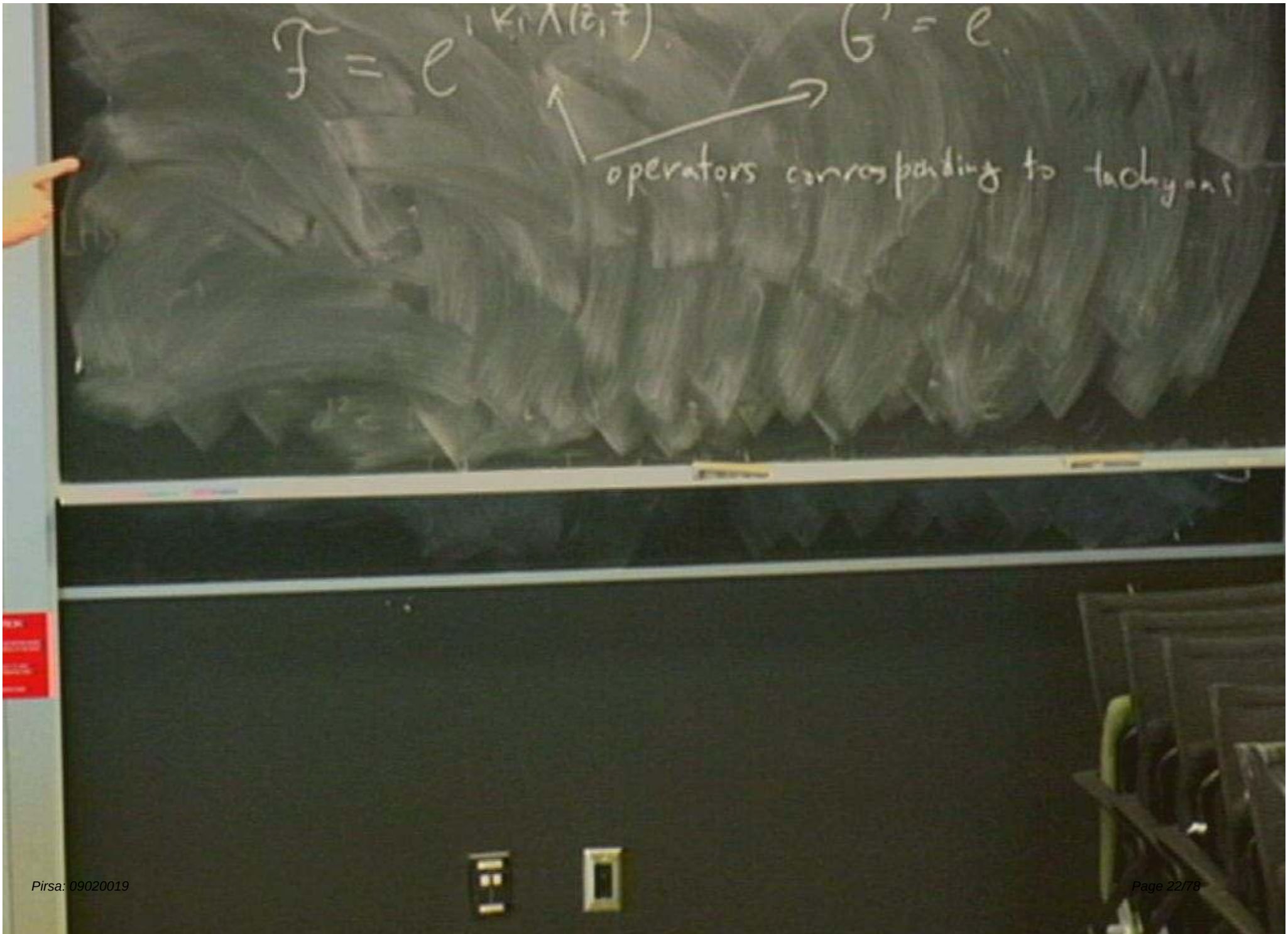
z z'

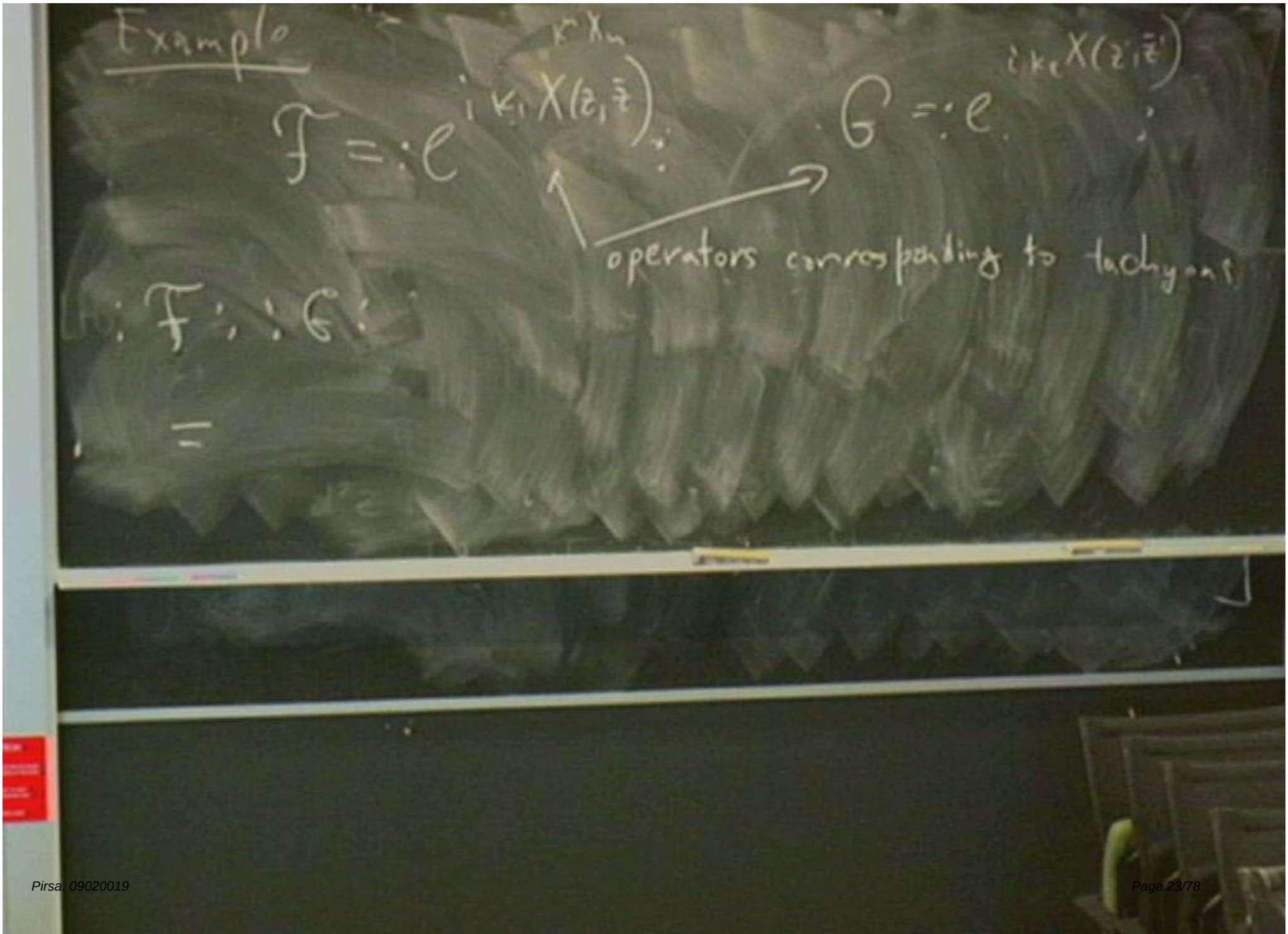
no singularities

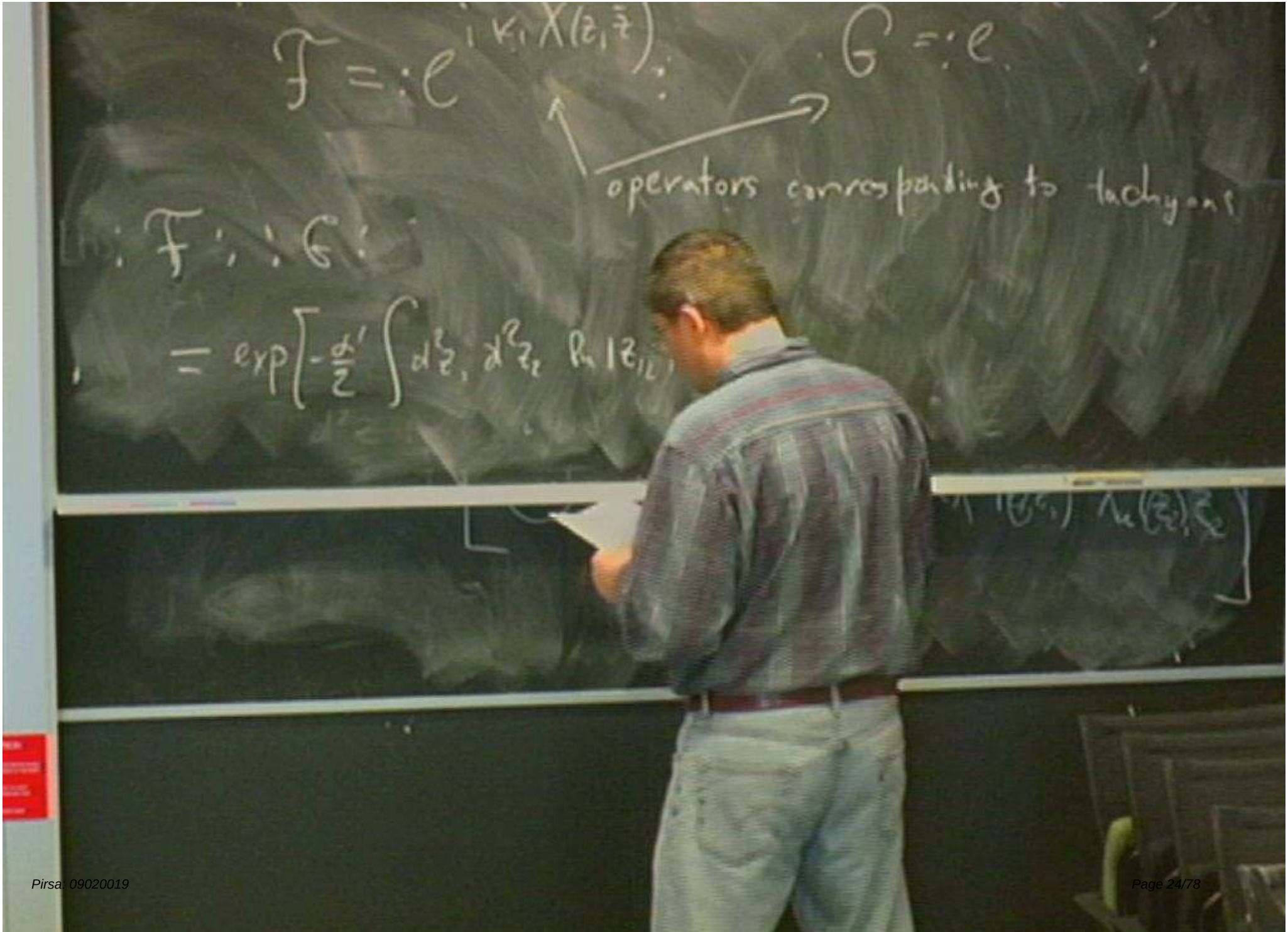
$$+ \frac{1}{2!} \left(-\frac{1}{2} \right)^2 4 \, \eta^{\mu\nu} \eta_{\mu\nu} \left(\partial\partial' \ln |z-z'| \right)^2 + 0$$

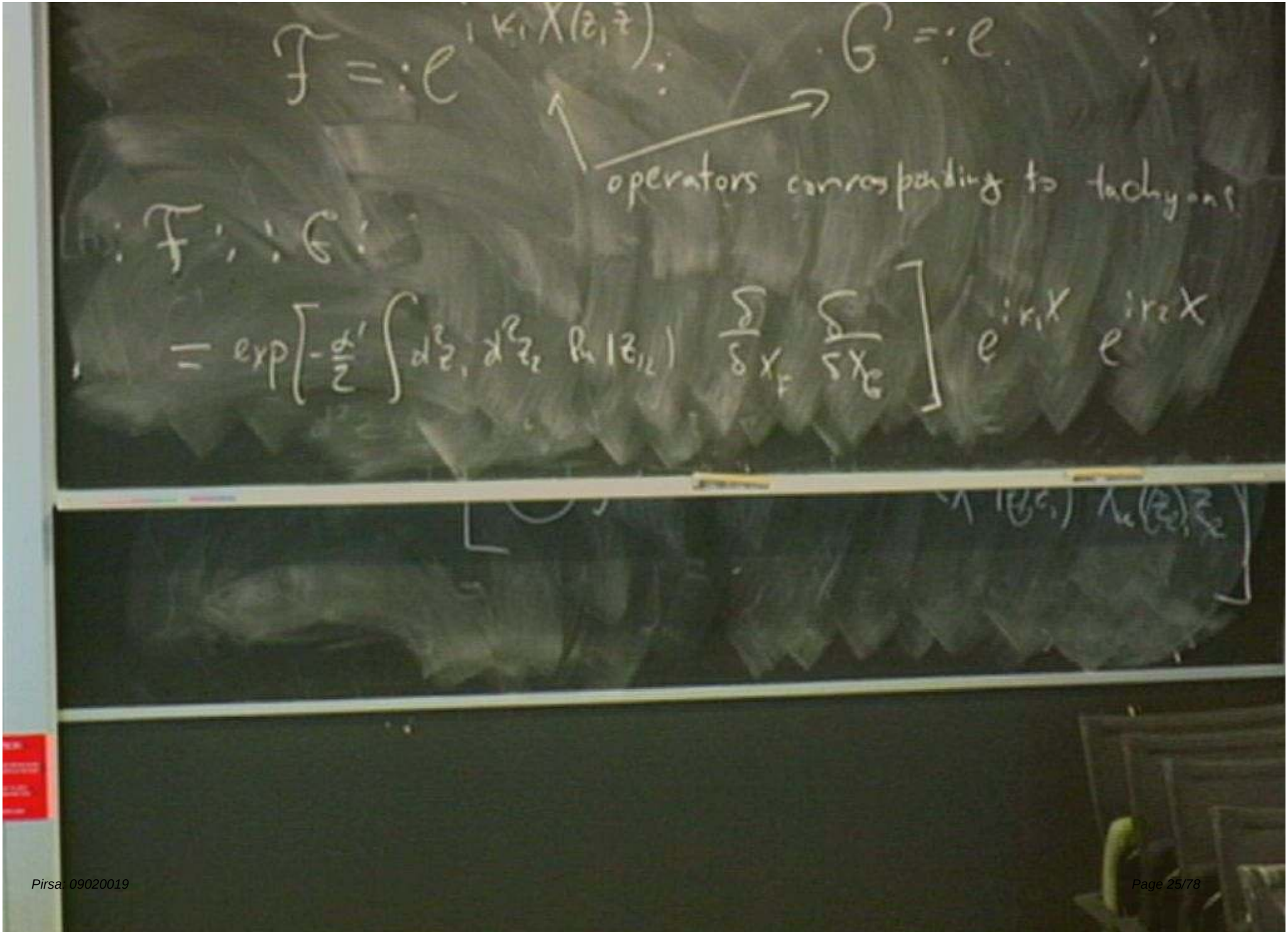
$$=: \partial^\mu X \partial X_\mu \; \partial' X^\nu \partial' X_\nu : + \left\{ \frac{(2)^2}{2} \frac{D}{(z-z')^4} - \frac{2}{(z-z')^2} : \partial X^\mu \partial X_\mu : \right\}$$


Example
$F = e^{i k_1 X(z,\bar{z})}$
$G = e^{i k_2 X(z',\bar{z}')}$


operators corresponding to tachyons


Example
$F =: e^{i k_1 X(z,\bar{z})}:$
$G =: e^{i k_2 X(z,\bar{z})}:$
operators corresponding to tachyons
$\langle\, : F : : G : \,\rangle$
=


operators corresponding to tachyons


$F =: e^{i k_1 X(z_1)} :$
$G =: e^{\quad} :$
operators corresponding to tachyons
$\langle 0 | F_1 \ldots G_2 | 0 \rangle$
$= \exp\left[-\frac{\alpha'}{2} \int d^2 z_1 \, d^2 z_2 \, R_{12}(z_{12}) \, \frac{\delta}{\delta X_1} \frac{\delta}{\delta X_2} \right] e^{i k_1 X} \; e^{i k_2 X}$

$$= \exp\left(-\frac{\alpha'}{2}\int d^2z_1\, d^2z_2\, R_{11}(z_{12})\, \frac{\delta}{\delta X_F}\frac{\delta}{\delta X_F}\right] e^{ik_1 X} e^{ik_2 X}$$

$$\frac{\delta}{\delta X_F}$$

$$= \exp\left(-\frac{\alpha'}{2}\int d^2z_1\, d^2z_2\, R_{ij}(z_{12})\, \frac{\delta}{\delta X_F}\, \frac{\delta}{\delta X_G}\right] e^{ik_1 X}\, e^{ik_2 X}$$

$$\frac{\delta}{\delta X_1}\left[e^{ikX}\right] = e^{ikX}$$

:

$$= \exp\left(-\frac{\alpha'}{2}\int d^2z_1\, d^2z_2\, P_{11}(z_{12})\, \frac{\delta}{\delta X_F}\,\frac{\delta}{\delta X_G}\right] e^{ik_1 X}\, e^{ik_2 X}$$

$$\frac{\delta}{\delta X^\mu(z_2)}\left[e^{ikX(z_1)}\right] = e^{ikX}\, ik^\mu\, \delta^2(z_1 - z_2)$$

$$= \exp\left(-\frac{\alpha'}{2}\int d^2z_1\, d^2z_2\, R_{\mu\nu}(z_{12})\, \frac{\delta}{\delta X_F}\, \frac{\delta}{\delta X_G}\right] e^{ik_1 X} \; e^{ik_2 X}$$

$$\frac{\delta}{\delta X_{\mu}(z_1)}\left[e^{ikX(z_2)}\right] = e^{ikX(z_2)}\, ik^{\mu}\, \delta^2(z_1 - z_2)$$

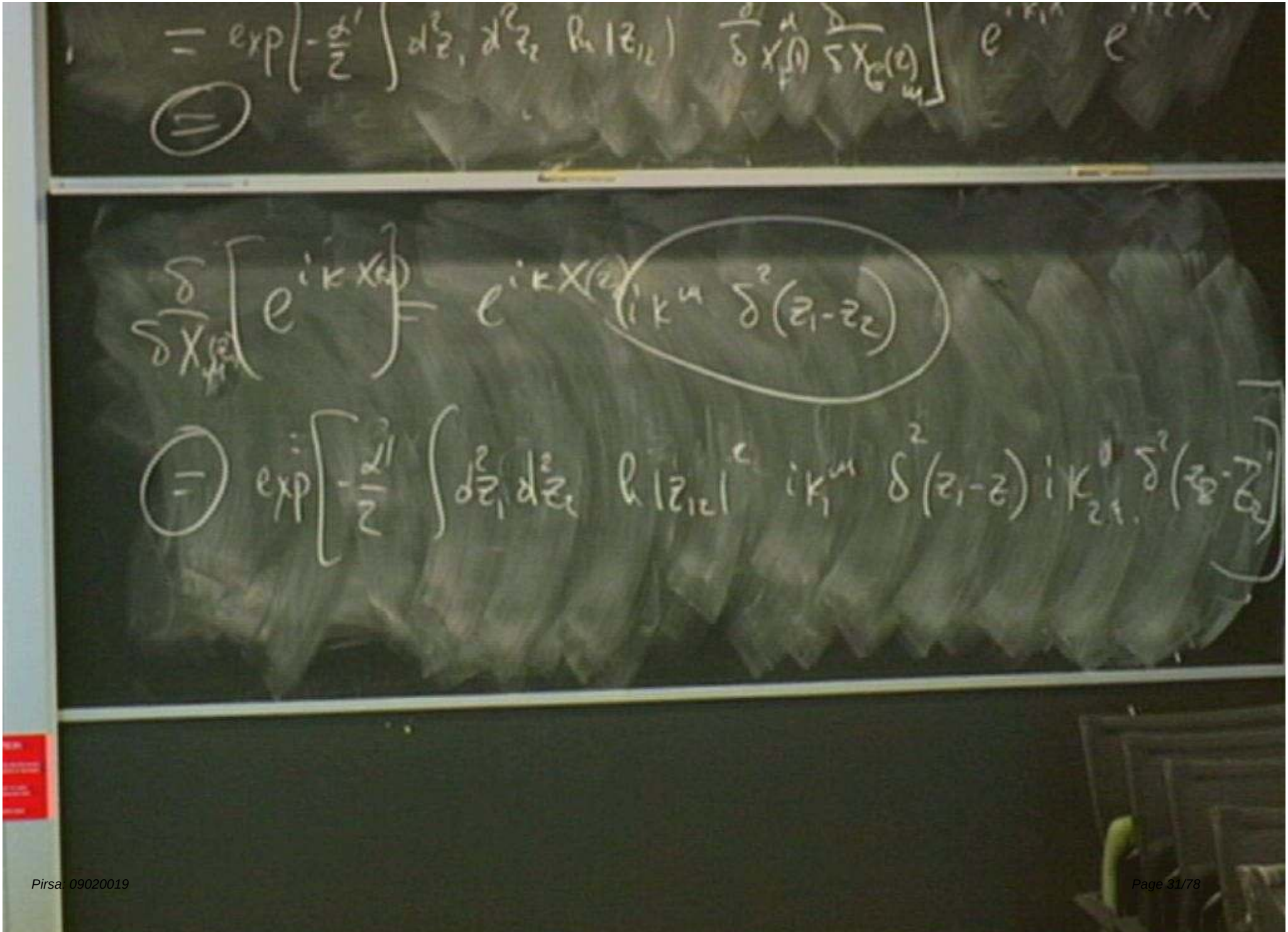
$$= \exp\left[-\frac{\alpha'}{2}\right.$$

$$= \exp\left(-\frac{\alpha'}{2}\int d^2z_1\, d^2z_2\, R_{\mu\nu}(z_{12})\, \frac{\delta}{\delta X_F}\, \frac{\delta}{\delta X_G}\right] e^{ik_1 X} \cdots$$

$$= \cdots$$

$$\frac{\delta}{\delta X_{\mu}(z_1)}\left[e^{ik X(z_2)}\right] = e^{ik X(z_2)}\, ik^{\mu}\, \delta^2(z_1 - z_2)$$

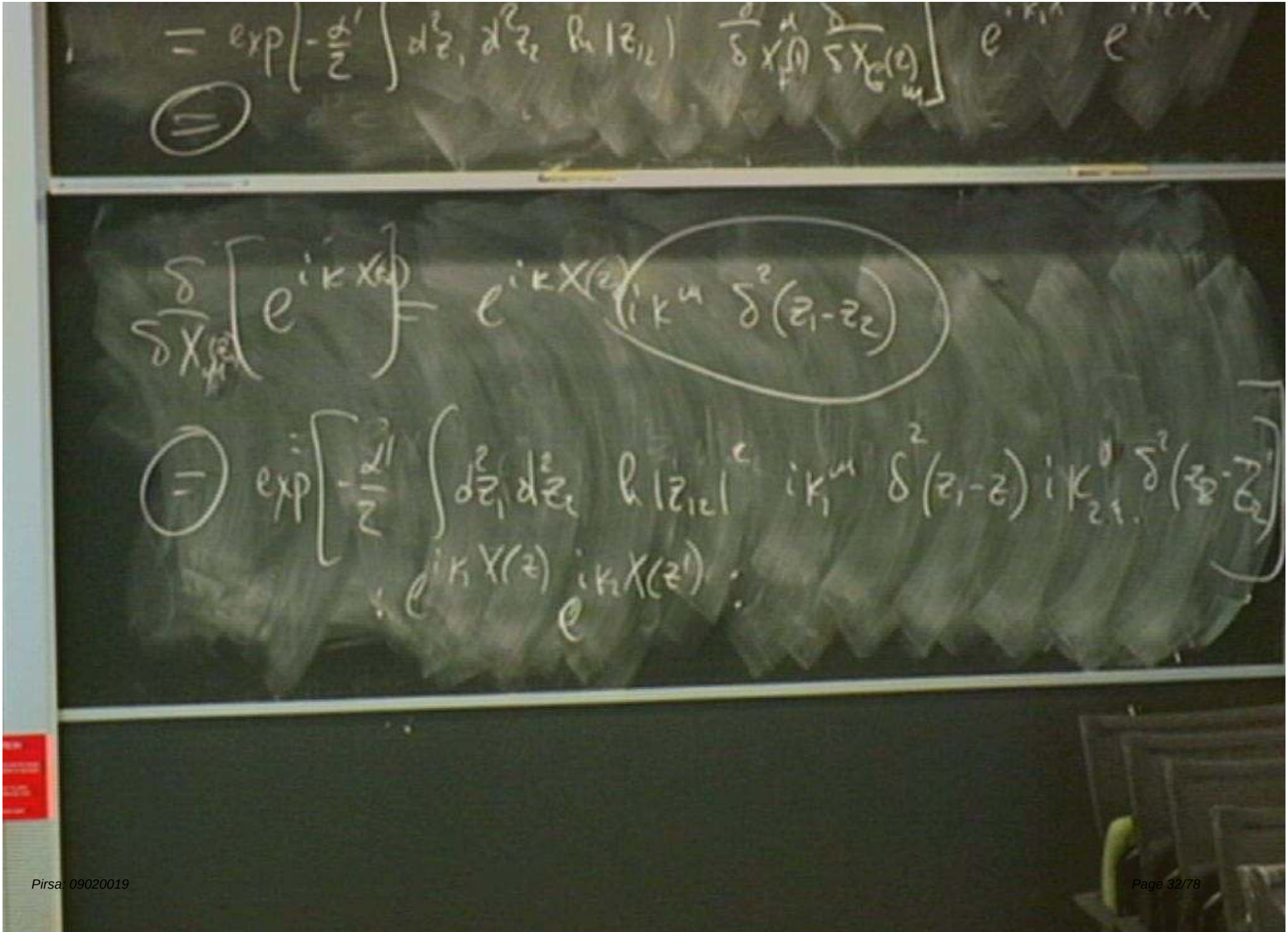
$$= \exp\left[-\frac{\alpha'}{2}\int d^2z_1\, d^2z_2\right.$$



$$= \exp\left(-\frac{\alpha'}{2}\int d^2z_1\, d^2z_2\, P_k(z_{12})\, \frac{\delta}{\delta X^\mu(z_1)}\frac{\delta}{\delta X_\mu(z_2)}\right] e^{ik_1 X} e^{ik_2 X}$$

$$\frac{\delta}{\delta X_\mu(z)}\left[e^{ikX(z_2)}\right] = e^{ikX(z_2)}\, ik^\mu\, \delta^2(z_1 - z_2)$$

$$= \exp\left[-\frac{\alpha'}{2}\int d^2z_1\, d^2z_2\, P_k(|z_{12}|)\, ik_1^\mu\, \delta^2(z_1 - z)\, ik_{2\mu}\, \delta^2(z_2 - z)\right]$$



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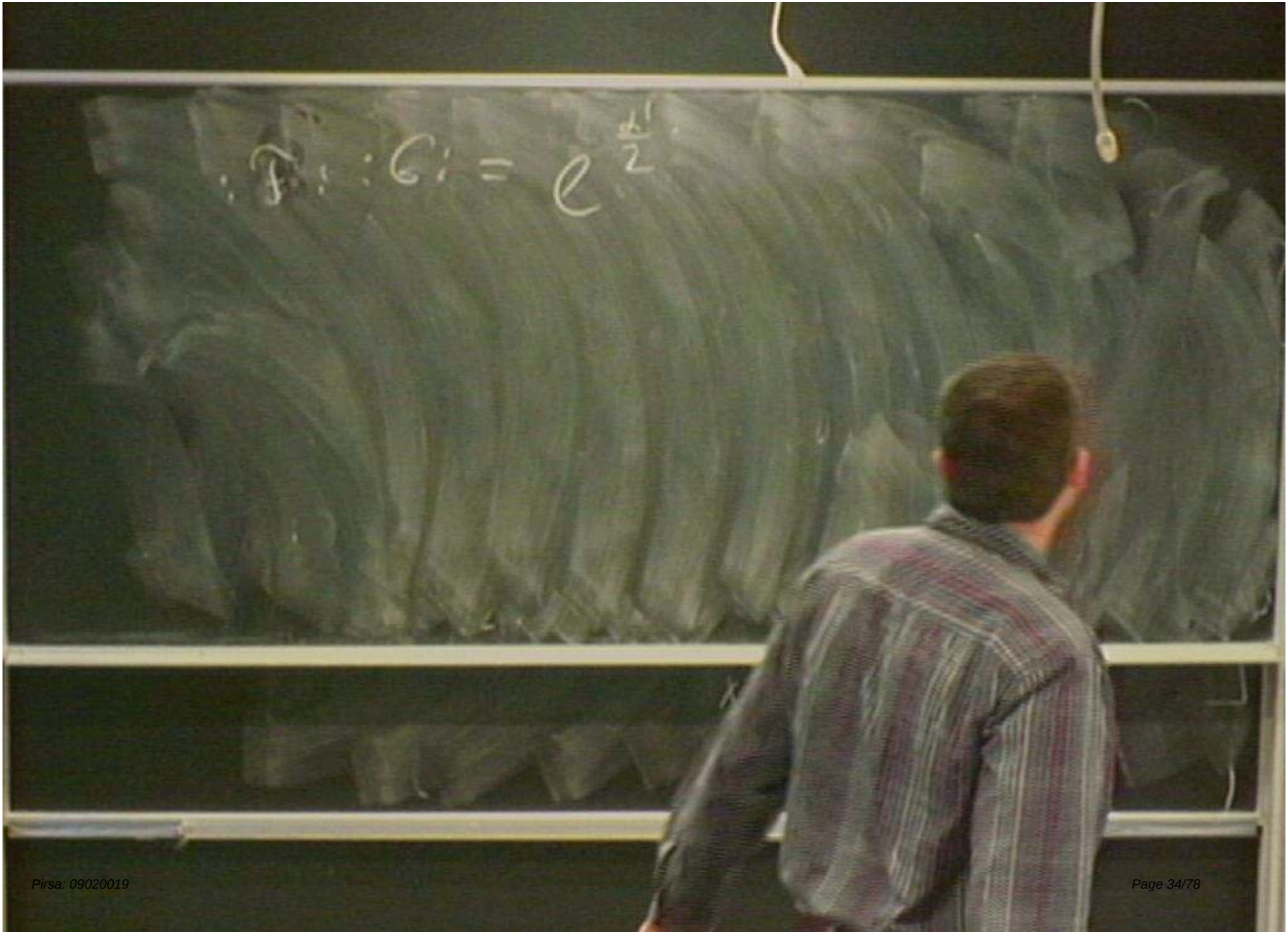
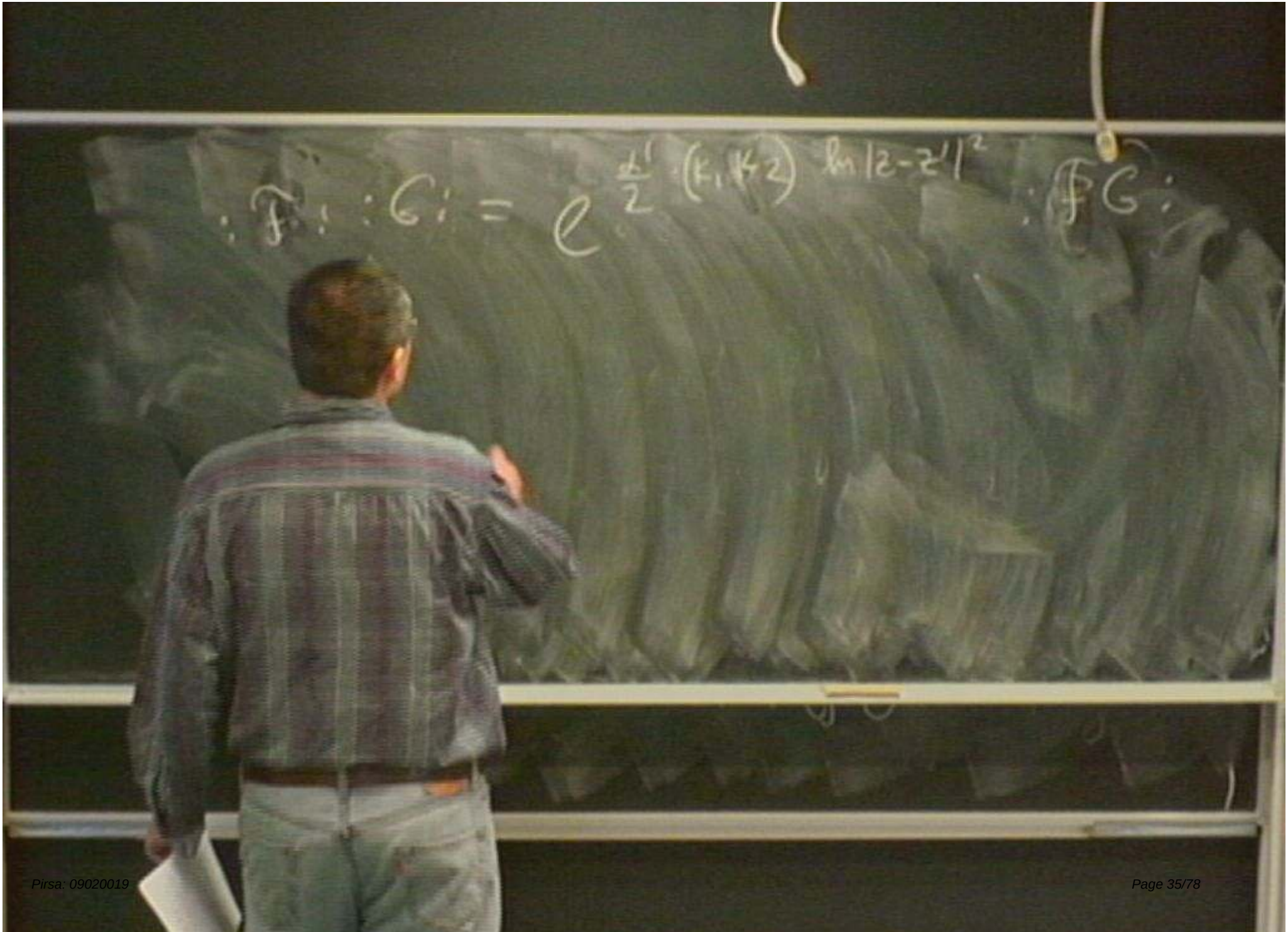
$$= \exp\left(-\frac{\alpha'}{2}\int d^2z_1\, d^2z_2\, h(|z_{12}|)\, \frac{\delta}{\delta X^\mu(z_1)}\frac{\delta}{\delta X_\mu(z_2)}\right] e^{ik_1 X} e^{ik_2 X}$$

$$\frac{\delta}{\delta X^\mu(z_1)}\left[e^{ikX(z_2)}\right] = e^{ikX(z_2)}\, ik^\mu\, \delta^2(z_1-z_2)$$

$$= \exp\left[-\frac{\alpha'}{2}\int d^2z_1\, d^2z_2\, h(|z_{12}|)\, ik_1^\mu\, \delta^2(z_1-z)\, ik_{2\mu}\, \delta^2(z_2-z')\right]$$

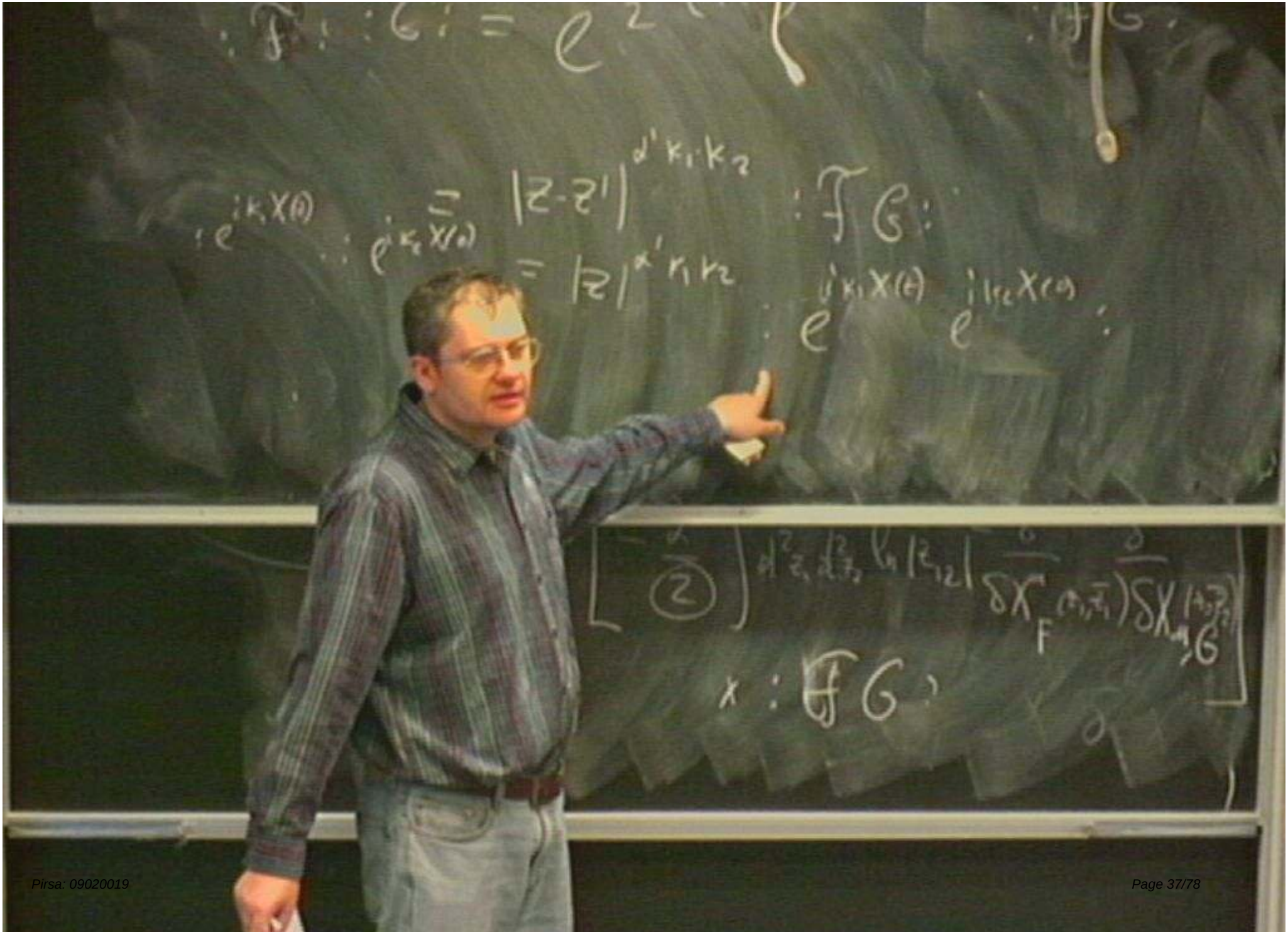
$$:e^{ik_1X(z)}\, e^{ik_2X(z')}:$$

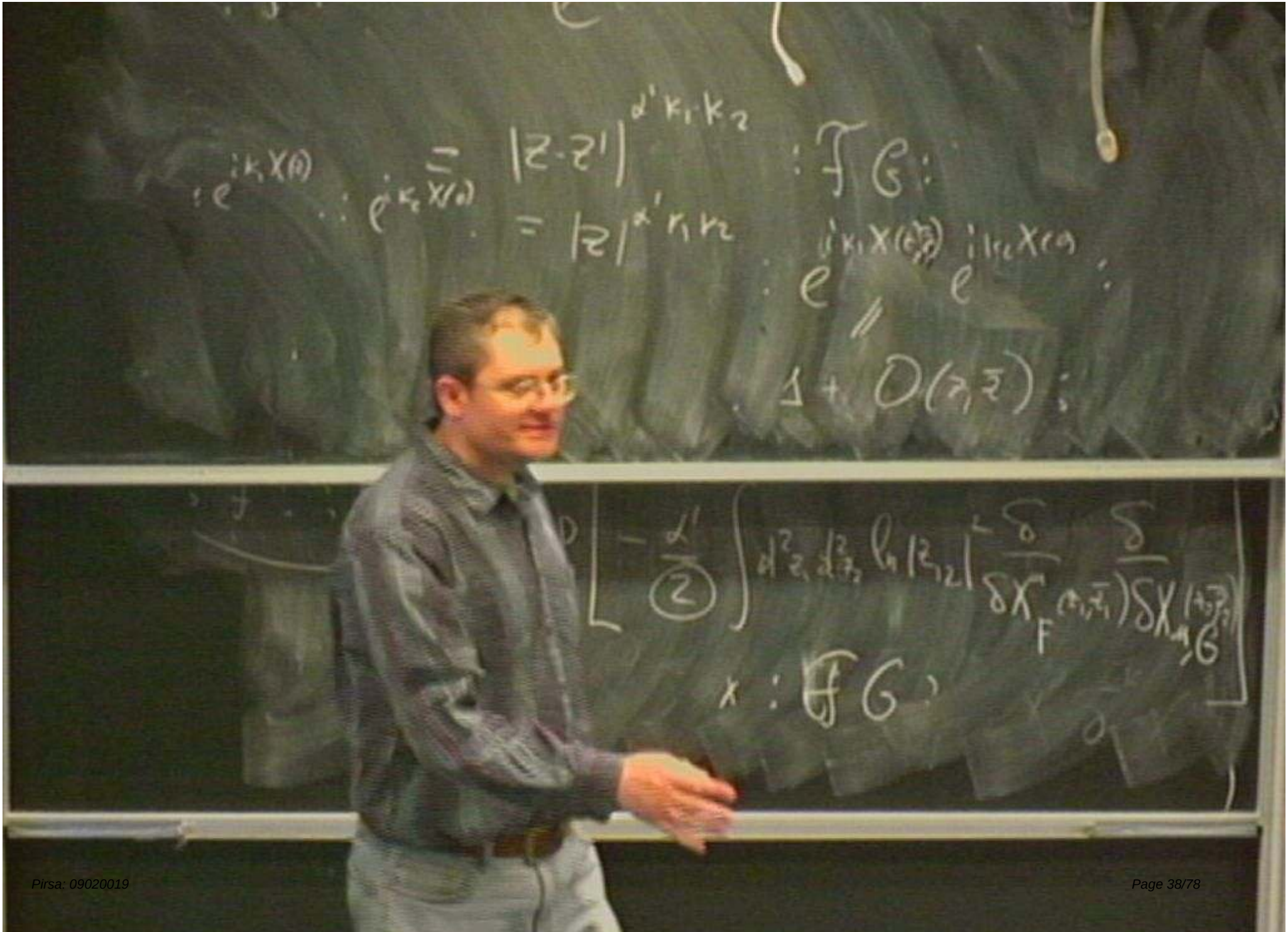

: T : : G : = e

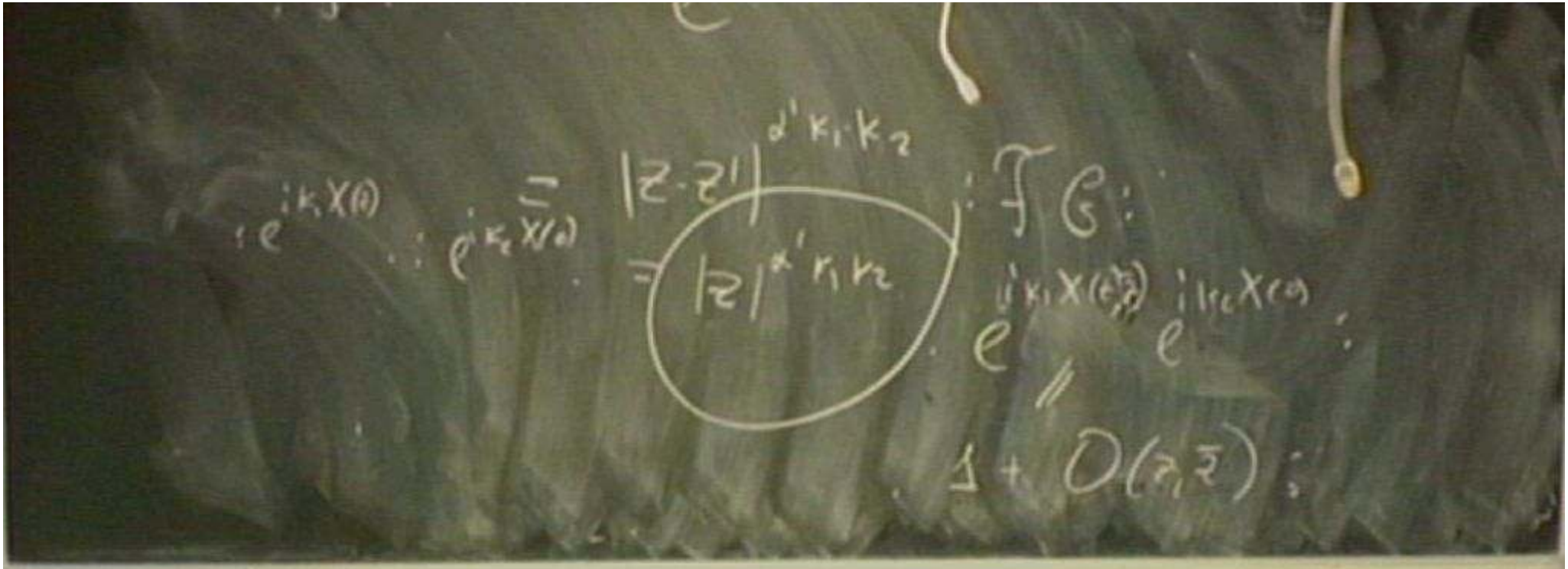

:Φ: :G: = e^{d'/2 (K1,K2) ln|z-z'|^2} :Φ G:

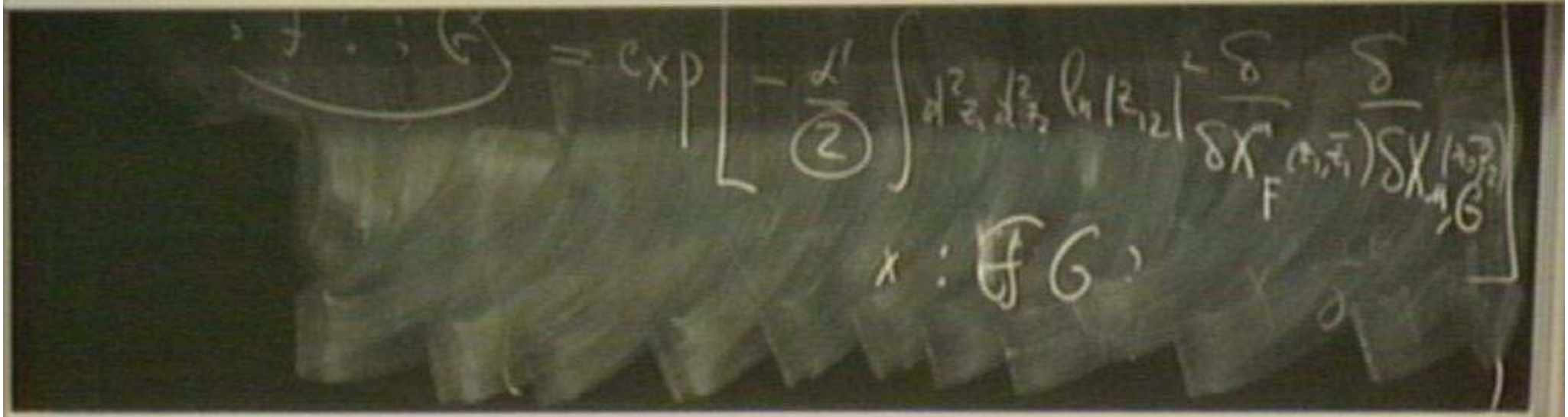
$$:\mathcal{F}::G: = e^{\frac{d^2}{2}(k_1 k_2)\ln|z-z'|^2} :\mathcal{F}G:$$

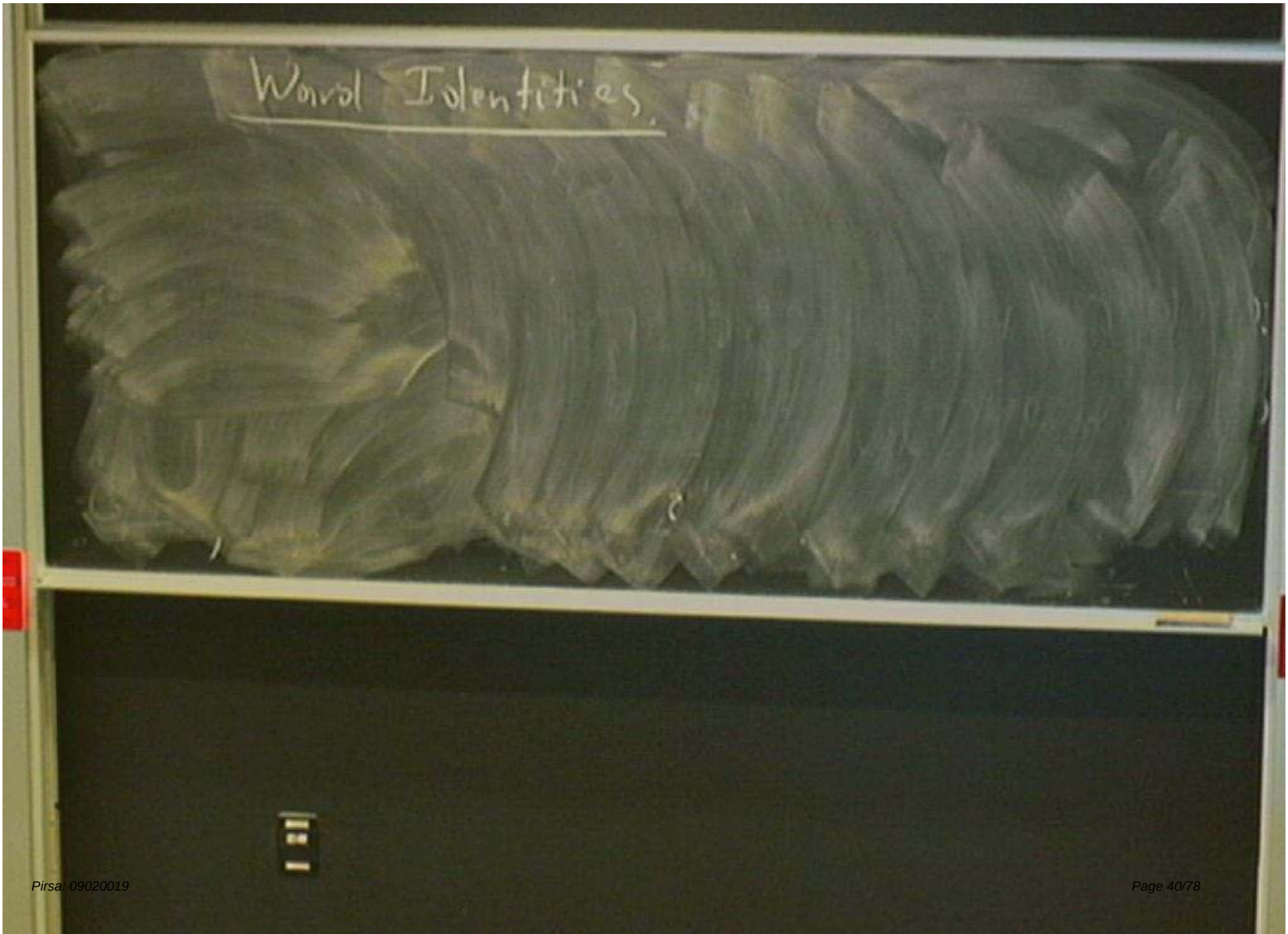
$$= |z-z'|^{d^2 k_1 \cdot k_2} :\mathcal{F}G:$$

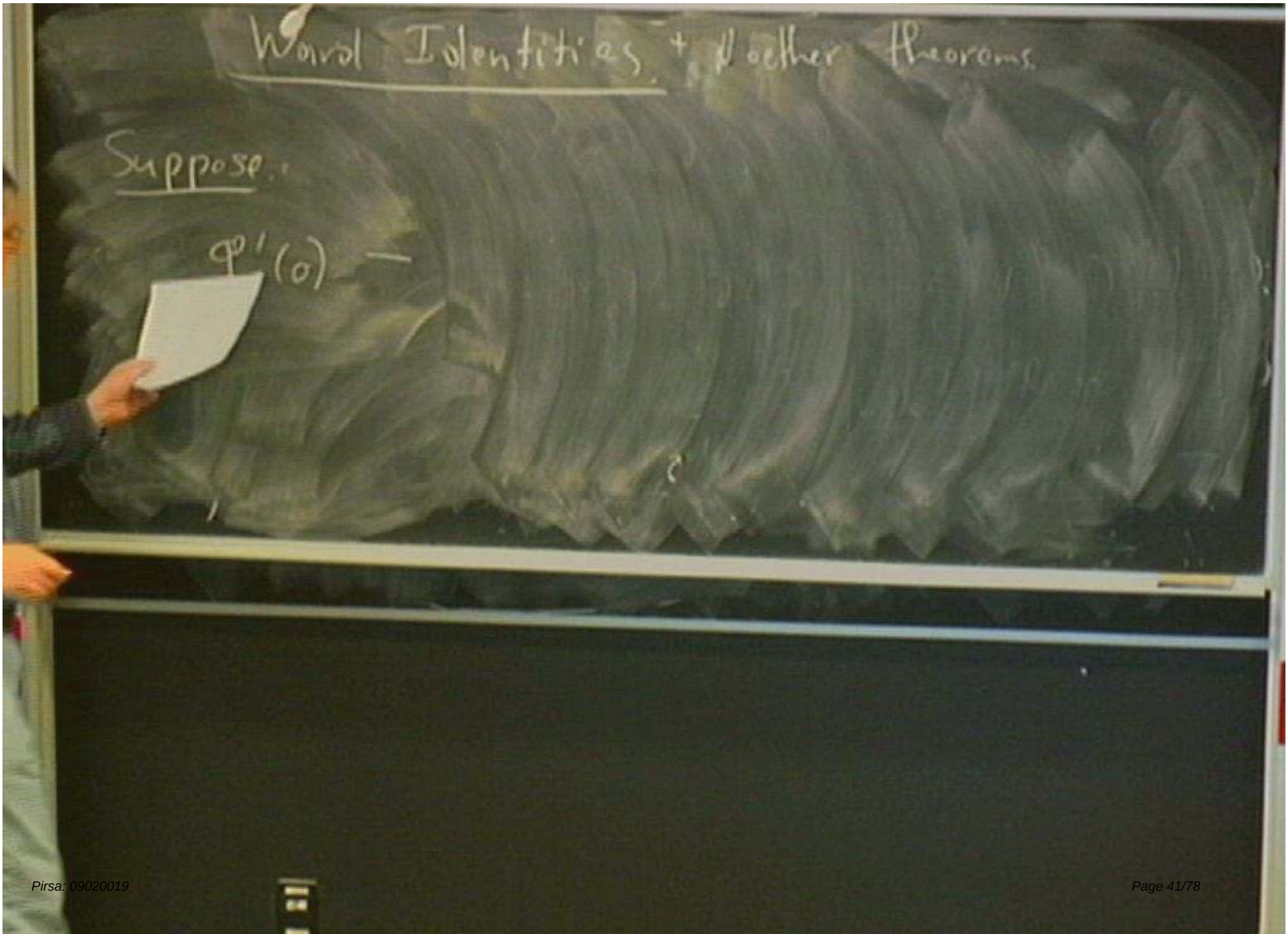



$:e^{ik_1X(z)}:\,:e^{ik_2X(z')}: = |z-z'|^{\alpha' k_1 k_2}\,:fg:$
$= |z|^{\alpha' k_1 k_2}$
$:e^{ik_1X(z)}\,e^{ik_2X(0)}:$
$\Delta + O(z,\bar{z})$
$\exp\left[-\frac{\alpha'}{2\pi}\int d^2z_1\, d^2z_2\, \ln|z_{12}|^2 \frac{\delta}{\delta X_F(z_1,\bar{z}_1)} \frac{\delta}{\delta X_G(z_2,\bar{z}_2)}\right]$
$x : fg :$


$: e^{ik_1 X(z)} :$
$: e^{ik_2 X(z')} :$
$= |z-z'|^{\alpha' k_1 k_2}$
$: F G :$
$e^{ik_1 X(z,\bar{z})}$
$e^{ik_2 X(0)}$
$\Delta + O(z,\bar{z})$


$: F : : G : = \exp\left[-\frac{\alpha'}{2} \int d^2 z_1 d^2 z_2 \, \ln|z_{12}|^2 \frac{\delta}{\delta X_F(z_1,\bar{z}_1)} \frac{\delta}{\delta X_G(z_2,\bar{z}_2)} \right]$
$\times : F G :$


Ward Identities


Ward Identities + Noether theorems
Suppose:
φ'(0) =

Ward Identities + Noether Theorems

Suppose:

$$\varphi'_\alpha(\sigma) = \varphi_\alpha(\sigma) + \varepsilon \cdot \delta\varphi_\alpha(\sigma)$$

Ward Identities + Noether theorems

Suppose: exact symmetry of my QFT

$$\varphi'_\alpha(\sigma) = \varphi_\alpha(\sigma) + \varepsilon \cdot \delta\varphi_\alpha(\sigma)$$

ε: Infinitesimal parameter of sym trans.

Suppose: exact symmetry of my QFT

$$\varphi'_\alpha(\sigma) = \varphi_\alpha(\sigma) + \varepsilon \cdot \delta\varphi_\alpha(\sigma)$$

ε: Infinitesimal parameter of sym trans.

$$\partial_a \varepsilon = 0$$

$\varphi_\alpha \to \varphi'_\alpha = \varphi_\alpha + \varepsilon \cdot \rho(\sigma) \cdot \delta\varphi_\alpha(\sigma)$

$\varphi_\alpha \to \varphi'_\alpha = \varphi_\alpha + \varepsilon \cdot \rho(\sigma) \cdot \delta\varphi_\alpha(\sigma)$

$\rho(\sigma)$ has a finite support

$\varphi_\alpha \to \varphi'_\alpha = \varphi_\alpha + \varepsilon \cdot \rho(\sigma) \cdot \delta\varphi_\alpha(\sigma)$

$\rho(\sigma)$ has a finite support

$$\int [d\varphi']\, e^{-S[\varphi']}$$

$\varphi_\alpha \to \varphi'_\alpha = \varphi_\alpha + \varepsilon \cdot \rho(\sigma) \cdot \delta\varphi_\alpha(\sigma)$

$\rho(\sigma)$ has a finite support

$$\int [d\varphi'] e^{-S[\varphi']} \overset{d\varepsilon=0}{=} \int [d\varphi] e^{-S}$$

$\varphi_\alpha \to \varphi'_\alpha = \varphi_\alpha + \varepsilon \cdot \rho(\sigma) \cdot \delta\varphi_\alpha(\sigma)$

$\rho(\sigma)$ has a finite support

$$\int [d\varphi'] e^{-S[\varphi']} \overset{J^2=1}{=} \int [d\varphi] e^{-S} \left[1 + \frac{i\varepsilon}{2\pi} \right.$$

$\varphi_\alpha \to \varphi'_\alpha = \varphi_\alpha + \varepsilon \cdot \rho(\sigma) \cdot \delta\varphi_\alpha(\sigma)$

$\rho(\sigma)$ has a finite support

$$\int [d\varphi']\, e^{-S[\varphi']} \overset{\text{def}}{=} \int [d\varphi]\, e^{-S} \left[1 + \frac{i\varepsilon}{2\pi} \int d^2\sigma \sqrt{-g}\, j^a(\sigma) \right]$$

$\varphi_\alpha \to \varphi'_\alpha = \varphi_\alpha + \varepsilon \cdot \rho(\sigma) \cdot \delta\varphi_\alpha(\sigma)$

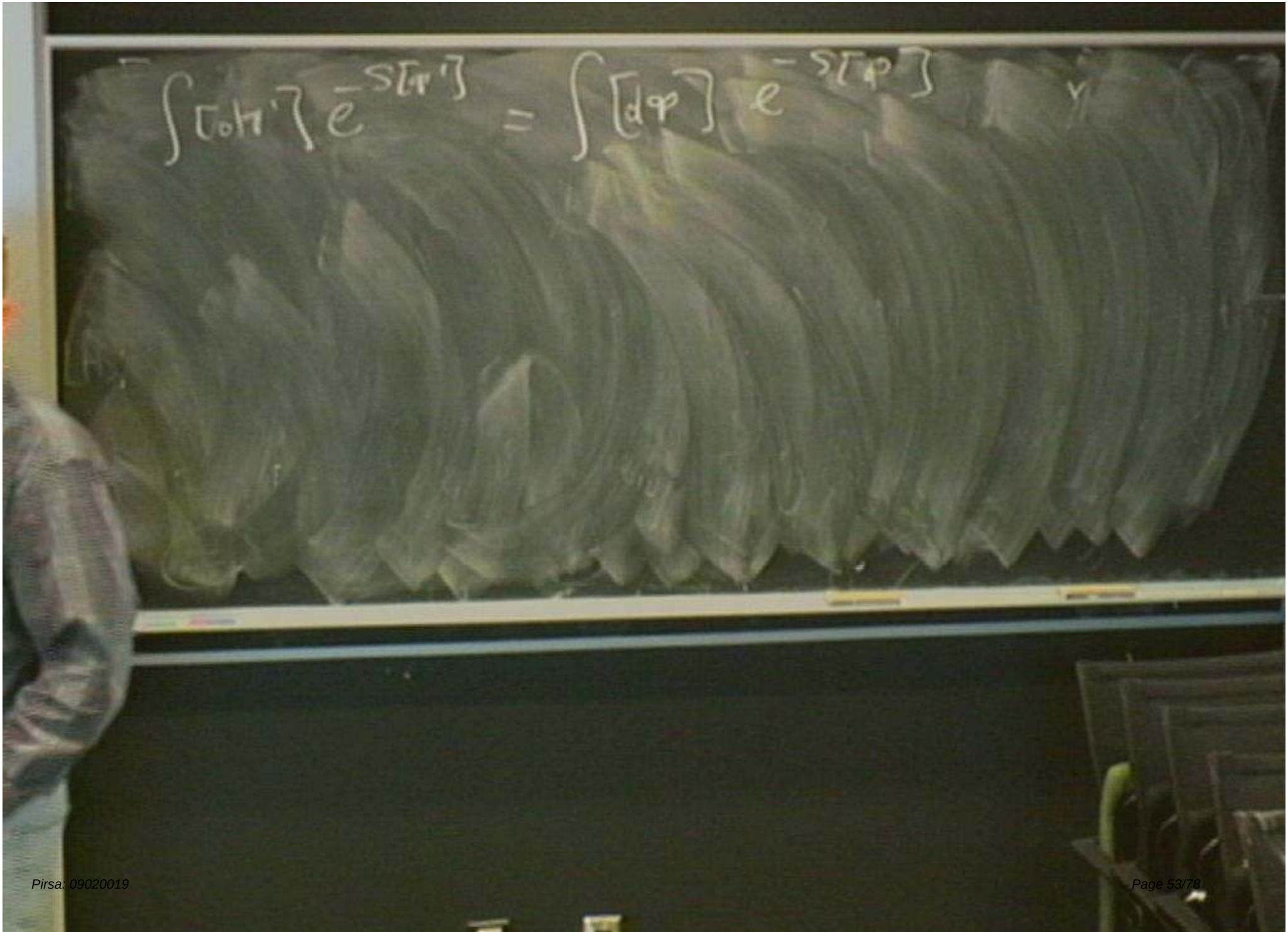
$\rho(\sigma)$ has a finite support

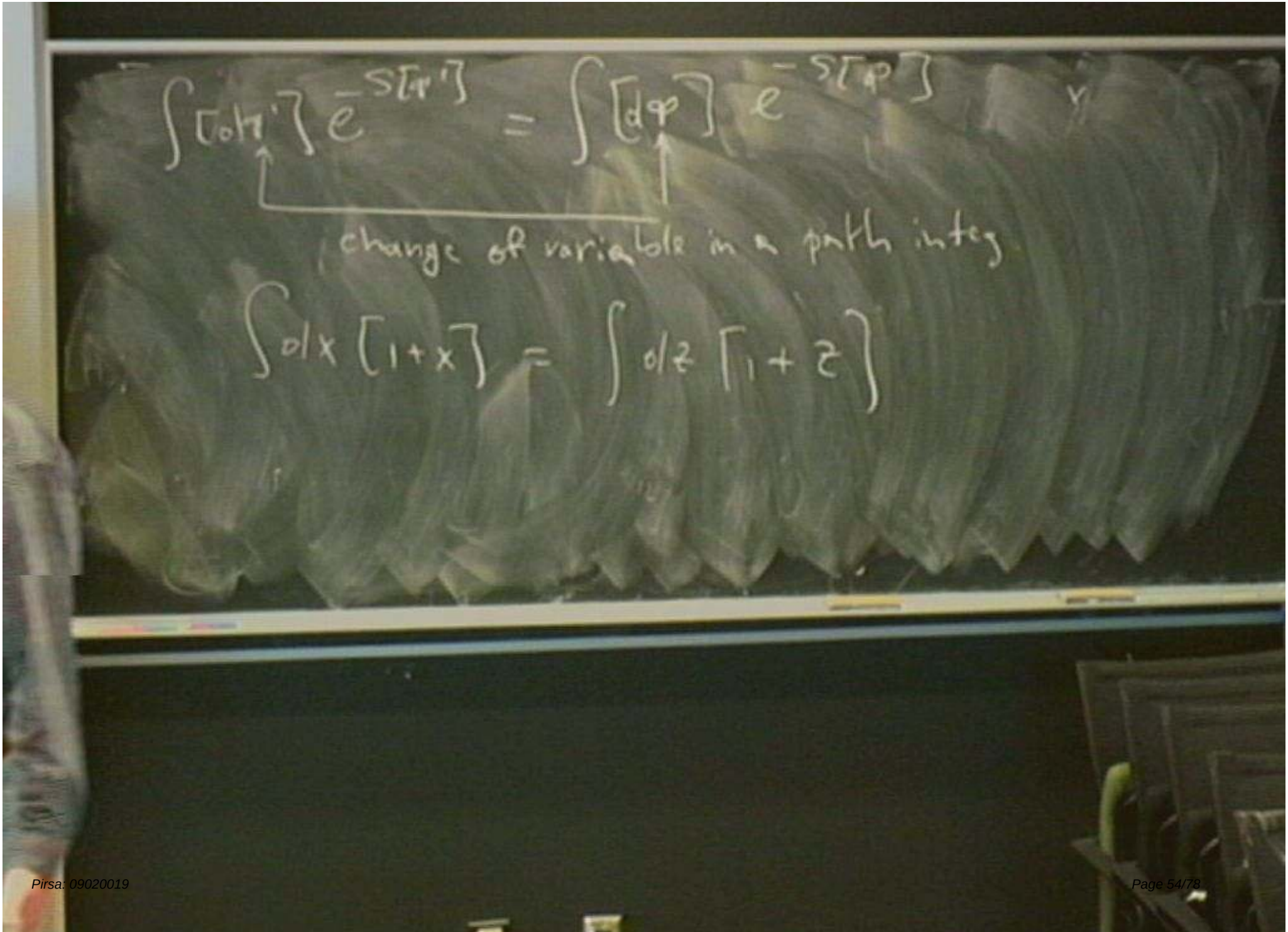
$$\int [d\varphi'] e^{-S[\varphi']} \overset{\text{def}}{=} \int [d\varphi] e^{-S} \left[1 + \frac{i\varepsilon}{2\pi} \int d^2\sigma \sqrt{g}\, j^\alpha(\sigma) \partial_\alpha \rho \right]$$

$\varphi_\alpha \to \varphi'_\alpha = \varphi_\alpha + \varepsilon \cdot \rho(\sigma) \cdot \delta\varphi_\alpha(\sigma)$

$\rho(\sigma)$ has a finite support

$$\int [d\varphi'] e^{-S[\varphi']} \overset{J=0}{=} \int [d\varphi] e^{-S} \left[1 + \frac{i\varepsilon}{2\pi} \int d^2\sigma \sqrt{g}\, j^a(\sigma) \partial_a \rho + O(\varepsilon^2) \right]$$

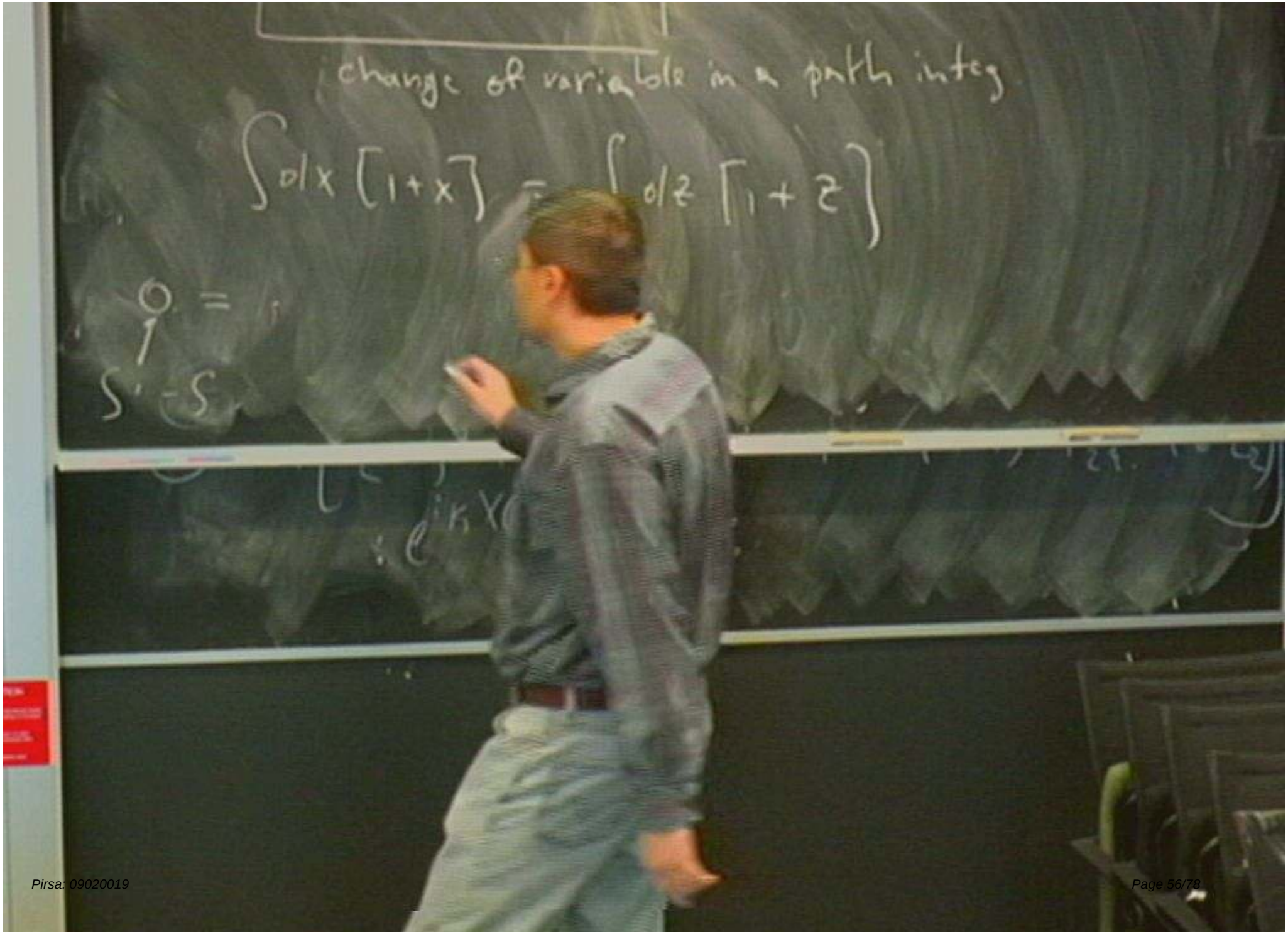

$\int [D\varphi'] e^{-S[\varphi']} = \int [d\varphi] e^{-S[\varphi]}$


$\int [d\varphi'] e^{-S[\varphi']} = \int [d\varphi] e^{-S[\varphi]}$
change of variable in a path integ
$\int dx \, [1+x] = \int dz \, [1+z]$

$\varphi_\alpha \to \varphi'_\alpha = \varphi_\alpha + \varepsilon \cdot \rho(\sigma) \cdot \delta\varphi_\alpha(\sigma)$

$\rho(\sigma)$ has a finite support

$$\int [d\varphi']\, e^{-S[\varphi']} = \int [d\varphi]\, e^{-S} \left[1 + \frac{i\varepsilon}{2\pi} \int d^2\sigma \, j^{(\alpha)} \delta\varphi_\alpha + O(\varepsilon^2) \right]$$

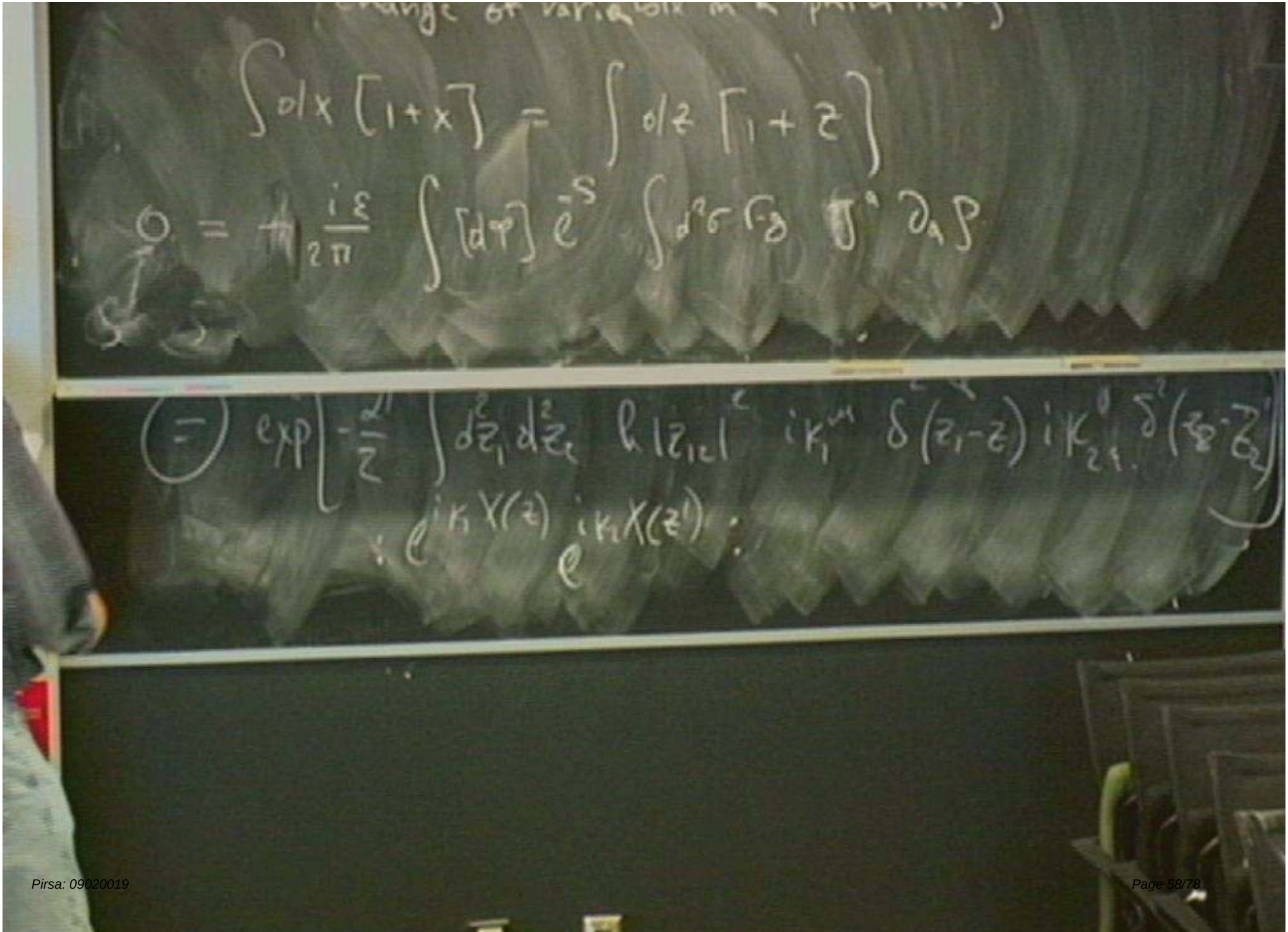

change of variable in a path integ
$\int \mathcal{D}x\,[1+x] = \int \mathcal{D}z\,[1+z]$

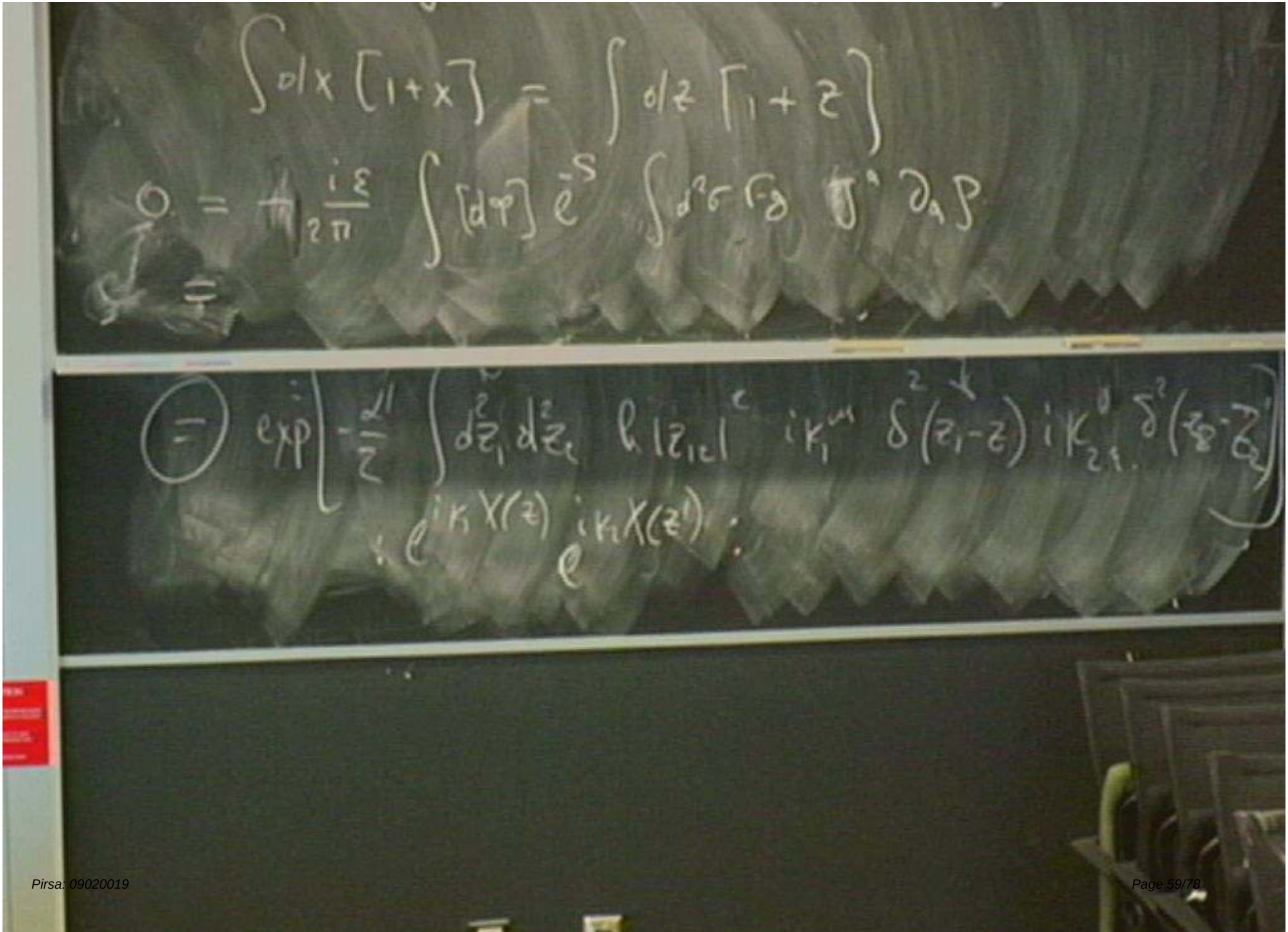
change of variable in a path integ

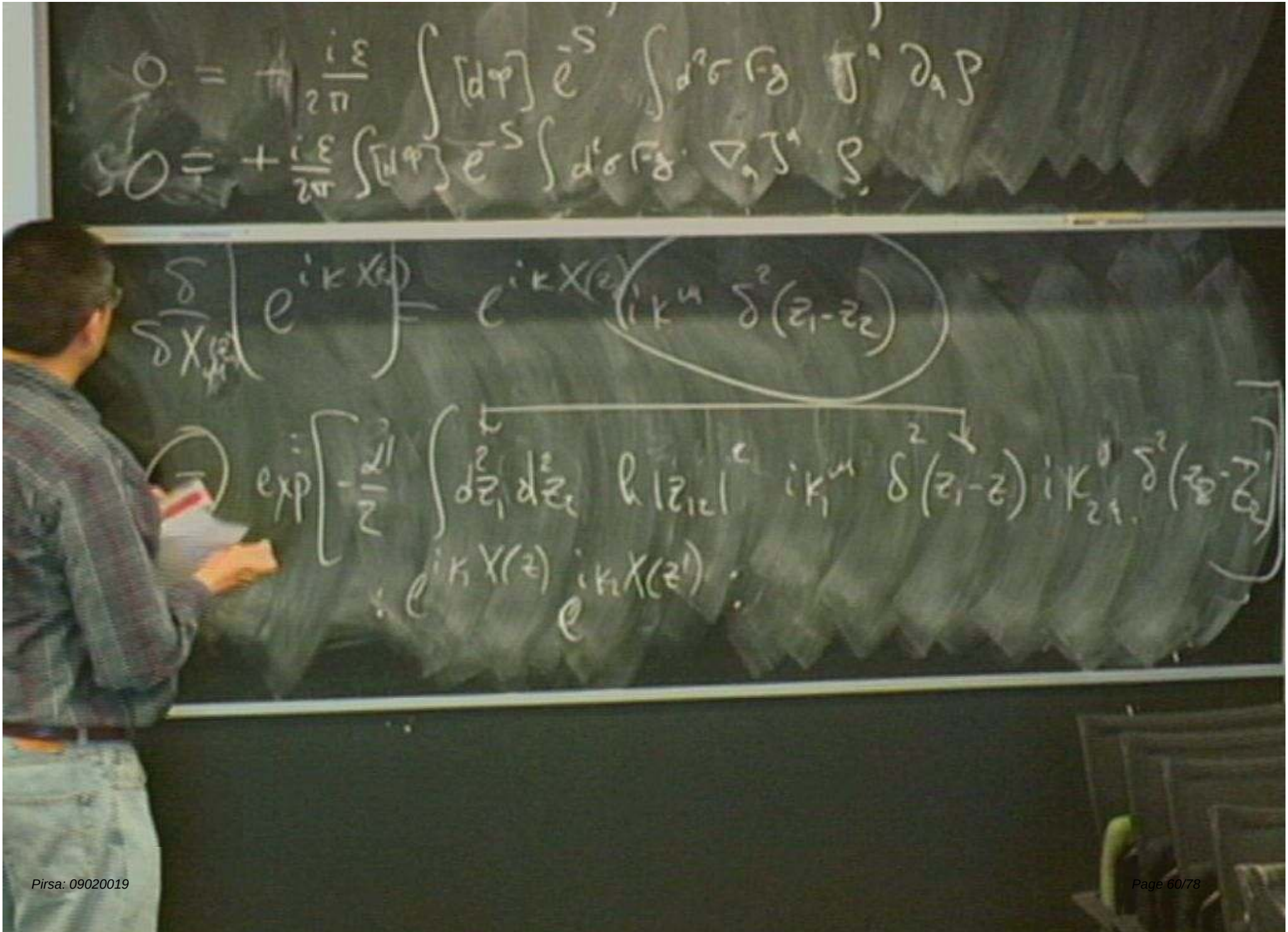
$$\int \mathcal{D}[x]\,[1+x] = \int \mathcal{D}[z]\,[1+z]$$

$$\mathcal{O} = \frac{i\varepsilon}{2\pi} \int [d\varphi]\, e^{-S}$$

$$: e^{ik_1 X(z)}\, e^{ik_2 X(z')} :$$


change of variable in a path integral
$\int d|x\,[1+x] = \int d|z\,[1+z]$
$= \frac{i\varepsilon}{2\pi} \int [d\tau]\, e^{-S} \int d^2\sigma\, \tau_\beta\, \Pi^\alpha\, \partial_\alpha \beta$
$= \exp\left[-\frac{\alpha'}{2} \int d^2z_1\, d^2z_2\, h(|z_{12}|)\, ik_1^\mu\, \delta(z_1 - z)\, ik_{2\mu}\, \delta^2(z_2 - z')\right]$
$: e^{ik_1 X(z)}\, e^{ik_2 X(z')} :$


$\int d^2x \left[1+x\right] = \int d^2z \left[1+z\right]$
$\int = \frac{i\varepsilon}{2\pi} \int [d\varphi]\, e^{-S} \int d^2\sigma$
$= \exp\left[-\frac{\alpha'}{2} \int d^2z_i\, d^2z_i \; h_i |z_{ii}|^{c} \; i k_i^{\mu} \; \delta^2(z_i - z) \; i k_{2\mu}^{0} \; \delta^2(z_2 - z') \right.$
$\left. : e^{i k_1 X(z)} \; e^{i k_2 X(z')} : \right]$



Pirsa: 09020019

$$\delta_\varepsilon \mathcal{O} = -\frac{\varepsilon}{2\pi i} \int d^2\sigma \sqrt{-g}\, \rho(\zeta) \langle \nabla_a J^a \rangle$$

$$\delta_\varepsilon \mathcal{O} = -\frac{\varepsilon}{2\pi i} \int d^2\sigma \sqrt{-g}\, \rho(\zeta) \langle \nabla_a J^a \rangle$$

$$0 = \int [d\varphi]\, e^{-S} \int d^2\sigma \sqrt{-g}\, \nabla_a J^a$$

$$0 = \frac{\varepsilon}{2\pi} \int [d\varphi]\, e^{-S} \int d^2\sigma \sqrt{-g}\, \nabla_a J^a \rho$$

$$0 = -\frac{\varepsilon}{2\pi i} \int d^2\sigma \sqrt{-g}\, \rho(\zeta) \langle \nabla_a J^a \rangle$$

$$\Downarrow$$

$\nabla_a J^a = 0$ as an operator statement

$$0 = \int [d\varphi]\, e^{-S} \int d^2\sigma \sqrt{-g}\, \nabla_a J^a \cdot \rho$$

$$0 = -\frac{\varepsilon}{2\pi i} \int d^2\sigma \sqrt{-g}\, \rho(\zeta) \langle \nabla_a J^a \rangle$$

$$\Downarrow$$

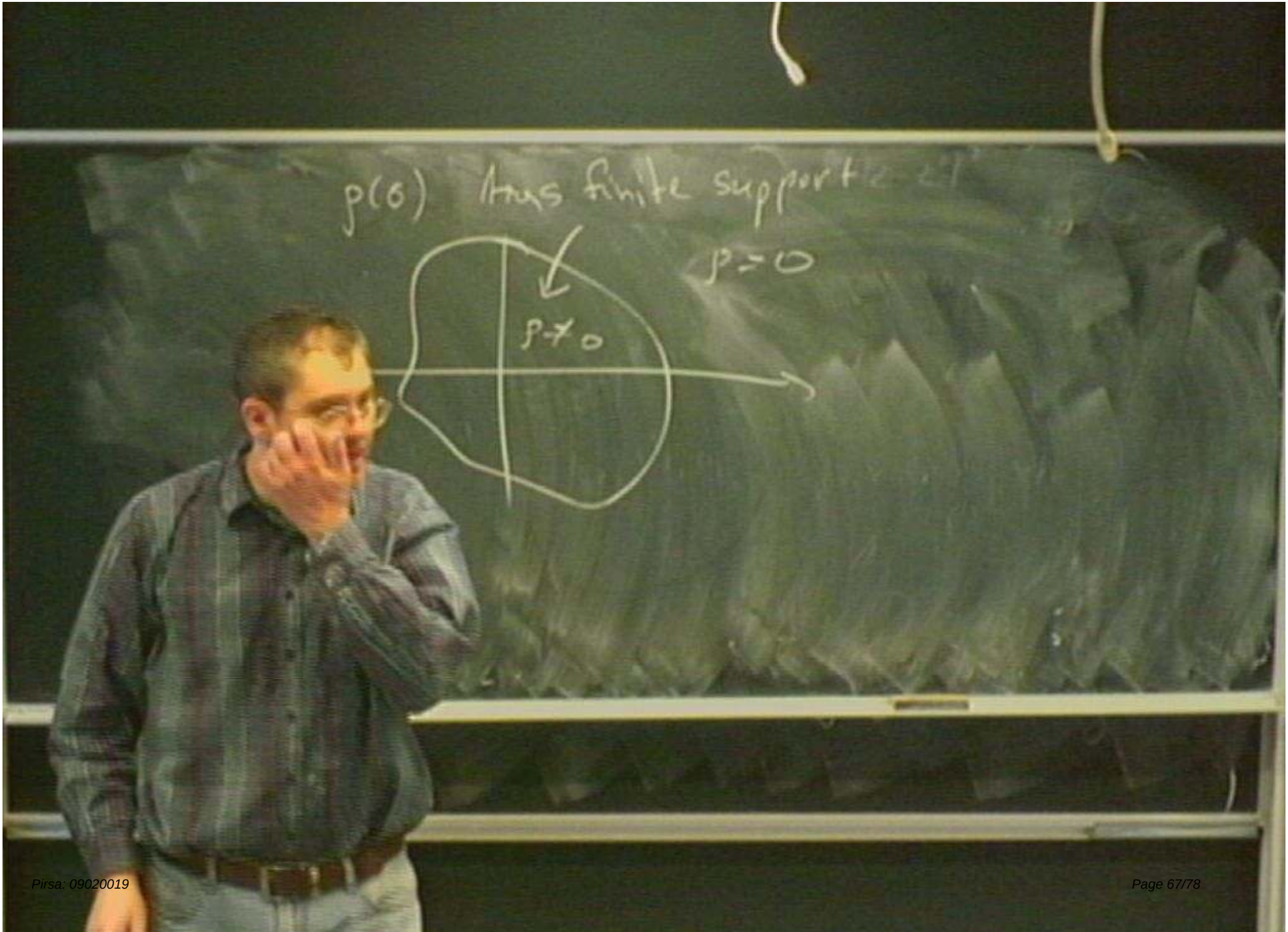
$$\nabla_a J^a = 0 \quad \text{as an operator statement}$$

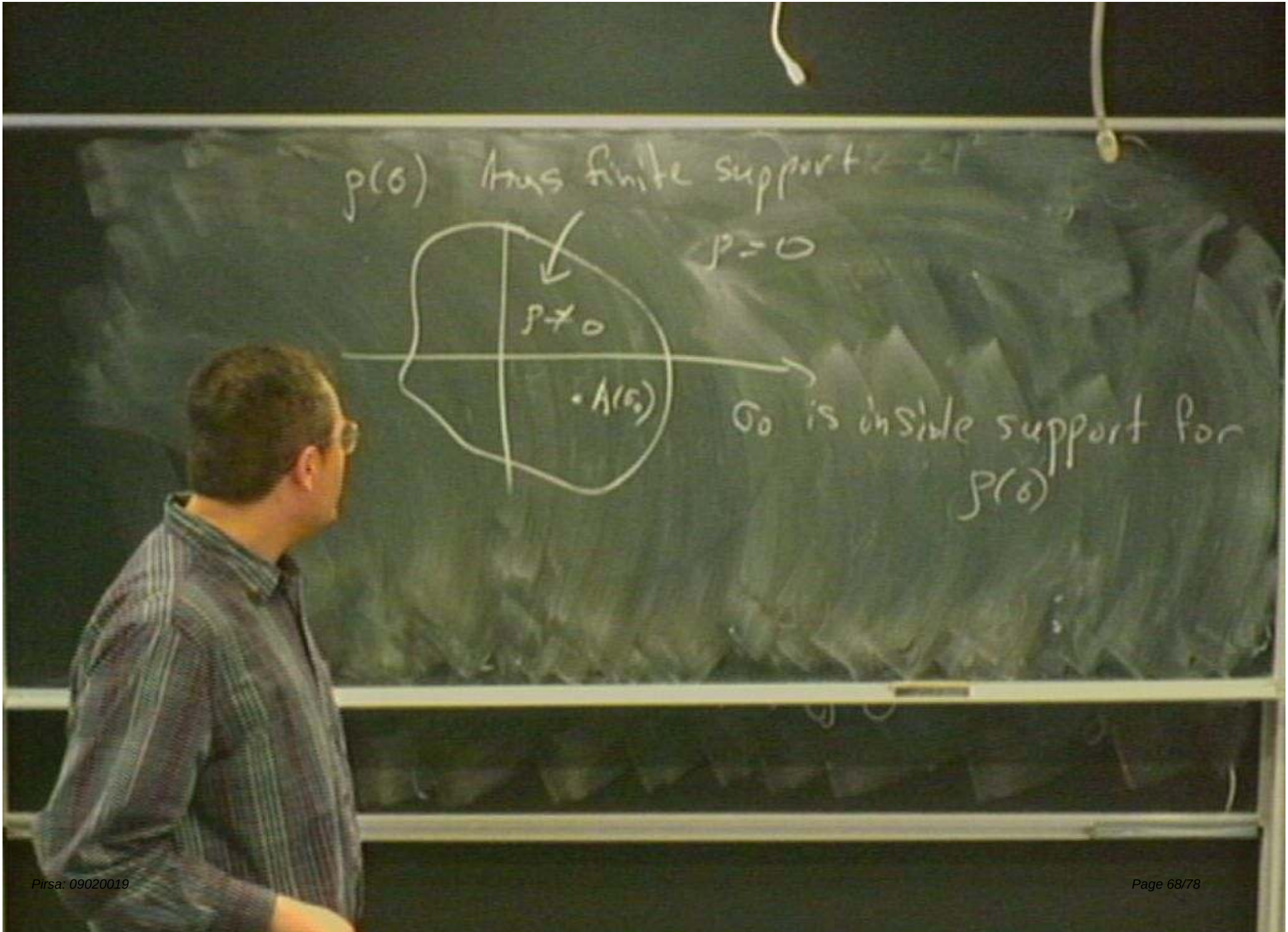
Sym trans.

$\varphi_x \to \varphi'_x = \varphi_x + \varepsilon \cdot \rho(\sigma) \cdot \delta\varphi_x(\sigma)$

$\rho(\sigma)$ has a finite support

$$\int [d\varphi]\, e^{-S[\varphi']} A(\varphi') = \int [d\varphi]\, e^{-S} \left[1 + \frac{i\varepsilon}{2\pi} \int d^2x \, j^{\mu}(x) \partial_{\mu} \varphi(x) + O(\varepsilon^2) \right]$$


p(0) has finite support
p=0
p≠0


p(0) has finite support
p=0
p≠0
A(G)
G0 is inside support for
p(0)

$\varphi_\alpha \to \varphi'_\alpha = \varphi_\alpha + \varepsilon \cdot \rho(\sigma) \cdot \delta\varphi_\alpha(\sigma)$; $A(\sigma) \to A'(\sigma) = A(\sigma) + \delta A$

$\rho(\sigma)$ has a finite support

$$\int [d\varphi']\, e^{-S[\varphi']} A(\sigma) = \int [d\varphi]\, e^{-S} \left[1 + \frac{i\varepsilon}{2\pi} \int d^2\sigma \sqrt{g}\, j^a(\sigma) \partial_a \rho + O(\varepsilon^2) \right]$$

$\varphi_\alpha \to \varphi'_\alpha = \varphi_\alpha + \varepsilon \cdot \rho(\sigma) \cdot \delta\varphi_\alpha(\sigma)$; $A(\sigma) \to A'(\sigma) = A(\sigma) + \delta A$

$\rho(\sigma)$ has a finite support

$$\int [d\varphi']\, e^{-S[\varphi']}\, A(\sigma) = \int [d\varphi]\, e^{-S}\left[A + \frac{i\varepsilon}{2\pi}\int d^2\sigma \sqrt{g}\, j(\sigma)\,\delta\varphi \right]$$

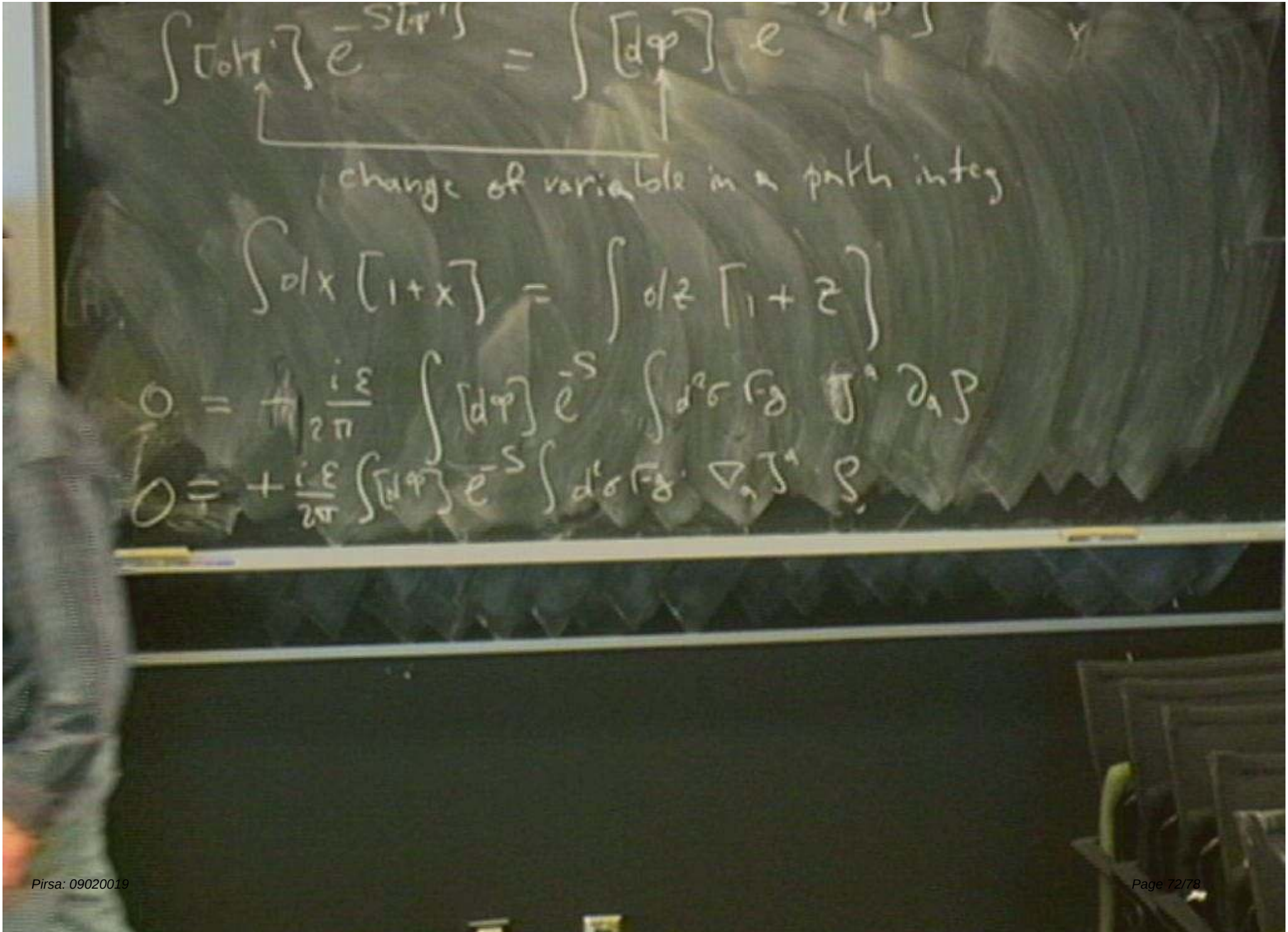
$$\delta A(\sigma) + O(\varepsilon^2)\Big]$$

$\varphi_\alpha \to \varphi'_\alpha = \varphi_\alpha + \varepsilon \cdot \rho(\sigma) \cdot \delta\varphi_\alpha(\sigma)$; $A(\sigma) \to A'(\sigma) = A(\sigma) + \delta A$

$\rho(\sigma)$ has a finite support

$\delta A \propto \varepsilon$

$$\int [d\varphi']\, e^{-S[\varphi']}\, A(\sigma)_{\{\varphi'\}} = \int [d\varphi]\, e^{-S}\left[A(\sigma)_{\{\varphi\}} + \frac{i\varepsilon}{2\pi}\int d^2\sigma \sqrt{-g}\, j^\alpha(\sigma)\,\partial_\alpha p \; \delta A(\sigma) + O(\varepsilon^2)\right]$$


change of variable in a path integ

$$0 = -\delta A(z) + \frac{i\varepsilon}{2\pi} \int [d\varphi]\, e^{-S} \int d^2\sigma \sqrt{-g}\; \Pi^a \nabla_a \rho$$

$$0 = -\delta A(z) + \frac{i\varepsilon}{2\pi} \int [d\varphi]\, e^{-S} \int d^2\sigma \sqrt{-g}\; \nabla_a J^a \rho$$

$$0 = -\frac{\varepsilon}{2\pi i} \int d^2\sigma \sqrt{-g}\; \rho(z) \; \langle \nabla_a J^a \rangle$$

$$\Downarrow$$

$$\nabla_a J^a = 0 \quad \text{as an operator statement}$$

$$0 = -\frac{\varepsilon}{2\pi i}\int d^2\sigma\,\sqrt{-g}\;\rho(\zeta)\;\langle\,\nabla_a T^a\,\rangle + \langle \delta A_0\rangle$$

$$\Downarrow$$

$\nabla_a T^a = 0$ as an operator statement

Symmetries.

$\varphi_\alpha \to \varphi'_\alpha = \varphi_\alpha + \varepsilon \cdot f(\sigma) \cdot \delta\varphi_\alpha(\sigma)$; $A(\sigma) \to A'(\sigma) = A(\sigma) + \delta A$

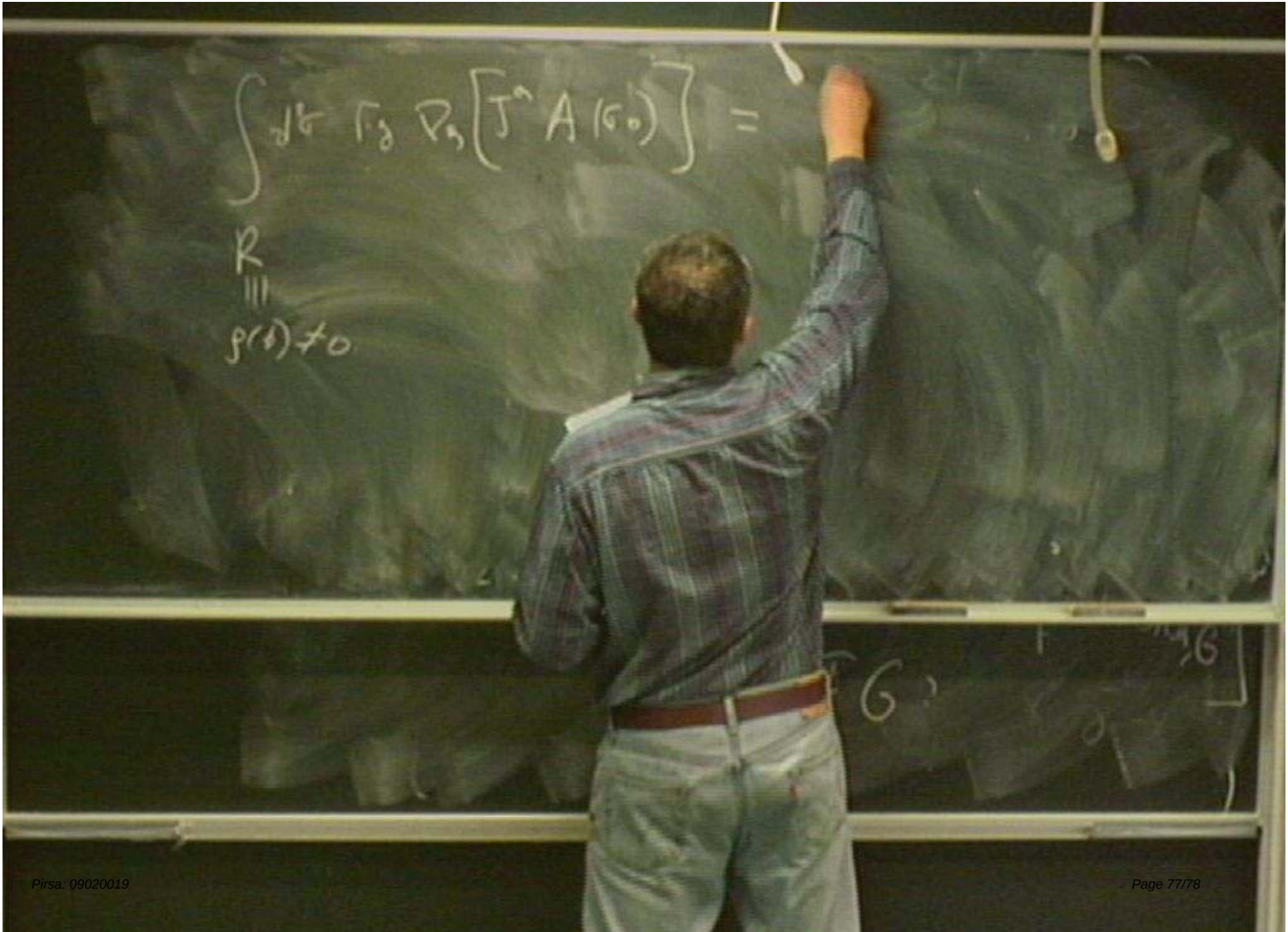
$\delta A = \varepsilon$

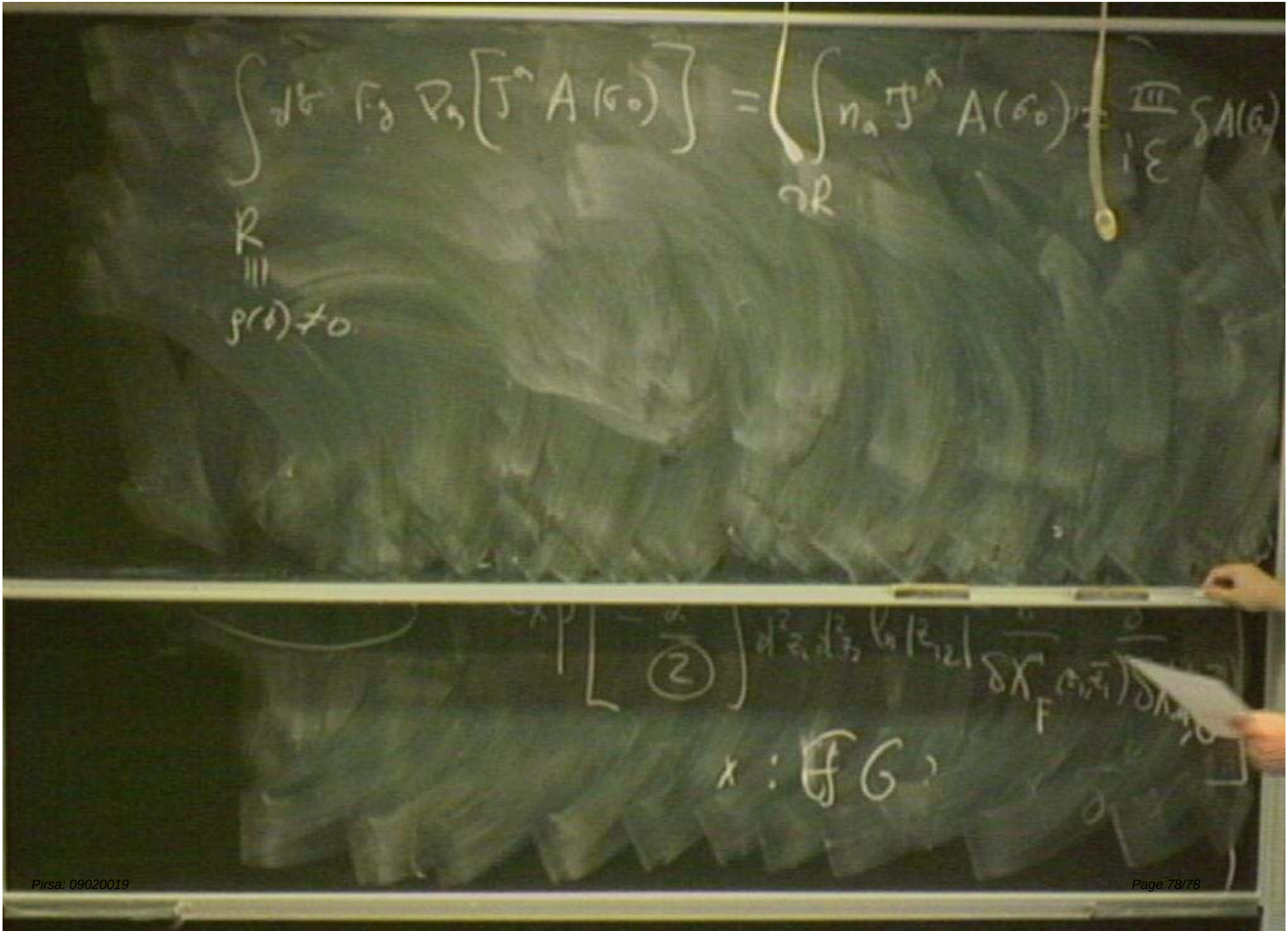
$f(\sigma)$ has a finite support

$$\int [d\varphi'] \, e^{-S[\varphi']} \, A(\sigma_0) = \int [d\varphi] \, e^{-S} \left[A(\sigma_0) + \frac{\varepsilon}{2\pi} \int d^2\sigma \, j^a(\sigma) \, \partial_a A(\sigma_0) + O(\varepsilon^2) \right]$$

$$0 = -\frac{\varepsilon}{2\pi i} \int d^2\sigma \sqrt{-g}\, \rho(\sigma) \langle A(\sigma) \nabla_a T^a \rangle + \langle \delta A(\sigma) \rangle$$

$$\nabla_a J^a A(\sigma_0) = \frac{1}{\sqrt{-g}} \delta^2(\sigma - \sigma_0) \frac{2\pi}{i\varepsilon} \sqrt{-g} A(\sigma)$$


R
ρ(t) ≠ 0


R
ρ(b) ≠ 0
∂R