Title: Introduction to the Bosonic String Part B

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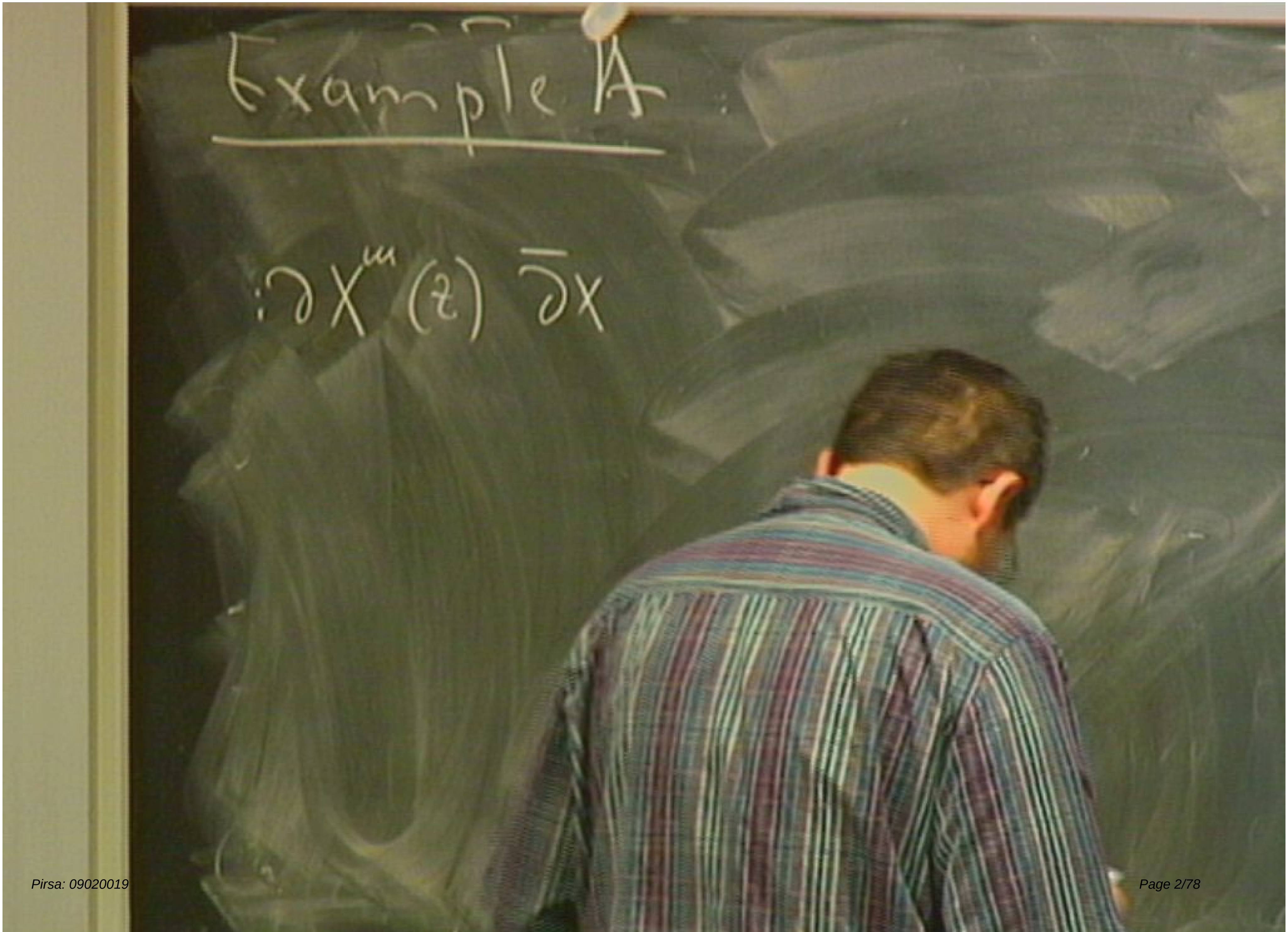
Abstract: This course provides a thorough introduction to the bosonic string based on the Polyakov path integral and conformal field theory. We introduce central ideas of string theory, the tools of conformal field theory, the Polyakov path integral, and the covariant quantization of the string. We discuss string interactions and cover the tree-level and one loop amplitudes. More advanced topics such as T-duality and D-branes will be taught as part of the course. The course is geared for M.Sc. and Ph.D. students enrolled in Collaborative Ph.D. Program in Theoretical Physics. Required previous course work: Quantum Field Theory (AM516 or equivalent). The course evaluation will be based on regular problem sets that will be handed in during the term. The primary text is the book: 'String theory. Vol. 1: An introduction to the bosonic string. J. Polchinski (Santa Barbara, KITP) . 1998. 402pp. Cambridge, UK: Univ. Pr. (1998) 402 p.' All interested students should contact Alex Buchel at abuchel@uwo.ca as soon as possible.



Example A

$$:\partial X^{\mu}(z)\,\bar{\partial}x$$

Example A :

$$:\partial X^{\mu}(z)\,\partial X_{\mu}(z):\ :\partial' X^{\nu}(z')\,\partial' X_{\nu}(z'):$$

Example A:

$$\underbrace{:\partial X^{m}(z)\,\partial X_{m}(z):}_{T'}\;:\partial' X^{\nu}(z')\,\partial' X_{\nu}(z'): =$$

Example A:

$$:\underbrace{\partial X^{\mu}(z)\,\partial X_{\mu}(z)}_{\text{related to stress energy tensor}}:\;:\partial' X^{\nu}(z')\,\partial' X_{\nu}(z'):\;=$$

$$=$$

Example A

$$\underbrace{:\partial X^{\mu}(z)\,\partial X_{\mu}(z):}_{\text{related to stress energy tensor}}\;:\partial' X^{\nu}(z')\,\partial' X_{\nu}(z'):\;=$$

$$= :\underset{\substack{\uparrow\\ z}}{\partial X^{\mu}\,\partial X_{\mu}}\;\underset{\substack{\uparrow\\ z'}}{\partial X^{\nu}\,\partial X_{\nu}}:$$

Example A

$$\underbrace{:\partial X^{\mu}(z)\,\partial X_{\mu}(z):}_{\text{related to stress energy tensor}}\ :\partial' X^{\nu}(z')\,\partial' X_{\nu}(z'):\ =$$

$$= \underbrace{:\underset{\substack{\uparrow\\ z}}{\partial X^{\mu}\,\partial X_{\mu}}\ \underset{\substack{\uparrow\\ z'}}{\partial X^{\nu}\,\partial X_{\nu}}:}_{\text{no singularities}}$$

traction = replace a pair of fields

Example A

$$\underbrace{:\partial X^{\mu}(z)\,\partial X_{\mu}(z):}_{\text{related to stress energy tensor}}\;:\partial' X^{\nu}(z')\,\partial' X_{\nu}(z'):\;=$$

$$=\underbrace{:\underset{\underset{z}{\uparrow}}{\partial X^{\mu}\partial X_{\mu}}\;\underset{\underset{z'}{\uparrow}}{\partial X^{\nu}\partial X_{\nu}}:}_{\text{no singularities}}\;-\frac{\alpha'}{2}\cdot$$

ntraction = replace a pair of fields

$\frac{1}{2}\alpha'\eta^{\mu_i\mu_j}\ln|z_{ij}|^2$

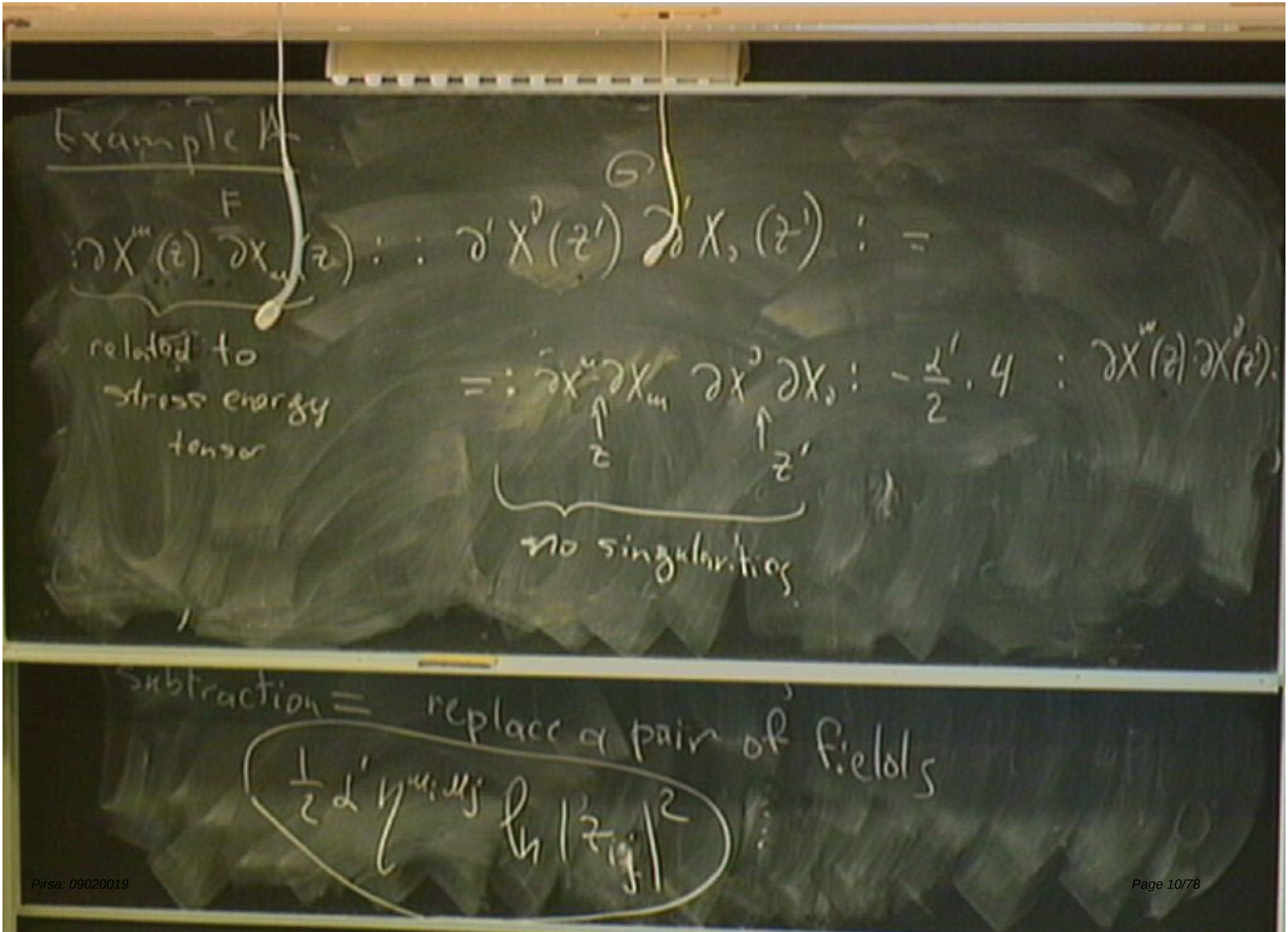
Example A

$$:\underbrace{\partial X^\mu(z)\,\overset{F}{\partial X_\mu}(z)}_{\text{related to stress energy tensor}}:\;:\partial' X^\nu(z')\,\overset{G}{\partial' X_\nu}(z'):\;=$$

$$=:\underbrace{\underset{\overset{\uparrow}{z}}{\partial X^\mu\,\partial X_\mu}\;\underset{\overset{\uparrow}{z'}}{\partial X^\nu\,\partial X_\nu}}_{\text{no singularities}}:\;-\frac{\alpha'}{2}\cdot 4$$

...traction = replace a pair of fields

$$\frac{1}{2}\alpha'\eta^{\mu_i\mu_j}\ln|z_{ij}|^2$$



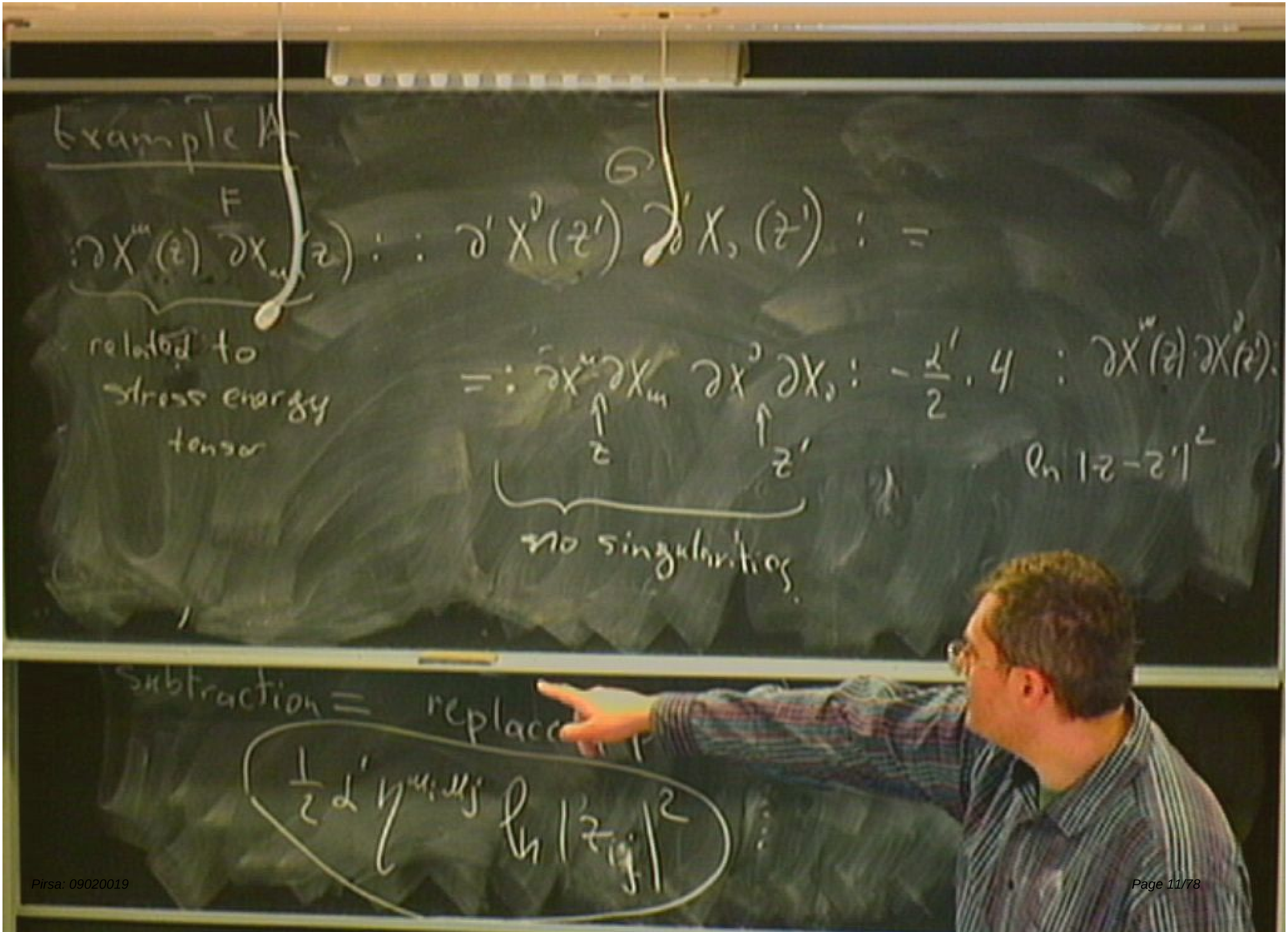
Example A

$$\underbrace{:\overset{F}{\partial X^{\mu}(z)\,\partial X_{\mu}(z)}:}_{\text{related to stress energy tensor}}\;:\overset{G}{\partial' X^{\nu}(z')\,\partial' X_{\nu}(z')}: =$$

$$= \underbrace{:\underset{\substack{\uparrow\\ z}}{\partial X^{\mu}\partial X_{\mu}}\;\underset{\substack{\uparrow\\ z'}}{\partial X^{\nu}\partial X_{\nu}}:}_{\text{no singularities}} - \frac{\alpha'}{2}\cdot 4\;:\partial X^{\mu}(z)\,\partial X^{\nu}(z'):$$

Subtraction = replace a pair of fields

$$\boxed{\frac{1}{2}\alpha'\eta^{\mu_i\mu_j}\ln|z_{ij}|^2}\;:$$



Example A

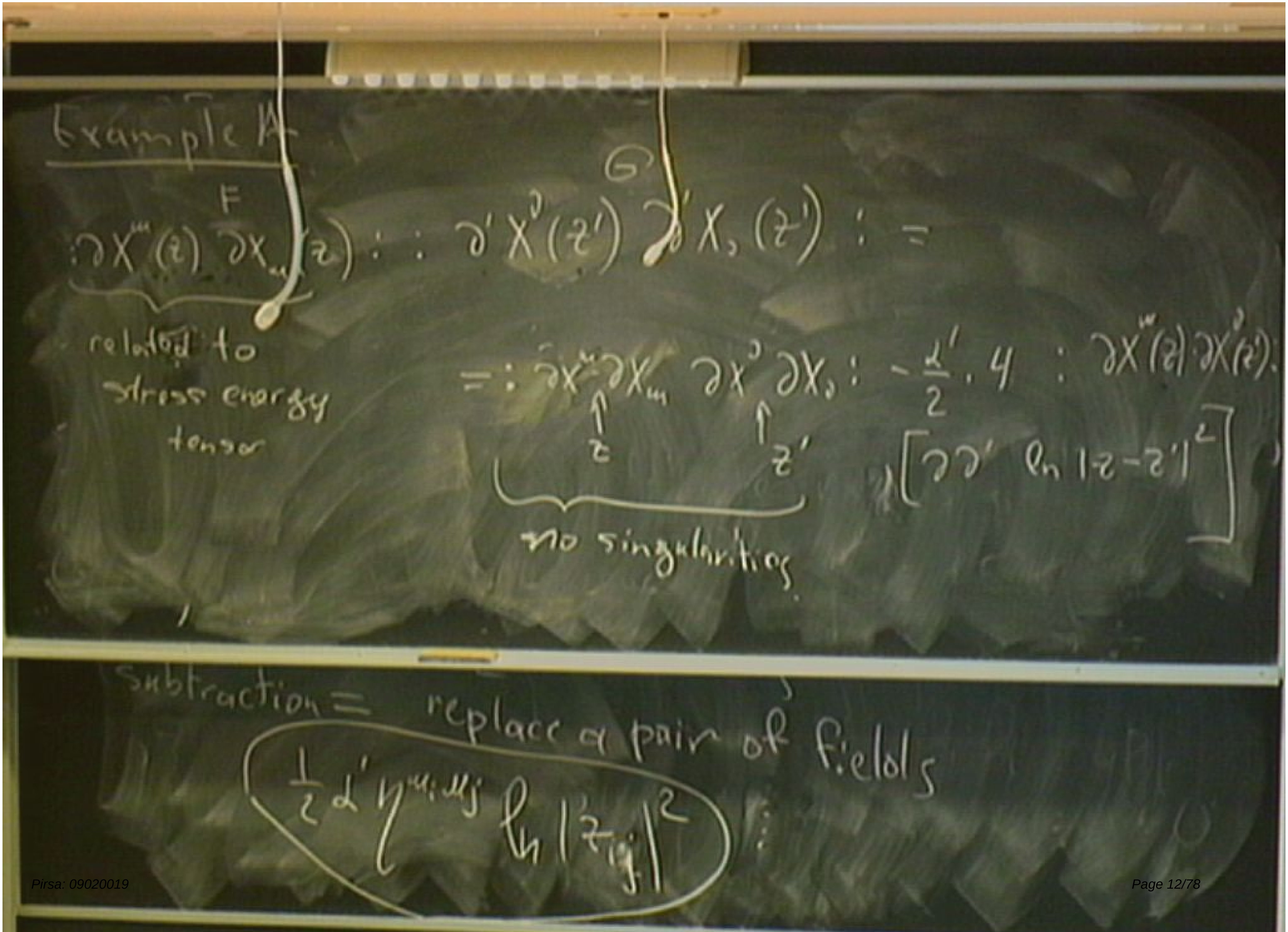
$$\underbrace{:\overset{F}{\partial X^{m}(z)\,\partial X_{m}(z)}:}_{\text{related to stress energy tensor}}\;:\partial' X^{\nu}(z')\,\partial' X_{\nu}(z'):\;=$$

$$=\underbrace{:\underset{\overset{\uparrow}{z}}{\partial X^{m}\partial X_{m}}\;\underset{\overset{\uparrow}{z'}}{\partial X^{\nu}\partial X_{\nu}}:}_{\text{no singularities}}\;-\;\frac{\alpha'}{2}\cdot 4\;:\partial X^{m}(z)\,\partial X^{\nu}(z'):$$

$\ln|z-z'|^2$

Subtraction = replace

$$\frac{1}{2}\alpha'\,\eta^{\mu_i\mu_j}\ln|z_{ij}|^2$$



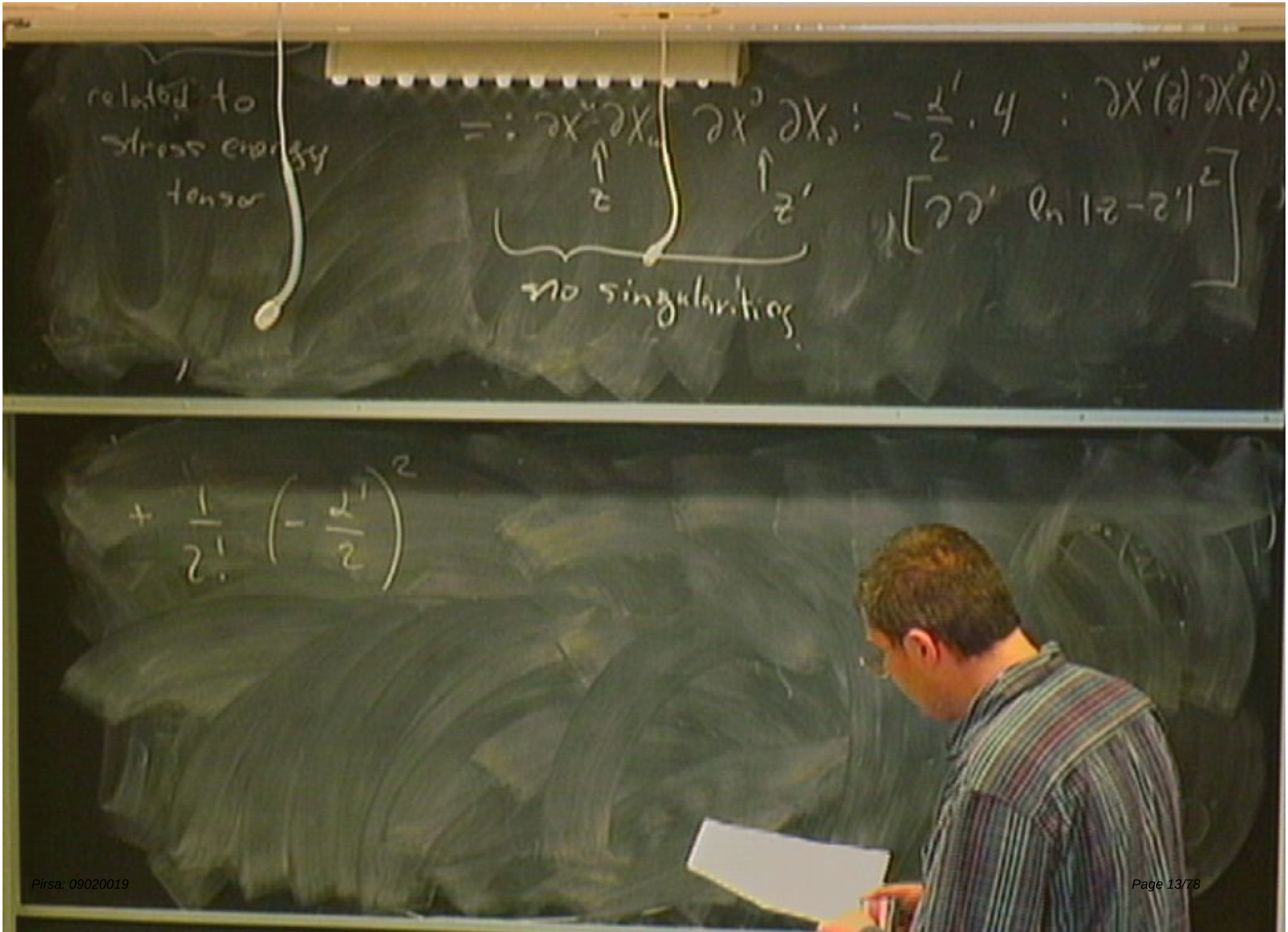
Example A

$$\underbrace{:\overset{F}{\partial X^{\mu}(z)\,\partial X_{\mu}(z)}:}_{\text{related to stress energy tensor}}\;:\overset{G}{\partial' X^{\nu}(z')\,\partial' X_{\nu}(z')}: =$$

$$= \underbrace{:\underset{\overset{\uparrow}{z}}{\partial X^{\mu}\partial X_{\mu}}\;\underset{\overset{\uparrow}{z'}}{\partial X^{\nu}\partial X_{\nu}}:}_{\text{no singularities}} - \frac{\alpha'}{2}\cdot 4\;:\partial X^{\mu}(z)\,\partial X^{\nu}(z'):\left[\partial\partial' \ln|z-z'|^2\right]$$

Subtraction = replace a pair of fields

$$\boxed{\frac{1}{2}\alpha'\eta^{\mu_i\mu_j}\ln|z_{ij}|^2}\;:$$



related to
stress energy
tensor

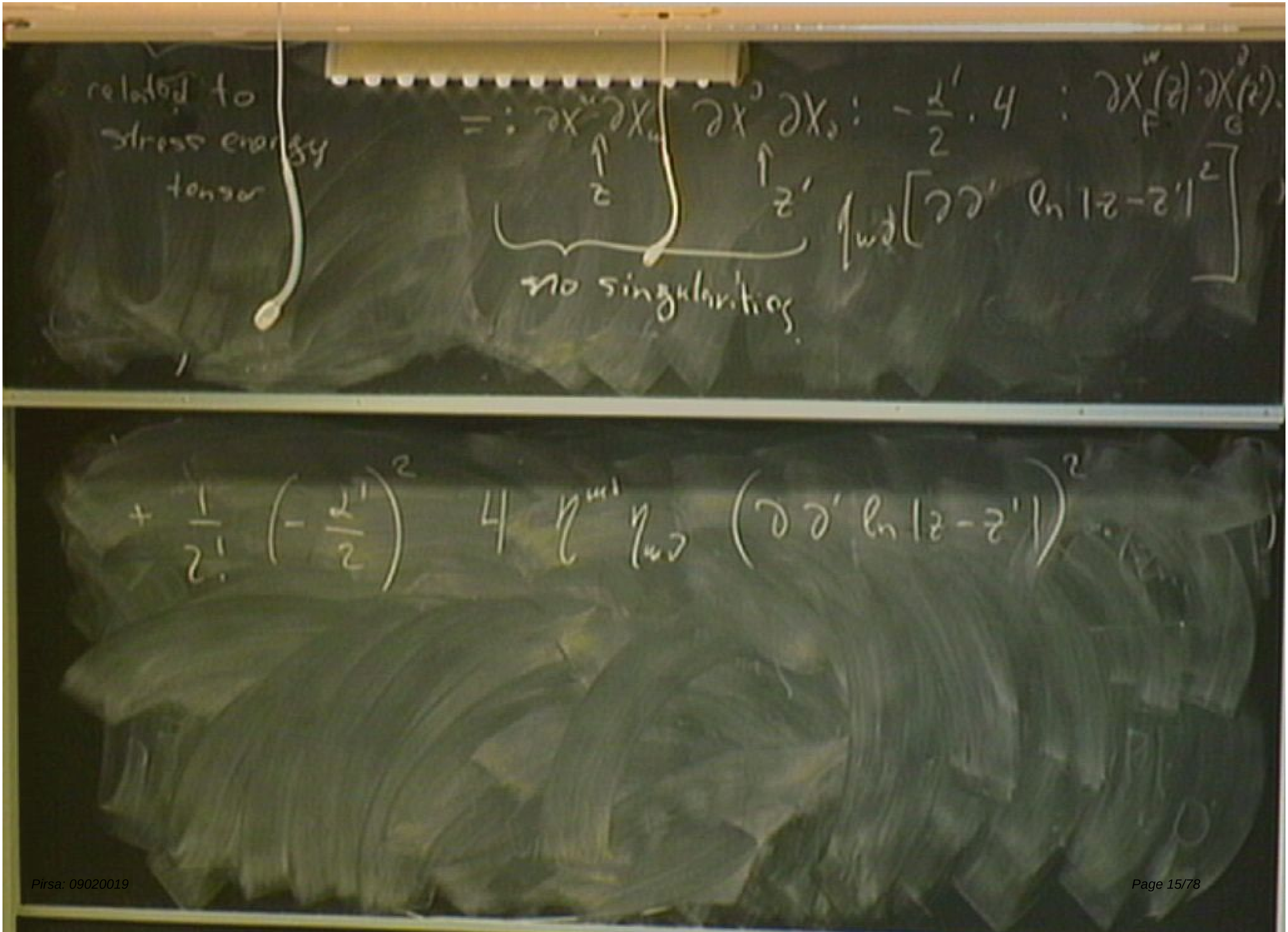
$$= \underbrace{:\underset{\substack{\uparrow \\ z}}{\partial X^\mu \partial X_\mu} \; \underset{\substack{\uparrow \\ z'}}{\partial X^\nu \partial X_\nu}:}_{\text{no singularities}} - \frac{\alpha'}{2}\cdot 4 \; :\partial X^\mu(z)\,\partial X^\nu(z'): \left[\partial\partial' \ln|z-z'|^2\right]$$

$$+ \frac{1}{2!}\left(-\frac{\alpha'}{2}\right)^2$$

related to
stress energy
tensor

$$= \underbrace{:\overset{\uparrow}{\underset{z}{\partial X^\mu \partial X_\mu}}\ \overset{\uparrow}{\underset{z'}{\partial X^\nu \partial X_\nu}}:}_{\text{no singularities}} - \frac{\alpha'}{2} \cdot 4 \ :\partial X^\mu(z)\, \partial X^\nu(z'): \ \text{1 w.d.}\left[\partial\partial' \ln|z-z'|^2\right]$$

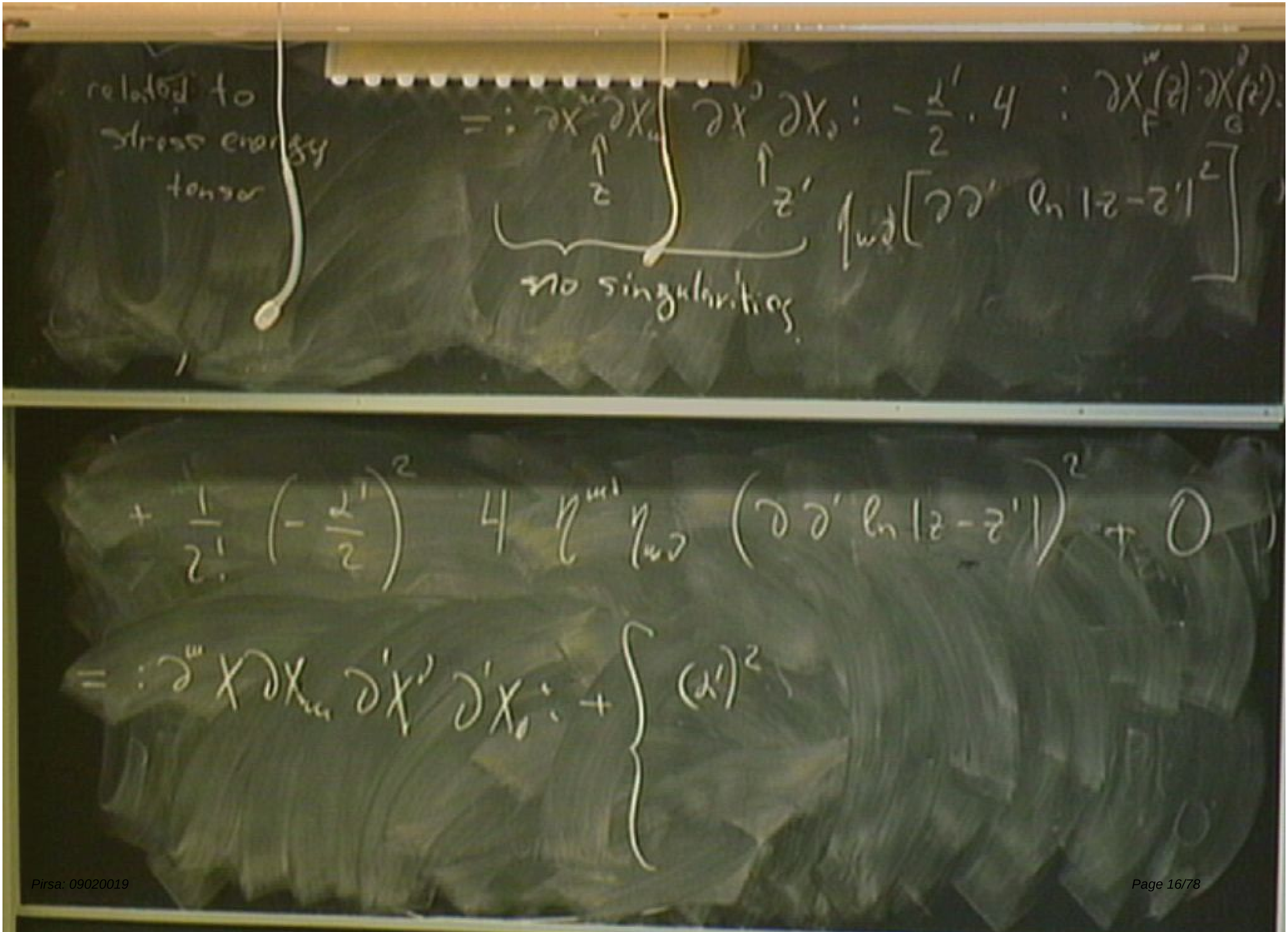
$$+ \frac{1}{2!}\left(-\frac{\alpha'}{2}\right)^2 \ 4$$



related to stress energy tensor

$$= \underbrace{:\partial X^{\mu}\underset{\substack{\uparrow\\ z}}{\partial X_{\mu}}\;\partial X^{\nu}\underset{\substack{\uparrow\\ z'}}{\partial X_{\nu}}:}_{\text{no singularities}} - \frac{\alpha'}{2}\cdot 4 \;:\partial X_F^{\mu}(z)\cdot\partial X_G^{\nu}(z'):\;\eta_{\mu\nu}\left[\partial\partial'\ln|z-z'|^2\right]$$

$$+\frac{1}{2!}\left(-\frac{\alpha'}{2}\right)^2 4\;\eta^{\mu\mu'}\eta_{\mu\nu}\left(\partial\partial'\ln|z-z'|\right)^2$$



related to stress energy tensor

$$= \underbrace{:\underset{\overset{\uparrow}{z}}{\partial X^\mu \partial X_\mu}\ \underset{\overset{\uparrow}{z'}}{\partial X^\nu \partial X_\nu}:}_{\text{no singularities}} - \frac{\alpha'}{2} \cdot 4 \ :\partial X^\mu_F(z) \cdot \partial X^\nu_G(z'): \ \eta_{\mu\nu}\left[\partial\partial' \ln|z-z'|^2\right]$$

$$+ \frac{1}{2!}\left(-\frac{\alpha'}{2}\right)^2 4 \ \eta^{\mu\nu}\eta_{\mu\nu}\left(\partial\partial' \ln|z-z'|\right)^2 + O($$

$$= :\partial^\mu X \partial X_\mu \partial' X^\nu \partial' X_\nu: + \Big\{ (\alpha')^2$$

related to stress energy tensor

$$= :\underbrace{\underset{\uparrow z}{\partial X^\mu \partial X_\mu} \; \underset{\uparrow z'}{\partial X^\nu \partial X_\nu}}_{\text{no singularities}}: - \frac{\alpha'}{2} \cdot 4 \; :\partial X^\mu_F(z) \cdot \partial X^\nu_G(z'): \; \eta_{\mu\nu}\left[\partial \partial' \ln|z-z'|^2\right]$$

$$+ \frac{1}{2!}\left(-\frac{\alpha'}{2}\right)^2 4 \; \eta^{\mu\nu}\eta_{\mu\nu}\left(\partial \partial' \ln|z-z'|\right)^2 + O$$

$$= :\partial^\mu X \partial X_\mu \partial' X^\nu \partial' X_\nu: + \left\{ (\alpha')^2 \, D \right.$$

related to stress energy tensor

$$= \underbrace{:\underset{\overset{\uparrow}{z}}{\partial X^\mu \partial X_\mu}\;\underset{\overset{\uparrow}{z'}}{\partial X^\nu \partial X_\nu}:}_{\text{no singularities}} - \frac{\alpha'}{2} \cdot 4 \; :\partial X^\mu_F(z) \cdot \partial X^\nu_G(z'): \; \eta_{\mu\nu} \left[\partial \partial' \ln |z-z'|^2 \right]$$

$$+ \frac{1}{2!} \left(-\frac{\alpha'}{2} \right)^2 4 \; \eta^{\mu\nu} \eta_{\mu\nu} \left(\partial \partial' \ln |z-z'| \right)^2 + O \;)$$

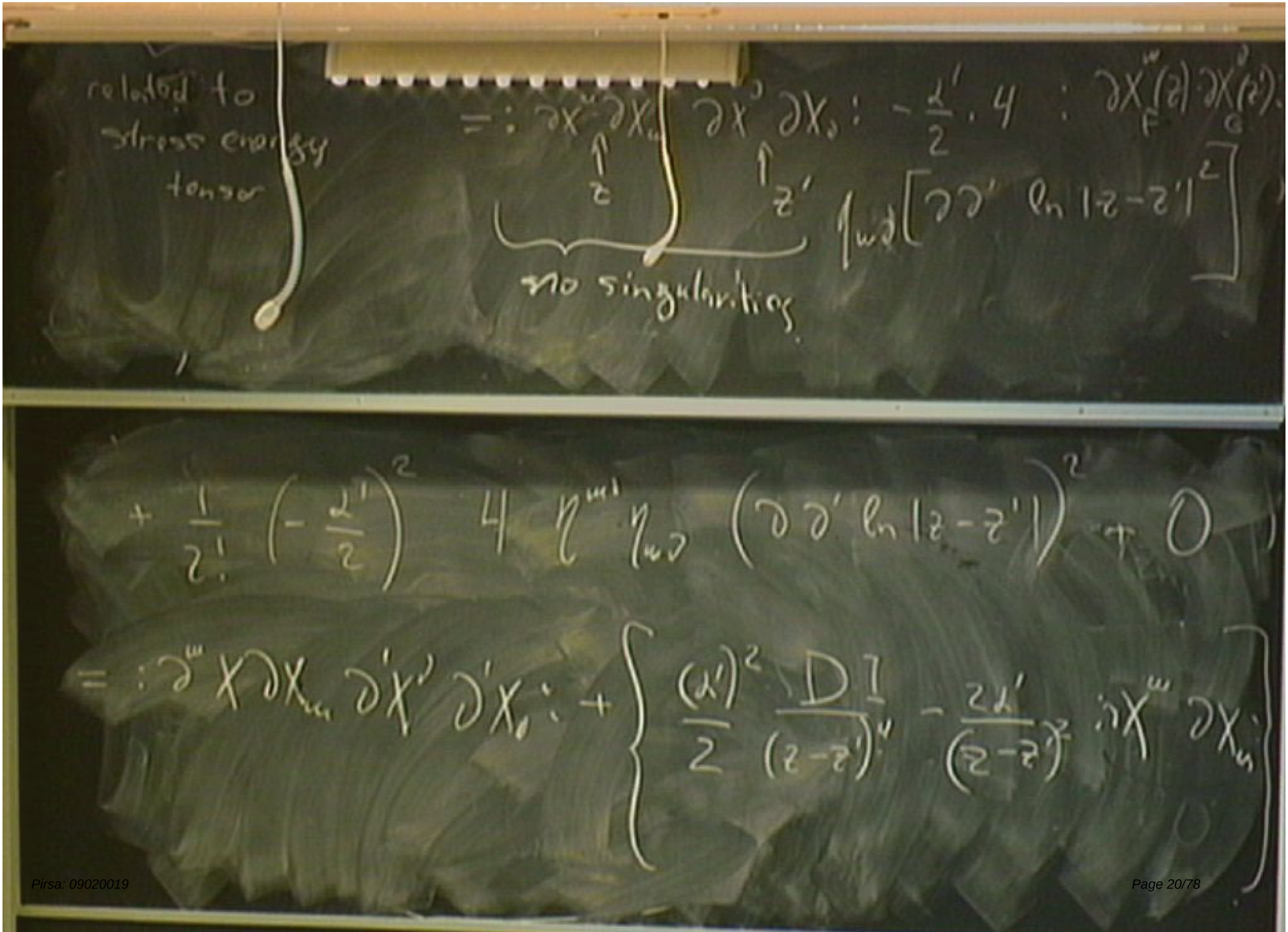
$$= \; :\partial^\mu X \partial X_\mu \partial' X^\nu \partial' X_\nu: + \left\{ \frac{(\alpha')^2}{2} \frac{D}{(z-z')^4} \right.$$

related to
stress energy
tensor

$$= :\underbrace{\overset{\uparrow}{\underset{z}{\partial X^\mu \partial X_\mu}} \; \overset{\uparrow}{\underset{z'}{\partial X^\nu \partial X_\nu}}}_{\text{no singularities}} : - \frac{\alpha'}{2} \cdot 4 \; :\partial X^\mu_F(z) \cdot \partial X^\nu_G(z'): \; \eta_{\mu\nu} \left[\partial\partial' \ln|z-z'|^2 \right]$$

$$+ \frac{1}{2!} \left(- \frac{\alpha'}{2} \right)^2 4 \; \eta^{\mu\nu} \eta_{\mu\nu} \left(\partial\partial' \ln|z-z'| \right)^2 + O \;)$$

$$= :\partial^\mu X \partial X_\mu \, \partial' X^\nu \partial' X_\nu : + \left\{ \frac{(\alpha')^2}{2} \frac{D}{(z-z')^4} - \frac{2\alpha'}{(z-z')^2} :\partial X^\mu \partial X_\mu: \right.$$



related to stress energy tensor

$$= \underbrace{:\overset{\uparrow}{\underset{z}{\partial X^\mu \partial X_\mu}}\;\overset{\uparrow}{\underset{z'}{\partial X^\nu \partial X_\nu}}:}_{\text{no singularities}} - \frac{\alpha'}{2}\cdot 4 \; :\partial X^\mu_F(z)\cdot\partial X^\nu_G(z'): \; \eta_{\mu\nu}\left[\partial\partial' \ln|z-z'|^2\right]$$

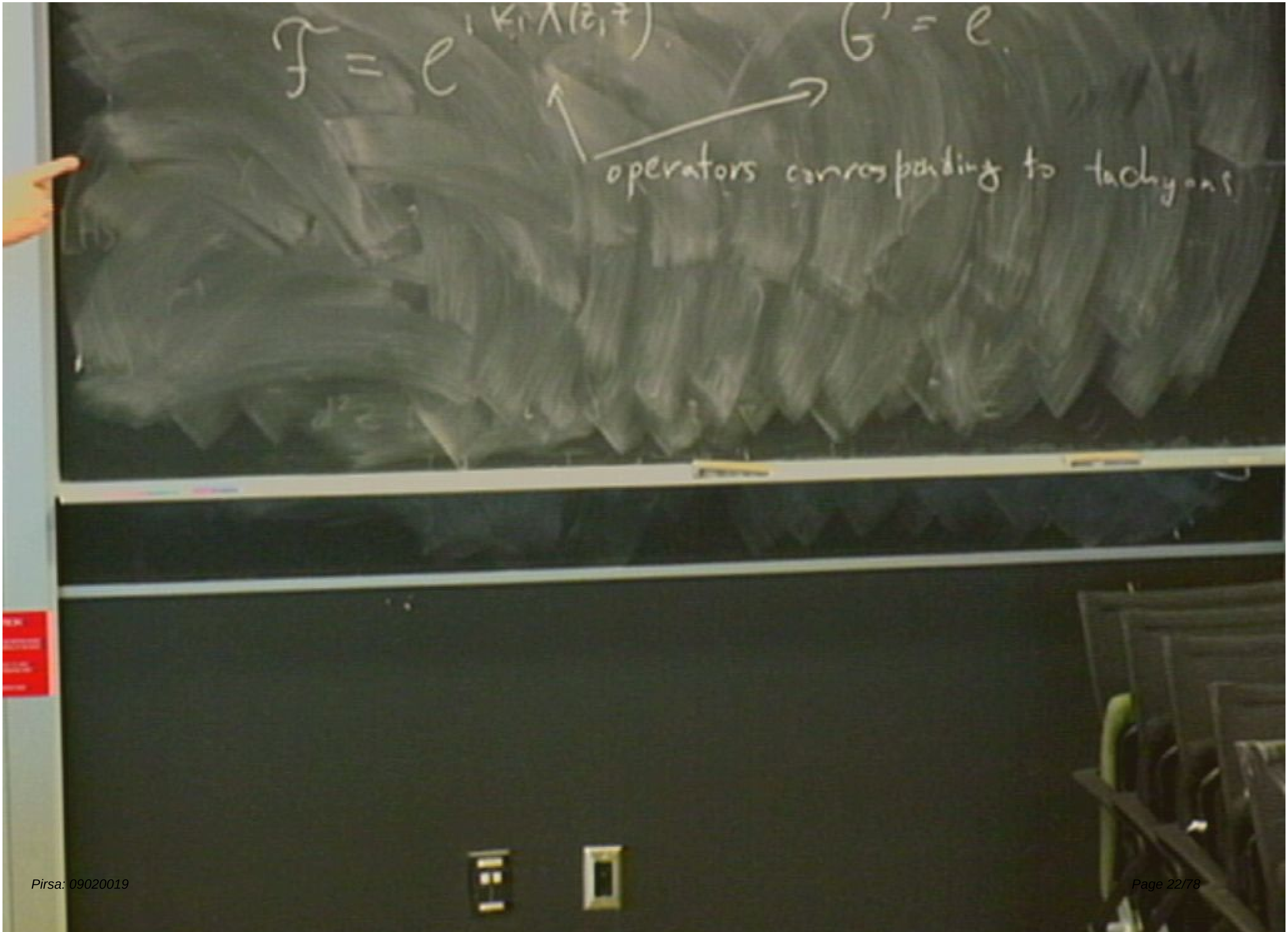
$$+\frac{1}{2!}\left(-\frac{\alpha'}{2}\right)^2 4\,\eta^{\mu\nu}\eta_{\mu\nu}\left(\partial\partial'\ln|z-z'|\right)^2 + O\;)$$

$$= :\partial^\mu X\partial X_\mu\, \partial' X^\nu\, \partial' X_\nu: + \left\{ \frac{(\alpha')^2}{2}\,\frac{D\,\mathbb{1}}{(z-z')^4} - \frac{2\alpha'}{(z-z')^2}\, :\partial X^\mu\, \partial X_\mu: \right\}$$

Example

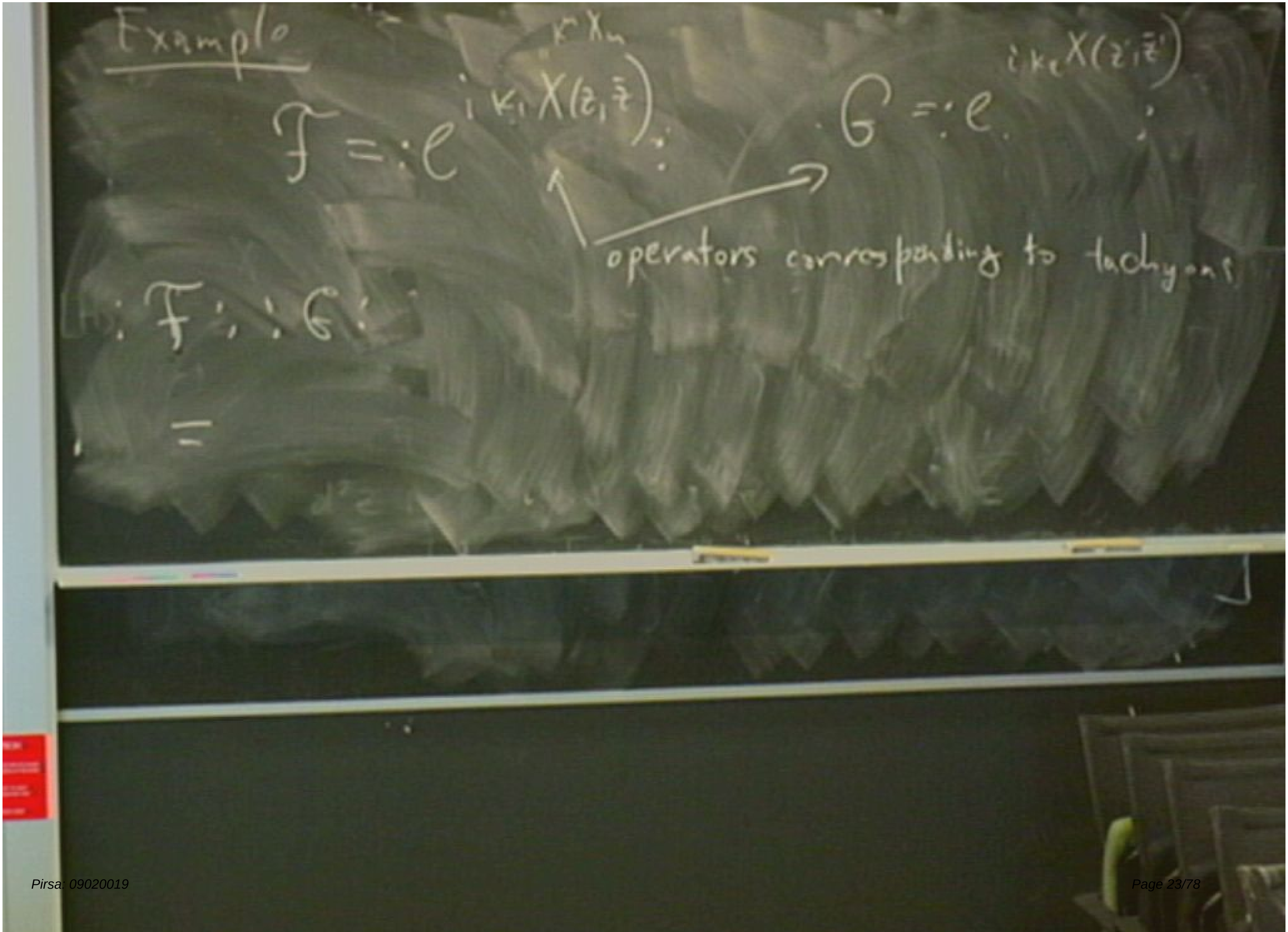
$k^\mu X_\mu$

$$\mathcal{F} = e^{i k_1 X(z, \bar{z})}. \qquad G = e^{i k_2 X(z', \bar{z}')}.$$



$\mathcal{T} = e^{i k_i \Lambda(z, \bar{z})}$. $G' = e$.

operators corresponding to tachyons



Example

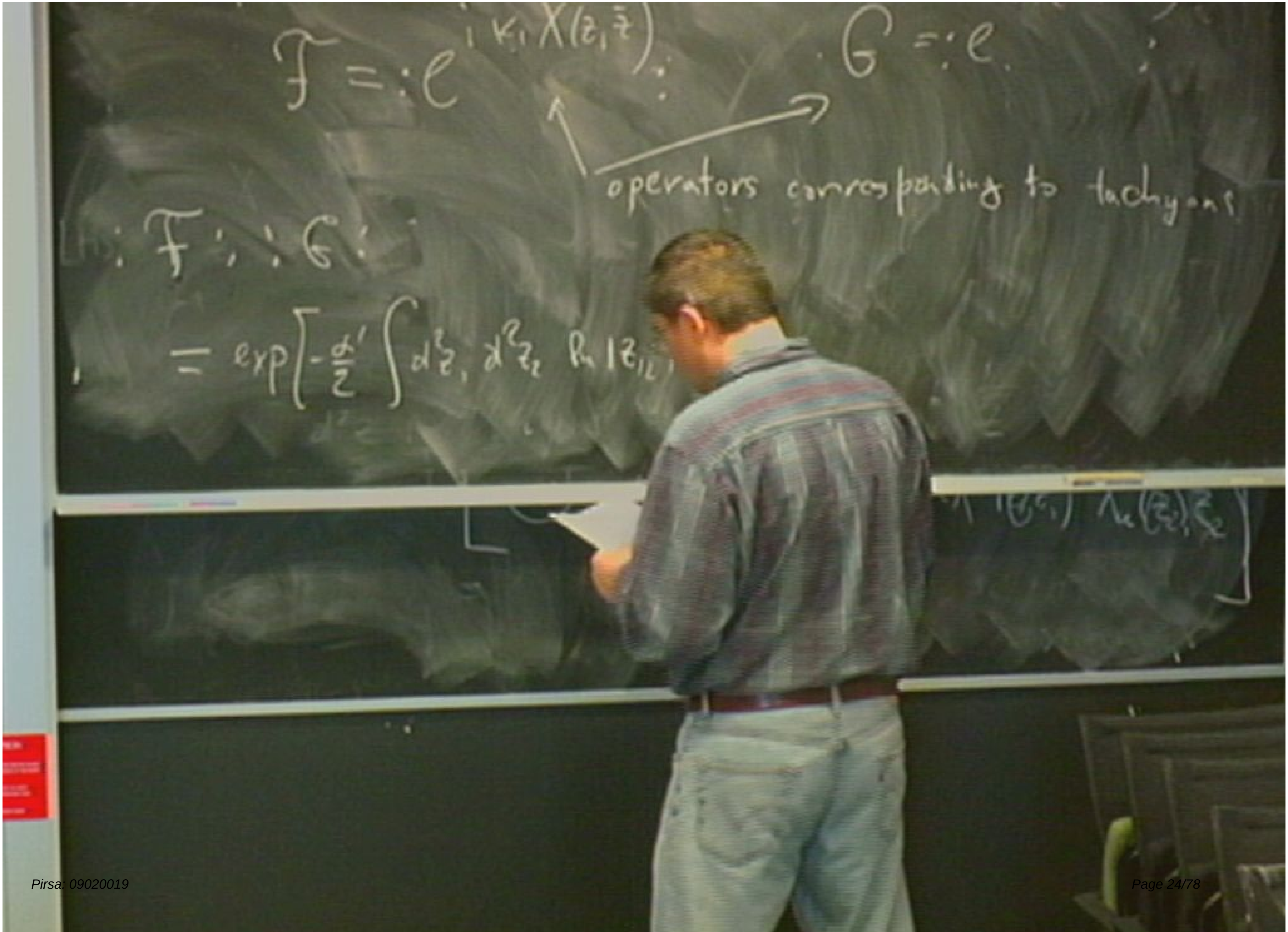
$k^\mu X_\mu$

$$\mathcal{F} = :e^{ik_1 X(z_1,\bar{z}_1)}: \qquad \mathcal{G} = :e^{ik_2 X(z_2,\bar{z}_2)}:$$

operators corresponding to tachyons

$$:\mathcal{F}:\,:\mathcal{G}:$$

$$=$$



$\mathcal{F} = :e^{i k_1 X(z_1 \bar{z})}:$ $\mathcal{G} = :e$:

operators corresponding to tachyons

$: \mathcal{F} : \; : \mathcal{G} :$

$= \exp\left[-\frac{\alpha'}{2} \int d^2 z_1 \, d^2 z_2 \, \ln |z_{12}|\right.$

$\Lambda(z_1, \bar{z}_1) \; \Lambda_2(\bar{z}_2, \bar{z}_2) \Big]$

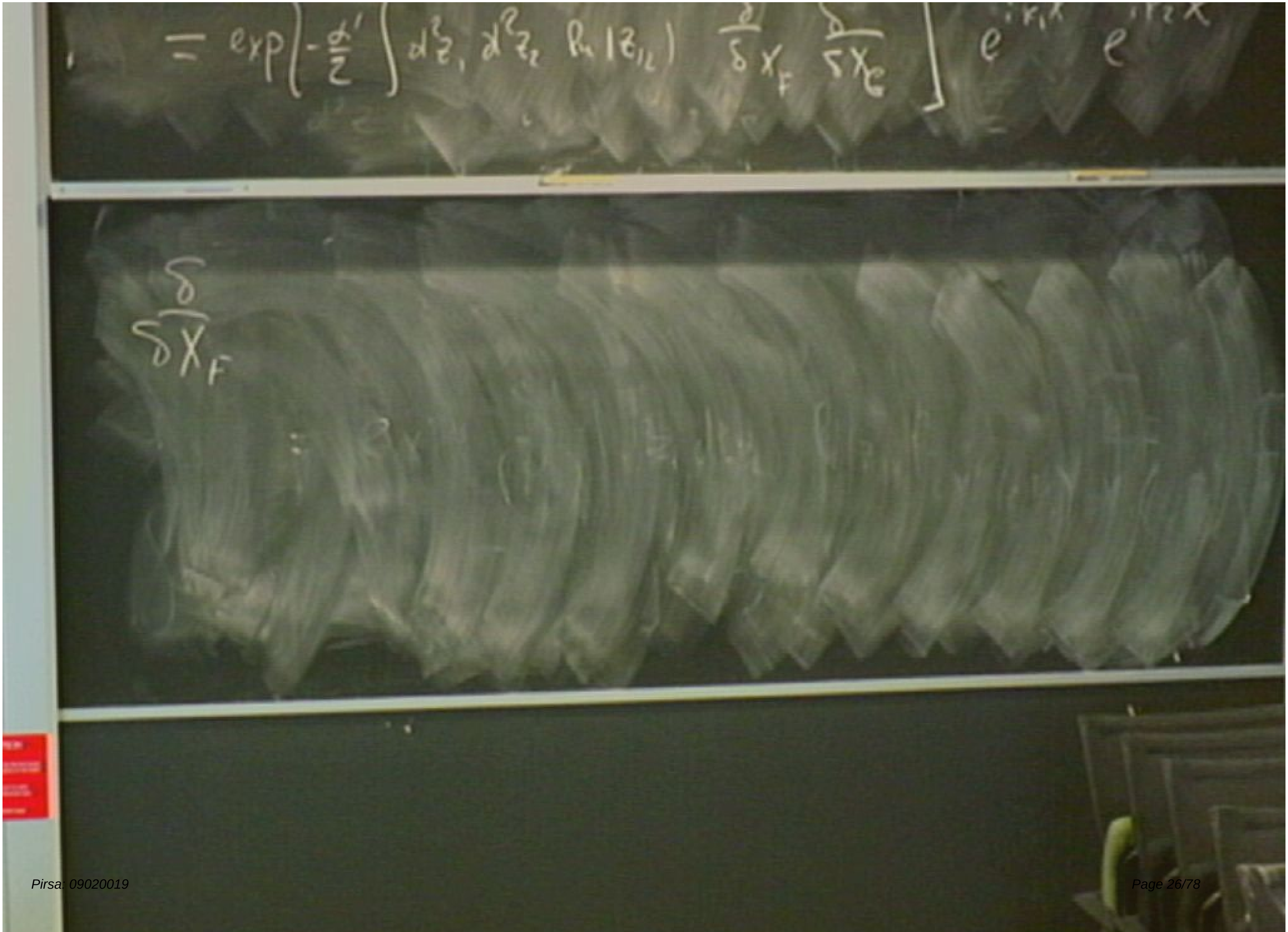
$$\mathcal{F} = :e^{i k_1 X(z_1 \bar{z})}: \qquad \mathcal{G} = :e\ :$$

operators corresponding to tachyons

$$:\mathcal{F}:\ :\mathcal{G}:$$

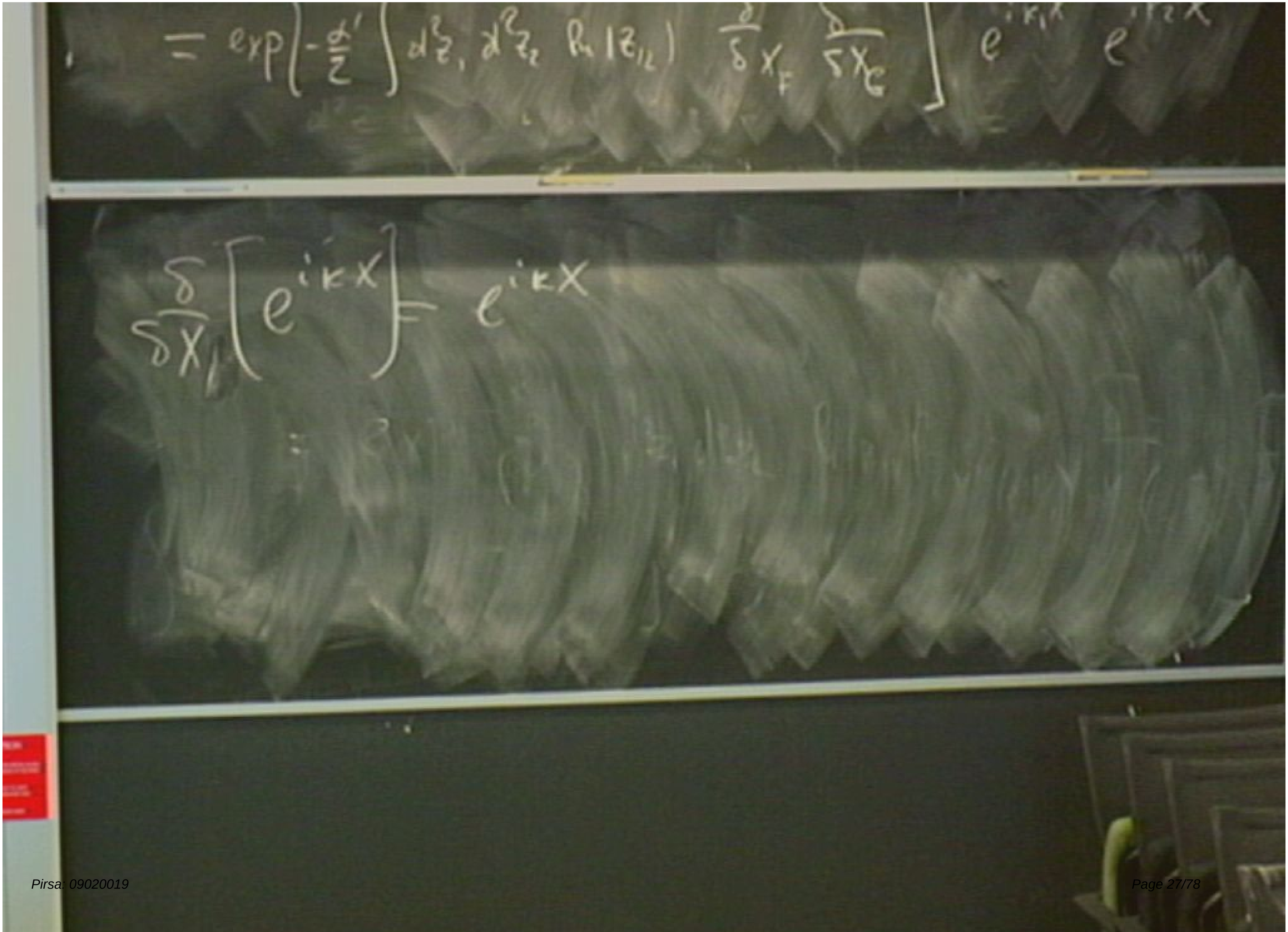
$$= \exp\left[-\frac{\alpha'}{2}\int d^2z_1\, d^2z_2\ \ln|z_{12}|\ \frac{\delta}{\delta X_F}\ \frac{\delta}{\delta X_G}\right] e^{ik_1X}\ e^{ik_2X}$$

$$X^\mu(z_1,\bar{z}_1)\ X_\mu(z_2,\bar{z}_2)$$



$$= \exp\Big(-\frac{\alpha'}{2}\int d^2z_1\, d^2z_2\, P_1(z_{12})\, \frac{\delta}{\delta X_F}\,\frac{\delta}{\delta X_G}\Big]\, e^{ik_1X}\, e^{ik_2X}$$

$$\frac{\delta}{\delta X_F}$$



$$= \exp\left[-\frac{\alpha'}{2}\int d^2z_1\, d^2z_2\, \ln|z_{12}|\ \frac{\delta}{\delta X_F}\,\frac{\delta}{\delta X_G}\right] e^{ik_1X}\, e^{ik_2X}$$

$$\frac{\delta}{\delta X}\left[e^{ikX}\right] = e^{ikX}$$

$$= \exp\left(-\frac{\alpha'}{2}\int d^2z_1\, d^2z_2\, \ln|z_{12}|\, \frac{\delta}{\delta X_F}\frac{\delta}{\delta X_G}\right]\, e^{ik_1X}\, e^{ik_2X}$$

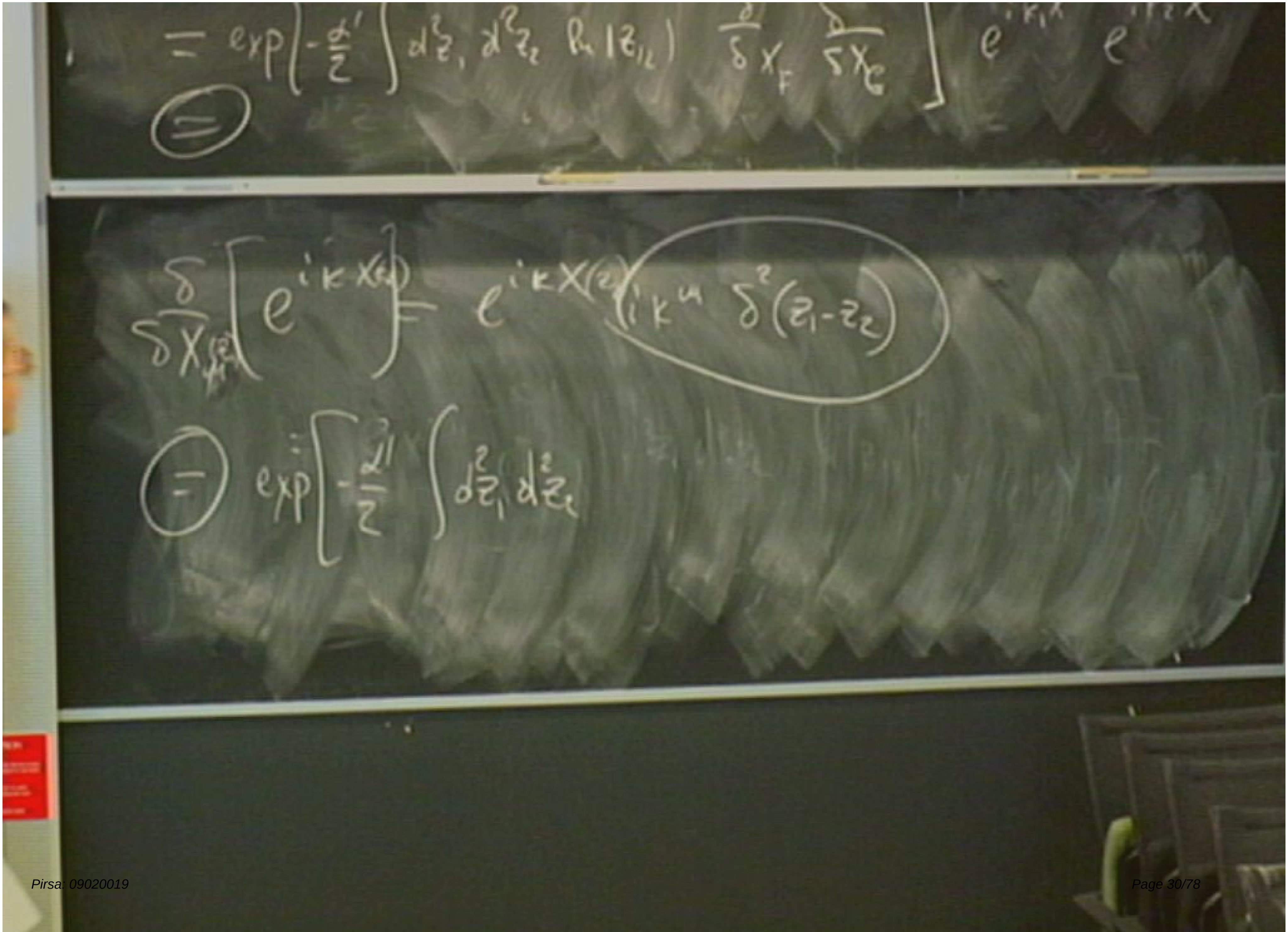
$$\frac{\delta}{\delta X_{\mu}(z_1)}\left[e^{ik\cdot X(z_2)}\right] = e^{ikX}\, ik^{\mu}\, \delta^2(z_1-z_2)$$

$$= \exp\left[-\frac{\alpha'}{2}\int d^2z_1\, d^2z_2\, \ln|z_{12}|\;\frac{\delta}{\delta X_F}\,\frac{\delta}{\delta X_G}\right] e^{ik_1 X}\; e^{ik_2 X}$$

$$\textcircled{=}$$

$$\frac{\delta}{\delta X_\mu(z_1)}\left[e^{ik\cdot X(z_2)}\right] = e^{ikX(z_2)}\,\boxed{ik^\mu\,\delta^2(z_1-z_2)}$$

$$\textcircled{=}\;:\exp\left[-\frac{\alpha'}{2}\right.$$



$$= \exp\left[-\frac{\alpha'}{2}\int d^2z_1\, d^2z_2\, \ln|z_{12}|\ \frac{\delta}{\delta X_F}\frac{\delta}{\delta X_G}\right] e^{ik_1X}\, e^{ik_2X}$$

$$\boxed{=}$$

$$\frac{\delta}{\delta X_\mu(z_1)}\left[e^{ikX(z_2)}\right] = e^{ikX(z_2)}\, \boxed{ik^\mu\, \delta^2(z_1-z_2)}$$

$$\boxed{=}\ \exp\left[-\frac{\alpha'}{2}\int d^2z_1\, d^2z_2\right.$$

$$= \exp\left[-\frac{\alpha'}{2}\int d^2z_1\, d^2z_2\, \ln|z_{12}|\, \frac{\delta}{\delta X^\mu(1)}\frac{\delta}{\delta X_\mu(2)}\right] e^{ik_1X}\, e^{ik_2X}$$

$$\textcircled{=}$$

$$\frac{\delta}{\delta X_\mu(z_1)}\left[e^{ikX(z_2)}\right] = e^{ikX(z_2)}\, \boxed{ik^\mu\, \delta^2(z_1-z_2)}$$

$$\textcircled{=} \exp\left[-\frac{\alpha'}{2}\int d^2z_1\, d^2z_2\, \ln|z_{12}|^2\, ik_1^\mu\, \delta^2(z_1-z)\, ik_{2\mu}\, \delta^2(z_2-z')\right]$$

$$= \exp\left[-\frac{\alpha'}{2}\int d^2z_1\, d^2z_2\, \ln|z_{12}|\; \frac{\delta}{\delta X^{\mu}_{}(1)}\frac{\delta}{\delta X_{\mu}(2)}\right] e^{ik_1X}\; e^{ik_2X}$$

$$\boxed{=}$$

$$\frac{\delta}{\delta X_{\mu}(z_1)}\left[e^{ikX(z_2)}\right] = e^{ikX(z_2)}\,\boxed{ik^{\mu}\,\delta^2(z_1-z_2)}$$

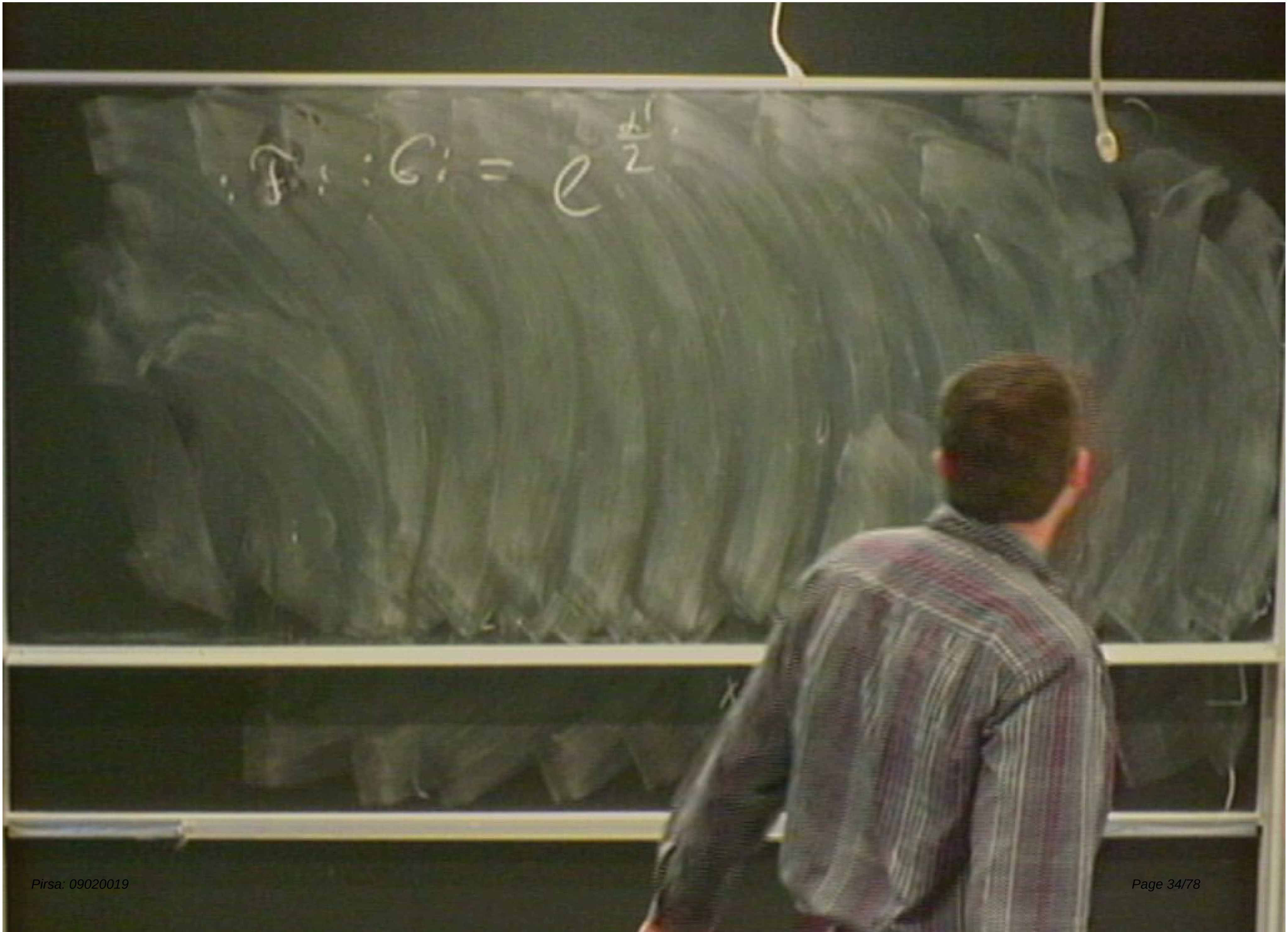
$$\boxed{=}\; :\exp\left[-\frac{\alpha'}{2}\int d^2z_1\, d^2z_2\, \ln|z_{12}|^2\; ik_1^{\mu}\,\delta^2(z_1-z)\, ik_{2\mu}\,\delta^2(z_2-z')\right] e^{ik_1X(z)}\, e^{ik_2X(z')}:$$

$$= \exp\left[-\frac{\alpha'}{2}\int d^2z_1\, d^2z_2\, \ln|z_{12}|\, \frac{\delta}{\delta X^\mu(z_1)}\frac{\delta}{\delta X_\mu(z_2)}\right] e^{ik_1X}\, e^{ik_2X}$$

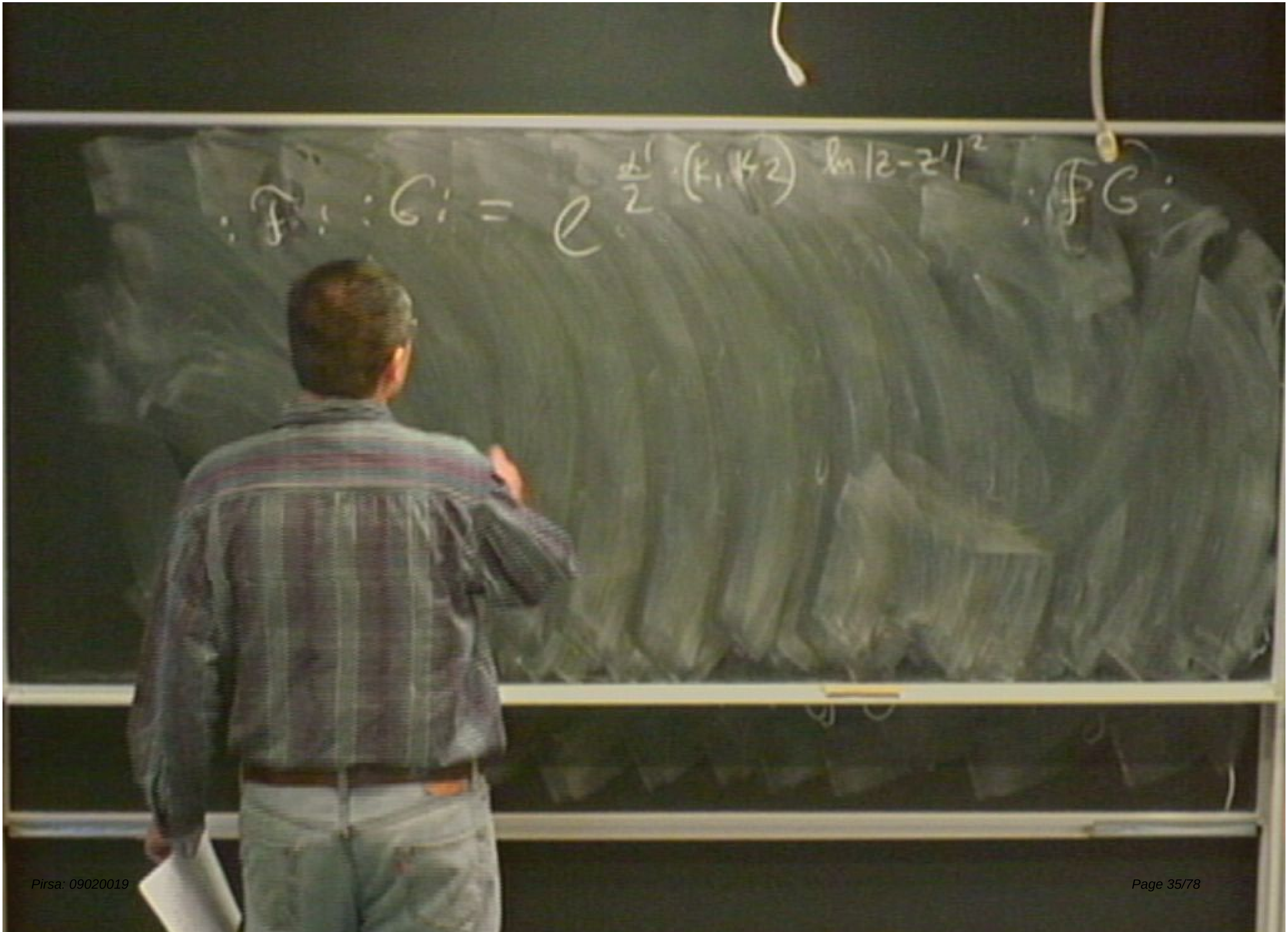
$$\boxed{=}$$

$$\frac{\delta}{\delta X_\mu(z_1)}\left[e^{ikX(z)}\right] = e^{ikX(z)}\, \boxed{ik^\mu\, \delta^2(z_1-z_2)}$$

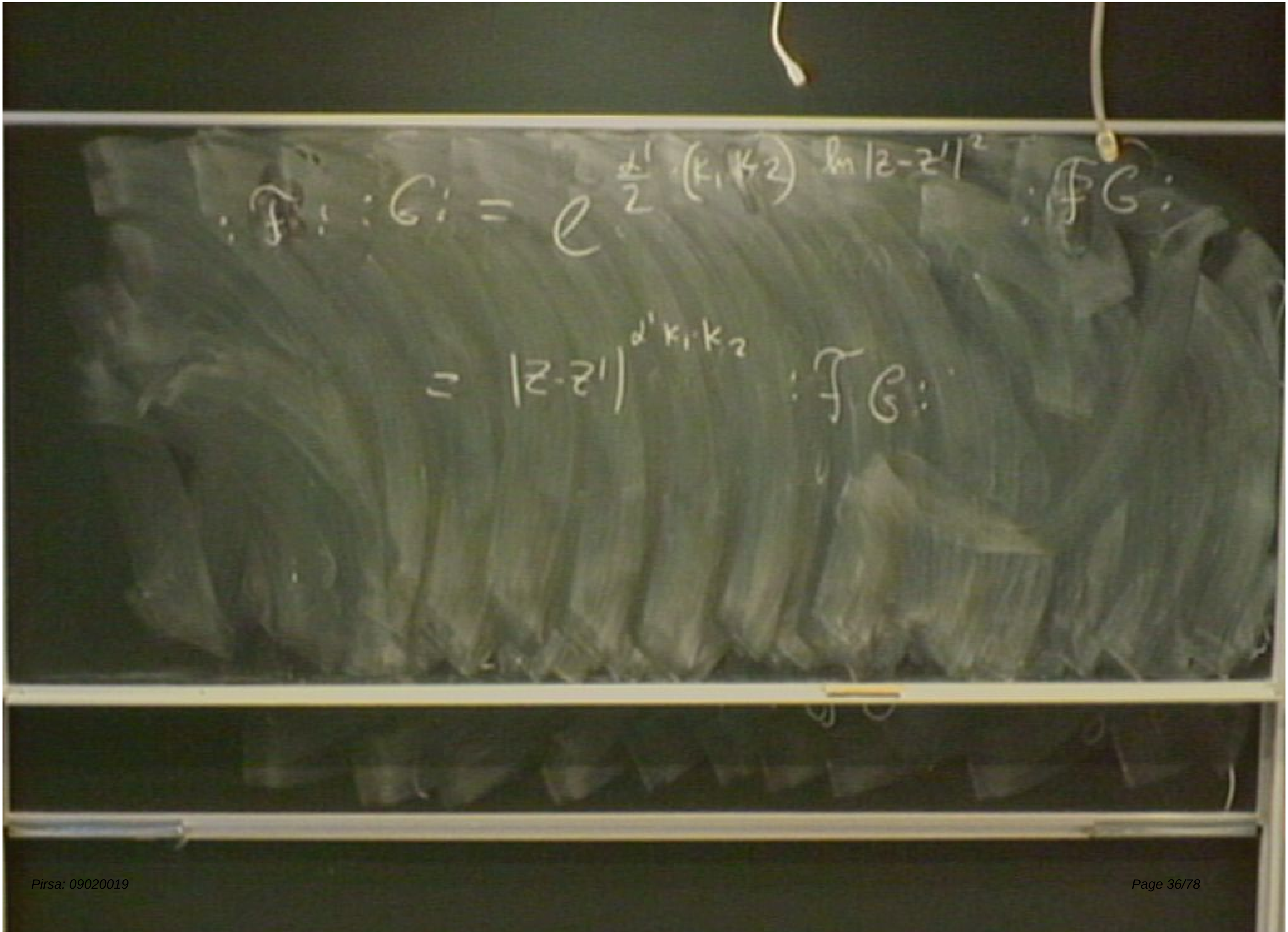
$$\boxed{=}\ \exp\left[-\frac{\alpha'}{2}\int d^2z_1\, d^2z_2\, \ln|z_{12}|^2\, ik_1^\mu\, \delta^2(z_1-z)\, ik_{2\mu}\, \delta^2(z_2-z')\right] : e^{ik_1X(z)}\, e^{ik_2X(z')} :$$



$$:f: \; :G: = e^{\frac{\alpha'}{2}}$$

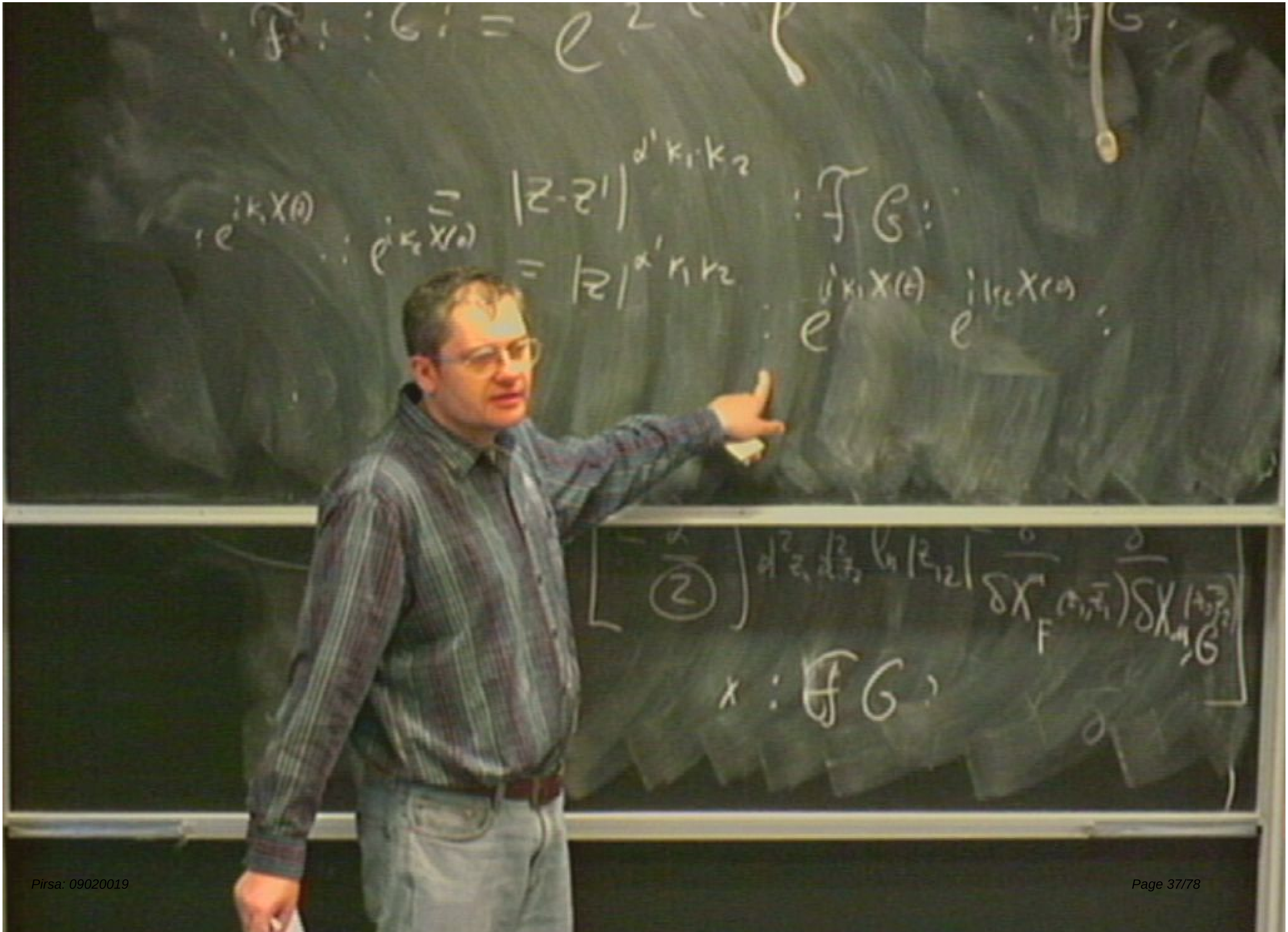


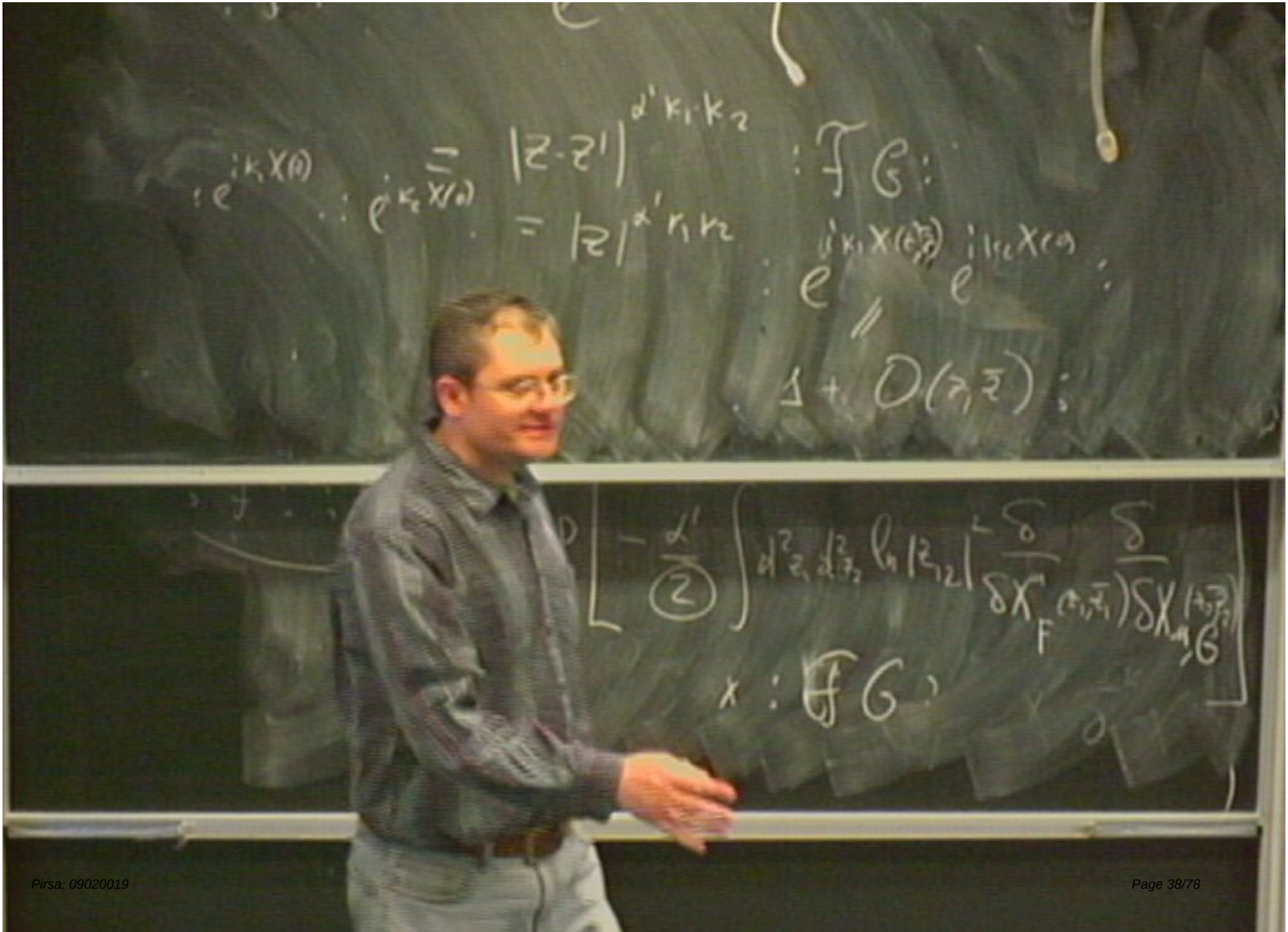
$$:F:\,:G: = e^{\frac{\alpha'}{2}(k_1 \cdot k_2)\ln|z-z'|^2}\,:FG:$$



$$:\mathcal{F}: \, :G: = e^{\frac{\alpha'}{2}(k_1 \cdot k_2) \ln|z-z'|^2} :\mathcal{F}G:$$

$$= |z-z'|^{\alpha' k_1 \cdot k_2} :\mathcal{F}G:$$


:F: :G: = e
:e^{ik_1 X(z)}: :e^{ik_2 X(0)}: = |z-z'|^{α' k_1·k_2} :F G:
= |z|^{α' k_1 k_2} :e^{ik_1 X(z)} e^{ik_2 X(0)}:
α'/2 ∫ d²z_1 d²z_2 ln|z_12| δ/δX_F(z_1, z̄_1) δ/δX_G(z_2, z̄_2)
× :FG:



$$:e^{ik_1X(z)}::e^{ik_2X(0)}: = |z-z'|^{\alpha' k_1\cdot k_2} :\mathcal{F}\,\mathcal{G}:$$

$$= |z|^{\alpha' k_1 k_2} :e^{ik_1X(z,\bar z)}\, e^{ik_2X(0)}:$$

$$/\!/$$

$$\Delta + \mathcal{O}(z,\bar z):$$

$$\exp\left[-\frac{\alpha'}{\boxed{2}}\int d^2z_1\, d^2z_2\, \ln|z_{12}|^2 \frac{\delta}{\delta X_F(z_1,\bar z_1)}\frac{\delta}{\delta X_{G}(z_2,\bar z_2)}\right]$$

$$\times :\mathcal{F}\,\mathcal{G}:$$

$$:e^{ik_1X(z)}::e^{ik_2X(0)}: = |z-z'|^{\alpha' k_1\cdot k_2} :\mathcal{F}\,\mathcal{G}:$$

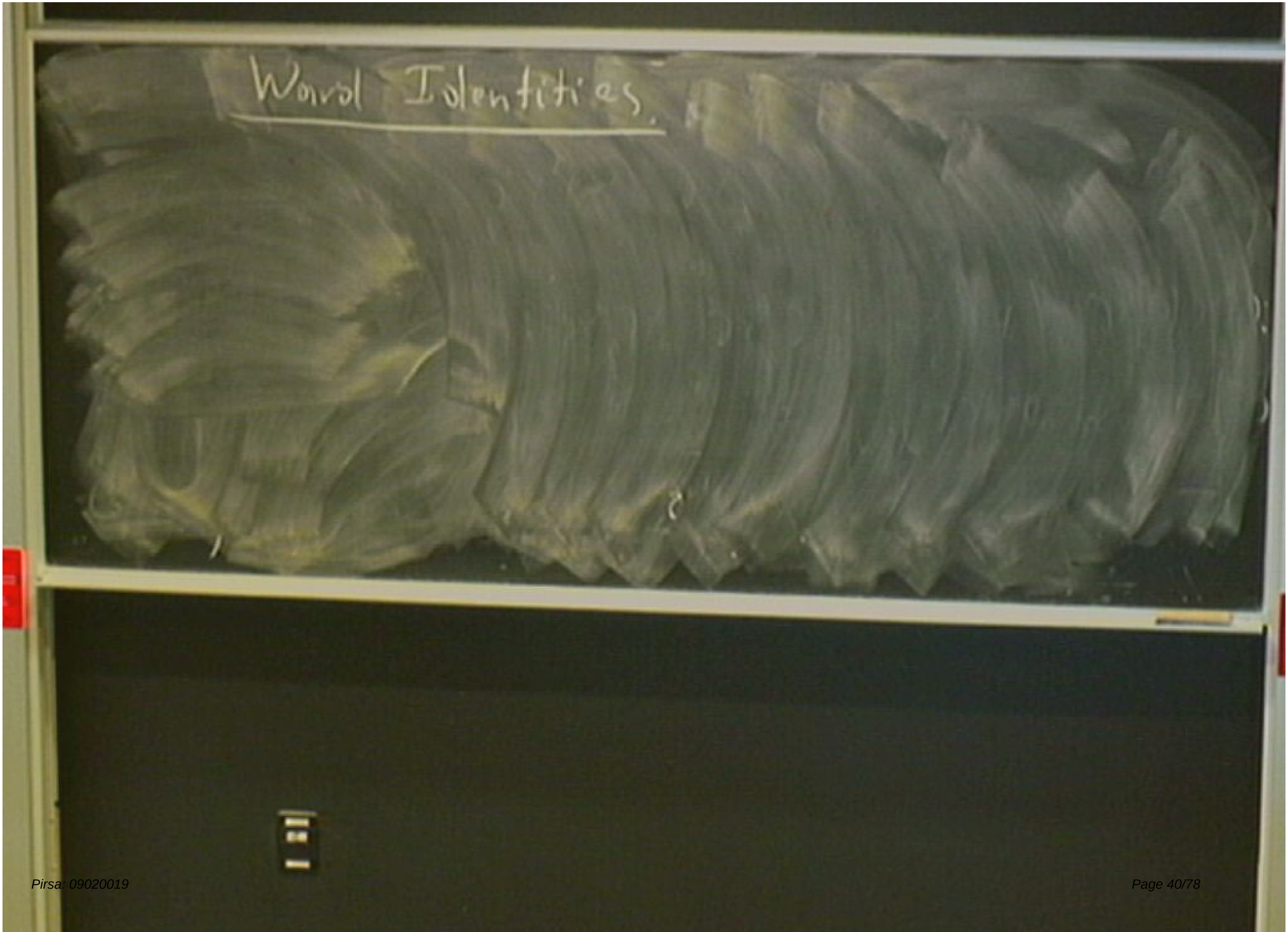
$$= \boxed{|z|^{\alpha' k_1 k_2}} :e^{ik_1X(z,\bar z)}\, e^{ik_2X(0)}:$$

$$\parallel$$

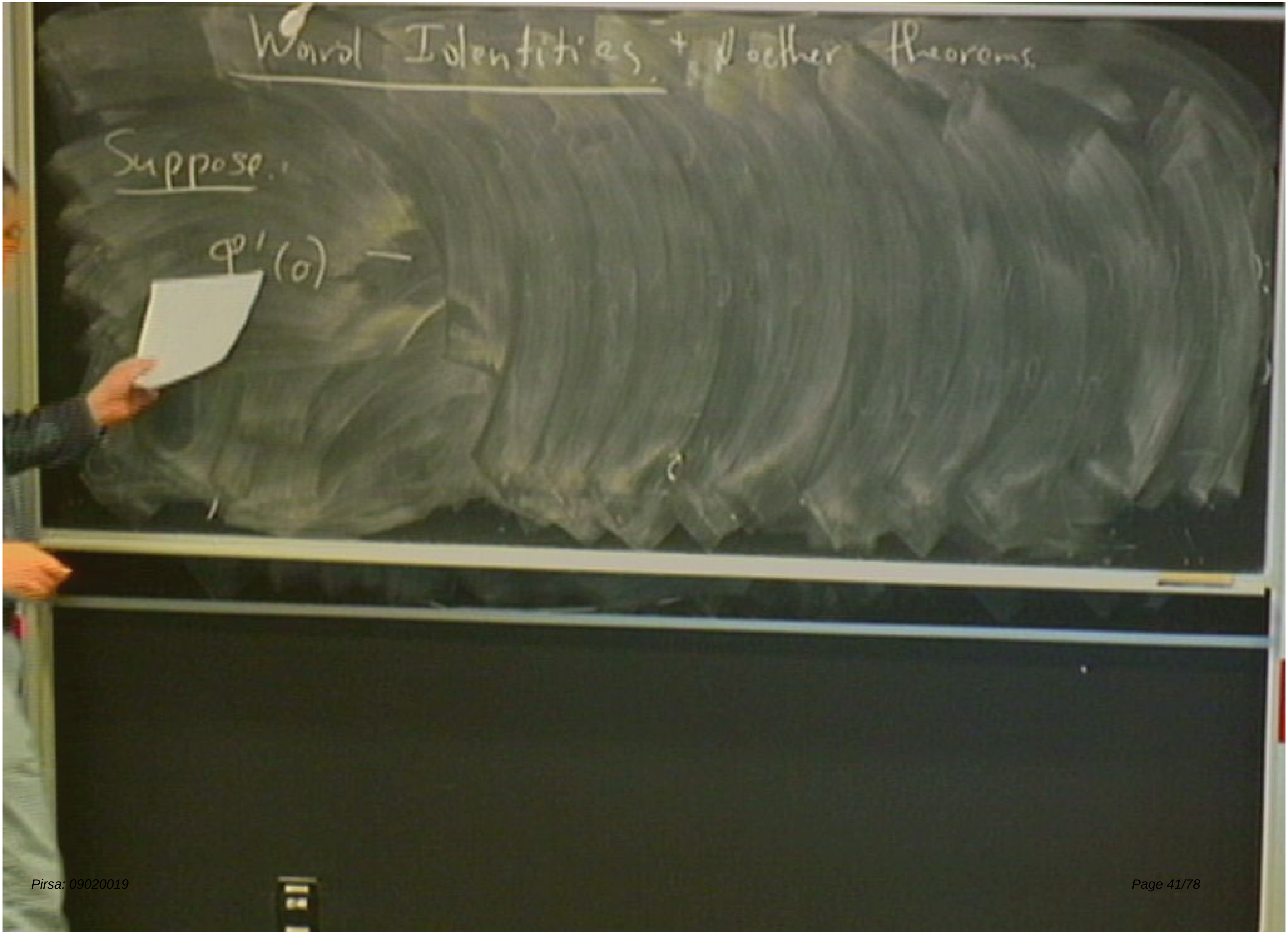
$$\Delta + \mathcal{O}(z,\bar z):$$

$$:\mathcal{F}::\mathcal{G}: = \exp\left[-\frac{\alpha'}{\boxed{2}}\int d^2z_1\, d^2z_2 \ln|z_{12}|^2 \frac{\delta}{\delta X^\mu_F(z_1,\bar z_1)}\frac{\delta}{\delta X_{\mu,G}(z_2,\bar z_2)}\right]$$

$$\times :\mathcal{F}\,\mathcal{G}:$$



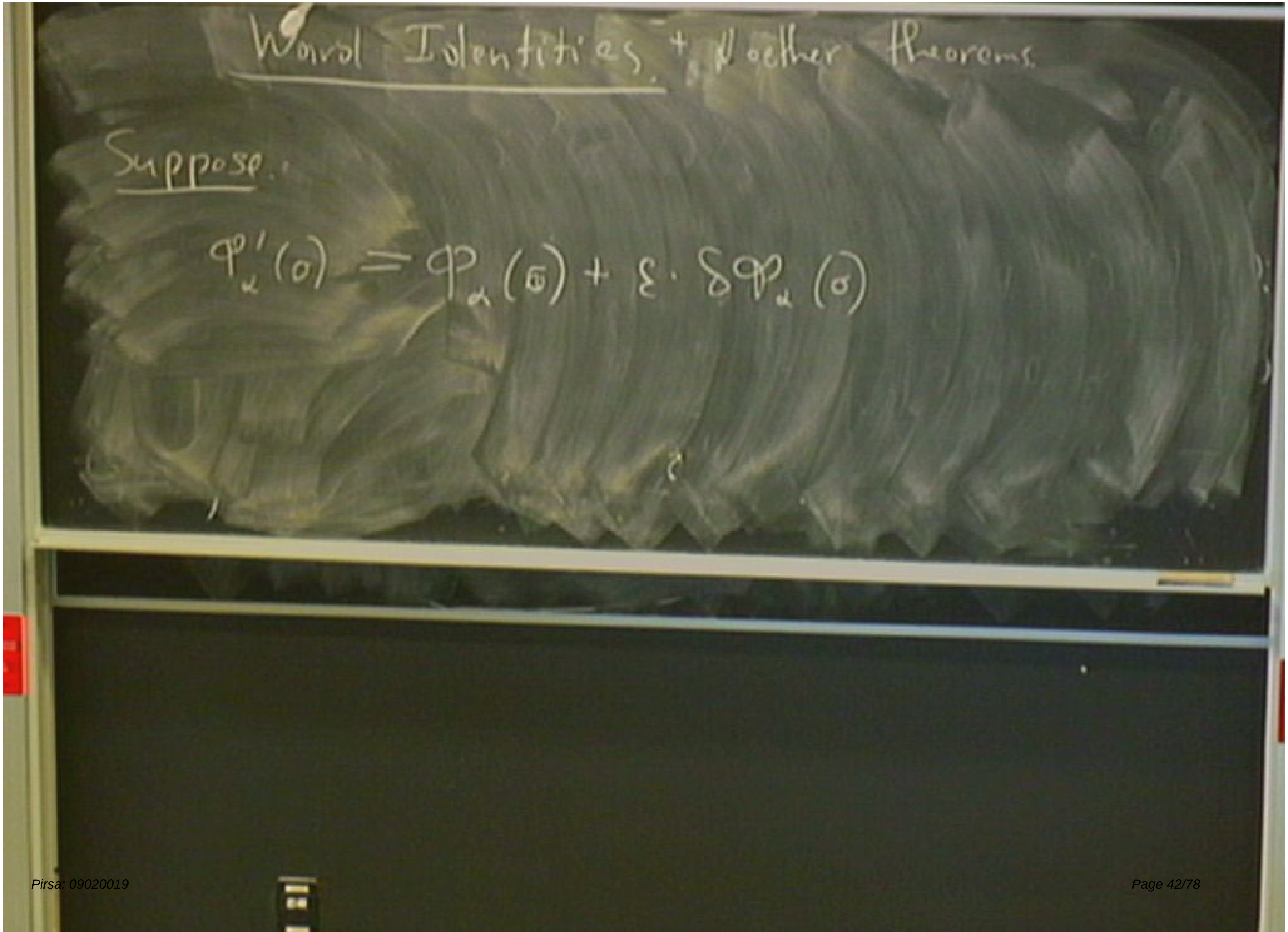
Ward Identities



Ward Identities + Noether theorems.

Suppose:

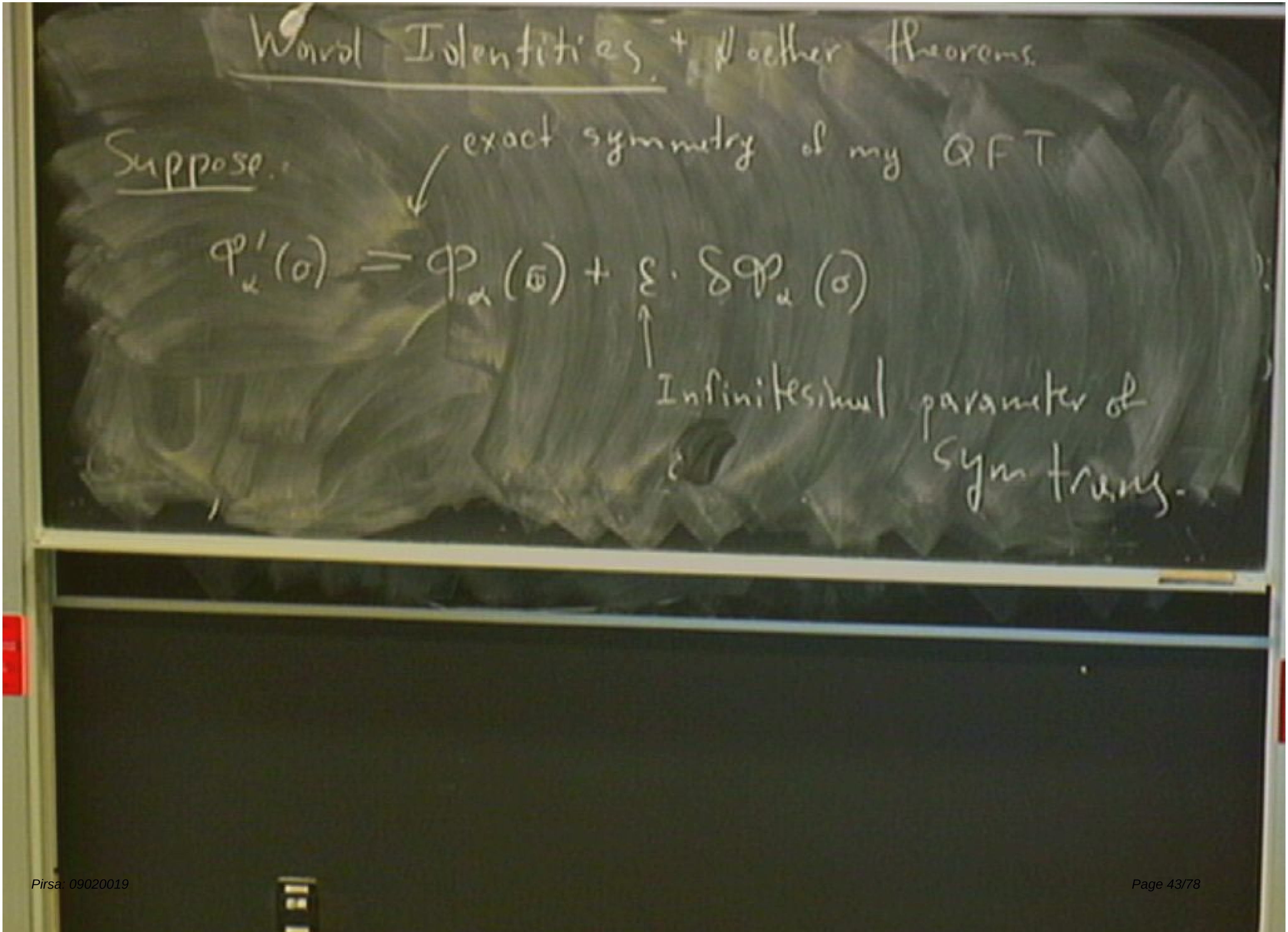
$\varphi'(0) -$



Ward Identities + Noether theorems

Suppose:

$$\varphi'_\alpha(\sigma) = \varphi_\alpha(\sigma) + \varepsilon \cdot \delta\varphi_\alpha(\sigma)$$



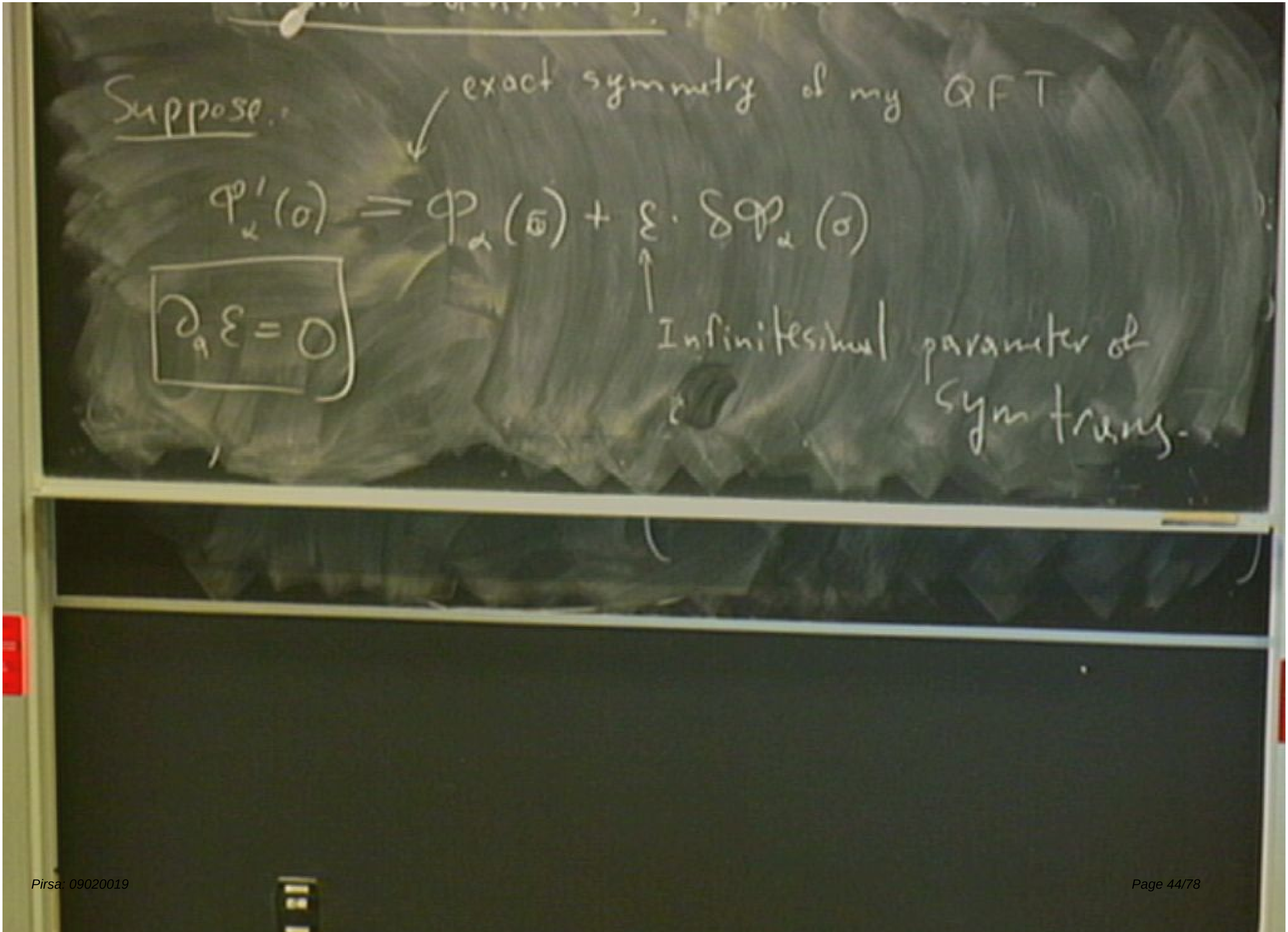
Ward Identities + Noether theorems

Suppose:

exact symmetry of my QFT

$$\varphi'_\alpha(\sigma) = \varphi_\alpha(\sigma) + \varepsilon \cdot \delta\varphi_\alpha(\sigma)$$

Infinitesimal parameter of sym trans.



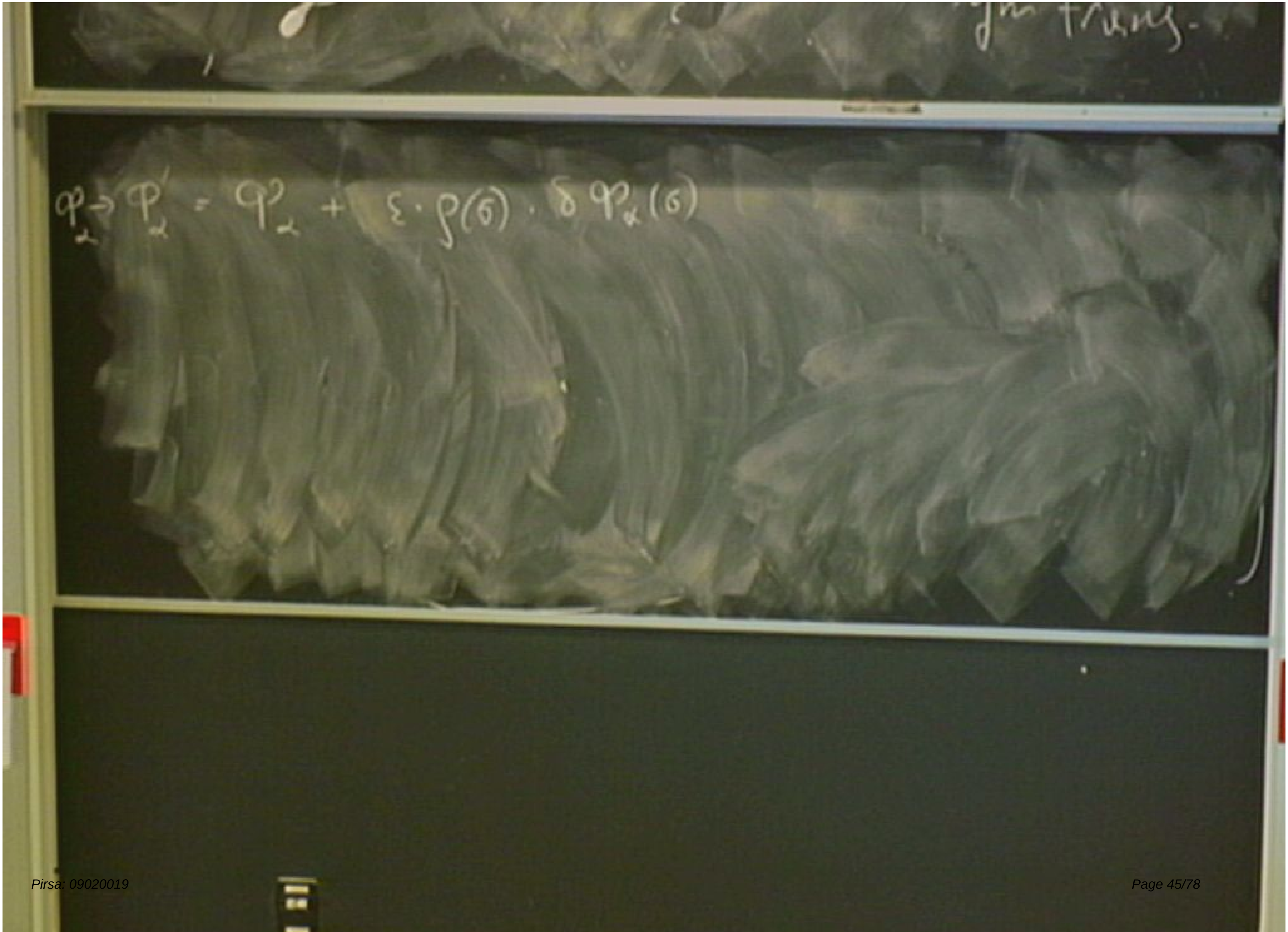
Suppose:

exact symmetry of my QFT

$$\varphi'_\alpha(\sigma) = \varphi_\alpha(\sigma) + \varepsilon \cdot \delta\varphi_\alpha(\sigma)$$

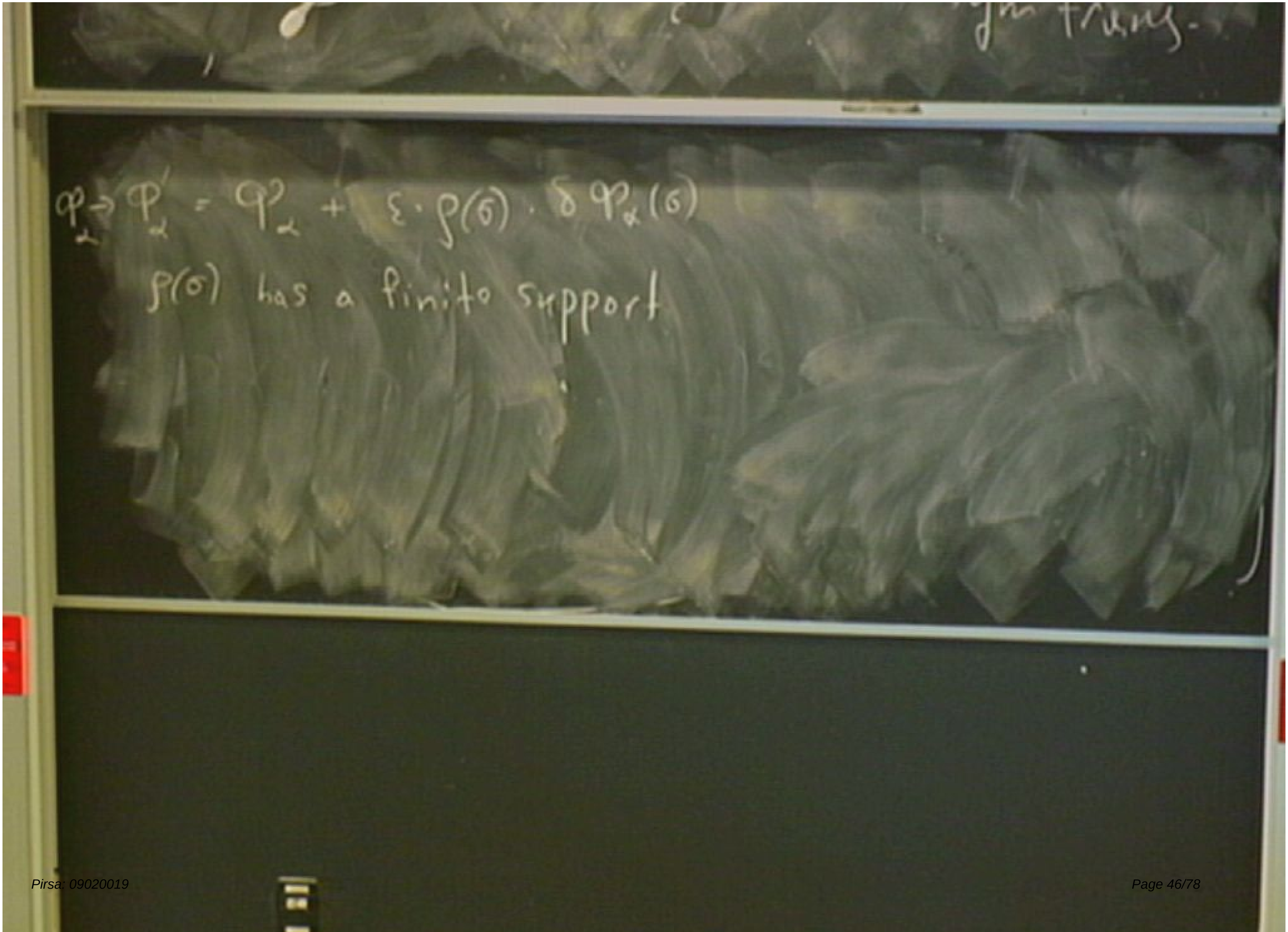
$$\boxed{\partial_a \varepsilon = 0}$$

Infinitesimal parameter of sym trans.



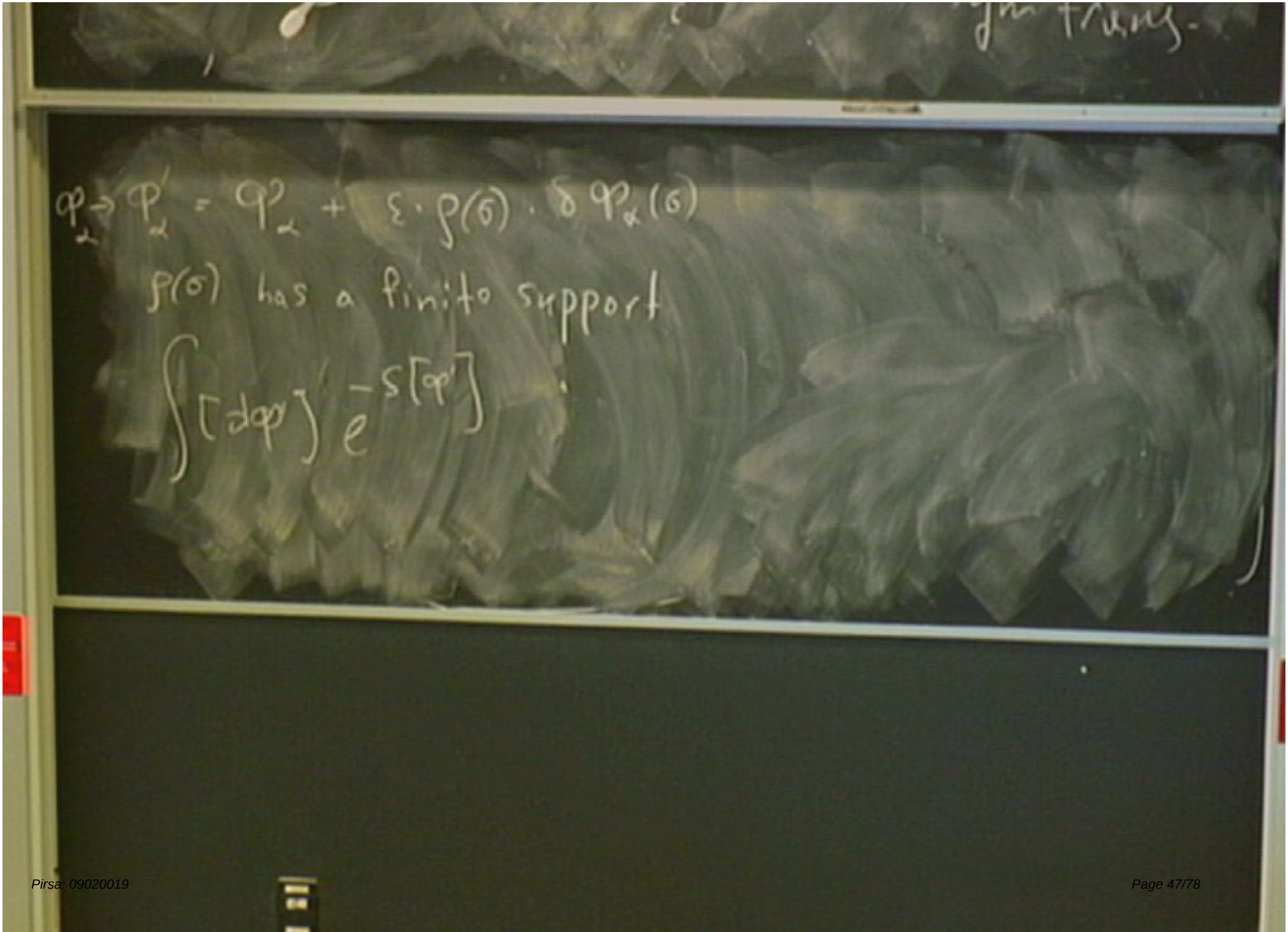
...gn trans.

$$\varphi_\alpha \to \varphi'_\alpha = \varphi_\alpha + \varepsilon \cdot \rho(\sigma) \cdot \delta\varphi_\alpha(\sigma)$$



$$\varphi_\alpha \to \varphi'_\alpha = \varphi_\alpha + \varepsilon \cdot \rho(\sigma) \cdot \delta\varphi_\alpha(\sigma)$$

$\rho(\sigma)$ has a finite support



thing.

$$\varphi_\alpha \to \varphi'_\alpha = \varphi_\alpha + \varepsilon \cdot \rho(\sigma) \cdot \delta\varphi_\alpha(\sigma)$$

$\rho(\sigma)$ has a finite support

$$\int [d\varphi]' \, e^{-S[\varphi']}$$

gm trans.

$$\varphi_\alpha \to \varphi'_\alpha = \varphi_\alpha + \varepsilon \cdot \rho(\sigma) \cdot \delta\varphi_\alpha(\sigma)$$

$\rho(\sigma)$ has a finite support

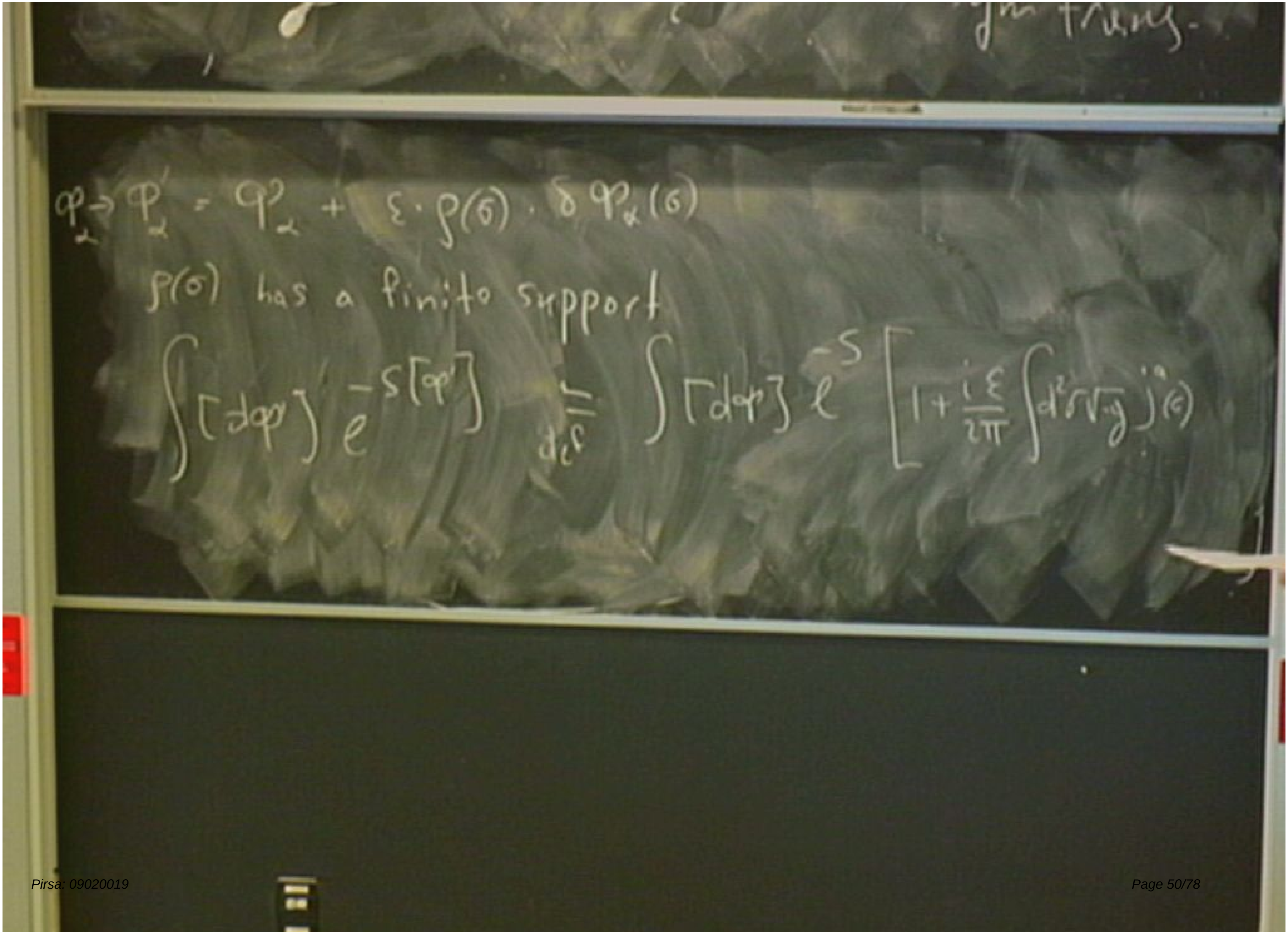
$$\int [d\varphi]' \, e^{-S[\varphi']} \underset{def}{=} \int [d\varphi] \, e^{-S}$$

gn trans.

$$\varphi_\alpha \to \varphi'_\alpha = \varphi_\alpha + \varepsilon \cdot \rho(\sigma) \cdot \delta\varphi_\alpha(\sigma)$$

$\rho(\sigma)$ has a finite support

$$\int [d\varphi'] \, e^{-S[\varphi']} \underset{\text{def}}{=} \int [d\varphi] \, e^{-S} \left[1 + \frac{i\varepsilon}{2\pi} \right.$$



$$\varphi_\alpha \to \varphi'_\alpha = \varphi_\alpha + \varepsilon \cdot \rho(\sigma) \cdot \delta\varphi_\alpha(\sigma)$$

$\rho(\sigma)$ has a finite support

$$\int [d\varphi'] \, e^{-S[\varphi']} \underset{\mathrm{def}}{=} \int [d\varphi] \, e^{-S} \left[1 + \frac{i\varepsilon}{2\pi} \int d^2\sigma \sqrt{-g} \, j^a(\sigma) \right.$$

$$\varphi_\alpha \to \varphi'_\alpha = \varphi_\alpha + \varepsilon \cdot \rho(\sigma) \cdot \delta \varphi_\alpha(\sigma)$$

$\rho(\sigma)$ has a finite support

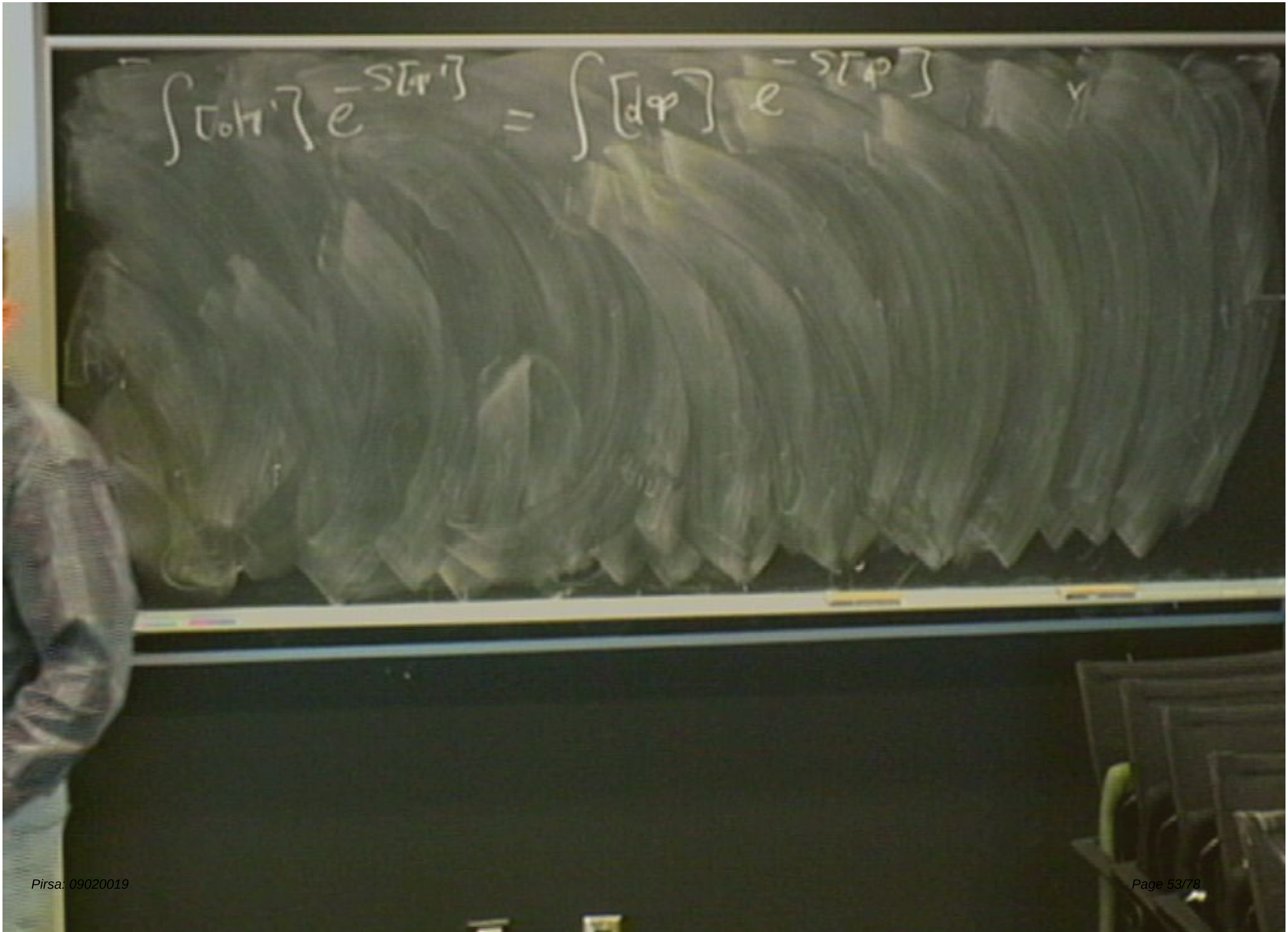
$$\int [d\varphi'] \, e^{-S[\varphi']} \underset{d.f.}{=} \int [d\varphi] \, e^{-S} \left[1 + \frac{i\varepsilon}{2\pi} \int d^2\sigma \sqrt{-g} \, j^a(\sigma) \, \partial_a \rho \right.$$

fixing.

$$\varphi_\alpha \to \varphi'_\alpha = \varphi_\alpha + \varepsilon \cdot \rho(\sigma) \cdot \delta\varphi_\alpha(\sigma)$$

$\rho(\sigma)$ has a finite support

$$\int [d\varphi'] \, e^{-S[\varphi']} \underset{\text{def}}{=} \int [d\varphi] \, e^{-S} \left[1 + \frac{i\varepsilon}{2\pi} \int \underbrace{d^2\sigma \sqrt{-g}} \, j^a(\sigma) \, \partial_a \rho + O(\varepsilon^2) \right]$$



$$\int [d\varphi'] \, e^{-S[\varphi']} = \int [d\varphi] \, e^{-S[\varphi]}$$

$$\int [d\varphi'] \, \bar{e}^{S[\varphi']} = \int [d\varphi] \, e^{-S[\varphi]}$$

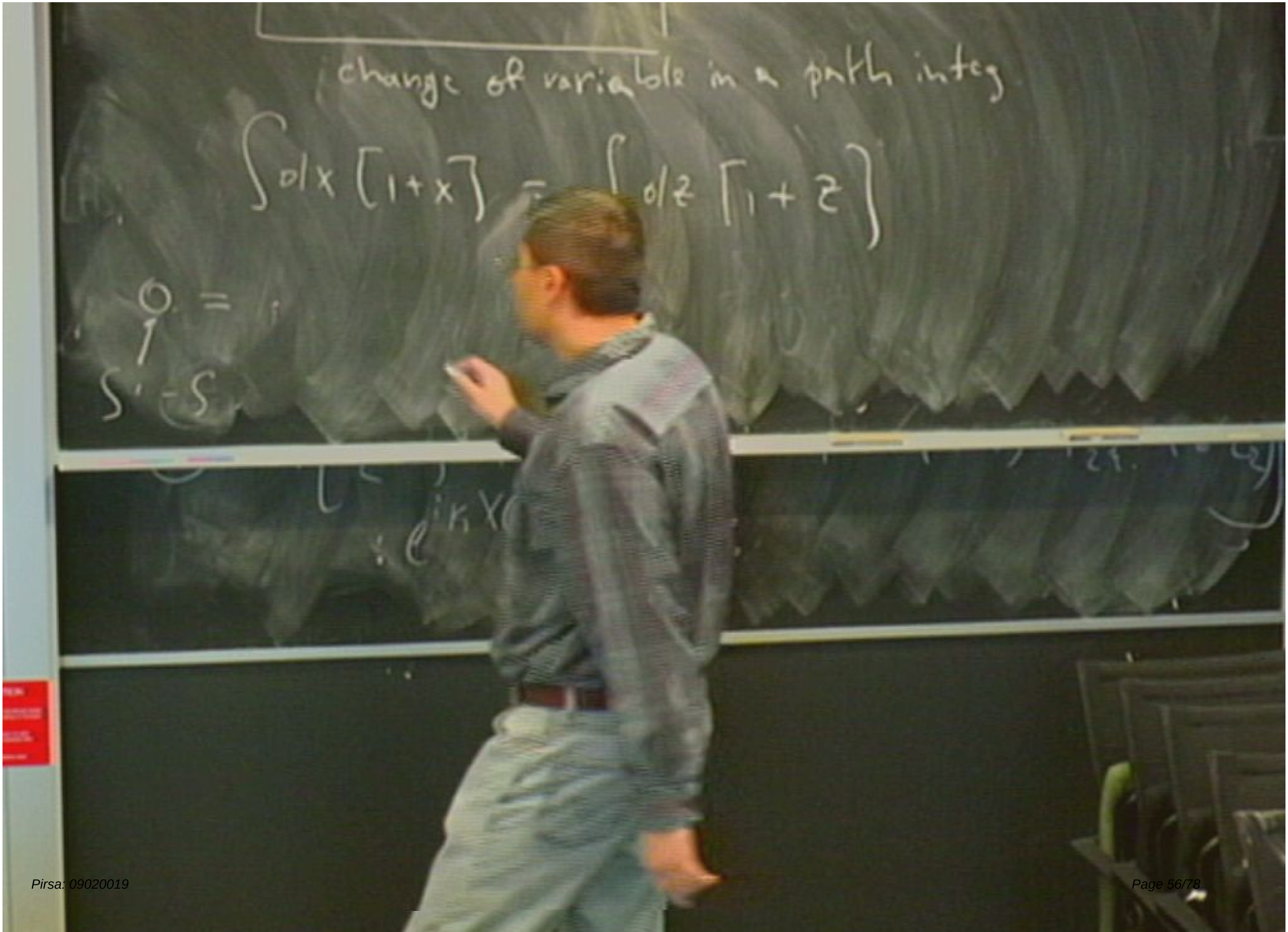
change of variable in a path integ

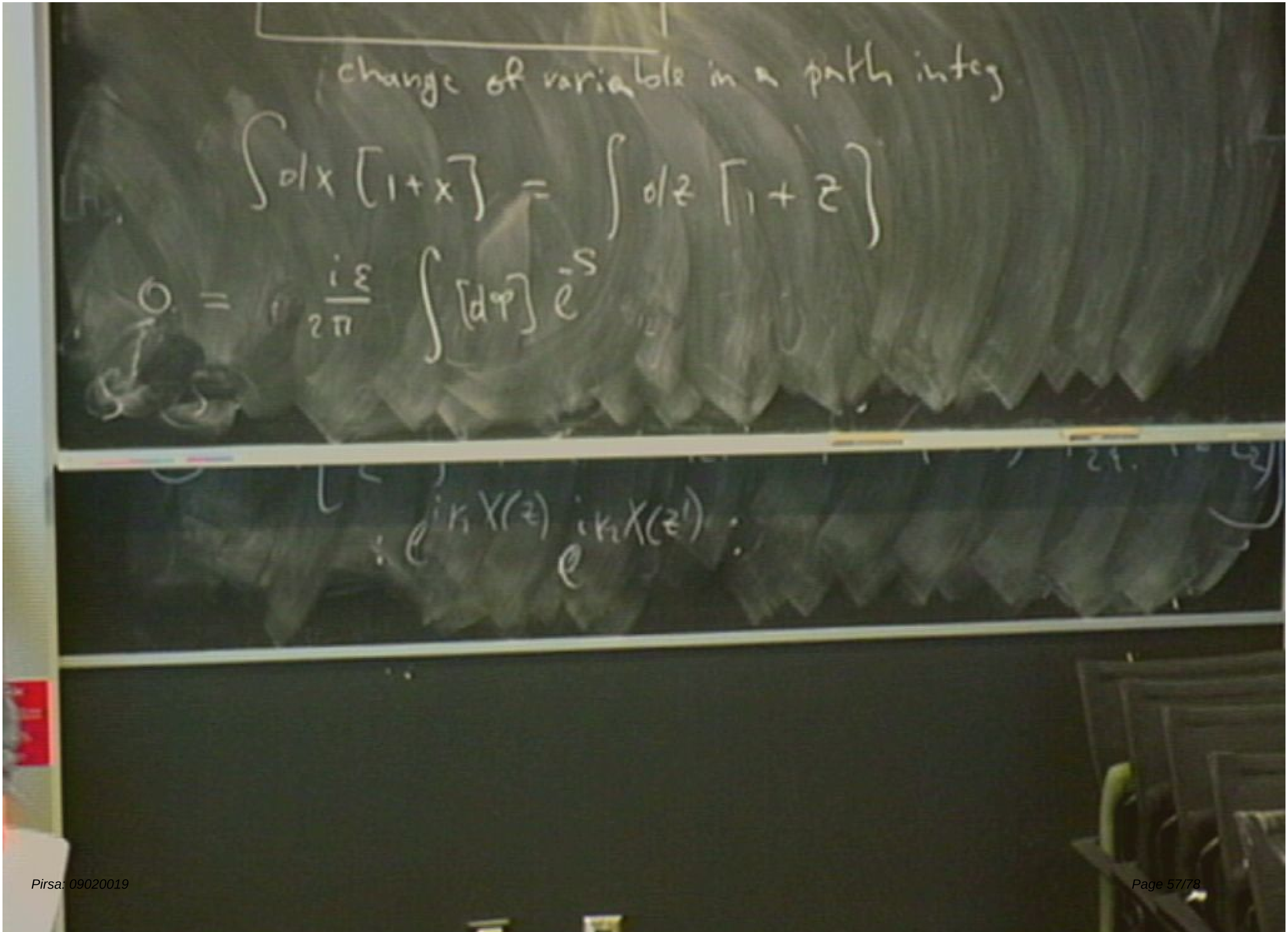
$$\int dx \, [1+x] = \int dz \, [1+z]$$

$$\varphi_\alpha \to \varphi'_\alpha = \varphi_\alpha + \varepsilon \cdot \rho(\sigma) \cdot \delta \varphi_\alpha(\sigma)$$

$\rho(\sigma)$ has a finite support

$$\underline{\int [d\varphi'] \, e^{-S[\varphi']}} = \int \underline{[d\varphi] \, e^{-S}} \left[1 + \frac{i\varepsilon}{2\pi} \int \underbrace{d^2\sigma \sqrt{g}} \, j^a(\sigma) \, \partial_a \rho + O(\varepsilon^2) \right]$$


change of variable in a path integ
$\int dx\,[1+x] = \int dz\,[1+z]$
$Q =$
$S' - S$
$e^{ik_1 X(}$

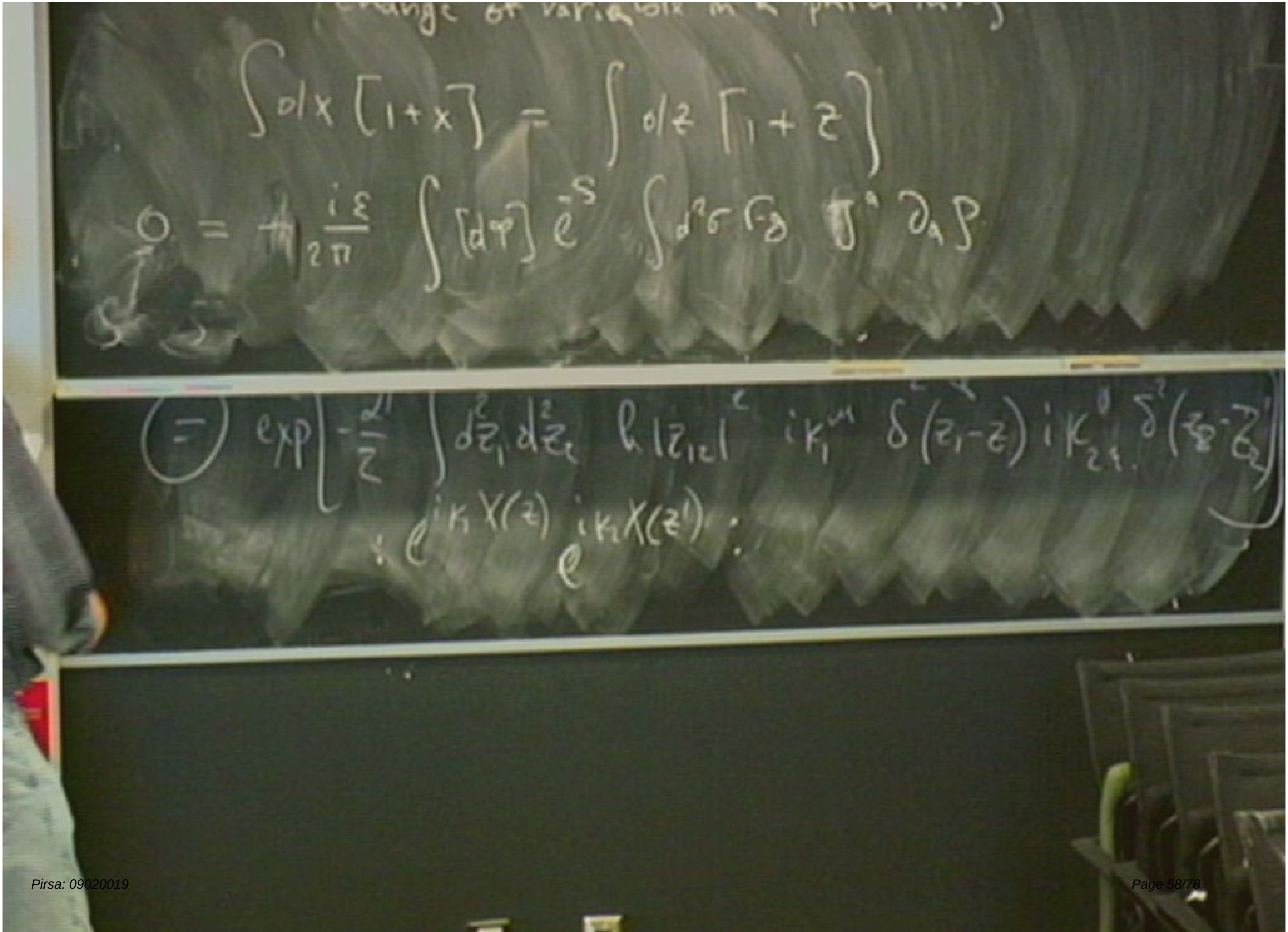


change of variable in a path integ

$$\int dx\,[1+x] = \int dz\,[1+z]$$

$$O = \frac{i\varepsilon}{2\pi}\int [d\varphi]\, e^{-S}$$

$$:e^{ik_1 X(z)}\, e^{ik_2 X(z')}:$$



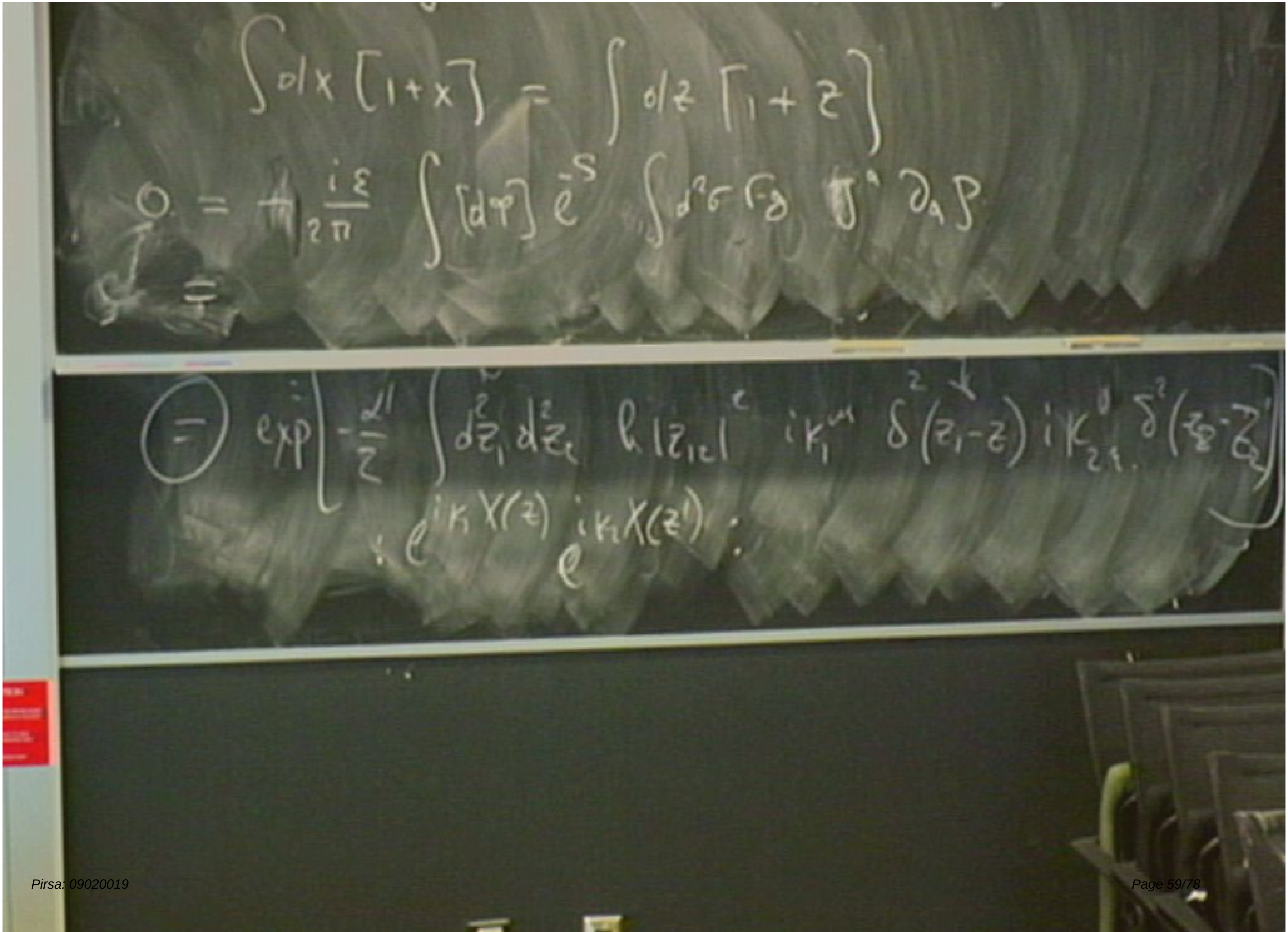
change of variable in a path integral

$$\int dx \left[1+x\right] = \int dz \left[1+z\right]$$

$$\delta = \frac{i\epsilon}{2\pi}\int [d\varphi]\, e^{-S} \int d^2\sigma \sqrt{g}\; j^a \partial_a \rho$$

$$\delta S$$

$$= \exp\left[-\frac{\alpha'}{2}\int d^2z_1\, d^2z_2\, \ln|z_{12}|^2\, ik_1^\mu\, \delta^2(z_1-z)\, ik_{2\nu}\, \delta^2(z_2-z')\right] :e^{ik_1X(z)}\, e^{ik_2X(z')}:$$



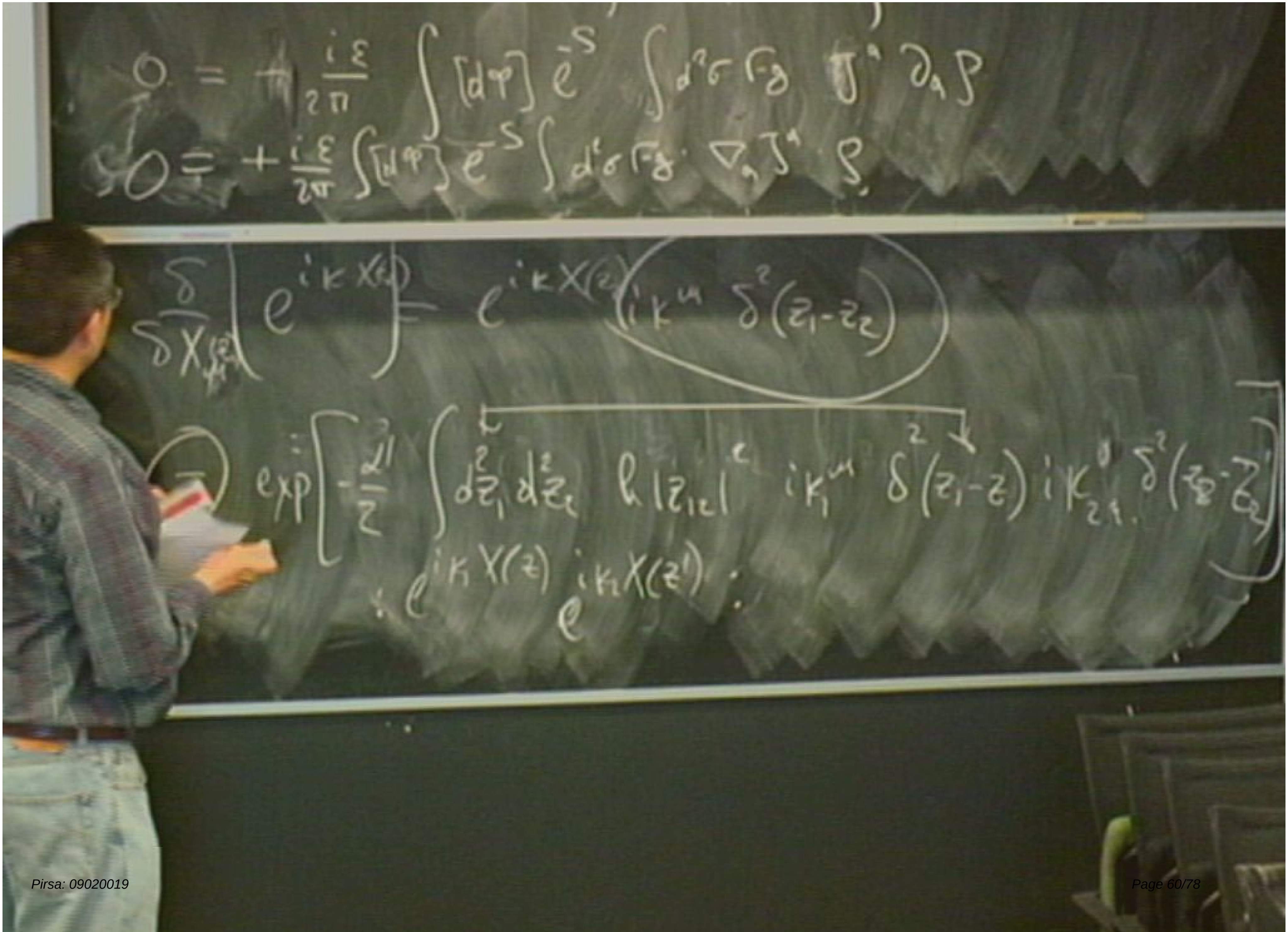
$$\int dx\,[1+x] = \int dz\,[1+z]$$

$$\mathcal{O} = \frac{i\varepsilon}{2\pi}\int [d\varphi]\, e^{-S}\int d^2\sigma\,\sqrt{-g}\;\rho^a\,\partial_a\rho$$

$$=$$

$$= \exp\left[-\frac{\alpha'}{2}\int d^2z_1\, d^2z_2\,\ln|z_{12}|^2\, ik_1^{\mu}\,\delta^2(z_1-z)\, ik_{2\nu}\,\delta^2(z_2-z')\right]$$

$$:e^{ik_1X(z)}\,e^{ik_2X(z')}:$$



$$0 = \frac{i\varepsilon}{2\pi}\int[d\varphi]\, e^{-S}\int d^2\sigma\sqrt{-g}\; J^a\,\partial_a\rho$$

$$0 = +\frac{i\varepsilon}{2\pi}\int[d\varphi]\, e^{-S}\int d^2\sigma\sqrt{-g}\;\nabla_a J^a\,\rho$$

$$\frac{\delta}{\delta X_\mu(z_2)}\left(e^{ikX(z_1)}\right) = e^{ikX(z_1)}\,\boxed{ik^\mu\,\delta^2(z_1-z_2)}$$

$$= \exp\left[-\frac{\alpha'}{2}\int d^2z_1\, d^2z_2\,\ln|z_{12}|^2\; ik_1^\mu\,\delta^2(z_1-z)\; ik_{2\nu}\,\delta^2(z_2-z')\right] :e^{ik_1X(z)}\, e^{ik_2X(z')}:$$

$$\mathcal{O} = \frac{\epsilon}{2\pi} \int [d\varphi]\, e \int d^2\sigma \sqrt{-g}\; J^a\, \partial_a \rho$$

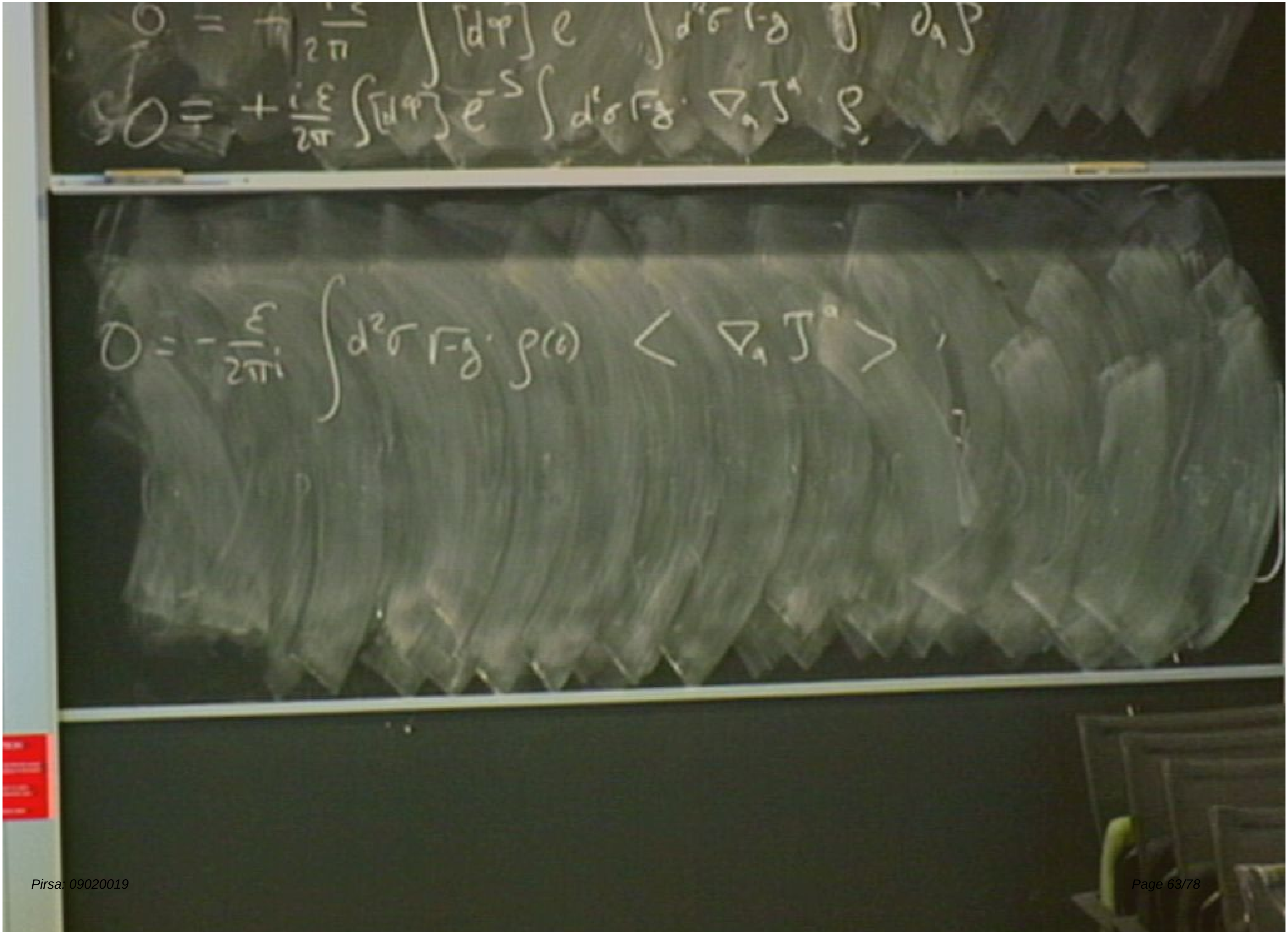
$$\mathcal{O} = +\frac{i\epsilon}{2\pi} \int [d\varphi]\, e^{-S} \int d^2\sigma \sqrt{-g} \cdot \nabla_a J^a \cdot \rho$$

$$\mathcal{O} = -\frac{\epsilon}{2\pi}$$

$$0 = \frac{\quad}{2\pi} \int [d\varphi]\, e \int d^2\sigma \sqrt{-g}\; J^a\; \partial_a \rho$$

$$0 = + \frac{i\varepsilon}{2\pi} \int [d\varphi]\, e^{-S} \int d^2\sigma \sqrt{-g} \cdot \nabla_a J^a \cdot \rho$$

$$0 = -\frac{\varepsilon}{2\pi i} \int d^2\sigma \sqrt{-g} \cdot \rho(\sigma) \left\langle \nabla_a J^a \right\rangle$$



$$0 = \frac{}{2\pi}\int[d\varphi]\,e\int d^2\sigma\sqrt{-g}\;J\;\partial_a\rho$$

$$\delta 0 = +\frac{i\epsilon}{2\pi}\int[d\varphi]\,e^{-S}\int d^2\sigma\sqrt{-g}\cdot\nabla_a J^a\,\rho,$$

$$0 = -\frac{\epsilon}{2\pi i}\int d^2\sigma\sqrt{-g}\cdot\rho(\sigma)\;\langle\nabla_a J^a\rangle$$

$$0 = \frac{}{2\pi} \int [d\varphi]\, e \int d^2\sigma \sqrt{-g}\; J\; \partial_a \rho$$

$$0 = + \frac{i\varepsilon}{2\pi} \int [d\varphi]\, e^{-S} \int d^2\sigma \sqrt{-g} \cdot \nabla_a J^a \cdot \rho$$

$$0 = -\frac{\varepsilon}{2\pi i} \int d^2\sigma \sqrt{-g} \cdot \rho(\sigma) \; \langle \nabla_a J^a \rangle$$

$$\Downarrow$$

$\nabla_a J^a = 0$ as an operator statement

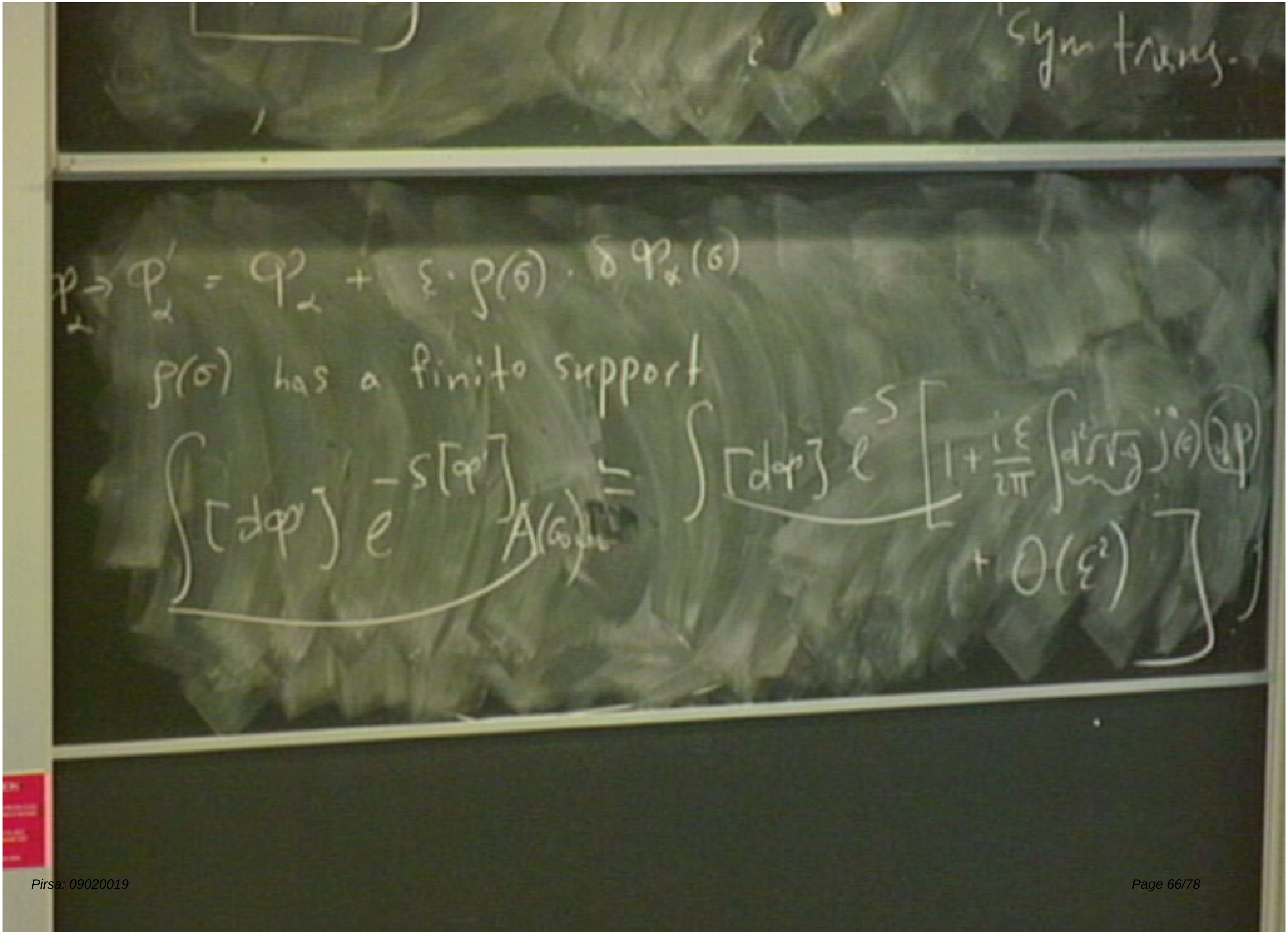
$$0 = \frac{}{2\pi}\int [d\varphi]\, e \int d^2\sigma \sqrt{-g}\; J\; \partial_a \rho$$

$$0 = +\frac{i\varepsilon}{2\pi}\int [d\varphi]\, e^{-S}\int d^2\sigma \sqrt{-g}\cdot \nabla_a J^a \cdot \rho,$$

$$0 = -\frac{\varepsilon}{2\pi i}\int d^2\sigma \sqrt{-g}\cdot \rho(\sigma) \;\langle \nabla_a J^a \rangle ,$$

$$\Downarrow$$

$\nabla_a J^a = 0$ as an operator statement

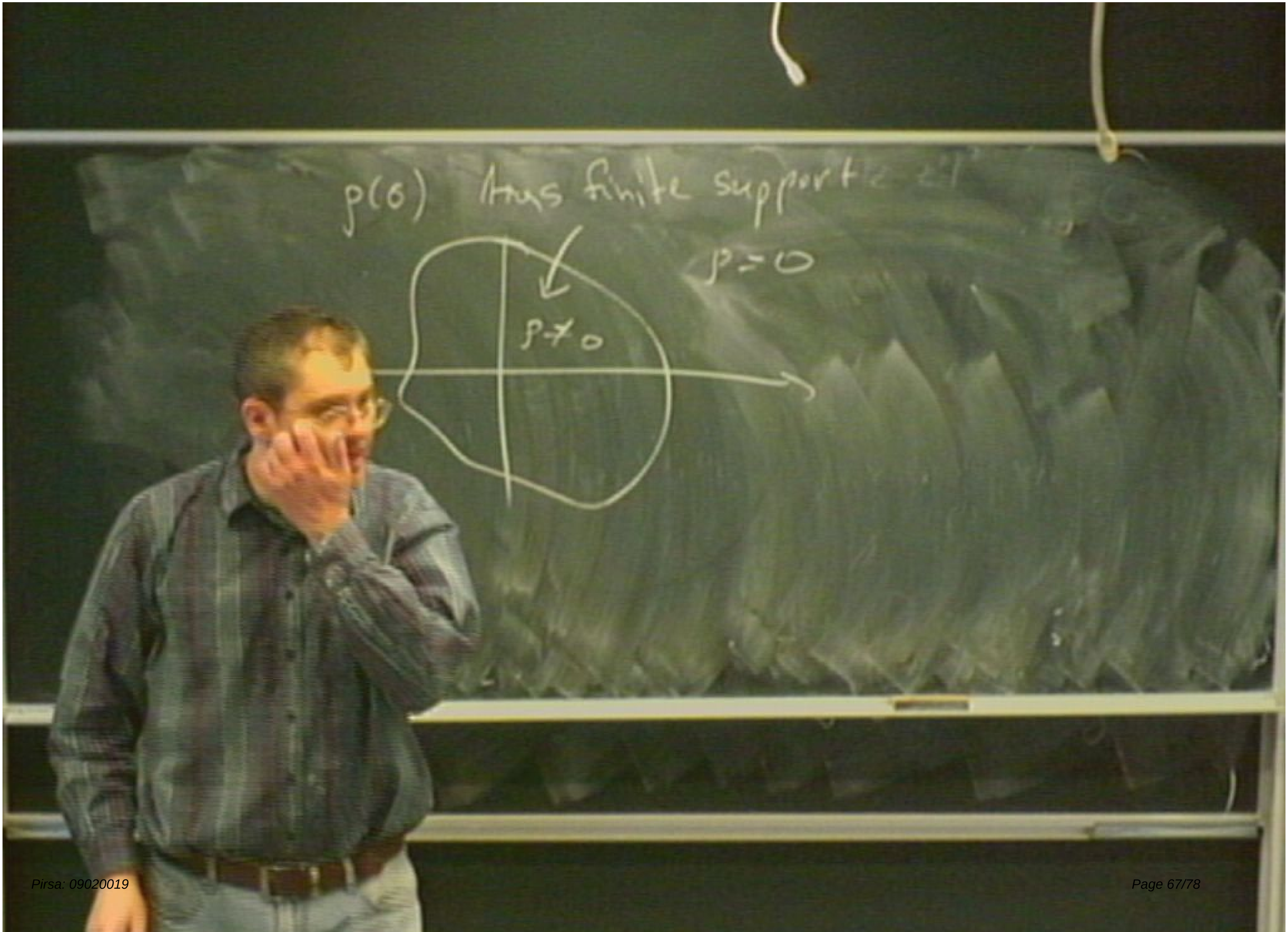


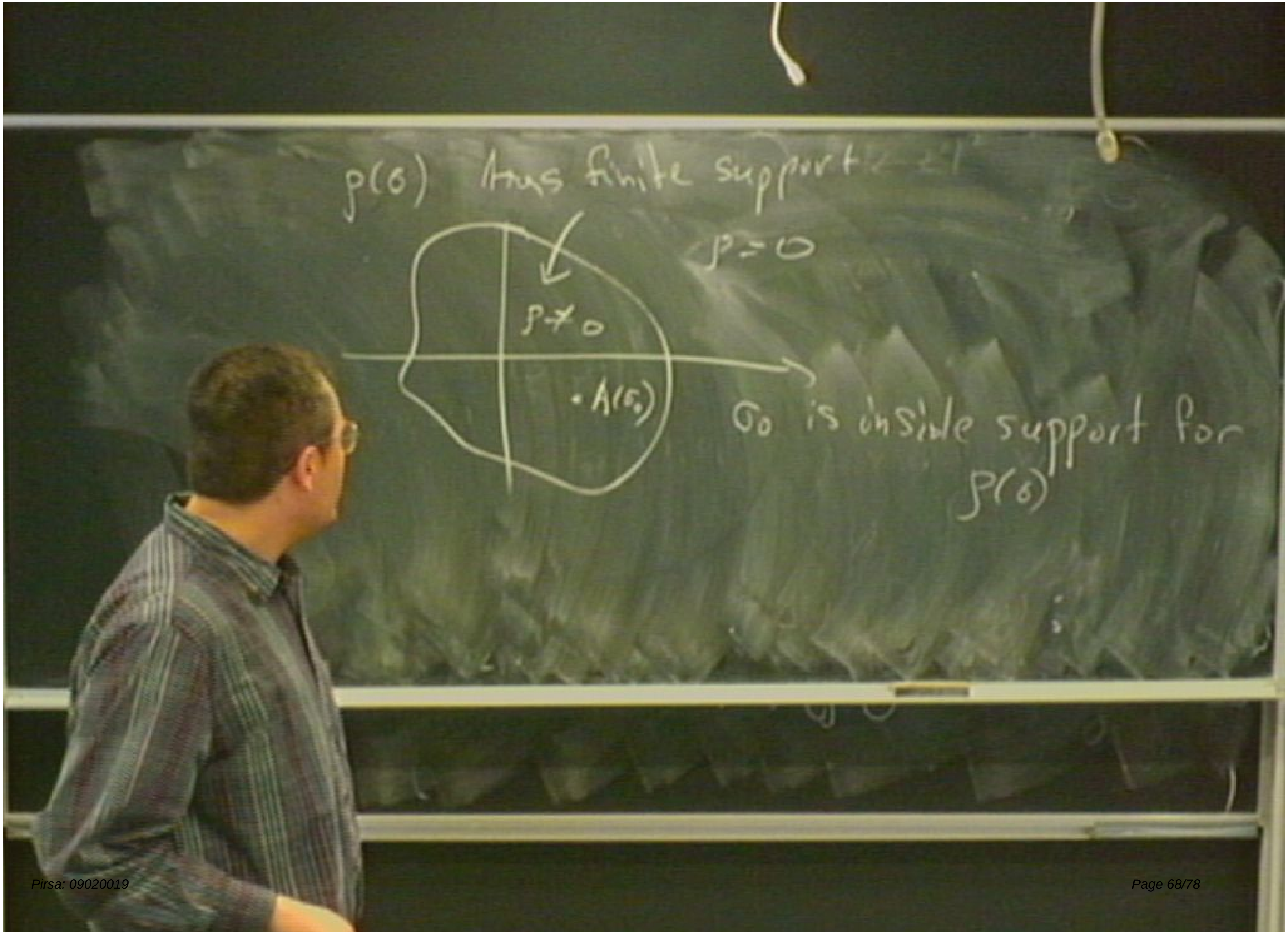
sym trans.

$$\varphi_\alpha \to \varphi'_\alpha = \varphi_\alpha + \xi \cdot \rho(\sigma) \cdot \delta\varphi_\alpha(\sigma)$$

$\rho(\sigma)$ has a finite support

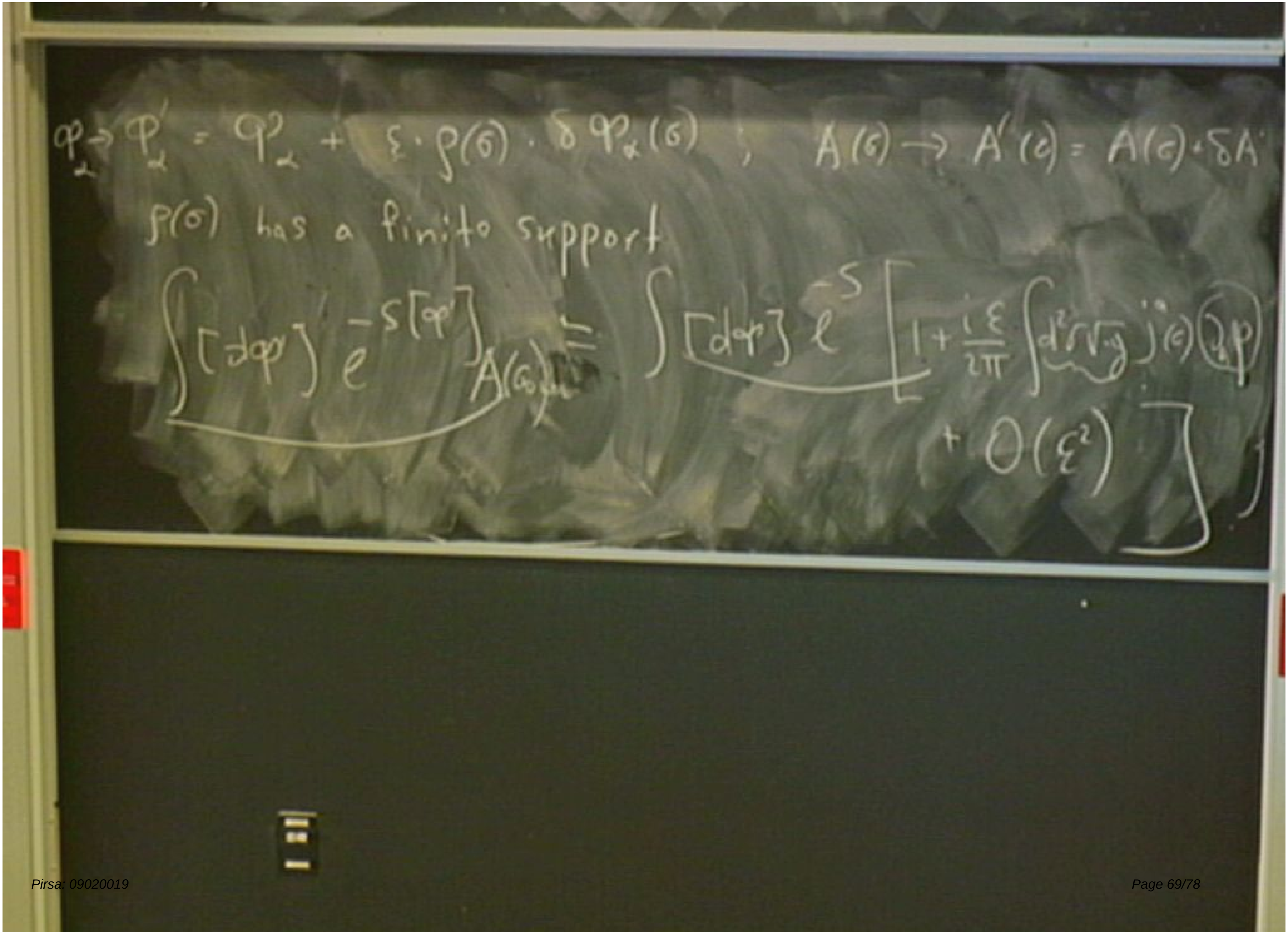
$$\underbrace{\int [d\varphi'] \, e^{-S[\varphi']} A(\sigma_0)} \simeq \int \underbrace{[d\varphi] \, e^{-S}} \left[1 + \frac{i\xi}{2\pi} \int \underbrace{d^2\sigma\sqrt{g}} \, j^a(\sigma) \boxed{\partial_a \rho} + O(\xi^2) \right]$$


ρ(σ) has finite support
ρ = 0
ρ ≠ 0



$\rho(\sigma)$ has finite support

σ_0 is inside support for $\rho(\sigma)$



$$\varphi_\alpha \to \varphi'_\alpha = \varphi_\alpha + \varepsilon \cdot \rho(\sigma) \cdot \delta\varphi_\alpha(\sigma) \quad ; \quad A(\sigma) \to A'(\sigma) = A(\sigma) + \delta A$$

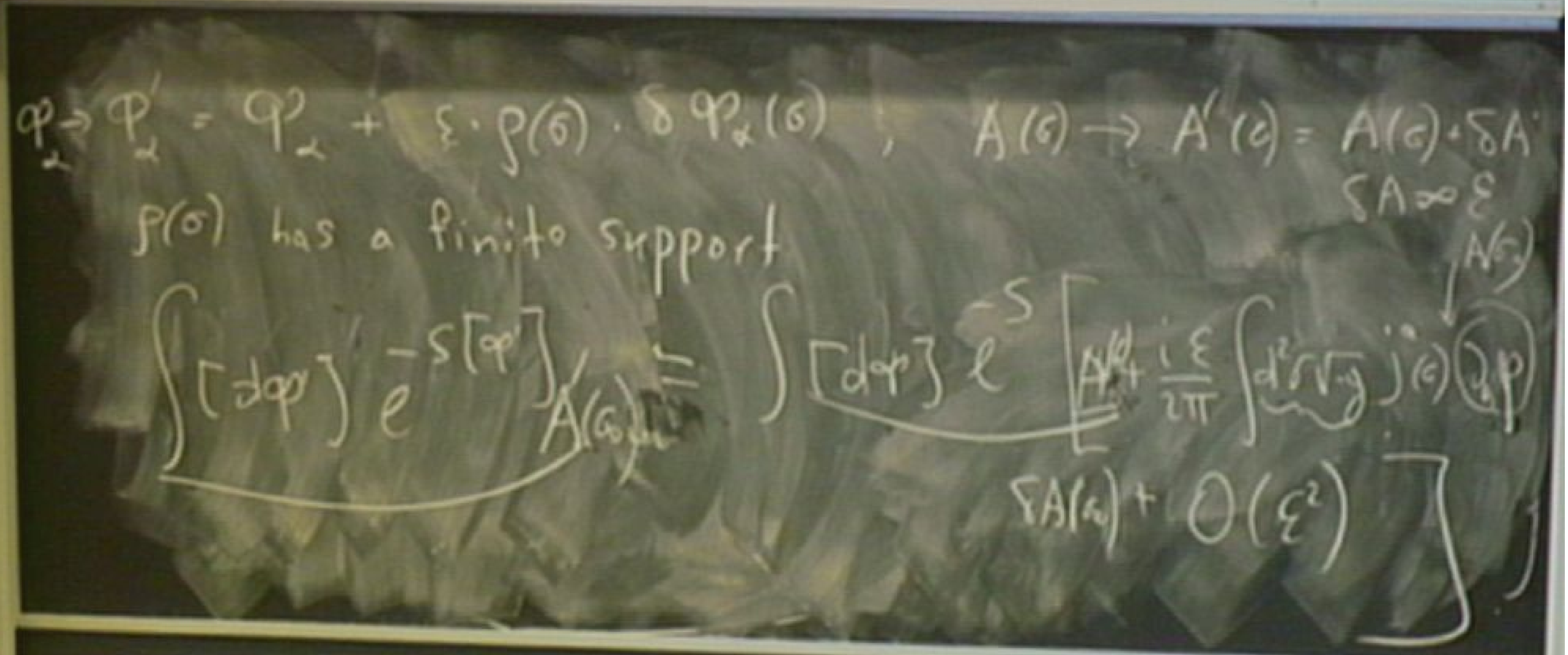
$\rho(\sigma)$ has a finito support

$$\underbrace{\int [d\varphi'] \, e^{-S[\varphi']} A(\sigma_0)}_{} \doteq \int \underbrace{[d\varphi] \, e^{-S}}_{} \left[1 + \frac{i\varepsilon}{2\pi} \int d^2\sigma \sqrt{-g} \, j^a(\sigma) \, \partial_a \rho + O(\varepsilon^2) \right]$$

$\varphi_\alpha \to \varphi'_\alpha = \varphi_\alpha + \varepsilon \cdot \rho(\sigma) \cdot \delta\varphi_\alpha(\sigma)$; $A(\sigma) \to A'(\sigma) = A(\sigma) + \delta A$

$\rho(\sigma)$ has a finito support

$$\underbrace{\int [d\varphi'] \, e^{-S[\varphi']} A(\sigma_0)} = \int \underbrace{[d\varphi] \, e^{-S} \Big[A} + \frac{i\varepsilon}{2\pi} \int \underbrace{d^2\sigma \sqrt{-g}} \, j^a(\sigma) \overset{A}{\boxed{\partial_a \rho}} + \delta A(\sigma_0) + O(\varepsilon^2) \Big]$$



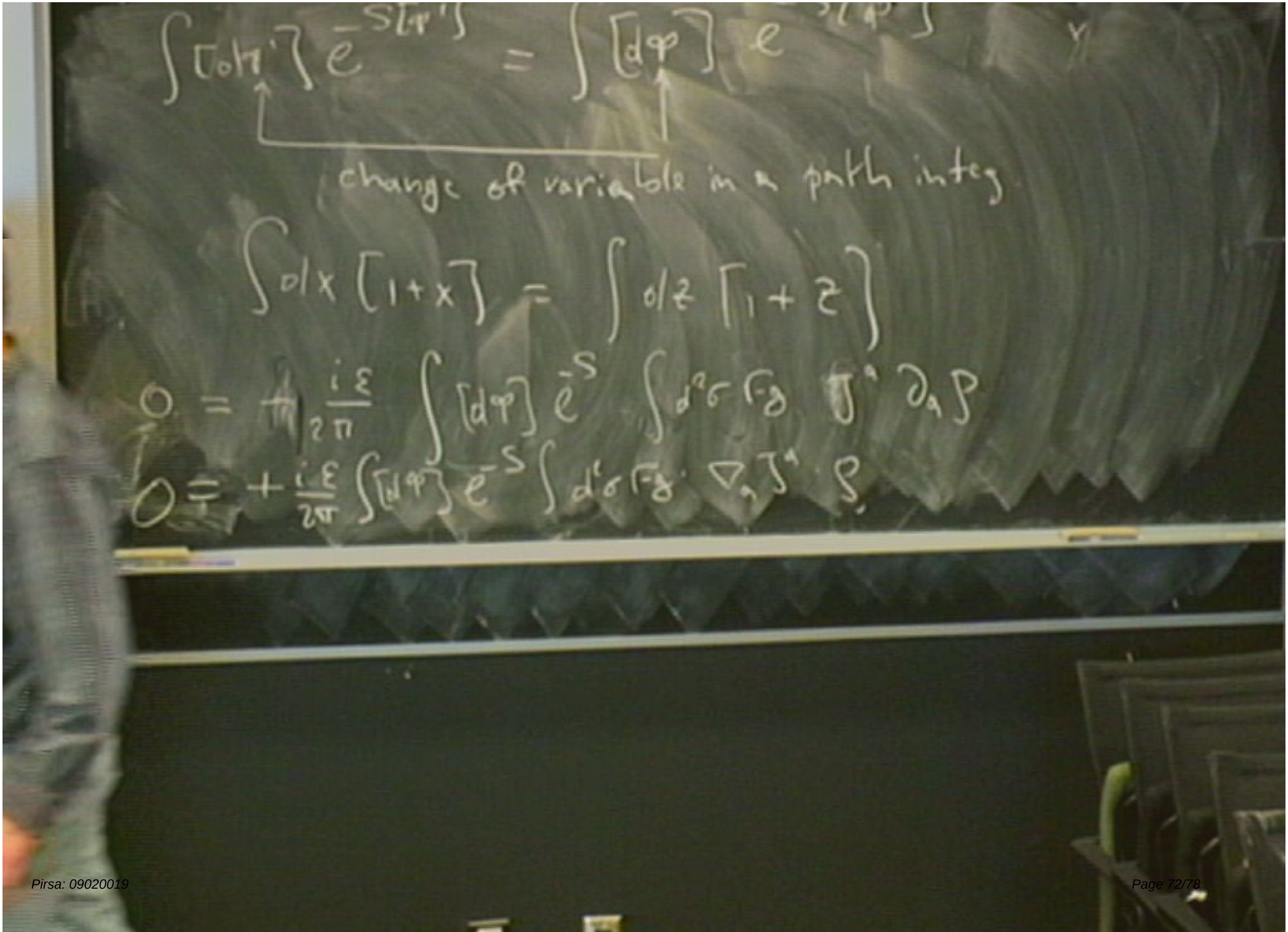
$\varphi_\alpha \to \varphi'_\alpha = \varphi_\alpha + \varepsilon \cdot \rho(\sigma) \cdot \delta\varphi_\alpha(\sigma)$; $A(\sigma) \to A'(\sigma) = A(\sigma) + \delta A$

$\delta A \propto \varepsilon$

$\rho(\sigma)$ has a finite support

$A(\sigma_0)$

$$\int [d\varphi']\, e^{-S[\varphi']} A'(\sigma_0) = \int [d\varphi]\, e^{-S} \Big[A(\sigma_0) + \frac{i\varepsilon}{2\pi} \int d^2\sigma \sqrt{-g}\, j^a(\sigma)\, \boxed{\partial_a \rho} + \delta A(\sigma_0) + O(\varepsilon^2) \Big]$$



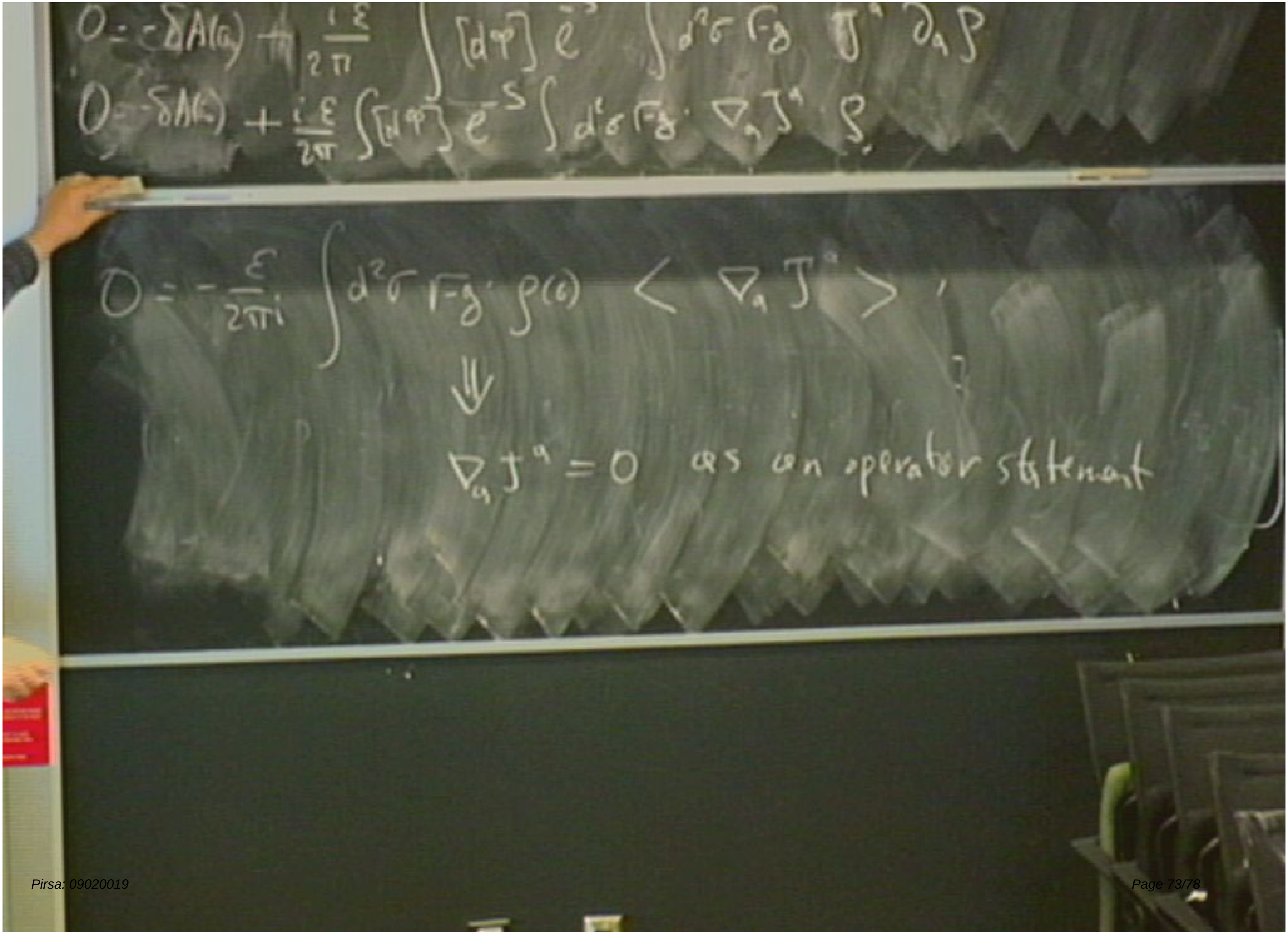
$$\int [d\varphi'] \, e^{-S[\varphi']} = \int [d\varphi] \, e^{-S[\varphi]}$$

change of variable in a path integ

$$\int dx \, [1 + x] = \int dz \, [1 + z]$$

$$0 = \frac{i\varepsilon}{2\pi} \int [d\varphi] \, e^{-S} \int d^2\sigma \sqrt{-g} \, J^a \, \partial_a \rho$$

$$0 = + \frac{i\varepsilon}{2\pi} \int [d\varphi] \, e^{-S} \int d^2\sigma \sqrt{-g} \, \nabla_a J^a \, \rho$$



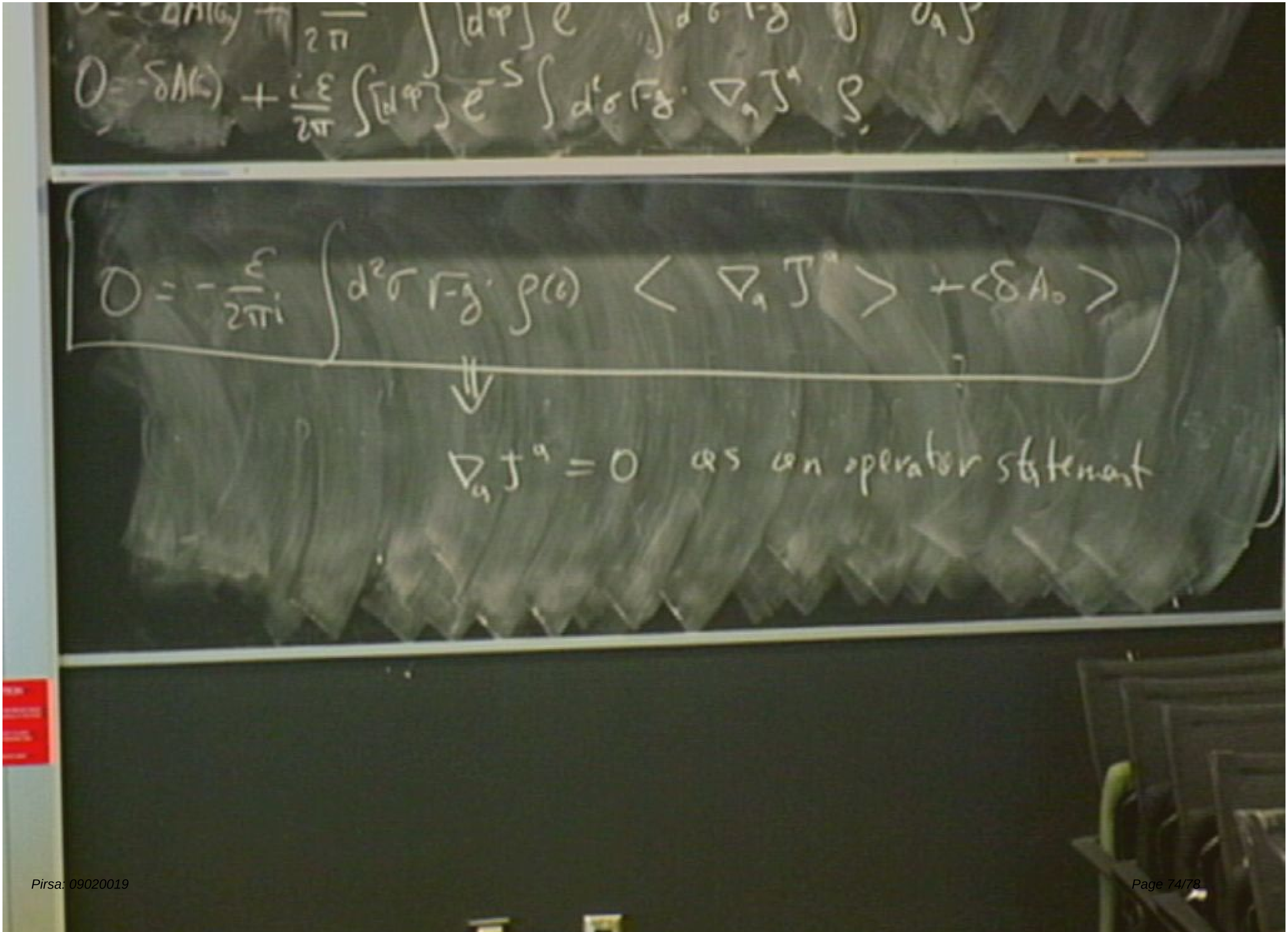
$$0 = -\delta A(\sigma_0) + \frac{i\varepsilon}{2\pi}\int [d\varphi]\, e^{-S} \int d^2\sigma \sqrt{-g}\; J^a\, \partial_a \rho$$

$$0 = -\delta A(\sigma_0) + \frac{i\varepsilon}{2\pi}\int [d\varphi]\, e^{-S} \int d^2\sigma \sqrt{-g}\; \nabla_a J^a\, \rho$$

$$0 = -\frac{\varepsilon}{2\pi i}\int d^2\sigma \sqrt{-g}\; \rho(\sigma)\, \langle \nabla_a J^a \rangle \,,$$

$$\Downarrow$$

$\nabla_a J^a = 0$ as an operator statement



$$\Delta\Lambda(\sigma_0) + \frac{}{2\pi}\int[d\varphi]\,e\int d^2\sigma\sqrt{-g}\quad\partial_a J$$

$$0 = -\delta\Lambda(\sigma_0) + \frac{i\,\varepsilon}{2\pi}\int[d\varphi]\,e^{-S}\int d^2\sigma\sqrt{-g}\cdot\nabla_a J^a\cdot\rho$$

$$\boxed{0 = -\frac{\varepsilon}{2\pi i}\int d^2\sigma\sqrt{-g}\cdot\rho(\sigma)\;\langle\nabla_a J^a\rangle + \langle\delta A_0\rangle}$$

$$\Downarrow$$

$\nabla_a J^a = 0$ as an operator statement

symmetry.

$$\varphi_\alpha \to \varphi'_\alpha = \varphi_\alpha + \epsilon \cdot \rho(\sigma) \cdot \delta\varphi_\alpha(\sigma) \; ; \quad A(\sigma) \to A'(\sigma) = A(\sigma) + \delta A$$

$$\delta A \propto \epsilon$$

$\rho(\sigma)$ has a finite support

$$\int [d\varphi'] e^{-S[\varphi']} A'(\sigma_0) \doteq \int [d\varphi] e^{-S} \left[A(\sigma_0) + \frac{\epsilon}{2\pi} \int d^2\sigma \sqrt{g}\, j^a(\sigma)\, \partial_a \rho \, A(\sigma_0) + \delta A(\sigma_0) + O(\epsilon^2) \right]$$

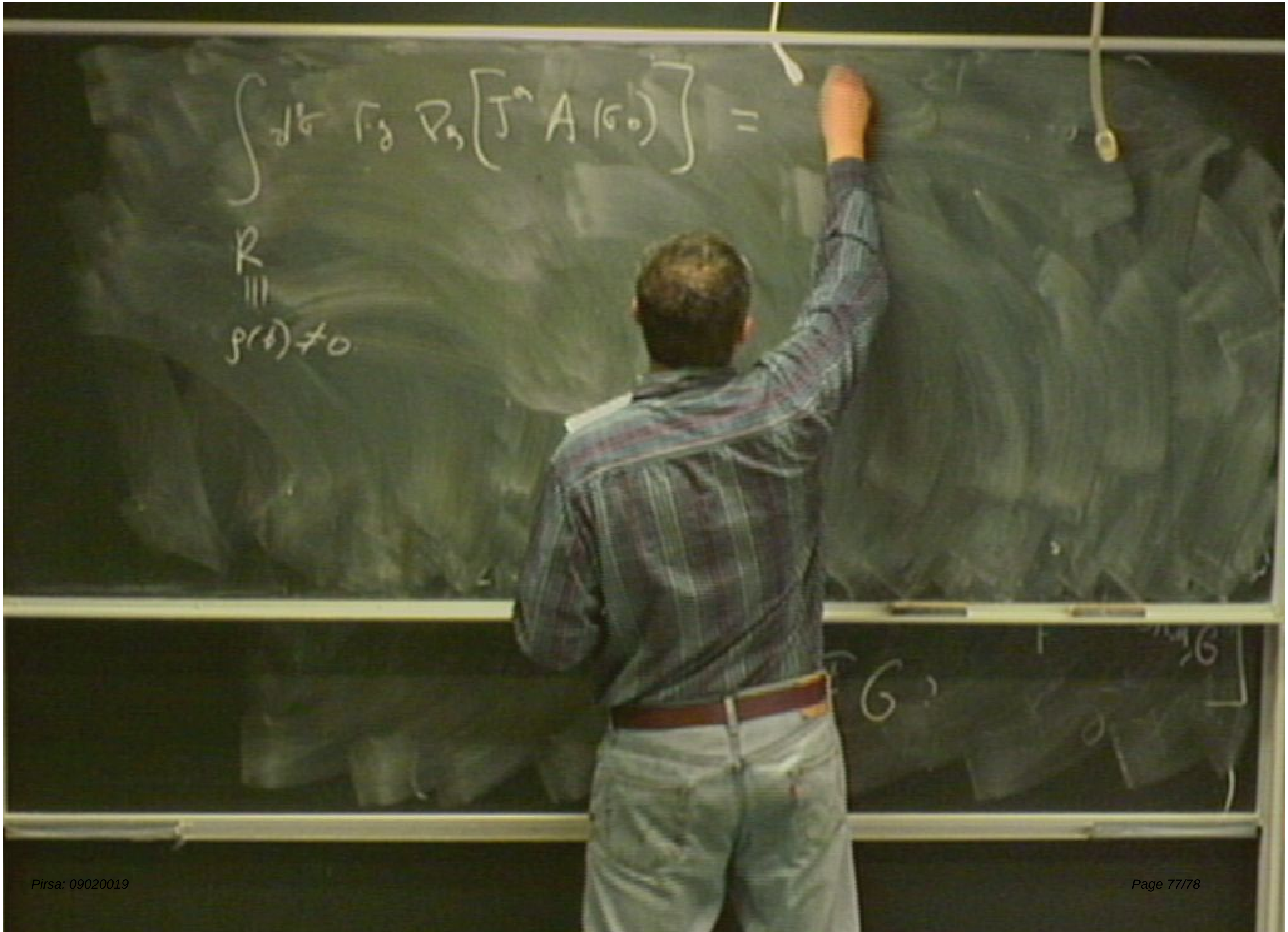
$$\frac{}{2\pi}\int[d\varphi]\,e \int d\sigma\sqrt{-g}\ \rho\ \partial_a J$$

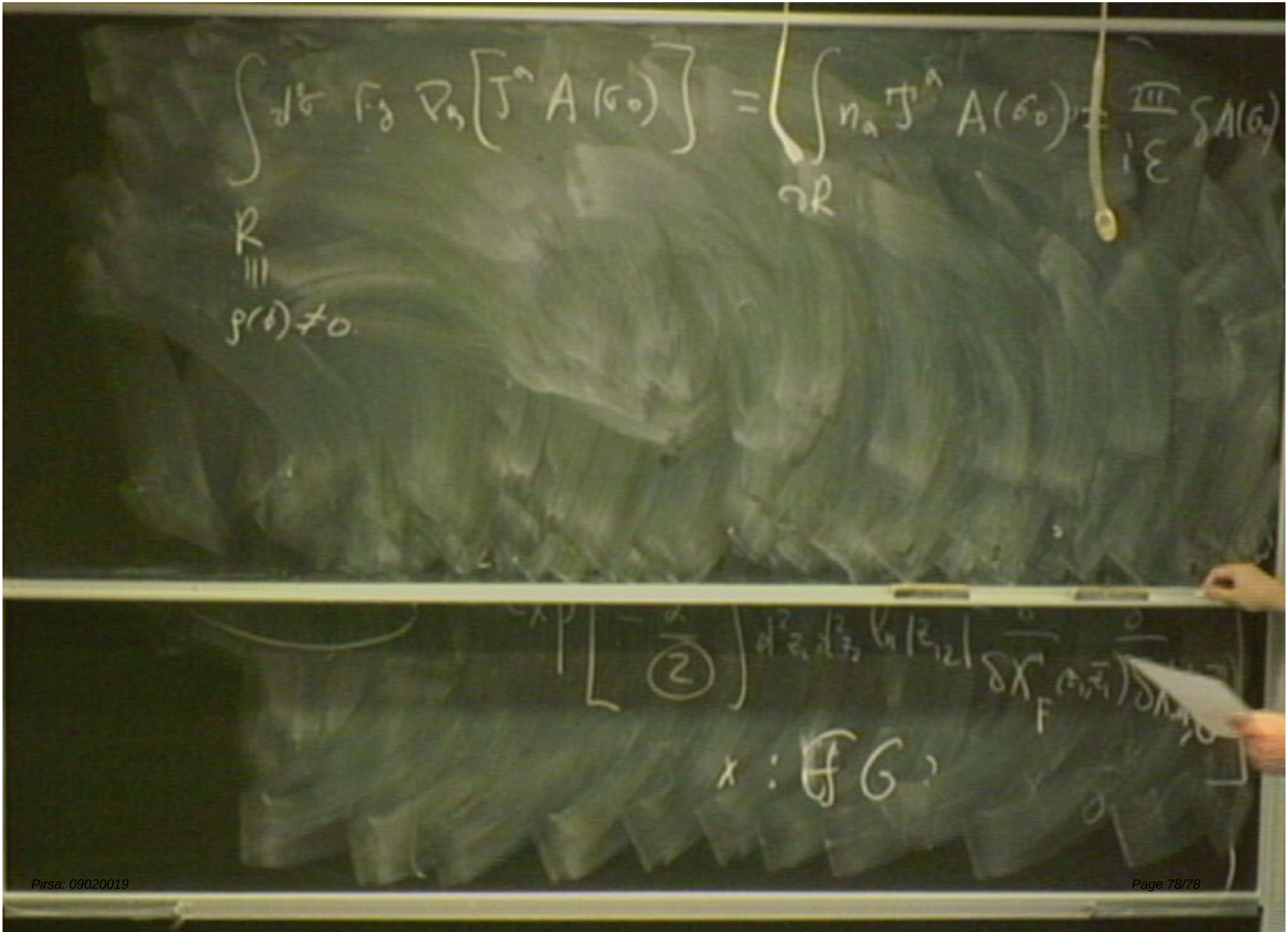
$$0 = -\delta A(\sigma_0) + \frac{i\varepsilon}{2\pi}\int[d\varphi]\,e^{-S}\int d^2\sigma\sqrt{-g}\cdot\nabla_a J^a\cdot\rho$$

$$\boxed{0 = -\frac{\varepsilon}{2\pi i}\int d^2\sigma\sqrt{-g}\cdot\rho(\sigma)\ \langle A(\sigma_0)\nabla_a J^a\rangle - \langle\delta A(\sigma_0)\rangle}$$

$$\parallel$$

$$\nabla_a J^a A(\sigma_0) = \frac{1}{\sqrt{-g}}\delta^2(\sigma-\sigma_0)\frac{2\pi}{i\varepsilon}\delta A(\sigma)$$


$\int d^6\! r_g \, D_g \left[J^a A(G_0) \right] =$
R
$|||$
$\rho(t) \neq 0$



$$\int_R d^2\sigma \sqrt{g}\, D_a\left[J^a A(\sigma_0)\right] = \int_{\partial R} n_a J^a A(\sigma_0) = \frac{\text{III}}{i\epsilon}\,\delta A(\sigma_0)$$

R

$|||$

$g(\sigma)\neq 0$

$$\exp\left[-\frac{\alpha'}{②}\int d^2z_1\, d^2z_2 \ln|z_{12}|\,\frac{\delta}{\delta X^\mu_F(z_1,\bar z_1)}\frac{\delta}{\delta X_{\mu F}(z_2,\bar z_2)}\right]$$

$x : G$?