

Title: Hot spacetime for cold atoms

Date: Feb 20, 2009 02:30 PM

URL: <http://pirsa.org/09020015>

Abstract: TBA

① We can apply G-G duality to NRCFT

② $\text{NRCFT}_z \longleftrightarrow \text{Schrödinger}$

① We can apply G - G duality to MRCFT

② $MRCFT_{\epsilon} \longleftrightarrow$ Schrödinger

③ $\begin{matrix} \textcircled{G} \\ \Delta, \epsilon \end{matrix}$

① We can apply G-G duality to MRCFT

② $MRCFT_{\mathbb{Z}} \leftrightarrow$ Schrödinger

③ $\mathbb{G}_{\Delta, \ell, \xi} \Rightarrow$ 2hol dimensions

② $MRCFT_{\mathbb{Z}} \iff$ Schrödinger

③ $\mathbb{G}_{\Delta, e} \Rightarrow 2 \text{ hol dimensions}$

④ Lots of examples (\exists proof for perthy)

⑤

④

Lots of examples (∃ proof for perthy)

⑤

Quantum ($\frac{1}{n}$) effect tractable

- ④ Lots of examples (Iprat + orthy)
- ⑤ Quantum ($\frac{1}{v}$) effect tractable

0807,

AA K Balasubramanian

J. McGreevy

- ④ Lots of examples (Iprat + partly)
- ⑤ Quantum ($\frac{1}{v}$) effect tractable

0807

AA

K Balasubramanian

J. McGreevy

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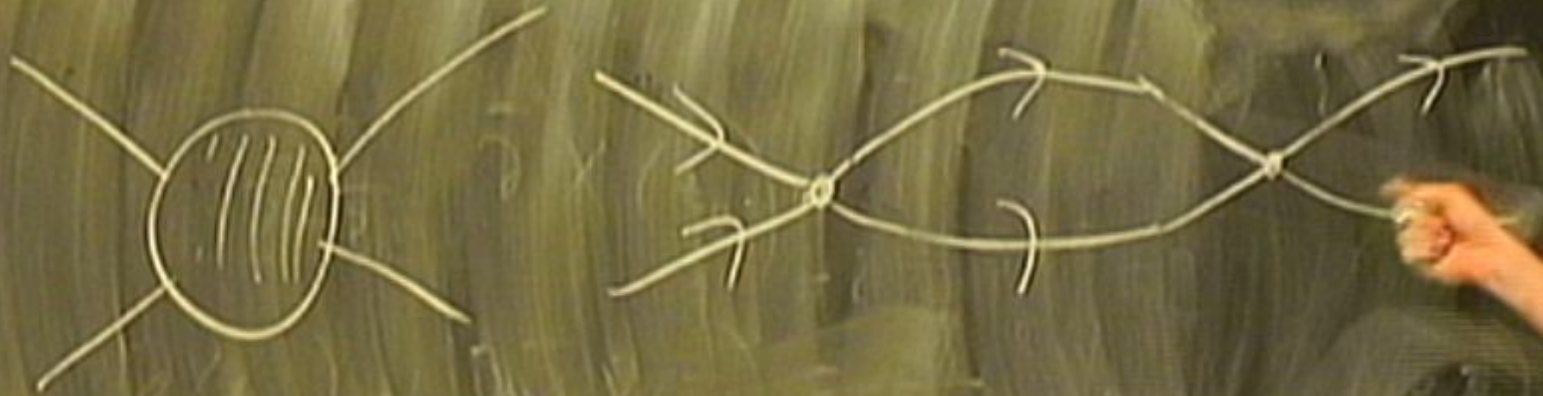
A Sub

S Vazquez

A Maloney

WIP

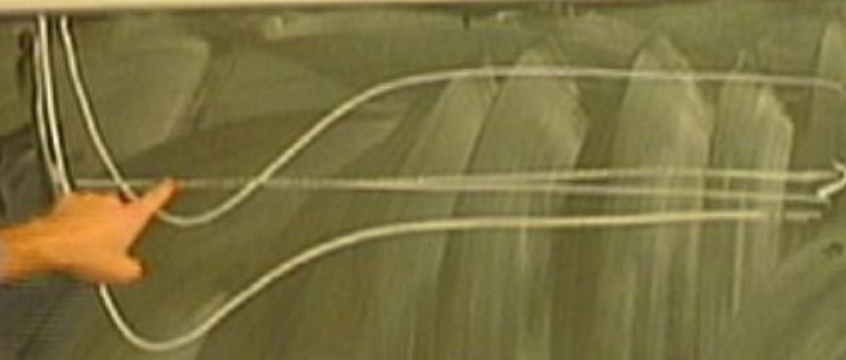
+ Students MIT



$$\omega \sim k^2$$

$$\omega^2 \sim k^4$$



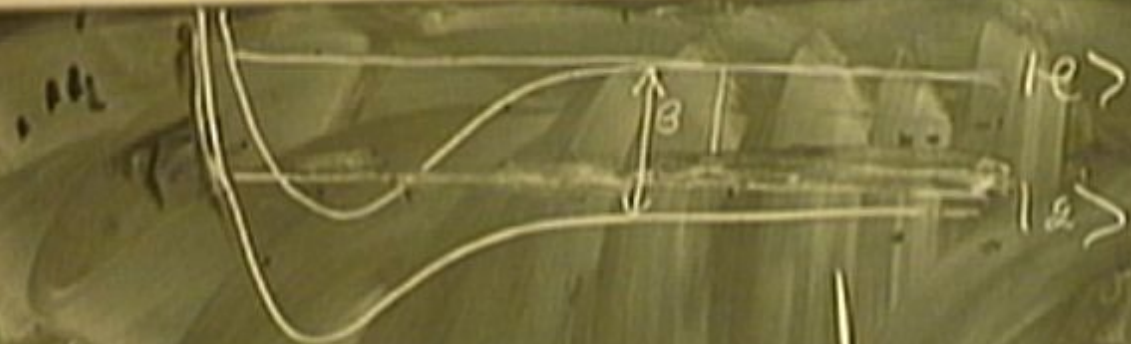






$a \rightarrow +\infty$
 <conformal>
 \mathbb{R}
 \mathbb{R}





$a \rightarrow \infty$
 < Conformal >
 NR

BEC BCS S

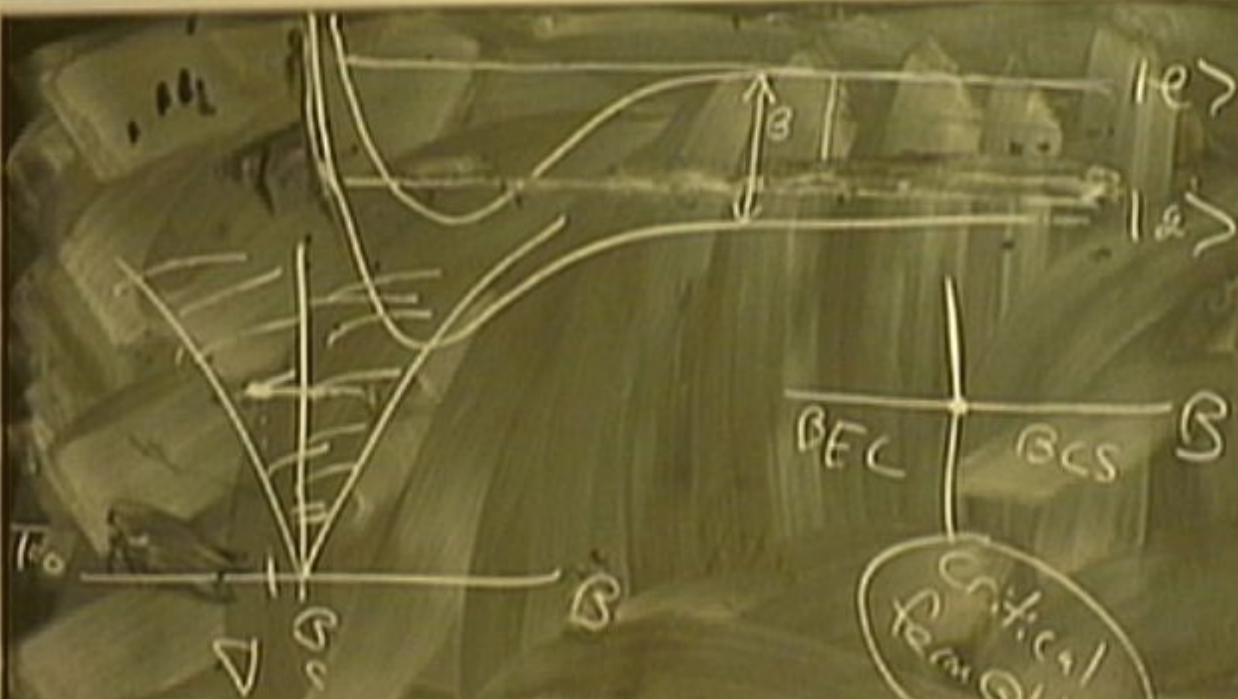
Critical fermion



$a \rightarrow \infty$
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 NR

BEC BCS β

Critical fermion

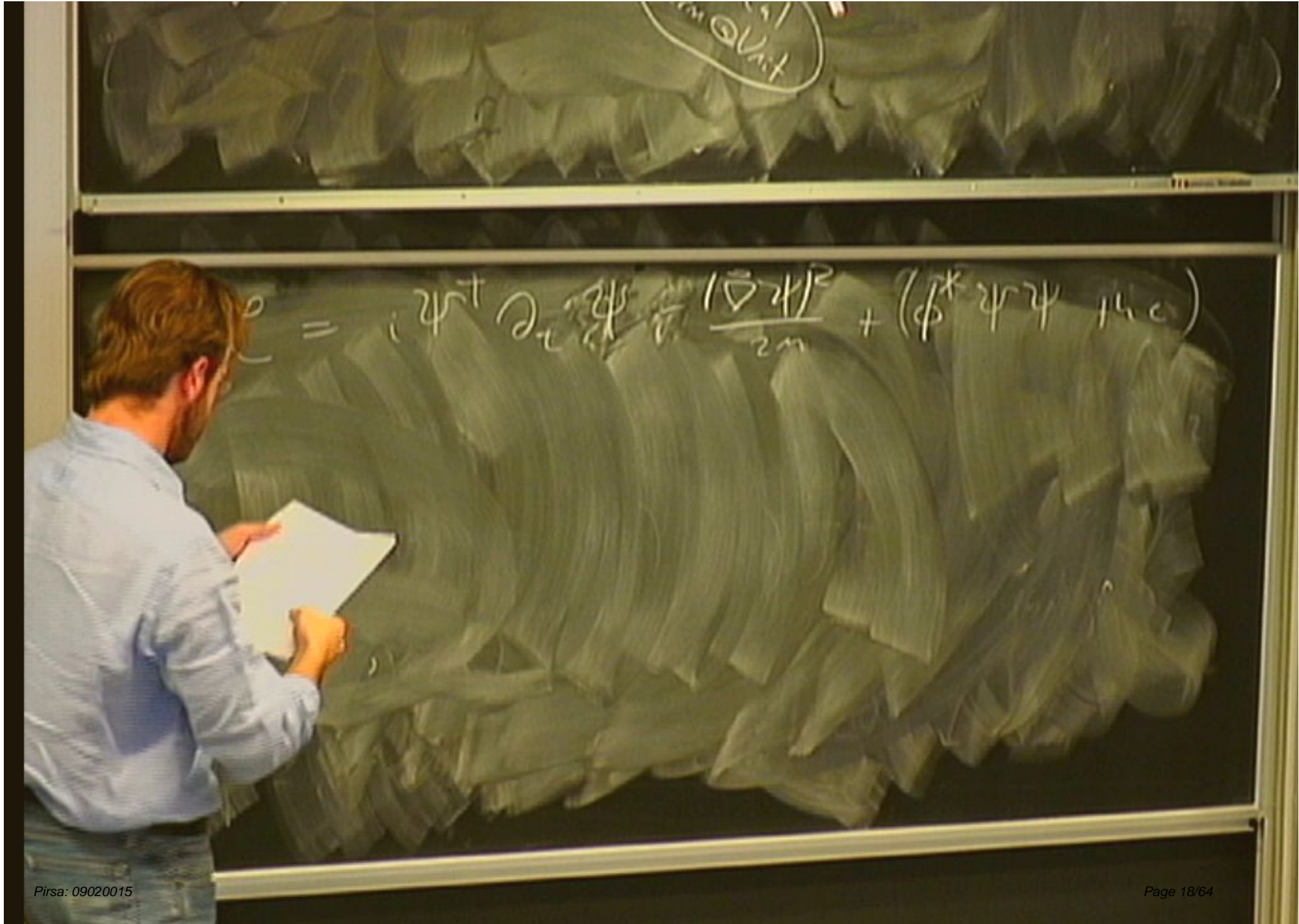


$a \rightarrow +\infty$
 < conformal >
 \mathbb{Z}_2 NR

BEC BCS B

Critical fermi @ V_{crit}





$$\mathcal{L} = i\psi^\dagger \partial_t \psi - \frac{\hbar^2 \nabla^2 \psi}{2m} + (\psi^\dagger \psi \mu + c)$$

$$i\partial_t \psi - \frac{\nabla^2 \psi}{2m} = 0$$

$$\mathcal{L} = i\psi^\dagger \partial_t \psi - \frac{\hbar^2 \nabla^2 \psi}{2m} + (\psi^\dagger \psi + \mu c)$$

$$i\partial_t \psi - \frac{\nabla^2 \psi}{2m} = 0$$

$$x \rightarrow \lambda x \quad t \rightarrow \lambda^2 t$$

$$\mathcal{L} = i\psi^\dagger \partial_t \psi - \frac{\hbar^2 \nabla^2 \psi}{2m} + (\psi^\dagger \psi \mu + c)$$

$$i\partial_t \psi - \frac{\nabla^2 \psi}{2m} = 0$$

$$\boxed{x \rightarrow \lambda x \quad t \rightarrow \lambda^2 t}$$

$$\mathcal{L} = i\psi^\dagger \partial_t \psi - \frac{\hbar^2 \nabla^2 \psi}{2m} + (\psi^\dagger \psi \mu, c)$$

$$i\partial_t \psi - \frac{\nabla^2 \psi}{2m} = 0$$

$$D: \boxed{x \rightarrow \lambda x \quad t \rightarrow \lambda^2 t}$$

$$N = \int \psi^\dagger \psi$$

$$C(x) = \frac{x}{1+ct} \quad t \rightarrow \frac{t}{1+ct}$$

Schrödinger Group

$\{P, M, H, K, N, D, C\}$

Schrödinger Group

$$\{P, M, H, K, N, D, C\}$$

$$[M, M] = M$$

$$[M, P] = P$$

$$[M, K] = K$$

$$[H, K] = P$$

$$[P, K] = 0$$

$$[D, H] = 2H$$

$$[D, P] = P$$

Schrödinger Group

Sch²

$$\{P, M, H, K, N, D, C\}$$

$$[M, M] = M \quad [M, P] = P$$

$$[M, K]$$

$$[H, K] = P$$

$$[P, K] = 0$$

$$[D, H] = z \cdot H$$

$$[D, P] = P$$

$$[D, K] = i(H + z)$$

Schrodinger Group

Sch^z

$\{P, M, H, K, N, D, C\}$

$$[M, M] = M \quad [M, P] = P \quad [M, K] = K$$

$$[H, K] = P \quad [P, K] = (2-z)N$$

$$[D, H] = zH \quad [D, P] = P \quad [D, K] = (1+z)K$$

$$\mathcal{L} = i\psi^\dagger \partial_t \psi - \frac{(\nabla \psi)^2}{2m} + (\phi^* \psi \psi + \text{h.c.})$$

$$i\partial_t \psi - \frac{\nabla^2 \psi}{2m} = 0$$

$$D: \boxed{x \rightarrow \lambda x \quad t \rightarrow \lambda^2 t}$$

$$N = \psi^\dagger \psi$$

$$C: x \rightarrow \frac{x}{1+ct} \quad t \rightarrow \frac{t}{1+ct}$$

$$\mathcal{L} = i\psi^\dagger \partial_t \psi - \frac{(\nabla \psi)^2}{2m} + (\phi^* \psi \psi + c)$$

$$i\partial_t \psi - \frac{\nabla^2 \psi}{2m} = 0$$

$$D: \left[x \rightarrow \lambda x \quad t \rightarrow \lambda^2 t \right]$$

$$N = \psi^\dagger \psi$$

$$C(x) = \frac{x}{1+ct} \quad t \rightarrow \frac{t}{1+ct} \quad \text{if } z=2$$

Schrodinger Group

Sch^z

$\{P, M, H, K, N, D, C\}$

$$[M, M] = M \quad [M, P] = P \quad [M, K] = K$$

$$[H, K] = P \quad [P, K] = (2-z)N \quad [D, N] = (2-z)N$$

$$[D, H] = zH \quad [D, P] = P \quad [D, K] = (1+z)K$$

Schrodinger Group

{P, M, H, K, N, D, C}

[M, M] = M

[M, P] = P

[M, K] = K

[H, K] = P

[P, K] = (2-z)N

[D, N] = (2-z)N

[D, H] = zH

[D, P] = P

[D, K] = i(1+z)K

Sch^z

[D, C] = -zC
z=2 [H, C] = -iD

Schrodinger Group

Sch^z

{P, M, H, K, N, D, C}

$[D, C] = -z \cdot C$
$[H, C] = -iD$

$[M, M] = M$

$[M, P] = P$

$[M, K] = -K$

$[H, K] = P$

$[P, K] = (z-z)N$

$[D, N] = (z-z)N$

$[D, H] = z \cdot H$

$[D, P] = P$

$[D, K] = i(1+z)K$

Schrodinger's Group

$\{P, M, H, K, N, D, C\}$

$$[M, M] = M$$

$$[M, P] = P$$

$$[M, K] = K$$

$$[H, K] = P$$

$$[P, K] = 0$$

$$[D, N] = 0$$

$$[D, H] = z \cdot H$$

$$[D, P] = P$$

$$[D, K] = i(H + z)K$$

Sch^z

$$[D, C] = -z \cdot C$$

$$z = -2 \quad [H, C] = -iD$$

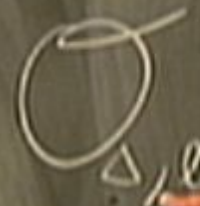
$$\mathcal{L} = i\psi^\dagger \partial_t \psi - \frac{(\nabla \psi)^2}{2m} + (\phi^* \psi \psi + \text{h.c.})$$

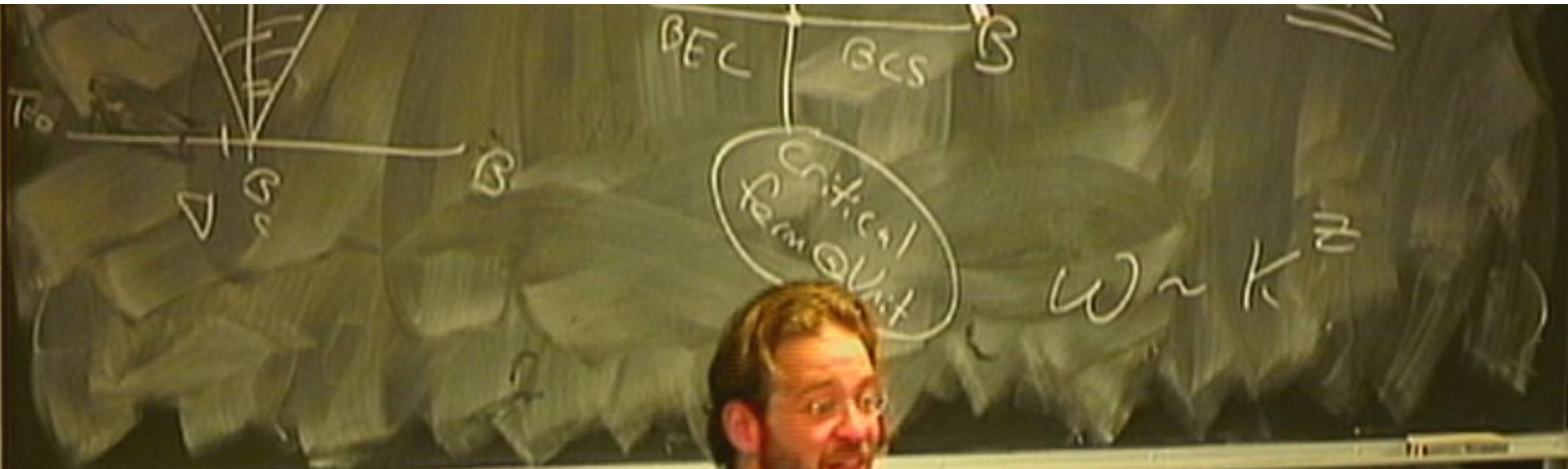
$$i\partial_t \psi - \frac{\nabla^2 \psi}{2m} = 0$$

$$D: \boxed{x \rightarrow \lambda x \quad t \rightarrow \lambda^2 t}$$

$$N = \psi^\dagger \psi$$

$$C: x \rightarrow \frac{x}{1+ct} \quad t \rightarrow \frac{t}{1+ct} \quad \text{if } z=2$$





$i\partial_t \psi - \dots = 0$
 $D: \boxed{x \rightarrow t}$
 $N = \dots$
 $C_0 x \frac{x}{1+ct}$
 $Z=2$
 $\frac{O_{\Delta, l}}{\dots}$

- (4) Lots of examples (Iprat to porthy)
- (5) Quantum ($\frac{1}{\hbar}$) effect tractable

$$\text{Cont. Group (dt)} = \text{Ison (X)}$$
$$\parallel$$
$$\text{Sch}^2 \qquad \text{Sch}^2$$

- ④ Lots of examples (Iprat for penthy)
- ⑤ Quantum ($\frac{1}{\hbar}$) effect tractable

NR Conf. Group (d+1) = Isom (X)

\parallel
Sch²

Sch²

D

= \mathcal{D}_r

N

=

- ④ Lots of examples \setminus (\exists proof for post-thy)
- ⑤ Quantum ($\frac{1}{\hbar}$) effect tractable

$$\text{NR Cont. Group (dt)} = \text{Ison}(X)$$

$$\parallel$$

$$\text{Sch}^2$$

$$\text{Sch}^2$$

$$D$$

$$\parallel$$

$$\partial_c$$

$$N$$

$$\parallel$$

$$\partial_{\mu} \partial_{\nu} = S$$



$$L^2(a, R)$$

$$(\hat{p}_t, \hat{p}_x \dots (\hat{p}_t + \hat{H})|\psi\rangle = 0$$

$$\partial_t \hat{\pi} \psi = 0, \int \mathcal{D}A^+ \int \mathcal{D}A^- |\psi\rangle \langle \psi|$$

AdS in light Cone

$$\frac{-2 dt dz + d\vec{x}^2 + dr^2}{r^2}$$

$L^2(a, R^2)$
 $(\hat{p}_t, \hat{p}_x \dots (\hat{x}_t + \hat{H})|\psi\rangle = 0$

AdS in light Cone

$$\frac{-2 dt dz + d\vec{x}^2 + dr^2}{r^2}$$

$D: \mathbb{R}^4 \rightarrow \lambda \mathbb{R}^4, x, t \rightarrow \lambda x, \lambda t$



$$L(a, R)$$

$$(\hat{p}_t, \hat{p}_x \dots (\hat{p}_t + \hat{H})|\psi\rangle = 0$$

$$\partial_t \hat{\pi} \psi = 0, \int \mathcal{D}\lambda^t \int \mathcal{D}\lambda^x |\psi\rangle \langle \lambda^t, \lambda^x|$$

AdS in light Cone

$$\frac{-2 dt d\vec{x} + dr^2}{r^2}$$

$$D: \mathbb{R}^4 \rightarrow \lambda^t, \begin{matrix} x \rightarrow \lambda^x \\ t \rightarrow \lambda^t \\ \vec{x} \rightarrow \lambda^i \end{matrix}$$



$L^2(a, R)$

$$(\hat{p}_t + \hat{H})\psi = 0$$

$$\partial_t \psi = 0, \quad \int \partial_t \psi = 0, \quad \int \partial_t \psi = 0$$

AdS in light Cone

$$\frac{-\beta dt^2}{r^4} - \frac{2 dt dz + d\vec{x}^2 + dr^2}{r^2}$$

$$D: M \rightarrow M, \quad \begin{aligned} x &\rightarrow \lambda x \\ t &\rightarrow \lambda t \\ z &\rightarrow z \end{aligned}$$



$$L^2(\mathbb{S}^1, \mathbb{R}^4)$$

$$(\hat{p}_t, \hat{p}_x \dots (\hat{p}_t + \hat{H})|\psi\rangle = 0$$

$$\partial_t \hat{\pi} \psi = 0, \int \mathcal{D}A^+ \int \mathcal{D}A^- |\psi\rangle \langle \psi|$$

AdS in light Cone

$$\frac{\beta dt^2}{r^2} - \frac{2 dt dz + d\vec{x}^2 + dr^2}{r^2}$$

$$D: \mathbb{R}^4 \rightarrow \mathbb{R}^4, \begin{cases} x \rightarrow \lambda x \\ t \rightarrow \lambda^2 t \\ z \rightarrow \lambda^2 z \end{cases}$$



$L^2(a, \mathbb{R}^n)$

$(\hat{p}_t, \hat{p}_x \dots (\hat{p}_t + \hat{H})\psi = 0$

$\partial_t \hat{H} \psi = 0, \int \mathcal{D}A^4 \int \mathcal{D}A^T |\psi\rangle \langle \psi|$

Sch^z

$$\frac{-\beta \dot{t}^2}{r^2} \quad \frac{-2 dt d\zeta + d\vec{x}^2 + dr^2}{r^2}$$

$D: \mathbb{R}^4 \rightarrow \mathbb{R}^4, \begin{matrix} x \rightarrow \lambda x \\ t \rightarrow \lambda^2 t \\ \zeta \rightarrow \lambda^2 \zeta \end{matrix}$

$L^2(\mathbb{R}^3, \mathbb{R}^3)$
 $(\hat{p}_t, \hat{p}_x \dots (\hat{p}_t + \hat{H})|\psi\rangle = 0$ $\partial_t \hat{\pi} \psi = 0$

Sch^z

$$\frac{-\beta dt^2}{r^2} - \frac{2 dt d\zeta + d\vec{x}^2 + dr^2}{r^2}$$

$D: \mathbb{R}^4 \rightarrow \mathbb{R}^4$, $x \rightarrow \lambda x$
 $t \rightarrow \lambda^2 t$
 $\zeta \rightarrow \lambda^2 \zeta$

$$\mathbb{N} = \partial_\zeta$$

$L^2(\mathbb{R}^3, \mathbb{R}^3)$
 $(\hat{p}_t, \hat{p}_t \dots (\hat{p}_t + \hat{H})|\psi\rangle = 0$ $\partial_t \hat{\pi} \psi = 0$

Sch^z

$$\frac{-\beta dt^2}{r^2} - \frac{2 dt d\zeta + d\vec{x}^2 + dr^2}{r^2}$$

$D: \mathbb{R}^4 \rightarrow \mathbb{R}^4$
 $x \rightarrow \lambda x$
 $t \rightarrow \lambda^2 t$
 $\zeta \rightarrow \lambda^2 \zeta$

$\mathbb{N} = \partial_3$

$K: x \rightarrow x - vt$
 $\zeta \rightarrow \zeta + \int (\vec{v} \cdot \vec{x})$

$L^2(\mathbb{R}^3, \mathbb{R}^3)$
 $(\hat{x}_t, \hat{p}_t) \dots (\hat{x}_t + \hat{H})|\psi\rangle = 0$

Sch^z

$$\frac{-\beta dt^2}{r^2} - \frac{2 dt d\zeta + d\vec{x}^2 + dr^2}{r^2}$$

$D: \mathbb{R}^4 \rightarrow \mathbb{R}^4$
 $x \rightarrow \lambda x$
 $t \rightarrow \lambda^2 t$
 $\zeta \rightarrow \lambda^2 \zeta$

$\vec{N} = \partial_\zeta$

$K: x \rightarrow x - vt, \zeta \rightarrow \zeta + \left(\vec{v} \cdot \vec{x} - \frac{v^2 t}{2} \right)$

$L^2(\mathbb{R}^3, \mathbb{K})$
 $(\hat{p}_t, \hat{p}_x \dots (\hat{x}_t + \hat{H})|\psi\rangle = 0$

Sch^z

$$\frac{-\beta^2 dt^2}{r^2} - \frac{2 dt d\zeta + d\vec{x}^2 + d\tau^2}{r^2}$$

$D: \mathbb{R}^4 \rightarrow \mathbb{R}^4$
 $x \rightarrow \lambda x$
 $t \rightarrow \lambda^2 t$
 $\zeta \rightarrow \lambda^2 \zeta$

$N = \partial_\zeta$

$K: x \rightarrow x - vt$
 $\zeta \rightarrow \zeta + \left(\vec{v} \cdot \vec{x} - \frac{v^2 t}{2} \right)$

- ④ Lots of examples (Iprat + orthy)
- ⑤ Quantum effect tractable

NR Cont. Group (dt1) = $I_{\text{son}}(X)$

\equiv
Sch²

Sch²

D

\equiv ∂_r

N

\equiv $\partial_{\theta} = \frac{1}{r} \partial_{\theta}$

∂_{θ}



$L^2(\mathbb{R}^3, \mathbb{R}^3)$
 $(\hat{x}_t, \hat{p}_t) \dots (\hat{x}_t + \hat{H})|\psi\rangle = 0$
 $\partial_t \hat{\pi} \psi = 0, \dots \int \mathcal{D}A^4 \int \mathcal{D}A^T |\psi\rangle \langle \psi|$

Sch $z \quad t \rightarrow \frac{1}{\Delta} t \quad \xi \rightarrow \Delta \xi$

$$\frac{-\beta \dot{t}^2}{r^2} \quad \frac{-2 dt d\xi + d\vec{x}^2 + dr^2}{r^2}$$

$D: \mathbb{R}^4 \rightarrow \mathbb{R}^4, \quad x \rightarrow \lambda x$
 $t \rightarrow \lambda^2 t$
 $\xi \rightarrow \lambda^2 \xi$

$$\mathbb{N} = \partial_\xi$$

$K: x \rightarrow x - vt, \quad \xi \rightarrow \xi + \left(\vec{v} \cdot \vec{x} - \frac{v^2}{2} t \right)$

$$L^2(a, \mathbb{R}^n) \quad \hat{p}_t, \hat{p}_z \dots \quad (\hat{p}_t + \hat{H})|\psi\rangle = 0 \quad \partial_t \hat{\Pi} \psi = 0 \quad \int \mathcal{D}A^t \int \mathcal{D}A^z |\psi\rangle \langle \psi|$$

Sch $z \rightarrow \frac{1}{\Delta} z$ $\xi \rightarrow \Delta \xi$

βL_3 is phys

$$-\beta dt^2 - 2 dt dz + dz^2$$

$D: \mathbb{R}^4 \rightarrow \mathbb{R}^4$, $x \rightarrow \lambda x$
 $t \rightarrow \lambda^2 t$
 $z \rightarrow \lambda^2 z$

$\xi \rightarrow \xi + \left(\vec{v} \cdot \vec{x} - \frac{v^2}{2} t \right)$

$L^2(\mathbb{R}^3, \mathbb{R}^3)$
 $(\hat{x}_t, \hat{p}_t) \dots (\hat{x}_t + \hat{H})|\psi\rangle = 0$ $\partial_t \hat{\pi} \psi = 0$

Sch z $t \rightarrow \frac{1}{\Delta} t$ $\xi \rightarrow \Delta \xi$ $\beta L_3 \rightarrow \text{phys}$

$$\frac{\beta dt^2}{r^2} - \frac{2 dt d\xi + d\vec{x}^2 + dr^2}{r^2}$$

$D: \mathbb{R}^4 \rightarrow \mathbb{R}^4$, $x \rightarrow \lambda x$
 $t \rightarrow \lambda^2 t$
 $\xi \rightarrow \lambda^2 \xi$

$\Phi \sim f^\Delta$
 σ_Δ
 $N = \partial_3$
 $K: x \rightarrow x - vt$, $\xi \rightarrow \xi + \left(\vec{v} \cdot \vec{x} - \frac{v^2 t}{2} \right)$

$$S[\phi_0] = \int_{\partial X} d^m x \left. \phi(x) \partial_n \phi(x) \right|_{r=\epsilon}$$

$$S[\phi_0] = \int d^4x \phi(x) \partial_r \phi(x) \Big|_{r=\epsilon}$$

$$h^{tt} = 0$$

$$h^{zz}$$

$$h \partial_t \phi + \left(\frac{\partial^2 \phi}{\partial x^2} \right)$$

$$[D, H] = z: H$$

$$[D, T] = -T$$

$$[D, K] = -K$$

$$S[\phi_0] = \int d^4x \phi(x) \partial_r \phi(x)$$

$$h^{tt} = 0$$

$$h^{zz}$$

$$h^{t1}$$

$$\langle \phi_\Delta \phi_\Delta \rangle = \dots$$

$$e^{-\frac{ex^2}{2|t|}}$$

$$\omega \sim k^2$$

① ✓ We can apply G-G duality to MRCFT

② ✓ $MRCFT_z \leftrightarrow$ Schrödinger

③ ✓ $\mathbb{S}_{\Delta, e} \Rightarrow$ 2 hol dimensions

④ Lots of examples (\exists proof for pentth)

⑤ Quantum ($\frac{1}{N}$) effect tractable

$$N = \mathcal{D}_{\Delta, e}$$

G, V

$$ds^2 = -2dvdr + ds_m^2 + A du^2$$

$$\Delta_m A = \alpha A$$

GV

$$ds^2 = \frac{dr^2}{1 - \frac{2M}{r}} + A du^2$$

$$\Delta A = 12\pi A$$

Melvin

G V

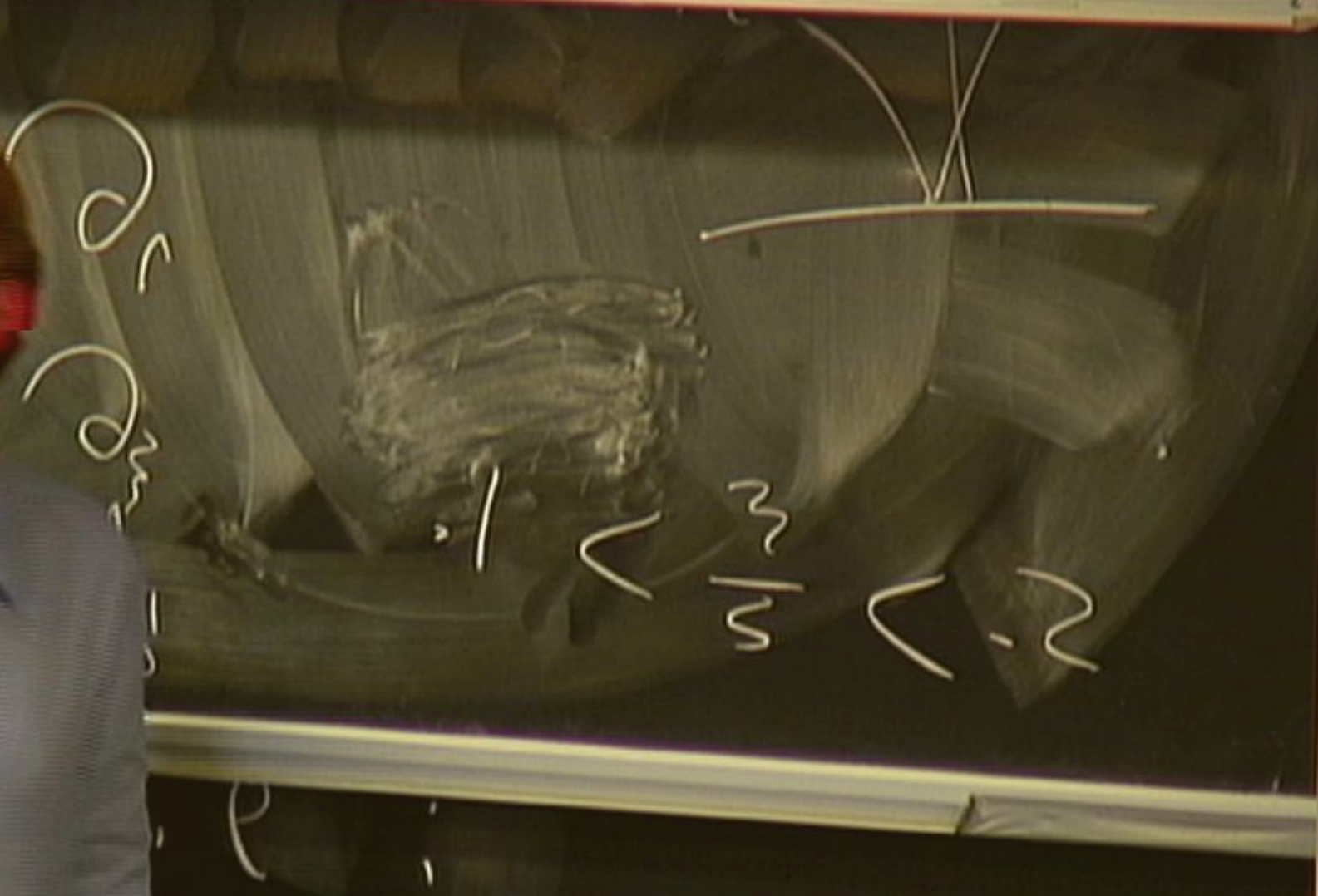
$$ds^2 = \left[\frac{dr^2}{1 - \frac{2M}{r}} + \left(\frac{A}{r^2} du^2 \right) \right]$$

GN

$$ds^2 = \frac{dr^2 + r^2 d\Omega^2}{r^2} + \left(\frac{A}{r^4} du^2 \right)$$

$$\boxed{\Phi(z+1) = e^{\beta J_2} \Phi(z)} \quad \mathbb{Z}(\mathcal{M}, A)$$





$$z = \frac{dv + dr + dr^2 + ds^2}{r^2} + \left(\frac{A - du}{r^2} \right)$$

$$\psi(z+1) = e^{g_{p^2} + \dots} \Phi(z)$$



$$\frac{dr^2 + ds^2}{r^2}$$

$$l^2 \Phi^2$$

$$+ l^2 = e^{g_{\mu\nu}} \Phi(z) \} z(M, A)$$

$$\frac{dr dv + dr^2}{r^2} + ds_{\text{hor}}^2 (N + 4\beta \frac{dr}{r}) \partial_t \Phi$$

$$+ L) = e^{g_{\text{hor}}^2} \Phi(z) \Big] z(M, A)$$