Title: Brane Tilings, M2 Branes and Chern Simons theories

Date: Feb 24, 2009 11:00 AM

URL: http://pirsa.org/09020013

Abstract: Brane Tilings are known to describe the largest known class of SCFT's in 3+1 dimensions. There is a well established formalism to find AdS_5 x SE_5 duals to these SCFT's and to compare results on both sides. This talk extends this formalism to 2+1 dimensional SCFT's, living on the world volume of M2 branes, which are dual to AdS_4 x SE_7 backgrounds of M theory. The SCFT's are quiver gauge theories with 4 supercharges (N=2 in 2+1 dimensions) and Chern Simons (CS) couplings. They admit a moduli space of vacuum configurations which is a CY4 cone over SE_7. The talk will go over the formalism and look at several examples in detail. The computation of scaling dimensions will be mentioned and relations to regular toric Fano 3-folds if time permits.

Brane Tilings, M2 Branes, CS Theories

Amihay Hanany

In collaboration with

Alberto Zaffaroni

Yang-Hui He

David Vegh

Sebastian Franco

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- Algebraic Geometry - Quiver Gauge Theories

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- M2 probing CY4 - Cone over H^7

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- If put c conditions on CS levels G-c dimensional sub - lattice of SCFT's

Recall in 3+1 dimensions

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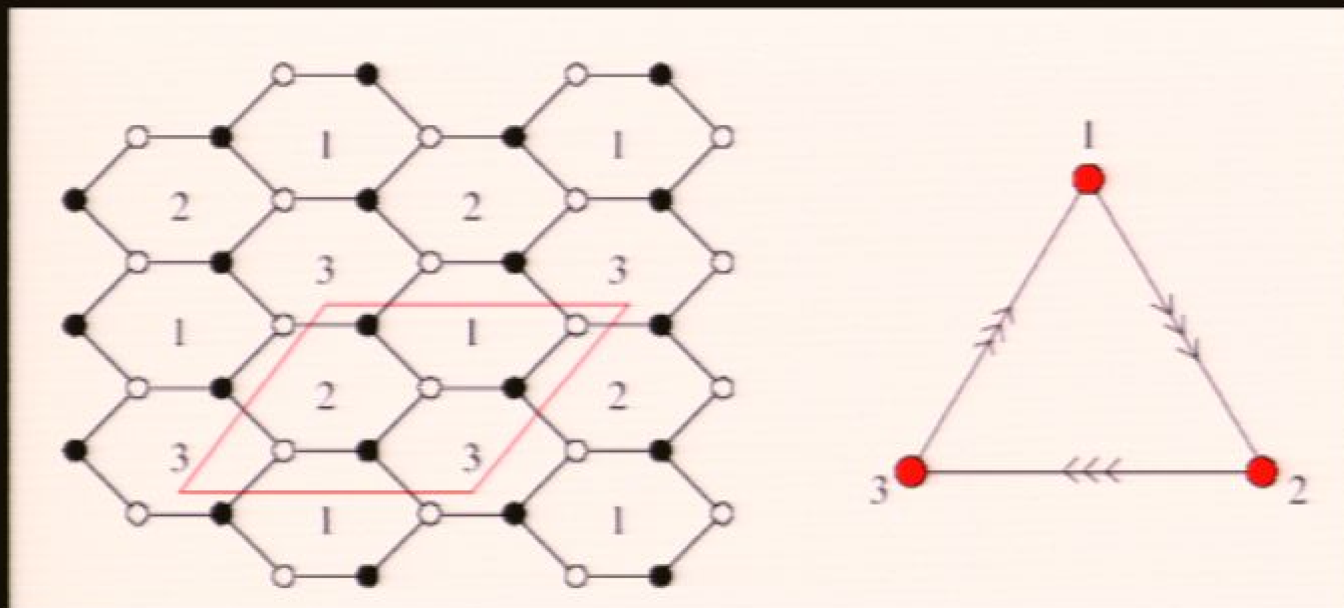
- $AdS_5 \times H^5$, H^5 Sasaki Einstein base of CY3
- Best description in terms of "Brane Tilings"



Periodic bipartite tiling

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Nathan Broomhead
Dimer Models and Calabi-Yau Algebras

Tiling - Quiver dictionary

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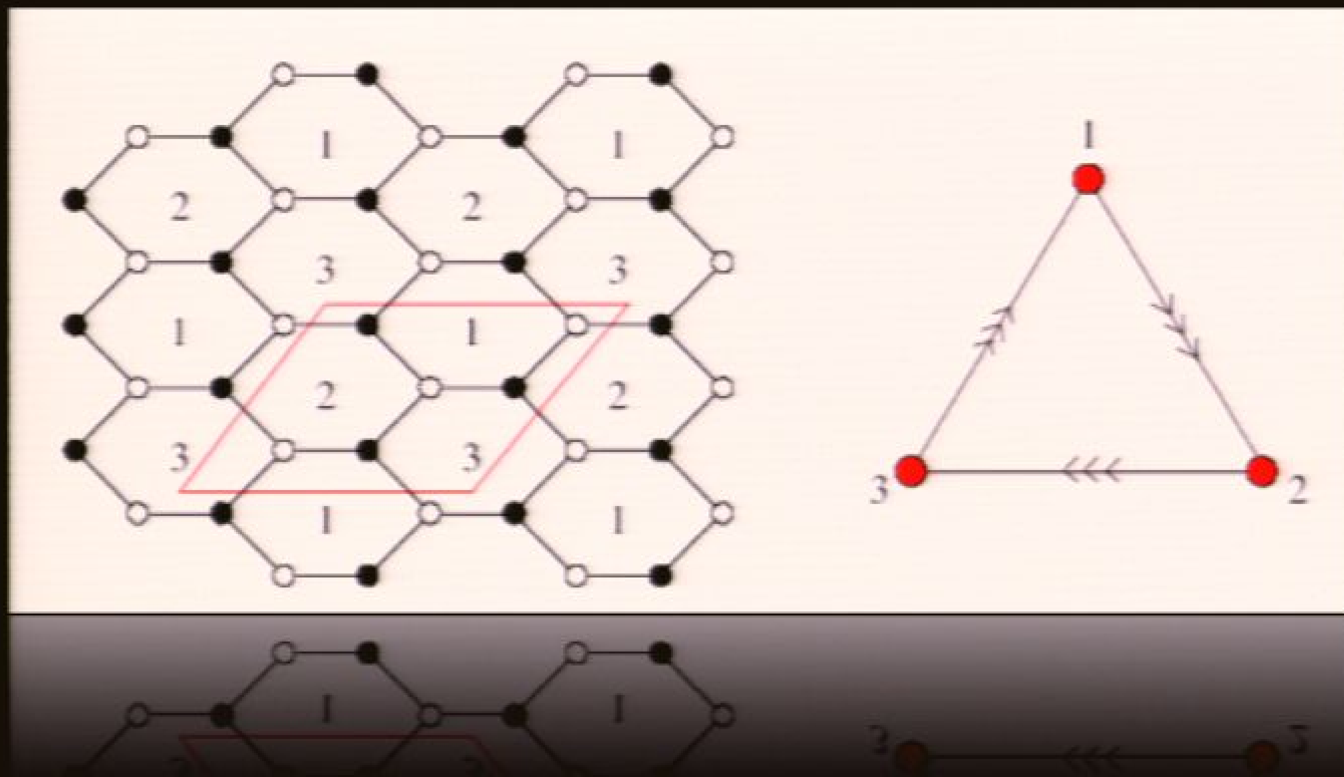
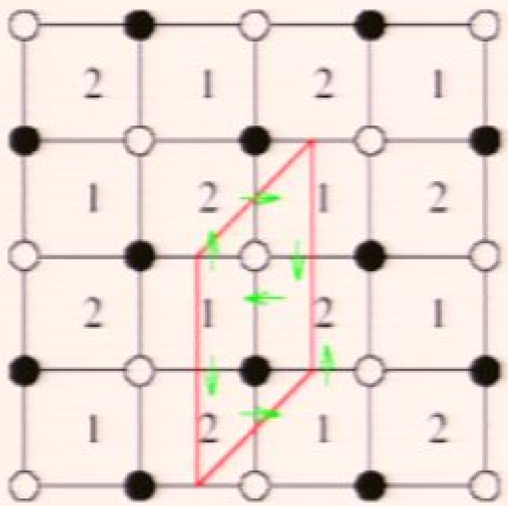
- k valent node - A k-th order interaction term in the superpotential



3 Hexagon tiling

Ex: Chessboard Tiling

$CY_6 = conifold$



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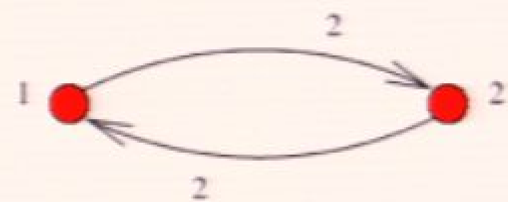

quiver

$$W = X_{12}^{(1)} X_{21}^{(1)} X_{12}^{(2)} X_{21}^{(2)} - X_{12}^{(1)} X_{21}^{(2)} X_{12}^{(2)} X_{21}^{(1)}$$

Comments

- Arrows are oriented in an alternating fashion

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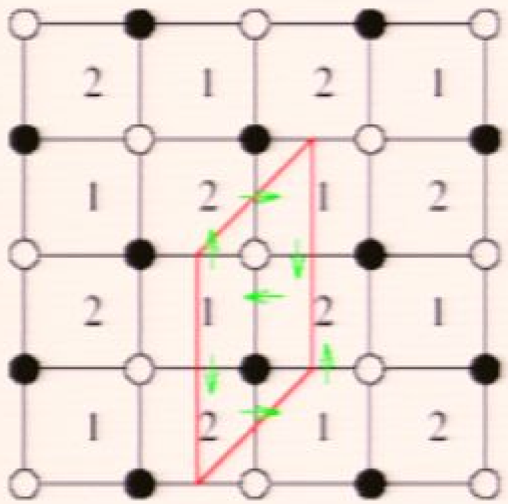

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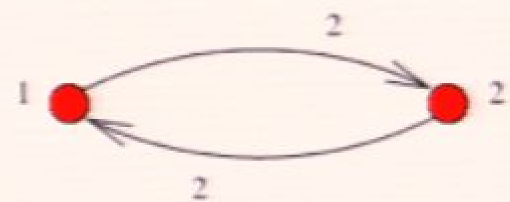

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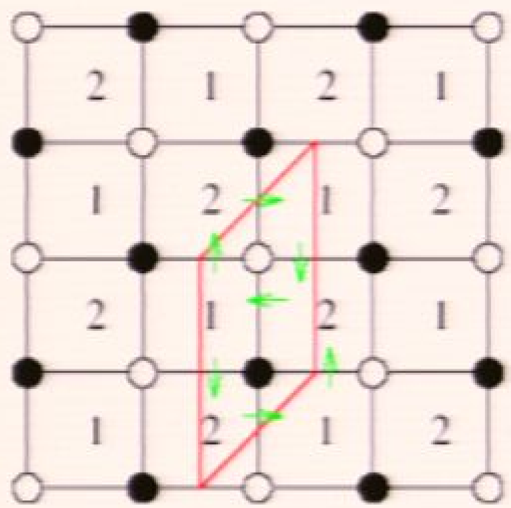

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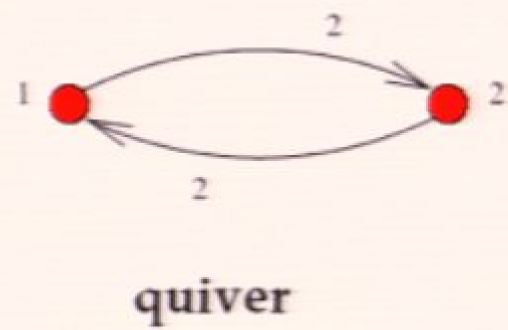

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- Graph is bi-partite: Nodes alternate between clockwise (white) and counterclockwise (black) orientations of arrows

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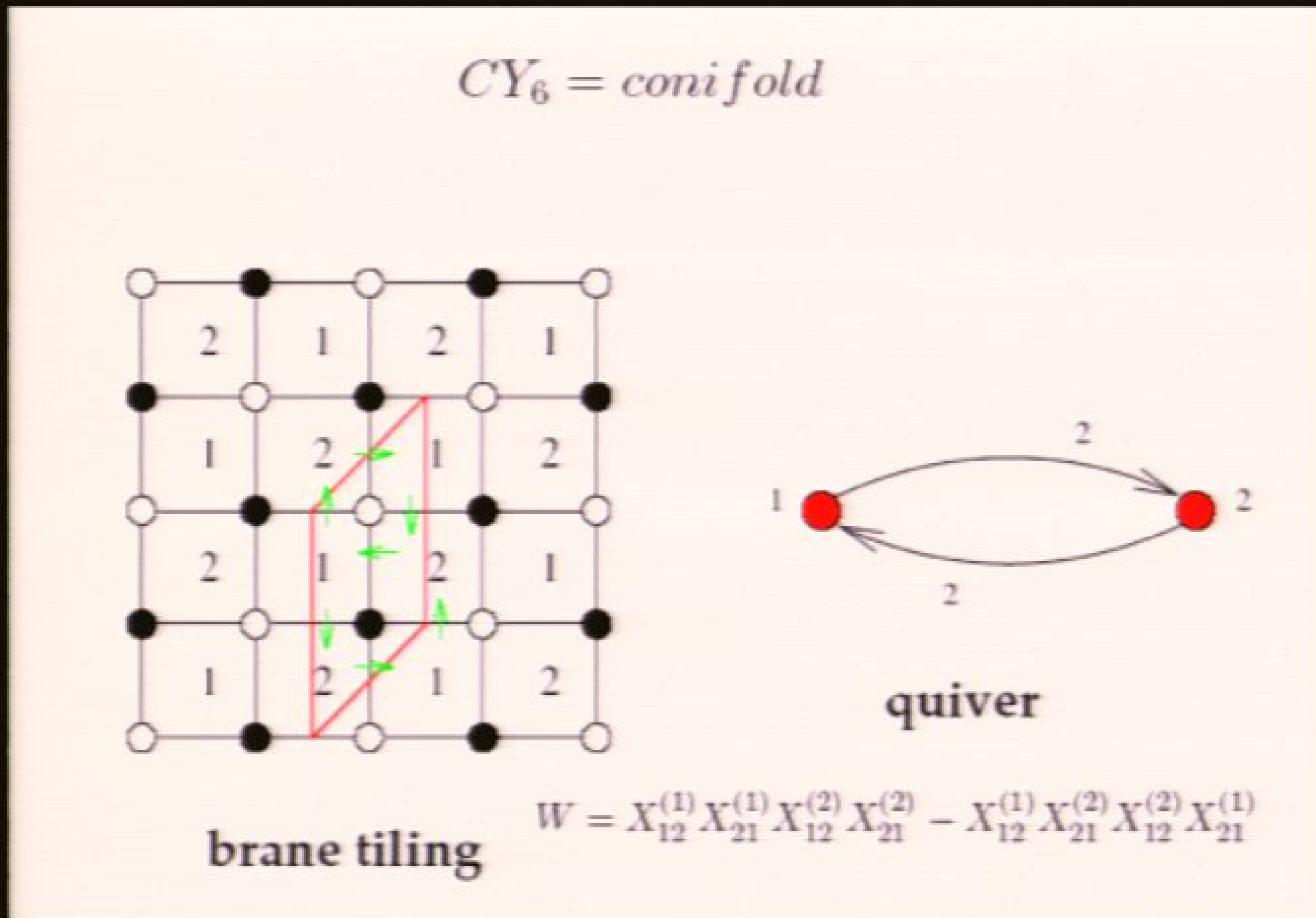

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- These rules define a unique Lagrangian in 3+1 dimensions

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- Given a 2d periodic, bipartite tiling with G tiles, add G CS levels, 1 for each tile.

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- Largest known family of SCFT's in 2+1d!

The 2+1d Lagrangian

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\begin{aligned}
& - \int d^4\theta \sum_{X_{ab}} X^\dagger_{ab} e^{-V_a} X_{ab} e^{V_b} \\
& + \; i \int d^4\theta \sum_{a=1}^{G} k_a \int_0^1 dt V_a \bar{D}^\alpha (e^{tV_a} D_\alpha e^{-tV_a}) \\
& + \int d^2\theta W(X_{ab}) + \text{c.c.}
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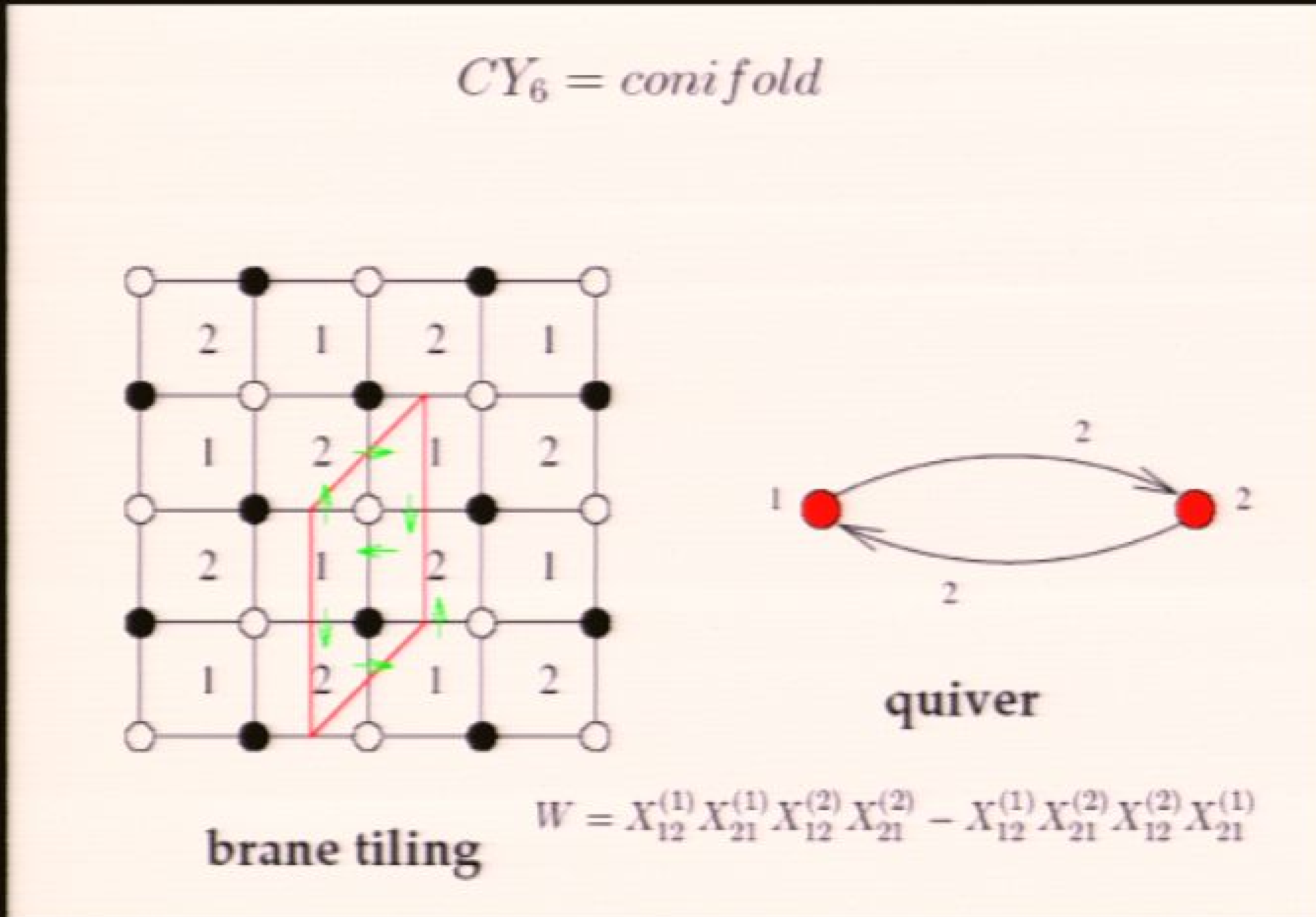

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$$C = \begin{pmatrix} 1 & 1 & 1 & \dots & 1 \\ k_1 & k_2 & k_3 & \dots & k_G \end{pmatrix} .$$

Vacuum Equations

$$
\begin{aligned}
\partial_{X_{ab}} W &= 0 \\
\iota_a(X) := \sum_{b=1}^{G} X_{ab} X_{ab}^{\dagger} - \sum_{c=1}^{G} X_{ca}^{\dagger} X_{ca} + [X_{aa}, X_{aa}^{\dagger}] &= 4 k_a \sigma_a \\
\sigma_a X_{ab} - X_{ab} \sigma_b &= 0
\end{aligned}
$$

Forward Algorithm

INPUT 1: Quiver $\rightarrow d_{G\times E} \rightarrow (Q_D)_{(G-2)\times c} = \ker(C)_{(G-2)\times G} \cdot \tilde{Q}_{G\times c}\ ;$

with $d_{G\times E} := \tilde{Q}_{G\times c} \cdot (P^T)_{c\times E}$

INPUT 2: CS Levels $\rightarrow C_{2\times G}$ $\nearrow$

$\nearrow$

INPUT 3: Superpotential $\rightarrow P_{E\times c} \rightarrow (Q_F)_{(c-G-2)\times c} = [\ker P]^t\ ;$

$\downarrow$

$$(Q_t)_{(c-4)\times c} = \begin{pmatrix} (Q_D)_{(G-2)\times c} \\ (Q_F)_{(c-G-2)\times c} \end{pmatrix} \rightarrow$$

OUTPUT: $(G_t)_{4\times c} = [\mathrm{Ker}(Q_t)]^t$

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\end{aligned}
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$$|DX_{ab}|^2$$

$$A_3 X - X A_3$$

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- First set of equations - F terms - Master Space
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- Third set - a new ingredient in 2+1d

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- Toric, singular non-compact CY cone of dim G+2

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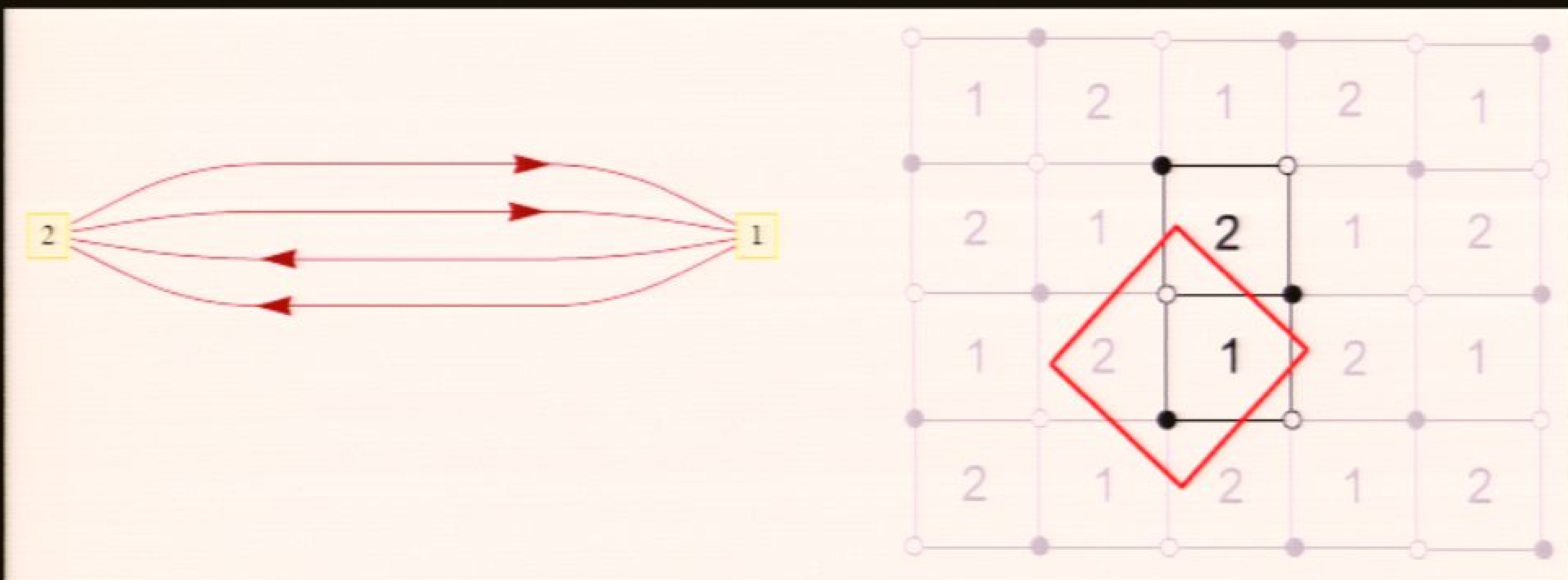

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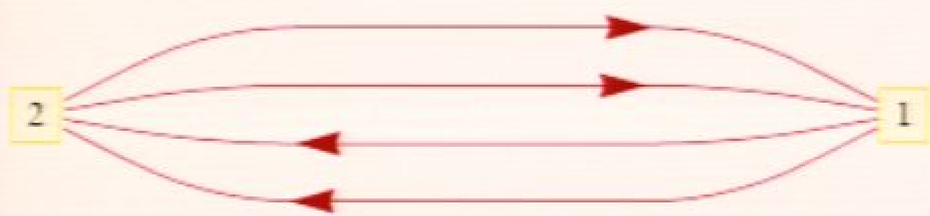

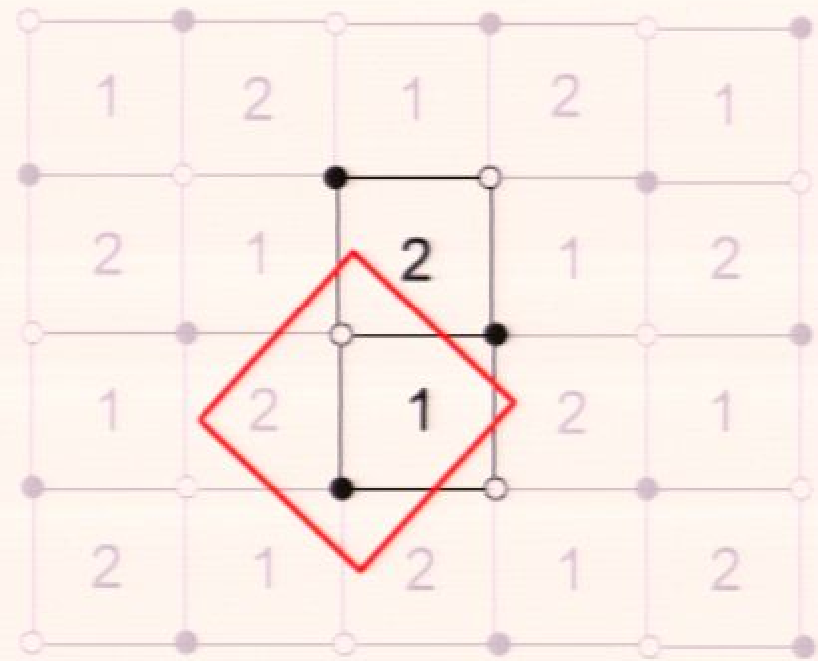

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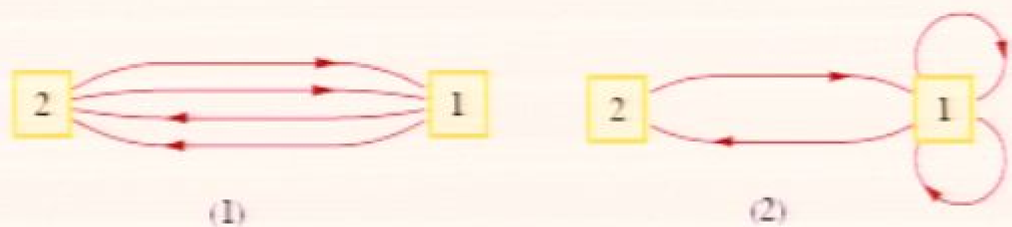
4 fields in the quiver



$$W_{(1)} = \mathrm{Tr}(X^1_{12}X^1_{21}X^2_{12}X^2_{21} - X^1_{12}X^2_{21}X^2_{12}X^1_{21}); \qquad W_{(2)} = \mathrm{Tr}(X_{12}[\phi^1_2, \phi^2_2]X_{21})$$

Figure 1: The quivers with 4 fields and 2 nodes. There are 2 solutions and the 2-term superpotentials are also given. The moduli space in both cases is just the trivial CY 4-fold $\mathbb{C}^4$.

1 hexagon; 1 double edge, G=2

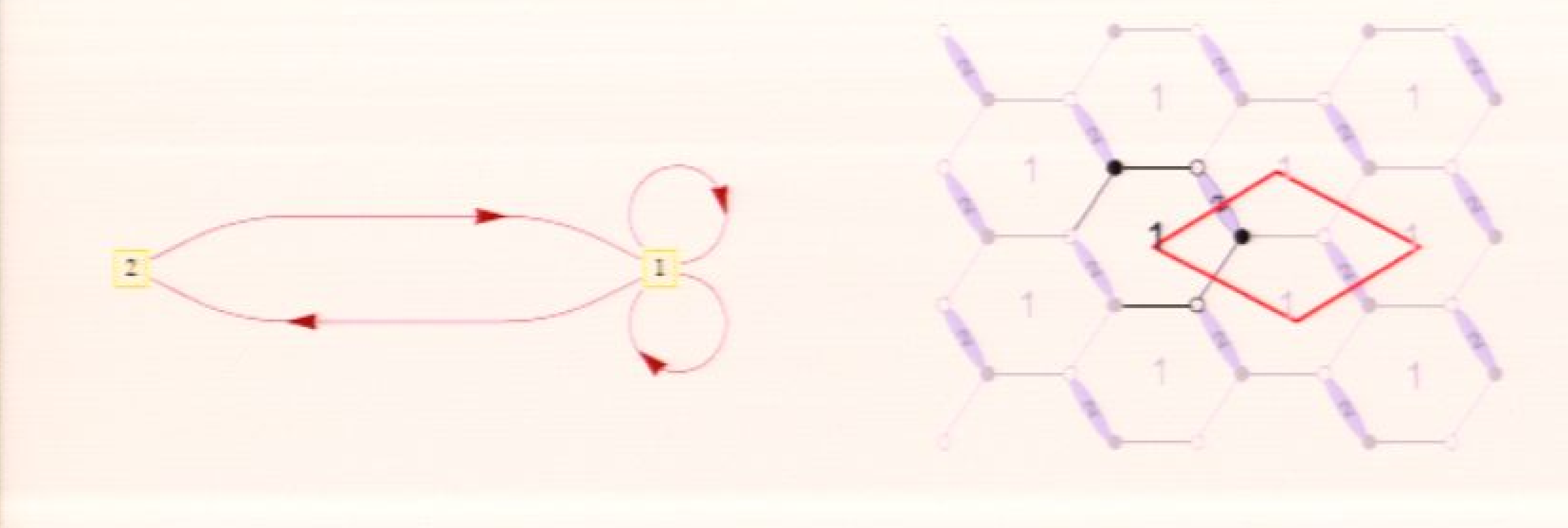
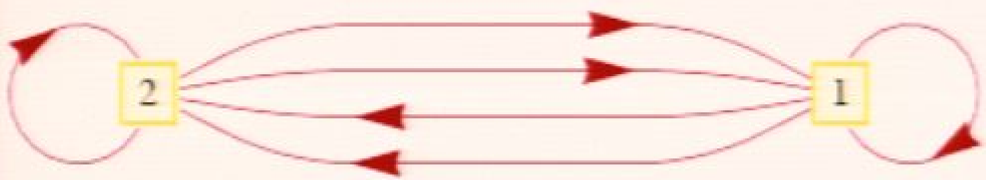
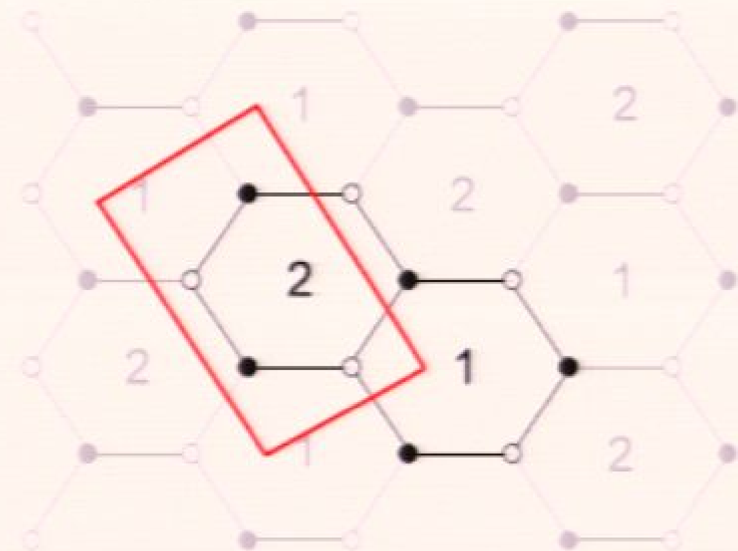


Figure 3: (i) Quiver diagram for phase 2 of the $\mathbb{C}^4$ theory. (ii) Tiling for phase 2 of the $\mathbb{C}^4$ theory.

2 hexagon tiling; (1-,1)
Conifold ($\mathscr{C}$) x $\mathbb{C}$





$$W = \phi_1(X^1_{12}X^2_{21} - X^2_{12}X^1_{21}) + \phi_2(X^1_{21}X^2_{12} - X^2_{21}X^1_{12})$$

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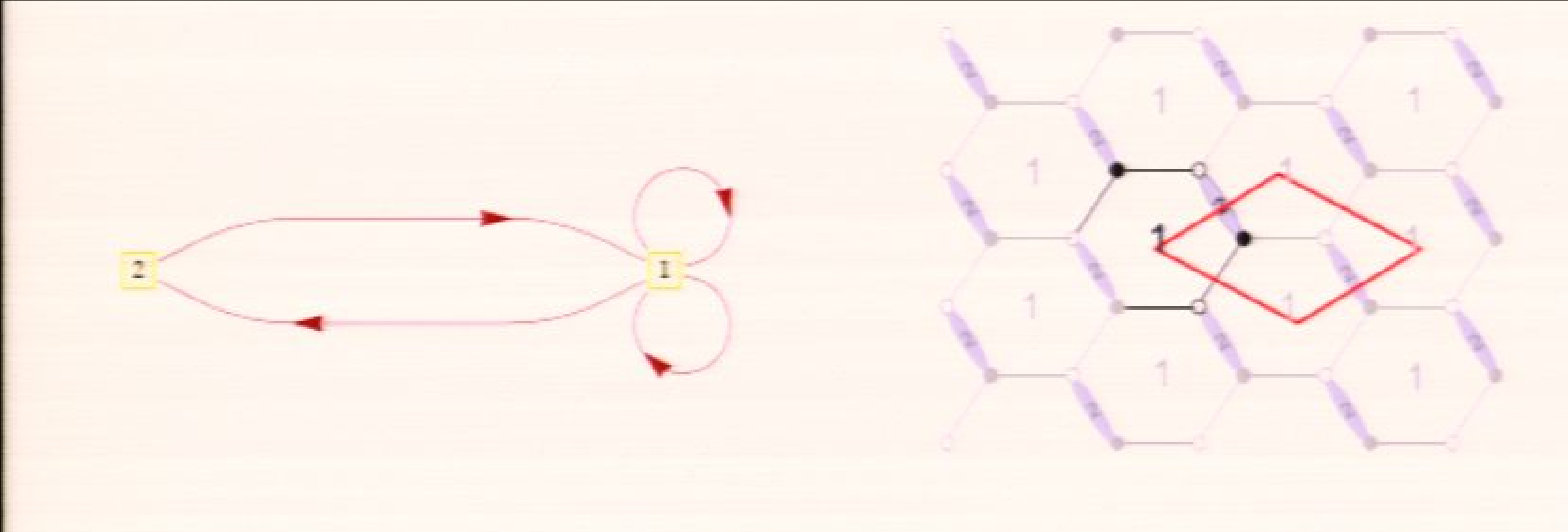

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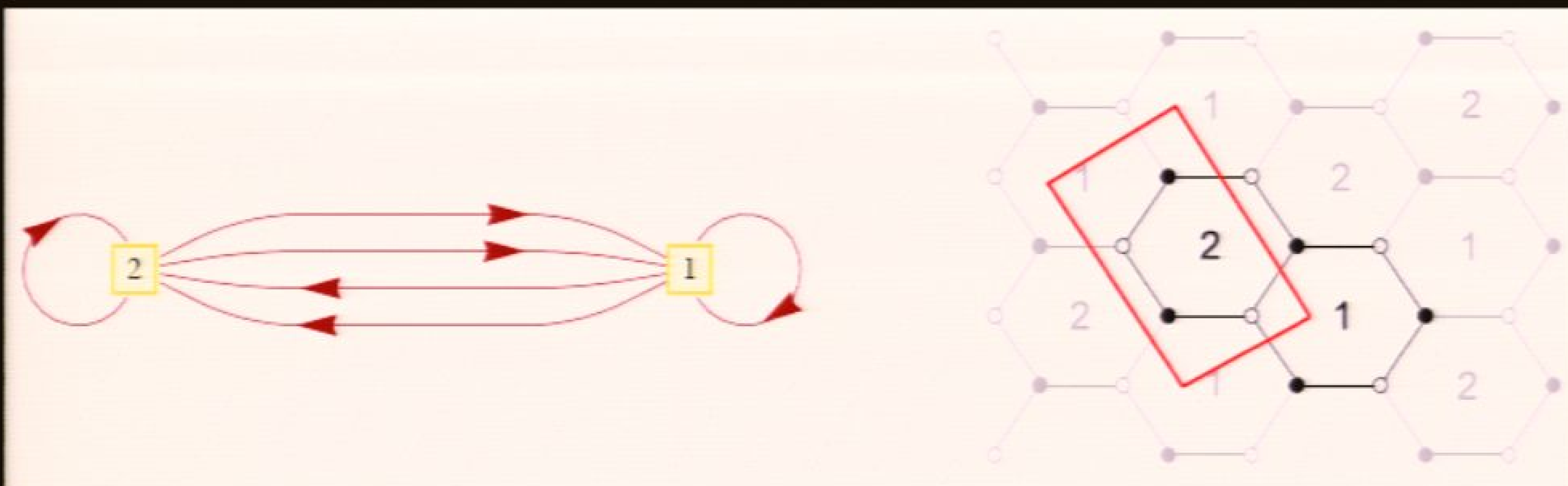

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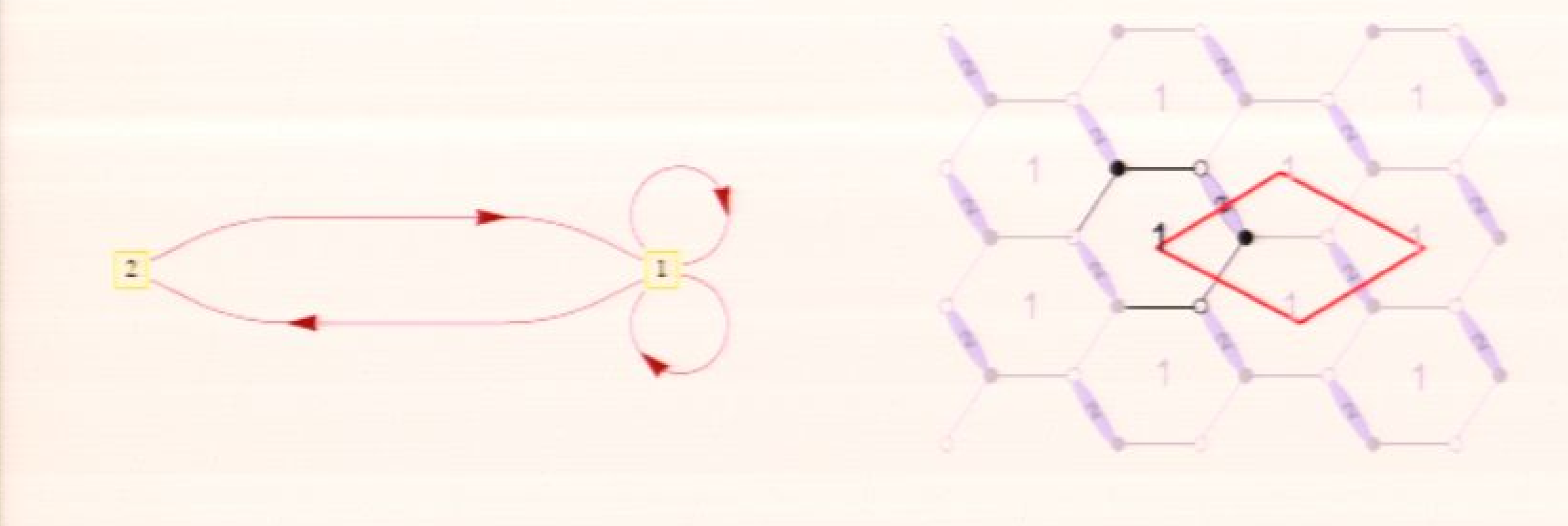

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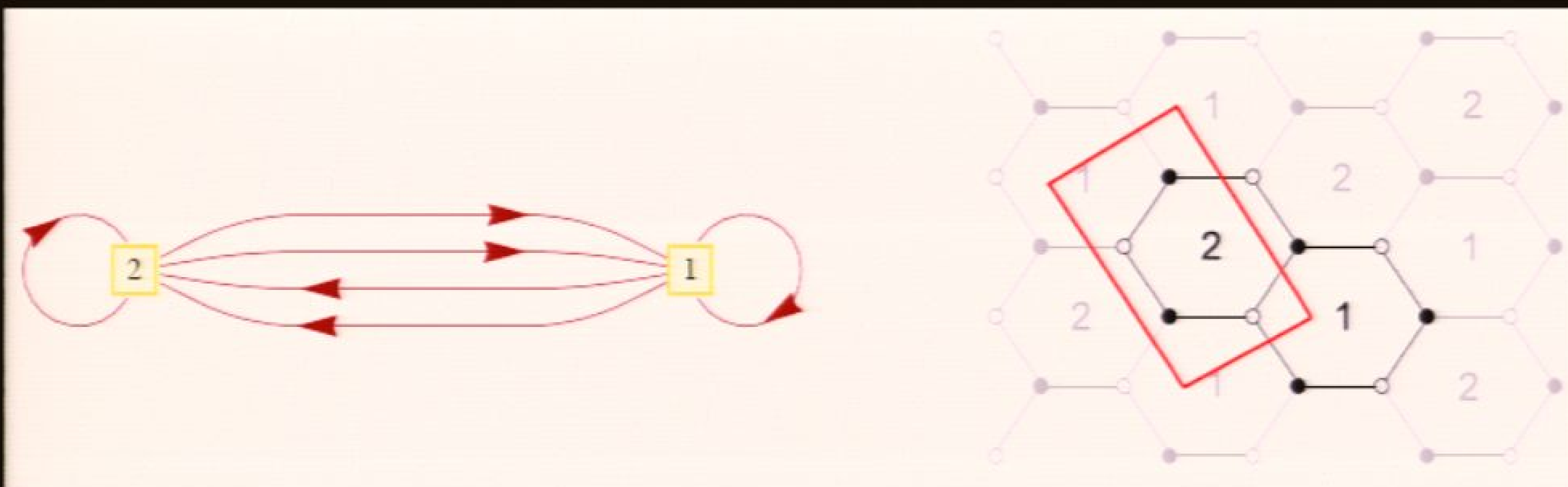

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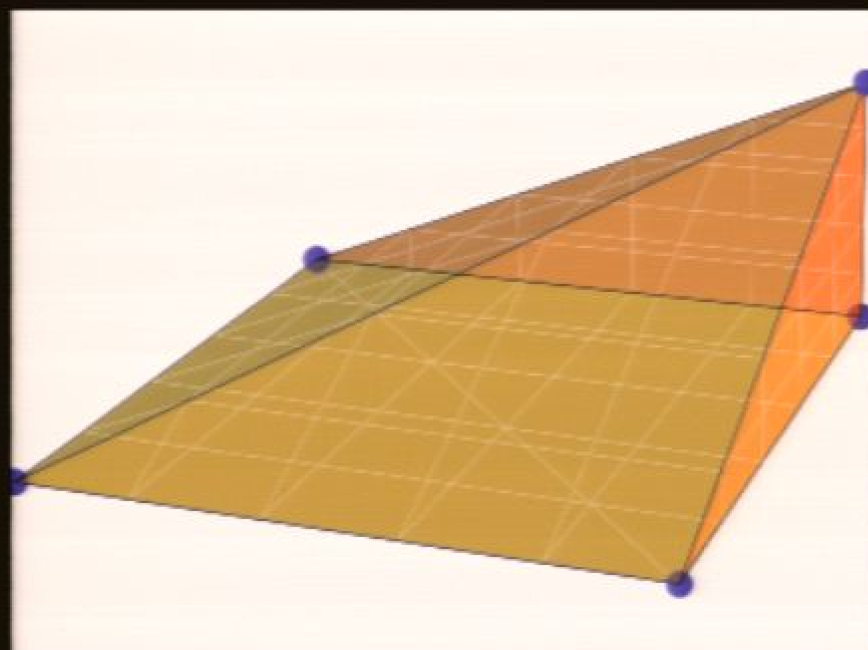
- Master space - 2+1d mesonic moduli space
- Non trivial scaling dimensions
- 1/2 for ϕ's, 3/4 for X's
- Non-trivial SCFT in the IR
- a test of AdS/CFT



Toric Diagram $\mathscr{C} \times \mathbb{C}$

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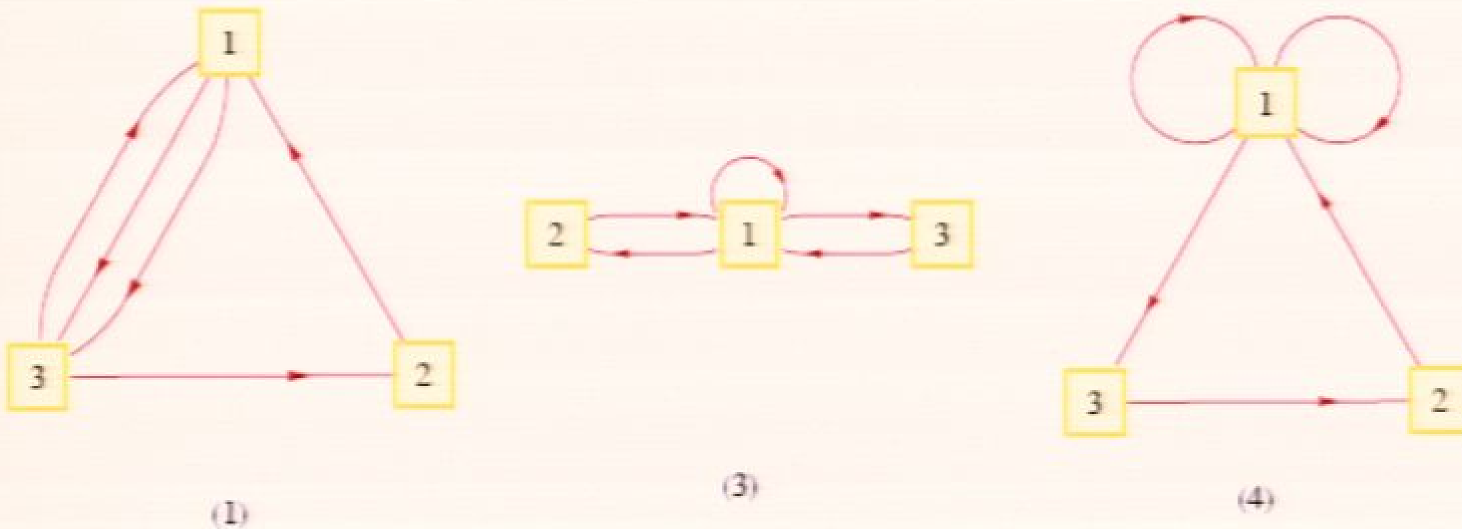
- Divide by complexified gauge group
- Moduli space: toric singular CY4 cone

5 fields in the Quiver Master space - $\mathbb{C}^5$



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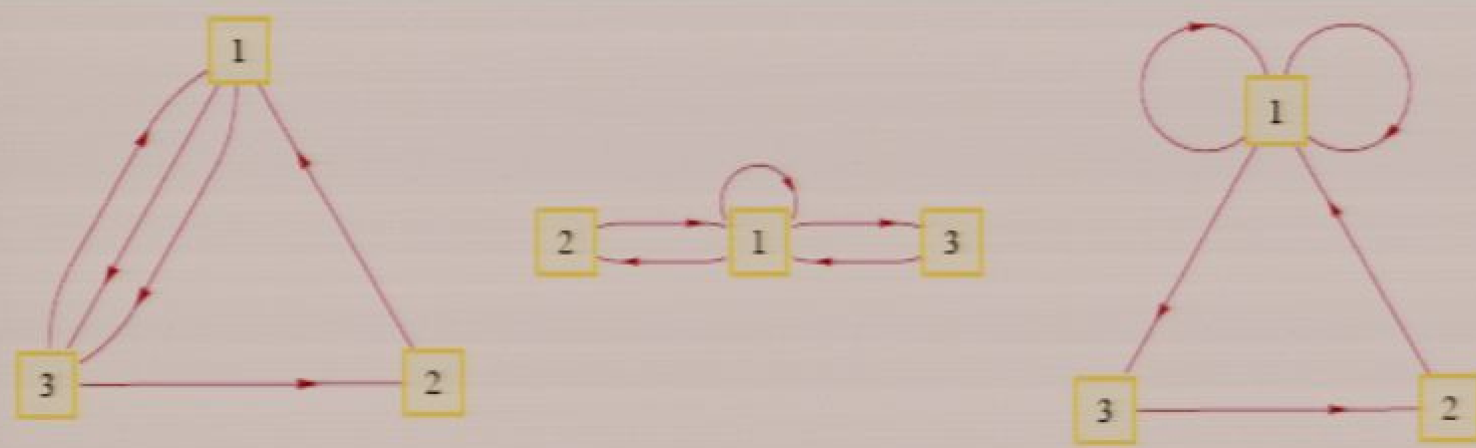
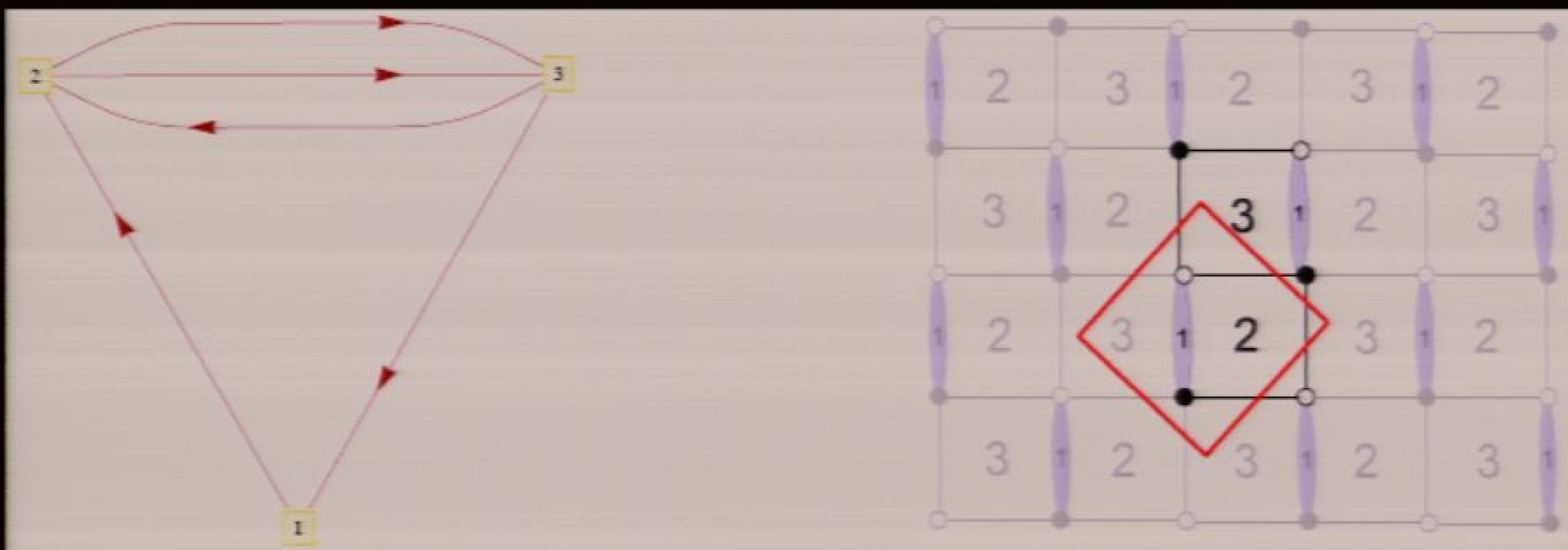


(1) (3) (4)

$$W_{(1)} = \mathrm{Tr}(X_{21}X_{13}^1X_{31}X_{13}^2X_{32} - X_{21}X_{13}^2X_{31}X_{13}^1X_{32})\ ;$$
$$W_{(3)} = \mathrm{Tr}(X_{21}\phi_1X_{13}X_{31}X_{12} - X_{21}X_{13}X_{31}\phi_1X_{12})\ ;$$
$$W_{(4)} = \mathrm{Tr}(X_{21}[\phi_1^1, \phi_1^2]X_{13}X_{32}).$$

Chessboard tiling; 1 double edge; (1,-1,0)



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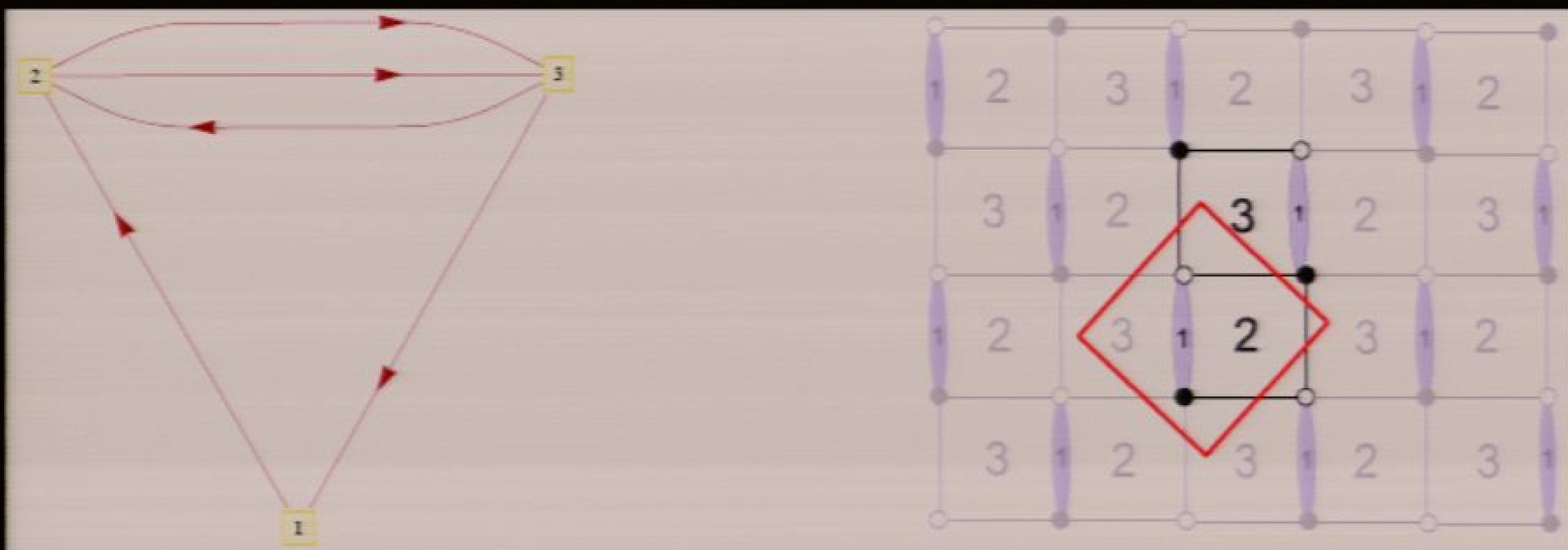
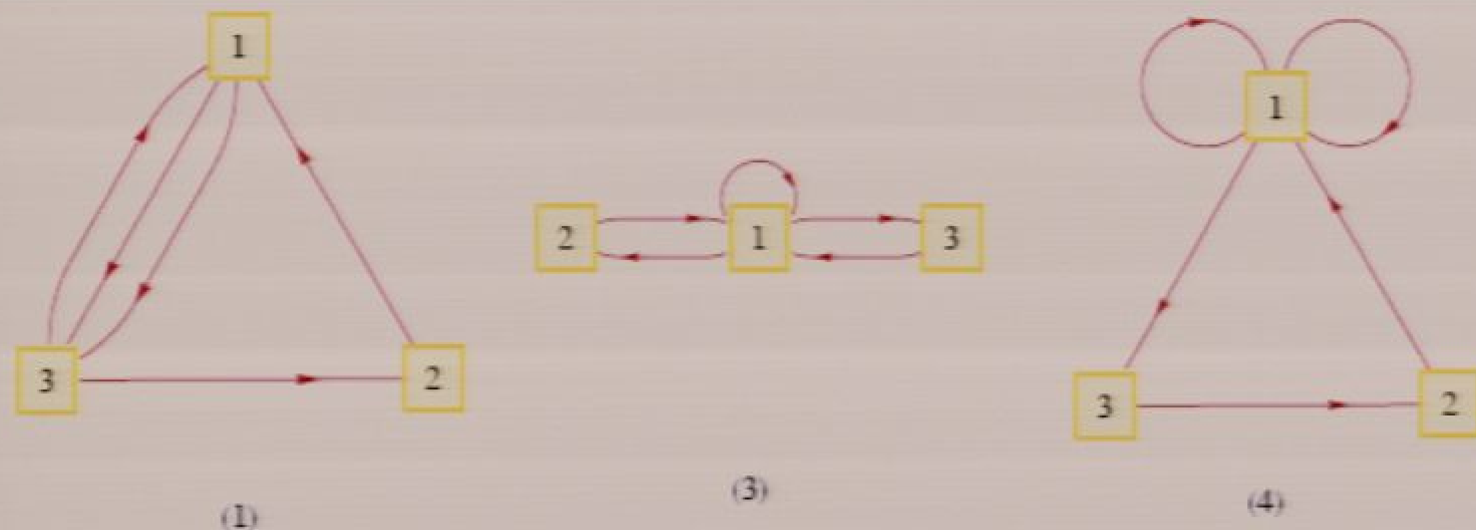


5 fields in the Quiver Master space - $\mathbb{C}^5$



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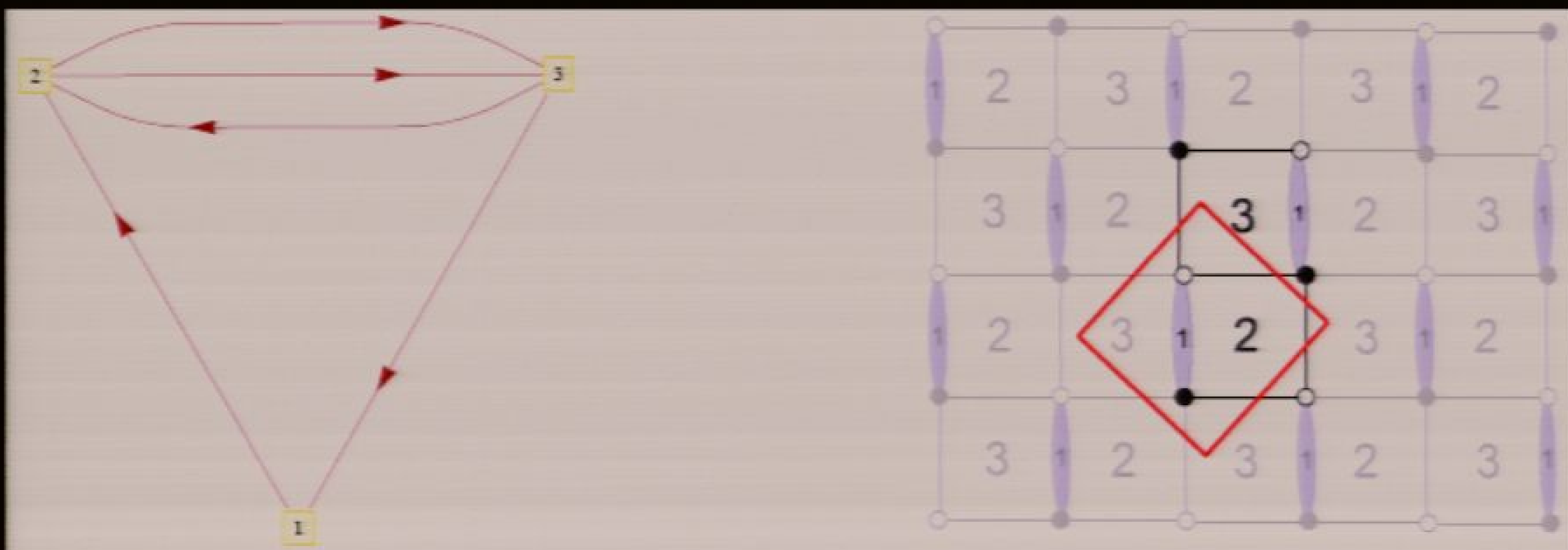

Chessboard tiling; 1 double edge; (1,-1,0)

- mesonic moduli space is conifold x $\mathbb{C}$

Chessboard tiling; 1 double edge; (1,-1,0)

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- mesonic moduli space is conifold x $\mathbb{C}$
- 1 dimensional baryonic moduli space

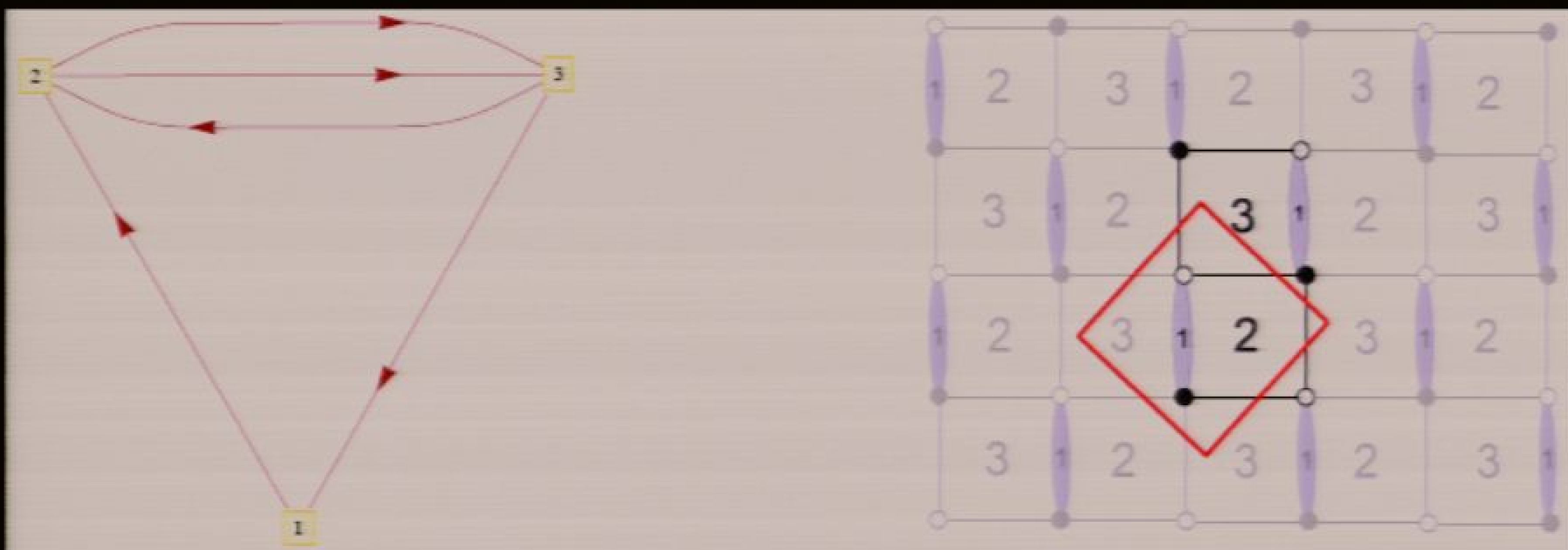
Chessboard tiling; 1 double edge; (1,-1,0)

- mesonic moduli space is conifold x $\mathbb{C}$
- 1 dimensional baryonic moduli space
- Combined mesonic baryonic space - $\mathbb{C}^5$

Chessboard tiling; 1 double edge; (1,-1,0)

- mesonic moduli space is conifold x $\mathbb{C}$
- 1 dimensional baryonic moduli space
- Combined mesonic baryonic space - $\mathbb{C}^5$
- Scaling dimensions 1/2 for X_{12}, 3/8 other

Chessboard tiling; 1 double edge; (1,-1,0)



Chessboard tiling; 1 double edge; (1,-1,0)

- mesonic moduli space is conifold x $\mathbb{C}$
- 1 dimensional baryonic moduli space
- Combined mesonic baryonic space - $\mathbb{C}^5$

Chessboard tiling; 1 double edge; (1,-1,0)

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- Combined mesonic baryonic space - $\mathbb{C}^5$
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Toric Duality
conifold x $\mathbb{C}$

- Two phases
- one with 2 tiles | one with 3 tiles

Toric Duality conifold x $\mathbb{C}$

- Two phases
- one with 2 tiles | one with 3 tiles
- Master space: mesonic | mesonic baryonic

Toric Duality
conifold x $\mathbb{C}$

- Two phases
- one with 2 tiles | one with 3 tiles
- Master space: mesonic | mesonic baryonic
- mesonic generators: linear | bi-linear

Global symmetry conifold x $\mathbb{C}$

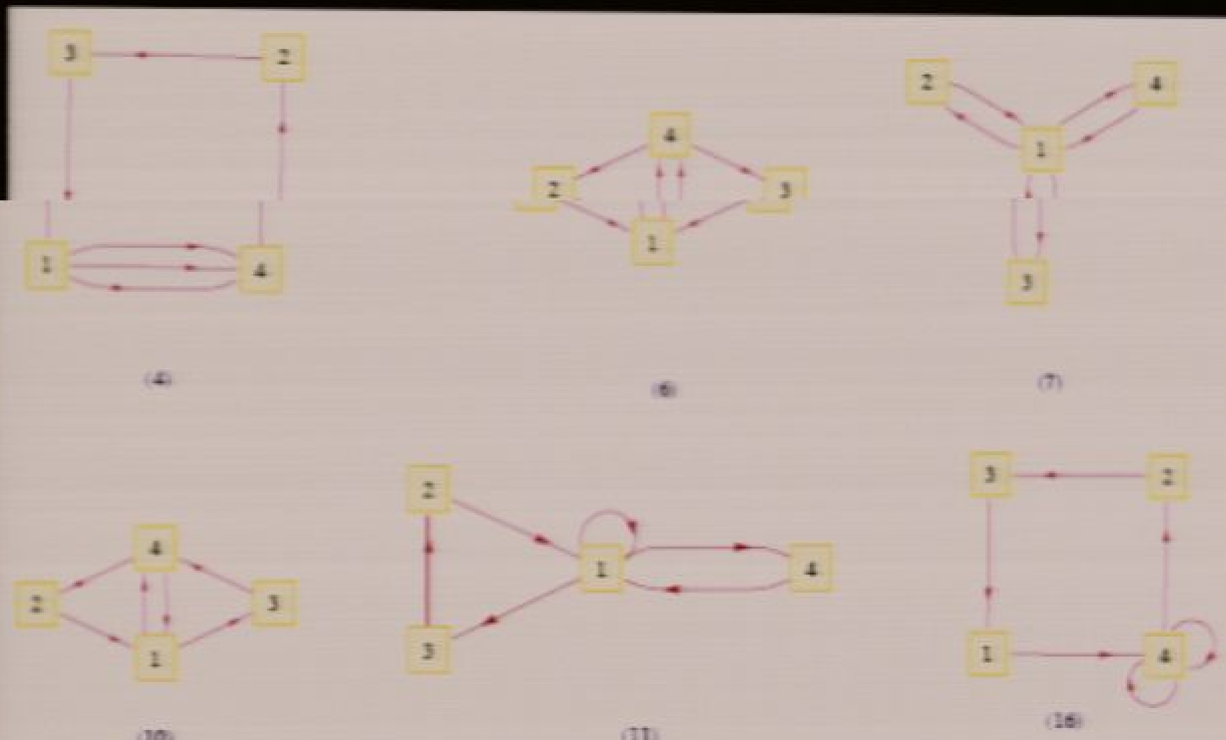
Hilbert Series
conifold x $\mathbb{C}$

Hilbert Series
conifold x $\mathbb{C}$

$$\frac{1}{(1-t_1x_1b)\left(1-\frac{t_1x_2}{b}\right)\left(1-\frac{t_1b}{x_1}\right)\left(1-\frac{t_1}{x_2b}\right)(1-t_2)}$$

Classification of 2+1d theories?

6 fields in the Quiver



$$W_{(4)} = \mathrm{Tr}(X_{31}X^1_{14}X_{41}X^2_{14}X_{42}X_{23} - X_{31}X^2_{14}X_{41}X^1_{14}X_{42}X_{23})\ ;$$
$$W_{(6)} = \mathrm{Tr}(X_{42}X_{21}(X^1_{14}X_{43}X_{31}X^2_{14} - X^2_{14}X_{43}X_{31}X^1_{14}))\ ;$$
$$W_{(7)} = \mathrm{Tr}(X_{12}X_{21}(X_{14}X_{41}X_{13}X_{31} - X_{13}X_{31}X_{14}X_{41}))\ ;$$
$$W_{(10)} = \mathrm{Tr}(X_{42}X_{21}X_{14}X_{41}X_{13}X_{34} - X_{42}X_{21}X_{13}X_{34}X_{41}X_{14})\ ;$$
$$W_{(11)} = \mathrm{Tr}(X_{32}X_{21}\phi_1X_{14}X_{41}X_{13} - X_{32}X_{21}X_{14}X_{41}\phi_1X_{13})\ ;$$
$$W_{(16)} = \mathrm{Tr}(X_{42}X_{23}X_{31}X_{14}[\phi^1_4, \phi^2_4])$$

G=2, E=4, Model I

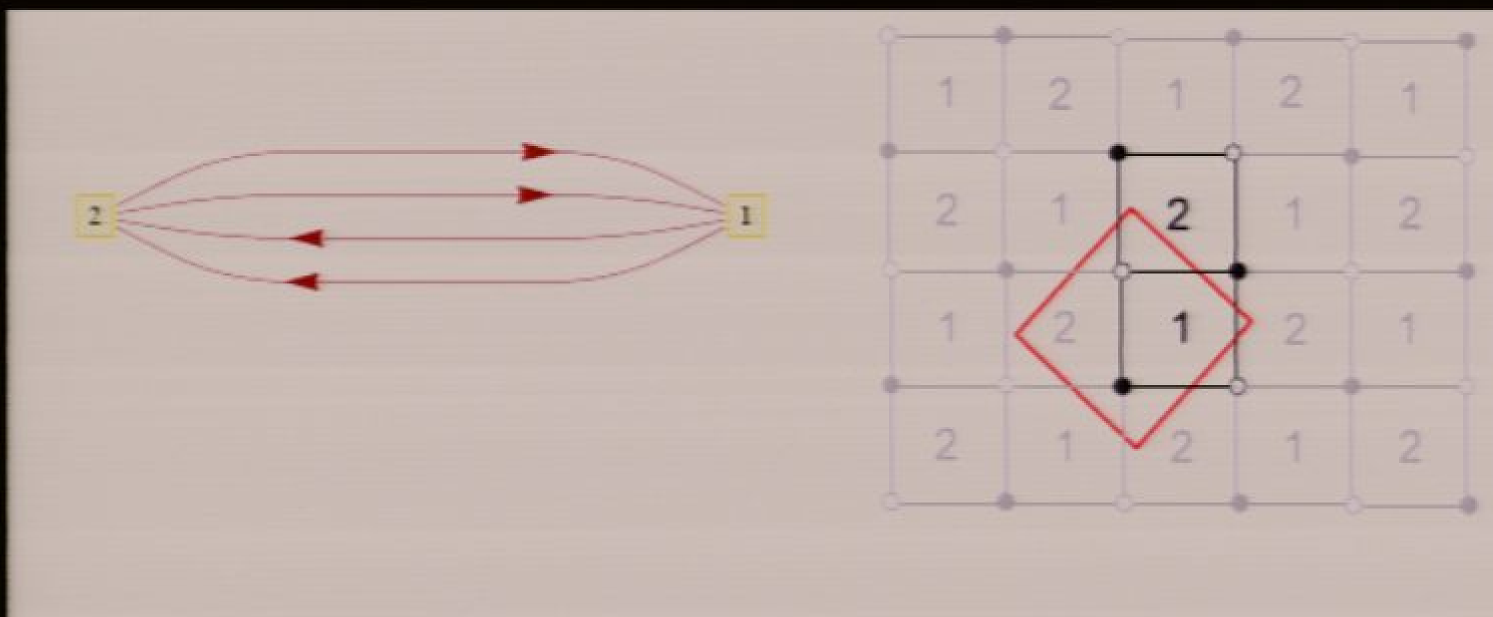


Figure 1: (i) Quiver diagram for the ABJM theory. (ii) Tiling for the ABJM theory.

G=4, E=6, Model IV

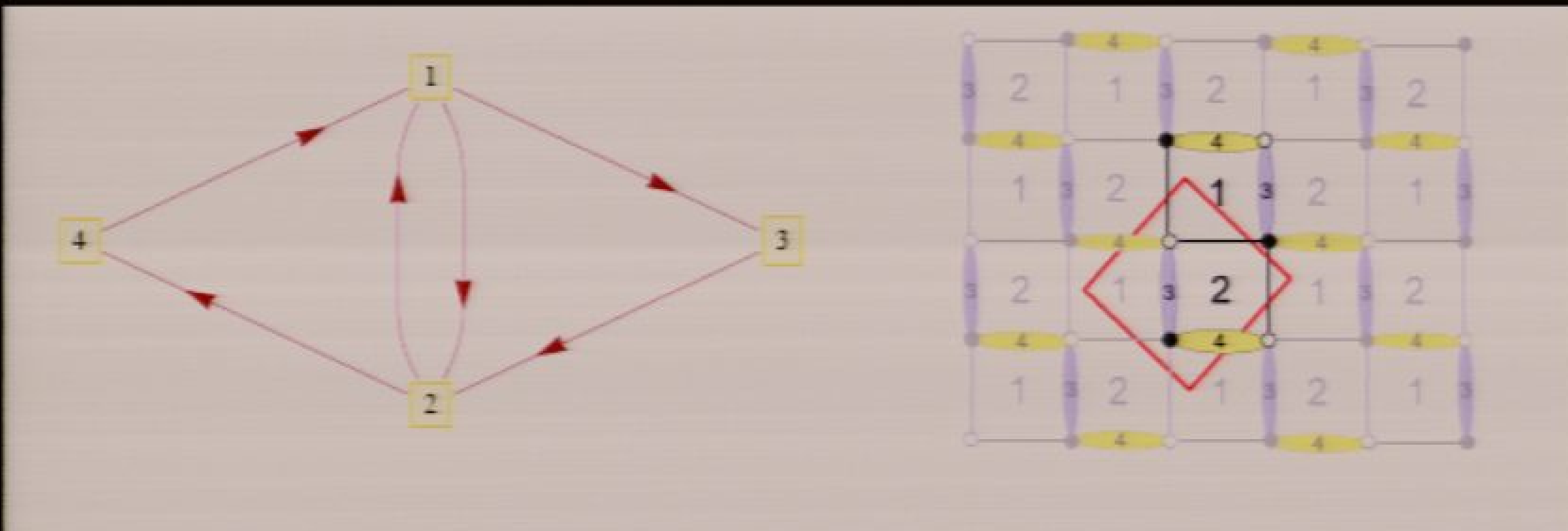


Figure 11: (i) Quiver diagram for phase 2 of the D_3 theory. (ii) Tiling for phase 2 of the D_3 theory.

Counting Quivers
1 Hexagon

$$
\begin{aligned}
f_1(t) &= \frac{1}{(1-t)(1-t^2)(1-t^3)} \\
&= 1 + t + 2t^2 + 3t^3 + \ldots
\end{aligned}
$$

Counting Quivers
Chessboard Tiling

$$\begin{aligned} f_2(t) &= \frac{1-t^6}{(1-t)(1-t^2)^2(1-t^3)(1-t^4)} \\ &= 1 + t + 3t^2 + 4t^3 + 8t^4 + \ldots \end{aligned}$$

Summary

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- Infinitely many quivers

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- Each represents a lattice of SCFT's in 2+1d

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