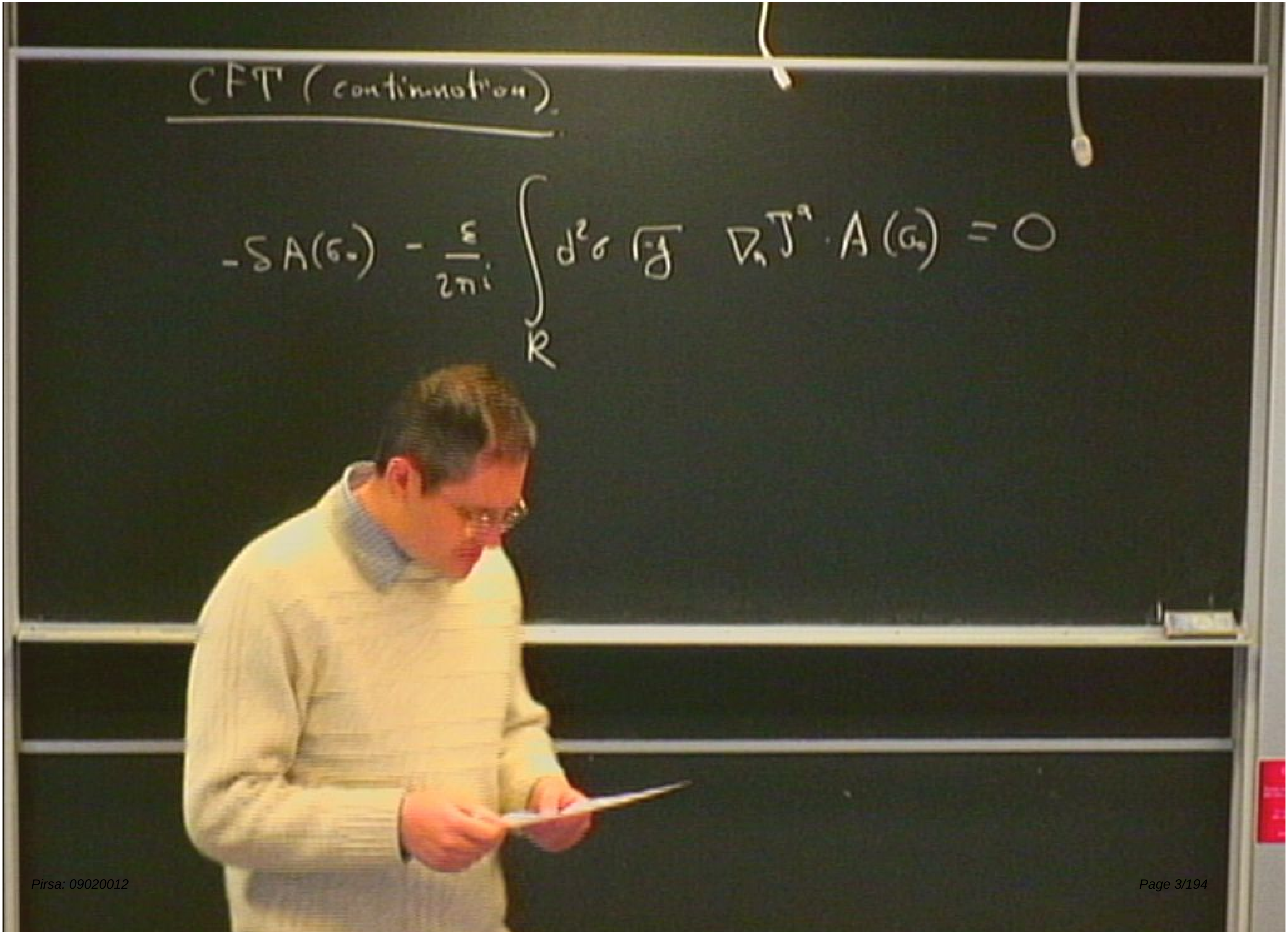
Title: Introduction to the Bosonic String

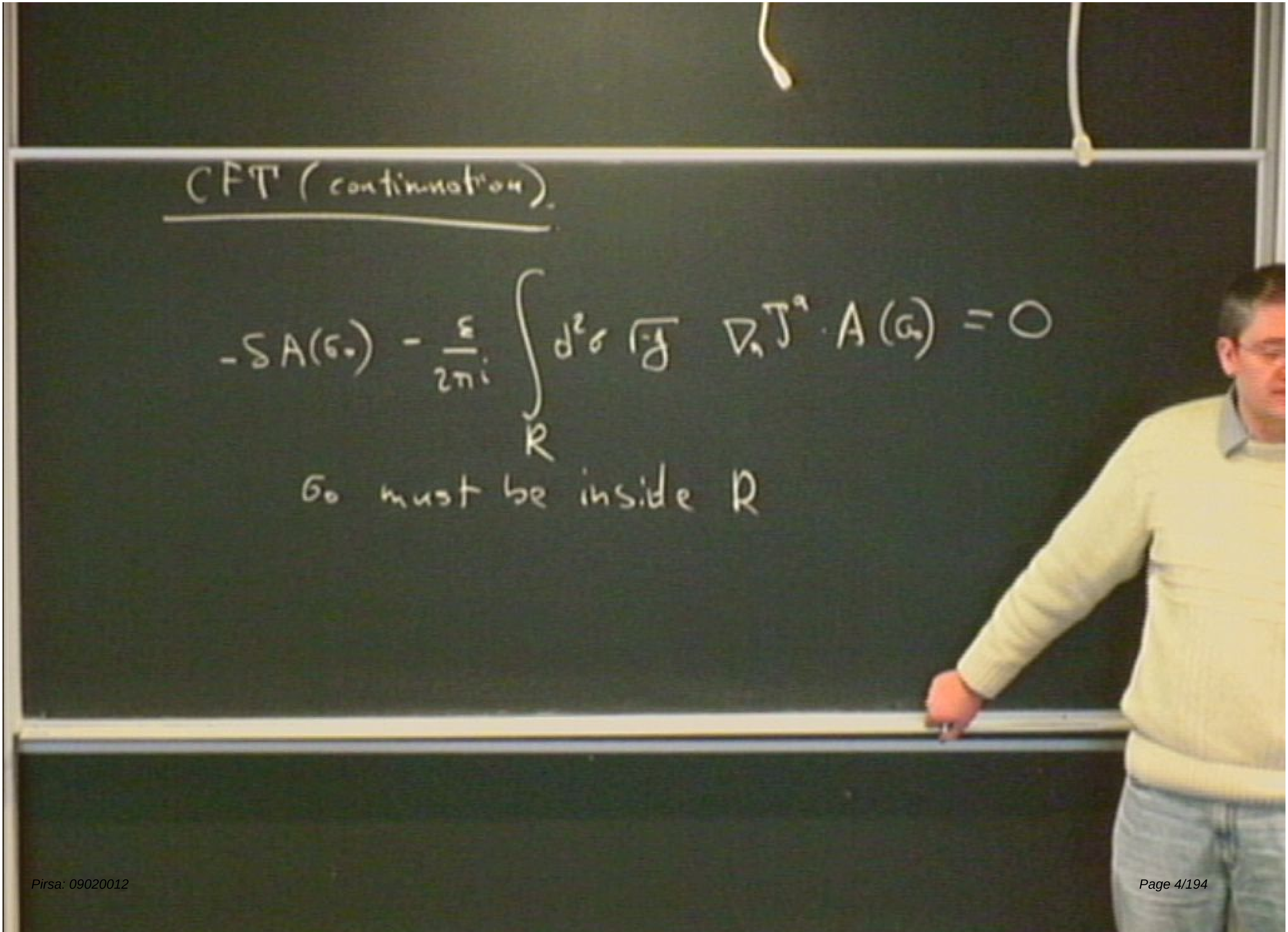
Date: Feb 20, 2009 10:00 AM

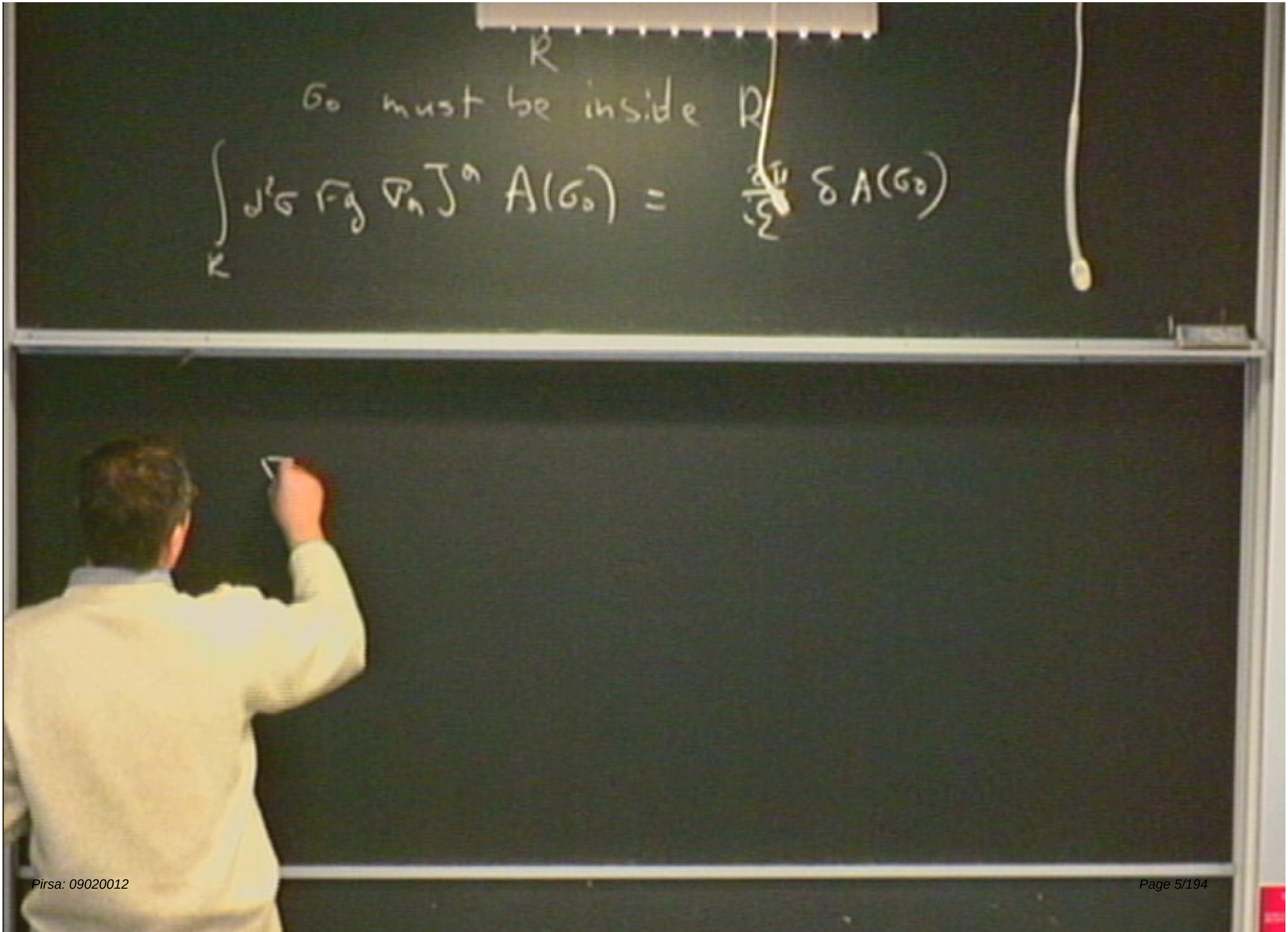
URL: http://pirsa.org/09020012

Abstract: This course provides a thorough introduction to the bosonic string based on the Polyakov path integral and conformal field theory. We introduce central ideas of string theory, the tools of conformal field theory, the Polyakov path integral, and the covariant quantization of the string. We discuss string interactions and cover the tree-level and one loop amplitudes. More advanced topics such as T-duality and D-branes will be taught as part of the course. The course is geared for M.Sc. and Ph.D. students enrolled in Collaborative Ph.D. Program in Theoretical Physics. Required previous course work: Quantum Field Theory (AM516 or equivalent). The course evaluation will be based on regular problem sets that will be handed in during the term. The primary text is the book: 'String theory. Vol. 1: An introduction to the bosonic string. J. Polchinski (Santa Barbara, KITP) . 1998. 402pp. Cambridge, UK: Univ. Pr. (1998) 402 p.' All interested students should contact Alex Buchel at abuchel@uwo.ca as soon as possible.

CFT (continuation).


CFT (continuation).
$-\delta A(\sigma_w) - \frac{\varepsilon}{2\pi i}\int_R d^2\sigma \sqrt{-g}\, \nabla_a J^a \cdot A(\sigma_w) = 0$


CFT (continuation).
$-\delta A(\sigma_0) - \frac{\epsilon}{2\pi i} \int_R d^2\sigma \sqrt{-g}\; \nabla_a J^a \cdot A(\sigma_0) = 0$
σ_0 must be inside R


R
σ0 must be inside R
$\int_R d^2\sigma \sqrt{g}\, \nabla_a J^a A(\sigma_0) =$
$\delta A(\sigma_0)$

R

σ_0 must be inside R

$$\int_R d^2\sigma \sqrt{-g}\, \nabla_a J^a A(\sigma_0) = \frac{2\pi}{i\xi}\, \delta A(\sigma_0)$$

$$\nabla_a J^a A(\sigma_0) = \frac{2\pi}{i\xi} \frac{1}{\sqrt{-g}}\, \delta^2(\sigma - \sigma_0)\, \delta A(\sigma).$$

R

σ_0 must be inside R

$$\int_R d^2\sigma \sqrt{-g}\, \underset{\uparrow \atop \sigma}{\nabla_a}\left[J^a(\sigma) A(\sigma_0)\right] = \frac{2\pi}{i\epsilon}\, \delta A(\sigma_0)$$

$$\nabla_a J^a A(\sigma_0) = \frac{2\pi}{i\epsilon} \frac{1}{\sqrt{-g}} \delta^2(\sigma - \sigma_0)\, \delta A(\sigma).$$

R

σ_0 must be inside R

$$\int_R d^2\sigma \sqrt{-g}\, \underset{\sigma}{\nabla_a}\left[J^a(\sigma) A(\sigma_0)\right] = \frac{2\pi}{i\xi}\, \delta A(\sigma_0)$$

$$\nabla_a J^a A(\sigma_0) = \frac{2\pi}{i\xi} \frac{1}{\sqrt{-g}} \delta^2(\sigma-\sigma_0)\ \delta A(\sigma).$$

$$\int_{\partial R} n_a J^a \cdot A(\sigma_0) = \frac{2\pi}{i\xi} \delta A(\sigma_0)$$

$$-\delta A(\sigma_0) + \frac{i\varepsilon}{2\pi}\int_R d^2\sigma \sqrt{-g}\, \nabla_a J^a\, A(\sigma_0) = 0$$

σ_0 must be inside R

$$\int_R d^2\sigma \sqrt{-g}\, \underset{\sigma}{\nabla_a}\left[J^a(\sigma) A(\sigma_0) \right] = \frac{2\pi}{i\varepsilon}\,\delta A(\sigma_0)$$

$$\left\{ \langle \nabla_a J^a \rangle = 0 \right.$$

$$\int_{\partial R} n_a J^a\, A(\sigma_0) = \frac{2\pi}{i\varepsilon}\,\delta A(\sigma_0)$$

$$\oint_{\partial R} \left(i_z \, dz - i_{\bar z} \, d\bar z \right) A(z_0, \bar z_0) = \frac{2\pi}{\epsilon} \, \delta A(z_0, \bar z_0)$$

$$\oint_{\partial R} \left(j_z \, dz - j_{\bar z} \, d\bar z \right) A(z_0, \bar z_0) = \frac{2\pi}{\epsilon} \delta A(z_0, \bar z_0)$$

$\Rightarrow$ LHS simplifies when

j_z is holomorphic $\bar\partial J_z = 0$

$$\oint_{\partial R} \left(j_z \, dz - j_{\bar z} \, d\bar z \right) A(z_0, \bar z_0) = \frac{2\pi}{\varepsilon} \delta A(z_0, \bar z_0)$$

$\Rightarrow$ LHS simplifies when

j_z – is holomorphic $\bar\partial j_z = 0$

$j_{\bar z}$ – is antiholomorphic $\partial j_{\bar z} = 0$

$$\oint_{\partial R} \left(j_z \, dz - j_{\bar z} \, d\bar z \right) A(z_0, \bar z_0) = \frac{2\pi}{\varepsilon} \, \delta A(z_0, \bar z_0)$$

$\Rightarrow$ LHS simplifies when

$2\pi i \left[\mathrm{Res} \; j_z A(z_0, \bar z_0) \right]$

j_z — is holomorphic $\bar\partial J_z = 0$

$j_{\bar z}$ — is antiholomorphic $\partial J_{\bar z} = 0$

$$\oint_{\partial R} \left(j_z \, dz - j_{\bar z} \, d\bar z \right) A(z_0, \bar z_0) = \frac{2\pi}{\epsilon} \, \delta A(z_0, \bar z_0)$$

$\Rightarrow$ LHS simplifies when

j_z is holomorphic $\bar\partial J_z = 0$

$j_{\bar z}$ — is antiholomorphic $\partial J_{\bar z} = 0$

$$2\pi i \left[\operatorname{Res} j_z A(z_0, \bar z_0) \right] + 2\pi i \left[\operatorname{Res} j_{\bar z} A \right]$$

$$\oint_{\partial R} \left(j_z\, dz - j_{\bar z}\, d\bar z \right) A(z_0, \bar z_0) = \frac{2\pi}{\varepsilon}\, \delta A(z_0, \bar z_0)$$

$\Rightarrow$ LHS simplifies when

j_z — is holomorphic $\bar\partial j_z = 0$

$j_{\bar z}$ — is antiholomorphic $\partial j_{\bar z} = 0$

$$2\pi i \left[\mathrm{Res}\; j_z A(z_0, \bar z_0) \right] + 2\pi i \left[\mathrm{Res}\; j_{\bar z} A \right] = \frac{2\pi}{\varepsilon}\, \delta A(z_0, \bar z_0)$$

j_z — is holomorphic $\bar\partial j_z = 0$

$j_{\bar z}$ — is antiholomorphic $\partial j_{\bar z} = 0$

$+ 2\pi i \left[\mathrm{Res}\, j_z A \right]$

$= \frac{2\pi}{\varepsilon} \delta A(z_0, \bar z_0)$

$f(z)$

$\mathrm{Res}\, f(z) \equiv \mathrm{coeff}$

j_z — is holomorphic $\bar\partial j_z = 0$

$j_{\bar z}$ — is antiholomorphic $\partial j_{\bar z} = 0$

$+ 2\pi i \left[\mathrm{Res}\, j_z A \right]$

$= \frac{2\pi}{\varepsilon} \delta A(z_0, \bar z_0)$

$f(z)$

$\mathrm{Res}\, f(z) \equiv$ coeff of $\frac{1}{z}$

$$f(z) = \sum_{-\infty}^{+\infty} \frac{f_n}{z^n} \Rightarrow \mathrm{Res}\, f(z) = f_1$$

j_z — is holomorphic $\bar\partial j_z = 0$

$j_{\bar z}$ — is antiholomorphic $\partial j_{\bar z} = 0$

$+ 2\pi i \left[\mathrm{Res}\, j_{\bar z} A \right]$

$= \frac{2\pi}{\varepsilon} \delta A(z_0, \bar z_0)$

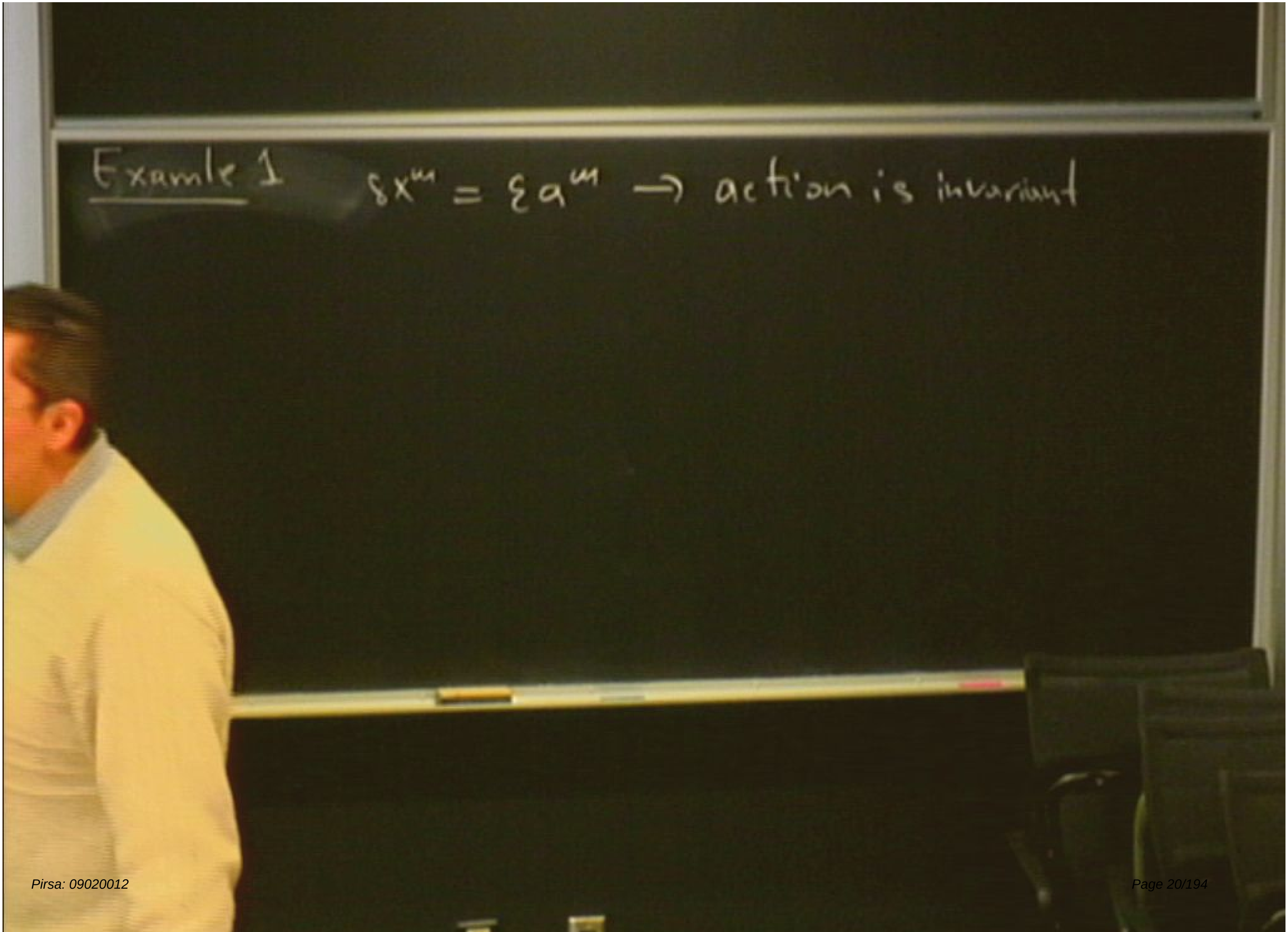
$f(z)$

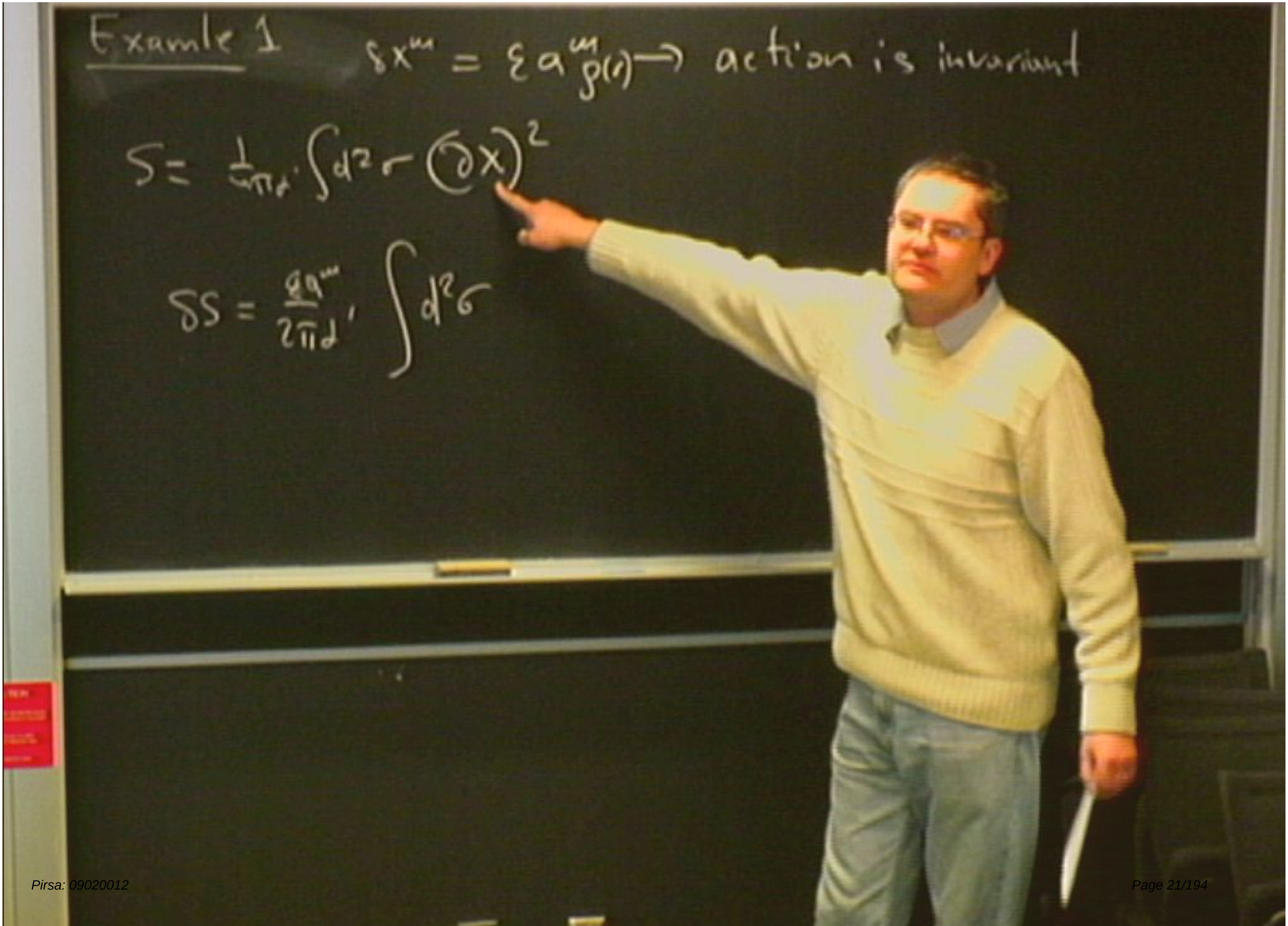
$\mathrm{Res}\, f(z) \equiv$ coeff of $\frac{1}{z}$

$$f(z) = \sum_{-\infty}^{+\infty} \frac{f_n}{z^n} \Rightarrow \mathrm{Res}\, f(z) = f_1$$

$$\mathrm{Res}_{z \to z_0} j_z A(z_0, \bar z_0) + \overline{\mathrm{Res}}_{\bar z \to \bar z_0} j_{\bar z} A(z_0, \bar z_0) = \frac{1}{i\varepsilon} \delta A(z_0, \bar z_0)$$

$$\bar\partial \partial X = 0.$$


Examle 1
$\delta x^\mu = \varepsilon a^\mu \rightarrow$ action is invariant


Examle 1
$\delta X^\mu = \varepsilon a^\mu \rho(\tau) \rightarrow$ action is invariant
$S = \frac{1}{4\pi\alpha'} \int d^2\sigma \, (\partial X)^2$
$\delta S = \frac{\varepsilon a^\mu}{2\pi\alpha'} \int d^2\sigma$

$$S = \frac{1}{4\pi\alpha'}\int d^2\sigma\, (\partial X)^2$$

$$\delta S = \frac{a\, a_\mu}{2\pi\alpha'}\int d^2\sigma\ \partial^a X^\mu\, \partial_a \rho$$

$$\delta S = \frac{\epsilon a_\mu}{2\pi\alpha'} \int d^2\sigma \; \partial^a X^\mu \, \partial_a \rho = -\frac{1}{2\pi} \int$$

$$[d\varphi'] \, e^{-S[\varphi']} = [d\varphi] \, e^{-S[\varphi]} \left[1 + \overbrace{\frac{i\epsilon}{2\pi} \int d^2\sigma \; j^a \nabla_a \rho}^{-\delta S} + O(\epsilon^2) \right]$$

$$\delta S = \frac{\varepsilon q_m}{2\pi\alpha'}\int d^2\sigma\ \partial^a X^m\ \partial_a \rho = -\frac{i\varepsilon}{2\pi}\int$$

$$[d\varphi']\, e^{-S[\varphi']} = [d\varphi]\, e^{-S[\varphi]} \left[1 + \frac{i\varepsilon}{2\pi} \underbrace{\int d^2\sigma\ j^a \nabla_a \rho}_{-\delta S} + O(\varepsilon^2) \right]$$

$$\frac{\varepsilon q_m}{2\pi\alpha'} = -\frac{i\varepsilon}{2\pi}$$

$$\delta S = \frac{\varepsilon a_m}{2\pi\alpha'} \int d^2\sigma \; \partial^a X^m \partial_a \rho$$

$$[d\varphi'] e^{-S[\varphi']} = [d\varphi] e^{-S[\varphi]} \left[1 + \frac{i\varepsilon}{2\pi} \int d^2\sigma \; j^a \nabla_a \rho + O(\varepsilon^2) \right]$$

$-\delta S$

$$\partial^a X \frac{\varepsilon a_m}{2\pi\alpha'} = \frac{-i\varepsilon}{2\pi} j^a$$

$$\delta S = \frac{\epsilon a_m}{2\pi\alpha'} \int d^2\sigma \; \partial^a X^m \; \partial_a \rho$$

$$[d\varphi'] \, e^{-S[\varphi']} = [d\varphi] \, e^{-S[\varphi]} \left[1 + \frac{i\epsilon}{2\pi} \int d^2\sigma \; j^a \partial_a \rho + O(\epsilon^2) \right]$$

$\overset{-\delta S}{\swarrow}$

$$\partial^a X_a \frac{\epsilon a_m}{2\pi\alpha'} = \frac{-i\epsilon}{2\pi} j^a \quad \Rightarrow \quad j^a_a = a_m j^m_a$$

$$\boxed{j_a^m = + \frac{i}{\alpha'} \partial_a X^m}$$

$$\delta S = \frac{\varepsilon a_m^{m}}{2\pi\alpha'} \int d^2\sigma \; \partial^a X^m \partial_a \rho$$

$$[d\varphi'] e^{-S[\varphi']} = [d\varphi] e^{-S[\varphi]} \left[1 + \frac{i\varepsilon}{2\pi} \int d^2\sigma \; j^a \partial_a \rho + O(\varepsilon^2) \right]$$

$-\delta S$

$$\partial^a X_{\mu} \frac{\varepsilon a_m}{2\pi\alpha'} = \frac{-i\varepsilon}{2\pi} J^a \Rightarrow J_a = a_m J^m_a$$

$$\boxed{J_a^m = + \frac{i}{\alpha'} \partial_a X^m} \Rightarrow \begin{array}{l} J_z^m = \frac{i}{\alpha'} \partial X^m \\ \tilde{J}_{\bar z}^m = \frac{i}{\alpha'} \bar\partial X^m \end{array}$$

$$\delta S = \frac{\epsilon a_m^m}{2\pi\alpha'} \int d^2\sigma \; \partial^a X^m \; \partial_a \rho$$

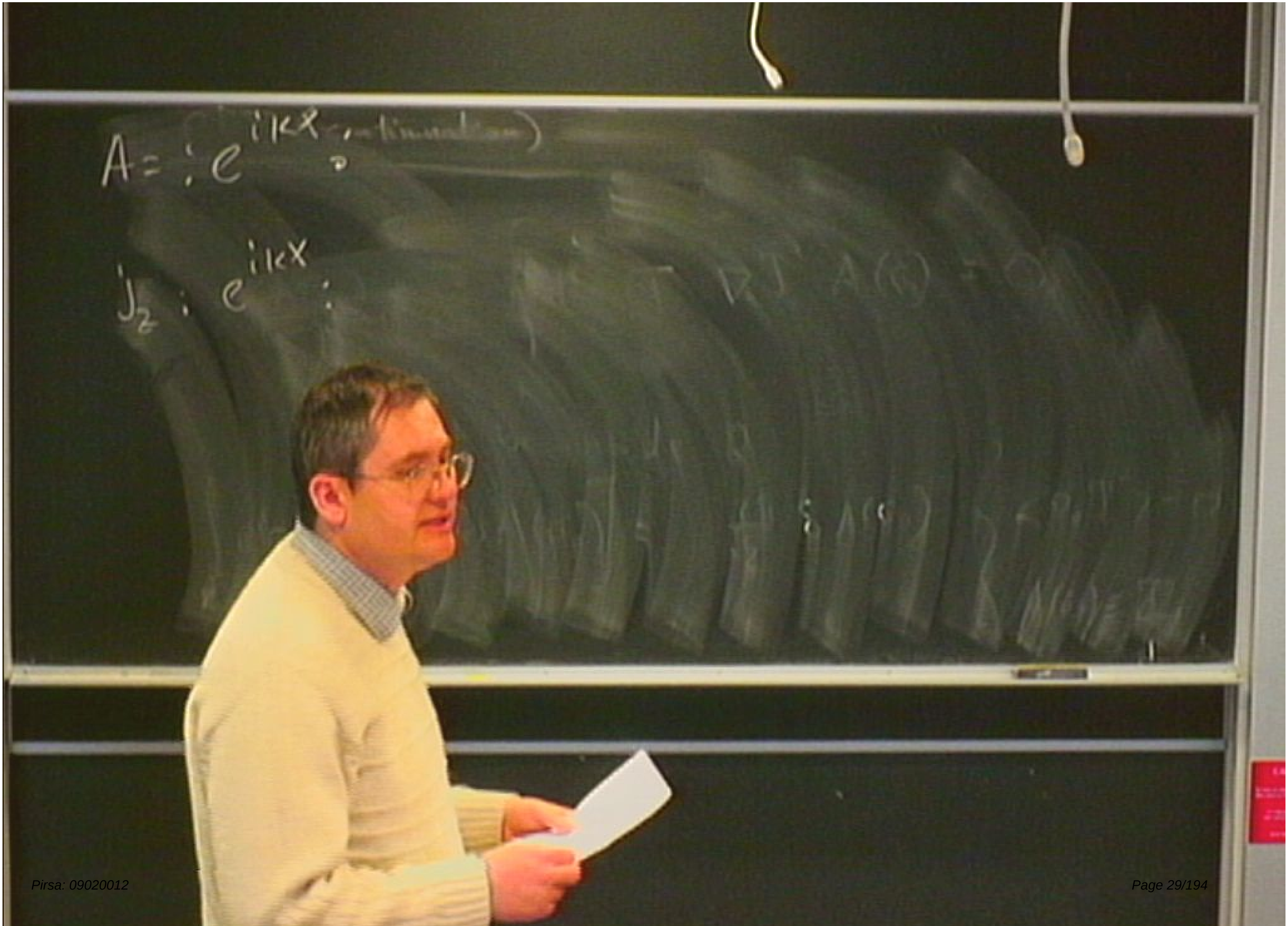
$$[d\varphi'] \, e^{-S[\varphi']} = [d\varphi] \, e^{-S[\varphi]} \left[1 + \frac{i\epsilon}{2\pi} \int d^2\sigma \; \overset{-\delta S}{j^a \nabla_a \rho} + O(\epsilon^2) \right]$$

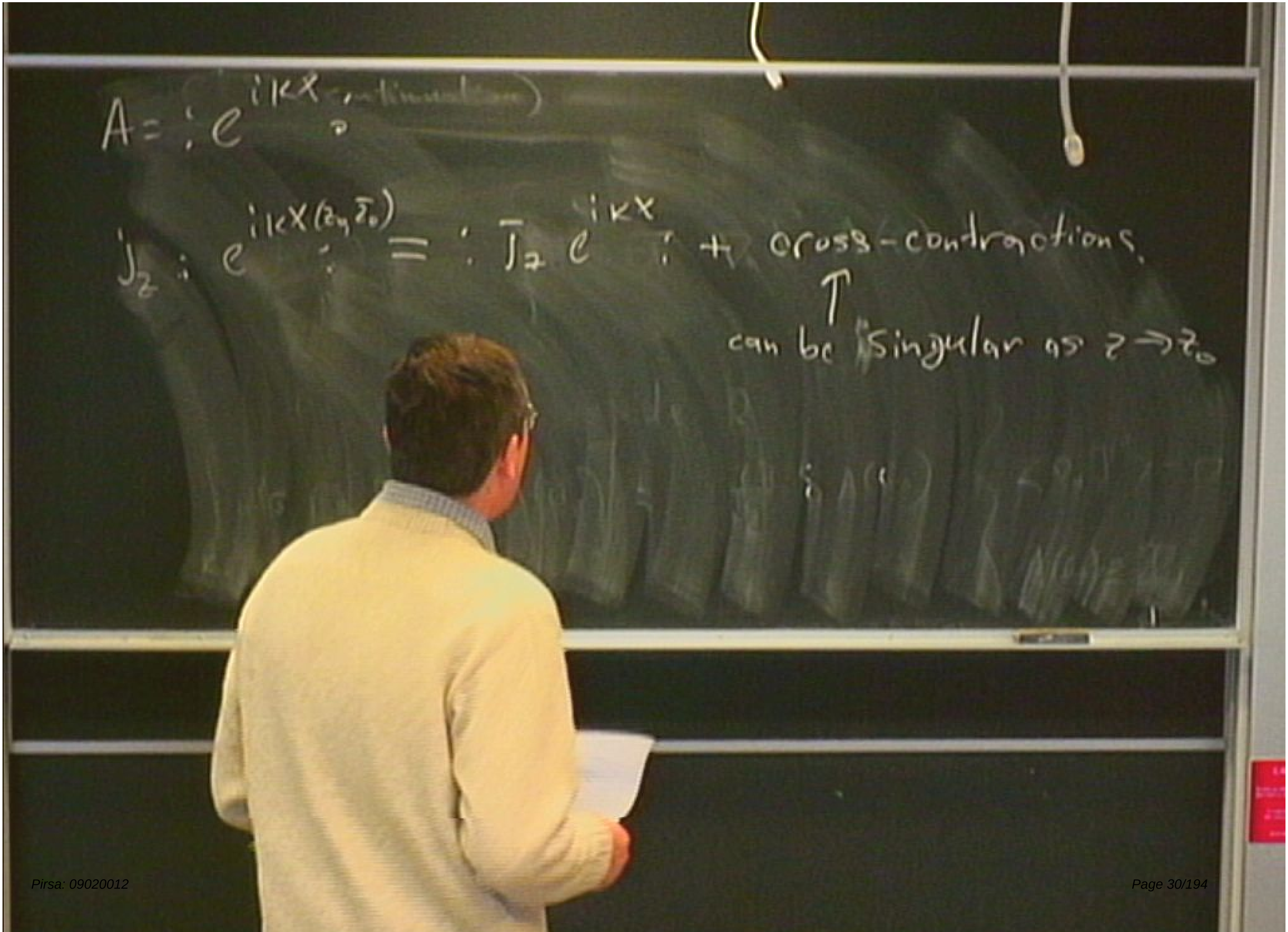
$$\partial^a X_a \frac{\epsilon a_m}{2\pi\alpha'} = \frac{-i\epsilon}{2\pi} J^a \quad \Rightarrow \quad J_a = a_m J^m_a$$

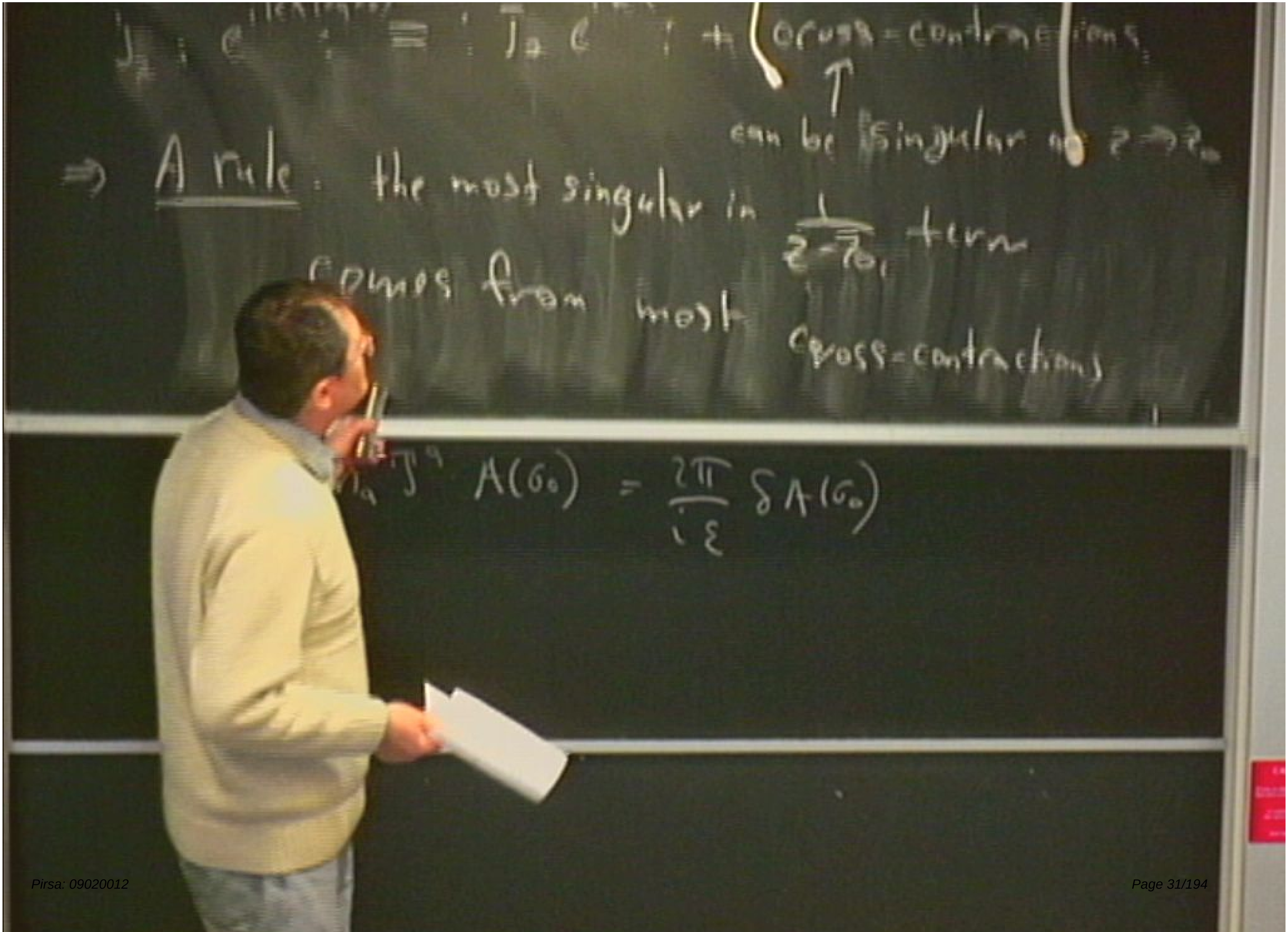
holomorphic

$$\boxed{J_a^m = + \frac{i}{\alpha'} \partial_a X^m} \Rightarrow \begin{array}{l} J_z^m = \frac{i}{\alpha'} \partial X^m \\ \tilde{J}_{\bar z}^m = \frac{i}{\alpha'} \bar\partial X^m \end{array}$$

antiholomorphic


A = : e^ikx
j_z : e^ikx :

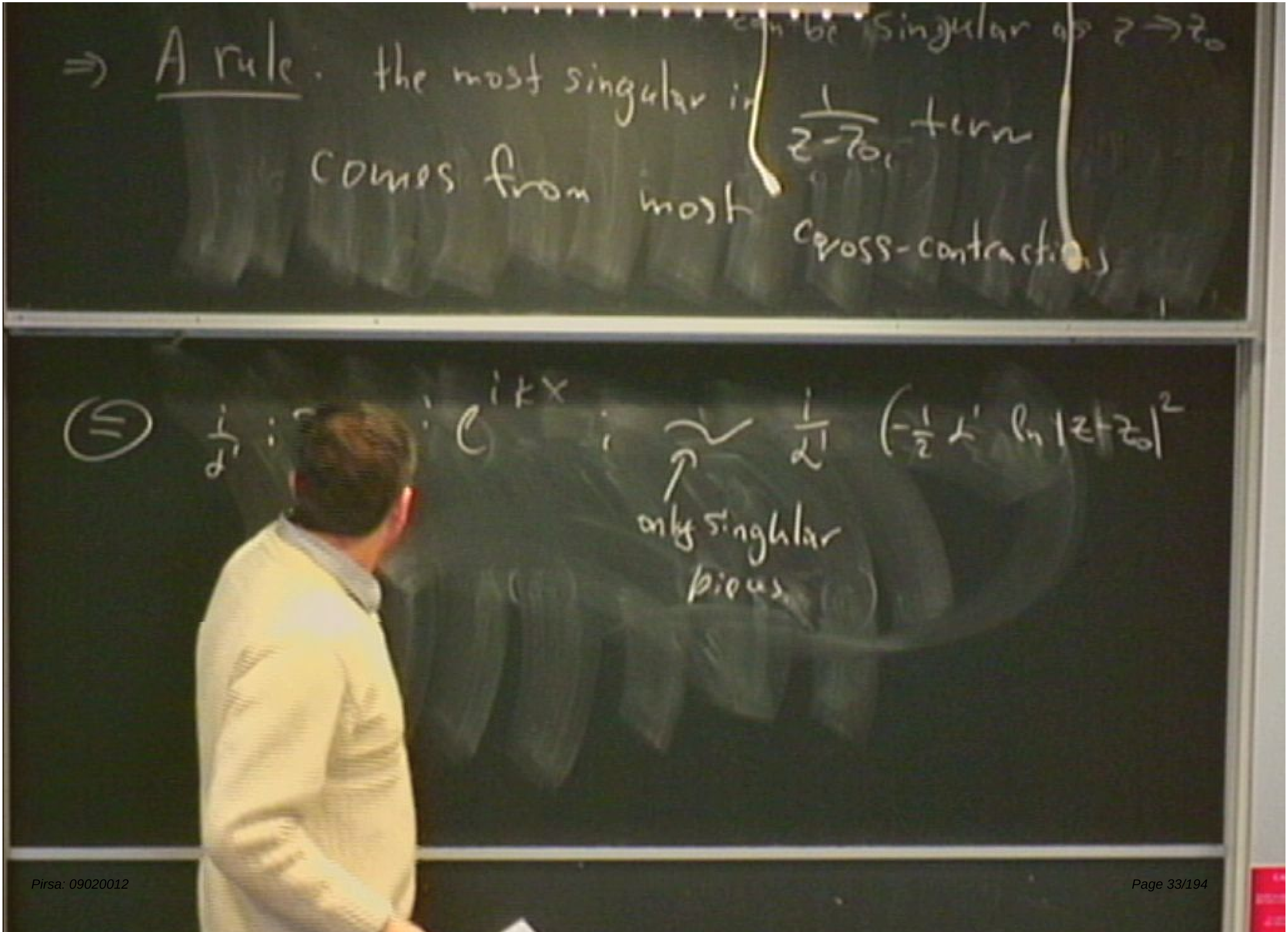

A = :e^{ikX}
j_z : e^{ikX(z_0, z̄_0)} : = : J_z e^{ikX} : + cross-contractions.
can be singular as z → z_0


cross-contractions
can be singular as z → z0
⇒ A rule: the most singular in 1/(z − z0) term
comes from most cross-contractions

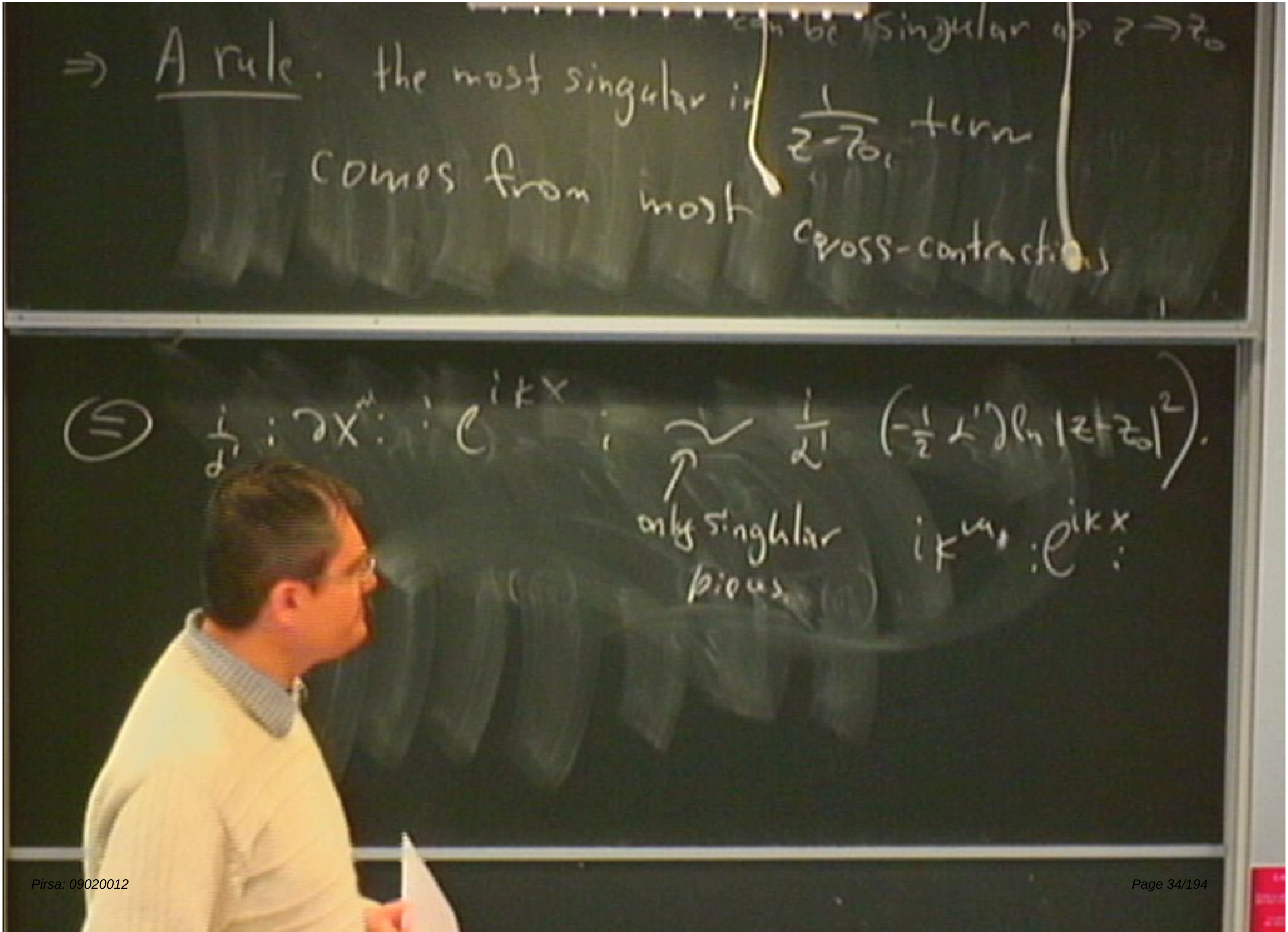
can be singular as $z \to z_0$

$\Rightarrow$ A rule: the most singular in $\frac{1}{z-z_{0i}}$ term comes from most cross-contractions

(5) $\frac{i}{\alpha'} : \partial X : : e^{ikX}$



can be singular as z → z0
⇒ A rule: the most singular in 1/(z − z0) term
comes from most cross-contractions
only singular pieces


can be singular as z → z_0
⇒ A rule: the most singular in 1/(z−z_0) term
comes from most cross-contractions
i/α' :∂X^μ: :e^{ikX}: ~ i/α' (−½ α' ∂ ln|z−z_0|²).
only singular piece
ik^μ :e^{ikx}:

can be singular as $z \to z_0$

$\Rightarrow$ A rule: the most singular in $\frac{1}{z-z_{0}}$ term comes from most cross-contractions

$$\Leftarrow \quad \frac{i}{\alpha'} :\partial X^{\mu}: \; :e^{ikX}: \quad \sim \frac{i}{\alpha'}\left(-\tfrac{1}{2}\alpha' \partial \ln|z-z_0|^2\right) \cdot \frac{\delta}{\delta X^{\nu}} \, ik^{\mu} :e^{ikX}:$$

only singular piece

$$= \frac{k^{\mu}}{2(z-z_0)} :e^{ikX}:$$

can be singular as $z \to z_0$

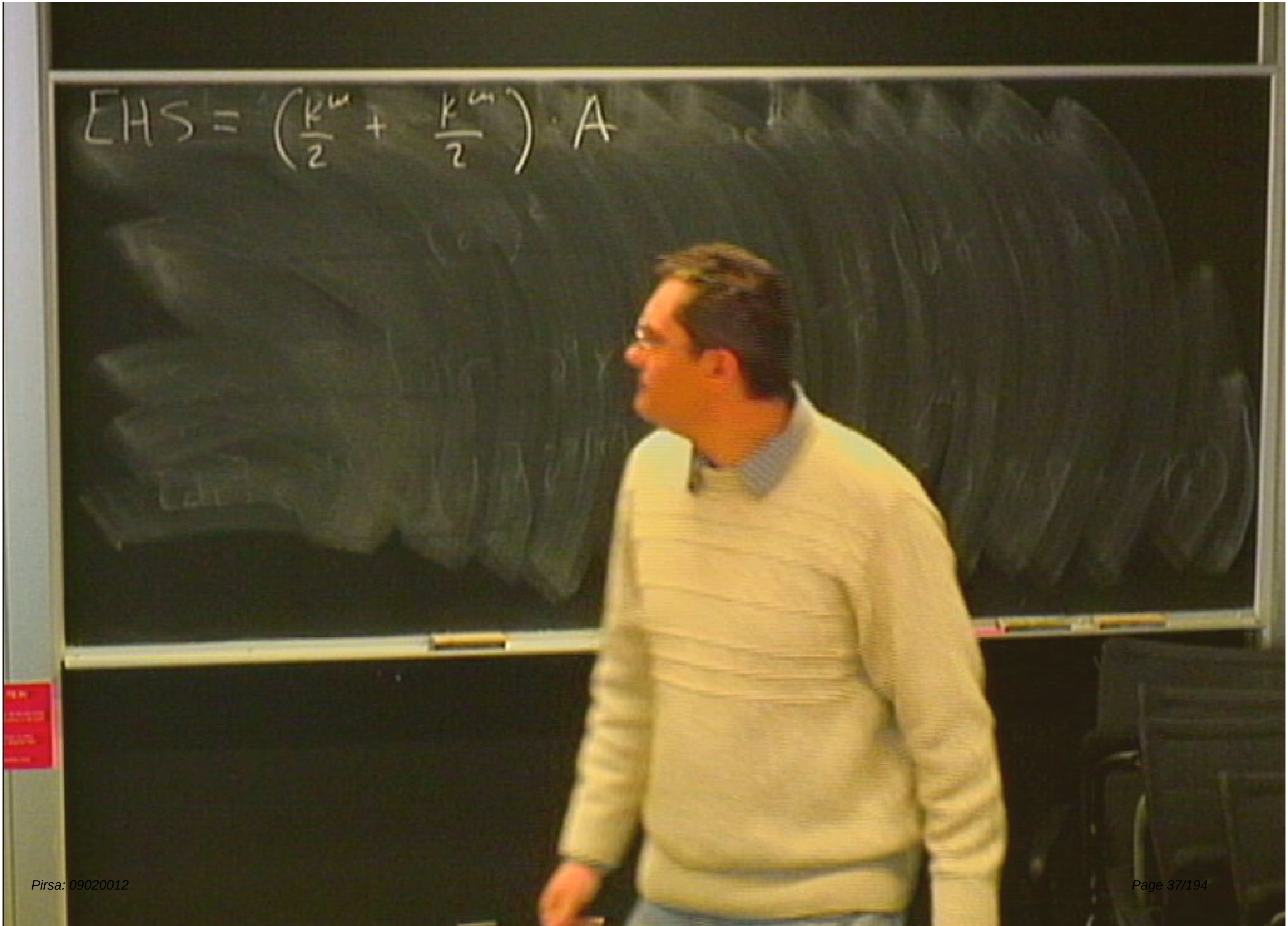
$\Rightarrow$ A rule. the most singular in $\frac{1}{z-z_0}$ term comes from most cross-contractions

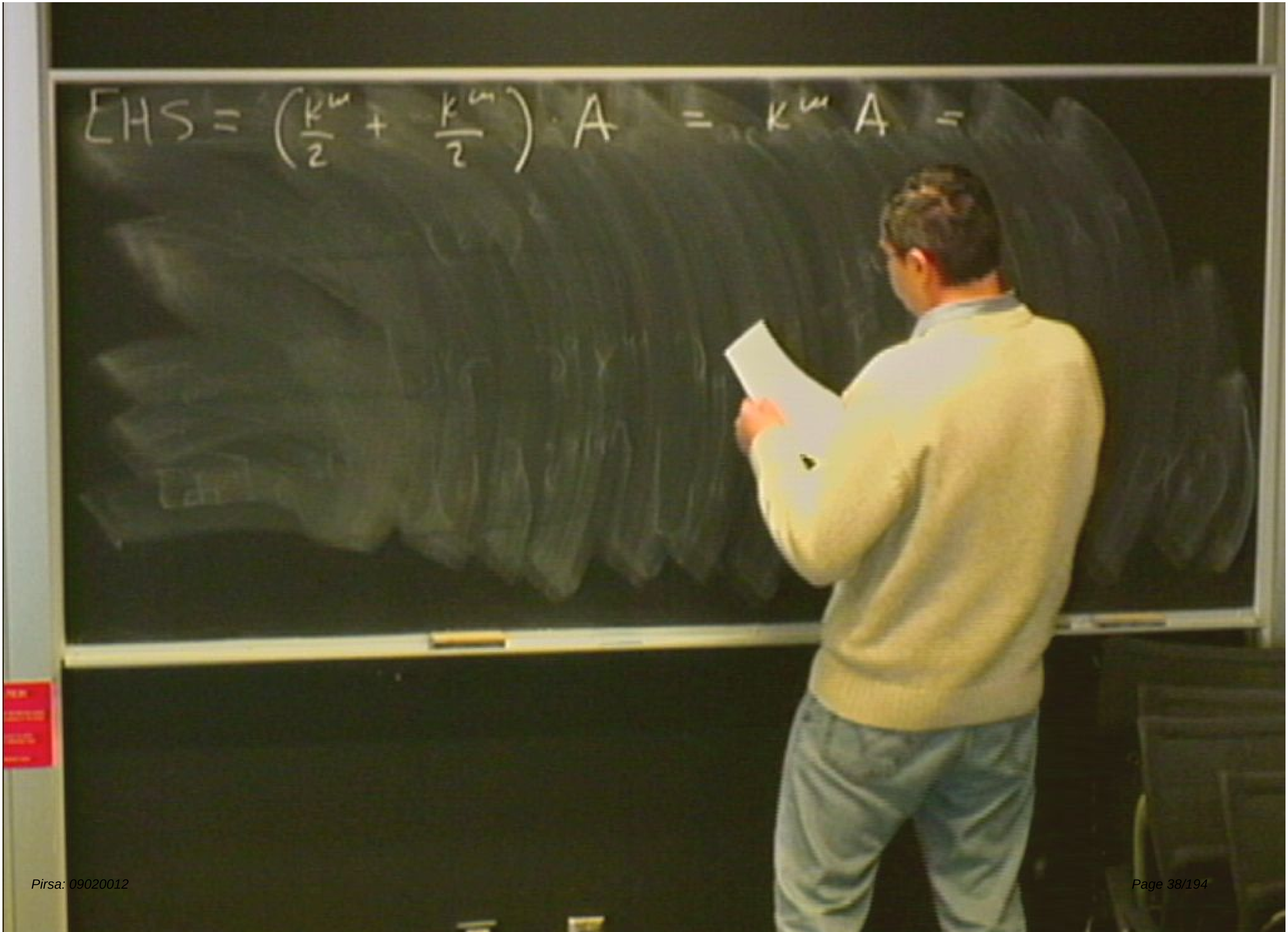
$$\frac{i}{\alpha'} :\partial X^\mu: :e^{ikx}: \sim \frac{i}{\alpha'}\left(-\frac{1}{2}\alpha'\right)\partial \ln|z-z_0|^2 \left(\frac{\delta}{\delta X^\nu}\right) ik^\mu :e^{ikx}:$$

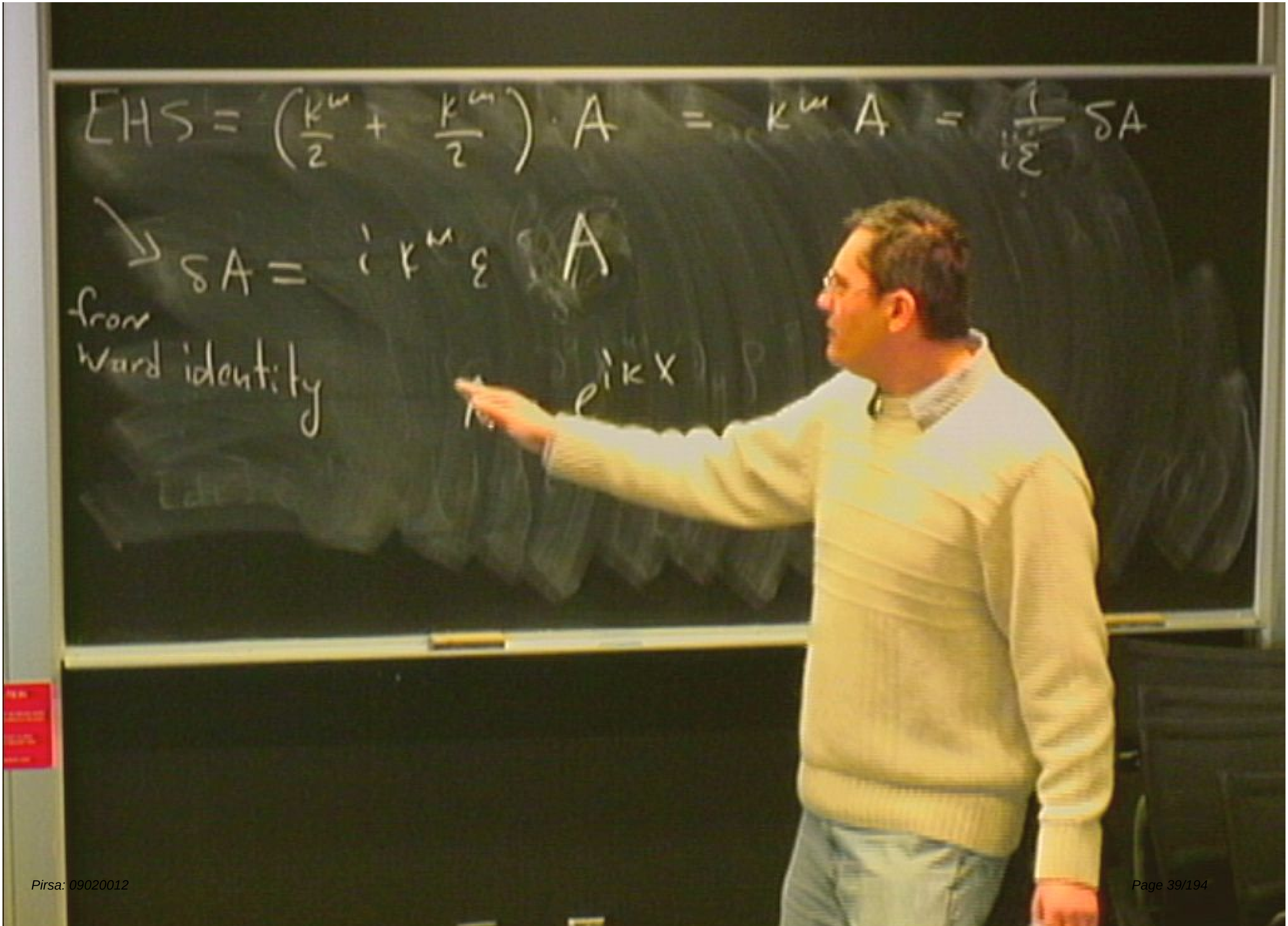
only singular piece

$$\frac{i}{\alpha'} :\bar\partial X^\mu: :e^{ikx}: = \frac{k^\mu}{2(z-z_0)} :e^{ikx}:$$

$$= \frac{k^\mu}{2(\bar z-\bar z_0)} :e^{ikx}:$$


EHS = (k^w/2 + k^m/2) · A


$EHS = \left(\frac{k^{uu}}{2} + \frac{k^{uu}}{2}\right) A = k^{uu} A =$


EHS = (k^μ/2 + k^μ/2) A = k^μ A = 1/(iε) δA
δA = i k^μ ε A
from
Ward identity
e^{ikx}

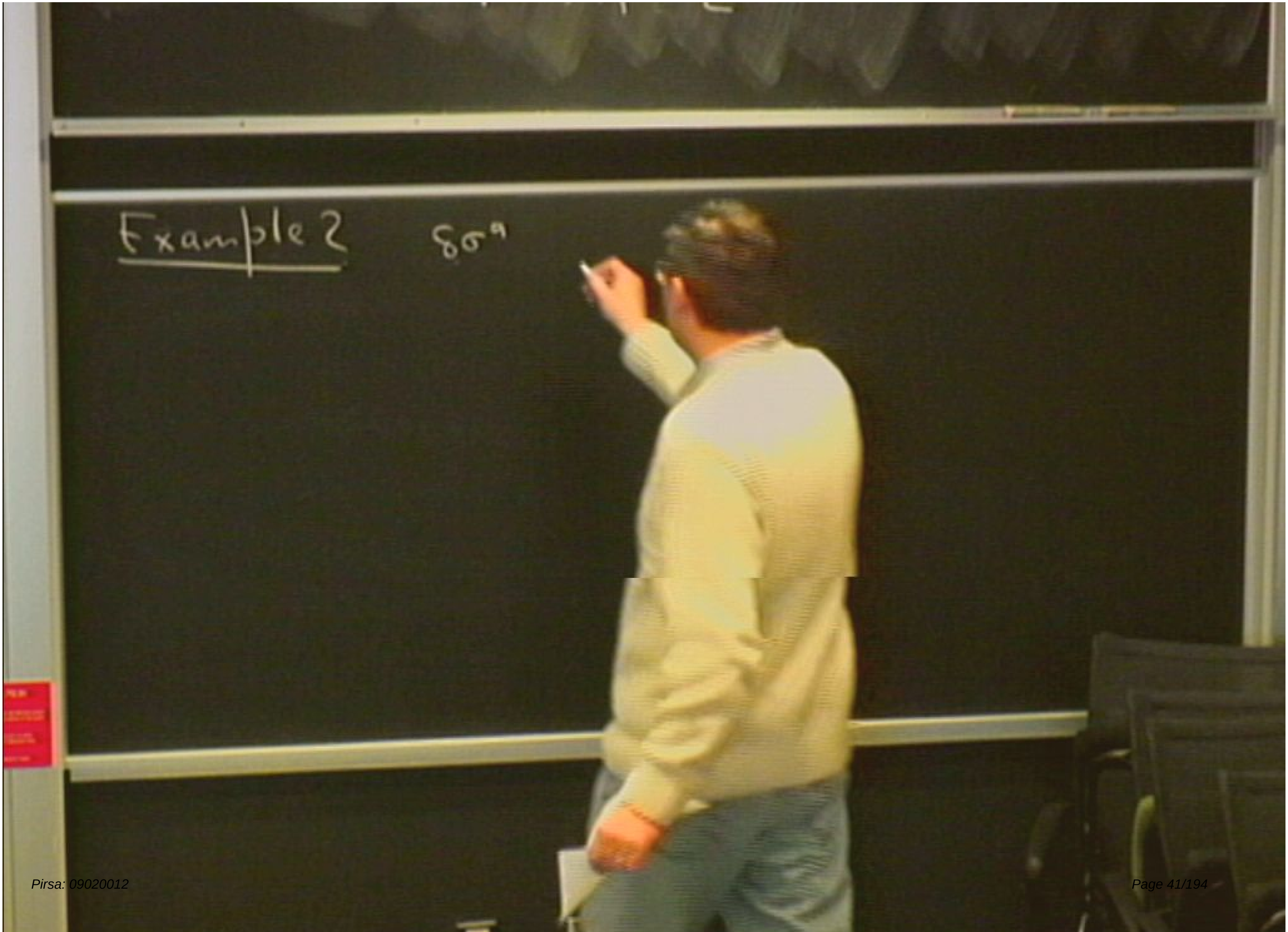
$$\text{LHS} = \left(\frac{k^\mu}{2} + \frac{k^\mu}{2}\right) A = k^\mu A = \frac{1}{i\varepsilon}\,\delta A$$

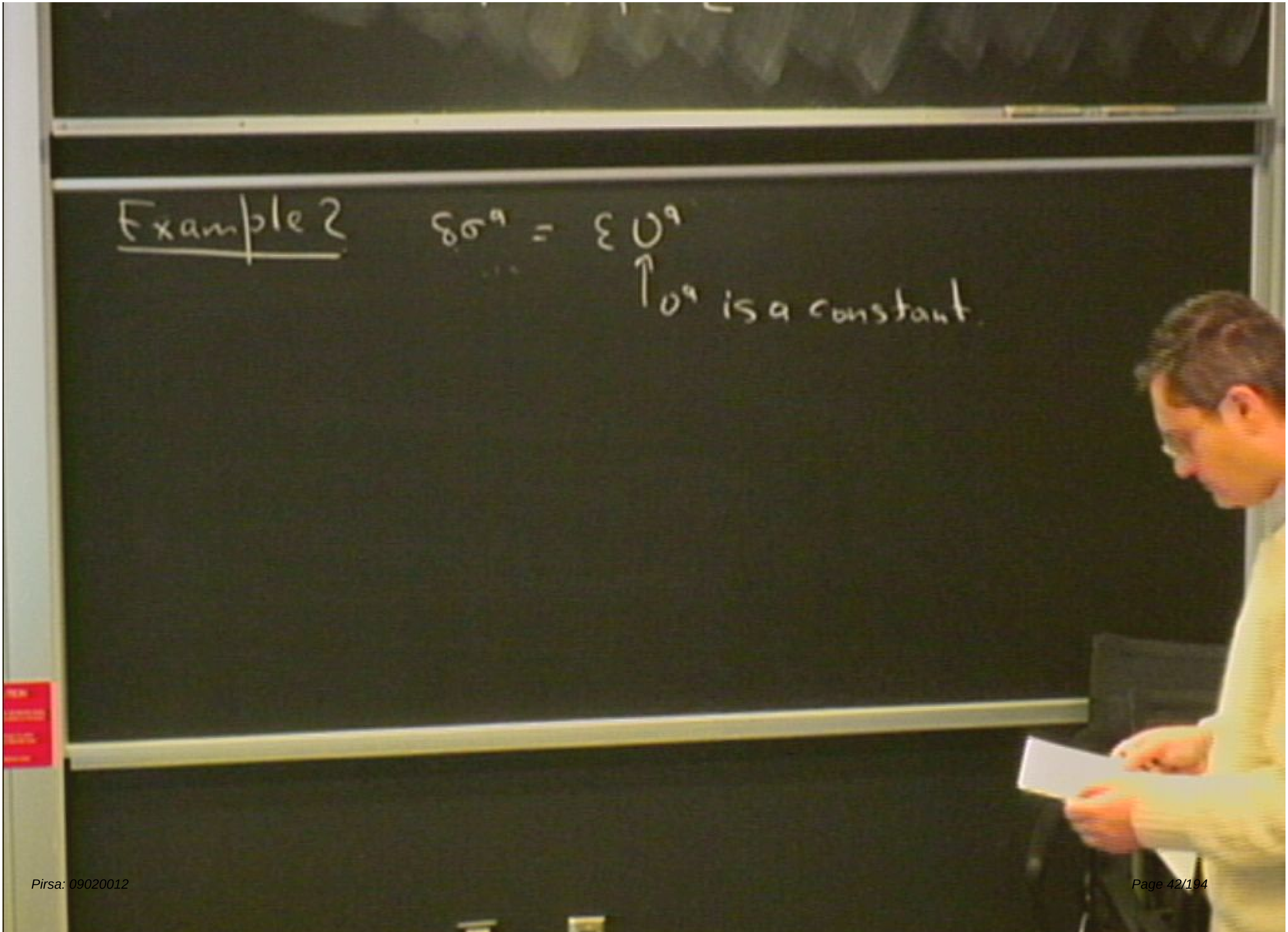
$$\searrow\ \delta A = i k^\mu \varepsilon A$$

from
Ward identity

$$A = e^{ikx} \longrightarrow e^{ikx + ik^\mu \varepsilon}$$

$$X^\mu \to X^\mu + \varepsilon$$


Example 2
δσ^a

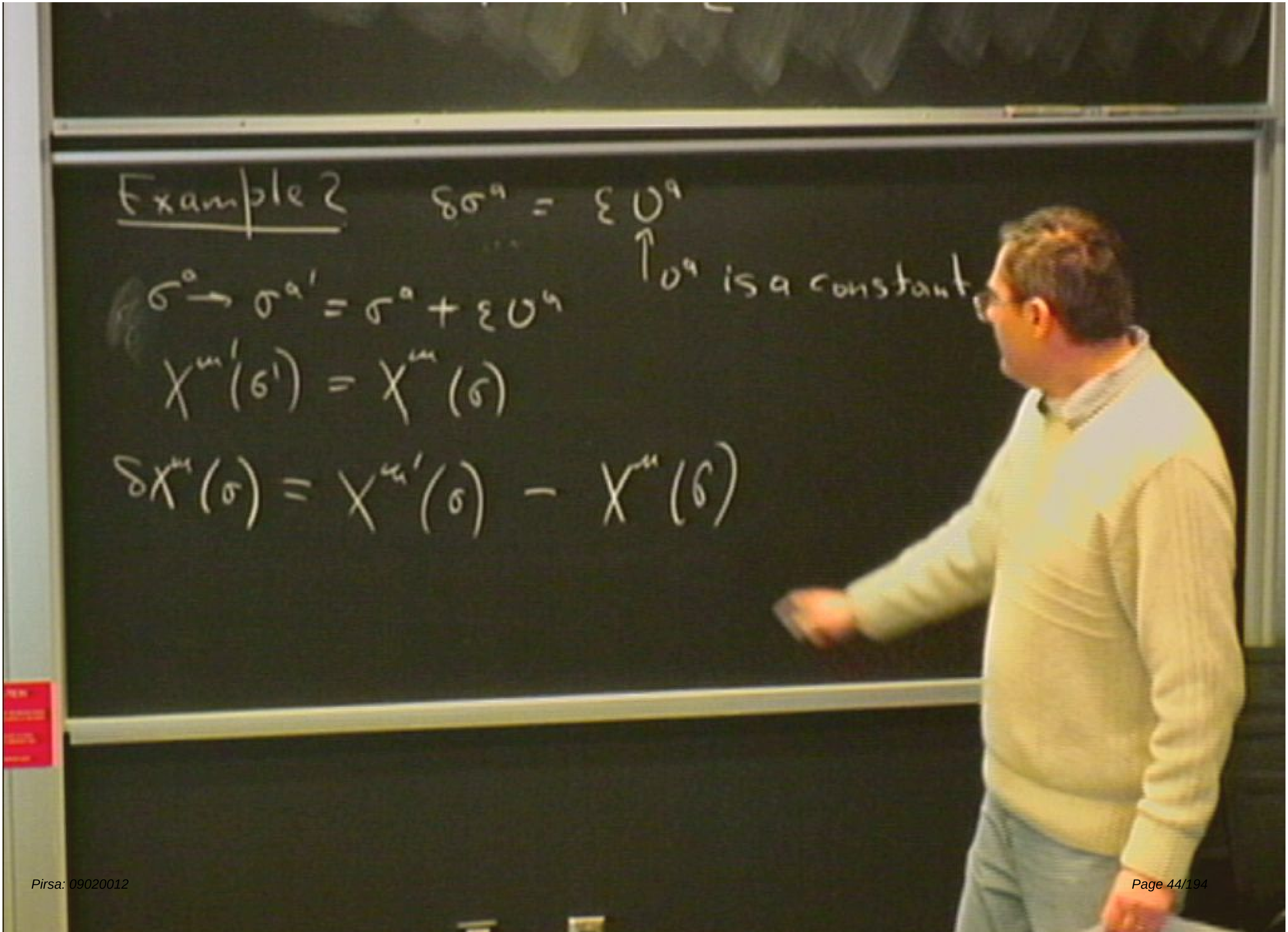

Example 2
$\delta\sigma^a = \varepsilon v^a$
v^a is a constant.

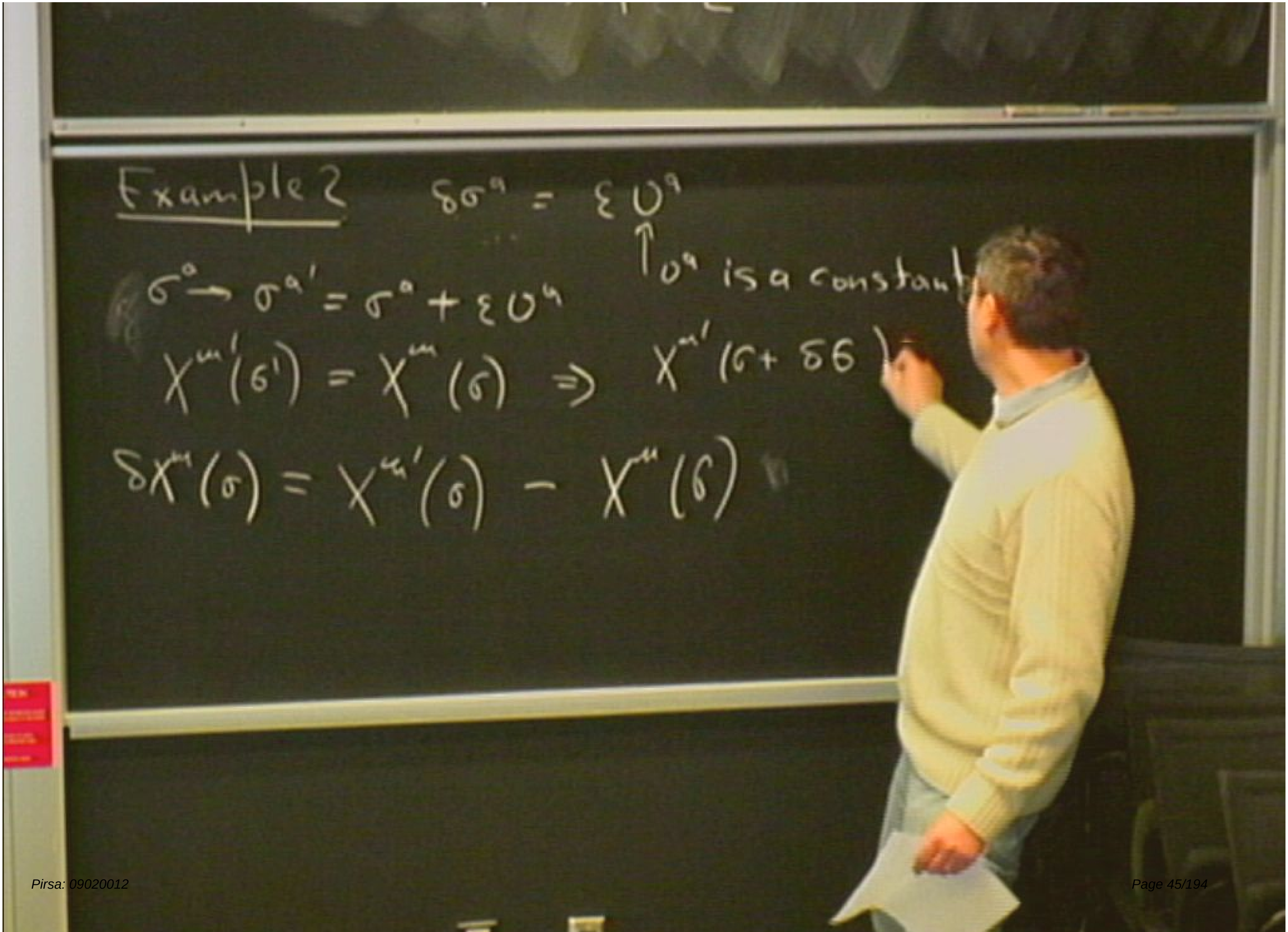
Example 2 $\quad \delta\sigma^a = \varepsilon\, v^a$

$\uparrow$ v^a is a constant.

$$\sigma^a \to \sigma^{a\prime} = \sigma^a + \varepsilon\, v^a$$

$$X^{m\prime}(\sigma') = X^m(\sigma)$$


Example 2 $\delta\sigma^a = \varepsilon\, v^a$
v^a is a constant
$\sigma^a \to \sigma^{a\prime} = \sigma^a + \varepsilon\, v^a$
$X^{\mu\prime}(\sigma') = X^\mu(\sigma)$
$\delta X^\mu(\sigma) = X^{\mu\prime}(\sigma) - X^\mu(\sigma)$


Example 2 $\delta\sigma^a = \varepsilon v^a$
v^a is a constant
$\sigma^a \to \sigma^{a\prime} = \sigma^a + \varepsilon v^a$
$X^{\mu\prime}(\sigma') = X^\mu(\sigma) \Rightarrow X^{\mu\prime}(\sigma + \delta\sigma)$
$\delta X^\mu(\sigma) = X^{\mu\prime}(\sigma) - X^\mu(\sigma)$

Example 2 $\quad \delta\sigma^a = \varepsilon v^a$

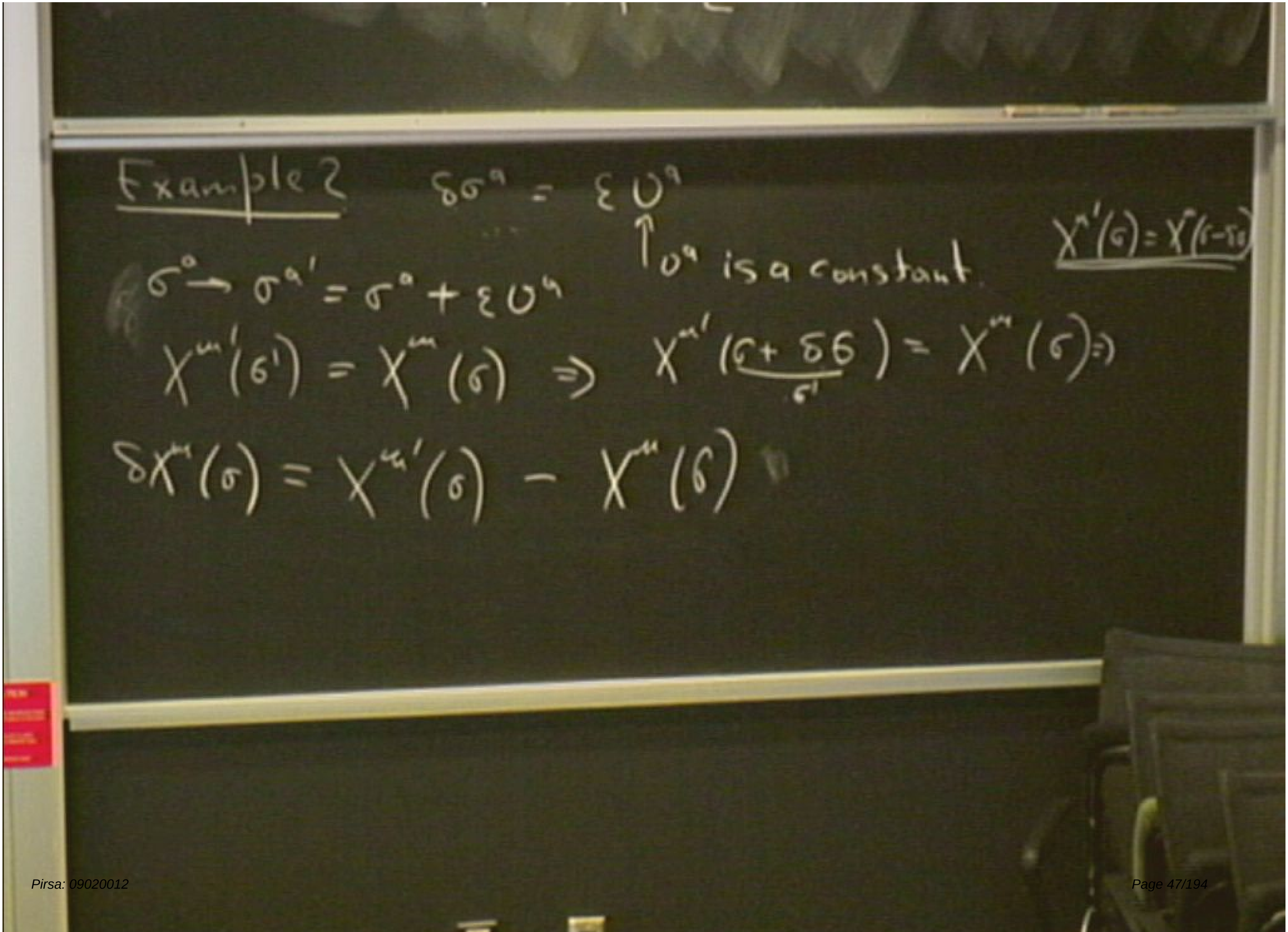
$\uparrow$ v^a is a constant

$$X^{\mu\prime}(\sigma) = X^{\mu}(\sigma - \delta\sigma)$$

$$\sigma^a \to \sigma^{a\prime} = \sigma^a + \varepsilon v^a$$

$$X^{\mu\prime}(\sigma') = X^{\mu}(\sigma) \Rightarrow X^{\mu\prime}(\sigma + \delta\sigma) = X^{\mu}(\sigma) \Rightarrow$$

$$\delta X^{\mu}(\sigma) = X^{\mu\prime}(\sigma) - X^{\mu}(\sigma)$$


Example 2 $\delta\sigma^a = \varepsilon v^a$
v^a is a constant
$X^{\mu\prime}(\sigma) = X^\mu(\sigma-\delta\sigma)$
$\sigma^a \to \sigma^{a\prime} = \sigma^a + \varepsilon v^a$
$X^{\mu\prime}(\sigma') = X^\mu(\sigma) \Rightarrow X^{\mu\prime}(\underbrace{\sigma+\delta\sigma}_{\sigma'}) = X^\mu(\sigma) \Rightarrow$
$\delta X^\mu(\sigma) = X^{\mu\prime}(\sigma) - X^\mu(\sigma)$

Example 2 $\delta\sigma^a = \varepsilon v^a$

$\uparrow$ v^a is a constant

$X^{\mu\prime}(\sigma) = X^\mu(\sigma - \delta\sigma)$

$$\sigma^a \to \sigma^{a\prime} = \sigma^a + \varepsilon v^a$$

$$X^{\mu\prime}(\sigma') = X^\mu(\sigma) \Rightarrow X^{\mu\prime}(\underbrace{\sigma + \delta\sigma}_{\sigma'}) = X^\mu(\sigma) \Rightarrow$$

$$\delta X^\mu(\sigma) = X^{\mu\prime}(\sigma) - X^\mu(\sigma)$$

$$= X^{\mu\prime}(\sigma^a - \varepsilon v^a) - X^\mu(\sigma) = -\varepsilon v^a \partial_a X^\mu$$

Example 2 $\delta\sigma^a = \varepsilon v^a$

$\uparrow v^a$ is a constant.

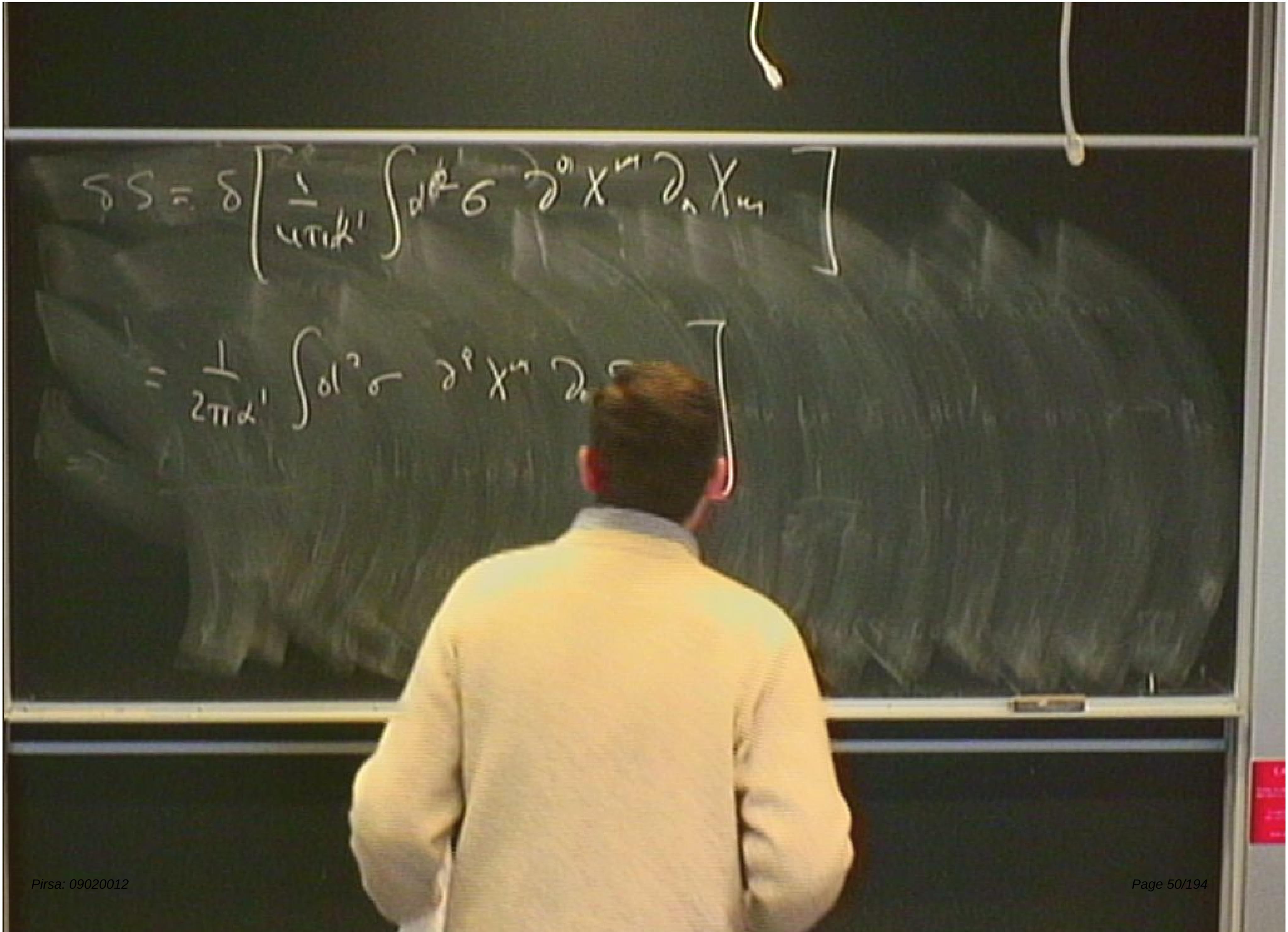
$\underline{X^{\mu\prime}(\sigma) = X^\mu(\sigma-\delta\sigma)}$

$$\sigma^a \to \sigma^{a\prime} = \sigma^a + \varepsilon v^a$$

$$X^{\mu\prime}(\sigma') = X^\mu(\sigma) \Rightarrow X^{\mu\prime}(\underbrace{\sigma+\delta\sigma}_{\sigma'}) = X^\mu(\sigma) \Rightarrow$$

$$\delta X^\mu(\sigma) = X^{\mu\prime}(\sigma) - X^\mu(\sigma)$$

$$= X^{\mu\prime}(\sigma^a - \varepsilon v^a) - X^\mu(\sigma) = \boxed{-\varepsilon v^a \partial_a X^\mu = \delta X^\mu}$$


$\delta S = \delta \left[\frac{1}{4\pi\alpha'} \int d^2\sigma \, \partial^a X^\mu \partial_a X_\mu \right]$
$= \frac{1}{2\pi\alpha'} \int d^2\sigma \, \partial^a X^\mu \partial_a$

$$\delta S = \delta \left[\frac{1}{4\pi\alpha'} \int d^2\sigma \, \partial^a X^\mu \partial_a X_\mu \right]$$

$$= \frac{1}{2\pi\alpha'} \int d^2\sigma \, \partial^a X^\mu \partial_a \delta X_\mu \Big]$$

$$\delta S = \delta\left[\frac{1}{4\pi\alpha'}\int d^2\sigma\, \partial^a X^\mu \partial_a X_\mu\right]$$

$$= \frac{1}{2\pi\alpha'}\int d^2\sigma\, \partial^b X^\mu \partial_b \delta X_\mu \Bigg] =$$

$$\delta X_\mu = -\varepsilon(\sigma)\, v^a \partial_a X_\mu$$

$$= \frac{k^\mu}{2(\bar{z}-\bar{z}_0)} :e^{ikX}:$$

$$\delta S = \delta\left(\frac{1}{4\pi\alpha'}\int d^2\sigma\ \partial^a X^\mu\ \partial_a X_\mu\right)$$

$$= \frac{1}{2\pi\alpha'}\int d^2\sigma\ \partial^b X^\mu\ \partial_b \delta X_\mu = -\frac{1}{2\pi\alpha'}\int d^2\sigma\ \partial^a X^\mu\ \partial_a \varepsilon\ v^b\ \partial_b X_\mu$$

$$\delta X_\mu = -\underline{\varepsilon(\sigma)}\ v^a\ \partial_a X_\mu$$

$$\frac{1}{\alpha'} :\bar\partial X^\mu: e^{ikx}: = \frac{k}{2(z-z_0)} :e^{ikX}:$$

$$= \frac{k^\mu}{2(\bar z-\bar z_0)} :e^{ikX}:$$

$$= \frac{1}{2\pi\alpha'}\int d^2\sigma\ \partial^a X^\mu \partial_a \delta X_\mu \Big] = -\frac{1}{2\pi\alpha'}\int d^2\sigma\ \partial^a X^\mu \partial_a \varepsilon\, v^b \partial_b X_\mu$$

$$\delta X_\mu = -\underline{\varepsilon(\sigma)}\, v^a \partial_a X_\mu \qquad - \frac{v^b}{2\pi\alpha'}\int d^2\sigma\ \partial^a X^\mu \partial_b \partial_a X^\mu \varepsilon$$

$$\frac{i}{\alpha'} : \bar\partial X^\mu : e^{ikx} : \; = \frac{k^\mu}{2(z-z_0)} : e^{ikx} :$$

$$= \frac{k^\mu}{2(\bar z-\bar z_0)} : e^{ikx} :$$

$$\delta X_\mu = -\underline{\varepsilon(\sigma)}\, \partial^a \underbrace{\partial_a X_\mu}$$

$$\frac{\partial^b}{2\pi\alpha'} \int d^2\sigma\, \partial X \cdots \partial_b \partial_a X \cdot \varepsilon$$

(5) $\frac{i}{\alpha'} :\partial X^\mu: \; :e^{ikx}: \;\sim\; \frac{i}{\alpha'} \left(-\frac{1}{2}\alpha' \partial \ln|z-z_0|^2\right).$

only singular pieces

$\frac{\delta}{\delta X^\nu}$ $\quad ik^\mu \, :e^{ikx}:$

$$\frac{i}{\alpha'} :\bar\partial X^\mu: \; :e^{ikx}: \;=\; \frac{k^\mu}{2(z-z_0)} :e^{ikx}: \;=\; \frac{k^\mu}{2(\bar z-\bar z_0)} :e^{ikx}:$$

$\delta X_\mu = -\varepsilon(\sigma)\, \partial^a \partial_a X_\mu$ $\quad \frac{1}{2\pi\alpha'}\int d^2\sigma\, \partial X \; \partial_b \partial_a X\, \varepsilon$

$$-\delta S = \frac{i}{2\pi}\int d^2\sigma \sqrt{-g}\; j^a\, \partial_a \varepsilon$$

$$\delta X_\mu = -\underline{\varepsilon(\sigma)}\, \rho^a \underbrace{\partial_a X_\mu}$$

$$\frac{\upsilon}{2\pi\alpha'}\int d^2\sigma\, \partial X \quad \partial_b \partial_a X\, \varepsilon$$

(2)

$$-\delta S = \frac{i}{2\pi}\int d^2\sigma \sqrt{-g}\; j^a\, \underline{\partial_a \varepsilon}$$

(2): $\partial^a X^\mu \partial_b \partial_a X^\mu \cdot \varepsilon = \frac{1}{2}\partial_b\left[\partial^a X^\mu \partial_a X_\mu\right]$

$\delta X_\mu = -\underline{\varepsilon(\sigma)}\,\sigma^a \partial_a X_\mu$

$\frac{\sigma}{2\pi\alpha'}\int d^2\sigma\, \partial X \quad \partial_b \partial_a X \cdot \varepsilon$

$$-\delta S = \frac{i}{2\pi}\int d^2\sigma \sqrt{-g}\; j^a\, \underline{\partial_a \varepsilon}$$

(2): $\partial^a X^\mu \partial_b \partial_a X_\mu \cdot \varepsilon = \frac{1}{2}\partial_b\left[\partial^a X^\mu \partial_a X_\mu\right]\cdot \varepsilon$

$\hookrightarrow -\frac{1}{2}(\partial X)^2 \cdot \partial_b \varepsilon$

$$J_a = \frac{v^b}{i\alpha'}\left(\partial_a X^m \partial_b X_m - \frac{1}{2}\delta_{ab}(\partial X)^2\right)$$

$$J_a = \frac{\partial^b}{i\alpha'}\left(\partial_a X^\mu \partial_b X_\mu - \frac{1}{2}\delta_{ab}(\partial X)^2\right)$$

$$T_{ab} = -\frac{1}{\alpha'}\left(\partial_a X^\mu \partial_b X_\mu - \frac{1}{2}\delta_{ab}(\partial X)^2\right)$$

$$J_a = \frac{v^b}{i\alpha'}\left(\partial_a X^\mu \partial_b X_\mu - \frac{1}{2}\delta_{ab}(\partial X)^2\right)$$

$$T_{ab} \stackrel{\text{def}}{=} -\frac{1}{\alpha'}\left(\partial_a X^\mu \partial_b X_\mu - \frac{1}{2}\delta_{ab}(\partial X)^2\right)$$

$$J_a = -\frac{1}{i} v^b T_{ab} = i v^b T_{ab}$$

In 4CFT.

$$T_{ab} = -\frac{1}{\alpha'} :\partial_a X^\mu \partial_b X_\mu - \frac{1}{2}\delta_{ab}(\partial X)^2:$$

In 4CFT

$$T_{ab} = -\frac{1}{\alpha'} : \partial_a X^\mu \partial_b X_\mu - \frac{1}{2}\delta_{ab}(\partial X)^2 :$$

$\Rightarrow$ Conformal inv

In 4CFT.

$$T_{ab} = -\frac{1}{\alpha'} : \partial_a X^{\mu} \partial_b X_{\mu} - \frac{1}{2} \delta_{ab} (\partial X)^2 :$$

$\Rightarrow$ Conformal inv

$$T_a{}^a = 0$$

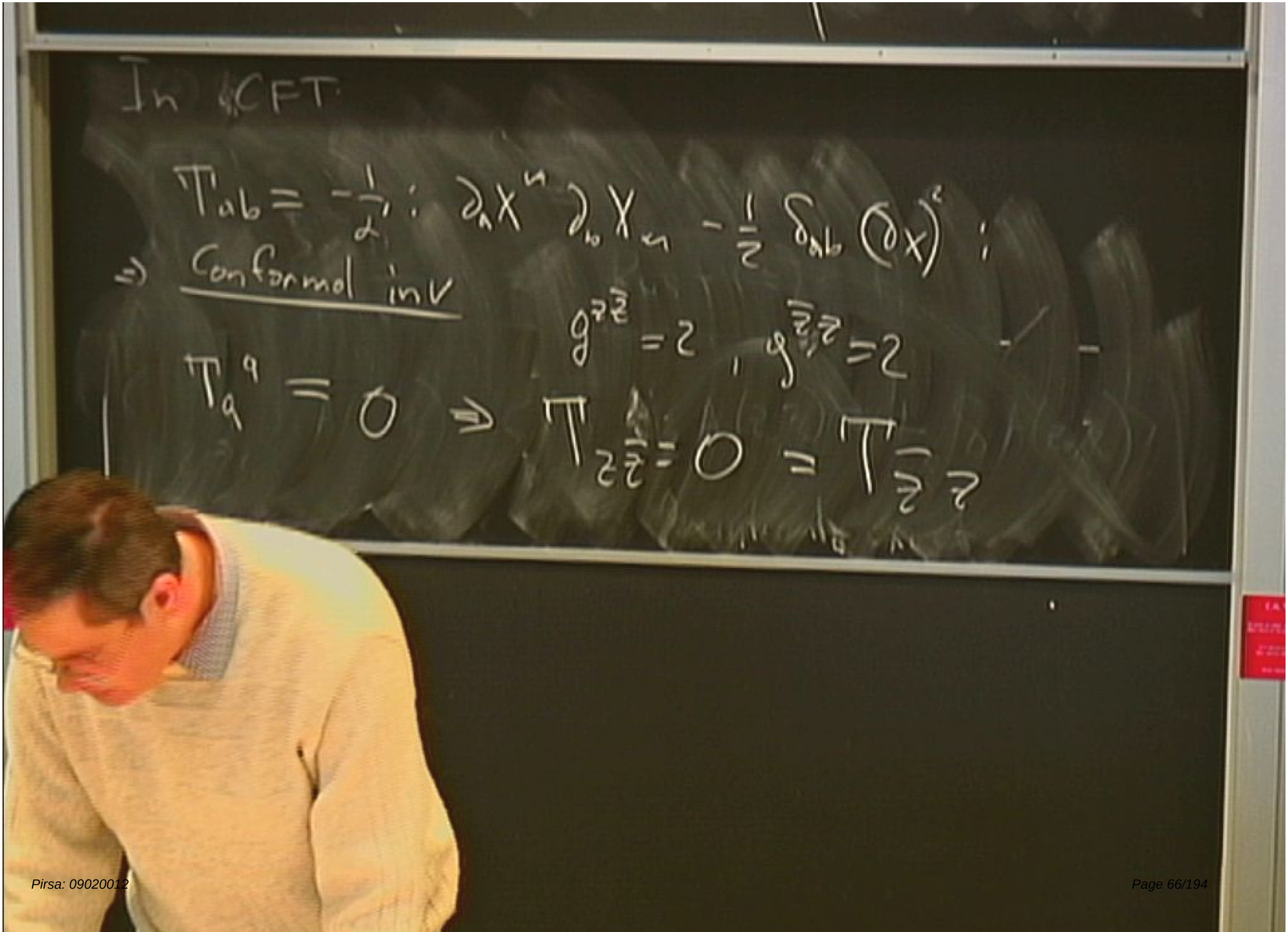
In 4CFT

$$T_{ab} = -\frac{1}{\alpha'} : \partial_a X^\mu \partial_b X_\mu - \frac{1}{2}\delta_{ab} (\partial X)^2 :$$

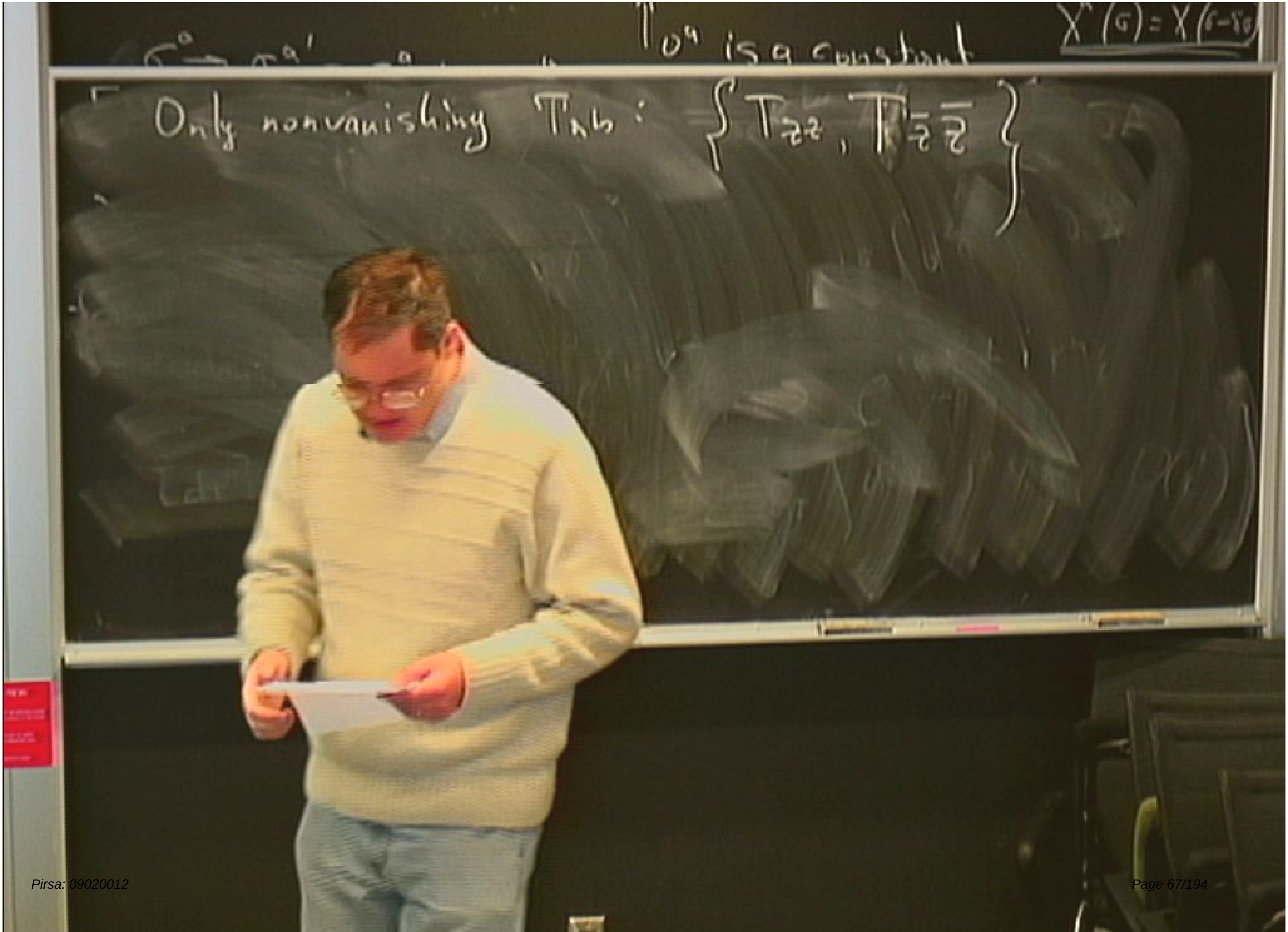
$\Rightarrow$ Conformal inv

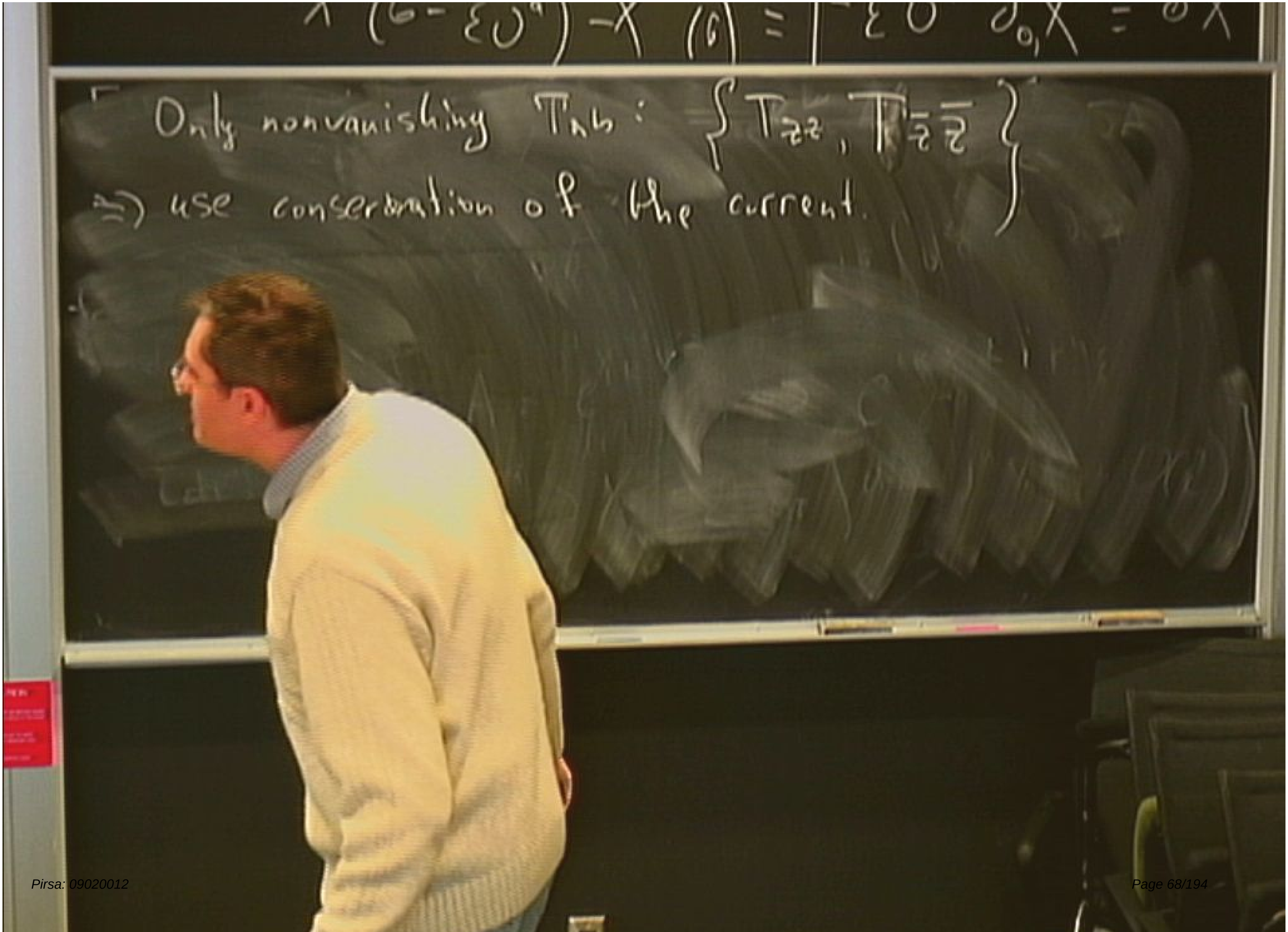
$$g^{z\bar{z}} = 2 \quad , \quad g^{\bar{z}z} = 2$$

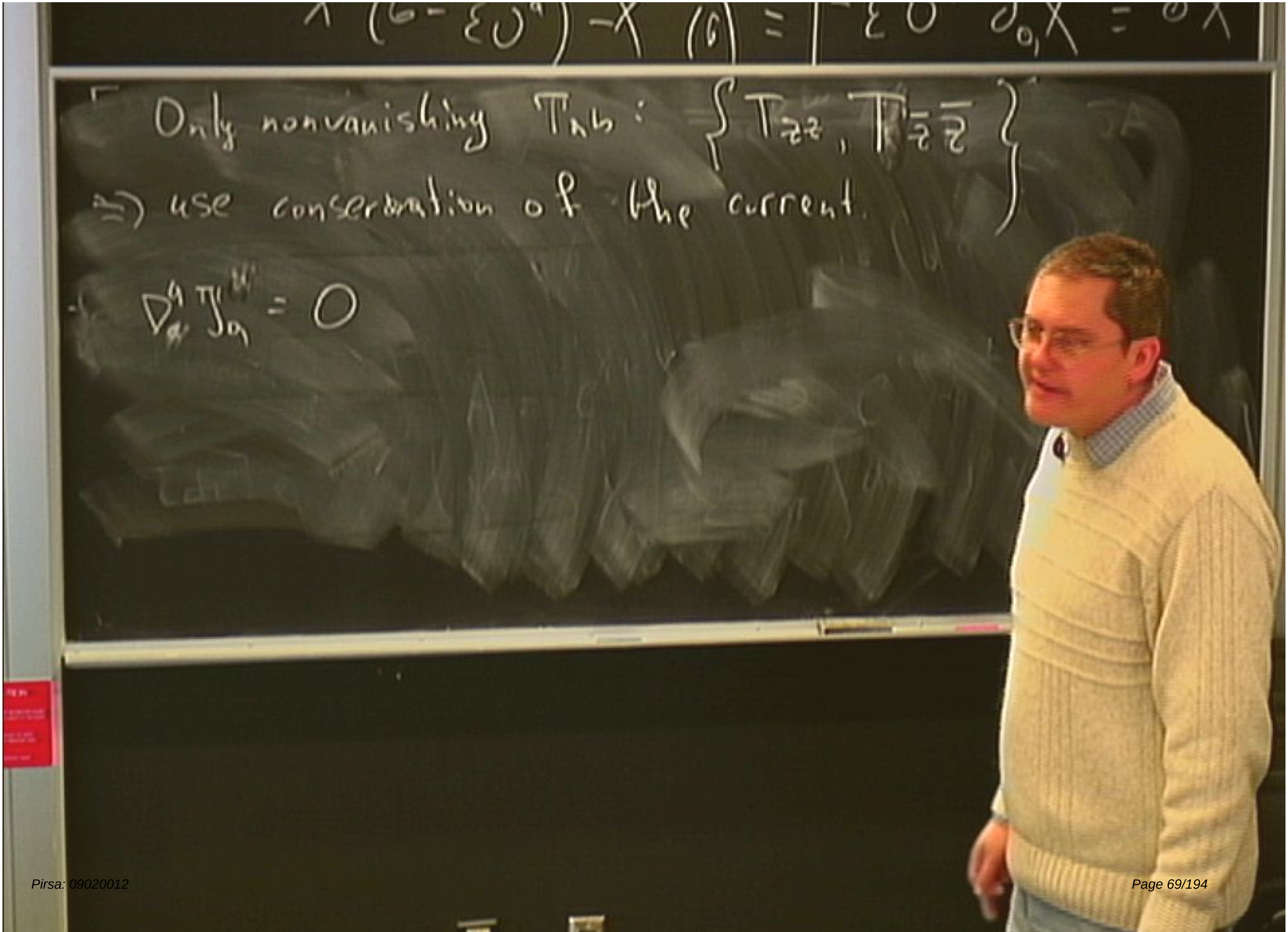
$$T_a{}^a = 0$$

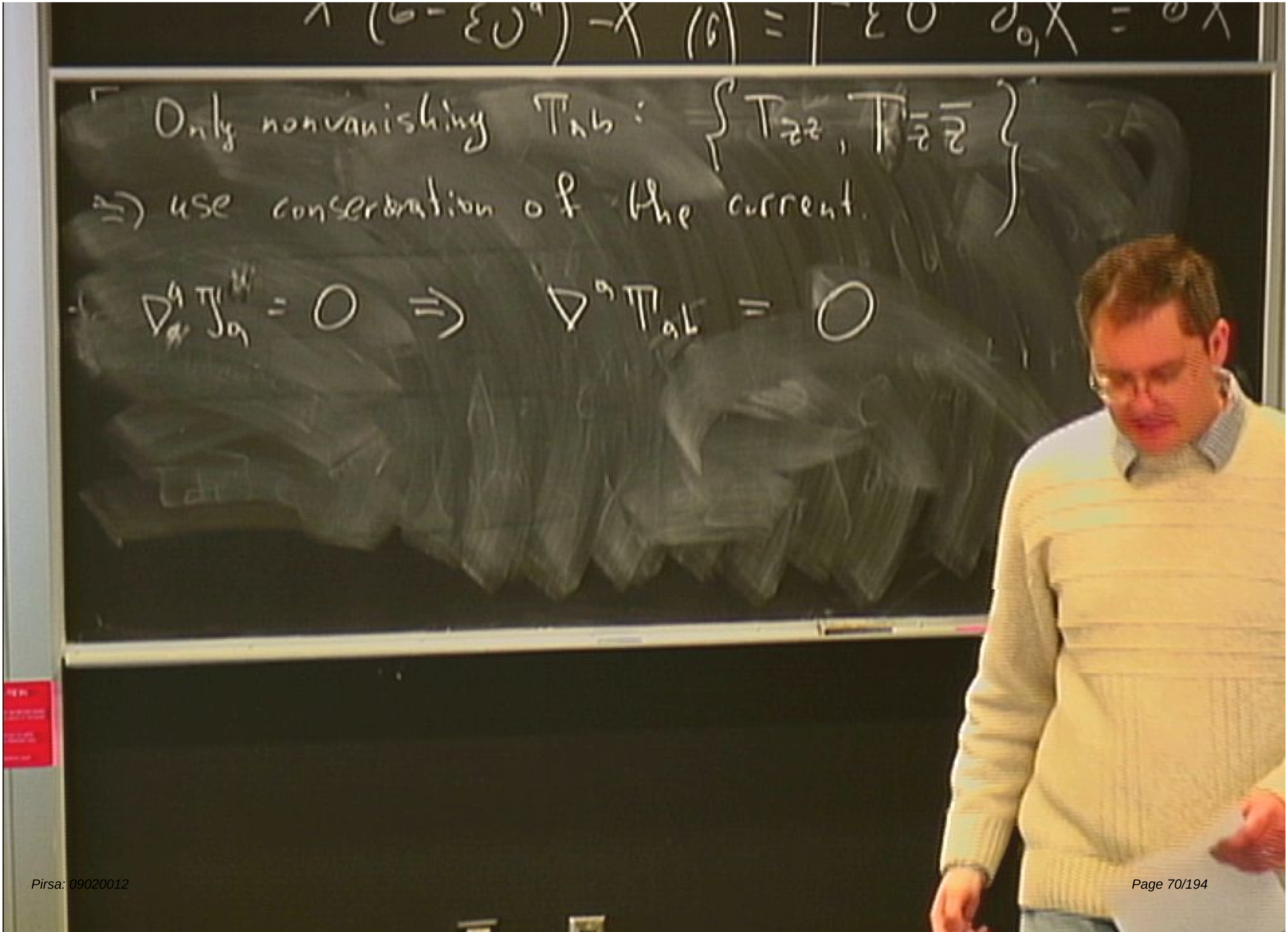


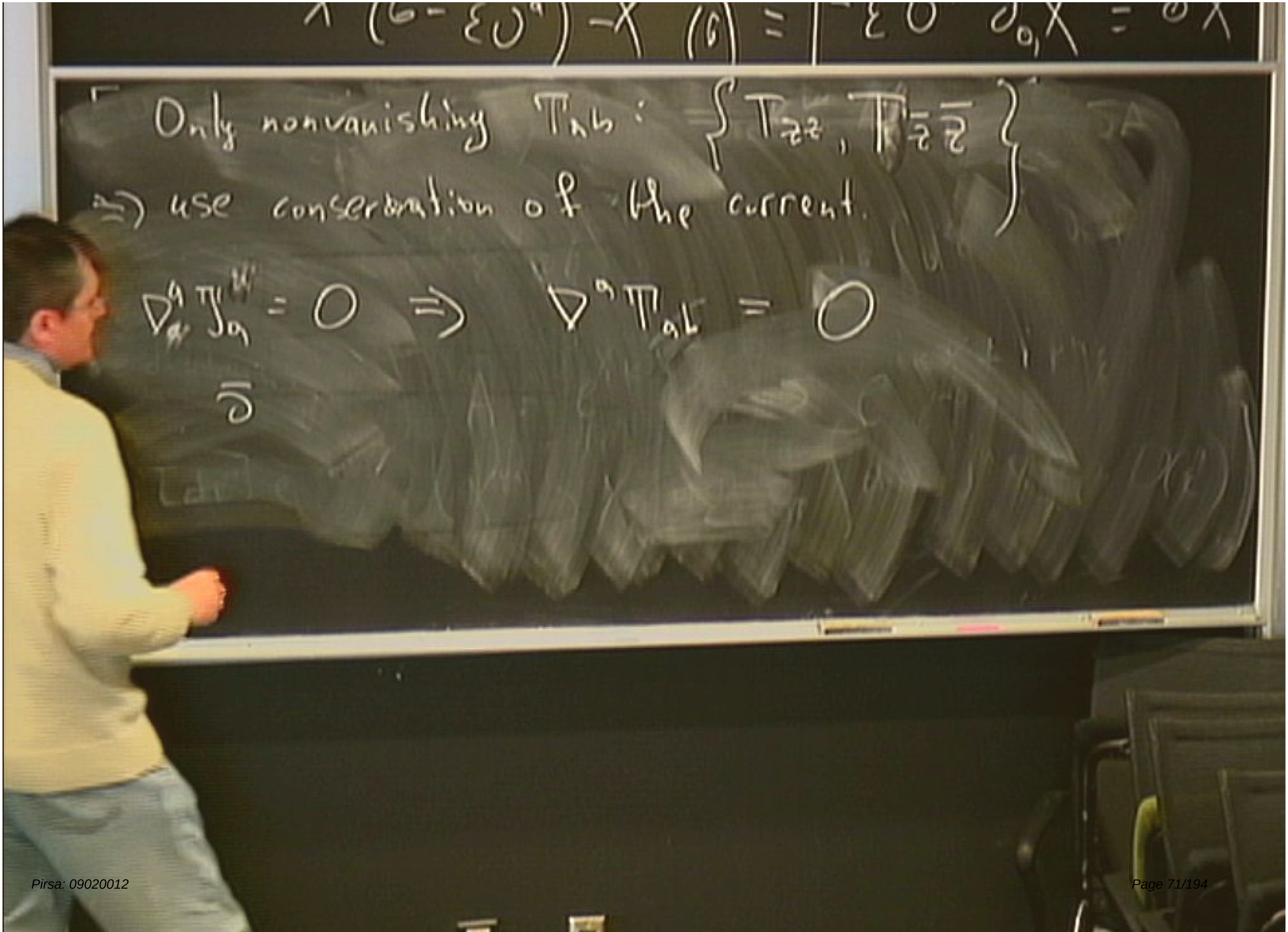
In CFT:
$T_{ab} = -\frac{1}{\alpha'} : \partial_a X^\mu \partial_b X_\mu - \frac{1}{2} \delta_{ab} (\partial X)^2 :$
$\Rightarrow$ Conformal inv
$g^{z\bar{z}} = 2$, $g^{\bar{z}z} = 2$
$T_a{}^a = 0 \Rightarrow T_{z\bar{z}} = 0 = T_{\bar{z}z}$


Only nonvanishing T_{ab} : $\{ T_{zz} , T_{\bar{z}\bar{z}} \}$


Only nonvanishing T_ab : { T_zz , T_z̄z̄ }
⇒ use conservation of the current.


Only nonvanishing T_{ab} : $\{ T_{zz}, T_{\bar{z}\bar{z}} \}$
$\Rightarrow$ use conservation of the current.
$\nabla^a T_{ab} = 0$


Only nonvanishing T_{ab} : $\{T_{zz}, T_{\bar{z}\bar{z}}\}$
⇒ use conservation of the current.
$\nabla^a J_a = 0 \Rightarrow \nabla^a T_{ab} = 0$


Only nonvanishing T_{ab} : $\{ T_{zz} , T_{\bar{z}\bar{z}} \}$
⇒ use conservation of the current.
$\nabla^a J_a = 0 \Rightarrow \nabla^a T_{ab} = 0$
$\bar{\partial}$

$$X(\sigma - \varepsilon \sigma^a) - X(\sigma) = -\varepsilon \sigma^a \partial_a X = \delta X$$

Only nonvanishing T_{ab} : $\{ T_{zz}, T_{\bar{z}\bar{z}} \}$

$\Rightarrow$ use conservation of the current.

$$\nabla^a J_a^{\mu} = 0 \Rightarrow \nabla^a T_{ab} = 0$$

$$\bar{\partial} T_{zb} + \partial$$

$$X^\mu(\sigma - \varepsilon\sigma^a) - X^\mu(\sigma) = -\varepsilon\sigma^a \partial_a X = \delta X$$

Only nonvanishing T_{ab} : $\left\{ T_{zz},\ T_{\bar z \bar z} \right\}$

$\Rightarrow$ use conservation of the current.

$$\nabla^a J_a = 0 \Rightarrow \nabla^a T_{ab} = 0$$

$$\bar\partial T_{zb} + \partial T_{\bar z b} = 0$$

$$X'(\sigma - \varepsilon \sigma^a) - X(\sigma) = -\varepsilon \sigma^a \partial_a X = \delta X$$

Only nonvanishing T_{ab} : $\{ T_{zz}, T_{\bar{z}\bar{z}} \}$

$\Rightarrow$ use conservation of the current.

$$\nabla^a J_a = 0 \Rightarrow \nabla^a T_{ab} = 0$$

$$\bar{\partial} T_{zb} + \partial T_{\bar{z}b} = 0$$

$b = z \Rightarrow \bar{\partial} T_{zz} = 0$

$b = \bar{z} \Rightarrow \partial T_{\bar{z}\bar{z}} = 0$

$\Rightarrow$ use conservation of the current.

$$\nabla_\mu J^\mu_a = 0 \Rightarrow \nabla^a T_{ab} = 0$$

$$\bar\partial T_{zb} + \partial T_{\bar z b} = 0 \qquad b = z \Rightarrow \bar\partial T_{zz} = 0$$

$$b = \bar z \Rightarrow \partial T_{\bar z \bar z} = 0$$

$$T_{zz} = T(z)$$

$$J^\mu_{\bar z} = \frac{i}{\alpha'} \bar\partial X^\mu$$

antiholomorphic

$\Rightarrow$ use conservation of the current.

$\nabla^a_\mu J^\mu_a = 0 \Rightarrow \nabla^a T_{ab} = 0$

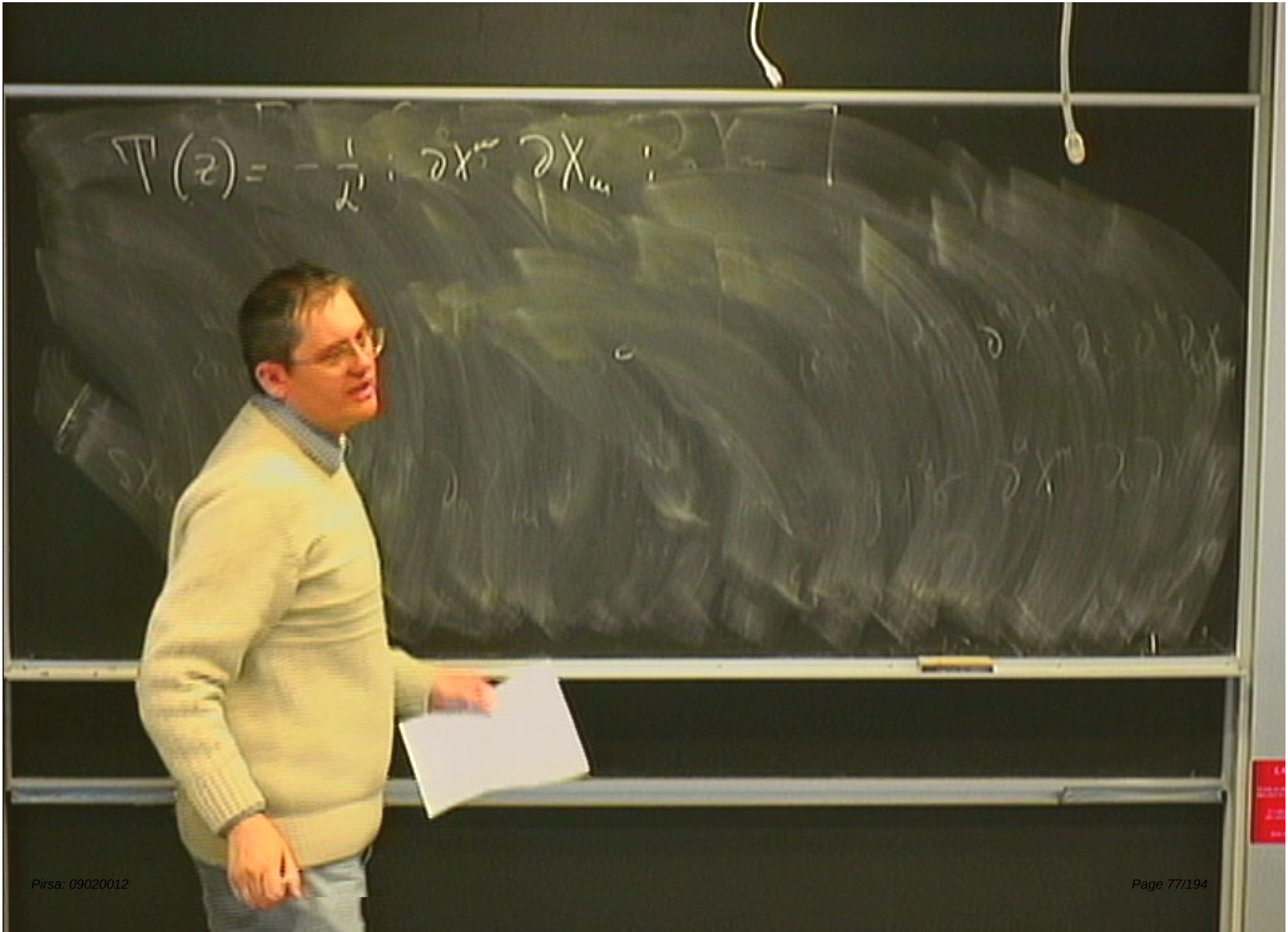
$\bar\partial T_{zb} + \partial T_{\bar z b} = 0$ / $b = z \Rightarrow \bar\partial T_{zz} = 0$

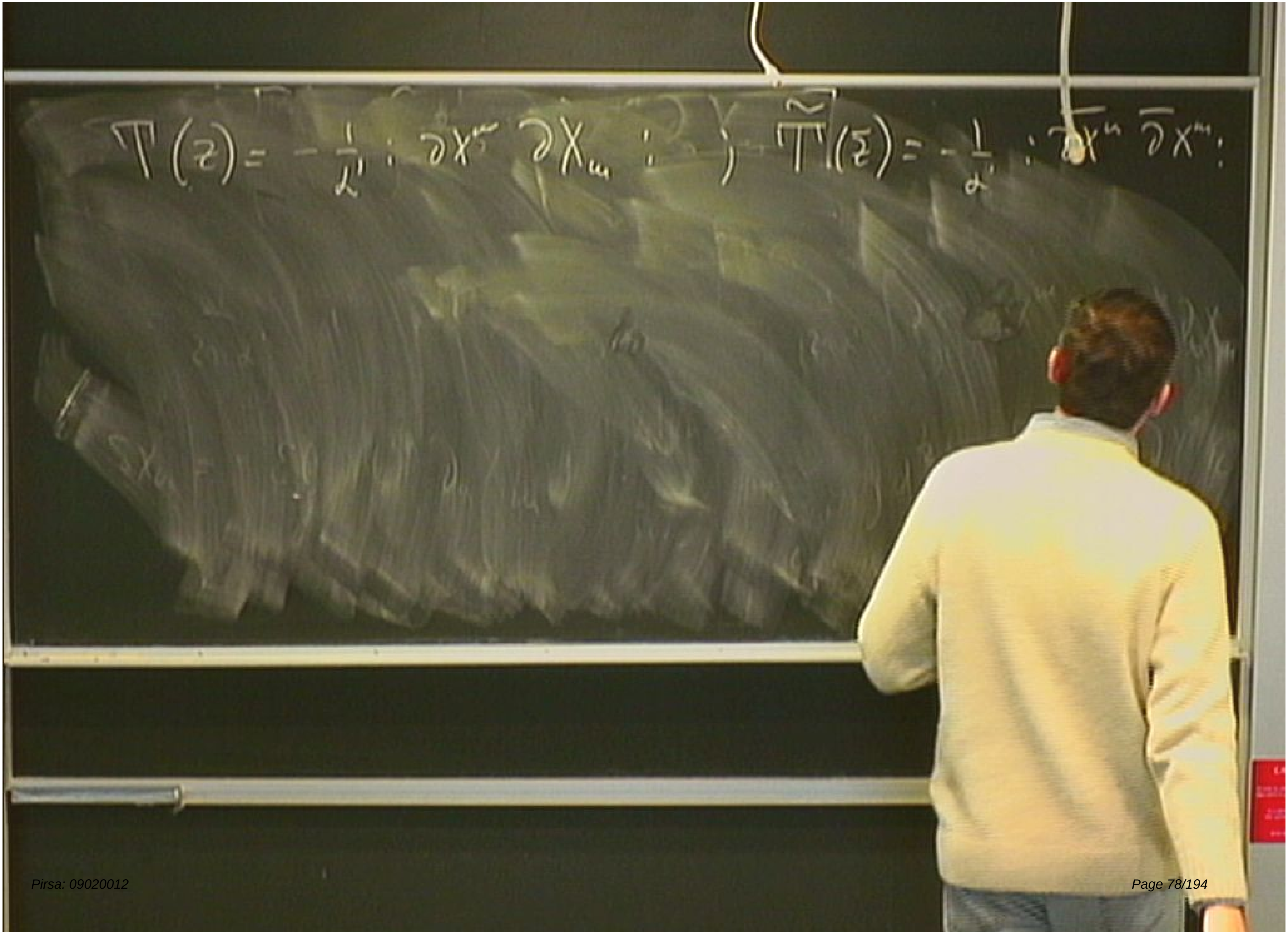
$b = \bar z \Rightarrow \partial T_{\bar z \bar z} = 0$

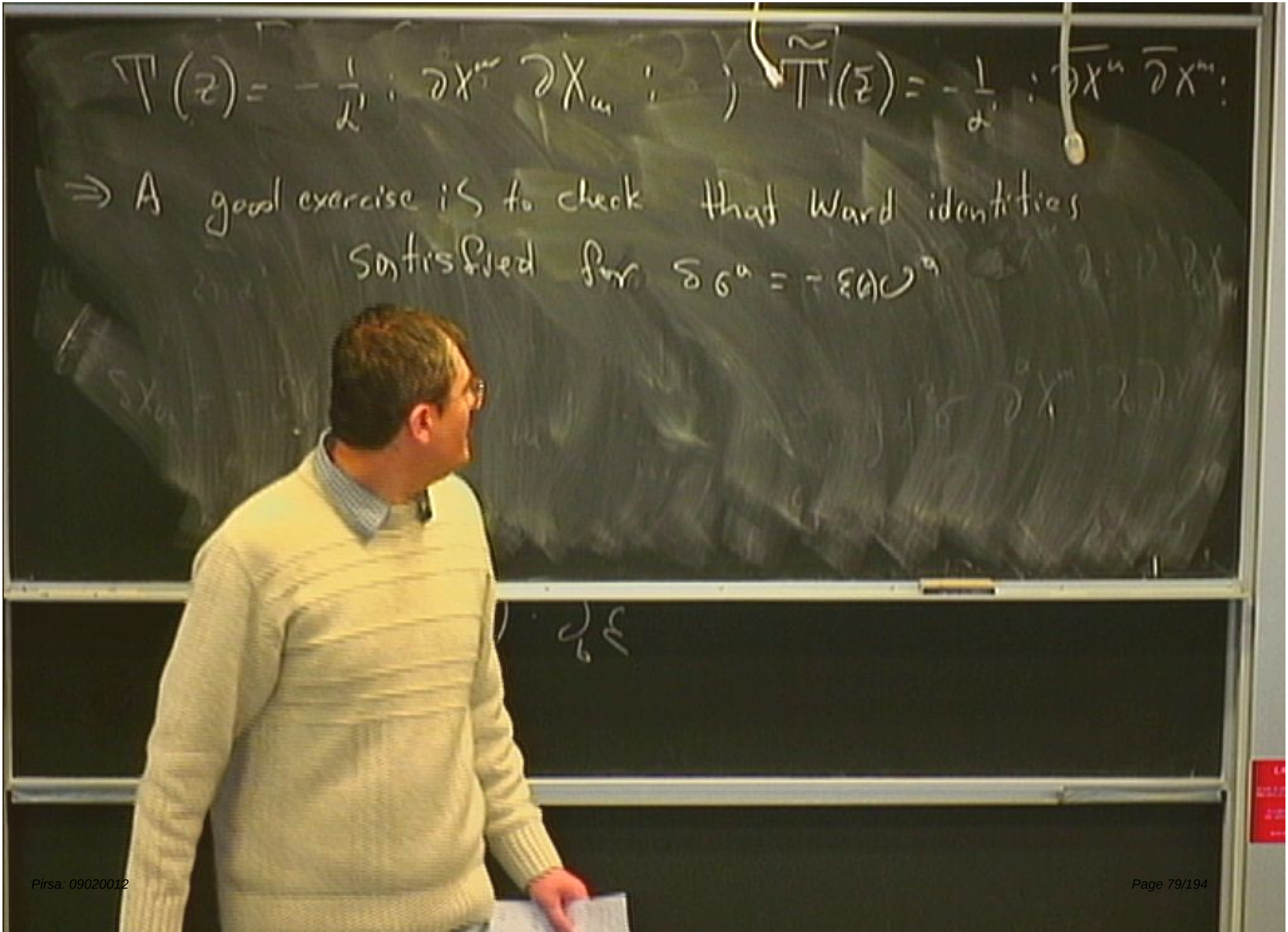
$T_{zz} = T(z)$ & $T_{\bar z \bar z} = \tilde T(\bar z)$

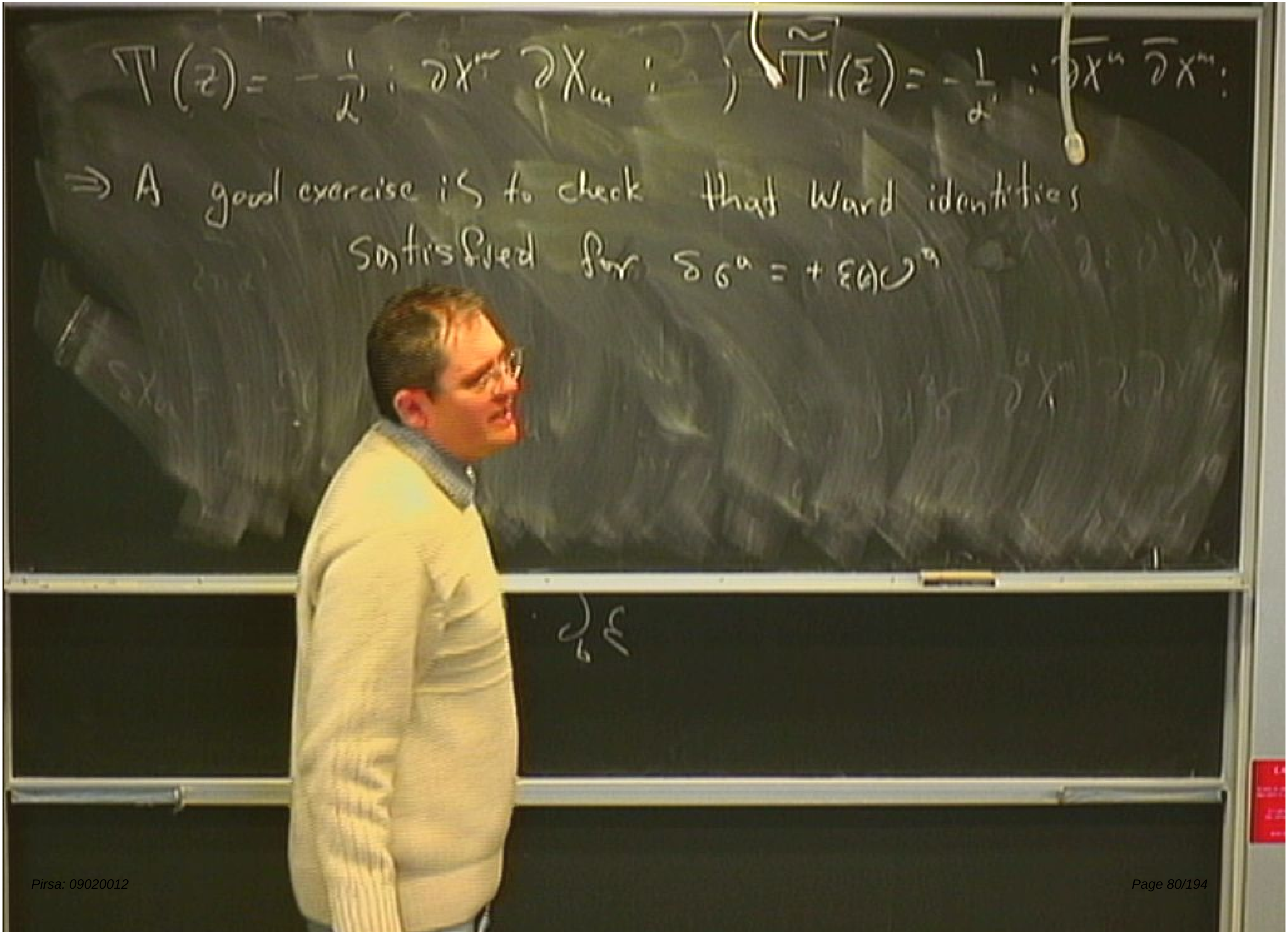
$J_{\bar z}^{\mu} = \frac{i}{\alpha'} \bar\partial X^\mu$

'antiholomorphic'


$T(z) = -\frac{1}{\alpha'} :\partial X^\mu \partial X_\mu:$


$T(z) = -\frac{1}{\alpha'} :\partial X^\mu \partial X_\mu: \quad \tilde{T}(\bar{z}) = -\frac{1}{\alpha'} :\bar{\partial} X^\mu \bar{\partial} X^\mu:$


$T(z) = -\frac{1}{\alpha'} :\partial X^{\mu} \partial X_{\mu}: \;,\; \tilde{T}(\bar{z}) = -\frac{1}{\alpha'} :\bar{\partial} X^{\mu} \bar{\partial} X_{\mu}:$
$\Rightarrow$ A good exercise is to check that Ward identities
satisfied for $\delta\sigma^a = -\epsilon(\sigma) v^a$


T(z) = -1/α' :∂X^μ ∂X_μ: ; T̃(z̄) = -1/α' :∂̄X^μ ∂̄X_μ:
⇒ A good exercise is to check that Ward identities
satisfied for δσ^a = + ε(σ)v^a

$$T(z) = -\frac{1}{\alpha'} :\partial X^\mu \partial X_\mu: \; ; \quad \tilde{T}(\bar z) = -\frac{1}{\alpha'} :\bar\partial X^\mu \bar\partial X_\mu:$$

$\Rightarrow$ A good exercise is to check that Ward identities satisfied for $\delta\sigma^a = +\varepsilon(\sigma) v^a$

$$j = i\, v^a T_{ab}$$

$$\Rightarrow -\frac{1}{2}(\partial X)^2 \cdot \partial_b \varepsilon$$

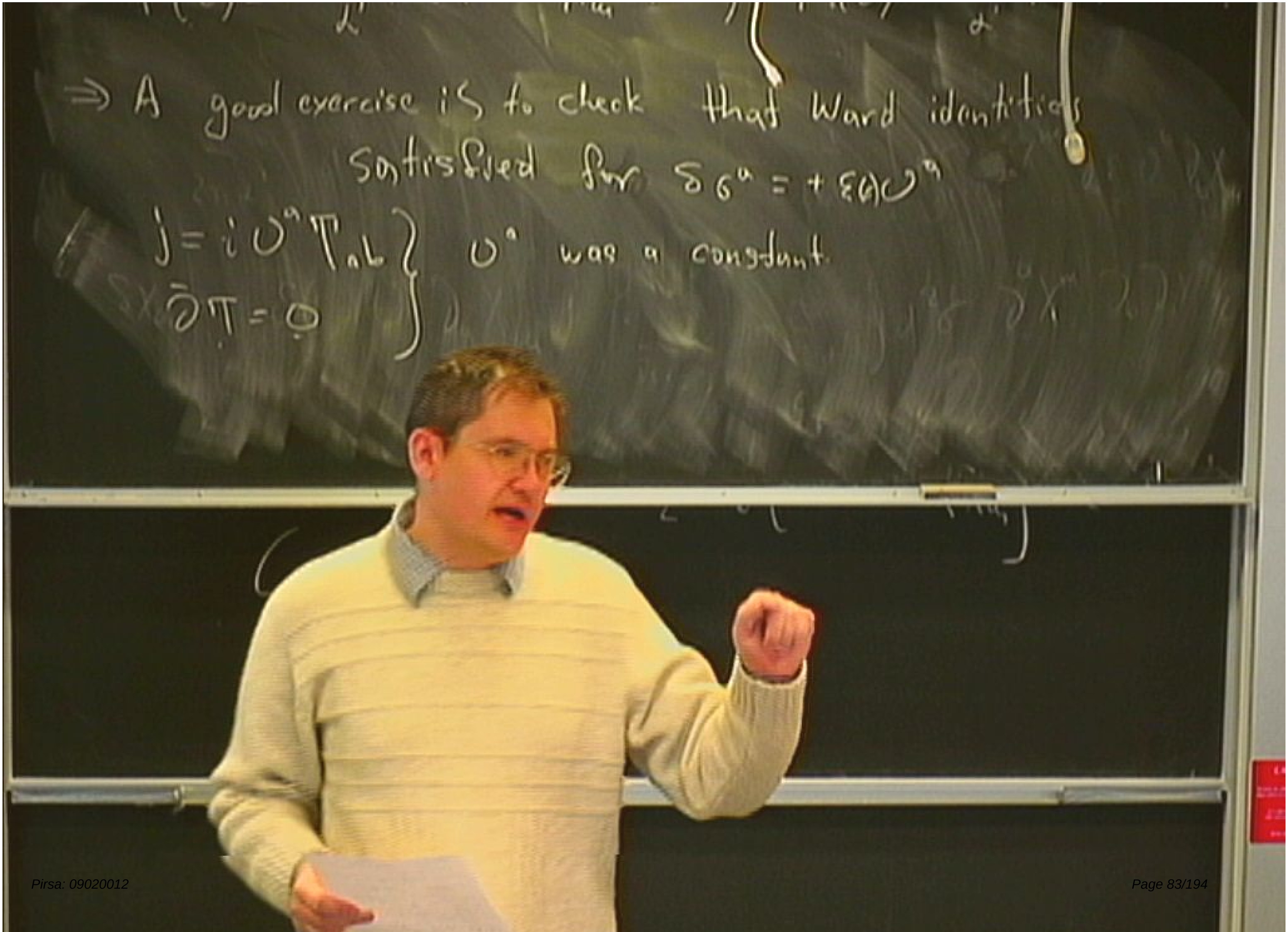
$$T(z) = -\frac{1}{\alpha'} : \partial X^{\mu} \partial X_{\mu} : \; ; \; \tilde{T}(\bar{z}) = -\frac{1}{\alpha'} : \bar{\partial} X^{\mu} \bar{\partial} X^{\mu} :$$

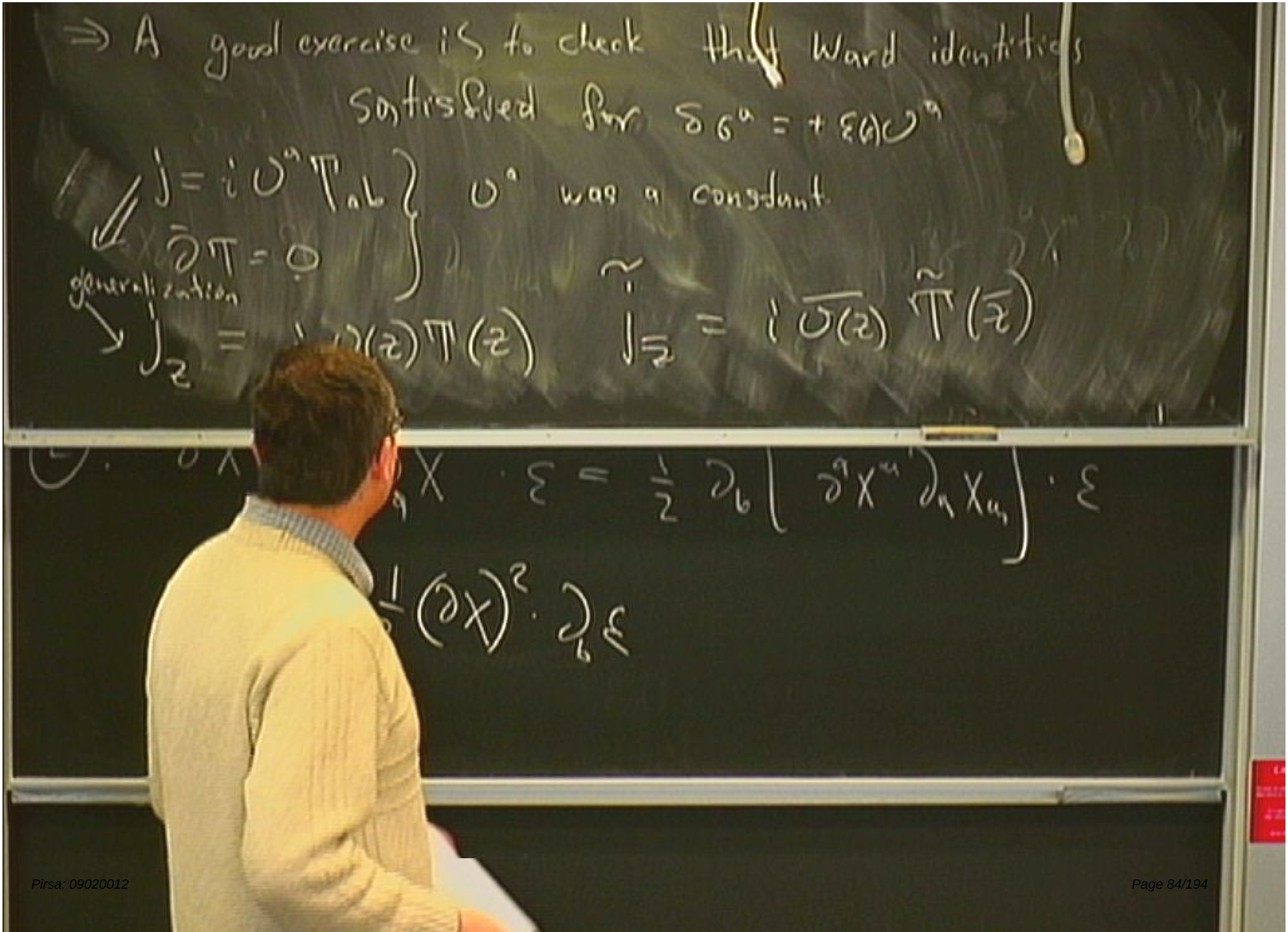
$\Rightarrow$ A good exercise is to check that Ward identities satisfied for $\delta \sigma^a = + \varepsilon(\sigma) v^a$

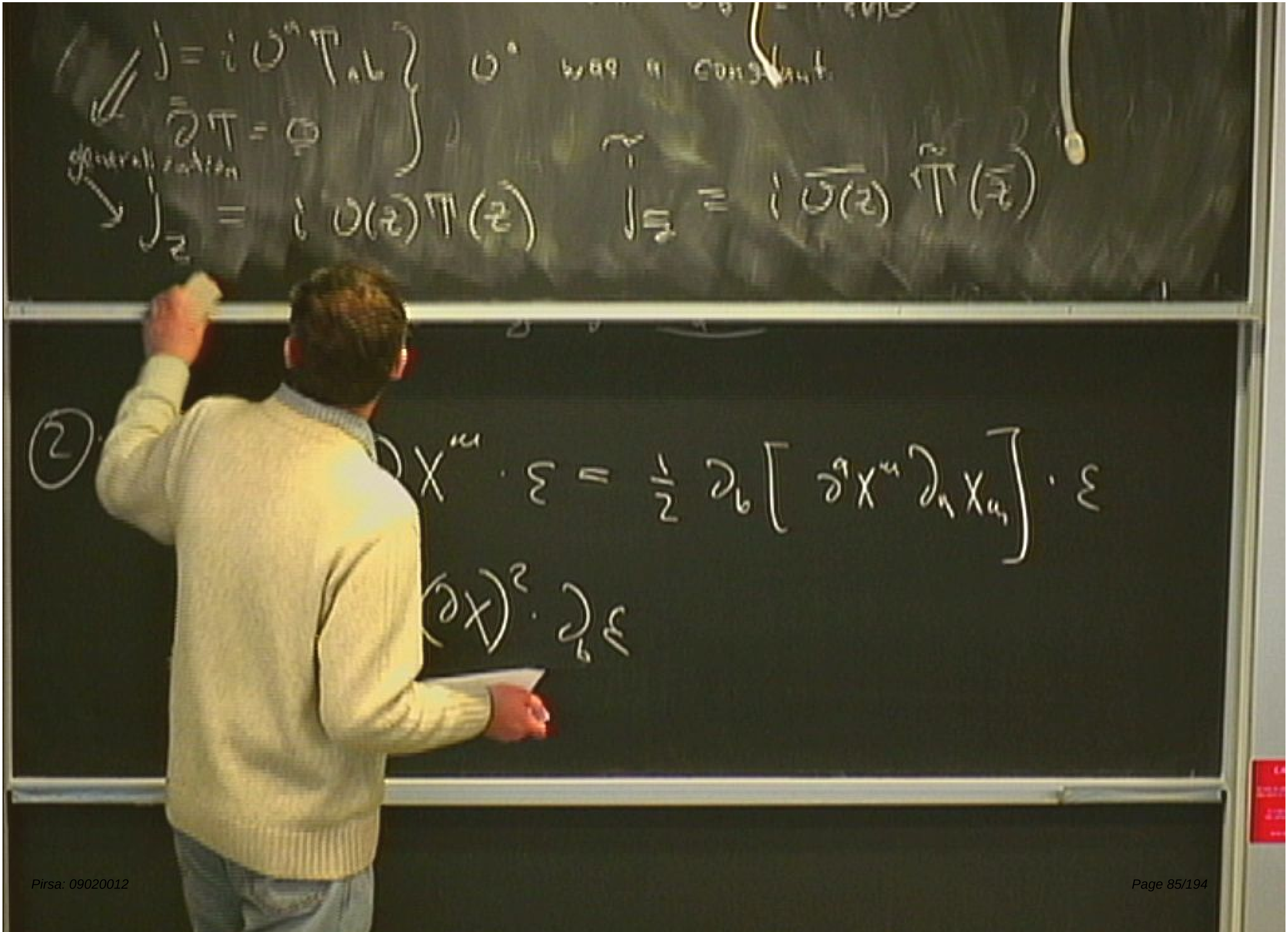
$j = i v^a T_{ab}$

$\bar{\partial} T = 0$

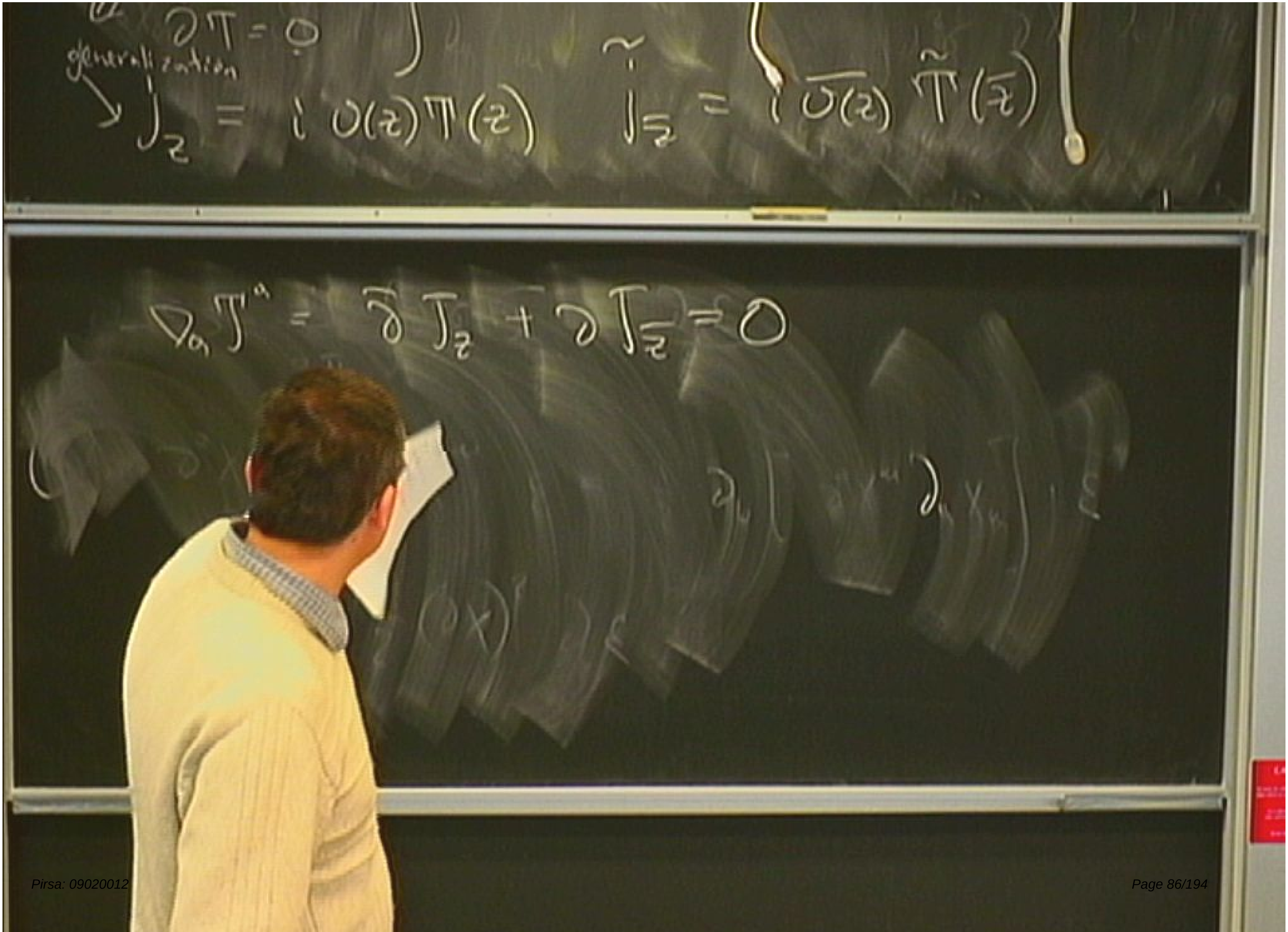
$\rightarrow \; -\frac{1}{2} (\partial X)^2 \cdot \partial_b \varepsilon$


⇒ A good exercise is to check that Ward identities
satisfied for δσ^a = + ε(σ)υ^a
j = i υ^a T_ab
υ^a was a constant.
∂T = 0


⇒ A good exercise is to check that Ward identities
satisfied for δσ^a = + ε(σ)v^a
j = i v^a T_ab
v^a was a constant
∂̄T = 0
generalization
j_z = i v(z) T(z)
j̃_z̄ = i v̄(z̄) T̃(z̄)



was a constant
generalization
②


∂T = 0
generalization
j_z = i v(z) T(z)
j̃_z̄ = i v̄(z̄) T̃(z̄)
∇_a j^a = ∂̄ j_z + ∂ j_z̄ = 0

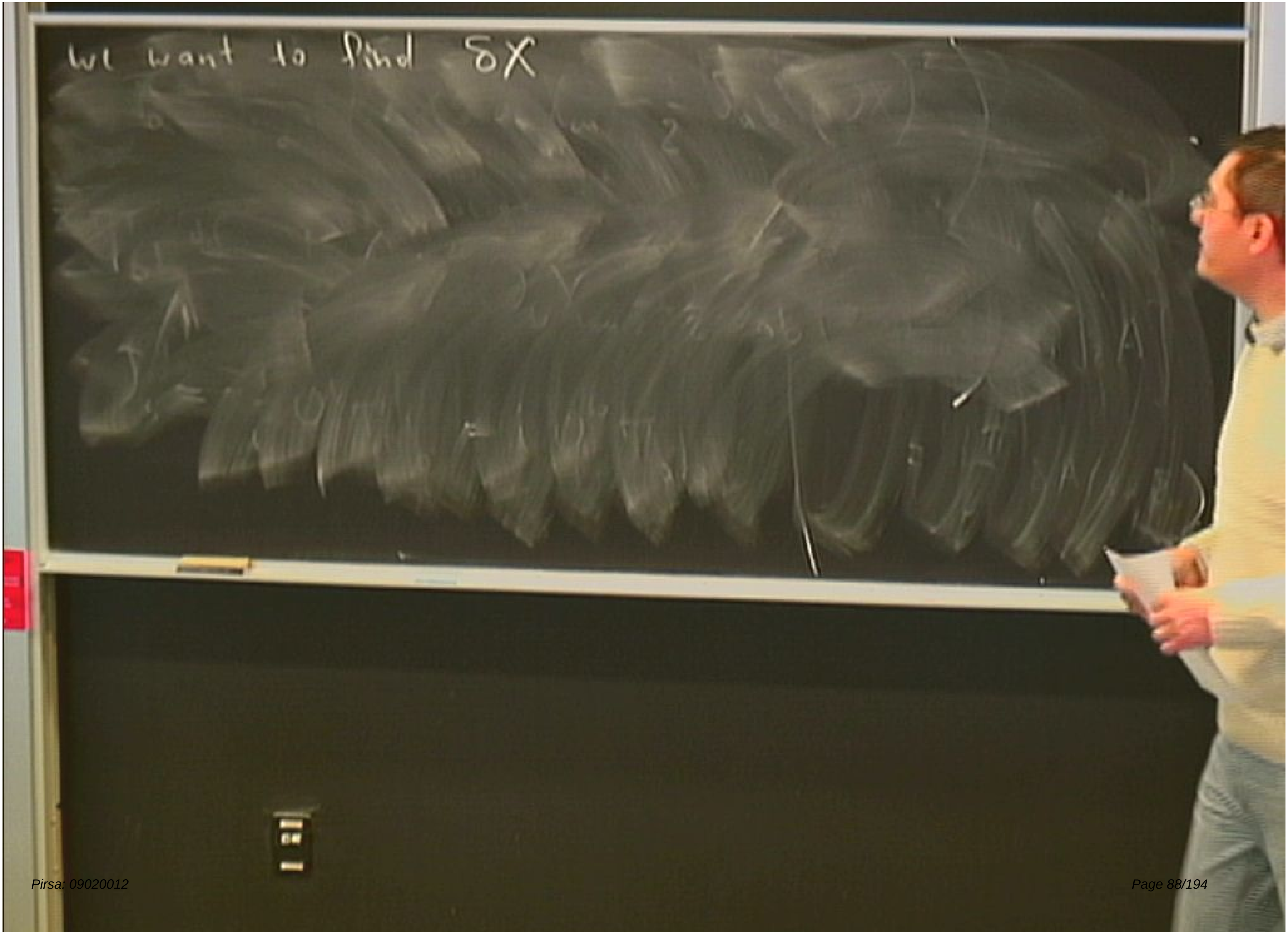
$\partial T = 0$

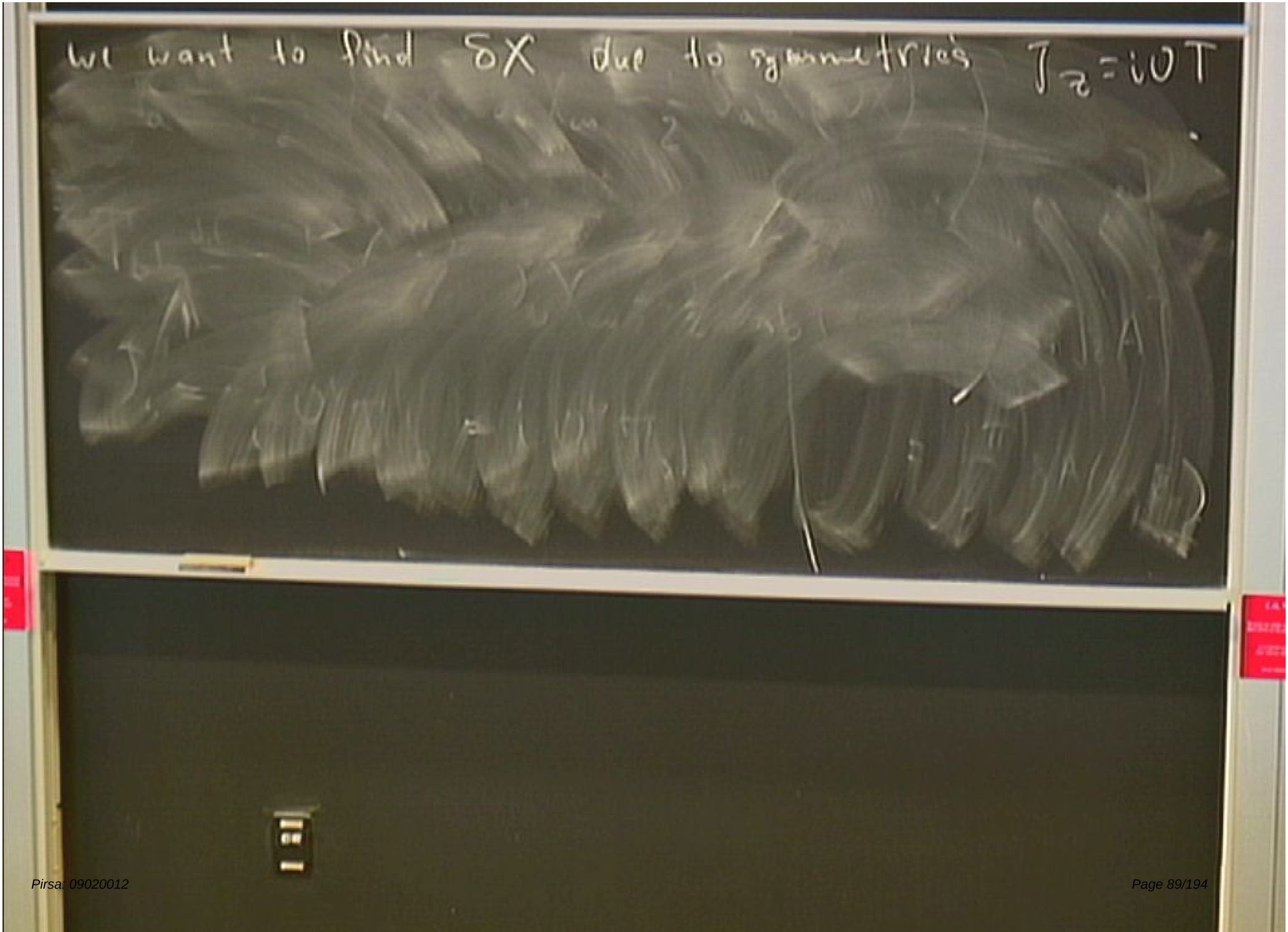
generalization

$\rightarrow j_z = i\, \upsilon(z) T(z) \qquad \tilde{j}_{\bar{z}} = i\, \overline{\upsilon(z)}\, \tilde{T}(\bar{z})$

$$\nabla_a J^a = \bar{\partial} J_z + \partial J_{\bar{z}} = 0$$

$\Rightarrow$ The new currents are conserved provided $\upsilon(z)$ is holomorphic


we want to find δX


we want to find δX due to symmetries $J_z = i\theta T$

we want to find δX due to symmetries $\quad J_{\bar z} = i \bar{\partial} T$

$$\operatorname*{Res}_{z\to z_0} j_z A + \overline{\operatorname*{Res}_{\bar z \to \bar z_0}} \, \tilde{j}_{\bar z} A = \frac{1}{i\varepsilon}\, \delta A$$

we want to find δX due to symmetries $\quad J_{\bar{z}} = i\partial T$

$$\operatorname*{Res}_{z \to z_0} j_z A + \operatorname*{\overline{Res}}_{\bar{z} \to \bar{z}_0} \tilde{j}_{\bar{z}} A = \frac{1}{i\varepsilon} \delta A$$

we want to find δX due to symmetries $J_{\bar{z}} = i\sigma T$

$$\operatorname*{Res}_{z \to z_0} j_z A + \operatorname*{\overline{Res}}_{\bar{z} \to \bar{z}_0} \tilde{j}_{\bar{z}} A = \frac{1}{i\epsilon} \delta A$$

$\Rightarrow A = X^{\mu}$

$$\operatorname{Res}_{z\to z_0} j_z A + \overline{\operatorname{Res}}_{\bar z\to \bar z_0} \tilde{j}_{\bar z} A = \frac{1}{i\varepsilon} \delta A$$

$$\Rightarrow A = X^{\mu}$$

$$j_z = i v \pi$$

$\Rightarrow A = X$

$j_{\bullet} = i\upsilon T$

$:T(z): \; :X^{\mu}(z_0, \bar{z}_0)$

$T_a{}^a = 0 \Rightarrow T_{z\bar{z}} = 0 = T_{\bar{z}z}$

$\Rightarrow A = X$

$j_a = i \partial T$

$:T(z): \; :X^{\mu}(z_0, \bar{z}_0): \; = -\frac{1}{\alpha'} :\partial X \partial X(z) \; : X^{\mu}(z_0, \bar{z}_0):$

$T_a{}^a = 0 \Rightarrow T_{z\bar{z}} = 0 = T_{\bar{z}z}$

$j_\bullet = i\partial T$

$:T(z):\; :X^\mu(z_0,\bar z_0): \sim -\frac{1}{\alpha'} :\partial X \partial X(z): \; :X^\mu(z_0,\bar z_0):$

$\sim -\left(\frac{1}{\alpha'}\right) \cdot$

$T_{ab} = \alpha' : \partial_a X \partial_b X_\mu - \frac{1}{2}\delta_{ab}(\partial X)^2 :$

$\Rightarrow$ Conformal inv

$g^{z\bar z} = 2, \quad g^{\bar z z} = 2$

$T_a{}^a = 0 \Rightarrow T_{z\bar z} = 0 = T_{\bar z z}$

$$:T(z):\ :X^{\mu}(z_0,\bar z_0): = -\frac{1}{\alpha'} :\partial X \partial X(z) : X^{\mu}(z_0,\bar z_0):$$

$$\sim -\left(\frac{1}{\alpha'}\right)\cdot\left(-\frac{\alpha'}{2}\ln|z-z_0|^2\right)$$

$$T_{ab} = -\frac{1}{\alpha'} :\partial_a X^{\mu}\partial_b X_{\mu} - \frac{1}{2}\delta_{ab}(\partial X)^2:$$

$\Rightarrow$ Conformal inv

$$g^{z\bar z} = 2,\quad g^{\bar z z} = 2$$

$$T_a{}^a = 0 \Rightarrow T_{z\bar z} = 0 = T_{\bar z z}$$

$$:T(z):\ :X(z_0,\bar z_0):\ = -\frac{1}{\alpha'} :\partial X \partial X(z) : \ :X^{m}(z_0,\bar z_0):$$

$$\sim -\left(\frac{1}{\alpha'}\right)\cdot\left(-\frac{\alpha'}{2}\ln|z-z_0|^2\right)$$

$$T_{ab} = -\frac{1}{\alpha'} :\partial_a X^{m} \partial_b X_m - \frac{1}{2}\delta_{ab}(\partial X)^2 :$$

$\Rightarrow$ Conformal inv

$$g^{z\bar z} = 2 \ , \ g^{\bar z z} = 2$$

$$T_a{}^a = 0 \Rightarrow T_{z\bar z} = 0 = T_{\bar z z}$$

$$:T(z):\ :X^{\mu}(z_0,\bar z_0): = -\frac{1}{\alpha'} :\partial X \partial X(z) : \ :X^{\mu}(z_0,\bar z_0):$$

$$\sim -\left(\frac{1}{\alpha'}\right)\cdot\left(-\frac{\alpha'}{2}\partial_z \ln (z-z_0)^2\right)\cdot 2$$

$$T_{ab} = -\frac{1}{\alpha'} :\partial_a X^{\mu}\partial_b X_{\mu} - \frac{1}{2}\delta_{ab}(\partial X)^2:$$

$\Rightarrow$ Conformal inv

$$g^{z\bar z} = 2,\quad g^{\bar z z} = 2$$

$$T_a{}^a = 0 \Rightarrow T_{z\bar z} = 0 = T_{\bar z z}$$

$$:T(z):\ :X^{\mu}(z_0,\bar z_0): = -\frac{1}{\alpha'} :\partial X \partial X(z): \ :X^{\mu}(z_0,\bar z_0):$$

$$\sim -\left(\frac{1}{\alpha'}\right)\cdot\left(-\frac{\alpha'}{2}\partial_z \ln(z-z_0)^2\right)\cdot 2\ :\partial X(z_0):$$

$$T_{ab} = - \quad \partial_a X^{\mu}\, \partial_b X_{\mu} - \frac{1}{2}\delta_{ab}(\partial X)^2 :$$

$\Rightarrow$ Confor

$$g^{z\bar z} = 2 , \quad g^{\bar z z} = 2$$

$$T_a{}^a$$

$$T_{z\bar z} = 0 = T_{\bar z z}$$

$$\sim \left(\frac{1}{\alpha'}\right)\left(-\frac{\alpha'}{2}\partial_z \ln(z-z_0)^2\right)\cdot 2 \; :\partial X(z_0):$$

$T' X'' \sim$

$$\sim -\left(\frac{1}{\alpha'}\right)\left(-\frac{\alpha'}{2}\partial_z \ln |z-z_0|^2\right)\cdot 2 \;:\partial X(z_0):$$

$$\Pi' X'' \sim \frac{1}{z-z_0}\,\partial X(z_0)$$

$$\sim \left(\frac{1}{\alpha'}\right)\left(-\frac{\alpha'}{2}\partial_z \ln(z-z_0)^2\right)\cdot 2 \;:\partial X(z_0):$$

$$T' X^m \sim \frac{1}{z-z_0}\,\partial X(z_0)$$

$$\tilde{T}\, X^m$$

$$\sim -\left(\frac{1}{\alpha'}\right)\left(-\frac{\alpha'}{2}\partial_z \ln(z-z_0)^2\right)\cdot 2 \; :\partial X(z_0):$$

$$T\, X^{\mu} \sim \frac{1}{z-z_0}\,\partial X(z_0)$$

$$\tilde{T}\, X^{\mu} \sim \frac{1}{\bar{z}-\bar{z}_0}\,\bar{\partial} X(\bar{z}_0)$$

$$:T(z): \; :X(z_0,\bar z_0): \; \sim \; -\frac{1}{\alpha'} \; \partial X \partial X(z) \; :X(z_0,\bar z_0):$$

$$\sim -\left(\frac{1}{\alpha'}\right)\left(-\frac{\alpha'}{2}\partial_z \ln (z-z_0)^2\right)\cdot 2 \; :\partial X(z_0):$$

$$T X^{\mu} \sim \frac{1}{z-z_0}\,\partial X(z_0)$$

$$\tilde{T} X^{\mu} \sim \frac{1}{\bar z - \bar z_0}\,\bar\partial X(\bar z_0)$$

$$\partial X(z_0)$$

$$\sim \left(\frac{1}{\alpha'}\right) \left(-\frac{\alpha'}{2}\partial_z \ln(z-z_0)^2\right) \cdot 2 \; :\partial X(z_0):$$

$$T' X^{\mu} \sim \frac{1}{z-z_0} \partial X(z_0)$$

$$\tilde{T} X^{\mu} \sim \frac{1}{\bar{z}-\bar{z}_0} \bar{\partial} X(\bar{z}_0)$$

$$i\theta \left(\partial X(z_0) + \bar{\partial} X(\bar{z}_0) \right)$$

$$\sim \left(\frac{1}{\alpha'}\right)\left(-\frac{\alpha'}{2}\partial_z \ln(z-z_0)^2\right)\cdot 2 \; :\partial X(z_0):$$

$$T X \sim \frac{1}{z-z_0}\,\partial X(z_0)$$

$$\tilde{T} X \sim \frac{1}{\bar{z}-\bar{z}_0}\,\bar{\partial} X(\bar{z}_0)$$

$$i\theta\,\partial X(z_0) + i\bar{\theta}\,\bar{\partial} X(\bar{z}_0)$$

$$\sim \left(\frac{1}{\alpha'}\right)\left(-\frac{\alpha'}{2}\partial_z \ln|z-z_0|^2\right)\cdot 2 \; :\partial X(z_0):$$

$$T\, X^\mu \sim \frac{1}{z-z_0}\,\partial X(z_0)$$

$$\tilde{T}\, X^\mu \sim \frac{1}{\bar z-\bar z_0}\,\bar\partial X(\bar z_0)$$

$$i\vartheta\, \partial X(z_0) + i\bar\vartheta\,\bar\partial X(\bar z_0) = \frac{1}{i\varepsilon}\,\delta X$$

$$\delta X^{\mu\nu} = - \varepsilon \, \mathcal{O}(z_0)$$

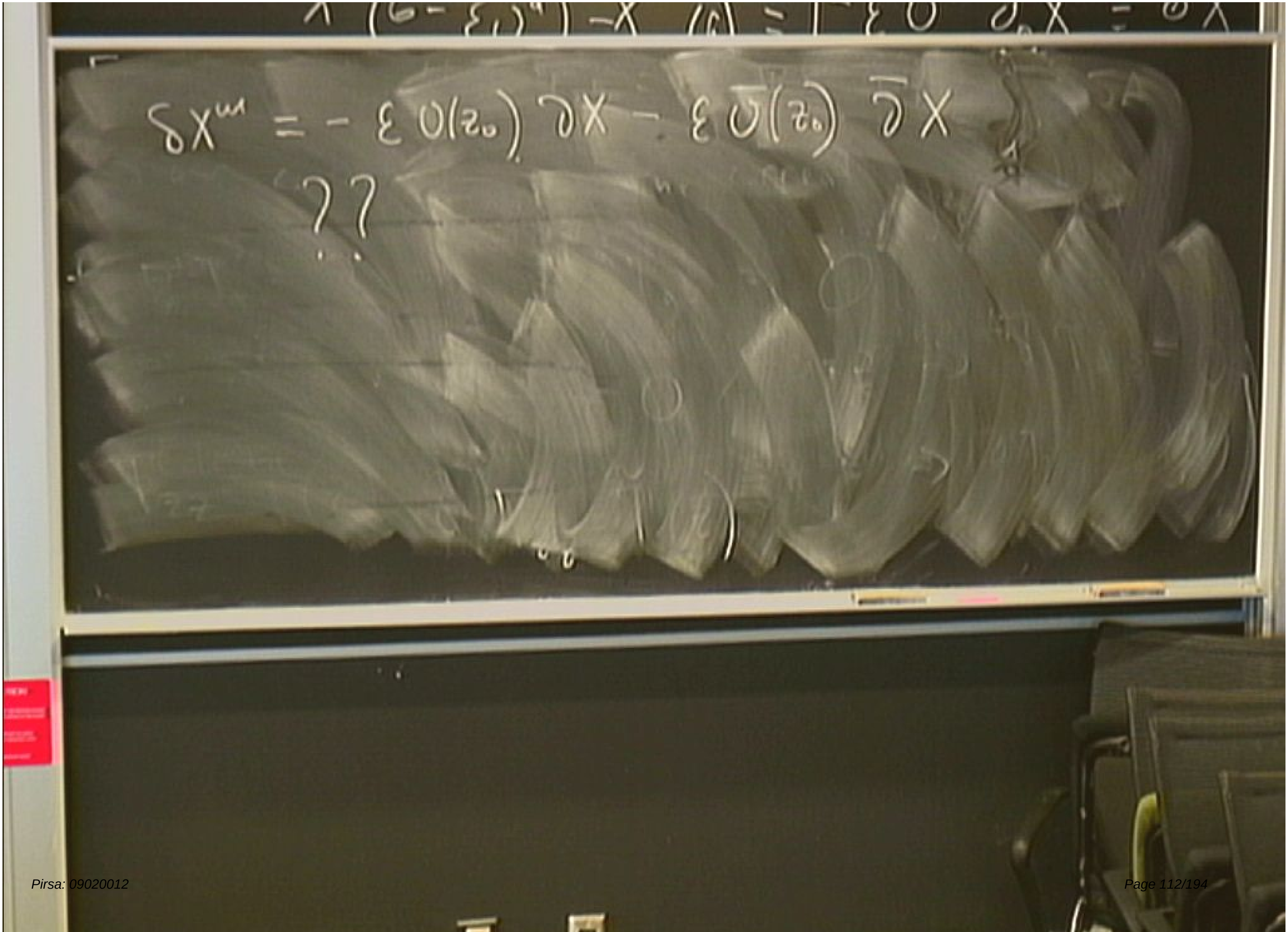
$$\sim -\left(\frac{1}{\alpha'}\right)\cdot\left(-\frac{\alpha'}{2}\partial_z \ln (z-z_0)^2\right)\cdot 2 \; :\partial X(z_0):$$

$$T' X^{m} \sim \frac{1}{z-z_0}\,\partial X(z_0)$$

$$\tilde{T}\, X^{m} \sim \frac{1}{\bar{z}-\bar{z}_0}\,\bar{\partial} X(\bar{z}_0)$$

$$i\,v(z)\,\partial X(z_0) + i\,\bar{v}(\bar{z})\,\bar{\partial} X(\bar{z}_0) = \frac{1}{i\varepsilon}\,\delta X$$

$$\delta X^{\mu} = -\,\varepsilon\, v(z_0)\,\partial X - \varepsilon\, \bar{v}(z_0)\,\bar{\partial} X$$


$\delta X^{\mu} = -\varepsilon\, v(z_0)\, \partial X - \varepsilon\, \bar{v}(\bar{z}_0)\, \bar{\partial} X$
??

$$\delta X^{\mu} = -\varepsilon\, v(z_0)\, \partial X - \bar{\varepsilon}\, \bar{v}(\bar{z}_0)\, \bar{\partial} X$$

??

$\varepsilon \to$

$$\delta X^{\mu} = -\varepsilon\, v(z_0)\, \partial X - \varepsilon\, \bar{v}(z_0)\, \bar{\partial} X$$

??

$$z_0 \to z_0 + \varepsilon\, v(z_0)$$

$$\delta X^{\mu}(z_\mu) = -\,\varepsilon\, v(z_\mu)\,\partial X - \varepsilon\,\bar{v}(\bar{z}_\mu)\,\bar{\partial} X$$

??

$$z_\mu \to z_\mu + \varepsilon\, v(z_\mu)$$

$$\delta X^{\mu}(z_\mu) = -\varepsilon\, v(z_\mu)\, \partial X - \varepsilon\, \bar{v}(\bar{z}_n)\, \bar{\partial} X$$

??

$$z_\mu \to z_\mu + \varepsilon\, v(z_\mu)$$

$$\delta X^{\mu} = X^{\mu\prime}(z', \bar{z}') - X^{\mu}(z, \bar{z}) = X^{\mu}(z - \varepsilon v, \bar{z} - \varepsilon \bar{v}) - X^{\mu}$$

anti holomorphic

$$\delta X^\mu(z_\mu) = -\varepsilon v(z_\mu)\,\partial X - \varepsilon \bar{v}(\bar{z}_\mu)\,\bar{\partial} X$$

??

$$z_\mu \to z_\mu + \varepsilon v(z_\mu)$$

$$\delta X^\mu = X^{\mu\prime}(z', \bar{z}') - X^\mu(z, \bar{z}) = X^\mu(z - \varepsilon v, \bar{z} - \varepsilon \bar{v}) - X^\mu$$

$$= -\varepsilon v\,\partial X - \varepsilon\,\bar{v}\,\bar{\partial} X$$

antiholomorphic

$$\delta X^{\mu}(z_n) = -\varepsilon\, v(z_n)\, \partial X - \varepsilon\, \bar{v}(\bar{z}_n)\, \bar{\partial} X$$

??

$$z_n \to z_n + \varepsilon\, v(z_n)$$

$$\delta X^{\mu} = X'^{\mu}(z', \bar{z}') - X^{\mu}(z, \bar{z}) = X^{\mu}(z - \varepsilon v, \bar{z} - \varepsilon \bar{v}) - X^{\mu}$$

$$= -\varepsilon\, v\, \partial X - \varepsilon\, \bar{v}\, \bar{\partial} X$$

anti holomorphic

$$\delta X^{\mu}(z_\mu) = -\varepsilon\, v(z_\mu)\, \partial X - \varepsilon\, \bar{v}(\bar{z}_\mu)\, \bar{\partial} X$$

??

$$z_\mu \to z_\mu + \varepsilon\, v(z_\mu) \longrightarrow \text{infinitesimal transf}$$

$$z \to z' = f(z)$$

$$\delta X^{\mu} = X^{\mu\prime}(z', \bar{z}') - X^{\mu}(z, \bar{z}) = X^{\mu}(z - \varepsilon v, \bar{z} - \varepsilon \bar{v}) - X^{\mu}$$

$$= -\varepsilon\, v\, \partial X - \varepsilon\, \bar{v}\, \bar{\partial} X$$

antiholomorphic

$$z_\mu \to z_\mu + \epsilon v(z_\mu)$$

$$z \to z' = z - \epsilon v(z)$$

$$\delta X^\mu = X^{\mu\prime}(z', \bar z') - X^\mu(z, \bar z) = X^\mu(z - \epsilon v, \bar z - \epsilon \bar v) - X^\mu$$

$$= -\epsilon v \partial X - \epsilon \bar v \bar\partial X$$

$$J^a \Rightarrow J_a^\mu = a_\mu J^\mu_a$$

holomorphic

$$\boxed{+ \frac{i}{\alpha'} \partial_a X^\mu} \Rightarrow J_z^\mu = \frac{i}{\alpha'} \partial X^\mu$$

$$\tilde J_{\bar z}^\mu = \frac{i}{\alpha'} \bar\partial X^\mu$$

antiholomorphic

$$= -\xi\, \omega\, \partial X - \bar{\xi}\, \bar{\omega}\, \bar{\partial} X)$$

$$f(z) = \xi\, z$$

$$= -\varepsilon\, v\, \partial X - \bar{\varepsilon}\, \bar{v}\, \bar{\partial} X)$$

$$f(z) = \xi\, z$$

If $|\xi| = 1 \rightarrow$ just a rotation

$$= -\xi \, \omega \, \partial X - \bar{\xi} \, \bar{\omega} \, \bar{\partial} X \,)$$

$$f(z) = \xi z$$

If $|\xi| = 1 \longrightarrow$ just a rotation

But in general: rot + scaling

$$= -\xi\, \omega\, \partial X - \bar{\xi}\, \bar{\omega}\, \bar{\partial} X)$$

$f(z) = \xi z$

If $|\xi| = 1 \rightarrow$ just a rotation

But in general: rot + scaling

$$\frac{y'}{y'^2} = \frac{y''}{y^2}$$

$$= -\xi\, \omega\, \partial X - \bar{\xi}\, \bar{\omega}\, \bar{\partial} X)$$

$f(z) = \xi z$

If $|\xi| = 1 \rightarrow$ just a rotation

But in general: rot + scaling

$$\frac{y'}{|y'|} = \frac{y^m}{|y|^2}$$

$$y^m \rightarrow \lambda \cdot y^m$$

$$= -\xi\, v\, \partial X - \bar{\xi}\,(\bar{v}\,\bar{\partial} X)$$

$$f(z) = \xi z$$

If $|\xi| = 1 \rightarrow$ just a rotation

But in general: rot + scaling

$$\frac{y'^{\,1}}{y'^{\,2}} = \frac{y^1}{y^2}$$ spatial conformal transf.

$$y^m \rightarrow \lambda \cdot y^m$$

$$= -\xi\, \omega\, \partial X - \bar{\xi}\, \bar{\omega}\, \bar{\partial} X$$

$f(z) = \xi z$

If $|\xi| = 1 \Rightarrow$ just a rotation

But in general: rot + scaling

$$\frac{y^{\mu\prime}}{\sqrt{y^{\prime 2}}} = \frac{y^{\mu}}{\sqrt{y^{2}}}$$ special conformal transf.

$y^{\mu} \to \lambda \cdot y^{\mu}$

$$\delta X^{\mu}(z_p) = -\varepsilon\, v(z_p)\, \partial X - \varepsilon\, \bar{v}(\bar{z}_p)\, \bar{\partial} X$$

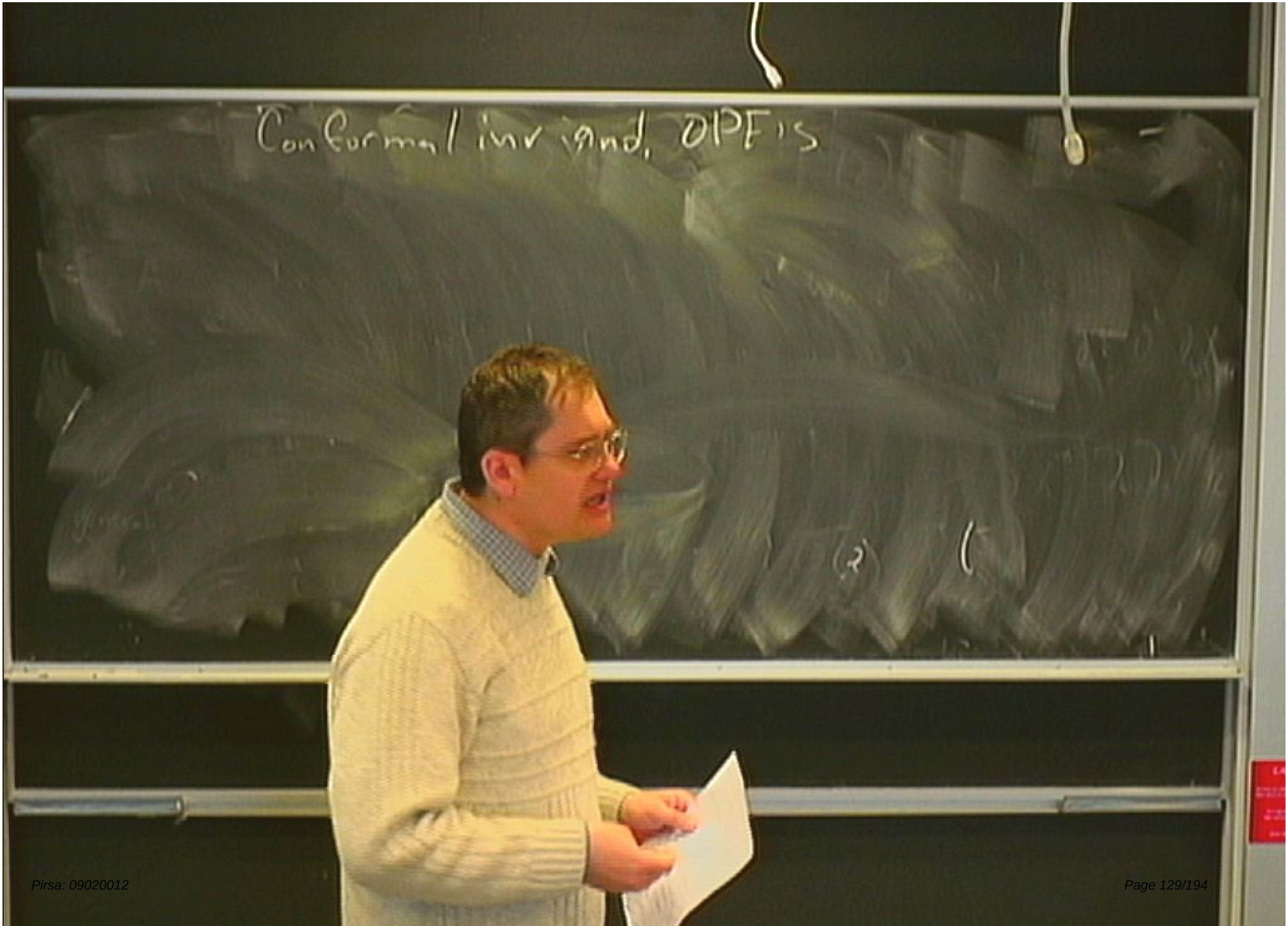
??

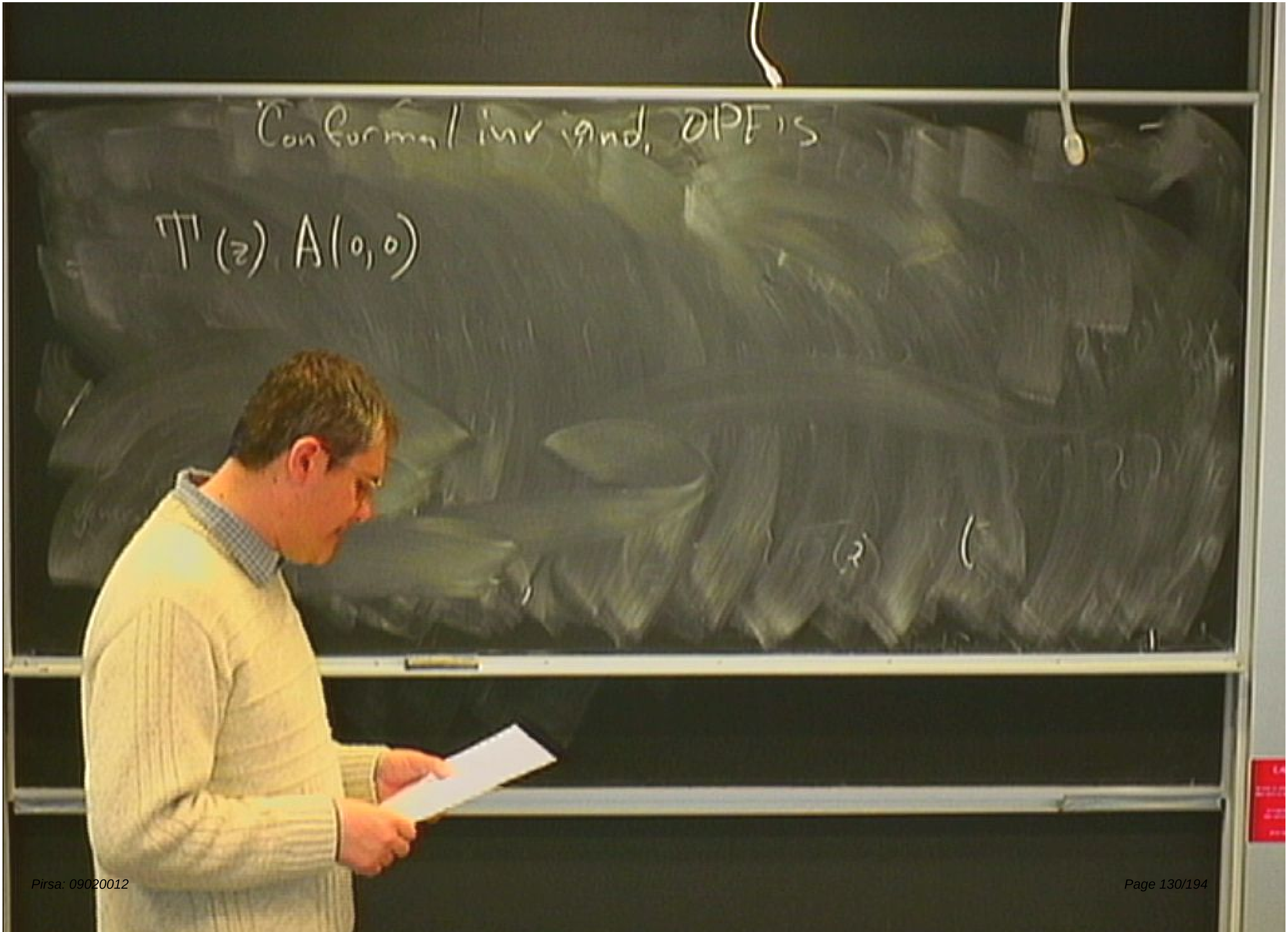
$$z_p \to z_p + \varepsilon\, v(z_p) \longrightarrow$$ infinitesimal transf $\longrightarrow$ is a holomorph

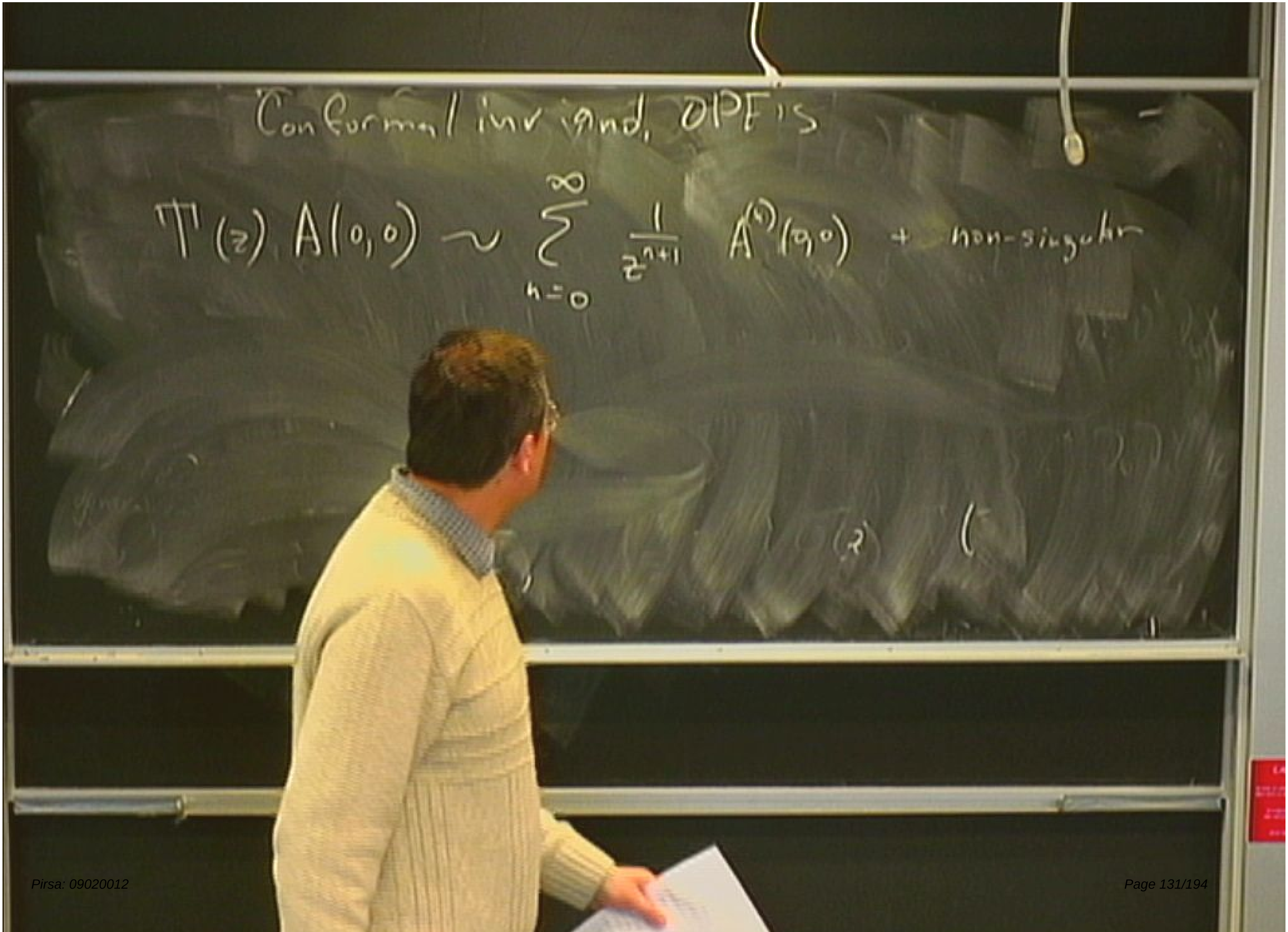
$$z \to z' = f(z)$$

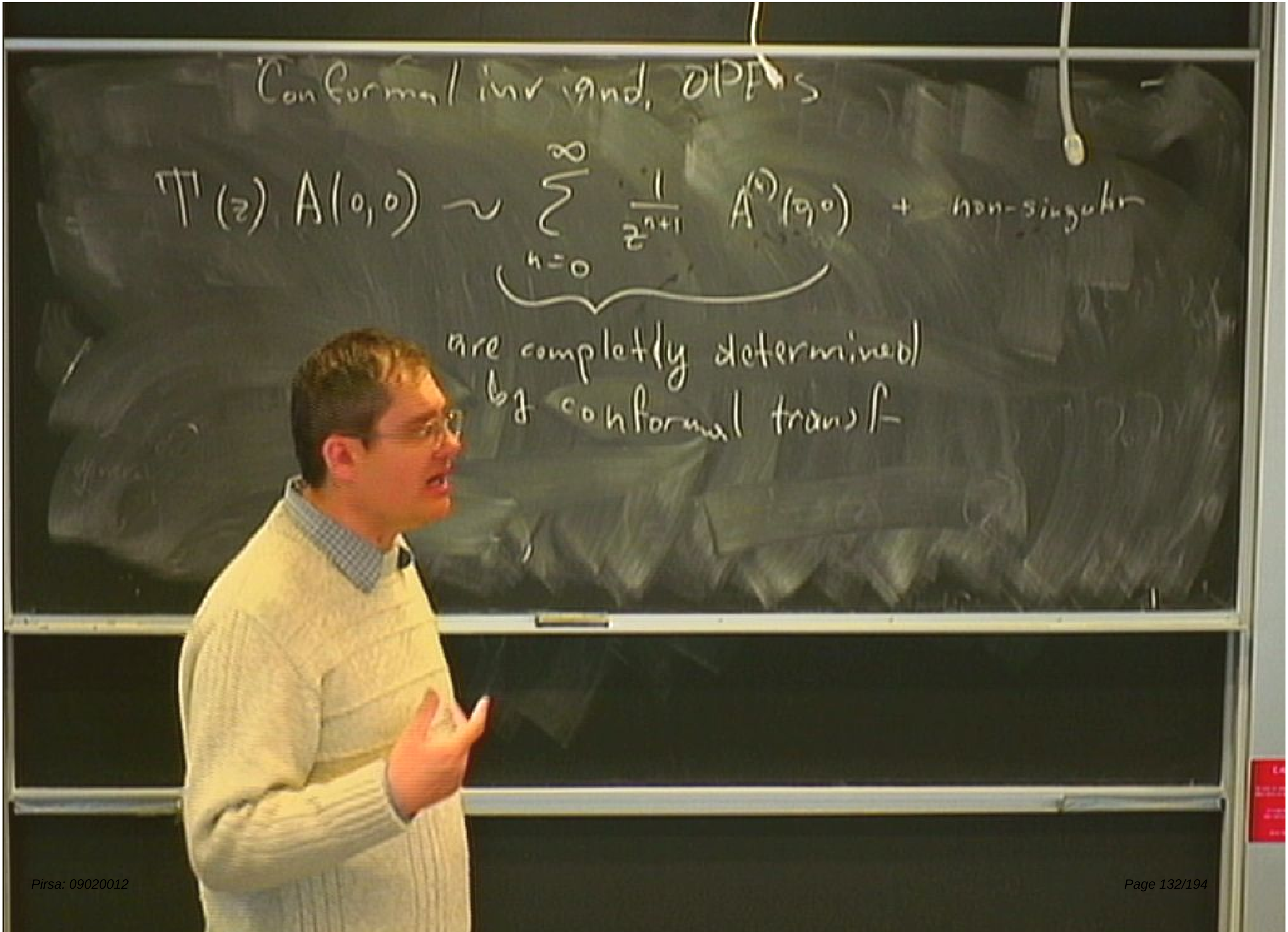
$$\delta X^{\mu} = X^{\mu\prime}(z', \bar{z}') - X^{\mu}(z, \bar{z}) = X^{\mu}(z - \varepsilon v, \bar{z} - \varepsilon \bar{v}) - X^{\mu}$$

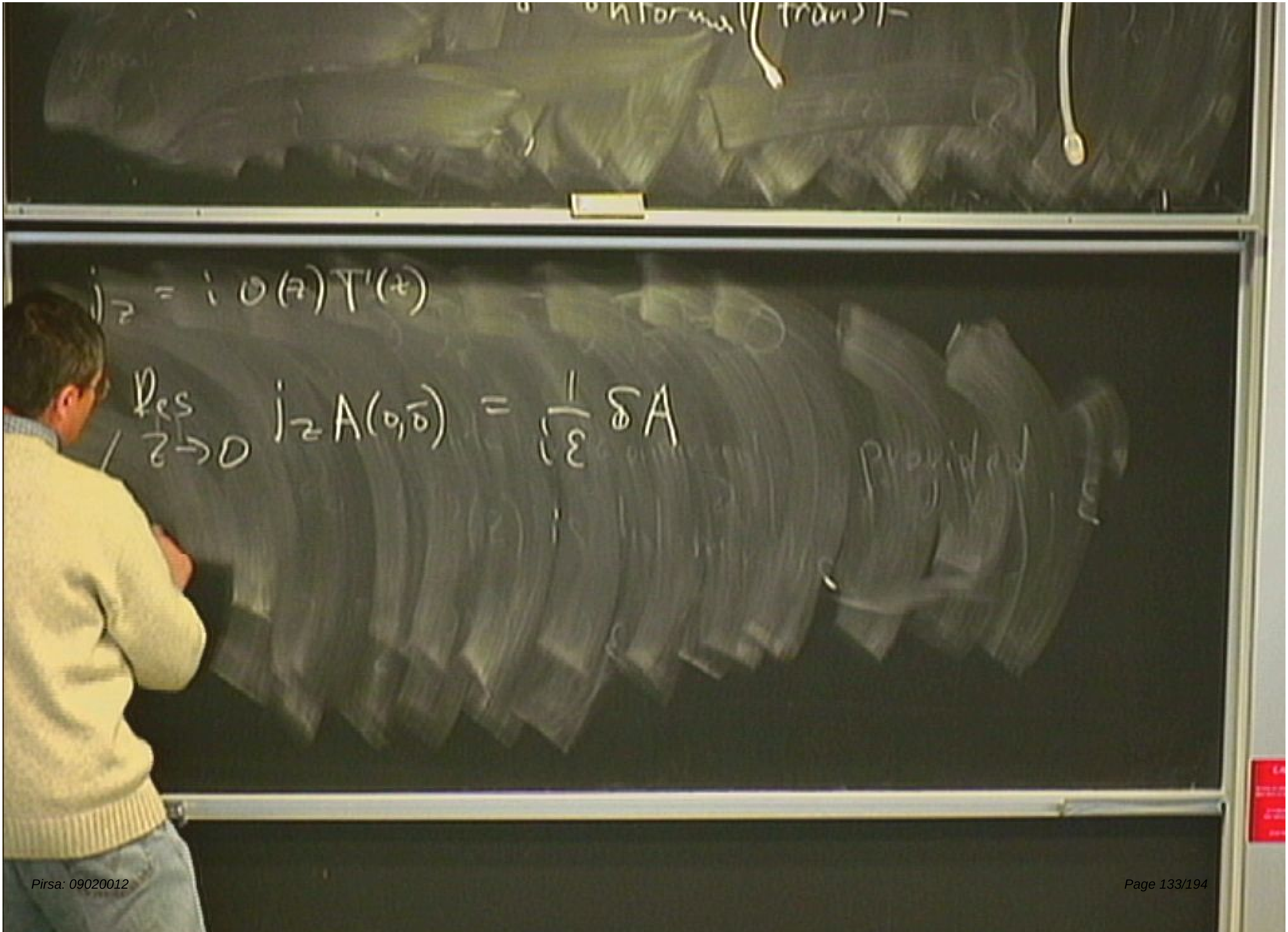
$$= -\varepsilon\, v\, \partial X - \varepsilon\, \bar{v}\, \bar{\partial} X$$

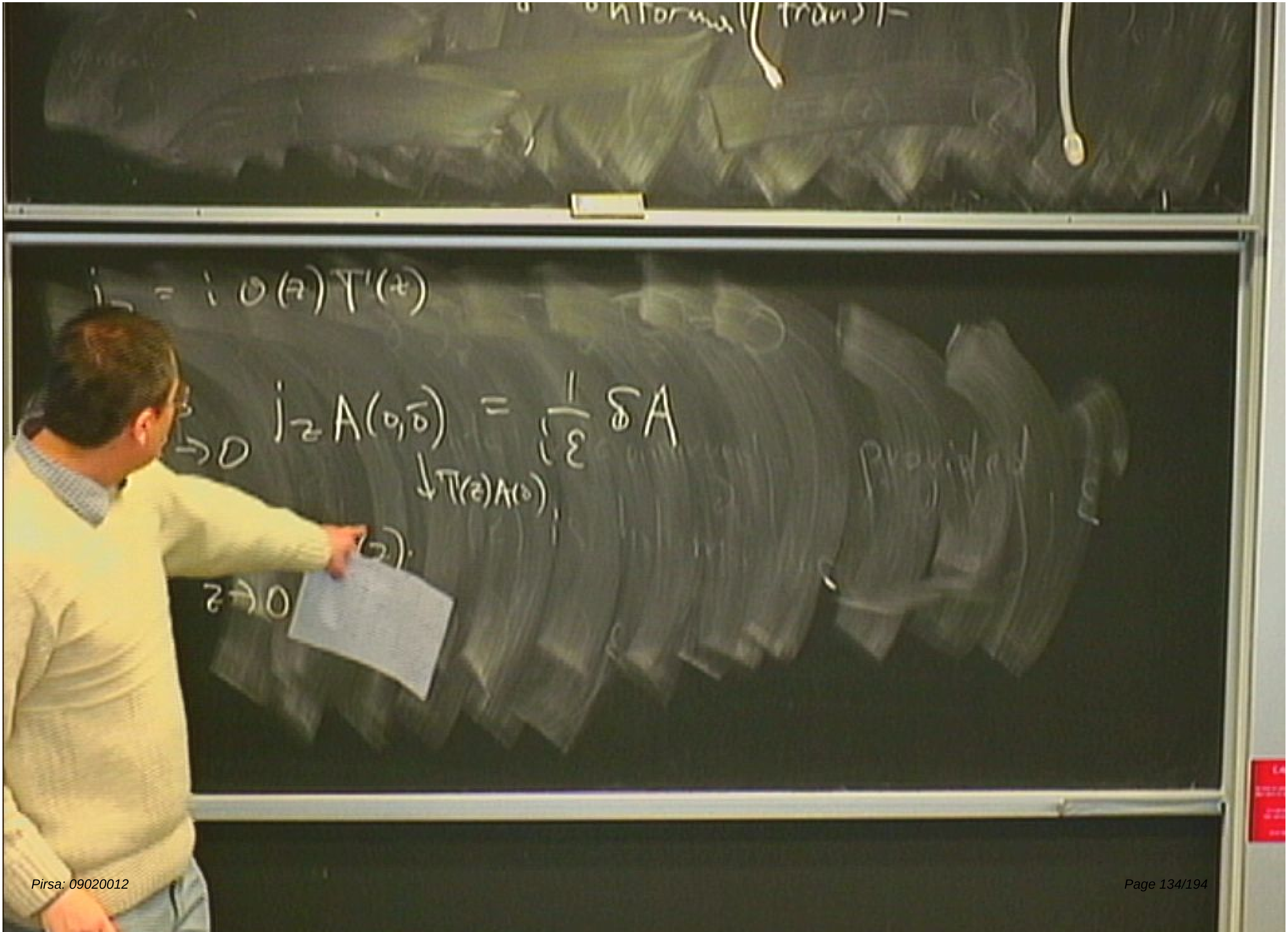

Conformal inv and, OPE's

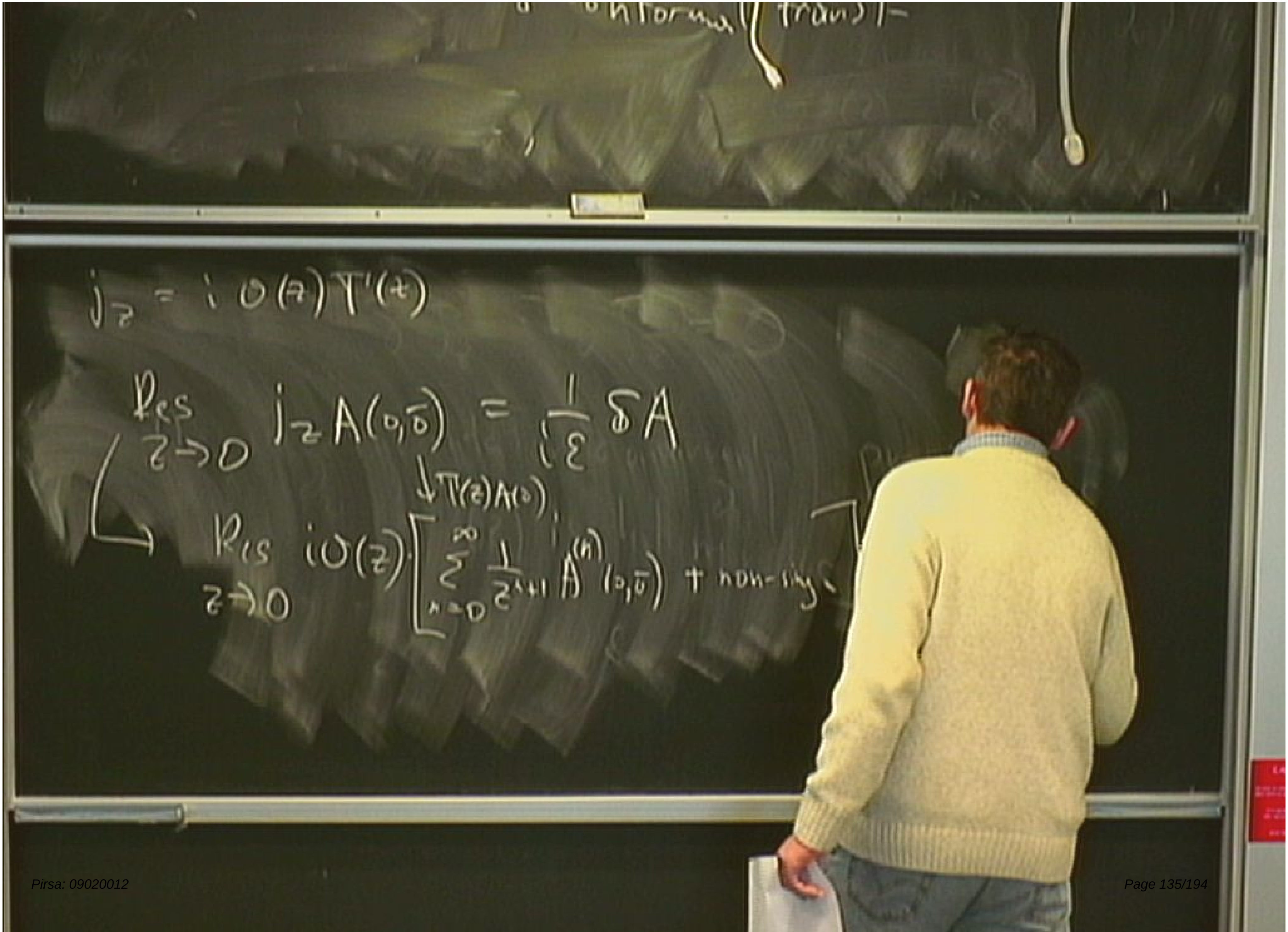

Conformal inv and, OPE's
T(z) A(0,0)

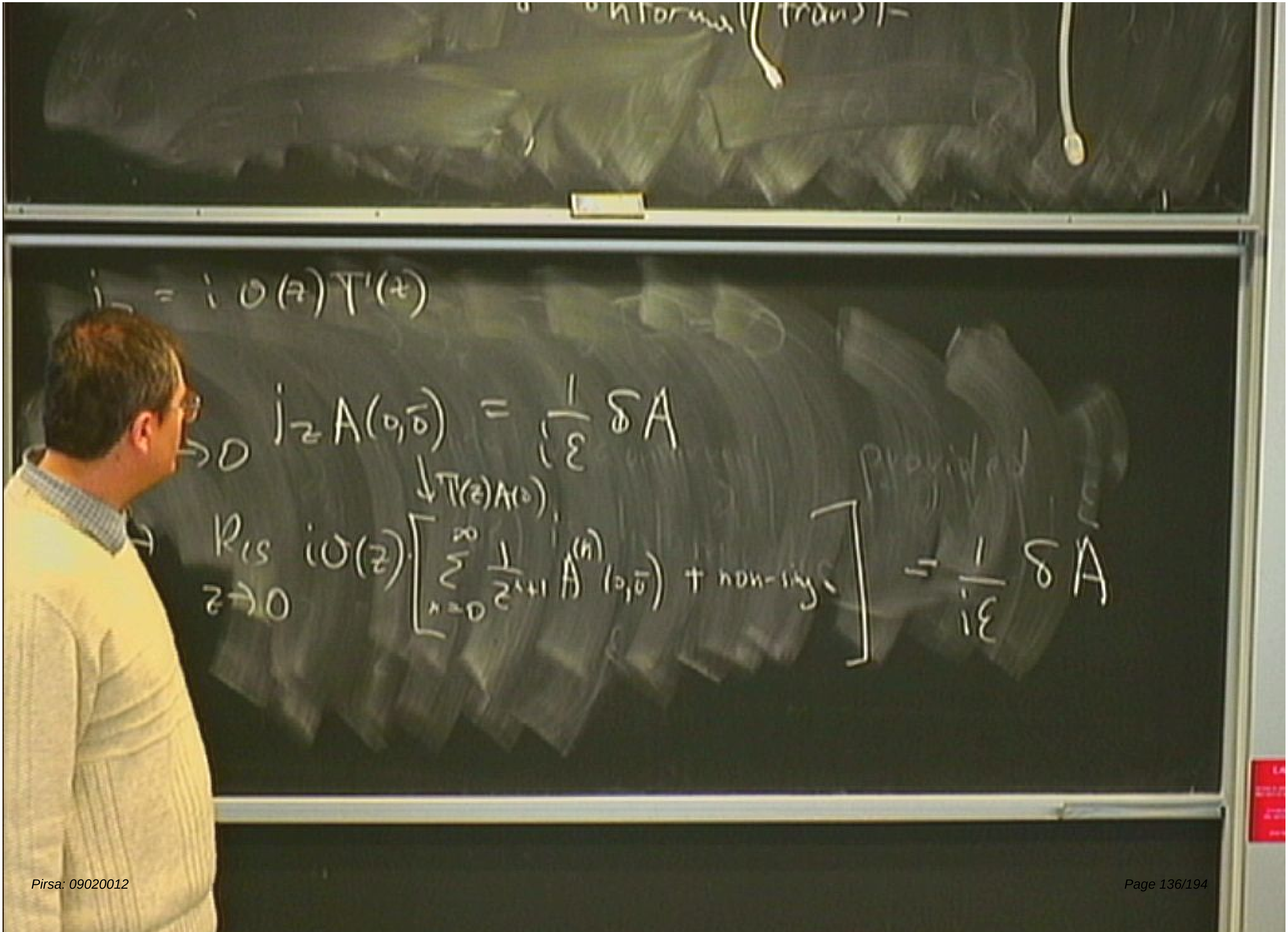

Conformal inv and OPE's
$T(z)\, A(0,0) \sim \sum_{n=0}^{\infty} \frac{1}{z^{n+1}} A^{(n)}(0,0) + \text{non-singular}$


Conformal inv and OPE's
$T(z)\, A(0,0) \sim \underbrace{\sum_{n=0}^{\infty} \frac{1}{z^{n+1}} A^{(n)}(0,0)}$ + non-singular
are completly determined
by conformal transf


j_z = i O(z) T'(z)
Res_{z→0} j_z A(0,0̄) = (1/iε) δA


j_z A(0, 0̄) = 1/(iε) δA


$j_z = i\,\mathcal{O}(z)\,T'(z)$
$\mathrm{Res}_{z\to 0}\; j_z A(0,\bar{0}) = \frac{1}{i\varepsilon}\,\delta A$
$T(z)A(0)$
$\mathrm{Res}_{z\to 0}\; i\,\mathcal{O}(z)\Big[\sum_{n=0}^{\infty} \frac{1}{z^{n+1}} A^{(n)}(0,\bar{0})$ + non-sing


onformal transf.
j_z = i O(z) T'(z)
j_z A(0,0̄) = 1/iε δA
provided
T(z)A(0)
Res iO(z) [Σ_{n=0}^∞ 1/z^{n+1} A^(n)(0,0̄) + non-sing] = 1/iε δA
z→0

... conformal transf. ...

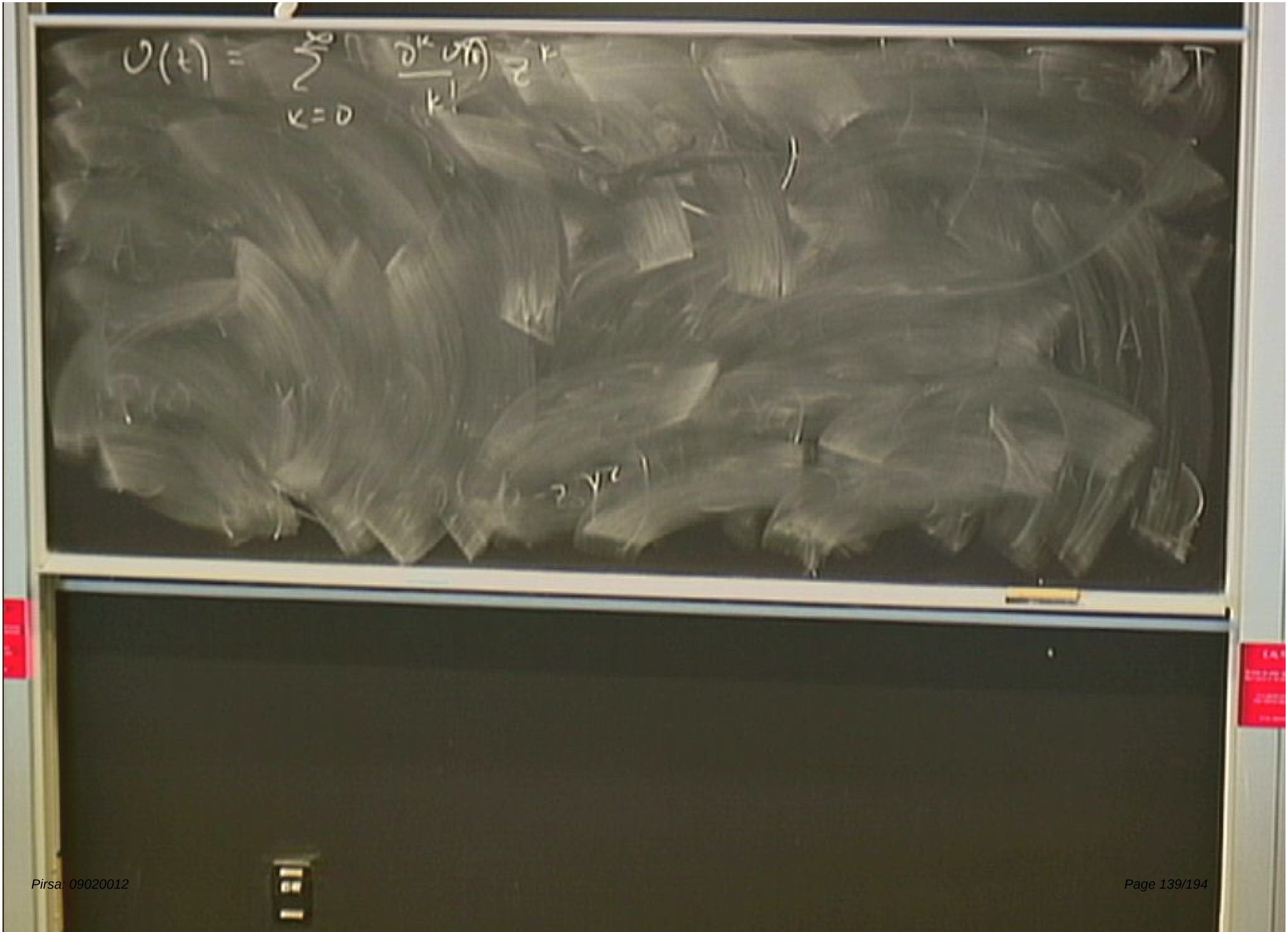
$$j_z = i\,\mathcal{O}(z)\,T'(z)$$

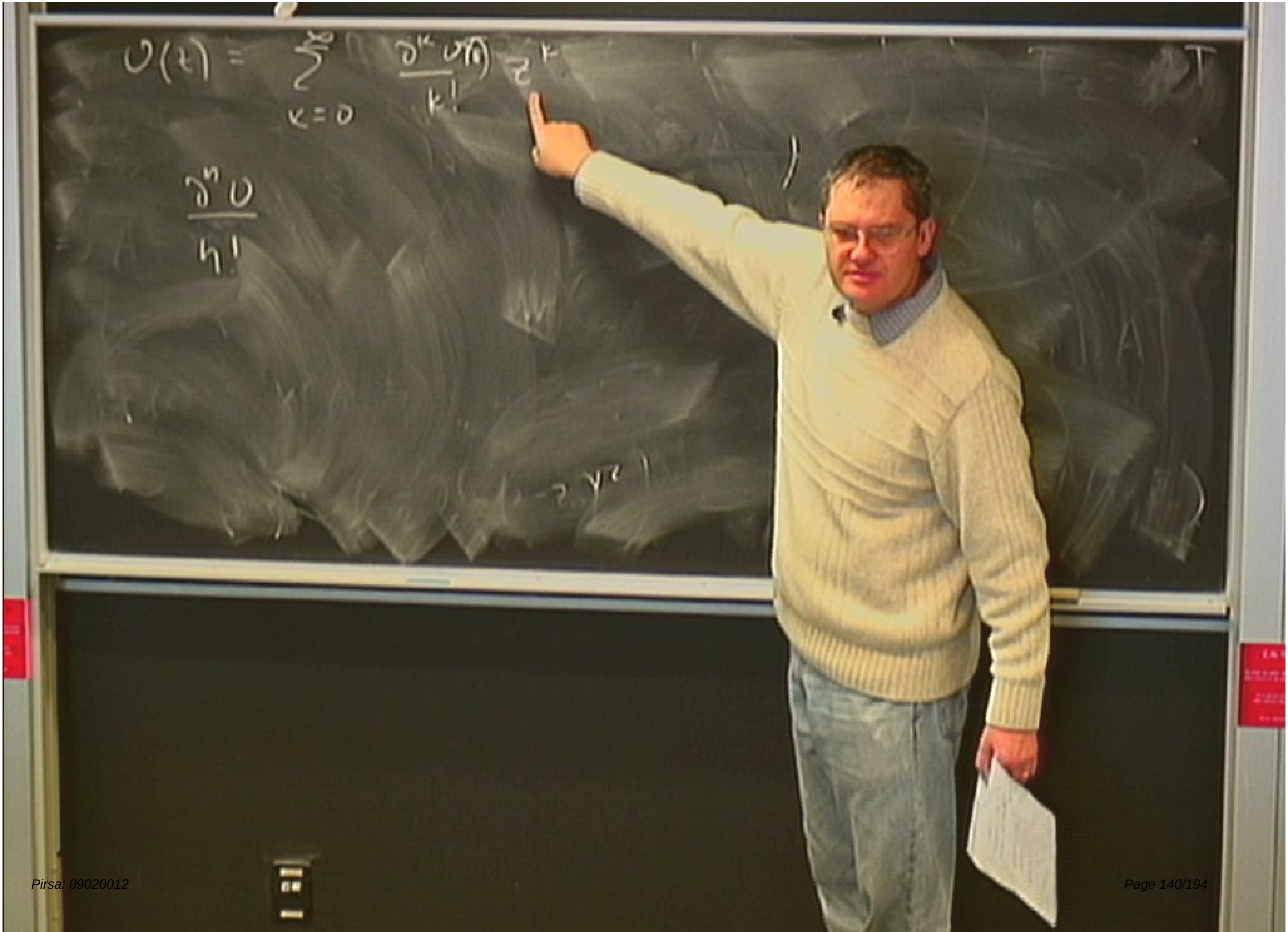
$$\operatorname*{Res}_{z\to 0}\ j_z A(0,\bar{0}) = \frac{1}{i\epsilon}\,\delta A$$

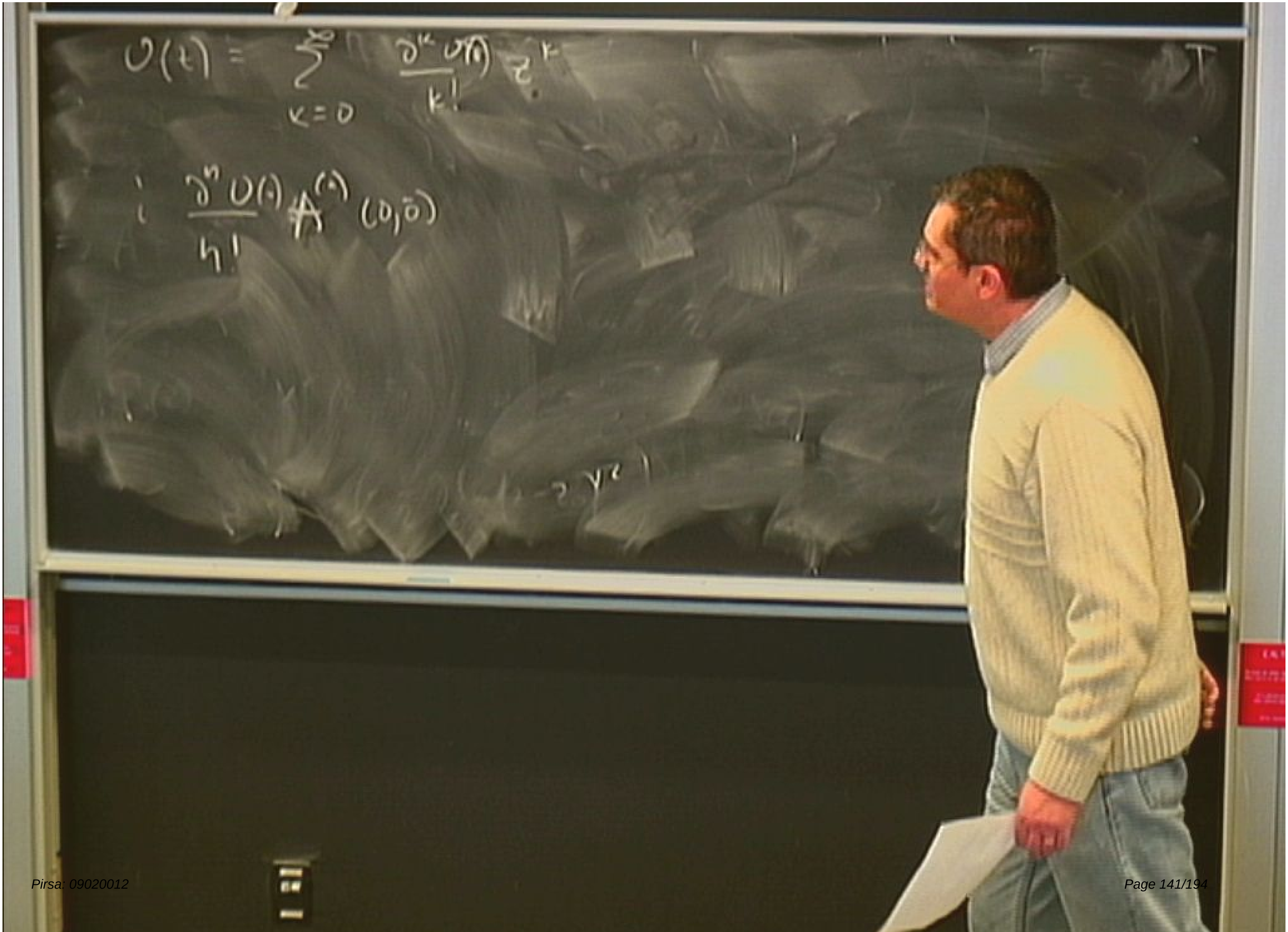
$$\downarrow T(z)A(0)$$

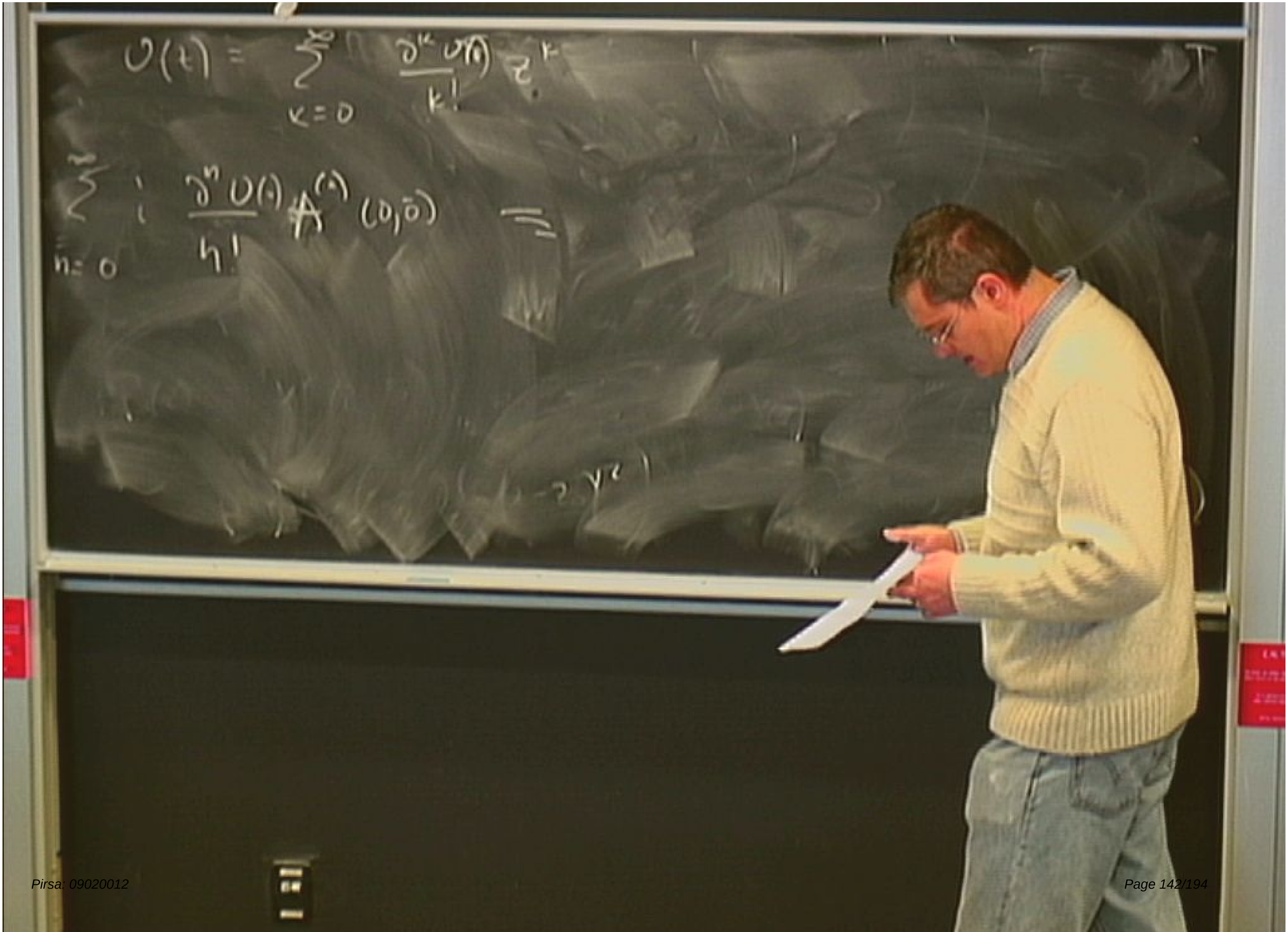
$$\operatorname*{Res}_{z\to 0}\ i\,\underline{\mathcal{O}(z)}\left[\sum_{n=0}^{\infty}\frac{1}{z^{n+1}}A^{(n)}(0,\bar{0}) + \cancel{\text{nonsing.}}\right] = \frac{1}{i\epsilon}\,\delta A$$

$$v(t) = \sum_{k=0}^{\infty} \frac{\partial^k v}{k!} t^k$$


$U(t) = \sum_{k=0}^{\infty} \frac{\partial^k U(0)}{k!} t^k$



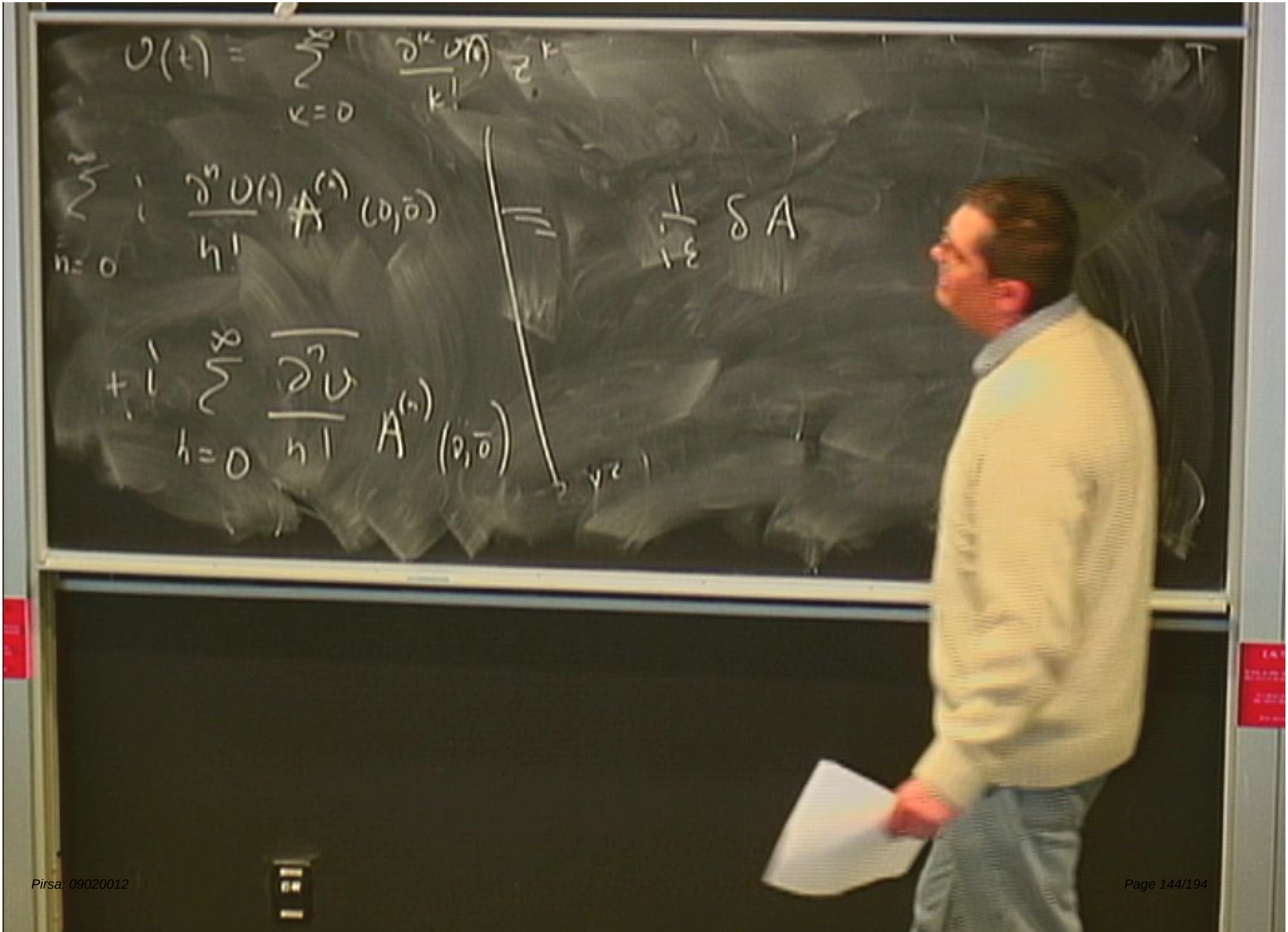

$\phi(z) = \sum_{k=0}^{\infty} \frac{\partial^k \phi(0)}{k!} z^k$
$\frac{\partial^n \phi(0)}{n!} A^{(n)}(0,\bar{0})$



U(z) = Σ ∂^k U(0) / k! z^k
k=0
Σ i ∂^n U(0) / n! A^(n) (0,0̄) =
n=0

$$U(z) = \sum_{k=0}^{\infty} \frac{\partial^k U(0)}{k!} z^k$$

$$\sum_{n=0}^{\infty} i \frac{\partial^n U(0)}{n!} A^{(n)}(0,\bar{0}) = \frac{1}{i\varepsilon}\delta A$$

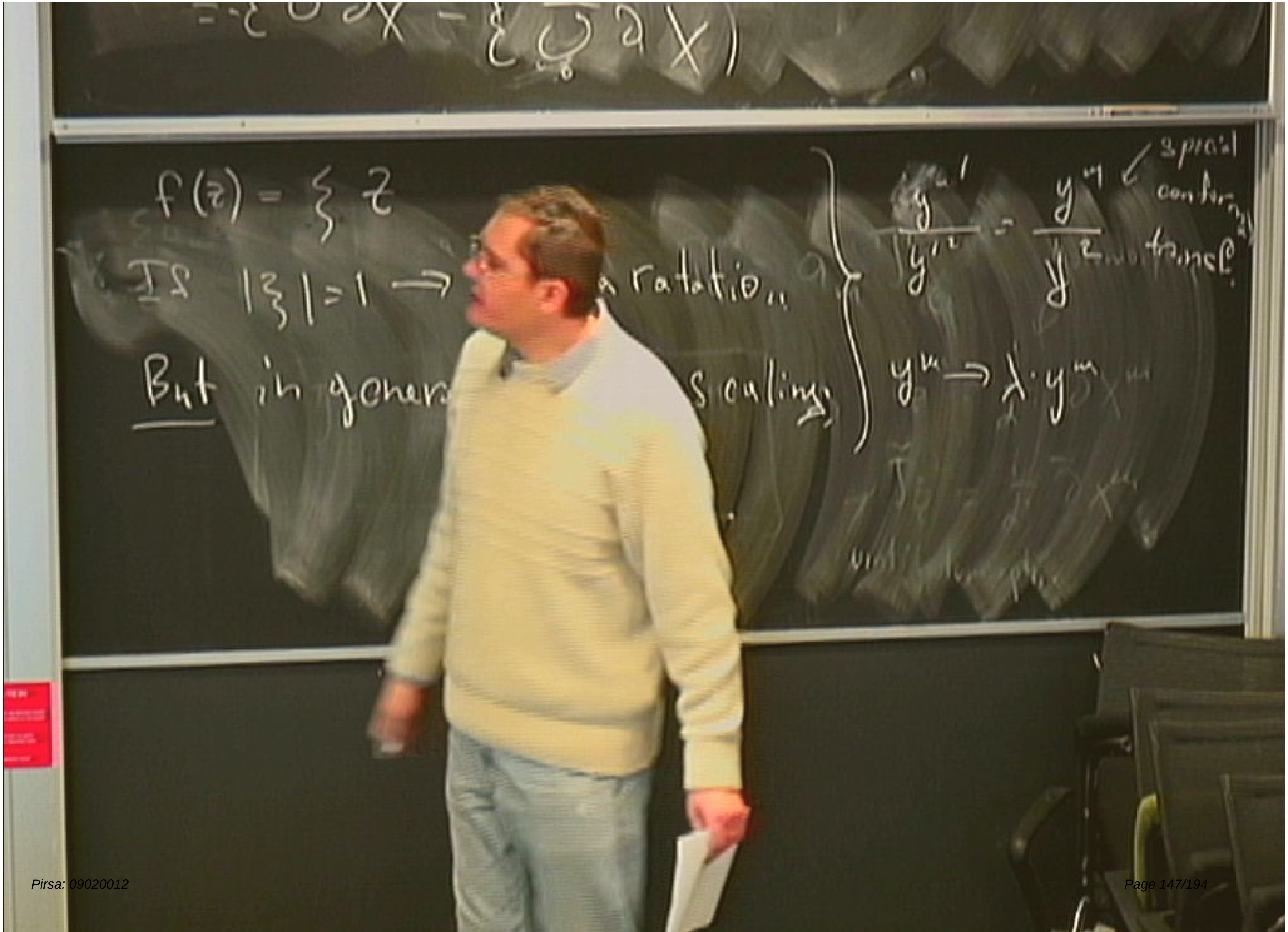


$$\upsilon(z) = \sum_{k=0}^{\infty} \frac{\partial^k \upsilon(0)}{k!} z^k \qquad z \to z + \varepsilon\, \upsilon(z)$$

$$\sum_{n=0}^{\infty} i\, \frac{\partial^n \upsilon(0)}{n!} A^{(n)}(0,\bar{0}) = \frac{1}{i\varepsilon}\, \delta A$$

$$+\, i \sum_{n=0}^{\infty} \frac{\overline{\partial^n \upsilon}}{n!} A^{(n)}(0,\bar{0})$$

Consider a basis of operators


f(z) = ξ z
If |ξ|=1 →
rotation
But in genera
scalings
y^m → λ·y^m
spatial
conformal
transf.

$f(z) = \xi z$

If $|\xi| = 1 \rightarrow$ just a rotation

But in general: rot + scalings

$\frac{y^{a\prime}}{\sqrt{y^{\prime 2}}} = \frac{y^a}{\sqrt{y^2}}$ spatial conformal transf.

$y^m \rightarrow \lambda \cdot y^m$

Consider a basis of operators which are eigenstates of rigit scale transformation.

Consider a basis of operators which are eigenstates of rigid scale transformation.

$$A(z, \bar{z}) \rightarrow$$

Consider a basis of operators which are eigenstates of rigit scale transformation.

$$A(z,\bar{z}) \rightarrow A'(z',\bar{z}') =$$

Consider a basis of operators which are eigenstates of rigit scale transformation.

$$A(z,\bar{z}) \rightarrow A'(z',\bar{z}') = \zeta^{-h}\,\bar{\zeta}^{-\tilde{h}}\,A(z,\bar{z})$$

Consider a basis of operators which are eigenstates of rigit scale transformation.

$$A(z,\bar{z}) \rightarrow A'(z',\bar{z}') = \xi^{-h}\,\bar{\xi}^{-\tilde{h}}\,A(z,\bar{z})$$

$$z \rightarrow z' = \xi z$$

Consider a basis of operators which are eigenstates of rigit scale transformation.

$$A(z,\bar{z}) \to A'(z',\bar{z}') = \zeta^{-h}\,\bar{\zeta}^{-\tilde{h}}\,A(z,\bar{z})$$

$z \to z' = \zeta z$ $\quad (h,\tilde{h})$ - conformal weights

Consider a basis of operators which are eigenstates of rigit scale transformation.

$$A(z,\bar{z}) \rightarrow A'(z',\bar{z}') = \xi^{-h}\,\bar{\xi}^{-\tilde{h}}\,A(z,\bar{z})$$

$h+\tilde{h}$ $\quad z \rightarrow z' = \xi z$ $\quad (h,\tilde{h})$ - conformal weights

Consider a basis of operators which are eigenstates of rigit scale transformation.

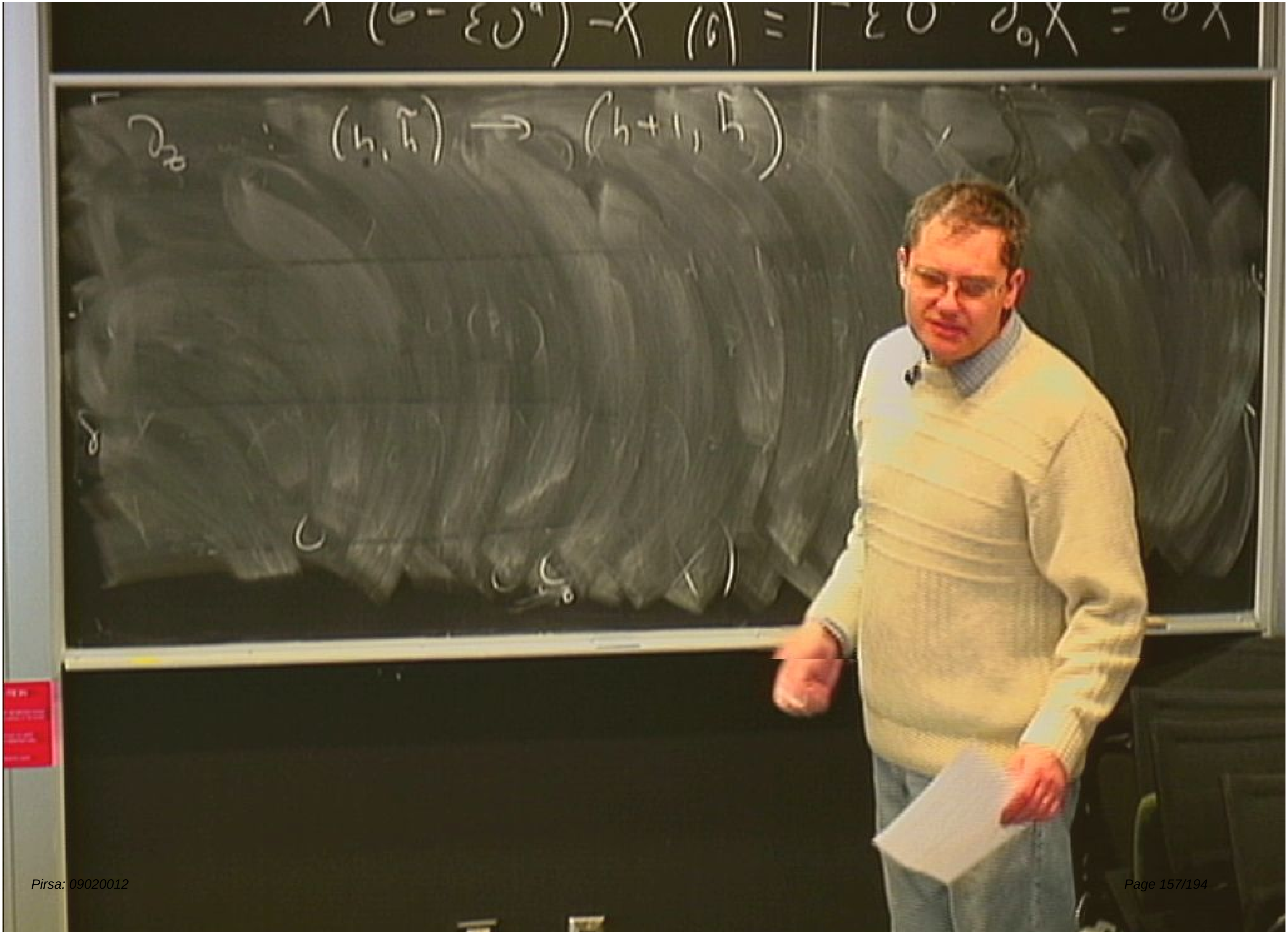
$$A(z,\bar{z}) \longrightarrow A'(z',\bar{z}') = \xi^{-h}\,\bar{\xi}^{-\tilde{h}}\,A(z,\bar{z})$$

$z \to z' = \xi z$

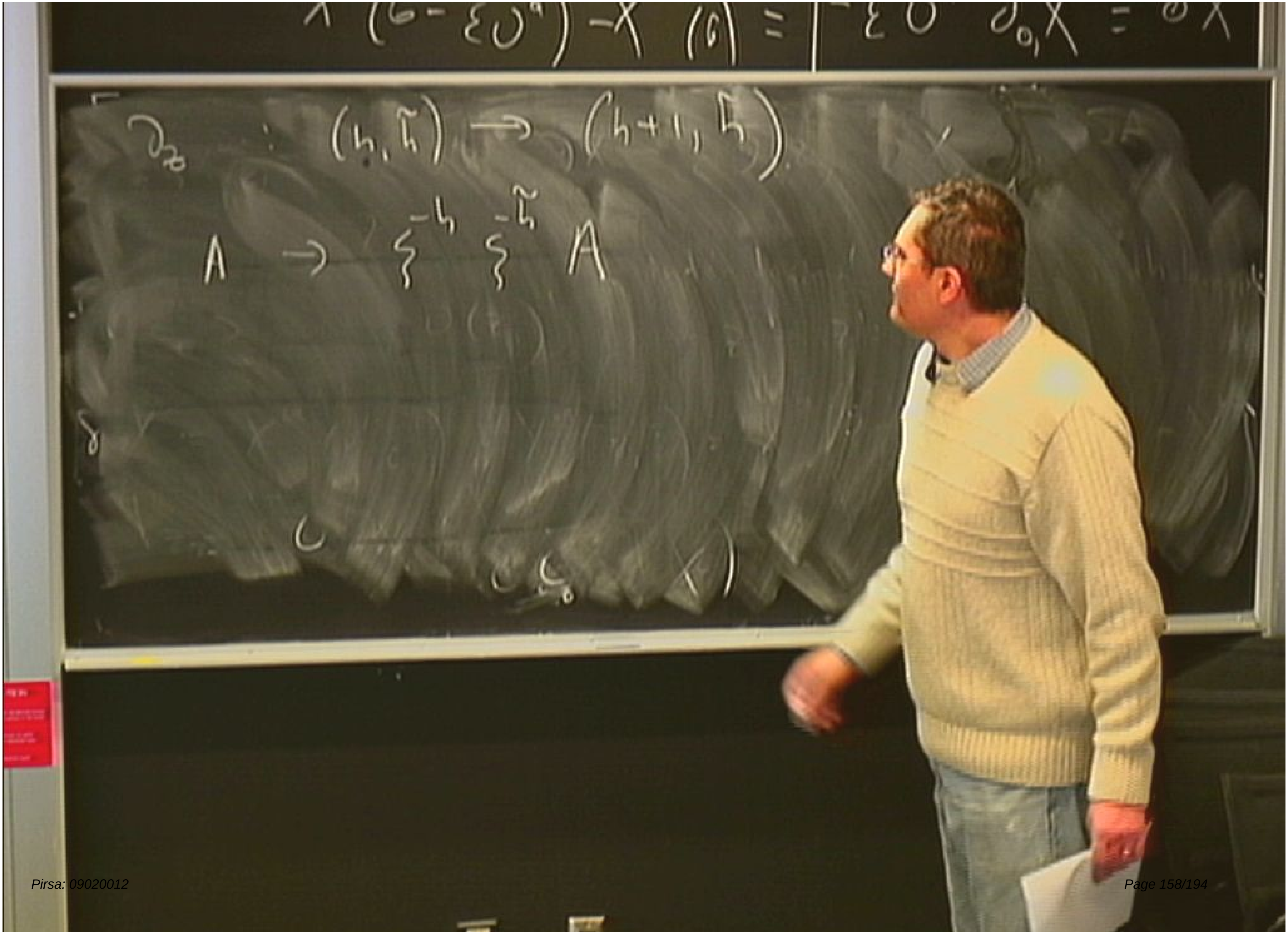
$h+\tilde{h}$ - dim of A

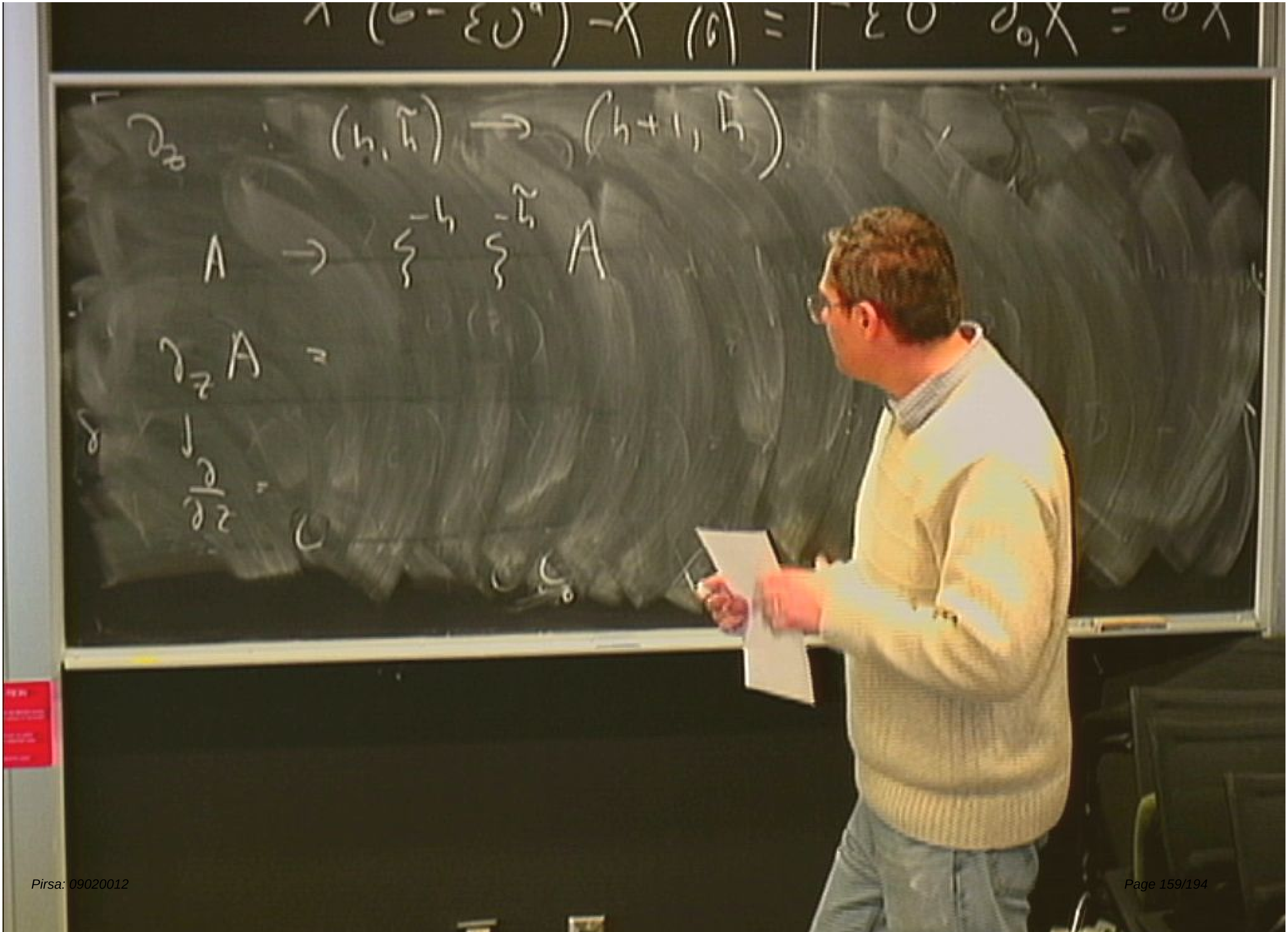
$h-\tilde{h}$ - spin

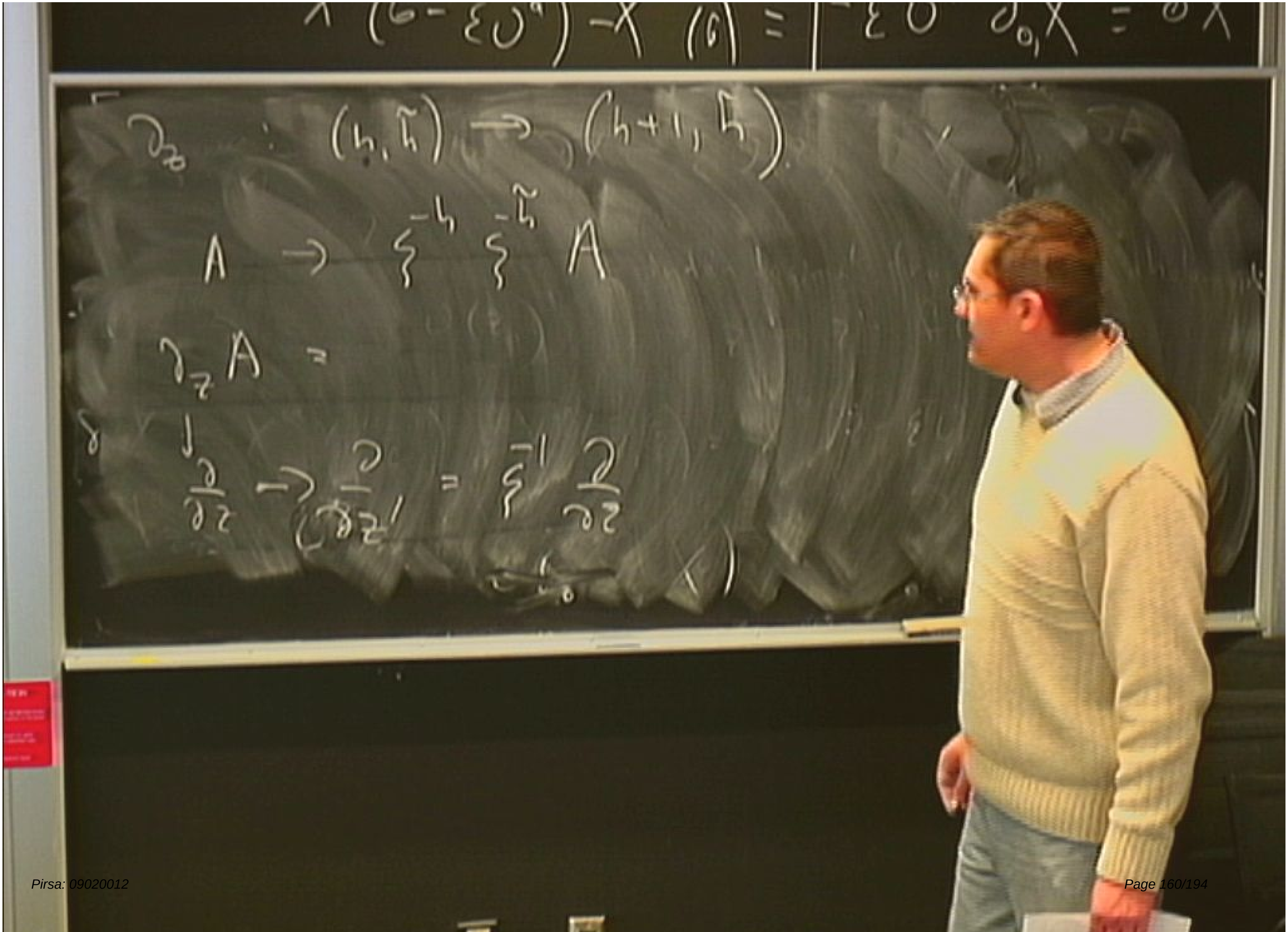
$(h,\tilde{h})$ - conformal weights

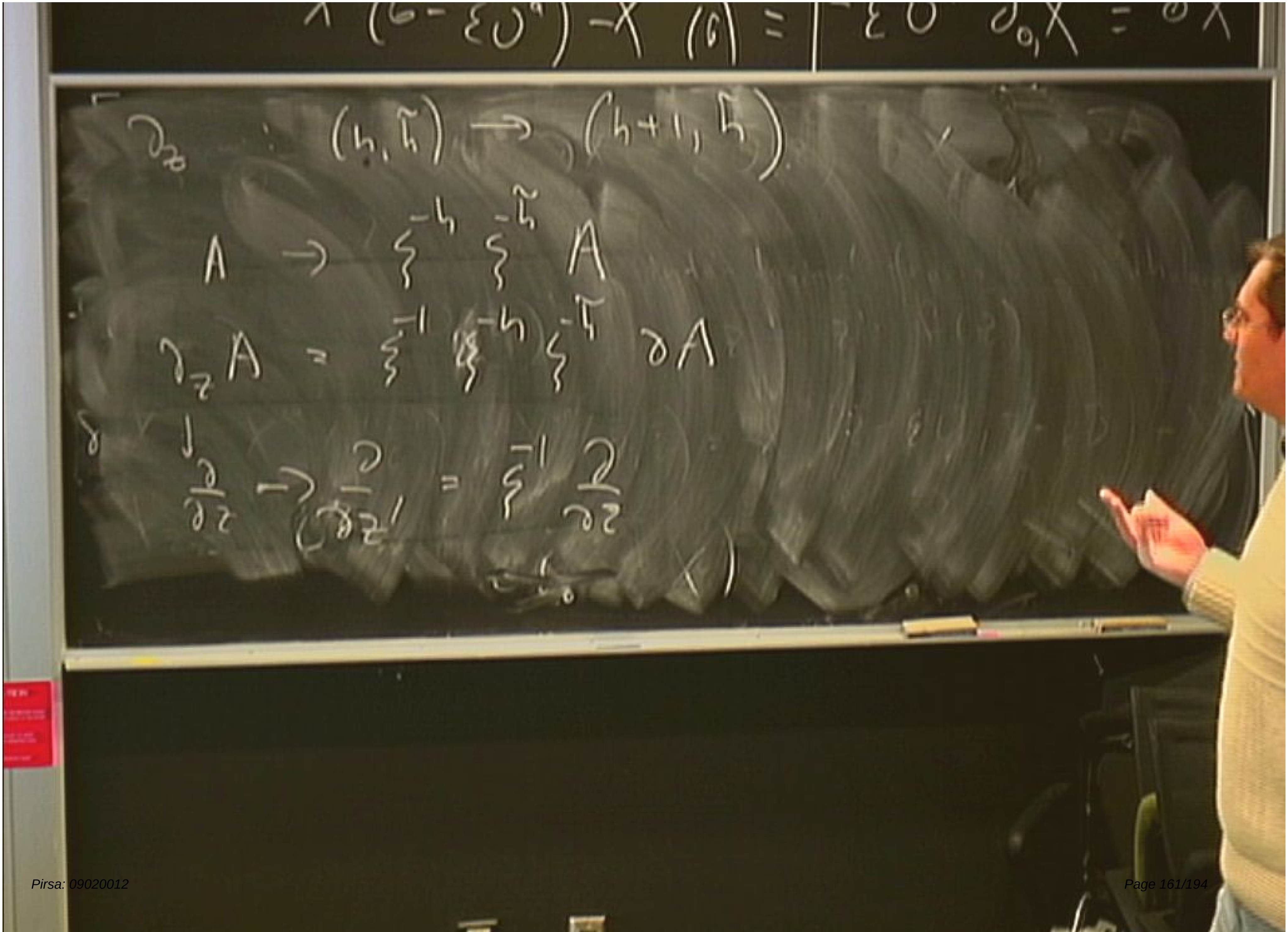


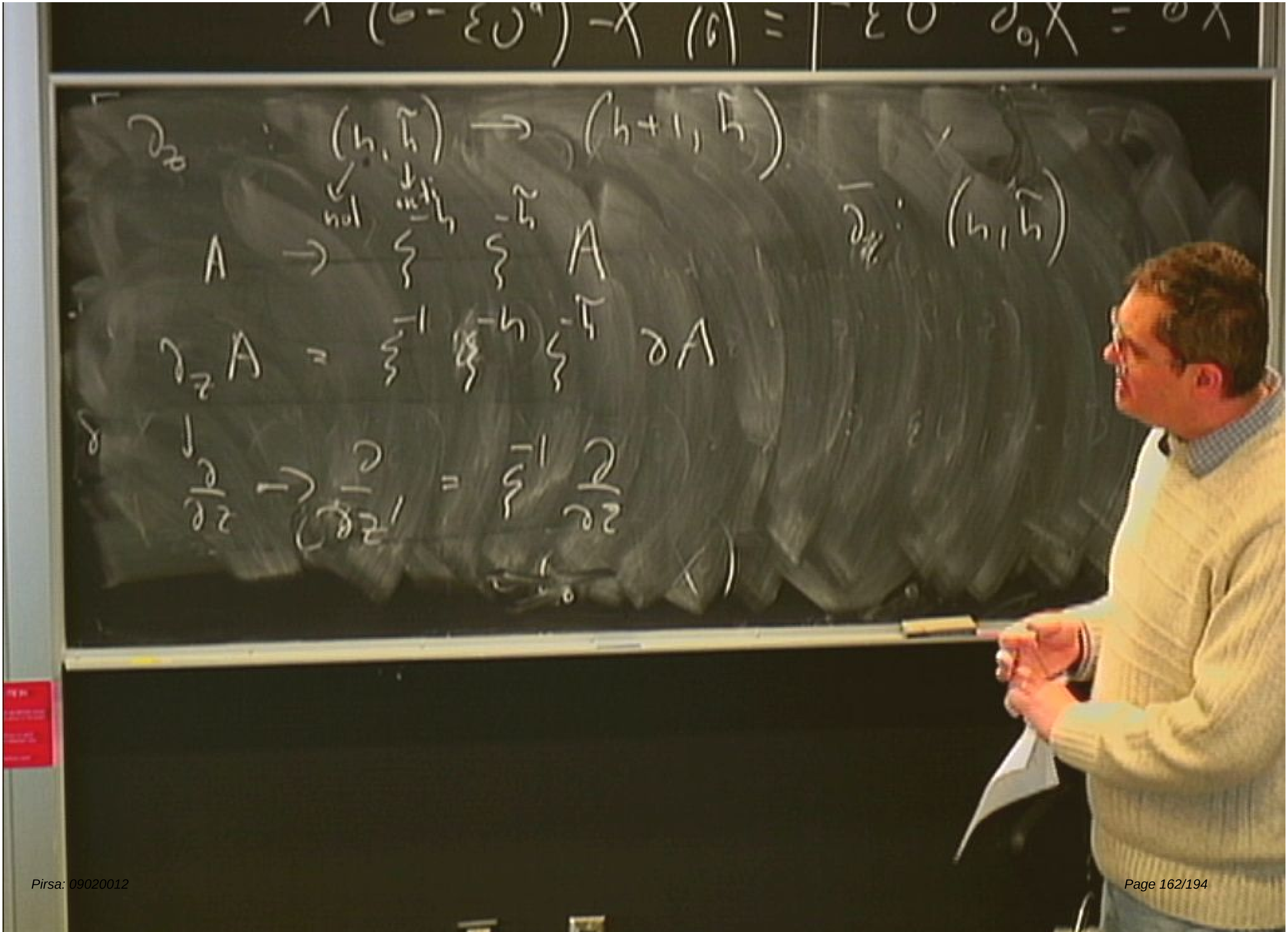
∂z : (h, h̃) → (h+1, h̃)


$\partial_z : (h, \tilde{h}) \to (h+1, \tilde{h})$
$A \to \xi^{-h} \bar{\xi}^{-\tilde{h}} A$


$\partial_{z_0} : (h, \tilde{h}) \to (h+1, \tilde{h})$
$A \to \xi^{-h} \bar{\xi}^{-\tilde{h}} A$
$\partial_z A =$


∂z : (h, h̃) → (h+1, h̃)
A → ξ^-h ξ̄^-h̃ A
∂z A =
∂/∂z → ∂/∂z' = ξ^-1 ∂/∂z


$\partial_z : (h, \tilde h) \to (h+1, \tilde h)$
$A \to \xi^{-h} \bar\xi^{-\tilde h} A$
$\partial_z A = \xi^{-1} \xi^{-h} \bar\xi^{-\tilde h} \partial A$
$\frac{\partial}{\partial z} \to \frac{\partial}{\partial z'} = \xi^{-1} \frac{\partial}{\partial z}$



$$X(\sigma - \varepsilon \sigma^{n}) - X(\sigma) = -\varepsilon \sigma^{n} \partial_{\sigma} X = \delta X$$

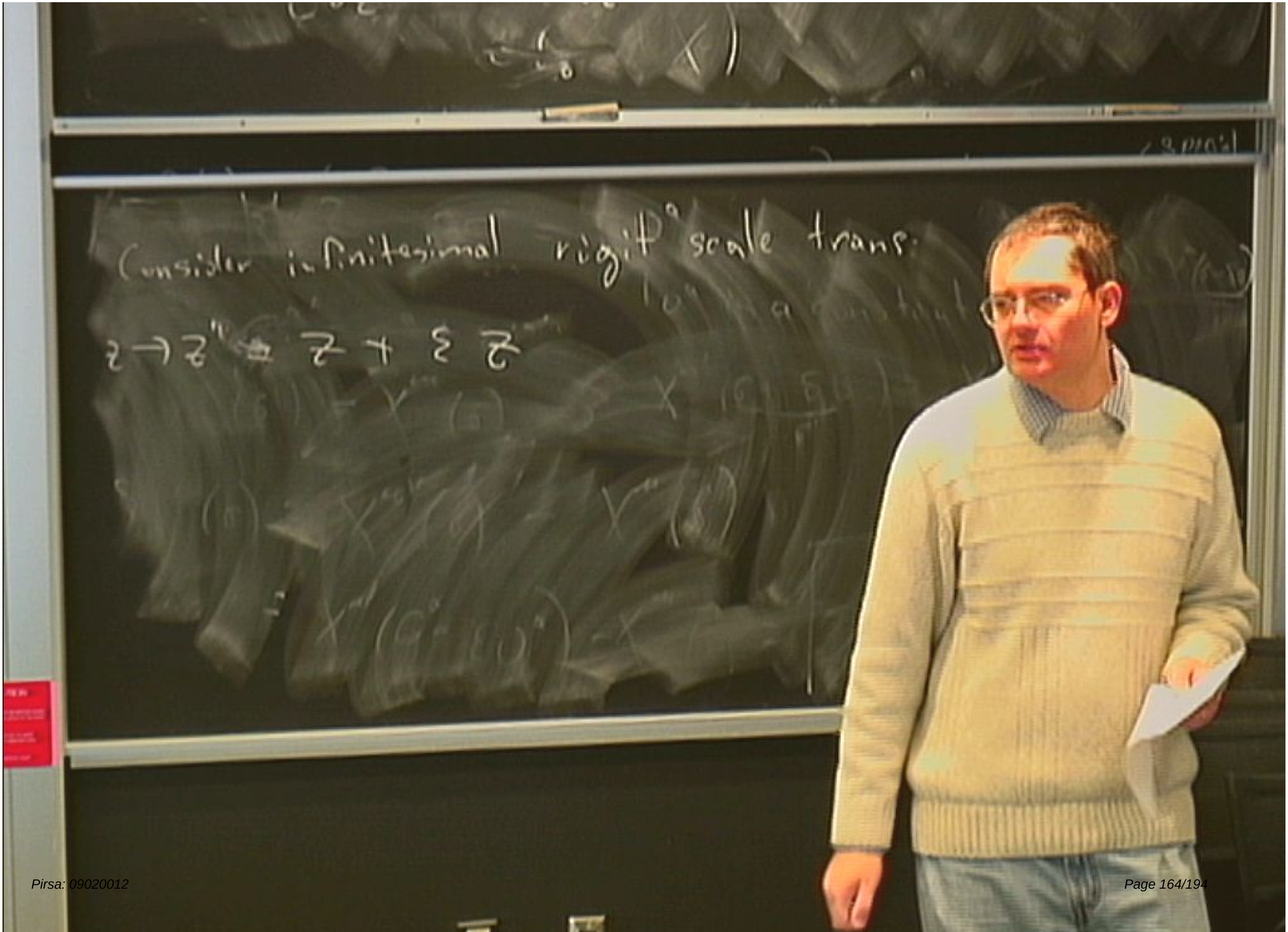
$$\partial_z : (h, \tilde{h}) \to (h+1, \tilde{h})$$

$$\bar{\partial}_{\bar{z}} : (h, \tilde{h}) \to (h, \tilde{h}+1)$$

$$A \to \zeta^{-h} \zeta^{-\tilde{h}} A$$

$$\partial_z A = \zeta^{-1} \zeta^{-h} \zeta^{-\tilde{h}} \partial A$$

$$\frac{\partial}{\partial z} \to \frac{\partial}{\partial z'} = \zeta^{-1} \frac{\partial}{\partial z}$$


Consider infinitesimal rigid scale trans:
z → z' = z + ε z

Consider infinitesimal rigid scale trans:

$$z \to z' = z + \varepsilon z \Rightarrow \zeta = (1+\varepsilon)$$

Consider infinitesimal rigid scale trans:

$$z \to z' = z + \varepsilon z \Rightarrow \xi = (1+\varepsilon)$$

$$O(\bar{z})$$

Consider infinitesimal rigid scale trans:

$$z \to z' = z + \varepsilon z \Rightarrow \xi = (1+\varepsilon)$$

$$U(z) = z$$

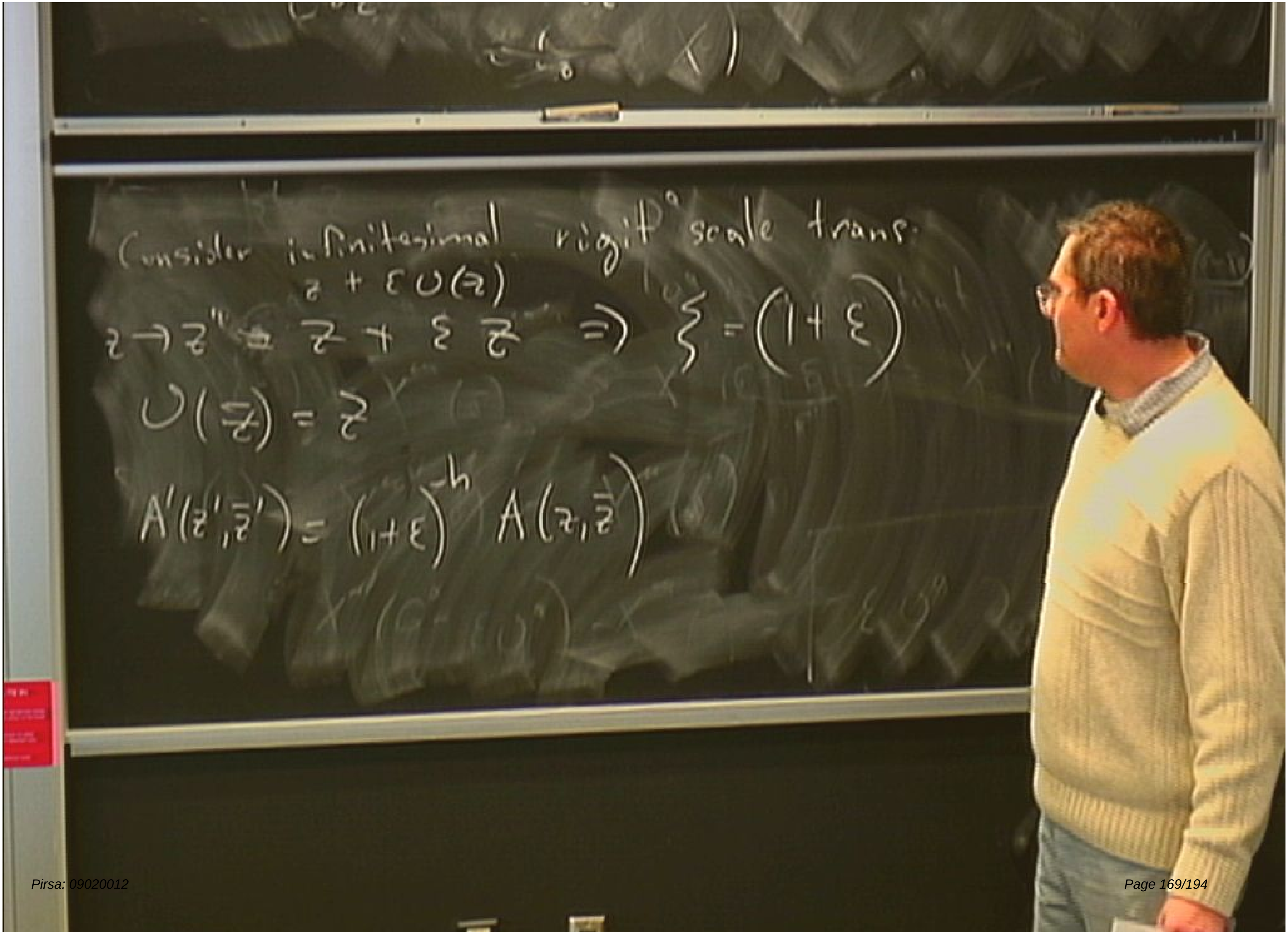
Consider infinitesimal rigid scale trans:

$$z + \varepsilon\, v(z)$$

$$z \to z' = z + \varepsilon z \Rightarrow \zeta = (1+\varepsilon)$$

$$v(z) = z$$

$$A'(z', \bar{z}') =$$


Consider infinitesimal rigid scale trans:
z + εU(z)
z → z' = z + ε z ⇒ ξ = (1+ε)
U(z) = z
A'(z', z̄') = (1+ε)^-h A(z, z̄)

Consider infinitesimal rigid° scale trans.

$z + \varepsilon\, v(z)$

$z \to z'' = z + \varepsilon z \Rightarrow \xi = (1+\varepsilon)$

$v(z) = z$

$A'(z', \bar{z}') = (1+\varepsilon)^{-h} A(z, \bar{z}) = (1 - \varepsilon h) A(z, \bar{z})$

Consider infinitesimal rigid scale trans:

$$z + \varepsilon\, v(z)$$

$$z \to z'' = z + \varepsilon z \Rightarrow \xi = (1+\varepsilon)$$

$$v(z) = z$$

$$A'(z',\bar{z}') = (1+\varepsilon)^{-h} A(z,\bar{z}) = (1-\varepsilon h)\, A(z,\bar{z})$$

$$\delta A = A'(z,\bar{z}) - A'(z,\bar{z})$$

$$= A$$

$$\delta A = A'(z,\bar{z}) - A'(z,\bar{z})$$

$$= A(z-\varepsilon z,\bar{z}) - h\varepsilon\, A(z,\bar{z}) - A(z,\bar{z})$$

$$=$$

$$\delta A = A'(z,\bar z) - A'(z,\bar z)$$

$$= A(z-\epsilon z,\bar z) - h\epsilon A(z,\bar z) - A(z,\bar z)$$

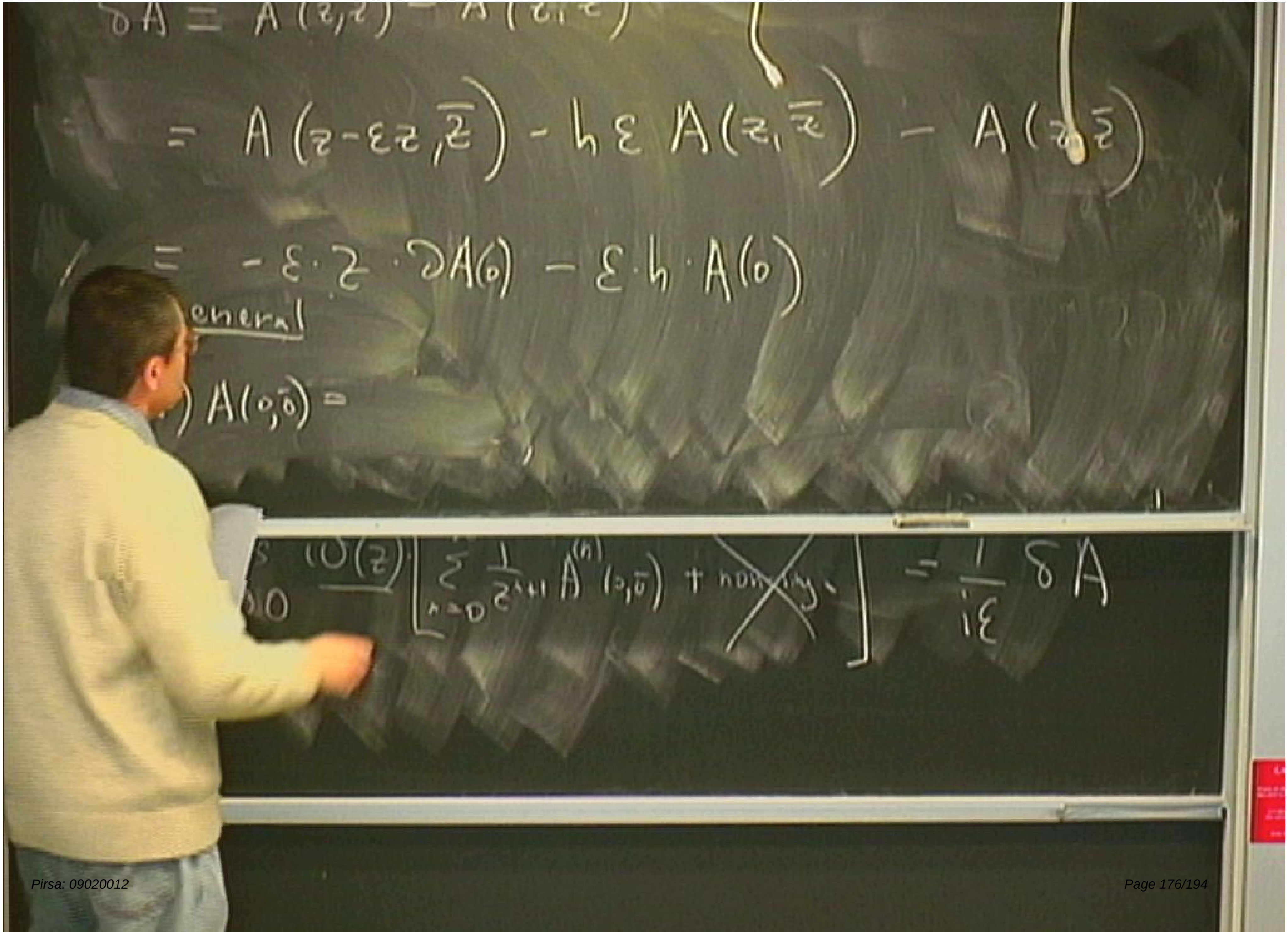
$$= -\epsilon \cdot z \cdot$$

$$\delta A \equiv A(z,\bar z) - A(z,\bar z)$$

$$= A(z-\varepsilon z,\bar z) - h\varepsilon A(z,\bar z) - A(z,\bar z)$$

$$= -\varepsilon\cdot z\cdot \partial A(0) - \varepsilon\cdot h\cdot A(0) =$$

$$\lim_{z\to 0} \underline{(O(z))}\left[\sum_{n=0} \frac{1}{z^{n+1}} A^{(n)}(0,\bar 0) + \cancel{\text{non sing.}}\right] = \frac{1}{i\varepsilon}\,\delta A$$



= A(z-εz, z̄) - hε A(z, z̄) - A(z, z̄)
= -ε·z·∂A(0) - ε·h·A(0)
A(0, 0̄) =
= 1/iε δA

$$= A(z - \varepsilon z, \bar{z}) - h\varepsilon A(z, \bar{z}) - A(z, \bar{z})$$

$$= -\varepsilon \cdot z \cdot \partial A(0) - \varepsilon \cdot h \cdot A(0)$$

In general

$$T(z) A(0,\bar{0}) \sim \sum_{n=0}^{\infty} \frac{1}{z^{n+1}} A^{(n)}(0,0) \Leftrightarrow$$

$$\hookrightarrow \operatorname{Res}_{z \to 0} \, i\underline{v(z)} \left[\sum_{n=0}^{\infty} \frac{1}{z^{n+1}} A^{(n)}(0,\bar{0}) + \cancel{\text{non-sing.}} \right] = \frac{1}{i\varepsilon} \delta A$$

$$= A(z-\varepsilon z, \bar{z}) - h\varepsilon A(z, \bar{z}) - A(z, \bar{z})$$

$$= -\varepsilon \cdot z \cdot \partial A(0) - \varepsilon \cdot h \cdot A(0)$$

<u>In general</u>

$$T(z)A(0,\bar{0}) \sim \sum_{n=0}^{\infty} \frac{1}{z^{n+1}} A^{(n)}(0,\bar{0}) \Leftrightarrow \delta A = -\varepsilon \sum_{n=0}^{\infty} \frac{1}{n!}\left[\partial^n \varepsilon\right] A^{(n)}$$

$$\hookrightarrow \operatorname*{Res}_{z\to 0}\, i\underline{\varepsilon(z)}\left[\sum_{n=0}^{\infty} \frac{1}{z^{n+1}} A^{(n)}(0,\bar{0}) + \cancel{\text{non sing.}}\right] = \frac{1}{i\varepsilon}\,\delta A$$

$$= A(z - \varepsilon z, \bar{z}) - h\varepsilon A(z, \bar{z}) - A(z, \bar{z})$$

$$= -\varepsilon \cdot z \cdot \partial A(0) - \varepsilon \cdot h \cdot A(0)$$

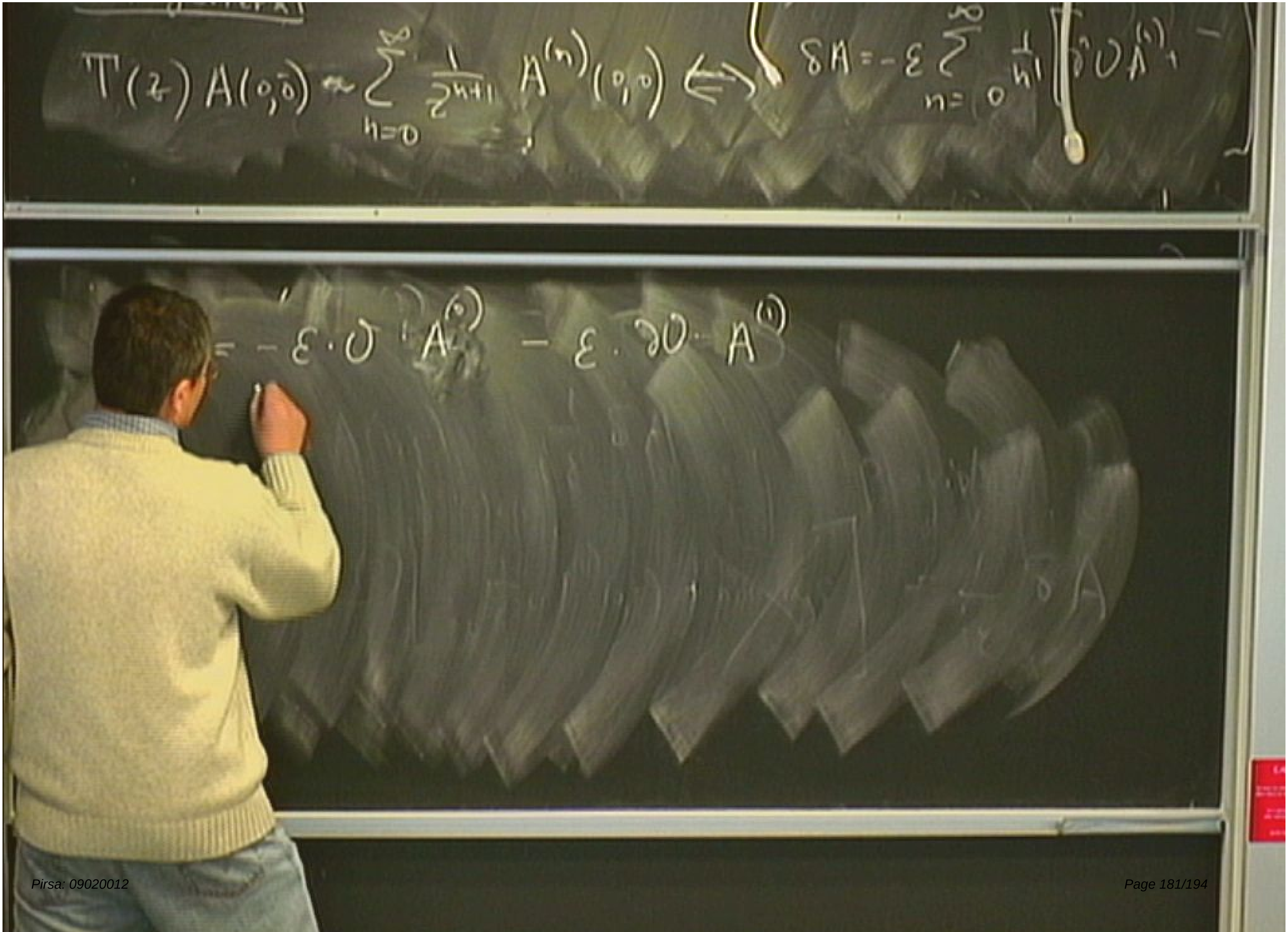
<u>In general</u>

$$T(z)\, A(0,\bar{0}) \sim \sum_{n=0}^{\infty} \frac{1}{z^{n+1}} A^{(n)}(0,\bar{0}) \Longleftrightarrow \delta A = -\varepsilon \sum_{n=0}^{\infty} \frac{1}{n!} \left[\partial^n \mho A^{(n)} + \ldots \right]$$

$$\hookrightarrow \operatorname*{Res}_{z \to 0}\, i\underline{\mho(z)} \left[\sum_{n=0}^{\infty} \frac{1}{z^{n+1}} A^{(n)}(0,\bar{0}) + \cancel{\text{non-sing.}} \right] = \frac{1}{i\varepsilon}\, \delta A$$

$$T(z)\,A(0,\bar{0}) \sim \sum_{n=0}^{\infty} \frac{1}{z^{n+1}}\, A^{(n)}(0,0) \Longleftrightarrow \delta A = -\varepsilon \sum_{n=0}^{\infty} \frac{1}{n!} \left[\partial^n \varepsilon\, A^{(n)} + \right.$$

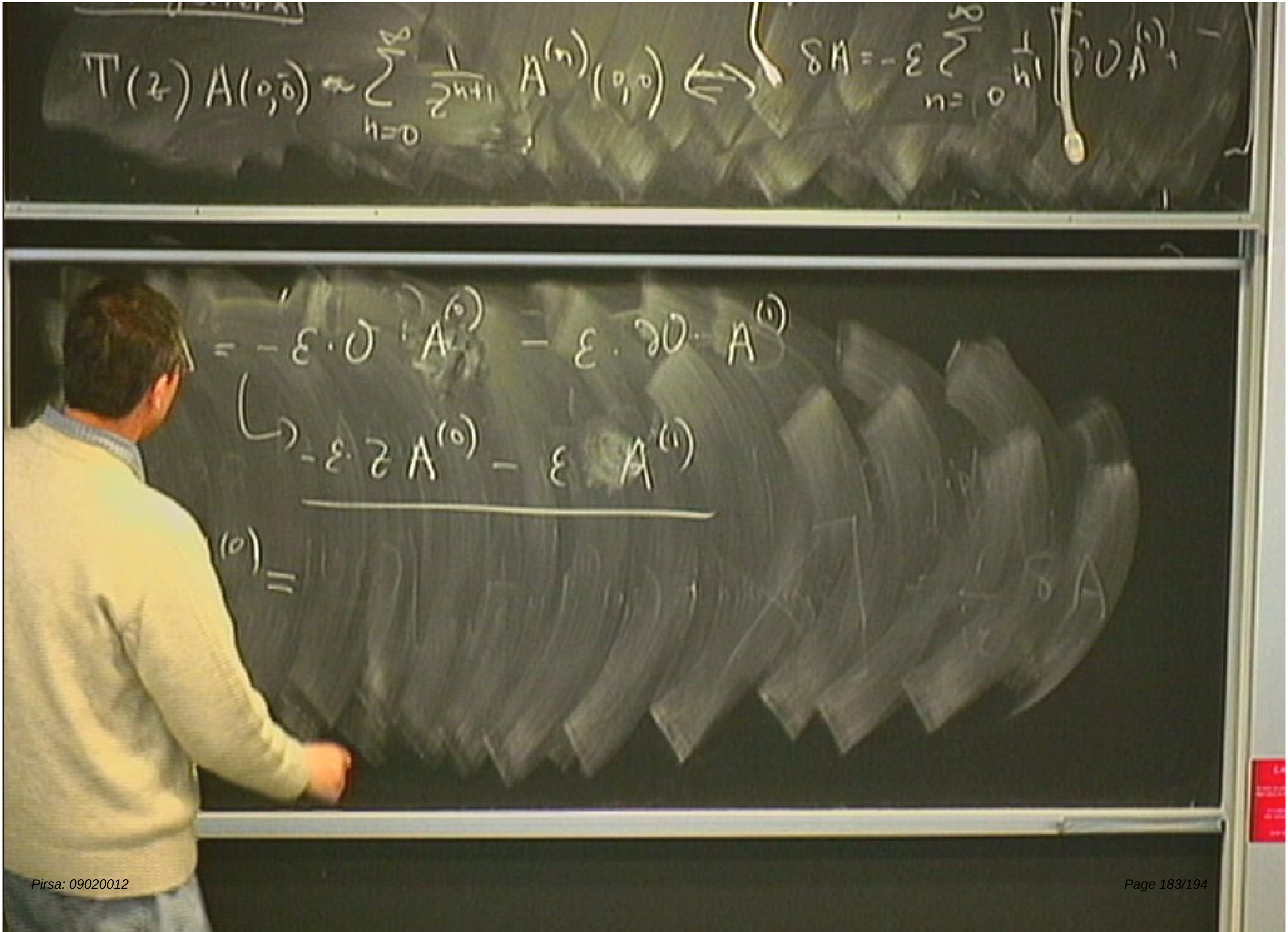
$\delta A =$


$T(z)\,A(0,0) \sim \sum_{n=0}^{\infty} \frac{1}{z^{n+1}} A^{(n)}(0,0) \Longleftrightarrow \delta A = -\varepsilon \sum_{n=0}^{\infty} \frac{1}{n!}$
$= -\varepsilon \cdot \partial A^{(0)} - \varepsilon \cdot \partial\partial \cdot A^{(1)}$

$$T(z)\,A(0,\bar{0}) \sim \sum_{n=0}^{\infty} \frac{1}{z^{n+1}} A^{(n)}(0,0) \iff \delta A = -\varepsilon \sum_{n=0}^{\infty} \frac{1}{n!} \left[\partial^n \mathcal{O}\, A^{(n)} \right.$$

$$\delta A = -\varepsilon \cdot \mathcal{O}\, A^{(0)} - \varepsilon \cdot \partial \mathcal{O} \cdot A^{(1)}$$

$$\hookrightarrow -\varepsilon \cdot z\, A^{(0)} - \varepsilon\, A^{(1)}$$

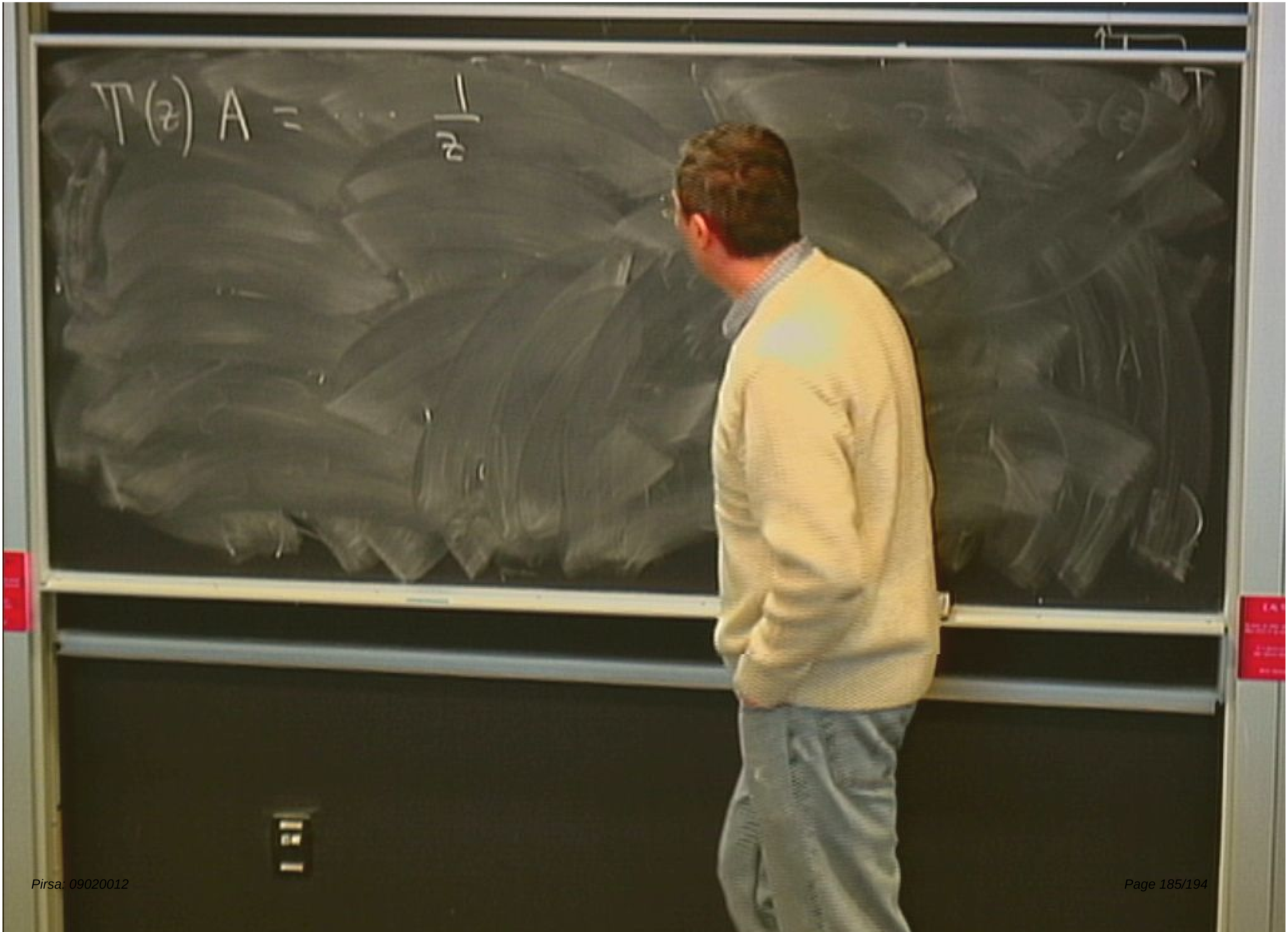

$T(z)\,A(0,\bar{0}) \sim \sum_{n=0}^{\infty} \frac{1}{z^{n+1}} A^{(n)}(0,0) \Longleftrightarrow \delta A = -\varepsilon \sum_{n=0}^{\infty} \frac{1}{n!} \partial^n \mathcal{O}\, A^{(n)} +$
$= -\varepsilon \cdot \mathcal{O}\, A^{(0)} - \varepsilon \cdot \partial\mathcal{O} \cdot A^{(1)}$
$\hookrightarrow -\varepsilon \cdot z\, A^{(0)} - \varepsilon\, A^{(1)}$
$^{(0)} =$

$$T(z)\,A(0,\bar{0}) \sim \sum_{n=0}^{\infty} \frac{1}{z^{n+1}}\, A^{(n)}(0,0) \iff \delta A = -\varepsilon \sum_{n=0}^{\infty} \frac{1}{n!}\, \partial^n \upsilon\, A^{(n)}$$

$$\delta A = -\varepsilon \cdot \upsilon\, A^{(0)} - \varepsilon \cdot \partial\upsilon \cdot A^{(1)}$$

$$\hookrightarrow -\varepsilon \cdot z\, A^{(0)} - \varepsilon\, A^{(1)}$$

$$A^{(0)} = \partial A \qquad A^{(1)} = h \cdot A$$


T(z) A = . . . 1/z

$$T(z)\,A = \cdots \frac{1}{z^2} A^{(1)} + \frac{1}{z} A^{(0)} + \cdots$$

$$T(z)\, A = \cdots \frac{1}{z^2} A^{(1)} + \frac{1}{z} A^{(0)} + \cdots$$

$$= \cdots \frac{h}{z^2} A + \frac{1}{z} \partial A$$

$$T(z)\, A = \cdots \frac{1}{z^2} A^{(0)} + \frac{1}{z} A^{(-1)} + \cdots$$

$$= \cdots\cdots \frac{h}{z^2} A + \frac{1}{z} \partial A + \cdots$$

$$= \ldots ?? \; \frac{h}{z^2} A + \frac{1}{z}\partial A + \ldots$$

Primary fields or tensor operators

$z \to z' = \xi z$ $\quad (h, \tilde{h})$ - conformal weights

$h + \tilde{h}$ - dim of A

$h - \tilde{h}$ - spin

$$= \ldots \frac{h}{z^2} A + \frac{1}{z} \partial A + \ldots$$

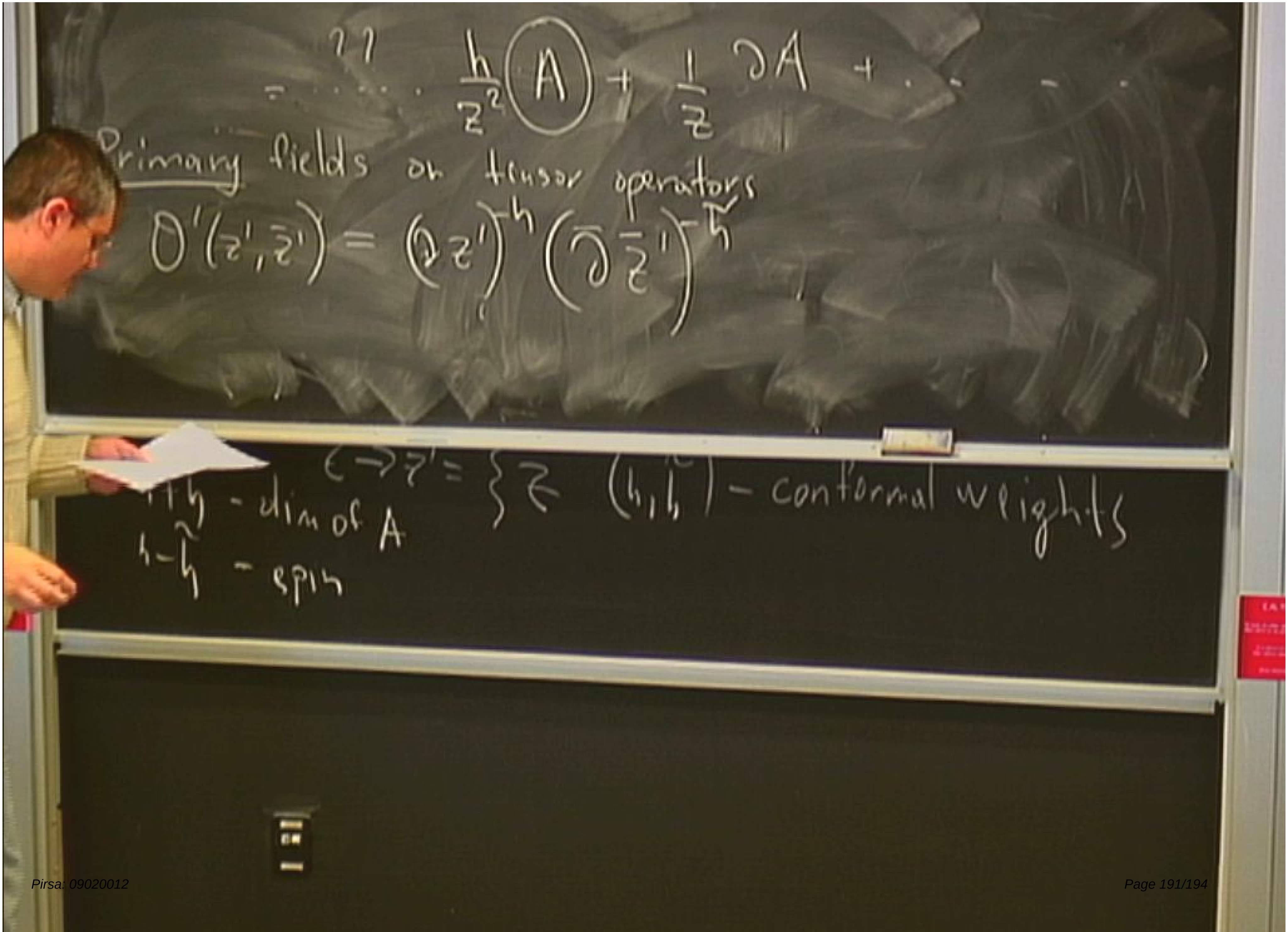
Primary fields or tensor operators

$$O'(z', \bar{z}') =$$

$z \to z' = \zeta z$ $\quad (h, \tilde{h})$ - conformal weights

$h + \tilde{h}$ - dim of A

$h - \tilde{h}$ - spin


?? = ... h/z^2 A + 1/z ∂A + ...
Primary fields or tensor operators
O'(z', z̄') = (∂z')^-h (∂̄z̄')^-h̃
z → z' = f(z)   (h, h̃) - conformal weights
h+h̃ - dim of A
h-h̃ - spin

$$?? \quad = \dots \frac{h}{z^2} A + \frac{1}{z}\partial A + \dots$$

Primary fields or tensor operators

$$\mathcal{O}'(z',\bar{z}') = (\partial z')^{-h} (\bar{\partial}\bar{z}')^{-\tilde{h}} \mathcal{O}(z,\bar{z})$$

$z \to z' = \zeta z$ $\quad (h,\tilde{h})$ - conformal weights

$h+\tilde{h}$ - dim of A

$h-\tilde{h}$ - spin

$\mathcal{O}'(z',\bar{z}') = (\partial z')^{h}(\partial \bar{z}')^{\tilde{h}}\, \mathcal{O}(z,\bar{z})$

$\hookrightarrow T(z)\,\mathcal{O} =$

... transformation.

$$A(z,\bar{z}) \longrightarrow A'(z',\bar{z}') = \xi^{-h}\,\bar{\xi}^{-\tilde{h}}\,A(z,\bar{z})$$

$z \to z' = \xi z$

$(h,\tilde{h})$ – conformal weights

$h+\tilde{h}$ - dim of A

$h-\tilde{h}$ - spin

$$\mathcal{O}'(z', \bar{z}') = (\partial z')^{h} (\partial \bar{z}')^{\tilde{h}}\, \mathcal{O}(z, \bar{z})$$

$$\hookrightarrow T(z)\mathcal{O} = \frac{h}{z^2}\mathcal{O} + \frac{1}{z}\partial\mathcal{O}$$

of rigid scale transformation.

$$A(z, \bar{z}) \longrightarrow A'(z', \bar{z}') = \xi^{-h}\, \bar{\xi}^{-\tilde{h}}\, A(z, \bar{z})$$

$h + \tilde{h}$ - dim of A $\qquad z \to z' = \xi z \qquad (h, \tilde{h})$ - conformal weights

$h - \tilde{h}$ - spin