

Title: Introduction to the Bosonic String

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Abstract: This course provides a thorough introduction to the bosonic string based on the Polyakov path integral and conformal field theory. We introduce central ideas of string theory, the tools of conformal field theory, the Polyakov path integral, and the covariant quantization of the string. We discuss string interactions and cover the tree-level and one loop amplitudes. More advanced topics such as T-duality and D-branes will be taught as part of the course. The course is geared for M.Sc. and Ph.D. students enrolled in Collaborative Ph.D. Program in Theoretical Physics. Required previous course work: Quantum Field Theory (AM516 or equivalent). The course evaluation will be based on regular problem sets that will be handed in during the term. The primary text is the book: 'String theory. Vol. 1: An introduction to the bosonic string. J. Polchinski (Santa Barbara, KITP) . 1998. 402pp. Cambridge, UK: Univ. Pr. (1998) 402 p.' All interested students should contact Alex Buchel at abuchel@uwo.ca as soon as possible.

CFT (continuation)

CFT (continuation)

$$-S A(\sigma_0) - \frac{\epsilon}{2\pi i} \int_{\mathcal{R}} d^2\sigma \sqrt{-g} \nabla_n J^n \cdot A(\sigma_0) = 0$$



CFT (continuation)

$$-SA(\sigma_0) - \frac{\epsilon}{2\pi i} \int_R d^2\sigma \sqrt{g} \nabla_a J^a \cdot A(\sigma) = 0$$

σ_0 must be inside R

R
 G_0 must be inside R

$$\int_K d^4x \sqrt{-g} \nabla_n J^n A(G_0) = \frac{\delta A}{\delta G} \delta A(G_0)$$



\mathbb{R}
 σ_0 must be inside \mathbb{R}

$$\int_{\mathbb{R}} d^4\sigma \sqrt{-g} \nabla_n J^n A(\sigma_0) = \frac{2\pi}{i\epsilon} \delta A(\sigma_0)$$

$$\nabla_n J^n A(\sigma) = \frac{2\pi}{i\epsilon} \frac{1}{\sqrt{-g}} \delta^2(\sigma - \sigma_0) \delta A(\sigma)$$

\mathbb{R}
 σ_0 must be inside \mathbb{R}

$$\int_{\mathbb{R}} d^d \sigma \sqrt{-g} \nabla_a \left[J^a(\sigma) A(\sigma_0) \right] = \frac{2\pi i}{\epsilon} \delta A(\sigma_0)$$

$$\nabla_a J^a(\sigma) = \frac{2\pi i}{\epsilon} \frac{1}{\sqrt{-g}} \delta^2(\sigma - \sigma_0) \delta A(\sigma)$$

R
 σ_0 must be inside R

$$\int_K d^4\sigma \sqrt{-g} \nabla_a \left[J^a(\sigma) A(\sigma_0) \right] = \frac{2\pi}{i\varepsilon} \delta A(\sigma_0)$$

$$\nabla_a J^a(\sigma) = \frac{2\pi}{i\varepsilon} \frac{1}{\sqrt{-g}} \delta^2(\sigma - \sigma_0) \delta A(\sigma)$$

$$\int_{\partial R} n_a J^a \cdot A(\sigma_0) = \frac{2\pi}{i\varepsilon} \delta A(\sigma_0)$$

$$-SA(\epsilon_0) + \frac{i\epsilon}{2\pi} \int_R d^2\sigma \sqrt{-g} \nabla_\alpha J^\alpha A(G_0) = 0$$

G_0 must be inside R

$$\int_R d^2\sigma \sqrt{-g} \nabla_\alpha \left[J^\alpha A(G_0) \right] = \frac{2\pi}{i\epsilon} \delta A(G_0) \quad \left\{ \langle \nabla_\alpha J^\alpha \rangle = 0 \right.$$

$$\int_{\partial R} n_\alpha J^\alpha A(G_0) = \frac{2\pi}{i\epsilon} \delta A(G_0)$$

$$\oint_{\partial R} (i_{\bar{z}} dz - i_{\bar{z}} d\bar{z}) A(z_0, \bar{z}_0) = \frac{2\pi}{\epsilon} \delta A(z_0, \bar{z}_0)$$

$$\oint_{\partial R} (i_{\bar{z}} dz - i_z d\bar{z}) A(z_0, \bar{z}_0) = \frac{2\pi}{\varepsilon} \delta A(z_0, \bar{z}_0)$$

\Rightarrow LHS simplifies when

$J_z \rightarrow$ is holomorphic $\bar{\partial} J_z = 0$

$$\oint_{\partial R} (i_{\bar{z}} dz - i_{\bar{z}} d\bar{z}) A(z_0, \bar{z}_0) = \frac{2\pi}{\epsilon} \delta A(z_0, \bar{z}_0)$$

\Rightarrow LHS simplifies when

$J_{\bar{z}}$ is holomorphic $\bar{\partial} J_{\bar{z}} = 0$

$J_{\bar{z}}$ is antiholomorphic $\partial J_{\bar{z}} = 0$

$$\oint_{\partial R} (j_z dz - j_{\bar{z}} d\bar{z}) A(z_0, \bar{z}_0) = \frac{2\pi}{\epsilon} \delta A(z_0, \bar{z}_0)$$

\Rightarrow LHS simplifies when.

$$2\pi i \left[\text{Res } j_z A(z_0, \bar{z}_0) \right]$$

j_z is holomorphic $\bar{\partial} j_z = 0$

$j_{\bar{z}}$ is antiholomorphic $\partial j_{\bar{z}} = 0$

$$\oint_{\partial R} (j_z dz - j_{\bar{z}} d\bar{z}) A(z_0, \bar{z}_0) = \frac{2\pi}{\epsilon} \delta A(z_0, \bar{z}_0)$$

\Rightarrow LHS simplifies when

j_z is holomorphic $\bar{\partial} j_z = 0$

$j_{\bar{z}}$ is antiholomorphic $\partial j_{\bar{z}} = 0$

$$2\pi i \left[\text{Res } j_z A(z_0, \bar{z}_0) \right]$$

$$+ 2\pi i \left[\text{Res } j_{\bar{z}} A \right]$$

$$\oint_{\partial R} (j_z dz - j_{\bar{z}} d\bar{z}) A(z_0, \bar{z}_0) = \frac{2\pi}{\varepsilon} \delta A(z_0, \bar{z}_0)$$

\Rightarrow LHS simplifies when.

j_z is holomorphic $\bar{\partial} j_z = 0$

$j_{\bar{z}}$ is antiholomorphic $\partial j_{\bar{z}} = 0$

$$2\pi i \left[\text{Res } j_z A(z_0, \bar{z}_0) \right]$$

$$+ 2\pi i \left[\text{Res } j_{\bar{z}} A \right]$$

$$= \frac{2\pi}{\varepsilon} \delta A(z_0, \bar{z}_0)$$

$J_z = 0$ holomorphic $\partial J_z = 0$

$J_{\bar{z}} = 0$ antiholomorphic $\partial \bar{J}_{\bar{z}} = 0$

$$+ 2\pi i [\text{Res } J_z A]$$

$$= \frac{2\pi i}{z} \delta A(z, \bar{z})$$

$f(z)$

$\text{Res } f(z) \equiv \text{coeff}$

$J_z = 0$ is holomorphic $\partial J_z = 0$

$J_{\bar{z}} = 0$ is antiholomorphic $\partial J_{\bar{z}} = 0$

$$+ 2\pi i [\text{Res } J_z A]$$

$$= \frac{2\pi i}{\varepsilon} \delta A(z_0, \bar{z}_0)$$

$f(z)$

$\text{Res } f(z) \equiv \text{coeff of } \frac{1}{z}$

$$f(z) = \sum_{-\infty}^{+\infty} \frac{f_n}{z^{n+1}} \Rightarrow \text{Res } f(z) = f_0$$

$J_z = 0$ is holomorphic $\partial J_z = 0$
 $J_{\bar{z}} = 0$ is antiholomorphic $\partial J_{\bar{z}} = 0$

$$\begin{aligned}
 &+ 2\pi i \left[\text{Res } J_{\bar{z}} A \right] \\
 &= \frac{2\pi i}{\varepsilon} \delta A(z_0, \bar{z}_0)
 \end{aligned}$$

$f(z)$
 $\text{Res } f(z) \equiv \text{coeff of } \frac{1}{z}$

$$f(z) = \sum_{-\infty}^{+\infty} \frac{f_n}{z^{n+1}} \Rightarrow \text{Res } f(z) = f_1$$

$$\text{Res}_{z \rightarrow z_0} J_z A(z_0, \bar{z}_0) + \overline{\text{Res}_{\bar{z} \rightarrow \bar{z}_0} J_{\bar{z}} A(z_0, \bar{z}_0)} = \frac{1}{i\varepsilon} \delta A(z_0, \bar{z}_0)$$

$$\nabla \varphi X = 0.$$

Example 1 $\delta x^{\mu} = \xi a^{\mu} \rightarrow$ action is invariant

Example 1 $g_{\mu\nu} = \xi a^{\mu\nu} g(r) \rightarrow$ action is invariant

$$S = \frac{1}{4\pi\alpha'} \int d^2\sigma (\partial X)^2$$

$$SS = \frac{g_{\mu\nu}}{2\pi\alpha'} \int d^2\sigma$$



$$S = \frac{1}{4\pi\alpha'} \int d^2\sigma (\partial X)^2$$

$$\delta S = \frac{g_{\alpha\beta}}{2\pi\alpha'} \int d^2\sigma \partial^\alpha X^\mu \partial_\alpha \delta X^\nu$$

$$\begin{aligned}
 \delta S &= \frac{g g_{\text{YM}}^2}{2\pi^2} \int d^2\sigma \partial^\alpha X^\mu \partial_\alpha \beta = -\frac{1}{2\pi} \int d^2\sigma \partial^\alpha X^\mu \partial_\alpha \beta \\
 [d\varphi] e^{-S[\varphi]} &= [d\varphi] e^{-S[\varphi]} \left[1 + \frac{i\varepsilon}{2\pi} \int d^2\sigma j^a \nabla_a \mathcal{P} + \mathcal{O}(\varepsilon^2) \right]
 \end{aligned}$$

$$\begin{aligned}
 \delta S &= \frac{\delta q_{\mu\nu}}{2\pi i} \int d^2\sigma \partial^\alpha X^\mu \partial_\alpha \beta = -\frac{i}{2\pi} \int d^2\sigma \partial^\alpha X^\mu \partial_\alpha \beta \\
 [d\varphi'] e^{-S[\varphi]} &= [d\varphi] e^{-S[\varphi]} \left[1 + \frac{i}{2\pi} \int d^2\sigma j^a \nabla_a \beta + \mathcal{O}(\varepsilon^2) \right]
 \end{aligned}$$

$$\frac{\delta q_{\mu\nu}}{2\pi i} = -\frac{i}{2\pi}$$

$$\begin{aligned}
 \delta S &= \frac{\delta q_m}{2\pi\alpha'} \int d^2\sigma \partial^a X^\mu \partial_a \beta \\
 [d\varphi^i] e^{-S[\varphi]} &= [d\tau] e^{-S[\tau]} \left[1 + \frac{i\varepsilon}{2\pi} \int d^2\sigma j^a \nabla_a \beta + \mathcal{O}(\varepsilon^2) \right]
 \end{aligned}$$

$$\delta X^\mu = \frac{\delta q_m}{2\pi\alpha'} = -\frac{i\varepsilon}{2\pi} j^a$$

$$\begin{aligned}
 \delta S &= \frac{g \alpha_m}{2\pi\alpha'} \int d^2\sigma \partial^a X^\mu \partial_a \beta \\
 [d\varphi] e^{-S[\varphi]} &= [d\varphi] e^{-S[\varphi]} \left[1 + \frac{i\varepsilon}{2\pi} \int d^2\sigma \left(\overset{\delta S}{j^a \nabla_a \beta} + \mathcal{O}(\varepsilon^2) \right) \right]
 \end{aligned}$$

$$\overset{\delta S}{X} \frac{\sum a_m}{2\pi\alpha'} = -\frac{i\varepsilon}{2\pi} j^a \Rightarrow \mathcal{T}_a^\mu = a_m \mathcal{T}_a^\mu$$

$$\boxed{\mathcal{T}_{0,1}^\mu = +\frac{i}{\alpha'} \partial_a X^\mu}$$

$$\delta S = \frac{\epsilon \alpha_m}{2\pi\alpha'} \int d^2\sigma \partial^\alpha X^m \partial_\alpha \beta$$

$$[d\varphi'] e^{-S[\varphi]} = [d\tau] e^{-S[\tau]} \left[1 + \frac{i\epsilon}{2\pi} \int d^2\sigma j^a \nabla_a \beta + \mathcal{O}(\epsilon^2) \right]$$

↖ -δS

$$\frac{\epsilon \alpha_m}{2\pi\alpha'} j^a = -\frac{i\epsilon}{2\pi} j^a \Rightarrow \mathcal{T}_a^m = \alpha_m \mathcal{T}_a^m$$

$$\mathcal{T}_a^m = +\frac{i}{\alpha'} \partial_a X^m$$

$$\Rightarrow \mathcal{T}_2^m = \frac{i}{\alpha'} \partial X^m$$

$$\tilde{\mathcal{T}}_2^m = \frac{i}{\alpha'} \bar{\partial} X^m$$

$$\begin{aligned}
 \delta S &= \frac{\alpha' g_{\mu\nu}}{2\pi\alpha'} \int d^2\sigma \partial^\alpha X^\mu \partial_\alpha \delta \rho \\
 [d\rho] e^{-S[\rho]} &= [d\rho] e^{-S[\rho]} \left[1 + \frac{i\varepsilon}{2\pi} \int d^2\sigma j^a \nabla_a \rho + \mathcal{O}(\varepsilon^2) \right]
 \end{aligned}$$

$\swarrow -\delta S$

$$\delta X^\mu \frac{\alpha' g_{\mu\nu}}{2\pi\alpha'} = -\frac{i\varepsilon}{2\pi} j^a \Rightarrow \mathcal{J}_a^\mu = g_{\mu\nu} \mathcal{T}_a^\nu$$

\swarrow holomorphic

$$\mathcal{T}_{0,1}^\mu = +\frac{i}{\alpha'} \partial_a X^\mu$$

$$\Rightarrow \mathcal{J}_2^\mu = \frac{i}{\alpha'} \partial X^\mu$$

$$\nearrow \tilde{\mathcal{J}}_2^\mu = \frac{i}{\alpha'} \bar{\partial} X^\mu$$

\swarrow antiholomorphic

$$A = \int e^{ikx} \dots$$

$$j_2 = e^{ikx}$$



$$A = : e^{iKX} :_{\tau} \text{ (normal ordered)}$$

$$j_2 : e^{iKX(z_0, \bar{z}_0)} : = : \bar{J}_2 e^{iKX} : + \text{cross-contractions}$$

↑
can be singular as $z \rightarrow z_0$

$J_+ \in \mathbb{C} \equiv J_+ \in \mathbb{C} + (\text{cross-contractions})$
 can be singular as $z \rightarrow z_0$

\Rightarrow A rule: the most singular in $\frac{1}{z-z_0}$ term
 comes from most cross-contractions

$$J_+^{-1} A(G_0) = \frac{2\pi}{i\varepsilon} \delta A(G_0)$$

\Rightarrow A rule. the most singular in $\frac{1}{z-z_0}$ term
 comes from most cross-contractions
 can be singular as $z \rightarrow z_0$

$\textcircled{\leq} \frac{1}{z} : \gamma X : : \mathcal{C}^{ikX}$

\Rightarrow A rule: the most singular in $\frac{1}{z-z_0}$ term
 comes from most cross-contractions
 can be singular as $z \rightarrow z_0$

\Rightarrow $\frac{1}{z}$; $i k x$; $\frac{1}{z}$; $(-\frac{1}{2} \ln |z+z_0|^2)$
 only singular pieces

\Rightarrow A rule. the most singular in $\frac{1}{z-z_0}$ term
 comes from most cross-contraction
 can be singular as $z \rightarrow z_0$

$\textcircled{\leq} \frac{i}{2} : \partial X^{\mu} : : e^{ikX} ; \quad \frac{i}{2} (-\frac{1}{2} \alpha') \ln |z-z_0|^2$
 only singular pieces $i k_{\mu} : e^{ikX} :$

\Rightarrow A rule. the most singular in $\frac{1}{z-z_0}$ term
 comes from most cross-contractions
 can be singular as $z \rightarrow z_0$

$\textcircled{=}$ $\frac{i}{2} : \partial X^\mu : : e^{ikX}$; $\frac{i}{2} (-\frac{1}{2} \alpha'^2) \ln |z-z_0|^2$
 $\int \frac{\delta X^\nu}{\delta X^\nu}$; only singular pieces ; $i k_\mu : e^{ikX}$
 $= \frac{k_\mu}{2(z-z_0)} : e^{ikX} :$

\Rightarrow A rule. the most singular in $\frac{1}{z-z_0}$ term
 comes from most cross-contractions
 can be singular as $z \rightarrow z_0$

$\Leftrightarrow \frac{i}{2} : \partial X^\mu : : e^{ikX} ; \quad \frac{i}{2} (-\frac{1}{2} \alpha') \ln |z+z_0|^2$
 only singular pieces
 $i k_\mu : e^{ikX} :$
 $\frac{i}{2} : \partial X^\mu : = \frac{k^\mu}{2(z-z_0)} : e^{ikX} :$
 $\frac{i}{2} : \bar{\partial} X^\mu : = \frac{k^\mu}{2(\bar{z}-\bar{z}_0)} : e^{ikX} :$

$$EHS = \left(\frac{k^{LH}}{2} + \frac{k^{LH}}{2} \right) \cdot A$$

$$EHS = \left(\frac{k^w}{2} + \frac{k^w}{2} \right) A = k^w A =$$



$$EHS = \left(\frac{k^\mu}{2} + \frac{k^\mu}{2} \right) \cdot A = k^\mu A = \frac{1}{i\varepsilon} \delta A$$

$\rightarrow \delta A = i k^\mu \varepsilon A$
 from Ward identity

$$e^{ikx}$$

$$EHS = \left(\frac{k^\mu}{2} + \frac{k^\mu}{2} \right) A = k^\mu A = \frac{1}{i\varepsilon} \delta A$$

from
Ward identity

$$\delta A = i k^\mu \varepsilon A$$

$$A = e^{i k X} \rightarrow e^{i k X + i k^\mu \varepsilon}$$

$$X^\mu \rightarrow X^\mu + \varepsilon$$

Example 2

85°

Example 2

$$\delta\sigma^a = \varepsilon U^a$$

↑ U^a is a constant.

Example 2

$$\delta\sigma^a = \varepsilon U^a$$

$\uparrow U^a$ is a constant.

$$\sigma^a \rightarrow \sigma^{a'} = \sigma^a + \varepsilon U^a$$

$$X^m(\sigma') = X^m(\sigma)$$

Example 2 $\delta\sigma^n = \varepsilon U^n$

$$\sigma^a \rightarrow \sigma^{a'} = \sigma^a + \varepsilon U^n$$

$$X^{uu'}(\sigma') = X^{uu}(\sigma)$$

$$\delta X^{uu}(\sigma) = X^{uu'}(\sigma) - X^{uu}(\sigma)$$

$\uparrow U^n$ is a constant

Example 2 $\delta\sigma^a = \varepsilon U^a$

$$\sigma^a \rightarrow \sigma^{a'} = \sigma^a + \varepsilon U^a$$

$\uparrow U^a$ is a constant

$$X^{u'}(\sigma') = X^u(\sigma) \Rightarrow X^{u'}(\sigma + \delta\sigma)$$

$$\delta X^u(\sigma) = X^{u'}(\sigma) - X^u(\sigma)$$

Example 2

$$\delta\sigma^a = \varepsilon U^a$$

$\uparrow U^a$ is a constant.

$$X^a(\sigma) = X^a(\sigma - \delta\sigma)$$

$$\sigma^a \rightarrow \sigma^{a'} = \sigma^a + \varepsilon U^a$$

$$X^{m'}(\sigma') = X^m(\sigma) \Rightarrow X^{m'}(\sigma + \delta\sigma) = X^m(\sigma) \Rightarrow$$

$$\delta X^m(\sigma) = X^{m'}(\sigma) - X^m(\sigma)$$

Example 2

$$\delta\sigma^a = \varepsilon U^a$$

$\uparrow U^a$ is a constant.

$$\underline{X^a(\sigma) = X^a(\sigma - \delta\sigma)}$$

$$\sigma^a \rightarrow \sigma^{a'} = \sigma^a + \varepsilon U^a$$

$$X^{a'}(\sigma') = X^a(\sigma) \Rightarrow X^{a'}(\underbrace{\sigma + \delta\sigma}_{\sigma'}) = X^a(\sigma) \Rightarrow$$

$$\delta X^a(\sigma) = X^{a'}(\sigma) - X^a(\sigma)$$

Example 2

$$\delta\sigma^a = \varepsilon U^a$$

\uparrow U^a is a constant.

$$\underline{X^m(\sigma) = X^m(\sigma - \delta\sigma)}$$

$$\sigma^a \rightarrow \sigma^{a'} = \sigma^a + \varepsilon U^a$$

$$X^m(\sigma') = X^m(\sigma) \Rightarrow X^m(\sigma + \frac{\delta\sigma}{\sigma'}) = X^m(\sigma)$$

$$\delta X^m(\sigma) = X^m(\sigma) - X^m(\sigma')$$

$$= X^m(\sigma - \varepsilon U^a) - X^m(\sigma) = -\varepsilon U^a \partial_{\sigma^a} X^m$$

Example 2

$$\delta\sigma^a = \varepsilon U^a$$

$\uparrow U^a$ is a constant.

$$\underline{X^a(\sigma) = X^a(\sigma - \delta\sigma)}$$

$$\sigma^a \rightarrow \sigma^{a'} = \sigma^a + \varepsilon U^a$$

$$X^{m'}(\sigma') = X^m(\sigma) \Rightarrow X^{m'}(\underbrace{\sigma + \delta\sigma}_{\sigma'}) = X^m(\sigma) \Rightarrow$$

$$\delta X^m(\sigma) = X^{m'}(\sigma) - X^m(\sigma)$$

$$= X^{m'}(\sigma^a - \varepsilon U^a) - X^m(\sigma) = \left[X^m_{\sigma^a} \varepsilon U^a - X^m \right]$$

$$\delta S = \delta \left[\frac{1}{4\pi\alpha'} \int d^2\sigma \partial^\alpha X^\mu \partial_\alpha X_\mu \right]$$

$$= \frac{1}{2\pi\alpha'} \int d^2\sigma \partial^\alpha X^\mu \delta X_\mu$$

$$\delta S = \delta \left[\frac{1}{4\pi\alpha'} \int d^2\sigma \partial^\alpha X^\mu \partial_\alpha X_\mu \right]$$

$$= \frac{1}{2\pi\alpha'} \int d^2\sigma \partial^\alpha X^\mu \partial_\alpha \delta X_\mu$$

$$\partial(\bar{7}-\bar{6}) \cdot \leftarrow$$

$$\delta S = \delta \left[\frac{1}{4\pi\alpha'} \int d^2\sigma \partial_a X^\mu \partial_b X_\mu \right]$$

$$= \frac{1}{2\pi\alpha'} \int d^2\sigma \partial_b X^\mu \partial_a \delta X_\mu =$$

$$\delta X_\mu = -\varepsilon(\sigma) \eta^{\mu\nu} \partial_\nu X_\mu$$

$$= \frac{k^\mu}{2(\bar{z}-z_0)} \cdot e^{ikX}$$

$$= \frac{1}{2\pi i} \int_{\sigma_1}^{\sigma_2} \int_{\sigma_1}^{\sigma_2} X^u \partial_b \delta X_u \left[= -\frac{1}{2\pi i} \int_{\sigma_1}^{\sigma_2} \int_{\sigma_1}^{\sigma_2} X^u \partial_a \delta X_u \right]$$

$$\delta X_u = -\frac{\varepsilon(\zeta)}{i} \partial_a X_u$$

$$\frac{1}{i} : \bar{\partial} X^u : e^{ikx} = \frac{k}{2(z-\bar{z}_0)} : e^{ikx} :$$

$$= \frac{k^u}{2(\bar{z}-\bar{z}_0)} : e^{ikx} :$$

$$= \frac{1}{2\pi\alpha'} \int d^2\sigma \partial_a X^\mu \partial_b X^\nu \delta X_\mu = -\frac{1}{2\pi\alpha'} \int d^2\sigma \partial_a X^\mu \partial_b X^\nu \epsilon \delta X_\mu$$

$$\delta X_\mu = -\frac{\epsilon(\sigma)}{2\pi\alpha'} \partial_a X^\mu$$

$$\frac{1}{2\pi\alpha'} \partial_a X^\mu e^{ikx} = \frac{k^\mu}{2(z-\bar{z}_0)} e^{ikx} = \frac{k^\mu}{2(\bar{z}-\bar{z}_0)} e^{ikx}$$

$$\partial \bar{X}_m = -\frac{\partial(\sigma)}{\partial \bar{z}} \partial \bar{X}_m$$

$$\frac{1}{2\pi i} \int d^2 \sigma \partial X^m \partial \bar{X}^n$$

$$\Leftrightarrow \frac{1}{2} : \partial X^m : = e^{ikX} ; \quad \frac{1}{2} \left(-\frac{1}{2} \alpha' \right) \ln |z+z_0|^2$$

only singular
poles

$$ik \alpha' : e^{ikX} :$$

$$\frac{1}{2} : \partial \bar{X}^m : = \frac{k^m}{2(z-z_0)} : e^{ikX} : = \frac{k^m}{2(\bar{z}-\bar{z}_0)} : e^{ikX} :$$

$$\delta X_{\mu} = -\frac{\epsilon(\sigma)}{2\pi\alpha'} \partial_a X_{\mu}$$

$$\left(\frac{0}{2\pi\alpha'} \right) \int d^2\sigma \partial_a X^{\mu} \partial_b X^{\nu} \epsilon$$

$$-\delta S = \frac{i}{2\pi} \int d^2\sigma \sqrt{-g} \partial_a \epsilon$$



$$\delta X_m = - \frac{\delta(\epsilon)}{2\pi} \partial_a X_m$$

$$\left(\frac{0}{2\pi} \right) \int d^2\sigma \partial_a X^m \partial_b X^m$$

②

$$-\delta S = \frac{i}{2\pi} \int d^2\sigma \sqrt{-g} \partial_a \epsilon$$

②: $\partial_a X^m \partial_b X^m \epsilon = \frac{1}{2} \partial_b [\partial_a X^m \partial_a X^m]$

$$\delta X_m = - \frac{\delta(\sigma)}{2\pi\alpha'} \partial_a X_m \quad \left(\frac{0}{2\pi\alpha'} \int d^2\sigma \partial X \partial_b \partial_a X \right) \quad \textcircled{2}$$

$$-\delta S = \frac{i\psi}{2\pi} \int d^2\sigma \sqrt{g} \partial_a \epsilon$$

$$\textcircled{2} \cdot \partial^a X^m \partial_b \partial_a X^m \cdot \epsilon = \frac{1}{2} \partial_b [g^a X^m \partial_a X^m] \cdot \epsilon$$

$$\rightarrow -\frac{1}{2} (\partial X)^2 \cdot \partial_b \epsilon$$

$$T_b = \frac{1}{i\alpha'} \left(\eta_a X^{\mu} \partial_b X_{\mu} - \frac{1}{2} \delta_{ab} (\partial X)^2 \right)$$

$$T_a = \frac{1}{i d^4} \left(\partial_a X^\mu \partial_b X_\mu - \frac{1}{2} \delta_{ab} (\partial X)^2 \right)$$

$$\mathbb{T}_{ab} = -\frac{1}{d^4} \left(\partial_a X^\mu \partial_b X_\mu - \frac{1}{2} \delta_{ab} (\partial X)^2 \right)$$

$$J_a = \frac{0^b}{i \alpha'} \left(\partial_a X^\mu \partial_b X_\mu - \frac{1}{2} \delta_{ab} (\partial X)^2 \right)$$

$$\Pi_{ab} \stackrel{\text{def}}{=} -\frac{1}{\alpha'} \left(\partial_a X^\mu \partial_b X_\mu - \frac{1}{2} \delta_{ab} (\partial X)^2 \right)$$

$$J_a = -\frac{1}{i} \dot{X}^\perp \Pi_{ab} = i \dot{X}^b \Pi_{ab}$$

In 4CFT.

$$T_{ab} = -\frac{1}{2} : \partial_a X^\mu \partial_b X_\mu - \frac{1}{2} \delta_{ab} (\partial X)^2 :$$

In 4CFT.

$$T_{ab} = -\frac{1}{2\alpha'} : \partial_a X^\mu \partial_b X_\mu - \frac{1}{2} \delta_{ab} (\partial X)^2 :$$

⇒ Conformal inv

In 4D CFT:

$$T_{ab} = -\frac{1}{2\alpha'} : \partial_a X^\mu \partial_b X_\mu - \frac{1}{2} \delta_{ab} (\partial X)^2 :$$

\Rightarrow Conformal inv

$$T_a^a = 0$$

In 4CFT.

$$T_{ab} = -\frac{1}{2\alpha'} : \partial_a X^\mu \partial_b X_\mu - \frac{1}{2} \delta_{ab} (\partial X)^2 :$$

\Rightarrow Conformal inv

$$T_a^a = 0$$

$$g^{\bar{z}\bar{z}} = 2, \quad g^{\bar{z}z} = 2$$

In CFT

$$T_{ab} = -\frac{1}{2} : \partial_a X^\mu \partial_b X_\mu - \frac{1}{2} \delta_{ab} (\partial X)^2 :$$

Conformal inv

$$g^{z\bar{z}} = 2, \quad g^{\bar{z}z} = 2$$

$$T_a^a = 0$$

\Rightarrow

$$T_{z\bar{z}} = 0 = T_{\bar{z}z}$$

$\sigma^0 \rightarrow \sigma^a$... σ^a is a constant $\chi(\sigma) = \chi(\sigma - i\epsilon)$

Only nonvanishing T_{ab} : $\{T_{zz}, T_{\bar{z}\bar{z}}\}$



$$\lambda (6 - \epsilon_0) - \lambda (6) = \epsilon_0 \lambda = 0 \lambda$$

Only nonvanishing T_{ab} : $\{T_{zz}, T_{\bar{z}\bar{z}}\}$
 \Rightarrow use conservation of the current.

$$\lambda (6 - \epsilon_0^4) - \lambda (6) = \epsilon_0^4 \lambda = 0$$

Only nonvanishing T_{ab} : $\{T_{zz}, T_{\bar{z}\bar{z}}\}$
 \Rightarrow use conservation of the current.

$$\nabla_a J^a = 0$$



$$\lambda (6 - \epsilon_0^2) - \lambda (6) = \epsilon_0^2 \lambda - \epsilon_0^2 \lambda = 0$$

Only nonvanishing T_{ab} : $\{T_{zz}, T_{\bar{z}\bar{z}}\}$
 \Rightarrow use conservation of the current.

$$\nabla_a J_a^W = 0 \Rightarrow \nabla_a T_{aL} = 0$$



$$\lambda (6 - \epsilon \partial^a) - \lambda (6) = \lambda \epsilon \partial^a \lambda = \epsilon \lambda$$

Only nonvanishing T_{ab} : $\{T_{zz}, T_{\bar{z}\bar{z}}\}$
 \Rightarrow use conservation of the current.

$$\partial_a \pi^a = 0 \Rightarrow \partial_a \pi_{ab} = 0$$

∂

$$\lambda (6 - \epsilon \mathcal{U}^a) - \lambda (0) = \epsilon \mathcal{U}^a \partial_0 \lambda = 0 \lambda$$

Only nonvanishing T_{ab} : $\{ T_{z\bar{z}}, T_{\bar{z}z} \}$
 \Rightarrow use conservation of the current.

$$\nabla_a \pi^a = 0 \Rightarrow \nabla_a \pi_{ab} = 0$$

$$\partial_z T_{z\bar{z}} + \partial_{\bar{z}} T_{\bar{z}z} = 0$$

$$\lambda (6 - \epsilon_0^2) - \lambda (6) = \epsilon_0^2 \lambda = 0 \Rightarrow \lambda = 0$$

Only nonvanishing T_{ab} : $\{T_{zz}, T_{\bar{z}\bar{z}}\}$
 \Rightarrow use conservation of the current.

$$\nabla_a J^a = 0 \Rightarrow \nabla_a T_{ab} = 0$$

$$\partial_{\bar{z}} T_{zb} + \partial_z T_{\bar{z}b} = 0$$

$$\lambda (b - \epsilon \partial^a) - \lambda (a) = \epsilon \partial^a \lambda = \partial_a \lambda = 0$$

Only nonvanishing T_{ab} : $\{T_{zz}, T_{\bar{z}\bar{z}}\}$
 \Rightarrow use conservation of the current.

$$\nabla_a J^a = 0 \Rightarrow \nabla_a T_{ab} = 0$$

$$\partial_z T_{zb} + \cancel{\partial_{\bar{z}} T_{\bar{z}b}} = 0 \quad \nearrow b=z \Rightarrow \partial_z T_{zz} = 0$$

$$b=\bar{z} \Rightarrow \partial_{\bar{z}} T_{\bar{z}\bar{z}} = 0$$

\Rightarrow use conservation of the current.

$$\nabla_a J^a = 0 \Rightarrow \nabla_a T_{ab} = 0$$

$$\partial T_{zb} + \cancel{\partial T_{\bar{z}b}} = 0 \quad \nearrow b=z \Rightarrow \partial T_{zz} = 0$$

$$T_{zz} = T(z) \quad b=\bar{z} \Rightarrow \partial T_{z\bar{z}} = 0$$

$$\nabla_{\bar{z}} = \frac{1}{2} \partial_{x^{\mu}}$$

antiholomorphic

\Rightarrow use conservation of the current.

$$\nabla_a J^a = 0 \Rightarrow \nabla_a T_{ab} = 0$$

$$\partial_{\bar{z}} T_{zb} + \partial_{\bar{z}} T_{\bar{z}b} = 0 \quad b=z \Rightarrow \partial_{\bar{z}} T_{zz} = 0$$

$$T_{zz} = T(z) \quad \& \quad T_{\bar{z}\bar{z}} = \tilde{T}(\bar{z}) \quad b=\bar{z} \Rightarrow \partial_{\bar{z}} T_{\bar{z}\bar{z}} = 0$$

$$T_{\bar{z}} = \frac{1}{2} \partial x_{\mu}$$

antiholomorphic

$$\mathbb{T}(z) = -\frac{1}{z} : \partial X_m : \partial X_m :$$



$$\mathbb{T}(z) = -\frac{1}{2} : \partial X^{\mu} \partial X_{\mu} : \quad \tilde{\mathbb{T}}(\bar{z}) = -\frac{1}{2} : \bar{\partial} X^{\mu} \bar{\partial} X_{\mu} :$$



$$T(z) = -\frac{1}{2} : \partial X^{\mu} \partial X_{\mu} : \quad \tilde{T}(\bar{z}) = -\frac{1}{2} : \bar{\partial} X^{\mu} \bar{\partial} X_{\mu} :$$

⇒ A good exercise is to check that Ward identities
 satisfied for $\delta G^{\mu} = -\epsilon \partial^{\mu} \psi^{\nu}$

$$T(z) = -\frac{1}{2} : \partial X^\mu \partial X_\mu : \quad \tilde{T}(\bar{z}) = -\frac{1}{2} : \bar{\partial} X^\mu \bar{\partial} X_\mu :$$

⇒ A good exercise is to check that Ward identities
 satisfied for $\delta g^{\mu\nu} = +\epsilon \eta^{\mu\nu}$



δg

$$T(z) = -\frac{1}{2} : \partial X^\mu \partial X_\mu : \quad ; \quad \tilde{T}(\bar{z}) = -\frac{1}{2} : \bar{\partial} X^\mu \bar{\partial} X_\mu :$$

⇒ A good exercise is to check that Ward identities
 satisfied for $\delta \sigma^a = +\epsilon \theta^a$

$$j = i \theta^a T_{ab}$$

$$\rightarrow -\frac{1}{2} (\partial X) \cdot \partial \epsilon$$

$$T(z) = -\frac{1}{2} : \partial X^\mu \partial X_\mu : \quad ; \quad \tilde{T}(\bar{z}) = -\frac{1}{2} : \bar{\partial} X^\mu \bar{\partial} X_\mu :$$

⇒ A good exercise is to check that Ward identities
 satisfied for $\delta \sigma^a = +\epsilon \eta \omega^a$

$$j = i \omega^a T_{ab}$$

$$\partial \pi = 0$$

$$\frac{1}{2} (\partial X) \cdot \partial \epsilon$$

\Rightarrow A good exercise is to check that Ward identities
satisfied for $\delta\phi^a = +\epsilon\theta^a$

$$\left. \begin{aligned} j &= i\theta^a \pi_{ab} \\ \partial_\mu \pi &= 0 \end{aligned} \right\} \theta^a \text{ was a constant.}$$

⇒ A good exercise is to check that Ward identities
 satisfied for $\delta G^{\mu\nu} = +\epsilon G^{\mu\nu}$

$$j = i U^{\mu\nu} \Pi_{ab} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} U^{\mu\nu} \text{ was a constant.}$$

generalization

$$j_z = i U(z) \Pi(z) \quad \tilde{j}_z = i \overline{U(z)} \tilde{\Pi}(z)$$

$$\epsilon = \frac{1}{2} \partial_b \left[\partial^a X^{\mu\nu} \partial_n X_{\mu\nu} \right] \cdot \epsilon$$

$$\frac{1}{2} (\partial X)^2 \cdot \partial_b \epsilon$$

$$\left. \begin{aligned}
 J &= i \int \psi^\dagger \not{\partial} \psi \\
 \bar{\partial} \Pi &= 0
 \end{aligned} \right\} \psi^\dagger \text{ was a constant.}$$

generalization

$$J_2 = i \int \psi^\dagger(z) \not{\partial} \psi(z) \quad \bar{J}_2 = i \int \psi^\dagger(\bar{z}) \not{\partial} \psi(\bar{z})$$

②

$$\delta X^\mu \cdot \epsilon = \frac{1}{2} \partial_b \left[\partial^a X^\mu \partial_a X^\mu \right] \cdot \epsilon$$

$$(\partial X)^2 \cdot \partial_b \epsilon$$

$$\partial \pi = 0$$

generalization

$$J_2 = i \int \sigma(z) \pi(z)$$

$$\tilde{J}_2 = \int \overline{\sigma(z)} \tilde{\pi}(z)$$

$$\nabla_a J^a = \bar{\partial} J_2 + \partial \tilde{J}_2 = 0$$

$$\partial \pi = 0$$

generalization

$$J_2 = i U(z) \pi(z)$$

$$\tilde{J}_2 = i \overline{U(z)} \tilde{\pi}(\bar{z})$$

$$\nabla_a J^a = \bar{\partial} J_2 + \partial \tilde{J}_2 = 0$$

\Rightarrow The new currents are conserved provided $U(z)$ is holomorphic

We want to find δX

We want to find δX due to symmetries $J_z = i\Omega T$

We want to find δX due to symmetries $T_z = i\partial_t$

$$\text{Re} \int_{z \rightarrow \bar{z}_0} j_z A + \overline{\text{Re} \int_{\bar{z} \rightarrow \bar{z}_0} \tilde{j}_z A} = \frac{1}{i\epsilon} \delta A$$

We want to find δX due to symmetries $T_z = iUT$

$$\text{Res}_{z \rightarrow z} j_z A + \overline{\text{Res}}_{\bar{z} \rightarrow \bar{z}_0} \tilde{j}_z A = \frac{1}{i\epsilon} \delta A$$

We want to find δX due to symmetries $T_z = i\partial_t$

$$\text{Res}_{z \rightarrow z_0} j_z A + \overline{\text{Res}}_{\bar{z} \rightarrow \bar{z}_0} \tilde{j}_z A = \frac{1}{i\varepsilon} \delta A$$

$$\Rightarrow A = X^{\omega}$$

$$P_{ez} jz A + \overline{P_{es}} \overline{jz} A = \frac{1}{i\epsilon} (\delta A)$$

$$\Rightarrow A = X^{\omega}$$

$$jz = i\nu\pi$$

$$\Rightarrow A = X$$

$$j = i \nu \pi$$

$$:\pi(z) : = X^{\mu}(z_0, \bar{z}_0)$$

$$\pi_a = 0$$

$$\Rightarrow$$

$$\pi$$

$$= 0$$

$$= \pi$$

$$z$$

$$\Rightarrow A = X$$

$$j_0 = i \nu \pi$$

$$:\pi(z) = : X^{\mu}(z_0, \bar{z}_0) : = -\frac{1}{2} : \partial X^{\mu} \partial X^{\nu}(z) : X^{\mu}(z_0, \bar{z}_0) :$$

$$\pi^{\mu\nu} = 0 \Rightarrow \pi_{z\bar{z}} = 0 = \pi_{\bar{z}z}$$

$$j_{\pi} = i \nu \pi$$

$$\pi(z) : X^m(z_0, \bar{z}_0) \sim \frac{1}{z} : \partial X \partial \bar{X}(z) : X^m(z_0, \bar{z}_0) \sim \left(\frac{1}{z} \right)$$

\Rightarrow Conformal inv

$$g^{z\bar{z}} = 2, \quad g^{\bar{z}z} = 2$$

$$\pi_a^a = 0 \Rightarrow \pi_{z\bar{z}} = 0 = \pi_{\bar{z}z}$$

$$T(z) = \frac{1}{2} \dot{X}^\mu \dot{X}_\mu \quad ; \quad X^\mu(z_0, \bar{z}_0) ;$$

$$\sim \left(\frac{1}{z} \right) \cdot \left(-\frac{z'}{2} \ln |z - z_0|^2 \right)$$

$$T_{ab} = -\frac{1}{2} \dot{X}^\mu \dot{X}_\mu - \frac{1}{2} \delta_{ab} (\partial X)^2 ;$$

Conformal inv

$$T_a^a = 0 \Rightarrow T_{z\bar{z}} = 0 = T_{\bar{z}z}$$

$$T(z) = \frac{1}{2} \dot{X}^\mu \dot{X}_\mu = \frac{1}{2} \dot{X}^\mu \dot{X}^\nu \eta_{\mu\nu} = \frac{1}{2} \dot{X}^\mu \dot{X}^\nu \partial_x \partial_{\bar{x}} X_\mu \partial_x \partial_{\bar{x}} X_\nu$$

$$\sim \left(\frac{1}{2} \right) \cdot \left(-\frac{1}{2} \ln|z-z_0|^2 \right)$$

$$T_{ab} = -\frac{1}{2} \dot{X}^\mu \dot{X}_\mu = -\frac{1}{2} \delta_{ab} (\partial X)^2$$

\Rightarrow Conformal inv

$$T_a^a = 0 \Rightarrow T_{z\bar{z}} = 0 = T_{\bar{z}z}$$

$$g^{z\bar{z}} = 2, \quad g^{\bar{z}z} = 2$$

$$T(z) = X^{\mu}(\tau_0, \bar{z}_0) \cdot \left(-\frac{1}{2} \partial_X \partial_{\bar{X}} \right) : X^{\mu}(\tau_0, \bar{z}_0) : \\ \sim \left(-\frac{1}{2} \right) \cdot \left(-\frac{1}{2} \partial_X^2 \ln |z - z_0|^2 \right) \cdot 2$$

$$T_{ab} = -\frac{1}{2} : \partial_a X^{\mu} \partial_b X_{\mu} - \frac{1}{2} \delta_{ab} (\partial X)^2 : \\ \Rightarrow \text{Conformal inv}$$

$$T_a^a = 0 \Rightarrow T_{z\bar{z}} = 0 = T_{\bar{z}z}$$

$$\begin{aligned}
 \Gamma(z) &: X^{\mu}(z_0, \bar{z}_0) : = -\frac{1}{2} : \partial X^{\mu} \partial X^{\nu}(z) : X^{\mu}(z_0, \bar{z}_0) : \\
 &\sim -\left(\frac{1}{2}\right) \cdot \left(-\frac{2}{2} \partial_{\bar{z}} \ln |z - z_0|^2\right) \cdot 2 : \partial X^{\mu}(z_0) :
 \end{aligned}$$

$$\Gamma_{ab} = -\partial_a X^{\mu} \partial_b X_{\mu} - \frac{1}{2} \delta_{ab} (\partial X)^2 :$$

\Rightarrow Conform

$$\Gamma_a^a$$

$$g^{z\bar{z}} = 2, \quad g^{\bar{z}z} = 2$$

$$\Gamma_{z\bar{z}} = 0 = \Gamma_{\bar{z}z}$$

$$\sim \begin{pmatrix} -\frac{1}{z} \\ -\frac{z'}{2} \partial \ln |z-z_0|^2 \end{pmatrix} \cdot 2 ; \partial X(z_0)$$

$$\pi X^m \sim$$

[The main body of the chalkboard is heavily scribbled out with dark chalk, obscuring most of the original text.]

$$\sim \left(\frac{1}{z} \right) \left(-\frac{z'}{2} \frac{\partial \ln(z-z_0)^2}{\partial z} \right) \cdot 2 ; \partial X(z_0)$$

$$\pi \cdot X''' \sim \frac{1}{z-z_0} \partial X(z_0)$$

$$\sim \begin{pmatrix} -\frac{1}{z} \\ -\frac{z'}{2} \frac{\partial \ln(z-z_0)^2}{z} \end{pmatrix} \cdot 2 ; \partial X(z_0)$$

$$\begin{aligned} \mathbb{E} X^m &\sim \frac{1}{z-z_0} \partial X(z_0) \\ \mathbb{E} X^m & \end{aligned}$$

$$\sim \begin{pmatrix} -\frac{1}{z} \\ -\frac{z'}{2} \partial_{\bar{z}} \ln |z-z_0|^2 \end{pmatrix} \cdot 2 ; \partial X(z_0)$$

$$\exists X^m \sim \frac{1}{z-z_0} \partial X(z_0)$$

$$\exists X^m \sim \frac{1}{\bar{z}-\bar{z}_0} \partial X(\bar{z}_0)$$

$$\sim \begin{pmatrix} -1 \\ 1 \end{pmatrix} \left(-\frac{z_1}{2} \frac{\partial f_n}{\partial z} (z-z_0)^2 \right) \cdot 2 ; \partial X(z_0)$$

$$\exists \exists X^m \sim \frac{1}{z-z_0} \partial X(z_0)$$

$$\exists \exists X^m \sim \frac{1}{z-z_0} \partial X(z_0)$$

$$\partial X(z_0)$$

$$\sim \begin{pmatrix} -1 \\ 1 \end{pmatrix} \left(-\frac{z_1}{2} \frac{\partial f_n}{\partial z} (z-z_0)^2 \right) \cdot 2 ; \partial X(z_0)$$

$$\exists X^m \sim \frac{1}{z-z_0} \partial X(z_0)$$

$$\exists X^m \sim \frac{1}{z-\bar{z}_0} \partial X(\bar{z}_0)$$

$$\partial X(z) + \bar{\partial} X(\bar{z})$$

$$\sim \begin{pmatrix} -\frac{1}{z} \\ -\frac{1}{z} \end{pmatrix} \left(-\frac{z'}{2} \frac{\partial^2 \chi}{\partial z^2} (z-z_0)^2 \right) \cdot 2 ; \partial \chi(z_0)$$

$$\mathbb{H} \chi''' \sim \frac{1}{z-z_0} \partial \chi(z_0)$$

$$\mathbb{H} \chi''' \sim \frac{1}{z-\bar{z}_0} \partial \chi(\bar{z}_0)$$

$$i \partial \chi(z) + i \bar{\partial} \chi(\bar{z})$$

$$\sim \begin{pmatrix} -\frac{1}{z} \\ -\frac{1}{z} \end{pmatrix} \left(-\frac{z'}{2} \frac{\partial}{\partial z} \ln |z-z_0|^2 \right) \cdot 2 ; \partial X(z_0)$$

$$\mathbb{E} X^m \sim \frac{1}{z-z_0} \partial X(z_0)$$

$$\mathbb{E} X^m \sim \frac{1}{z-\bar{z}_0} \partial X(\bar{z}_0)$$

$$X \delta_{(z)} = \partial X(z) + i \bar{\partial} X(\bar{z})$$

$$\delta X^m = -\epsilon \psi(z_0)$$

$$\sim \begin{pmatrix} -1 \\ 1 \end{pmatrix} \cdot \left(-\frac{z'}{2} \frac{\partial \ln |z-z_0|^2}{\partial z} \right) \cdot 2 ; \partial X(z_0)$$

$$\Re X''' \sim \frac{1}{z-z_0} \partial X(z_0)$$

$$\Im X''' \sim \frac{1}{\bar{z}-\bar{z}_0} \bar{\partial} X(\bar{z}_0)$$

$$i \partial(z) \partial X(z) + i \bar{\partial}(\bar{z}) \bar{\partial} X(\bar{z}) = \frac{1}{z} \delta X$$

$$\delta X^m = -\epsilon \bar{U}(z_0) \partial X - \epsilon \bar{U}(z_0) \bar{\partial} X$$

$$\delta X^m = -\epsilon \sigma(z_0) \partial X - \epsilon \bar{\sigma}(z_0) \bar{\partial} X$$

??

$$\delta X^m = -\epsilon \sigma(z_0) \partial X - \epsilon \bar{\sigma}(z_0) \bar{\partial} X$$

??

$z \rightarrow$

$$\delta X^m = -\varepsilon \psi(z_0) \partial X - \varepsilon \bar{\psi}(z_0) \bar{\partial} X$$

??

$$z_0 \rightarrow z_0 + \varepsilon \psi(z_0)$$

$$\delta X^w(z_n) = -\epsilon U(z_n) \partial X - \epsilon \bar{U}(z_n) \bar{\partial} X$$

??

$$z_n \rightarrow z_n + \epsilon U(z_n)$$

$$\delta X^m(z) = -\epsilon \psi(z) \partial X - \epsilon \bar{\psi}(z) \bar{\partial} X$$

??

$$z \rightarrow z + \epsilon \psi(z)$$

$$\delta X^m = X^m(z + \epsilon \psi, \bar{z} + \epsilon \bar{\psi}) - X^m(z, \bar{z}) = X^m(z + \epsilon \psi, \bar{z} + \epsilon \bar{\psi}) - X^m(z, \bar{z})$$

unit holomorphic

$$\delta X^m(z) = -\epsilon \psi(z) \partial X - \epsilon \bar{\psi}(z) \bar{\partial} X$$

??

$$z \rightarrow z + \epsilon \psi(z)$$

$$\delta X^m = X^m(z + \epsilon \psi, \bar{z} + \epsilon \bar{\psi}) - X^m(z, \bar{z}) = X^m(z - \epsilon \psi, \bar{z} - \epsilon \bar{\psi}) - X^m(z, \bar{z})$$

$$= -\epsilon \psi \partial X - \epsilon \bar{\psi} \bar{\partial} X$$

unit holomorphic

$$\delta X^m(z_n) = -\epsilon \psi(z_n) \partial X - \epsilon \bar{\psi}(z_n) \bar{\partial} X$$

??

$$z_n \rightarrow z_n + \epsilon \psi(z_n)$$

$$\delta X^m = X^m(z'_n, \bar{z}'_n) - X^m(z_n, \bar{z}_n) = X^m(z_n - \epsilon \psi, \bar{z}_n - \epsilon \bar{\psi}) - X^m(z_n, \bar{z}_n)$$

$$= -\epsilon \psi \partial X - \epsilon \bar{\psi} \bar{\partial} X$$

anti-holomorphic

$$\delta X^m(z) = -\epsilon \psi(z) \partial X - \epsilon \bar{\psi}(z) \bar{\partial} X$$

??

infinitesimal transform

$$z \rightarrow z + \epsilon \psi(z)$$

$$z \rightarrow z' = f(z)$$

$$\delta X^m = X^m(z', \bar{z}') - X^m(z, \bar{z}) = X^m(z - \epsilon \psi, \bar{z} - \epsilon \bar{\psi}) - X^m(z, \bar{z})$$

$$= -\epsilon \psi \partial X - \epsilon \bar{\psi} \bar{\partial} X$$

holomorphic

$$z \rightarrow z + \varepsilon \psi(z)$$

$$\delta X^m = X^m(z + \varepsilon \psi, \bar{z} + \varepsilon \bar{\psi}) - X^m(z, \bar{z}) = \varepsilon \left(\psi \partial_z X^m + \bar{\psi} \partial_{\bar{z}} X^m \right)$$

$$J^a \Rightarrow T_a^m = a_m^i T_i^m \quad \text{holomorphic}$$

$$\left[\partial_a X^m + \frac{i}{2} \epsilon_{ab} \partial_b X^m \right]$$

$$\Rightarrow T_z^m = \frac{i}{2} \partial_z X^m$$

$$\tilde{T}_{\bar{z}}^m = \frac{i}{2} \partial_{\bar{z}} X^m$$

antiholomorphic

$$= -\varepsilon \partial \bar{z} - \varepsilon \bar{\partial} z$$

$$f(z) = \zeta z$$



$$= -\varepsilon \cup \partial X - \varepsilon \bar{\cup} \bar{\varepsilon} X$$

$$f(z) = \xi z$$

IS $|\xi| = 1 \rightarrow$ just a rotation.

$$= -\varepsilon \partial \bar{x} - \varepsilon \bar{\partial} x$$

$$f(z) = \zeta z$$

IS $|\zeta| = 1 \rightarrow$ just a rotation.

But in general: rot + scaling.

$$= -\varepsilon \omega \partial \chi - \varepsilon \bar{\omega} \bar{\partial} \bar{\chi}$$

$$f(z) = \zeta z$$

If $|\zeta| = 1 \rightarrow$ just a rotation.

But in general: rot + scaling.

$$\frac{y^{-1}}{y^{1/2}} = \frac{y^{-1/2}}{y^{1/2}}$$

$$= -\varepsilon \cup \varepsilon X - \varepsilon \bar{\alpha} \bar{\alpha} X$$

$$f(z) = \zeta z$$

If $|\zeta| = 1 \rightarrow$ just a rotation.

But in general: rot + scaling.

$$\frac{y^1}{y^2} \rightarrow \frac{y^1}{y^2}$$

$$y^m \rightarrow \lambda \cdot y^m$$

$$= -\varepsilon \cup \partial X - \varepsilon \bar{\partial} \bar{a} X$$

$$f(z) = \zeta z$$

IS $|\zeta| = 1 \rightarrow$ just a rotation.

But in general: rot + scaling

$$\frac{y^u}{y^v} \rightarrow \frac{y^u}{\lambda y^v}$$

special conformal trans!

$$y^u \rightarrow \lambda \cdot y^u$$

$$= -\varepsilon \cup \partial X - \varepsilon \bar{\cup} \bar{\partial} X$$

$$f(z) = \zeta z$$

If $|\zeta| = 1 \rightarrow$ just a rotation.

But in general: rot + scaling.

$$\frac{y^u}{y^v} = \frac{y^u}{z} \quad \left\{ \begin{array}{l} \text{special} \\ \text{conformal} \\ \text{transf.} \end{array} \right.$$

$$y^u \rightarrow \lambda \cdot y^u$$

$$\delta X^m(z) = -\varepsilon \psi(z) \partial X - \varepsilon \bar{\psi}(z) \bar{\partial} X$$

??

$$z \rightarrow z + \varepsilon \psi(z)$$

infinitesimal transform $z \rightarrow z' = f(z)$ is holomorphic

$$\delta X^m = X^m(z', \bar{z}') - X^m(z, \bar{z}) = X^m(z - \varepsilon \psi, \bar{z} - \varepsilon \bar{\psi}) - X^m(z, \bar{z})$$

$$= -\varepsilon \psi \partial X - \varepsilon \bar{\psi} \bar{\partial} X$$

Conformal inv and, DPE's



Conformal inv and, OPE's

$$T(z) \sim A(0,0)$$

Conformal inv and, OPE's

$$\mathbb{T}(z) A(0,0) \sim \sum_{n=0}^{\infty} \frac{1}{z^{n+1}} A^{(n)}(0) + \text{non-singular}$$

Conformal inv and, OPE's

$$\mathbb{T}(z) A(0,0) \sim \underbrace{\sum_{n=0}^{\infty} \frac{1}{z^{n+1}} A^{(n)}(0)}_{\text{non-singular}}$$

are completely determined
by conformal transf-

uniform (trans)

$$j_z = j_0(z) \Upsilon'(z)$$

$$\lim_{\Delta z \rightarrow 0} j_z A(\Delta z) = \frac{1}{\epsilon} \delta A$$

... formula (trans) -

$$j_1 = \rho(z) \Upsilon'(z)$$

$$\lim_{z \rightarrow 0} j_2 A(z, \bar{z}) = \frac{1}{\epsilon} \delta A$$

$\downarrow \Upsilon(z) A(z)$

$z \rightarrow 0$



uniform (trans)

$$j_z = i \psi(z) \psi'(z)$$

$$\text{Res}_{z \rightarrow 0} j_z A(0, \bar{0}) = \frac{1}{i \varepsilon} \delta A$$

$\downarrow \psi(z) A(0)$

$$\text{Res}_{z \rightarrow 0} i \psi(z) \left[\sum_{n=0}^{\infty} \frac{1}{z^{n+1}} A^{(n)}(0, \bar{0}) \right] + \text{non-sing.}$$

uniform (trans)

$$j = i \omega(z) \Upsilon'(z)$$

$$\rightarrow 0 \quad j_z A(0, \bar{0}) = \frac{1}{i\epsilon} \delta A$$

$\downarrow \Upsilon(z) A(0)$

$$\rightarrow \text{Res}_{z \rightarrow 0} i \omega(z) \left[\sum_{n=0}^{\infty} \frac{1}{z^{n+1}} A^{(n)}(0, \bar{0}) + \text{non-sing.} \right] = \frac{1}{i\epsilon} \delta A$$

uniform (trans) -

$$j_z = i\omega(z)T'(z)$$

$$\text{Res}_{z \rightarrow 0} j_z A(0, \bar{0}) = \frac{1}{i\epsilon} \delta A$$

$$\text{Res}_{z \rightarrow 0} i\omega(z) \left[\sum_{n=0}^{\infty} \frac{1}{z^{n+1}} A^{(n)}(0, \bar{0}) + \text{non-pole} \right] = \frac{1}{i\epsilon} \delta A$$

$$U(t) = \sum_{k=0}^{\infty} \frac{\partial^k U}{\partial t^k} \frac{t^k}{k!}$$

$\frac{\partial^k U}{\partial t^k}$

$$U(t) = \sum_{k=0}^{\infty} \frac{\partial^k U(0)}{k!} t^k$$

-2.42

$$U(t) = \sum_{k=0}^{\infty} \frac{\partial^k U(0)}{k!} t^k$$

$$\frac{\partial^3}{\partial t^3}$$



$$U(x) = \sum_{k=0}^{\infty} \frac{\partial^k U(x)}{k!} x^k$$

$$\frac{\partial^n U(x)}{n!} \Big|_{(0,0)}$$

$$U(t) = \sum_{k=0}^{\infty} \frac{\partial^k U(0)}{k!} t^k$$

$$\sum_{n=0}^{\infty} \frac{\partial^n U(0)}{n!} A^{(n)}(0,0) =$$

$\rightarrow \gamma^2$

$$U(z) = \sum_{k=0}^{\infty} \frac{\partial^k U(z)}{\partial z^k} z^k$$

$$\sum_{n=0}^{\infty} \frac{\partial^n U(z)}{h^n} A^{(n)}(0,0) = \frac{1}{1-\varepsilon} \delta A$$

$\rightarrow \gamma^2$

$$U(z) = \sum_{k=0}^{\infty} \frac{\partial^k U(z)}{k!} z^k$$

$$\sum_{n=0}^{\infty} \frac{\partial^n U(z)}{n!} A^{(n)}(0,0) = \frac{1}{1-\varepsilon} \delta A$$

$$+ i \sum_{h=0}^{\infty} \frac{\partial^h U}{h!} A^{(h)}(0,0)$$

$$U(z) = \sum_{k=0}^{\infty} \frac{\partial^k U(z)}{\partial z^k} \frac{z^k}{k!}$$

$$z \rightarrow z + \epsilon U(z)$$

$$\sum_{n=0}^{\infty} \frac{\partial^n U(z)}{n!} A^{(n)}(0,0) = \frac{1}{1-\epsilon} \delta A$$

$$+ i \sum_{h=0}^{\infty} \frac{\partial^h U}{h!} A^{(h)}(0,0)$$

Consider a basis of operators

$$f(z) = \zeta z$$

Is $|\zeta|=1 \rightarrow$ a rotation

But in general scaling

$$\frac{y^{n+1}}{y^n} = \frac{y^{n+1}}{z^n} \leftarrow \text{special conformal transform?}$$

$$y^n \rightarrow \lambda \cdot y^n$$

$$f(z) = \zeta z$$

IS $|\zeta|=1 \rightarrow$ just a rotation.

But in general: rot + scaling

$$\frac{y^u}{y^v} = \frac{y^u}{z^v} \leftarrow \text{special conformal transform!}$$

$$y^u \rightarrow \lambda \cdot y^u$$

Consider a basis of operators which are eigenstates of right scale transformation.

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$$A(z, \bar{z}) \rightarrow$$

Consider a basis of operators which are eigenstates of rigid scale transformation.

$$A(z, \bar{z}) \rightarrow A'(z', \bar{z}') =$$

Consider a basis of operators which are eigenstates of right scale transformation.

$$A(z, \bar{z}) \rightarrow A'(z', \bar{z}') = \zeta^{-h} \bar{\zeta}^{-\tilde{h}} A(z, \bar{z})$$

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$$z \rightarrow z' = \xi z$$

Consider a basis of operators which are eigenstates of right scale transformation.

$$A(z, \bar{z}) \rightarrow A'(z', \bar{z}') = \underbrace{\zeta^{-h}}_{\zeta} \underbrace{\bar{\zeta}^{-\tilde{h}}}_{\bar{\zeta}} A(z, \bar{z})$$

$z \rightarrow z' = \zeta z \quad (h, \tilde{h})$ - conformal weights

Consider a basis of operators which are eigenstates of right scale transformation.

$$A(z, \bar{z}) \rightarrow A'(z', \bar{z}') = \underbrace{z^{-h}}_{h+\tilde{h}} \underbrace{\bar{z}^{-\tilde{h}}}_{\tilde{h}} A(z, \bar{z})$$

$z \rightarrow z' = \zeta(z) \quad (h, \tilde{h})$ - conformal weights

Consider a basis of operators which are eigenstates of right scale transformation.

$$A(z, \bar{z}) \rightarrow A'(z', \bar{z}') = \underbrace{\zeta^{-h}}_{\substack{h+\tilde{h} \\ \text{dim of } A}} \underbrace{\bar{\zeta}^{-\tilde{h}}}_{\substack{h-\tilde{h} \\ \text{spin}}} A(z, \bar{z})$$

$$z \rightarrow z' = \zeta z \quad (h, \tilde{h}) - \text{conformal weights}$$

$$\lambda (0 - \varepsilon \varepsilon^h) - X (0) = \dots$$

$$(h, \tilde{h}) \rightarrow (h+1, \tilde{h})$$

$$A \rightarrow \xi^{-h} \xi^{-\tilde{h}} A$$



$$X(0) = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} - X(0) = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\mathcal{L}_2 \quad (h, \tilde{h}) \rightarrow (h+1, \tilde{h})$$

$$A \rightarrow \zeta^{-h} \zeta^{-\tilde{h}} A$$

$$\mathcal{L}_2 A =$$

$$\rightarrow \frac{e^{i\tilde{h}}}{\zeta^2}$$

$$\lambda (0 - \varepsilon_0) - X (0) = \dots$$

$$\gamma_2 \quad (h, \tilde{h}) \rightarrow (h+1, \tilde{h})$$

$$A \rightarrow \xi^{-h} \xi^{-\tilde{h}} A$$

$$\gamma_2 A =$$

$$\frac{\partial}{\partial z} \left(\frac{\partial}{\partial \tilde{z}} \right) = \xi^{-1} \frac{\partial}{\partial z}$$

Consider infinitesimal rigid^a scale trans.
 $z \rightarrow z'' \approx z + \varepsilon z$



Consider infinitesimal rigid^o scale trans.

$$z \rightarrow z'' \Rightarrow z + \epsilon z \Rightarrow \zeta = (1 + \epsilon)$$

Consider infinitesimal rigid^o scale trans.

$$z \rightarrow z'' = z + \epsilon z \Rightarrow \zeta = (1 + \epsilon)$$

$$U(z)$$

Consider infinitesimal rigid scale trans.

$$z \rightarrow z' = z + \epsilon z \Rightarrow \zeta = (1 + \epsilon)$$

$$\zeta(z) = z$$

Consider infinitesimal rigid^a scale trans.

$$z \rightarrow z'' = z + \varepsilon U(z) \Rightarrow \zeta = (1 + \varepsilon)$$

$$U(z) = z$$

$$A'(z', \bar{z}') =$$

Consider infinitesimal rigid^o scale trans.
 $z \rightarrow z'' = z + \varepsilon U(z) \Rightarrow \zeta = (1 + \varepsilon)$

$$U(z) = z$$

$$A'(z', \bar{z}') = (1 + \varepsilon)^{-h} A(z, \bar{z})$$

Consider infinitesimal rigid^a scale trans.

$$z \rightarrow z'' = z + \varepsilon U(z) \Rightarrow \zeta = (1 + \varepsilon)$$

$$U(\bar{z}) = \bar{z}$$

$$A'(z', \bar{z}') = (1 + \varepsilon)^{-h} A(z, \bar{z}) = (1 - \varepsilon h) A(z, \bar{z})$$

Consider infinitesimal rigid^a scale trans.

$$z \rightarrow z'' = z + \varepsilon U(z) \Rightarrow \zeta = (1 + \varepsilon)$$

$$U(\bar{z}) = \bar{z}$$

$$A'(z', \bar{z}') = (1 + \varepsilon)^{-h} A(z, \bar{z}) = (1 - \varepsilon h) A(z, \bar{z})$$

$$\delta A = A'(z, \bar{z}) - A'(z_1, \bar{z})$$

$$= A$$

$$\delta A = A'(z, \bar{z}) - A'(z_1, \bar{z}_1)$$

$$= A(z - \epsilon z, \bar{z}) - h \epsilon A(z_1, \bar{z}_1) - A(z, \bar{z})$$

$$\delta A = A'(z, \bar{z}) - A'(z, \bar{z})$$

$$A(z - \varepsilon z, \bar{z}) - h \varepsilon A'(z, \bar{z}) - A(z, \bar{z})$$

$$- \varepsilon \cdot z$$

$$\delta A = A(z, z) - A(z_1, z_1)$$

$$= A(z - \epsilon z, \bar{z}) - h \epsilon A(z, \bar{z}) - A(z, \bar{z})$$

$$= -\epsilon \cdot z \cdot \partial A(z) - \epsilon \cdot h \cdot A(z)$$

$$A \delta \frac{1}{z} = \left[\frac{1}{z} + \epsilon \frac{1}{z^2} \right] + \dots$$

$$\delta A = A(z, z) - A(z, \bar{z})$$

$$= A(z - \epsilon z, \bar{z}) - h \epsilon A(z, \bar{z}) - A(z, \bar{z})$$

$$= -\epsilon \cdot z \cdot \partial A(z) - \epsilon \cdot h \cdot A(z)$$

general

$$A(0, 0) =$$



$$A \delta \frac{1}{i\epsilon} = \left[\sum_{n=0}^{\infty} \frac{1}{z^{n+1}} A^{(n)}(0, \bar{0}) + \text{non-} \dots \right]$$

$$= A(z - \varepsilon z, \bar{z}) - h \varepsilon A(z, \bar{z}) - A(z, \bar{z})$$

$$= -\varepsilon \cdot z \cdot \partial A(0) - \varepsilon \cdot h \cdot A(0)$$

In general

$$\mathbb{T}(z) A(0, \bar{0}) \approx \sum_{h=0}^{\infty} \frac{1}{z^{h+1}} A^{(h)}(0, \bar{0}) \Leftrightarrow$$

$$\begin{aligned} \left. \begin{array}{l} \text{Res} \\ z \rightarrow 0 \end{array} \right\} i \mathbb{O}(z) \left[\sum_{h=0}^{\infty} \frac{1}{z^{h+1}} A^{(h)}(0, \bar{0}) + \text{non-pole} \right] &= \frac{1}{i \varepsilon} \delta A \end{aligned}$$

$$= A(z - \epsilon z, \bar{z}) - h \epsilon A(z, \bar{z}) - A(z, \bar{z})$$

$$= -\epsilon \cdot z \cdot \partial A(0) - \epsilon \cdot h \cdot A(0)$$

In general

$$T(z) A(0,0) \approx \sum_{n=0}^{\infty} \frac{1}{z^{n+1}} A^{(n)}(0,0) \Leftrightarrow \delta A = -\epsilon \sum_{n=0}^{\infty} \frac{1}{n!} \left[\partial^n \partial A^{(n)} \right]$$

$$\lim_{z \rightarrow 0} \text{Res } iU(z) \left[\sum_{n=0}^{\infty} \frac{1}{z^{n+1}} A^{(n)}(0,0) + \text{non-poly.} \right] = \frac{1}{i\epsilon} \delta A$$

$$= A(z - \epsilon z, \bar{z}) - h \epsilon A(z, \bar{z}) - A(z, \bar{z})$$

$$= -\epsilon \cdot z \cdot \partial A(0) - \epsilon \cdot h \cdot A(0)$$

In general

$$T(z) A(0, \bar{0}) \approx \sum_{n=0}^{\infty} \frac{1}{z^{n+1}} A^{(n)}(0, \bar{0}) \Leftrightarrow \delta A = -\epsilon \sum_{n=0}^{\infty} \frac{1}{n!} \left[\partial^n \partial A^{(n)} \right]$$

$$\lim_{z \rightarrow 0} i \partial(z) \left[\sum_{n=0}^{\infty} \frac{1}{z^{n+1}} A^{(n)}(0, \bar{0}) + \text{non-ly.} \right] = \frac{1}{i \epsilon} \delta A$$

$$T(z) A(0, \bar{0}) \sim \sum_{h=0}^{\infty} \frac{1}{z^{h+1}} A^{(h)}(0, \bar{0}) \Leftrightarrow$$

$$\delta A = -\varepsilon \sum_{n=0}^{\infty} \frac{1}{z^{n+1}} \left[\partial_{\bar{z}} A^{(n)} \right]$$

$$\delta A =$$

$$T(z) A(0,0) \approx \sum_{h=0}^{\infty} \frac{1}{z^{h+1}} A^{(h)}(0,0) \Leftrightarrow \delta H = -\varepsilon \sum_{n=0}^{\infty} \frac{1}{z^{n+1}} \partial \cdot A^{(n)}$$

$$= -\varepsilon \cdot \partial \cdot A^{(0)} - \varepsilon \cdot \partial \cdot A^{(1)}$$

$$T(z) A(0, \bar{0}) \sim \sum_{h=0}^{\infty} \frac{1}{z^{h+1}} A^{(h)}(0, \bar{0}) \Leftrightarrow$$

$$\delta H = -\varepsilon \sum_{n=0}^{\infty} \frac{1}{z^{n+1}} \partial A^{(n)}$$

$$= -\varepsilon \cdot \partial A^{(0)} - \varepsilon \cdot \partial A^{(1)}$$

$$\hookrightarrow -\varepsilon z A^{(0)} - \varepsilon A^{(1)}$$

$$T(z) A(0,0) \approx \sum_{h=0}^{\infty} \frac{1}{z^{h+1}} A^{(h)}(0,0)$$

$$\delta H = -\varepsilon \sum_{n=0}^{\infty} \frac{1}{z^{n+1}} \partial A^{(n)}$$

$$= -\varepsilon \cdot \partial A^{(0)} - \varepsilon \cdot \partial A^{(1)}$$

$$\hookrightarrow -\varepsilon \cdot z \partial A^{(0)} - \varepsilon A^{(1)}$$

$$A^{(0)} =$$

$$T(z) A(0,0) \sim \sum_{h=0}^{\infty} \frac{1}{z^{h+1}} A^{(h)}(0,0) \Leftrightarrow$$

$$\delta A = -\varepsilon \sum_{n=0}^{\infty} \frac{1}{z^{n+1}} \partial A^{(n)}$$

$$\delta A = -\varepsilon \cdot \partial A^{(0)} - \varepsilon \cdot \partial A^{(1)}$$

$$\hookrightarrow -\varepsilon \cdot z A^{(0)} - \varepsilon A^{(1)}$$

$$A^{(0)} = \partial A \quad A^{(1)} = h \cdot A$$

$$\prod(z) A = \frac{1}{z}$$

$$\mathbb{T}(z) A = \dots + \frac{1}{z^2} A^{(2)} + \frac{1}{z} A^{(1)} + \dots$$

$$\mathbb{T}(z) A = \dots \frac{1}{z^2} A^{(1)} + \frac{1}{z} A^{(0)} + \dots$$

$$= \dots \frac{h}{z^2} A + \frac{1}{z} \partial A$$

$$\mathbb{T}(z) A = \dots \frac{1}{z^2} A^{(1)} + \frac{1}{z} A^{(0)} + \dots$$

$$\dots \frac{1}{z^2} \textcircled{A} + \frac{1}{z} \partial A + \dots$$



$$= \dots \cdot \frac{\hbar}{2^2} \textcircled{A} + \frac{1}{2} \partial A + \dots$$

Primary fields or tensor operators

$h+h$ - dim of A
 $h-h$ - spin
 (h, h) - conformal weights

$$= \dots \cdot \frac{h}{z^2} \textcircled{A} + \frac{1}{z} \partial A + \dots$$

Primary fields or tensor operators

$$O'(z', \bar{z}') =$$

$z \rightarrow z' = \left\{ \begin{array}{l} z \\ \bar{z} \end{array} \right.$ (h, \bar{h}) - conformal weights
 $h + \bar{h}$ - dim of A
 $h - \bar{h}$ - spin

$$= \dots \cdot \frac{h}{z^2} \circlearrowleft A + \frac{1}{z^2} \partial A + \dots$$

Primary fields or tensor operators

$$D'(z', \bar{z}') = \left(\frac{z}{z'} \right)^{h'} \left(\frac{\bar{z}}{\bar{z}'} \right)^{\bar{h}'}$$

$z \rightarrow z' = \left\{ \begin{matrix} z \\ \bar{z} \end{matrix} \right\}$ (h, \bar{h}) - conformal weights
 h - dim of A
 \bar{h} - spin

$$= \dots \cdot \frac{h}{z^2} \circlearrowleft A + \frac{1}{z} \partial A + \dots$$

Primary fields or tensor operators

$$\mathcal{O}'(z', \bar{z}') = \left(\frac{z}{z'} \right)^{-h} \left(\frac{\bar{z}}{\bar{z}'} \right)^{-\tilde{h}} \mathcal{O}(z, \bar{z})$$

$h + \tilde{h}$ - dim of A
 $h - \tilde{h}$ - spin

$z \rightarrow z' = \dots$ (h, \tilde{h}) - conformal weights

$$\mathcal{O}(z', \bar{z}') = \mathcal{O}(z) \left(\frac{\partial \bar{z}'}{\partial \bar{z}} \right)^{h'} \mathcal{O}(z, \bar{z})$$

$$\rightarrow \mathcal{T}(z) \mathcal{O} =$$

or right scale transformation.

$$A(z, \bar{z}) \rightarrow A'(z', \bar{z}') = \zeta^{-h} \bar{\zeta}^{-\tilde{h}} A(z, \bar{z})$$

$z \rightarrow z' = \zeta z$ (h, \tilde{h}) - conformal weights
 $h + \tilde{h}$ - dim of A
 $h - \tilde{h}$ - spin

$$\mathbb{T}(z) \circ = \frac{h}{z^2} \circ + \frac{i\alpha}{z} \circ$$

or rig. + scale transformation.

$$A(z, \bar{z}) \rightarrow A'(z', \bar{z}') = \underbrace{\zeta^{-h}}_{\sim} \underbrace{\bar{\zeta}^{-\tilde{h}}}_{\sim} A(z, \bar{z})$$

$\zeta \rightarrow \zeta' = \zeta$ (h, \tilde{h}) - conformal weights
 $h + \tilde{h}$ - dim of A
 $h - \tilde{h}$ - spin