Title: Introduction to the Bosonic String

Date: Feb 13, 2009 10:00 AM

URL: http://pirsa.org/09020011

Abstract: This course provides a thorough introduction to the bosonic string based on the Polyakov path integral and conformal field theory. We introduce central ideas of string theory, the tools of conformal field theory, the Polyakov path integral, and the covariant quantization of the string. We discuss string interactions and cover the tree-level and one loop amplitudes. More advanced topics such as T-duality and D-branes will be taught as part of the course. The course is geared for M.Sc. and Ph.D. students enrolled in Collaborative Ph.D. Program in Theoretical Physics. Required previous course work: Quantum Field Theory (AM516 or equivalent). The course evaluation will be based on regular problem sets that will be handed in during the term. The primary text is the book: 'String theory. Vol. 1: An introduction to the bosonic string. J. Polchinski (Santa Barbara, KITP) . 1998. 402pp. Cambridge, UK: Univ. Pr. (1998) 402 p.' All interested students should contact Alex Buchel at abuchel@uwo.ca as soon as possible.


Conformal Field Theory
(CFT

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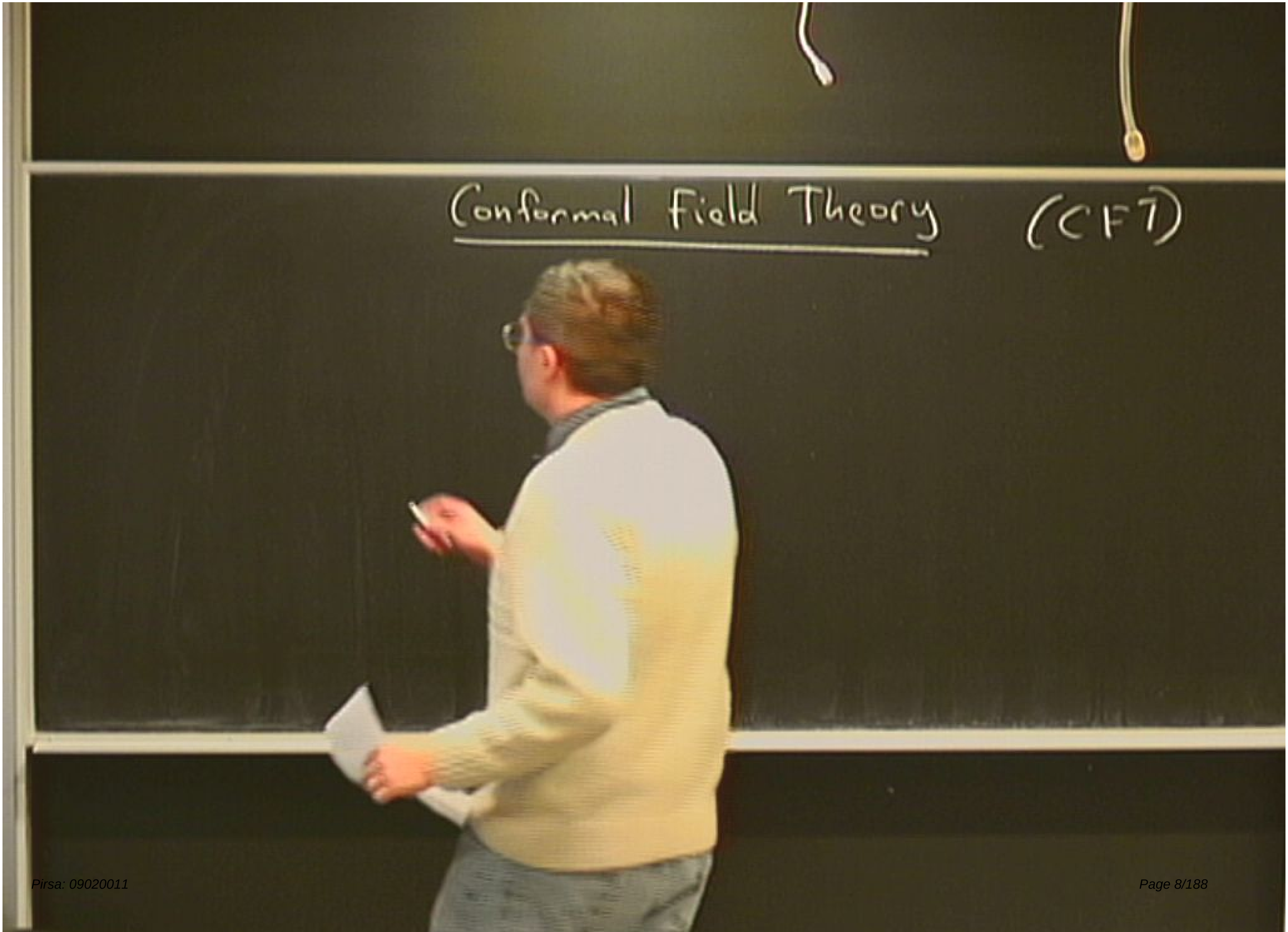
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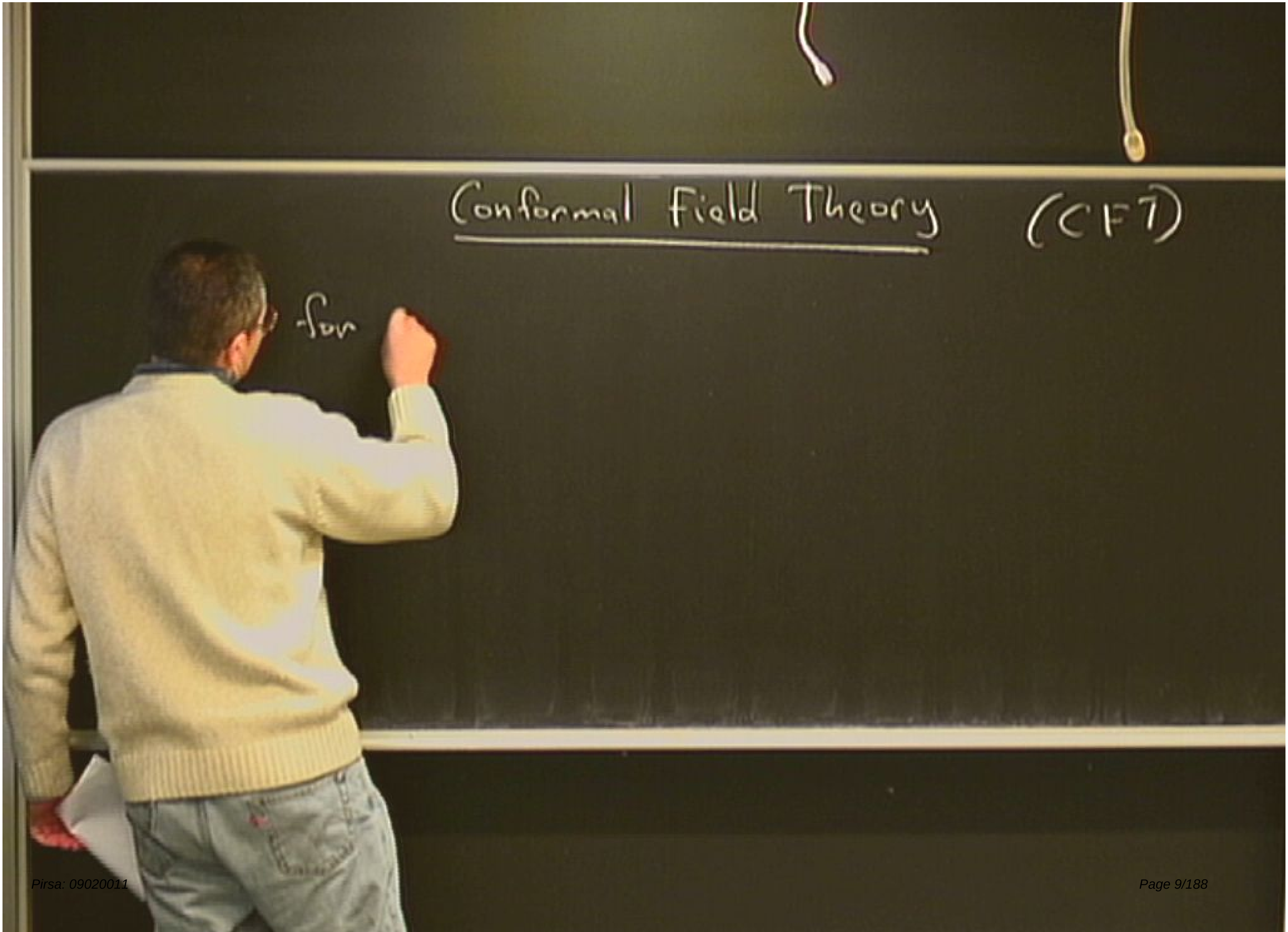
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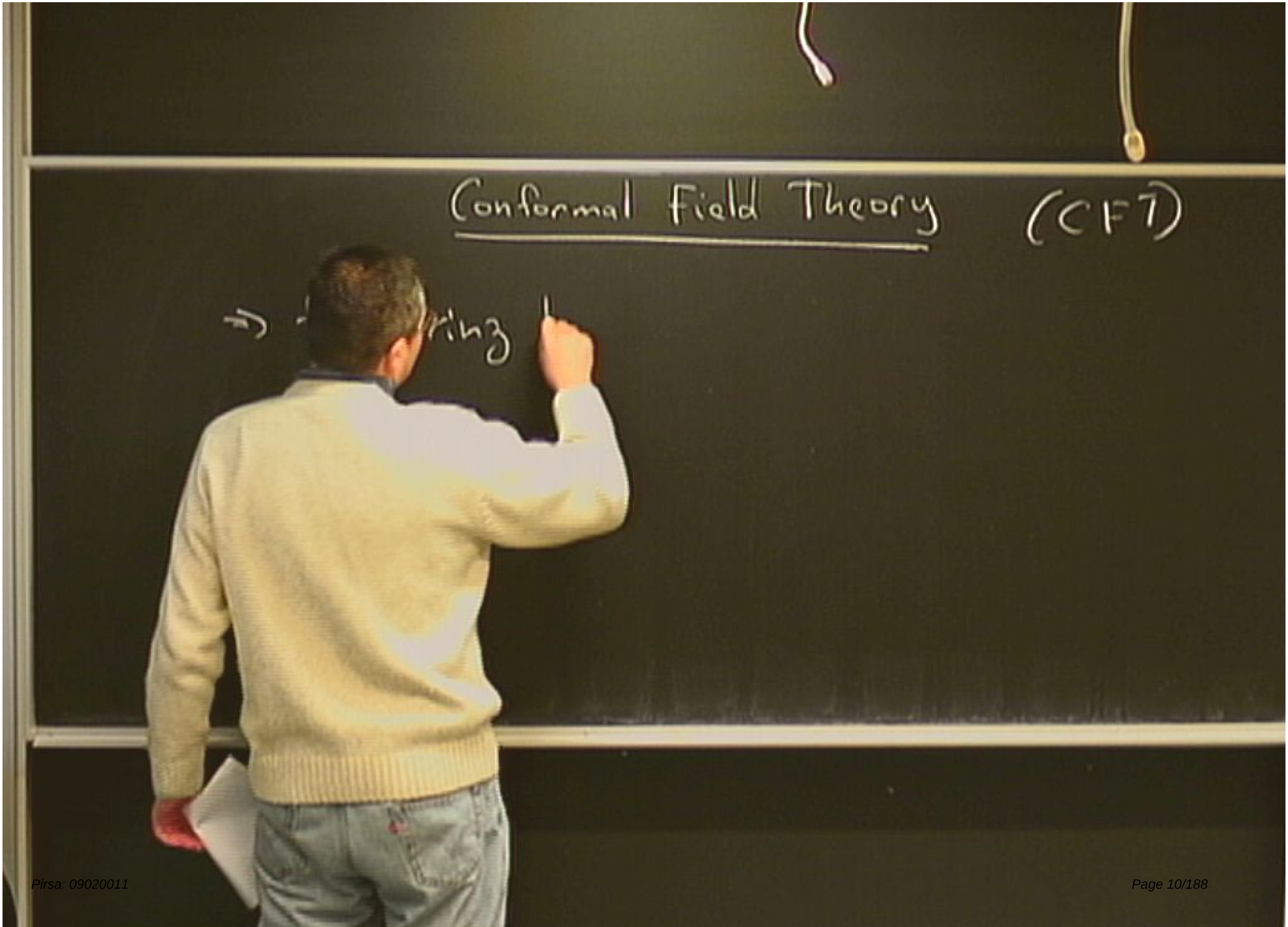
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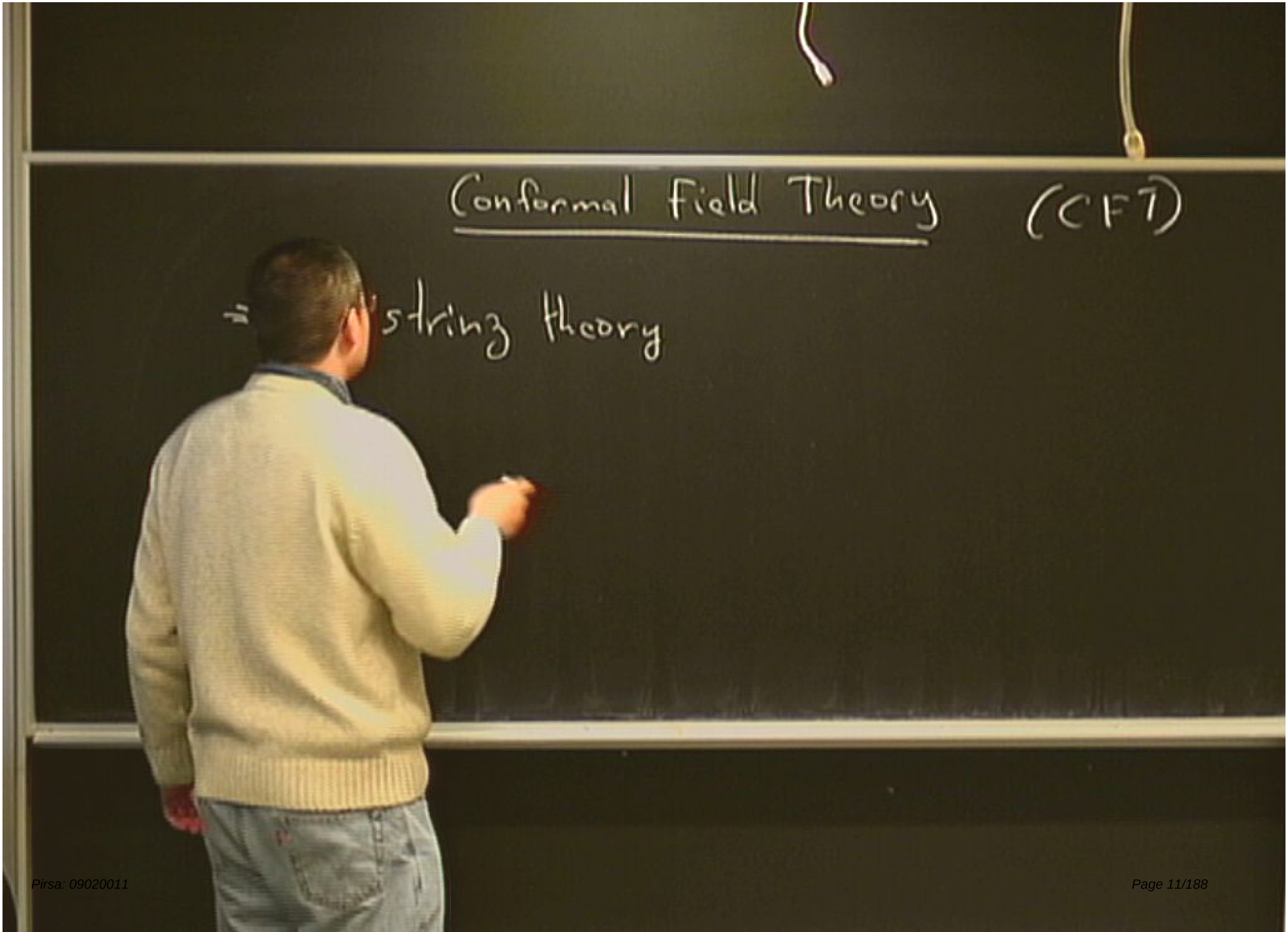
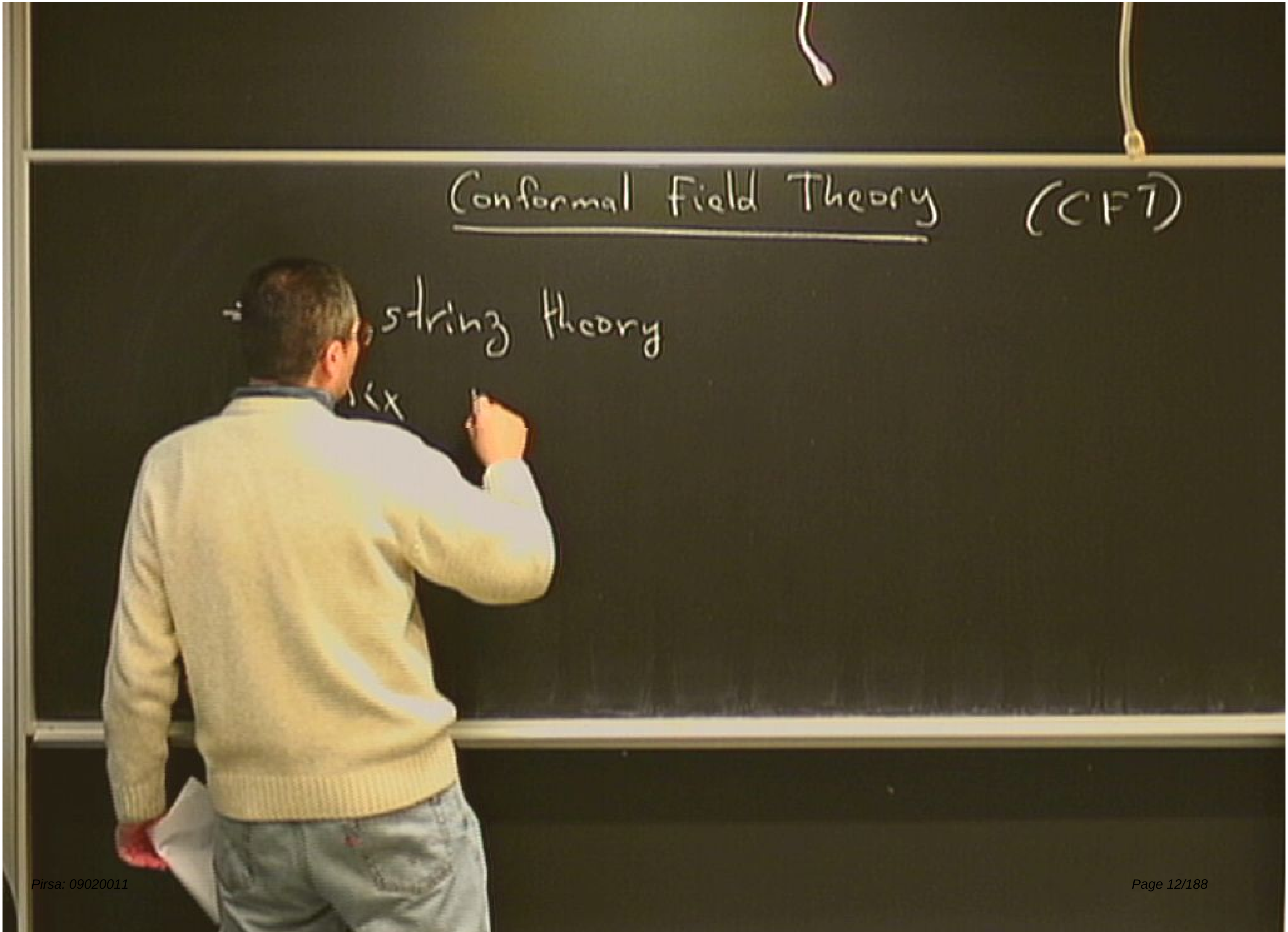

Conformal Field Theory (CFT)
string theory


Conformal Field Theory (CFT)
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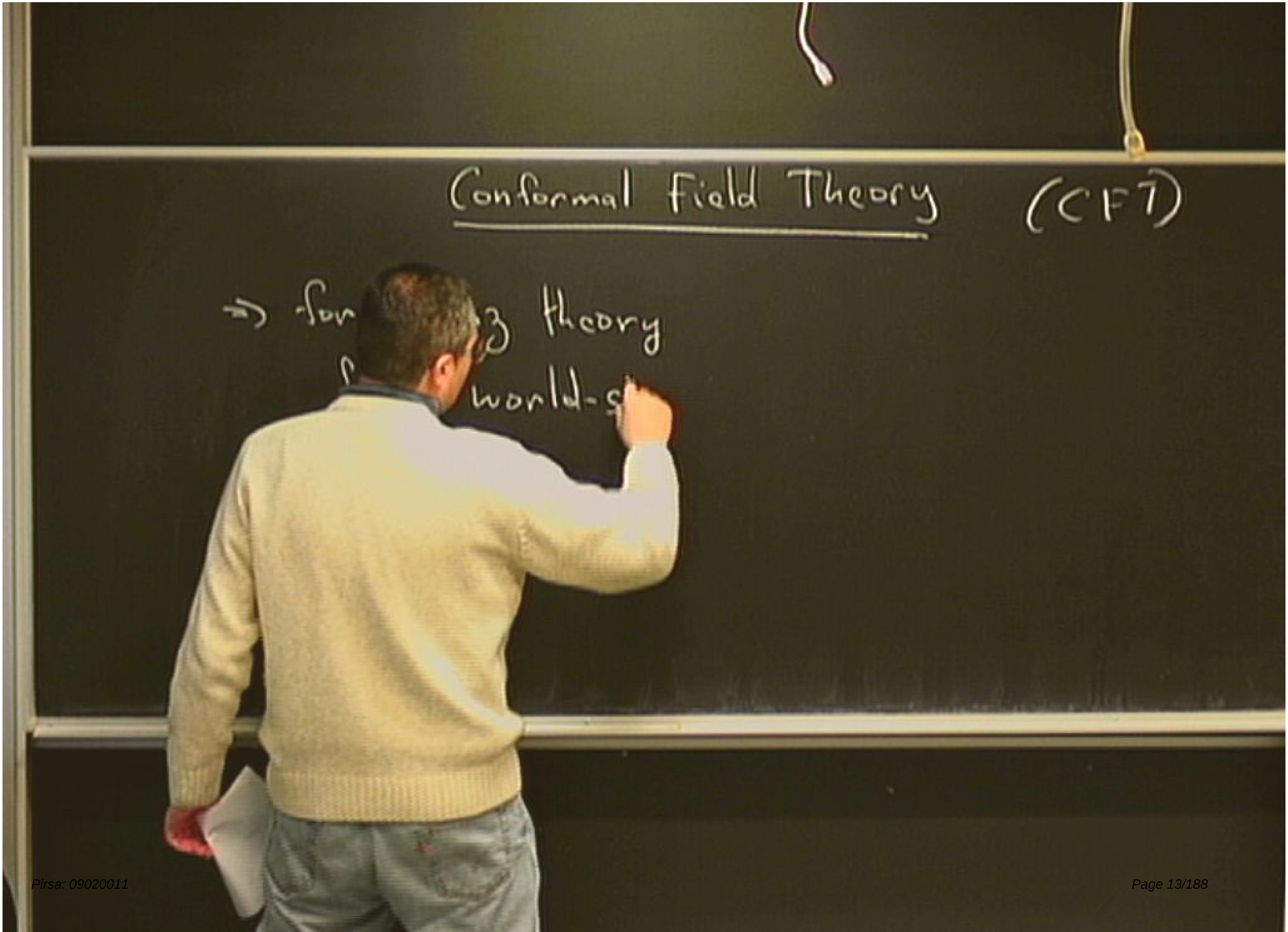
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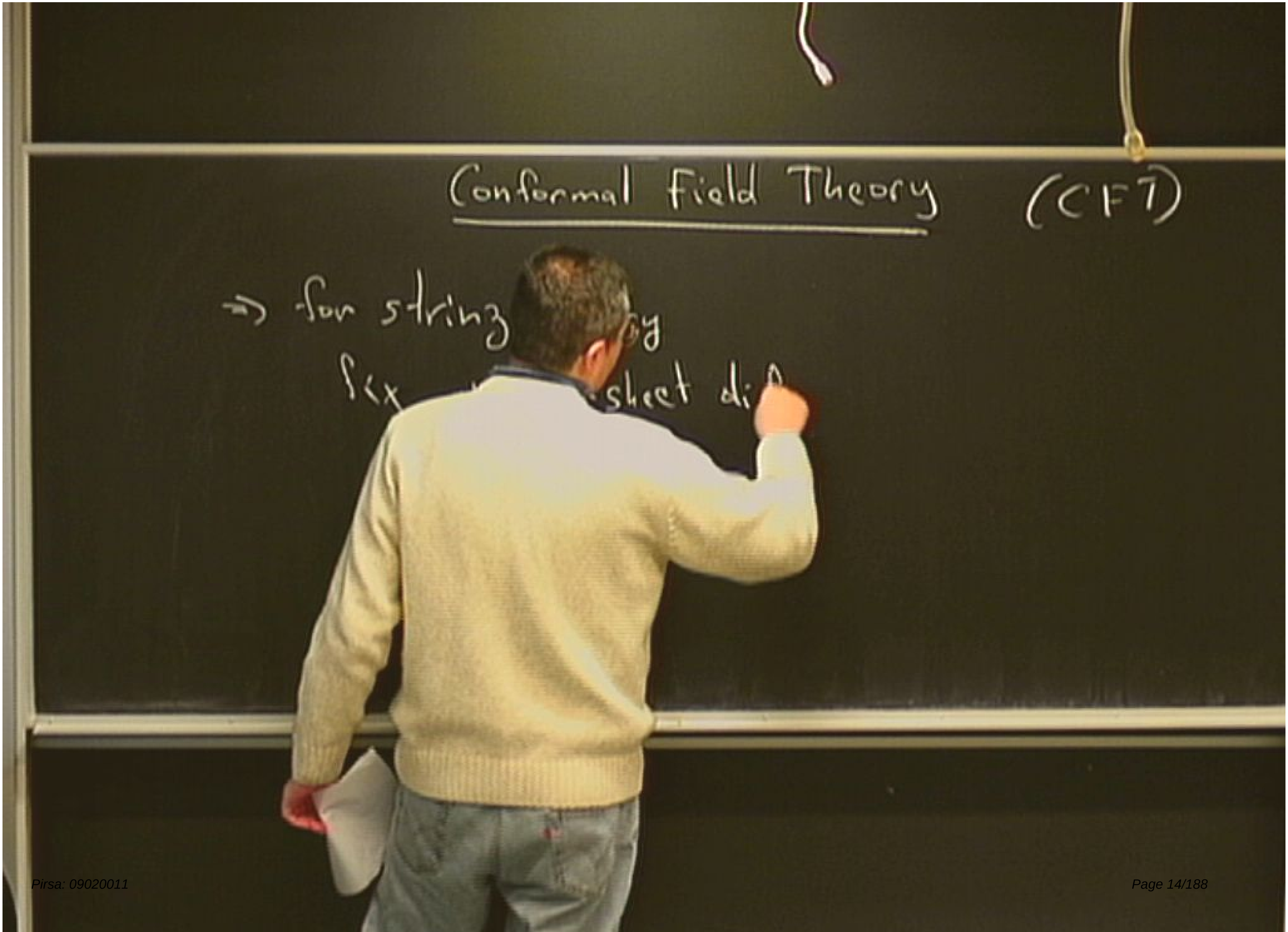
Conformal Field Theory (CFT)
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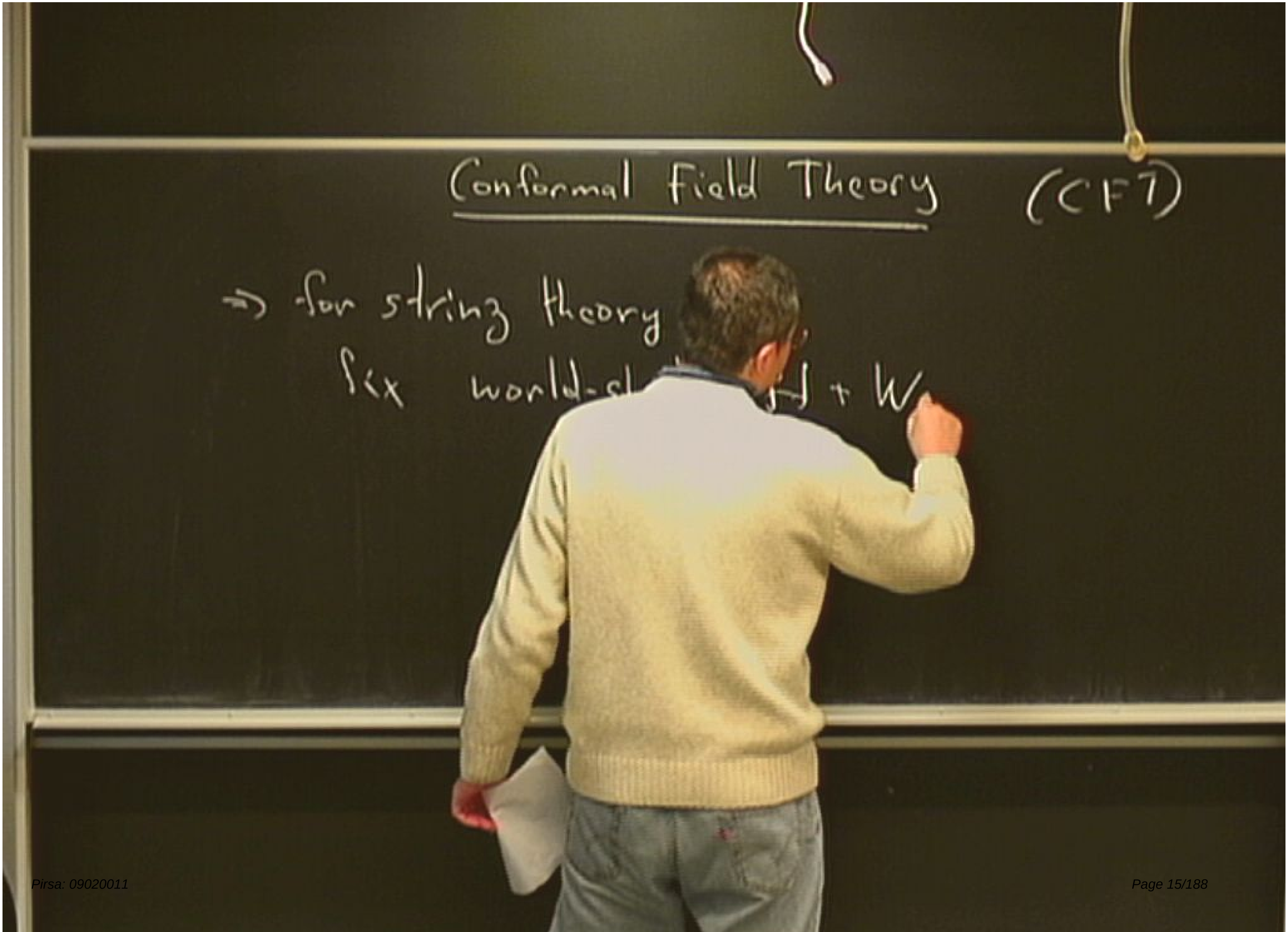
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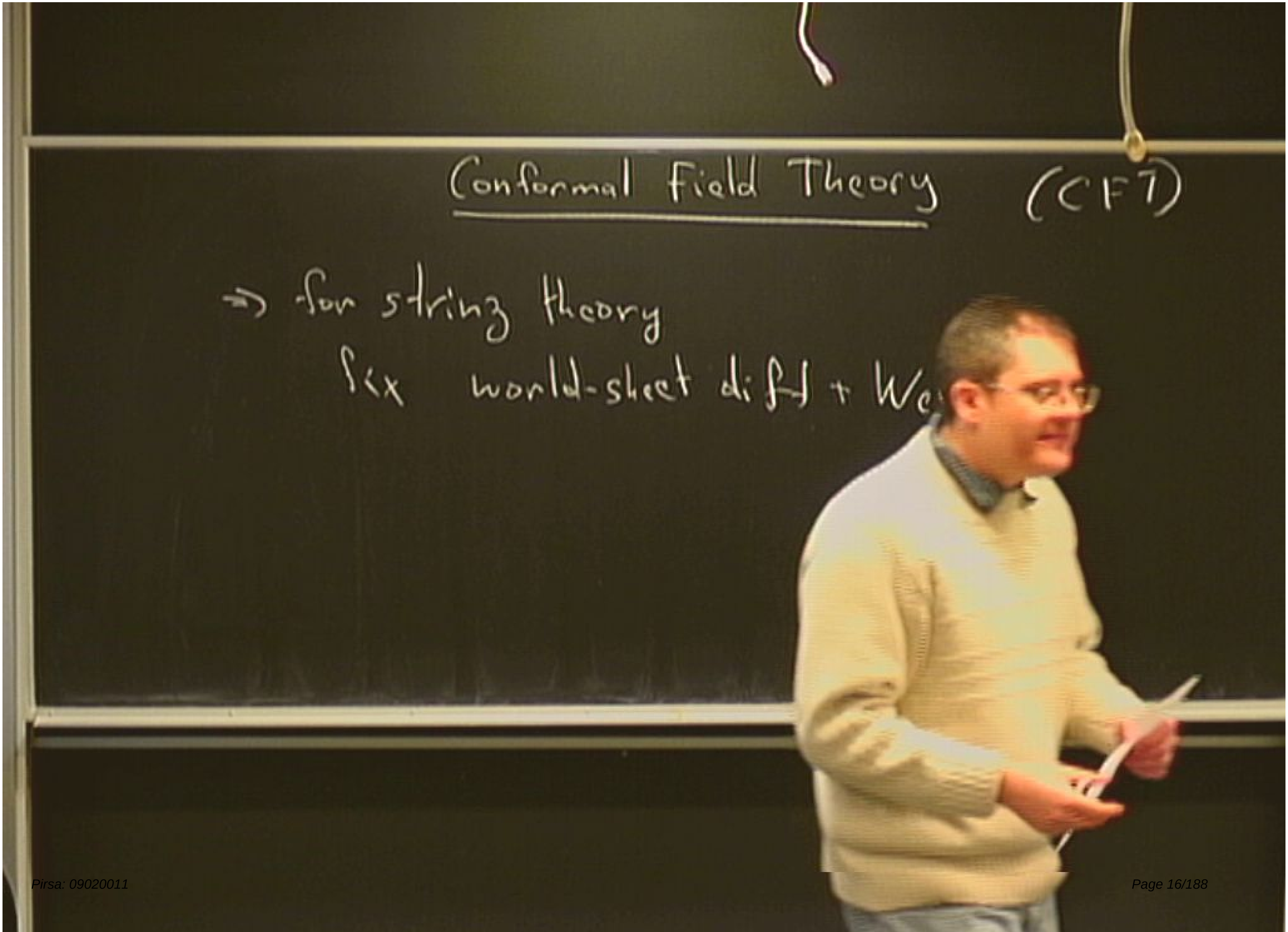
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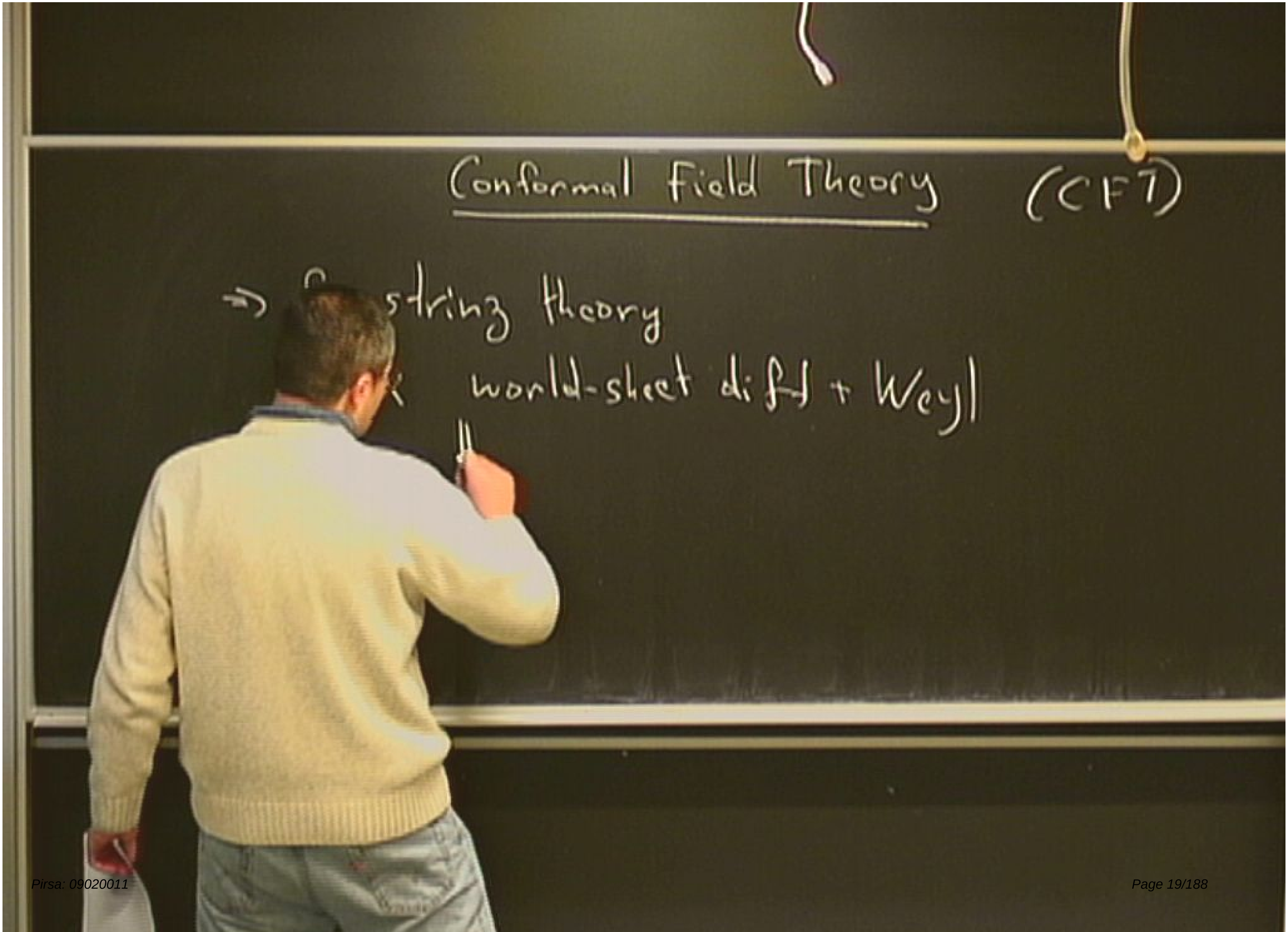
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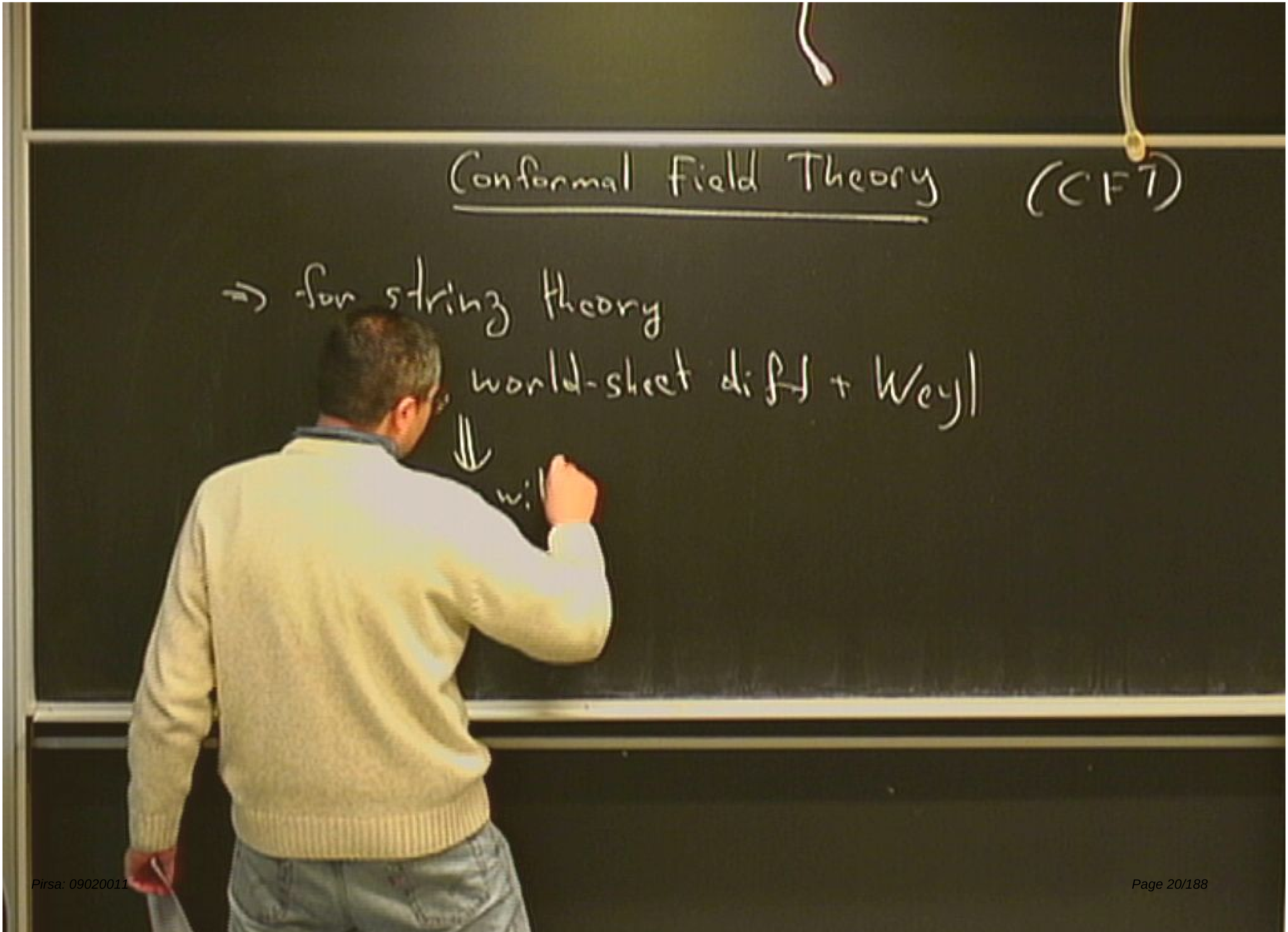
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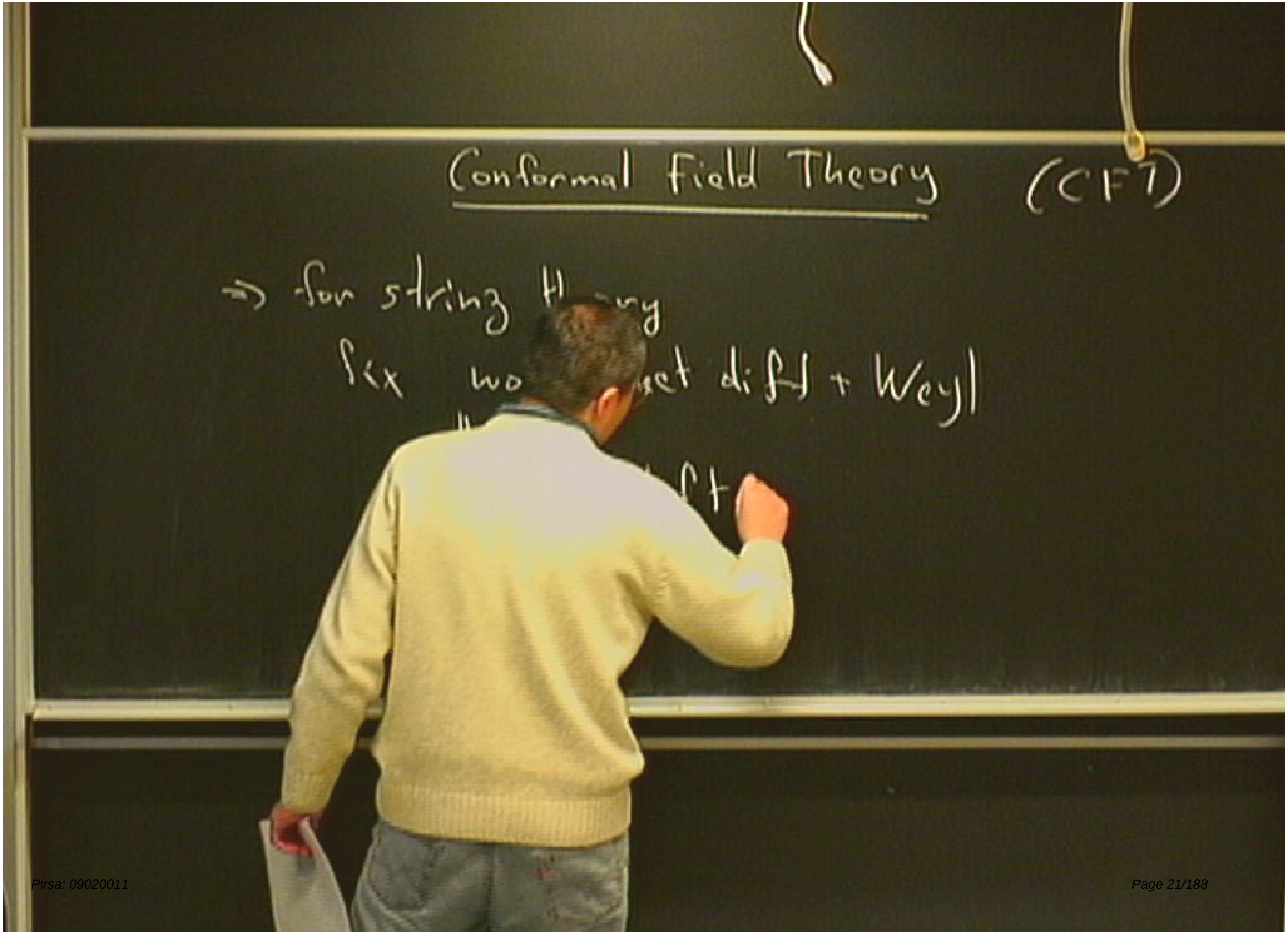

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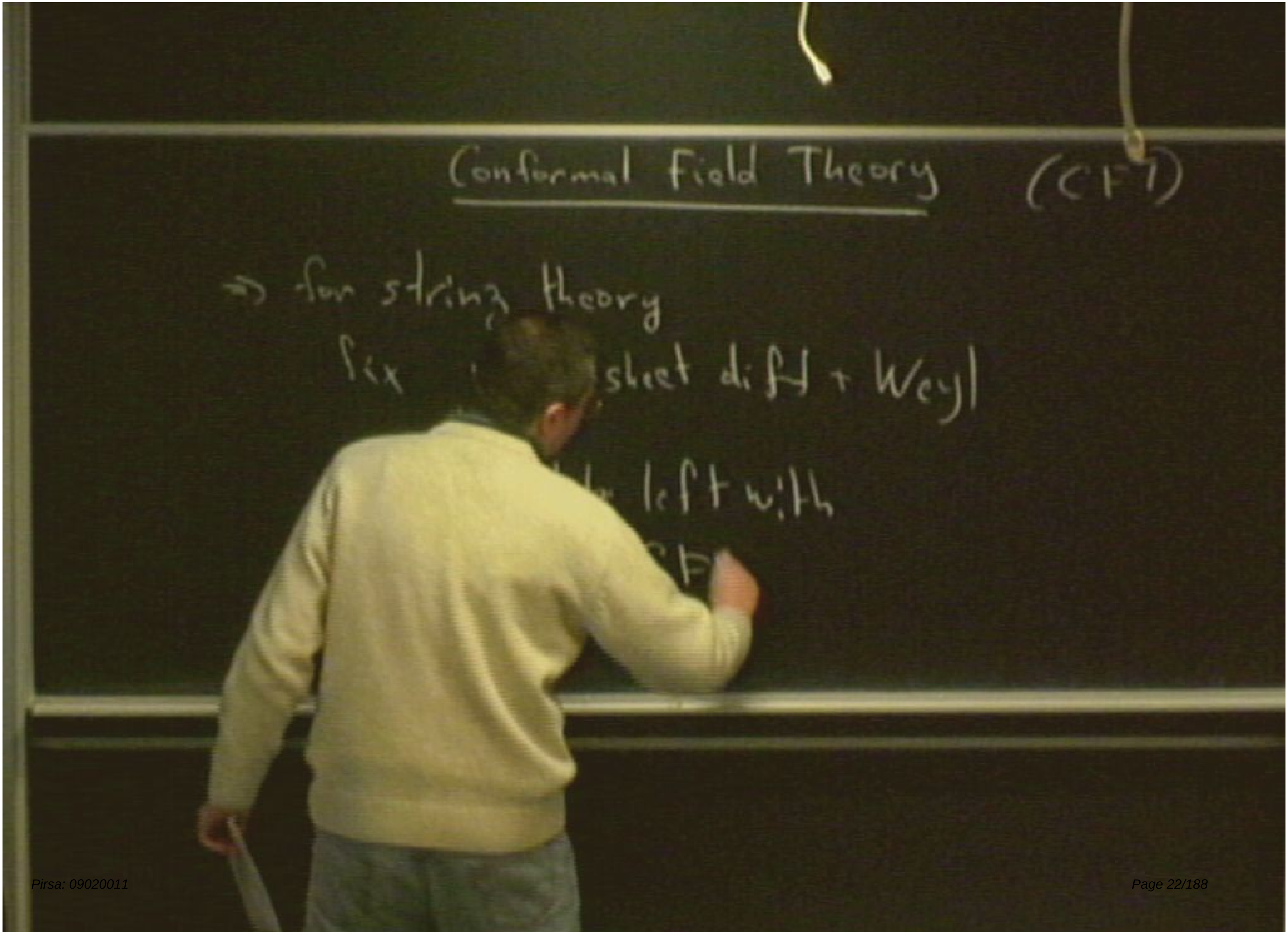
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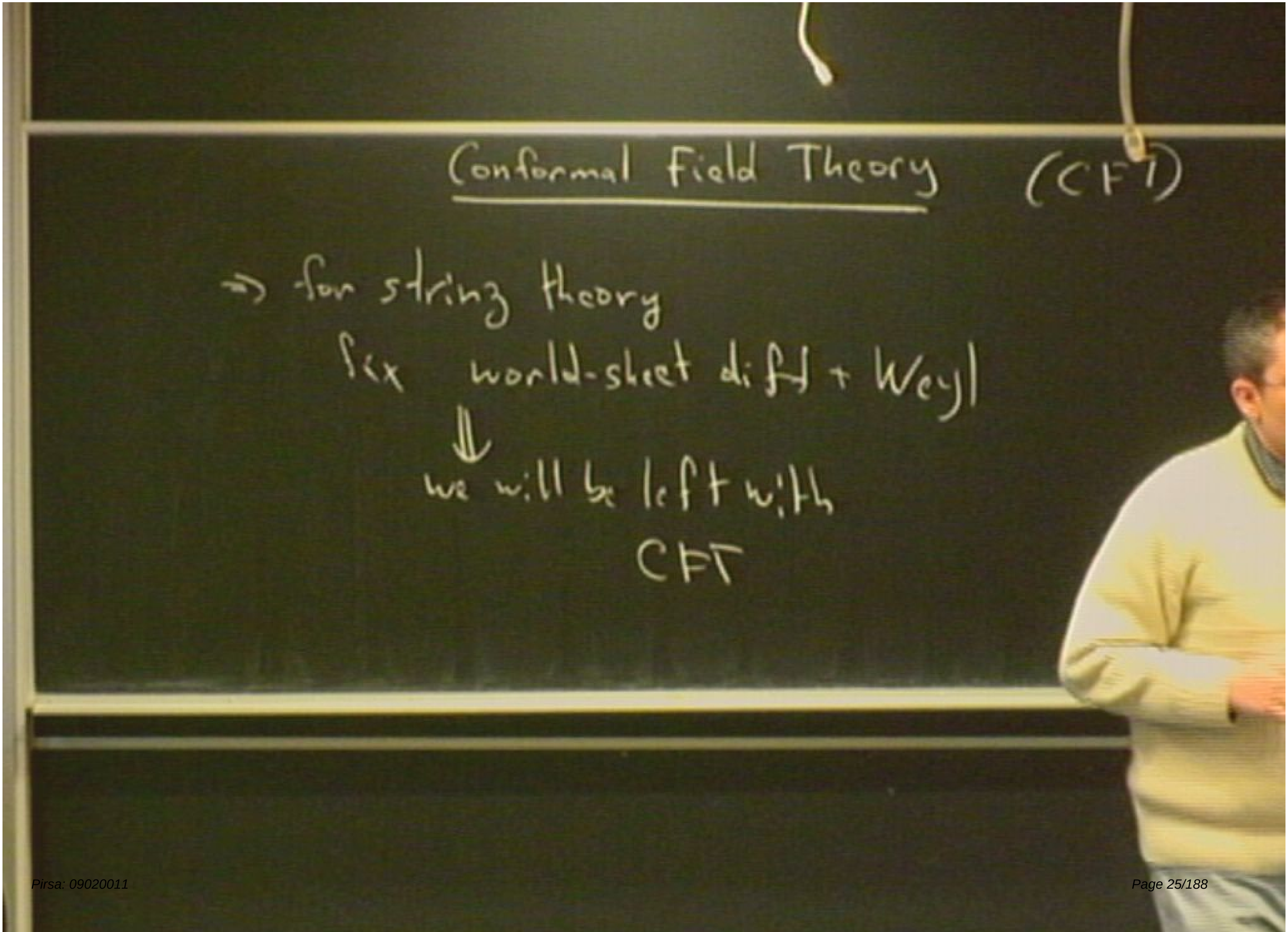
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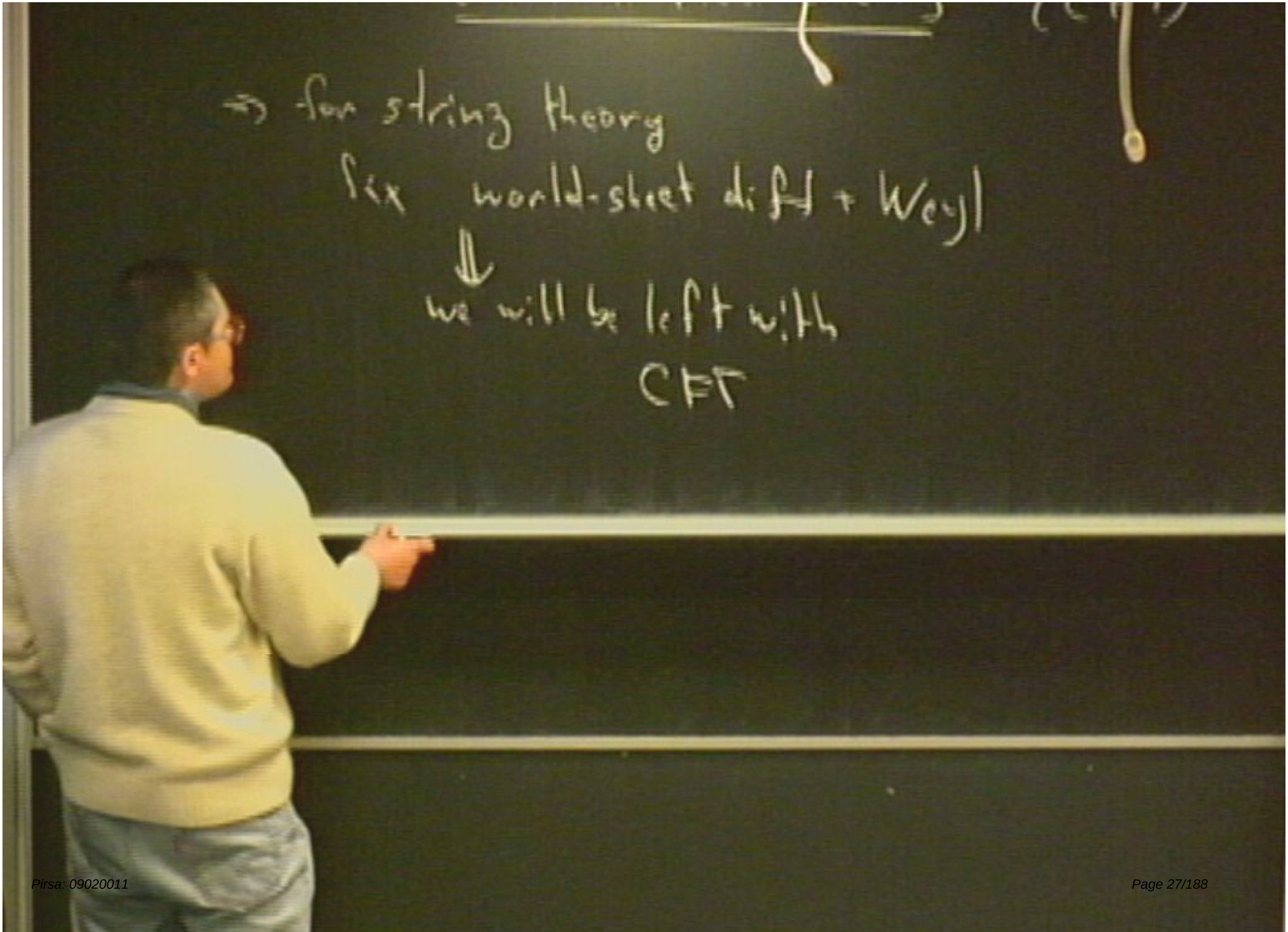
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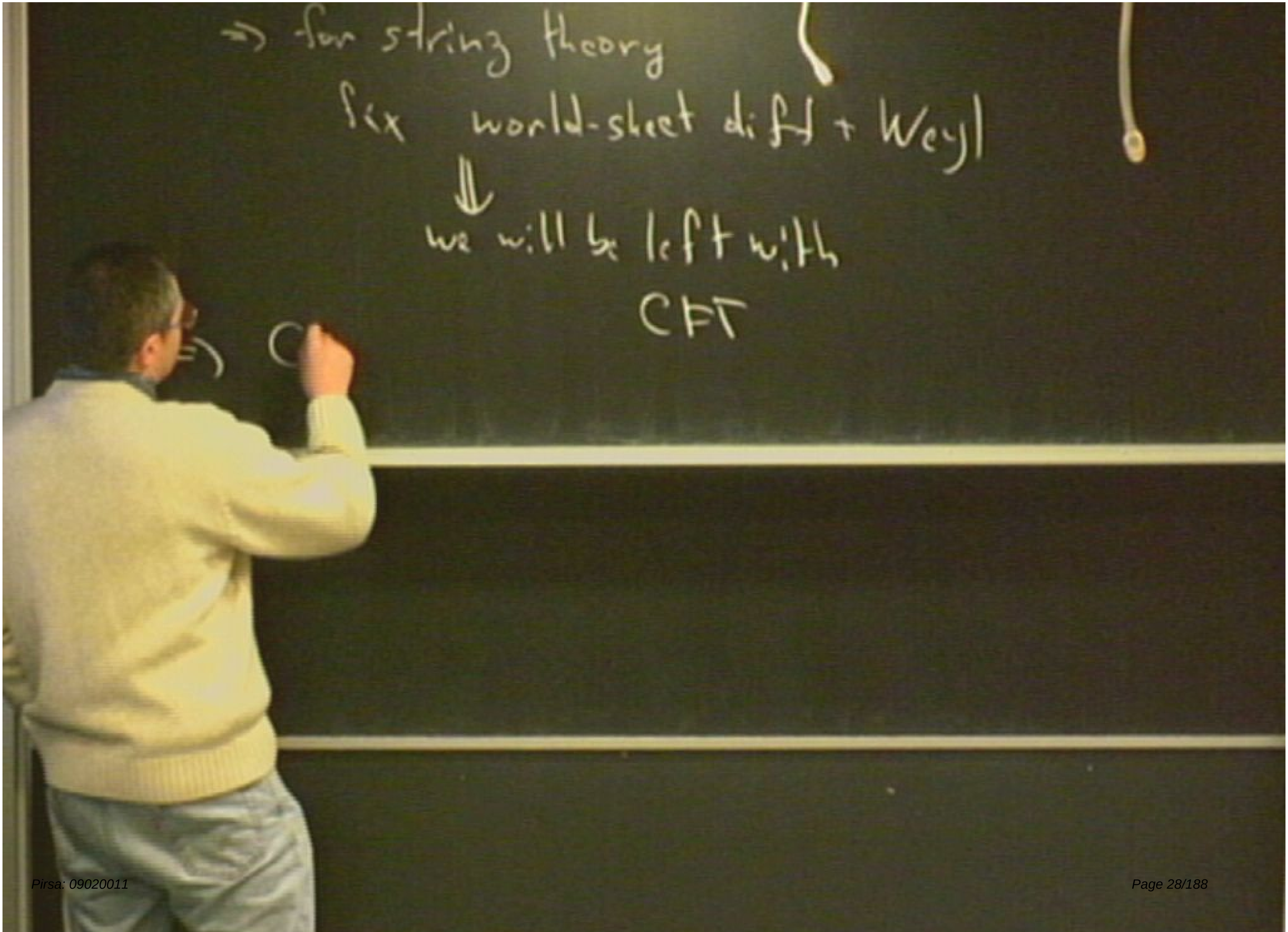
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we will be left with
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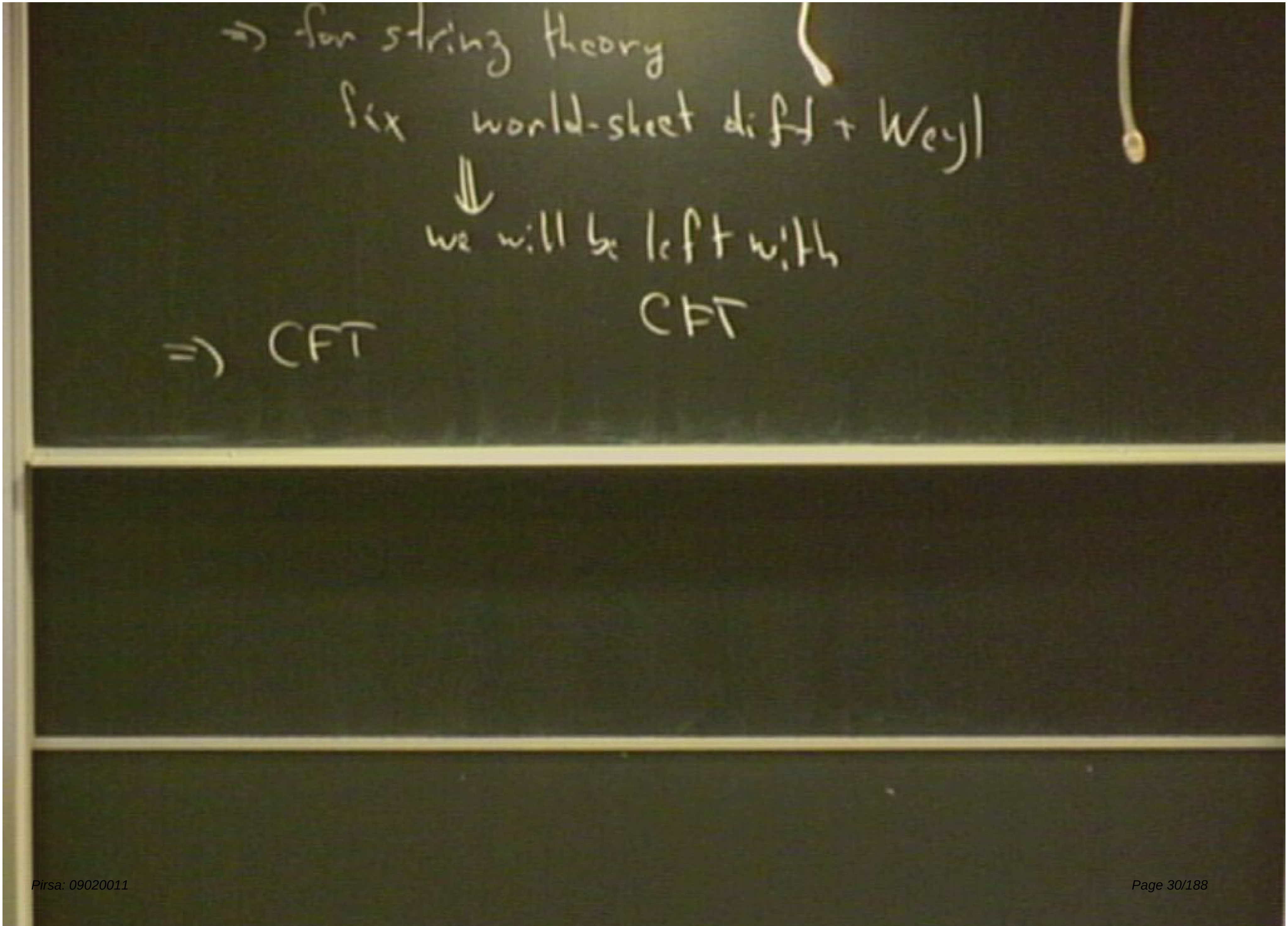
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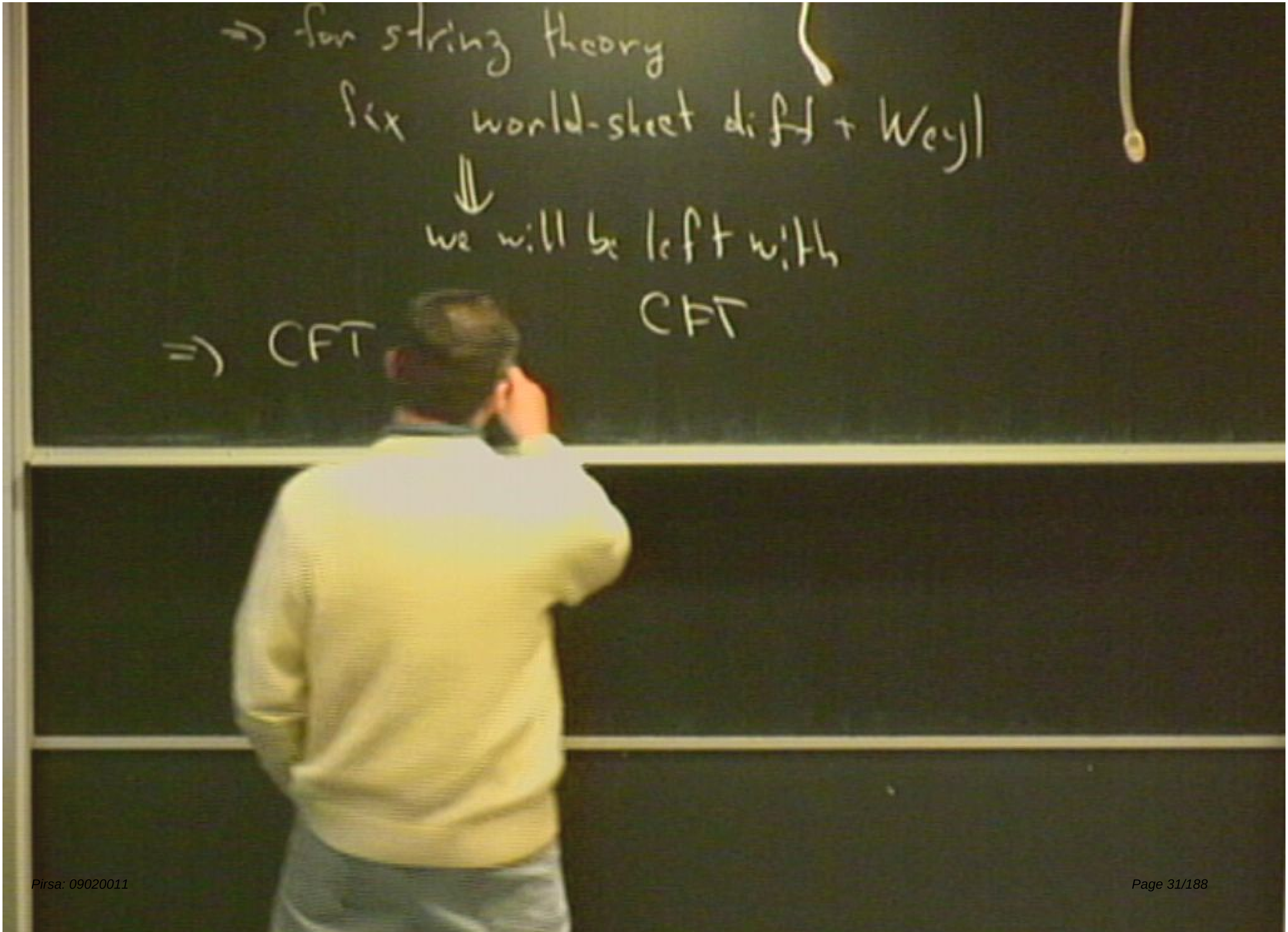
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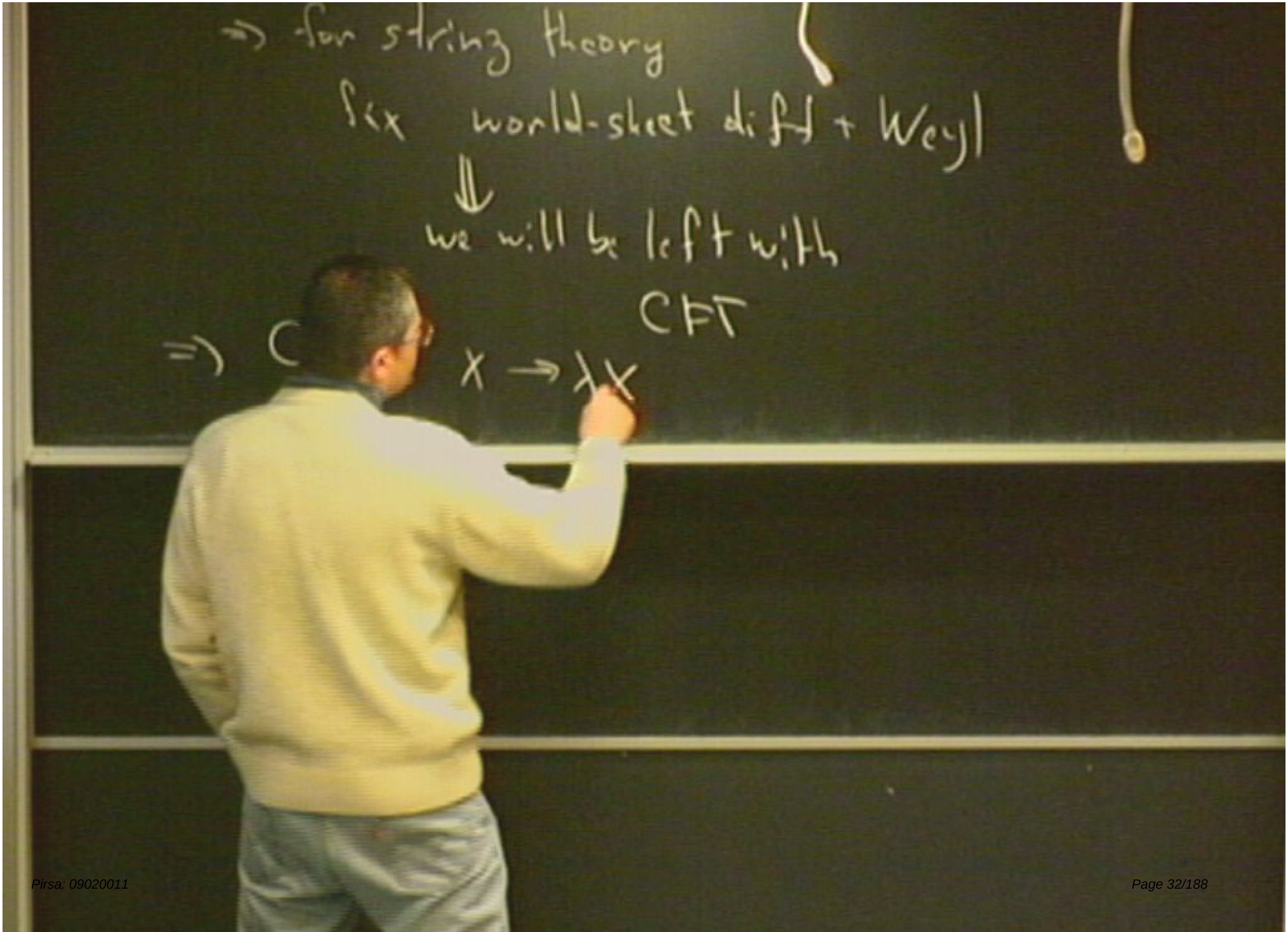
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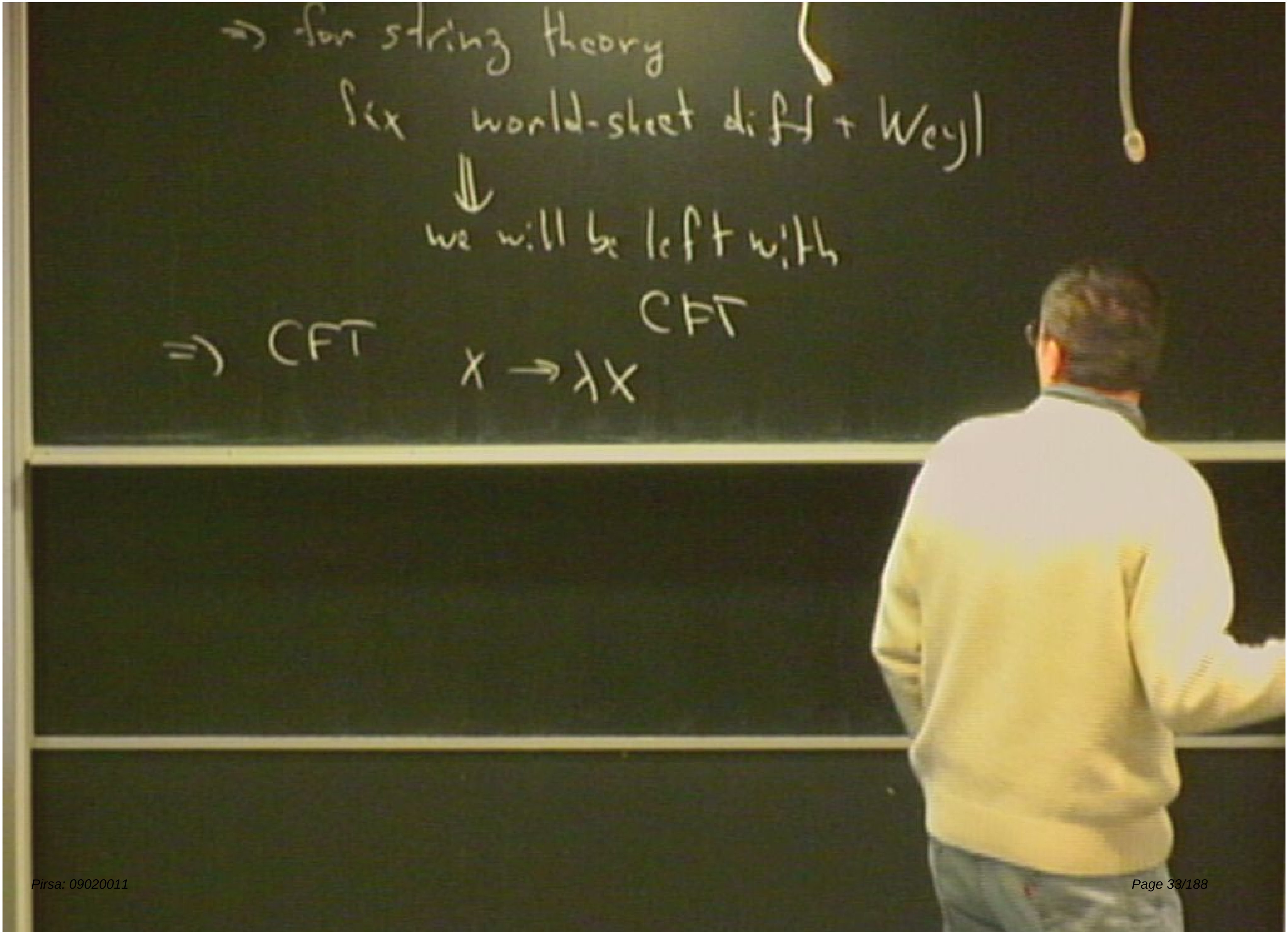
X → λX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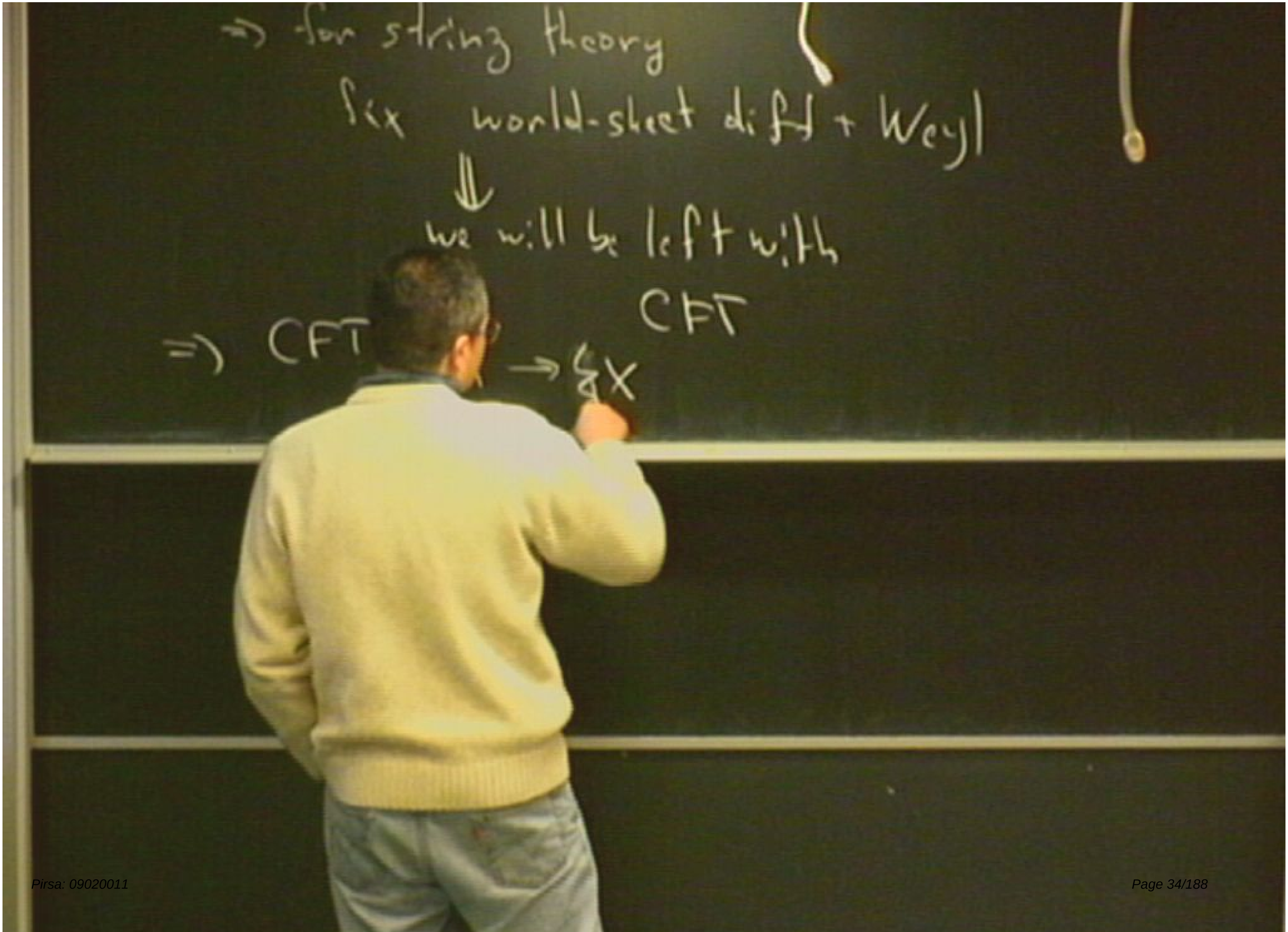
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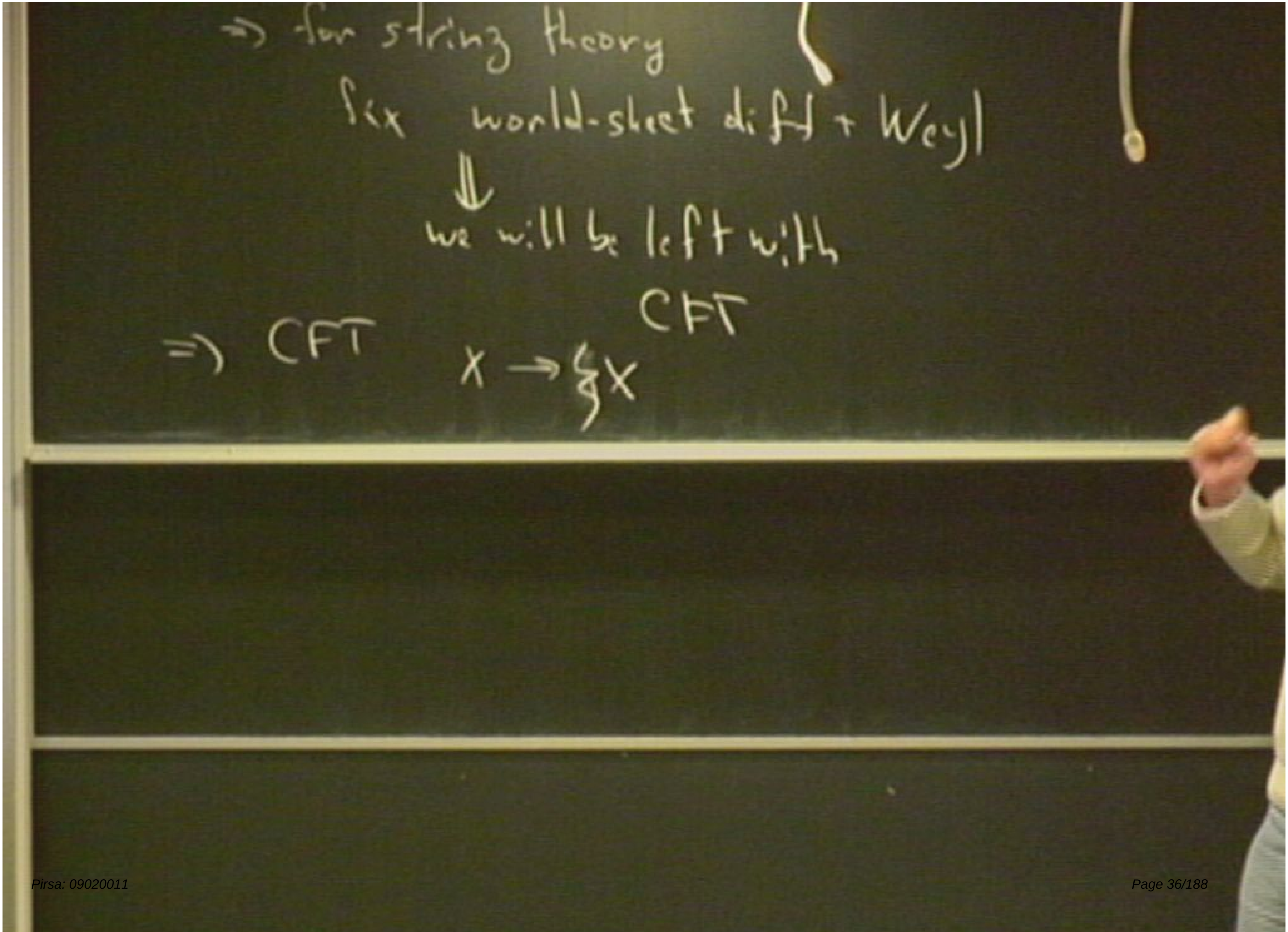
⇒ CFT $\quad X \to \xi X$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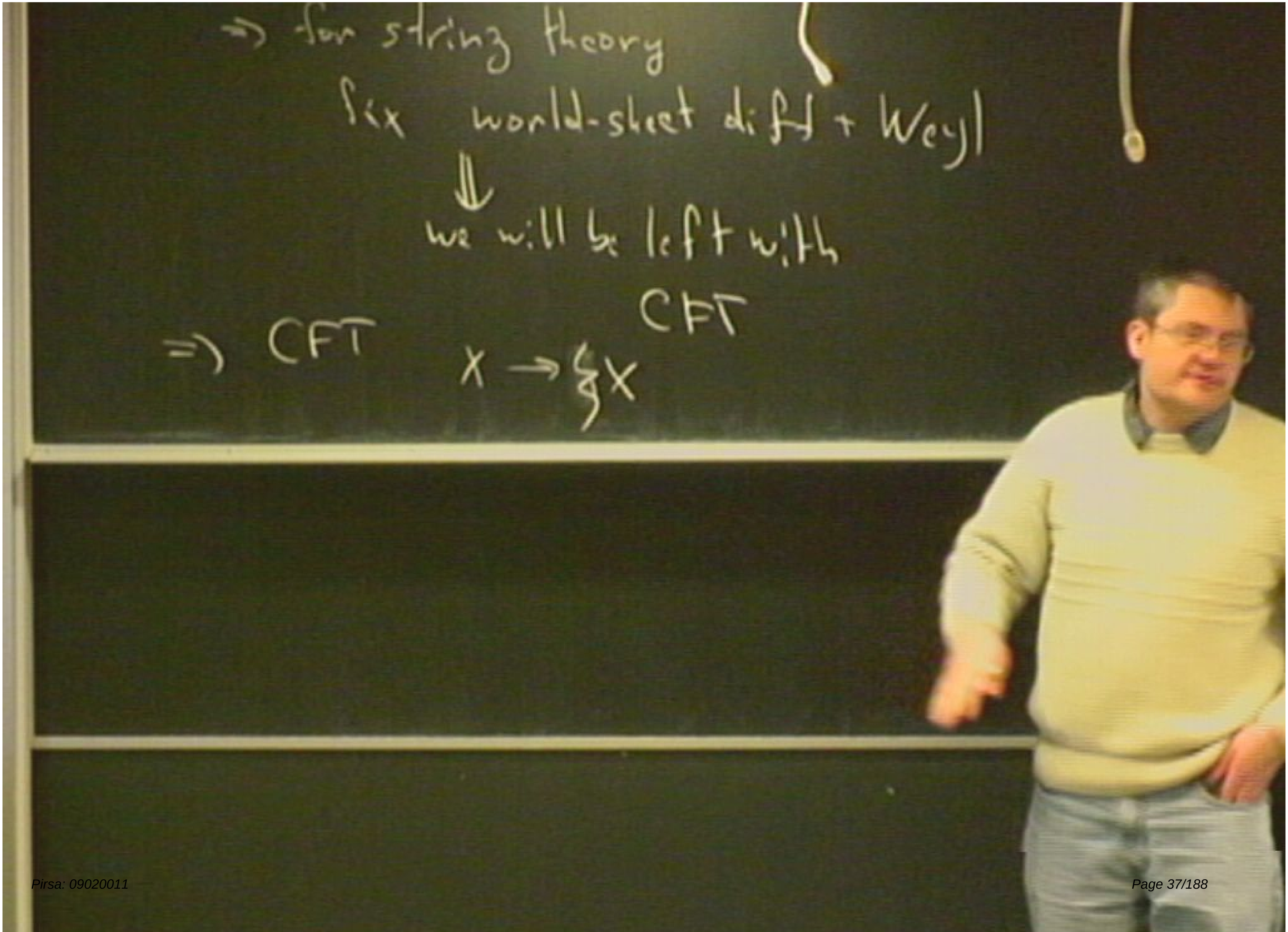
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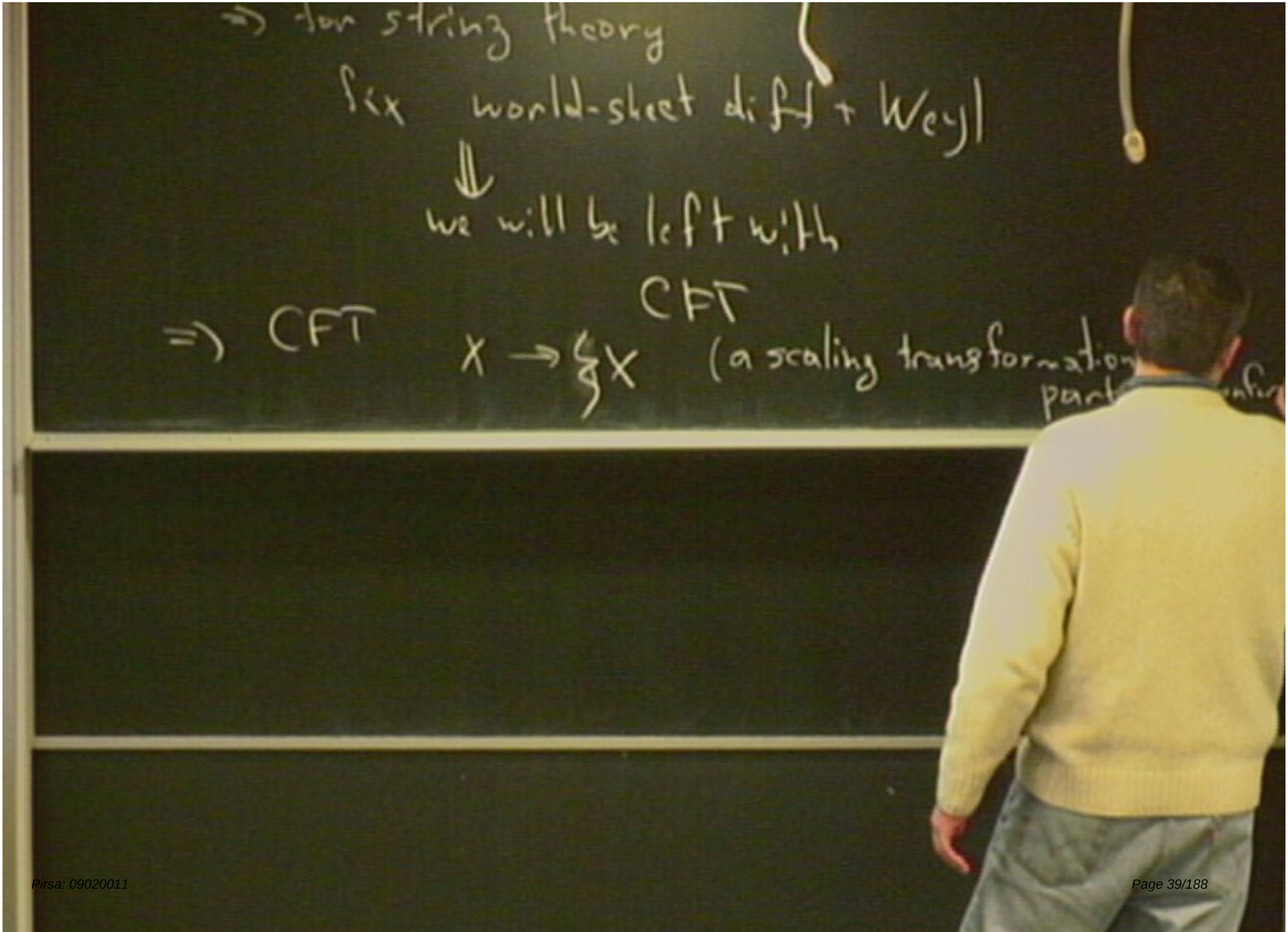
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CFTs show up in condensed matter @

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CFTs show up in condensed matter @ 2^{nd} order phase transitions

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CFTs show up in condensed matter @ 2^{nd} order phase transitions

Notes

"Applied CFT" Paul Ginsparg

D - massless scalars in 2-dim. (toy CFT).

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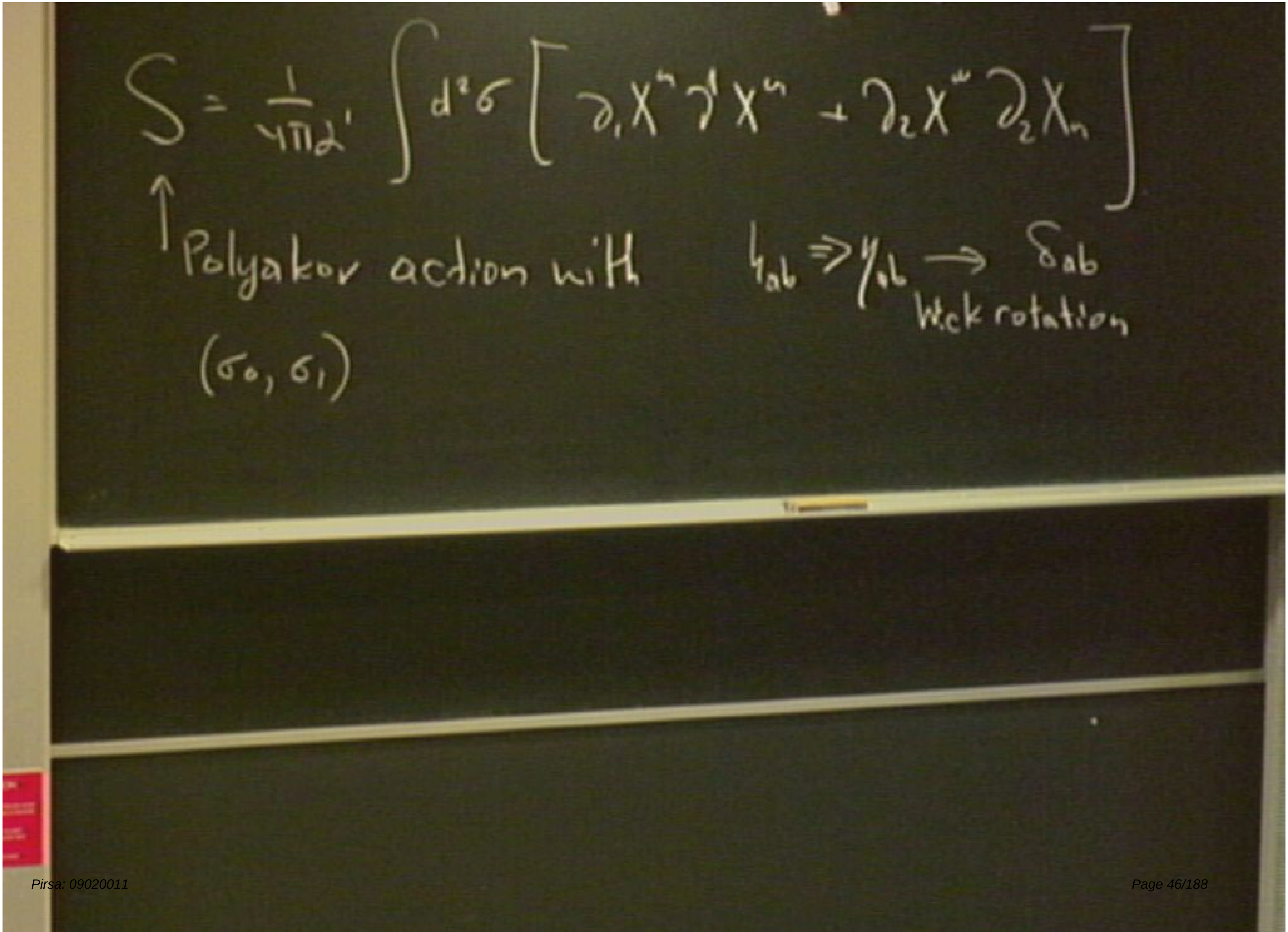
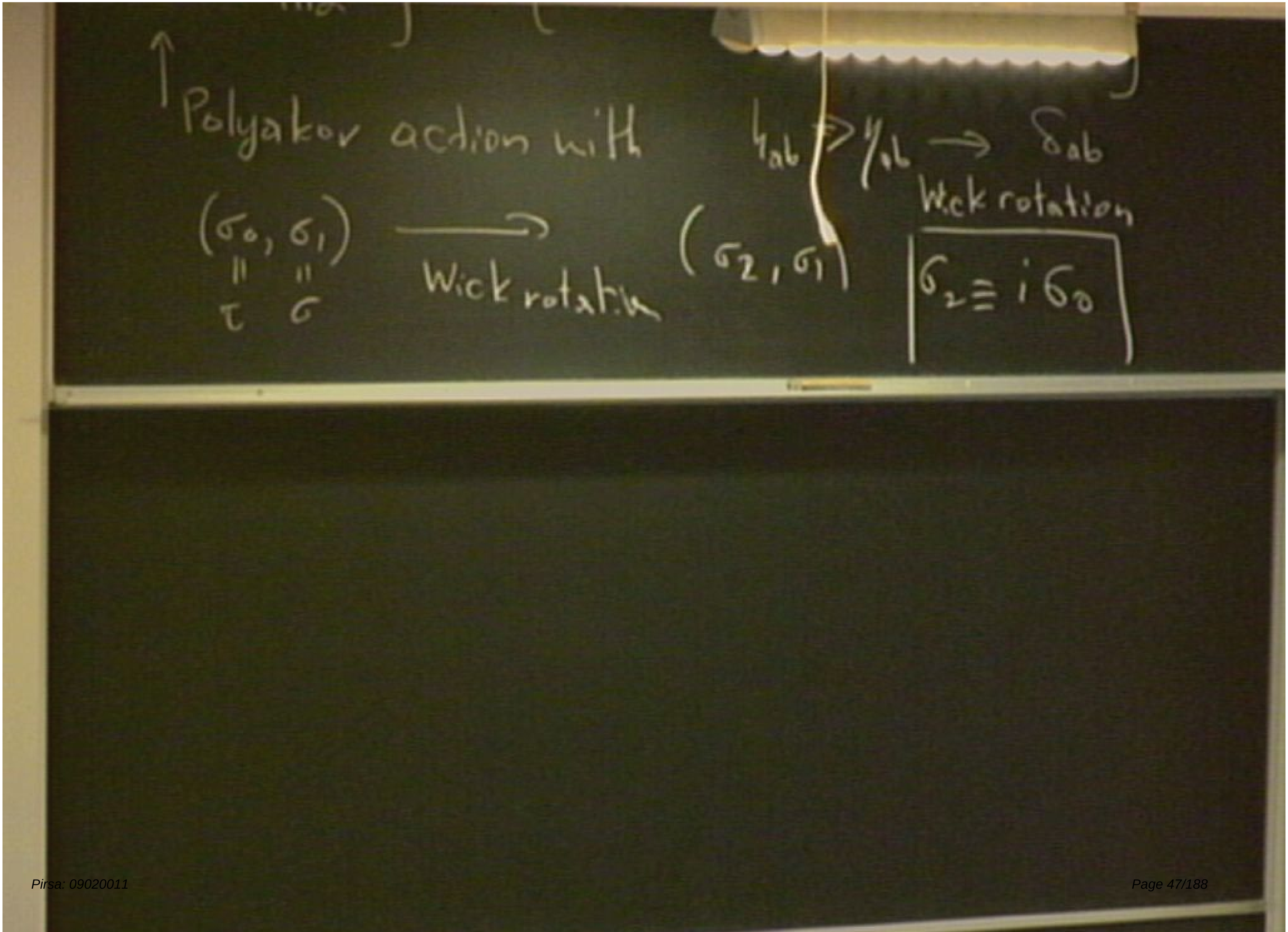
$$S = \frac{1}{4\pi\alpha'}\int d^2\sigma \left[\partial_1 X^\mu \partial^1 X^\mu + \partial_2 X^\mu \partial_2 X_\mu \right]$$

D - massless scalars in 2-dim. (toy CFT).

$$S = \frac{1}{4\pi\alpha'} \int d^2\sigma \left[\partial_1 X^\mu \partial^1 X^\mu + \partial_2 X^\mu \partial_2 X_\mu \right]$$

↑ Polyakov action with $h_{ab} \Rightarrow \eta_{ab}$


$S = \frac{1}{4\pi\alpha'} \int d^2\sigma \left[\partial_1 X^\mu \partial^1 X^\mu + \partial_2 X^\mu \partial_2 X_\mu \right]$
Polyakov action with
$h_{ab} \Rightarrow \eta_{ab} \longrightarrow \delta_{ab}$
Wick rotation
(σ_0, σ_1)


Polyakov action with $h_{ab} \Rightarrow \eta_{ab} \rightarrow \delta_{ab}$
Wick rotation
$(\sigma_0, \sigma_1) \longrightarrow (\sigma_2, \sigma_1)$
$\tau \quad \sigma$
Wick rotation
$\sigma_2 \equiv i\sigma_0$

τ σ Wick rotation (σ_2, σ_1) $\sigma_2 \equiv i\sigma_0$

$z = \sigma^1 + i\sigma^2$

$\bar{z} = \sigma^1 - i\sigma^2$

$\partial_z =$

Polyakov action with $\gamma_{ab} \Rightarrow \eta_{ab} \longrightarrow \delta_{ab}$

Wick rotation

$(\sigma_0, \sigma_1) \xrightarrow{\text{Wick rotation}} (\sigma_2, \sigma_1)$

$\sigma_0 = \tau$, $\sigma_1 = \sigma$

$$\sigma_2 \equiv i\sigma_0$$

$$z = \sigma^1 + i\sigma^2 \qquad \partial \equiv \partial_z = \frac{1}{2}(\partial_1 - i\partial_2)$$

$$\bar{z} = \sigma^1 - i\sigma^2 \qquad \bar{\partial} = \partial_{\bar{z}} = \frac{1}{2}(\partial_1 + i\partial_2)$$

$$\partial z = 1 \qquad \bar{\partial} z = 0 \qquad \ldots$$

(σ^0, σ^1)

$\;\;\parallel \quad \parallel$

$\;\;\tau \quad\; \sigma$

Wick rotation (σ_2, σ_1) $\quad \sigma_2 \equiv i\sigma_0$

$$z = \sigma^1 + i\sigma^2 \qquad \partial \equiv \partial_z = \frac{1}{2}(\partial_1 - i\partial_2)$$

$$\bar{z} = \sigma^1 - i\sigma^2 \qquad \bar{\partial} = \partial_{\bar{z}} = \frac{1}{2}(\partial_1 + i\partial_2)$$

$$\partial z = 1 \qquad \bar{\partial} z = 0 \quad \ldots$$

z ↑ holomorphic

$\bar{z}$ ↑ antiholomorphic

$$\partial = (\partial^{1}, \partial^{2})$$

$$\partial^{z} = \partial^{1} + i\partial_{2}$$

$$\partial^{\bar{z}} = \partial^{1} - i\partial_{2}$$

$$v = (v^1, v^2)$$

$$v^z = v^1 + i v_2$$

$$v^{\bar z} = v^1 - i v_2$$

$$\delta_{ab} \rightarrow \begin{pmatrix} g_{zz} & g_{z\bar z} \\ g_{z\bar z} & g_{\bar z \bar z} \end{pmatrix}$$

$$v^{z} = v^{1} - i v_{2}$$

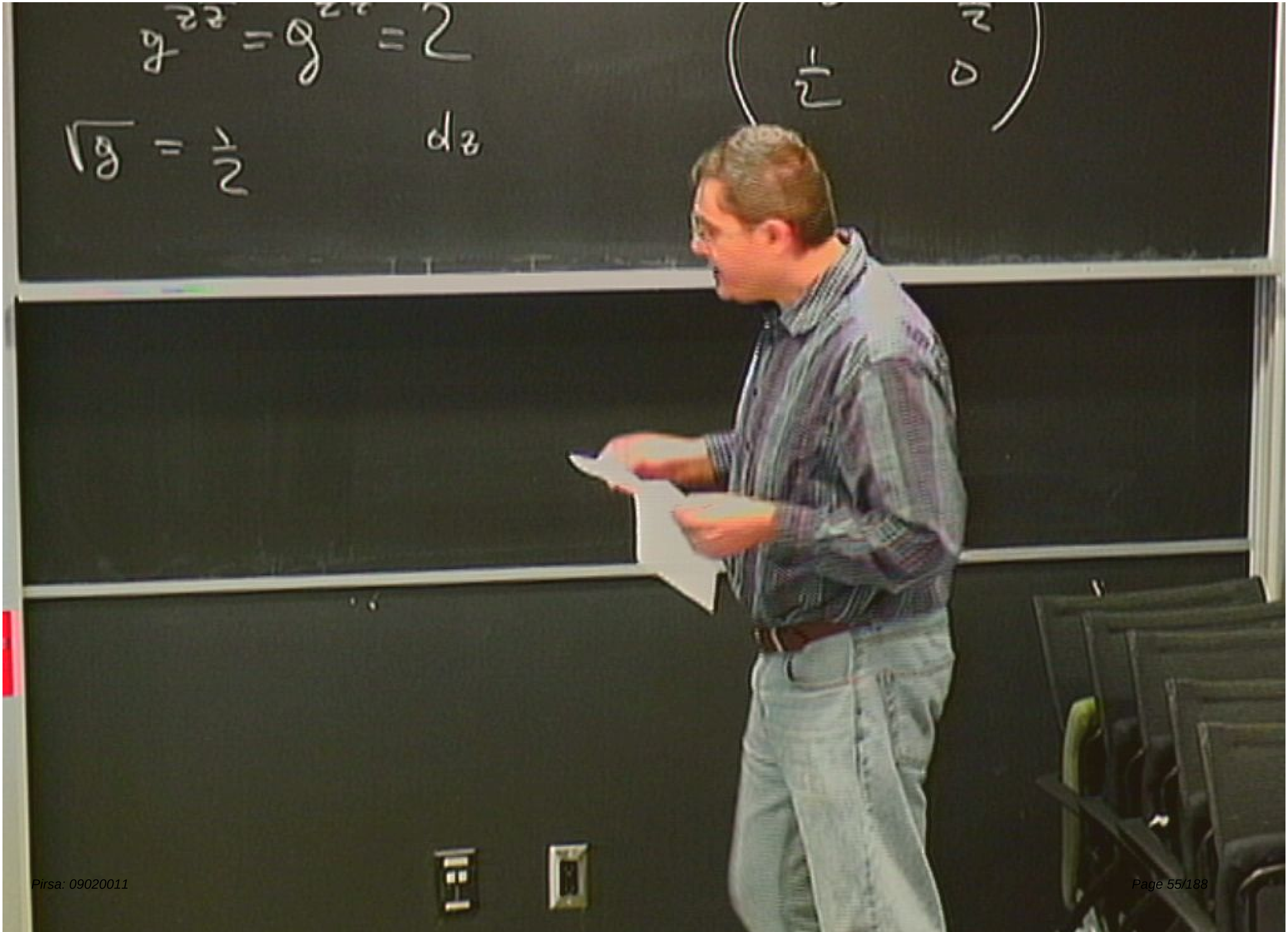
$$\parallel$$

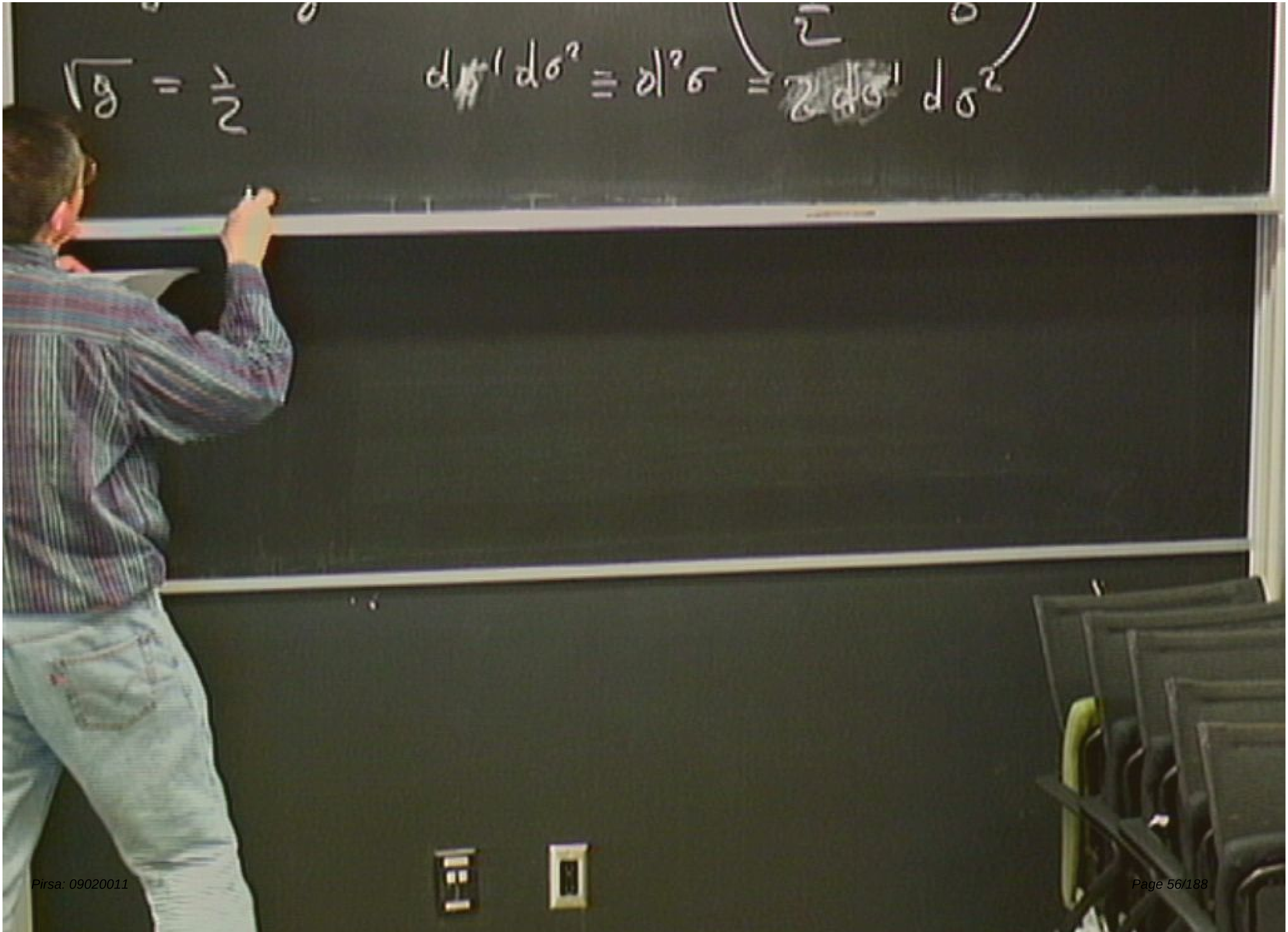
$$\begin{pmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{pmatrix}$$

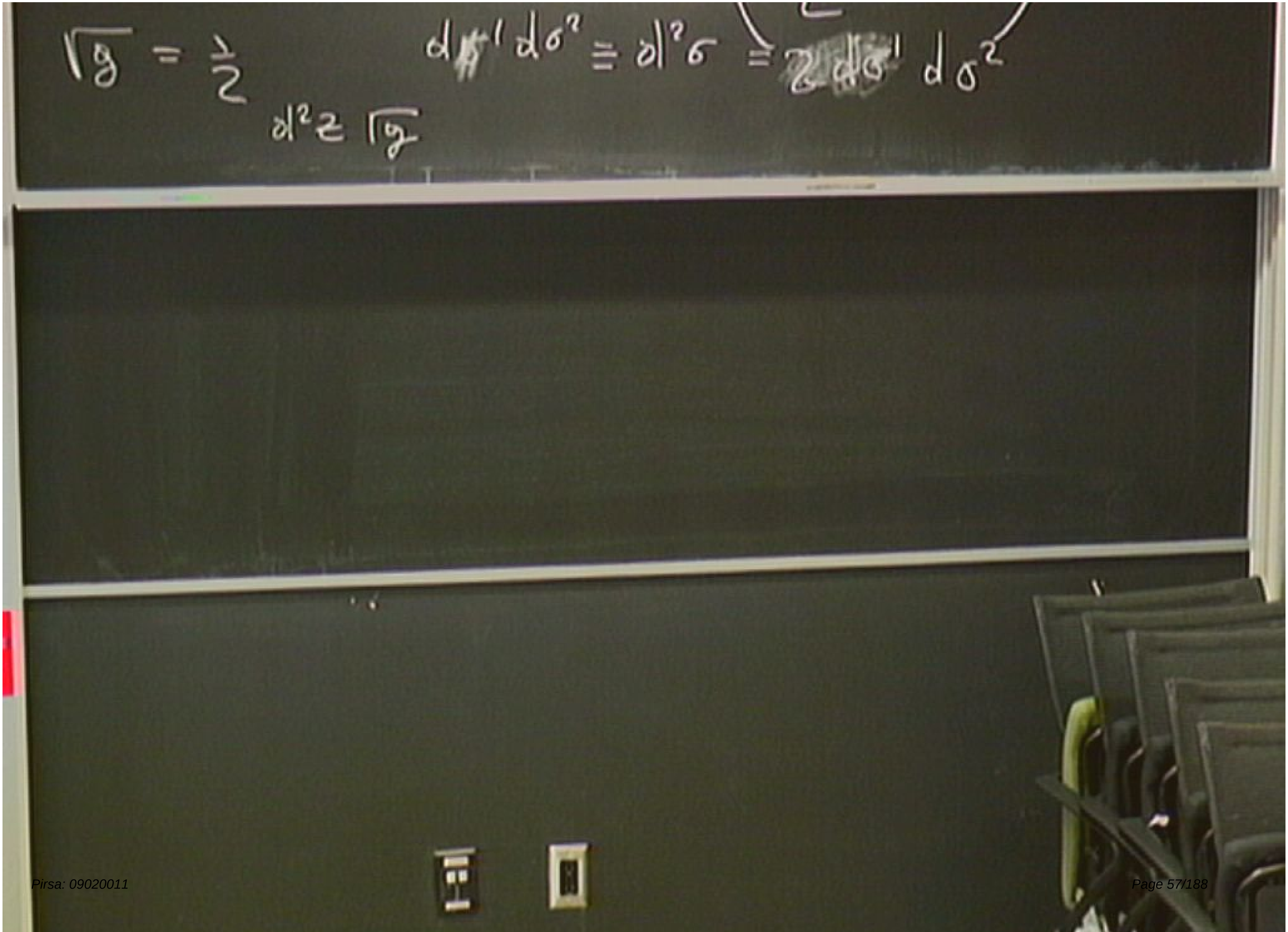
$$\partial^{z} = \partial^{1} - i\partial_{2}$$

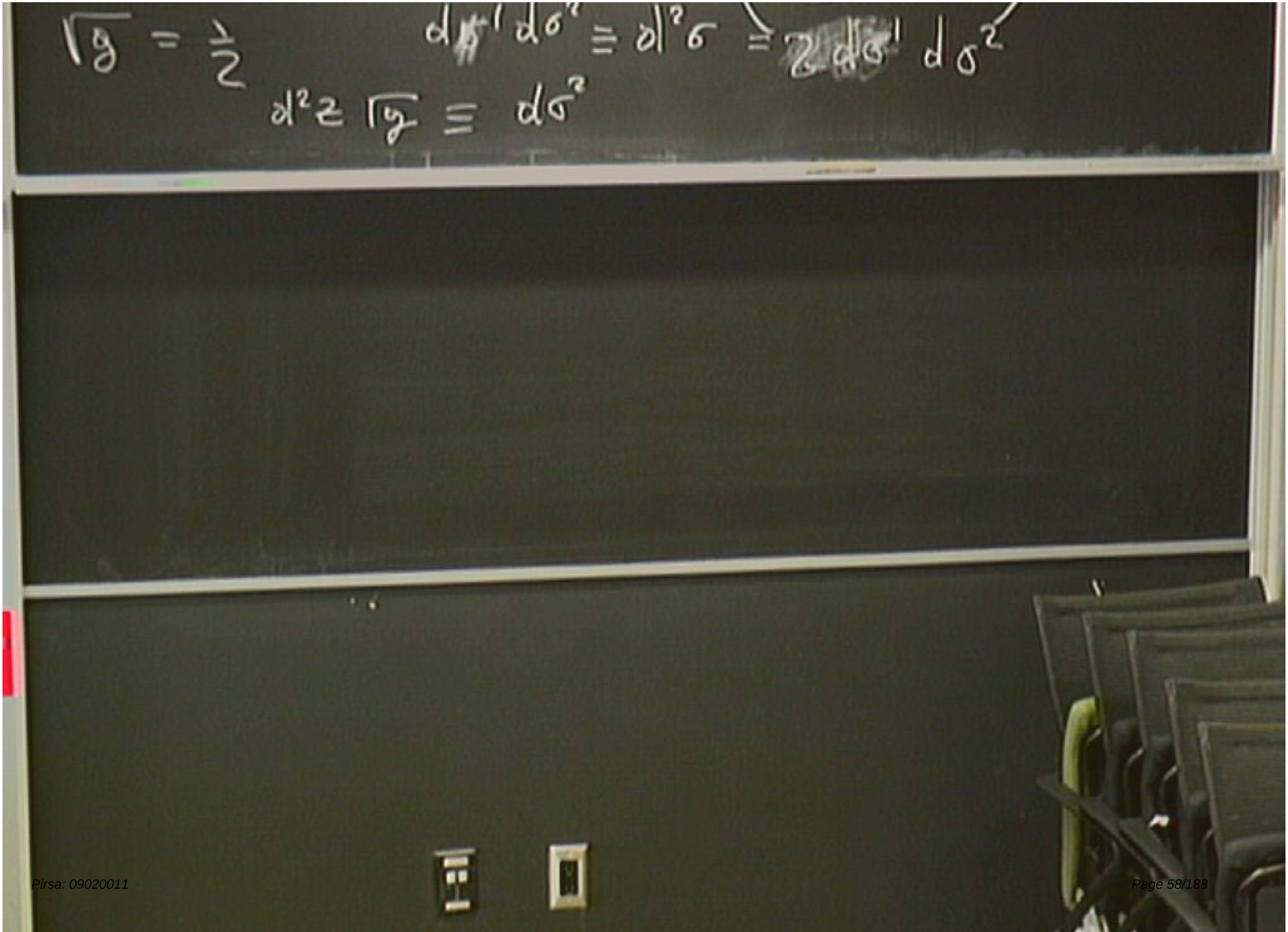
$$g^{z\bar{z}} = g^{\bar{z}z} = 2$$

$$= \begin{pmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{pmatrix}$$


$\sqrt{g} = \frac{1}{2}$
dz


√g = 1/2
dσ¹ dσ² ≡ d²σ ≡ dσ¹ dσ²


$\sqrt{g} = \frac{1}{2}$
$d^2z \sqrt{g}$
$d\sigma^1 d\sigma^2 \equiv d^2\sigma \equiv d\sigma^1 d\sigma^2$


√g = 1/2
dσ¹ dσ² ≡ d²σ ≡ dσ¹ dσ²
d²z √g ≡ dσ²

$$\sqrt{g} = \frac{1}{2} \qquad d\sigma^1 d\sigma^2 \equiv d^2\sigma = 2\, d\sigma^1\, d\sigma^2$$

$$d^2 z \sqrt{g} \equiv d\sigma^2$$

$$\int d^2\sigma\ \delta^2(\sigma_1, \sigma_2) = \int d^2\sigma\ \delta(\sigma_1)\,\delta(\sigma_2) = 1$$

$$\int d^2 z\ \delta^2(z, \bar{z}) = 1 \Rightarrow \delta^2(z, \bar{z}) = \frac{1}{2}\,\delta^2(\sigma_1, \sigma_2)$$

$$\sqrt{g} = \frac{1}{2} \qquad d\sigma^1 d\sigma^2 \equiv d^2\sigma = 2\, d\sigma^1\, d\sigma^2$$

$$d^2 z \sqrt{g} \equiv d\sigma^2$$

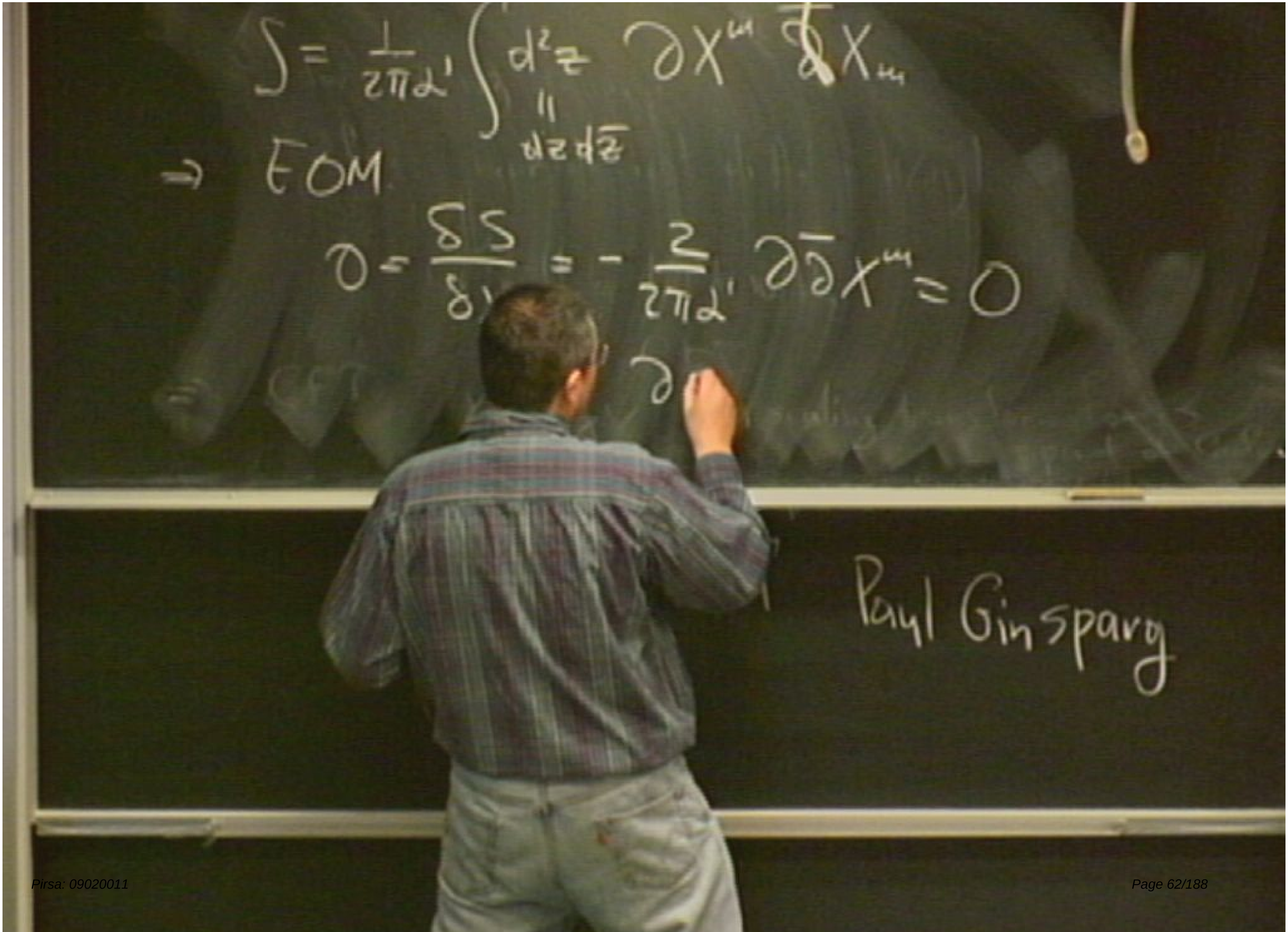
$$\int d^2\sigma\ \delta^2(\sigma_1, \sigma_2) = \int d^2\sigma\ \delta(\sigma_1)\,\delta(\sigma_2) = 1$$

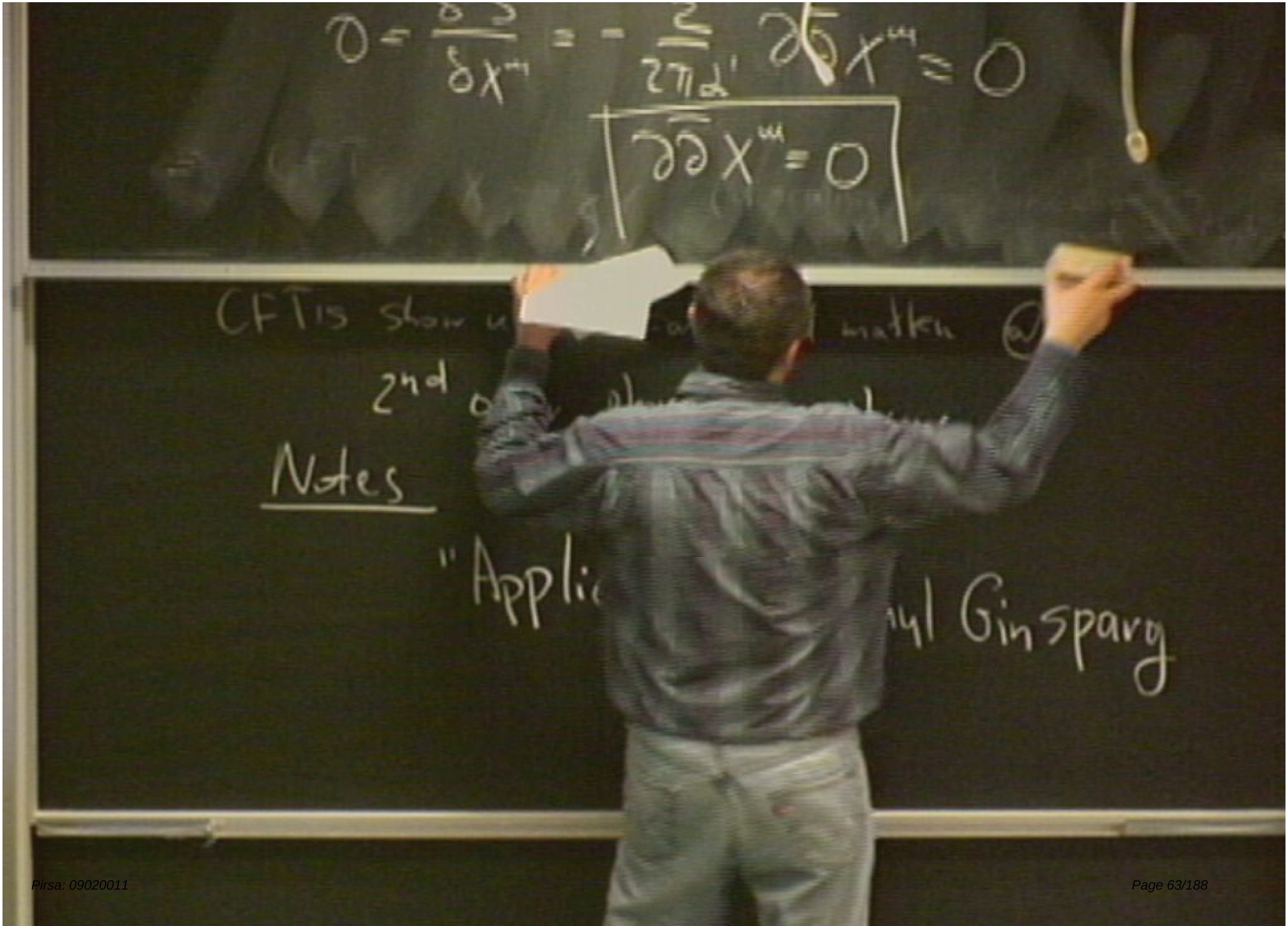
$$\int d^2 z\ \delta^2(z, \bar{z}) = 1 \Rightarrow \delta^2(z, \bar{z}) = \frac{1}{2}\delta^2(\sigma_1, \sigma_2)$$

Divergence theorem.

In above notation

$$S = \frac{1}{2\pi\alpha'}\int d^2z\ \partial X^\mu\, \bar{\partial} X_\mu$$


$S = \frac{1}{2\pi\alpha'} \int d^2z \; \partial X^\mu \bar{\partial} X_\mu$
$d^2z = dz\, d\bar{z}$
$\Rightarrow$ EOM
$0 = \frac{\delta S}{\delta X} = -\frac{2}{2\pi\alpha'} \partial\bar{\partial} X^\mu = 0$
Paul Ginsparg


$0 = \frac{\delta S}{\delta X^\mu} = -\frac{2}{2\pi\alpha'} \partial\bar{\partial} X^\mu = 0$
$\partial\bar{\partial} X^\mu = 0$
CFTs show
matter
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Notes
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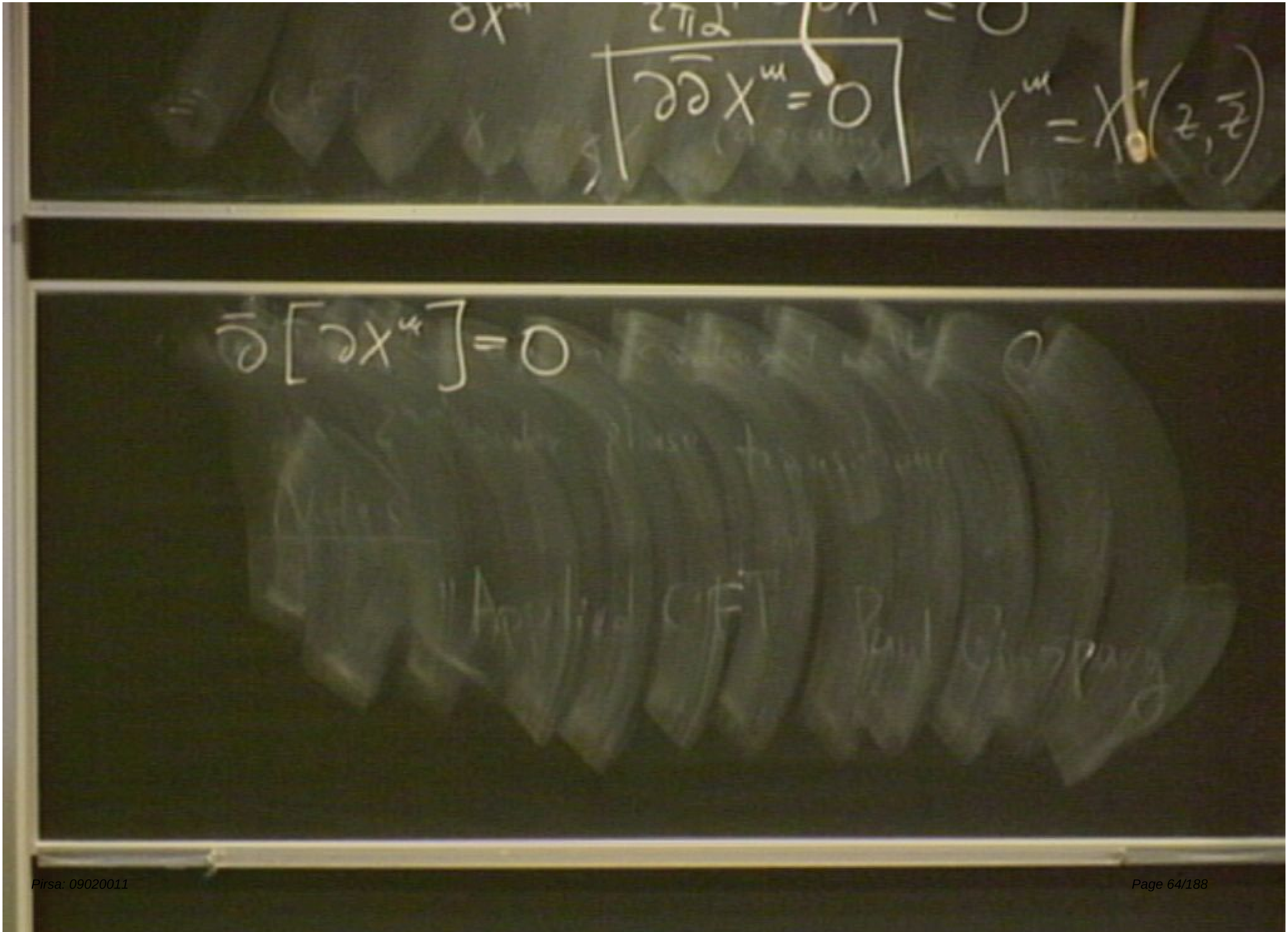


$\partial\bar{\partial}X^{\mu}=0$
$X^{\mu}=X^{\mu}(z,\bar{z})$
$\bar{\partial}[\partial X^{\mu}]=0$
"Applied CFT" Paul Ginsparg

$$\partial\bar{\partial} X^\mu = 0$$

$$X^\mu = X^\mu(z, \bar{z})$$

$$\bar{\partial}\left[\partial X^\mu\right] = 0$$

$$\partial X^\mu = \partial X^\mu(z)$$

← a holomorphic function.

$$\bar{\partial} X^\mu = \bar{\partial} X^\mu(\bar{z})$$

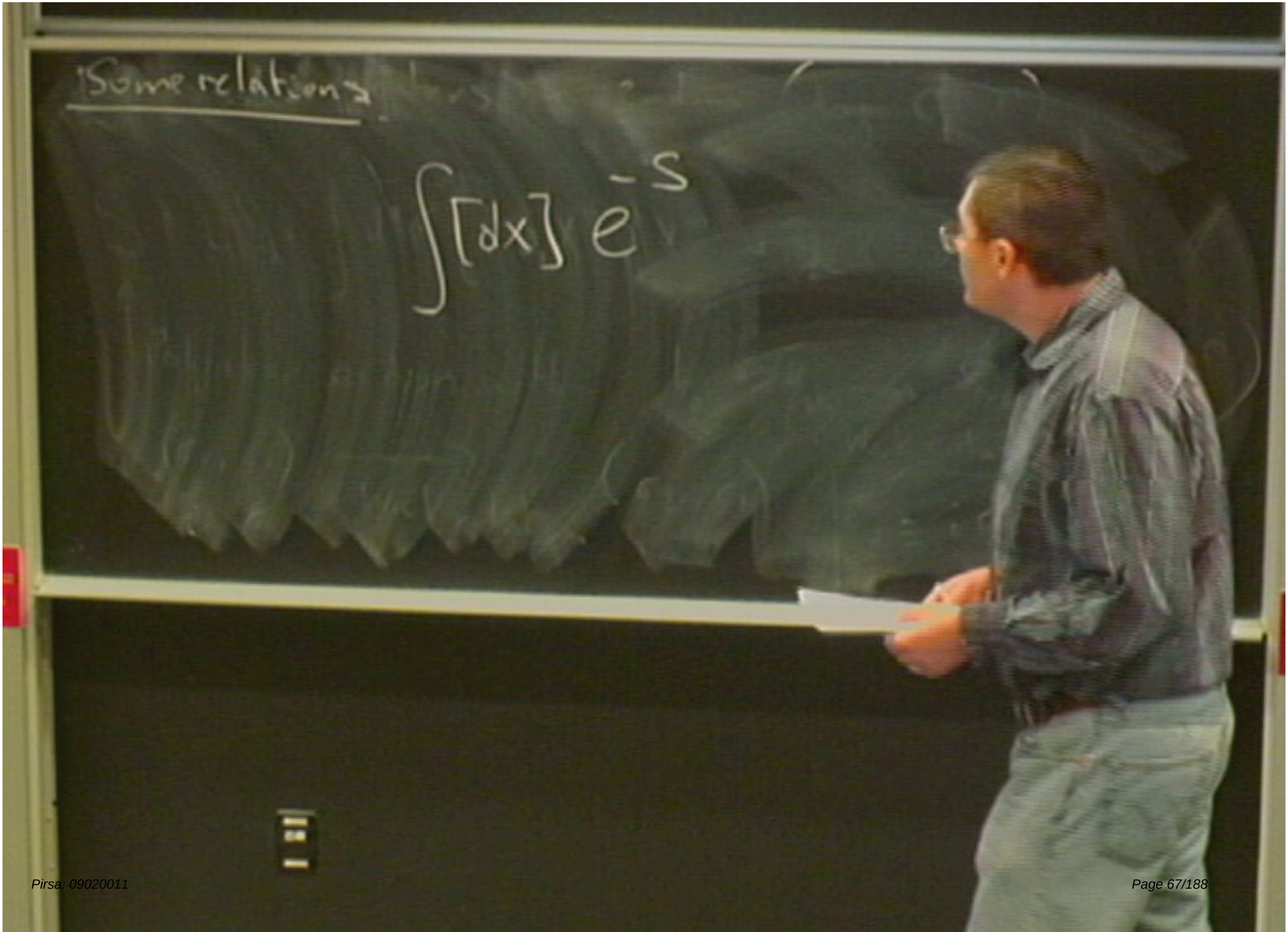
$$\boxed{\partial\bar{\partial} X^\mu = 0} \qquad X^\mu = X^\mu(z,\bar{z})$$

$$\bar{\partial}\left[\partial X^\mu\right] = 0$$

← a holomorphic function.

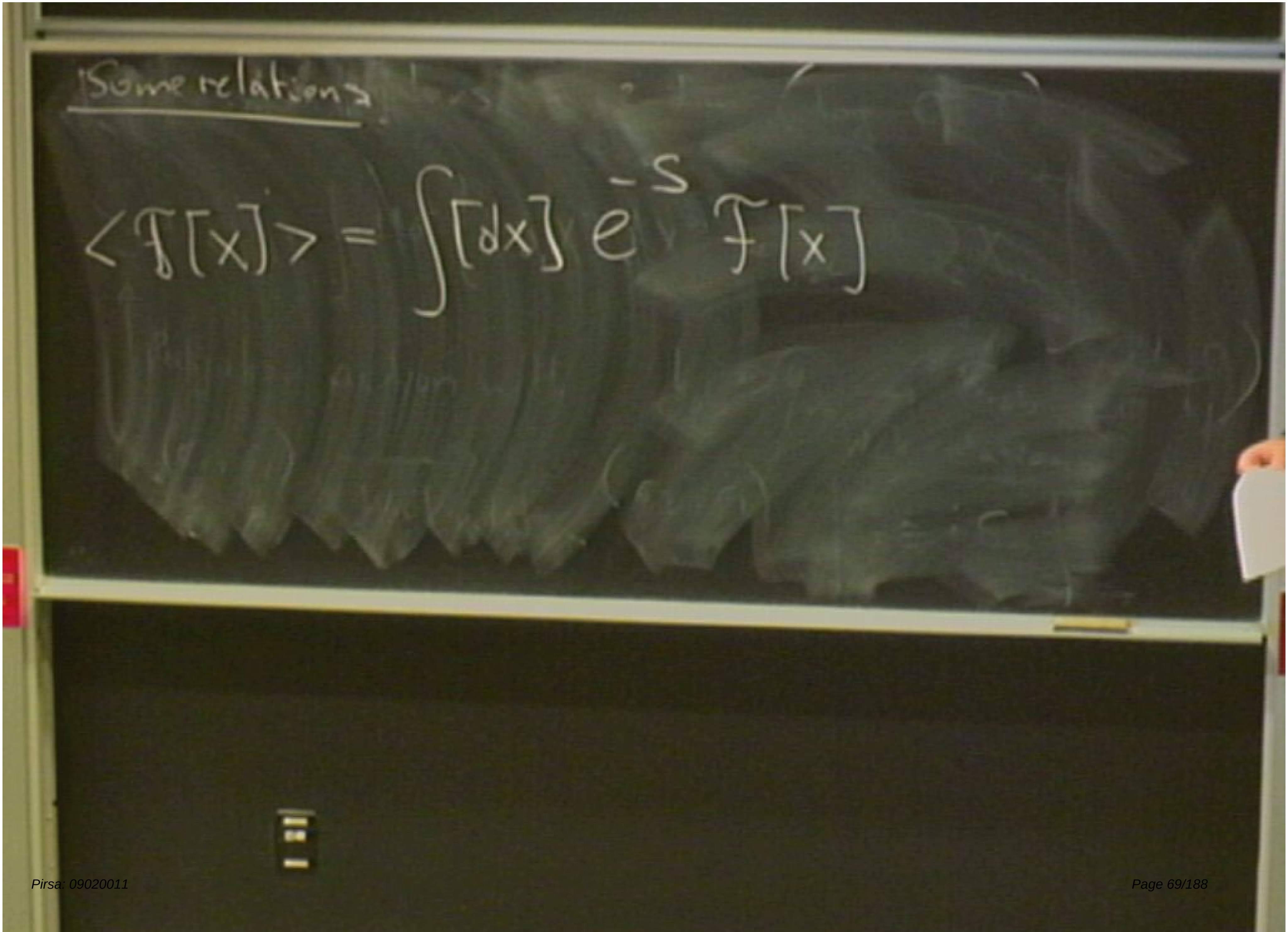
$$\partial X^\mu = \partial X^\mu(z)$$

$$\partial\left[\bar{\partial} X^\mu\right] = 0 \qquad \bar{\partial} X^\mu = \bar{\partial} X^\mu(\bar{z})$$


Some relations
$\int [dx]\, e^{-S}$

Some relations

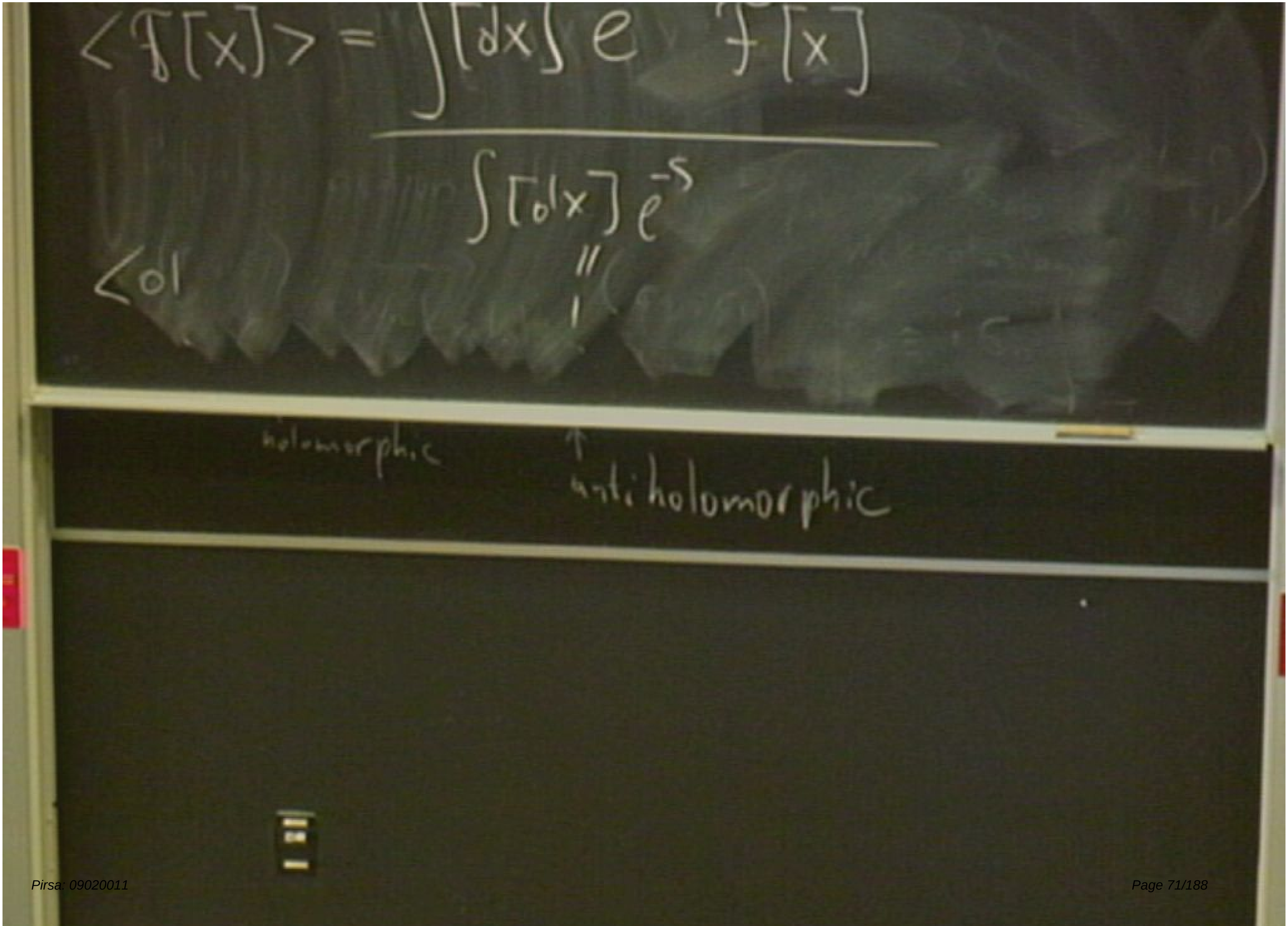
$$\langle f[x] \rangle = \int [dx]\, e^{-S}$$

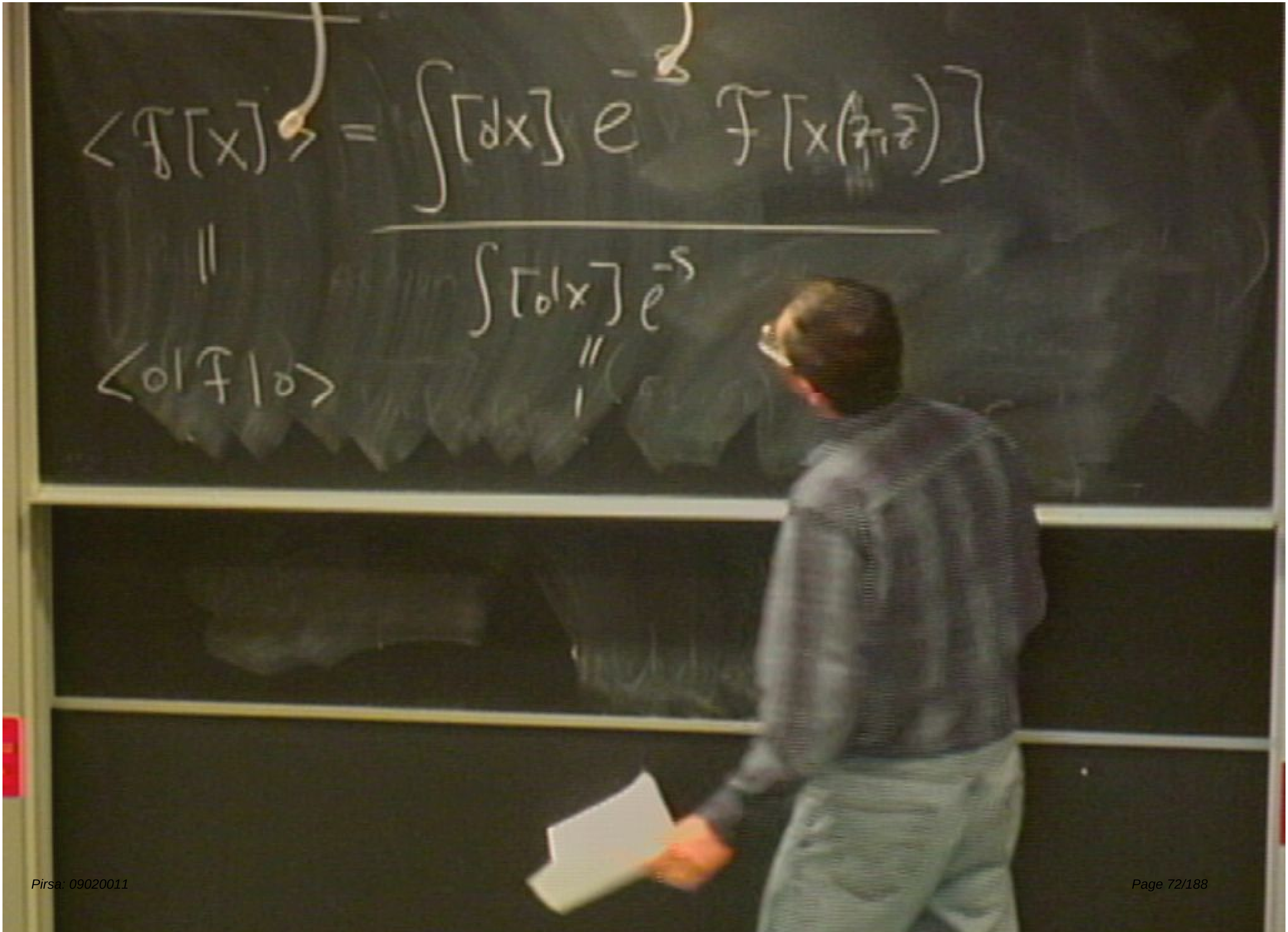

Some relations
$\langle \mathcal{F}[x] \rangle = \int [dx]\, e^{-S}\, \mathcal{F}[x]$

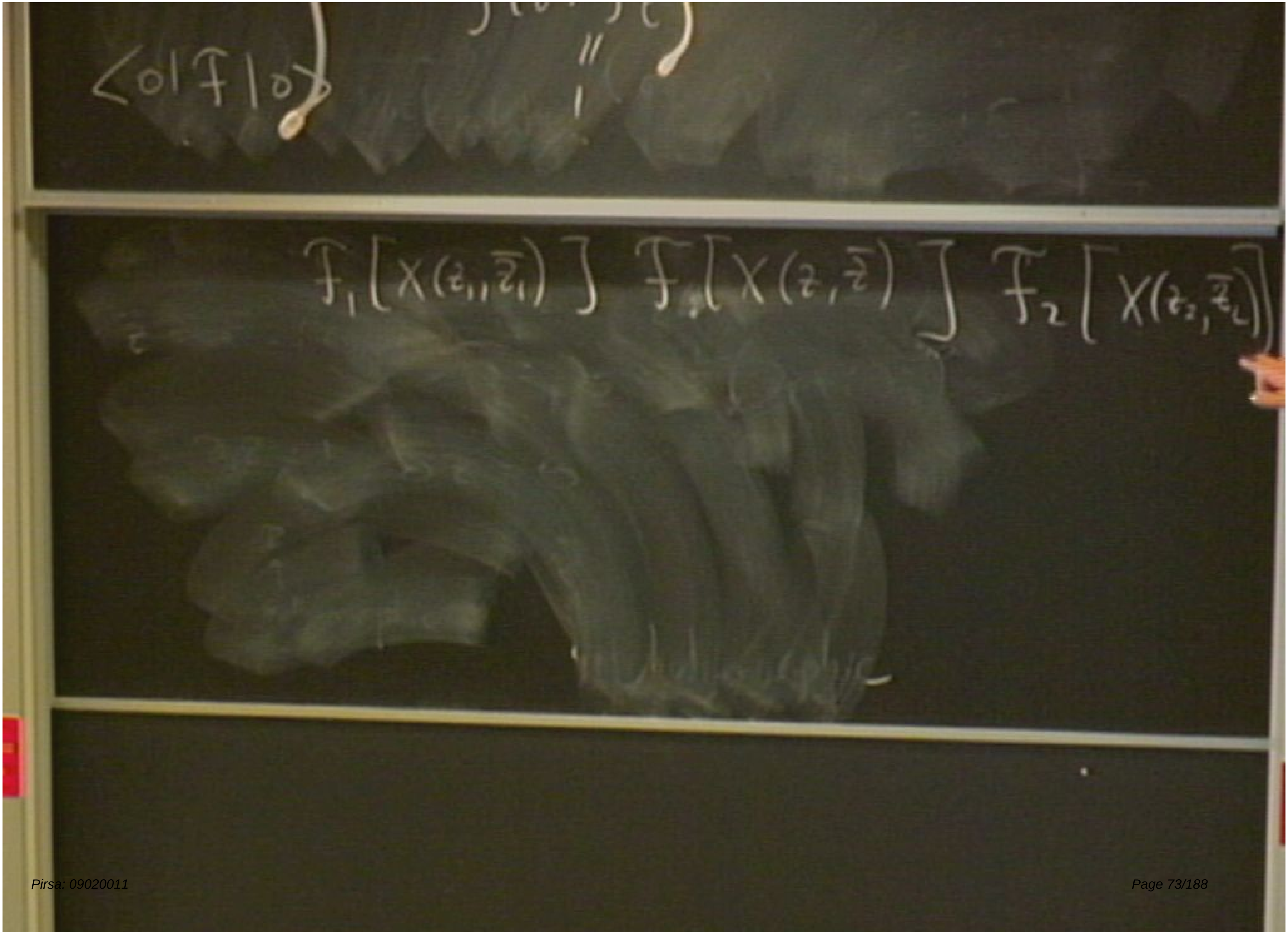
Some relations

$$\langle F[x] \rangle = \frac{\int [dx]\, e^{-S}\, F[x]}{\int [dx]\, e^{-S}}$$

$$\int [dx]\, e^{-S} = 1$$


$\langle f[x] \rangle = \dfrac{\int [dx]\, e^{-S} f[x]}{\int [dx]\, e^{-S}}$
$= 1$
$\langle 0|$
holomorphic
antiholomorphic


<f[x]> = ∫[dx] e F[x(τ1,τ2)]
∫[dx] e-S
<0|F|0>



<0|F|0>
F1[X(z1, z̄1)] F[X(z, z̄)] F2[X(z2, z̄2)]

$\langle 0 | F | 0 \rangle$

$$\int d[X] e^{-S} F_1[X(z_1,\bar{z}_1)] \; F[X(z,\bar{z})] \; F_2[X(z_2,\bar{z}_2)]$$

$$\langle 0 | F | 0 \rangle$$

$$= 1$$

$$\int d[X]\, e^{-S}\, F_1\left[X(z_1,\bar z_1)\right]\, F\left[X(z,\bar z)\right]\, F_2\left[X(z_2,\bar z_2)\right]$$

$$= \langle 0 | F_1 F F_2 | 0 \rangle$$

$$\langle 0 | F | 0 \rangle$$

$$= 1$$

$$\int d[X]\, e^{-S} F_1[X(z_1,\bar z_1)]\, F[X(z,\bar z)]\, F_2[X(z_2,\bar z_2)]$$

$$= \langle 0 | F_1 F F_2 | 0 \rangle$$

$\langle 0|F|0\rangle$

$$= 1$$

$$\int d[X]\, e^{-S}\, F_1[X(z_1,\bar z_1)]\; F[X(z,\bar z)]\; F_2[X(z_2,\bar z_2)]$$

$$= \langle 0|F_1 F F_2|0\rangle$$

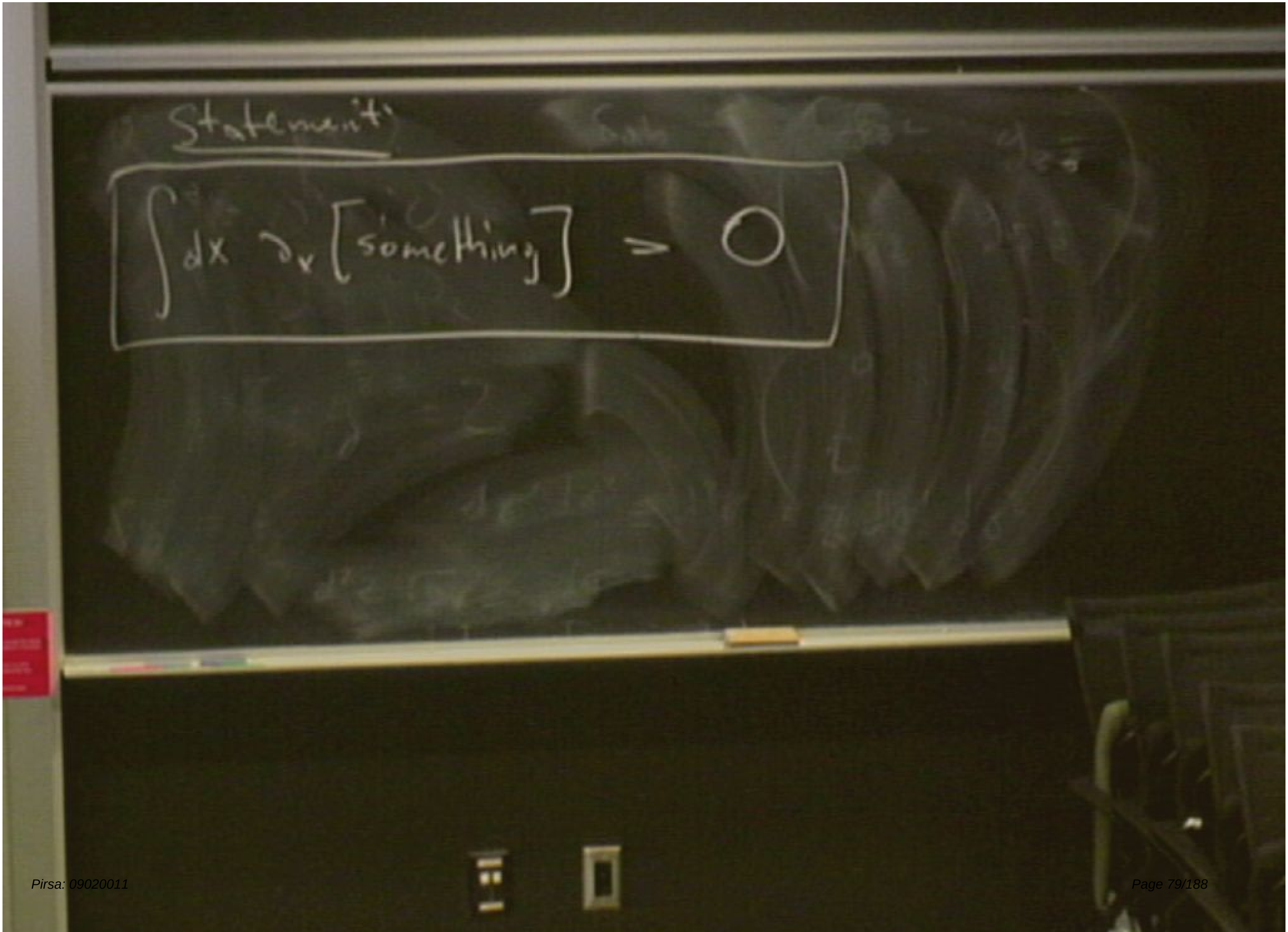
$$= \langle 1|F|2\rangle$$

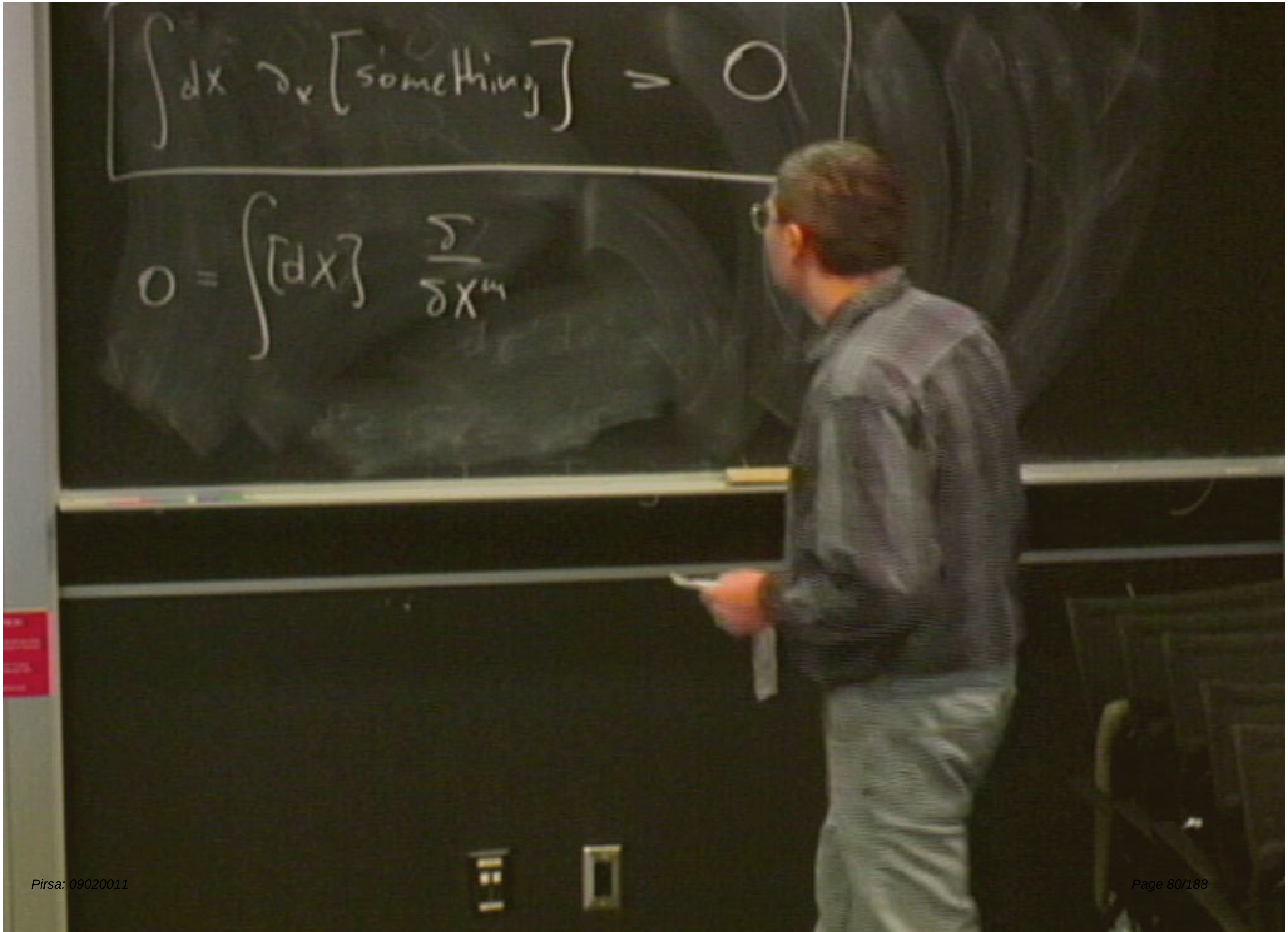
$$\langle 0 | F | 0 \rangle \qquad = 1$$

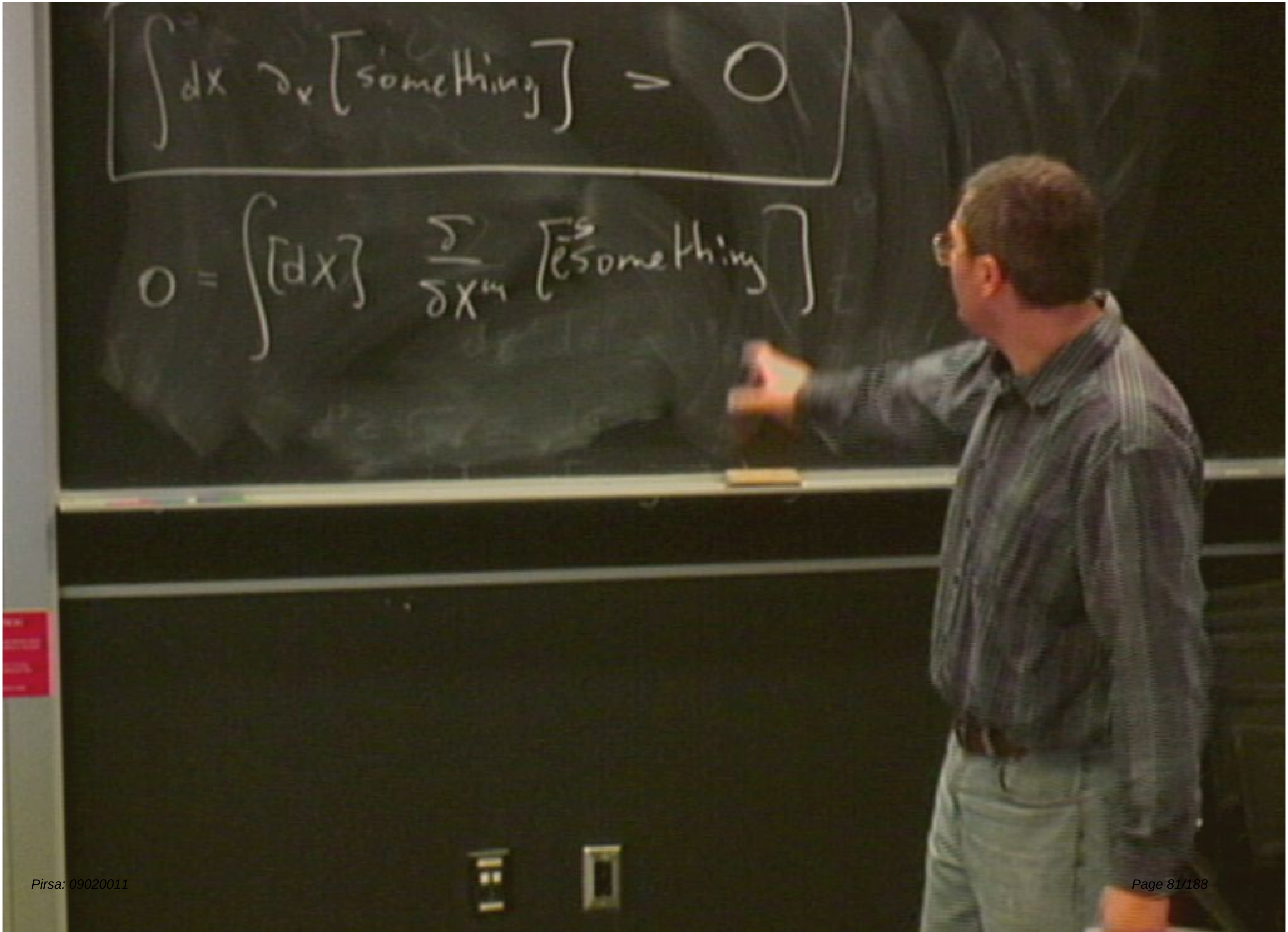
$$\int d[X]\, e^{-S}\, F_1[X(z_1,\bar z_1)]\; F[X(z,\bar z)]\; F_2[X(z_2,\bar z_2)]$$

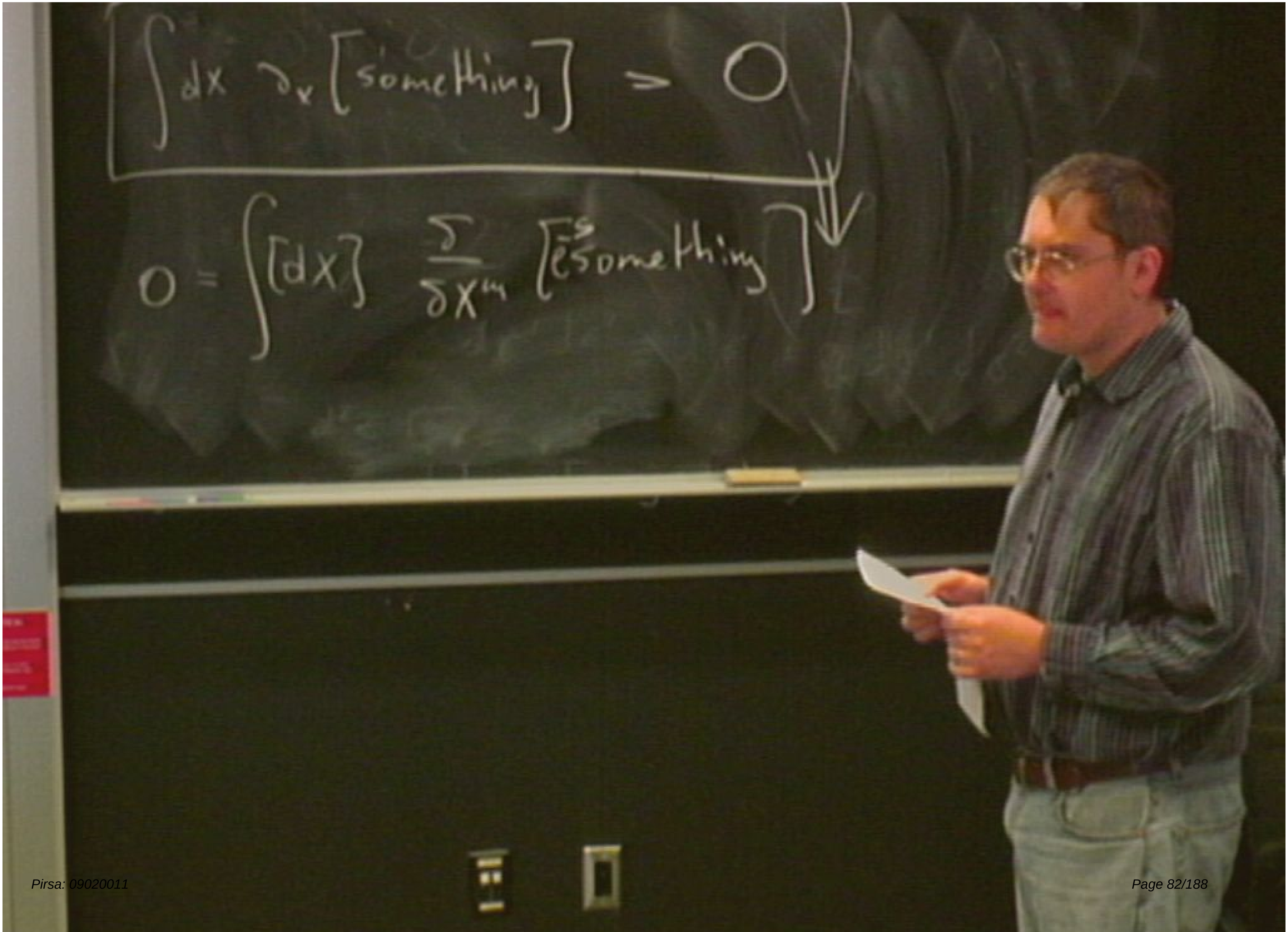
$$= \langle 0 | F_1 F F_2 | 0 \rangle$$

$$= \langle 1 | F | 2 \rangle \qquad |2\rangle = F_2 |0\rangle$$


Statement:
$\int dx \, \partial_x [\text{something}] = 0$

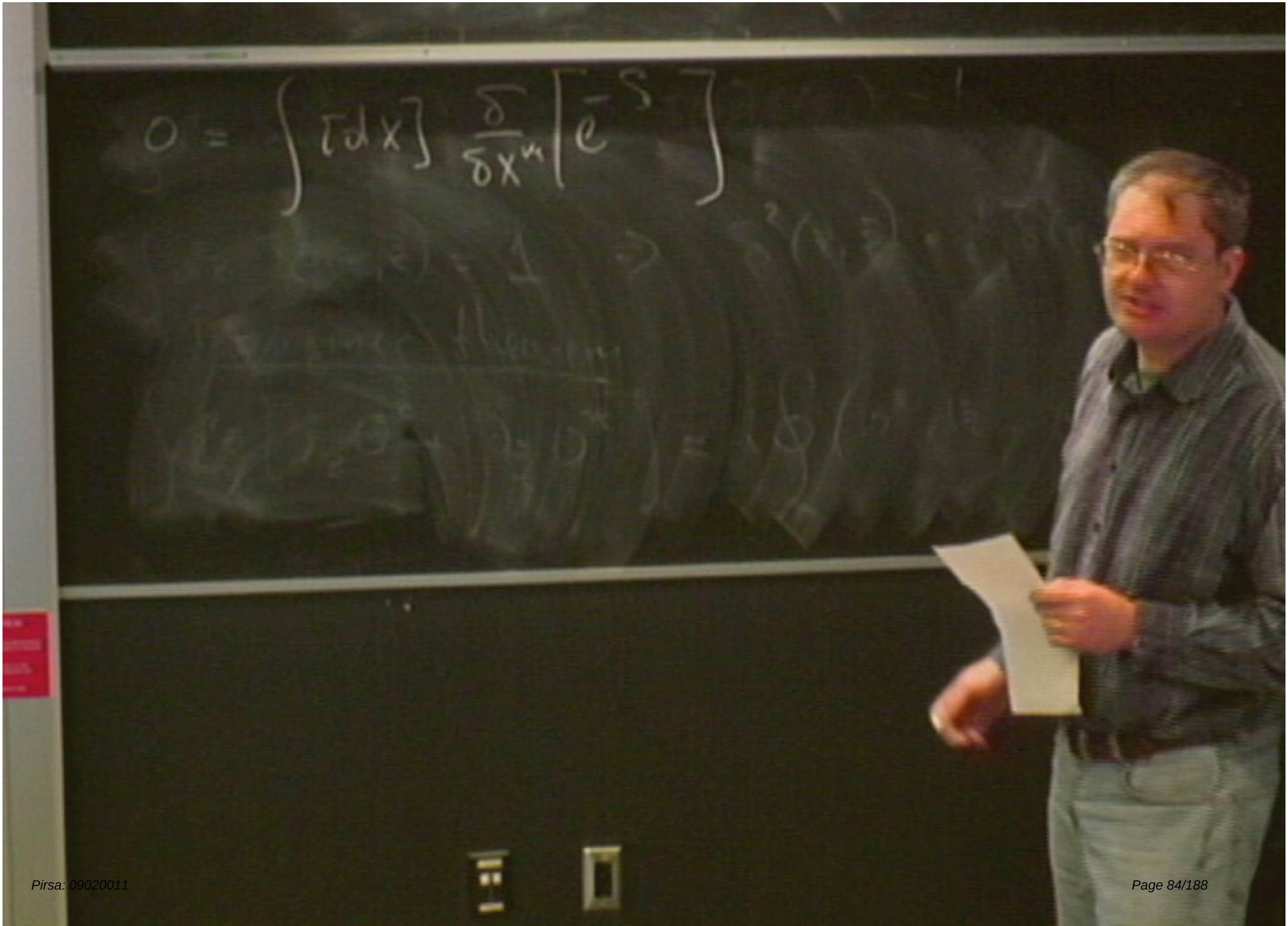

$\int dx\ \partial_x [\text{something}] = 0$
$0 = \int [dX]\ \frac{\delta}{\delta X^m}$


∫dx ∂x [something] → 0
0 = ∫[dx] δ/δx^m [something]


∫dx ∂x [something] = 0
0 = ∫[dX] δ/δX^μ [e^S something]

$$0 = \int [dX] \frac{\delta}{\delta X^{\mu}} \left[e^{-S} \text{something} \right]$$

$$\int d^2z \left(\partial_z O^z + \partial_{\bar{z}} O^{\bar{z}} \right) = i \oint \left(O^z d\bar{z} - O^{\bar{z}} dz \right)$$


$0 = \int [dx] \frac{\delta}{\delta x^\mu} [e^{-S}]$

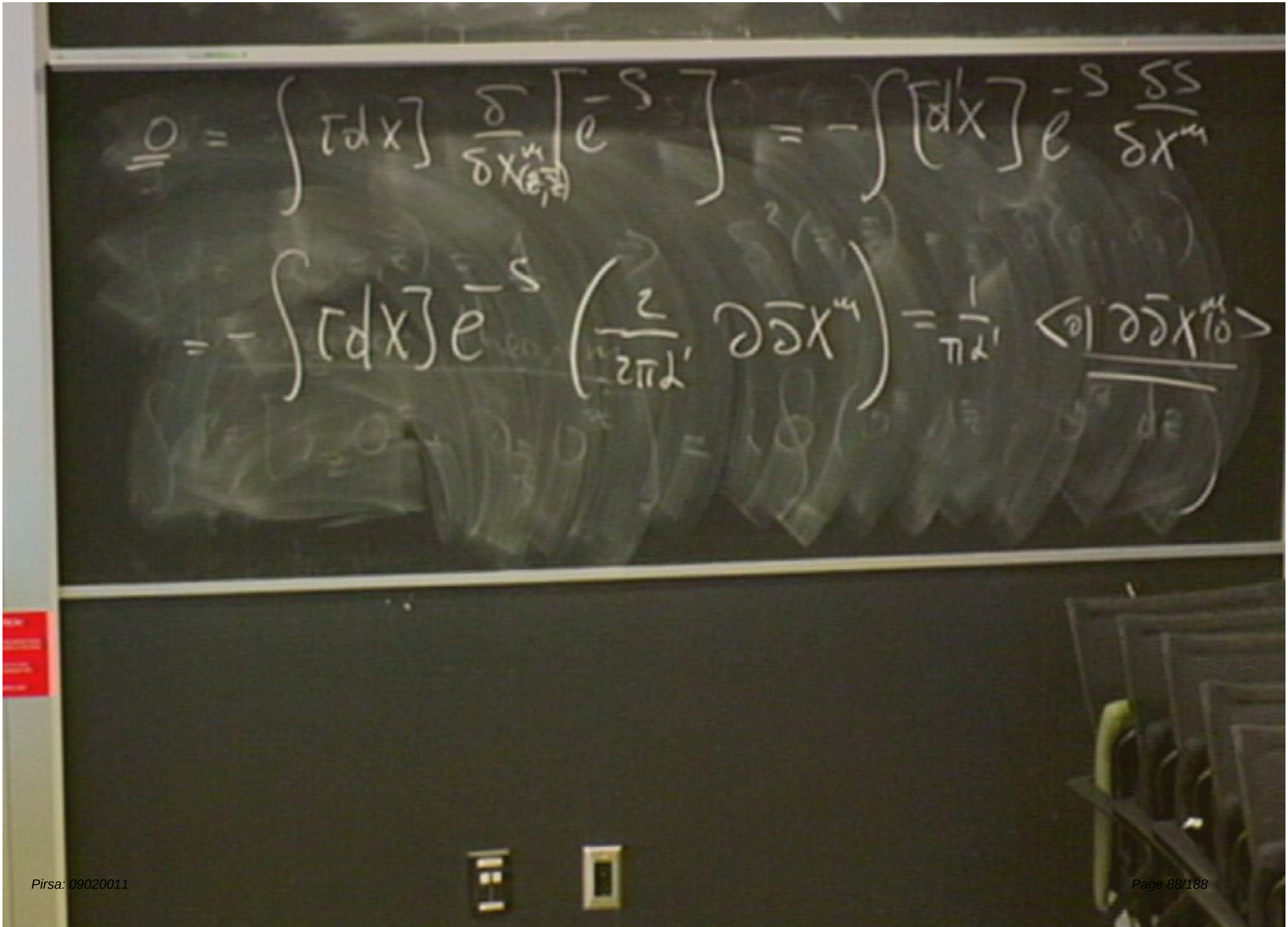
$$0 = \int [dx] \frac{\delta}{\delta x^n}\left[e^{-S}\right] = -\int [dx]\, e^{-S} \frac{\delta S}{\delta x^n}$$

$$0 = \int [dx] \frac{\delta}{\delta x^\mu}\left[e^{-S}\right] = -\int [dx]\, e^{-S} \frac{\delta S}{\delta x^\mu}$$

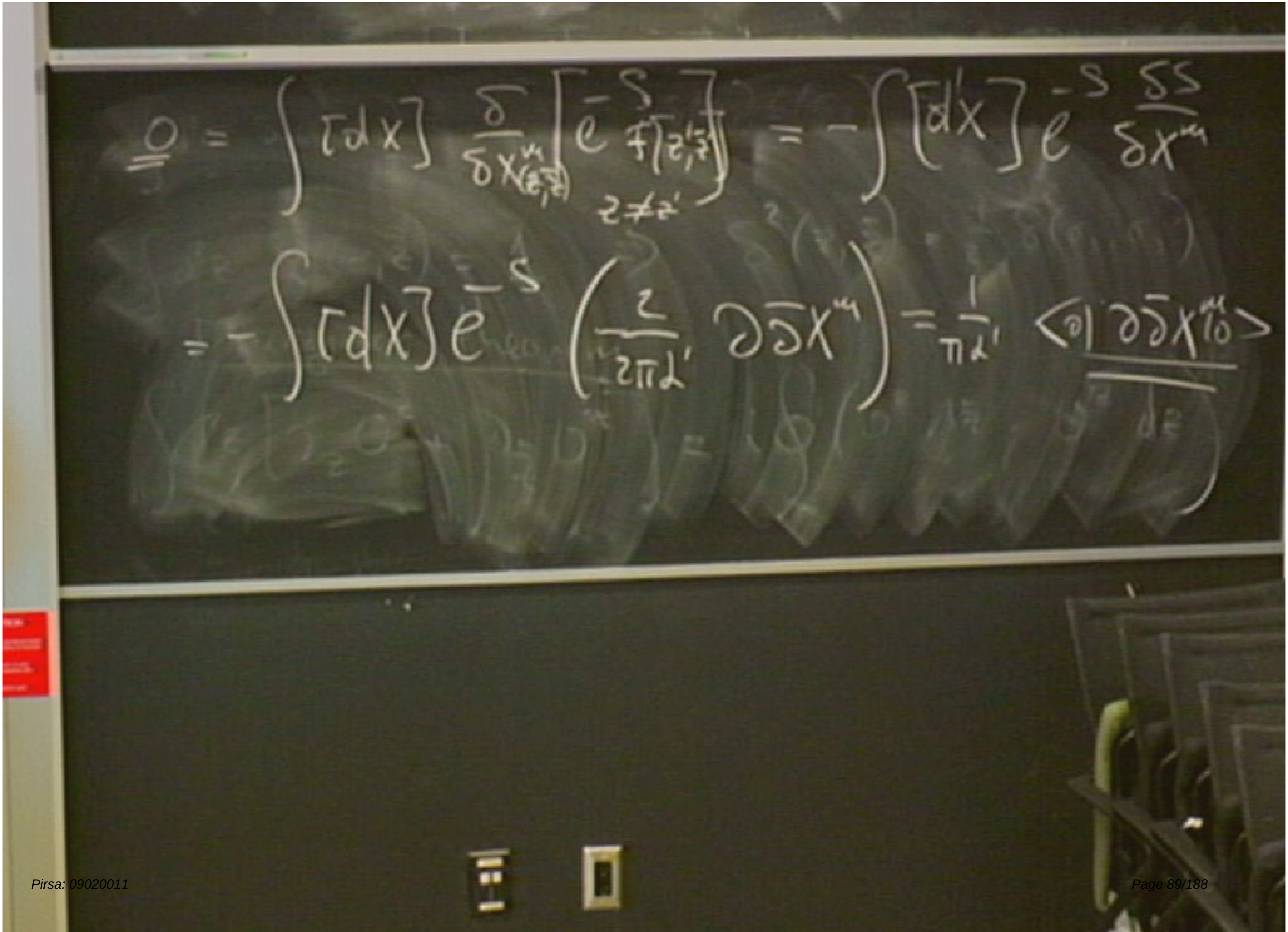
$$= -\int [dx]\, e^{-S}$$

$$0 = \int [dX] \frac{\delta}{\delta X^\mu}\left[e^{-S}\right] = -\int [dX]\, e^{-S} \frac{\delta S}{\delta X^\mu}$$

$$= -\int [dX]\, e^{-S} \left(\frac{2}{2\pi\alpha'} \partial\bar{\partial} X^\mu\right)$$



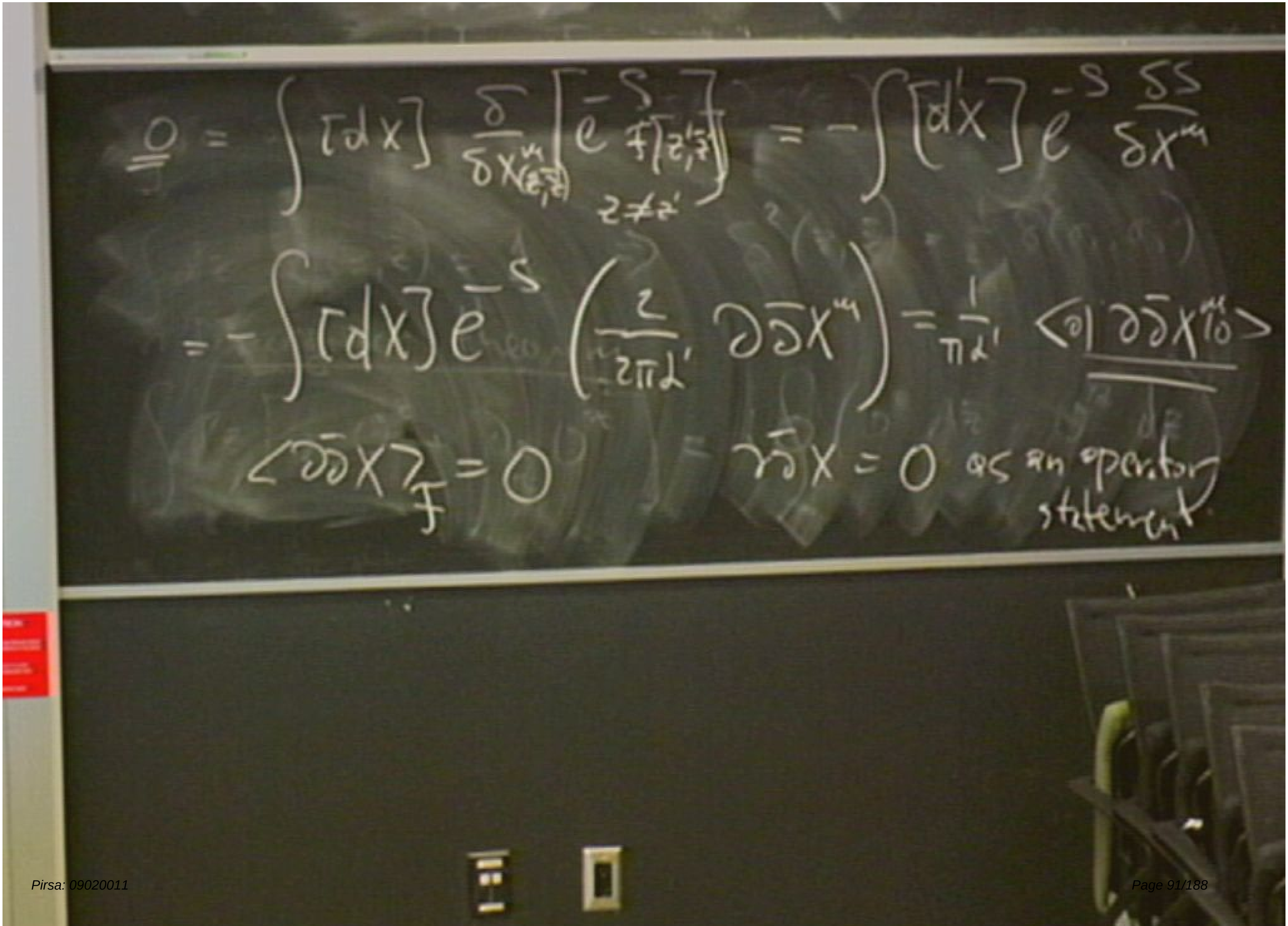
$0 = \int [dX] \frac{\delta}{\delta X^\mu(z,\bar z)} \left[e^{-S} \right] = -\int [dX]\, e^{-S} \frac{\delta S}{\delta X^\mu}$
$= -\int [dX]\, e^{-S} \left(\frac{2}{2\pi\alpha'} \partial\bar\partial X^\mu \right) = \frac{1}{\pi\alpha'} \langle 0 | \partial\bar\partial X^\mu(z) | 0 \rangle$

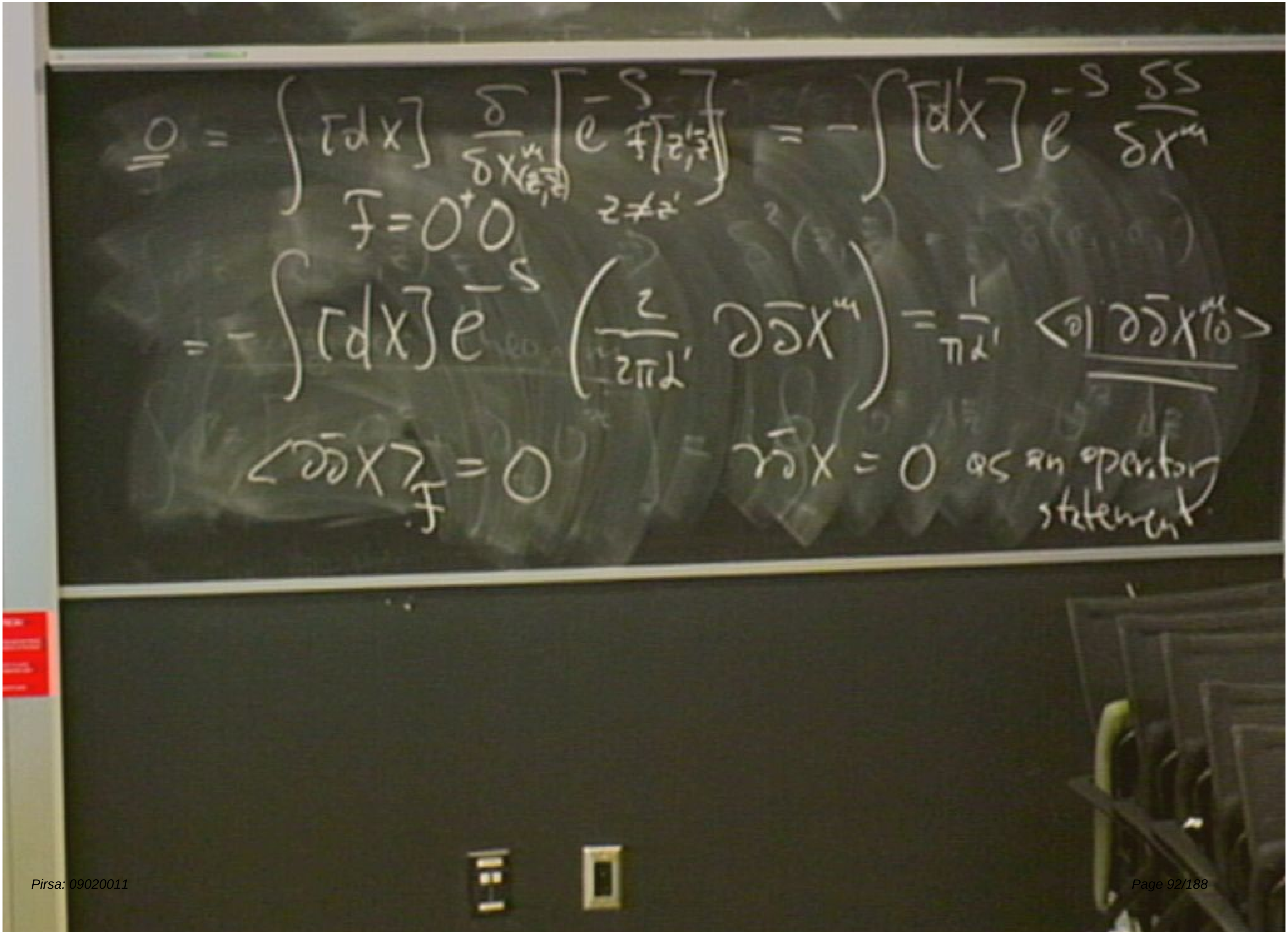

$0 = \int [dX] \frac{\delta}{\delta X^\mu(z,\bar z)} \left[e^{-S} f(z',\bar z') \right]_{z \neq z'} = -\int [dX] e^{-S} \frac{\delta S}{\delta X^\mu}$
$= -\int [dX] e^{-S} \left(\frac{2}{2\pi\alpha'} \partial\bar\partial X^\mu \right) = \frac{1}{\pi\alpha'} \langle 0 | \partial\bar\partial X^\mu | 0 \rangle$

$$0 = \int [dX] \frac{\delta}{\delta X^\mu(z,\bar z)} \left[e^{-S} f[z',\bar z'] \right]_{z \neq z'} = -\int [dX]\, e^{-S} \frac{\delta S}{\delta X^\mu}$$

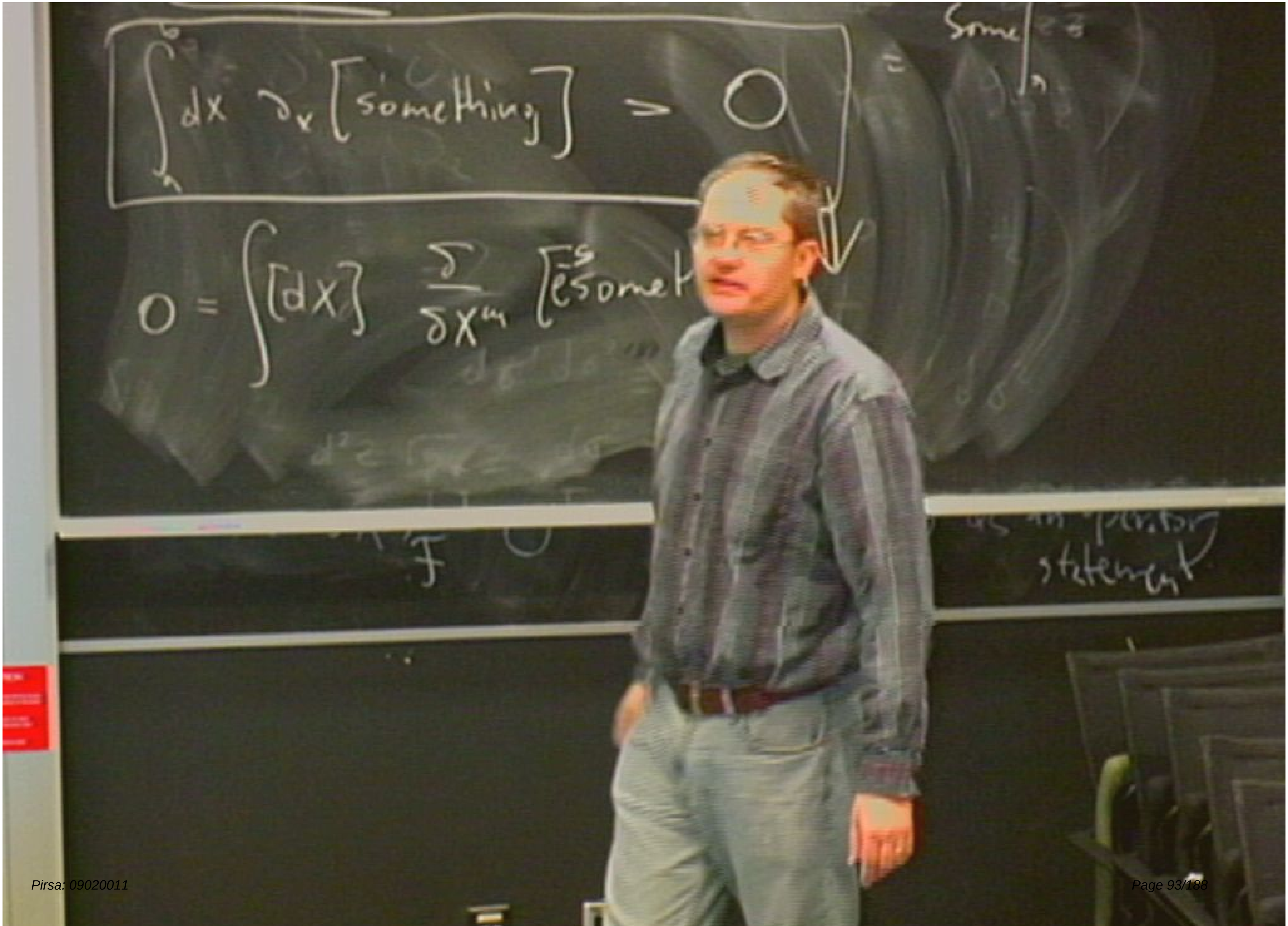
$$= -\int [dX]\, e^{-S} \left(\frac{2}{2\pi\alpha'} \partial\bar\partial X^\mu \right) = \frac{1}{\pi\alpha'} \langle 0 | \partial\bar\partial X^\mu | 0 \rangle$$

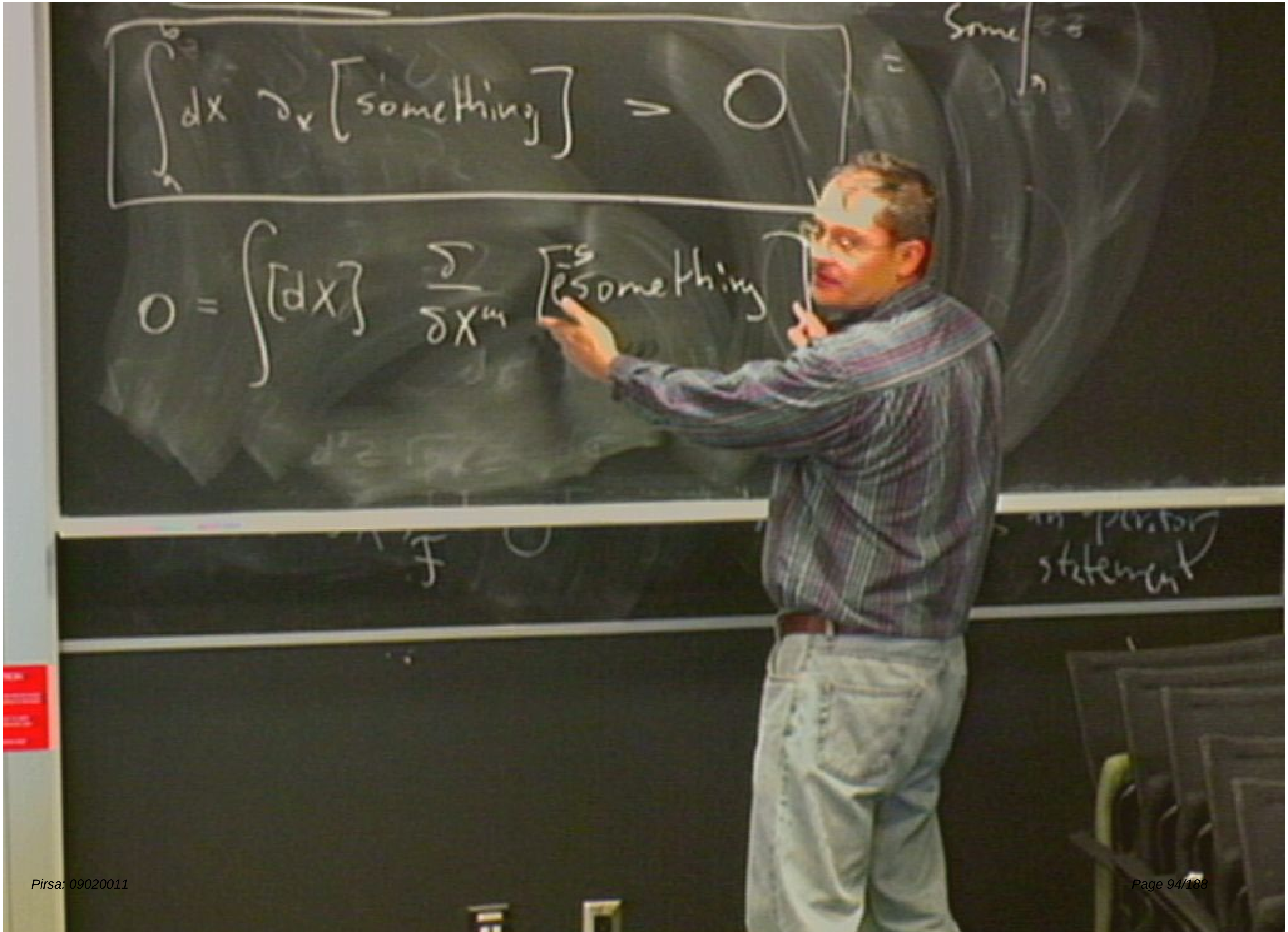
$$\langle \partial\bar\partial X \rangle_f = 0$$

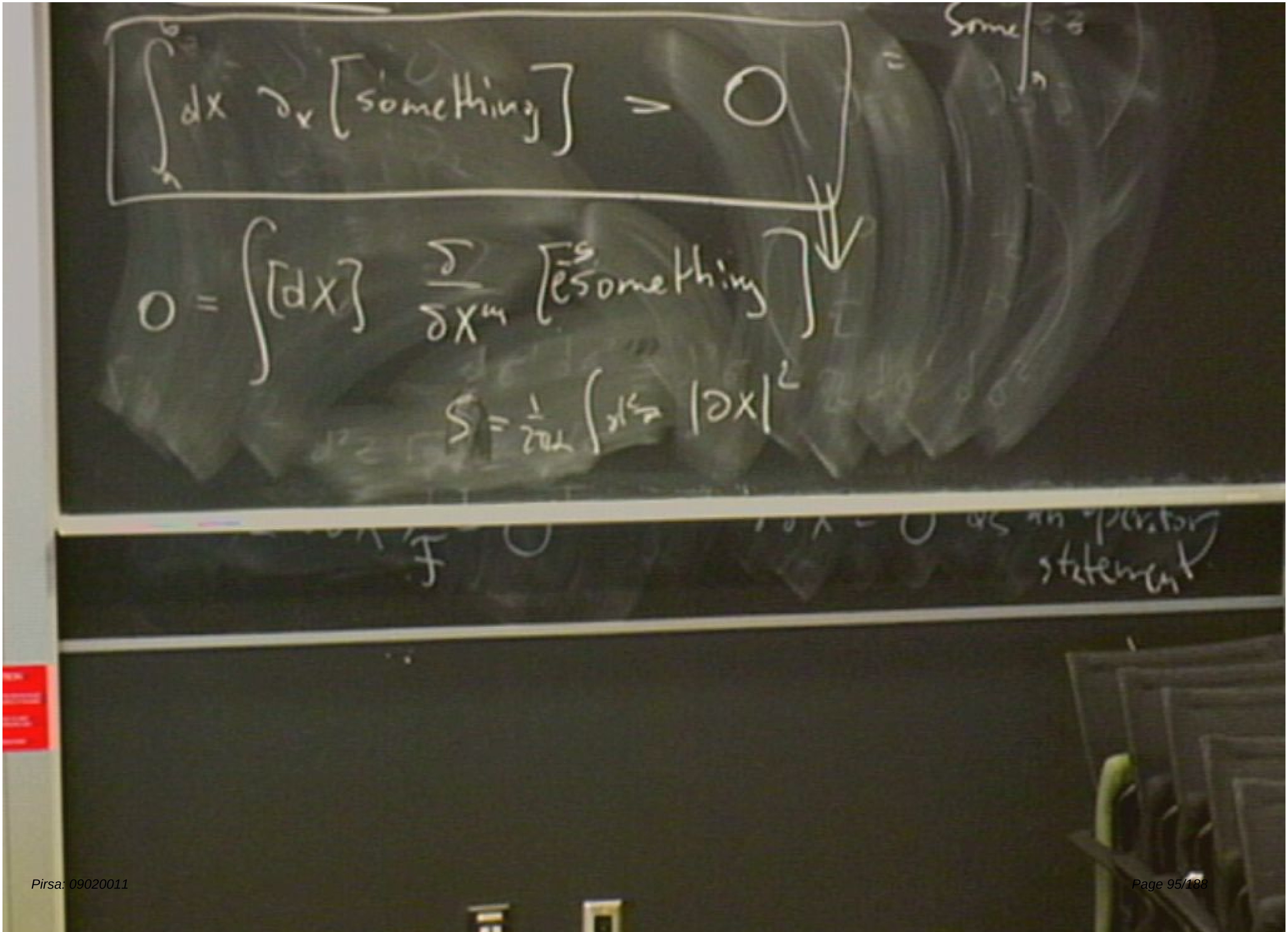

$0 = \int [dx] \frac{\delta}{\delta X^\mu(z,\bar z)} \left[e^{-S} \mathcal{F}[z',\bar z'] \right]_{z \neq z'} = -\int [dx]\, e^{-S} \frac{\delta S}{\delta X^\mu}$
$= -\int [dx]\, e^{-S} \left(\frac{2}{2\pi\alpha'} \partial\bar\partial X^\mu \right) = \frac{1}{\pi\alpha'} \langle 0 | \partial\bar\partial X^\mu | 0 \rangle$
$\langle \partial\bar\partial X \rangle_{\mathcal{F}} = 0$
$\partial\bar\partial X = 0$ as an operator statement

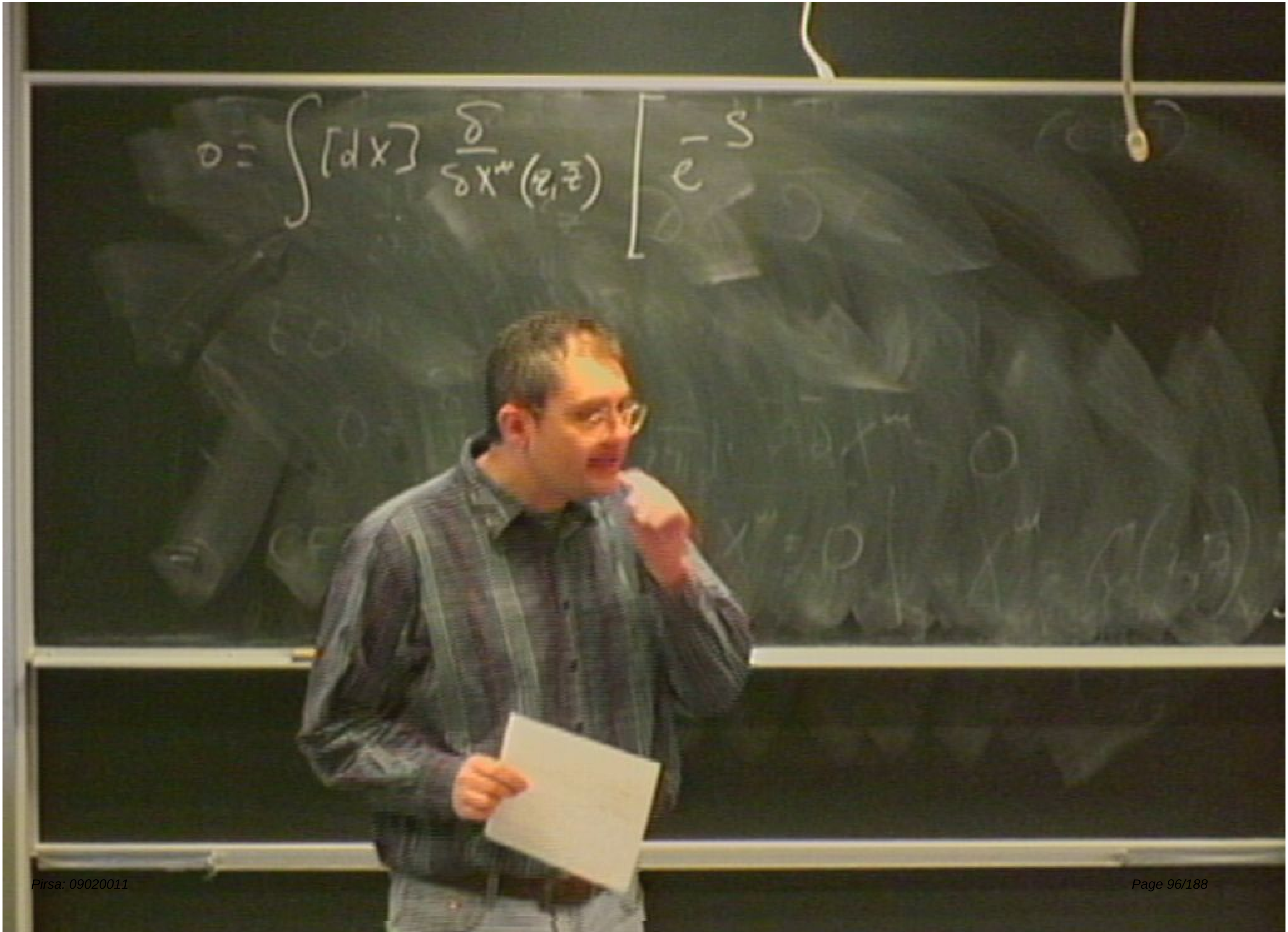


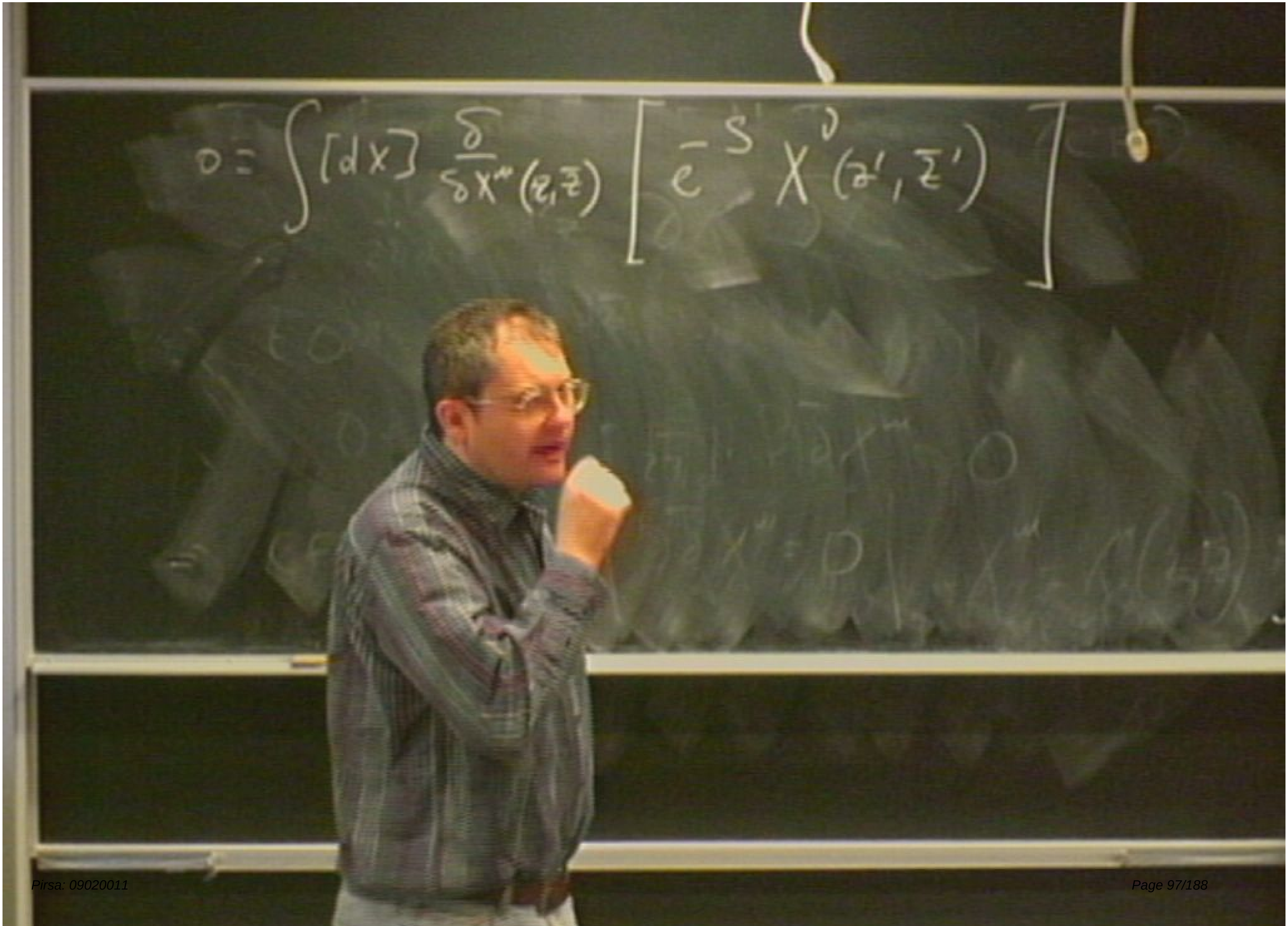
$0 = \int [dx] \frac{\delta}{\delta X^\mu(z,\bar z)} \left[e^{-S} F[z',\bar z'] \right]_{z\neq z'} = -\int [dx]\, e^{-S} \frac{\delta S}{\delta X^\mu}$
$F = O^\dagger O$
$= -\int [dx]\, e^{-S} \left(\frac{2}{2\pi\alpha'} \partial\bar\partial X^\mu \right) = \frac{1}{\pi\alpha'} \langle 0 | \partial\bar\partial X^\mu | 0 \rangle$
$\langle \partial\bar\partial X \rangle_F = 0$
$\partial\bar\partial X = 0$ as an operator statement

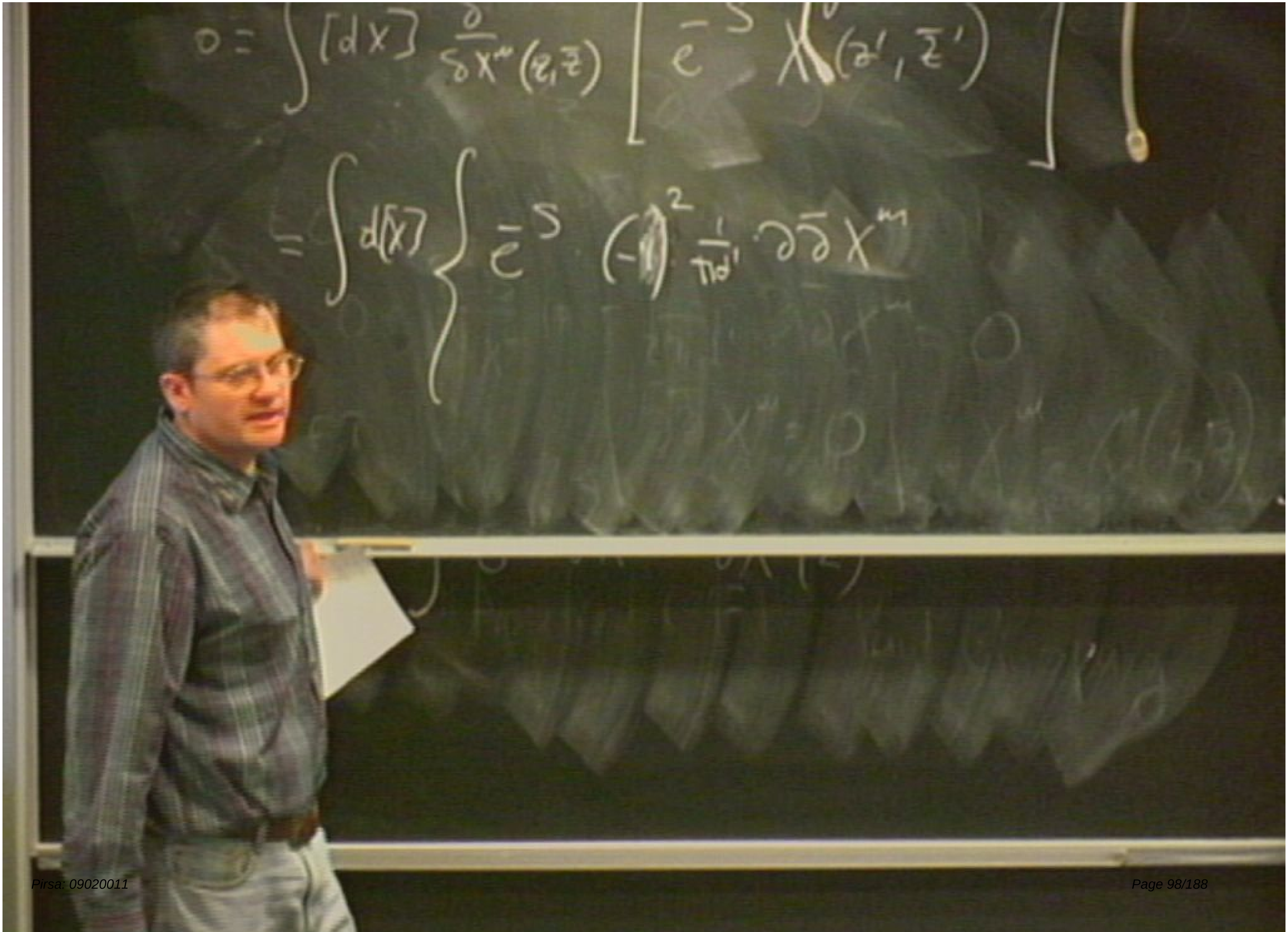

something
statement

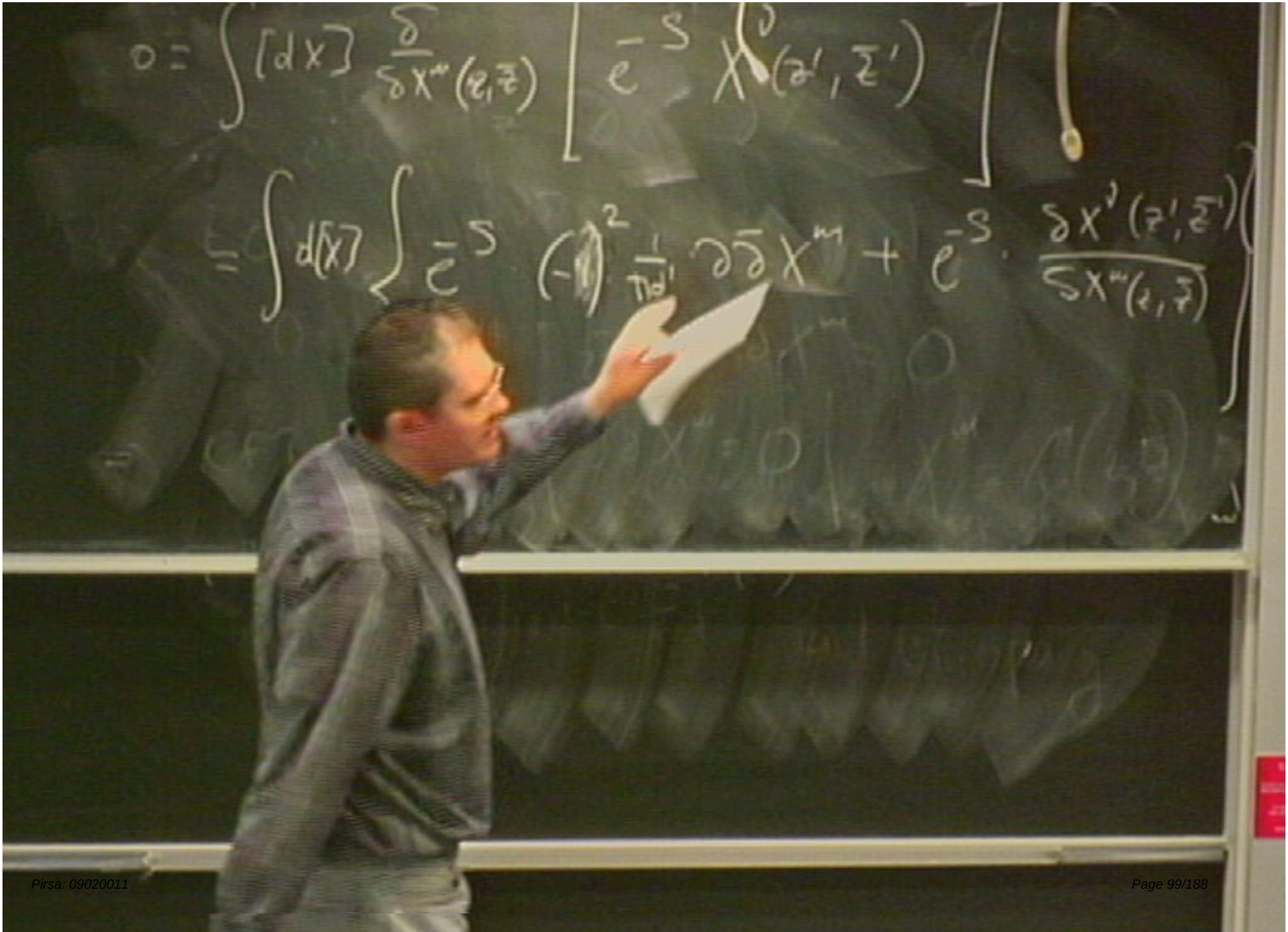

∫dx ∂ₓ[something] = 0
0 = ∫[dX] δ/δX^m [something]
statement

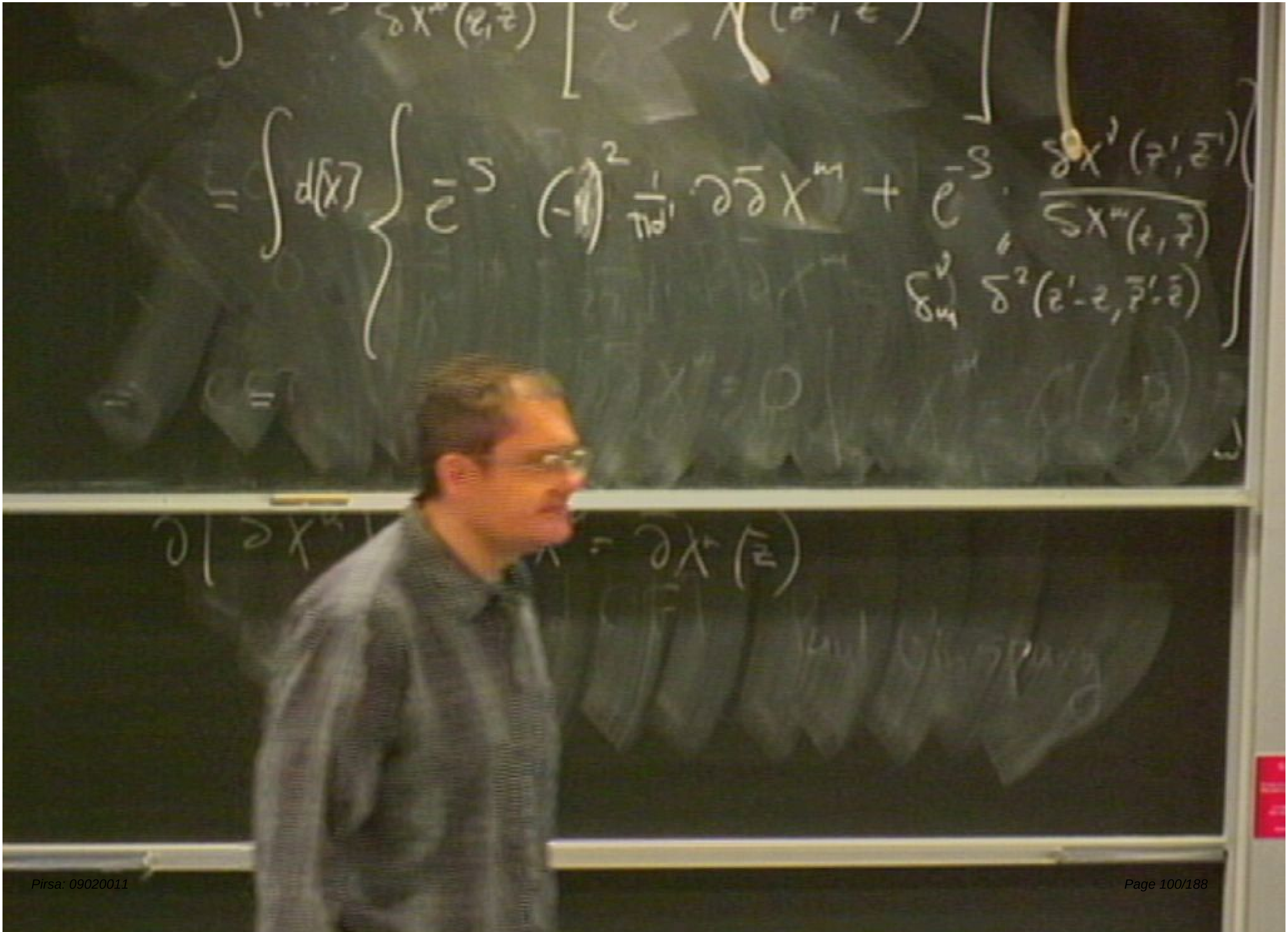

∫ dx ∂x [something] = 0
0 = ∫[dX] δ/δX^μ [e^-S something]
S = 1/2πα' ∫ d²σ |∂X|²
Some
statement

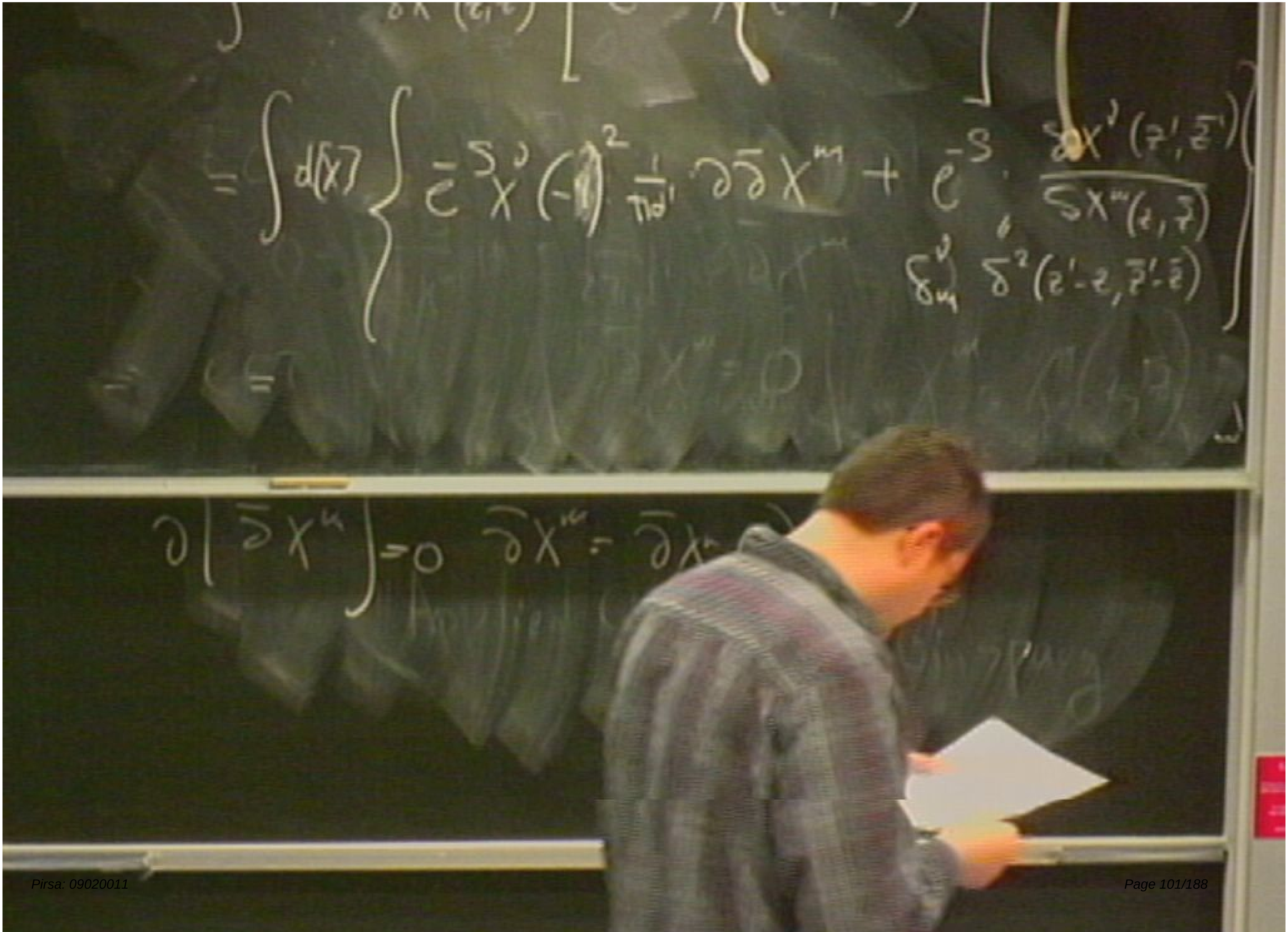

$0 = \int [dX] \frac{\delta}{\delta X^\mu(z,\bar{z})} \Big[e^{-S}$

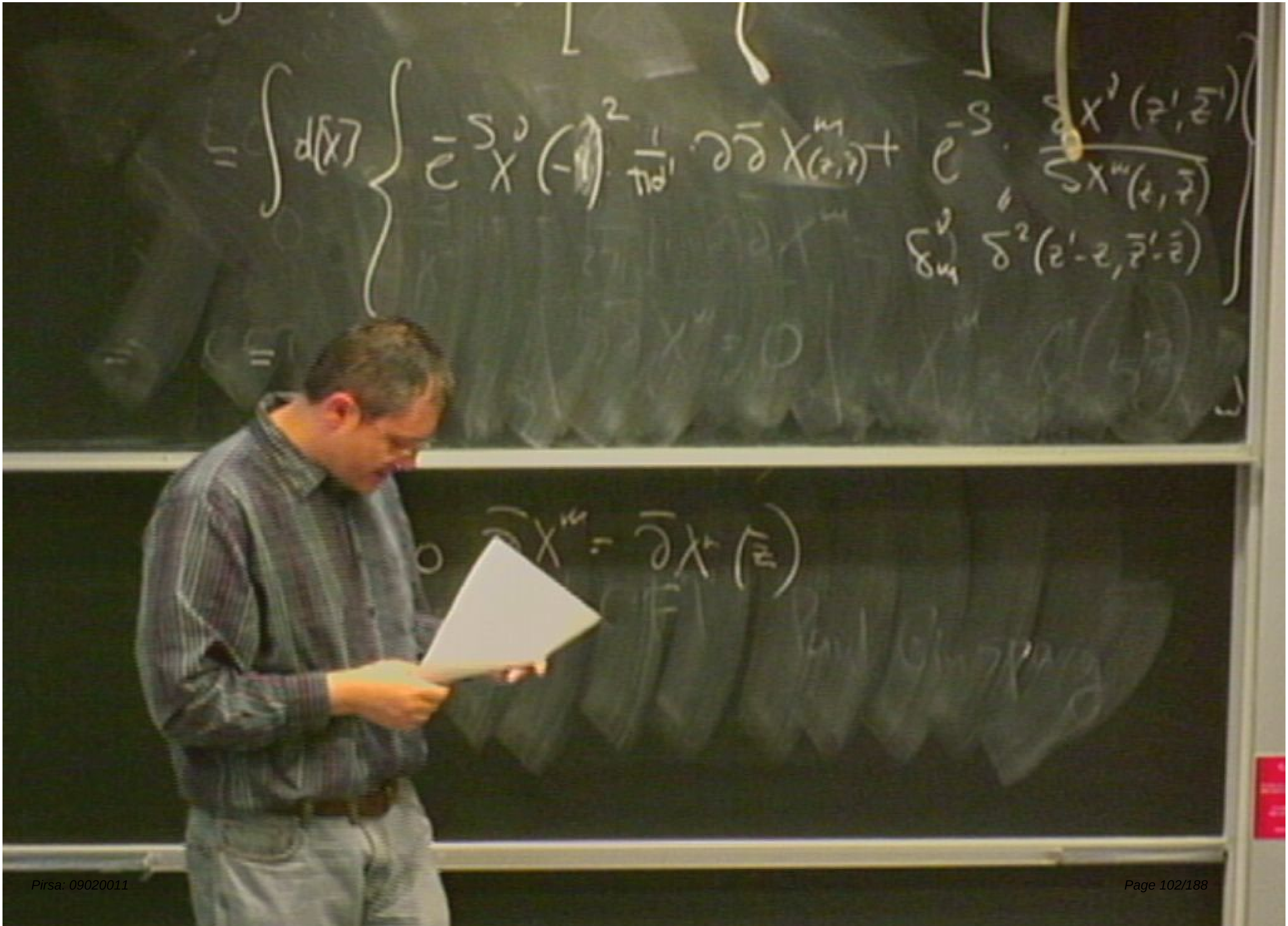

$0 = \int [dX] \frac{\delta}{\delta X^\mu(z,\bar{z})} \left[e^{-S} X^\nu(z',\bar{z}') \right]$

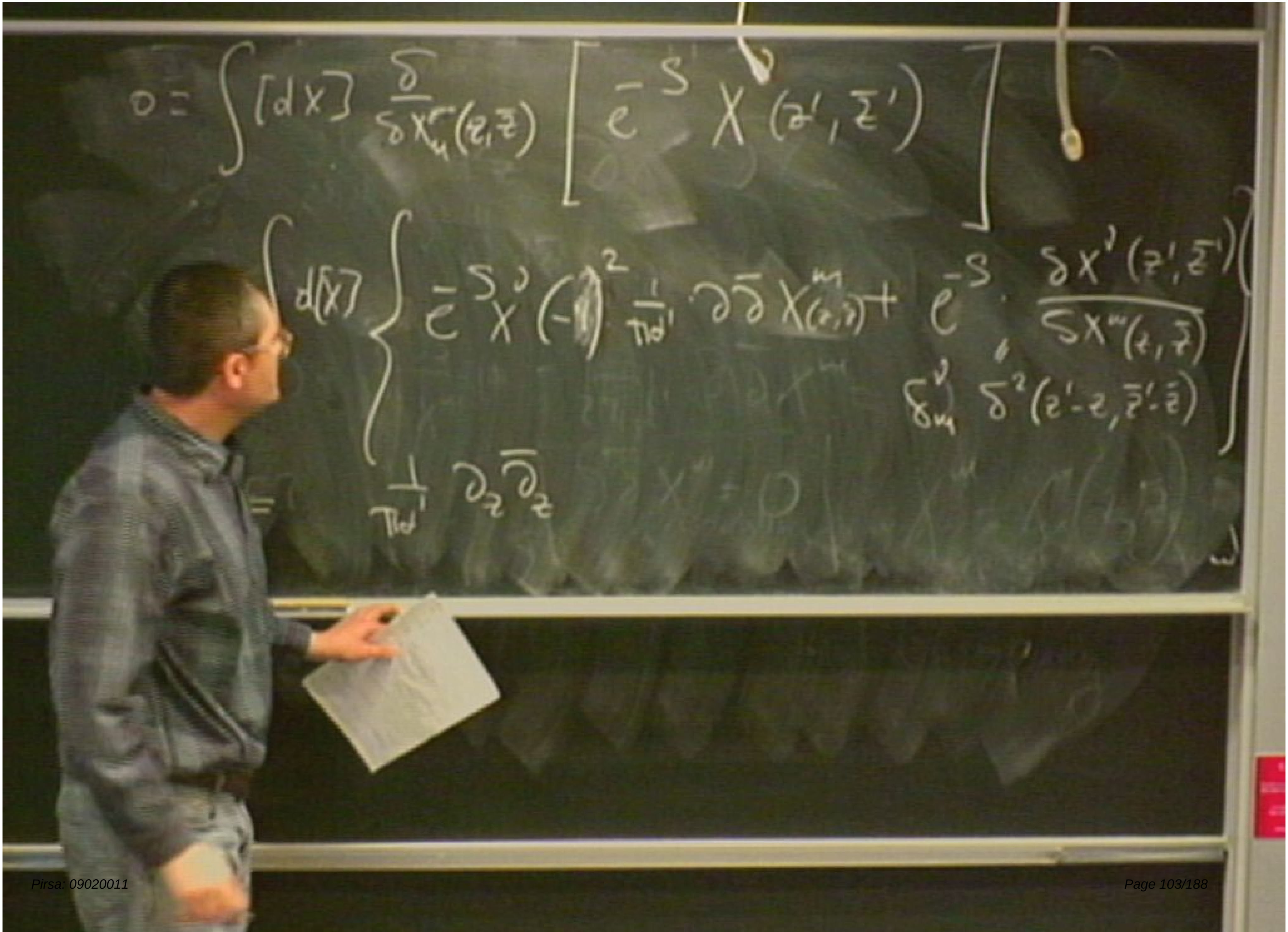

0 = ∫ [dX] δ/δX^μ(z,z̄) [e^{-S} X^ν(z′,z̄′)]
= ∫ d[X] { e^{-S} (-1)^2 1/πα′ ∂∂̄X^μ

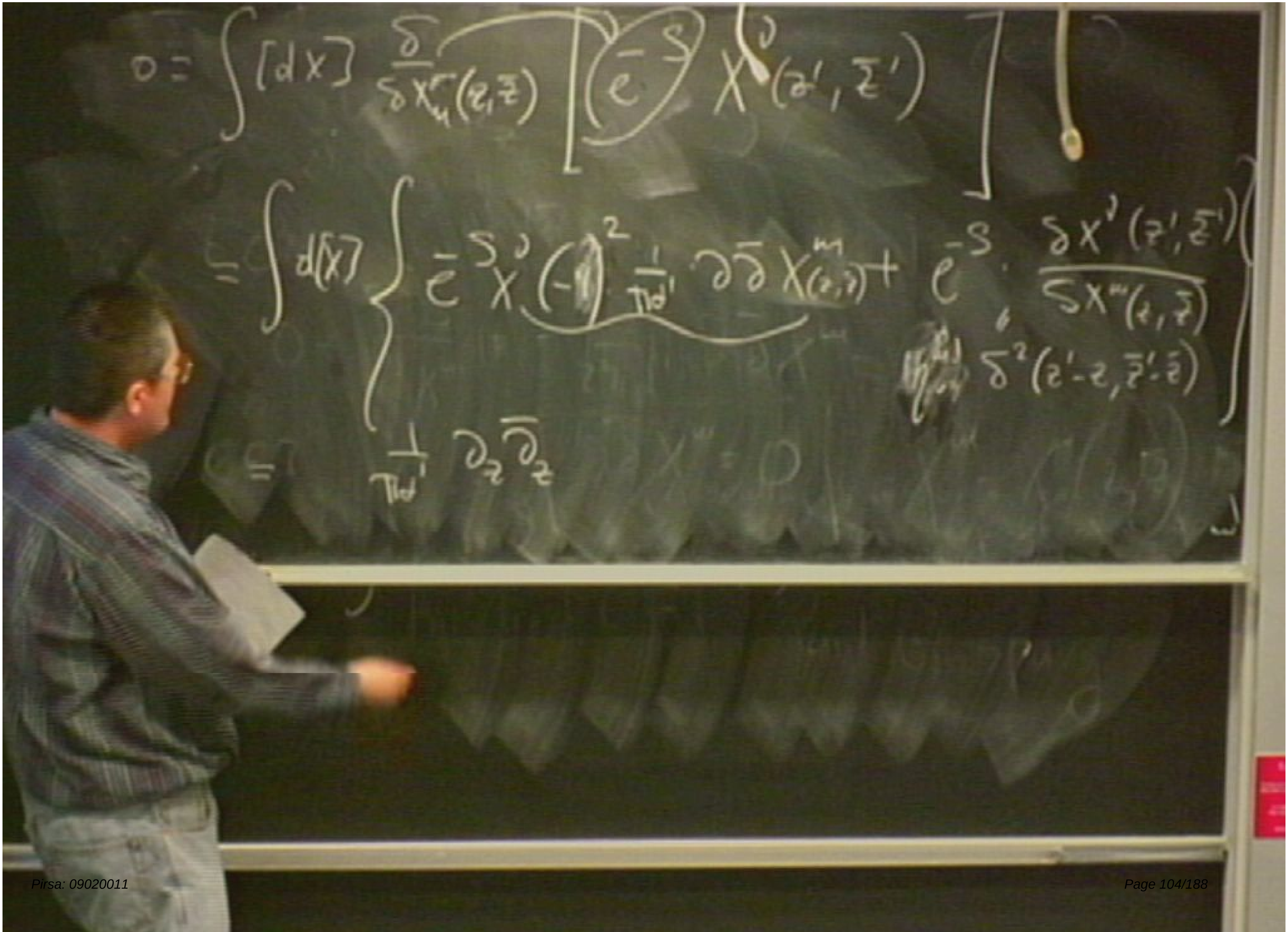

$0 = \int [dX] \frac{\delta}{\delta X^\mu(z,\bar z)} \left[e^{-S} X^\nu(z',\bar z') \right]$
$= \int d[X] \left\{ e^{-S} (-)^2 \frac{1}{\pi\alpha'} \partial\bar\partial X^\mu + e^{-S} \cdot \frac{\delta X^\nu(z',\bar z')}{\delta X^\mu(z,\bar z)} \right\}$








$0 = \int [dX] \frac{\delta}{\delta X^m(z,\bar z)} \left[e^{-S} X^{\nu}(z',\bar z') \right]$
$\int d[X] \left\{ e^{-S} X^{\nu} (-1)^2 \frac{1}{\pi\alpha'} \partial\bar\partial X^m(z,\bar z) + e^{-S} \frac{\delta X^{\nu}(z',\bar z')}{\delta X^m(z,\bar z)} \right\}$
$\delta^{\nu}_m \, \delta^2(z'-z, \bar z'-\bar z)$
$\frac{1}{\pi\alpha'} \partial_z \bar\partial_{\bar z}$


0 = ∫[dX] δ/δX^μ(z,z̄) [e^{-S} X^ν(z',z̄')]
= ∫d[X] { e^{-S} X^ν (-1)^2 1/πα' ∂∂̄X^μ(z,z̄) + e^{-S} δX^ν(z',z̄')/δX^μ(z,z̄)
η^{μν} δ^2(z'-z, z̄'-z̄) }
1/πα' ∂_z ∂̄_z

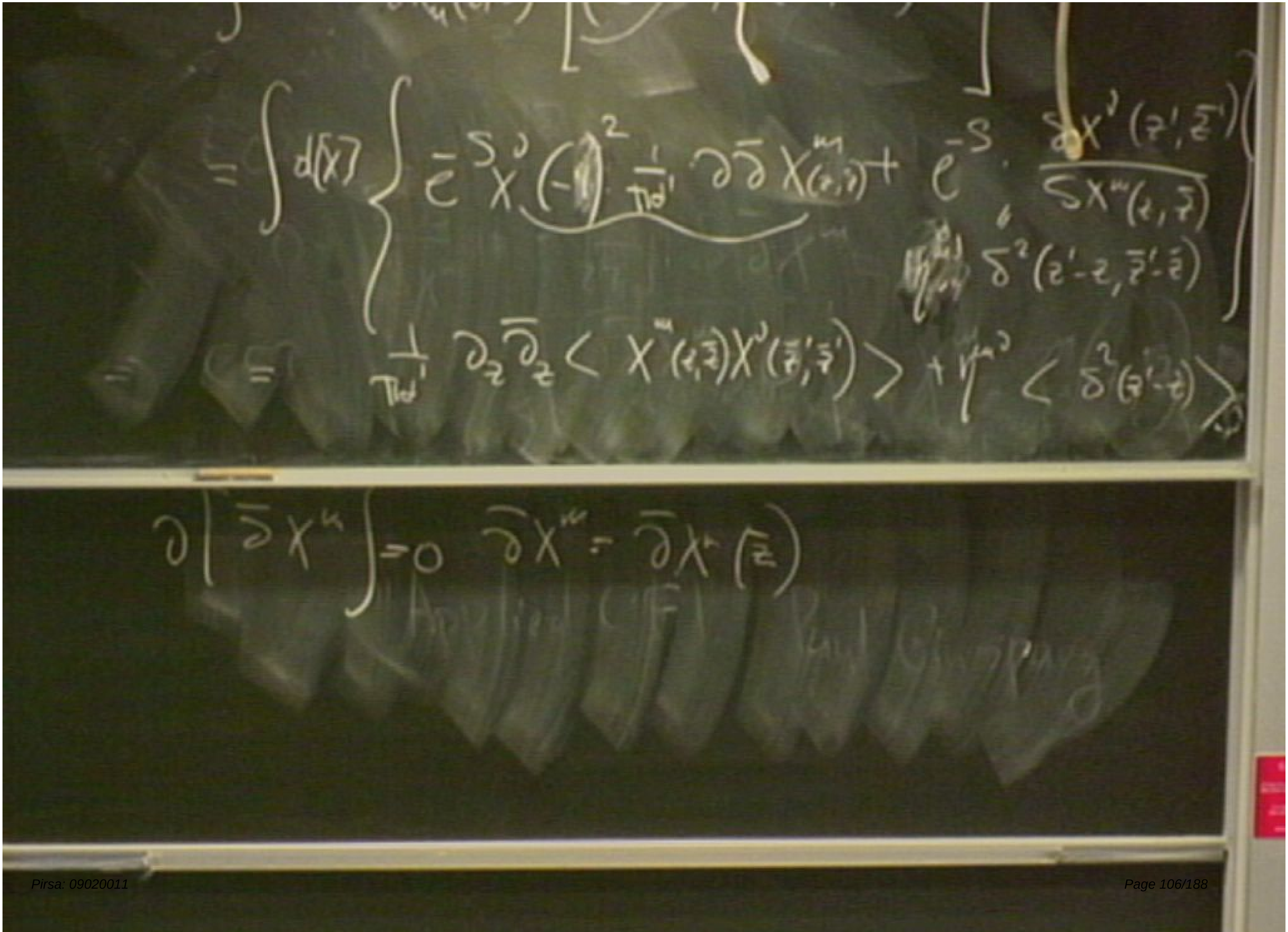
$$\int [dX] \; \frac{\delta}{\delta X_\mu(z,\bar z)} \left[e^{-S} X^\nu(z',\bar z') \right]$$

$$= \int d[X] \left\{ e^{-S} X^\nu (-1)^2 \frac{1}{\pi\alpha'} \partial\bar\partial X^\mu(z,\bar z) + e^{-S} \frac{\delta X^\nu(z',\bar z')}{\delta X^\mu(z,\bar z)} \right\}$$

$$\frac{\delta X^\nu(z',\bar z')}{\delta X^\mu(z,\bar z)} = \eta^{\mu\nu} \delta^2(z'-z, \bar z'-\bar z)$$

$$= \frac{1}{\pi\alpha'} \partial_z \bar\partial_{\bar z} \langle X^\mu(z,\bar z) X^\nu(z',\bar z') \rangle$$

$$\partial \left(\bar\partial X^\mu \right) = 0 \qquad \bar\partial X^\mu = \bar\partial X^\mu(\bar z)$$


= ∫ d[X] { e^{-S} X^ν (−1)^2 (1/πα') ∂∂̄ X^μ(z,z̄) + e^{-S} δX^ν(z',z̄') / δX^μ(z,z̄) }
η^{μν} δ^2(z'−z, z̄'−z̄)
= (1/πα') ∂_z ∂̄_z̄ < X^μ(z,z̄) X^ν(z̄',z') > + η^{μν} < δ^2(z'−z) >
∂(∂̄X^μ) = 0   ∂̄X^μ = ∂̄X^μ(z̄)
Applied CFT   Paul Ginsparg

$$= \frac{1}{\pi\alpha'}\partial_z\bar{\partial}_{\bar z}\langle X^\mu(z,\bar z)X^\nu(z',\bar z')\rangle + \eta^{\mu\nu}\langle\delta^2(z'-z)\rangle$$

$$-\frac{1}{\pi\alpha'}\partial_z\partial_{\bar z}X^\mu(z,\bar z)X^\nu(z',\bar z') = -\eta^{\mu\nu}\delta^2(z-z')$$

$$= \frac{1}{\pi\alpha'}\partial_z\bar{\partial}_{\bar z}\langle X^\mu(z,\bar z)X^\nu(z',\bar z')\rangle + \eta^{\mu\nu}\langle\delta^2(z'-z)\rangle$$

$$-\frac{1}{\pi\alpha'}\partial_z\partial_{\bar z}X^\mu(z,\bar z)X^\nu(z',\bar z') = -\eta^{\mu\nu}\delta^2(z-z')$$

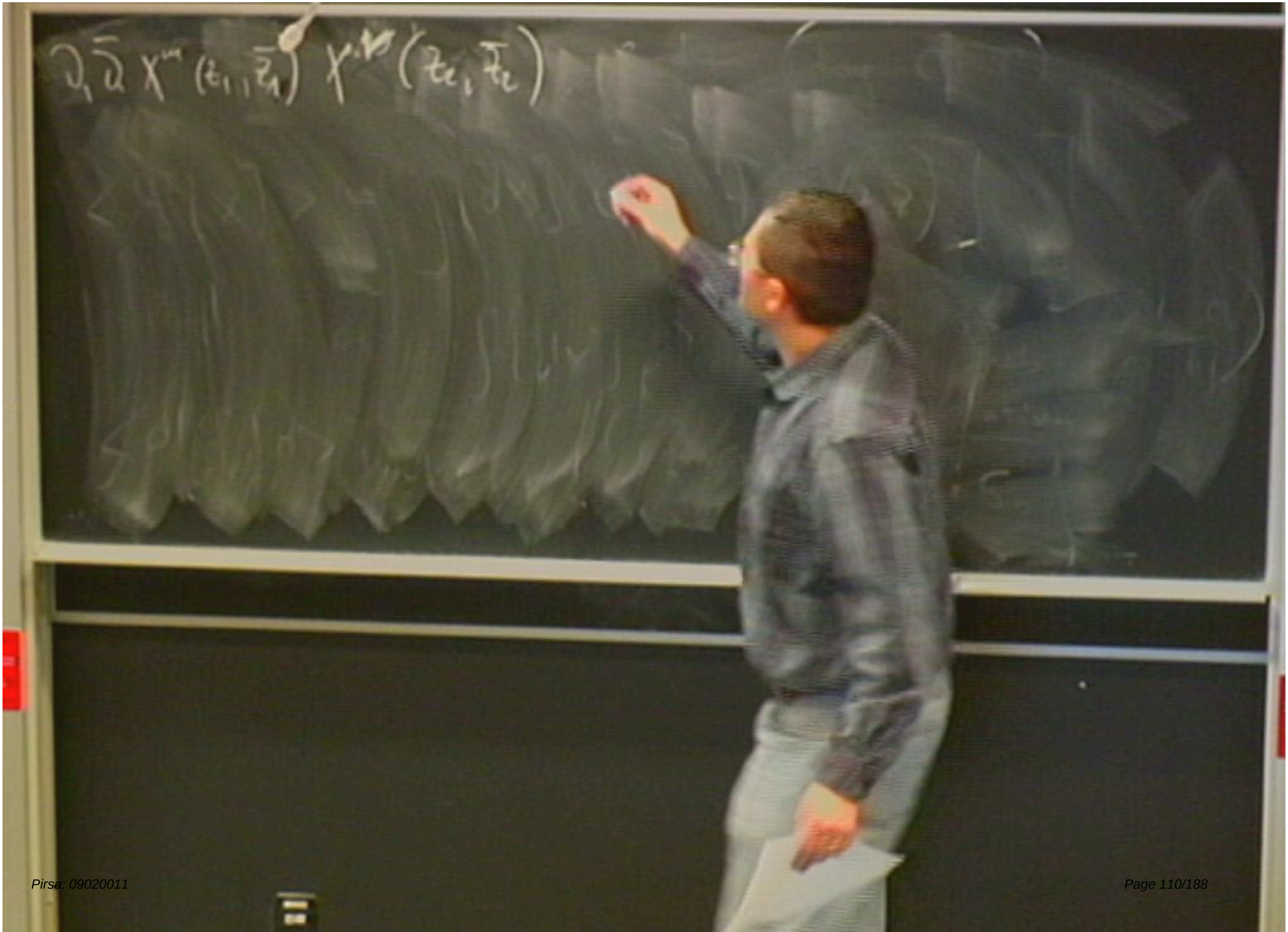
Normal ordering

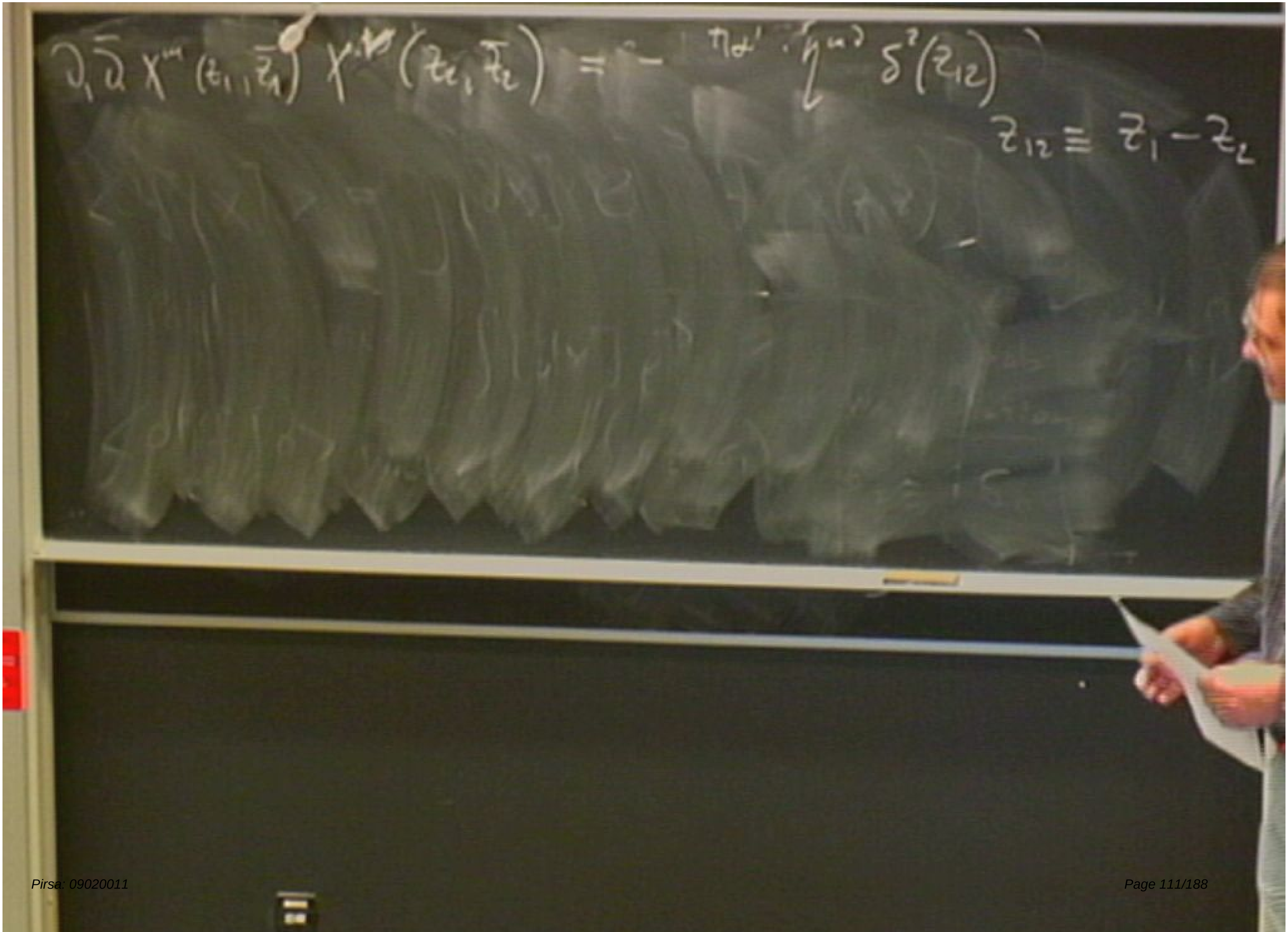
$$= \frac{1}{\pi\alpha'} \partial_z \bar{\partial}_{\bar z} \langle X^\mu(z,\bar z) X^\nu(z',\bar z') \rangle + \eta^{\mu\nu} \langle \delta^2(z'-z) \rangle$$

Normal ordering

$$\partial\bar{\partial} X^\mu \Big|_{\text{classically}} = 0$$

$$\partial\bar{\partial} X^\mu \Big|_{Q} \neq 0$$

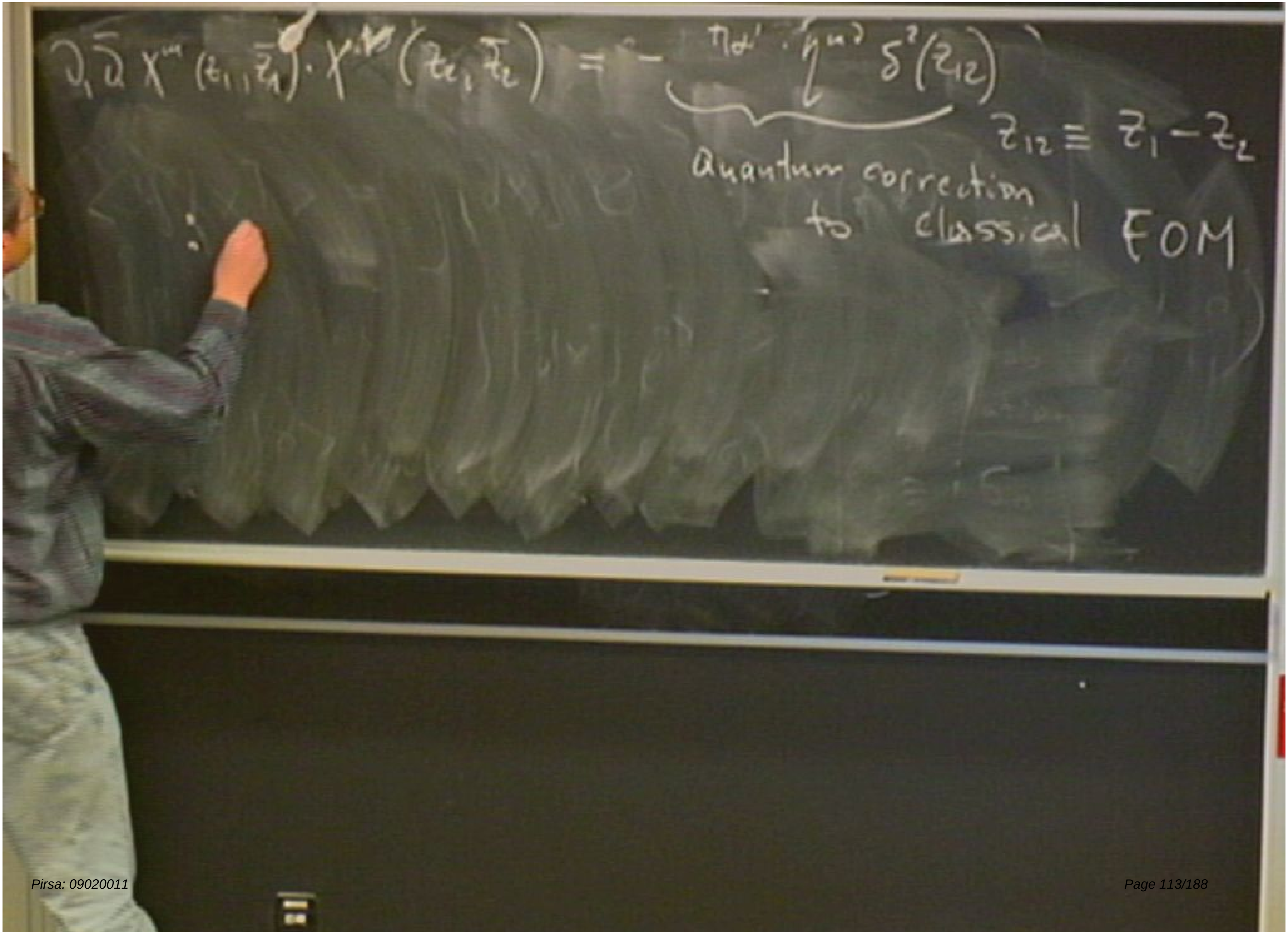

$\partial_1 \bar{\partial}_2 X^{\mu}(z_1, \bar{z}_1)\, X^{\nu}(z_2, \bar{z}_2)$


$\partial_1 \bar{\partial}_1 X^{\mu}(z_1, \bar{z}_1) X^{\nu}(z_2, \bar{z}_2) = -\pi\alpha' \eta^{\mu\nu} \delta^2(z_{12})$
$z_{12} \equiv z_1 - z_2$

$$\partial_1 \bar{\partial}_1 X^{\mu}(z_1, \bar{z}_1)\, X^{\nu}(z_2, \bar{z}_2) = \underbrace{-\pi\alpha' \, \eta^{\mu\nu} \, \delta^2(z_{12})}$$

$z_{12} \equiv z_1 - z_2$

Quantum correction to classical EOM


$\partial_1 \bar{\partial} X^{\mu}(z_1, \bar{z}_1) \cdot X^{\nu}(z_2, \bar{z}_2) = - \eta^{\mu\nu} \delta^2(z_{12})$
$z_{12} \equiv z_1 - z_2$
Quantum correction
to classical EOM

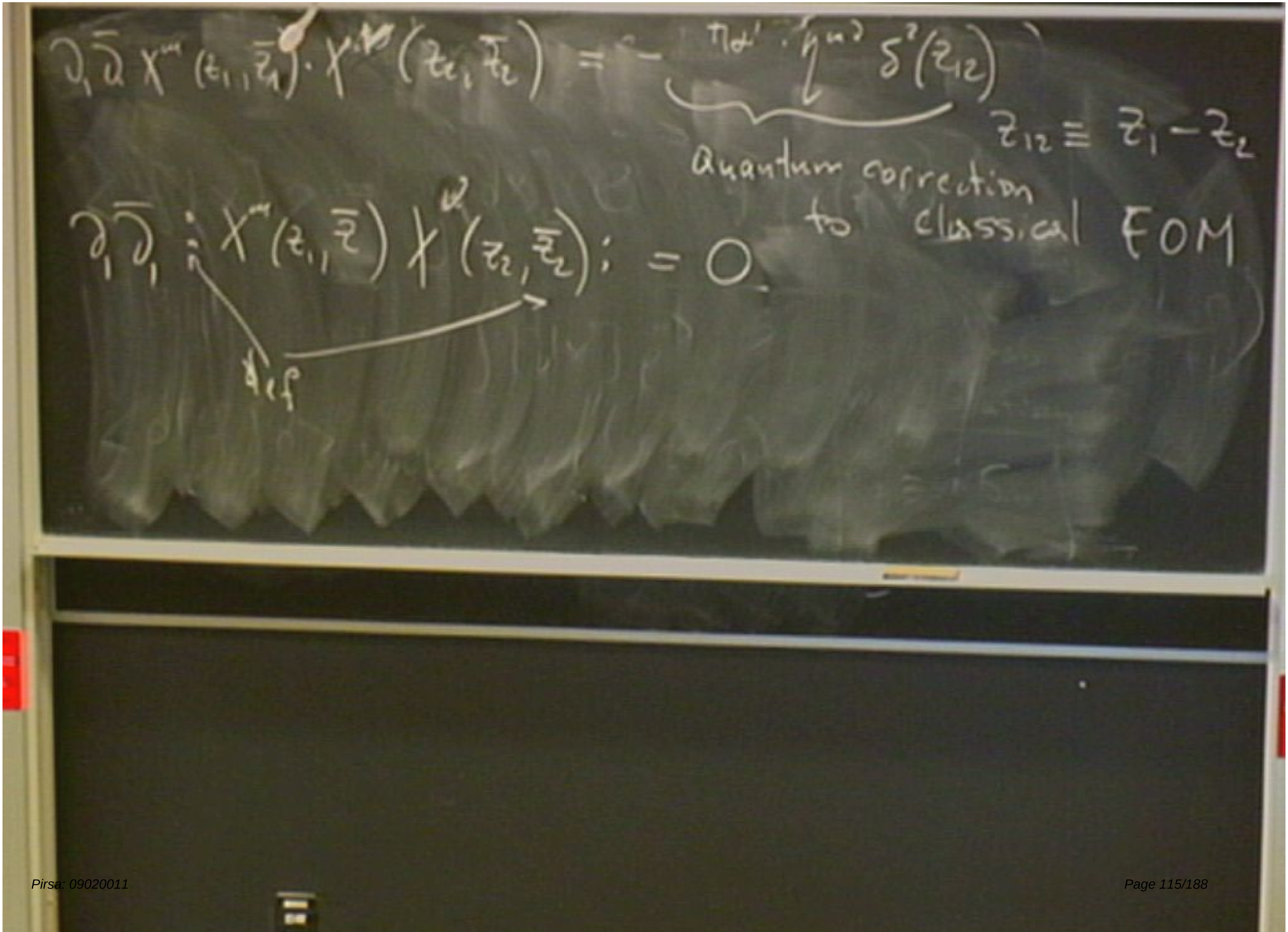
$$\partial_1 \bar{\partial}_1 X^\mu(z_1, \bar{z}_1) \cdot X^\nu(z_2, \bar{z}_2) = -\pi\alpha' \, \eta^{\mu\nu} \, \delta^2(z_{12})$$

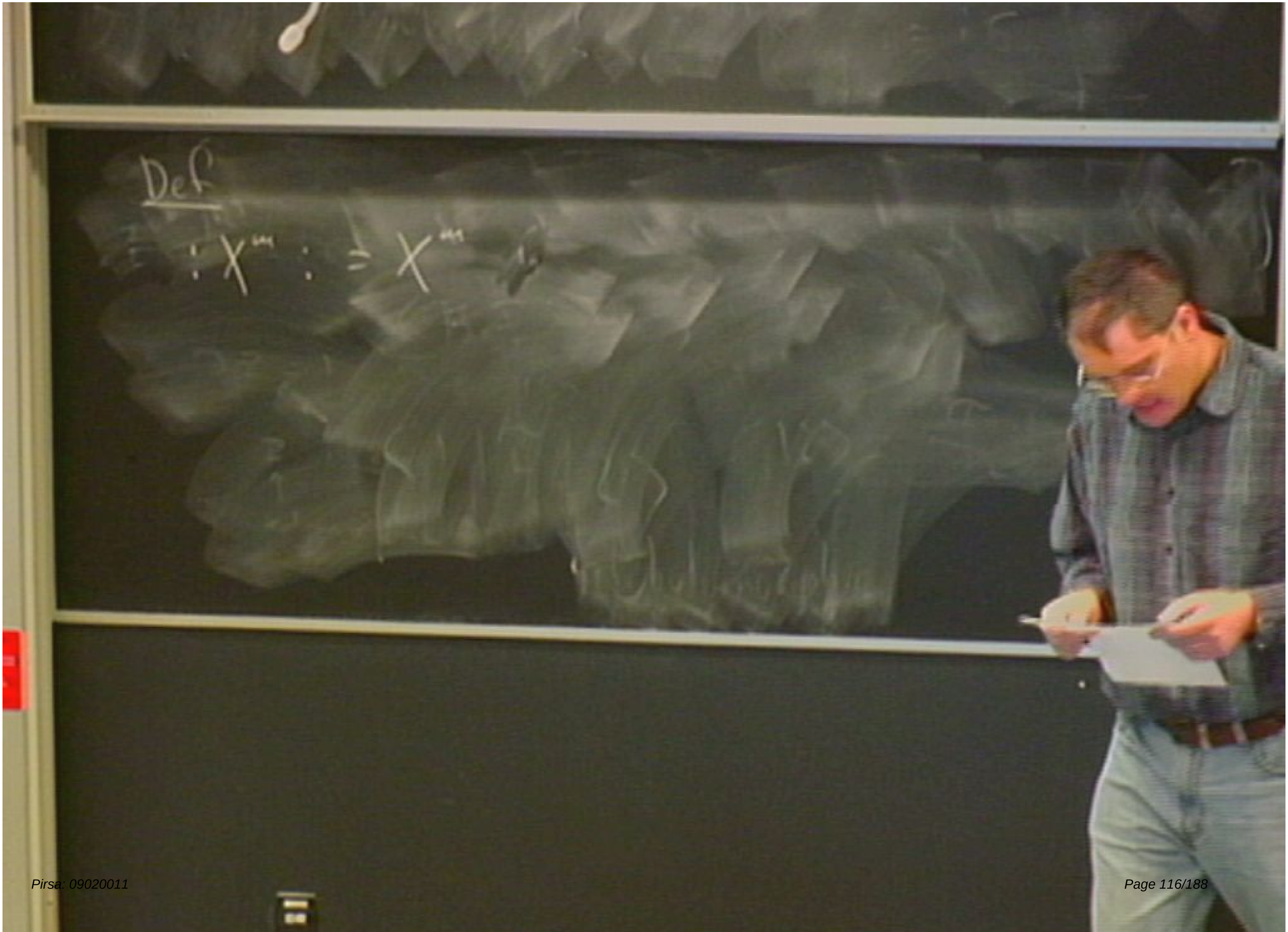
$z_{12} \equiv z_1 - z_2$

Quantum correction to classical EOM

$$:X^\mu(z_1, \bar{z})\, X^\nu(z_2, \bar{z}_2):$$

def


$\partial_1 \bar\partial X^\mu(z_1,\bar z_1) \cdot X^\nu(z_2,\bar z_2) = -\pi\alpha' \eta^{\mu\nu} \delta^2(z_{12})$
$z_{12} \equiv z_1 - z_2$
Quantum correction to classical EOM
$\partial_1 \bar\partial_1 :X^\mu(z_1,\bar z_1) X^\nu(z_2,\bar z_2): = 0$
def


Def
:X^m: = X^m


Def
$:X^{\mu}: = X^{\mu}$
$:X^{\mu} X^{\nu}: = X^{\mu} X^{\nu}$

Def

$$:X^\mu: = X^\mu$$

$$:X^\mu X^\nu: = X^\mu X^\nu + \frac{\alpha'}{2}\eta^{\mu\nu} \ln|z_{12}|^2$$

Def

$$:X^\mu: = X^\mu$$

$$:X^\mu X^\nu: = X^\mu X^\nu + \frac{\alpha'}{2}\eta^{\mu\nu}\ln|z_{12}|^2$$

$$\partial_1\bar{\partial}_1 :X^\mu X^\nu: = -\pi\alpha'$$

Def

$$:X^\mu: = X^\mu$$

$$:X^\mu X^\nu: = X^\mu X^\nu + \frac{\alpha'}{2}\eta^{\mu\nu}\ln|z_{12}|^2$$

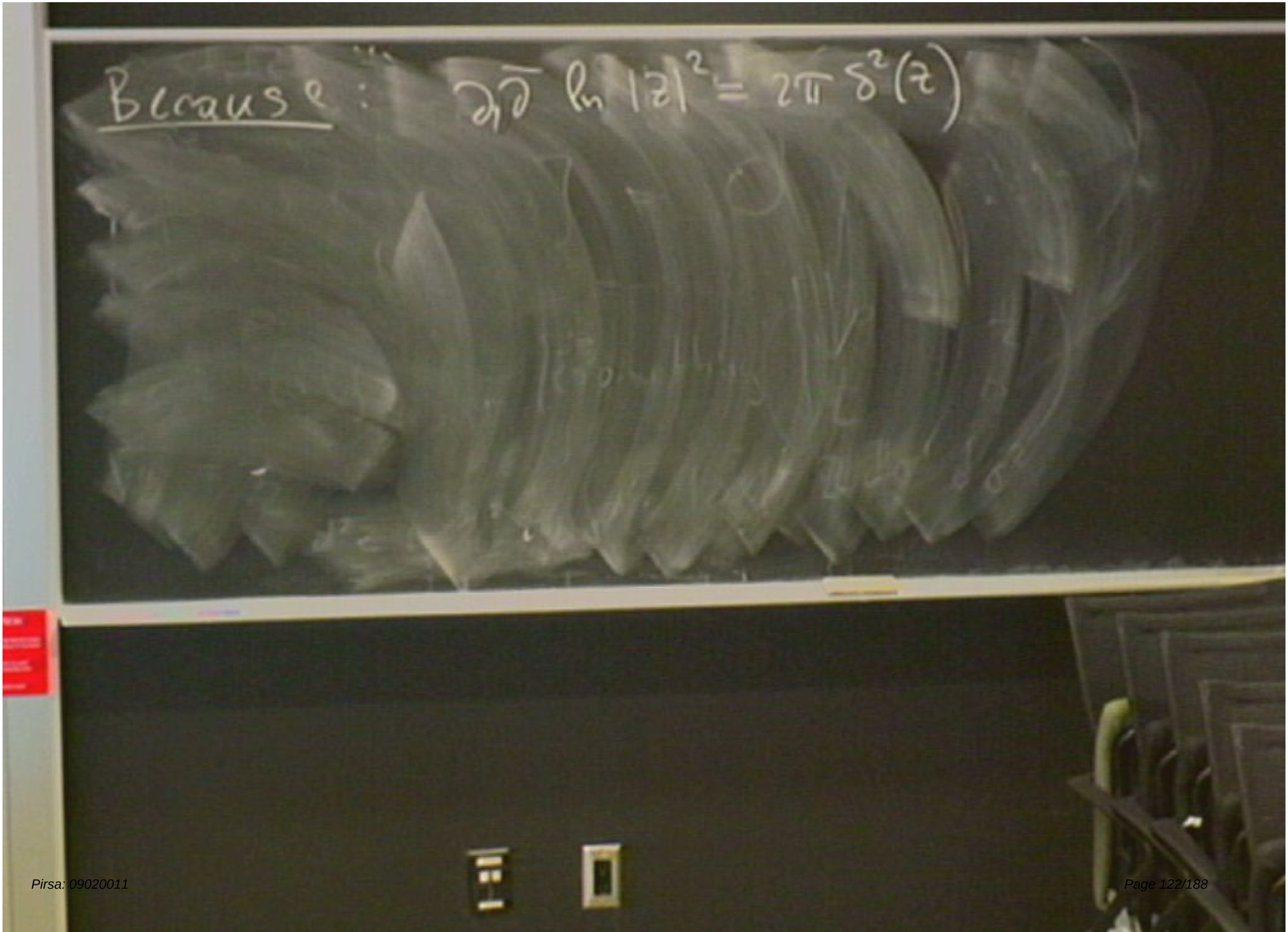
$$\partial_1\bar{\partial}_1 :X^\mu X^\nu: = -\pi\alpha'\eta^{\mu\nu}\delta^2(z_{12})$$

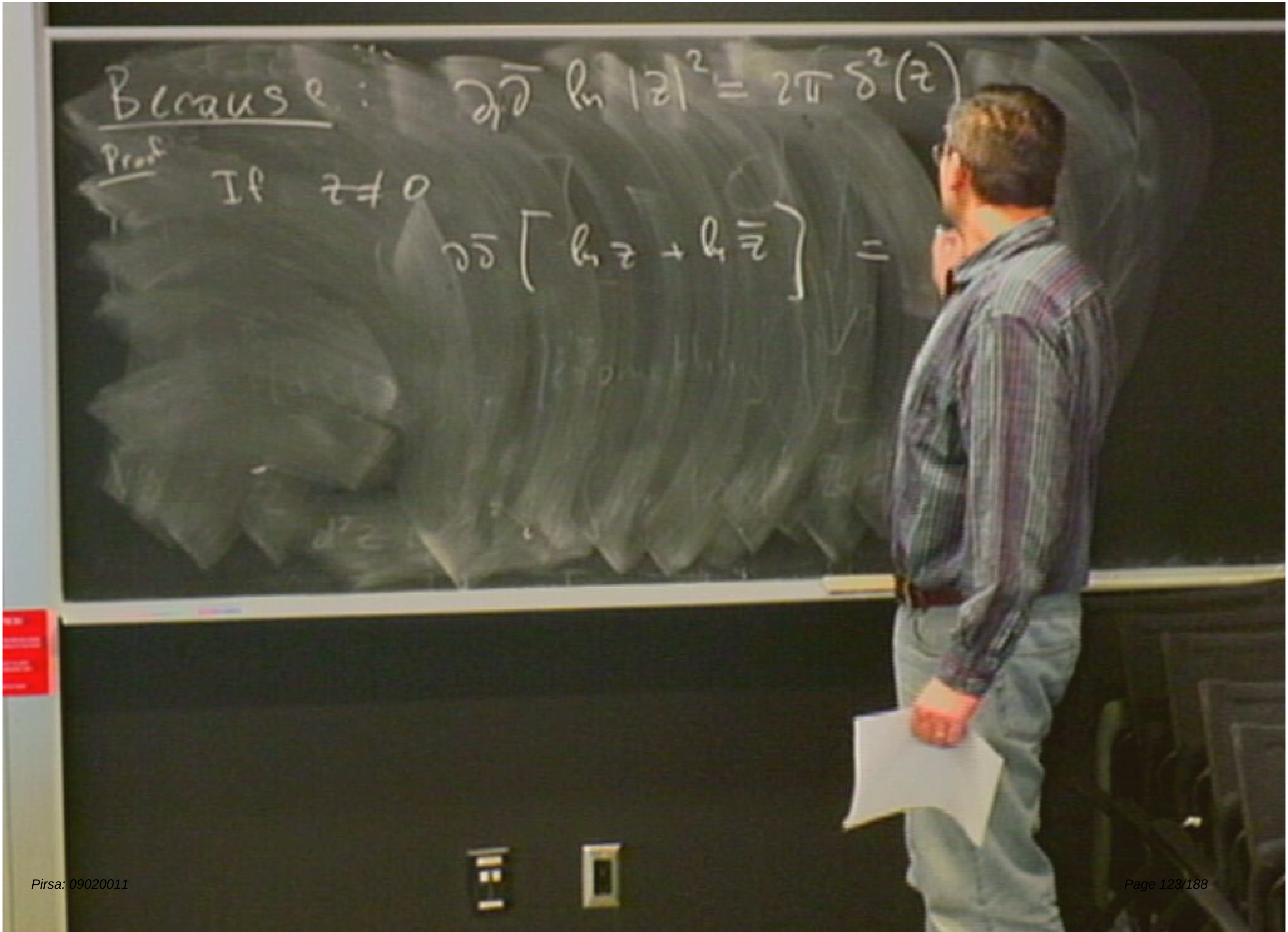
Def

$$:X^\mu: = X^\mu$$

$$:X^\mu X^\nu: = X^\mu X^\nu + \frac{\alpha'}{2}\eta^{\mu\nu}\ln|z_{12}|^2$$

$$\partial_1\bar{\partial}_1 :X^\mu X^\nu: = -\pi\alpha'\eta^{\mu\nu}\delta^2(z_{12}) + \frac{\alpha'}{2}\eta^{\mu\nu}\partial_1\bar{\partial}_1 \ln|z_{12}|^2$$


Because: $\partial\bar{\partial} \ln |z|^2 = 2\pi \delta^2(z)$


Because : $\partial\bar{\partial} \ln |z|^2 = 2\pi \delta^2(z)$
Proof
If $z \neq 0$
$\partial\bar{\partial} [\ln z + \ln \bar{z}] =$

Because: $\partial\bar{\partial} \ln |z|^2 = 2\pi \delta^2(z)$

Proof

If $z \neq 0$

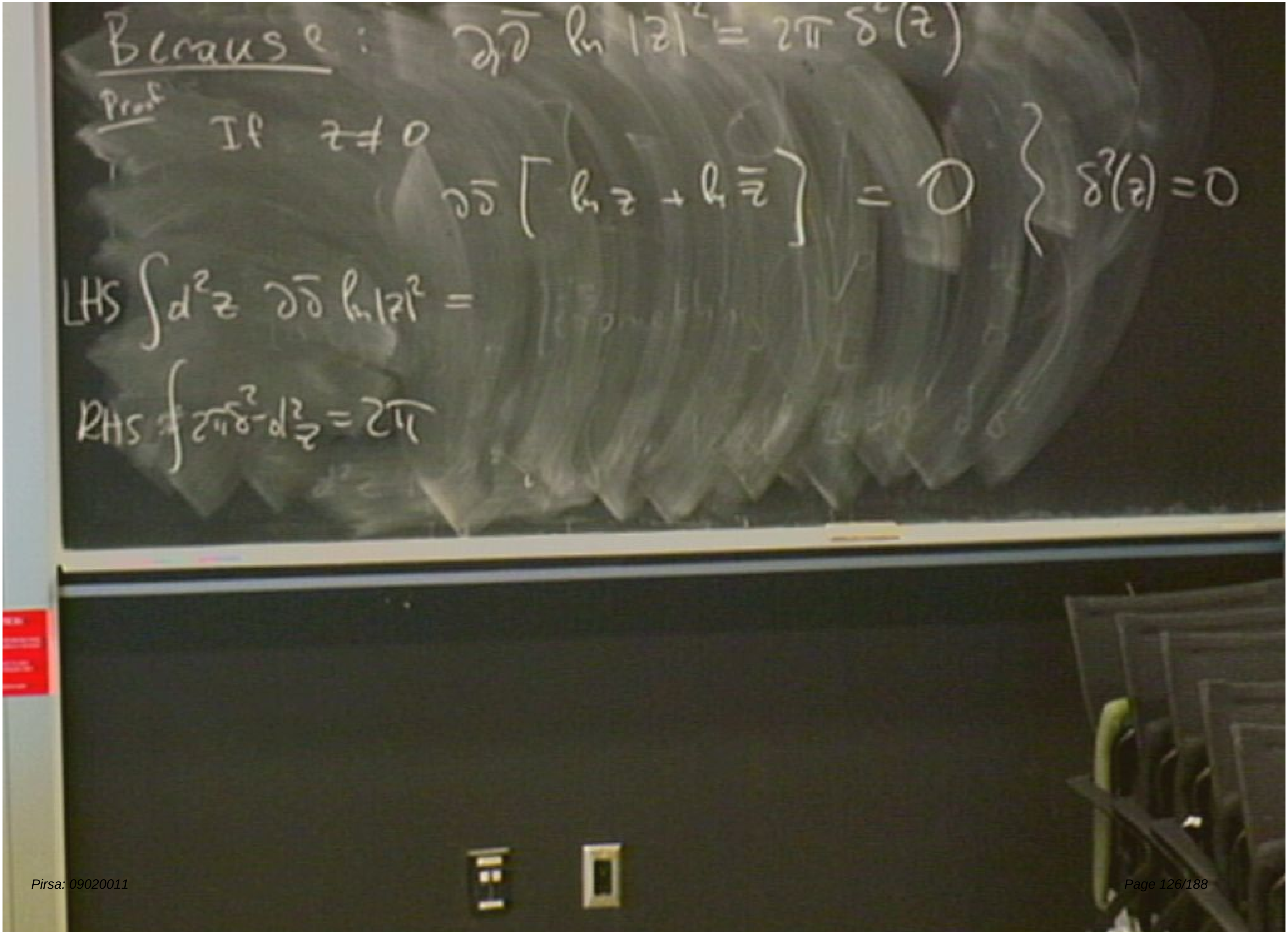
$$\partial\bar{\partial}\left[\ln z + \ln \bar{z}\right] = 0 \quad \Big\} \quad \delta^2(z) = 0$$

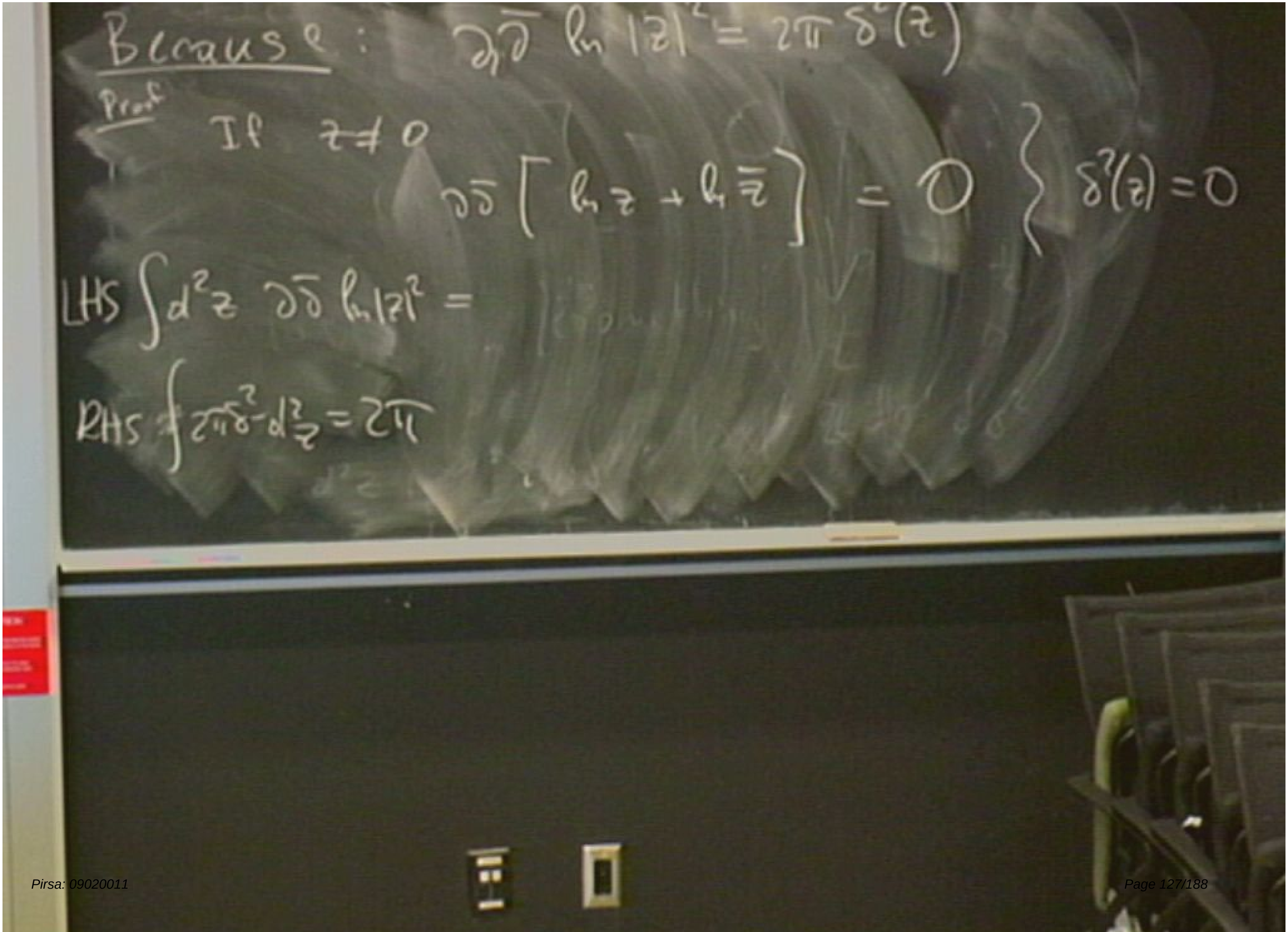
<u>Because</u>: $\partial\bar{\partial} \ln |z|^2 = 2\pi \delta^2(z)$

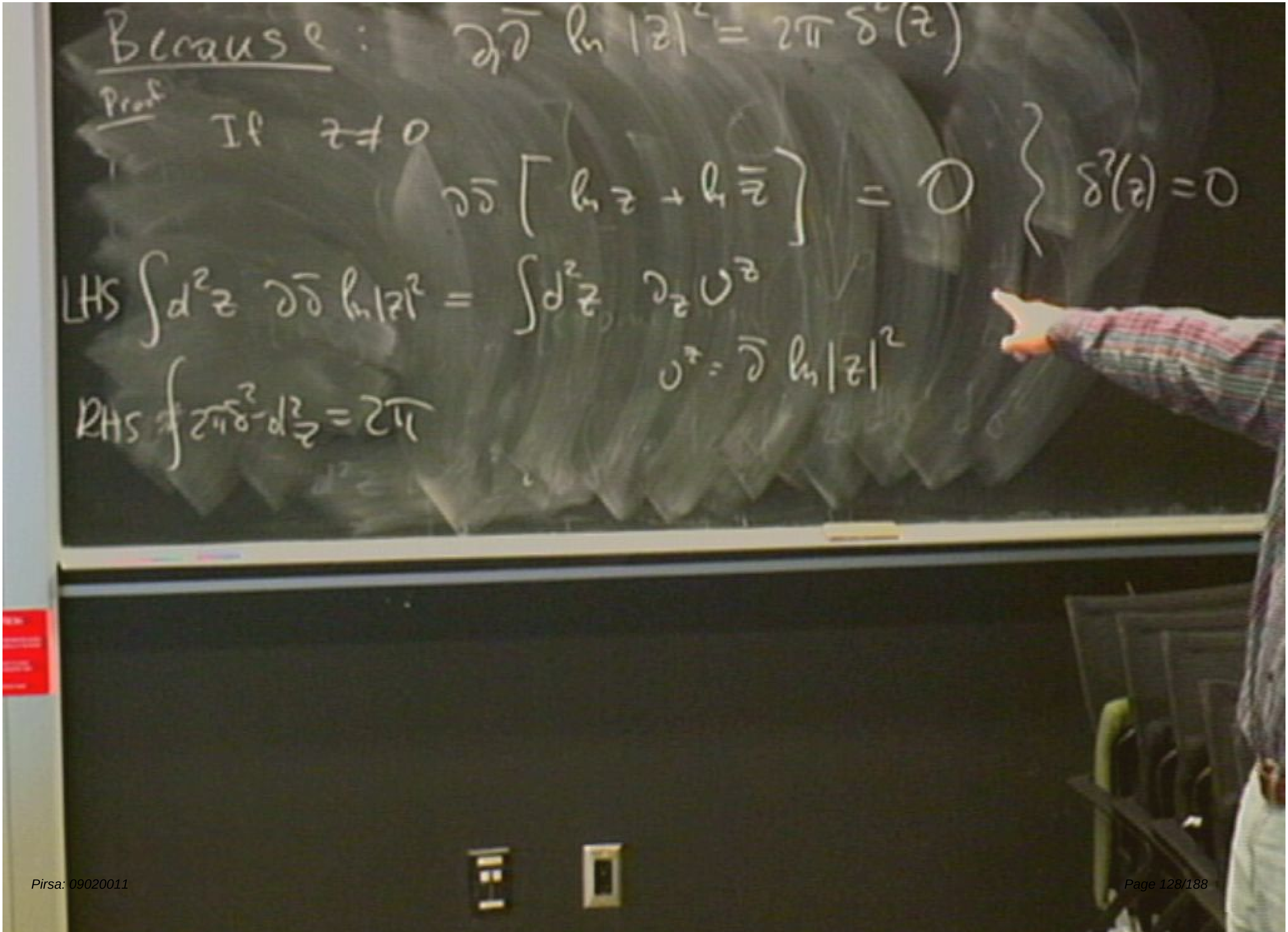
<u>Proof</u> If $z \neq 0$

$$\partial\bar{\partial}\left[\ln z + \ln \bar{z}\right] = 0 \quad \Big\} \quad \delta^2(z) = 0$$

$$\int d^2z \; \partial\bar{\partial} \ln|z|^2 =$$


Because: ∂∂̄ ln |z|² = 2π δ²(z)
Proof If z ≠ 0
∂∂̄ [ln z + ln z̄] = 0 } δ²(z) = 0
LHS ∫d²z ∂∂̄ ln|z|² =
RHS ∫2πδ² d²z = 2π


Because: ∂∂̄ ln |z|² = 2π δ²(z)
Proof
If z ≠ 0
∂∂̄ [ln z + ln z̄] = 0
δ²(z) = 0
LHS ∫d²z ∂∂̄ ln|z|² =
RHS ∫2π δ² d²z = 2π


Because: $\partial\bar{\partial} \ln|z|^2 = 2\pi\delta^2(z)$
Proof
If $z \neq 0$
$\partial\bar{\partial}[\ln z + \ln \bar{z}] = 0$
$\delta^2(z) = 0$
LHS $\int d^2z\, \partial\bar{\partial} \ln|z|^2 = \int d^2z\, \partial_z v^z$
$v^z = \bar{\partial} \ln|z|^2$
RHS $\int 2\pi\delta^2 d^2z = 2\pi$

Because: $\partial\bar{\partial} \ln|z|^2 = 2\pi\,\delta^2(z)$

Proof

If $z \neq 0$

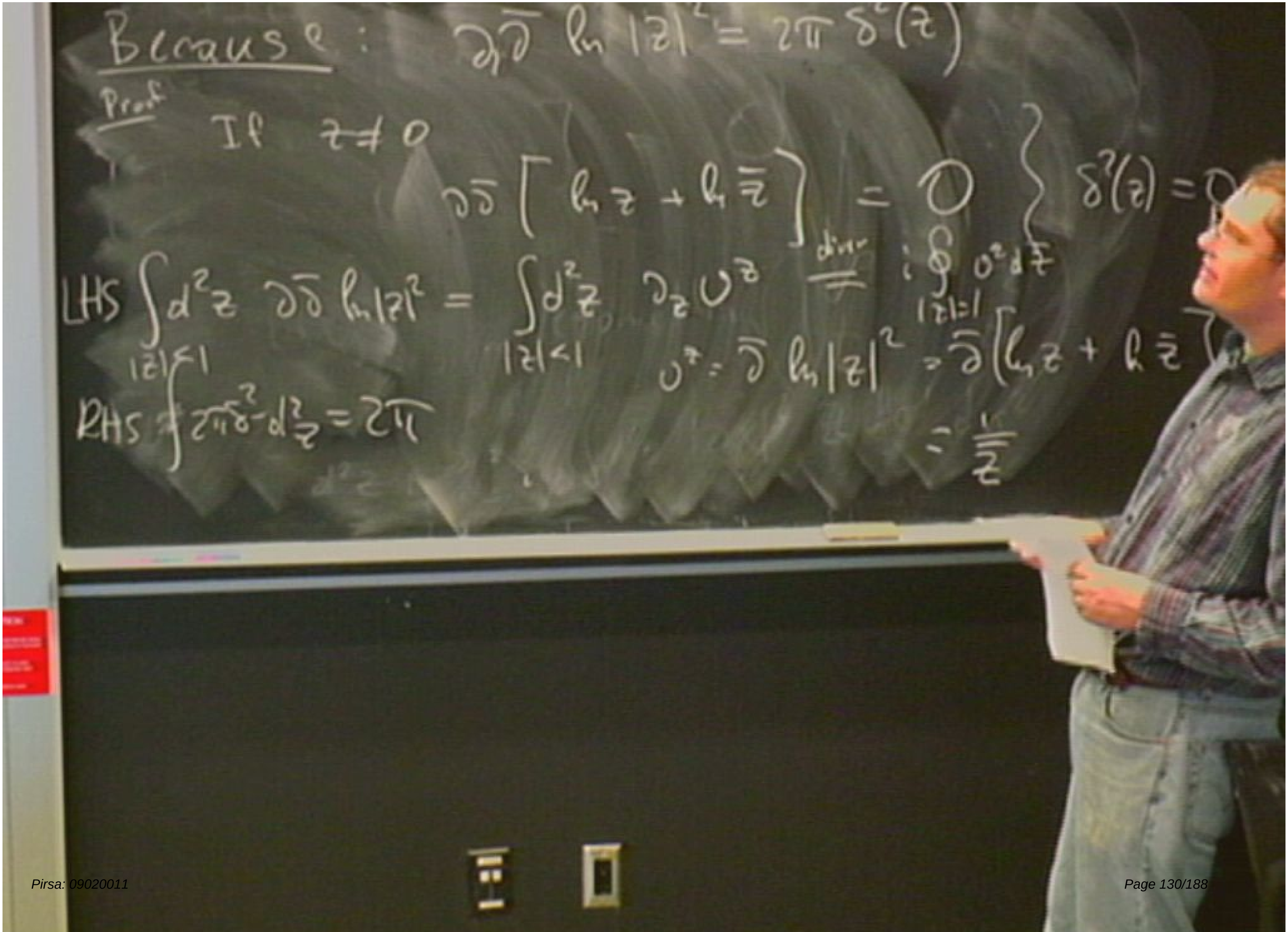
$\partial\bar{\partial}\left[\ln z + \ln \bar{z}\right] = 0 \quad \Big\} \quad \delta^2(z) = 0$

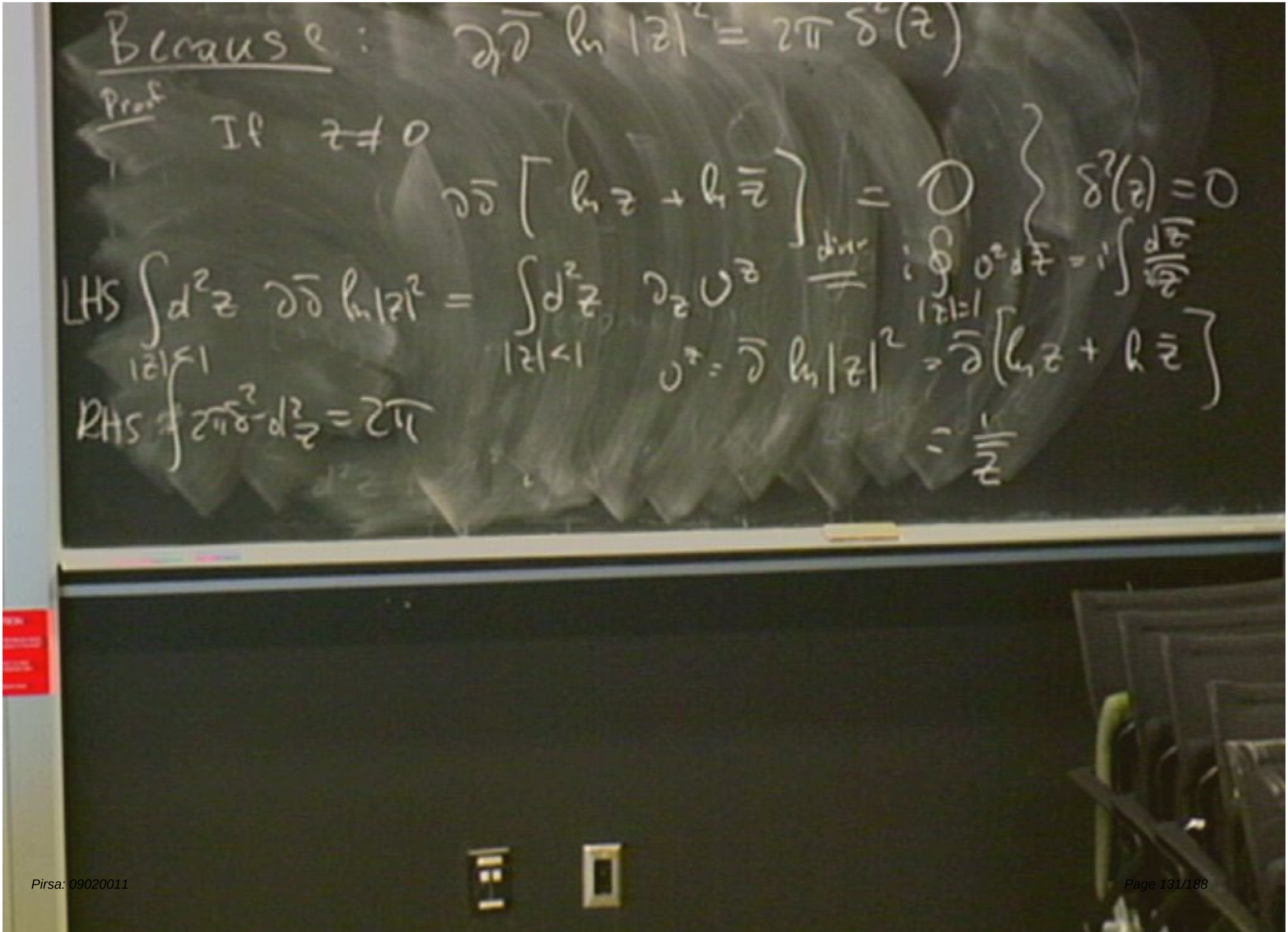
LHS $\int d^2z\, \partial\bar{\partial} \ln|z|^2 = \int d^2z\, \partial_z v^z$

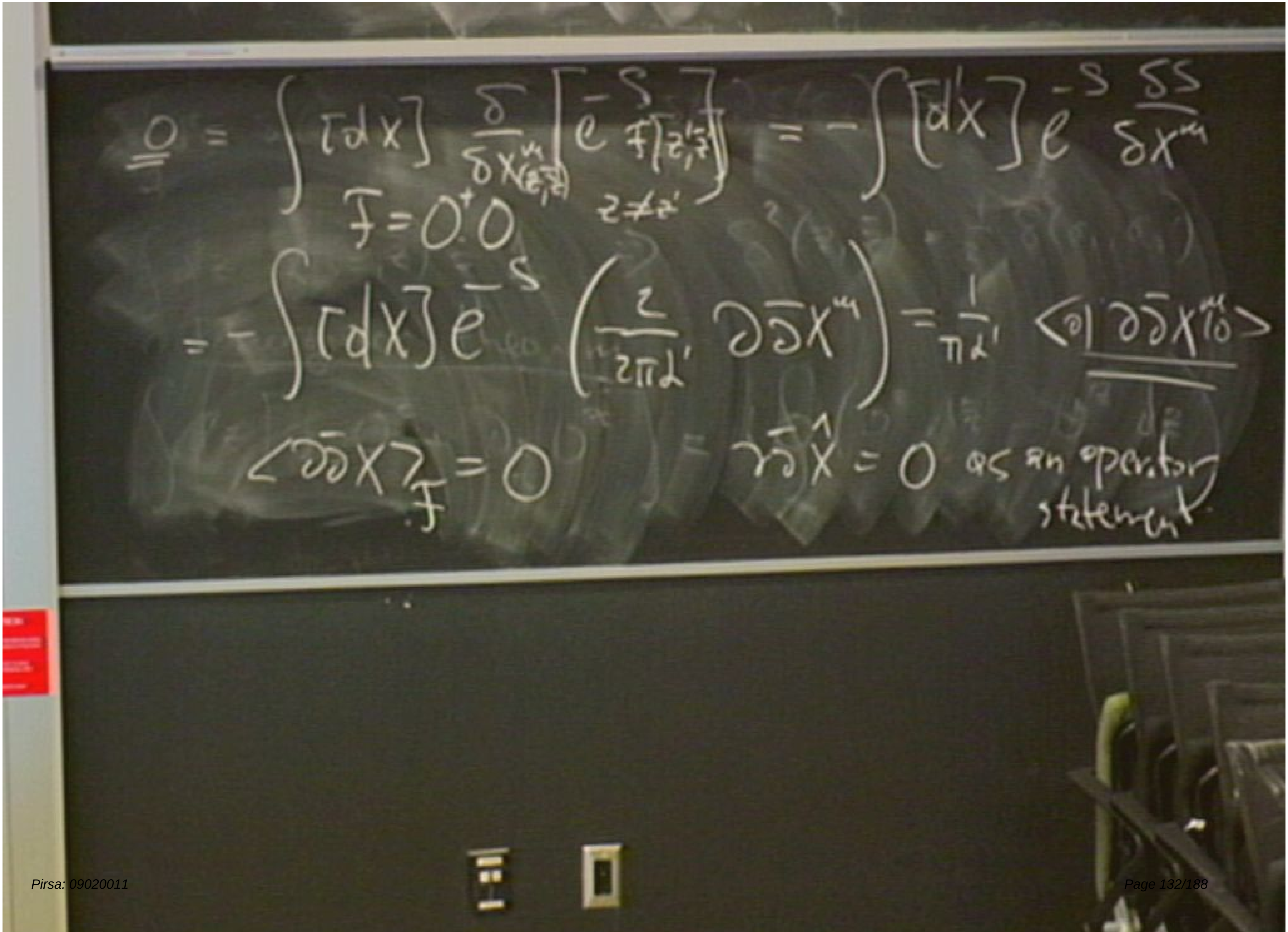
$v^z = \bar{\partial} \ln|z|^2 = \bar{\partial}\left[\ln z + \ln \bar{z}\right]$

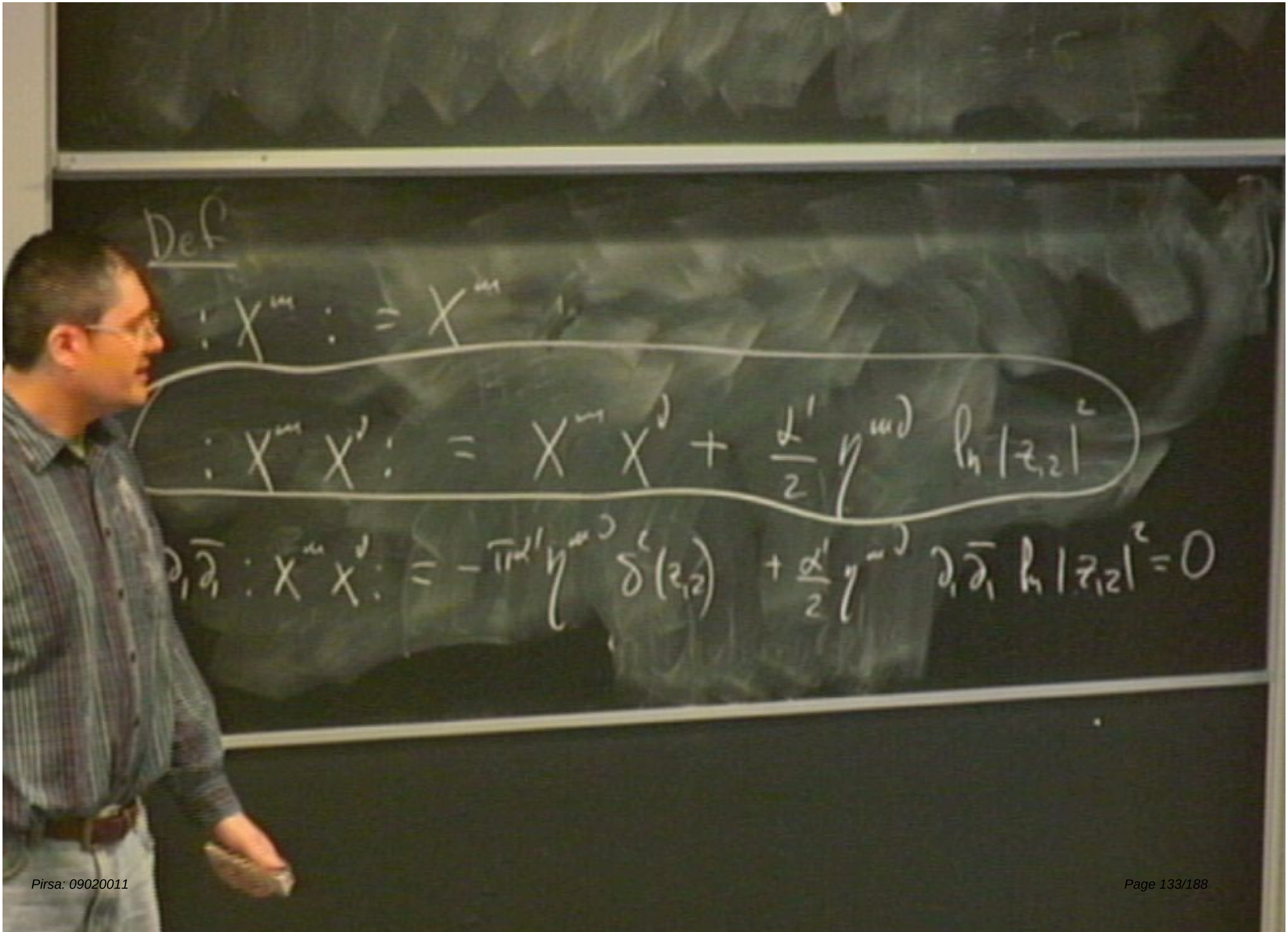
$= \frac{1}{\bar{z}}$

RHS $\int 2\pi\,\delta^2\, d^2z = 2\pi$


Because : $\partial\bar{\partial} \ln |z|^2 = 2\pi \delta^2(z)$
Proof
If $z \neq 0$
$\partial\bar{\partial} [\ln z + \ln \bar{z}] = 0$ } $\delta^2(z) = 0$
LHS $\int_{|z|<1} d^2z \, \partial\bar{\partial} \ln|z|^2 = \int_{|z|<1} d^2z \, \partial_z v^z \overset{\text{div}}{=} i \oint_{|z|=1} v^z d\bar{z}$
$v^z = \bar{\partial} \ln|z|^2 = \bar{\partial} (\ln z + \ln \bar{z})$
$= \frac{1}{\bar{z}}$
RHS $\int 2\pi \delta^2 d^2z = 2\pi$

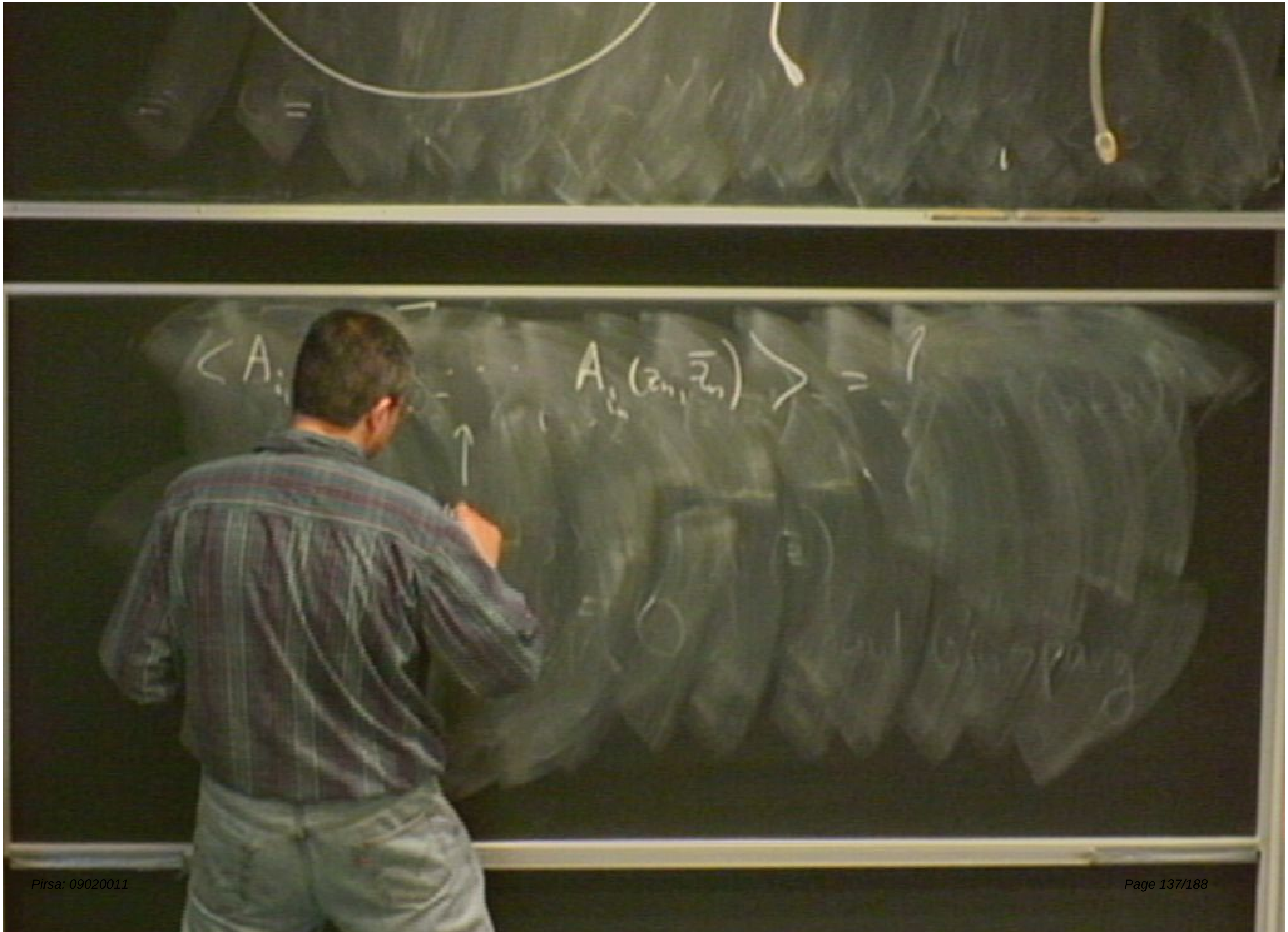

Because: ∂∂̄ ln |z|² = 2π δ²(z)
Proof
If z ≠ 0
∂∂̄ [ln z + ln z̄] = 0 } δ²(z) = 0
LHS ∫ d²z ∂∂̄ ln|z|² = ∫ d²z ∂_z v^z̄ =div= i ∮ v^z dz̄ = i ∫ dz̄/z̄
|z|≤1 |z|<1 |z|=1
v^z = ∂̄ ln|z|² = ∂̄ [ln z + ln z̄]
= 1/z̄
RHS ∫ 2π δ² d²z = 2π

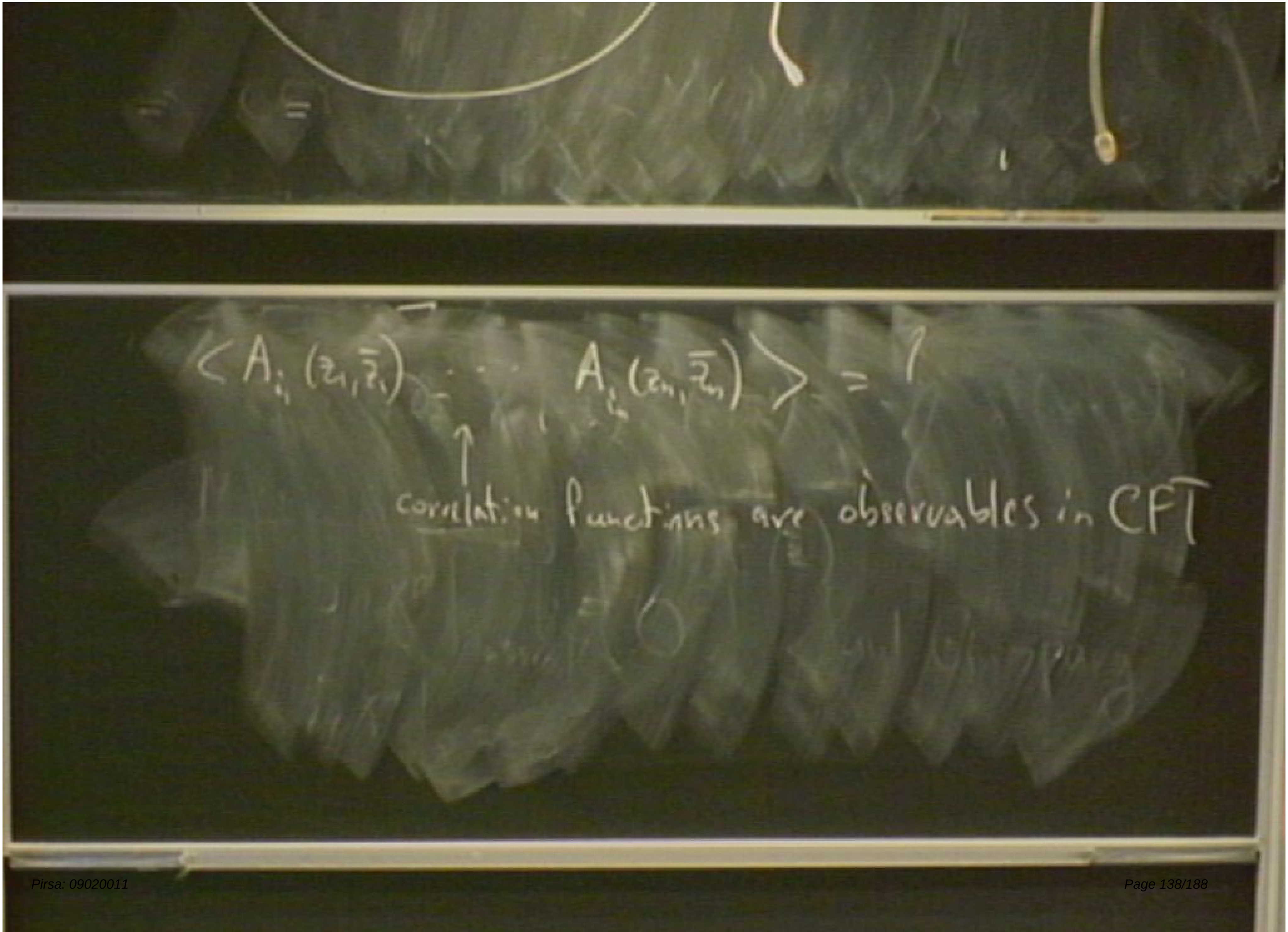

$0 = \int [dX] \frac{\delta}{\delta X^\mu(z,\bar z)} \left[e^{-S} \mathcal{F}[z',\bar z'] \right] = -\int [dX] e^{-S} \frac{\delta S}{\delta X^\mu}$
$\mathcal{F} = \mathcal{O}^+ \mathcal{O}$
$z \neq z'$
$= -\int [dX] e^{-S} \left(\frac{2}{2\pi\alpha'} \partial\bar\partial X^\mu \right) = \frac{1}{\pi\alpha'} \langle 0 | \partial\bar\partial X^\mu | 0 \rangle$
$\langle \partial\bar\partial X \rangle_{\mathcal{F}} = 0$
$\partial\bar\partial \hat{X} = 0$ as an operator statement

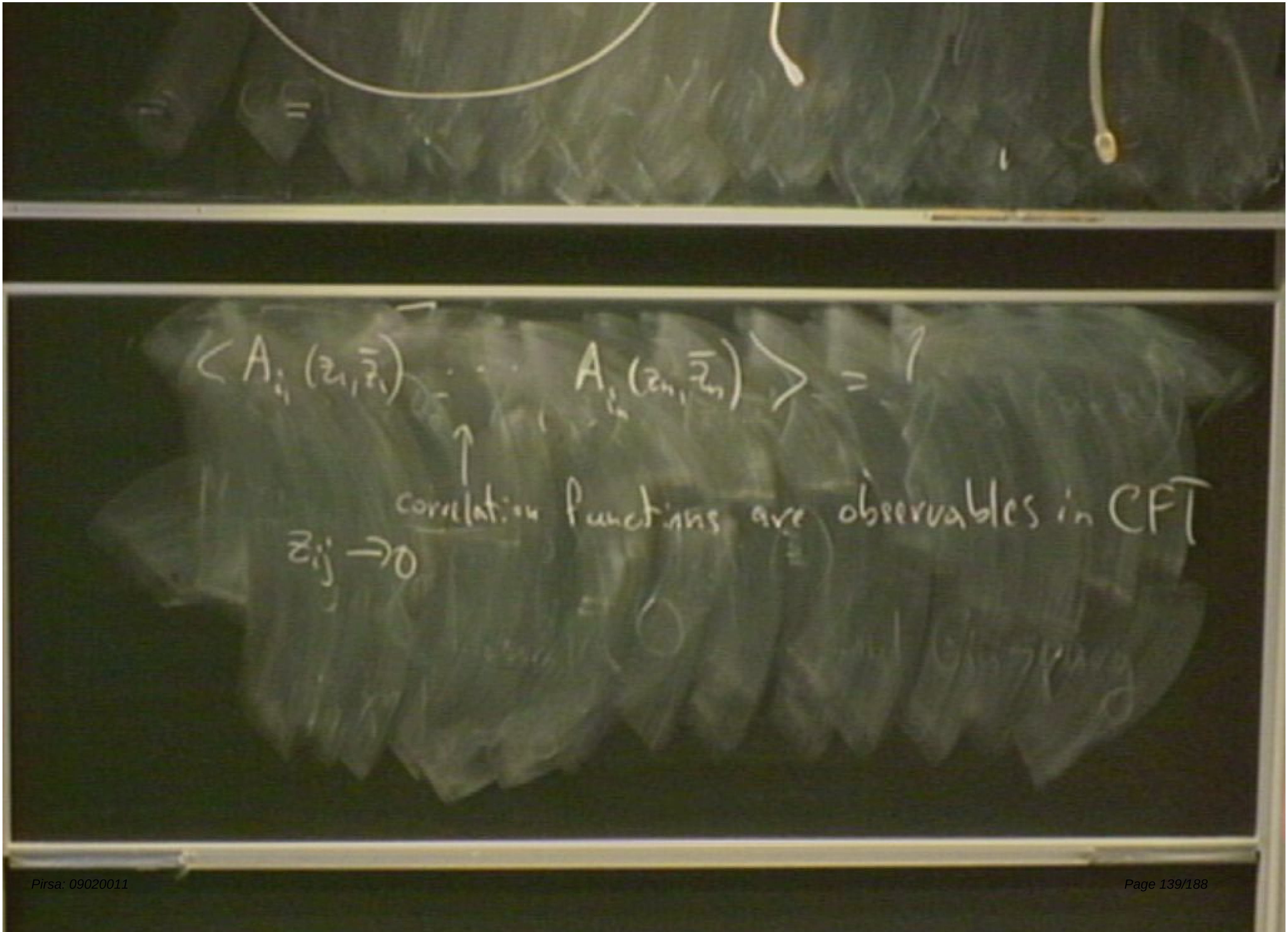

Def
$:X^\mu: = X^\mu$
$:X^\mu X^\nu: = X^\mu X^\nu + \frac{\alpha'}{2}\eta^{\mu\nu} \ln|z_{12}|^2$
$\partial_1\bar\partial_1 :X^\mu X^\nu: = -\pi\alpha'\eta^{\mu\nu}\delta^2(z_{12}) + \frac{\alpha'}{2}\eta^{\mu\nu}\partial_1\bar\partial_1 \ln|z_{12}|^2 = 0$

X^{μ}
X^{ν}


X
X


$\langle A_{i_1} \cdots A_{i_n}(z_n, \bar{z}_n) \rangle = ?$


$\langle A_{i_1}(z_1,\bar{z}_1) \cdots A_{i_n}(z_n,\bar{z}_n) \rangle = ?$
correlation functions are observables in CFT



$\langle A_{i_1}(z_1,\bar{z}_1) \cdots A_{i_n}(z_n,\bar{z}_n) \rangle = ?$
correlation functions are observables in CFT
$z_{ij} \to 0$

$$\langle A_{i_1}(z_1,\bar{z}_1) \cdots A_{i_n}(z_n,\bar{z}_n) \rangle = ?$$

↑ correlation functions are observables in CFT

$z_{ij} \to 0$

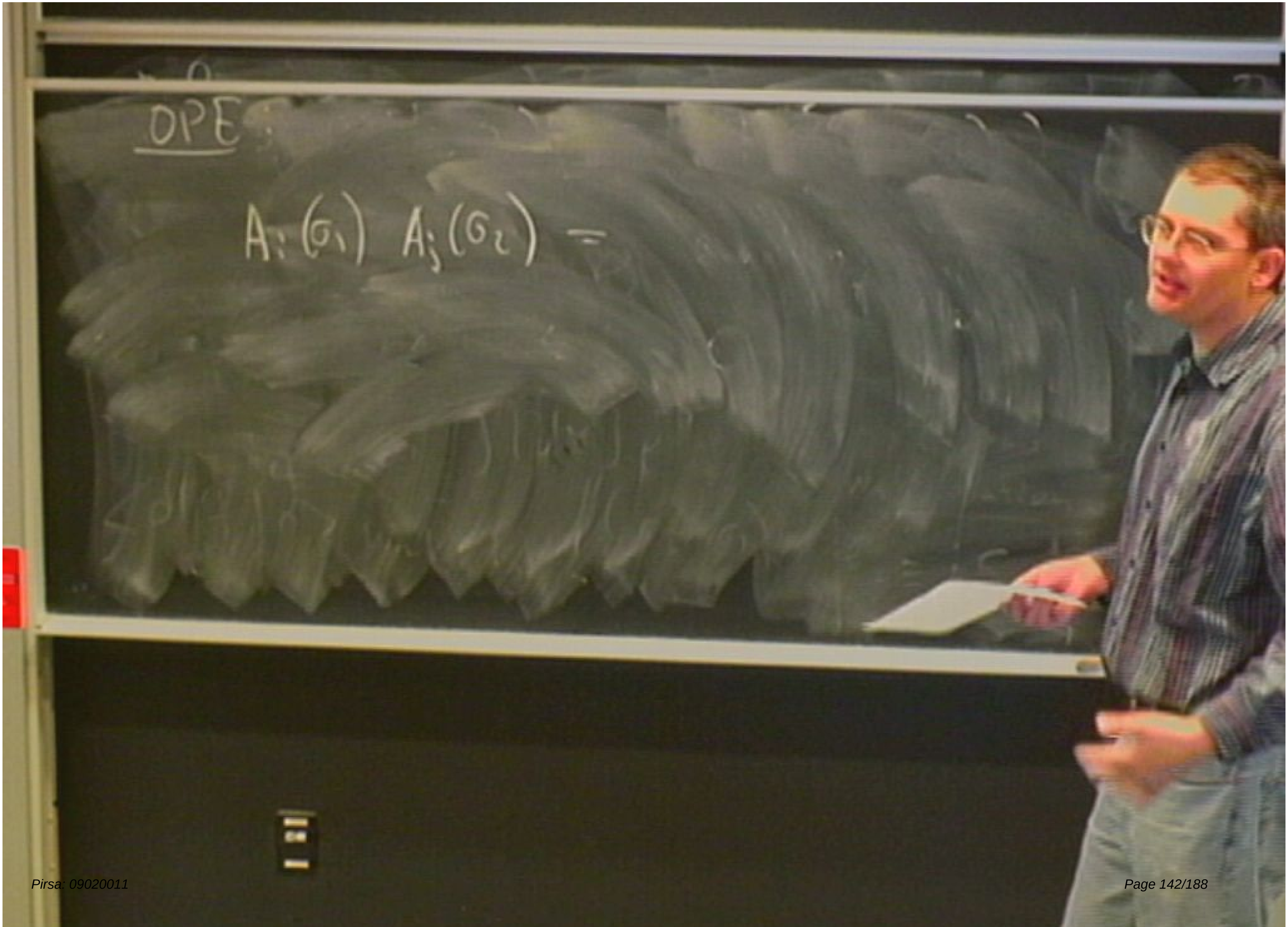
↑ interested in singularities

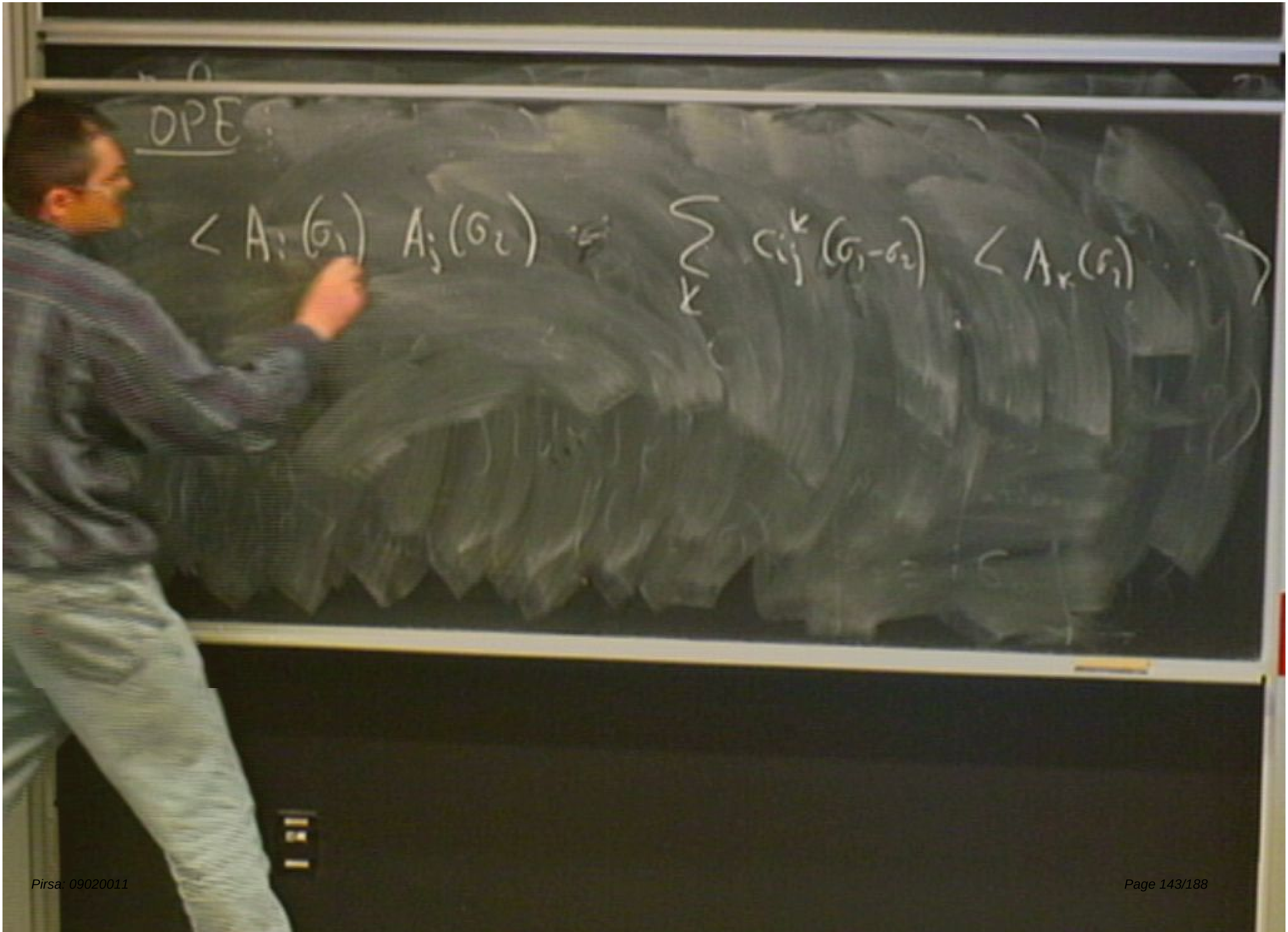
Def

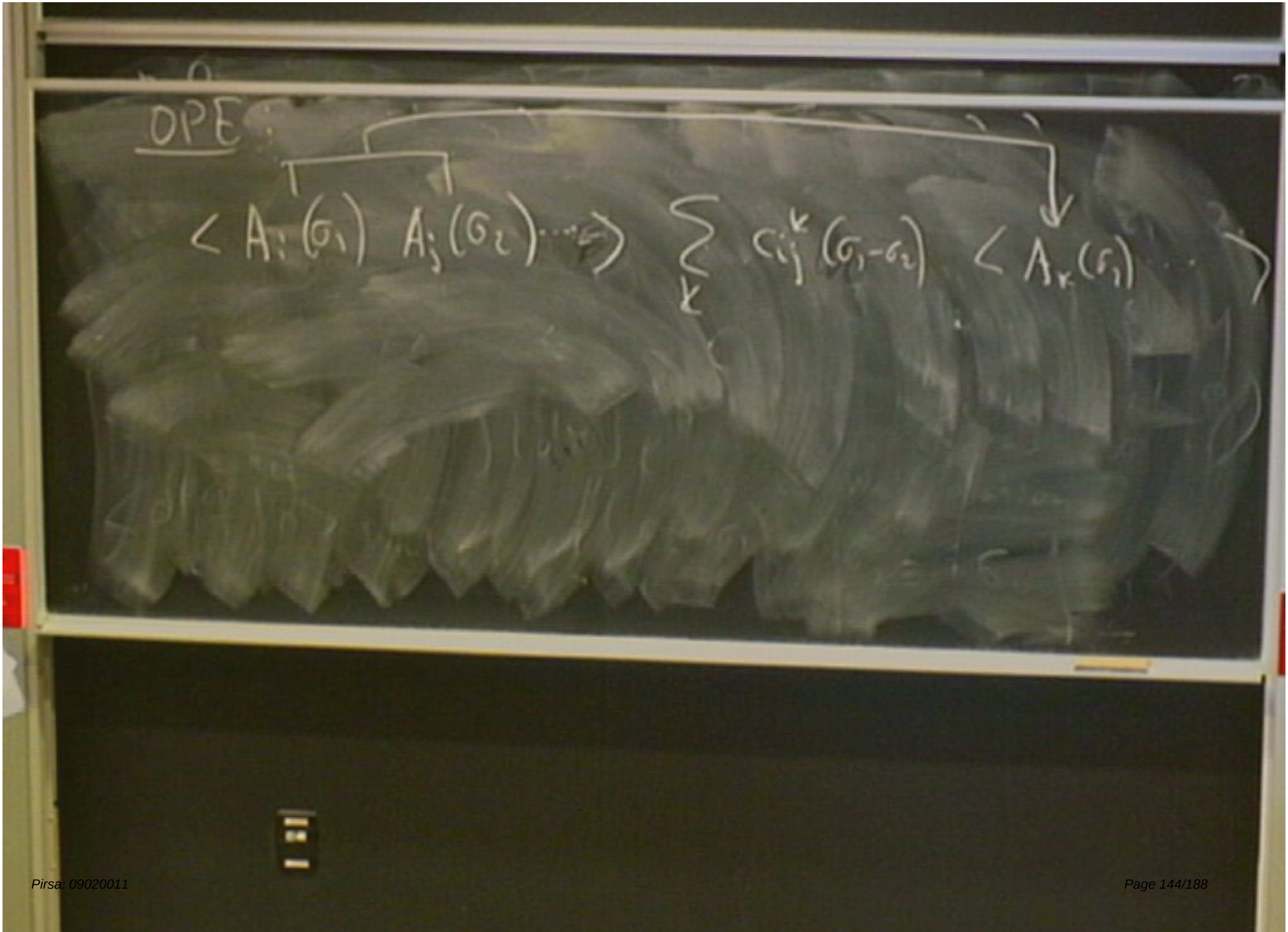
$$:X^\mu: = X^\mu$$

$$:X^\mu X^\nu: = X^\mu X^\nu + \frac{\alpha'}{2}\eta^{\mu\nu} \ln|z_{12}|^2$$

$$\partial_1\bar{\partial}_1 :X^\mu X^\nu: = -\pi\alpha'\eta^{\mu\nu}\delta^2(z_{12}) + \frac{\alpha'}{2}\eta^{\mu\nu}\partial_1\bar{\partial}_1 \ln|z_{12}|^2 = 0$$


OPE:
A_i(σ_1) A_j(σ_2) =

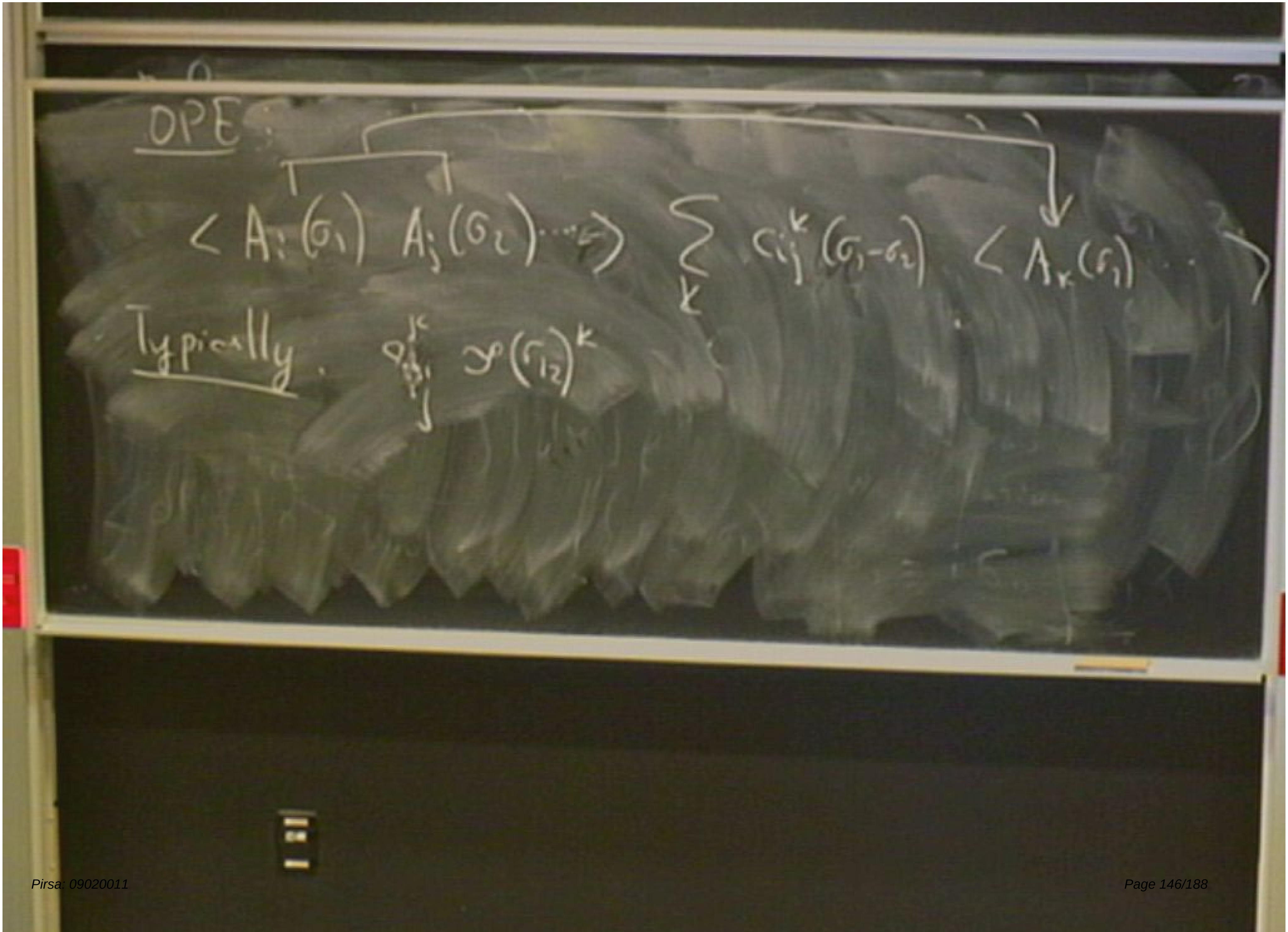

OPE :
< A_i(σ_1) A_j(σ_2) ~ Σ_k c_ij^k(σ_1-σ_2) < A_k(σ_1) ... >

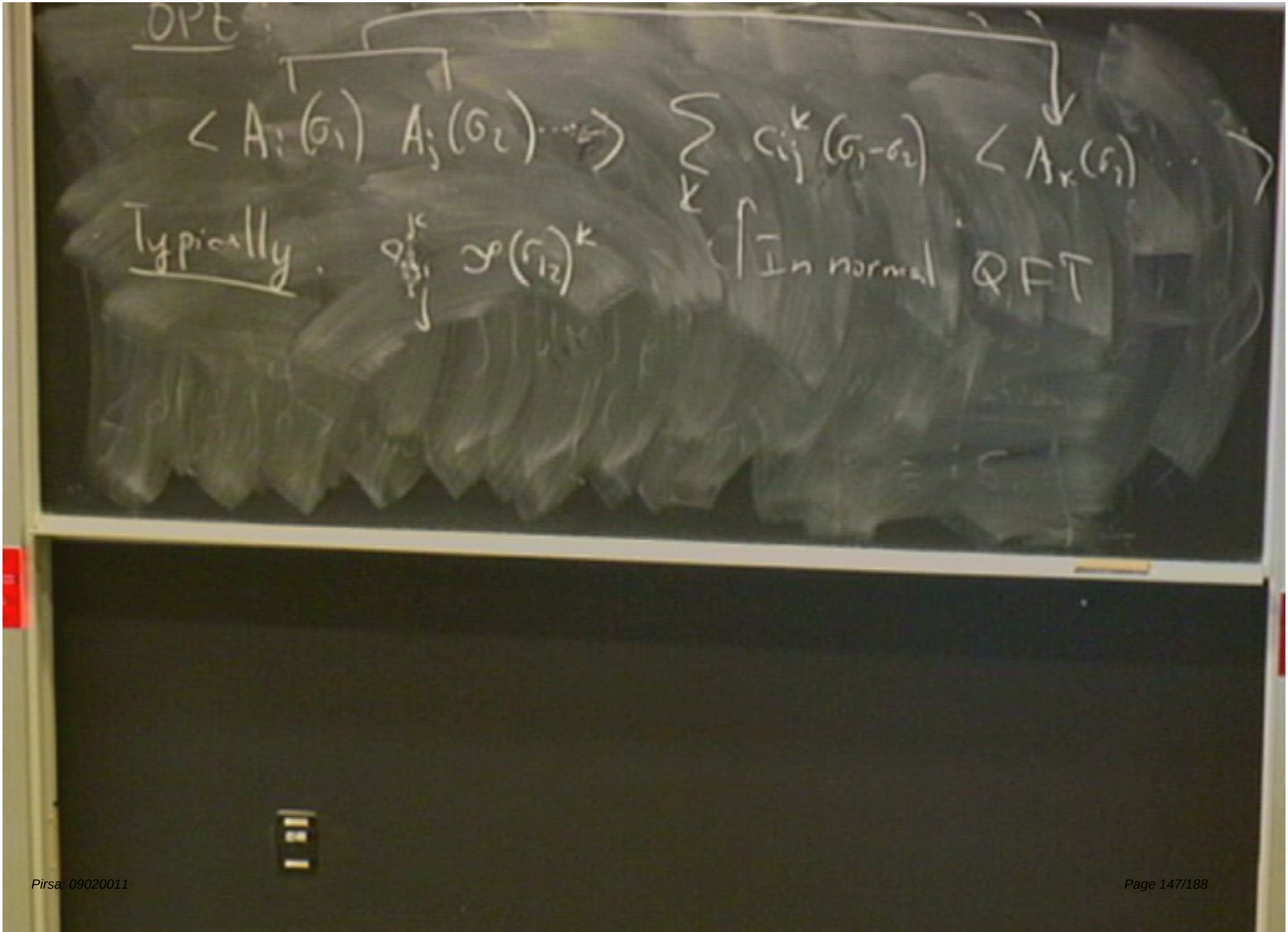

OPE:
$\langle A_i(\sigma_1)\, A_j(\sigma_2) \cdots \rangle \rightarrow \sum_k c_{ij}^k(\sigma_1-\sigma_2) \langle A_k(\sigma_1) \cdots \rangle$

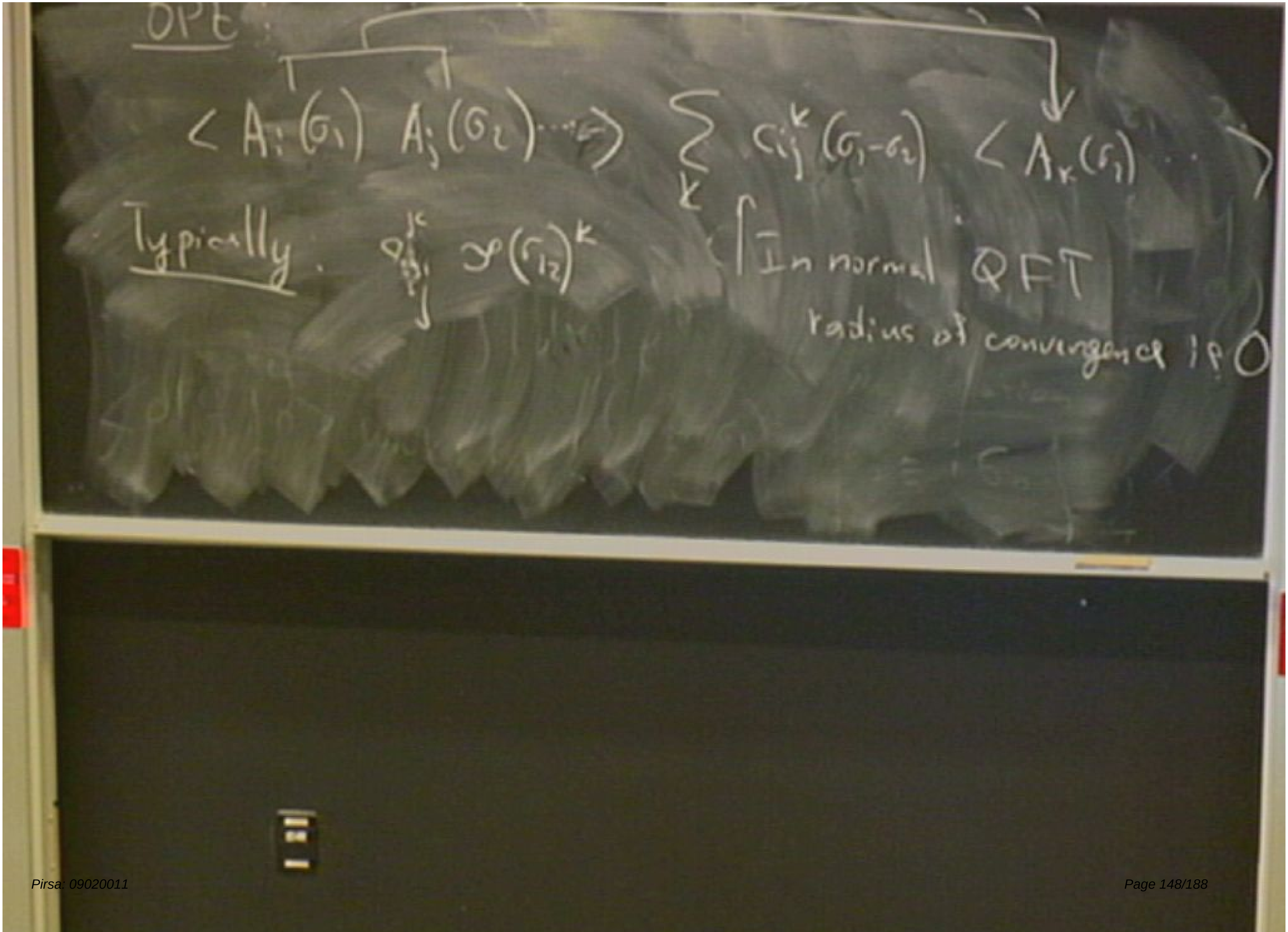
OPE :

$$\langle A_i(\sigma_1)\, A_j(\sigma_2) \cdots \rangle \longrightarrow \sum_k c_{ij}^{k}(\sigma_1 - \sigma_2) \langle A_k(\sigma_1) \cdots \rangle$$

Typically


OPE:
< A_i(σ_1) A_j(σ_2) ... > → Σ_k c_ij^k(σ_1 − σ_2) < A_k(σ_1) ... >
Typically


OPE:
< A_i(σ_1) A_j(σ_2) ... > → Σ_k c_ij^k (σ_1 - σ_2) < A_k(σ_2) ... >
Typically: c_ij^k ~ (σ_12)^k
In normal QFT


OPE :
< A_i(σ_1) A_j(σ_2) ... > → Σ_k c_ij^k(σ_1 − σ_2) < A_k(σ_1) ... >
Typically : c_ij^k ~ (σ_12)^k
In normal QFT
radius of convergence is 0

Typically: $\partial^{k}_{z_1} \; \mathcal{O}(r_{12})^{k}$

In normal QFT radius of convergence is 0

In CFTs OPEs have finite radius of convergence!

$$\partial_1 \bar{\partial}_1 :X^\mu X^\nu: = -\pi\alpha' \eta^{\mu\nu} \delta^2(z_{12}) + \frac{\alpha'}{2}\eta^{\mu\nu} \partial_1 \bar{\partial}_1 \ln |z_{12}|^2 = 0$$

Typically: $\partial^{k}_{\phi_i} \mathcal{S}(r_{12})^k$

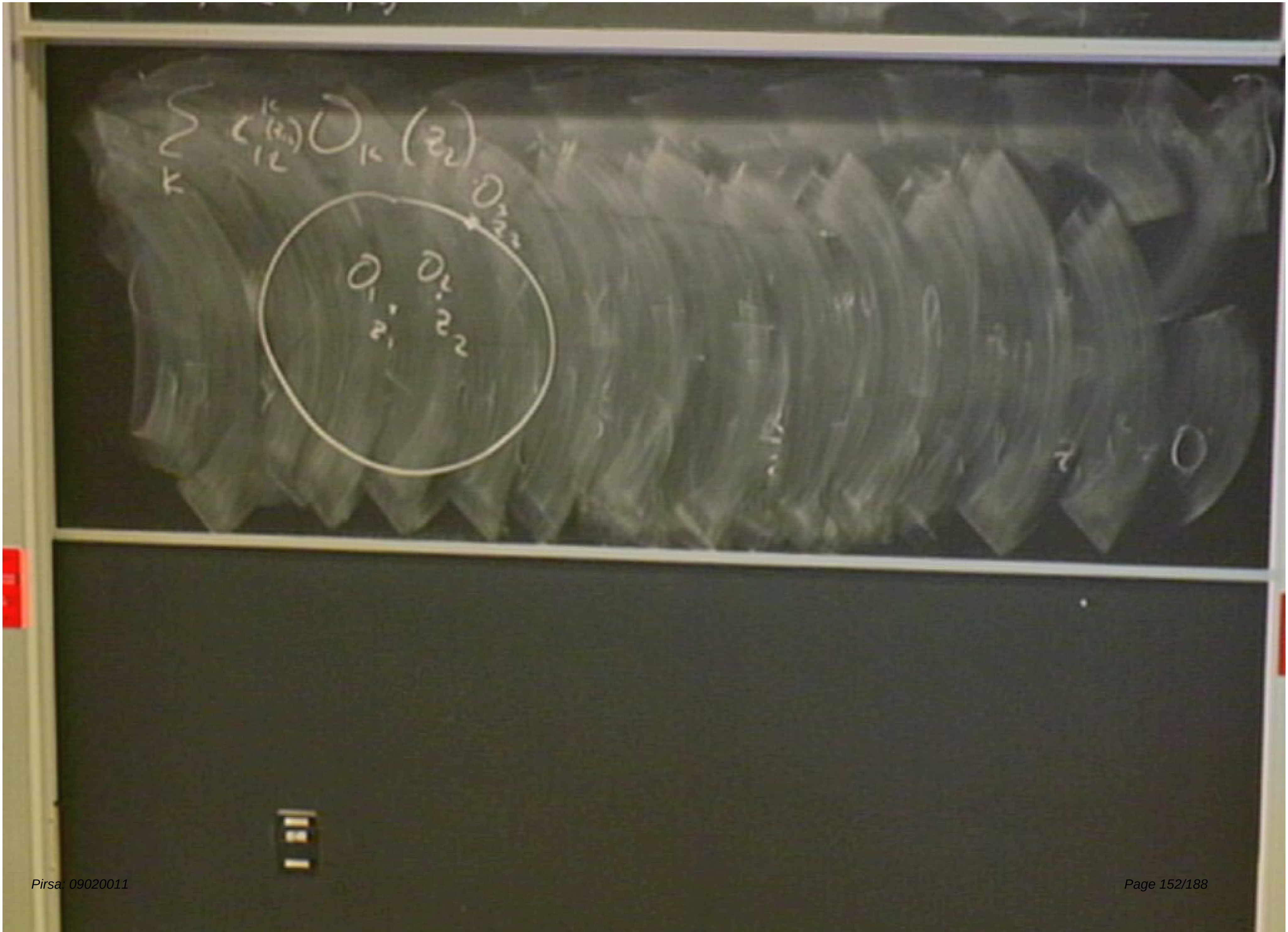
In normal QFT radius of convergence is 0

In CFT's OPEs have finite radius of convergence

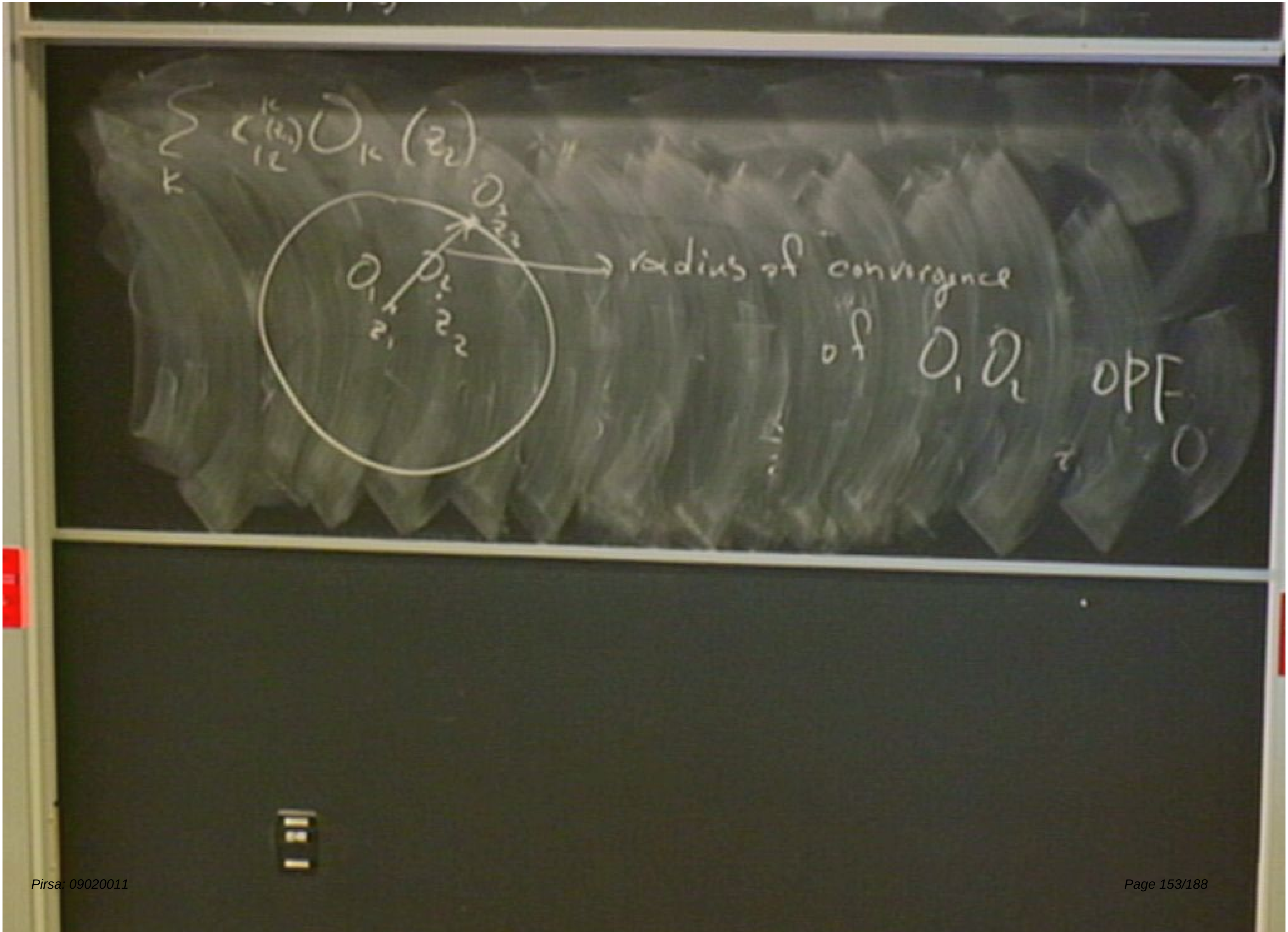
$O_1(z_1,\bar{z}_1)\,O_2(z_2,\bar{z}_2) = $

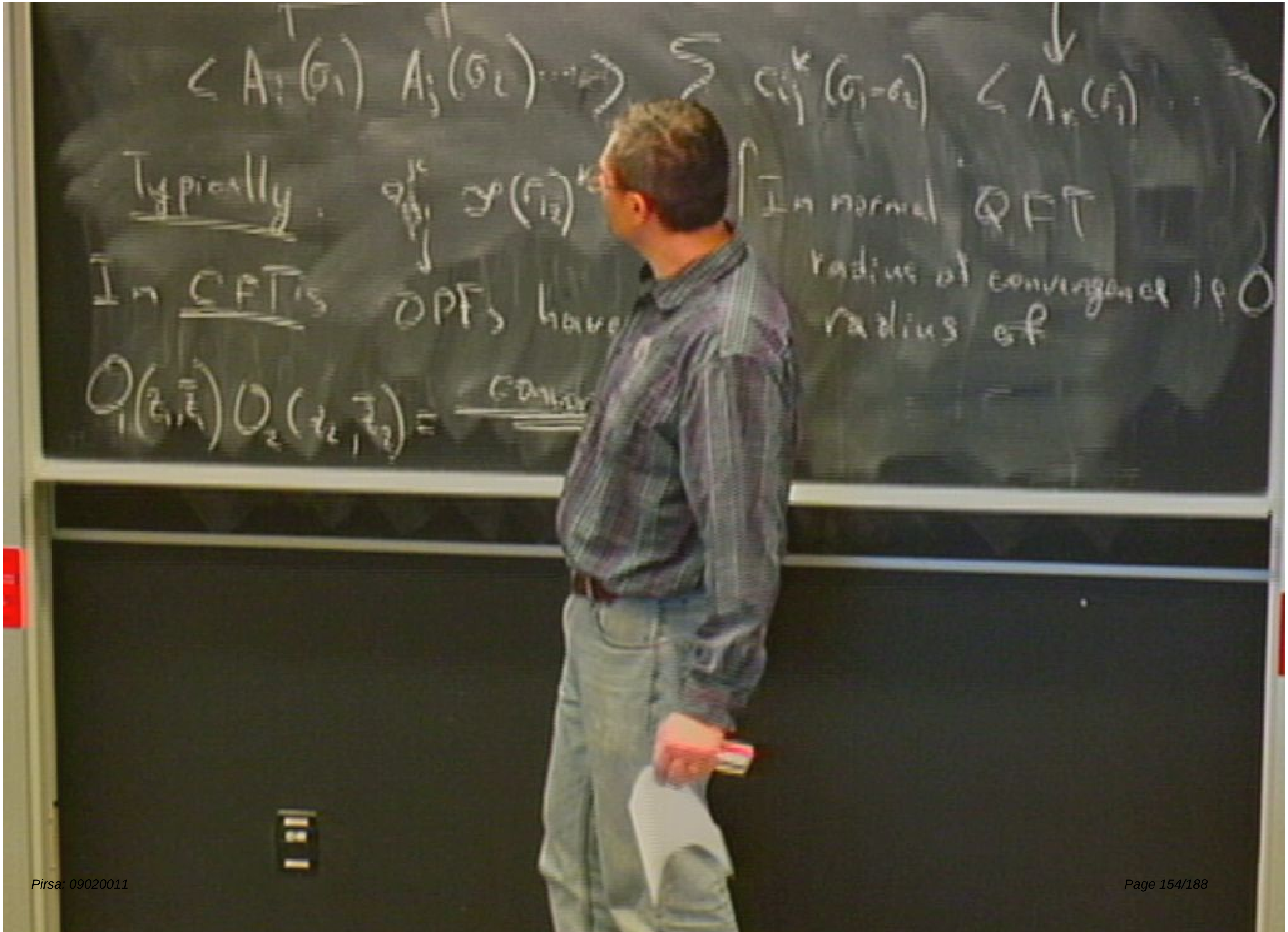
$$\partial_1\bar{\partial}_1 :X^\mu X^\nu: = -\pi\alpha'\eta^{\mu\nu}\delta^2(z_{12}) + \frac{\alpha'}{2}\eta^{\mu\nu}\partial_1\bar{\partial}_1 \ln|z_{12}|^2 = 0$$

$$\sum c_{12}^{k}(z_{12})\, \mathcal{O}_{k}(z_2)$$



$\sum_k c_{12}^{k}(z_{12}) O_k(z_2)$
O_1 O_2
z_1 z_2


radius of convergence
of O1 O2 OPE


Typically
In normal QFT
radius of convergence is 0
In CFTs OPEs have
radius of

$\langle A_i(\sigma_1)\, A_j(\sigma_2) \cdots B \rangle \left(\sum_k c_{ij}^k(\sigma_1-\sigma_2)\, \langle A_k(\sigma_2)\, B \rangle \right)$

Typically. $c_{ij}^k \sim (r_{12})^k$

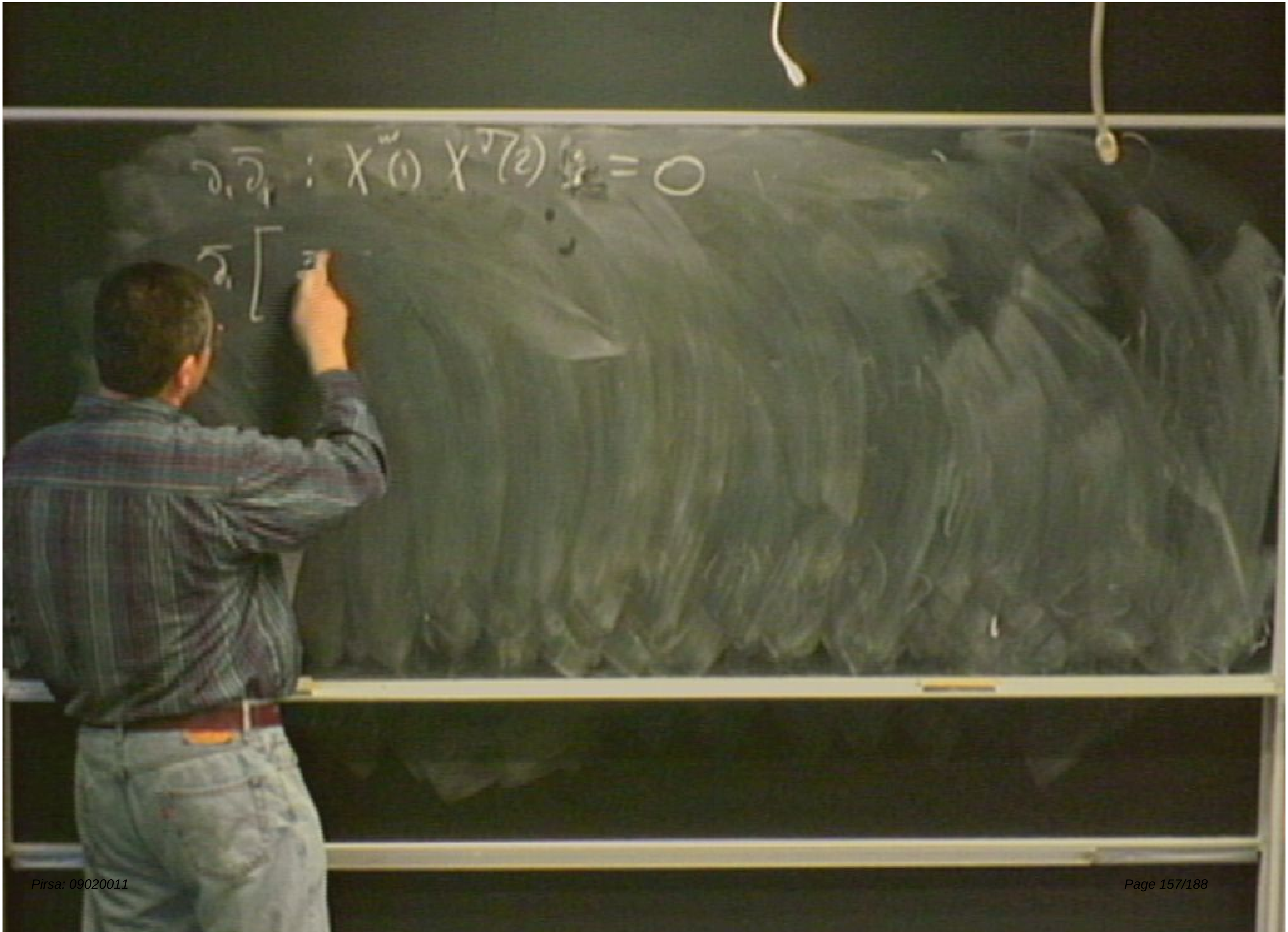
In normal QFT

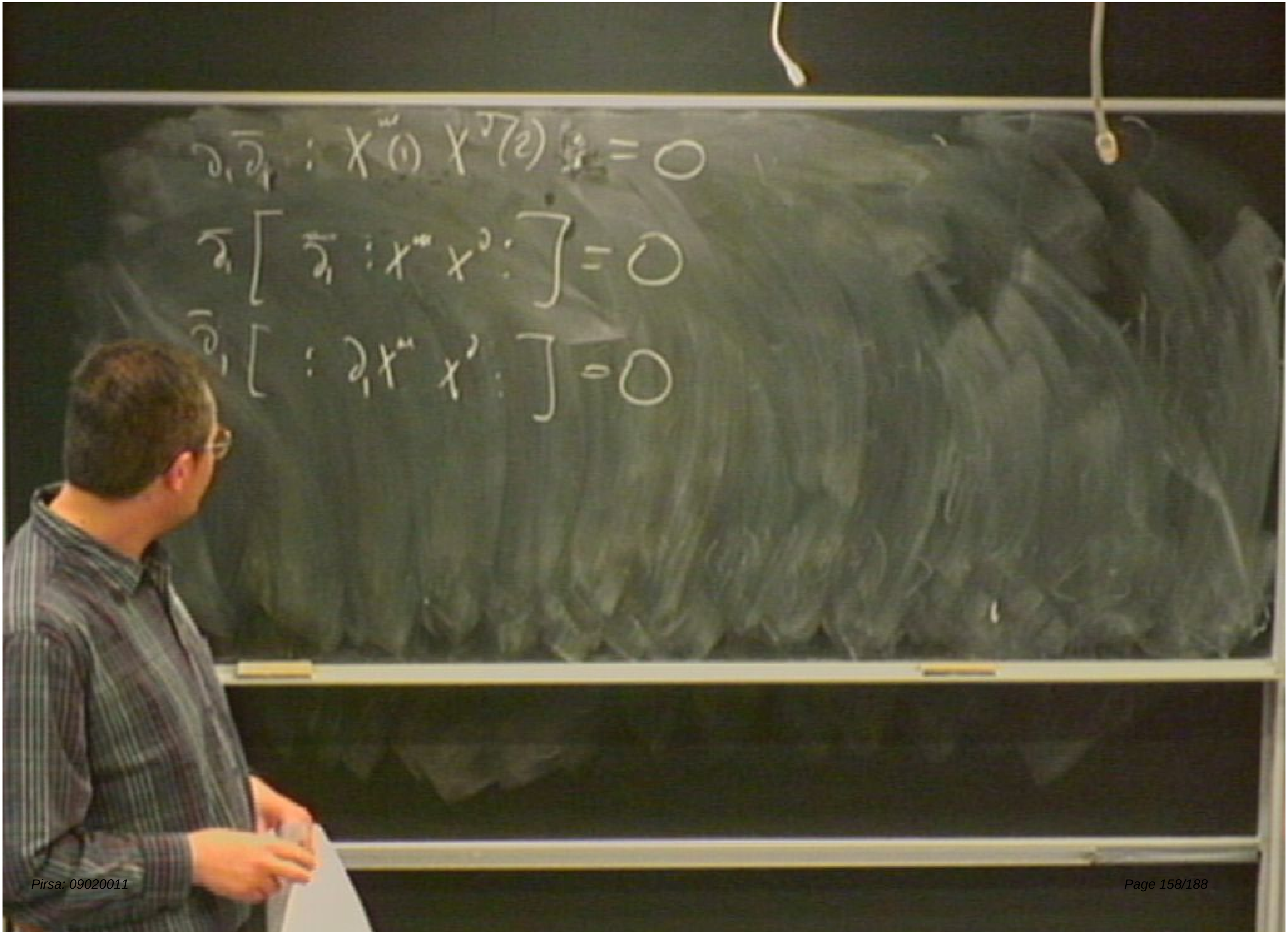
radius of convergence is 0

In CFTs OPEs have finite radius of convergence

$O_1(z_1,\bar{z}_1)\, O_2(z_2,\bar{z}_2) =$ convergent!

$\partial_1 \bar{\partial}_1 : \chi(1)\, \chi(2) = 0$



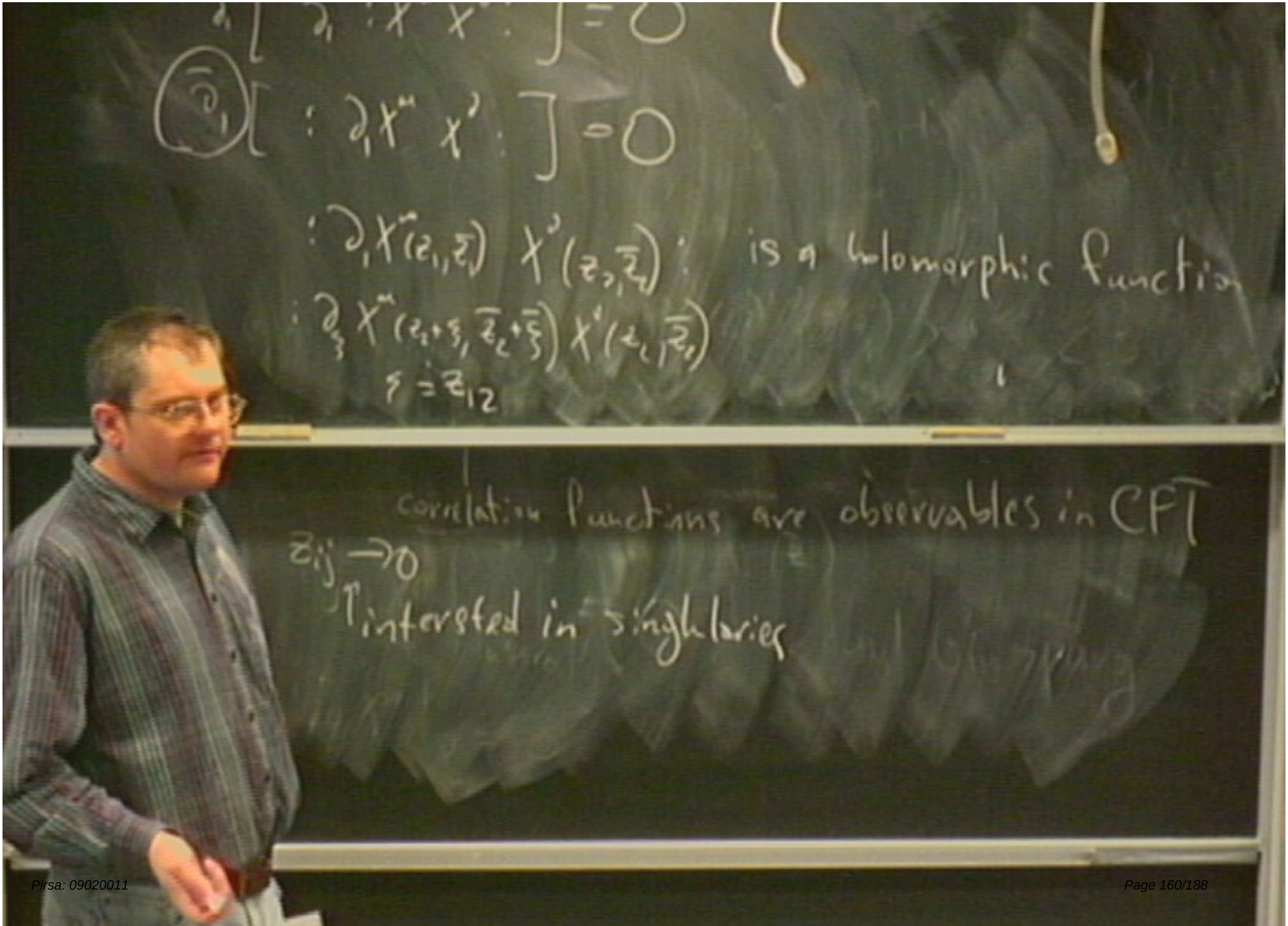

$\partial_1 \bar{\partial}_1 : X^{\mu}(1)\, X^{\nu}(2) : = 0$
$\partial_1 [\, \bar{\partial}_1 : X^{\mu} X^{\nu} : \,] = 0$
$\bar{\partial}_1 [\, : \partial_1 X^{\mu}\, X^{\nu} : \,] = 0$

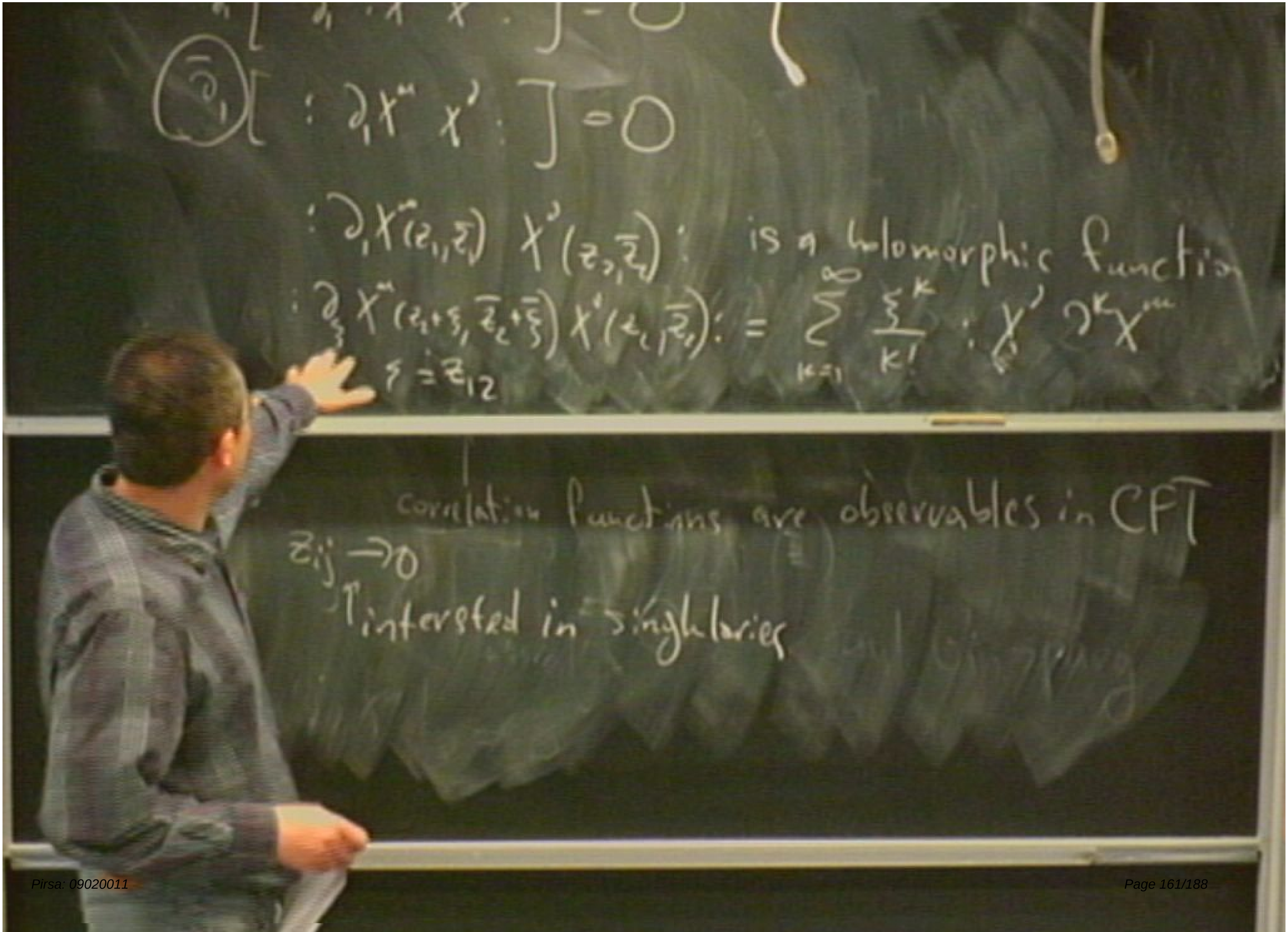
$$\partial_1 \bar{\partial}_1 : X^{\mu}(1)\, X^{\nu}(2) : = 0$$

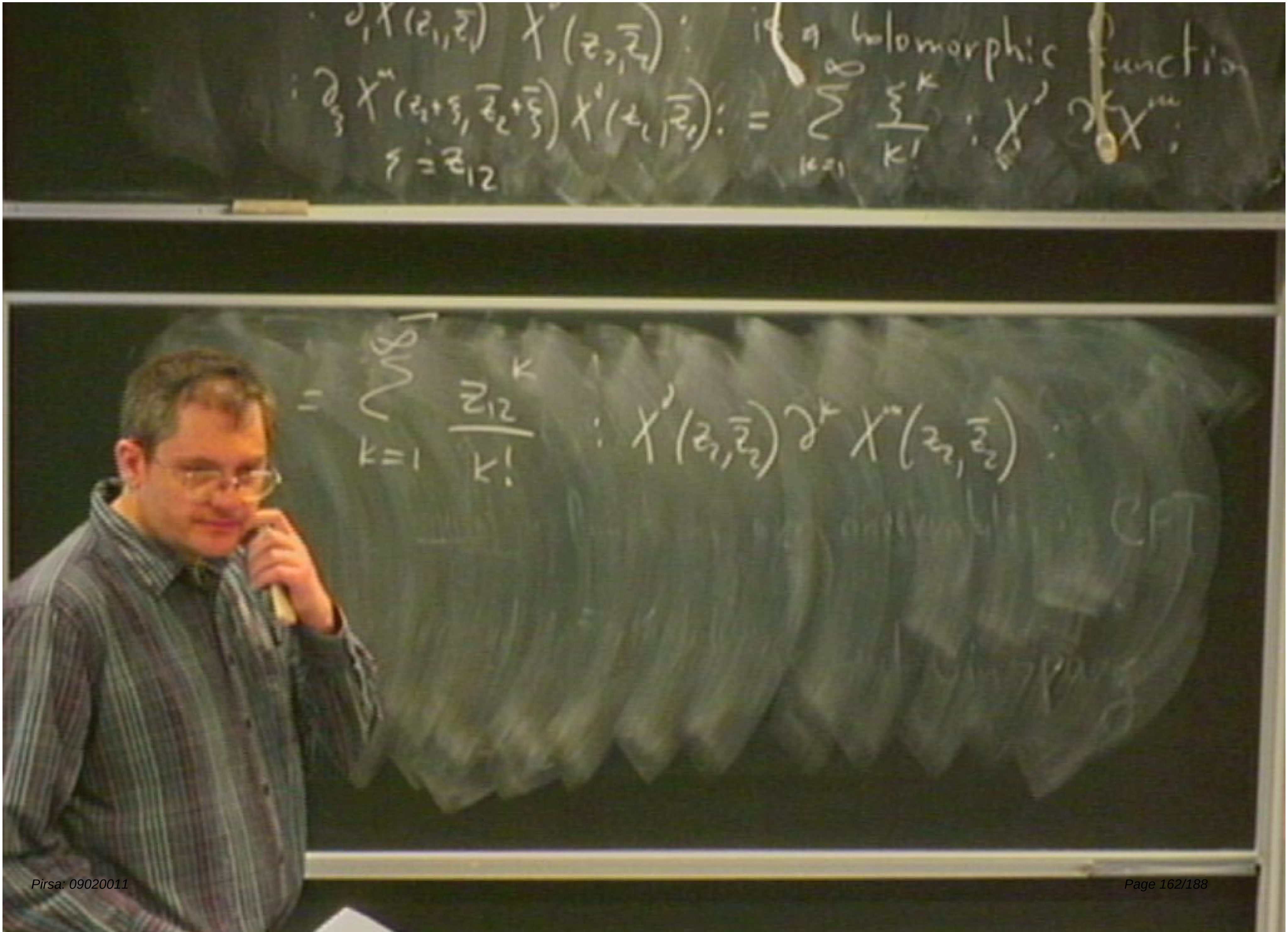
$$\partial_1 \left[\bar{\partial}_1 : X^{\mu} X^{\nu} : \right] = 0$$

$$\bar{\partial}_1 \left[: \partial_1 X^{\mu}\, X^{\nu} : \right] = 0$$

$: \partial_1 X^{\mu}(z_1, \bar{z}_1)\, X^{\nu}(z_2, \bar{z}_2) :$ is a holomorphic function


is a holomorphic function
correlation functions are observables in CFT
interested in singularities


$\bar{\partial}_1 [: \partial_1 X^\mu X^\nu :] = 0$
$: \partial_1 X^\mu (z_1, \bar{z}_1) X^\nu (z_2, \bar{z}_2) :$ is a holomorphic function
$: \partial_\xi X^\mu (z_2 + \xi, \bar{z}_2 + \bar{\xi}) X^\nu (z_2, \bar{z}_2) : = \sum_{k=1}^{\infty} \frac{\xi^k}{k!} : X^\nu \partial^k X^\mu$
$\xi = z_{12}$
correlation functions are observables in CFT
$z_{ij} \to 0$
interested in singularities


is a holomorphic function

$\partial_1 X(z_1,\bar z_1) \; X^\nu(z_2,\bar z_2):$ is a holomorphic function

$$:\partial_\xi X^\mu(z_2+\xi, \bar z_2+\bar\xi)\, X^\nu(z_1,\bar z_1): \; = \sum_{k=1}^{\infty} \frac{\xi^k}{k!} :X^\nu \partial^k X^\mu:$$

$\xi = z_{12}$

$$= \sum_{k=1}^{\infty} \frac{z_{12}^k}{k!} :X^\nu(z_2,\bar z_2)\, \partial^k X^\mu(z_2,\bar z_2):$$

$$:X^\mu(z_1,\bar z_1)\, X^\nu(z_2,\bar z_2): \; = \; :X^\mu(z_2,\bar z_2)\, X^\nu(z_2,\bar z_2):$$

$$\bar\partial_1 \left[:\partial_1 X^\mu X^\nu: \right] = 0 \qquad \partial_1 \left[:\bar\partial_1 X^\mu X^\nu: \right] = 0$$

$:\partial_1 X^\mu(z_1,\bar z_1)\, X^\nu(z_2,\bar z_2):$ is a holomorphic func

$$:\partial_\xi X^\mu(z_2+\xi, \bar z_2+\bar\xi)\, X^\nu(z_2,\bar z_2): = \sum_{k=1}^{\infty} \frac{\xi^k}{k!} :X^\nu \partial^k X^\mu:$$

$$\xi = z_{12}$$

$$= \sum_{k=1} \frac{z_{12}^k}{k!} :X^\nu(z_2,\bar z_2)\, \partial^k X^\mu(z_2,\bar z_2):$$

$$:X^\mu(z_1,\bar z_1)\, X^\nu(z_2,\bar z_2): = :X^\mu(z_2,\bar z_2)\, X^\nu(z_2,\bar z_2):$$

$:\partial_1 X^\mu(z_1,\bar z_1)\, X^\nu(z_2,\bar z_2):$ is a holomorphic function

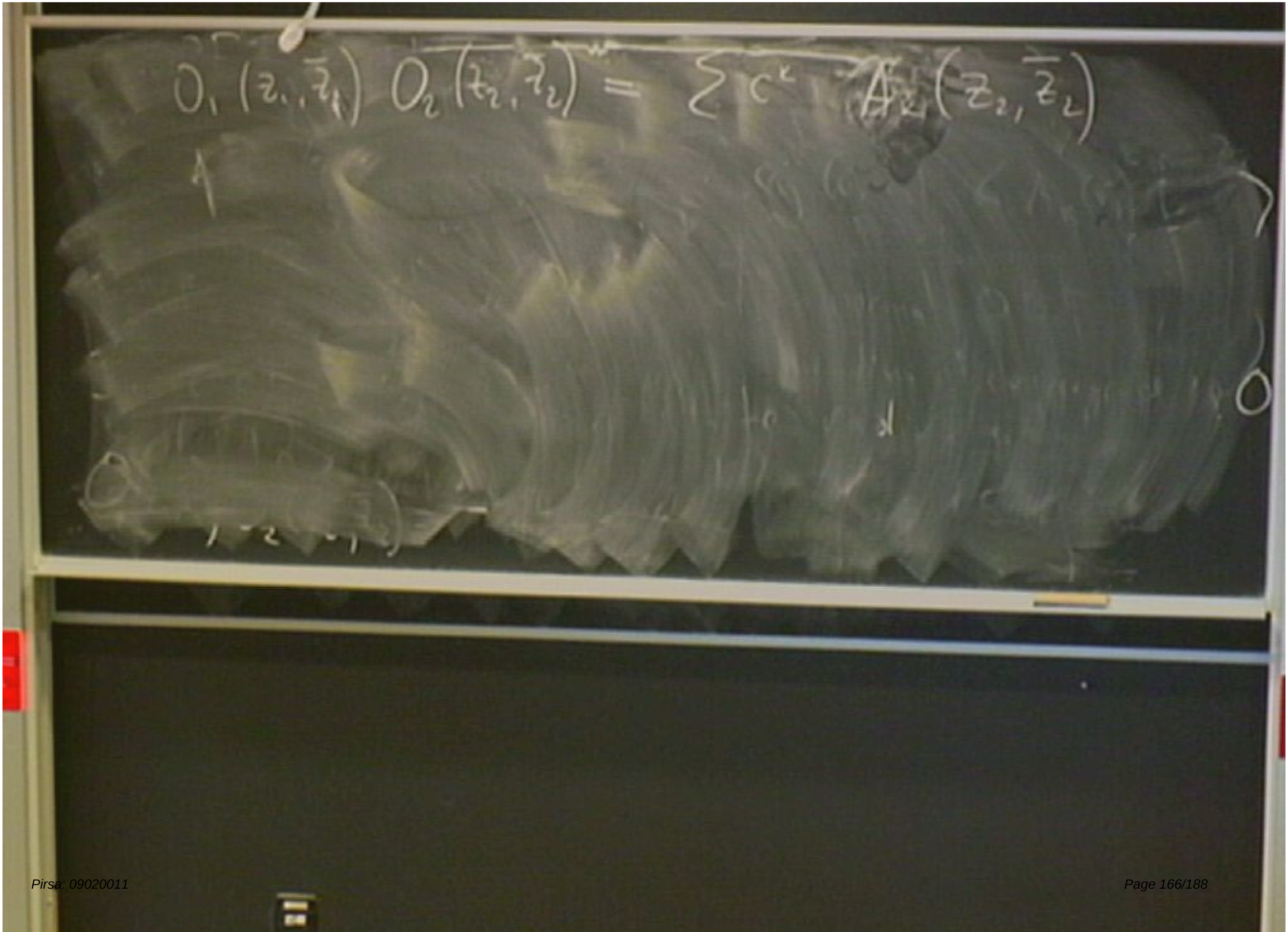
$$:\partial_\xi X^\mu(z_2+\xi, \bar z_2+\bar\xi)\, X^\nu(z_2,\bar z_2): \;=\; \sum_{k=1}^{\infty} \frac{\xi^k}{k!} :X^\nu \partial^k X^\mu:$$

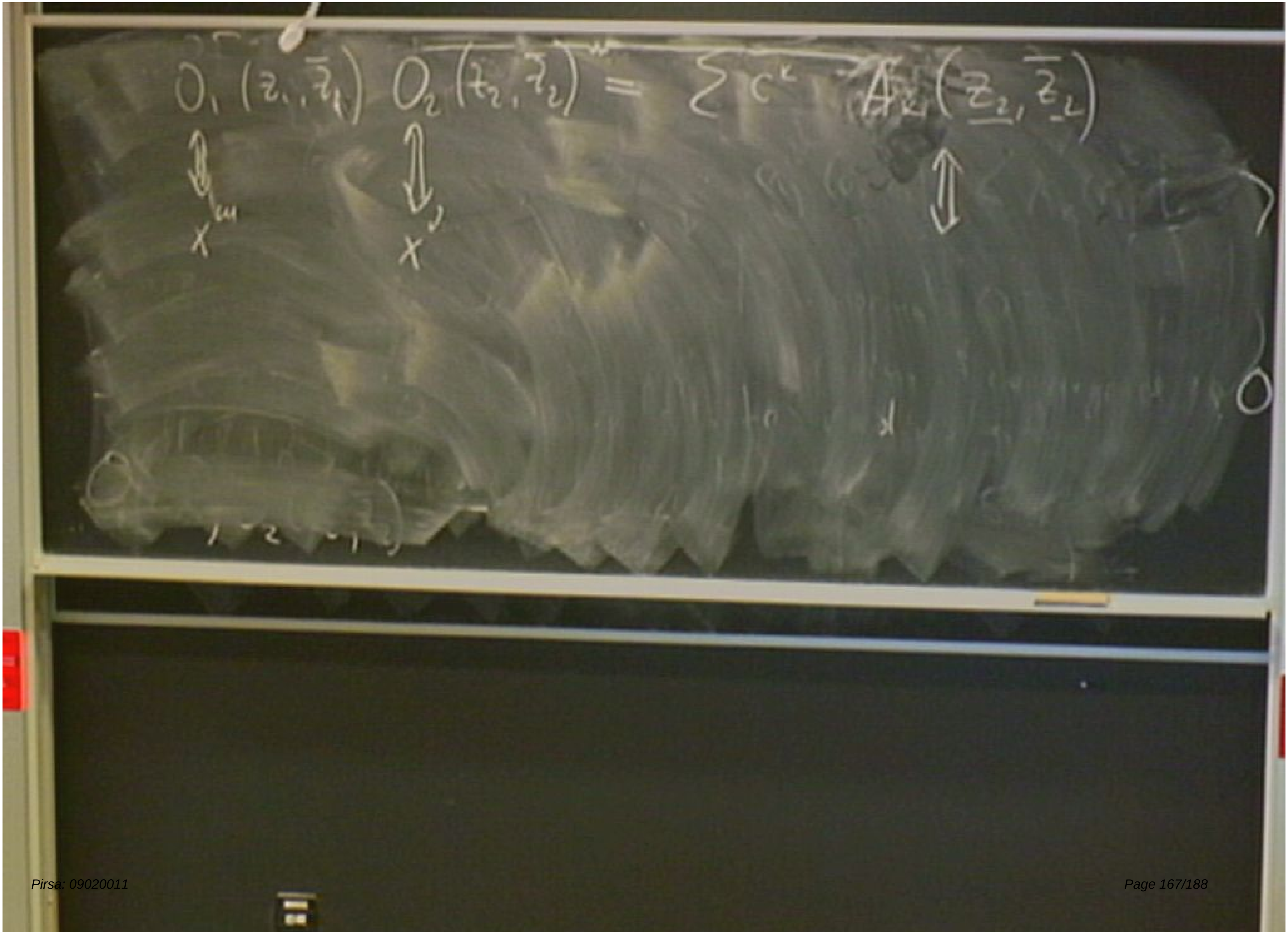
$\xi = z_{12}$

$$= \sum_{k=1}^{\infty} \frac{z_{12}^k}{k!} :X^\nu(z_2,\bar z_2)\, \partial^k X^\mu(z_2,\bar z_2):$$

$$:X^\mu(z_1,\bar z_1)\, X^\nu(z_2,\bar z_2): \;=\; :X^\mu(z_2,\bar z_2)\, X^\nu(z_2,\bar z_2):$$

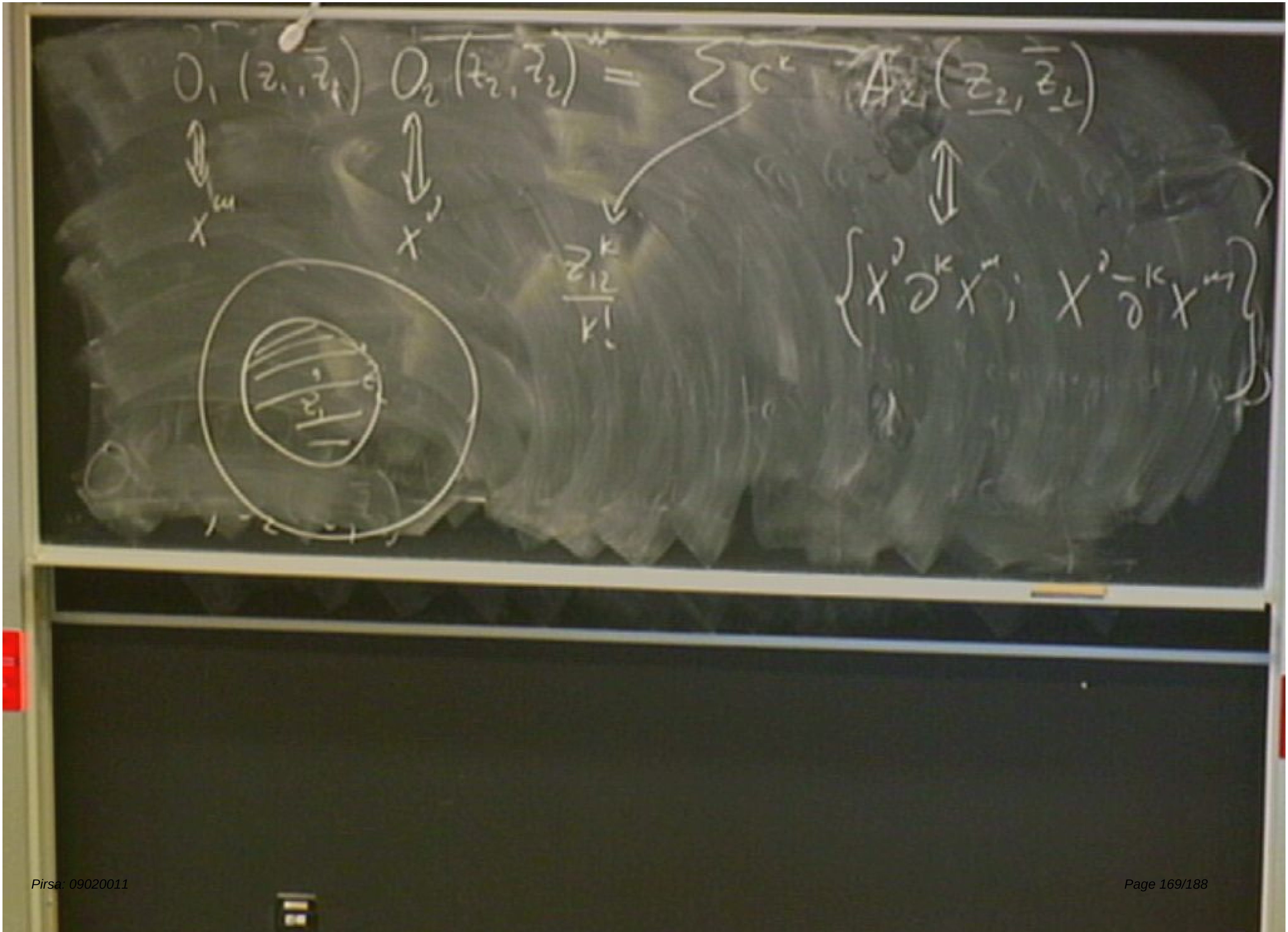
$$+ \sum_{k=1}^{\infty} \left\{ \frac{z_{12}^k}{k!} :X^\nu \partial^k X^\mu: + \frac{\bar z_{12}^k}{k!} :X^\nu \bar\partial^k X^\mu: \right.$$


O1 (z1, z̄1) O2 (z2, z̄2) = Σ c^k Ak (z2, z̄2)

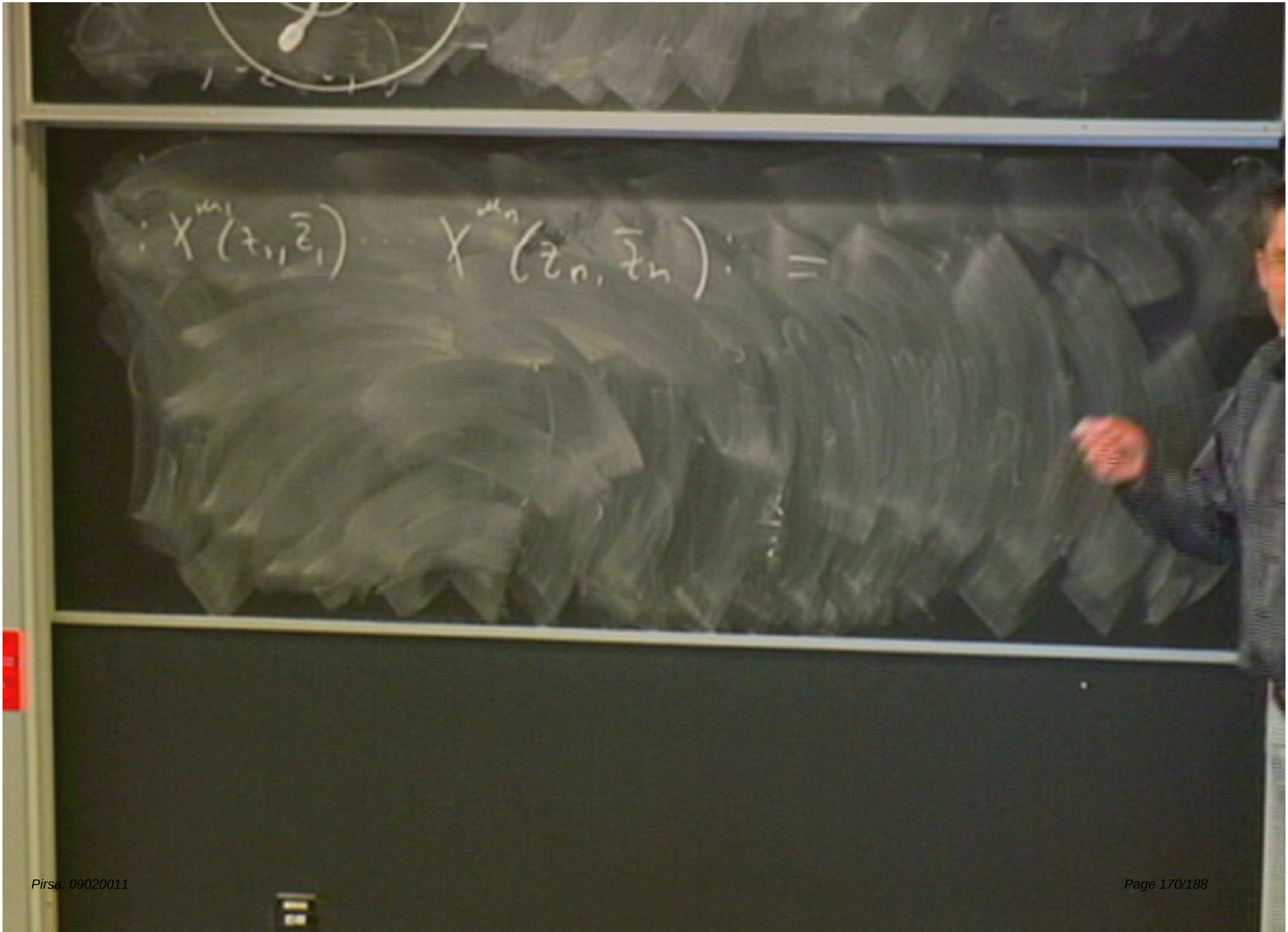

O1 (z1, z̄1) O2 (z2, z̄2) = Σ C^k Ak (z2, z̄2)
X
X

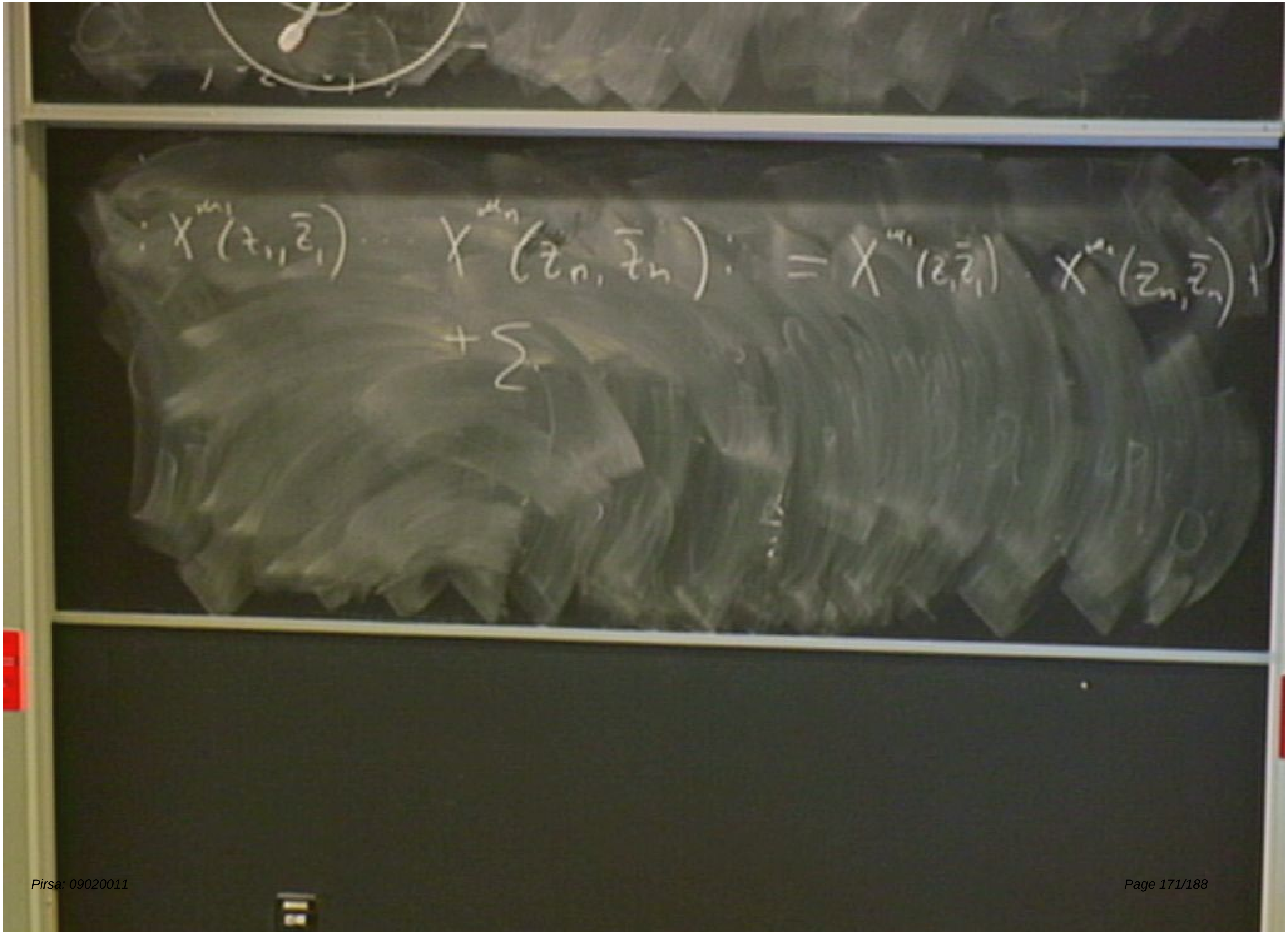
$$O_1(z_1,\bar{z}_1)\, O_2(z_2,\bar{z}_2) = \sum c^k A_k(z_2,\bar{z}_2)$$

$$O_1 \Updownarrow X^\mu \qquad O_2 \Updownarrow X^\nu \qquad A_k \Updownarrow \left\{ X^\nu \partial^k X^\mu ;\ X^\nu \bar{\partial}^k X^\mu \right\}$$



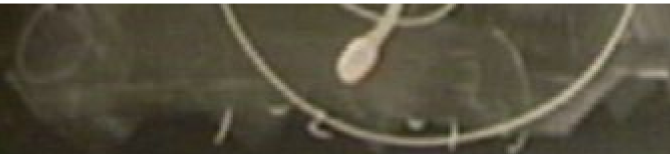
O1 (z1, z̄1) O2 (z2, z̄2) = Σ C^k A_k (z2, z̄2)
X^μ
X^ν
z12^k / k!
{X^ν ∂^k X^μ ; X^ν ∂̄^k X^μ}


:X^μ1(z1, z̄1) ... X^μn(zn, z̄n): =


:X^μ1(z1, z̄1) ⋯ X^μn(zn, z̄n): = X^μ1(z1, z̄1) ⋯ X^μn(zn, z̄n) +
+ Σ

$$:X^{\mu_1}(z_1,\bar{z}_1)\cdots X^{\mu_n}(z_n,\bar{z}_n): \;=\; X^{\mu_1}(z_1,\bar{z}_1)\cdots X^{\mu_n}(z_n,\bar{z}_n) + \sum \text{subtractions}$$

subtraction =

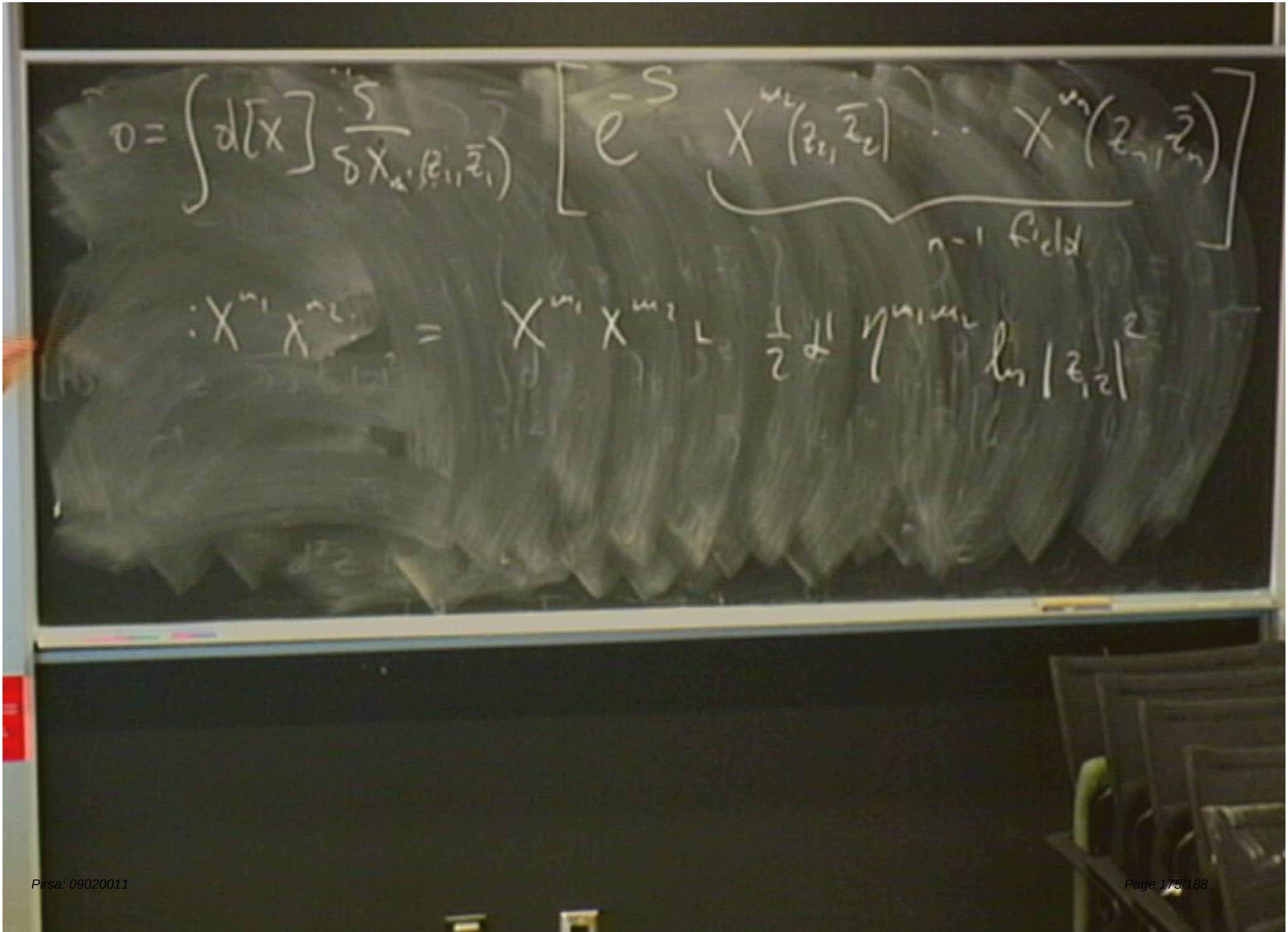


$$:X^{\mu_1}(z_1,\bar z_1)\cdots X^{\mu_n}(z_n,\bar z_n): \;=\; X^{\mu_1}(z_1,\bar z_1)\cdots X^{\mu_n}(z_n,\bar z_n) + \sum \text{subtractions}$$

subtraction = replace a pair of fields

$$\frac{1}{2}\alpha' \eta^{\mu_i\mu_j} \ln|z_{ij}|^2$$

$$0 = \int d[X] \, \frac{\delta}{\delta X_{\mu_1}(z_1, \bar{z}_1)} \left[e^{-S} \, X^{\mu_2}(z_2, \bar{z}_2) \cdots X^{\mu_n}(z_n, \bar{z}_n) \right]$$

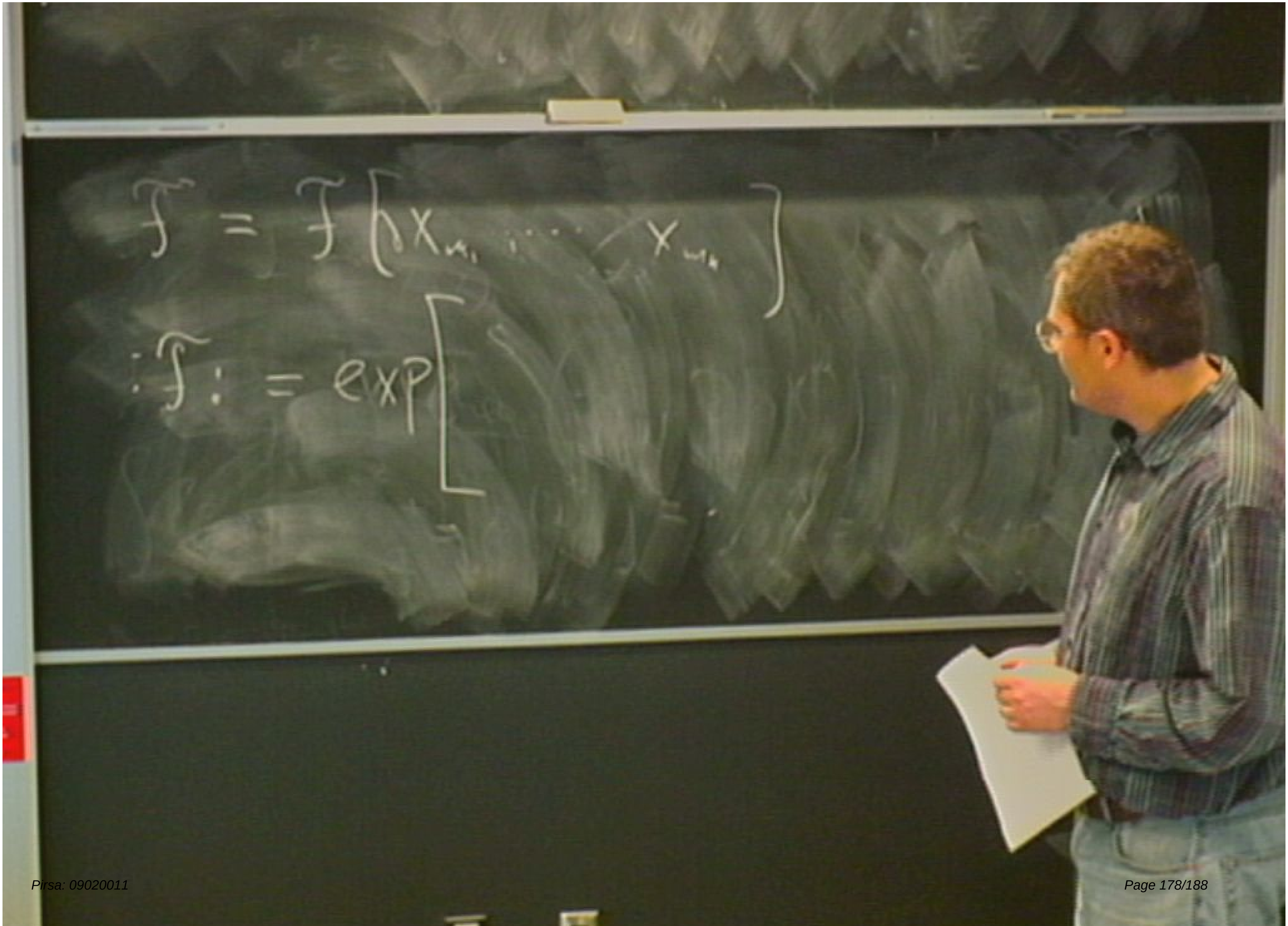

$0 = \int d[X] \frac{\delta}{\delta X_{\mu_1}(z_1,\bar z_1)} \left[e^{-S} X^{\mu_2}(z_2,\bar z_2) \cdots X^{\mu_n}(z_n,\bar z_n) \right]$
n-1 field
$:X^{\mu_1} X^{\mu_2}: = X^{\mu_1} X^{\mu_2} + \frac{1}{2}\alpha' \eta^{\mu_1\mu_2} \ln|z_{12}|^2$

$$:X^{\mu_1}(z_1,\bar{z}_1)\cdots X^{\mu_n}(z_n,\bar{z}_n): \; = X^{\mu_1}(z_1,\bar{z}_1)\cdots X^{\mu_n}(z_n,\bar{z}_n) + \sum \text{subtractions}$$

subtraction = replace a pair of fields

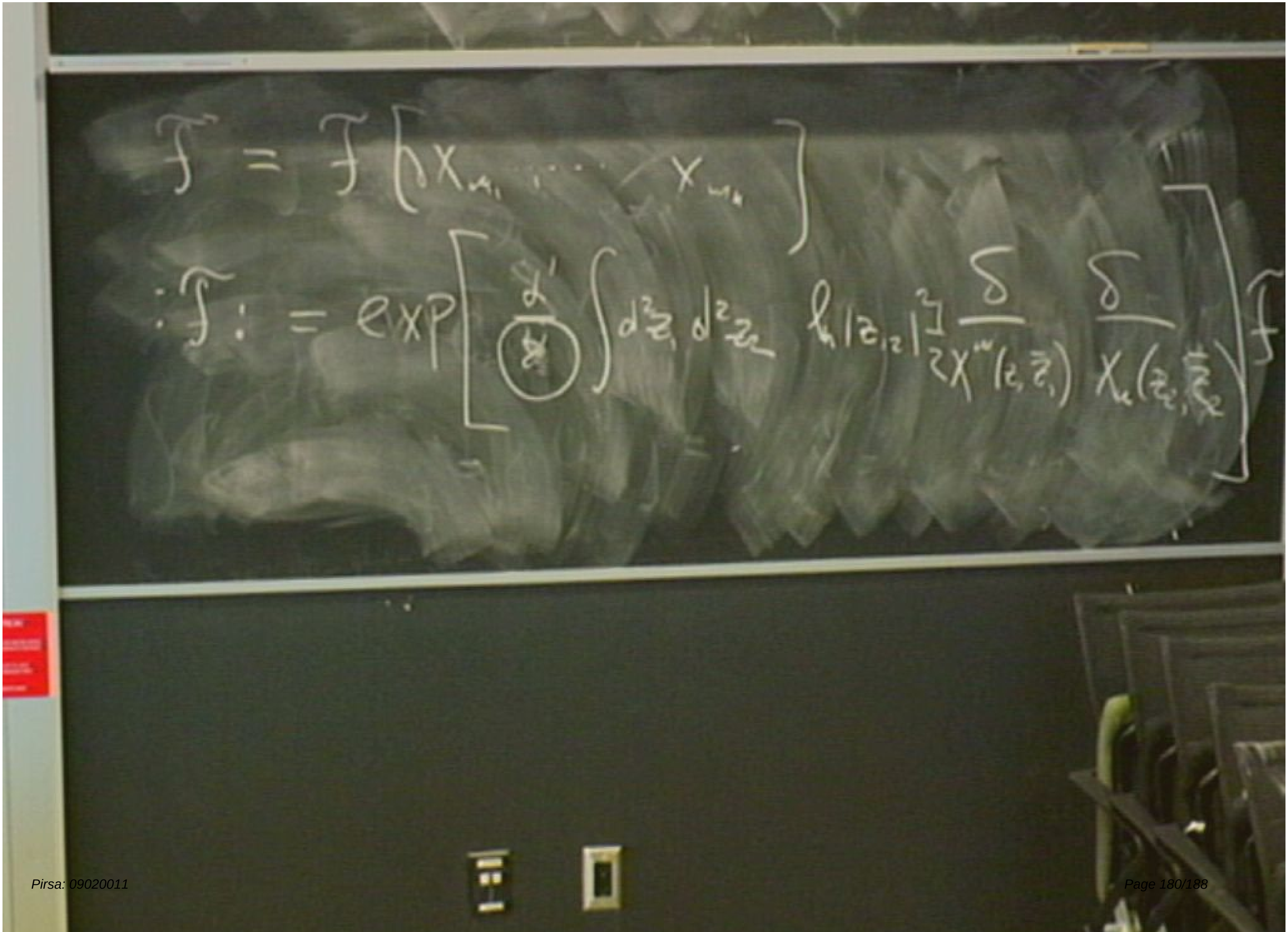
$$\frac{1}{2}\alpha' \eta^{\mu_i \mu_j} \ln |z_{ij}|^2$$


F = F [X_{w_1}, ..., X_{w_k}]

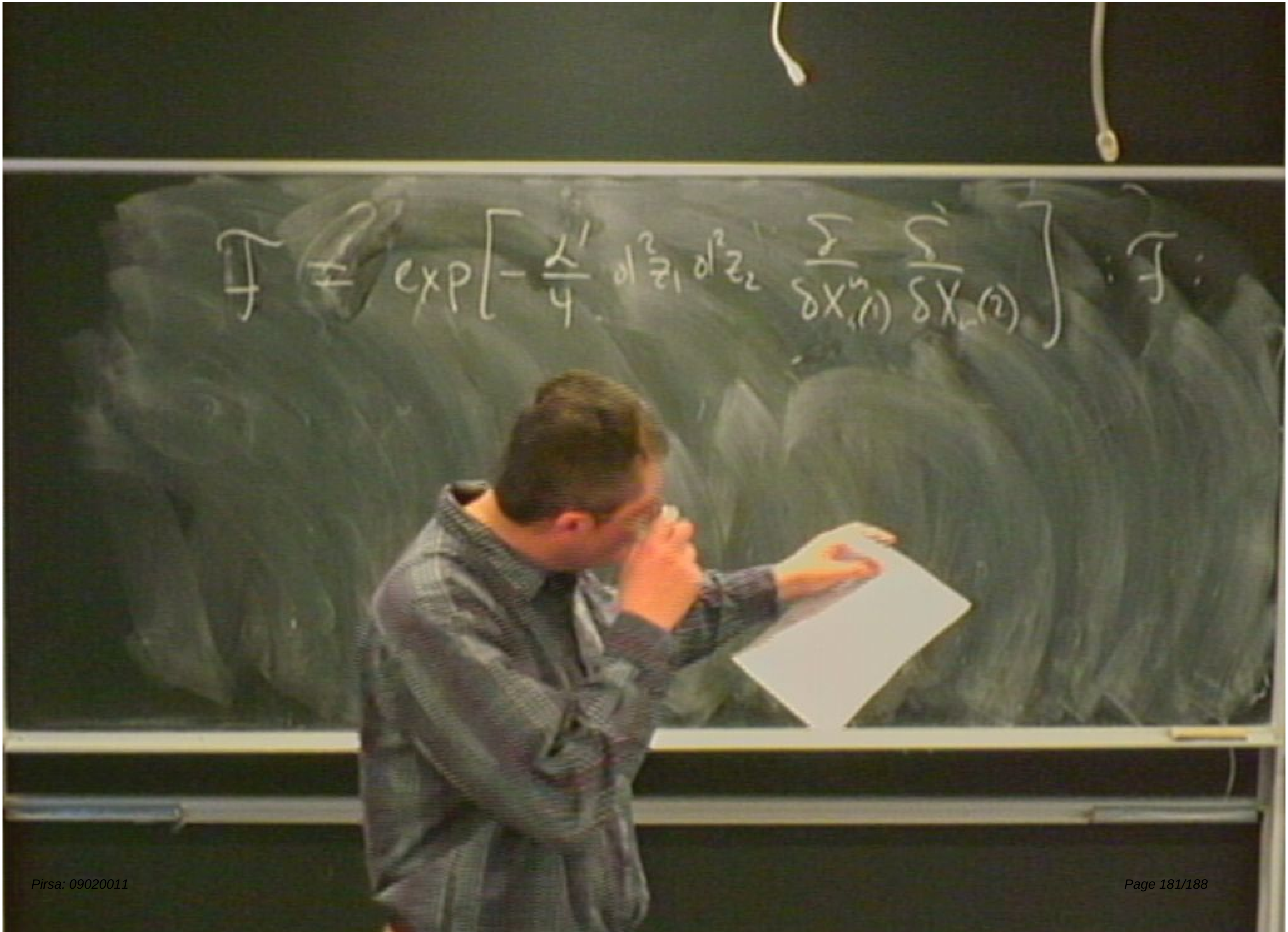

F = F[X_{μ1}, ..., X_{μk}]
:F: = exp[

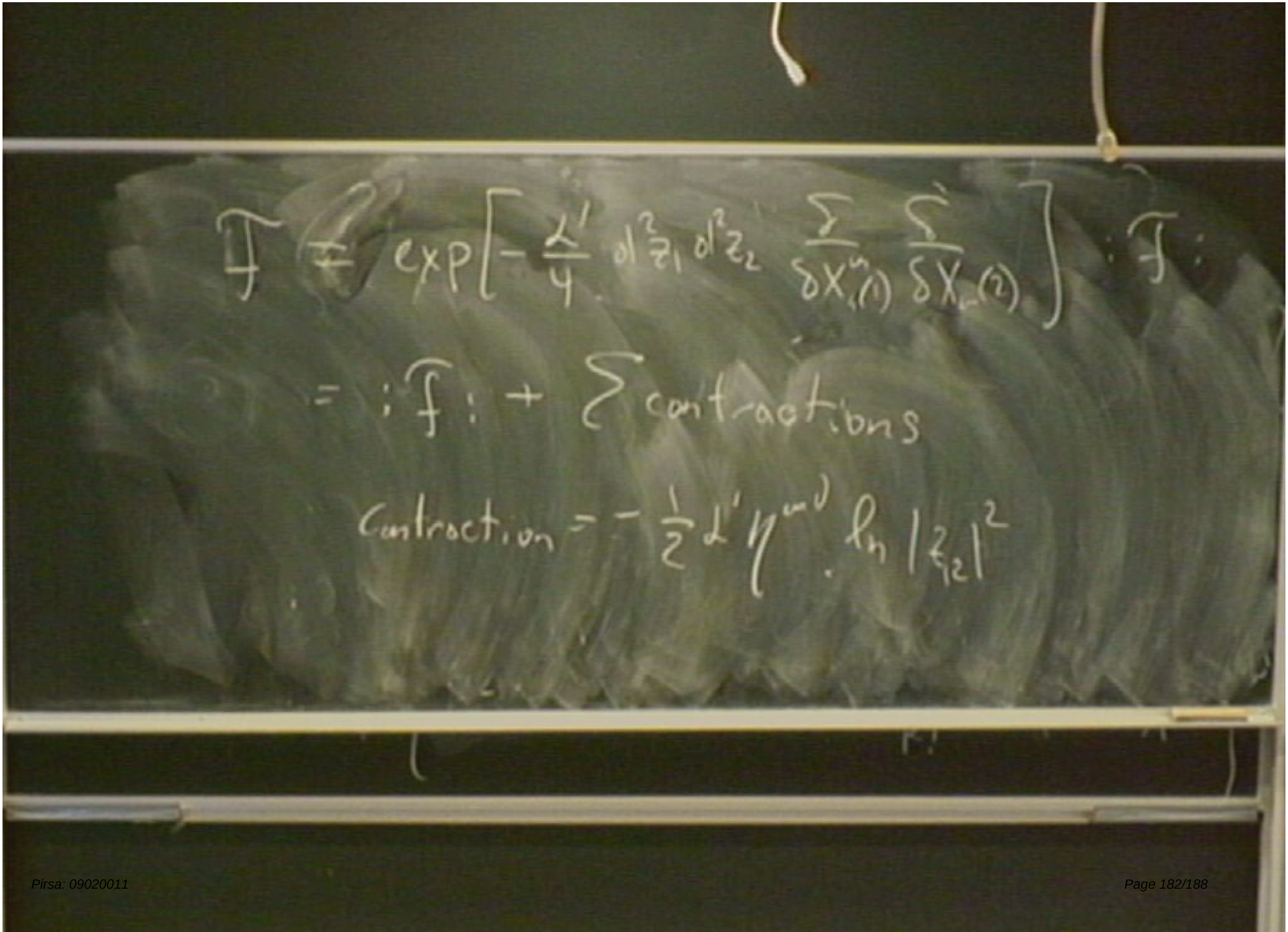
$$\mathcal{F} = \mathcal{F}[X_{\mu_1}, \ldots, X_{\mu_k}]$$

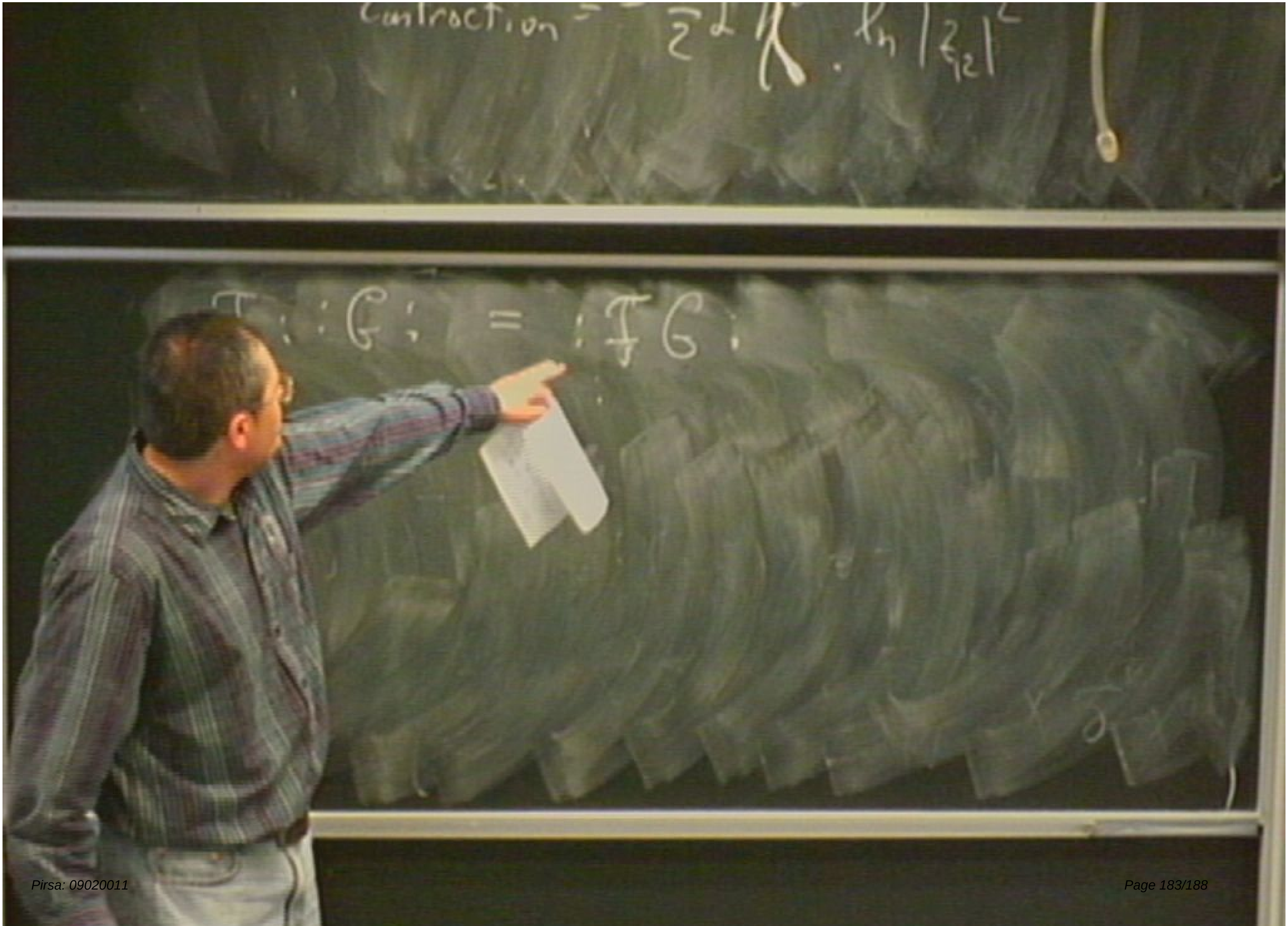
$$:\mathcal{F}: = \exp\left[\frac{\alpha'}{4} \int d^2 z_1\, d^2 z_2 \, \ln |z_{12}|^2 \, \frac{\delta}{\delta X^{\mu}(z_1, \bar{z}_1)} \, \frac{\delta}{\delta X_{\mu}(z_2, \bar{z}_2)} \right] \mathcal{F}$$

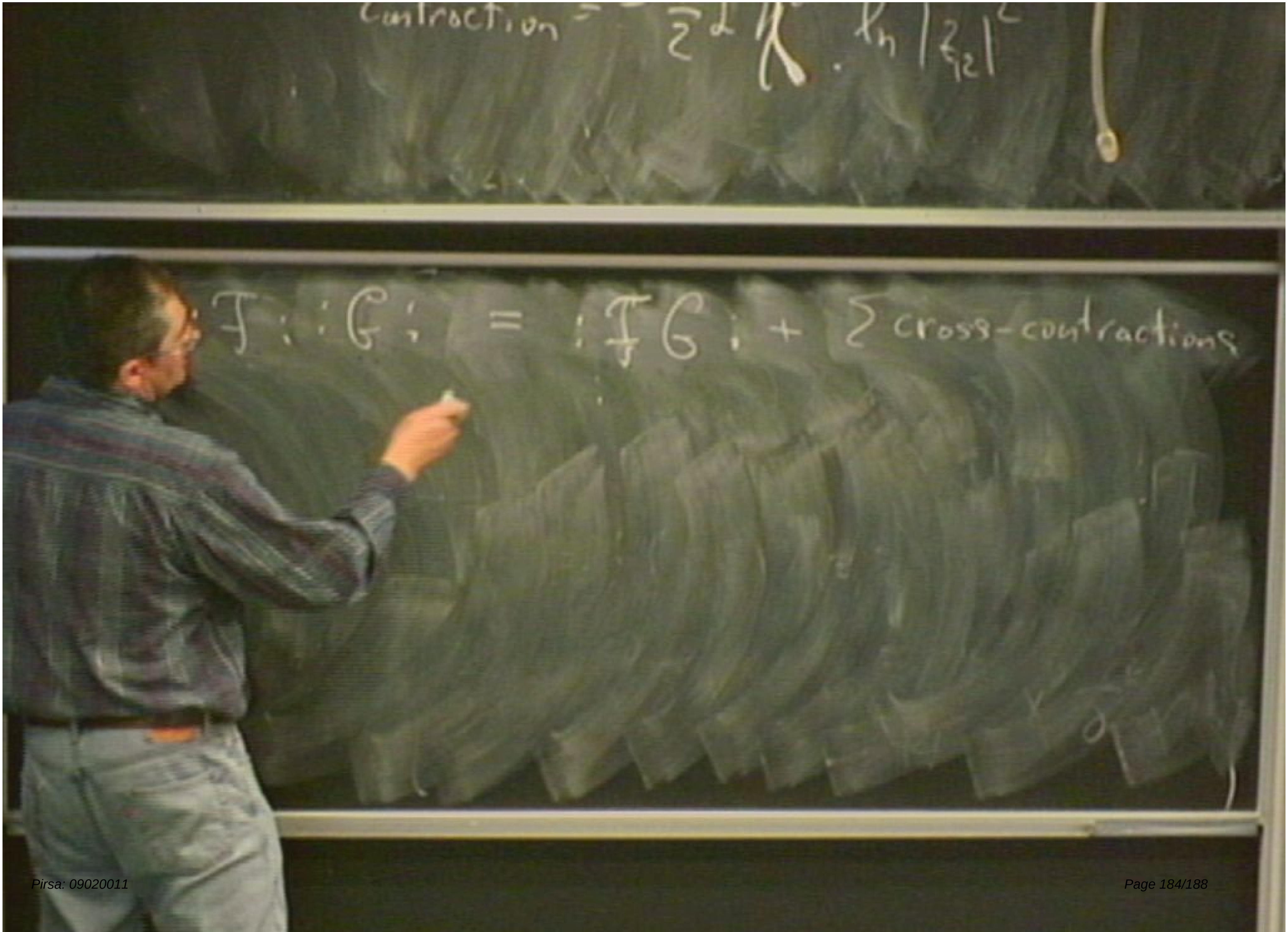


$\tilde{F} = F[X_{\mu_1}, \dots, X_{\mu_n}]$
$:\tilde{F}: = \exp\left[\alpha' \int d^2z_1\, d^2z_2 \ln|z_{12}|^2 \frac{\delta}{\delta X^{\mu}(z_1,\bar{z}_1)} \frac{\delta}{\delta X_{\mu}(z_2,\bar{z}_2)} \right] \tilde{F}$


F = exp[- α'/4 d²z₁ d²z₂ δ/δX^μ(1) δ/δX_μ(2)] :F:


$\mathcal{F} = \exp\left[-\frac{\alpha'}{4} \int d^2z_1 \, d^2z_2 \, \frac{\delta}{\delta X^{\mu}(1)} \frac{\delta}{\delta X_{\mu}(2)} \right] :\mathcal{F}:$
$= :f: + \sum$ contractions
contraction $= -\frac{1}{2} \alpha' \eta^{\mu\nu} \ln |z_{12}|^2$


contraction


contraction
: F : : G : = : F G : + Σ cross-contractions

contraction $= -\frac{\alpha'}{2}\eta \cdot \ln|z_{12}|^2$

$$:\mathcal{F}:\,:\mathcal{G}: \;=\; :\mathcal{F}\mathcal{G}: + \sum \text{cross-contractions}$$

$$:\mathcal{F}:\,:\mathcal{G}: \;=\; \exp\left[-\frac{\alpha'}{2}\int d^2z_1\, d^2z_2\, \ln|z_{12}|^2 \right.$$

contraction $= -\frac{\alpha'}{2} \eta^{\mu\nu} \ln |z_{12}|^2$

$$:\mathcal{F}: \, :\mathcal{G}: \; = \; :\mathcal{F}\mathcal{G}: + \sum \text{cross-contractions}$$

$$:\mathcal{F}: \, :\mathcal{G}: = \exp\left[-\frac{\alpha'}{2} \int d^2z_1 \, d^2z_2 \ln |z_{12}|^2 \frac{\delta}{\delta X^\mu_F(z_1,\bar{z}_1)} \frac{\delta}{\delta \right.$$


contraction
cross-contractions

contraction $= -\frac{\alpha'}{2} \eta^{\mu\nu} \ln |z_{12}|^2$

$$:\mathcal{F}:\,:\mathcal{G}: = :\mathcal{F}\mathcal{G}: + \sum \text{cross-contractions}$$

$$:\mathcal{F}:\,:\mathcal{G}: = \exp\left[-\frac{\alpha'}{2}\int d^2z_1\, d^2z_2 \ln|z_{12}|^2 \frac{\delta}{\delta X^\mu_F(z_1,\bar z_1)} \frac{\delta}{\delta X_{\mu,G}(z_2,\bar z_2)}\right]$$

$$\times :\mathcal{F}\mathcal{G}:$$