Title: Warped Wilson Line Inflation

Date: Feb 17, 2009 02:00 PM

URL: http://pirsa.org/09020005

Abstract: I will discuss the possibility that a 'Wilson line' degree of freedom can play the role of an inflaton in a warped flux compactification, in the context of the DBI inflationary scenario. I will show how warped DBI Wilson line inflation offers an attractive alternative to ordinary (position field) DBI inflation, inasmuch as observational and theoretical constraints get considerably relaxed. Thus, besides the large non-Gaussianities produced in DBI scenarios, Wilson lines allow for an observable amount of gravitational waves, within consistent approximations.

Warped Wilson Line Inflation

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and
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Based on: arXiv:0810.5001
JCAP01(2009)045

In collaboration with:
A. Avgoustidis

Outline

- D-brane Inflation:
  - i) Slow roll: unwarped, warped
  - ii) DBI Inflation: "fast roll"
    - Position field DBI (warped)
    - Problems in position DBI inflation
- Warped Wilson Line Inflation
  - Motivation
  - Wilson lines and D-branes
  - Wilson line DBI inflation

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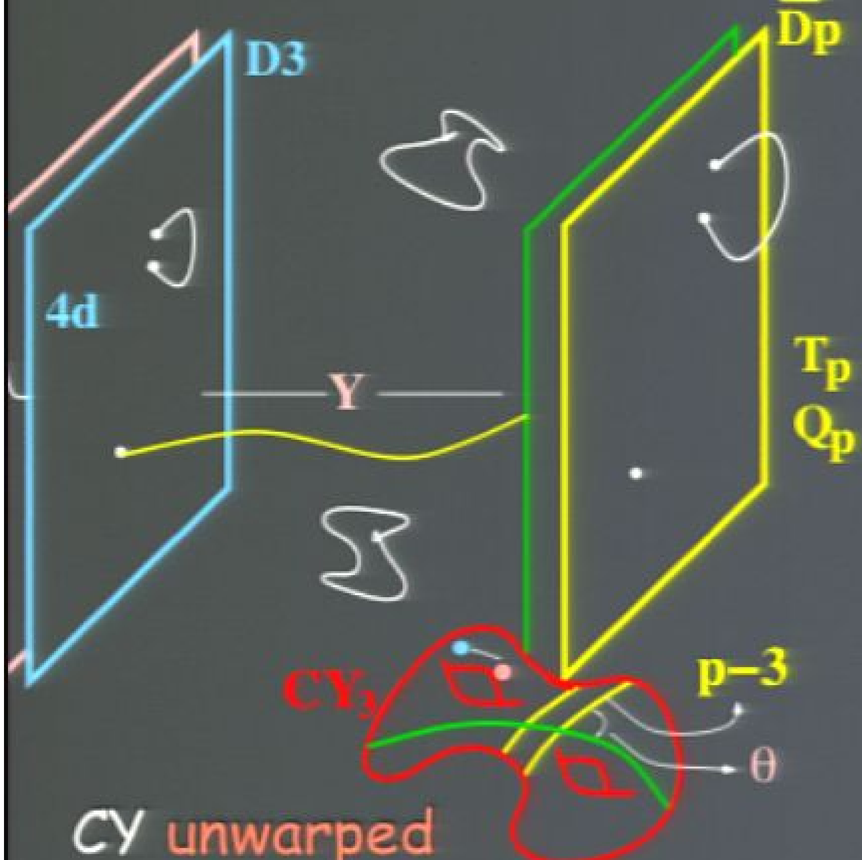
D-Brane inflation.

- Consider type II string theory (unwarped) compactification

$$M_4 \times X_6$$



CY unwarped compactification

- A pair of Dp-branes expanding the full M_4 and wrapping (p-3) dimensions in the compact space X_6.

- The Dp-branes are separated by a distance Y in the compact space.

- Open string modulus associated to brane position acts as the inflaton.

['98 Dvali-Tye]

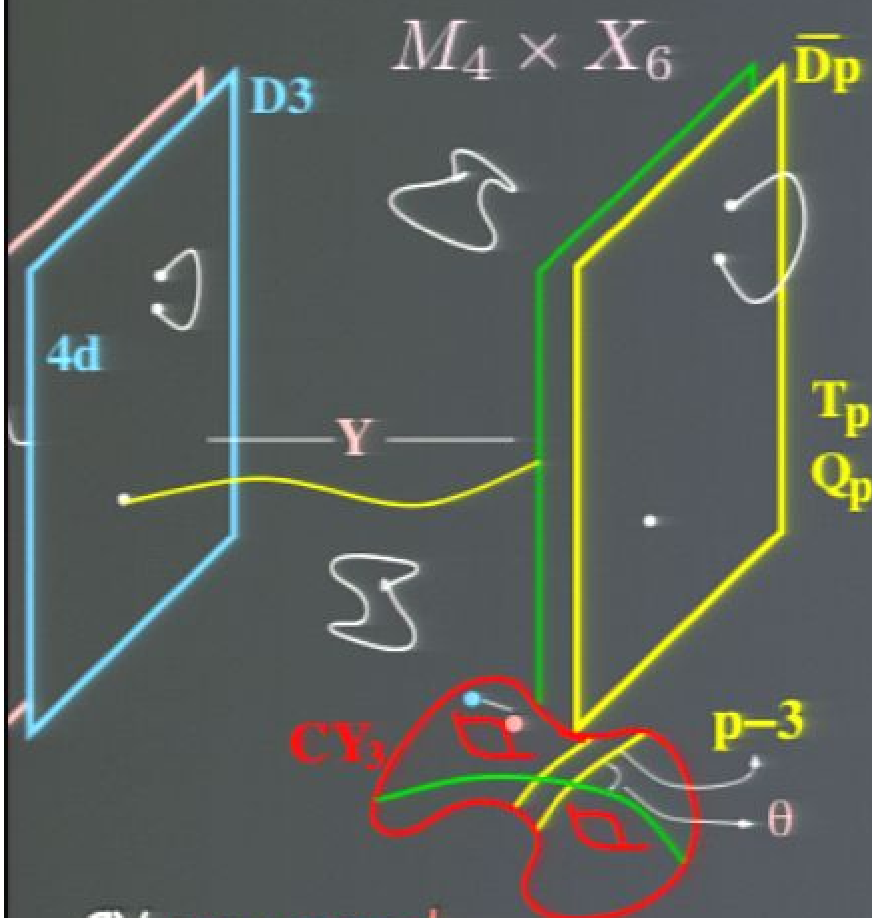
- The potential for the 4D scalar field $V(Y)$ associated to brane position can be calculated.

- If the system preserves susy $V(Y) = 0$

D-Brane Inflation

(unwarped)

Break susy slightly: interaction potential is generated $V(Y) \neq 0$



- Require flat potential: slow roll inflation.

$$\epsilon \equiv \frac{M_P^2}{2}\left(\frac{V'}{V}\right)^2 \ll 1$$

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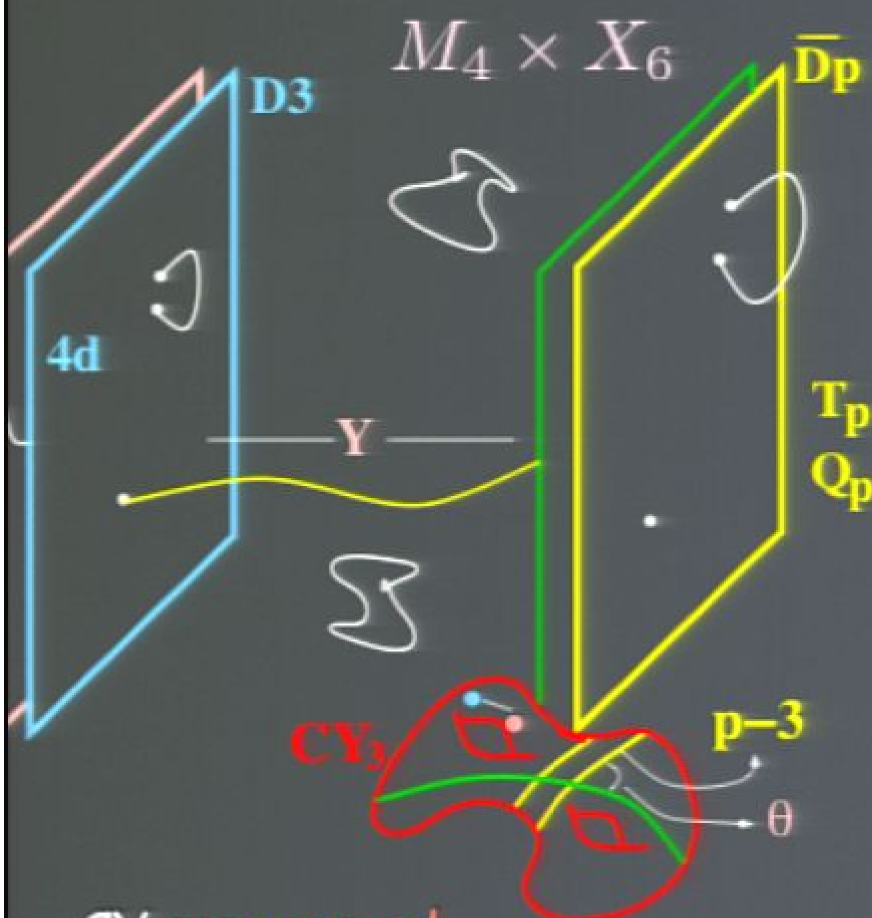

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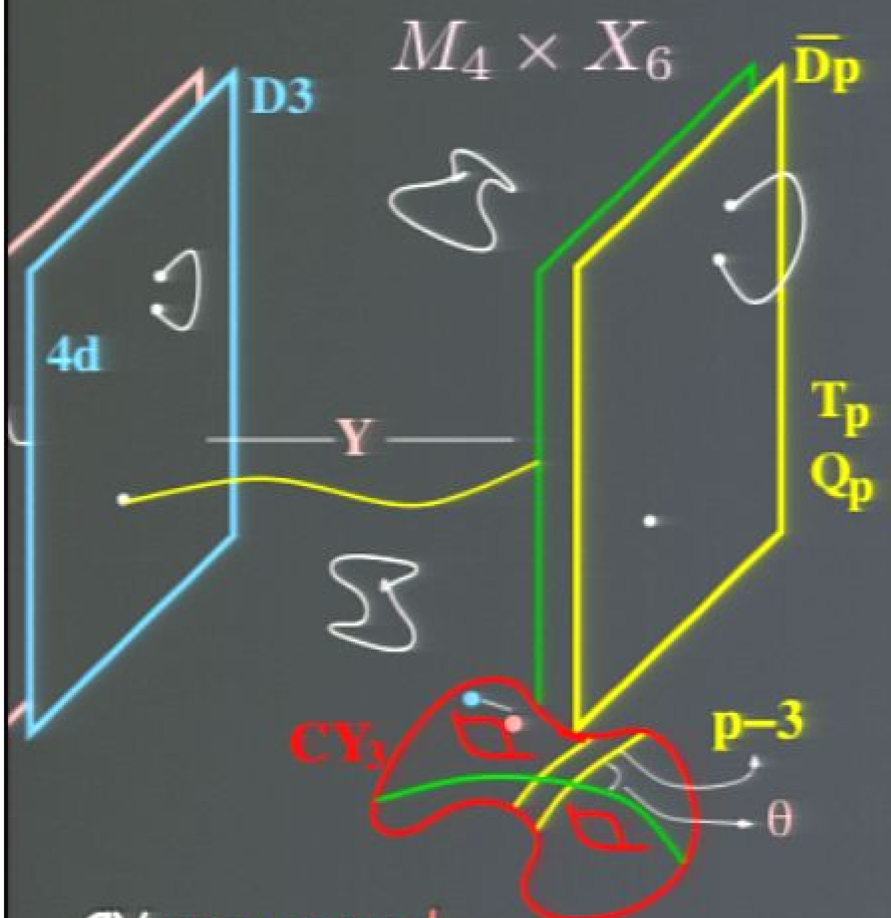

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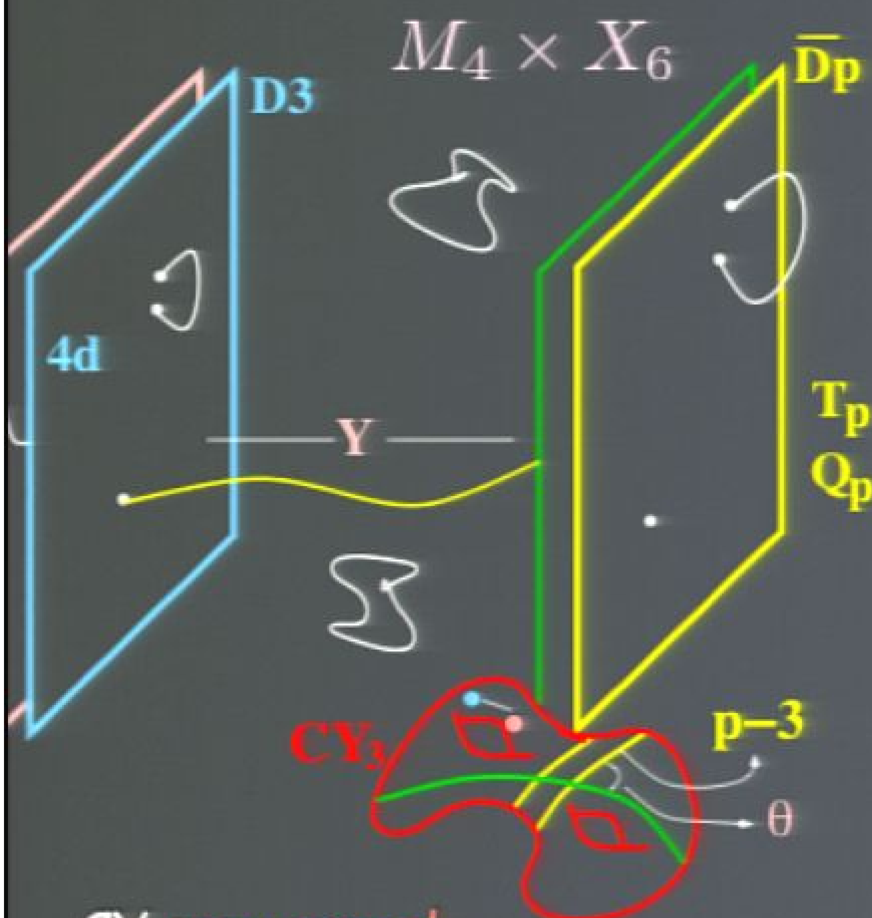

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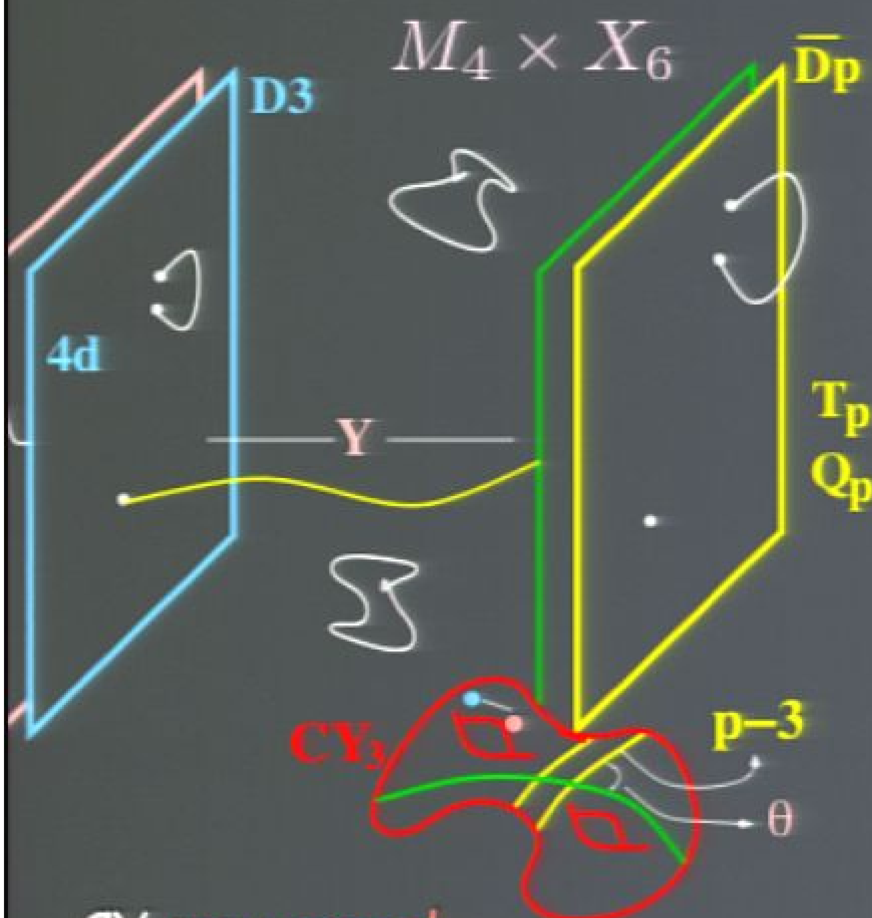

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D-Brane Inflation
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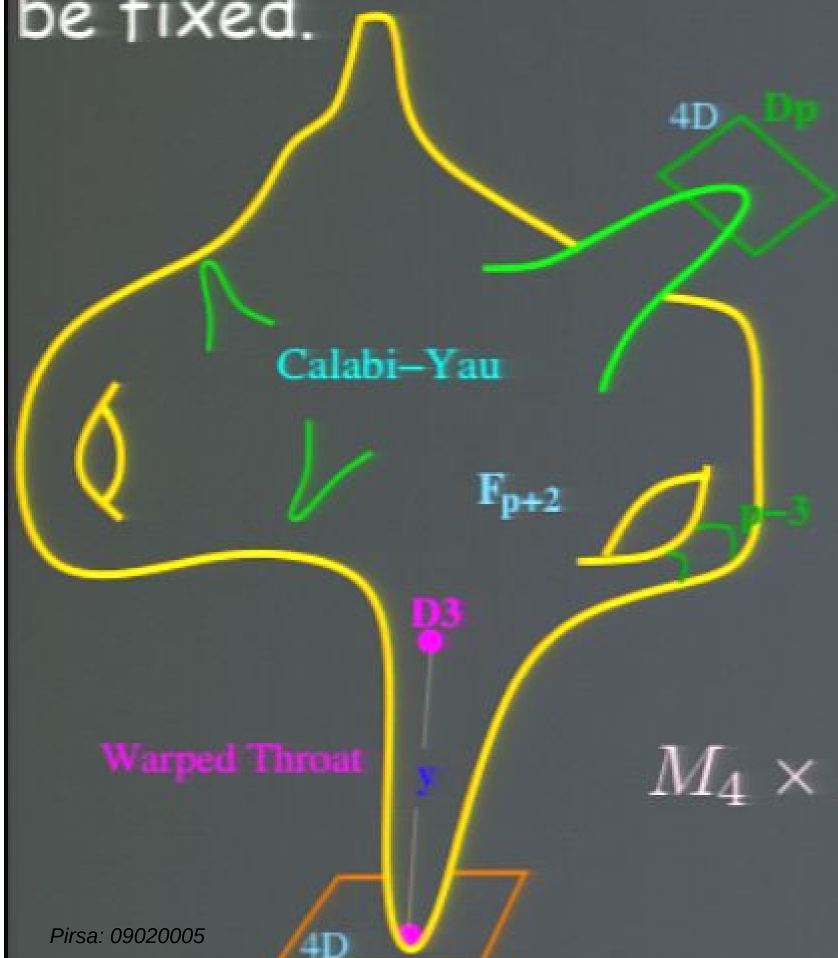

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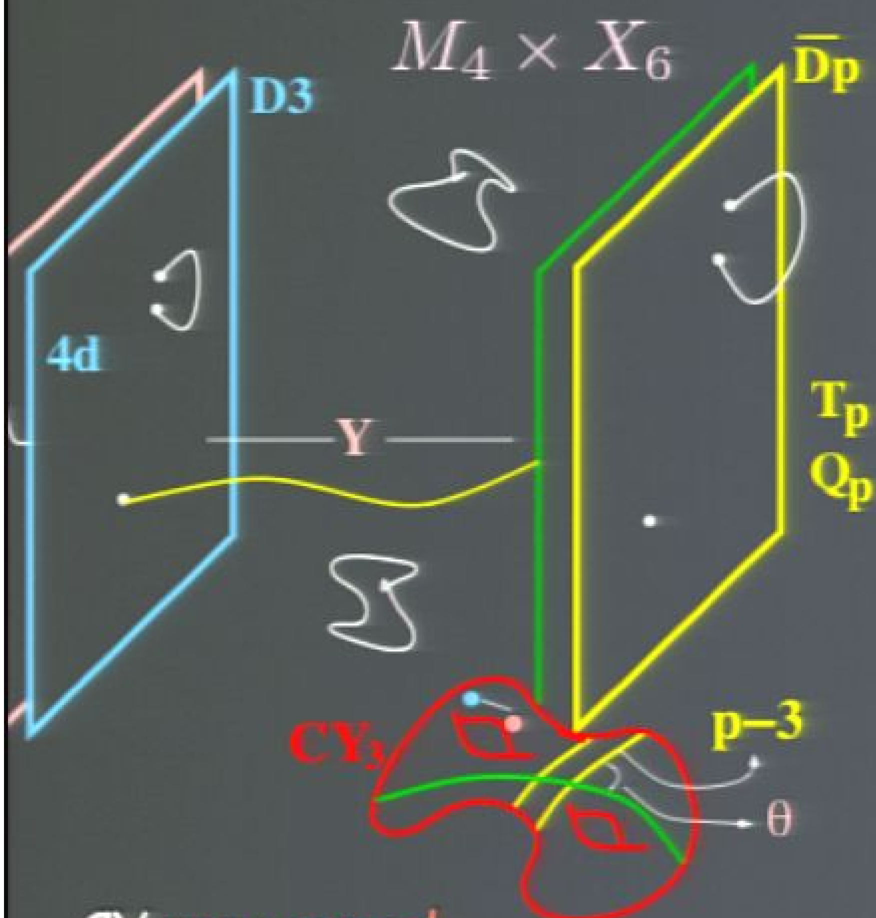

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➡ Brane-antibrane $D\overline{D}$ $(\theta = \pi)$ [Burgess et.al. '01]

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CY unwarped compactification

All other moduli are fixed (stabilised)

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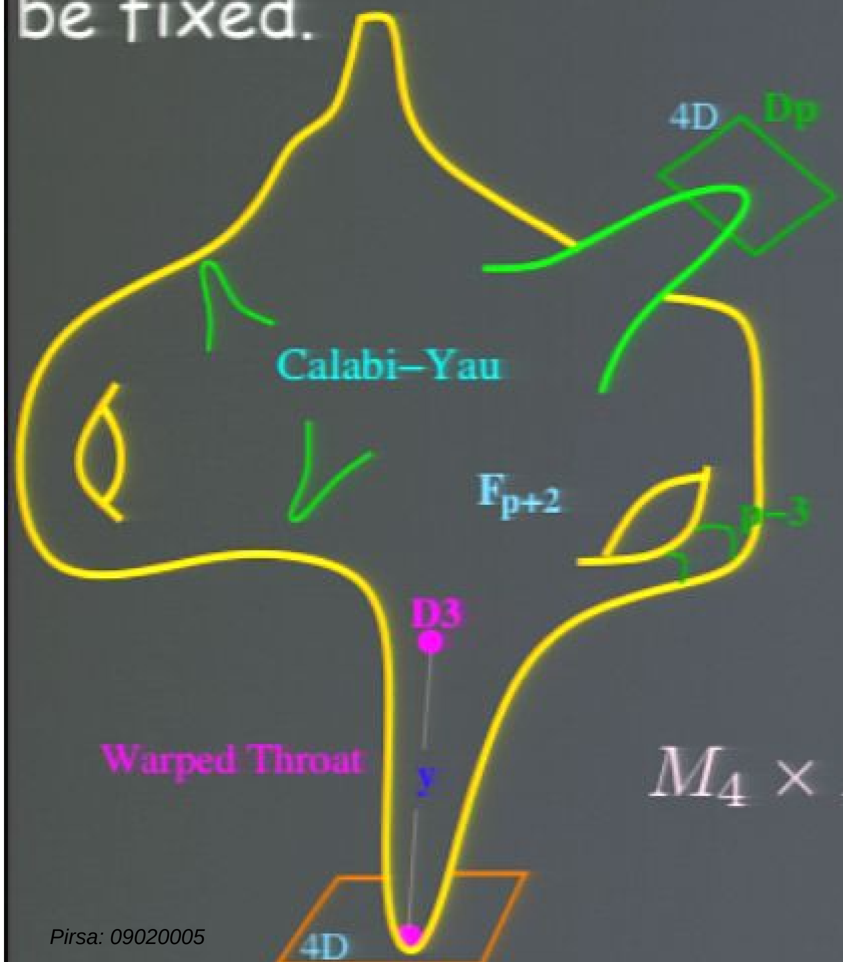

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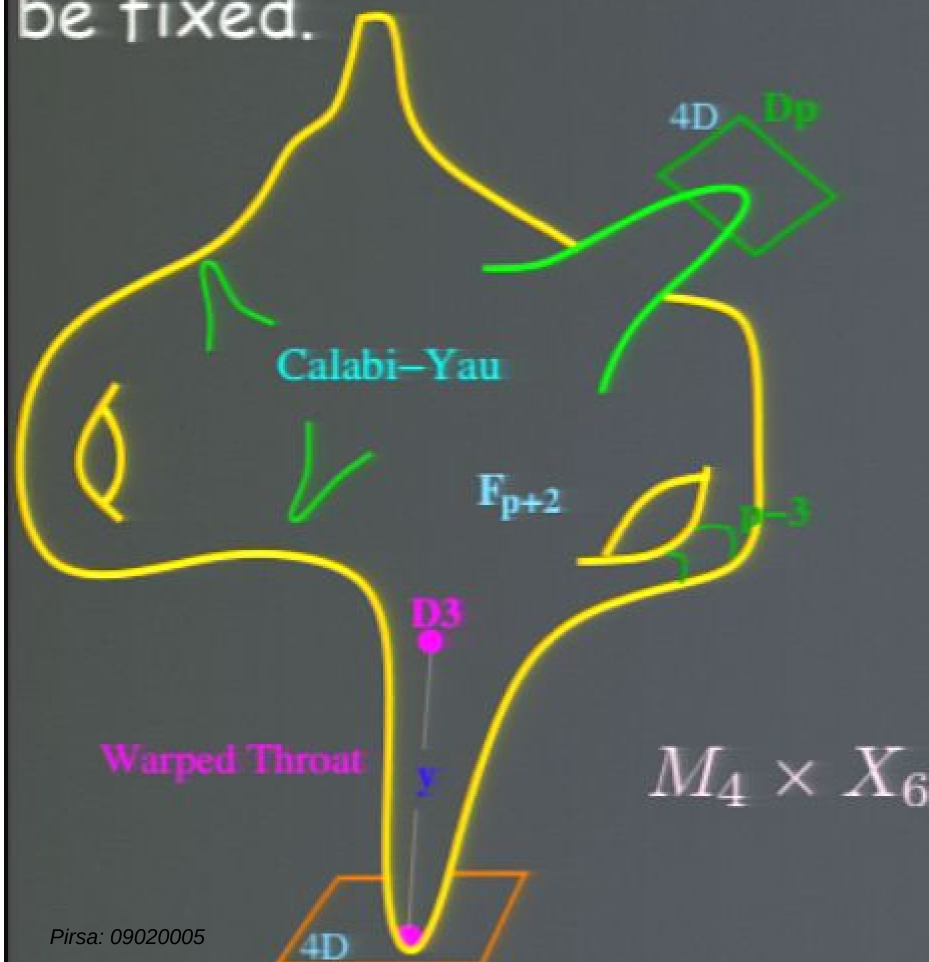

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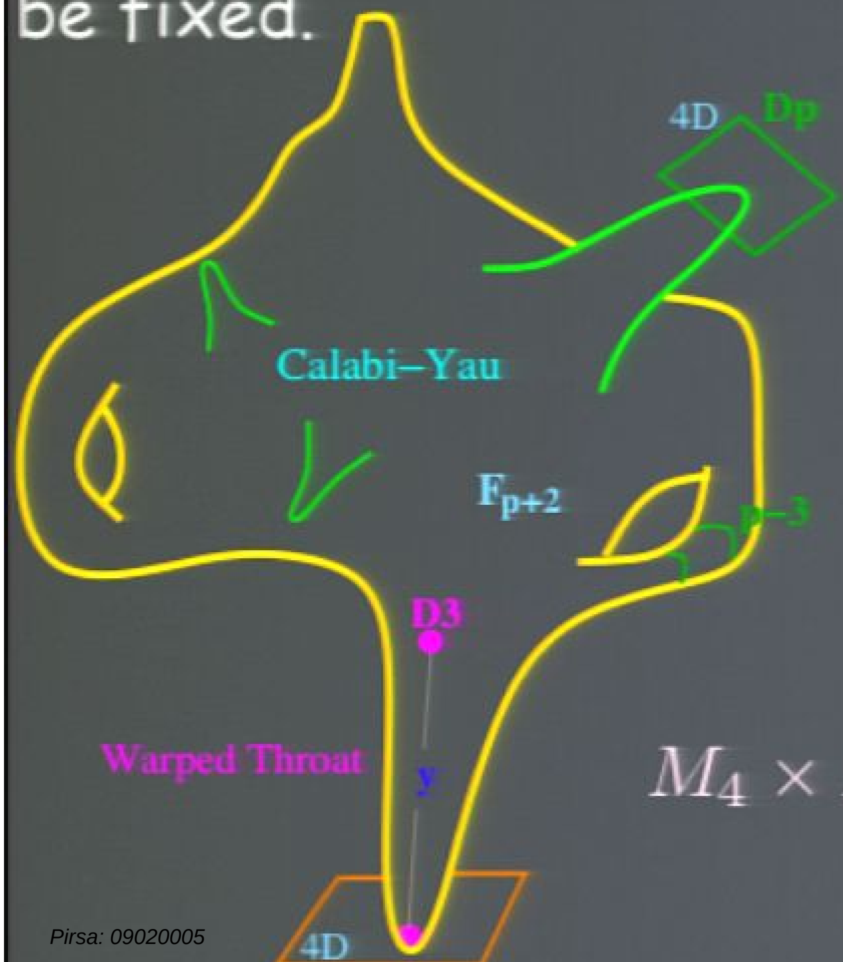

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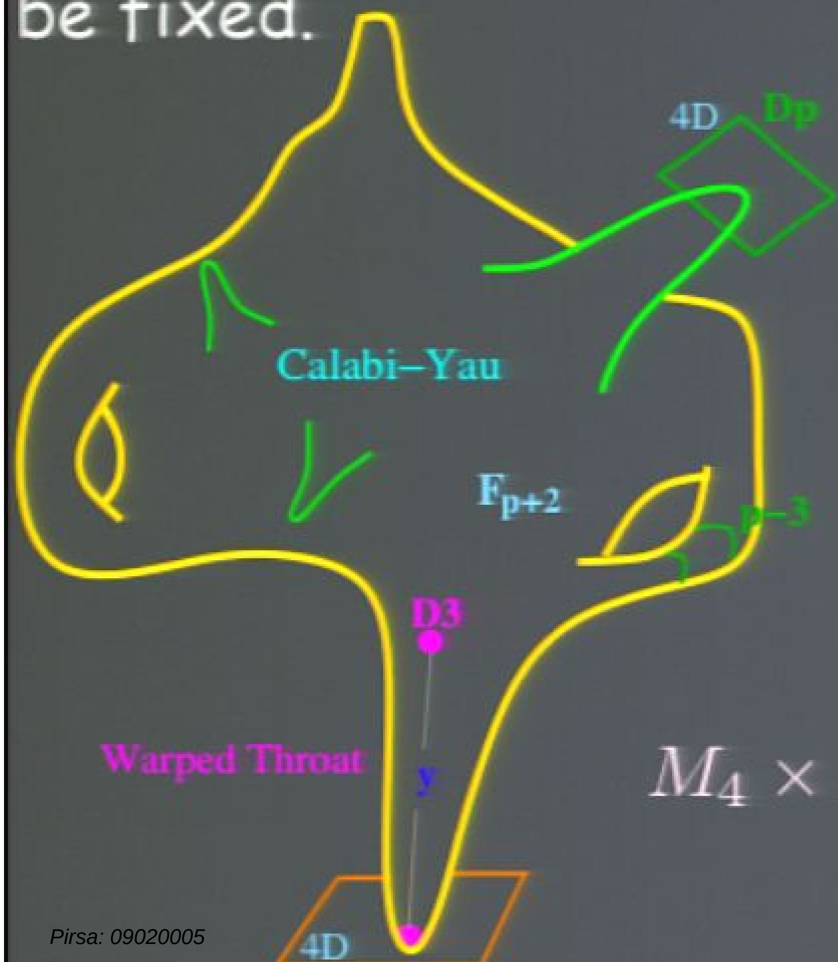

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η problem!

D-Brane Inflation: DBI scenario

['04 Silverstein-Tong]

- For a probe Dp-brane moving along radial direction in a warped geometry with fluxes (F_{n+2}).

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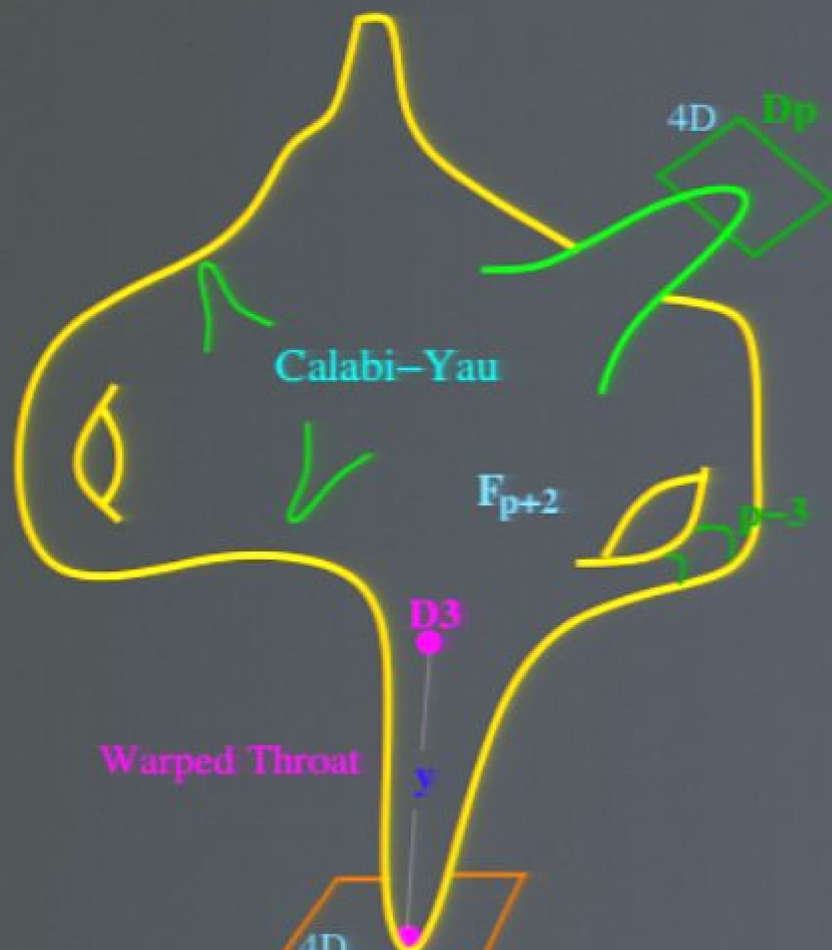

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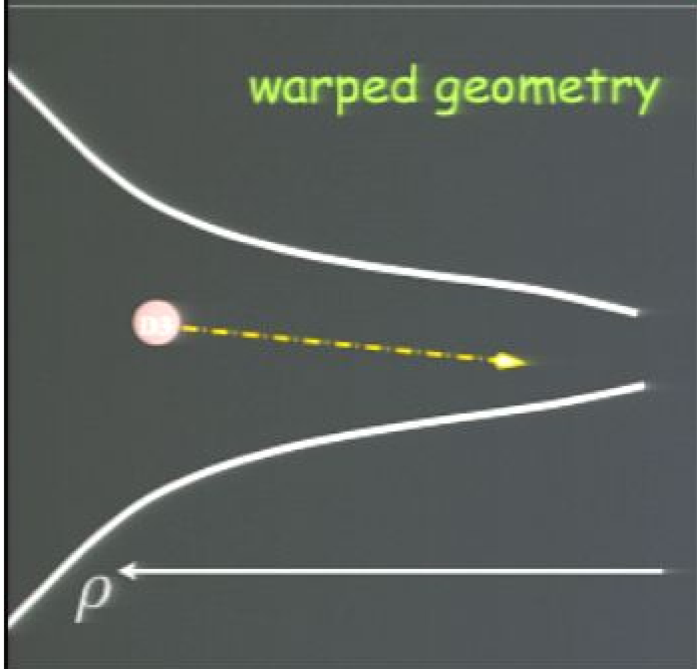
- In a warped background, these kinetic terms, combined with strong warping can give rise to accelerating trajectories, even for steep potentials ("fast roll", no η problem).

DBI scenario



- Consider D3-brane moving along radial direction in a warped geometry in type IIB (e.g $AdS_5 \times S_5$, Klebanov-Strassler).

$$ds_{10}^2 = h^{-1/2}(\rho)\, dx_\mu dx^\mu + h^{1/2}(\rho) ds_6^2$$

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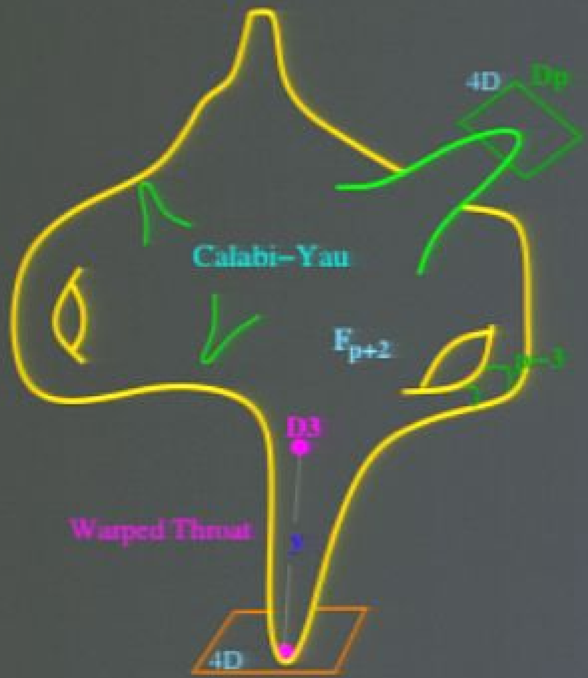

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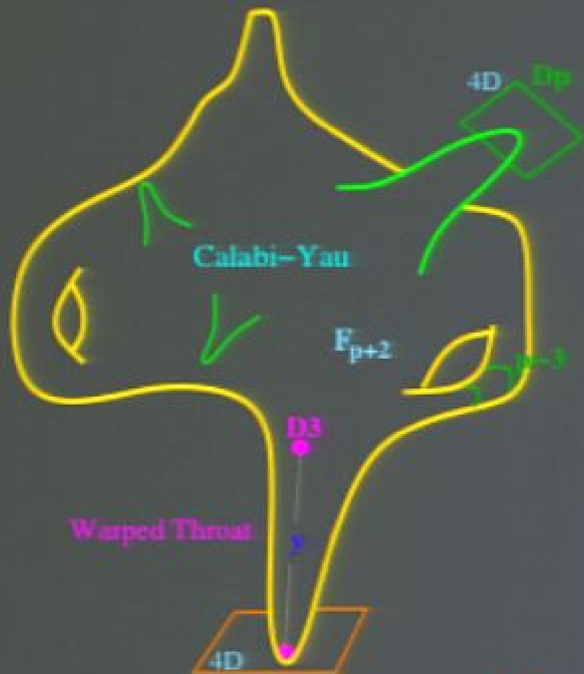

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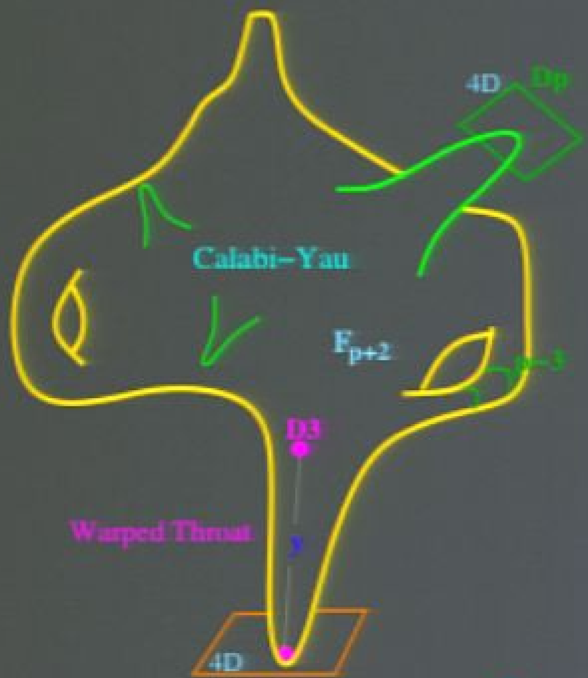

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α' = string scale
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$N = F_5$ flux units
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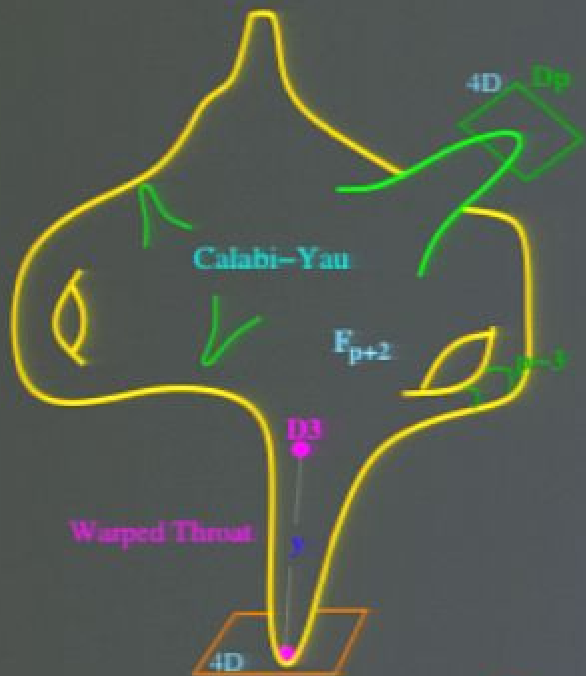

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  => small curvatures

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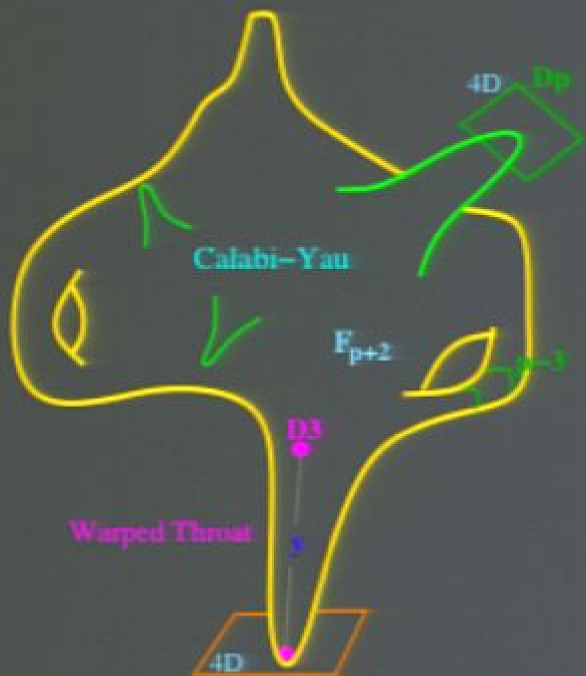

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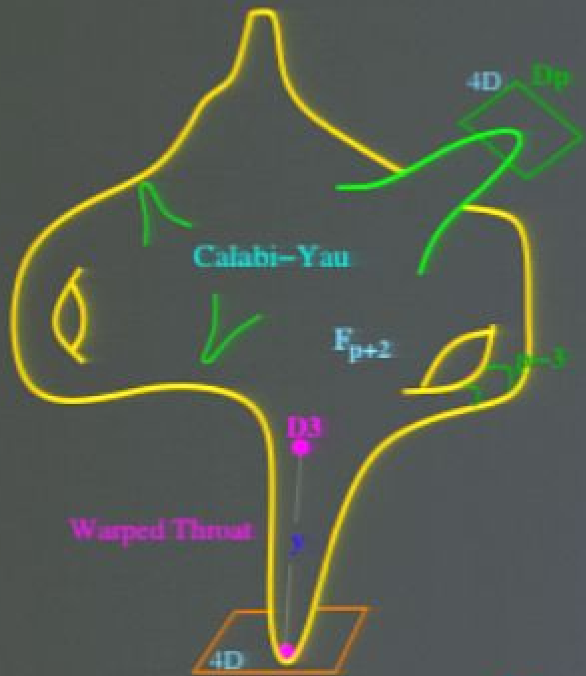
DBI scenario



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- String perturbation regime => $g_s \ll 1$
- Probe brane approximation: single D3

Coupling to gravity: DBI Inflation

- Consider 4D action:

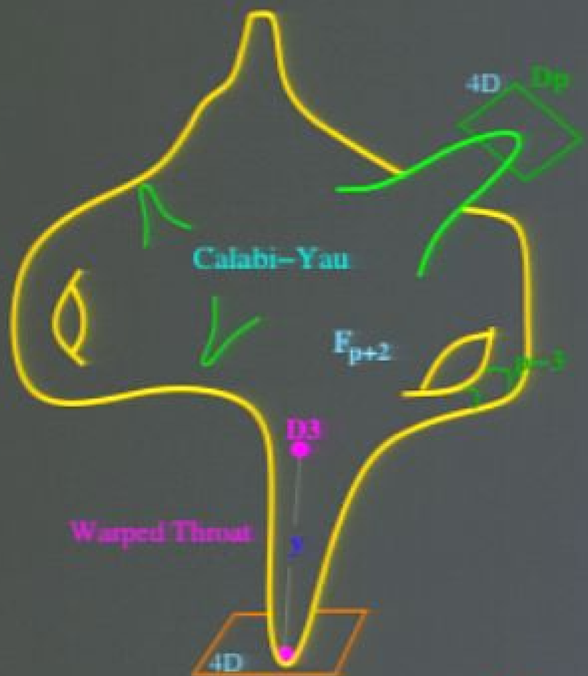
$$\int d^4x\sqrt{-g}\left\{\frac{M_{Pl}^2}{2}R-\left[h^{-1}\sqrt{1-h\,\dot{\phi}^2}-q\,h^{-1}+V(\phi)\right]\right\}$$

$$\phi=\sqrt{T_3}\,\rho$$
(normalised field)

$$M_{Pl}^2=V_6/\kappa_{10}^2$$
$$\kappa_{10}^2=(2\pi)^7g_s^2\alpha'^4$$

Due to speed limit, $\dot{\phi}^2<h^{-1}$, when the warp factor becomes large and $\dot{\phi}\ll 1$, potential term dominates and (single field) inflation can occur!

DBI scenario



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Take Friedman-Robertson-Walker 4D metric:

$$ds_4^2 = -dt^2 + a^2(t)\,dx_i dx^i$$

Friedmann equation $\quad H^2 = \frac{1}{3M_{Pl}^2} E \qquad H = \frac{\dot{a}}{a}$

Energy density and pressure are controlled by the parameter γ

$$E = \frac{(\gamma - 1)}{h} + V\ ; \qquad P = \frac{(1-\gamma^{-1})}{h} - V \qquad \boxed{\gamma = \frac{1}{\sqrt{1 - h\,\dot{\phi}^2}}}$$

Scalar field equation

$$\frac{1}{a^3}\frac{d}{dt}\left[a^3\dot{\phi}\,\gamma\right] = \gamma\,\left(\gamma^{-1} - q\right)^2 \frac{h'}{2\,h^2} - V'$$

Accelerating solutions if $\ddot{a} > 0$: $\quad \frac{\ddot{a}}{a} = H^2\,(1 - \epsilon) \qquad \epsilon = -\frac{\dot{H}}{H^2}$

Inflationary parameter: $\Rightarrow \quad \epsilon < 1$

Cosmological Perturbations

[Garriga-Mukhanov; Chen-Huang-Kachru-Shiu; Lidsey-Seery]

- For a general action of the form:

$$S_4 = \frac{M_{Pl}^2}{2}\int d^4x\sqrt{-g}\,R - \int d^4x\sqrt{-g}\,P(X,\phi) \qquad X = -\frac{1}{2}\partial^\nu\phi\partial_\nu\phi$$

- Inflationary parameters:

$$\epsilon \equiv -\frac{\dot{H}}{H^2} \qquad \eta \equiv \frac{\dot{\epsilon}}{\epsilon H} \qquad s \equiv \frac{\dot{c}_s}{c_s H}$$

- Perturbation amplitudes & spectra:

$$P_S^2 = \frac{1}{8\pi^2 M_{Pl}^2}\frac{H^2}{c_s\,\epsilon} \qquad 1 - n_s = 4\epsilon - 2\eta + 2s$$

$$n_t = -2\epsilon$$

$$P_T^2 = \frac{2}{\pi^2}\frac{H^2}{M_{Pl}^2} \qquad r \equiv P_T^2/P_S^2 = -8c_s\,n_t = 16\,c_s\,\epsilon$$

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For DBI:

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- Nonlinearity parameter

Consistency bounds

✓ Brane moves very fast: $h\dot{\phi}^2 \sim 1 \Longrightarrow \gamma \gg 1 \Longrightarrow c_S^2 = \gamma^{-2} \ll 1$

➡ non-Gaussianities -> bound on $\gamma \leq \boxed{21}$ $\left[f_{NL} = -\frac{1}{3}\left(\frac{1}{c_S^2} - 1\right)\right]$

✓ Total 6D volume > throat volume (field range) [Baumann-McAllister]

$$\left(\frac{\Delta\phi}{M_{Pl}}\right)^2 < \frac{4}{N}$$

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- Nonlinearity parameter $f_{NL} = -\frac{1}{\ldots}\left(\frac{1}{\ldots} - 1\right)$

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$$r < \frac{32\pi^3}{(\Delta\mathcal{N})^6 Vol(X_5)} P_S^2$$

Backreaction: [Easson, Gregory, Mota, Tasinato, Zavala '07]

✓ Check that the probe brane does not warp the internal geometry as it moves near to the speed of light. Use toy exact sol: $R^9 \times S^1$

✓ As the D-brane moves on the S^1 a horizon forms and becomes larger as the brane moves faster. The proper energy of the D3 also increases.

✓ Probe approximation means that this curvature is always less than the ambient curvature.

D3 on $R^9 \times S^1$

$$\Longrightarrow \quad \gamma - 1 \ll g_s^{-1} R^4 \ell_s^{-4} = g_s M^2 \qquad g_s M \gg 1$$

More general configurations

★ Consider higher dimensional Dp-branes wrapped over cycles of the throat. The larger the volume of the cycles, the larger the field range for the same size of the extra dimensions. . .

i) D5 wrapping a 2-cycle in the internal manifold.

[Kobayashi, Mukohyama, Kinoshita '07; Becker, Leblond, Shandera '07]

ii) D7 brane wrapping 4-cycle.

[Kobayashi, Mukohyama, Kinoshita '07]

. . . However, higher dimensional branes have a larger backreaction and this makes (almost) impossible to have a valid probe brane approximation through the inflationary era. LH bound is not improved [Alabidi-Lidsey '08].

★ Spinflation. In general brane moves in six (internal) directions. Angular motion can help in a multifield inflationary scenario, but not enough.

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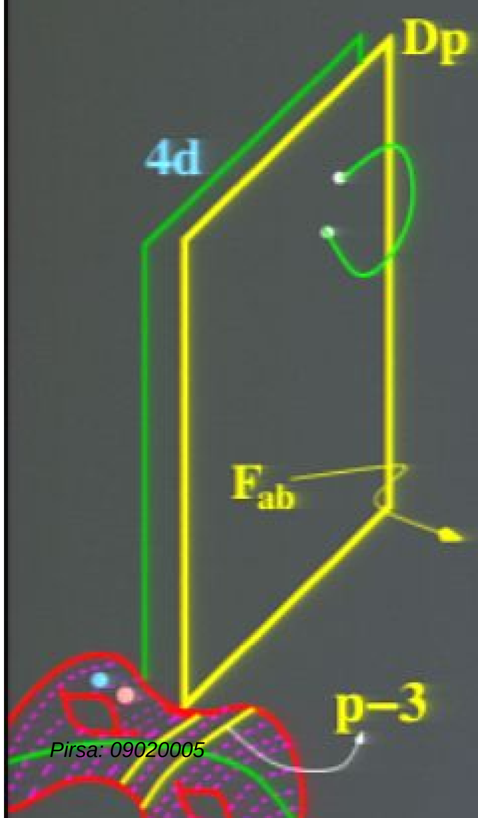
Wilson Lines and D-branes

★ Wilson lines (WL) are open string moduli in type II string theory. Arise naturally in string theory compactifications.

★ They correspond to background values of gauge fields that can be turned on in spaces of non-trivial homotopy (topology): $\gamma = \partial C$

$F_2 = dA_1\,,$ WL: $U_\gamma = e^{\oint_\gamma A}$ If $\pi_1[C] \neq I$

$F = 0\,, \quad U_\gamma \neq 1$ and $A = \lambda\, dx\,, \quad \lambda = \text{const.}$



★ They appear on the world volumes of Dp-branes:

$F_{\mu\nu}$ ⟶ 4d magnetic field

F_{ij} ⟶ internal magnetic field (breaks susy)

$F_{\mu i}$ ⟶ Wilson line $F_{\mu i} = \partial_\mu A_i(x^\mu)$

=> scalar field from 4d point of view

Wilson Lines and Inflation I

★ In type II theories T-duality relates:

Wilson Lines ↔ Brane Separations
Magnetic Field ↔ Brane Angles

★ "Angled D-brane inflation" already studied. Inflaton is driven by the separation between the branes, while brane angles break susy (potential is generated):
[Bellido, Rabadan, Zamora '01; Gomez-Reino, Zavala '02] .

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Can be discussed in a DBI set up!

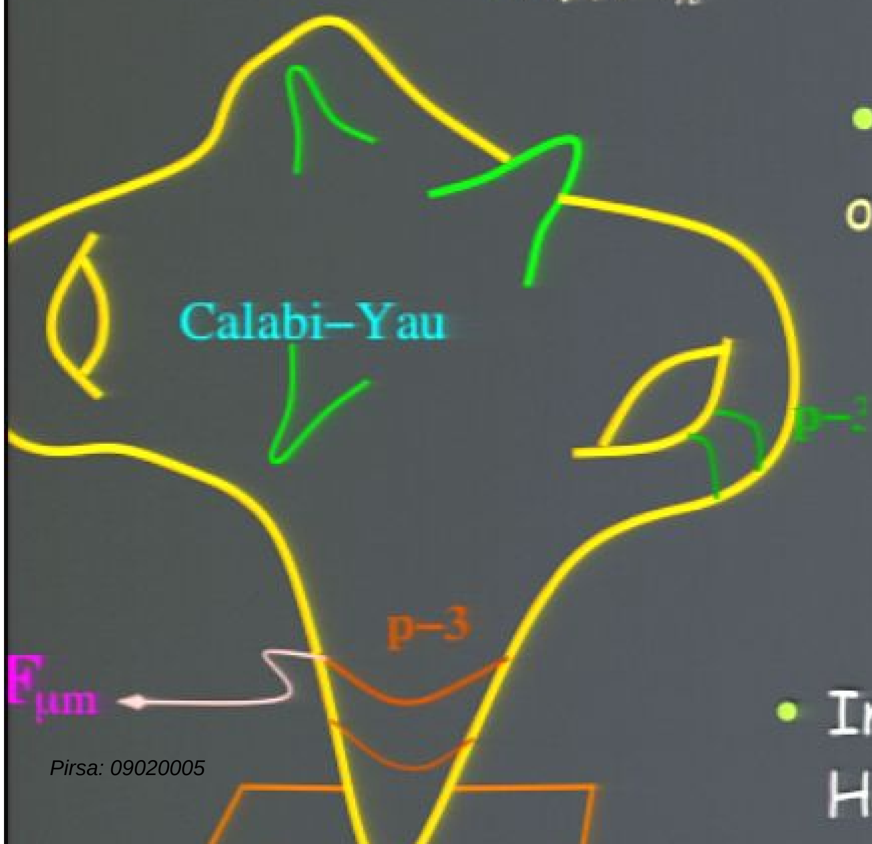
Wilson Line DBI Inflation

- Consider a Dp-brane (p>3) in warped compactification (in type IIB). All types of fluxes turned on. D-brane action is described by

$$S_{DBI} = -\mu_p \int d\xi^{p+1} e^{\frac{(p-3)}{4}\phi} \sqrt{-det(\gamma_{ab} + e^{-\frac{\phi}{2}} \mathcal{F}_{ab})}$$

$$S_{WZ} = q\,\mu_p \int_{\mathcal{W}_{p+1}} \sum_n \mathcal{C}_n \wedge e^{\mathcal{F}}$$

where $\mathcal{F}_{ab} = B_{ab} + 2\pi\alpha' F_{ab}$



- Turn on Wilson lines and magnetic flux on its world volume, Fab.

$$F_{\mu m} = \partial_\mu A_m \qquad F_{mn} = \text{const.}$$

$$A_m = \frac{\lambda_m}{L_m} \qquad \lambda \in [-\pi, \pi]$$

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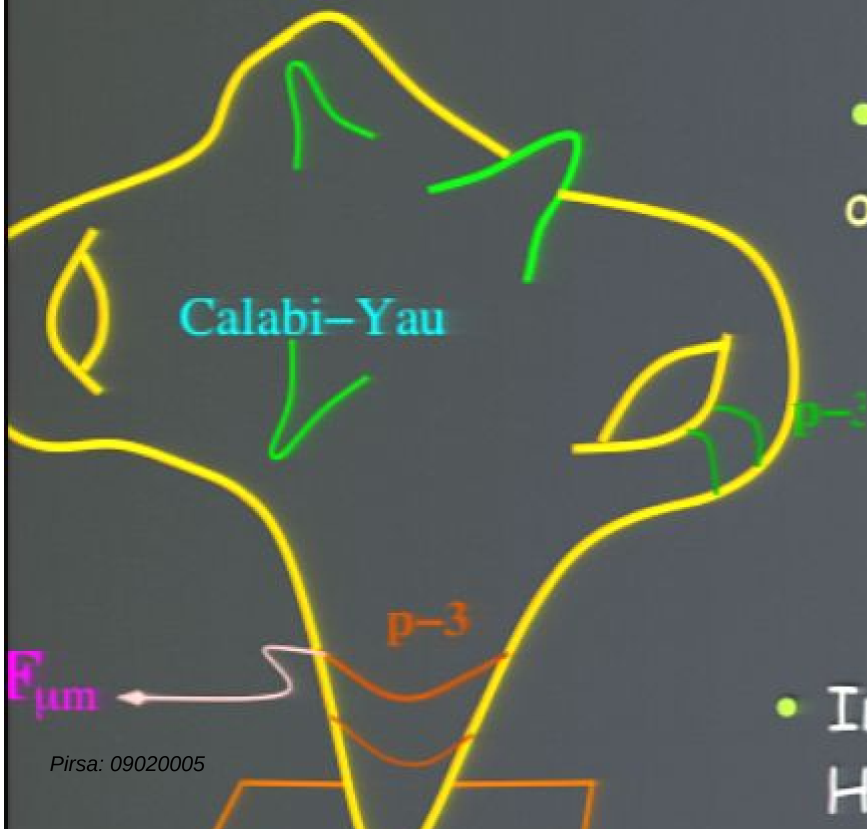

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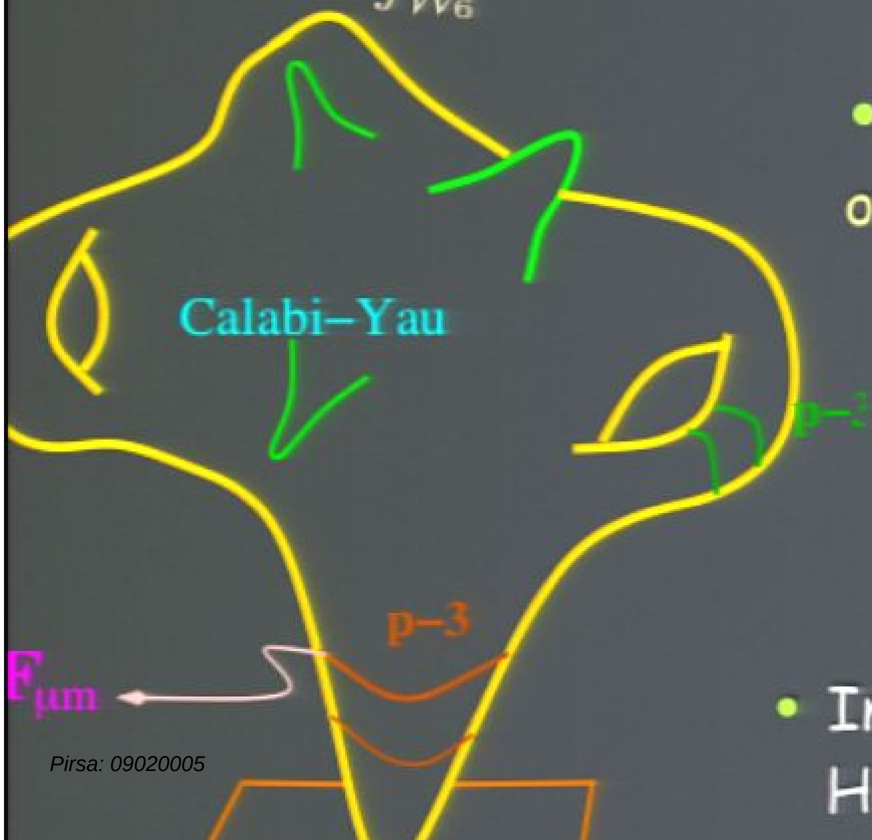

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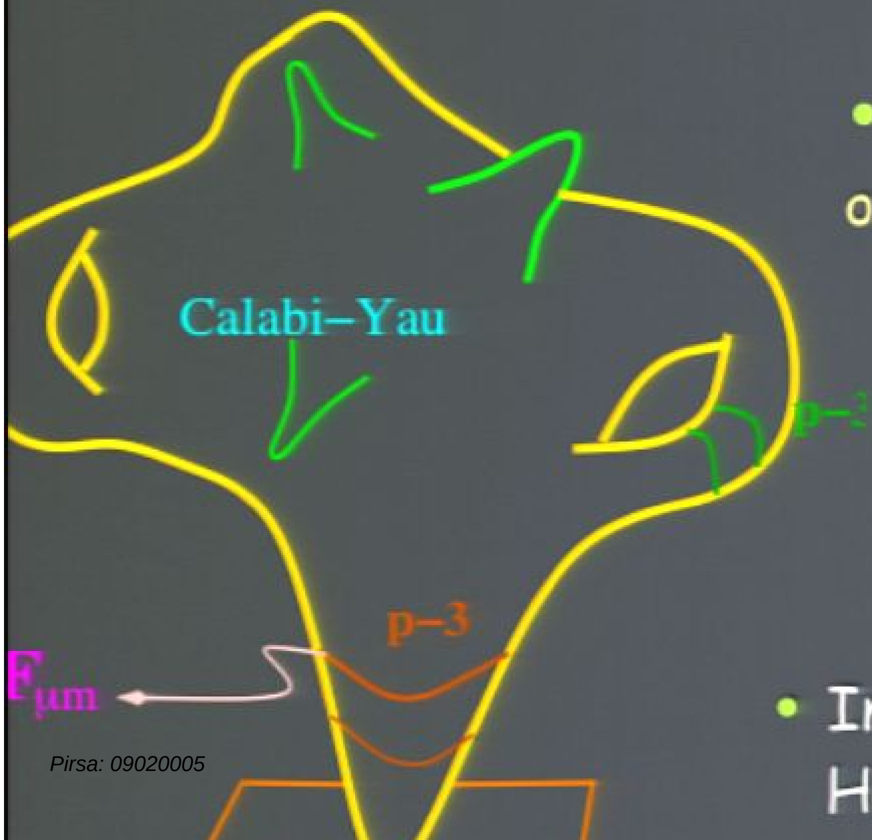

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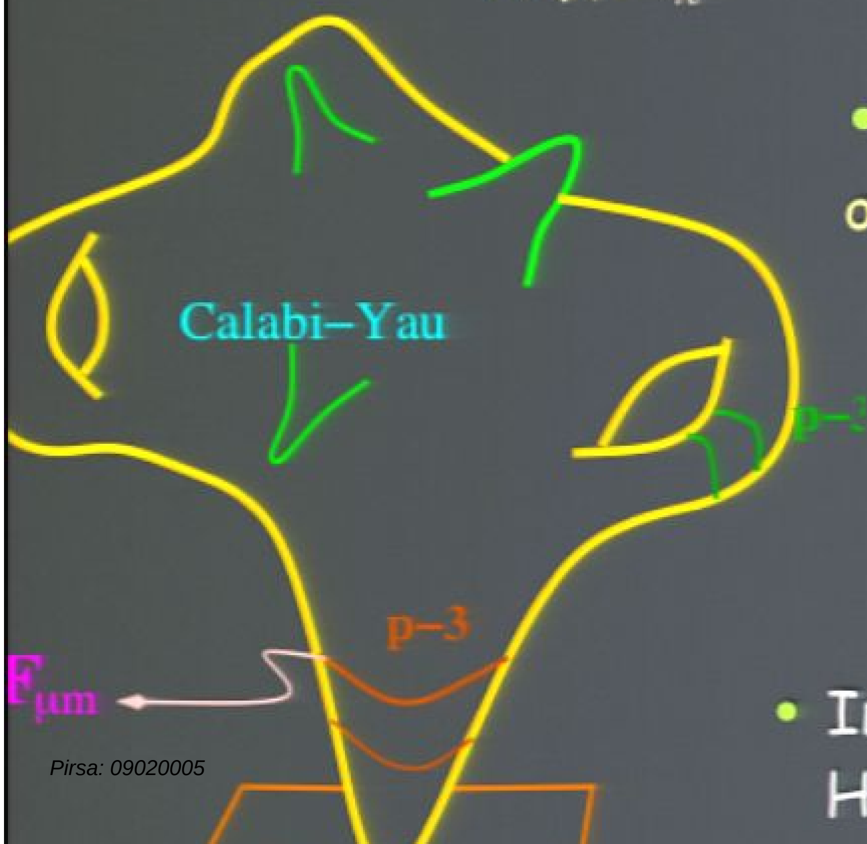

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$$S_{DBI} = -\mu_p \int d^{p-3}\xi \, e^{\frac{(p-3)}{4}\phi} \, h^{-1} \sqrt{\det(E_{mn})} \int d^4x \sqrt{-g} \sqrt{1 + h^{1/2}(2\pi\alpha')^2 e^{-\phi} \, (\partial A)^2}$$

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$$E_{mn} = \gamma_{mn} + e^{-\frac{\phi}{2}}(\mathcal{B}_{mn} + 2\pi\alpha' F_{mn}) \qquad (\partial A)^2 = \partial_\mu A_m \partial^\mu A_n g^{mn}$$

Field normalisation

$$f_0 = h^{-1}\mu_p \int d^{p-3}\xi\, e^{\frac{(p-3)}{4}\phi}\sqrt{\det(E_{mn})} \qquad \chi = \left[\frac{(2\pi\alpha')^2 e^{-\phi}}{L^2} f_0\, g^{xx}\right]^{1/2}\lambda$$

Four dimensional action:

$$S_4 = \frac{M_{Pl}^2}{2}\int d^4x\sqrt{-g}\, R - \int d^4x\sqrt{-g}\left[f_0\sqrt{1 - f_0^{-1}\,\dot{\chi}^2} + V(\chi) + F_{0_p}\right]$$

$$\int d^4x\sqrt{-g}\left\{\frac{M_{Pl}^2}{2}R - \left[h^{-1}\sqrt{1 - h\,\dot{\phi}^2} - q\,h^{-1} + V(\phi)\right]\right\}$$

$$S_{5WZ} = q\,\mu_5 \int d^4x\sqrt{-g}\,\left[M\eta_6(y) + N h^{-1}(y)\,(b(y) + 2\pi\alpha'\, f_2)\right]$$

Wilson Line DBI Inflation

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$$h > 1 \quad \Leftrightarrow \quad f < 1$$

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Four dimensional metric is of FRW form:

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Four dimensional metric is of FRW form:

$$ds_4^2 = -dt^2 + a^2(t)\,dx_i dx^i$$

Friedmann equation $H^2 = \frac{1}{3M_{Pl}^2} E$ $H = \frac{\dot{a}}{a}$

Energy density and pressure are controlled by the parameter γ

$E = f_0\gamma - q\,F_p + V\,;$ $P = q\,F_p - f_0\gamma^{-1} - V$ $\gamma = \frac{1}{\sqrt{1 - f_0^{-1}\dot{\chi}^2}}$

Scalar field equation

$$\frac{1}{a^3}\frac{d}{dt}\left[a^3\gamma\,\dot{\chi}\right] = -V'$$

Accelerating solutions if $\ddot{a} > 0$: $\frac{\ddot{a}}{a} = H^2\,(1-\epsilon)$

$\epsilon = -\frac{\dot{H}}{H^2}$ $\Rightarrow \quad \epsilon < 1$

Cosmological Perturbations

[Garriga-Mukhanov; Chen-Huang-Kachru-Shiu; Lidsey-Seery]

- For a general action of the form:

$$S_4 = \frac{M_{Pl}^2}{2}\int d^4x\sqrt{-g}\,R - \int d^4x\sqrt{-g}\,P(X,\phi) \qquad X = -\frac{1}{2}\partial^\nu\phi\partial_\nu\phi$$

- Inflationary parameters:

$$\epsilon \equiv -\frac{\dot{H}}{H^2} \qquad \eta \equiv \frac{\dot{\epsilon}}{\epsilon H} \qquad s \equiv \frac{\dot{c}_s}{c_s H}$$

$$c_s^2 = \frac{dP}{dE} = \gamma^{-1}$$

- Perturbation amplitudes & spectra:

$$P_S^2 = \frac{1}{8\pi^2 M_{Pl}^2}\frac{H^2}{c_s\,\epsilon} \qquad 1 - n_s = 4\epsilon - 2\eta + 2s$$

$$n_t = -2\epsilon$$

$$P_T^2 = \frac{2}{\pi^2}\frac{H^2}{M_{Pl}^2} \qquad r \equiv P_T^2/P_S^2 = -8c_s\,n_t = 16\,c_s\,\epsilon$$

- Nonlinearity parameter $f_{NL} = -\frac{1}{\cdot}$ …

Consistency bounds in WLDBI

✓ Field range (WL is not related to 6D volume)

$$\left(\frac{\Delta\chi}{M_{Pl}}\right)^2 < 2^8\pi^{11}\, g^{\chi\chi} g_s\, \frac{\tilde{f}_0}{l^2\,\mathcal{V}_6}$$

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Lidsey-Huston:

$$I) \quad r > \frac{4(1-n_s)}{\sqrt{1-3f_{NL}}};$$

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$$II) \quad \frac{f_0}{M_{Pl}^4} = \frac{P_S^2\,\pi^2\, r^2}{16}\left(1 - \frac{1}{3f_{NL}}\right)$$

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$$r < 0.25$$

Depends on the geometry,
(background and D-brane used)

$$\tilde{f}_0, \quad g^{\chi\chi}$$

and scales

$$l, \quad \mathcal{V}_6, \quad g_s$$

- Estimated values in KS-like warped geometry:

$$\tilde{f}_0 \sim \frac{(g_s M)^{\frac{(p-7)}{2}} \mathcal{V}_{p-3}}{(2\pi)^p} \left(\frac{\ell_s}{\varepsilon^{2/3}}\right)^{p-7} \frac{\mathcal{B}_0^{(p-3)/2}}{I_0}$$

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$$0.16 < r < 0.24$$

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Summary

★ We have studied the implications of having a different open string modulus driving inflation in warped compactification: Wilson Line

★ For single field case, Wilson line has different origin, thus giving rise to different field range and consistent bounds for r!

★ Besides large non-Gaussian effects, observable gravitational wave spectrum can be generated.

★ Backreaction issues can also be relaxed, in contrast with position DBI inflation. (Brane is not moving!).

Future directions

★ We have taken a phenomenological approach. Construct concrete geometrical setup where relevant questions can be answered.

★ More general configuration involves more fields: Brane position as well as WL play a roll (this should be similar to Spinflation). Perturbations analysis changes.

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Accelerating solutions if $\ddot a > 0$:

$$\frac{\ddot a}{a} = H^2 (1 - \epsilon)$$

$$\epsilon = - \frac{\dot H}{H^2} \qquad \Rightarrow \quad \epsilon < 1$$

Wilson Lines and Inflation I

★ In type II theories T-duality relates:

Wilson Lines ↔ Brane Separations	
Magnetic Field ↔ Brane Angles	

★ "Angled D-brane inflation" already studied. Inflaton is driven by the separation between the branes, while brane angles break susy (potential is generated):
[Bellido, Rabadan, Zamora '01; Gomez-Reino, Zavala '02] .

★ Can we identify a WL with the inflaton(s) in warped scenario?
[Warped Wilson line <=> warped brane at angles inflation]

Can be discussed in a DBI set up!

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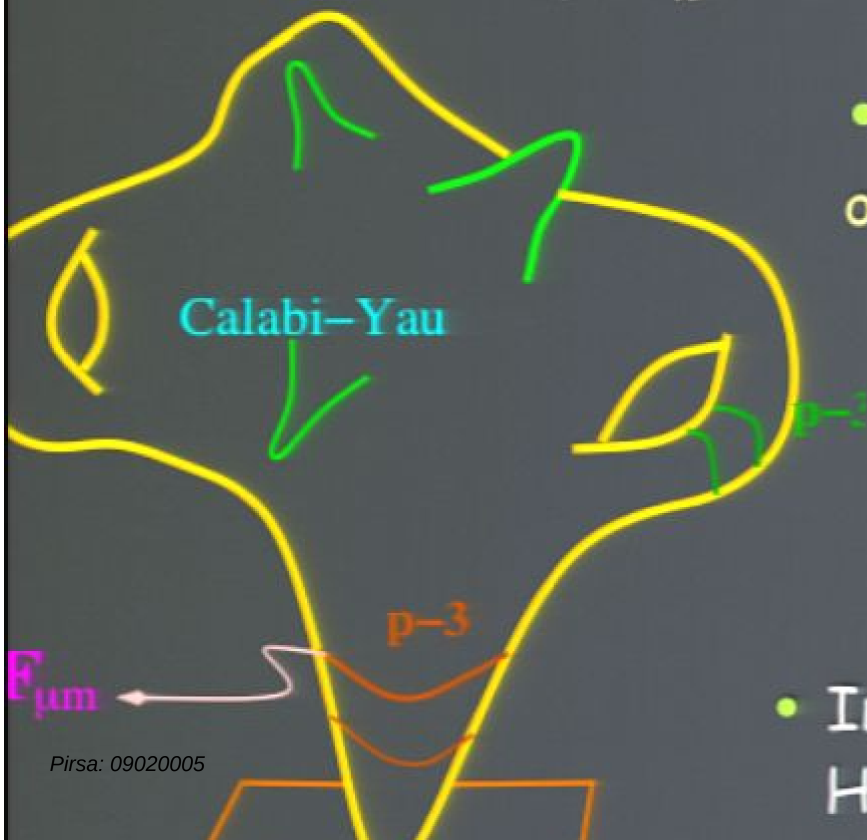
Wilson Line DBI Inflation

- Consider a Dp-brane (p>3) in warped compactification (in type IIB). All types of fluxes turned on. D-brane action is described by

$$S_{DBI} = -\mu_p \int d\xi^{p+1} e^{\frac{(p-3)}{4}\phi} \sqrt{-det(\gamma_{ab} + e^{-\frac{\phi}{2}} \mathcal{F}_{ab})}$$

$$S_{WZ} = q\,\mu_p \int_{\mathcal{W}_{p+1}} \sum_n \mathcal{C}_n \wedge e^{\mathcal{F}}$$

where $\mathcal{F}_{ab} = B_{ab} + 2\pi\alpha' F_{ab}$



- Turn on Wilson lines and magnetic flux on its world volume, Fab.

$$F_{\mu m} = \partial_\mu A_m \qquad F_{mn} = \text{const.}$$

$$A_m = \frac{\lambda_m}{L_m} \qquad \lambda \in [-\pi, \pi]$$

- In principle, position & WL play a role. Here, single DBI Wilson Line inflation.