Title: Braneworld Black Holes

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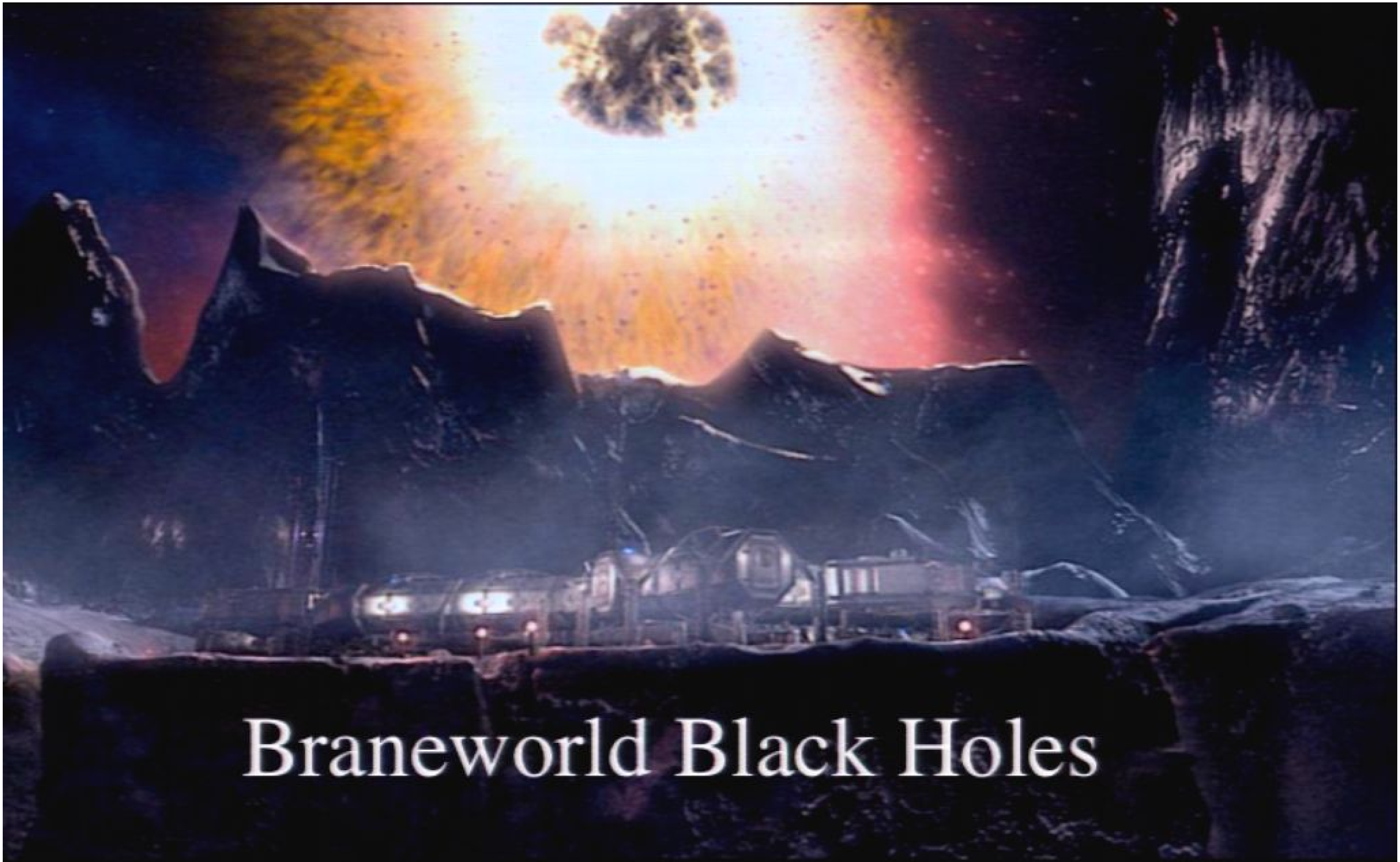
Abstract: Braneworlds are a fascinating way of hiding extra dimensions by confining ourselves to live on a brane. One particular model (Randall-Sundrum) has a link to string theory via living in anti de Sitter space. I'll describe how the ads/cft correspondence has been used to claim that a braneworld black hole would tell us how Hawking radiation back reacts on spacetime, thus solving one of the outstanding problems of quantum gravity - the ultimate fate of an evaporating black hole. I'll review evidence for this conjecture, ending with some recent work that shows it may be problematic.



Braneworld Black Holes

Ruth Gregory

Durham Centre for Particle Theory

Outline

- Braneworlds - a brief review
- Brane gravity and cosmology
- Black holes and extra dimensions
- Brane black hole problems
- Holographic conjecture
- Karch Randall braneworld black holes

RG, Simon Ross, Robin Zegers: JHEP 0809:029,2008 [arXiv:0802.2037 hep-th]

Why extra Dimensions?

Like Einstein we seem obsessed with finding a “unified” description of nature.

The Standard Model works well for particle physics, but Gravity seems stubbornly classical.

AE: “*God does not play at dice*”
NB: “*Stop telling God what to do!*”

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Kaluza and Klein

suggested a geometric way of unifying Gravity and Electromagnetism 90 years ago, by adding an extra dimension to spacetime - a small circle, which geometrically represented the U(1) gauge group of electromagnetism in a very direct fashion.

$$\mathbf{G}^{(5)} \longrightarrow \underbrace{\mathbf{G}^{(4)} + \Phi}_{\text{Scalar-tensor gravity}} + \underbrace{\mathbf{A}_{(\mathbf{em})}}_{\text{Electromagnetism}}$$

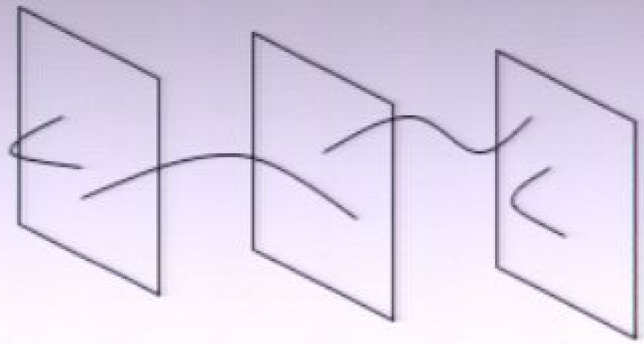
What is a Braneworld?

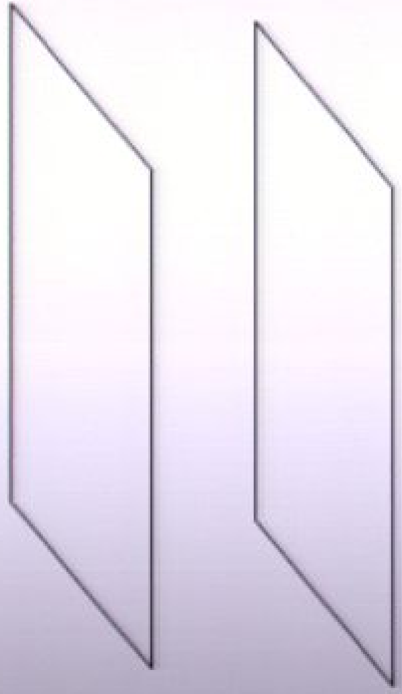
Simply put, a braneworld is a slice through spacetime on which we live. We cannot (easily) see the extra dimensions perpendicular to our slice, all of our standard physics is confined. We can however deduce those extra dimensions by carefully monitoring the behaviour of gravity…

…like Mr. Game and Watch - we are flatlanders.

Branes are very natural in string theory, which requires 10 or 11 dims

Open strings can end on hypersurfaces called D-branes





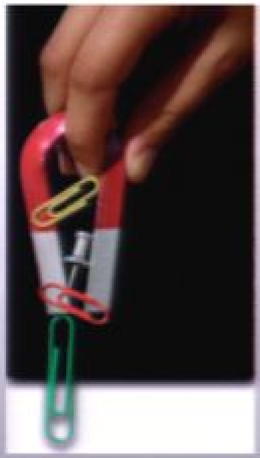
Heterotic M-theory has the 11th dimension a finite interval with matter confined at each end, only a limited number of fields propagating in the bulk.

Extra dimensions have to be hidden. Until a few years ago, it was assumed that this was like the Kaluza-Klein picture: compactification on small scales……

……but then an old idea resurfaced motivated by:

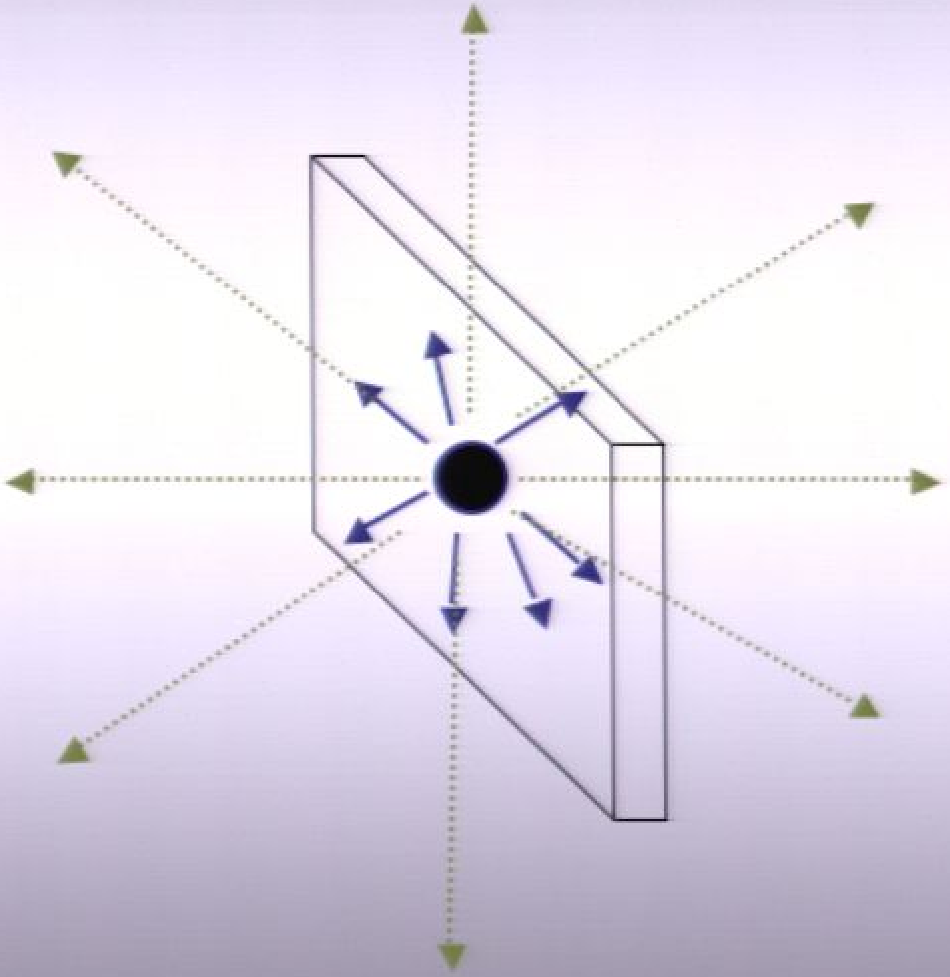
THE HIERARCHY PROBLEM

— or, why is gravity so weak?



Gravity is weak because it feels all of spacetime, brane and bulk, whereas electromagnetism is confined to the brane.

Braneworld gravity allows us to get very different behaviour on large, as well as small, scales.



Braneworlds vs. Kaluza-Klein

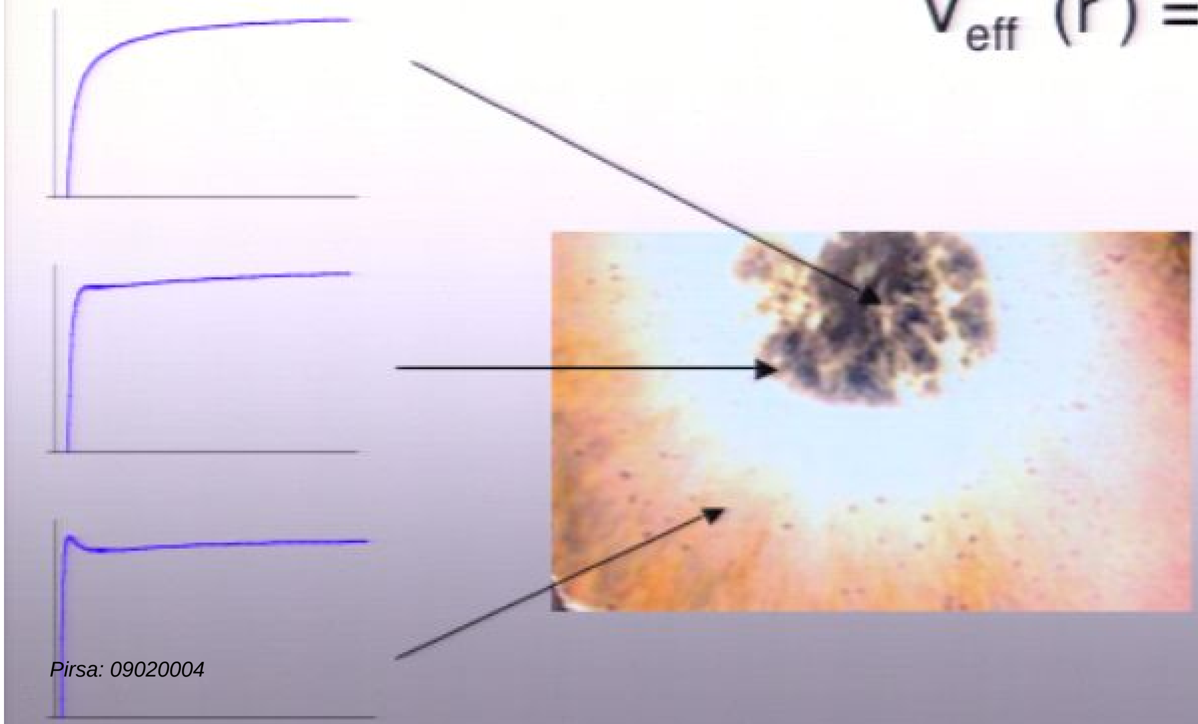
- Extra dimensions large
- Localized in extra-D
- Geometry curves into bulk - a **warped compactification**

- Extra dimension small
- Smeared over circle
- Geometry independent of extra dimension

Why Black Holes?

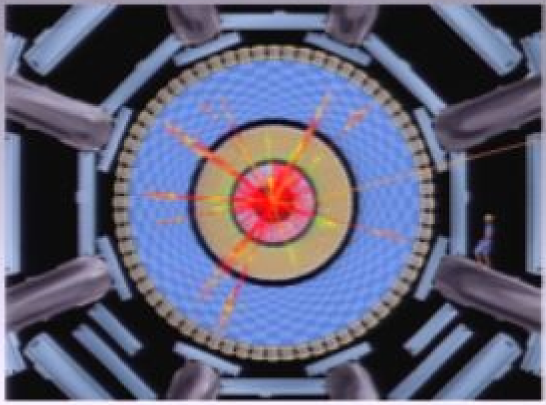
- Astrophysics: Perhaps large black holes can probe large extra dimensions?

$$V_{\text{eff}}(r) = 1 - 2\frac{GM}{r} + \frac{h^2}{r^2} - 2\frac{GMh^2}{r^3}$$



Accretion discs affected by strong gravity?

Why Black Holes?



- LHC physics: Perhaps small black holes can be produced in collisions?

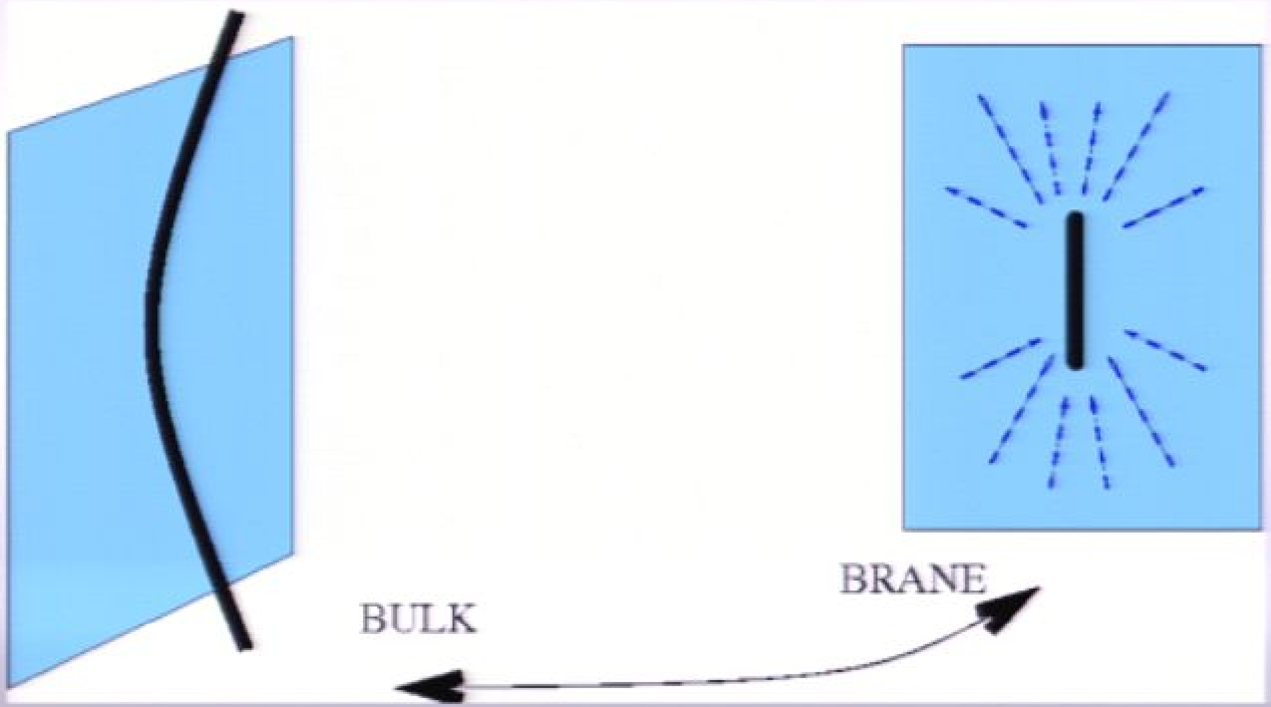
Hoop Conjecture: Horizon forms if mass concentrated within r_s.

$$E^{-1} < r_s \sim \frac{1}{M_D}\left(\frac{E}{M_D}\right)^{(2+n)/(1+n)}$$

Most studies assume an exact higher dimensional Schwarzschild soln.

Why Black Holes?

- String theory / Quantum Gravity:



★ Perhaps through ads/cft, we can get insight into quantum corrected black holes?

Black Holes in 4D

$$ds^2 = \left(1 - \frac{2GM}{c^2r}\right) dt^2 - \left(1 - \frac{2GM}{c^2r}\right)^{-1} dr^2 - r^2 d\Omega^2$$

Black holes radiate: $\boxed{kT = \frac{h\,c^3}{8\pi GM}}$

Have Entropy: $\boxed{S = \frac{A\,k\,c^3}{4hG}}$

Surface gravity: $\boxed{\kappa = 1/4GM}$

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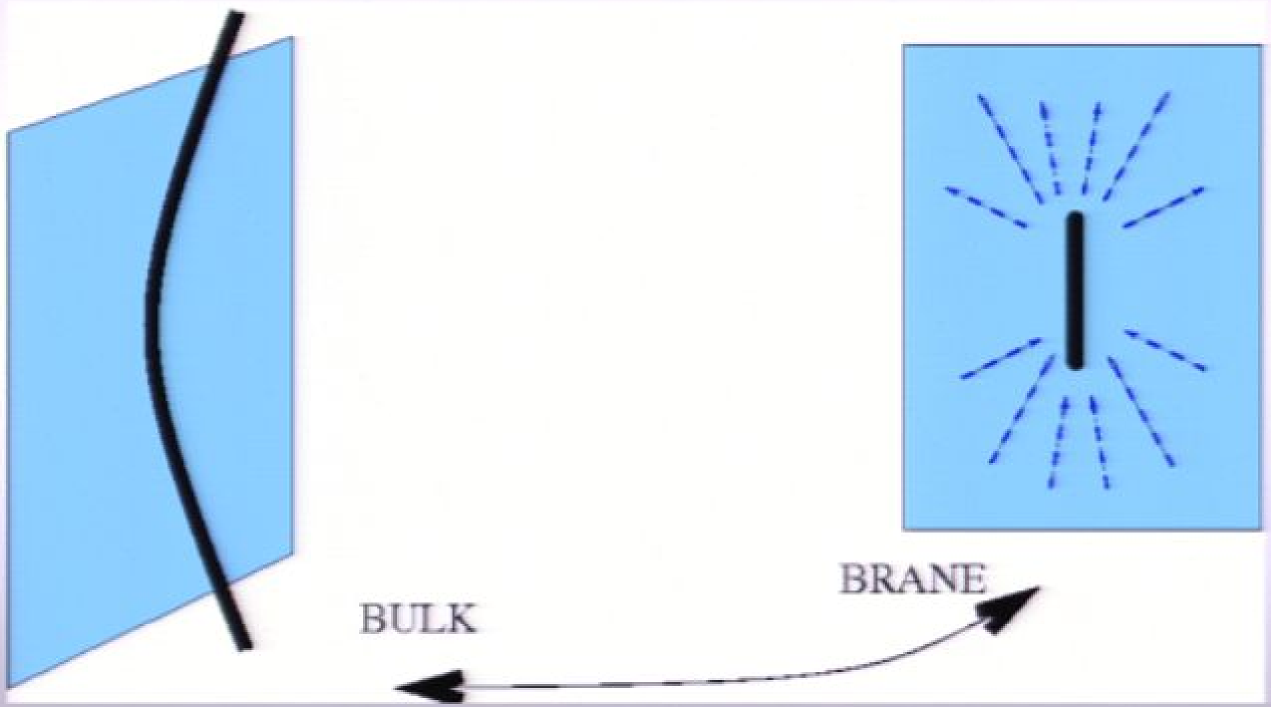

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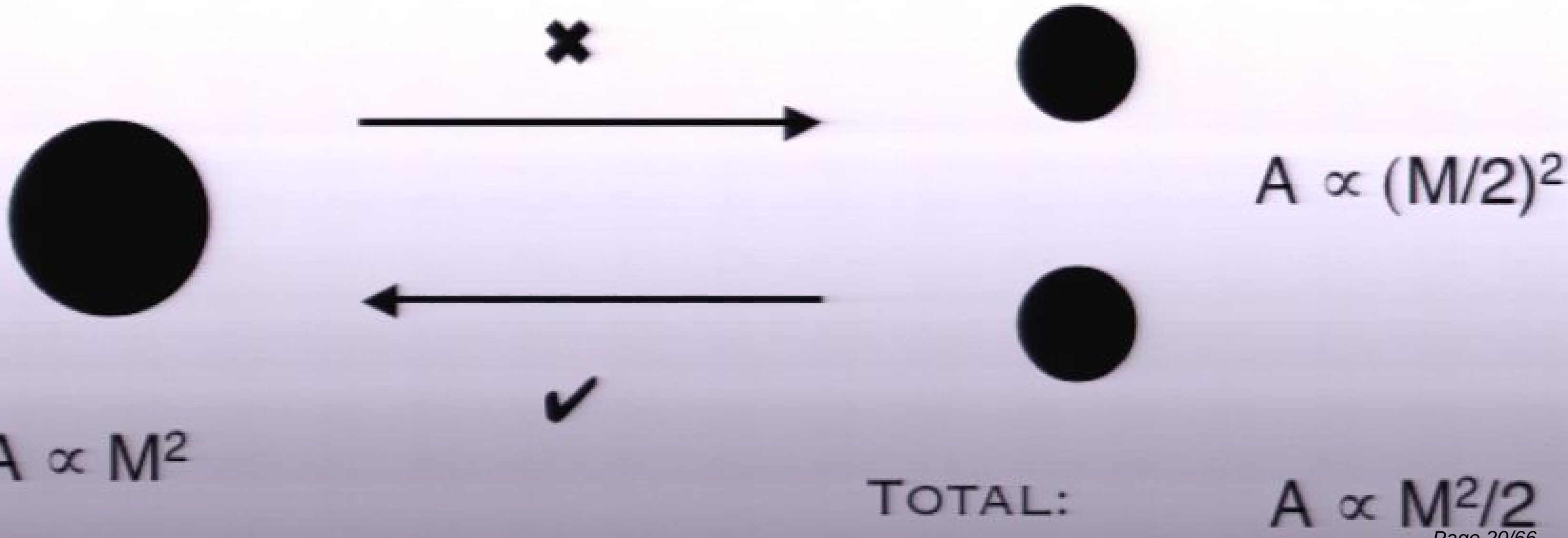
Black Hole Theorems

In 4D, we know that black holes are topologically spherical, are characterized by relatively few "quantum numbers", that, classically, the area of the event horizon must increase, and that black holes cannot bifurcate.



Kaluza Klein black holes

The simplest KK black hole is the Schwarzschild black string, obtained by adding a flat direction to the 4D Schwarzschild metric.

The full family of KK black "holes" include boosting along the 5th direction (electric) or having the S^1 Hopf fibred over the two-spheres surrounding the black hole (magnetic).

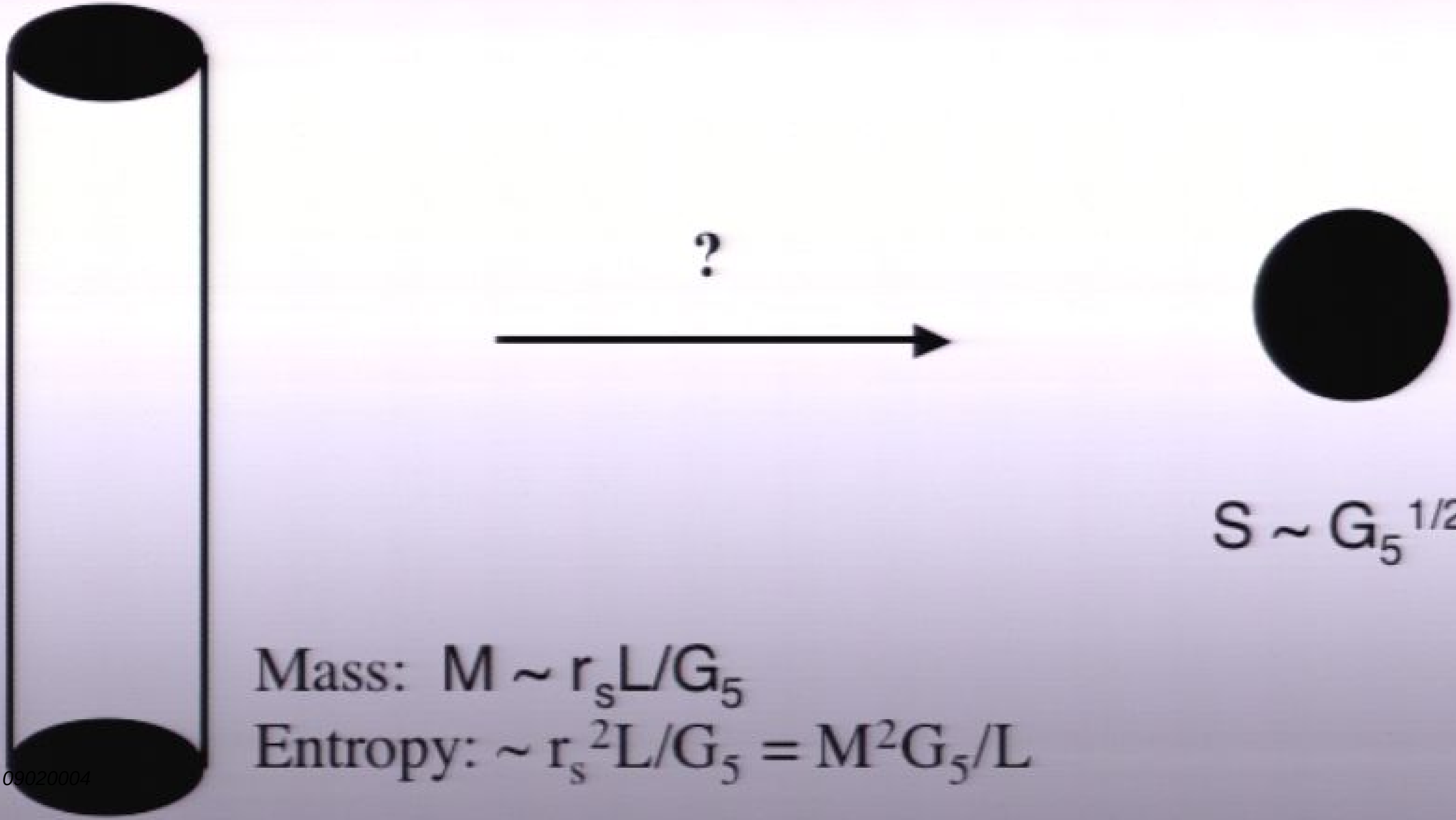
These are *all* uniform in the 5th dimension.

String theory has more fields, more dimensions, but the basic solutions are much the same.

(Gibbons & Wiltshire, Gibbons & Maeda, Horowitz & Strominger)

KK black hole theorems?

What happens to the black hole theorems? Uniqueness? Event horizon topology? *Stability*?



$S \sim G_5^{1/2} M^{3/2}$

Mass: $M \sim r_s L/G_5$
Entropy: $\sim r_s^2 L/G_5 = M^2 G_5/L$

Brane black holes:

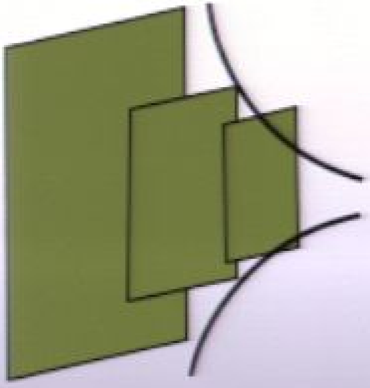
- Braneworld black holes should reflect the strongly localized nature of the brane.
- What is the shape of the horizon?
- How does it extend in the bulk?
- Codimensionality of the brane is a problem. (Nonlinearity and spin structure of gravity.)

Look at in the context of a specific model - Randall Sundrum.

Randall-Sundrum

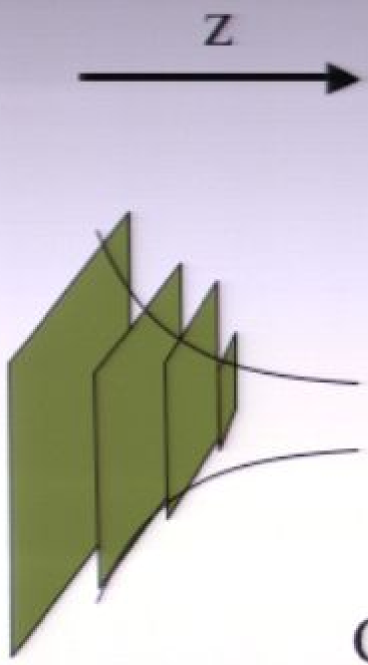
The most compelling braneworld is a very simple model, based on branes in anti-de-Sitter (ads) space.

ADS = spacetime with a *negative* cosmological constant!



Geometry away from the braneworld is strongly *warped*.

z (the extra dimension)



$$\boxed{ds^2 = e^{-2k|z|} [\eta_{\mu\nu} dx^\mu dx^\nu] - dz^2}$$

Warp factor

Flat spacetime

Extra dimension

Geometry away from the braneworld is strongly *warped*. This example has a negative cosmological constant in the bulk, and is symmetric around the brane -
Randall-Sundrum, *PRL* **83,** 3370&4690 (1999)

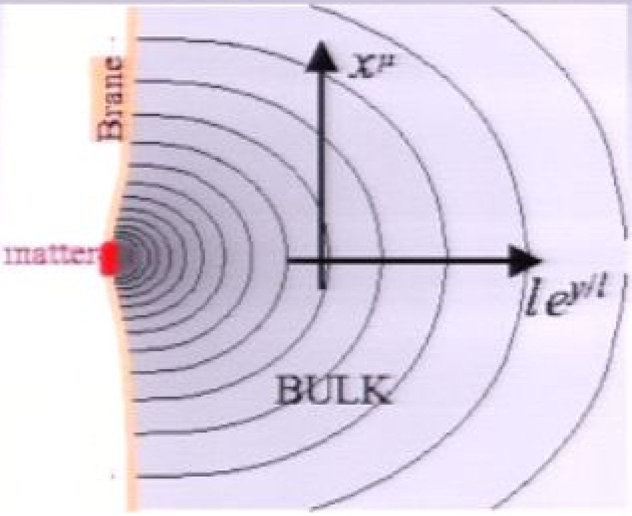
RS Gravity (perturbative): $g_{ab} \longrightarrow g_{ab} + h_{ab}$

To find out how gravity behaves, we perturb the metric:

$$h_{\mu\nu} = \underset{\substack{\Big\uparrow \\ \text{TTF/4D}}}{\chi_{\mu\nu}} - k^{-1} f_{,\mu\nu} - 2k \underset{\substack{\Big\uparrow \\ \text{brane bending}}}{f} \eta_{\mu\nu}$$

The $\chi_{\mu\nu}$ consists of massless and massive 4D modes, which couple to a matter source via $T_{\mu\nu} - T\eta_{\mu\nu}/3$: i.e. as a massive spin 2 particle. The f-term represents a large gauge transformation between the brane and infinity - or motion of the brane. It is a massless 4D field, coupling to T.

Working out the details, we find there are two main pieces contributing to brane gravity: the bulk modes, which behave like massive gravity for the brane, and a brane bending mode, which combines with the bulk massless mode to give



FOUR DIMENSIONAL EINSTEIN GRAVITY

Garriga & Tanaka, *Phys Rev Lett* **84**, 2778 (2000).

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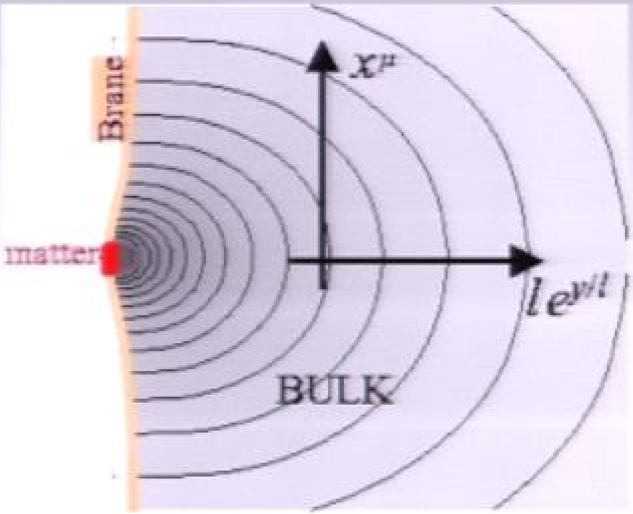

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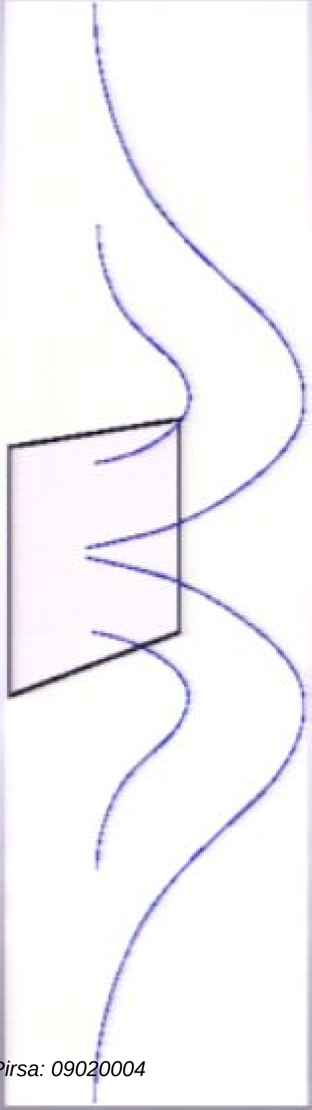
Randall-Sundrum gravity



We can visualize Randall-Sundrum gravity as lines of force spreading out from the brane, but being "pulled back" onto the brane by the negatively curved bulk. At small scales, we can see some effect of the bulk, as force lines are spreading out off the brane, but at large scales, more and more of these force lines return, giving us a standard Newton's Law.

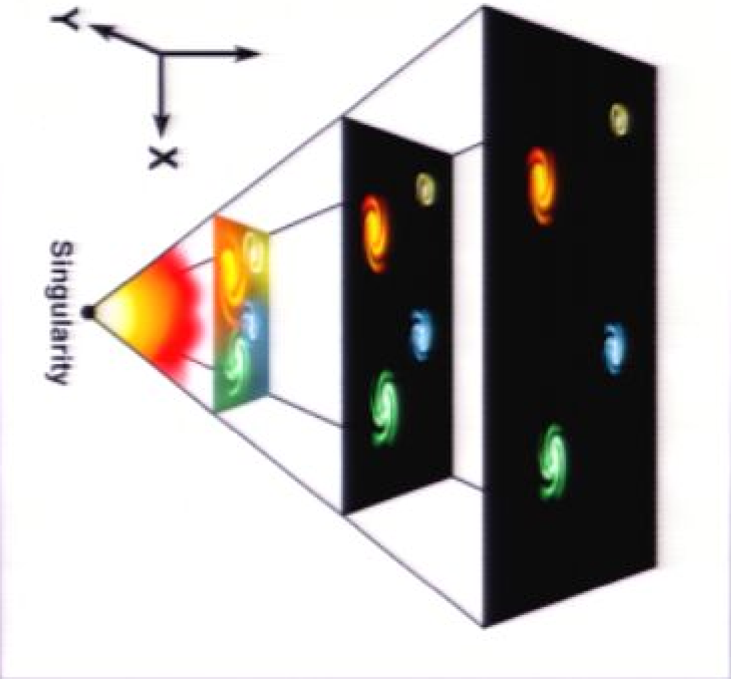
$$V(r) = \frac{GM}{r}\left(1 + \frac{2}{3k^2r^2}\right)$$

Cosmological Braneworlds

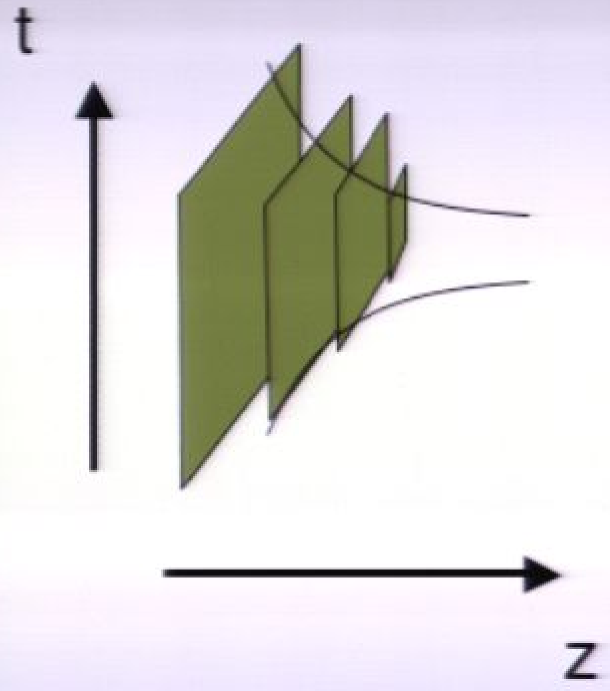
Are there surfaces of lower dimensionality, which have the interpretation of an expanding universe?

Usually, homogeneity and isotropy of the universe give us a very simple FRW description:

$$\boxed{ds^2 = dt^2 - a^2(t)\, d\mathbf{x}^2_{\kappa}}$$



Braneworld Cosmology



Our Universe can now not only evolve in time, but the geometry warps into the extra dimension. This means that although all we see is our scale factor evolving in time, it also evolves in space off the brane, and we must solve for this as well.

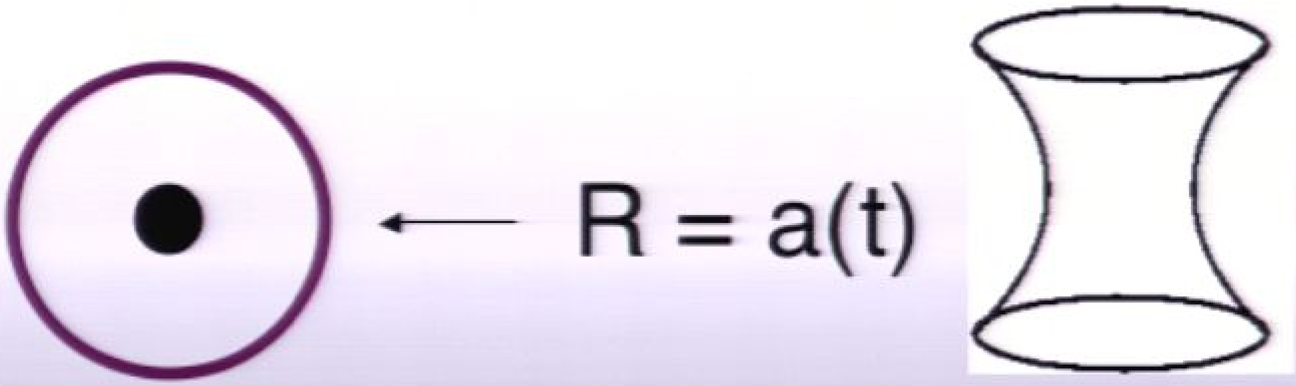
$$\boxed{ds^2 = A^2(t,z)[dt^2 - dz^2] - B^2(t,z)\, d\mathbf{x}^2}$$

$$E = E_0 + \rho$$
$$T = E_0 - p$$

Because we want a brane-FRW universe, we now no longer have the energy equal to tension on the brane.

Cosmology as Geometry

However, because there is sufficient symmetry (essentially only one functional degree of freedom) this is an integrable system, and the spacetime away from the brane is totally known (a black hole) and the cosmological brane is simply a slice equidistant from the horizon!



An accelerating Universe

The trajectory in the black hole spacetime *defines* the scale factor.

Bowcock, Charmousis, RG: hep-th/0007177

Modified Cosmology

From the point of view of the braneworld observer (us!) we can ask what the cosmological equations are. These are given by General Relativity in the higher dimension rewritten in terms of physical quantities on the brane.

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{\kappa}{a^2} = \frac{8\pi G\rho}{3} + \left(\frac{G\rho}{\lambda}\right)^2 + \frac{\mu}{a^4}$$

Modified
Friedman
Equation

“Non-conventional” ρ^2 term.

Black hole: dark radiation term.

Binetruy, Deffayet, Langlois: hep-th/9910219

Braneworlds and Holography

Notice the effect of the bulk black hole on the brane: μ/a^4
This is the same as a radiation dominated Universe, although our brane is empty! However, black holes radiate, and a QFT will be in a thermal state in a black hole background.

Classical
Bulk
Gravity

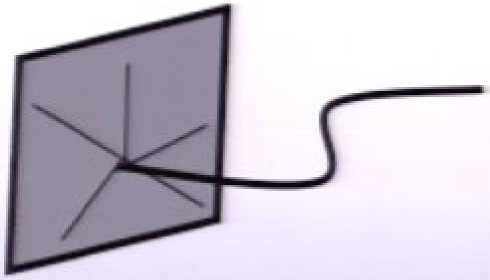
↔

Quantum
Corrected
Brane gravity

Gubser, Verlinde, Duff….

Branes in String Theory

String theory is full of branes, extended object charged under RR or NSNS fields present in the low energy effective theory.



D-branes in string theory are surfaces on which open strings can end. The charges carried by the ends of the strings give gauge fields confined to the D-brane.

D-branes and adS

The D-branes can also be associated with extended charged solutions in supergravity, natural cousins of the domain walls or vortices on which braneworlds were originally based. For example, the D3 brane is sourced by the self-dual 5-form in IIB theory, and has a near horizon limit of $adS_5 \times S^5$, giving a natural link to the RS model.

$$ds^2 = f^{-1/2}\, dx^2_{||} + f^{1/2}\, [\, dr^2 + r^2\, d\Omega^2_5\,]$$

$$f = 1 + 4\pi g N \alpha'^2 / r^4$$

Randall-Sundrum ADS/CFT:

As $r \rightarrow 0$:

$$ds^2 = (r/L)^2\, dx^2_{||} + L^2\, dr^2/r^2 + L^2\, d\Omega^2_V$$

$$L^4 = 4\pi g N \alpha'^2$$ *($L = k^{-1}$ is the AdS length scale)*

In AdS/CFT, we let $N \rightarrow \infty$, and $l_s \rightarrow 0$. We compute Newton's constant from the 10D metric

$$G_5 \hbar = (2\pi)^3\, g_s^2\, \alpha'^4 / L^5 = \pi L^3 / (2N^2)$$

Thus $L^2 = \hbar N^2 G_4 / \pi$, and the corrections to the Newtonian potential correspond exactly to the 1-loop corrections to the graviton propagator.

Cosmological black hole:

For the brane cosmology, the bulk black hole has a Hawking temperature $\pi T_H = \mu^{1/4} L^{-3/2}$ which implies

$$\frac{\mu}{R^4} = \pi^4 L^2 T^4 = \frac{8 \pi G_4}{3} \rho_{cft} (3/4)$$

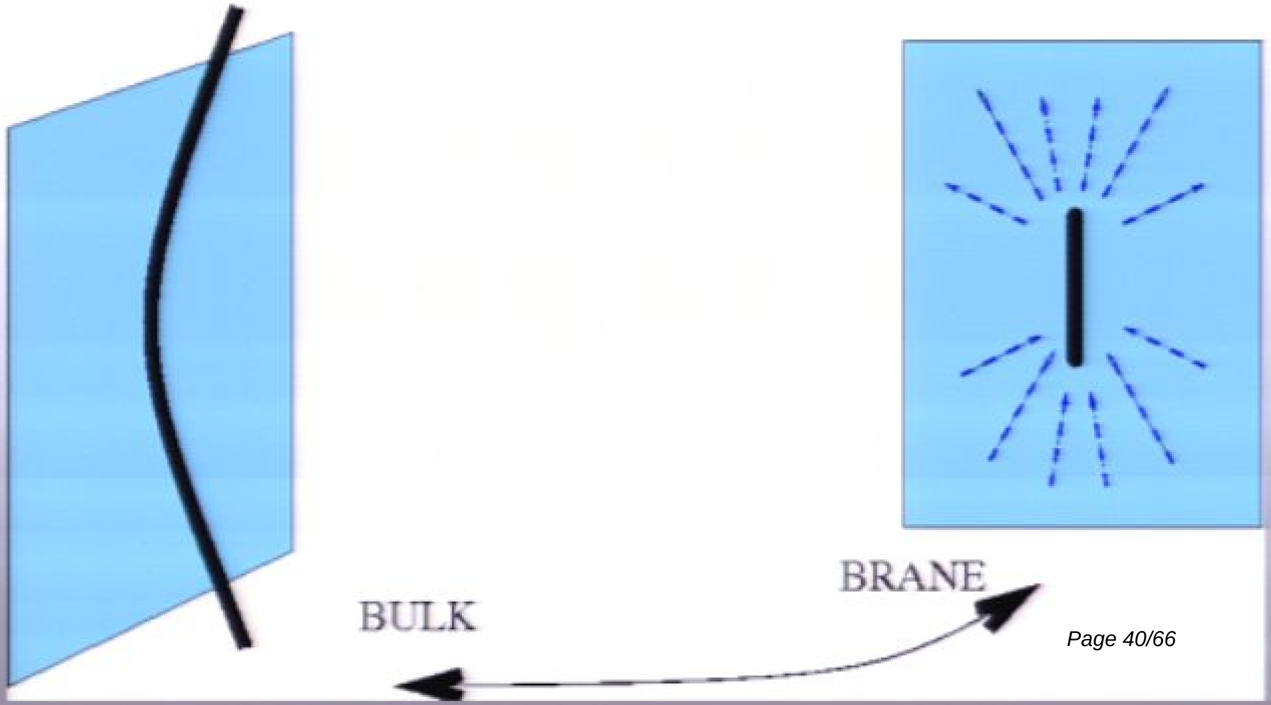
Where ρ_{cft} is the energy of a CFT at finite temperature, T, and the temperature is adjusted from the bulk value to the comoving brane value.

Bulk to Brane Black Hole?

What if we displace the bulk black hole a little, and allow it to move towards the brane, then from the brane point of view this might correspond to collapsing radiation, formation of a black hole, and subsequent evaporation.

Hawking radiation might simply be the result of a bulk dynamical process.

Tanaka: gr-qc/0203082



Holographic Conjecture

Emparan, Fabbri, Kaloper: hep-th/0206155

From both linearized and cosmological brane gravity, there is evidence to support the **Holographic Conjecture**: that a classical bulk solution corresponds to a quantum corrected braneworld solution.
However, this conjecture relies on the fact that N^2h is finite as $h \rightarrow 0$.

- ***All*** the N^2 states have to be accessible.

Fitzpatrick, Randall, Wiseman: hep-th/0608208

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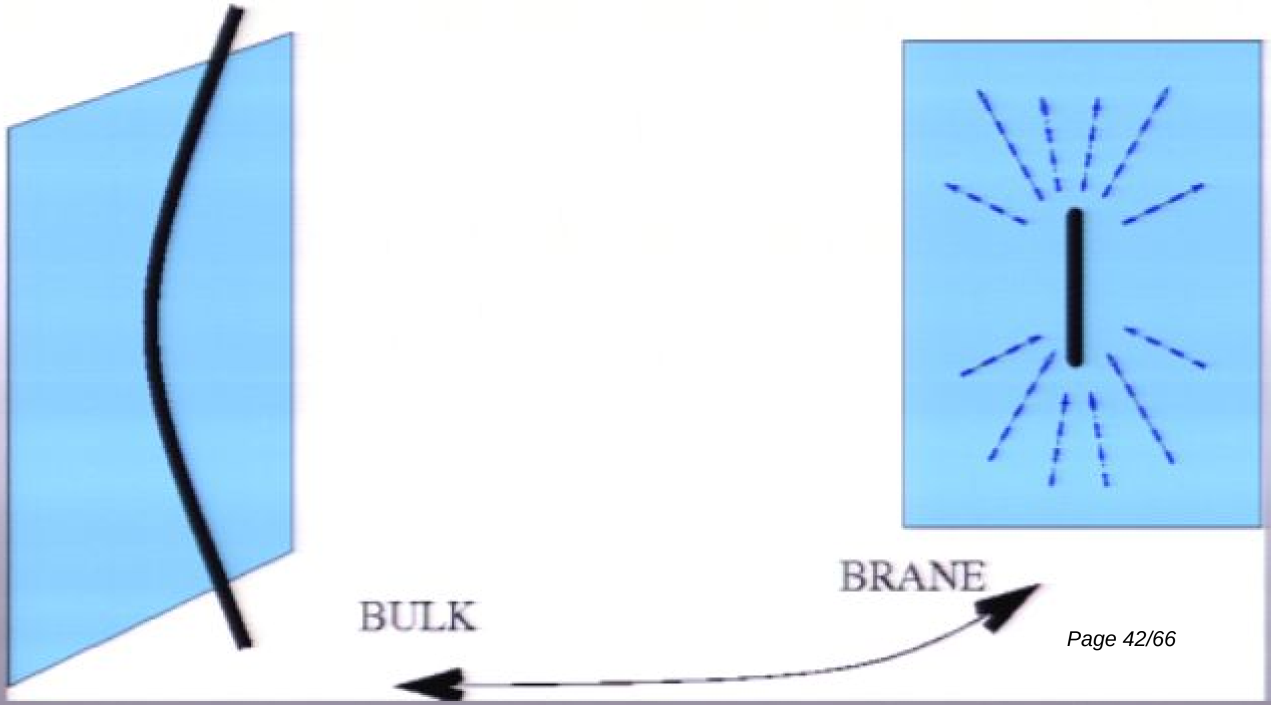

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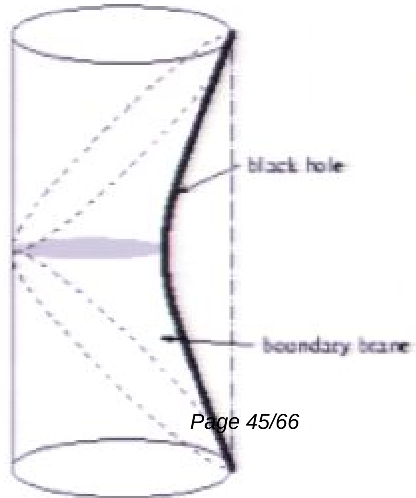
Bulk Horizon - Regular or Singular?

Static or Time dependent?

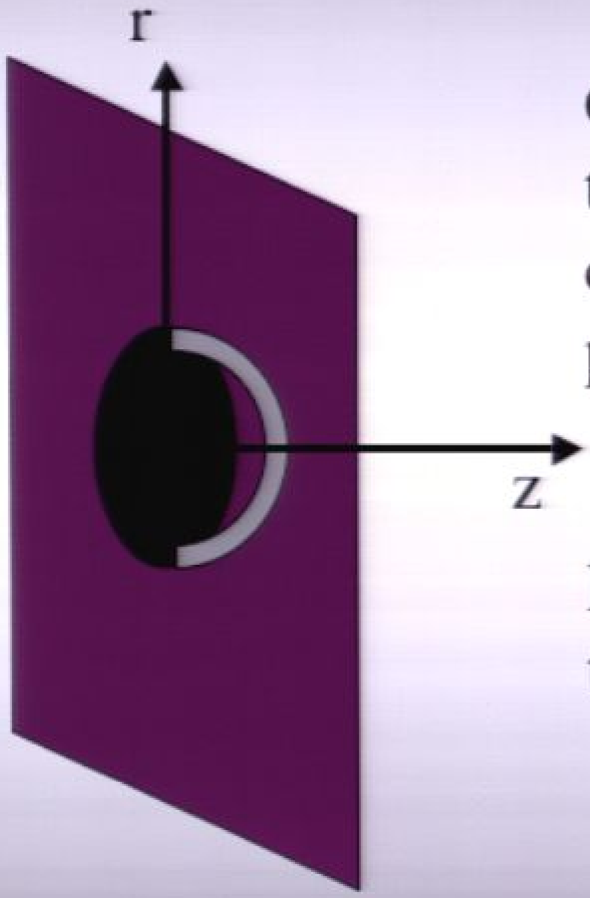
Radiating black hole, standard picture.

Boulware vacuum - singular horizon.

Depends on the **C-metric**



Why is this hard to answer?



GR is a highly nonlinear theory. This is a two-dimensional problem, but unlike cosmology, has an additional field which prevents integrability.

Essentially, there is too much freedom in the bulk to know what is happening there.

Charmousis, RG: gr-qc/0306069

Different Approaches

- Numerical solution of eqns
- Brane-only approach

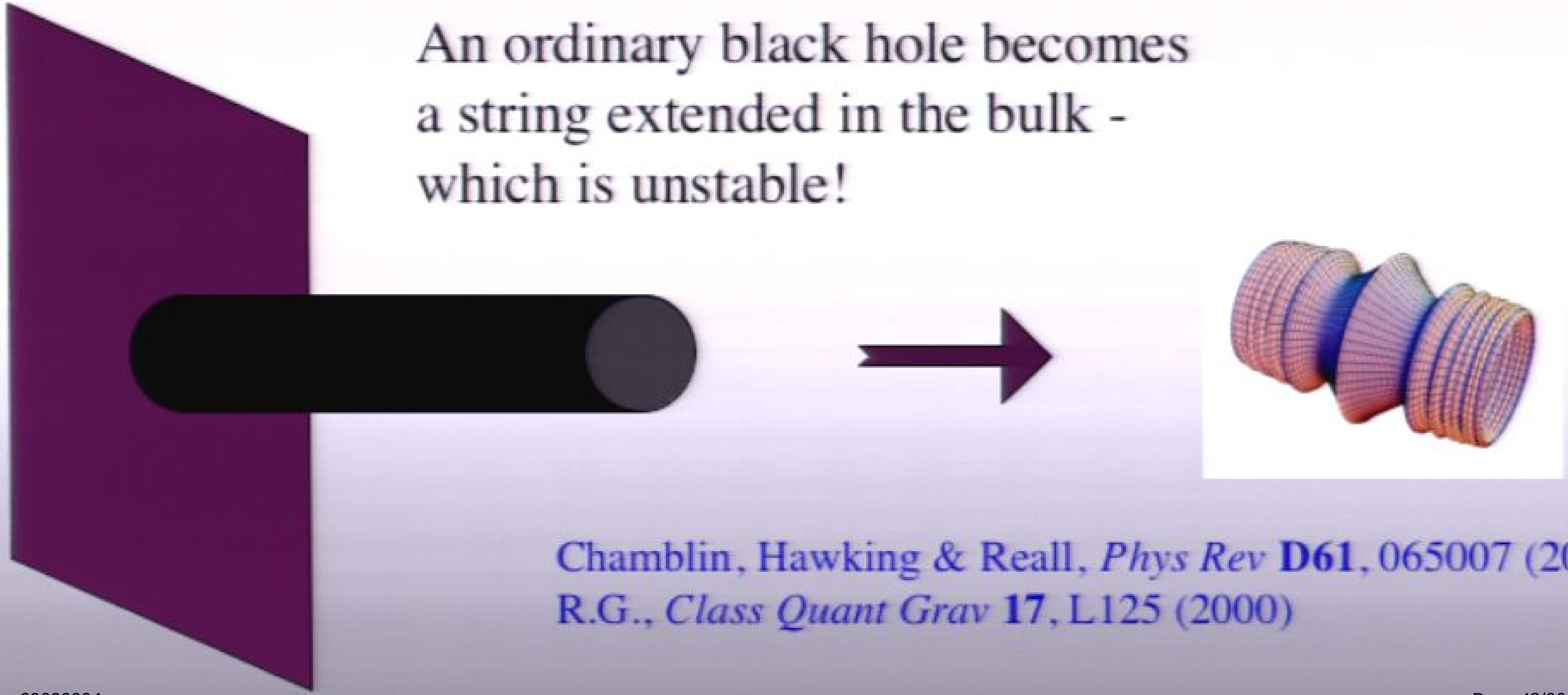
  This is based on the intrinsic brane eqns: $G_{\mu\nu} = \mathcal{E}_{\mu\nu}$ where $\mathcal{E}$ is the "Weyl term" - a projection of the bulk curvature onto the brane.

- Bulk approach

  Take a known bulk and try to construct a sensible brane star or black hole within it.

The Black String

A brane black hole could give the quantum corrected black hole.



An ordinary black hole becomes
a string extended in the bulk -
which is unstable!

Chamblin, Hawking & Reall, *Phys Rev* **D61**, 065007 (2000)
R.G., *Class Quant Grav* **17**, L125 (2000)

Thermodynamic = GL instability?

It is tempting to equate the classical black string instability to the thermodynamic instability of the quantum black hole, however the timescales are different:

For the instability must integrate over the unstable KK modes

$$\delta M(v) = L \int^{m^*} dm\, M_0(2G_N M_0 \Omega(m) - 1/2)\, e^{\Omega v}\, u_m(0)$$

Fabbri & Procopio: 0704.3728 [hep-th]

Approximating $\Omega(m) \sim m^2/M - m/2$ gives

$$\delta \dot{M}(v) \approx L^{3/2} \int^{1/(2GM)} \sqrt{m}\,(2m^2 - mM - 1/2)(2m^2 - mM)$$

$$\propto \frac{1}{G_N} \frac{L^{3/2}}{(G_N M)^{3/2}}$$

Whereas for Hawking radiation: $\dfrac{dM}{dt} = \dfrac{1}{G_N} \dfrac{L^2}{G_N^2 M^2}$

Testing Holography

Obviously, it is difficult to get an exact RS solution - but is there any other way we can test holography?

RS branes are fine-tuned, $8\pi G_5 E=6k$. We can de-tune to let $8\pi G_5 E < 6k$, this is a Karch-Randall brane.

Holography is less well understood, but perhaps for close to critical tension, we can get some useful input.

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$$\delta \dot{M}(v) \approx L^{3/2} \int^{1/(2GM)} \sqrt{m}\, (2m^2 - mM - 1/2)(2m^2 - mM)$$
$$\propto \frac{1}{G_N} \frac{L^{3/2}}{(G_N M)^{3/2}}$$

Whereas for Hawking radiation: $\frac{dM}{dt} = \frac{1}{G_N} \frac{L^2}{G_N^2 M^2}$

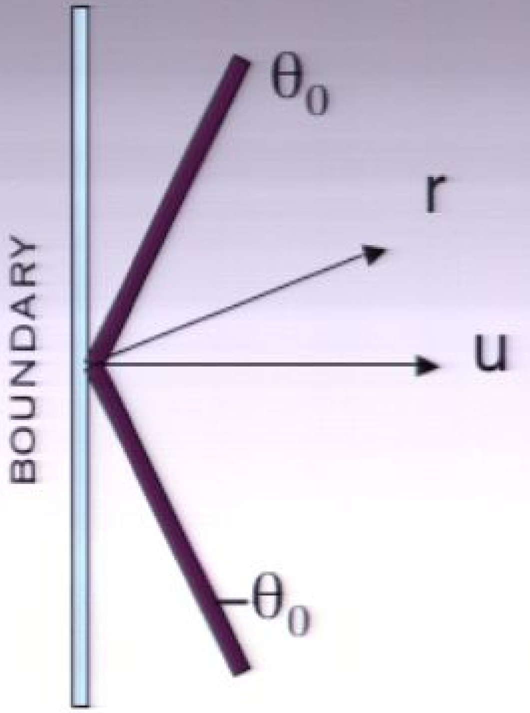
Testing Holography

Obviously, it is difficult to get an exact RS solution - but is there any other way we can test holography?

RS branes are fine-tuned, $8\pi G_5 E=6k$. We can de-tune to let $8\pi G_5 E <6k$, this is a Karch-Randall brane.

Holography is less well understood, but perhaps for close to critical tension, we can get some useful input.

Karch Randall branes



Start with adS: $$ds^2 = \frac{1}{k^2u^2}[dt^2 - d\mathbf{x}^2 - du^2]$$

Change co-ords:

$$ds^2 = \frac{1}{k^2\cos^2\theta}\left[\frac{1}{r^2}[dt^2 - d\mathbf{y}^2 - du^2] - d\theta^2\right]$$

A KR brane sits at $\theta = \pm\theta_0$, and has tension $8\pi G_5 E = 6k \sin\theta_0$

and induced metric $$ds^2 = \frac{1}{r^2\underbrace{k^2\cos^2\theta_0}_{k_4{}^2}}[dt^2 - d\mathbf{y}^2 - dr^2]$$

Karch-Randall black string

A Karch Randall black string is like the CHR black string, but because the KR branes cutoff the bulk near the boundary, two positive tension branes can have a black string running between them which goes through the bulk and has no bulk horizon singularity.

As for any black string, this has a classical instability at a characteristic wavelength.

But, these strings have *finite* length, so if the black string is massive enough - it is stable.

Classical:Quantum comparison

Black holes in AdS are similar to black holes in a box, there are two possible black holes solutions radiating at a particular temperature, one small, and one large mass. The large mass black hole is thermodynamically stable, the small one is not.

This appears consistent with the bulk classical instability.

However, if the large black hole is QM stable, we can calculate the back reaction of the radiation on the spacetime at weak coupling, and compare with the (strong coupling) classical holographic result.

Classical renormalization of $T_{\mu\nu}$

The classical braneworld black string solution is a pure Sch-adS black hole from the brane point of view. But there is a renormalization of the stress tensor.

$$E_{KR} = E_{RS} + \lambda \qquad \Lambda_4 = 8\,\pi\, G_4\, \lambda_{\text{eff}}$$

$$\uparrow \qquad\qquad\qquad \uparrow$$

"bare" cosmological const

"measured" cos const

To compute the renormalization of λ, need to know the precise relation between G_4 and G_5, which is different for KR:

$$G_4 = \frac{4\pi G_5}{3} E_{KR}\, G_5$$

(RG & A. Padilla)

Holographic Result:

Substituting this in gives the general result, which in the $\lambda \rightarrow 0$ limit gives the “strong coupling” result:

$$< T_{\mu\nu} > \; = \; -(\lambda - \lambda_{\text{eff}})\, g_{\mu\nu} = \; \lambda_{\text{ren}}\, g_{\mu\nu}$$

$$\lambda_{\text{ren}} = -\frac{3}{8\pi G_5} \; \frac{(\, 2k^2 - k_4{}^2 - 2k(k^2 - k_4{}^2)^{1/2}\,)}{(k^2 - k_4{}^2)^{1/2}}$$

$$\sim \; -\frac{3\, k_4{}^4}{32\pi G_5} \quad \sim \; -\frac{3\hbar N^2}{16\pi^2 L^3}\; k_4{}^4$$

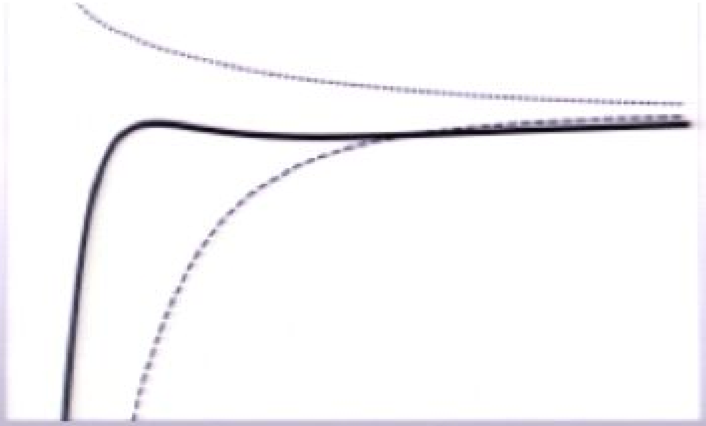
Quantum back-reaction (weak coupling)

RG, Simon Ross, Robin Zegers: 0802.2037 [hep-th]

At weak coupling, can use Page's heat kernel approximation, the energy momentum has three independent functions:

$$< T^{\mu}{}_{\nu} > \propto T_1(\delta^{\mu}{}_{\nu} - 4\, \delta^{\mu}{}_{0}\, \delta^{0}{}_{\nu}) + 3\, T_2\, \delta^{\mu}{}_{0}\, \delta^{0}{}_{\nu} + T_3\, \delta^{\mu}{}_{r}\, \delta^{r}{}_{\nu}$$

This is clearly not proportional to the metric!



Thus it would appear holography is not consistent with expected back reaction, and access to the N^2 states disappears in the RS limit.

Summary

❑ Brane black holes are an interesting open problem

❑ Implications for astrophysics, LHC physics and quantum gravity

❑ AdS/CFT has an interplay with RS branes, as yet poorly understood

❑ The holographic conjecture is that classical brane/bulk solutions give quantum corrected black holes.

❑ Evidence from Karch-Randall branes seems to contradict this conjecture.

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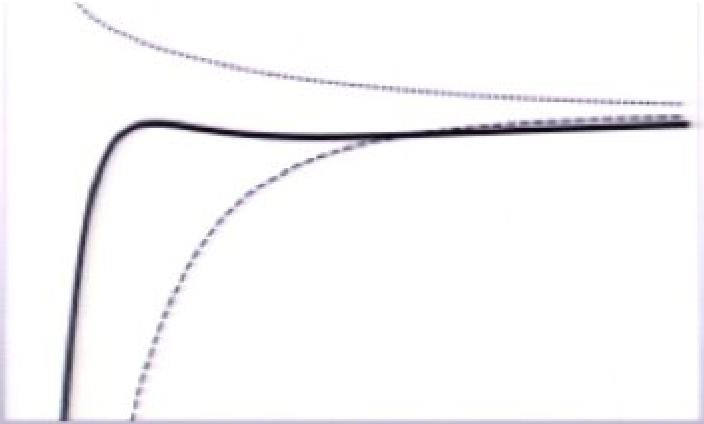

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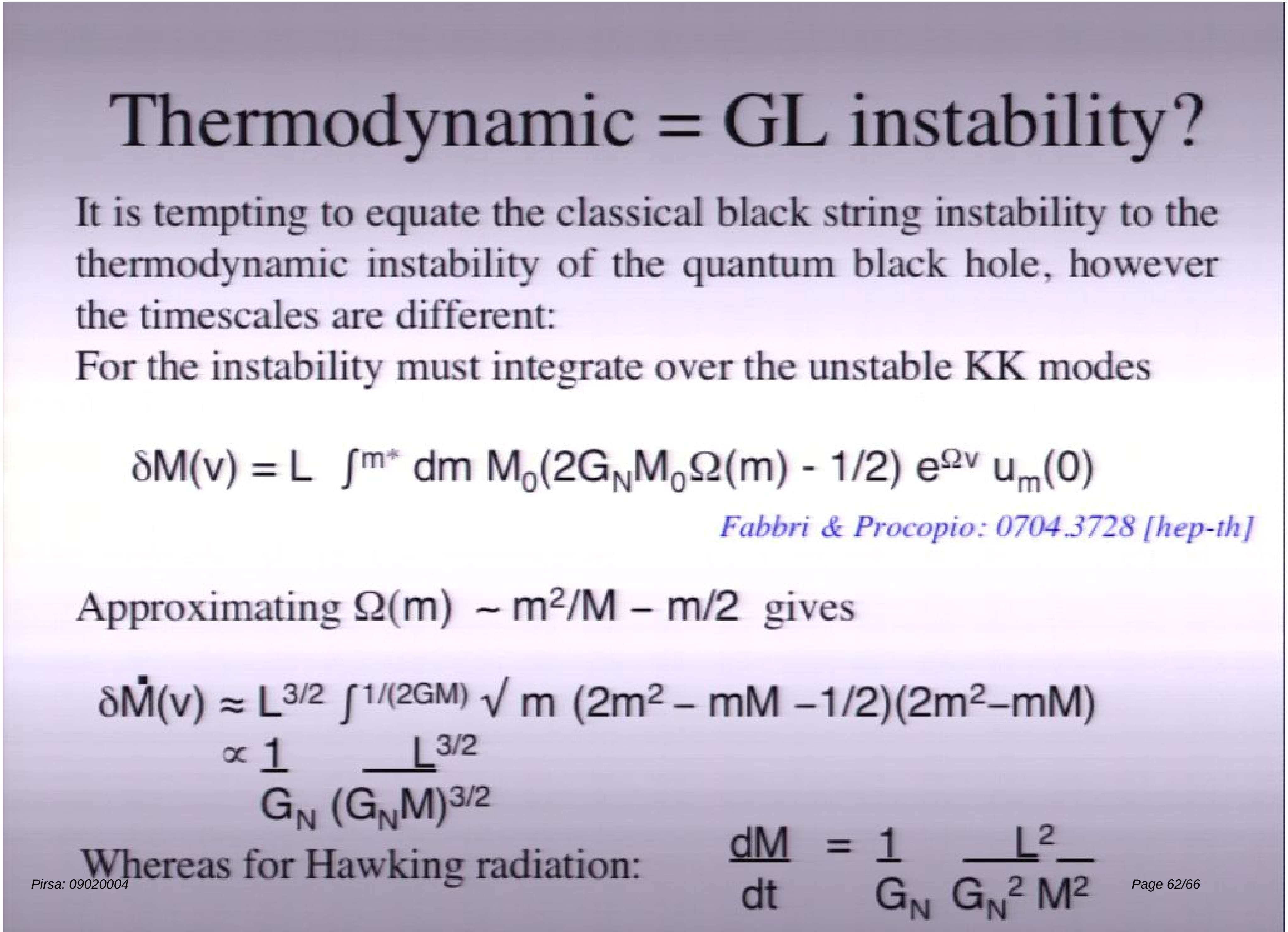

Thermodynamic = GL instability?

It is tempting to equate the classical black string instability to the thermodynamic instability of the quantum black hole, however the timescales are different:

For the instability must integrate over the unstable KK modes

$$\delta M(v) = L \int^{m^*} dm\, M_0(2G_N M_0 \Omega(m) - 1/2)\, e^{\Omega v}\, u_m(0)$$

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Holographic Conjecture

Emparan, Fabbri, Kaloper: hep-th/0206155

From both linearized and cosmological brane gravity, there is evidence to support the **Holographic Conjecture**: that a classical bulk solution corresponds to a quantum corrected braneworld solution.
However, this conjecture relies on the fact that N^2h is finite as $h \rightarrow 0$.

o ***All*** the N^2 states have to be accessible.

Fitzpatrick, Randall, Wiseman: hep-th/0608208

D-branes and adS

The D-branes can also be associated with extended charged solutions in supergravity, natural cousins of the domain walls or vortices on which braneworlds were originally based. For example, the D3 brane is sourced by the self-dual 5-form in IIB theory, and has a near horizon limit of $adS_5 \times S^5$, giving a natural link to the RS model.

$$ds^2 = f^{-1/2}\, dx^2_{||} + f^{1/2}\, [\, dr^2 + r^2\, d\Omega^2_5\,]$$

$$f = 1 + 4\pi g N \alpha'^2 / r^4$$

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Randall-Sundrum ADS/CFT:

As $r \rightarrow 0$:

$$ds^2 = (r/L)^2 \, dx^2_{||} + L^2 \, dr^2/r^2 + L^2 \, d\Omega^2_V$$

$$L^4 = 4\pi g N \alpha'^2$$ *($L = k^{1}$ is the AdS length scale)*

In AdS/CFT, we let $N \rightarrow \infty$, and $l_s \rightarrow 0$. We compute Newton's constant from the 10D metric

$$G_5 \hbar = (2\pi)^3 \, g_s^2 \, \alpha'^4 \, / \, L^5 = \pi L^3 / \, (2N^2)$$

Thus $L^2 = \hbar \, N^2 \, G_4 \, /\pi$, and the corrections to the Newtonian potential correspond exactly to the 1-loop corrections to the graviton propagator.