Title: The Strong Gravity Theorem: a model-independent inequality in quantum gravity

Date: Jan 27, 2009 11:00 AM

URL: http://pirsa.org/09010040

Abstract: We derive a universal upper bound on the weight of the lowest primary operator in any two-dimensional conformal field theory with a given central charge. Translated into gravitational language using the AdS/CFT dictionary, our result proves rigorously that the lightest massive state in any theory of 3D gravity and matter with negative cosmological constant can be no heavier than a particular function the cosmological constant and the Planck scale. For a large AdS space, the lower bound approaches the mass of the lightest BTZ black hole. The derivation applies at finite central charge and does not rely on an asymptotic expansion at large central charge, or on any use of semiclassical or bulk physics. Neither does our proof rely on any special property of the CFT such as supersymmetry or holomorphic factorization. The only assumptions are unitarity, modular invariance, and a discrete spectrum. Our proof firmly demonstrates for the first time that there exists a universal center-of-mass energy beyond which a theory of 'pure' quantum gravity can never consistently be extended.

The Strong Gravity Theorem: a model-independent inequality in quantum gravity

Simeon Hellerman

based on :

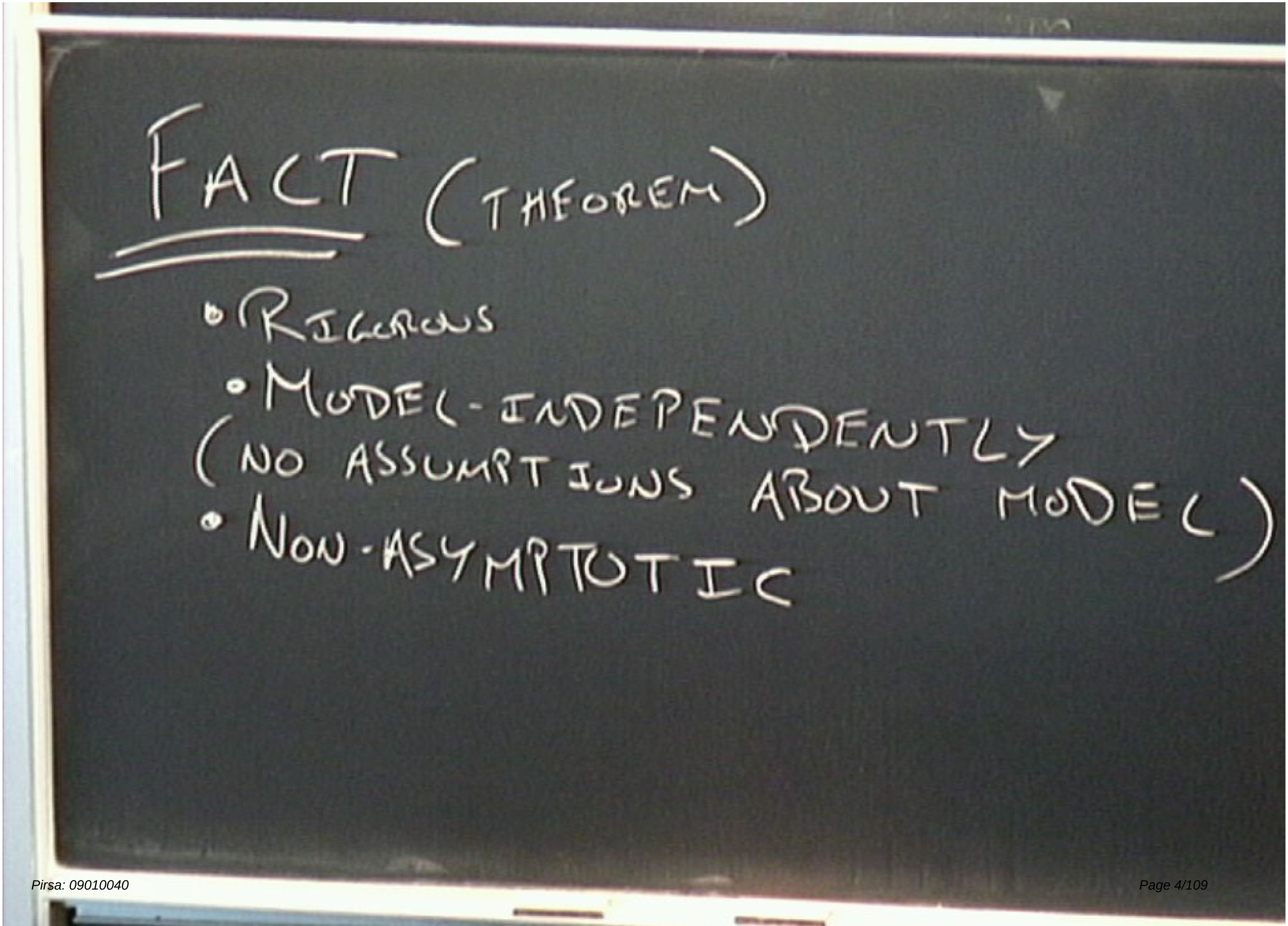
S.H., to appear

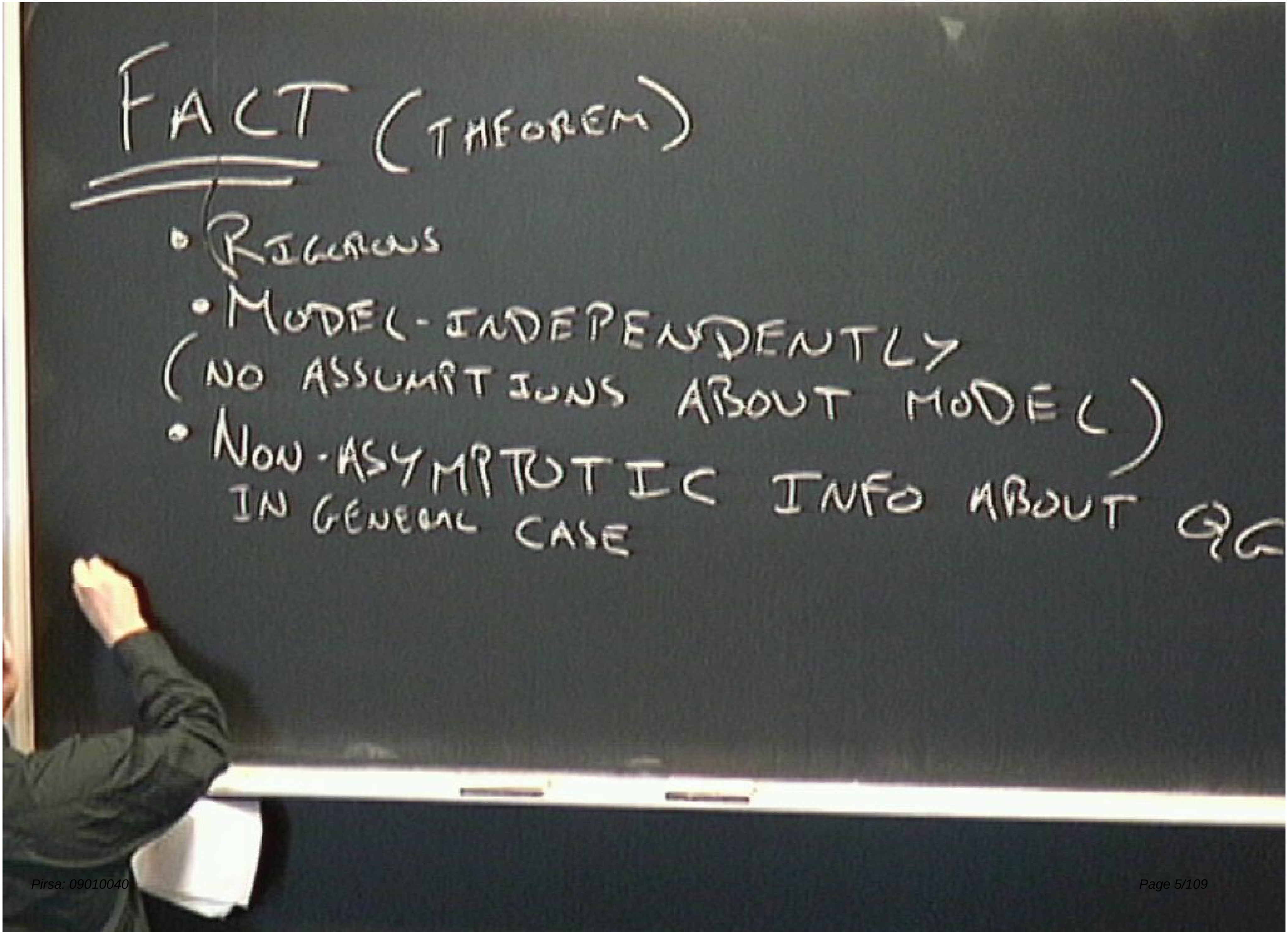
Perimeter Institute, Waterloo, ON, 27 Jan 2009

FACT (THEOREM)

- RIGOROUS
- MODEL-INDEPENDENTLY

(NO ASSUMPTIONS ABOUT MODEL)


FACT (THEOREM)
• RIGOROUS
• MODEL-INDEPENDENTLY
(NO ASSUMPTIONS ABOUT MODEL)
• NON-ASYMPTOTIC


FACT (THEOREM)
• RIGOROUS
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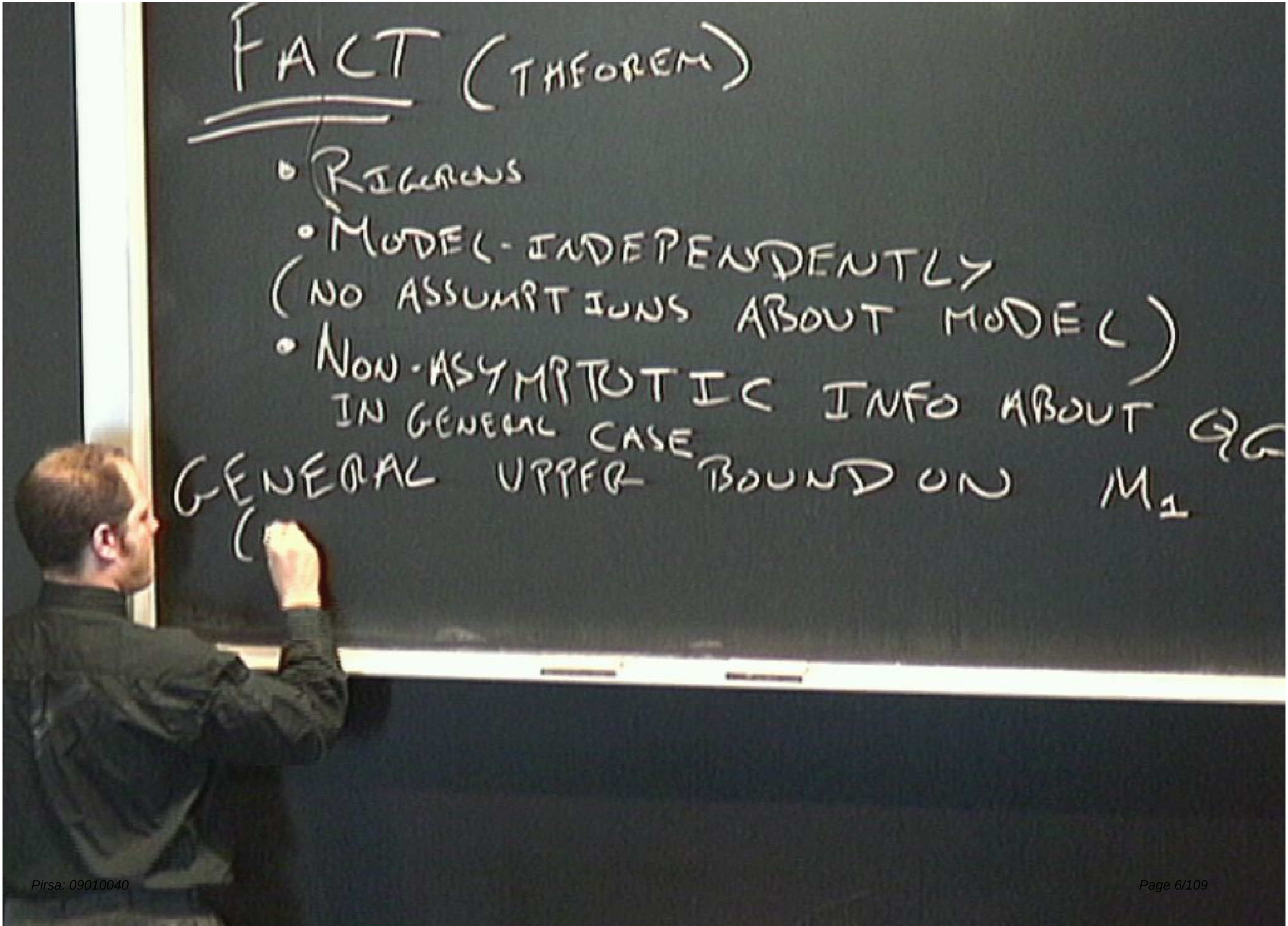
• NON-ASYMPTOTIC INFO ABOUT QG
IN GENERAL CASE


FACT (THEOREM)
• RIGOROUS
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IN GENERAL CASE
GENERAL UPPER BOUND ON M1
(

FACT (THEOREM)

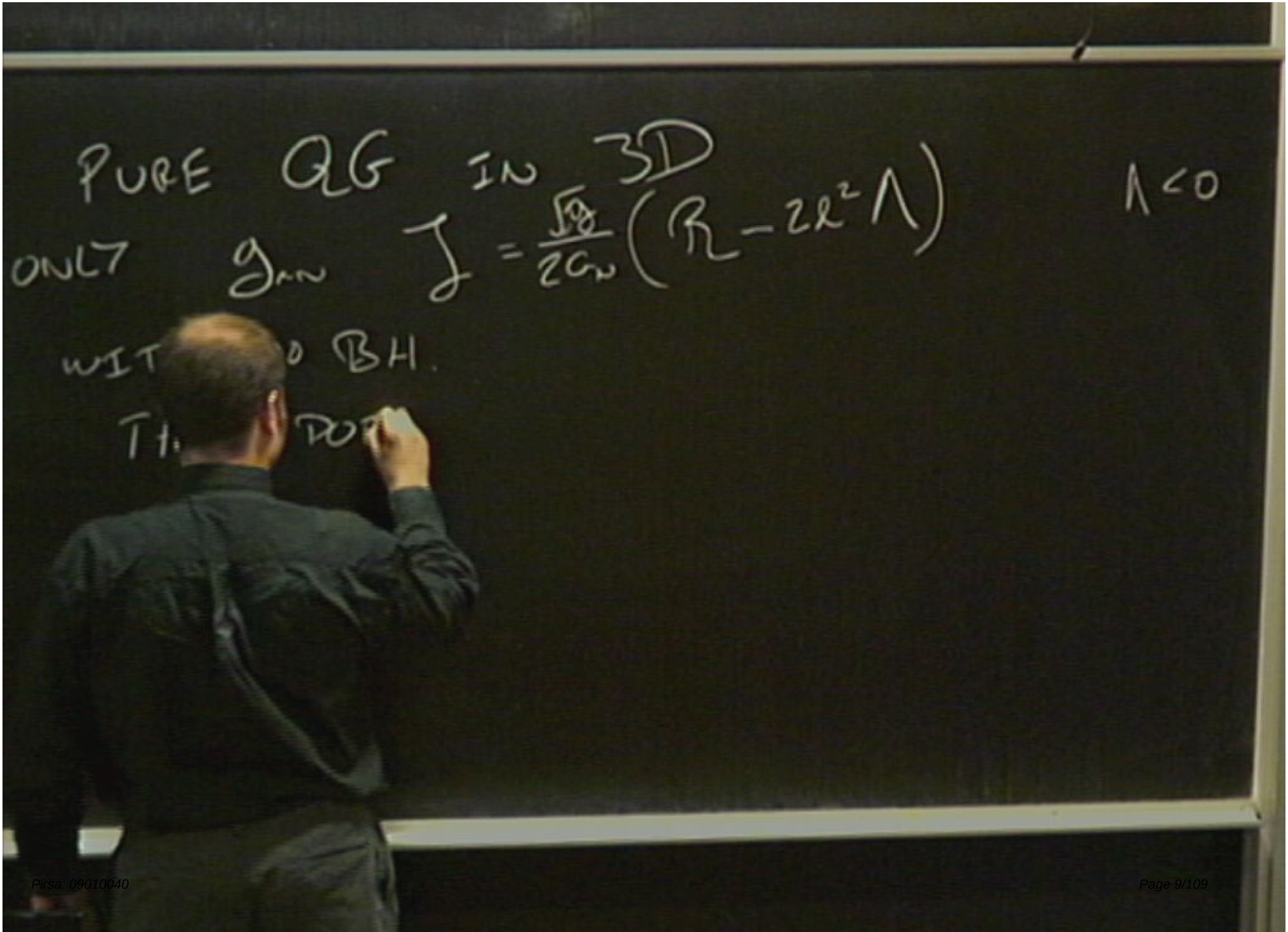
- RIGOROUS
- MODEL-INDEPENDENTLY (NO ASSUMPTIONS ABOUT MODEL)
- NON-ASYMPTOTIC INFO ABOUT QG IN GENERAL CASE

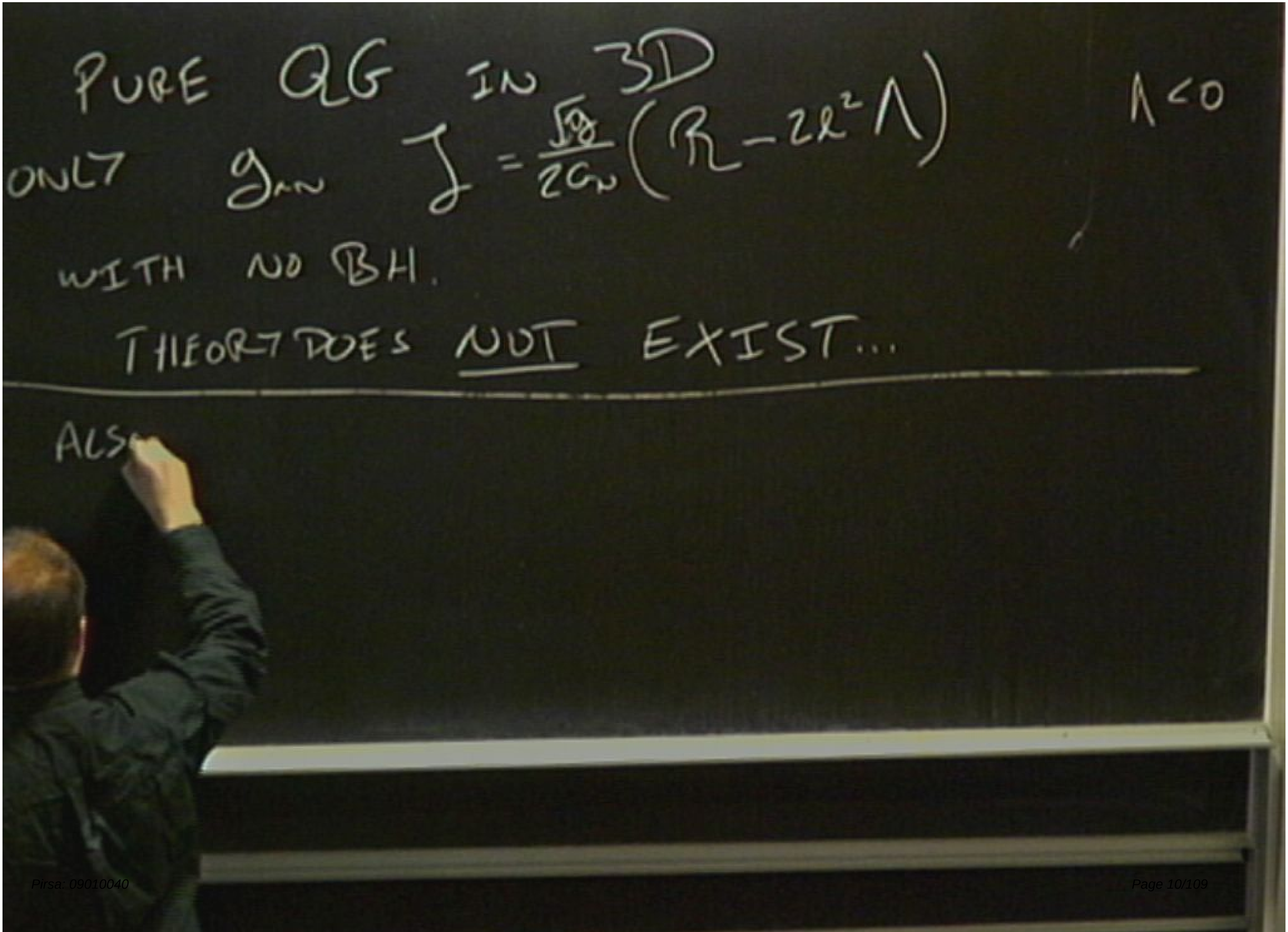
GENERAL UPPER BOUND ON M_1
(MASS OF LIGHTEST MASSIVE STATE)
WITH $G_N > 0$ AND $\Lambda \leq 0$

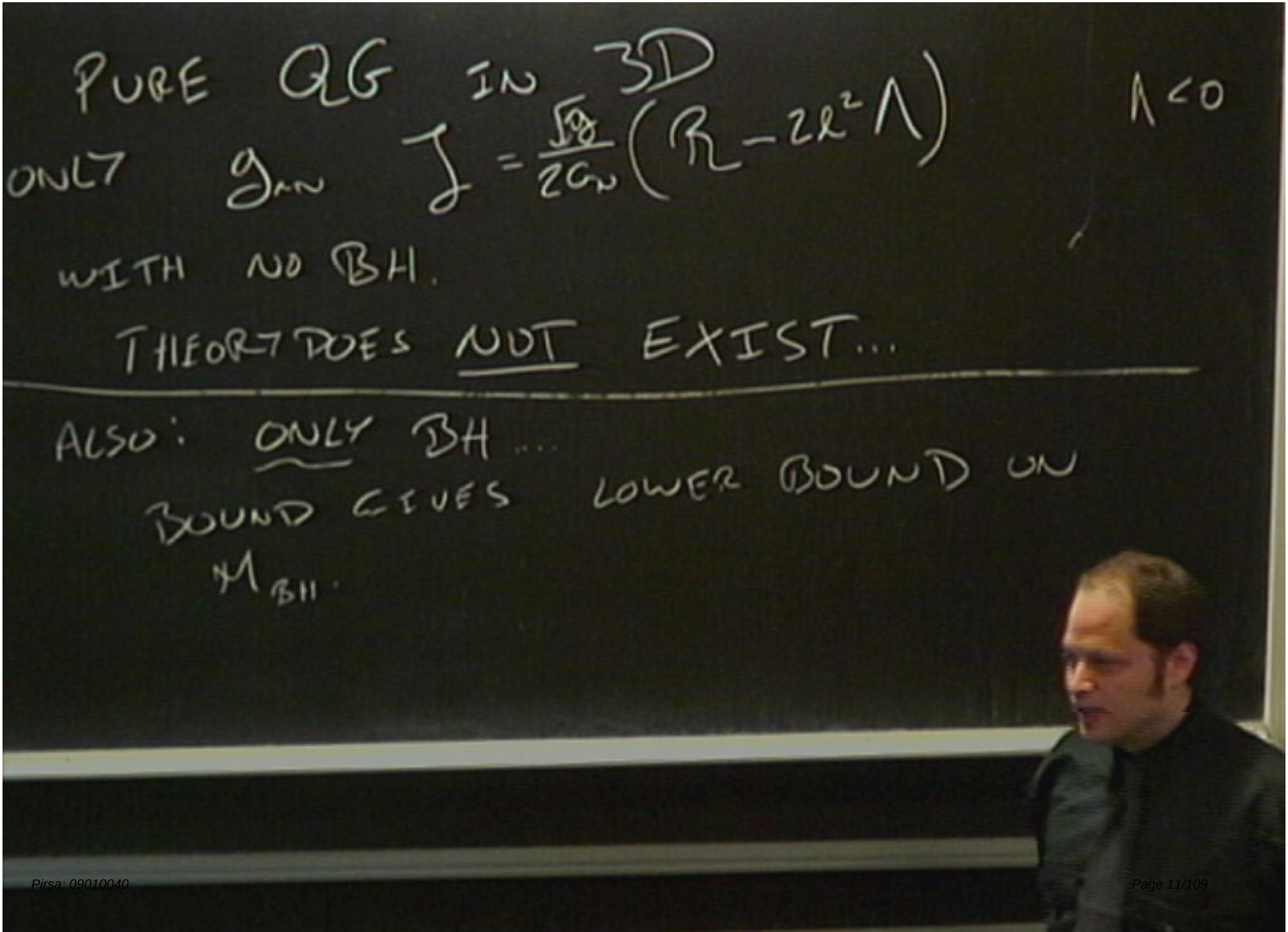
PURE QG IN 3D

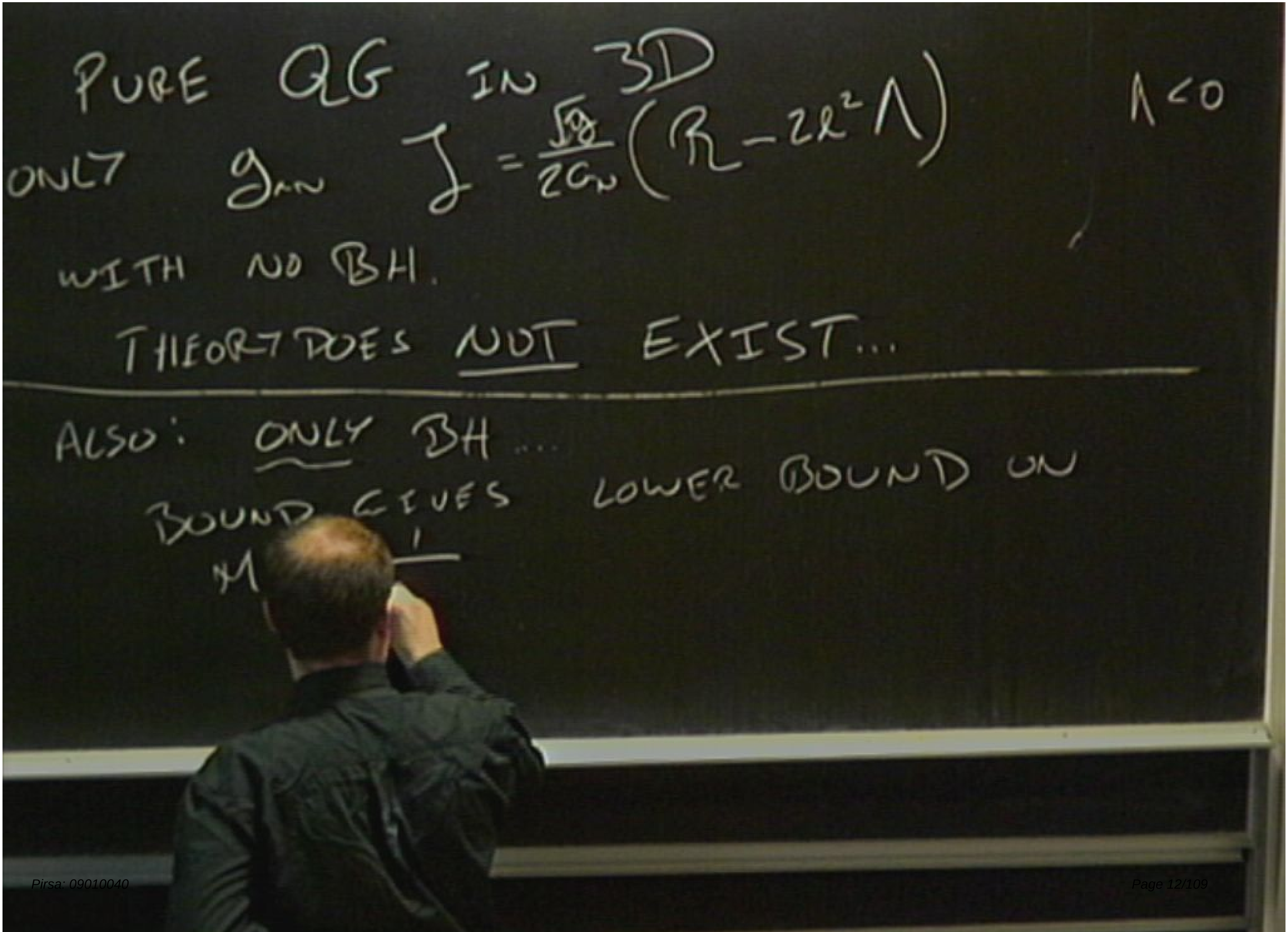
ONLY $g_{\mu\nu}$ $\mathcal{I} = \frac{\sqrt{g}}{2G_N}\left(R - 2\ell^2\Lambda\right)$

WITH NO BH.


PURE QG IN 3D
Λ<0
ONLY $g_{\mu\nu}$
$I = \frac{\sqrt{g}}{2G_N}\left(R - 2\ell^2\Lambda\right)$
BH.


PURE QG IN 3D
ONLY g
I = √g / 2G (R − 2l²Λ)
Λ<0
WITH NO BH.
THEORY DOES NOT EXIST...


PURE QG IN 3D
ONLY g_μν I = √g/2G_N (R − 2ℓ²Λ)
Λ<0
WITH NO BH.
THEORY DOES NOT EXIST...
ALSO: ONLY BH ...
BOUND GIVES LOWER BOUND ON
M_BH.


PURE QG IN 3D
Λ<0
ONLY g_μν I = √g/2G_N (R − 2ℓ²Λ)
WITH NO BH.
THEORY DOES NOT EXIST...
ALSO: ONLY BH ...
BOUND GIVES LOWER BOUND ON
M

PURE QG IN 3D $\Lambda < 0$

ONLY $g_{\mu\nu}$ $I = \frac{\sqrt{g}}{2G_N}\left(R - 2\ell^2 \Lambda\right)$

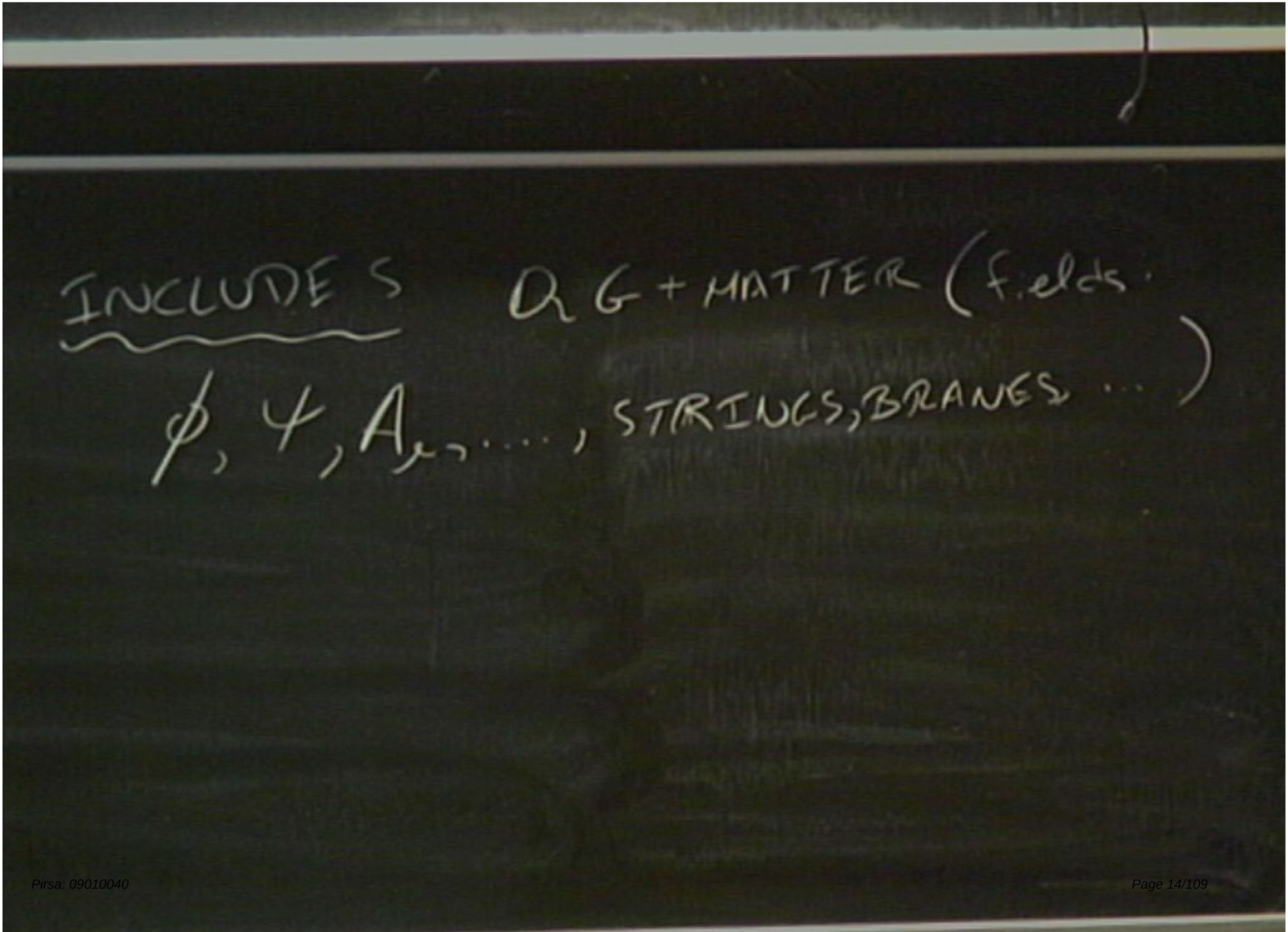
WITH NO BH.

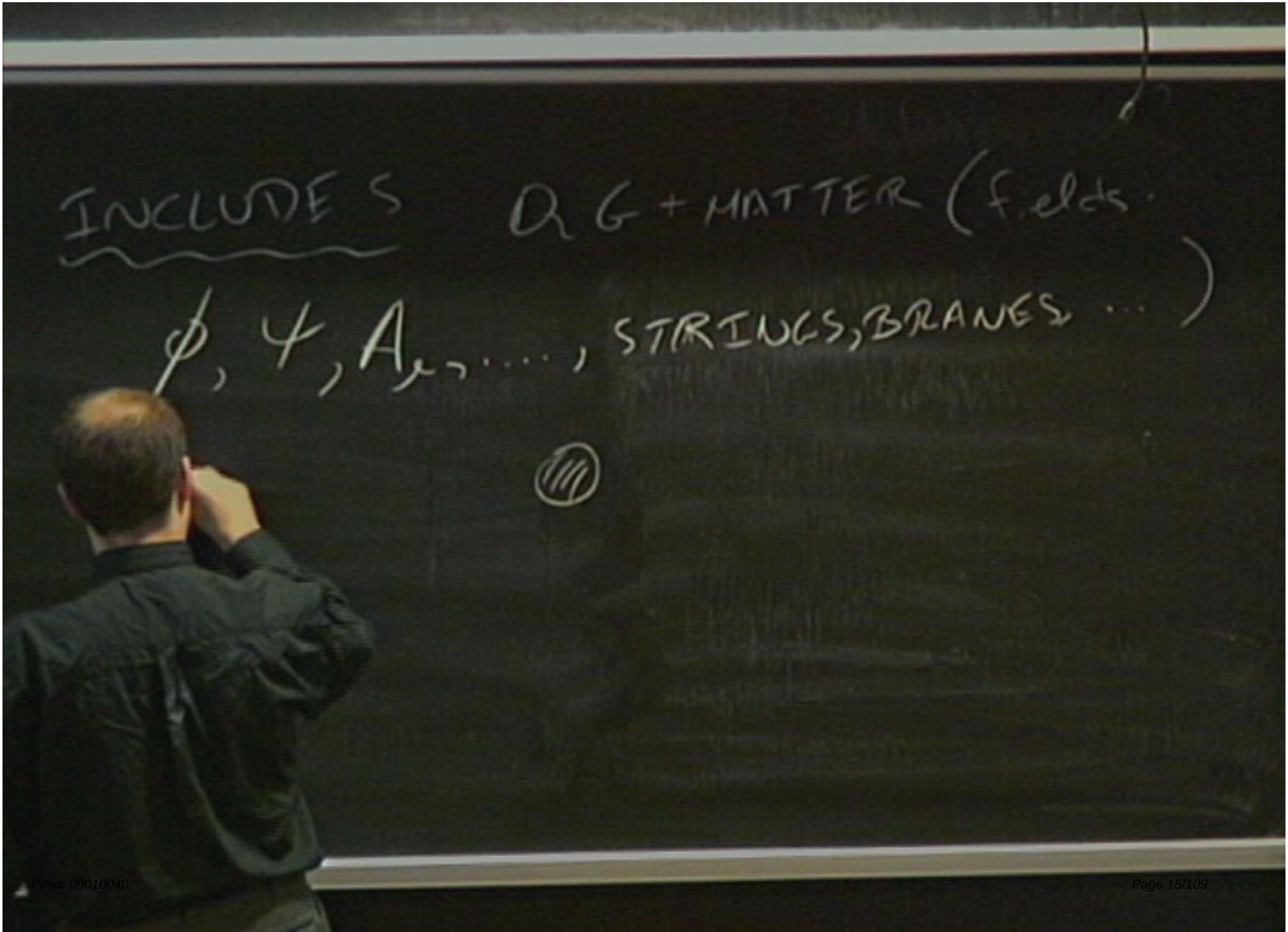
THEORY DOES NOT EXIST...

ALSO: ONLY BH ....

BOUND GIVES LOWER BOUND ON

$M_{BH} = \frac{1}{4G_N}$.


INCLUDES QG + MATTER (fields:
φ, ψ, A_μ, ..., STRINGS, BRANES ...)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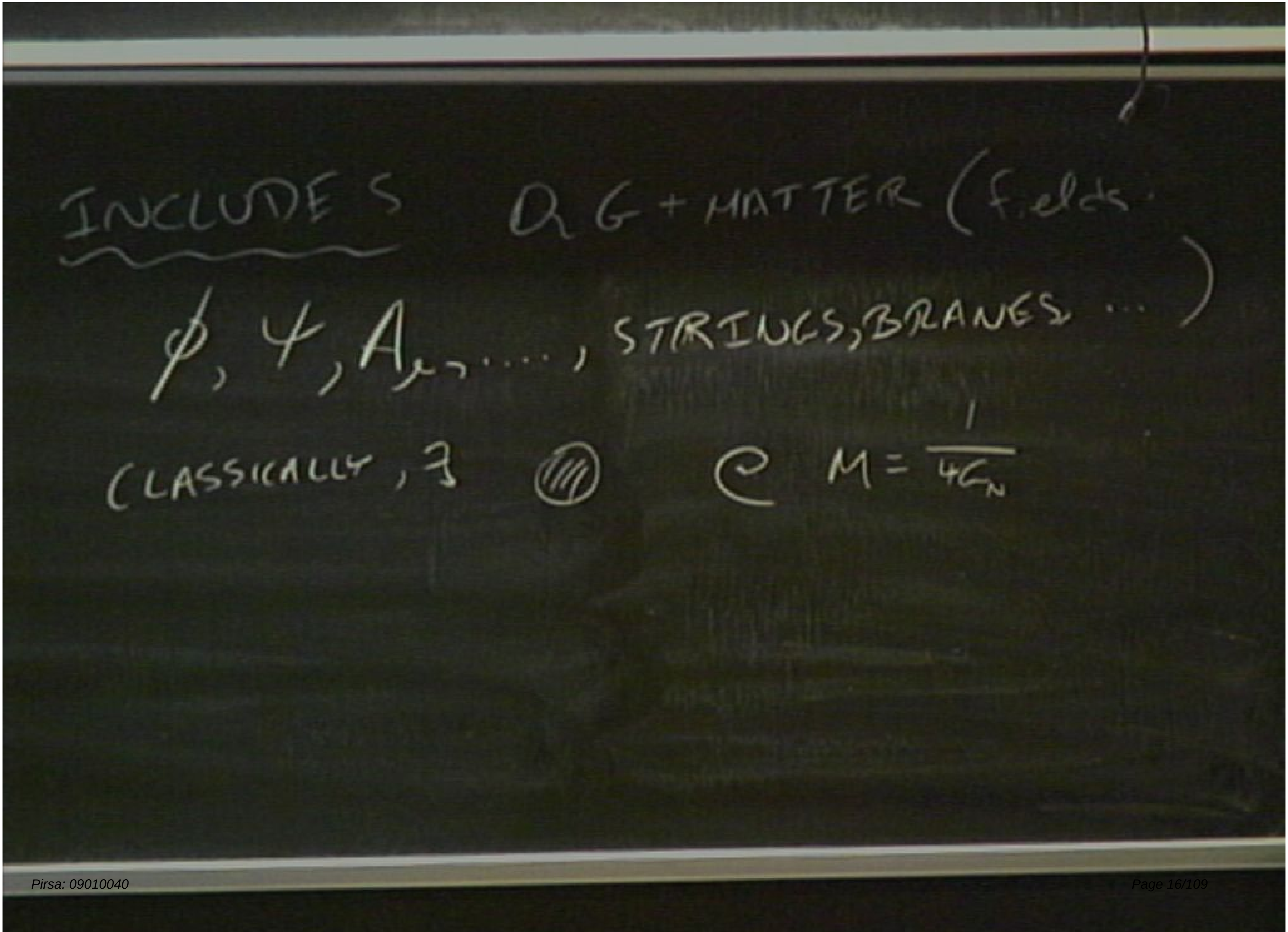
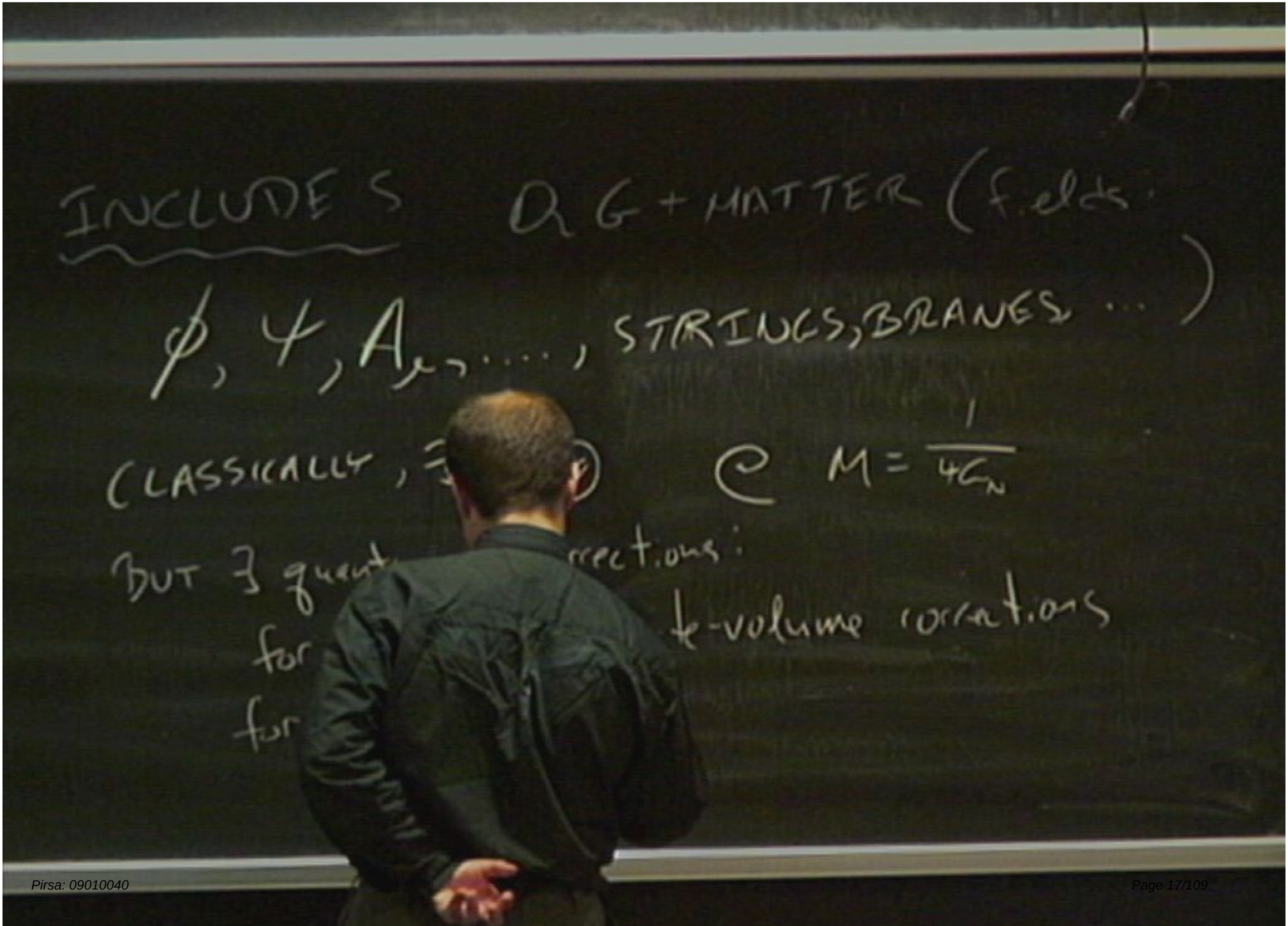
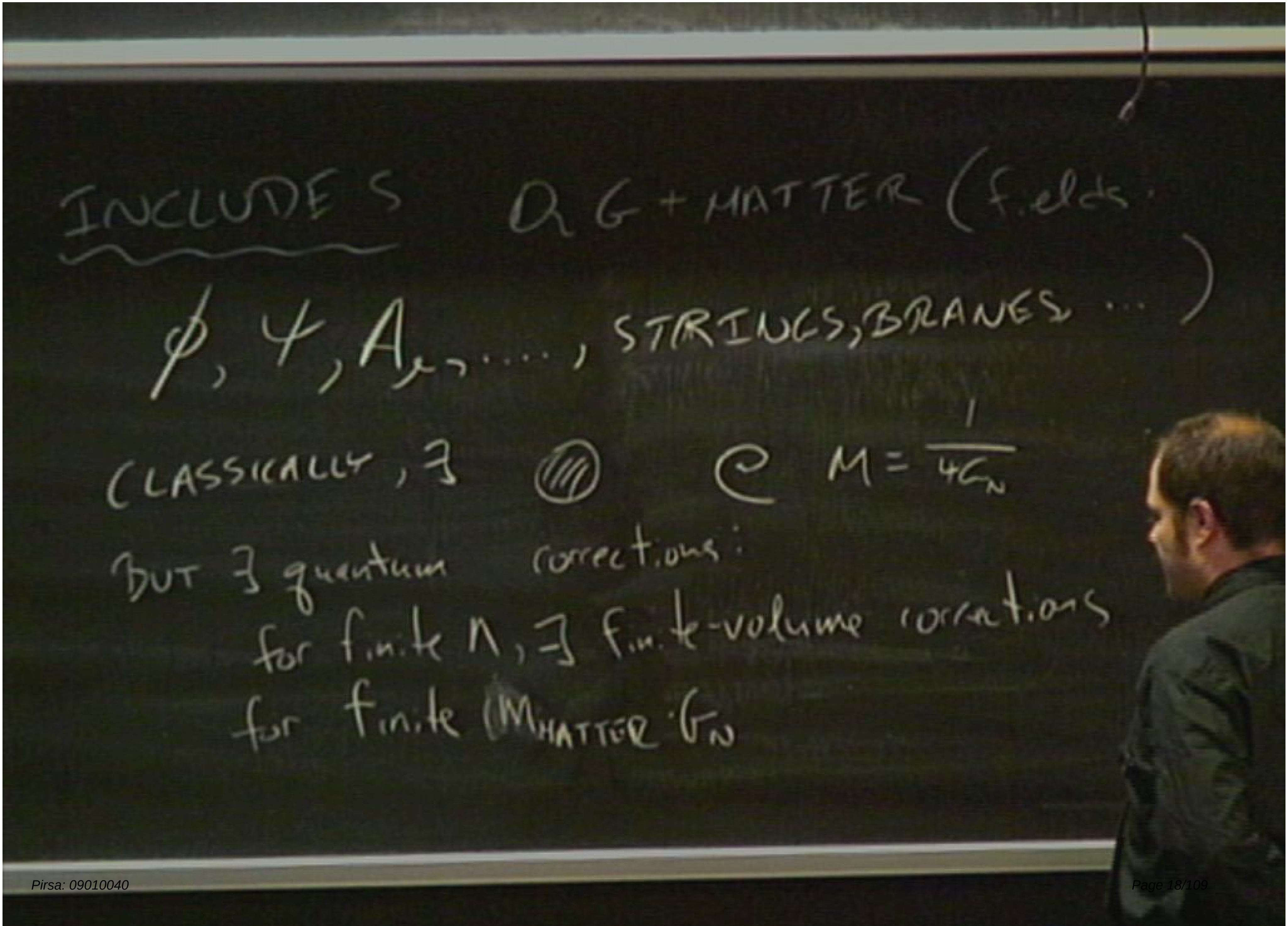
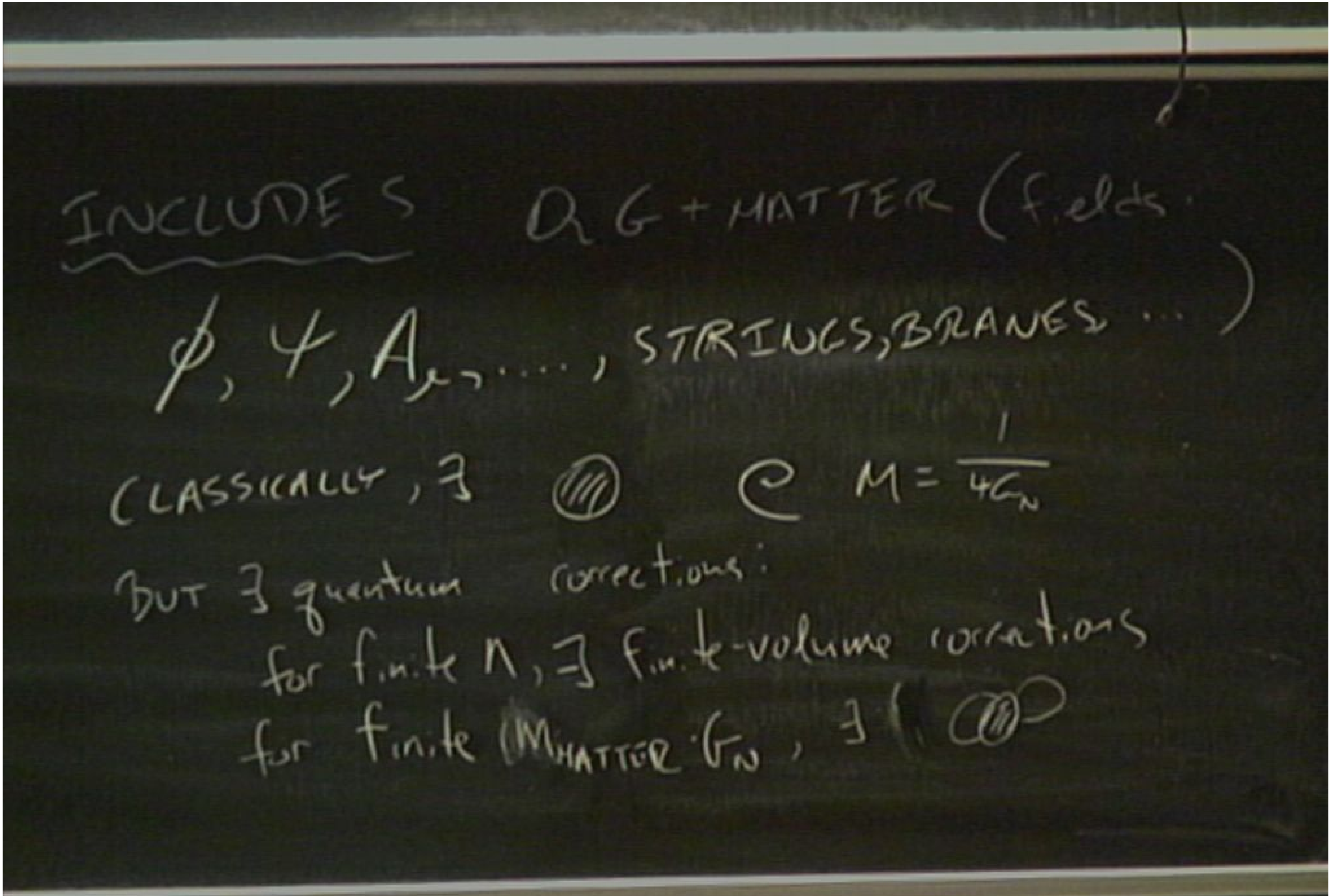

INCLUDES QG + MATTER (fields
φ, ψ, A_μ, ..., STRINGS, BRANES ...)
CLASSICALLY, ∃ @ M = 1/4G_N


INCLUDES
QG + MATTER (fields
STRINGS, BRANES ...)
CLASSICALLY,
M = 1/4G_N
BUT ∃
-volume corrections
for
for


INCLUDES QG + MATTER (fields
ϕ, ψ, A_μ, ..., STRINGS, BRANES ...)
CLASSICALLY, ∃ @ $M = \frac{1}{4G_N}$
BUT ∃ quantum corrections:
for finite Λ, ∃ finite-volume corrections
for finite $M_{MATTER} \cdot G_N$


INCLUDES QG + MATTER (fields:
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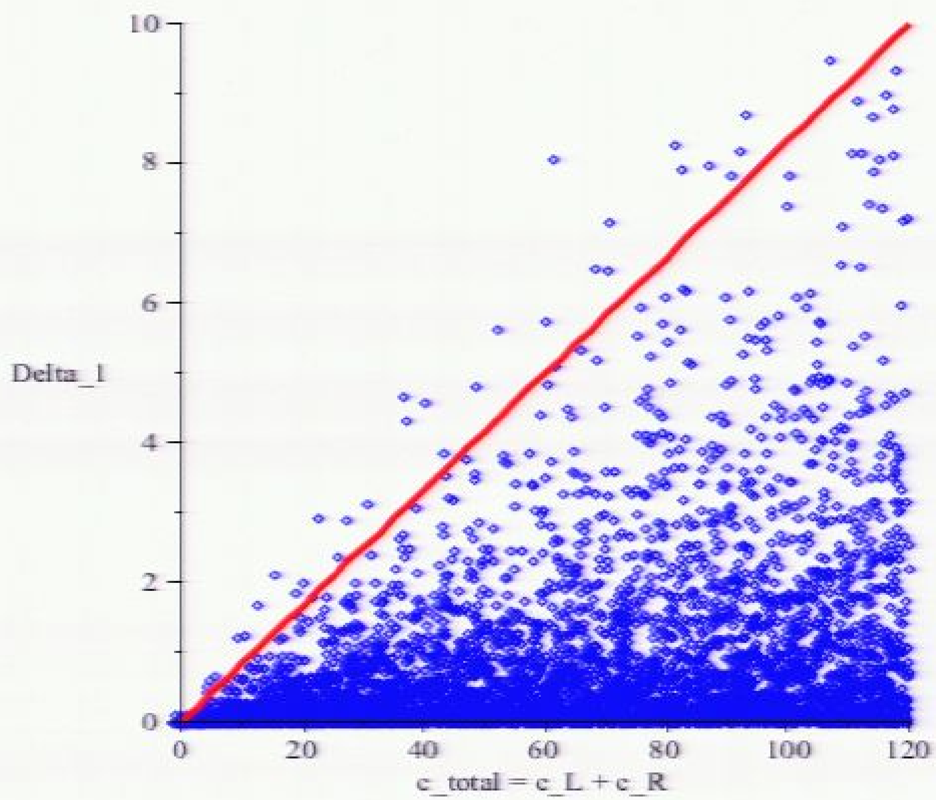
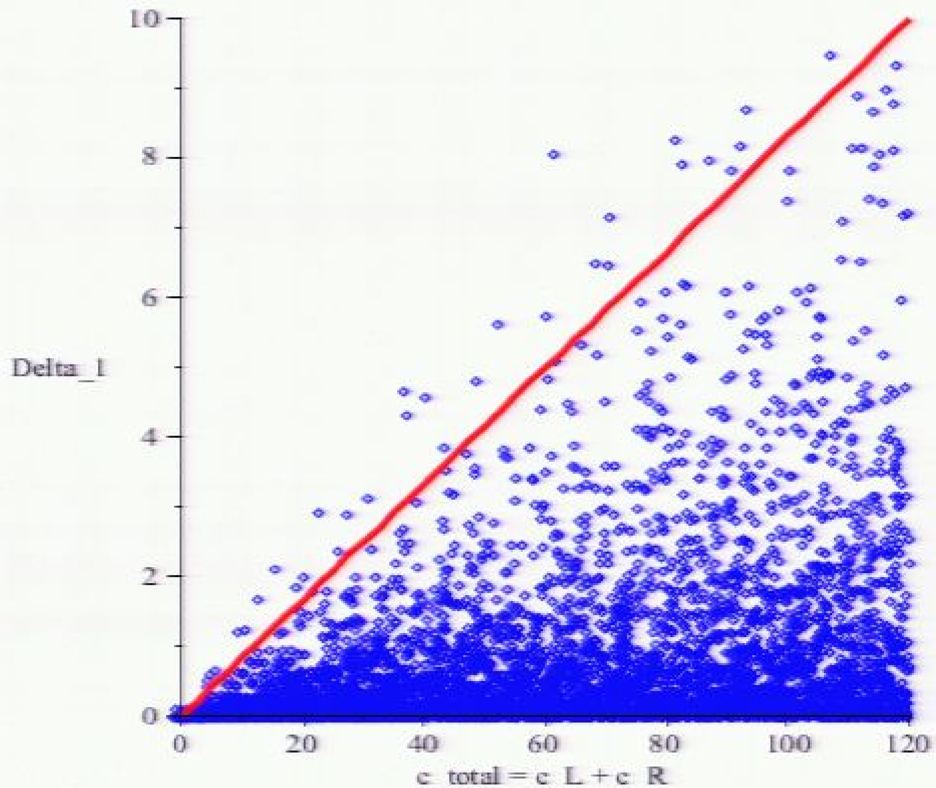
CLASSICALLY, ∃ @ M = 1/4G_N
BUT ∃ quantum corrections:
for finite Λ, ∃ finite-volume corrections
for finite M_MATTER · G_N, ∃

Does the landscape of 2D CFT look like this?



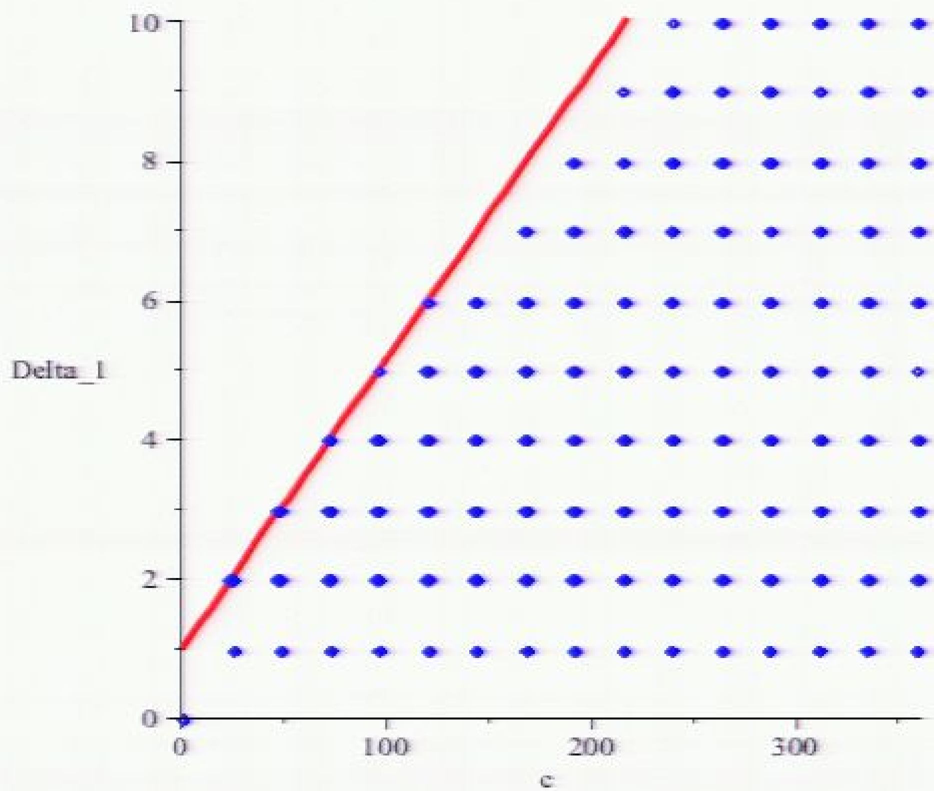
One logical possibility is that there is no "sharp" upper bound on Δ_1, just a random distribution that falls off quickly above $\Delta_1 \simeq \frac{c+\tilde{c}}{12}$.

Does the landscape of 2D CFT look like this?



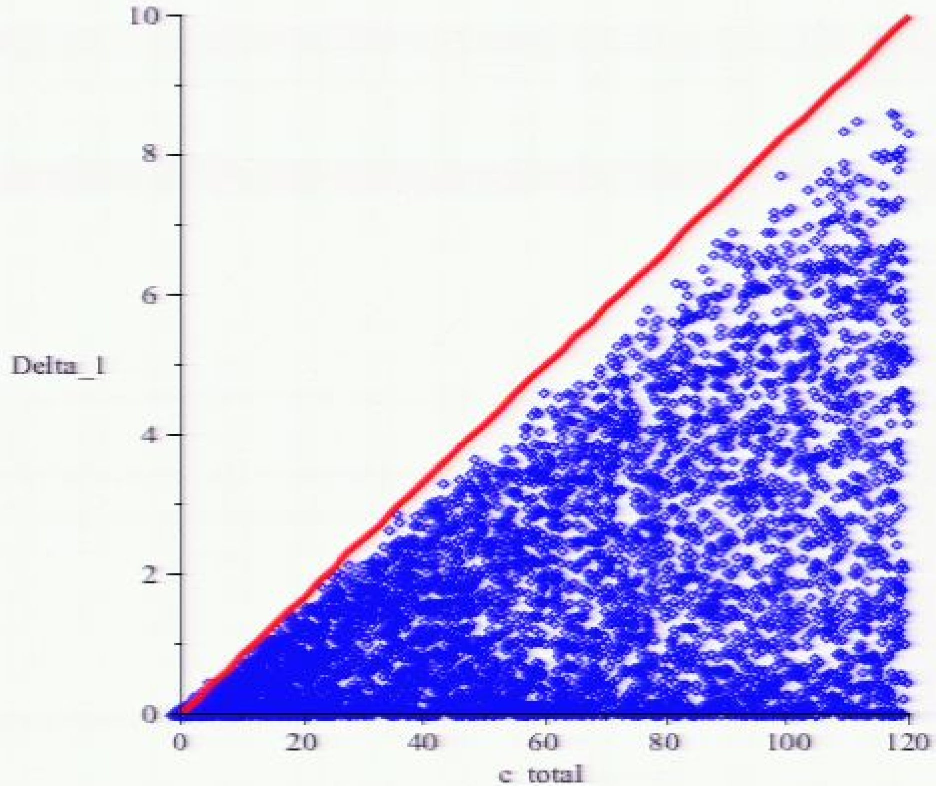
One logical possibility is that there is no "sharp" upper bound on Δ_1, just a random distribution that falls off quickly above $\Delta_1 \simeq \frac{c+\tilde{c}}{12}$.

The landscape of holomorphically factorized 2D CFT looks like this

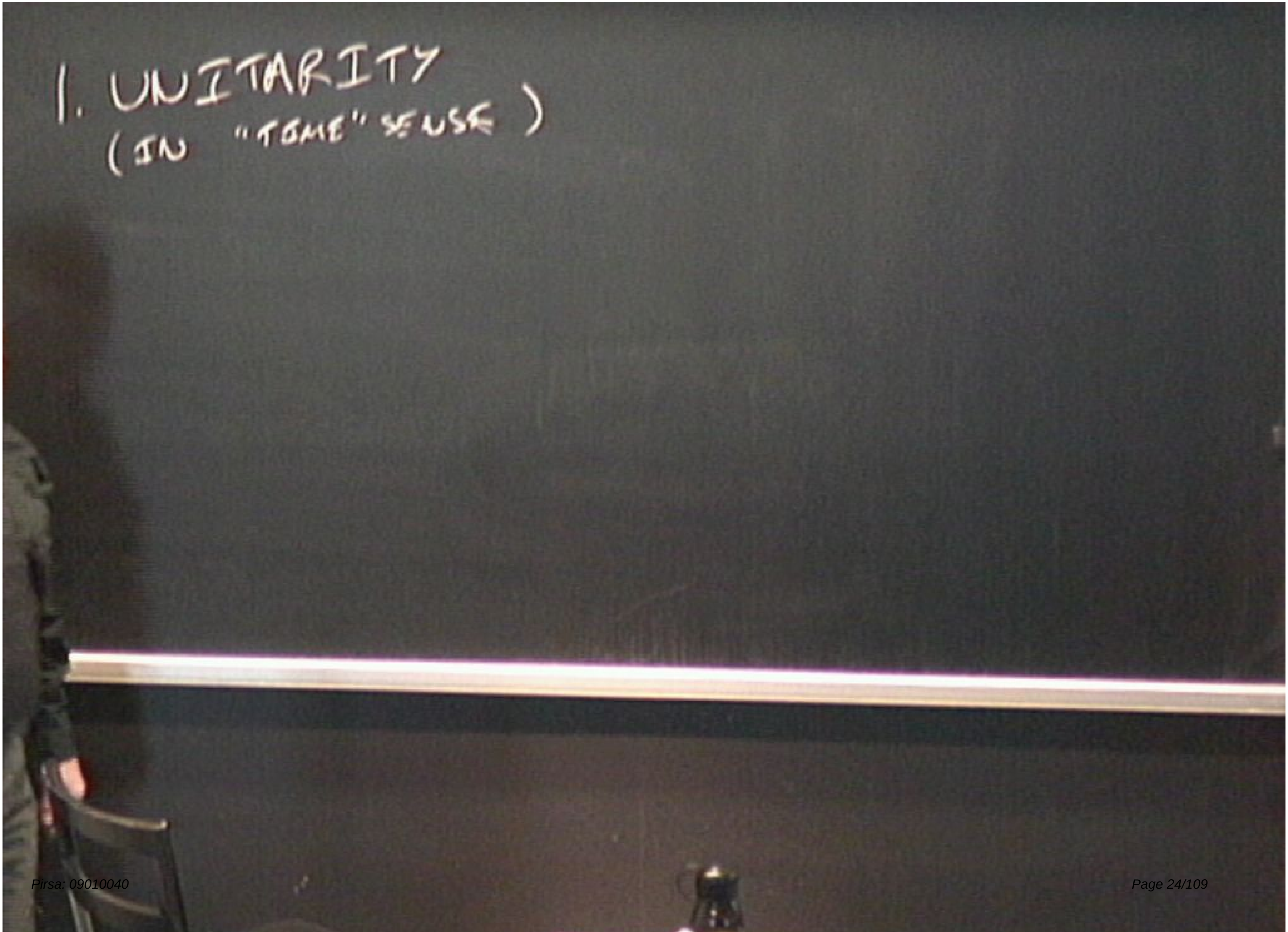


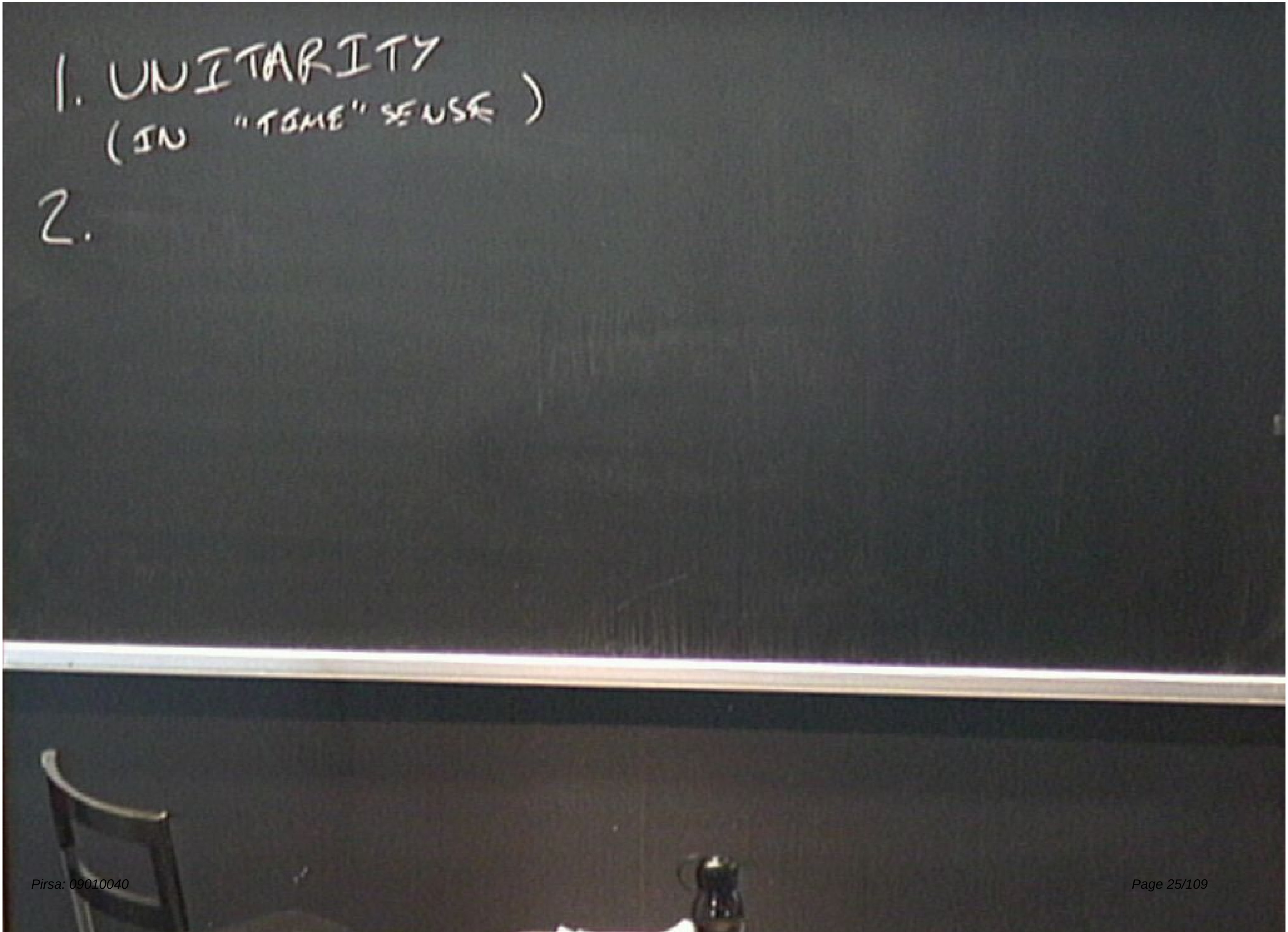
We know for a fact that the landscape of holomorphically factorized CFT looks something like this. In this case, the red line lies at $\Delta_1 = 1 + \frac{c+\tilde{c}}{24}$.

We'll show that the full landscape of 2D CFT looks like this...



Here, the red line lies at $\Delta_1 = \frac{3}{2\pi} + \frac{c+\tilde{c}}{12}$.


1. UNITARITY
(IN "TIME" SENSE)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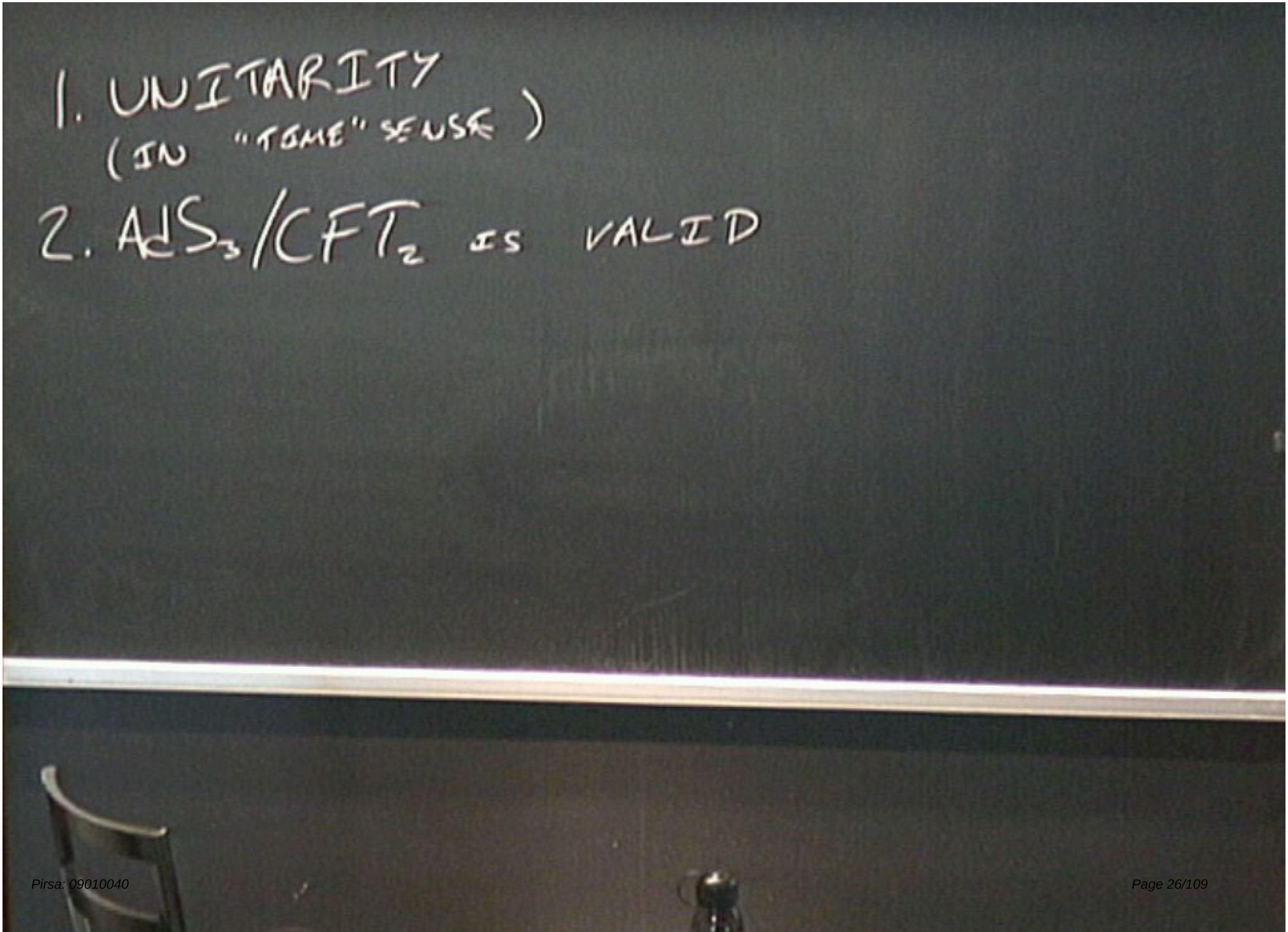
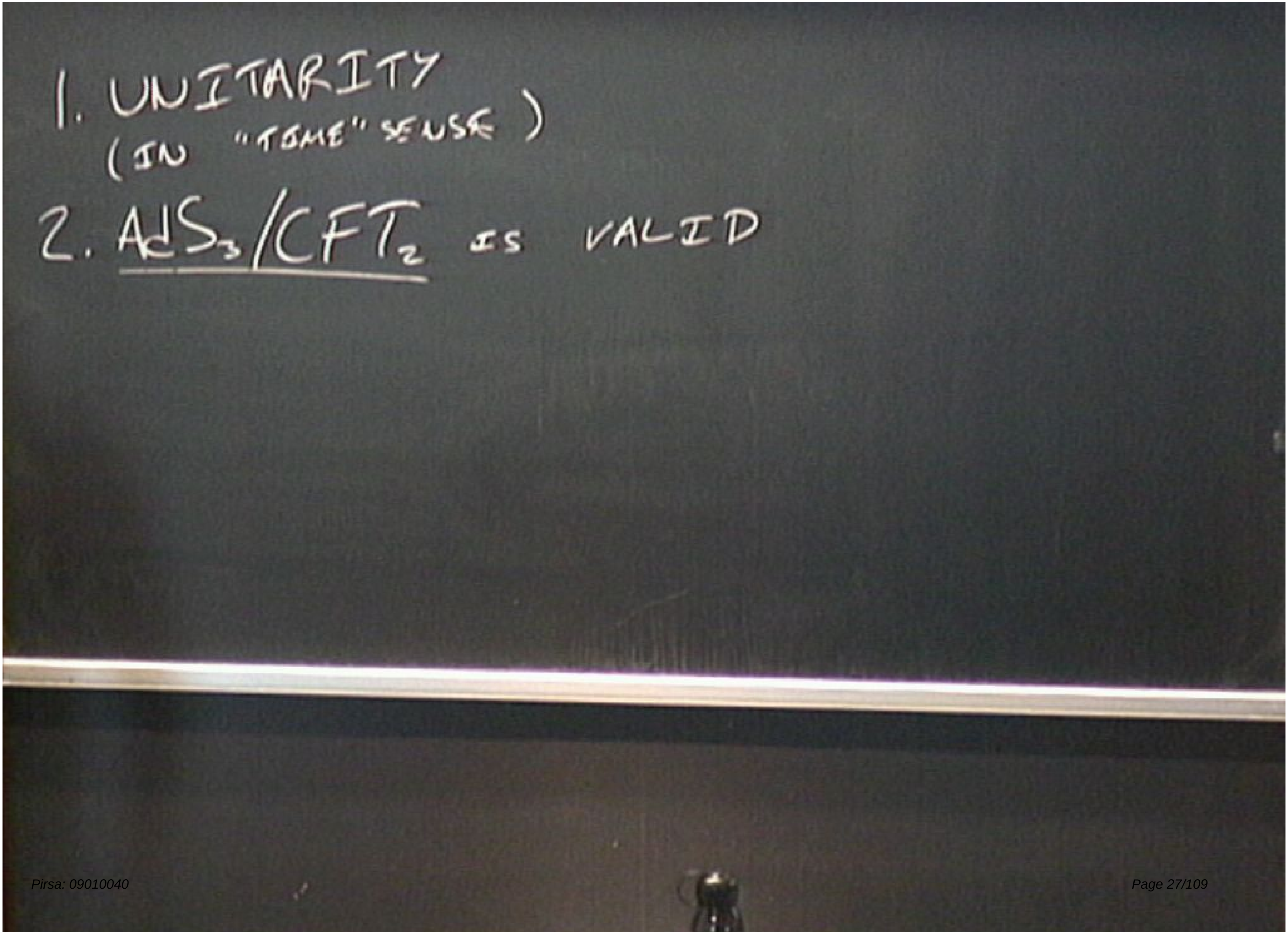
(IN "TIME" SENSE)
2.


1. UNITARITY
(IN "TIME" SENSE)
2. AdS_3/CFT_2 IS VALID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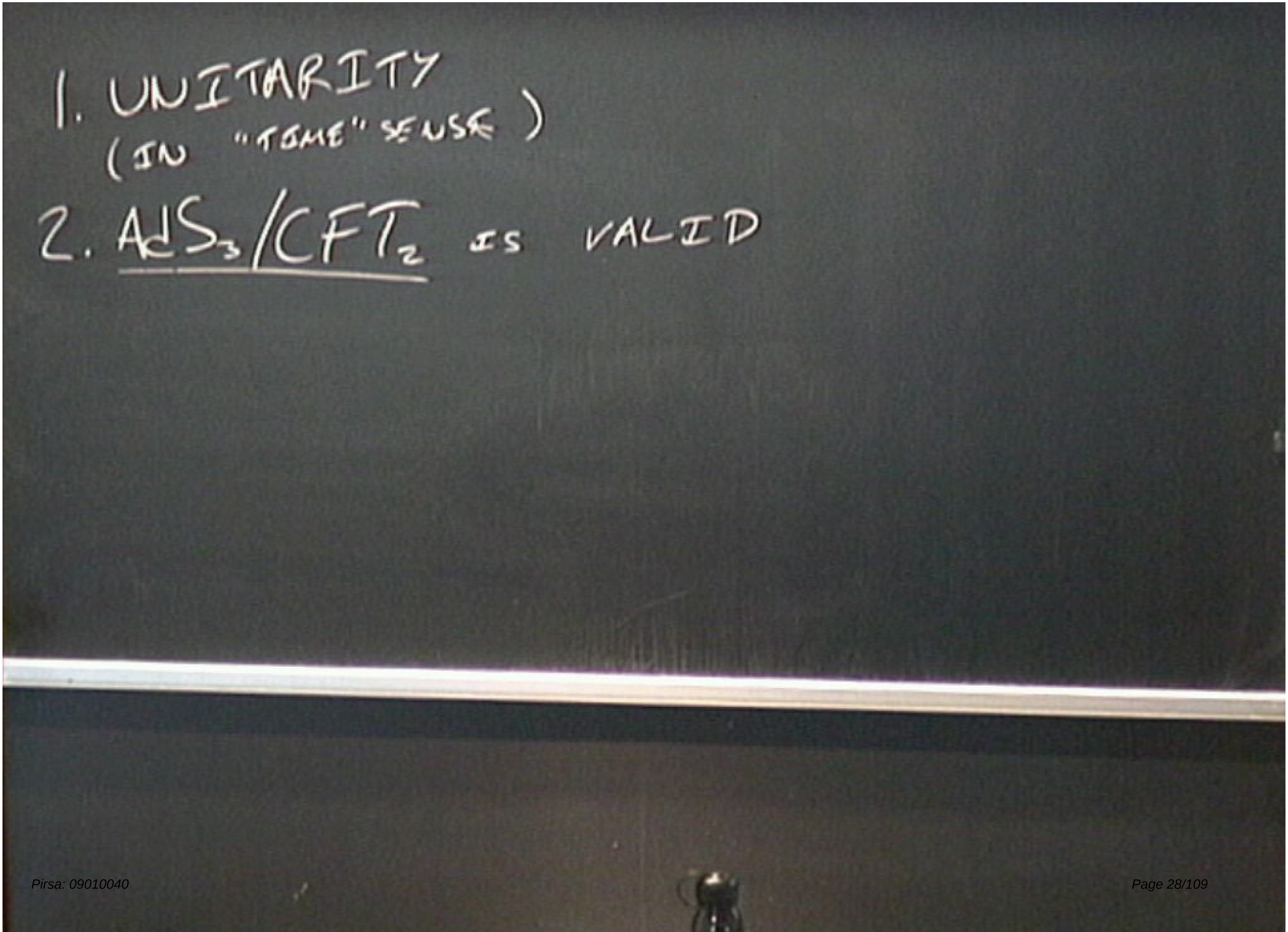
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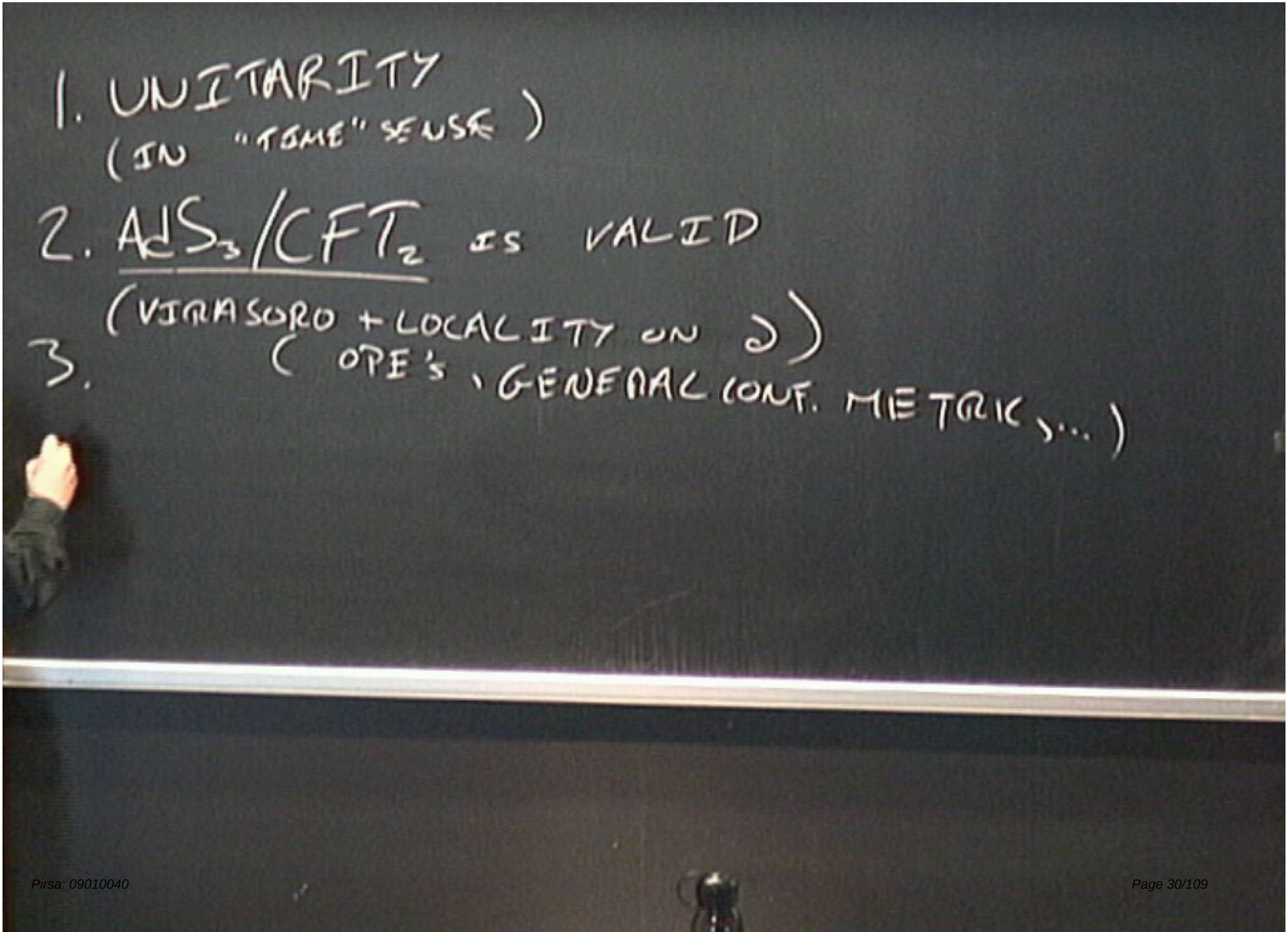
1. UNITARITY
(IN "TIME" SENSE)

2. $\underline{AdS_3/CFT_2}$ IS VALID
(VIRASORO + LOCALITY ON ∂)


1. UNITARITY
(IN "TIME" SENSE)
2. AdS3/CFT2 IS VALID
(VIRASORO + LOCALITY ON ∂)
3. (OPE's , GENERAL CONF. METRIC, ...)

1. UNITARITY
(IN "TIME" SENSE)

2. AdS_3/CFT_2 IS VALID
(VIRASORO + LOCALITY ON ∂)

3. MODULAR INVARIANCE
(OPE'S , GENERAL CONF. METRIC, ...)

1. UNITARITY
(IN "TIME" SENSE)

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(VIRASORO + LOCALITY ON ∂)
(OPE's, GENERAL CONF. METRIC, ...)

3. MODULAR INVARIANCE

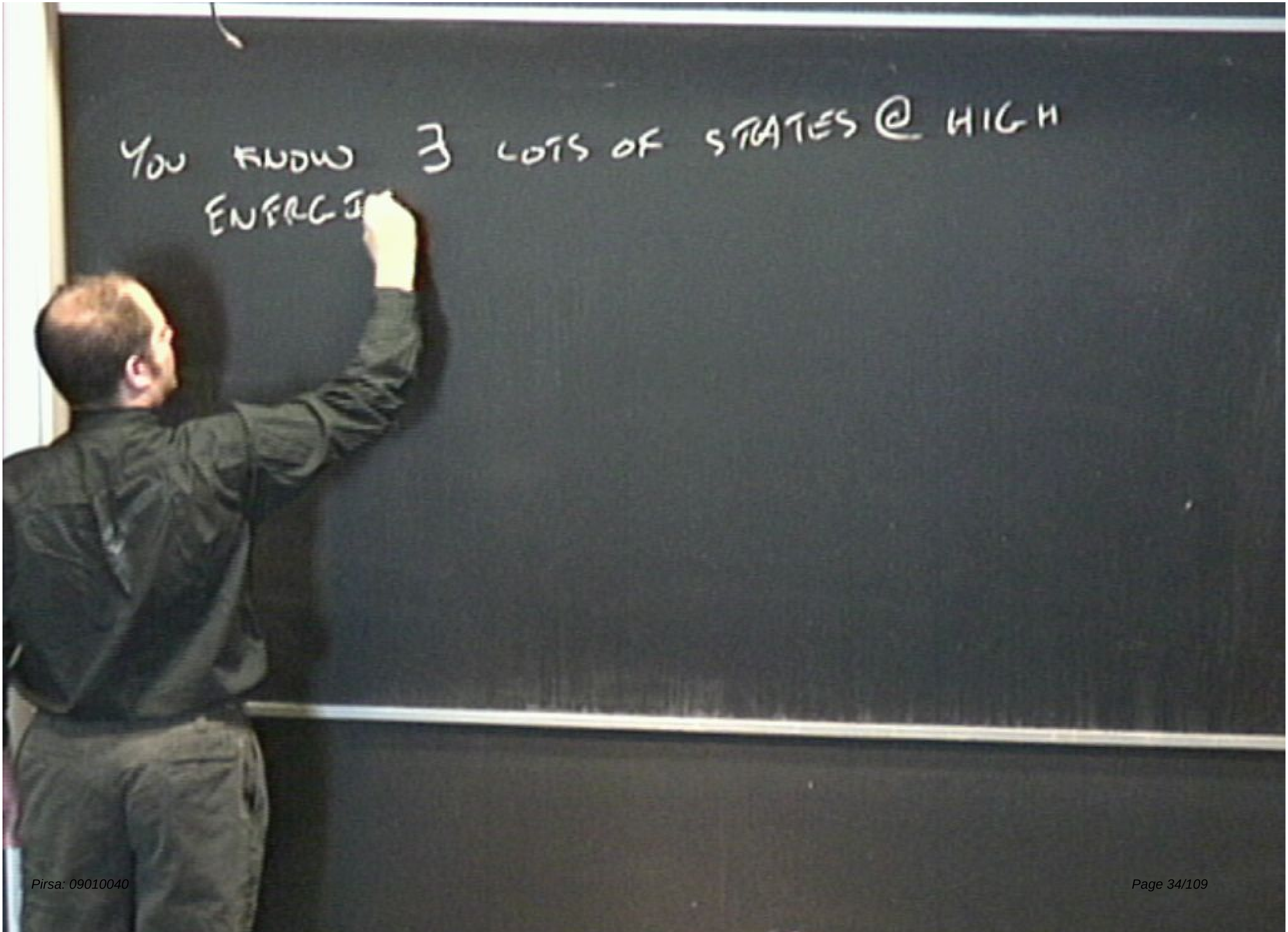
(VIRASORO + LOCALITY ON ∂)

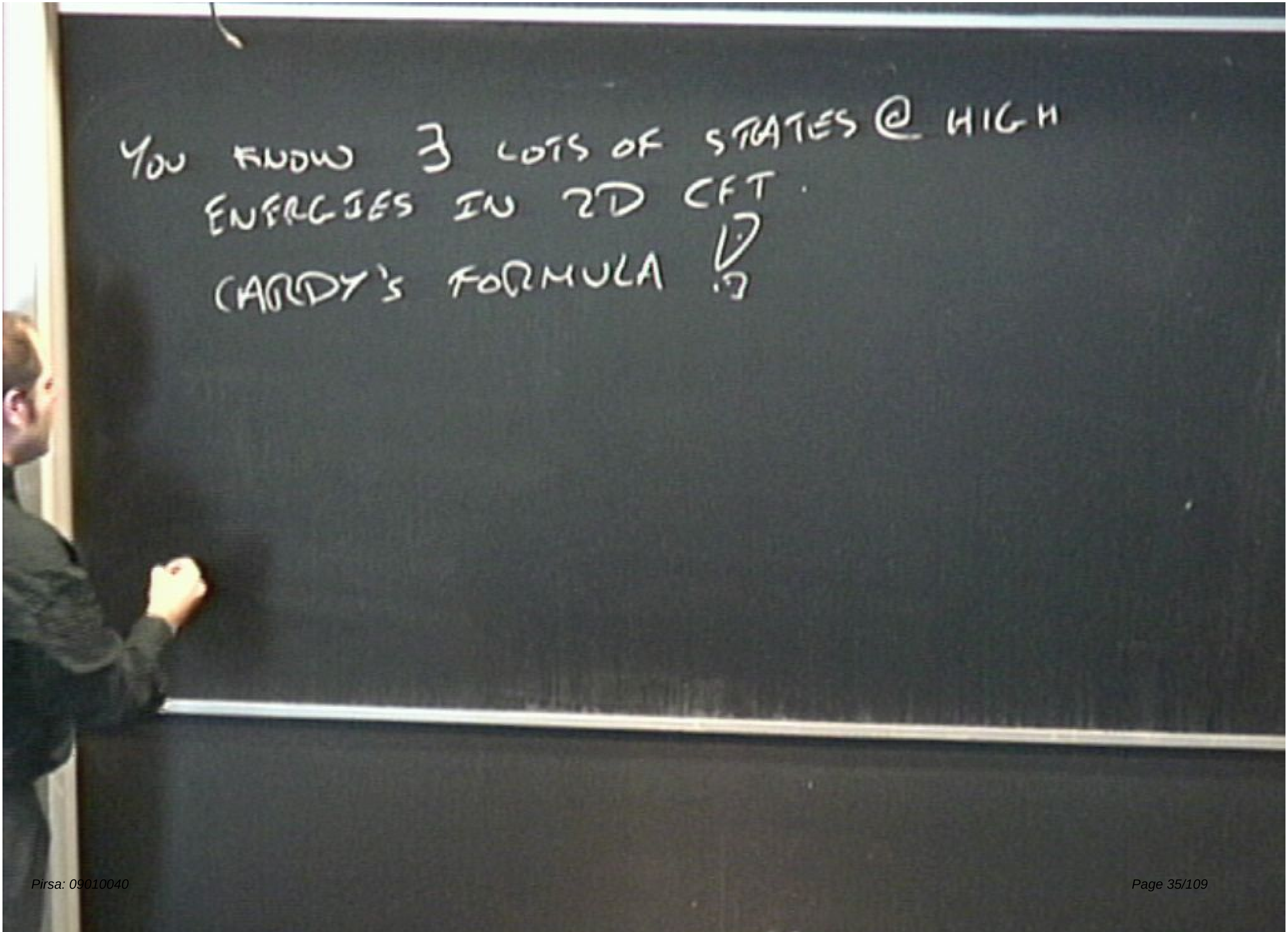
(OPE's , GENERAL CONF. METRIC , ...)

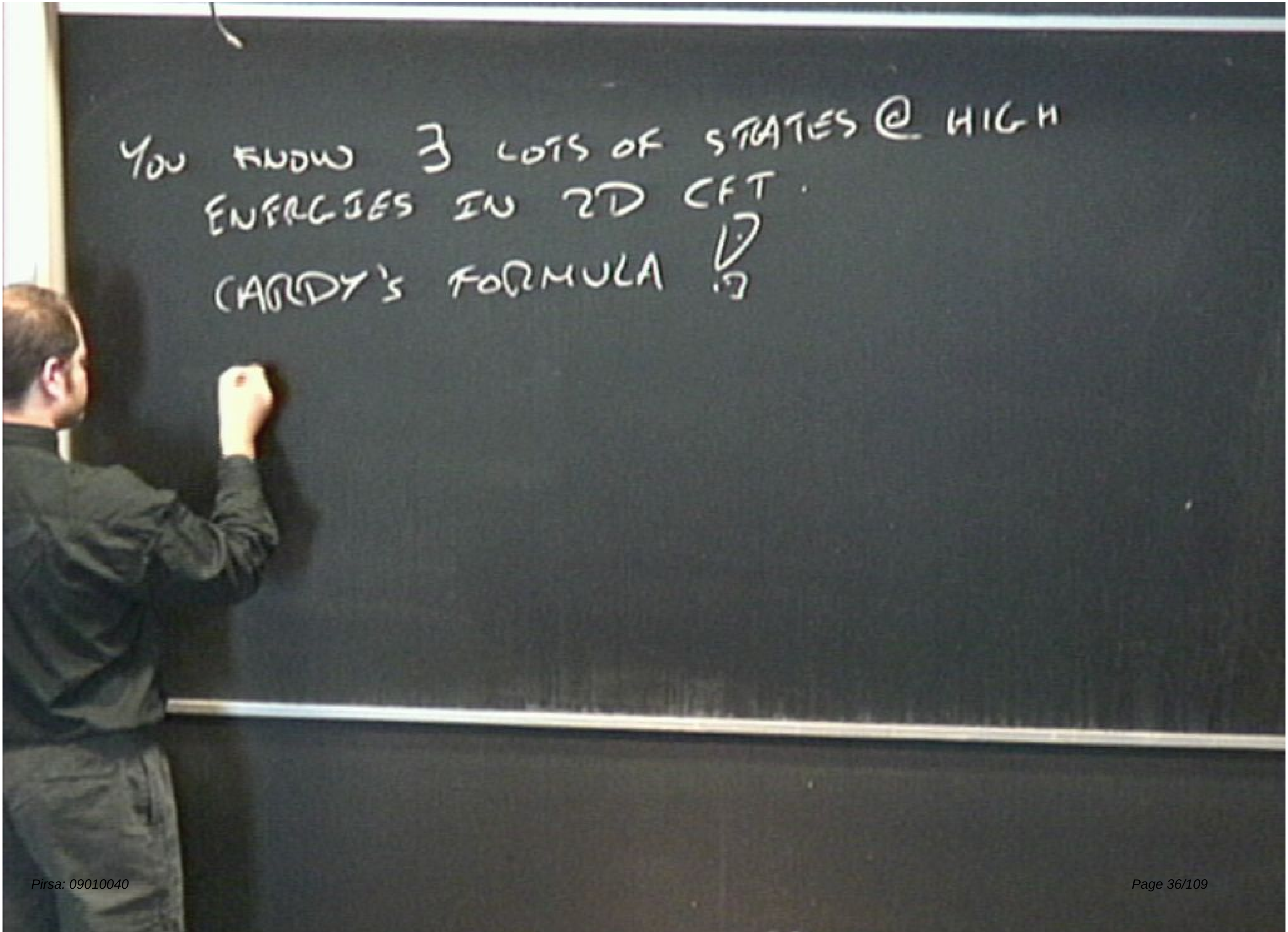
3. MODULAR INVARIANCE

4. DISCRETE OF SPECTRUM

I WILL


YOU KNOW ∃ LOTS OF STATES @ HIGH
ENERG


YOU KNOW ∃ LOTS OF STATES @ HIGH
ENERGIES IN 2D CFT.
CARDY'S FORMULA !?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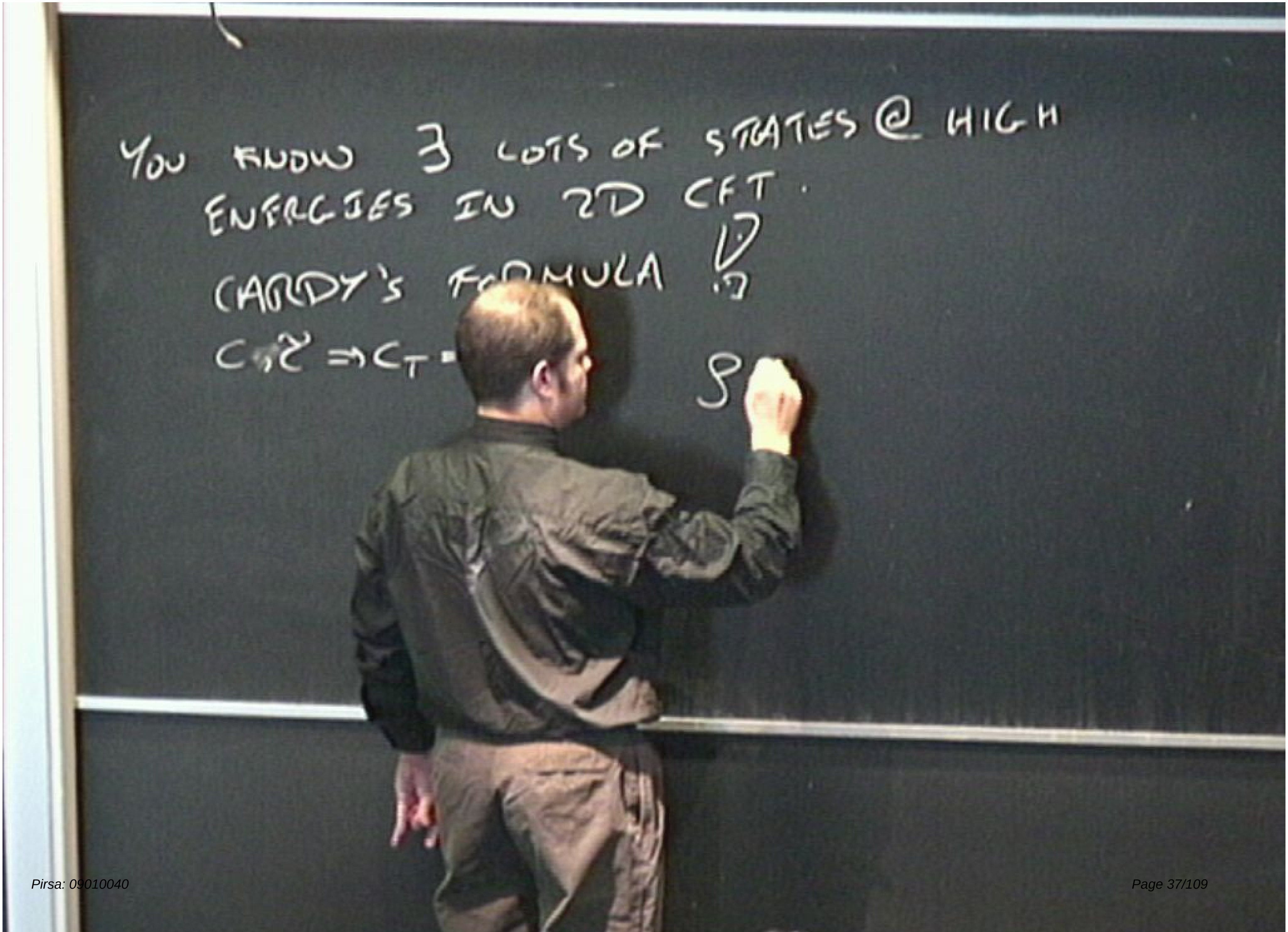
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YOU KNOW ∃ LOTS OF STATES @ HIGH ENERGIES IN 2D CFT.

CARDY'S FORMULA !!

$c, \tilde{c} \Rightarrow c_T = c + \tilde{c}$

$$\frac{dN}{dE} \sim e^{\left(\sqrt{\frac{c_T}{6} E}\right)}$$

YOU KNOW $\exists$ LOTS OF STATES @ HIGH ENERGIES IN 2D CFT.

CARDY'S FORMULA !

$c, \tilde{c} \Rightarrow c_T = c + \tilde{c}$

$$\frac{dN}{dE} \sim e^{\left(\sqrt{\frac{c_T}{6} E}\right)}$$ AT HIGH E.

YOU KNOW ∃ LOTS OF STATES @ HIGH ENERGIES IN 2D CFT.

CARDY'S FORMULA !

$c, \tilde{c} \Rightarrow c_T = c + \tilde{c}$

$\beta = \frac{k_B}{T}$

$Z(\beta)$

$\frac{dN}{dE} \sim e\left(\sqrt{\frac{c_T}{6} E}\right)$ AT HIGH E.

YOU KNOW $\exists$ LOTS OF STATES @ HIGH ENERGIES IN 2D CFT.

CARDY'S FORMULA !!

$c, \tilde{c} \Rightarrow c_T = c + \tilde{c}$

$\beta = \frac{k_B}{T}$

$$Z(\beta) = \sum_n e(-\beta E_n)$$

$$\frac{dN}{dE} \sim e\left(\sqrt{\frac{c_T}{6} E}\right) \text{ AT HIGH } E.$$

$$Z(\beta) = Z\left(\frac{4\pi^2}{\beta}\right)$$

YOU KNOW ∃ LOTS OF STATES @ HIGH ENERGIES IN 2D CFT.

CARDY'S FORMULA !?

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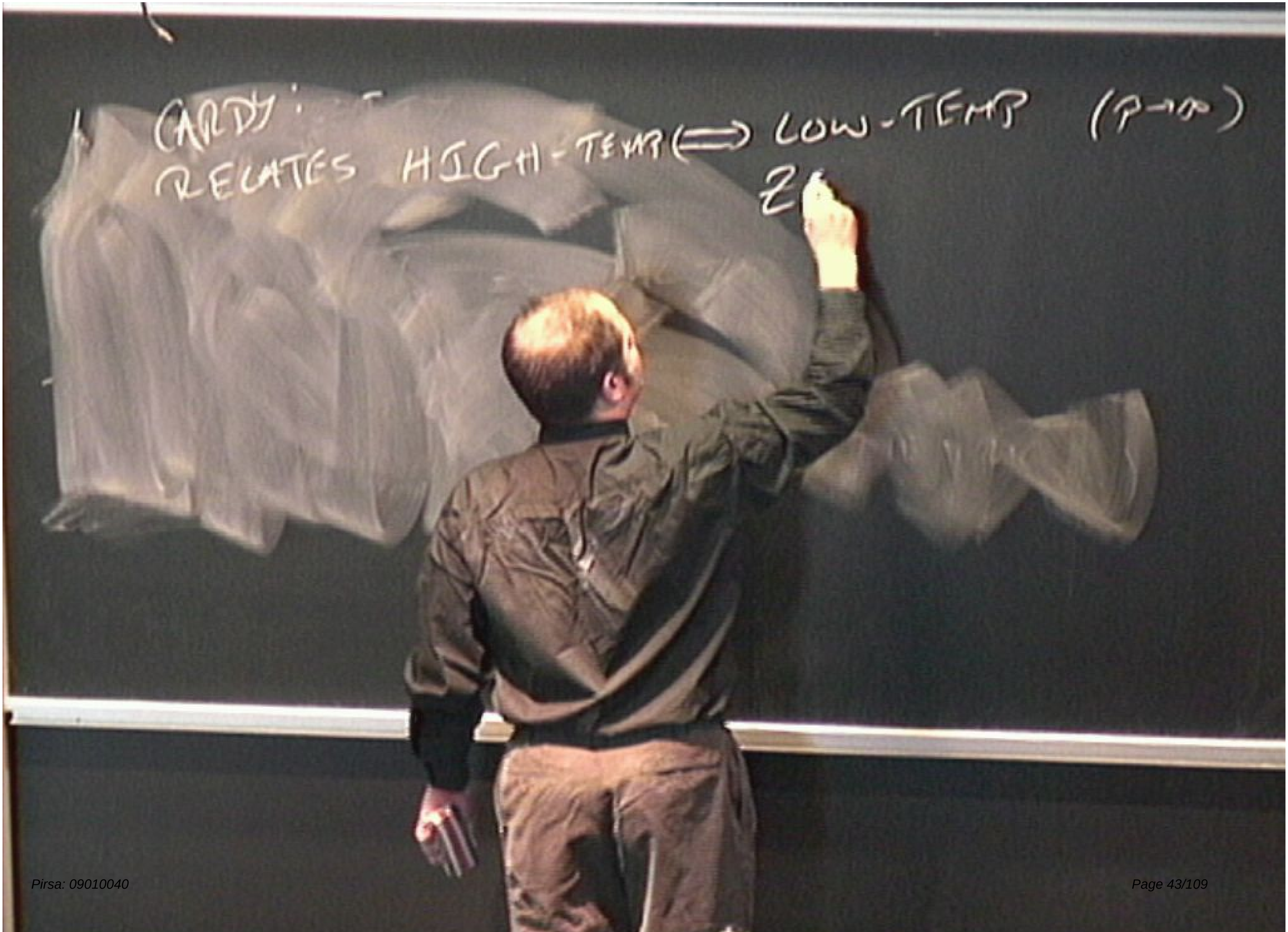
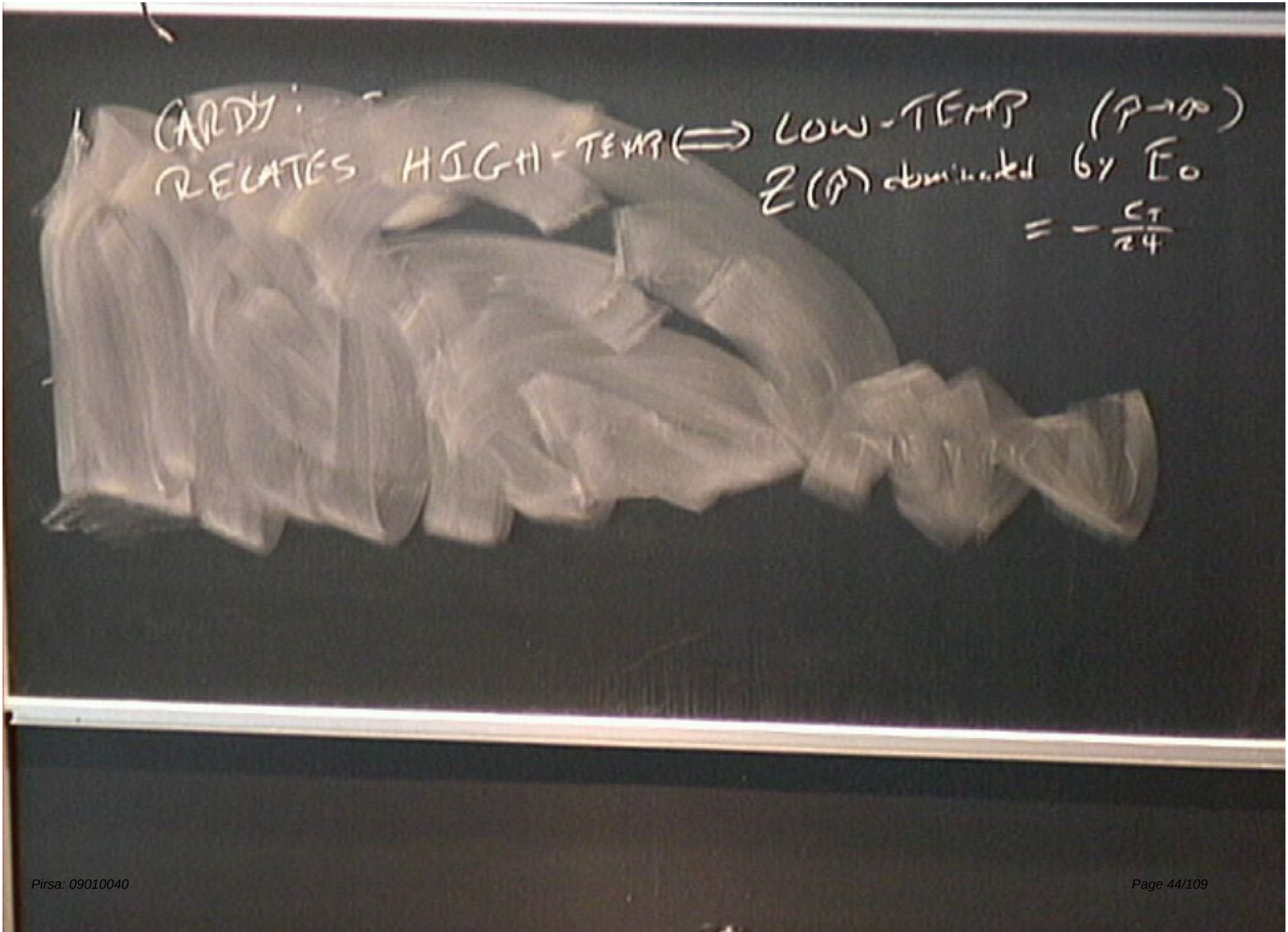
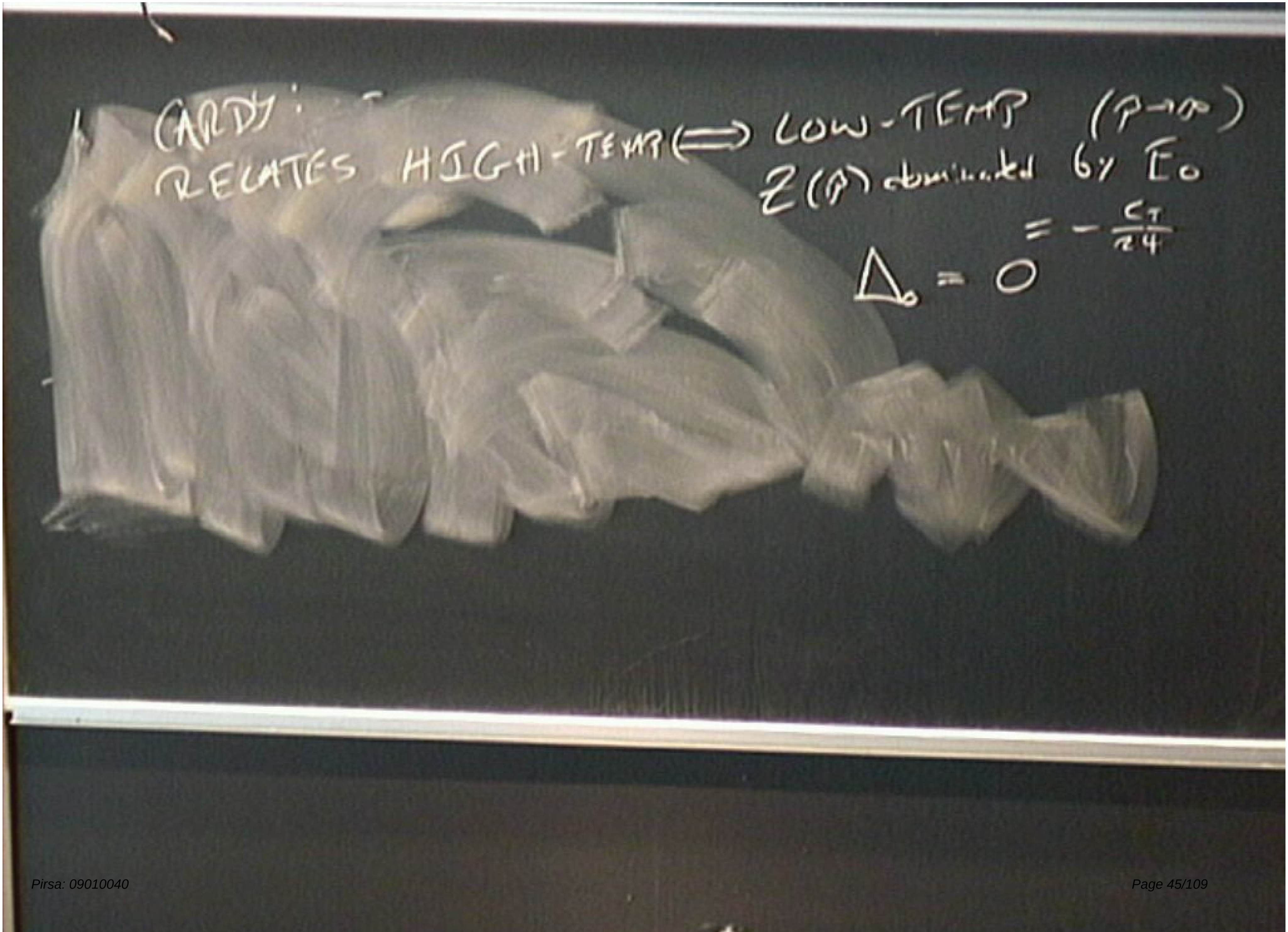
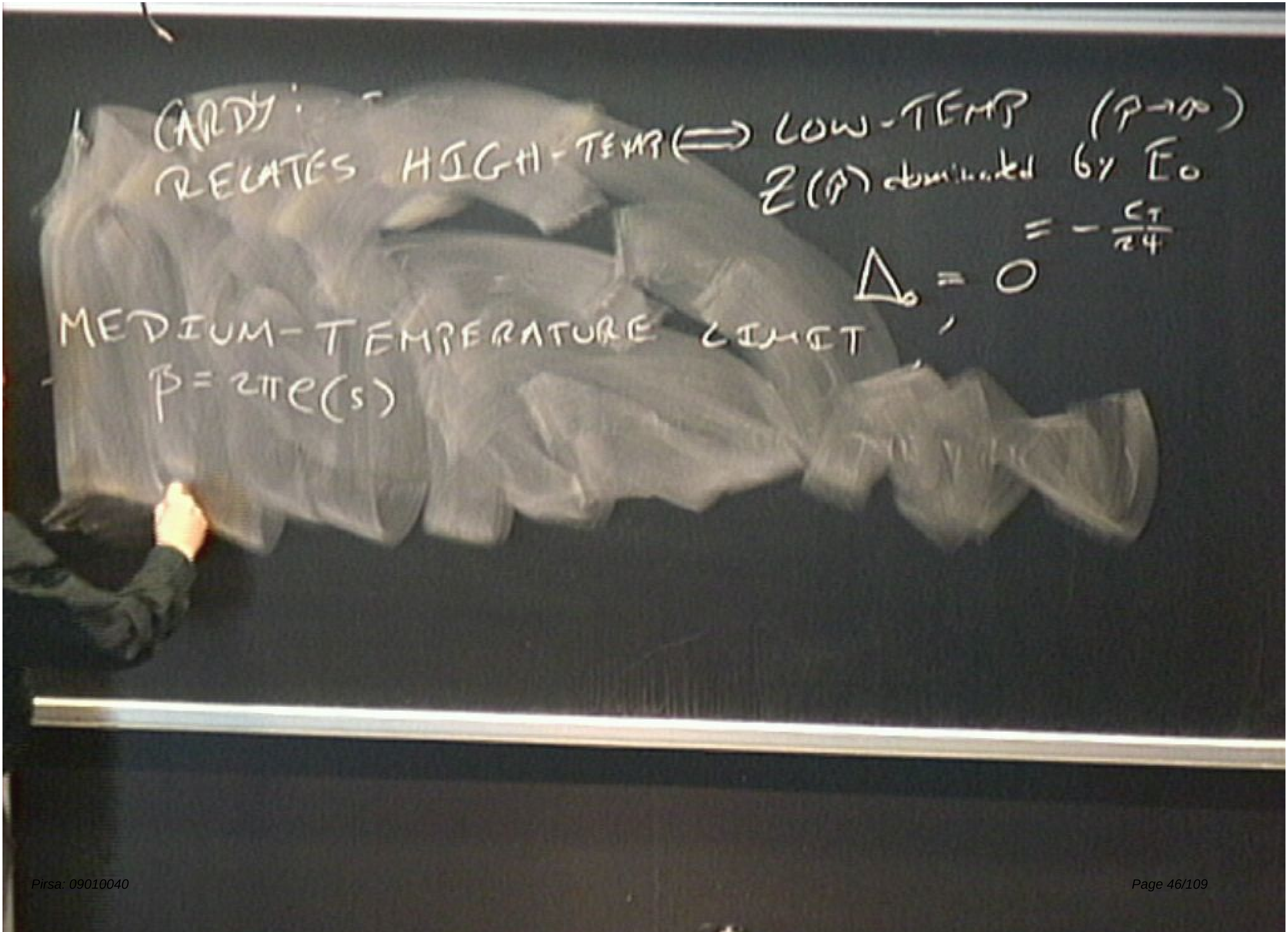

CARDY:
RELATES HIGH-TEMP ⟺ LOW-TEMP (β→∞)
Z


CARDY:
RELATES HIGH-TEMP ⟺ LOW-TEMP (β→∞)
Z(β) dominated by E0
= − cT/24


CARDY:
RELATES HIGH-TEMP ⟺ LOW-TEMP (β→∞)
Z(β) dominated by $E_0 = -\frac{c_T}{24}$
$\Delta_0 = 0$


CARDY:
RELATES HIGH-TEMP ⟺ LOW-TEMP (β→∞)
Z(β) dominated by E0
= − c/24
Δ0 = 0
MEDIUM-TEMPERATURE LIMIT
β = 2πe(s)

CARDY:

RELATES HIGH-TEMP $\Longleftrightarrow$ LOW-TEMP $(\beta \to \infty)$

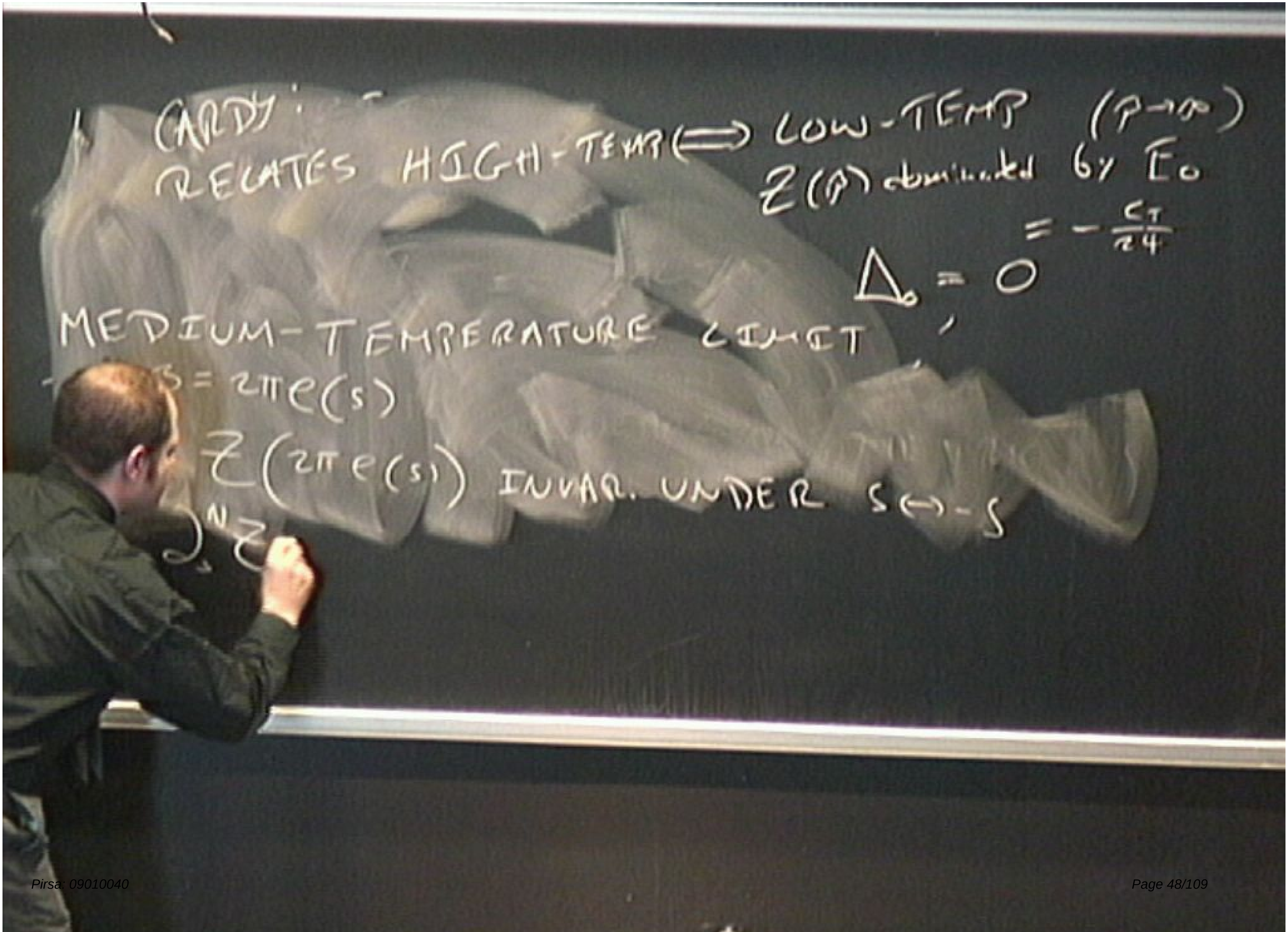
$Z(\beta)$ dominated by $E_0 = -\frac{c_T}{24}$

$\Delta_0 = 0$

MEDIUM-TEMPERATURE LIMIT

$\beta = 2\pi e(s)$

$Z(2\pi e(s))$ INVAR. UNDER $s \leftrightarrow -s$


CARDY:
RELATES HIGH-TEMP ⟺ LOW-TEMP (β→∞)
Z(β) dominated by E₀
= − c_T / 24
Δ₀ = 0
MEDIUM-TEMPERATURE LIMIT
β = 2πe(s)
Z(2πe(s)) INVAR. UNDER s ↔ −s

CARDY:
RELATES HIGH-TEMP $\Longleftrightarrow$ LOW-TEMP ($\beta \to \infty$)

$Z(\beta)$ dominated by $E_0 = -\frac{c_T}{24}$

$\Delta_0 = 0$

MEDIUM-TEMPERATURE LIMIT

$\beta = 2\pi e(s)$

$Z(2\pi e(s))$ INVAR. UNDER $s \leftrightarrow -s$

$\partial_s^N Z(2\pi e(s)) = 0$ FOR N ODD!

CARDY:

RELATES HIGH-TEMP $\Longleftrightarrow$ LOW-TEMP $(\beta \to \infty)$

$Z(\beta)$ dominated by E_0 $= -\frac{c_T}{24}$

$\Delta_0 = 0$

MEDIUM-TEMPERATURE LIMIT

$\beta = 2\pi e(s)$

$Z(2\pi e(s))$ INVAR. UNDER $s \leftrightarrow -s$

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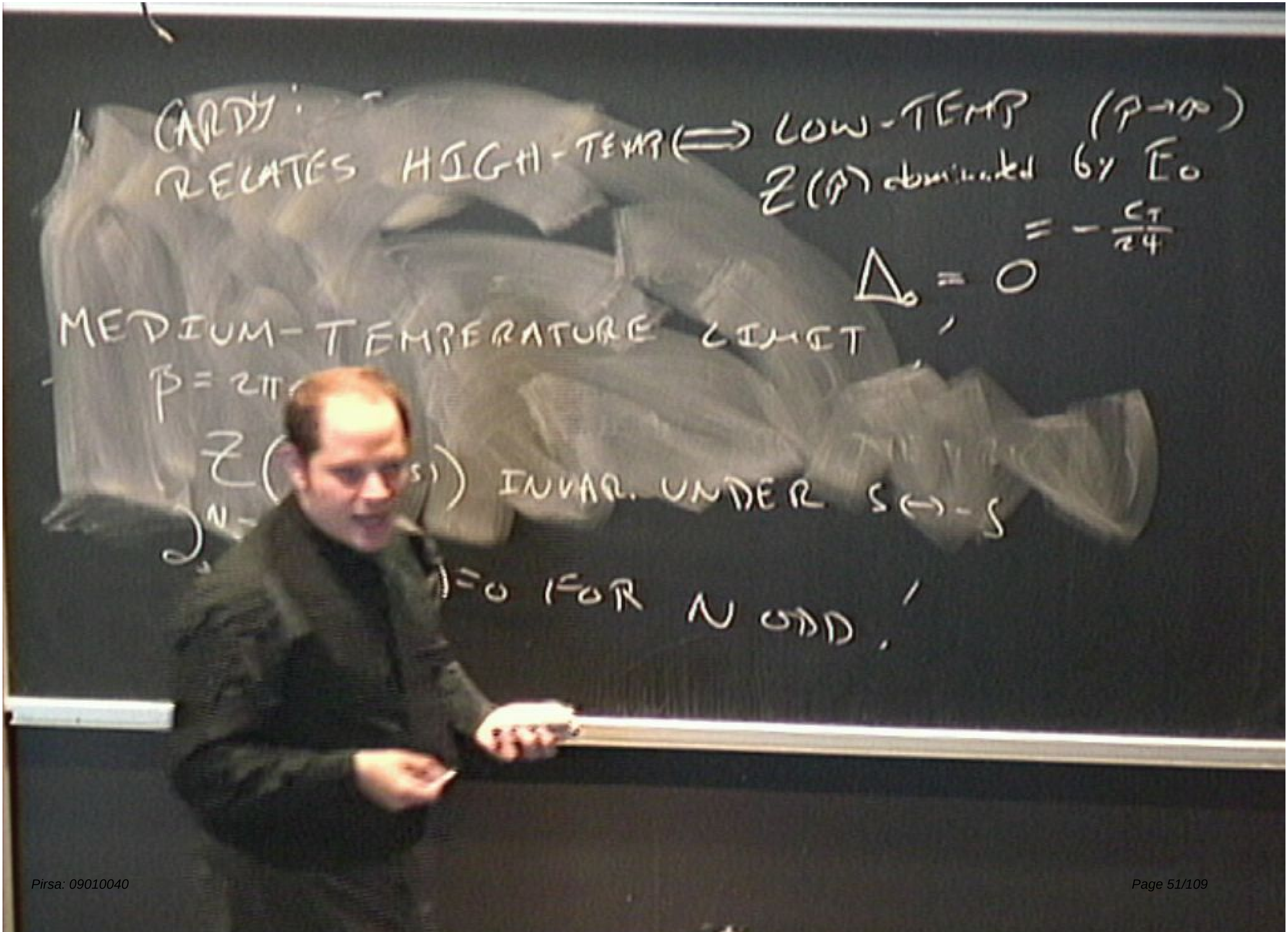

CARDY:
RELATES HIGH-TEMP ⟺ LOW-TEMP (β→∞)
Z(β) dominated by E₀ = − c_T/24
Δ₀ = 0
MEDIUM-TEMPERATURE LIMIT
β = 2π
INVAR. UNDER S ↔ −S
= 0 FOR N ODD!

CARDY:
RELATES HIGH-TEMP $\Longleftrightarrow$ LOW-TEMP $(\beta \to \infty)$

$Z(\beta)$ dominated by $E_0 = -\frac{c_T}{24}$

$\Delta_0 = 0$

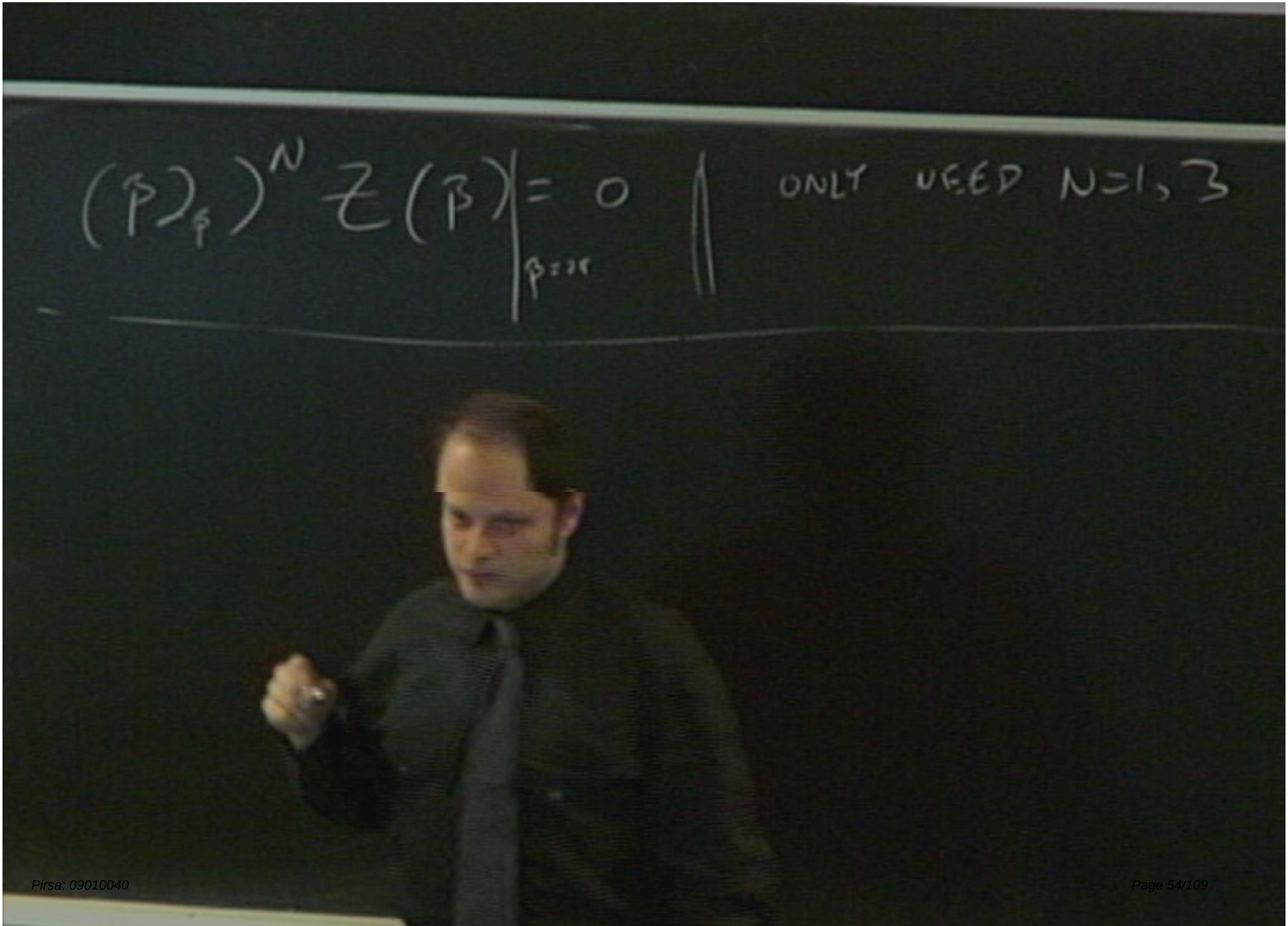
MEDIUM-TEMPERATURE LIMIT

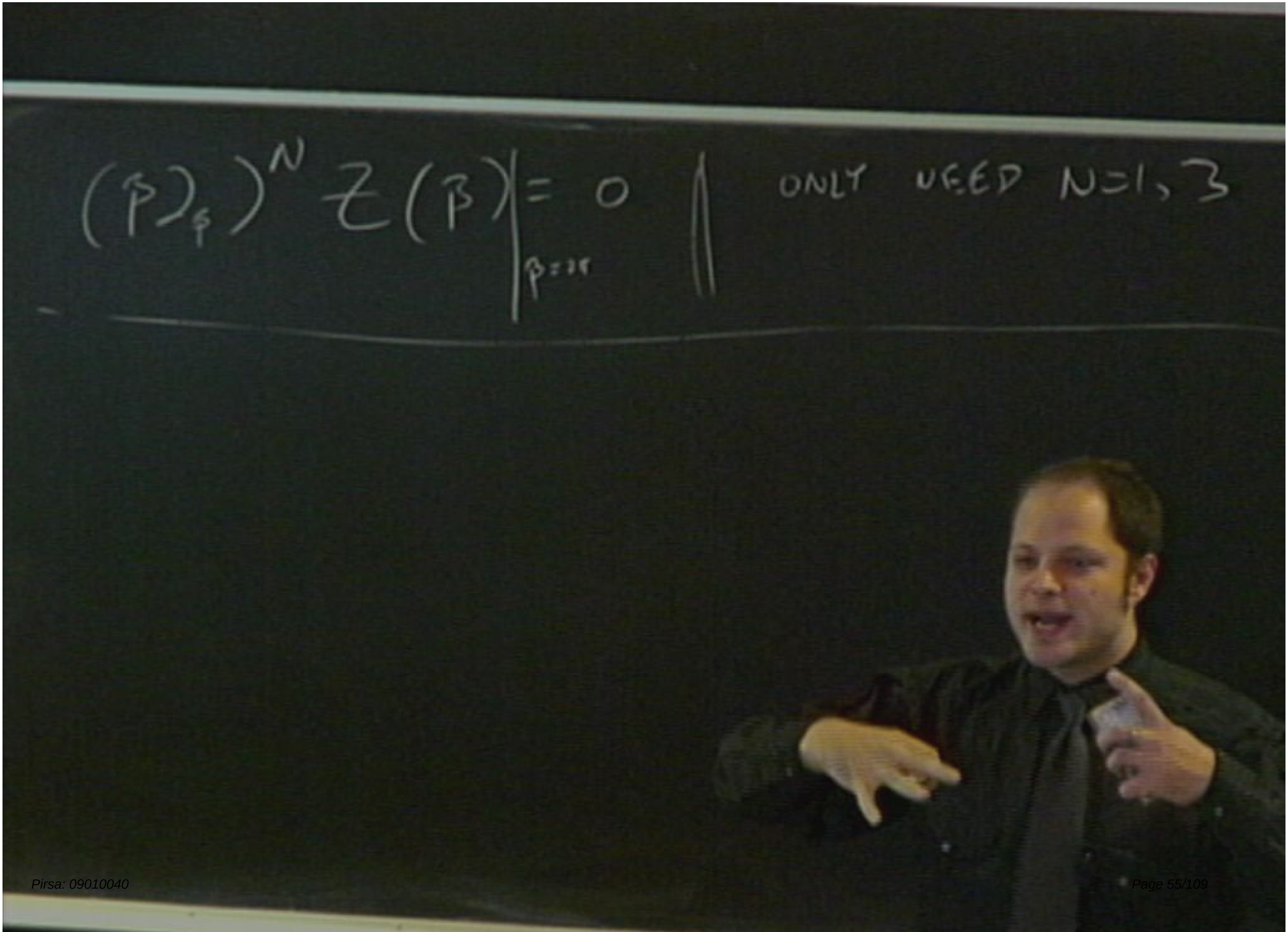
$\beta = 2\pi e(s)$

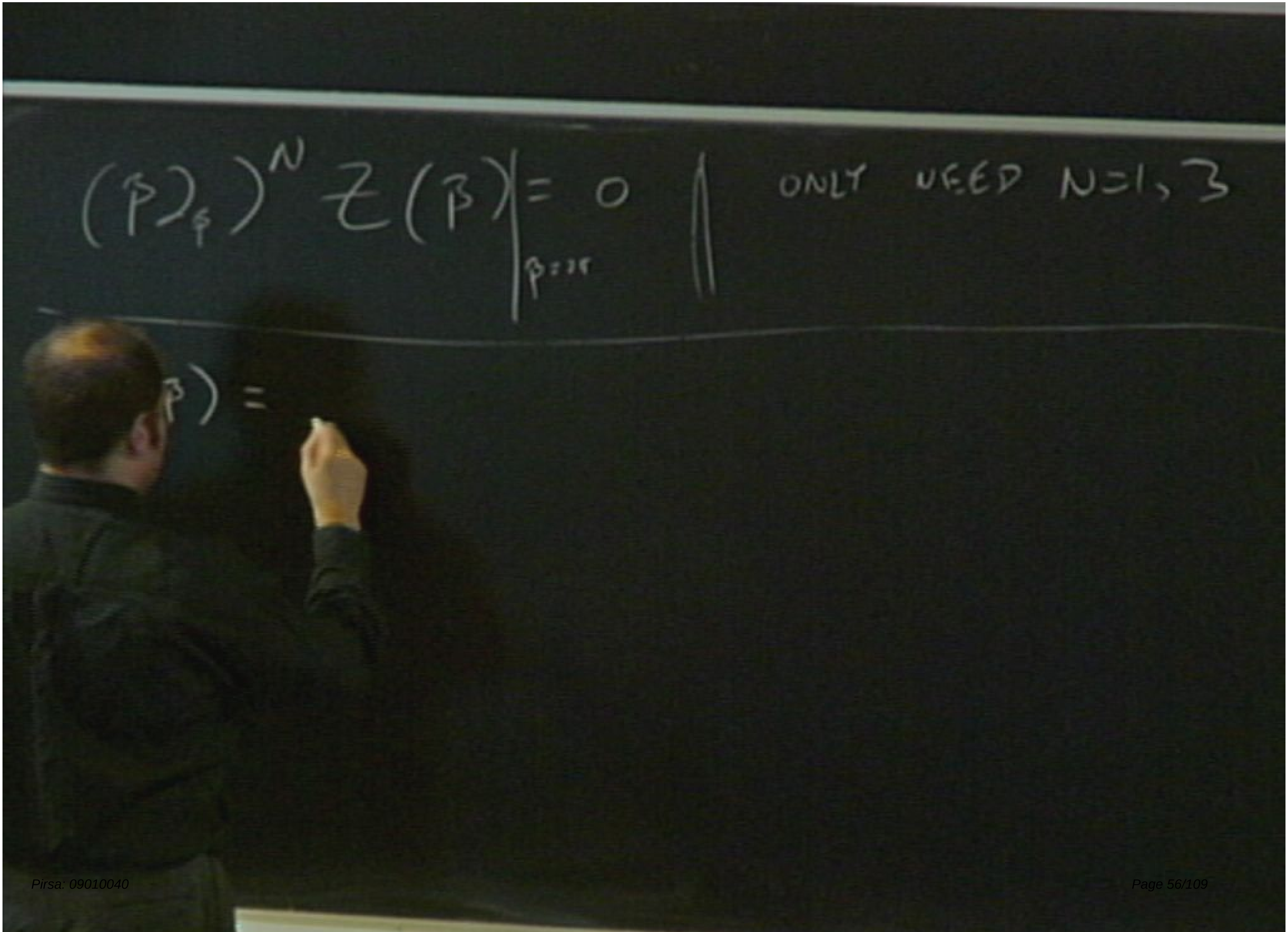
$Z(2\pi e(s))$ INVAR. UNDER $s \leftrightarrow -s$

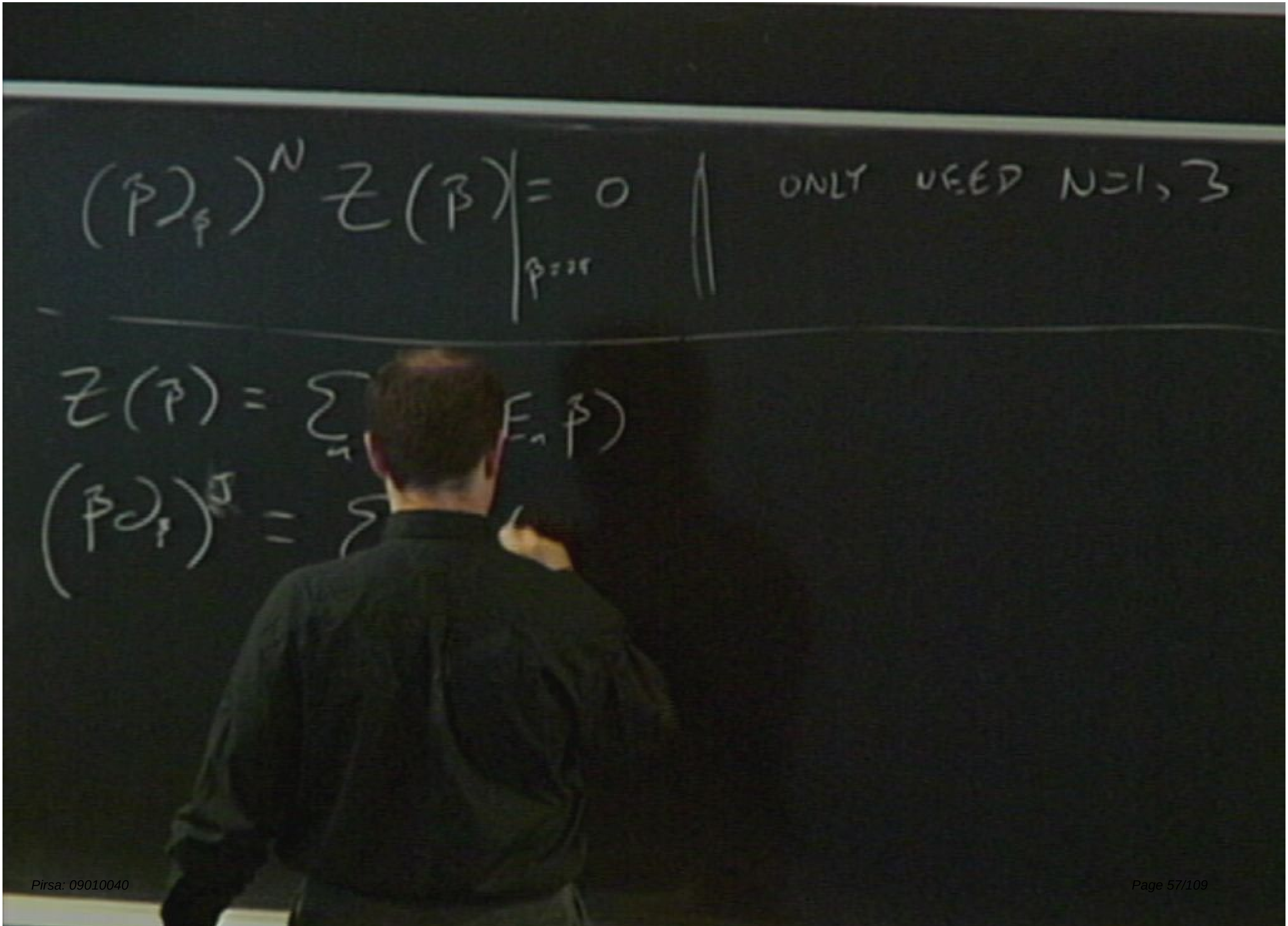
$\partial_s^N Z(2\pi e(s)) = 0$ FOR N ODD!

$$\left(\partial_\beta\right)^N Z(\beta)\Big|_{\beta=2\pi} = 0$$


ONLY NEED N=1, 3


(∂_β)^N Z(β)|_{β=2π} = 0
ONLY NEED N=1, 3


ONLY NEED N=1, 3



(β∂_β)^N Z(β)| = 0
β=2π
ONLY NEED N=1, 3
Z(β) = Σ

$$(\beta\partial_\beta)^N Z(\beta)\Big|_{\beta=2\pi} = 0$$

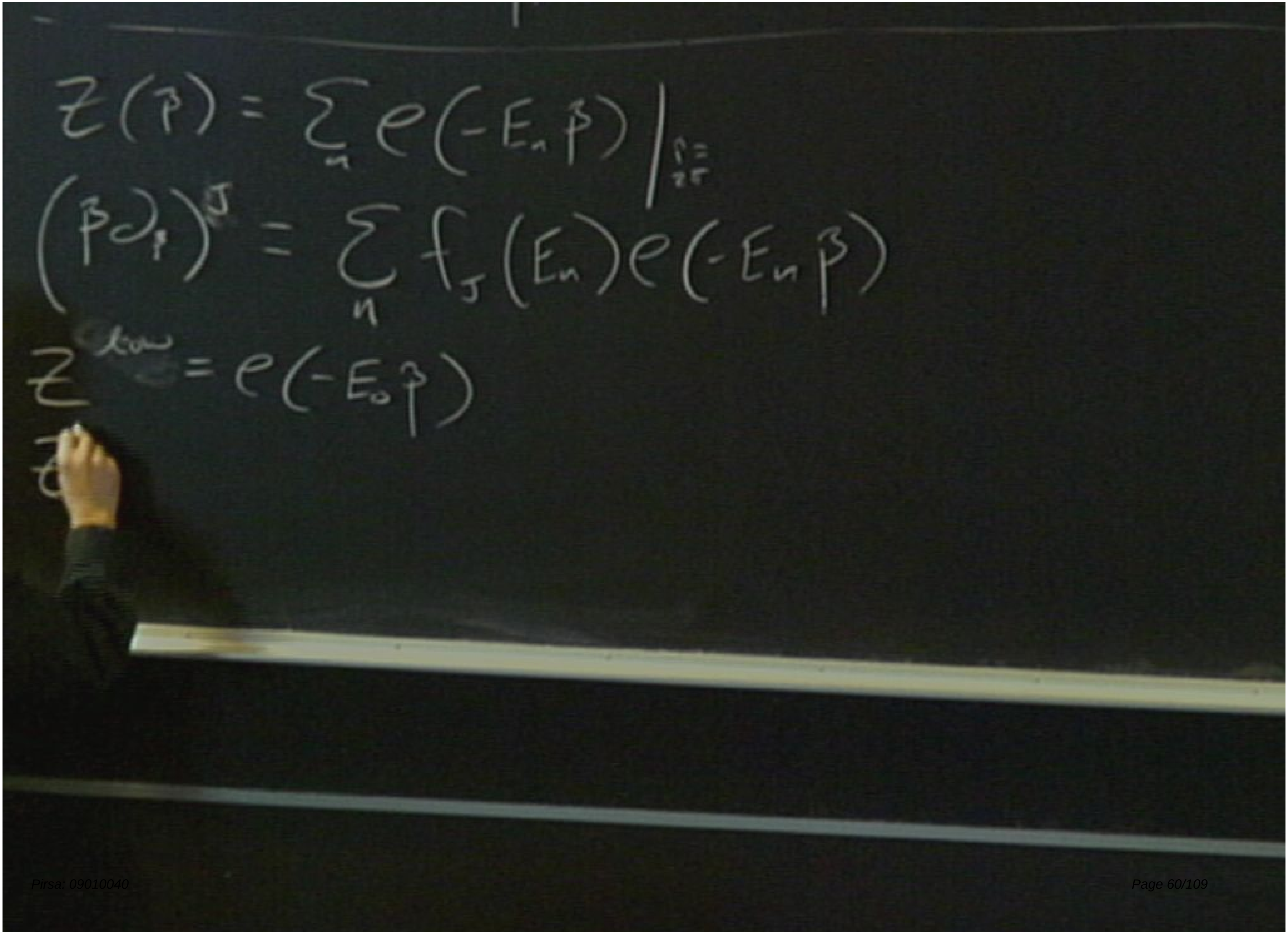
ONLY NEED N=1,3

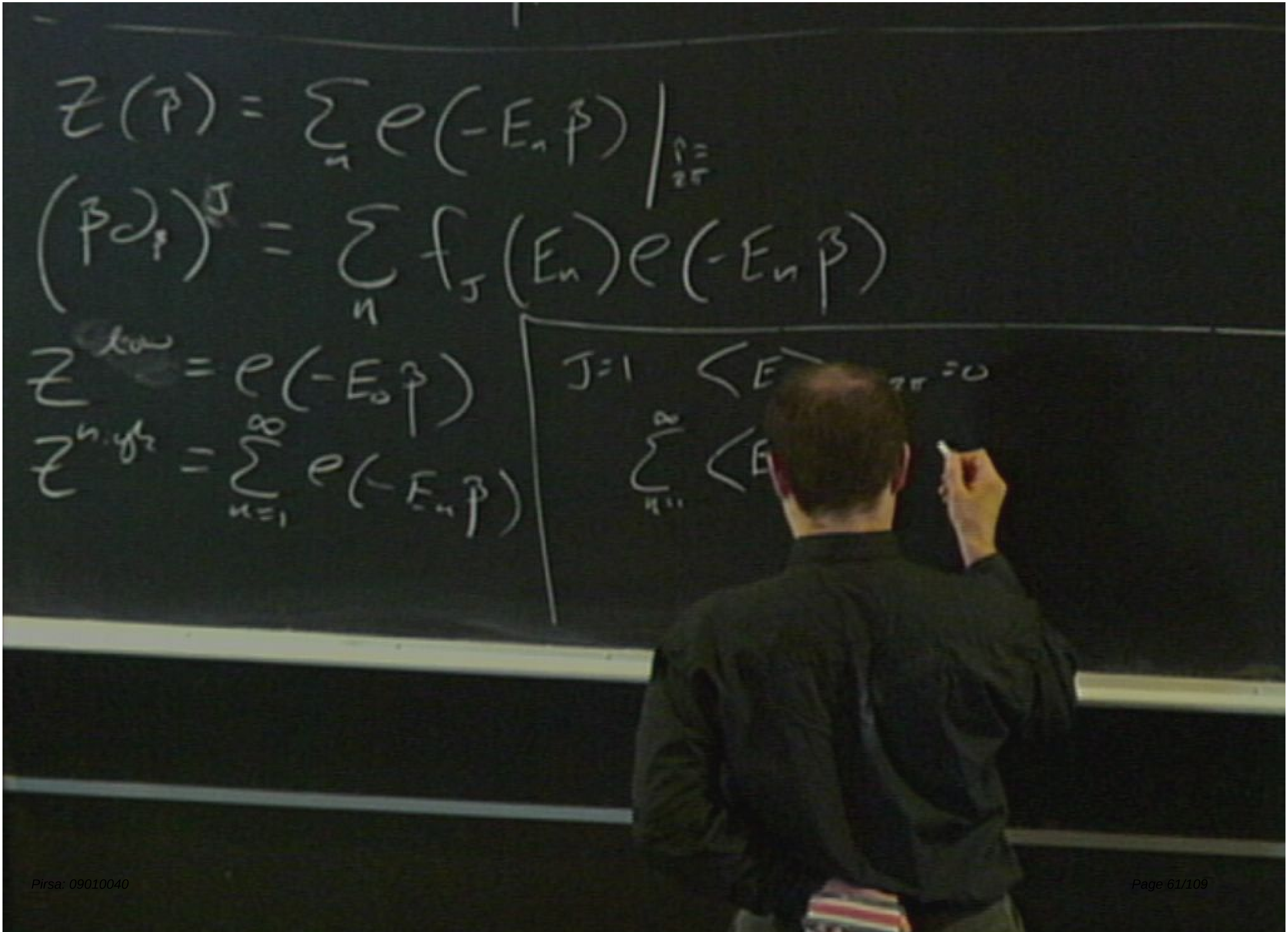
$$Z(\beta) = \sum_n e(-E_n\beta)\Big|_{2\pi}$$

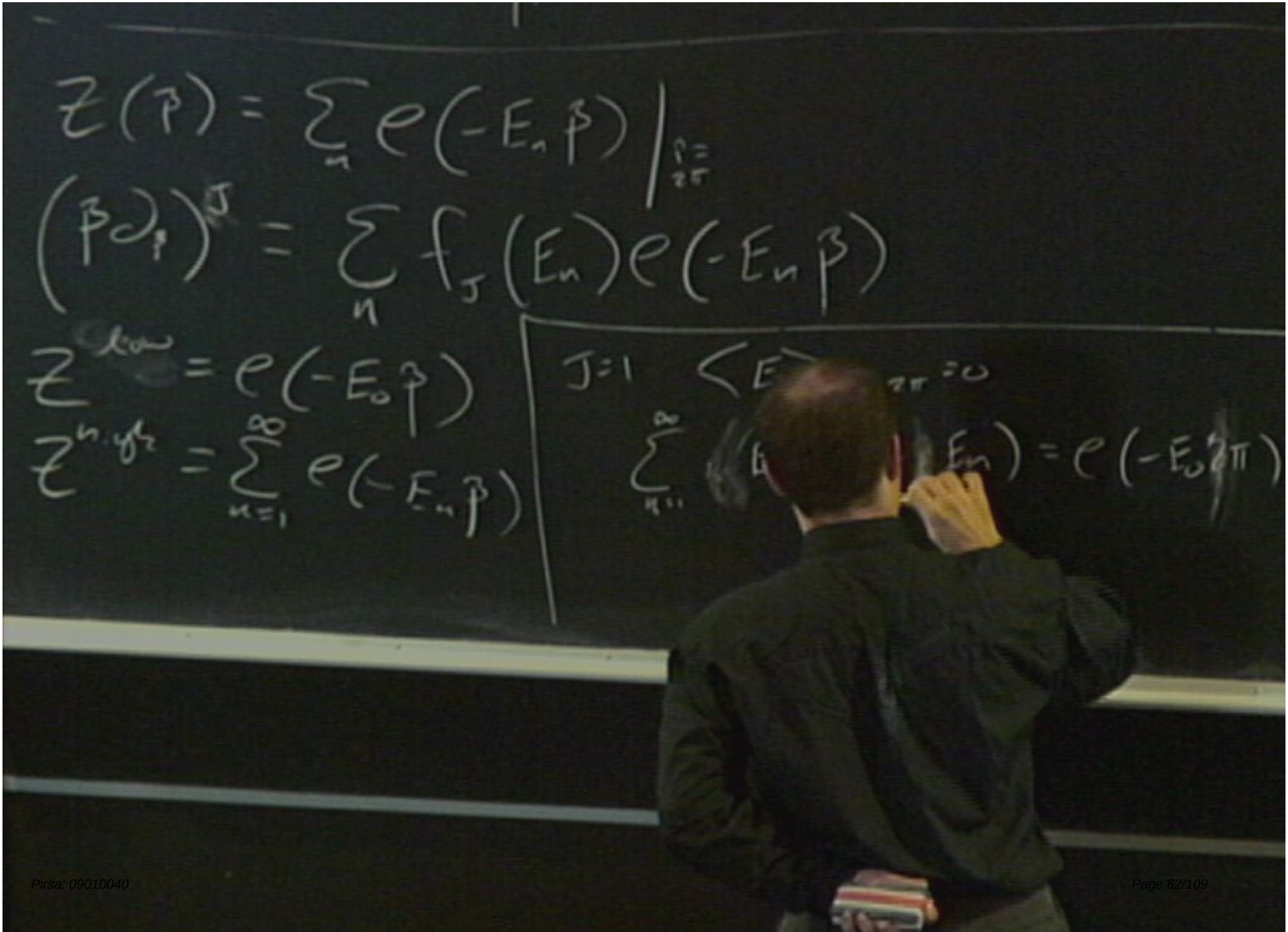
$$(\beta\partial_\beta)^J = \sum_n f_J(E_n)\, e(-E_n\beta)$$

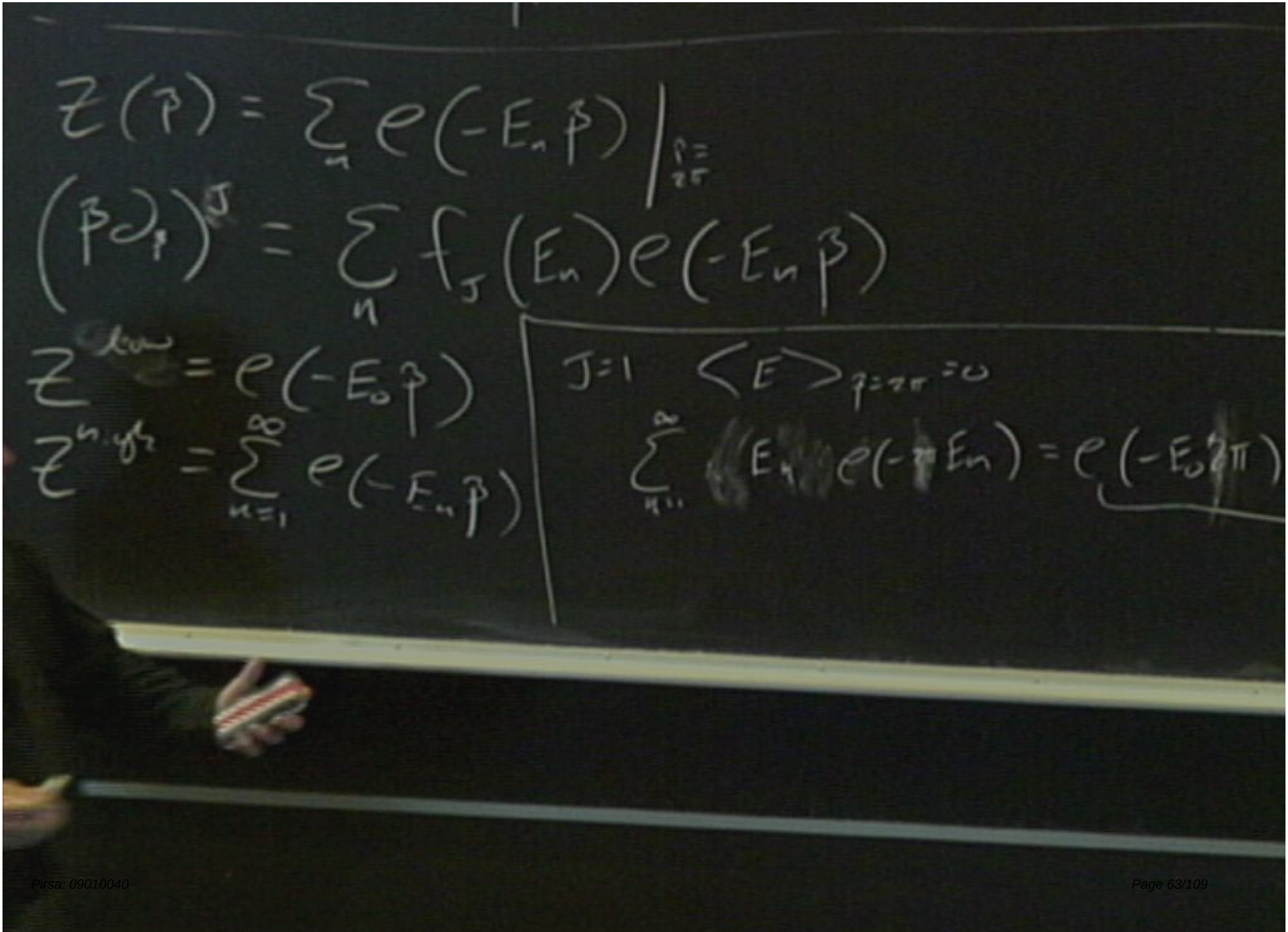
$$Z(\beta) = \sum_n e(-E_n \beta) \Big|_{\beta = 2\pi}$$

$$(\beta \partial_\beta)^J = \sum_n f_J(E_n)\, e(-E_n \beta)$$


Z(β) = Σ_n e(−E_n β) |_{β=2π}
(β∂_β)^J = Σ_n f_J(E_n) e(−E_n β)
Z^low = e(−E_0 β)


Z(β) = Σ_n e(-E_n β) | β=2π
(β∂_β)^J = Σ_n f_J(E_n) e(-E_n β)
Z^low = e(-E_0 β)
Z^n.ph = Σ_{n=1}^∞ e(-E_n β)
J=1
=0
Σ_{n=1}^∞


$Z(\beta) = \sum_n e(-E_n \beta) \Big|_{\beta = 2\pi}$
$(\beta \partial_\beta)^J = \sum_n f_J(E_n) e(-E_n \beta)$
$Z^{low} = e(-E_0 \beta)$
$Z^{n.gh} = \sum_{n=1}^{\infty} e(-E_n \beta)$
$J=1$


$Z(\beta) = \sum_n e(-E_n \beta) \Big|_{\beta = 2\pi}$
$(\beta \partial_\beta)^J = \sum_n f_J(E_n) e(-E_n \beta)$
$Z^{low} = e(-E_0 \beta)$
$Z^{high} = \sum_{n=1}^{\infty} e(-E_n \beta)$
$J=1 \quad \langle E \rangle_{\beta=2\pi} = 0$
$\sum_{n=1}^{\infty} E_n e(-2\pi E_n) = e(-E_0 2\pi)$

$$(\beta\partial_\beta)^N Z(\beta)\Big|_{\beta=2\pi} = 0$$

ONLY NEED $N=1,3$

$$Z(\beta) = \sum_n e(-E_n\beta)\Big|_{\beta=2\pi}$$

$$(\beta\partial_\beta)^J = \sum_n f_J(E_n)\, e(-E_n\beta)$$

$$Z^{low} = e(-E_0\beta)$$

$$Z^{high} = \sum_{n=1}^{\infty} e(-E_n\beta)$$

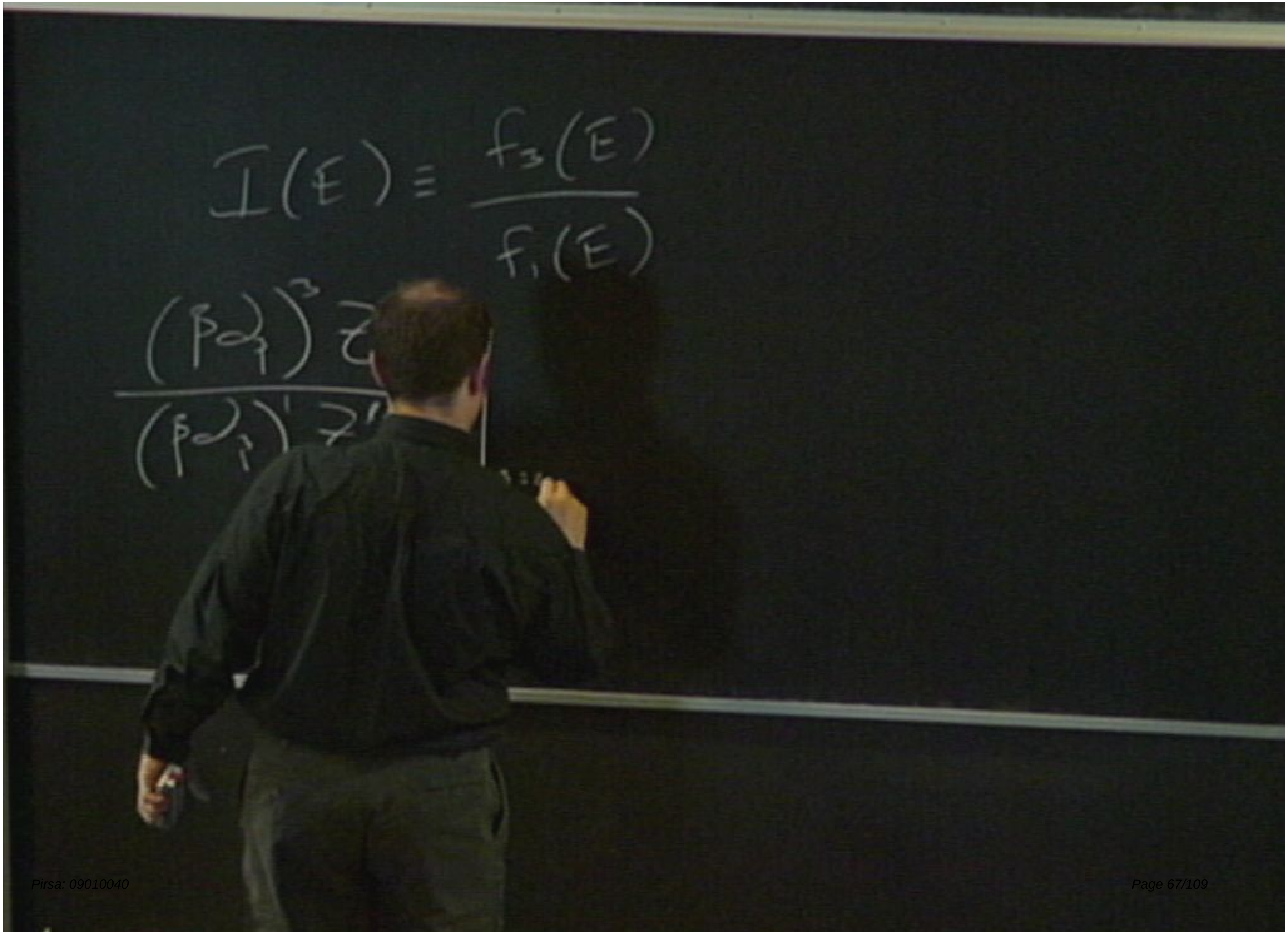
$J=1$ $\quad \langle E\rangle_{\beta=2\pi} = 0$ $\quad \sim E_0^3$

$$\sum_{n=1}^{\infty} E_n\, e(-2\pi E_n) = e(-E_0 2\pi)\,\frac{c+\tilde{c}}{24}$$

E_n^3

$$I(E) \equiv \frac{f_3(E)}{f_1(E)}$$

$$I(E) \equiv \frac{f_3(E)}{f_1(E)}$$



$I(E) \equiv \frac{f_3(E)}{f_1(E)}$
$\frac{(\beta\alpha_1)^3 Z}{(\beta\alpha_1)^1 Z^1}$

$$I(E) \equiv \frac{f_3(E)}{f_1(E)} = (2\pi E)^2 - 3(2\pi E) + 1$$

$$\left.\frac{(\beta\partial_\beta)^3 Z^{\text{low}}}{(\beta\partial_\beta)^1 Z^{\text{low}}}\right|_{\beta=2\pi} = \left.\frac{(\beta\partial_\beta)^3 Z^{\text{high}}}{(\beta\partial_\beta)^1 Z^{\text{high}}}\right|_{\beta=2\pi}$$

$$I(E) \equiv \frac{f_3(E)}{f_1(E)} = (2\pi E)^2 - 3(2\pi E) + 1$$

$$\left.\frac{(\beta\partial_\beta)^3 Z^{low}}{(\beta\partial_\beta)^1 Z^{low}}\right|_{\beta=2\pi} = \left.\frac{(\beta\partial_\beta)^3 Z^{high}}{(\beta\partial_\beta)^1 Z^{high}}\right|_{\beta=2\pi}$$

$$\parallel$$

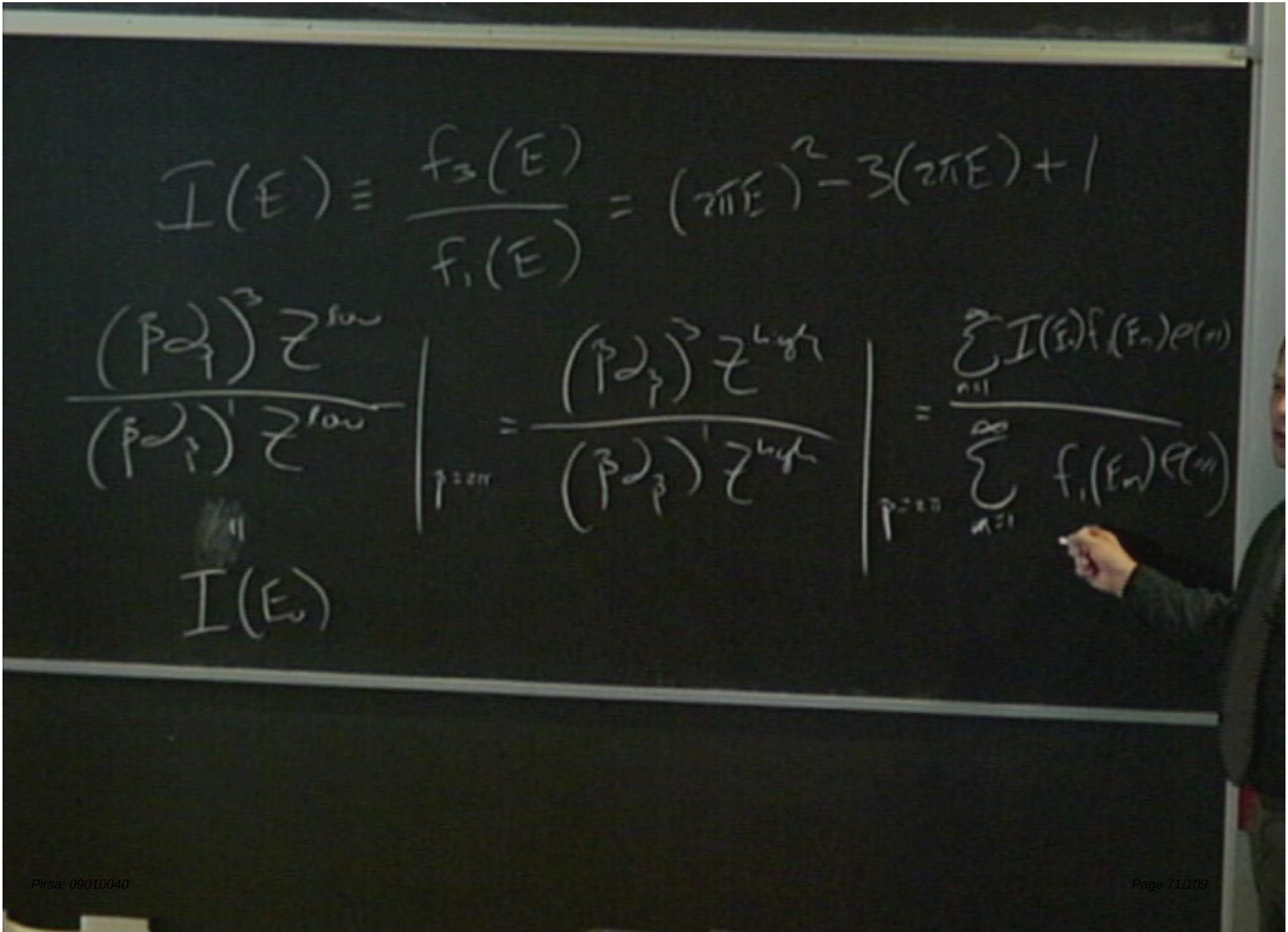
$$I(E_0)$$

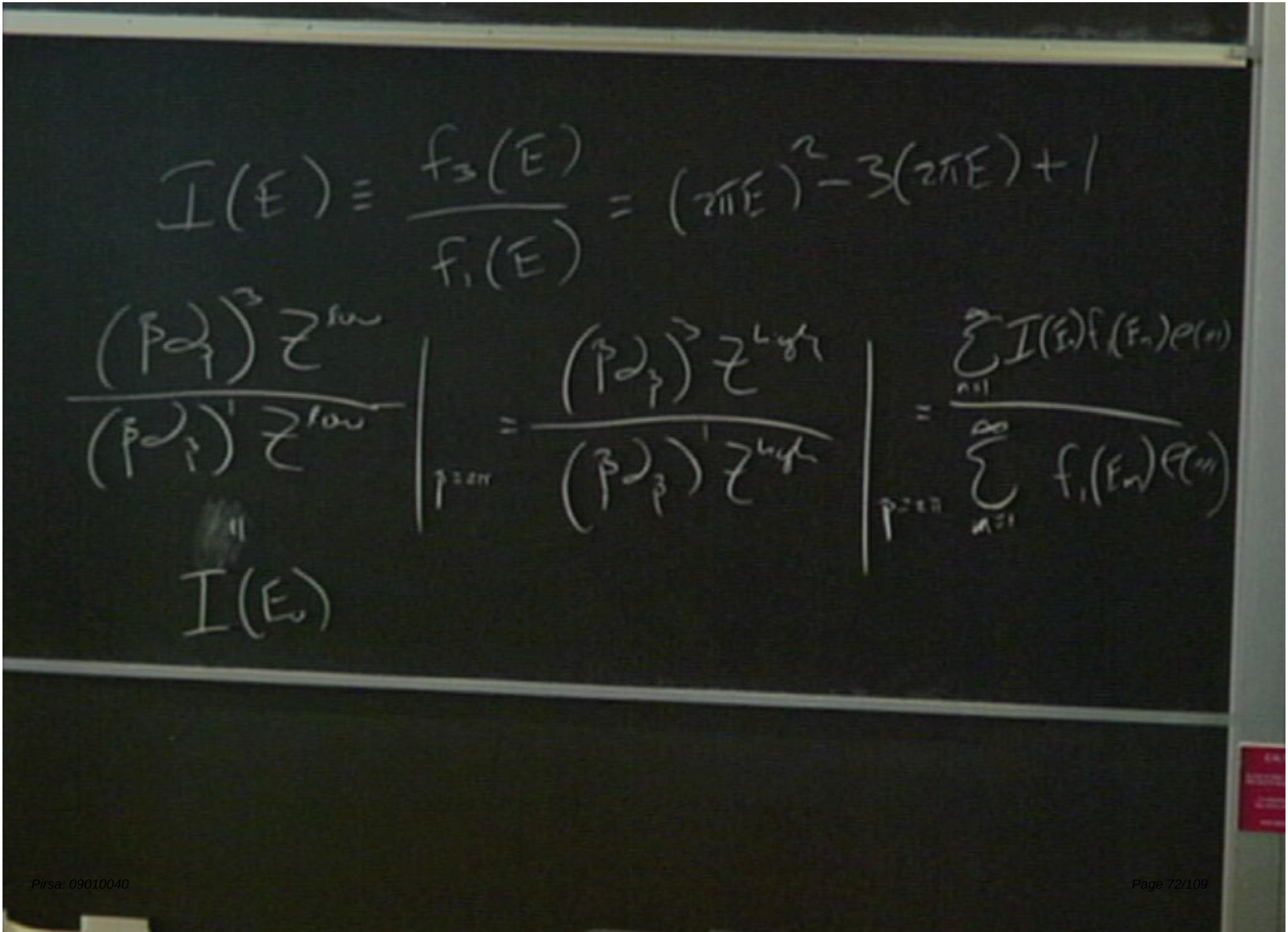
$$I(E) \equiv \frac{f_3(E)}{f_1(E)} = (2\pi E)^2 - 3(2\pi E) + 1$$

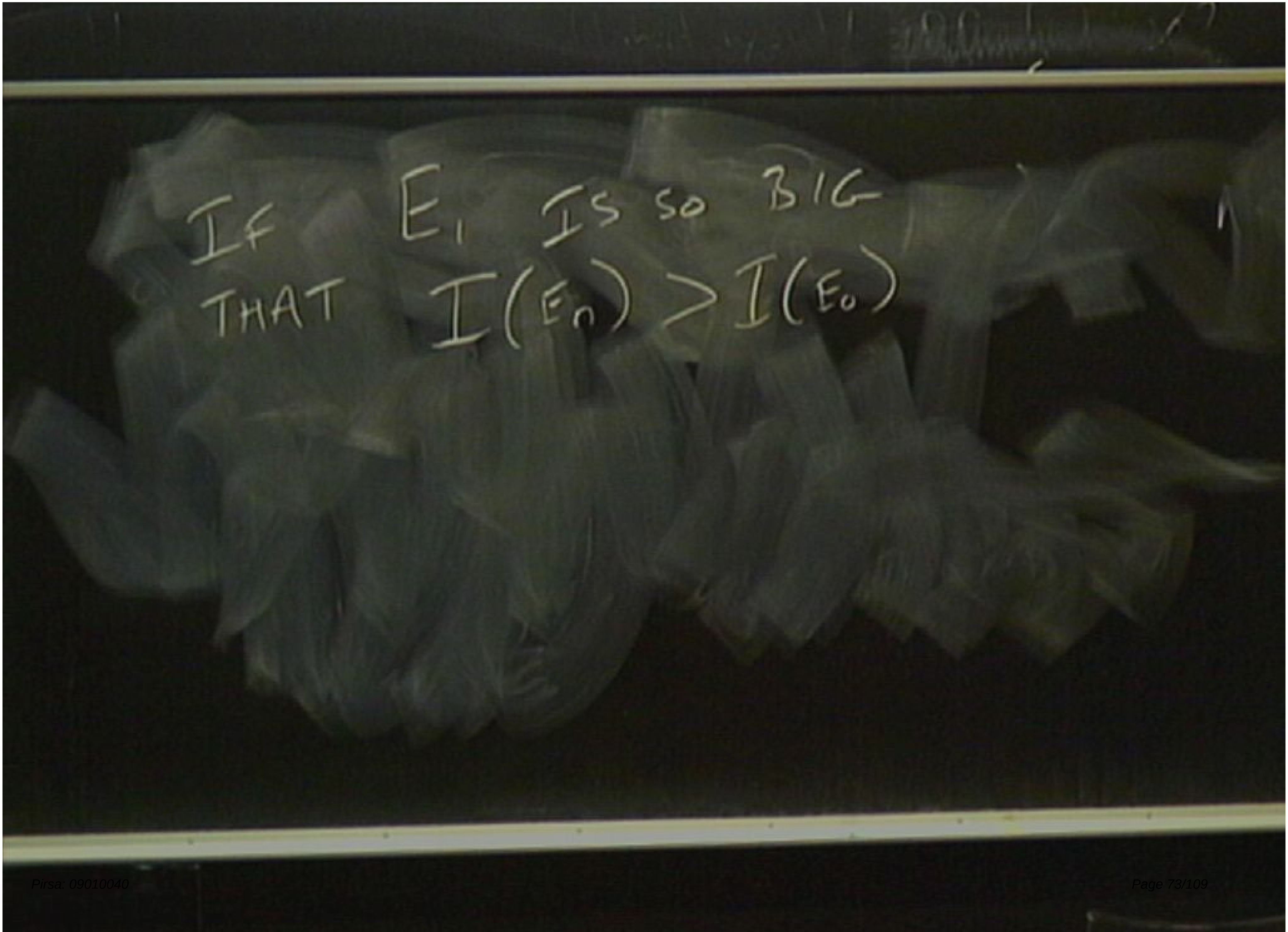
$$\left.\frac{(\beta\partial_\beta)^3 Z^{low}}{(\beta\partial_\beta)^1 Z^{low}}\right|_{\beta=2\pi} = \left.\frac{(\beta\partial_\beta)^3 Z^{high}}{(\beta\partial_\beta)^1 Z^{high}}\right|_{\beta=2\pi} =$$

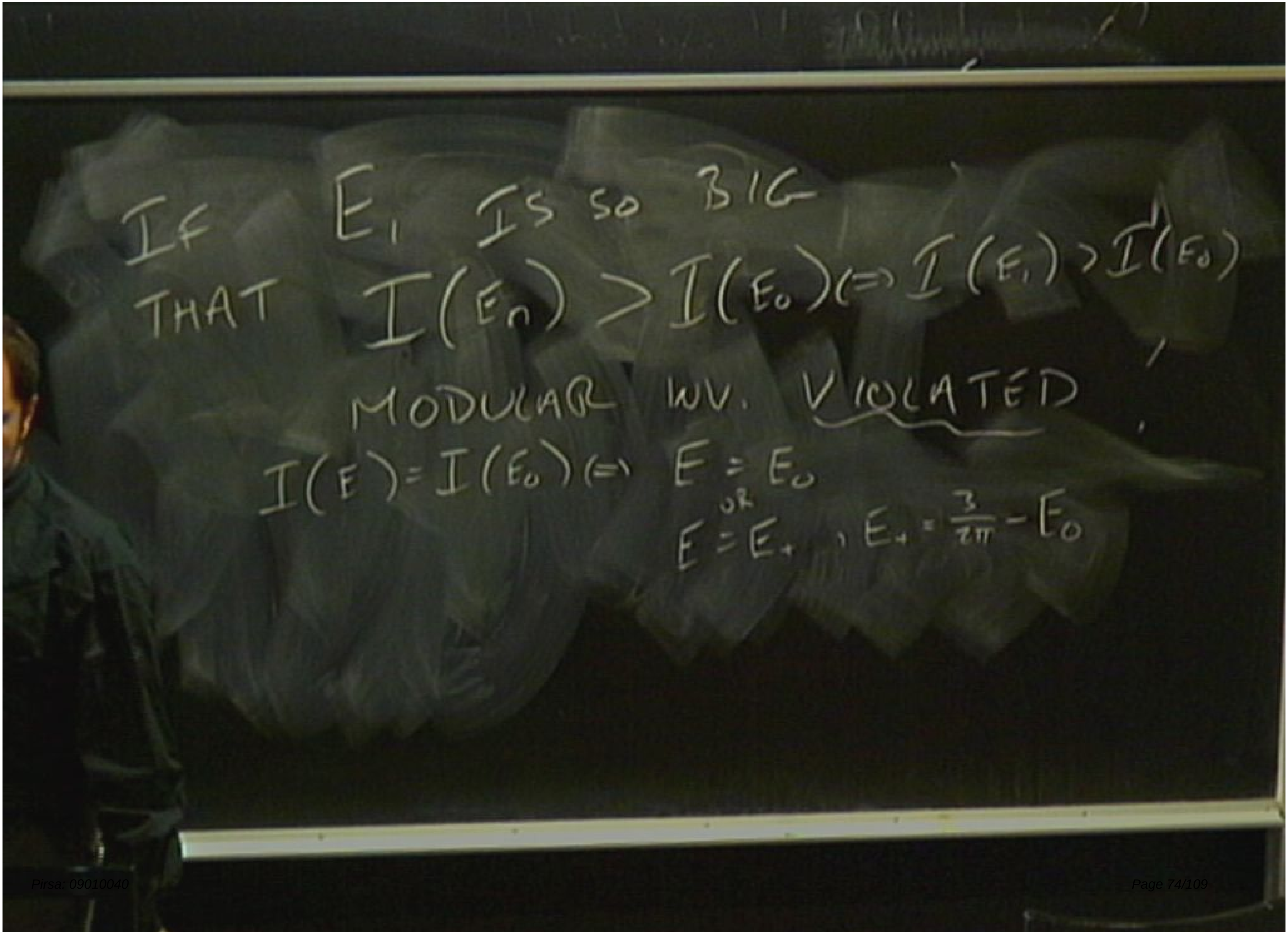
$$\|$$

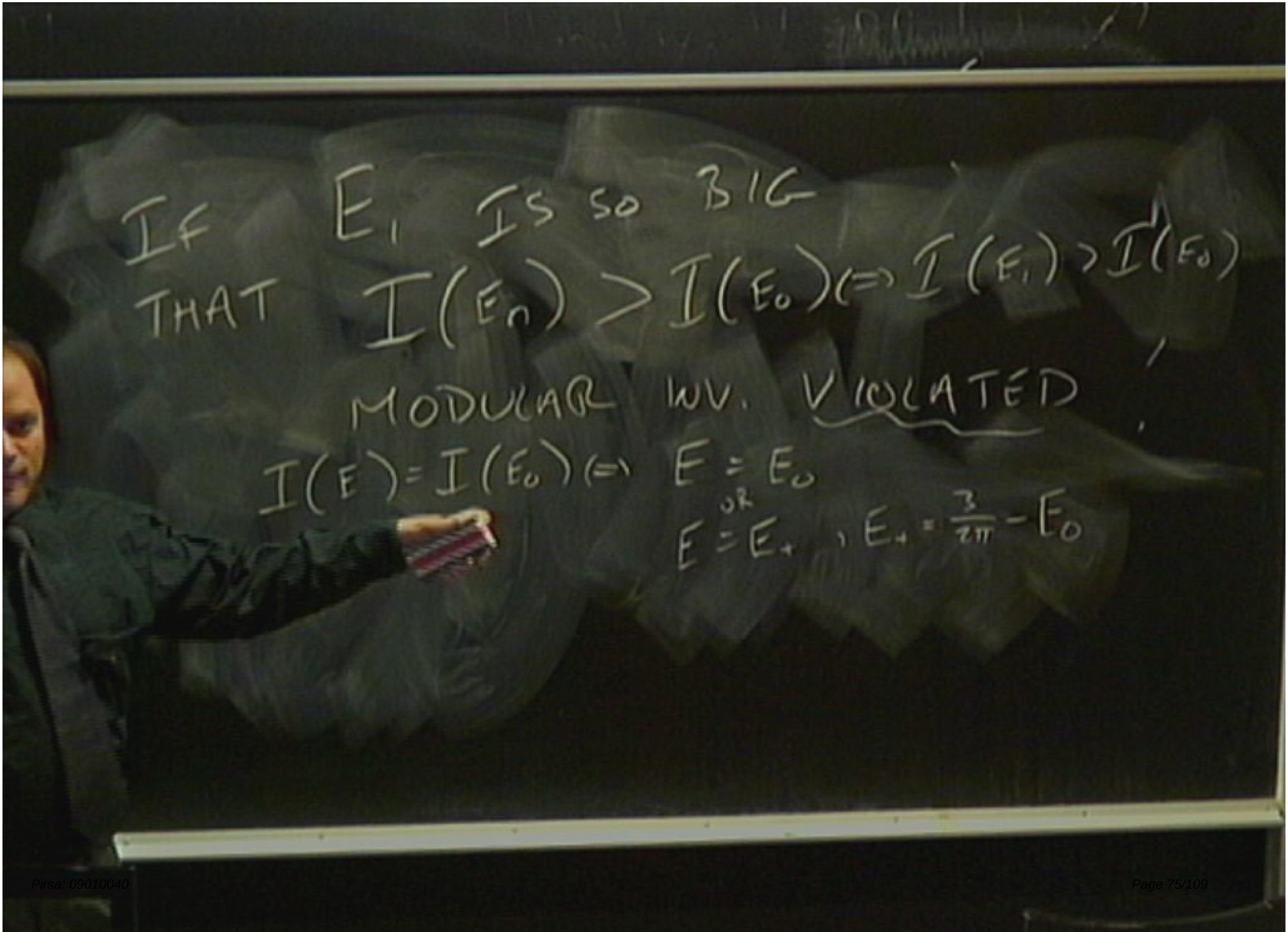
$$I(E_0)$$


$I(E) \equiv \frac{f_3(E)}{f_1(E)} = (2\pi E)^2 - 3(2\pi E) + 1$
$\left.\frac{(\beta \partial_\beta)^3 Z^{low}}{(\beta \partial_\beta)^1 Z^{low}}\right|_{\beta=2\pi} = \left.\frac{(\beta \partial_\beta)^3 Z^{high}}{(\beta \partial_\beta)^1 Z^{high}}\right|_{\beta=2\pi} = \frac{\sum_{n=1}^{\infty} I(E_n) f_1(E_n) \rho(n)}{\sum_{n=1}^{\infty} f_1(E_n) \rho(n)}$
$\parallel$
$I(E_0)$


$I(E) \equiv \frac{f_3(E)}{f_1(E)} = (2\pi E)^2 - 3(2\pi E) + 1$
$\left.\frac{(\beta\lambda_\beta)^3 Z^{low}}{(\beta\lambda_\beta)^1 Z^{low}}\right|_{\beta=2\pi} = \left.\frac{(\beta\lambda_\beta)^3 Z^{high}}{(\beta\lambda_\beta)^1 Z^{high}}\right|_{\beta=2\pi} = \frac{\sum_{n=1}^{\infty} I(E_n) f_1(E_n) \rho(n)}{\sum_{n=1}^{\infty} f_1(E_n) \rho(n)}$
$= I(E_0)$


IF E_1 IS SO BIG
THAT $I(E_n) > I(E_0)$


IF E1 IS SO BIG
THAT I(En) > I(E0) ⟺ I(E1) > I(E0)
MODULAR INV. VIOLATED
I(E) = I(E0) ⟺ E = E0
OR
E = E+ , E+ = 3/2π − E0


IF E1 IS SO BIG
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MODULAR INV. VIOLATED
I(E) = I(E0) ⟺ E = E0
OR
E = E+ , E+ = 3/2π − E0

IF E_1 IS SO BIG

THAT $I(E_n) > I(E_0) \Leftrightarrow I(E_1) > I(E_0)$

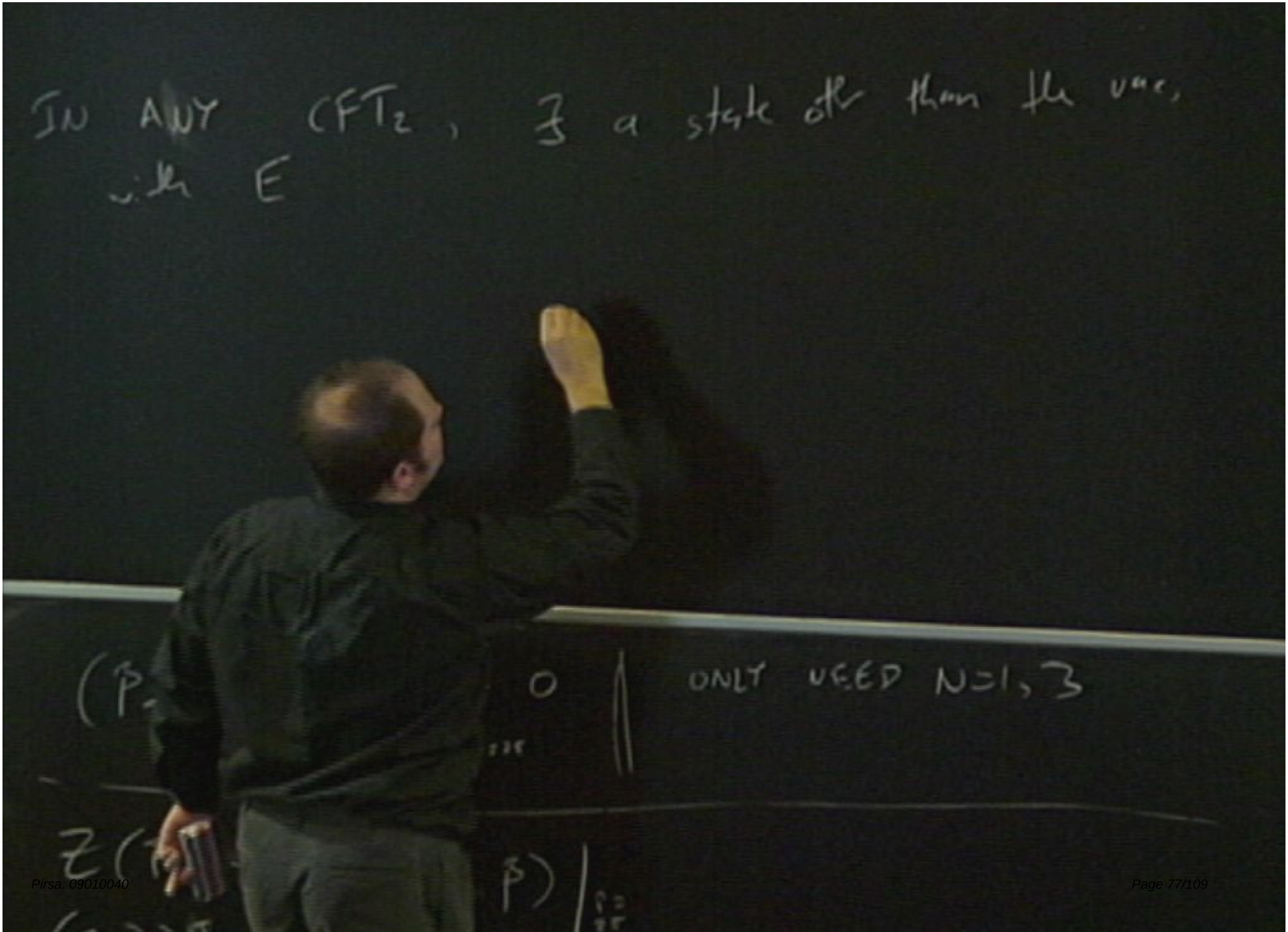
MODULAR INV. VIOLATED!

$I(E_1) = I(E_0) \Leftrightarrow E = E_0$

OR

$E = E_+ \,,\; E_+ = \frac{3}{2\pi} - E_0$

$I(E_n) \geq I(E_0) \Leftrightarrow E_1 > E_+$


IN ANY CFT2, ∃ a state other than the vac,
with E
ONLY NEED N=1,3

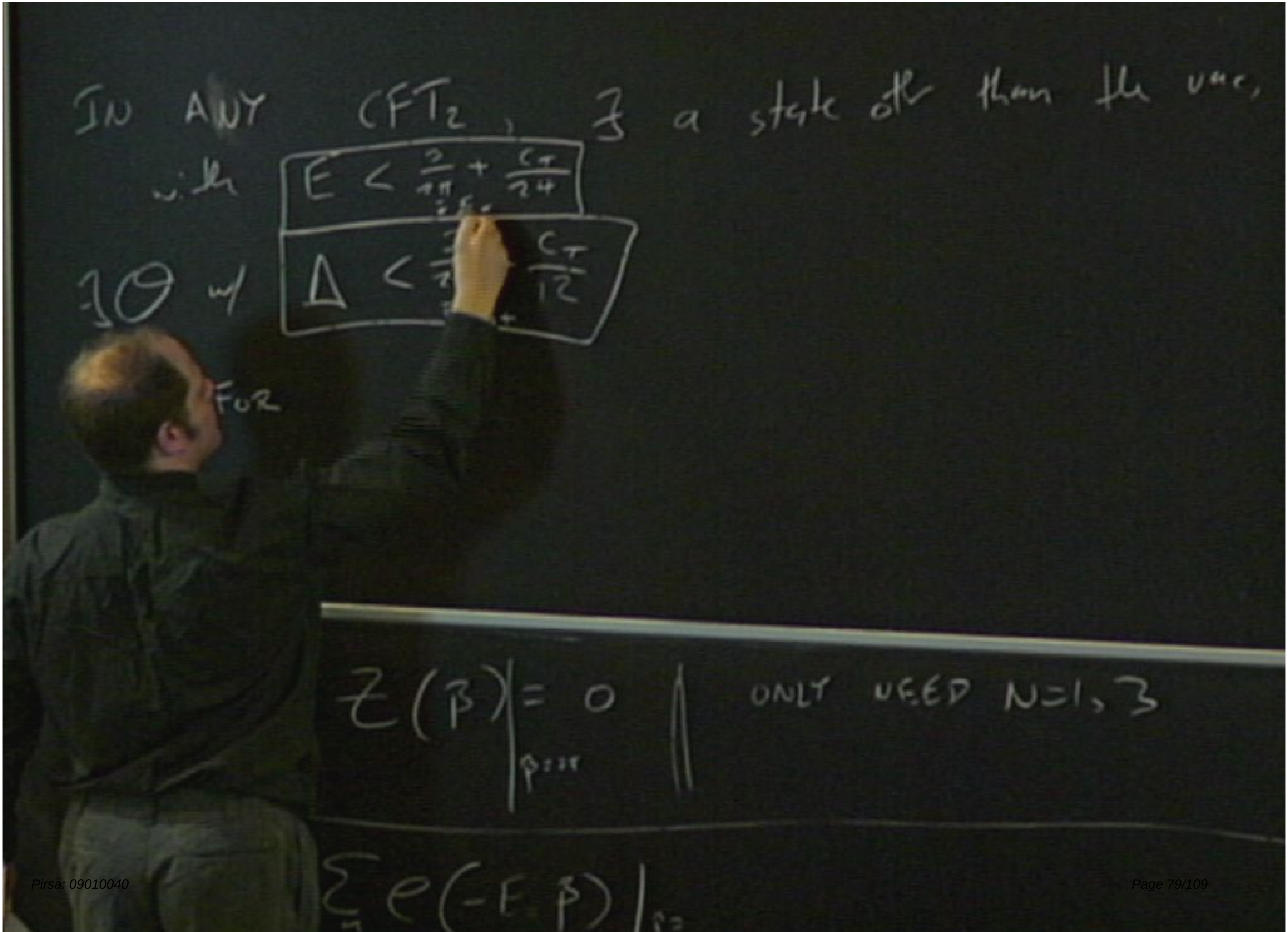
IN ANY CFT_2, $\exists$ a state other than the vac, with $\boxed{E < \frac{3}{4\pi} + \frac{c_T}{24}}$

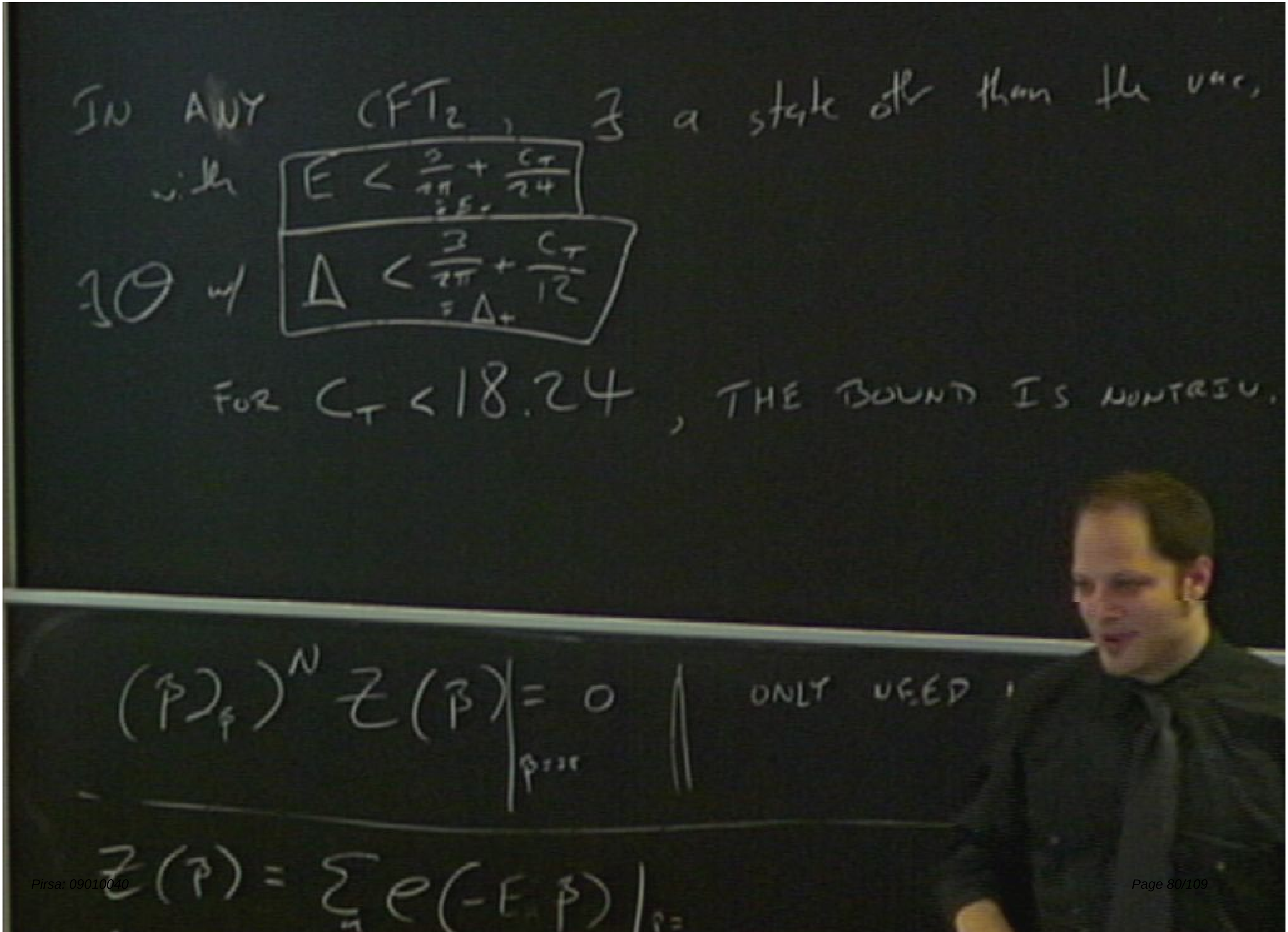
$\exists \mathcal{O}$ w/ $\boxed{\Delta < \frac{3}{2\pi} + \frac{c_T}{12}}$

$$\left((\beta \partial_\beta)^N Z(\beta) \right)\Big|_{\beta = 2\pi} = 0$$

ONLY NEED $N = 1, 3$

$$Z(\beta) = \sum_n e(-E_n \beta)\Big|_{\beta = 2\pi}$$


IN ANY CFT2, ∃ a state other than the vac,
with E < + cT/24
1O w/ Δ < cT/12
FOR
Z(β) = 0
ONLY NEED N=1, 3


IN ANY CFT$_2$, ∃ a state other than the vac, with $E < \frac{3}{4\pi} + \frac{c_T}{24}$
∃ O w/ $\Delta < \frac{3}{2\pi} + \frac{c_T}{12}$
FOR $c_T < 18.24$, THE BOUND IS NONTRIV.
$(\beta\partial_\beta)^N Z(\beta)\Big|_{\beta=2\pi} = 0$
ONLY USED
$Z(\beta) = \sum e(-E_n\beta)$

IN ANY CFT_2, $\exists$ a state other than the vac, with $E < \frac{3}{2\pi} + \frac{c_T}{24} \equiv E_+$

$\exists\, \mathcal{O}$ w/ $\Delta < \frac{3}{2\pi} + \frac{c_T}{12} \equiv \Delta_+$

FOR $c_T < 18.24$, THE BOUND IS NONTRIV.

$$(\beta\partial_\beta)^N Z(\beta)\Big|_{\beta=2\pi} = 0$$

ONLY USED $N=1,3$

$$Z(\beta) = \sum_n e(-E_n \beta)\Big|_{\beta=}$$

IN ANY CFT_2, $\exists$ a state other than the vac, with $E < \frac{3}{2\pi} + \frac{c_T}{24}$ $= E_+$

$\exists \mathcal{O}$ w/ $\Delta < \frac{3}{2\pi} + \frac{c_T}{12}$ $= \Delta_+$

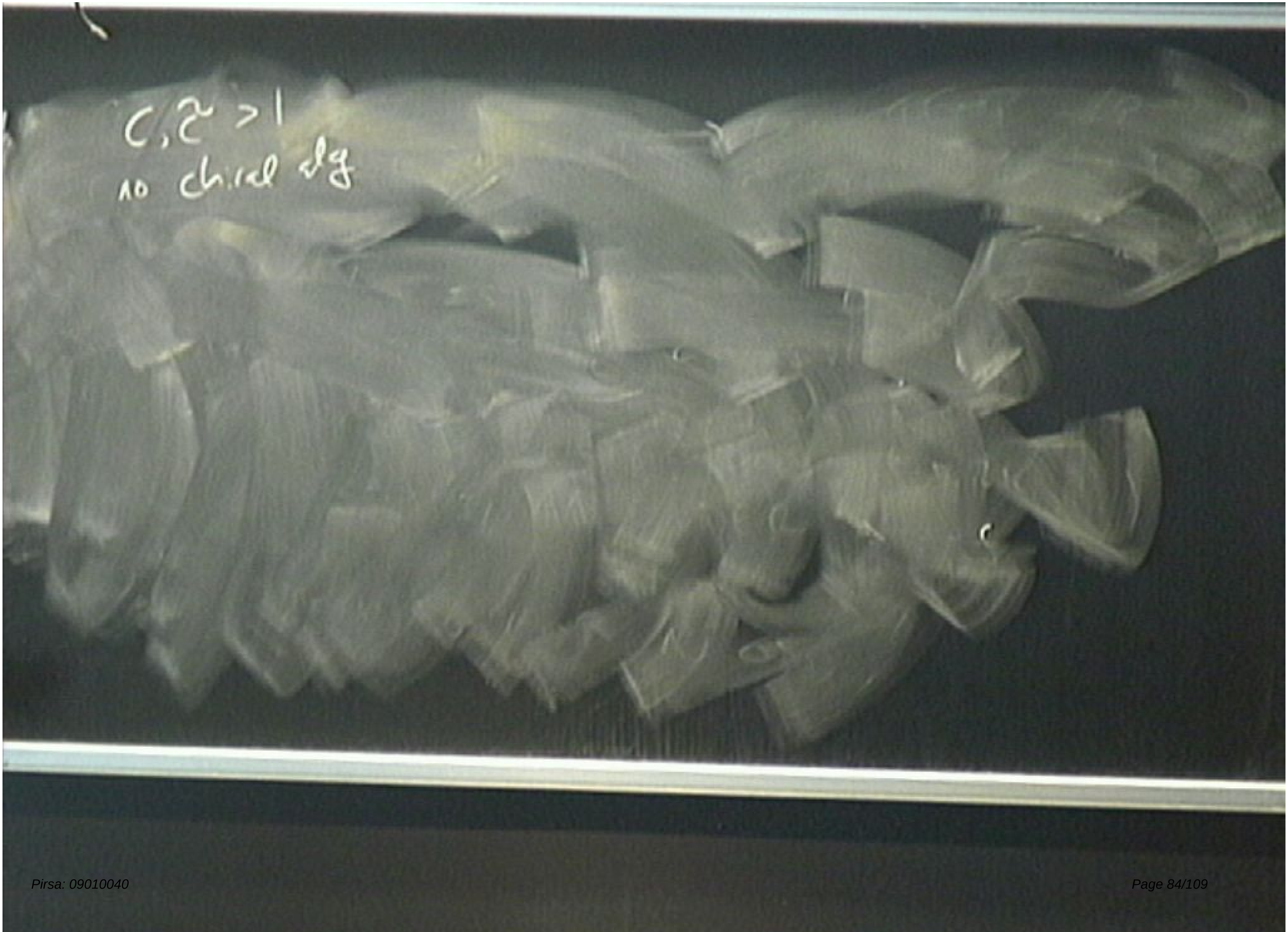
$\frac{3}{2\pi} \sim \frac{1}{2}$

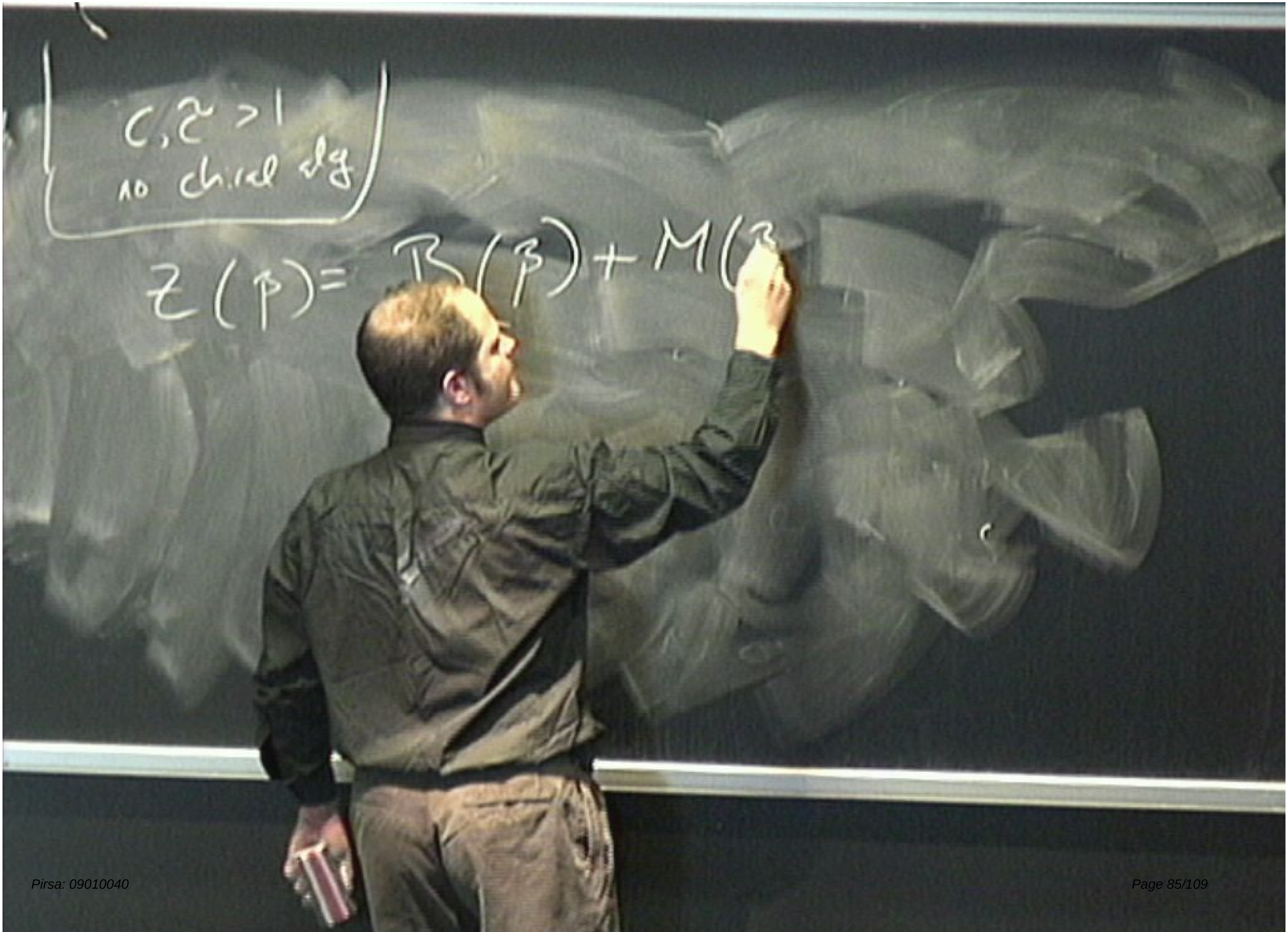
FOR $c_T < 18.24$, THE BOUND IS NONTRIV.

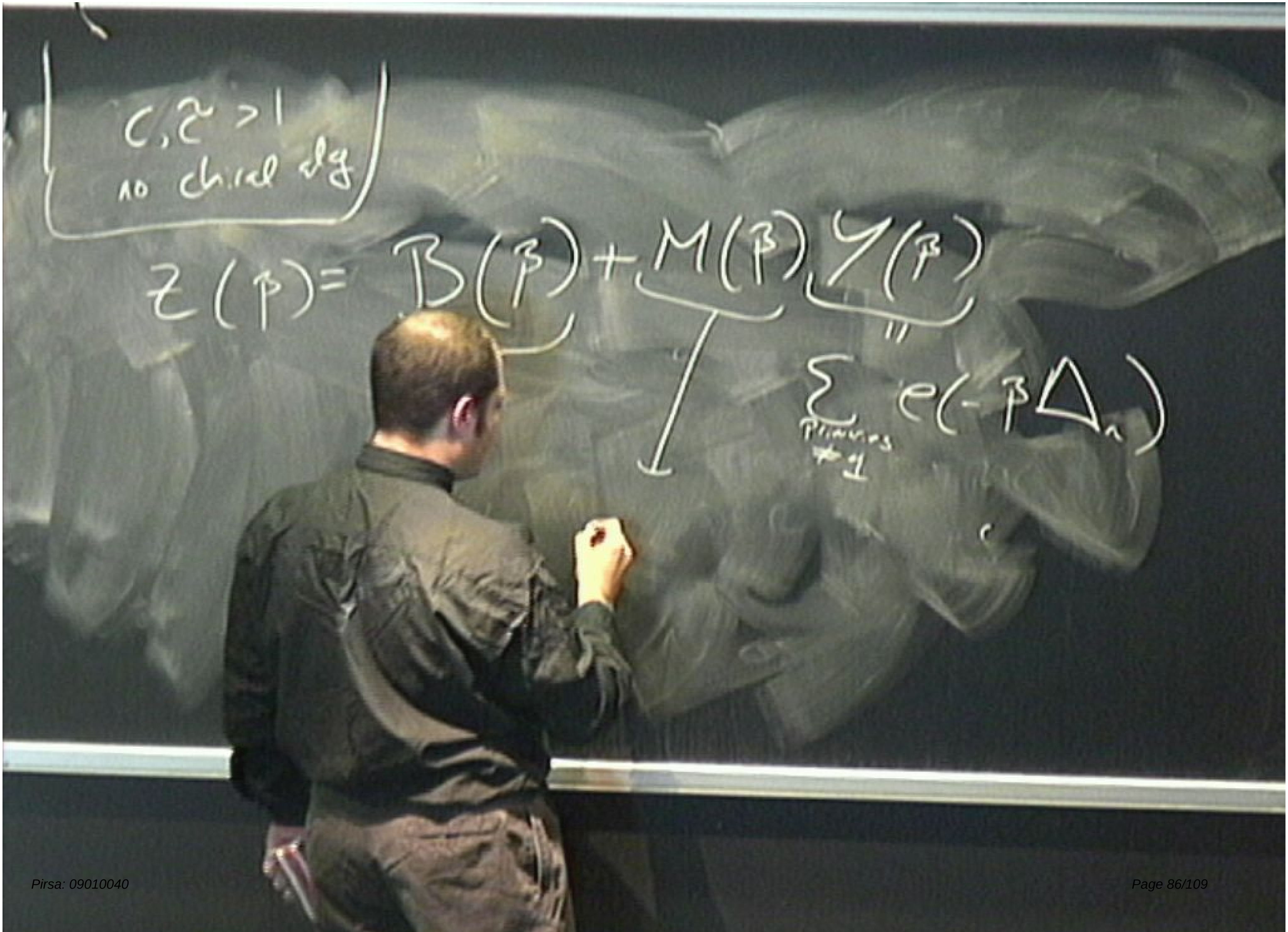
$$\left.(\beta\partial_\beta)^N Z(\beta)\right|_{\beta=2\pi} = 0$$

ONLY USED N=1, 3


c,z >1


c, c̃ > 1
no chiral alg


c, c̄ > 1
no chiral alg
Z(β) = B(β) + M(β


c, c̃ > 1
no chiral alg
Z(β) = B(β) + M(β) χ(β)
Σ e(-βΔ_n)
primaries

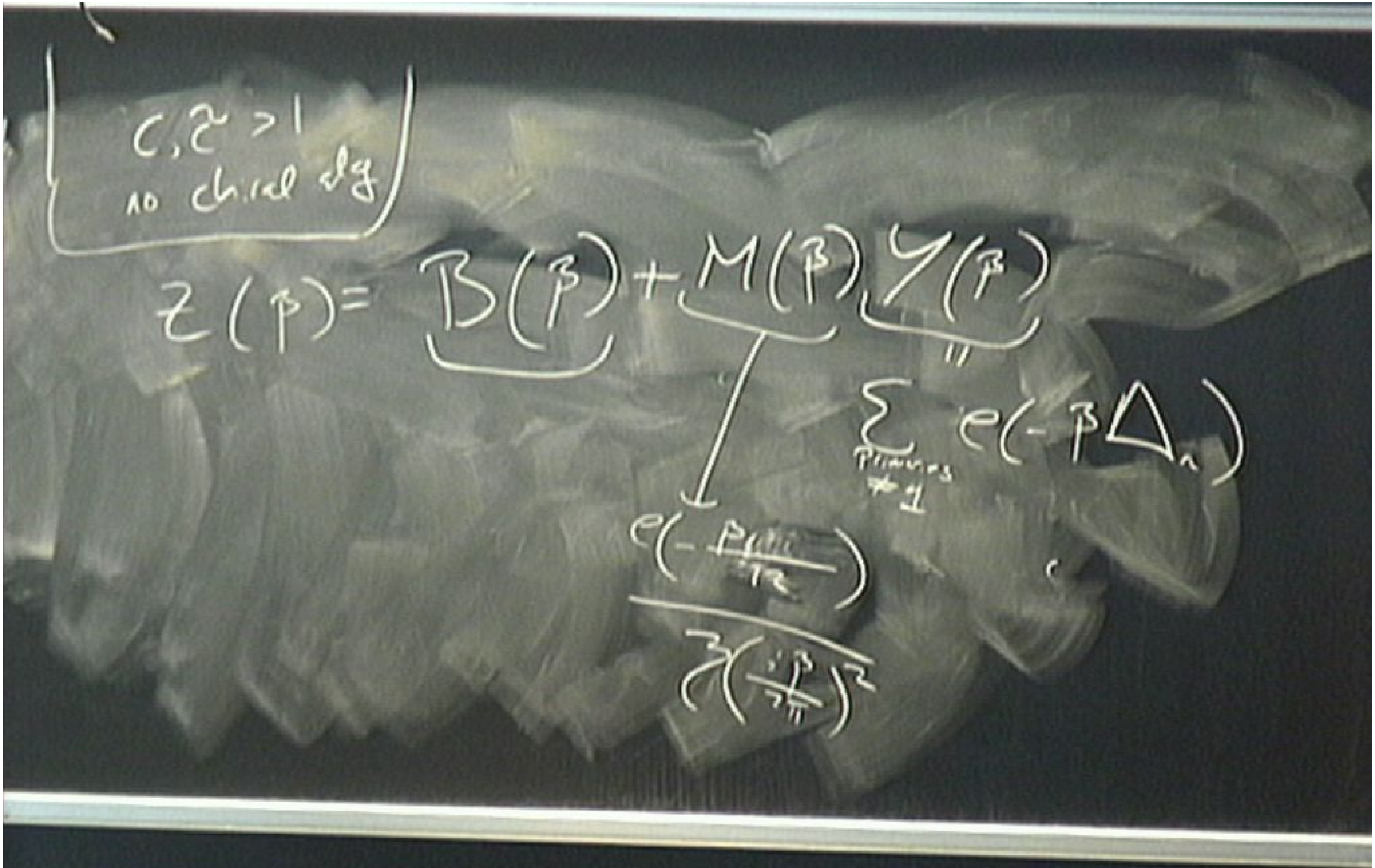
$c, \bar{c} > 1$

no chiral alg

$$Z(\beta) = B(\beta) + M(\beta)\, Y(\beta)$$

$$Y(\beta) = \sum_{\substack{\text{primaries} \\ \neq 1}} e(-\beta \Delta_n)$$

$$M(\beta) = \frac{e\left(-\frac{\beta(c-1)}{12}\right)}{\eta\left(\frac{i\beta}{2\pi}\right)^2}$$


c, c̄ > 1
no chiral alg
Z(β) = B(β) + M(β) Y(β)
Σ e(-βΔ_n)
primaries
≠ 1

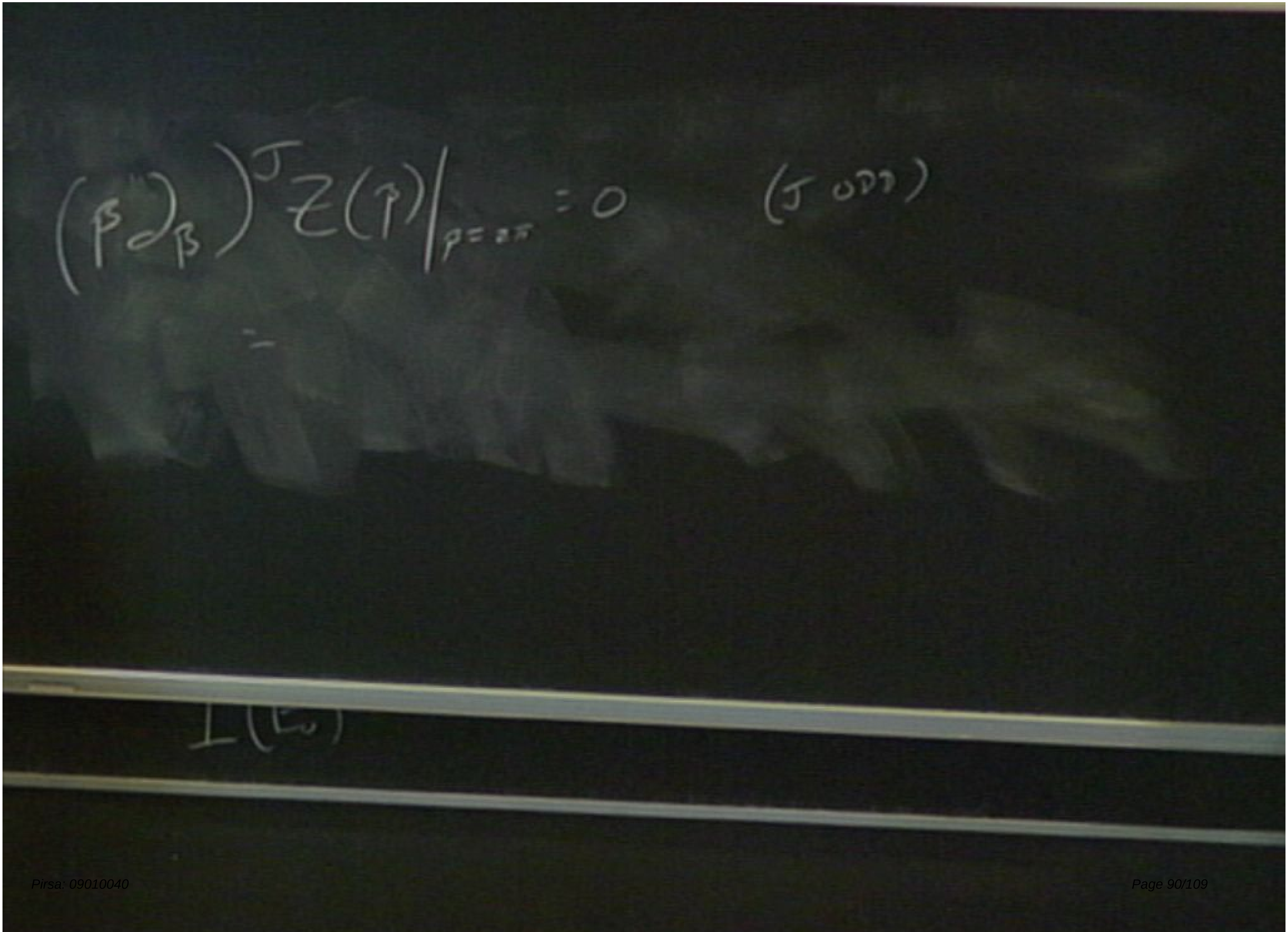
$c, \tilde{c} > 1$
no chiral alg

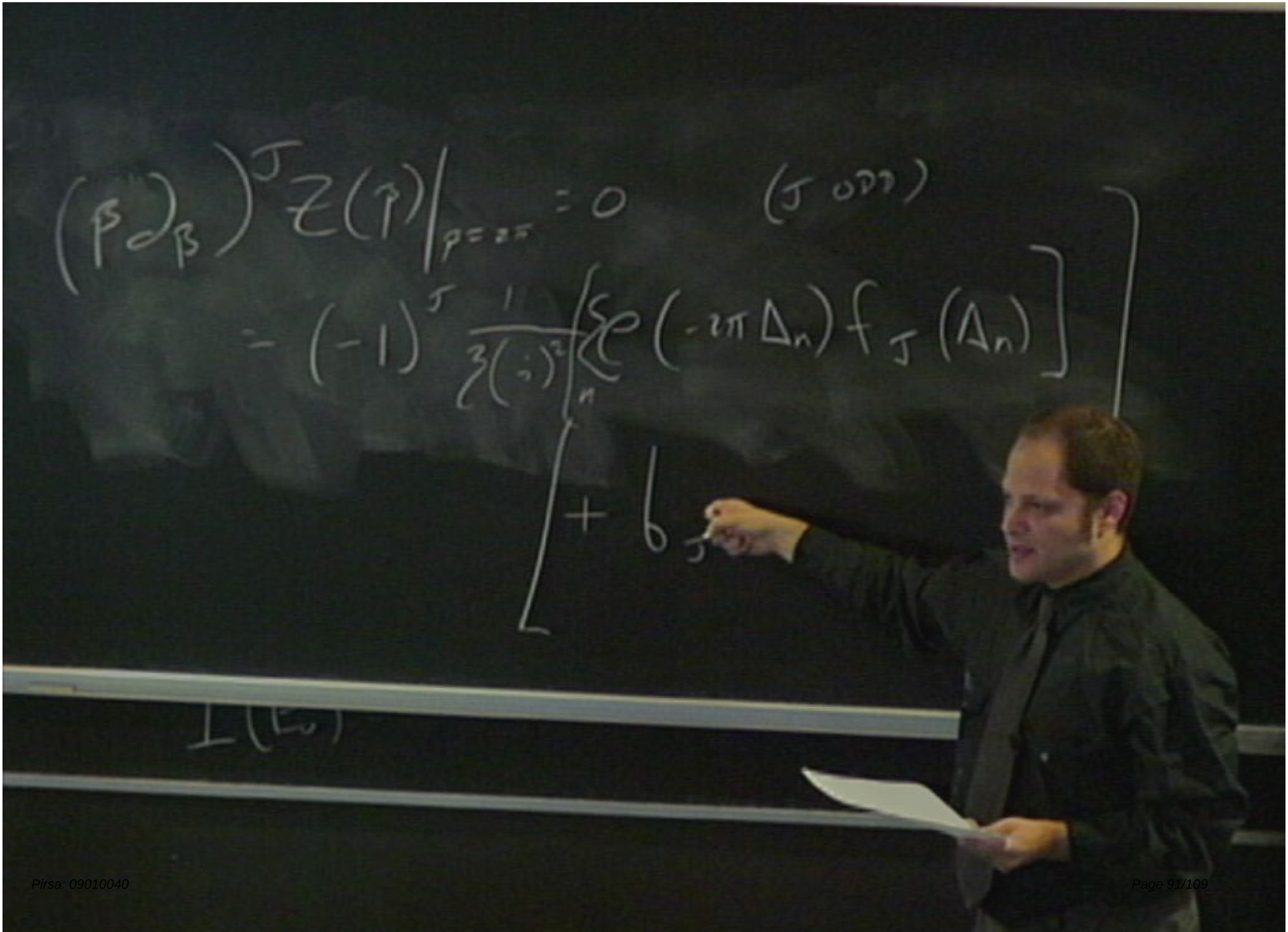
$$Z(\beta) = B(\beta) + M(\beta)\, Y(\beta)$$

$$B(\beta) = (1 - e(-\beta))^{2} M(\beta)$$

$$M(\beta) = \frac{e\left(-\frac{\beta \ldots}{12}\right)}{\eta\left(\frac{i\beta}{2\pi}\right)^{2}}$$

$$Y(\beta) = \sum_{\substack{\text{primaries} \\ \neq 1}} e(-\beta \Delta_n)$$

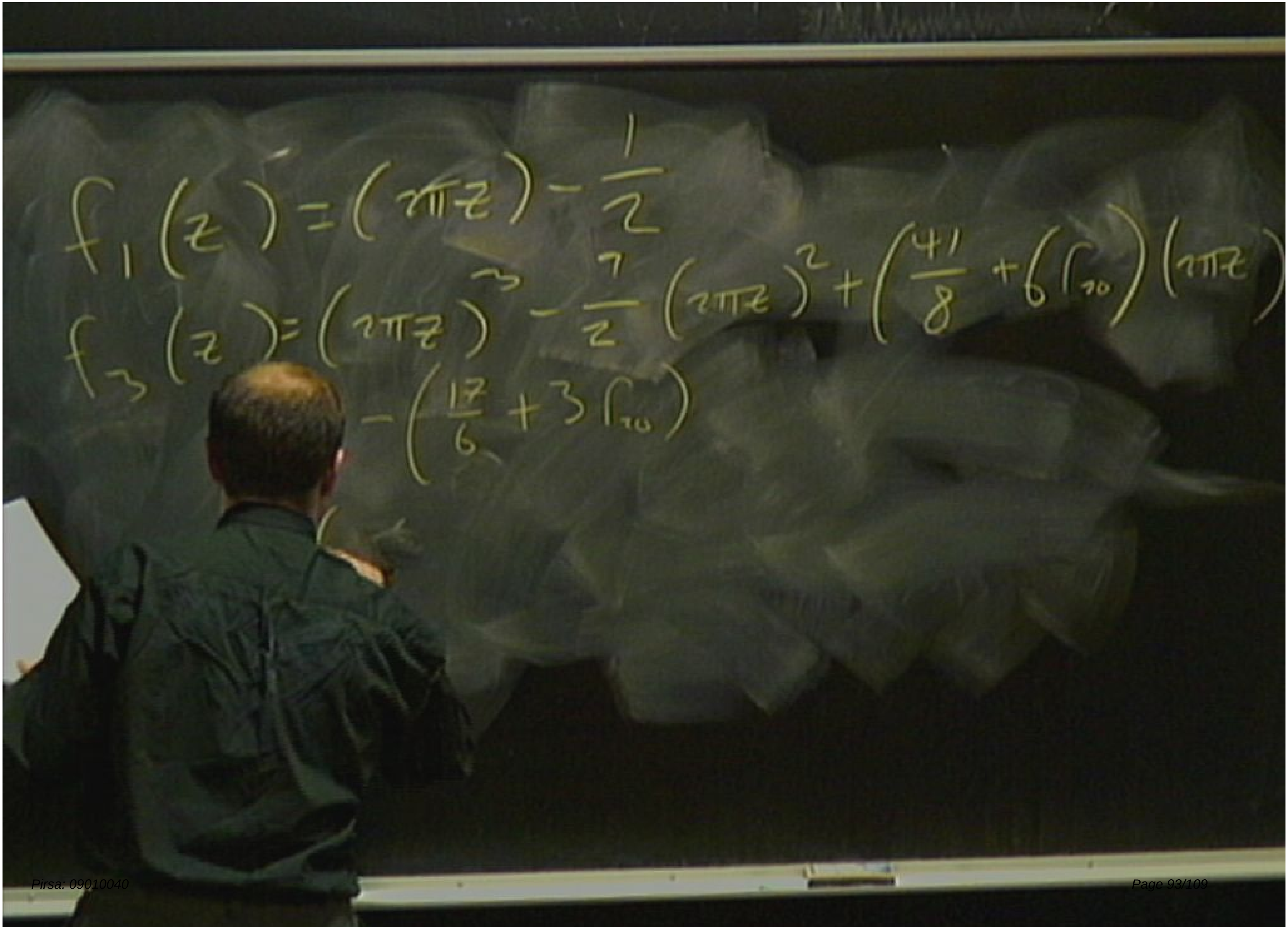

$(\beta\partial_\beta)^J Z(\beta)|_{\beta=2\pi} = 0$ (J odd)
$I(E_0)$


(β∂_β)^J Z(β)|_{β=2π} = 0 (J odd)
= (−1)^J 1/ζ(·/n)^2 [Σ_n e(−2π Δ_n) f_J (Δ_n)]
+ b_J

$$\left(\beta\partial_\beta\right)^J Z(\beta)\Big|_{\beta=2\pi} = 0 \qquad (J \text{ odd})$$

$$= (-1)^J \frac{1}{\zeta(\cdot)^2}\left[\left[\sum_n e(-2\pi\Delta_n) f_J(\Delta_n)\right] + b_J\right]$$

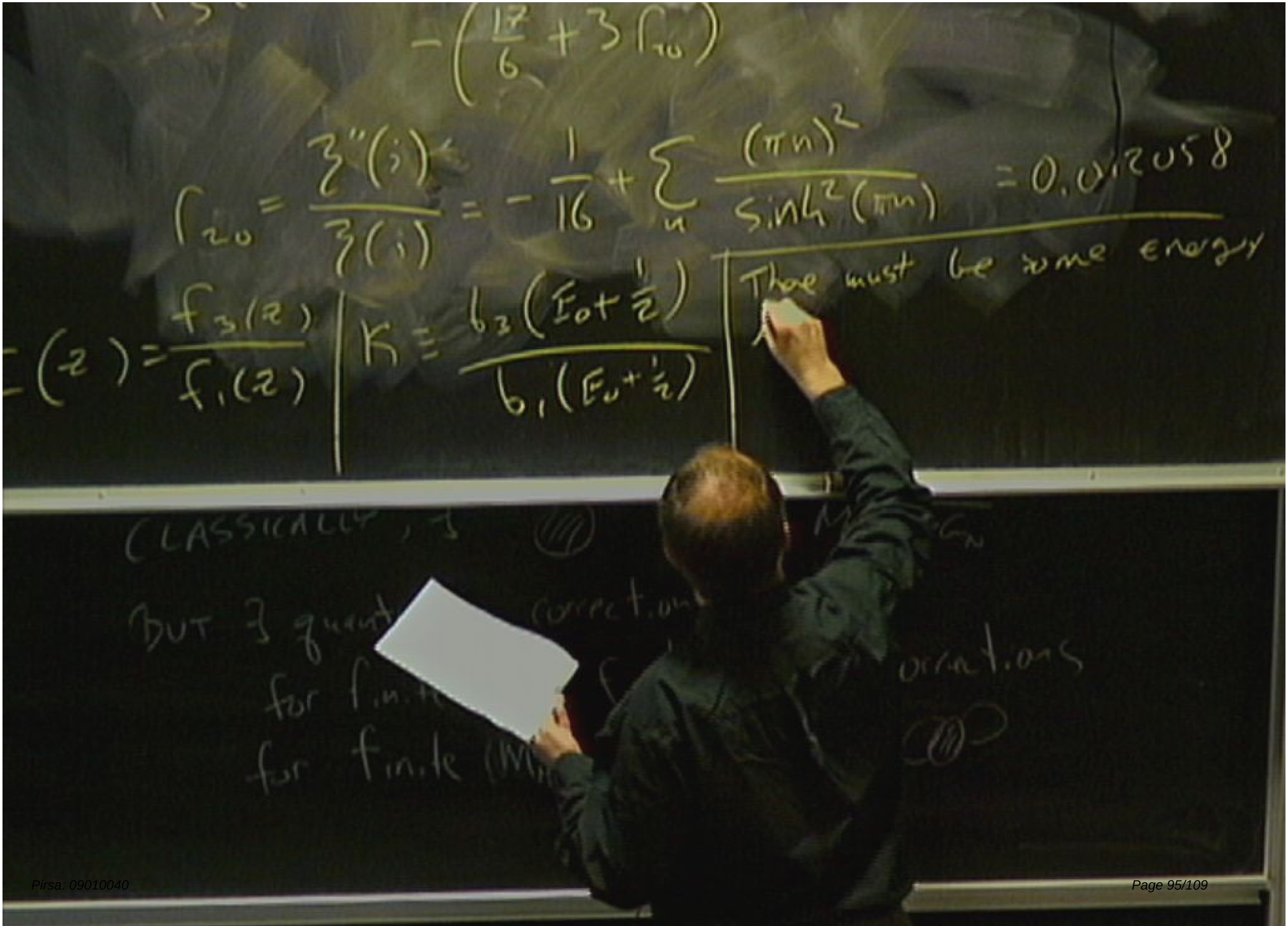
$I(E_0)$


f1(z) = (2πz)^(-1/2)
f3(z) = (2πz)^(-3) - 7/2 (2πz)^2 + (41/8 + 6ζ10)(2πz)
- (17/6 + 3ζ10)

$$f_1(z) = (2\pi z) - \frac{1}{2}$$

$$f_3(z) = (2\pi z)^3 - \frac{7}{2}(2\pi z)^2 + \left(\frac{41}{8} + 6\,\Gamma_{20}\right)(2\pi z) - \left(\frac{17}{6} + 3\,\Gamma_{20}\right)$$

$$\Gamma_{20} = \frac{\zeta''(i)}{\zeta(i)} = -\frac{1}{16} + \sum_n \frac{(\pi n)^2}{\sinh^2(\pi n)} = 0.012058$$



There must be some energy
CLASSICALLY
BUT ∃ quant
correction
corrections
for finite
= 0.012058

$$-\left(\frac{17}{6}+3\Gamma_{20}\right)$$

$$\Gamma_{20} = \frac{\zeta''(i)}{\zeta(i)} = -\frac{1}{16} + \sum_n \frac{(\pi n)^2}{\sinh^2(\pi n)} = 0.012058$$

$$I(\tau) = \frac{f_3(\tau)}{f_1(\tau)}$$

$$K \equiv \frac{b_3(E_0+\frac{1}{2})}{b_1(E_0+\frac{1}{2})}$$

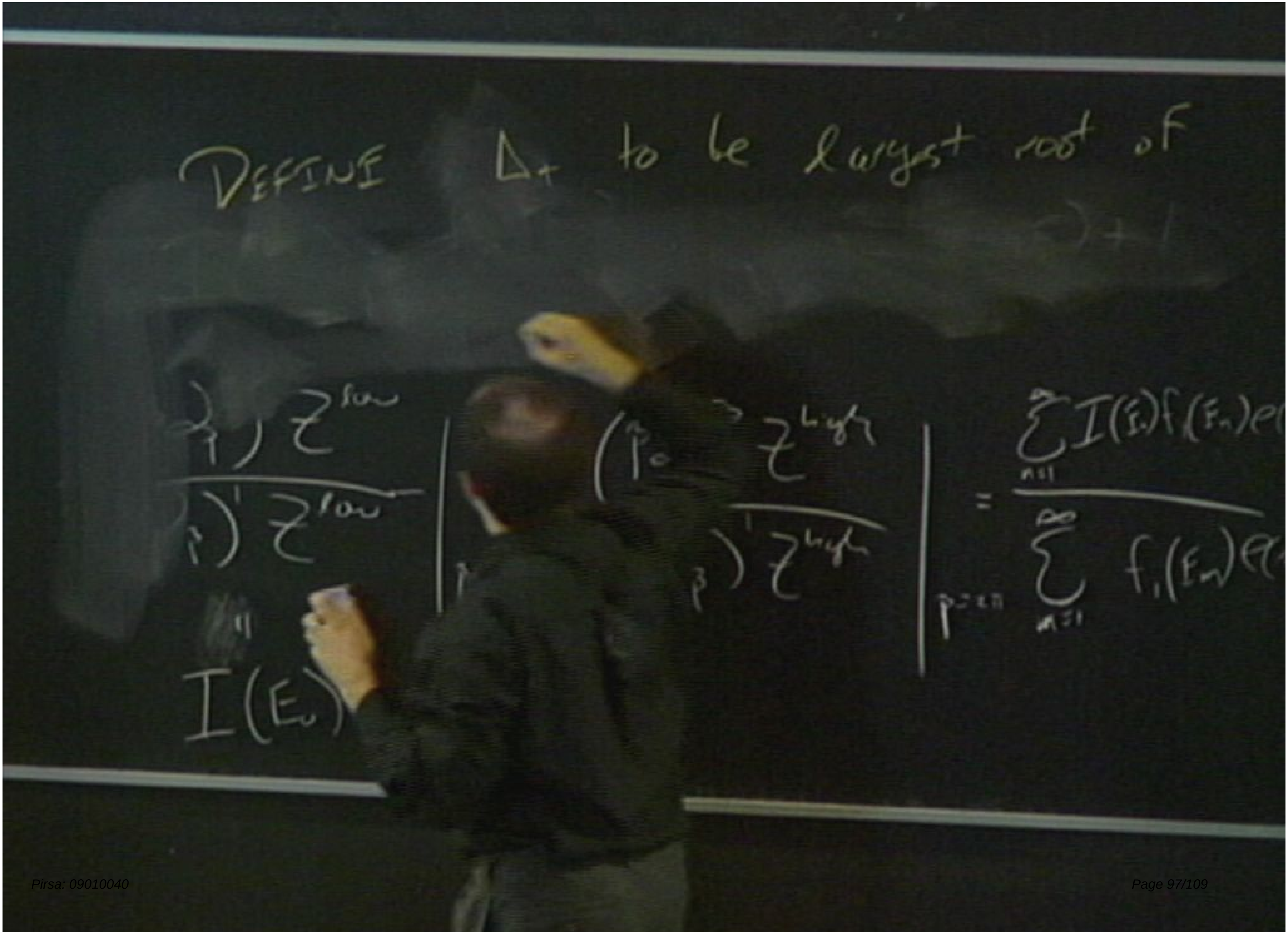
There must be some energy level $E_1 > E_0$ s.t. $I(E_1) < K$.

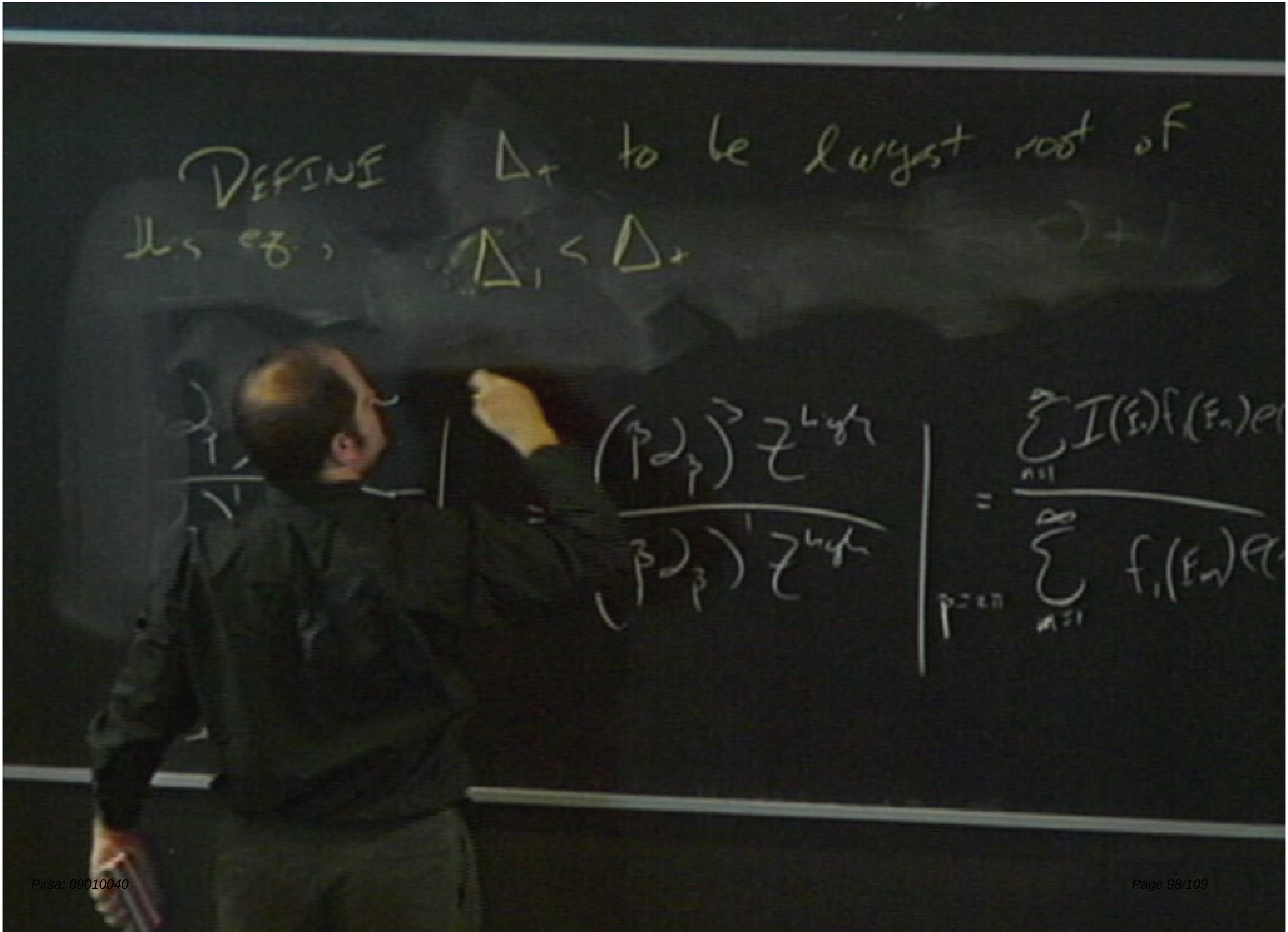
CLASSICALLY, ∃ … @ $M = \frac{\ }{4G_N}$

BUT ∃ quantum corrections:

for finite Λ, ∃ finite-volume corrections

for finite $M_{MATTER} G_N$, ∃ …


DEFINE Δ+ to be largest root of

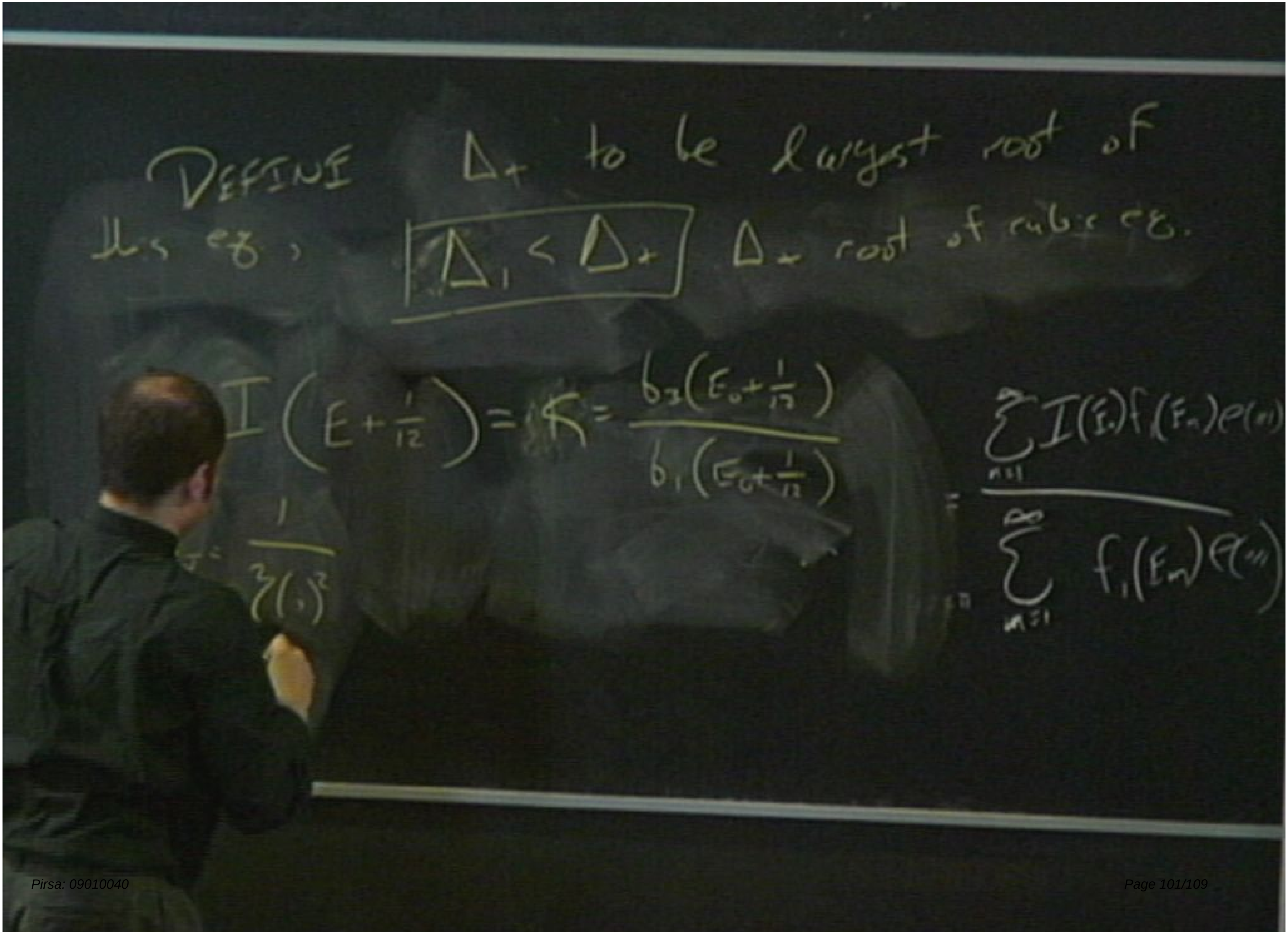

DEFINE Δ+ to be largest root of
this eq., Δ1 < Δ+

DEFINE Δ_+ to be largest root of this eq., $\boxed{\Delta_1 < \Delta_+}$ Δ_+ root of cubic eq.

$$= \frac{\sum_{n=1}^{\infty} I(E_n) f_1(E_n) e(\ldots}{\sum_{n=1}^{\infty} f_1(E_n) e(\ldots}$$

DEFINE Δ_* to be largest root of this eq., $\boxed{\Delta_1 < \Delta_*}$ Δ_* root of cubic eq.

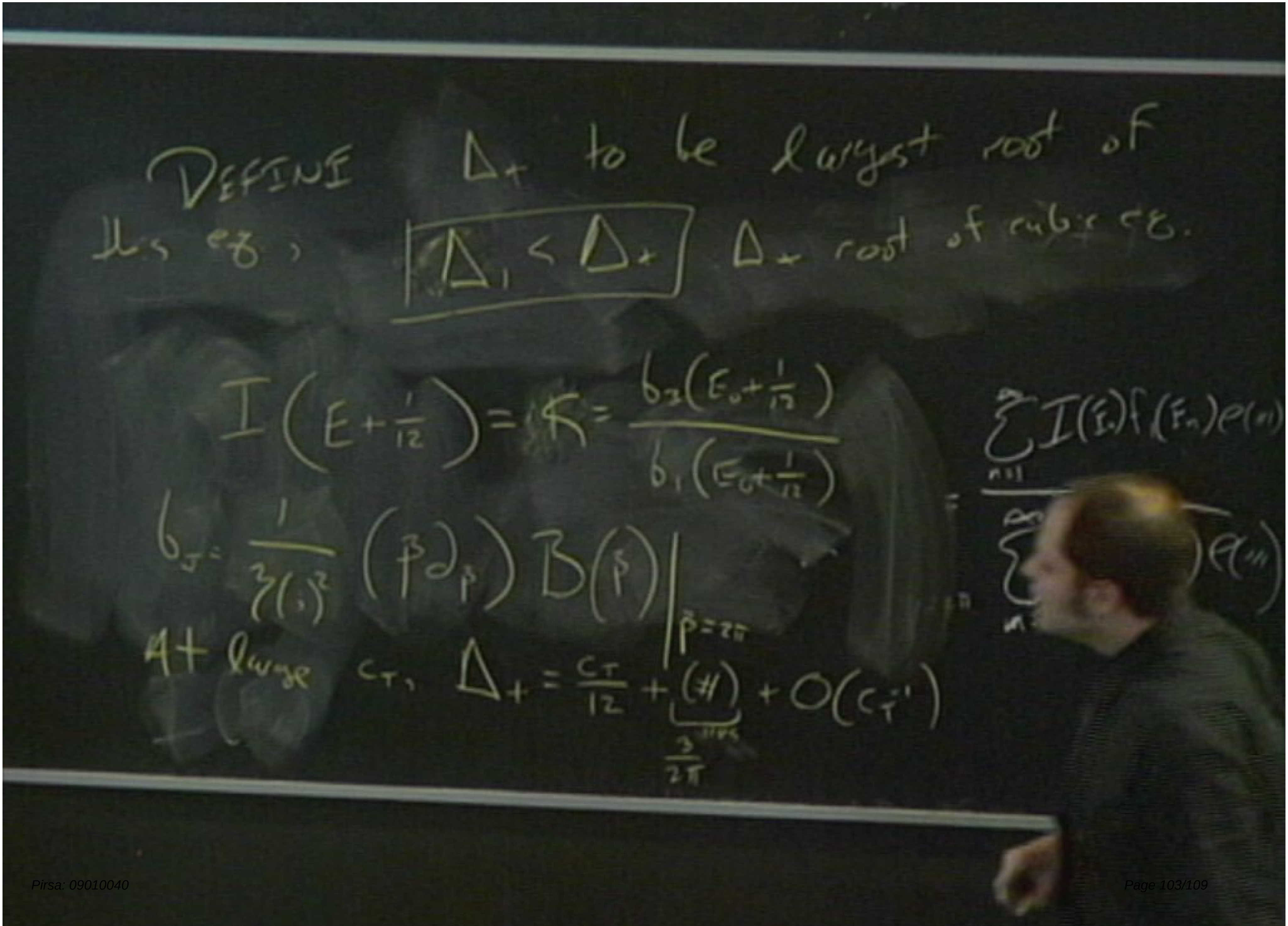
$$I\left(E+\frac{1}{12}\right) = R = \frac{b_2\left(E_0+\frac{1}{12}\right)}{b_1\left(E_0+\frac{1}{12}\right)} = \frac{\sum_{n=1}^{\infty} I(E_n) f_1(E_n) e(n)}{\sum_{n=1}^{\infty} f_1(E_n) e(n)}$$


DEFINE Δ+ to be largest root of
this eq., Δ1 < Δ+ Δ+ root of cubic eq.
I(E + 1/12) = K = b2(E0 + 1/12) / b1(E0 + 1/12)

DEFINE Δ_+ to be largest root of this eq., $\boxed{\Delta_1 < \Delta_+}$ Δ_* root of cubic eq.

$$I\left(E+\frac{1}{12}\right) = K = \frac{b_3\left(E_0+\frac{1}{12}\right)}{b_1\left(E_0+\frac{1}{12}\right)} = \frac{\sum_{n=1}^{\infty} I(E_n)\, f_1(E_n)\, \rho(n)}{\sum_{n=1}^{\infty} f_1(E_n)\, \rho(n)}$$

$$b_j = \frac{1}{\zeta(3)^2}\left(\beta\partial_\beta\right) B(\beta)\Big|_{\beta=2\pi}$$

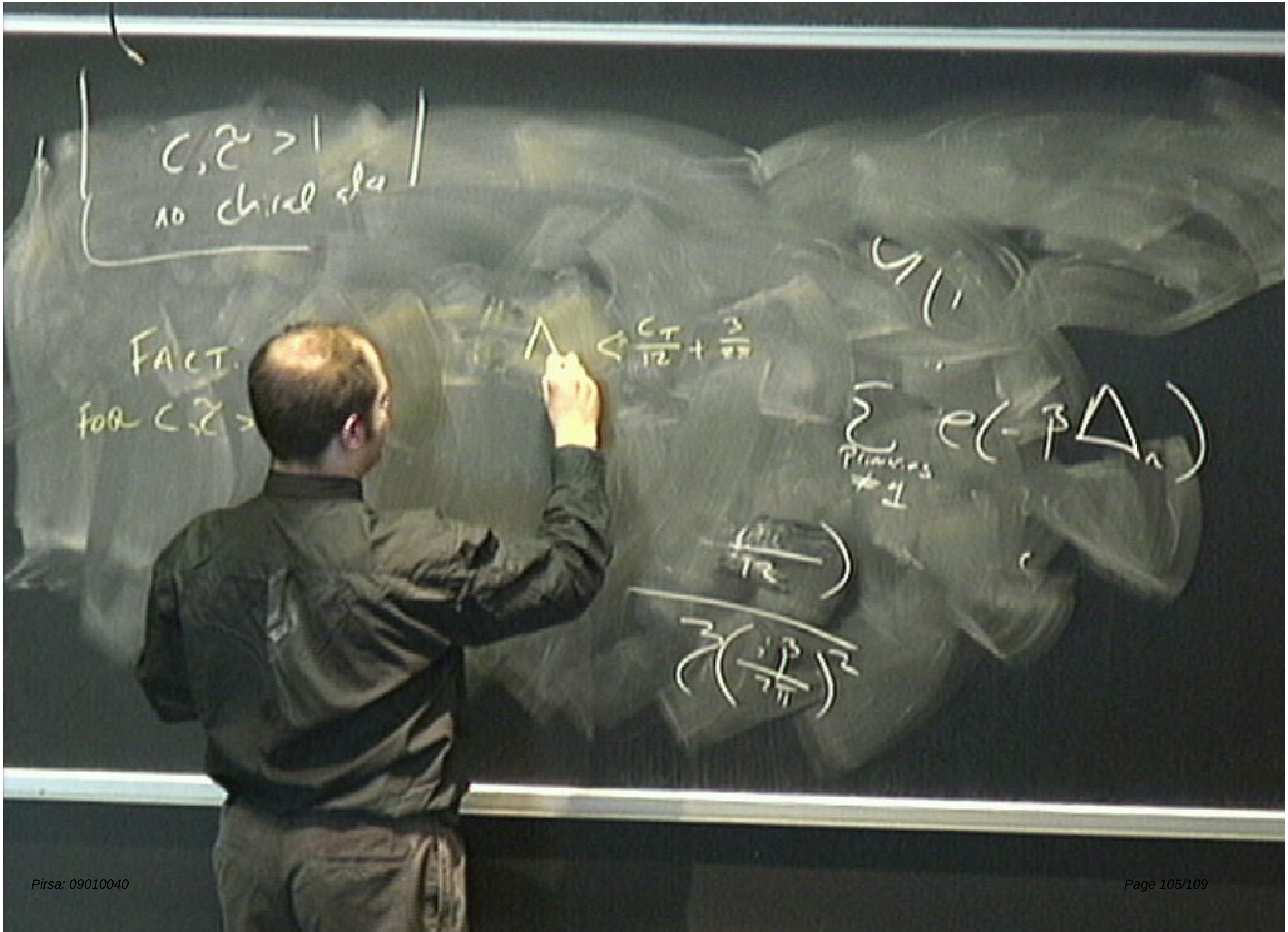

DEFINE Δ+ to be largest root of
this eq., Δ1 < Δ+ Δ* root of cubic eq.
I(E + 1/12) = K = b3(E0 + 1/12) / b1(E0 + 1/12)
bJ = 1/ζ(1)^2 (β∂β) B(β) | β=2π
At large cT, Δ+ = cT/12 + (#) + O(cT^-1)
3/2π

DEFINE Δ_+ to be largest root of this eq., $\boxed{\Delta_1 < \Delta_+}$ Δ_+ root of cubic eq.

$$I\left(E+\frac{1}{12}\right) = \mathcal{R} = \frac{b_2\left(E_0+\frac{1}{12}\right)}{b_1\left(E_0+\frac{1}{12}\right)} = \frac{\sum_{n=1}^{\infty} I(E_n) f_1(E_n)\rho(n)}{\sum_{n=1}^{\infty} f_1(E_n)\rho(n)}$$

$$b_j = \frac{1}{\zeta(j)^2}\left(\beta\partial_\beta\right) B(\beta)\Big|_{\beta=2\pi}$$

At large c_T, $\Delta_+ = \frac{c_T}{12} + \underbrace{(\#)}_{\frac{3}{2\pi}} + O(c_T^{-1})$


c, c̃ > 1
FACT.
Primaries ≠ 1

$c, \tilde{c} > 1$

no chiral alg

FACT: $\frac{c_T}{12} + \frac{3}{2\pi} + .03 \leq \Delta_+ \leq \frac{c_T}{12} + \frac{3}{2\pi}$

FOR $c, \tilde{c} > 1$

$$\sum_{\substack{\text{primaries} \\ \neq 1}} e(-\beta \Delta_n)$$

$c, \tilde{c} > 1$
no chiral alg

FACT: $\frac{c_T}{12} + \frac{3}{2\pi} + .03 < \Delta_+ < \frac{c_T}{12} + \frac{3}{2\pi}$

FOR $c, \tilde{c} > 1$

$c, \tilde{c} > 1$

no chiral alg

FACT: $\frac{c_T}{12} + \frac{3}{2\pi} + .03 < \Delta_+ < \frac{c_T}{12} + \frac{3}{2\pi}$

FOR $c, \tilde{c} > 1$

$$E_+ = \frac{c_T}{24} + \frac{3}{2\pi}$$
$$= \frac{c}{12} + \frac{3}{2\pi}$$

FOR $c, \bar{c} > 1$

$$E_+ = \frac{c_T}{24} + \frac{3}{2\pi}$$

$$= \frac{c}{12} + \frac{3}{2\pi}$$

$$\exists \quad M < \frac{1}{4G} + O\left(\frac{1}{\ell_{AdS}}\right)$$

$$\longrightarrow M_{BTZ}$$