Title: The Kerr/CFT Correspondence

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Abstract: Holography is usually applied to black holes that are supersymmetric, charged, or living in higher dimensions. The astrophysical Kerr black holes that have been observed in the sky have none of these nice properties, and AdS/CFT does not apply. Nevertheless, by studying the symmetries of the near horizon region, I will show that extreme Kerr black holes are holographically dual to a two-dimensional conformal field theory. The U(1) isometry of the near horizon region extends asymptotically to a Virasoro algebra. We compute the central charge semiclassically, and find in particular that the observed black hole GRS 1915+105 is approximately dual to a CFT with central charge 10^79. The microstate counting of the dual CFT correctly reproduces the Bekenstein-Hawking entropy. I will also discuss generalizations to a wide variety of extreme black holes in assorted theories, the possibility of moving away from the external limit, and potential applications of the correspondence.

The Kerr/CFT Correspondence

Tom Hartman
Harvard University

0809.4266 with Guica, Song, and Strominger
0811.4393 with Murata, Nishioka, and Strominger

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Black Holes and Quantum Physics Workshop
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Motivation

Question

AdS/CFT has led to many insights about black holes, but does not apply to the black holes observed in the Universe.

Can we apply holography to these black holes?

Kerr Black Holes

- Overview
- Asymptotic Symmetries
- Thermodynamics
- Applications & More

- 4d rotating black hole

  $J \le M^2$

- Extremal limit

  $J = M^2$ "chiral"

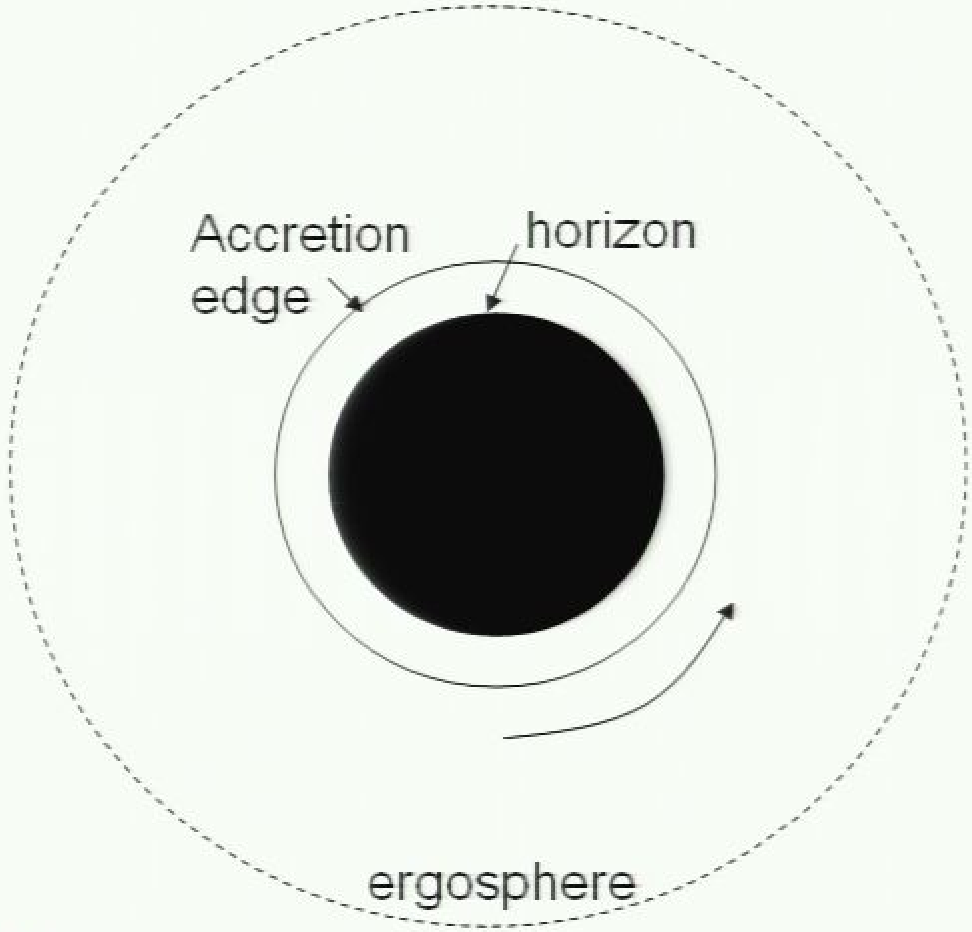
- GRS 1915+105

  $J > .98 M^2$

  McClintock et al. 2006

- Bekenstein-Hawking Entropy

  $$S_{\text{ext}} = \frac{\text{Area}}{4} = 2\pi J$$



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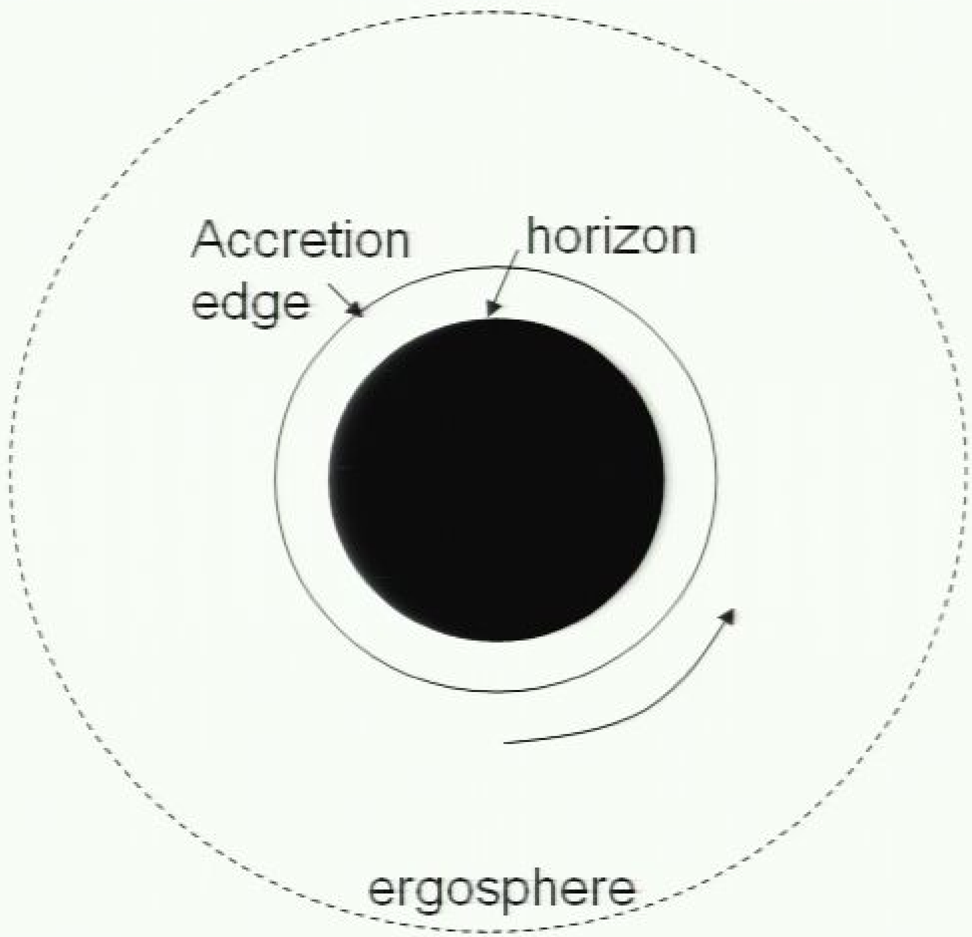
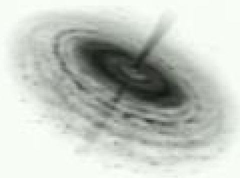




Punch line

- Main Result

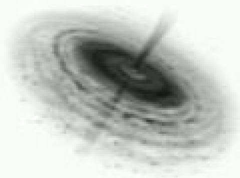
  Near the horizon of an extremal Kerr black hole, (any) consistent theory of quantum gravity is dual to a 2D conformal field theory.

  Central charge: $c = 12\,J$

- Derivation: states transform under a Virasoro algebra (ie in representations of the 2d conformal group)

- Applies to astrophysical black holes (and more)

- Things we don't need
  - Charge
  - Anti de Sitter space (AdS)
  - Extra dimensions
  - Supersymmetry
  - String theory



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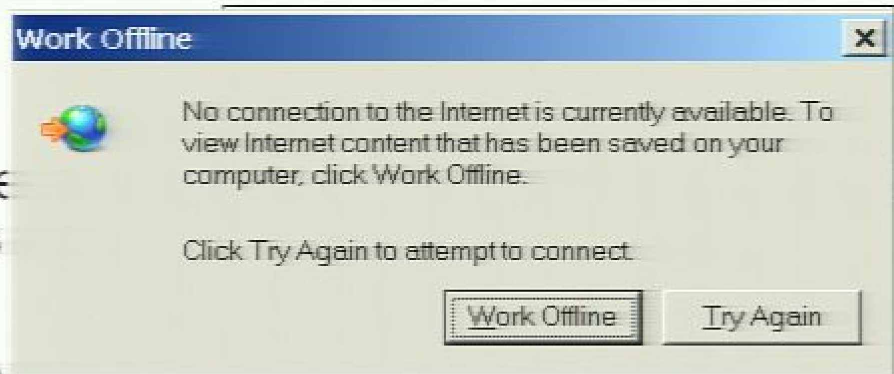

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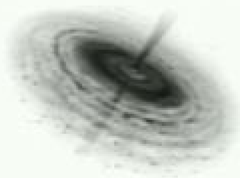

Outline

- Overview ✓
- Asymptotic Symmetries
- Entropy
- Generalizations and applications
    - Charge
    - Anti de Sitter space (AdS)
    - Extra dimensions
    - Supersymmetry
    - String theory

AdS_3/CFT_2

- Explains every entropy calculation in string theory, eg entropy of 5d black holes Strominger, Vafa '95

- But, complexities of string theory are not needed
  Strominger '97

- Brown & Henneaux ('86) showed *quantum gravity on AdS_3 is dual to a CFT with central charge*

$$c = \frac{3\ell}{2G}$$

ℓ = AdS radius
G = Newton constant

- Method: Asymptotic Symmetry Group (ASG)

Near horizon extreme Kerr (NHEK)

Near horizon limit:

$$ds^2 = 2J\Omega^2 \left(\underbrace{-r^2 dt^2 + \frac{dr^2}{r^2}}_{AdS_2} + d\theta^2 + \Lambda^2 (d\phi + r dt)^2 \right)$$

Bardeen, Horowitz '99

$\Omega^2, \Lambda^2 =$ functions of θ

$\phi \sim \phi + 2\pi$

Isometries:

$U(1)_L$ rotating ϕ

$SL(2,R)_R$ acting on the AdS_2

Asymptotic Symmetries I

- Near horizon metric

$$ds^2 = 2J\Omega^2\left(-r^2dt^2 + \frac{dr^2}{r^2} + d\theta^2 + \Lambda^2(d\phi + r dt)^2\right)$$

- Asymptotic Symmetry Group [example: $U(1)$ gauge theory]

$$\text{ASG} = \frac{\text{Allowed symmetries}}{\text{Symmetries vanishing at infinity}}$$

- Computing the ASG
  - Impose boundary conditions
  - Find allowed symmetries
  - Compute the charges
  - Check everything is well defined (finite charges)
  - Compute Dirac brackets

Asymptotic Symmetries II

- ★ Impose boundary conditions, $g_{tt} \sim \mathcal{O}(r^2)$, etc.
- ★ Find allowed diffeos:

$$\begin{aligned} \zeta &= \epsilon(\phi)\partial_\phi - r\epsilon(\phi)\partial_r \\ \zeta_t &= \partial_t \end{aligned}$$

- ★ Generators ζ_n with $\epsilon_n = e^{in\phi}$ satisfy a Virasoro algebra,

$$i\{\zeta_m, \zeta_n\}_{L.B.} = (m-n)\zeta_{m+n}$$

- ★ Associated charges $Q_n(g_{\mu\nu})$

Determined by action

$$\delta Q(\zeta, g) = \int_{\partial\Sigma} k[\zeta, g, \delta g]$$

- ★ Supplemental boundary condition $M^2 = J$ (extremality)

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Central charge

★ Compute Dirac brackets

$$\{Q_m, Q_n\}_{D.B.} = \delta_n Q_m$$

★ Result is the Virasoro algebra,

$$i\{Q_m, Q_n\}_{D.B.} = (m-n)Q_{m+n} + \frac{c}{12}(m^3 - m)\delta_{m+n}$$

⇒ quantum gravity on NHEK is holographically dual to a 2d CFT with

$$c = 12\ J$$

$$\mathrm{GRS}\ 1915{+}105 \to c \sim 10^{79}$$

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Outline

- Overview ✓
- Asymptotic Symmetries ✓
- Entropy
  - Cardy formula

$$S = \frac{\pi^2}{3} c T$$

- Generalizations and applications

Temperature

- At extremality, first law becomes

$$0 = T_H dS = dM - \Omega_H dJ - \Phi dQ$$

- So define conjugate potentials for extremal variations

$$dS = \frac{dJ}{T_L} + \frac{dQ}{T_e}$$

- For Kerr,

$$S = 2\pi J \to T_L = \frac{1}{2\pi}$$

- Quantum state on extreme Kerr has density matrix

$$\rho = e^{-\hat{J}/T_L}$$

Entropy

Plug central charge and temperature

$$c_L = 12J$$
$$T_L = \frac{1}{2\pi}$$

into the Cardy formula

$$S_{CFT} = \frac{\pi^2}{3} c_L T_L$$

$$S_{CFT} = \frac{2\pi J}{\hbar} = \frac{\text{Area}}{4} = S_{macro}$$

Assumptions

- For Kerr/CFT ("quantum gravity on NHEK is a CFT"), only assumption is:
  - A consistent UV completion of quantum gravity on NHEK exists
- For entropy, using the Cardy formula assumes:
  - Modular invariance
  - Sufficient but not necessary condition:

$$T \gg c \quad (\text{ie, } \tfrac{1}{2\pi} \gg 10^{79})$$

Uh-oh.

Same thing happens in string theory, but is explained by highly twisted sectors. Does something similar happen here?

Maybe – the mass gap is very small ~ $1/M^3$. This suggests an effective description with small c, large T. More on this later.

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Plug central charge and temperature

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What can we compute?

Other black holes

- 4d Kerr Guica, TH, Song, Strominger
- Higher dimensions Lu, Mei, Pope
- Asymptotic AdS various papers
- Charge TH, Murata, Nishioka, Strominger
- String theory (D0-D6, D1-D5, NS5) and Supergravity
  Azeyanagi, Ogawa, Terashima
  Nakayama
  Chow, Cvetic, Lu, Pope
  Lu, Mei, Pope, Vazquez-Poritz
  Chen, Wang

Greybody Factors

★ Extreme Kerr has $T_H = 0$, but it decays via superradiance into modes

$$\Phi \sim e^{im\phi - i\omega t} S_\ell(\theta) R(r)$$

with

$$0 < \omega < m\Omega_H$$

For small ω,

$$\text{Decay rate} = \Gamma_\ell(\omega) \sim (\omega - m\Omega_H)^{2\ell+1}$$

★ This is a two-point function in the CFT

similar to:
Maldacena, Strominger '97

$$\Gamma \sim \int dx^+ dx^- e^{-i\omega_R t - i\omega_L \phi} < \mathcal{O}\mathcal{O} >$$

★ Large ω? Gravity: Teukolsky & Press, '74;

CFT: work in progress!
w/ W. Song and A. Strominger

$$\Gamma = \frac{\sinh^2 2\pi\delta}{\cosh^2 \pi(m-\delta) + \cosh^2 \pi(m+\delta) + 2\cos 2\pi\sigma \cosh \pi(m+\delta) \cosh \pi(m-\delta)}$$

$\delta \equiv$ function of m, ℓ, M

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What about SL(2,R)?

AdS$_3$ (Brown-Henneaux)

Exact: $SL(2,R)_L \times SL(2,R)_R$

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Asymptotic: $\text{Virasoro} \times \text{Virasoro}$

Kerr

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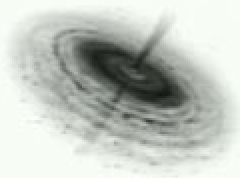

What about SL(2,R)?

Near-extremal entropy

★ The zero mode of $\mathrm{SL}(2,\mathrm{R})_R$ is

$$\zeta_0 = \partial_t$$

★ Writing this in terms of the original Kerr coordinates suggests

$$Q_0 \sim M^2 - J \equiv E_R$$

★ If we assume

$$c_R = c_L = 12J$$

then the Cardy formula gives the correct near extremal entropy,

$$S_{CFT} = 2\pi J + 2\pi\sqrt{\frac{c_R}{6}E_R} + \cdots$$

★ Summary: We have only found the chiral left half of the CFT in Kerr/CFT, but we suspect that there are also right-movers which account for the entropy away from extremality

"U-duality"

- 5D 3-charge black hole

$$S = 2\pi\sqrt{n_1 n_2 n_3}$$

- String theory U-duality changes c, T with $S \propto cT$ fixed

- 5d Kerr (or 4d Kerr-Newman) has near horizon isometries

$$SL(2,R)_R \times U(1)_\phi \times U(1)_\psi$$

Lu, Mei, Pope

TH, Murata, Nishioka, Strominger

- Two consistent choices of boundary conditions:
  - First choice: $U(1)_\phi \to$ Virasoro with central charge

$$c_\phi \sim J_\phi$$

  - Second choice: $U(1)_\psi \to$ Virasoro with central charge

$$c_\psi \sim J_\psi$$

- *Either* choice gives the correct entropy!

Conclusion

- **Summary: Gravity on extreme Kerr is a CFT.**
  - Nothing exotic is necessary (but exotic black holes work too)
  - Applies to astrophysical black holes, eg GRS 1915+105

- **Open questions**
  - Beyond extremality
  - What can we calculate with the CFT?
    - greybody factors?
    - astrophysics (accretion, X-ray emission, etc.)?

- ***Really* open questions**
  - What is the CFT?
  - What/where are the microstates?