Title: Introduction to the Bosonic String 3B

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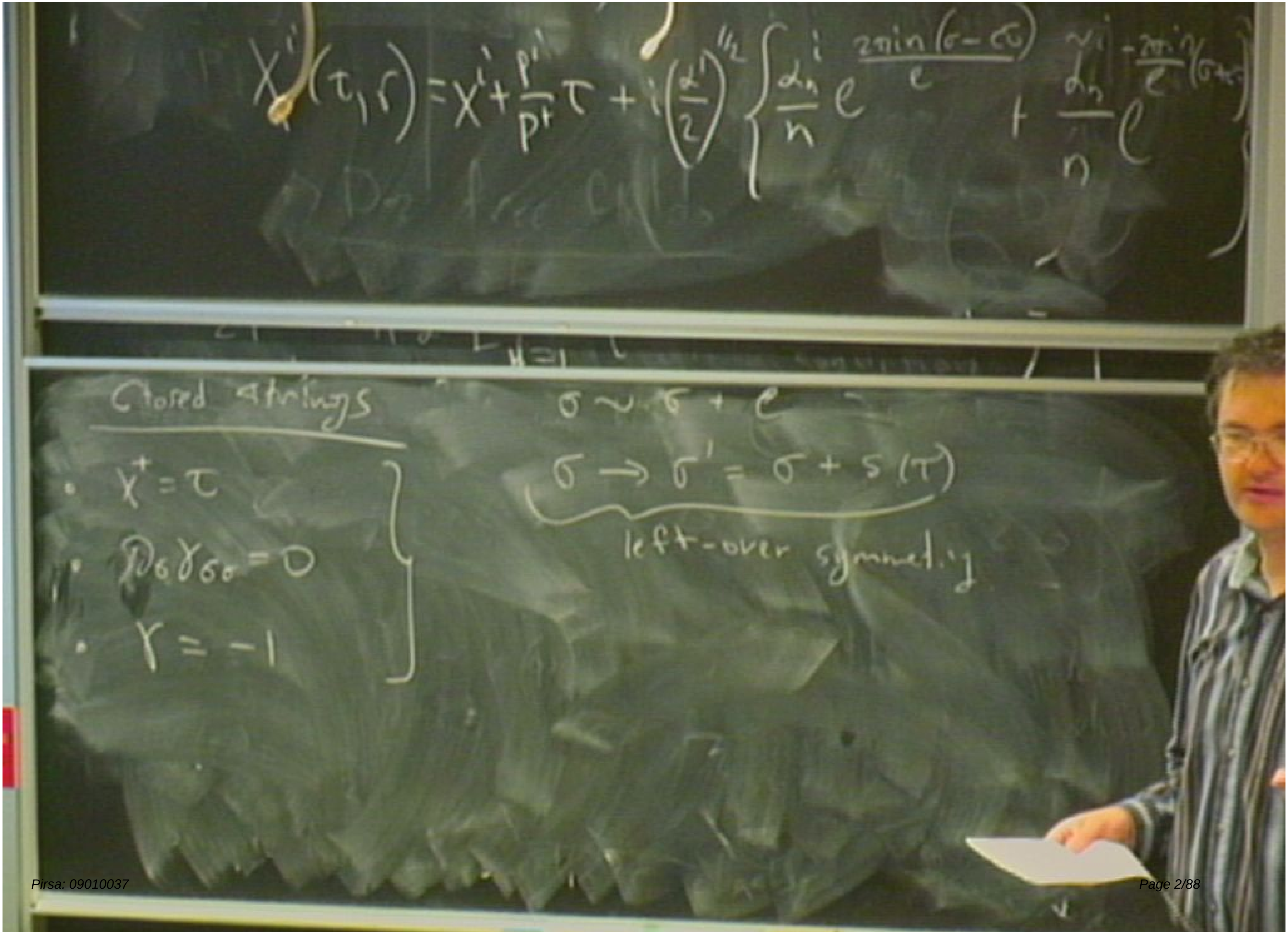
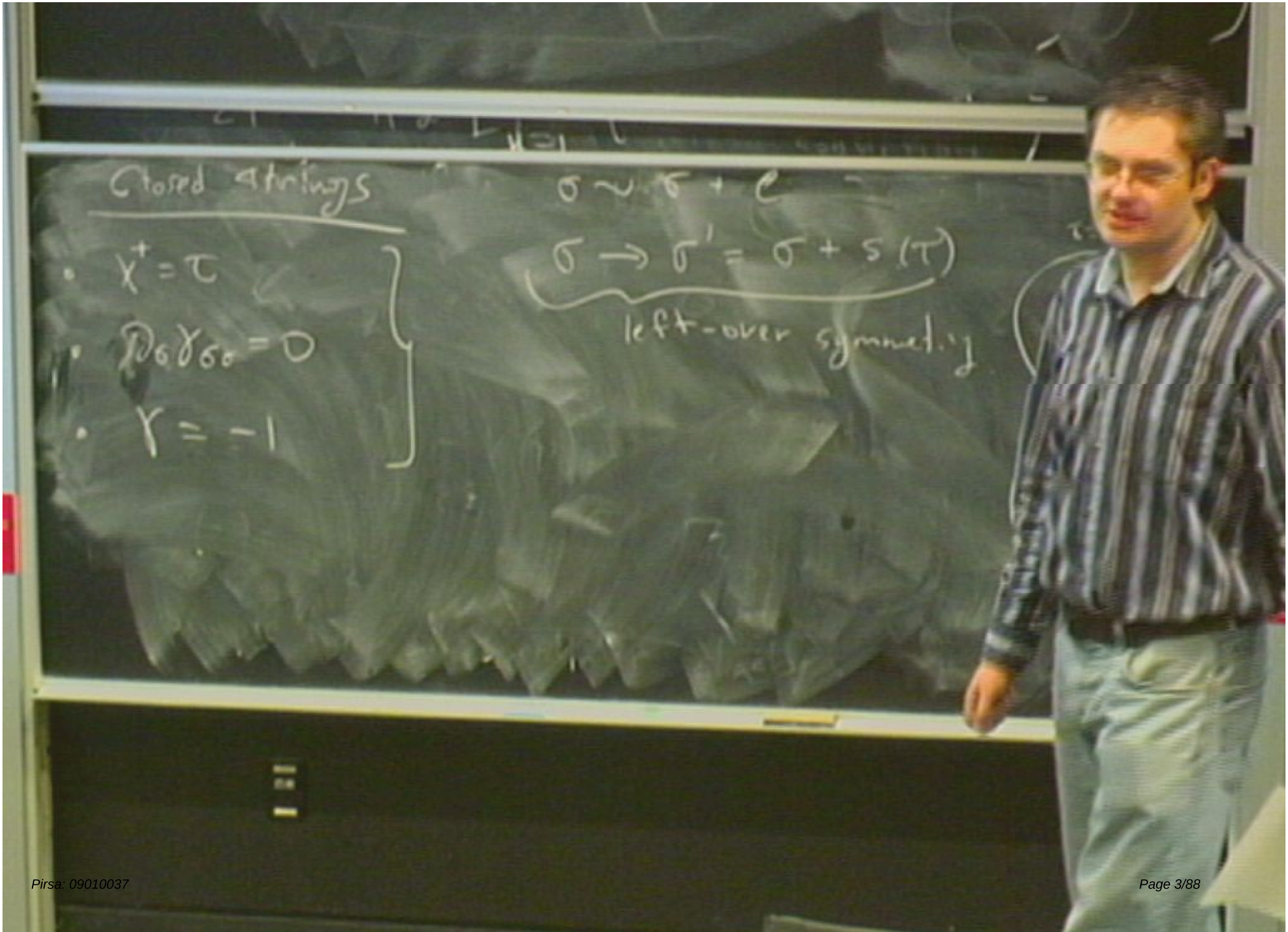
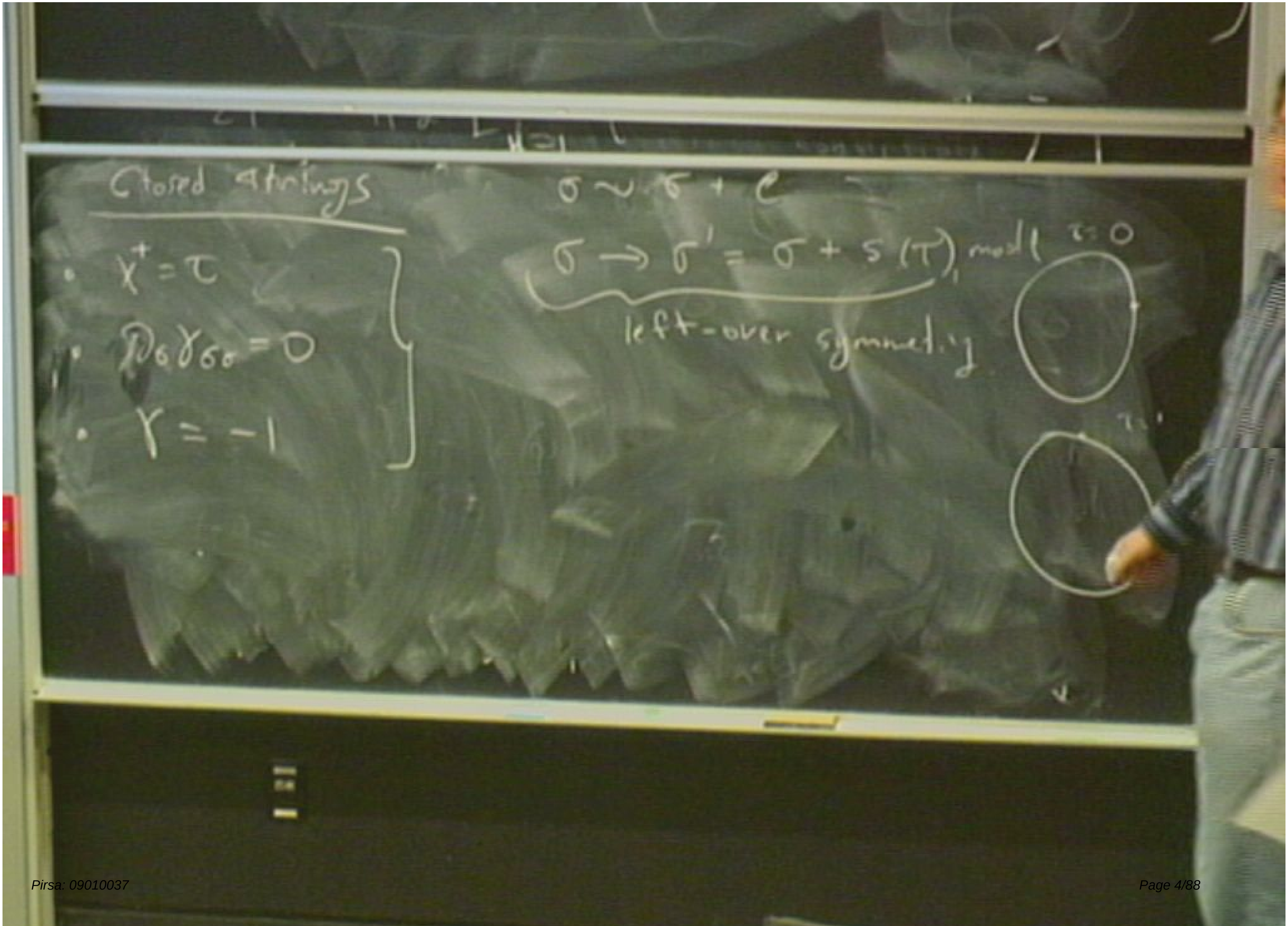
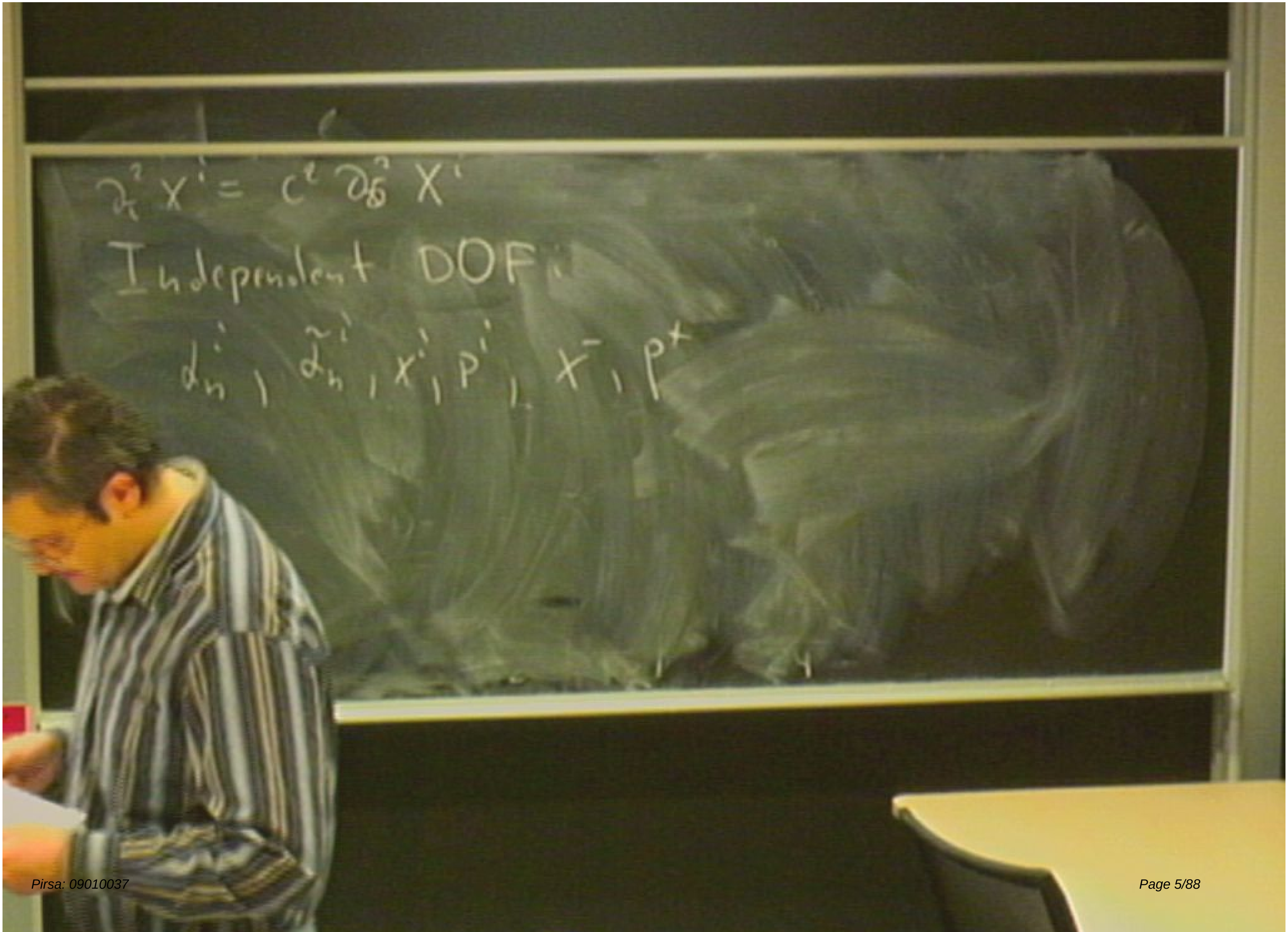
URL: http://pirsa.org/09010037

Abstract: This course provides a thorough introduction to the bosonic string based on the Polyakov path integral and conformal field theory. We introduce central ideas of string theory, the tools of conformal field theory, the Polyakov path integral, and the covariant quantization of the string. We discuss string interactions and cover the tree-level and one loop amplitudes. More advanced topics such as T-duality and D-branes will be taught as part of the course. The course is geared for M.Sc. and Ph.D. students enrolled in Collaborative Ph.D. Program in Theoretical Physics. Required previous course work: Quantum Field Theory (AM516 or equivalent). The course evaluation will be based on regular problem sets that will be handed in during the term. The primary text is the book: 'String theory. Vol. 1: An introduction to the bosonic string. J. Polchinski (Santa Barbara, KITP) . 1998. 402pp. Cambridge, UK: Univ. Pr. (1998) 402 p.' All interested students should contact Alex Buchel at abuchel@uwo.ca as soon as possible.


Closed strings
X+ = τ
∂σ γσσ = 0
γ = −1
σ ~ σ + ℓ
σ → σ' = σ + s(τ)
left-over symmetry


Closed strings
X⁺ = τ
∂σ γστ = 0
γ = −1
σ ∼ σ + ℓ
σ → σ' = σ + s(τ)
left-over symmetry


Closed strings
X+ = τ
∂σ γσσ = 0
γ = −1
σ ~ σ + ℓ
σ → σ' = σ + s(τ), mod ℓ
left-over symmetry
τ=0


$\partial_t^2 X^i = c^2 \partial_\sigma^2 X^i$
Independent DOF:
$\alpha_n^i, \tilde{\alpha}_n^i, x^i, p^i, x^-, p^+$

closed $X^i(\tau, \sigma+\ell) = X^i(\tau,\sigma)$

$$X^i(\tau,\sigma) = x^i + \frac{p^i}{p^+}\tau + i\left(\frac{\alpha'}{2}\right)^{1/2}\left\{\frac{\alpha_n^i}{n} e^{\frac{2\pi i n(\sigma - c\tau)}{\ell}} + \frac{\tilde{\alpha}_n^i}{n} e^{-\frac{2\pi i n}{\ell}(\sigma + c\tau)}\right\}$$

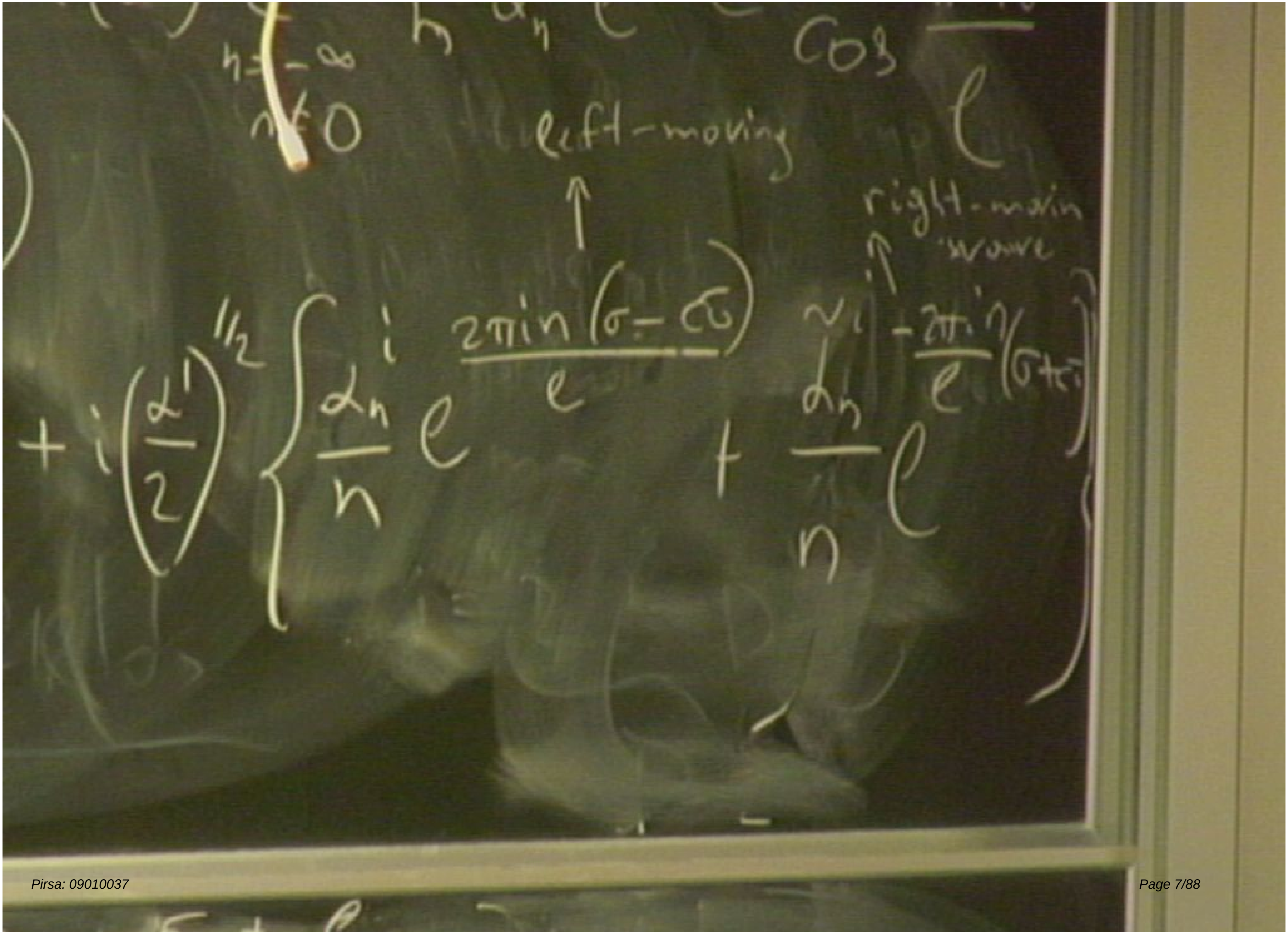
Closed strings

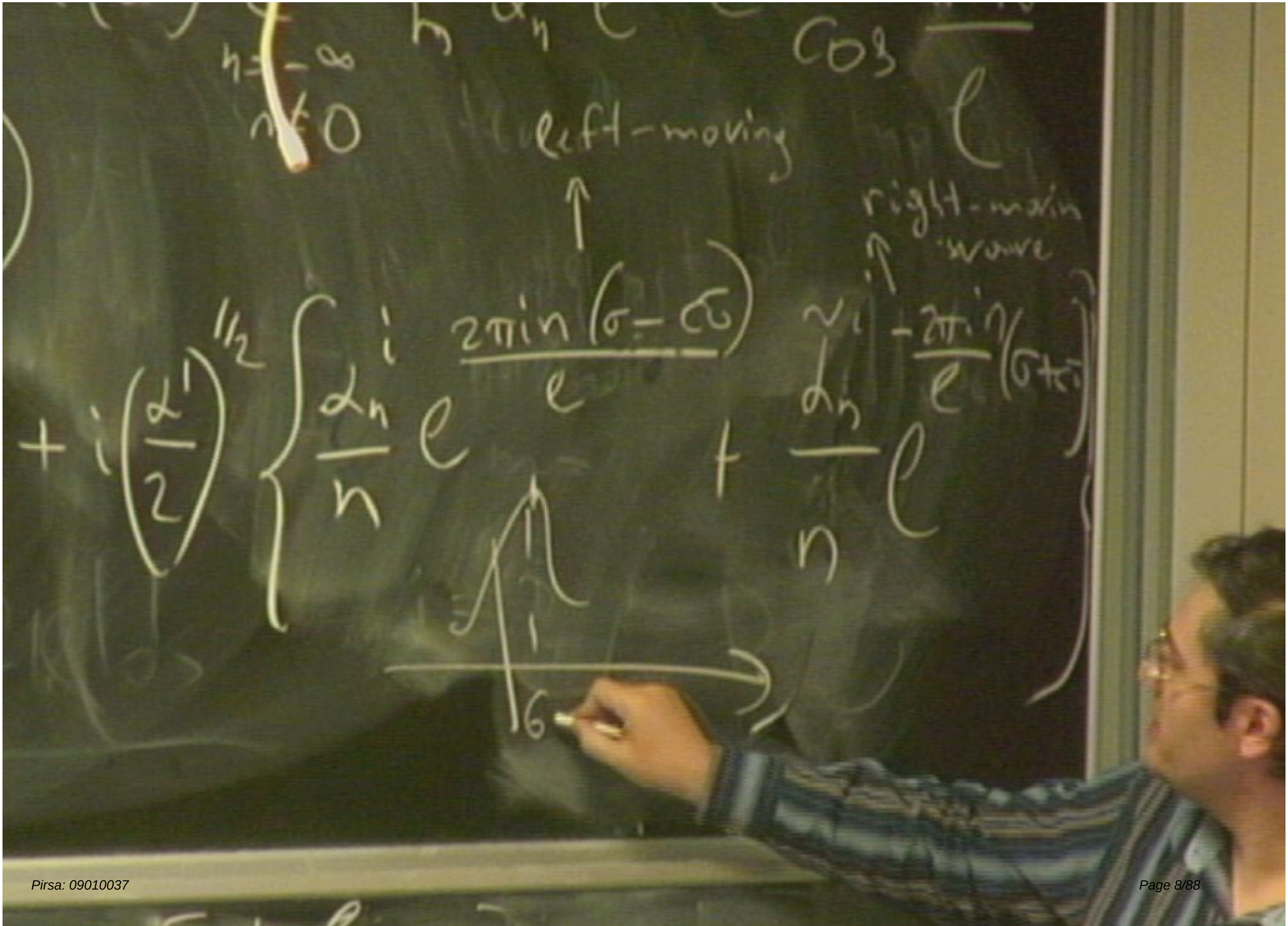
- $X^+ = \tau$
- $\partial_\sigma \gamma_{\sigma\sigma} = 0$
- $\gamma = -1$

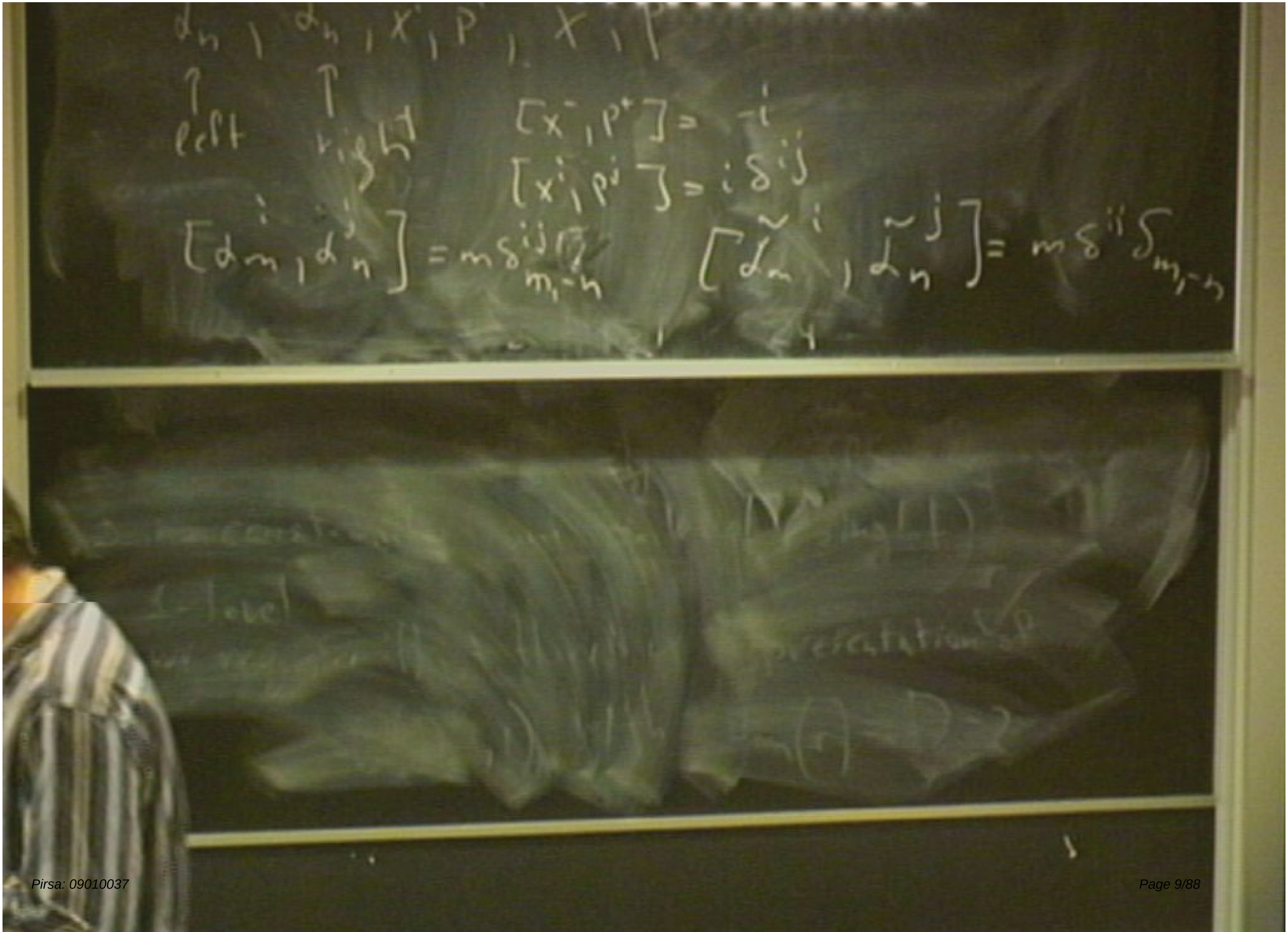
$\sigma \sim \sigma + \ell$

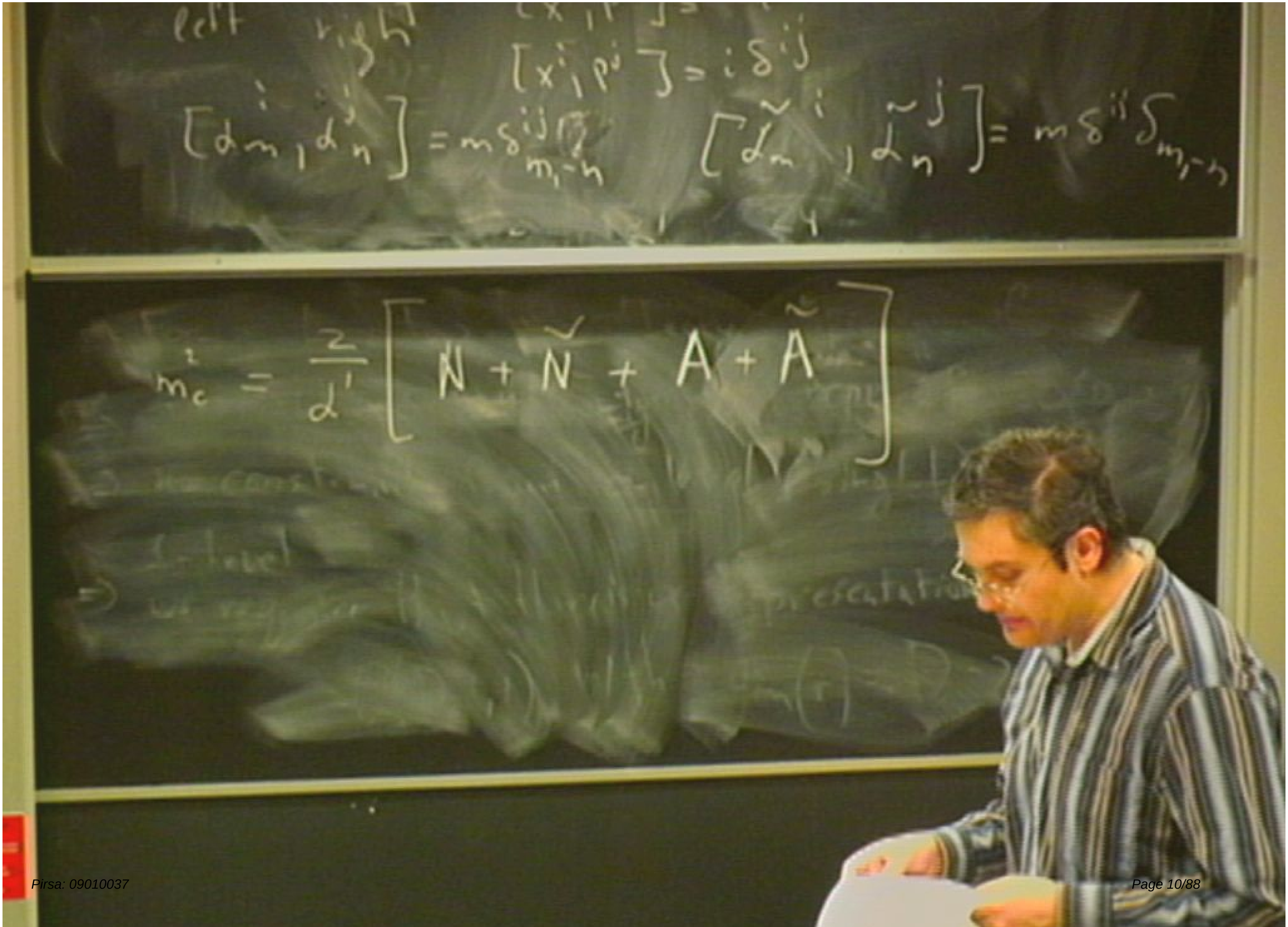
$\sigma \to \sigma' = \sigma + s(\tau)$, mod ℓ

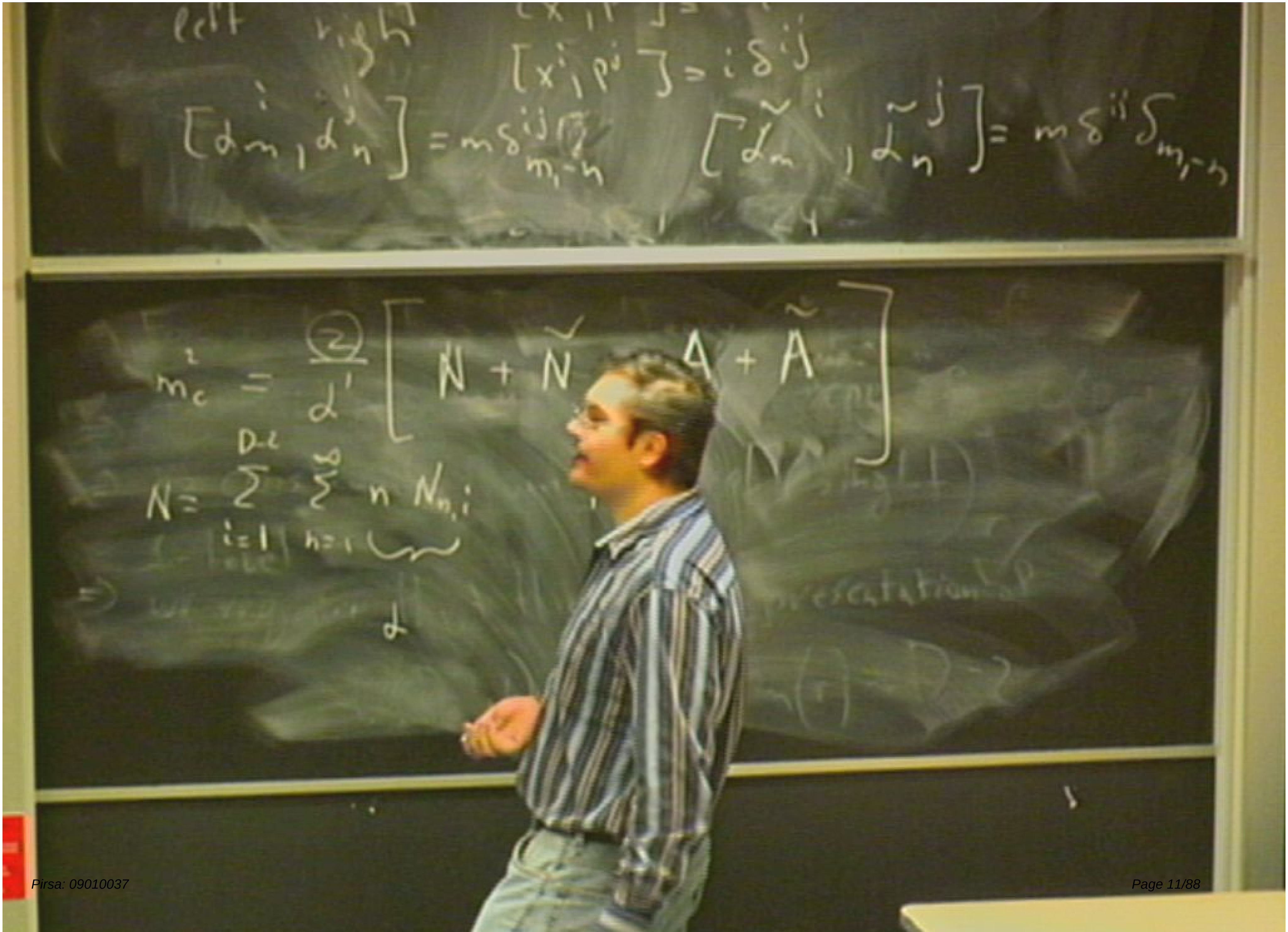
left-over symmetry


left-moving
right-movin
wave


left-moving
right-movin
wave


α_n , α̃_n , x^- , p^+ , x^i , p^i
left right
[x^- , p^+] = -i
[x^i , p^j] = i δ^ij
[α^i_m , α^j_n] = m δ^ij δ_m,-n
[α̃^i_m , α̃^j_n] = m δ^ij δ_m,-n


left right
[x^i, p^j] = i δ^ij
[α_m^i, α_n^j] = m δ^ij δ_{m,-n}
[α̃_m^i, α̃_n^j] = m δ^ij δ_{m,-n}
m_c^2 = 2/α' [N + Ñ + A + Ã]



left right

$$[x^i, p^j] = i\delta^{ij}$$

$$[\alpha_m^i, \alpha_n^j] = m\delta^{ij}\delta_{m,-n} \qquad [\tilde{\alpha}_m^i, \tilde{\alpha}_n^j] = m\delta^{ij}\delta_{m,-n}$$

$$m_c^2 = \frac{2}{\alpha'}\left[N + \tilde{N} + A + \tilde{A}\right]$$

$$N = \sum_{i=1}^{D-2}\sum_{n=1}^{\infty} n \underbrace{N_{n,i}}_{\alpha_{-n}^i \alpha_n^i} \qquad \tilde{N} = \sum_{i=1}^{D-2}\sum_{n=1}^{\infty} n \underbrace{\tilde{N}_{n,i}}_{\tilde{\alpha}_{-n}^i \tilde{\alpha}_{n}^i}$$

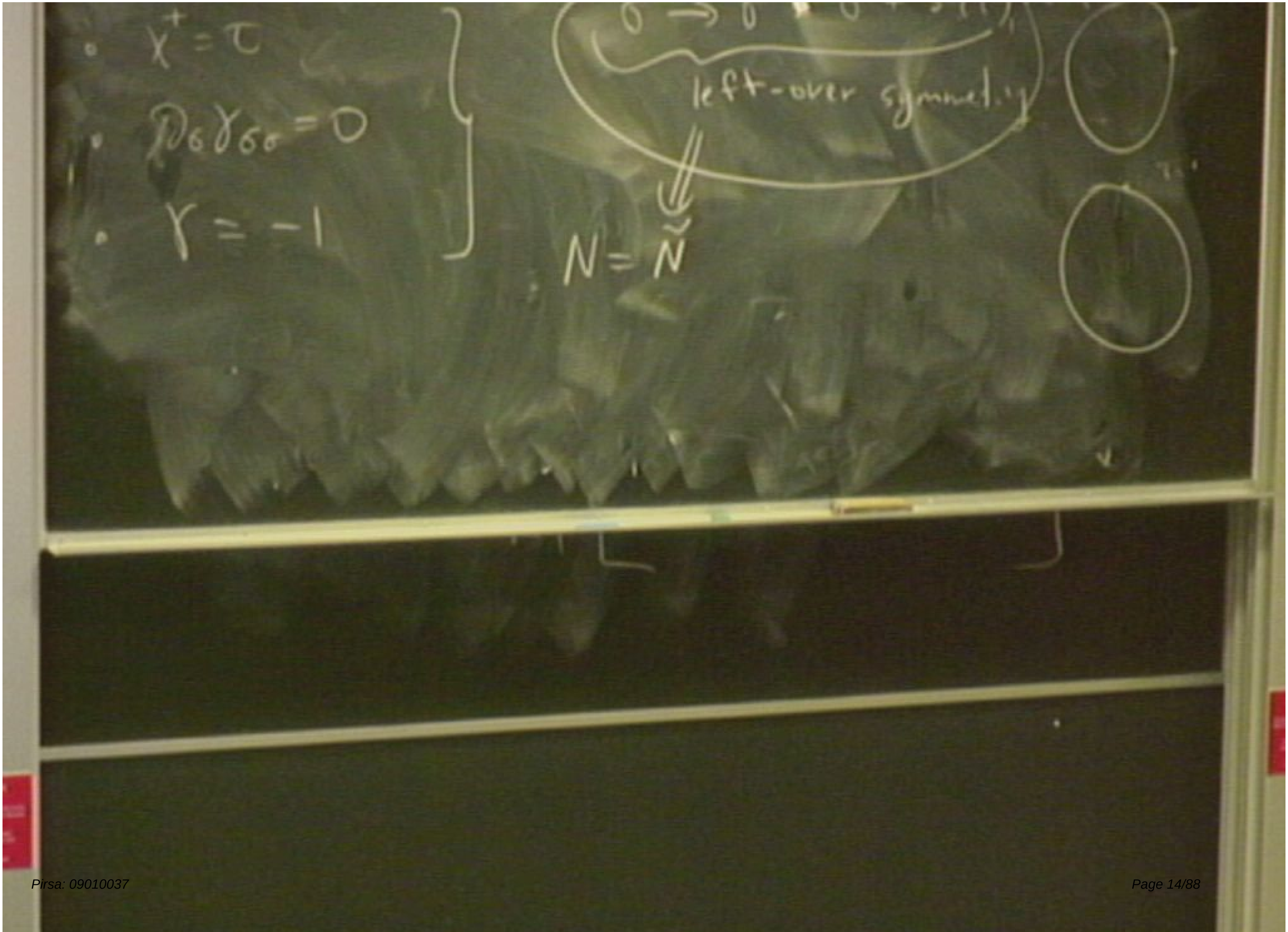
left right

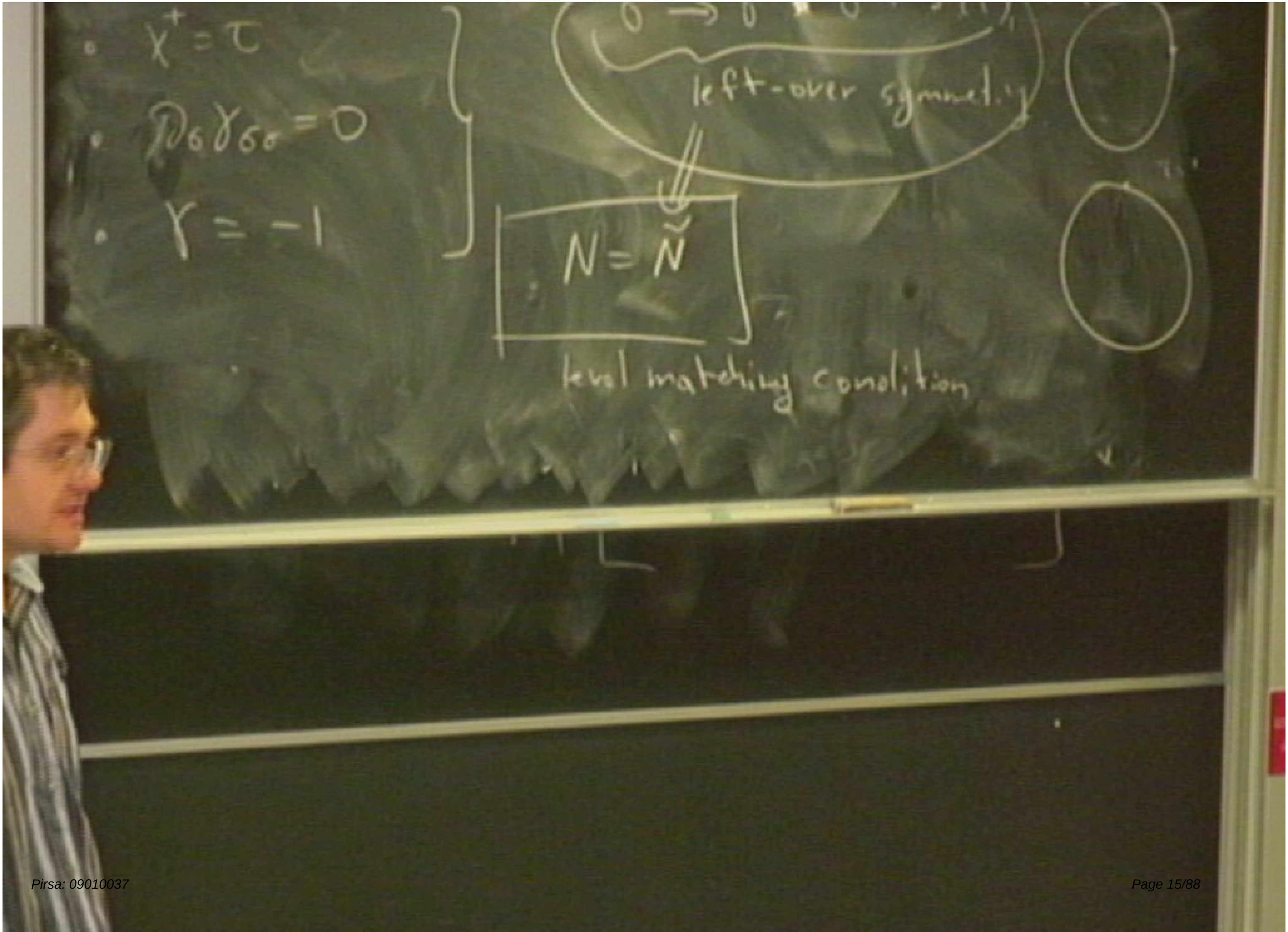
$$[x^i, p^j] = i\delta^{ij}$$

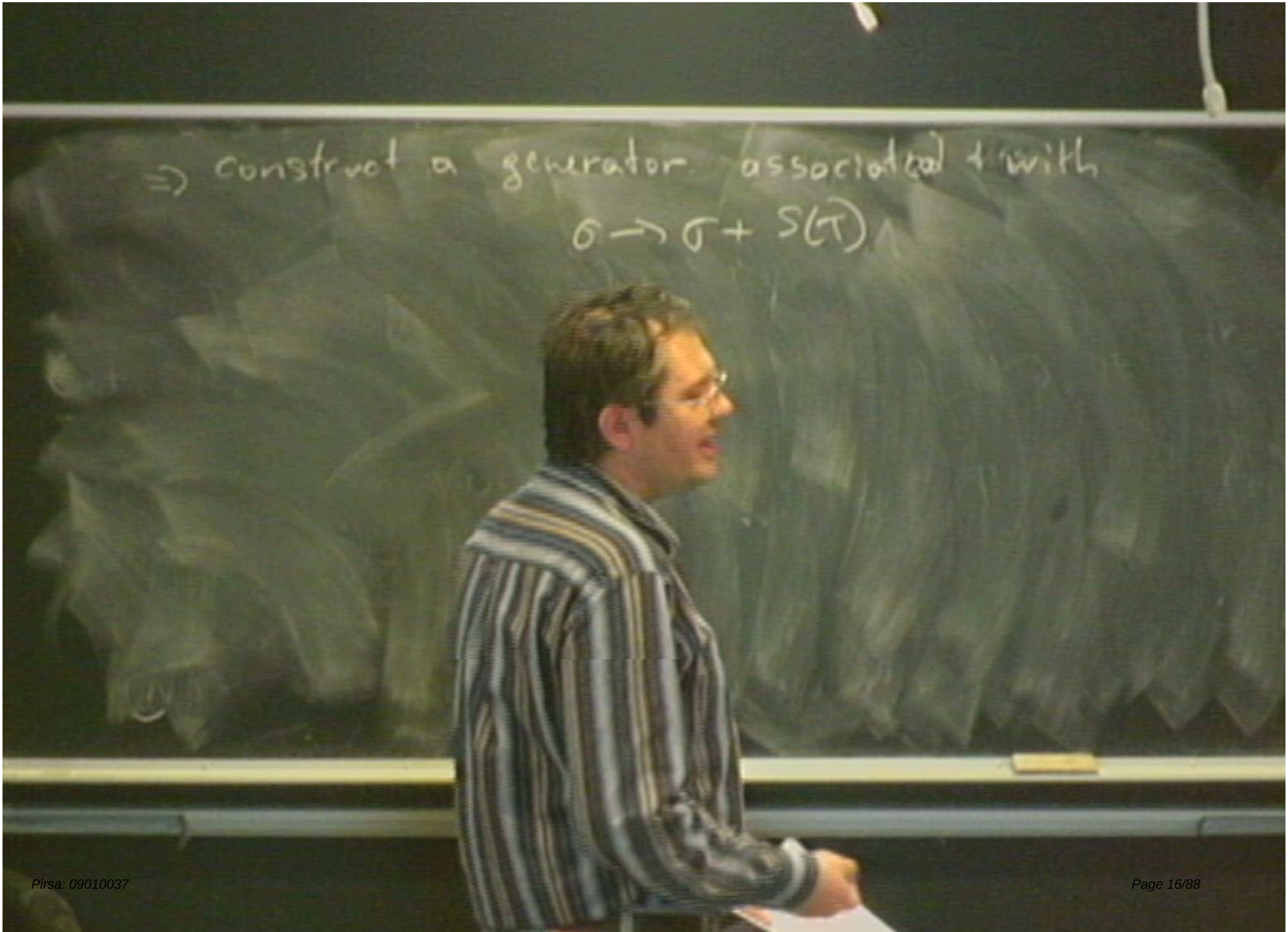
$$[\alpha^i_m, \alpha^j_n] = m\delta^{ij}\delta_{m,-n} \qquad [\tilde{\alpha}^i_m, \tilde{\alpha}^j_n] = m\delta^{ij}\delta_{m,-n}$$

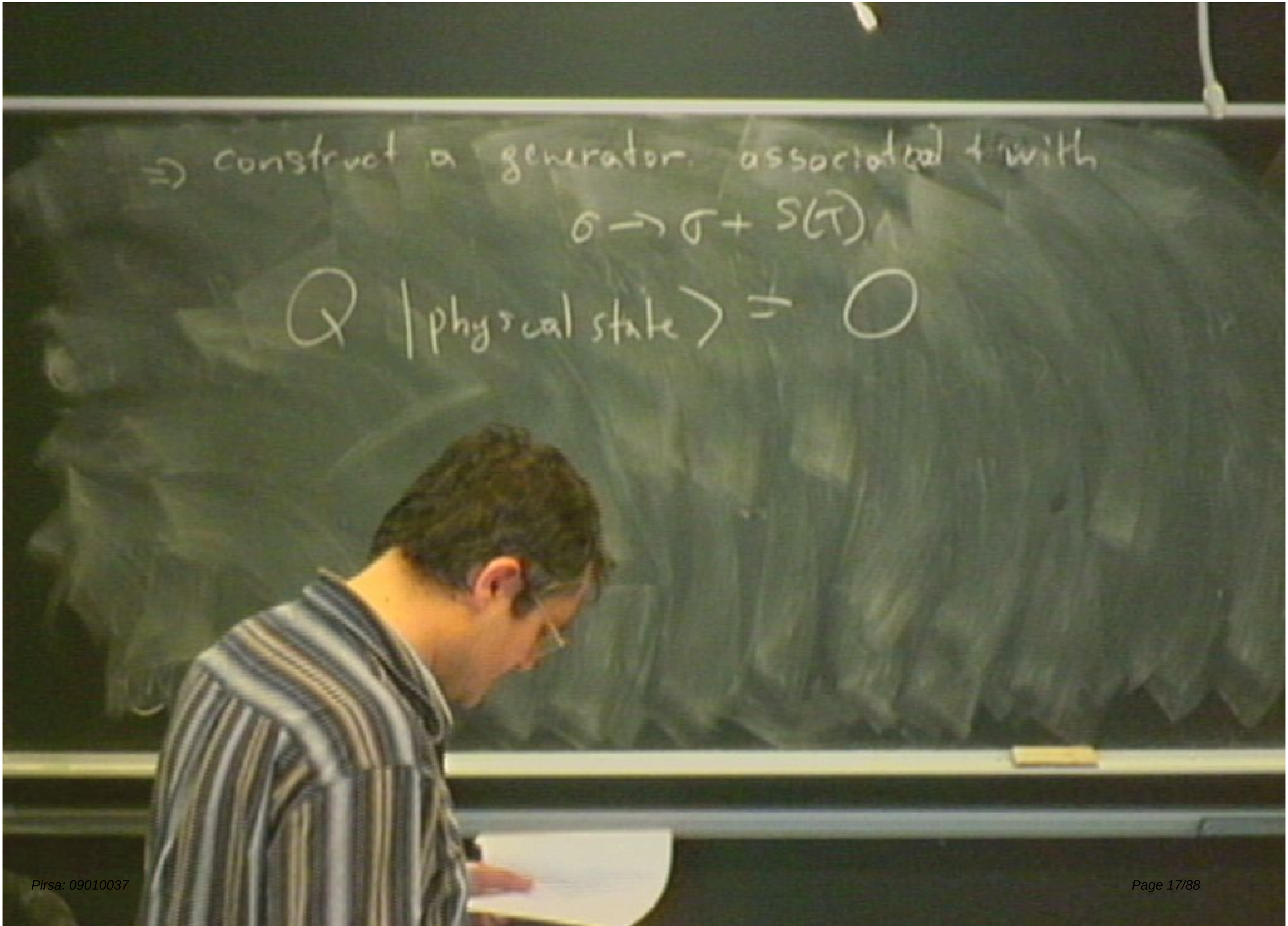
$$m_c^2 = \frac{2}{\alpha'}\left[N + \tilde{N} + A + \tilde{A}\right] \Rightarrow A = \tilde{A} = \frac{2-D}{24}$$

$$N = \sum_{i=1}^{D-2}\sum_{n=1}^{\infty} n \underbrace{N_{n,i}}_{\alpha^i_{-n}\alpha^i_n} \qquad \tilde{N} = \sum_{i=1}^{D-2}\sum_{n=1}^{\infty} n \underbrace{\tilde{N}_{n,i}}_{\tilde{\alpha}^i_{-n}\tilde{\alpha}^i_n}$$

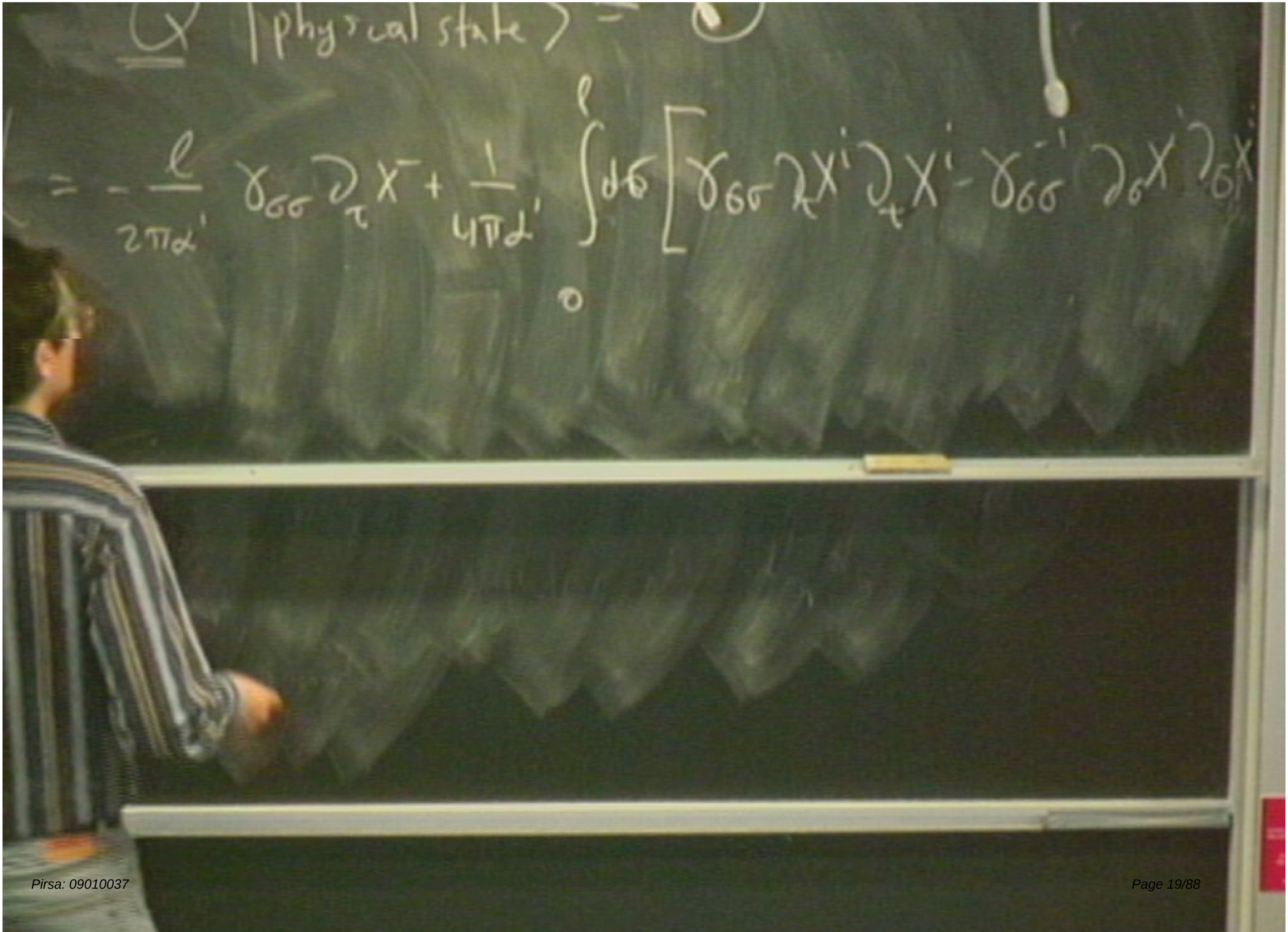

x+ = τ
γ = −1
left-over symmetry
N = Ñ


x⁺ = τ
γ = −1
left-over symmetry
N = Ñ
level matching condition

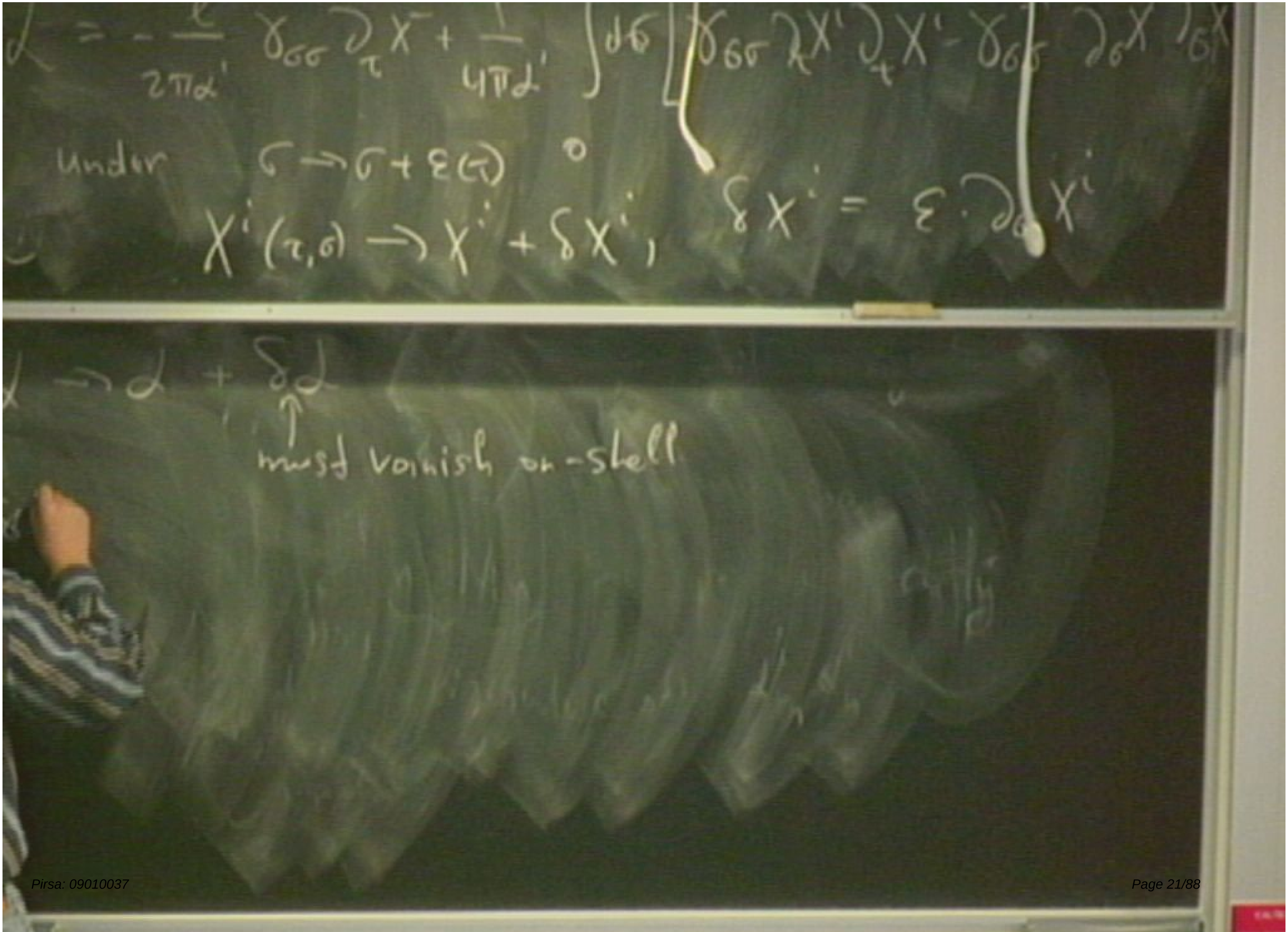

⇒ construct a generator associated with
σ → σ + S(τ)

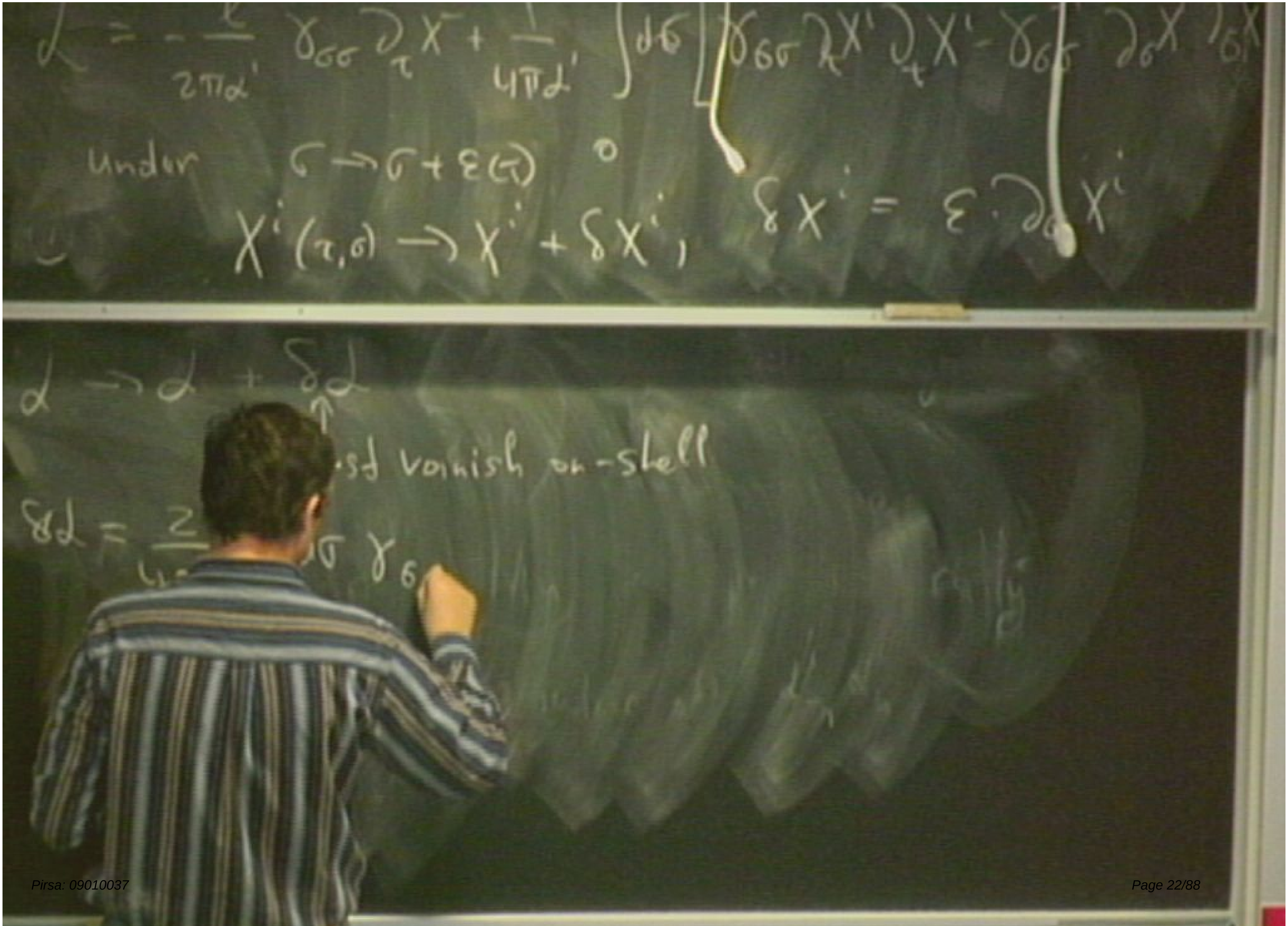

⇒ construct a generator associated with
σ → σ + δ(τ)
Q |physical state> ≐ 0


Q |physical state> = 0
L = - l/(2πα') γσσ ∂τ X⁻ + 1/(4πα') ∫₀^l dσ [γσσ ∂τ Xⁱ ∂τ Xⁱ - γσσ⁻¹ ∂


Q |physical state⟩ = 0
= − ℓ/(2πα') γσσ ∂τ X⁻ + 1/(4πα') ∫₀^ℓ dσ [γσσ ∂τ Xⁱ ∂τ Xⁱ − γσσ⁻¹ ∂σ Xⁱ ∂σ Xⁱ


$Q\,|\text{physical state}\rangle = 0$
$L = -\frac{l}{2\pi\alpha'}\gamma_{\sigma\sigma}\partial_\tau X^- + \frac{1}{4\pi\alpha'}\int_0^l d\sigma\,[\gamma_{\sigma\sigma}\partial_\tau X^i\partial_\tau X^i - \gamma^{\sigma\sigma}\partial_\sigma X^i\partial_\sigma X^i$
under $\sigma \to \sigma + \varepsilon(\tau)$
$X^i(\tau,\sigma) \to X^i + \delta X^i, \quad \delta X^i = \varepsilon\,\partial_\sigma X^i$


under σ → σ + ε(τ)
X^i(τ,σ) → X^i + δX^i , δX^i = ε ∂_σ X^i
must vanish on-shell


under σ → σ + ε(τ)
X^i(τ,σ) → X^i + δX^i , δX^i = ε ∂_σ X^i
vanish on-shell

$$\mathcal{L} = -\frac{\ell}{2\pi\alpha'}\gamma_{\sigma\sigma}\partial_\tau X^- + \frac{1}{4\pi\alpha'}\int d\sigma\left[\gamma_{\sigma\sigma}\partial_\tau X^i\partial_\tau X^i - \gamma_{\sigma\sigma}^{-1}\partial_\sigma X^i\partial_\sigma X^i\right]$$

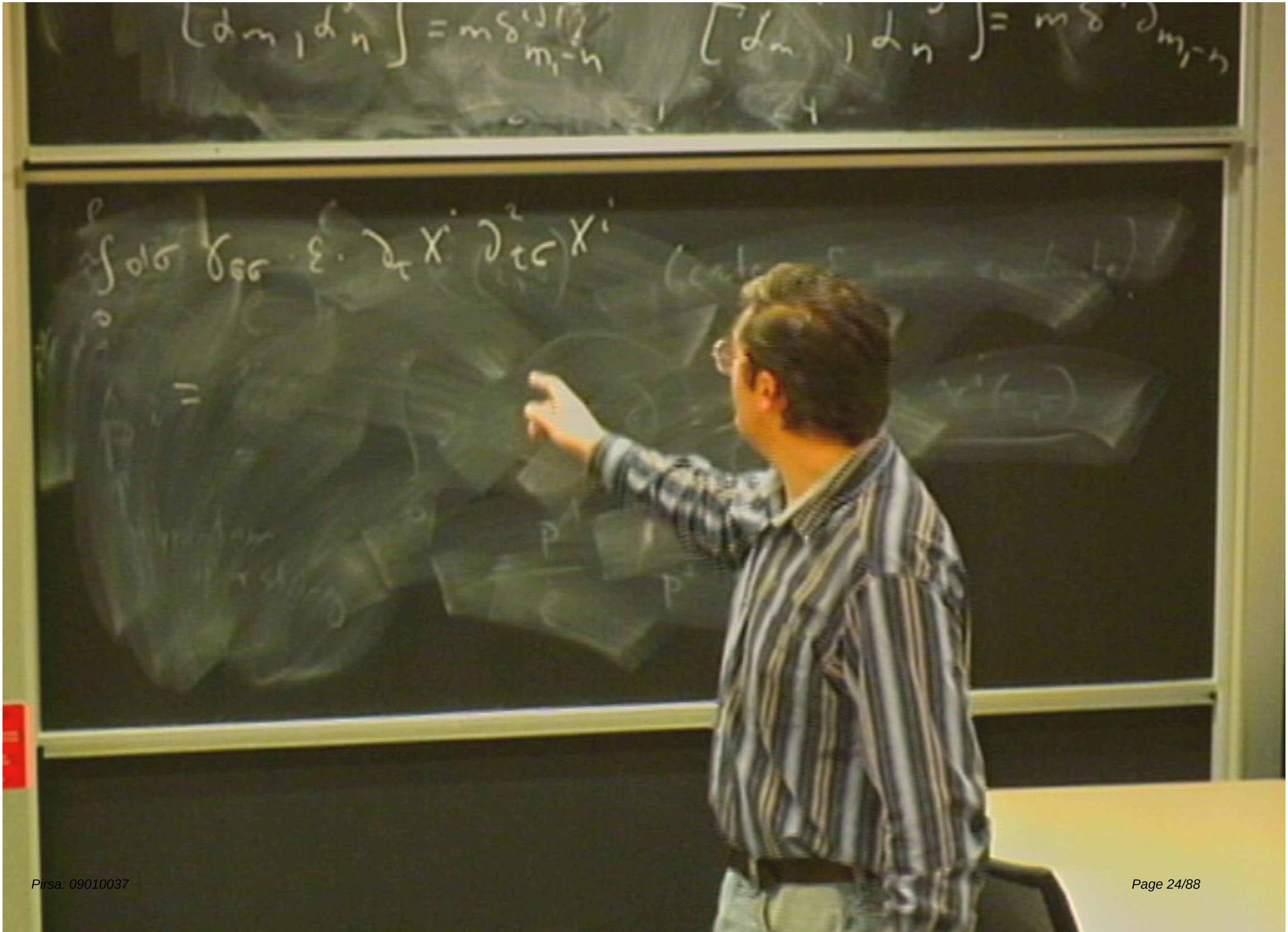
under $\sigma \to \sigma + \varepsilon(\tau)$

$$X^i(\tau,\sigma) \to X^i + \delta X^i, \quad \delta X^i = \varepsilon(\tau)\partial_\sigma X^i$$

$$\mathcal{L} \to \mathcal{L} + \delta\mathcal{L}$$

↑ must vanish on-shell

$$\delta\mathcal{L} = \frac{2}{4\pi\alpha'}\int_0^\ell d\sigma\left[\gamma_{\sigma\sigma}\partial_\tau X^i\partial^2_{\tau\sigma}X^i\cdot\varepsilon + \gamma_{\sigma\sigma}\partial_\tau X^i\partial_\tau\varepsilon\cdot\partial_\sigma X^i - \gamma_{\sigma\sigma}^{-1}\partial_\sigma X^i\partial^2_\sigma X^i\,\varepsilon\right]$$



$[\alpha_m, \alpha_n] = m\delta^{ij}\delta_{m,-n}$ $\qquad [\alpha_m, \alpha_n] = m\delta\,\delta_{m,-n}$

$$\int_0^\ell d\sigma\, \gamma_{\sigma\sigma}\, \epsilon\, \partial_\tau X^i\, \partial^2_{\tau\sigma} X^i$$

(center of ...)

$$= \gamma_{\sigma\sigma}\, \epsilon \int_0^\ell d\sigma\, \tfrac{1}{2}\, \partial_\sigma \left[\partial_\tau X^i\, \partial_\tau X^i \right]$$

$$[\alpha_m, \alpha_n^j] = m\delta^{ij}\delta_{m,-n} \qquad [\tilde{\alpha}_m, \tilde{\alpha}_n] = m\delta^{ij}\delta_{m,-n}$$

$$= \gamma_{\sigma\sigma}\,\varepsilon \int_0^{\ell} d\sigma\, \frac{1}{2}\, \partial_\sigma\left[\partial_\tau X^i\, \partial_\tau X^i\right] = 0$$

$$\gamma_{\sigma\sigma}^{-1}\,\varepsilon \int_0^{\ell} d\sigma\, \frac{1}{2}\, \partial_\sigma\left[\partial_\sigma X^i\, \partial_\sigma X^i\right]$$

$$\delta L = \frac{1}{2\pi\alpha'} \int_0 d\sigma \, \gamma_{\sigma\sigma} \, \partial_\tau \varepsilon \, \partial_\tau X^i \, \partial_\sigma X^i$$

$$\delta S_p =$$

$$\delta \mathcal{L} = \frac{1}{2\pi\alpha'} \int_0 d\sigma \, \gamma_{\sigma\sigma} \, \partial_\tau \varepsilon \, \partial_\tau X^i \, \partial_\sigma X^i$$

$$\delta S_p = \int d\tau \, \delta \mathcal{L} = \int d\tau \; \varepsilon \, \partial_\tau Q$$

$$Q = -\frac{1}{2\pi\alpha'}$$

$$\delta \mathcal{L} = \frac{1}{2\pi\alpha'} \int_0^{\ell} d\sigma \, \gamma_{\sigma\sigma} \, \partial_\tau \varepsilon \, \partial_\tau X^i \partial_\sigma X^i$$

$$\delta S_p = \int d\tau \, \delta \mathcal{L} = \int d\tau \, \varepsilon \, \partial_\tau Q$$

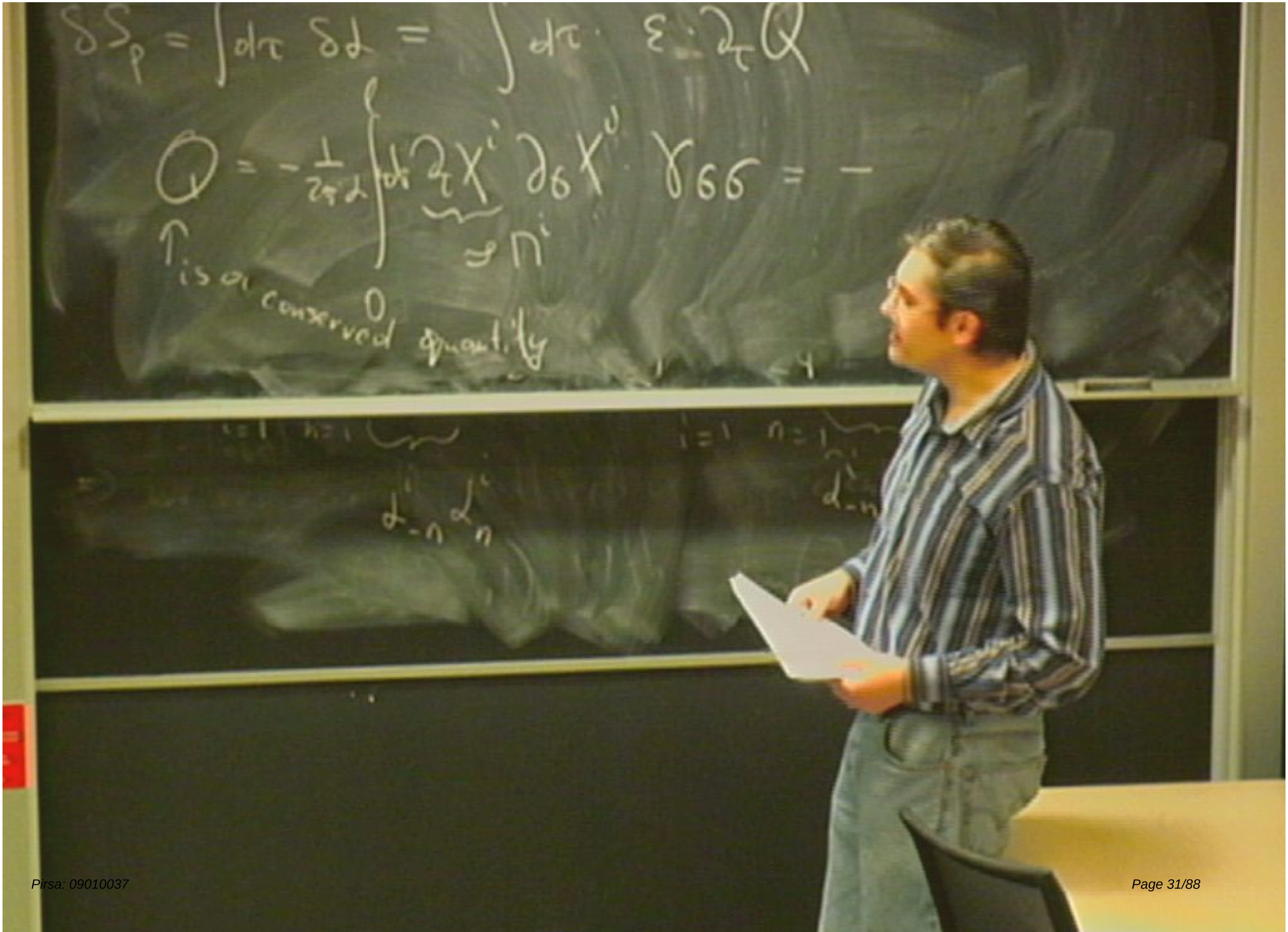
$$Q = -\frac{1}{2\pi\alpha'} \int_0^{\ell} d\sigma \, \partial_\tau X^i \partial_\sigma X^i \, \gamma_{\sigma\sigma}$$

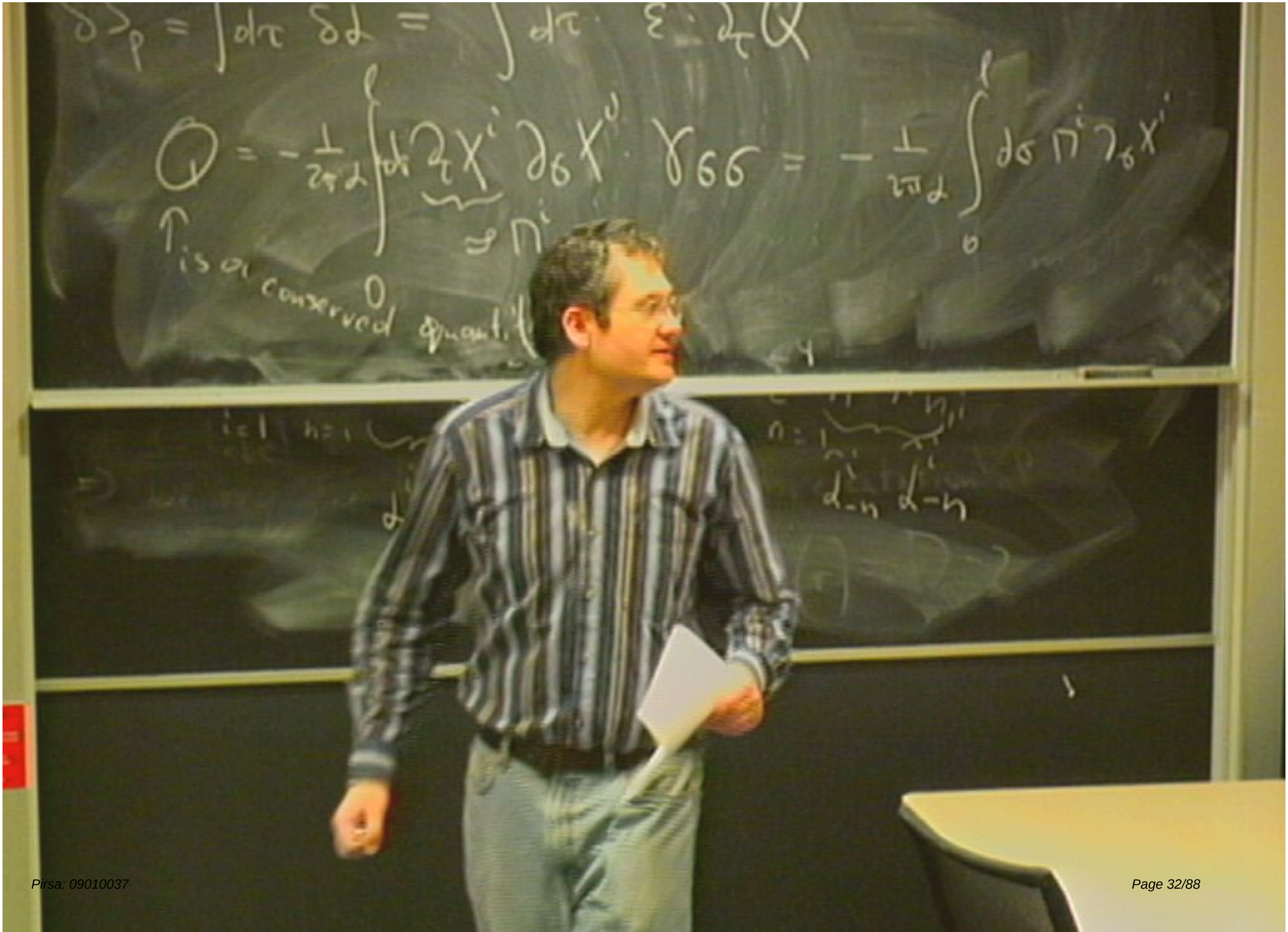
$$\delta S_p = \int d\tau \, \delta L = \int d\tau \cdot \varepsilon \, \partial_\tau Q$$

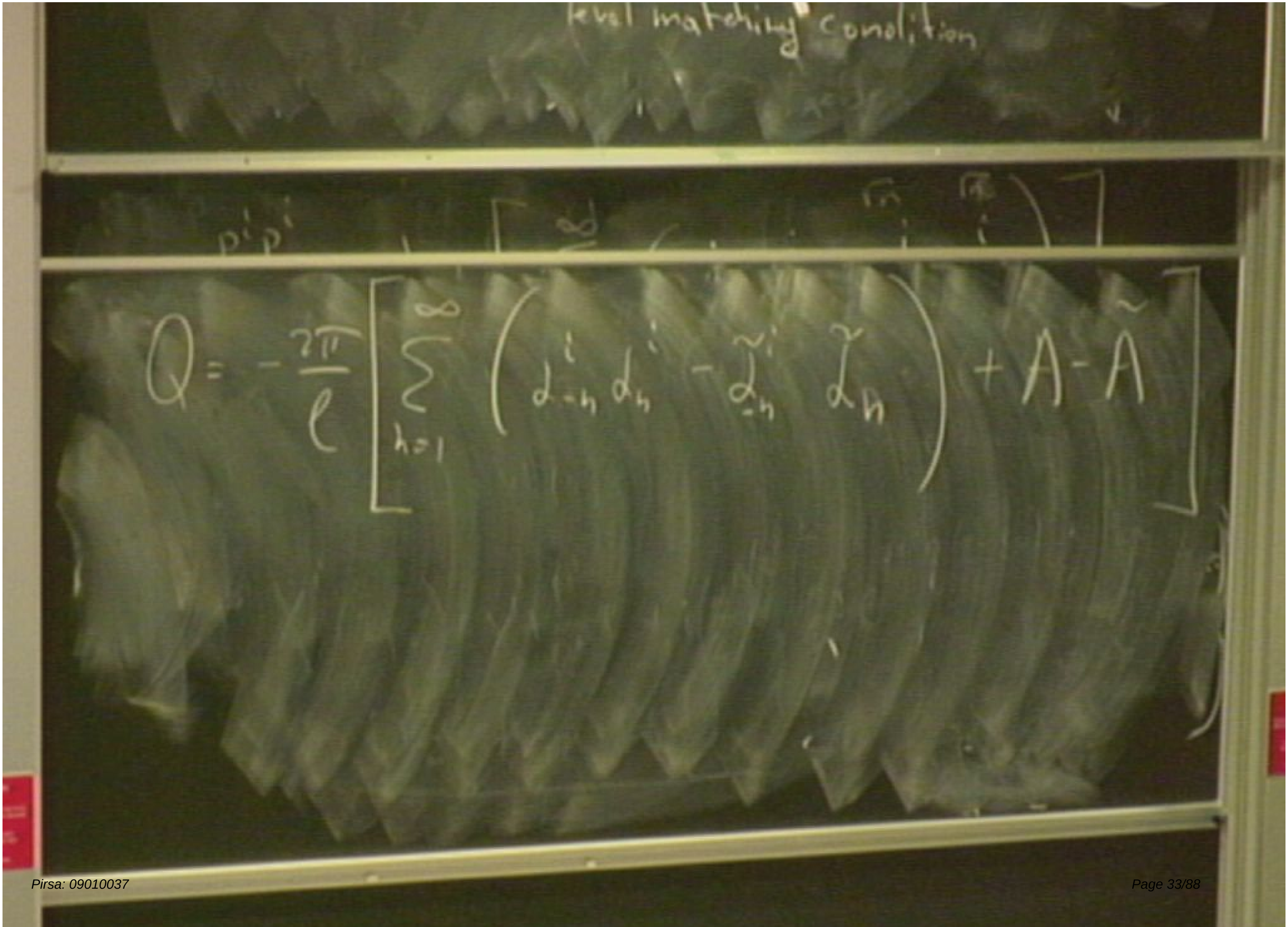
$$Q = -\frac{1}{2\pi\alpha} \int_0^\ell d\sigma \, \partial_\tau X^i \, \partial_\sigma X^i \, \gamma_{\sigma\sigma}$$

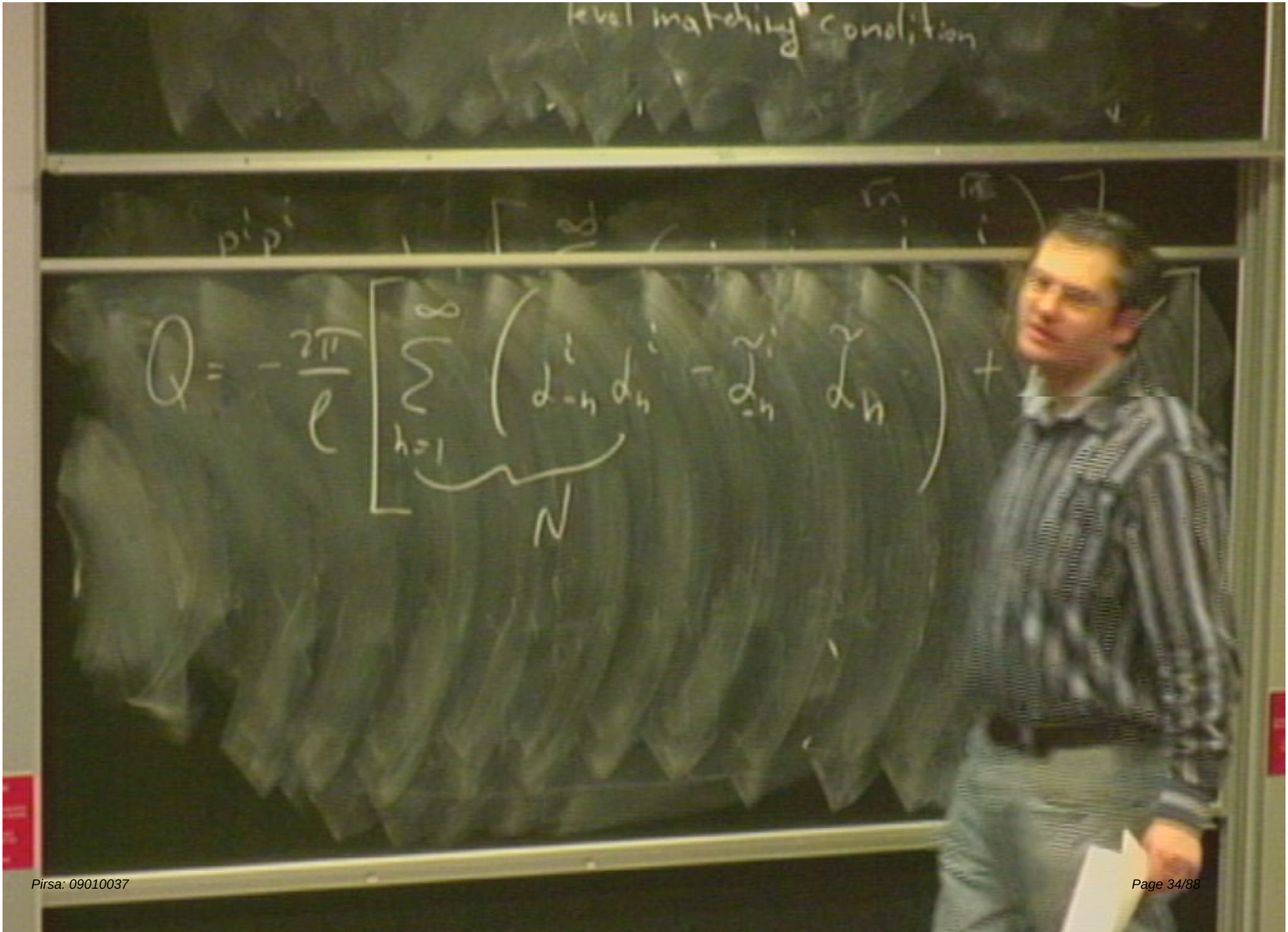
↑ is a conserved quantity

$\alpha^i_{-n} \alpha^i_n$ $\tilde{\alpha}^i_{-n} \alpha^i_{-n}$


δS_p = ∫dτ δL = ∫dτ · ε · ∂_τ Q
Q = − 1/(2πα') ∫dσ ∂_τ X^i ∂_σ X^i · γ_σσ = −
is a conserved quantity


is a conserved quantity

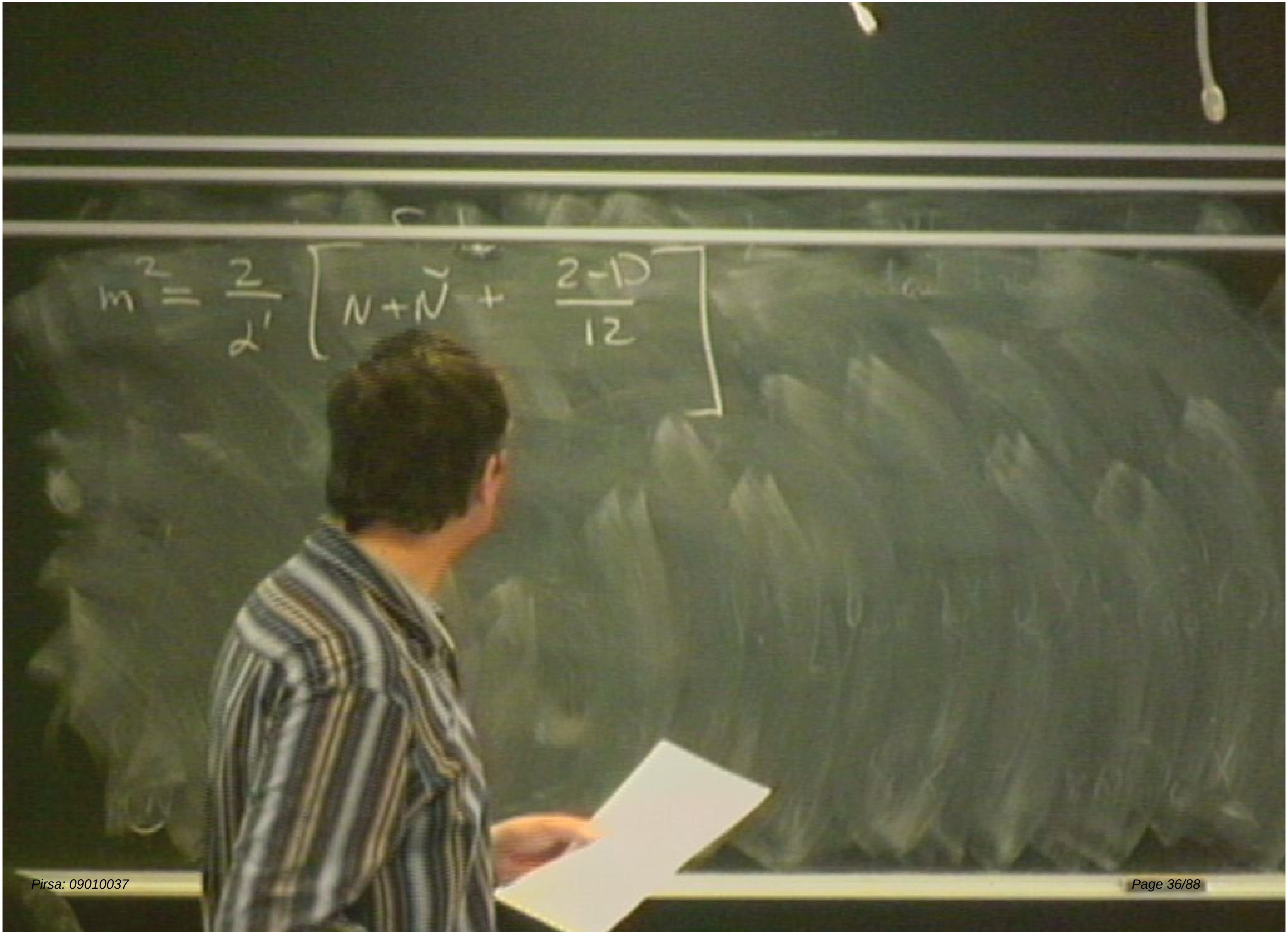

level matching condition
$Q = -\frac{2\pi}{\ell}\left[\sum_{n=1}^{\infty}\left(\alpha^i_{-n}\alpha^i_n - \tilde{\alpha}^i_{-n}\tilde{\alpha}^i_n\right) + A - \tilde{A}\right]$


level matching condition
$Q = -\frac{2\pi}{\ell}\left[\sum_{n=1}^{\infty}\left(\alpha^i_{-n}\alpha^i_n - \tilde{\alpha}^i_{-n}\tilde{\alpha}^i_n\right) + \right.$
N

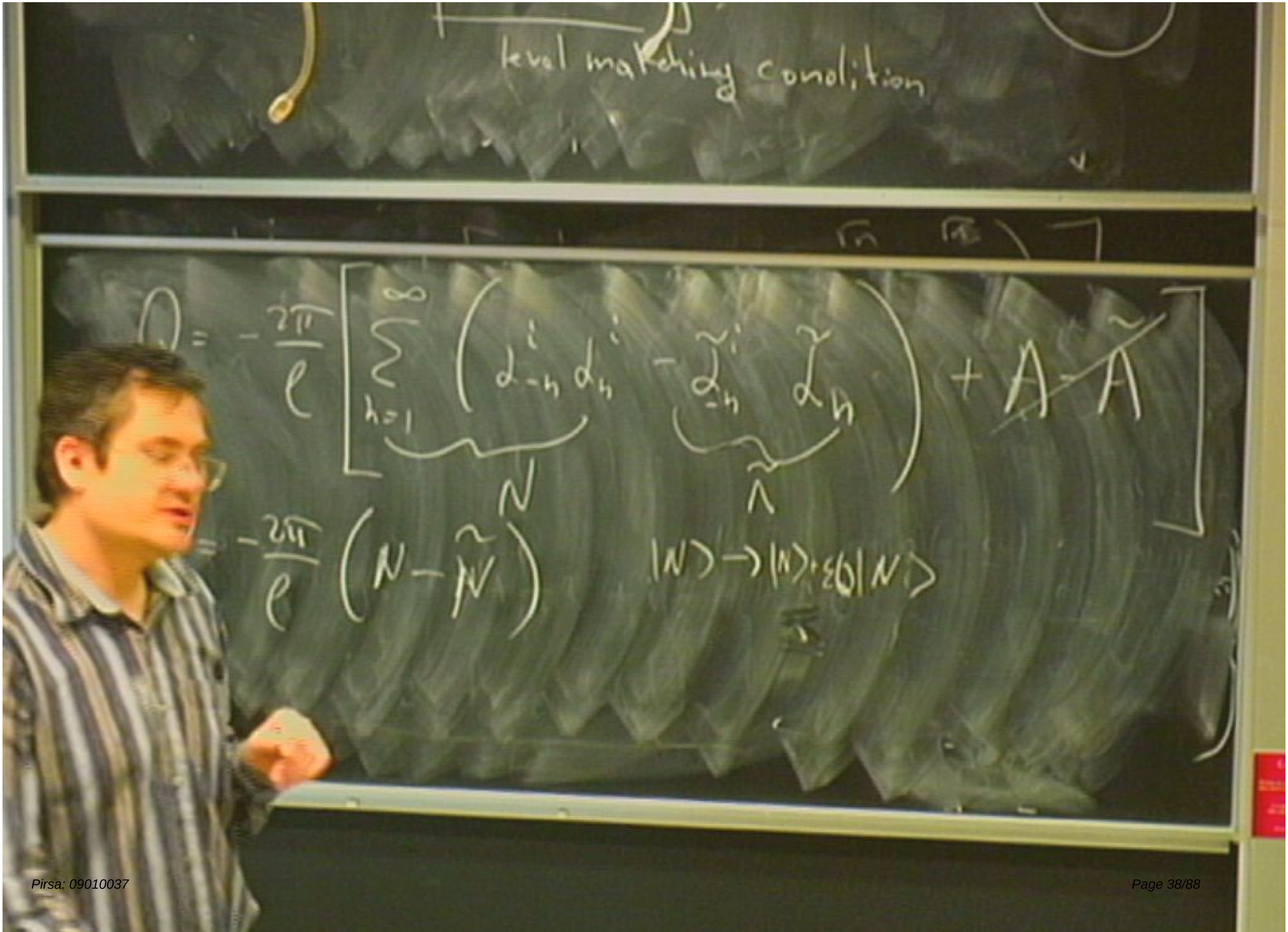
level matching condition

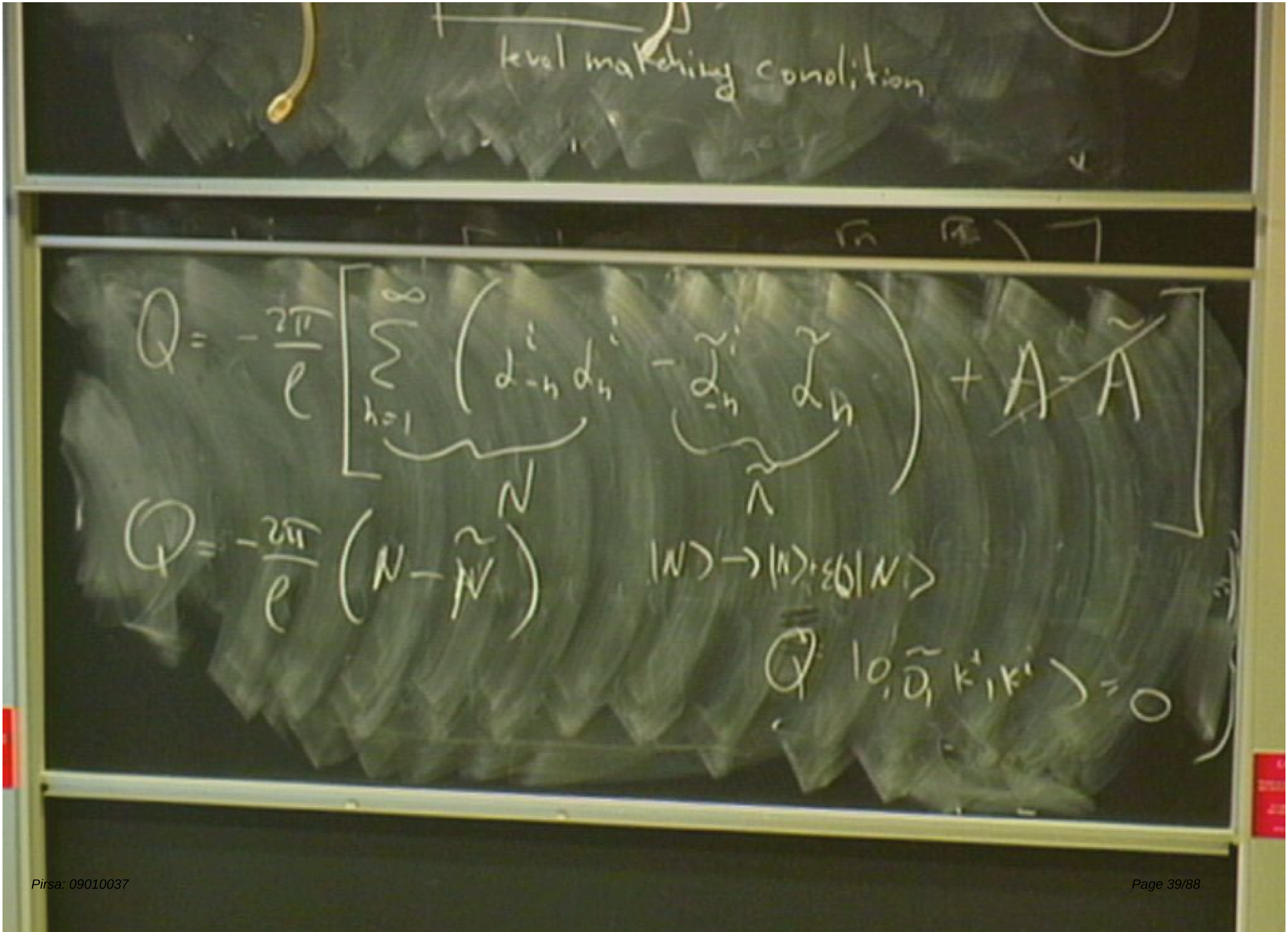
$$Q = -\frac{2\pi}{\ell}\left[\underbrace{\sum_{n=1}^{\infty}\alpha^i_{-n}\alpha^i_n}_{N} - \underbrace{\tilde{\alpha}^i_{-n}\tilde{\alpha}^i_n}_{\tilde{N}} + A - \tilde{A}\right]$$

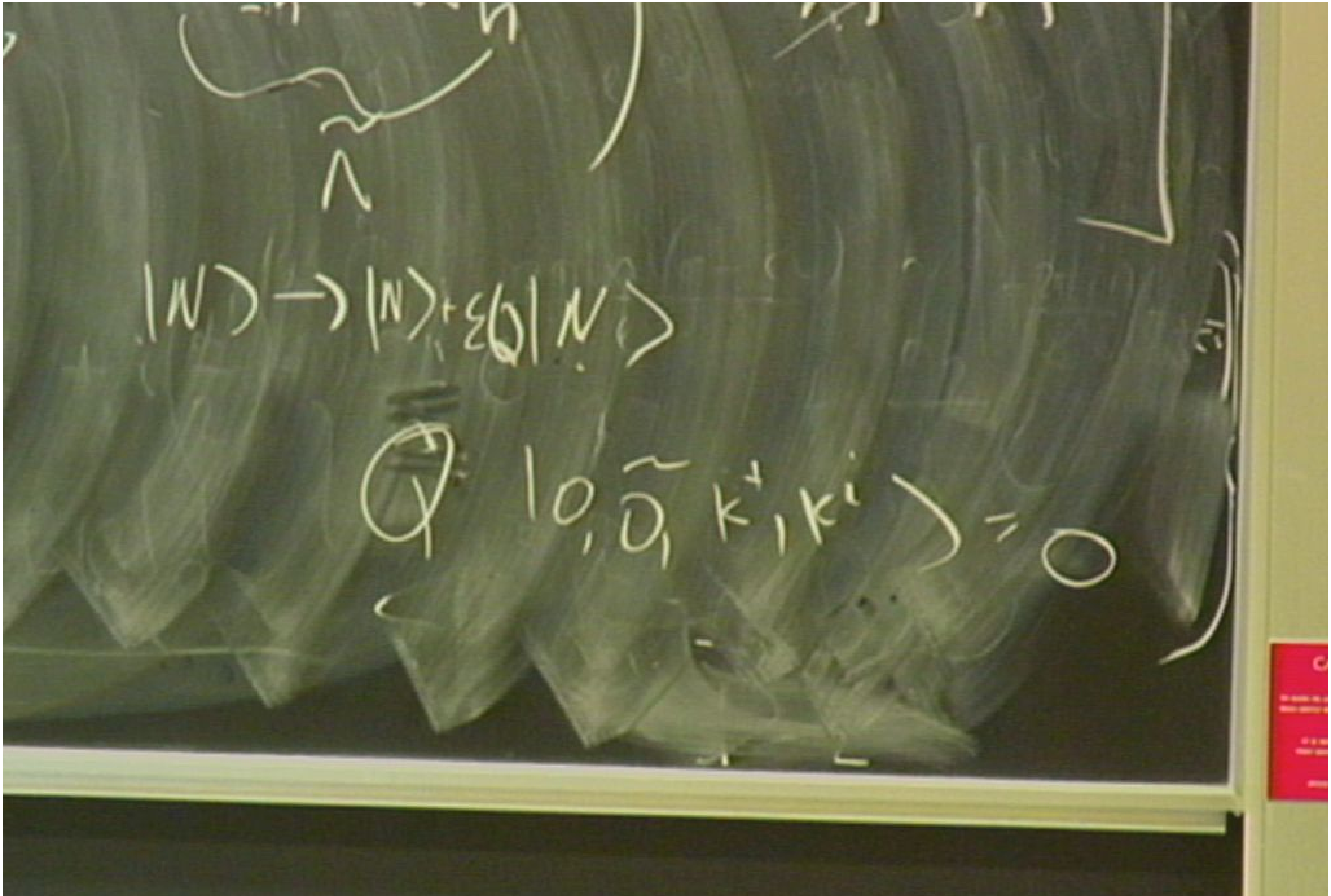
$$Q = -\frac{2\pi}{\ell}\left(N - \tilde{N}\right)$$

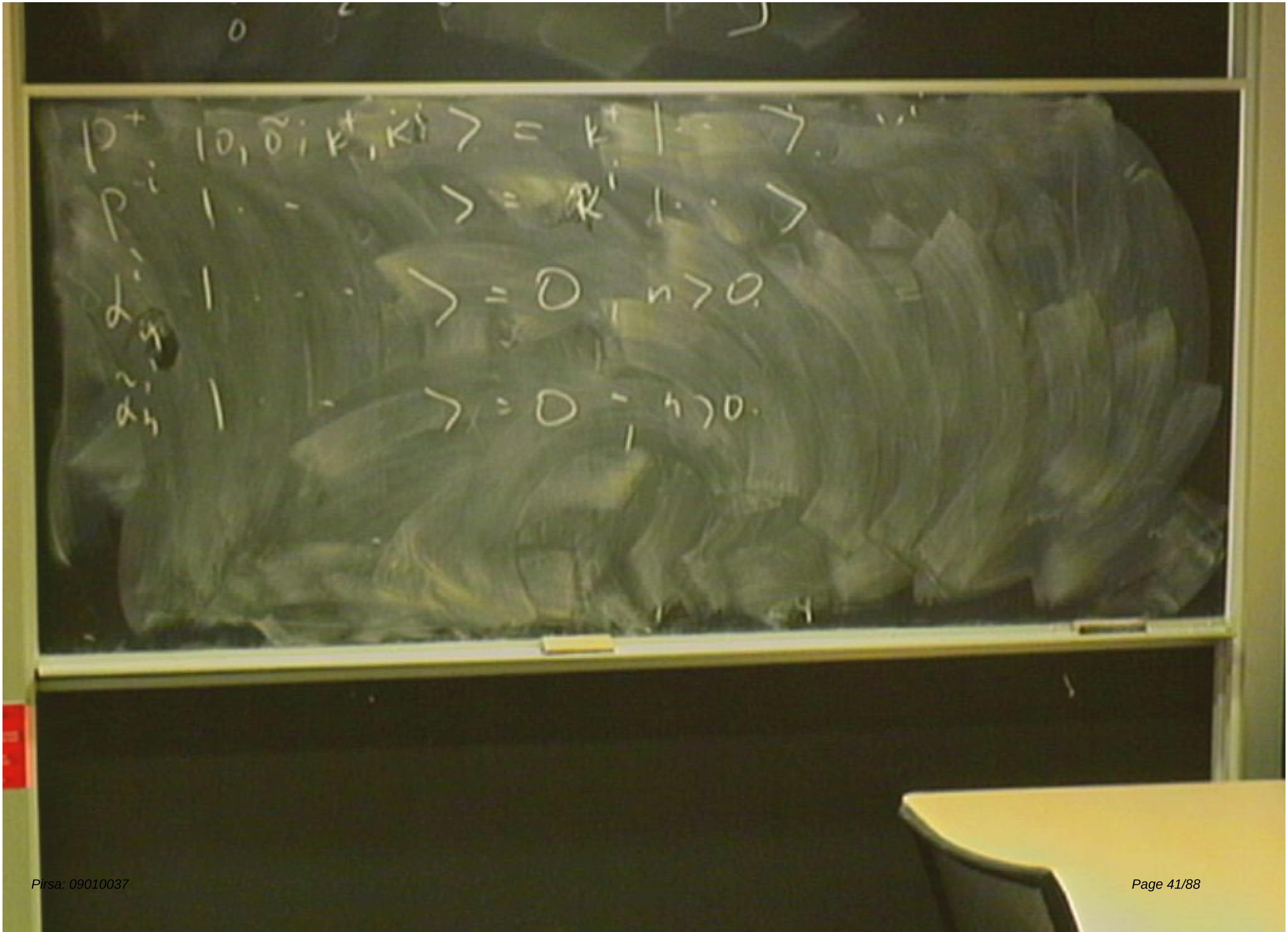

$m^2 = \frac{2}{\alpha'}\left[N + \tilde{N} + \frac{2-D}{12}\right]$

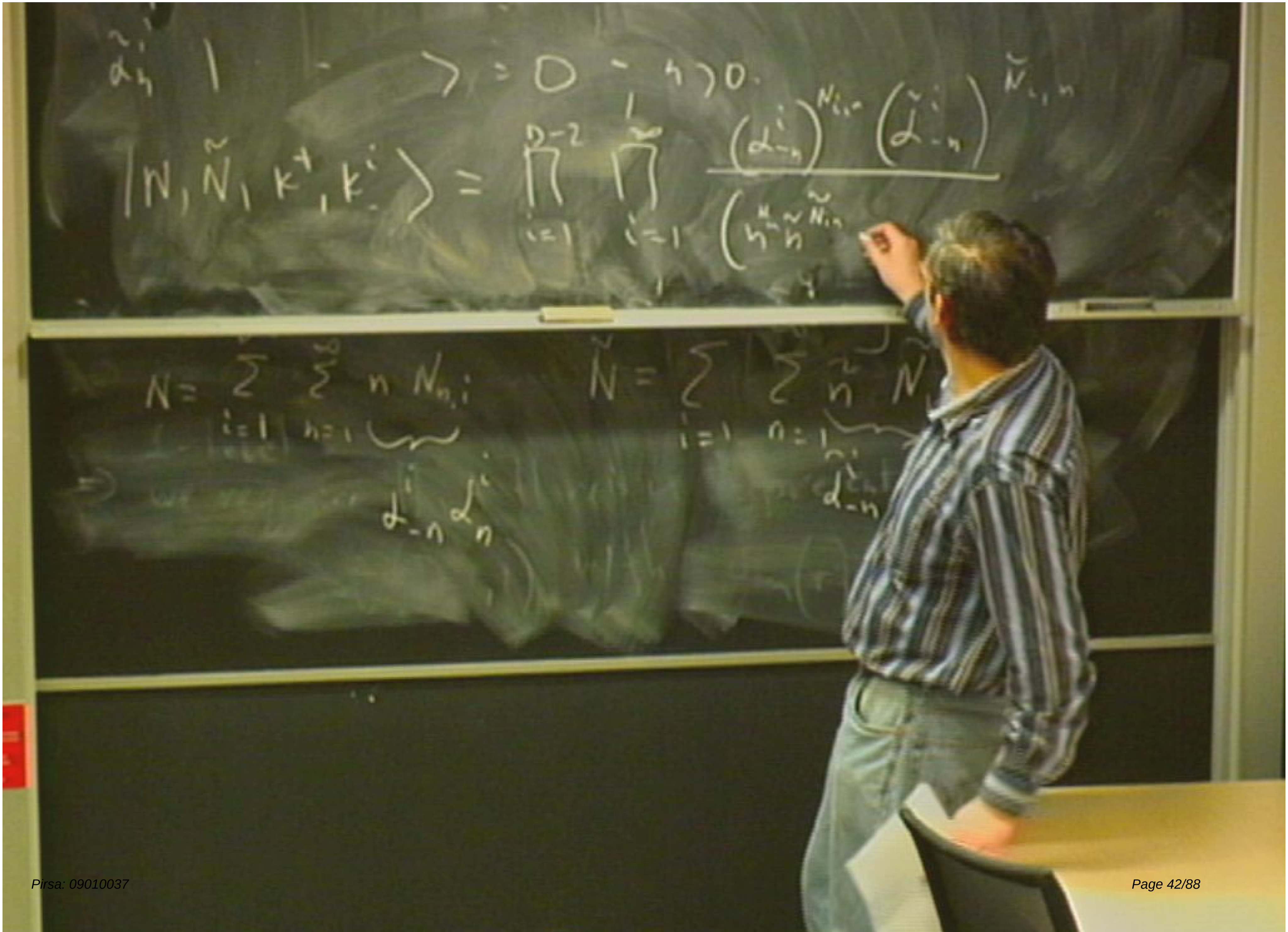
$$m^2 = \frac{2}{\alpha'}\left[N + \tilde{N} + \frac{2-D}{12}\right], \quad N = \tilde{N}$$

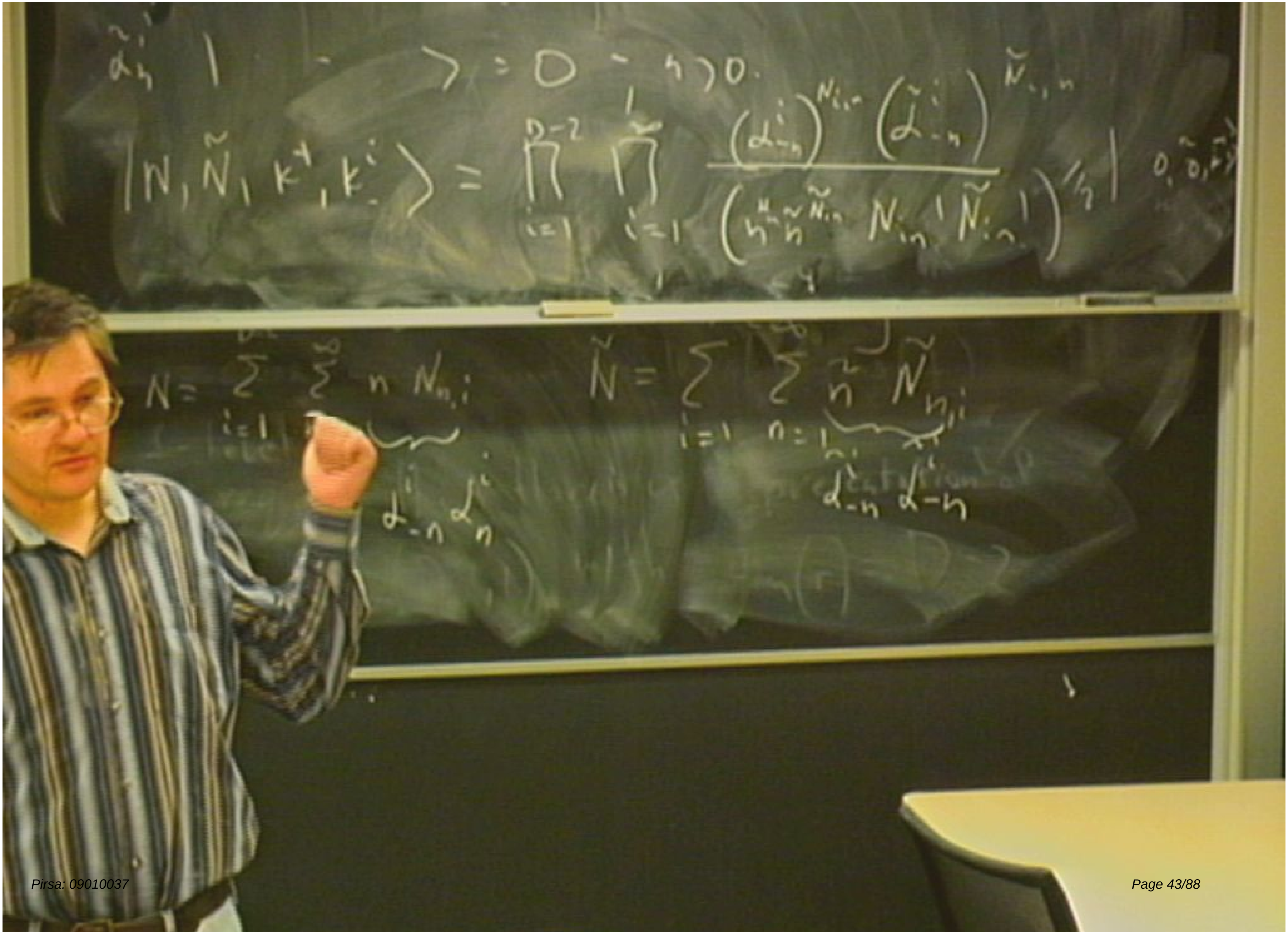

level matching condition
$0 = -\frac{2\pi}{\ell}\left[\sum_{n=1}^{\infty}\left(\alpha^i_{-n}\alpha^i_n - \tilde{\alpha}^i_{-n}\tilde{\alpha}^i_n\right) + A - \tilde{A}\right]$
N
$\tilde{N}$
$= -\frac{2\pi}{\ell}\left(N - \tilde{N}\right)$
$|N\rangle \to |N\rangle + \varepsilon Q |N\rangle$


level matching condition
$Q = -\frac{2\pi}{\ell}\left[\sum_{n=1}^{\infty}\left(\alpha^i_{-n}\alpha^i_n - \tilde{\alpha}^i_{-n}\tilde{\alpha}^i_n\right) + A - \tilde{A}\right]$
N
$\tilde{N}$
$Q = -\frac{2\pi}{\ell}\left(N - \tilde{N}\right)$
$|N\rangle \to |N\rangle + \varepsilon Q|N\rangle$
$Q|0, \tilde{0}, k^+, k^i\rangle = 0$


|N⟩ → |N⟩ + εQ |N⟩
Q |0, 0̃, k⁺, k⁻⟩ = 0


p^+ |0, 0̃; k^+, k^i > = k^+ | . . >
p^i | . . > = k^i | . . >
α^i_n | . . . > = 0, n > 0
α̃^i_n | . . > = 0, n > 0.


|N, Ñ, k+, ki⟩ =
N = Σ Σ n N_n,i
i=1 n=1
Ñ = Σ Σ ñ Ñ
i=1 n=1



$$m^2 = \frac{2}{\alpha'}\left[N + \tilde{N} + \frac{2-D}{12}\right], \quad N = \tilde{N}$$

$\Rightarrow$ <u>0-level</u>

$$N = \tilde{N} = 0$$

$$m^2 = \frac{2-D}{6\alpha'}$$

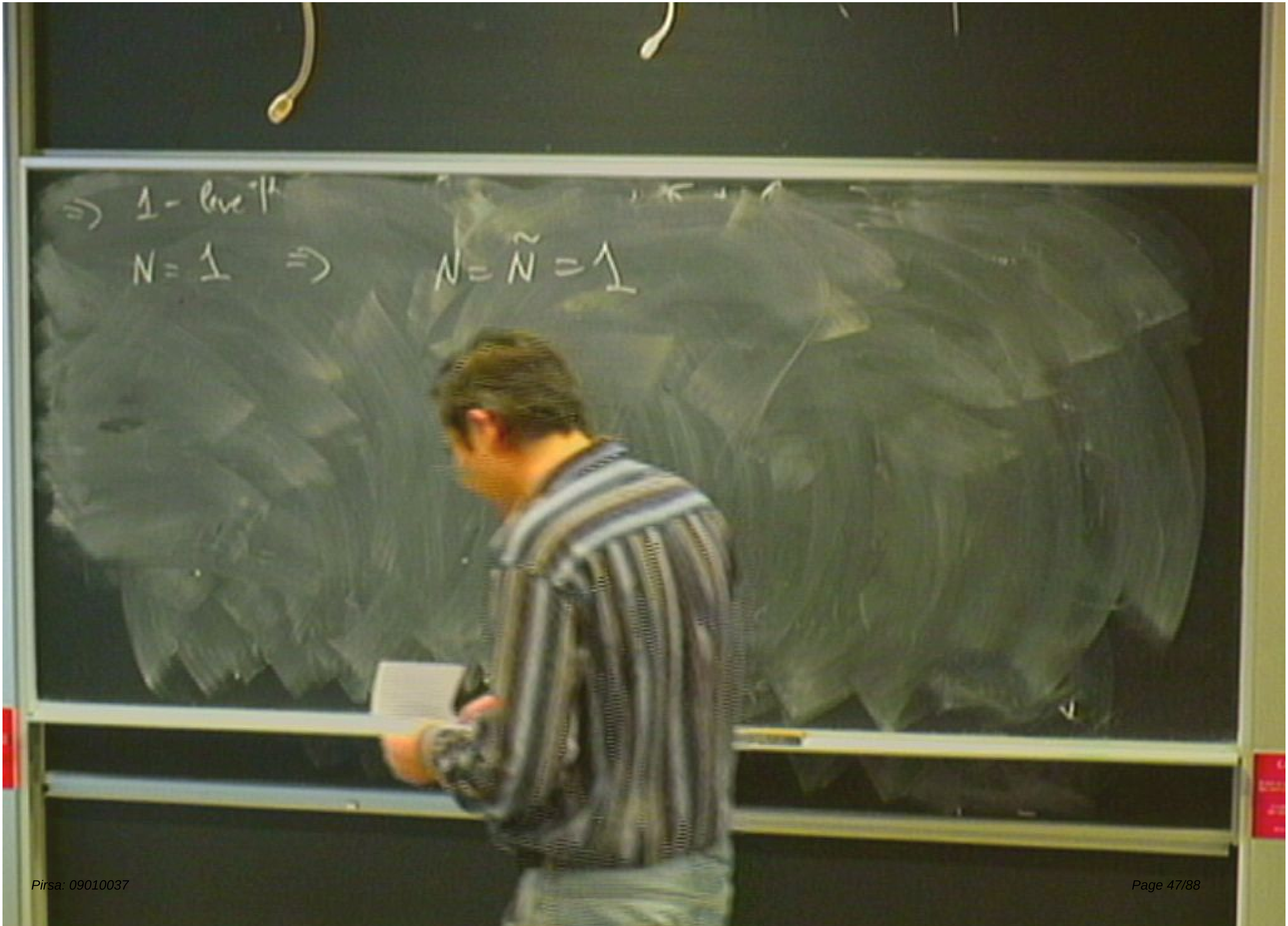
$$m^2 = \frac{2}{\alpha'}\left[N + \tilde{N} + \frac{2-D}{12}\right], \quad N = \tilde{N}$$

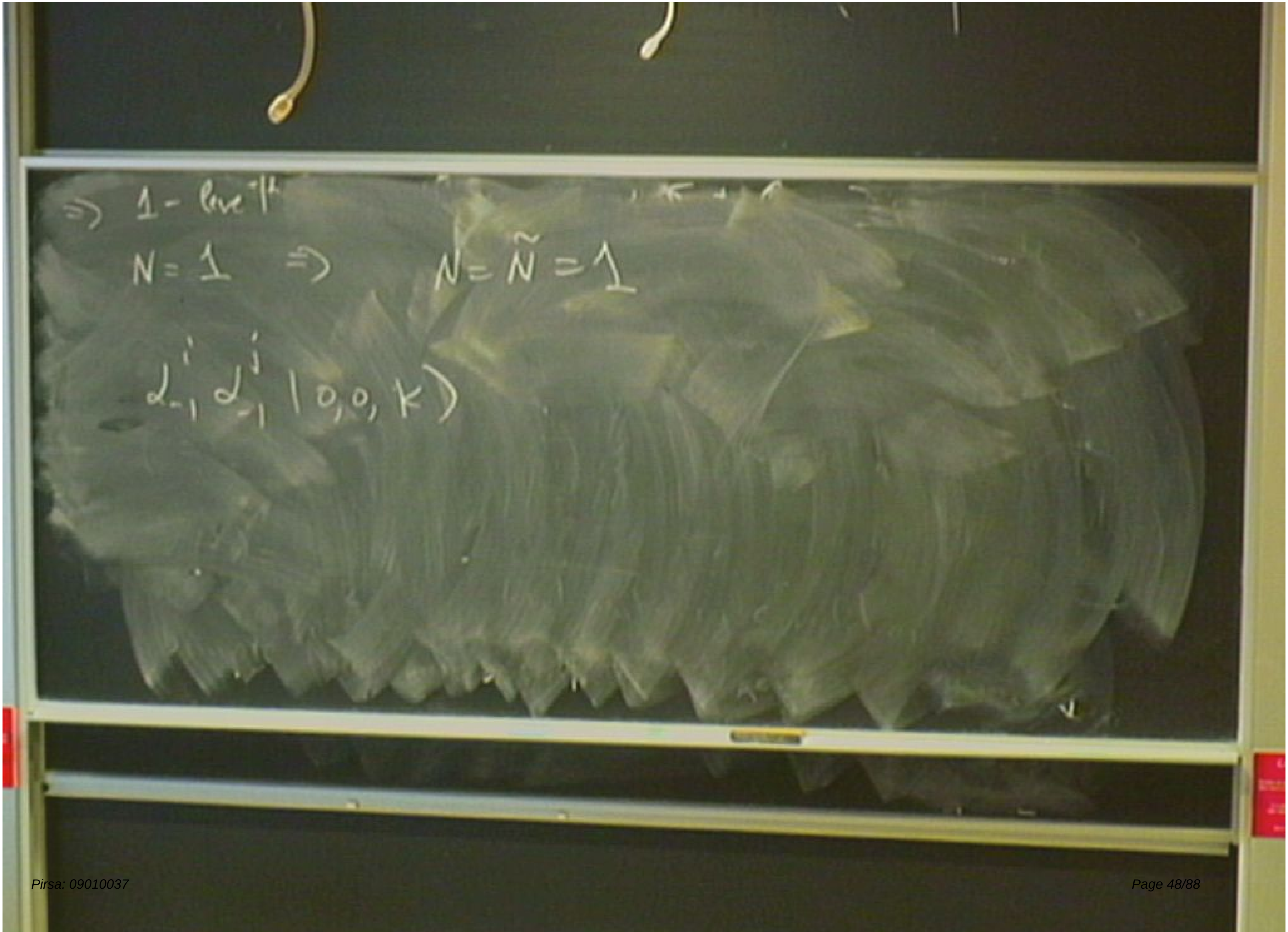
$\Rightarrow$ <u>0-level</u>

$$N = \tilde{N} = 0$$

$$m^2 = \frac{2-D}{6\alpha'} \quad \text{is a tachyon} \quad \underline{D > 2}$$


N = 1

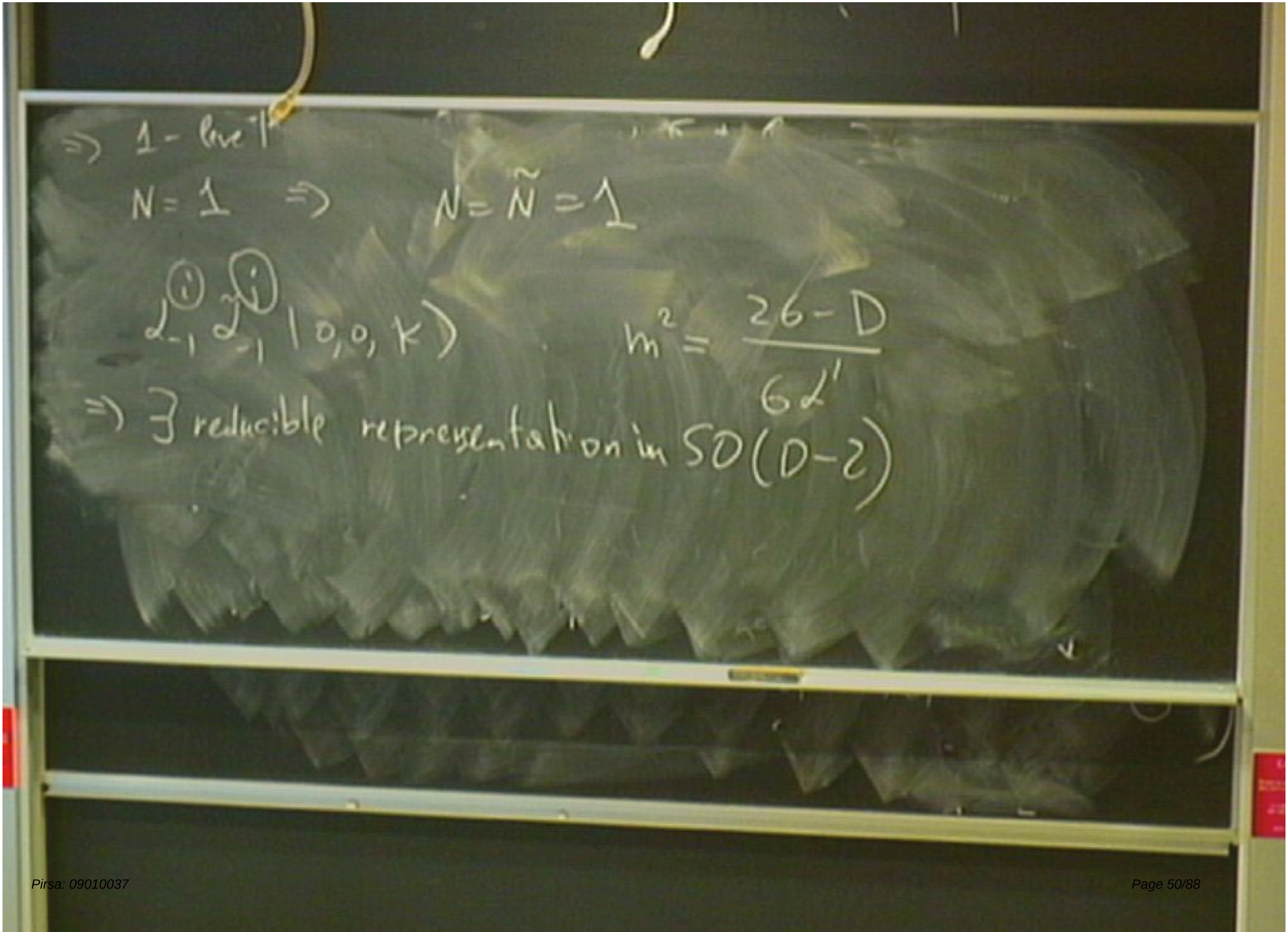

N = 1 ⇒ N = Ñ = 1


$N = 1 \Rightarrow \tilde{N} = N = 1$
$\alpha^i_{-1} \alpha^j_{-1} |0,0,k\rangle$

$\Rightarrow$ 1 - level

$N = 1 \Rightarrow N = \tilde{N} = 1$

$\alpha^i_{-1}\,\alpha^j_{-1}\,|0,0,k\rangle \qquad m^2 = \frac{26-D}{6\alpha'}$


⇒ 1 - level
N = 1 ⇒ $N = \tilde{N} = 1$
$\alpha^{i}_{-1} \tilde{\alpha}^{i}_{-1} |0,0,k\rangle$
$m^2 = \frac{26-D}{6\alpha'}$
⇒ ∃ reducible representation in SO(D-2)

⇒ 1 - level

$N = 1 \Rightarrow N = \tilde{N} = 1$

$\alpha_{-1}^{i} \tilde{\alpha}_{-1}^{j} |0,0,k\rangle \qquad m^2 = \dfrac{26 - D}{6\alpha'}$

⇒ ∃ reducible representation in $SO(D-2)$ $\Big\}$ $\underline{D = 26}$

$Q\,|0, \tilde{0}, k^i, k^i\rangle = 0$

$N = 1 \Rightarrow \quad N = \tilde{N} = 1$

$\alpha^{i}_{-1} \tilde{\alpha}^{j}_{-1} |0,0,k\rangle \qquad m^2 = \frac{26-D}{6\alpha'}$

$\Rightarrow \exists$ reducible representation in $SO(D-2)$ $\quad \Big\} \quad \underline{D=26}$

e^{ij}

$Q = -\frac{2\pi}{\ell}\left(N - \tilde{N}\right) \qquad |N\rangle \to |N\rangle + \epsilon Q |N\rangle$

$Q \, |0, \tilde{0}, k^{+}, k^{i}\rangle = 0$

$N = 1$ $\quad$ $\tilde{N} = N = 1$

$\alpha^{i}_{-1}\tilde{\alpha}^{j}_{-1}|0,0,k\rangle \qquad m^2 = \frac{26-D}{6\alpha'}$

$D = 26$

$\Rightarrow \exists$ reducible representation in $SO(D-2)$

$$e^{ij} = \frac{1}{2}\left(e^{ij} + e^{ji} - \frac{2}{D-2}\delta^{ij}e^{kk}\right)$$

$$Q = -\frac{2\pi}{\ell}\left(N - \tilde{N}\right) \qquad |N\rangle \to |N\rangle + \epsilon Q|N\rangle$$

$$Q|0,\tilde{0},k^{+},k^{i}\rangle = 0$$

$N = 1$ $\qquad$ $N = \tilde{N} = 1$

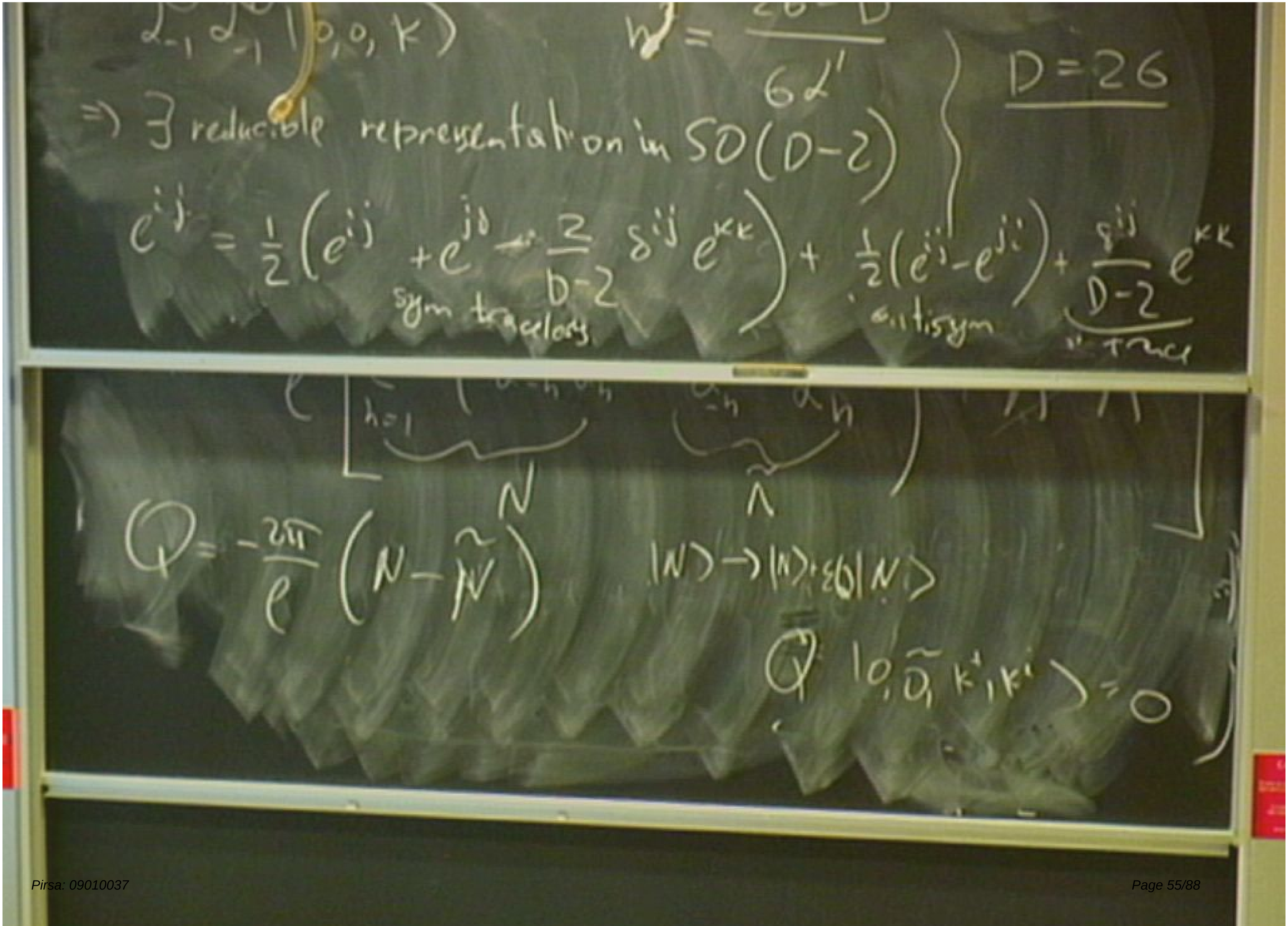
$$\alpha_{-1}^{i}\,\tilde{\alpha}_{-1}^{j}\,|0,0,k\rangle \qquad m^2 = \frac{26-D}{6\alpha'} \qquad \underline{D = 26}$$

$\Rightarrow$ $\exists$ reducible representation in $SO(D-2)$

$$e^{ij} = \frac{1}{2}\left(e^{ij} + e^{ji} - \frac{2}{D-2}\,\delta^{ij} e^{kk}\right) + \frac{1}{2}\left(e^{ij} - e^{ji}\right) + \frac{\delta^{ij}}{D-2}\, e^{kk}$$

$$Q = -\frac{2\pi}{\ell}\left(N - \tilde{N}\right) \qquad |N\rangle \to |N\rangle + \epsilon Q|N\rangle$$

$$Q\,|0, \tilde{0}, k^{+}, k^{i}\rangle = 0$$


D=26
⇒ ∃ reducible representation in SO(D−2)
sym traceless
antisym
trace
|N⟩ → |N⟩ + ε Q |N⟩

$\Rightarrow \exists$ reducible representation in $SO(D-2)$ $\qquad 6\alpha'$ $\qquad \underline{D=26}$

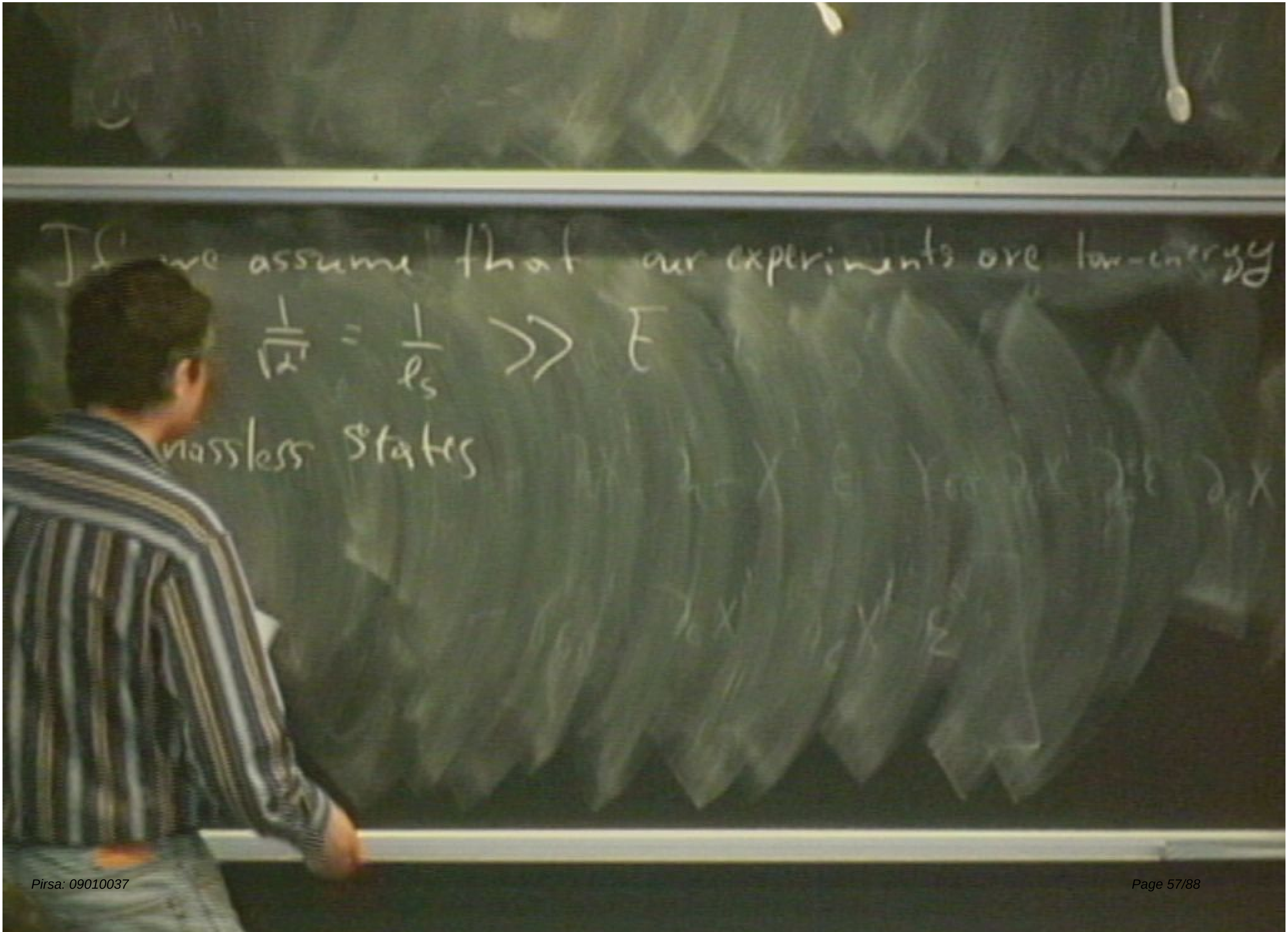
$$e^{ij} = \frac{1}{2}\left(e^{ij} + e^{ji} - \frac{2}{D-2}\delta^{ij} e^{kk}\right) + \frac{1}{2}\left(e^{ij} - e^{ji}\right) + \frac{\delta^{ij}}{D-2} e^{kk}$$

graviton — sym traceless — antisym — trace

$$Q = -\frac{2\pi}{\ell}\left[\sum_{n=1}\left(\underbrace{\alpha_{-n}\alpha_n}_{N} - \underbrace{\tilde{\alpha}_{-n}\tilde{\alpha}_n}_{\tilde{N}}\right) + A - \tilde{A}\right]$$

$$Q = -\frac{2\pi}{\ell}\left(N - \tilde{N}\right) \qquad |N\rangle \to |N\rangle + \varepsilon Q |N\rangle$$

$$\bar{Q}\, |0, \tilde{0}, k^+, k^i\rangle = 0$$


If we assume that our experiments are low-energy
1/√α' = 1/ℓs >> E
massless states


If we assume that our experiments are low-energy
1/√α' = 1/ℓs >> E
⇒ massless states
open string

If we assume that our experiments are low-energy

$$\frac{1}{\sqrt{\alpha'}} = \frac{1}{\ell_s} \gg E$$

$\Rightarrow$ massless states

<u>open string</u>: A_μ

<u>closed string</u>: $g_{\mu\nu}$, $B_{\mu\nu}$

If we assume that our experiments are low-energy

$$\frac{1}{\sqrt{\alpha'}} = \frac{1}{\ell_s} \gg E$$

$\Rightarrow$ massless states

open string: A_μ

closed string: $g_{\mu\nu}$, $B_{\mu\nu}$, ϕ

If we assume that our experiments are low-energy

$$\frac{1}{\sqrt{\alpha'}} = \frac{1}{\ell_s} \gg E \qquad F = dA$$

$\Rightarrow$ massless states

open string $\rightarrow$ A_μ

closed string: $g_{\mu\nu}$, $B_{\mu\nu}$, φ

If we assume that our experiments are low-energy

$$\frac{1}{\sqrt{\alpha'}} = \frac{1}{\ell_s} \gg E \qquad F = dA$$

$\Rightarrow$ massless states

open string → $A_\mu + \partial_\mu X$

closed string: $g_{\mu\nu}, B_{\mu\nu}, \phi$

If we assume that our experiments are low-energy

$$\frac{1}{\sqrt{\alpha'}} = \frac{1}{\ell_s} \gg E$$

$$F = dA$$

$$A \to A + d\chi$$

$\Rightarrow$ massless states

open string: $A_\mu + \partial_\mu \chi$

$B \to B + d$

closed string: $g_{\mu\nu}$, $B_{\mu\nu}$, φ

If we assume that our experiments are low-energy

$$\frac{1}{\sqrt{\alpha'}} = \frac{1}{\ell_s} \gg E$$

$F = dA$

$A \to A + d\chi$

$\Rightarrow$ massless states

open string: $A_\mu + \partial_\mu \chi$

$B \to B + d$

closed string: $g_{\mu\nu}$, $B_{\mu\nu}$, ϕ

$F_{(3)} = dB$

If we assume that our experiments are low-energy

$$\frac{1}{\sqrt{\alpha'}} = \frac{1}{\ell_s} \gg E$$

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$$A \to A + d\chi$$

$\Rightarrow$ massless states

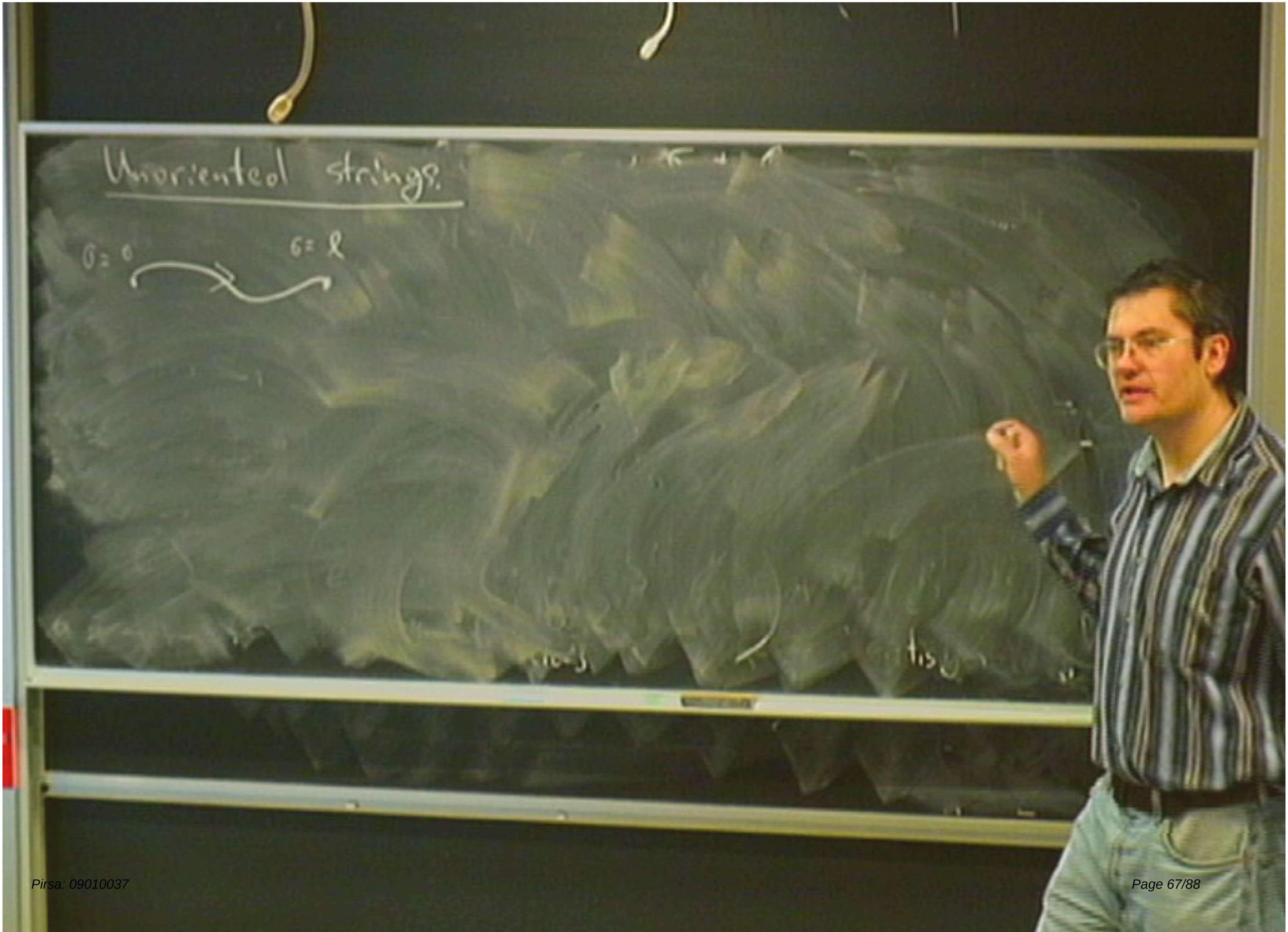
open string: A_μ

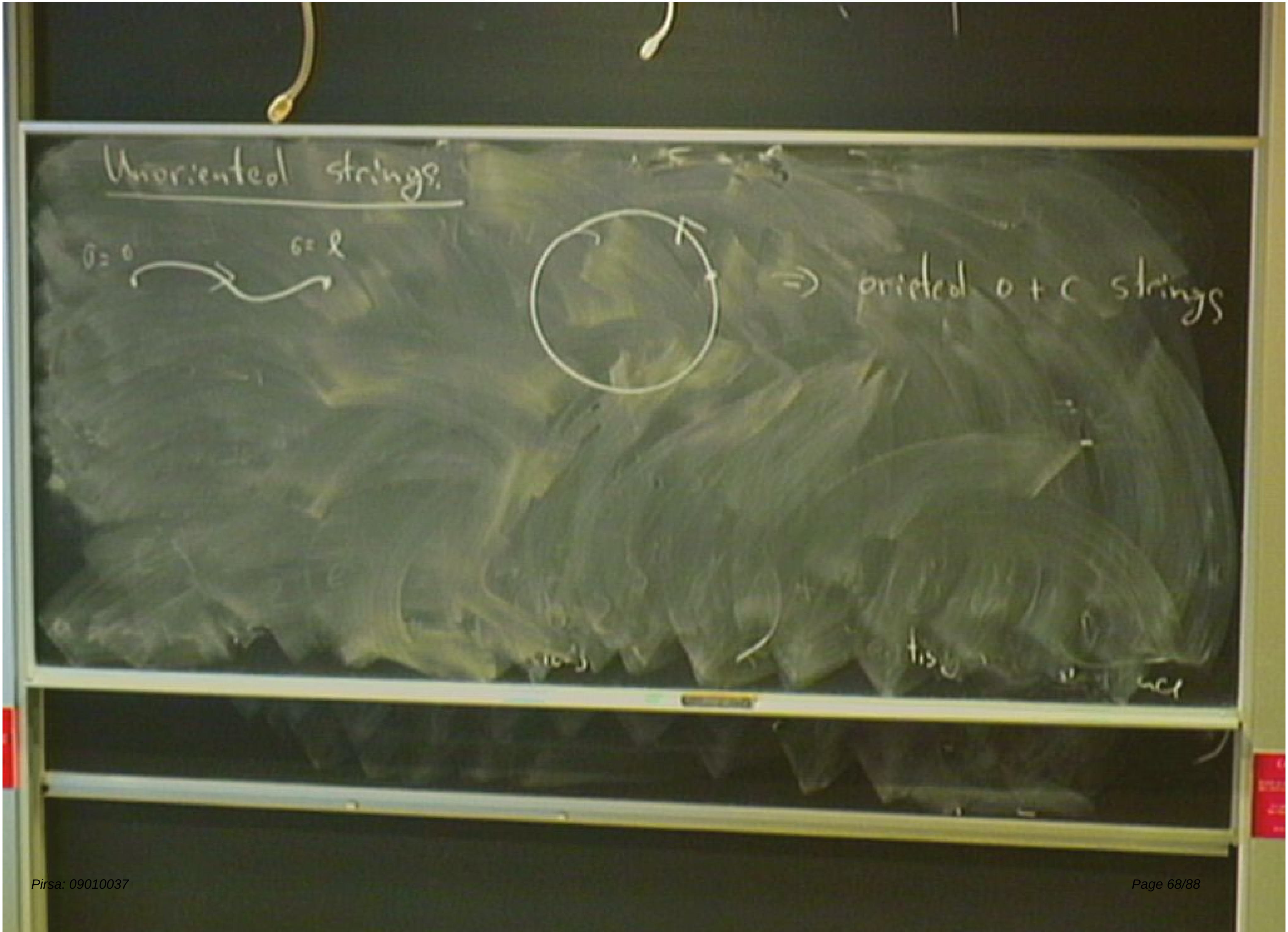
closed string: $g_{\mu\nu}$, $B_{\mu\nu}$, ϕ

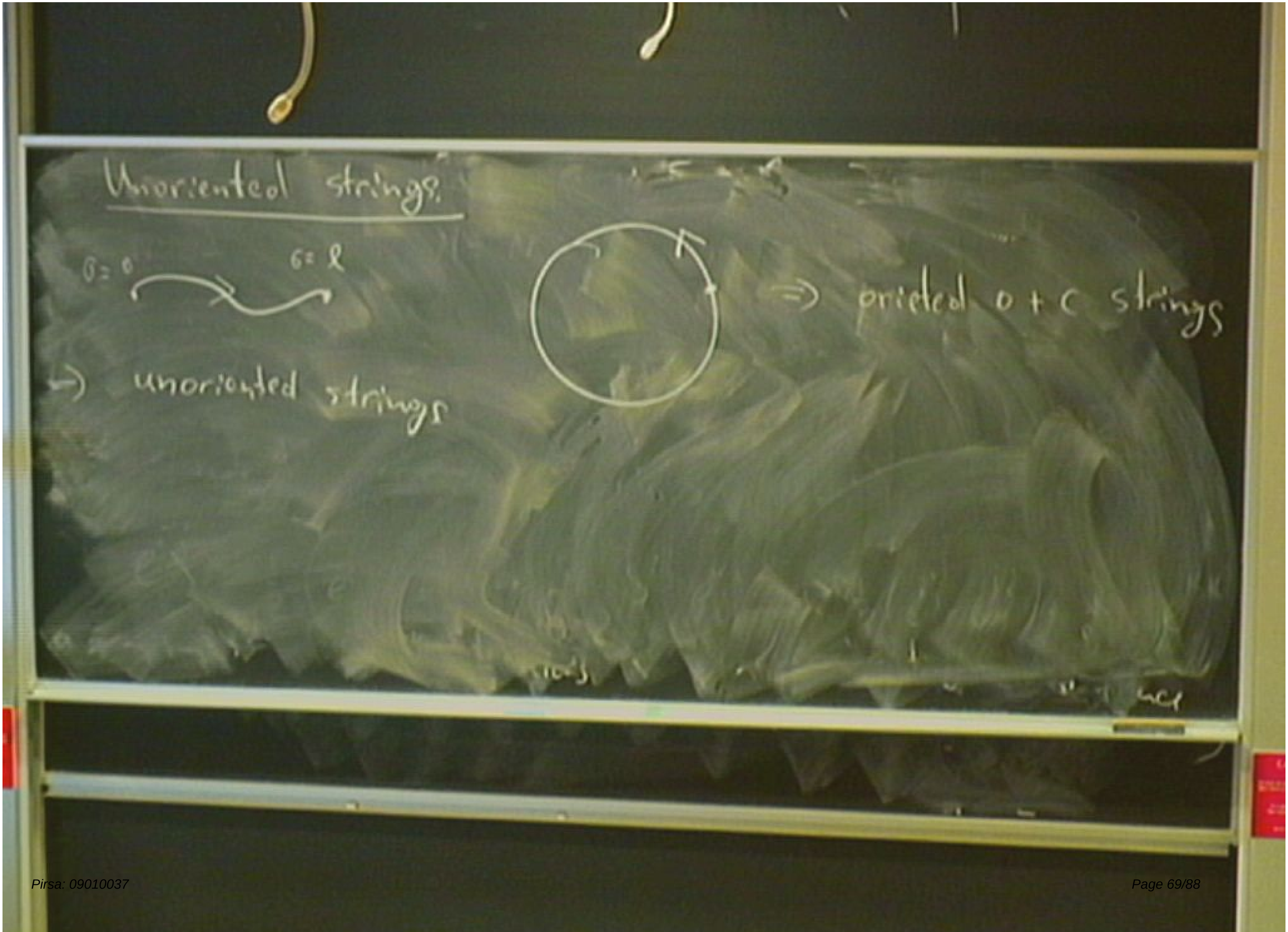
$$B \to B + d\Lambda$$

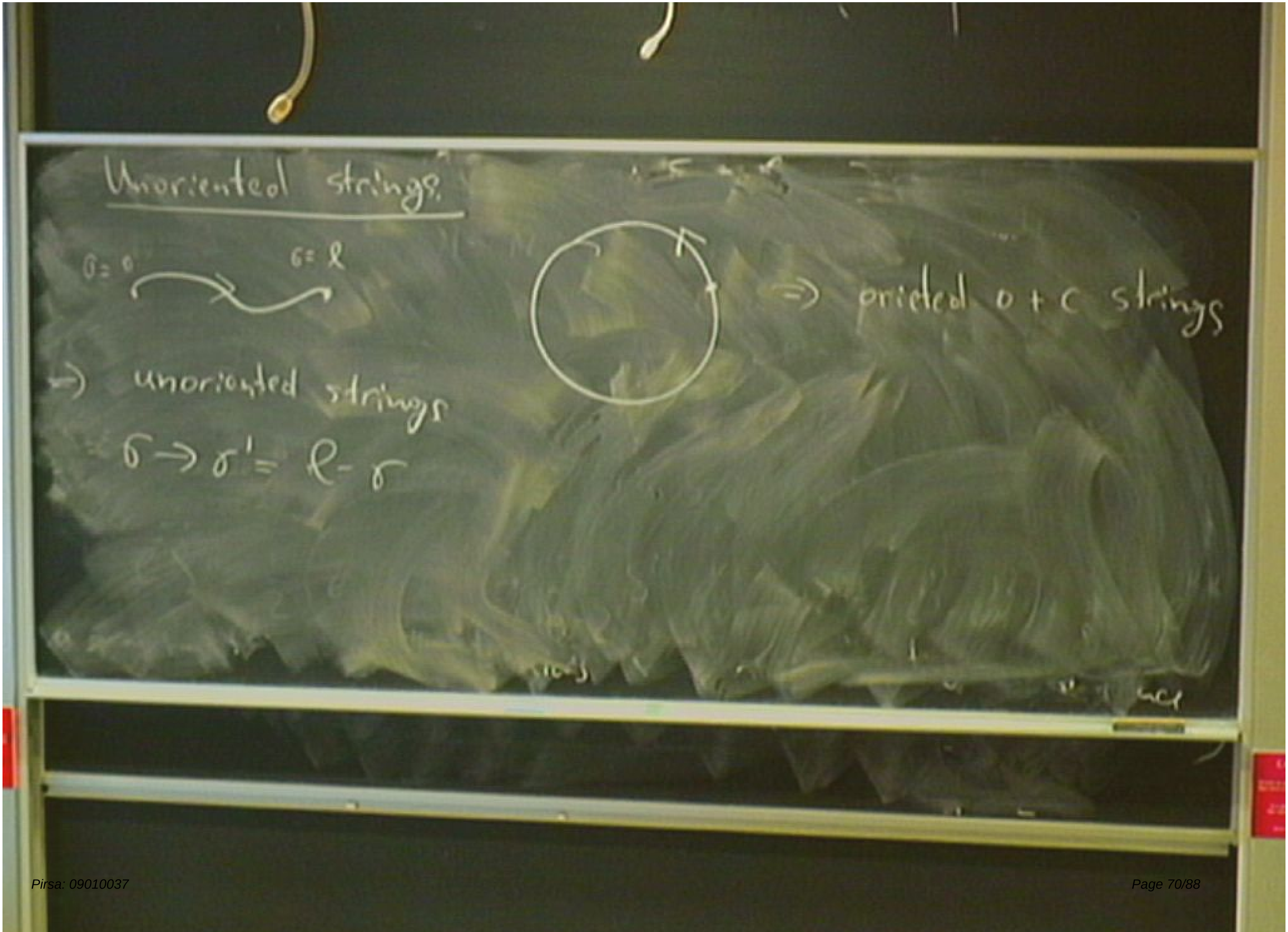
$$F_{(3)} = dB$$


Unoriented strings:


Unoriented strings:
σ = 0
σ = ℓ


Unoriented strings.
σ = 0
σ = ℓ
⇒ oriented o + c strings


Unoriented strings:
σ=0
σ=ℓ
⇒ unoriented strings
⇒ oriented o + c strings


Unoriented strings:
σ = 0
σ = ℓ
⇒ unoriented strings
σ → σ' = ℓ − σ
⇒
o + c strings

Unoriented strings.

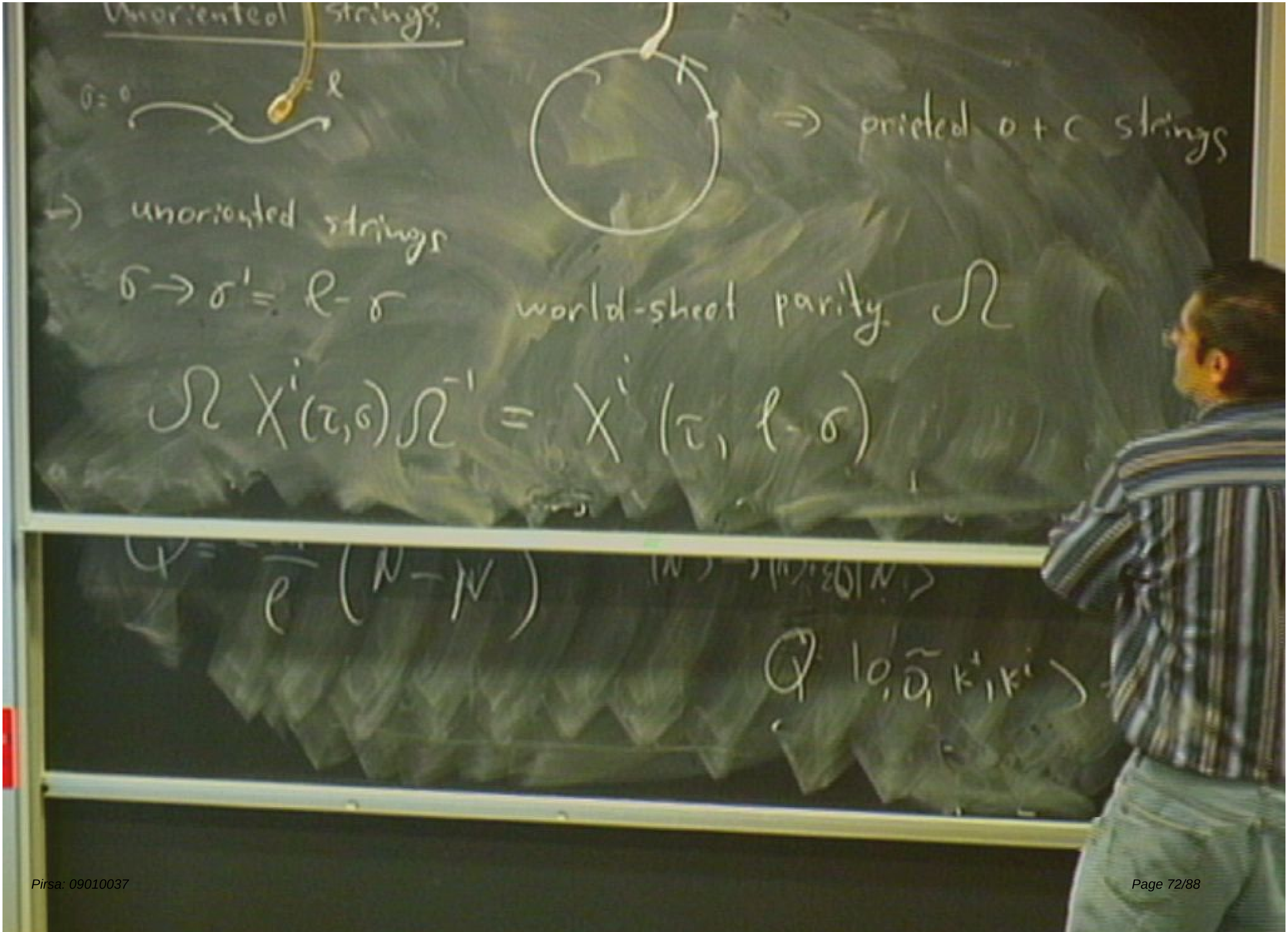
⇒ oriented o + c strings

⇒ unoriented strings

$\sigma \to \sigma' = \ell - \sigma$ world-sheet parity Ω

$P = \frac{\ldots}{\ell}(N - \tilde{N})$

$Q\,|0, \tilde{0}, k^{+}, k^{i}\rangle = 0$


Unoriented strings.
σ = 0
ℓ
⇒ o + c strings
⇒ unoriented strings
σ → σ' = ℓ - σ
world-sheet parity Ω
Ω X^i(τ,σ) Ω^-1 = X^i(τ, ℓ - σ)

$\sigma \to \sigma' = \ell - \sigma$ world-sheet parity Ω

$$\Omega X^i(\tau,\sigma)\Omega^{-1} = X^i(\tau, \ell - \sigma)$$

$\Omega \alpha_n^i$

$\sigma \to \sigma' = \ell - \sigma$ world-sheet parity Ω

$$\Omega X^i(\tau,\sigma)\Omega^{-1} = X^i(\tau, \ell - \sigma)$$

$$\Omega \alpha_n^i \Omega^{-1} = (-1)^n \alpha_n^i$$

$\sigma \to \sigma' = \ell - \sigma$ world-sheet parity Ω

$$\Omega X^i(\tau,\sigma)\Omega^{-1} = X^i(\tau, \ell - \sigma)$$

$$\Omega \alpha_n^i \Omega^{-1} = (-1)^n \alpha_n^i \quad \text{(open strings)}$$

closed $\Omega \alpha_n^i \Omega^{-1} =$

$$\sigma \to \sigma' = \ell - \sigma \qquad \text{world-sheet parity } \Omega$$

$$\Omega X^i(\tau,\sigma)\Omega^{-1} = X^i(\tau, \ell - \sigma)$$

$$\Omega \alpha_n^i \Omega^{-1} = (-1)^n \alpha_n^i \qquad \text{(open strings)}$$

closed

$$\Omega \alpha_n^i \Omega^{-1} = \tilde{\alpha}_n^i$$

$$\Omega \tilde{\alpha}_n^i \Omega^{-1} = \alpha_n^i$$

$\sigma \to \sigma' = \ell - \sigma$ world-sheet parity Ω

$$\Omega X^i(\tau,\sigma)\Omega^{-1} = X^i(\tau, \ell-\sigma)$$

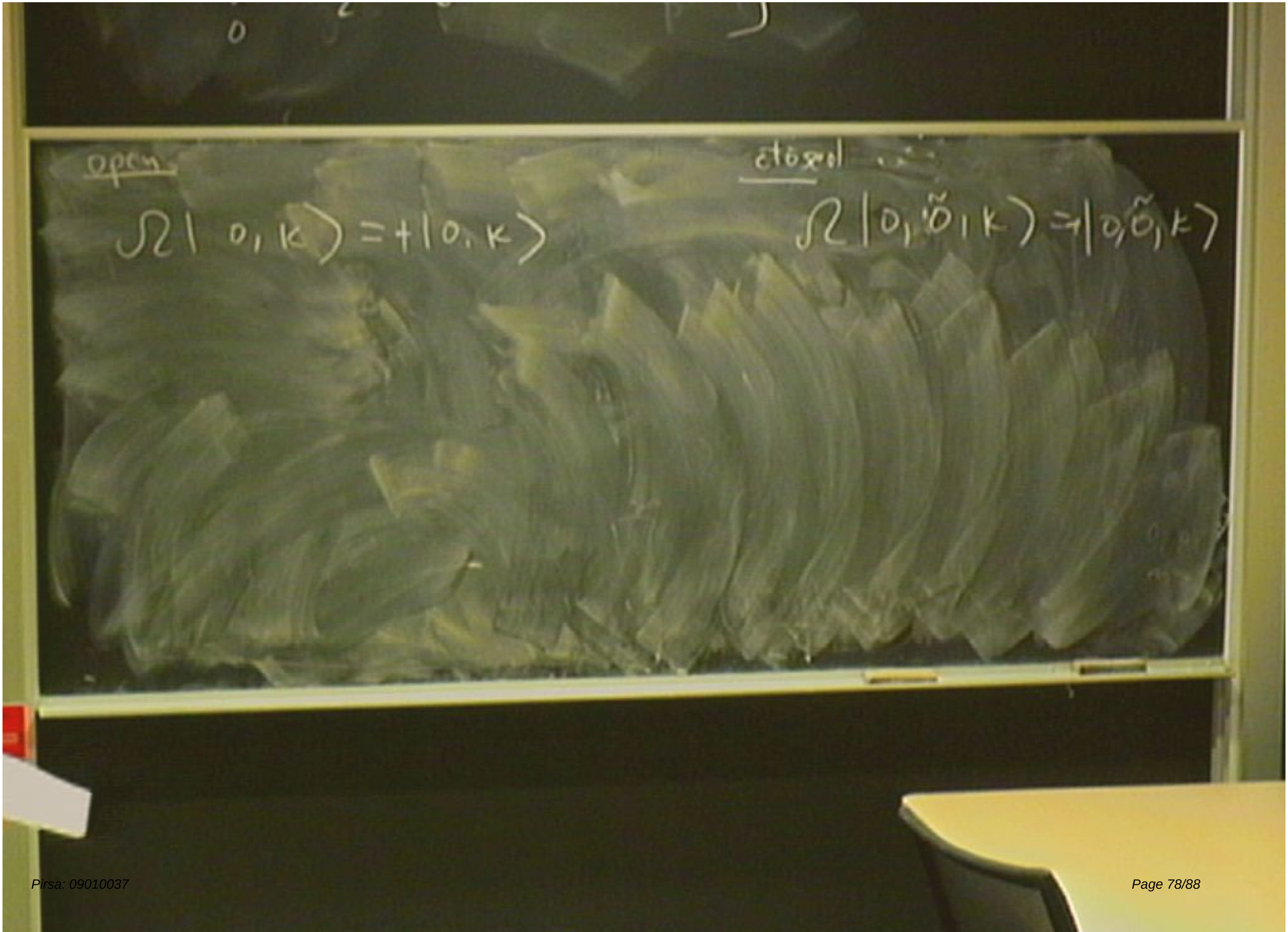
[illegible]

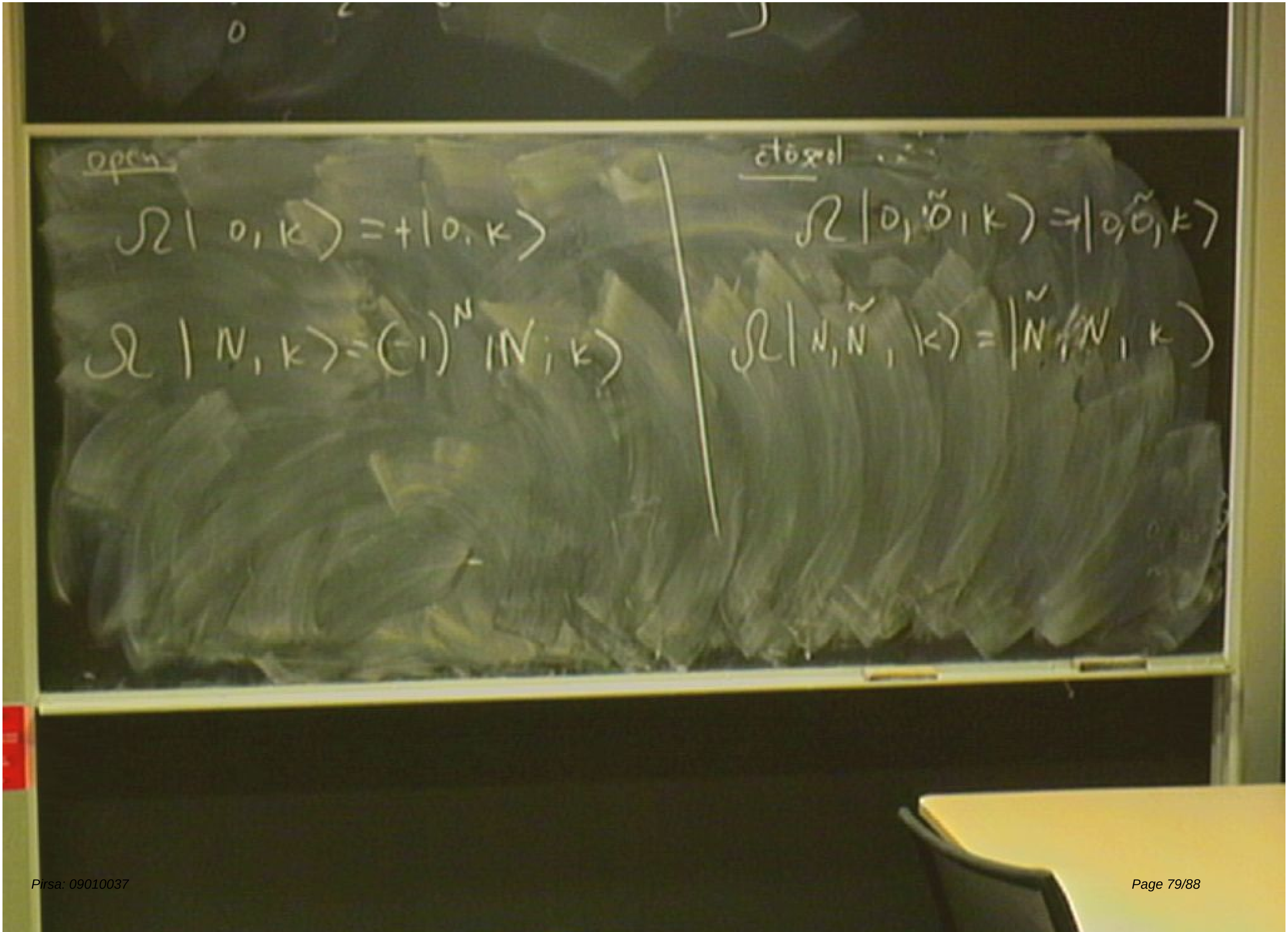
$$\Omega \alpha_n^i \Omega^{-1} = (-1)^n \alpha_n^i \quad \text{(open strings)}$$

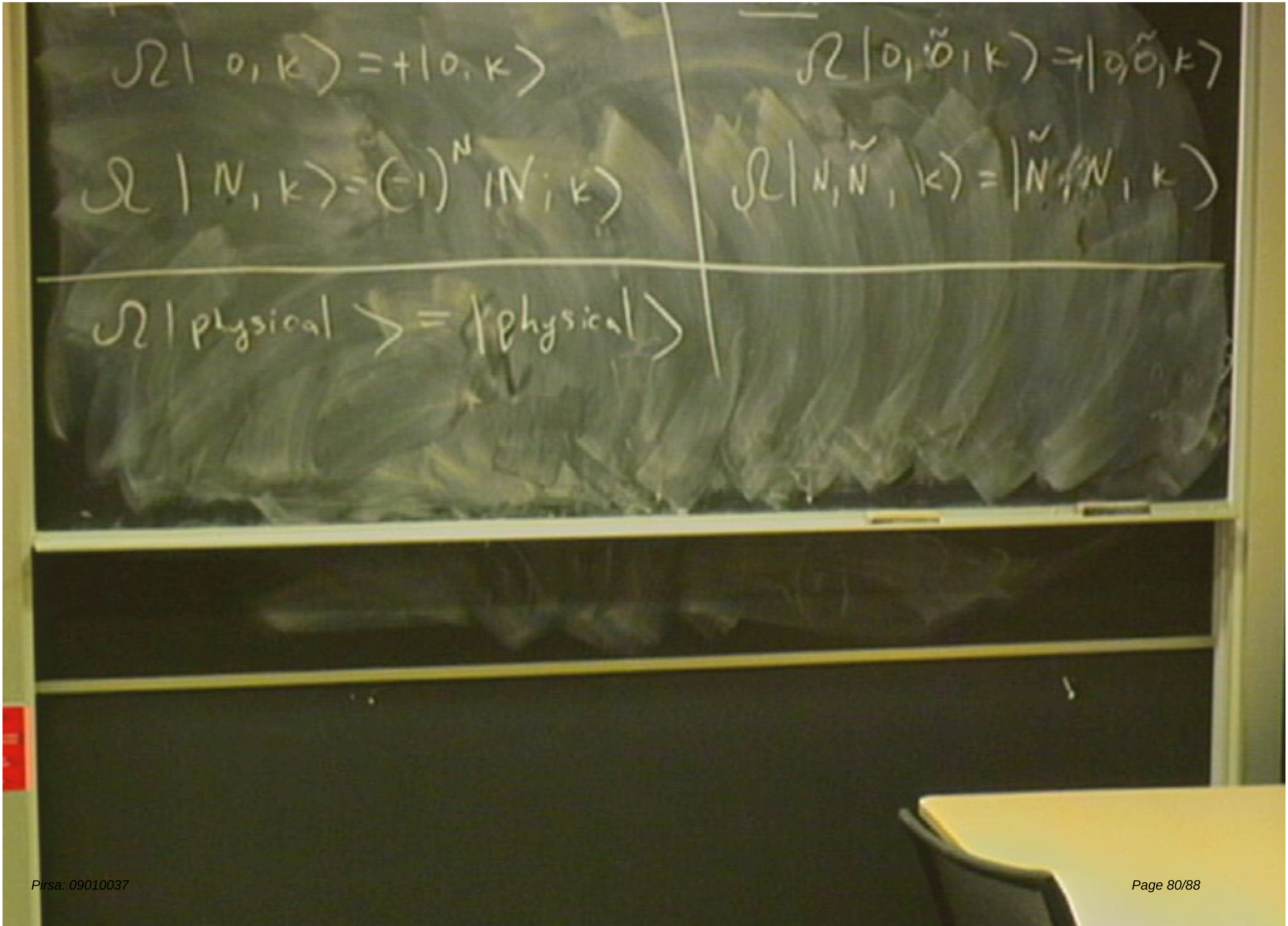
closed

$$\Omega \alpha_n^i \Omega^{-1} = \tilde{\alpha}_n^i$$

$$\Omega \tilde{\alpha}_n^i \Omega^{-1} = \alpha_n^i \qquad \Omega^2 = 1$$


open
Ω| 0, k⟩ = +|0, k⟩
closed
Ω|0, õ, k⟩ = −|0, õ, k⟩


open
Ω| 0, k⟩ = +|0, k⟩
Ω | N, k⟩ = (-1)^N |N; k⟩
closed
Ω|0, Õ, k⟩ = +|0, Õ, k⟩
Ω|N, Ñ, k⟩ = |Ñ, N, k⟩


Ω| 0, k⟩ = +|0, k⟩
Ω| N, k⟩ = (-1)^N |N, k⟩
Ω|0, Õ, k⟩ = |0, Õ, k⟩
Ω|N, Ñ, k⟩ = |Ñ, N, k⟩
Ω| physical⟩ = |physical⟩

$$\Omega|0,k\rangle = +|0,k\rangle \qquad \Omega|0,\tilde{0},k\rangle = +|0,\tilde{0},k\rangle$$

$$\Omega|N,k\rangle = (-1)^N|N,k\rangle \qquad \Omega|N,\tilde{N},k\rangle = |\tilde{N},N,k\rangle$$

$$\Omega|\text{physical}\rangle = |\text{physical}\rangle$$

$\Rightarrow$ open unorient strings A_m is projected out

$\Omega\,|N,k\rangle = (-1)^{N}\,|N;k\rangle$ | $\Omega\,|N,\tilde{N},k\rangle = |\tilde{N},N,k\rangle$

$\Omega\,|\text{physical}\rangle = |\text{physical}\rangle$

⇒ open unorient strings: A_m is projected out

⇒ closed unoriented string

$\alpha^{i}_{-n}\alpha^{i}_{n}$ $\qquad$ $\alpha^{i}_{-n}\tilde{\alpha}^{i}_{-n}$

$\Omega |N, k\rangle = (-1)^{N} |N; k\rangle$ | $\Omega |N, \tilde{N}, k\rangle = |\tilde{N}, N, k\rangle$

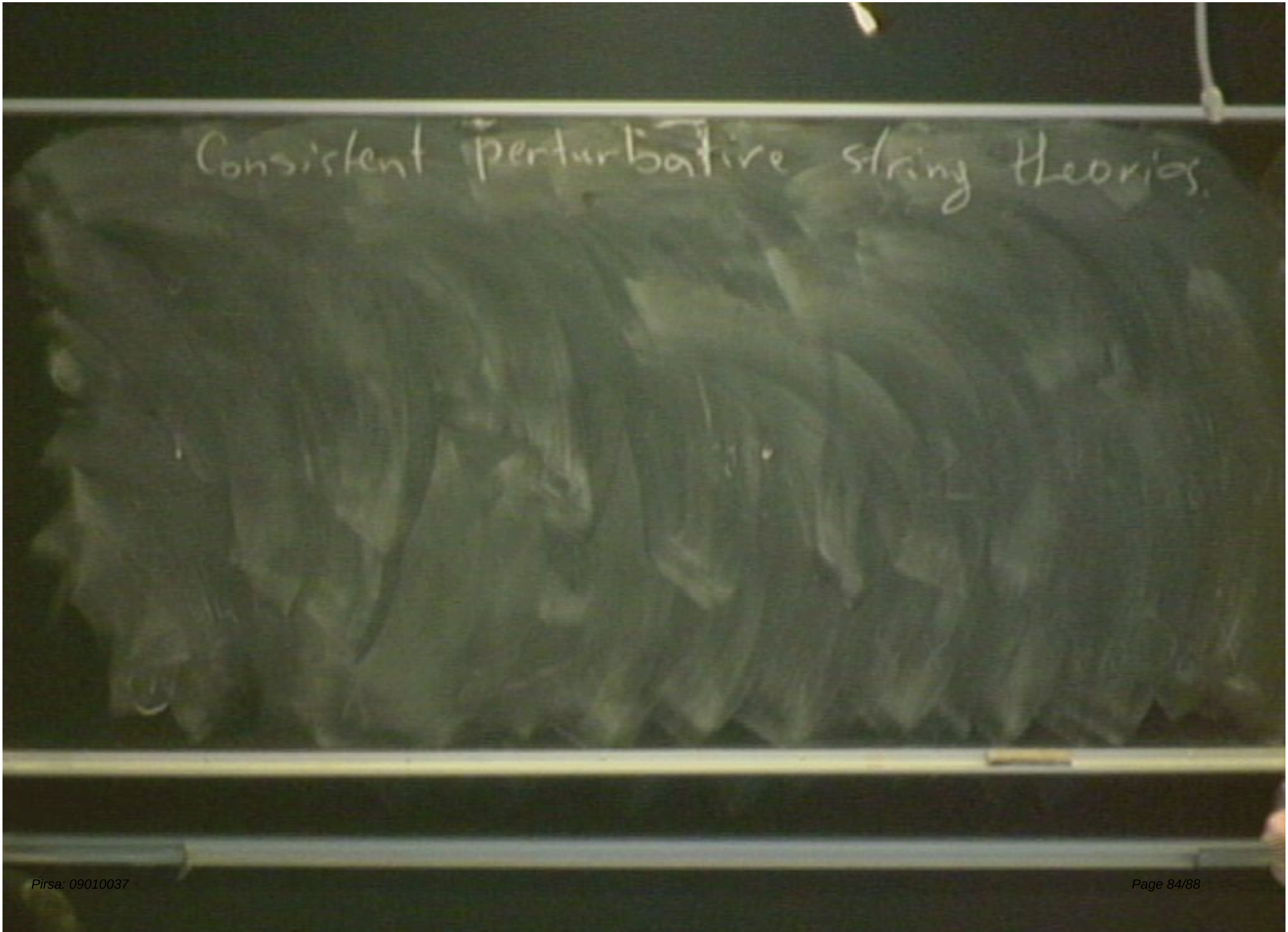
$\Omega |\text{physical}\rangle = |\text{physical}\rangle$

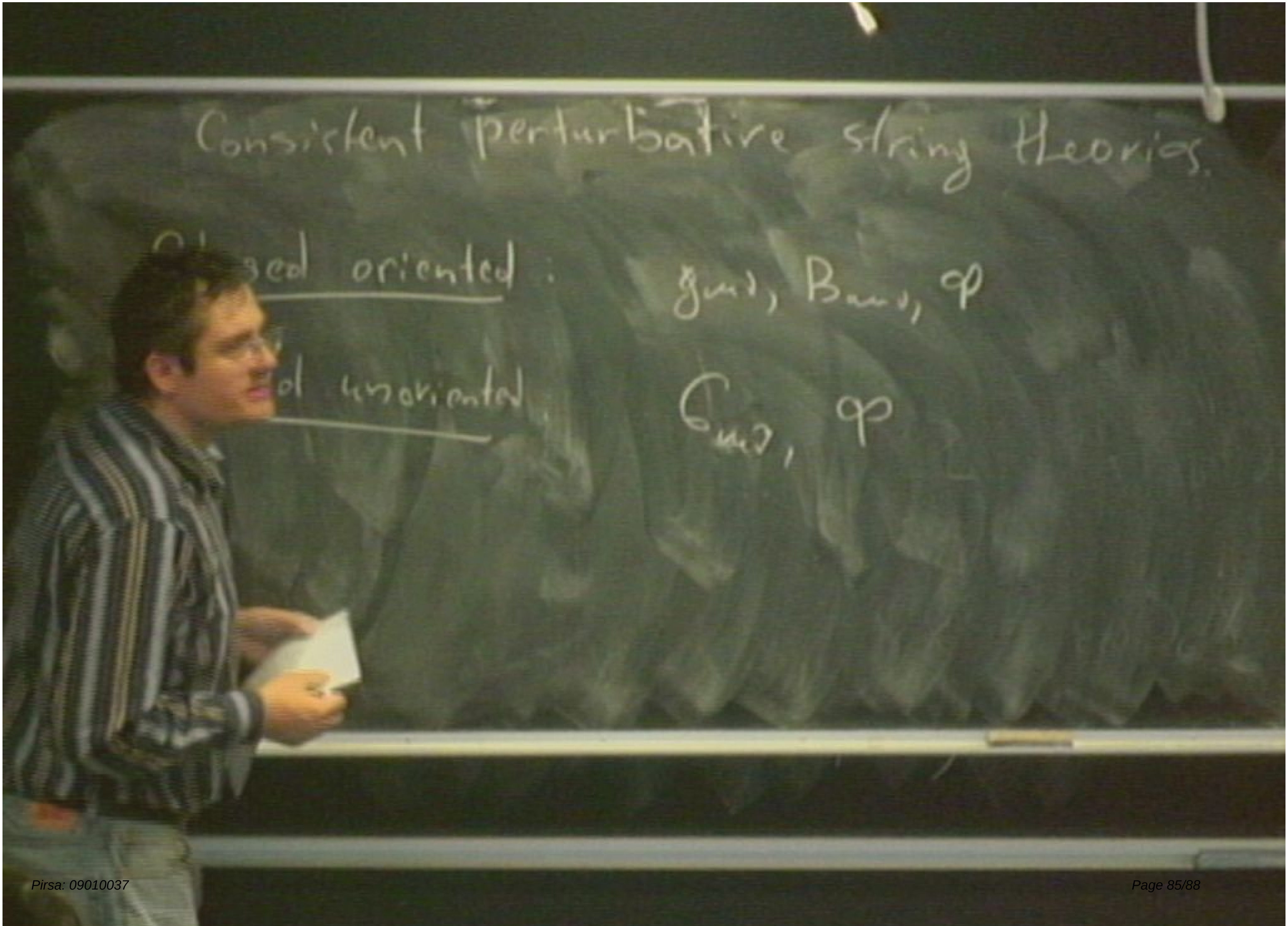
⇒ open unorient strings A_m is projected out

⇒ closed unoriented string B_{mn} is projected out

$i=1 \quad n=1$ $\qquad$ $i=1 \quad n=1$

$\alpha^i_{-n} \alpha^i_n$ $\qquad$ $\tilde{\alpha}^i_{-n} \alpha^i_{-n}$


Consistent perturbative string theories.


Consistent perturbative string theories.
oriented :
$g_{\mu\nu}$, $B_{\mu\nu}$, φ
unoriented
$G_{\mu\nu}$, φ

Consistent perturbative string theories.

Closed oriented : $g_{\mu\nu}$, $B_{\mu\nu}$, φ

Closed unoriented : $G_{\mu\nu}$, φ


Closed unoriented
Closed + open oriented
open

$g_{\mu\nu}, B_{\mu\nu}, \varphi$

Closed unoriented: $G_{\mu\nu}, \varphi$

Closed + open oriented: $g_{\mu\nu}, B_{\mu\nu}, \varphi, A_\mu$

Closed + open unoriented: $G_{\mu\nu}, \varphi$

open string: A_μ

Closed string: $g_{\mu\nu}, B_{\mu\nu}, \varphi$

$B \to B +$

$F_{(3)} = dB$