Title: Introduction to the Bosonic String Part B

Date: Jan 23, 2009 12:00 PM

URL: http://pirsa.org/09010036

Abstract: This course provides a thorough introduction to the bosonic string based on the Polyakov path integral and conformal field theory. We introduce central ideas of string theory, the tools of conformal field theory, the Polyakov path integral, and the covariant quantization of the string. We discuss string interactions and cover the tree-level and one loop amplitudes. More advanced topics such as T-duality and D-branes will be taught as part of the course. The course is geared for M.Sc. and Ph.D. students enrolled in Collaborative Ph.D. Program in Theoretical Physics. Required previous course work: Quantum Field Theory (AM516 or equivalent). The course evaluation will be based on regular problem sets that will be handed in during the term. The primary text is the book: 'String theory. Vol. 1: An introduction to the bosonic string. J. Polchinski (Santa Barbara, KITP) . 1998. 402pp. Cambridge, UK: Univ. Pr. (1998) 402 p.' All interested students should contact Alex Buchel at abuchel@uwo.ca as soon as possible.

$$X^+ = \tau$$

$$\partial_\sigma \gamma_{\sigma\sigma} = 0$$

$$(-\gamma)^{1/2} = 1$$

$$\gamma_{\alpha\beta} = \begin{bmatrix} \gamma_{\tau\tau}(\tau,\sigma) & \gamma_{\tau\sigma}(\tau,\sigma) \\ \gamma_{\tau\sigma}(\tau,\sigma) & \gamma_{\sigma\sigma}(\tau) \end{bmatrix}$$

$X^0 = \tau$

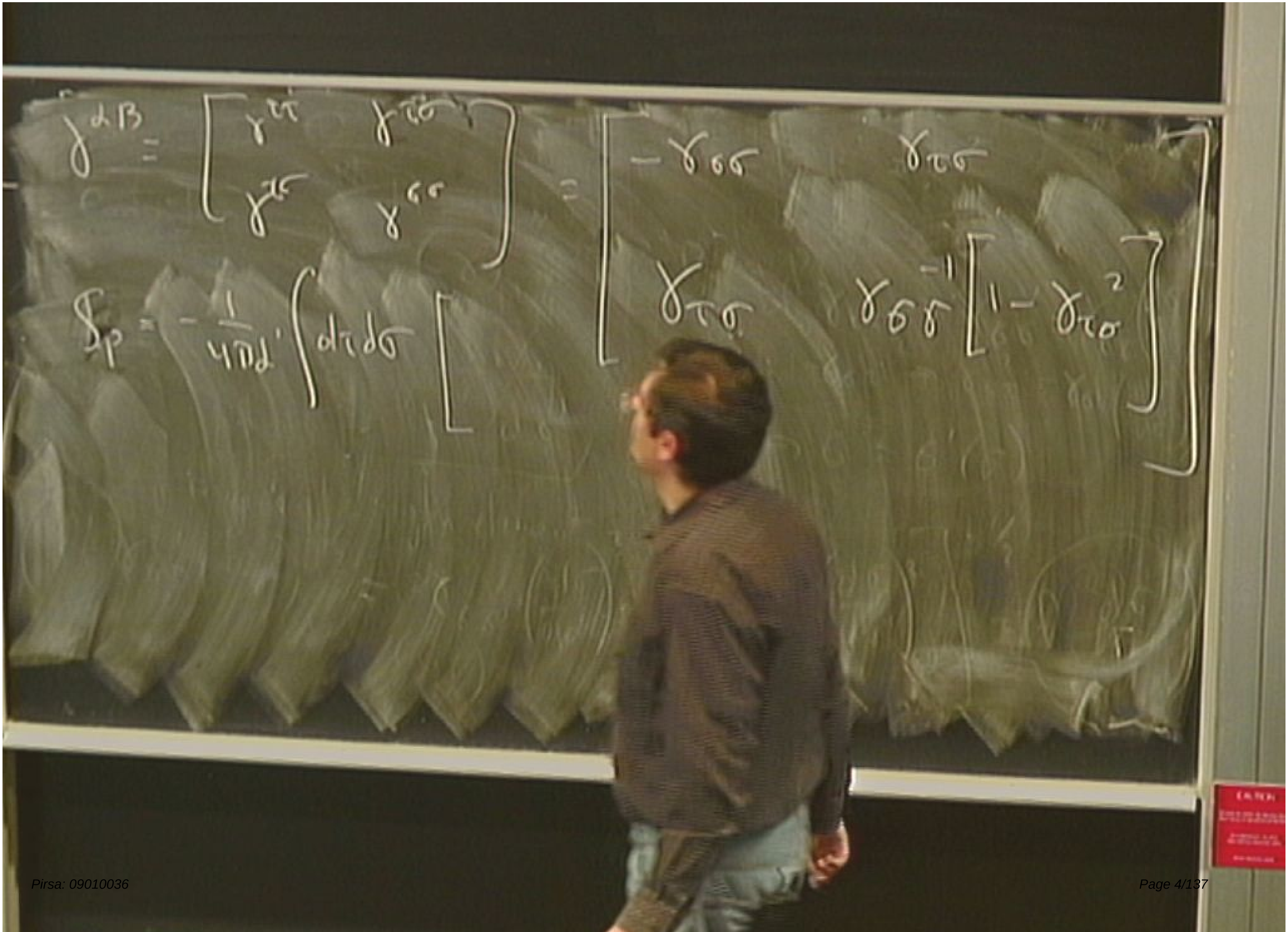
$\partial_\sigma \gamma_{\sigma\sigma} = 0$

$(-\gamma)^{1/2} = 1$

$$\gamma_{\alpha\beta} = \begin{bmatrix} \gamma_{\tau\tau}(\tau,\sigma) & \gamma_{\tau\sigma}(\tau,\sigma) \\ \gamma_{\tau\sigma}(\tau,\sigma) & \gamma_{\sigma\sigma}(\tau) \end{bmatrix}$$

$\det \gamma_{\alpha\beta} = -1 \Rightarrow \gamma_{\sigma\sigma}\gamma_{\tau\tau} - \gamma_{\tau\sigma}^2 = -1$

$\gamma_{\tau\tau} = \gamma_{\sigma\sigma}^{-1}\left[\gamma_{\tau\sigma}^2 - 1\right]$


$\gamma^{\alpha\beta} = \begin{bmatrix} \gamma^{\tau\tau} & \gamma^{\tau\sigma} \\ \gamma^{\sigma\tau} & \gamma^{\sigma\sigma} \end{bmatrix} = \begin{bmatrix} -\gamma_{\sigma\sigma} & \gamma_{\tau\sigma} \\ \gamma_{\tau\sigma} & \gamma_{\sigma\sigma}^{-1}\left[1-\gamma_{\tau\sigma}^2\right] \end{bmatrix}$
$S_p = -\frac{1}{4\pi\alpha'}\int d\tau\, d\sigma \Big[$

$$\gamma^{\alpha\beta} = \begin{bmatrix} \gamma^{\tau\tau} & \gamma^{\tau\sigma} \\ \gamma^{\sigma\tau} & \gamma^{\sigma\sigma} \end{bmatrix} = \begin{bmatrix} -\gamma_{\sigma\sigma} & \gamma_{\tau\sigma} \\ \gamma_{\tau\sigma} & \gamma_{\sigma\sigma}^{-1}\left[1 - \gamma_{\tau\sigma}^{2}\right] \end{bmatrix}$$

$$S_p = -\frac{1}{4\pi\alpha'}\int d\tau\, d\sigma \left[\sqrt{-\gamma}\,\gamma^{\alpha\beta}\,\partial_\alpha X^\mu\, \partial_\beta X_\mu\right]$$

$$\gamma^{\alpha\beta} = \begin{bmatrix} \gamma^{\tau\tau} & \gamma^{\tau\sigma} \\ \gamma^{\tau\sigma} & \gamma^{\sigma\sigma} \end{bmatrix} = \begin{bmatrix} -\gamma_{\sigma\sigma} & \gamma_{\tau\sigma} \\ \gamma_{\tau\sigma} & \gamma_{\sigma\sigma}^{-1}\left[1-\gamma_{\tau\sigma}^{2}\right] \end{bmatrix}$$

$$S_p = -\frac{1}{4\pi\alpha'}\int d\tau\, d\sigma \left[\sqrt{-\gamma}\, \gamma^{\alpha\beta}\, \partial_\alpha X^\mu \partial_\beta X_\mu\right]$$

$$= -\frac{1}{4\pi\alpha'}\int d\tau\, d\sigma \left[\right.$$

$$\gamma^{\alpha\beta} = \begin{bmatrix} \gamma^{\tau\tau} & \gamma^{\tau\sigma} \\ \gamma^{\tau\sigma} & \gamma^{\sigma\sigma} \end{bmatrix} = \begin{bmatrix} -\gamma_{\sigma\sigma} & \gamma_{\tau\sigma} \\ \gamma_{\tau\sigma} & \gamma_{\sigma\sigma}^{-1}\left[1-\gamma_{\tau\sigma}^{2}\right] \end{bmatrix}$$

$$S_p = -\frac{1}{4\pi\alpha'}\int d\tau\, d\sigma \left[\sqrt{-\gamma}\,\gamma^{\alpha\beta}\,\partial_\alpha X^\mu\,\partial_\beta X_\mu\right]$$

$$= -\frac{1}{4\pi\alpha'}\int d\tau\, d\sigma \left[-\gamma\right.$$

$$\gamma^{\alpha\beta} = \begin{bmatrix} \gamma^{\tau\tau} & \gamma^{\tau\sigma} \\ \gamma^{\sigma\tau} & \gamma^{\sigma\sigma} \end{bmatrix} = \begin{bmatrix} -\gamma_{\sigma\sigma} & \gamma_{\tau\sigma} \\ \gamma_{\tau\sigma} & \gamma_{\sigma\sigma}^{-1}\left[1-\gamma_{\tau\sigma}^{2}\right] \end{bmatrix}$$

$$S_P = -\frac{1}{4\pi\alpha'}\int d\tau\, d\sigma \Big[\underbrace{\sqrt{-\gamma}}_{=1}\, \gamma^{\alpha\beta}\, \partial_\alpha X^\mu\, \partial_\beta X_\mu\Big]$$

$$= -\frac{1}{4\pi\alpha'}\int d\tau\, d\sigma \Big[-\gamma_{\sigma\sigma}\, \partial_\tau X^\mu\, \partial$$

$$\gamma^{\alpha\beta} = \begin{bmatrix} \gamma^{\tau\tau} & \gamma^{\tau\sigma} \\ \gamma^{\sigma\tau} & \gamma^{\sigma\sigma} \end{bmatrix} = \begin{bmatrix} -\gamma_{\sigma\sigma} & \gamma_{\tau\sigma} \\ \gamma_{\tau\sigma} & \gamma_{\sigma\sigma}^{-1}\left[1-\gamma_{\tau\sigma}^{2}\right] \end{bmatrix}$$

$$S_p = -\frac{1}{4\pi\alpha'}\int d\tau\, d\sigma \left[\underbrace{\sqrt{-\gamma}}_{=1}\, \gamma^{\alpha\beta}\, \partial_\alpha X^\mu\, \partial_\beta X_\mu\right]$$

$$= -\frac{1}{4\pi\alpha'}\int d\tau\, d\sigma \Big[-\gamma_{\sigma\sigma}\, \partial_\tau X^\mu\, \partial_\tau X_\mu + 2\gamma_{\tau\sigma}\, \partial_\tau X^\mu\, \partial_\sigma X_\mu$$

$$\gamma^{\tau\sigma} \qquad \gamma^{\sigma\sigma}$$

$$\gamma_{\tau\sigma} \qquad \gamma_{\sigma\sigma}^{-1}\left[1-\gamma_{\tau\sigma}^{2}\right]$$

$$S_P = -\frac{1}{4\pi\alpha'}\int d\tau\, d\sigma \left[\sqrt{-\gamma}\,\gamma^{\alpha\beta}\,\partial_\alpha X^\mu\,\partial_\beta X_\mu\right]$$

$$= -\frac{1}{4\pi\alpha'}\int d\tau\, d\sigma \Big[-\gamma_{\sigma\sigma}\,\partial_\tau X^\mu\,\partial_\tau X_\mu + 2\,\gamma_{\tau\sigma}\,\partial_\tau X^\mu\,\partial_\sigma X_\mu + \gamma_{\sigma\sigma}^{-1}\left(1-\gamma_{\tau\sigma}^{2}\right)\partial_\sigma X^\mu\,\partial_\sigma X_\mu\Big]$$

$$\gamma_{\tau\sigma}^2 = -1$$

$$\gamma_{\tau\sigma}^2 - 1$$

$$X^\mu = \{X^+, X^-, X^i\}$$

$\ldots_{\tau\sigma}^2 = -1$

$\gamma_{\tau\sigma}^2 - 1 \,]$

$$X^\mu = \{ X^+ , X^- , X^i \}$$

$$\partial_\tau X^+ = 1 \qquad \partial_\sigma X^+ = 0$$

$\gamma_{\tau\sigma}^2 = -1$

$\gamma_{\tau\sigma}^2 - 1$

$$X^\mu = \left\{ X^+ , X^- , X^i \right\}$$

$$\partial_\tau X^+ = 1 \qquad \partial_\sigma X^+ = 0$$

$\Rightarrow$ collect i, j pieces

$$+\gamma_{\sigma\sigma}\left(1-\gamma_{\tau\sigma}\right)\partial_\sigma X^m \partial_\sigma X_m \Big]$$

$$X^m = \left\{ X^+,\, X^-,\, X^i \right\}$$

$$\partial_\tau X^+ = 1 \qquad \partial_\sigma X^+ = 0$$

$\Rightarrow$ collect i, j pieces

$$-\gamma_{\sigma\sigma}\,\partial_\tau X^i\,\partial_\tau X^i + 2\,\cdots\,\partial_\tau X^i\,\partial_\sigma X^i$$

$$+$$

$$+ \gamma_{\sigma\sigma}^{-1}\left(1-\gamma_{\tau\sigma}^2\right)\partial_\sigma X^m \partial_\sigma X_m \Big]$$

$$X^m = \left\{ X^+, X^-, X^i \right\}$$

$$\partial_\tau X^+ = 1 \qquad \partial_\sigma X^+ = 0$$

$\Rightarrow$ collect i, j pieces

$$-\gamma_{\sigma\sigma}\partial_\tau X^i \partial_\tau X^i + 2\gamma_{\tau\sigma}\partial_\tau X^i \partial_\sigma X^i$$

$$+ \gamma_{\sigma\sigma}^{-1}\left(1-\gamma_{\tau\sigma}^2\right)\partial_\sigma$$

$$+ \gamma_{\sigma\sigma}^{-1} \left(1 - \gamma_{\tau\sigma}^2\right) \partial_\sigma X^\mu \partial_\sigma X_\mu \Big]$$

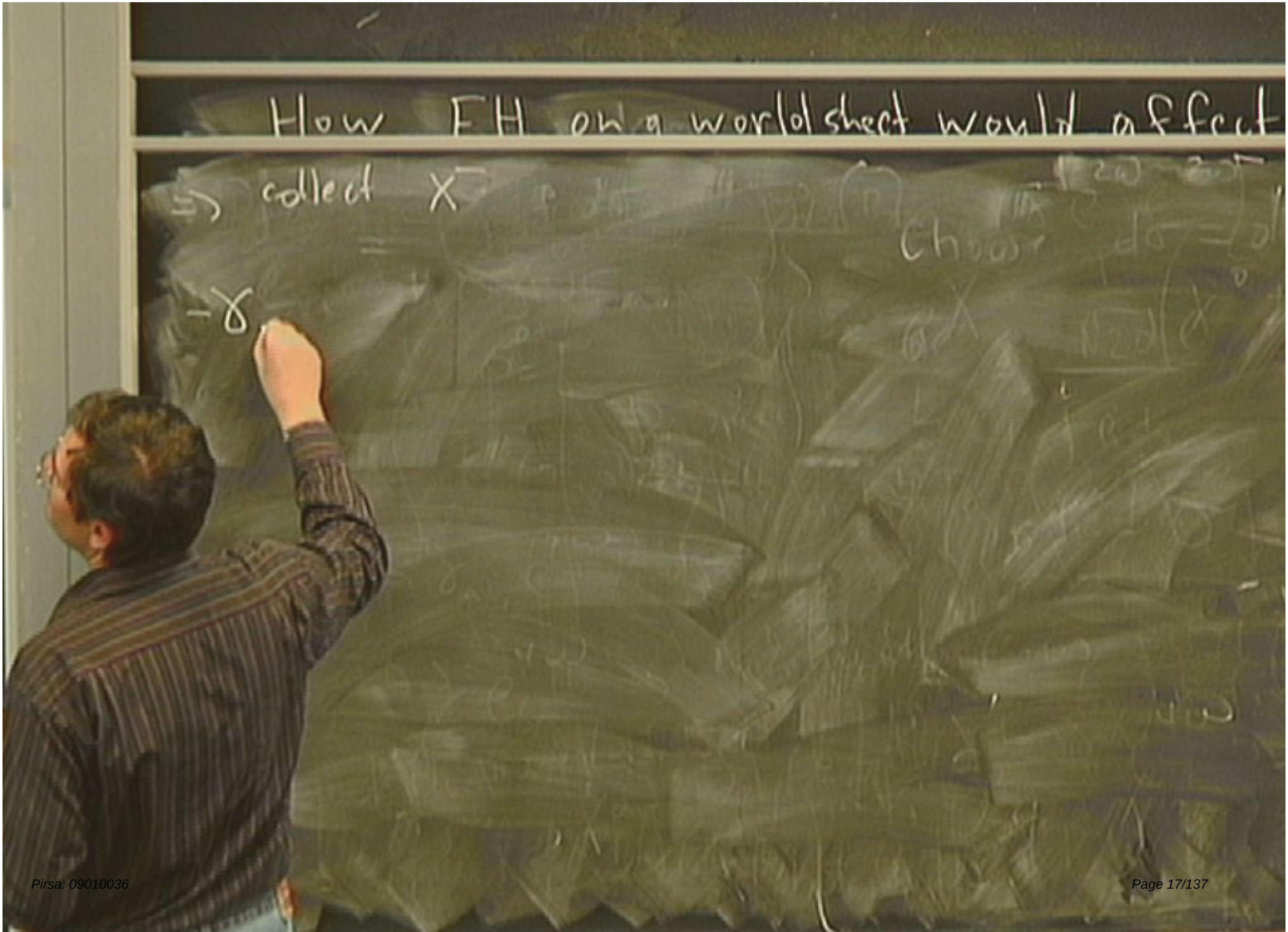
$$X^\mu = \left\{ X^+ , X^- , X^i \right\}$$

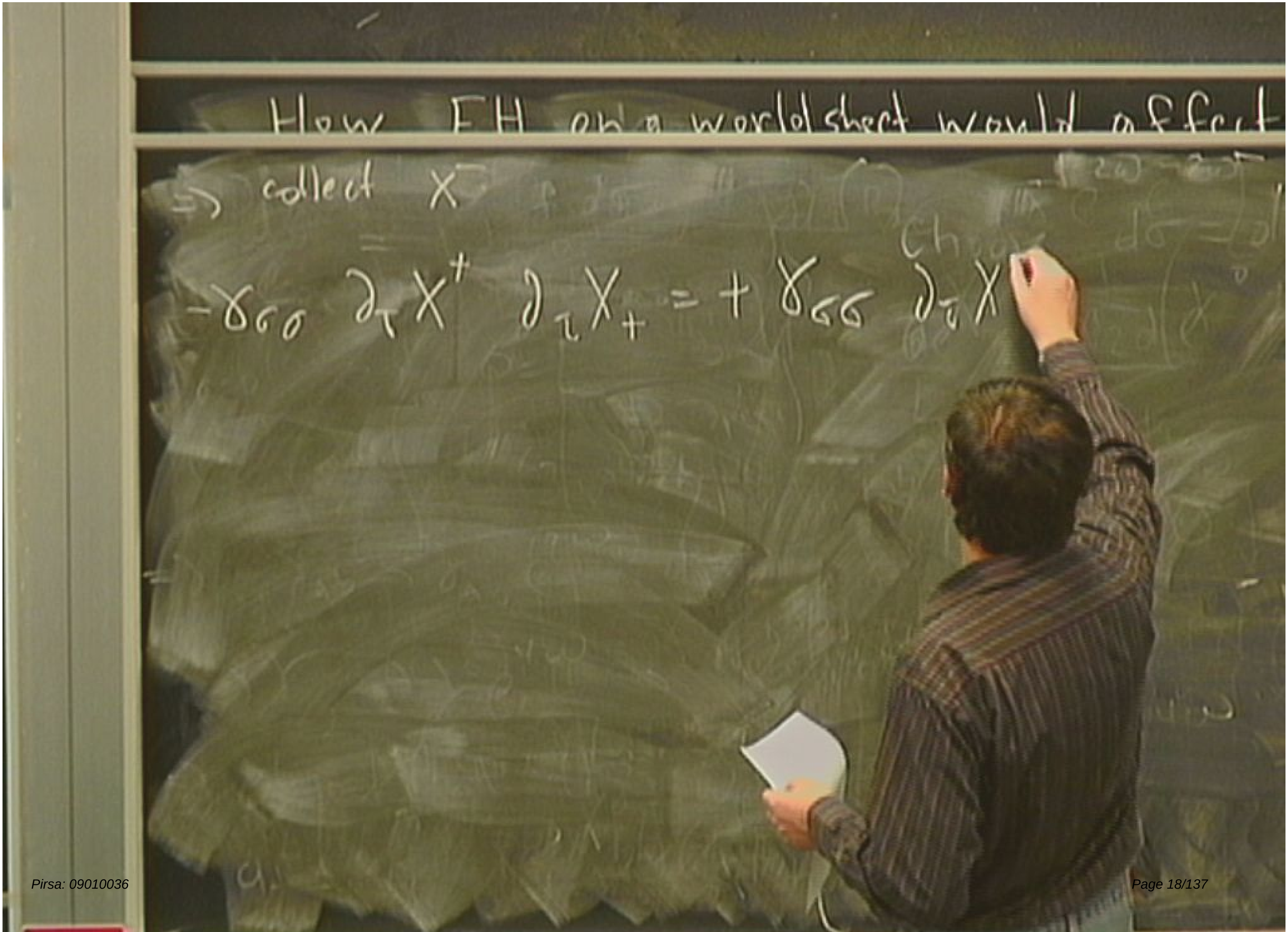
$$\partial_\tau X^+ = 1 \quad \partial_\sigma X^+ = 0$$

$\Rightarrow$ collect i, j pieces

$$- \gamma_{\sigma\sigma} \partial_\tau X^i \partial_\tau X^i + 2 \gamma_{\tau\sigma} \partial_\tau X^i \partial_\sigma X^i$$

$$+ \gamma_{\sigma\sigma}^{-1} \left(1 - \gamma_{\tau\sigma}^2\right) \partial_\sigma X^i \partial_\sigma X^i$$

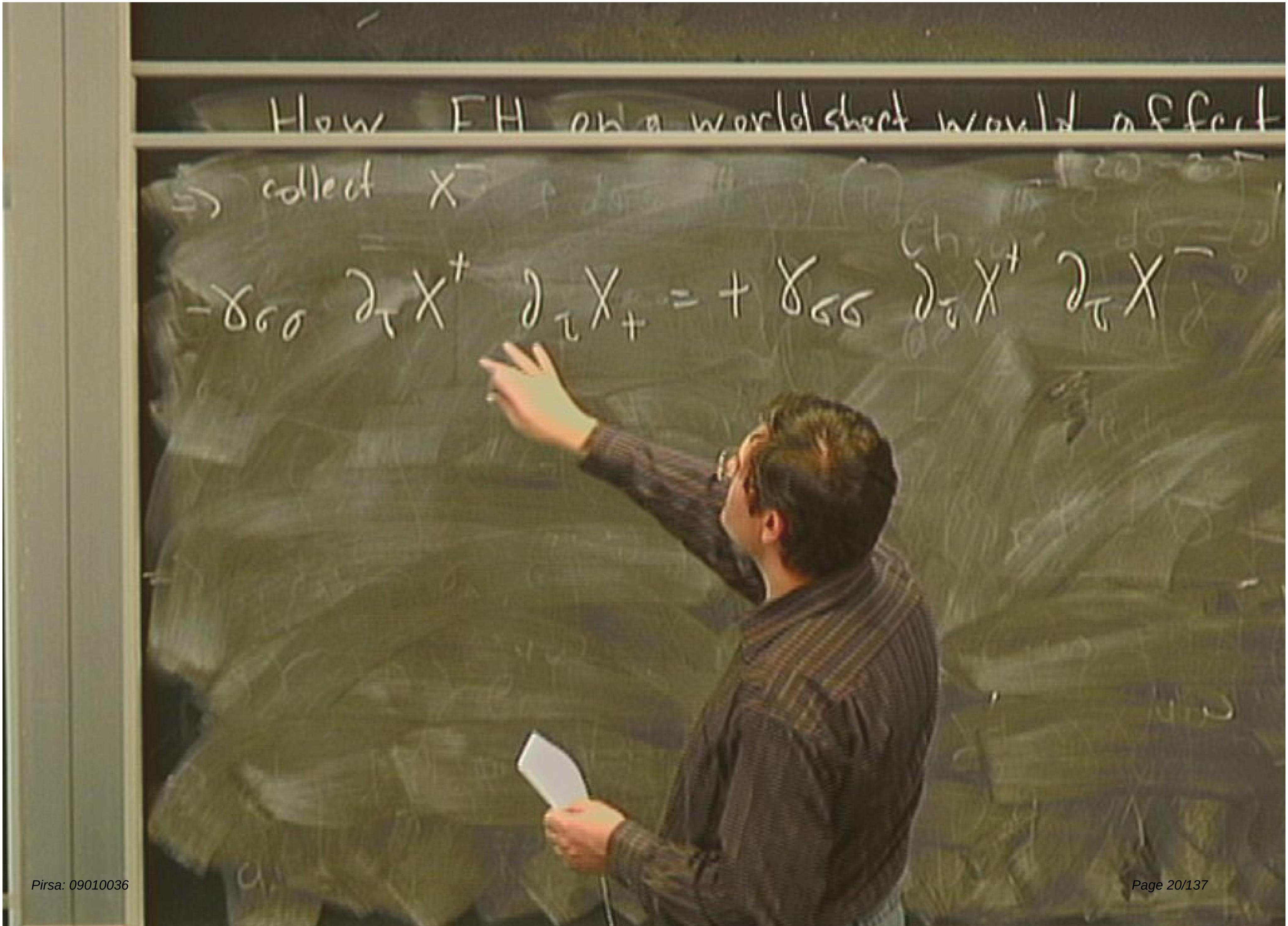

How FH on a worldsheet would affect
⇒ collect X⁻
Choose


How FH on a worldsheet would affect
⇒ collect X⁻
−γσσ ∂τ X⁺ ∂τ X₊ = + γσσ ∂τ X

How FH on a worldsheet would affect

$\Rightarrow$ collect X^-

$$-\gamma_{\sigma\sigma}\,\partial_\tau X^+\,\partial_\tau X_+ = +\gamma_{\sigma\sigma}\,\partial_\tau X^+\,\partial_\tau X$$


How FH on a worldsheet would affect
⇒ collect X^-
$-\gamma_{\sigma\sigma}\,\partial_\tau X^+\,\partial_\tau X_+ = +\gamma_{\sigma\sigma}\,\partial_\tau X^+\,\partial_\tau X^-$

How EH on a worldsheet would affect

$\Rightarrow$ collect X^-

$$-\gamma_{\sigma\sigma}\,\partial_\tau X^+\,\partial_\tau X_+ = +\gamma_{\sigma\sigma}\,\partial_\tau X^+\,\partial_\tau X^-$$

$\oplus$

$$-\gamma_{\sigma\sigma}\,\partial_\tau X^-\,\partial_\tau X_- = +\gamma_{\sigma\sigma}\,\partial_\tau X^-\,\partial_\tau X^+$$

How FH on a worldsheet would affect

$\Rightarrow$ collect X^-

$$-\gamma_{\sigma\sigma}\,\partial_\tau X^+\,\partial_\tau X_+ = +\gamma_{\sigma\sigma}\,\partial_\tau X^+\,\partial_\tau X^-$$

$\oplus$

$$-\gamma_{\sigma\sigma}\,\partial_\tau X^-\,\partial_\tau X_- = +\gamma_{\sigma\sigma}\,\partial_\tau X^-\,\partial_\tau X^+$$

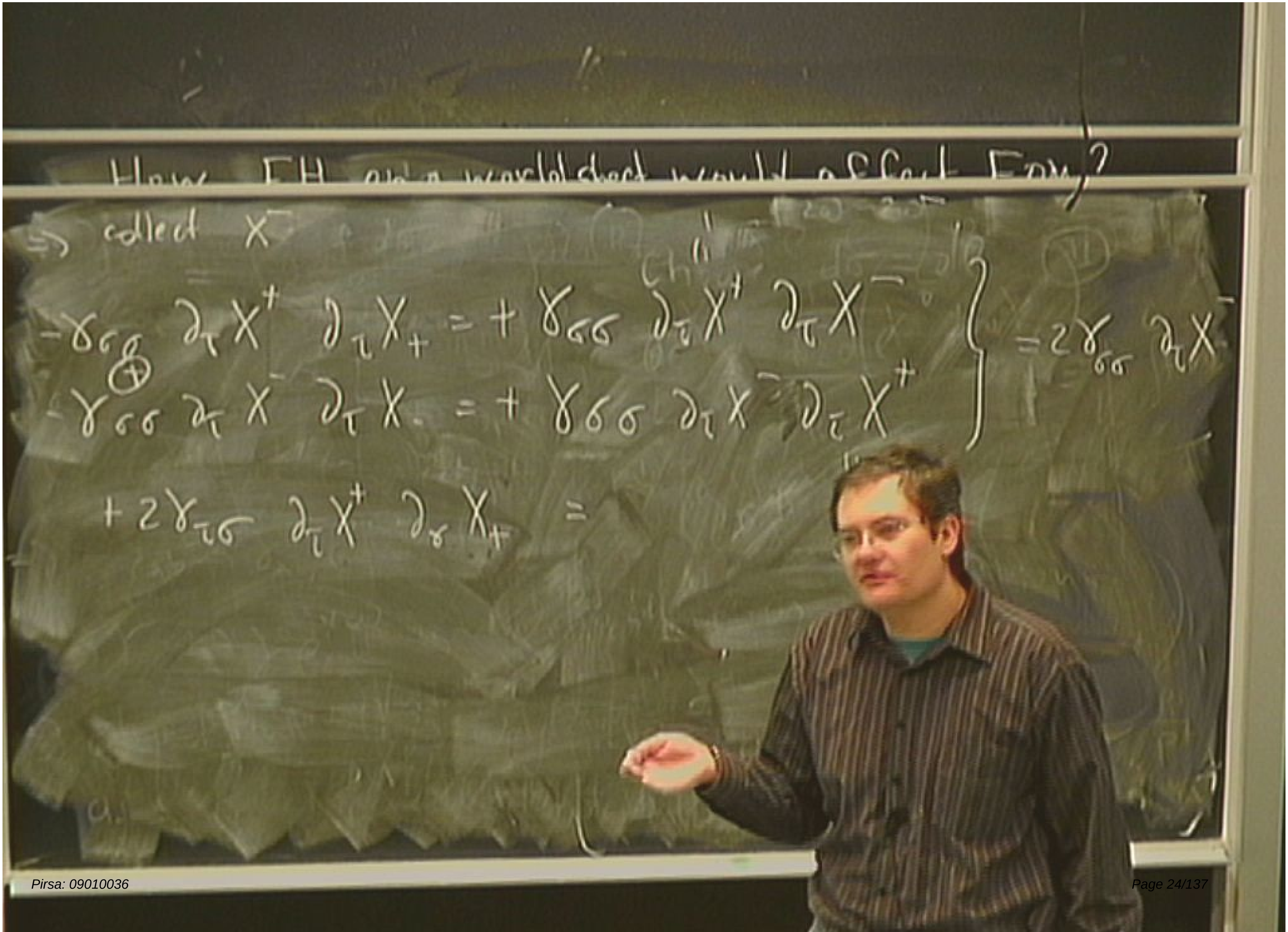
How EH on a worldsheet would affect Eom?

$\Rightarrow$ collect X^-

$$-\gamma_{\sigma\sigma}\,\partial_\tau X^+\,\partial_\tau X_+ = +\gamma_{\sigma\sigma}\,\partial_\tau X^+\,\partial_\tau X^-$$

$$-\gamma_{\sigma\sigma}\,\partial_\tau X^-\,\partial_\tau X_- = +\gamma_{\sigma\sigma}\,\partial_\tau X^-\,\partial_\tau X^+$$

$$= 2\gamma_{\sigma\sigma}\,\partial_\tau X^-$$


⇒ collect X⁻
−γσσ ∂τ X⁺ ∂τ X₊ = + γσσ ∂τ X⁺ ∂τ X⁻
−γσσ ∂τ X⁻ ∂τ X₋ = + γσσ ∂τ X⁻ ∂τ X⁺
= 2 γσσ ∂τ X⁻
+ 2 γτσ ∂τ X⁺ ∂σ X₊ =

How EH on a worldsheet would affect Eqn?

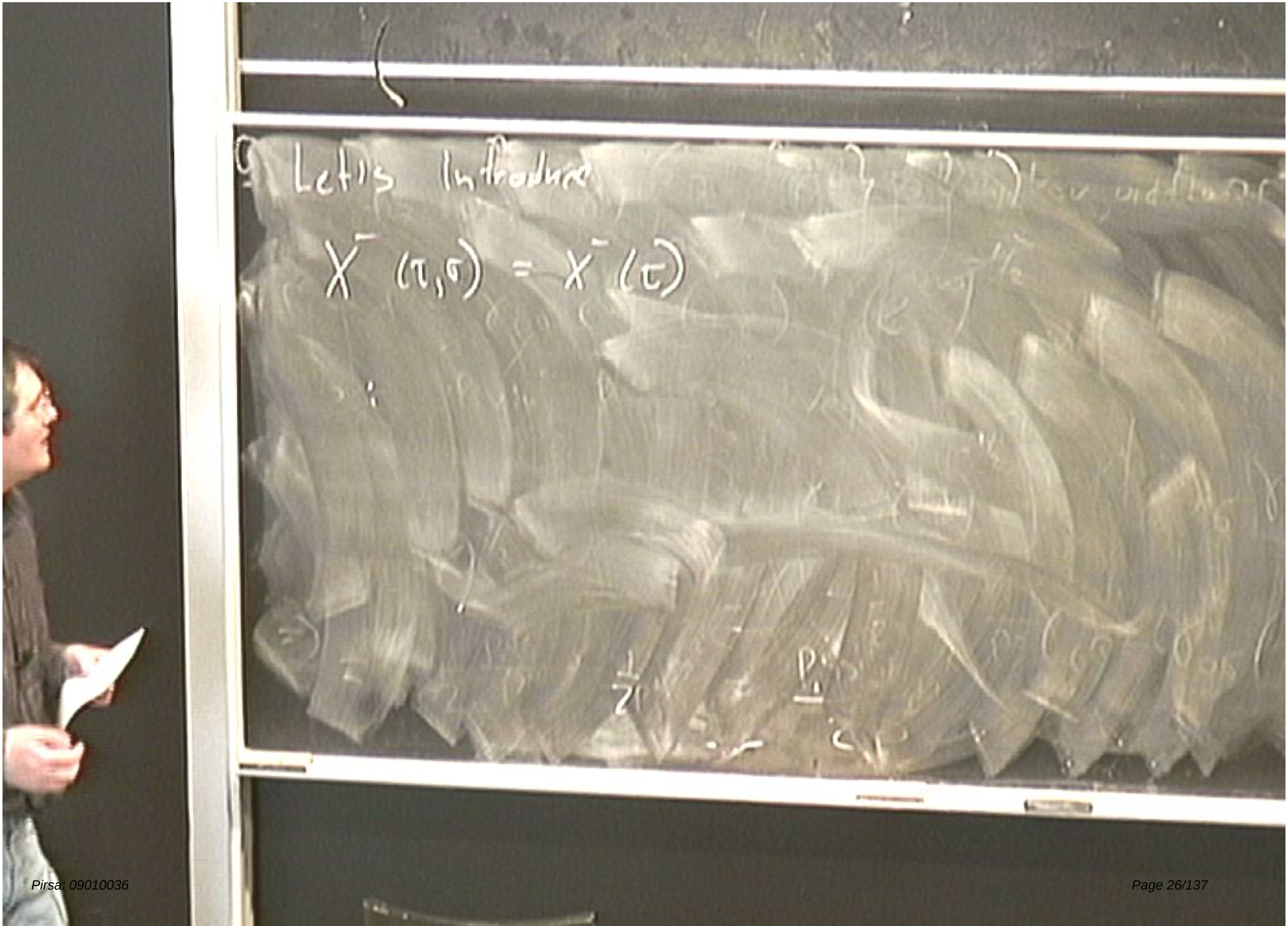
$\Rightarrow$ collect X^-

$$-\gamma_{\sigma\sigma}\, \partial_\tau X^+\, \partial_\tau X_+ = +\gamma_{\sigma\sigma}\, \partial_\tau X^+\, \partial_\tau X^-$$

$$-\gamma_{\sigma\sigma}\, \partial_\tau X^-\, \partial_\tau X_- = +\gamma_{\sigma\sigma}\, \partial_\tau X^-\, \partial_\tau X^+$$

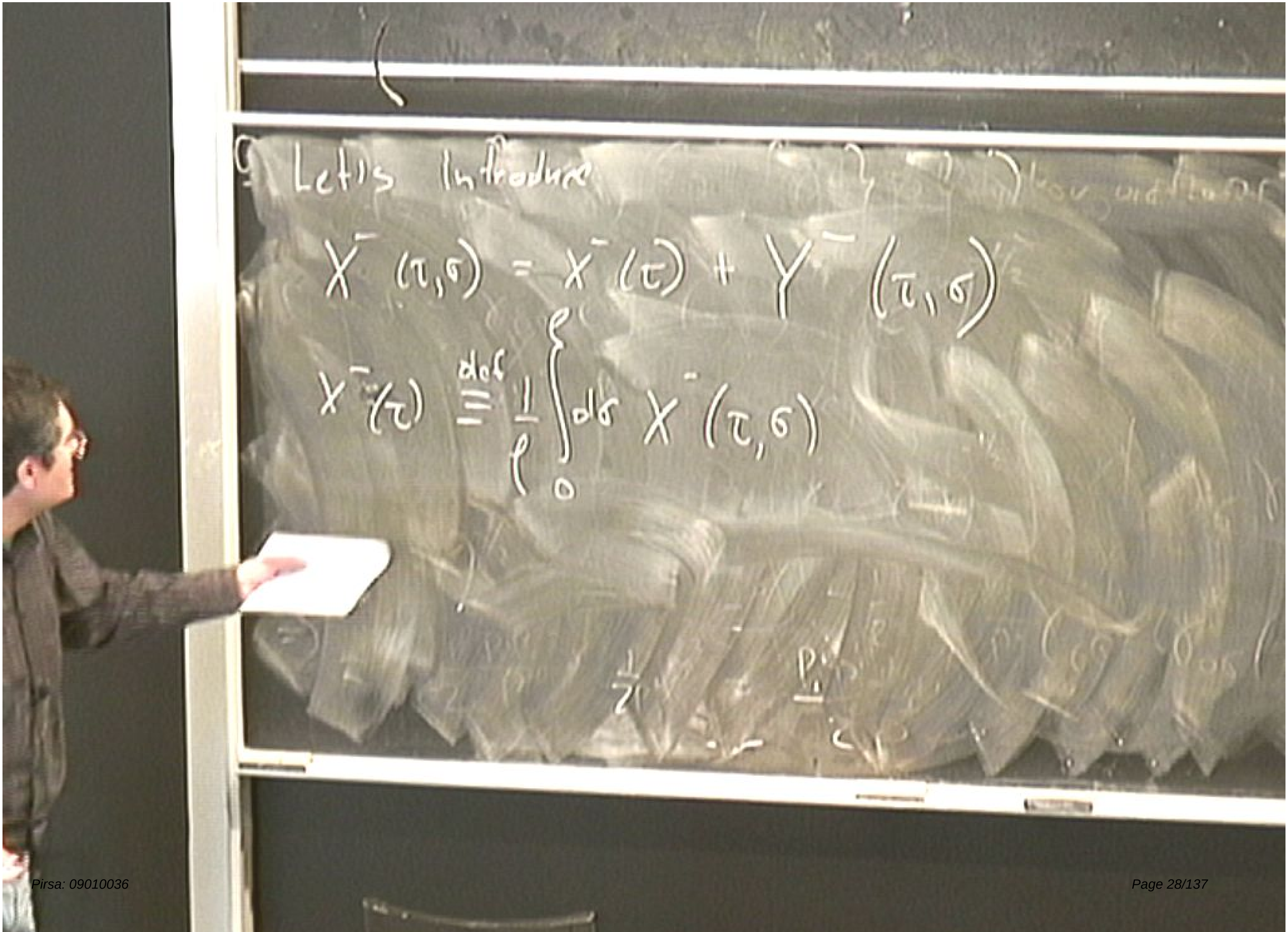
$$= 2\gamma_{\sigma\sigma}\, \partial_\tau X$$

$$+2\gamma_{\tau\sigma}\, \partial_\tau X^+\, \partial_\sigma X_+ = -2\gamma_{\tau\sigma}\, \partial_\sigma X^-$$


Let's introduce
$X^-(\tau,\sigma) = x^-(\tau)$

Let's Introduce

$$X^-(\tau,\sigma) = x^-(\tau) + Y^-(\tau,\sigma)$$


Let's Introduce
$X^-(\tau,\sigma) = x^-(\tau) + Y^-(\tau,\sigma)$
$x^-(\tau) \stackrel{def}{\equiv} \frac{1}{\ell}\int_0^\ell d\sigma\, X^-(\tau,\sigma)$

Let's Introduce

$$X^-(\tau,\sigma) = x^-(\tau) + Y^-(\tau,\sigma)$$

$$x^-(\tau) \overset{\text{def}}{\equiv} \frac{1}{\ell}\int_0^\ell d\sigma\, X^-(\tau,\sigma) \Rightarrow \int_0^\ell d\sigma\, Y^- = 0$$

Let's introduce

$$X^-(\tau,\sigma) = x^-(\tau) + Y^-(\tau,\sigma)$$

$$x^-(\tau) \stackrel{\text{def}}{=} \frac{1}{\ell}\int_0^\ell d\sigma\, X^-(\tau,\sigma) \Rightarrow \int_0^\ell d\sigma\, Y^-(\tau,\sigma) = 0$$

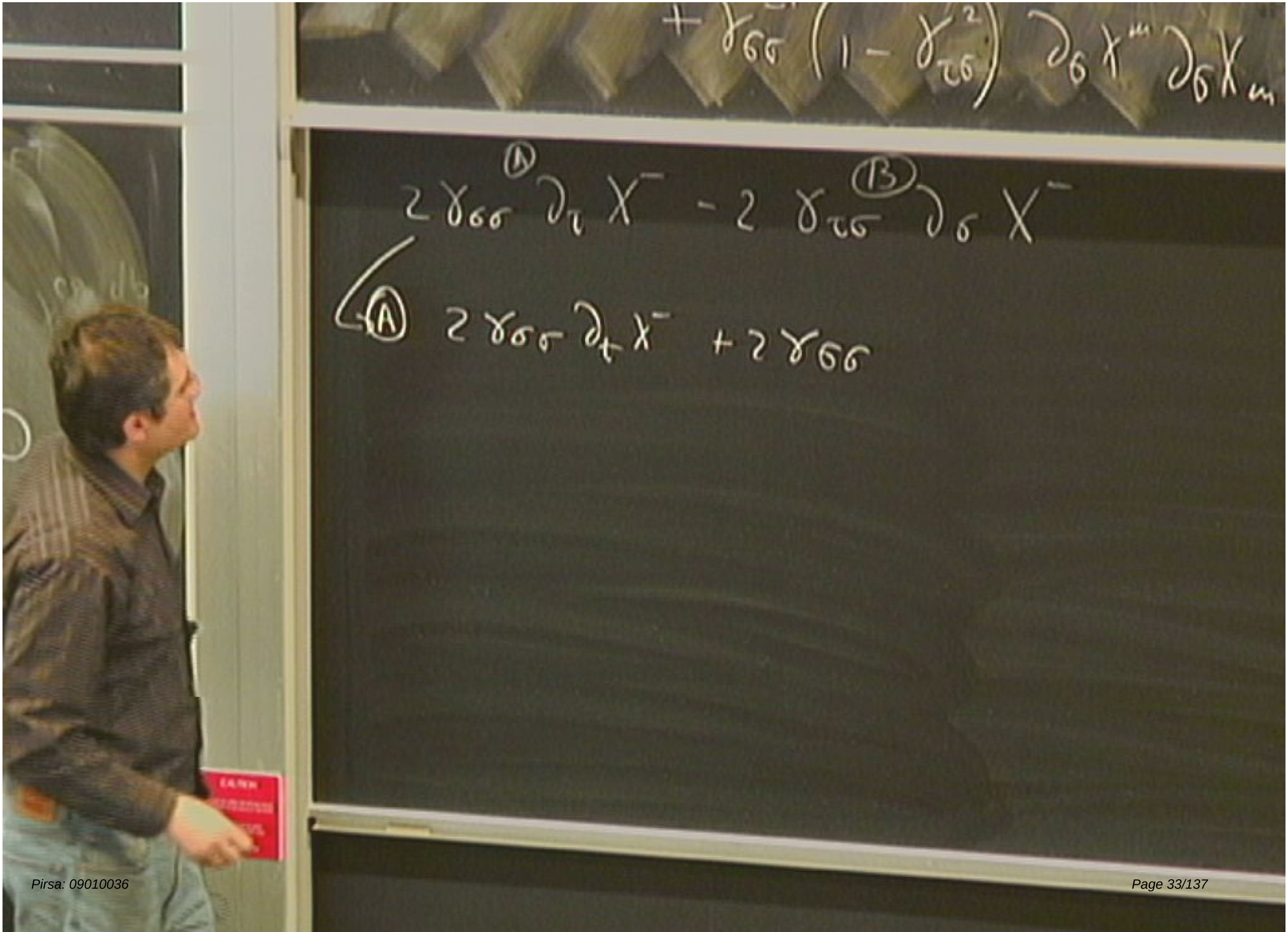
Let's introduce

$$X^-(\tau,\sigma) = x^-(\tau) + Y^-(\tau,\sigma)$$

$$x^-(\tau) \stackrel{\text{def}}{\equiv} \frac{1}{\ell}\int_0^\ell d\sigma\, X^-(\tau,\sigma) \Rightarrow \int_0^\ell d\sigma\, Y^-(\tau,\sigma) = 0$$

$$+ \gamma_{\sigma\sigma} \left(1 - \gamma_{\tau\sigma}^2\right) \partial_\sigma X^m \partial_\sigma X_m$$

$$2 \gamma_{\sigma\sigma} \partial_\tau X^- - 2 \gamma_{\tau\sigma} \partial_\sigma X^-$$


$+\gamma_{\sigma\sigma}\left(1-\gamma_{\tau\sigma}^{2}\right)\partial_\sigma X^{m}\partial_\sigma X_{m}$
A
$2\gamma_{\sigma\sigma}\partial_\tau X^{-}-2\gamma_{\tau\sigma}\partial_\sigma X^{-}$
B
A $2\gamma_{\sigma\sigma}\partial_\tau X^{-}+2\gamma_{\sigma\sigma}$

$$\partial_\sigma X_m$$

$$2\gamma_{\sigma\sigma}\,\partial_\tau X^- - 2\,\gamma_{\tau\sigma}\,\partial_\sigma X^-$$

$$(A)\quad 2\gamma_{\sigma\sigma}\,\partial_t X^- + \underbrace{2\gamma_{\sigma\sigma}\,\partial_\tau Y^-}_{\text{does not matter}}$$

$$2\gamma_{\sigma\sigma}\partial_\tau X^- - 2\gamma_{\tau\sigma}\partial_\sigma X^-$$

$$A\quad 2\gamma_{\sigma\sigma}\partial_\tau X^- + \underbrace{2\gamma_{\sigma\sigma}\partial_\tau Y^-}_{\text{does not matter}}$$

$$\int_{-\infty}^{+\infty} d\tau \int_0^{\ell} d\sigma$$

$$2\gamma_{\sigma\sigma}\,\partial_\tau X^- - 2\gamma_{\tau\sigma}\,\partial_\sigma X^-$$

$$2\gamma_{\sigma\sigma}\,\partial_\tau X^- + \underbrace{2\gamma_{\sigma\sigma}\,\partial_\tau Y^-}_{\text{does not matter}}$$

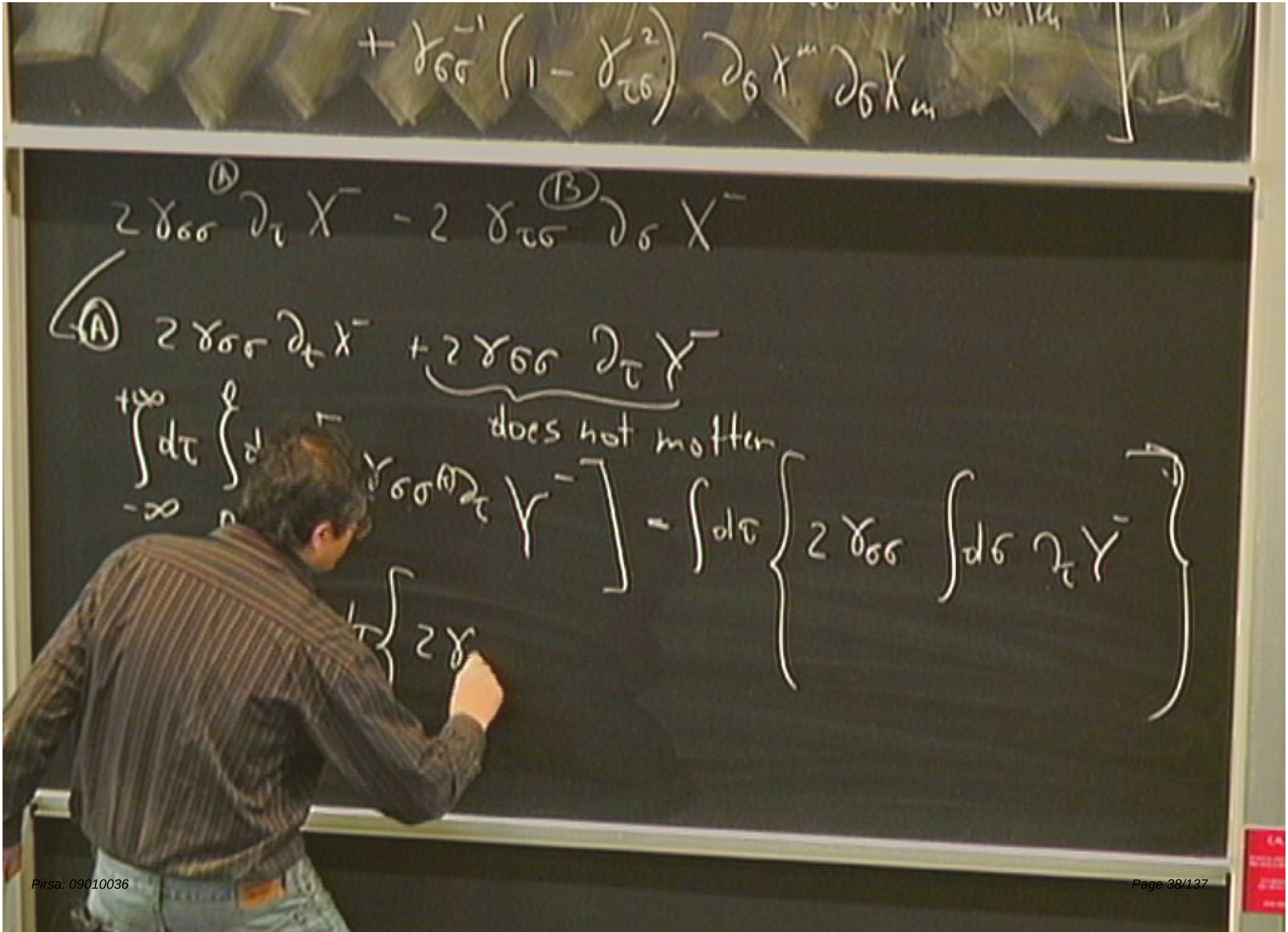
$$\int_{-\infty}^{+\infty} d\tau \int_0^{\ell} d\sigma \left[2\gamma_{\sigma\sigma}^{(1)}\,\partial_\tau Y^-\right] =$$

$$\partial_\sigma X^{m} \quad \partial_\sigma X_m$$

$$2\,\gamma_{\sigma\sigma}^{\;A}\,\partial_\tau X^- - 2\,\gamma_{\tau\sigma}^{\;B}\,\partial_\sigma X^-$$

$$A \quad 2\,\gamma_{\sigma\sigma}\,\partial_t X^- + \underbrace{2\,\gamma_{\sigma\sigma}\,\partial_\tau Y^-}_{\text{does not matter}}$$

$$\int_{-\infty}^{+\infty} d\tau \int_0^{\ell} d\sigma \left[2\,\gamma_{\sigma\sigma}^{(1)}\,\partial_\tau Y^- \right] - \int d\tau \left\{ 2\,\gamma_{\sigma\sigma} \right.$$


$+ \gamma_{\sigma\sigma}^{-1}\left(1-\gamma_{\tau\sigma}^{2}\right)\partial_\sigma X^{m}\,\partial_\sigma X_{m}$
$2\gamma_{\sigma\sigma}\,\partial_\tau X^{-} - 2\,\gamma_{\tau\sigma}\,\partial_\sigma X^{-}$
A
B
$2\gamma_{\sigma\sigma}\,\partial_t X^{-} + 2\gamma_{\sigma\sigma}\,\partial_\tau Y^{-}$
does not matter
$- \int d\tau \left\{ 2\gamma_{\sigma\sigma} \int d\sigma\, \partial_\tau Y^{-} \right\}$

$$+\gamma_{\sigma\sigma}^{-1}\left(1-\gamma_{\tau\sigma}^{2}\right)\partial_\sigma X^{m}\partial_\sigma X_{m}\Big]$$

$$\overset{A}{2\gamma_{\sigma\sigma}\partial_\tau X^-} - \overset{B}{2\gamma_{\tau\sigma}\partial_\sigma X^-}$$

$$A\quad 2\gamma_{\sigma\sigma}\partial_\tau X^- + \underbrace{2\gamma_{\sigma\sigma}\partial_\tau Y^-}_{\text{does not matter}}$$

$$\int_{-\infty}^{+\infty} d\tau \int_0^{\ell} d\sigma \left[2\gamma_{\sigma\sigma}(\tau)\partial_\tau Y^-\right] = \int d\tau \left\{2\gamma_{\sigma\sigma}\int d\sigma\, \partial_\tau Y^-\right\}$$

$$= \int d\tau \Big\{ 2\gamma_{\sigma\sigma}$$

$$+\gamma_{\sigma\sigma}^{-1}\left(1-\gamma_{\tau\sigma}^{2}\right)\partial_\sigma X^{\mu}\partial_\sigma X_{\mu}\Big]$$

$$\overset{(A)}{2\gamma_{\sigma\sigma}\partial_\tau X^-}-\overset{(B)}{2\gamma_{\tau\sigma}\partial_\sigma X^-}$$

$$(A)\quad 2\gamma_{\sigma\sigma}\partial_\tau x^- + \underbrace{2\gamma_{\sigma\sigma}\partial_\tau Y^-}_{\text{does not matter}}$$

$$\int_{-\infty}^{+\infty}d\tau\int_0^{\ell}d\sigma\left[2\gamma_{\sigma\sigma}(\tau)\partial_\tau Y^-\right]=\int d\tau\left\{2\gamma_{\sigma\sigma}\int d\sigma\,\partial_\tau Y^-\right\}$$

$$=\int d\tau\left\{2\gamma_{\sigma\sigma}\partial_\tau\right.$$

$$+\gamma_{\sigma\sigma}^{-1}\left(1-\gamma_{\tau\sigma}^{2}\right)\partial_\sigma X^m \partial_\sigma X_m \Big]$$

$$2\gamma_{\sigma\sigma}\,\partial_\tau X^- \;-\; 2\,\gamma_{\tau\sigma}\,\partial_\sigma X^-$$

A, B

A: $$2\gamma_{\sigma\sigma}\,\partial_\tau X^- + \underbrace{2\gamma_{\sigma\sigma}\,\partial_\tau Y^-}_{\text{does not matter}}$$

$$\int_{-\infty}^{+\infty} d\tau \int_0^{\ell} d\sigma \left[2\gamma_{\sigma\sigma}(\tau)\,\partial_\tau Y^-\right] = \int d\tau \left\{ 2\gamma_{\sigma\sigma} \int d\sigma\, \partial_\tau Y^- \right\}$$

$$= \int d\tau \left\{ 2\gamma_{\sigma\sigma}\,\partial_\tau \int_0^{\ell} d\sigma\, Y^- \right\}$$

$$+\gamma_{\sigma\sigma}^{-1}\left(1-\gamma_{\tau\sigma}^{2}\right)\partial_\sigma X^m \partial_\sigma X_m$$

$$2\gamma_{\sigma\sigma}\,\partial_\tau X^- - 2\,\gamma_{\tau\sigma}\,\partial_\sigma X^-$$

A B

A $$2\gamma_{\sigma\sigma}\partial_\tau X^- + \underbrace{2\gamma_{\sigma\sigma}\,\partial_\tau Y^-}_{\text{does not matter}}$$

$$\int_{-\infty}^{+\infty} d\tau \int_0^{\ell} d\sigma \left[2\gamma_{\sigma\sigma}(\tau)\partial_\tau Y^-\right] = \int d\tau \left\{2\gamma_{\sigma\sigma}\int d\sigma\, \partial_\tau Y^-\right\}$$

$$= \int d\tau \left\{2\gamma_{\sigma\sigma}\partial_\tau \underbrace{\int_0^{\ell} d\sigma\, Y^-}_{=0}\right\}$$

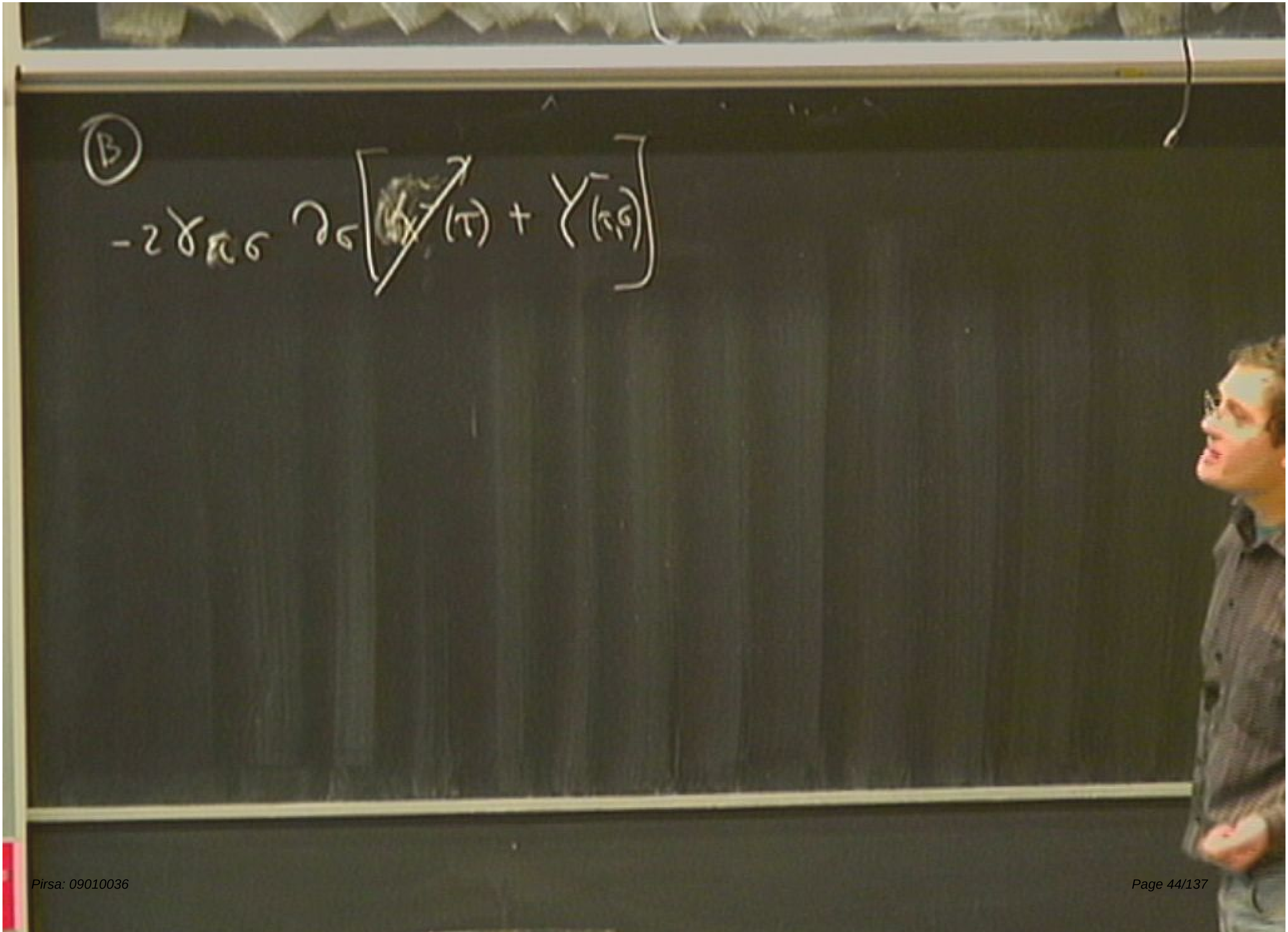
$$+\gamma_{\sigma\sigma}^{-1}\left(1-\gamma_{\tau\sigma}^{2}\right)\partial_\sigma X^m \partial_\sigma X_m \Big]$$

$$\overset{\text{A}}{2\gamma_{\sigma\sigma}\partial_\tau X^-} - \overset{\text{B}}{2\gamma_{\tau\sigma}\partial_\sigma X^-}$$

$$\text{A} \quad 2\gamma_{\sigma\sigma}\partial_\tau X^- + \underbrace{\cancel{2\gamma_{\sigma\sigma}\partial_\tau Y^-}}_{\text{does not matter}}$$

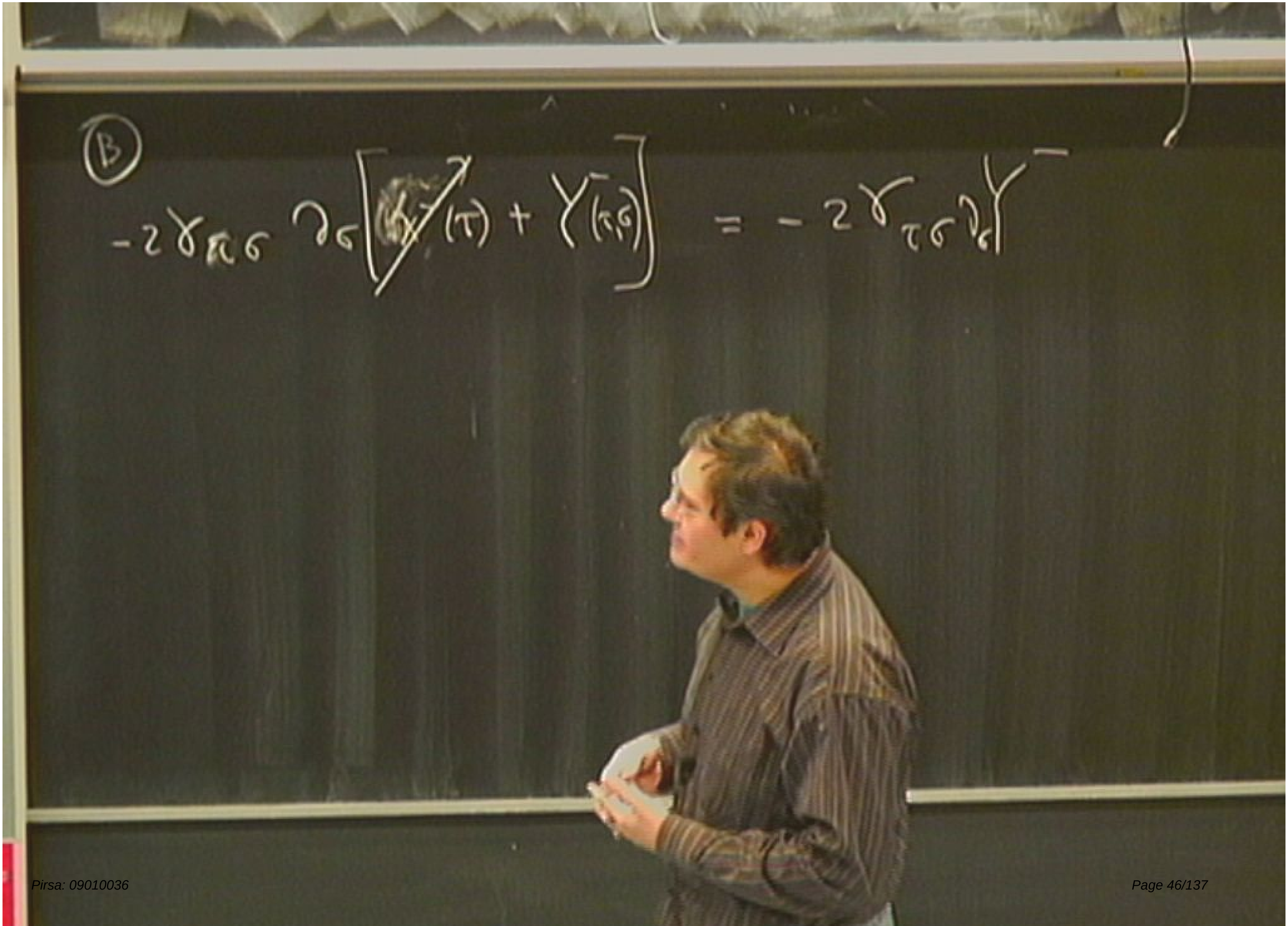
$$\int_{-\infty}^{+\infty} d\tau \int_0^{\ell} d\sigma \left[2\gamma_{\sigma\sigma}(\tau)\partial_\tau Y^-\right] = \int d\tau \left\{2\gamma_{\sigma\sigma}\int d\sigma\, \partial_\tau Y^-\right\}$$

$$= \int d\tau \left\{2\gamma_{\sigma\sigma}\partial_\tau \underbrace{\int_0^{\ell} d\sigma\, Y^-}_{=0}\right\} = 0$$


B
$-2\gamma_{\pi\sigma}\,\partial_\sigma\left[\cancel{\gamma}(\tau) + \gamma(\pi,\sigma)\right]$

(B)

$$-2\gamma_{\tau\sigma}\,\partial_\sigma\left[\cancel{Y^-}(\tau) + Y^-(\tau,\sigma)\right] = -2\gamma_{\tau\sigma}\,Y^-$$


B
$-2\gamma_{\tau\sigma}\,\partial_\sigma\left[\cancel{}(\tau) + Y^{-}(\tau,\sigma)\right] = -2\gamma_{\tau\sigma}\,\partial_\sigma Y^{-}$

B

$$-2\gamma_{\tau\sigma}\,\partial_\sigma\left[\cancel{\Upsilon^-}(\tau) + \Upsilon^-(\tau,\sigma)\right] = -2\gamma_{\tau\sigma}\,\partial_\sigma \Upsilon^-$$

$$+\gamma_{\sigma\sigma}^{-1}\left(1-\gamma_{\tau\sigma}^{2}\right)\partial_\sigma X^m \partial_\sigma X_m \Big]$$

$$2\gamma_{\sigma\sigma}\overset{(A)}{\partial_\tau X^-} - 2\gamma_{\tau\sigma}\overset{(B)}{\partial_\sigma X^-}$$

$$(A)\quad 2\gamma_{\sigma\sigma}\partial_t X^- + \underbrace{\cancel{2\gamma_{\sigma\sigma}}\,\partial_\tau Y^-(\tau,\sigma)}_{\text{does not matter}}$$

$$\int_{-\infty}^{+\infty} d\tau \int_0^\ell d\sigma \left[2\gamma_{\sigma\sigma}\partial_\tau Y^-\right] - \int d\tau \left\{ 2\gamma_{\sigma\sigma}\int d\sigma\, \partial_\tau Y^- \right\}$$

$$= \int d\tau \left\{ 2\gamma_{\sigma\sigma}\partial_\tau \int_0^\ell d\sigma \overset{0}{\overset{\shortparallel}{Y^-}} \right\} = 0$$

(B)

$$-2\gamma_{\tau\sigma}\,\partial_\sigma\left[\cancel{y^-(\tau)} + Y^-(\tau,\sigma)\right] = -2\gamma_{\tau\sigma}\,\partial_\sigma Y^-$$

Y^- is not a dynamical field, but rather is a sophisticated Lag multiplier.

(B)

$$-2\gamma_{\tau\sigma}\,\partial_\sigma\left[\cancel{h_Y}(\tau) + Y^-(\tau,\sigma)\right] = -2\gamma_{\tau\sigma}\,\partial_\sigma Y^-$$

Y^- is not a dynamical field, but rather is a sofisticated Lag multiplier.

$\Rightarrow$ It will

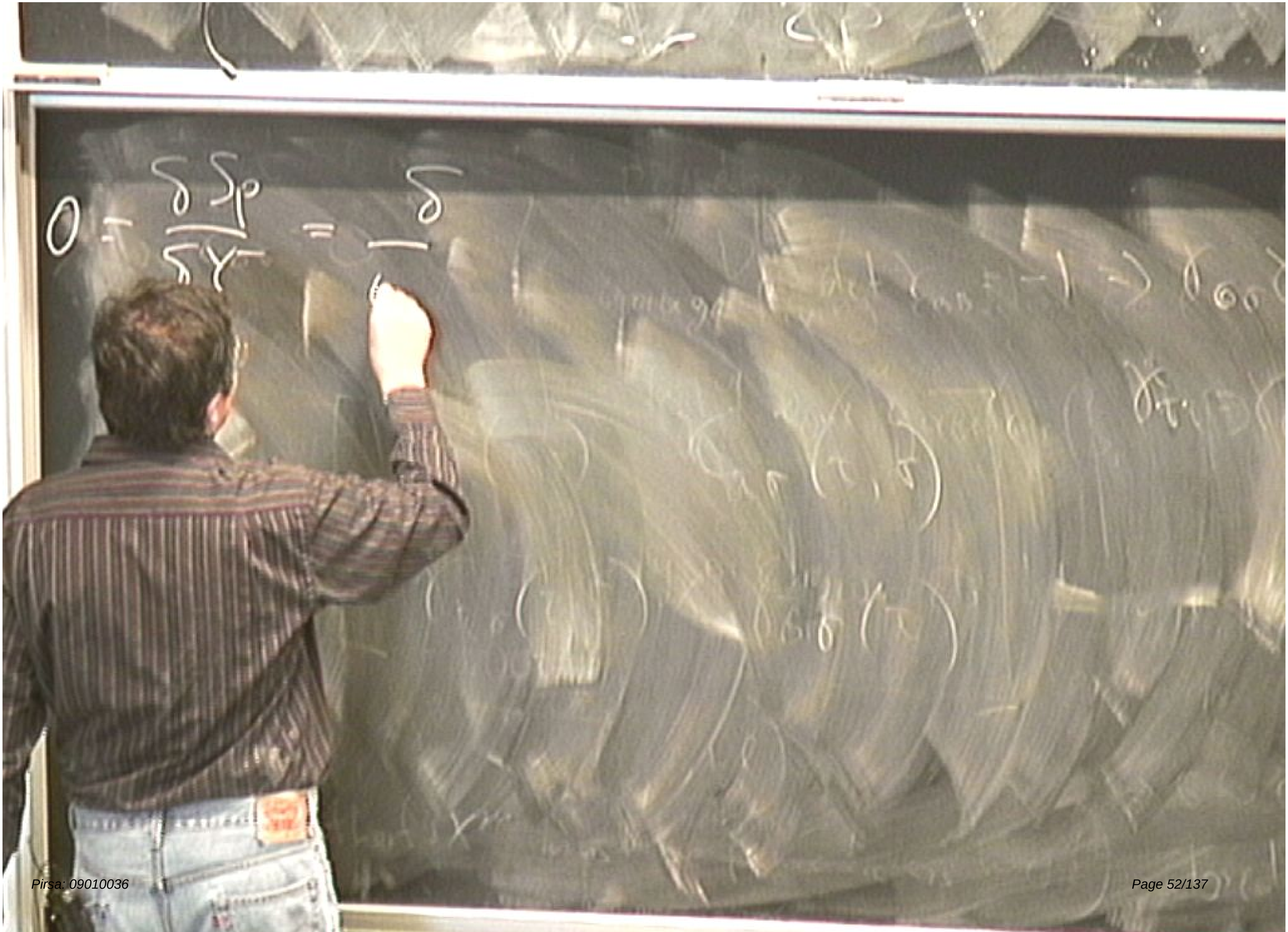
(B)

$$-2\gamma_{\tau\sigma}\,\partial_\sigma\left[\cancel{b^{-}(\tau)} + Y^{-}(\tau,\sigma)\right] = -2\gamma_{\tau\sigma}\partial_\sigma Y^{-}$$

Y^{-} is not a dynamical field, but rather

is a sofisticated Lag multiplier.

$\Rightarrow$ It will lead us to $\boxed{\gamma_{\tau\sigma}(\tau,\sigma) = 0}$



$$0 = \delta S_P = \delta \int d\tau\, d\sigma$$

$$0 = \delta S_P = \delta \int d\tau \, d\sigma$$

$$0 = \delta S_P = \delta \int d\tau\, d\sigma \left(-2\gamma_{\tau\sigma}\, \partial_\sigma\, \delta Y^- \right)$$

$$0 = \delta S_P = \int d\tau\, d\sigma \left(-2\, \gamma_{\tau\sigma}\, \partial_\sigma \delta Y^- \right)$$

$$= \int d\tau\, d\sigma\, (+2) \left[\partial_\sigma \gamma_{\sigma\sigma} \right] \delta Y^-$$

$$0 = \delta S_P = \int d\tau\, d\sigma \left(-2\,\gamma_{\tau\sigma}\,\partial_\sigma \delta Y^-\right)$$

$$= \int d\tau\, d\sigma\, (+2)\left[\partial_\sigma \gamma_{\tau\sigma}\right] \delta Y^- \; + \; -2\int d\tau\, \gamma_{\tau\sigma}\, \delta Y^- \Big|_{\sigma=0}^{\sigma=\ell}$$

$$0 = \delta S_p = \int d\tau\, d\sigma \left(-2\gamma_{\tau\sigma}\, \partial_\sigma \delta Y^-\right)$$

$$= \int d\tau\, d\sigma\, (+2)\left[\partial_\sigma \gamma_{\tau\sigma}\right] \delta Y^- + -2\int d\tau\, \gamma_{\tau\sigma}\, \delta Y^- \Big|_{\sigma=0}^{\sigma=\ell}$$

N Boundary condition in L-C gauge.

$$\partial^\sigma X^{i\mu} = 0$$

$$0 = \delta S_P = \int d\tau\, d\sigma \left(-2\gamma_{\tau\sigma}\,\partial_\sigma \delta Y^-\right)$$

$$= \int d\tau\, d\sigma\, (+2)\left[\partial_\sigma \gamma_{\tau\sigma}\right]\delta Y^- + -2\int d\tau\, \gamma_{\tau\sigma}\,\delta Y^-\Big|_{\sigma=0}^{\sigma=l}$$

$\hookrightarrow$ N Boundary condi[...] l-c gauge.

$$\partial^\sigma X^\mu = [\ldots]\ \gamma^{\sigma\sigma}\partial_\sigma X^\mu + \gamma^{\sigma\tau}\partial_\tau X^\mu$$

$$0 = \delta S_p = \int d\tau\, d\sigma \left(-2\, \gamma_{\tau\sigma}\, \partial_\sigma \delta Y^- \right)$$

$$= \int d\tau\, d\sigma\, (+2) \left[\partial_\sigma \gamma_{\tau\sigma} \right] \delta Y^- \quad + \quad -2 \int d\tau\, \gamma_{\tau\sigma}\, \delta Y^- \Big|_{\sigma=0}^{\sigma=\ell}$$

$\hookrightarrow$ N Boundary condition in L-C gauge.

$$\partial^\sigma X^\mu = 0 = \gamma^{\sigma\sigma} \partial_\sigma X^\mu + \gamma^{\sigma\tau} \partial_\tau X^\mu$$

$$= \gamma_{\tau\sigma}\, \partial_\tau X^\mu + \gamma_{\tau\sigma}^{-1} \left(1 - \gamma_{\tau\sigma}^2 \right) \partial_\sigma X^\mu = 0$$

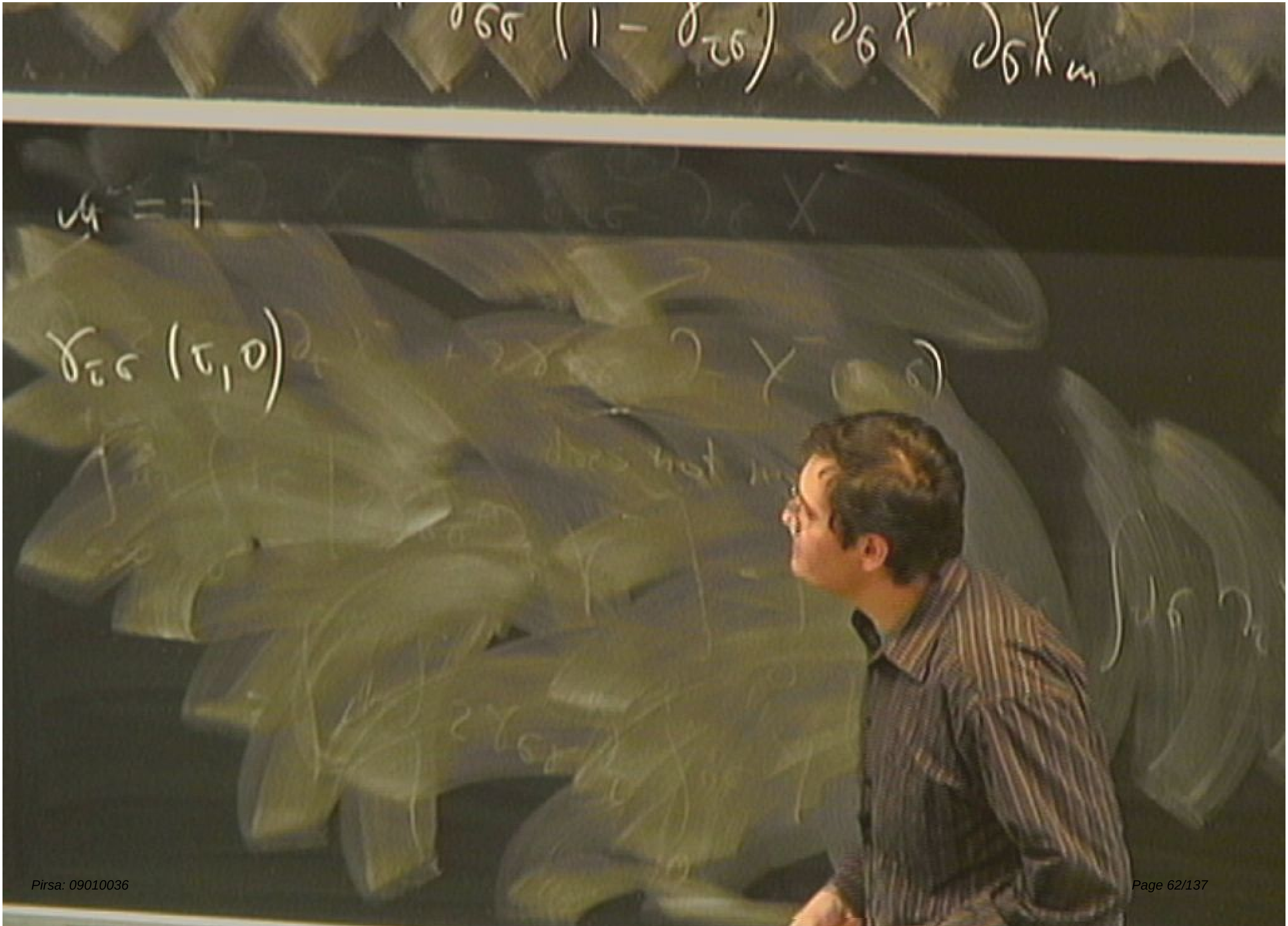
$$0 = \delta S_P = \int d\tau \, d\sigma \left(-2\gamma_{\tau\sigma}\, \partial_\sigma \delta Y^-\right)$$

$$= \int d\tau \, d\sigma \,(+2)\left[\partial_\sigma \gamma_{\tau\sigma}\right] \delta Y^- + -2\int d\tau \, \gamma_{\tau\sigma}\, \delta Y^- \Big|_{\sigma=0}^{\sigma=\ell}$$

↳ N Boundary condition in L-C gauge.

$$\partial^\sigma X^\mu = 0 = \gamma^{\sigma\sigma}\partial_\sigma X^\mu + \gamma^{\sigma\tau}\partial_\tau X^\mu$$

$$= \gamma_{\tau\sigma}\partial_\tau X^\mu + \gamma_{\tau\sigma}^{-1}\left(1-\gamma_{\tau\sigma}^2\right)\partial_\sigma X^\mu \Big|_{\sigma=0,\ell} = 0$$



$$+ \gamma_{\sigma\sigma}^{-1}\left(1-\gamma_{\tau\sigma}^{2}\right)\partial_\sigma X^\mu \partial_\sigma X_\mu \Big]$$

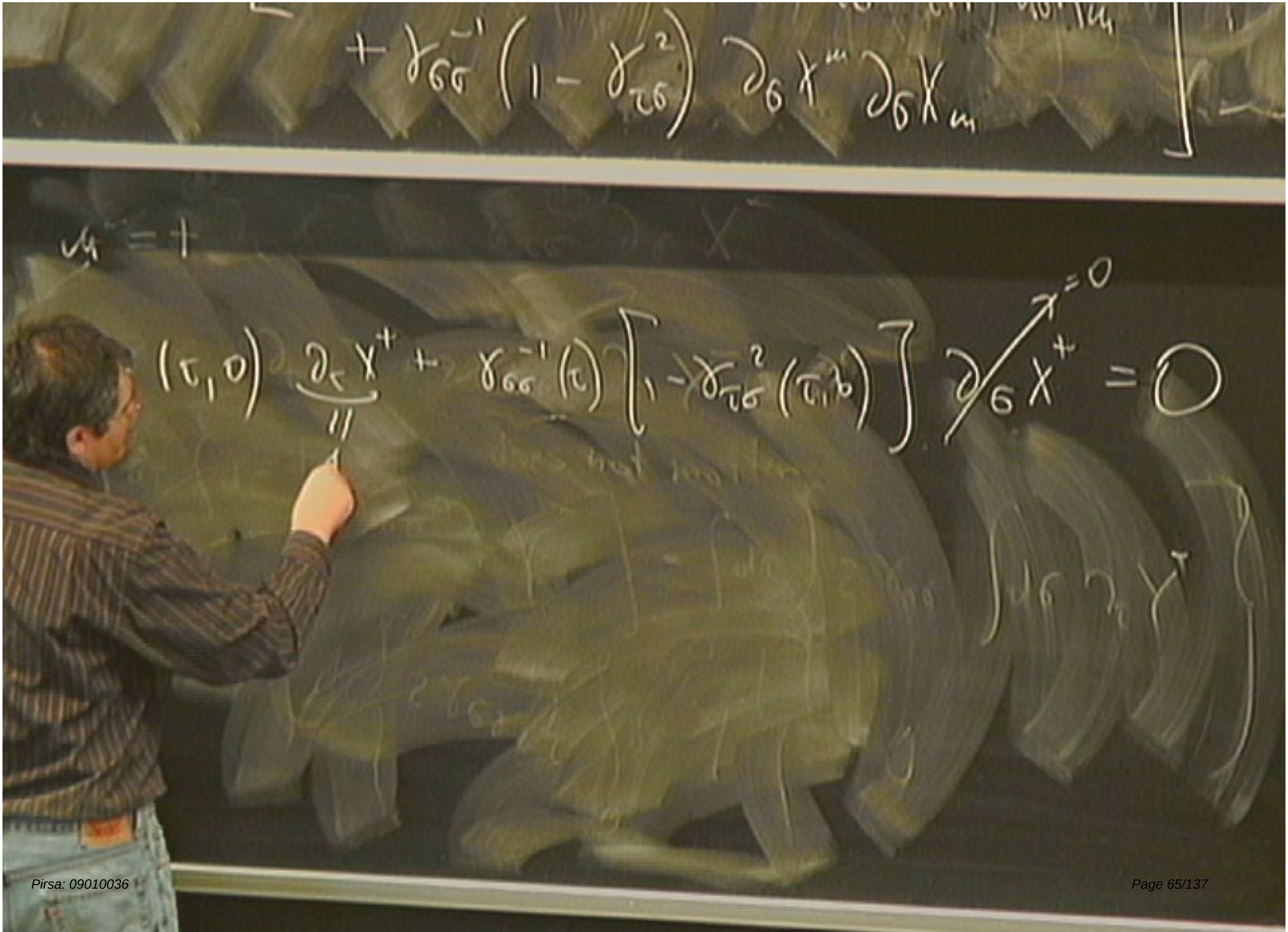
$\mu = +$

$$\gamma_{\tau\sigma}(\tau,\sigma)\,\partial_\tau X^+ + \gamma_{\sigma\sigma}^{-1}(\tau)\left[1-\gamma_{\tau\sigma}^{2}(\tau,\sigma)\right]\partial_\sigma X^+ = 0$$

$$+\gamma_{\sigma\sigma}^{-1}\left(1-\gamma_{\tau\sigma}^{2}\right)\partial_\sigma X^\mu\,\partial_\sigma X_\mu\Big]$$

$\mu = +$

$$\gamma_{\tau\sigma}(\tau,0)\,\partial_\tau X^+ + \gamma_{\sigma\sigma}^{-1}(\tau)\left[1-\gamma_{\tau\sigma}^{2}(\tau,0)\right]\overbrace{\cancel{\partial_\sigma X^+}}^{=0} = 0$$



$+ \gamma_{\sigma\sigma}^{-1}\left(1-\gamma_{\tau\sigma}^{2}\right)\partial_\sigma X^{\mu}\partial_\sigma X_{\mu}$
$\mu = +$
$(\tau,0)\ \partial_\tau X^{+} + \gamma_{\sigma\sigma}^{-1}(\tau)\left[1-\gamma_{\tau\sigma}^{2}(\tau,0)\right]\cancel{\partial_\sigma X^{+}} = 0$
$=0$

$$+\gamma_{\sigma\sigma}^{-1}\left(1-\gamma_{\tau\sigma}^{2}\right)\partial_\sigma X^{\mu}\partial_\sigma X_{\mu}\Big]$$

$\mu = +$

$$\gamma_{\tau\sigma}(\tau,0)\underbrace{\partial_\tau X^{+}}_{\substack{\parallel \\ 1}}+\gamma_{\sigma\sigma}^{-1}(\tau)\left[1-\gamma_{\tau\sigma}^{2}(\tau,0)\right]\cancelto{0}{\partial_\sigma X^{+}}=0$$

$$\int d\sigma\, \partial_\tau X^{+}$$

$$+\gamma_{\sigma\sigma}^{-1}\left(1-\gamma_{\tau\sigma}^{2}\right)\partial_\sigma X^m \partial_\sigma X_m \Big]$$

$\mu = +$

$$\gamma_{\tau\sigma}(\tau,0)\ \underbrace{\partial_\tau X^+}_{=1} + \gamma_{\sigma\sigma}^{-1}(\tau)\left[1-\gamma_{\tau\sigma}^{2}(\tau,0)\right]\ \overbrace{\cancel{\partial_\sigma X^+}}^{=0} = 0$$

$\gamma_{\tau\sigma}(\tau,0)$

$\int d\sigma\, \partial_\tau X^+$

$$+ \gamma_{\sigma\sigma}^{-1}\left(1 - \gamma_{\tau\sigma}^{2}\right) \partial_\sigma X^\mu \partial_\sigma X_\mu \Big]$$

$\mu = +$

$$\gamma_{\tau\sigma}(\tau, 0)\, \underbrace{\partial_\tau X^+}_{= 1} + \gamma_{\sigma\sigma}^{-1}(\tau)\left[1 - \gamma_{\tau\sigma}^{2}(\tau, 0)\right] \cancel{\partial_\sigma X^+}^{=0} = 0$$

$$\boxed{\gamma_{\tau\sigma}(\tau, 0) = \gamma_{\tau\sigma}(\tau, \ell) = 0}$$

$$\left\{ \int d\sigma\, \partial_\tau X^+ \right\}$$

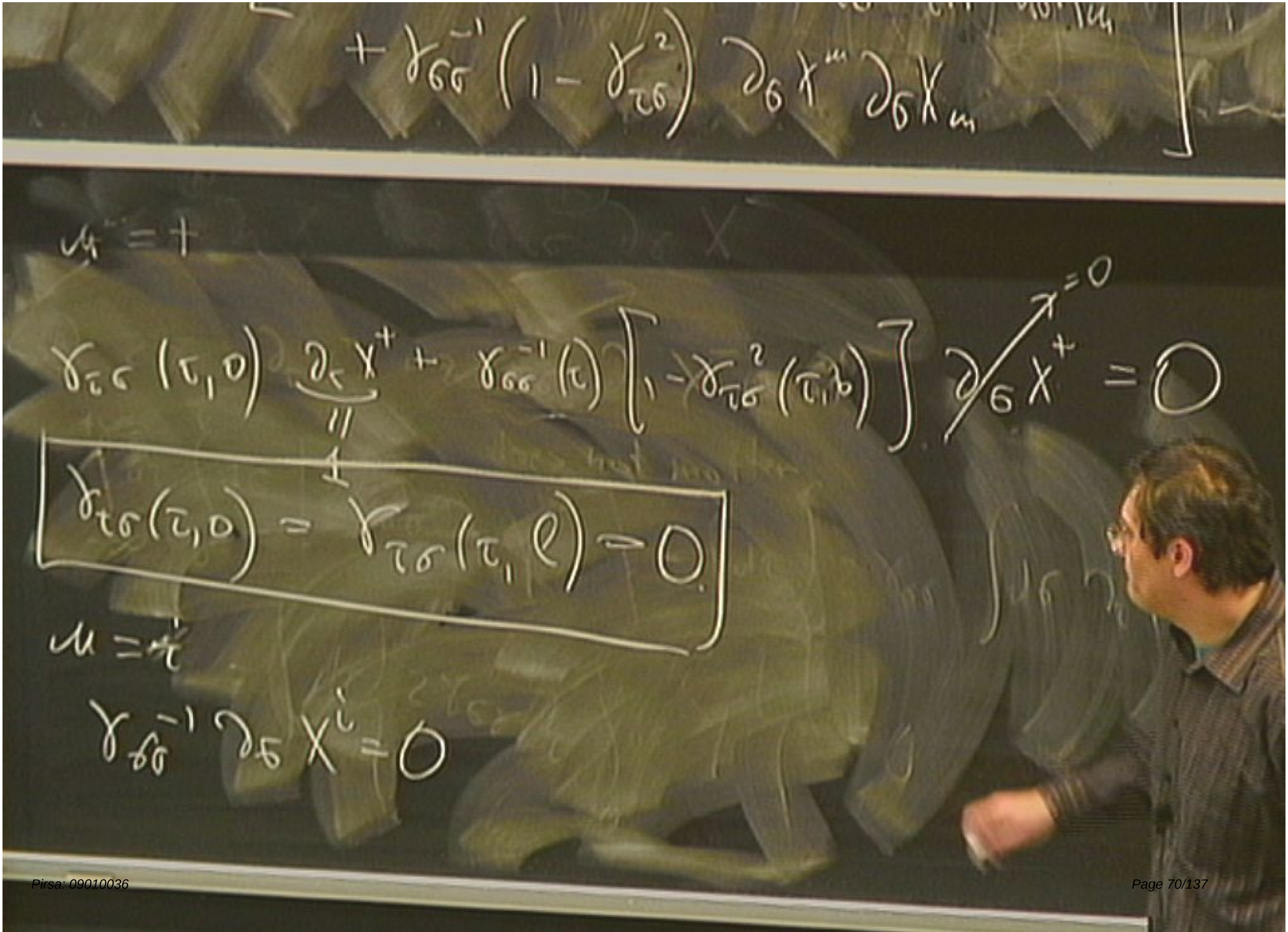
$$0 = \delta S_P = \int d\tau\, d\sigma \left(-2\gamma_{\tau\sigma}\, \partial_\sigma \delta Y^- \right)$$

$$= \int d\sigma\, d\sigma\, (+2) \left[\partial_\sigma \gamma_{\tau\sigma} \right] \delta Y^- + -2 \int d\tau\, \gamma_{\tau\sigma}\, \delta Y \Big|_{\sigma=0}^{\sigma=\ell}$$

$\hookrightarrow$ N Boundary condition in L-C gauge.

$$\partial^\sigma X^{i\mu} \Big|_{\sigma=0,\ell} = 0 = \gamma^{\sigma\sigma} \partial_\sigma X^\mu + \gamma^{\sigma\tau} \partial_\tau X^\mu$$

$$= \cancel{\gamma_{\tau\sigma} \partial_\tau X^\mu} + \gamma_{\sigma\sigma}^{-1} \left(1 - \gamma_{\tau\sigma}^2\right) \partial_\sigma X^\mu \Big|_{\sigma=0,\ell} = 0$$


$+\gamma_{\sigma\sigma}^{-1}\left(1-\gamma_{\tau\sigma}^{2}\right)\partial_\sigma X^\mu \partial_\sigma X_\mu \Big]$
$\mu = +$
$\gamma_{\tau\sigma}(\tau,0)\,\underbrace{\partial_\tau X^+}_{=1} + \gamma_{\sigma\sigma}^{-1}(\tau)\left[1-\gamma_{\tau\sigma}^{2}(\tau,0)\right]\,\cancel{\partial_\sigma X^+}^{=0} = 0$
$\boxed{\gamma_{\tau\sigma}(\tau,0) = \gamma_{\tau\sigma}(\tau,\ell) = 0}$
$\mu = i$
$\gamma_{\sigma\sigma}^{-1}\partial_\sigma X^i = 0$

$$+\gamma_{\sigma\sigma}^{-1}\left(1-\gamma_{\tau\sigma}^{2}\right)\partial_\sigma X^m \partial_\sigma X_m \Big]$$

$m=+$

$$\underbrace{\gamma_{\tau\sigma}(\tau,0)\,\partial_\tau X^+}_{=1} + \gamma_{\sigma\sigma}^{-1}(\tau)\Big[1-\gamma_{\tau\sigma}^{2}(\tau,0)\Big]\,\cancel{\partial_\sigma X^+}^{=0} = 0$$

$$\boxed{\gamma_{\tau\sigma}(\tau,0) = \gamma_{\tau\sigma}(\tau,\ell) = 0}$$

$m=i$

$$\gamma_{\sigma\sigma}^{-1}\,\partial_\sigma X^i\Big|_{\sigma=0,\ell} = 0 \qquad \boxed{\partial_\sigma X^i\Big|_{\sigma=0,\ell} = 0}$$

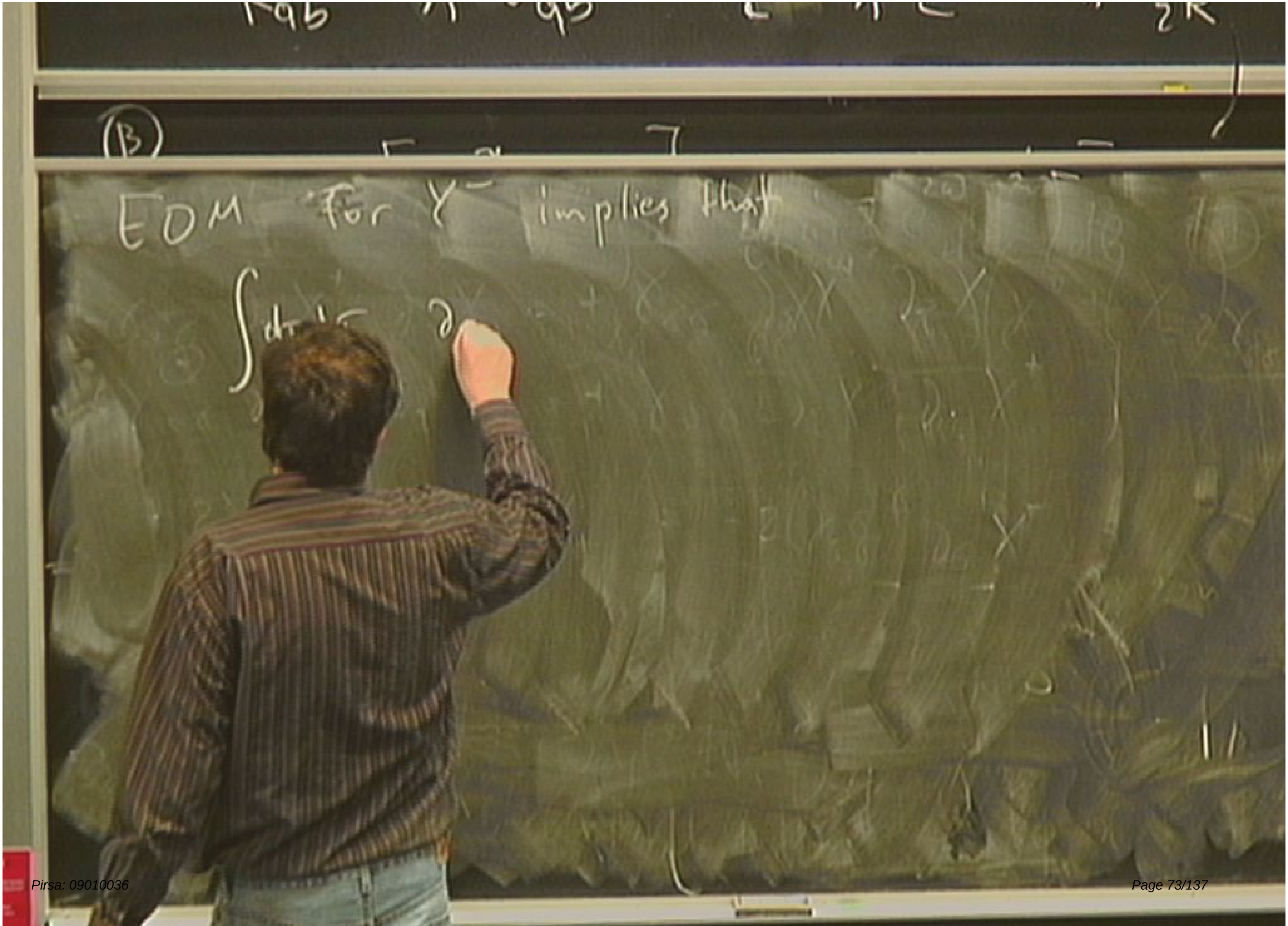
$$0 = \delta S_P = \int d\tau\, d\sigma \left(-2\gamma_{\tau\sigma}\,\partial_\sigma \delta Y^-\right)$$

$$= \int d\sigma\, d\sigma\, (+2)\left[\partial_\sigma \gamma_{\tau\sigma}\right]\delta Y^- + -2\,\cancel{\int d\tau\, \gamma_{\tau\sigma}\,\delta Y^-}\Big|_{\sigma=0}^{\sigma=\ell}$$

Boundary condition in l-c gauge.

$$\partial^\sigma X^\mu\Big|_{\sigma=0,\ell} = 0 = \gamma^{\sigma\sigma}\partial_\sigma X^\mu + \gamma^{\sigma\tau}\partial_\tau X^\mu$$

$$= \cancel{\gamma_{\tau\sigma}\,\partial_\tau X^\mu} + \gamma_{\sigma\sigma}^{-1}\left(1-\gamma_{\tau\sigma}^2\right)\partial_\sigma X^\mu\Big|_{\sigma=0,\ell} = 0$$


EOM for X⁻ implies that

EOM for Y^- implies that

$$\int d\tau\, d\sigma \;\partial_\sigma \Pi_{\tau\sigma}\; \delta Y^- = 0$$

EOM for Y^- implies that

$$\int d\tau\, d\sigma \;\; \partial_\sigma \gamma_{\tau\sigma} \;\; \delta Y^-(\tau,\sigma) = 0$$

$$\Downarrow$$

$$\partial_\sigma \gamma_{\tau\sigma} = 0$$

(B)

EOM for Y^- implies that

$$\int d\tau\, d\sigma \;\; \partial_\sigma \gamma_{\tau\sigma} \;\; \delta Y^-(\tau,\sigma) = 0$$

$$\Downarrow$$

$$\partial_\sigma \gamma_{\tau\sigma} = 0$$

wrong!

EOM for Y^- implies that

$$\int d\tau\, d\sigma \;\; \partial_\sigma \mathcal{P}_{\tau\sigma} \;\; \delta Y^-(\tau,\sigma) = 0$$

$$\Downarrow$$

$$\partial_\sigma \mathcal{P}_{\tau\sigma} = 0$$

wrong!

EOM for Y^- implies that

$$\int d\tau\, d\sigma \;\; \partial_\sigma \gamma_{\tau\sigma} \;\; \delta Y^-(\tau,\sigma) = 0$$

$$\Downarrow$$

$$\partial_\sigma \gamma_{\tau\sigma} = 0$$

wrong!

$$\int Y^- d\sigma = 0$$

$$\int \delta Y^- d\sigma = 0$$

EOM for Y^- implies that

$$\int d\tau\, d\sigma \;\; \partial_\sigma \gamma_{\tau\sigma} \;\; \delta Y^-(\tau,\sigma) = 0$$

$$\Downarrow$$

$$\partial_\sigma \gamma_{\tau\sigma} = 0$$ wrong!

$$\partial_\sigma \gamma_{\tau\sigma} = \text{const}(\tau)$$

$$\int Y^- d\sigma = 0$$

$$\int \delta Y^- d\sigma = 0$$

EOM for Y^- implies that

$$\int d\tau\, d\sigma \; \partial_\sigma \gamma_{\tau\sigma} \; \delta Y^-(\tau,\sigma) = 0$$

$$\Downarrow$$

$$\partial_\sigma \gamma_{\tau\sigma} = 0$$

wrong!

$$\int Y^- d\sigma = 0$$

$$\int \delta Y^- d\sigma = 0$$

$$\partial_\sigma \gamma_{\tau\sigma} = \text{const}(\tau)$$

$$\partial_\sigma^2 \gamma_{\tau\sigma} = 0$$

EOM for Y^- implies that

$$\int d\tau\, d\sigma \quad \partial_\sigma \gamma_{\tau\sigma} \quad \delta Y^-(\tau,\sigma) = 0$$

$$\int Y^- d\sigma = 0$$

$$\int \delta Y^- d\sigma = 0$$

$$\Downarrow$$

$$\partial_\sigma \gamma_{\tau\sigma} = 0$$ wrong!

$$\partial_\sigma \gamma_{\tau\sigma} = \text{const}(\tau)$$

$$\partial_\sigma^2 \gamma_{\tau\sigma} = 0 \qquad \gamma_{\tau\sigma}(\tau,0) = 0 \qquad \gamma_{\tau\sigma}(\tau,\ell) = 0$$

$F_{ab} \;\; \Lambda \; g_{ab} \qquad \mathcal{L} \;\; \Lambda \; \mathcal{L} \qquad \Lambda \; {}_2R$

(B)

EOM for Y^- implies that

$$\int Y^- d\sigma = 0$$

$$\int d\tau \, d\sigma \;\; \partial_\sigma \gamma_{\tau\sigma} \;\; \delta Y^-(\tau,\sigma) = 0$$

$$\int \delta Y^- d\sigma = 0$$

$$\Downarrow$$

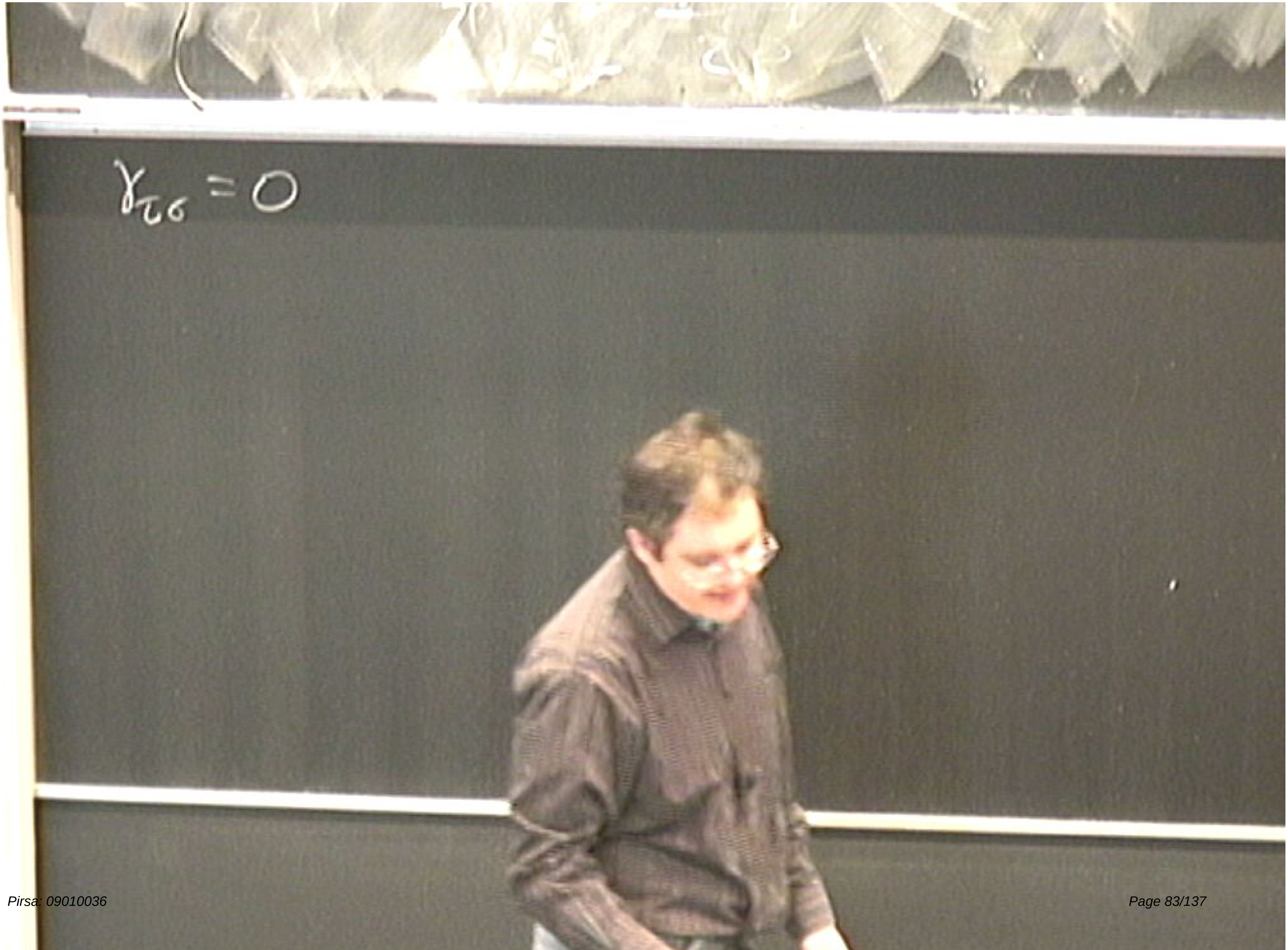
$$\partial_\sigma \gamma_{\tau\sigma} = 0 \qquad \underline{\text{wrong}}$$

$$\partial_\sigma \gamma_{\tau\sigma} = \text{const}(\tau)$$

$$\partial_\sigma^2 \gamma_{\tau\sigma} = 0 \qquad \gamma_{\tau\sigma}(\tau,0) = 0 \qquad \gamma_{\tau\sigma}(\tau,\ell) = 0$$

$$\Downarrow$$

$$\gamma_{\tau\sigma}(\tau,\sigma) = 0$$


$\gamma_{\tau\sigma} = 0$

$$\gamma_{\tau\sigma} = 0$$

$$\gamma_{\tau\sigma} = 0$$

$$X^{-}(\tau)\,,\ \gamma_{\sigma\sigma}(\tau)$$

$\gamma_{\tau\sigma} = 0$

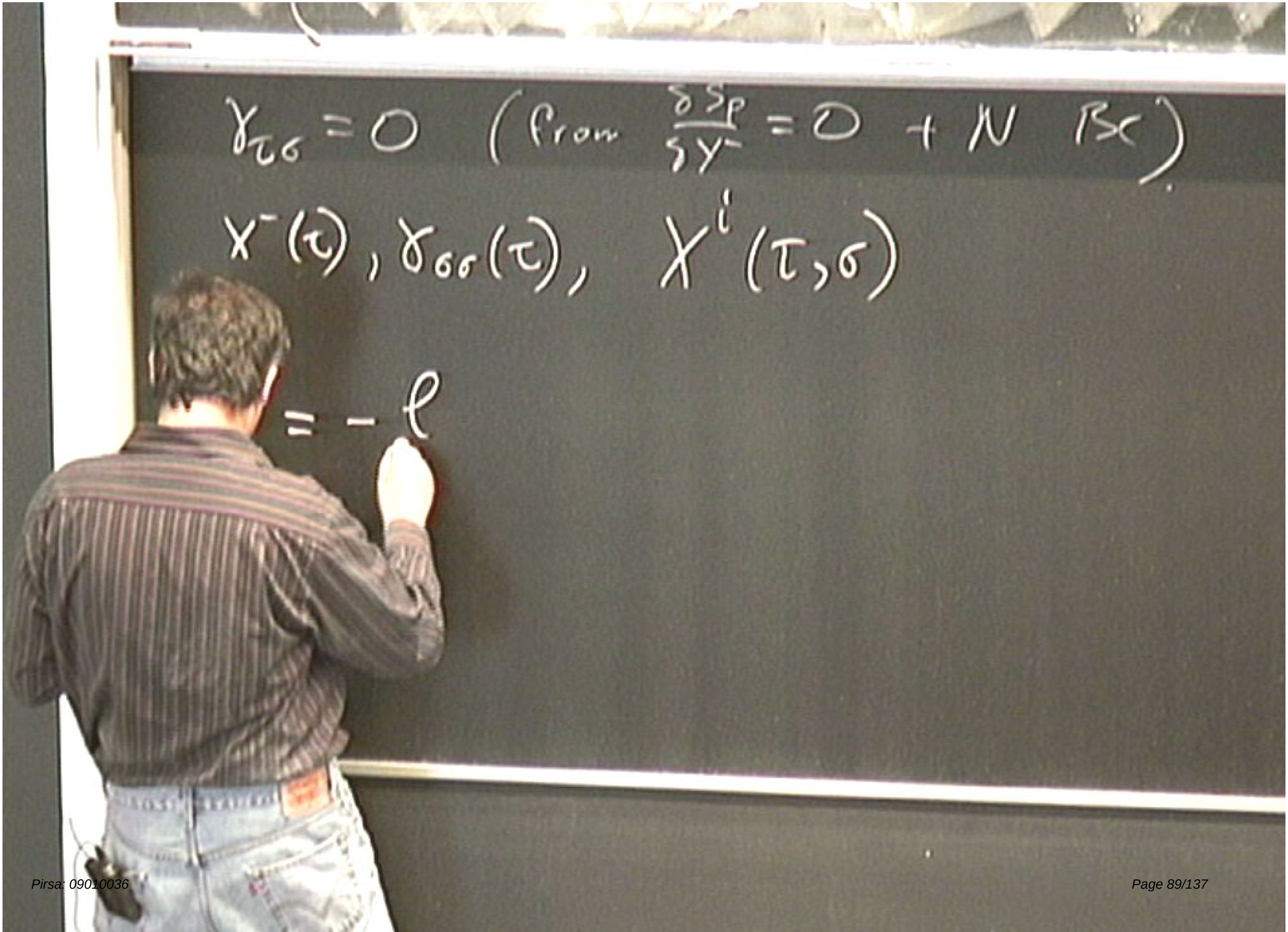
$X^-(\tau), \gamma_{\sigma\sigma}(\tau), \quad X^i(\tau,\sigma)$

$$\gamma_{\tau\sigma} = 0$$

$$X^-(\tau),\ \gamma_{\sigma\sigma}(\tau),\quad X^i(\tau,\sigma)$$

$\gamma_{\tau\sigma} = 0$ $\left(\text{from } \frac{\delta S_p}{\delta \gamma^-} = 0 \right.$

$X^-(\tau)$, $\gamma_{\sigma\sigma}(\tau)$, $X^i(\tau,\sigma)$


$\gamma_{\tau\sigma} = 0 \quad \left(\text{from } \frac{\delta S_P}{\delta Y^-} = 0 + N \ BC \right)$
$X^-(\tau), \ \gamma_{\sigma\sigma}(\tau), \quad X^i(\tau, \sigma)$
$= - \ell$

$$\gamma_{\tau\sigma} = 0 \quad \left(\text{from } \frac{\delta S_P}{\delta Y^-} = 0 + N \ \text{BC} \right)$$

$$X^-(\tau),\ \gamma_{\sigma\sigma}(\tau),\ X^i(\tau,\sigma)$$

$$L = -\frac{\ell}{2\pi\alpha'}\gamma_{\sigma\sigma}(\tau)\,\partial_\tau X^- + \frac{1}{4\pi\alpha'}\int_0^{\ell} d\sigma \ \ldots \partial X^i \,\partial_\tau X^i$$

$$\gamma_{\tau\sigma} = 0 \quad \left(\text{from } \frac{\delta S_P}{\delta \gamma^{-}} = 0 + N \text{ BC}\right).$$

$$X^-(\tau),\ \gamma_{\sigma\sigma}(\tau),\ X^i(\tau,\sigma)$$

$$L = -\frac{\ell}{2\pi\alpha'}\gamma_{\sigma\sigma}(\tau)\,\partial_\tau X^- + \frac{1}{4\pi\alpha'}\int_0^\ell d\sigma \left[\gamma_{\sigma\sigma}\,\partial_\tau X^i\,\partial_\tau X^i - \gamma_{\sigma\sigma}^{-1}\,\partial_\sigma X^i\,\partial_\sigma X^i\right]$$

$$\gamma_{\tau\sigma} = 0 \quad \left(\text{from } \frac{\delta S_P}{\delta Y^-} = 0 + N \text{ BC} \right).$$

$$X^-(\tau),\ \gamma_{\sigma\sigma}(\tau),\quad X^i(\tau,\sigma)$$

$$L = -\frac{\ell}{2\pi\alpha'}\gamma_{\sigma\sigma}(\tau)\,\partial_\tau X^- + \frac{1}{4\pi\alpha'}\int_0^\ell d\sigma \left[\gamma_{\sigma\sigma}\,\partial_\tau X^i\,\partial_\tau X^i - \gamma_{\sigma\sigma}^{-1}\,\partial_\sigma X^i\,\partial_\sigma X^i \right]$$

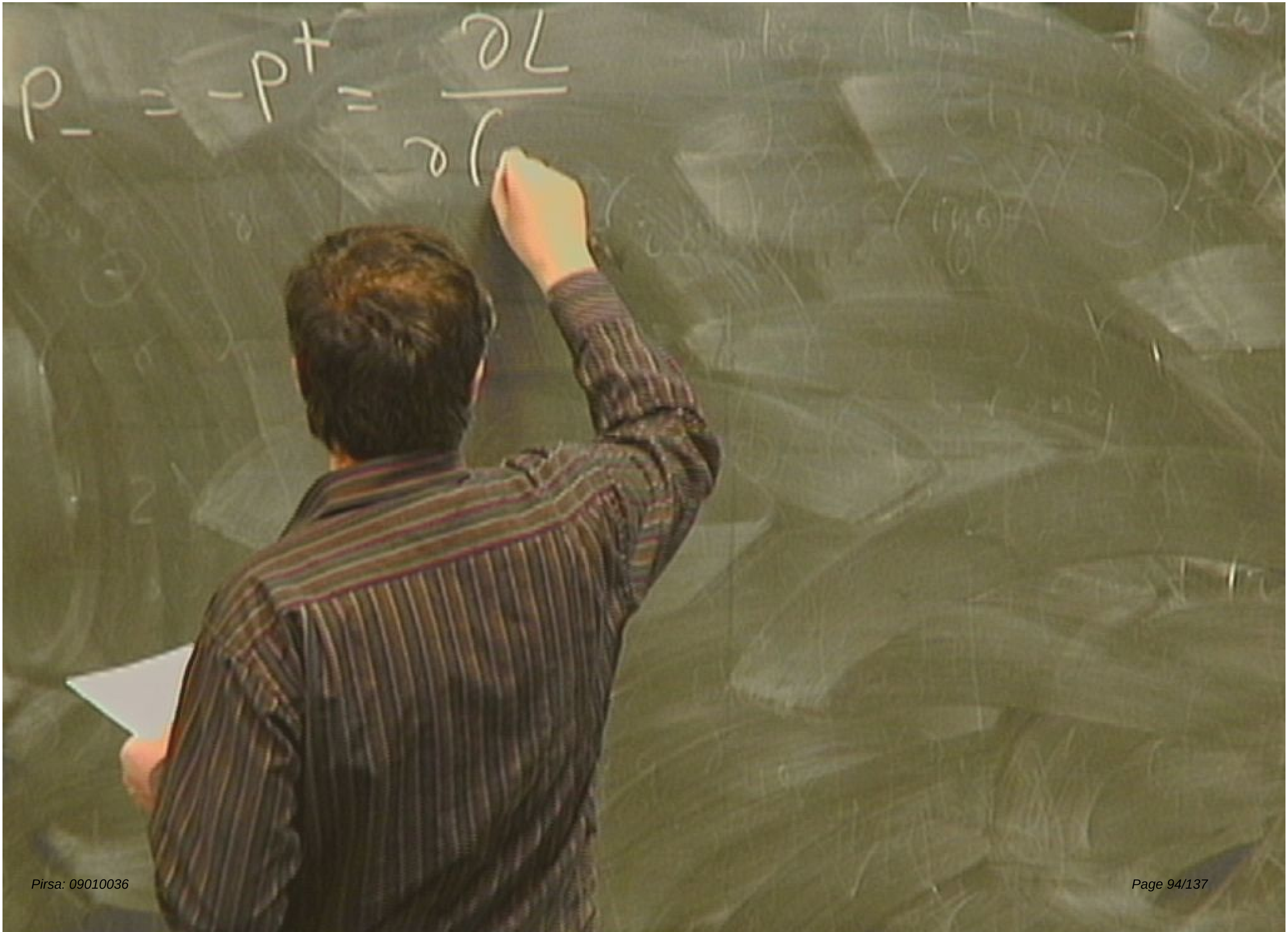
$$S_{l.c.} = \int d\tau\, L$$

$\gamma_{\tau\sigma} = 0 \quad \left(\text{from } \frac{\delta S_P}{\delta Y^-} = 0 + N \ \text{BC} \right)$

$x^-(\tau), \ \gamma_{\sigma\sigma}(\tau), \quad X^i(\tau,\sigma)$

$$L = -\frac{\ell}{2\pi\alpha'} \gamma_{\sigma\sigma}(\tau) \, \partial_\tau x^-(\tau) + \frac{1}{4\pi\alpha'} \int_0^\ell d\sigma \left[\gamma_{\sigma\sigma} \partial_\tau X^i \partial_\tau X^i - \gamma_{\sigma\sigma}^{-1} \partial_\sigma X^i \partial_\sigma X^i \right]$$

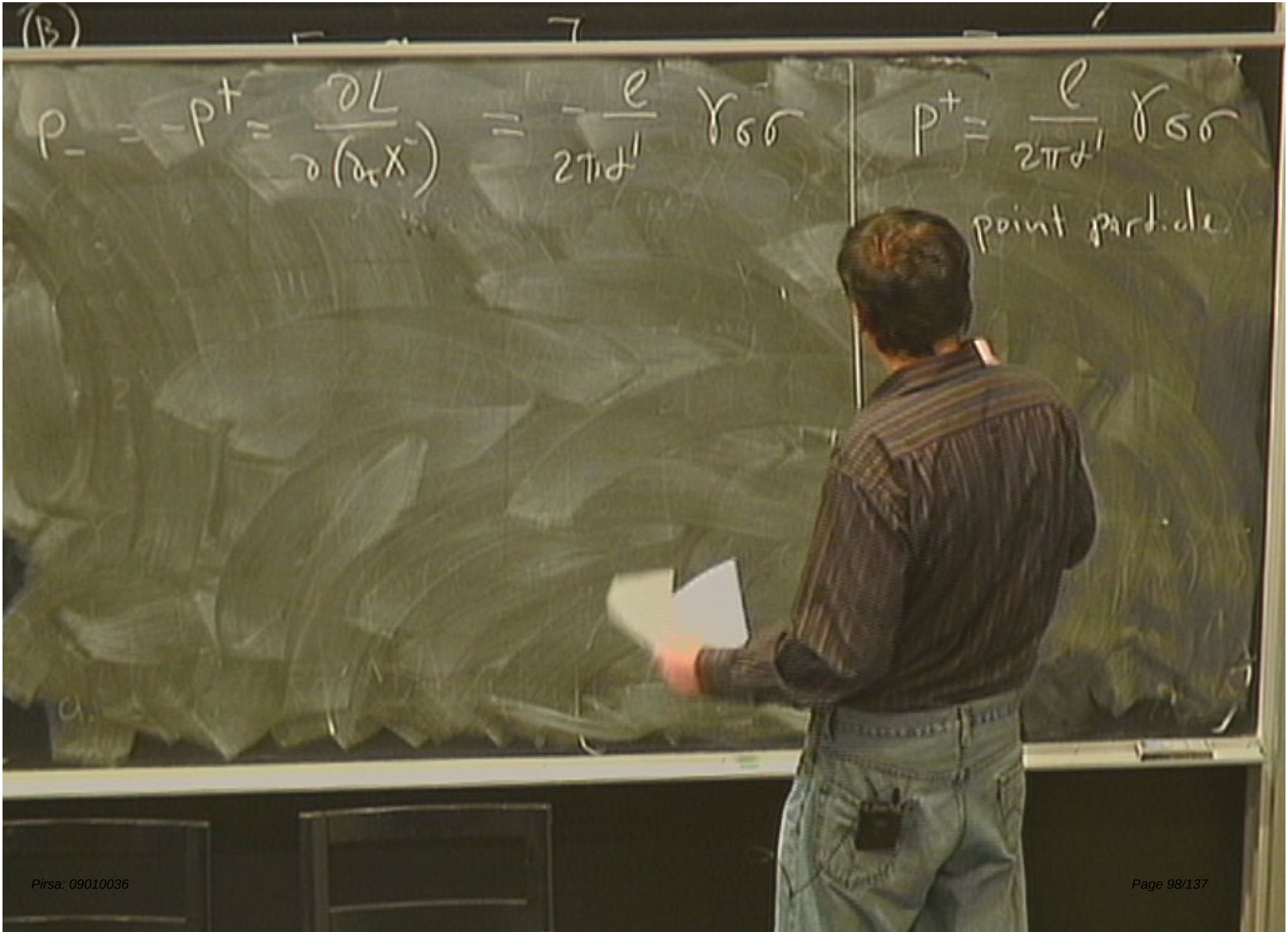
$$S_{l.c.} = \int d\tau \, L$$


$p_- = -p^+ = \frac{\partial L}{\partial (}$

$$p_- = -p^+ = \frac{\partial L}{\partial(\partial_t X^-)}$$

$$P_- = -P^+ = \frac{\partial L}{\partial(\partial_t X^-)}$$

$$P_- = -P^+ = \frac{\partial L}{\partial(\partial_t X^-)} = -\frac{e}{2\pi\alpha'}\gamma_{\sigma\sigma}$$

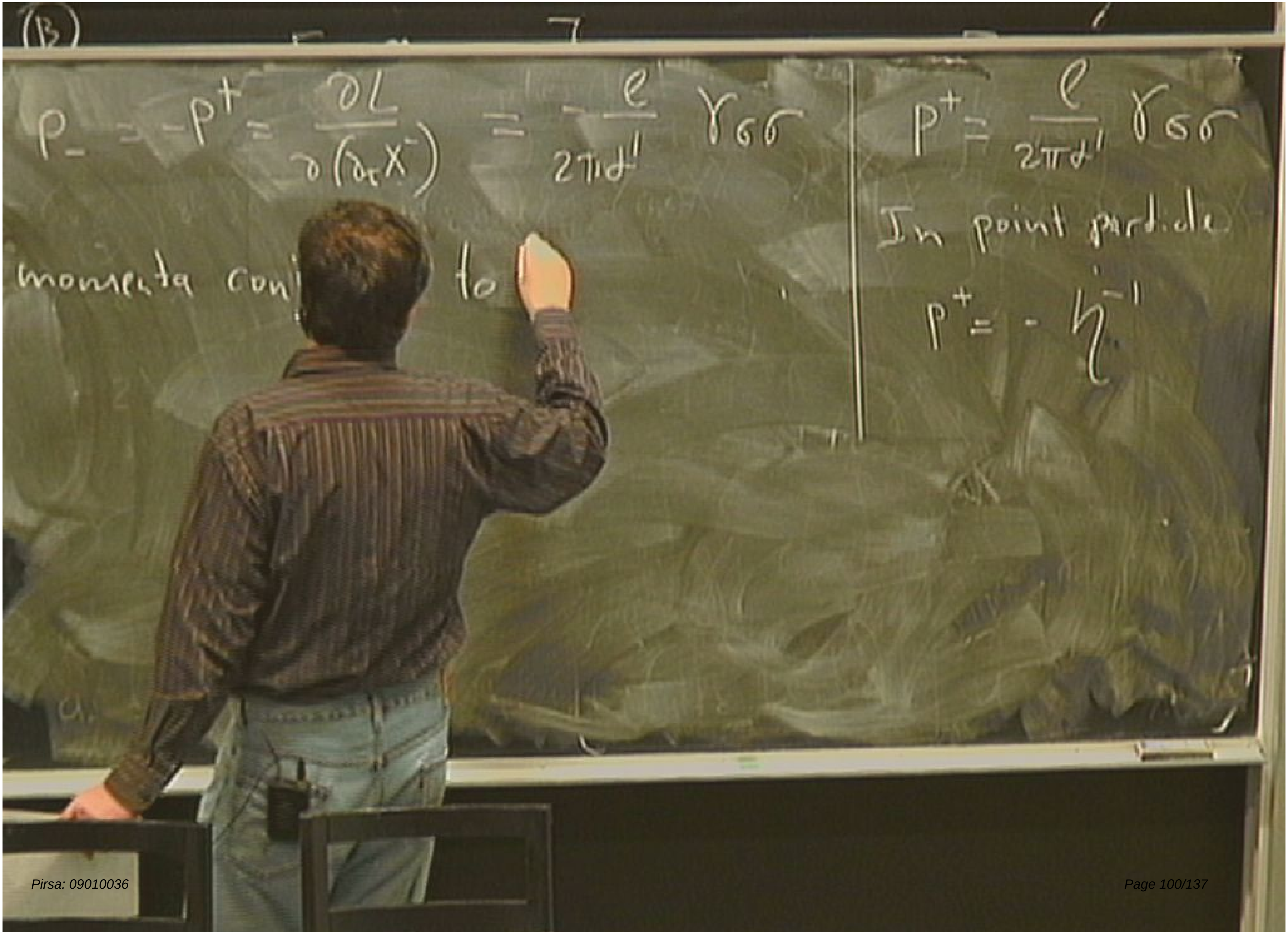

$p_- = -p^+ = \frac{\partial L}{\partial(\partial_\tau X^-)} = -\frac{\ell}{2\pi\alpha'}\gamma_{\sigma\sigma}$
$p^+ = \frac{\ell}{2\pi\alpha'}\gamma_{\sigma\sigma}$
point particle

$$p_- = -p^+ = \frac{\partial L}{\partial(\partial_\tau X^-)} = -\frac{\ell}{2\pi\alpha'}\gamma_{\sigma\sigma}$$

$$p^+ = \frac{\ell}{2\pi\alpha'}\gamma_{\sigma\sigma}$$

In point particle

$$p^+ = -\dot{\eta}^{-1}$$



$p_- = -p^+ = \frac{\partial L}{\partial(\partial_\tau X^-)} = -\frac{\ell}{2\pi\alpha'}\gamma_{\sigma\sigma}$
$p^+ = \frac{\ell}{2\pi\alpha'}\gamma_{\sigma\sigma}$
In point particle
$p^+ = -\dot{h}^{-1}$
momenta con to

$$p_- = -p^+ = \frac{\partial L}{\partial(\partial_\tau X^-)} = -\frac{\ell}{2\pi\alpha'}\gamma_{\sigma\sigma}$$

$$p^+ = \frac{\ell}{2\pi\alpha'}\gamma_{\sigma\sigma}$$

In point particle

$$p^+ = -\dot{\eta}^{-1}$$

...menta conjugate to $X^i(\tau, \sigma)$

$$p_- = -p^+ = \frac{\partial L}{\partial(\partial_\tau X^-)} = -\frac{\ell}{2\pi\alpha'}\,\gamma_{\sigma\sigma}$$

momenta conjugate to $X^i(\tau,\sigma)$

$$\Pi^i = \Pi_i =$$

$$p^+ = \frac{\ell}{2\pi\alpha'}\,\gamma_{\sigma\sigma}$$

In point particle

$$p^+ = -\dot{\eta}^{-1}$$

$$p_- = -p^+ = \frac{\partial L}{\partial(\partial_\tau X^-)} = -\frac{\ell}{2\pi\alpha'}\gamma_{\sigma\sigma}$$

momenta conjugate to $X^i(\tau,\sigma)$

$$\Pi^i = \Pi_i = \frac{\partial L}{\partial\, \partial_\tau X^i}$$

$$p^+ = \frac{\ell}{2\pi\alpha'}\gamma_{\sigma\sigma}$$

In point particle

$$p^+ = -\dot{\eta}^{-1}$$

$$p_- = -p^+ = \frac{\partial L}{\partial(\partial_\tau X^-)} = -\frac{\ell}{2\pi\alpha'}\gamma_{\sigma\sigma}$$

$$p^+ = \frac{\ell}{2\pi\alpha'}\gamma_{\sigma\sigma}$$

In point particle

$$p^+ = -\dot{\eta}^{-1}$$

momenta conjugate to $X^i(\tau,\sigma)$

$$\Pi^i = \Pi_i = \frac{\delta L}{\partial\,\partial_\tau X^i} = \frac{2}{4\pi\alpha'}\gamma_{\sigma\sigma}\,\partial_\tau X^i$$

$$p_- = -p^+ = \frac{\partial L}{\partial(\partial_\tau X^-)} = -\frac{\ell}{2\pi\alpha'}\gamma_{\sigma\sigma}$$

$$p^+ = \frac{\ell}{2\pi\alpha'}\gamma_{\sigma\sigma}$$

In point particle

$$p^+ = \dot{\eta}^{-1}$$

momenta conjugate to $X^i(\tau,\sigma)$

$$\Pi^i = \Pi_i = \frac{\delta L}{\delta\,\partial_\tau X^i} = \frac{1}{2\pi\alpha'}\gamma_{\sigma\sigma}\,\partial_\tau X^i =$$

$$p_- = -p^+ = \frac{\partial L}{\partial(\partial_\tau X^-)} = -\frac{\ell}{2\pi\alpha'}\gamma_{\sigma\sigma}$$

$$p^+ = \frac{\ell}{2\pi\alpha'}\gamma_{\sigma\sigma}$$

In point particle

$$p^+ = -\dot{\eta}^{-1}$$

momenta conjugate to $X^i(\tau,\sigma)$

$$\Pi^i = \Pi_i = \frac{\delta L}{\delta\,\partial_\tau X^i} = \frac{1}{2\pi\alpha'}\gamma_{\sigma\sigma}\,\partial_\tau X^i = \frac{p^+}{\ell}\,\partial_\tau X^i$$

$$\partial_\tau X^i = \frac{\ell}{p^+}\Pi^i$$

$$p_- = -p^+ = \frac{\partial L}{\partial(\partial_\tau X^-)} = -\frac{\ell}{2\pi\alpha'}\gamma_{\sigma\sigma}$$

$$p^+ = \frac{\ell}{2\pi\alpha'}\gamma_{\sigma\sigma}$$

In point particle

$$p^+ = -\eta^{-1}$$

momenta conjugate to $X^i(\tau,\sigma)$

$$\Pi^i = \Pi_i = \frac{\delta L}{\partial \partial_\tau X^i} = \frac{1}{2\pi\alpha'}\gamma_{\sigma\sigma}\,\partial_\tau X^i = \frac{p^+}{\ell}\,\partial_\tau X^i$$

$$\boxed{\partial_\tau X^i = \frac{\ell}{p^+}\Pi^i}$$

$$H = P_- \partial_\tau X^-$$

$$H = P_- \partial_\tau X^- + \int_0^\ell d\sigma\, \Pi_i\, \partial_\tau X^i$$

$$H = P_- \, D_\tau X^- + \int_0^\ell d\sigma \, \Pi_i \, D_\tau X^i - L$$

$$H = P_- \partial_\tau X^- + \int_0^\ell d\sigma \, \Pi_i \, \partial_\tau X^i - L$$

$$\partial_\tau X^-$$

$$H = P_- \partial_\tau X^- + \int_0^\ell d\sigma \, \Pi_i \partial_\tau X^i - L$$

$$= P_- \partial_\tau X^-$$

$$H = P_- \partial_\tau X^- + \int_0^\ell d\sigma\, \Pi_i \partial_\tau X^i - L$$

$$= P_- \partial_\tau X^- + \int_0^\ell d\sigma$$

$\frac{1}{2\ell}$ $\frac{P_i}{2}$

$$H = P_- \partial_\tau X^- + \int_0^\ell d\sigma\, \Pi_i \partial_\tau X^i - L$$

$$= P_- \partial_\tau X^- + \int_0^\ell d\sigma\, \frac{\Pi_i \cdot \Pi^i \ell}{p^+}$$

$$L = -\frac{\ell}{2\pi\alpha'}\gamma_{\sigma\sigma}(\tau)\,\partial_\tau \bar{x}(\tau) + \frac{1}{4\pi\alpha'}\int_0^\ell d\sigma\left[\gamma_{\sigma\sigma}\,\partial_\tau X^i\,\partial_\tau X^i - \gamma_{\sigma\sigma}^{-1}\,\partial_\sigma X^i\,\partial_\sigma X^i\right]$$

$$S_{1\bullet} = \int d\tau\, L$$

$$L = -\frac{\ell}{2\pi\alpha'}\gamma_{\sigma\sigma}(\tau)\,\partial_\tau X^-(\tau) + \frac{1}{4\pi\alpha'}\int_0^\ell d\sigma\left[\gamma_{\sigma\sigma}\,\partial_\tau X^i\,\partial_\tau X^i - \gamma_{\sigma\sigma}^{-1}\,\partial_\sigma X^i\,\partial_\sigma X^i\right]$$

$$S_1 = \int d\tau\, L$$

$$x^-(\tau),\ \gamma_{\sigma\sigma}(\tau),\ X^i(\tau,\sigma)$$

$$p_-$$

$$L = \left(-\frac{\ell}{2\pi\alpha'}\gamma_{\sigma\sigma}(\tau)\right)\partial_\tau x^-(\tau) + \frac{1}{4\pi\alpha'}\int_0^\ell d\sigma\left[\gamma_{\sigma\sigma}\,\partial_\tau X^i\,\partial_\tau X^i - \gamma_{\sigma\sigma}^{-1}\,\partial_\sigma X^i\,\partial_\sigma X^i\right]$$

$$\int d\tau\, L$$

$$= P_- \partial_t X^- + \int_0^{\ell} d\sigma \, \frac{\Pi_i \cdot \Pi^i \, \ell}{p^+} - P_- \partial_t X^-$$

p_-

$$= \left(\frac{\ell}{\pi\alpha'}\gamma_{\sigma\sigma}(\tau)\right)\partial_\tau \bar{X}(\tau) + \frac{1}{4\pi\alpha'}\int_0^{\ell} d\sigma \left[\gamma_{\sigma\sigma}\,\partial_\tau X^i \partial_\tau X^i - \gamma_{\sigma\sigma}^{-1}\,\partial_\sigma X^i \partial_\sigma X^i\right]$$

$$= \int dt \, L$$

$$= P_- \partial_\tau X^- + \int_0^\ell d\sigma \, \frac{\Pi_i \cdot \Pi^i \, \ell}{p^+} - P_- \partial_\tau X^-$$

$$- \frac{1}{\pi\alpha'}$$

$$L = \left(-\frac{\ell}{2\pi\alpha'} \gamma_{\sigma\sigma}(\tau)\right) \partial_\tau X^-(\tau) + \frac{1}{4\pi\alpha'} \int_0^\ell d\sigma \left[\gamma_{\sigma\sigma} \, \partial_\tau X^i \, \partial_\tau X^i - \gamma_{\sigma\sigma}^{-1} \, \partial_\sigma X^i \, \partial_\sigma X^i \right]$$

$$S_{l.c.} = \int d\tau \, L$$

$$= P_- \partial_\tau X^- + \int_0^\ell d\sigma \, \frac{\Pi_i \cdot \Pi^i \, \ell}{p^+} - P_- \partial_\tau X^-$$

$$- \frac{1}{4\pi\alpha'} \int_0^\ell d\sigma \, \frac{2\pi\alpha' p^+}{\ell}$$

$$L = \left(- \frac{\ell}{2\pi\alpha'} \gamma_{\sigma\sigma}(\tau) \right) \partial_\tau X^-(\tau) + \frac{1}{4\pi\alpha'} \int_0^\ell d\sigma \left[\gamma_{\sigma\sigma} \, \partial_\tau X^i \, \partial_\tau X^i - \gamma_{\sigma\sigma}^{-1} \, \partial_\sigma X^i \, \partial_\sigma X^i \right]$$

$$S = \int d\tau \, L$$

$$= P_- \partial_\tau X^- + \int_0^\ell d\sigma \, \frac{\Pi_i \cdot \Pi^i \, \ell}{p^+} - P_- \partial_\tau X^-$$

$$- \frac{1}{4\pi\alpha'} \int_0^\ell d\sigma \, \frac{2\pi\alpha' p^+}{\ell}$$

$$\gamma_{\sigma\sigma}$$

$$L = \left(-\frac{\ell}{2\pi\alpha'} \gamma_{\sigma\sigma}(\tau)\right) \partial_\tau X^-(\tau) + \frac{1}{4\pi\alpha'} \int_0^\ell d\sigma \left[\gamma_{\sigma\sigma} \partial_\tau X^i \partial_\tau X^i - \gamma_{\sigma\sigma}^{-1} \partial_\sigma X^i \partial_\sigma X^i \right]$$

$$S_{l.c.} = \int d\tau \, L$$

$$= P_- \partial_\tau X^- + \int_0^\ell d\sigma \, \frac{\Pi_i \cdot \Pi^i \, \ell}{p^+} - P_- \partial_\tau X^-$$

$$- \frac{1}{4\pi\alpha'} \int_0^\ell d\sigma \, \underset{\gamma_{\sigma\sigma}}{\underset{\parallel}{\frac{2\pi\alpha' p^+}{\ell}}} \, \frac{\Pi^i \Pi^i \ell^2}{(p^+)^2} + \frac{1}{4\pi\alpha'}$$

$$= \left(-\frac{\ell}{2\pi\alpha'} \gamma_{\sigma\sigma}(\tau)\right) \partial_\tau X^-(\tau) + \frac{1}{4\pi\alpha'} \int_0^\ell d\sigma \left[\gamma_{\sigma\sigma} \, \partial_\tau X^i \partial_\tau X^i - \gamma_{\sigma\sigma}^{-1} \, \partial_\sigma X^i \partial_\sigma X^i \right]$$

$$\int dt \, L$$

$$= P_- \partial_\tau X^- + \int_0^\ell d\sigma \, \frac{\Pi_i \cdot \Pi^i \ell}{p^+} - P_- \partial_\tau X^-$$

$$- \frac{1}{4\pi\alpha'} \int_0 d\sigma \, \underbrace{\frac{2\pi\alpha' p^+}{\ell}}_{\gamma_{\sigma\sigma}} \, \frac{\Pi^i \Pi^i \ell^2}{(p^+)^2} + \frac{1}{4\pi\alpha'} \int_0 d\sigma$$

$$L = \left(-\frac{\ell}{2\pi\alpha'} \gamma_{\sigma\sigma}(\tau)\right) \partial_\tau X^-(\tau) + \frac{1}{4\pi\alpha'} \int_0 d\sigma \left[\gamma_{\sigma\sigma} \partial_\tau X^i \partial_\tau X^i - \gamma_{\sigma\sigma}^{-1} \partial_\sigma X^i \partial_\sigma X^i \right]$$

$$S_{l.c.} = \int d\tau \, L$$

$$= P_- \partial_\tau X^- + \int_0^\ell d\sigma \, \frac{\Pi_i \cdot \Pi^i \ell}{p^+} - P_- \partial_\tau X^-$$

$$- \frac{1}{4\pi\alpha'} \int_0^\ell d\sigma \, \underbrace{\frac{2\pi\alpha' p^+}{\ell}}_{= \gamma_{\sigma\sigma}} \frac{\Pi^i \Pi^i \ell^2}{(p^+)^2} + \frac{1}{4\pi\alpha'} \int_0^\ell d\sigma \, \frac{\ell}{2\pi\alpha' p^+}$$

$$L = \left(-\frac{\ell}{2\pi\alpha'} \gamma_{\sigma\sigma}(\tau)\right) \partial_\tau X^-(\tau) + \frac{1}{4\pi\alpha'} \int_0^\ell d\sigma \Big[\gamma_{\sigma\sigma} \partial_\tau X^i \partial_\tau X^i$$

$$- \gamma_{\sigma\sigma}^{-1} \partial_\sigma X^i \partial_\sigma X^i \Big]$$

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$$L = \left(-\frac{\ell}{2\pi\alpha'} \gamma_{\sigma\sigma}(\tau)\right) \partial_\tau X^-(\tau) + \frac{1}{4\pi\alpha'} \int_0^\ell d\sigma \left[\gamma_{\sigma\sigma} \partial_\tau X^i \partial_\tau X^i - \gamma_{\sigma\sigma}^{-1} \partial_\sigma X^i \partial_\sigma X^i \right]$$

$$S_{l.c.} = \int d\tau \, L$$

$$\partial X^\mu\Big|_{\sigma=0,\ell} = \gamma^{\sigma\sigma}\,\partial_\sigma X^\mu + \gamma^{\sigma\tau}\,\partial_\tau X^\mu$$

$$= \cancel{\gamma_{\tau\sigma}\,\partial_\tau X^\mu} + \gamma_{\tau\sigma}^{-1}\left(1-\gamma_{\tau\sigma}^2\right)\partial_\sigma X^\mu\Big| = 0$$

$$H = P_-\,\partial_\tau X^- + \int_0^\ell d\sigma\; \Pi_i\,\partial_\tau X^i - L$$

$$= \cancel{P_-\,\partial_\tau X^-} + \int_0^\ell d\sigma\; \frac{\Pi_i \cdot \Pi^i\,\ell}{p^+} - \cancel{P_-\,\partial_\tau X^-} -$$

$$- \frac{1}{4\pi\alpha'}\int_0^\ell d\sigma\; \underbrace{\frac{2\pi\alpha' p^+}{\ell}}_{=\,\gamma_{\sigma\sigma}}\; \frac{\Pi^i\Pi^i\,\ell^2}{(p^+)^2} + \frac{1}{4\pi\alpha'}\int_0^\ell d\sigma\; \frac{\ell}{2\pi\alpha' p^+}\;\partial_\sigma X^i\,\partial_\sigma X^i$$

$$\partial X^{-}(\tau, \sigma)\big|_{\sigma=0,\ell} = \gamma^{\sigma\sigma}\partial_\sigma X^{-} + \gamma^{\sigma\tau}\partial_\tau X^{-}$$

$$= \cancel{\gamma_{\tau\sigma}\partial_\tau X^{-}} + \gamma_{\sigma\sigma}^{-1}(1-\gamma_{\tau\sigma}^2)\,\partial_\sigma X^{-}\Big| = 0$$

$$H = P_-\partial_\tau X^- + \int_0^\ell d\sigma\, \Pi_i \partial_\tau X^i - L$$

$$= \cancel{P_-\partial_\tau X^-} + \int_0^\ell d\sigma\, \frac{\Pi_i \cdot \Pi^i \ell}{p^+} - \cancel{P_-\partial_\tau X^-}$$

$$- \frac{1}{2}\int_0^\ell d\sigma\, \underset{\underset{\gamma_{\sigma\sigma}}{\parallel}}{\frac{2\pi\alpha' p^+}{\ell}}\, \frac{\Pi^i\Pi^i \ell^2}{(p^+)^2} + \frac{1}{4\pi\alpha'}\int_0^\ell d\sigma\, \frac{\ell}{2\pi\alpha' p^+}\, \partial_\sigma X^i \partial_\sigma X^i$$

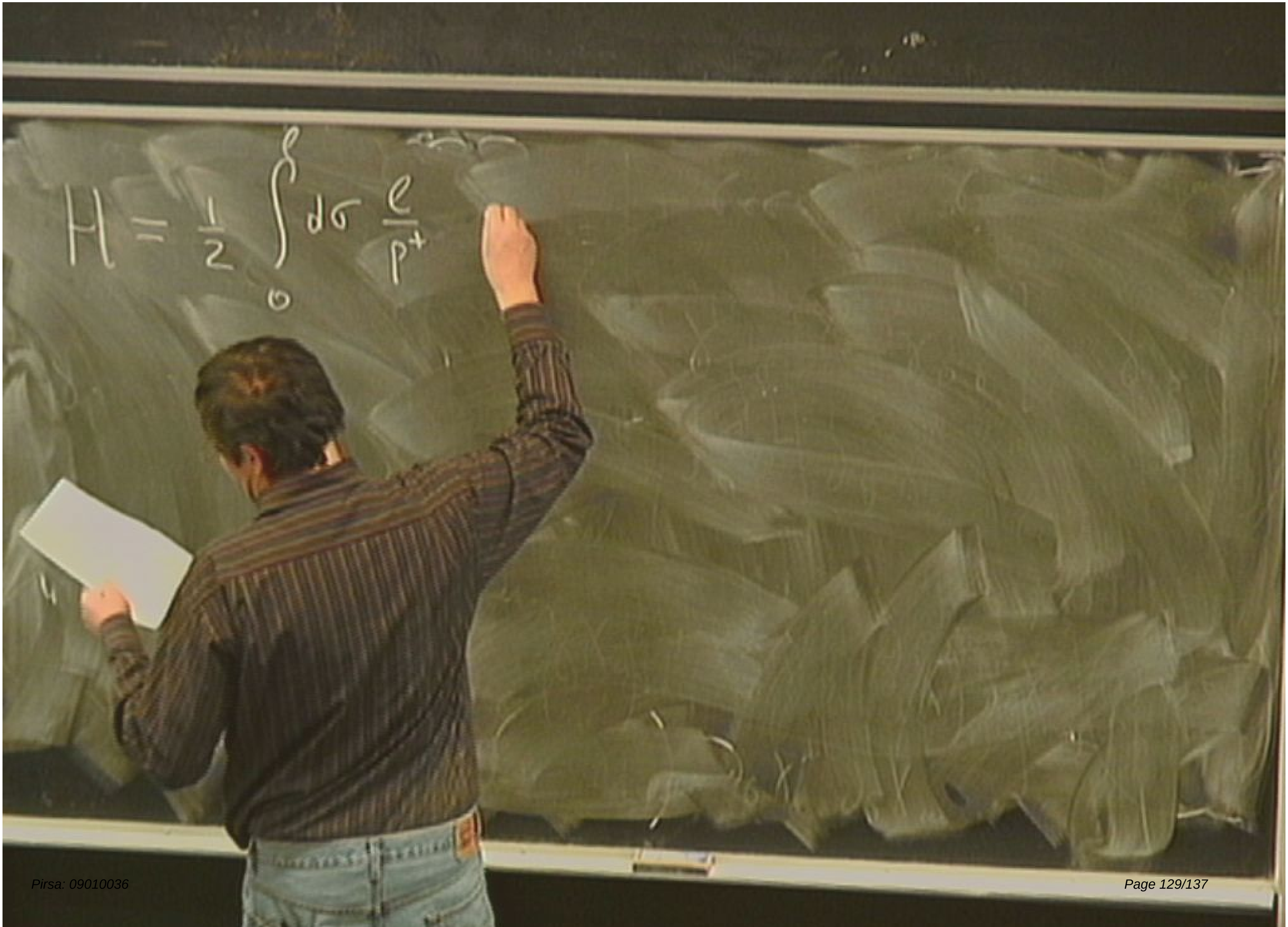
$$\partial X^{+}(\ldots)\Big|_{\sigma=0,\ell} = \gamma^{\sigma\sigma}\,\partial_\sigma X^{\mu} + \gamma^{\sigma\tau}\,\partial_\tau X^{\mu}$$

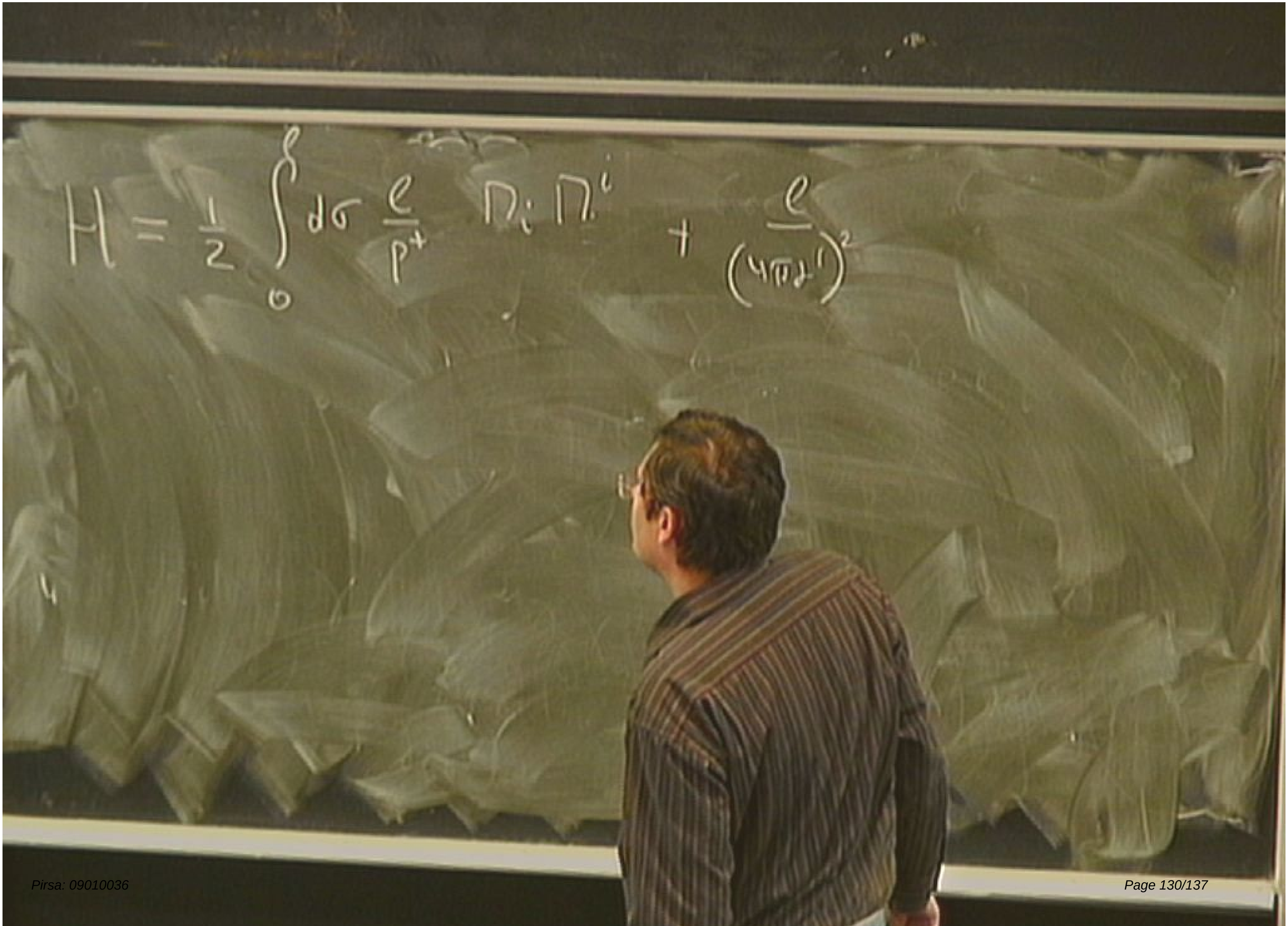
$$= \cancel{\gamma_{\tau\sigma}\,\partial_\tau X^{\mu}} + \gamma_{\sigma\sigma}^{-1}\left(1-\gamma_{\tau\sigma}^{2}\right)\partial_\sigma X^{\mu}\Big| = 0$$

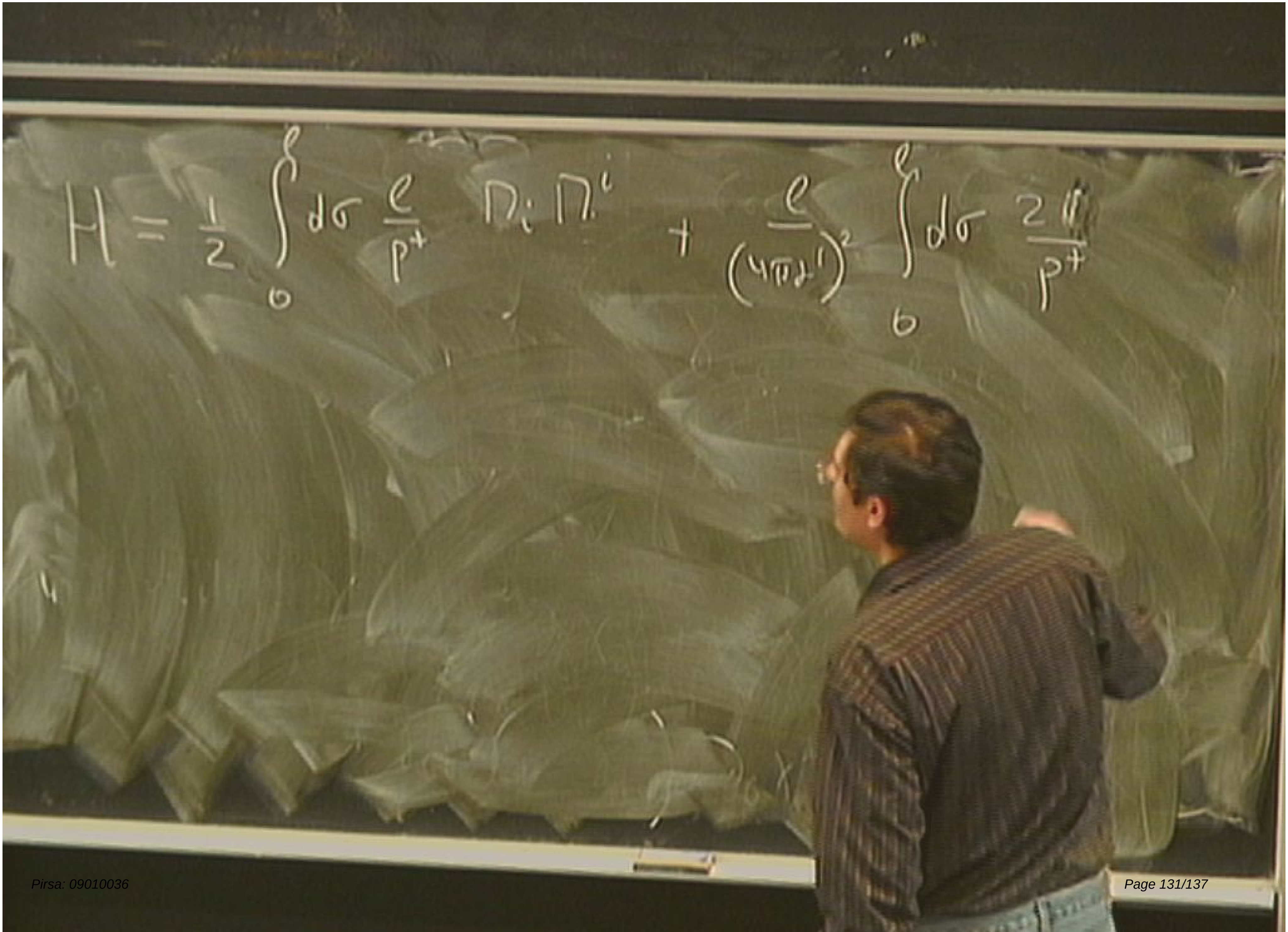
$$H = P_-\,\partial_\tau X^- + \int_0^\ell d\sigma\,\Pi_i\,\partial_\tau X^i - L$$

$$= \cancel{P_-\,\partial_\tau X^-} + \int_0^\ell d\sigma\,\frac{\Pi_i\cdot\Pi^i\,\ell}{p^+} - \cancel{P_-\,\partial_\tau X^-}$$

$$- \frac{1}{\ldots}\int_0^\ell d\sigma \underbrace{\ldots}_{\gamma_{\sigma\sigma}} \frac{\Pi^i\Pi^i\,\ell}{(p^+)} + \frac{1}{4\pi\alpha'}\int_0^\ell d\sigma\,\frac{\ell}{2\pi\alpha' p^+}\,\partial_\sigma X^i\,\partial_\sigma X^i$$


H = 1/2 ∫ dσ ℓ/p+


$H = \frac{1}{2}\int_0 d\sigma \, \frac{e}{p^+} \, \Pi_i \Pi^i + \frac{e}{(4\pi\alpha')^2}$


$H = \frac{1}{2} \int_0^{\ell} d\sigma \frac{\ell}{p^+} \Pi_i \Pi^i + \frac{\ell}{(4\pi\alpha')^2} \int_0^{\ell} d\sigma \frac{2}{p^+}$

$$H = \frac{1}{2}\int_0^{\ell} d\sigma \, \frac{\ell}{p^+} \, \Pi_i \Pi^i + \frac{\ell}{(4\pi\alpha')^2}\int_0^{\ell} d\sigma \, \frac{2\ell}{p^+} \, \partial_\sigma X^i \partial_\sigma X^i$$

$$H = \frac{1}{2}\int_0^{\ell} d\sigma \, \frac{c}{p^+} \, \Pi_i \Pi^i \; + \; \frac{c}{(4\pi\alpha')^2}\int_0^{\ell} d\sigma \, \frac{2\pi}{p^+} \, \partial_\sigma X^i \partial_\sigma X^i$$

$\Rightarrow$ From

$$H = \frac{1}{2}\int_0^{\ell} d\sigma \, \frac{e}{p^+} \, \Pi_i \Pi^i + \frac{e}{(4\pi\alpha')^2}\int_0^{\ell} d\sigma \, \frac{2}{p^+} \, \partial_\sigma X^i \partial_\sigma X^i$$

$\Rightarrow$ From HJ EOM

$\partial_\tau P^+ = 0 \qquad P^+ = \text{const}$

$$H = \frac{1}{2}\int_0^{\ell} d\sigma \, \frac{e}{p^+} \, \Pi_i \Pi^i + \frac{e}{(4\pi\alpha')^2}\int_0^{\ell} d\sigma \, \frac{2\pi}{p^+} \, \partial_\sigma X^i \partial_\sigma X^i$$

$\Rightarrow$ From H J EOM

$\partial_\tau p^+ = 0 \qquad p^+ = \text{const}$

$$H = \frac{e}{4\pi\alpha' p^+}\int_0^{\ell} d\sigma \, \Big[\qquad \Big]$$

$$H = \frac{1}{2}\int_0^{\ell} d\sigma \, \frac{e}{p^+} \, \Pi_i \Pi^i + \frac{e}{(4\pi\alpha')^2}\int_0^{\ell} d\sigma \, \frac{2}{p^+} \, \partial_\sigma X^i \partial_\sigma X^i$$

$\Rightarrow$ From H's EOM

$\partial_\tau P^+ = 0$ $\qquad p^+ = \text{const}$

$$H = \frac{e}{4\pi\alpha' p^+}\int_0^{\ell} d\sigma \Big[\, 2\pi\alpha' \, \Pi_i \Pi^i$$

$$H = \frac{1}{2}\int_0^{\ell} d\sigma \, \frac{e}{p^+} \, \Pi_i \Pi^i + \frac{e}{(4\pi\alpha')^2}\int_0^{\ell} d\sigma \, \frac{2}{p^+} \, \partial_\sigma X^i \partial_\sigma X^i$$

$\Rightarrow$ From HJ EOM

$$\partial_\tau p^+ = 0 \qquad p^+ = \text{const}$$

$$H = \frac{e}{4\pi\alpha' p^+}\int_0^{\ell} d\sigma \left[2\pi\alpha' \, \Pi_i \Pi^i + \frac{1}{2\pi\alpha'} \partial_\sigma X^i \partial_\sigma X^i \right]$$