

Title: Introduction to the Bosonic String Part B

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Abstract: This course provides a thorough introduction to the bosonic string based on the Polyakov path integral and conformal field theory. We introduce central ideas of string theory, the tools of conformal field theory, the Polyakov path integral, and the covariant quantization of the string. We discuss string interactions and cover the tree-level and one loop amplitudes. More advanced topics such as T-duality and D-branes will be taught as part of the course. The course is geared for M.Sc. and Ph.D. students enrolled in Collaborative Ph.D. Program in Theoretical Physics. Required previous course work: Quantum Field Theory (AM516 or equivalent). The course evaluation will be based on regular problem sets that will be handed in during the term. The primary text is the book: 'String theory. Vol. 1: An introduction to the bosonic string. J. Polchinski (Santa Barbara, KITP) . 1998. 402pp. Cambridge, UK: Univ. Pr. (1998) 402 p.' All interested students should contact Alex Buchel at [abuchel@uwo.ca](mailto:abuchel@uwo.ca) as soon as possible.

$$X^\tau = \tau$$

$$\partial_\sigma \gamma_{\sigma\sigma} = 0$$

$$(-\gamma)^{1/2} = 1$$

$$\gamma_{\alpha\beta} = \begin{bmatrix} \gamma_{\tau\tau}(\tau, \sigma) & \gamma_{\tau\sigma}(\tau, \sigma) \\ \gamma_{\tau\sigma}(\tau, \sigma) & \gamma_{\sigma\sigma}(\tau) \end{bmatrix}$$

$$X^1 = \tau$$

$$\partial_\alpha \gamma_{\alpha\beta} = 0$$

$$(-\gamma)^{1/2} = 1$$

$$\gamma_{\alpha\beta} = \begin{bmatrix} \gamma_{\tau\tau}(\tau, \sigma) & \gamma_{\tau\sigma}(\tau, \sigma) \\ \gamma_{\sigma\tau}(\tau, \sigma) & \gamma_{\sigma\sigma}(\tau) \end{bmatrix}$$

$$\det \gamma_{\alpha\beta} = -1 \Rightarrow \gamma_{\sigma\sigma} \gamma_{\tau\tau} - \gamma_{\tau\sigma}^2 = -1$$

$$\gamma_{\tau\tau} = \gamma_{\sigma\sigma}^{-1} [\gamma_{\tau\sigma}^2 - 1]$$

$$\gamma_{AB} = \begin{bmatrix} \gamma_{tt} & \gamma_{t\sigma} \\ \gamma_{t\sigma} & \gamma_{\sigma\sigma} \end{bmatrix} = \begin{bmatrix} -\gamma_{\sigma\sigma} & \gamma_{\tau\sigma} \\ \gamma_{\tau\sigma} & \gamma_{\sigma\sigma}^{-1} [1 - \gamma_{\tau\sigma}^2] \end{bmatrix}$$

$$S_p = -\frac{1}{4\pi d'} \int dt d\sigma$$



$$\delta_{AB} = \begin{bmatrix} \gamma_{tt} & \gamma_{t\alpha} \\ \gamma_{t\alpha} & \gamma_{\alpha\beta} \end{bmatrix} = \begin{bmatrix} -\gamma_{00} & \gamma_{0\alpha} \\ \gamma_{0\alpha} & \gamma_{\alpha\beta} \left[ 1 - \gamma_{00}^2 \right] \end{bmatrix}$$

$$\delta_p = -\frac{1}{\sqrt{|\gamma|}} \int dt d\alpha d\beta \left[ \sqrt{|\gamma|} \gamma_{AB} \partial_\alpha X^A \partial_\beta X^B \right]$$

$$\gamma^{\alpha\beta} = \begin{bmatrix} \gamma^{tt} & \gamma^{t\sigma} \\ \gamma^{t\sigma} & \gamma^{\sigma\sigma} \end{bmatrix} = \begin{bmatrix} -\gamma_{\sigma\sigma} & \gamma_{\tau\sigma} \\ \gamma_{\tau\sigma} & \gamma_{\sigma\sigma}^{-1} [1 - \gamma_{\tau\sigma}^2] \end{bmatrix}$$

$$S_p = -\frac{1}{4\pi\alpha'} \int dt d\tau d\sigma \left[ \sqrt{-\gamma} \gamma^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X_\mu \right]$$

$$= -\frac{1}{4\pi\alpha'} \int dt d\tau d\sigma \left[ \dots \right]$$

$$\gamma_{\alpha\beta} = \begin{bmatrix} \gamma_{\alpha\alpha} & \gamma_{\alpha 0} \\ \gamma_{\alpha 0} & \gamma_{00} \end{bmatrix} = \begin{bmatrix} -\gamma_{00} & \gamma_{00} \\ \gamma_{00} & \gamma_{00}^{-1} [1 - \gamma_{00}^2] \end{bmatrix}$$

$$S_p = -\frac{1}{4\pi\alpha'} \int dt d\tau d\sigma \left[ \sqrt{-\gamma} \gamma^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X_\mu \right]$$

$$= -\frac{1}{4\pi\alpha'} \int dt d\tau d\sigma \left[ -\gamma \right]$$

$$\gamma_{\alpha\beta} = \begin{bmatrix} \gamma_{\alpha\alpha} & \gamma_{\alpha 0} \\ \gamma_{\alpha 0} & \gamma_{00} \end{bmatrix} = \begin{bmatrix} -\gamma_{00} & \gamma_{00} \\ \gamma_{00} & \gamma_{00}^{-1} [1 - \gamma_{00}^2] \end{bmatrix}$$

$$S_p = -\frac{1}{4\pi\alpha'} \int dt d\sigma \left[ \sqrt{-\gamma} \gamma^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X_\mu \right]$$

$$= -\frac{1}{4\pi\alpha'} \int dt d\sigma \left[ -\gamma_{00} \partial_\alpha X^\mu \partial_\alpha X_\mu \right]$$



$$\gamma_{\alpha\beta} = \begin{bmatrix} \gamma_{\alpha\alpha} & \gamma_{\alpha 0} \\ \gamma_{\alpha 0} & \gamma_{00} \end{bmatrix} = \begin{bmatrix} -\gamma_{00} & \gamma_{00} \\ \gamma_{00} & \gamma_{00}^{-1} [1 - \gamma_{00}^2] \end{bmatrix}$$

$$S_p = -\frac{1}{4\pi\alpha'} \int dt d\sigma \left[ \sqrt{-\gamma} \gamma^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X_\mu \right]$$

$$= -\frac{1}{4\pi\alpha'} \int dt d\sigma \left[ -\gamma_{00} \partial_\tau X^\mu \partial_\tau X_\mu + 2\gamma_{00} \partial_\tau X^\mu \partial_\sigma X_\mu \right]$$

$$\begin{aligned}
 S_p &= -\frac{1}{4\pi\alpha'} \int d\tau d\sigma \left[ \gamma_{\tau\sigma} \gamma^{\mu\nu} \partial_\alpha X^\mu \partial_\beta X^\nu \right] \\
 &= -\frac{1}{4\pi\alpha'} \int d\tau d\sigma \left[ -\gamma_{\sigma\tau} \partial_\tau X^\mu \partial_\sigma X_\mu + 2\gamma_{\tau\sigma} \partial_\tau X^\mu \partial_\sigma X_\mu \right. \\
 &\quad \left. + \gamma_{\sigma\sigma}^{-1} (1 - \gamma_{\tau\tau}^2) \partial_\sigma X^\mu \partial_\sigma X_\mu \right]
 \end{aligned}$$

$$\left[ \begin{array}{c} \sigma_{11}^2 \\ \sigma_{12}^2 \end{array} \right]$$

$$X^{in} = \{ X^+, X^-, X^i \}$$

*[The rest of the chalkboard contains heavily scribbled-out text, likely representing a derivation or a set of equations that has been obscured.]*

**CAUTION**  
 SURFACE AFTER  
 RECENT USE IS  
 EXTREMELY HOT

$$X^{\text{un}} = \{X^+, X^-, X^i\}$$

$$\partial_c X^+ = 1 \quad \partial_b X^+ = 0$$

$$\left[ \begin{array}{c} \alpha_{\tau\tau}^2 \\ \alpha_{\tau\sigma}^2 \\ \alpha_{\sigma\sigma}^2 \end{array} \right]$$

$$X^{in} = \{X^+, X^-, X^i\}$$

$$\partial_c X^+ = 1 \quad \partial_\sigma X^+ = 0$$

$\Rightarrow$  collect  $i, j$  pieces

$$+ \gamma_{00} (1 - \gamma_{00}) \partial_0 X^m \partial_0 X_m$$

$$X^m = \{ X^+, X^-, X^i \}$$

$$\partial_\tau X^+ = 1 \quad \partial_\sigma X^+ = 0$$

$\Rightarrow$  collect  $i, j$  pieces

$$- \gamma_{00} \partial_\tau X^i \partial_\tau X^i + 2 \gamma_{0i} \partial_\tau X^i \partial_0 X^i + \gamma_{ij} \partial_0 X^i \partial_0 X^j$$

$$+ \gamma_{66} (1 - \gamma_{26}) \partial_6 X^m \partial_6 X_m$$

$$X^m = \{ X^+, X^-, X^i \}$$

$$\partial_t X^+ = 1 \quad \partial_6 X^+ = 0$$

$\Rightarrow$  collect  $i, j$  pieces

$$- \gamma_{66} \partial_t X^i \partial_t X^i + 2 \gamma_{26} \partial_t X^i \partial_6 X^i + \gamma_{66}^{-1} (1 - \gamma_{26}^2) \partial_6 X^i \partial_6 X^i$$

$$+ \gamma_{00} (1 - \gamma_{00}) \partial_0 X^m \partial_0 X_m$$

$$X^m = \{X^+, X^-, X^i\}$$

$$\partial_\tau X^+ = 1 \quad \partial_\sigma X^+ = 0$$

$\Rightarrow$  collect  $i, j$  pieces

$$- \gamma_{00} \partial_\tau X^i \partial_\tau X^i + 2 \gamma_{00} \partial_\tau X^i \partial_\sigma X^i + \gamma_{00}^{-1} (1 - \gamma_{00}^2) \partial_\sigma X^i \partial_\sigma X^i$$



How FH on a worldsheet would affect

$\Rightarrow$  collect  $X^-$

$-\delta$

How FH on a worldsheet would affect

⇒ collect  $X^-$

$$-\gamma_{00} \partial_\tau X^+ \partial_\tau X_+ = + \gamma_{00} \partial_\sigma X^- \partial_\sigma X_+$$

How FH on a worldsheet would affect

→ collect  $X^-$

$$-\delta_{00} \partial_\tau X^+ \partial_\tau X_+ = + \delta_{00} \partial_\tau X^+ \partial_\tau X$$

How FH on a worldsheet would affect

→ collect  $X^-$

$$-\delta_{00} \partial_\tau X^+ \partial_\tau X^+ = + \delta_{00} \partial_\tau X^+ \partial_\tau X^-$$

How EH on a worldsheet would affect

→ collect  $X^-$

$$-\gamma_{\sigma\sigma} \partial_\tau X^+ \partial_\tau X_- = + \gamma_{\sigma\sigma} \partial_\tau X^+ \partial_\tau X^-$$
$$-\gamma_{\sigma\sigma} \partial_\tau X^- \partial_\tau X_+ = + \gamma_{\sigma\sigma} \partial_\tau X^- \partial_\tau X^+$$

How EH on a worldsheet would affect

→ collect  $X^-$

$$-\gamma_{\sigma\sigma} \partial_\tau X^+ \partial_\tau X^- = + \gamma_{\sigma\sigma} \partial_\tau X^+ \partial_\tau X^-$$

$$-\gamma_{\sigma\sigma} \partial_\tau X^- \partial_\tau X^+ = + \gamma_{\sigma\sigma} \partial_\tau X^- \partial_\tau X^+$$

How EH on a worldsheet would affect Eqn?

⇒ collect  $X^-$

$$\left. \begin{aligned}
 -\gamma_{\sigma\sigma} \partial_\tau X^+ \partial_\tau X^+ &= + \gamma_{\sigma\sigma} \partial_\tau X^+ \partial_\tau X^- \\
 -\gamma_{\sigma\sigma} \partial_\tau X^- \partial_\tau X^- &= + \gamma_{\sigma\sigma} \partial_\tau X^- \partial_\tau X^+
 \end{aligned} \right\} = 2\gamma_{\sigma\sigma} \partial_\tau X^-$$

How EH on a worldsheet would affect Eqn?

⇒ collect  $X^-$

$$\left. \begin{aligned} -\gamma_{\sigma\sigma} \partial_\tau X^+ \partial_\tau X_- &= +\gamma_{\sigma\sigma} \partial_\tau X^+ \partial_\tau X^- \\ -\gamma_{\sigma\sigma} \partial_\tau X^- \partial_\tau X_+ &= +\gamma_{\sigma\sigma} \partial_\tau X^- \partial_\tau X^+ \end{aligned} \right\} = 2\gamma_{\sigma\sigma} \partial_\tau X^-$$

$$+ 2\gamma_{\tau\sigma} \partial_\tau X^+ \partial_\sigma X_+ =$$





How  $\Gamma_H$  on a worldsheet would affect Eq.?

$\Rightarrow$  collect  $X^-$

$$\left. \begin{aligned}
 -\gamma_{\alpha\beta} \partial_\tau X^+ \partial_\tau X^- &= + \gamma_{\alpha\beta} \partial_\tau X^+ \partial_\tau X^- \\
 -\gamma_{\alpha\beta} \partial_\tau X^- \partial_\tau X^+ &= + \gamma_{\alpha\beta} \partial_\tau X^- \partial_\tau X^+
 \end{aligned} \right\} = 2\gamma_{\alpha\beta} \partial_\tau X^+ \partial_\tau X^-$$

$$+ 2\gamma_{\alpha\beta} \partial_\tau X^+ \partial_\tau X^- = - 2\gamma_{\alpha\beta} \partial_\tau X^- \partial_\tau X^+$$

Let's introduce

$$X^-(\pi, \sigma) = X^-(\tau)$$

Let's introduce

$$X^-(\tau, \sigma) = X^-(\tau) + Y^-(\tau, \sigma)$$

Let's introduce

$$X^-(\tau, \sigma) = X^-(\tau) + Y^-(\tau, \sigma)$$

$$X^-(\tau) \stackrel{\text{def}}{=} \int_0^{\tau} d\sigma X^-(\tau, \sigma)$$

Let's introduce

$$\bar{X}(\tau, \sigma) = \bar{X}(\tau) + \bar{Y}(\tau, \sigma)$$

$$\bar{X}(\tau) \stackrel{\text{def}}{=} \int_0^{\tau} d\sigma \bar{X}(\tau, \sigma) \Rightarrow \int_0^{\tau} d\sigma \bar{Y} = 0$$

Let's introduce

$$\bar{X}(\tau, \sigma) = \bar{X}(\tau) + \bar{Y}(\tau, \sigma)$$

$$\bar{X}(\tau) \stackrel{\text{def}}{=} \frac{1}{\sigma} \int_0^\sigma \bar{X}(\tau, \sigma) \Rightarrow \int_0^\sigma \bar{X}(\tau, \sigma)$$

Let's introduce

$$X^-(\tau, \sigma) = X^-(\tau) + Y^-(\tau, \sigma)$$

$$X^-(\tau) \stackrel{\text{def}}{=} \int_0^{\tau} d\sigma X^-(\tau, \sigma) \Rightarrow \int_0^{\tau} d\sigma Y^-(\tau, \sigma) = 0$$

$$+ \delta_{\sigma\sigma} (1 - \delta_{\sigma\sigma}^2) \partial_{\sigma} X^m \partial_{\sigma} X_m$$

$$2\delta_{\sigma\sigma} \partial_{\sigma} X^{-} - 2\delta_{\sigma\sigma} \partial_{\sigma} X^{-}$$

CAUTION  
DO NOT TOUCH  
THIS BOARD  
OR YOU WILL  
BE IN TROUBLE



$$+ \gamma_{\sigma\sigma} (1 - \gamma_{\sigma\sigma}^2) \partial_\sigma X^m \partial_\sigma X_m$$

$$2\gamma_{\sigma\sigma} \textcircled{A} \partial_t X^- - 2\gamma_{\sigma\sigma} \textcircled{B} \partial_\sigma X^-$$

$$\textcircled{A} 2\gamma_{\sigma\sigma} \partial_t X^- + 2\gamma_{\sigma\sigma}$$

$$2\gamma_{\sigma\sigma} \textcircled{A} \partial_\tau X^- - 2\gamma_{\tau\sigma} \textcircled{B} \partial_\sigma X^-$$

$$\textcircled{A} 2\gamma_{\sigma\sigma} \partial_\tau X^- + \underbrace{2\gamma_{\sigma\sigma} \partial_\tau X^-}_{\text{does not matter}}$$

$$2\gamma_{\sigma\sigma} \textcircled{A} \partial_\tau X^- - 2\gamma_{\tau\sigma} \textcircled{B} \partial_\sigma X^-$$

$$\textcircled{A} 2\gamma_{\sigma\sigma} \partial_\tau X^- + \underbrace{2\gamma_{\sigma\sigma} \partial_\tau X^-}_{\text{does not matter}}$$

$$\int_{-\infty}^{+\infty} d\tau \int_0^l d\sigma$$

$$2\gamma_{\sigma\sigma} \textcircled{A} \partial_\tau X^- - 2\gamma_{\tau\sigma} \textcircled{B} \partial_\sigma X^-$$

$$\textcircled{A} \left[ 2\gamma_{\sigma\sigma} \partial_\tau X^- + \underbrace{2\gamma_{\sigma\sigma} \partial_\tau X^-}_{\text{does not matter}} \right]$$

$$\int_{-\infty}^{+\infty} d\tau \int_0^l d\sigma \left[ 2\gamma_{\sigma\sigma} \partial_\tau X^- \right]$$

$$2\gamma_{\sigma\sigma} \overset{\textcircled{A}}{\partial_\tau} X^- - 2\gamma_{\tau\sigma} \overset{\textcircled{B}}{\partial_\sigma} X^-$$

$$\textcircled{A} \quad 2\gamma_{\sigma\sigma} \partial_\tau X^- + \underbrace{2\gamma_{\sigma\sigma} \partial_\tau X^-}_{\text{does not matter}} - \int d\tau \left\{ 2\gamma_{\sigma\sigma} \right.$$

$$\int_{-\infty}^{+\infty} d\tau \int_0^l d\sigma \left[ 2\gamma_{\sigma\sigma} \partial_\tau X^- \right]$$

$$+ \gamma_{\sigma\sigma}^{-1} (1 - \gamma_{\tau\sigma}^2) \partial_\sigma X^m \partial_\sigma X_m$$

$$2\gamma_{\sigma\sigma} \partial_\tau X^- - 2\gamma_{\tau\sigma} \partial_\sigma X^-$$

$$\textcircled{A} 2\gamma_{\sigma\sigma} \partial_\tau X^- + 2\gamma_{\sigma\sigma} \partial_\tau X^-$$

does not matter

$$\int_{-\infty}^{+\infty} d\tau \int d\sigma \left[ \gamma_{\sigma\sigma} \partial_\tau X^- \right] - \int d\sigma \left\{ 2\gamma_{\sigma\sigma} \int d\sigma \partial_\tau X^- \right\}$$

$$+ \gamma_{\sigma\sigma}^{-1} (1 - \gamma_{\tau\sigma}^2) \partial_\sigma X^m \partial_\sigma X_m$$

$$2\gamma_{\sigma\sigma} \overset{(A)}{\partial_\tau} X^- - 2\gamma_{\tau\sigma} \overset{(B)}{\partial_\sigma} X^-$$

$$\textcircled{A} \quad 2\gamma_{\sigma\sigma} \partial_\tau X^- + 2\gamma_{\sigma\sigma} \partial_\tau X^-$$

$$\int_{-\infty}^{+\infty} d\tau \int_0^l d\sigma \left[ 2\gamma_{\sigma\sigma} \partial_\tau X^- \right] - \int d\sigma \left\{ 2\gamma_{\sigma\sigma} \int d\sigma \partial_\tau X^- \right\}$$

does not matter  
on top or bottom

$$= \int d\tau \left\{ 2\gamma_{\sigma\sigma} \right\}$$

$$+ \gamma_{\sigma\sigma}^{-1} (1 - \gamma_{\tau\sigma}^2) \partial_\sigma X^m \partial_\sigma X_m$$

$$2\gamma_{\sigma\sigma} \overset{(A)}{\partial_\tau} X^- - 2\gamma_{\tau\sigma} \overset{(B)}{\partial_\sigma} X^-$$

$$\begin{aligned} & \textcircled{A} \quad 2\gamma_{\sigma\sigma} \partial_\tau X^- + 2\gamma_{\sigma\sigma} \partial_\tau X^- \\ & \int_{-\infty}^{+\infty} d\tau \int_0^l d\sigma \left[ \underbrace{2\gamma_{\sigma\sigma} \partial_\tau X^-}_{\text{does not matter}} \right] - \int d\sigma \left\{ 2\gamma_{\sigma\sigma} \int d\sigma \partial_\tau X^- \right\} \\ & = \int d\tau \left\{ 2\gamma_{\sigma\sigma} \partial_\tau \right\} \end{aligned}$$



$$+ \gamma_{\sigma\sigma}^{-1} (1 - \gamma_{\tau\sigma}^2) \partial_\sigma X^m \partial_\sigma X_m$$

$$2\gamma_{\sigma\sigma} \partial_\tau X^- - 2\gamma_{\tau\sigma} \partial_\sigma X^-$$

$$\textcircled{A} \quad 2\gamma_{\sigma\sigma} \partial_\tau X^- + 2\gamma_{\tau\sigma} \partial_\sigma X^-$$

$$\int_{-\infty}^{+\infty} d\tau \int_0^l d\sigma \left[ 2\gamma_{\sigma\sigma} \partial_\tau X^- \right] - \int d\sigma \left\{ 2\gamma_{\sigma\sigma} \int d\sigma \partial_\tau X^- \right\}$$

does not matter  
matter for soap

$$= \int d\tau \left\{ 2\gamma_{\sigma\sigma} \partial_\tau \int_0^l d\sigma X^- \right\}$$

$$+ \gamma_{00}^{-1} (1 - \gamma_{00}^2) \partial_0 X^m \partial_0 X_m$$

$$2\gamma_{00} \partial_t X^- \quad \text{(A)} \quad - 2\gamma_{00} \partial_0 X^- \quad \text{(B)}$$

$$\text{(A)} \quad 2\gamma_{00} \partial_t X^- + 2\gamma_{00} \partial_t X^-$$

$$\int_{-\infty}^{+\infty} dt \int_0^1 dx \left[ 2\gamma_{00} \partial_t X^- \right] - \int_0^1 dx \left[ 2\gamma_{00} \int_0^1 dx \partial_t X^- \right]$$

does not matter

$$= \int dt \left\{ 2\gamma_{00} \int_0^1 dx \left[ \partial_t X^- \right] \right\} = 0$$

$$+ \gamma_{\sigma\sigma}^{-1} (1 - \gamma_{\tau\sigma}^2) \partial_\sigma X^m \partial_\sigma X_m$$

$$2\gamma_{\sigma\sigma} \partial_\tau X^- \quad - \quad 2\gamma_{\tau\sigma} \partial_\sigma X^-$$

$$\textcircled{A} \quad 2\gamma_{\sigma\sigma} \partial_\tau X^- + \cancel{2\gamma_{\tau\sigma} \partial_\sigma X^-}$$

does not matter

$$\int_{-\infty}^{+\infty} d\tau \int_0^{\rho} d\sigma \left[ 2\gamma_{\sigma\sigma} \partial_\tau X^- \right] - \int_0^{\rho} d\sigma \left\{ 2\gamma_{\sigma\sigma} \int_0^{\rho} d\sigma \partial_\tau X^- \right\} = 0$$

$$= \int d\tau \left\{ 2\gamma_{\sigma\sigma} \partial_\tau \int_0^{\rho} d\sigma \right\} = 0$$

(B)

$$-2\gamma\pi\sigma \partial\sigma \left[ \cancel{\gamma}(\pi) + \gamma(\pi, \sigma) \right]$$

(B)

$$-2\gamma_{\tau\sigma} \partial_\sigma \left[ \cancel{\gamma^\mu(\tau)} + Y^-(\tau, \sigma) \right] = -2\gamma_{\tau\sigma} Y^-$$

(B)

$$-2\gamma\pi\sigma \partial_\sigma \left[ \cancel{\psi}(\pi) + \psi(\pi) \right] = -2\gamma\pi\sigma \partial_\sigma \psi$$



(B)

$$-2\gamma\pi\sigma \partial_\sigma \left[ \cancel{\gamma}(\pi) + \gamma(\pi) \right] = -2\gamma\pi\sigma \partial_\sigma \gamma$$

$$+ \gamma_{\sigma\sigma}^{-1} (1 - \gamma_{\sigma\sigma}^2) \partial_\sigma X^m \partial_\sigma X_m$$

$$2\gamma_{\sigma\sigma} \partial_\tau X^- - 2\gamma_{\tau\sigma} \partial_\sigma X^-$$

$$\textcircled{A} \quad 2\gamma_{\sigma\sigma} \partial_\tau X^- + \cancel{2\gamma_{\sigma\sigma} \partial_\tau X^- (\tau, \sigma)}$$

does not matter

$$\int_{-\infty}^{+\infty} dt \int_0^l dx \left[ 2\gamma_{\sigma\sigma} \partial_\tau X^- \right] - \int dx \left\{ 2\gamma_{\sigma\sigma} \int dx \partial_\tau X^- \right\}$$

$$= \int dt \left\{ 2\gamma_{\sigma\sigma} \int_0^l dx \partial_\tau X^- \right\} = 0$$



(B)

$$-2\gamma_{\tau\sigma} \partial_\sigma \left[ \cancel{\gamma}(\tau) + Y^-(\tau, \sigma) \right] = -2\gamma_{\tau\sigma} \partial_\sigma Y^-$$

$Y^-$  is not a dynamical field, but rather  
is a sophisticated Lag multiplier.

(B)

$$-2\gamma_{\tau\sigma} \partial_{\sigma} \left[ \cancel{\gamma(\tau)} + Y(\tau) \right] = -2\gamma_{\tau\sigma} \partial_{\sigma} Y^{-}$$

$Y^{-}$  is not a dynamical field, but rather

is a sophisticated Lag multiplier.

$\Rightarrow$  It will

(B)

$$-2\gamma_{\tau\sigma} \partial_\sigma \left[ \cancel{\gamma(\tau)} + Y^-(\tau, \sigma) \right] = -2\gamma_{\tau\sigma} \partial_\sigma Y^-$$

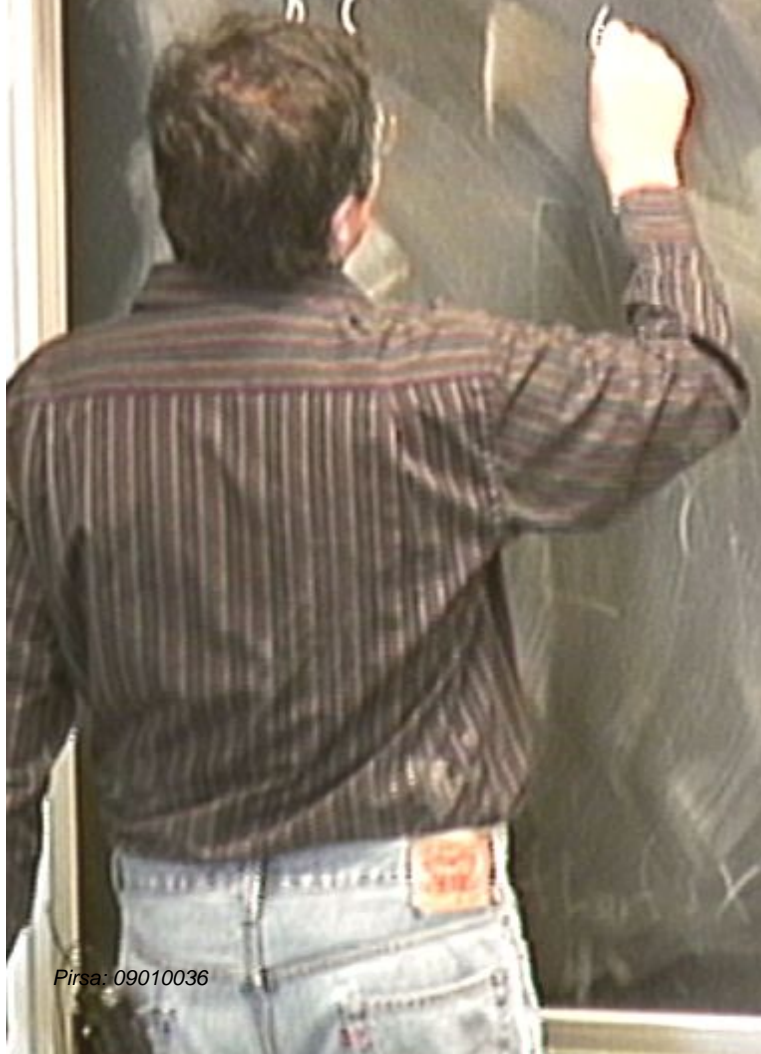
$Y^-$  is not a dynamical field, but rather

is a sophisticated Lag multiplier.

$\Rightarrow$  It will lead us to

$$\gamma_{\tau\sigma}(\tau, \sigma) = 0$$

$$0 = \frac{\delta S_p}{\delta Y} = -\delta$$



$$0 = \delta S_p = \delta \int dt \pi \dot{\sigma}$$

$$0 = \delta S_p = \delta \int dt \pi d\sigma$$

$$0 = \delta S_p = \delta \int d\tau d\sigma \left( -2\gamma_{\tau\sigma} \partial_\sigma \delta Y^- \right)$$

$$0 = \delta S_p = \int d\tau d\sigma \left( -2 \gamma_{\tau\sigma} \partial_\sigma \delta Y^- \right)$$

$$= \int d\sigma \partial_\sigma \left( +2 \gamma_{\tau\sigma} \right) \delta Y^-$$



$$\begin{aligned}
 0 - \delta S_p &= \int d\tau d\sigma \left( -2 \gamma_{\tau\sigma} \partial_\sigma \delta Y^- \right) \\
 &= \int d\sigma d\tau \left( +2 \right) \left( \partial_\sigma \delta Y^- \right) + \left. -2 \int d\tau \gamma_{\tau\sigma} \delta Y^- \right|_{\sigma=0}^{\sigma=l}
 \end{aligned}$$

$$\begin{aligned}
 0 = \delta S_p &= \int d\tau d\sigma \left( -2\gamma_{\tau\sigma} \partial_\sigma \delta Y^- \right) \\
 &= \int d\sigma \alpha \left( +2 \right) \left[ \partial_\sigma \delta Y^- \right] + \left. -2 \int d\tau \gamma_{\tau\sigma} \delta Y^- \right|_{\sigma=0}^{\sigma=\ell} \\
 N & \text{ Boundary condition in L-C gauge.} \\
 \delta x_{\mu} &= 0
 \end{aligned}$$

$$\begin{aligned}
 0 = \delta S_p &= \int d\tau d\sigma \left( -2\delta\tau\sigma \partial_\sigma \delta Y^- \right) \\
 &= \int d\sigma \left[ \delta\tau \partial_\sigma \delta Y^- \right]_{\sigma=0}^{\sigma=1} + \int d\tau \delta\tau \delta Y^- \Big|_{\sigma=0}^{\sigma=1} \\
 \hookrightarrow N & \text{ Boundary condition} \\
 & \text{L-c gauge} \\
 & \delta g_{\mu\nu} = \delta X^\mu + \delta X^\nu
 \end{aligned}$$

$$0 = \delta S_p = \int d\tau d\sigma \left( -2\gamma_{\tau\sigma} \partial_\sigma \delta Y^- \right)$$

$$= \int d\sigma \left( +2 \left[ \partial_\sigma \delta Y^- \right] \right) + \left. -2 \int d\tau \gamma_{\tau\sigma} \delta Y^- \right|_{\sigma=0}^{\sigma=\ell}$$

Boundary condition in L-C gauge.

$$0 = \delta X^m = 0 = \gamma_{\tau\sigma} \partial_\sigma X^m + \gamma_{\tau\sigma} \partial_\sigma X^m$$

$$= \gamma_{\tau\sigma} \partial_\sigma X^m + \gamma_{\tau\sigma} \partial_\sigma X^m$$

$$\begin{aligned}
 \delta S_p &= \int d\tau d\sigma \left( -2\gamma_{\tau\sigma} \partial_\sigma \delta Y^- \right) \\
 &= \int d\sigma d\tau \left( +2 \right) \left[ \partial_\sigma \partial_{\tau\sigma} \right] \delta Y^- + \left. -2 \int d\tau \gamma_{\tau\sigma} \delta Y^- \right|_{\sigma=0}^{\sigma=l}
 \end{aligned}$$

$\hookrightarrow$  N Boundary condition in L-C gauge.

$$\begin{aligned}
 \delta X^m = 0 &= \gamma_{\tau\sigma} \partial_\sigma X^m + \gamma_{\sigma\tau} \partial_\tau X^m \\
 &= \gamma_{\tau\sigma} \partial_\tau X^m + \gamma_{\sigma\tau} \partial_\sigma X^m
 \end{aligned}$$

$$\partial_{\tau} (1 - \delta_{\tau\tau}) \partial_{\delta} X \partial_{\delta} X_m$$

$$u_4 = +$$

$$\delta_{\tau\tau} (\tau, 0)$$

$$+ \gamma_{\sigma\sigma}^{-1} (1 - \gamma_{\tau\sigma}^2) \partial_\sigma X^m \partial_\sigma X_m$$

$$u_4 = +$$

$$\gamma_{\sigma\sigma}(\tau, 0) \partial_\sigma X^+ + \gamma_{\sigma\sigma}^{-1}(\tau) \left[ 1 - \gamma_{\tau\sigma}^2(\tau, 0) \right] \partial_\sigma X^+ = 0$$

$$+ \gamma_{\sigma\sigma}^{-1} (1 - \gamma_{\tau\sigma}^2) \partial_\sigma X^m \partial_\sigma X_m$$

$$u_4 = +$$

$$\gamma_{\sigma\sigma}(\tau, 0) \partial_\sigma X^+ + \gamma_{\sigma\sigma}^{-1}(\tau) \left[ 1 - \gamma_{\tau\sigma}^2(\tau, 0) \right] \partial_\sigma X^+ = 0$$



$$+ \gamma_{\sigma\sigma}^{-1} (1 - \gamma_{\tau\sigma}^2) \partial_{\sigma} X^m \partial_{\sigma} X_m$$

$$u_4 = +$$

$$(\tau, 0) \frac{\partial_{\tau} X^+}{=} + \gamma_{\sigma\sigma}^{-1}(\tau) \left[ 1 - \gamma_{\tau\sigma}^2(\tau, 0) \right] \cancel{\partial_{\sigma} X^+} = 0$$

$$+ \gamma_{\sigma\sigma}^{-1} (1 - \gamma_{\tau\sigma}^2) \partial_{\sigma} X^m \partial_{\sigma} X_m$$

$$u_4 = +$$

$$\gamma_{\tau\sigma}(\tau, 0) \underbrace{\frac{\partial_{\tau} X^+}{1}} + \gamma_{\sigma\sigma}^{-1}(\tau) \left[ 1 - \gamma_{\tau\sigma}^2(\tau, 0) \right] \cancel{\partial_{\sigma} X^+} = 0$$

$$+ \gamma_{\sigma\sigma}^{-1} (1 - \gamma_{\tau\sigma}^2) \partial_{\sigma} X^m \partial_{\sigma} X_m$$

$$u_4 = +$$

$$\gamma_{\sigma\sigma}(\tau, 0) \underbrace{\partial_{\sigma} X^+}_{=1} + \gamma_{\sigma\sigma}^{-1}(\tau) \left[ 1 - \gamma_{\tau\sigma}^2(\tau, 0) \right] \cancel{\partial_{\sigma} X^+} = 0$$

$$\gamma_{\tau\sigma}(\tau, 0)$$

$$+ \gamma_{\sigma\sigma}^{-1} (1 - \gamma_{\tau\sigma}^2) \partial_{\sigma} X^m \partial_{\sigma} X_m$$

$$u_4 = +$$

$$\gamma_{\tau\sigma}(\tau, 0) \frac{\partial_{\tau} X^+}{=} + \gamma_{\sigma\sigma}^{-1}(\tau) \left[ 1 - \gamma_{\tau\sigma}^2(\tau, 0) \right] \cancel{\partial_{\sigma} X^+} = 0$$

$$\boxed{\gamma_{\tau\sigma}(\tau, 0) = \gamma_{\tau\sigma}(\tau, \ell) = 0}$$

$$\begin{aligned}
 0 = \delta S_p &= \int d\tau d\sigma \left( -2 \delta\tau\sigma \partial_\sigma \delta Y^- \right) \\
 &= \int d\sigma \left( +2 \partial_\sigma \delta\tau\sigma \right) \delta Y^- + \left. -2 \int d\tau \delta\tau\sigma \delta Y^- \right|_{\sigma=0}^{\sigma=l}
 \end{aligned}$$

Boundary condition in L-C gauge.

$$\begin{aligned}
 \delta X^m &= \delta\tau \partial_\sigma X^m + \delta\sigma \partial_\sigma X^m \\
 &= \delta\tau \partial_\sigma X^m + \delta\sigma \partial_\sigma X^m \\
 &= \delta\tau \partial_\sigma X^m + \delta\sigma \partial_\sigma X^m
 \end{aligned}$$

$$+ \gamma_{\sigma\sigma}^{-1} (1 - \gamma_{\tau\sigma}^2) \partial_{\sigma} X^m \partial_{\sigma} X_m$$

$$u = +$$

$$\gamma_{\tau\sigma}(\tau, 0) \frac{\partial_{\tau} X^+}{=} + \gamma_{\sigma\sigma}^{-1}(\tau) \left[ 1 - \gamma_{\tau\sigma}^2(\tau, 0) \right] \cancel{\partial_{\sigma} X^+} = 0$$

$$\boxed{\gamma_{\tau\sigma}(\tau, 0) = \gamma_{\tau\sigma}(\tau, \ell) = 0}$$

$$u = -$$

$$\gamma_{\sigma\sigma}^{-1} \partial_{\sigma} X^i = 0$$

$$+ \gamma_{\sigma\sigma}^{-1} (1 - \gamma_{\tau\sigma}^2) \partial_{\sigma} X^m \partial_{\sigma} X_m$$

$$u = +$$

$$\gamma_{\tau\sigma}(\tau, 0) \frac{\partial_{\tau} X^+}{=} + \gamma_{\sigma\sigma}^{-1}(\tau) \left[ 1 - \gamma_{\tau\sigma}^2(\tau, 0) \right] \cancel{\partial_{\sigma} X^+} = 0$$

$$\boxed{\gamma_{\tau\sigma}(\tau, 0) = \gamma_{\tau\sigma}(\tau, \ell) = 0}$$

$$u = -$$

$$\gamma_{\sigma\sigma}^{-1} \partial_{\sigma} X^i \Big|_{\sigma=0, \ell} = 0$$

$$\boxed{\partial_{\sigma} X^i \Big|_{\sigma=0, \ell} = 0}$$

$$0 = \delta S_p = \int d\tau d\sigma \left( -2\gamma_{20} \partial_\sigma \delta Y^- \right)$$

$$= \int d\sigma d\tau \left( +2 \right) \left( \partial_\sigma \delta Y^- \right) +$$

Boundary condition in L-C gauge.

$$\delta X^m|_{\sigma=0} = \gamma^{\sigma\tau} \partial_\sigma X^m + \gamma^{\sigma\tau} \partial_\tau X^m$$

$$= \cancel{\gamma^{\sigma\tau} \partial_\sigma X^m} + \gamma^{\sigma\tau} \partial_\tau X^m$$



195 195 2K

(B)

EDM for  $\gamma$  implies that

Substituting  $\lambda$

(B)

EDM for  $Y^-$  implies that

$$\int dt d\sigma \partial_\sigma \mathcal{L} \delta Y^- = 0$$

(B)

EDM for  $\gamma^-$  implies that

$$\int dt d\sigma \partial_\sigma \mathcal{H}_{\tau\sigma} + \delta Y(\pi, \sigma) = 0$$

$\Downarrow$

$$\partial_\sigma \mathcal{H}_{\tau\sigma} = 0$$

(B)

EDM For  $\gamma^-$  implies that

$$\int dt d\sigma \partial_\sigma \delta \gamma_{\tau\sigma} \delta \gamma(\pi, \sigma) = 0$$

$\Downarrow$

$$\partial_\sigma \delta \gamma_{\tau\sigma} = 0$$

wrong!

(B)

EDM for  $\gamma^-$  implies that

$$\int dt d\sigma \delta \mathcal{L}_{\sigma} \delta \gamma(\pi, \sigma) = 0$$

$\Downarrow$

$$\delta \mathcal{L}_{\sigma} = 0$$

wrong!

$\tau_{ab}$   $\tau_{ab}$   $\tau_{ab}$   $\tau_{ab}$   $\tau_{ab}$   $\tau_{ab}$   $\tau_{ab}$   $\tau_{ab}$

(B)

EDM For  $Y^-$  implies that

$$\int Y^- d\sigma = 0$$

$$\int d\tau d\sigma \delta Y^- = 0$$

$$\int \delta Y^- d\sigma = 0$$

$\implies$

$$\delta Y^- = 0$$

wrong!

$\tau_{ab}$   $\tau_{ab}$   $\tau_{ab}$   $\tau_{ab}$   $\tau_{ab}$   $\tau_{ab}$   $\tau_{ab}$   $\tau_{ab}$

(B)

EDM For  $Y^-$  implies that

$$\int d\tau d\sigma \partial_\sigma \tau_{\tau\sigma} \delta Y^{\tau\sigma} = 0$$

$$\int Y^- d\sigma = 0$$

$$\int \delta Y^- d\sigma = 0$$

$\Downarrow$

$$\partial_\sigma \tau_{\tau\sigma} = 0$$

wrong!

$$\partial_\sigma \tau_{\tau\sigma} = \text{const}(\tau)$$

$\tau_{ab}$   $\tau_{ab}$   $\tau_{ab}$   $\tau_{ab}$   $\tau_{ab}$   $\tau_{ab}$   $\tau_{ab}$   $\tau_{ab}$

(B)

EDM for  $\gamma^-$  implies that

$$\int \gamma^- d\sigma = 0$$

$$\int d\tau d\sigma \partial_\sigma \gamma_{\tau\sigma} = 0$$

$$\int \delta \gamma^- d\sigma = 0$$

$\Downarrow$

$\partial_\sigma \gamma_{\tau\sigma} = 0$  wrong!

$\partial_\sigma \gamma_{\tau\sigma} = \text{const}(\tau)$

$\partial_\sigma^2 \gamma_{\tau\sigma} = 0$



$\tau_{ab} \quad \tau_{ab} \quad \tau_{ab} \quad \tau_{ab} \quad \tau_{ab} \quad \tau_{ab} \quad \tau_{ab}$

(B)

EDM For  $\gamma^-$  implies that

$$\int \gamma^- d\sigma = 0$$

$$\int d\tau d\sigma \partial_\sigma \gamma_{\tau\sigma} \delta \gamma_{\tau\sigma} = 0$$

$$\int \delta \gamma^- d\sigma = 0$$

$\Downarrow$

$$\partial_\sigma \gamma_{\tau\sigma} = 0$$

wrong!

$$\partial_\sigma \gamma_{\tau\sigma} = \text{const}(\tau)$$

$$\partial_\sigma^2 \gamma_{\tau\sigma} = 0$$

$$\delta_{\tau\sigma}(z, 0) = 0$$

$$\delta_{\tau\sigma}(\tau, \theta) = 0$$

(B)

EDM for  $\gamma^-$  implies that

$$\int \gamma^- d\sigma = 0$$

$$\int d\tau d\sigma \partial_\sigma \gamma_{\tau\sigma} \delta \gamma_{\tau\sigma} = 0$$

$$\int \delta \gamma^- d\sigma = 0$$

$\Downarrow$

$$\partial_\sigma \gamma_{\tau\sigma} = 0$$

wrong!

$$\partial_\sigma \gamma_{\tau\sigma} = \text{const}(\tau)$$

|   |                                 |   |
|---|---------------------------------|---|
| $\partial_\sigma^2 \gamma_{\tau\sigma} = 0$ | $\gamma_{\tau\sigma}(z, 0) = 0$ | $\gamma_{\tau\sigma}(\tau, \theta) = 0$ |
| $\Downarrow$                                |                                 |   |
| $\gamma_{\tau\sigma}(\tau, \sigma) = 0$     |                                 |   |

$$\gamma_{\tau\tau} = 0$$

$$\gamma_{20} = 0$$

$$\gamma_{\tau\sigma} = 0$$

$$X^-(\tau), \gamma_{\sigma\sigma}(\tau)$$

$$\gamma_{\tau\sigma} = 0$$

$$X^-(\tau), \gamma_{\sigma\sigma}(\tau), X^i(\tau, \sigma)$$

$$\gamma_{\tau\sigma} = 0$$

$$X^-(\tau), \gamma_{\sigma\sigma}(\tau), X^i(\tau, \sigma)$$

$$\gamma_{\tau\sigma} = 0 \quad (\text{from } \frac{\delta S_P}{\delta y^-} = 0)$$

$$X^-(\tau), \quad \gamma_{\sigma\sigma}(\tau), \quad X^i(\tau, \sigma)$$



$$\gamma_{\tau\sigma} = 0 \quad (\text{from } \frac{\delta S_P}{\delta y^-} = 0 + N \text{ Bc})$$

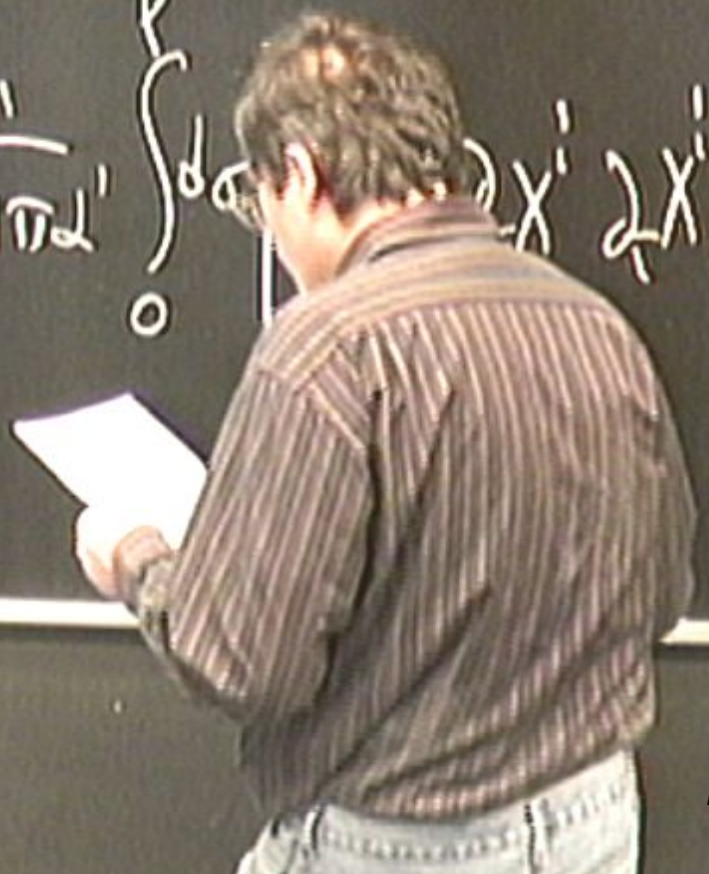
$$X^-(\tau), \gamma_{\sigma\sigma}(\tau), X^i(\tau, \sigma)$$

$$= -\ell$$

$$\gamma_{\tau\sigma} = 0 \quad (\text{from } \frac{\delta S_P}{\delta Y} = 0 + N \text{ BC})$$

$$X^-(\tau), \gamma_{\sigma\sigma}(\tau), X^i(\tau, \sigma)$$

$$L = -\frac{\ell}{2\pi\alpha'} \gamma_{\sigma\sigma}(\tau) \partial_\tau X^- + \frac{1}{4\pi\alpha'} \int_0^\ell d\sigma \partial_\tau X^i \partial_\tau X^i$$



$$\gamma_{\tau\sigma} = 0 \quad (\text{from } \frac{\delta S_P}{\delta y} = 0 + N \text{ BC})$$

$$X^-(\tau), \gamma_{\sigma\sigma}(\tau), X^i(\tau, \sigma)$$

$$L = -\frac{\ell}{2\pi\alpha'} \gamma_{\sigma\sigma}(\tau) \partial_\tau X^- + \frac{i}{4\pi\alpha'} \int_0^\ell d\sigma \left[ \gamma_{\sigma\sigma} \partial_\tau X^i \partial_\tau X^i - \gamma_{\sigma\sigma}^{-1} \partial_\sigma X^i \partial_\sigma X^i \right]$$

$$\gamma_{\tau\sigma} = 0 \quad (\text{from } \frac{\delta S_P}{\delta y} = 0 + N \text{ Bc})$$

$$X^-(\tau), \gamma_{\sigma\sigma}(\tau), X^i(\tau, \sigma)$$

$$L = -\frac{e}{2\pi\alpha'} \gamma_{\sigma\sigma}(\tau) \partial_\tau X^- + \frac{1}{4\pi\alpha'} \int_0^e d\sigma \left[ \gamma_{\sigma\sigma} \partial_\tau X^i \partial_\tau X^i - \gamma_{\sigma\sigma}^{-1} \partial_\sigma X^i \partial_\sigma X^i \right]$$

$$S_P = \int d\tau L$$

$$\gamma_{\tau\sigma} = 0 \quad (\text{from } \frac{\delta S_P}{\delta y} = 0 + N \text{ Bc})$$

$$X^-(\tau), \gamma_{\sigma\sigma}(\tau), X^i(\tau, \sigma)$$

$$L = -\frac{\ell}{2\pi\alpha'} \gamma_{\sigma\sigma}(\tau) \partial_\tau X^- + \frac{i}{4\pi\alpha'} \int_0^\ell d\sigma \left[ \gamma_{\sigma\sigma} \partial_\tau X^i \partial_\tau X^i - \gamma_{\sigma\sigma}^{-1} \partial_\sigma X^i \partial_\sigma X^i \right]$$

$$S_P = \int d\tau L$$

$$p_- = -p^+ = \frac{\partial L}{\partial \dot{q}}$$

$$p_- = -p^+ = \frac{\partial L}{\partial (\dot{x}^-)}$$

$$p_- = -p^+ = \frac{\partial L}{\partial (\dot{x}^-)}$$



$$p_- = -p^+ = \frac{\partial L}{\partial (\partial_t X^-)} = -\frac{e}{2\pi\alpha'} \gamma_{00}$$

(B)

$$p_- = -p^+ = \frac{\partial L}{\partial (\partial_t X^-)} = -\frac{e}{2\pi\alpha'} \gamma_{55}$$

$$p^+ = \frac{e}{2\pi\alpha'} \gamma_{55}$$

point particle

(B)

$$p_- = -p^+ = \frac{\partial L}{\partial (\partial_t X^-)} = -\frac{e}{2\pi\alpha'} \gamma_{00}$$

$$p^+ = \frac{e}{2\pi\alpha'} \gamma_{00}$$

In point particle

$$p^+ = -\hbar^{-1}$$

(B)

$$p_- = -p^+ = \frac{\partial L}{\partial (\partial_t X^-)} = -\frac{e}{2\pi\alpha'} \gamma_{\phi\phi}$$

$$p^+ = \frac{e}{2\pi\alpha'} \gamma_{\phi\phi}$$

momenta conserved to

In point particle

$$p^+ = -\hbar^{-1}$$

(B)

$$p_- = -p^+ = \frac{\partial L}{\partial (\partial_\tau X^-)} = -\frac{e}{2\pi\alpha'} \gamma_{\sigma\sigma}$$

$$p^+ = \frac{e}{2\pi\alpha'} \gamma_{\sigma\sigma}$$

momenta conjugate to  $X^\mu(\tau, \sigma)$

In point particle

$$p^+ = -\hbar^{-1}$$

(B)

$$p_- = -p^+ = \frac{\partial L}{\partial (\partial_\tau X^-)} = -\frac{e}{2\pi\alpha'} \gamma_{\sigma\sigma}$$

$$p^+ = \frac{e}{2\pi\alpha'} \gamma_{\sigma\sigma}$$

momenta conjugate to  $X^i(\tau, \sigma)$

In point particle

$$p^+ = -\hbar^{-1}$$

$$\Pi^i = \Pi_i =$$

(B)

$$p_- = -p^+ = \frac{\partial L}{\partial (\partial_\tau X^-)} = -\frac{e'}{2\pi\alpha'} \gamma_{\sigma\sigma}$$

$$p^+ = \frac{e}{2\pi\alpha'} \gamma_{\sigma\sigma}$$

momenta conjugate to  $X^i(\tau, \sigma)$

In point particle

$$\Pi^i = \Pi_i = \frac{\partial L}{\partial \dot{X}^i}$$

$$p^+ = -\hbar^{-1}$$

(B)

$$p_- = -p^+ = \frac{\partial L}{\partial (\partial_\tau X^-)} = -\frac{e}{2\pi\alpha'} \gamma_{00}$$

$$p^+ = \frac{e}{2\pi\alpha'} \gamma_{00}$$

momenta conjugate to  $X^i(\tau, \sigma)$

In point particle

$$p^+ = -\hbar^{-1}$$

$$\Pi^i = \Pi_i = \frac{\delta L}{\delta \partial_\tau X^i} = \frac{2}{4\pi\alpha'} \gamma_{00} \partial_\tau X^i$$



(B)

$$p_- = -p^+ = \frac{\partial L}{\partial (\partial_\tau X^-)} = -\frac{e}{2\pi\alpha'} \gamma_{00}$$

$$p^+ = \frac{e}{2\pi\alpha'} \gamma_{00}$$

momenta conjugate to  $X^i(\tau, \sigma)$

In point particle

$$p^+ = -\hbar^{-1}$$

$$\Pi^i = \Pi_i = \frac{\delta L}{\delta \partial_\tau X^i} = \frac{e}{2\pi\alpha'} \gamma_{0i} \partial_\tau X^i$$

(B)

$$p_- = -p^+ = \frac{\partial L}{\partial (\partial_\tau X^-)} = -\frac{e}{2\pi\alpha'} \gamma_{\sigma\sigma}$$

$$p^+ = \frac{e}{2\pi\alpha'} \gamma_{\sigma\sigma}$$

momenta conjugate to  $X^i(\tau, \sigma)$

In point particle

$$p^+ = -\hbar^{-1}$$

$$\Pi^i = \Pi_i = \frac{\delta L}{\delta \partial_\tau X^i} = \frac{e}{2\pi\alpha'} \gamma_{\sigma\sigma} \partial_\tau X^i = \frac{p^+}{e} \partial_\tau X^i$$

$$\partial_\tau X^i = \frac{e}{p^+} \Pi^i$$

(B)

$$p_- = -p^+ = \frac{\partial L}{\partial (\partial_\tau X^-)} = -\frac{e}{2\pi\alpha'} \gamma_{\sigma\sigma}$$

$$p^+ = \frac{e}{2\pi\alpha'} \gamma_{\sigma\sigma}$$

momenta conjugate to  $X^i(\tau, \sigma)$

In point particle

$$p^+ = -\hbar^{-1}$$

$$\Pi^i = \Pi_i = \frac{\delta L}{\delta \partial_\tau X^i} = \frac{e}{2\pi\alpha'} \gamma_{\sigma\sigma} \partial_\tau X^i = \frac{p^+}{e} \partial_\tau X^i$$

$$\partial_\tau X^i = \frac{e}{p^+} \Pi^i$$

$$H = P - \partial_t X^{-}$$

$\frac{1}{2}$

$\frac{1}{2}$

$$H = P_- \partial_t X^- + \int_0^{\ell} d\sigma \left( \frac{1}{2} \dot{X}^i \partial_t X^i \right)$$

$$\frac{1}{2}$$

$$\frac{1}{2}$$

$$H = P_- \partial_\tau X^- + \int_0^{2\pi} d\sigma \left( \frac{1}{2} \dot{X}^i \partial_\tau X^i - \mathcal{L} \right)$$

=

$\frac{1}{2}$

$\frac{1}{2}$

$$H = P \partial_t X^- + \int_0^l d\sigma \left( \frac{1}{2} \dot{X}^i \partial_\sigma X^i \right) - L$$

$$\partial_t X^-$$

$$\frac{1}{2}$$

$$\frac{1}{2}$$

$$H = P \partial_t X^- + \int_0^L d\sigma \left( \Pi_i \partial_t X^i \right) - L$$

$$= P \partial_t X^-$$



$$H = P_- \partial_\tau X^- + \int_0^{\ell} d\sigma \left( \frac{1}{2} \dot{X}^i \partial_\tau X^i \right) - L$$

$$= P_- \partial_\tau X^- + \int_0^{\ell} d\sigma$$

$$H = P_- \partial_\tau X^- + \int_0^{\ell} d\sigma \Pi_i \partial_\tau X^i - L$$

$$= P_- \partial_\tau X^- + \int_0^{\ell} d\sigma \frac{\Pi_i \cdot \Pi^i \ell}{p^+}$$

$$L = - \frac{\rho}{2\pi\alpha'} \delta_{\sigma\sigma}(\tau) \partial_\tau \bar{X}^\mu + \frac{1}{4\pi\alpha'} \int_0^\rho d\sigma \left[ \gamma_{\sigma\sigma} \partial_\tau X^\mu \partial_\tau X^\mu - \gamma_{\sigma\sigma}^{-1} \partial_\sigma X^\mu \partial_\sigma X^\mu \right]$$

$$S_{10} = \int dt L$$

$$L = \left( -\frac{p}{2\pi\alpha'} \delta_{\sigma\sigma}(\tau) \right) \partial_\tau X^\mu + \frac{1}{4\pi\alpha'} \int_0^{\ell} d\sigma \left[ \dot{X}^\mu \dot{X}^\nu \delta_{\mu\nu} - \dot{X}^\mu \partial_\sigma X^\nu \partial_\sigma X^\mu \right]$$

$$S_{1-} = \int dt L$$

$\frac{1}{2\pi} \int_{-\pi}^{\pi} \dots$   $\frac{1}{4\pi} \int_{-\pi}^{\pi} \dots$

$\lambda(t), \delta\sigma\sigma(t), \lambda(t, \sigma)$

$P_z$

$$L = \left( -\frac{\ell}{2\pi\alpha'} \gamma_{\sigma\sigma}(t) \right) \partial_c \bar{X}^a + \frac{1}{4\pi\alpha'} \int_0^\ell d\sigma \left[ \gamma_{\sigma\sigma} \partial_\tau X^i \partial_\tau X^i - \gamma_{\sigma\sigma}^{-1} \partial_\sigma X^i \partial_\sigma X^i \right]$$

$\int_{-\pi}^{\pi} dt L$

$$= P_2 \partial_t X^- + \int d\sigma \frac{P_2 \cdot P_2^\dagger e}{P_2^\dagger} - P_2 \partial_t X^-$$

$$= \left( \frac{e}{\pi \alpha'} \delta_{\sigma\sigma}(\tau) \right) \partial_\tau X^\mu + \frac{1}{4\pi \alpha'} \int_0^{\ell} d\sigma \left[ \begin{aligned} & \gamma_{\sigma\sigma} \partial_\tau X^\mu \partial_\sigma X^\mu \\ & - \gamma_{\sigma\sigma}^{-1} \partial_\sigma X^\mu \partial_\sigma X^\mu \end{aligned} \right]$$

$$\mathcal{L} = \int d\tau L$$

$$= P_2 \partial_t X^- + \int_0^{\ell} d\sigma \frac{P_i \cdot P^i e}{P^+} - P_2 \partial_t X^-$$

$$L = \left( \frac{e}{2\pi\alpha'} \delta_{\sigma\sigma}(\tau) \right) \partial_\tau X^\mu + \frac{1}{4\pi\alpha'} \int_0^{\ell} d\sigma \left[ \gamma_{\sigma\sigma} \partial_\tau X^i \partial_\sigma X^i - \gamma_{\sigma\sigma}^{-1} \partial_\sigma X^i \partial_\sigma X^i \right]$$

$$S_1 = \int d\tau L$$

$$\begin{aligned}
 &= P_0 \partial_t X^- + \int_0^p d\sigma \frac{N_i \cdot N^i e}{p^+} - P_0 \partial_t X^- \\
 &- \frac{1}{4\pi\alpha'} \int_0^p d\sigma \frac{2\pi\alpha' p^+}{e}
 \end{aligned}$$

$$\begin{aligned}
 &= \left( -\frac{1}{2\pi\alpha'} \delta_{\sigma\sigma}(\tau) \right) \partial_\tau X^- + \frac{1}{4\pi\alpha'} \int_0^p d\sigma \left[ \dot{X}_\sigma^i \dot{X}_\sigma^i - \dot{X}_\sigma^- \dot{X}_\sigma^- \right] \\
 &= \int d\tau L
 \end{aligned}$$



$$= P_- \partial_t X^- + \int_0^{\rho} d\sigma \frac{P_i \cdot P^i e}{P^+} - P_- \partial_t X^-$$

$$- \frac{1}{4\pi\alpha'} \int_0^{\rho} d\sigma \frac{2\pi\alpha' P^+}{e} = \gamma_{\sigma\sigma}$$

$$L = \left( \frac{1}{2\pi\alpha'} \delta_{\sigma\sigma}(\tau) \right) \partial_\sigma X^\mu + \frac{1}{4\pi\alpha'} \int_0^{\rho} d\sigma \left[ \dot{X}^\mu \dot{X}^\mu - \gamma_{\sigma\sigma}^{-1} \partial_\sigma X^\mu \partial_\sigma X^\mu \right]$$

$$S_{10} = \int dt L$$

$$= P_+ \partial_t X^- + \int_0^p d\sigma \frac{\Pi_i \cdot \Pi^i e}{p^+} - P_- \partial_t X^-$$

$$- \frac{1}{4\pi\alpha'} \int_0^p d\sigma \frac{2\pi\alpha' p^+}{e} \frac{\Pi^i \Pi^i e^2}{(p^+)^2} + \frac{1}{4\pi\alpha'}$$

$\delta\sigma\sigma$

$$= \left( \frac{1}{2\pi\alpha'} \delta_{\sigma\sigma}(\tau) \right) \partial_\sigma X^\mu + \frac{1}{4\pi\alpha'} \int_0^p d\sigma \left[ \begin{aligned} & \gamma_{\sigma\sigma} \partial_\sigma X^i \partial_\sigma X^i \\ & - \gamma_{\sigma\sigma}^{-1} \partial_\sigma X^i \partial_\sigma X^i \end{aligned} \right]$$

$$= P_0 \partial_t X^- + \int_0^{\rho} d\sigma \frac{\Pi_i \cdot \Pi^i e}{p^+} - P_0 \partial_t X^-$$

$$- \frac{1}{4\pi\alpha'} \int_0^{\rho} d\sigma \frac{2\pi\alpha' p^+}{e} \frac{\Pi^i \Pi^i e^2}{(p^+)^2} + \frac{1}{4\pi\alpha'} \int_0^{\rho} d\sigma$$

$$L = \left( \frac{1}{2\pi\alpha'} \delta_{\sigma\sigma}(\tau) \right) \partial_\tau X^- + \frac{1}{4\pi\alpha'} \int_0^{\rho} d\sigma \left[ \dot{X}^i \dot{X}^i \dot{X}^2 \dot{X}^2 \dot{X}^2 \dot{X}^2 \right]$$

$$S_{10} = \int dt L$$

$$\begin{aligned}
 &= P_0 \partial_t X^- + \int_0^{\rho} d\sigma \frac{\Pi_i \cdot \Pi^i e}{p^+} - P_0 \partial_t X^- \\
 &- \frac{1}{4\pi\alpha'} \int_0^{\rho} d\sigma \frac{2\pi\alpha' p^+}{e} \frac{\Pi^i \Pi^i e^2}{(p^+)^2} + \frac{1}{4\pi\alpha'} \int_0^{\rho} d\sigma \frac{e}{2\pi\alpha' p^+}
 \end{aligned}$$

$$\begin{aligned}
 L &= \left( \frac{1}{2\pi\alpha'} \delta_{\sigma\sigma}(\tau) \right) \partial_\tau X^{\bar{0}} + \frac{1}{4\pi\alpha'} \int_0^{\rho} d\sigma \left[ \dot{X}^{\bar{0}} \dot{X}^{\bar{0}} - \dot{X}^i \dot{X}^i \right] \\
 S_{10} &= \int d\tau L
 \end{aligned}$$

$$\begin{aligned}
 &= P_+ \partial_t X^- + \int_0^p d\sigma \frac{\Pi_i \cdot \Pi^i e}{p^+} - P_- \partial_t X^- \\
 &- \frac{1}{4\pi\alpha'} \int_0^p d\sigma \frac{2\pi\alpha' p^+}{e} \frac{\Pi^i \Pi^i e^2}{(p^+)^2} + \frac{1}{4\pi\alpha'} \int_0^p d\sigma \frac{e}{2\pi\alpha' p^+} \partial_\sigma X^i \partial_\sigma X^i
 \end{aligned}$$

$$\begin{aligned}
 L &= \left( \frac{1}{2\pi\alpha'} \delta_{\sigma\sigma}(\tau) \right) \partial_\tau X^- + \frac{1}{4\pi\alpha'} \int_0^p d\sigma \left[ \gamma_{\sigma\sigma} \partial_\tau X^i \partial_\tau X^i \right. \\
 S_{10} &= \int dt L \left. - \gamma_{\sigma\sigma} \partial_\sigma X^i \partial_\sigma X^i \right]
 \end{aligned}$$

$$0 = \left( \frac{\partial \mathcal{L}}{\partial X^{\mu}} \right)_{\sigma=0} + \delta^{\mu\nu} \left( \frac{\partial \mathcal{L}}{\partial X^{\nu}} \right)_{\sigma=0} = 0$$

$$H = P_- \dot{X}^- + \int_0^{\ell} d\sigma \Pi_i \dot{X}^i - \mathcal{L}$$

$$= \cancel{P_- \dot{X}^-} + \int_0^{\ell} d\sigma \frac{\Pi_i \cdot \Pi^i e}{p^+} - \cancel{P_- \dot{X}^-}$$

$$= \frac{1}{4\pi\alpha'} \int_0^{\ell} d\sigma \frac{e}{2\pi\alpha' p^+} \frac{\Pi^i \Pi^i e^2}{(p^+)^2} + \frac{1}{4\pi\alpha'} \int_0^{\ell} d\sigma \frac{e}{2\pi\alpha' p^+} \partial_{\sigma} X^i \partial_{\sigma} X^i$$

$$0 = \left( X_0 \partial_\tau + X_0^m \partial_\sigma + X_0^m \partial_\tau \right) \dots$$

$$= \cancel{X_0^2 \partial_\tau} + \dots + \dots$$

$$H = P_- \partial_\tau X^- + \int_0^l d\sigma \Pi_i \partial_\tau X^i - L$$

$$= \cancel{P_- \partial_\tau X^-} + \int_0^l d\sigma \frac{\Pi_i \cdot \Pi^i e}{p^+} - \cancel{P_- \partial_\tau X^-}$$

$$= \frac{1}{2\pi\alpha'} \int_0^l d\sigma \frac{p^+}{e} \frac{\Pi^i \Pi^i e^2}{(p^+)^2} + \frac{1}{4\pi\alpha'} \int_0^l d\sigma \frac{e}{2\pi\alpha' p^+} \partial_\sigma X^i \partial_\sigma X^i$$

$$0 = \left( X_0 \partial_0 + X_1 \partial_1 + X_2 \partial_2 + X_3 \partial_3 \right) \left( \frac{1}{2} \dot{X}^2 - \frac{1}{2} X'^2 \right)$$

$$H = P_- \dot{X}^- + \int_0^{\ell} d\sigma \Pi_i \dot{X}^i - L$$

$$= \cancel{P_- \dot{X}^-} + \int_0^{\ell} d\sigma \frac{\Pi_i \cdot \Pi^i e}{p^+} - \cancel{P_- \dot{X}^-}$$

$$= \frac{1}{2\pi\alpha'} \int_0^{\ell} d\sigma \left( \frac{1}{p^+} \Pi_i \Pi^i e + \frac{1}{4\pi\alpha'} \int_0^{\ell} d\sigma \frac{e}{2\pi\alpha' p^+} \partial_0 X^i \partial_0 X^i \right)$$



$$H = \frac{1}{2} \int_0^{\infty} d\sigma \frac{\partial^2}{\partial \tau^2}$$

$$H = \frac{1}{2} \int_{\sigma} d\sigma \frac{e}{p^+} \Pi_i \Pi^i + \frac{e}{(4\pi\alpha')^2}$$

$$H = \frac{1}{2} \int_0^{\ell} d\sigma \frac{e}{p^+} \Pi_i \Pi^i + \frac{e}{(4\pi\alpha')^2} \int_0^{\ell} d\sigma \frac{2\ell}{p^+}$$

$$H = \frac{1}{2} \int_0^{\ell} d\sigma \frac{e}{p^+} \dot{N}_i \dot{N}_i + \frac{e}{(4\pi\alpha')^2} \int_0^{\ell} d\sigma \frac{2\ell}{p^+} \partial_0 X^i \partial_0 X^i$$

$$H = \frac{1}{2} \int_0^{\ell} d\sigma \frac{e}{p^+} \dot{N}_i \dot{N}_i + \frac{e}{(4\pi\alpha')^2} \int_0^{\ell} d\sigma \frac{2\ell}{p^+} \partial_0 X^i \partial_0 X^i$$

⇒ From

$$H = \frac{1}{2} \int_0^l d\sigma \frac{e}{p^+} \pi_i \pi^i + \frac{e}{(4\pi\alpha')^2} \int_0^l d\sigma \frac{2\alpha'}{p^+} \partial_0 X^i \partial_0 X^i$$

$\Rightarrow$  From HT EOM  
 $\partial_\tau p^+ = 0$        $p^+ = \text{const}$

$$H = \frac{1}{2} \int_0^{\ell} d\sigma \frac{e}{p^+} \dot{X}^i \dot{X}^i + \frac{e}{(4\pi\alpha')^2} \int_0^{\ell} d\sigma \frac{2T}{p^+} \partial_0 X^i \partial_0 X^i$$

⇒ From HT EOM

$$\partial_\tau p^+ = 0$$

$$p^+ = \text{const}$$

$$H = \frac{e}{4\pi\alpha' p^+} \int_0^{\ell} d\sigma$$

$$H = \frac{1}{2} \int_0^{\ell} d\sigma \frac{e}{p^+} \Pi_i \Pi^i + \frac{e}{(4\pi\alpha')^2} \int_0^{\ell} d\sigma \frac{2\alpha'}{p^+} \partial_0 X^i \partial_0 X^i$$

⇒ From HT EOM

$$\partial_\tau p^+ = 0$$

$$p^+ = \text{const}$$

$$H = \frac{e}{4\pi\alpha' p^+} \int_0^{\ell} d\sigma \left[ 2\pi\alpha' \Pi_i \Pi^i \right]$$



$$H = \frac{1}{2} \int_0^{\ell} d\sigma \frac{e}{p^+} \Pi_i \Pi^i + \frac{e}{(4\pi\alpha')^2} \int_0^{\ell} d\sigma \frac{2\alpha'}{p^+} \partial_0 X^i \partial_0 X^i$$

⇒ From HT EOM

$$\partial_\tau p^+ = 0$$

$$p^+ = \text{const}$$

$$H \approx \frac{e}{4\pi\alpha' p^+} \int_0^{\ell} d\sigma \left[ 2\pi\alpha' \Pi_i \Pi^i + \frac{1}{2\pi\alpha'} \partial_0 X^i \partial_0 X^i \right]$$