Title: Introduction to the Bosonic String Part B

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Abstract: This course provides a thorough introduction to the bosonic string based on the Polyakov path integral and conformal field theory. We introduce central ideas of string theory, the tools of conformal field theory, the Polyakov path integral, and the covariant quantization of the string. We discuss string interactions and cover the tree-level and one loop amplitudes. More advanced topics such as T-duality and D-branes will be taught as part of the course. The course is geared for M.Sc. and Ph.D. students enrolled in Collaborative Ph.D. Program in Theoretical Physics. Required previous course work: Quantum Field Theory (AM516 or equivalent). The course evaluation will be based on regular problem sets that will be handed in during the term. The primary text is the book: 'String theory. Vol. 1: An introduction to the bosonic string. J. Polchinski (Santa Barbara, KITP) . 1998. 402pp. Cambridge, UK: Univ. Pr. (1998) 402 p.' All interested students should contact Alex Buchel at abuchel@uwo.ca as soon as possible.

- Poincare inv
- reparam inv

$\Rightarrow$ integrate out γ_{ab}

$$\frac{\delta S}{\delta \gamma_{ab}} = 0$$

• Poincare inv

• reparam inv

⇒ integrate out γ_{ab}

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M^{ab}

. Poincare inv

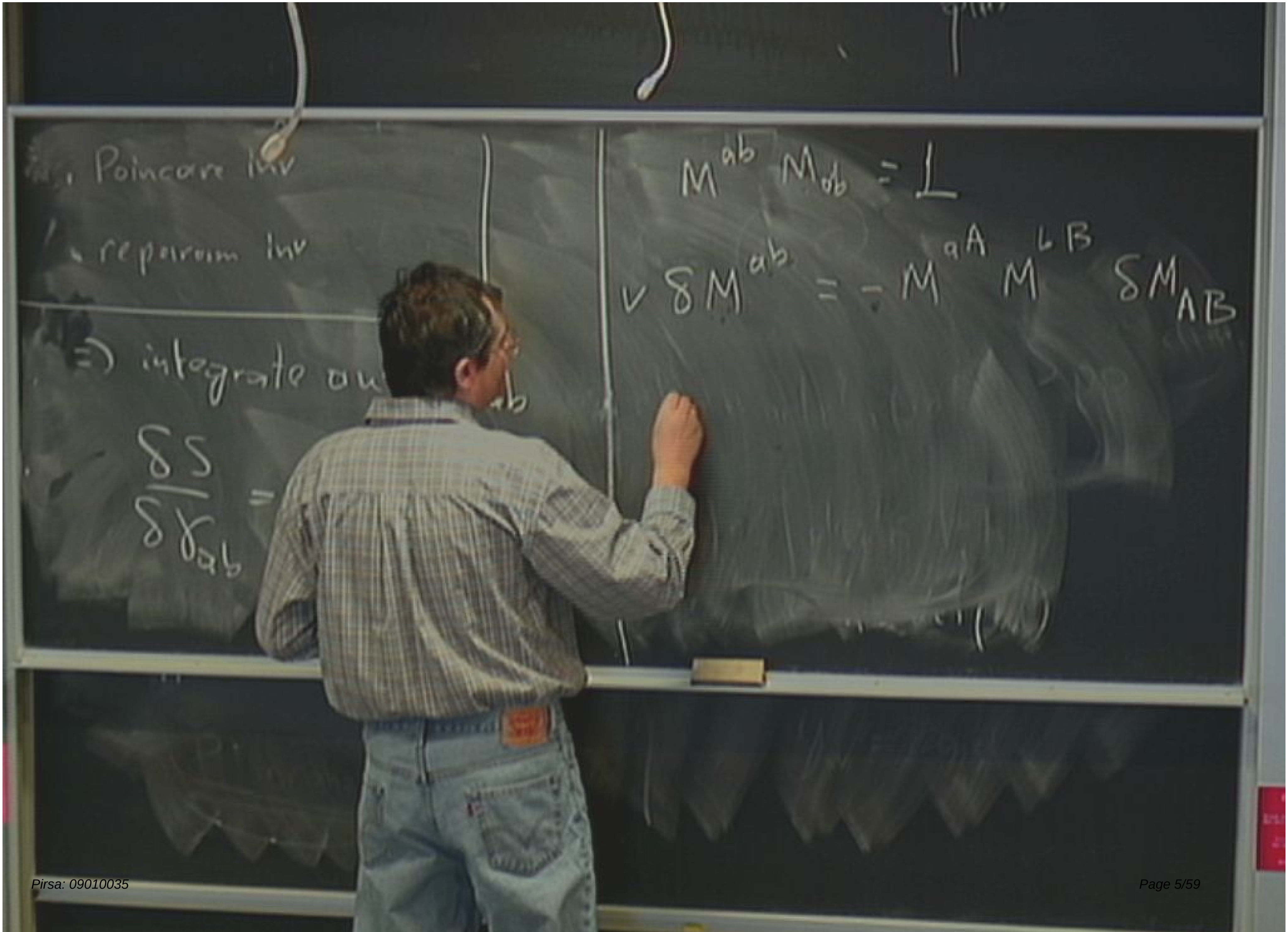
. reparam inv

$\Rightarrow$ integrate out γ_{ab}

$$\frac{\delta S}{\delta \gamma_{ab}} = 0$$

$$M^{ab} M_{ab} = L$$

$$\delta M^{ab} = - M^{aA} M^{bB} \delta M_{AB}$$


Poincare inv
reparam inv
=) integrate ou
δS / δγab =
M^ab Mab = L
δM^ab = - M^aA M^bB δMAB

• Poincare inv

• reparam inv

⇒ integrate out γ_{ab}

$$\frac{\delta S}{\delta \gamma_{ab}} = 0$$

$$M^{ab} M_{ab} = 1$$

$$\delta M^{ab} = - M^{aA} M^{bB} \delta M_{AB}$$

$$M = \det(M_{ab})$$

$$\delta M = M M^{ab} \delta M_{ab}$$

$$\ln\left[\det(M_{ab})\right] = \mathrm{Tr}\,\ln M_{ab}$$

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choose my basis.

$$M_{ab} = \begin{pmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_d \end{pmatrix}$$

$$\delta \ln\left[\det(M_{ab})\right] = \delta \,\mathrm{Tr}\, \ln M_{ab}$$

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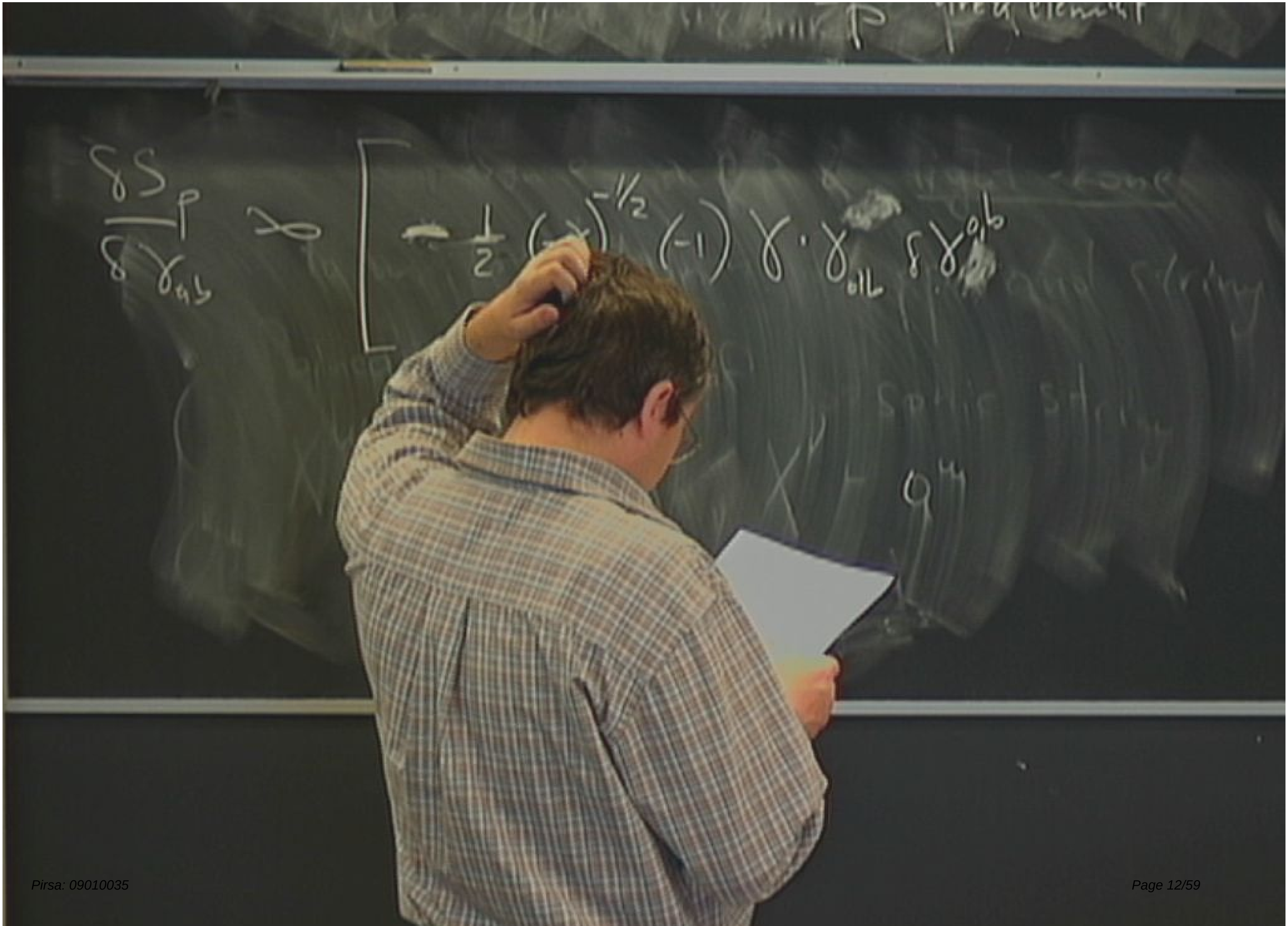
choose my basis.

$$M_{ab} = \begin{pmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \ddots & \\ & & & \lambda_d \end{pmatrix}$$

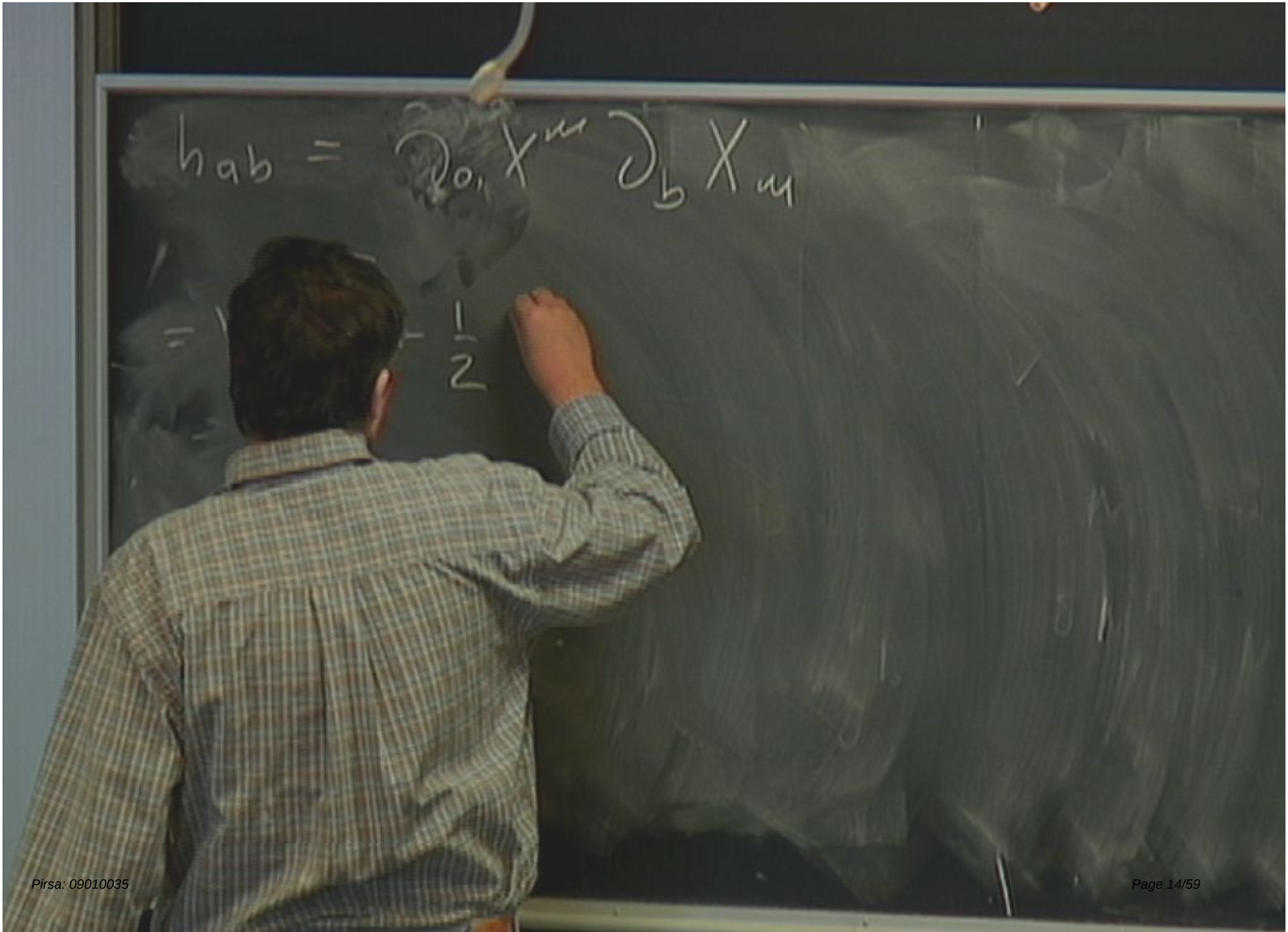
$$M^{-1}\delta M = M^{ab}\delta M_{ab}$$


δS_P / δγ_ab ∝ [1/2 (−γ)^{−1/2} (−1) γ · γ_ab δγ^ab

M S_p invariant area element

$$\frac{\delta S_p}{\delta \gamma_{ab}} \propto \left[-\frac{1}{2}(-\gamma)^{+1/2} \gamma_{cd}\, \delta\gamma^{cd}\, \partial_a X^\mu \partial_b X_\mu \gamma^{ab} + \sqrt{-\gamma}\, \partial_c X^\mu \partial_d X_\mu\, \delta\gamma^{cd} \right]$$

$$= \sqrt{-\gamma}\left[-\frac{1}{2} \gamma^{ab} \partial_a X^\mu \partial_b X_\mu \gamma_{cd} + \partial_c X^\mu \partial_d X_\mu \right] \delta\gamma^{cd}$$


$h_{ab} = \partial_a X^\mu \partial_b X_\mu$
$\frac{1}{2}$

$$h_{ab} = \partial_a X^\mu \partial_b X_\mu$$

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$$\frac{\delta S}{\delta \gamma^{cd}}$$

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$$\frac{\delta S}{\delta \gamma^{cd}} = 0 \Rightarrow \quad h_{cd} = \frac{1}{2}\gamma_{cd}\,\gamma^{ab} h_{ab}$$

$$h_{ab} = \partial_a X^\mu \partial_b X_\mu$$

$$= \sqrt{-\gamma}\left[-\frac{1}{2}\gamma^{ab} h_{ab}\gamma_{cd} + h_{cd}\right]\delta\gamma^{cd}$$

$$\frac{\delta S}{\delta\gamma^{cd}} = 0 \implies h_{cd} = \frac{1}{2}\gamma_{cd}\,\gamma^{ab} h_{ab}$$

$$h = \frac{1}{4}\left(\gamma^{ab} h_{ab}\right)^2 \gamma$$

$$\int_{PP}\left[\tau'(\tau)\right] = \int_{PP}[\tau]\left[\quad\right] D\tau$$

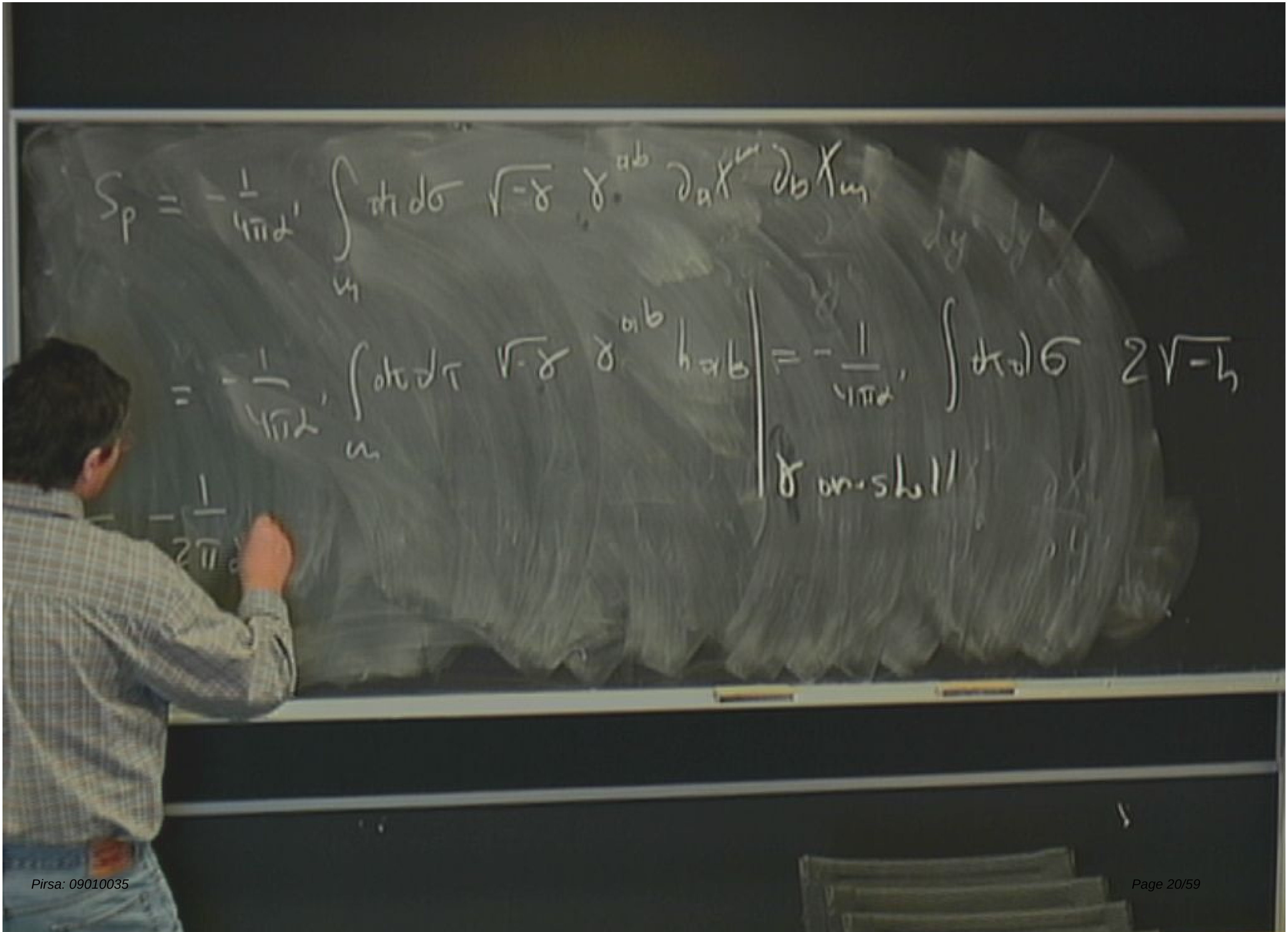
$$h_{ab} = \partial_a X^\mu \partial_b X_\mu$$

$$= \sqrt{-\gamma}\left[-\frac{1}{2}\gamma^{ab} h_{ab}\gamma_{cd} + h_{cd}\right]\delta\gamma^{cd}$$

$$\frac{\delta S}{\delta \gamma^{cd}} = 0 \Rightarrow \quad h_{cd} = \frac{1}{2}\gamma_{cd}\,\gamma^{ab} h_{ab}$$

$$h = \frac{1}{4}\left(\gamma^{ab}h_{ab}\right)^2 \gamma \Rightarrow \sqrt{-h} = \frac{1}{2}\gamma^{ab}h_{ab}\sqrt{-\gamma}$$

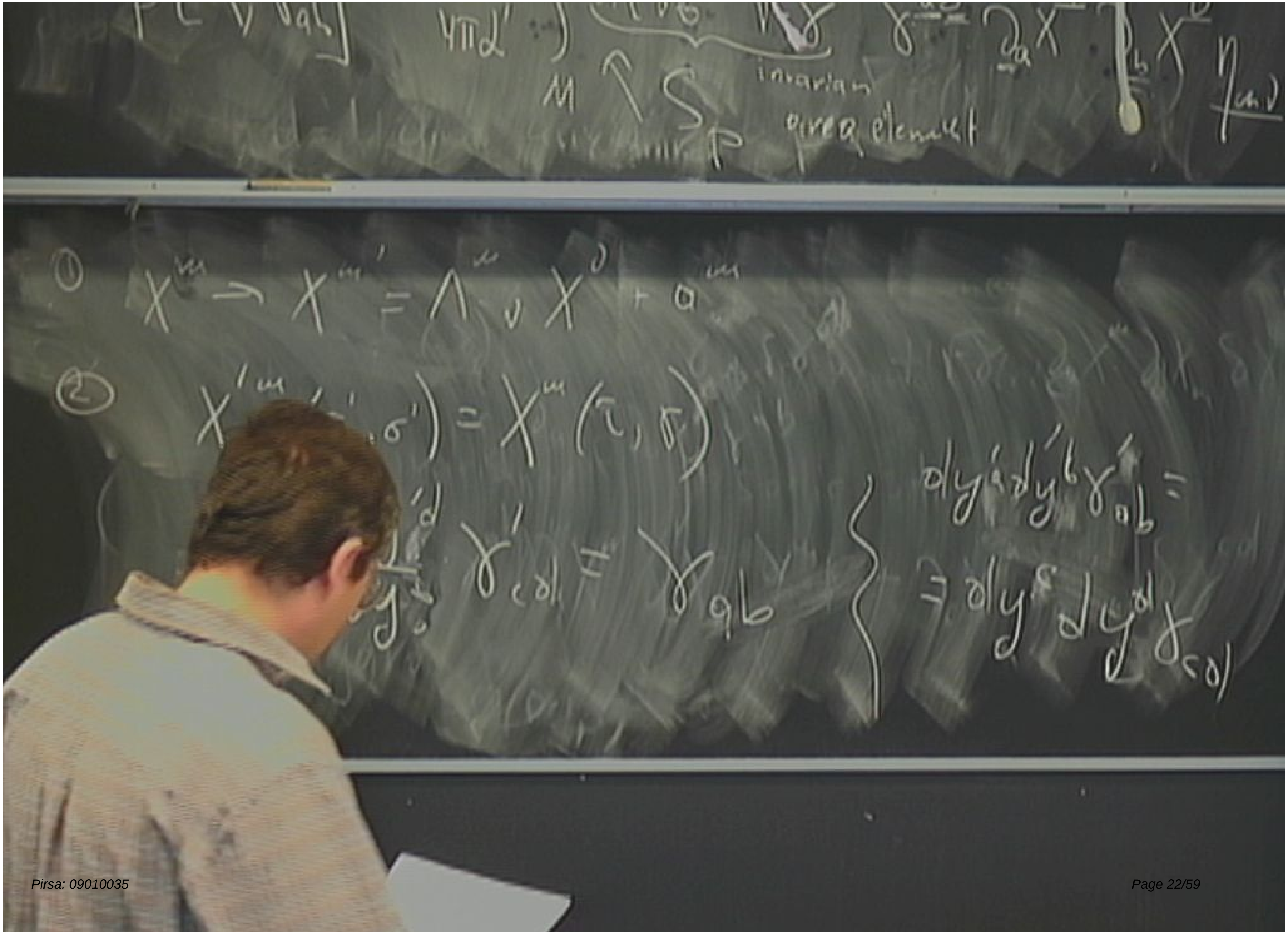
$$S_{pp}[\tau'(\tau)] = S_{pp}[\tau][\quad] \, d\tau$$


$S_p = -\frac{1}{4\pi\alpha'} \int_M d\tau\, d\sigma \sqrt{-\gamma}\, \gamma^{ab} \partial_a X^\mu \partial_b X_\mu$
$= -\frac{1}{4\pi\alpha'} \int_M d\tau\, d\tau \sqrt{-\gamma}\, \gamma^{ab} h_{ab} \Big|_{\gamma \text{ on-shell}} = -\frac{1}{4\pi\alpha'} \int d\tau\, d\sigma\, 2\sqrt{-h}$
$= -\frac{1}{2\pi}$

$$S_p = -\frac{1}{4\pi\alpha'} \int_M d\tau\, d\sigma\, \sqrt{-\gamma}\, \gamma^{ab}\, \partial_a X^\mu\, \partial_b X_\mu$$

$$= -\frac{1}{4\pi\alpha'} \int_M d\tau\, d\tau\, \sqrt{-\gamma}\, \gamma^{ab}\, h_{ab} \Big|_{\gamma \text{ on-shell}} = -\frac{1}{4\pi\alpha'} \int d\tau\, d\sigma\; 2\sqrt{-h}$$

$$= -\frac{1}{2\pi\alpha'} \int d\tau\, d\sigma\, \sqrt{-h} = S_{NG}$$


invariant
area element
① X^μ → X^μ' = Λ^μ_ν X^ν + a^μ
② X'^μ(τ', σ') = X^μ(τ, σ)
γ'_cd = γ_ab
dy'^a dy'^b γ'_ab = dy^c dy^d γ_cd

M $\int$ invariant area element

① $X^\mu \to X^{\mu'} = \Lambda^\mu{}_\nu X^\nu + a^\mu$

② $X'^\mu(\tau', \sigma') = X^\mu(\tau, \sigma)$

$$\frac{\partial y'^c}{\partial y^a} \frac{\partial y'^d}{\partial y^b} \gamma'_{cd} = \gamma_{ab}$$

$$\left\{ \begin{array}{l} dy'^a dy'^b \gamma'_{ab} = \\ = dy^c dy^d \gamma_{cd} \end{array} \right.$$

5 * 2-dimentional Weyl Inv.

$$X'^{\mu}(\tau,\sigma) = X^{\mu}(\tau,\sigma)$$

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$$\gamma'_{ab} = e^{2\omega}\gamma_{ab}$$

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$$\gamma'_{ab} = e^{2\omega}\gamma_{ab} \qquad \omega = \omega(\tau,\sigma)$$

$$\sqrt{-\gamma} \Rightarrow \sqrt{-\gamma'} = \sqrt{e^{4\omega}}\sqrt{-\gamma} = e^{2\omega}\sqrt{-\gamma}$$

$$\gamma^{ab} \to \gamma'^{ab} = e^{-2\omega}\gamma^{ab}$$

$$\sqrt{-\gamma}\,\gamma^{ab} \to \sqrt{-\gamma'}\,\gamma'^{ab} =$$

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$$\sqrt{-\gamma}\,\gamma^{ab} \to \sqrt{-\gamma'}\,\gamma'^{ab} =$$

$\Rightarrow$ we can write S_p for d-dim object.

$$\tilde{S}_p = -T \int d\xi_1 \cdots d\xi_d \sqrt{-\gamma}\, \gamma^{ab}\, \partial_a X^{\mu}\, \partial_b X_{\mu}$$

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$\hookrightarrow$ is not inv under Weyl transf

unless $d = 2$

$$\sqrt{-\gamma} \to e^{dw} \sqrt{-\gamma}$$

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↳ is not inv under Weyl transf
unless $d = 2$

$$\sqrt{-\gamma} \to e^{dw} \sqrt{-\gamma}$$

$$\gamma^{ab} \to e^{-2w} \gamma^{ab}$$

⇒ we can write S_p for d-dim object.

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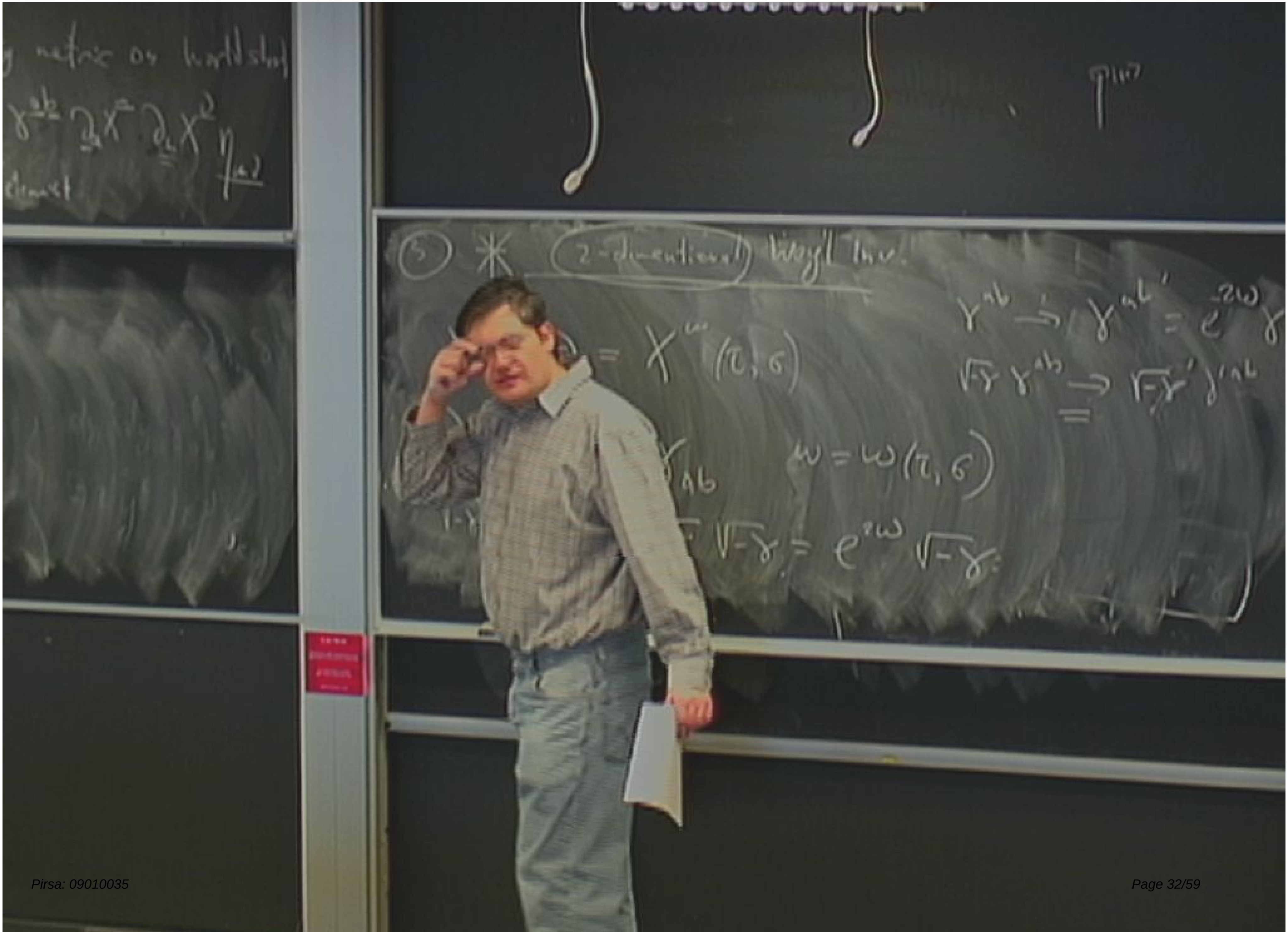
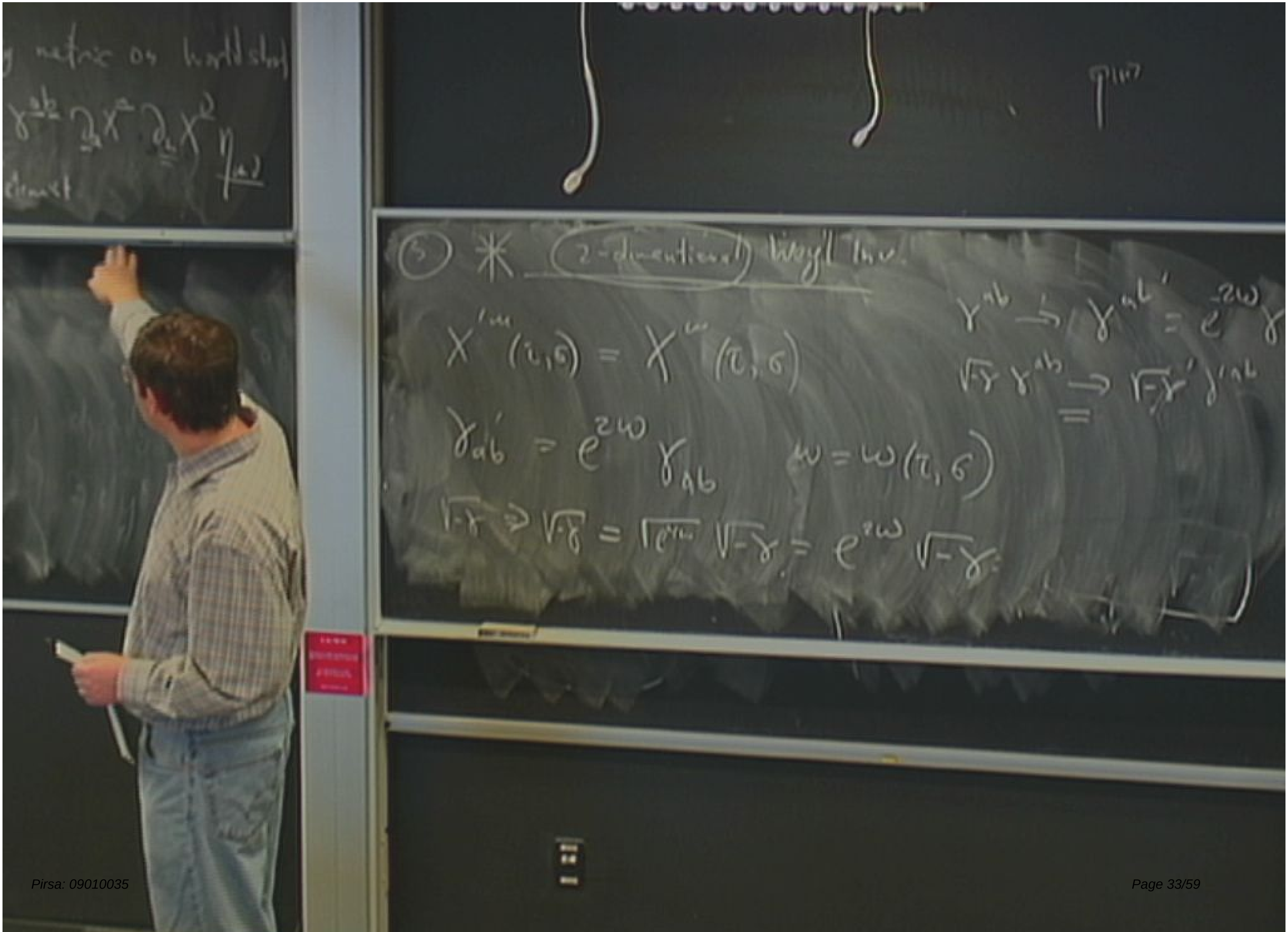
↳ is not inv under Weyl transf

unless $d = 2$

$$\sqrt{-\gamma} \to e^{d\omega} \sqrt{-\gamma}, \qquad \sqrt{-\gamma}\,\gamma^{ab} \to \sqrt{-\gamma}\,\gamma^{ab}\, e^{(d-\frac{1}{2})\omega}$$

$$\gamma^{ab} \to e^{-2\omega} \gamma^{ab}$$




metric on worldsheet
$\gamma^{ab} \partial_a X^\mu \partial_b X^\nu \eta_{\mu\nu}$
element
(3) * 2-dimentional Weyl Inv.
$X'^\mu(\tau,\sigma) = X^\mu(\tau,\sigma)$
$\gamma'_{ab} = e^{2\omega} \gamma_{ab} \quad \omega = \omega(\tau,\sigma)$
$\sqrt{-\gamma} \to \sqrt{-\gamma'} = \sqrt{e^{4\omega}} \sqrt{-\gamma} = e^{2\omega} \sqrt{-\gamma}$
$\gamma^{ab} \to \gamma'^{ab} = e^{-2\omega} \gamma^{ab}$
$\sqrt{-\gamma}\,\gamma^{ab} \Rightarrow \sqrt{-\gamma'}\,\gamma'^{ab}$

invariant

area element

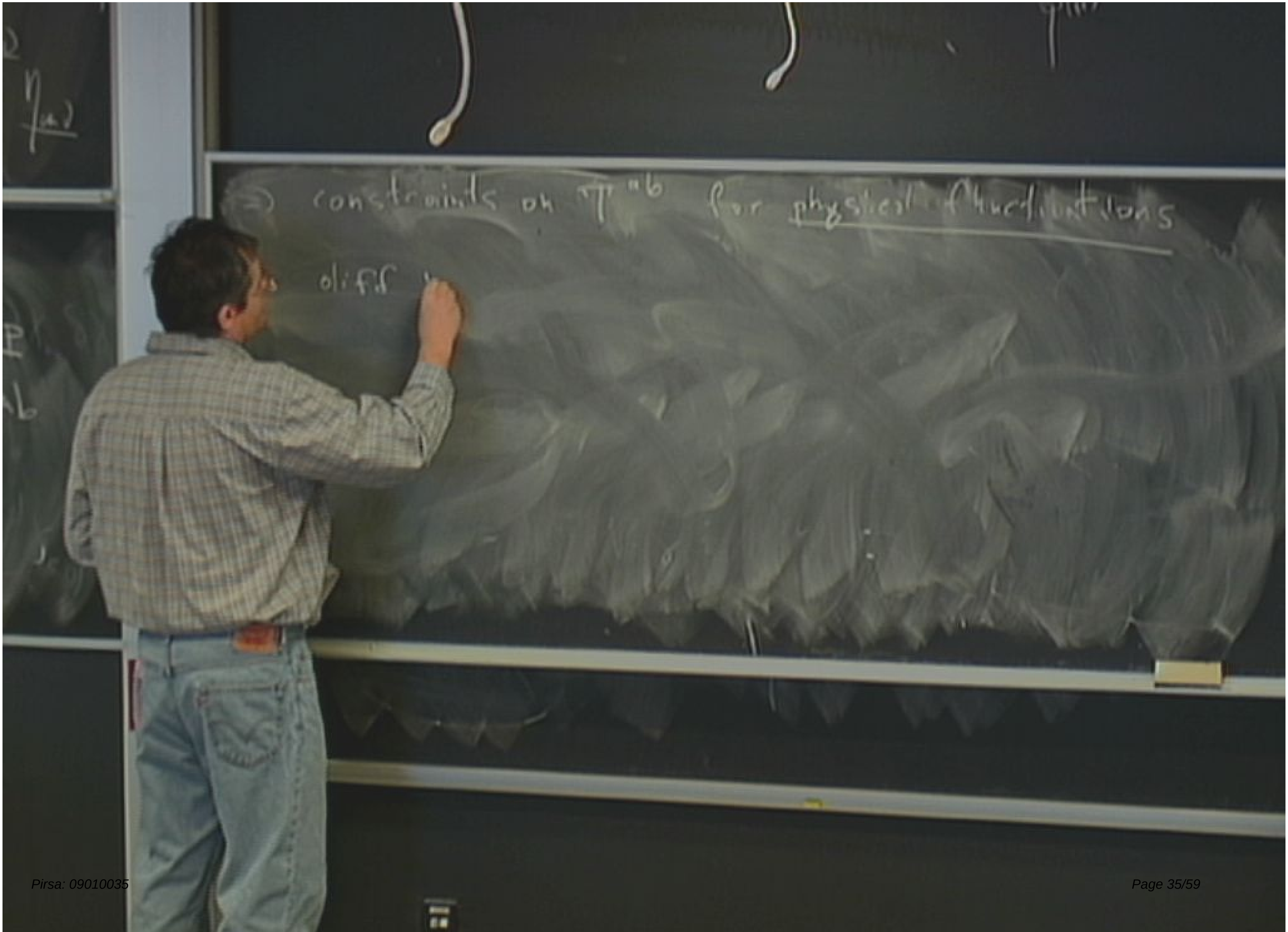
M S_P

In general in QFT

$$T^{ab} = \frac{2}{\sqrt{-\gamma}} \frac{\delta S}{\delta \gamma_{ab}}$$

In string theory

$$T^{ab}_{\eta} = \frac{-4\pi}{\sqrt{\gamma}} \frac{\delta S_P}{\delta \gamma_{ab}}$$


constraints on T^ab for physical fluctuations
diff

$\Rightarrow$ constraints on T^{ab} for physical fluctuations

- diff inv on worldsheet implies that $\nabla_a T^{ab} = 0$
- weyl inv implies that

$\Rightarrow$ constraints on T^{ab} for physical fluctuations

- diff inv on worldsheet implies that
  $\nabla_a T^{ab} = 0$
- weyl inv implies that
  $T_a{}^a = \gamma^{ab} T_{ab} = 0$

invariant area element

$$\nabla_i f_{abc} = \partial_i f_{abc} - \Gamma^{\lambda}_{ia} f_{\lambda bc} - \quad -$$

$$\nabla_i f^a = \partial_i f^a + \Gamma^a_{i\lambda} f^\lambda$$

$$\nabla_i f = \partial_i f$$

invariant

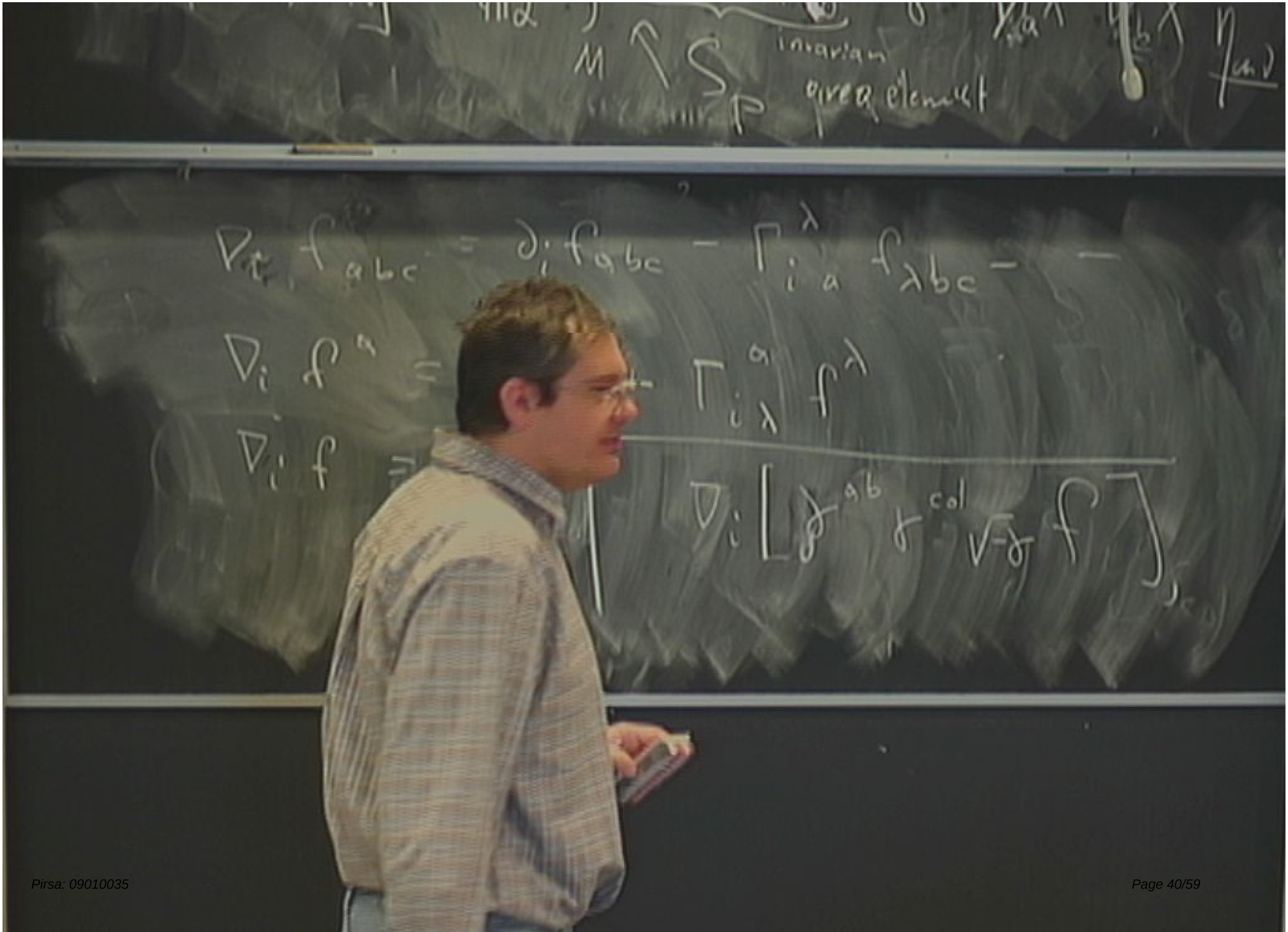
area element

$$\nabla_i f_{abc} = \partial_i f_{abc} - \Gamma^{\lambda}_{ia} f_{\lambda bc} - \cdots$$

$$\nabla_i f^a = \partial_i f^a + \Gamma^{a}_{i\lambda} f^{\lambda}$$

$$\nabla_i f = \partial_i f$$

$\partial_i [\ ]$


invariant
area element

M invariant

area element

$$\nabla_i f_{abc} = \partial_i f_{abc} - \Gamma^{\lambda}_{i\,a} f_{\lambda bc} - \cdots - \cdots$$

$$\nabla_i f^a = \partial_i f^a + \Gamma^{a}_{i\lambda} f^\lambda$$

$$\nabla_i f = \partial_i f$$

$$= \nabla_i \left[\gamma^{ab} \gamma^{cd} \sqrt{-\gamma}\, f \right]$$

$$\gamma^{ab} \gamma^{cd} \sqrt{-\gamma}\, \nabla_i f$$

invariant

M

given element

$$\nabla_i f_{abc} = \partial_i f_{abc} - \Gamma_{ia}^{\lambda} f_{\lambda bc} - \ldots -$$

$$\nabla_i f^a = \partial_i f^a + \Gamma_{i\lambda}^{a} f^\lambda$$

$$\nabla_i f = \partial_i f$$

$$= \nabla_i \left[\gamma^{ab} \gamma^{cd} \sqrt{-g}\, f \right]$$

$$\gamma^{ab} \gamma^{cd} \sqrt{-g}\, \nabla_i f$$

$\Rightarrow$ for physical fluctuation

$$\frac{\delta S}{\delta \gamma_{ab}} = 0$$

⇒ for physical fluctuation

$$\frac{\delta S}{\delta \gamma_{ab}} = 0$$

Under weyl transf

$$\gamma_{ab} \to \gamma_{ab} e^{2\omega}$$

⇒ for physical fluctuation

$$\frac{\delta S_p}{\delta \gamma_{ab}} = 0$$

Under weyl transf

$$\gamma_{ab} \to \gamma_{ab} e^{2\omega} \; : \; S_p \to S_p \Rightarrow \frac{\delta S_p}{\delta \omega} = 0$$

$\Rightarrow$ for physical fluctuations

$$\frac{\delta S_p}{\delta \gamma_{ab}} = 0$$

Under weyl transf.

$$\gamma_{ab} \to \gamma_{ab} e^{2\omega} \; : \; S_p \to S_p \Rightarrow \frac{\delta S_p}{\delta \omega} = 0$$

$$\frac{\delta}{\delta \omega} = \frac{\delta}{\delta \gamma_{ab}}$$

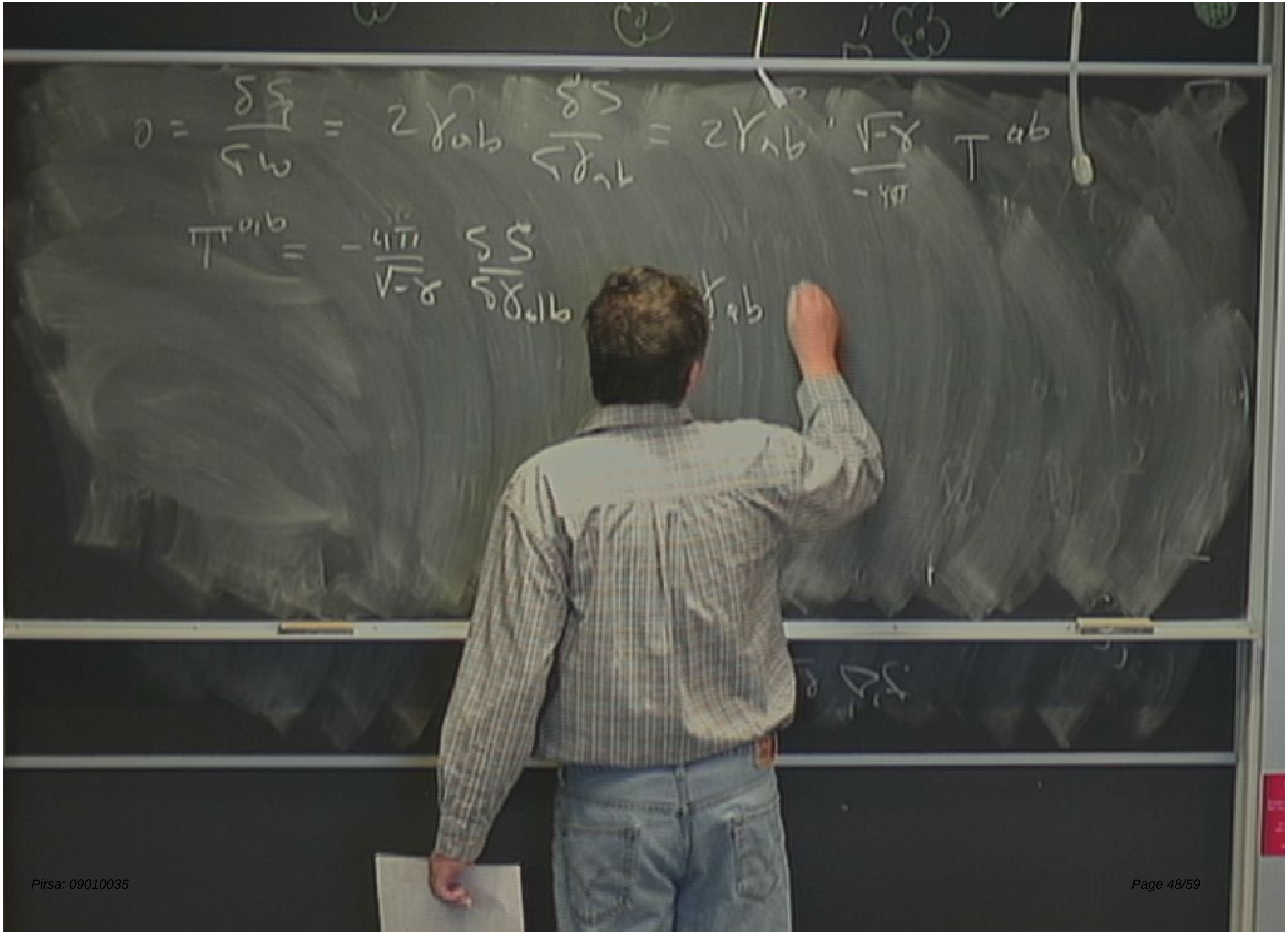
$\Rightarrow$ for physical fluctuations

$$\frac{\delta S_p}{\delta \gamma_{ab}} = 0$$

Under weyl transf

$$\gamma_{ab} \to \gamma_{ab} e^{2\omega} : \quad S_p \to S_p \quad \Rightarrow \quad \frac{\delta S_p}{\delta \omega} = 0$$

$$\frac{\delta}{\delta \omega} = 2\gamma_{ab} \frac{\delta}{\delta \gamma_{ab}}$$


$0 = \frac{\delta S}{\delta \omega} = 2\gamma_{ab} \frac{\delta S}{\delta \gamma_{ab}} = 2\gamma_{ab} \frac{\sqrt{-\gamma}}{-4\pi} T^{ab}$
$T^{ab} = -\frac{4\pi}{\sqrt{-\gamma}} \frac{\delta S}{\delta \gamma_{ab}}$
γ_{ab}

$$0 = \frac{\delta S}{\delta \omega} = 2\gamma_{ab} \frac{\delta S}{\delta \gamma_{ab}} = 2\gamma_{ab} \frac{\sqrt{-\gamma}}{-4\pi} T^{ab}$$

$$T^{ab} = -\frac{4\pi}{\sqrt{-\gamma}} \frac{\delta S}{\delta \gamma_{ab}}$$

$$\boxed{\gamma_{ab} T^{ab} = 0}$$

$\Rightarrow$ inv under world-sheet diff.

$S_P \Rightarrow f_P$

$$\frac{\delta S_P}{\delta \gamma_{ab}} = 0$$ provided that $\delta\gamma_{ab}$ is due to

$\Rightarrow$ inv under world-sheet diff

$S_p \Rightarrow f$

$$\frac{\delta S_p}{\delta \gamma_{ab}} = 0$$ provided that $\delta\gamma_{ab}$ is due to change of coordinates

y_a

$\Rightarrow$ inv under world-sheet diff

$S_P \Rightarrow f$

$\frac{\delta S_P}{\delta \gamma_{ab}} = 0$ provided that $\delta\gamma_{ab}$ is due to change of coordinates

$y_a \Rightarrow$

⇒ inv under world-sheet diff

$S_p \rightarrow S_p$

$$\frac{\delta S_p}{\delta \gamma_{ab}} = 0$$ provided that $\delta\gamma_{ab}$ is due to change of coordinates

$$y^a_{\parallel} \rightarrow y^a_{\parallel} + \xi^a(\tau,\sigma)$$

$\Rightarrow$ inv under

$S_P \Rightarrow S_P$

$\frac{\delta S_P}{\delta \gamma_{ab}} = 0$ provided that $\delta\gamma_{ab}$ is due to change of coordinates

$y^a_H \Rightarrow y^a_H + \xi^a(\tau, \sigma)$

$\gamma_{ab} \Rightarrow \gamma_{ab} - \nabla_a \xi_b - \nabla_b \xi_a$

$S'_{pp}[\tau'(\tau)] = S'_{pp}[\tau] \quad [\quad]' \frac{d\tau'}{d\tau}$

$y^a \to y^a + \xi^a(\tau,\sigma)$

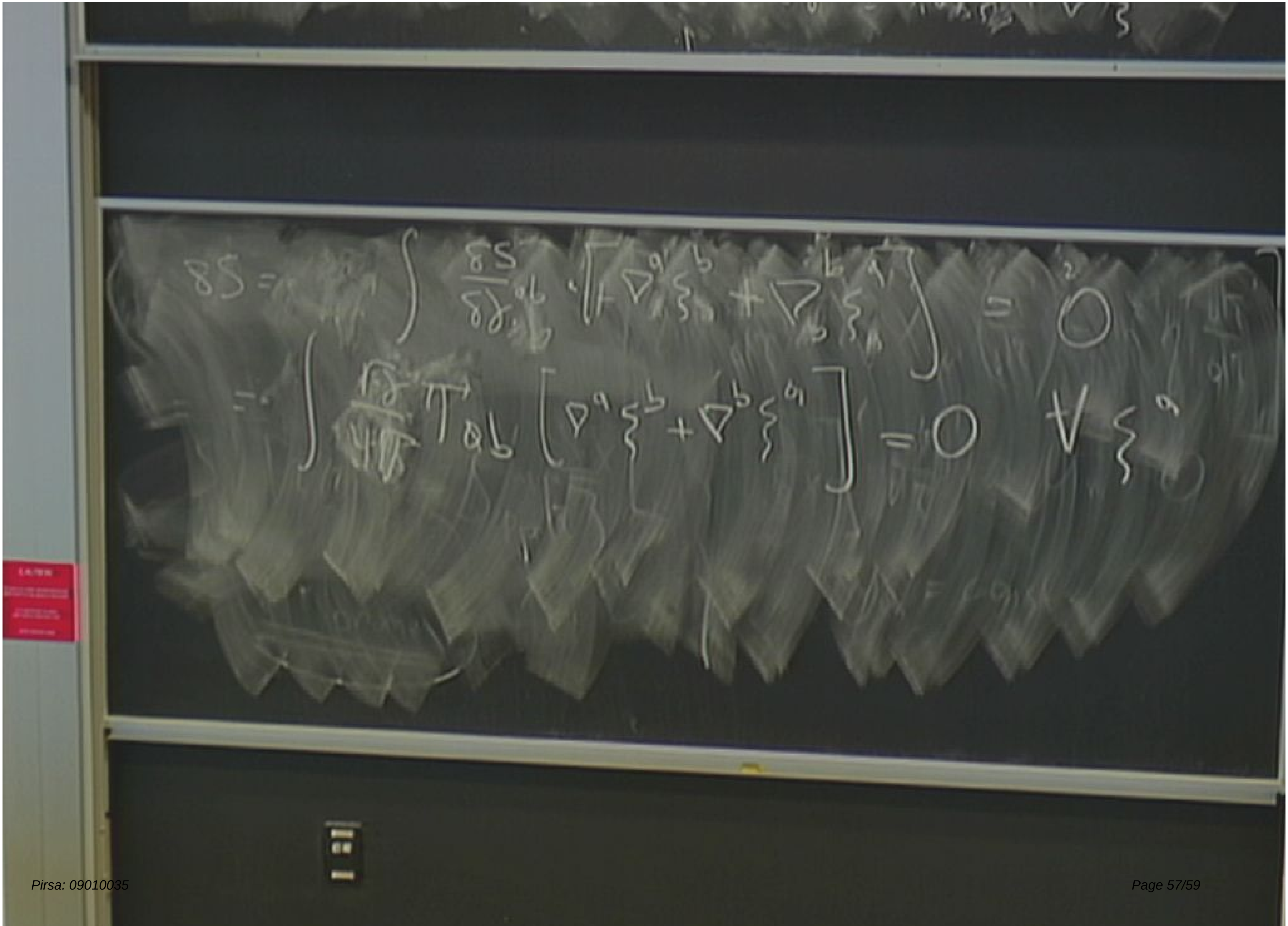
$\gamma_{ab} \to \gamma_{ab} - \nabla_a \xi_b - \nabla_b \xi_a \Rightarrow \delta\gamma_{ab} = -\nabla_a \xi_b - \nabla_b \xi_a$

$\delta\gamma^{ab} = +\nabla^a \xi^b + \nabla^b \xi^a$

$\delta S = -\frac{1}{4\pi\alpha'}\int$

$$\delta S = \int \frac{\delta S}{\delta g_{ab}} \left[\nabla^a \xi^b + \nabla^b \xi^a \right] = 0$$

$$= \int \frac{\sqrt{-g}}{\sqrt{-g}} T_{ab}$$


$\delta S = \int \frac{\delta S}{\delta g_{ab}} \left[\nabla^a \xi^b + \nabla^b \xi^a\right] = 0$
$= \int \sqrt{-g}\, T_{ab} \left[\nabla^a \xi^b + \nabla^b \xi^a\right] = 0 \quad \forall\, \xi^a$

$$\delta S = \int \frac{\delta S}{\delta g^{ab}} \left[\nabla^a \xi^b + \nabla^b \xi^a \right] = 0$$

$$= \int_M \frac{\sqrt{-g}}{4\pi} T_{ab} \left[\nabla^a \xi^b + \nabla^b \xi^a \right] = 0 \quad \forall \xi^a$$

$$= -2 \int_M \frac{\sqrt{-g}}{4\pi} \left[\nabla^a T_{ab} \right]$$

$$\delta S = \int \frac{\delta S}{\delta g_{ab}} \left[\nabla^a \xi^b + \nabla^b \xi^a \right] = 0$$

$$= \int_M \frac{\sqrt{-g}}{4\pi} T_{ab} \left[\nabla^a \xi^b + \nabla^b \xi^a \right] = 0 \quad \forall \, \xi^a$$

$$= -2 \int_M \frac{\sqrt{-g}}{4\pi} \left[\nabla^a T_{ab} \right] \xi^b = 0 \qquad \boxed{\nabla^a T_{ab} = 0}$$