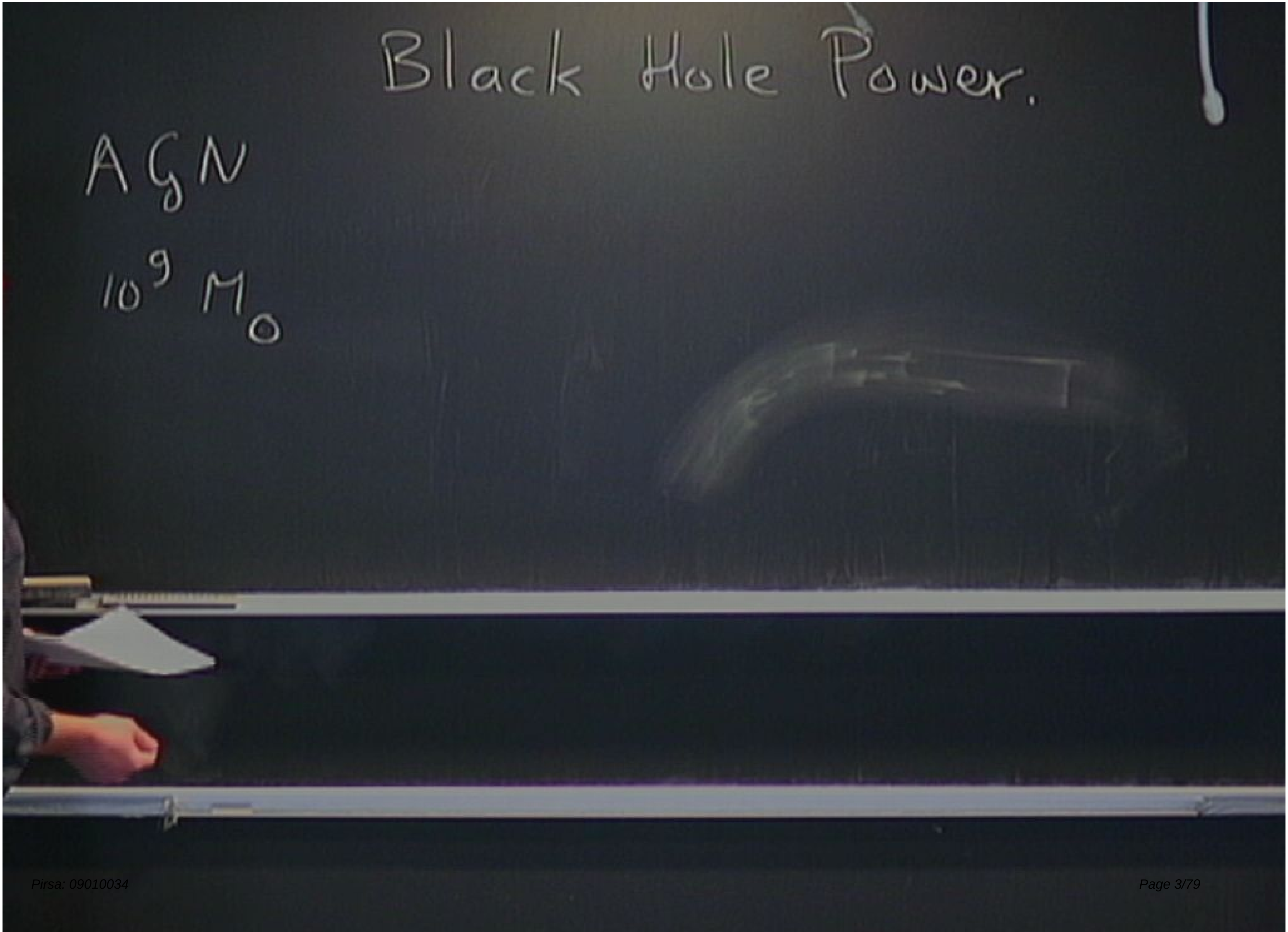
Title: On Energy Extraction from Rotating Black Holes

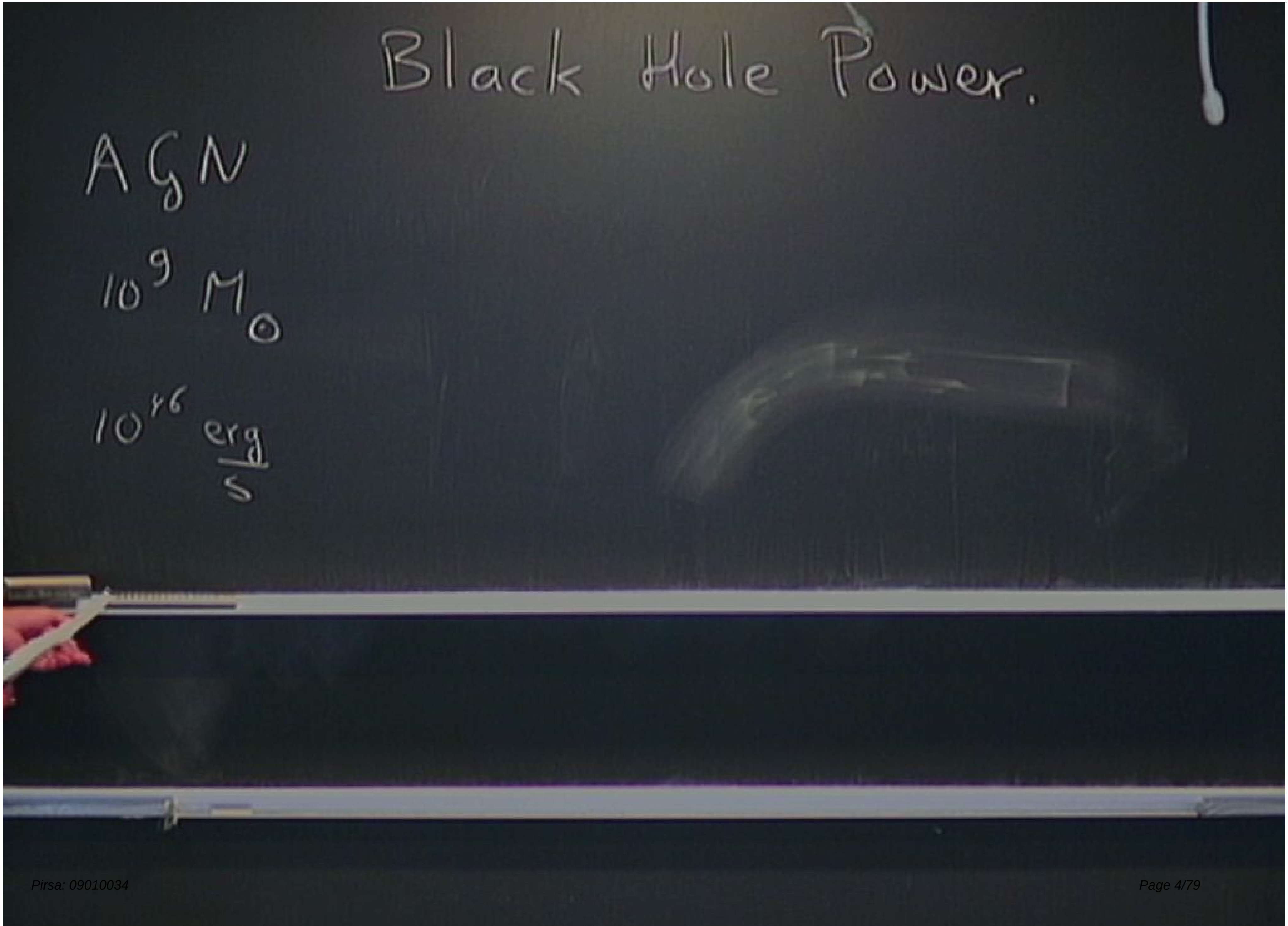
Date: Jan 25, 2009 10:30 AM

URL: http://pirsa.org/09010034

Abstract: In this talk, I start with a short review of the available mechanisms for extracting energy from rotating black holes, in particular superradiance and the Blandford-Znajek process. I then describe how these mechanisms may be realized and studied via the AdS/CFT and Kerr/CFT correspondence.

Black Hole Power.


Black Hole Power.
AGN
$10^9 M_\odot$



Black Hole Power.
AGN
$10^9 M_\odot$
10^{46} erg/s

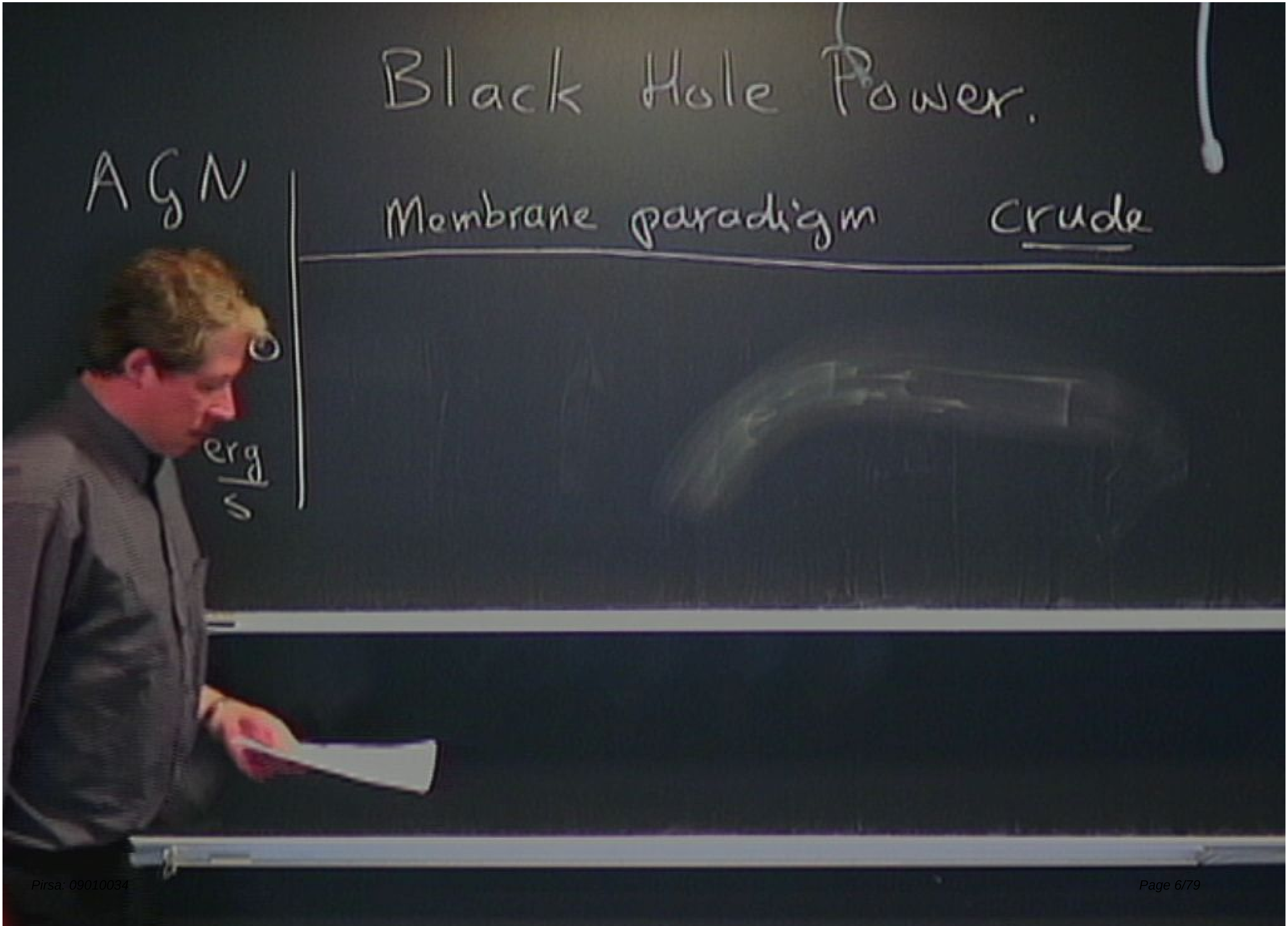
AGN

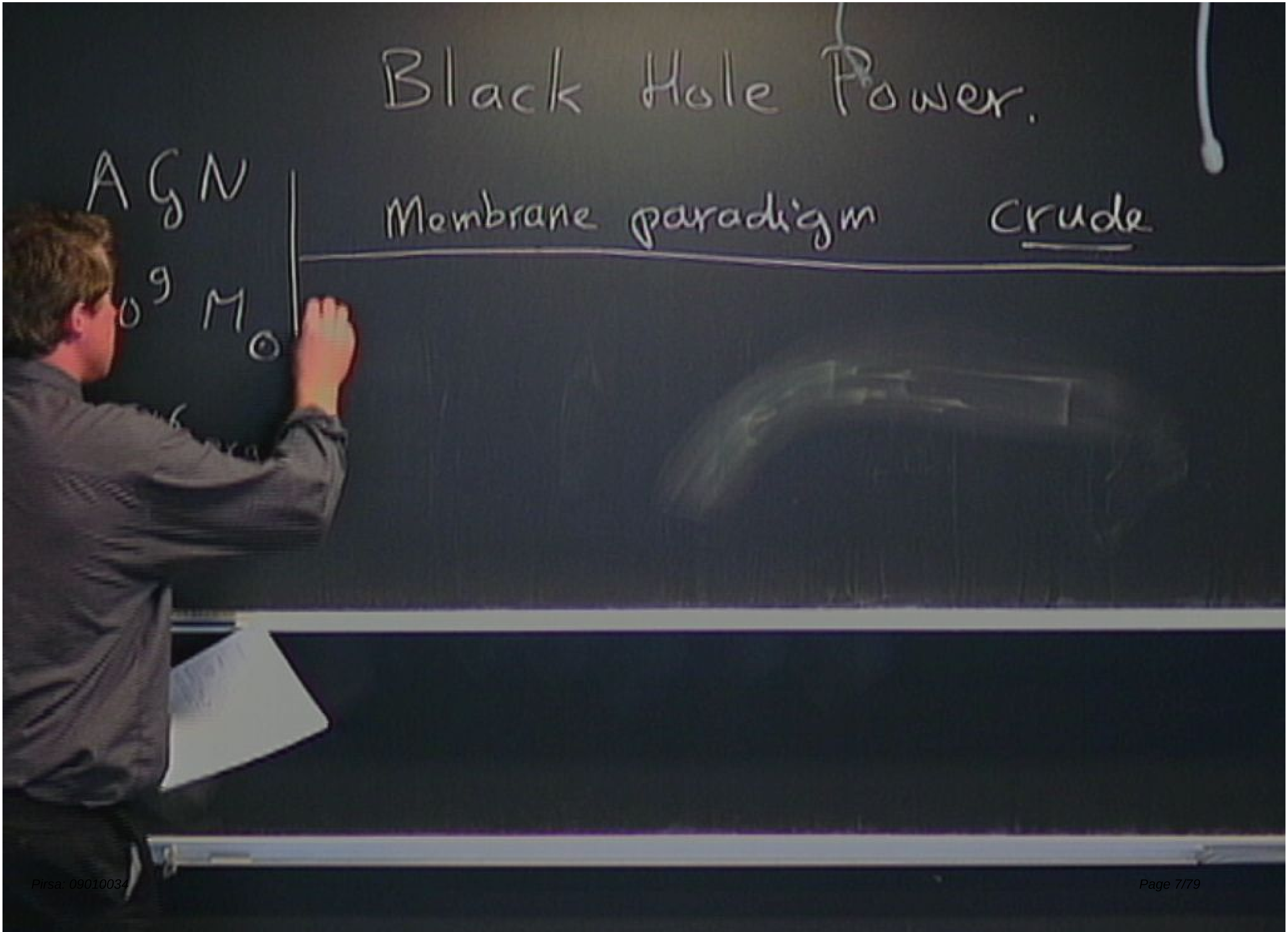
$10^9 M_\odot$

$10^{46} \frac{erg}{s}$

Membrane paradigm

Crude


Black Hole Power.
AGN
Membrane paradigm
crude
erg
s


Black Hole Power.
AGN
Membrane paradigm
crude

Black Hole Power.

AGN

$10^9 M_\odot$

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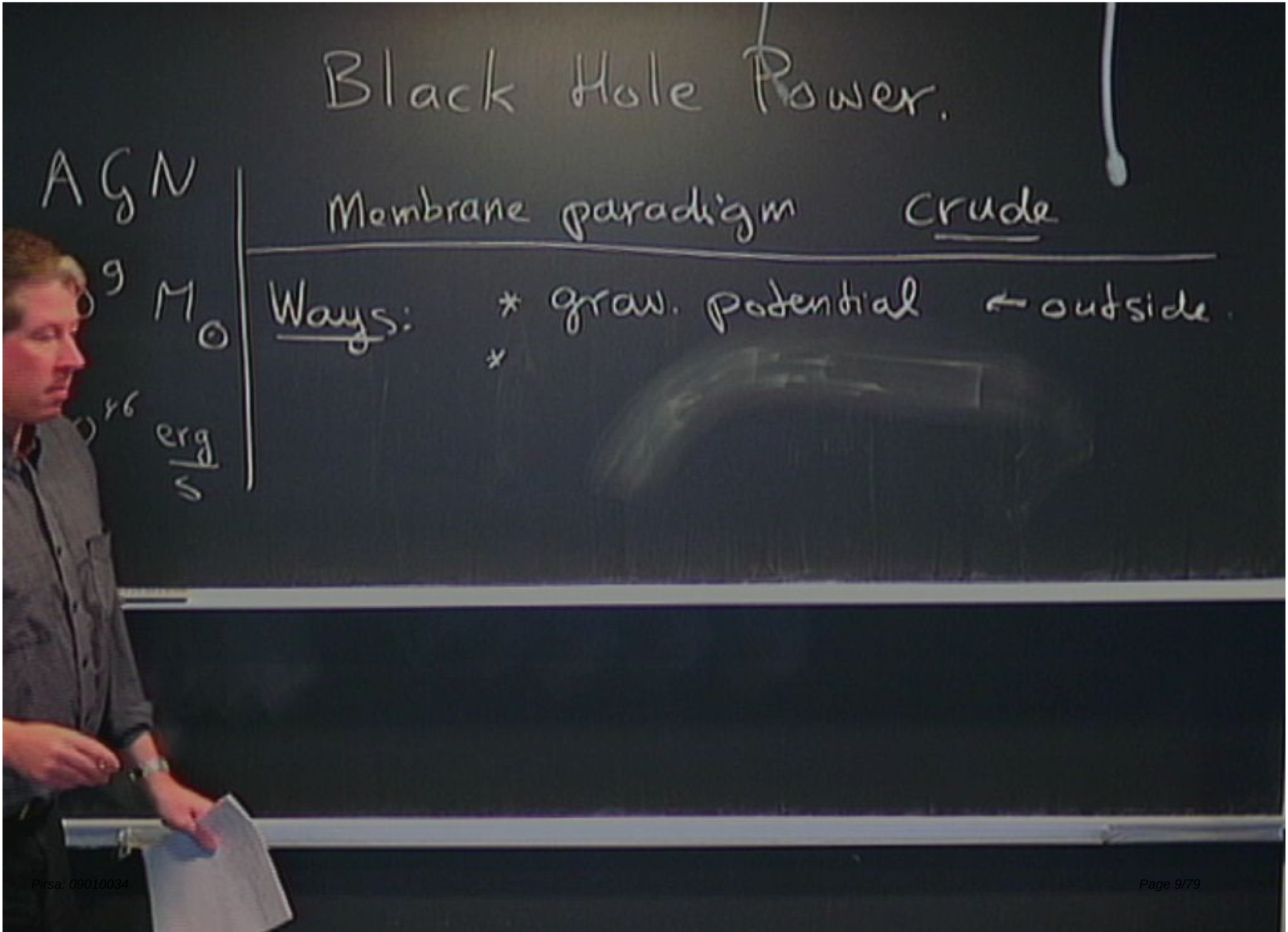
Membrane paradigm   crude

Ways: * grav. potential


Black Hole Power.
AGN
Membrane paradigm crude
Ways: * grav. potential ← outside
*
erg
s

Black Hole Power.

AGN

$10^9 M_\odot$

$10^{46} \frac{erg}{s}$

Membrane paradigm   crude

Ways:
* grav. potential ← outside
* Penrose proces
* superradiance
*

Korr

Black Hole Power.

AGN

$10^9 M_\odot$

$10^{46} \frac{erg}{s}$

Membrane paradigm — crude

Ways:

Kerr

* grav. potential ← outside.
* Penrose proces
* superradiance
* Blandford-Znajek
*

Black Hole Power.

AGN

$10^9 M_\odot$

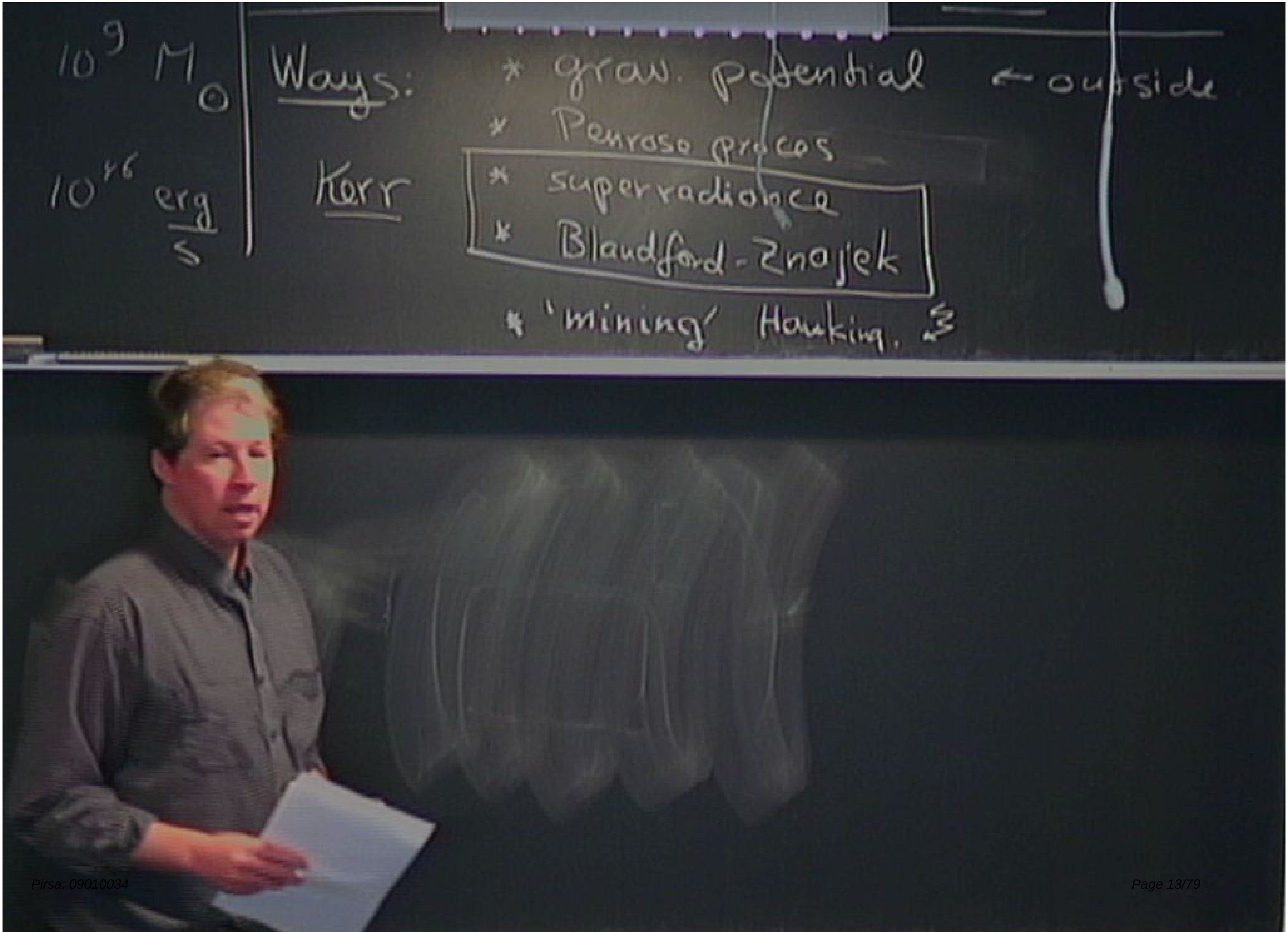
$10^{46} \frac{erg}{s}$

Membrane paradigm — crude

Ways:

Kerr

* grav. potential ← outside.
* Penrose process
* superradiance
* Blandford - Znajek
* 'mining' Hawking. ?


10^9 M_⊙
10^46 erg/s
Ways:
* grav. potential
← outside
* Penrose process
Kerr
* superradiance
* Blandford-Znajek
* 'mining' Hawking.

$10^{46}\ \frac{erg}{s}$

Kerr

* Penrose process
* superradiance
* Blandford-Znajek
* 'mining' Hawking. ?

Question:

$10^{46}\ \frac{erg}{s}$

Kerr

* Penrose process
* superradiance
* Blandford-Znajek
* 'mining' Hawking.

Question: Choose favorite M_{BH}, J
make a BH power plant.

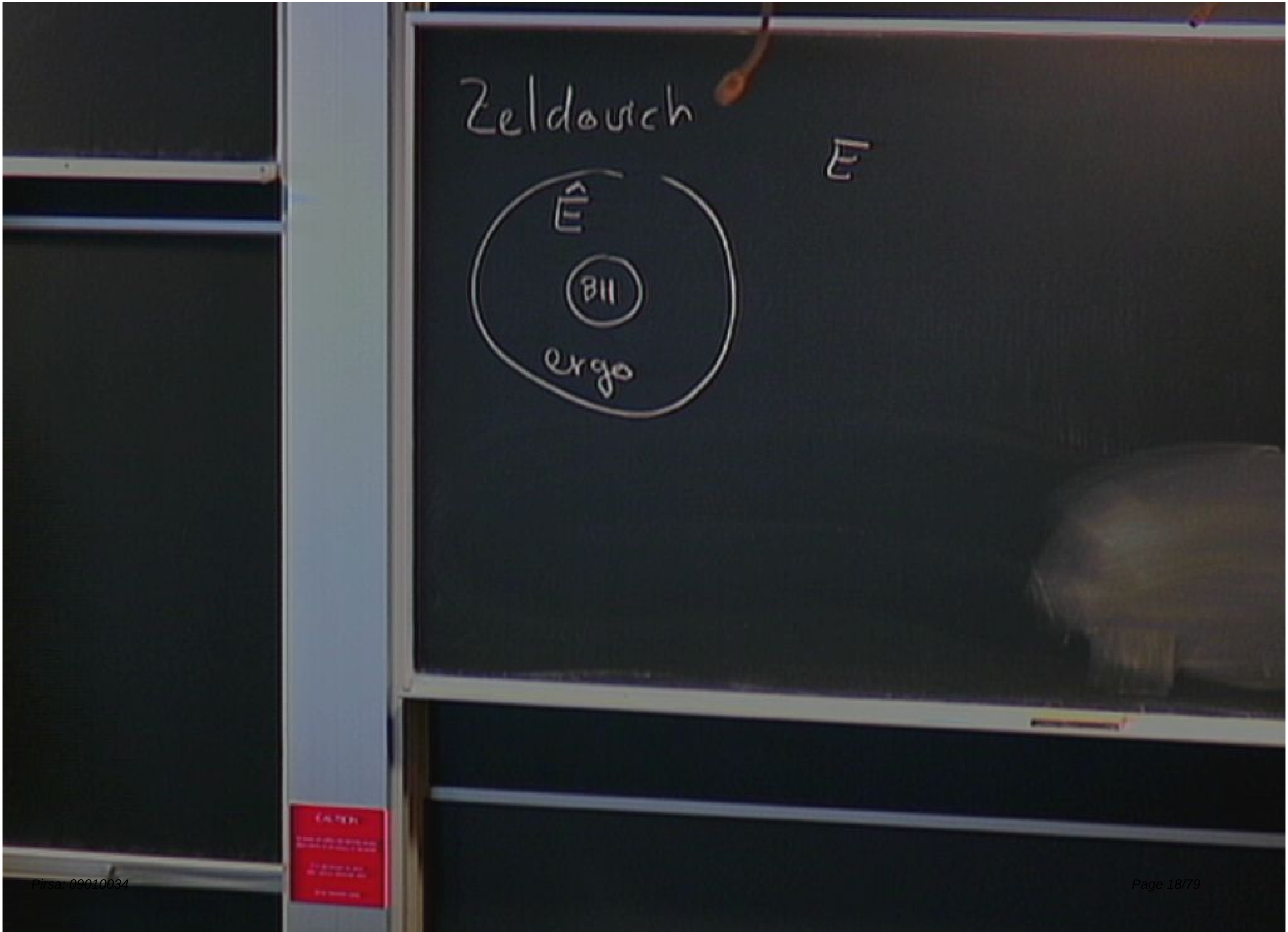
10^{46} $\frac{erg}{s}$

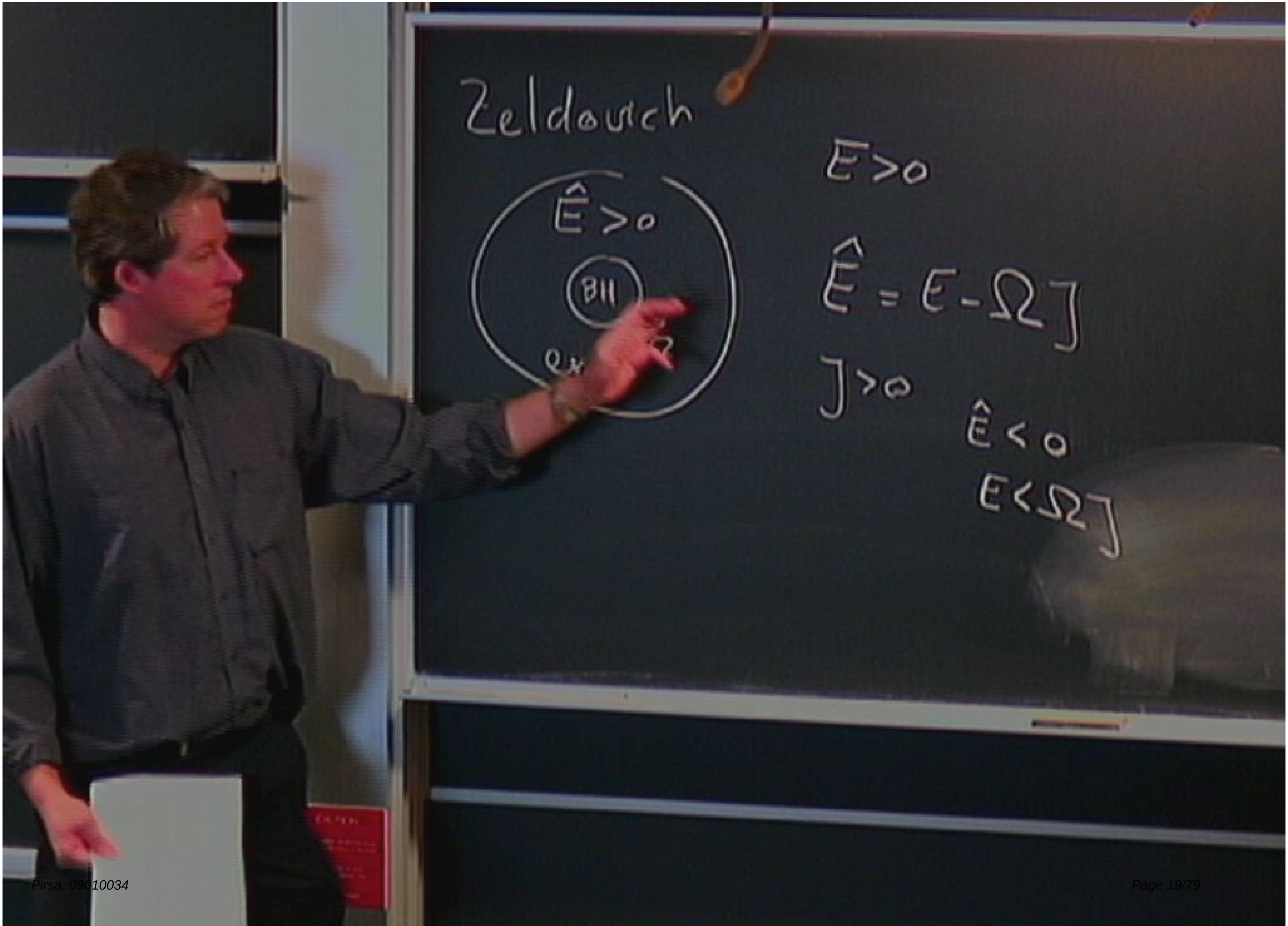
Kerr

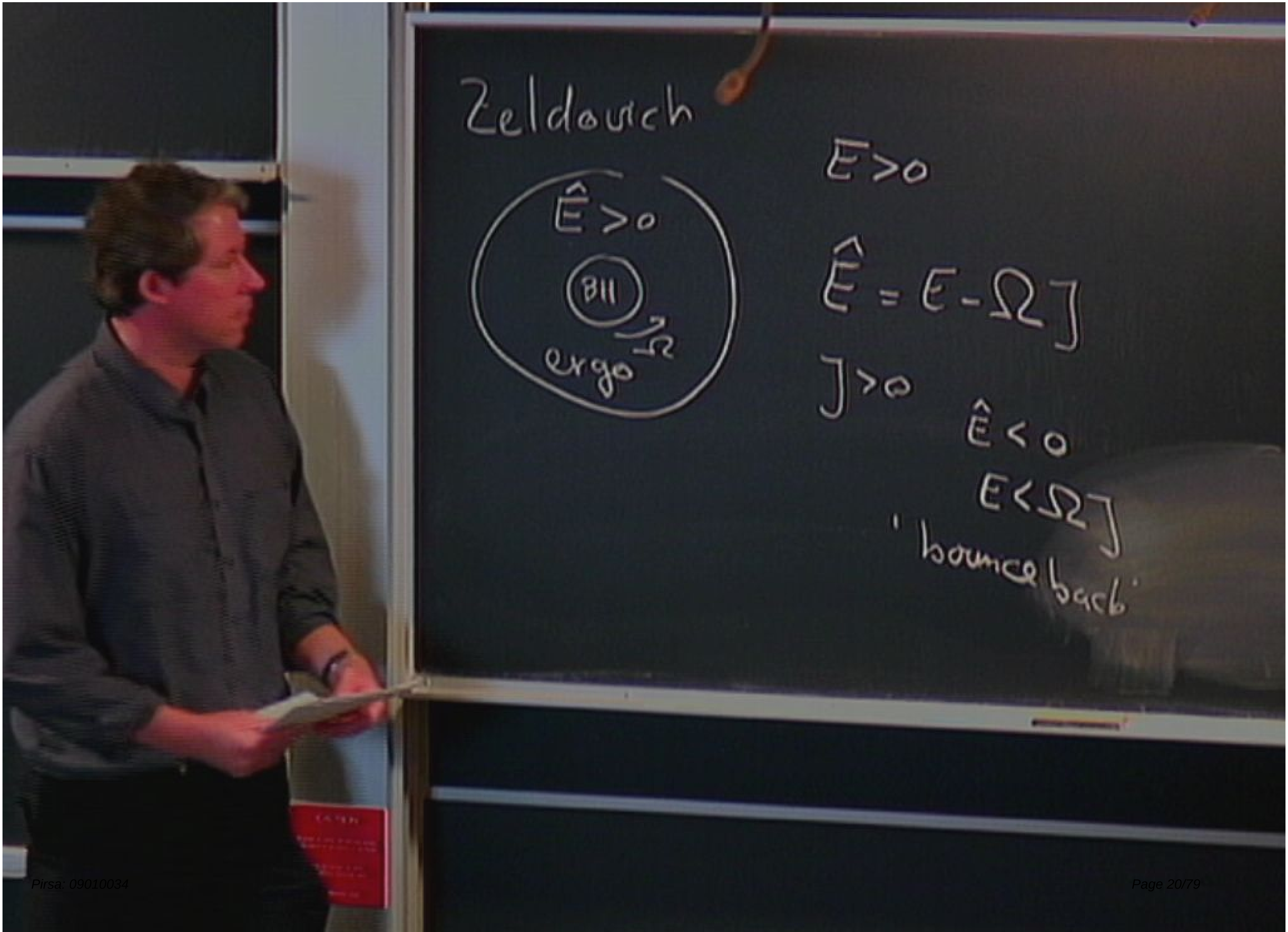
* Penrose process
* superradiance
* Blandford - Znajek
* 'mining' Hawking.

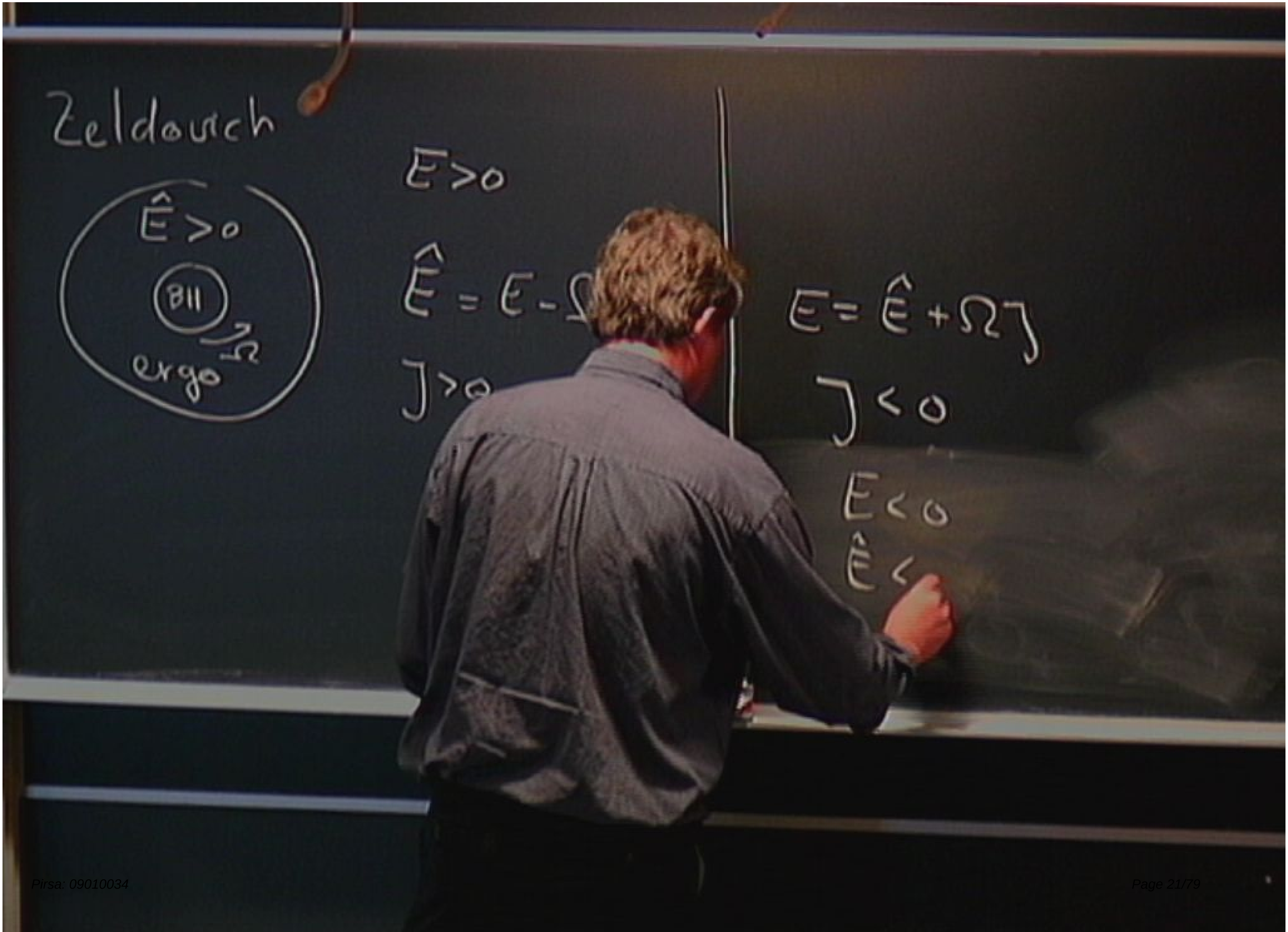
Question: Choose favorite M_{BH}, J
make a BH power plant.


Zeldovich
BH
ergo

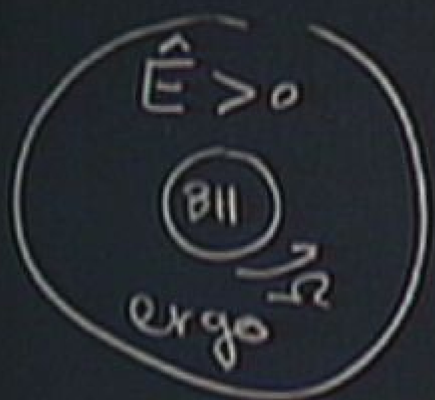

Zeldovich
Ê
BH
ergo
E


Zeldovich
Ê > 0
BH
E > 0
Ê = E − ΩJ
J > 0
Ê < 0
E < ΩJ


Zeldovich
Ê>0
BH
Ω
ergo
E>0
Ê = E-ΩJ
J>0
Ê<0
E<ΩJ
'bounce back'


Zeldovich
Ê>0
BH
ergo
Ω
E>0
J>0
E = Ê + ΩJ
J<0
E<0

Zeldovich



$E > 0$

$\hat{E} = E - \Omega J$

$J > 0$

$\hat{E} < 0$

$E < \Omega J$

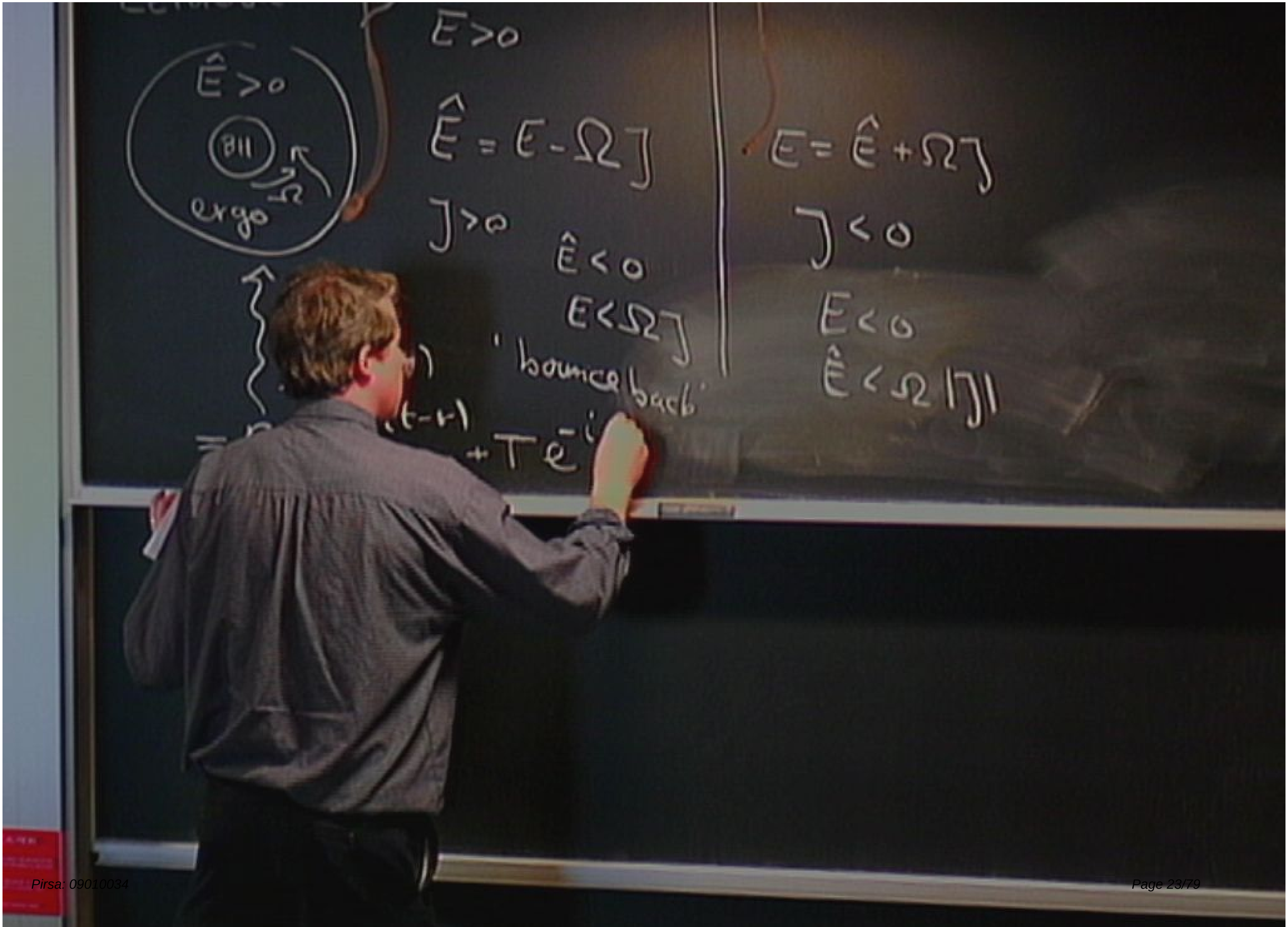
'bounce back'

$E = \hat{E} + \Omega J$

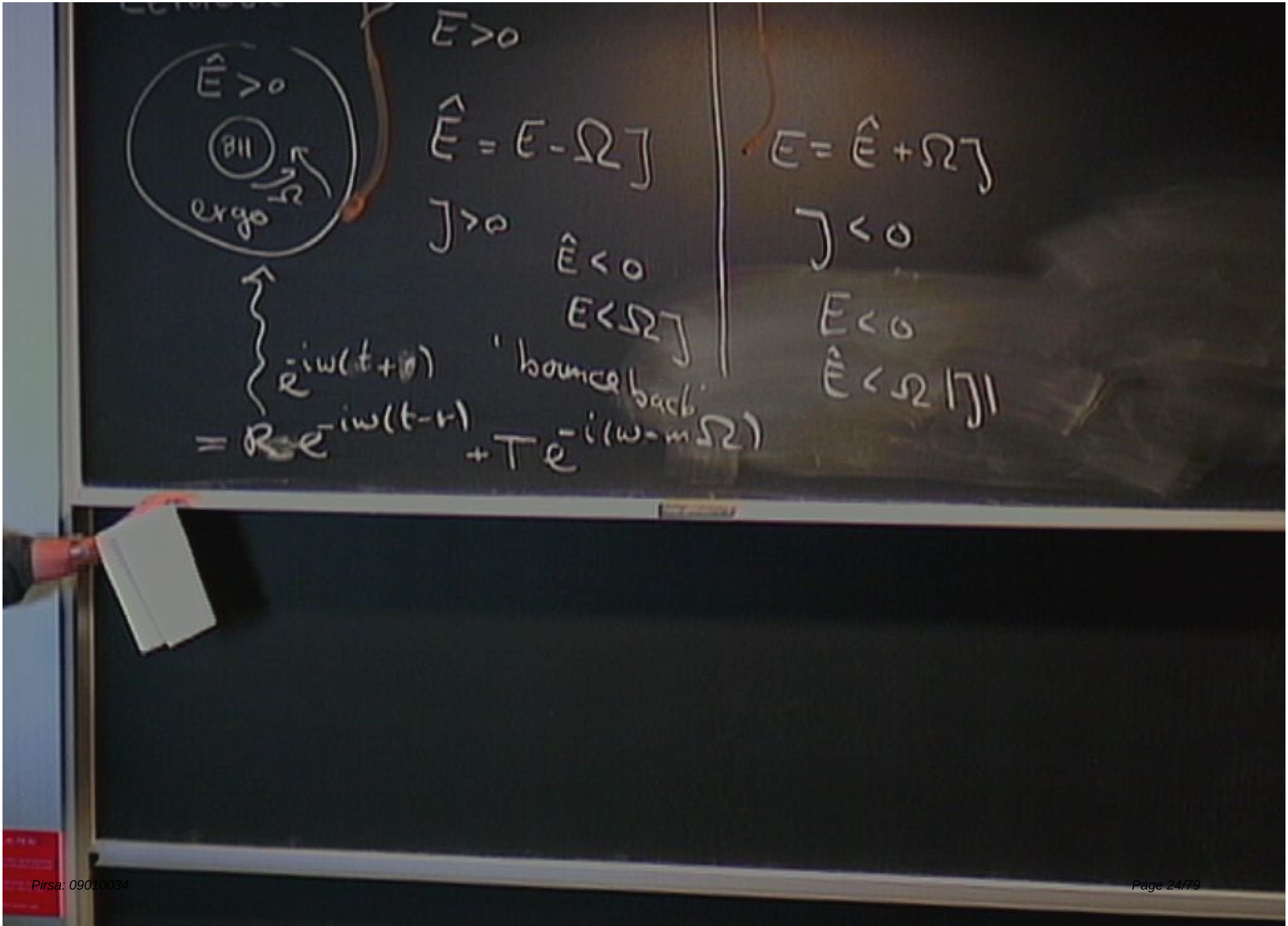
$J < 0$

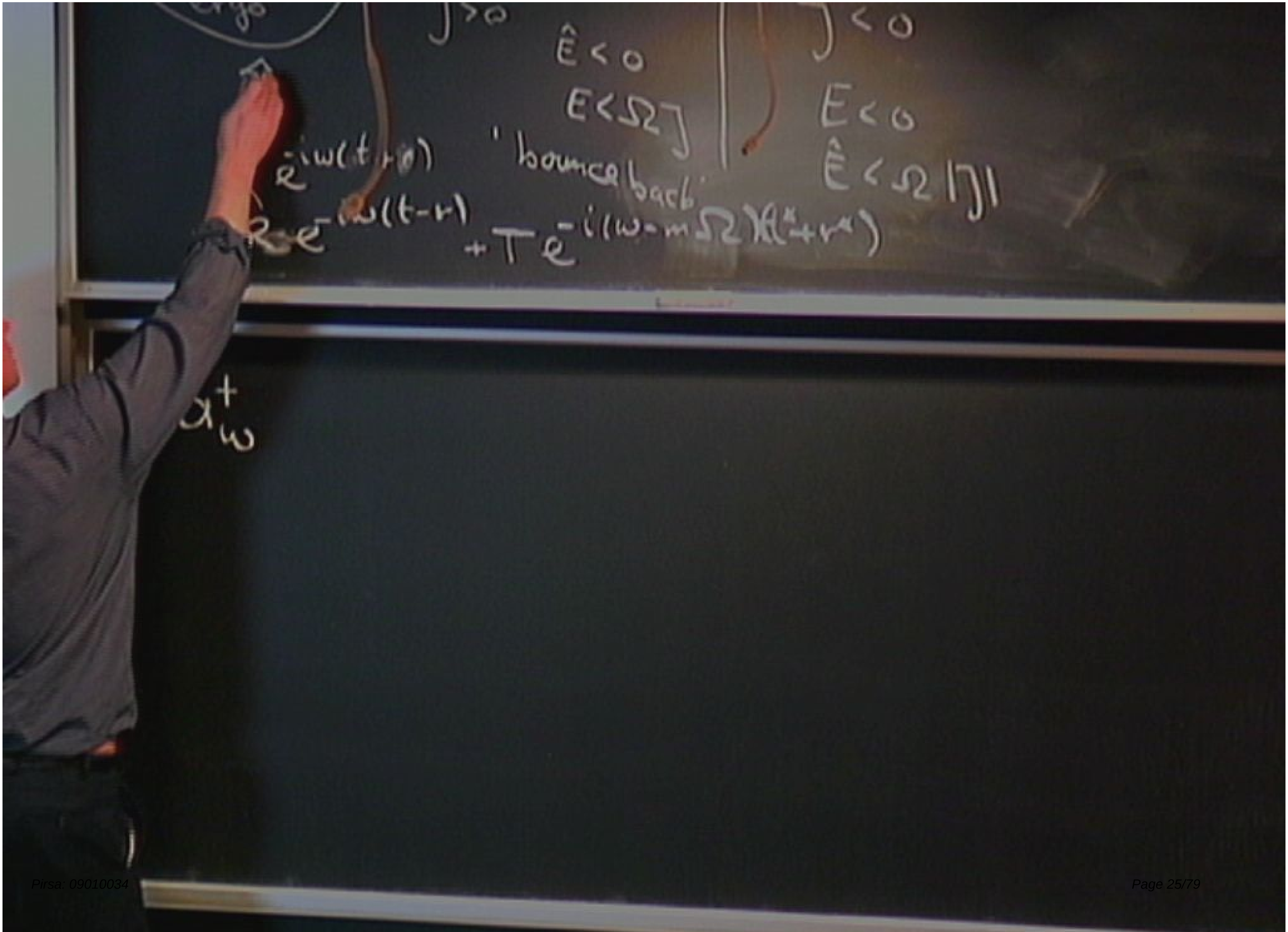
$E < 0$

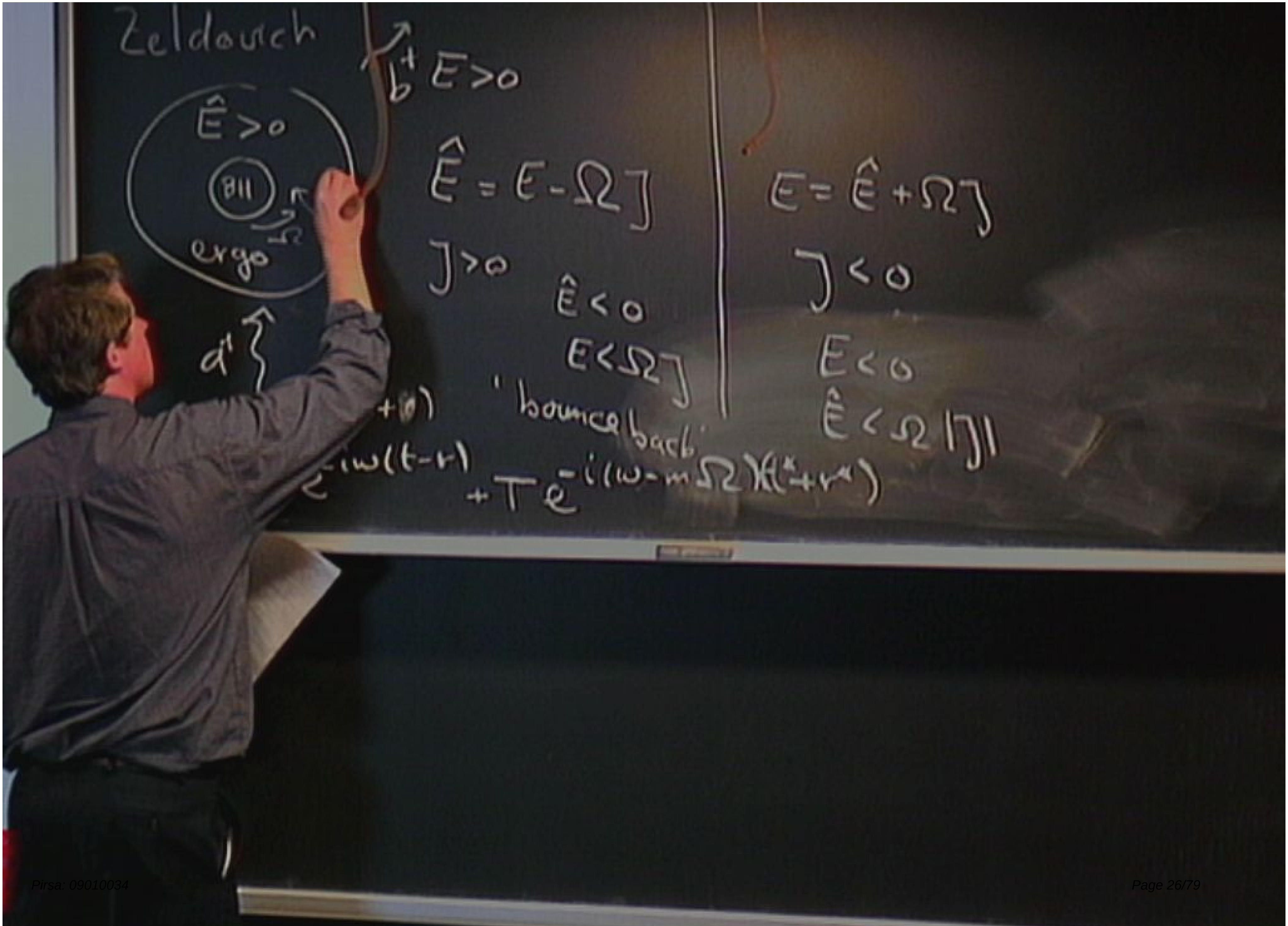
$\hat{E} < \Omega |J|$

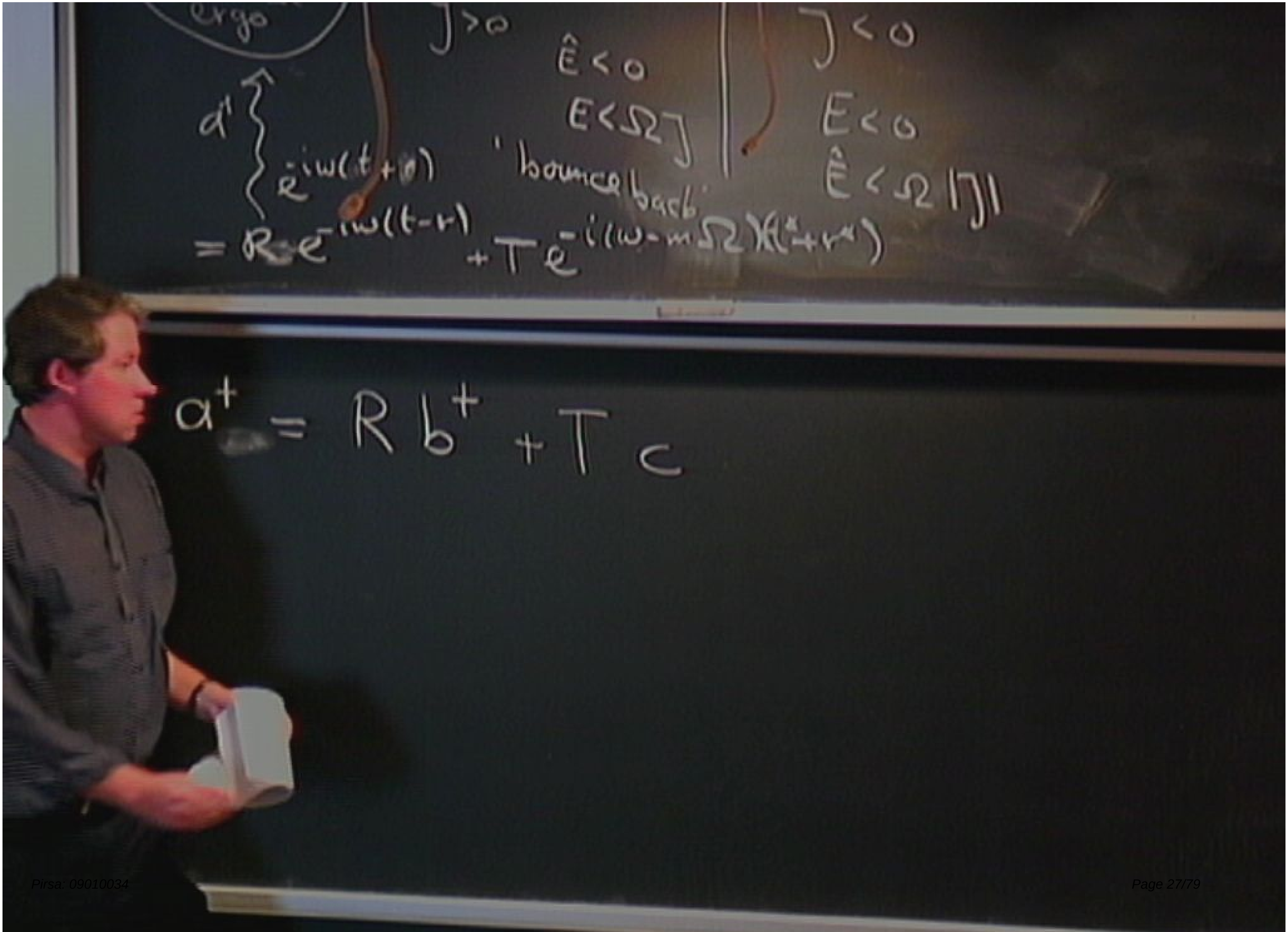


E>0
Ê>0
BH
ergo
Ω
Ê = E − ΩJ
J>0
Ê<0
E<ΩJ
'bounce back'
E = Ê + ΩJ
J<0
E<0
Ê<Ω|J|


$E > 0$
$\hat{E} > 0$
BH
ergo
Ω
$\hat{E} = E - \Omega J$
$E = \hat{E} + \Omega J$
$J > 0$
$J < 0$
$\hat{E} < 0$
$E < \Omega J$
$E < 0$
$\hat{E} < \Omega |J|$
$e^{-i\omega(t+r)}$
'bounce back'
$= R e^{-i\omega(t-r)} + T e^{-i(\omega - m\Omega)}$


J > 0
Ê < 0
E < Ω J
e^{-iω(t+r)}
'bounce back'
J < 0
E < 0
Ê < Ω|J|
R e^{-iω(t-r)} + T e^{-i(ω-mΩ)(t*+r*)}
a_ω^+


Zeldovich
b† E>0
Ê>0
BH
ergo
a†
Ê = E − ΩJ
J>0
Ê<0
E<ΩJ
'bounceback'
E = Ê + ΩJ
J<0
E<0
Ê<Ω|J|
e^{−iω(t−r)}
+T e^{−i(ω−mΩ)(t*+r*)}


ergo
J > 0
Ê < 0
E < ΩJ
J < 0
E < 0
Ê < Ω|J|
a†
e^{-iω(t+r)}
'bounce back'
= R e^{-iω(t-r)} + T e^{-i(ω-mΩ)(t*+r*)}
a⁺ = R b⁺ + T c

$J > 0$ $\qquad \hat{E} < 0$ $\qquad E < \Omega J$ $\qquad | \qquad J < 0$ $\qquad E < 0$ $\qquad \hat{E} < \Omega |J|$

$a^\dagger$

$e^{-i\omega(t+r)}$ 'bounce back'

$$= R e^{-i\omega(t-r)} + T e^{-i(\omega - m\Omega)(t^* + r^*)}$$

$$a^+ = R\, b^+ + T\, c$$

annihilates an $E<0$ particle.
inside ergo.

$J > 0$ $\hat{E} < 0$ $E < \Omega J$ 'bounce back'

$J < 0$ $E < 0$ $\hat{E} < \Omega |J|$

a^{+}

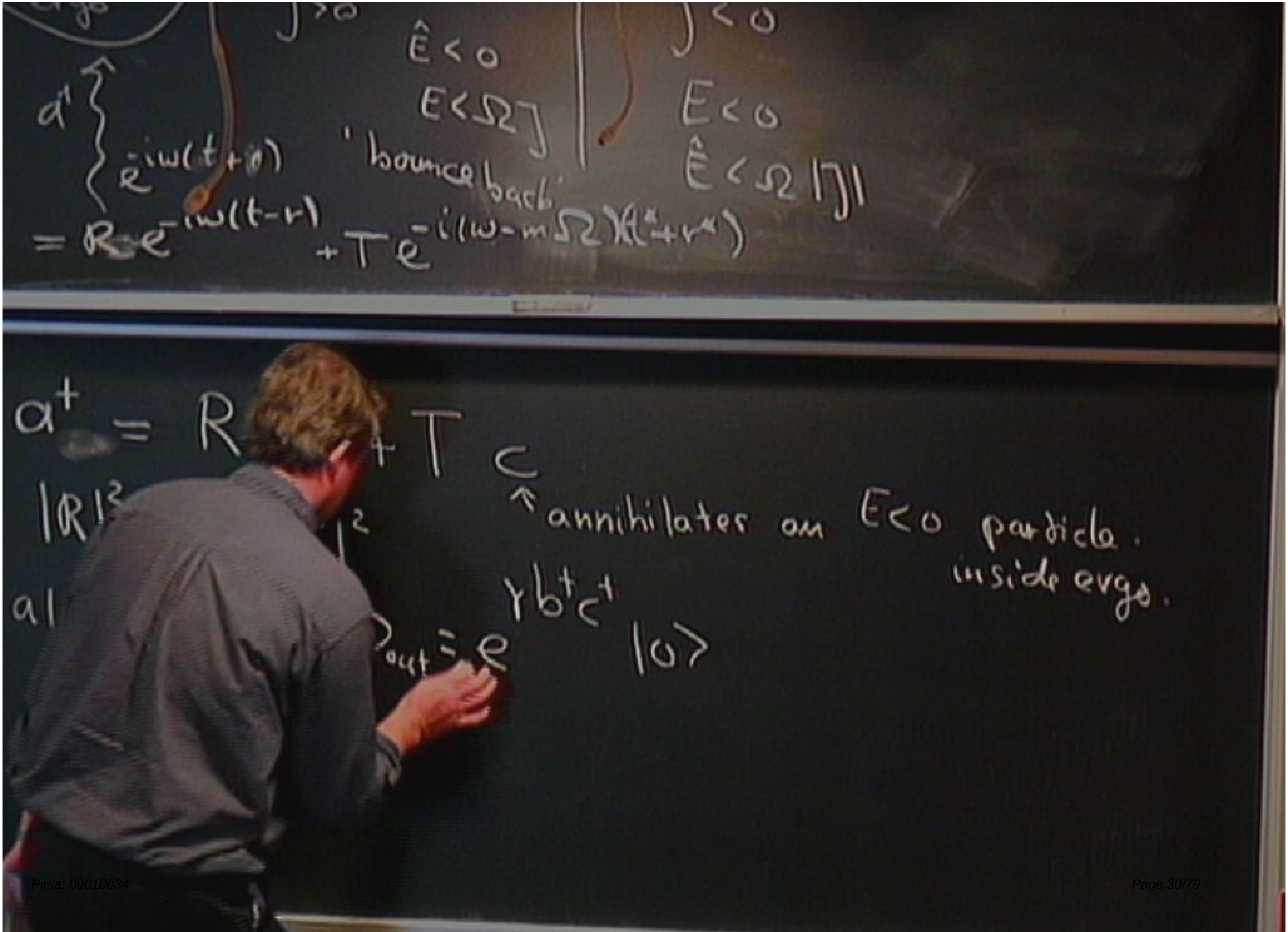
$e^{-i\omega(t+r)}$

$= R e^{-i\omega(t-r)} + T e^{-i(\omega - m\Omega)(t^{*}+r^{*})}$

$a^{+} = R\, b^{+} + T\, c$

$|R|^2 = 1 + |T|^2$

annihilates an $E<0$ particle.
inside ergo.


Ê < 0
E < Ω
E < 0
Ê < Ω |J|
'bounce back'
annihilates an E<0 particle.
inside ergo.
|0>

ergo $J>0$ $\hat{E}<0$ $E<\Omega J$ | $J<0$ $E<0$ $\hat{E}<\Omega|J|$

$a^\dagger$ $e^{-i\omega(t+r)}$ 'bounce back'

$$= R e^{-i\omega(t-r)} + T e^{-i(\omega - m\Omega)(t^*+r^*)}$$

$$a^+ = R\, b^+ + T\, c$$

c: annihilates an $E<0$ particle inside ergo.

$$|R|^2 = 1 + |T|^2$$

$a|0\rangle_{in} = 0$

$$|0\rangle_{in} = e^{\gamma b^+ c^+}|0\rangle_{out}$$

$J > 0$ $\hat{E} < 0$ $J < 0$

$a^{\dagger}$ $E < \Omega J$ $E < 0$

$e^{-i\omega(t+r)}$ 'bounce back' $\hat{E} < \Omega |J|$

$= R e^{-i\omega(t-r)} + T e^{-i(\omega - m\Omega)(t^* + r^*)}$

$a^{+} = R\, b^{+} + T\, c$

$|R|^2 = 1 + |T|^2$

annihilates an $E<0$ particle. inside ergo.

$a|0\rangle_{in} = 0$

$|0\rangle_{in} = e^{\gamma b^{+} c^{+}} |0\rangle_{out}$

$J > 0$

$\hat{E} < 0$

$E < \Omega J$

$J < 0$

$E < 0$

$\hat{E} < \Omega |J|$

$\to a^{\dagger}$

$e^{-i\omega(t+r)}$ 'bounce back'

$= R\, e^{-i\omega(t-r)} + T e^{-i(\omega - m\Omega)(t^* + r^*)}$

$$a^{\dagger} = R\, b^{\dagger} + T\, c$$

c annihilates an $E<0$ particle inside ergo.

$$|R|^2 = 1 + |T|^2$$

$a|0\rangle_{in} = 0$

$$|0\rangle_{in} = e^{\gamma b^{\dagger} c^{\dagger}} |0\rangle_{out}$$

$\langle in|\, b^{\dagger} b\, |in\rangle$

$J > 0$ $\qquad \hat{E} < 0 \qquad E < \Omega J$ $\qquad J < 0 \qquad E < 0 \qquad \hat{E} < \Omega |J|$

$a^\dagger \quad e^{-i\omega(t+r)}$ 'bounce back'

$$= R e^{-i\omega(t-r)} + T e^{-i(\omega - m\Omega)(t^* + r^*)}$$

$$a^\dagger = R\, b^\dagger + T\, c$$

annihilates an $E<0$ particle. inside ergo.

$$|R|^2 = 1 + |T|^2$$

$a|0\rangle_{in} = 0$

$$|0\rangle_{in} = e^{\gamma b^\dagger c^\dagger} |0\rangle_{out}$$

$$\langle n | b^\dagger b | n \rangle = |R|^2 n + |T|^2$$

$J > 0$

$\hat{E} < 0$

$E < \Omega J$

$J < 0$

$E < 0$

$\hat{E} < \Omega |J|$

$\to a^\dagger \; \{ \; e^{-i\omega(t+r)}$ 'bounce back'

$= R e^{-i\omega(t-r)} + T e^{-i(\omega - m\Omega)(t^* + r^*)}$

$$a^\dagger = R\, b^\dagger + T\, c$$

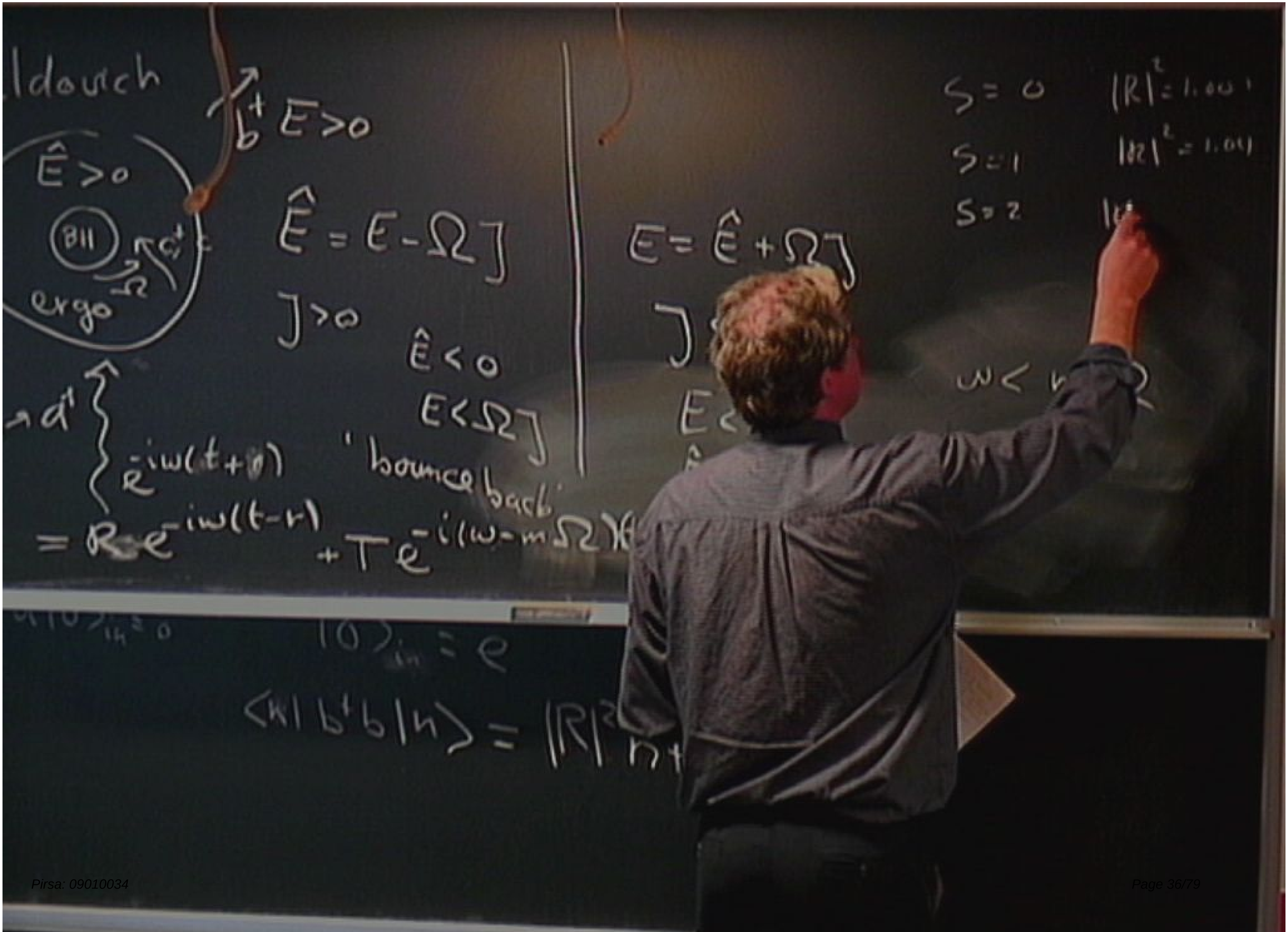
$\uparrow$ annihilates an $E<0$ particle inside ergo.

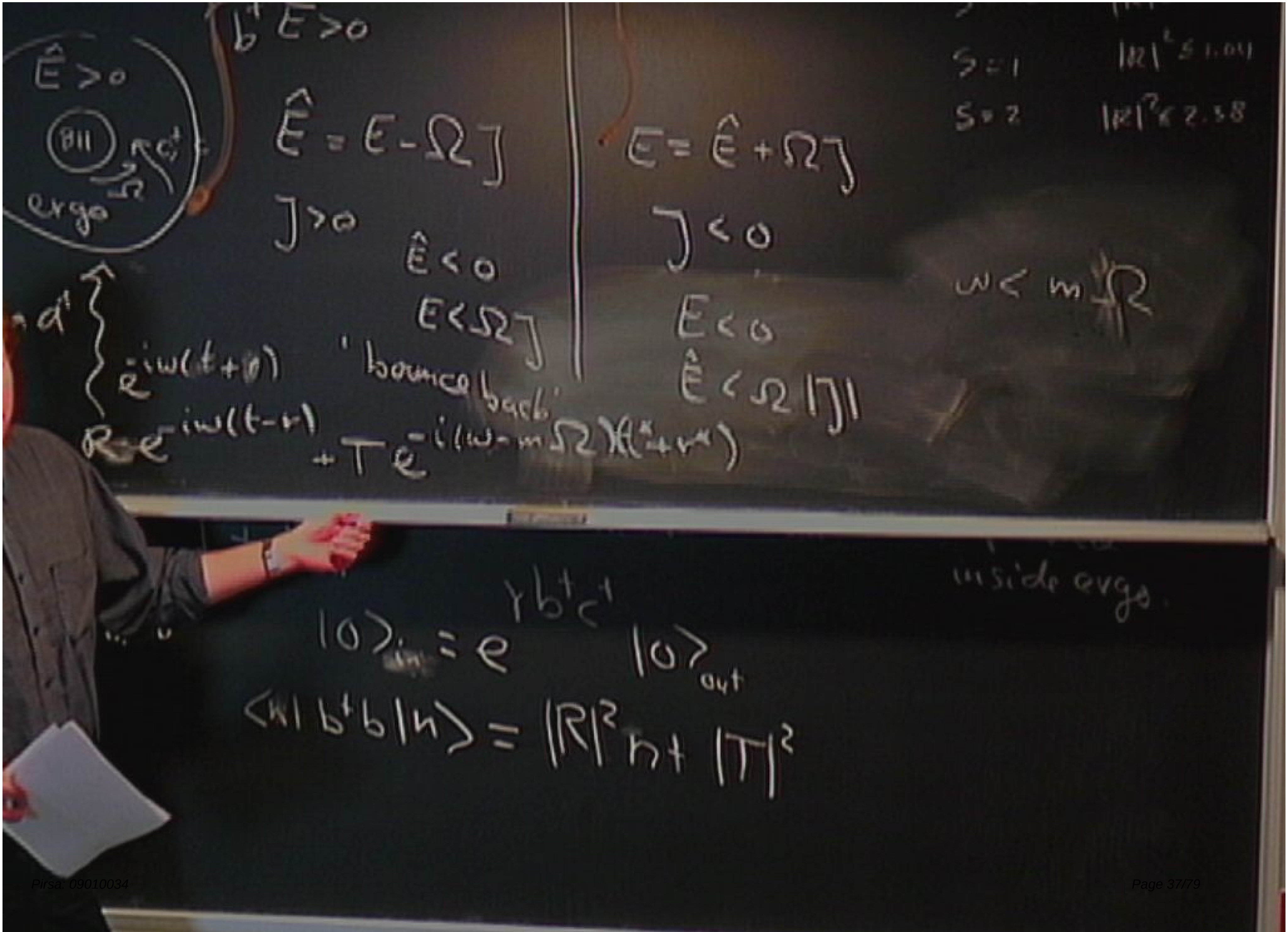
$$|R|^2 = 1 + |T|^2$$

$a|0\rangle_{in} = 0$

$$|0\rangle_{in} = e^{\gamma b^\dagger c^\dagger} |0\rangle_{out}$$

$$\langle n| b^\dagger b |n\rangle = |R|^2 n + |T|^2$$


E>0
Ê>0
BH
ergo
Ê = E - ΩJ
J>0
Ê<0
E<ΩJ
'bounce back'
E = Ê + ΩJ
s = 0
s = 1
s = 2
ω < mΩ


$b^{\dagger}$ $E>0$
$\hat{E}>0$
BH
ergo
$\hat{E} = E - \Omega J$
$J>0$
$\hat{E}<0$
$E<\Omega J$
'bounce back'
$E = \hat{E} + \Omega J$
$J<0$
$E<0$
$\hat{E}<\Omega|J|$
$s=1$ $|R|^2 \leq 1.04$
$s=2$ $|R|^2 \leq 2.38$
$\omega < m\Omega$
$e^{-i\omega(t+r)}$
$Re^{-i\omega(t-r)}$
$+Te^{-i(\omega-m\Omega)(t^*+r^*)}$
inside ergo.
$|0\rangle_{in} = e^{\gamma b^{\dagger}c^{\dagger}} |0\rangle_{out}$
$\langle n| b^{\dagger}b|n\rangle = |R|^2 n + |T|^2$

$$\frac{\partial}{\partial \ell} = \frac{\partial}{\partial t} - \Omega \frac{\partial}{\partial \varphi}$$

$$\frac{\partial}{\partial \tilde{t}} = \frac{\partial}{\partial t} - \Omega \frac{\partial}{\partial \varphi}$$

$$= R e^{-i\omega(t-r)} + T e^{-i(\omega - m\Omega)(t^* + r^*)}$$

$$a^\dagger = R\, b^\dagger + T\, c$$

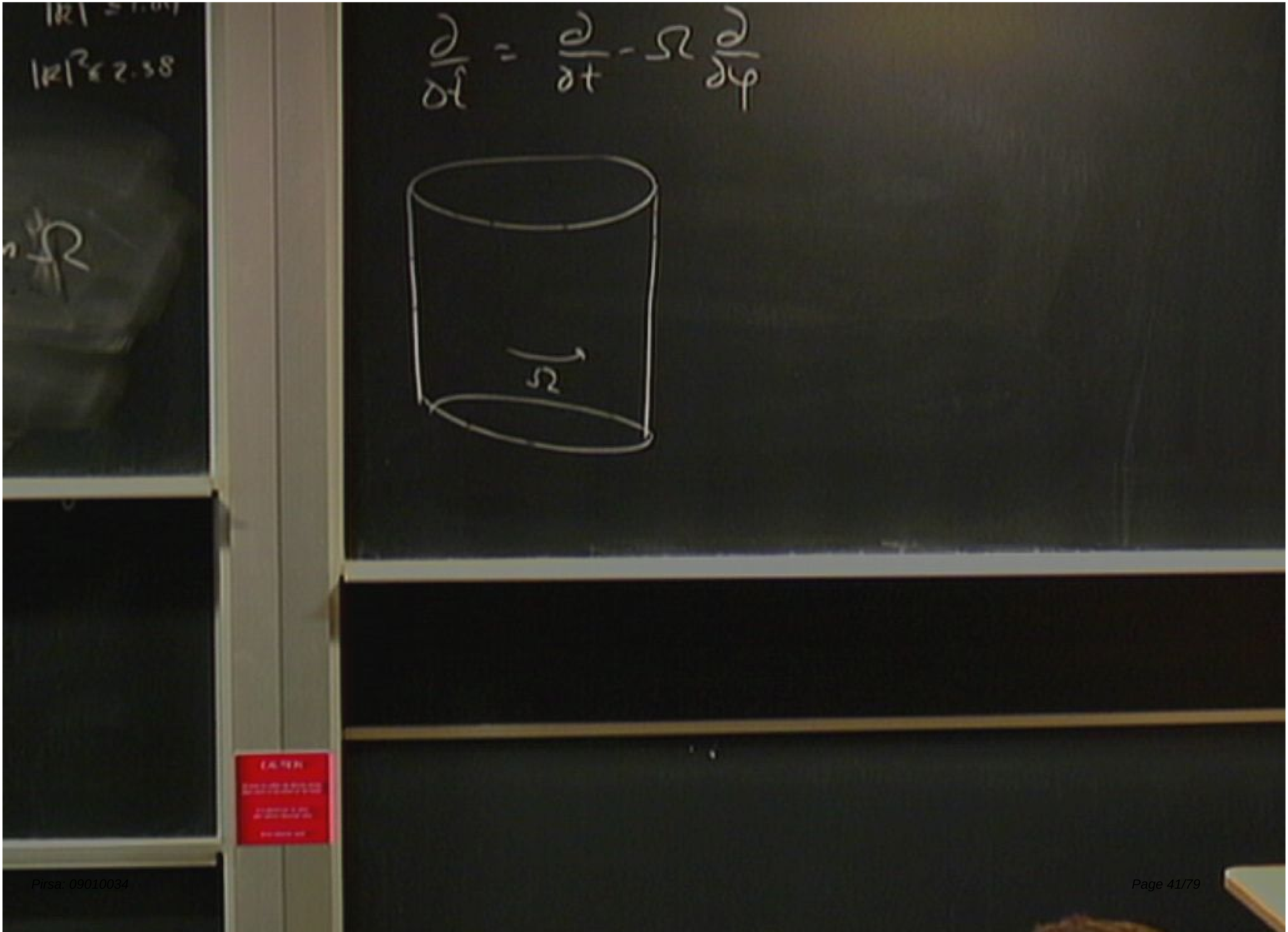
c annihilates an $E<0$ particle. inside ergo.

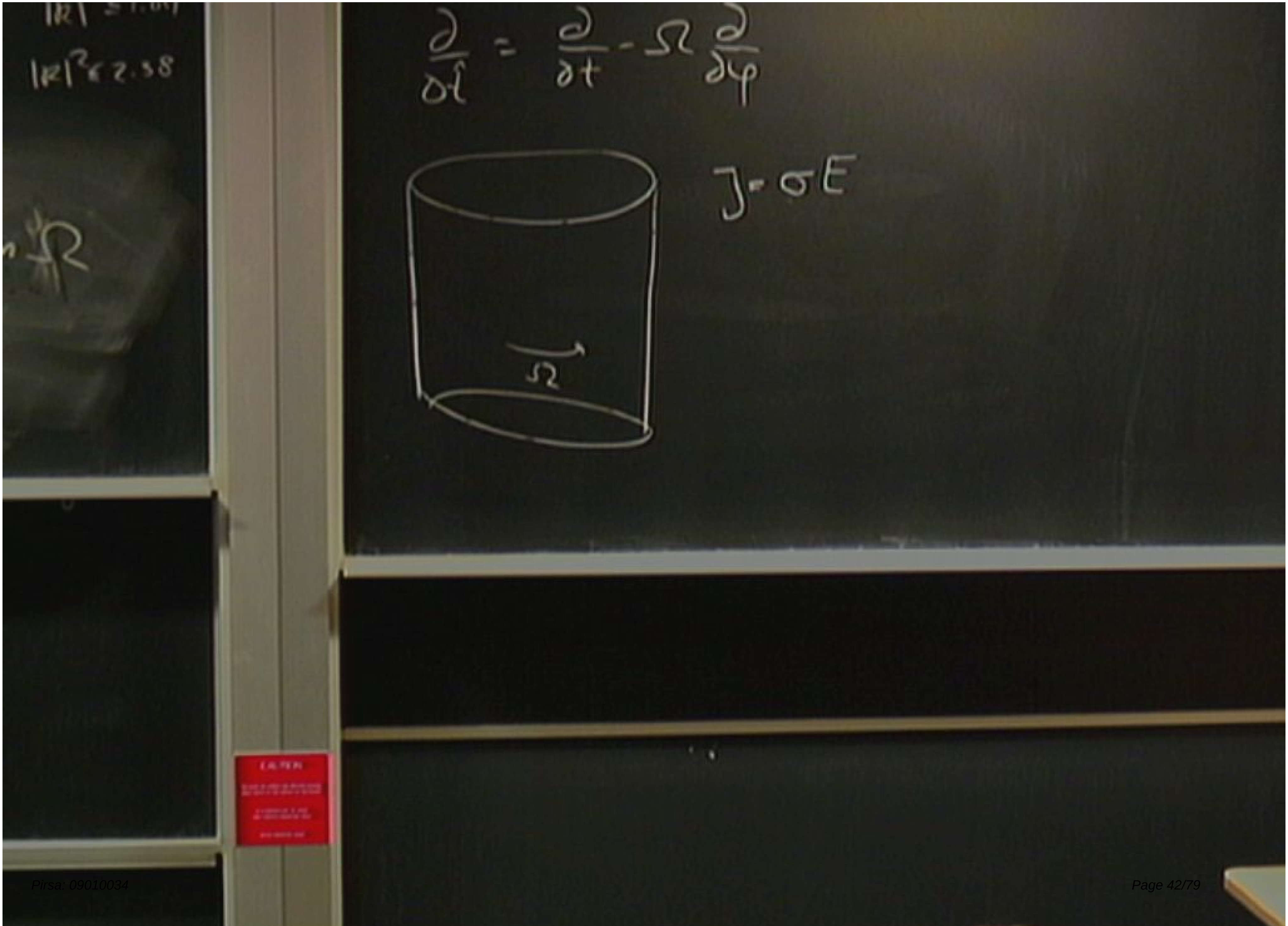
$$|R|^2 = 1 + |T|^2$$

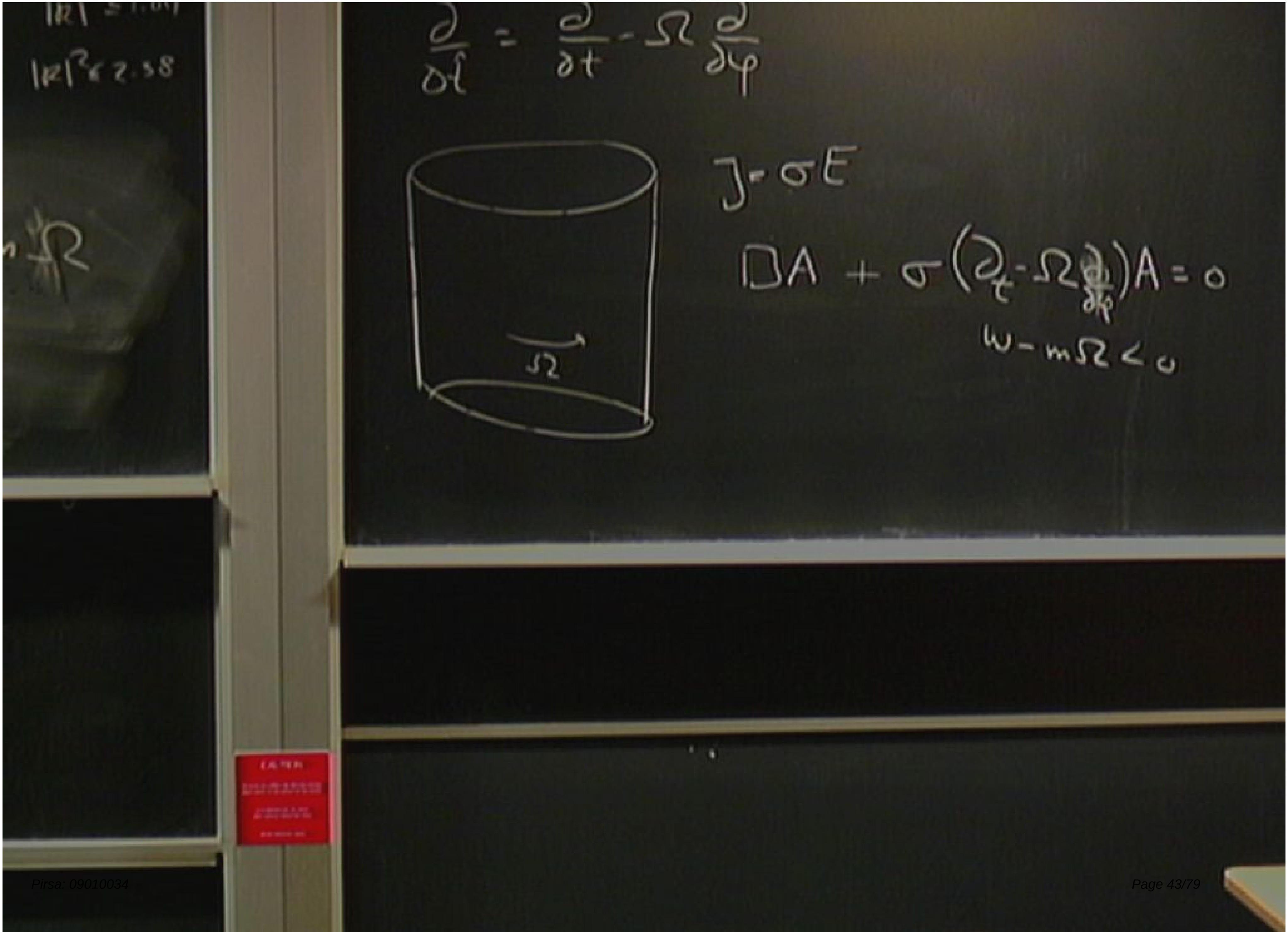
$a|0\rangle_{in} = 0$

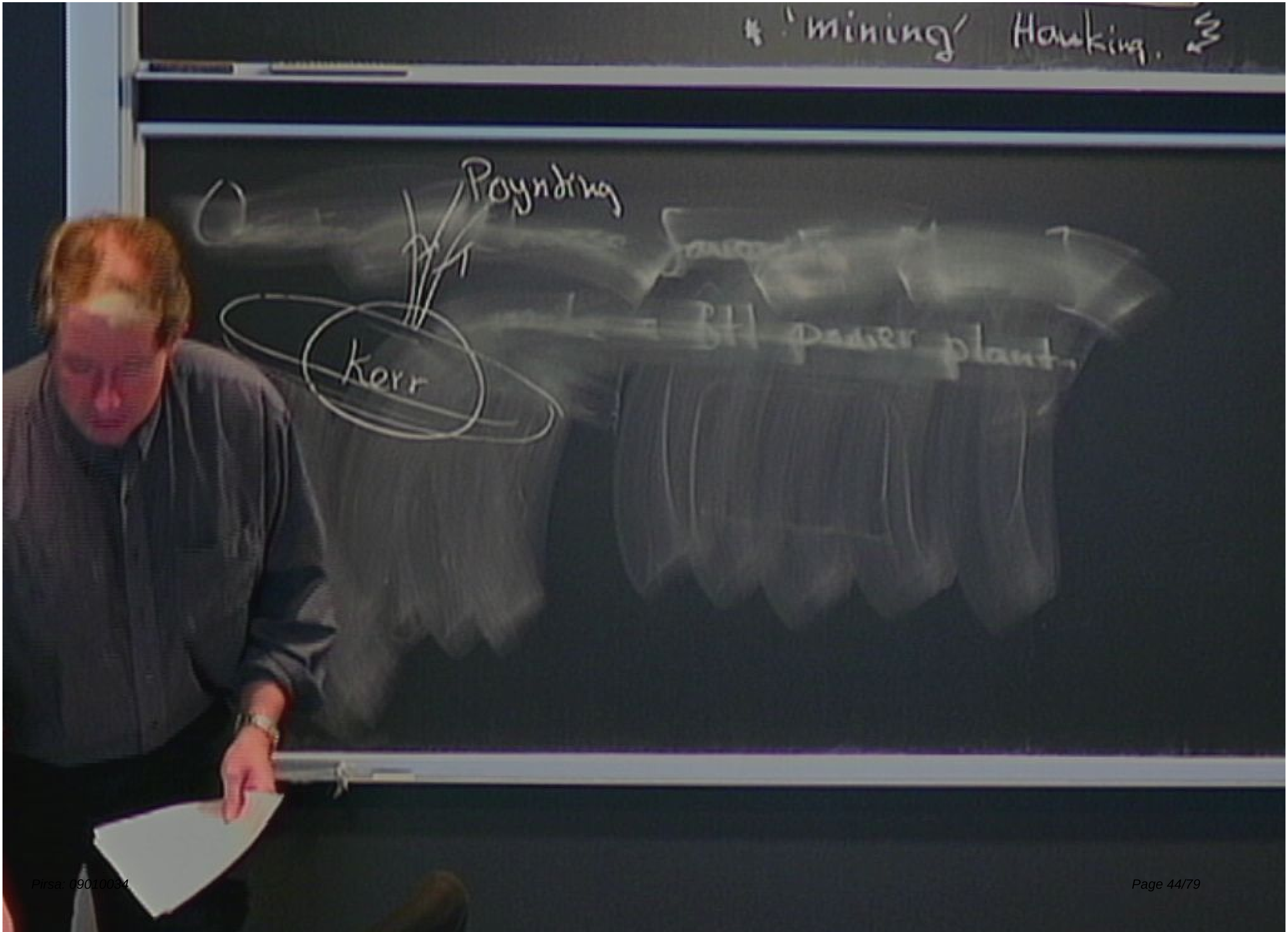
$$|0\rangle_{in} = e^{\gamma b^\dagger c^\dagger} |0\rangle_{out}$$

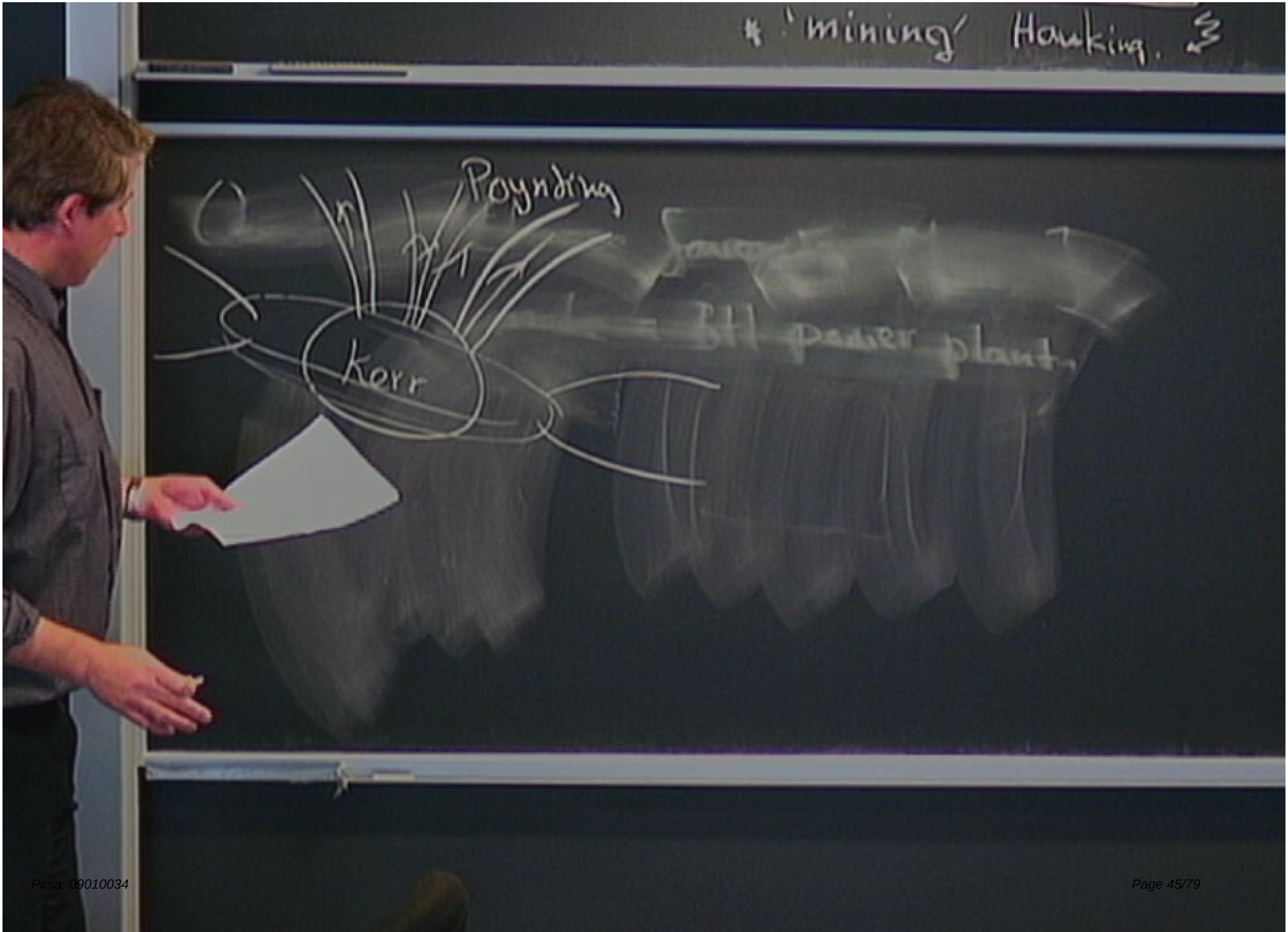
$$\langle n|\, b^\dagger b\, |n\rangle = |R|^2 n + |T|^2$$

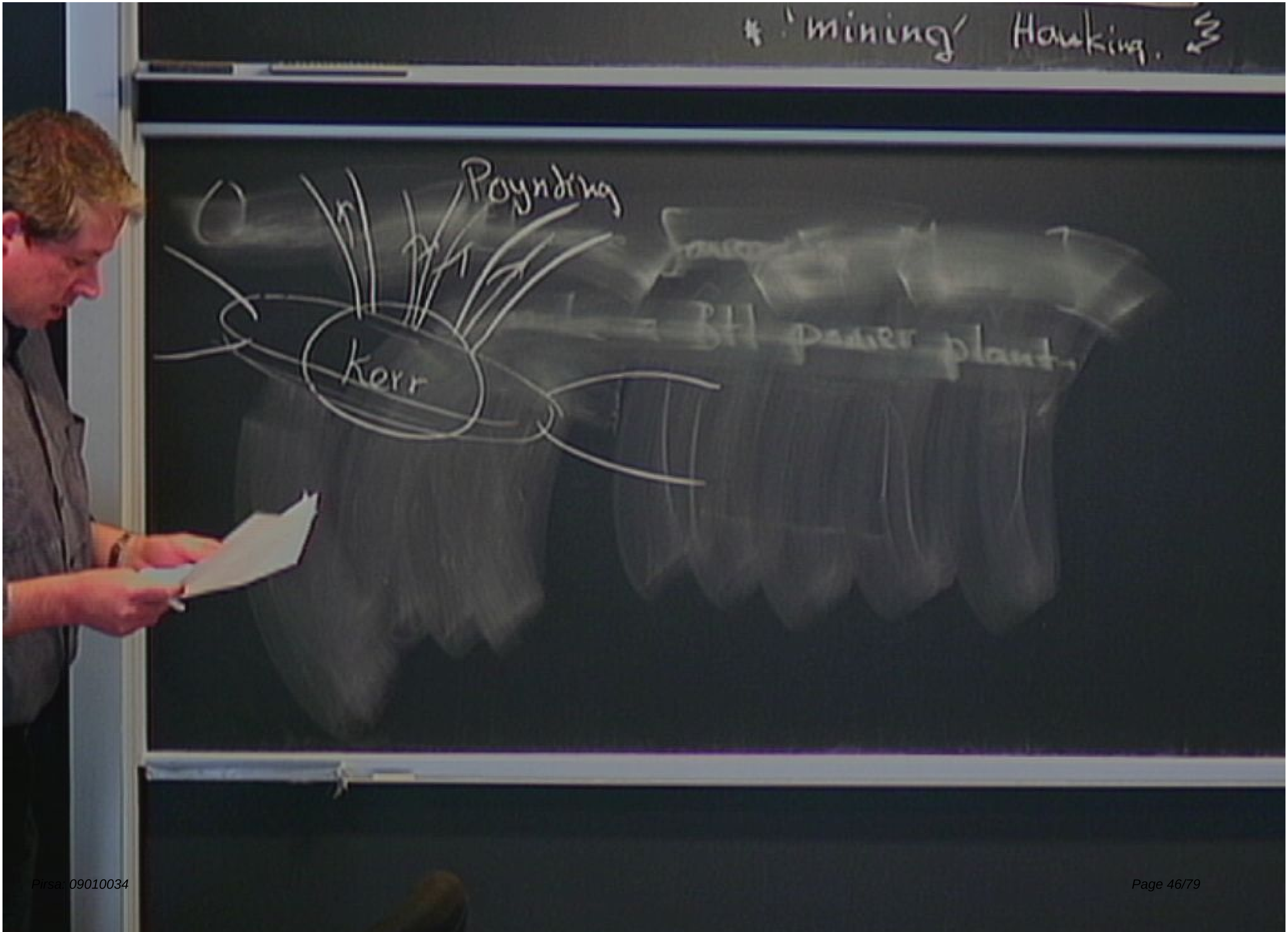

$|R|^2 \leq 2.38$
$\frac{\partial}{\partial \tilde{t}} = \frac{\partial}{\partial t} - \Omega \frac{\partial}{\partial \varphi}$
Ω


∂/∂t̃ = ∂/∂t − Ω ∂/∂φ
J = σE
Ω
|R|² ≤ 2.38

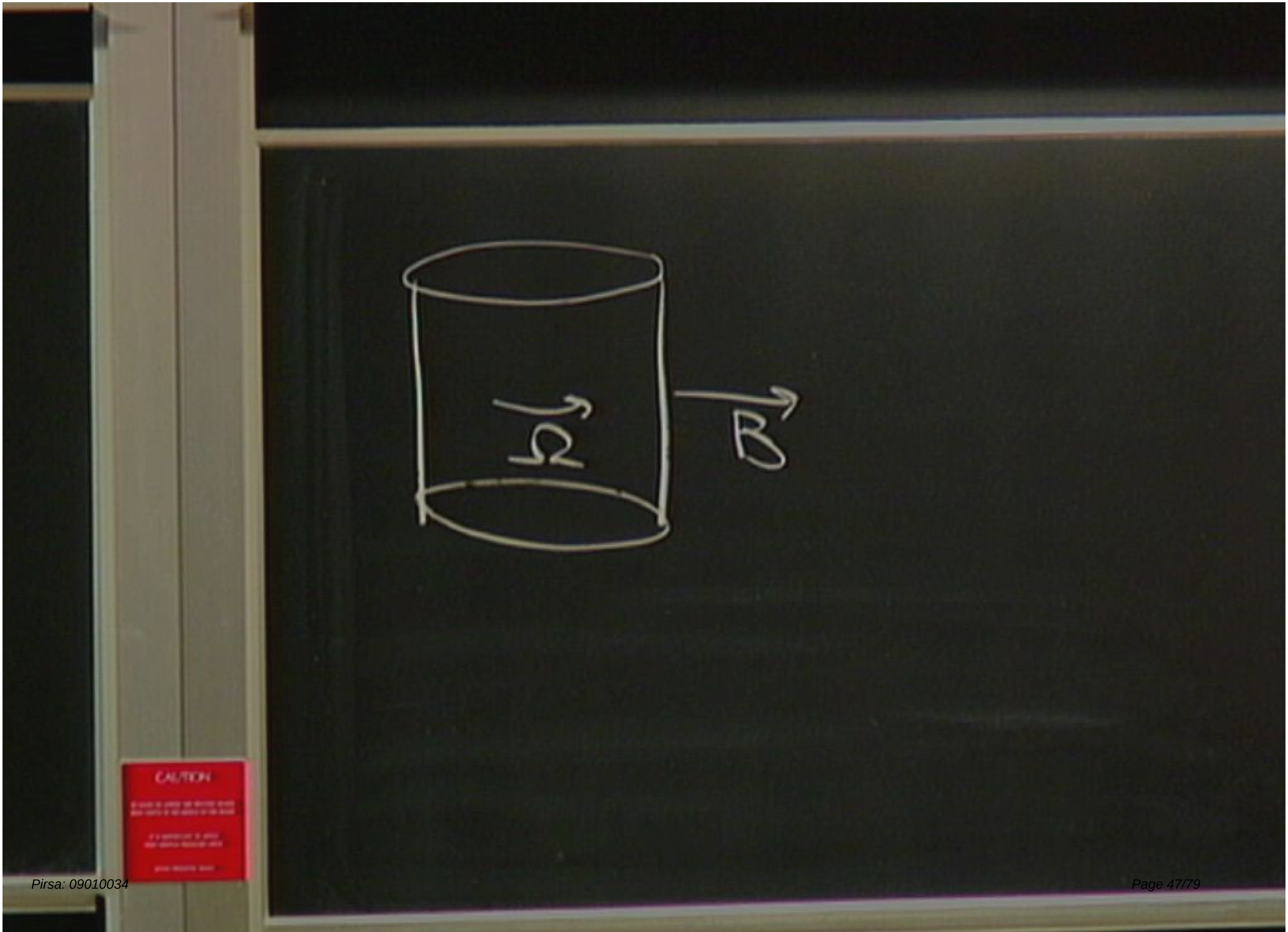

$|R|^2 \leq 2.58$
$\frac{\partial}{\partial \tilde{t}} = \frac{\partial}{\partial t} - \Omega \frac{\partial}{\partial \varphi}$
Ω
$J = \sigma E$
$\Box A + \sigma(\partial_t - \Omega \frac{\partial}{\partial \varphi})A = 0$
$\omega - m\Omega < 0$

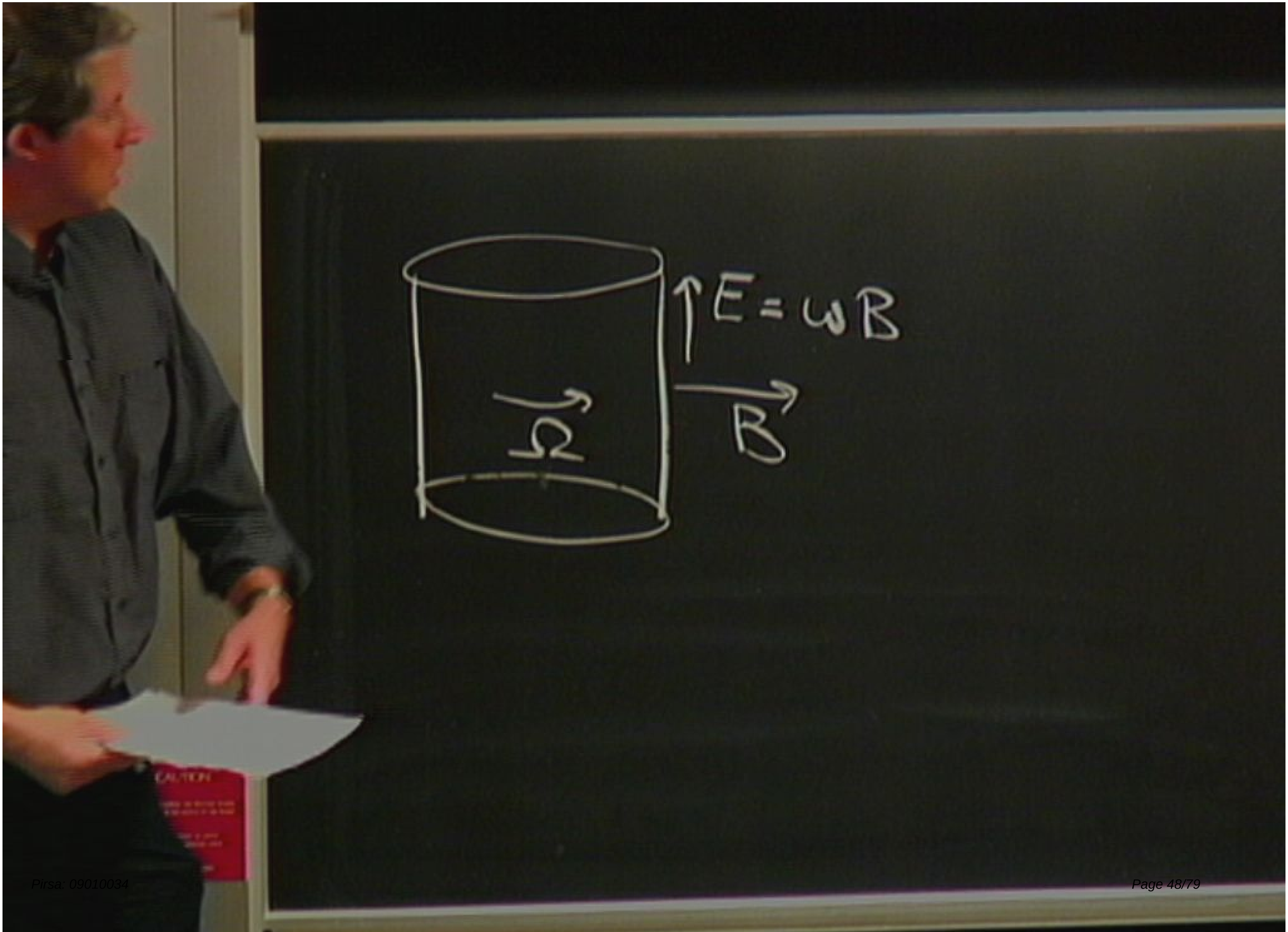

* 'mining' Hawking.
Poynting
Kerr
BH power plant.

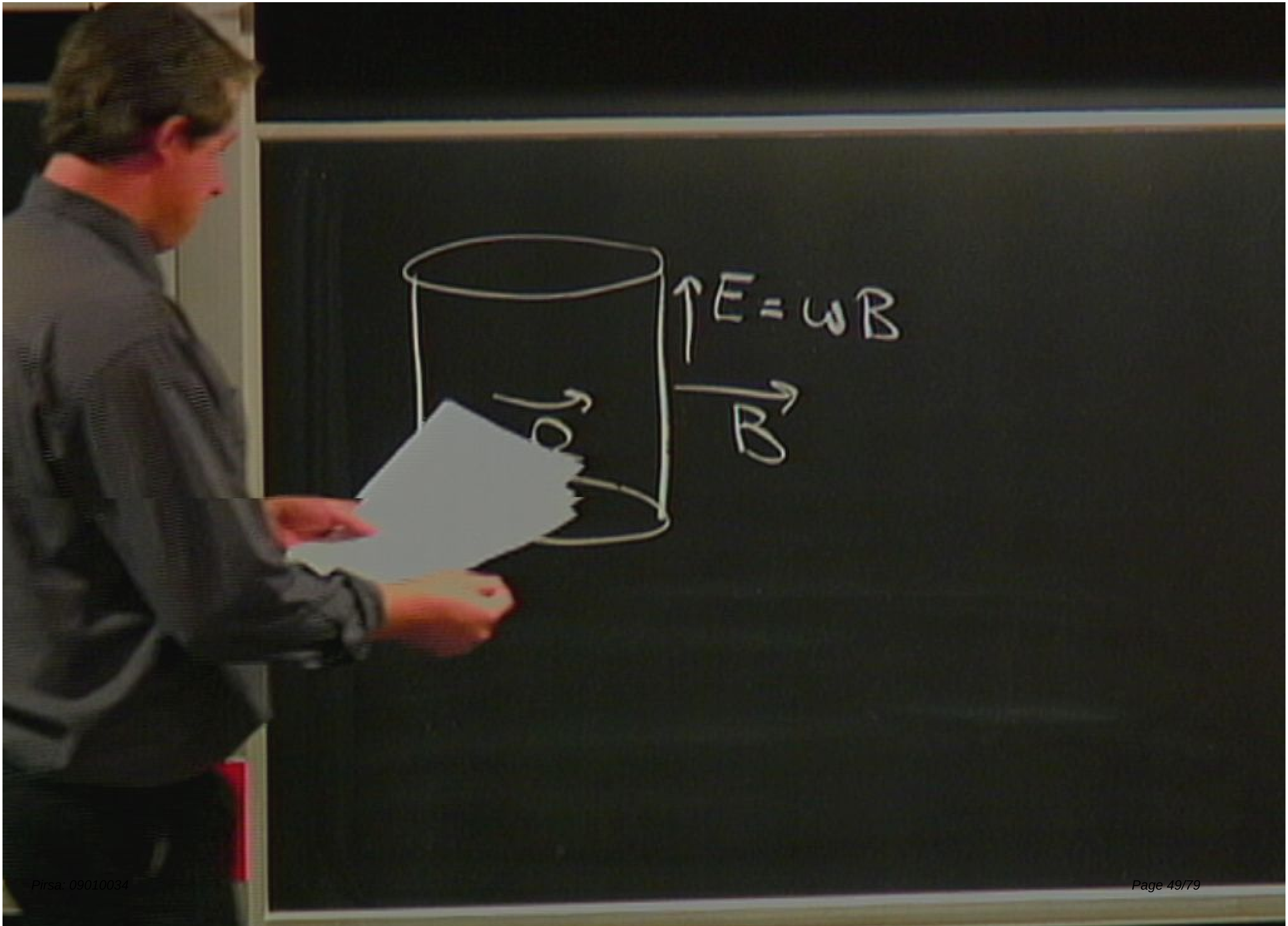

* 'mining' Hawking.
Poynting
Kerr
BH power plant.



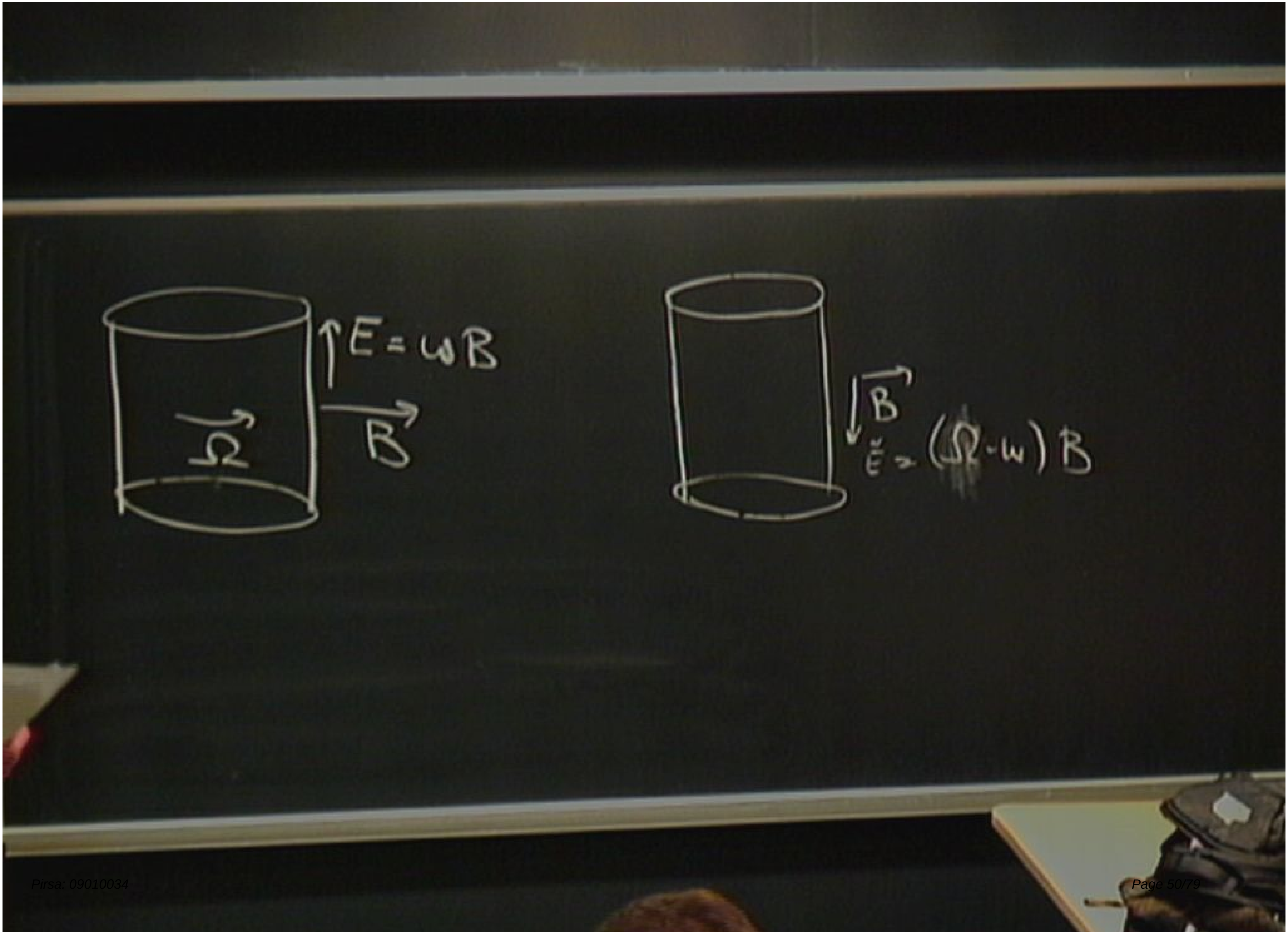
* 'mining' Hawking.
Poynting
Kerr
BH power plant.

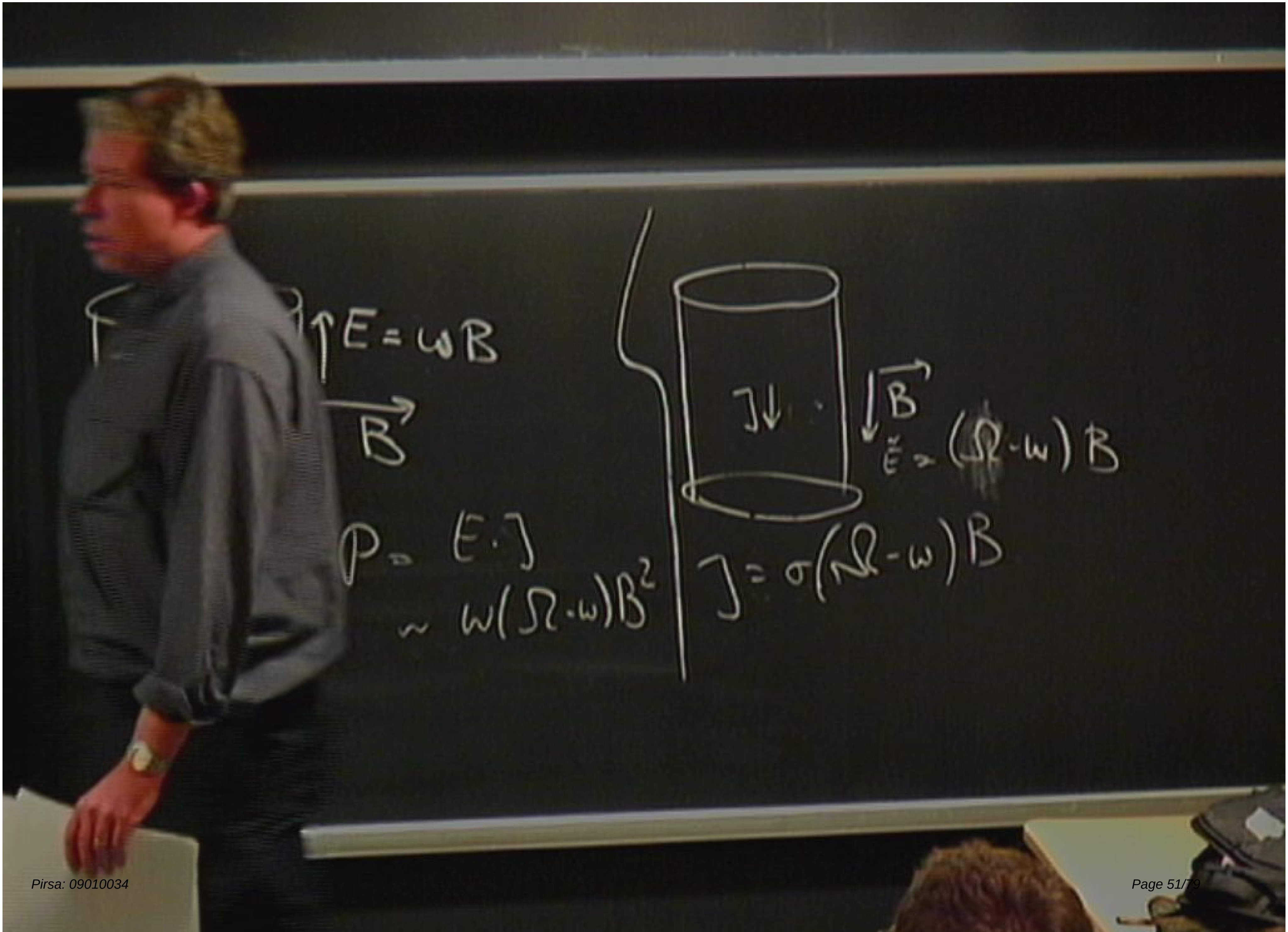

$\vec{\Omega}$
$\vec{B}$
CAUTION

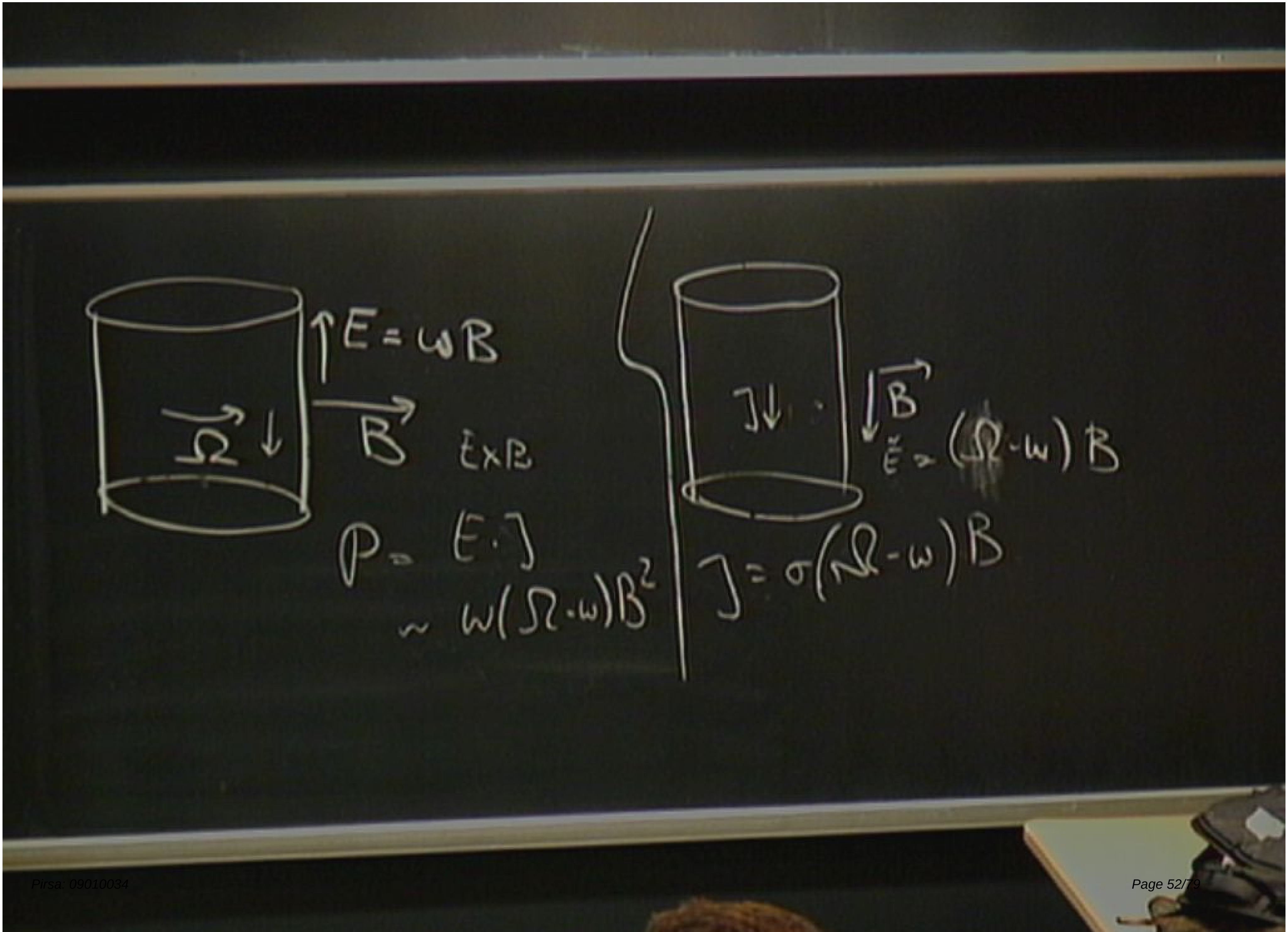

E = ωB
Ω
B

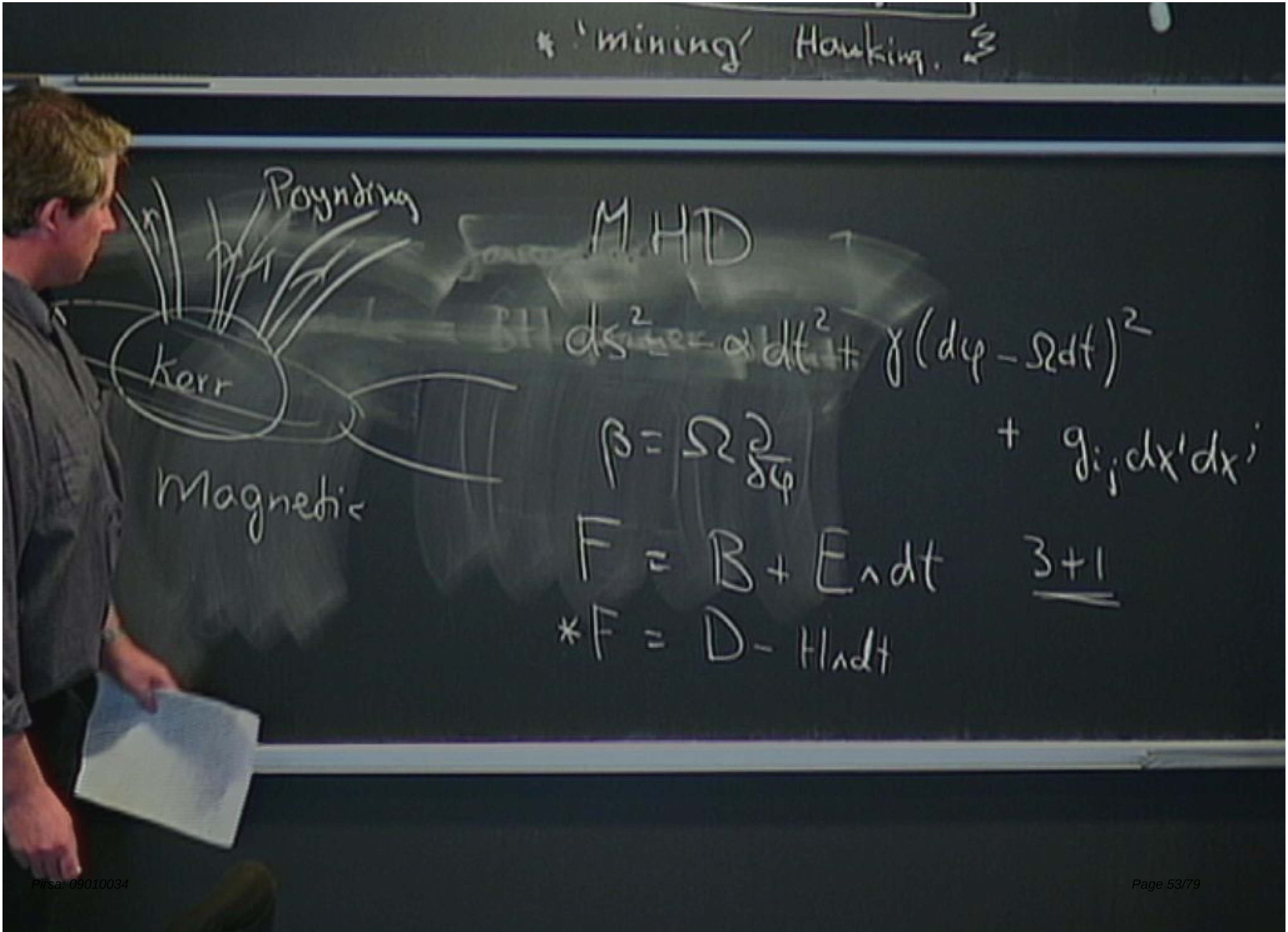


E = ωB
ω
B

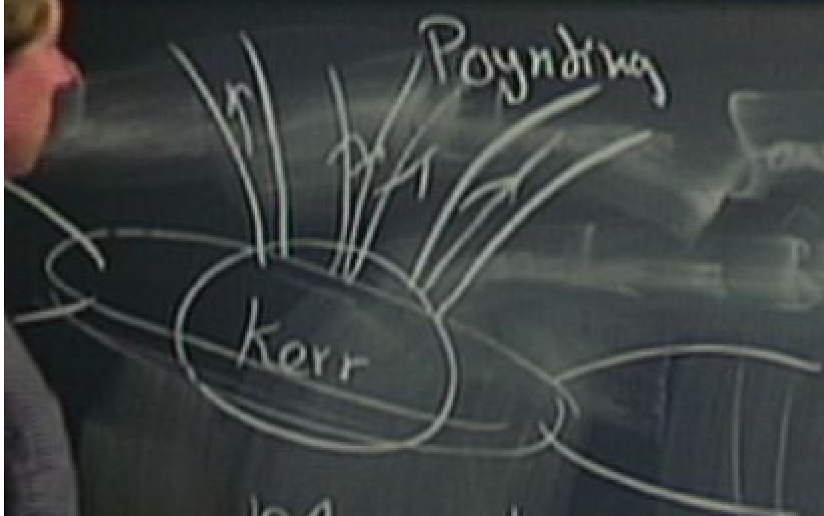

E = ωB
Ω
B
B
E = (Ω - ω) B


E = ωB
B
J
B
E ≈ (Ω - ω) B
P = E · J
~ ω(Ω - ω)B²
J = σ(Ω - ω) B


E = ωB
B
Ω
E×B
P = E·J
~ ω(Ω-ω)B²
J
B
E = (Ω-ω)B
J = σ(Ω-ω)B


* 'mining' Hawking.
Poynting
Kerr
Magnetic
MHD
ds² = -α dt² + γ(dφ - Ω dt)²
β = Ω ∂/∂φ
+ g_ij dx^i dx^j
F = B + E∧dt
3+1
*F = D - H∧dt

* 'mining' Hawking.



MHD

$$ds^2 = -\alpha\, dt^2 + \gamma\,(d\varphi - \Omega\, dt)^2 + g_{ij}\, dx^i dx^j$$

$$\beta = \Omega \frac{\partial}{\partial \varphi}$$

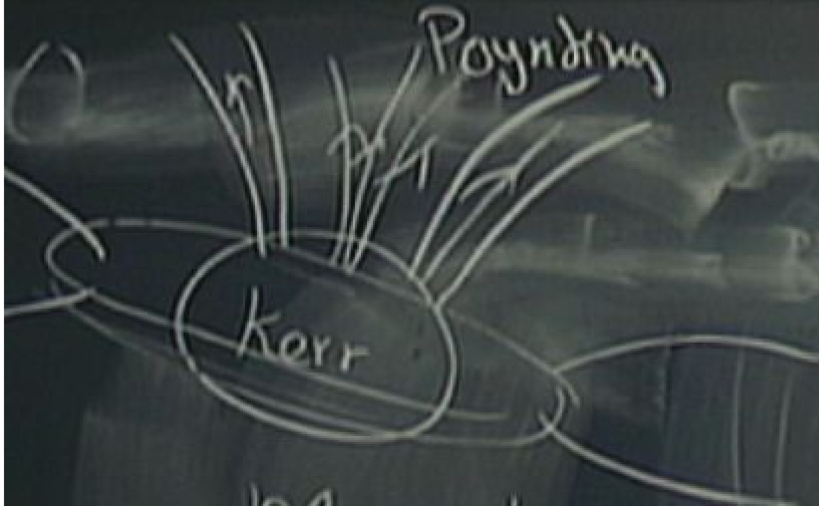
$$F = B + E \wedge dt \qquad \underline{3+1}$$

$$*F = D - H \wedge dt$$

Magnetic

$$\mathcal{L} = \int (E \cdot D - B \cdot H)$$

* 'mining' Hawking. ξ



MHD

$$ds^2 = -\alpha\, dt^2 + \gamma (d\varphi - \Omega dt)^2 + g_{ij}\, dx^i dx^j$$

$$\beta = \Omega \frac{\partial}{\partial \varphi}$$

Magnetic

$$S = \int (E\cdot D - B\cdot H)$$

$$F = B + E \wedge dt \qquad \underline{3+1}$$

$$*F = D - H \wedge dt$$

$= R e^{\ldots} \qquad + T e^{-i(\omega - m\Omega \ldots)}$

Force free $\quad F_{\mu\nu} J^{\nu} = 0$

$\left\{ e^{-i\omega(t+r)} \right.$ bounce back $\quad E < \Omega |J|$

$= R\, e^{-i\omega(t-r)} + T\, e^{-i(\omega - m\Omega)(t+r^*)}$

Force free $\quad F_{\mu\nu} J^\nu = 0 \quad\quad E \cdot B = 0$

$E = -\, i_V$

$V = \omega \frac{\partial}{\partial \phi}$

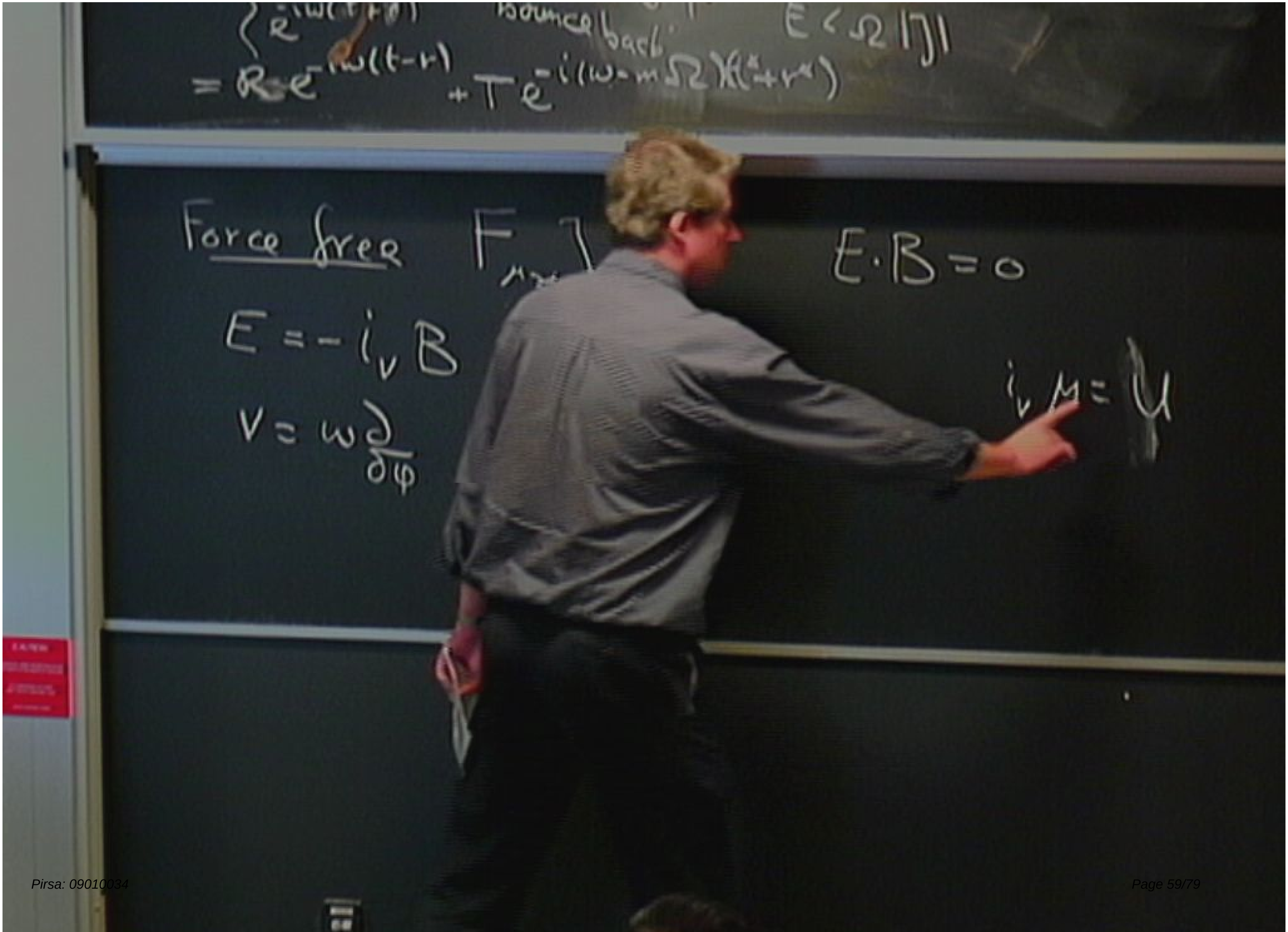
$\{ e^{-i\omega(t+r^*)}$ bounce back $E < \Omega |J|$

$$= R e^{-i\omega(t-r^*)} + T e^{-i(\omega - m\Omega)(t+r^*)}$$

Force free $F_{\mu\nu} J^\nu = 0$ $E \cdot B = 0$

$$E = - i_v B$$

$$v = \omega \frac{\partial}{\partial \varphi}$$


bounce back
E < Ω|J|
= R e^{-iω(t-r)} + T e^{-i(ω-mΩ)(t*+r*)}
Force free
E·B = 0
E = - i_v B
V = ω ∂/∂φ
i_v M = ψ

$\left\{ e^{-i\omega(t+r)} \right.$ bounce back $\quad E < \Omega |J|$

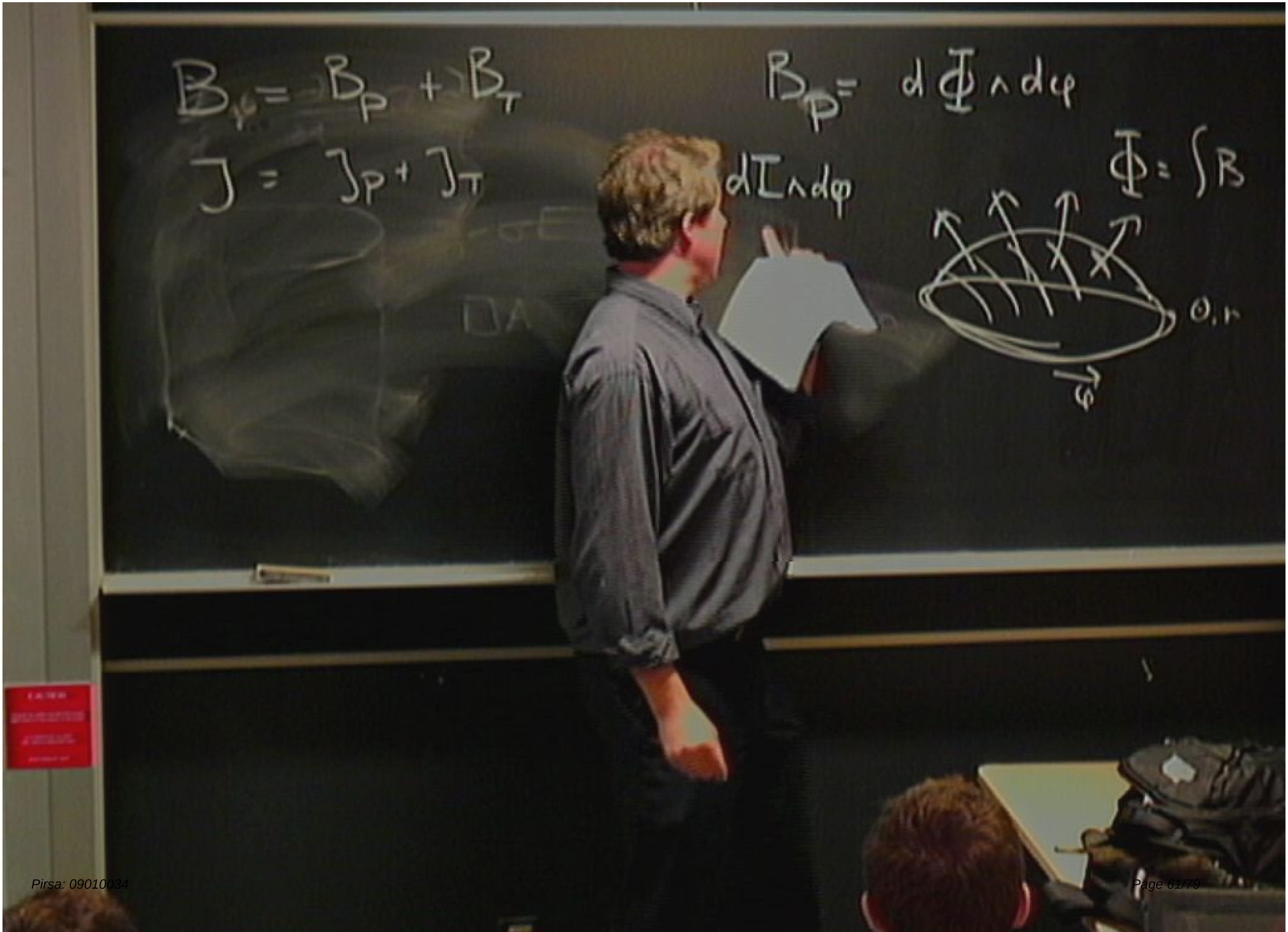
$= R e^{-i\omega(t-r)} + T e^{-i(\omega - m\Omega)(t^{*} + r^{*})}$

Force free $\quad F_{\mu\nu} J_{\nu} = 0 \qquad E \cdot B = 0$

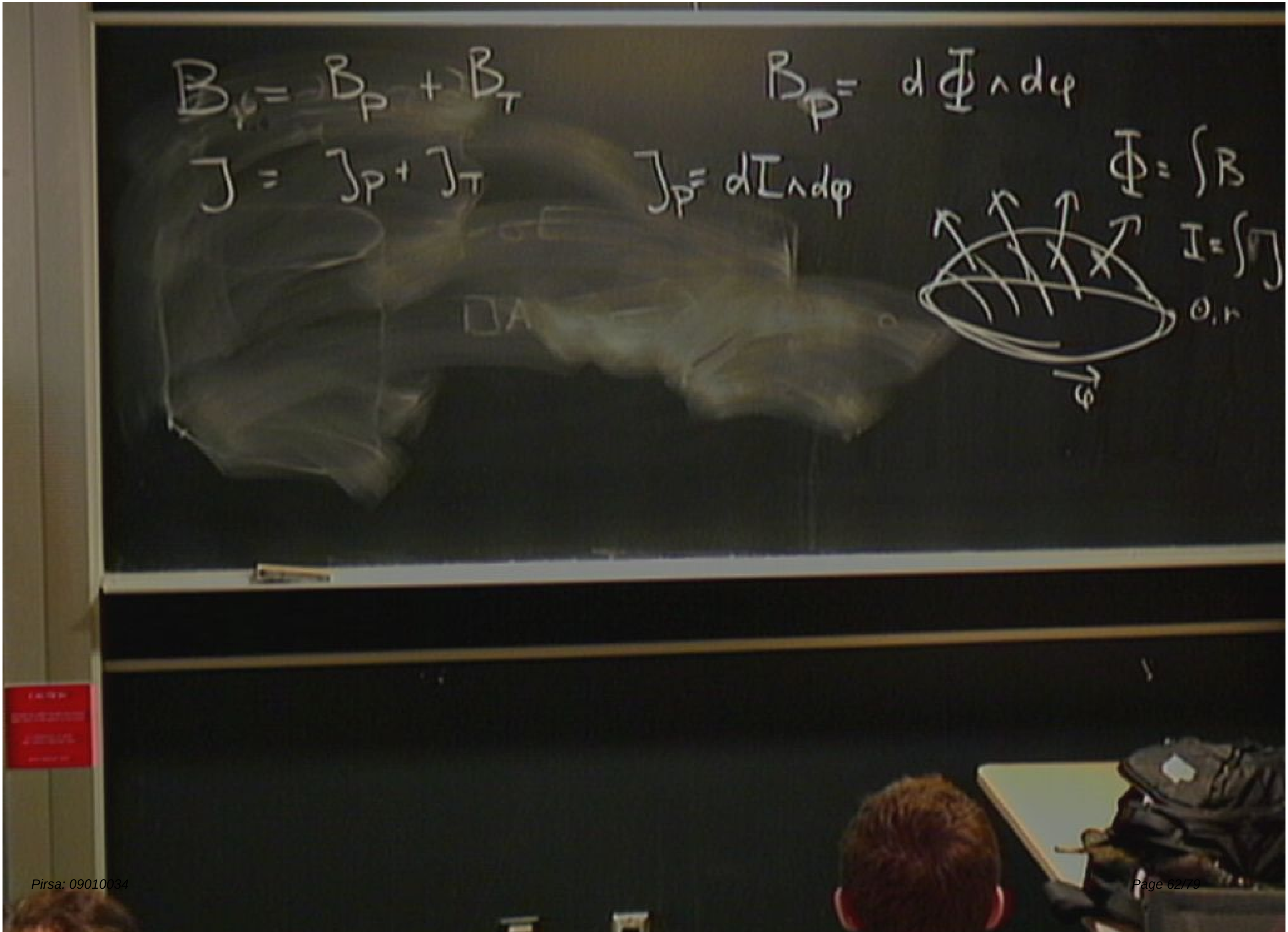
$E = - i_{v} B$

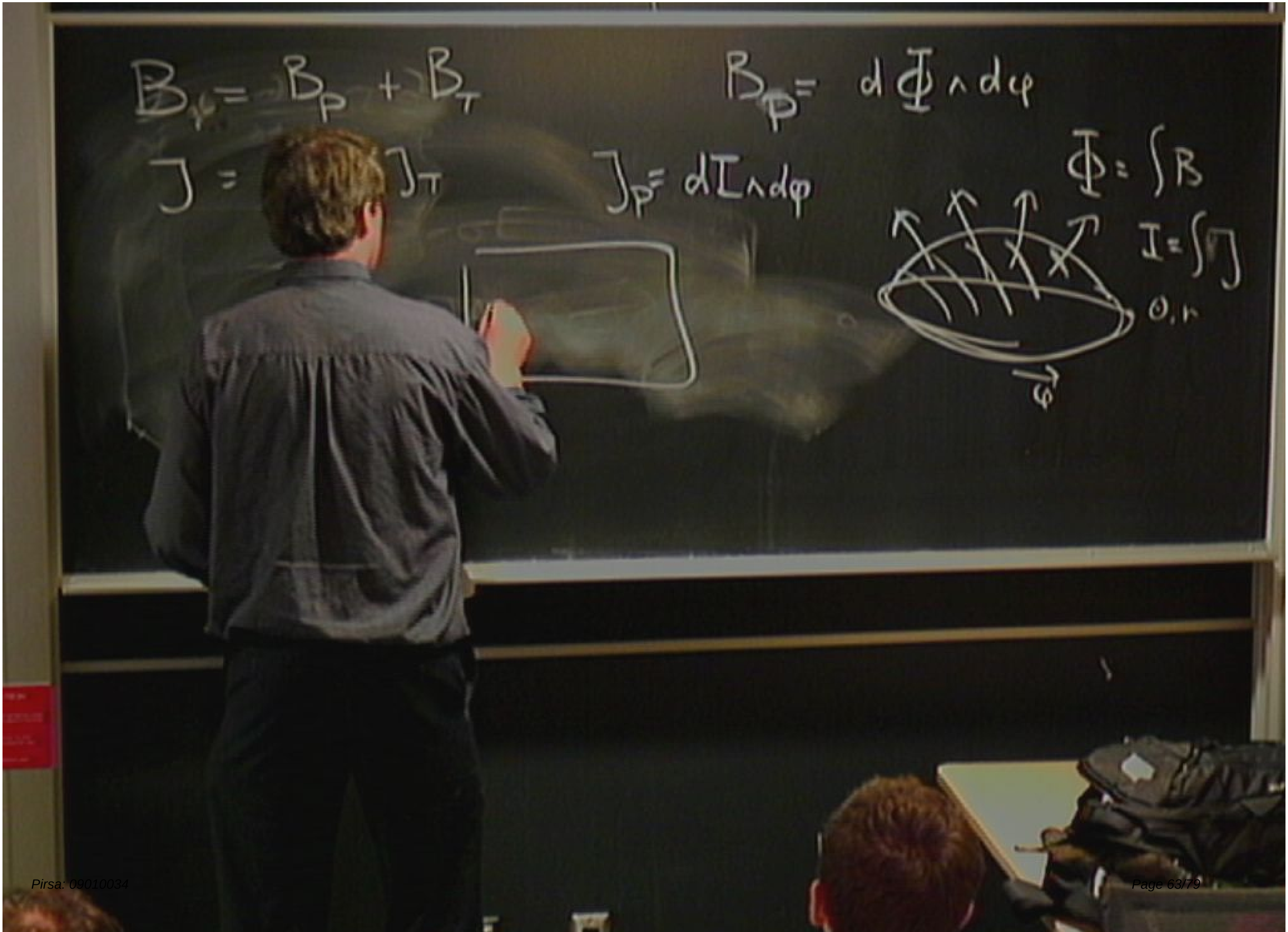
$J - \rho U = f B \qquad i_{v} \mu = U$

$v = \omega \frac{\partial}{\partial \phi}$



$B_\phi = B_P + B_T$
$J = J_P + J_T$
$B_P = d\Phi \wedge d\varphi$
$d I \wedge d\phi$
$\Phi = \int B$


$B = B_P + B_T$
$B_P = d\Phi \wedge d\varphi$
$J = J_P + J_T$
$J_P = dI \wedge d\varphi$
$\Phi = \int B$
$I = \int J$
$0, r$
$\vec{\varphi}$



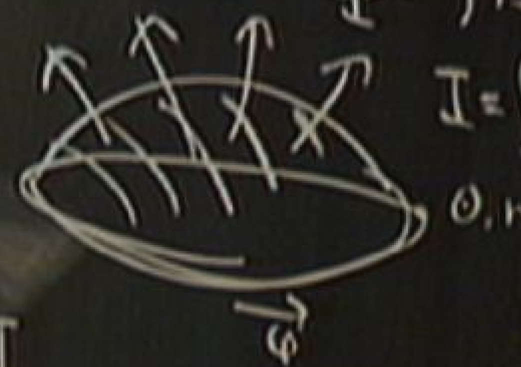
$B_\psi = B_P + B_T$
$B_P = d\Phi \wedge d\varphi$
$J =$
J_T
$J_P = dI \wedge d\varphi$
$\Phi = \int B$
$I = \int J$
$0, r$
$\vec{\varphi}$

$$B_{tot} = B_P + B_T \qquad B_P = d\Phi \wedge d\varphi$$

$$J = J_P + J_T \qquad J_P = dI \wedge d\varphi$$

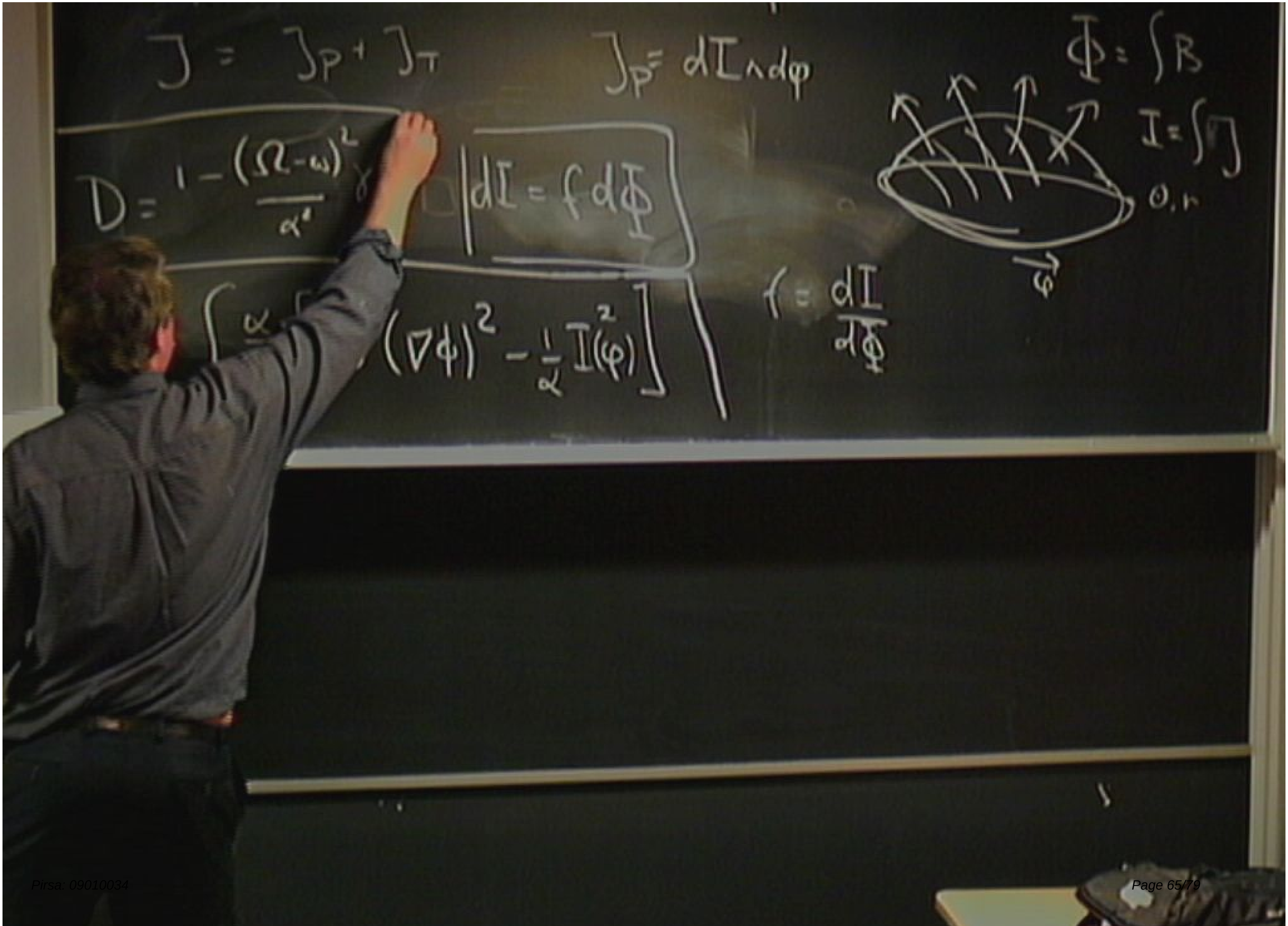
$$\Phi = \int B$$

$$I = \int J$$



$$\boxed{dI = f\, d\Phi}$$

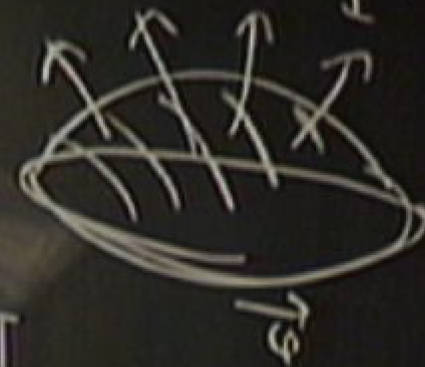
$$f = \frac{dI}{d\Phi}$$


J = J_P + J_T
J_P = dI ∧ dφ
Φ = ∫B
I = ∫J
dI = f dΦ
f = dI/dΦ

$$J = J_P + J_T \qquad J_P = dI \wedge d\varphi$$

$$\Phi = \int B$$

$$I = \int J$$



$$\boxed{D = \frac{1 - (\Omega - \omega)^2 \gamma^2}{\alpha^2}} \qquad \boxed{dI = f\, d\Phi}$$

$$S = \int \frac{\alpha}{\gamma^2}\left[D\,(\nabla\phi)^2 - \frac{1}{\alpha} I^2(\varphi) \right]$$

$$f = \frac{dI}{d\Phi}$$

$$S = \int \frac{\alpha}{\gamma^2}\left[D\,(\nabla\phi)^2 - \frac{1}{\alpha}\,I^2(\phi)\right]$$

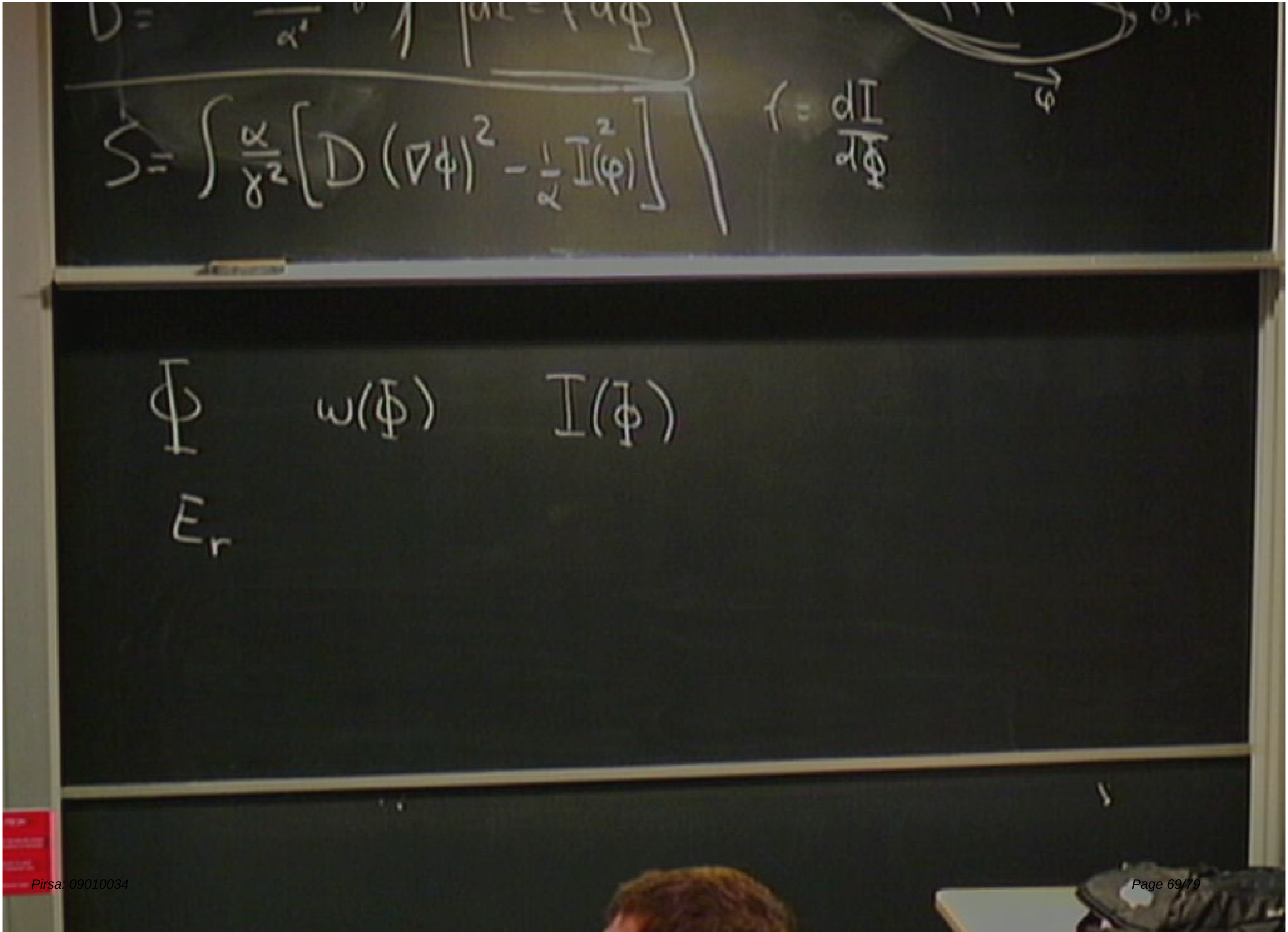
$$f = \frac{dI}{d\Phi}$$

Φ $\omega(\Phi)$

$$S = \int \frac{\alpha}{\gamma^2}\left[D\,(\nabla\phi)^2 - \frac{1}{\alpha}\,I^2(\phi)\right]$$

$$f = \frac{dI}{d\Phi}$$

$$\Phi \qquad \omega(\Phi) \qquad I(\Phi)$$


$S = \int \frac{\alpha}{\gamma^2}\left[D(\nabla\phi)^2 - \frac{1}{2}I^2(\phi)\right]$
$f = \frac{dI}{d\Phi}$
Φ $\omega(\Phi)$ $I(\Phi)$
E_r

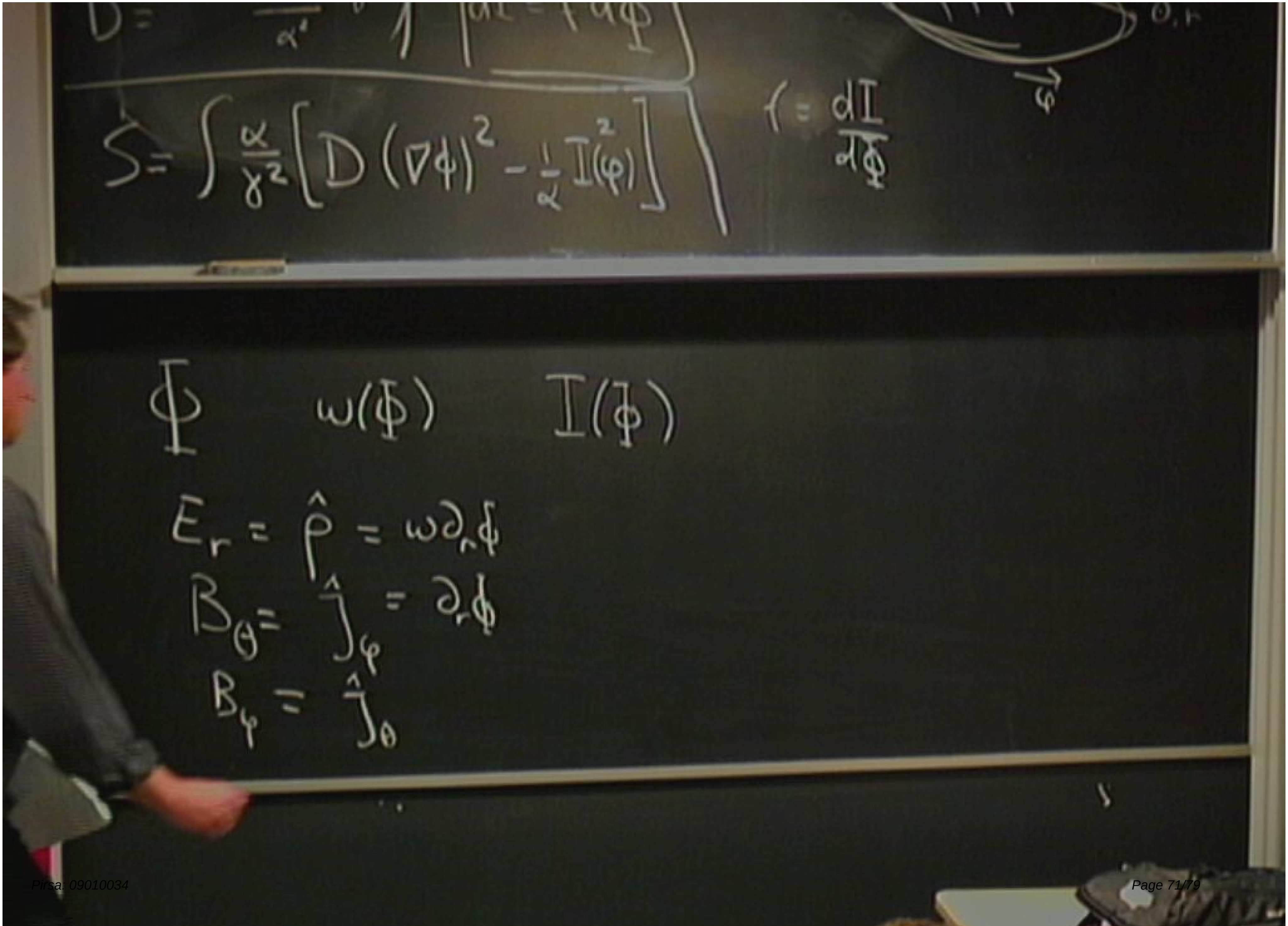
$$S = \int \frac{\alpha}{\gamma^2}\left[D(\nabla\phi)^2 - \frac{1}{\alpha} I^2(\phi)\right]$$

$$f = \frac{dI}{d\Phi}$$

Φ $\omega(\Phi)$ $I(\Phi)$

$E_r = \hat{p}$

$B_\theta = J$


$S = \int \frac{\alpha}{\gamma^2}\left[D(\nabla\phi)^2 - \frac{1}{\alpha} I^2(\phi)\right]$
$f = \frac{dI}{d\Phi}$
$\Phi \quad \omega(\Phi) \quad I(\Phi)$
$E_r = \hat{\rho} = \omega \partial_r \phi$
$B_\theta = \hat{j}_\varphi = \partial_r \phi$
$B_\varphi = \hat{j}_\theta$

$$S = \int \frac{\alpha}{\gamma^2}\left[D\,(\nabla\phi)^2 - \frac{1}{\alpha}\,I^2(\phi)\right] \qquad f = \frac{dI}{d\Phi}$$

$\vec{\varphi}$ θ, r

$$\Phi \qquad \omega(\Phi) \qquad I(\Phi)$$

$$E_r = \hat{\rho} = \omega\,\partial_r\Phi$$

$$B_\theta = \hat{j}_\varphi = \partial_r\Phi$$

$$B_\varphi = \hat{j}_\theta = I$$

external B_r

$$E_\theta = \omega B_r$$

$$E_\varphi = 0$$

$$S = \int \frac{\alpha}{\gamma^2}\left[D\,(\nabla\phi)^2 - \frac{1}{\alpha} I^2(\phi)\right]$$

$$f = \frac{dI}{d\Phi}$$

Φ $\omega(\Phi)$ $I(\Phi)$

$$E_r = \hat{\rho} = \omega\,\partial_r\phi$$
$$B_\theta = \hat{j}_\varphi = \partial_r\phi$$
$$B_\varphi = \hat{j}_\theta = I$$

external B_r

$$E_\theta = \omega B_r$$
$$E_\varphi = 0$$

$$S = \int \frac{\alpha}{\gamma^2}\left[D(\nabla\phi)^2 - \frac{1}{\alpha} I^2(\phi)\right]$$

$$f = \frac{dI}{d\Phi}$$

$\vec{\varphi}$

$\uparrow E = \omega B$

$\vec{B}$

$E \times B$

$P = E \cdot J$

$\sim \omega(\Omega - \omega)B^2$

$\downarrow \vec{B}$

$\vec{E} = (\Omega - \omega) B$

$J = \sigma(\Omega - \omega) B$

$$S = \int \frac{\alpha}{\gamma^2}\left[D(\nabla\phi)^2 - \frac{1}{\alpha}I^2(\phi)\right]$$

$$f = \frac{dI}{d\phi}$$

$\uparrow E = \omega B$

$\vec{B}$

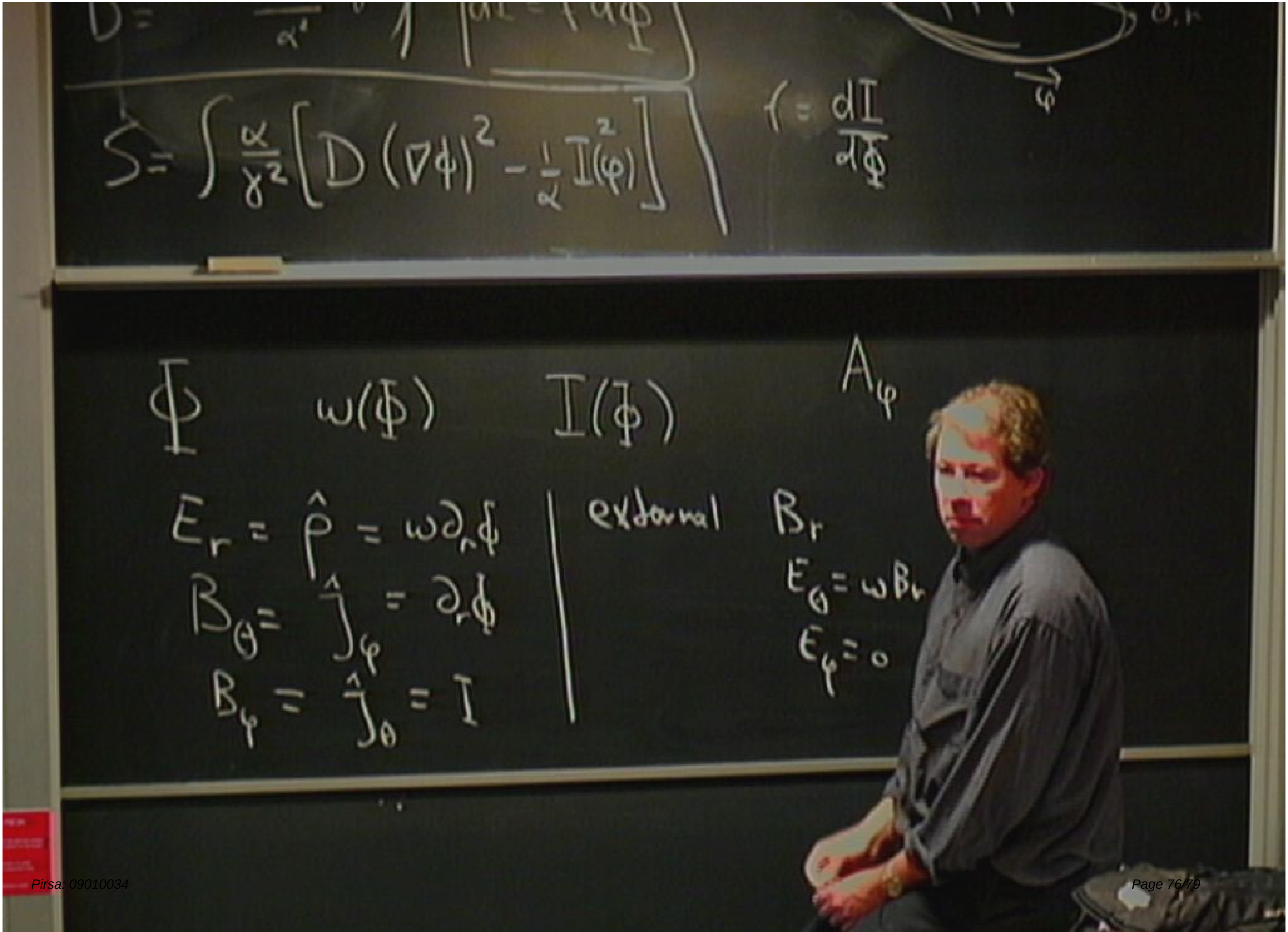
$E \times B$

$$P = E \cdot J$$
$$\sim \omega(\Omega - \omega)B^2$$

$\vec{B}$

$$\vec{E} = (\Omega - \omega)B$$

$$J = \sigma(\Omega - \omega)B$$


$S = \int \frac{\alpha}{\gamma^2}\left[D(\nabla\phi)^2 - \frac{1}{\alpha}I^2(\phi)\right]$
$f = \frac{dI}{d\Phi}$
Φ $\omega(\Phi)$ $I(\Phi)$ A_φ
$E_r = \hat{\rho} = \omega\partial_r\phi$
$B_\theta = \hat{j}_\varphi = \partial_r\phi$
$B_\varphi = \hat{j}_\theta = I$
external B_r
$E_\theta = \omega B_r$
$E_\varphi = 0$

$dI = f\, d\Phi$

$$S = \int \frac{\alpha}{\gamma^2}\left[D(\nabla\phi)^2 - \frac{1}{\alpha} I^2(\varphi)\right] \qquad f = \frac{dI}{d\Phi}$$

Φ $\qquad \omega(\Phi)$ $\qquad I(\Phi)$ $\qquad A_\varphi = \Phi$

$$\langle e^{\int A_\varphi J_\varphi} \rangle$$

$$\partial_r \Lambda_0 = \boxed{E_r = \hat{\rho}} = \omega\, \partial_r \phi$$

$$B_\theta = \hat{j}_\varphi = \partial_r \phi$$

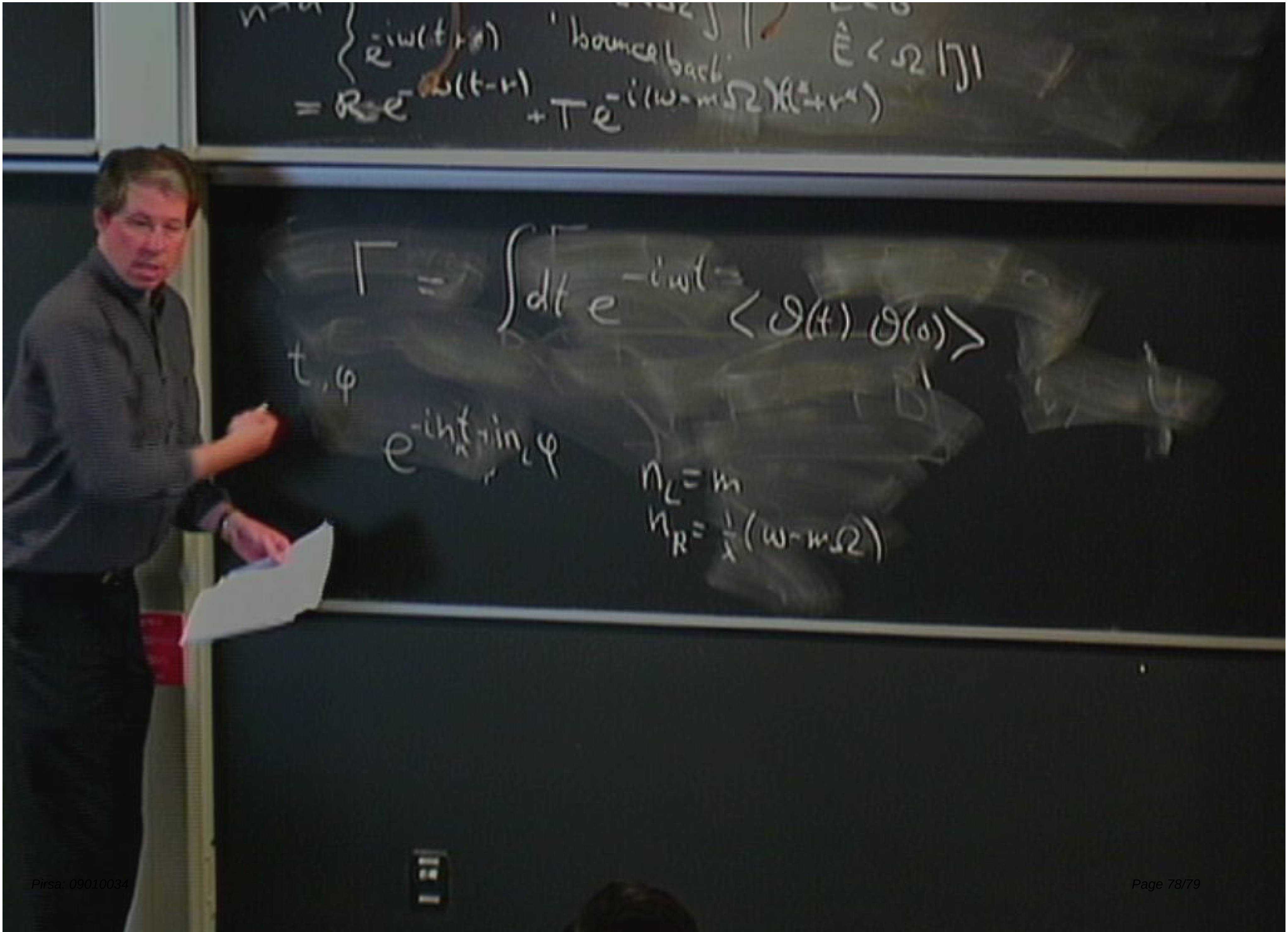
$$B_\varphi = \hat{j}_\theta = I$$

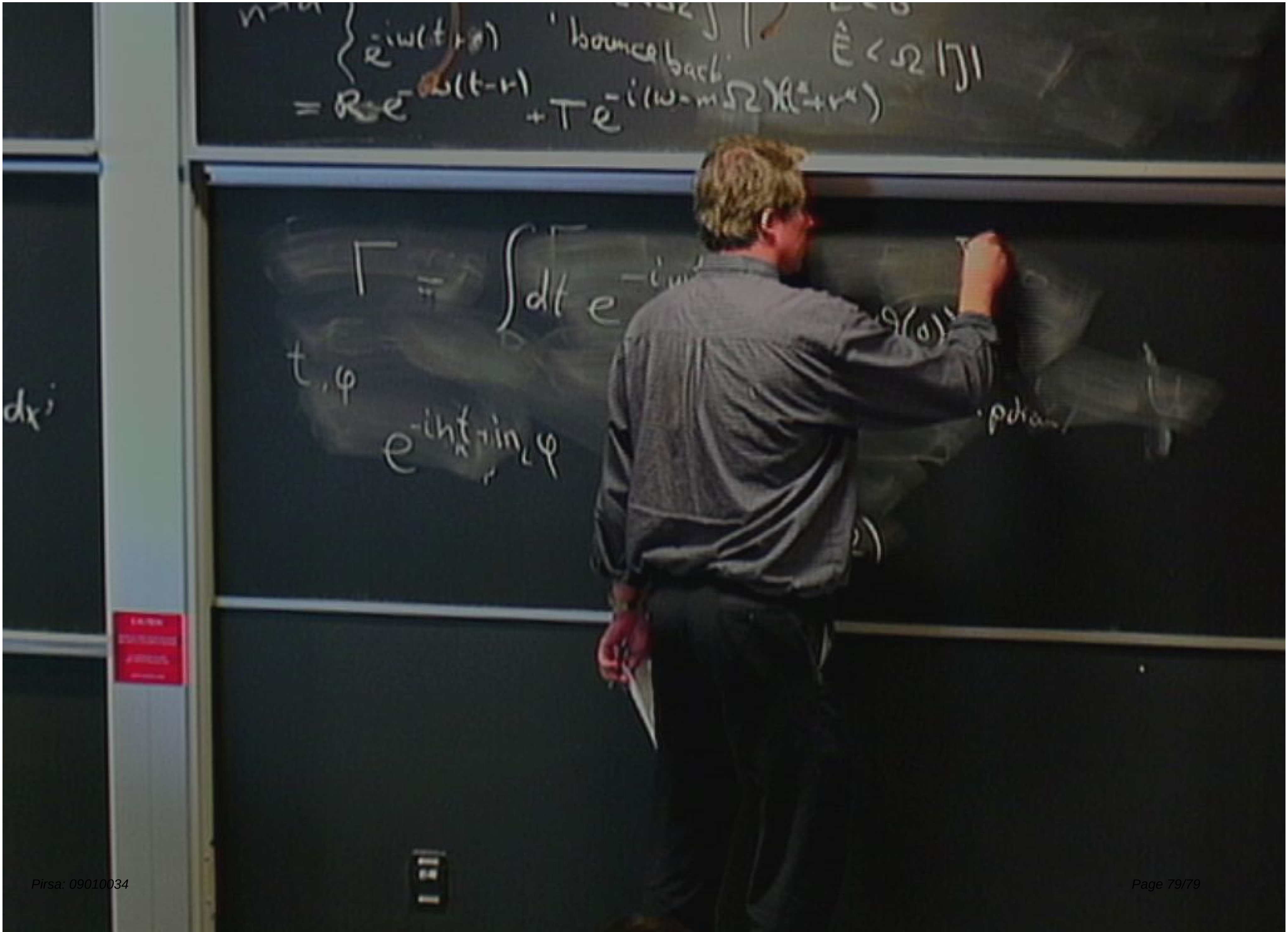
external B_r

$E_\theta = \omega B_r$

$E_\varphi = 0$

$$\partial_r A_\varphi \sim \frac{\delta}{\delta A_\varphi}$$




'bounce back'