Title: Black Holes as Fast Scramblers

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Abstract: We consider the problem of how fast a quantum system can scramble (thermalize) information, given that the interactions are between bounded clusters of degrees of freedom. Based on previous work, we conjecture: 1) The most rapid scramblers take a time logarithmic in the number of degrees of freedom. 2)Matrix quantum mechanics (systems whose degrees of freedom are n by n matrices) saturate the bound. 3) Black holes are the fastest scramblers in nature. The conjectures are based on the principle of black hole complementarily, quantum information theory, and the study of black holes in string theory. This talk is based on Y. Sekino and L. Susskind, arXiv:0808.2096 [hep-th].

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(Okayama Institute for Quantum Physics)

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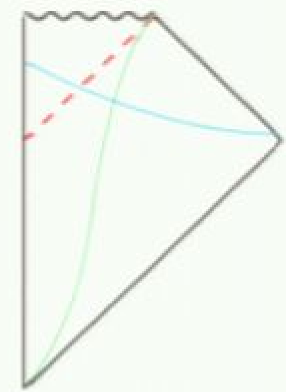
- We make conjectures on "scrambling" of information:

1. Fastest scramblers in nature take time ~log N to scramble information over the whole system, (assuming interaction involves finite clusters of d.o.f.) .
2. Black holes saturate the bound.
3. Matrix quantum mechanics saturate the bound.

- Based on
  - Black hole complementarity
  - Hayden and Preskill's work
  - Black holes in string theory

Black hole information puzzle

- Black hole created by gravitational collapse:



- If we believe in local field theory, the d.o.f. on spatially separated points are independent:

$$\mathcal{H}(\Sigma) = \mathcal{H}(\Sigma_{in}) \otimes \mathcal{H}(\Sigma_{out})$$

- Outside observer can only access d.o.f. outside BH.

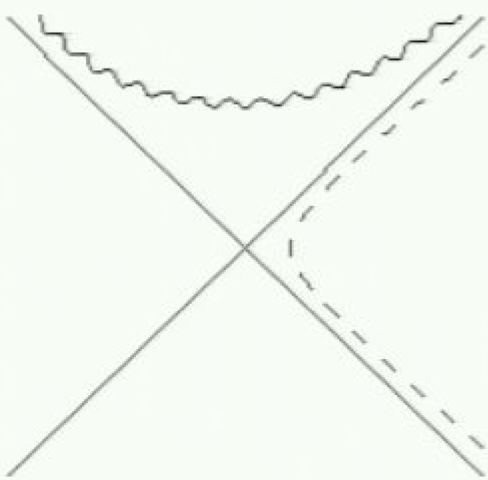
Evaporation of black hole

- Semi-classical analysis tells us that:
  - Black hole emits Hawking (thermal) radiations.
  - Eventually evaporates away.
  - Final state: $\mathcal{H}(\Sigma_{out})$ with thermal excitations?

- It appears that a pure state evolves into a mixed state (violating unitarity).

Complementarity

- Consistent quantum theory should be defined in a region that a single observer can access.
  - From outside: BH is a hot membrane on the "stretched horizon" which absorbs matter and emits Hawking radiation.
  - From inside: matter falls into a BH.



- These viewpoints are "complementary" (different descriptions of the same phenomenon).

According to complementarity,

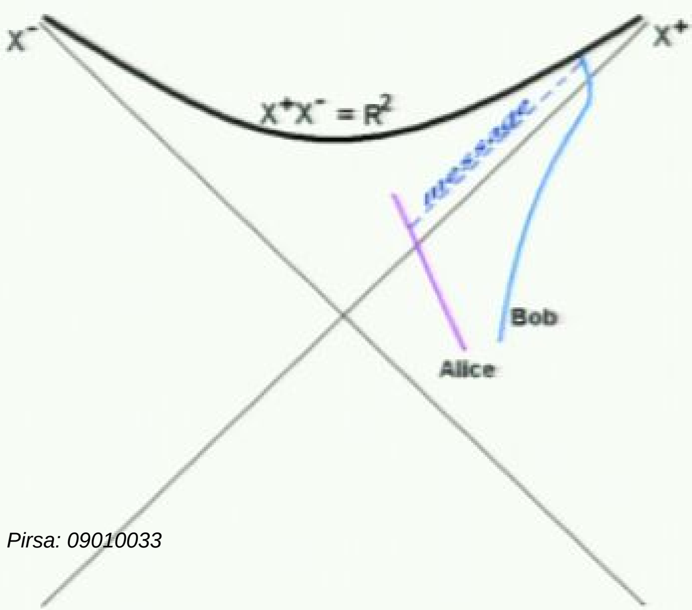
- Hawking radiations carry information on the quantum states that have fallen into the BH.
- Formation and evaporation of BH is a unitary process.

- String theory supports complementarity:
  In AdS/CFT, boundary theory is manifestly unitary.

- What are its consequences?

Potential trouble

- Do we have copies of the same quantum state (matter inside horizon and Hawking radiation) ?
- We have to make sure that a single observer cannot make a copy of a quantum state.



Thought experiment (Susskind-Thorlacius, '94; Preskill):

- Alice falls into the BH.
- Bob collects Hawking radiations, decodes Alice's info and jumps in, and receive message from Alice.

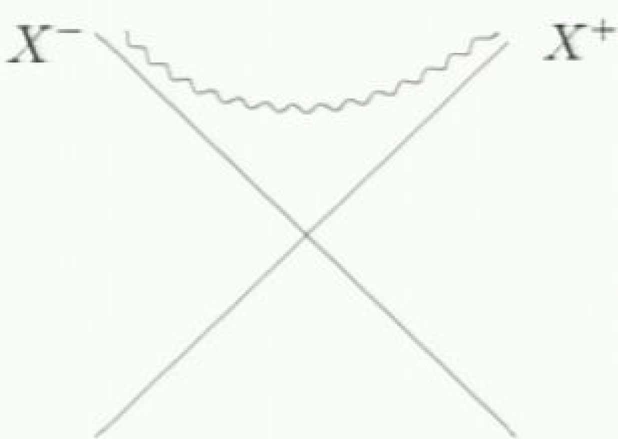
- Schwarzschild black hole

$$ds^2 = -\left(1 - \frac{2GM}{r}\right) dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2 d\Omega^2$$

- Horizon radius: R=2GM.

Entropy: $S = \frac{A}{4G} = \frac{\pi R^2}{G}$ Temperature: $T = \frac{1}{4\pi R}$

- Geometry near the horizon: Rindler space



$$\begin{aligned} ds^2 &= -\rho^2 d\omega^2 + d\rho^2 \\ &= -dX^+ dX^- \end{aligned}$$

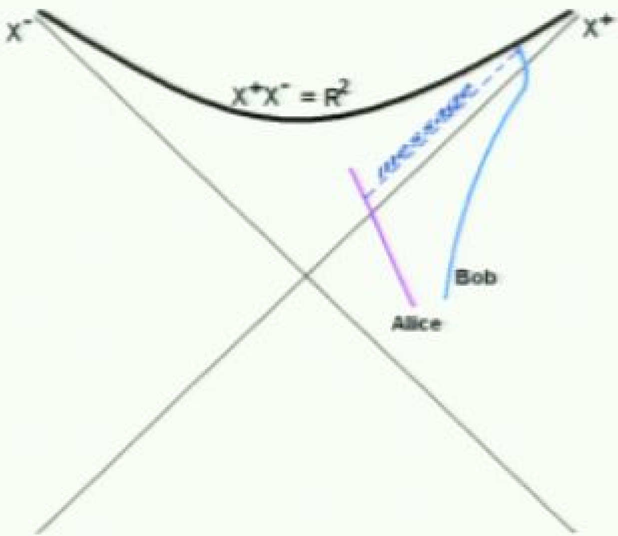
$$(X^\pm = \pm \rho e^{\pm\omega})$$

Rindler time: (time in unit of 1/T)

$$\omega = (2\pi T)t$$

Singularity at $X^+X^- = R^2$

- Bob stays at $\rho = R$
- After time $\omega_{\rm ret}$ he jumps into the BH. $(X^+ = R\exp(\omega_{\rm ret}))$
- He hits the horizon at $X^- < R\exp(-\omega_{\rm ret})$.
- If Alice wants that her message reaches Bob before he hits the horizon, she has to send it sooner than $\Delta X^- = \Delta\tau = R\exp(-\omega_{\rm ret})$ after crossing the horizon.



- Assume Alice's energy is less than the BH mass. From the uncertainty principle, she cannot send the message sooner than $\Delta\tau = \dfrac{1}{M} = \dfrac{2G}{R}$

- To prevent cloning, we need $\omega_{\rm ret} > \log R$

Estimate for the "retrieval time"

(How many Hawking photons should Bob collect in order to decode Alice's info?)

- Basic fact (Page, '93):
  Suppose a system (in a pure state) consists of subsystems A and B. How much info does A have?

Wave function: $\psi(\alpha, \beta)$ $\quad \alpha \in \mathcal{H}(A), \quad \beta \in \mathcal{H}(B)$

Density matrix on A: $(\rho_A)_{\alpha\alpha'} = \sum_\beta \psi(\alpha, \beta)^* \psi(\alpha', \beta)$

Entanglement entropy: $S_A = -\mathrm{Tr}(\rho_A \log \rho_A)$

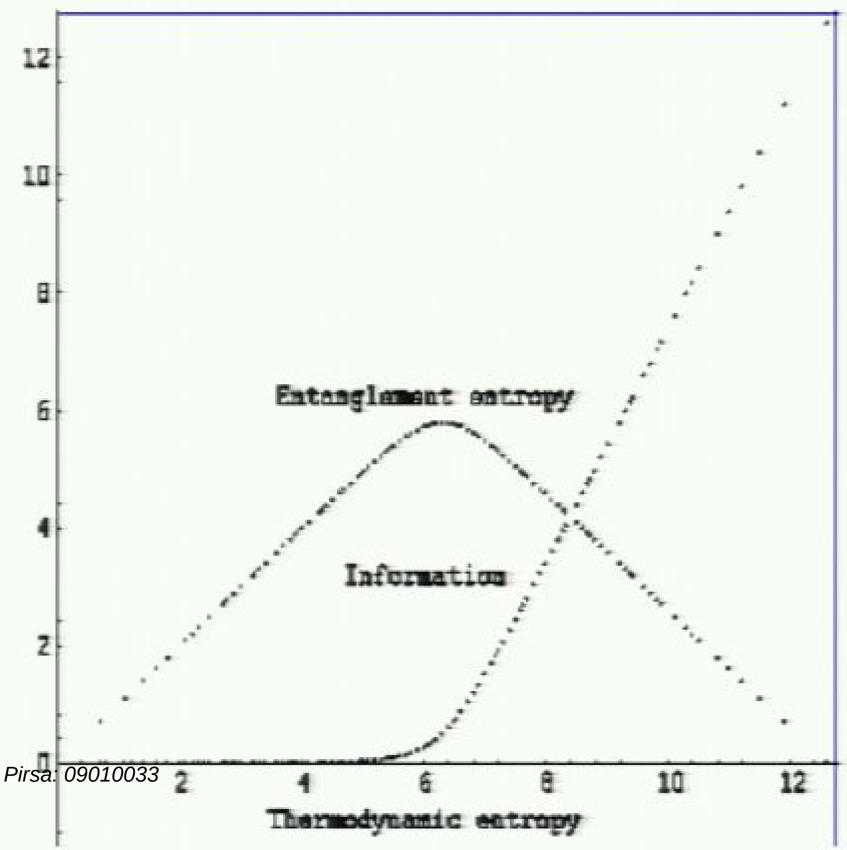
Information in a subsystem:

$$I_A = S_{\max} - S_A, \quad (S_{\max} = \dim(\mathcal{H}_A))$$

- Average over possible pure states (unit vectors in 2^N dimensional Hilbert space) with Haar measure on $U(2^N)$
- Result: When subsystem A is smaller than half the whole system, there is almost no info:

$$S_A = m - O(e^{2m-N})$$

$$2^m = \dim(\mathcal{H}_A), \quad 2^N = \dim(\mathcal{H}_{\text{total}})$$



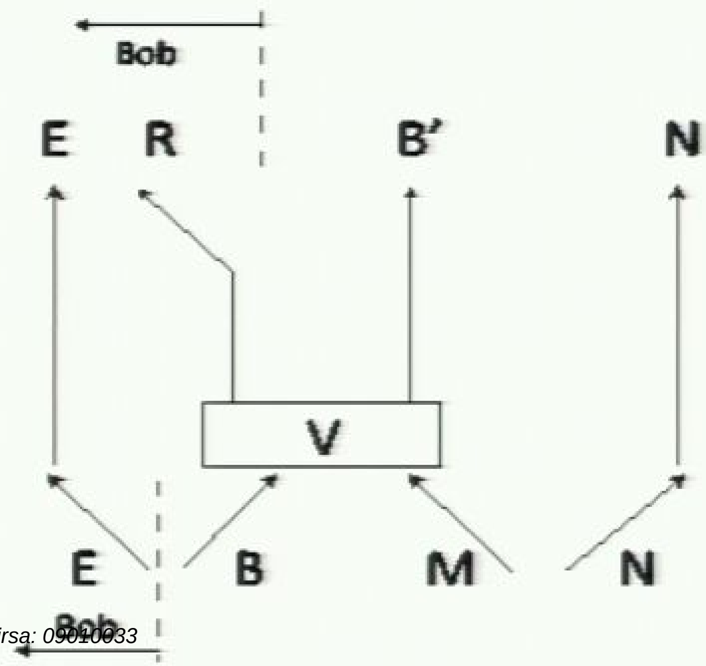
- The "halfway point" of evaporation:

$$-\frac{dM}{dt} \sim T^2 \sim \frac{1}{M^2} \quad \Rightarrow t_{\text{evap}} \sim M^3$$

This is late enough to prevent cloning.

Smarter way to recover information (Hayden-Preskill, '07)

- Suppose Bob has been collecting all the Hawking photons (since the BH has formed). Then Alice jumps into BH. How many additional bits should Bob collect to decode Alice's message?



E: previously emitted Hawking rad.

B: black hole (Assume dim(E)>dim(B))

M: Alice's message (k bits)

N: reference system (maximally entangled with M)

<u>V: random unitary transformation on BM</u>

R: additionally emitted Hawking rad.

B': black hole after emitting R

- For typical unitary V, subsystems B' and N are almost decoupled when slightly more than k bits are emitted.

$$\int dV \| \rho^{NB'}(V) - \rho(V) \otimes \rho^{B'}_{\max} \|_1^2 \leq 2^{-2(s-k)}$$

(s: bits in Hawking rad.)

- This means the system RE is almost maximally correlated with N after k bits of Hawking radiations are emitted. (i.e. Bob obtains Alice's info almost immediately.)

- In the above argument, V was assumed to be completely random.
- The time scale relevant for the info retrieval will be the time needed for this to be established (i.e. M to be mixed ("scrambled") with B).

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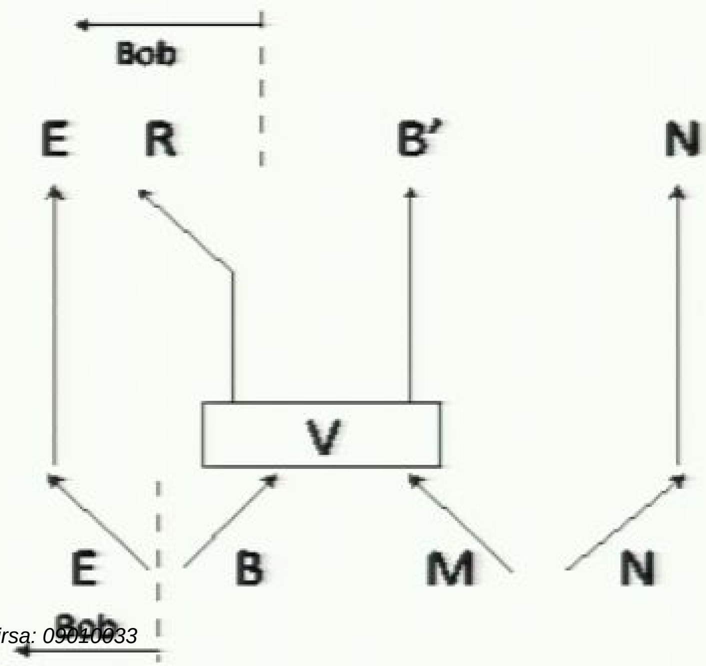

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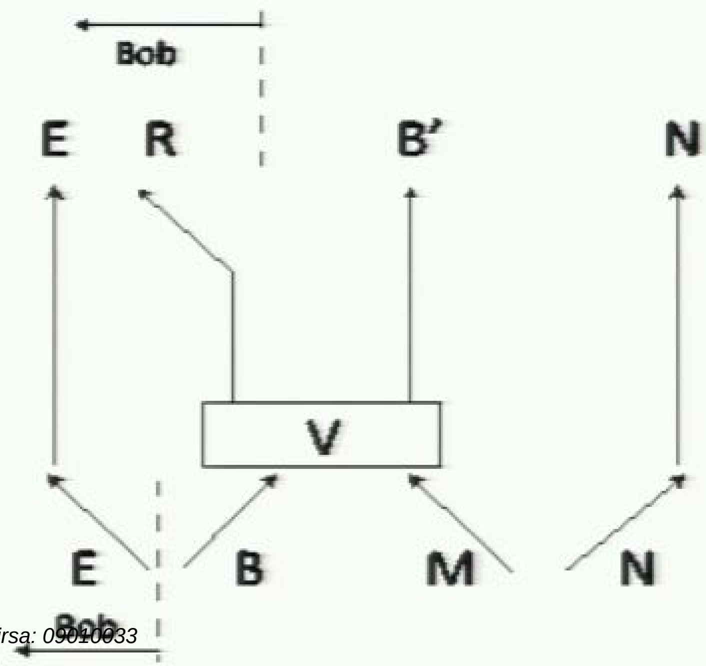

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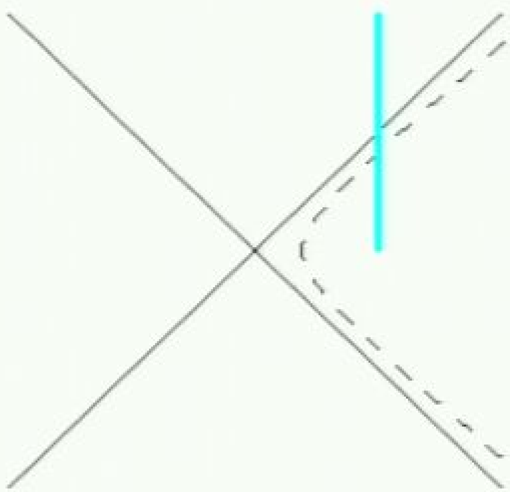
Estimate for the scrambling time

- Consider a charged particle falling into a BH.



Rindler space (near horizon):

$$\begin{aligned} ds^2 &= -\rho^2 d\omega^2 + d\rho^2 + dx_i^2 \\ &= -dt^2 + dz^2 + dx_i^2 \end{aligned}$$

$$(z = \rho \cosh \omega, \quad t = \rho \sinh \omega)$$

Electric field of the point particle:

$$E_z = \frac{e(z - z_0)}{[(z - z_0)^2 + x_i^2]^{3/2}} = \frac{1}{\rho} E_\rho$$

- "Membrane paradigm": surface charge density induced on the stretched horizon $(\rho_0 = \ell_s)$

$$\sigma = \frac{1}{4\pi\rho_0} E_\rho$$

- At late (Rindler) time, surface charge density is

$$\sigma \sim \frac{e}{4\pi(\ell_s e^{\omega})^2[1+(x_i e^{-\omega}/\ell_s)^2]^{3/2}}$$

  - charge spreads exponentially $\Delta x \sim \ell_s e^{\omega}$

- Thus, time needed for the perturbation to spread over the whole horizon (scrambling time) is

$$\omega_* = \log R/\ell_s \sim \log S$$

  - Note that this is fast. Usually, diffusion takes time

$\omega_* \sim N^{2/d}$ (N: # of d.o.f.; d: spatial dimension)

Conjectures

1. Fastest scramblers in nature take time ~log N to scramble information over the whole system, (assuming interaction involves finite clusters of d.o.f.)

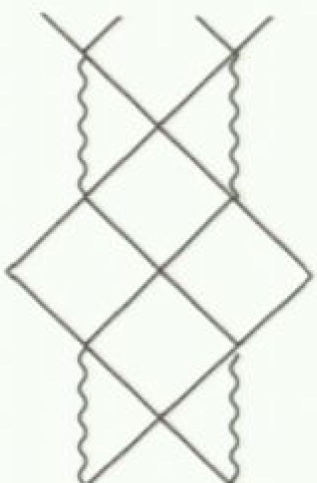
2. Black holes saturate the bound.

3. Matrix quantum mechanics (dual to the BH) saturates the bound.

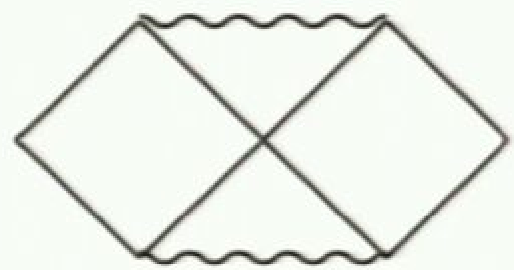
D0-brane black hole

- (9+1)-dim BH, charged under RR 1-form
- Remark on the causal structure:

Naively, there are inner and outer horizons



Including back reaction, causal structure will be similar to Schwarzschild



- Metric (in the "decoupling limit")

$$ds^2 = \alpha' \left[-\left(\frac{g_{\rm YM}^2 n}{U^7}\right)^{-1/2} \left(1 - \frac{U_0^7}{U^7}\right) dt^2 + \left(\frac{g_{\rm YM}^2 n}{U^7}\right)^{1/2} \left\{ \left(1 - \frac{U_0^7}{U^7}\right)^{-1} dU^2 + U^2 d\Omega_8^2 \right\} \right].$$

  - Charge (number of the D0-branes): n
  - Energy (mass above extremality), Entropy, temperature

$$E \sim \frac{U_0^7}{g_{\rm YM}^4}, \qquad S \sim \frac{n^2 U_0^{9/2}}{(g_{\rm YM}^2 n)^{3/2}}, \qquad T \sim \frac{U_0^{5/2}}{(g_{\rm YM}^2 n)^{1/2}},$$

  - String coupling:

$$e^{\phi} \sim \frac{1}{n} \left(\frac{g_{\rm YM}^2 n}{U^3}\right)^{7/4}$$

  - Time scale for evaporation (approach to extremality) : $t_{\rm evap} \sim n E^{2/7} / g_{YM}^{6/7}$

- Classical gravity analysis: valid when string coupling and curvature are small at the horizon.

$$1 \ll (g_{\rm YM}^2 n/U_0^3) \ll n^{4/7}$$

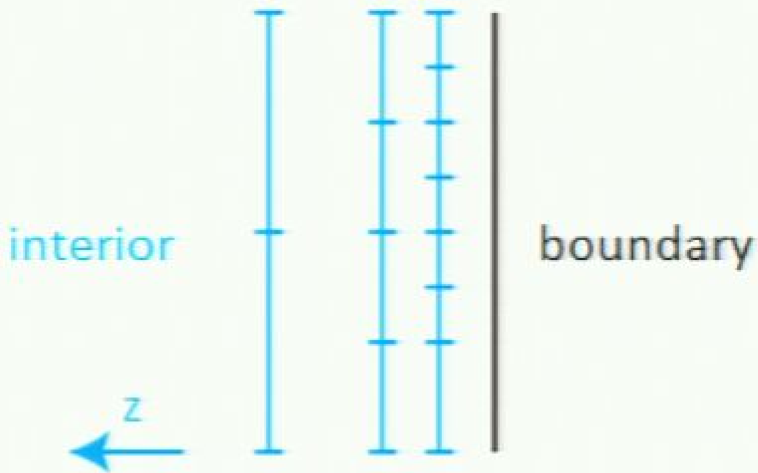
  – BH with fixed temperature, large entropy limit
  – Gauge theory in the ‘t Hooft limit

- Geometry near the horizon is Rindler.
- Scrambling time (in unit of inverse temperature):

$$\omega_* = \log(R_{\rm hor}/\ell_s) = \log(g_{\rm YM}^2 n/U_0^3)^{1/4} \sim C \log n$$

AdS/CFT

- (A "cell" of size $R_{\rm AdS}$) = (cut-off cell in boundary theory)



$$ds^2 = R_{\rm AdS}^2 \frac{dz^2 + dx^2}{z^2}$$

UV cut-off : $\Delta x = \epsilon$

$\leftrightarrow$ IR cut-off: $z_{\rm IR} = \epsilon$

$$({\rm Area}) = R_{\rm AdS}^4 = g_s n \ell_s^4, \qquad \left(S = ({\rm Area})/\ell_P^4 = n^2\right)$$

- AdS black hole (large $R_{\rm hor} > R_{\rm AdS}$, stable BH):

Time for a perturbation to spread over the size $R_{\rm AdS}$:

$$\omega_* \sim \log(R_{\rm AdS}/\ell_s) \sim C \log n$$

Matrix theory

- (0+1) D SYM: Lagrangian is (schematically)

$$L = \mathrm{Tr} \sum_a \dot{X}^a \dot{X}^a - \mathrm{Tr} \sum_{ab} [X^a, X^b]^2$$

$$X^a : n \times n \text{ matrices}, \ N = n \times n\text{: total number of d.o.f.}$$

- Localized perturbation has angular momentum

$$\ell \sim R_s \sim (g_{\mathrm{YM}}^2 n / U_0^3)^{1/4}$$

Corresponding operator: $\mathcal{O}_{\mathrm{pert}} \sim \mathrm{Tr}(\underbrace{X \cdots X}_{\ell})$

- Basic picture of scrambling: At each time step, # of element "connected" to the perturbed element grows by a factor of 4 (due to the quartic interaction). (All matrix elements are "next to each other.")

Toward testing the conjecture

- One measure of being scrambled:
  - BH: thermal ground state
  - Perturbation at t=0: $\mathcal{O}_{\mathrm{pert}}(0)$
  - After time t, if system is scrambled, we will have

$$\langle \mathcal{O}_a(t)\mathcal{O}_{\mathrm{pert}}(0)\rangle_\beta \sim \langle \mathcal{O}_a(t)\rangle_\beta$$

    for some set of $\{\mathcal{O}_a\}$
    (operators involving small subspace of matrix elements)

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