Title: Black holes as mirrors

Date: Jan 24, 2009 02:30 PM

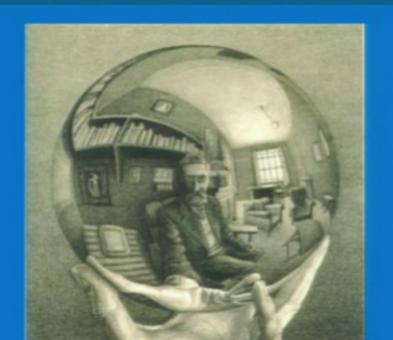
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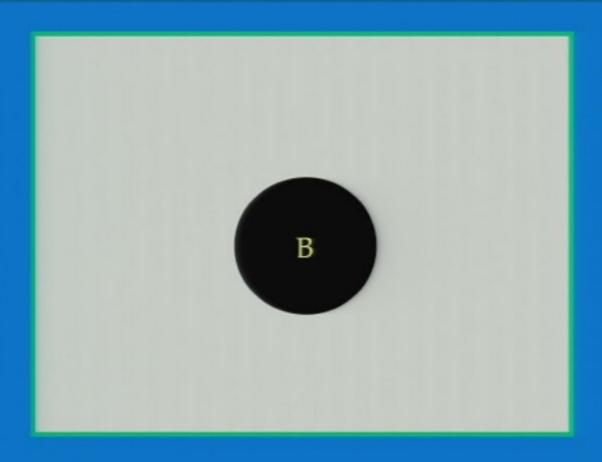
Abstract: I'll discuss information retrieval from evaporating black holes, assuming that the internal dynamics of a black hole is unitary and rapidly mixing, and assuming that the retriever has unlimited control over the emitted Hawking radiation. If the evaporation of the black hole has already proceeded past the 'half-way' point, where half of the initial entropy has been radiated away, then additional quantum information deposited in the black hole is revealed in the Hawking radiation very rapidly. Information deposited prior to the half-way point remains concealed until the half-way point, and then emerges quickly. These conclusions hold because typical local quantum circuits are efficient encoders for quantum error-correcting codes that nearly achieve the capacity of the quantum erasure channel. The resulting estimate of a black hole's information retention time, based on speculative dynamical assumptions, is just barely compatible with the black hole complementary hypothesis. (Joint work with John Preskill).

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Black Holes as Mirrors

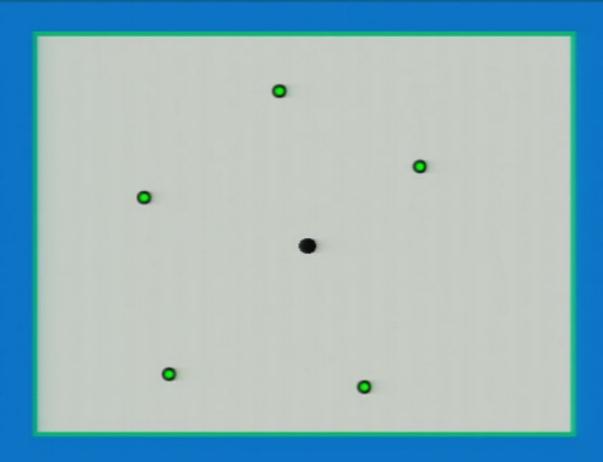
Patrick Hayden (McGill University) John Preskill (Caltech)





Pure state

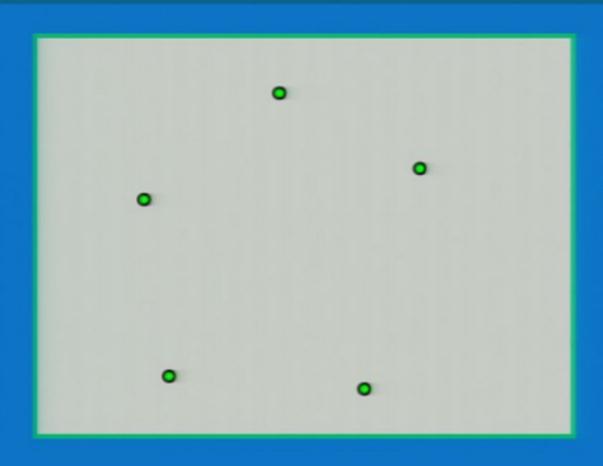
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Pure state

Thermal Hawking radiation

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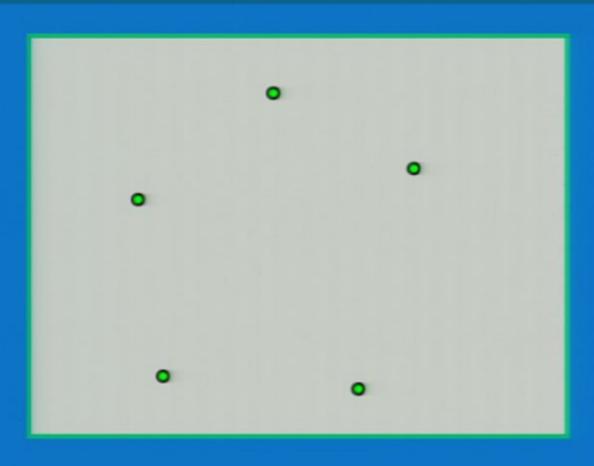


Pure state

Thermal Hawking radiation

Radiation but no black hole

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Pure state

Thermal Hawking radiation

Radiation but no black hole

Finer Question: Does information dropped into a black hole ever come out?

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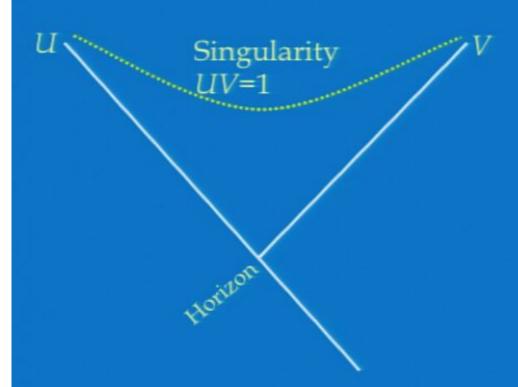
Yes: Copies of information exist "simultaneously" inside and outside the black hole.



Cloning is unverifiable by an experiment

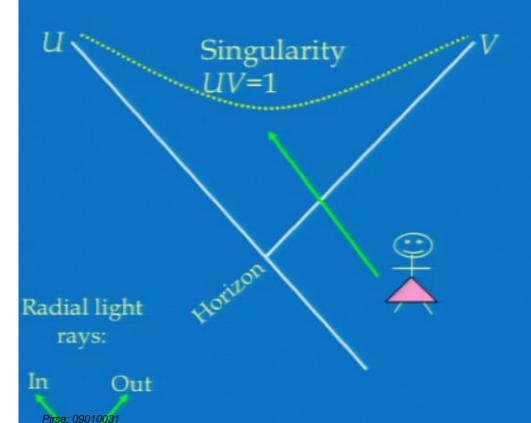
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Cloning is unverifiable by an experiment

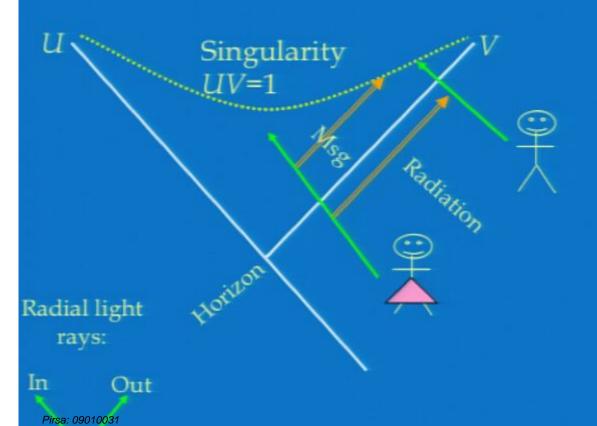


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Cloning is unverifiable by an experiment

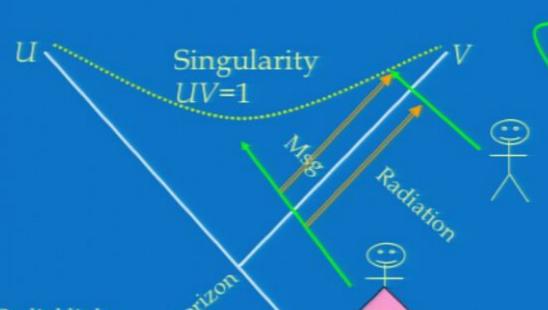


Cloning is unverifiable by an experiment



Cloning is unverifiable by an experiment

Bob's delay (Sch. time)



 Alice proper time to release msg

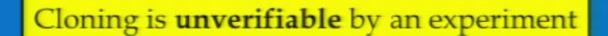
$$au_{Al} = Cr_s \exp\left(\frac{-\Delta t}{r_s}\right)$$

Schwarzschild radius

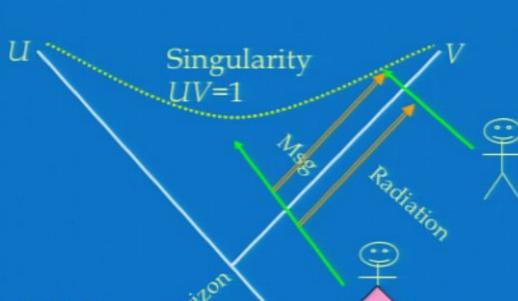
Radial light rays:

In Out

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Bob's delay (Sch. time)



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$$au_{Al} = Cr_s \exp\left(\frac{-\Delta t}{r_s}\right)$$

Schwarzschild radius

Require expt using sub-Planck energy:

Radial light rays:

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In

Out



$$t_{info} > C'r_s \log r_s$$

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Entanglement Entropy $S(\phi_{out})$

 $A_{init} - A$

Radiated bits

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t_{info} via Page ansatz

Random $|\phi\rangle \in \mathcal{H}_{in} \otimes \mathcal{H}_{out} \cong \mathbb{C}^{d^2}$

Area $A \sim \log \dim \mathcal{H}_{in}$

Entanglement Entropy

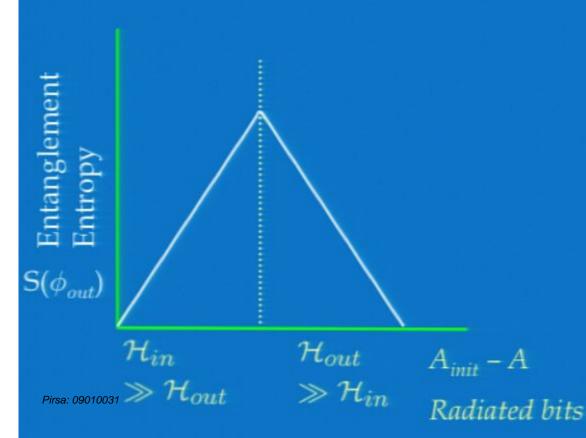
 $S(\phi_{out})$

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 $A_{init} - A$

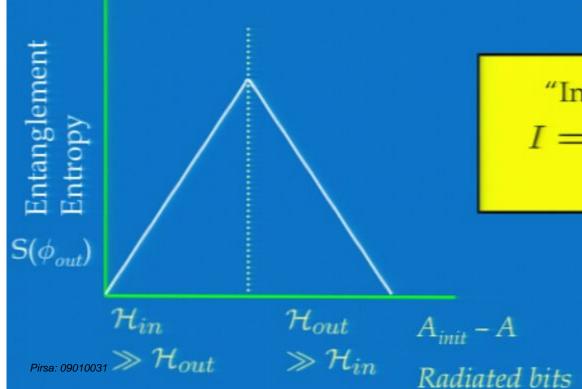
Radiated bits

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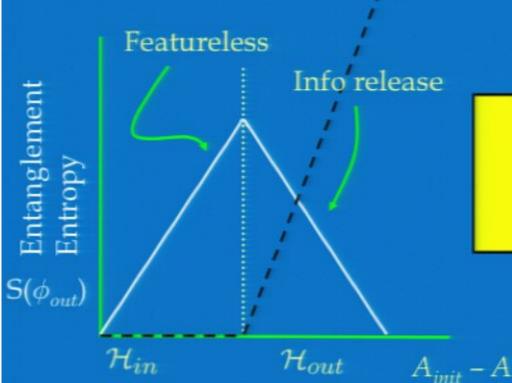
Area $A \sim \log \dim \mathcal{H}_{in}$

"Information" in the radiation:

$$I = \log \dim \mathcal{H}_{out} - S(\phi_{out})$$



Radiated bits



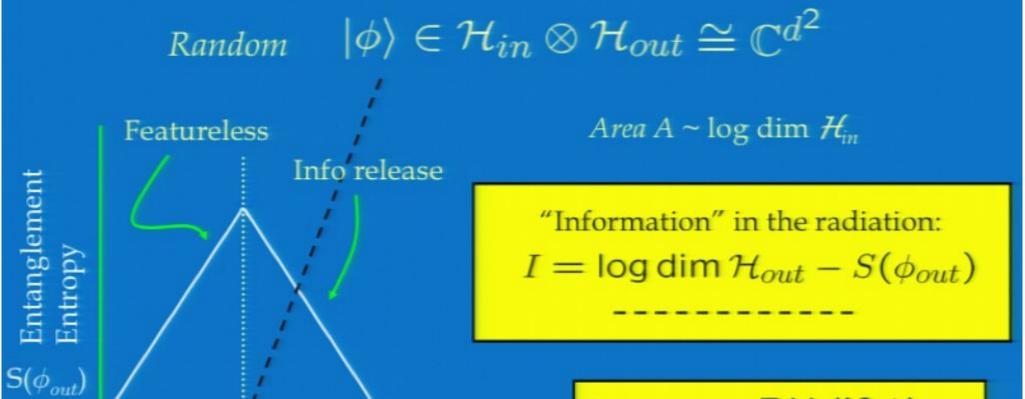
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 $\gg \mathcal{H}_{in}$

Area $A \sim \log \dim \mathcal{H}_{in}$

"Information" in the radiation:

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 $A_{init} - A$ $\gg \mathcal{H}_{in}$ Radiated bits

 \mathcal{H}_{out}

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BH lifetime Page 24/124

$$t_{info} > C'r_s \log r_s$$

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$$t_{info} > C'r_s \log r_s$$

Page estimate: $t_{info} \sim r_s^3$

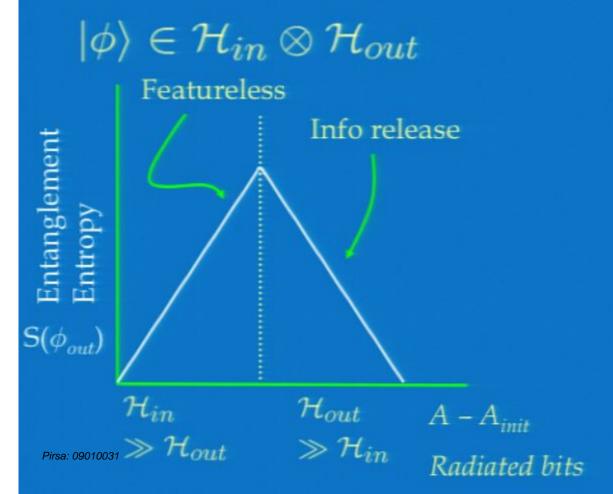
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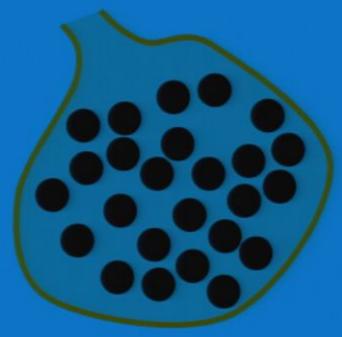
$$t_{info} > C'r_s \log r_s$$

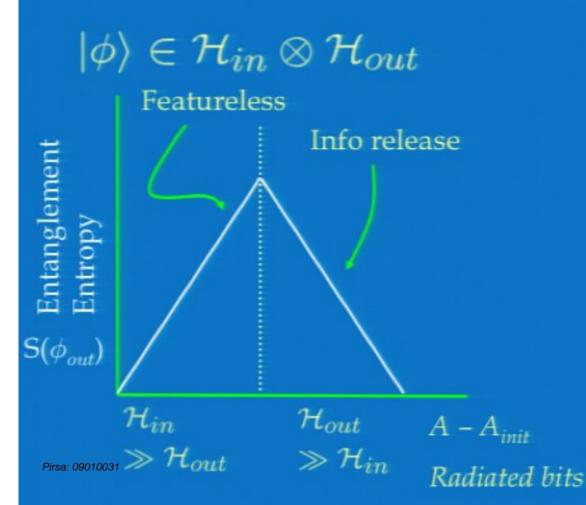
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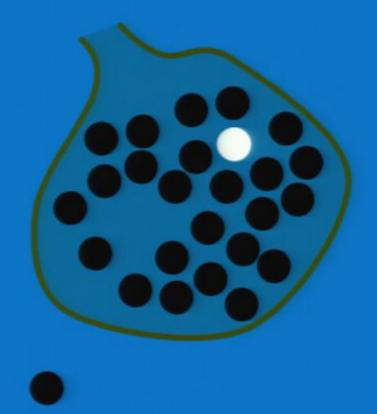
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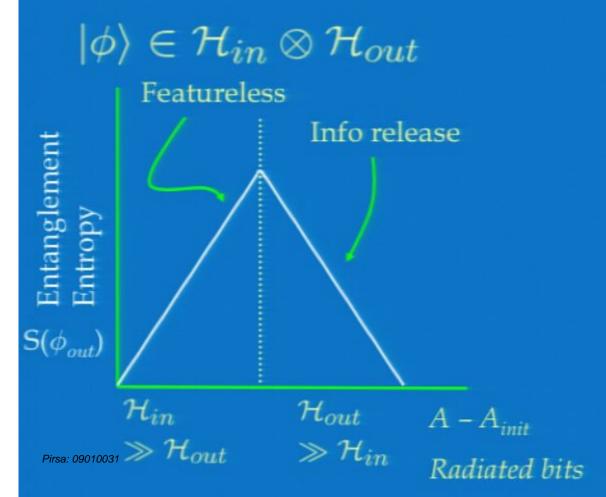
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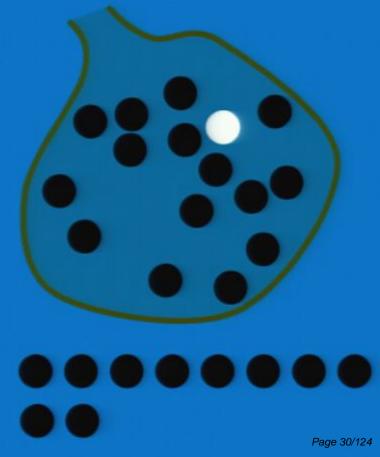


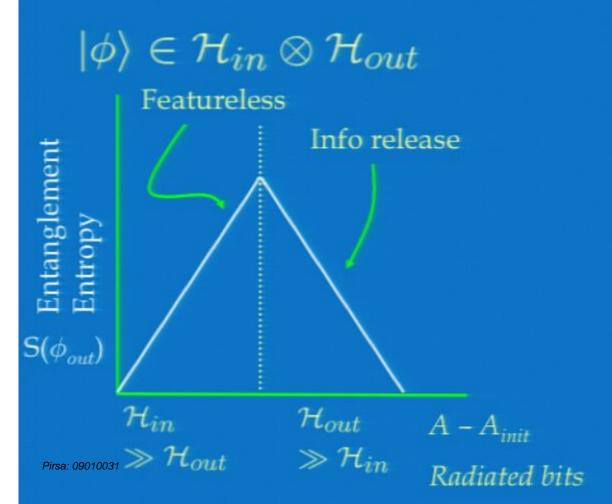


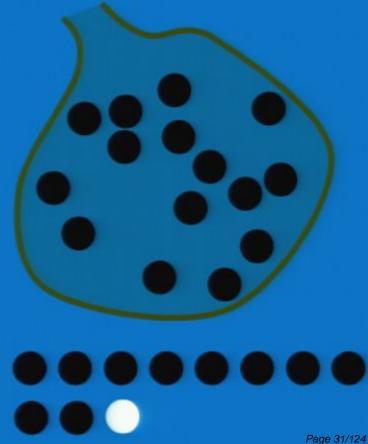






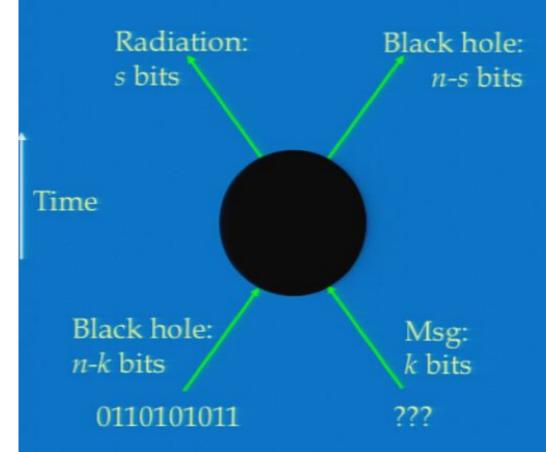




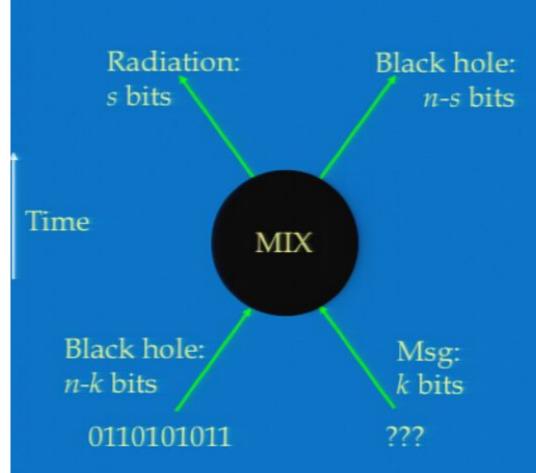




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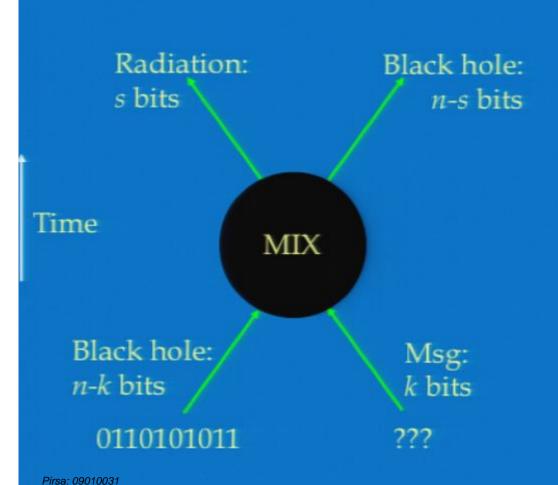


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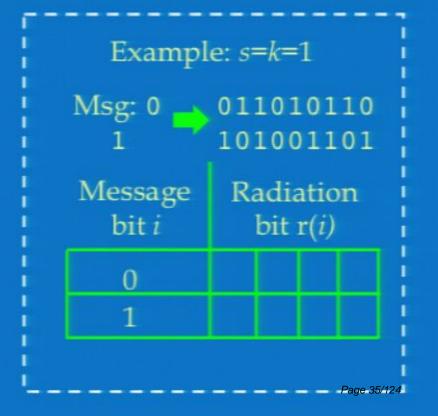


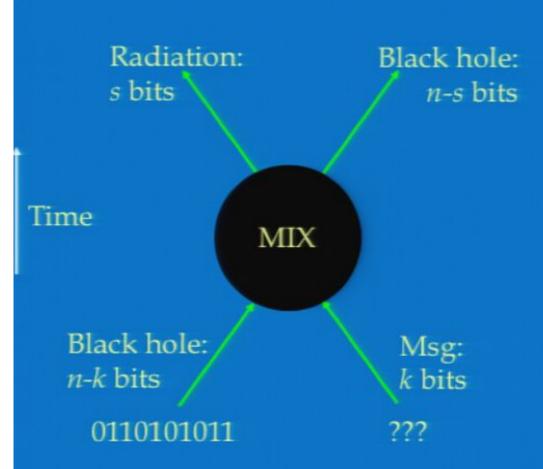
Assume MIX is a *random* permutation on 2^n bit strings.

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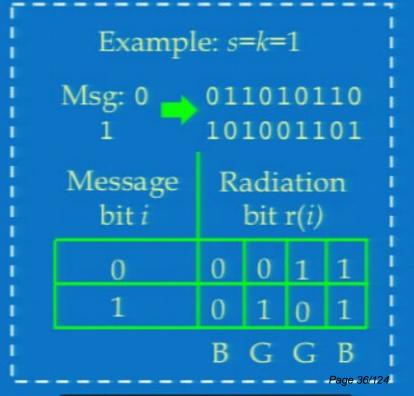


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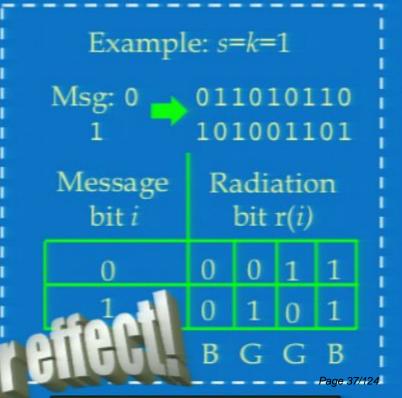
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In annual D cok-s

Classical Model



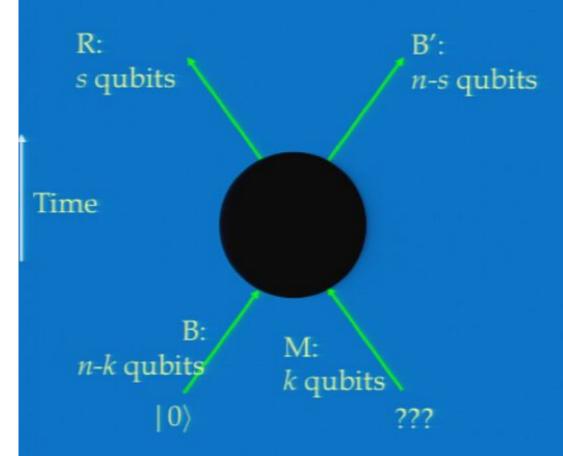
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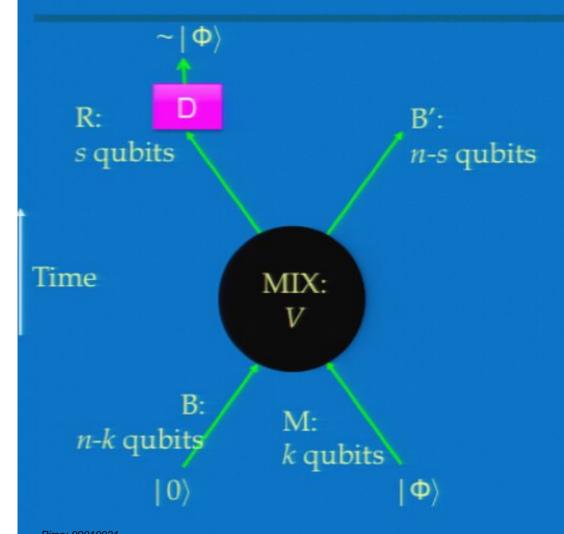


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In a new l. D. o 2k-s

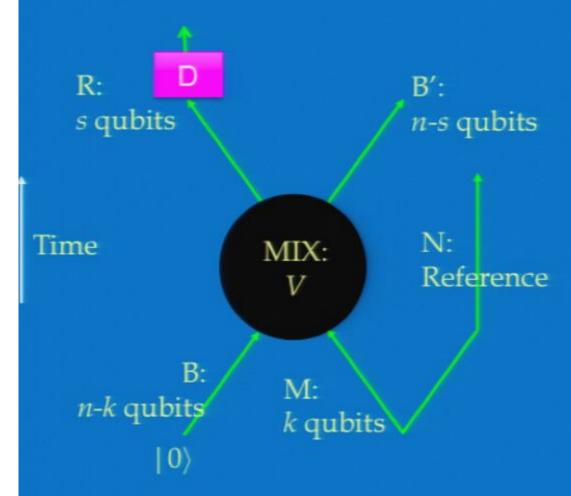
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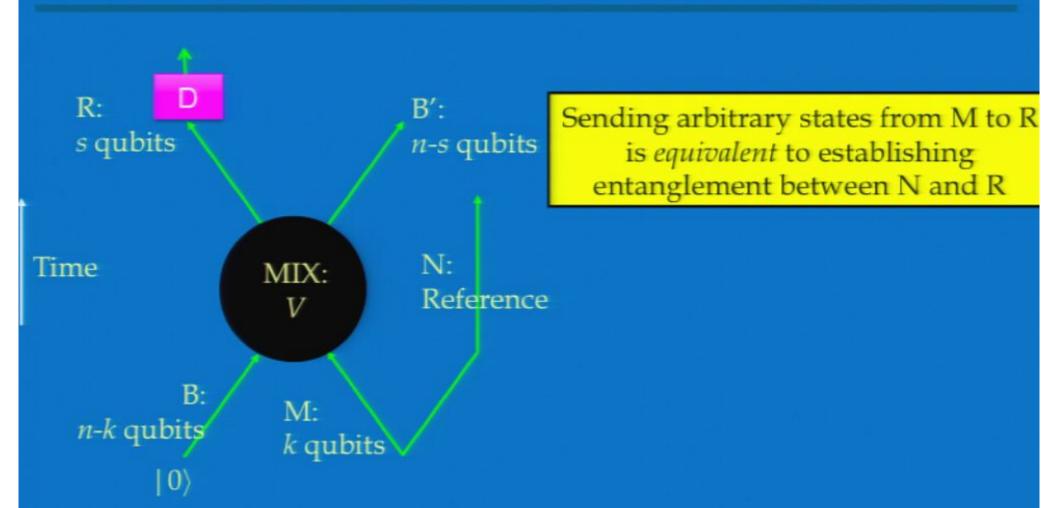


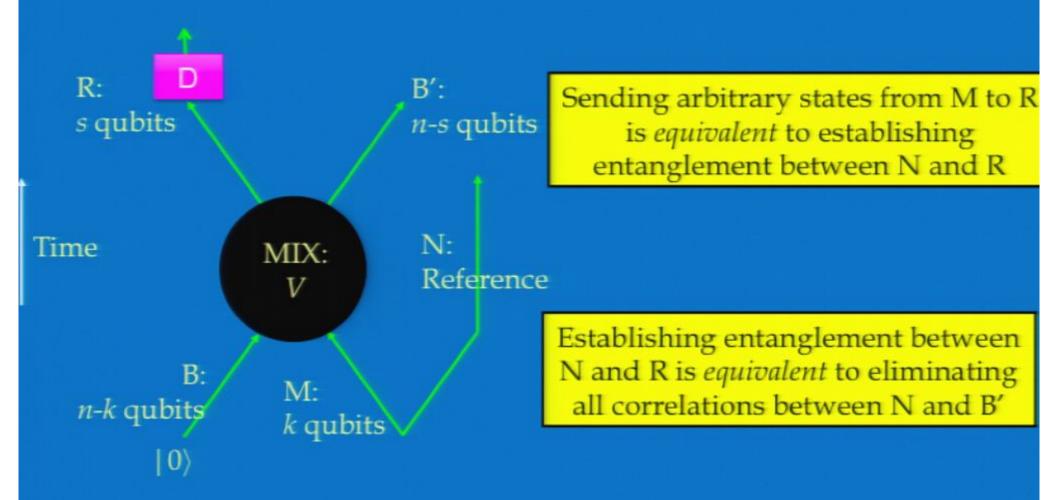
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Purification: $|\Psi\rangle_{\chi\chi}$ is a purification of ρ_{χ} if $\mathrm{tr}_{\chi} |\psi\rangle\langle\psi| = \rho_{\chi}$.



Purification: $|\Psi\rangle_{\chi \gamma}$ is a purification of ρ_{χ} if $\mathrm{tr}_{\gamma} |\psi\rangle\langle\psi| = \rho_{\chi}$.

Purifications are essentially unique.

(Up to local unitaries on the purifying space.)

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Purification: $|\Psi\rangle_{XY}$ is a purification of ρ_X if $tr_Y |\psi\rangle\langle\psi| = \rho_X$.

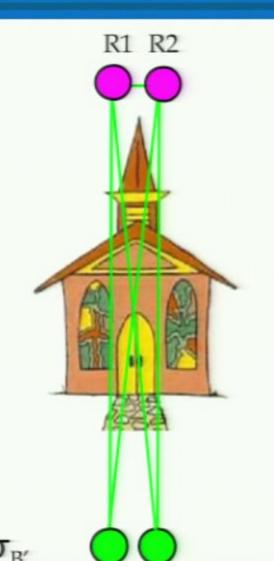


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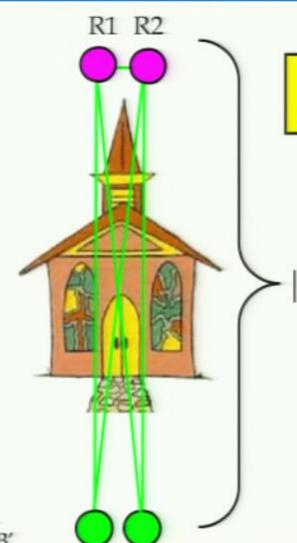
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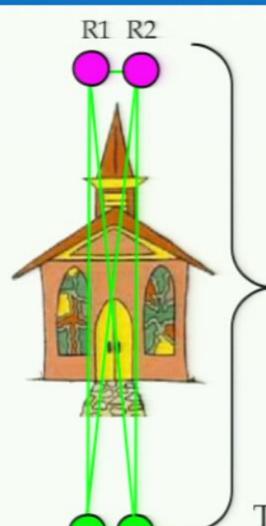
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 $|\xi_{NB'R}\rangle$

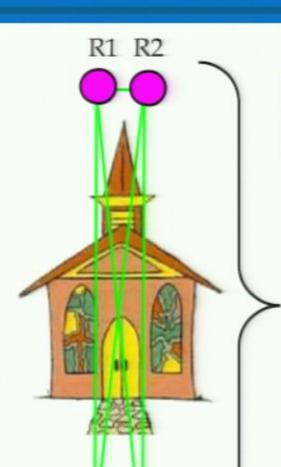


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 $|\xi_{NB'R}\rangle$

$$\operatorname{Tr}_{R} \xi_{NB'R} = \rho_{N} \otimes \sigma_{B'}$$

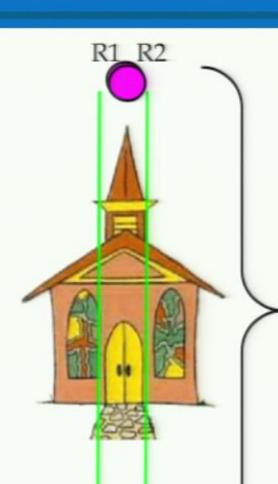


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$$|\xi_{NB'R}\rangle$$
 =(id_{NB'} \otimes U_R) $|\phi_{NR1}\rangle$ $|\psi_{B'R2}\rangle$

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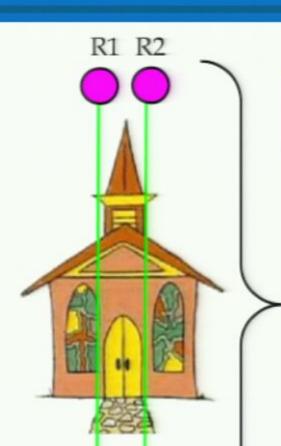


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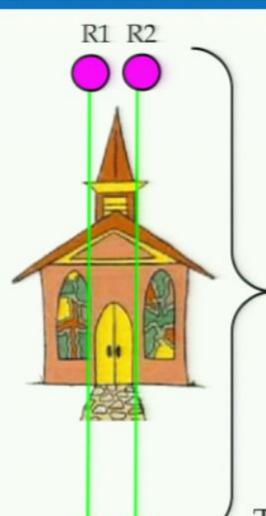


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Purification: $|\Psi\rangle_{XY}$ is a purification of ρ_X if $tr_Y |\psi\rangle\langle\psi| = \rho_X$.

 $|\phi_{NR1}\rangle |\psi_{B'R2}\rangle$

Purifications are essentially unique.

(Up to local unitaries on the purifying space.)

$$\operatorname{Tr}_{R} \xi_{NB'R} = \rho_{N} \otimes \sigma_{B'}$$

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R1 R2

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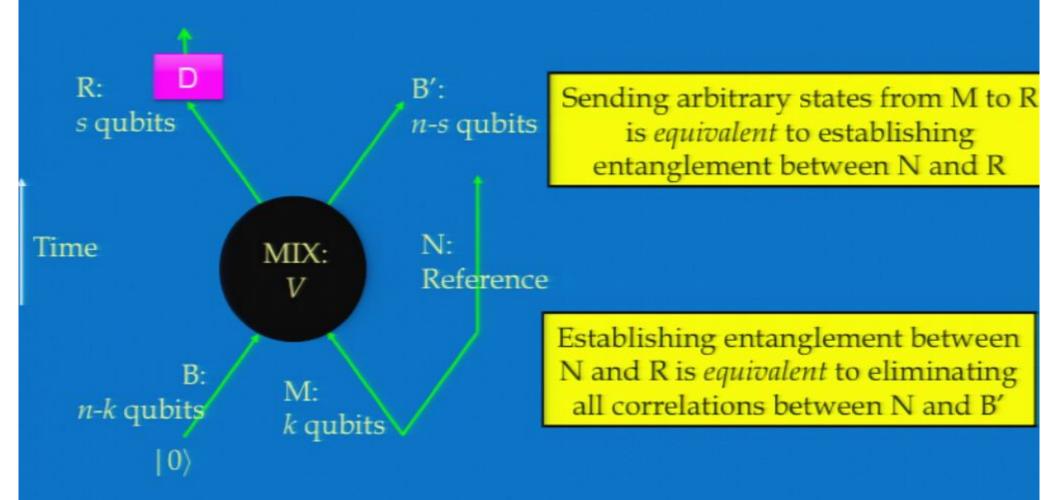
 $|\phi_{NR1}\rangle |\psi_{B'R2}\rangle = (id_{NB'}\otimes U_R^{-1}) |\xi_{NB'R}\rangle$

Purifications are essentially *unique*.

(Up to local unitaries on the purifying space.)

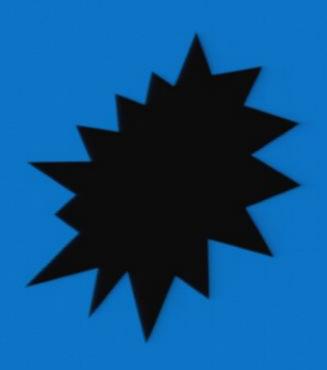
$$\operatorname{Tr}_{R} \xi_{NB'R} = \rho_{N} \otimes \sigma_{B'}$$

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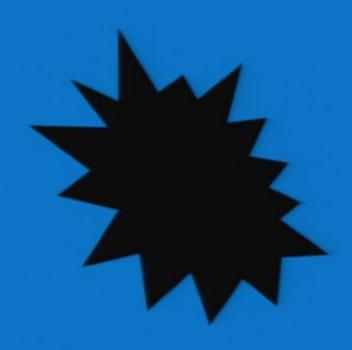




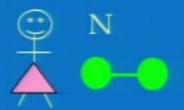
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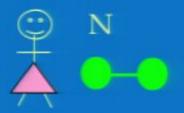


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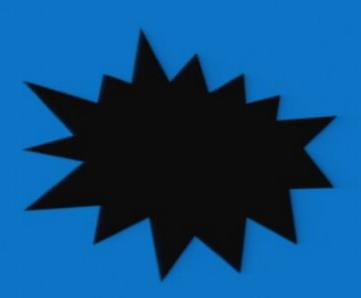
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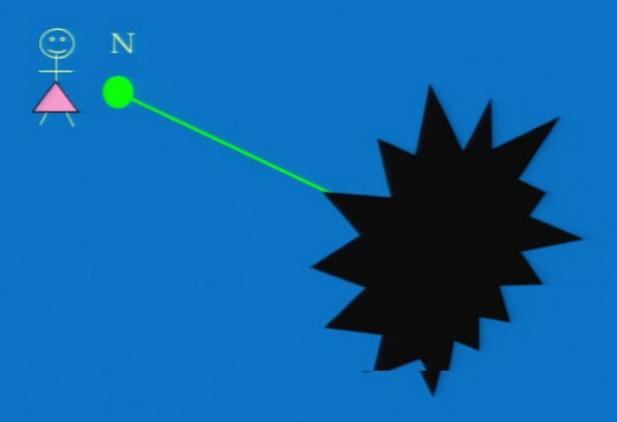


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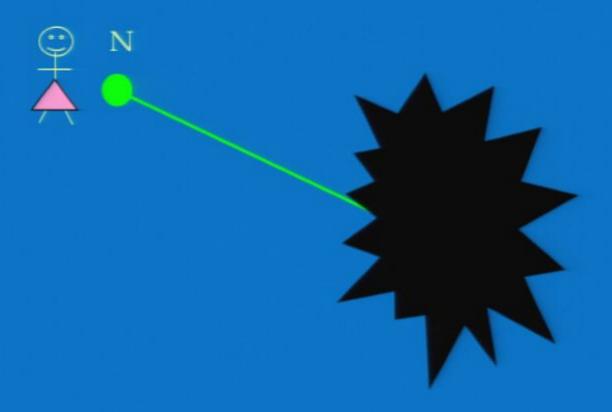




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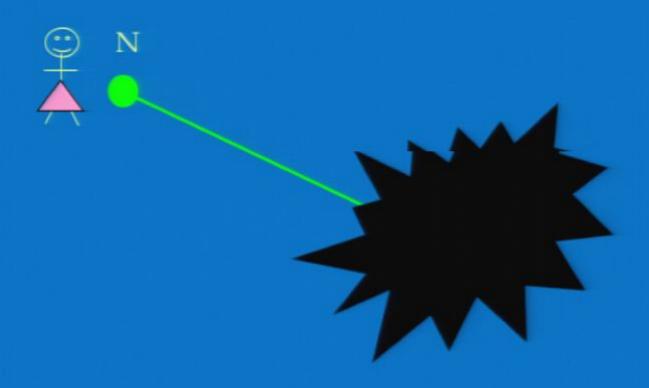


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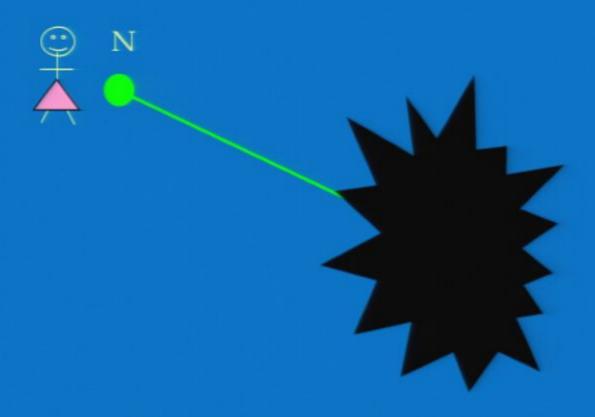
How long until entanglement with N escapes?

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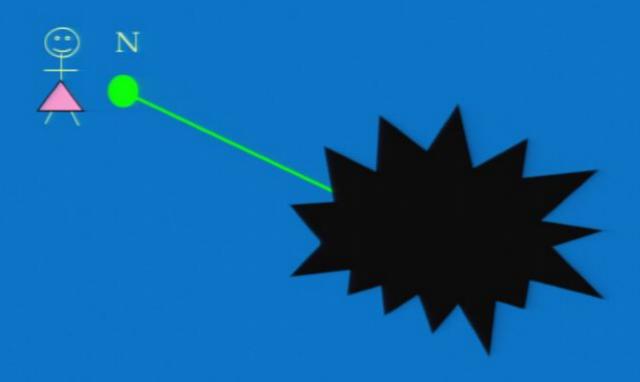


How long until entanglement with N escapes?

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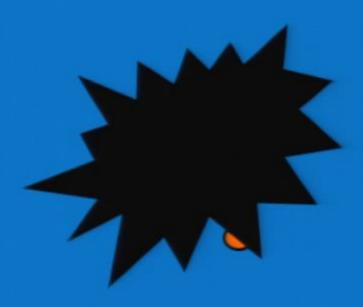


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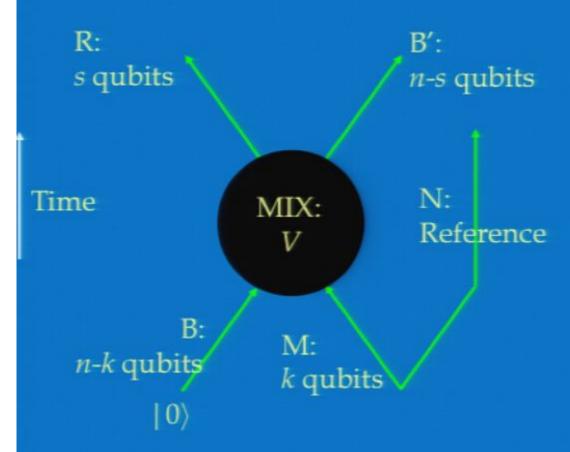


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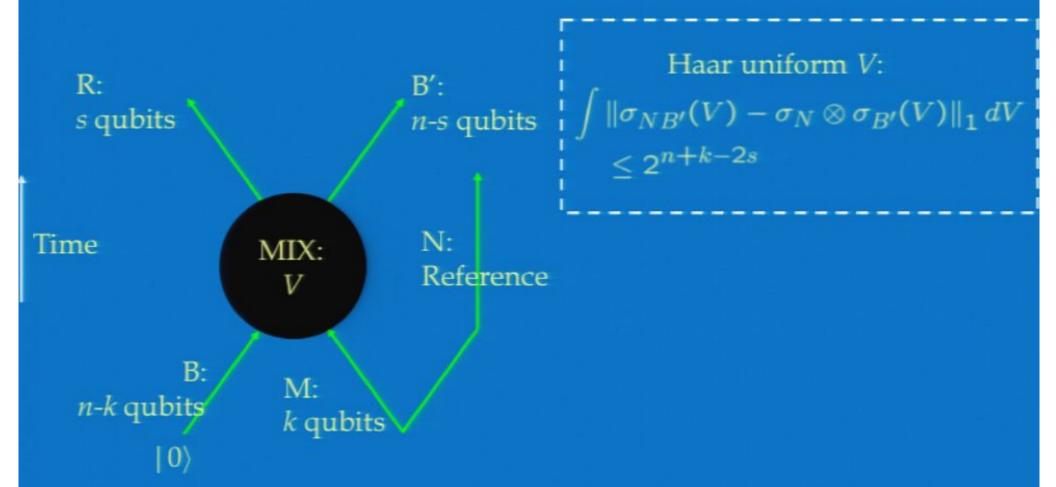


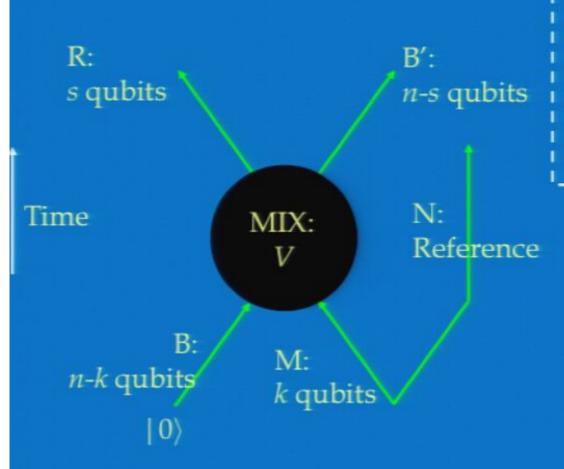


How long until entanglement with N escapes?



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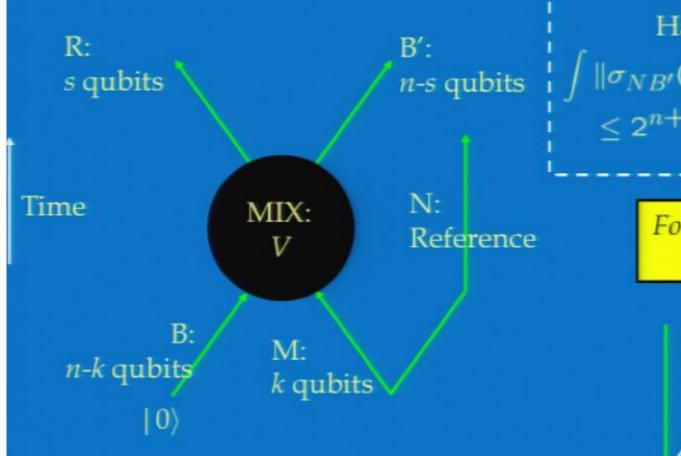




Haar uniform V: $\int \|\sigma_{NB'}(V) - \sigma_N \otimes \sigma_{B'}(V)\|_1 dV$

 $\leq 2^{n+k-2k}$

For good decoupling: $s \gg (n+k)/2$



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Haar uniform V:

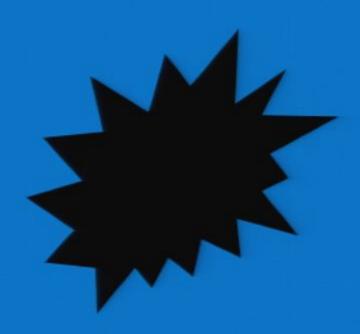
$$\int \|\sigma_{NB'}(V) - \sigma_N \otimes \sigma_{B'}(V)\|_1 dV$$

$$< 2^{n+k-2s}$$

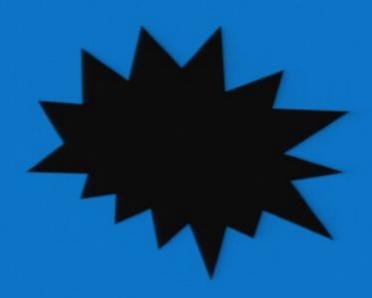
For good decoupling: $s \gg (n+k)/2$



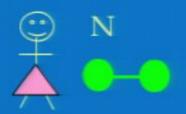
Another modest experiment



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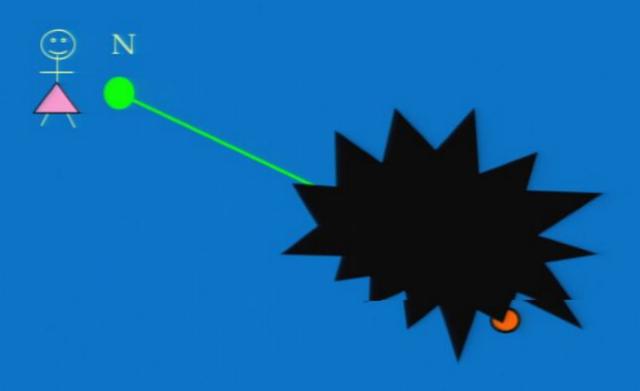


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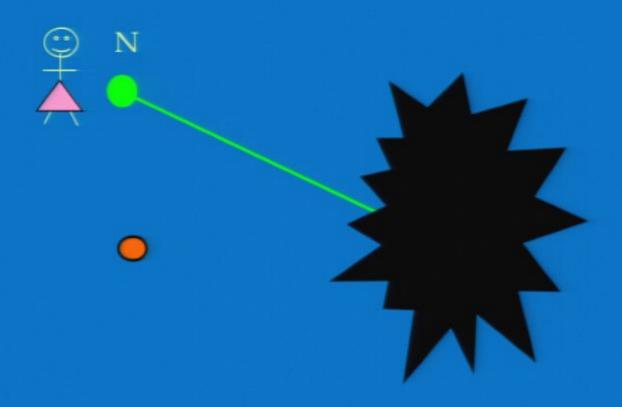




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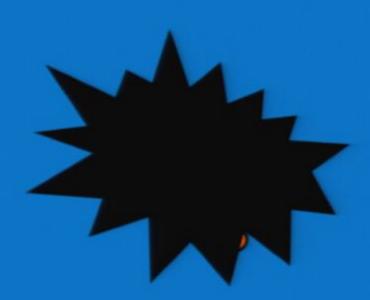


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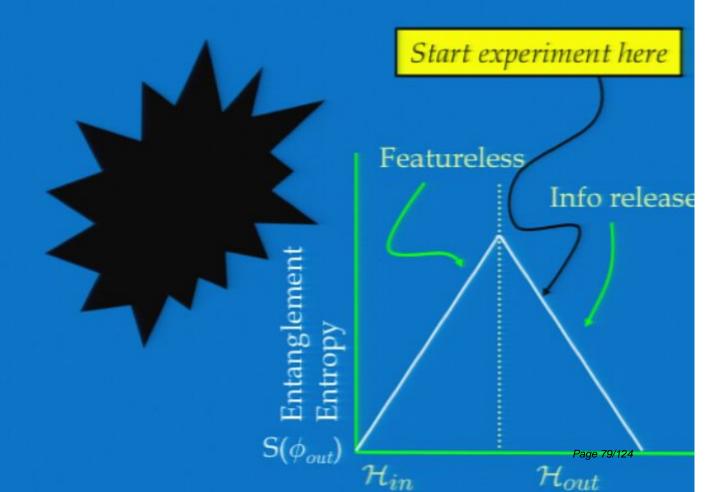
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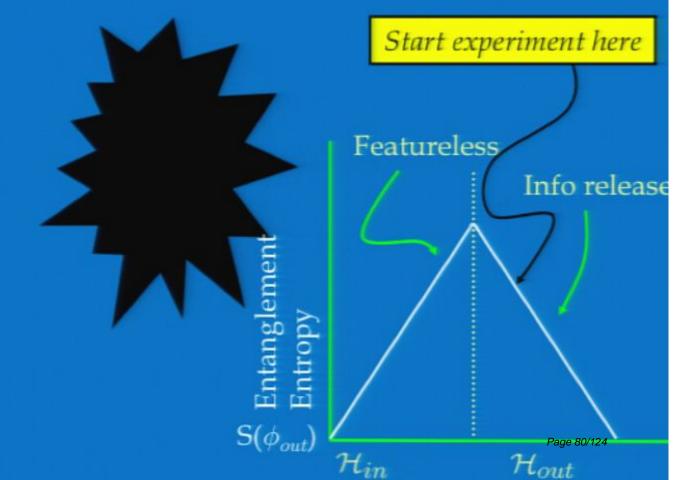
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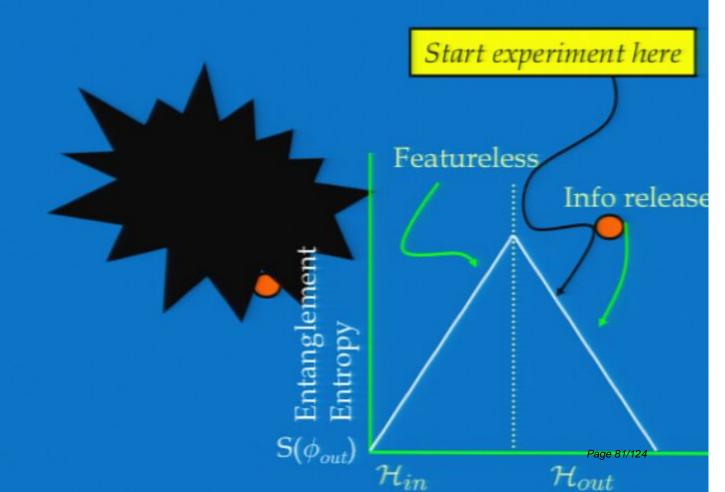


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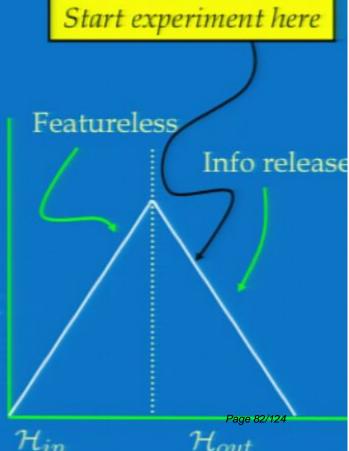






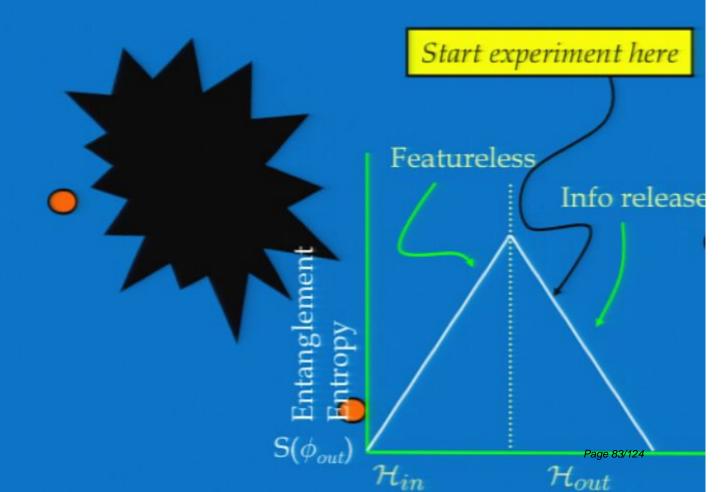




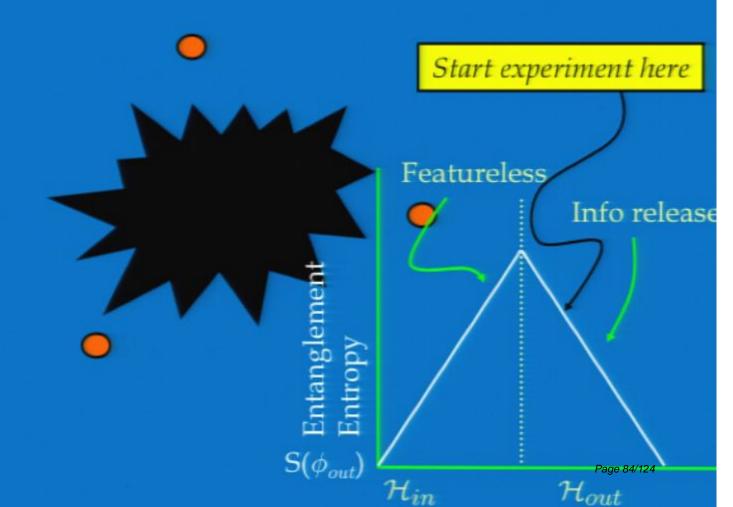


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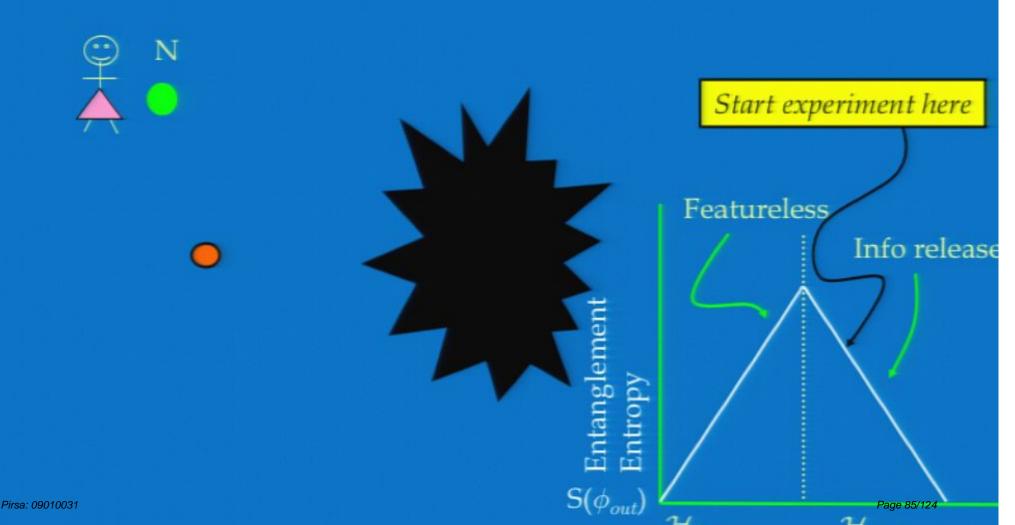


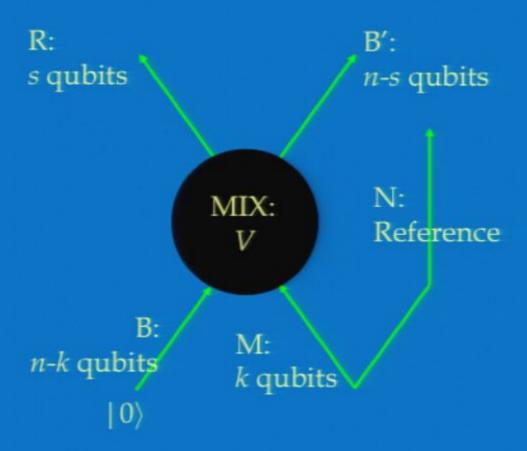




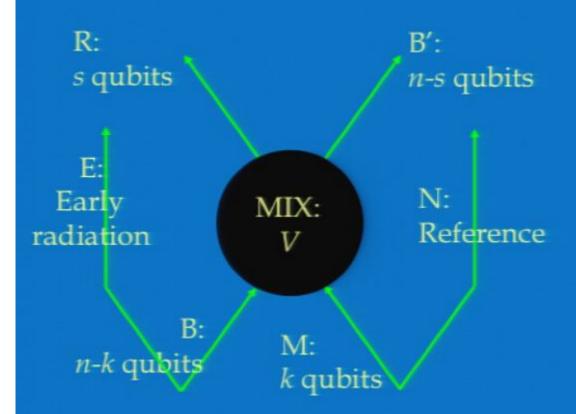


Pirsa: 09010031

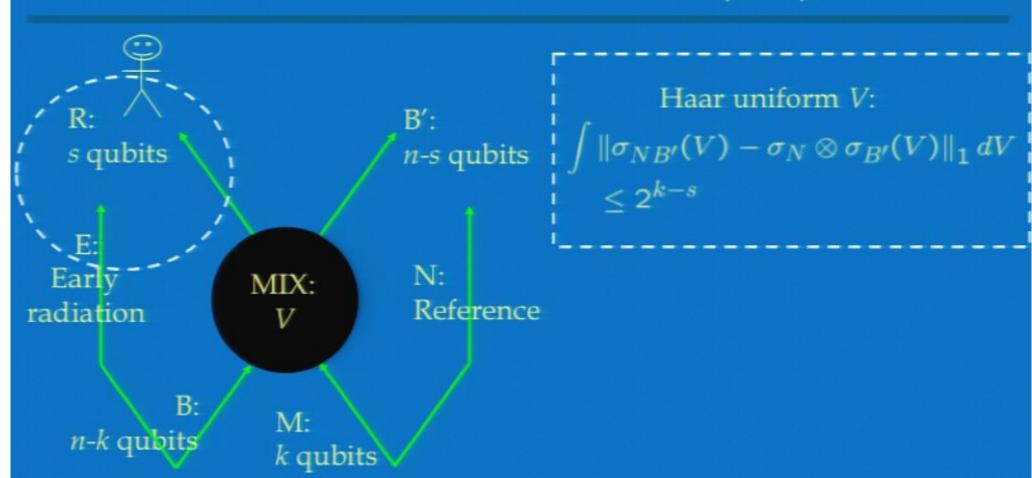




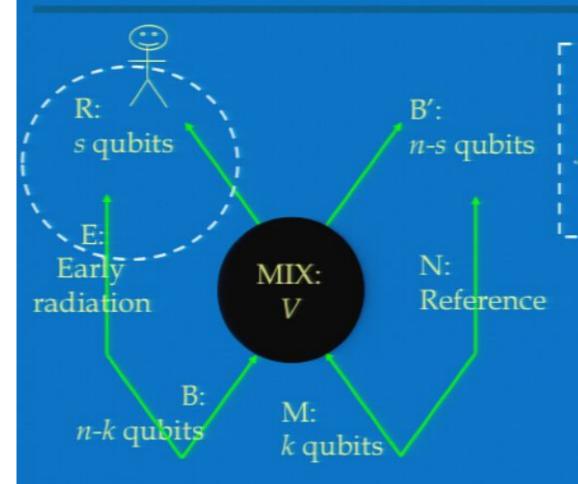
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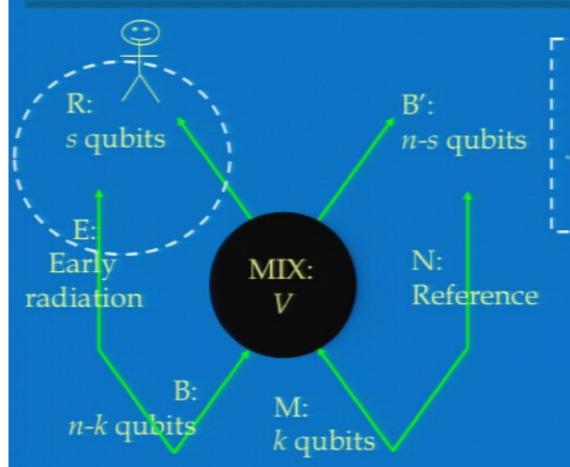
Haar uniform V:

$$\int \|\sigma_{NB'}(V) - \sigma_N \otimes \sigma_{B'}(V)\|_1 dV$$

$$\leq 2^{k-s}$$

For good decoupling: $s\gg k$





Haar uniform V:

$$\int \|\sigma_{NB'}(V) - \sigma_N \otimes \sigma_{B'}(V)\|_1 dV$$

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For good decoupling: $s \gg k$



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Quasinormal mode ringdown ~ r_s

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- Quasinormal mode ringdown ~ r_s
- Spreading of a charge over stretched horizon ~ r_s log r_s

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- Spreading of a charge over stretched horizon ~ r_s log r_s
- AdS/CFT ~ 1/T_{Hawking} (~ r_s in Schwars.)

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- Quasinormal mode ringdown ~ r_s
- Spreading of a charge over stretched horizon ~ r_s log r_s
- AdS/CFT ~ 1/T_{Hawking} (~ r_s in Schwars.)

Suggestive, but our question is more demanding:

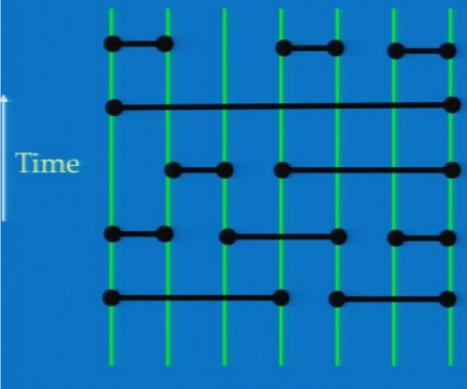
$$\int \|\sigma_{NB'}(V) - \sigma_N \otimes \sigma_{B'}(V)\|_1 dV \leq 2^{k-s}$$

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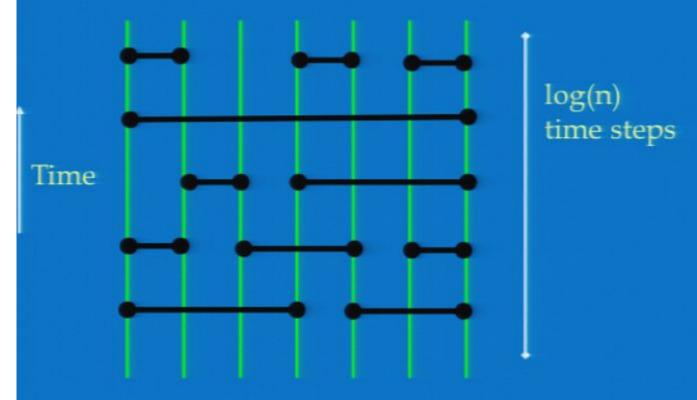
Haar random V would take exponential time

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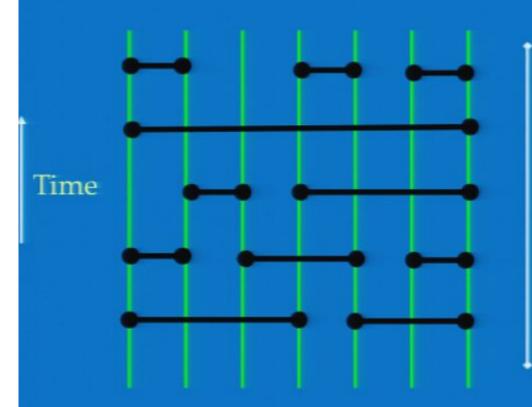
Haar random V would take exponential time



Haar random V would take **exponential** time



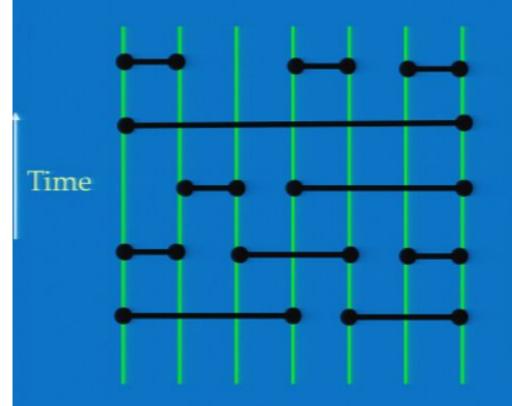
Haar random V would take exponential time



log(n) time steps Cheat sheet: $n \sim A$ $A \sim r_s^2$

What is a time step?

Haar random V would take exponential time

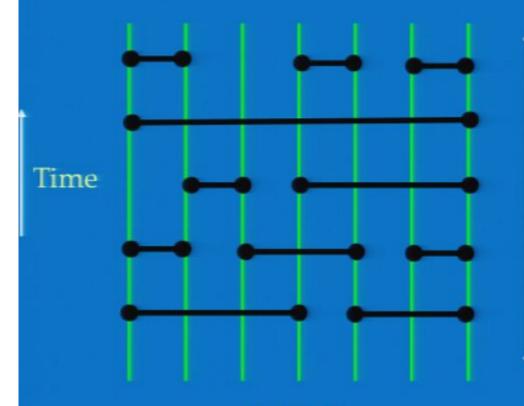


log(n) time steps Cheat sheet: $n \sim A$ $A \sim r_s^2$

What is a time step?

1 Planck time at 1 proper Planck length from the horizon

Haar random V would take exponential time



log(n) time steps Cheat sheet: $n \sim A$ $A \sim r_s^2$

What is a time step?

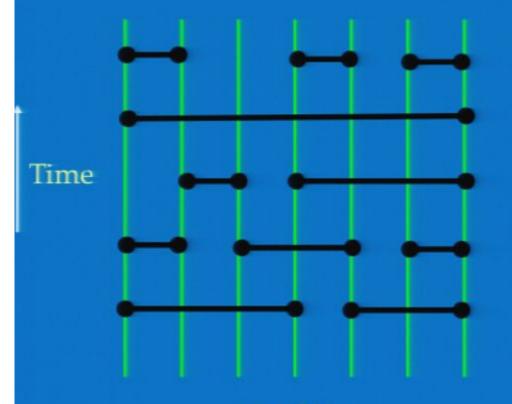
1 Planck time at 1 proper Planck length from the horizon

Thermalization in local time $log(A) \sim log(r_s)$

n qubits

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Haar random V would take **exponential** time



log(n) time steps Cheat sheet: $n \sim A$ $A \sim r_s^2$

What is a time step?

1 Planck time at 1 proper Planck length from the horizon

Thermalization in local time $log(A) \sim log(r_s)$

n qubits

Schwarzschild observer: Page 102/124(rs)

Black Hole Complementarity Consistency Condition

$$t_{info} > C'r_s \log r_s$$

New estimate: $t_{info} \sim r_s \log r_s$

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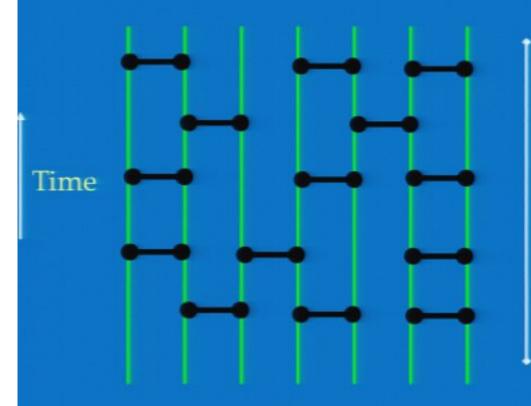
Black Hole Complementarity Consistency Condition

$$t_{info} > C'r_s \log r_s$$

New estimate: $t_{info} \sim r_s \log r_s$

 M_{\odot}

Haar random V would take **exponential** time



r_s log(n) time steps Cheat sheet: $n \sim A$ $A \sim r_s^2$

What is a time step?

1 Planck unit of Schwarzschild time

Black Hole Complementarity Consistency Condition

$$t_{info} > C'r_s \log r_s$$

New estimate: $t_{info} \sim r_s \log r_s$



 M_{\odot}

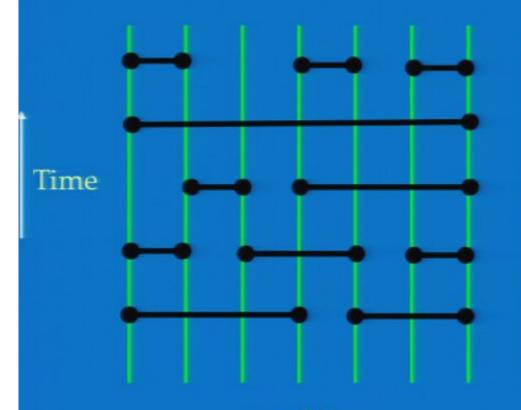
Black Hole Complementarity Consistency Condition

$$t_{info} > C'r_s \log r_s$$

New estimate: $t_{info} \sim r_s \log r_s$

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Haar random V would take exponential time



log(n) time steps Cheat sheet: $n \sim A$ $A \sim r_s^2$

What is a time step?

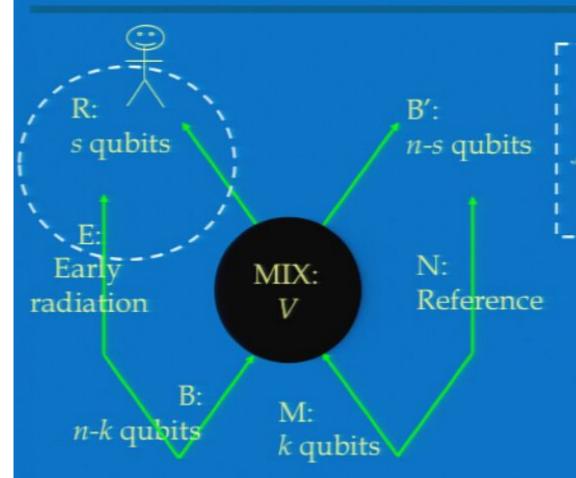
1 Planck time at 1 proper Planck length from the horizon

Thermalization in local time $log(A) \sim log(r_s)$

n qubits

Schwarzschild observer: fsgel08/124(rs)

Quantum Model (v2)



Haar uniform V:

$$\int \|\sigma_{NB'}(V) - \sigma_N \otimes \sigma_{B'}(V)\|_1 dV$$

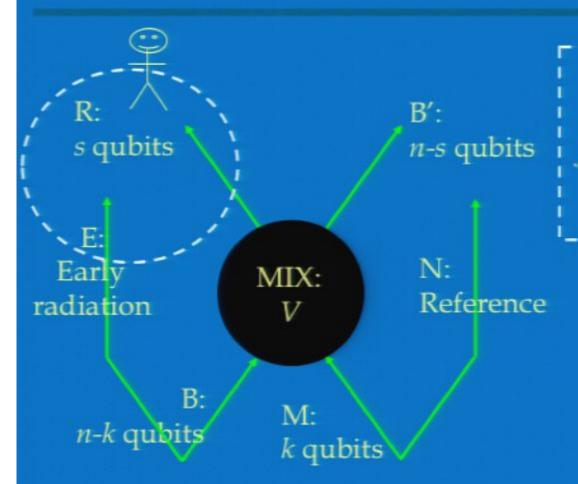
$$\leq 2^{k-s}$$

For good decoupling: $s \gg k$



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Quantum Model (v2)



Haar uniform V:

$$\int \|\sigma_{NB'}(V) - \sigma_N \otimes \sigma_{B'}(V)\|_1 dV$$

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For good decoupling: $s\gg k$



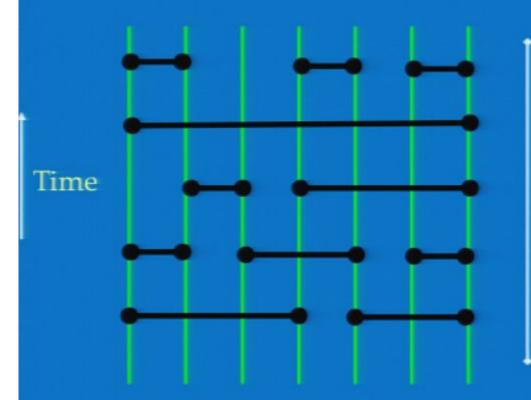
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Thermalization time of a Schwarzschild black hole

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Haar random V would take exponential time

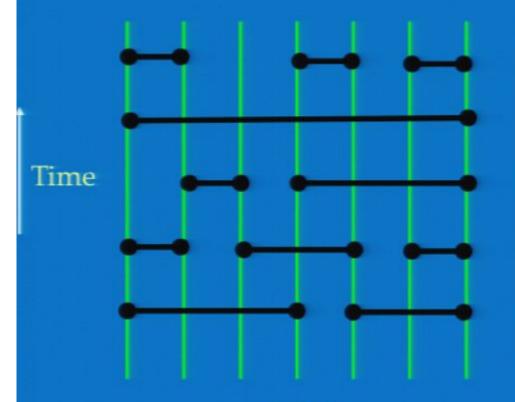


log(n) time steps Cheat sheet: $n \sim A$ $A \sim r_s^2$

What is a time step?

n qubits

Haar random V would take exponential time



log(n) time steps Cheat sheet: $n \sim A$ $A \sim r_s^2$

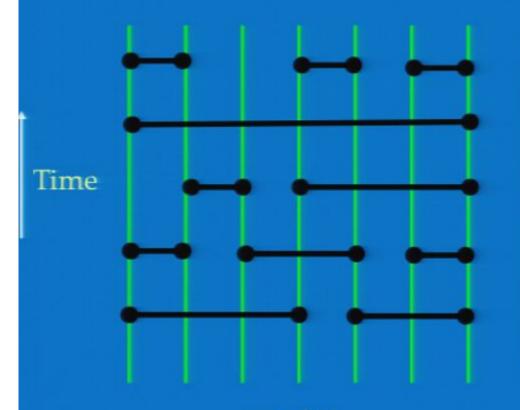
What is a time step?

1 Planck time at 1 proper Planck length from the horizon

Thermalization in local time $log(A) \sim log(r_s)$

n qubits

Haar random V would take exponential time



log(n) time steps Cheat sheet: $n \sim A$ $A \sim r_s^2$

What is a time step?

1 Planck time at 1 proper Planck length from the horizon

Thermalization in local time $log(A) \sim log(r_s)$

n qubits

Schwarzschild observer: 1° 114/124 (rs)

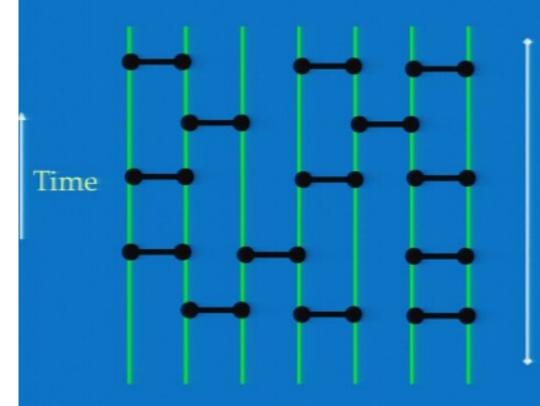
Black Hole Complementarity Consistency Condition

$$t_{info} > C'r_s \log r_s$$

New estimate: $t_{info} \sim r_s \log r_s$

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Haar random V would take **exponential** time



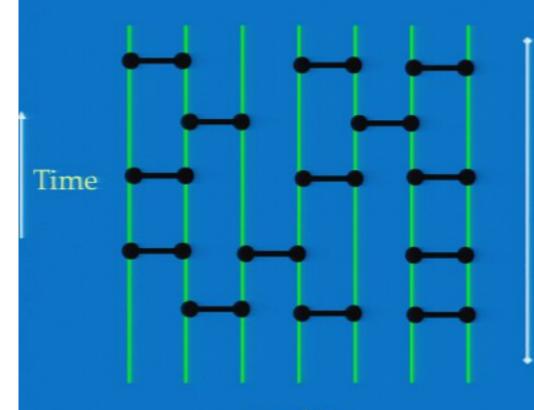
r_s log(n) time steps Cheat sheet: $n \sim A$ $A \sim r_s^2$

What is a time step?

1 Planck unit of Schwarzschild time

n qubits

Haar random V would take exponential time



r_s log(n) time steps Cheat sheet: $n \sim A$ $A \sim r_s^2$

What is a time step?

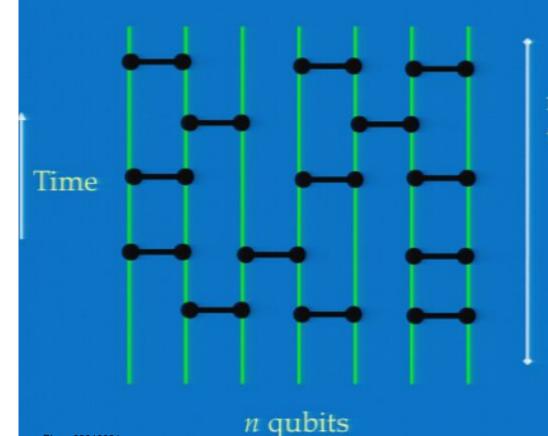
1 Planck unit of Schwarzschild time

Slowdown by light crossing time to compensate for local gates

n qubits

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Haar random V would take exponential time



r_s log(n) time steps Cheat sheet: $n \sim A$ $A \sim r_s^2$

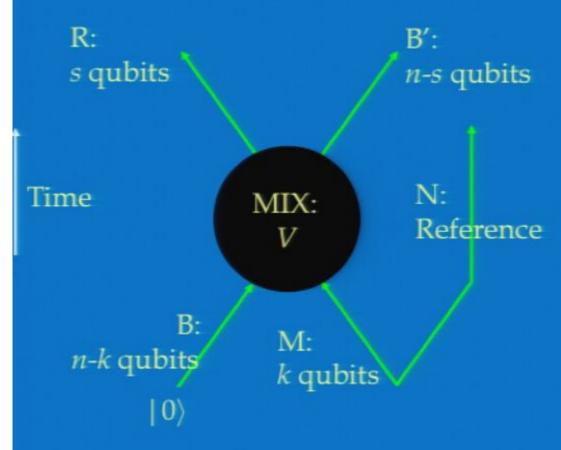
What is a time step?

1 Planck unit of Schwarzschild time

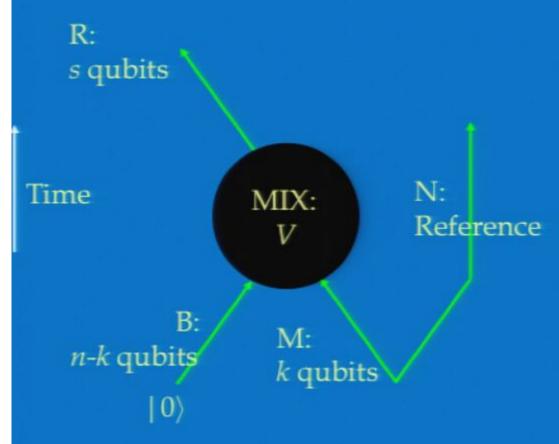
Slowdown by light crossing time to compensate for local gates

Thermalization in local time $r_s \log(A) \sim r_s \log(r_s^{Page})^{18/124}$

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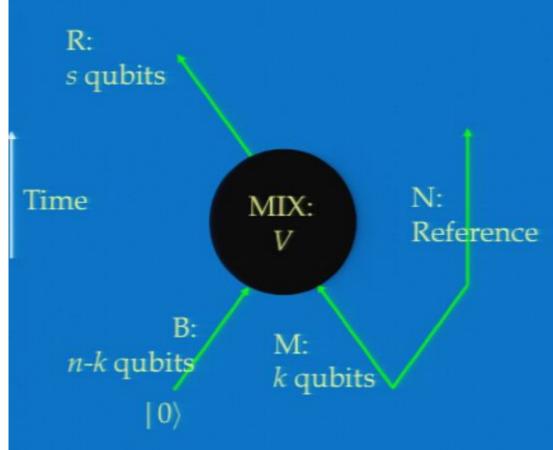


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n qubits go in, but only s qubits come out

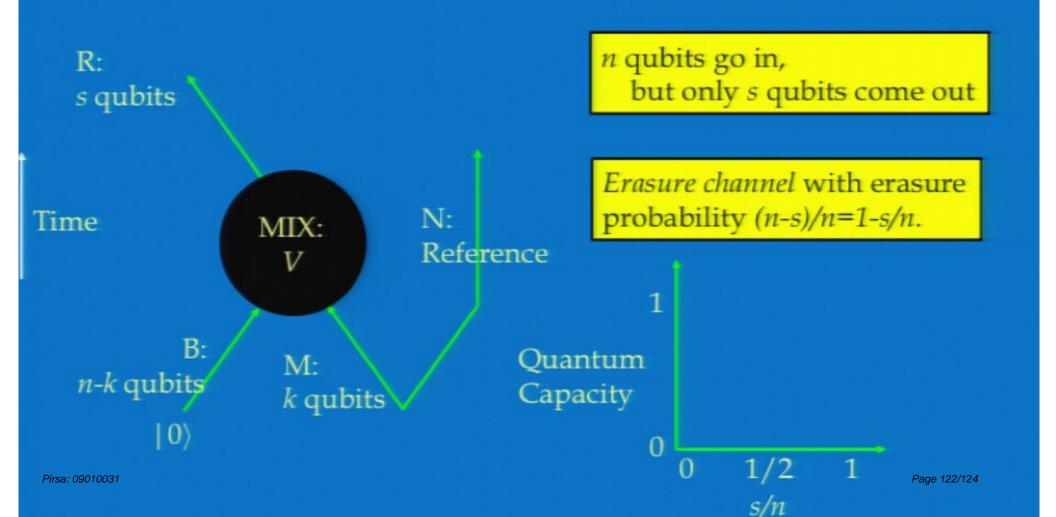
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n qubits go in, but only s qubits come out

Erasure channel with erasure probability (n-s)/n=1-s/n.

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Summary

- For black holes maximally entangled with their Hawking radiation, t_{info} is determined by the time scale for thermalization
- Our best estimates are that this is just barely compatible with the black hole complementarity hypothesis

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What next?

- Consistency of BH complementarity in other geometries
- Decoding complexity
- Thermalization
 - Study convergence of random circuits [Harrow-Lo 2007]
 - Study convergence for toy nonlocal, nonintegrable Hamiltonian QM systems [H-Lashkari-Osbourne]
 - Study thermalization in stringy models of black holes (matrix theory) [Sekino-Susskind]

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