Title: Unitarity and Holography in Gravitational Physics

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Abstract: Because the gravitational Hamiltonian is a pure boundary term on-shell, asymptotic gravitational fields store information in a manner not possible in local field theories. Two properties follow from this purely gravitational behavior. The first, 'Boundary Unitarity,' holds under AdS-like boundary conditions. This is the statement that the algebra of boundary observables is independent of time; i.e., that the algebra of boundary observables at any one time t_1 in fact coincides with the algebra of boundary observables at any other time t_2. As a result, any information available at the boundary at time t_1 remains available at any other time t_2. The second, 'Perturbative Holography,' holds under either AdS-like or asymptotically flat boundary conditions. In the AdS context, it is the statement that the algebra of boundary observables at any time t includes all perturbative observables anywhere in the spacetime. In the asymptotically flat context, Perturbative Holography is that statement that the algebra of observables on I^+ within any neighborhood of i^0 contains all perturbative observables. Perturbative Holography holds about any classical solution with a regular past infinity; i.e., spacetimes which collapse to form classical black holes are explicitly allowed. We derive the above properties and discuss their implications for information in black hole evaporation.

Holography and Unitarity in Gravitational Physics

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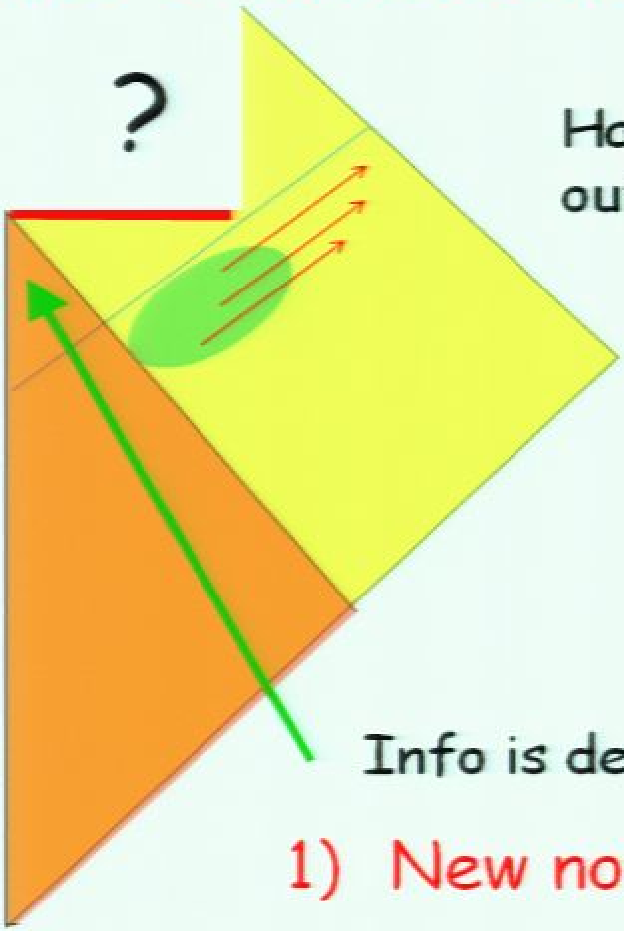
PI Black Hole Conference

arXiv: 0808.2842 & 0808.2845

Info Paradox in a Nutshell



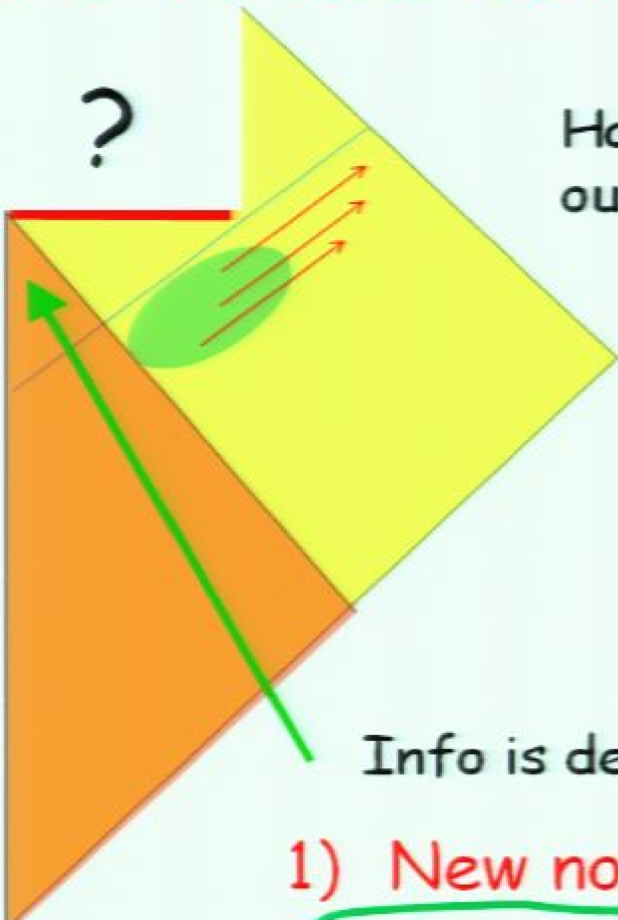
Hawking radiation forms outside the black hole.

Is the info transferred to the Hawking radiation and, if so, how?

Info is deep inside the black hole.

1) New non-locality or causality violation?

Info Resolution in a Nutshell



Hawking radiation forms outside the black hole.

Equivalent info is stored outside.

Just GR & QM! No need for stringy effects or other new non-localities.

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1) New non-locality or causality violation?
2) Infalling info is stored outside?

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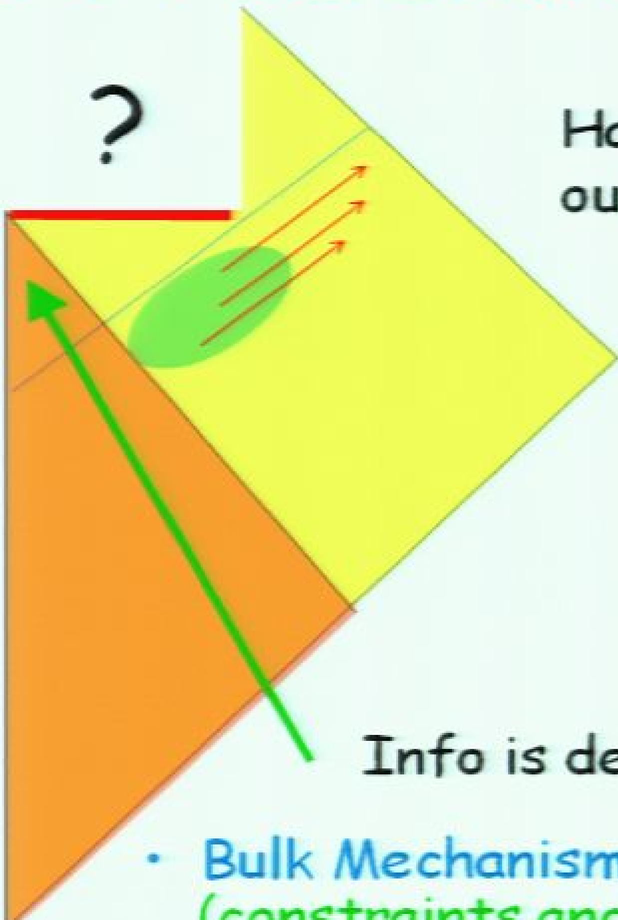

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- Bulk Mechanism for Holography: Gravitational Gauss Law (constraints and gauge invariance).

On-shell Hamiltonian is a pure boundary term!
(See also Balasubramanian, Marolf, and Rozali, but this time consider entire observable algebras...)

Outline:

I. AdS Boundary Unitarity:

$A_{\text{Bndy Obs}}(t_2) = A_{\text{Bndy Obs}}(t_1)$, for all t_1, t_2

II. Ads Perturbative Holography:

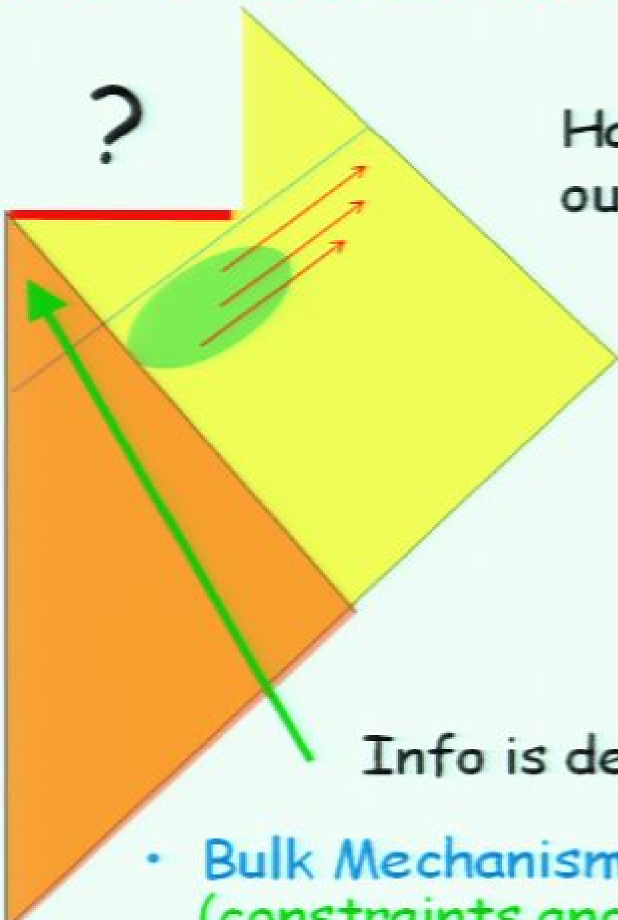
$A_{\text{Pert Obs}}(\text{all } t) = A_{\text{Bndy Obs}}(t_1)$, for any t_1

III. As. Flat Pert. Holography and BH evaporation

$A_{\text{Pert Obs(all t)}} = A_{\text{Obs}}$ (i^0 and early I^+: $u < u_0$)

IV. Summary

Info Resolution in a Nutshell



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IV. Summary

The role of QM

Technical Result:

Similar for QM & CM

$O \in A$, generated by $A_1, A_2, A_3 \ldots$

1. Holds classically for Poisson Algebra
2. Holds in QM with usual notion of algebra

Analogy: $J_z \in A$, generated by J_x, J_y

Physical Interpretation

CM: Measurements of $J_x, J_y, \ldots$ may tell us nothing about J_z!

QM: Information about O can be obtained by measuring $A_1, A_2, \ldots$

(E.g., Suppose an ensemble of identically prepared spins. Find J_z as follows:

For half, measure J_x and then J_y.

For other half, measure J_y and then J_x.)

Outline:

I. AdS Boundary Unitarity:

$A_{Bndy\ Obs}(t_2) = A_{Bndy\ Obs}(t_1)$, for all t_1, t_2

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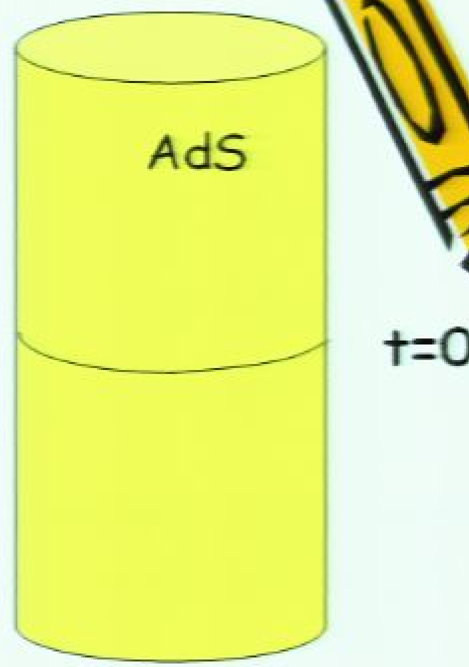
I. AdS Boundary Unitarity



E.g., Einstein-Hilbert + scalars

Fix Bndy @ z=0. In Fefferman-Graham Gauge:

$$ds^2 = z^{-2}\left(dz^2 + g_{ij}(x,z)\, dx^i dx^j\right), \qquad x^i = t, x, y.$$

$$g_{ij} = g_{ij}^{(0)} + z\, g_{ij}^{(1)} + z^2 g_{ij}^{(2)} + \ldots$$

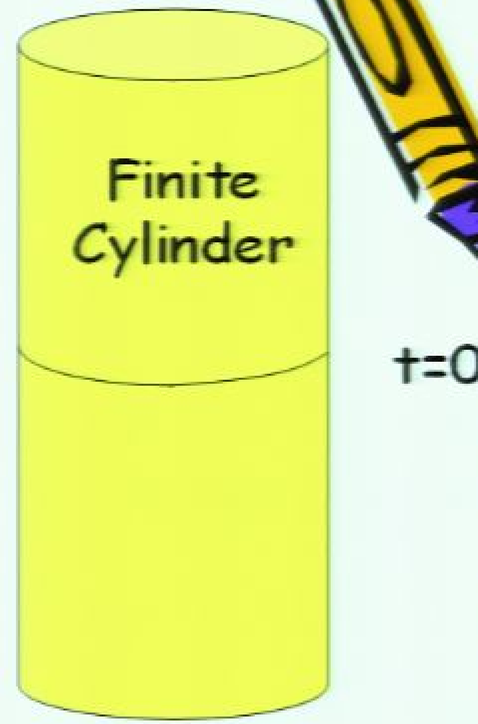
Fix $g_{ij}^{(0)}$ and corresp "Dirichlet data" for scalars. Simplest case: Choose Stationary BCs.

Recall: Both $T_{ij}(x)$ and $\phi_N(x)$ are boundary *observables*. Diffeos that are gauge must vanish too fast at infinity to affect them.

Let $A_{\text{Bndy Obs}}(t)$ be the (Poisson) algebra generated by such Bndy Obs at time t.

Note: $H := \int_{\text{Bndy Cut w/ } t = \text{const}} T_{ij}\, \xi^i\, \underline{n}^i \, dA \in A_{\text{Bndy Obs}}(t)$ for each t.

"Boundary Unitarity," part 2:



$H = H(t) \in A_{\text{bndy obs}}(t)$

E.g., $H := \int_{t=\text{const}} T_{ij}\, \xi^i\, \underline{n}^i \; dA$

Note: For any *observable* O,

$$\partial_t O\,(t) = -\,i\,[O(t)\,,H]$$

3. *Suppose* * that we can exponentiate H to define $e^{iH\Delta t}$

Then, $O\,(t_2) = e^{-iH\Delta t}\, O(t_1)\, e^{iH\Delta t}$

I.e., expresses any Bndy Obs at t_2
in terms of Bndy Fields ϕ_N, $T_{ij,}$ at any other t_1.

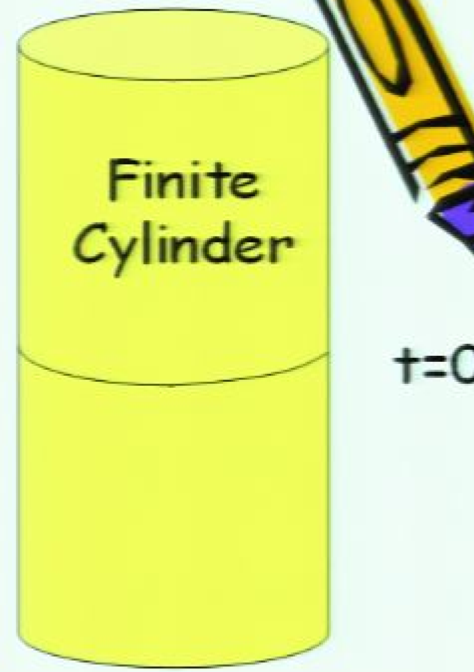
⇒ $A_{\text{bndy obs}}(t_1) = A_{\text{bndy obs}}(t_2)$ "Boundary Unitarity!"

In QM, information present on the Bndy at any one time t_1 remains present at any other time t_2.

Comment on Assumption:

For any *observable* O, $\partial_t O(t) = -i\,[O(t), H]$

3. *Suppose* * that we can exponentiate H to define $e^{iH\Delta t}$



Classical Interpretation on space of smooth metrics:

Assumes long-time existence of solutions to EOMs, at least in some neighborhood of the Bndy.

I.e., form of "Cosmic Censorship."

QM interpretation:

Assumes quantum Hamiltonian can still be built from ϕ_N, T_{ij}, but that Quantum Gravity "resolves any singularities".

Appears consistent w/ both String Theory (AdS/CFT) & LQG.

Time-Dependent Case:

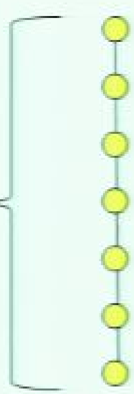
For any *observable* O, $\partial_t O(t) = -i\,[O(t), H(t)]$

Idea: Use $U(t_1,t_2) = P \exp\left(-i \int dt\, H(t)\right)$, and express in terms of fields at just t_1 to write

$$O(t_2) = U(t_2,t_1)\, O(t_1)\, U(t_1,t_2) \in A_{\text{bndy Obs}}(t_1)$$

Requires a limiting construction:

$\Delta t = N\varepsilon$



$$O(t_N) = O(t_{N-1}) + i\varepsilon\,[O(t_{N-1}), H(t_{N-1})] + O(\varepsilon^2)$$

$$O(t_{N-1}) = O(t_{N-2}) + i\varepsilon\,[O(t_{N-2}), H(t_{N-2})] + O(\varepsilon^2)$$

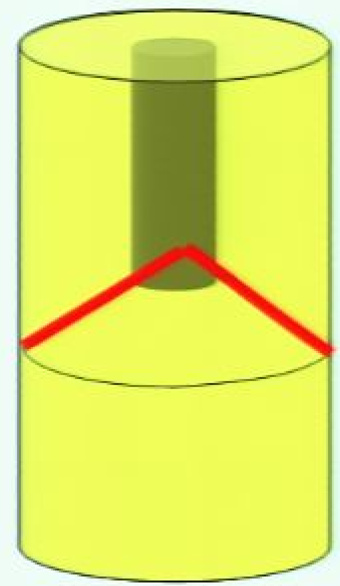
Etc. Take $\varepsilon \to 0$ with Δt fixed.

II. AdS Perturbative Holography

Summary of Above: Any info ever present in the Bndy Fields remains encoded in Bndy Fields.

Q: Is this everything? Or is there more info "in the bulk."

A: Maybe, but not in perturbation theory.



Consider perturbation theory abt some classical solution which is largely empty before t=0.

(Though need not remain empty for time-dep BCs. E.g., can make a black hole.)

At linearized level, any h_{ab}, ϕ can be written (up to gauge) in terms of Bndy observables at early times by solving EOMs.

(Related to Holmgren's Uniqueness Thm.)

→ Remains true at any order in perturbation theory.

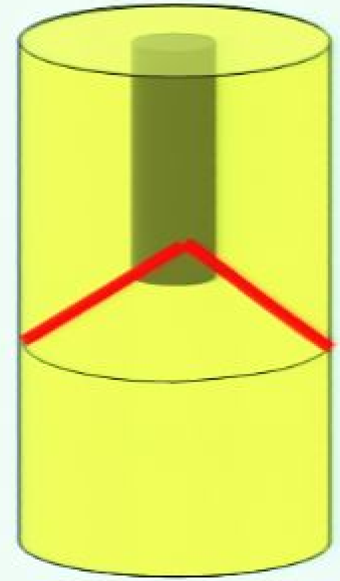
Perturbative Holography

So, *any* perturbative observable can be written in terms of Bndy Observables at early times by solving EOMs.

$$A_{\text{All Pert Obs}} = A_{\text{Bndy Obs}}(\text{all } t < 0)$$

But in *gravity*, at any order beyond the linearized theory, the Hamiltonian can again be written as a boundary term!

(I.e., Gauss' Law gives a useful measure of the energy.)



Above Bndy Unitarity Argument ⇒

$$A_{\text{Bndy Obs}}(\text{all } t < 0) = A_{\text{Bndy Obs}}(\text{any single } t)$$

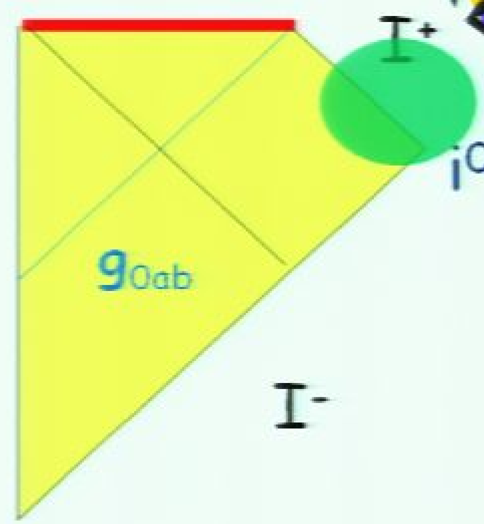
⇒ $A_{\text{All Pert Obs}} = A_{\text{Bndy Obs}}(\textit{any}\text{ single } t)$

"Perturbative Holography"

III. Comments on As Flat case

1. Perturbative Holography:

Consider a collapsing black hole background g_{0ab} in pure Einstein-Hilbert gravity.

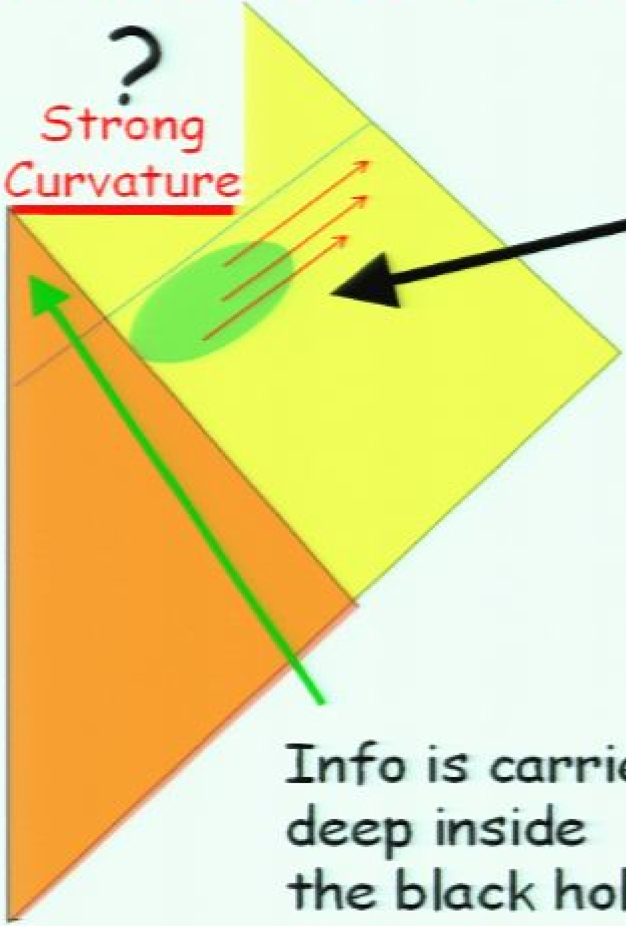


Claim: A complete set of perturbative observables is available on I^+ in any neighborhood of i^0.

2. *Suggests* Unitary S-matrix, with info imprinted in Hawking radiation (next slide).

Basic Mechanism: Constraints and local energy conservation!

Cartoon of BH evaporation



Info is carried deep inside the black hole.

Suppose physics far from strong coupling region is essentially perturbative.

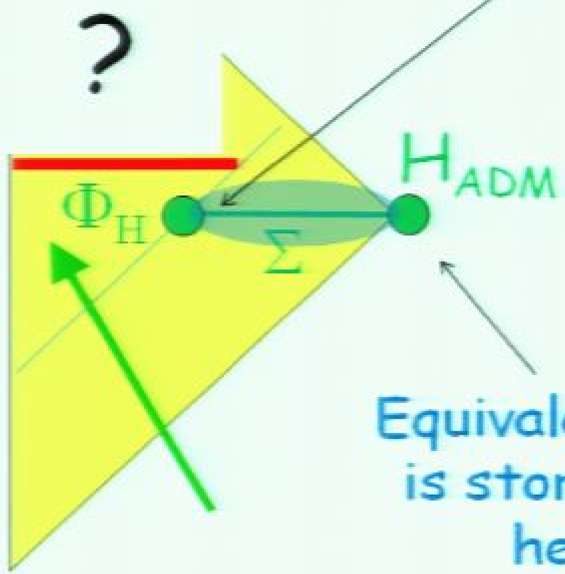
Then perturbative holography implies that all info is encoded in asymptotic fields g_{ab}, especially H_{ADM}.

But constraints relate H_{ADM} to $T^{Hawking}{}_{ab}$ and a surface term "Gauss Law Grav. Flux" Φ_H at the horizon.

$$H_{ADM} - \Phi_H(h) = \int_\Sigma T^{Hawking}{}_{ab}(h)$$

Cartoon of Black Hole Evaporation 2

Remaining info is stored here!



Equivalent info is stored out here

Info carried inside by infalling matter.

$H_{ADM} - \Phi_H(h) \sim \int_\Sigma T_{ab}(h)$

Info shared between Φ_H and T_{ab}.

$\Phi_H(h) \to 0$ as BH evaporates.
$\Rightarrow$ info transferred *locally* to T_{ab}.

Indeed, once evaporation is complete, constraint implies $H_{ADM} \sim \int_\Sigma T_{ab}(h)$.

I.e., info fully transferred to Hawking radiation.