Title: Matrix models for the black hole information paradox

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Abstract: I'll discuss some large N quantum mechanical theories that are toy models for eternal black holes in AdS via gauge/gravity duality. They can be used to study the classical limit and quantum corrections in gravity, and their roles in the information paradox. We demonstrate that such large N models can exhibit late time fall-off of a two-point function. By computing higher genus corrections explicitly, we argue that the fall-off, and thus information loss, persist even after perturbative gravity corrections are included.

Matrix models for the black hole information paradox

Takuya Okuda, Perimeter Institute
Joint work with N. Iizuka and J. Polchinski

Maldacena's paradox for eternal black holes

Two matrix models for black holes

- Exponential decay of a two-point function (Iizuka&Polchinski)
- Perturbative 1/N corrections do not restore information (Iizuka, TO, &Polchinski)

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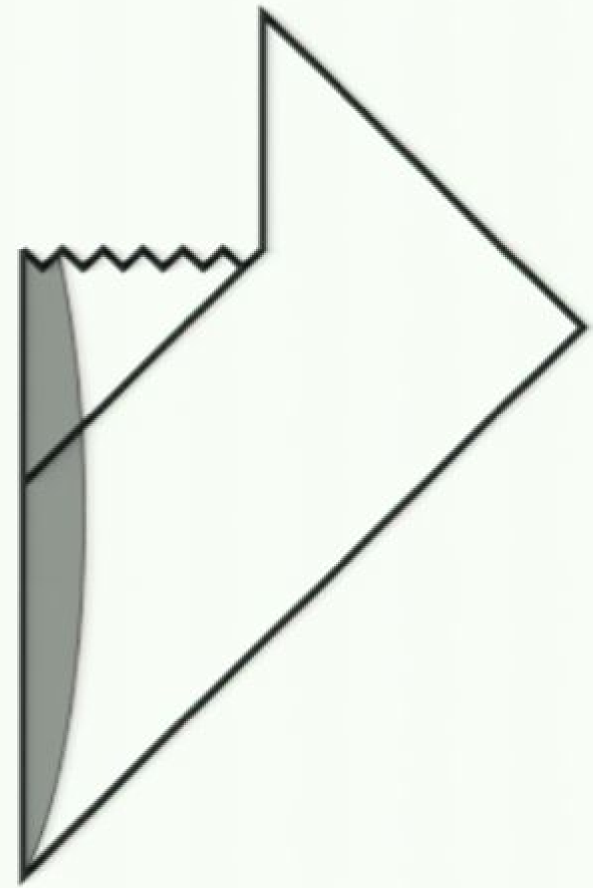
Hawking argued that information is lost

Collapsing matter produces a black hole.

A black hole in Minkowski space evaporates by emitting radiation.

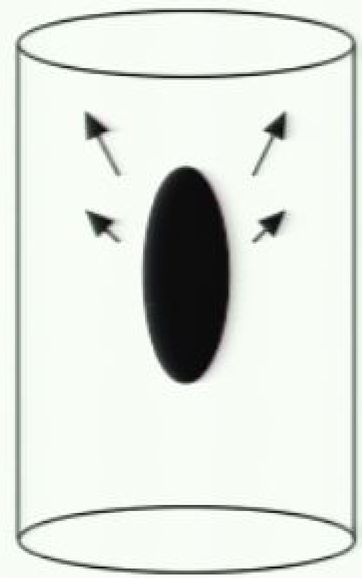
Semiclassically, radiation is thermal, and there is no phase correlation.

Information seems to be lost.



AdS/CFT convinced many of us that information is preserved

A **small black hole** in AdS evaporates, and the process is described, in principle, by a unitary evolution in gauge theory.



Information must be preserved.

There have to be phase correlations. We want to understand **where Hawking's argument breaks down**.

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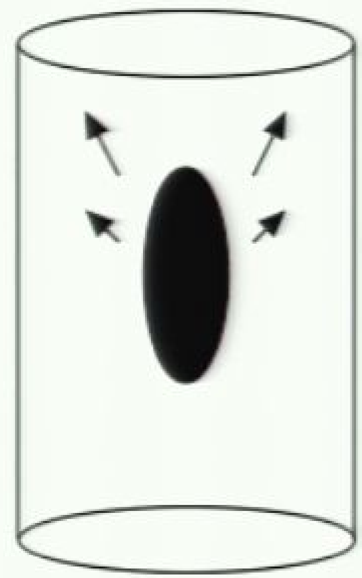

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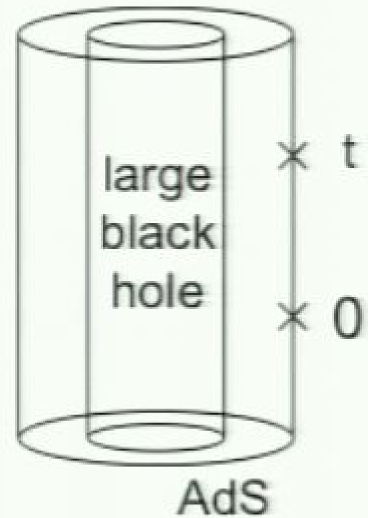
Mardacena's ``paradox'' is a cleaner set-up

A **large black hole** in AdS does not evaporate.

``Paradox'': A two-point correlation function <O(t)O(0)> shows **exponential fall-off** $e^{-\Gamma t}$ in AdS. Information lost at large N. But in gauge theory, there must be **recurrences** (discrete spectrum). Information restored at finite N.



It's not a true paradox: **Exponential decay** at large N, **recurrences** at finite N.

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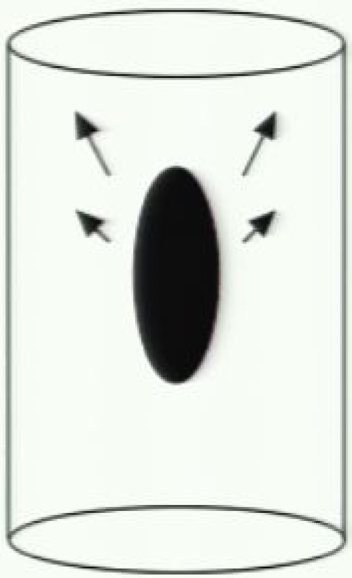

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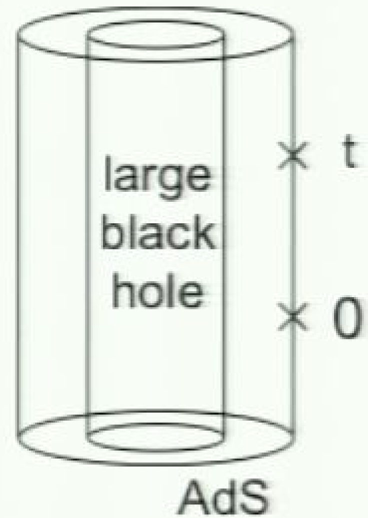

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Questions in gravity are difficult

Questions:

Which correction in gravity restores recurrences?
(Perturbative loop correction or non-perturbative?)

Is it the secondary saddle point in Euclidean gravity?
(Maldacena, Hawking)

$G_{\mu\nu}$

Questions in gauge theory are easier

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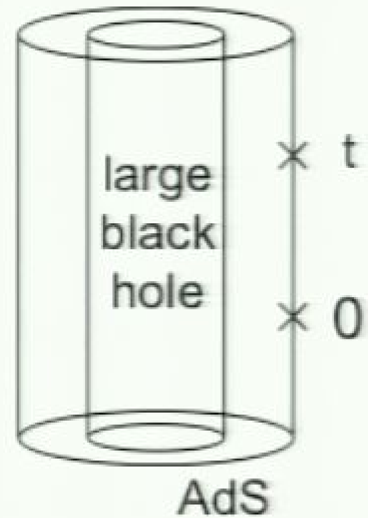

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Ask the questions in toy matrix models

Physical theories with known gravity duals, like N=4 Yang -Mills and D0-brane quantum mechanics, are still too difficult.

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Two toy models
(Matrix quantum mechanics)

Goals:
1. Exponential decay (Iizuka and Polchinski)
2. 1/N corrections (Iizuka, TO and Polchinski)

Iizuka and Polchinski studied a cubic model to demonstrate exponential decay

The simplest possible model:

One adjoint field $A_{ij},\ A^\dagger_{ij}$

One fundamental field $a_i,\ a^\dagger_i$

Choose the Hamiltonian

$$H = mA^\dagger_{ij}A_{ji} + Ma^\dagger_i a_i + ga^\dagger_i(A + A^\dagger)_{ij}a_j$$

Consider the two-point function at finite T

$$e^{iM(t-t')}\left\langle \mathrm{T}\, a_i(t)a^\dagger_j(t')\right\rangle_T \equiv \delta_{ij}G(T, t-t')$$

Take $M \to \infty$ to get thermal ensemble of free adjoints

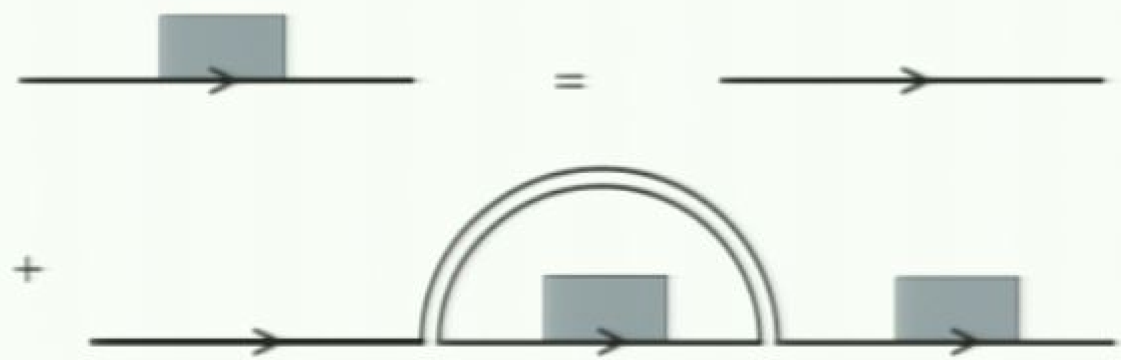
Recurrences at finite N are automatic in matrix quantum mechanics

Demonstration of exponential decay in a two-point function at large N is non-trivial.

They worked in the planar approximation.

The key tool is the planar Schwinger-Dyson equation

Graphically, the SD equation is given as



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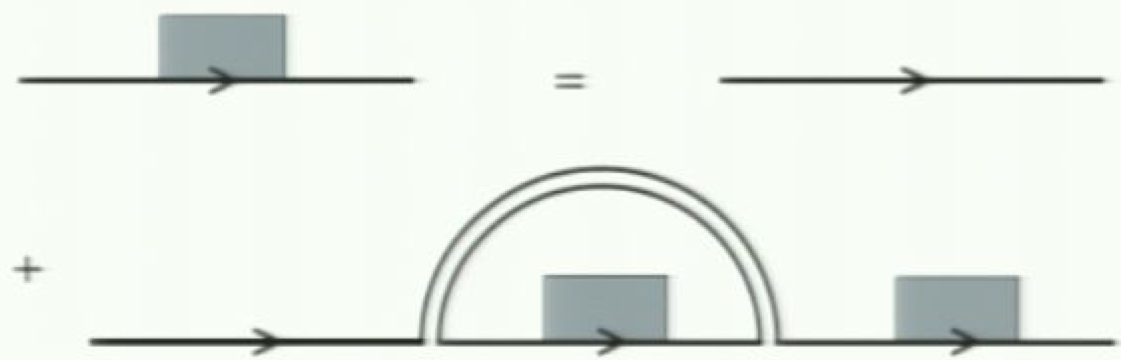

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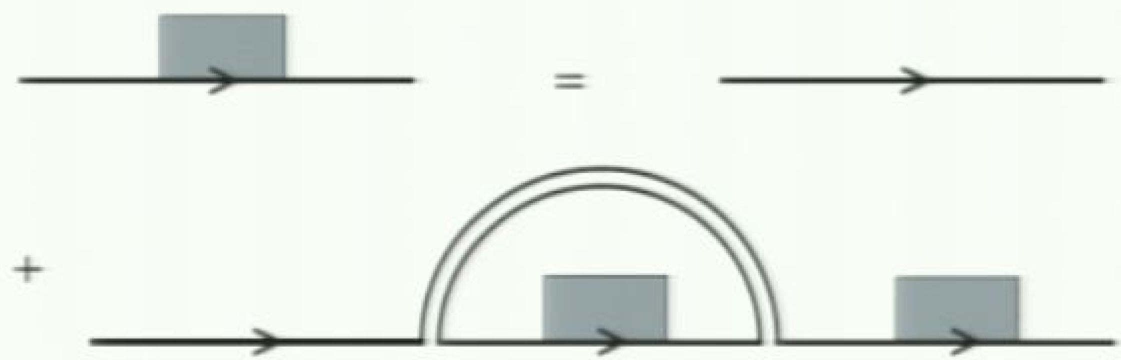

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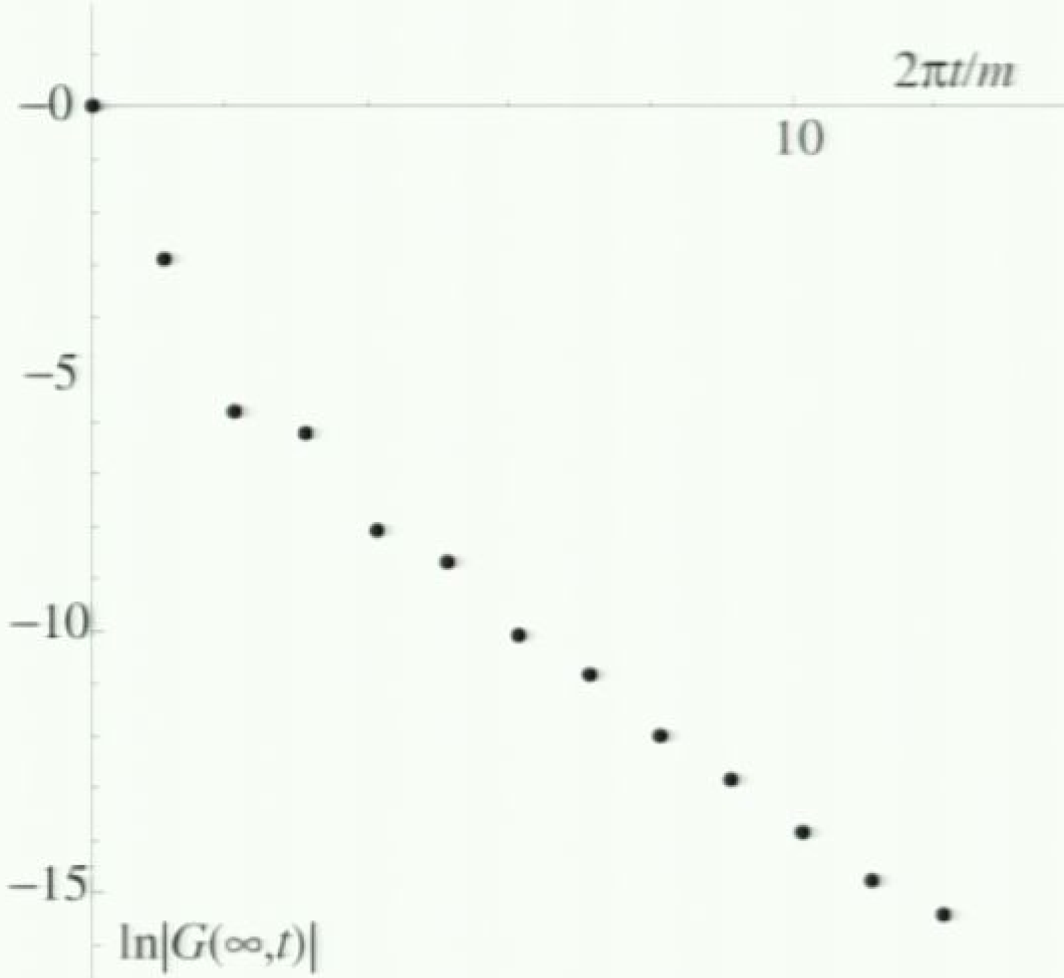
They demonstrated exponential decay at high temperature



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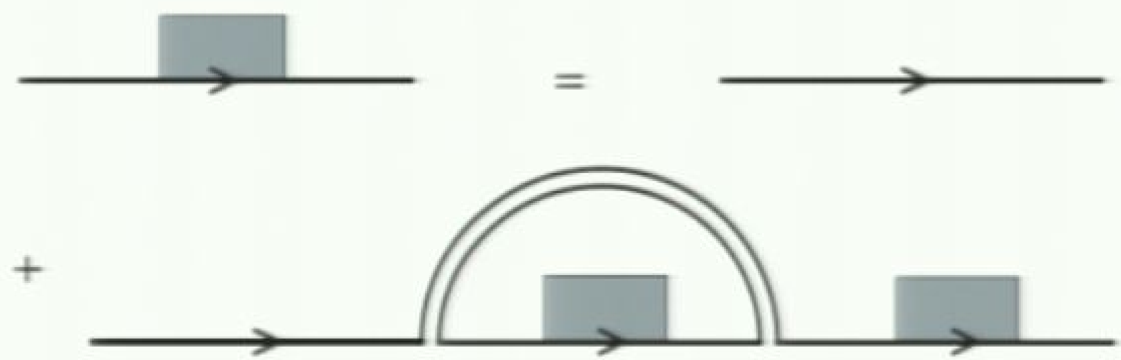

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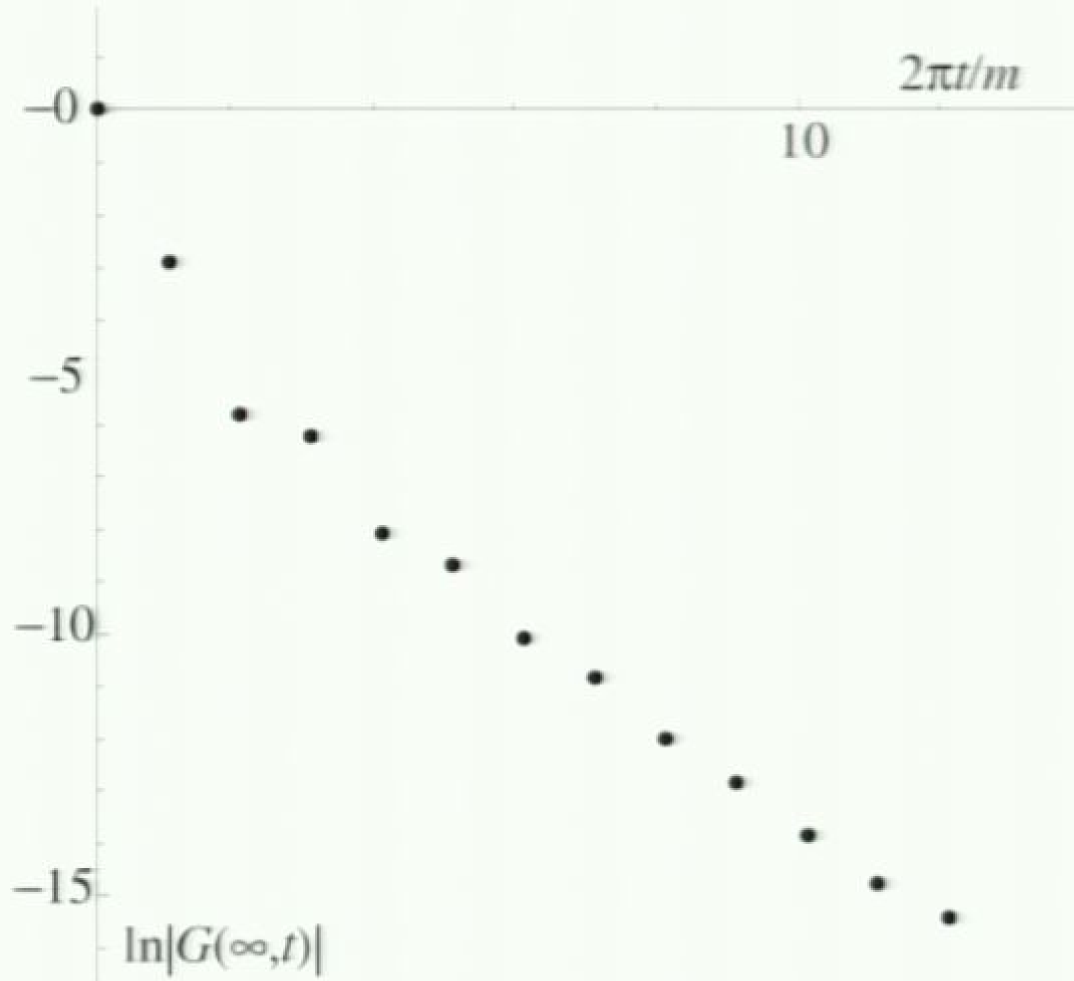

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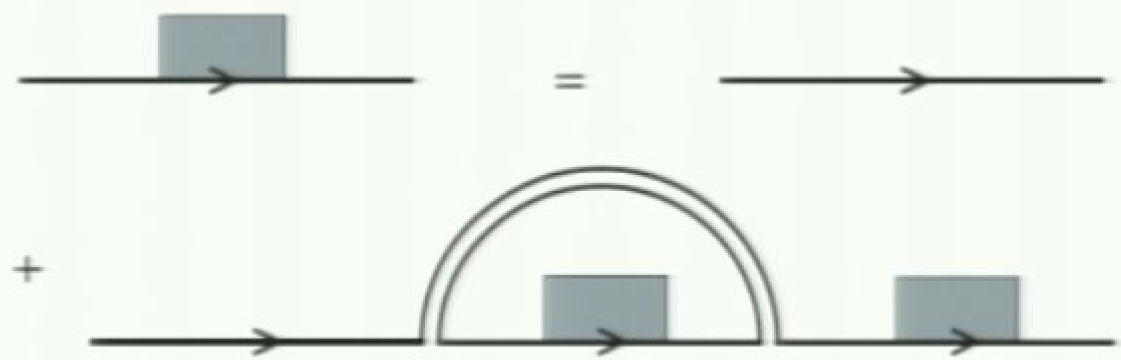

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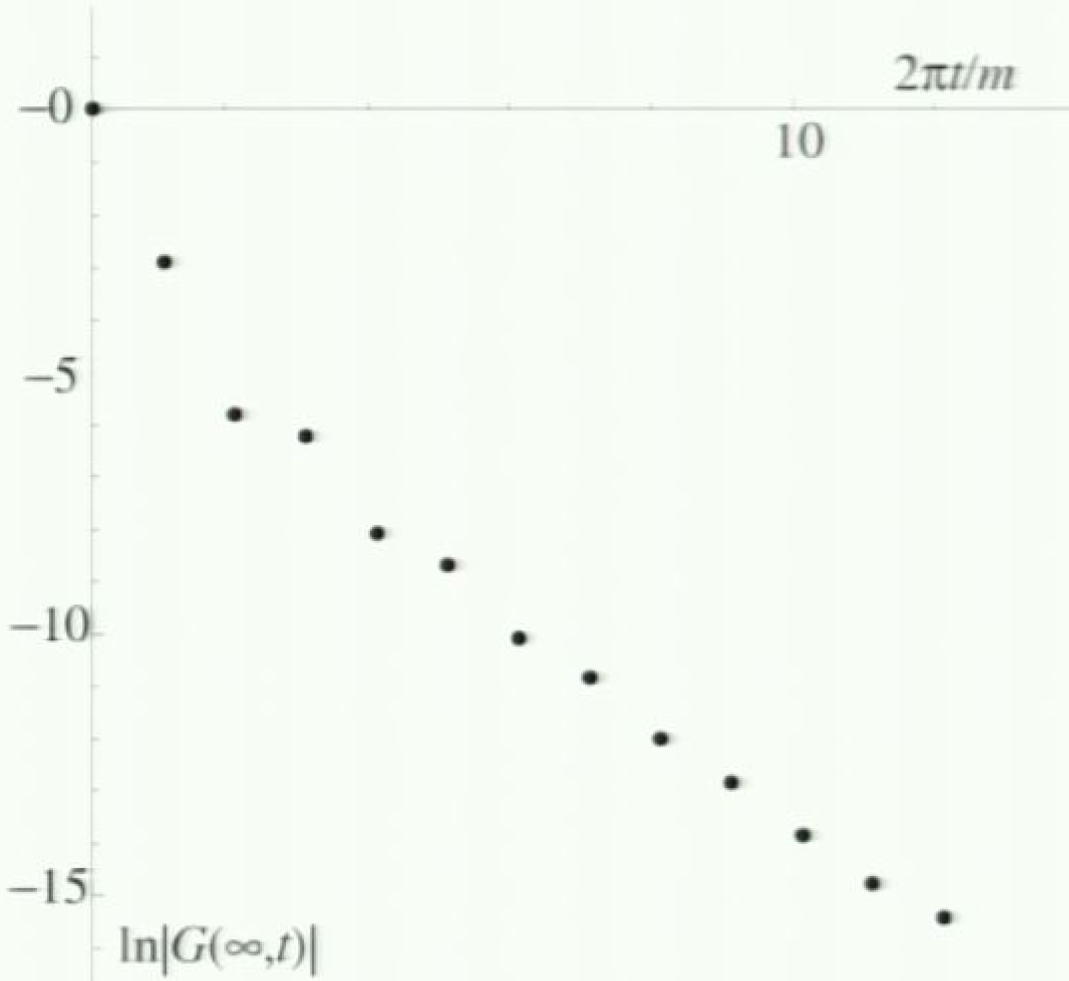


Mini-summary for the cubic model

(Iizuka and Polchinski)

Demonstrated exponential decay at high temperature.

Numerical results.

Analytic solutions desired.

Study a quartic model to compute perturbative 1/N corrections (Iizuka, TO, Polchinski)

The Hamiltonian is

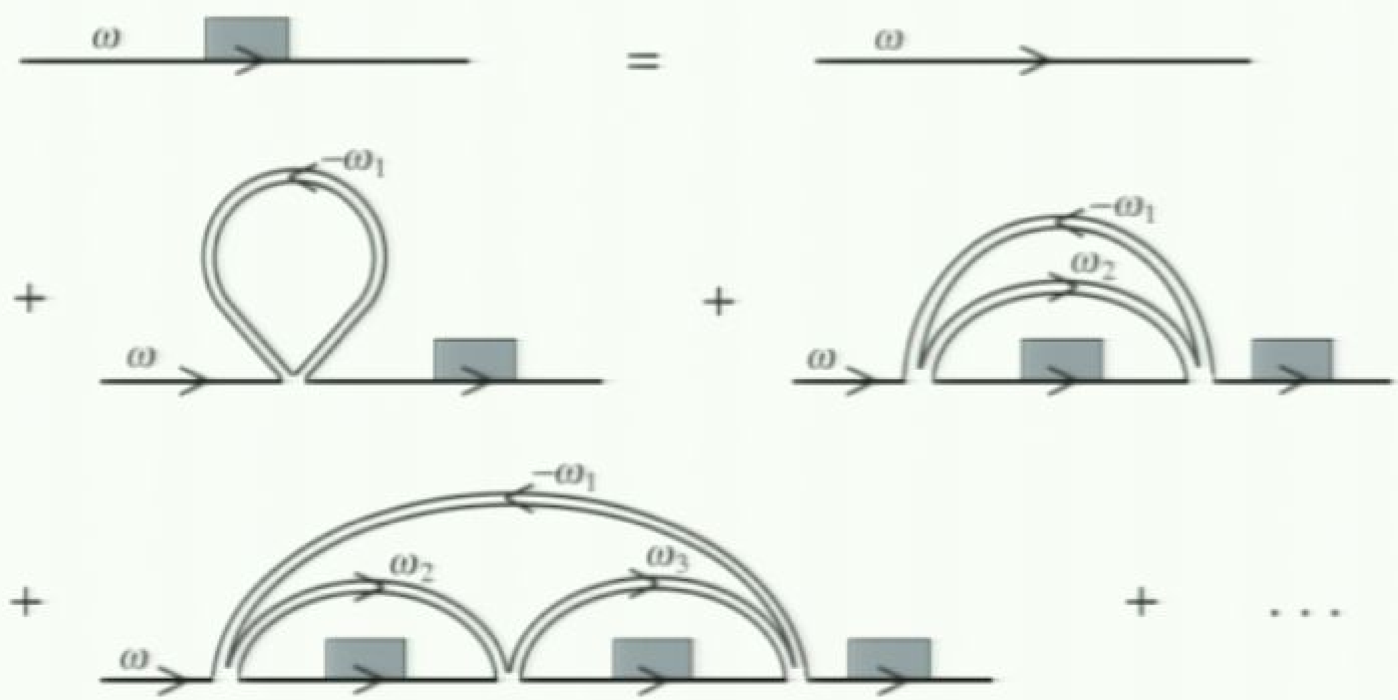
$$H = mA^{\dagger}_{ij}A_{ji} + Ma^{\dagger}_{i}a_{i} + H_{\text{int}}$$

$$H_{\text{int}} = -hq_{li}Q_{il} \ , \quad Q_{il} = A^{\dagger}_{ij}A_{jl}$$
$$q_{li} = -a^{\dagger}_{i}a_{l}$$

The model has a large amount of symmetries, and is not generic.

We used Feynman diagrams and the SD equations to compute corrections

Genus zero



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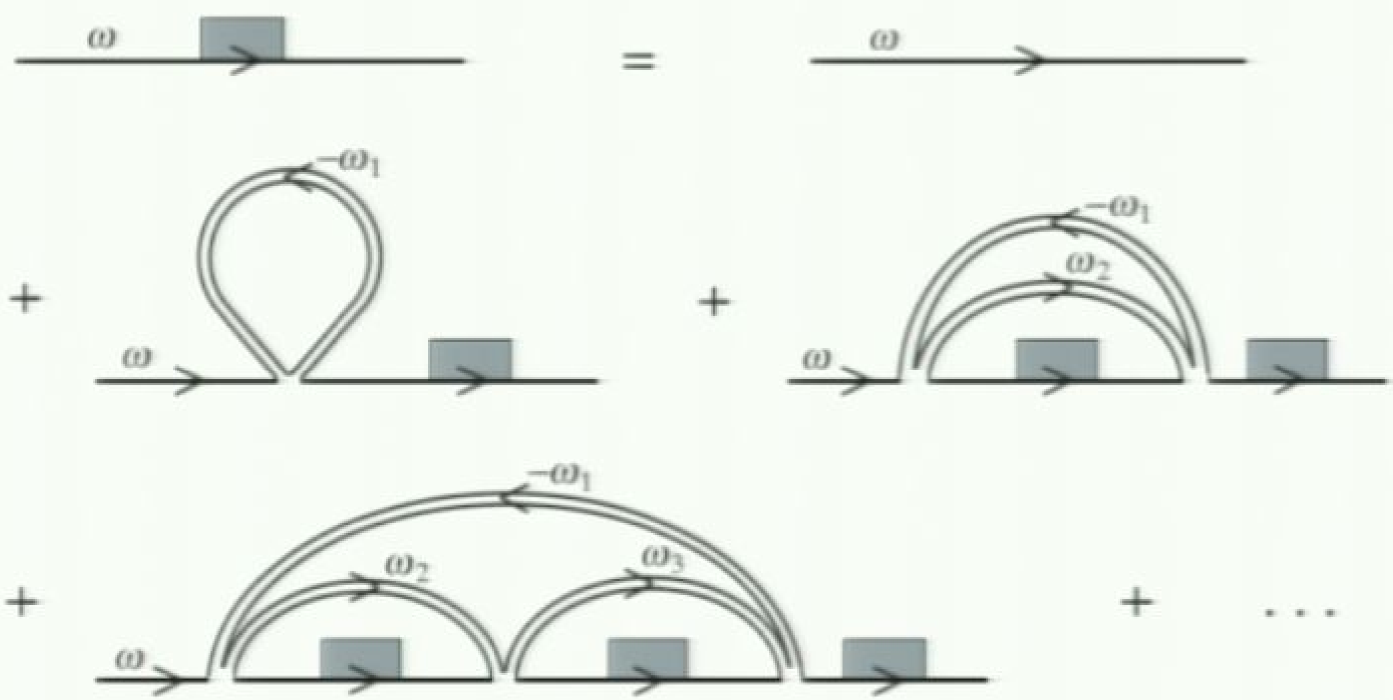

Continuous spectrum remains after a 1/N correction is included

Genus zero: a branch cut

$$\tilde{G}^{(0)}(T,\omega) = \frac{i(1-y)}{2\omega\lambda}\left(\lambda + \omega - \sqrt{(\omega-\omega_+)(\omega-\omega_-)}\right)$$

Genus one: still a branch cut = continuous spectrum
=> No recurrence.

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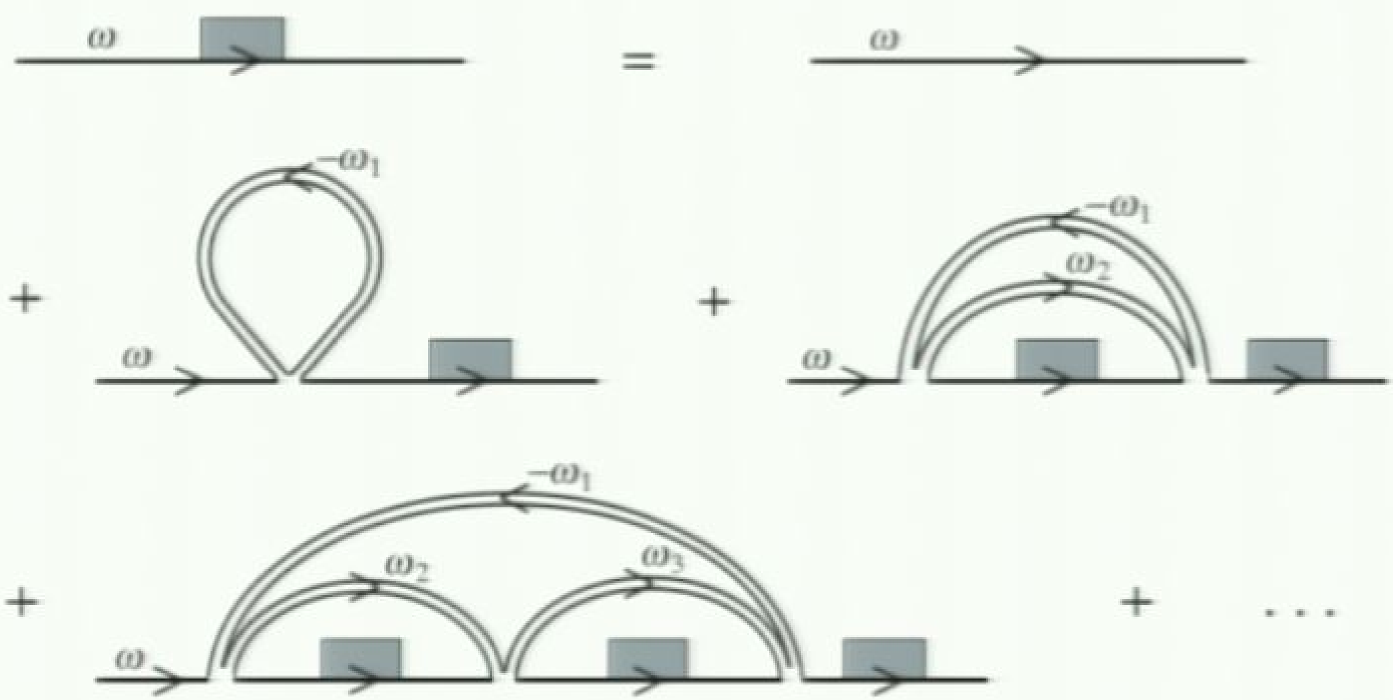

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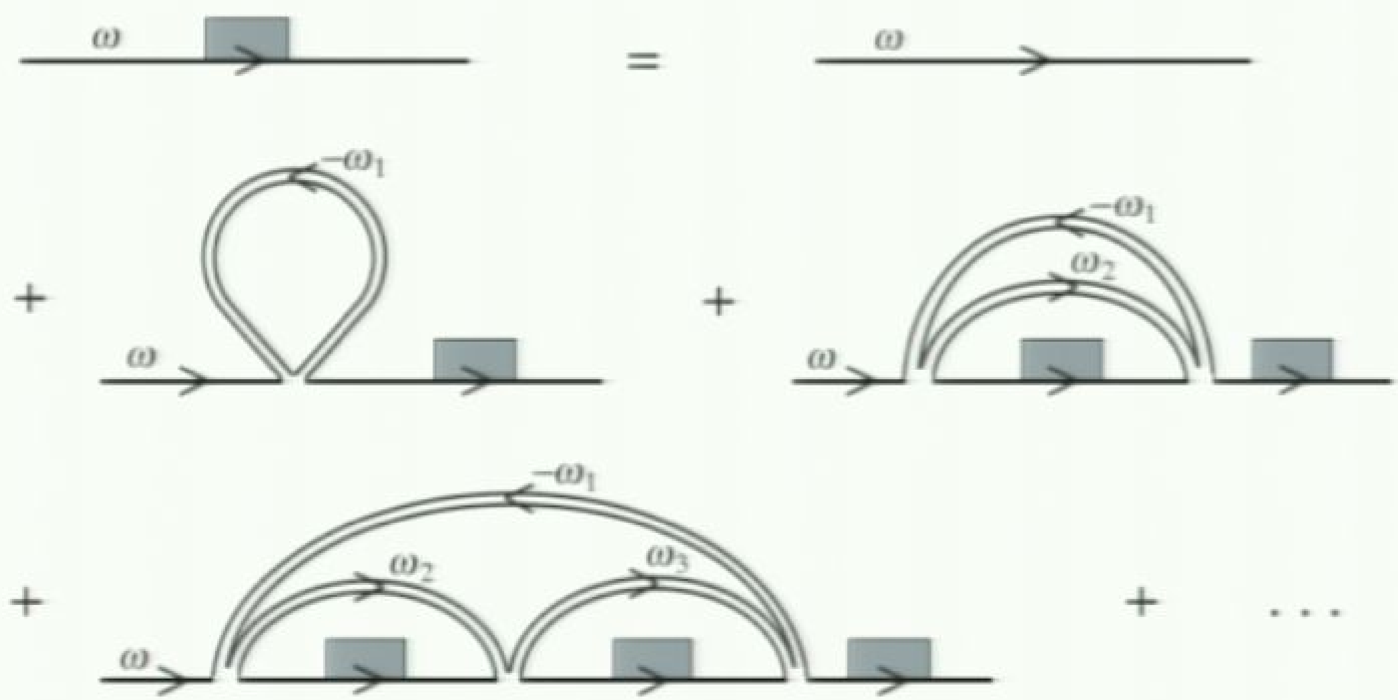

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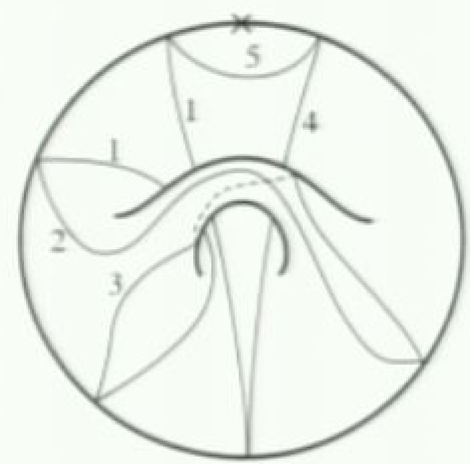
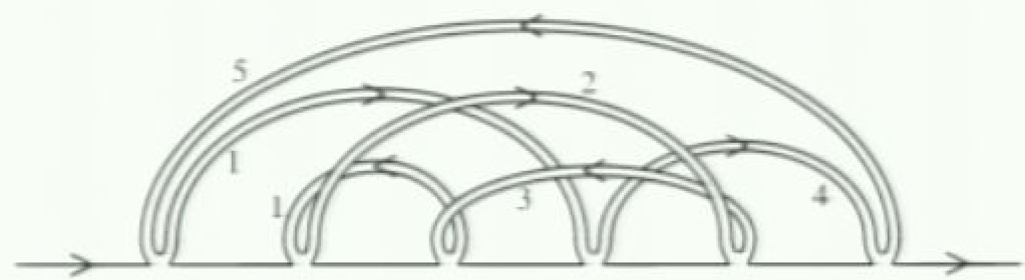
The diagrams for genus one can be enumerated.

The genus zero and one SD equations can be solved.





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2. Loop equations can be used to compute 1/N corrections

For any operator $\mathcal{O}_{ji}$, the following equation holds.

$$\langle \mathcal{O}_{ji} A_{ij} \rangle = \frac{y}{1-y} \langle [A_{ij}, \mathcal{O}_{ji}] \rangle \qquad y = e^{-m/T}$$

This relation can be used to compute

$$NG(t) = \theta(t) \left\langle \mathrm{Tr}\, e^{-ihQt} \right\rangle \; ,$$

$$N\tilde{G}(\omega) = \left\langle \mathrm{Tr} \frac{i}{\omega - hQ} \right\rangle \; .$$

Loop equations are similar to the bulk equations of motion

For example, we can derive the relation (loop equation)

$$\langle \mathrm{Tr}\, Q \, \mathrm{Tr}\, Q \rangle = \frac{N^2 y}{(1-y)^2} + \langle \mathrm{Tr}\, Q \rangle^2$$

between U(N) invariant correlators. Metrics in the bulk are also invariant under the boundary gauge Transformations.

Genus zero and and higher contributions to G(t) can be computed using the loop equations.

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3. The correlator can be rewritten as a sum over Young tableaux

The charge-charge interaction can be written as a sum of quadratic Casimirs:

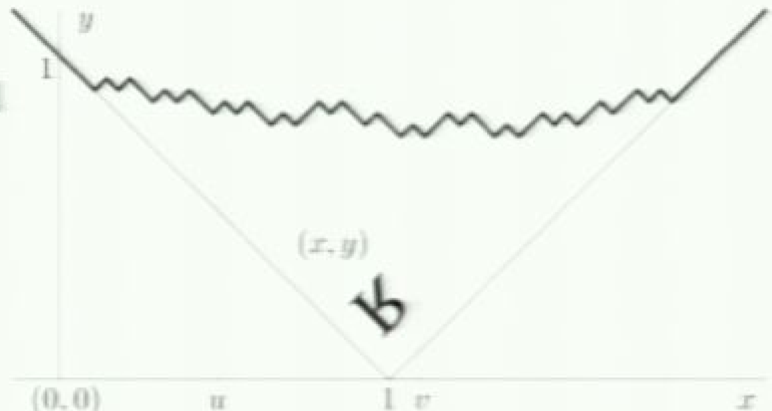
$$2q \cdot Q = (q + Q)^2 - q^2 - Q^2$$

The spectrum can be found by decomposing the Hilbert space into representations R of U(N).

$$-i\tilde{G}(\omega) = (1 - y)^{N^2} \sum_R y^{|R|} (\dim R)^2 \, \Omega_R(\omega)$$

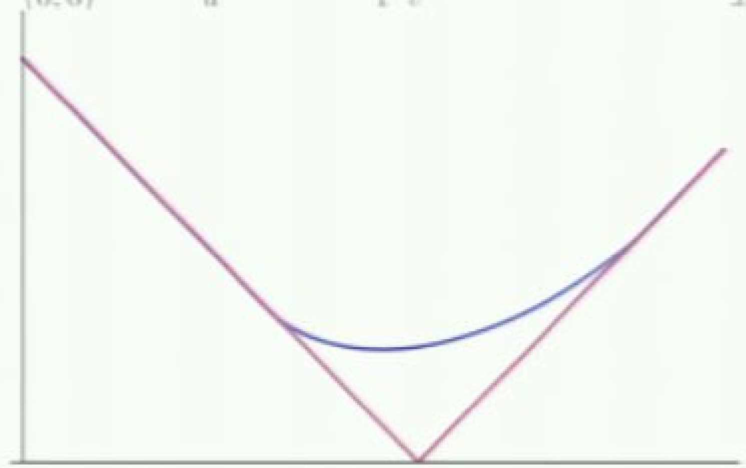
The sum can be approximated by a functional integral, and then a saddle point

R :representation=Young tableau



The summand is expressed in terms of the shape function.

In the large N limit, the sum becomes a functional integral.



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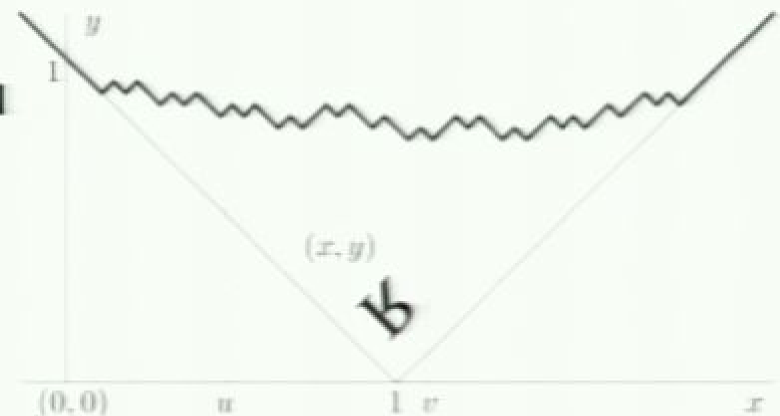

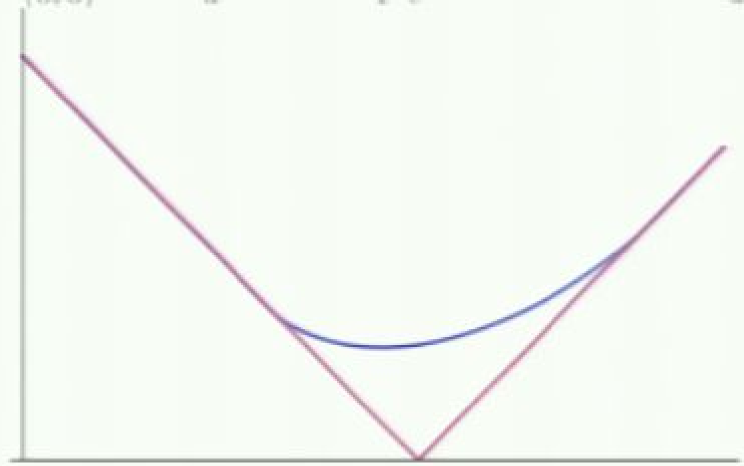

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$$2q \cdot Q = (q+Q)^2 - q^2 - Q^2$$

The spectrum can be found by decomposing the Hilbert space into representations R of U(N).

$$-i\tilde{G}(\omega) = (1-y)^{N^2} \sum_R y^{|R|} (\dim R)^2 \, \Omega_R(\omega)$$

The sum can be approximated by a functional integral, and then a saddle point

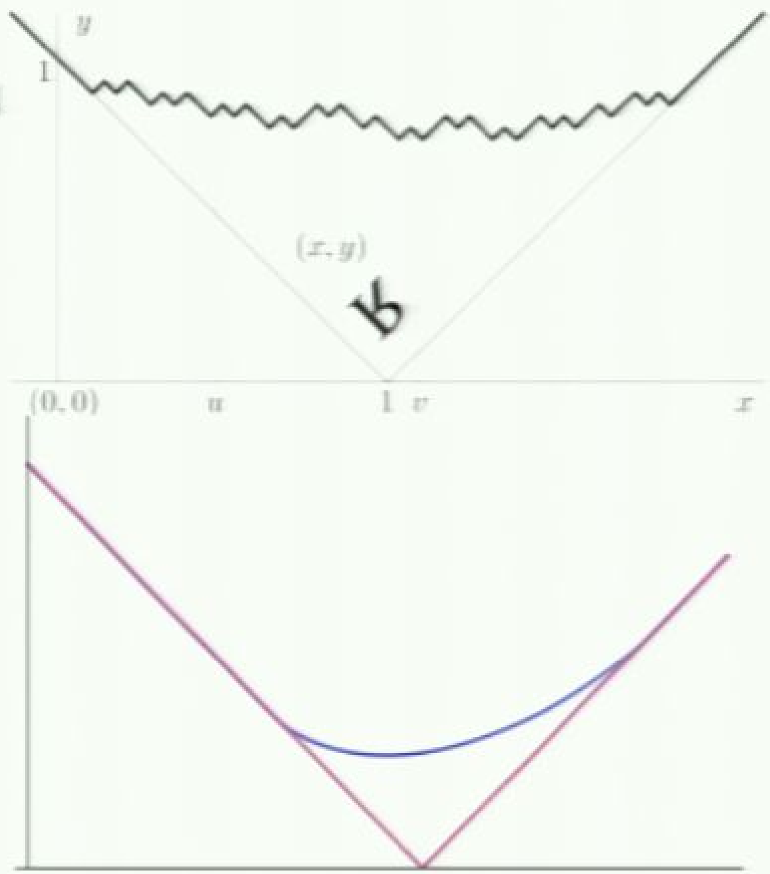
R :representation=Young tableau

The summand is expressed in terms of the shape function.

In the large N limit, the sum becomes a functional integral.

The saddle = typical tableau

➔ Genus zero correlator



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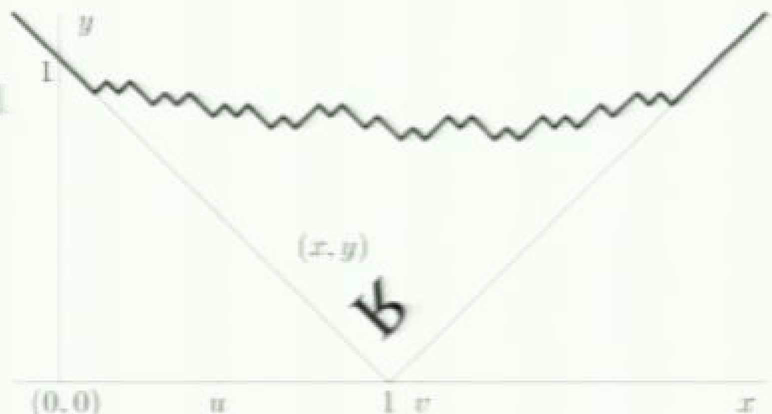

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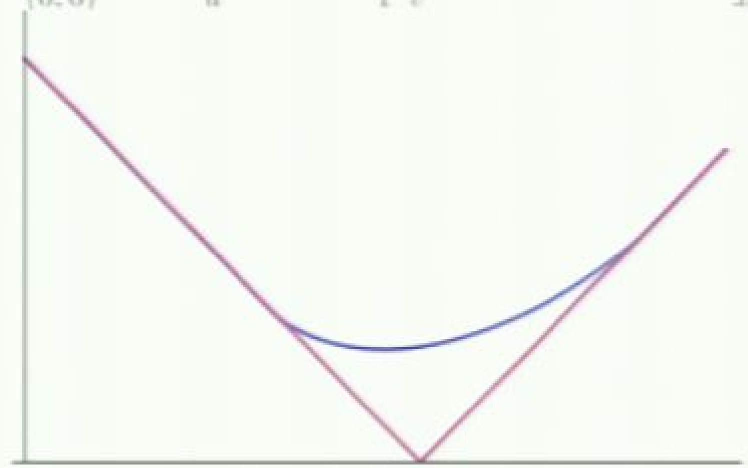

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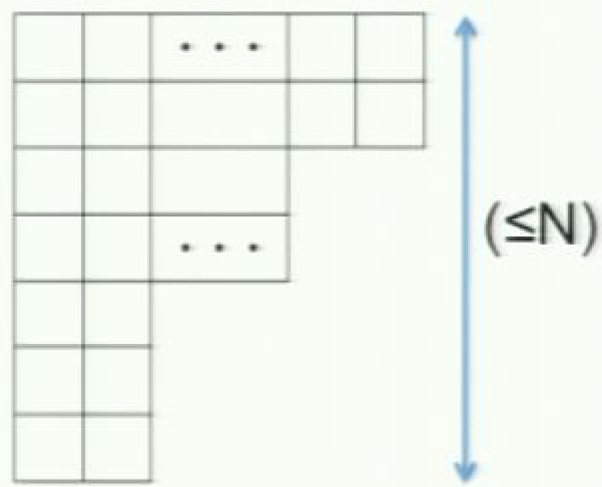
The typical tableau is an example of emergent geometry

In the large N (thermodynamic) limit an effective geometry appears.

There is a bound (≤N) on the number of rows, so the size of the Hilbert space is smaller than the unbounded case. (**Stringy exclusion principle**)



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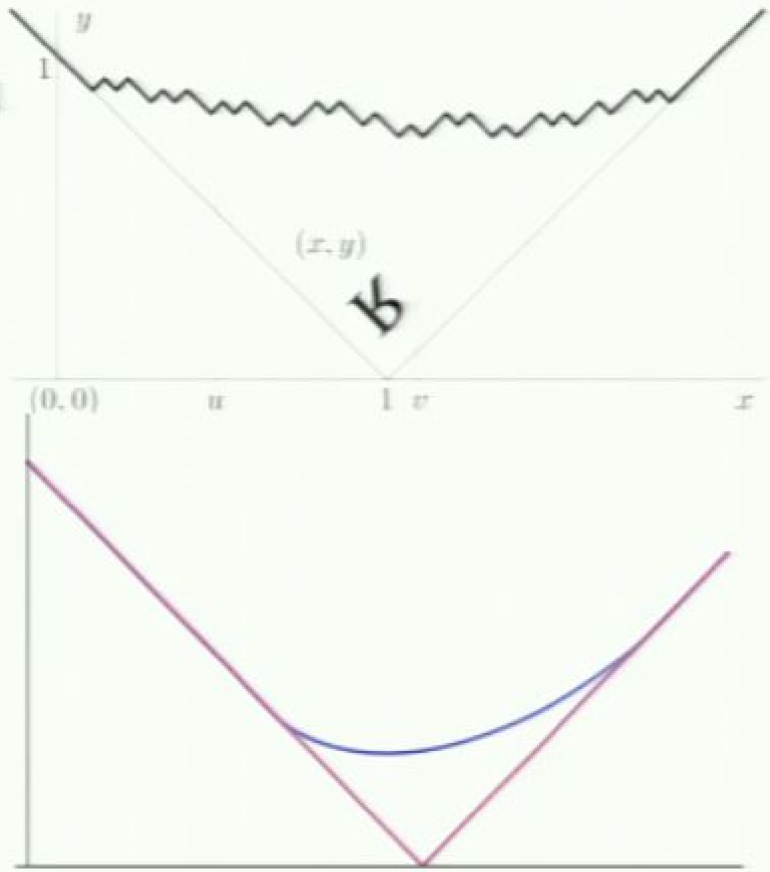

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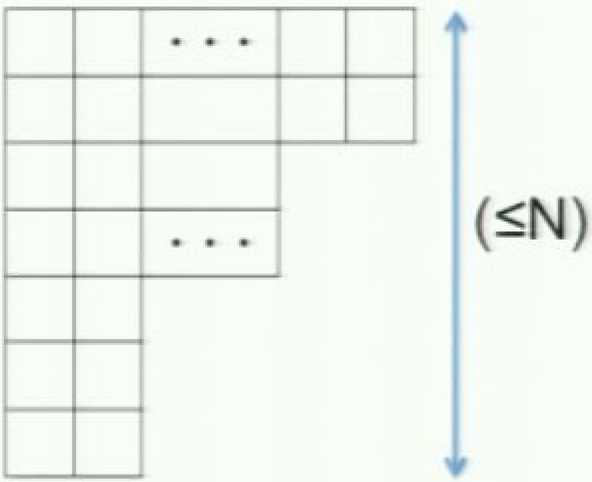

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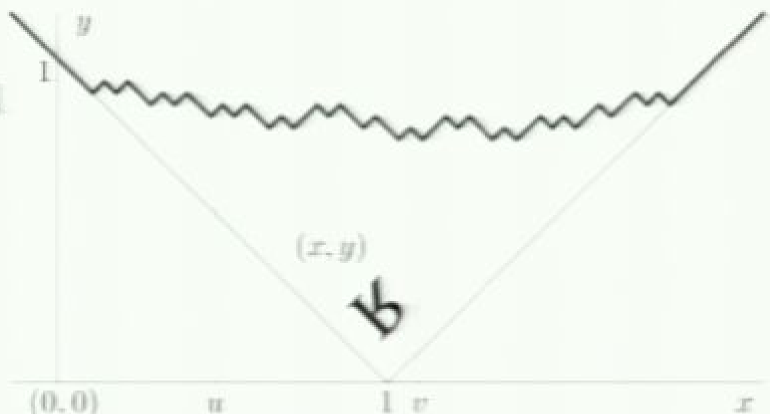

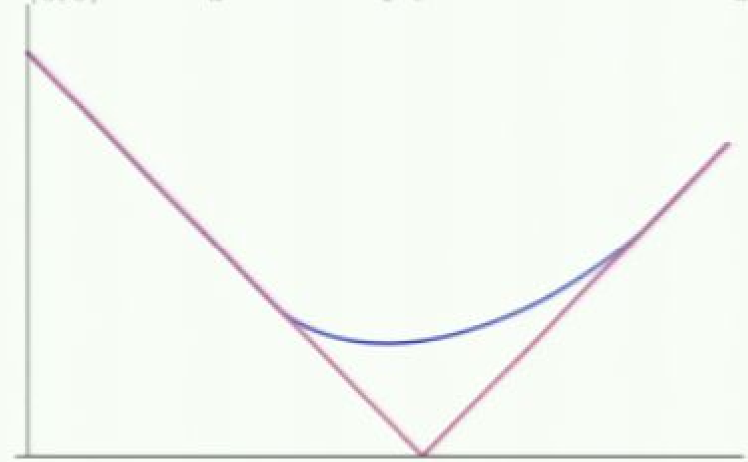

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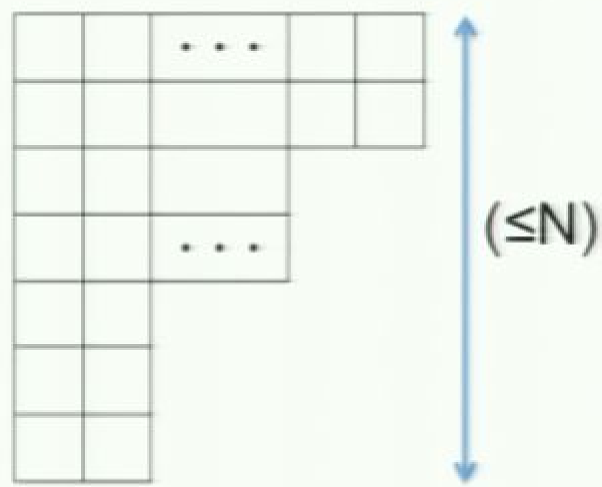

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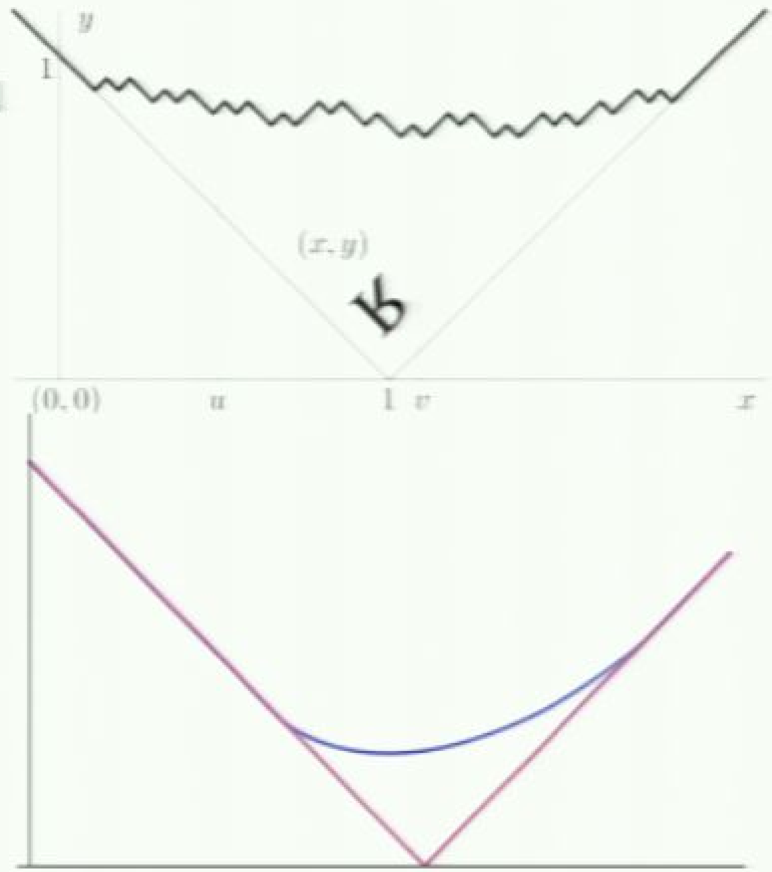

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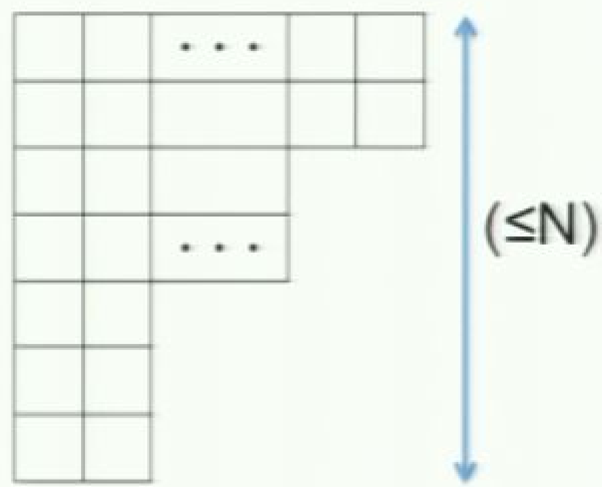

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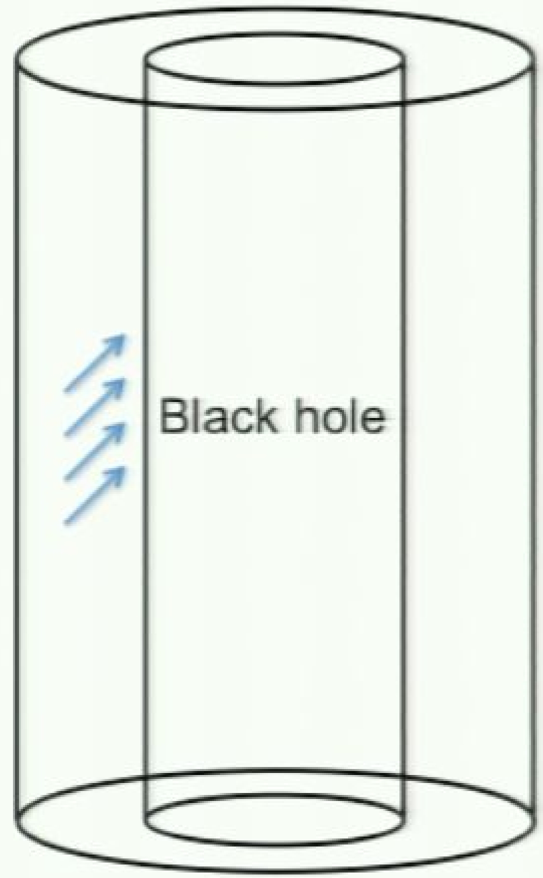

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However, only e^S of them can be independent in quantum theory.

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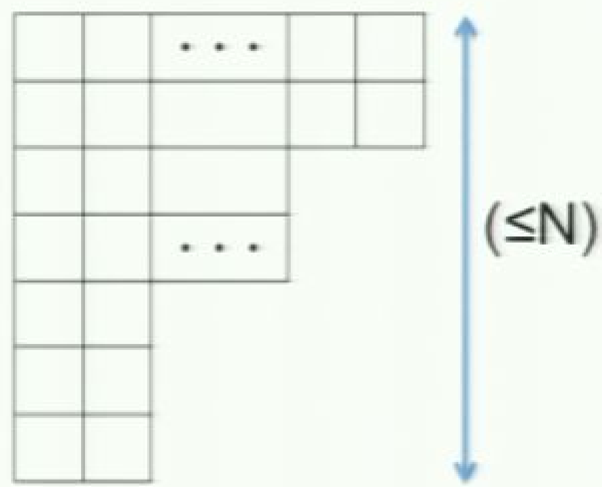

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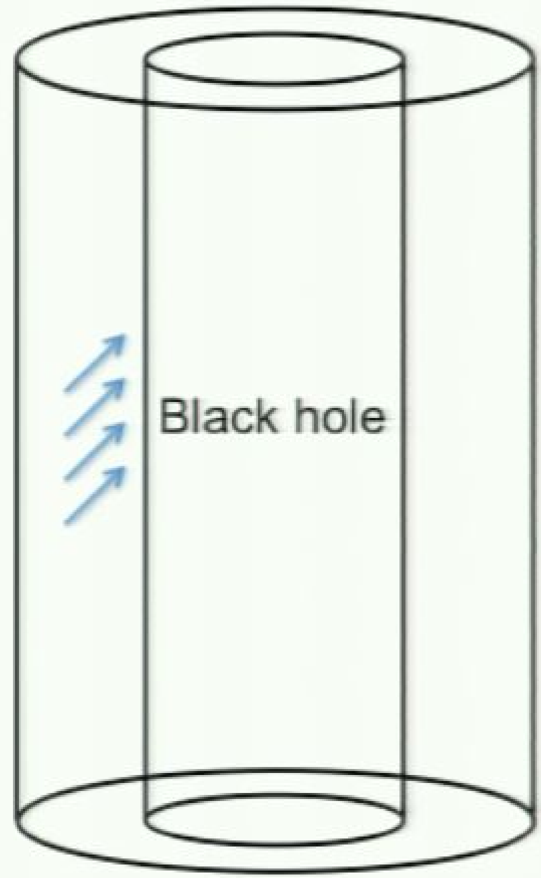

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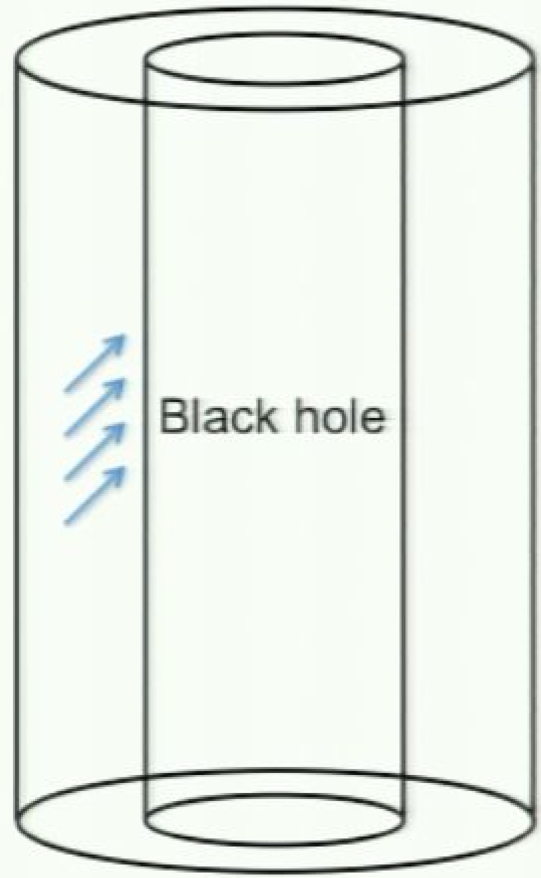

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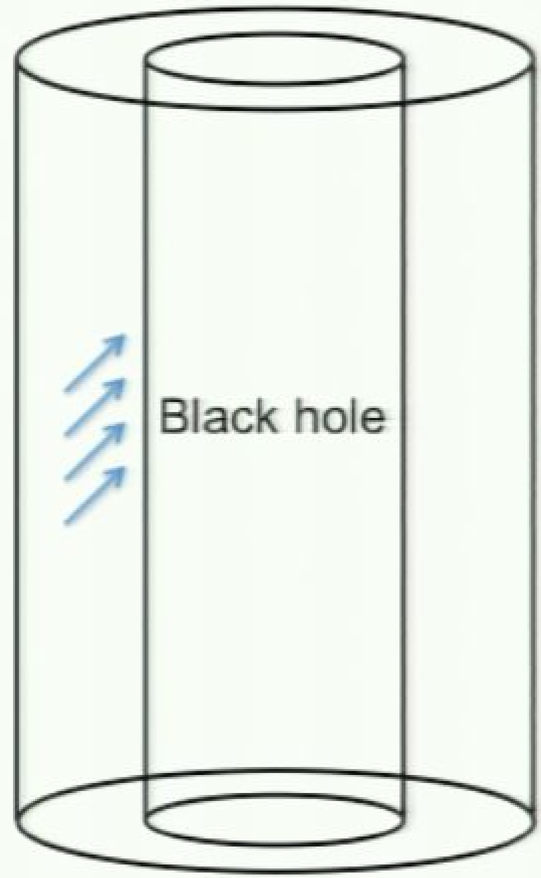

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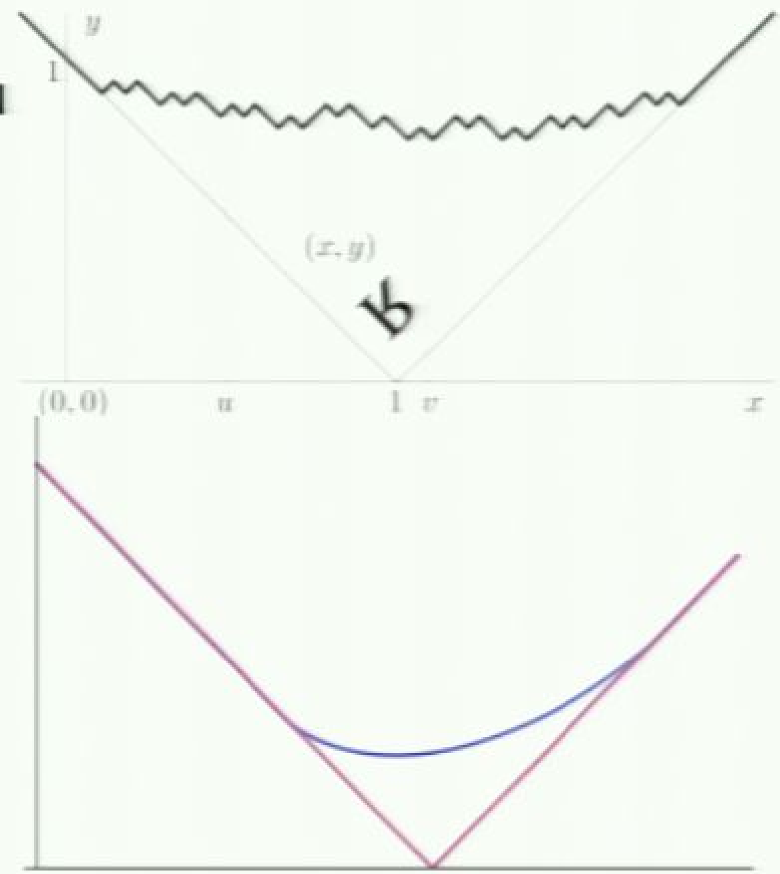

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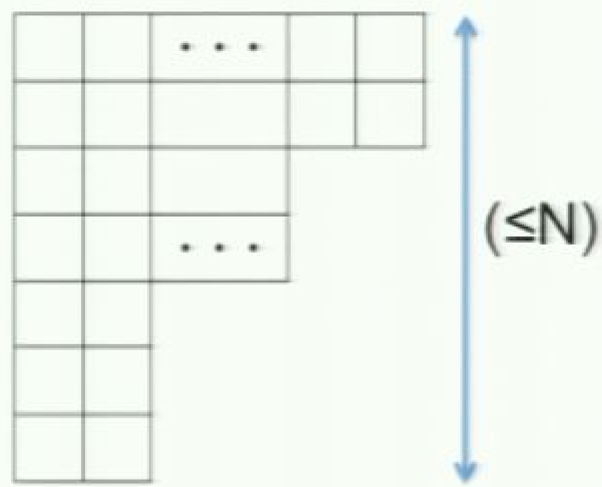

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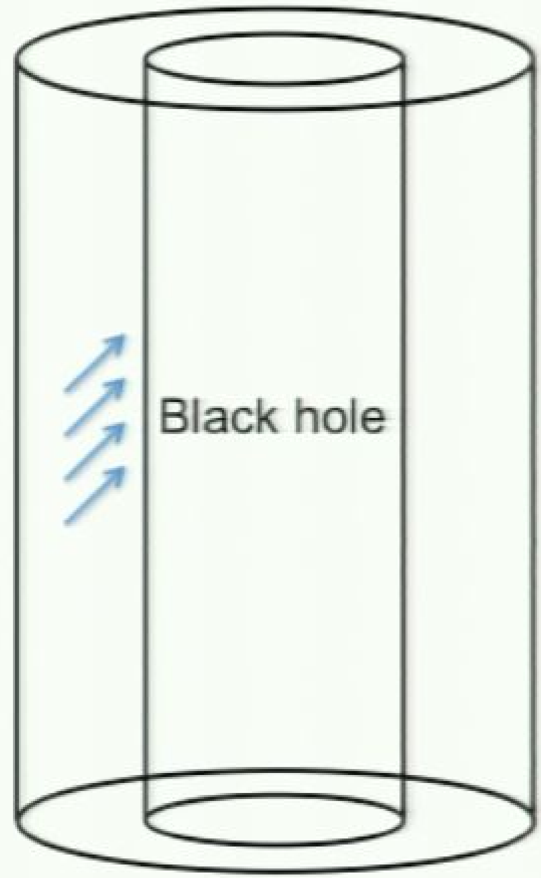

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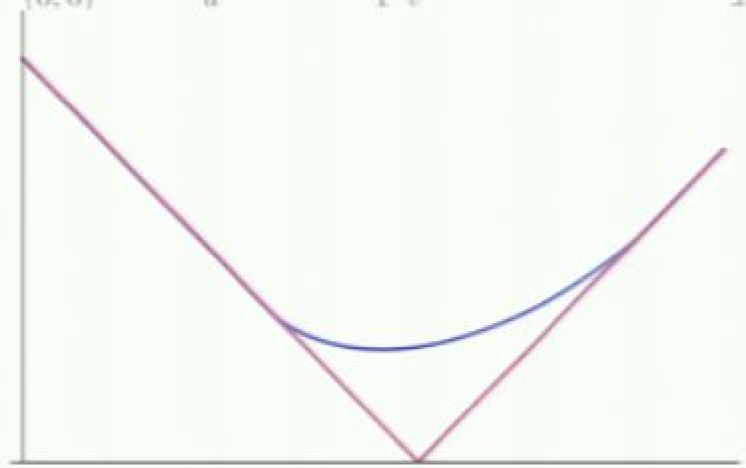

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2. Loop equations can be used to compute 1/N corrections

For any operator $\mathcal{O}_{ji}$, the following equation holds.

$$\langle \mathcal{O}_{ji} A_{ij} \rangle = \frac{y}{1-y} \langle [A_{ij}, \mathcal{O}_{ji}] \rangle \qquad y = e^{-m/T}$$

This relation can be used to compute

$$NG(t) = \theta(t) \left\langle \mathrm{Tr}\, e^{-ihQt} \right\rangle \ ,$$

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Continuous spectrum remains after a 1/N correction is included

Genus zero: a branch cut

$$\tilde{G}^{(0)}(T,\omega) = \frac{i(1-y)}{2\omega\lambda}\left(\lambda+\omega-\sqrt{(\omega-\omega_+)(\omega-\omega_-)}\right)$$

Genus one: still a branch cut = continuous spectrum
=> No recurrence.

$$\tilde{G}^{(1)}(T,\omega) = \frac{iy^2x_0^3(1-x_0)^4(1-x_0[1-y])}{(1-2x_0+x_0^2[1-y])^4(\omega[1-x_0]^2-\lambda_y y)}$$

$$x_0 = -i\lambda_y\tilde{G}^{(0)}(\omega) \qquad \lambda_y = \frac{\lambda}{1-y} = \frac{hN}{1-y}$$

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What restores information?

Maldacena and Hawking conjectured that the sum over geometries (saddle points) restores information.

A known saddle point is the thermal AdS, which has the same boundary as the AdS black hole (Hawking & Page).

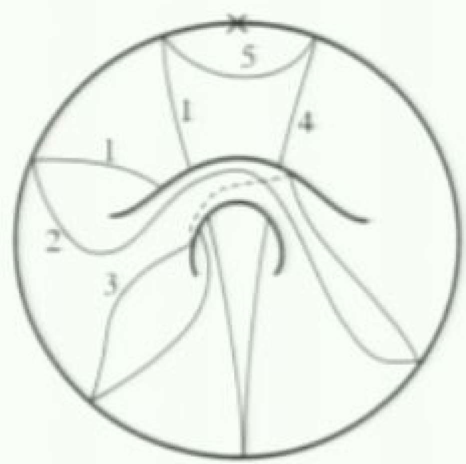
The thermal AdS is expected to be realized as a saddle in the gauge field integral = Polyakov loop. (Aharony et al.)

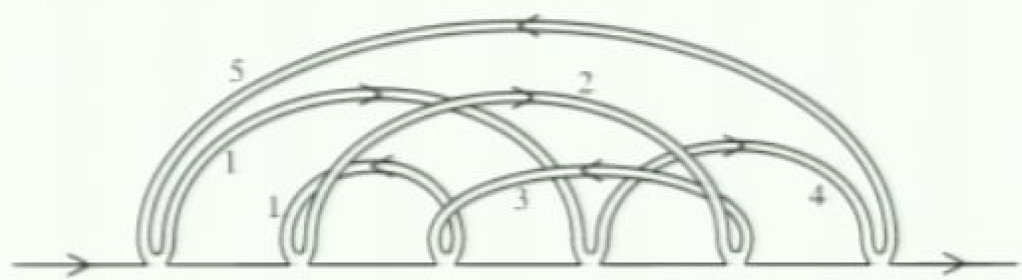
$$\int [DA_\tau] P e^{\oint_0^\beta d\tau A_\tau} \dots$$

We used Feynman diagrams and the SD equations to compute corrections

The diagrams for genus one can be enumerated.

The genus zero and one SD equations can be solved.





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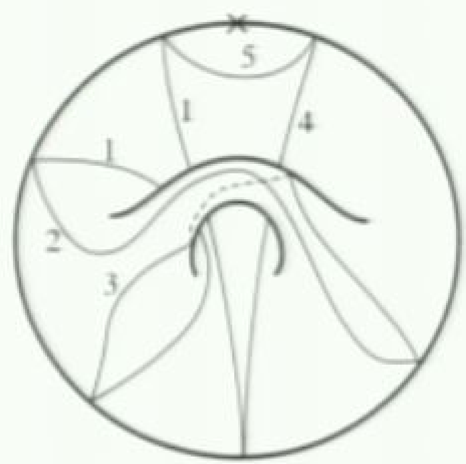

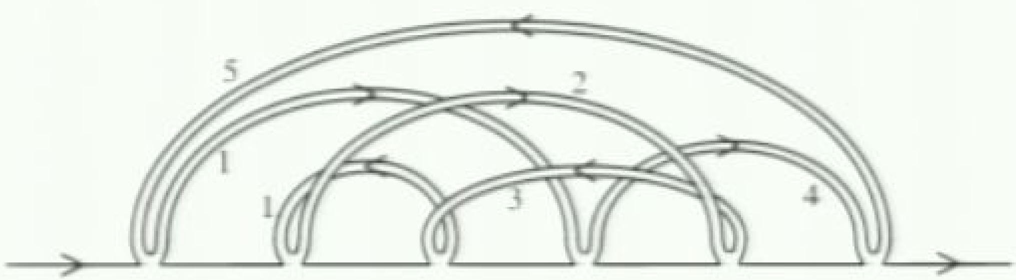

Study a quartic model to compute perturbative 1/N corrections (Iizuka, TO, Polchinski)

The Hamiltonian is

$$H = mA^{\dagger}_{ij}A_{ji} + Ma^{\dagger}_{i}a_{i} + H_{\text{int}}$$

$$H_{\text{int}} = -hq_{li}Q_{il} \ , \quad Q_{il} = A^{\dagger}_{ij}A_{jl}$$

$$q_{li} = -a^{\dagger}_{i}a_{l}$$

The model has a large amount of symmetries, and is not generic.

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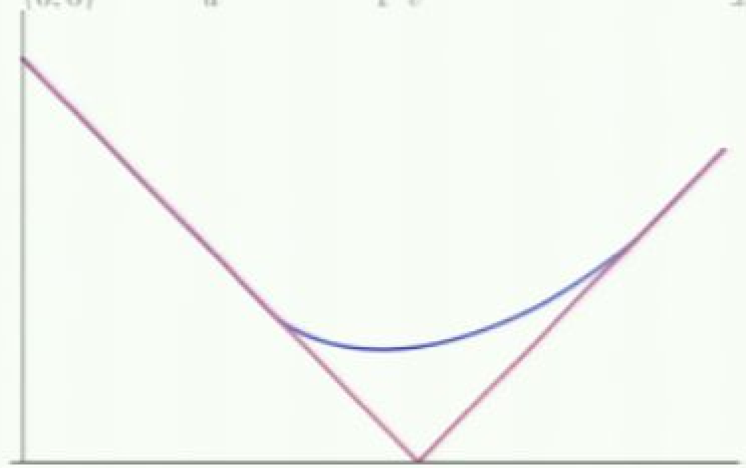

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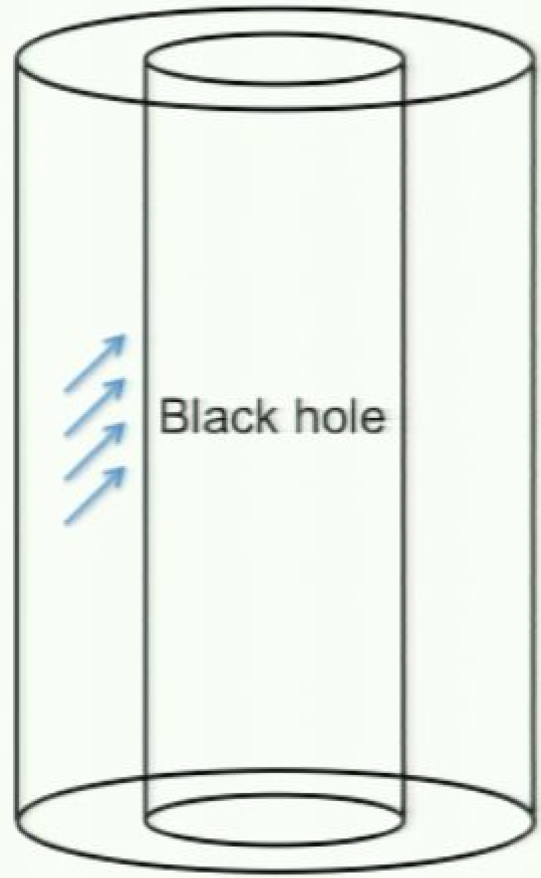

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We didn't gauge the U(N) symmetry, so the second saddle is not the reason for information restoration.

There are other works arguing against the conjecture. (Barbon, Rabinovic, Birmingham, Sachs, Solodukhin, Kleban, Porrati, Rabadan, ...)

Summary and conclusions

Iizuka and Polchinski demonstrated exponential decay in a large N matrix model.

Perturbative 1/N corrections do not resolve information loss, which requires non-perturbative effects.

Our models are examples where recurrences are restored without extra saddle points.

The subtlety in the large N limit of matrix quantum mechanics may be responsible for quantum behaviors of black holes.

Open problems and future directions

Analytic understanding of the exponential decay.

Matrix models for other problems. For example, look for a model with the largest $\Gamma \sim \log N^2$ (c.f. later talks by Hayden and Sekino).

Models with gravity duals (c.f. Nishimura's talk)

Exponential decay for open strings in the AdS black hole background.