

Title: Schwarzschild radius and black hole thermodynamics with alpha' corrections from simulations of SUSY matrix quantum mechanics

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Abstract: We present new results from our Monte Carlo simulation of SUSY matrix quantum mechanics with 16 supercharges at finite temperature. The internal energy can be fitted nicely to the behavior predicted from the dual black hole thermodynamics including the alpha' corrections. The temporal Wilson loop can also be predicted from the gravity side, and it is directly related to the Schwarzschild radius of the dual black hole geometry. Our results for the Wilson loop indeed confirm this prediction up to subleading terms anticipated from the alpha' corrections on the gravity side. All these results give us strong support and a firm basis for the idea to use matrix model simulations to study quantum gravity.

# Schwarzschild Radius and Black Hole Thermodynamics with $\alpha'$ Corrections from Simulations of SUSY Matrix Quantum Mechanics

Jun Nishimura (KEK)

Talk at “Black Holes and Quantum Physics” workshop,  
Perimeter Institute, Jan.24, '09

Ref.) Anagnostopoulos-Hanada- J.N.-Takeuchi, PRL 100 ('08) 021601  
Hanada-Miwa-J.N.-Takeuchi, arXiv:0811.2081[hep-th]  
Hanada-Hyakutake-J.N.-Takeuchi, arXiv:0811.3102[hep-th]

- based on collaborations with

Konstantinos Anagnostopoulos  
(National Technical University, Athens, Greece)

Masanori Hanada (Weizmann Inst., Israel)

Yoshifumi Hyakutake (Osaka Univ.)

Akitsugu Miwa  
(U. of Tokyo, Komaba  
→ Harish-Chandra Research Inst., India)

Shingo Takeuchi (KEK → APCTP, Korea)

# 0. Introduction

# Simulating Quantum Black Holes

"quantum black hole"

or

microscopic description  
of black holes

requires

superstring theory



gauge/string  
duality ('98-)

strongly coupled gauge theory

Monte Carlo simulations  
analogous to lattice QCD

□ Anagnostopoulos-Hanada-J.N.-Takeuchi, PRL 100 ('08)  
021601 [arXiv:0707.4454]

NEW

CERN COURIER, March 2008

KEK

## Superstrings reveal the interior structure of a black hole

A research group at KEK has succeeded in calculating the state inside a black hole using computer simulations based on superstring theory. The calculations confirmed for the first time that the temperature dependence of the energy inside a black hole agrees with the power-law behavior expected from calculations based on Stephen Hawking's theory of black-hole radiation.

The result demonstrates that the behaviour of elementary particles in a collection of strings in superstring theory can explain thermodynamical properties of black holes.

In 1974, Stephen Hawking at Cambridge showed theoretically that black holes are not entirely black. A black hole in fact emits light and particles from its surface, so that it shrinks little by little. Since then, physicists have suspected that black holes should have a certain interior structure, but they have been unable to describe the state inside a black hole using general relativity, as the curvature of space-time becomes so large towards the centre of the hole that quantum effects make the theory no longer applicable. Superstring theory, however, offers the possibility of bringing together general relativity and quantum mechanics in a consistent manner, so many theoretical physicists have been investigating whether this theory can describe the interior of a black hole.

Jun Nishimura and colleagues at KEK established a method that effectively treats the oscillation of elementary strings depending on their frequency. They used the Hitachi SR1000 model K1 supercomputer installed at KEK in March 2006 to calculate the thermodynamical behaviour of the collection of strings inside a black hole. The results showed that as the temperature



The Hitachi SR1000 model K1 supercomputer calculates the interior structure of a black hole. It provides a peak performance of 2.15 teraflops peak. (Courtesy of KEK)

decreased, the simulation reproduced behavior of a black hole as predicted by Hawking's theory (figure 1).

This demonstrates that the mysterious thermodynamical properties of black holes can be explained by a collection of strings fluctuating inside. The result also indicates that superstring theory will develop further to play an important role in solving problems such as the evaporation of black holes and the state of the early universe.

### Further reading:

K.S. Anagnostopoulos et al. 2008 Phys. Rev. Lett. **100** 021601.

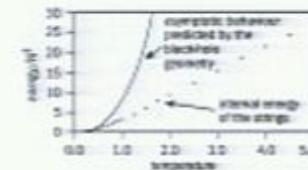


Fig. 1. Plot of the entropy of the collection of strings against the temperature. The solid line represents the behavior of the black hole predicted by Hawking's theory. The results agree in the lower temperature regime, where the calculation based on general relativity becomes valid.

# A basis of gauge/gravity duality

Maldacena ('97)

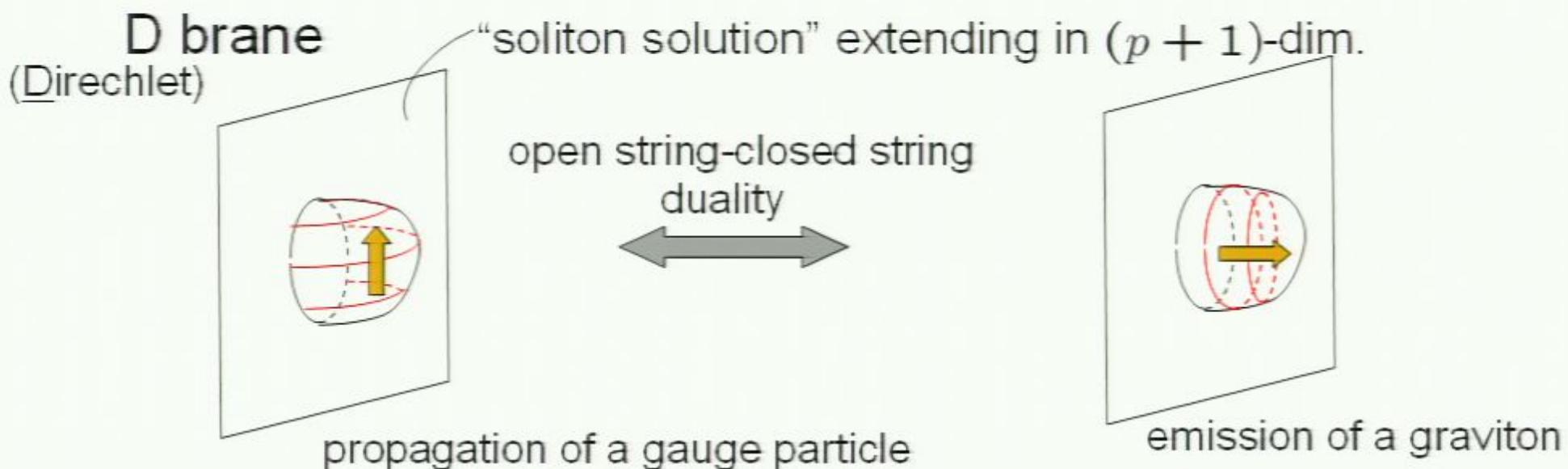


## ■ superstring

{ open string  
closed string

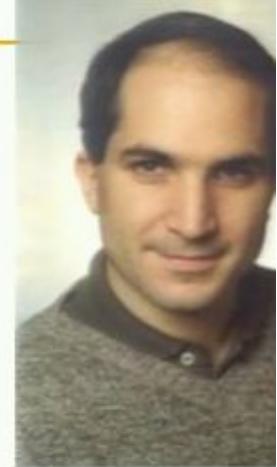
massless modes

gauge particles,...  
graviton, dilaton,...



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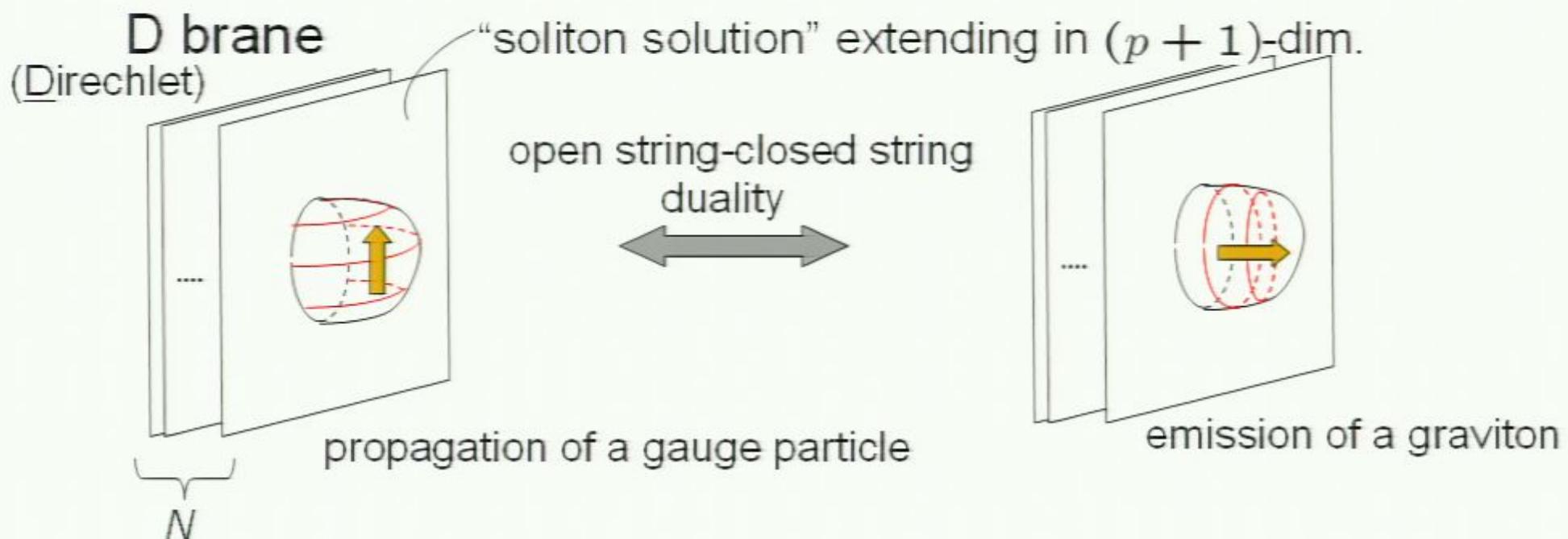


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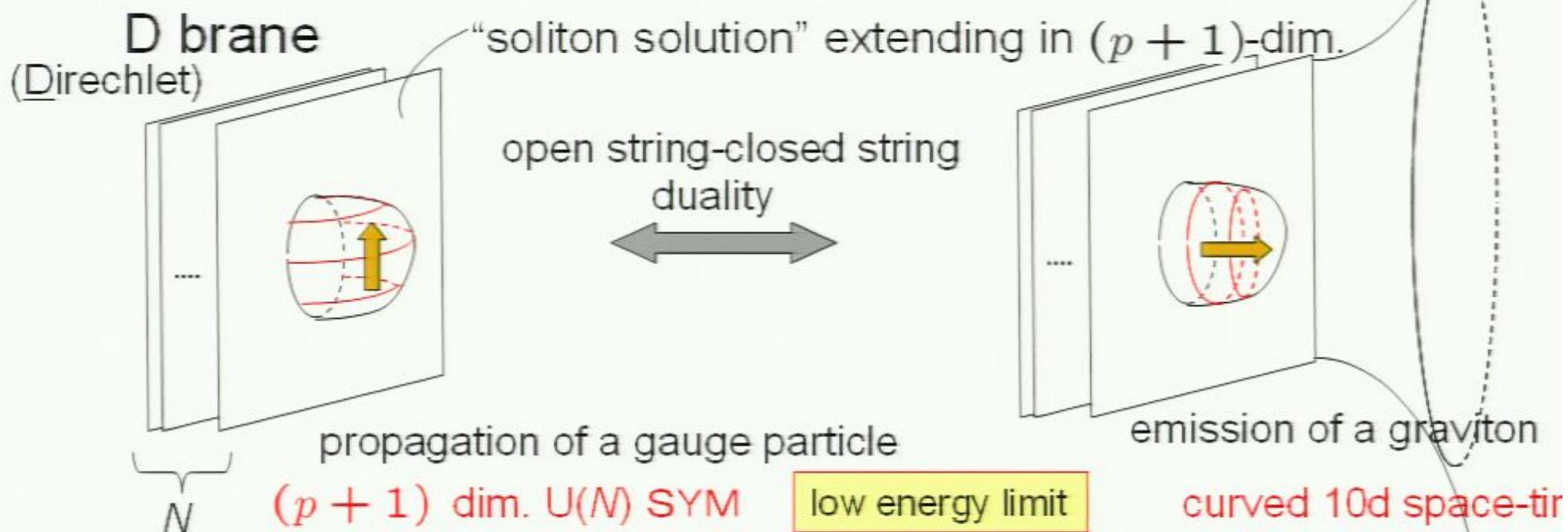


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# Gauge-gravity duality for D0-brane system

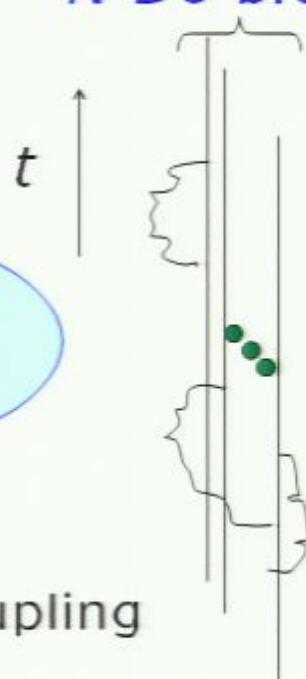
type IIA superstring

$N$  D0 branes

1d  $U(N)$  SUSY  
gauge theory

at finite  $T$

$\lambda$  : 't Hooft coupling



Itzhaki-Maldacena-Sonnenschein  
-Yankielowicz ('98)

horizon

black 0-brane solution  
in type IIA SUGRA

near-extremal black hole

In the decoupling limit, the D0 brane system describes the black hole **microscopically**.

large  $N$  and large  $\lambda$   $\rightarrow$  SUGRA description : valid

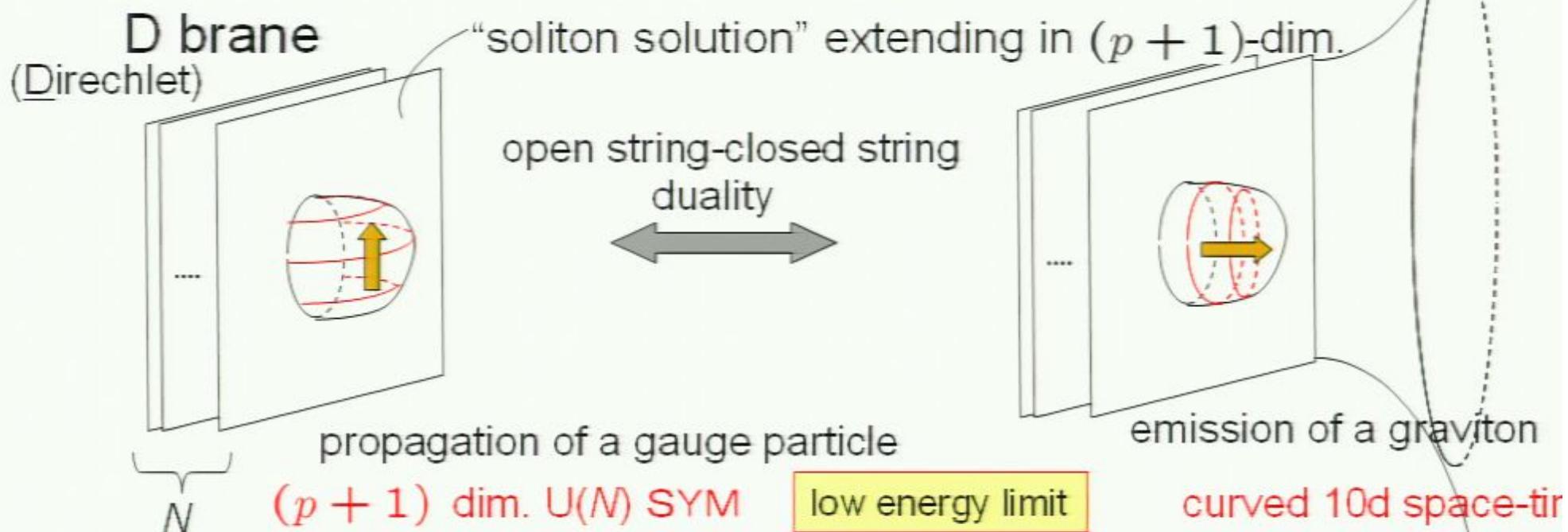
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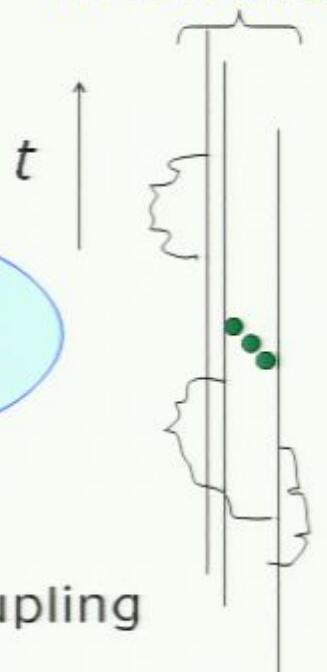
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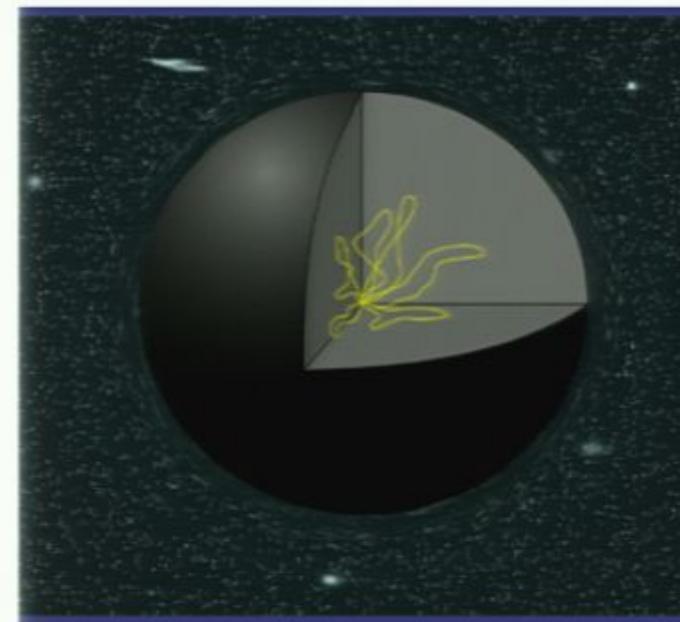
## black hole thermodynamics

Anagnostopoulos-Hanada-J.N.-Takeuchi ('08)

$$\frac{1}{N^2} \left( \frac{E}{\lambda^{1/3}} \right) = \underbrace{\frac{9}{14} \left\{ 4^{13} 15^2 \left( \frac{\pi}{7} \right)^{14} \right\}^{1/5}}_{7.41} \left( \frac{T}{\lambda^{1/3}} \right)^{14/5}$$

including  $\alpha'$  corrections

Hanada-Hyakutake-J.N.-Takeuchi, arXiv:0811.3102



## Schwarzschild radius from Wilson loop

Hanada-Miwa-J.N.-Takeuchi, arXiv:0811.2081[hep-th]

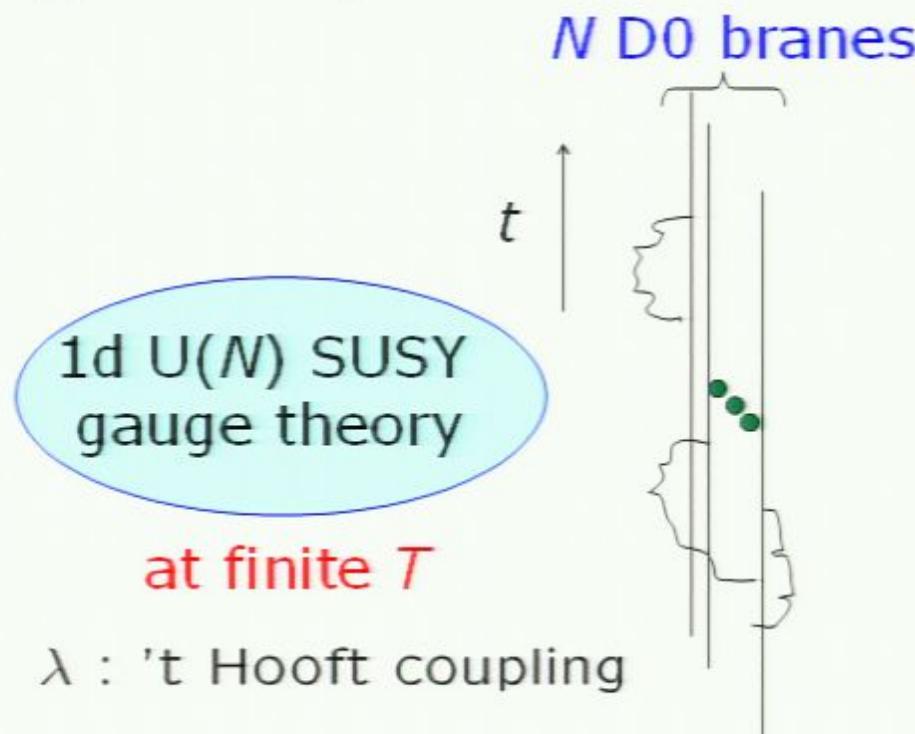
$$W \equiv \text{tr } \mathcal{P} \exp \left[ i \int_0^\beta dt \{ A(t) + i X_9(t) \} \right] \sim \exp \left( \frac{\beta R_{\text{Sch}}}{2\pi\alpha'} \right)$$

$$\ln W = \frac{\beta R_{\text{Sch}}}{2\pi\alpha'} = \underbrace{\frac{1}{2\pi} \left\{ \frac{16\sqrt{15}\pi^{7/2}}{7} \right\}^{2/5}}_{1.89} \left( \frac{T}{\lambda^{1/3}} \right)^{-3/5}$$

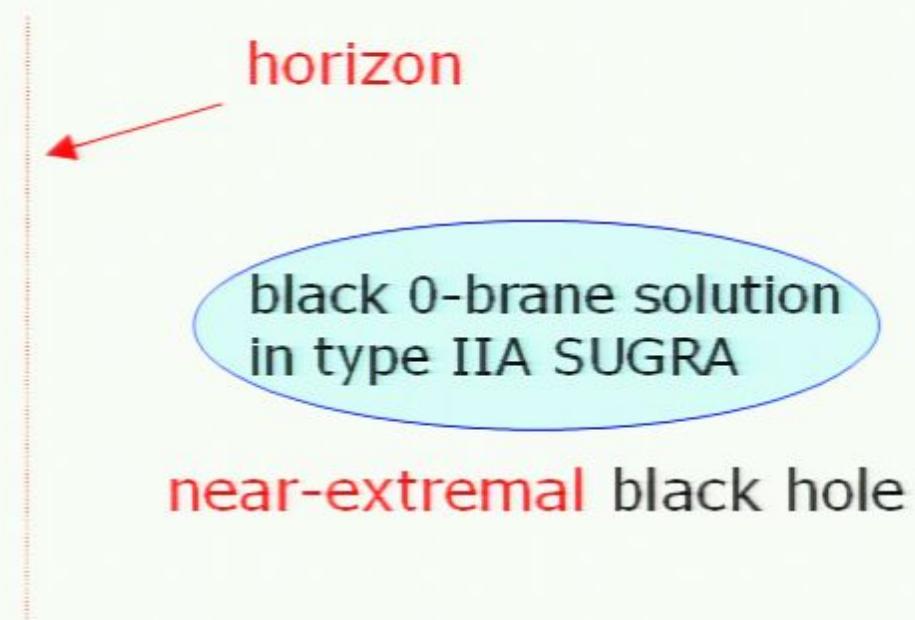
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the **fuzzball** picture !

# Gauge-gravity duality for D0-brane system

type IIA superstring



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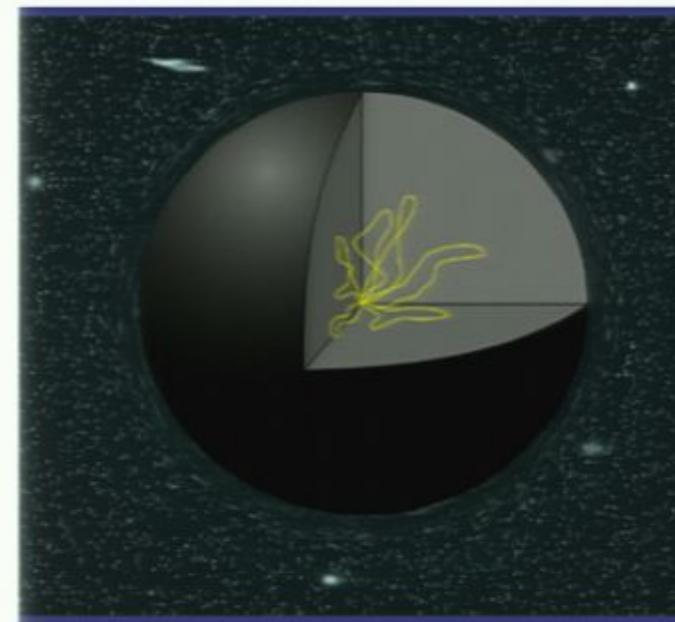
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1. Simulating SUSY matrix QM with 16 supercharges
2. Dual gravity description and black hole thermodynamics
3. Higher derivative corrections to black hole thermodynamics from SUSY QM
4. Schwarzschild radius from Wilson loop
5. Summary

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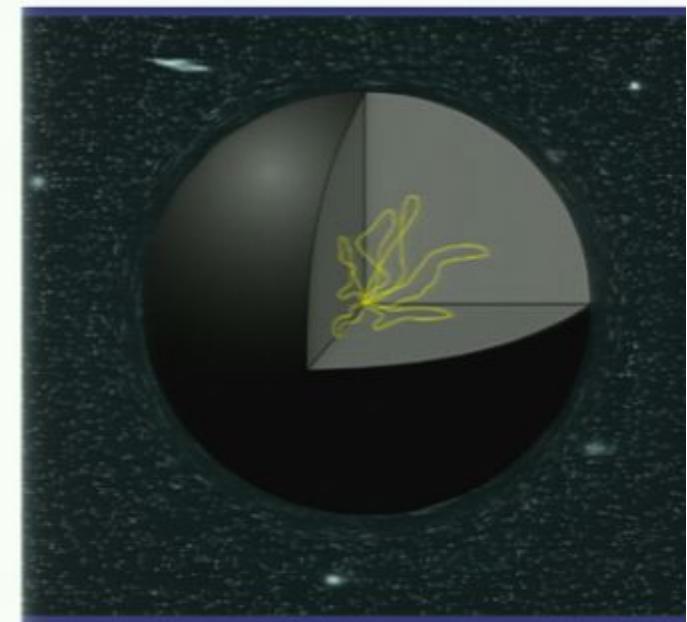
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# 1. Simulating SUSY QM with 16 supercharges

# SUSY matrix QM with 16 supercharges

$$S_b = \frac{N}{\lambda} \int_0^\beta dt \operatorname{tr} \left\{ \frac{1}{2} (DX_i(t))^2 - \frac{1}{4} [X_i(t), X_j(t)]^2 \right\}$$
$$S_f = \frac{N}{\lambda} \int_0^\beta dt \operatorname{tr} \left\{ \frac{1}{2} \psi_\alpha D\psi_\alpha - \frac{1}{2} \psi_\alpha (\gamma_i)_{\alpha\beta} [X_i, \psi_\beta] \right\}$$

1d gauge theory

$$D = \partial_t - i [A(t), \cdot]$$

$$\begin{cases} X_j(t) & (j = 1, \dots, 9) \\ \psi_\alpha(t) & (\alpha = 1, \dots, 16) \end{cases} \quad \begin{matrix} \text{p.b.c.} \\ \text{anti p.b.c.} \end{matrix}$$

$T = \beta^{-1}$  temperature

$\lambda = g^2 N$  't Hooft coupling

$$\lambda_{\text{eff}} = \frac{\lambda}{T^3}$$

$\lambda = 1$  (without loss of generality)

|        |   |  |                          |
|--------|---|--|--------------------------|
| low T  | → | strongly coupled   | dual gravity description |
| high T | → | non-zero modes : weakly coupled (high T exp)<br>(zero modes : integrated non-perturbatively) |                          |

Kawahara-J.N.-Takeuchi,  
JHEP 0712 (2007) 103, arXiv:0710.2188[hep-th]

# Fourier-mode simulation respecting SUSY maximally

Hanada-J.N.-Takeuchi, PRL 99 (07) 161602 [arXiv:0706.1647]

$$X_i(t) = \sum_{n=-\Lambda}^{\Lambda} \tilde{X}_{i,n} e^{i\omega nt} \quad \omega = \frac{2\pi}{\beta}$$

Note: Gauge symmetry can be fixed non-perturbatively in 1d.

- static diagonal gauge :

$$A(t) = \frac{1}{\beta} \text{diag}(\alpha_1, \dots, \alpha_N)$$

$$S_{\text{FP}} = - \sum_{a < b} 2 \ln \left| \sin \frac{\alpha_a - \alpha_b}{2} \right|$$

- residual gauge symmetry :

$$\begin{cases} \tilde{X}_{i,n}^{ab} \mapsto \tilde{X}_{i,n-\nu_a+\nu_b}^{ab} \\ \alpha_a \mapsto \alpha_a + 2\pi\nu_a \end{cases}$$

$$g(t) = \text{diag}(e^{i\omega\nu_1 t}, \dots, e^{i\omega\nu_N t})$$

$$X_i \mapsto g X_i g^\dagger$$

$$A \mapsto g A g^\dagger + i g \partial_t g^\dagger$$

should be fixed by imposing  $-\pi < \alpha_a \leq \pi$

c.f.) lattice approach : Catterall-Wiseman, PRD78 (08) 041502

Earlier works based on Gaussian approximation

Kabat-Lifschytz-Lowe, PRL 86 (2001) 1426

## 2. Dual gravity description and black hole thermodynamics

Anagnostopoulos-Hanada- J.N.-Takeuchi, PRL 100  
('08) 021601 [arXiv:0707.4454]

# Dual gravity description

After taking the decoupling limit :  $\alpha' \rightarrow 0$

$$U \equiv \frac{r}{\alpha'} , \quad \lambda \equiv g_s N \alpha'^{-3/2} \quad (\text{fixed})$$

$$f(U) \equiv \frac{U^{7/2}}{\sqrt{d_0 \lambda}} \left\{ 1 - \left( \frac{U_0}{U} \right)^7 \right\}$$

$$ds^2 = \alpha' \left\{ f(U) dt^2 + \frac{1}{f(U)} dU^2 + \sqrt{d_0 \lambda} U^{-3/2} d\Omega_{(8)}^2 \right\}$$

range of validity:  $N^{-10/21} \ll \frac{T}{\lambda^{1/3}} \ll 1$

## Black hole thermodynamics

$$\left\{ \begin{array}{l} \text{Hawking temperature :} \\ \\ \text{Bekenstein-Hawking entropy :} \end{array} \right. \quad \begin{aligned} \frac{T}{\lambda^{1/3}} &= \frac{7}{16\sqrt{15}\pi^{7/2}} \left( \frac{U_0}{\lambda^{1/3}} \right)^{5/2} \\ S &= \frac{1}{28\sqrt{15}\pi^{7/2}} N^2 \left( \frac{U_0}{\lambda^{1/3}} \right)^{9/2} \end{aligned}$$

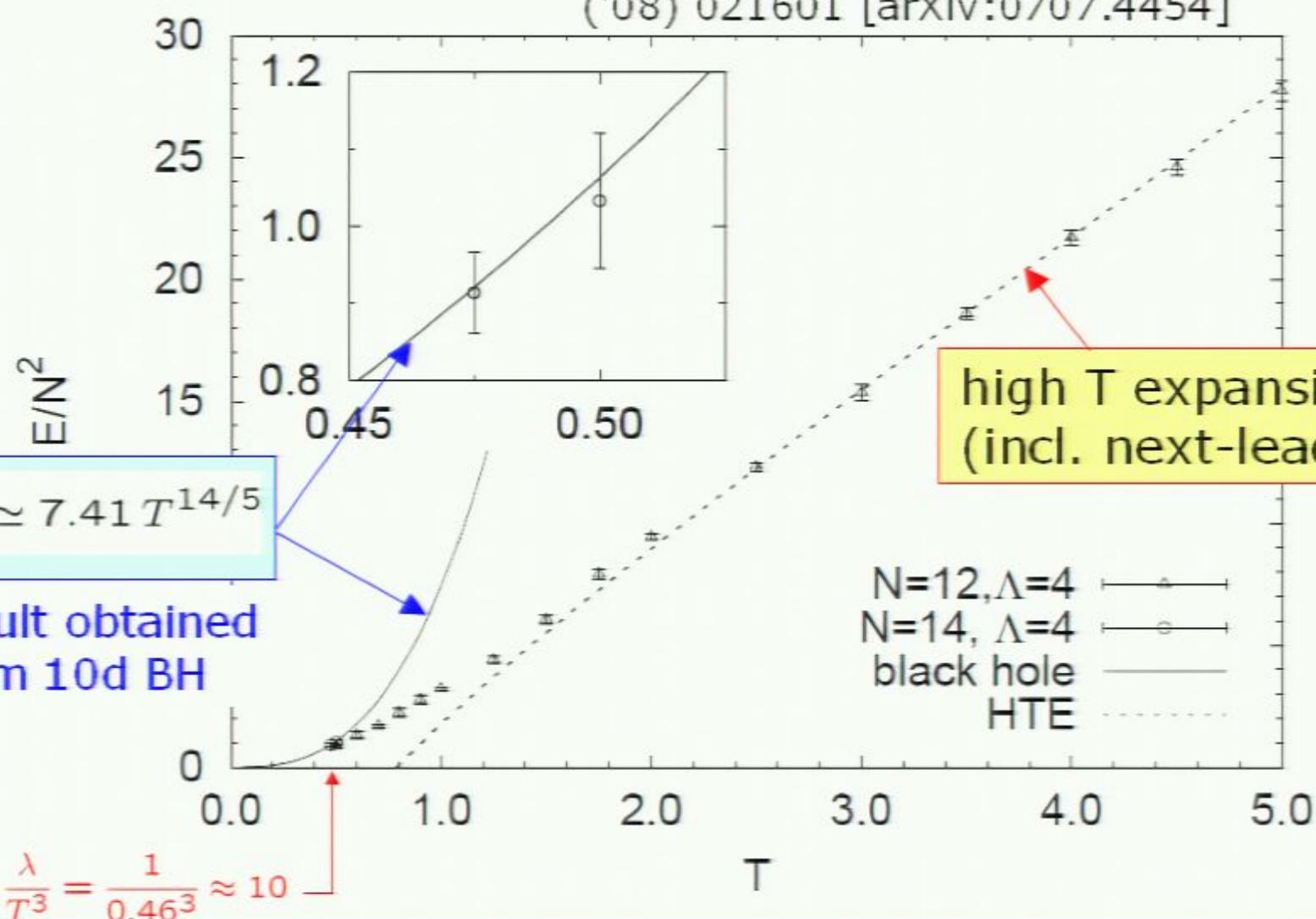
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# Result: Internal energy

$$E = \frac{\partial}{\partial \beta} (\beta \mathcal{F})$$

free energy

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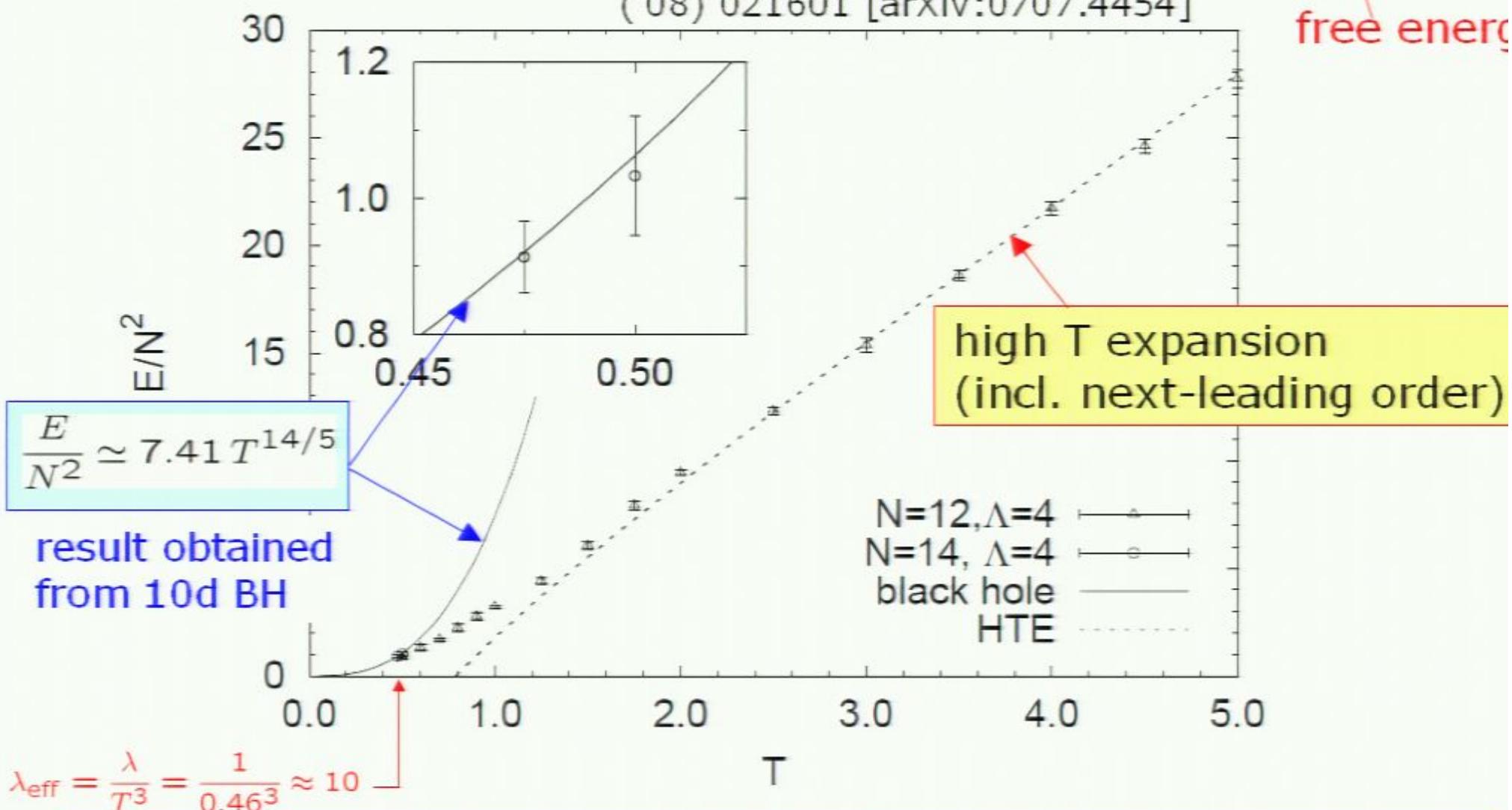
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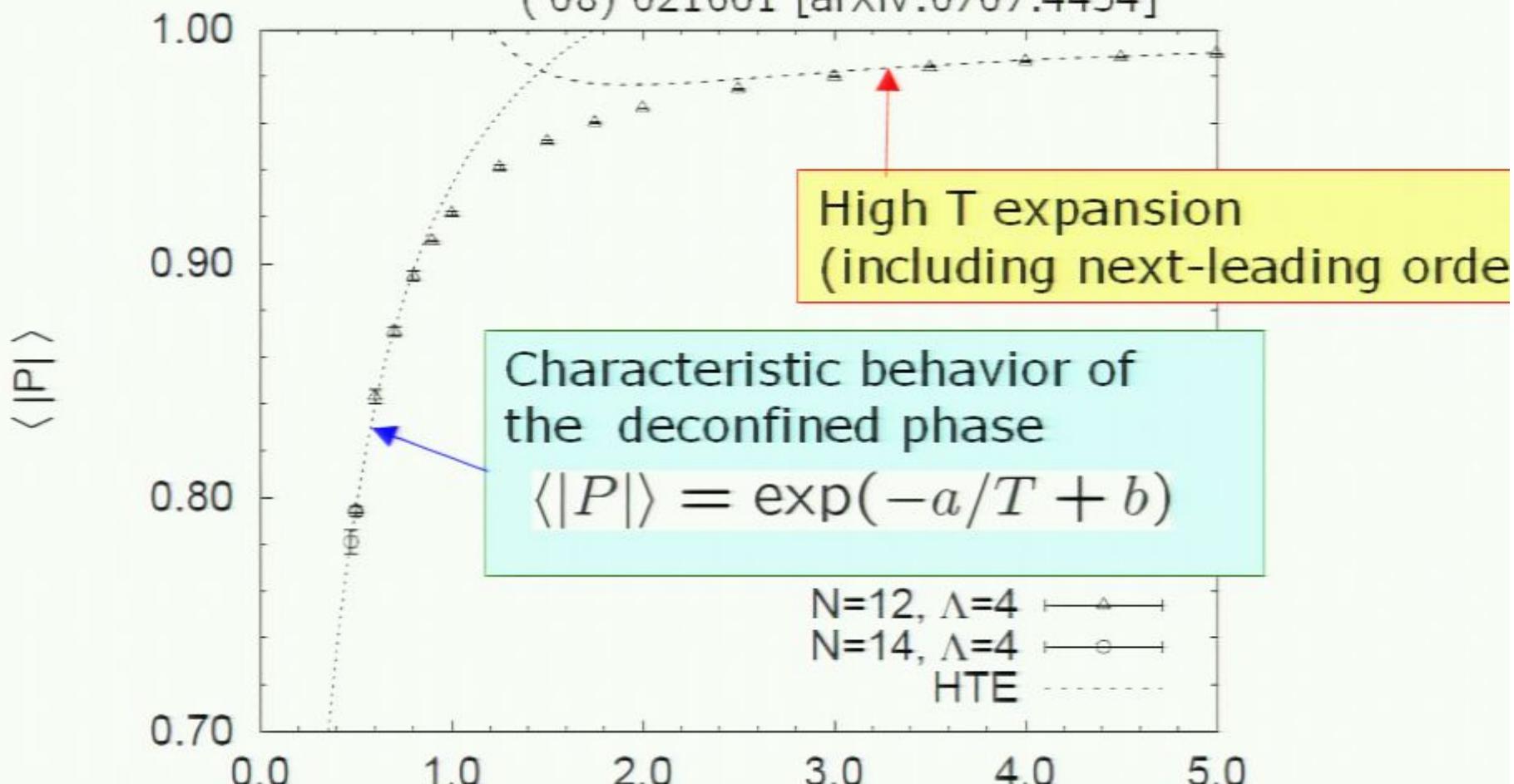
Anagnostopoulos-Hanada- J.N.-Takeuchi, PRL 100 ('08) 021601 [arXiv:0707.4454]

free energy



# Result: Polyakov line

Anagnostopoulos-Hanada- J.N.-Takeuchi, PRL 100  
('08) 021601 [arXiv:0707.4454]



no phase transition unlike in bosonic case  $T_c$

→ consistent with analyses on the gravity size (Barbon et al., Aharony et al.)

### 3. Higher derivative corrections to black hole thermodynamics from SUSY QM

Hanada-Hyakutake-J.N.-Takeuchi, arXiv:0811.3102[hep-th]

# $\alpha'$ corrections to type IIA SUGRA action

low energy effective action of type IIA superstring theory

➡ tree-level scattering amplitudes of the massless modes

leading term : type IIA SUGRA action

$$\mathcal{S}_{(0)} = \frac{1}{16\pi G_N} \int d^{10}x \sqrt{-g} \left\{ e^{-2\phi} (R + 4\partial_\mu\phi\partial^\mu\phi) - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} \right\}$$

$$G_N \sim \alpha'^4 g_s^2$$

explicit calculations of 2-pt and 3-pt amplitudes

$$\Rightarrow \mathcal{S}_{(1)} = \mathcal{S}_{(2)} = 0$$

4-pt amplitudes

$$\Rightarrow \mathcal{S}_{(3)} = \frac{\alpha'^3}{16\pi G_N} \int d^{10}x \sqrt{-g} \left\{ e^{-2\phi} \mathcal{R}^4 + \dots \right\}$$

Complete form is yet to be determined,  
but we can still make a dimensional analysis.

# Black hole thermodynamics with $\alpha'$ corrections

curvature radius of the dual geometry

$$\rho^2 \sim \left( \frac{\lambda^{1/3}}{U_0} \right)^{3/2} \alpha'$$

$\alpha'$  corrections

$$\rightarrow \frac{\alpha'}{\rho^2} \sim \left( \frac{U_0}{\lambda^{1/3}} \right)^{3/2} \sim \left( \frac{T}{\lambda^{1/3}} \right)^{3/5} \quad \frac{T}{\lambda^{1/3}} \sim \left( \frac{U_0}{\lambda^{1/3}} \right)^{5/2}$$

corrections at  $\alpha'^3$  order gives

$$\frac{1}{N^2} \frac{E}{\lambda^{1/3}} = 7.41 \left( \frac{T}{\lambda^{1/3}} \right)^{14/5} \left\{ 1 + \boxed{a \left( \frac{T}{\lambda^{1/3}} \right)^{9/5}} \right\}$$

More careful treatment leads to the same conclusion.  
(Hanada-Hyakutake-J.N.-Takeuchi,arXiv:0811.3102)

Setting  $\lambda = 1$ ,

$$\frac{E}{N^2} = 7.41 T^{14/5} - C T^{23/5}$$

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$$\rightarrow \frac{\alpha'}{\rho^2} \sim \left( \frac{U_0}{\lambda^{1/3}} \right)^{3/2} \sim \left( \frac{T}{\lambda^{1/3}} \right)^{3/5} \quad \frac{T}{\lambda^{1/3}} \sim \left( \frac{U_0}{\lambda^{1/3}} \right)^{5/2}$$

corrections at  $\alpha'^3$  order gives

$$\frac{1}{N^2} \frac{E}{\lambda^{1/3}} = 7.41 \left( \frac{T}{\lambda^{1/3}} \right)^{14/5} \left\{ 1 + \boxed{a \left( \frac{T}{\lambda^{1/3}} \right)^{9/5}} \right\}$$

More careful treatment leads to the same conclusion.  
(Hanada-Hyakutake-J.N.-Takeuchi,arXiv:0811.3102)

Setting  $\lambda = 1$ ,

$$\frac{E}{N^2} = 7.41 T^{14/5} - C T^{23/5}$$

# $\alpha'$ corrections to type IIA SUGRA action

low energy effective action of type IIA superstring theory

← tree-level scattering amplitudes of the massless modes

leading term : type IIA SUGRA action

$$\mathcal{S}_{(0)} = \frac{1}{16\pi G_N} \int d^{10}x \sqrt{-g} \left\{ e^{-2\phi} (R + 4\partial_\mu\phi\partial^\mu\phi) - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} \right\}$$

$$G_N \sim \alpha'^4 g_s^2$$

explicit calculations of 2-pt and 3-pt amplitudes

$$\Rightarrow \mathcal{S}_{(1)} = \mathcal{S}_{(2)} = 0$$

4-pt amplitudes

$$\Rightarrow \mathcal{S}_{(3)} = \frac{\alpha'^3}{16\pi G_N} \int d^{10}x \sqrt{-g} \left\{ e^{-2\phi} \mathcal{R}^4 + \dots \right\}$$

Complete form is yet to be determined,  
but we can still make a dimensional analysis.

# Black hole thermodynamics with $\alpha'$ corrections

curvature radius of the dual geometry

$$\rho^2 \sim \left( \frac{\lambda^{1/3}}{U_0} \right)^{3/2} \alpha'$$

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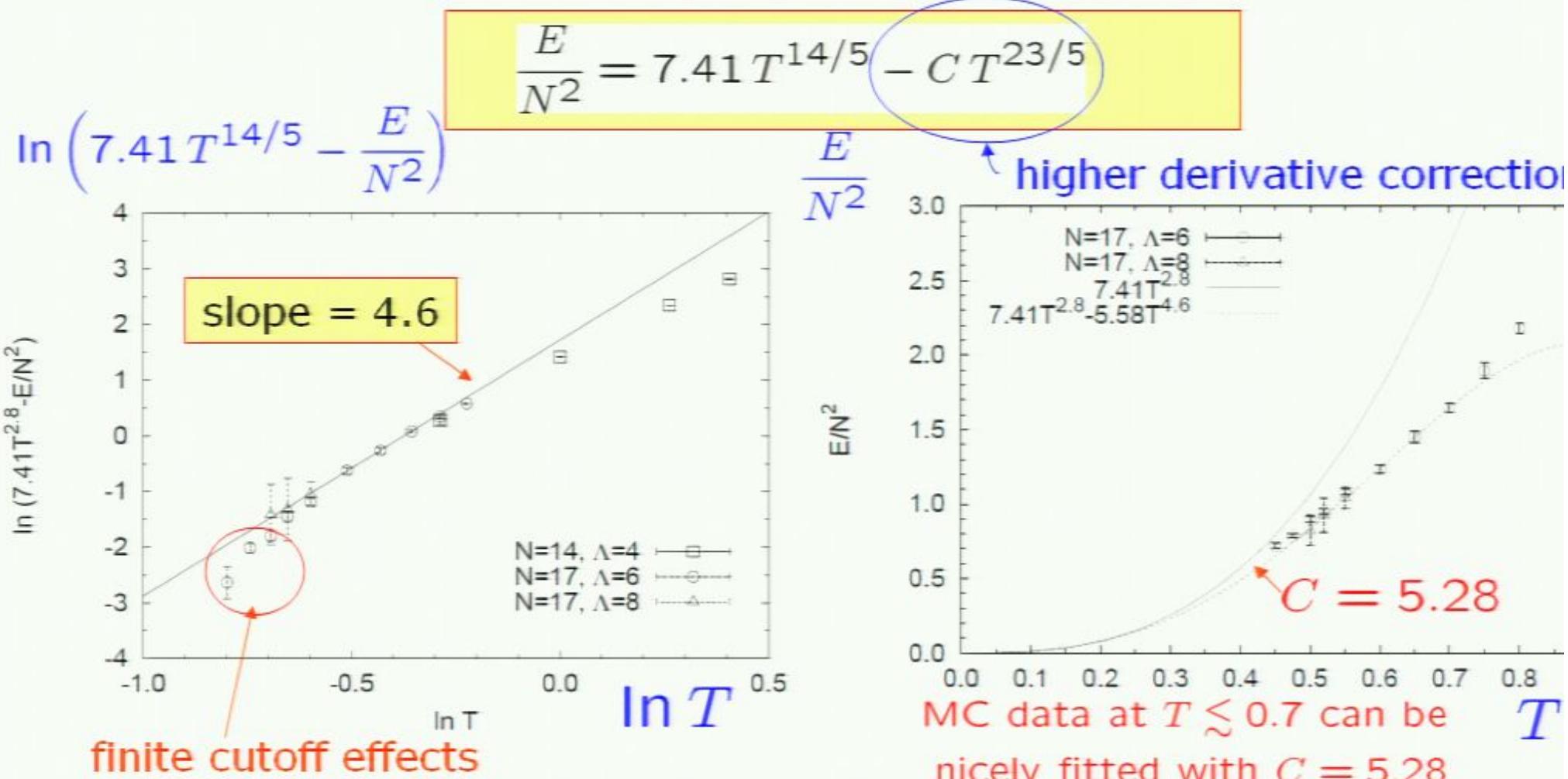
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# Higher derivative corrections to black hole thermodynamics from SUSY QM

Hanada-Hyakutake-J.N.-Takeuchi, arXiv:0811.3102[hep-th]

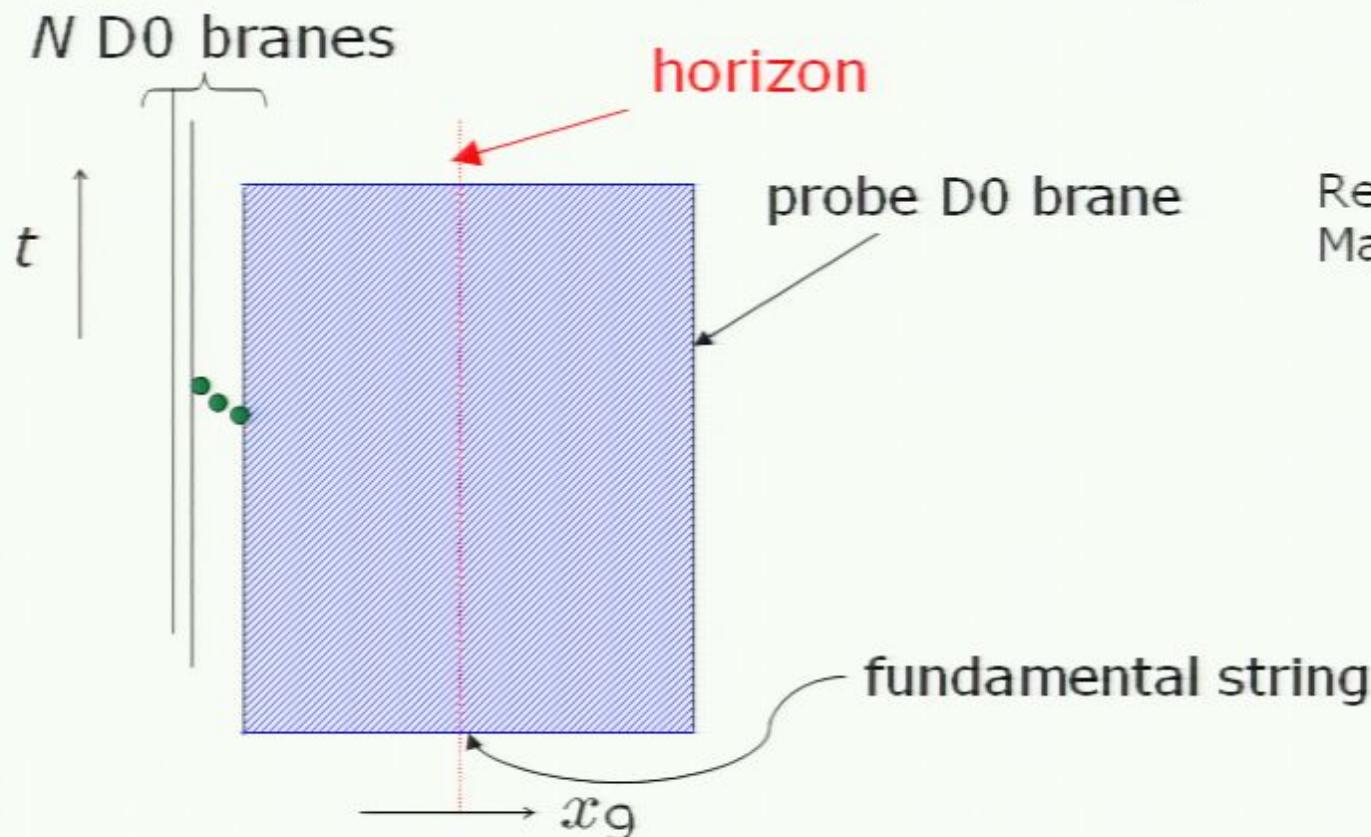


### 3. Schwarzschild radius from Wilson loop

Hanada-Miwa-J.N.-Takeuchi , arXiv:0811.2081[hep-th]

# Calculation of Wilson loop

Hanada-Miwa-J.N.-Takeuchi, arXiv:0811.2081[hep-th]



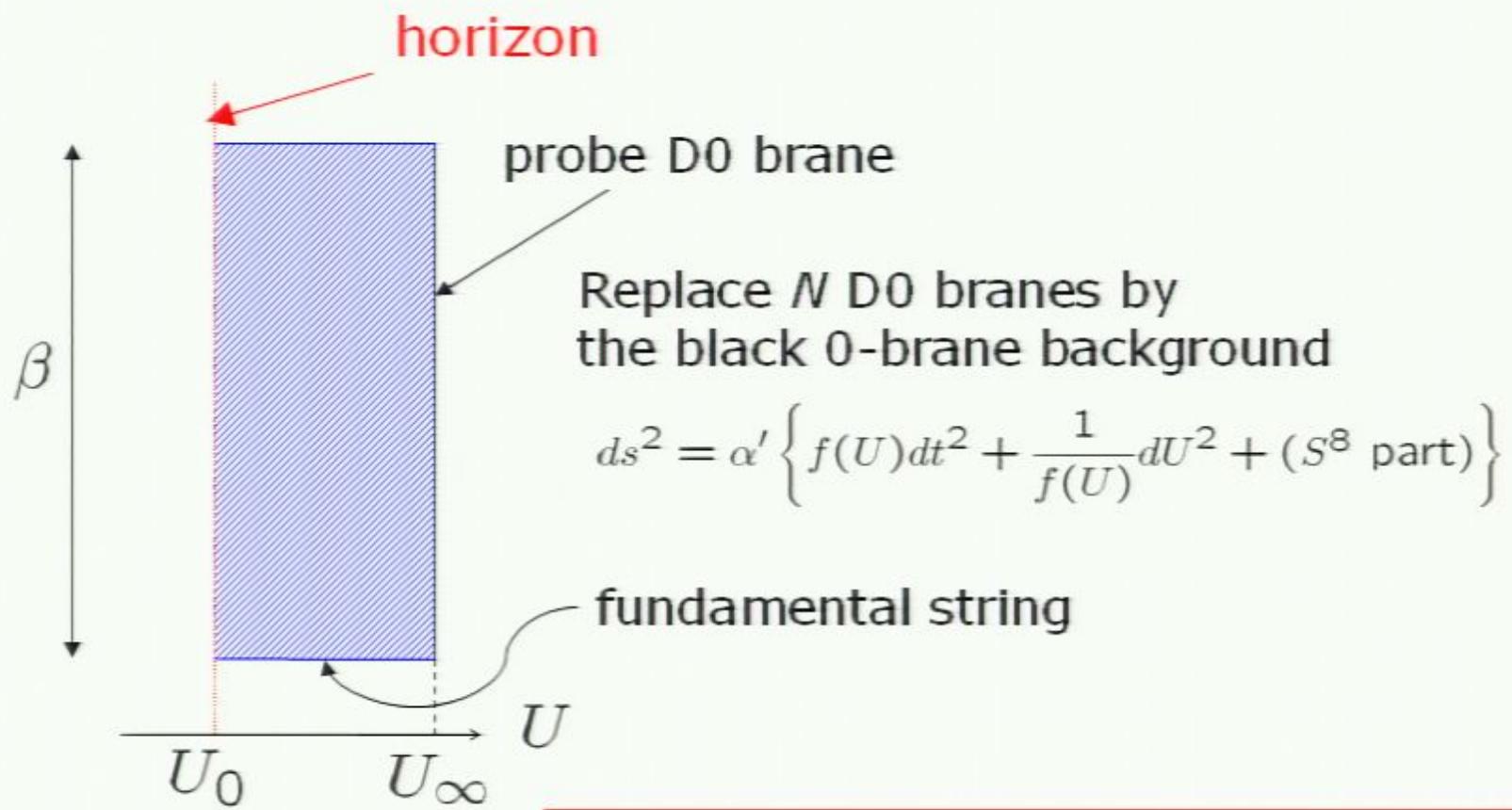
Rey-Yee ('98),  
Maldacena ('98)

gauge theory side :

propagation of a test particle  
coupled to  $A(t)$  and  $X_9(t)$

$$W = \text{tr } \mathcal{P} \exp \left[ i \int_0^\beta dt \{ A(t) + i X_9(t) \} \right]$$

## Calculation of Wilson loop (cont'd)



gravity theory side :

propagation of the string  
in the b.g. geometry

string action for the minimal surface

$$S_{\text{String}} = \frac{1}{2\pi} \beta (U_\infty - U_0)$$

## Calculation of Wilson loop (cont'd)

$$W e^{-M\beta} = e^{-S_{\text{string}}} \quad \text{at large } N \text{ and large } \lambda$$

perimeter-law suppression factor  
due to propagation of a particle with mass  $M$

$$S_{\text{string}} = \frac{1}{2\pi} \beta (U_\infty - U_0)$$

$$\log W - \boxed{\beta M} = \frac{\beta U_0}{2\pi} - \boxed{\frac{\beta U_\infty}{2\pi}}$$

natural to identify

more sophisticated justification  
a la Drukker-Gross-Ooguri ('99)

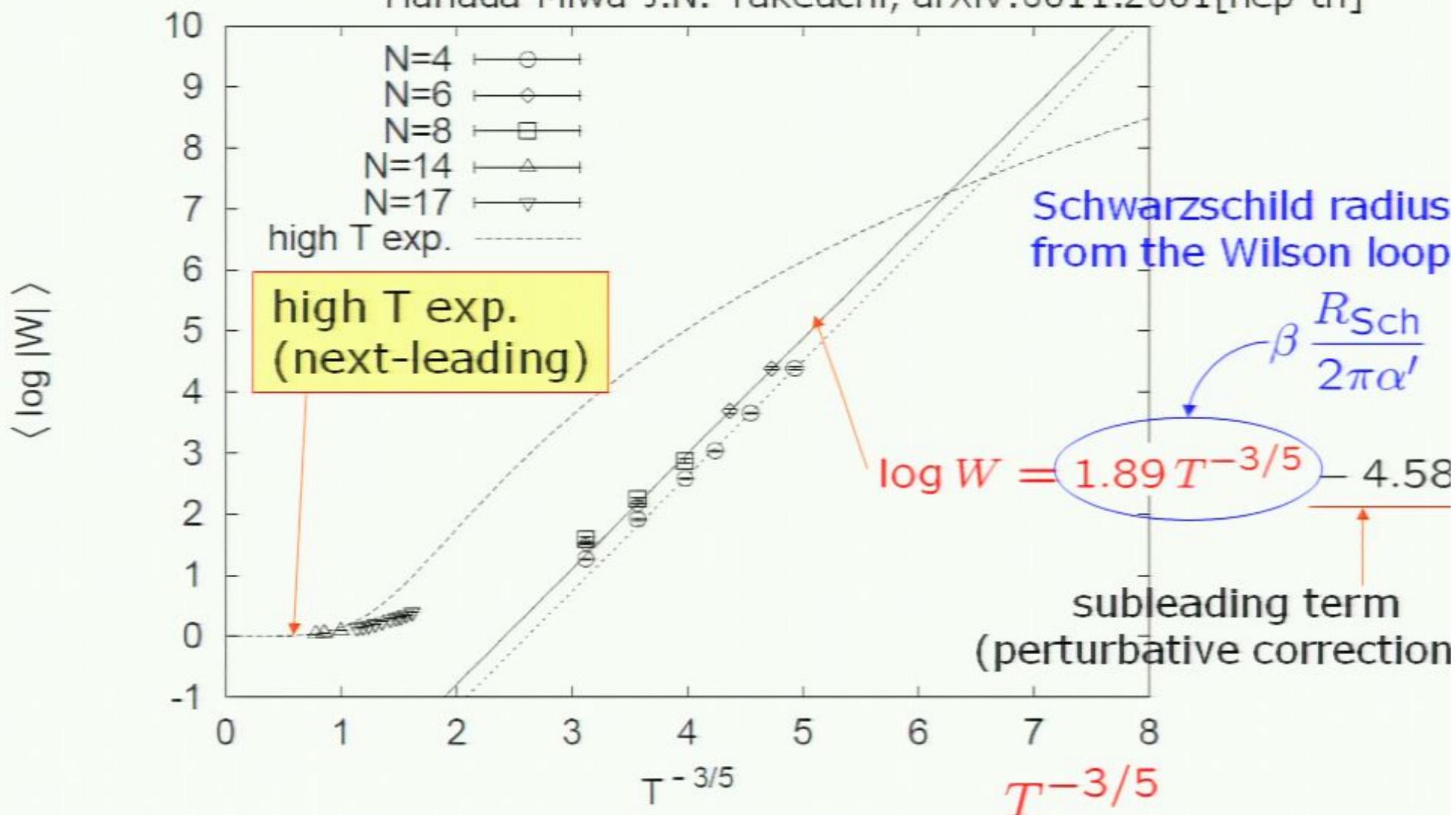


$$\log W = \frac{\beta U_0}{2\pi} = \frac{\beta R_{\text{Sch}}}{2\pi\alpha'} = \underbrace{\frac{1}{2\pi} \left\{ \frac{16\sqrt{15}\pi^{7/2}}{7} \right\}^{2/5}}_{1.89} \left( \frac{T}{\lambda^{1/3}} \right)^{-3/5}$$

# Results: Wilson loop

$$W = \text{tr } \mathcal{P} \exp \left[ i \int_0^\beta dt \{ A(t) + i X_9(t) \} \right]$$

Hanada-Miwa-J.N.-Takeuchi, arXiv:0811.2081[hep-th]



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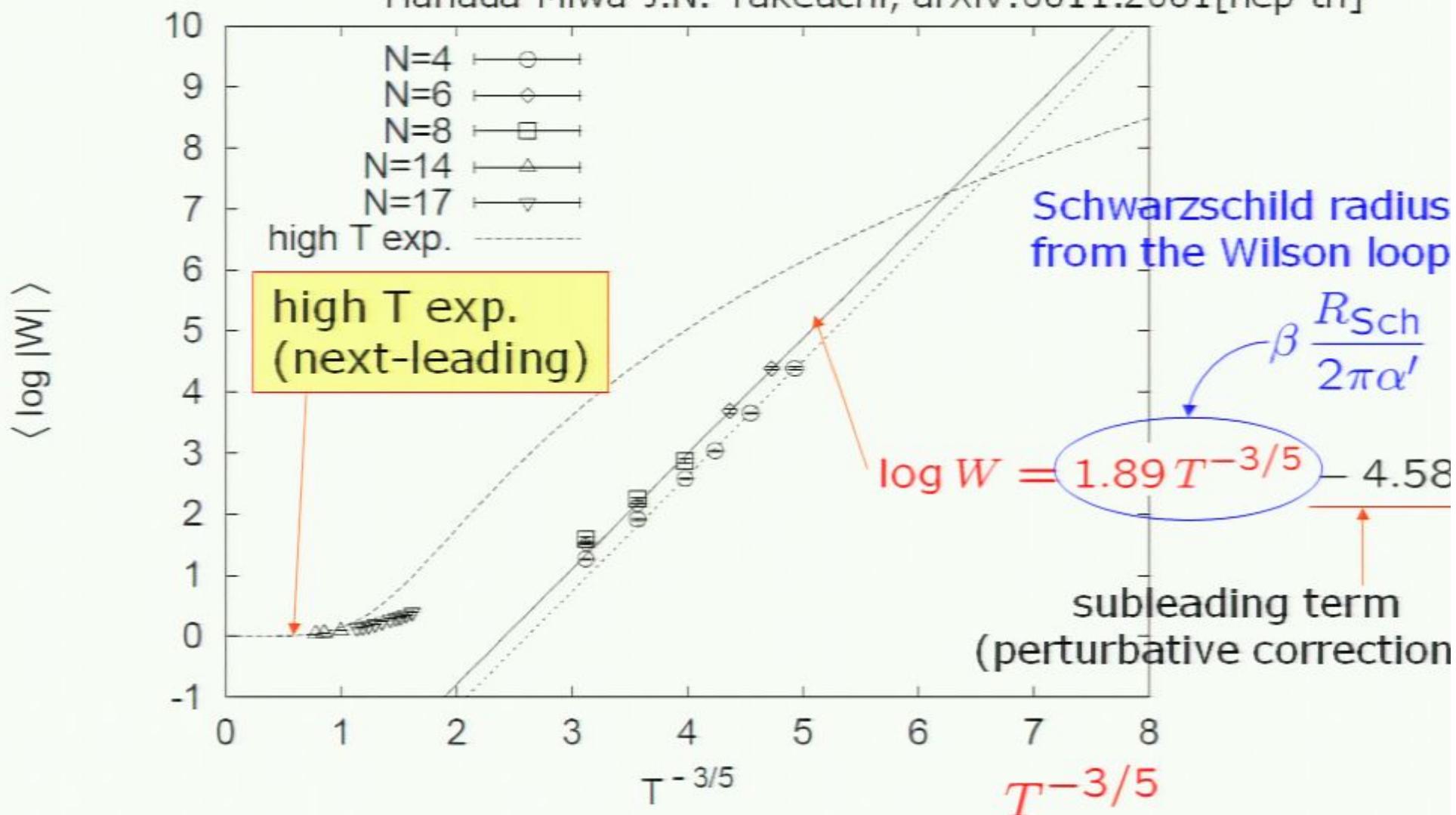


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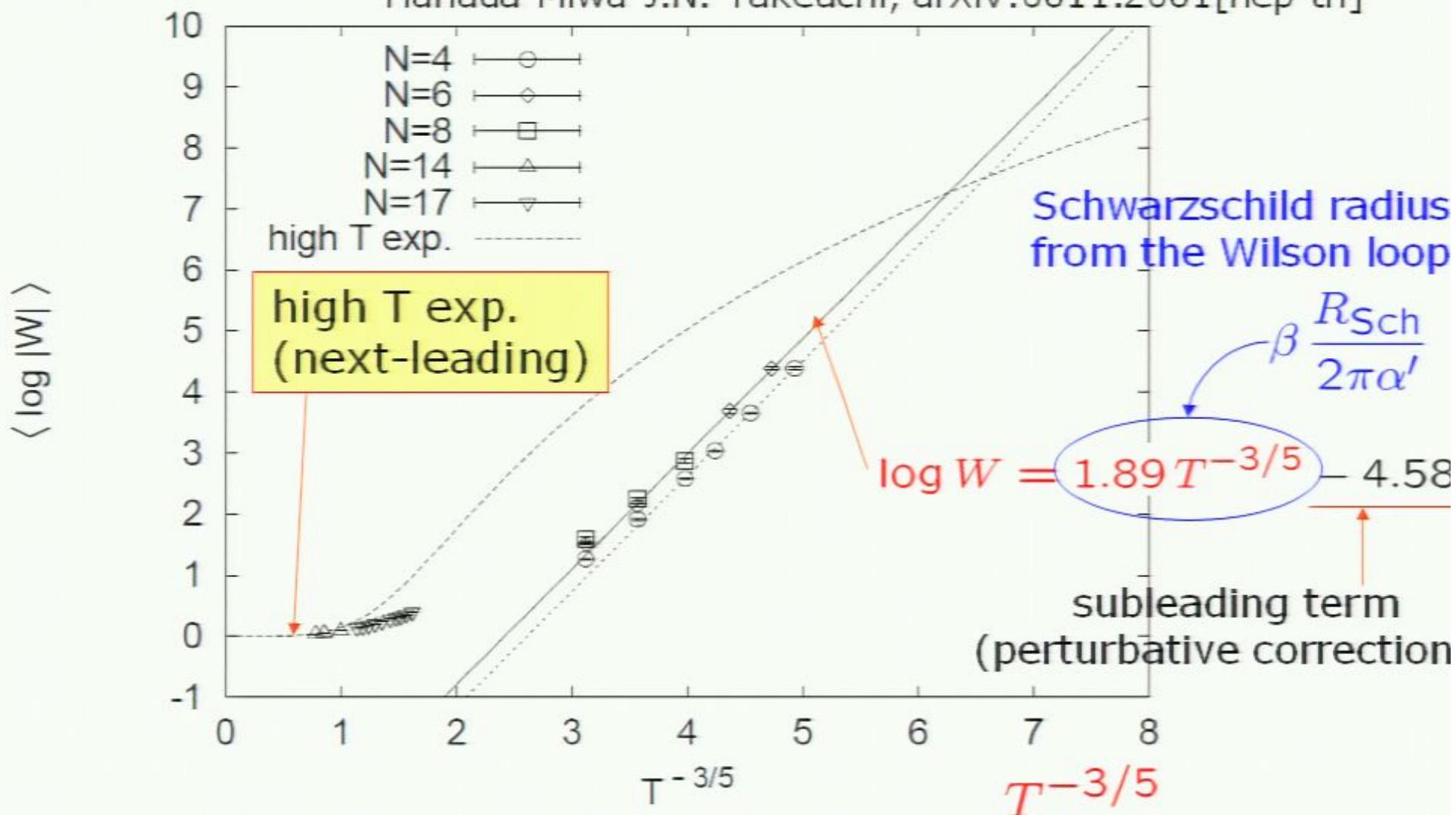


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## 5. Summary

# Summary and future prospects

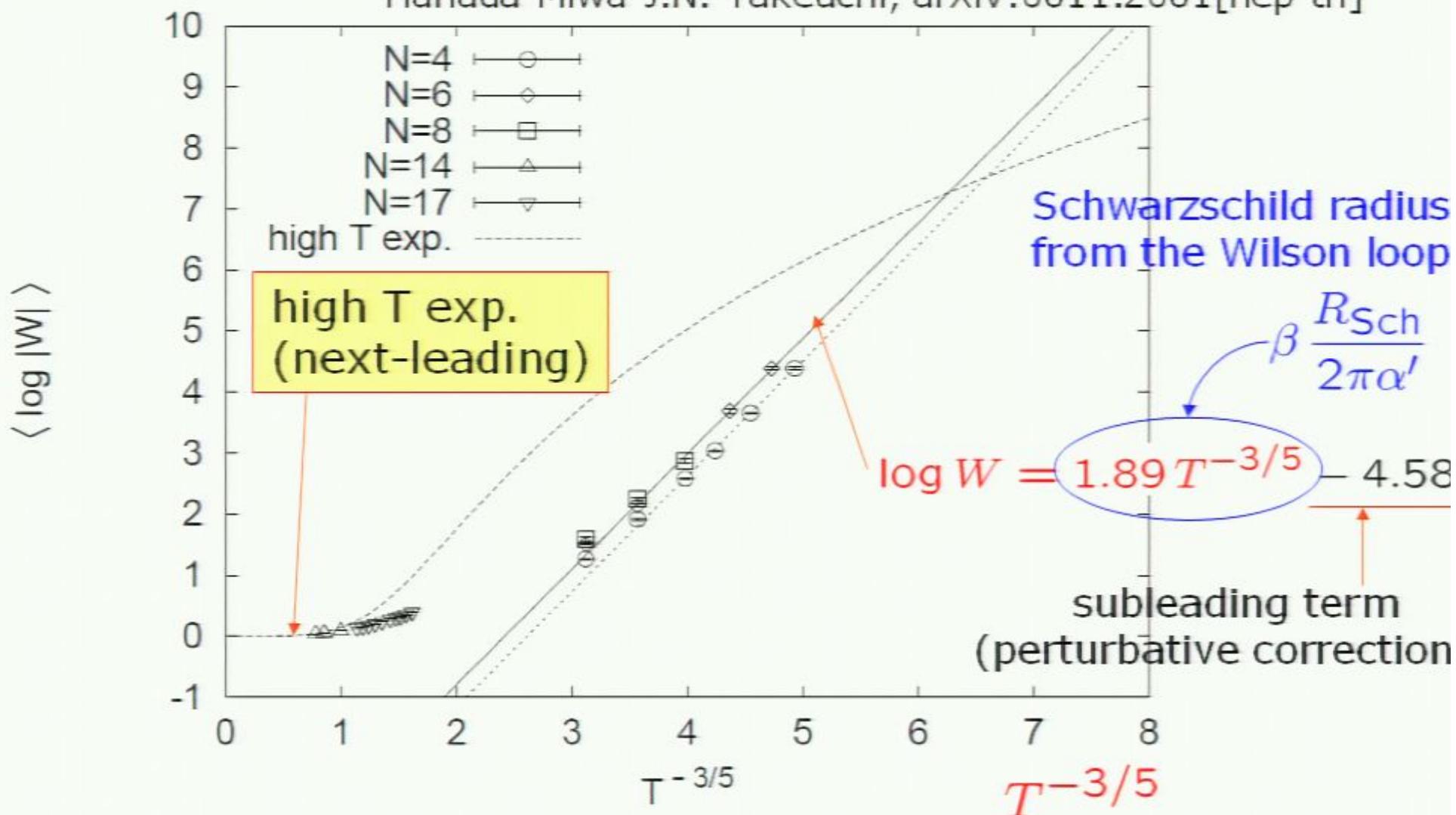
- Monte Carlo studies of supersymmetric large N gauge theories
  - powerful method for superstring theory
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Schwarzschild radius reproduced from Wilson loop
- a highly nontrivial check of the duality microscopic origin of the black hole thermodynamics including higher derivative corrections !

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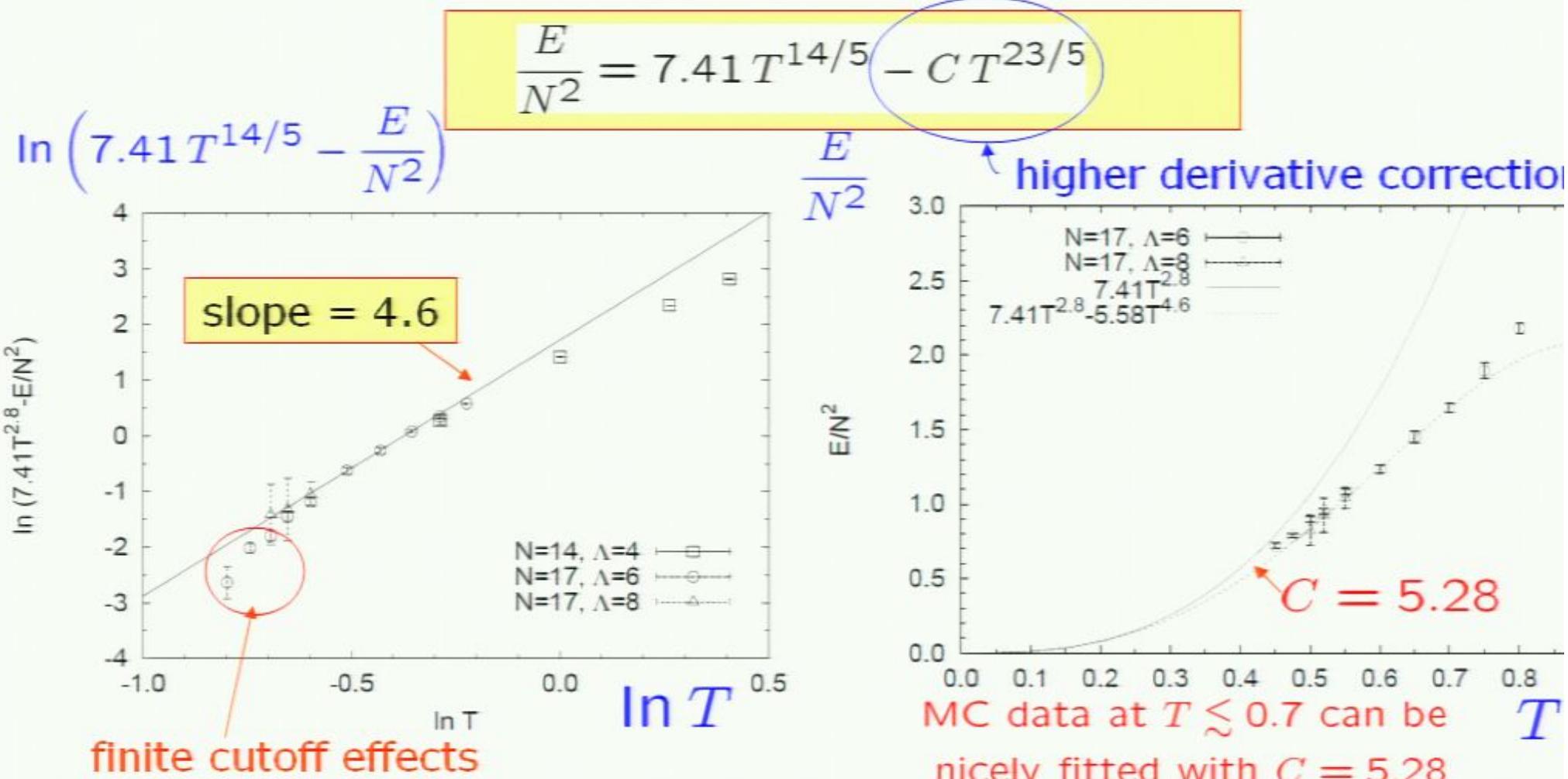


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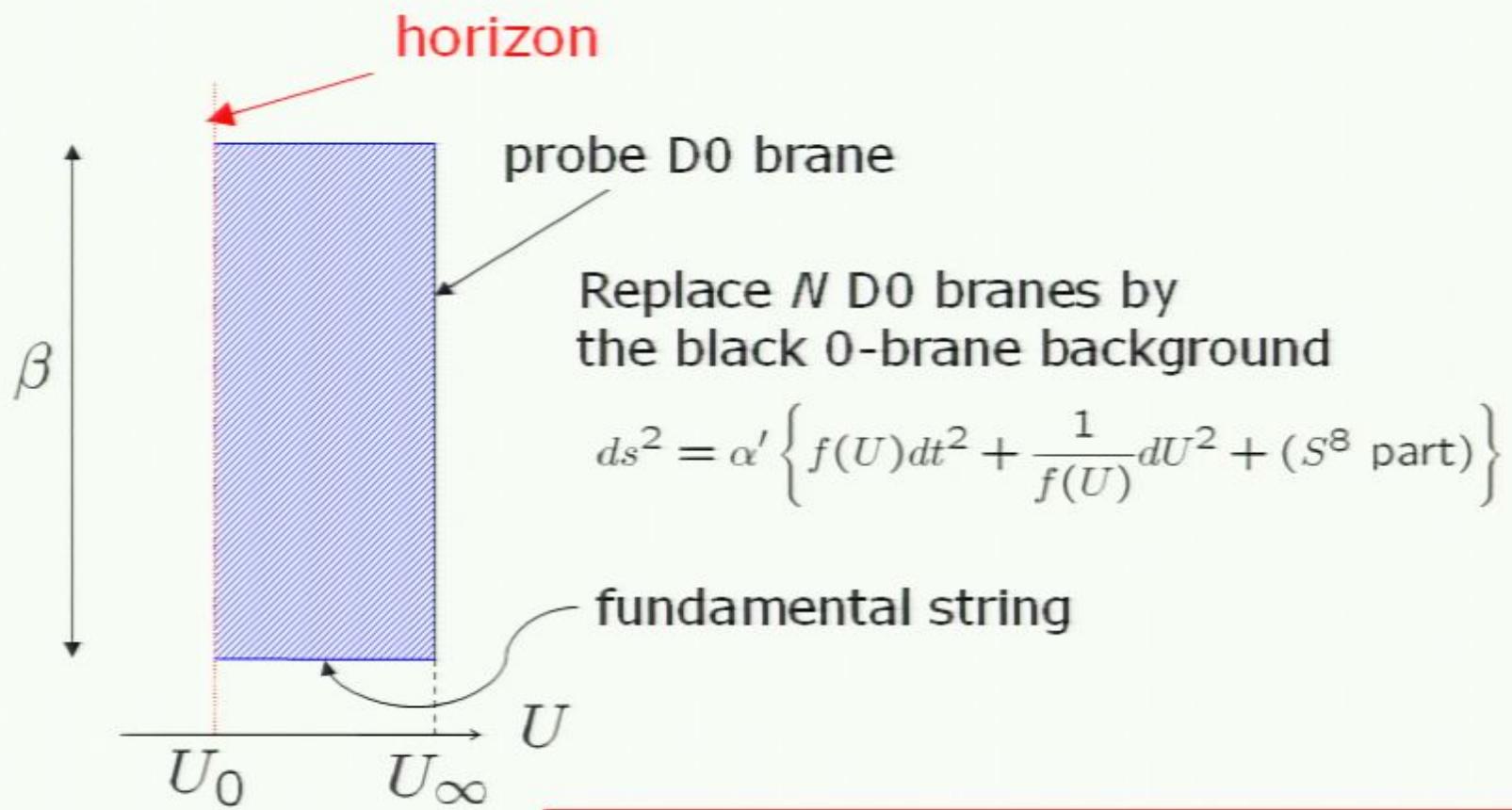
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# Simulating Quantum Universe

Nonperturbative formulations of superstring/M theory

Matrix Theory

Banks-Fischler-Shenker-Susskind ('97)

Type IIB matrix model

Ishibashi-Kawai-Kitazawa-Tsuchiya ('97)

Matrix String Theory

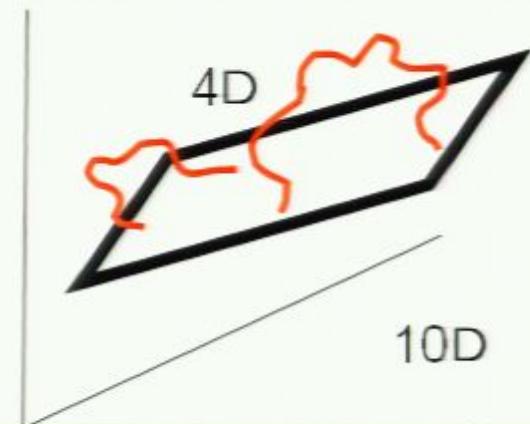
Dijkgraaf-Verlinde-Verlinde ('97)

How does our **4d space-time** appear from **10d (11d) space-time**?

e.g.)  $S = N \text{tr} \left\{ -\frac{1}{4} [X_\mu, X_\nu]^2 + \frac{1}{2} \psi_\alpha (\Gamma_\mu)_{\alpha\beta} [X_\mu, \psi_\beta] \right\}$

$$\text{SO}(10) \xrightarrow[\text{SSB}]{} \text{SO}(4) ?$$

Eigenvalue distribution of  $X_\mu$



Gaussian expansion method

J.N.-Sugino ('01),  
Kawai et al. ('01), ...

Monte Carlo simulation, in progress