

Title: LQG Black Holes

Date: Jan 23, 2009 10:30 AM

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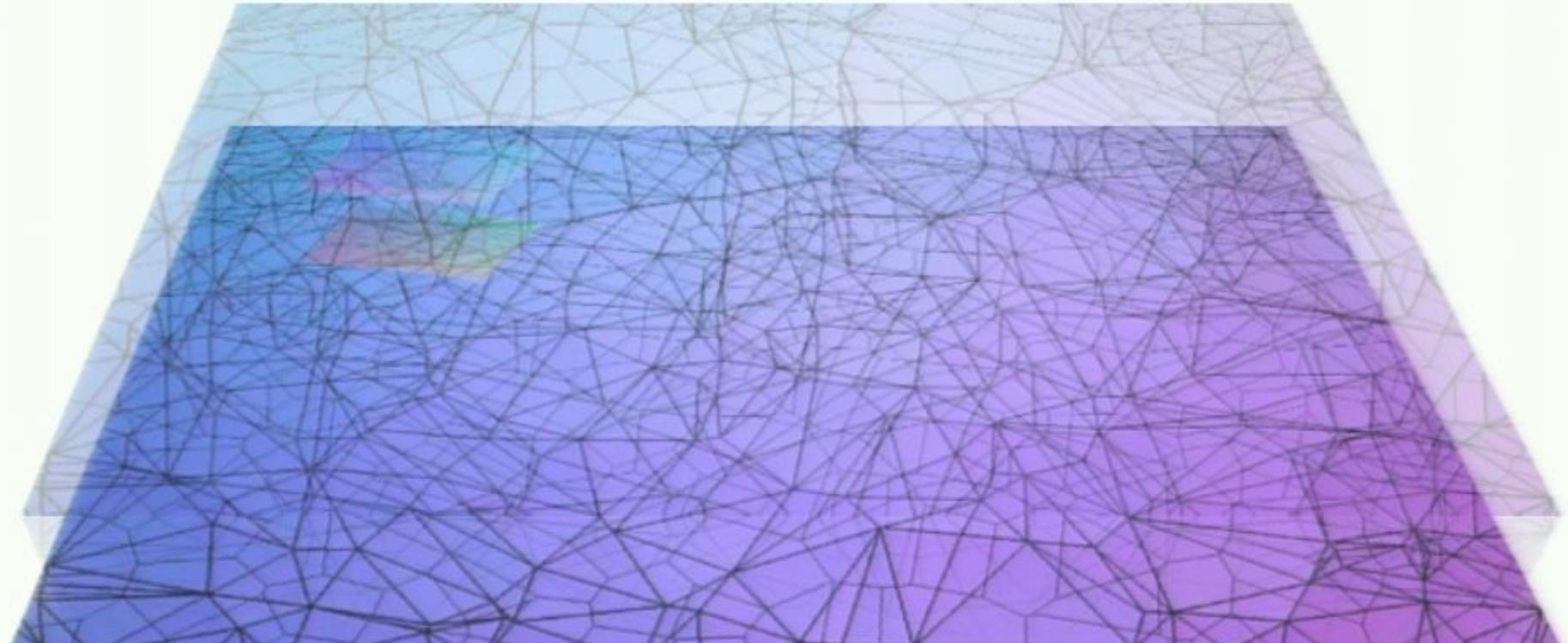
Abstract: In this talk we introduce loop quantum gravity and we apply the theory to the black hole singularity problem. The Schwarzschild black hole solution inside the event horizon coincides with the Kantowski-Sachs space-time and we can study a simple spherically symmetric mini-superspace model. We show the classical black hole singularity is controlled by the quantum theory and the space-time can be dynamically extended beyond the classical singularity. We consider a semiclassical analysis of the black hole in LQG and we focus our attention on the space-time structure. The semiclassical solution is regular everywhere and similar to the Reissner-Nordstrom metric. The LQG black hole metric interpolates between two asymptotically flat regions, the ' $r=\infty$ ' region and the ' $r=0$ ' region. The metric is self-dual in the sense it is invariant under the symmetry ($r \rightarrow 1/r$) which relates small and large distances. We study the thermodynamics of the semiclassical solution.

LQG Black Holes

Leonardo Modesto

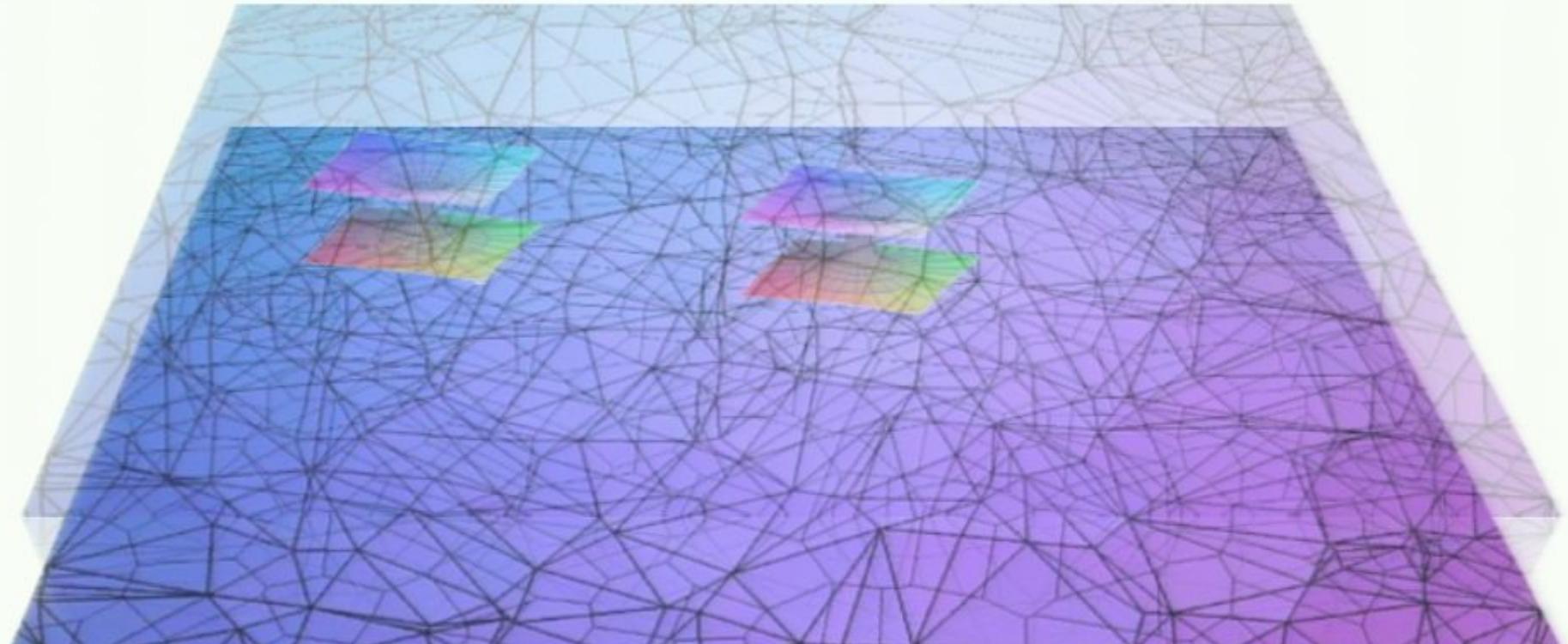
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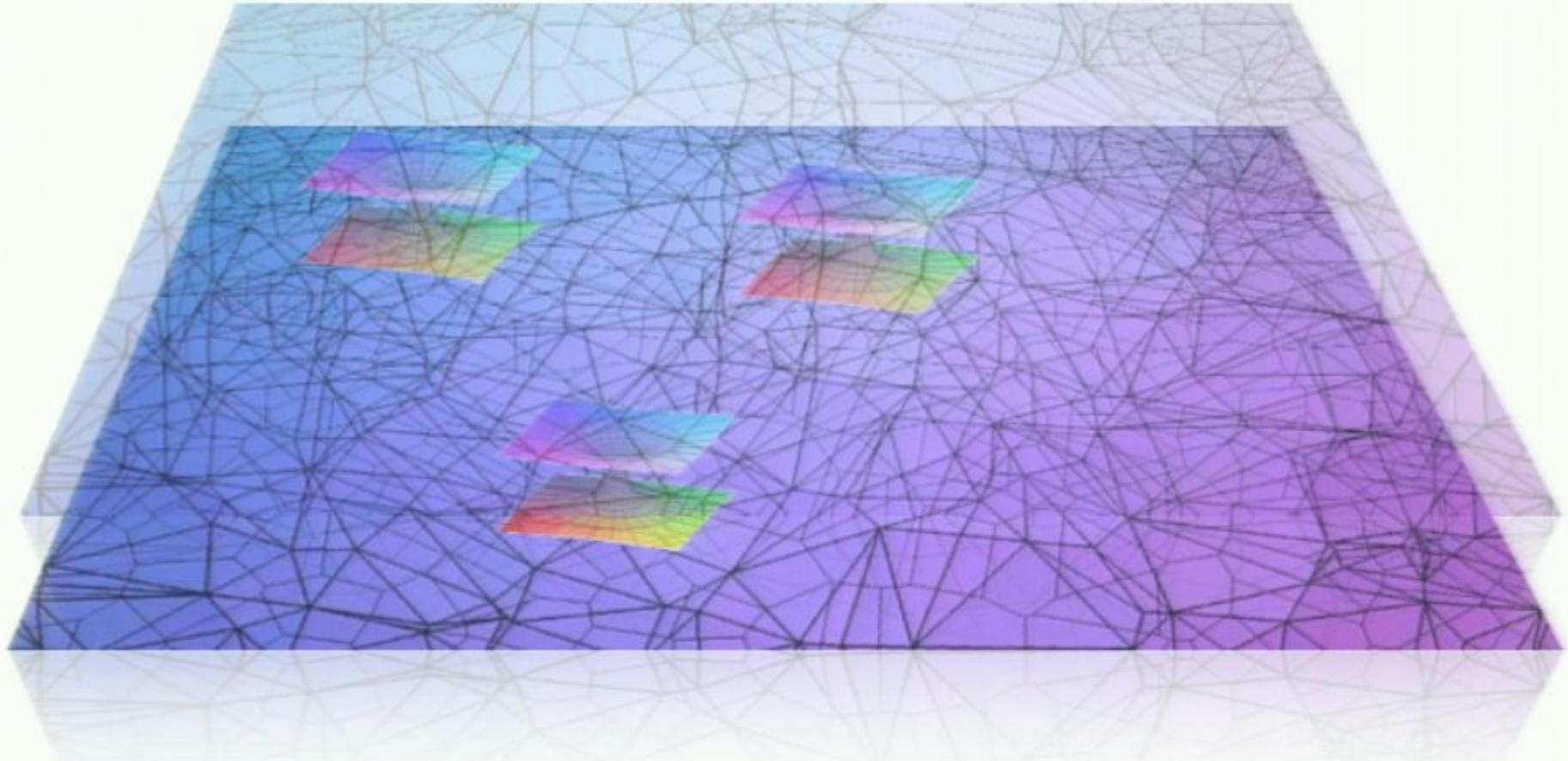
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OUTLINE

Loop quantum gravity.

*Non singular black hole,
Loop quantum black hole.*

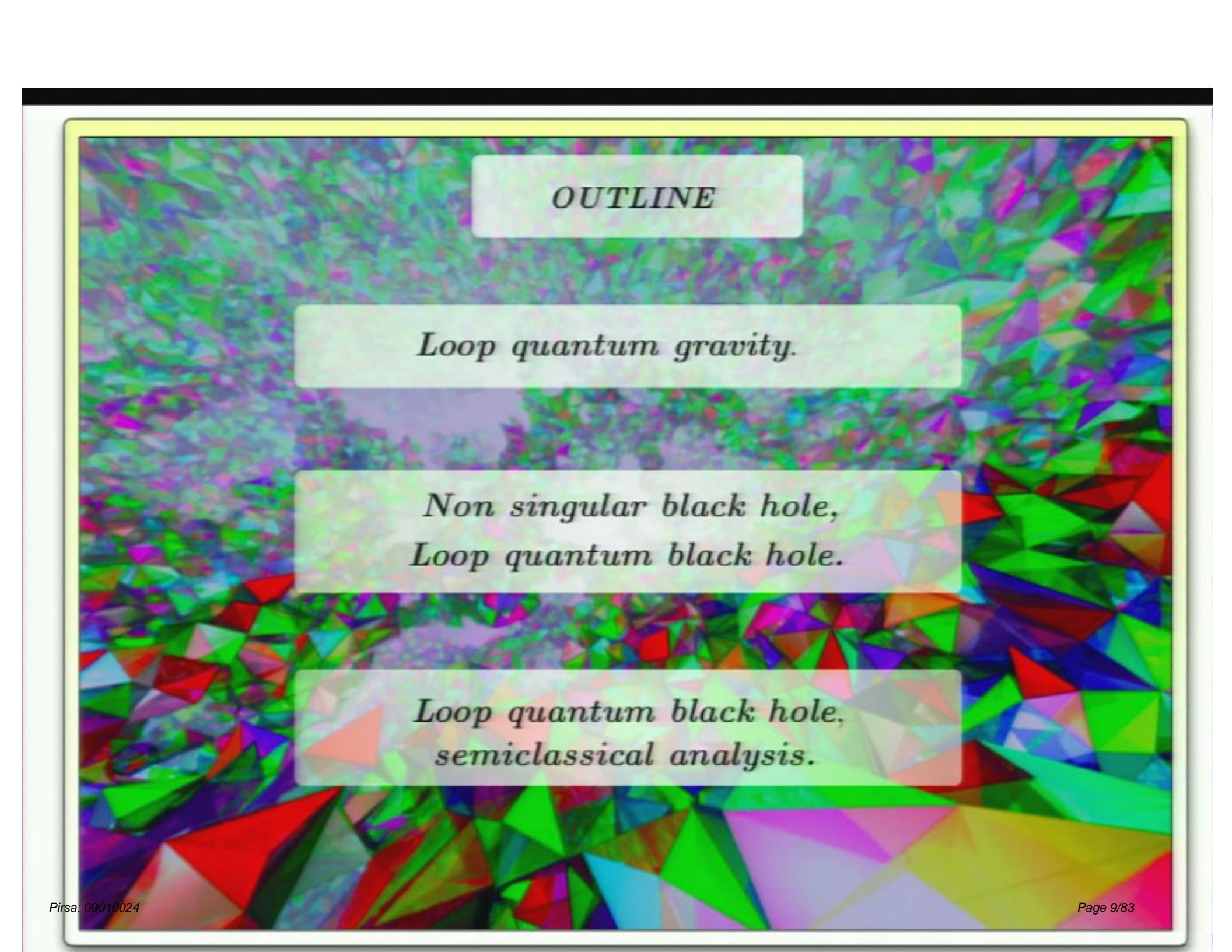
*Loop quantum black hole,
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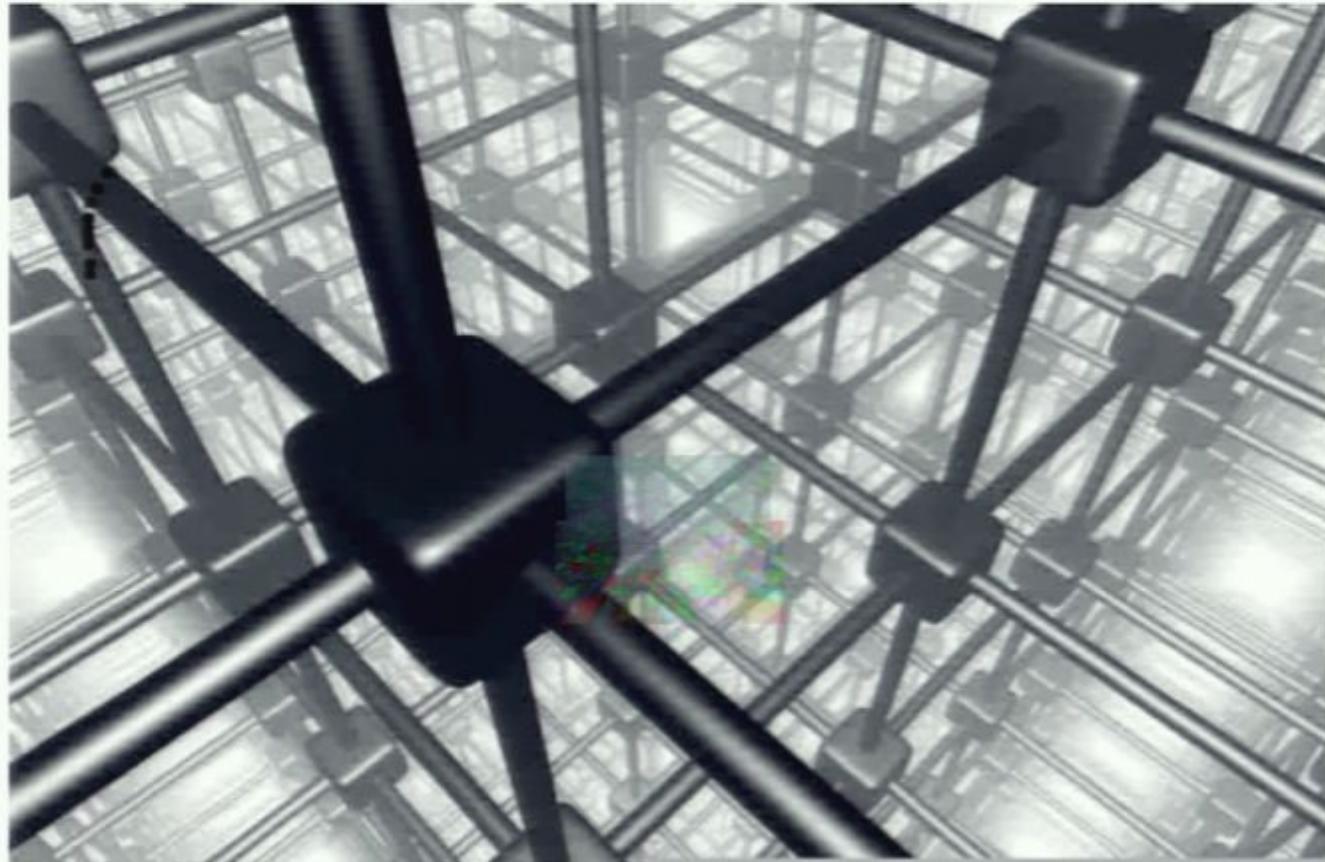


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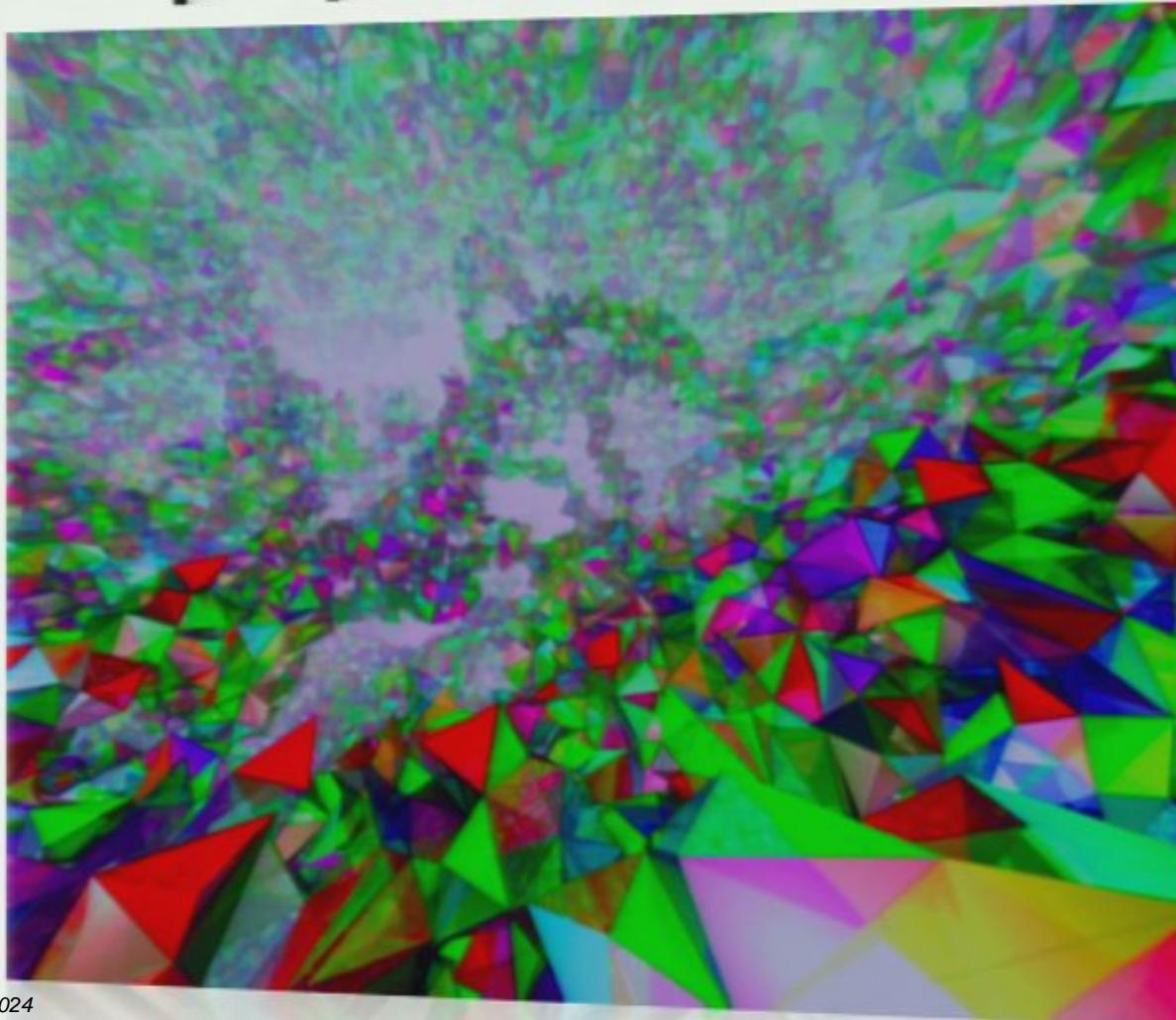
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Loop quantum gravity

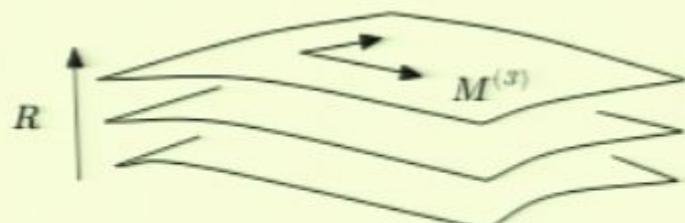


Ashtekar variables

Basic Formalism of Loop Quantum Gravity

Space-time foliation

$$M^{(4)} \sim M^{(3)} \times R$$



Phase space

$$(q_{ab}, K_{ab})$$

We introduce the triads $e_a^i(x)$

$$q_{ab}(x) = e_a^i(x) e_b^j(x)$$

Phase space

$$\begin{aligned} (E_i^a(x), K_a^i(x)), E_a^i &:= e e_a^i \\ K_a^i(x) &:= K_{ab} E_j^b \delta^{ij} \end{aligned}$$

Ashtekar variables

The transition to the connection variables is made using a canonical transformation

$$A_a^i(x) = \Gamma_a^i(x) + \gamma K_a^i(x)$$

$$\Gamma_a^i = -\frac{1}{2} \epsilon_k^{ij} e_j^b \left(\partial_{[a} e_{b]}^k + \delta^{kl} \delta_{ms} e_l^c e_a^m \partial_b e_c^s \right)$$

New canonical pair on the phase space

$$(A_a^i(x), E_i^a(x))$$

$$\{E_i^a(x), E_j^b(y)\} = 0 \quad \{A_a^i(x), A_b^j(y)\} = 0 \quad \{A_a^i(x), E_j^b(y)\} = \gamma G \delta_j^i \delta_a^b \delta^3(x, y)$$

General Re

The constraints in terms of Ashtekar variables

Gauss constraint

$$G_i(E_j^a, K_a^j) := \epsilon_{ijk} E^{aj} K_a^k$$

Diffeomorphism constraint

$$V^a(E_i^a, A_a^i) := E_i^a F_{ab}^i - (1 + \gamma^2) K_a^i G_i$$

Hamiltonian constraint

$$S(E_i^a, A_a^j) := \frac{E_i^a E_j^b}{\sqrt{\det(E)}} \left(\epsilon_k^{ij} F_{ab}^k - 2(1 + \gamma^2) K_{[a}^i K_{b]}^j \right)$$

Geometric operators

Area operator

$$A_S[E_i^a] := \int_S \sqrt{E_i^a E_j^b \delta^{ij} n_a n_b} d\sigma^1 d\sigma^2$$

Volume operator

$$V_B := \int_B \sqrt{\frac{1}{3!} \epsilon_{abc} E_i^a E_j^b E_k^c \epsilon^{ijk}} d^3 \sigma$$

Quantum Gravity

Quantum Einstein's equations

$$\widehat{G}_i(A, E)|\Psi\rangle := \widehat{D_a} \widehat{E_i^a} |\Psi\rangle = 0,$$

$$\widehat{V}_a(A, E)|\Psi\rangle := \widehat{E_i^a} \widehat{F_{ab}^i}(A) |\Psi\rangle = 0,$$

$$\widehat{S}(A, E)|\Psi\rangle := [\sqrt{\det E}^{-1} \widehat{E_i^a E_j^b F_{ab}^{ij}}(A) + \dots] |\Psi\rangle = 0.$$

Kinematical Hilbert space

Holonomy : $h_e[A] = P \exp - \int_e A \in SU(2)$

$$h_e[A] = h_{e_1}[A] h_{e_2}[A]$$



Under a gauge transformation : $h'_e[A] = g(x(0)) h_e[A] g^{-1}(x(1))$

Given $\phi \in \text{Diff}(\Sigma)$ **we have :** $h_e[\phi^* A] = h_{\phi^{-1}(e)}[A]$

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Spin networks

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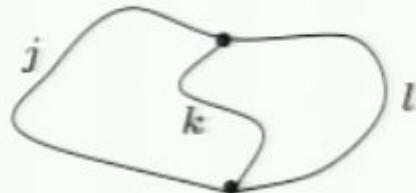
Cylindrical functions (Cyl $_{\gamma}$) : $\gamma = \{ \text{edges } e \subset \Sigma \mid \text{meeting at the end points} \},$

$$\psi_{\gamma,f} \in Cyl_{\gamma}, \quad f : SU(2)^{N_e} \rightarrow \mathbb{C} : \quad \psi_{\gamma,f} := f(h_{e_1}, h_{e_2}, \dots, h_{e_n}).$$

$$s_{\gamma, \{j_e\}, \{\iota_n\}}[A] = \bigotimes_{n \in \gamma} \iota_n \bigotimes_{e \in \gamma} {}^{(j_e)}\Pi(h_e[A])$$

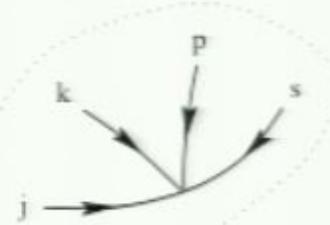
$$\text{Cyl} = \cup_{\gamma} \text{Cyl}_{\gamma}$$

$$\Theta_{e_1 \cup e_2 \cup e_3}^{j,k,l}[A] = {}^{(j)}\Pi(h_{e_1}[A])_{m_1 n_1} {}^{(k)}\Pi(h_{e_2}[A])_{m_2 n_2} {}^{(l)}\Pi(h_{e_3}[A])_{m_3 n_3} \iota^{m_1 m_2 m_3} \iota^{n_1 n_2 n_3}$$



$$\Theta_{e_1 \cup e_2 \cup e_3}^{1,1/2,1/2}[A] = {}^1\Pi(h_{e_1}[A])^{ij} {}^{1/2} \Pi(h_{e_2}[A])_{AB} {}^{1/2} \Pi(h_{e_3}[A])_{CD} \sigma_i^{AC} \sigma_j^{BD}.$$

$${}^{(j)}\Pi(h_{e_1}[A])_{m_1 n_1} {}^{(k)}\Pi(h_{e_2}[A])_{m_2 n_2} {}^{(p)}\Pi(h_{e_3}[A])_{m_3 n_3} {}^{(s)}\Pi(h_{e_4}[A])_{m_4 n_4} \iota^{n_1 n_2 n_3 n_4}$$



$$\iota^{n_1 n_2 n_3 n_4} \in j \otimes k \otimes p \otimes s$$

Scalar Product

Inner product

$$\langle \psi_{\gamma, f}, \psi_{\gamma', g} \rangle = \int \prod_{e \in \Gamma_{\gamma\gamma'}} dh_e \overline{f(h_{e_1}, \dots h_{e_{N_e}})} g(h_{e_1}, \dots h_{e_{N_e}})$$

$\Gamma_{\gamma\gamma'}$ is any graph such that both $\gamma \subset \Gamma_{\gamma\gamma'}$ and $\gamma' \subset \Gamma_{\gamma\gamma'}$.

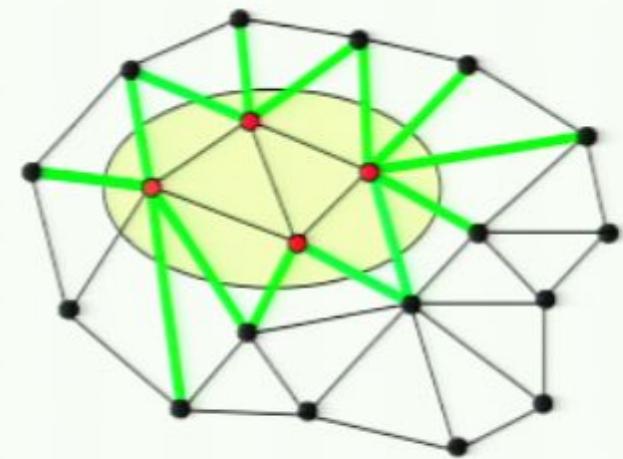
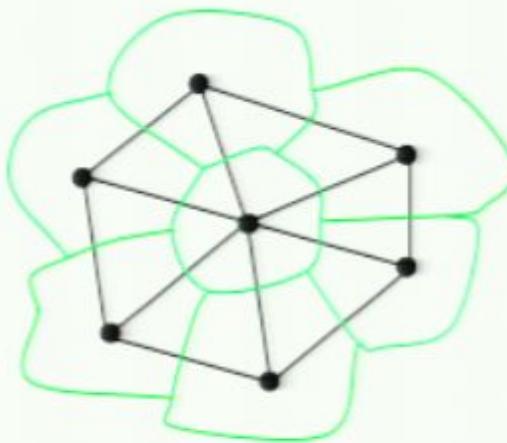
The kinematical Hilbert space H_{kin} is the Cauchy completion of the space of cylindrical functions Cyl in the Ashtekar-Lewandowski measure,

$$\psi = \sum_{n=1}^{\infty} a_n \psi_n , \quad ||\psi||^2 = \sum_{n=1}^{\infty} |a_n|^2 ||\psi_n||^2 < \infty.$$

Diff-Invariant States

Area spectrum

$$\hat{\mathbf{A}}_S |\psi\rangle = 8\pi\ell_p^2 \gamma \sum_p \sqrt{j_p(j_p + 1)} |\psi\rangle$$



$$V(R) |\gamma, j_l, i_1, \dots, i_N\rangle = \sqrt{(16\pi G)^3} V_{i_n}^{i'_n} |\gamma, j_l, i_1, \dots, i'_n, \dots, i_N\rangle$$

Hamiltonian Constraint

Quantization of the scalar constraint

$$S^E(N) = \int_{\Sigma} d^3x \ N \frac{E_i^a E_j^b}{\sqrt{\det(E)}} \epsilon^{ij} {}_k F_{ab}^k,$$

$$\frac{E_i^b E_j^c}{\sqrt{\det(E)}} \epsilon^{ijk} \epsilon_{abc} = \frac{4}{\kappa \gamma} \{ A_a^k, V \} , \quad V = \int \sqrt{\det(E)} \text{ volume of } \Sigma.$$

$$S^E(N) = \int_{\Sigma} dx^3 \ N \ \epsilon^{abc} \delta_{ij} F_{ab}^i \{ A_c^j, V \} .$$

Given an infinitesimal loop α_{ab} on the ab -plane with coordinate area ϵ^2 ,

$$h_{\alpha_{ab}}[A] - h_{\alpha_{ab}^I}[A] = \epsilon^2 F_{ab}^i \tau_i + O(\epsilon^4) , \quad h_{e_a}^{-I}[A] \{ h_{e_a}[A], V \} = \epsilon \{ A_a^i, V \} + O(\epsilon^2) ,$$

e_a is a path along the a -coordinate of coordinate length ϵ .



$$S^E(N) = \lim_{\epsilon \rightarrow 0} \sum_I N_I \ \epsilon^{abc} \text{Tr} \left[(h_{\alpha_{ab}^I}[A] - h_{\alpha_{ab}^I}^{-I}[A]) h_{e_c^I}^{-I}[A] \{ h_{e_c^I}[A], V \} \right] .$$

Action on Spin Networks

The quantum constraint can formally be written :

$$S^E(N) = \lim_{\epsilon \rightarrow 0} \sum_I N_I \epsilon^{abc} \text{Tr} \left[(\hat{h}_{\alpha_{ab}^I}[A] - \hat{h}_{\alpha_{ab}^I}^{-1}[A]) \hat{h}_{e_c^I}[A] \left[\hat{h}_{e_c^I}[A], \hat{V} \right] \right]$$

The regulated quantum scalar constraint acts only on spin network nodes

$$\hat{S}_\epsilon(N) \psi_{\gamma,f} = \sum_{n \in \gamma} N_n \hat{S}_\epsilon^n \psi_{\gamma,f} ,$$

where \hat{S}_ϵ^n acts only on the node $n \in \gamma$ and N_n

is the value of the lapse $N(x)$ at the node.

$$E_i^a(x) = -i\hbar 8\pi G_N \frac{\delta}{\delta A_a^i} h_e(A) = \int ds \dot{e}(s)^a \delta^3(e(s), x) h_{e_1}(A) \tau_i h_{e_2}(A)$$

The action on four valent nodes can be written as :

$$\begin{aligned} \hat{S}_\epsilon^n & \quad \text{Diagram: Four edges meeting at a central node labeled } n. \\ &= \sum_{op} S_{jklm, opq} \quad \text{Diagram: Four edges meeting at a central node labeled } n, \text{ with two additional edges } p \text{ and } q \text{ connecting to the same node.} \\ & \quad + \\ &+ \sum_{op} S_{jlmk, opq} \quad \text{Diagram: Four edges meeting at a central node labeled } n, \text{ with two additional edges } p \text{ and } q \text{ connecting to different nodes.} \\ & \quad + \sum_{op} S_{jmkl, opq} \quad \text{Diagram: Four edges meeting at a central node labeled } n, \text{ with two additional edges } p \text{ and } q \text{ connecting to different nodes.} \end{aligned}$$

Avoiding Black Hole Singularity in LQG

't Hooft Quantum Black Hole

The Schwarzschild solution inside the horizon

$$ds^2 = -\frac{dT^2}{\left(\frac{2MG_N}{T} - 1\right)} + \left(\frac{2MG_N}{T} - 1\right)dr^2 + T^2(\sin^2 \theta d\phi^2 + d\theta^2)$$

$$T \in]0, 2MG_N[, \quad r \in]-\infty, +\infty[.$$

The Kantowski-Sachs space-time ($R \times R \times S^2$) :

$$ds^2 = -N^2(t)dt^2 + a^2(t)dr^2 + b^2(t)(\sin^2 \theta d\phi^2 + d\theta^2)$$

$$R^{\mu\nu\rho\sigma\delta}R_{\mu\nu\rho\sigma\delta} = \frac{48G_N^2 M^2}{b(t)^6}$$

$$\int_{\gamma}^{\gamma'} \omega = C$$

$$A = \dots \tau \circ d\alpha$$

$$= \sin(\theta) T_\phi - i \Gamma_0 \cos(\theta) \sin(\theta)$$

H

13.1.es

$$z\bar{G_N}$$

Rescale var. $\sim L_0$ $\sim L_0$

$$ib = \dim_{\mathbb{C}} \frac{D_b^2}{\dim_{\mathbb{C}} [L]^2} \sim \beta_2 |\sin^2 \ell|$$

K.S. Classical Theory

$$A = \tilde{c}(t)\tau_3 dx + \tilde{b}(t)\tau_2 d\theta - \tilde{b}(t)\tau_1 \sin \theta d\phi + \tau_3 \cos \theta d\phi,$$

$$E = \tilde{p}_c(t)\tau_3 \sin \theta \frac{\partial}{\partial x} + \tilde{p}_b(t)\tau_2 \sin \theta \frac{\partial}{\partial \theta} - \tilde{p}_b(t)\tau_1 \frac{\partial}{\partial \phi}.$$

Hamiltonian constraint :

$$\mathcal{C}_H = \int d^3x N e^{-1} [\epsilon_{ijk} F_{ab}^i E_j^a E_k^b - 2(1 + \gamma^2) K_{[a}^i K_{b]}^j E_i^a E_j^b], \quad E^{ai} = (\text{dete}) e^{ai}.$$

$$\mathcal{C}_H = -\frac{N}{2G_N \gamma^2} \left[(b^2 + \gamma^2) \frac{p_b \text{sgn}(p_c)}{\sqrt{|p_c|}} + 2bc \sqrt{|p_c|} \right], \quad \mathcal{C}^a \equiv 0, \quad \mathcal{G}^i \equiv 0.$$

Rescaled variables : $b = \bar{b}$, $c = L_0 \bar{c}$, $p_b = L_0 \bar{p}_b$, $p_c = \bar{p}_c$. $[c] = L^0$, $[p_c] = L^2$, $[b] = L^0$, $[p_b] = L^2$.

$$V = 4\pi \sqrt{|p_c|} p_b,$$

$$g_{ab} = \text{diag} \left(\frac{p_b^2}{|p_c| L_0^2}, |p_c|, |p_c| \sin^2 \theta \right) = \text{diag} \left(a^2, b^2, b^2 \sin^2 \theta \right)$$

Classical phase space

Canonical pairs : $(b, p_b), (c, p_c)$.

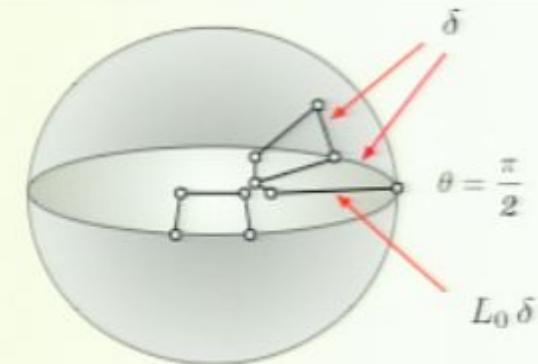
Symplectic structure : $\{c, p_c\} = 2\gamma G_N, \{b, p_b\} = \gamma G_N$.

Holonomies

$$h_1(A) = e^{\delta c \tau_3},$$

$$h_I = \exp \int A_I^i \tau_i d\lambda \quad \rightarrow \quad h_2(A) = e^{\delta b \tau_2},$$

$$h_3(A) = e^{-\delta b \tau_1}.$$



$$\text{Field straight : } F_{ab}^i \tau_i = {}^0 \omega_a^i {}^0 \omega_b^j \left(\frac{h_i^{(\delta)} h_j^{(\delta)} h_i^{(\delta)-1} h_j^{(\delta)-1}}{\delta^2} \right).$$

Hamiltonian Constraint

$$\mathcal{C}_H = \frac{-N}{(8\pi G_N)^2 \gamma^3 \delta^3} \text{Tr} \left[\sum_{ijk} \epsilon^{ijk} h_i^{(\delta)} h_j^{(\delta)} h_i^{(\delta)-1} h_j^{(\delta)-1} h_k^{(\delta)} \left\{ h_k^{(\delta)-1}, V \right\} + 2\gamma^2 \delta^2 \tau_3 h_1^{(\delta)} \left\{ h_1^{(\delta)-1}, V \right\} \right]$$

Quantum Theory

QUANTUM THEORY

$$H = H_{p_b} \otimes H_{p_c} = L^2(R^2_{Bohr}).$$

Basis in Hilbert space :

$$|\mu\rangle \otimes |\tau\rangle \rightarrow \langle b|\mu\rangle \otimes \langle c|\tau\rangle = e^{i\mu b/2} \otimes e^{i\tau c/2}, \quad \langle \mu_1, \tau_1 | \mu_2, \tau_2 \rangle = \delta_{\mu_1, \mu_2} \delta_{\tau_1, \tau_2}.$$

$$LQG : \left(\hat{E}, \hat{h}_\gamma \right) \rightarrow LQBH : \left(\hat{p}_b, \hat{p}_c, \hat{h}_x^{(\mu)}, \hat{h}_{\theta, \phi}^{(\tau)} \right).$$

$$\hat{p}_b = -i\gamma l_P^2 \frac{\partial}{\partial b}, \quad \hat{p}_c = -2i\gamma l_P^2 \frac{\partial}{\partial c}.$$

$$\hat{p}_b |\mu, \tau\rangle = \frac{1}{2}\gamma l_P^2 \mu |\mu, \tau\rangle, \quad \hat{p}_c |\mu, \tau\rangle = \gamma l_P^2 \tau |\mu, \tau\rangle.$$

$$Volume\ operator : \hat{V} = 4\pi |\hat{p}_b| \sqrt{|\hat{p}_c|}.$$

$$Volume\ spectrum : V_{\mu, \tau} = 2\pi \gamma^{3/2} |\mu| \sqrt{|\tau|} l_P^3.$$

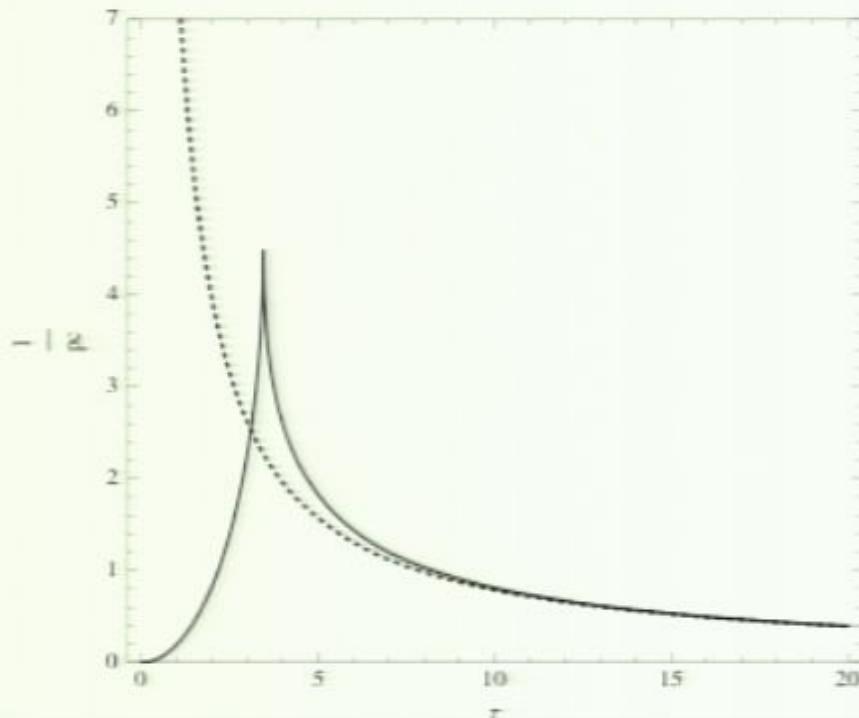
Spatial Section Curvature

Spatial Curvature

$${}^{(3)}R = \frac{2}{|p_c|}$$

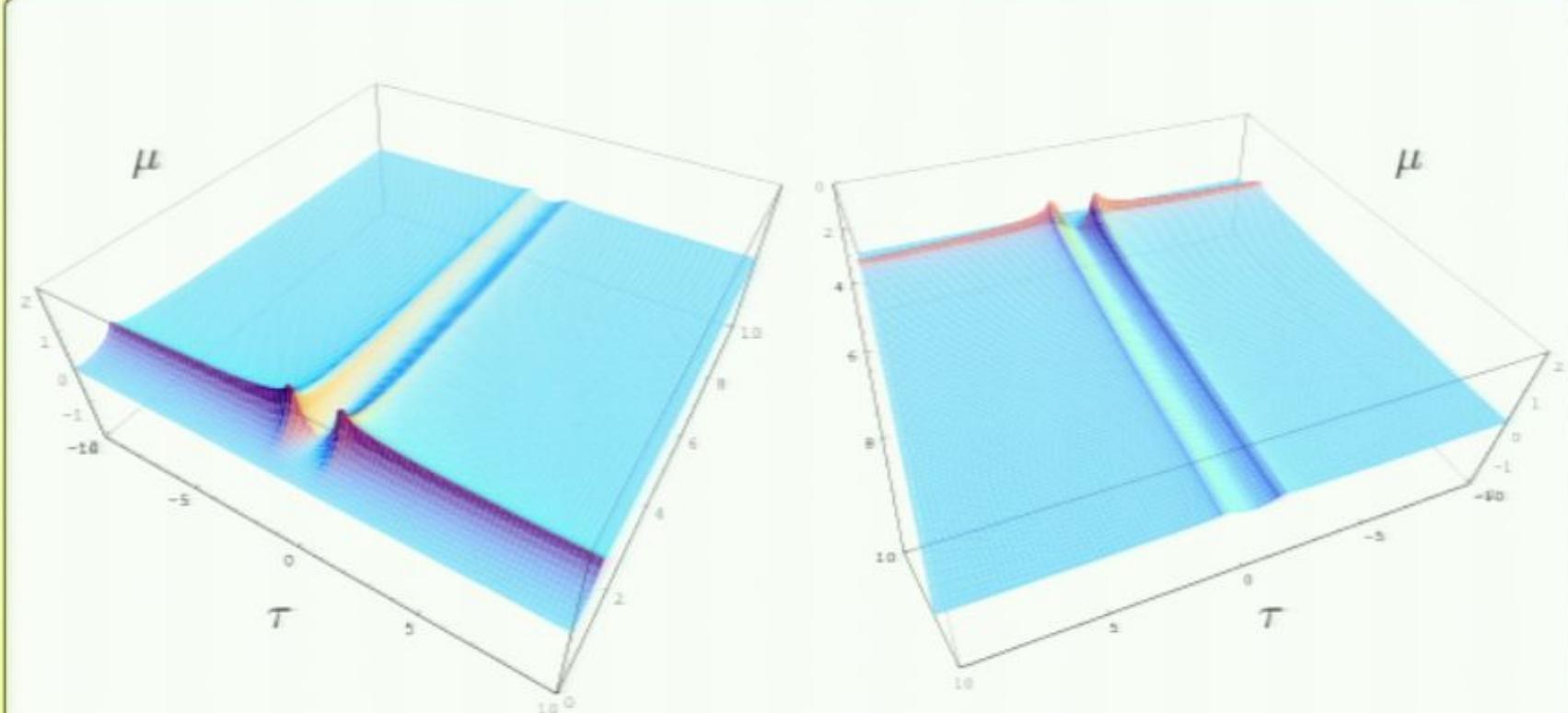
$$\frac{1}{|p_c|} = \left(\frac{1}{\gamma G_N} \{c, \sqrt{|p_c|}\} \right)^2 = \lim_{\delta \rightarrow 0} \left(\frac{3}{\gamma G_N \delta j(j+1)(2j+1)} \text{Tr} \left[\tau_3^{(j)} e^{c \delta \tau_3^{(j)}} \{e^{-c \delta \tau_3^{(j)}}, \sqrt{|p_c|}\} \right] \right)^2.$$

$$\widehat{\frac{1}{|p_c|}}_j |\mu, \tau\rangle = \left(\frac{3}{\sqrt{\gamma} l_P \delta j(j+1)(2j+1)} \sum_{k=-j}^j [k(\sqrt{|\tau|} - \sqrt{|\tau - 2k\delta|})] \right)^2 |\mu, \tau\rangle$$



Inverse Volume Spectrum

Inverse volume operator spectrum



Quantum Dynamics

Quantum Dynamics

The solutions of the Hamiltonian constraint are in C^*
dual of the dense subspace C of the kinematical space H_{kin} .

General state in C^* : $\langle \Psi | = \sum_{\mu, \tau} \Psi_\mu^\tau \langle \mu, \tau |$.

The constraint eq. $\hat{H}|\Psi\rangle = 0$ gives a difference eq.

for the coefficients Ψ_μ^τ :

$$\mathcal{O}_+(\tau)(\Psi_{\mu+1}^{\tau+1} - \Psi_{\mu-1}^{\tau+1}) - \mathcal{O}_-(\tau)(\Psi_{\mu+1}^{\tau-1} - \Psi_{\mu-1}^{\tau-1}) + \mathcal{O}_0(\tau)(m_+(\mu)\Psi_{\mu+2}^\tau - \kappa m_0(\mu)\Psi_\mu^\tau + m_-(\mu)\Psi_{\mu-2}^\tau) = 0$$

$$\mathcal{O}_+(\tau) = \sqrt{|\tau+1|} + \sqrt{|\tau|}$$

$$\mathcal{O}_-(\tau) = \sqrt{|\tau-1|} + \sqrt{|\tau|}$$

$$\mathcal{O}_0(\tau) = \sqrt{|\tau+1/2|} - \sqrt{|\tau-1/2|}$$

$$m_+(\mu) = \mu + 1, \quad m_-(\mu) = \mu - 1, \quad m_0(\mu) = \mu.$$

$$\kappa = 2(1 + \gamma^2 \delta^2), \quad \delta = 1/2, \quad \mu > 2.$$

Semiclassical Analysis

Semiclassical Analysis

$$\begin{aligned}\mathcal{C}_H &= \frac{-N}{(8\pi G_N)^2 \gamma^3 \delta^3} \text{Tr} \left[\sum_{ijk} \epsilon^{ijk} h_i^{(\delta_i)} h_j^{(\delta_j)} h_i^{(\delta_i)-1} h_j^{(\delta_j)-1} h_k^{(\delta)} \left\{ h_k^{(\delta)-1}, V \right\} + 2\gamma^2 \delta^2 \tau_3 h_1^{(\delta)} \left\{ h_1^{(\delta)-1}, V \right\} \right] \\ &= -\frac{N}{2G_N \gamma^2} \left\{ 2 \frac{\sin \delta c}{\delta} \frac{\sin(\sigma(\delta) \delta b)}{\delta} \sqrt{|p_c|} + \left(\frac{\sin^2(\sigma(\delta) \delta b)}{\delta^2} + \gamma^2 \right) \frac{p_b \operatorname{sgn}(p_c)}{\sqrt{|p_c|}} \right\}.\end{aligned}$$

$$\delta_i = (\delta c, \sigma(\delta) \delta b, \sigma(\delta) \delta b), \quad \sigma(\delta) = 1/\sqrt{1 + \gamma^2 \delta^2}.$$

Hamilton eq. motion :

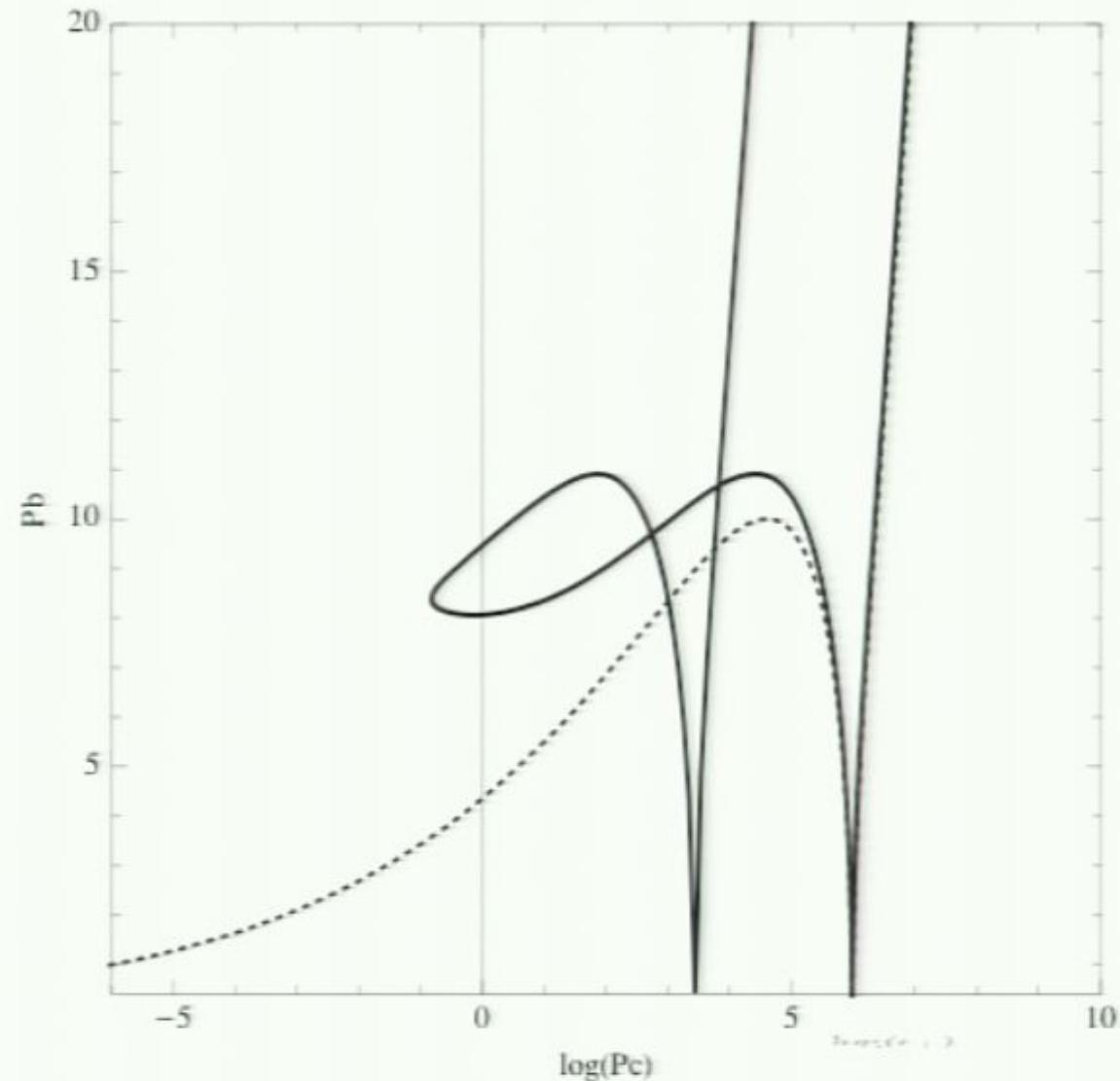
$$\begin{aligned}N &= (\gamma \sqrt{|p_c|} \operatorname{sgn}(p_c) \delta) / (\sin \sigma(\delta) \delta b), \quad \mathcal{C}_H = 0, \\ \dot{c} &= -2 \frac{\sin \delta c}{\delta}, \quad \dot{p}_c = 2p_c \cos \delta c, \quad \dot{b} = -\frac{1}{2} \left(\frac{\sin \sigma(\delta) \delta b}{\delta} + \frac{\gamma^2 \delta}{\sin \sigma(\delta) \delta b} \right), \quad \dot{p}_b = \frac{\sigma(\delta)}{2} \cos \sigma(\delta) \delta b \left(1 - \frac{\gamma^2 \delta^2}{\sin^2 \sigma(\delta) \delta b} \right) p_b.\end{aligned}$$

Solutions :

$$c(t) = \frac{2}{\delta} \arctan \left(\mp \frac{\gamma \delta m p_b^0}{2t^2} \right), \quad p_c(t) = \pm \frac{1}{t^2} \left[\left(\frac{\gamma \delta m p_b^0}{2} \right)^2 + t^4 \right], \quad \cos \sigma(\delta) \delta b(t) = \rho(\delta) \begin{bmatrix} 1 - \left(\frac{2m}{t} \right) \mathcal{P}(\delta) \\ 1 + \left(\frac{2m}{t} \right) \mathcal{P}(\delta) \end{bmatrix}, \quad p_b(t) = p_b(c(t), p_c(t), b(t)).$$

$$\rho(\delta) = \sqrt{1 + \gamma^2 \delta^2}, \quad \mathcal{P}(\delta) = \frac{\sqrt{1 + \gamma^2 \delta^2} - 1}{\sqrt{1 + \gamma^2 \delta^2} + 1}.$$

$$p_b = p_b(\log(p_c))$$



Metric form of the solution

$$ds^2 = -N^2(t)dt^2 + X^2(t)dx^2 + Y^2(t)(d\theta^2 + \sin\theta d\phi^2),$$

$$N^2(t) = \frac{\gamma^2 \delta^2 |p_c(t)|}{t^2 \sin^2 \sigma(\delta) \delta b}, \quad X^2(t) = \frac{p_b^2(t)}{L_0^2 |p_c(t)|}, \quad Y^2(t) = |p_c(t)|.$$

Schwarzschild coordinates : $(t \leftrightarrow x \equiv r)$

$$-N^2(t) \rightarrow g_{rr}(r),$$

$$X^2(t) \rightarrow g_{tt}(r),$$

$$Y^2(t) \rightarrow g_{\theta\theta}(r) = g_{\phi\phi}/\sin^2\theta.$$

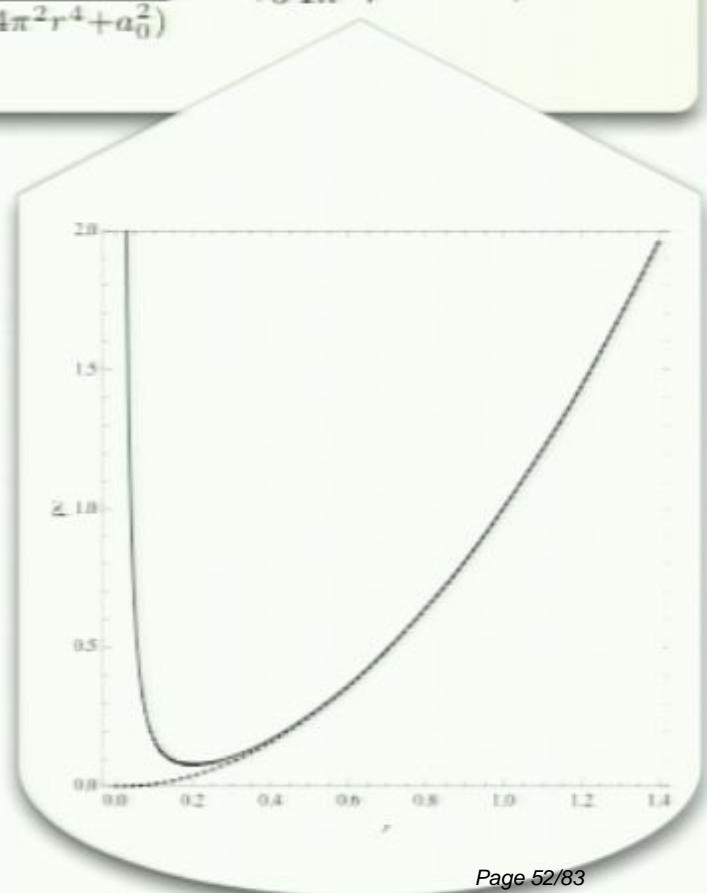
LQG BLACK HOLE

$$ds^2 = -\frac{64\pi^2(r-r_+)(r-r_-)(r+r_+\mathcal{P}(\delta))^2}{64\pi^2r^4+a_0^2}dt^2 + \frac{dr^2}{\frac{64\pi^2(r-r_+)(r-r_-)r^4}{(r+r_+\mathcal{P}(\delta))^2(64\pi^2r^4+a_0^2)}} + \left(\frac{a_0^2}{64\pi^2r^2} + r^2\right)d\Omega^{(2)}.$$

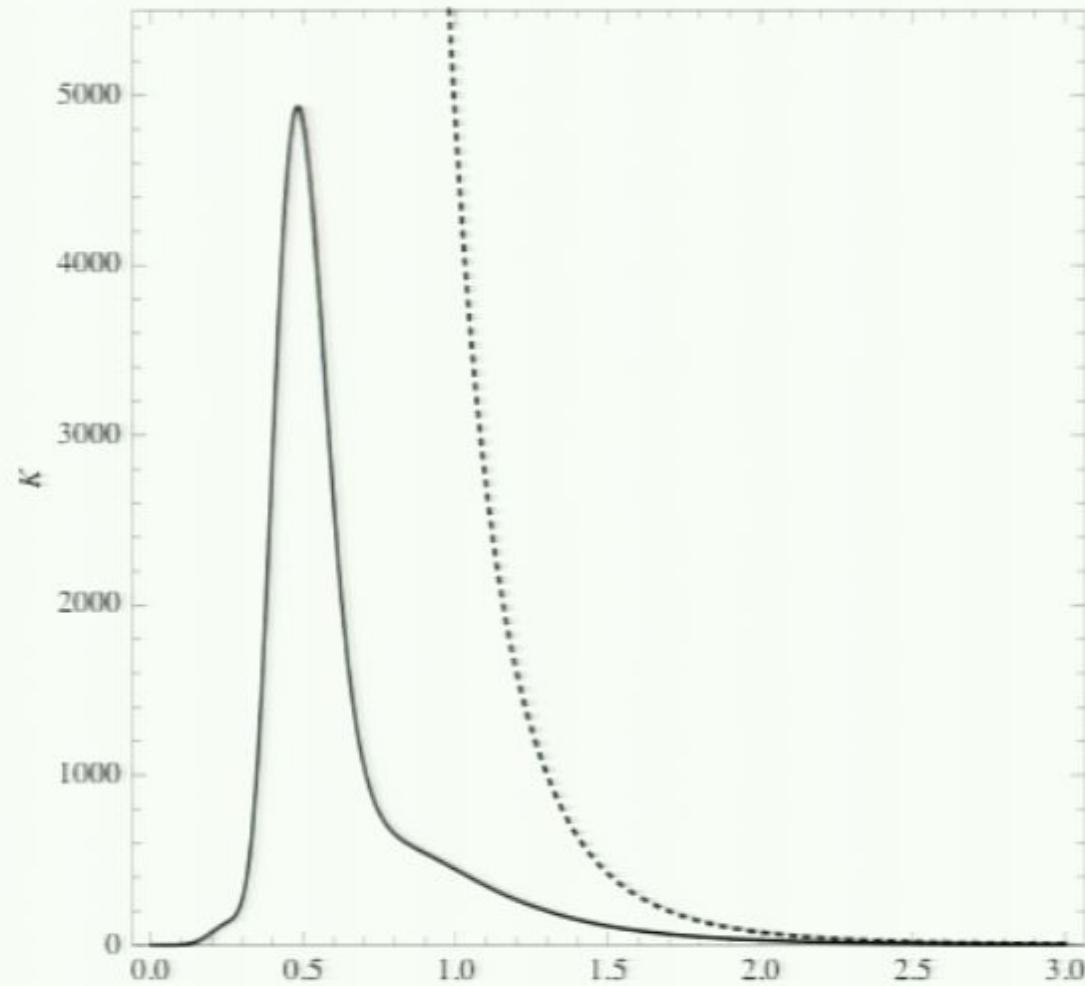


$$r_+ = 2m, \quad a_0 = 2\sqrt{3}\pi\gamma l_P^2,$$

$$r_- = 2m\mathcal{P}(\delta)^2 = 2m \left(\frac{2 + \gamma^2\delta^2 - 2\sqrt{1 + \gamma^2\delta^2}}{2 + \gamma^2\delta^2 + 2\sqrt{1 + \gamma^2\delta^2}} \right).$$



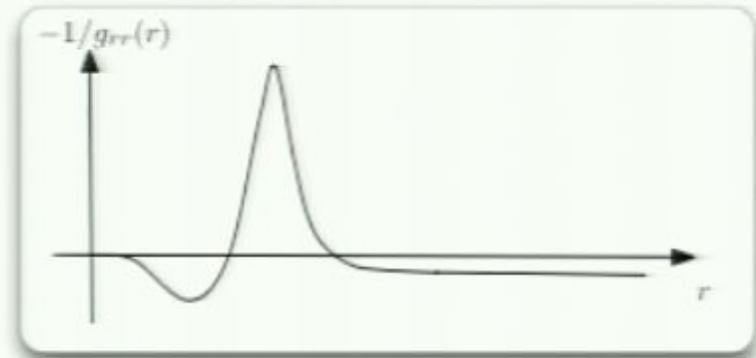
Kretschmann Invariant



THE LQG METRIC

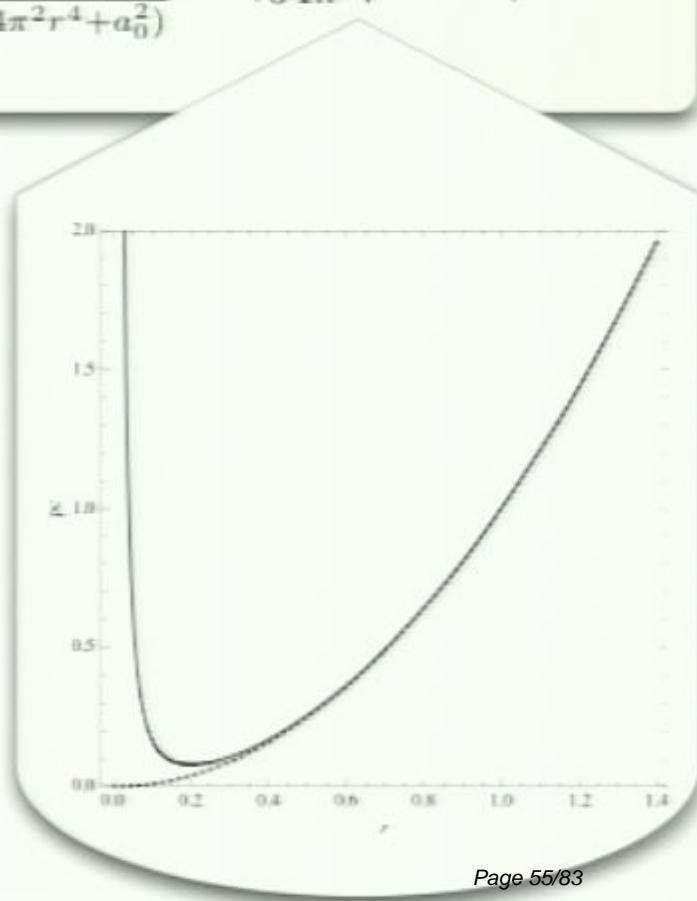
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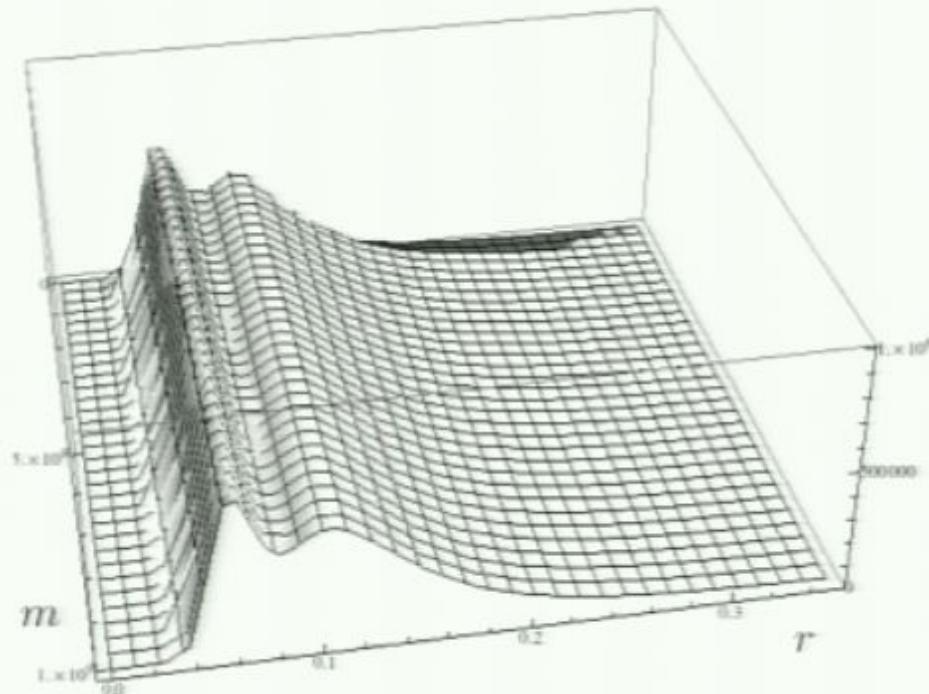


Kretschmann Invariant

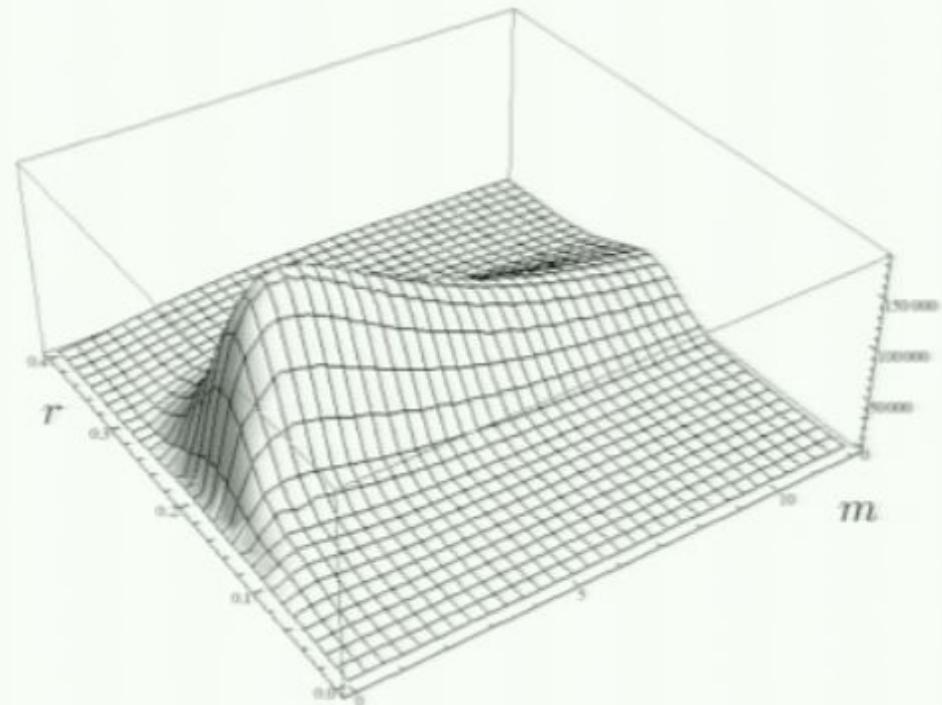
Kretschmann Invariant

$K(m, r)$

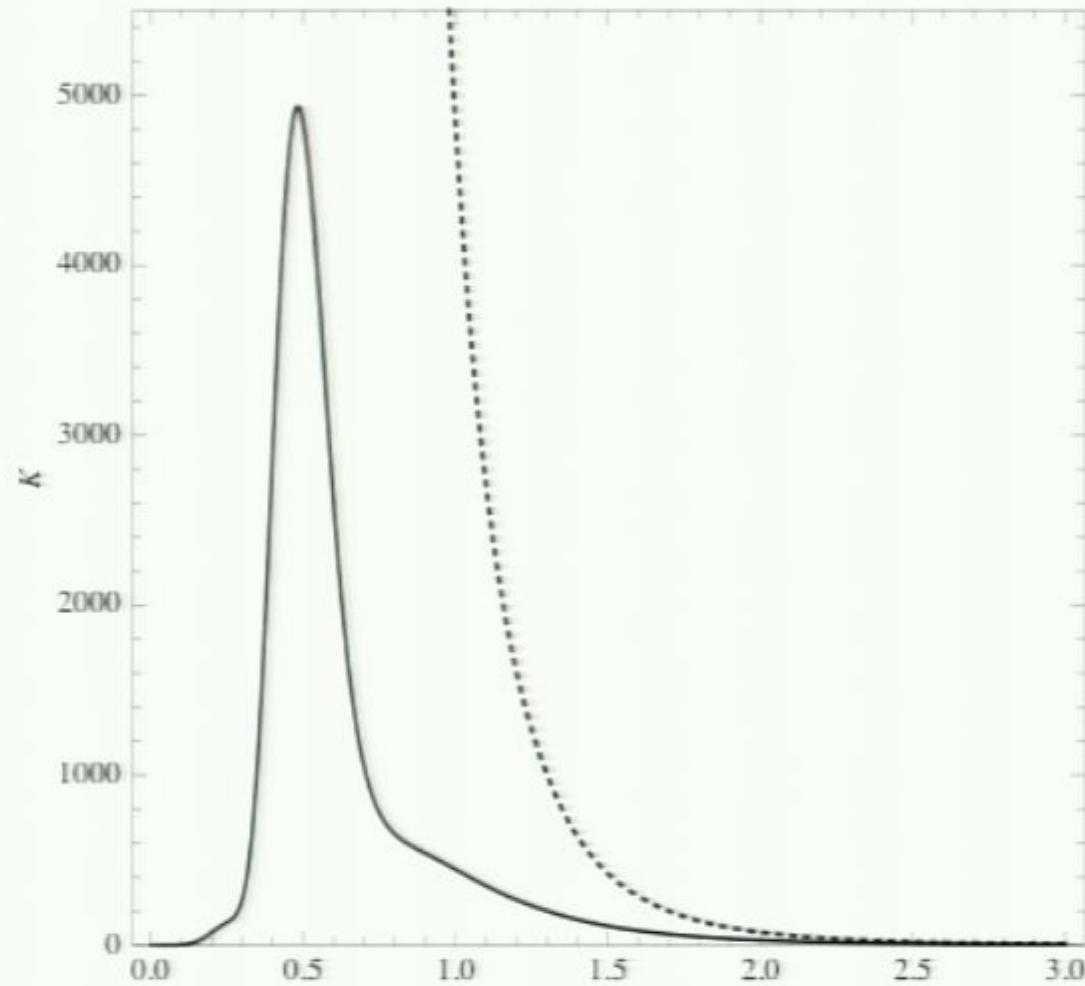
$K(m, r)$



$K(m, r)$



Kretschmann Invariant

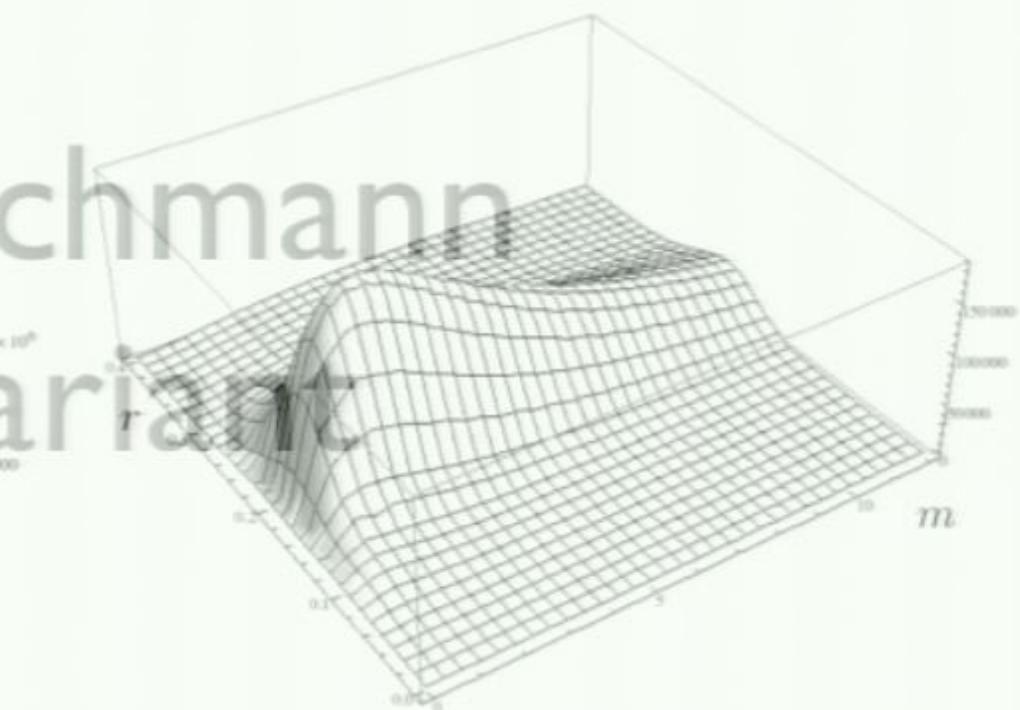
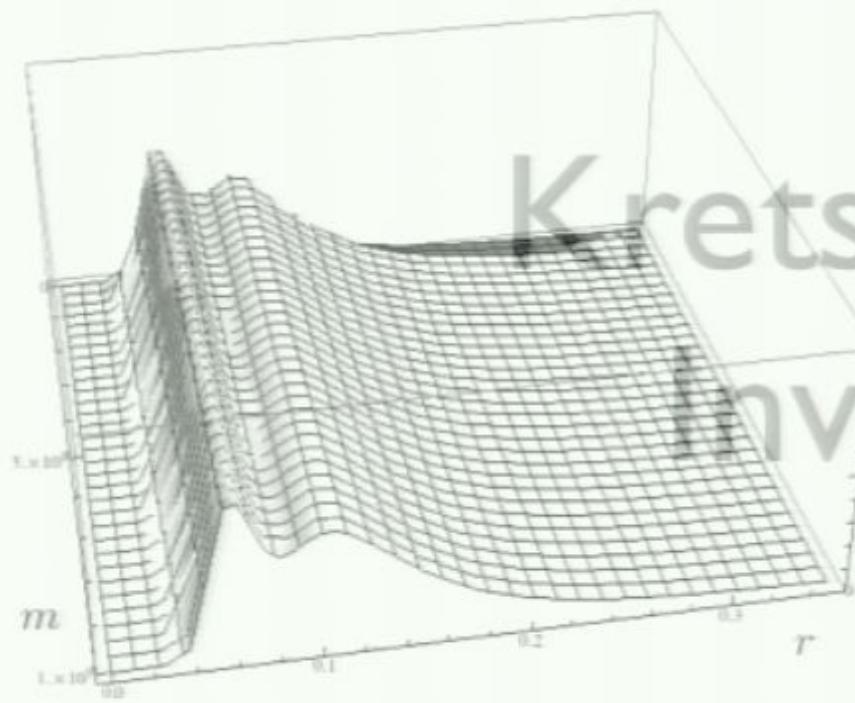


Kretschmann Invariant

$K(m, r)$

$K(m, r)$

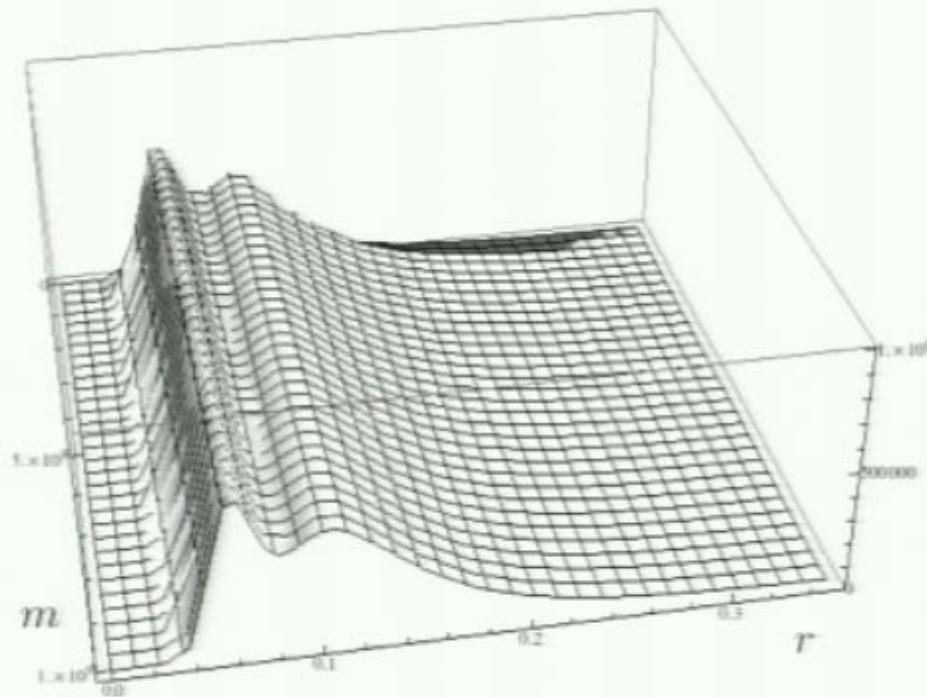
$K(m, r)$



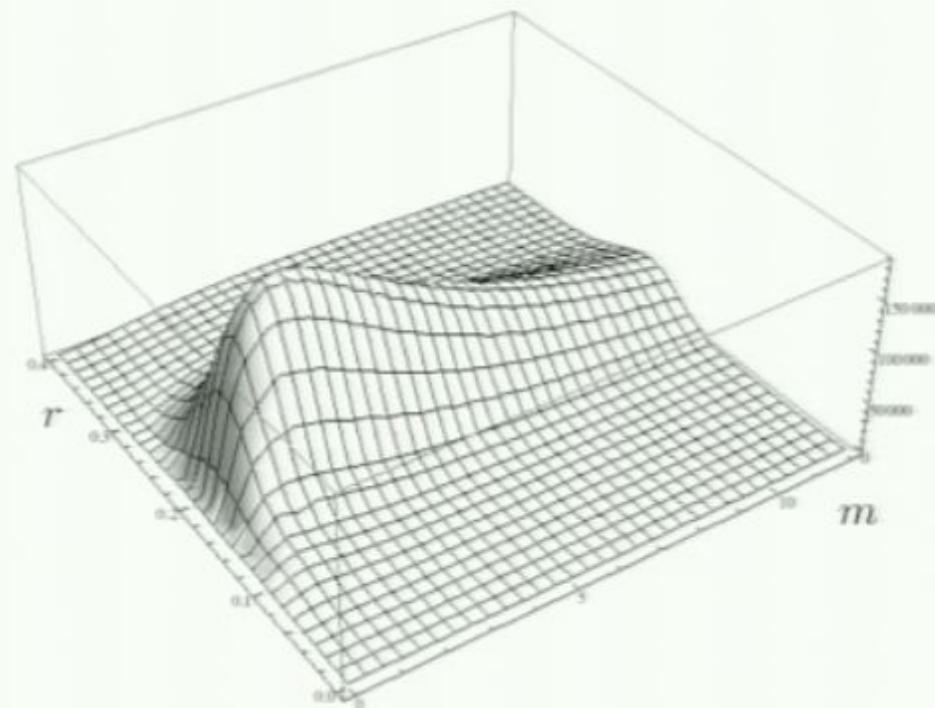
Kretschmann Invariant

$K(m, r)$

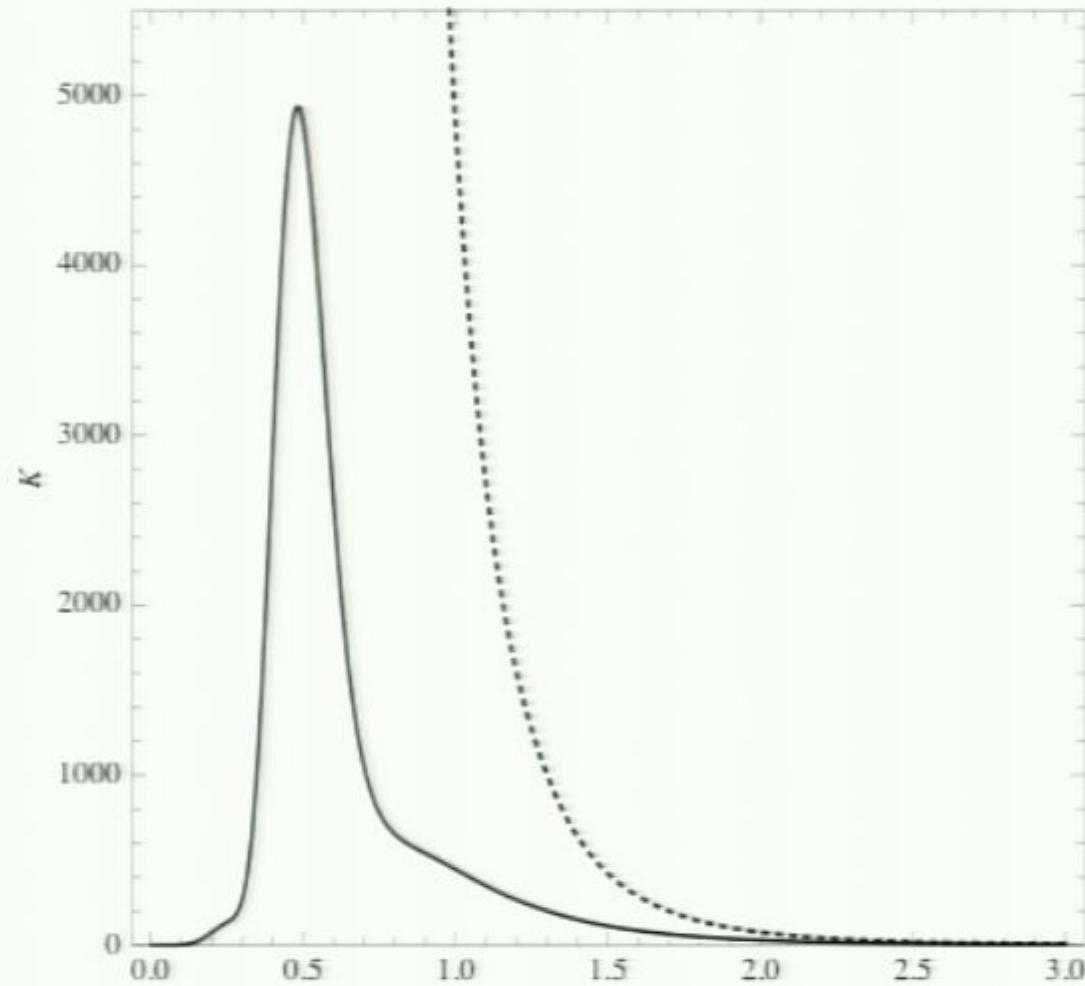
$K(m, r)$



$K(m, r)$



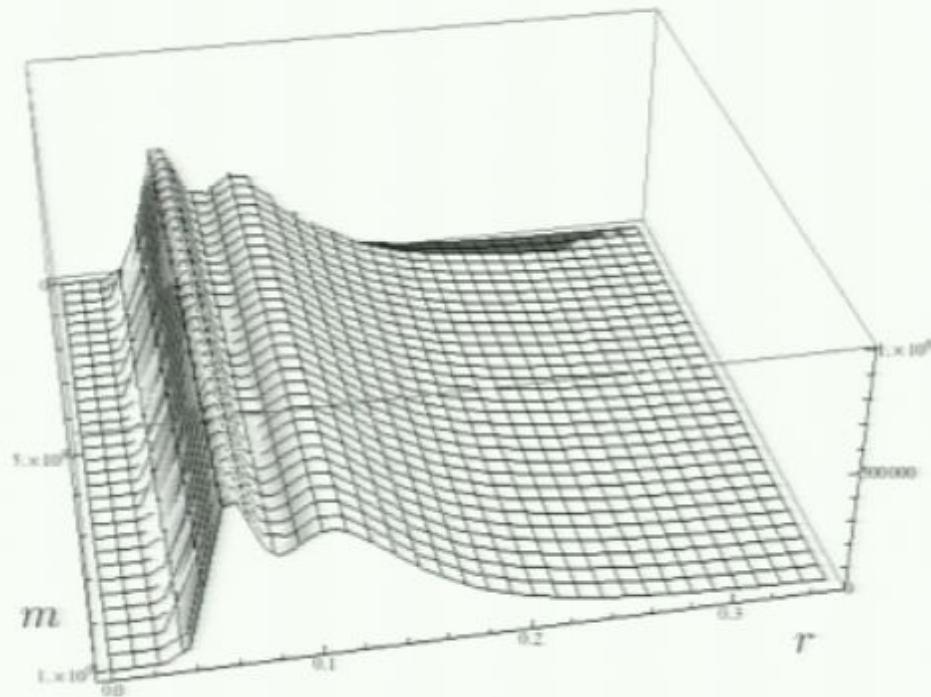
Kretschmann Invariant



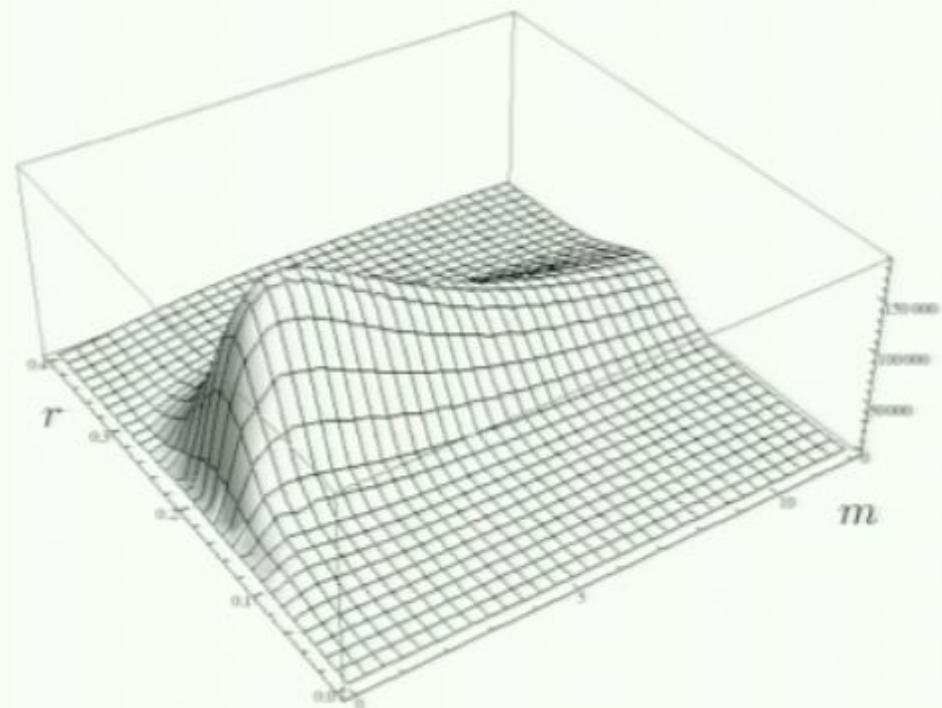
Kretschmann Invariant

$K(m, r)$

$K(m, r)$



$K(m, r)$



Schwarzschild Core

Schwarzschild Core in $r \sim 0$

For $r \sim 0$ & $R = 1/r\sqrt{c}$:

$$ds^2 = -(a - b r)dt^2 + \frac{dr^2}{cr^4 - dr^5} + \frac{d\Omega^{(2)}}{cr^2}, \quad a, b, c, d(m, a_0, \delta, \gamma)$$

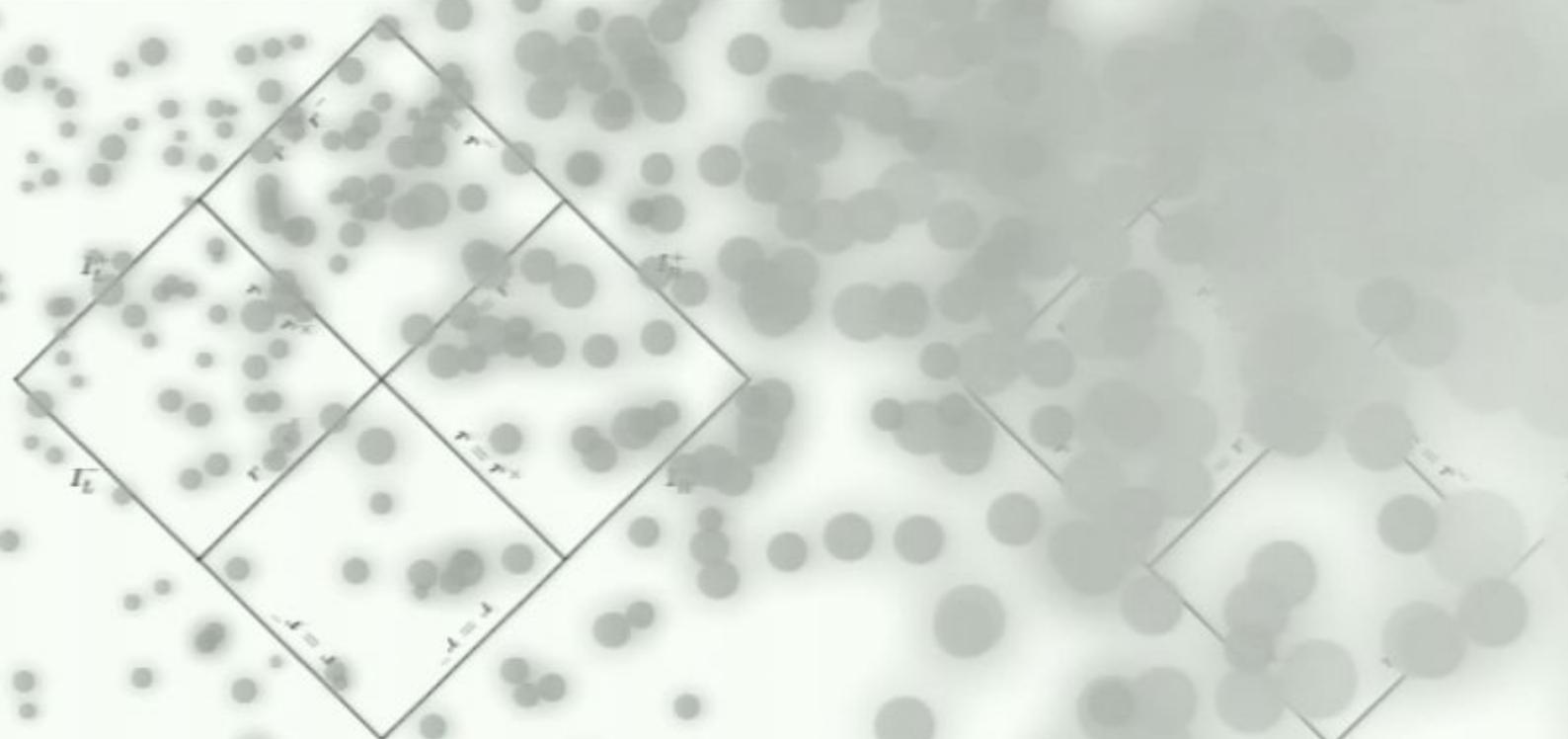


$$ds^2 = -\left(1 - \frac{2M}{R}\right)dt^2 + \frac{dR^2}{1 - \frac{2M}{R}} + R^2 d\Omega^{(2)}, \quad M \sim \frac{a_0}{2m\gamma^4\delta^4}.$$

Causal structure & Carter – Penrose Diagrams

Tortoise coordinate

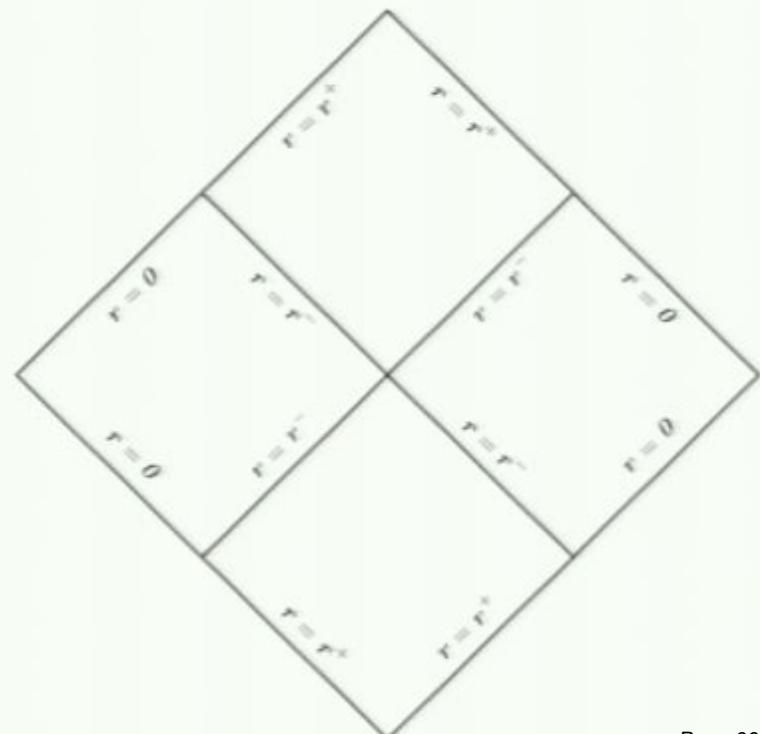
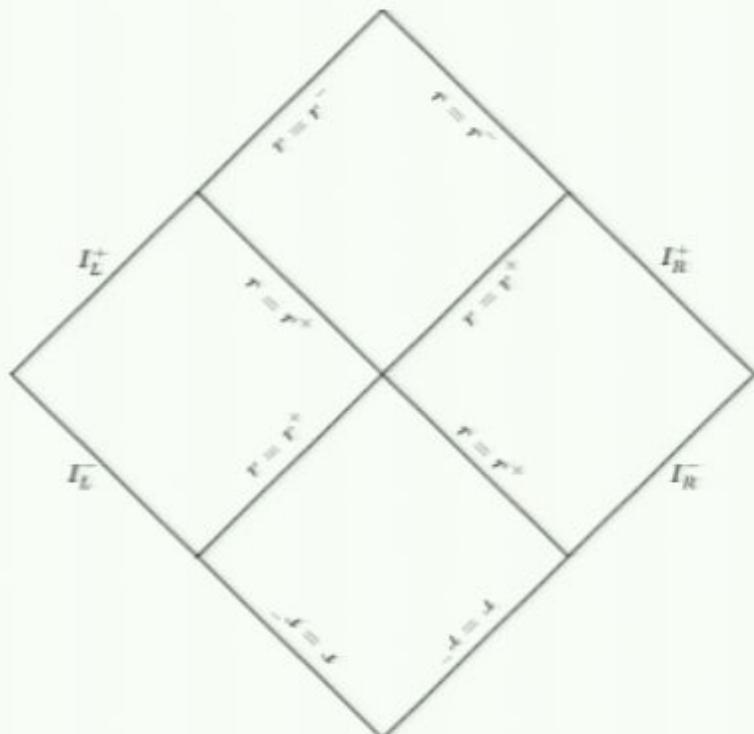
$$r^* = \frac{1}{512\pi^2} \left[-\frac{2a_0^2}{\mathcal{P}(\delta)^2 m^2 r} + 512\pi r + \frac{a_0^2 (\mathcal{P}(\delta)^2 + 1)}{\mathcal{P}(\delta)^4 m^3} \log(r) \right] - \frac{a_0^2 + 1024\pi^2 m^4}{(\mathcal{P}(\delta)^2 - 1)m^3} \log|r - r_+| + \frac{a_0^2}{m}$$

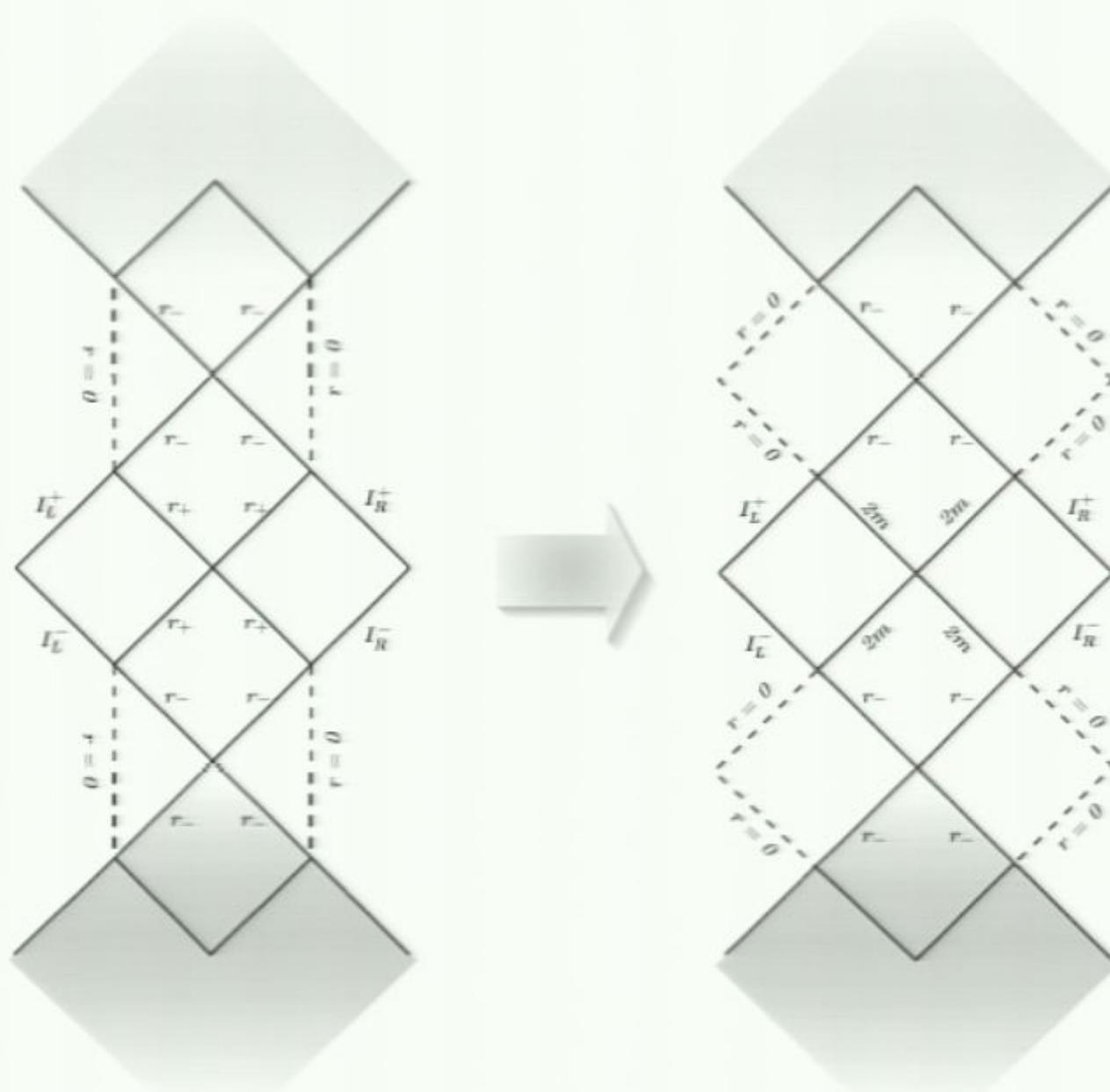


Causal structure & Carter – Penrose Diagrams

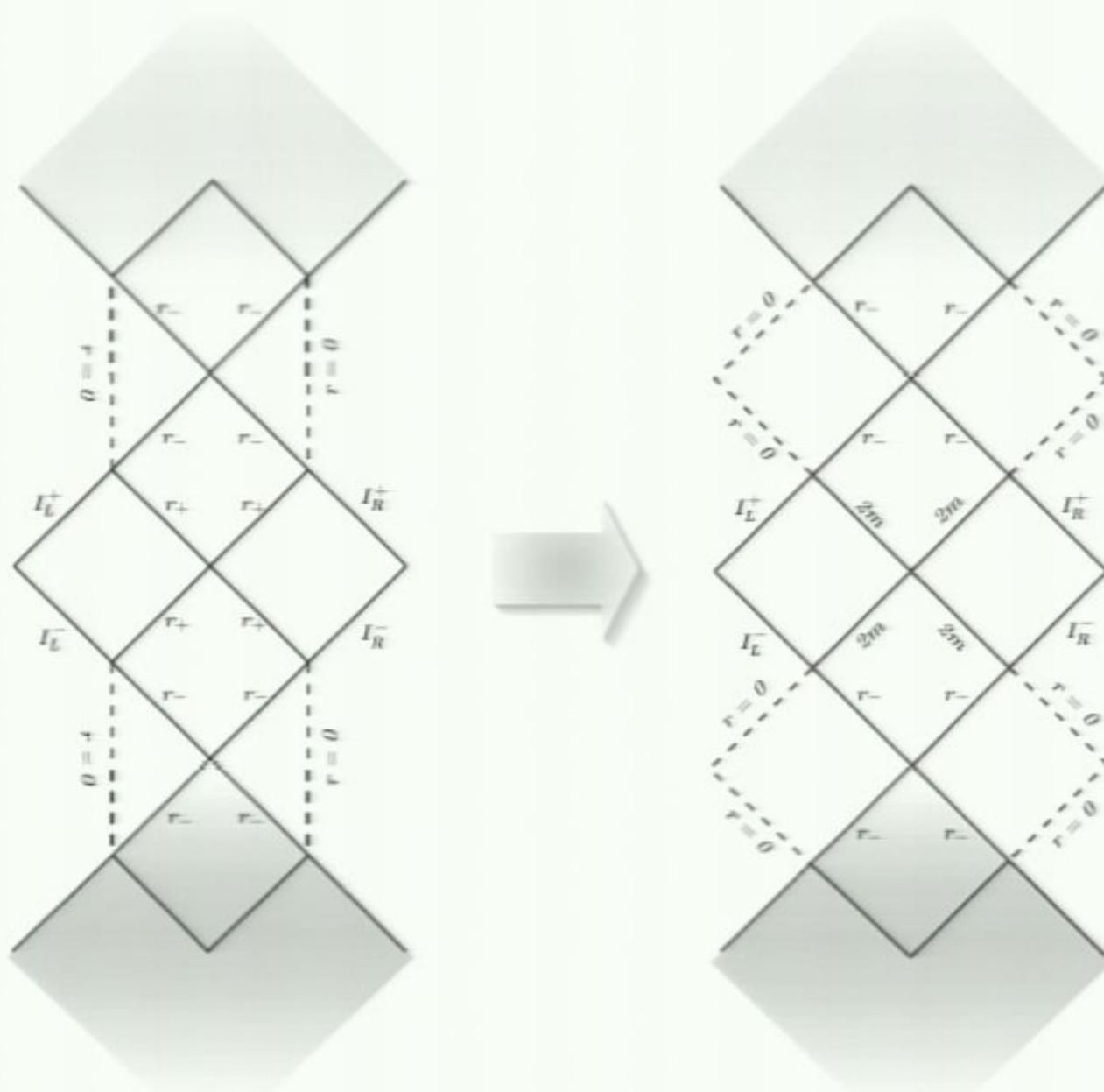
Tortoise coordinate :

$$r^* = \frac{1}{512\pi^2} \left[-\frac{2a_0^2}{\mathcal{P}(\delta)^2 m^2 r} + 512\pi^2 r + \frac{a_0^2(\mathcal{P}(\delta)^2 + 1)}{\mathcal{P}(\delta)^4 m^3} \log(r) - \frac{a_0^2 + 1024\pi^2 m^4}{(\mathcal{P}(\delta)^2 - 1)m^3} \log|r - r_+| + \frac{a_0^2 + 1024\pi^2 \mathcal{P}(\delta)^4 m^4}{(\mathcal{P}(\delta)^2 - 1)\mathcal{P}(\delta)^4 m^3} \log|r - r_-| \right].$$





Self-Duality

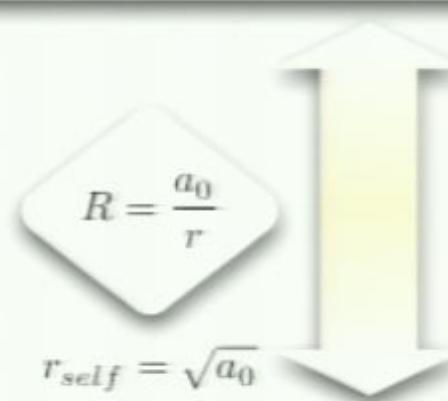


Self-Duality

SELF – DUALITY

The Metric :

$$ds^2 = -\frac{(r - r_+)(r - r_-)(r + r_\star)^2}{r^4 + a_0^2} dt^2 + \frac{dr^2}{\frac{(r - r_+)(r - r_-)r^4}{(r + r_\star)^2(r^4 + a_0^2)}} + \left(\frac{a_0^2}{r^2} + r^2\right) d\Omega^{(2)}.$$



$$r_+ \rightarrow R_- = \frac{a_0}{r_+} = \frac{a_0}{2m},$$

$$r_- \rightarrow R_+ = \frac{a_0}{r_-} = \frac{a_0}{2m\mathcal{P}(\delta)^2},$$

$$r_\star \rightarrow R_\star = \frac{a_0}{r_\star} = \frac{a_0}{2m\mathcal{P}(\delta)}.$$

The Dual Metric :

$$ds^2 = -\frac{(R - R_+)(R - R_-)(R + R_\star)^2}{R^4 + a_0^2} dt^2 + \frac{dR^2}{\frac{(R - R_+)(R - R_-)R^4}{(R + R_\star^d)^2(R^4 + a_0^2)}} + \left(\frac{a_0^2}{R^2} + R^2\right) d\Omega^{(2)}.$$

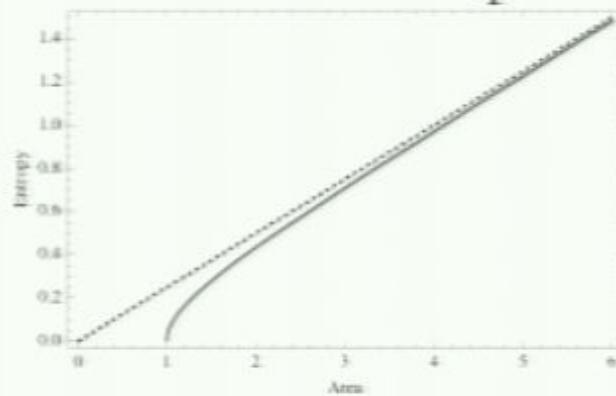
Thermodynamics

THERMODYNAMICS

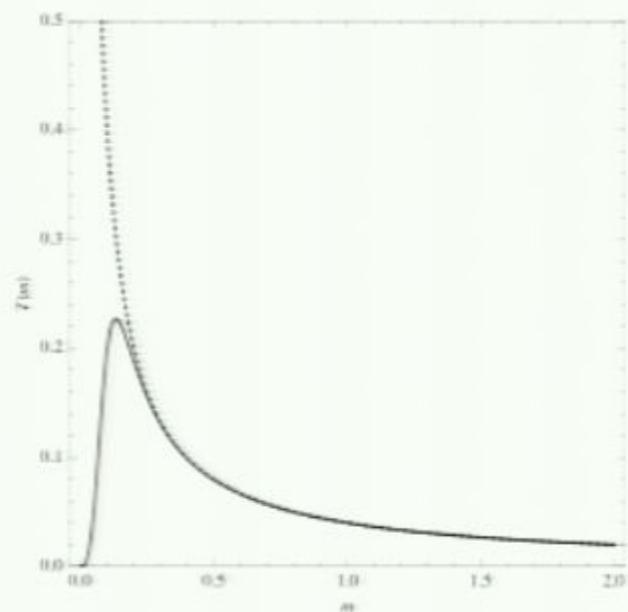
Temperature :

$$T = \kappa/2\pi , \quad T(m) = \frac{128\pi\sigma(\delta)\sqrt{\Omega(\delta)} m^3}{1024\pi^2 m^4 + a_0^2}.$$

Entropy : $S = \frac{\sqrt{A^2 - a_0^2}}{4}$.



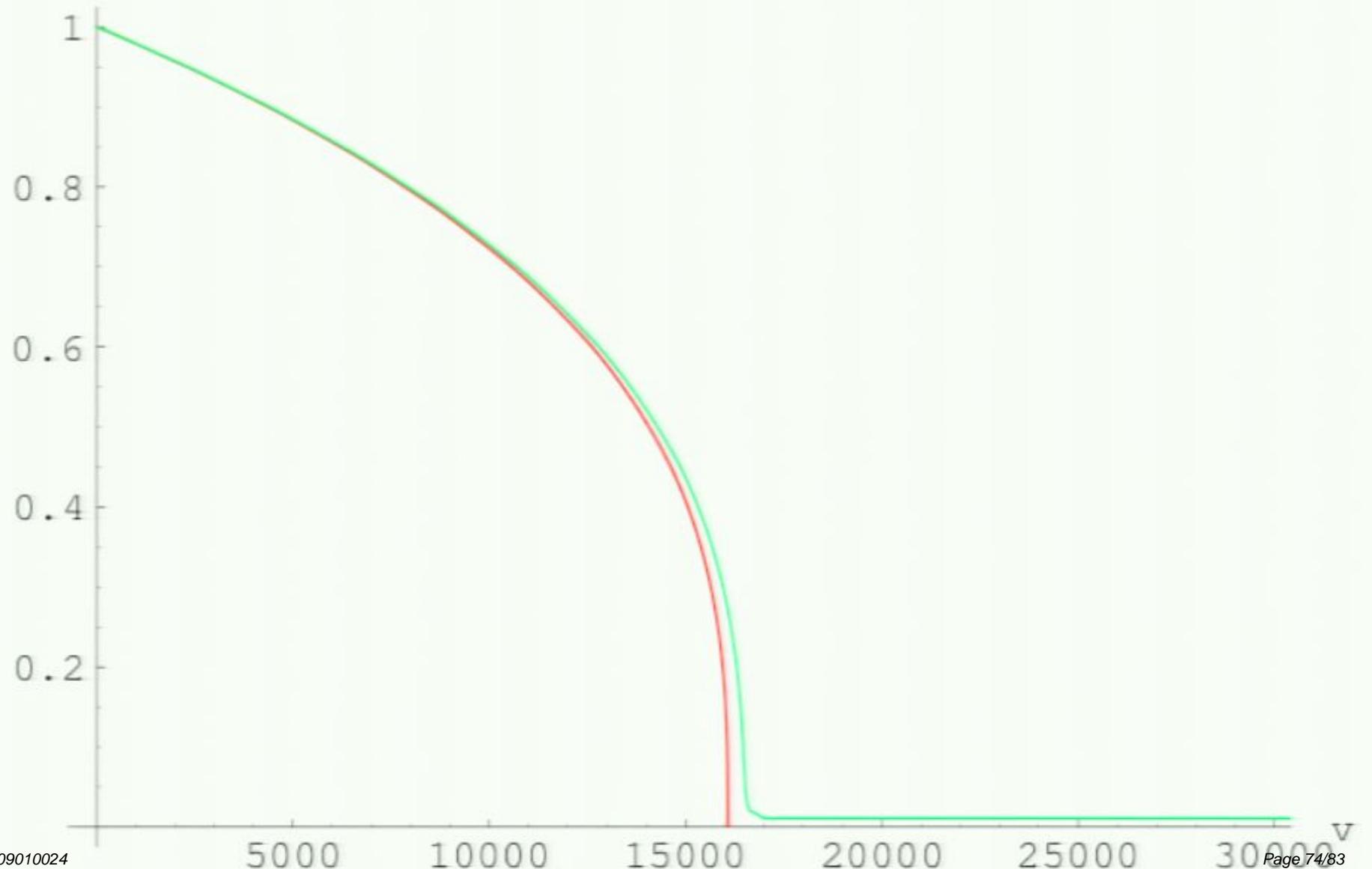
$$ds^2 = -g(r)dv^2 + 2\sqrt{\frac{g(r)}{f(r)}} dr dv + h^2(r)d\Omega^{(2)}.$$



Evaporation :

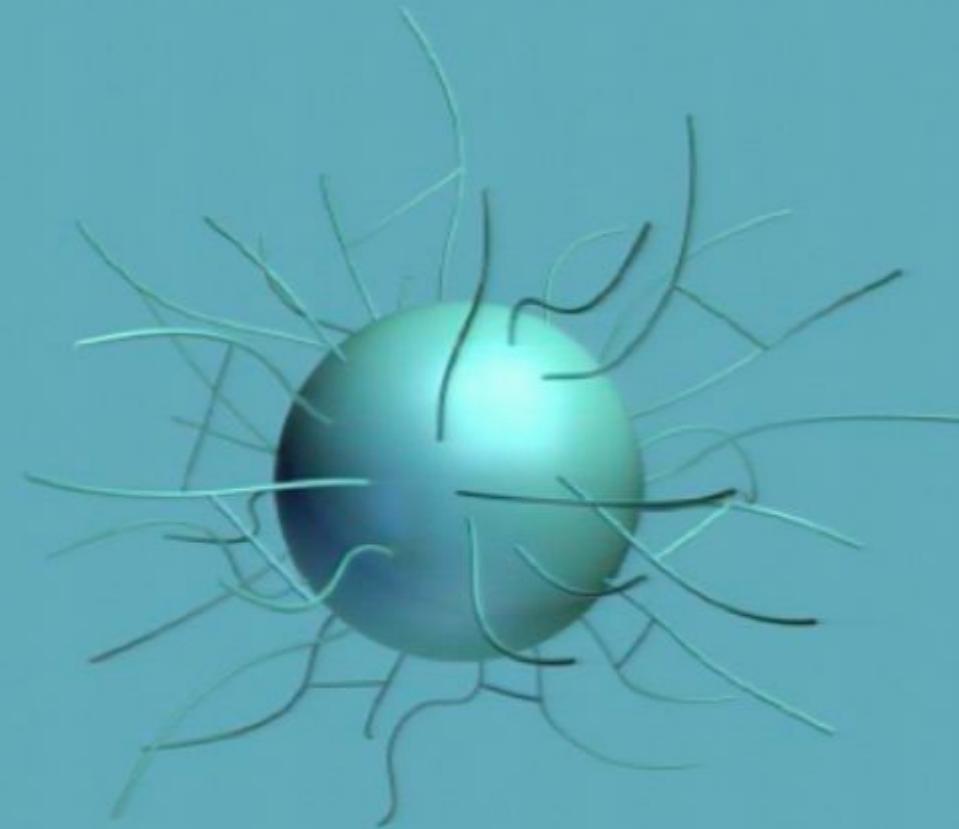
$$-\frac{dm(v)}{dv} = \mathcal{L}[m(v)] , \quad \mathcal{L}(m) = \frac{4194304 m^{10} \pi^3 \alpha \sigma^4 \Omega^2}{(a_0^2 + 1024 m^4 \pi^2)^3}.$$

$m[v]$

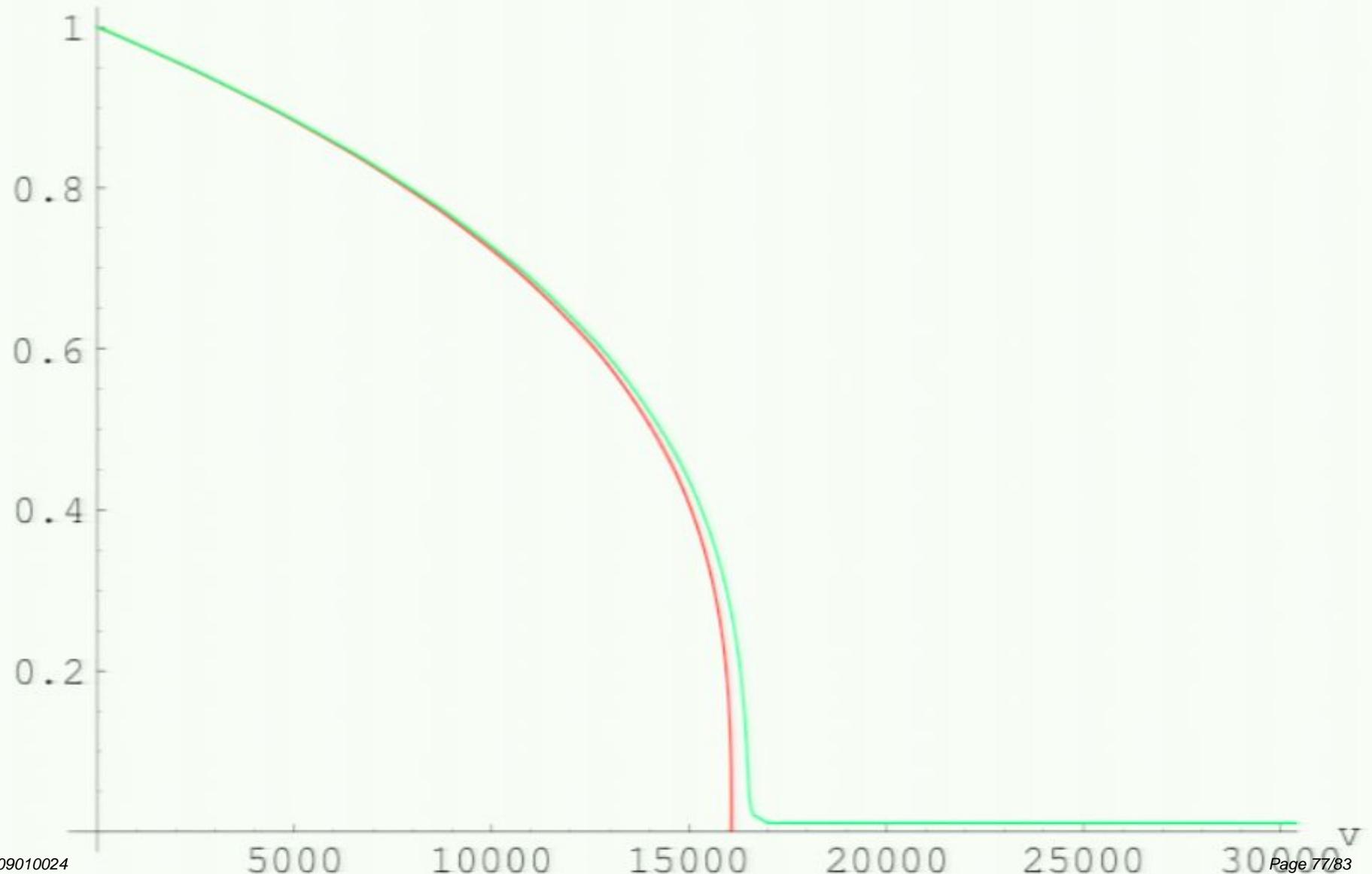


Black Hole Entropy

Black Hole Entropy



$m[v]$

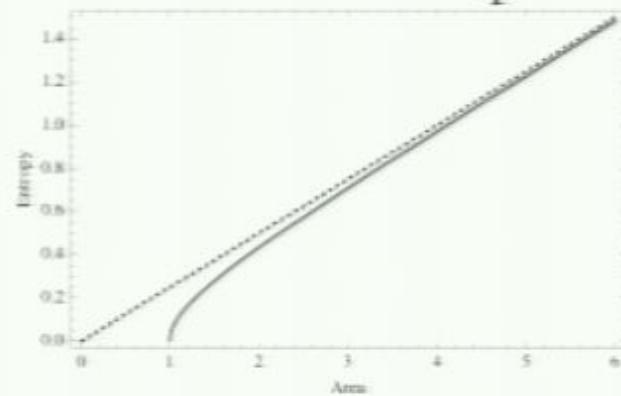


THERMODYNAMICS

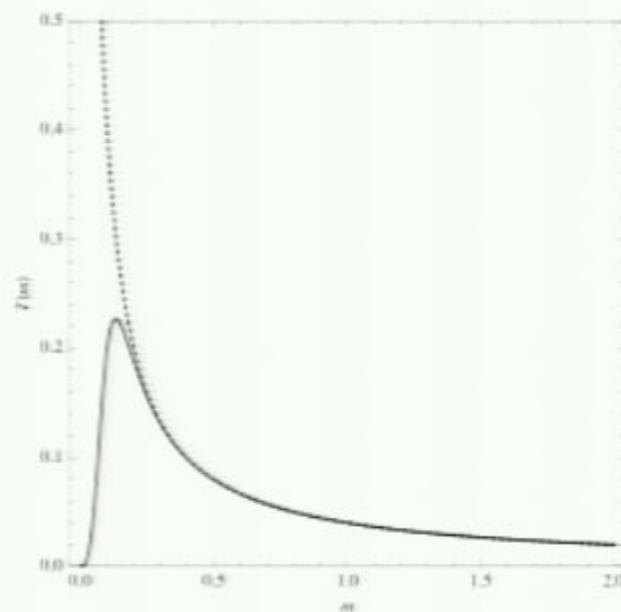
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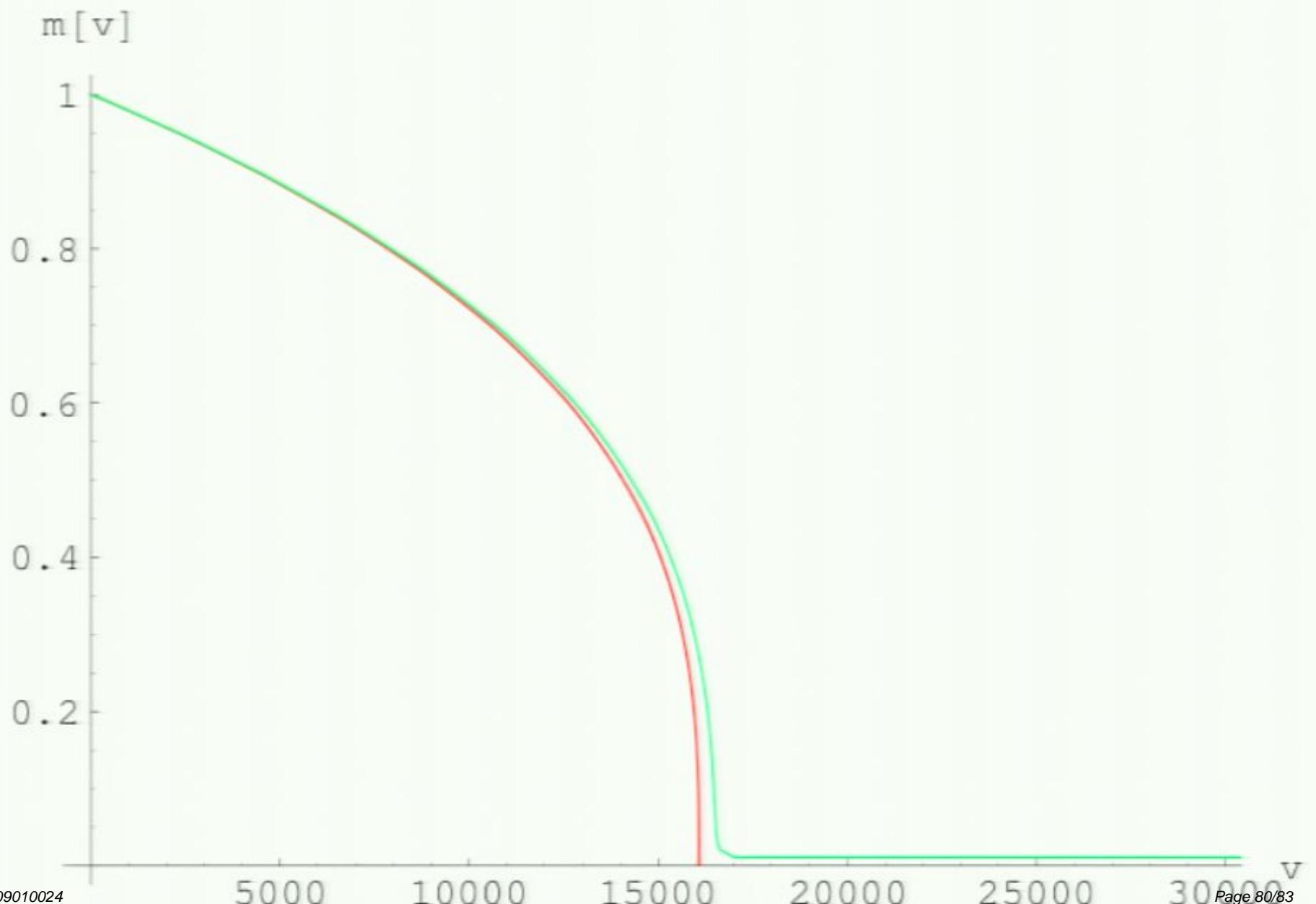


$$ds^2 = -g(r)dv^2 + 2\sqrt{\frac{g(r)}{f(r)}}drdv + h^2(r)d\Omega^{(2)}.$$



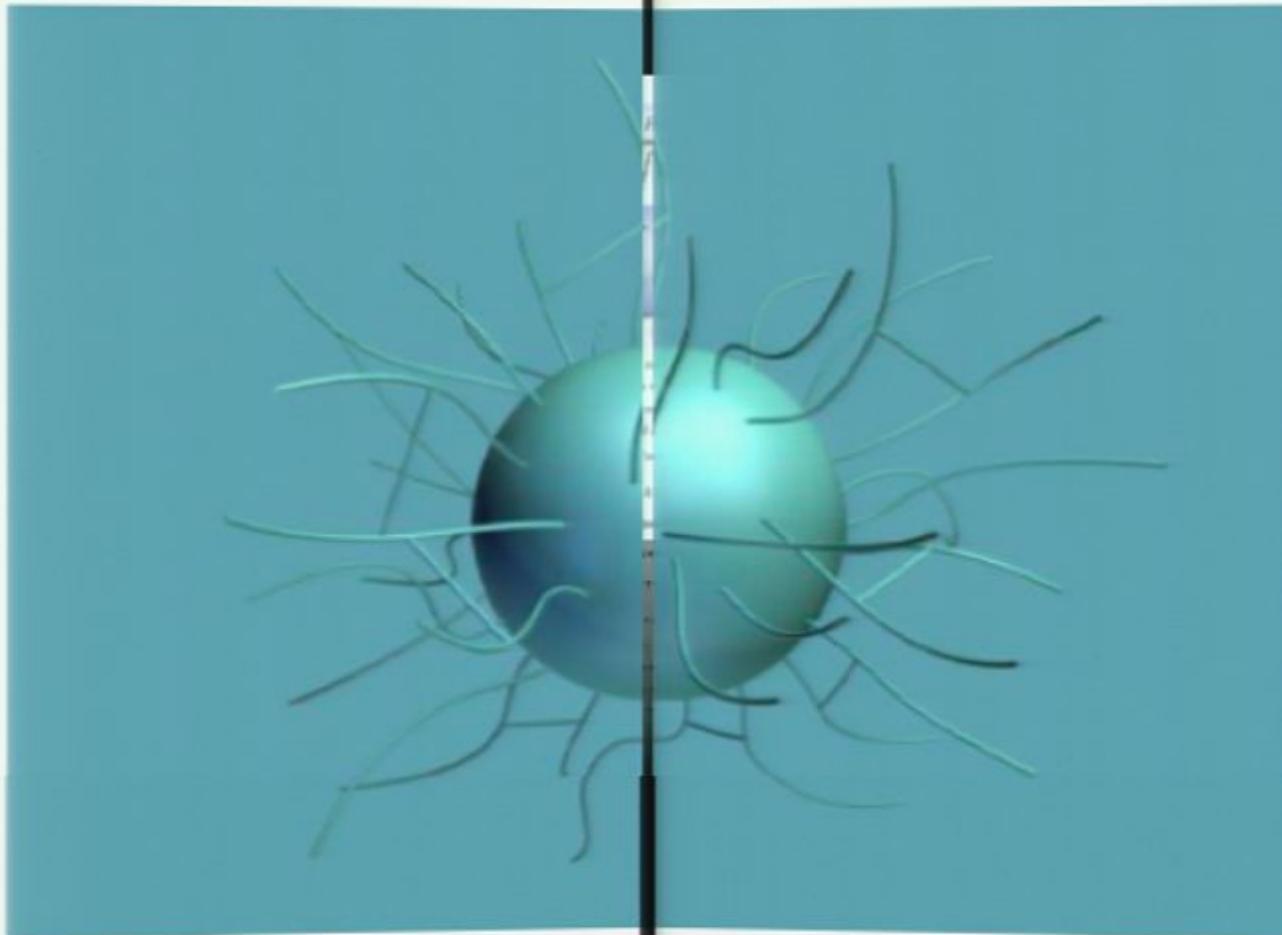
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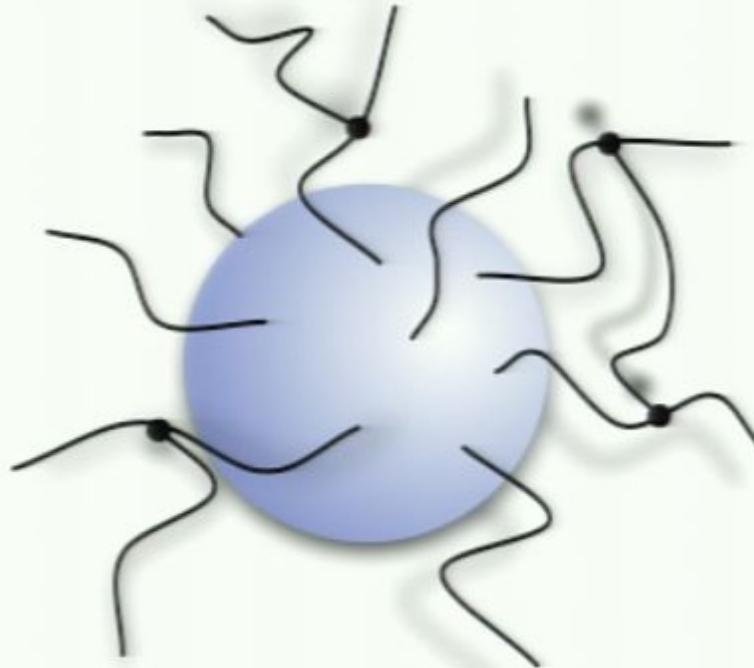


Black Hole Entropy

Black Hole Entropy



ENTROPY :



The spin network pierces the event horizon surface.

The basic idea is as follows :

The entropy is given by the logarithm of the number of LQG states that give the surface a fixed area, A : $S = \kappa \ln N(A)$

A possible "End" on the horizon : vector $\in \mathcal{H}_j$. $N(A) = \sum_{\text{all sets}(A)} \dim(\otimes_i \mathcal{H}_{j_i})$.

$$S = \frac{\ln(2) \kappa}{4\pi\gamma\sqrt{3}\hbar G_N} A \quad \rightarrow \quad \gamma = \frac{\ln(2)}{\pi\sqrt{3}}.$$