

Title: Thermal instability of the toric code in the Hamiltonian setting and implications for topological quantum computing

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Abstract: In topological quantum computation, a quantum algorithm is performed by braiding and fusion of certain quasi-particles called anyons. Therein, the performed quantum circuit is encoded in the topology of the braid. Thus, small inaccuracies in the world-lines of the braided anyons do not adversely affect the computation. For this reason, topological quantum computation has often been regarded as error-resilient per se, with no need for quantum error-correction. However, newer work [1], [2] shows that even topological computation is plagued with (small) errors. As a consequence, it requires error-correction, too, and in the scaling limit causes a poly-logarithmic overhead similar to systems without topological error-correction. I will discuss Nussinov and Ortiz' recent result [2] that the toric code is not fault-tolerant in a Hamiltonian setting, and outline its potential implications for topological quantum computation in general. [1] Nayak, C., Simon, S. H., Stern, A. et al. Non-Abelian anyons and topological quantum computation. Rev. Mod. Phys. 80, 1083-1159 (2008). [2] Z. Nussinov and G. Ortiz, arXiv:0709.2717 (condmat)



On topological quantum computation

Robert Raussendorf,

University of British Columbia

Perimeter Institute, Jan 19, 2009



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Outline

1. Brief review of topological quantum computation
2. The debate on the stability of the toric code & its likely implication for topological QC in general



Topological quantum computation

Sankar Das Sarma, Michael Freedman, and Chetan Nayak

The search for a large-scale, error-free quantum computer is reaching an intellectual junction at which semiconductor physics, knot theory, string theory, anyons, and quantum Hall effects are all coming together to produce quantum immunity.

Part I: Topological Quantum Computation



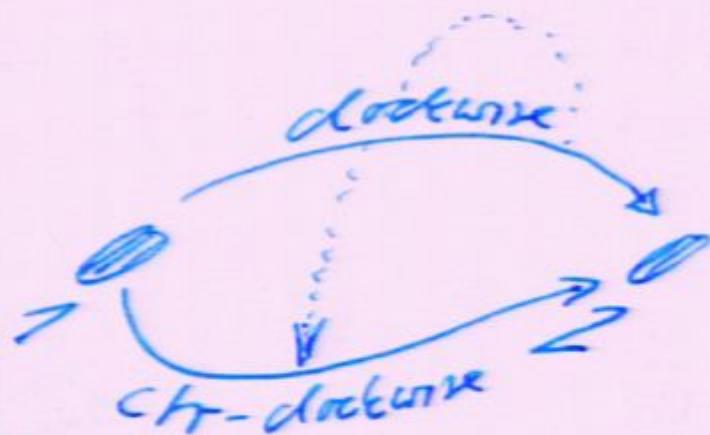
$|4\rangle \rightarrow |4\rangle$

Bosons

$|4\rangle \rightarrow |4\rangle$

Fermions

Are other exchange relations possible?



- In dimension $d > 2$, clockwise and counter-clockwise exchange can be continuously deformed into another.

$d > 2$: No.

$d=2$: Yes

$d=1$: ambig.

Is more than exchange waves possible?

$$| \overset{1}{\circ} \overset{2}{\circ} \overset{3}{\circ} \overset{\dots}{\circ} \overset{n}{\circ} \rangle$$

$U_1 \quad U_2$



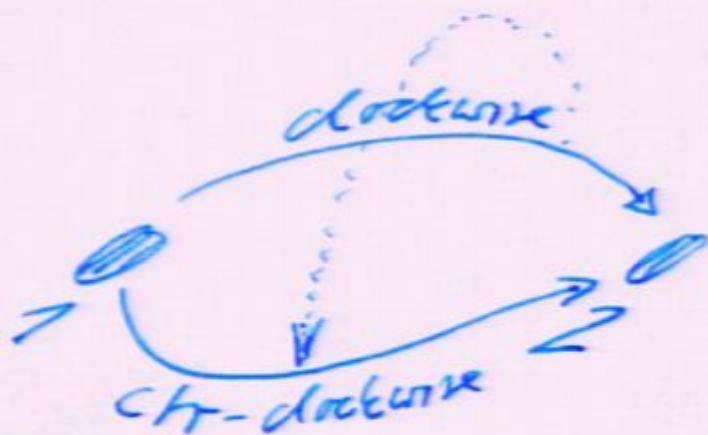
yield joint Hilbert space of dim k .

$$| 14 \rangle \rightarrow U_1 | 14 \rangle \rightarrow U_2 | 14 \rangle$$

$$[U_1, U_2] \neq 0 \quad \underline{\text{can happen}}$$

← then have non-abelian anyons

Are other exchange relations possible?



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$d > 2$: No.

$d = 2$: Yes

$d = 1$: ambig.

Is more than exchange waves possible?

$$| \underbrace{\begin{array}{ccccccc} 1 & 2 & 3 & 0 & 0 & 0 & \dots & n \\ \textcirclearrowleft & \textcirclearrowleft & \textcirclearrowleft & \textcirclearrowleft & \textcirclearrowleft & \textcirclearrowleft & \dots & \textcirclearrowleft \\ u_1 & u_2 & & & & & & \end{array}}_{\text{Yield joint field wave of form } k} \rangle$$

Yield joint field wave of form k .

$$| 14 \rangle \rightarrow u_1 | 14 \rangle \rightarrow u_2 u_1 | 14 \rangle$$

$$[u_1, u_2] \neq 0 \quad \underline{\text{can happen}}$$

∴ then have non-abelian anyons

An anyon quantum computer



$|1\rangle \rightarrow U_1 |1\rangle \rightarrow \dots$

Model for
anyon QC

elementary quantum
gate by exchange
of anyons.

"unwrapping" is prerequisite for "unfolding"

Constraint on U_1, U_2

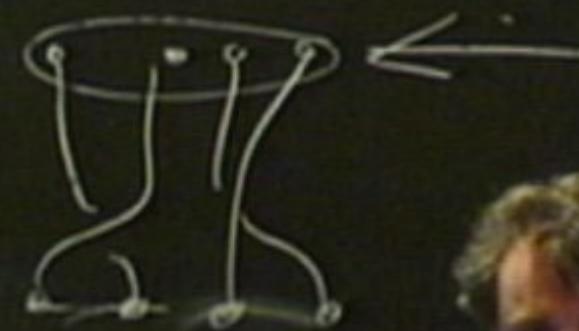
$$U_1 \circ U_2 = U_2 \circ U_1$$

Yang
Baxter
relation

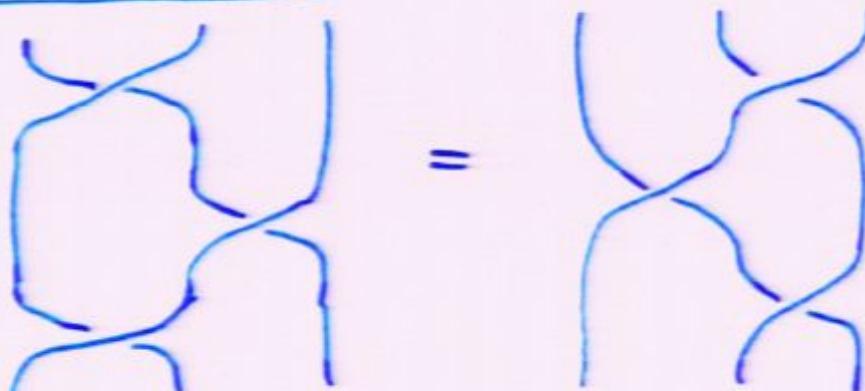
- Why?: Because two braids can be continuously deformed into another by a coherent same operation







Constraints on U_1, U_2

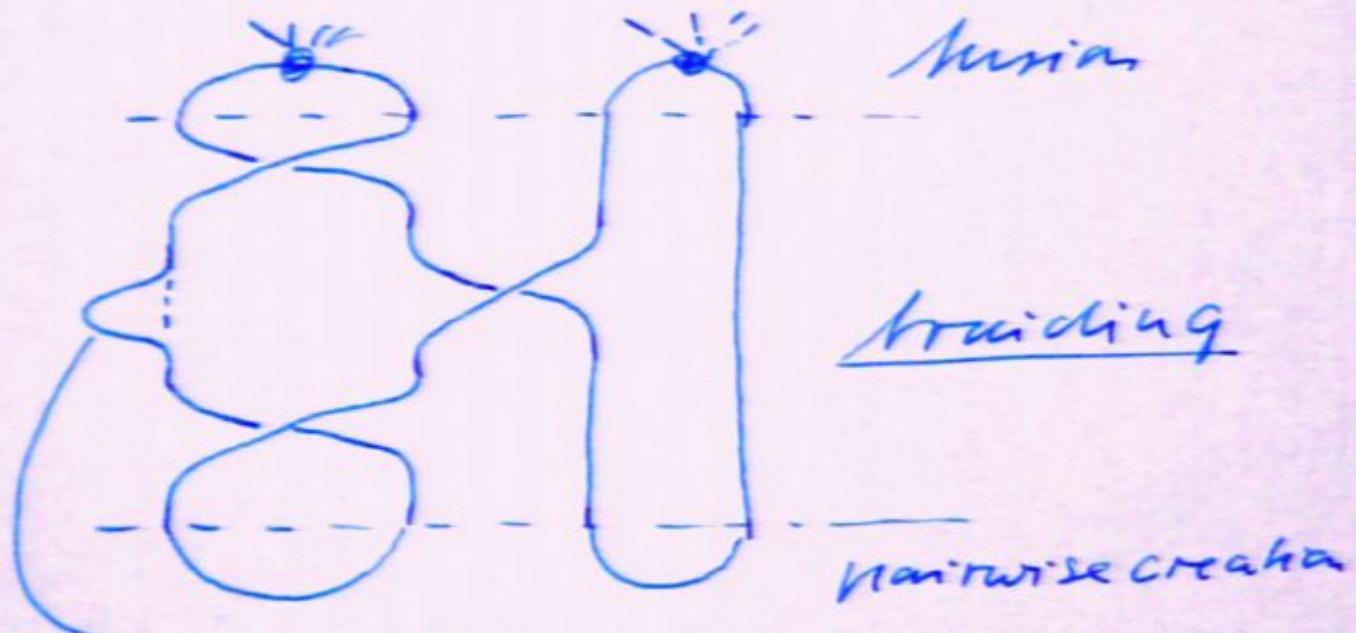


$$U_1 \circ U_2 = U_2 \circ U_1$$

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- Why?: Because two braids can be continuously deformed into another
 ↪ coherent same operation

Steps of TQC

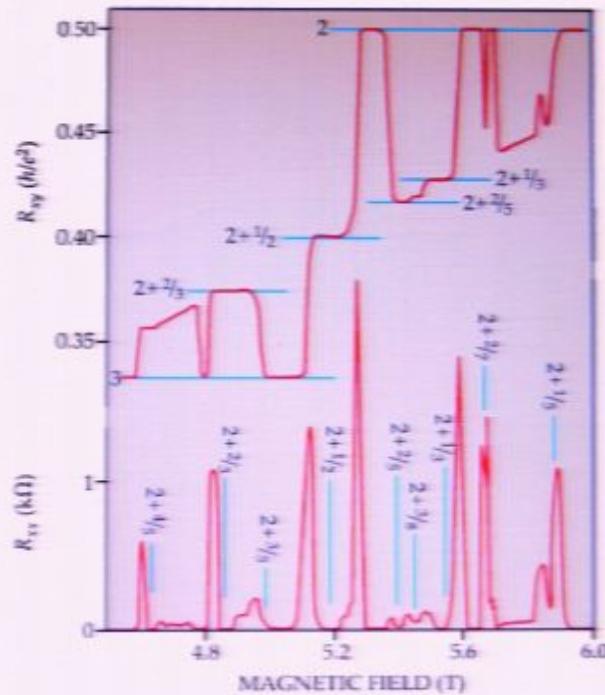


Operation depends only on
topology of anyon paths.

Detours don't matter

Robustness of TQC against
small perturbations.

But, **do** anyons really exist?



- Anyons occur as low-energy excitations in a 2D electron gas, at plateaus of the fractional quantum Hall effect.
- Both abelian and non-abelian anyons predicted.

- Filling fraction $\nu = 5/2$:

Theory: predict non-abelian anyons
as excitations above ground state

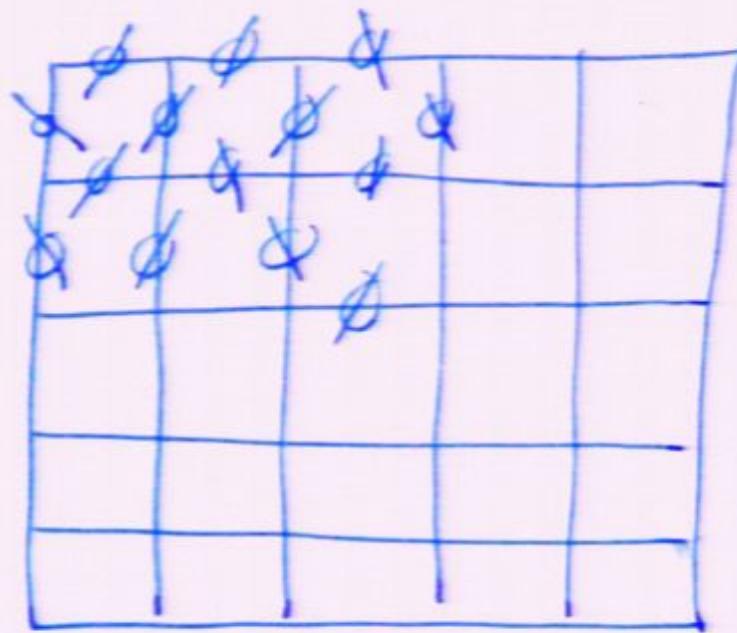
Experiment: Hall plateau roughly seen,
non-abelian statistics not yet
confirmed

- Filling fraction $\nu = 12/5$:

Theory: predict nonabelian Luminous

Experiment: Plateau sometimes seen,
but more often not.

Other systems: open lattices



Example: Tonic code \leftarrow

* AKitagawa (1997)

Non-Abelian Anyons and Topological Quantum Computation

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Topological quantum computation has recently emerged as one of the most exciting approaches to constructing a fault-tolerant quantum computer. The proposal relies on the existence of topological states of matter whose quasiparticle excitations are neither bosons nor fermions, but are particles known as *Non-Abelian anyons*, meaning that they obey non-Abelian braiding statistics. Quantum information is stored in states with multiple quasiparticles, which have a topological degeneracy. The unitary gate operations which are necessary for quantum computation are carried out by braiding quasiparticles, and then measuring the multi-quasiparticle states. The fault-tolerance of a topological quantum computer arises from the non-local encoding of the states of the quasiparticles, which makes them immune to errors caused by local perturbations. To date, the only such topological states thought to have been found in nature are fractional quantum Hall states, most prominently the $\nu = 5/2$ state, although several other prospective candidates have been proposed in systems as disparate as ultra-cold atoms in optical lattices and thin film superconductors. In this review article, we describe current research in this field, focusing on the general theoretical concepts of non-Abelian statistics as it relates to topological quantum computation, on understanding non-Abelian quantum Hall states, on proposed experiments to detect non-Abelian anyons, and on proposed architectures for a topological quantum computer. We address both the mathematical underpinnings of topological quantum computation and the physics of the subject using the $\nu = 5/2$ fractional quantum Hall state as the archetype of a non-Abelian topological state enabling fault-tolerant quantum computation.

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I. INTRODUCTION

In recent years, physicists' understanding of the quantum

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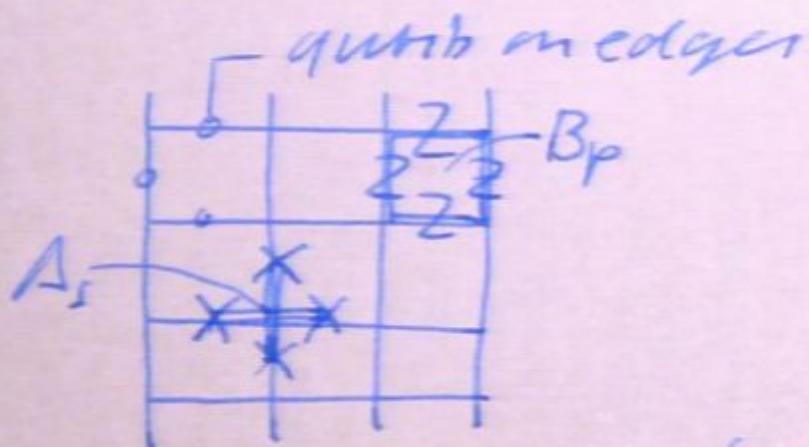
C. Universal Topological Quantum Computation

D. Errors

In recent years, physicists' understanding of the quantum

Part II: The debate on the stability of the toric code

Surface code in a lattice*



$$[A_s, B_p] = 0$$

$$[A_s, A_{s'}] = 0$$

$$[B_p, B_{p'}] = 0$$

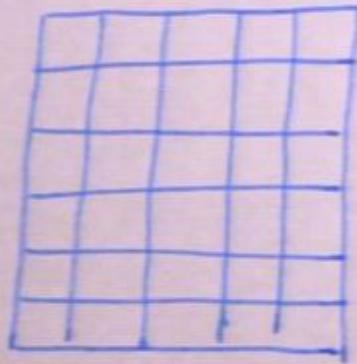
Hamiltonian: $H_K = -g \left(\sum_s A_s + \sum_p B_p \right)$

Ground state manifold:

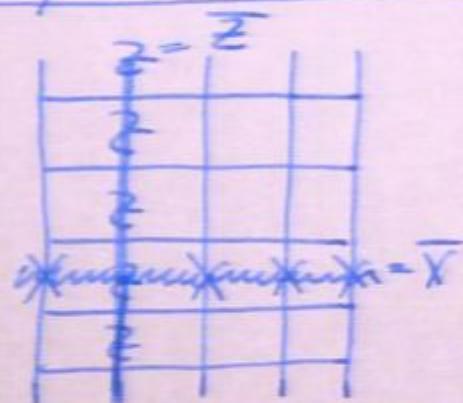
$$A_s |14\rangle = |14\rangle, B_p |14\rangle = |14\rangle, H_{sys}$$

*: A. Kitaev (1997)

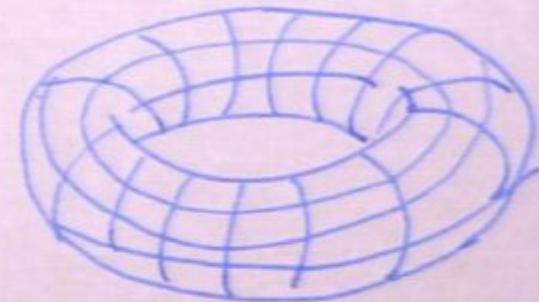
How many qubits are encoded?



0



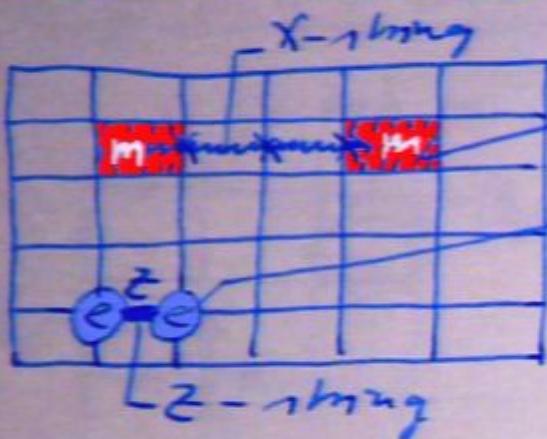
1



2

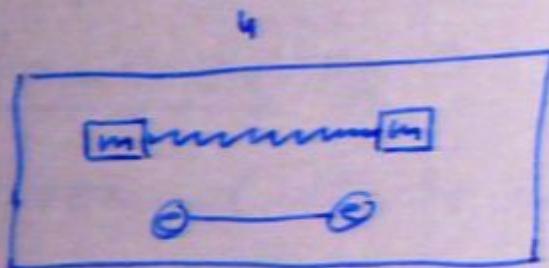
- depends only on topology

Excitations



$$\langle B_p \rangle = -1$$

$$\langle A_s \rangle = -1$$



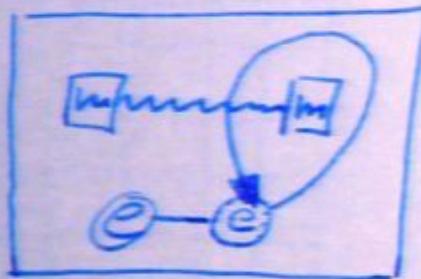
Excited state

$$|4me\rangle$$

• has energy 4g above GS

- "electric" (e) and "magnetic" (m) quasi-particles

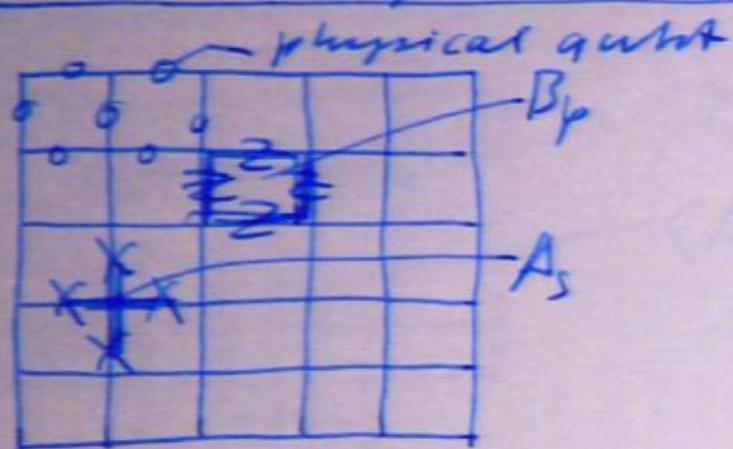
Double exchange



$$|4_{me}\rangle \rightarrow |4_{me}\rangle$$

What is the stability of the logic code?

Another way to use the toric code



Code space \mathcal{K}_c

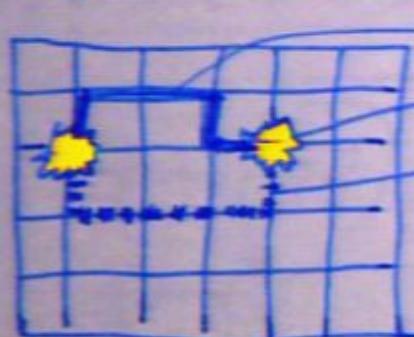
$$|1\bar{4}\rangle \in \mathcal{K}_c$$

$$A_5 |1\bar{4}\rangle = |1\bar{4}\rangle$$

$$B_p |1\bar{4}\rangle = |1\bar{4}\rangle, \text{ hps}$$

- No Hamiltonian
- Measure stabilizer operators
over and over again
to collect syndrome history & correct

What's topological here?



$$E = Z(c)$$
$$\langle A_S \rangle = -1$$
$$E' = Z(c')$$

$$E \cong E'$$

Non-trivial
syndrome at
endpoints of
error chain

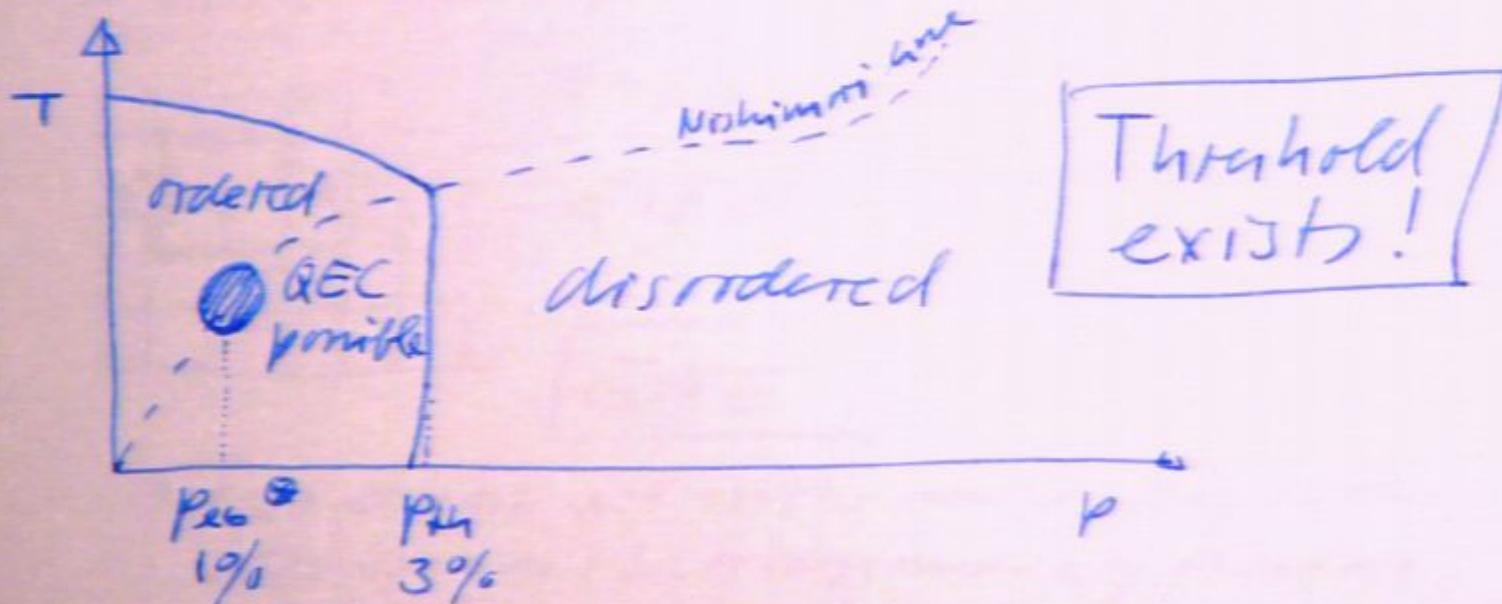
- Two errors act in the same way on the code space if corresponding chains differ by surface boundary only.

↳ homological equivalence

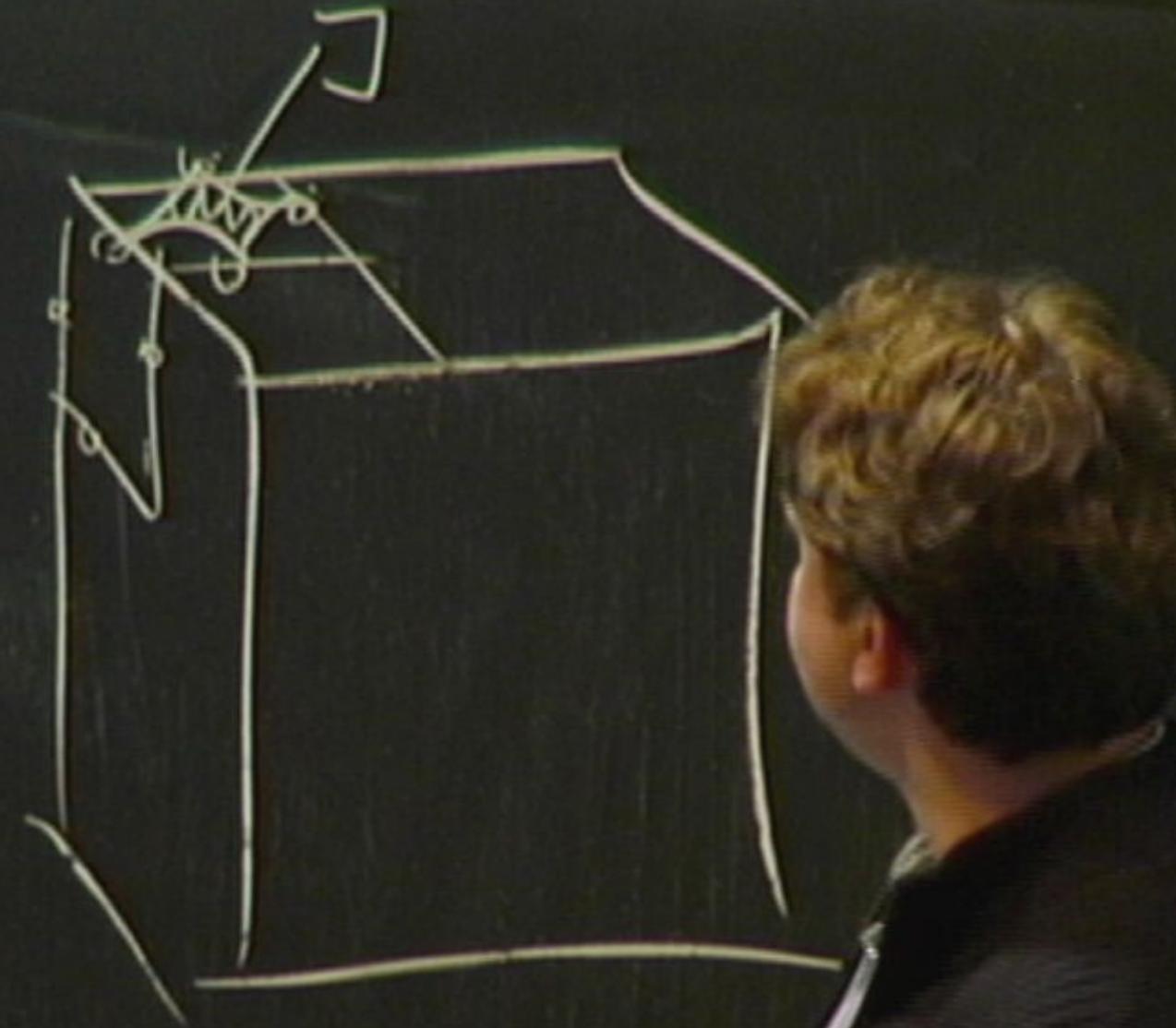
Error-correction by stabilizer measurement ⁽¹⁾

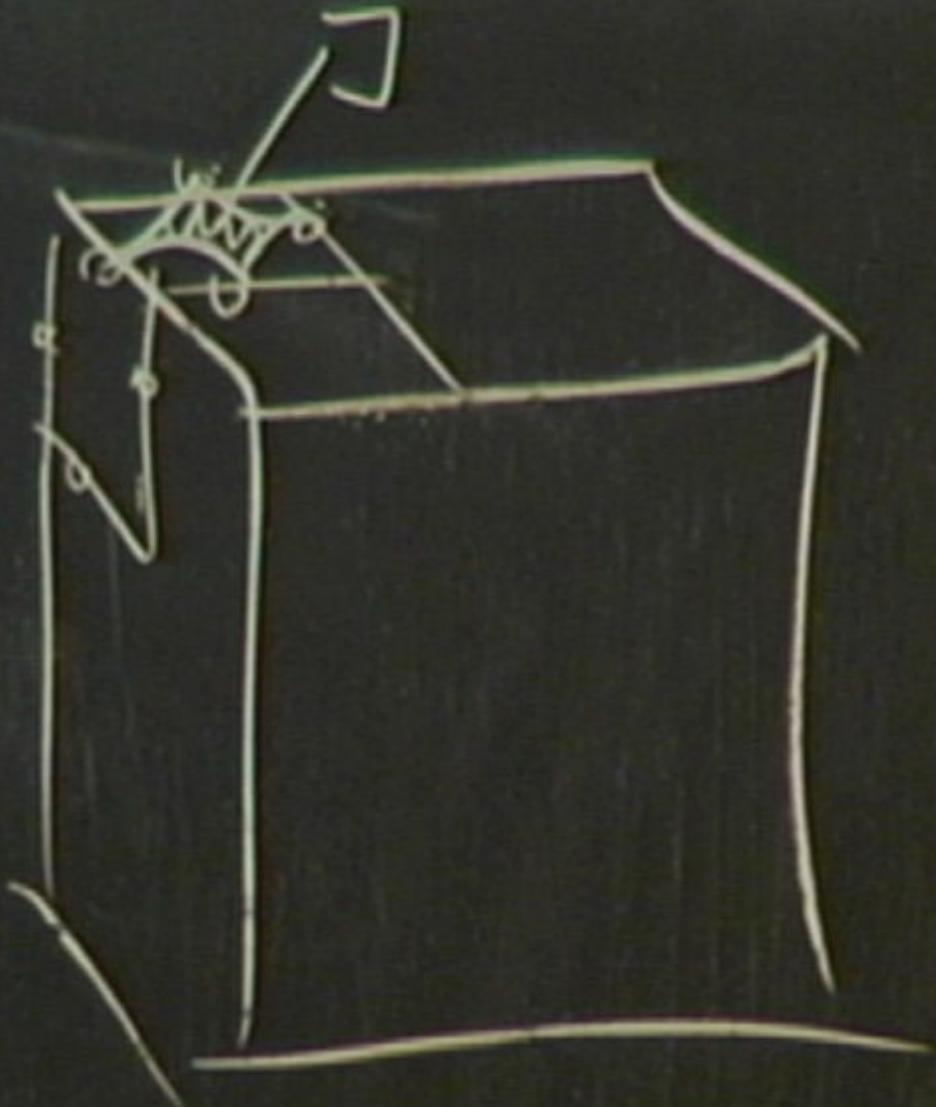
- leads to fault-tolerant quantum memory
- Can be related to Stat Mech-model,
the "random plaquette \mathbb{Z}_2 -gauge model":
(RPMI)
- Dennis, Kitaev, Landahl, Preskill (2001).

Phase diagram of the RRGM

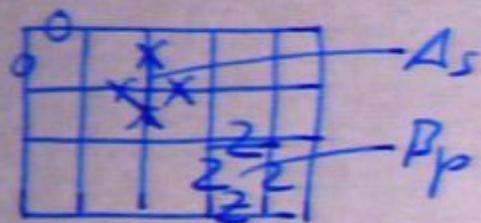


- phase diagram numerical
- $P_{c5} = 1.04\%$ analytical lower bound to threshold
- Dennis et al. (2001)





Back to Hamiltonian setting:



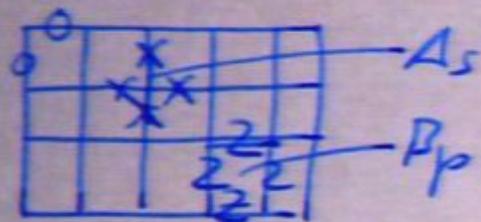
$$H = -g \left(\sum_S A_S + \sum_P B_P \right)$$

• How does the autocorrelation

$$G_{\bar{X}}(\epsilon) := \langle \bar{X}(0) \bar{X}(\epsilon) \rangle$$

decay with time?

Back to Hamiltonian setting:

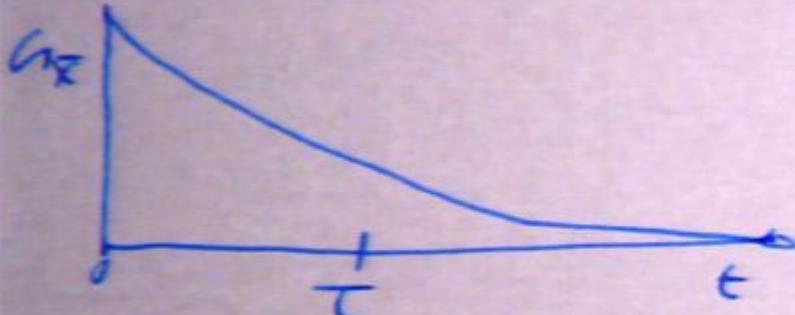


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• How does the autocorrelation

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At intermediate times:
 $G_X \sim e^{-t/\tau}$

Characteristic
time scale:

$$\tau = \frac{\pi}{4\chi(1-\tanh 2J)}$$

χ : "dode speed" of system, J : inverse temp.

No phase transition for toric code
at finite T

Nurmiro and Ortiz, cond-mat/0709.2717

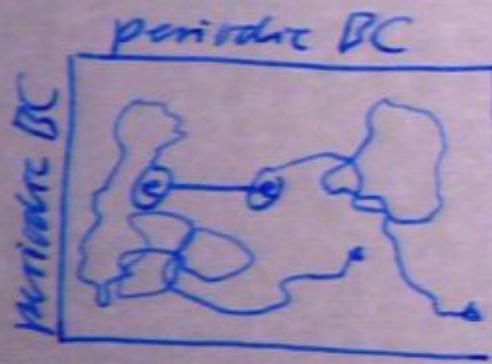
Toric code at finite T:

$$\text{char.time: } \tau = \frac{\pi}{4X(1-\tanh 2J)}$$

- depends strongly on temperature τ/J
 - does not depend on lattice size
 - ↳ Cannot increase storage time by increasing lattice size.
- \Rightarrow This quantum memory is not fault-tolerant.

Numerov2 oraz, cond-mat/0709.2712

An explanation



Quasiparticles Θ

- are pairwise created by thermal excitation
- drift around (RW)
- recombine

- $\Theta\Theta$ pair canns move if in closer non-trivial loop before recombination.

Harmful process

- Thermal excitation

↳ Drift

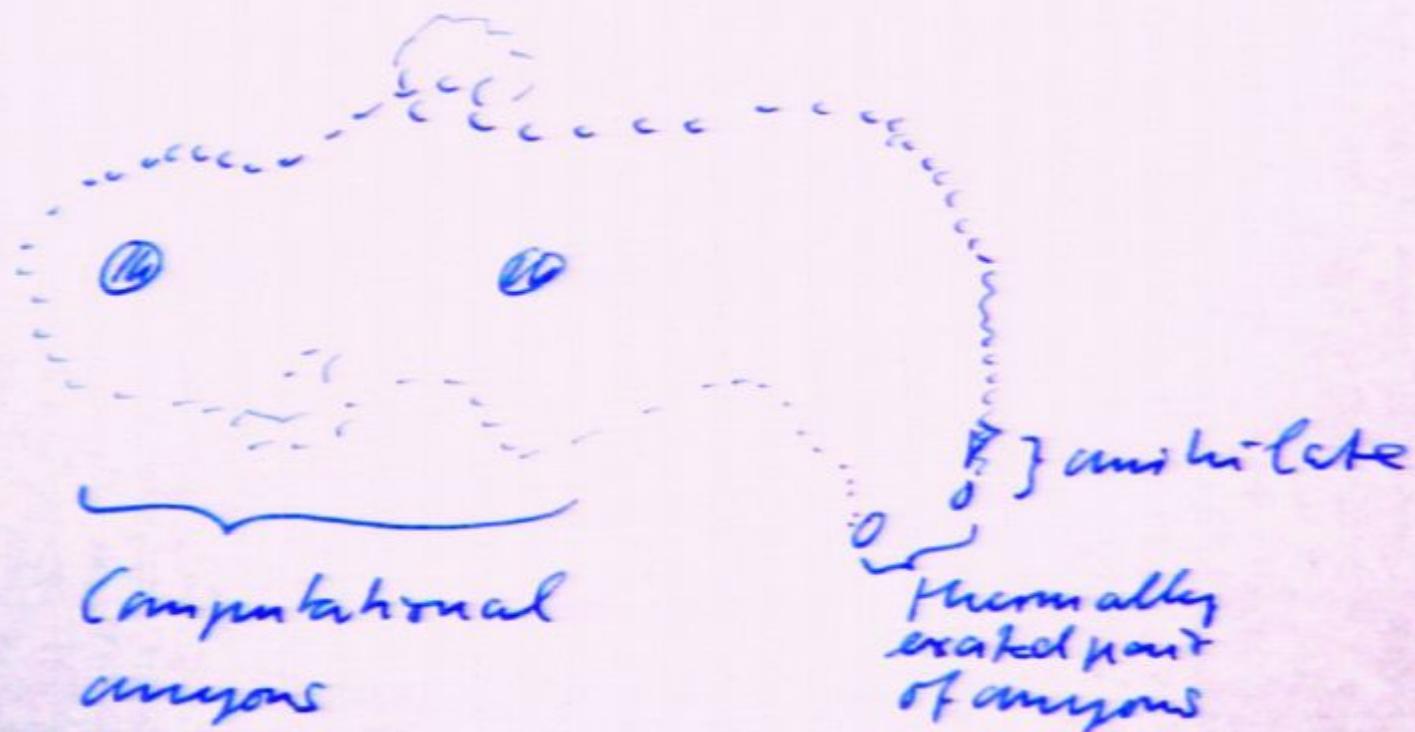
↳ Recombination

= not particular to logic code

= rather: Consequence of Hamiltonian
scattering

↳ Source of error in any scattering
for topological QC

Error source for TQC



Conclusion :

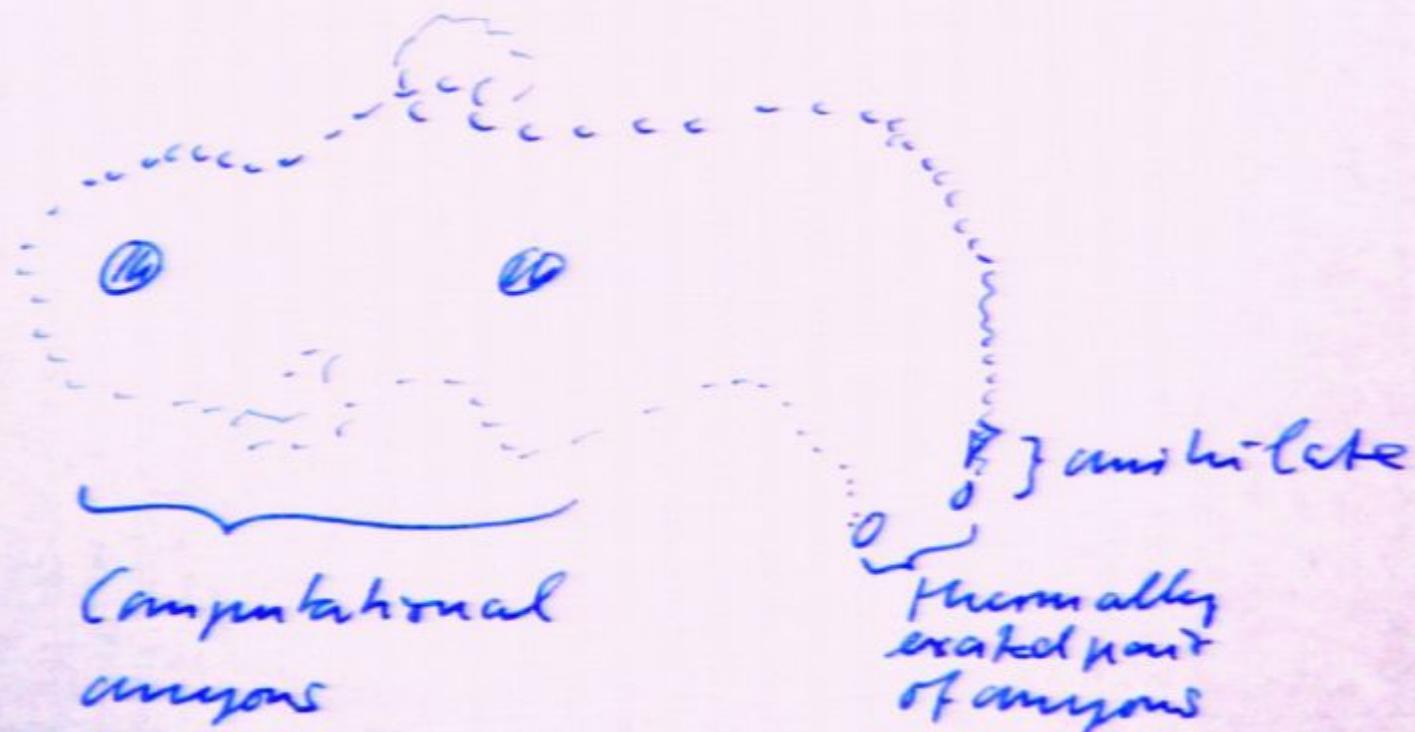
The circuit model and topological QC

- both need quantum error-correction
- both carry poly-log overhead.

$$\overset{!}{O}' \sim O \cdot \log^4 O$$

size of m-
coded circuit size of bare
circuit

Error source for TQC



Conclusion :

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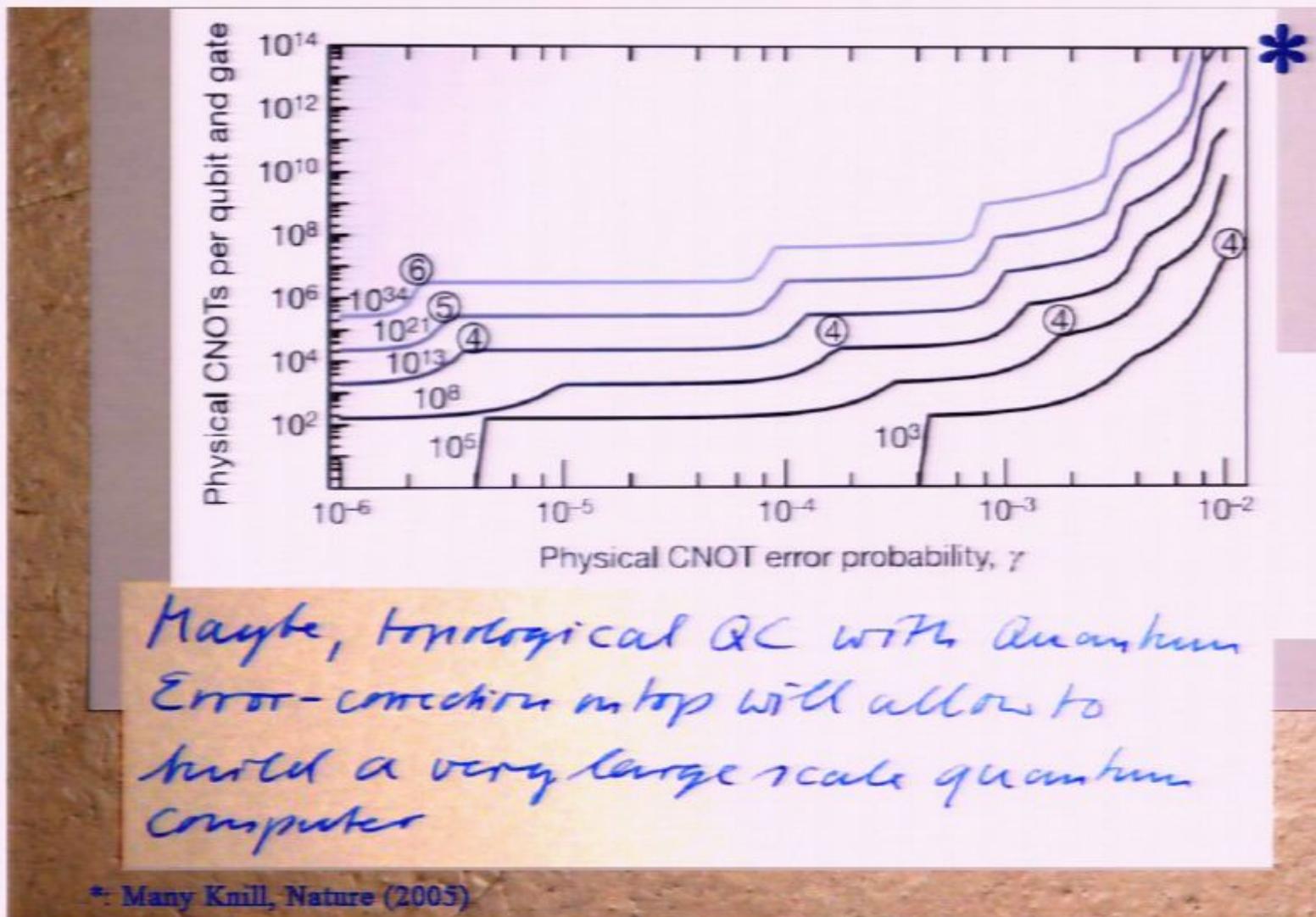
In topological QC:

$$p \sim e^{-\Delta/T}$$

p: error-probability, Δ : gap, T: temperature

↳ very small error-rates admirable.

Conclusion, part II



In topological QC:

$$p \sim e^{-\Delta/T}$$

p: error-probability, Δ : gap, T: temperature

↳ very small error-rates admirable.

Conclusion :

The circuit model and topological QC

- both need quantum error-correction
- both carry poly-log overhead.

$$\text{size of m-} \quad \sim \quad \text{size of bare}$$

O' $O \cdot \log^k O$

coded circuit circuit

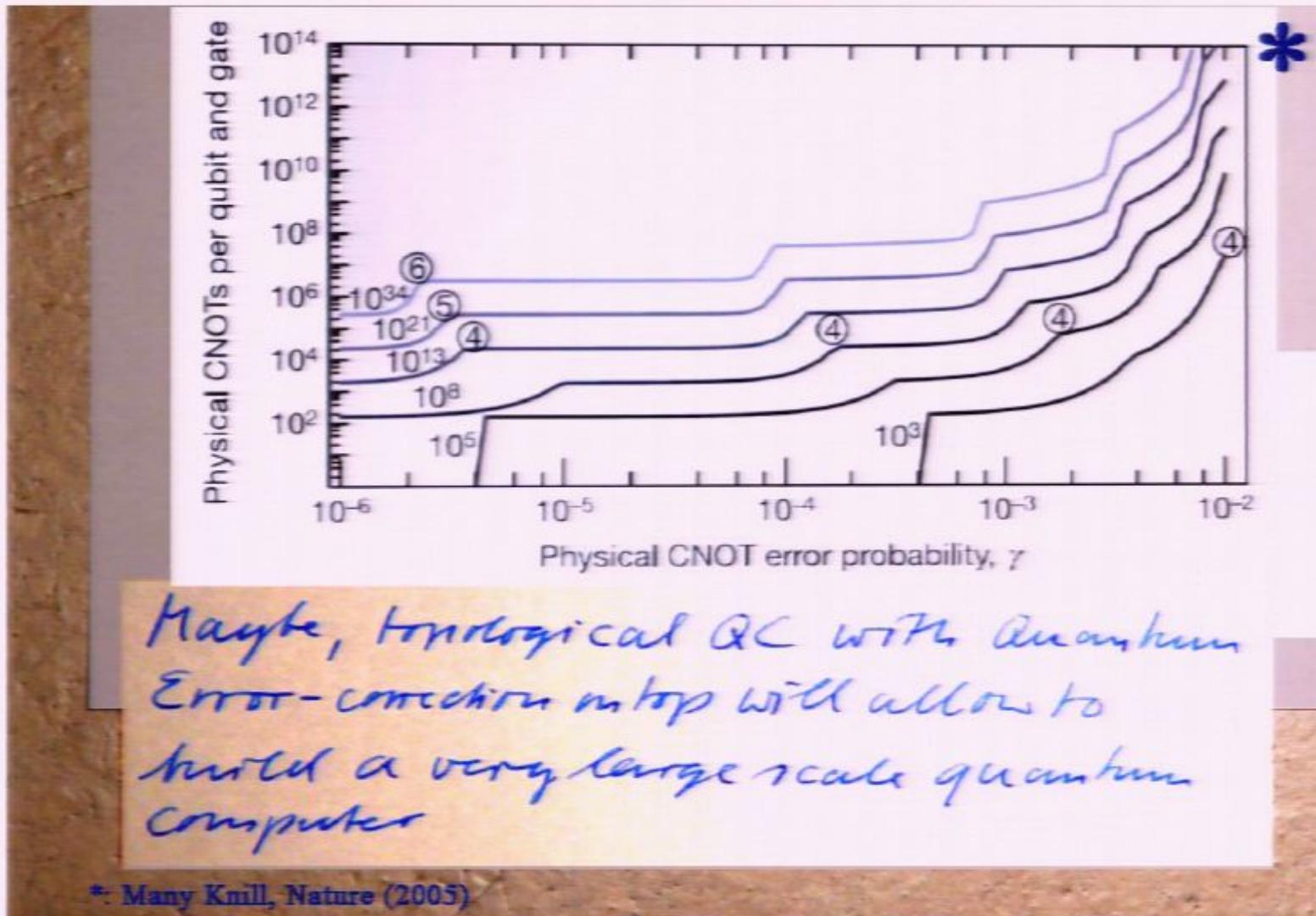
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Conclusion:

The circuit model and topological QC

- both need quantum error-correction
- both carry poly-log overhead.

$$\text{size of m-coded circuit} \sim O \cdot \log^4 O \quad \text{size of bare circuit}$$