Title: Introduction to the Bosonic String 3A

Date: Jan 30, 2009 10:00 AM

URL: http://pirsa.org/09010015

Abstract: This course provides a thorough introduction to the bosonic string based on the Polyakov path integral and conformal field theory. We introduce central ideas of string theory, the tools of conformal field theory, the Polyakov path integral, and the covariant quantization of the string. We discuss string interactions and cover the tree-level and one loop amplitudes. More advanced topics such as T-duality and D-branes will be taught as part of the course. The course is geared for M.Sc. and Ph.D. students enrolled in Collaborative Ph.D. Program in Theoretical Physics. Required previous course work: Quantum Field Theory (AM516 or equivalent). The course evaluation will be based on regular problem sets that will be handed in during the term. The primary text is the book: 'String theory. Vol. 1: An introduction to the bosonic string. J. Polchinski (Santa Barbara, KITP) . 1998. 402pp. Cambridge, UK: Univ. Pr. (1998) 402 p.' All interested students should contact Alex Buchel at abuchel@uwo.ca as soon as possible.

$\Rightarrow$ light-cone analysis of S_p

- $\tau = X^+$
- $\partial_\sigma \gamma_{\sigma\sigma} = 0$
- $\gamma = -1$

$\Rightarrow$ light-cone analysis of S_p

- $\tau = X^+$
- $\partial_\sigma \gamma_{\sigma\sigma} = 0$
- $\gamma = -1$

} can always be made locally

$$H_0 = \frac{\ell}{4\pi\alpha' p^+} \int_0^{\ell} d\sigma \left[2\pi\alpha' \, \Pi_i \Pi^i + \frac{1}{2\pi\alpha'} \partial_\sigma X^i \partial_\sigma X^i \right]$$

$$H_0 = \frac{\ell}{4\pi\alpha' p^+} \int_0^{\ell} d\sigma \left[2\pi\alpha' \, \Pi_i \Pi^i + \frac{1}{2\pi\alpha'} \partial_\sigma X^i \partial_\sigma X^i \right]$$

$$X^- = X^-(\tau) \,, \quad \Pi_i = \Pi_i(\tau,\sigma)$$

$$X_i = X_i(\tau,\sigma).$$

$$\partial_\tau X^-$$

$$H_0 = \frac{\ell}{4\pi\alpha' p^+} \int_0^{\ell} d\sigma \left[2\pi\alpha' \, \Pi_i \Pi^i + \frac{1}{2\pi\alpha'} \partial_\sigma X^i \partial_\sigma X^i \right]$$

$$X^- = X^-(\tau) \,, \quad \Pi_i = \Pi_i(\tau, \sigma)$$

$$p^+ \qquad X_i = X_i(\tau, \sigma).$$

$$\partial_\tau X^- = \frac{\partial H}{\partial P_-}$$

$$H_0 = \frac{\ell}{4\pi\alpha' p^+} \int_0^\ell d\sigma \left[2\pi\alpha' \, \Pi_i \Pi^i + \frac{1}{2\pi\alpha'} \partial_\sigma X^i \partial_\sigma X^i \right]$$

$x^- = x^-(\tau)$, $\Pi_i = \Pi_i(\tau, \sigma)$

p^+ $\qquad X_i = X_i(\tau, \sigma)$.

$$\partial_\tau X^- = \frac{\partial H}{\partial P_-}$$

$$= - \frac{\partial H}{\partial P^+}$$

$$H_0 = \frac{\ell}{4\pi\alpha' p^+} \int_0^\ell d\sigma \left[2\pi\alpha' \, \Pi_i \Pi^i + \frac{1}{2\pi\alpha'} \partial_\sigma X^i \partial_\sigma X^i \right]$$

$X^- = X^-(\tau)$, $\Pi_i = \Pi_i(\tau, \sigma)$

p^+ $\qquad X_i = X_i(\tau, \sigma).$

$$\Rightarrow \partial_\tau X^- = \frac{\partial H}{\partial P_-}$$

$$= -\frac{\partial H}{\partial P^+} = \frac{H}{p^+}$$

$$H_0 = \frac{\ell}{4\pi\alpha' p^+} \int_0^{\ell} d\sigma \left[2\pi\alpha' \, \Pi_i \Pi^i + \frac{1}{2\pi\alpha'} \partial_\sigma X^i \partial_\sigma X^i \right]$$

$X^- = X^-(\tau)$, $\Pi_i = \Pi_i(\tau, \sigma)$

p^+ $\quad X_i = X_i(\tau, \sigma)$.

$$\partial_\tau X^- = \frac{\partial H}{\partial P_-} = -\frac{\partial H}{\partial P^+} = \frac{H}{p^+}$$

$$\partial_\tau p^+ = -\partial_\tau P_- = \frac{\partial H}{\partial X^-} = 0 \Rightarrow p^+ \text{ is conserved quantity.}$$

quantity.

$$\partial_\tau X^i = \frac{\delta H}{\delta \Pi^i} \; ; \quad \partial_\tau \Pi^i = -\frac{\delta H}{\delta X^i}$$

quantity.

$$\partial_\tau X^i = \frac{\delta H}{\delta \Pi^i}$$

$$= 2\pi\alpha' c\, \Pi^i$$

$$c = \frac{\ell}{2\pi\alpha' p^+}$$

$$\partial_\tau \Pi^i = -\frac{\delta H}{\delta X^i}$$

$$\partial_\tau \Pi_- = \frac{\partial}{\partial X^-} = 0 \Rightarrow \Pi^- \text{ is conserved quantity}$$

$$\partial_\tau X^i = \frac{\delta H}{\delta \Pi^i}$$

$$= 2\pi\alpha' c\, \Pi^i$$

$$c = \frac{\ell}{2\pi\alpha' p^+}$$

$$\partial_\tau \Pi^i = -\frac{\delta H}{\delta X^i}$$

$$= -\frac{c}{2}\,\frac{1}{2\pi\alpha'}\cdot 2$$

$$\ldots = \partial_\tau P_- = \frac{\ldots}{\partial X^-} = 0 \Rightarrow P^+ \text{ is conserved}$$

quantity.

$$\partial_\tau X^i = \frac{\delta H}{\delta \Pi^i} = 2\pi\alpha' c\, \Pi^i$$

$$c = \frac{\ell}{2\pi\alpha' p^+}$$

$$\partial_\tau \Pi^i = -\frac{\delta H}{\delta X^i} = (-)^2 \frac{c}{2} \frac{1}{2\pi\alpha'} \cdot 2\, \partial_\sigma^2 X^i = \frac{c}{2\pi\alpha'} \partial_\sigma^2 X^i$$

$$\partial_\tau \Pi_- = \frac{\ \ }{\partial X^-} = 0 \Rightarrow \Pi^- \text{ is conserved quantity.}$$

$$\partial_\tau X^i = \frac{\delta H}{\delta \Pi^i}$$

$$= 2\pi\alpha' c\, \Pi^i$$

$$c = \frac{\ell}{2\pi\alpha' p^+}$$

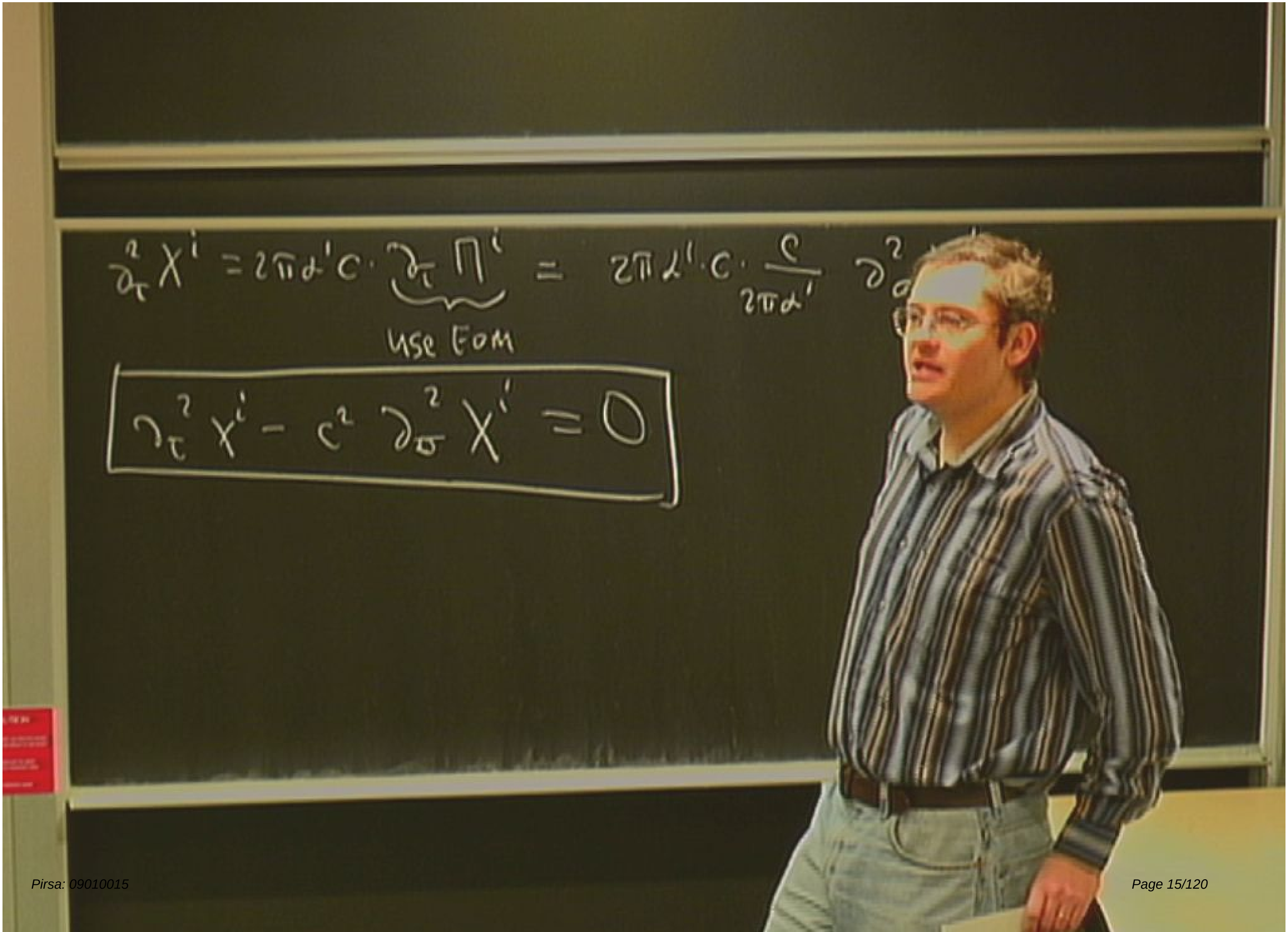
$$\partial_\tau \Pi^i = -\frac{\delta H}{\delta X^i}$$

$$= (-)\frac{2c}{2}\frac{1}{2\pi\alpha'}\cdot 2\, \partial_\sigma^2 X^i$$

$$= \frac{c}{2\pi\alpha'}\partial_\sigma^2 X^i$$

$$i = 2, \cdots D-1$$

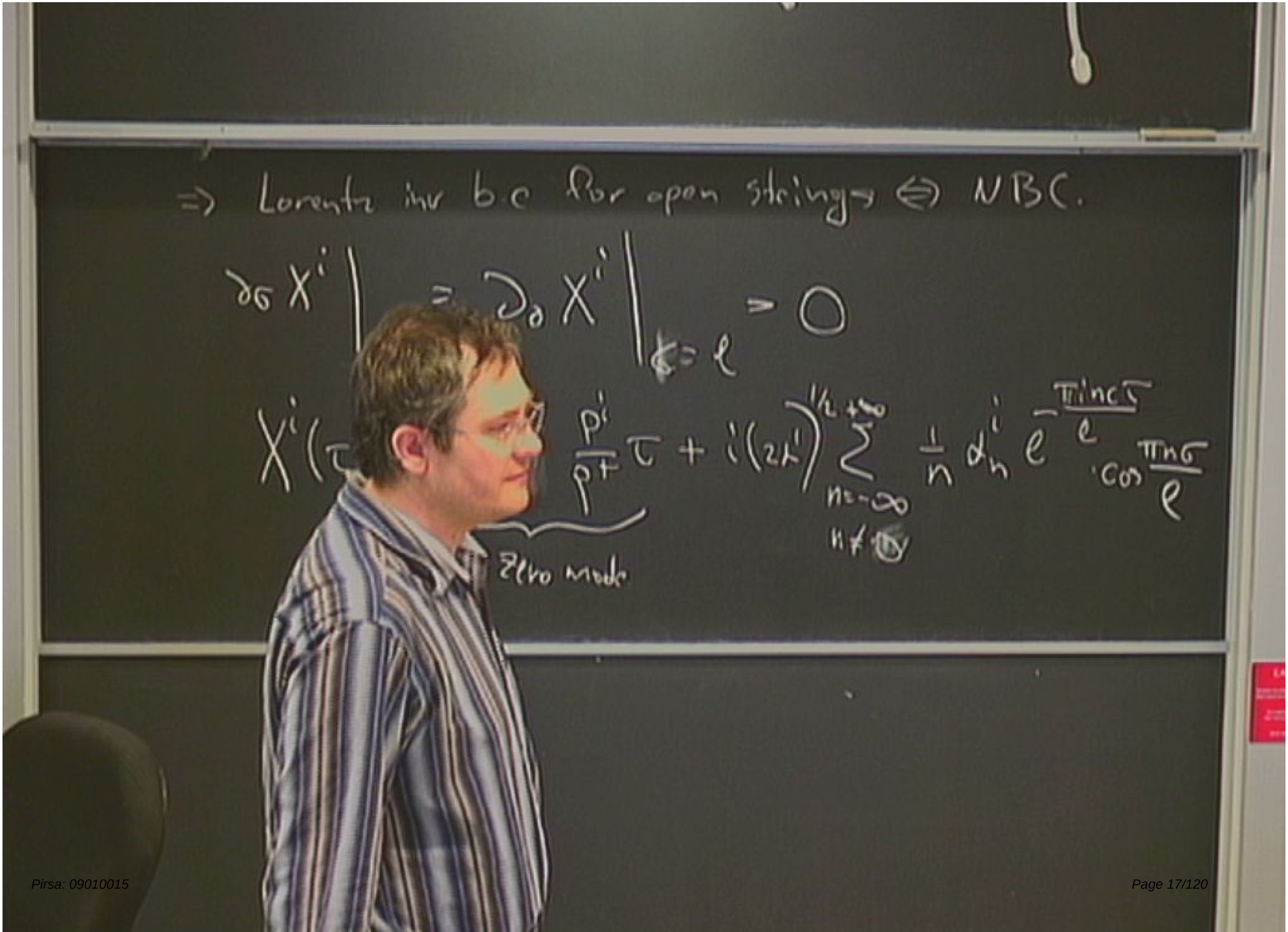
$\Rightarrow$ D-2 free fields


$\partial_\tau^2 X^i = 2\pi\alpha' c \cdot \partial_\tau \Pi^i = 2\pi\alpha' \cdot c \cdot \frac{c}{2\pi\alpha'} \partial_\sigma^2$
use EOM
$\partial_\tau^2 X^i - c^2 \partial_\sigma^2 X^i = 0$

$$\partial_\tau^2 X^i = 2\pi\alpha' c \cdot \underbrace{\partial_\tau \Pi^i}_{\text{use EoM}} = 2\pi\alpha' \cdot c \cdot \frac{c}{2\pi\alpha'} \partial_\sigma^2 X^i$$

$$\boxed{\partial_\tau^2 X^i - c^2 \partial_\sigma^2 X^i = 0}$$

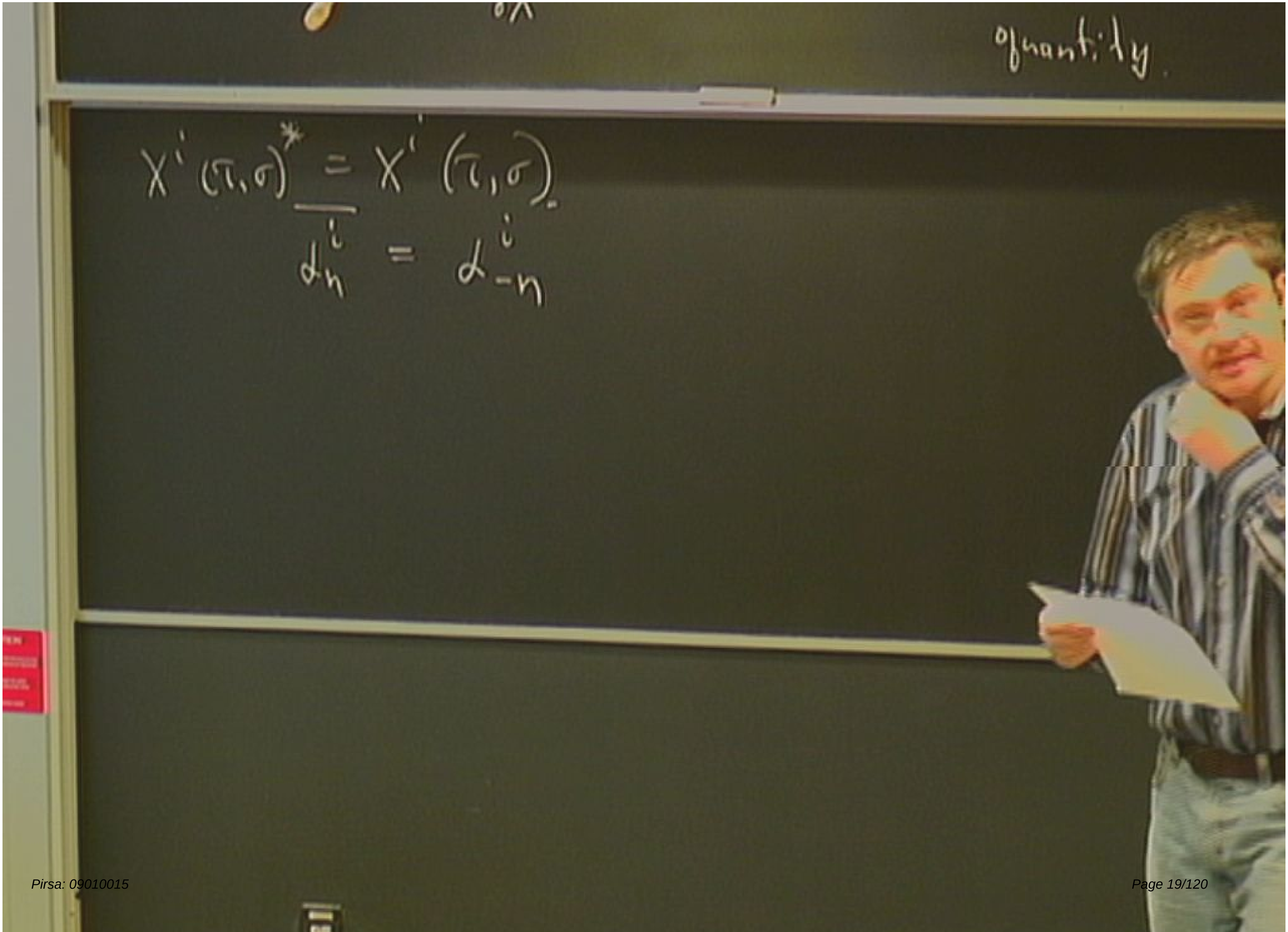
↑ wave equation with c being the speed.

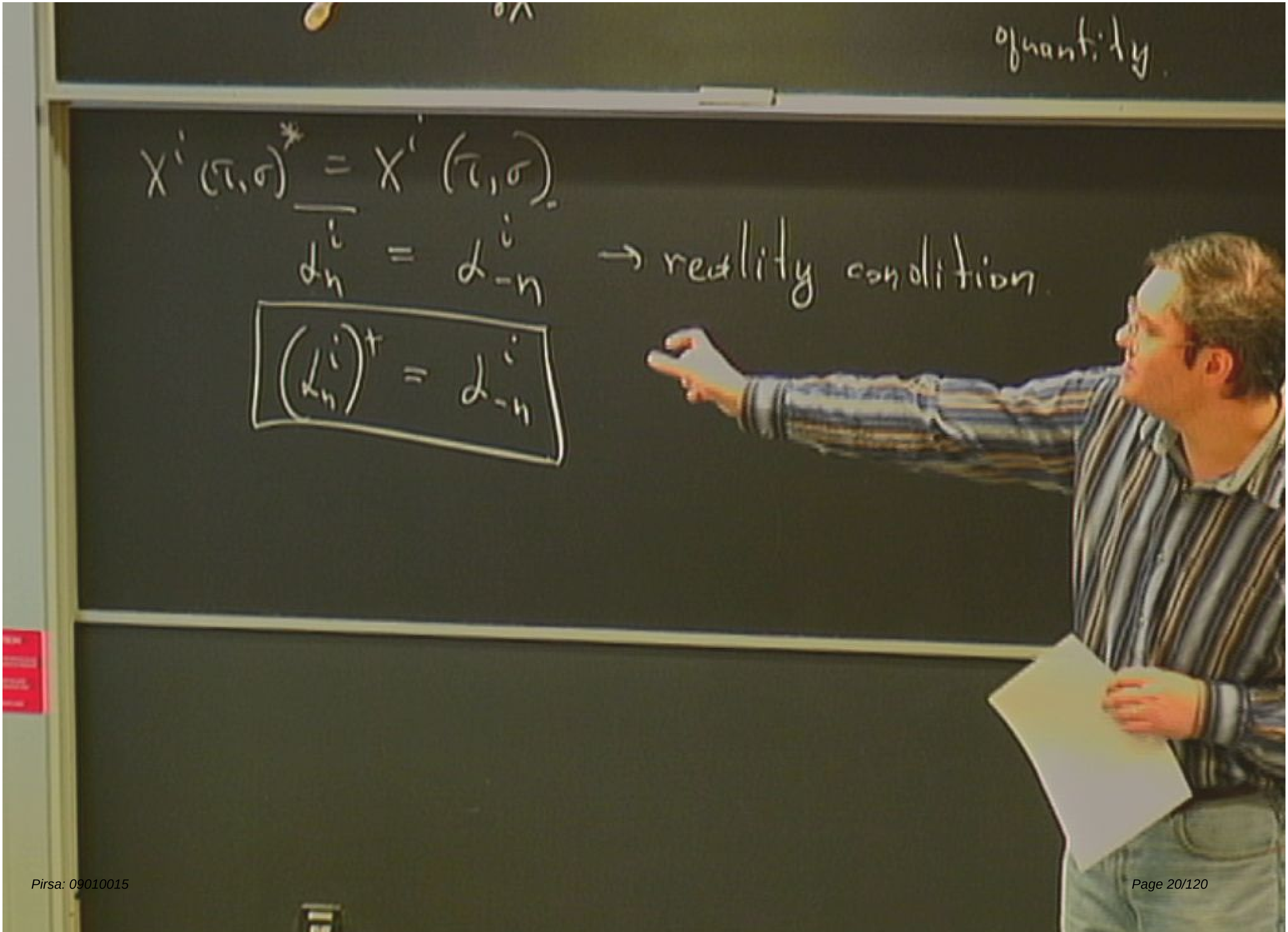

⇒ Lorentz inv b.c for open strings ⟺ NBC.
zero mode

$\Rightarrow$ Lorentz inv b.c for open strings $\Leftrightarrow$ NBC.

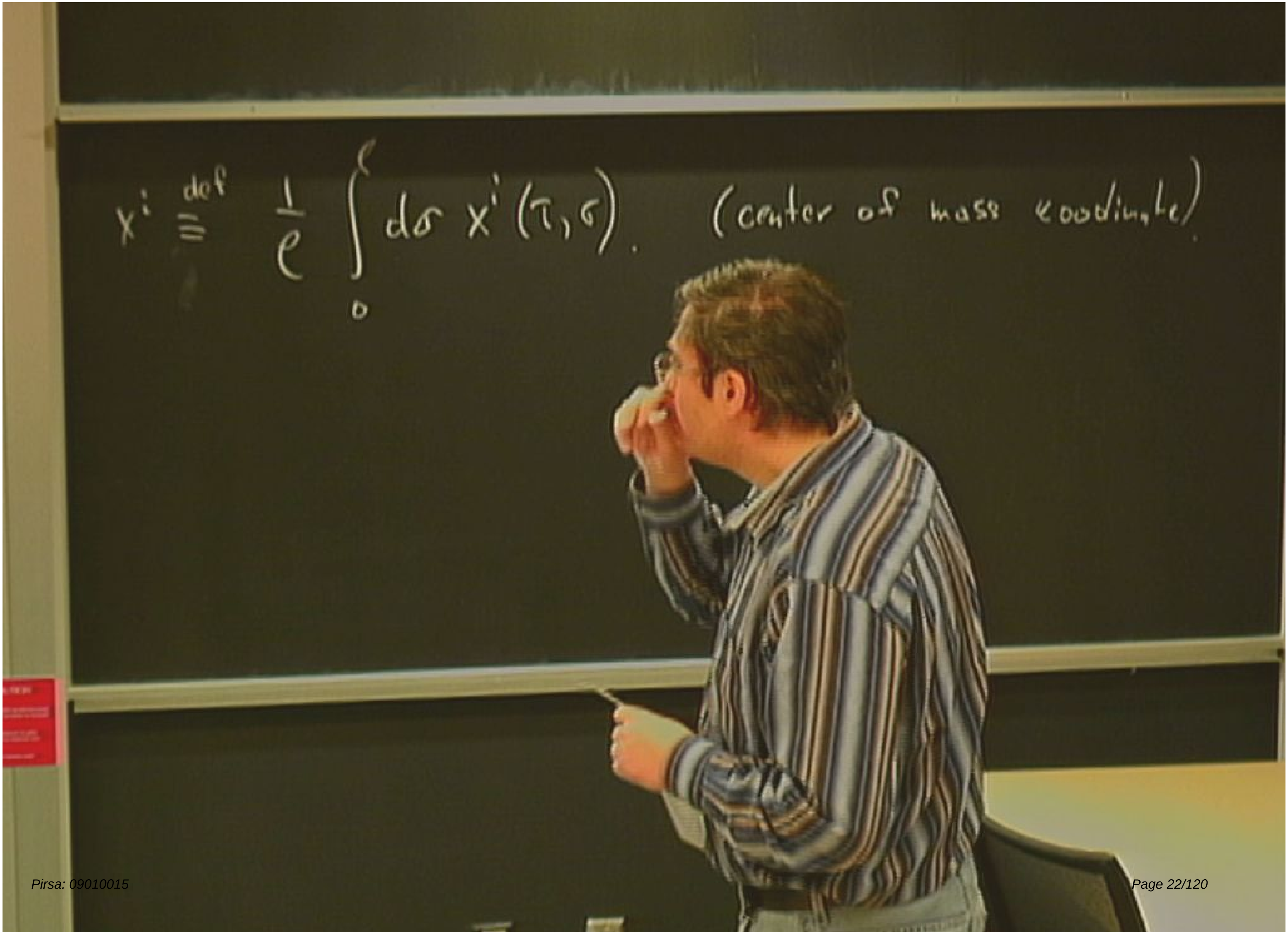
$$\partial_\sigma X^i\Big|_{\sigma=0} = \partial_\sigma X^i\Big|_{\sigma=\ell} = 0$$

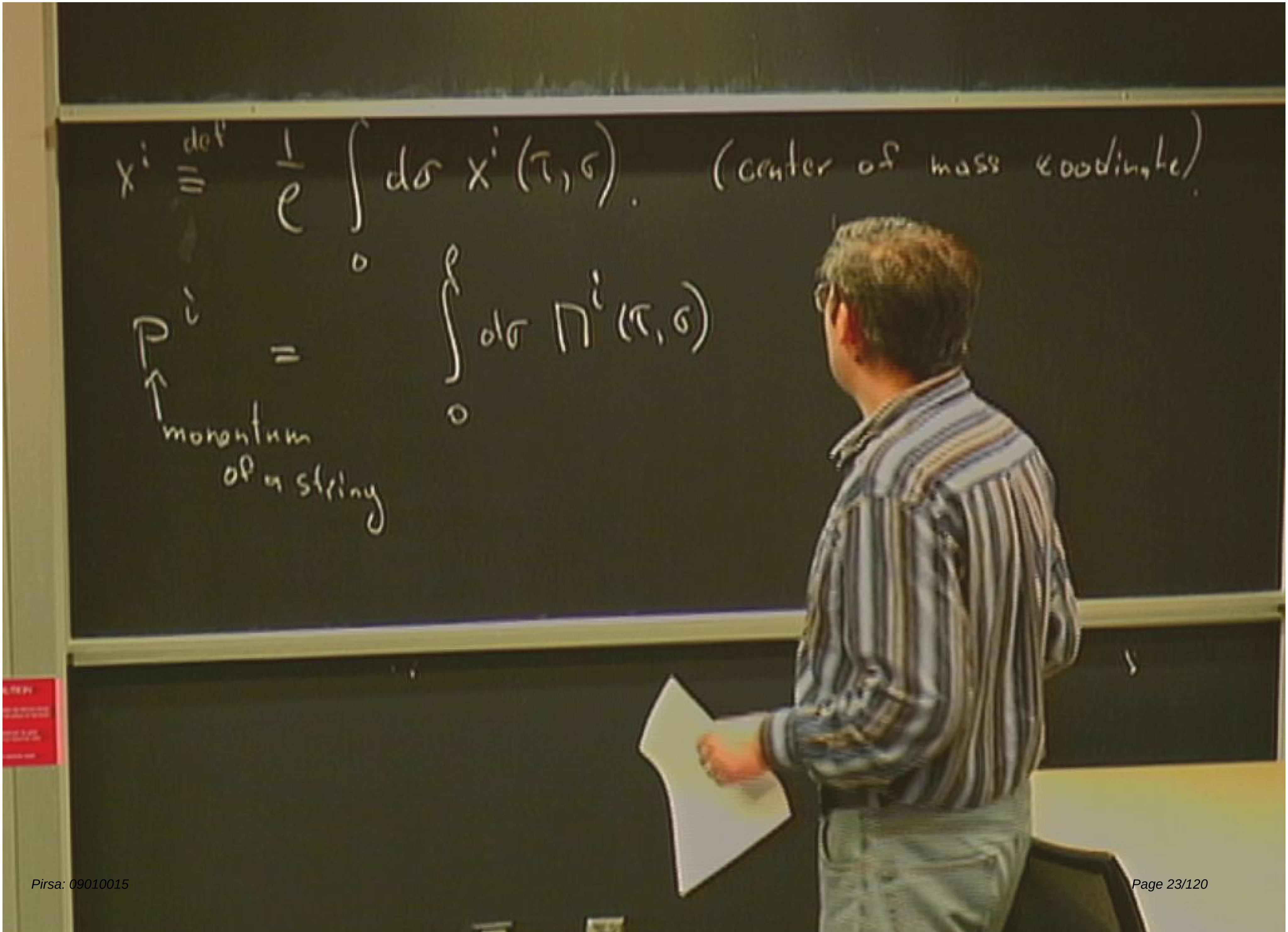
$$X^i(\tau,\sigma) = \underbrace{x^i + \frac{p^i}{p^+}\tau}_{\text{Zero mode}} + i(2\lambda')^{1/2}\sum_{\substack{n=-\infty \\ n\neq 0}}^{+\infty} \frac{1}{n}\alpha_n^i\, e^{-\frac{\pi i n c\tau}{\ell}} \cdot \cos\frac{\pi n\sigma}{\ell}$$


quantity
$X^i(\tau,\sigma)^* = X^i(\tau,\sigma)$
$\alpha_n^i = \alpha_{-n}^i$


quantity
$X^i(\tau,\sigma)^* = X^i(\tau,\sigma)$
$\alpha_n^{i*} = \alpha_{-n}^i$ → reality condition
$(\alpha_n^i)^\dagger = \alpha_{-n}^i$

$$x^i \stackrel{\text{def}}{=} \frac{1}{\ell}\int_0^{\ell} d\sigma\, x^i(\tau,\sigma).$$


$x^i \overset{\text{def}}{\equiv} \frac{1}{\ell}\int_0^{\ell} d\sigma\, x^i(\tau,\sigma).$ (center of mass coordinate).


$x^i \stackrel{\text{def}}{=} \frac{1}{\ell} \int_0^{\ell} d\sigma \, x^i(\tau,\sigma)$. (center of mass coordinate)
$P^i = \int_0^{\ell} d\sigma \, \Pi^i(\tau,\sigma)$
momentum of a string

$$\partial_\tau P^+ = -\partial_\tau P_- = \frac{\partial H}{\partial X^-} = 0 \Rightarrow p^+ \text{ is conserved quantity}$$

$$\partial_\tau X^i = \frac{\delta H}{\delta \Pi^i} = 2\pi\alpha' c\, \Pi^i$$

$$c = \frac{\ell}{2\pi\alpha' p^+}$$

$$\partial_\tau \Pi^i = -\frac{\delta H}{\delta X^i} = (-)^2 \frac{c}{2} \frac{1}{2\pi\alpha'} \cdot 2\, \partial_\sigma^2 X^i = \frac{c}{2\pi\alpha'} \partial_\sigma^2 X^i$$

$i = 2, \cdots D-1$

$\Rightarrow$ $D-2$ free fields

$$x^i \stackrel{\text{def}}{\equiv} \frac{1}{\ell} \int_0^{\ell} d\sigma \, X^i(\tau,\sigma). \quad \text{(center of mass coordinate)}.$$

$$p^i = \int_0^{\ell} d\sigma \, \Pi^i(\tau,\sigma) = \frac{p^+}{\ell} \int_0^{\ell} d\sigma \, \partial_\tau X^i(\tau,\sigma)$$

momentum of a string

$$x^i \equiv \frac{1}{\ell}\int_0^{\ell} d\sigma\, X^i(\tau,\sigma). \qquad \text{(center of mass coordinate).}$$

$$\hat{p}^i = \int_0^{\ell} d\sigma\, \Pi^i(\tau,\sigma) = \frac{p^+}{\ell}\int_0^{\ell} d\sigma\, \partial_\tau X^i(\tau,\sigma).$$

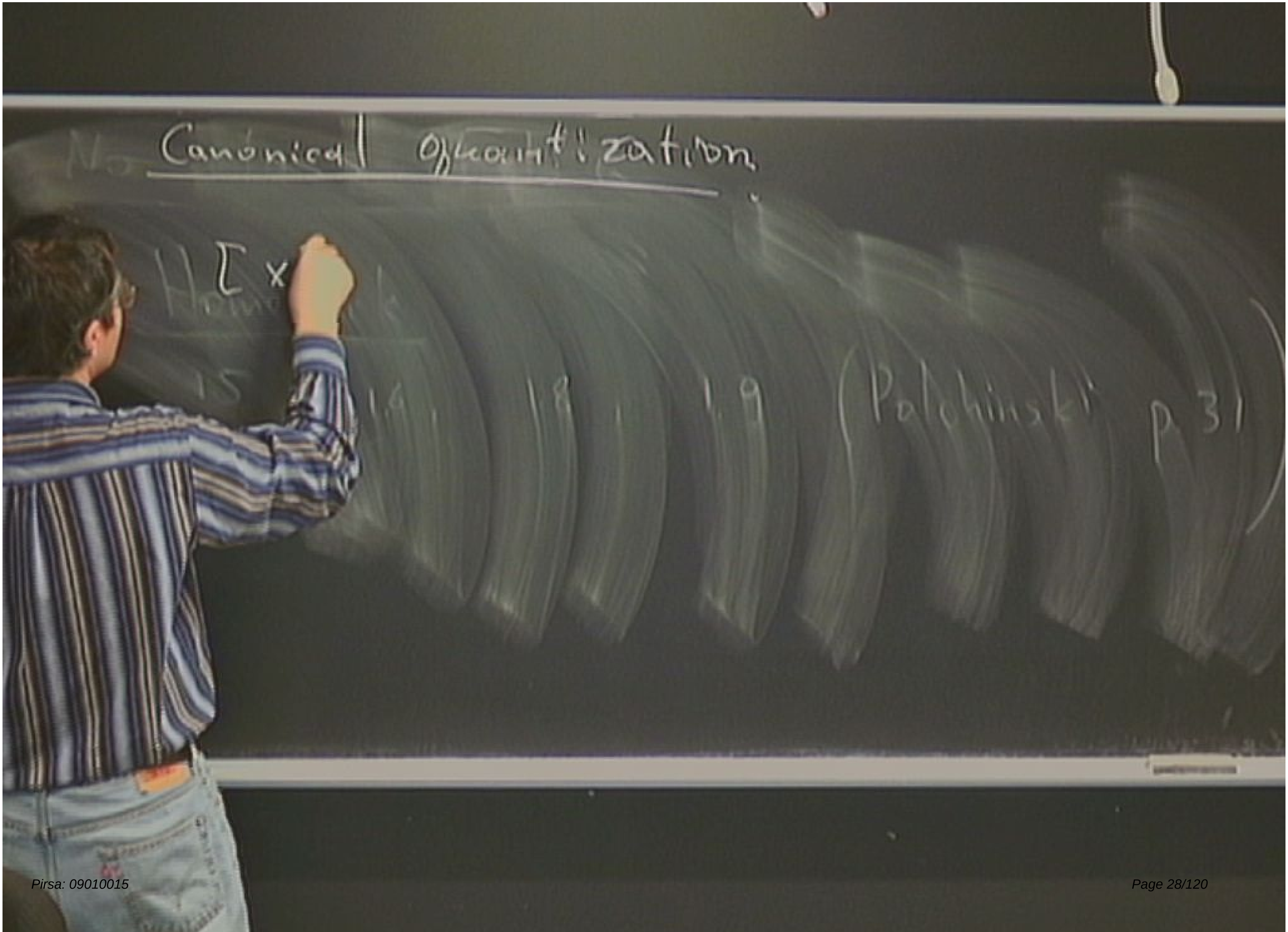
$$= \frac{p^+}{\ell}\cdot\frac{p^i}{p^+}\cdot\ell$$

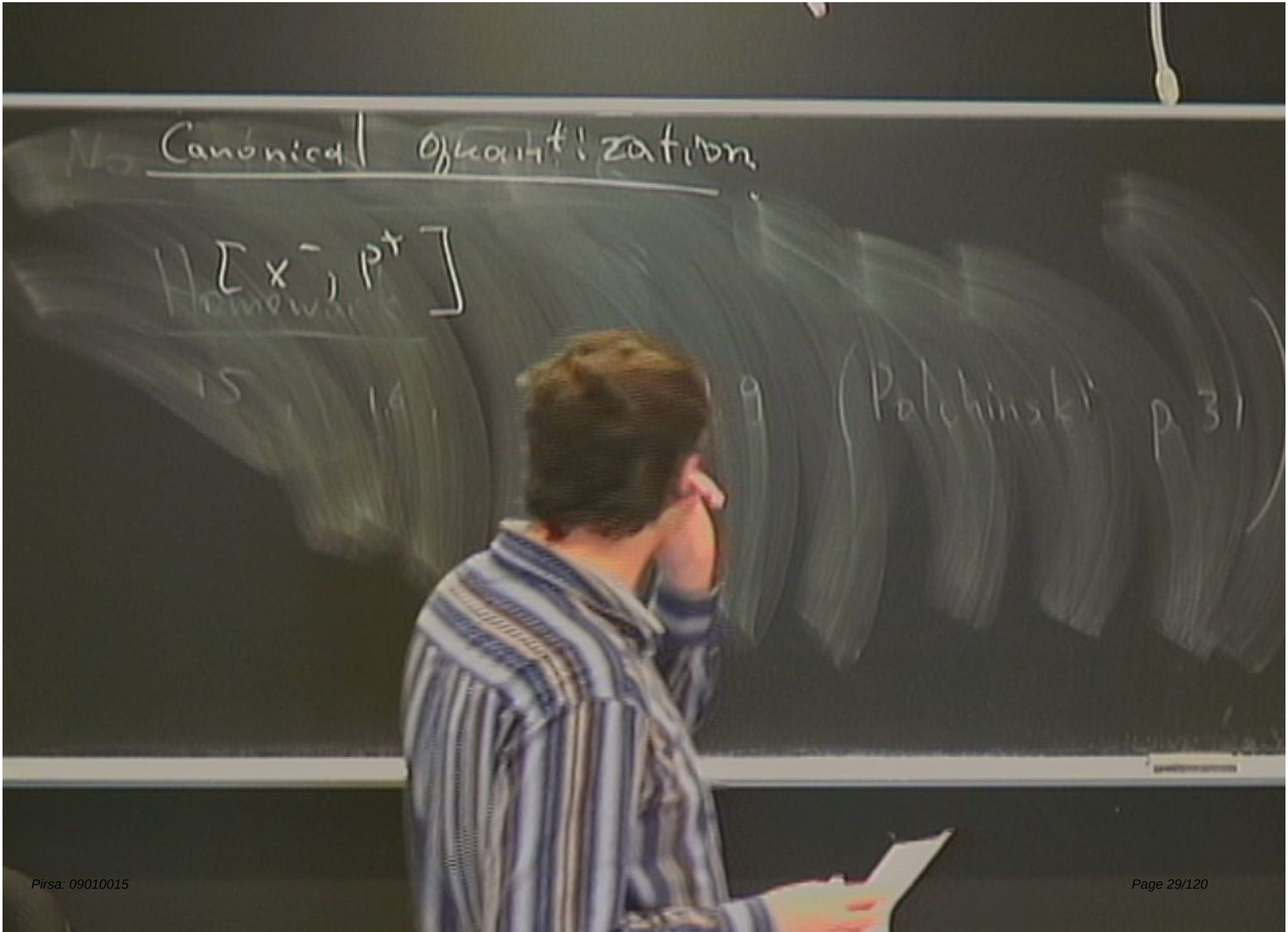
momentum of a string

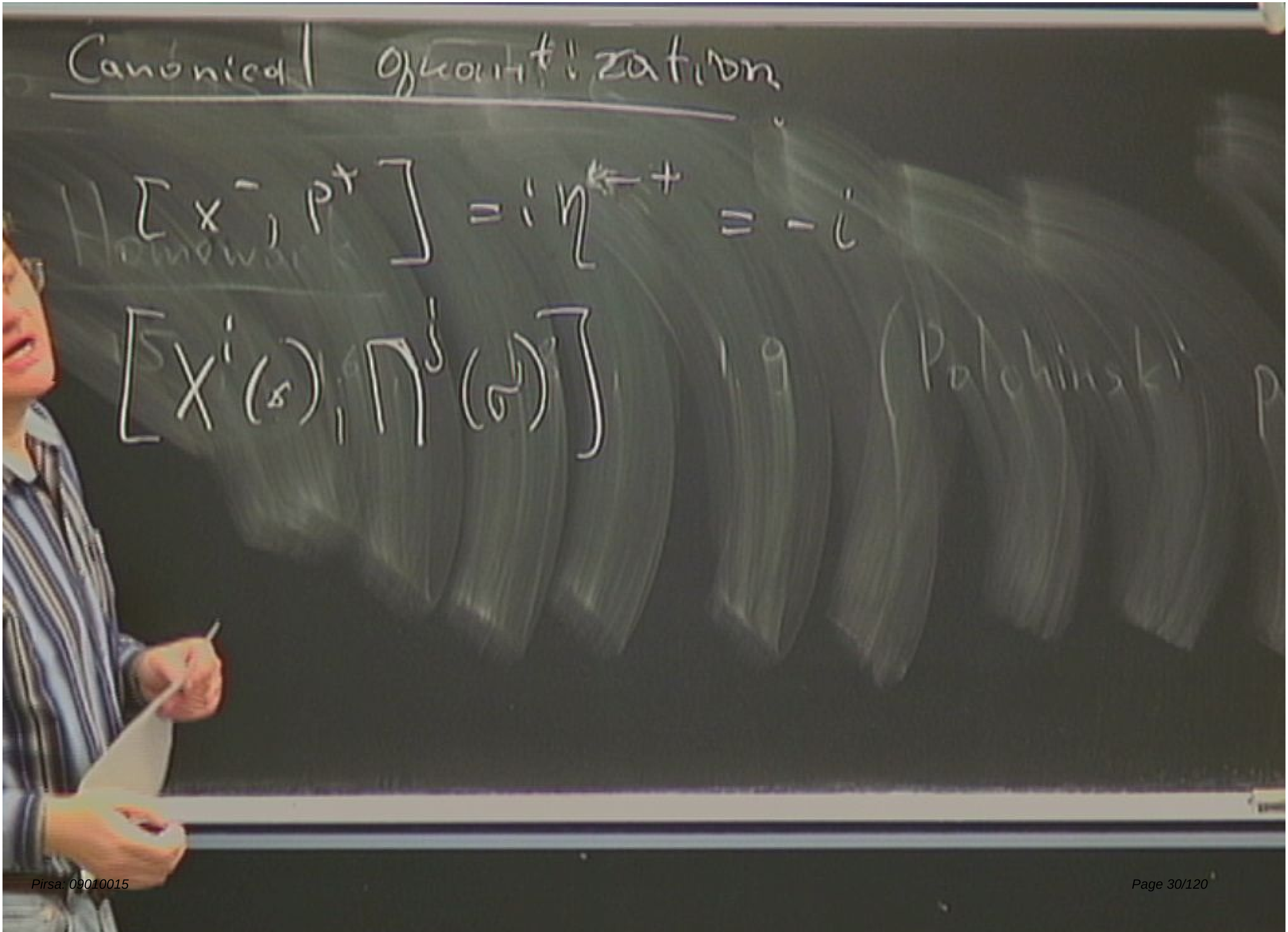
Lorentz inv b.c for open strings $\Leftrightarrow$ NBC.

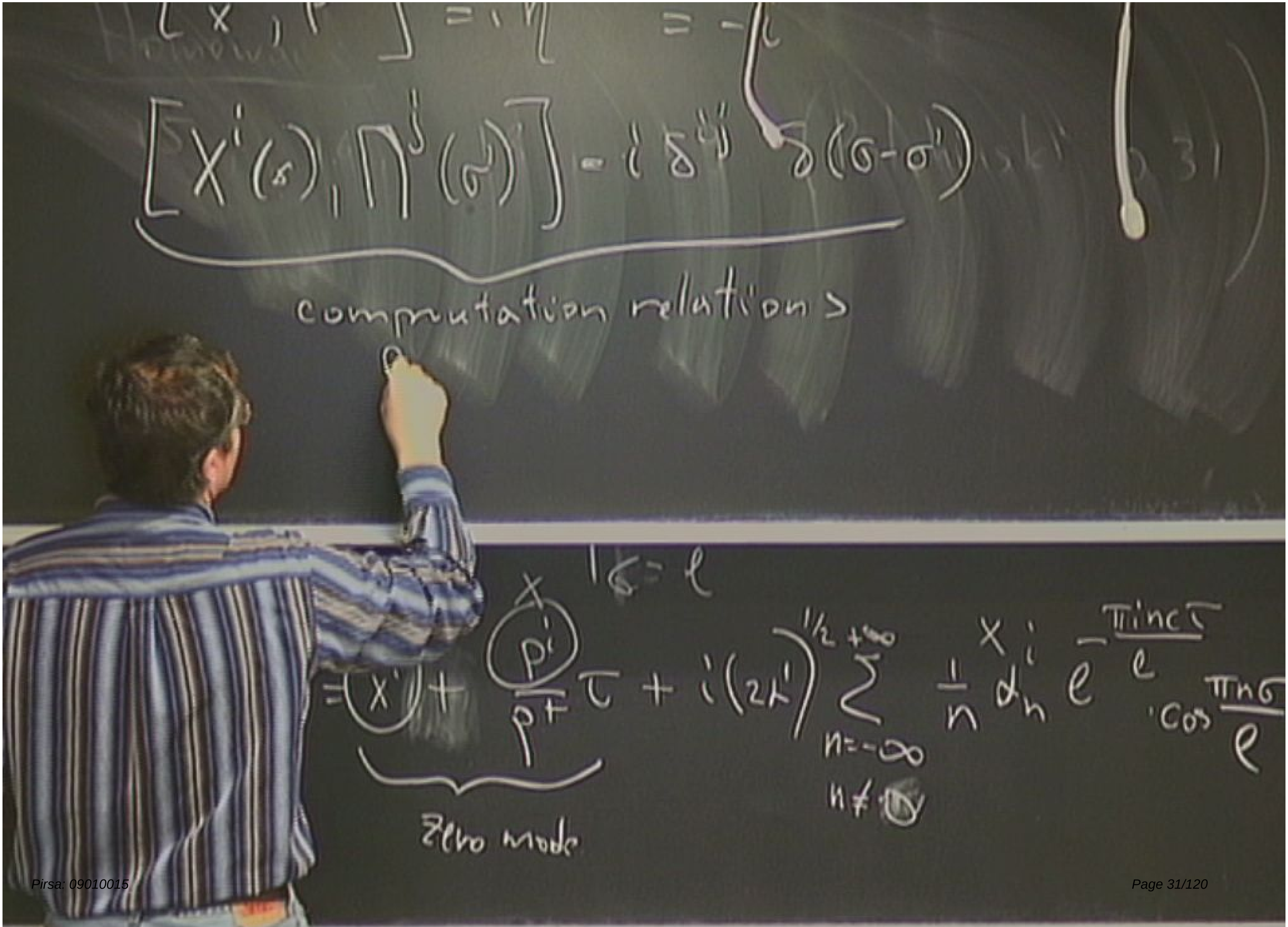
$$\partial_\sigma X^i \Big|_{\sigma=0} = \partial_\sigma X^i \Big|_{\sigma=\ell} = 0$$

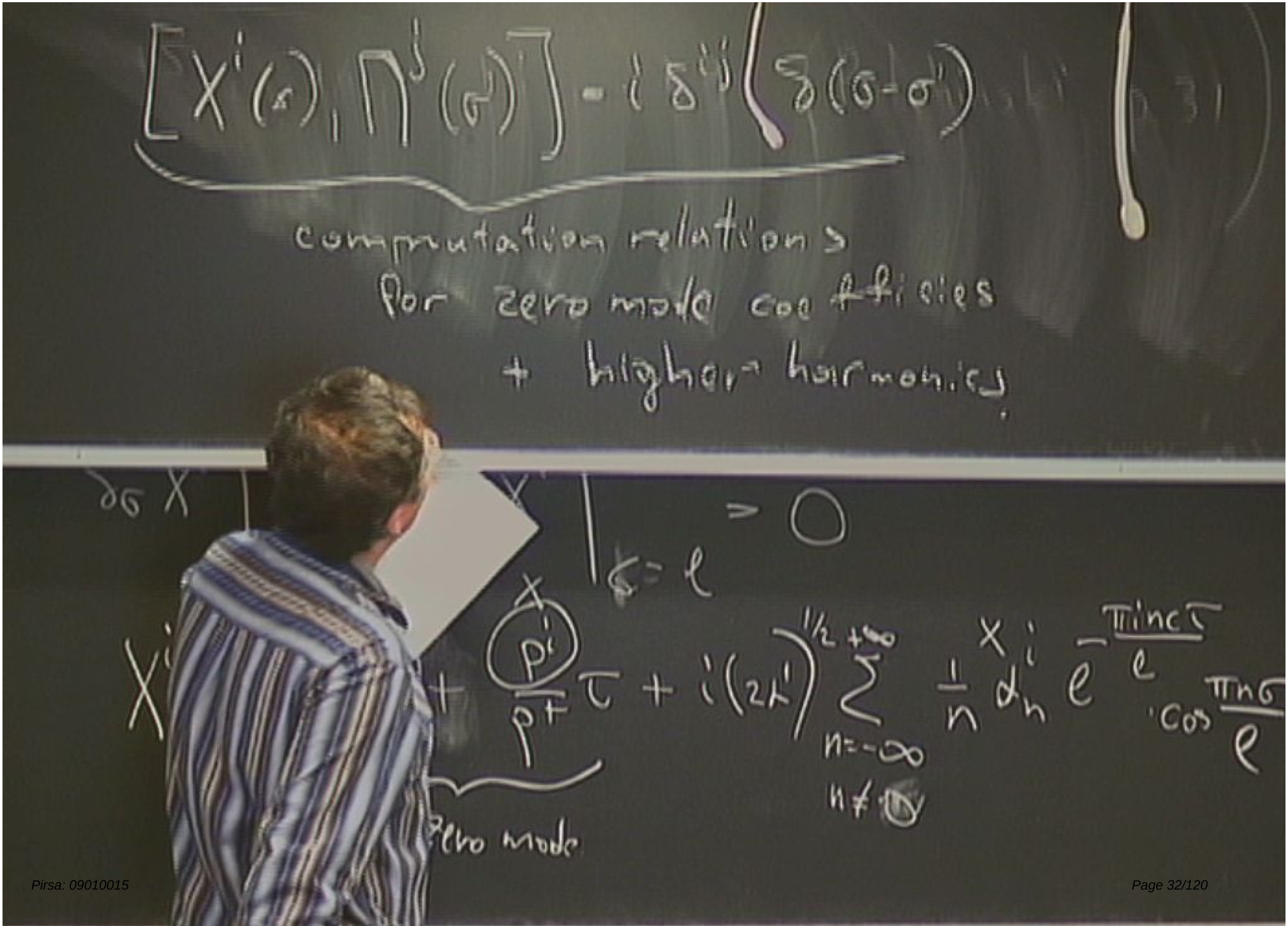
$$X^i(\tau,\sigma) = \underbrace{x^i + \frac{p^i}{p^+}\tau}_{\text{Zero mode}} + i(2\alpha')^{1/2} \sum_{\substack{n=-\infty \\ n\neq 0}}^{+\infty} \frac{1}{n} \alpha_n^i \, e^{-\frac{\pi i n c \tau}{\ell}} \cdot \cos\frac{\pi n \sigma}{\ell}$$


Canonical quantization
(Polchinski p 31)


Canonical quantization
$[x^-, p^+]$
(Polchinski p 31)


Canonical quantization
$[x^-, p^+] = i\eta^{-+} = -i$
$[X^i(\sigma), \Pi^j(\sigma')]$
(Polchinski


[X^i(σ), Π^j(σ')] = i δ^ij δ(σ-σ')
computation relations
|σ=ℓ
= x^i + p^i/p^+ τ + i(2α')^1/2 Σ_{n=-∞, n≠0}^{+∞} 1/n α_n^i e^{-πinτ/ℓ} cos πnσ/ℓ
Zero mode


[X^i(σ), Π^j(σ')] = iδ^{ij} δ(σ-σ')
commutation relations
for zero mode coefficies
+ higher harmonics
∂σ X^i |σ=ℓ = 0
X^i = x^i + p^i/p^+ τ + i(2α')^{1/2} Σ_{n=-∞, n≠0}^{+∞} 1/n α_n^i e^{-πinc τ/ℓ} cos πnσ/ℓ
zero mode

Canonical Quantization

$[x^-, p^+] = i\eta^{-+} = -i$

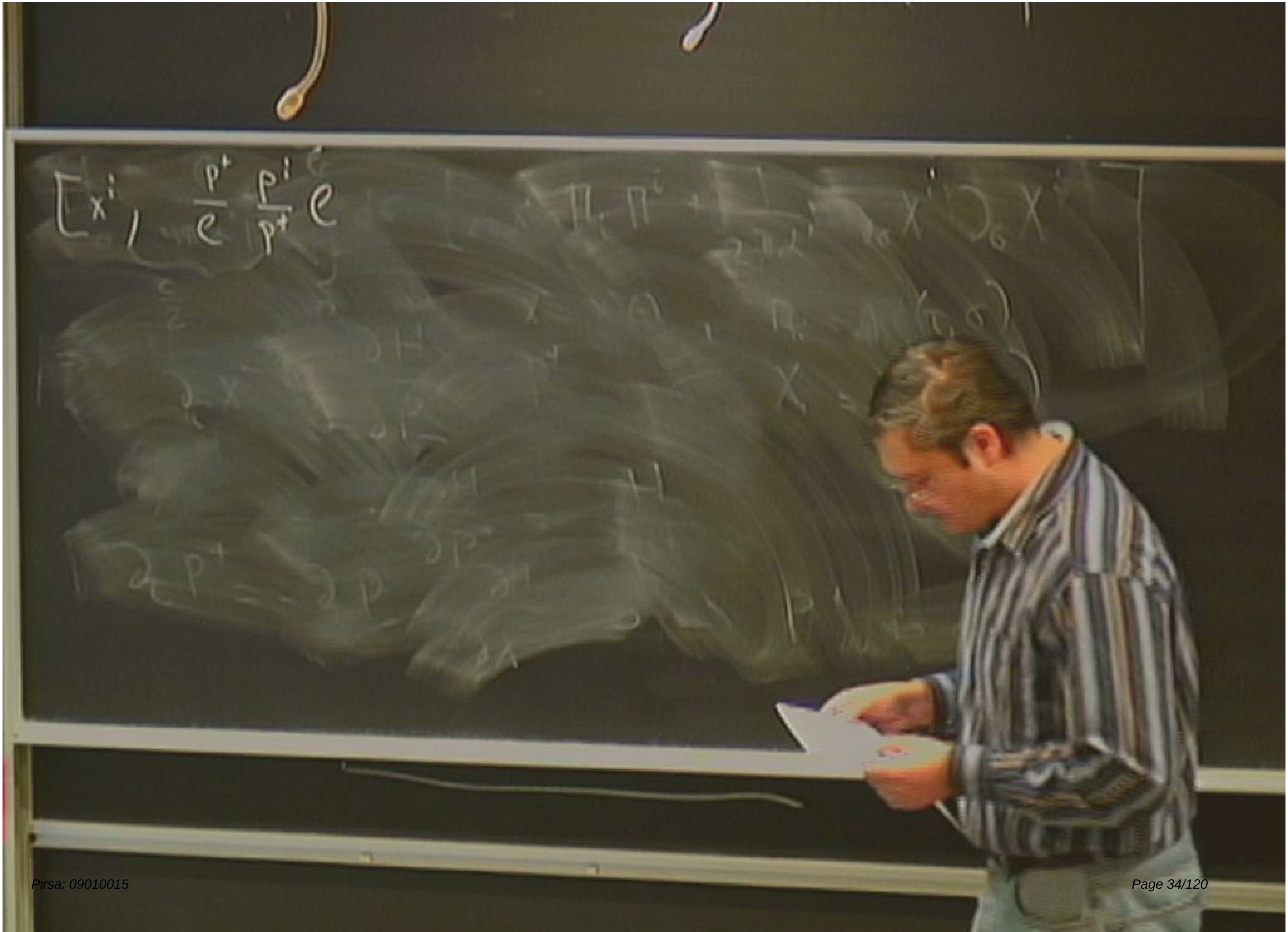
$$\underbrace{[X^i(\sigma), \Pi^j(\sigma')] = i\delta^{ij}\,\delta(\sigma-\sigma')}$$

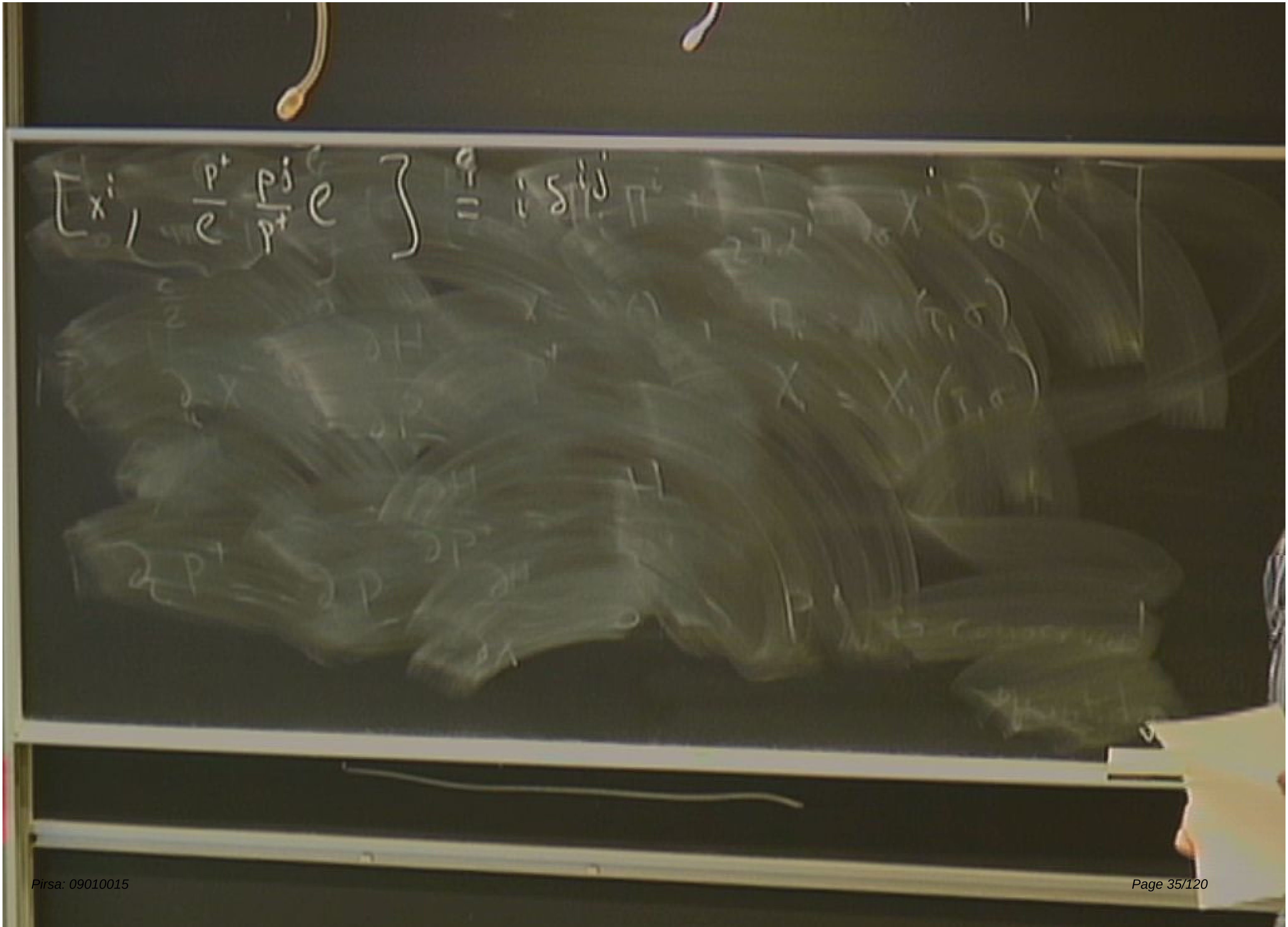
commutation relations
for zero mode coefficies
+ higher harmonics

$[X^i(\sigma), X^j(\sigma')] =$

$n \neq 0$

Zero mode




$[x^i, \frac{p^+}{e}\frac{p^j}{p^+}e] = i\delta^{ij}$

$$\left[x^i, \frac{p^+}{e}\frac{p^j}{p^+}e\right] \stackrel{?}{=} i\delta^{ij}$$

↑ zero mode

$$[x^i, p^j] = i\delta^{ij} \rightarrow \text{a point particle}$$

$$\left[x^i, \frac{p^+}{e}\frac{p^j}{p^+} e\right] \overset{?}{=} i\delta^{ij}\,\mathbb{1}$$

↑ zero mode

$$[x^i, p^j] = i\delta^{ij} \rightarrow \text{a point particle}$$

$$[\alpha_m^i, \alpha_n^j] = m\,\delta^{ij}\,\delta_{m,-n}$$

$$\boxed{[\alpha^i_m, \alpha^j_n] = m\,\delta^{ij}\,\delta_{m,-n}} \qquad m \neq 0$$

Simple harmonic oscillator algebra

$$X^i(\tau,\sigma)^* = X^i(\tau,\sigma)$$

free fields

$$\boxed{[\alpha^i_m, \alpha^j_n] = m\,\delta^{ij}\,\delta_{m,-n}} \qquad m \neq 0$$

Simple harmonic oscillator algebra

$\Rightarrow$ fix m

$$\alpha^i_m = \sqrt{m}\, a$$

$$\alpha^i_{-m}$$

$$\boxed{[\alpha^i_m, \alpha^j_n] = m\,\delta^{ij}\,\delta_{m,-n}}$$

$m \neq 0$

Simple harmonic oscillator algebra

$\Rightarrow$ fix m, i

$\alpha^i_m = \sqrt{m}\, a$

$\alpha^i_{-m} = \sqrt{m}\, a^+$

$[a, a^+] = 1$

$$\boxed{[\alpha_m^i, \alpha_n^j] = m\,\delta^{ij}\,\delta_{m,-n}}$$ $m \neq 0$

Simple harmonic oscillator algebra

$\Rightarrow$ fix m, i

$\alpha_m^i = \sqrt{m}\, a$

$\alpha_{-m}^i = \sqrt{m}\, a^+$

$[a, a^+] = 1$

$\Rightarrow$ a Polyakov string is

- center of mass motion
- infinite set of harmonic oscillators

$m =$

Simple harmonic oscillator algebra

$\Rightarrow$ fix m, i

$$\alpha^i_m = \sqrt{m}\, a$$

$$\alpha^i_{-m} = \sqrt{m}\, a^+$$

$$[a, a^+] = 1$$

$\Rightarrow$ a Polyakov string is

- center of mass motion
- infinite set of harmonic oscillator $m = 1, \ldots, \infty$

$$i = \underbrace{2, \ldots, D-1}_{D-2}$$

D-2 free fields

Hilbert space of a single string states.

Hilbert space of a single string states.

$|k^+, k^i\rangle$ $k^+, k^i \rightarrow$ eigenvalues of p^+, p^i

$\underbrace{|k^+, k^i\rangle}_{\text{same as PP labels}}$

$$p^+ |k^+, k^i\rangle = k^+ |k^+, k^i\rangle$$
$$p^i |k^+, k^i\rangle = k^i |k^+, k^i\rangle$$

Hilbert space of a single string states.

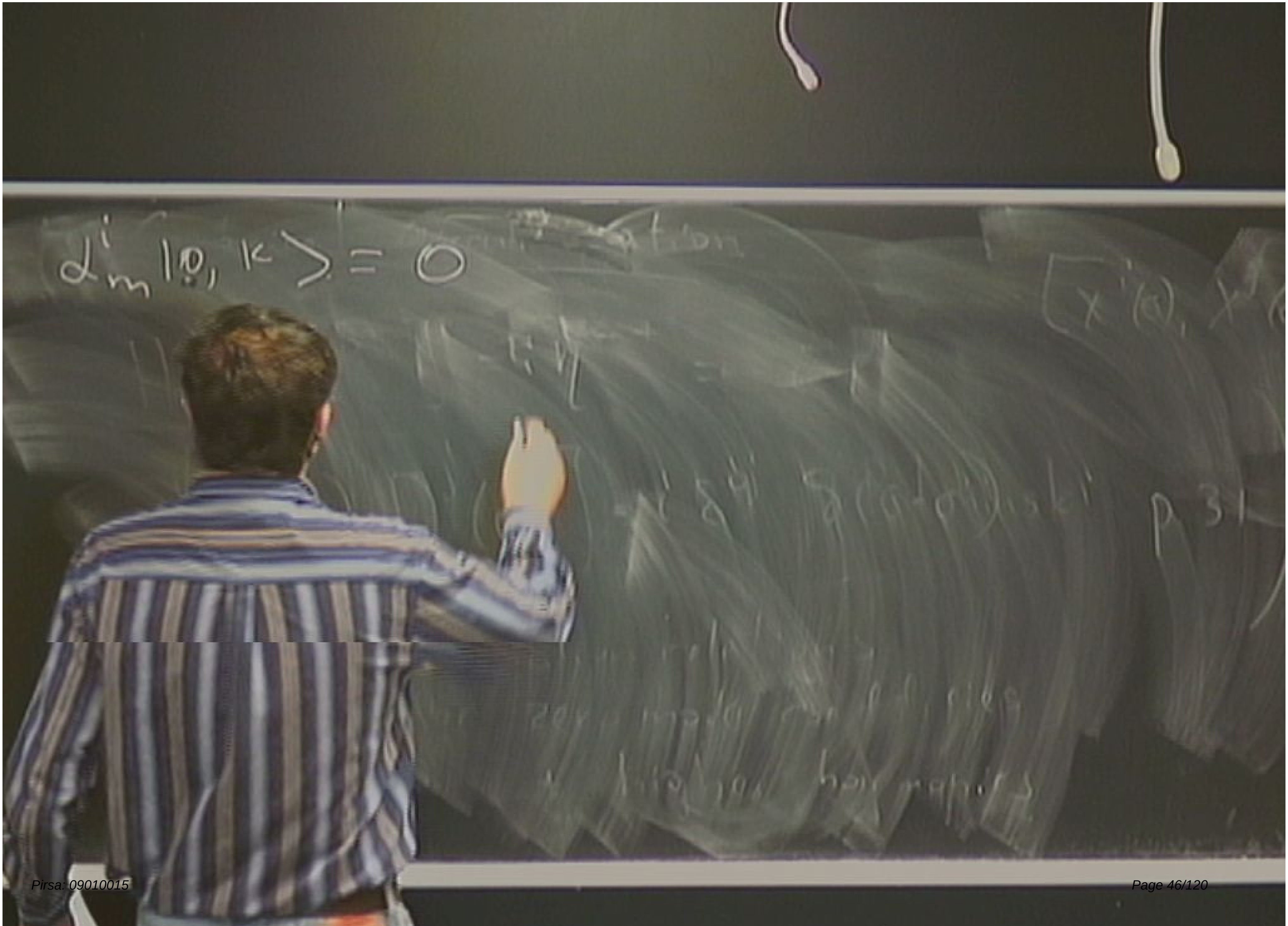
$\underbrace{|\overbrace{k^+, k^i}^{=k}\rangle}_{\text{same as PP labels}}$

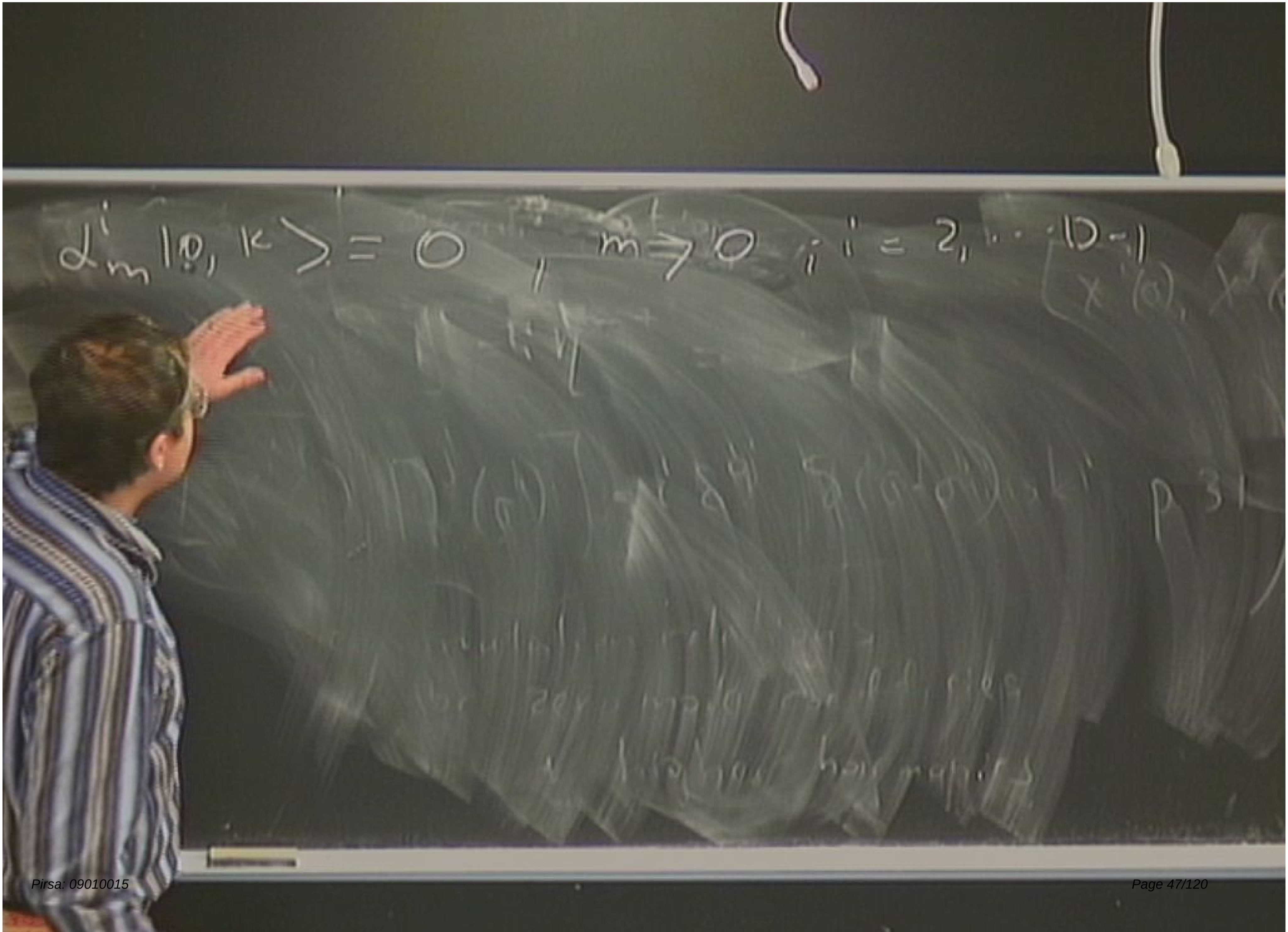
$k^+, k^i \rightarrow$ eigenvalues of p^+, p^i

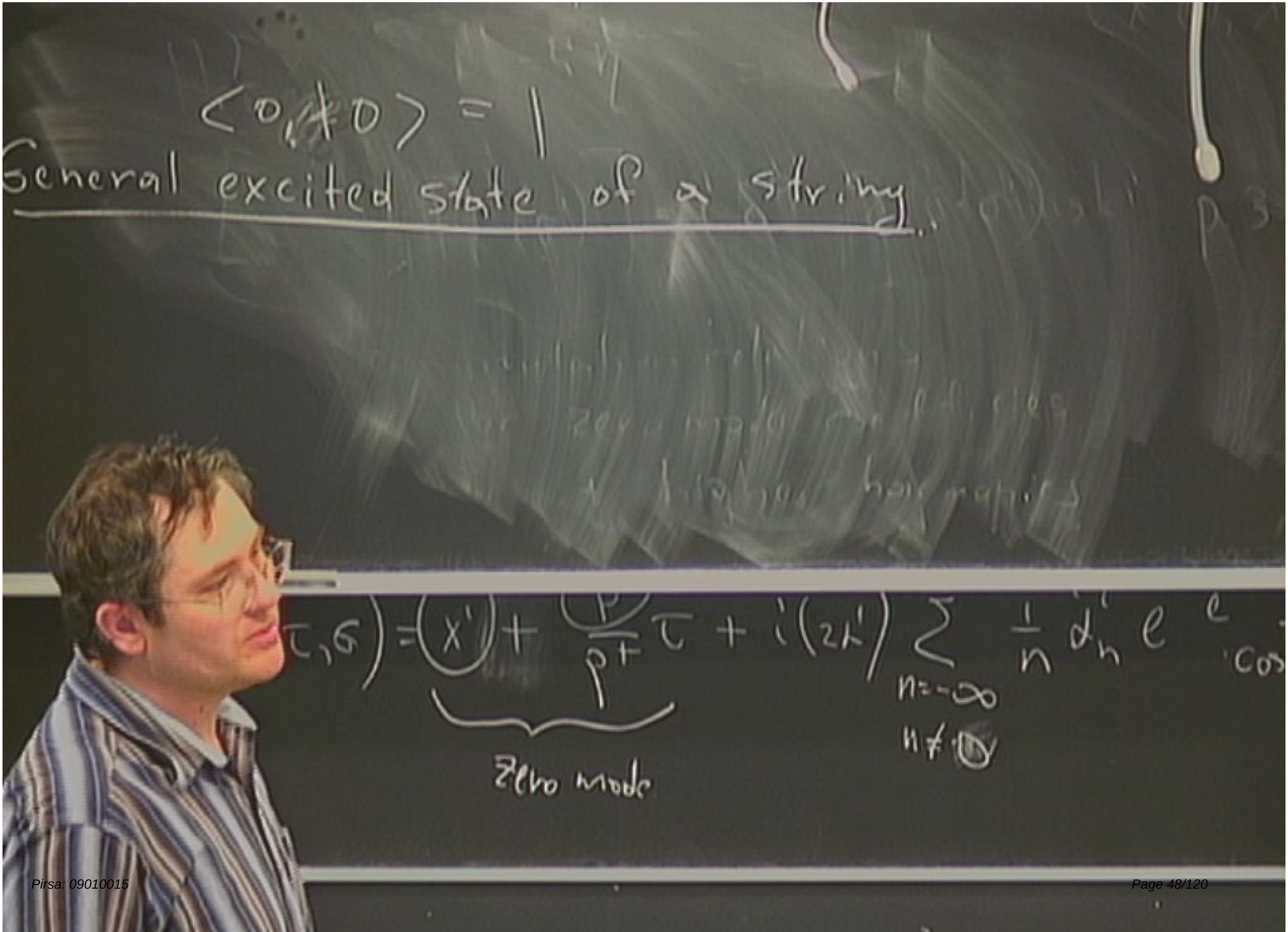
ground state of a string

$p^+ |k^+, k^i\rangle = k^+ |k^+, k^i\rangle$

$p^i |k^+, k^i\rangle = k^i |k^+, k^i\rangle$


$\alpha^i_m |0, k\rangle = 0$


$\alpha^i_m |0, k\rangle = 0$, $m \geq 0$, $i = 2, \dots D-1$


General excited state of a string
Zero mode
n=-∞
n≠0

$$\langle 0, k | 0 \rangle = 1$$

General excited state of a string

$$|N; k\rangle = \left(\alpha^i_{-n}\right)^{N_{i,n}}$$

$$= \underbrace{x^i + \frac{p^i}{p^+}\tau}_{\text{zero mode}} + i(2\alpha') \sum_{\substack{n=-\infty \\ n \neq 0}} \frac{1}{n} \alpha^i_n e \ \cos$$

$$\langle 0, k | 0 \rangle = 1$$

General excited state of a string

$$|N; k\rangle = \left(\alpha^i_{-n}\right)^{N_{i,n}}$$

$$\to (a^\dagger)^n |0\rangle$$

$$\underbrace{\quad + \frac{p^i}{p^+}\tau}_{\text{zero mode}} + i(2\alpha') \sum_{\substack{n=-\infty \\ n \neq 0}}^{+\infty} \frac{1}{n} \alpha^i_n e^{\ldots}$$

$$\langle 0, \cancel{\ } 0\rangle = 1$$

General excited state of a string

$$|N;k\rangle = \left(\alpha^i_{-n}\right)^{N_{i,n}}$$

(a^{+})

$$X^i(\tau,\sigma) = \underbrace{x^i + \frac{p^i}{p^+}\tau}_{\text{Zero mode}} + i(2\alpha')^{1/2}\sum_{\substack{n=-\infty \\ n\neq 0}}^{+\infty} \frac{1}{n}\alpha^i_n e^{-\ldots} \cos$$

$\langle 0, 0 \rangle = 1$

General excited state of a string.

$$|N; k\rangle = \prod_{i=2}^{D-1} \prod_{n=1}^{\infty} \left(\frac{(\alpha^i_{-n})^{N_{i,n}}}{n^{N_{i,n}} N_{i,n}!} \right)^{1/2} |0; k\rangle$$

$$\frac{(a^+)^N |0\rangle}{(N!)^{1/2}}$$

$$X^i(\tau,\sigma) = \underbrace{x^i + \frac{p^i}{p^+}\tau}_{\text{zero mode}} + i(2\alpha')^{1/2} \sum_{\substack{n=-\infty \\ n \neq 0}}^{+\infty} \frac{1}{n} \alpha^i_n e^{-\frac{\pi i n c \tau}{\ell}} \cdot \cos$$

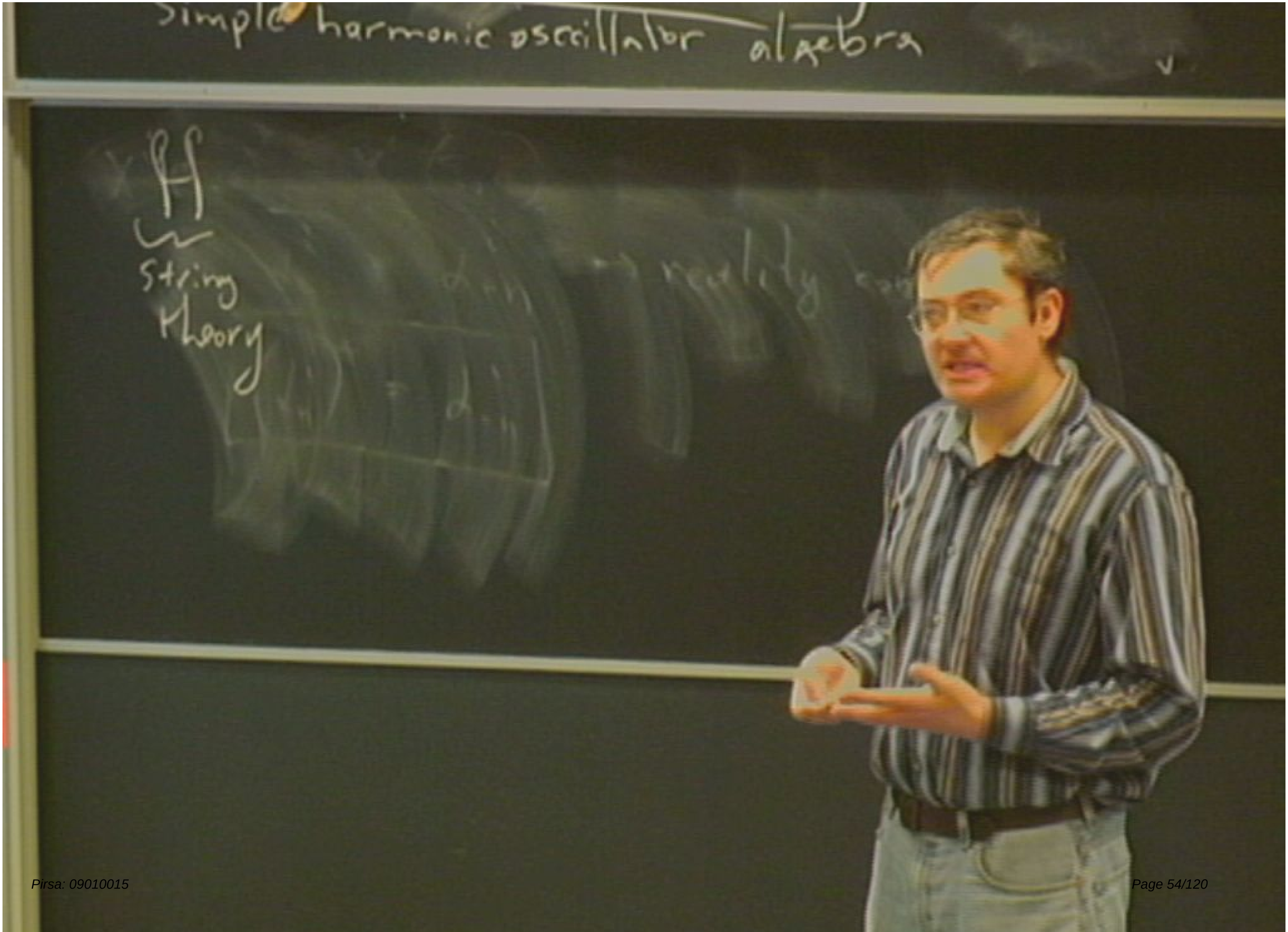
$0, \neq 0\rangle = 1$

...cited state of a string

$$= \prod_{i=2}^{D-1} \prod_{n=1}^{\infty} \frac{\left(\alpha^{i}_{-n}\right)^{N_{i,n}}}{\left(n^{N_{i,n}}\, N_{i,n}!\right)^{1/2}} \, |0; k\rangle$$

$N_{i,n}$ are occupation #s of (i, n) mode

$$(\tau,\sigma) = \underbrace{x^{i} + \frac{p^{i}}{p^{+}}\tau}_{\text{zero mode}} + i(2\alpha')\sum_{\substack{n=-\infty \\ n\neq 0}} \frac{1}{n}\alpha^{i}_{n}\, e \;\; e \; \cos\frac{\pi n \sigma}{\ell}$$



simple harmonic oscillator algebra
string theory
reality

simple harmonic oscillator algebra

$$\underbrace{H}_{\text{string theory}} = H_0 + H_1 + H_2 + \cdots$$

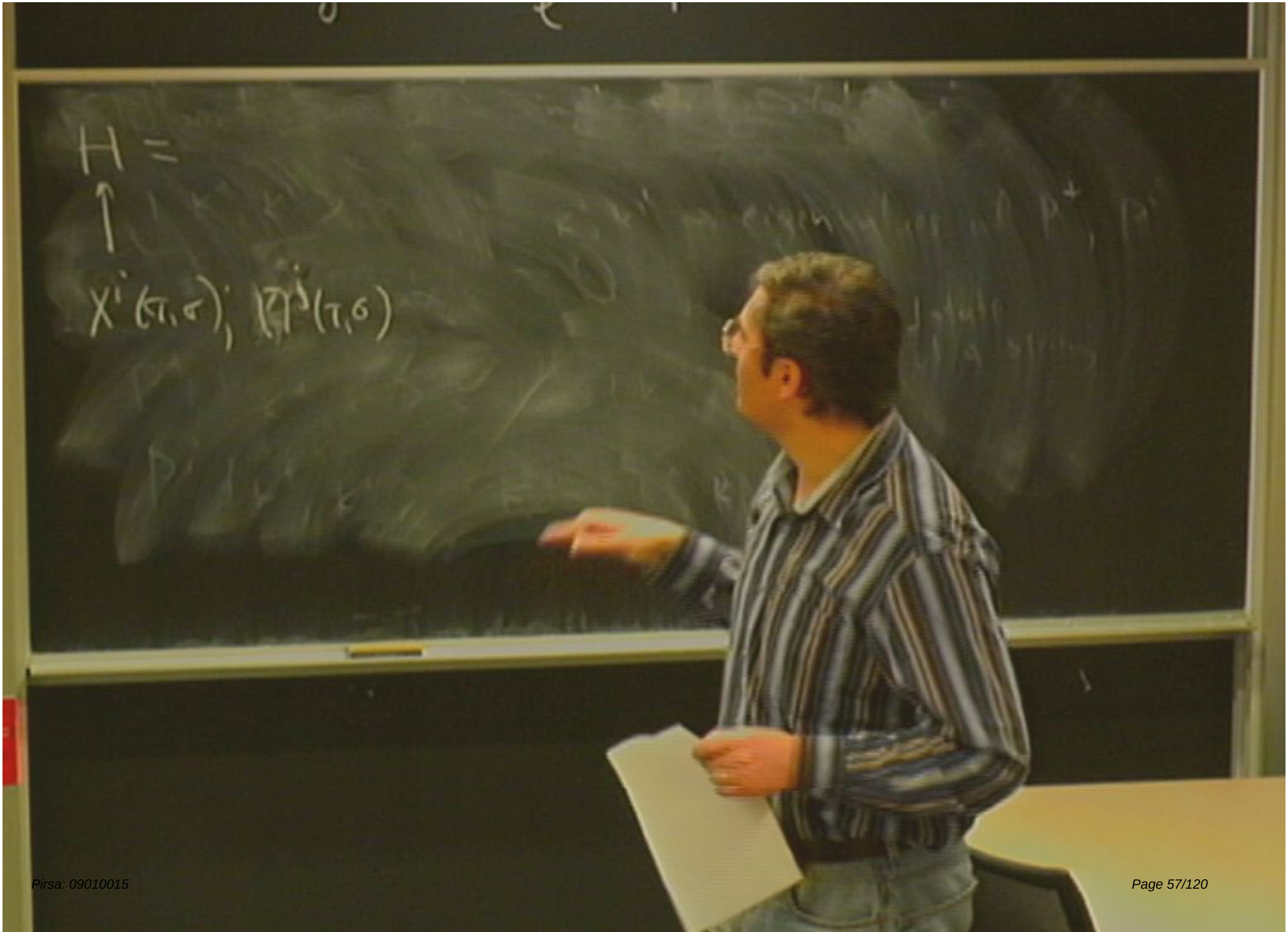
just discussion

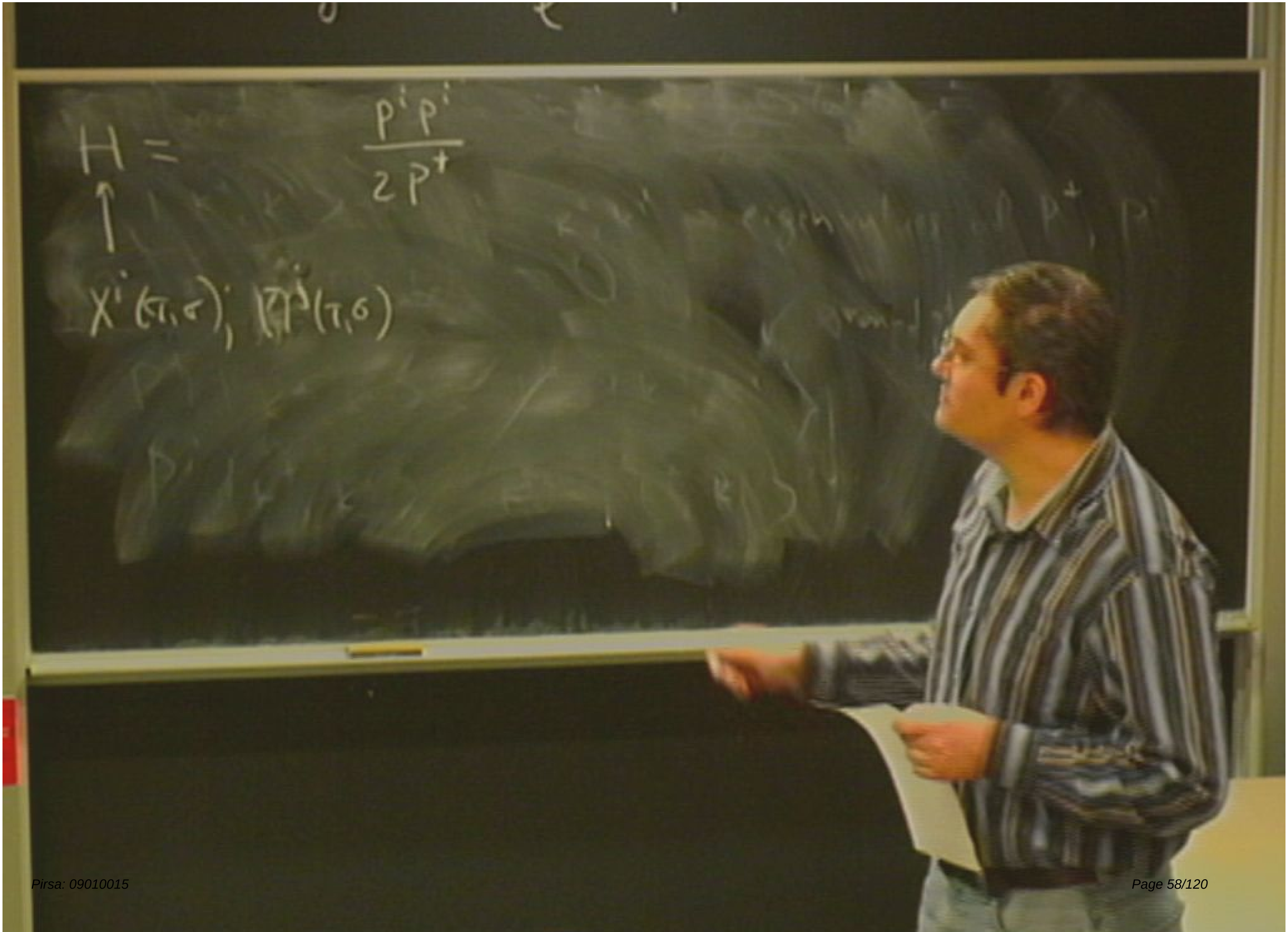
simple harmonic oscillator algebra

$$\underbrace{\mathcal{H}}_{\text{string theory}} = H_0 \oplus H_1 \oplus H_2 + \cdots$$

H_1 ↑ just discussion

$$H_n = \left(\otimes H_1\right)^n$$


H =
X^i(τ,σ); P^j(τ,σ)


H = p^i p^i / 2p^+
X^i(τ,σ); P^i(τ,σ)

$$H = \frac{p^i p^i}{2p^+} + \frac{1}{2p^+\alpha'}\left(\sum_{i=1}^{\infty} \underbrace{\alpha_{-n}^i \alpha_n^i}_{\text{implicit summation over } i}\right)$$

$\uparrow$

$X^i(\tau,\sigma)$; $P^i(\tau,\sigma)$

$$H = \frac{p^i p^i}{2p^+} + \frac{1}{2p^+\alpha'}\Big(\sum_{i=1}^{\infty} \underbrace{\alpha_{-n}^i \alpha_n^i}_{\text{implicit summation over } i} + \underbrace{A}_{\text{Quantum effect}}\Big)$$

$X^i(\tau,\sigma);\ P^i(\tau,\sigma)$

$$H = \frac{p^i p^i}{2p^+} + \frac{1}{2p^+\alpha'}\left(\sum_{i=1}^{\infty} \underbrace{\alpha^i_{-n}\alpha^i_n}_{\text{implicit summation over } i} + A\right)$$

Quantum effect

$X^i(\tau,\sigma)$; $P^i(\tau,\sigma)$

$\Rightarrow \{D, A\}$ - free parameters.

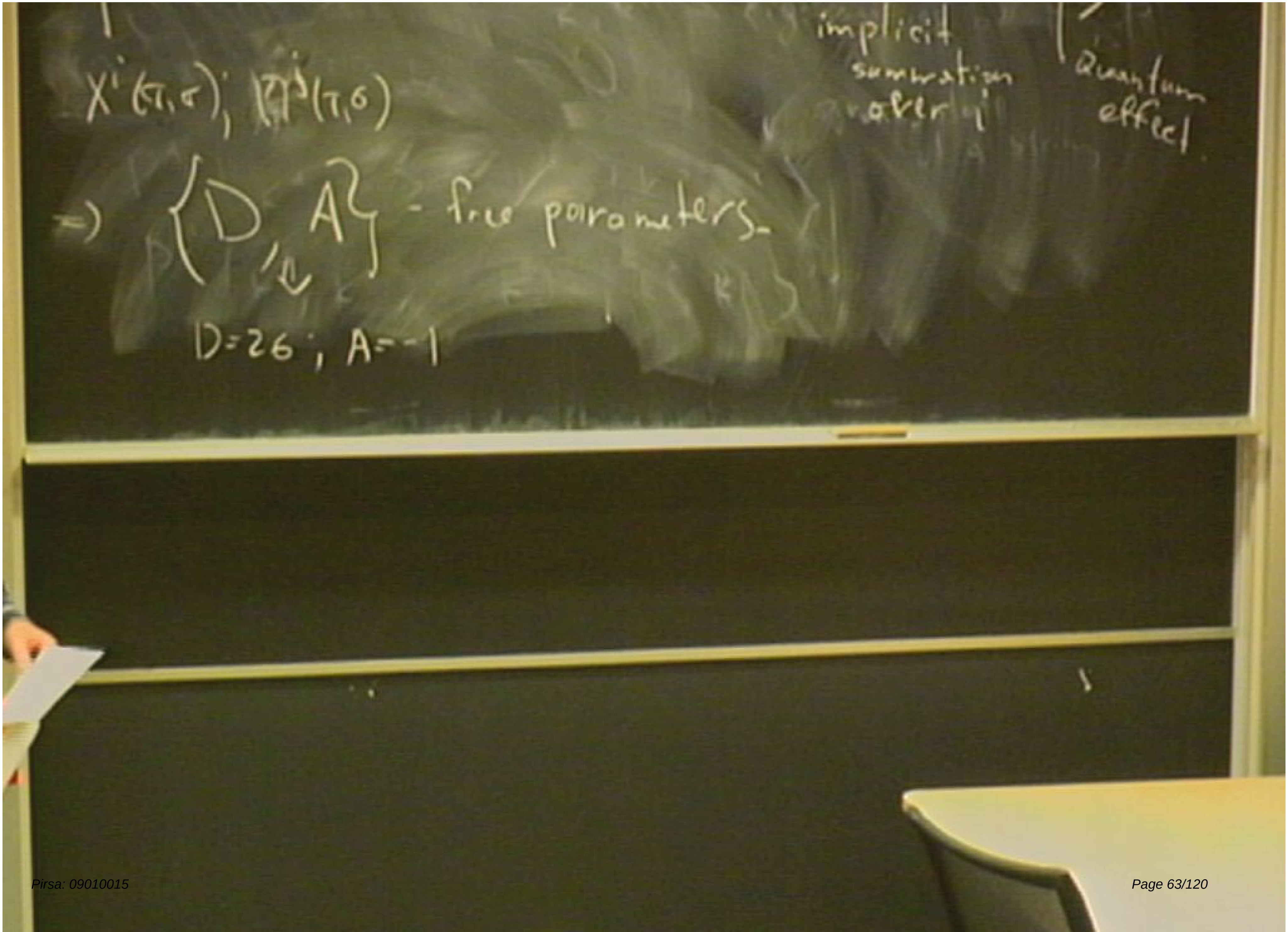
$X^i(\tau,\sigma)$; $P^j(\tau,\sigma)$

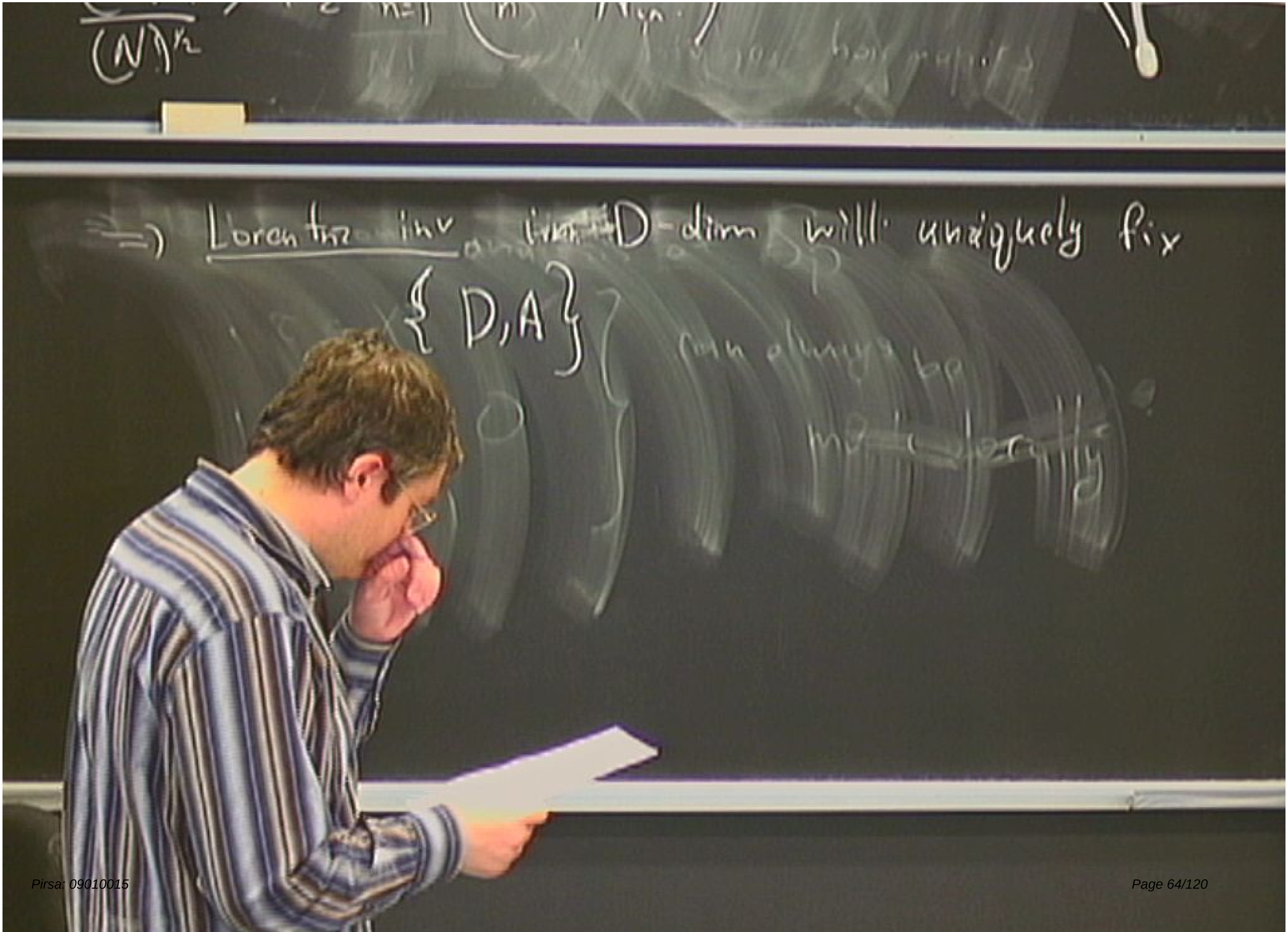
implicit summation over i

Quantum effect

$\Rightarrow \{D, A\}$ - free parameters

$D = 26$


implicit
summation
over i
Quantum
effect
$X^i(\tau,\sigma)$; $P^j(\tau,\sigma)$
$\Rightarrow$ {D, A} - free parameters.
D=26; A=-1


(N)^½
⟹ Lorentz inv in D-dim will uniquely fix
{D, A}

$\frac{}{(N)^{1/2}}$

$\Rightarrow$ Lorentz inv in D-dim will uniquely fix

$\{D, A\}$

$\Rightarrow$ $[M^{\mu\nu}, P^{\lambda}] = \ldots$

generators of Lorentz transform..

$\underbrace{\qquad D-1}_{D-2}$

Talk about A

$$H = \frac{1}{2}\hbar\omega\left[a^{\dagger}a + A\right]$$

$\underbrace{\epsilon_1 \cdots D-1}_{D-2}$

Talk about A

$H = \hbar\omega\left[a^\dagger a + A\right] \Leftrightarrow$ we must have $E = \hbar\omega\left[n + \frac{1}{2}\right]$

$a^\dagger a\,|N\rangle = n\,|N\rangle$

D-1

D-2

Talk about A

$H = \hbar\omega\left[a^\dagger a + A\right] \Longleftrightarrow$

we must have

$E = \hbar\omega\left[n + \frac{1}{2}\right]$

$a^\dagger a\,|N\rangle = n|N\rangle$

$[a, a^\dagger] = 1$

$\frac{\hbar\omega}{2}\left[a^\dagger a + a a^\dagger\right]$

$$\frac{\hbar\omega}{2}\left[a^+a + a^-a^+\right] \Rightarrow \frac{\hbar\omega}{2}\left[2a^+a + 1\right]$$

$$\mathcal{H} = H_0 \oplus H_1 \oplus H_2 + \cdots$$

string theory

just discussion

$$H_n = \text{sym}\left(\otimes H_1\right)^n$$

$$H = \frac{p^i p^i}{2p^+} + \frac{1}{4p^+\alpha'}\left[\sum_{n=1}^{\infty}\left(\alpha_{-n}^i\alpha_n^i + \alpha_n^i\alpha_{-n}^i\right)\right]$$

$$H = \frac{p^i p^i}{2p^+} + \frac{1}{4p^+\alpha'}\left[\sum_{n=1}^{\infty}\left(\alpha^i_{-n}\alpha^i_n + \alpha^i_n\alpha^i_{-n}\right)\right]$$

$$= \frac{p^i p^i}{2p^+} + \frac{1}{2p^+\alpha'}\sum_{n=1}^{\infty}\left[\alpha^i_{-n}\alpha^i_n\right.$$

$$H = \frac{p^i p^i}{2p^+} + \frac{1}{4p^+\alpha'}\left[\sum_{n=1}^{\infty}\left(\alpha^i_{-n}\alpha^i_n + \alpha^i_n\alpha^i_{-n}\right)\right]$$

$$= \frac{p^i p^i}{2p^+} + \frac{1}{2p^+\alpha'}\sum_{n=1}^{\infty}\left[\alpha^i_{-n}\alpha^i_n + \frac{1}{2}(D-2)\right]$$

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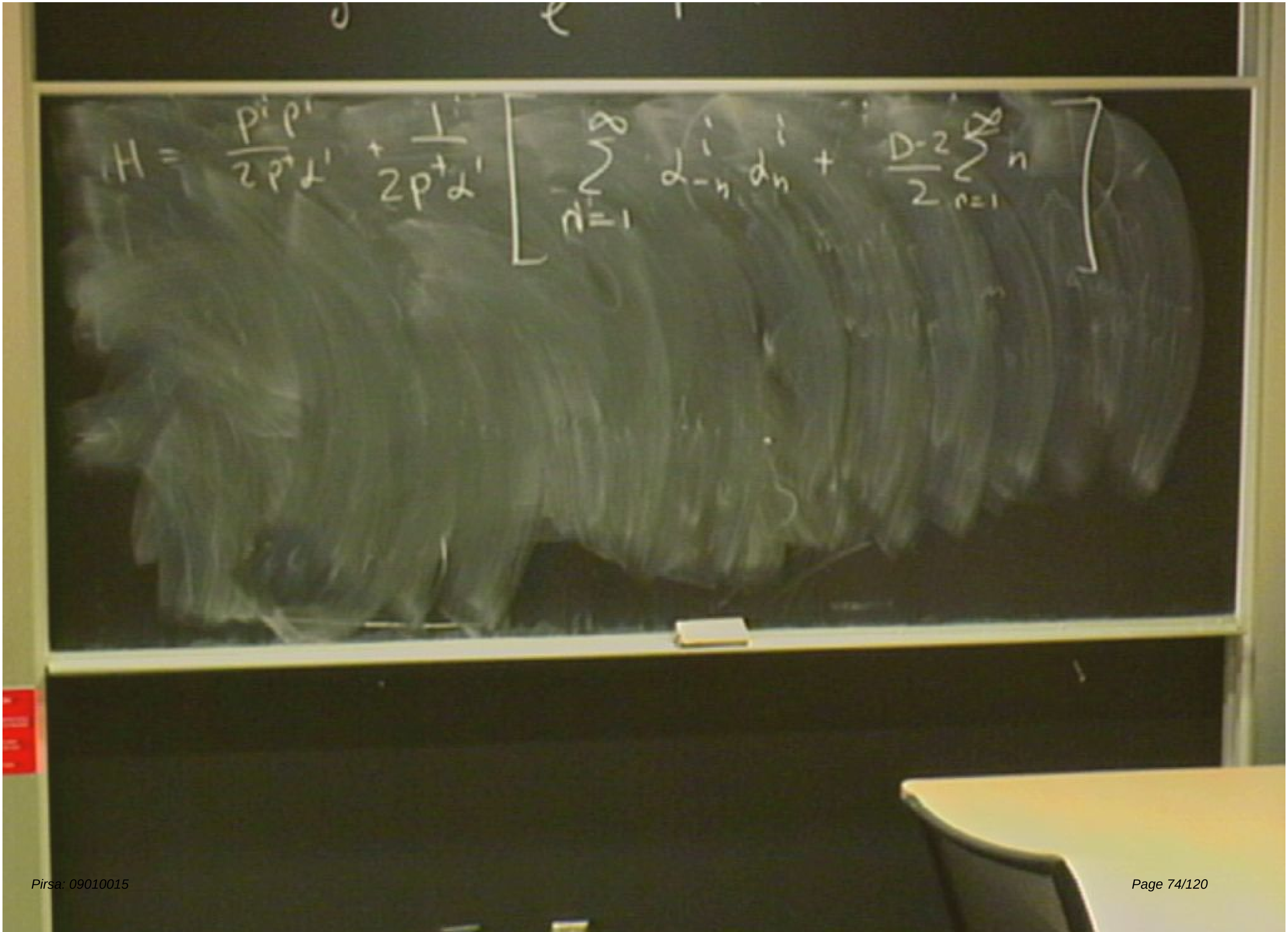
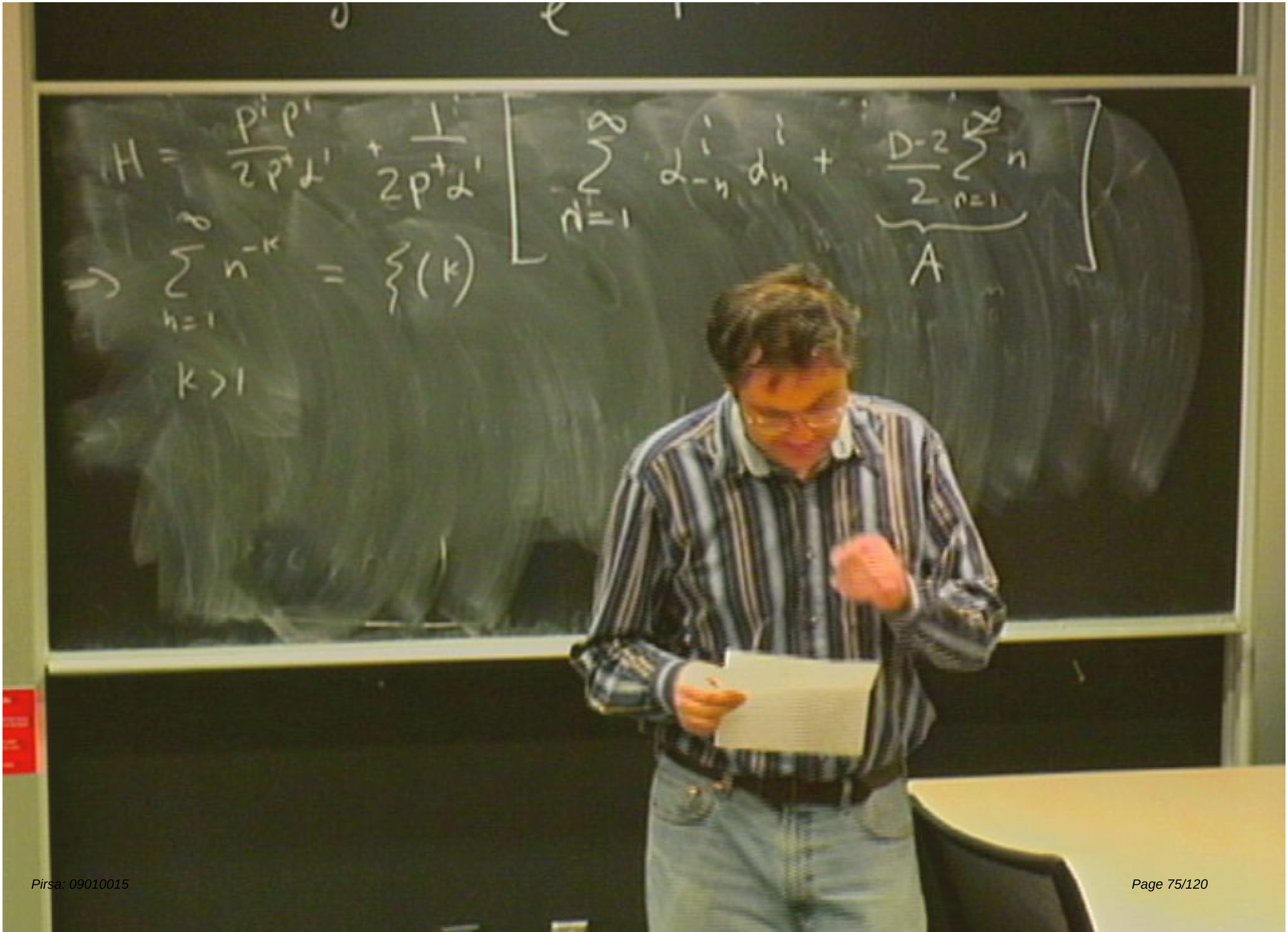

$H = \frac{p^i p^i}{2p^+ \alpha'} + \frac{1}{2p^+ \alpha'} \left[\sum_{n=1}^{\infty} \alpha^i_{-n} \alpha^i_n + \frac{D-2}{2} \sum_{n=1}^{\infty} n \right]$


$H = \frac{p^i p^i}{2p^+\alpha'} + \frac{1}{2p^+\alpha'}\left[\sum_{n=1}^{\infty} \alpha^i_{-n}\alpha^i_n + \underbrace{\frac{D-2}{2}\sum_{n=1}^{\infty} n}_{A}\right]$
$\Rightarrow \sum_{n=1}^{\infty} n^{-k} = \zeta(k)$
$k > 1$

$$H = \frac{p^i p^i}{2p^+\alpha'} + \frac{1}{2p^+\alpha'}\left[\sum_{n=1}^{\infty} \alpha^i_{-n}\alpha^i_n + \underbrace{\frac{D-2}{2}\sum_{n=1}^{\infty} n}_{A}\right]$$

$$\rightarrow \sum_{n=1}^{\infty} n^{-k} = \zeta(k)$$

$k > 1$ Rieman zeta function

$$A = \frac{D-2}{2}\,\zeta(-1)$$

$$H = \frac{p^i p^i}{2p^+ \alpha'} + \frac{1}{2p^+ \alpha'} \left[\sum_{n=1}^{\infty} \alpha_{-n}^i \alpha_n^i + \underbrace{\frac{D-2}{2} \sum_{n=1}^{\infty} n}_{A} \right]$$

$$\rightarrow \sum_{n=1}^{\infty} n^{-k} = \zeta(k)$$

$k > 1$ Rieman zeta function

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 \dots \quad |x| < 1$$

$$A = \frac{D-2}{2} \zeta(-1)$$

analytical continuation

$$\Rightarrow \sum_{n=1} n^{-k} = \zeta(k)$$

$k > 1$ Rieman zeta function

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 \ldots \quad |x| < 1$$

analytical continuation

$$A = \frac{D-2}{2}\,\zeta(-1) = \frac{D-2}{2}\left(-\frac{1}{12}\right) = \frac{2-D}{24}$$

$$= \frac{D-2}{2}\left(-\frac{1}{12}\right) = \frac{2-D}{24}$$

$$1+2+3+\cdots = -\frac{1}{12}$$

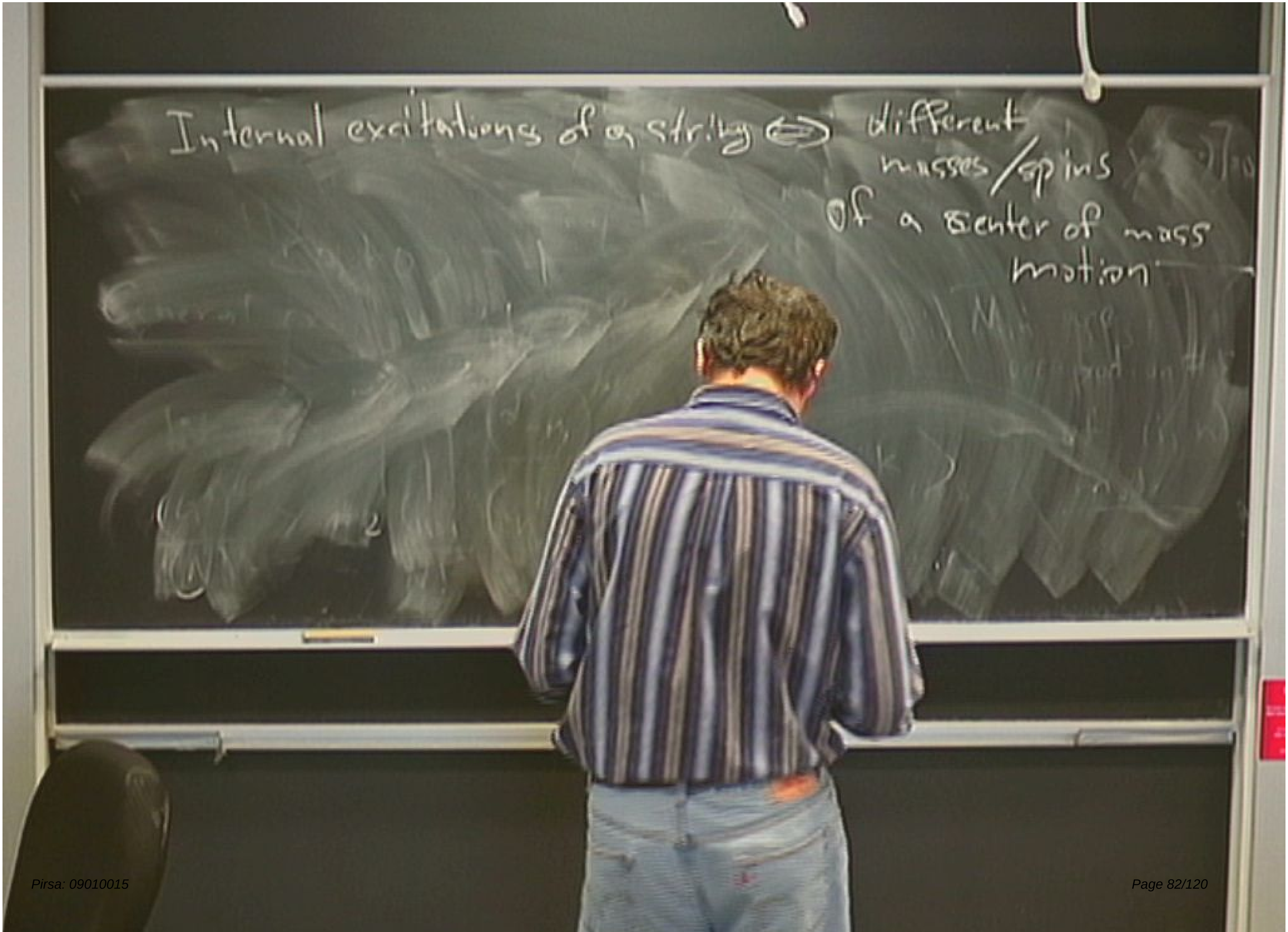
$$\frac{D-2}{2}\left(-\frac{1}{12}\right) = \frac{2-D}{24}$$

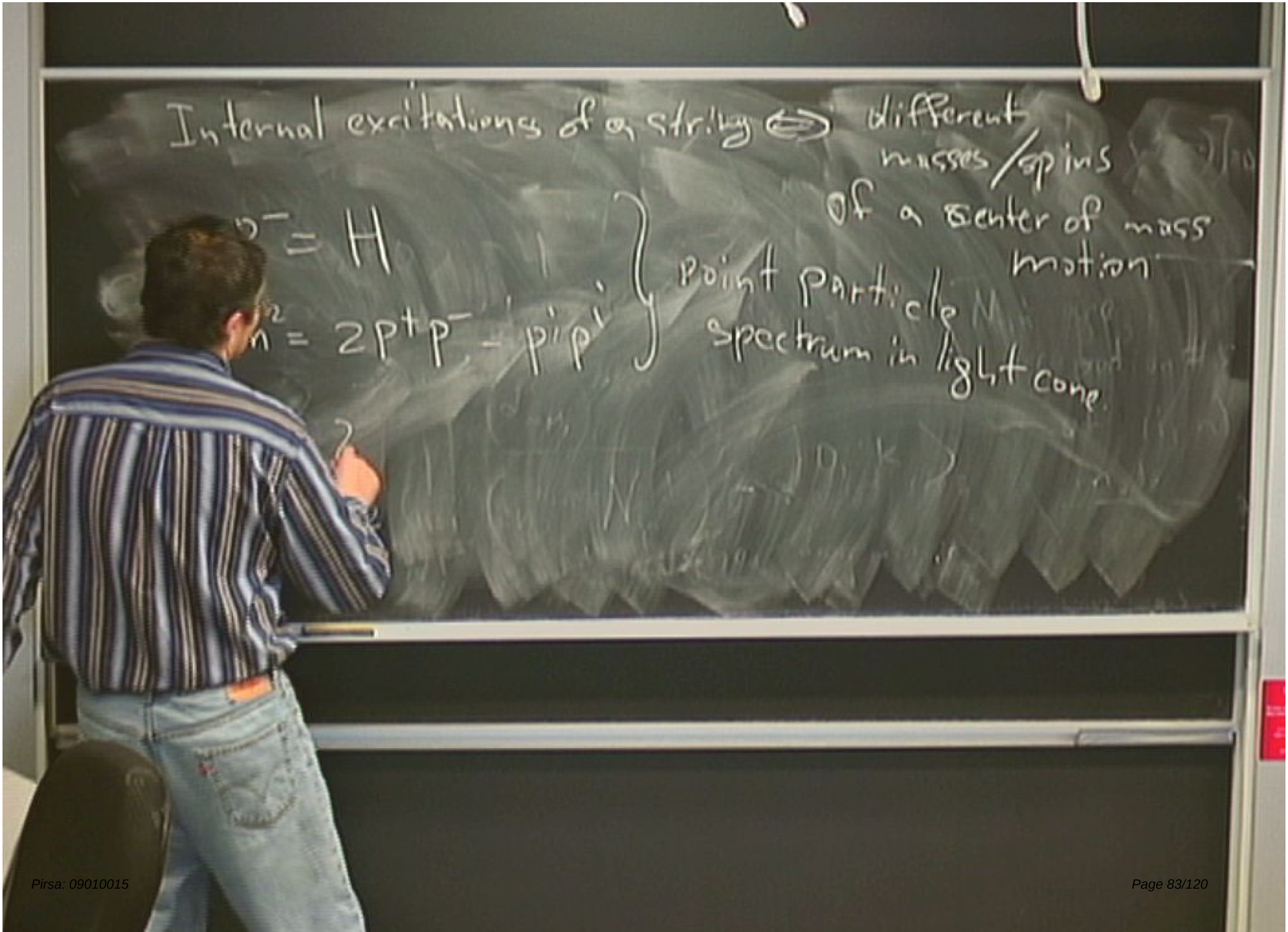
$$1+2+3+\cdots = -\frac{1}{12}$$

$\Rightarrow$ The spectrum of string excitations is

<u>Lorentz inv!</u>


Internal excitations of a string ⟺ different
masses/spins


Internal excitations of a string ⟺ different masses/spins
of a center of mass motion


Internal excitations of a string ⟺ different masses/spins of a center of mass motion
p⁻ = H
m² = 2p⁺p⁻ − pⁱpⁱ
point particle spectrum in light cone

Internal excitations of a string $\Leftrightarrow$ different masses / spins of a center of mass motion

$$\left.\begin{aligned} p^- &= H \\ m^2 &= 2p^+p^- - p^ip^i \end{aligned}\right\} \text{point particle spectrum in light cone}$$

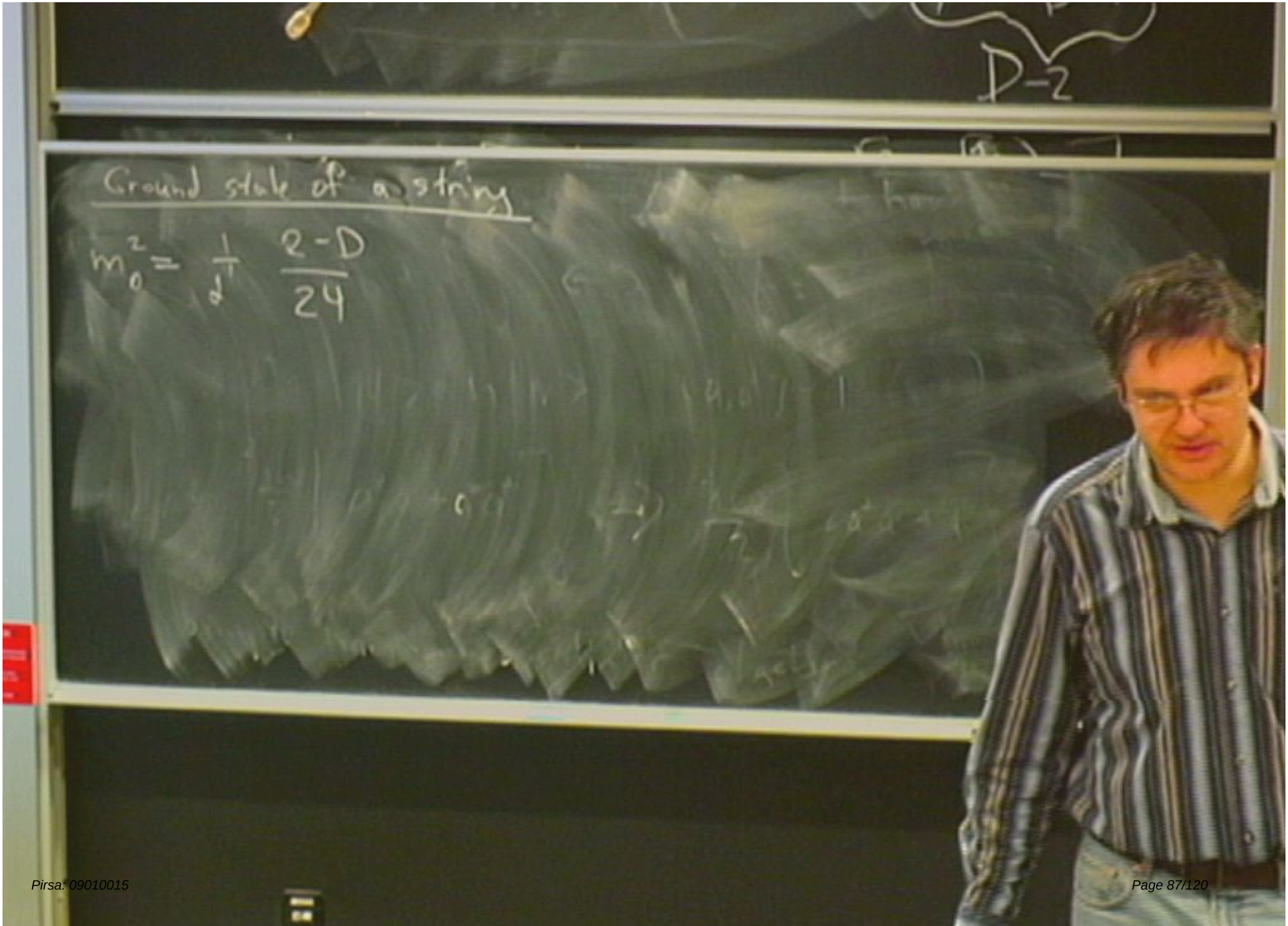
$$= 2p^+H - p^ip^i$$

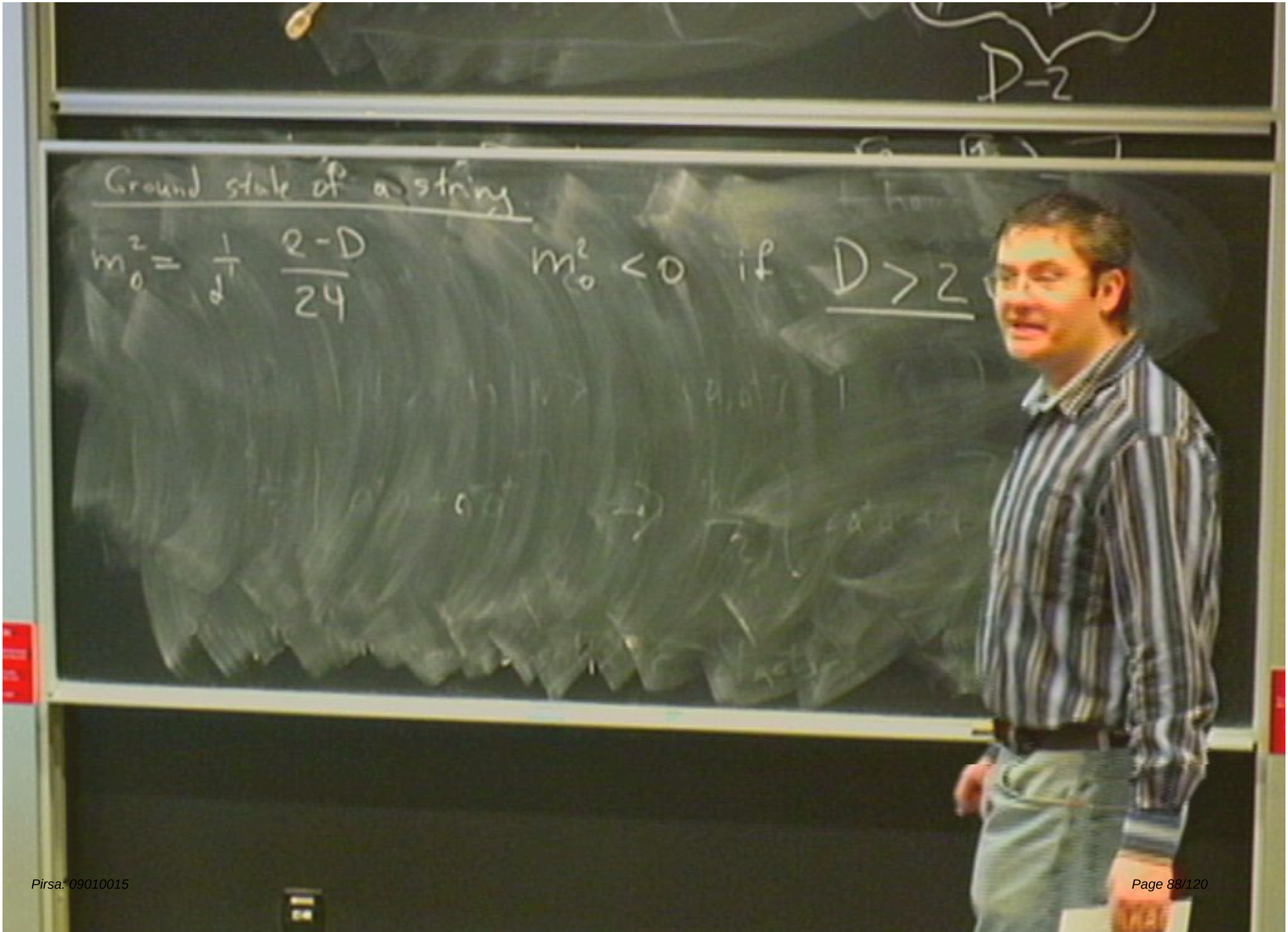
$$m^2 = \frac{1}{\alpha'}\left(N + \frac{2-D}{24}\right)$$

$$= \sum_{i=2}^{D-1} \sum_{n=1}^{\infty} \underbrace{n \, N_{in}}_{\text{eigenvalues of } \alpha^i_{-n}\alpha^i_n}$$

$$m^2 = \frac{1}{\alpha'}\left(N + \frac{2-D}{24}\right)$$

$$N = \sum_{i=2}^{D-1} \sum_{n=1}^{\infty} \underbrace{n\, N_{in}}_{\text{eigenvalues of } \alpha_{-n}^{i} \alpha_{n}^{i}}$$


D-2
Ground state of a string
$m_0^2 = \frac{1}{\alpha'} \frac{2-D}{24}$


D-2
Ground state of a string
$m_0^2 = \frac{1}{\alpha'} \frac{2-D}{24}$
$m_0^2 < 0$ if $D > 2$

$D-2$

Ground state of a string

$m_0^2 = \frac{1}{\alpha'} \frac{2-D}{24}$ $\quad m_0^2 < 0$ if $\underline{D > 2}$

a QFT

$D-2$

Ground state of a string

$$m_0^2 = \frac{1}{\alpha'}\,\frac{2-D}{24} \qquad m_0^2 < 0 \text{ if } \underline{D > 2}$$

a QFT

$$\mathcal{L} = -\frac{1}{2}(\partial\varphi)^2 - V(\varphi)$$

$D-2$

Ground state of a string

$$m_0^2 = \frac{1}{\alpha'}\,\frac{2-D}{24} \qquad m_0^2 < 0 \text{ if } \underline{D>2}$$

a QFT

$$\mathcal{L} = -\frac{1}{2}(\partial\varphi)^2 - V(\varphi) \qquad V(\varphi) \text{ better be bounded from below.}$$

Ground state of a string

$$m_0^2 = \frac{1}{\alpha'}\,\frac{2-D}{24} \qquad m_0^2 < 0 \text{ if } \underline{D>2}$$

a QFT

$$\mathcal{L} = -\frac{1}{2}(\partial\varphi)^2 - V(\varphi)$$

$V(\varphi)$ better be bounded from below.

$$V(\varphi) = \frac{m^2\varphi^2}{2}$$

Ground state of a string

$$m_0^2 = \frac{1}{\alpha'} \frac{2-D}{24}$$ a tachyon

$m_0^2 < 0$ if $D > 2$

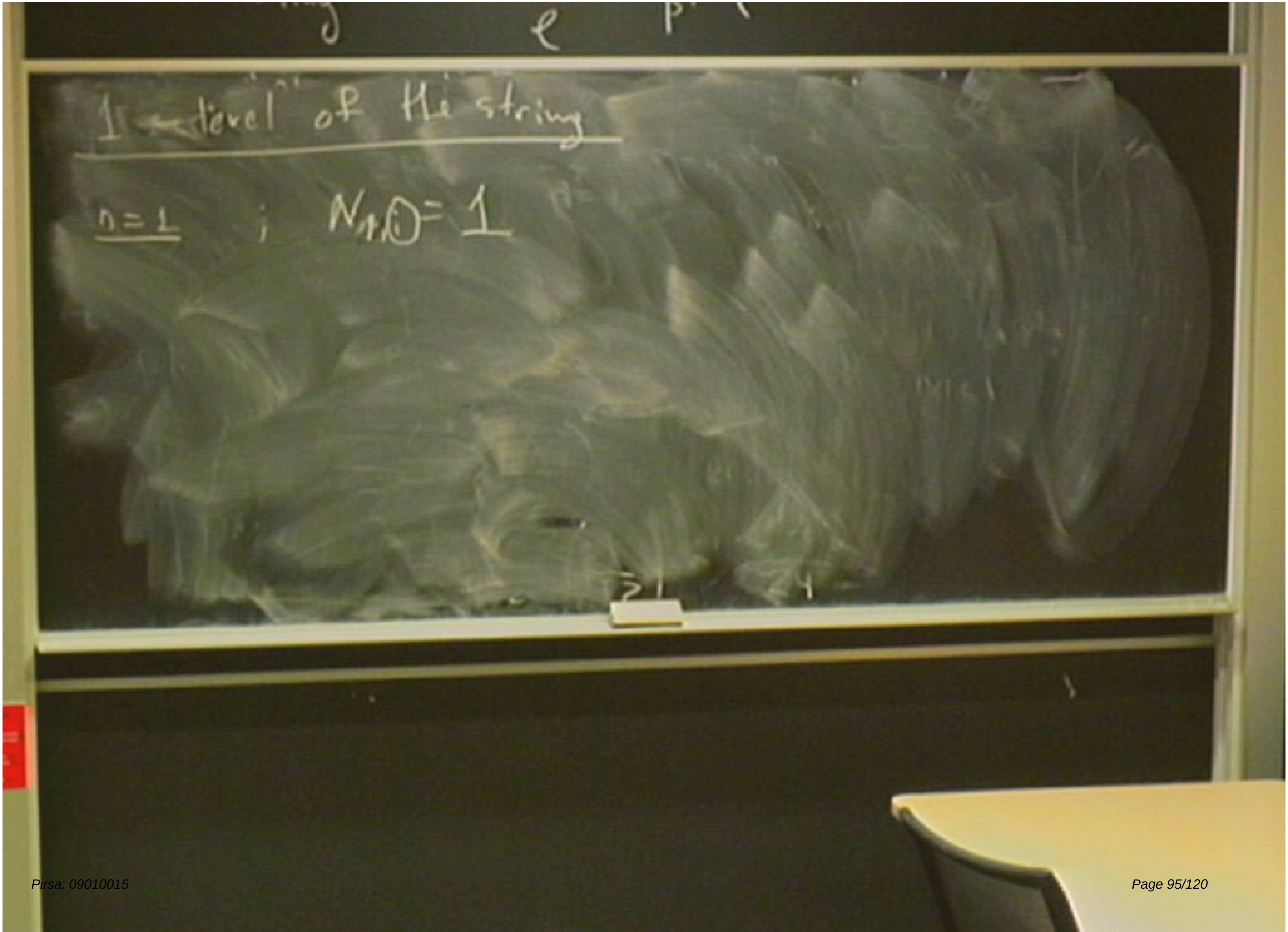
a QFT

$$\mathcal{L} = -\frac{1}{2}(\partial\varphi)^2 - V(\varphi)$$

$V(\varphi)$ better be bounded from below.

$$V(\varphi) = \frac{m^2\varphi^2}{2}$$


1st level of the string
n = 1


1st "level" of the string
n = 1 ; N1,1 = 1

For a Lorentz inv theory spectrum of excitations must form a repr of $SO(D_i-1,1)$

$\Rightarrow$ no constraint from o-level (singlet)

For a Lorentz inv theory spectrum of excitations must form a repr of $SO(D_0-1,1)$

$\Rightarrow$ no constraint from 0-level (singlet)

1-level

$\Rightarrow$

$\underline{n=1}$; $N_{1,D} = 1$

$$m_1^2 = \frac{1}{\alpha'}\left[1 + \frac{2-D}{24}\right] = \frac{1}{\alpha'}\,\frac{[26-D]}{24}$$

$\Rightarrow$ no constraint from 0-level (singlet)

1-level

$\Rightarrow$

$\underline{n=1}$; $N_{1,(1)} = 1$

$$m_1^2 = \frac{1}{\alpha'}\left[1 + \frac{2-D}{24}\right] = \frac{1}{\alpha'}\frac{[26-D]}{24}$$

degeneracy = $D-2$

$\Rightarrow$ no constraint from 0-level (singlet)

1-level

$\Rightarrow$

$$m_1^2 = \frac{1}{\alpha'}\left[1 + \frac{2-D}{24}\right] = \frac{1}{\alpha'}\frac{[26-D]}{24}$$

degeneracy $= D-2$

$\Rightarrow$ excitations must form a repr of $SO(D-1,1)$

$\Rightarrow$ no constraint from 0-level (singlet)

1-level

$\Rightarrow$ we require that there is a representation

$SO(D-1,1)$ with

$$m_1^2 = \frac{1}{\alpha'}\left[1 + \frac{2-D}{24}\right] = \frac{1}{\alpha'}\frac{\left[26-D\right]}{24}$$

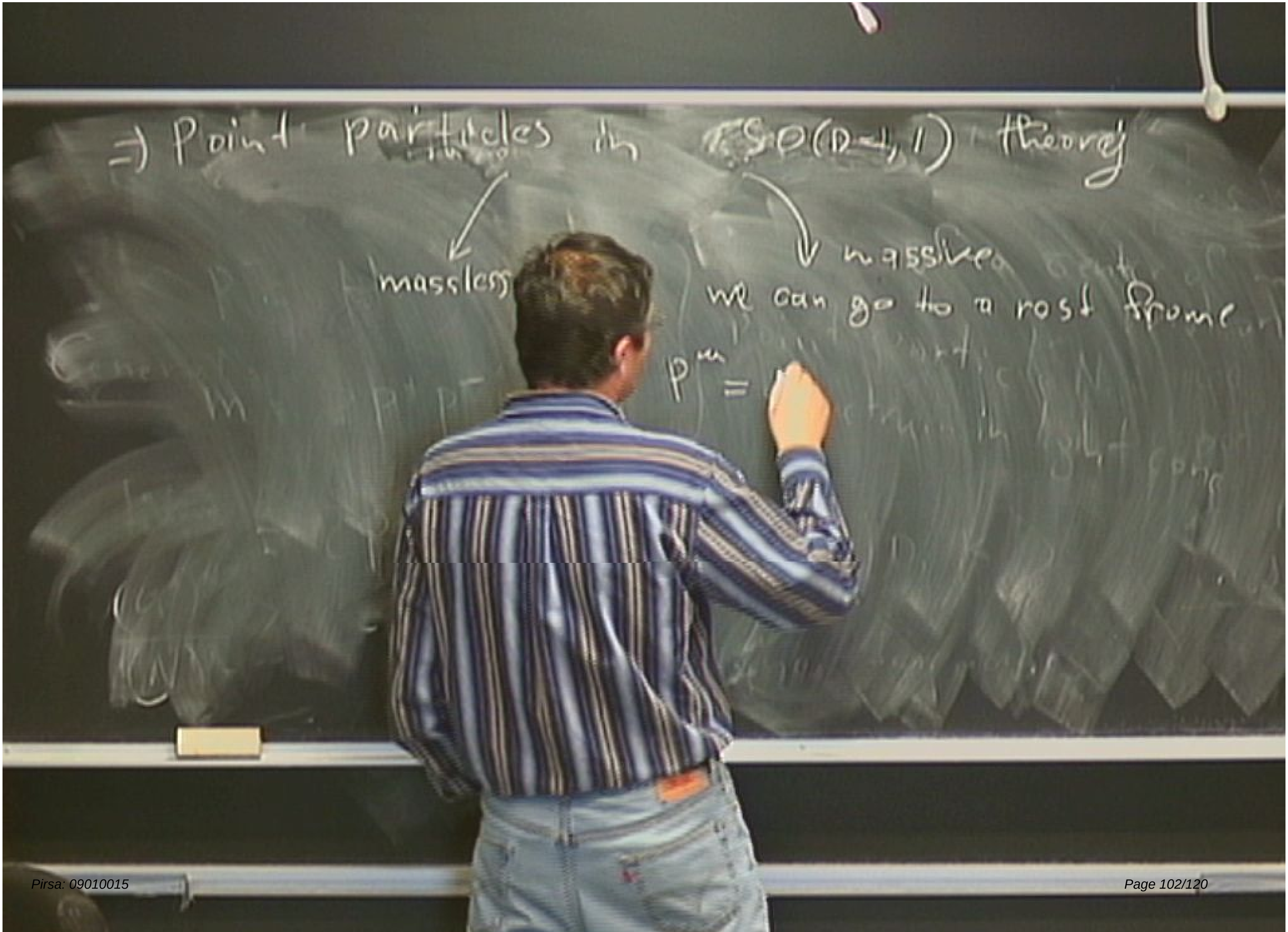
degeneracy $= D-2$

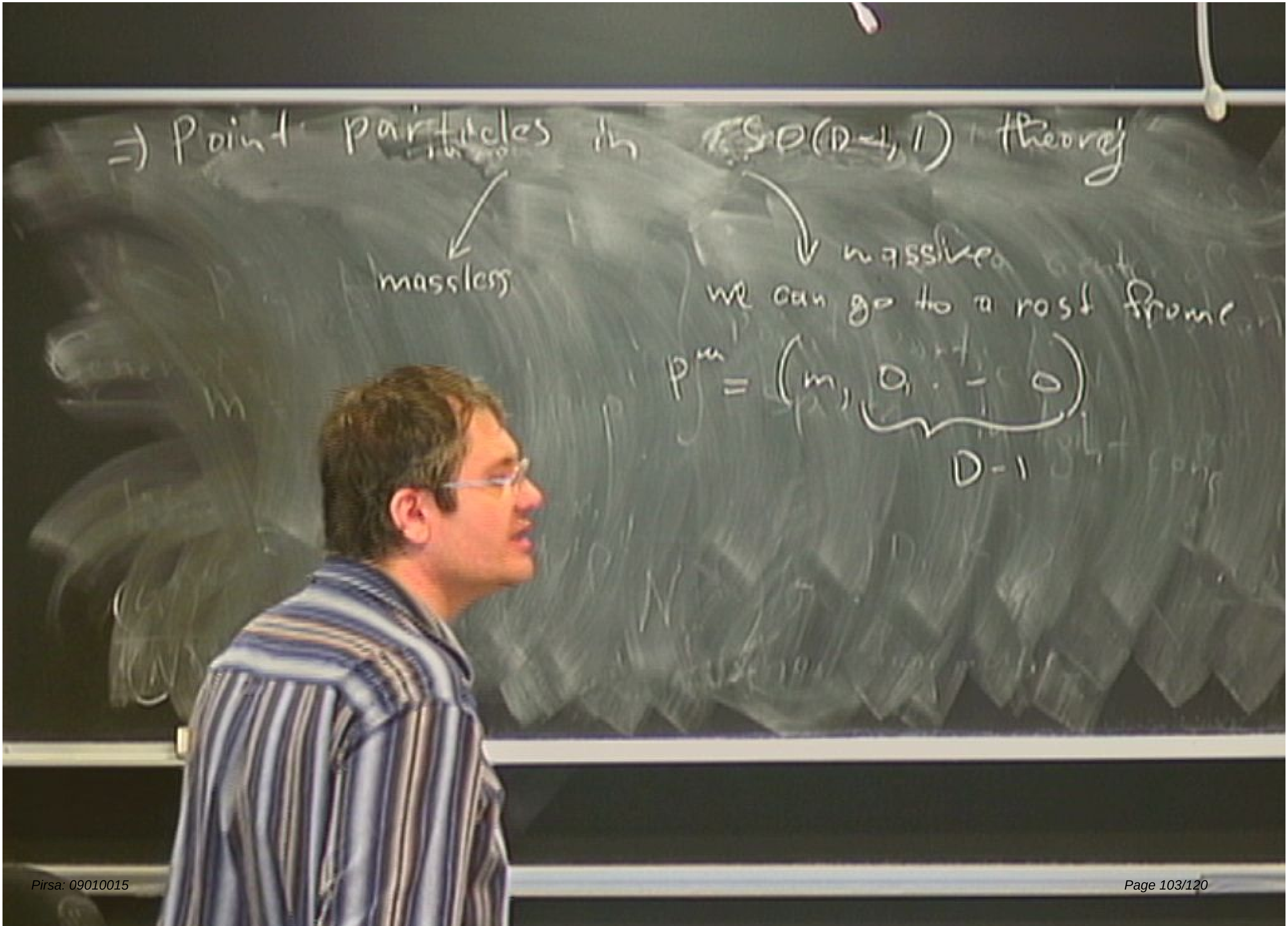
excitations must form a repr of $SO(D-1,1)$

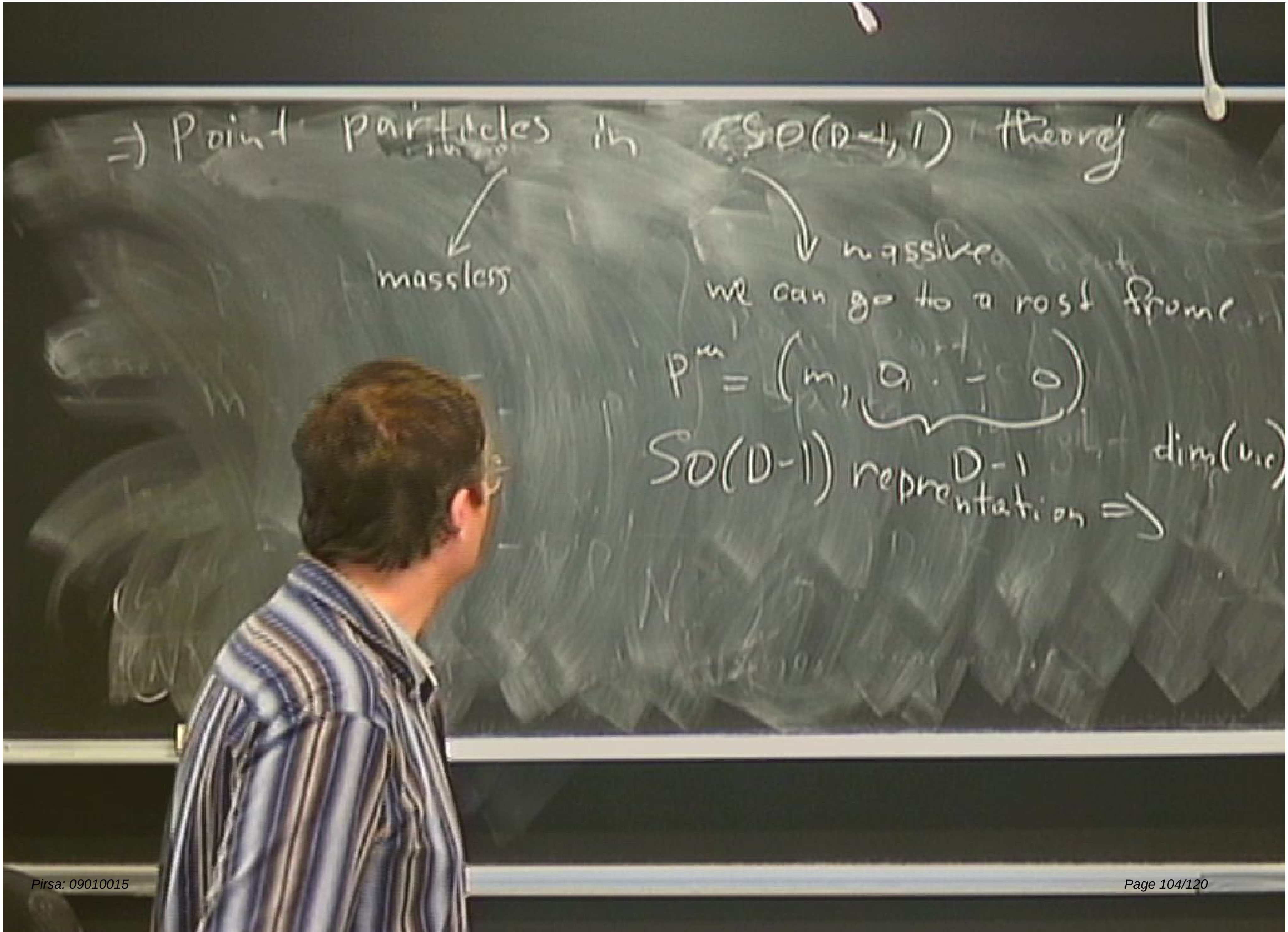
$\Rightarrow$ no constraint from 0-level (singlet)

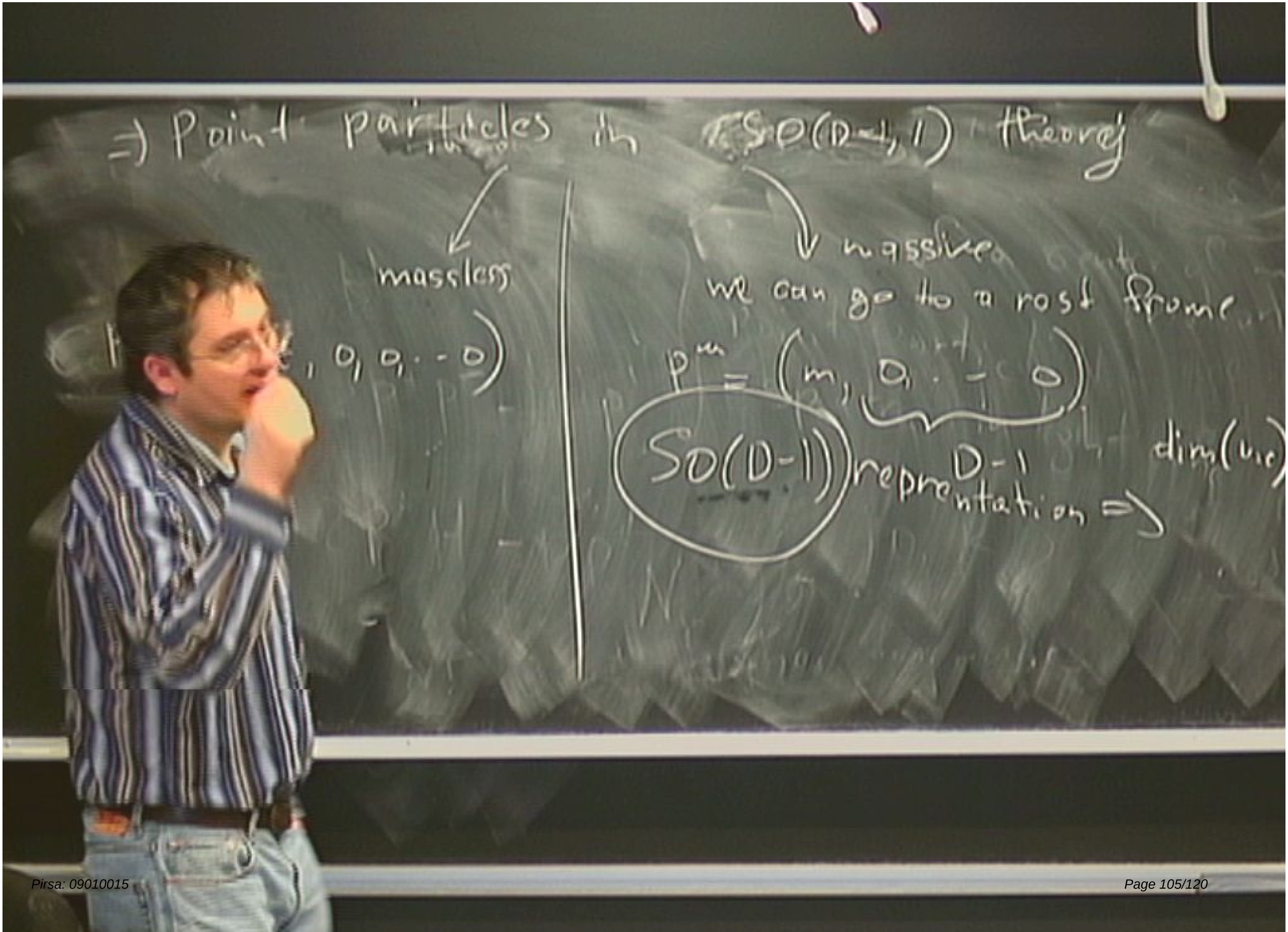
1-level

$\Rightarrow$ we require that there is a representation r of $SO(D-1,1)$ with $\dim(r) = D-2$


⇒ Point particles in SO(D-1,1) theory
massless
massive
we can go to a rest frame
p^μ =


⇒ Point particles in SO(D-1,1) theory
massless
massive
we can go to a rest frame
$p^\mu = (m, \underbrace{0, \cdots, 0}_{D-1})$


⇒ Point particles in SO(D-1,1) theory
massless
massive
we can go to a rest frame
p^μ = (m, 0, ... 0)
D-1
SO(D-1) representation ⇒
dim(


⇒ Point particles in SO(D-1,1) theory
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we can go to a rest frame
$p^\mu = (m, 0, \cdots 0)$
D-1
SO(D-1) representation ⇒

$\Rightarrow$ Point particles in $SO(D-1,1)$ theory

massless

$$p^\mu = (E, -E, \underbrace{0, 0, \cdots 0})$$

$SO(D-1,1) \supset$

massive

we can go to a rest frame

$$p^\mu = (m, \underbrace{0, \cdots 0})$$

$SO(D-1)$ representation $\Rightarrow$ $D-1$

dim(v.c)

$\Rightarrow$ Point particles in $SO(D-1,1)$ theory

massless

$$p^\mu = (E, -E, \underbrace{0, 0, \cdots 0})$$

$$SO(D-1,1) \supset SO(D-2)$$

$$\dim(\mathrm{vec}\,V) = \underline{\underline{D-2}}$$

massive

we can go to a rest frame

$$p^\mu = (m, \underbrace{0, \cdots 0})$$

$SO(D-1)$ representation $\overset{D-1}{}$ $\Rightarrow$ $\dim(\mathrm{vec})$

$\Rightarrow$ Because at level 1 we have $(D-2)$ states

$\Rightarrow$ $m_1^2 = 0$

⇒ Because at level 1 we have (D−2) states

⇒ $m_1^2 = 0 = \frac{1}{2}[26 - D] = 0 \Rightarrow \boxed{D = 26}$

$\Rightarrow$ Point particles in $SO(D-1,1)$ theory

massless

$$p^\mu = \Big(E, -E, \underbrace{0, 0, \cdots 0}\Big)$$

$$SO(D-1,1) \supset SO(D-2)$$

$$\dim(\text{vec}V) = D-2$$

massive

we can go to a rest frame

$$p^\mu = \Big(m, \underbrace{0, \cdots 0}\Big)$$

$SO(D-1)$ representation $\Rightarrow$ $D-1$ $\dim(\text{vec})$

$\Rightarrow$ Point particles in $SO(D-1,1)$ theory

massless

$$p^\mu = (E, -E, \underbrace{0, 0, \cdots 0})$$

$$SO(D-1,1) \supset SO(D-2)$$

$$\dim(\text{vec}\,V) = D-2$$

massive

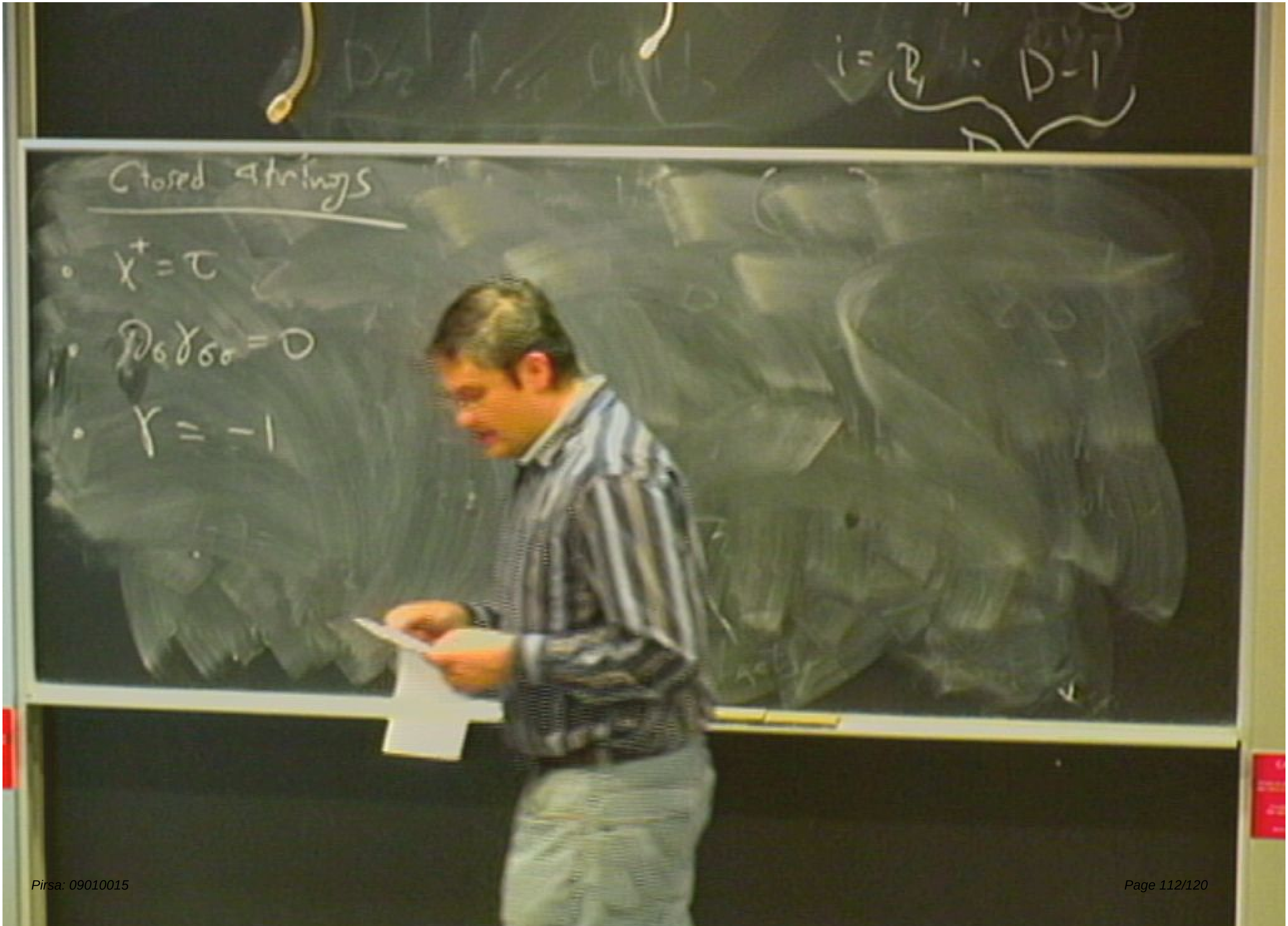
we can go to a rest frame

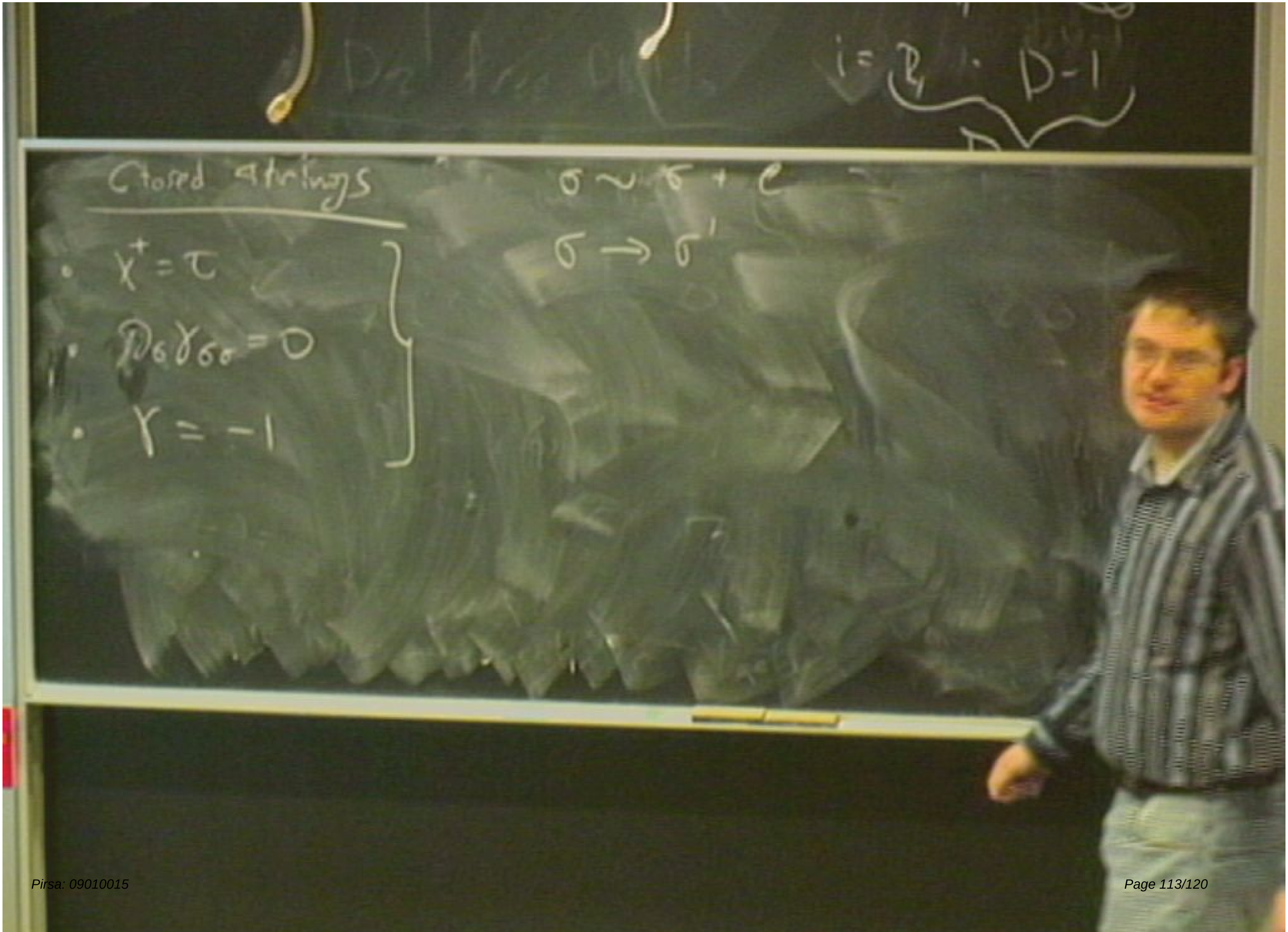
$$p^\mu = (m, \underbrace{0, \cdots 0})$$

$SO(D-1)$ representation $\Rightarrow$ $D-1$

$\dim(\text{v.c}$

A_μ


Closed strings
$X^+ = \tau$
$\partial_\sigma \gamma_{\sigma\sigma} = 0$
$\gamma = -1$
$i = 2, \ldots, D-1$


i = 2, .. D-1
Closed strings
σ ~ σ + c
σ → σ'
X+ = τ
∂σ γσσ = 0
γ = -1

$i = 2, \dots, D-1$

Closed strings

$\sigma \sim \sigma + c$

$\sigma \to \sigma' = \sigma + s(\tau)$

- $X^+ = \tau$
- $\partial_\sigma \gamma_{\sigma\sigma} = 0$
- $\gamma = -1$

$i = 2, \ldots, D-1$

Closed strings

- $X^+ = \tau$
- $\partial_\sigma \gamma_{\sigma\sigma} = 0$
- $\gamma = -1$

$\sigma \sim \sigma + c$

$\underbrace{\sigma \to \sigma' = \sigma + s(\tau)}_{\text{left-over symmetry}}$

open

$$X^i(\tau,\sigma) = x^i + \frac{p^i}{p^+}\tau + i(2\alpha')^{1/2}\sum_{\substack{n=-\infty \\ n\neq 0}}^{+\infty} \frac{1}{n}\alpha_n^i e^{-\frac{i\pi n c\tau}{\ell}} \cos\frac{\pi n\sigma}{\ell}$$

open

$$X^i(\tau,\sigma) = x^i + \frac{p^i}{p^+}\tau + i(2\alpha')^{1/2}\sum_{\substack{n=-\infty \\ n\neq 0}}^{+\infty} \frac{1}{n}\alpha_n^i \, e^{-\frac{i\pi n c\tau}{\ell}} \cos\frac{\pi n\sigma}{\ell}$$

closed

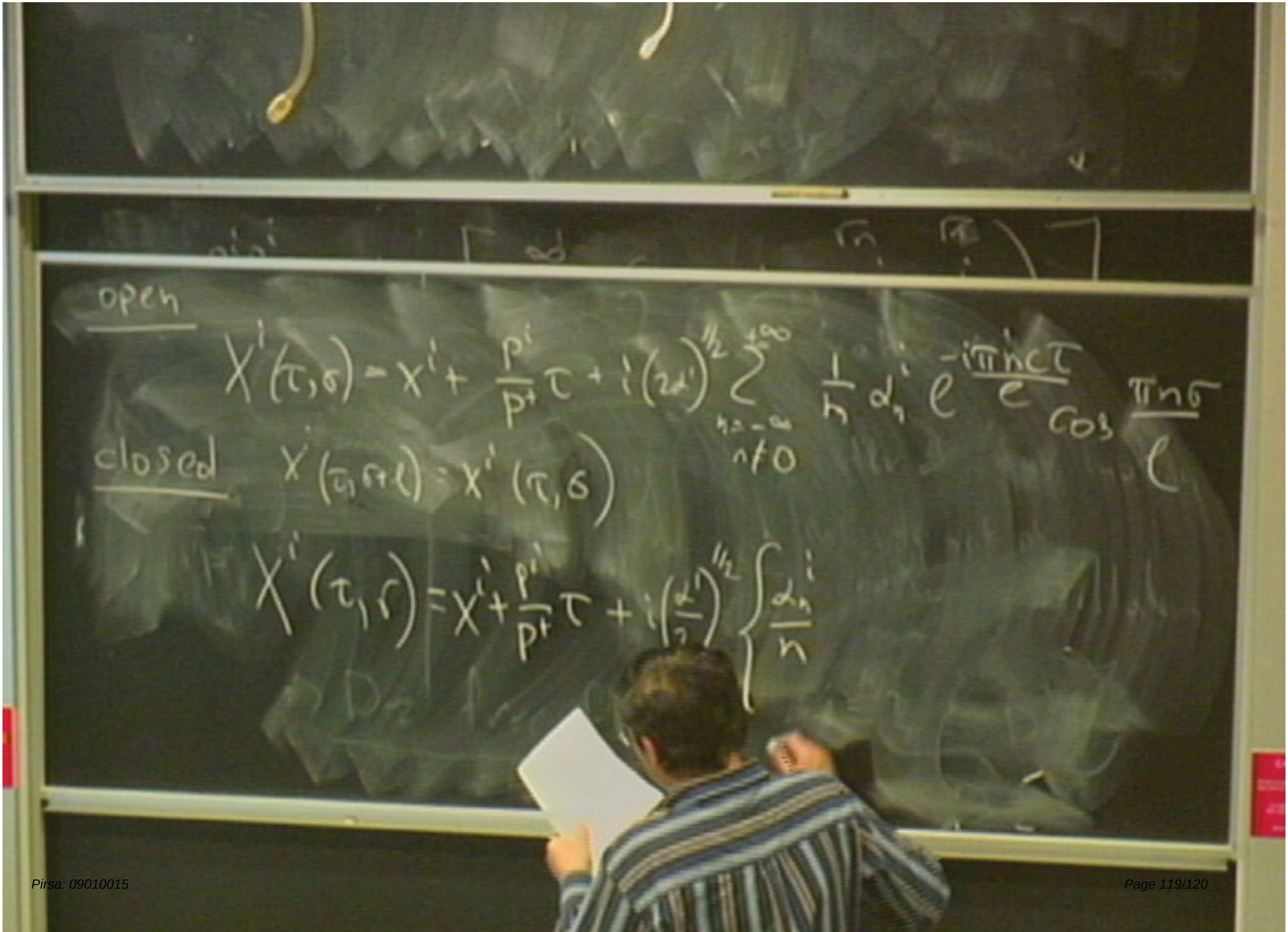
$$X^i(\tau,\sigma) =$$

open

$$X^i(\tau,\sigma) = x^i + \frac{p^i}{p^+}\tau + i(2\alpha')^{1/2}\sum_{\substack{n=-\infty \\ n\neq 0}}^{+\infty} \frac{1}{n}\alpha_n^i e^{-\frac{i\pi n c\tau}{\ell}} \cos\frac{\pi n\sigma}{\ell}$$

closed

$$X^i(\tau,\sigma) = x^i + \frac{p^i}{p^+}\tau$$


open
$X^i(\tau,\sigma) = x^i + \frac{p^i}{p^+}\tau + i(2\alpha')^{1/2}\sum_{n=-\infty,\, n\neq 0}^{+\infty} \frac{1}{n}\alpha_n^i e^{-\frac{i\pi n c\tau}{\ell}} \cos\frac{\pi n\sigma}{\ell}$
closed
$X^i(\tau,\sigma+\ell) = X^i(\tau,\sigma)$
$X^i(\tau,\sigma) = x^i + \frac{p^i}{p^+}\tau + i\left(\frac{\alpha'}{2}\right)^{1/2}\left\{\frac{\alpha_n^i}{n}\right.$

open

$$X^i(\tau,\sigma) = x^i + \frac{p^i}{p^+}\tau + i(2\alpha')^{1/2}\sum_{\substack{n=-\infty \\ n\neq 0}}^{+\infty} \frac{1}{n}\alpha_n^i\, e^{-\frac{i\pi n c\tau}{\ell}}\cos\frac{\pi n\sigma}{\ell}$$

closed

$$X^i(\tau,\sigma+\ell) = X^i(\tau,\sigma)$$

$$X^i(\tau,\sigma) = x^i + \frac{p^i}{p^+}\tau + i\left(\frac{\alpha'}{2}\right)^{1/2}\left\{\frac{\alpha_n^i}{n}\, e^{\frac{2\pi i n(\sigma - c\tau)}{\ell}} + \frac{\tilde{\alpha}_n^i}{n}\, e^{-\frac{2\pi i n}{\ell}(\sigma + c\tau)}\right.$$