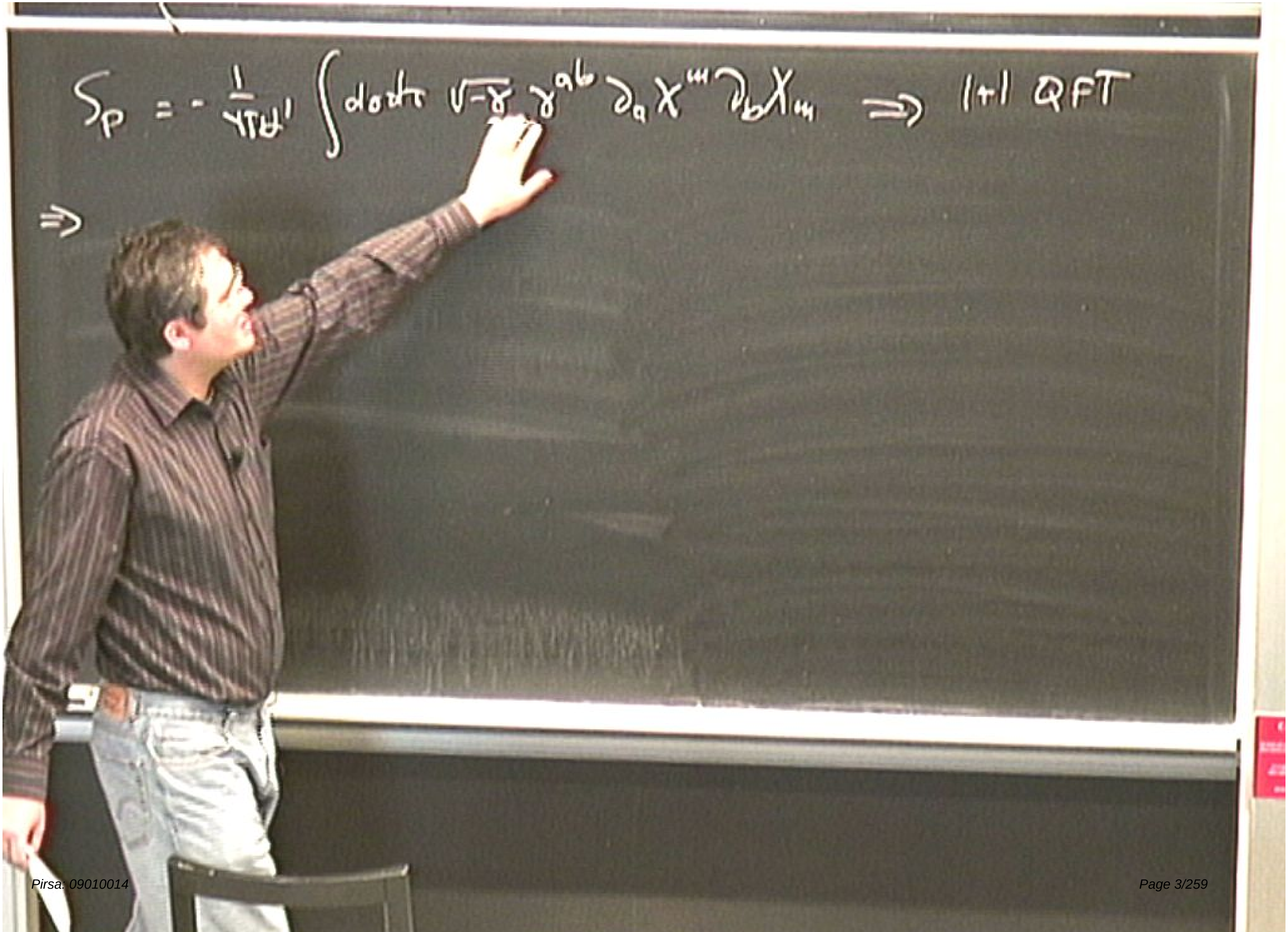
Title: Introduction to the Bosonic String

Date: Jan 23, 2009 10:00 AM

URL: http://pirsa.org/09010014

Abstract: This course provides a thorough introduction to the bosonic string based on the Polyakov path integral and conformal field theory. We introduce central ideas of string theory, the tools of conformal field theory, the Polyakov path integral, and the covariant quantization of the string. We discuss string interactions and cover the tree-level and one loop amplitudes. More advanced topics such as T-duality and D-branes will be taught as part of the course. The course is geared for M.Sc. and Ph.D. students enrolled in Collaborative Ph.D. Program in Theoretical Physics. Required previous course work: Quantum Field Theory (AM516 or equivalent). The course evaluation will be based on regular problem sets that will be handed in during the term. The primary text is the book: 'String theory. Vol. 1: An introduction to the bosonic string. J. Polchinski (Santa Barbara, KITP) . 1998. 402pp. Cambridge, UK: Univ. Pr. (1998) 402 p.' All interested students should contact Alex Buchel at abuchel@uwo.ca as soon as possible.

$$S_P = -\frac{1}{4\pi\alpha'}\int d\sigma\, d\tau\, \sqrt{-\gamma}\,\gamma^{ab}\,\partial_a X^{\mu}\,\partial_b X_{\mu}$$


$S_P = -\frac{1}{4\pi\alpha'}\int d\sigma d\tau \sqrt{-\gamma}\,\gamma^{ab}\partial_a X^\mu \partial_b X_\mu \Rightarrow$ 1+1 QFT
$\Rightarrow$

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$\Rightarrow$

$$S_P = -\frac{1}{4\pi\alpha'}\int d\sigma\, d\tau\, \sqrt{-\gamma}\, \gamma^{ab}\, \partial_a X^{\mu}\, \partial_b X_{\mu} \implies 1+1 \text{ QFT}$$

$$\implies T^{ab} = -\frac{4\pi}{\sqrt{-\gamma}}\,\frac{\delta S}{\delta\gamma_{ab}}$$

$$\implies$$

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$\Rightarrow$ 2-dim diffeo inv.

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$$\Rightarrow \quad \gamma_{ab} \to e^{2\omega}\gamma_{ab}$$

$$T_a{}^a = 0$$

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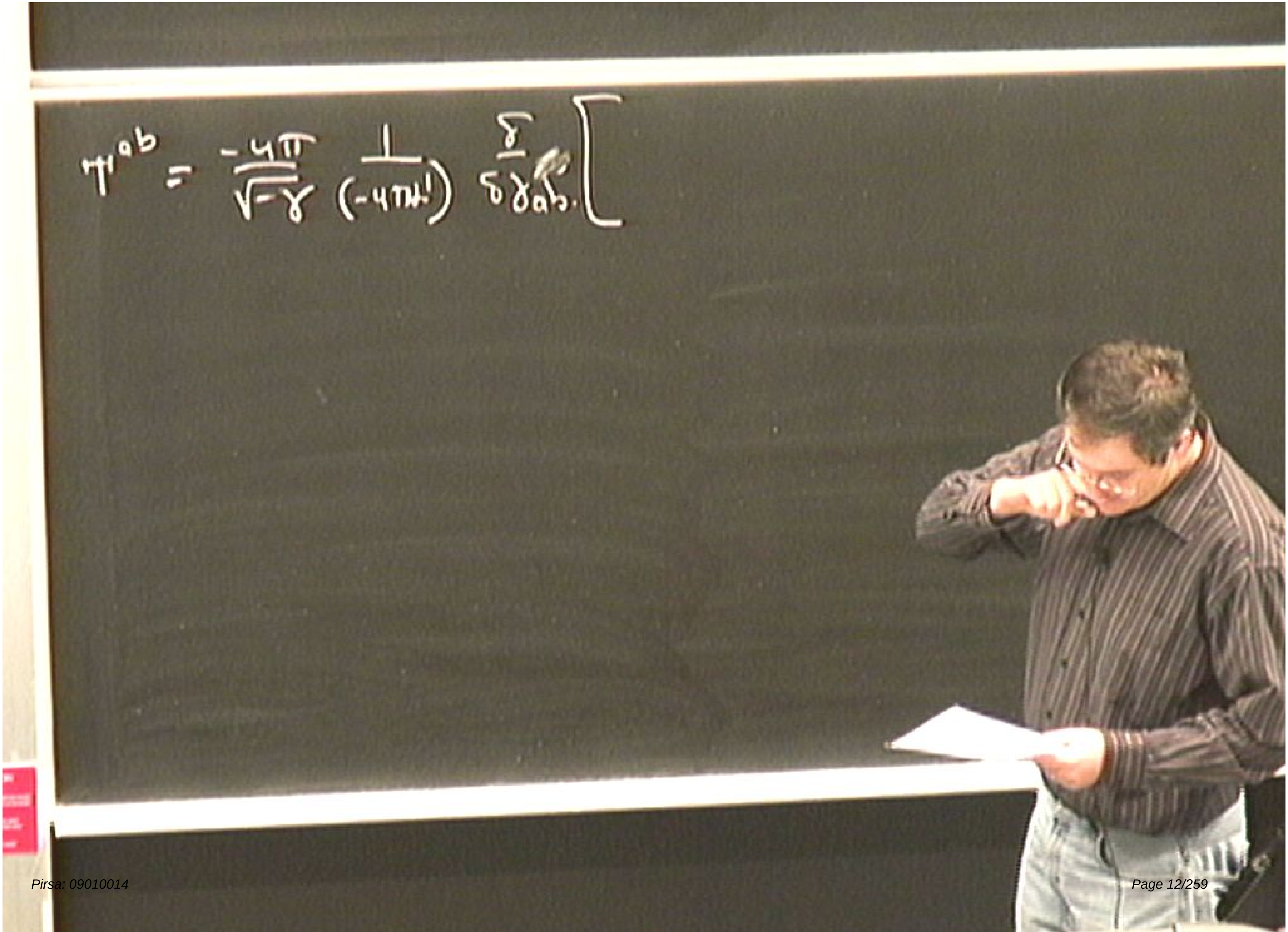
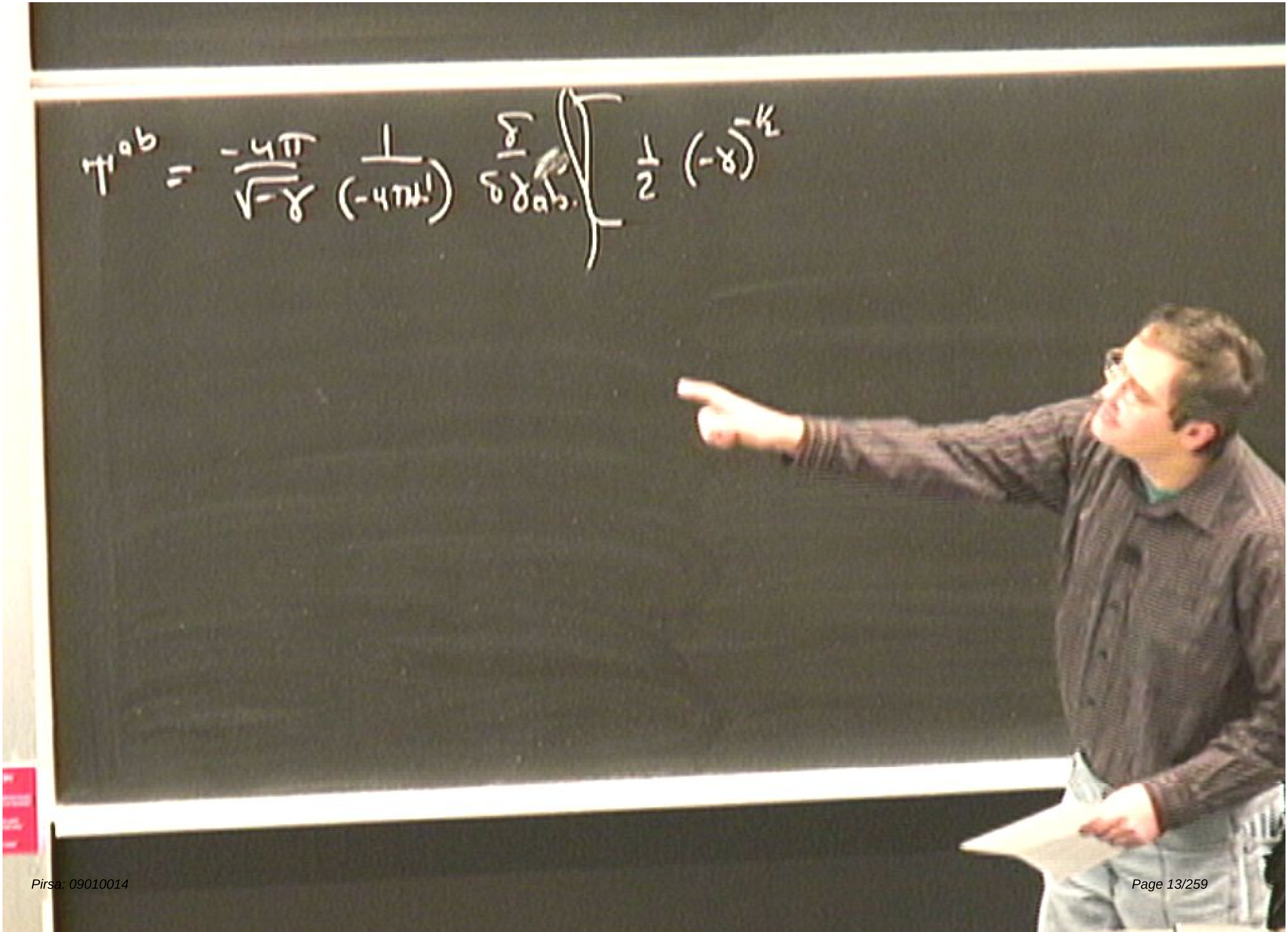

$\pi^{ab} = \frac{-4\pi}{\sqrt{-\gamma}} \frac{1}{(-4\pi\mu!)} \frac{\delta}{\delta\gamma_{ab}} \Big[$



$\pi^{ab} = \frac{-4\pi}{\sqrt{-\gamma}} \frac{1}{(-4\pi i!)} \frac{\delta}{\delta \gamma_{ab}} \int \Big[\frac{1}{2} (-\gamma)^{-1/2}$

$$\pi^{ab} = \frac{-4\pi}{\sqrt{-\gamma}} \frac{1}{(-4\pi i\hbar)} \frac{\delta}{\delta\gamma_{ab}} \left\{ \frac{1}{2} (-\gamma)^{-1/2} \gamma^{ab} \delta\gamma_{ab} \right.$$

$$S_P = -\frac{1}{4\pi\alpha'}\int d\sigma d\tau \sqrt{-\gamma}\,\underbrace{\gamma^{ab}\partial_a X^{\mu}\partial_b X_{\mu}}_{(\partial X)^2} \Rightarrow 1+1 \text{ QFT}$$

$$\Rightarrow \quad T^{ab} = -\frac{4\pi}{\sqrt{-\gamma}}\frac{\delta S}{\delta\gamma_{ab}}$$

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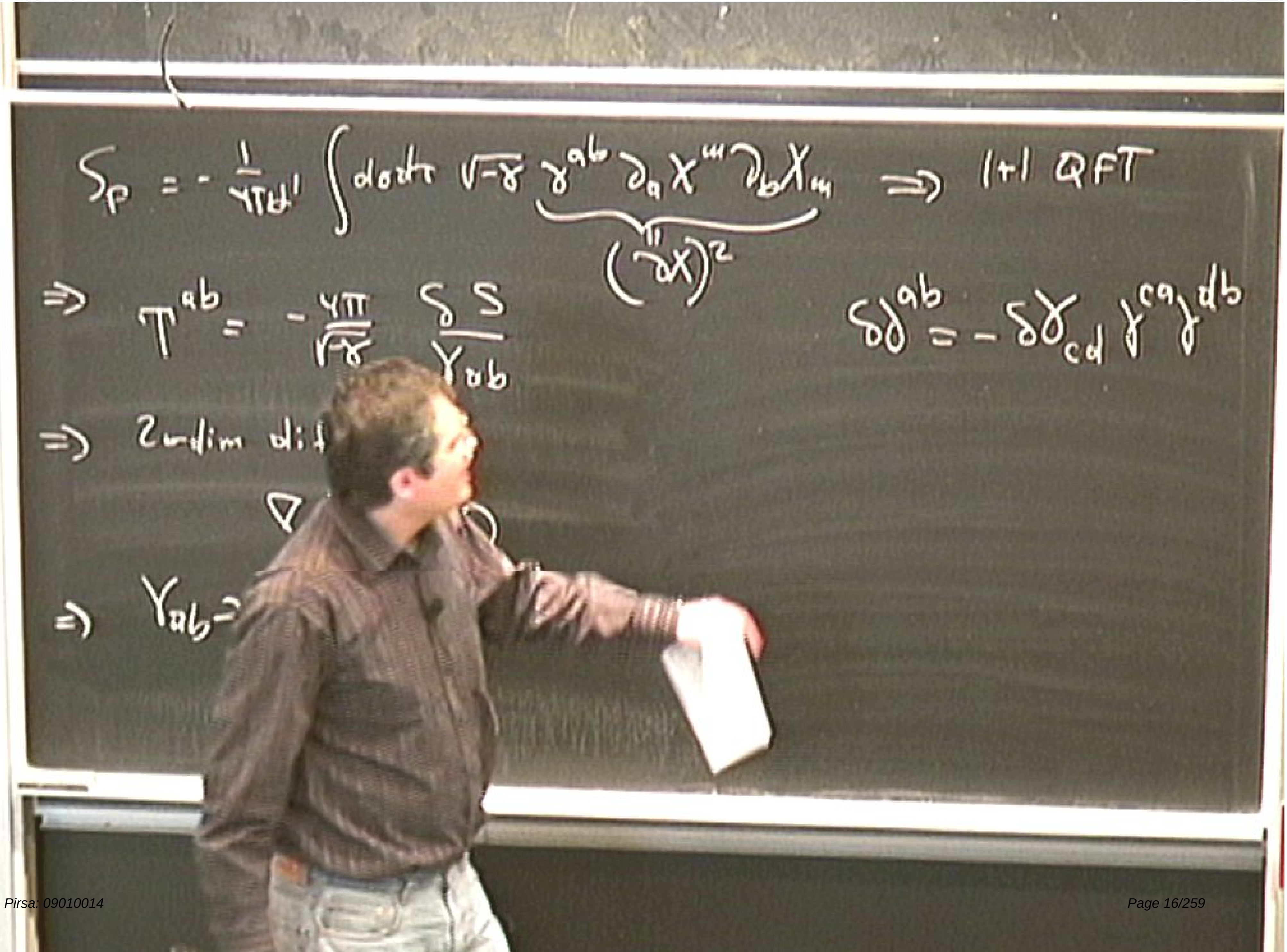
$S_P = -\frac{1}{4\pi\alpha'} \int d\sigma d\tau \sqrt{-\gamma}\, \underbrace{\gamma^{ab} \partial_a X^m \partial_b X_m}_{(\partial X)^2} \Rightarrow$ 1+1 QFT
$\Rightarrow T^{ab} = -\frac{4\pi}{\sqrt{-\gamma}} \frac{\delta S}{\gamma_{ab}}$
$\delta\gamma^{ab} = -\delta\gamma_{cd}\gamma^{ca}\gamma^{db}$
$\Rightarrow$ 2-dim dif
∇
$\Rightarrow \gamma_{ab} =$

$$S_P = -\frac{1}{4\pi\alpha'}\int d\sigma d\tau \sqrt{-\gamma}\, \underbrace{\gamma^{ab}\partial_a X^{\mu}\partial_b X_{\mu}}_{(\partial X)^2} \Rightarrow 1+1 \text{ QFT}$$

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$$\nabla_a T^{ab}\Big|_{X^\mu \text{ satisfies EOM}} = 0$$

$$\Rightarrow \quad \gamma_{ab} \to e^{2\omega}\gamma_{ab}$$

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$$\pi^{ab} = \frac{-4\pi}{\sqrt{-\gamma}} \frac{1}{(-4\pi\lambda')} \frac{\delta}{\delta\gamma_{ab}} \int \left[\frac{1}{2} (-\gamma)^{-1/2} \gamma^{ab} \delta\gamma_{ab} (\partial X)^2 - \sqrt{-\gamma} \right.$$

$$\delta\gamma_{ab}.\int\left[\frac{1}{2}(-\gamma)^{-1/2}\gamma^{ab}\delta\gamma_{ab}(\partial X)^2-\sqrt{-\gamma}\,\partial^a X^{\mu}\partial^b X_{\mu}\,\delta\gamma_{ab}\right]$$

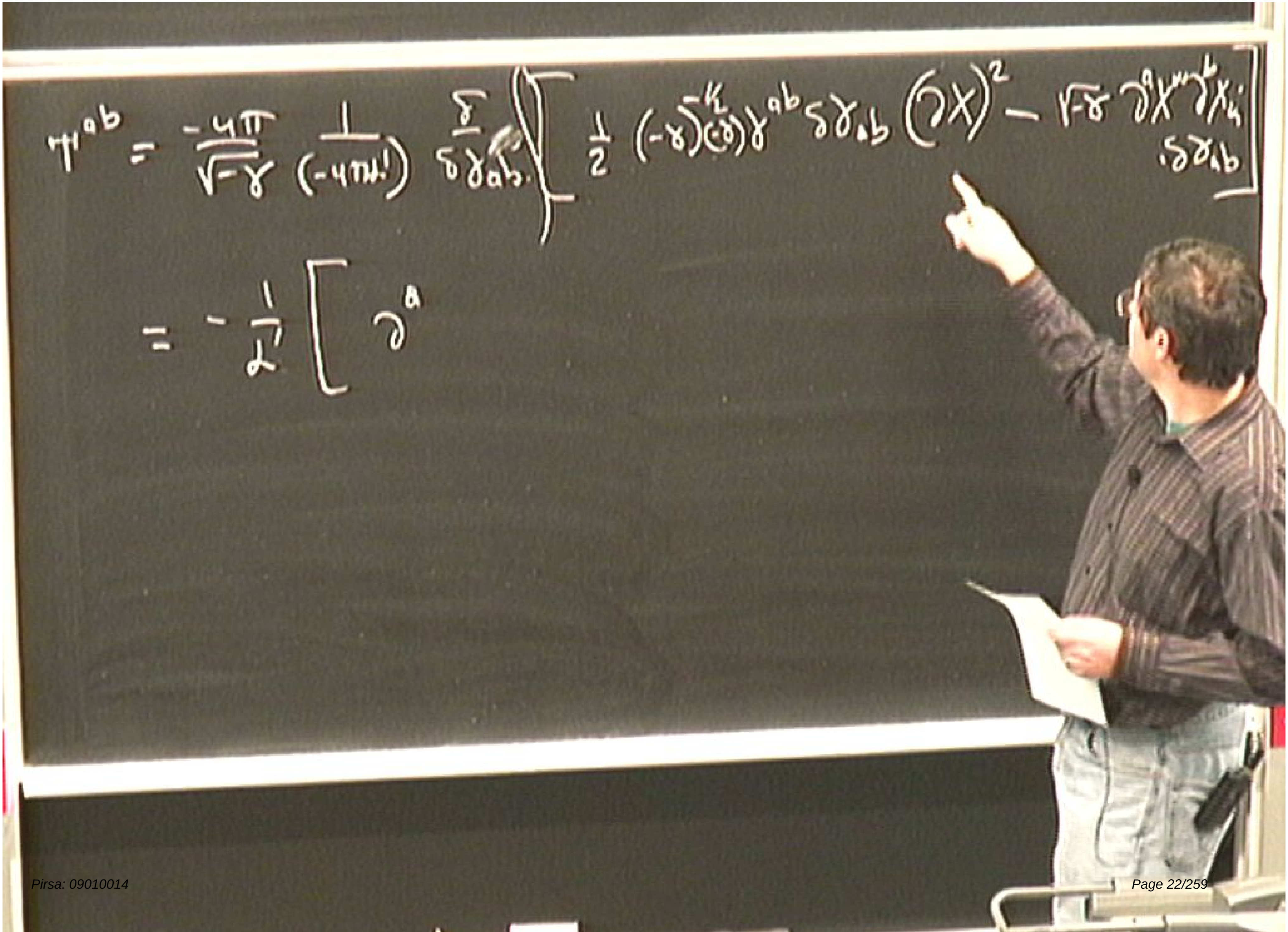
$$\delta\gamma_{ab} \int \left[\frac{1}{2} (-\gamma)^{-1/2} (-\gamma) \gamma^{ab} \delta\gamma_{ab} (\partial X)^2 - \sqrt{-\gamma}\, \partial^a X^\mu \partial^b X_\mu \, \delta\gamma_{ab} \right]$$

$\partial^a X^m \partial_b X_m \Longrightarrow$ 1+1 QFT

$X)^2$

$$\delta\gamma^{ab} = -\delta\gamma_{cd}\,\gamma^{ca}\gamma^{db}$$

$$\delta\gamma = \gamma\cdot\gamma^{ab}\,\delta\gamma_{ab}$$


$\pi^{ab} = \frac{-4\pi}{\sqrt{-\gamma}} \frac{1}{(-4\pi\alpha')} \frac{\delta}{\delta\gamma_{ab}} \int \left[\frac{1}{2} (-\gamma)^{-1/2} (-\gamma) \gamma^{ab} \delta\gamma_{ab} (\partial X)^2 - \sqrt{-\gamma}\, \partial X^\mu \partial X_\mu \, \delta\gamma_{ab} \right]$
$= -\frac{1}{\alpha'} \left[\partial^a \right.$

$$T^{ab} = \frac{-4\pi}{\sqrt{-\gamma}} \frac{1}{(-4\pi\alpha')} \frac{\delta}{\delta\gamma_{ab}} \int \left[\frac{1}{2} (-\gamma)^{-1/2} (-\gamma) \gamma^{ab} \delta\gamma_{ab} (\partial X)^2 - \sqrt{-\gamma}\, \partial^a X^m \partial^b X_m \, \delta\gamma_{ab} \right]$$

$$= -\frac{1}{\alpha'} \left[\partial^a X^m \partial^b X_m - \frac{1}{2} \gamma^{ab} \partial_c X^m \partial^c X_m \right]$$

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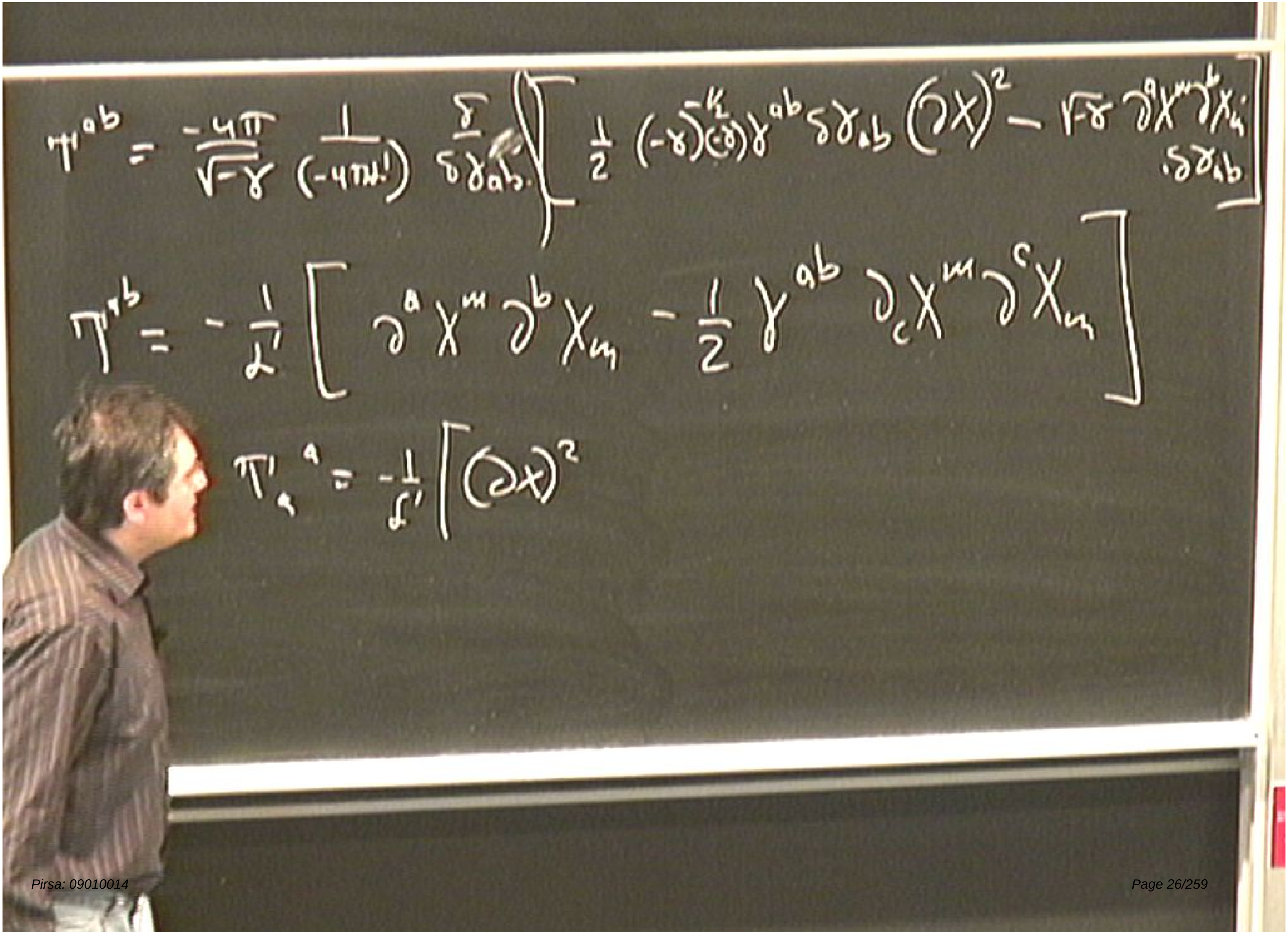

$T'^{ab} = \frac{-4\pi}{\sqrt{-\gamma}} \frac{1}{(-4\pi\alpha')} \frac{\delta}{\delta\gamma_{ab}} \left[\frac{1}{2}(-\gamma)^{-1/2}(-\gamma)\gamma^{ab}\delta\gamma_{ab}(\partial X)^2 - \sqrt{-\gamma}\,\partial^a X^m \partial^b X_m \delta\gamma_{ab} \right]$

$T^{ab} = -\frac{1}{\alpha'}\left[\partial^a X^m \partial^b X_m - \frac{1}{2}\gamma^{ab}\partial_c X^m \partial^c X_m \right]$

$T'_a{}^a = -\frac{1}{\alpha'}\left[(\partial X)^2 \right.$

$$\pi^{ab} = \frac{-4\pi}{\sqrt{-\gamma}} \frac{1}{(-4\pi\alpha')} \frac{\delta}{\delta\gamma_{ab}} \int \left[\frac{1}{2} (-\gamma)^{-1/2} (-\gamma) \gamma^{ab} \delta\gamma_{ab} (\partial X)^2 - \sqrt{-\gamma}\, \partial^a X^m \partial^b X_m \, \delta\gamma_{ab} \right]$$

$$\pi^{ab} = -\frac{1}{\alpha'} \left[\partial^a X^m \partial^b X_m - \frac{1}{2} \gamma^{ab} \partial_c X^m \partial^c X_m \right]$$

$$\pi'_a{}^a = -\frac{1}{\alpha'} \left[(\partial X)^2 - \frac{1}{2} \cdot \gamma^{ab} \gamma_{ab} (\partial X)^2 \right] = 0$$

Dynamical fields:

$$\gamma_{ab}\,,\quad X^{(\mu)}$$

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$$\gamma_{ab}\,,\; X^{\mu}$$

$$0 = \frac{\delta S}{\delta \gamma_{ab}} =$$

Dynamical fields:

$$\gamma_{ab}\ ,\ X^{\mu}$$

$$0 = \frac{\delta S}{\delta \gamma_{ab}} = T^{ab}$$

Dynamical fields:

$\gamma_{ab}\,,\; X^{\mu}$

EOM:

$$0 = \frac{\delta S}{\delta \gamma_{ab}} = \boxed{T^{ab} = 0}$$

Dynamical fields:

γ_{ab}, X^{μ}

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$$\delta S_p = -\frac{1}{4\pi\alpha'} \int d\tau\, d\sigma$$

Dynamical fields:

EOM: γ_{ab}, X^{μ}

$$0 = \frac{\delta S}{\delta \gamma_{ab}} = \boxed{T^{ab} = 0}$$

$$\delta S_p = -\frac{2}{4\pi\alpha'} \int d\tau\, d\sigma \,\sqrt{-\gamma}\,\gamma^{ab}\, \nabla_a X^{\mu} \cdot \nabla_b \delta X_{\mu}$$

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$=$

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$$= +\frac{2}{4\pi\alpha'} \int d\tau\, d\sigma\, \nabla_b \left[\sqrt{-\gamma}\,\gamma^{ab}\, \nabla_a X^{\mu}\right] \delta X_{\mu}$$

Dynamical fields:

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$$= +\frac{2}{4\pi\alpha'} \int d\tau\, d\sigma\, \nabla_b \left[\sqrt{-\gamma}\,\gamma^{ab} \nabla_a X^{\mu} \right] \delta X_{\mu} + BT.$$

Dynamical fields:

EOM: γ_{ab}, X^{μ}

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$$\Rightarrow \nabla_b \left[\sqrt{-\gamma}\, \gamma^{ab}\, \nabla_a X^{\mu} \right] = 0$$

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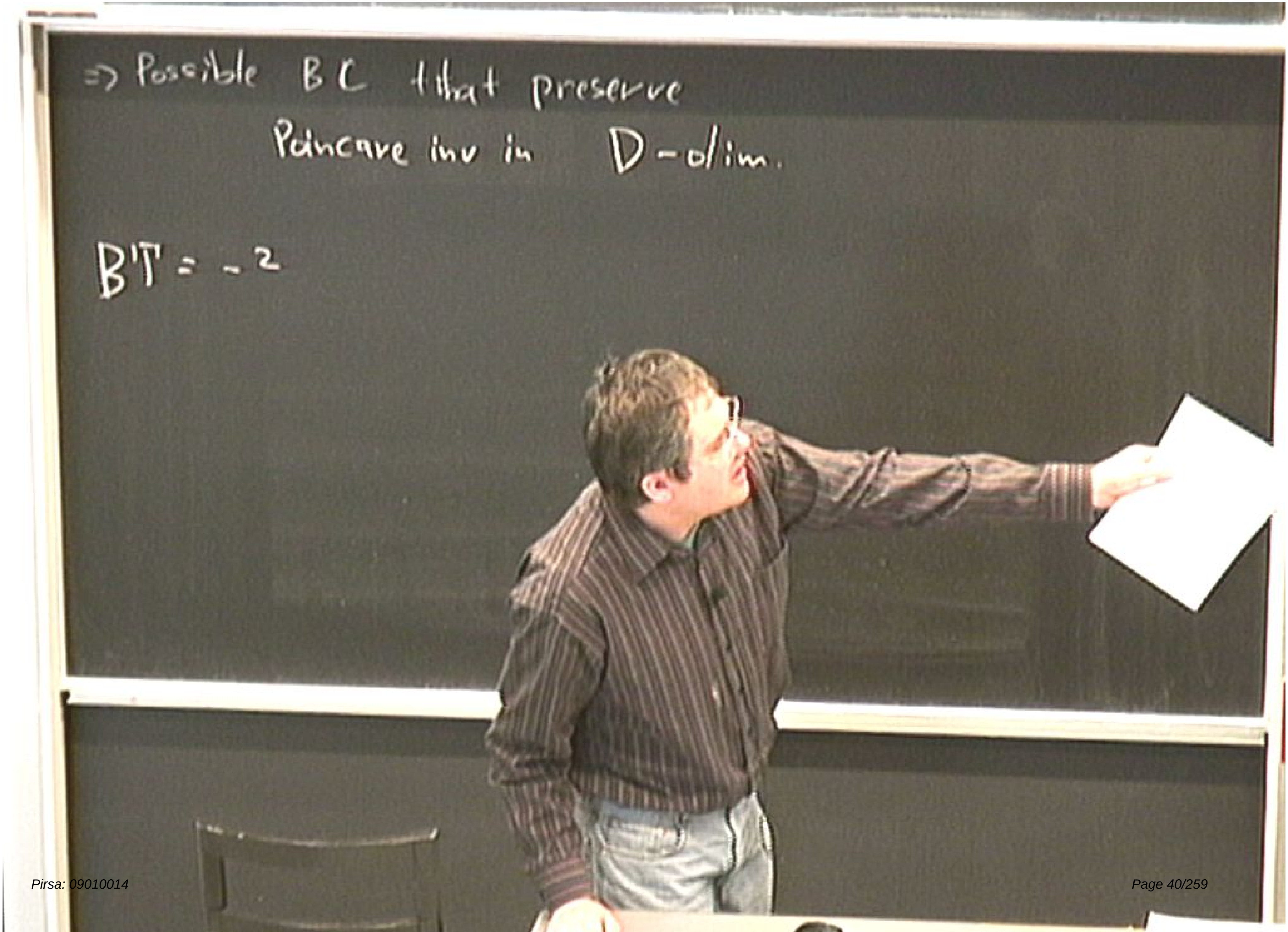
$$\Rightarrow \nabla_b \left[\sqrt{-\gamma}\,\gamma^{ab} \nabla_a X^{\mu}\right] = 0 \Rightarrow \boxed{\sqrt{-\gamma}\,\Box X^{\mu} = 0}$$

$\Rightarrow$ Possible BC that preserve

Poincare inv in D-dim.


⇒ Possible BC that preserve
Poincare inv in D-dim.
B'T = _2

⇒ Possible BC that preserve

Poincare inv in D-dim.

$$P^{T} = -\frac{2}{4\pi\alpha'}\int_{-\infty}^{+\infty} d\tau$$

⇒ Possible BC that preserve Poincare inv in D-dim.

$$B'T = -\frac{2}{4\pi\alpha'}\int_{-\infty}^{\infty} d\tau \;\sqrt{-\gamma}\;\gamma$$

⇒ Possible BC that preserve

Poincare inv in D-dim.

$$B'T = -\frac{2}{4\pi\alpha'}\int_{-\infty}^{\infty} d\tau \, \sqrt{-\gamma}\,\gamma^{\sigma a}\,\partial_a X^{\mu} \cdot \delta X_{\mu}$$

$\Rightarrow$ Possible BC that preserve

Poincare inv in D-dim.

$$B'T = -\frac{2}{4\pi\alpha'} \int_{-\infty}^{\infty} d\tau \, \sqrt{-\gamma} \, \gamma^{\sigma a} \, \partial_a X^{\mu} \cdot \delta X_{\mu} \Big|_{\sigma=0}^{\sigma=\ell}$$

Dynamical fields: γ_{ab}, X^μ

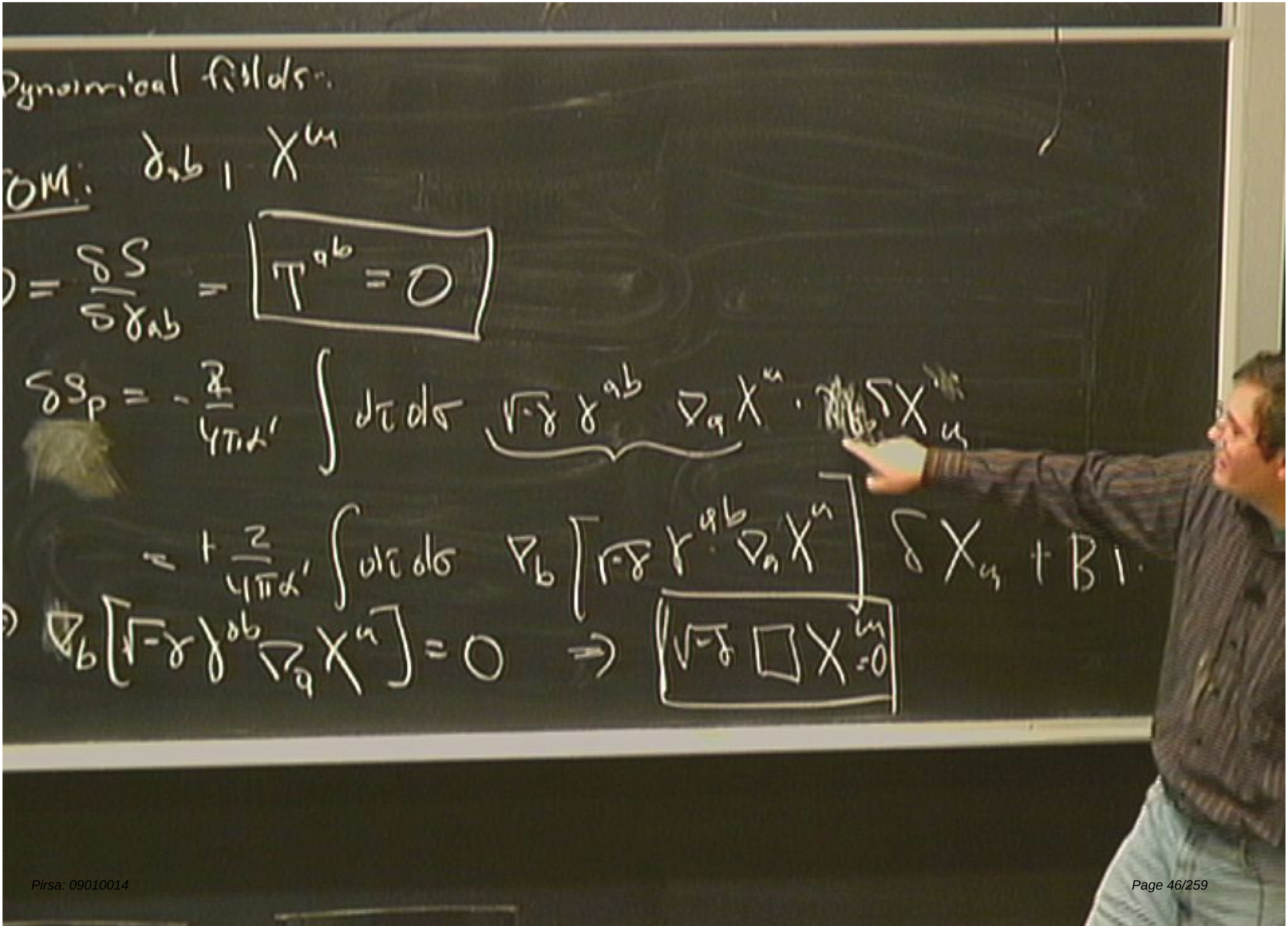
EOM:

$$0 = \frac{\delta S}{\delta \gamma_{ab}} \Rightarrow \boxed{T^{ab} = 0}$$

$$\delta S_P = -\frac{2}{4\pi\alpha'} \int d\tau\, d\sigma\, \sqrt{-\gamma}\,\gamma^{ab}\,\nabla_a X^\mu \cdot \nabla_b \delta X_\mu$$

$$= +\frac{2}{4\pi\alpha'} \int d\tau\, d\sigma\, \nabla_b\left[\sqrt{-\gamma}\,\gamma^{ab}\,\nabla_a X^\mu\right] \delta X_\mu + BT.$$

$$\Rightarrow \nabla_b\left[\sqrt{-\gamma}\,\gamma^{ab}\,\nabla_a X^\mu\right] = 0 \Rightarrow \boxed{\sqrt{-\gamma}\,\Box X^\mu = 0}$$


Dynamical fields:
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$0 = \frac{\delta S}{\delta \gamma_{ab}} \Rightarrow \boxed{T^{ab} = 0}$
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$= +\frac{2}{4\pi\alpha'} \int d\tau\, d\sigma\, \nabla_b \left[\sqrt{-\gamma}\,\gamma^{ab}\, \nabla_a X^\mu\right] \delta X_\mu + B.T.$
$\Rightarrow \nabla_b \left[\sqrt{-\gamma}\,\gamma^{ab}\, \nabla_a X^\mu\right] = 0 \Rightarrow \boxed{\sqrt{-\gamma}\,\Box X^\mu = 0}$

$\Rightarrow$ Possible BC that preserve

Poincare inv in D-dim.

$$B'T = -\frac{2}{4\pi\alpha'}\int_{-\infty}^{+\infty} d\tau\, \sqrt{-\gamma}\, \gamma^{\sigma a}\, \partial_a X^{\mu} \cdot \delta X_{\mu} \Bigg|_{\sigma=0}^{\sigma=\ell}$$

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$\Rightarrow$

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Poincare inv in D-dim.

$$B'T = -\frac{2}{4\pi\alpha'} \int_{-\infty}^{+\infty} d\tau \, \sqrt{-\gamma} \, \underbrace{\gamma^{\sigma a} \partial_a X^\mu}_{=0} \cdot \delta X_\mu \Bigg|_{\sigma=0}^{\sigma=\ell}$$

$\Rightarrow \partial^\sigma X^\mu(\tau, 0) = 0 \quad \partial^\sigma X^\mu(\tau, \ell) = 0$

$\Rightarrow$ Possible BC that preserve

Poincare inv in D-dim.

$$BT = -\frac{2}{4\pi\alpha'} \int_{-\infty}^{\infty} d\tau \, \sqrt{-\gamma} \, \underbrace{\gamma^{\sigma a}}_{=0} \partial_a X^\mu \cdot \delta X_\mu \Big|_{\sigma=0}^{\sigma=\ell}$$

$$\Rightarrow \boxed{\partial^\sigma X^\mu(\tau, 0) = 0 \quad \partial^\sigma X^\mu(\tau, \ell) = 0} \quad N \text{ Boundary conditions}$$

$\Rightarrow$ Possible BC that preserve

Poincare inv in D-dim.

$$B'T = -\frac{2}{4\pi\alpha'}\int_{-\infty}^{\infty} d\tau \, \sqrt{-\gamma}\, \underbrace{\gamma^{\sigma a}\partial_a X^{\mu}}_{\overset{\shortparallel}{0}} \cdot \delta X_{\mu} \Big|_{\sigma=0}^{\sigma=\ell}$$

$$\Rightarrow \boxed{\partial^{\sigma} X^{\mu}(\tau,0)=0 \quad \partial^{\sigma} X^{\mu}(\tau,\ell)=0}$$ N Boundary conditions for an open string

$\Rightarrow$

$\Rightarrow$ Possible BC that preserve
Poincare inv in D-dim.

$$B'T = -\frac{2}{4\pi\alpha'}\int_{-\infty}^{\infty} d\tau \, \sqrt{-\gamma}\, \underbrace{\gamma^{\sigma a}}_{=0}\partial_a X^\mu \cdot \delta X_\mu \Big|_{\sigma=0}^{\sigma=\ell}$$

$$\Rightarrow \boxed{\partial^\sigma X^\mu(\tau, 0) = 0 \quad \partial^\sigma X^\mu(\tau, \ell) = 0}$$ N Boundary conditions for an open string

$\Rightarrow \quad X^\mu(\tau, \sigma) =$

$\Rightarrow$ Possible BC that preserve

Poincare inv in D-dim.

$$B'T = -\frac{2}{4\pi\alpha'}\int_{-\infty}^{\infty} d\tau \, \underbrace{\sqrt{-\gamma}\,\gamma^{\sigma a}}_{=0}\, \partial_a X^\mu \cdot \delta X_\mu \Big|_{\sigma=0}^{\sigma=\ell}$$

$$\Rightarrow \boxed{\partial^\sigma X^\mu(\tau,0) = 0 \quad \partial^\sigma X^\mu(\tau,\ell) = 0}$$ N Boundary conditions for an open string

$$X^\mu(\tau,\sigma) = X^\mu(\tau,\sigma+\ell)$$

$\Rightarrow$ Possible BC that preserve

Poincare inv in D-dim.

$$BT = -\frac{2}{4\pi\alpha'}\int_{-\infty}^{\infty} d\tau \, \sqrt{-\gamma}\, \underbrace{\gamma^{\sigma a}\partial_a X^\mu}_{\overset{\parallel}{0}} \cdot \delta X_\mu \Bigg|_{\sigma=0}^{\sigma=\ell}$$

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$$\Rightarrow \quad X^\mu(\tau,\sigma) = X^\mu(\tau,\sigma+\ell)$$

(note that δX^μ must be periodic as well!)

$\Rightarrow$ Possible BC that preserve

Poincare inv in D-dim.

$$BT = -\frac{2}{4\pi\alpha'}\int_{-\infty}^{\infty} d\tau \sqrt{-\gamma}\, \underbrace{\gamma^{\sigma a}}_{\overset{\shortparallel}{0}} \partial_a X^\mu \cdot \delta X_\mu \Big|_{\sigma=0}^{\sigma=\ell}$$

$\Rightarrow$ $\boxed{\partial^\sigma X^\mu(\tau,0)=0 \quad \partial^\sigma X^\mu(\tau,\ell)=0}$ N Boundary conditions for an open string

$\Rightarrow$ $X^\mu(\tau,\sigma) = X^\mu(\tau,\sigma+\ell)$

(note that δX^μ must be periodic as well!)

$\Rightarrow$ Possible BC that preserve

Poincare inv in D-dim.

$$\delta I = -\frac{2}{4\pi\alpha'}\int_{-\infty}^{\infty} d\tau \sqrt{-\gamma}\,\underbrace{\gamma^{\sigma a}}_{=0}\partial_a X^\mu \cdot \delta X_\mu \Big|_{\sigma=0}^{\sigma=\ell}$$

$\Rightarrow$ $\boxed{\partial^\sigma X^\mu(\tau,0)=0 \quad \partial^\sigma X^\mu(\tau,\ell)=0}$ N Boundary conditions for an open string

$\Rightarrow$ $X^\mu(\tau,\sigma) = X^\mu(\tau,\sigma+\ell)$

(note that δX^μ must be periodic as well) $\Rightarrow$

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Poincare inv in D-dim.

$$-\frac{2}{4\pi\alpha'}\int_{-\infty}^{\infty} d\tau \sqrt{-\gamma}\, \underbrace{\gamma^{\sigma a}}_{0} \partial_a X^\mu \cdot \delta X_\mu \Big|_{\sigma=0}^{\sigma=\ell}$$

$$\boxed{X^\mu(\tau,0)=0 \quad \partial^\sigma X^\mu(\tau,\ell)=0}$$ N Boundary conditions for an open string

$$X^\mu(\tau,\sigma) = X^\mu(\tau,\sigma+\ell)$$

$\Rightarrow \sigma \sim \sigma+\ell$

(... δX^μ must be periodic as well) Closed strings

$\Rightarrow$ Possible BC that preserve

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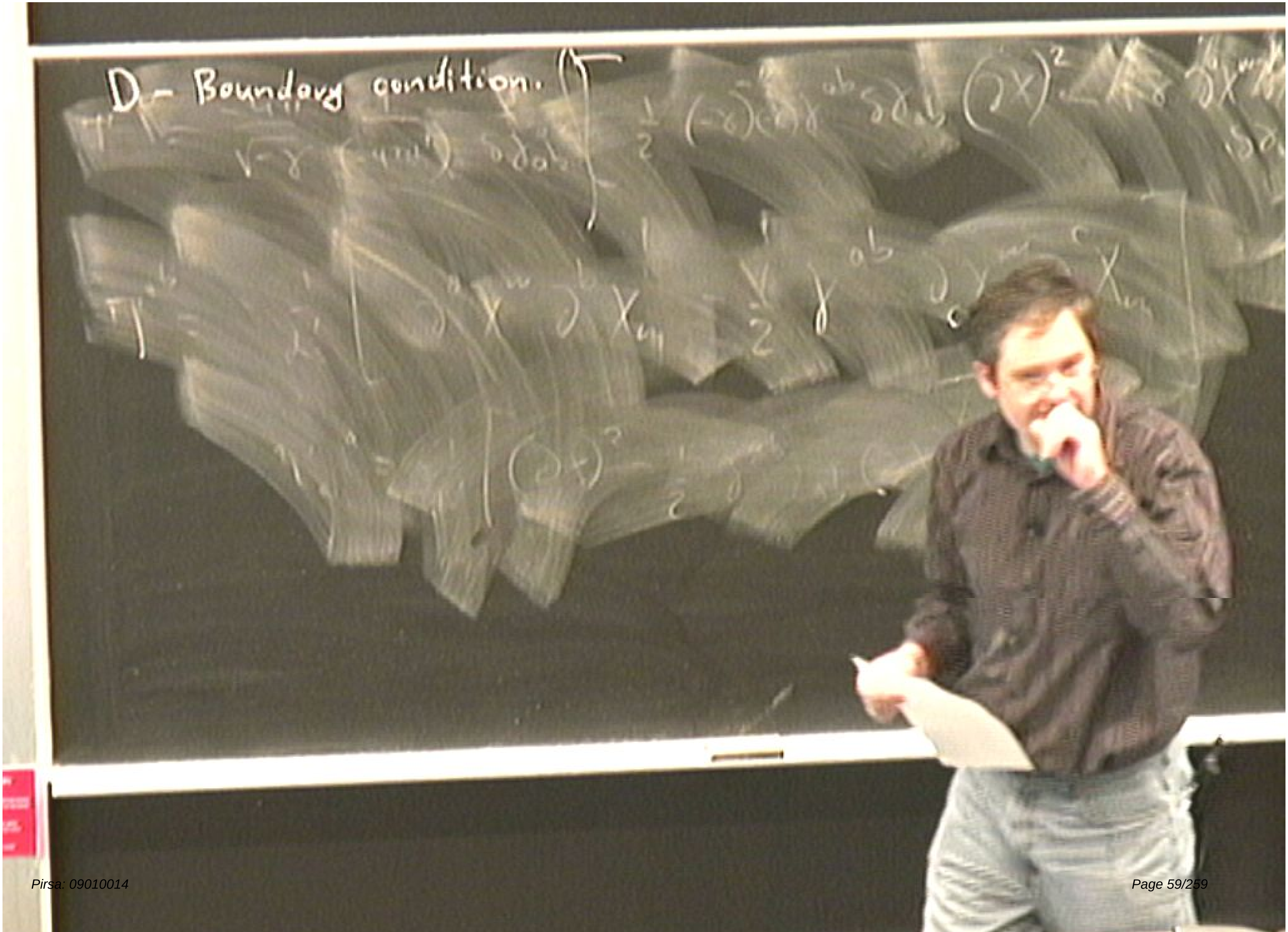
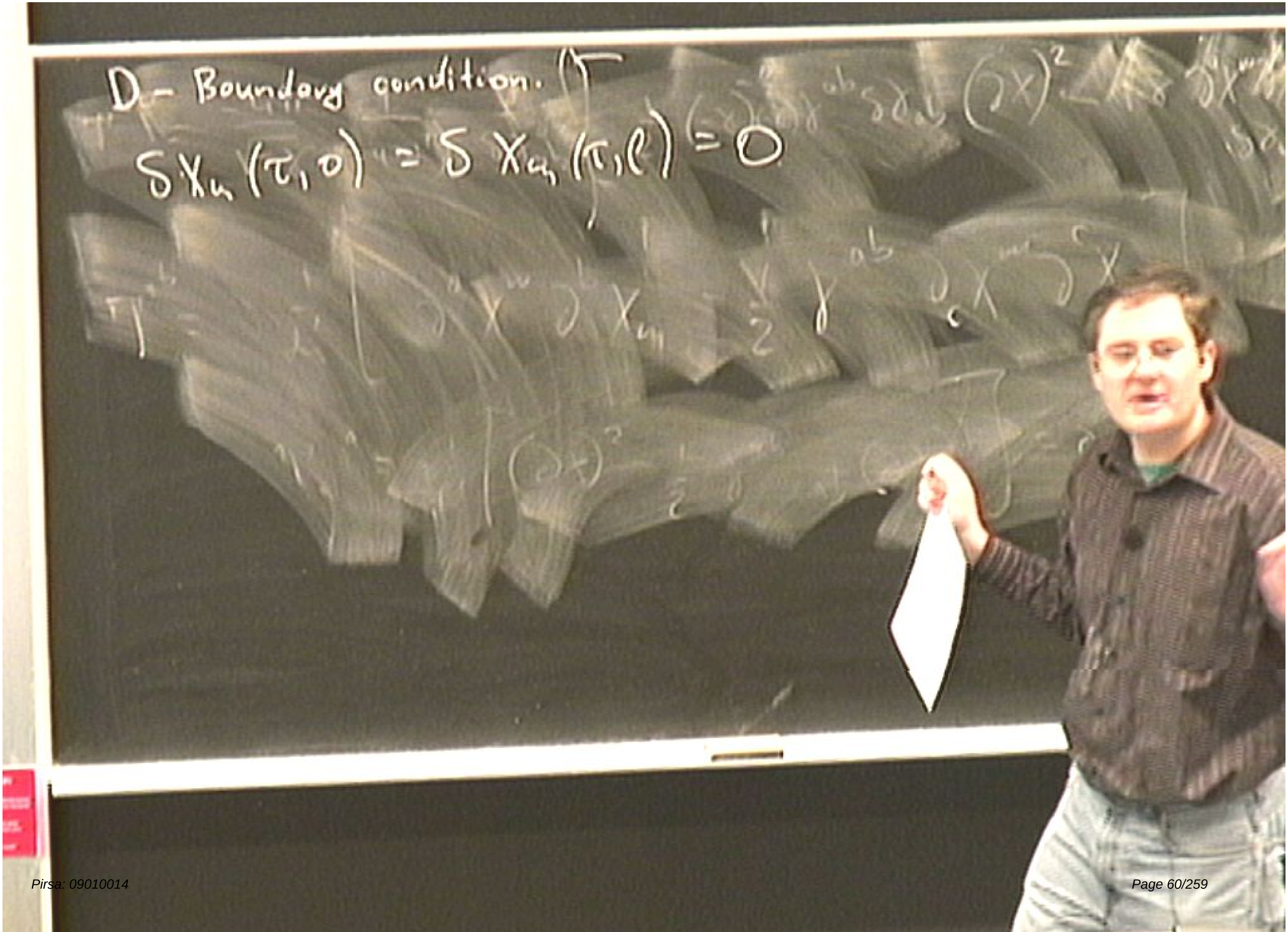
$$BT = -\frac{2}{4\pi\alpha'} \int_{-\infty}^{\infty} d\tau \, \sqrt{-\gamma} \, \underbrace{\gamma^{\sigma a}}_{=0} \partial_a X^\mu \cdot \delta X_\mu \Bigg|_{\sigma=0}^{\sigma=\ell}$$

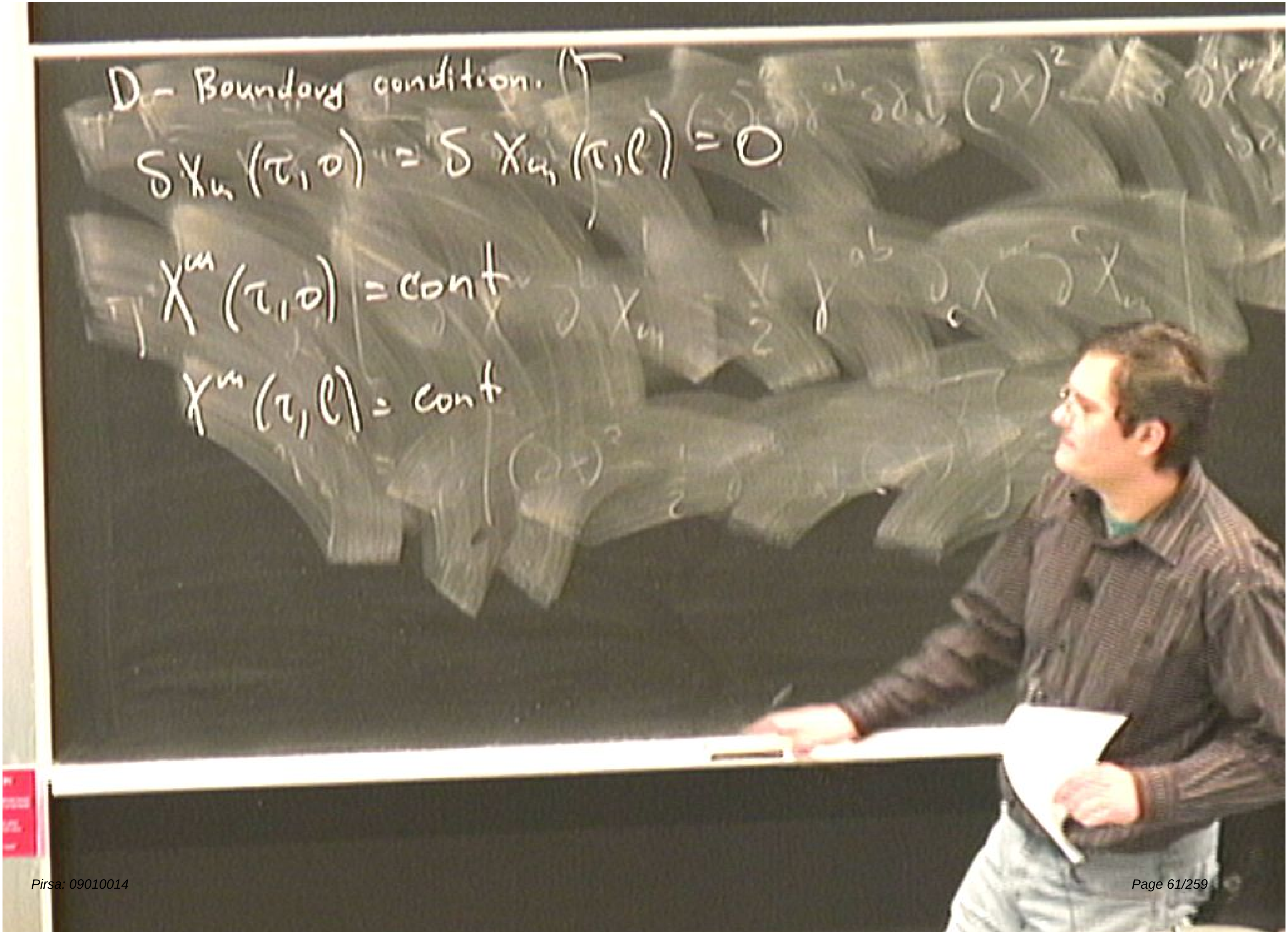
$$\Rightarrow \boxed{\partial^\sigma X^\mu(\tau, 0) = 0 \quad \partial^\sigma X^\mu(\tau, \ell) = 0}$$ N Boundary conditions for an open string

$$\Rightarrow \quad X^\mu(\tau, \sigma) = X^\mu(\tau, \sigma + \ell)$$

(note that δX^μ must be periodic as well!) $\Rightarrow \sigma \sim \sigma + \ell$ Closed strings


D - Boundary condition.


D - Boundary condition.
$\delta X_m(\tau,0) = \delta X_m(\tau,\ell) = 0$


D - Boundary condition.
$\delta X_m(\tau,0) = \delta X_m(\tau,\ell) = 0$
$X^m(\tau,0) = \text{cont}$
$X^m(\tau,\ell) = \text{cont}$

$$T_a{}^a = 0$$

$\Rightarrow$ Possible BC that preserve Poincare inv in D-dim.

$$BT = -\frac{2}{4\pi\alpha'}\int_{-\infty}^{\infty} d\tau \sqrt{-\gamma}\, \underbrace{\gamma^{\sigma a}\partial_a X^\mu}_{=\,0} \cdot \delta X_\mu \Big|_{\sigma=0}^{\sigma=\ell}$$

$$\Rightarrow \boxed{\partial^\sigma X^\mu(\tau, 0) = 0 \quad \partial^\sigma X^\mu(\tau, \ell) = 0}$$ N Boundary conditions for an open string

$$\rightarrow \quad X^\mu(\tau, \sigma) = X^\mu(\tau, \sigma + \ell)$$ $\Rightarrow \sigma \sim \sigma + \ell$

(note that δX^μ must be periodic as well!) Closed strings

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N Boundary conditions for an open string

$$X^\mu(\tau,\sigma) = X^\mu(\tau,\sigma+\ell)$$

(note that δX^μ must be periodic as well!) $\Rightarrow \sigma \sim \sigma+\ell$ Closed strings

$$T_a{}^a = 0$$

$\Rightarrow$ Possible BC that preserve Poincare inv in D-dim.

$$BT = -\frac{2}{4\pi\alpha'} \int_{-\infty}^{\infty} d\tau \left(\sqrt{-\gamma}\, \underbrace{\gamma^{\sigma a}}_{=0} \partial_a X^\mu\, \delta X_\mu \right) \Bigg|_{\sigma=0}^{\sigma=\ell}$$

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$$\rightarrow \quad X^\mu(\tau, \sigma) = X^\mu(\tau, \sigma + \ell)$$

(note that δX^μ must be periodic as well!) $\Rightarrow \sigma \sim \sigma + \ell$ Closed strings

D - Boundary condition.

$$\delta X_m(\tau, 0) = \delta X_m(\tau, \ell) = 0$$

$$X^m(\tau, 0) = \text{cont}$$

$$X^m(\tau, \ell) = \text{cont}$$

$$T_a^{\;a} = 0$$

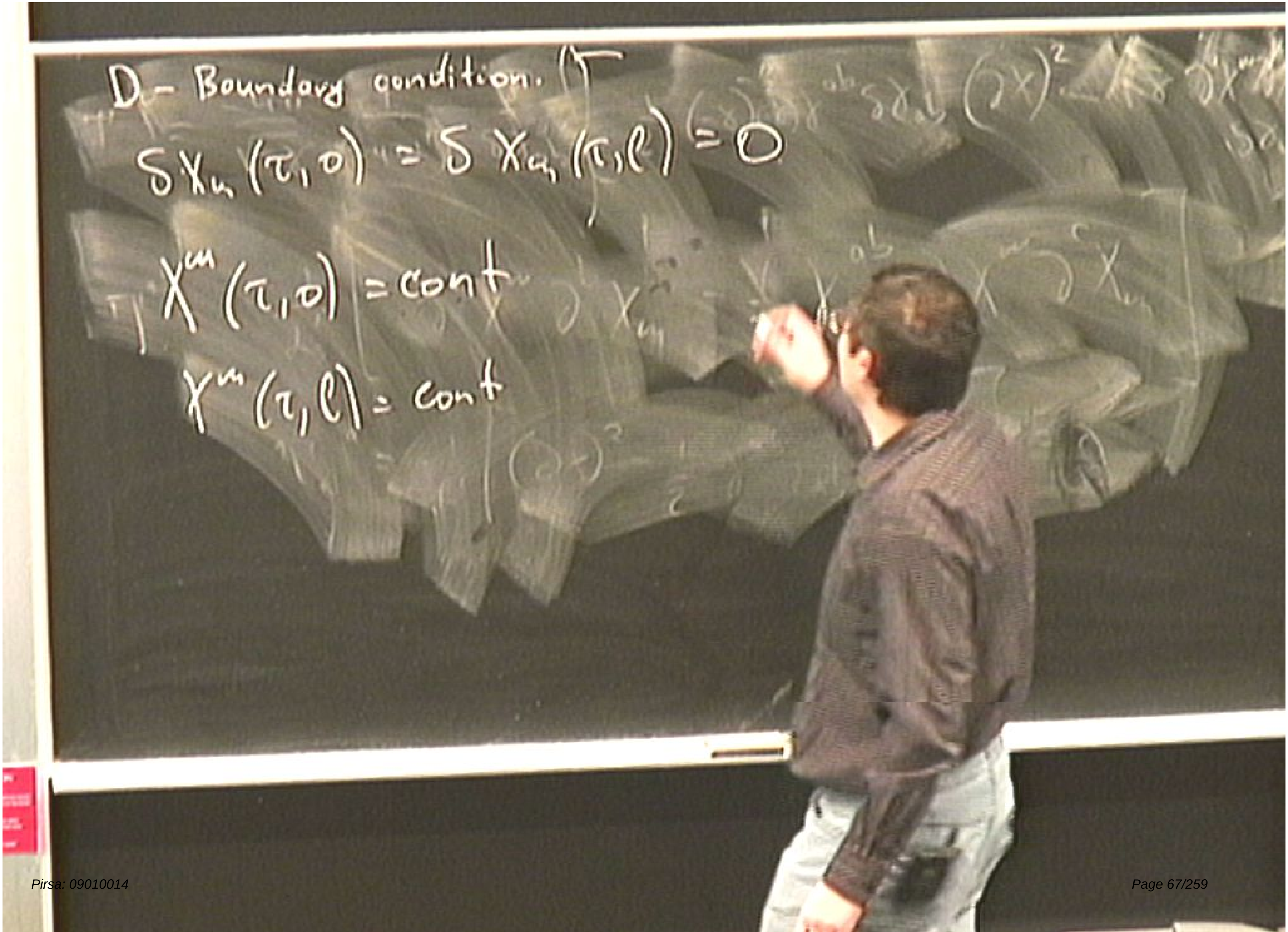
$\Rightarrow$ Possible BC that preserve Poincare inv in D-dim

$$B'T = -\frac{2}{4\pi\alpha'}\int_{-\infty}^{\infty} d\tau \left(\underbrace{\sqrt{-\gamma}\,\gamma^{\sigma a}\,\partial_a X^\mu}_{=0}\,\delta X_\mu \right)\Bigg|_{\sigma=0}^{\sigma=\ell}$$

$$\Rightarrow \boxed{\partial^\sigma X^\mu(\tau,0) = 0 \quad \partial^\sigma X^\mu(\tau,\ell) = 0}$$ N Boundary conditions for an open string

$$\rightarrow \quad X^\mu(\tau,\sigma) = X^\mu(\tau,\sigma+\ell)$$

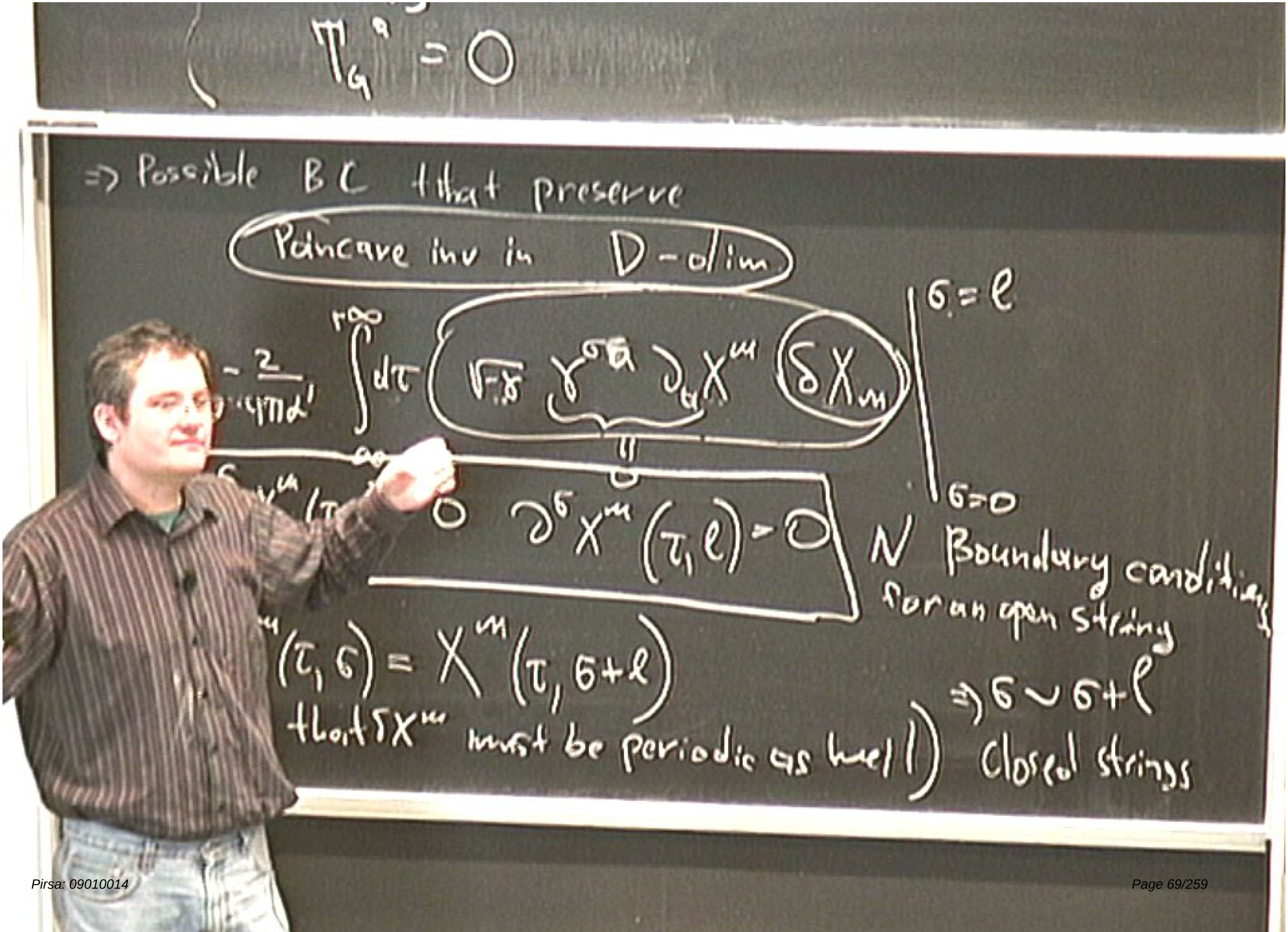
(note that δX^μ must be periodic as well!) $\Rightarrow \sigma \sim \sigma + \ell$ Closed strings


D - Boundary condition.
$\delta X_m(\tau,0) = \delta X_m(\tau,l) = 0$
$X^m(\tau,0) = \text{cont}$
$X^m(\tau,l) = \text{cont}$

D - Boundary condition.

$$\left.\begin{array}{l} \delta X_m(\tau,0) = \delta X_m(\tau,\ell) = 0 \\ \\ X^m(\tau,0) = \text{cont} \\ X^m(\tau,\ell) = \text{cont} \end{array}\right\}$$

Breaks Poincare invariance



Π_σ^a = 0
=> Possible BC that preserve
Poincare inv in D-dim
-2/(4πα') ∫_{-∞}^{∞} dτ (√-γ γ^{σa} ∂_a X^μ δX_μ) |_{σ=0}^{σ=ℓ}
∂^σ X^μ(τ,ℓ) = 0
N Boundary conditions for an open string
(τ,σ) = X^μ(τ,σ+ℓ)
that δX^μ must be periodic as well !)
=> σ ~ σ+ℓ
Closed strings

$$T_a{}^a = 0$$

$\Rightarrow$ Possible BC that preserve

Poincare inv in D-dim

$$B'T = -\frac{2}{4\pi\alpha'}\int_{-\infty}^{\infty} d\tau \left(\underbrace{\sqrt{-\gamma}\,\gamma^{\sigma a}\,\partial_a X^\mu}_{=\,0}\ \delta X_\mu \right)\Bigg|_{\sigma=0}^{\sigma=\ell}$$

$$\Rightarrow \boxed{\partial^\sigma X^\mu(\tau,0) = 0 \quad \partial^\sigma X^\mu(\tau,\ell) = 0}$$ N Boundary conditions for an open string

$$\Rightarrow X^\mu(\tau,\sigma) = X^\mu(\tau,\sigma+\ell)$$

(note that δX^μ must be periodic as well!) $\Rightarrow \sigma \sim \sigma + \ell$ Closed strings

$$BT = -\frac{T}{4\pi\alpha'}\int_{\mathcal{M}} d\tau\, d\sigma\; \nabla_b \left[\sqrt{-\gamma}\, \gamma^{ab}\, \partial_a X^{\mu}\, \delta X_{\mu} \right]$$

$$BT = -\frac{T}{4\pi\alpha'} \int_{\mathcal{M}} d\tau\, d\sigma\, \nabla_b \left[\sqrt{-\gamma}\, \gamma^{ab}\, \partial_a X^{\mu}\, \delta X_{\mu} \right]$$

$$BT = -\frac{1}{4\pi\alpha'} \int_{\mathcal{M}} d\tau\, d\sigma\, \nabla_b \left[\sqrt{-\gamma}\, \gamma^{ab}\, \partial_a X^{\mu}\, \delta X_{\mu} \right]$$

$$BT = -\frac{T}{4\pi\alpha'} \int_{\mathcal{M}} d\tau\, d\sigma\, \nabla_b \left[\sqrt{-\gamma}\, \gamma^{ab}\, \partial_a X^{\mu}\, \delta X_{\mu} \right]$$

$$\int \nabla_a$$

$$BT = -\frac{T}{4\pi\alpha'} \int_{M} d\tau\, d\sigma\ \nabla_b \left[\sqrt{-\gamma}\, \gamma^{ab}\, \partial_a X^{\mu}\, \delta X_{\mu} \right]$$

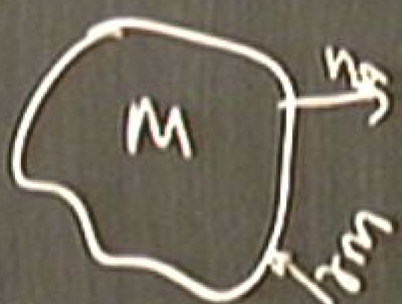
$$\int_{M} \nabla_a V^a$$

$$BT = -\frac{T}{4\pi\alpha'} \int_{\mathcal{M}} d\tau\, d\sigma\, \nabla_b \left[\sqrt{-\gamma}\, \gamma^{ab}\, \partial_a X^{\mu}\, \delta X_{\mu} \right]$$

$$\int_{\mathcal{M}} \nabla_a V^a = \int_{\partial \mathcal{M}} n_a V^a$$

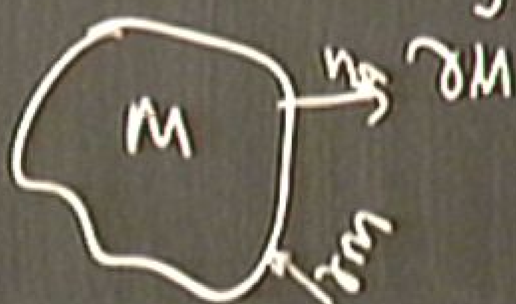
$$BT = -\frac{T}{4\pi\alpha'}\int_{M} d\tau\, d\sigma\, \nabla_b\left[\sqrt{-\gamma}\,\gamma^{ab}\,\partial_a X^{\mu}\,\delta X_{\mu}\right]$$

$$\int_{M} \nabla_a V^a = \int_{\partial M} n_a V^a$$



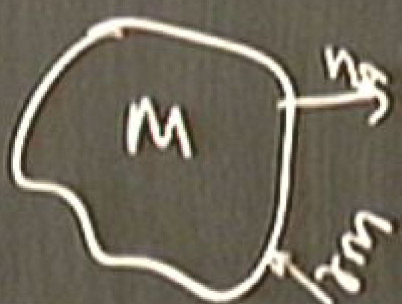
$$BT \equiv -\frac{T}{4\pi\alpha'}\int_{M} d\tau\, d\sigma\; \nabla_b\left[\sqrt{-\gamma}\,\gamma^{ab}\,\partial_a X^{\mu}\,\delta X_{\mu}\right]$$

$$\int_{M} \nabla_a V^a = \int_{\partial M} n_a V^a$$



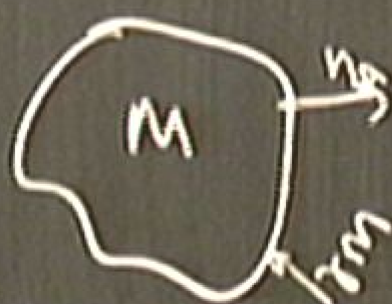
$$BT = -\frac{T}{4\pi\alpha'} \int_{M} d\tau\, d\sigma\, \nabla_b \left[\sqrt{-\gamma}\, \gamma^{ab}\, \partial_a X^{\mu}\, \delta X_{\mu} \right]$$

$$\int_{M} \nabla_a V^a = \int_{\partial M} n_a V^a \qquad = -\frac{1}{2\pi\alpha'} \int_{\partial M}$$



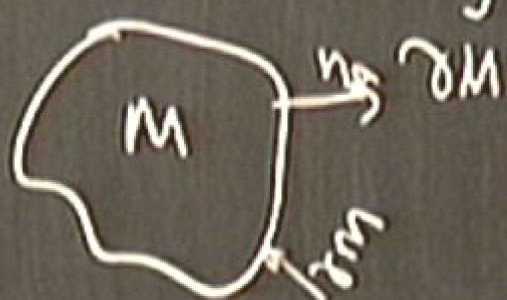
$$BT = -\frac{T}{4\pi\alpha'}\int_{M} d\tau\, d\sigma\, \nabla_b\left[\sqrt{-\gamma}\,\gamma^{ab}\,\partial_a X^{\mu}\,\delta X_{\mu}\right]$$

$$\int_{M} \nabla_a V^a = \int_{\partial M} n_a V^a \qquad = -\frac{1}{2\pi\alpha'}\int_{\partial M} n_b\, \partial^b X^{\mu}$$



$$BT = -\frac{T}{4\pi\alpha'} \int_{M} d\tau\, d\sigma\, \nabla_b \left[\sqrt{-\gamma}\, \gamma^{ab}\, \partial_a X^{\mu}\, \delta X_{\mu} \right] = -\frac{1}{2\pi\alpha'} \int_{\partial M} n_b\, \partial^b X^{\mu}\, \delta X_{\mu}$$

$$\int_{M} \nabla_a V^a = \int_{\partial M} n_a V^a$$

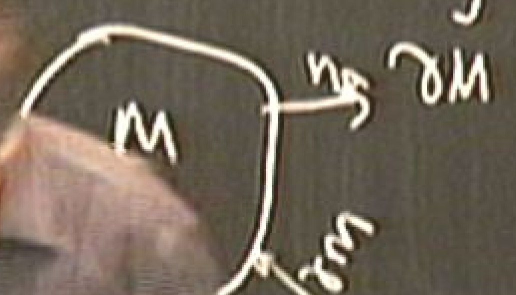


$$\boxed{n_b\, \partial^b X^{\mu} = 0}$$

$$\mathrm{BT} = -\frac{1}{4\pi\alpha'}\int_M d\tau\, d\sigma\, \nabla_b\left[\sqrt{-\gamma}\,\gamma^{ab}\,\partial_a X^\mu\,\delta X_\mu\right]$$

$$\int \nabla_a V^a = \int_{\partial M} n_a V^a$$

$$= -\frac{1}{2\pi\alpha'}\int_{\partial M} n_b\,\partial^b X^\mu\,\delta X_\mu$$



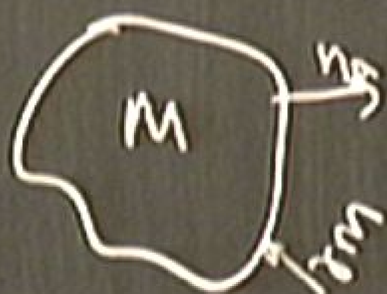
$$\boxed{n_b\,\partial^b X^\mu = 0}$$

→ N Boundary condition for open string.

$$BT = -\frac{T}{4\pi\alpha'}\int_{M} d\tau\, d\sigma\, \nabla_b\left[\sqrt{-\gamma}\,\gamma^{ab}\,\partial_a X^\mu\, \delta X_\mu\right]$$

$$= -\frac{1}{2\pi\alpha'}\int_{\partial M} n_b\, \partial^b X^\mu\, \delta X_\mu$$

$$\int_{M} \nabla_a V^a = \int_{\partial M} n_a V^a$$

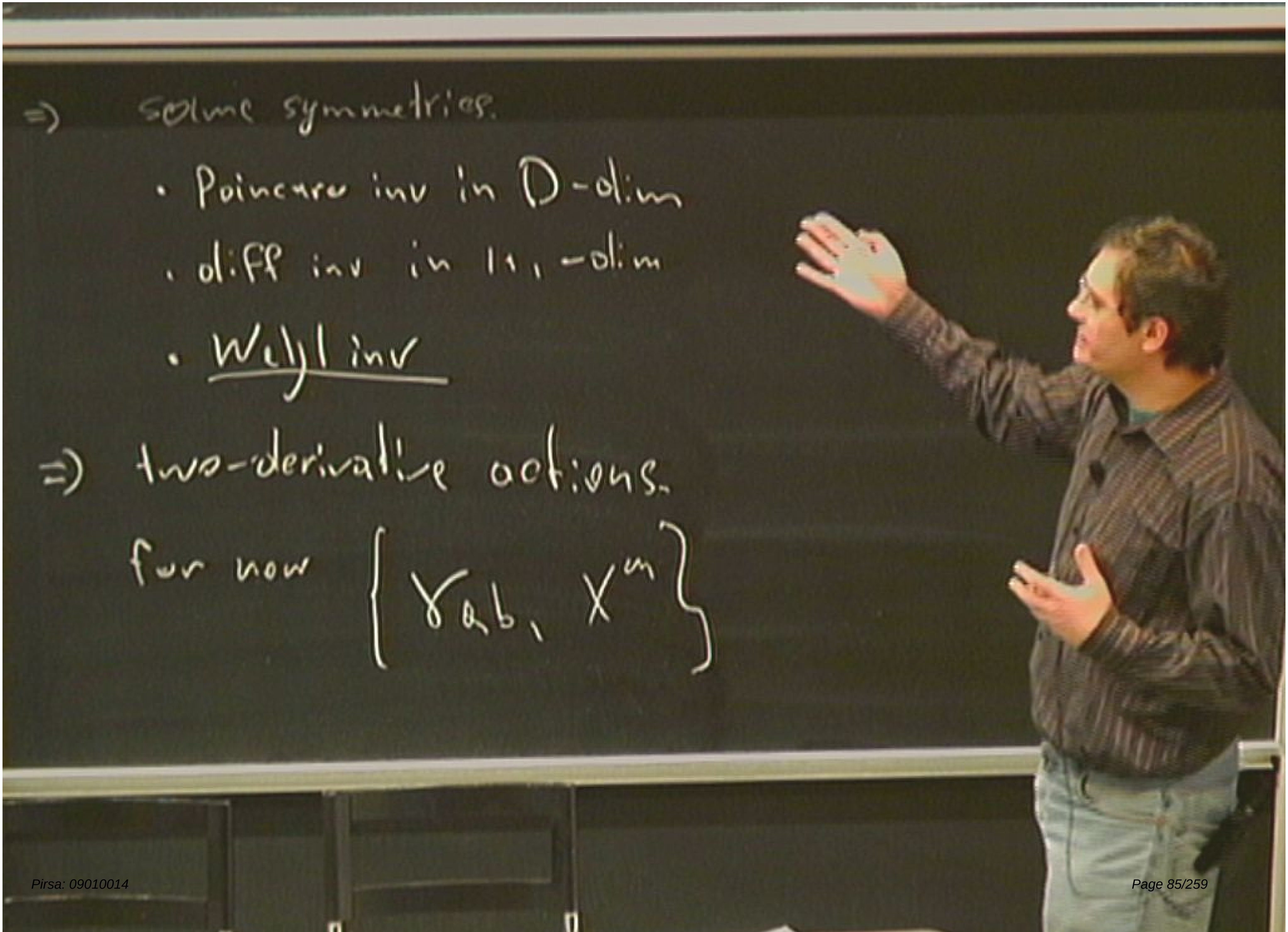


$$\boxed{n_b\, \partial^b X^\mu = 0}$$

$\rightarrow$ N Boundary condition for open string.

$\Rightarrow$ some symmetries.

- Poincare inv in D-dim
- diff inv in 1+1-dim
- Weyl inv

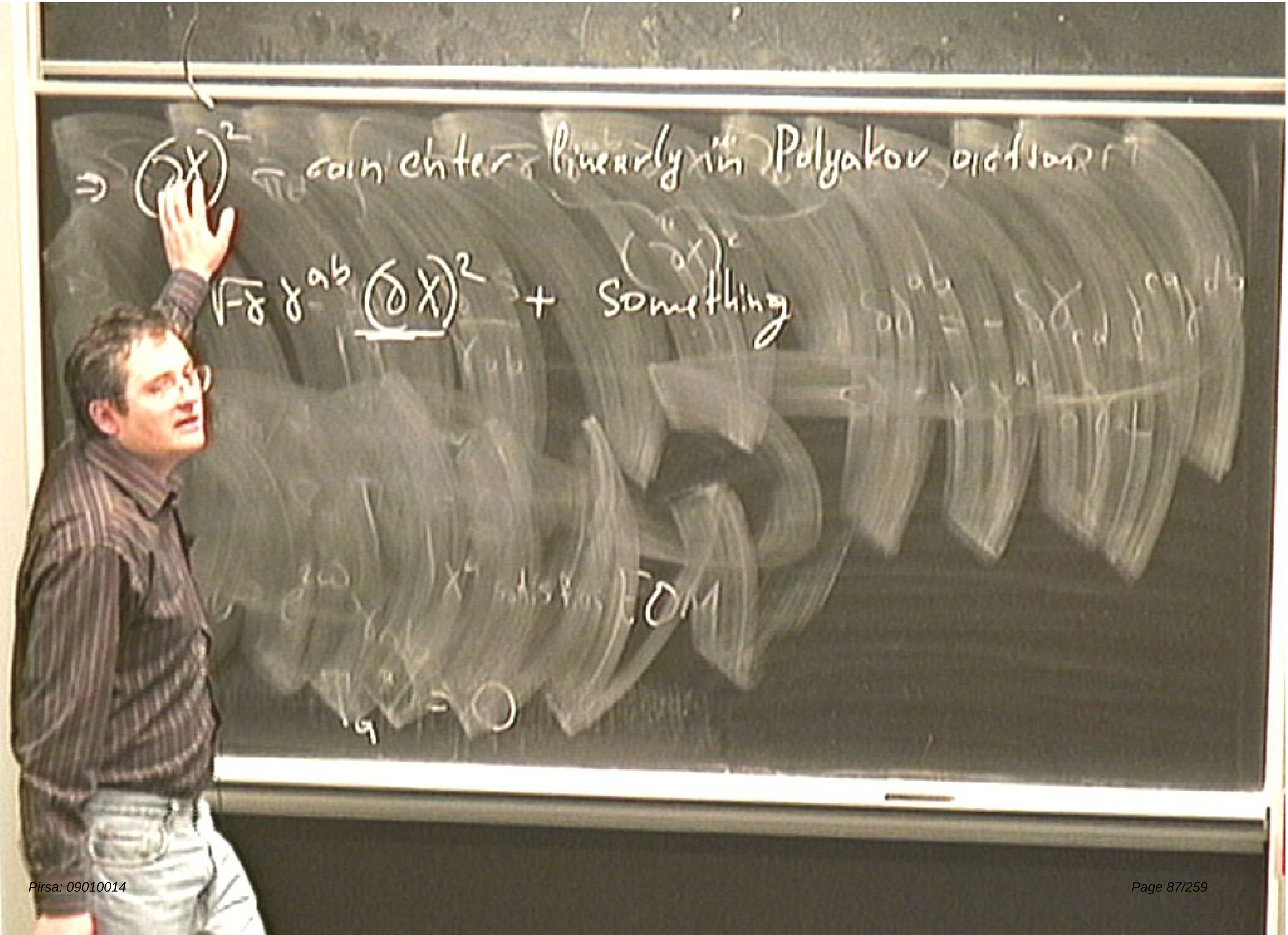

⇒ some symmetries.
• Poincare inv in D-dim
• diff inv in 1+1 -dim
• Weyl inv
⇒ two-derivative actions.
for now {γab, X^m}

⇒ solve symmetries.

- Poincare inv in D-dim
- diff inv in 1+1-dim
- Weyl inv

⇒ two-derivative actions.

for now $\{ \gamma_{ab}, X^{m} \}$


⇒ (∂X)² can enter linearly in Polyakov action
√-γ γ^ab (∂X)² + something
EOM

$\Rightarrow (\partial X)^2$ can enter linearly in Polyakov action

$$\sqrt{-\gamma}\,\gamma^{ab}(\partial X)^2 + \text{something}[\gamma_{ab}]$$

$\Rightarrow$ X^μ satisfies EOM

$= 0$

$\Rightarrow (\partial X)^2$ can enter linearly in Polyakov action

$$\sqrt{-\gamma}\,\gamma^{ab}(\partial X)^2 + \text{something}[\gamma_{ab}]$$

$\Rightarrow$

$\Rightarrow (\partial X)^2$ can enter linearly in Polyakov action

$$\sqrt{-\gamma}\,\gamma^{ab}(\partial X)^2 + \text{something}[\gamma_{ab}]$$

$\hookrightarrow$ must be geometric invariants

$\Rightarrow (\partial X)^2$ can enter linearly in Polyakov action

$$\sqrt{-\gamma}\,\gamma^{ab}\,\underline{(\partial X)^2} + \text{Something}[\gamma_{ab}]$$

$\hookrightarrow$ must be geometric invariants

$\sqrt{-\gamma}$, R

$\Rightarrow (\partial X)^2$ can enter linearly in Polyakov action

$$\sqrt{-\gamma}\,\gamma^{ab}(\partial X)^2 + \text{something}[\gamma_{ab}]$$

$\hookrightarrow$ must be geometric invariants

$\sqrt{-\gamma}$, R

(note that δX^{μ} must be periodic as well) Closed strings

$\Rightarrow (\partial X)^2$ can enter linearly in Polyakov action

$$\sqrt{-\gamma}\, \gamma^{ab} \underline{(\partial X)^2} + \text{Something}\left[\gamma_{ab}\right]$$

$\hookrightarrow$ must be geometric invariants

$\sqrt{-\gamma}$, R

$\Rightarrow \int d\tau d\sigma \sqrt{-\gamma}\, R$ potential new term.

(note that δX^μ must be periodic as well) Closed stri

$\Rightarrow (\partial X)^2$ — coincides linearly in Polyakov action

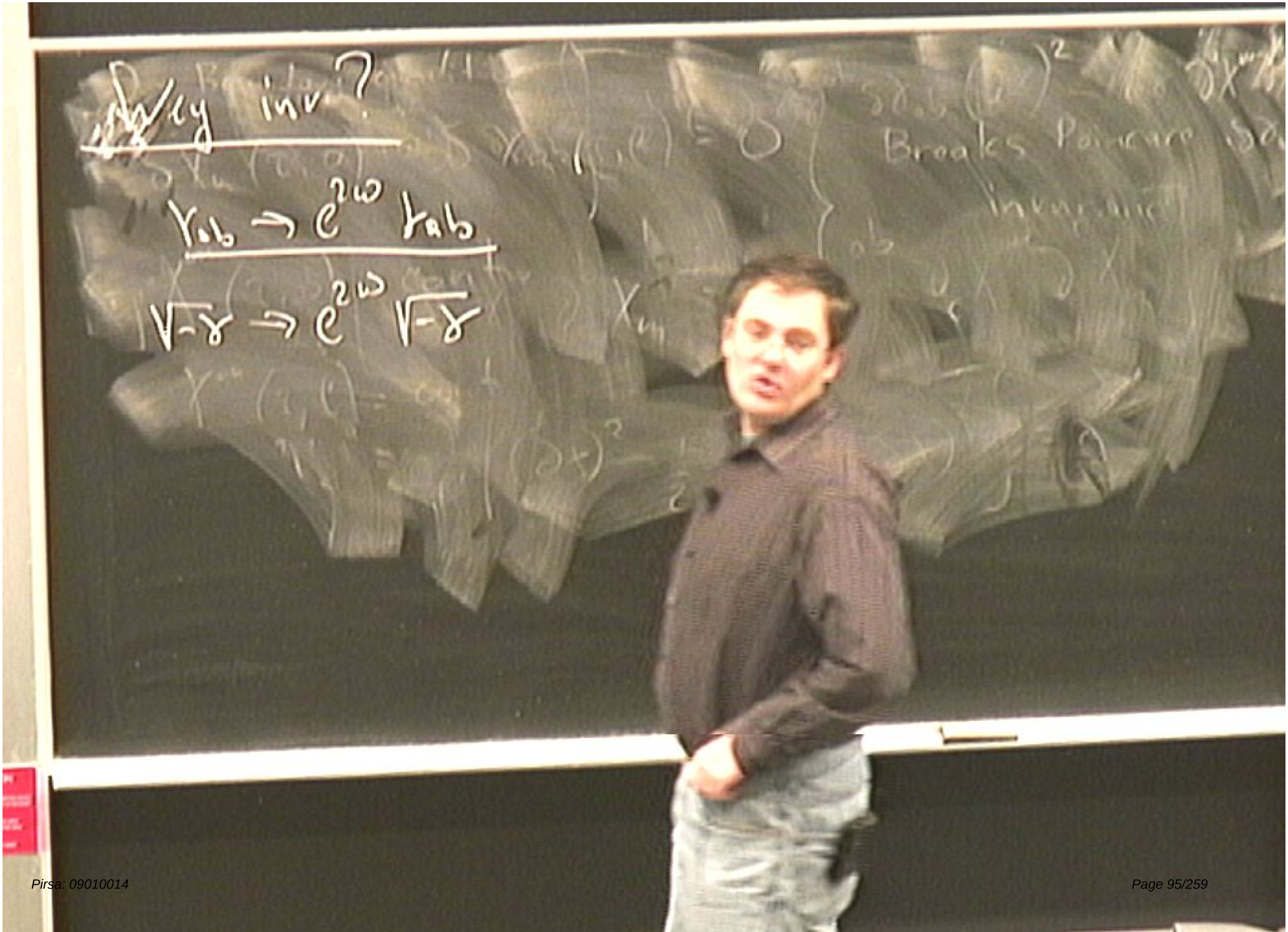
$\sqrt{-\gamma}\,\gamma^{ab}\underline{(\partial X)^2} + \text{something}[\gamma_{ab}]$

$\hookrightarrow$ must be geometric invariants

$\sqrt{-\gamma}$, R

$\Rightarrow \int d\tau\, d\sigma \sqrt{-\gamma}\, R$ potential new term.

(note that δX^μ must be periodic as well!) Closed strings


Weyl inv?
$\gamma_{ab} \to e^{2\omega} \gamma_{ab}$
$\sqrt{-\gamma} \to e^{2\omega} \sqrt{-\gamma}$

Weyl inv?

$\gamma_{ab} \to e^{2\omega} \gamma_{ab}$

$\sqrt{-\gamma} \to e^{2\omega} \sqrt{-\gamma}$

Breaks Poincaré

Why inv?

$$\gamma_{ab} \to e^{2\omega} \gamma_{ab}$$

$$\sqrt{-\gamma} \to e^{2\omega} \sqrt{-\gamma}$$

$S^2 \to$ of radius 1

Weyl inv?

$$\gamma_{ab} \to e^{2\omega} \gamma_{ab}$$

$$\sqrt{-\gamma} \to e^{2\omega} \sqrt{-\gamma}$$

$S^2 \to$ of radius 1

$R = 2$

Why inv?

$$\gamma_{ab} \to e^{2\omega}\gamma_{ab}$$

$$\sqrt{-\gamma} \to e^{2\omega}\sqrt{-\gamma}$$

$S^2 \to$ of radius 1

$$R = \frac{2}{1}$$

radius of the sphere $\to L$

$$\frac{2}{L^2}$$

Weyl inv?

$$\underline{\gamma_{ab} \to e^{2\omega} \gamma_{ab}}$$

$$\sqrt{-\gamma} \to e^{2\omega} \sqrt{-\gamma}$$

$S^2 \to$ of radius 1

$$R = \frac{2}{1}$$

radius of the sphere $\to L$

$$R = \frac{2}{L^2}$$

$$g_{\mu\nu} \to L^2 g_{\mu\nu}$$

Why inv?

$\gamma_{ab} \to e^{2\omega} \gamma_{ab}$

$\sqrt{-\gamma} \to e^{2\omega} \sqrt{-\gamma}$

$R \to R \cdot e^{-2\omega}$

$S^2 \to$ of radius 1

$R = \frac{2}{1}$

radius of the sphere $\to L$

$R = \frac{2}{L^2}$

$g_{\mu\nu} \to L^2 g_{\mu\nu}$

Weyl inv?

$$\gamma_{ab} \to e^{2\omega}\gamma_{ab}$$

$$\sqrt{-\gamma} \to e^{2\omega}\sqrt{-\gamma}$$

$$\to R \to R \cdot e^{-2\omega}$$

If $\partial\omega = 0$

$S^2 \to$ of radius 1

$$R = \frac{2}{1}$$

radius of the sphere $\to L$

$$R = \frac{2}{L^2}$$

$$g_{\mu\nu} \to L^2 g_{\mu\nu}$$

Weyl inv?

$\gamma_{ab} \to e^{2\omega} \gamma_{ab}$

$\sqrt{-\gamma} \to e^{2\omega} \sqrt{-\gamma}$

$\to R \to e^{-2\omega} \left[R \right.$

If $\partial_i \omega = 0$

$\omega = \omega(\tau, \sigma)$

$S^2 \to$ of radius 1

$R = \frac{2}{1}$

radius of the sphere $\to L$

$R = \frac{2}{L^2}$

$g_{\mu\nu} \to L^2 g_{\mu\nu}$

Weyl inv?

$$\gamma_{ab} \to e^{2\omega} \gamma_{ab}$$

$$\sqrt{-\gamma} \to e^{2\omega} \sqrt{-\gamma}$$

$$R \to e^{-2\omega} \left[R - 2(d-1) \Box \omega - (d-2)(d-1) (\nabla \omega)^2 \right]$$

If $\partial_\tau \omega = 0$

$$\omega = \omega(\tau, \sigma)$$

$S^2 \to$ of radius 1

$$R = \frac{2}{1}$$

radius of the sphere $\to L$

$$R = \frac{2}{L^2}$$

$$g_{\mu\nu} \to L^2 g_{\mu\nu}$$

Weyl inv?

$\gamma_{ab} \to e^{2\omega} \gamma_{ab}$

$\sqrt{-\gamma} \to e^{2\omega} \sqrt{-\gamma}$

$R \to e^{-2\omega} \left[R - 2(d-1) \Box \omega - \cancel{(d-2)(d-1)(\nabla\omega)^2} \right]$

If $R\omega = 0$

$\omega = \omega(\tau, \sigma)$

$d = 2$

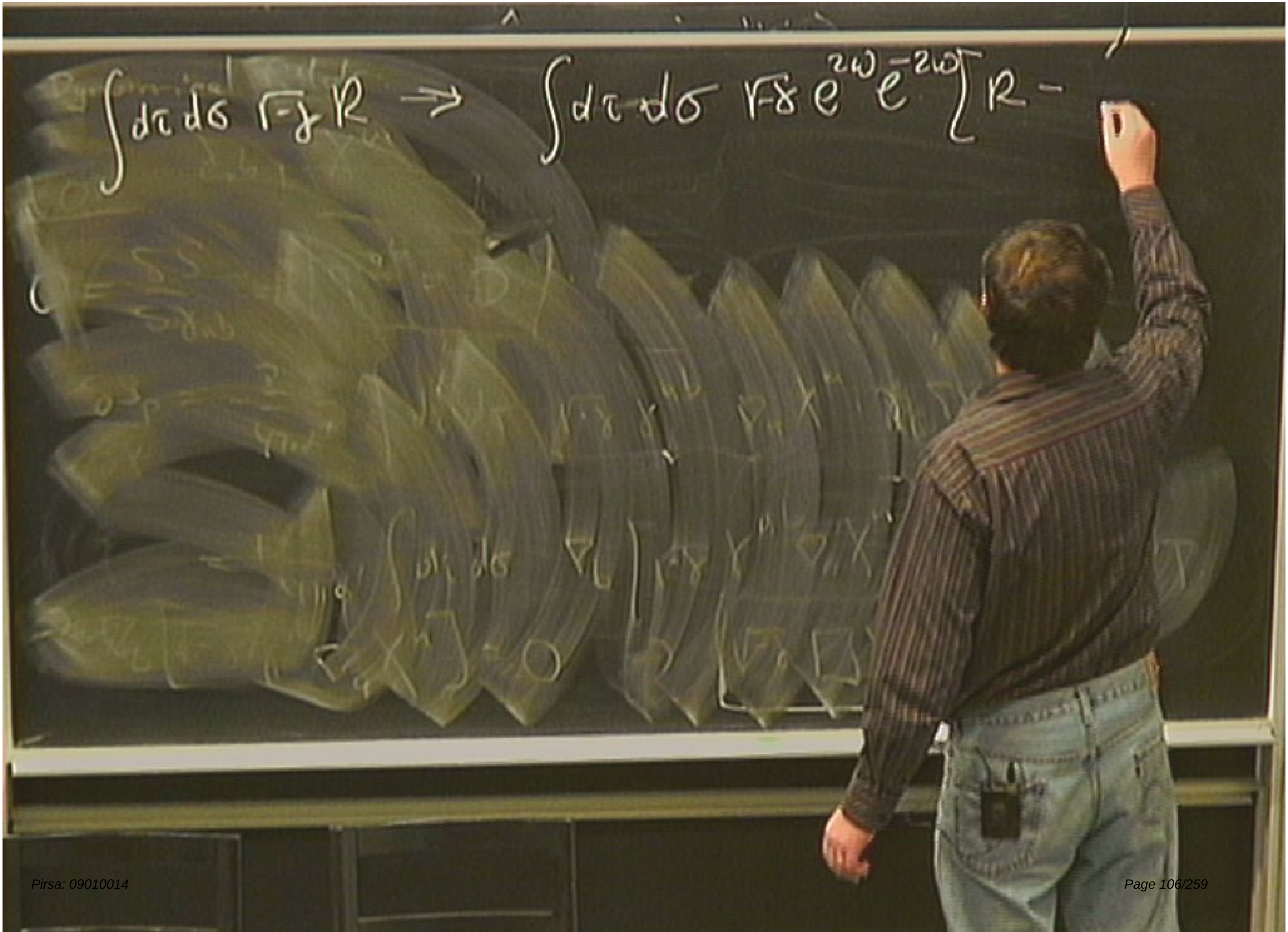
$S^2 \to$ of radius 1

$R = \frac{2}{1}$

radius of the sphere $\to L$

$R = \frac{2}{L^2}$

$g_{\mu\nu} \to L^2 g_{\mu\nu}$


$\int d\tau\, d\sigma \sqrt{-g}\, R \rightarrow \int d\tau\, d\sigma \sqrt{-\gamma}\, e^{2\omega} e^{-2\omega} \left[R - \right.$

$$\int d\tau\, d\sigma\, \sqrt{-g}\, R \rightarrow \int_{M} d\tau\, d\sigma\, \sqrt{-\gamma}\, e^{2\omega} e^{-2\omega} \left[R - 2 \Box \omega \right]$$

$$\int d\tau\, d\sigma \sqrt{-g}\, R \rightarrow \int_{\mathcal{M}} d\tau\, d\sigma \sqrt{-g}\, \cancel{e^{2\omega}}\, \cancel{e^{-2\omega}} \left[R - 2\, \Box \omega \right]$$

Extra term : $-2 \int d\tau\, d\sigma \sqrt{-g}\, \Box \omega$

$$\int d\tau\, d\sigma \sqrt{-g}\, R \Rightarrow \int_{M} d\tau\, d\sigma \sqrt{-\gamma}\, \cancel{e^{2\omega}}\, \cancel{e^{-2\omega}} \left[R - 2 \Box \omega \right]$$

Extra term: $-2 \int d\tau\, d\sigma \sqrt{-\gamma}\, \nabla_a \nabla^a \omega$

$$\int d\tau\, d\sigma \sqrt{-g}\, R \;\rightarrow\; \int_{\mathcal{M}} d\tau\, d\sigma \sqrt{-g}\, \cancel{e^{2\omega}}\, \cancel{e^{-2\omega}} \left[R - 2\,\Box\,\omega \right]$$

Extra term: $-2 \int d\tau d\sigma \sqrt{-g}\, \nabla_a \nabla^a \omega = -2 \int d\tau d\sigma \cdot \nabla_a \left[\sqrt{-g}\, \nabla^a \omega \right]$

$$\int d\tau\, d\sigma \sqrt{-\gamma}\, R \Rightarrow \int_{\mathcal{M}} d\tau\, d\sigma \sqrt{-\gamma}\, \cancel{e^{2\omega}}\, \cancel{e^{-2\omega}} \left[R - 2\,\Box\,\omega \right]$$

<u>Extra term</u>: $-2 \int d\tau\, d\sigma \sqrt{-\gamma}\, \nabla_a \nabla^a \omega = -2 \int d\tau d\sigma \cdot$

$$\nabla_a \left[\sqrt{-\gamma}\, \nabla^a \omega \right] = -2 \int_{\partial \mathcal{M}}$$

$$\int d\tau\, d\sigma\, \sqrt{-g}\, R \Rightarrow \int_{M} d\tau\, d\sigma\, \sqrt{-g}\, \cancel{e^{2\omega}}\, \cancel{e^{-2\omega}} \left[R - 2\,\Box\omega \right]$$

Extra term: $-2\int d\tau\, d\sigma\, \sqrt{-g}\, \nabla_a \nabla^a \omega = -2\int d\tau\, d\sigma\, \nabla_a \left[\sqrt{-g}\, \nabla^a \omega \right] = -2 \int_{\partial M} n_a \cdot \left(\sqrt{-g}\, \nabla^a \omega \right)$

$$\int d\tau\, d\sigma \sqrt{-\gamma}\, R \Rightarrow \int_{M} d\tau\, d\sigma \sqrt{-\gamma}\, \cancel{e^{2\omega}}\, \cancel{e^{-2\omega}} \left[R - 2 \Box \omega \right]$$

Extra term: $-2 \int d\tau\, d\sigma \sqrt{-\gamma}\, \nabla_a \nabla^a \omega = -2 \int d\tau\, d\sigma \cdot \nabla_a \left[\sqrt{-\gamma}\, \nabla^a \omega \right] = -2 \int_{\partial M} n_a \cdot \left(\sqrt{-\gamma}\, \nabla^a \omega \right)$

$= 0$

$$\int d\tau\, d\sigma \sqrt{-g}\, R \Rightarrow \int_{M} d\tau\, d\sigma \sqrt{-g}\, \cancel{e^{2\omega}}\, \cancel{e^{-2\omega}} \left[R - 2\,\Box\,\omega \right]$$

Extra term: $-2 \int d\tau\, d\sigma \sqrt{-g}\, \nabla_a \nabla^a \omega = -2 \int d\tau\, d\sigma\, \nabla_a \left[\sqrt{-g}\, \nabla^a \omega \right] = -2 \int_{\partial M} n_a \cdot \left(\sqrt{-g}\, \nabla^a \omega \right)$

$= 0$ closed string.

$$\int d\tau\, d\sigma \sqrt{-g}\, R \Rightarrow \int_{\mathcal{M}} d\tau\, d\sigma \sqrt{-g}\, \cancel{e^{2\omega}}\, \cancel{e^{-2\omega}} \left[R - 2\,\Box\,\omega \right]$$

<u>Extra term</u>: $-2\int d\tau d\sigma \sqrt{-g}\, \nabla_a \nabla^a \omega = -2\int d\tau d\sigma\, \nabla_a \left[\sqrt{-g}\, \nabla^a \omega\right] = -2\int_{\partial\mathcal{M}} n_a \cdot \left(\sqrt{-g}\, \nabla^a \omega\right)$

$= 0$ closed string. $(\partial M = 0)$

$$\int d\tau\, d\sigma \sqrt{-g}\, R \rightarrow \int_{M} d\tau\, d\sigma \sqrt{-g}\, \cancel{e^{2\omega}}\, \cancel{e^{-2\omega}} \left[R - 2\, \Box \omega \right]$$

Extra term: $-2 \int d\tau\, d\sigma \sqrt{-g}\, \nabla_a \nabla^a \omega = -2 \int d\tau\, d\sigma \cdot \nabla_a \left[\sqrt{-g}\, \nabla^a \omega \right] = -2 \int_{\partial M} n_a \cdot \left(\sqrt{-g}\, \nabla^a \omega \right)$

$= 0$ closed string. $(\partial M = 0)$

$$\int d\tau\, d\sigma \sqrt{-g}\, R \to \int_M d\tau\, d\sigma \sqrt{-g}\, \cancel{e^{2\omega}}\, \cancel{e^{-2\omega}} \left[R - 2 \Box \omega \right]$$

Extra term: $-2 \int d\tau d\sigma \sqrt{-g}\, \nabla_a \nabla^a \omega = -2 \int d\tau d\sigma\, \nabla_a \left[\sqrt{-g}\, \nabla^a \omega \right] = -2 \int_{\partial M} n_a \cdot \left(\sqrt{-g}\, \nabla^a \omega \right)$

$= 0$ closed string $(\partial M = 0)$

→ possible (but nontrivial) to generalize to open strings

$$\int d\tau\, d\sigma \sqrt{-\gamma}\, R \Rightarrow \int_{\mathcal{M}} d\tau\, d\sigma \sqrt{-\gamma}\, \cancel{e^{2\omega}}\, \cancel{e^{-2\omega}} \left[R - 2\,\Box\,\omega \right]$$

Extra term: $-2\int d\tau d\sigma \sqrt{-\gamma}\, \nabla_a \nabla^a \omega = -2\int d\tau d\sigma\, \nabla_a \left[\sqrt{-\gamma}\, \nabla^a \omega \right] = -2 \int_{\partial \mathcal{M}} n_a \cdot \left(\sqrt{-\gamma}\, \nabla^a \omega \right)$

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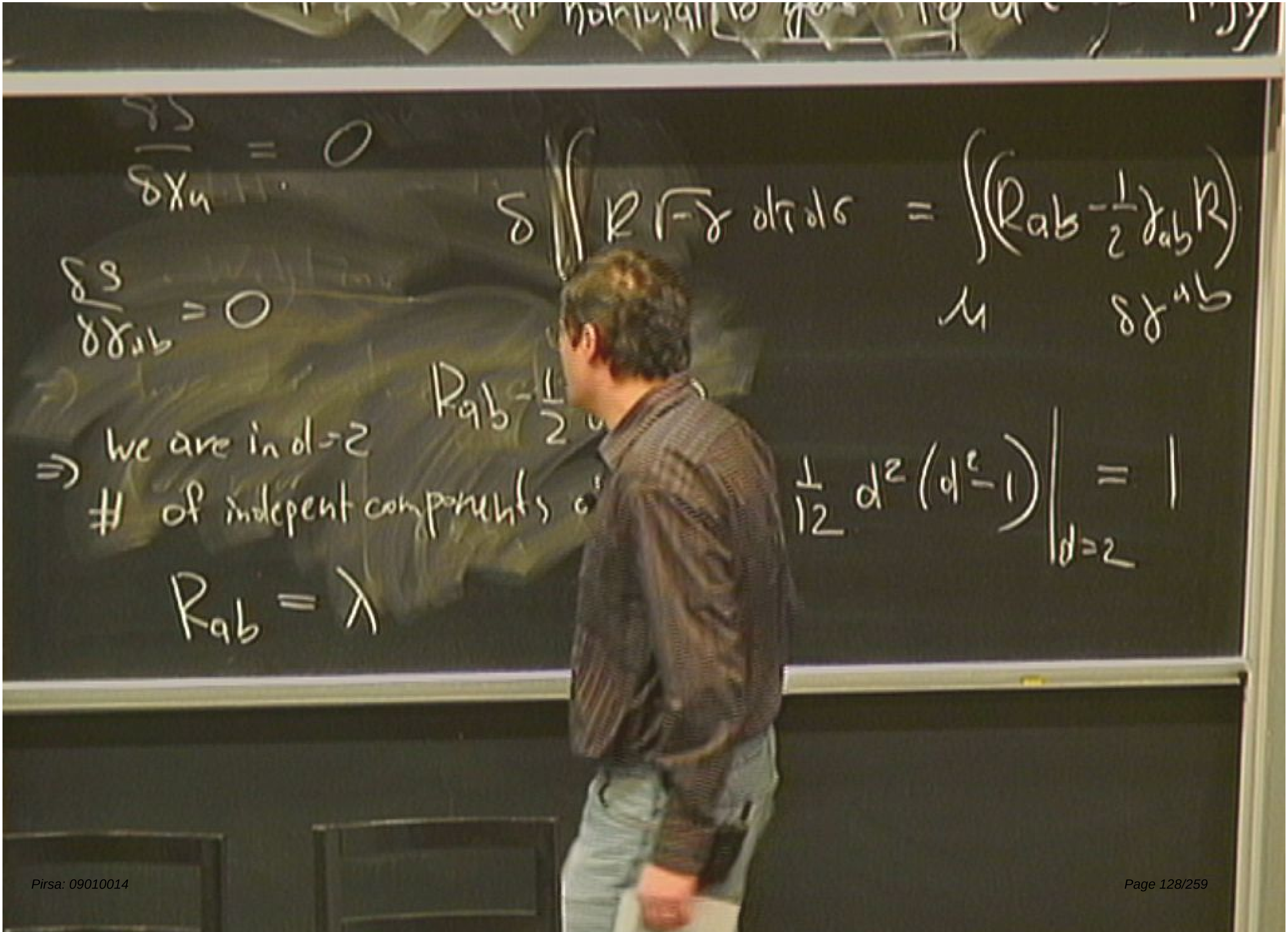
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# of indepent components of
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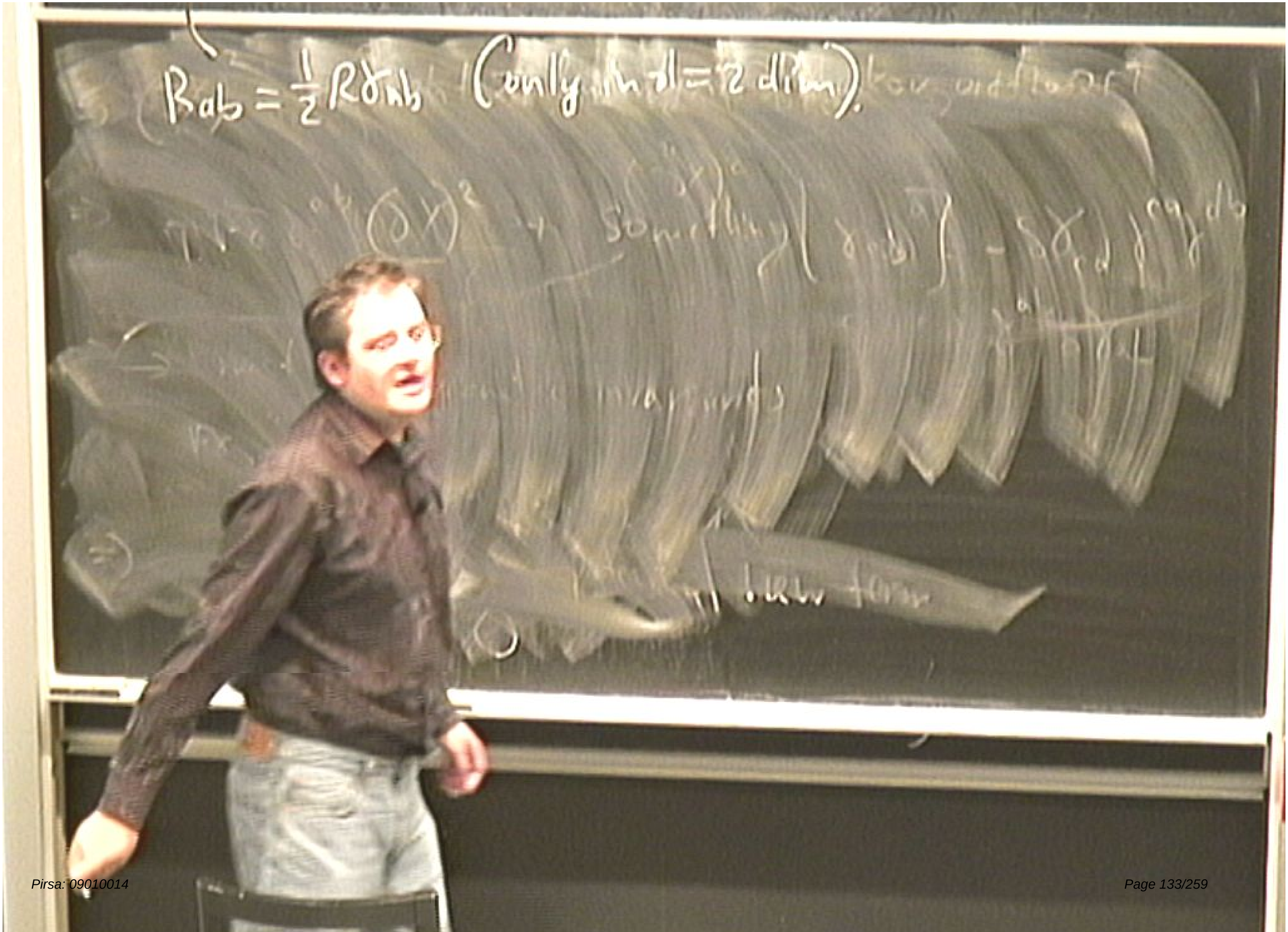
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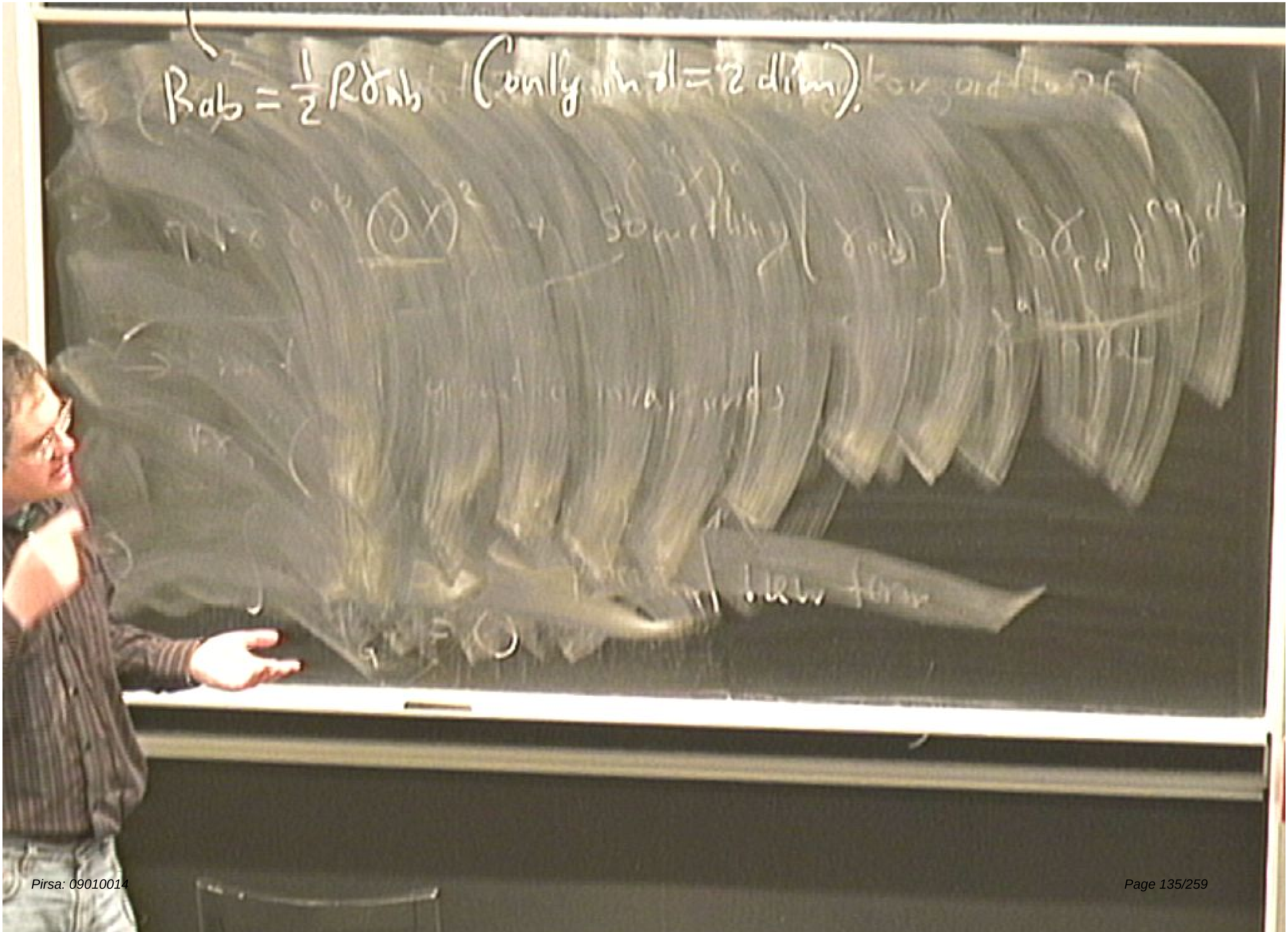
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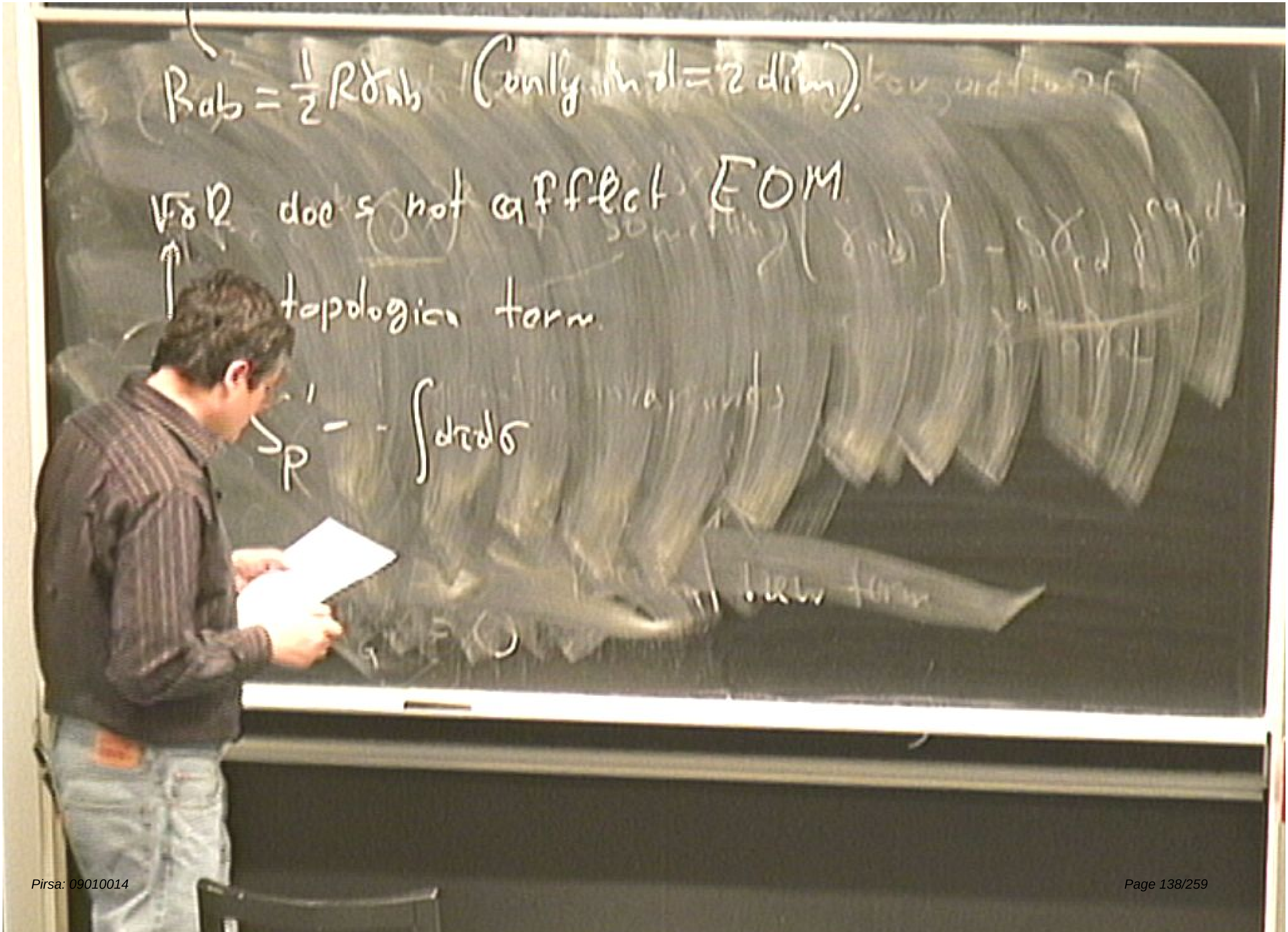
Rab = 1/2 R γab (only in d=2 dim).
√γ R does not affect EOM
topological term
S_R' = ∫dτdσ
new term

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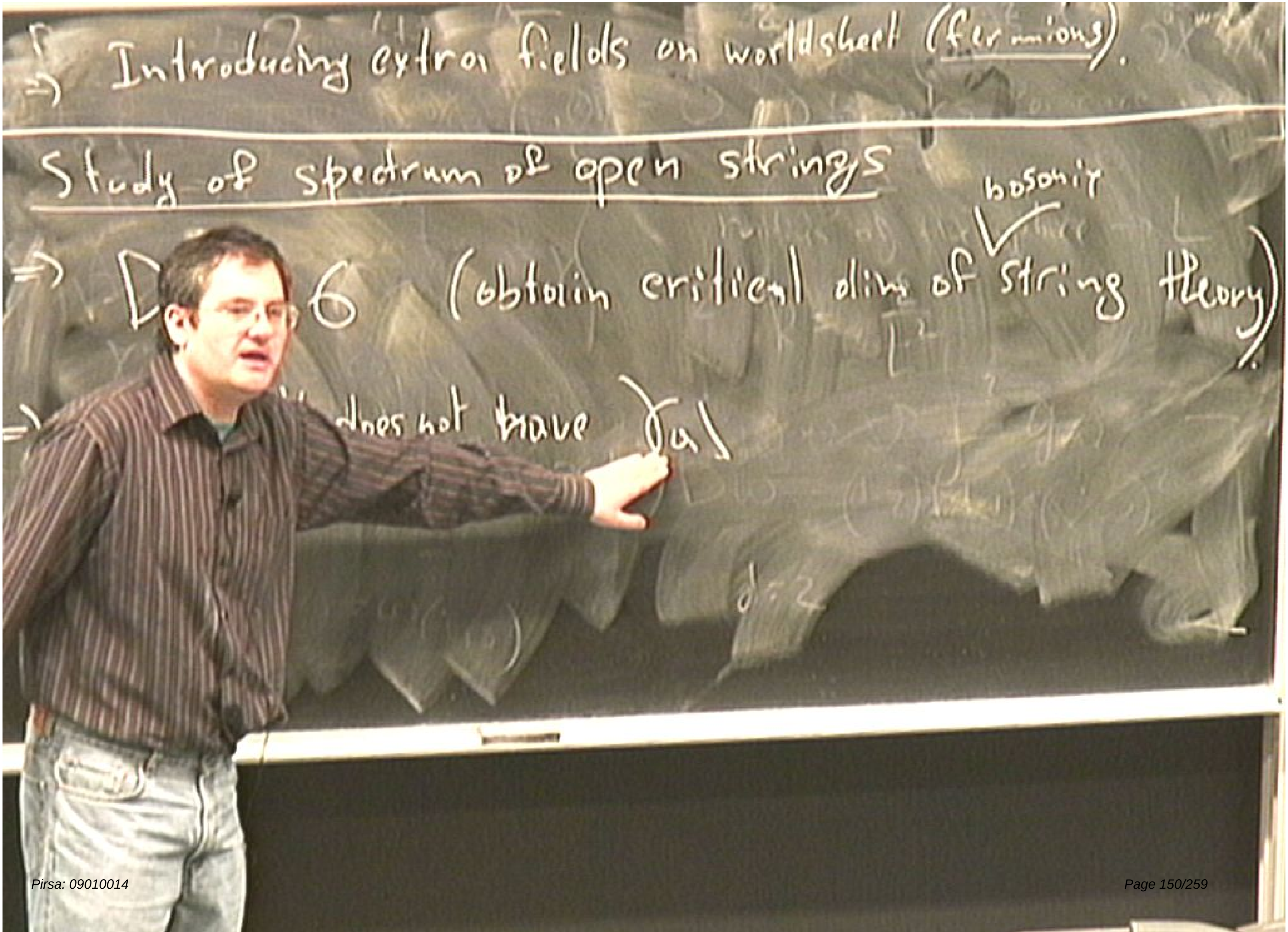
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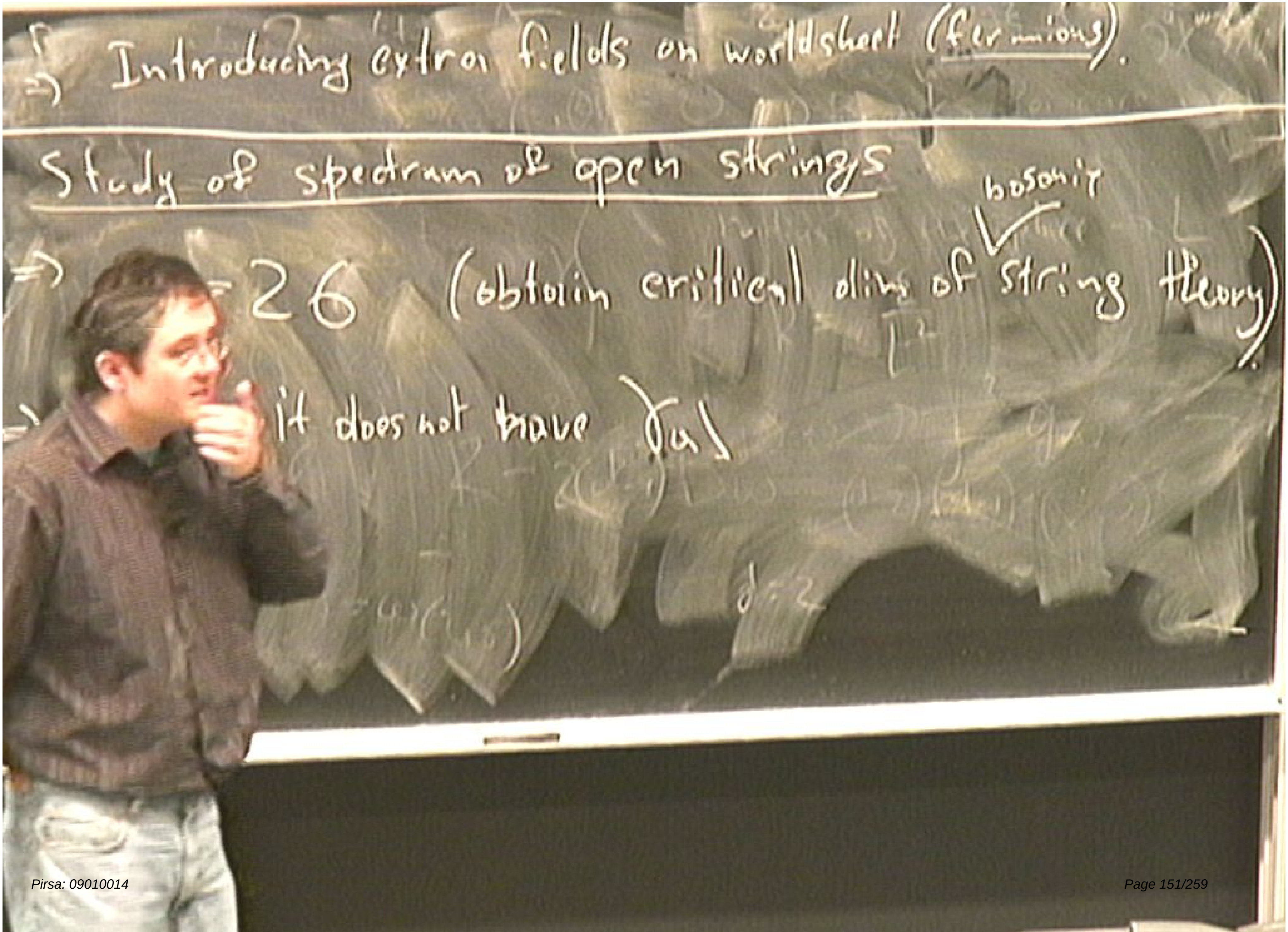
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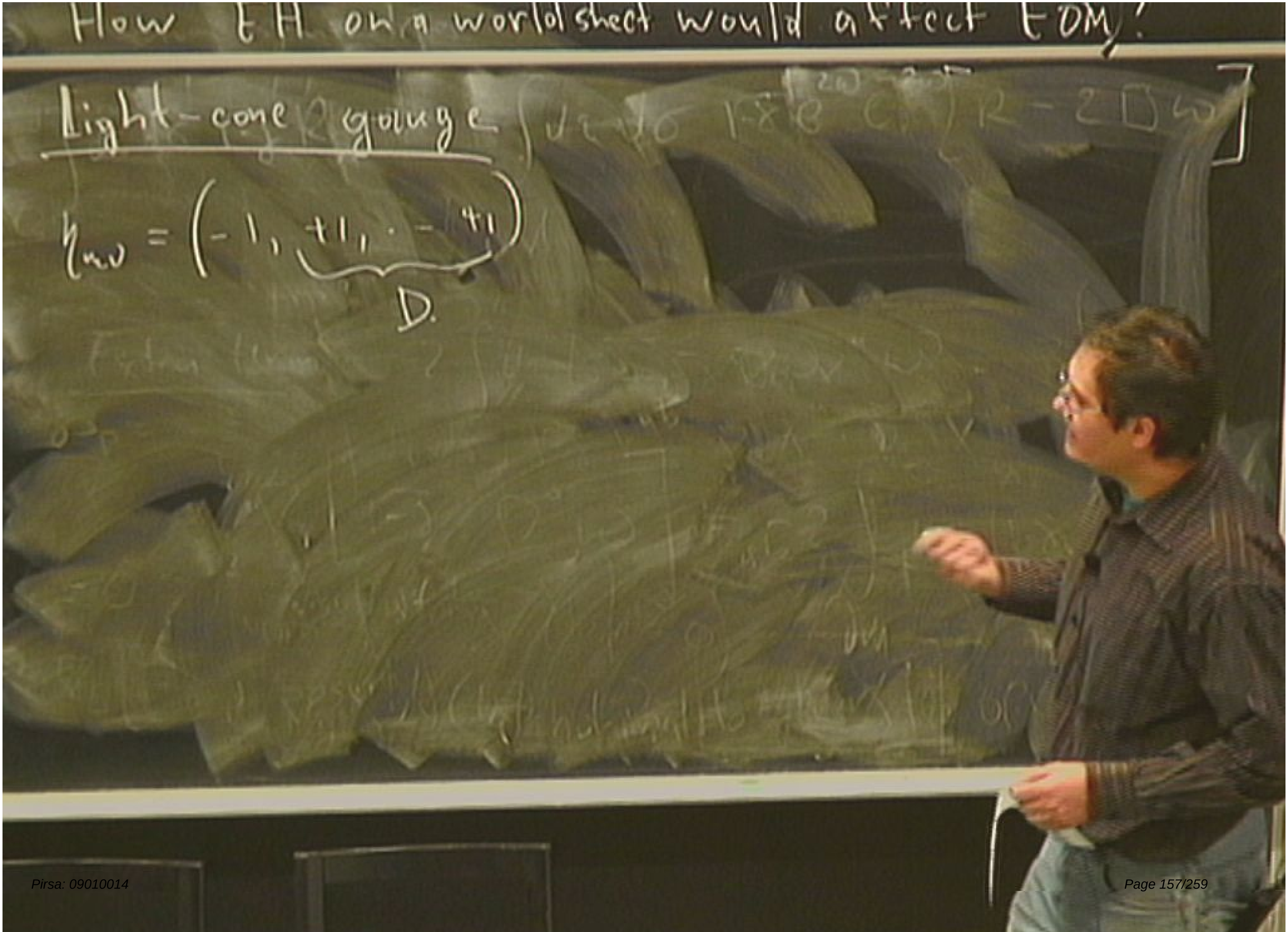
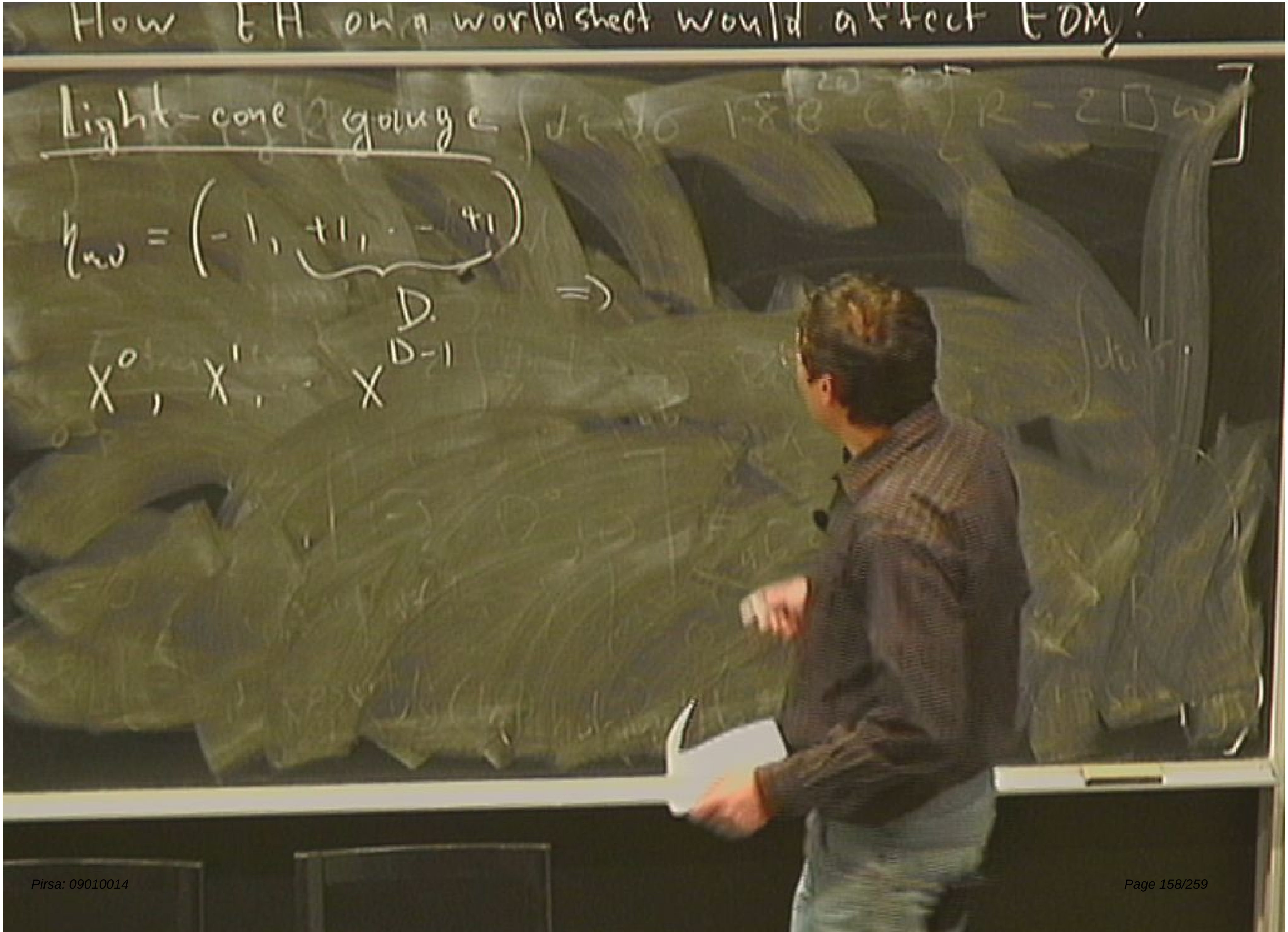

How ... on a worldsheet would affect EOM?
light-cone gauge
$\eta_{\mu\nu} = (-1, \underbrace{+1, \dots +1}_{D})$


How EH on a worldsheet would affect EOM?
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D
=)
X^0, X^1, . . X^{D-1}

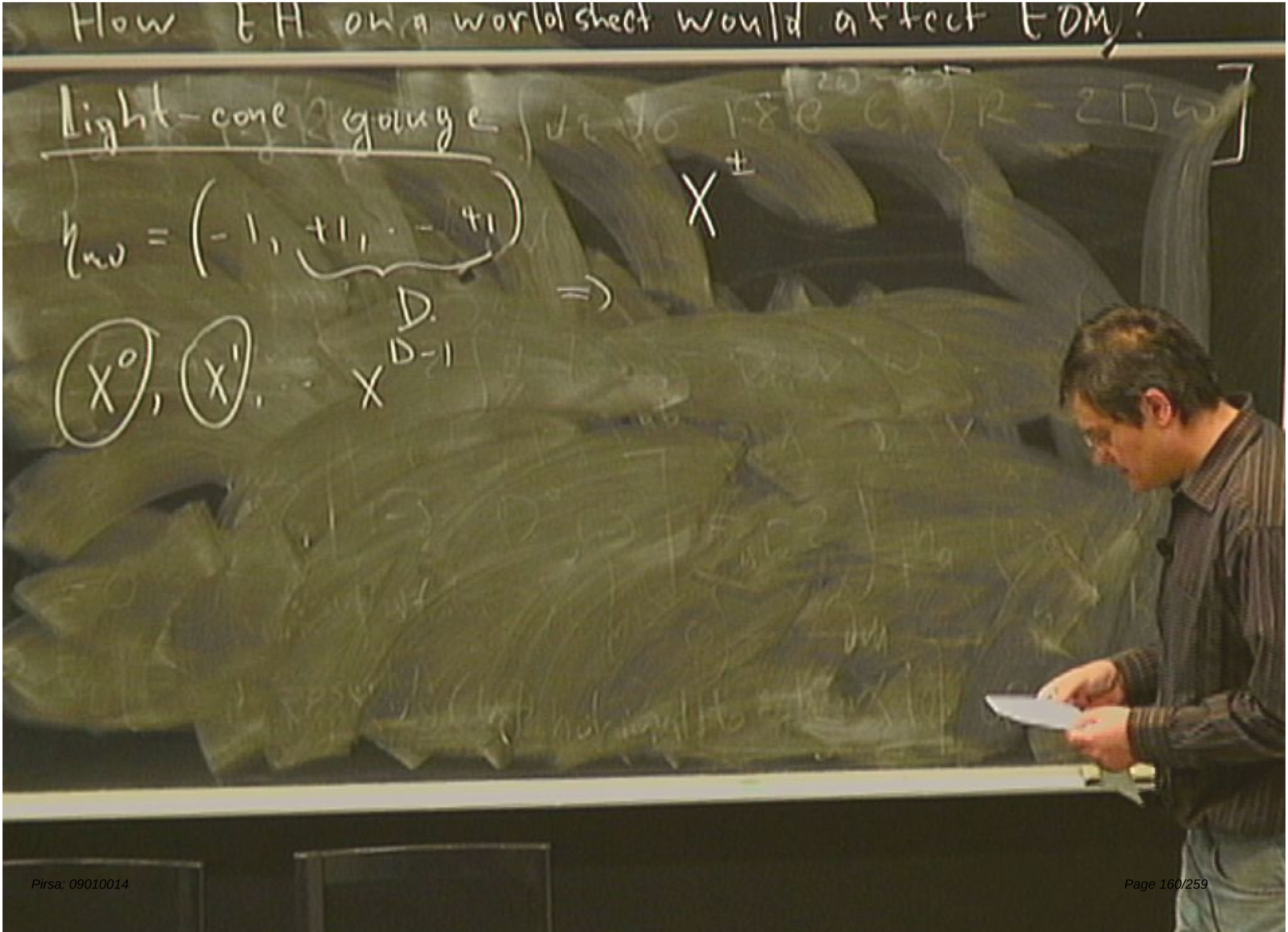
How EH on a worldsheet would affect EOM?

Light-cone gauge

$$\eta_{\mu\nu} = (-1, \underbrace{+1, \cdots +1}_{D})$$

$$\Rightarrow X^{\pm}$$

$$X^0, X^1, \cdots X^{D-1}$$


How tH on a worldsheet would affect EOM?
Light-cone gauge
η_μν = (-1, +1, ... +1)
D
X^0, X^1, ... X^(D-1)
=>
X^±

How EH on a worldsheet would affect EOM?

Light-cone gauge

$$\eta_{\mu\nu} = (-1, \underbrace{+1, \cdots +1}_{D})$$

$$X^0, X^1, \cdots X^{D-1}$$

$\Rightarrow$

$$X^{\pm} = \frac{1}{\sqrt{2}}\left(X^0 \pm X^1\right)$$

How GA on a worldsheet would affect EOM?

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$X^{0}, X^{1}, \cdots X^{D-1}$

$a^{\mu} b_{\mu}$

How EH on a worldsheet would affect EOM?

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$$a^{\mu} b_{\mu} = -a^0 b^0 + a^1 b^1 + \cdots + a^{D-1} b^{D-1}$$

$$=$$

How EA on a worldsheet would affect EOM?

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Light-cone gauge

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QM
{t, x}

QM

$$\{t, x\} \xrightarrow[\text{variable}]{\text{conjugate}} \{H, P\}$$

QM

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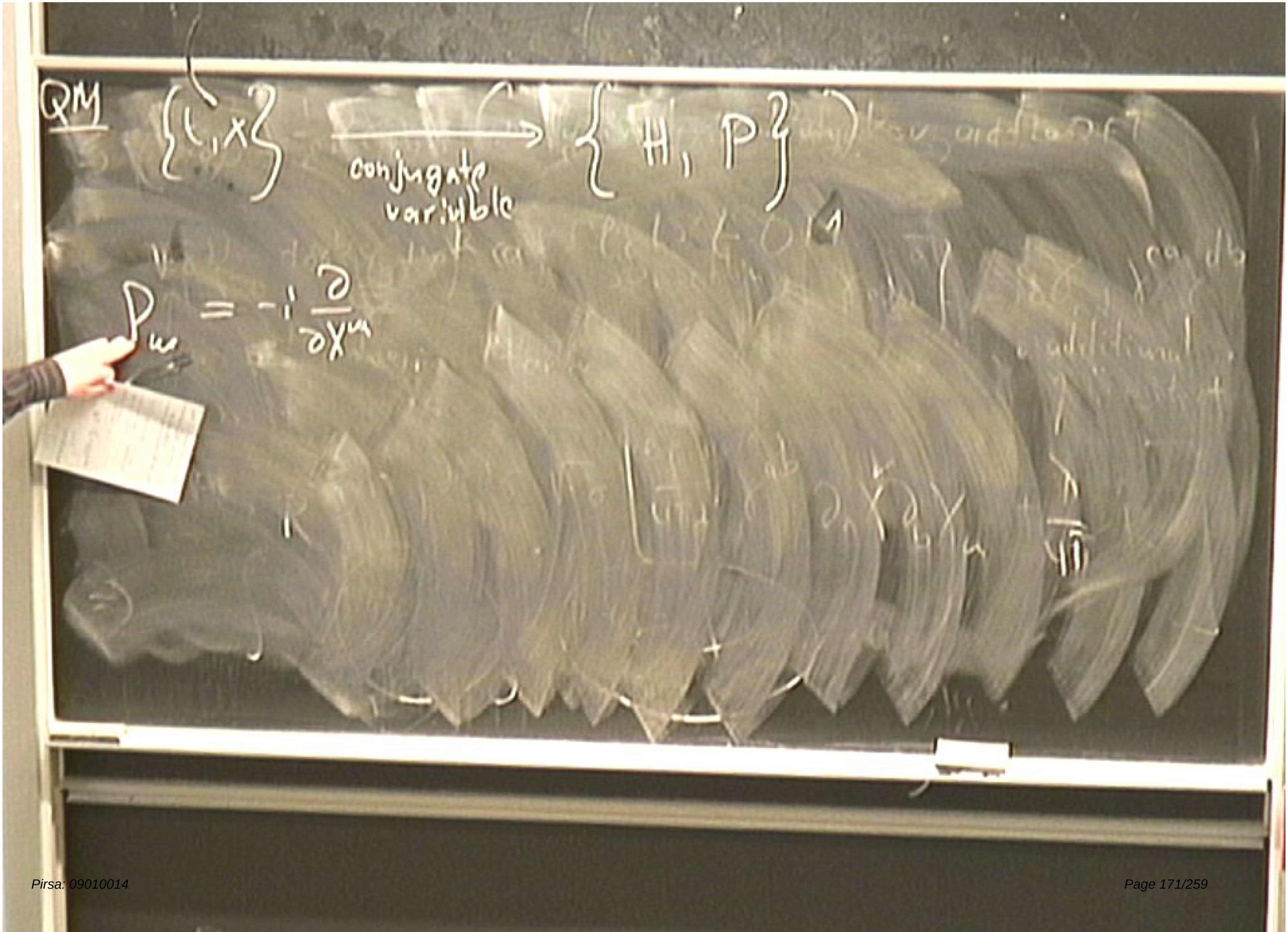
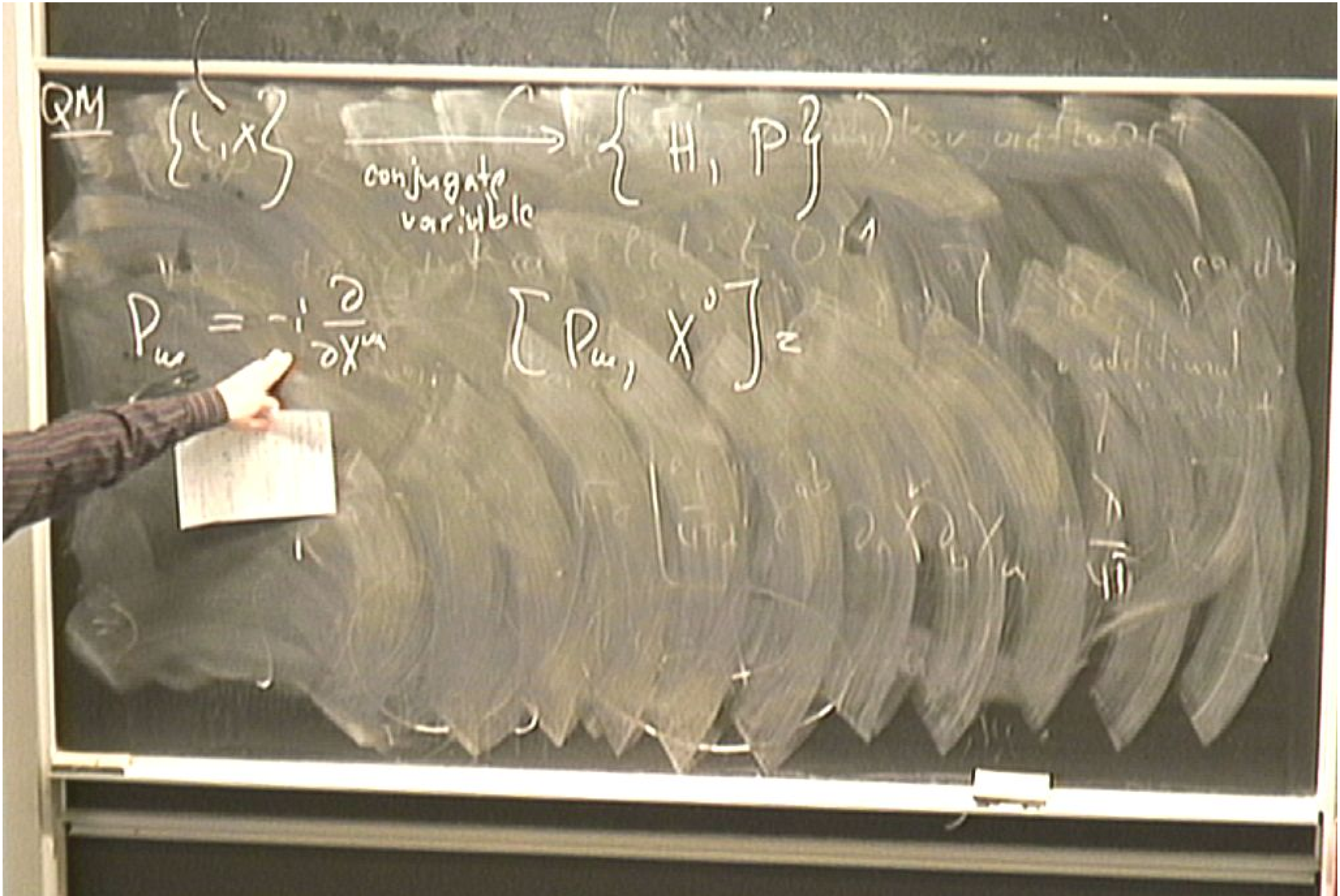
$$P_w =$$


QM
{t, x}
conjugate variable
{H, P}
$P_\mu = -i \frac{\partial}{\partial x^\mu}$


QM
{t, x} → {H, P}
conjugate variable
P_μ = -i ∂/∂x^μ
[P_μ, x^0] =

QM

$$\{t, x\} \xrightarrow[\text{conjugate variable}]{} \{H, P\}$$

$$P_\mu = -i\frac{\partial}{\partial x^\mu} \qquad [P_\mu, X^\nu] = -i\,\delta_\mu^\nu$$

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QM

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QM $\{t, x\} \xrightarrow[\text{conjugate variable}]{} \{H, P\}$

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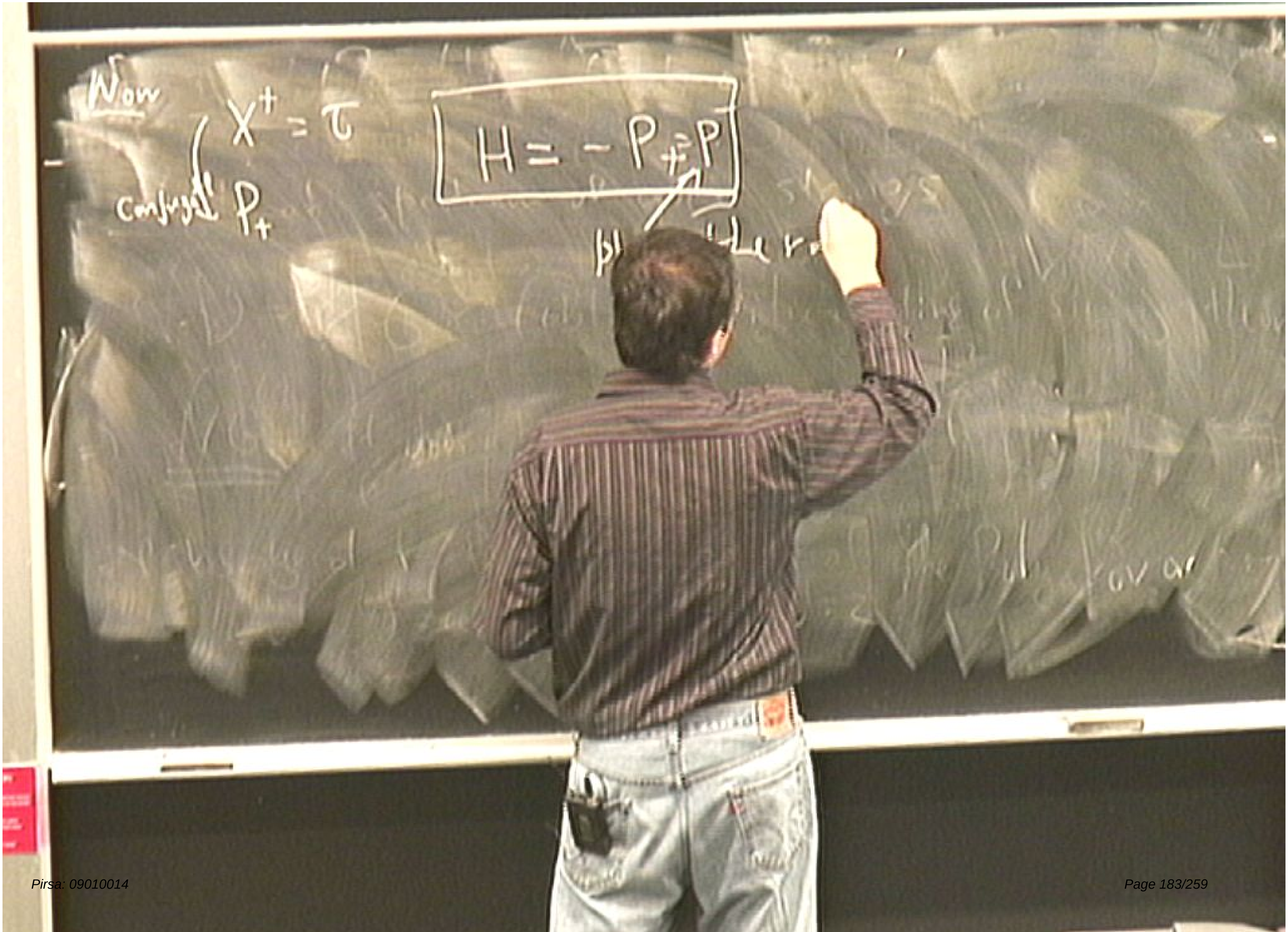
$H = i\frac{\partial}{\partial t}$

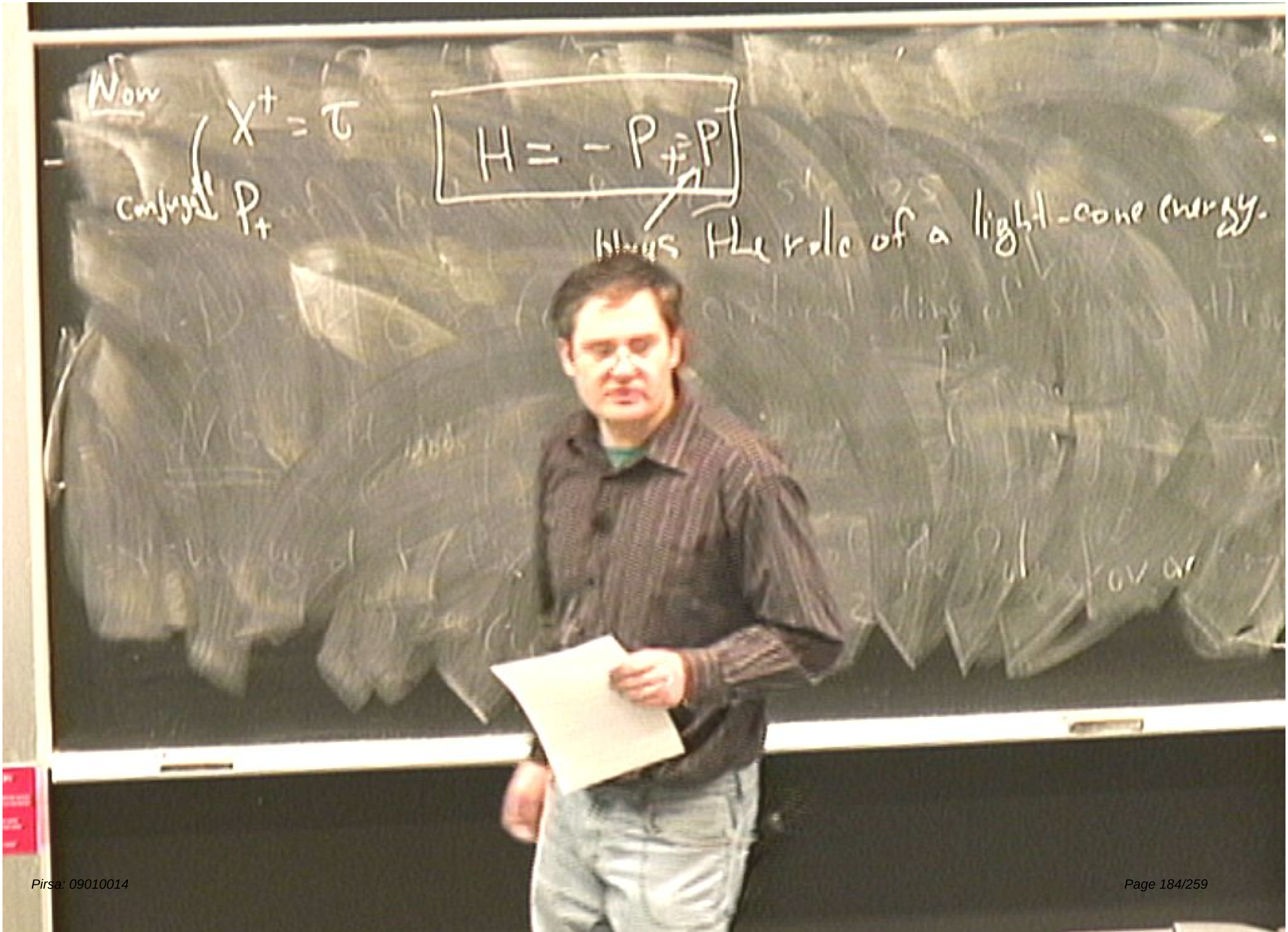
$$\boxed{H = -P_0}$$

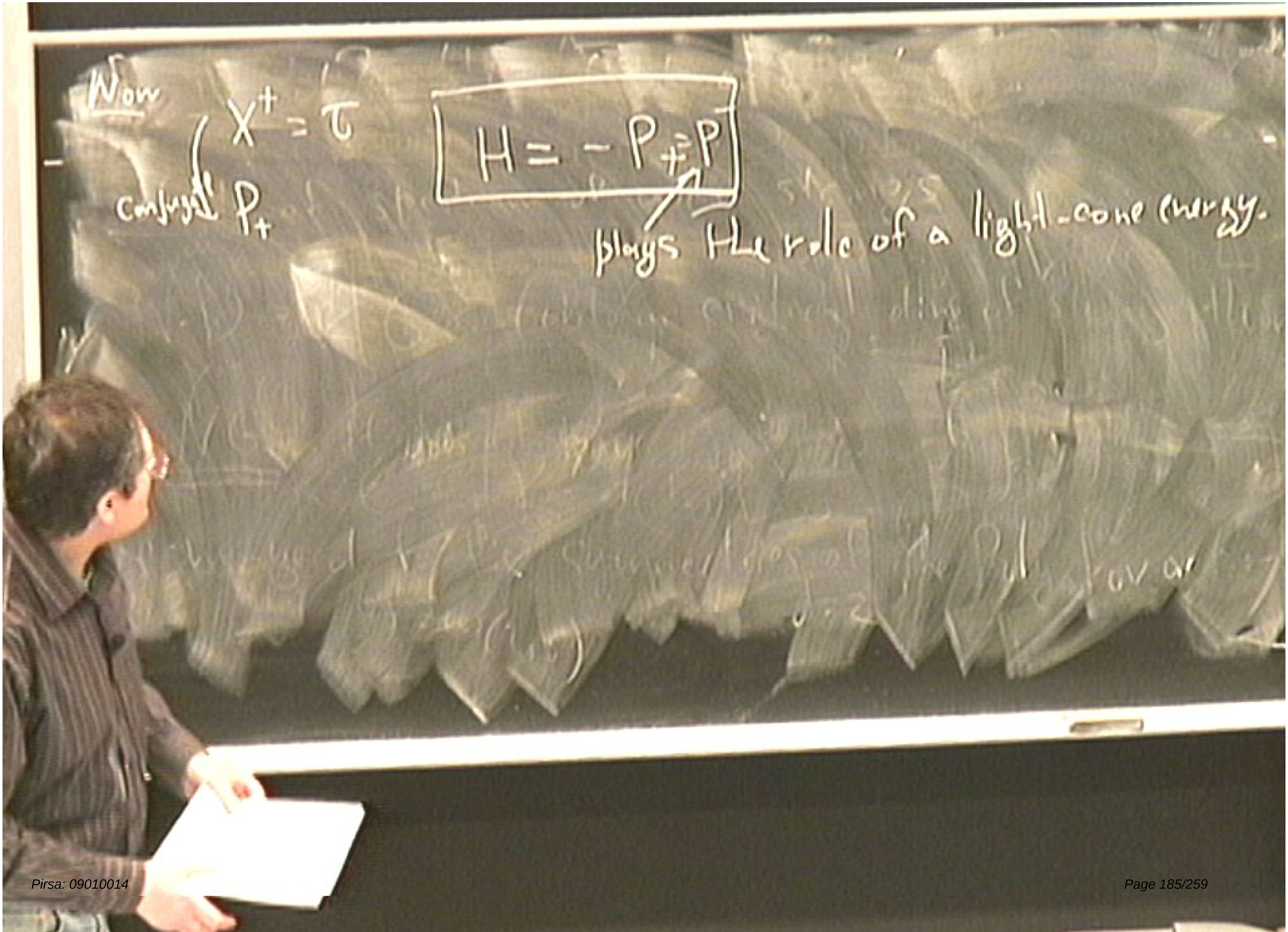
Now $\left(X^+ = \tau \right.$

conjugate P_+

$$\boxed{H = -P_+ = P^-}$$


Now
X+ = τ
P+
H = − P+ = P−


Now
X⁺ = τ
conjugate P₊
H = − P₊ = P⁻
plays the role of a light-cone energy.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Now

$$\begin{cases} X^+ = \tau \\ \text{conjugate } P_+ \end{cases} \qquad \boxed{H = -P_+ = P^-}$$

plays the role of a light-cone energy.

$$S'_{pp} = \frac{1}{2} \int d\tau \left[\eta^{-1} \dot{X}^m \dot{X}_m - \eta m^2 \right]$$

Now

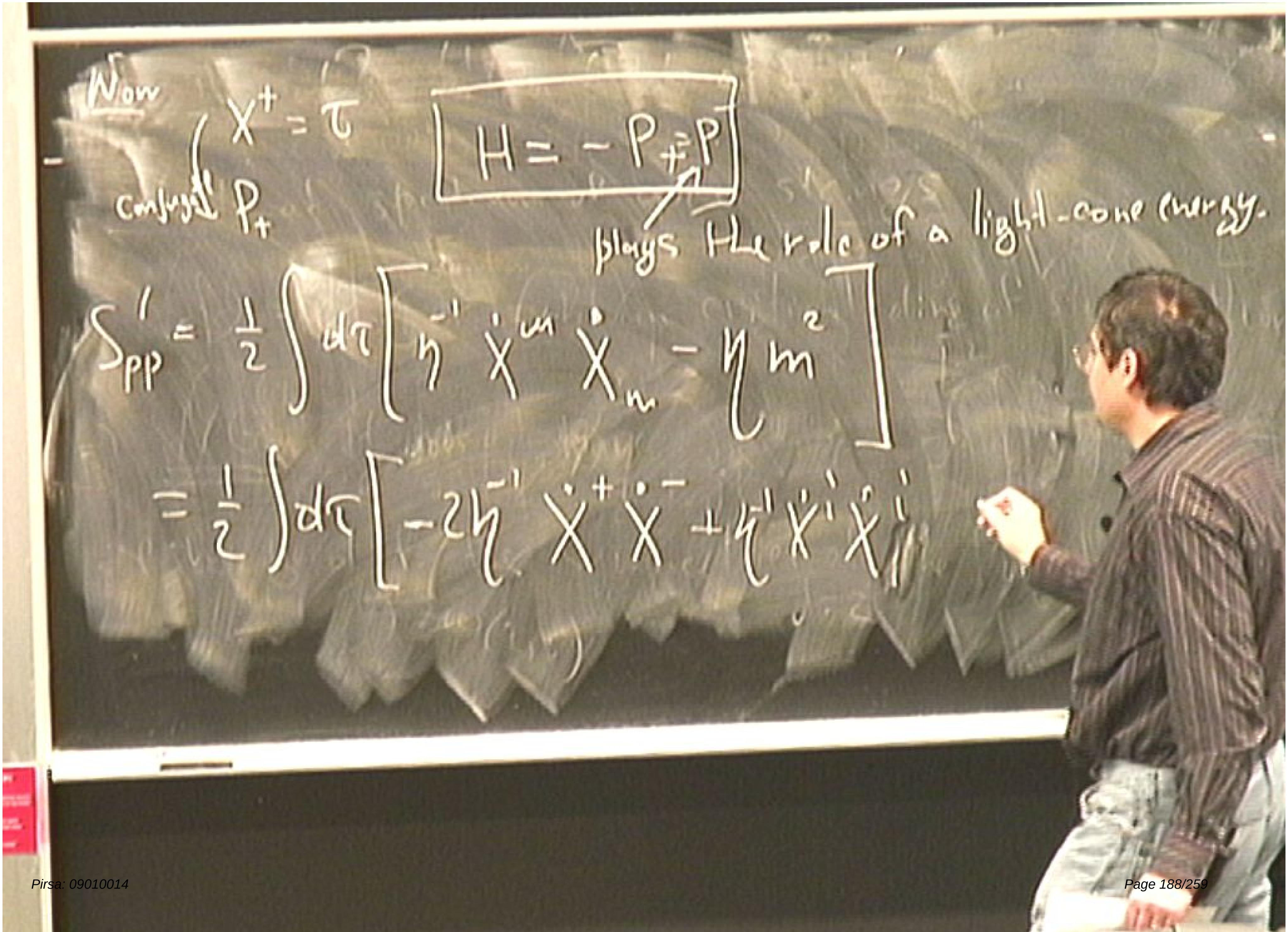
$$X^+ = \tau \qquad \boxed{H = -P_+ = P^-}$$

conjugate P_+

plays the role of a light-cone energy.

$$S'_{pp} = \frac{1}{2}\int d\tau \left[\eta^{-1} \dot{X}^m \dot{X}_m - \right]$$

$$= \frac{1}{2}\int d\tau \left[-2\eta^{-1} \dot{X}^+ \dot{X}^- \right.$$


Now
$X^+ = \tau$
$H = -P_+ = P^-$
conjugate P_+
plays the role of a light-cone energy.
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Now

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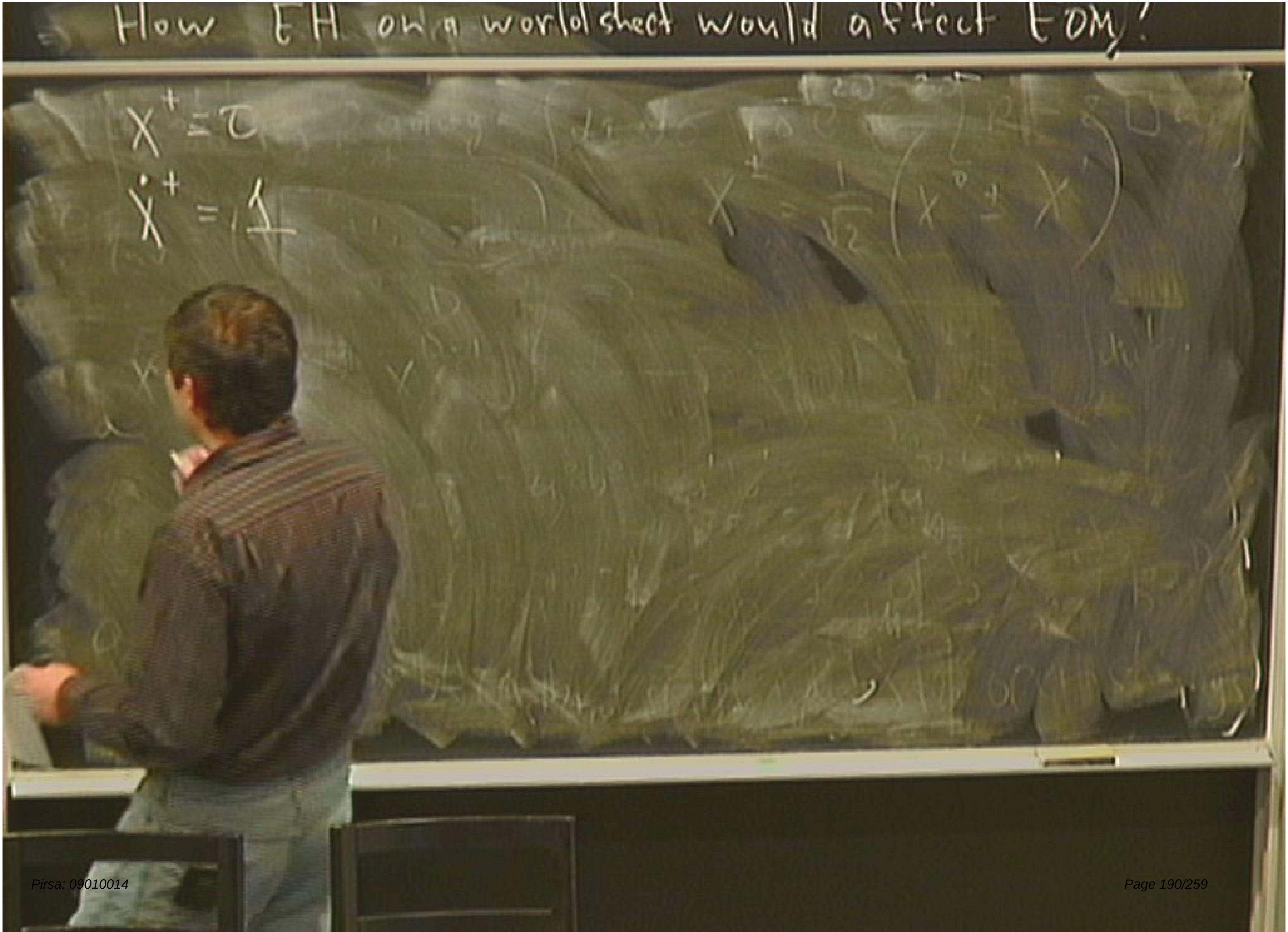
How EH on a worldsheet would affect EOM?
$X^+ = \tau$
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$X^{\pm} = \frac{1}{\sqrt{2}}(X^0 \pm X^1)$

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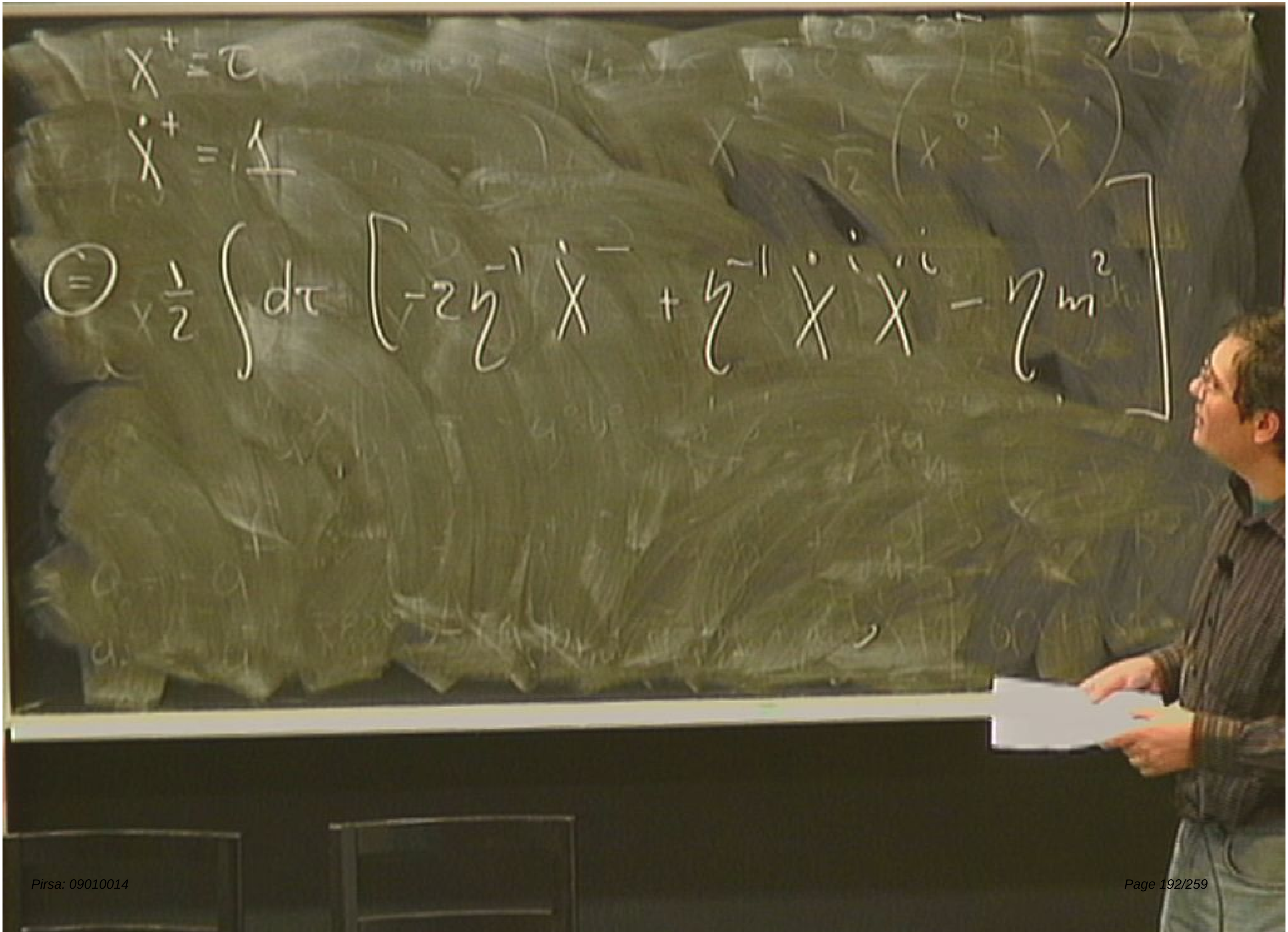
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Naively: η, X^i

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$$S = \frac{1}{2}\int d\tau \left[-2\eta^{-1}\dot{X}^- + \eta^{-1}\dot{X}^i\dot{X}^i - \eta m^2\right]$$

Naively: η, X^i

$$P_\eta =$$

$$X^+ = \tau \qquad X^{\pm} = \frac{1}{\sqrt{2}}\left(X^0 \pm X^1\right)$$

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Naively: $\eta,\ X^i$

$$P_\mu = \frac{\partial L}{\partial \dot{X}^\mu}$$

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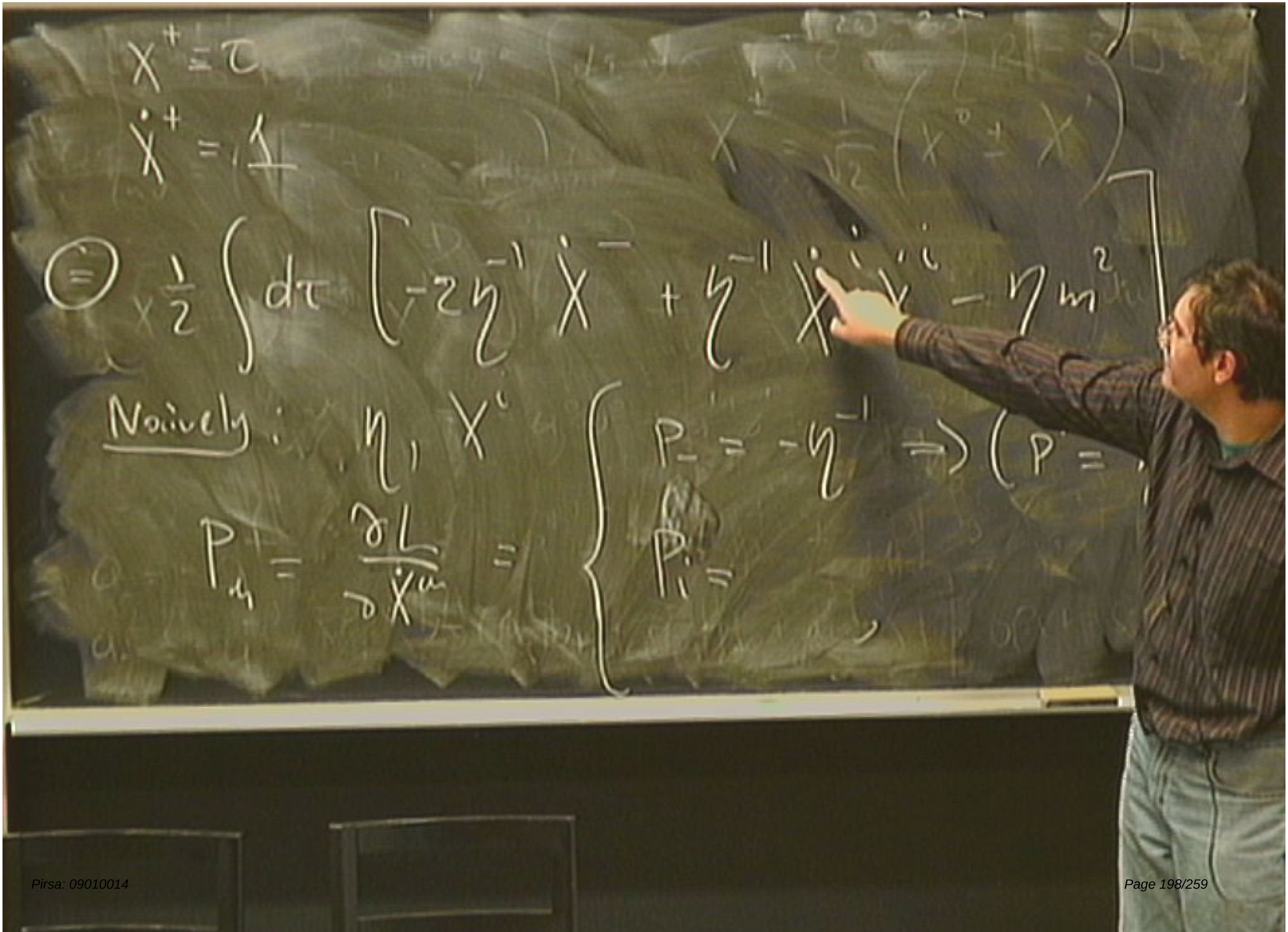

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$$\dot{X}^{-} + \eta^{-1}\dot{X}^{i}\dot{X}^{i} - \eta m^{2}\Big]$$

$$\begin{cases} P_{-} = -\eta^{-1} \Rightarrow \left(P^{+} = \eta^{-1}\right), \\ P_{i} = \eta^{-1}\dot{X}^{i} \Rightarrow \dot{X}^{i} = \eta \cdot P_{i} = \frac{P_i}{P^{+}} \end{cases}$$

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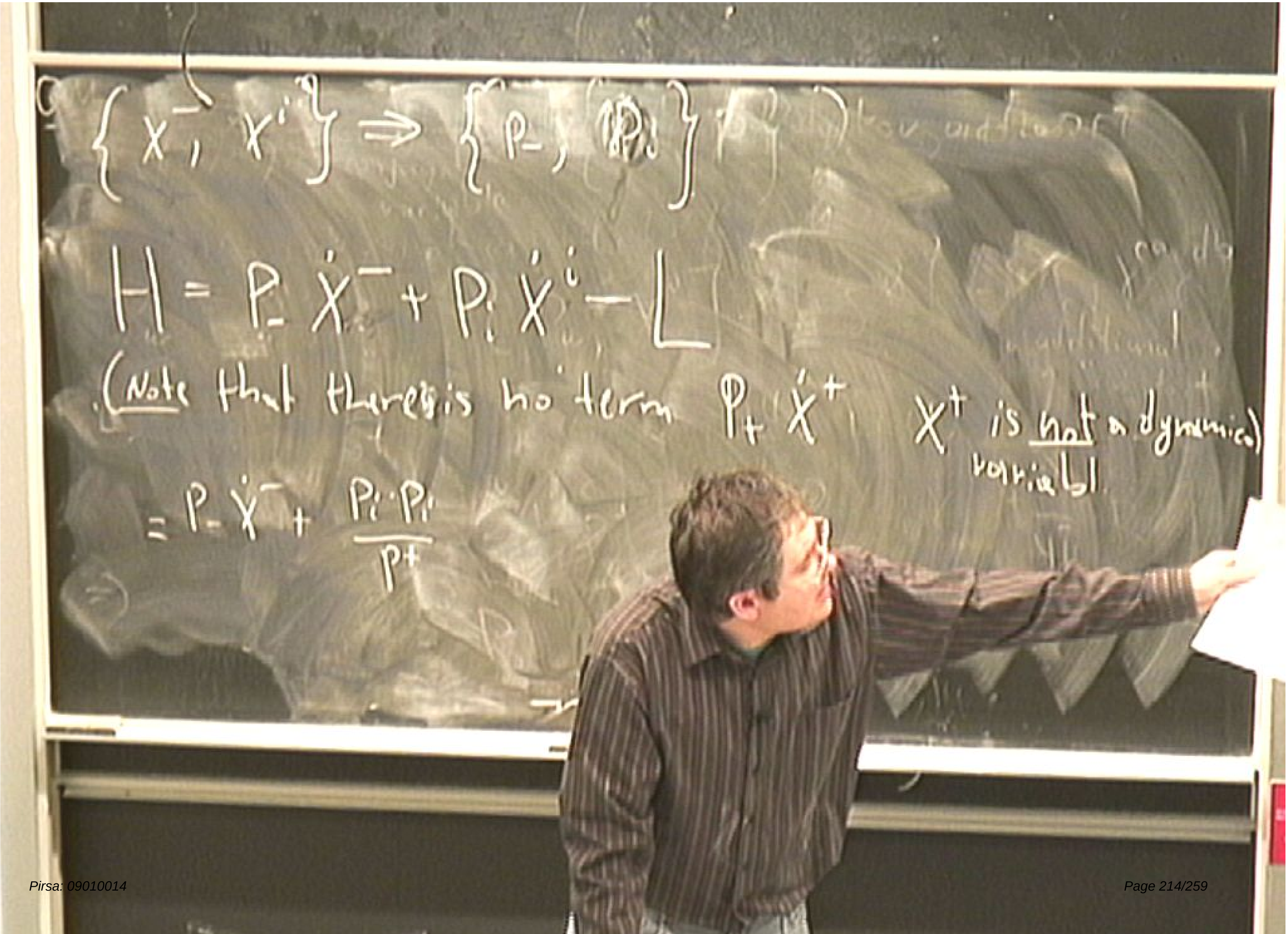
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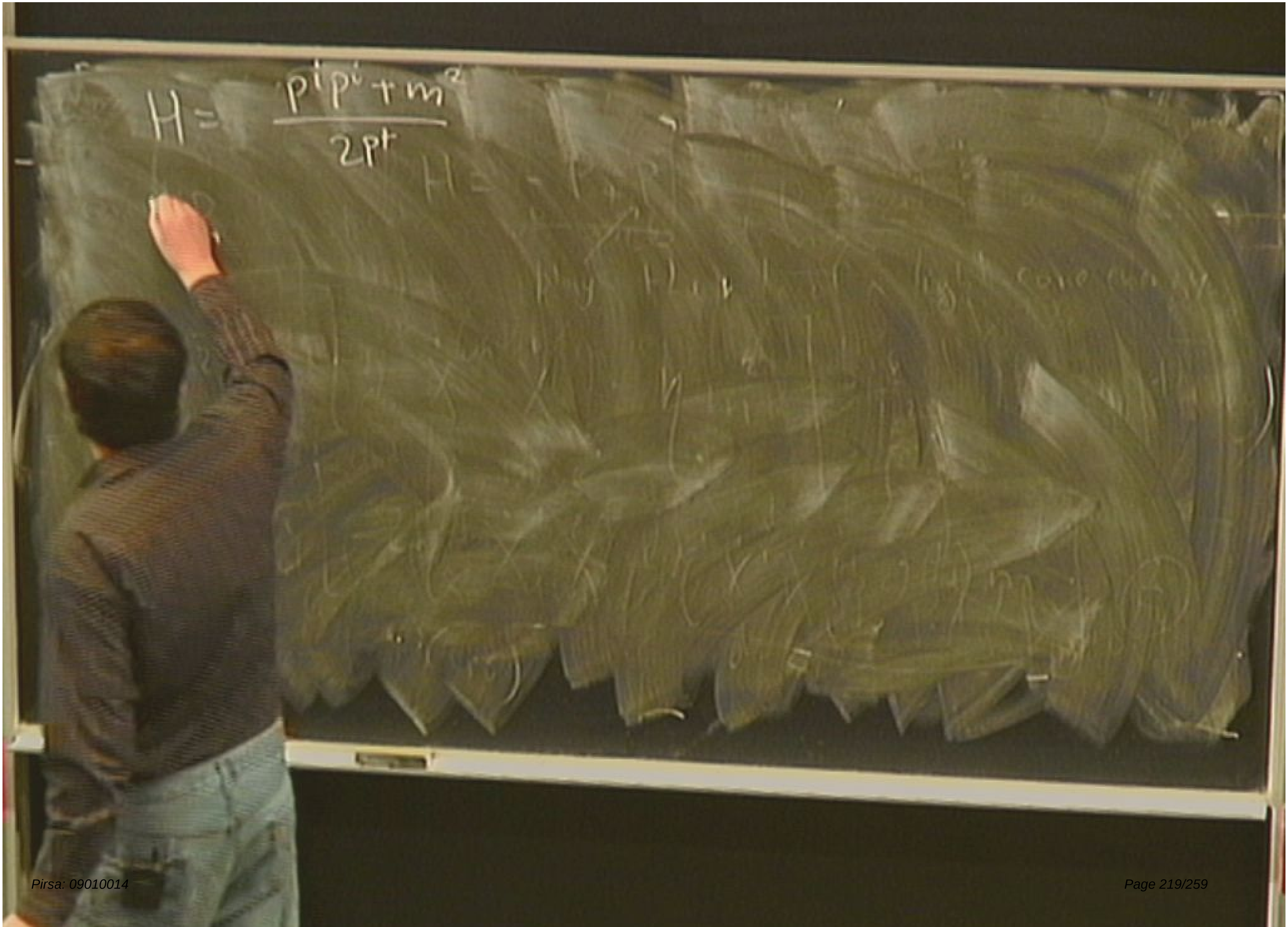
$$= \frac{1}{2} \frac{p_i p_i}{p^+} + \frac{1}{2p^+} m^2 = \frac{p_i p^i + m^2}{2p^+}$$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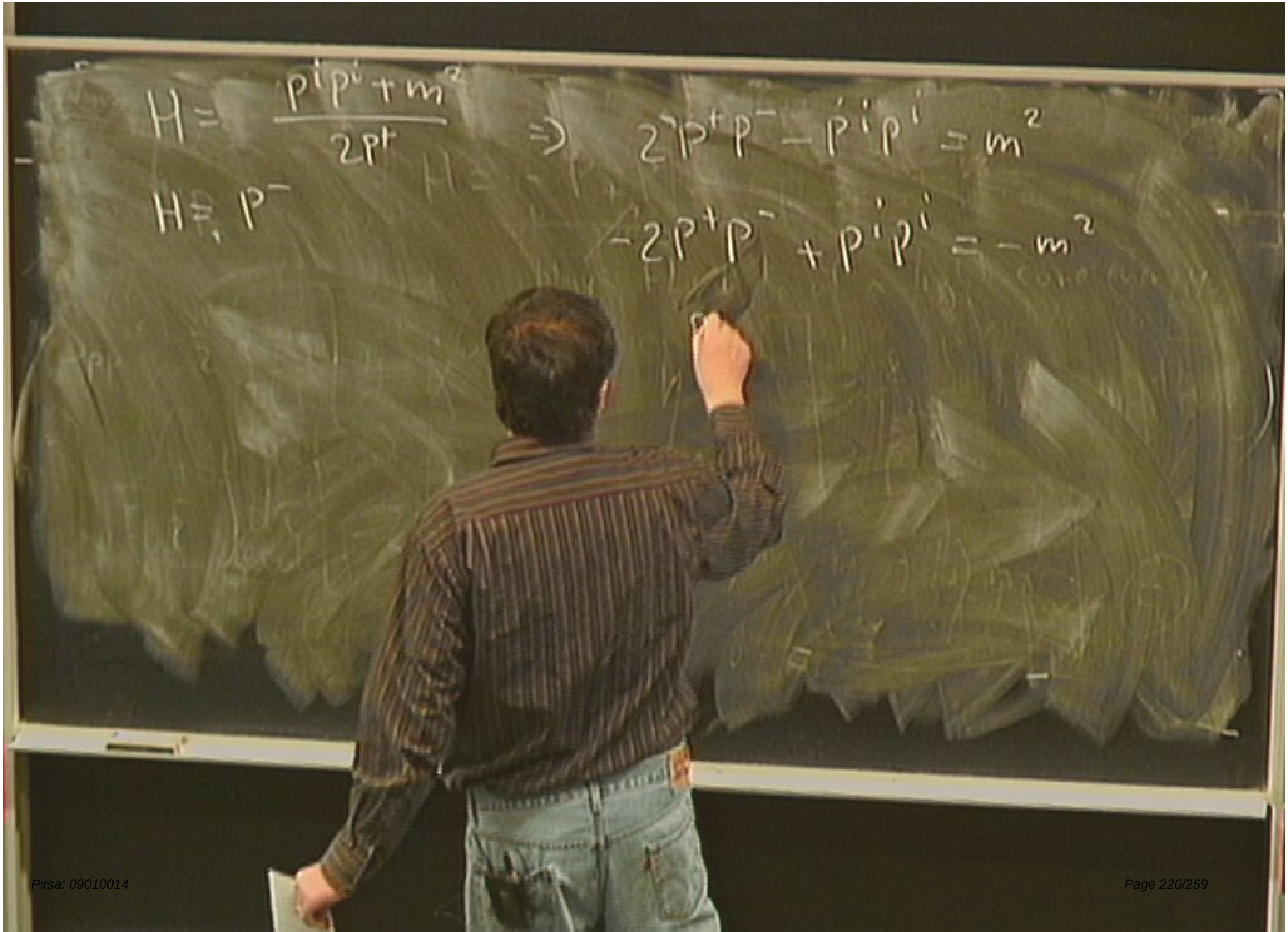

$H = \frac{p^i p^i + m^2}{2p^+}$ $\Rightarrow$ $2p^+p^- - p^ip^i = m^2$
$H \doteq p^-$
$-2p^+p^- + p^ip^i = -m^2$

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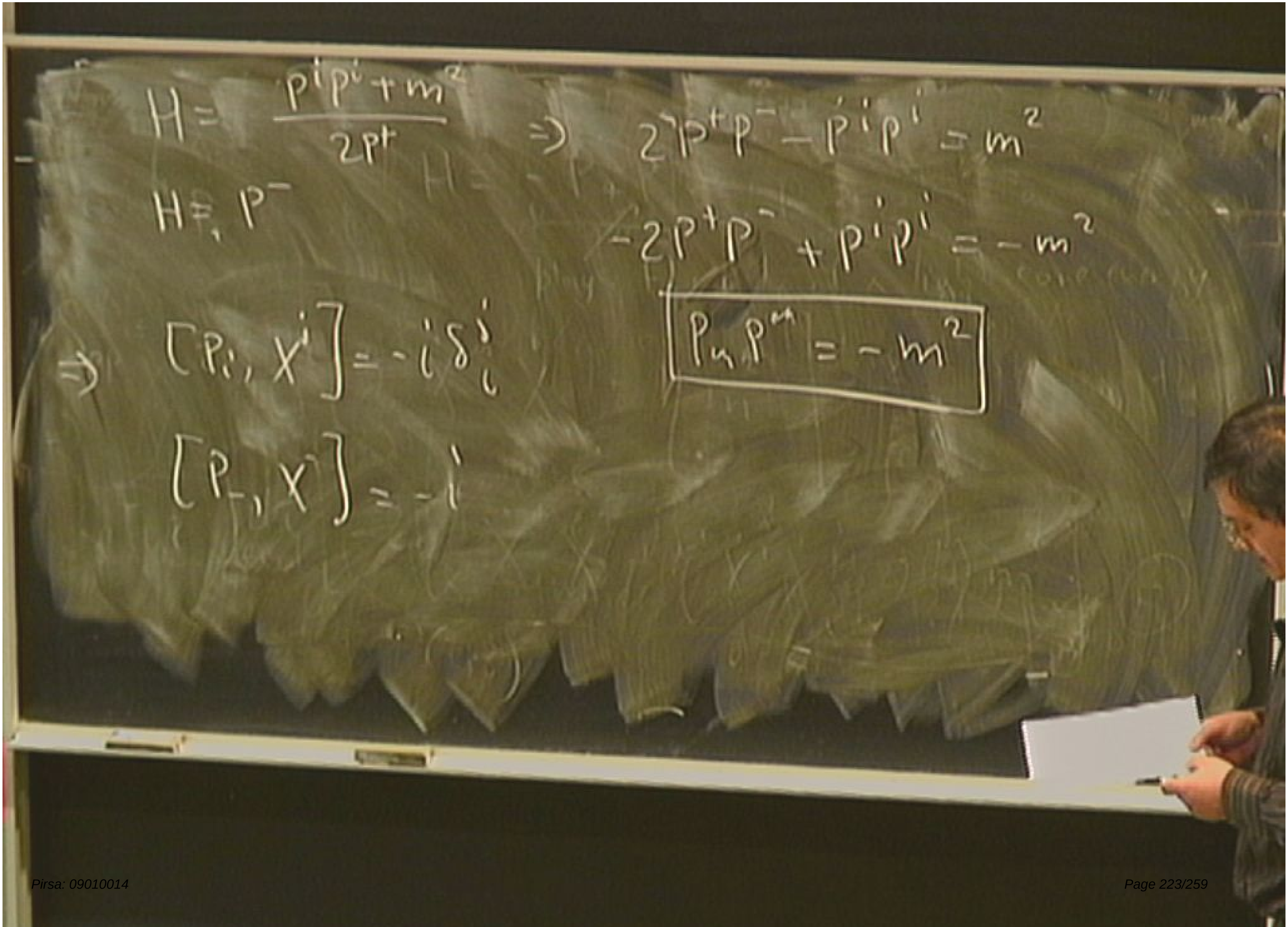
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$$\underbrace{|k_-, k^i\rangle}_{\text{momentum eigenvalues}}$$

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$$-2p^+p^- + p^ip^i = -m^2$$

$$\Rightarrow [P_i, X^j] = -i\delta^j_i \qquad \boxed{p_\mu p^\mu = -m^2}$$

$$[P_-, X^-] = -i \qquad \underbrace{|k_-, k^i\rangle}$$

momentum eigenvalues

Do the same for the string (P).

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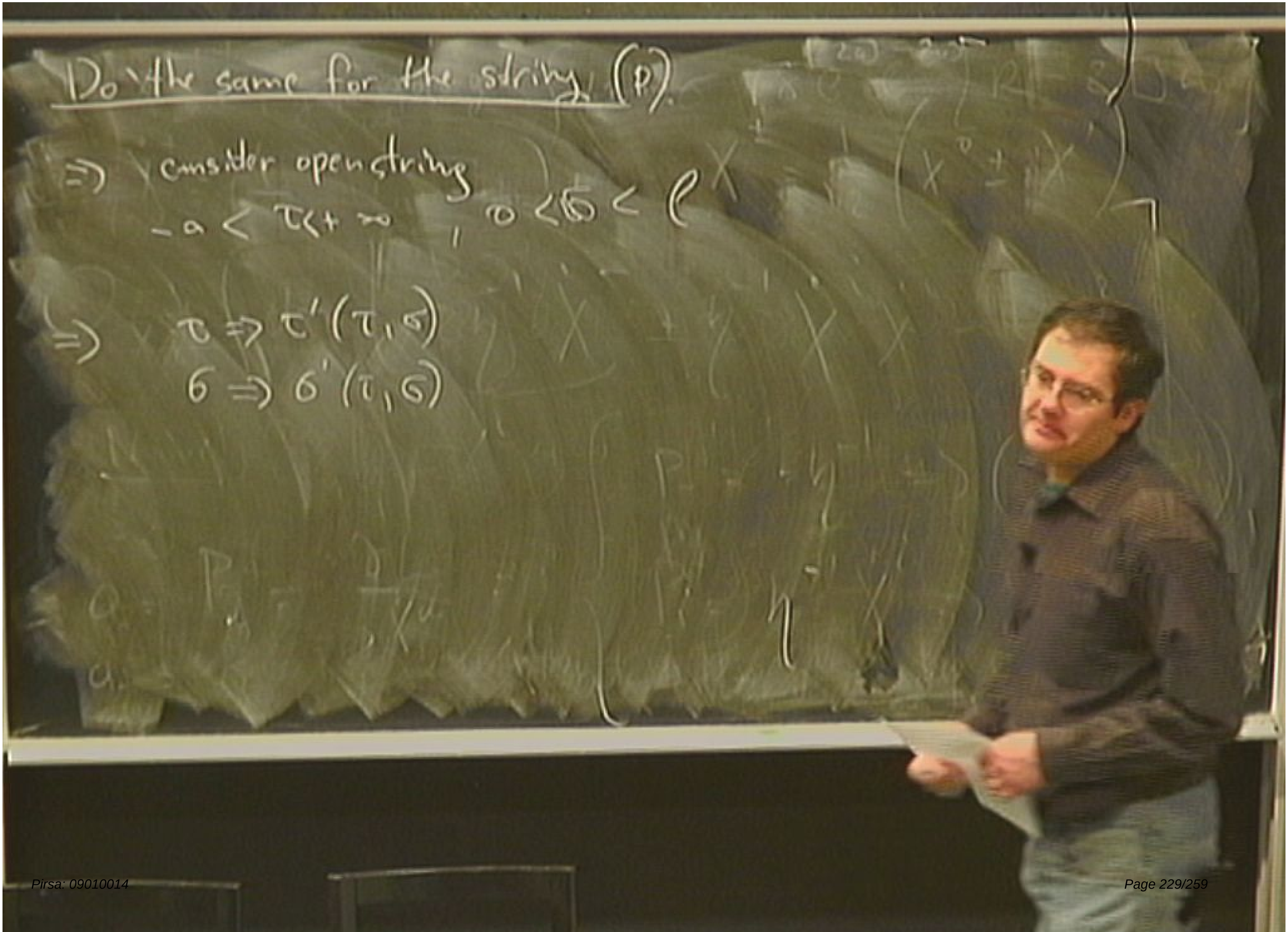
$\Rightarrow$ consider open string

$-\infty < \tau < +\infty\,,\quad 0 < \sigma < \ell$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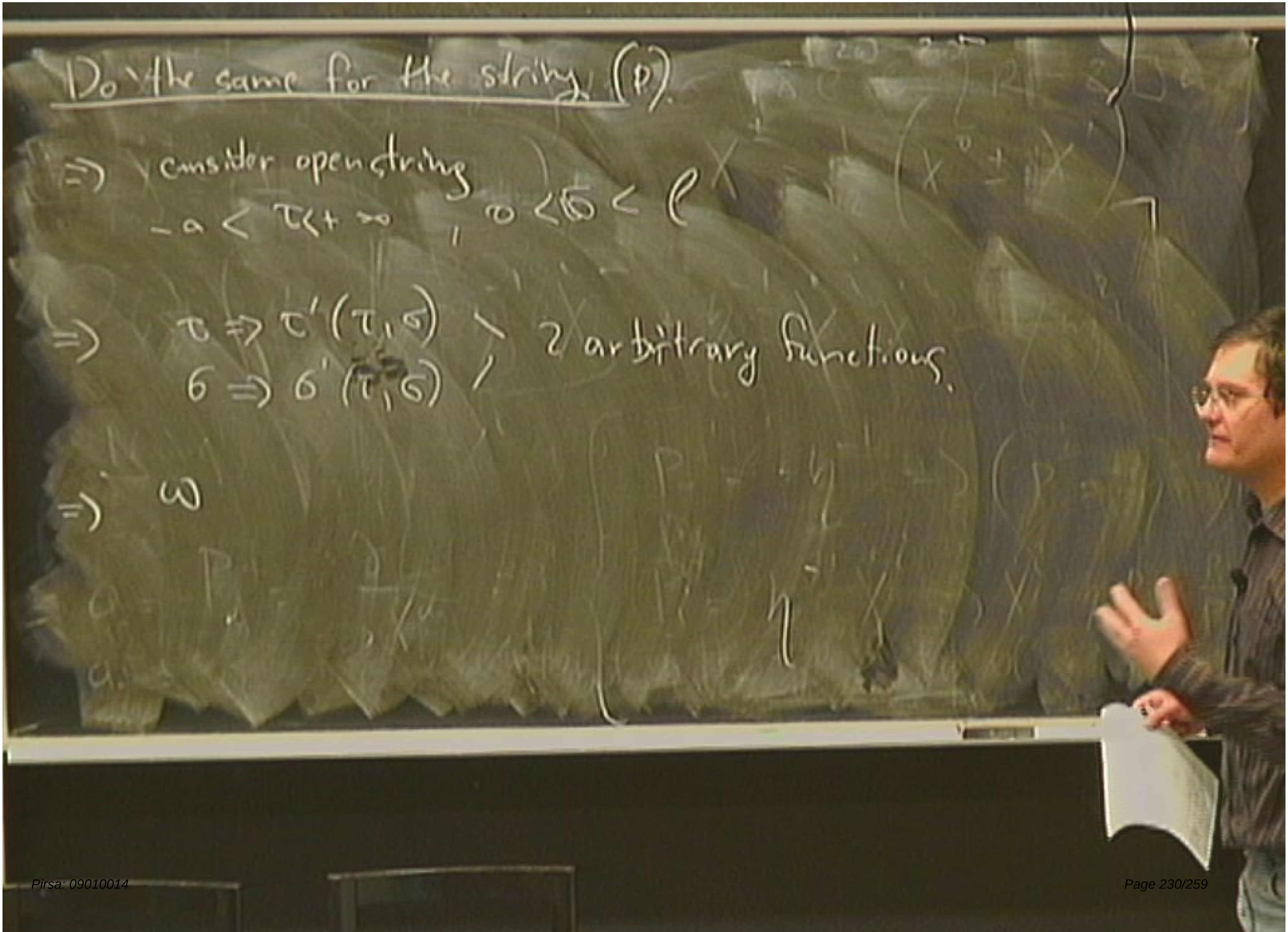
⇒ $\tau \Rightarrow \tau'(\tau, \sigma)$
$\sigma \Rightarrow \sigma'(\tau, \sigma)$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⇒ ω

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$\Rightarrow$ $\left.\begin{array}{l} \tau \Rightarrow \tau'(\tau,\sigma) \\ \sigma \Rightarrow \sigma'(\tau,\sigma) \end{array}\right\}$ 2 arbitrary functions.

$\Rightarrow$ $\omega(\tau,\sigma) \longrightarrow$ +1 more

$\Rightarrow$ 3 conditions to fix diffeo + + Weyl

$$\left.\begin{array}{l} X^+ = \tau \\ \partial_\sigma \gamma_{\sigma\sigma} = 0 \\ \gamma = -1 \end{array}\right\}$$ gauge fixing condition

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gauge fixing condition

$\Rightarrow$ light cone gauge for P

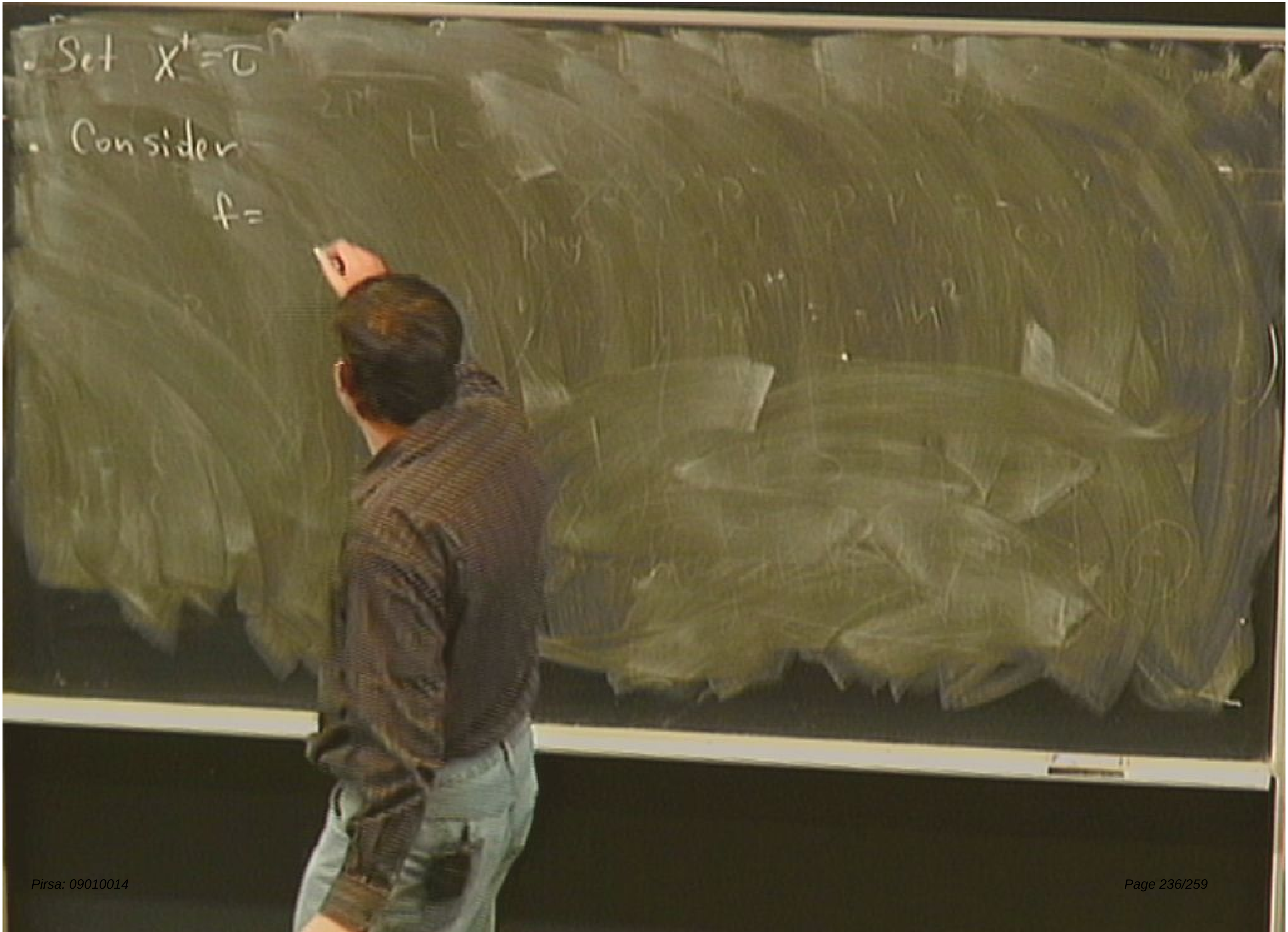
$\Rightarrow$ that we can always choose light

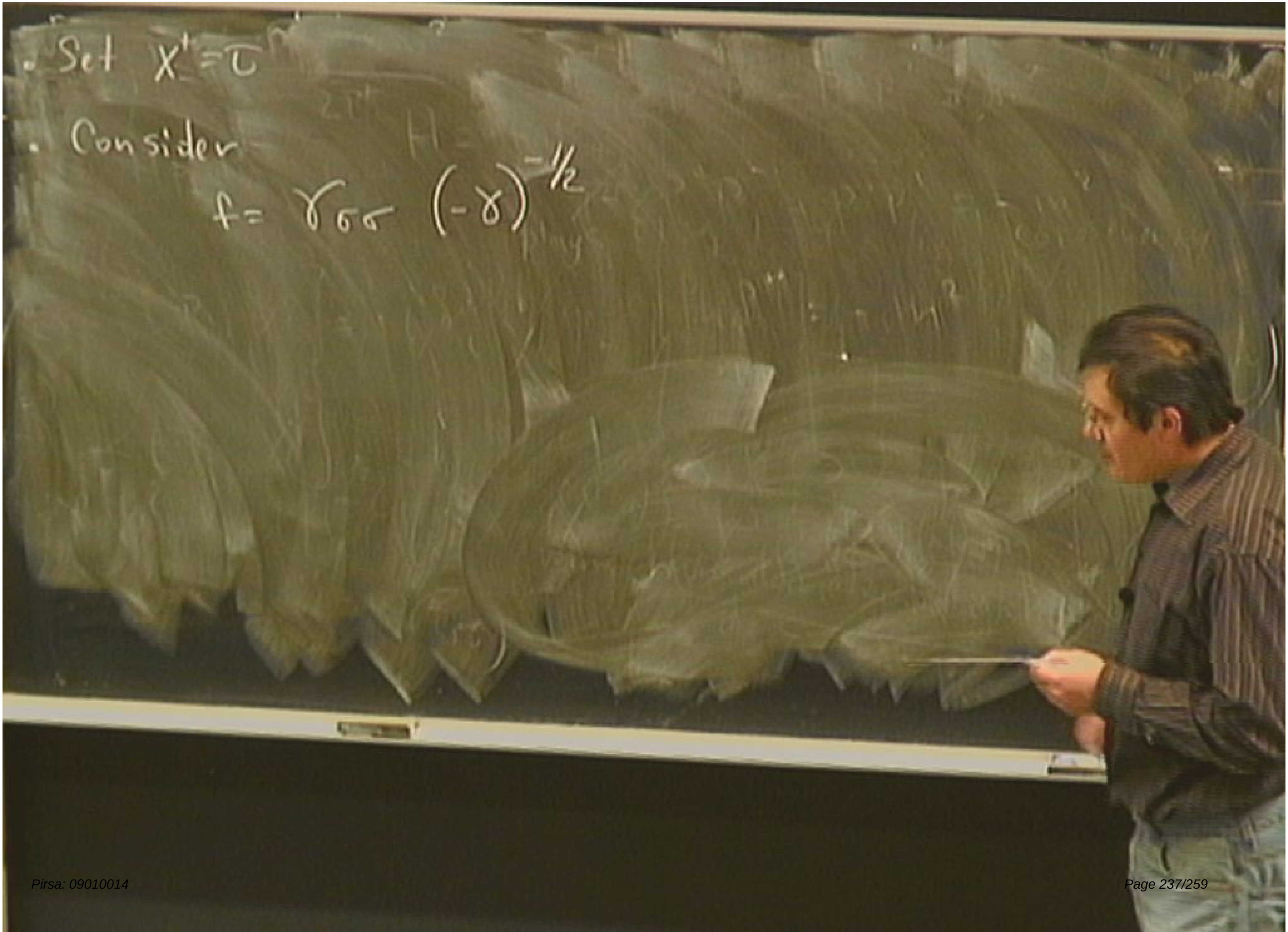
$$\left.\begin{array}{l} X^+ = \tau \\ \partial_\sigma \gamma_{\sigma\sigma} = 0 \\ \gamma = -1 \end{array}\right\}$$

gauge fixing condition

$\Rightarrow$ light cone gauge for P

$\Rightarrow$ that we can always choose light-cone fixing locally.


• Set $X^1 = \tau$
• Consider
$f =$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$f = \gamma_{\sigma\sigma} (-\gamma)^{-1/2}$

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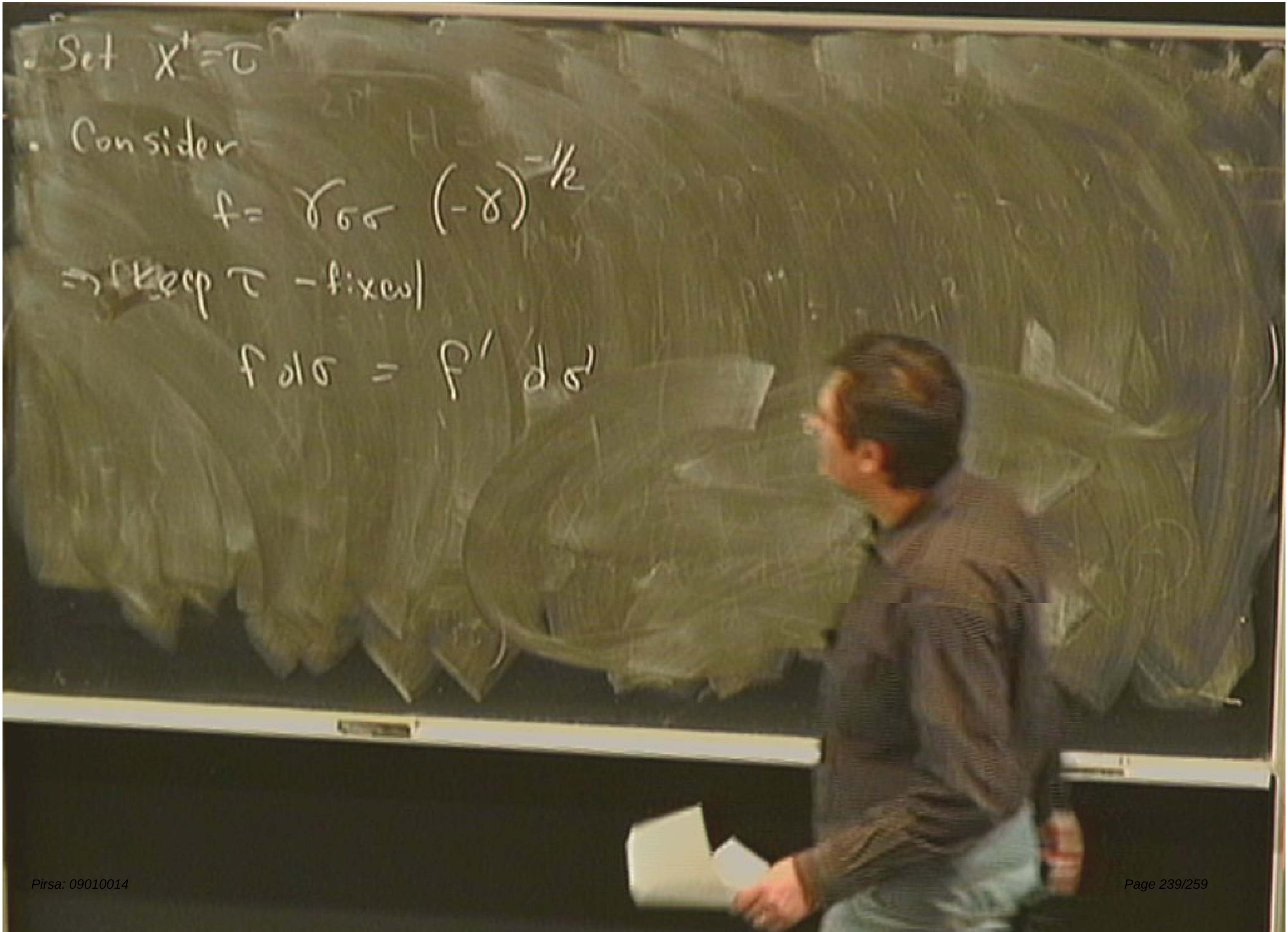
$\Rightarrow$ Keep τ - fixed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$f \, d\sigma = f' \, d\sigma'$

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$$\underbrace{f\, d\sigma}_{\text{invariant length element}} = f'\, d\sigma'$$

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$$\underbrace{f\, d\sigma}_{} = f'\, d\sigma'$$

invariant length element

$$f' = \gamma_{\sigma\sigma}' (-\gamma')^{-1/2} =$$

$$\gamma_{\sigma\sigma} \to \gamma_{\sigma\sigma}' = \gamma_{\sigma\sigma}\left(\frac{d\sigma}{d\sigma'}\right)^2$$

$$\gamma_{\sigma\tau} \to \gamma_{\sigma\tau}' = \gamma_{\sigma\tau}\left(\frac{d\sigma}{d\sigma'}\right)$$

$$\sigma \to \sigma' = \sigma'(\sigma)$$

- Set $X^1 = \tau$
- Consider

$$f = \gamma_{\sigma\sigma} (-\gamma)^{-1/2}$$

$\Rightarrow$ Keep τ - fixed

$$f \, d\sigma = f' \, d\sigma'$$

invariant length element

$$\gamma_{\tau\tau} \to \gamma_{\tau\tau}' = \gamma_{\tau\tau}$$

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$$\gamma_{\tau\tau} \to \gamma'_{\tau\tau} = \gamma_{\tau\tau}$$

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Set $X^1 = \tau$

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invariant length element

$$\gamma_{\tau\tau} \to \gamma_{\tau\tau}' = \gamma_{\tau\tau}$$

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$$\sigma \to \sigma' = \sigma'(\sigma)$$

$$f' = \gamma_{\sigma\sigma}' (-\gamma')^{-1/2} = \gamma_{\sigma\sigma} \left(\frac{d\sigma}{d\sigma'}\right)^2 (-\gamma)^{-1/2} \left[\left(\frac{d\sigma}{d\sigma'}\right)^2\right]^{-1/2}$$

Set $X^1 = \tau$

Consider

$$f = \gamma_{\sigma\sigma} (-\gamma)^{-1/2}$$

$\Rightarrow$ Keep τ - fixed

$$\boxed{f\, d\sigma = f'\, d\sigma'}$$

invariant length element

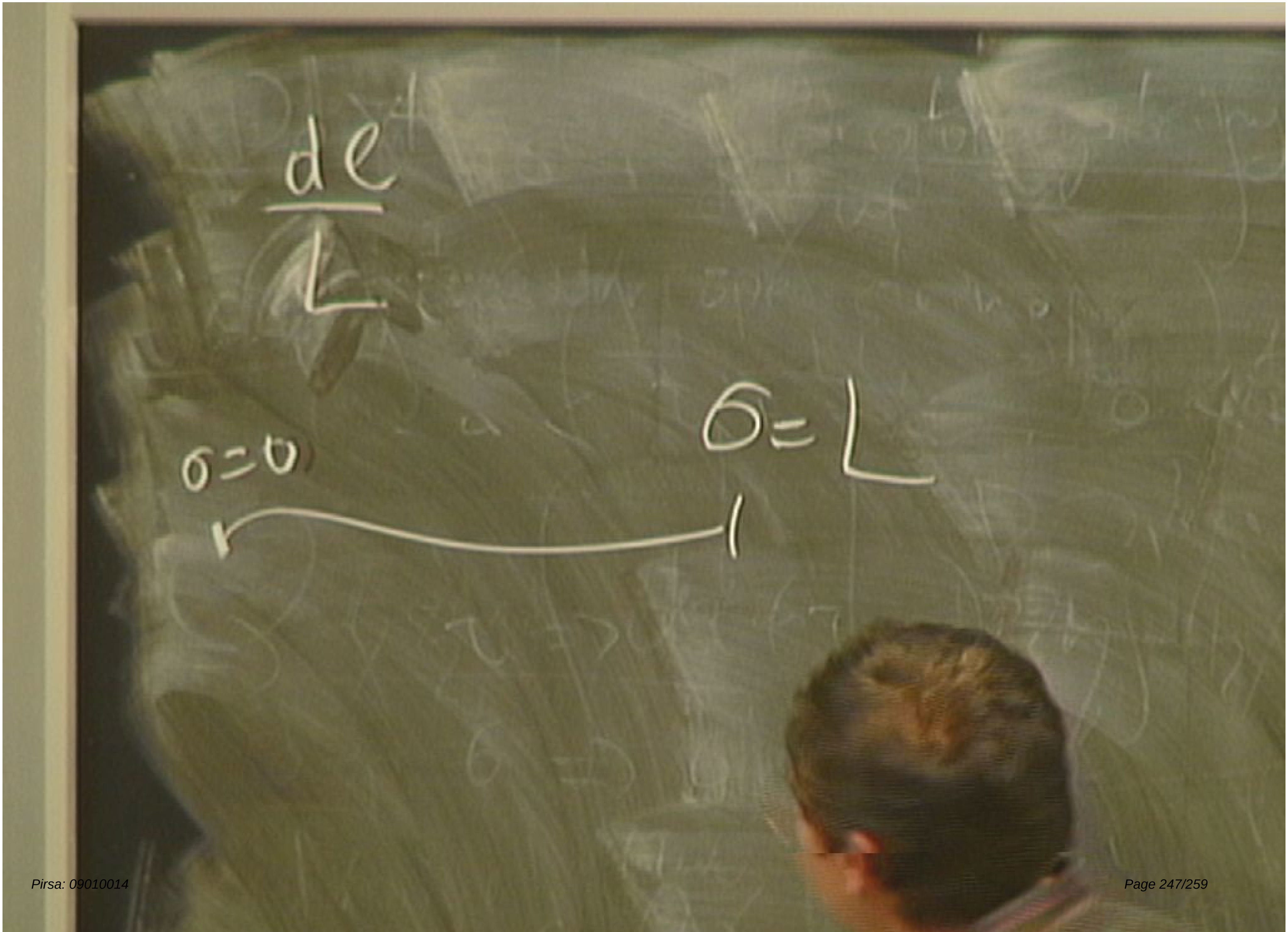
$$\gamma_{\tau\tau} \to \gamma_{\tau\tau}' = \gamma_{\tau\tau}$$

$$\gamma_{\sigma\sigma} \to \gamma_{\sigma\sigma}' = \gamma_{\sigma\sigma} \left(\frac{d\sigma}{d\sigma'}\right)^2$$

$$\gamma_{\sigma\tau} \to \gamma_{\sigma\tau}' = \gamma_{\sigma\tau} \left(\frac{d\sigma}{d\sigma'}\right)$$

$$\sigma \to \sigma' = \sigma'(\sigma)$$

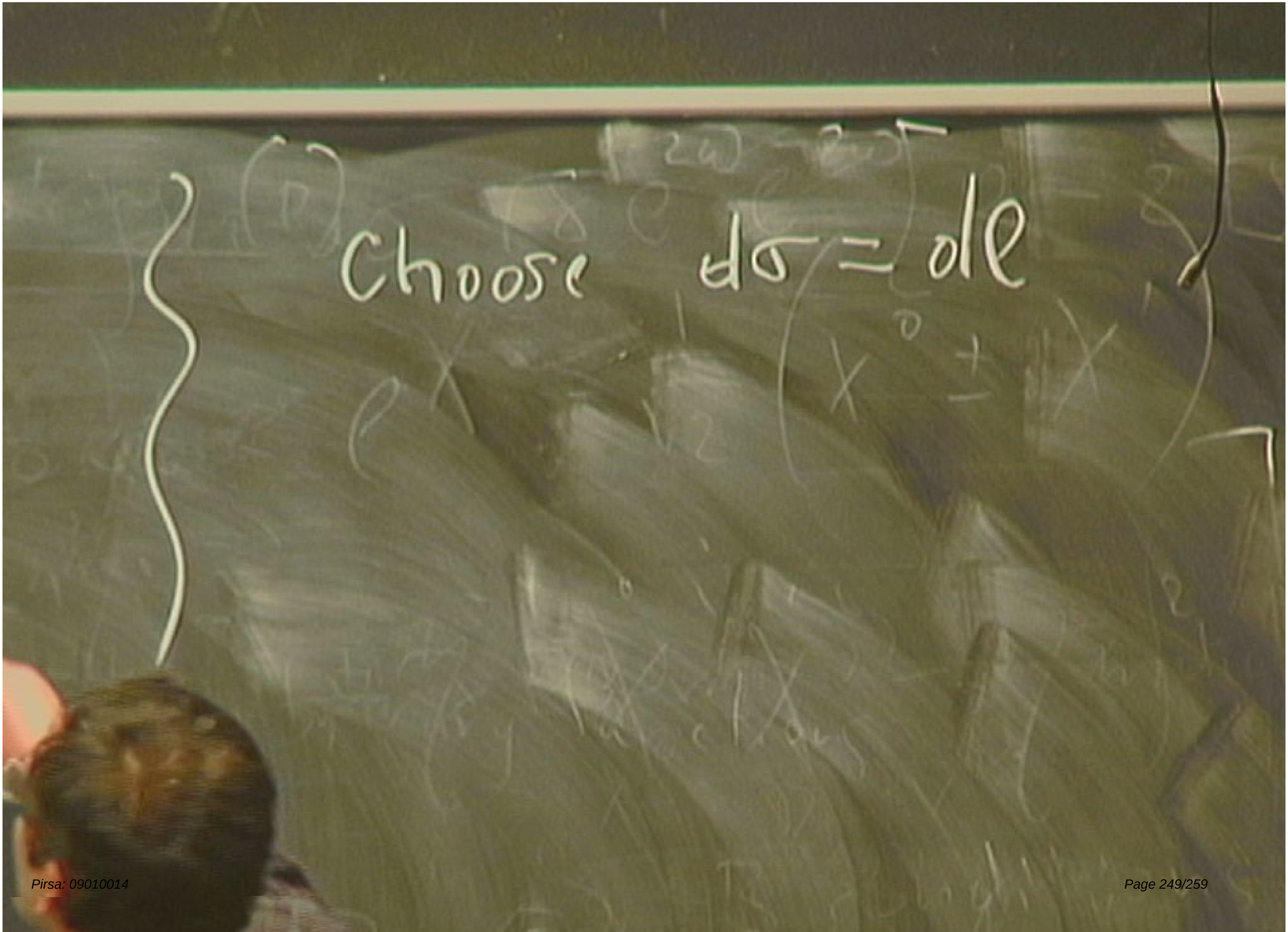
$$f' = \gamma_{\sigma\sigma}' (-\gamma')^{-1/2} = \gamma_{\sigma\sigma} \left(\frac{d\sigma}{d\sigma'}\right)^2 (-\gamma)^{-1/2} \left[\left(\frac{d\sigma}{d\sigma'}\right)^2\right]^{-1/2} = f \frac{d\sigma}{d\sigma'}.$$


$\frac{d\ell}{L}$
$\sigma = 0$
$\sigma = L$

$$\frac{d\ell}{L} = \frac{f\,d\sigma}{\int_0^L f\,d\sigma}$$

$\sigma = 0$

$\sigma = L$


Choose $d\sigma = d\ell$

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$$\frac{d\ell}{L} = \frac{f\, d\ell}{\int_0^L f\, d\sigma}$$

Choose $d\sigma = d\ell$ (II)

$$\frac{d\ell}{L} = \frac{f\, d\ell}{\int_0^L f\, d\sigma} \Rightarrow f = 1$$

Choose $d\sigma = d\ell$ (II)

$$\frac{d\ell}{L} = \frac{f\, d\ell}{\int_0^L f\, d\sigma} \Rightarrow f = \frac{1}{L}\int_0^L f\, d\sigma$$

Choose $d\sigma = d\ell$ (II)

$$\frac{d\ell}{L} = \frac{f\, d\ell}{\int_0^L f\, d\sigma} \Rightarrow f = \frac{1}{L}\int_0^L f\, d\sigma$$

Choose $d\sigma = d\ell$ II

$$\frac{d\ell}{L} = \frac{f\,d\ell}{\int^{L} f\,d\sigma} \Rightarrow f = \frac{1}{L}\int_0^L f\,d\sigma$$

$$\boxed{\partial_\sigma f = 0}$$

Choose $d\sigma = d\ell$ (II)

$$\frac{d\ell}{L} = \frac{f\, d\ell}{\int^{L} f\, d\sigma} \Rightarrow f = \frac{1}{L}\int_0^L f\, d\sigma$$

$$\boxed{\partial_\sigma f = 0}$$

$$\frac{d\ell}{L} = \frac{f\, d\sigma}{\int_0 f\, d\sigma}$$

$\sigma = L$

$\sigma = 0$

Choo

$$\frac{d\ell}{L} =$$

$$\Rightarrow \quad \gamma_{ab} \to \gamma_{ab}\, e^{2\omega}$$

γ

$$\frac{d\ell}{L} = \frac{f\, d\sigma}{\int_0^{L} f\, d\sigma}$$

$\sigma = 0$ $\qquad\qquad$ $\sigma = L$

Choose $d\sigma = d\ell$ (II)

$$\frac{d\ell}{L} = \frac{f\, d\ell}{\int f\, d\sigma} \Rightarrow f = \frac{1}{L}\int_0^{L} f\, d\sigma$$

$$\boxed{\partial_\sigma f = 0}$$

$$\Rightarrow \gamma_{ab} \to \gamma_{ab}\, e^{2\omega}$$

$\gamma \to \gamma e^{4\omega} \Rightarrow$ we can choose ω so that

$$\gamma' = \gamma e^{4\omega} = -1$$

$$\frac{d\ell}{L} = \frac{f\, d\sigma}{\int_0^L f\, d\sigma}$$

$\sigma = 0$ $\qquad$ $\sigma = L$

Choose $d\sigma = d\ell$ (II)

$$\frac{d\ell}{L} = \frac{f\, d\ell}{\int_0^L f\, d\sigma} \Rightarrow f = \frac{1}{L}\int_0^L f\, d\sigma$$

$$\boxed{\partial_\sigma f = 0}$$

$\Rightarrow \gamma_{ab} \to \gamma_{ab}\, e^{2\omega}$

$\gamma \leftrightarrow \gamma e^{4\omega} \Rightarrow$ we can choose ω so that $\gamma' = \gamma e^{4\omega} = -1$

$\Rightarrow$ Show that f is weyl inv.

$f = \gamma_{\sigma\sigma}\,(-\gamma)^{-1/2}$

$f \to f' = \gamma_{\sigma\sigma}\,\cancel{e^{2\omega}}\,(-\gamma)^{-1/2}\,\cancel{(e^{4\omega})}^{-1/2}$

<u>So</u>: Fixing diffeo + Weyl.

$\partial_\tau f = \boxed{0} = \partial_\sigma\left[\gamma_{\sigma\sigma}(-\gamma)^{-1/2}\right] = \boxed{\partial_\sigma \gamma_{\sigma\sigma}}$

$(-\gamma)^{-1/2} = 1$

$\gamma_{\sigma\sigma} = \gamma_{\sigma\sigma}(\tau)$