

Title: Introduction to the Bosonic String

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Abstract: This course provides a thorough introduction to the bosonic string based on the Polyakov path integral and conformal field theory. We introduce central ideas of string theory, the tools of conformal field theory, the Polyakov path integral, and the covariant quantization of the string. We discuss string interactions and cover the tree-level and one loop amplitudes. More advanced topics such as T-duality and D-branes will be taught as part of the course. The course is geared for M.Sc. and Ph.D. students enrolled in Collaborative Ph.D. Program in Theoretical Physics. Required previous course work: Quantum Field Theory (AM516 or equivalent). The course evaluation will be based on regular problem sets that will be handed in during the term. The primary text is the book: 'String theory. Vol. 1: An introduction to the bosonic string. J. Polchinski (Santa Barbara, KITP) . 1998. 402pp. Cambridge, UK: Univ. Pr. (1998) 402 p.' All interested students should contact Alex Buchel at [abuchel@uwo.ca](mailto:abuchel@uwo.ca) as soon as possible.

$$S_p = -\frac{1}{4\pi G} \int d^4x \sqrt{-g} g^{ab} \partial_a \chi^m \partial_b \chi_m$$

$$S_P = -\frac{1}{4\pi\alpha'} \int d\sigma d\tau \sqrt{-g} \gamma^{ab} \partial_a X^m \partial_b X_m \Rightarrow \text{1+1 QFT}$$

$\Rightarrow$

$$S_P = -\frac{1}{4\pi\alpha'} \int d\sigma d\tau \sqrt{-g} \gamma^{ab} \partial_a X^m \partial_b X_m \Rightarrow \text{1+1 QFT}$$

$\Rightarrow$

$$S_P = -\frac{1}{4\pi\alpha'} \int d\sigma d\tau \sqrt{-g} \gamma^{ab} \partial_a X^\mu \partial_b X_\mu \Rightarrow \text{1+1 QFT}$$

$$\Rightarrow \mathbb{F}^{ab} = -\frac{4\pi\alpha'}{\sqrt{-g}} \frac{\delta S}{\delta \gamma_{ab}}$$

$\Rightarrow$

$$S_P = -\frac{1}{4\pi\alpha'} \int d\sigma d\tau \sqrt{-g} \gamma^{ab} \partial_a X^\mu \partial_b X_\mu \Rightarrow \text{1+1 QFT}$$

$$\Rightarrow \mathbb{T}^{ab} = -\frac{4\pi}{\sqrt{-g}} \frac{\delta S}{\delta \gamma_{ab}}$$

$\Rightarrow$  2-dim diffeo inv.

$$S_P = -\frac{1}{4\pi\alpha'} \int d\sigma d\tau \sqrt{-g} g^{ab} \partial_a X^\mu \partial_b X_\mu \Rightarrow \text{1+1 QFT}$$

$$\Rightarrow T^{ab} = -\frac{4\pi}{\sqrt{-g}} \frac{\delta S}{\delta g_{ab}}$$

$\Rightarrow$  2-dim diffeo inv.

$$\nabla_a T^{ab} = 0$$

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$X^\mu$  satisfies EOM



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$$S_P = -\frac{1}{4\pi\alpha'} \int d\sigma d\tau \sqrt{-g} \gamma^{ab} \partial_a X^\mu \partial_b X_\mu \Rightarrow \text{1+1 QFT}$$

$$\Rightarrow \mathbb{T}^{ab} = -\frac{4\pi}{\sqrt{-g}} \frac{\delta S}{\delta \gamma_{ab}}$$

$\Rightarrow$  2-dim diffeo inv.

$$\nabla_a \mathbb{T}^{ab} = 0$$

$$\Rightarrow \gamma_{ab} \rightarrow e^{2\omega} \gamma_{ab} \quad \left. \begin{array}{l} \\ \end{array} \right\} X^\mu \text{ satisfies EOM}$$

$$\mathbb{T}_a^a = 0$$

$$S_P = -\frac{1}{4\pi\alpha'} \int d\sigma d\tau \sqrt{-g} \gamma^{ab} \partial_a X^\mu \partial_b X_\mu \Rightarrow \text{1+1 QFT}$$

$$\Rightarrow \mathbb{T}^{ab} = -\frac{4\pi}{\sqrt{-g}} \frac{\delta S}{\delta \gamma_{ab}}$$

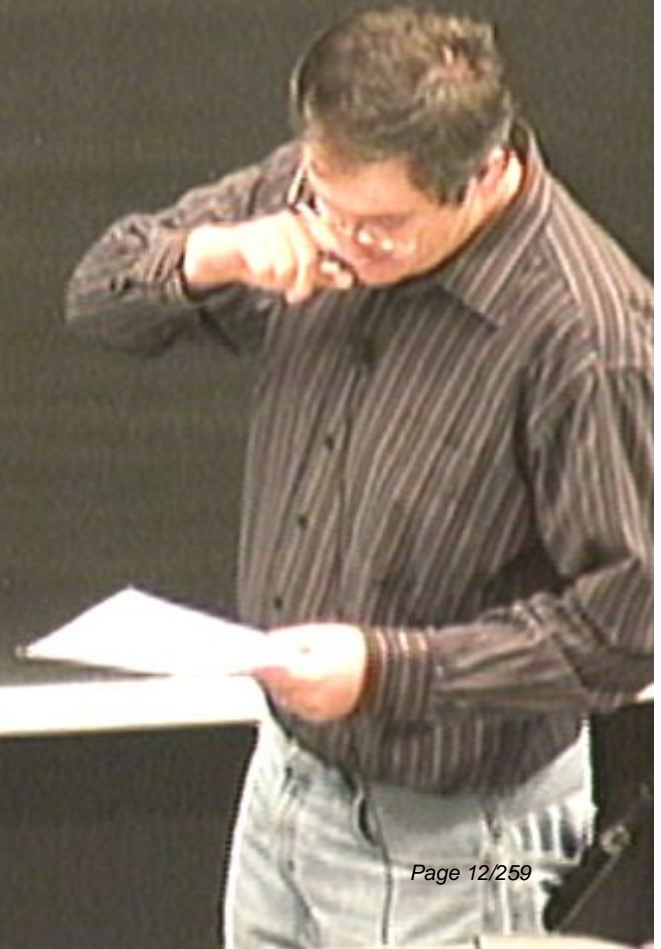
$\Rightarrow$  2-dim diff invar.

$$\nabla_a \mathbb{T}^{ab} = 0$$

$$\Rightarrow \gamma_{ab} \rightarrow e^{2\omega} \gamma_{ab} \quad X^\mu \text{ satisfies EOM}$$

$$\mathbb{T}_a^a = 0$$

$$\pi^{ab} = \frac{-4\pi}{\sqrt{-\gamma}} \left( \frac{1}{-4\pi} \right) \frac{\delta}{\delta g_{ab}} \left[ \right]$$



$$\Gamma_{ab} = \frac{-4\pi}{\sqrt{-g}} \frac{1}{(-4\pi i)} \int \frac{\delta}{\delta g_{ab}} \left[ \frac{1}{2} (-g)^{-\frac{1}{2}} \right]$$



$$\Gamma_{ab} = \frac{-4\pi}{\sqrt{-g}} \frac{1}{(-4\pi i)} \int \frac{\delta}{\delta \gamma_{ab}} \left[ \frac{1}{2} (-g)^{-\frac{1}{2}} \gamma^{ab} \delta \gamma_{ab} \right]$$

$$S_P = -\frac{1}{4\pi\alpha'} \int d\sigma d\tau \sqrt{-g} \gamma^{ab} \underbrace{\partial_a X^\mu \partial_b X_\mu}_{(\partial X)^2} \Rightarrow \text{1+1 QFT}$$

$$\Rightarrow \mathbb{T}^{ab} = -\frac{4\pi}{\sqrt{-g}} \frac{\delta S}{\delta g_{ab}}$$

$\Rightarrow$  2-dim diffeo inv.

$$\nabla_a \mathbb{T}^{ab} = 0$$

$$\Rightarrow Y_{ab} \rightarrow e^{2\omega} \gamma_{ab} \quad X^\mu \text{ satisfies EOM}$$

$$\mathbb{T}_a^a = 0$$

$$S_P = -\frac{1}{4\pi\alpha'} \int d\sigma d\tau \sqrt{-g} \gamma^{ab} \underbrace{\partial_a X^\mu \partial_b X_\mu}_{(\partial X)^2} \Rightarrow 1+1 \text{ QFT}$$

$$\Rightarrow T^{ab} = -\frac{4\pi\alpha'}{\sqrt{-g}} \frac{\delta S}{\delta \gamma_{ab}}$$

$$\delta \gamma^{ab} = -\delta \gamma_{cd} \gamma^{ca} \gamma^{db}$$

$\Rightarrow$  2-dim dif

$\Delta$

$$\Rightarrow Y_{ab} =$$



$$S_P = -\frac{1}{4\pi\alpha'} \int d\sigma d\tau \sqrt{-g} \underbrace{\gamma^{ab} \partial_a X^\mu \partial_b X_\mu}_{(\partial X)^2} \Rightarrow \text{1+1 QFT}$$

$$\Rightarrow T^{ab} = -\frac{4\pi}{\sqrt{-g}} \frac{\delta S}{\delta \gamma_{ab}}$$

$$\delta \gamma^{ab} = -\delta \gamma_{cd} \gamma^{ca} \gamma^{db}$$

$\Rightarrow$  2-dim diffeo inv.

$$\nabla_a T^{ab} = 0$$

$$\Rightarrow \gamma_{ab} \rightarrow e^{2\omega} \gamma_{ab} \quad X^\mu \text{ satisfies EOM}$$

$$T_a^a = 0$$

$$\Gamma_{ab} = \frac{-4\pi}{\sqrt{-g}} \frac{1}{(-4\pi i)} \frac{\delta}{\delta g_{ab}} \left[ \frac{1}{2} (-g)^{-\frac{1}{2}} g^{ab} \delta g_{ab} (2X)^2 - \sqrt{-g} \right]$$

$$\left. \begin{aligned} & \delta_{ab} \cdot \left[ \frac{1}{2} (-\gamma)^{-1/2} \gamma^{ab} \delta \gamma_{ab} (\partial X)^2 - \sqrt{-\gamma} \delta \gamma^{ab} \partial X^a \partial X^b \right] \end{aligned} \right\}$$

$$\left. \begin{array}{l} \delta_{ab} \\ \delta_{ab} \end{array} \right\} \frac{1}{2} (-\gamma)^{-\frac{1}{2}} (\gamma)^{\frac{1}{2}} \delta^{ab} \delta \delta_{ab} (\partial X)^2 - \sqrt{-\gamma} \delta^{ab} \partial X^a \partial X^b \delta \delta_{ab}$$

$\delta^m_b \chi_m \Rightarrow |+\rangle$  QFT

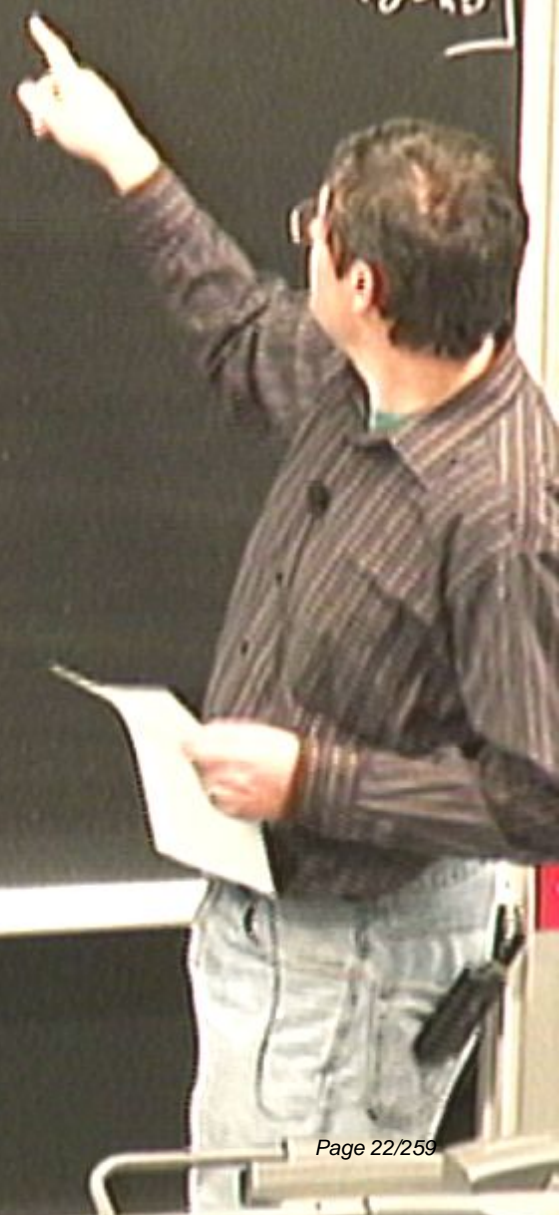
$\delta^2$

$$\delta \delta^{ab} = -\delta \delta_{cd} \delta^{ca} \delta^{db}$$

$$\delta \delta = \delta \cdot \delta^{ab} \delta \delta_{ab}$$

$$\pi_{ab} = \frac{-4\pi}{\sqrt{-g}} \left( \frac{1}{-4\pi k} \right) \frac{\delta}{\delta g_{ab}} \left[ \frac{1}{2} (-g)^{-\frac{1}{2}} g^{ab} g_{ab} (cX)^2 - \sqrt{-g} g^{abcd} \chi_{;c} \chi_{;d} \right]$$

$$= -\frac{1}{2} \left[ \partial^a \right]$$



$$\begin{aligned}
 T_{ab} &= \frac{-4\pi}{\sqrt{-g}} \left( \frac{1}{-4\pi k} \right) \frac{\delta}{\delta g_{ab}} \left[ \frac{1}{2} (-g)^{-\frac{1}{2}} g^{ab} \delta g_{ab} (\partial X)^2 - \sqrt{-g} \partial^a X^m \partial^b X_n \right. \\
 &\quad \left. \delta g_{ab} \right] \\
 &= -\frac{1}{2} \left[ \partial^a X^m \partial^b X_m - \frac{1}{2} g^{ab} \partial_c X^m \partial^c X_m \right]
 \end{aligned}$$

$$T^{ab} = \frac{-4\pi}{\sqrt{-g}} \left( \frac{1}{-4\pi} \right) \delta \delta_{ab} \left[ \frac{1}{2} (-g)^{-\frac{1}{2}} \delta^{ab} \delta \delta_{ab} (\partial X)^2 - \sqrt{-g} \frac{\partial X^m \partial X^b}{\delta \delta_{ab}} \right]$$

$$T^{ab} = -\frac{1}{2} \left[ \partial^a X^m \partial^b X_m - \frac{1}{2} g^{ab} \partial_c X^m \partial^c X_m \right]$$





$$T^{ab} = \frac{-4\pi}{\sqrt{-g}} \left( \frac{1}{-4\pi M^2} \right) \delta \delta_{ab} \left[ \frac{1}{2} (-g)^{-\frac{1}{2}} g^{ab} \delta \delta_{ab} (\partial X)^2 - \sqrt{-g} \partial^a X^m \partial^b X_n \delta \delta_{ab} \right]$$

$$T^{ab} = -\frac{1}{2} \left[ \partial^a X^m \partial^b X_m - \frac{1}{2} g^{ab} \partial_c X^m \partial^c X_m \right]$$



$$\pi^{ab} = \frac{-4\pi}{\sqrt{-\gamma}} \left( \frac{1}{-4\pi\kappa} \right) \frac{\delta}{\delta\gamma^{ab}} \left[ \frac{1}{2} (-\gamma)^{-\frac{\kappa}{2}} \gamma^{ab} \delta\gamma_{ab} (\partial X)^2 - \sqrt{-\gamma} \frac{\partial X^m \partial X^b}{\delta\gamma_{ab}} \right]$$

$$\pi^{ab} = -\frac{1}{2\kappa} \left[ \partial^a X^m \partial^b X_m - \frac{1}{2} \gamma^{ab} \partial_c X^m \partial^c X_m \right]$$

$$\pi_a^a = -\frac{1}{2\kappa} \left[ (\partial X)^2 \right]$$



$$\pi^{ab} = \frac{-4\pi}{\sqrt{-g}} \left( \frac{1}{-4\pi M} \right) \delta \delta_{ab} \left[ \frac{1}{2} (-g)^{-\frac{1}{2}} \delta^{ab} \delta \delta_{ab} (\partial X)^2 - \sqrt{-g} \frac{\partial X^m \partial X^n}{\delta \delta_{ab}} \right]$$

$$\pi^{ab} = -\frac{1}{2} \left[ \partial^a X^m \partial^b X_m - \frac{1}{2} g^{ab} \partial_c X^m \partial^c X_m \right]$$

$$\pi_a^a = -\frac{1}{2} \left[ (\partial X)^2 - \frac{1}{2} g^{ab} g_{ab} (\partial X)^2 \right] = 0$$

Dynamical fields:

$$\partial_\mu, X^\mu$$

Dynamical fields:

$\delta_{ab}, \chi^{\mu}$

$$0 = \frac{\delta S}{\delta \delta_{ab}} =$$

Dynamical fields:

$\delta_{ab}, \chi^{\mu}$

$$0 = \frac{\delta S}{\delta \gamma_{ab}} = T^{ab}$$

Dynamical fields:

EOM:  $\delta_{ab}, \chi^{\mu}$

$$0 = \frac{\delta S}{\delta \gamma_{ab}} = \boxed{T^{ab} = 0}$$

Dynamical fields:

EOM:  $\delta_{ab}, \chi^{\mu}$

$$0 = \frac{\delta S}{\delta \delta_{ab}} = \boxed{T^{ab} = 0}$$

$$\delta S_p = -\frac{1}{4\pi\alpha'} \int d\tau d\sigma$$



Dynamical fields:

EOM:  $\delta_{ab}, X^{\mu}$

$$0 = \frac{\delta S}{\delta \gamma_{ab}} = \boxed{T^{ab} = 0}$$

$$\delta S_p = -\frac{2}{4\pi\alpha'} \int d\tau d\sigma \sqrt{-\gamma} \gamma^{ab} \nabla_a X^{\mu} \cdot \nabla_b X^{\nu}$$

Dynamical fields:

EOM:  $\delta_{ab}, X^u$

$$0 = \frac{\delta S}{\delta \gamma_{ab}} = \boxed{T^{ab} = 0}$$

$$\delta S_p = -\frac{2}{4\pi\alpha'} \int d\tau d\sigma \sqrt{-\gamma} \gamma^{ab} \nabla_a X^u \cdot \nabla_b X^u$$

Dynamical fields:

EOM:  $\delta_{ab}, X^u$

$$0 = \frac{\delta S}{\delta \gamma_{ab}} = \boxed{T^{ab} = 0}$$

$$\begin{aligned} \delta S_p &= -\frac{2}{4\pi\alpha'} \int d\tau d\sigma \underbrace{\sqrt{-g} \gamma^{ab}} \nabla_a X^u \cdot \nabla_b \delta X^u \\ &= +\frac{2}{4\pi\alpha'} \int d\tau d\sigma \nabla_b \left[ \sqrt{-g} \gamma^{ab} \nabla_a X^u \right] \delta X^u \end{aligned}$$

Dynamical fields:

FOM:  $\delta_{ab}, X^u$

$$0 = \frac{\delta S}{\delta \gamma_{ab}} = \boxed{T^{ab} = 0}$$

$$\delta S_P = -\frac{2}{4\pi\alpha'} \int d\tau d\sigma \underbrace{\sqrt{-g} g^{ab}} \nabla_a X^u \cdot \nabla_b \delta X^u$$

$$+ \frac{2}{4\pi\alpha'} \int d\tau d\sigma \nabla_b \left[ \sqrt{-g} g^{ab} \nabla_a X^u \right] \delta X^u + \text{BT.}$$

Dynamical fields:

FOM:  $\delta_{ab}, X^{\mu}$

$$0 = \frac{\delta S}{\delta \gamma_{ab}} = \boxed{T^{ab} = 0}$$

$$\delta S_P = -\frac{2}{4\pi\alpha'} \int d\tau d\sigma \underbrace{\sqrt{-\gamma} \gamma^{ab}} \nabla_a X^{\mu} \cdot \nabla_b \delta X^{\nu}$$

$$= +\frac{2}{4\pi\alpha'} \int d\tau d\sigma \nabla_b \left[ \sqrt{-\gamma} \gamma^{ab} \nabla_a X^{\mu} \right] \delta X_{\mu} + \text{BT.}$$

$$\Rightarrow \nabla_b \left[ \sqrt{-\gamma} \gamma^{ab} \nabla_a X^{\mu} \right] = 0$$

Dynamical fields:

FOM:  $\delta_{ab}, X^u$

$$0 = \frac{\delta S}{\delta \gamma_{ab}} = \boxed{T^{ab} = 0}$$

$$\delta S_P = -\frac{2}{4\pi\alpha'} \int d\tau d\sigma \underbrace{\sqrt{-g} \gamma^{ab}} \nabla_a X^u \cdot \nabla_b \delta X^u$$

$$= +\frac{2}{4\pi\alpha'} \int d\tau d\sigma \nabla_b \left[ \sqrt{-g} \gamma^{ab} \nabla_a X^u \right] \delta X^u + \text{BT.}$$

$$\Rightarrow \nabla_b \left[ \sqrt{-g} \gamma^{ab} \nabla_a X^u \right] = 0 \Rightarrow \boxed{\sqrt{-g} \square X^u = 0}$$

$\Rightarrow$  Possible BC that preserve  
Poincaré inv in  $D$ -dim.

$\Rightarrow$  Possible BC that preserve  
Poincare inv in  $D$ -dim.

$$B^T = -2$$





$\Rightarrow$  Possible BC that preserve  
Poincare inv in D-dim.

$$D_{11} = -\frac{2}{4\pi\alpha'} \int_{-\infty}^{\infty} d\tau$$

$\Rightarrow$  Possible BC that preserve  
Poincare inv in D-dim.

$$B^{\prime T} = -\frac{2}{4\pi\alpha'} \int_{-\infty}^{\infty} dt \sqrt{-g} \gamma$$

$\Rightarrow$  Possible BC that preserve  
Poincare inv in D-dim.

$$B^{\text{IT}} = -\frac{2}{4\pi\alpha'} \int_{-\infty}^{\infty} d\tau \sqrt{-g} \gamma^{\sigma\alpha} \partial_{\alpha} X^m \cdot \delta X_m$$

⇒ Possible BC that preserve

Poincare inv in D-dim.

$$B' \Gamma = -\frac{2}{4\pi\alpha'} \int_{-\infty}^{\infty} d\tau \sqrt{-g} \gamma^{\sigma\alpha} \partial_\alpha X^m \cdot \delta X_m \quad \left. \begin{array}{l} \sigma = \ell \\ \sigma = 0 \end{array} \right\}$$

Dynamical fields:

OM:  $\delta_{ab}, X^u$

$$0 = \frac{\delta S}{\delta \gamma_{ab}} = \boxed{T^{ab} = 0}$$

$$\delta S_P = -\frac{2}{4\pi\alpha'} \int dt d\sigma \underbrace{\sqrt{-\gamma} \gamma^{ab}}_{\text{metric}} \nabla_a X^u \cdot \delta X^u$$

$$= +\frac{2}{4\pi\alpha'} \int dt d\sigma \nabla_b \left[ \sqrt{-\gamma} \gamma^{ab} \nabla_a X^u \right] \delta X_u + B$$

$$\nabla_b \left[ \sqrt{-\gamma} \gamma^{ab} \nabla_a X^u \right] = 0 \Rightarrow \boxed{\sqrt{-\gamma} \square X^u = 0}$$

Dynamical fields:

OM:  $\delta_{ab}, X^u$

$$0 = \frac{\delta S}{\delta \gamma_{ab}} = \boxed{T^{ab} = 0}$$

$$\delta S_P = -\frac{2}{4\pi\alpha'} \int dt d\sigma \underbrace{\sqrt{-\gamma} \gamma^{ab}} \nabla_a X^u \cdot \delta X^u$$

$$+ \frac{2}{4\pi\alpha'} \int dt d\sigma \nabla_b \left[ \sqrt{-\gamma} \gamma^{ab} \nabla_a X^u \right] \delta X_u + B1.$$

$$\nabla_b \left[ \sqrt{-\gamma} \gamma^{ab} \nabla_a X^u \right] = 0 \Rightarrow \boxed{\sqrt{-\gamma} \square X^u = 0}$$

⇒ Possible BC that preserve

Poincare inv in D-dim.

$$B' T = -\frac{2}{4\pi\alpha'} \int_{-\infty}^{\infty} dt \sqrt{-g} \gamma^{\sigma\alpha} \partial_\alpha X^m \cdot \delta X_m$$

$$\left. \begin{array}{l} \sigma = \ell \\ \sigma = 0 \end{array} \right\}$$

⇒ Possible BC that preserve

Poincare inv in D-dim.

$$B\Gamma = -\frac{2}{4\pi\alpha'} \int_{-\infty}^{\infty} d\tau \sqrt{-g} \underbrace{\gamma^{\sigma\alpha} \partial_{\alpha} X^{\mu}}_{=0} \cdot \delta X_{\mu} \quad \left. \begin{array}{l} \sigma = l \\ \sigma = 0 \end{array} \right\}$$

⇒



⇒ Possible BC that preserve

Poincare inv in D-dim.

$$B^{\text{IT}} = -\frac{2}{4\pi\alpha'} \int_{-\infty}^{\infty} d\tau \sqrt{-g} \underbrace{\gamma^{\sigma\alpha} \partial_{\alpha} X^{\mu}}_{=0} \cdot \delta X_{\mu\nu}$$

$$\Rightarrow \partial^{\sigma} X^{\mu}(\tau, 0) = 0 \quad \partial^{\sigma} X^{\mu}(\tau, \ell) = 0$$

$$\left. \begin{array}{l} \sigma = \ell \\ \sigma = 0 \end{array} \right\}$$

⇒ Possible BC that preserve

Poincare inv in D-dim.

$$B\Gamma = -\frac{2}{4\pi\alpha'} \int_{-\infty}^{\infty} d\tau \sqrt{-g} \underbrace{\gamma^{\sigma\alpha}}_{=} \partial_{\alpha} X^{\mu} \cdot \delta X_{\mu}$$

$\sigma = l$   
 $\sigma = 0$

⇒

$\partial^{\sigma} X^{\mu}(\tau, 0) = 0 \quad \partial^{\sigma} X^{\mu}(\tau, l) = 0$

N Boundary conditions

⇒ Possible BC that preserve

Poincare inv in  $D$ -dim.

$$BT = -\frac{2}{4\pi\alpha'} \int_{-\infty}^{\infty} d\tau \sqrt{-g} \underbrace{g^{\sigma\alpha}}_{=} \partial_{\alpha} X^{\mu} \cdot \delta X_{\mu}$$

⇒

$$\partial^{\sigma} X^{\mu}(\tau, 0) = 0 \quad \partial^{\sigma} X^{\mu}(\tau, \ell) = 0$$

$\sigma = \ell$   
 $\sigma = 0$   
N Boundary conditions  
for an open string

⇒ Possible BC that preserve

Poincare inv in  $D$ -dim.

$$B\Gamma = -\frac{2}{4\pi\alpha'} \int_{-\infty}^{\infty} d\tau \sqrt{-g} \underbrace{g^{\sigma\alpha}}_{=} \partial_{\alpha} X^{\mu} \cdot \delta X_{\mu}$$

$$\Rightarrow \boxed{\partial^{\sigma} X^{\mu}(\tau, 0) = 0 \quad \partial^{\sigma} X^{\mu}(\tau, \ell) = 0}$$

$$\rightarrow X^{\mu}(\tau, \sigma) =$$

$\sigma = \ell$   
 $\sigma = 0$   
N Boundary conditions  
for an open string

⇒ Possible BC that preserve

Poincare inv in  $D$ -dim.

$$B\Gamma = -\frac{2}{4\pi\alpha'} \int_{-\infty}^{\infty} d\tau \sqrt{-g} \underbrace{g^{\sigma\alpha}}_{=} \partial_{\alpha} X^{\mu} \cdot \delta X_{\mu}$$

$$\Rightarrow \boxed{\partial^{\sigma} X^{\mu}(\tau, 0) = 0 \quad \partial^{\sigma} X^{\mu}(\tau, \ell) = 0}$$

$$X^{\mu}(\tau, \sigma) = X^{\mu}(\tau, \sigma + \ell)$$

$\sigma = \ell$   
 $\sigma = 0$   
N Boundary conditions  
for an open string

⇒ Possible BC that preserve  
Poincare inv in D-dim.

$$B\Gamma = -\frac{2}{4\pi\alpha'} \int_{-\infty}^{\infty} d\tau \sqrt{-g} \underbrace{\gamma^{\sigma\alpha}}_{\parallel} \partial_{\alpha} X^{\mu} \cdot \delta X_{\mu}$$

$$\Rightarrow \boxed{\partial^{\sigma} X^{\mu}(\tau, 0) = 0 \quad \partial^{\sigma} X^{\mu}(\tau, \ell) = 0}$$

$\sigma = \ell$   
 $\sigma = 0$   
N Boundary conditions  
for an open string

$$\rightarrow X^{\mu}(\tau, \sigma) = X^{\mu}(\tau, \sigma + \ell)$$

(note that  $\delta X^{\mu}$  must be periodic as well)

⇒ Possible BC that preserve

Poincare inv in  $D$ -dim.

$$B\Gamma = -\frac{2}{4\pi\alpha'} \int_{-\infty}^{\infty} d\tau \sqrt{-g} \underbrace{g^{\sigma\alpha}}_{\parallel} \partial_{\alpha} X^{\mu} \cdot \delta X_{\mu}$$

$$\Rightarrow \boxed{\partial^{\sigma} X^{\mu}(\tau, 0) = 0 \quad \partial^{\sigma} X^{\mu}(\tau, \ell) = 0}$$

$\sigma=0$   
 $\sigma=\ell$   
N Boundary conditions  
for an open string

$$\rightarrow X^{\mu}(\tau, \sigma) = X^{\mu}(\tau, \sigma + \ell)$$

(note that  $\delta X^{\mu}$  must be periodic as well)

⇒ Possible BC that preserve  
Poincare inv in D-dim.

$$DIP = -\frac{2}{4\pi\alpha'} \int_{-\infty}^{\infty} d\tau \sqrt{-g} \underbrace{\gamma^{\sigma\alpha}}_{\parallel} \partial_{\alpha} X^{\mu} \cdot \delta X_{\mu}$$

$$\partial^{\sigma} X^{\mu}(\tau, 0) = 0 \quad \partial^{\sigma} X^{\mu}(\tau, \ell) = 0$$

$\left. \begin{array}{l} \sigma = \ell \\ \sigma = 0 \end{array} \right\} N \text{ Boundary conditions for an open string}$

⇒  $X^{\mu}(\tau, \sigma) = X^{\mu}(\tau, \sigma + \ell)$   
 (note that  $\delta X^{\mu}$  must be periodic as well) ⇒



⇒ Possible BC that preserve  
Poincare inv in D-dim.

$$-\frac{2}{4\pi\alpha'} \int_{-\infty}^{\infty} d\tau \sqrt{-g} \underbrace{\gamma^{\sigma\alpha} \partial_\alpha X^\mu}_{\parallel} \cdot \delta X_\mu \quad \left. \begin{array}{l} \sigma=l \\ \sigma=0 \end{array} \right\}$$

$$X^\mu(\tau, 0) = 0 \quad \partial^\sigma X^\mu(\tau, l) = 0$$

N Boundary conditions  
for an open string

$$X^\mu(\tau, \sigma) = X^\mu(\tau, \sigma + l)$$

if  $\delta X^\mu$  must be periodic as well ⇒  $\sigma \sim \sigma + l$   
closed strings

⇒ Possible BC that preserve

Poincare inv in D-dim.

$$B\Gamma = -\frac{2}{4\pi\alpha'} \int_{-\infty}^{\infty} d\tau \sqrt{-g} \underbrace{g^{\sigma\alpha}}_{\parallel} \partial_{\alpha} X^{\mu} \cdot \delta X_{\mu}$$

$\sigma = l$

$$\Rightarrow \boxed{\partial^{\sigma} X^{\mu}(\tau, 0) = 0 \quad \partial^{\sigma} X^{\mu}(\tau, l) = 0}$$

$\sigma = 0$

N Boundary conditions  
for an open string

$$\rightarrow X^{\mu}(\tau, \sigma) = X^{\mu}(\tau, \sigma + l)$$

(note that  $\delta X^{\mu}$  must be periodic as well) ⇒  $\sigma \sim \sigma + l$   
closed strings

D - Boundary condition.

$\frac{1}{2} (-x)(-y) \delta_{ab} \delta_{ab} (\partial x)^2$

*[The rest of the chalkboard contains heavily scribbled-out mathematical content, including various symbols like  $\partial x$ ,  $\delta_{ab}$ , and  $\delta_{ab}$ .]*



D - Boundary condition. (↑)

$$\delta X_n(\tau, 0) = \delta X_n(\tau, \ell) = 0$$

D - Boundary condition.  $\uparrow$

$$\delta X_m(\tau, 0) = \delta X_m(\tau, \ell) = 0$$

$$X_m(\tau, 0) = \text{const}$$

$$X_m(\tau, \ell) = \text{const}$$

$$\mathbb{T}_4^a = 0$$

⇒ Possible BC that preserve  
Poincare inv in D-dim.

$$B\Gamma = -\frac{2}{4\pi\alpha'} \int d\tau \sqrt{-g} \underbrace{\gamma^{\sigma\tau}}_{=} \partial_\sigma X^\mu \cdot \delta X_\mu$$

$$\Rightarrow \left[ \begin{array}{l} \partial_\sigma X^\mu(\tau, 0) = 0 \\ \partial^\sigma X_\mu(\tau, \ell) = 0 \end{array} \right]$$

$\sigma = \ell$   
 $\sigma = 0$   
N Boundary conditions  
for an open string

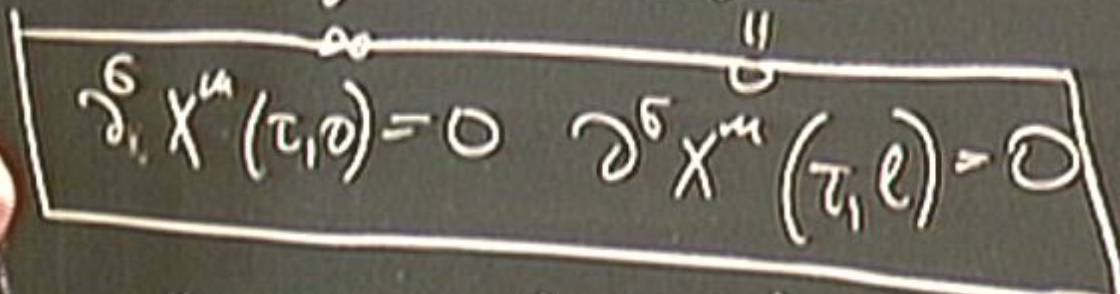
$$\rightarrow X^\mu(\tau, \sigma) = X^\mu(\tau, \sigma + \ell)$$

(note that  $\delta X^\mu$  must be periodic as well) ⇒  $\sigma \sim \sigma + \ell$   
closed strings

$$\mathbb{T}_4^a = 0$$

⇒ Possible BC that preserve  
Poincare inv in D-dim.

$$B^T = -\frac{2}{4\pi\alpha'} \int_{-\infty}^{\infty} d\tau \sqrt{-g} \underbrace{\gamma^{\sigma\tau}}_{=} \partial_\sigma X^\mu \cdot \delta X_\mu$$



σ = l  
σ = 0

N Boundary conditions  
for an open string

$$X^\mu(\tau, \sigma) = X^\mu(\tau, \sigma + l)$$

(note that  $\delta X^\mu$  must be periodic as well) ⇒  $\sigma \sim \sigma + l$   
closed strings

$$\mathbb{T}_4^a = 0$$

⇒ Possible BC that preserve  
Poincare inv in D-dim.

$$B^T = -\frac{2}{4\pi\alpha'} \int_{-\infty}^{\infty} d\tau \left( \sqrt{-g} \gamma^{\sigma\tau} \partial_\sigma X^\mu \delta X_\mu \right) \Big|_{\sigma=0}^{\sigma=l}$$

$$\Rightarrow \partial_\sigma X^\mu(\tau, 0) = 0 \quad \partial^\sigma X_\mu(\tau, l) = 0$$

N Boundary conditions  
for an open string

$$\rightarrow X^\mu(\tau, \sigma) = X^\mu(\tau, \sigma + l)$$

(note that  $\delta X^\mu$  must be periodic as well) ⇒  $\sigma \sim \sigma + l$   
closed strings



D - Boundary condition.  $\uparrow$

$$\delta X_m(\tau, 0) = \delta X_m(\tau, \ell) = 0$$

$$\uparrow X''(\tau, 0) = \text{cont}$$

$$X''(\tau, \ell) = \text{cont}$$

$$\mathbb{T}_4^a = 0$$

⇒ Possible BC that preserve

Poincare inv in D-dim.

$$B\Gamma = -\frac{2}{4\pi\alpha'} \int_{-\infty}^{\infty} d\tau \left( \sqrt{-g} \gamma^{\sigma\tau} \partial_\sigma X^\mu \delta X_\mu \right) \Big|_{\sigma=0}^{\sigma=l}$$

$$\Rightarrow \boxed{\partial_\sigma X^\mu(\tau, 0) = 0 \quad \partial^\sigma X_\mu(\tau, l) = 0}$$

N Boundary conditions for an open string

$$\rightarrow X^\mu(\tau, \sigma) = X^\mu(\tau, \sigma + l)$$

(note that  $\delta X^\mu$  must be periodic as well) ⇒  $\sigma \sim \sigma + l$  closed strings

D - Boundary condition.  $\uparrow$

$$\delta X_m(\tau, 0) = \delta X_m(\tau, l) = 0$$

$$\uparrow X_m(\tau, 0) = \text{cont}$$

$$X_m(\tau, l) = \text{cont}$$

D - Boundary condition.

$$\delta X_{\mu}(\tau, 0) = \delta X_{\mu}(\tau, \ell) = 0$$

$$X^{\mu}(\tau, 0) = \text{cont}$$

$$X^{\mu}(\tau, \ell) = \text{cont}$$

Breaks Poincare  
invariance

$$\pi_4^0 = 0$$

⇒ Possible BC that preserve

Poincare inv in D-dim

$$-\frac{2}{4\pi\alpha'} \int_{-\infty}^{\infty} d\tau \left( \sqrt{-g} \gamma^{\sigma\alpha} \partial_\alpha X^\mu \delta X_\mu \right)$$

$$\partial^\sigma X^\mu(\tau, \ell) = 0$$

$\sigma = \ell$   
 $\sigma = 0$   
 N Boundary conditions for an open string

$$X^\mu(\tau, \sigma) = X^\mu(\tau, \sigma + \ell)$$

that  $\delta X^\mu$  must be periodic as well ⇒  $\sigma \sim \sigma + \ell$   
 closed strings

$$\mathbb{T}_4^a = 0$$

⇒ Possible BC that preserve

Poincare inv in D-dim

$$B\Gamma = -\frac{2}{4\pi\alpha'} \int d\tau \left( \sqrt{-g} \gamma^{\sigma\tau} \partial_\sigma X^m \delta X_m \right) \Big|_{\sigma=0}^{\sigma=l}$$

$$\Rightarrow \boxed{\partial_\sigma X^m(\tau, 0) = 0 \quad \partial^\sigma X_m(\tau, l) = 0}$$

N Boundary conditions for an open string

$$\rightarrow X^m(\tau, \sigma) = X^m(\tau, \sigma + l)$$

(note that  $\delta X^m$  must be periodic as well) ⇒  $\sigma \sim \sigma + l$  closed strings

$$BT = - \frac{E}{4\pi\alpha'} \int_M d^4x \sqrt{-g} \nabla_b \left[ \sqrt{-g} g^{ab} \partial_a X^{\mu} \delta X_{\mu} \right]$$

$$BT \equiv - \frac{E}{4\pi d^1} \int_M d\sigma \nabla_b \left[ \sqrt{-g} g^{ab} \partial_a X^{\mu} \delta X_{\mu} \right]$$



$$BT \equiv - \frac{E}{4\pi d^2} \int_{\mathcal{M}} \nabla_b [ \sqrt{-g} g^{ab} \partial_a X^c \delta X_c ]$$

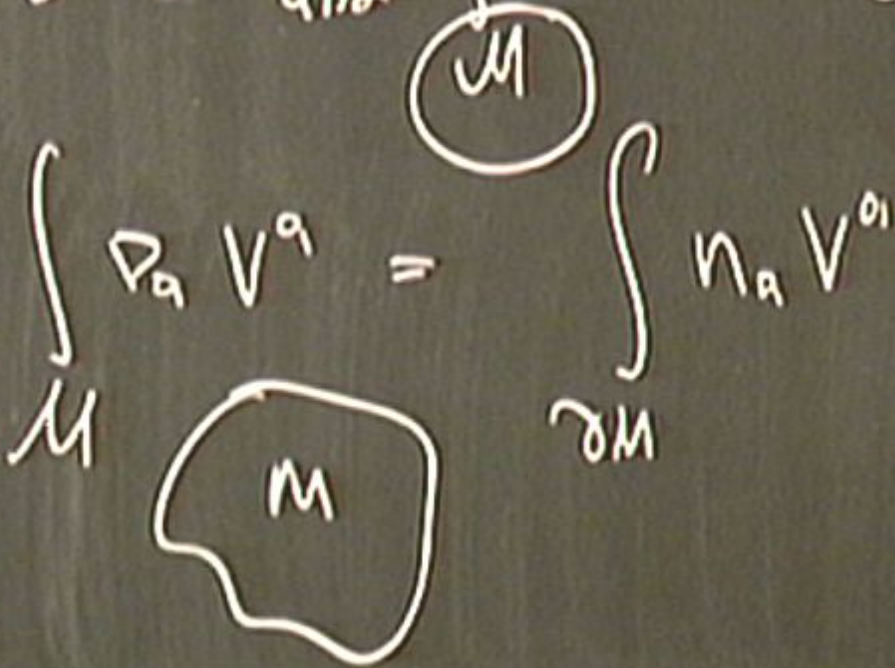
$$BT = - \frac{E}{4\pi d^1} \int_{\mathcal{M}} \nabla_b [ \sqrt{-g} g^{ab} \partial_a X^{\mu} \delta X_{\mu} ]$$

$$\int \mathcal{D}_g$$

$$BT \equiv - \frac{E}{4\pi d^1} \int_{\mathcal{M}} \text{div} \, d\sigma \, \nabla_b \left[ \sqrt{-g} \, g^{ab} \partial_a X^u \delta X_u \right]$$

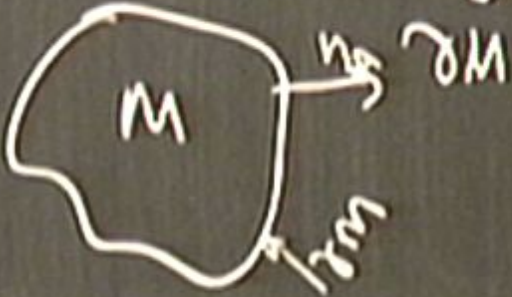
$$\int_{\mathcal{M}} \nabla_a V^a$$

$$BT \equiv - \frac{E}{4\pi d^1} \int_{\partial M} d\sigma \nabla_b [ \sqrt{-g} g^{ab} \partial_a X^c \delta X_c ]$$

$$\int_M \partial_a V^a = \int_{\partial M} n_a V^a$$


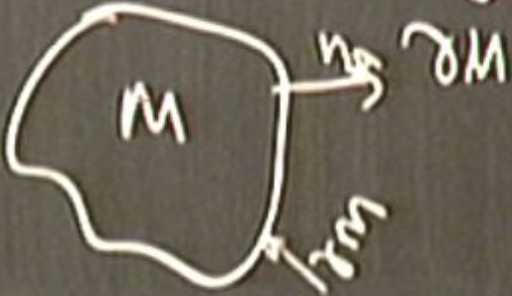


$$BT \equiv - \frac{E}{4\pi d^1} \int_{\mathcal{M}} d^4x \nabla_b [\sqrt{-g} g^{ab} \partial_a X^c \delta X_c]$$

$$\int_{\mathcal{M}} \partial_a V^a = \int_{\partial \mathcal{M}} n_a V^a$$


The diagram shows a manifold  $\mathcal{M}$  represented as an irregular blob. Its boundary is labeled  $\partial \mathcal{M}$ . A normal vector  $n_a$  is shown as an arrow pointing outwards from the boundary. The manifold itself is labeled  $\mathcal{M}$  inside the blob.

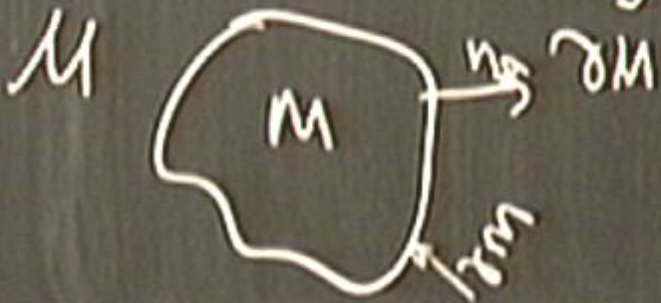
$$BT \equiv - \frac{E}{4\pi d^1} \int_{\partial M} d\sigma_a \nabla_b \left[ \sqrt{-g} g^{ab} \partial_a X^{\mu} \delta X_{\mu} \right]$$

$$\int_M \partial_a V^a = \int_{\partial M} n_a V^a$$


The diagram shows a manifold  $M$  represented as an irregular blob. The boundary is labeled  $\partial M$ . A normal vector  $n_a$  is shown as an arrow pointing outwards from the boundary. The manifold itself is labeled  $M$  inside the blob.

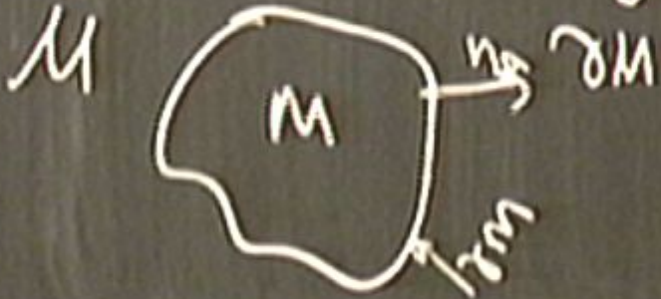
$$BT \text{ (E)} - \frac{1}{4\pi d'} \int_{\partial M} d\sigma \nabla_b \left[ \sqrt{-g} g^{ab} \partial_a X^{\mu} \delta X_{\mu} \right]$$

$$\int_M \partial_a V^a = \int_{\partial M} n_a V^a \quad \text{(E)} - \frac{1}{2\pi d'} \int_{\partial M}$$



$$BT \text{ (E)} - \frac{1}{4\pi d'} \int \text{div } d\sigma \nabla_b \left[ \sqrt{-g} g^{ab} \partial_a \chi^c \delta \chi_c \right]$$

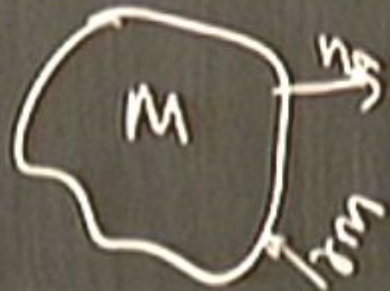
$$\int_M \partial_a V^a = \int_{\partial M} n_a V^a \quad \text{(E)} - \frac{1}{2\pi d'} \int_{\partial M} n_b \partial^b \chi^c$$





$$BT \text{ (E)} - \frac{1}{4\pi d'} \int d\sigma \nabla_b [\sqrt{-g} g^{ab} \partial_a X^m \delta X_m]$$

$$\int_M \nabla_a V^a = \int_{\partial M} n_a V^a \quad \text{(E)} - \frac{1}{2\pi d'} \int_{\partial M} n_b \partial^b X^m \delta X_m$$



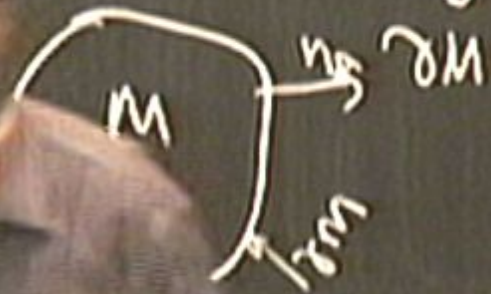
$$h_b \partial^b X^m = 0$$

$$\text{BT } \textcircled{=} - \frac{1}{4\pi\alpha'} \int_{\mathcal{M}} d\sigma^a d\sigma^b \nabla_b \left[ \sqrt{-g} g^{ab} \partial_a X^\mu \delta X_\mu \right]$$



$$\int \partial_a V^a = \int n_a V^a$$

$$\textcircled{=} - \frac{1}{2\pi\alpha'} \int n_b \partial^b X^\mu \delta X_\mu$$

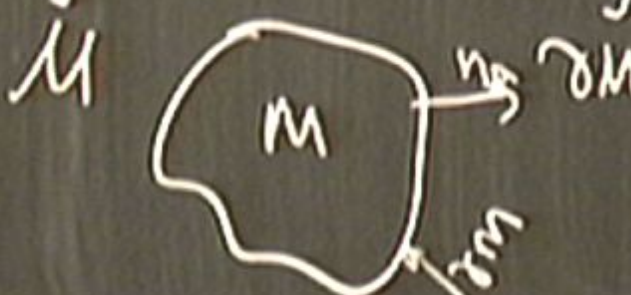


$$h_b \partial^b X^\mu = 0$$

→ N Boundary condition for open string.

$$\text{BT } \textcircled{=} - \frac{1}{4\pi\alpha'} \int d\sigma d\tau \nabla_b \left[ \sqrt{-g} g^{ab} \partial_a X^\mu \delta X_\mu \right]$$

$$\int_M \nabla_a V^a = \int_{\partial M} n_a V^a \quad \textcircled{=} - \frac{1}{2\pi\alpha'} \int n_b \partial^b X^\mu \delta X_\mu$$



$$h_b \partial^b X^\mu = 0$$

→  $N$  Boundary condition for open string.

⇒ some symmetries.

- Poincaré inv in  $D$ -dim
- diff inv in  $1+1$ -dim
- Weyl inv

⇒ solve symmetries.

- Poincare inv in  $D$ -dim
- diff inv in  $1+1$ -dim
- Weyl inv

⇒ two-derivative actions.

for now  $\left\{ \gamma_{ab}, X^m \right\}$



⇒ solve symmetries.

- Poincare inv in  $D$ -dim
- diff inv in  $1+1$ -dim
- Weyl inv

⇒ two-derivative actions.

for now  $\left\{ \gamma_{ab}, X^m \right\}$

$\Rightarrow (\partial X)^2$  can enter linearly in Polyakov action

$$\sqrt{-g} g^{ab} (\partial X)^2 + \text{something}$$



$\Rightarrow (\partial X)^2$  can enter linearly in Polyakov action

$$\sqrt{-g} \delta^{ab} (\partial X)^2 + \text{something}[\delta_{ab}]$$

$\Rightarrow$   $X^4$  satisfies EOM

$$\square = 0$$



$\Rightarrow (\partial X)^2$  can enter linearly in Polyakov action

$$\sqrt{-g} \gamma^{ab} (\partial X)^2 + \text{something}[\gamma_{ab}]$$



$\Rightarrow (\partial X)^2$  - can enter linearly in Polyakov action

$$\sqrt{-g} \gamma^{ab} (\partial X)^2 + \text{something}[\gamma_{ab}]$$

$\rightarrow$  must be geometric invariants

$\Rightarrow (\partial X)^2$  can enter linearly in Polyakov action

$$\sqrt{-g} \gamma^{ab} (\partial X)^2 + \text{something}[\gamma_{ab}]$$

$\hookrightarrow$  must be geometric invariants

$\mathbb{R}, \mathbb{R}$

$\Rightarrow (\partial X)^\mu$  - can enter linearly in Polyakov action

$$\sqrt{-g} \gamma^{ab} (\partial X)^2 + \text{something}[\gamma_{ab}]$$

$\Rightarrow$  must be geometric invariants

$\mathbb{R}, \mathbb{R}$

(note that  $X^\mu$  must be periodic as well)  $\Rightarrow \sigma \sim \sigma + 1$  closed strings

$\Rightarrow (\partial X)^\mu$  - can enter linearly in Polyakov action

$$-\frac{1}{2} \eta^{\alpha\beta} (\partial_\alpha X^\mu)(\partial_\beta X^\mu) + \text{something}[\gamma_{ab}]$$

$\Rightarrow$  must be geometric invariants

$F_8, \mathbb{R}$

$\Rightarrow \int d\tau d\sigma F_8 \mathbb{R}$  potential new term.

(note that  $X^\mu$  must be periodic as well)  $\Rightarrow \sigma \sim \sigma + 2\pi$  closed string

$\Rightarrow (\partial X)^2$  can enter linearly in Polyakov action

$$\sqrt{-g} \delta^{ab} (\partial X)^2 + \text{something}[\delta_{ab}]$$

$\Rightarrow$  must be geometric invariant

$$\sqrt{-g}, \textcircled{R}$$

$\Rightarrow$   $\int d\tau d\sigma \sqrt{-g} R$  potential new term.

(note that  $X^\mu$  must be periodic as well)  $\Rightarrow$  closed strings

Why inv?

$$\underline{Y_{ab} \rightarrow e^{2i\omega} k_{ab}}$$

$$\sqrt{-g} \rightarrow e^{2\omega} \sqrt{-g}$$

$\gamma_{ab}(\tau, \sigma) = 0$  }  $\text{Brokes Poincare}$   
 $\text{invariant}$   
 $\frac{1}{2} \dot{x}^\mu \dot{x}^\mu$   
 $(2x)^2$



Why inv?

$$\frac{Y_{ab} \rightarrow e^{2i\omega} k_{ab}}{\sqrt{-g} \rightarrow e^{2i\omega} \sqrt{-g}}$$

$$\sqrt{-g} \rightarrow e^{2i\omega} \sqrt{-g}$$



$S^2$

Breaks Poincare  
invariant



Why inv?

$$Y_{ab} \rightarrow e^{2\omega} Y_{ab}$$

$$\sqrt{-g} \rightarrow e^{2\omega} \sqrt{-g}$$

$S^2 \rightarrow$  of radius 1

Breaks Poincaré

invariance

Why inv?

$$\frac{Y_{ab} \rightarrow e^{2i\omega} k_{ab}}{\sqrt{-g} \rightarrow e^{2i\omega} \sqrt{-g}}$$

$$\sqrt{-g} \rightarrow e^{2i\omega} \sqrt{-g}$$

$S^2 \rightarrow$  of radius 1

$$R = 2$$

Why inv?

$$Y_{ab} \rightarrow e^{2\omega} \gamma_{ab}$$

$$\sqrt{-g} \rightarrow e^{2\omega} \sqrt{-\gamma}$$

$S^2 \rightarrow$  of radius  $\frac{1}{2}$

$$R = \frac{2}{\frac{1}{2}}$$

radius of the sphere  $\rightarrow L$

$$\frac{2}{\frac{1}{2}}$$

Why inv?

$$\gamma_{ab} \rightarrow e^{2\omega} \gamma_{ab}$$

$$\sqrt{-g} \rightarrow e^{2\omega} \sqrt{-g}$$

$S^2 \rightarrow$  of radius  $\frac{1}{2}$

$$R = \frac{2}{L^2}$$

radius of the sphere  $\rightarrow L$

$$R = \frac{2}{L^2}$$

$$g_{\mu\nu} \rightarrow L^2 g_{\mu\nu}$$

Why inv?

$$\gamma_{ab} \rightarrow e^{2\omega} \gamma_{ab}$$

$$\sqrt{-g} \rightarrow e^{2\omega} \sqrt{-g}$$

$$\rightarrow R \rightarrow R \cdot e^{-2\omega}$$

$S^2 \rightarrow$  of radius  $\frac{L}{2}$

$$R = \frac{2}{L^2}$$

radius of the sphere  $\rightarrow L$

$$R = \frac{2}{L^2}$$

$$g_{\mu\nu} \rightarrow L^2 g_{\mu\nu}$$

Why inv?

$$\underline{Y_{ab} \rightarrow e^{2\omega} \lambda_{ab}}$$

$$\sqrt{-g} \rightarrow e^{2\omega} \sqrt{-\lambda}$$

$$\rightarrow R \rightarrow R \cdot e^{-2\omega}$$

If  $R_{;\omega} = 0$

$S^2 \rightarrow$  of radius  $\frac{1}{2}$

$$R = \frac{2}{\frac{1}{2}}$$

radius of the sphere  $\rightarrow L$

$$R = \frac{2}{L^2}$$

$$g_{\mu\nu} \rightarrow L^2 g_{\mu\nu}$$

Why inv?

$$\gamma_{ab} \rightarrow e^{2\omega} \gamma_{ab}$$

$$\sqrt{-g} \rightarrow e^{2\omega} \sqrt{-g}$$

$\rightarrow R \rightarrow e^{-2\omega} [R]$   
If  $R_{,\omega} = 0$   
 $\omega = \omega(\tau, \sigma)$

$S^2 \rightarrow$  of radius  $\frac{1}{2}$

$$R = \frac{2}{\frac{1}{2}}$$

radius of the sphere  $\rightarrow L$

$$R = \frac{2}{L^2}$$

$$g_{\mu\nu} \rightarrow L^2 g_{\mu\nu}$$

Why inv?

$$\gamma_{ab} \rightarrow e^{2\omega} \gamma_{ab}$$

$$\sqrt{-g} \rightarrow e^{2\omega} \sqrt{-g}$$

$\rightarrow R \rightarrow e^{-2\omega} \left[ R - 2(d-1) \Delta \omega - (d-2)(d-1) (\nabla \omega)^2 \right]$   
 If  $R_{,\omega} = 0$   
 $\omega = \omega(\tau, \sigma)$

$S^2 \rightarrow$  of radius  $\frac{1}{2}$

$R = \frac{2}{\frac{1}{2}}$  is invariant

radius of the sphere  $\rightarrow L$

$$R = \frac{2}{L^2}$$

$g_{\mu\nu} \rightarrow L^2 g_{\mu\nu}$



Why inv?

$$\gamma_{ab} \rightarrow e^{2\omega} \gamma_{ab}$$

$$\sqrt{-g} \rightarrow e^{2\omega} \sqrt{-g}$$

$\rightarrow R \rightarrow e^{-2\omega} \left[ R - 2(d-1) \Delta \omega - (d-2)(d-1) (\nabla \omega)^2 \right]$   
 If  $R_{,\omega} = 0$   
 $\omega = \omega(\tau, \sigma)$

$S^2 \rightarrow$  of radius  $\frac{1}{2}$

$$R = \frac{2}{\frac{1}{2}}$$

radius of the sphere  $\rightarrow L$

$$R = \frac{2}{L^2}$$

$g_{\mu\nu} \rightarrow L^2 g_{\mu\nu}$   
 $(d-2)(d-1) (\nabla \omega)^2$

$$d=2$$

$$\int_{-\infty}^{\infty} dx \delta(x) f(x) \mathbb{R} \rightarrow \int_{-\infty}^{\infty} dx \delta(x) f(x) e^{2i\omega x} e^{-2i\omega x} \mathbb{R}$$



$$\int_{\mathcal{M}} d\tau d\sigma \sqrt{-g} R \rightarrow \int d\tau d\sigma \sqrt{-g} e^{2\omega} e^{-2\omega'} [R - 2\Box\omega]$$

$\mathcal{M}$

*[The following text is heavily obscured by diagonal brush strokes and is largely illegible. It appears to contain mathematical derivations or notes related to the equation above.]*



$$\int d\tau d\sigma \sqrt{-g} R \rightarrow \int d\tau d\sigma \sqrt{-g} e^{\frac{2\omega}{\ell}} e^{-\frac{2\omega}{\ell}} [R - 2' \square \omega]$$

$\mathcal{M}$

Extra term  $-2 \int d\tau d\sigma \sqrt{-g} \nabla_a \nabla^a \omega$

*[Faded handwritten notes and scribbles, including some boxed equations]*

$$\int d\tau d\sigma \sqrt{-g} R \Rightarrow \int d\tau d\sigma \sqrt{-g} e^{\frac{2\omega}{l}} e^{-\frac{2\omega}{l}} [R - 2' \square \omega]$$

Extra term  $-2 \int d\tau d\sigma \sqrt{-g} \nabla_a \nabla^a \omega = -2 \int d\tau d\sigma$

$$\nabla_a [\sqrt{-g} \nabla^a \omega]$$

$$\int d\tau d\sigma \sqrt{-g} R \Rightarrow \int d\tau d\sigma \sqrt{-g} e^{\frac{2\omega}{\ell}} e^{-\frac{2\omega}{\ell}} [R - 2\epsilon'(\omega)]$$

$\mathcal{M}$

Extra term  $-2 \int d\tau d\sigma \sqrt{-g} \nabla_a \nabla^a \omega = -2 \int d\tau d\sigma \dots$

$$\nabla_a [\sqrt{-g} \nabla^a \omega] = -2 \int \dots$$

$$\int d\tau d\sigma \sqrt{-g} R \Rightarrow \int d\tau d\sigma \sqrt{-g} e^{\frac{2\omega}{l}} e^{-\frac{2\omega}{l}} [R - 2\epsilon'(\omega)]$$

$\mathcal{M}$

Extra term  $-2 \int d\tau d\sigma \sqrt{-g} \nabla_a \nabla^a \omega = -2 \int d\tau d\sigma \dots$

$$\nabla_a [\sqrt{-g} \nabla^a \omega] = -2 \int_{\mathcal{M}} n_a \cdot (\sqrt{-g} \nabla^a \omega)$$



$$\int d\tau d\sigma \sqrt{-g} R \Rightarrow \int d\tau d\sigma \sqrt{-g} e^{\frac{2\omega}{\ell}} e^{-\frac{2\omega}{\ell}} [R - 2' \square \omega]$$

$\mathcal{M}$

Extra term  $-2 \int d\tau d\sigma \sqrt{-g} \nabla_a \nabla^a \omega = -2 \int d\tau d\sigma \dots$

$$= 0 \quad \nabla_a [\sqrt{-g} \nabla^a \omega] = -2 \int_{\mathcal{M}} n_a \cdot (\sqrt{-g} \nabla^a \omega)$$

$$\int d\tau d\sigma \sqrt{-g} R \rightarrow \int d\tau d\sigma \sqrt{-g} e^{\frac{2\omega}{\alpha'}} e^{-2\omega} [R - 2' \square \omega]$$

$\mathcal{M}$

Extra term  $-2 \int d\tau d\sigma \sqrt{-g} \nabla_a \nabla^a \omega = -2 \int d\tau d\sigma \dots$

$$\nabla_a [\sqrt{-g} \nabla^a \omega] = -2 \int_{\mathcal{M}} n_a \cdot (\sqrt{-g} \nabla^a \omega)$$

$= 0 \rightarrow \int_{\text{closed string}}$

$$\int d\tau d\sigma \sqrt{-g} R \Rightarrow \int d\tau d\sigma \sqrt{-g} e^{\frac{2\omega}{\alpha'}} e^{-2\omega} [R - 2\alpha' \square \omega]$$

$\mathcal{M}$

Extra term  $-2 \int d\tau d\sigma \sqrt{-g} \nabla_a \nabla^a \omega = -2 \int d\tau d\sigma$

$$\nabla_a [\sqrt{-g} \nabla^a \omega] = -2 \int_{\mathcal{M}} n_a \cdot (\sqrt{-g} \nabla^a \omega)$$

$= 0$   $\int_{\text{closed string}} (\partial M = 0)$   $\int_{\mathcal{M}}$   $(\sqrt{-g} \nabla^a \omega)$

$$\int d\tau d\sigma \sqrt{-g} R \rightarrow \int d\tau d\sigma \sqrt{-g} e^{\frac{2\omega}{\alpha'}} e^{-2\omega} [R - 2\alpha' \square \omega]$$

$\mathcal{M}$

Extra term  $-2 \int d\tau d\sigma \sqrt{-g} \nabla_a \nabla^a \omega = -2 \int d\tau d\sigma \dots$

$$\nabla_a [\sqrt{-g} \nabla^a \omega] = -2 \int_{\mathcal{M}} n_a (\sqrt{-g} \nabla^a \omega)$$

$= 0$   $\int_{\text{closed string}} (\partial M = 0)$   $\int_{\mathcal{M}}$   $(\sqrt{-g} \nabla^a \omega)$

$$\int d\tau d\sigma \sqrt{-g} R \rightarrow \int d\tau d\sigma \sqrt{-g} e^{\frac{2\omega}{\alpha'}} e^{-\frac{2\omega}{\alpha'}} [R - \alpha' \square \omega]$$

$\mathcal{M}$

Extra term  $-2 \int d\tau d\sigma \sqrt{-g} \nabla_a \nabla^a \omega = -2 \int d\tau d\sigma$

$$\nabla_a [\sqrt{-g} \nabla^a \omega] = -2 \int_{\partial \mathcal{M}} n_a (\sqrt{-g} \nabla^a \omega)$$

$= 0$   $\int_{\text{closed string}} (\partial \mathcal{M} = 0)$   $\int_{\partial \mathcal{M}}$   $\int_{\text{BT}}$

$\rightarrow$  possible (but natural to gener to open strings)



How EH on a worldsheet would affect EDM?

~~... Poincaré invariance  
... shift invariance  
... locality  
... Lorentz invariance  
... parity  
... time reversal  
... charge conjugation  
... CP  
... CPT~~





How EH on a worldsheet would affect EDM?

$$\frac{\delta S}{\delta X_\mu} = 0$$

$$\frac{\delta S}{\delta X_{\mu\nu}} = 0$$

How EH on a worldsheet would affect EOM?

$$\frac{\delta S}{\delta X_\mu} = 0$$

$$\frac{\delta S}{\delta \gamma_{ab}} = 0$$

$$\delta \int_M R \sqrt{-\gamma} d^2x = \int_M \left( R_{ab} - \frac{1}{2} \gamma_{ab} R \right) \delta \gamma^{ab}$$

How EH on a worldsheet would affect EOM?

$$\frac{\delta S}{\delta X_\mu} = 0$$

$$\frac{\delta S}{\delta \gamma_{ab}} = 0$$

$$\delta \int_{\mathcal{M}} R \sqrt{-\gamma} d^2 \sigma = \int_{\mathcal{M}} \left( R_{ab} - \frac{1}{2} \gamma_{ab} R \right) \delta \gamma^{ab}$$

$$R_{ab} - \frac{1}{2} \gamma_{ab} R$$

$$\frac{\delta S}{\delta X_\mu} = 0$$

$$\frac{\delta S}{\delta \gamma_{ab}} = 0$$

$\Rightarrow$  we are in  $d=2$

$$\delta \int_{\mathcal{M}} R \sqrt{-g} \, d^d x = \int_{\mathcal{M}} \left( R_{ab} - \frac{1}{2} \gamma_{ab} R \right) \delta \gamma^{ab}$$

$$R_{ab} - \frac{1}{2} \gamma_{ab} R$$

$$\frac{\delta S}{\delta X_\mu} = 0$$

$$\frac{\delta S}{\delta \gamma_{ab}} = 0$$

$$\delta \int \mathcal{L} \sqrt{-g} \, d^d x = \int \left( R_{ab} - \frac{1}{2} \gamma_{ab} R \right) \delta \gamma^{ab}$$

$\Rightarrow$  We are in  $d=2$   
 $R_{ab} - \frac{1}{2} \gamma_{ab} R$   
 $\#$  of independent components of  $R_{ab}$

$$\frac{1}{2} d^2 (d^2 - 1)$$

$$\frac{\delta S}{\delta x_\mu} = 0$$

$$\frac{\delta S}{\delta \gamma_{ab}} = 0$$

$$\delta \int R \sqrt{-g} \, d^d x = \int \left( R_{ab} - \frac{1}{2} \gamma_{ab} R \right) \delta \gamma^{ab} \, d^d x$$

$$R_{ab} - \frac{1}{2} \gamma_{ab} R$$

⇒ we are in  $d=2$

# of independent components of  $R_{ab}$

$$\frac{1}{12} d^2 (d^2 - 1) \Big|_{d=2} =$$

$$\frac{\delta S}{\delta X_\mu} = 0$$

$$\frac{\delta S}{\delta \gamma_{ab}} = 0$$

$$\delta \int \mathcal{L} \sqrt{-g} \, d^d x = \int \left( R_{ab} - \frac{1}{2} \gamma_{ab} R \right) \delta \gamma^{ab}$$

$$R_{ab} - \frac{1}{2} \gamma_{ab} R$$

⇒ we are in  $d=2$

# of independent components of  $R_{ab}$

$$\frac{1}{12} d^2 (d-1) \Big|_{d=2} = 1$$

$$\sum_s \delta x_{\mu} = 0$$

$$\sum_s \delta x_{ab} = 0$$

$$\delta \int R \sqrt{-g} \, d^d x = \int \left( R_{ab} - \frac{1}{2} g_{ab} R \right) \delta x^{ab}$$

$\Rightarrow$  we are in  $d=2$   
# of independent components of

$$R_{ab} = \lambda$$

$$\frac{1}{12} d^2 (d^2 - 1) \Big|_{d=2} = 1$$



$$\delta \chi_n = 0$$

$$\delta \int \delta \chi_{ab} = 0$$

$$\delta \int R \sqrt{-g} \, d^d x = \int \left( R_{ab} - \frac{1}{2} \delta_{ab} R \right) \delta g^{ab}$$

$$R_{ab} - \frac{1}{2} \delta_{ab} R$$

$\Rightarrow$  We are in  $d=2$   
# of independent components of  $R_{ab}$

$$\frac{1}{12} d^2 (d^2 - 1) \Big|_{d=2} = 1$$

$$R_{ab} = \lambda \delta_{ab}$$

$$\frac{\delta S}{\delta \chi_a} = 0$$

$$\frac{\delta S}{\delta \gamma_{ab}} = 0$$

$$\delta \int \sqrt{-g} \, dt \, d^3x = \int \left( R_{ab} - \frac{1}{2} \gamma_{ab} R \right) \delta \gamma^{ab}$$

$$R_{ab} - \frac{1}{2} \gamma_{ab} R$$

⇒ we are in  $d=2$   
 # of independent components of  $R_{ab}$

$$\frac{1}{12} d^2(d-1) = 1$$

$$R_{ab} = \lambda \gamma_{ab} \quad R = \lambda \cdot 2$$

$$\frac{\delta S}{\delta \chi_{\mu}} = 0$$

$$\frac{\delta S}{\delta \gamma_{ab}} = 0$$

$$\delta \int d^d x \sqrt{-g} \mathcal{L} = \int d^d x \left( R_{ab} - \frac{1}{2} \gamma_{ab} R \right) \delta \gamma^{ab}$$

$$R_{ab} - \frac{1}{2} \gamma_{ab} R$$

⇒ we are in  $d=2$

# of independent components of  $R_{ab}$

$$\frac{1}{12} d^2 (d^2 - 1) \Big|_{d=2} = 1$$

$$R_{ab} = \lambda \gamma_{ab} \quad \mathcal{L} = \lambda \cdot 2 \Rightarrow \lambda = \frac{1}{2} R$$

$$\delta \chi_n = 0$$

$$\delta \chi_{ab} = 0$$

$$\delta \int R \sqrt{-g} \, d^d x = \int \left( R_{ab} - \frac{1}{2} g_{ab} R \right) \delta g^{ab} \, d^d x$$

$$R_{ab} - \frac{1}{2} g_{ab} R$$

⇒ we are in  $d=2$

# of independent components of  $R_{ab}$

$$\frac{1}{2} d^2 (d^2 - 1) \Big|_{d=2} = 1$$

$$R_{ab} = \lambda g_{ab} \quad R = \lambda \cdot 2 \Rightarrow \lambda = \frac{1}{2} R$$

$$R_{ab} = \frac{1}{2} R \delta_{ab} \quad (\text{only in } d=2 \text{ dim})$$

$\nabla_{\mu} \nabla_{\nu} \delta_{\alpha\beta} = 0$   
 $\nabla_{\mu} \delta_{\alpha\beta} = 0$   
something  $\delta_{ab}$  -  $\delta_{ab}$   $\nabla_{\mu} \delta_{\alpha\beta}$

invariance

new form

$$\frac{\delta S}{\delta x_a} = 0$$

$$\frac{\delta S}{\delta \gamma_{ab}} = 0$$

$$\delta \int \sqrt{-g} R \, d^d x = \int \left( R_{ab} - \frac{1}{2} \gamma_{ab} R \right) \delta \gamma^{ab} \, d^d x$$

$$R_{ab} - \frac{1}{2} \gamma_{ab} R = 0$$

⇒ we are in  $d=2$

# of independent components of  $R_{ab}$

$$\frac{1}{2} d^2 (d-1) \Big|_{d=2} = 1$$

$$R_{ab} = \lambda \gamma_{ab} \quad R = \lambda \cdot 2 \Rightarrow \lambda = \frac{1}{2} R$$

$$R_{ab} = \frac{1}{2} R \delta_{ab} \quad (\text{only in } d=2 \text{ dim})$$

$$\nabla^2 \phi + \text{something}(\delta_{ab}) - \delta_{ab} \dots$$

metric invariance

in this form



$$R_{ab} = \frac{1}{2} R \delta_{ab} \quad (\text{only in } d=2 \text{ dim})$$

$\sqrt{-g}$  does not affect EOM

new form



$$R_{ab} = \frac{1}{2} R \delta_{ab} \quad (\text{only in } d=2 \text{ dim})$$

$\sqrt{g} R$  does not affect EOM

↑ is a topological term

$$R_{ab} = \frac{1}{2} R \delta_{ab} \quad (\text{only in } d=2 \text{ dim})$$

$\sqrt{g} R$  does not affect EOM

↑  
topological term

$$S_P = \int d^2x \, \dots$$

$$R_{ab} = \frac{1}{2} R \delta_{ab} \quad (\text{only in } d=2 \text{ dim})$$

$\sqrt{-g}$  does not affect EOM

$\int \sqrt{-g} R$  is a topological term

$$S_P' = \int d^d x \sqrt{-g} \left[ \frac{1}{4\pi\alpha'} \delta^{ab} \partial_a X^\mu \partial_b X_\mu + \frac{1}{4\pi\alpha'} R \right]$$

$$R_{ab} = \frac{1}{2} R \delta_{ab} \quad (\text{only in } d=2 \text{ dim})$$

$\sqrt{-g}$  does not affect EOM

$\uparrow$  is a topological term

additional "constant"

$$S_P' = \int d^2x \sqrt{-g} \left[ \frac{1}{4\pi\alpha'} \delta^{ab} \partial_a X^\mu \partial_b X_\mu + \frac{1}{4\pi\alpha'} R \right]$$

new term

$$R_{ab} = \frac{1}{2} R \delta_{ab} \quad (\text{only in } d=2 \text{ dim})$$

$\sqrt{g} R$  does not affect EOM

↑ is a topological term.

↑ additional "constant"

$$S'_P = \int d^2x \sqrt{g} \left[ \frac{1}{4\pi\alpha'} \delta^{ab} \partial_a X^\mu \partial_b X_\mu + \frac{1}{4\pi\alpha'} R \right]$$

new term

$$R_{ab} = \frac{1}{2} R \delta_{ab} \quad (\text{only in } d=2 \text{ dim})$$

$\sqrt{-g}$  does not affect EOM

$\uparrow$  is a topological term.

additional "constant"

$$S_P = \int d^2x \sqrt{-g} \left[ \frac{1}{4\pi\alpha'} \delta^{ab} \partial_a X^\mu \partial_b X_\mu + \frac{1}{4\pi\alpha'} R \right]$$



$$R_{ab} = \frac{1}{2} R \delta_{ab} \quad (\text{only in } d=2 \text{ dim})$$

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$$S_P = \int d^2x \sqrt{-g} \left[ \frac{1}{4\pi\alpha'} \delta^{ab} \partial_a X^\mu \partial_b X_\mu + \frac{1}{4\pi\alpha'} R \right]$$



→ Introducing extra fields on worldsheet (fermions).

*[The rest of the chalkboard contains very faint and mostly illegible handwritten notes, including mathematical symbols like  $\omega$ ,  $\psi$ ,  $\gamma$ , and  $\delta$ .]*



⇒ Introducing extra fields on worldsheet (Fermions).

*[The rest of the chalkboard contains very faint and mostly illegible handwritten notes, including mathematical symbols like  $\omega$ ,  $\psi$ , and  $\gamma$ .]*

→ Introducing extra fields on worldsheet (Fermions).

Study of spectrum of open strings

*[The rest of the page is heavily scribbled out with chalk, making the text illegible.]*

→ Introducing extra fields on worldsheet (fermions).

Study of spectrum of open strings

→ D

radius of the sphere  $\rightarrow L$

$$\frac{2}{1.2}$$

$$d=2$$

⇒ Introducing extra fields on worldsheet (fermions).

Study of spectrum of open strings

⇒  $D = 26$  (obtain critical dim of string theory) <sup>bosonic</sup>

→ Introducing extra fields on worldsheet (fermions).

Study of spectrum of open strings

→  $D = 26$  (obtain critical dim of string theory) <sup>bosonic</sup>

→ NG

⇒ Introducing extra fields on worldsheet (fermions).

Study of spectrum of open strings

⇒  $D = 6$  (obtain critical dim of string theory) <sup>bosonic</sup>

⇒ does not have tachyon

$d = 2$

→ Introducing extra fields on worldsheet (fermions).

Study of spectrum of open strings

→  $-26$  (obtain critical dim of string theory) <sup>bosonic</sup>

it does not have  $\alpha$

→ Introducing extra fields on worldsheet (fermions).

Study of spectrum of open strings

→  $D = 26$  (obtain critical dim of <sup>bosonic</sup> string theory)

→ NG it does not have  $\alpha'$



⇒ Introducing extra fields on worldsheet (fermions).

Study of spectrum of open strings

⇒  $D = 26$  (obtain critical dim of string theory) <sup>bosonic</sup>

⇒ NG it does not have  $(\alpha)$

⇒ we need to fix symmetries of the Polyakov action.

⇒ Introducing extra fields on worldsheet (fermions).

Study of spectrum of open strings

⇒  $D = 26$  (obtain critical dim of <sup>bosonic</sup> string theory)

⇒ NG it does not have  $(\alpha)$

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Study of spectrum of open strings

⇒  $D = 26$  (obtain critical dim of string theory) <sup>bosonic</sup>

⇒ NG it does not have  $(\alpha)$   $(\tau, \sigma, \delta \alpha)$

⇒ we need to fix symmetries of the Polyakov action.

⇒ Introducing extra fields on worldsheet (fermions).

Study of spectrum of open strings

⇒  $D = 26$  (obtain critical dim of string theory) <sup>bosonic</sup>

⇒ NG it does not have  $(\alpha)$   $(\tau, \sigma, \delta \tau, \delta \sigma)$

⇒ we need to fix symmetries of the Polyakov action.

How E.H. on a worldsheet would affect EDM?

light-cone gauge

$$\eta_{\mu\nu} = \left( -1, \underbrace{+1, \dots, +1}_D \right)$$

How E.H. on a worldsheet would affect EOM?

light-cone gauge

$$\eta_{\mu\nu} = \begin{pmatrix} -1, & \underbrace{+1, \dots, +1} \end{pmatrix}$$

$$X^0, X^1, \dots, X^{D-1}$$

$\Rightarrow$

How E.H. on a worldsheet would affect EDM?

light-cone gauge

$$\eta_{\mu\nu} = \begin{pmatrix} -1, & \underbrace{+1, \dots, +1} \end{pmatrix}$$

$$X^0, X^1, \dots, X^{D-1}$$

$$X^\pm$$

$\Rightarrow$

How E.H. on a worldsheet would affect EDM?

light-cone gauge

$$\eta_{\mu\nu} = (-1, \underbrace{+1, \dots, +1})$$

$$\circlearrowleft X^0, \circlearrowleft X^1, \dots, X^{D-1}$$

$$X^\pm$$

$\Rightarrow$



How E.H. on a worldsheet would affect EOM?

light-cone gauge

$$\eta_{\mu\nu} = (-1, \underbrace{+1, \dots, +1})$$

$$X^\pm = \frac{1}{\sqrt{2}} (X^0 \pm X^1)$$

$\Rightarrow$

$$\underbrace{X^0, X^1}_{\text{D.}}, X^{D-1}$$

How EH on a worldsheet would affect EDM?

Light-cone gauge

$$\eta_{uv} = (-1, \underbrace{+1, \dots, +1})$$

$$X^\pm = \frac{1}{\sqrt{2}} (X^0 \pm X^1)$$

$$\Rightarrow \textcircled{X^0}, \textcircled{X^1}, \dots, X^{D-1}$$

$$a^u | b_u$$

How tH on a worldsheet would affect EDM?

Light-cone gauge

$$\eta_{\mu\nu} = (-1, \underbrace{+1, \dots, +1})$$

$$X^\pm = \frac{1}{\sqrt{2}} (X^0 \pm X^1)$$

$$\textcircled{X^0}, \textcircled{X^1}, \dots, X^{D-1}$$

$$a^\mu | b_\mu = -a^0 b^0 + a^1 b^1 + \dots + a^{D-1} b^{D-1}$$

∴

How EH on a worldsheet would affect EDM?

Light-cone gauge

$$\eta_{\mu\nu} = \begin{pmatrix} -1, & \underbrace{+1, \dots, +1} \end{pmatrix}$$

$$X^\pm = \frac{1}{\sqrt{2}} (X^0 \pm X^1)$$

$$\circlearrowleft X^0, \circlearrowleft X^1, \dots, X^{D-1} \Rightarrow$$

$$a^\mu b_\mu = -a^0 b^0 + a^1 b^1 + \dots + a^{D-1} b^{D-1}$$

$$= -a^+ b^- - a^- b^+ + a^i b^i \quad i=1, \dots, D-1$$

# Light-cone gauge

$$\eta_{\mu\nu} = (-1, \underbrace{+1, \dots, +1})$$

$$X^\pm = \frac{1}{\sqrt{2}} (X^0 \pm X^1)$$

$$\circlearrowleft X^0, \circlearrowleft X^1, \dots, X^{D-1}$$

$$a^\mu b_\mu = -a^0 b^0 + a^1 b^1 + \dots + a^{D-1} b^{D-1}$$

$$a_- = -a^+ \quad \Rightarrow \quad -a^+ b^- - a^- b^+ + a^i b^i \quad i=2, \dots, D-1$$

$$K_{ab} = \lambda \delta_{ab} \quad L = \lambda Z \Rightarrow \lambda = \frac{1}{2} R$$

# Light-cone gauge

$$\eta_{\mu\nu} = (-1, \underbrace{+1, \dots, +1})$$

$$X^\pm = \frac{1}{\sqrt{2}} (X^0 \pm X^1)$$

$\Rightarrow$

$$\textcircled{X^0}, \textcircled{X^1}, \dots, X^{D-1}$$

$$a^\mu b_\mu = -a^0 b^0 + a^1 b^1 + \dots + a^{D-1} b^{D-1}$$

$$a_- = -a^+$$

$$a_+ = -a^-$$

$$= -a^+ b^- - a^- b^+ + a^i b^i \quad i=2, \dots, D-1$$

$$= a^+ b_+ + a^- b_- + a^i b_i$$

$$K_{ab} = \lambda \delta_{ab} \quad L = \lambda Z \Rightarrow \lambda = \frac{1}{2} R$$

# Light-cone gauge

$$\eta_{\mu\nu} = (-1, \underbrace{+1, \dots, +1})$$

$$X^\pm = \frac{1}{\sqrt{2}} (X^0 \pm X^1)$$

$\Rightarrow$

$$\textcircled{X^0}, \textcircled{X^1}, \dots, X^{D-1}$$

$$a^\mu b_\mu = -a^0 b^0 + a^1 b^1 + \dots + a^{D-1} b^{D-1}$$

$$a_- = -a^+$$

$$a_+ = -a^-$$

$$= -a^+ b^- - a^- b^+ + a^i b^i$$

$$= a^+ b_+ + a^- b_- + a^i b_i$$

$i=2, \dots, D-1$

$$K_{ab} = \lambda \delta_{ab}$$

$$L = \lambda Z \Rightarrow \lambda = \frac{1}{2} R$$





QM

$$\{L, X\}$$

conjugate  
variable

$$\{H, P\}$$

how did it get

*[The rest of the chalkboard contains very faint and mostly illegible handwritten notes, including some mathematical symbols and phrases like "additional" and "constant".]*

QM

$$\{L, X\}$$

conjugate  
variable

$$\{H, P\}$$

$$P_{LX} =$$

QM

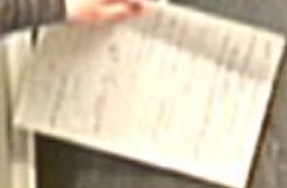
$$\{L, X\}$$

conjugate  
variable

$$\{H, P\}$$

can add to  $H$

$$P_{in} = -i \frac{\partial}{\partial x}$$



QM

$$\{L, X\}$$

conjugate  
variable

$$\{H, P\}$$

can add to set

$$P_m = -i \frac{\partial}{\partial x^m}$$

$$[P_m, X^0] = 0$$

additional  
constant

QM

$$\{L, X\}$$

conjugate  
variable

$$\{H, P\}$$

$$P_m = -i \frac{\partial}{\partial x^m}$$

$$[P_m, X^n] = -i \delta_m^n$$

QM

$$\{L, X\}$$

conjugate  
variable

$$\{H, P\}$$

$$P_m = -i \frac{\partial}{\partial x^m}$$

$$[P_m, X^{\nu}] = -i \delta_m^{\nu}$$

QM

$$\{L, X\}$$

conjugate  
variable

$$\{H, P\}$$

$$P_{\mu} = -i \frac{\partial}{\partial x^{\mu}}$$

$$[P_{\mu}, X^{\nu}] = -i \delta_{\mu}^{\nu}$$

$$\hookrightarrow P_0 = -i \frac{\partial}{\partial t}$$

QM

$$\{L, X\}$$

conjugate  
variable

$$\{H, P\}$$

$$P_u = -i \frac{\partial}{\partial x^u}$$

$$[P_u, X^v] = -i \delta_u^v$$

$$\hookrightarrow P_0 = -i \frac{\partial}{\partial t}$$





QM

$$\{L, X\}$$

conjugate  
variable

$$\{H, P\}$$

any) for, add to P.E.T

$$P_u = -i \frac{\partial}{\partial x^u}$$

$$[P_u, X^v] = -i \delta_u^v$$

$$\hookrightarrow P_0 = -i \frac{\partial}{\partial t}$$

$$i \frac{\partial \psi}{\partial t} = H \psi$$

QM

$$\{L, X\}$$

conjugate variable

$$\{H, P\}$$

have additional

$$P_u = -i \frac{\partial}{\partial x_u}$$

$$[P_u, X^u] = -i \delta_u^u$$

$$\hookrightarrow P_0 = -i \frac{\partial}{\partial t}$$

$$H = i \frac{\partial \Psi}{\partial t}$$

QM

$$\{L, X\}$$

conjugate variable

$$\{H, P\}$$

$$P_u = -i \frac{\partial}{\partial x^u}$$

$$[P_u, X^v] = -i \delta_u^v$$

$$\hookrightarrow P_0 = -i \frac{\partial}{\partial t}$$

$$i \frac{\partial \psi}{\partial t} = H \psi$$

$$H = i \frac{\partial}{\partial t}$$

QM

$$\{L, X\}$$

conjugate variable

$$\{H, P\}$$

for additional

$$P_u = -i \frac{\partial}{\partial x^u}$$

$$[P_u, X^v] = -i \delta_u^v$$

$$\hookrightarrow P_0 = -i \frac{\partial}{\partial t}$$

$$i \frac{\partial}{\partial t} = H$$

$$H = i \frac{\partial}{\partial t}$$

$$H = -P_0$$

QM

$$\{L, X\}$$

conjugate variable

$$\{H, P\}$$

canonical transformation

$$P_u = -i \frac{\partial}{\partial X^u}$$

$$[P_u, X^v] = -i \delta_u^v$$

$$\hookrightarrow P_0 = -i \frac{\partial}{\partial t}$$

$$i \frac{\partial \psi}{\partial t} = H \psi$$

$$H = i \frac{\partial}{\partial t}$$

$$H = -P_0$$

Now

$$X^+ = \tau$$

conjugate  $P_+$

$$H = -P_+ P_-$$

Now

$$X^+ = \tau$$

confused  $P_+$

$$H = -P_+ P$$

Here

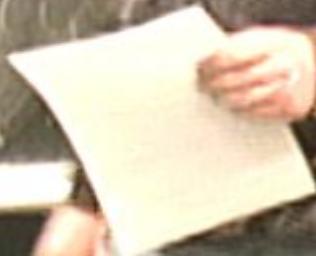
Now

$$X^+ = \tau$$

confused  $P_+$

$$H = -P_+ P_-$$

Thus the role of a light-cone energy.





Now

$$X^+ = \tau$$

confused  $P_+$

$$H = -P_+ P_-$$

plays the role of a light-cone energy.

Now

$$X^+ = \tau$$

confused  $P_+$

$$H = -P_+ P_-$$

plays the role of a light-cone energy.

$$S_{pp} = \frac{1}{2} \int d\tau \left[ \eta^{-1} \dot{X}^m \dot{X}_m - \eta m^2 \right]$$

Now

$$X^+ = \tau$$

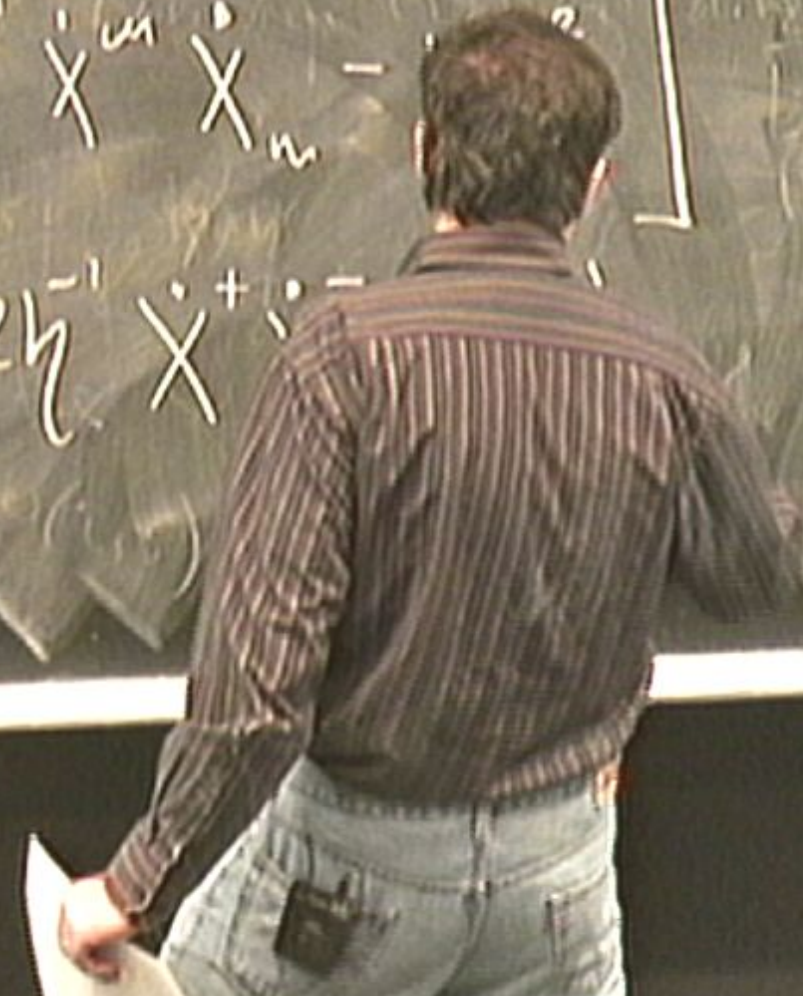
confused  $P_+$

$$H = -P_+ P_-$$

plays the role of a light-cone energy.

$$S_{pp} = \frac{1}{2} \int d\tau \left[ \eta^{-1} \dot{X}^{\mu} \dot{X}_{\mu} - \dots \right]$$

$$= \frac{1}{2} \int d\tau \left[ -2\eta^{-1} \dot{X}^+ \dot{X}^- - \dots \right]$$



Now

$$X^+ = \tau$$

confused  $P_+$

$$H = -P_+ P_-$$

plays the role of a light-cone energy.

$$S'_{pp} = \frac{1}{2} \int d\tau \left[ \eta^{-1} \dot{X}^m \dot{X}_m - \eta m^2 \right]$$

$$= \frac{1}{2} \int d\tau \left[ -2\eta^{-1} \dot{X}^+ \dot{X}^- + \eta^i \dot{X}^i \dot{X}^i \right]$$



Now

$$X^+ = \tau$$

conjugate  $P_+$

$$H = -P_+ P_-$$

plays the role of a light-cone energy.

$$S'_{pp} = \frac{1}{2} \int d\tau \left[ \eta^{-1} \dot{X}^m \dot{X}_m - \eta m^2 \right]$$

$$= \frac{1}{2} \int d\tau \left[ -2\eta^{-1} \dot{X}^+ \dot{X}^- + \eta^i \dot{X}^i \dot{X}^i - \eta m^2 \right]$$

⇒ How EH on a worldsheet would affect EOM?

$$X^+ = \tau$$

$$\dot{X}^+ = 1$$

$$X^+ = \frac{1}{\sqrt{2}} (X^0 + X^1)$$

How EH on a worldsheet would affect EDM?

$$X^+ = \tau$$

$$\dot{X}^+ = 1$$

$$\textcircled{=} \frac{1}{2} \int d\tau \left[ -2\eta^{-1} \dot{X}^- + \eta^{-1} \dot{X}^i \dot{X}^i - \eta m^2 \right]$$

$$X^+ = \tau$$

$$\dot{X}^+ = 1$$

$$\textcircled{=} \frac{1}{2} \int d\tau \left[ -2\eta^{-1} \dot{X}^- + \eta^{-1} \dot{X}^i \dot{X}^i - \eta m^2 \right]$$



$$X^+ = \tau$$

$$\dot{X}^+ = 1$$

$$\textcircled{=} \frac{1}{2} \int d\tau \left[ -2\eta^{-1} \dot{X}^- + \eta^{-1} \dot{X}^i \dot{X}^i - \eta m^2 \right]$$

Naively:  $\eta, X^i$

$$x^+ = \tau$$

$$\dot{x}^+ = 1$$

$$\textcircled{=} \frac{1}{2} \int d\tau \left[ -2\eta^{-1} \dot{x}^- + \eta^{-1} \dot{x}^i \dot{x}^i - \eta m^2 \right]$$

Naively:  $\eta, x^i$

$$P_{\eta} =$$

$$X^+ = \tau$$

$$\dot{X}^+ = 1$$

$$\textcircled{=} \frac{1}{2} \int d\tau \left[ -2\eta^{-1} \dot{X}^- + \eta^{-1} \dot{X}^i \dot{X}^i - \eta m^2 \right]$$

Naively:  $\eta, X^i$

$$P_{\eta} = \frac{\delta L}{\delta \dot{X}^{\eta}}$$

$$X^+ = \tau$$

$$\dot{X}^+ = 1$$

$$\textcircled{=} \frac{1}{2} \int d\tau \left[ -2\eta^{-1} \dot{X}^- + \eta^{-1} \dot{X}^i \dot{X}^i - \eta m^2 \right]$$

Naively:  $\eta, X^i$  }  $P_- =$

$$P_{+i} = \frac{\partial L}{\partial \dot{X}^i} =$$

$$X^+ = \tau$$

$$\dot{X}^+ = 1$$

$$\textcircled{=} \frac{1}{2} \int d\tau \left[ -2\eta^{-1} \dot{X}^- + \eta^{-1} \dot{X}^i \dot{X}^i - \eta m^2 \right]$$

Naively:  $\eta, X^i$   $\left\{ \begin{array}{l} P_- = -\eta^{-1} \Rightarrow (P^+ = \eta^{-1}) \\ \dots \end{array} \right.$

$$P_{\mu} = \frac{\delta L}{\delta \dot{X}^{\mu}} =$$

$$X^+ = \tau$$

$$\dot{X}^+ = 1$$

$$\textcircled{=} \frac{1}{2} \int d\tau \left[ -2\eta^{-1} \dot{X}^- + \eta^{-1} \dot{X}^i \dot{X}^i - \eta m^2 \right]$$

Naively:  $\eta, X^i$

$$P_{\eta} = \frac{\partial L}{\partial \dot{\eta}} = \left\{ \begin{array}{l} P_{-} = -\eta^{-1} \Rightarrow (P = \\ P_{i} = \dot{X}^i \end{array} \right.$$

$$X^+ = \tau$$

$$\dot{X}^+ = \underline{1}$$

$$\textcircled{=} \frac{1}{2} \int d\tau \left[ -2\eta^{-1} \dot{X}^- + \eta^{-1} \dot{X}^i \dot{X}^i - \eta m^2 \right]$$

Naively:  $\eta, X^i$   $\left\{ \begin{array}{l} P_- = -\eta^{-1} \Rightarrow (\dot{\tau} = \eta^{-1}) \\ P_i = \dots \end{array} \right.$

$$P_{\mu} = \frac{\partial L}{\partial \dot{X}^{\mu}} = \dots$$

$$X^+ = \tau$$

$$\dot{X}^+ = 1$$

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$$P_{\mu} = \frac{\partial L}{\partial \dot{X}^{\mu}}$$

$$\{x^-, x^+\} \Rightarrow \{P, \mathbb{P}\}$$

$$\{x^-, x^i\} \Rightarrow \{P_-, P_i\}$$

$$H = P_- \dot{x}^- + P_i \dot{x}^i - L$$

Now

$$X^+ = \tau$$

conjugate  $P_+$

$$H = -P_+ P_-$$

plays the role of a light-cone energy.

$$S'_{pp} = \frac{1}{2} \int d\tau \left[ \eta^{-1} \dot{X}^{\mu} \dot{X}_{\mu} - \eta m^2 \right]$$
$$= \frac{1}{2} \int d\tau \left[ -2\eta^{-1} \dot{X}^+ \dot{X}^- + \eta^{-1} \dot{X}^i \dot{X}^i - \eta m^2 \right] \quad (1)$$

$$\{x^-, x^i\} \Rightarrow \{P_-, P_i\}$$

$$H = P_- \dot{x}^- + P_i \dot{x}^i - L$$

Note that there is no term  $P_+ \dot{x}^+$

$$\{x^-, x^i\} \Rightarrow \{P_-, P_i\}$$

$$H = P_- \dot{x}^- + P_i \dot{x}^i - L$$

(Note that there is no term  $P_+ \dot{x}^+$   $x^+$  is not a dynamical variable.)

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(Note that there is no term  $P_+ \dot{x}^+$   $x^+$  is not a dynamical variable.)



$$\ddot{X} + \eta^{-1} X \dot{X} - \eta m^2 X = 0$$

$$\left\{ \begin{array}{l} P_- = -\eta^{-1} \Rightarrow (P^+ = \eta^{-1}) \\ P_i = \eta^{-1} \dot{X}^i \Rightarrow \dot{X}^i = \eta \cdot P_i = \frac{P_i}{\eta} \end{array} \right.$$

$$\{x^-, x^i\} \Rightarrow \{p_-, p_i\}$$

$$H = p_- \dot{x}^- + p_i \dot{x}^i - L$$

(Note that there is no term  $p_+ \dot{x}^+$ ,  $x^+$  is not a dynamical variable.)

$$= p_- \dot{x}^- + \frac{p_i \cdot p_i}{p^+}$$

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$$= P_- \dot{x}^- + \frac{P_i \cdot P_i}{P_+} + \eta^{-1} \dot{x}^- - \frac{1}{2} P_+ \frac{P_i \cdot P_i}{P_+^2} + \frac{1}{2} \eta m^2$$

$$\{x^-, x^i\} \Rightarrow \{p_-, p_i\}$$

$$H = p_- \dot{x}^- + p_i \dot{x}^i - L$$

(Note that there is no term  $p_+ \dot{x}^+$   $x^+$  is not a dynamical variable.)

$$= p_- \dot{x}^- + \frac{p_i \cdot p_i}{p_+} - p_- \dot{x}^- - \frac{1}{2} p_+^2 \frac{p_i \cdot p_i}{p_+^2} + \frac{1}{2} \eta m^2$$



$$\{x^-, x^i\} \Rightarrow \{p_-, p_i\}$$

$$H = p_- \dot{x}^- + p_i \dot{x}^i - L$$

(Note that there is no term  $p_+ \dot{x}^+$   $x^+$  is not a dynamical variable.)

$$= p_- \dot{x}^- + \frac{p_i \cdot p_i}{p^+} - p_- \dot{x}^- - \frac{1}{2} p^+ \frac{p_i \cdot p_i}{p^+ 2} + \frac{1}{2} \eta m^2$$

$$= \frac{1}{2} \frac{p_i \cdot p_i}{p^+} +$$

$$\{x^-, x^i\} \Rightarrow \{p_-, p_i\}$$

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$$= p_- \dot{x}^- + \frac{p_i p_i}{p_+} - p_- \dot{x}^- - \frac{1}{2} p_+^2 \frac{p_i p_i}{p_+^2} + \frac{1}{2} \eta m^2$$

$$= \frac{1}{2} \frac{p_i p_i}{p_+} + \frac{1}{2} p_+ m^2 = \frac{p_i p_i + p_+^2 m^2}{2 p_+}$$

$$H = \frac{p_i p_i + m^2}{2pt}$$

$$H =$$

$$H = \frac{p^i p^i + m^2}{2p^+}$$

$$\Rightarrow 2p^+ p^- - p^i p^i = m^2$$

$$H \approx p^-$$

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$$H = p^-$$

$$-2p^+ p^- + p^i p^i = -m^2$$

$$\Rightarrow [p_i, x^i] = -i\delta_i^i$$

$$p_+ p^- = -m^2$$

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$$[p_-, x] = -i \quad |k_-, k^i\rangle$$



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momentum eigenvalues

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$$\Rightarrow 2p^+ p^- - p^i p^i = m^2$$

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$$-2p^+ p^- + p^i p^i = -m^2$$

$$\Rightarrow [p_i, x^i] = -i \delta_i^i$$

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momentum eigenvalues

Do the same for the string (p)

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⇒ consider open string

$$-a < \tau < +\infty, \quad 0 < \xi < \ell$$

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$\Rightarrow$  consider open string  
 $-a < \tau < +\infty, \quad 0 < \sigma < \ell$

$\Rightarrow \tau \Rightarrow \tau'(\tau, \sigma)$   
 $\sigma \Rightarrow \sigma'(\tau, \sigma)$

Do the same for the string  $(P)$ .

$\Rightarrow$  consider open string  
 $-a < \tau < +\infty$  ,  $0 < \sigma < \ell$

$\Rightarrow$   $\tau \Rightarrow \tau'(\tau, \sigma)$  ,  $\sigma \Rightarrow \sigma'(\tau, \sigma)$  / 2 arbitrary functions.

$\Rightarrow$   $\omega$

Do the same for the string (p)

$\Rightarrow$  consider open string  
 $-a < \tau < a$ ,  $0 < \sigma < \ell$

$\Rightarrow$   $\tau \Rightarrow \tau'(\tau, \sigma)$   
 $\sigma \Rightarrow \sigma'(\tau, \sigma)$  } 2 arbitrary functions.

$\Rightarrow$   $\omega(\epsilon, \sigma) \rightarrow +1$  more

$\Rightarrow$  3 conditions  
to fix diffeo +  
+ Weyl

$X^+$

$$\int_{\Omega} X''(z, \theta) - 0 \quad \partial^{\theta} X''(z, \theta) = 0$$

$$X''(z, \theta) \quad Y''(z, \theta)$$

(note that  $X''$  must be zero on the boundary)



$$X^+ = \tau$$

$$\partial_\sigma \gamma_{\sigma\tau} = 0$$

$$\gamma_{\tau\tau} = -1$$

} gauge fixing condition

$$\begin{aligned} X^{\mu\nu}(\tau, \sigma) &= 0 & \partial_\sigma X^{\mu\nu}(\tau, \sigma) &= 0 \\ X^{\mu\nu}(\tau, \sigma) &= Y^{\mu\nu}(\tau, \sigma) \end{aligned}$$

$$X^+ = \tau$$

$$\partial_\sigma \gamma_{\sigma\sigma} = 0$$

$$\gamma = -1$$

} gauge fixing condition

→ light cone gauge for P

⇒ that we can always choose light

$$X^+ = \tau$$

$$\partial_\sigma \gamma_{\sigma\sigma} = 0$$

$$\gamma = -1$$

} gauge fixing condition

→ light cone gauge for P

⇒ that we can always choose light-cone fixing  
locally.

• Set  $X' = \mathbb{C}$

• Consider

$$f =$$

• Set  $X' = \tau$

• Consider

$$f = \gamma_{\sigma\sigma} (-\gamma)^{-1/2}$$

• Set  $X' = \tau$

• Consider

$$f = \gamma_{\sigma\sigma} (-\gamma)^{-1/2}$$

→ (keep  $\tau$  - fixed)

• Set  $x^* = \tau$

• Consider

$$f = \gamma \sigma \sigma (-\delta)^{-1/2}$$

$\Rightarrow$  (keep  $\tau$  - fixed)

$$f d\sigma = f' d\sigma'$$

• Set  $x' = \tau$

• Consider

$$f = \gamma_{\sigma\sigma} (-\gamma)^{-1/2}$$

⇒ (keep  $\tau$  - fixed)

$$f d\sigma = f' d\sigma'$$

invariant length element



Set  $x' = \tau$

Consider

$$f = \gamma_{\sigma\sigma} (-\dot{\sigma})^{-1/2}$$

$\Rightarrow$  (keep  $\tau$  - fixed)

$$f d\sigma = f' d\sigma'$$

invariant length element

$$f' = \gamma_{\sigma'\sigma'} (-\dot{\sigma}')^{-1/2} =$$

$$\begin{aligned} \gamma_{\sigma\sigma} &\rightarrow \gamma_{\sigma'\sigma'} = \gamma_{\sigma\sigma} \left(\frac{d\sigma}{d\sigma'}\right)^2 \\ \dot{\sigma} &\rightarrow \dot{\sigma}' = \dot{\sigma} \left(\frac{d\sigma}{d\sigma'}\right) \end{aligned}$$

$$\sigma \rightarrow \sigma' = \sigma'(\sigma)$$

• Set  $x' = \tau$

• Consider

$$f = \gamma_{\sigma\sigma} (-\gamma)^{-1/2}$$

⇒ (keep  $\tau$  - fixed)

$$f d\sigma = f' d\sigma'$$

invariant length element

$$f' = \gamma_{\sigma'\sigma'} (-\gamma')^{-1/2} =$$

$$\gamma_{\tau\tau} \rightarrow \gamma_{\tau\tau}' = \gamma_{\tau\tau}$$

$$\gamma_{\sigma\sigma} \rightarrow \gamma_{\sigma\sigma}' = \gamma_{\sigma\sigma} \left(\frac{d\sigma}{d\sigma'}\right)^2$$

$$\gamma_{\sigma\tau} \rightarrow \gamma_{\sigma\tau}' = \gamma_{\sigma\tau} \left(\frac{d\sigma}{d\sigma'}\right)$$

$$\sigma \rightarrow \sigma' = \sigma'(\sigma)$$

Set  $x' = \tau$

Consider

$$f = \gamma_{\sigma\sigma} (-\dot{\sigma})^{-1/2}$$

$\Rightarrow$  (keep  $\tau$  - fixed)

$$f d\sigma = f' d\sigma'$$

invariant length element  $z$

$$f' = \gamma_{\sigma\sigma'} (-\dot{\sigma}')^{-1/2} = \gamma_{\sigma\sigma} \left( \frac{d\sigma}{d\sigma'} \right)$$

$$\gamma_{\tau\tau} \rightarrow \gamma_{\tau\tau}' = \gamma_{\tau\tau}$$

$$\gamma_{\sigma\sigma} \rightarrow \gamma_{\sigma\sigma}' = \gamma_{\sigma\sigma} \left( \frac{d\sigma}{d\sigma'} \right)^2$$

$$\gamma_{\sigma\tau} \rightarrow \gamma_{\sigma\tau}' = \gamma_{\sigma\tau} \left( \frac{d\sigma}{d\sigma'} \right)$$

$$\sigma \rightarrow \sigma' = \sigma'(\sigma)$$

Set  $x' = 0$

Consider

$$f = \gamma_{\sigma\sigma} (-\gamma)^{-1/2}$$

⇒ (keep  $\tau$  - fixed)

$$f d\sigma = f' d\sigma'$$

invariant length element  $z$

$$f' = \gamma_{\sigma\sigma'} (-\gamma')^{-1/2} = \gamma_{\sigma\sigma} \left( \frac{d\sigma}{d\sigma'} \right) (-\gamma) \left[ \left( \frac{d\sigma}{d\sigma'} \right)^2 \right]$$

$$\gamma_{\tau\tau} \rightarrow \gamma_{\tau\tau}' = \gamma_{\tau\tau}$$

$$\gamma_{\sigma\sigma} \rightarrow \gamma_{\sigma\sigma}' = \gamma_{\sigma\sigma} \left( \frac{d\sigma}{d\sigma'} \right)^2$$

$$\gamma_{\sigma\tau} \rightarrow \gamma_{\sigma\tau}' = \gamma_{\sigma\tau} \left( \frac{d\sigma}{d\sigma'} \right)$$

$$\sigma \rightarrow \sigma' = \sigma'(\sigma)$$

Set  $x' = \tau$

Consider

$$f = \gamma_{\sigma\sigma} (-\gamma)^{-1/2}$$

$\Rightarrow$  (keep  $\tau$  - fixed)

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invariant length element  $z$

$$f' = \gamma_{\sigma\sigma'} (-\gamma')^{-1/2} = \gamma_{\sigma\sigma} \left( \frac{d\sigma}{d\sigma'} \right) (-\gamma) \left[ \left( \frac{d\sigma}{d\sigma'} \right)^2 \right]^{-1/2}$$

$$\gamma_{\tau\tau} \rightarrow \gamma'_{\tau\tau} = \gamma_{\tau\tau}$$

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$$\sigma \rightarrow \sigma' = \sigma'(\sigma)$$

Set  $x' = \tau$

Consider

$$f = \gamma_{\sigma\sigma} (-\gamma)^{-1/2}$$

⇒ (keep  $\tau$  - fixed)

$$f d\sigma = f' d\sigma'$$

invariant length element  $z$

$$f' = \gamma_{\sigma\sigma'} (-\gamma')^{-1/2} = \gamma_{\sigma\sigma} \left( \frac{d\sigma}{d\sigma'} \right) (-\gamma) \left[ \left( \frac{d\sigma}{d\sigma'} \right)^2 \right]^{-1/2} \left( f \frac{d\sigma}{d\sigma'} \right)$$

$$\gamma_{\tau\tau} \rightarrow \gamma'_{\tau\tau} = \gamma_{\tau\tau}$$

$$\gamma_{\sigma\sigma} \rightarrow \gamma'_{\sigma\sigma} = \gamma_{\sigma\sigma} \left( \frac{d\sigma}{d\sigma'} \right)^2$$

$$\gamma_{\sigma\tau} \rightarrow \gamma'_{\sigma\tau} = \gamma_{\sigma\tau} \left( \frac{d\sigma}{d\sigma'} \right)$$

$$\sigma \rightarrow \sigma' = \sigma'(\sigma)$$

$$\frac{de}{L}$$

$$\sigma = 0$$

$$\sigma = L$$



$$\frac{de}{L} = \frac{f d\sigma}{\int_0^L f d\sigma}$$

$$\sigma = 0$$

$$\sigma = L$$





Choose  $d\sigma = dl$

Choose  $d\sigma = dl$

$$\frac{dl}{L} = \frac{f dl}{\int_0^L f d\sigma}$$

Choose  $d\sigma = dl$



$$\frac{dl}{L} = \frac{f dl}{\int_0^L f d\sigma}$$

$$\Rightarrow f = \frac{1}{L}$$

Choose  $d\sigma = dl$

(II)

$$\frac{dl}{L} = \frac{f dl}{\int_0^L f d\sigma}$$

$$\Rightarrow f = \frac{1}{L} \int_0^L f d\sigma$$

Choose  $d\sigma = dl$

Ⓜ

$$\frac{dl}{L} = \frac{f dl}{\int_0^L f d\sigma}$$

$$f = \frac{\int_0^L f d\sigma}{L}$$

Choose  $d\sigma = dl$

II

$$\frac{dl}{L} = \frac{f dl}{\int_0^L f d\sigma}$$

$$\Rightarrow f = \frac{1}{L} \int_0^L f d\sigma$$

$$\delta\sigma f = 0$$

Choose  $d\sigma = dl$

II

$$\frac{dl}{L} = \frac{f dl}{\int_0^L f d\sigma}$$

$$\Rightarrow f = \frac{1}{L} \int_0^L f d\sigma$$

$$\delta\sigma f = 0$$

$$\frac{de}{L} = \frac{f d\sigma}{\int_0^L f d\sigma}$$

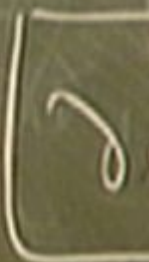
$$\sigma=0$$

$$\sigma=L$$



Choo

$$\frac{de}{L} =$$



$$\Rightarrow \delta_{ab} \rightarrow \delta_{ab} e^{2i\omega}$$

$\gamma$



$$\frac{de}{L} = \frac{f dr}{\int_0^L f dr}$$

$\sigma=0$        $\sigma=L$

Choose  $ds = de$  (II)

$$\frac{de}{L} = \frac{f de}{\int_0^L f dr} \Rightarrow f = \frac{f}{L} \int_0^L f dr$$

$$\partial_\sigma f = 0$$

$$\Rightarrow \delta_{ab} \rightarrow \delta_{ab} e^{2\omega}$$

$$\gamma \rightarrow \gamma e^{4\omega}$$

$\Rightarrow$  we can choose  $\omega$  so that

$$\delta' = \gamma e^{4\omega} = -1$$

$$\frac{de}{L} = \frac{f ds}{\int_0^L f ds}$$

$\sigma=0$        $\sigma=L$

Choose  $ds = de$  (I)

$$\frac{de}{L} = \frac{f de}{\int_0^L f ds} \Rightarrow f = \frac{1}{L} \int_0^L f ds$$

$$\nabla_\sigma f = 0$$

$$\Rightarrow \gamma_{ab} \rightarrow \gamma_{ab} e^{2\omega}$$

$\gamma \rightarrow \gamma e^{4\omega} \Rightarrow$  we can choose  $\omega$  so that

$\Rightarrow$  Show that  $f$  is Weyl inv.  $d' = \gamma e^{4\omega} = -1$

$$f = \gamma_{55} (-\gamma)^{-1/2}$$

$$f \rightarrow f' = \gamma_{55} e^{2\psi} (-\gamma)^{-1/2} (e^{4\psi})^{1/2}$$

So: Fixing diffeo  $\rightarrow$  Weyl.

$$\square \partial f = \square = \partial_0 \left[ \gamma_{55} (-\gamma)^{-1/2} \right] = \left[ \partial_0 \gamma_{55} \right]$$

$$(-\gamma)^{-1/2} = 1$$

$$\gamma_{55} = \gamma_{55}(\tau)$$