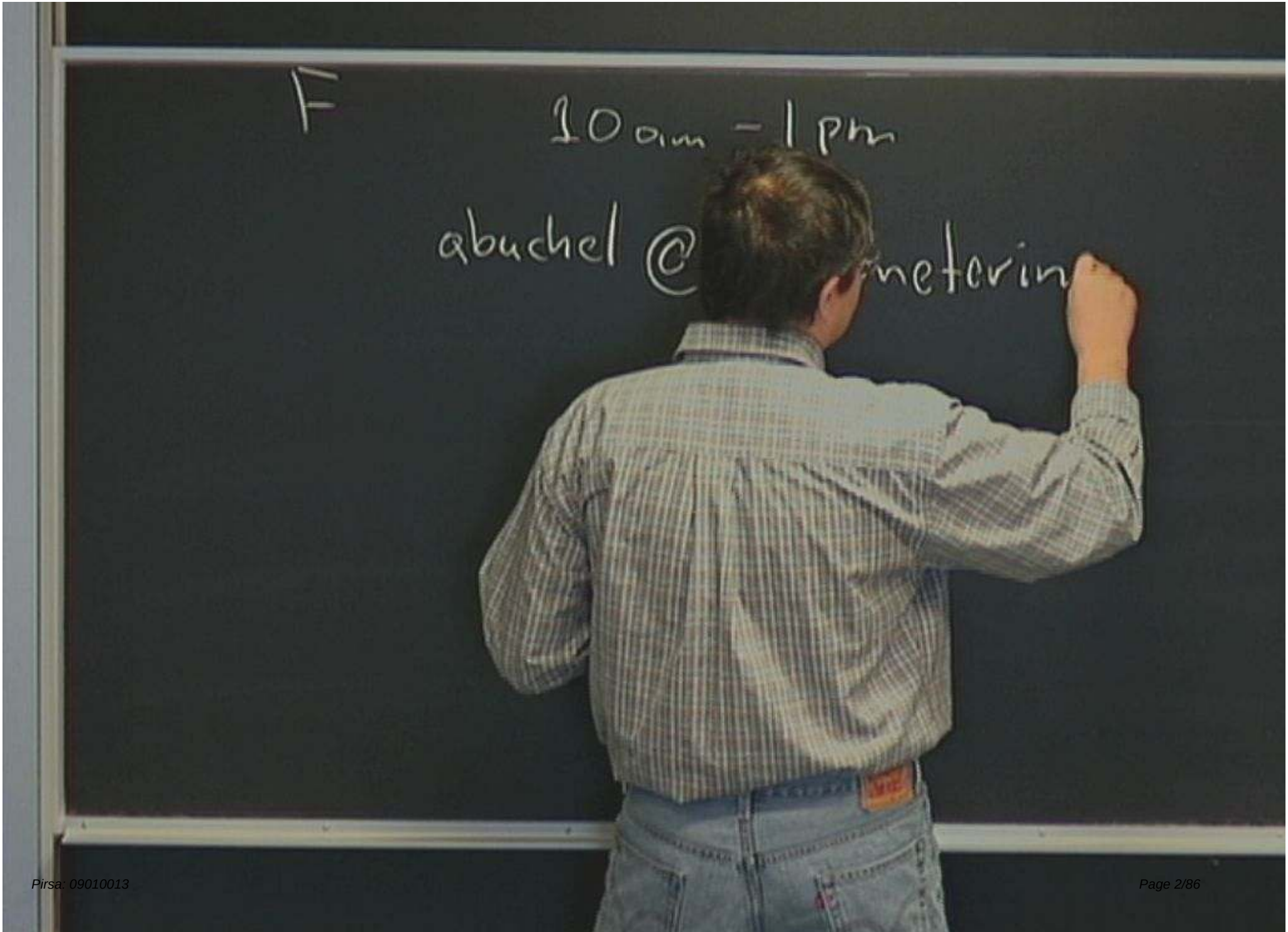
Title: Introduction to the Bosonic String

Date: Jan 16, 2009 10:00 AM

URL: http://pirsa.org/09010013

Abstract: This course provides a thorough introduction to the bosonic string based on the Polyakov path integral and conformal field theory. We introduce central ideas of string theory, the tools of conformal field theory, the Polyakov path integral, and the covariant quantization of the string. We discuss string interactions and cover the tree-level and one loop amplitudes. More advanced topics such as T-duality and D-branes will be taught as part of the course. The course is geared for M.Sc. and Ph.D. students enrolled in Collaborative Ph.D. Program in Theoretical Physics. Required previous course work: Quantum Field Theory (AM516 or equivalent). The course evaluation will be based on regular problem sets that will be handed in during the term. The primary text is the book: 'String theory. Vol. 1: An introduction to the bosonic string. J. Polchinski (Santa Barbara, KITP) . 1998. 402pp. Cambridge, UK: Univ. Pr. (1998) 402 p.' All interested students should contact Alex Buchel at abuchel@uwo.ca as soon as possible.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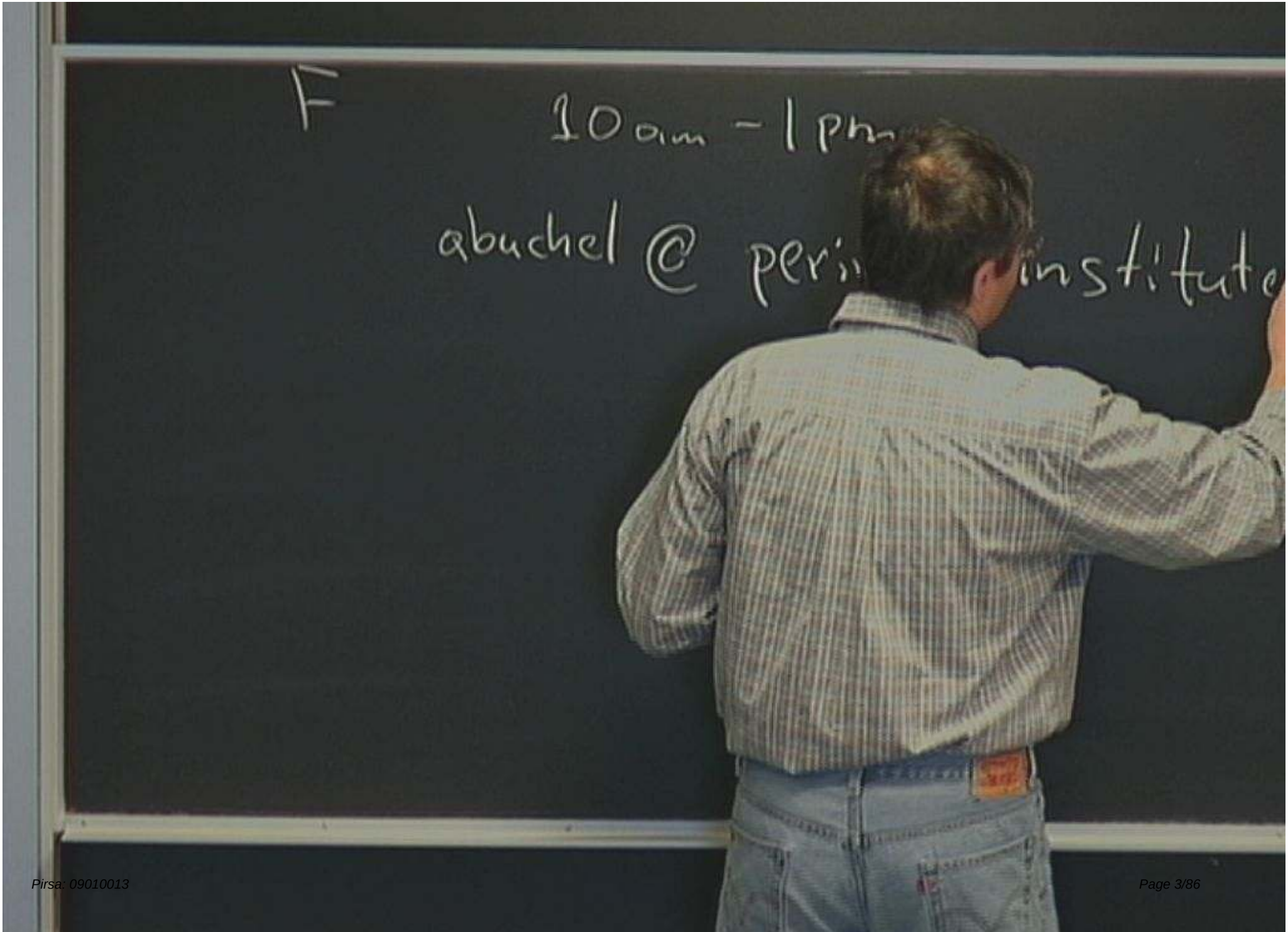
abuchel @ meteorin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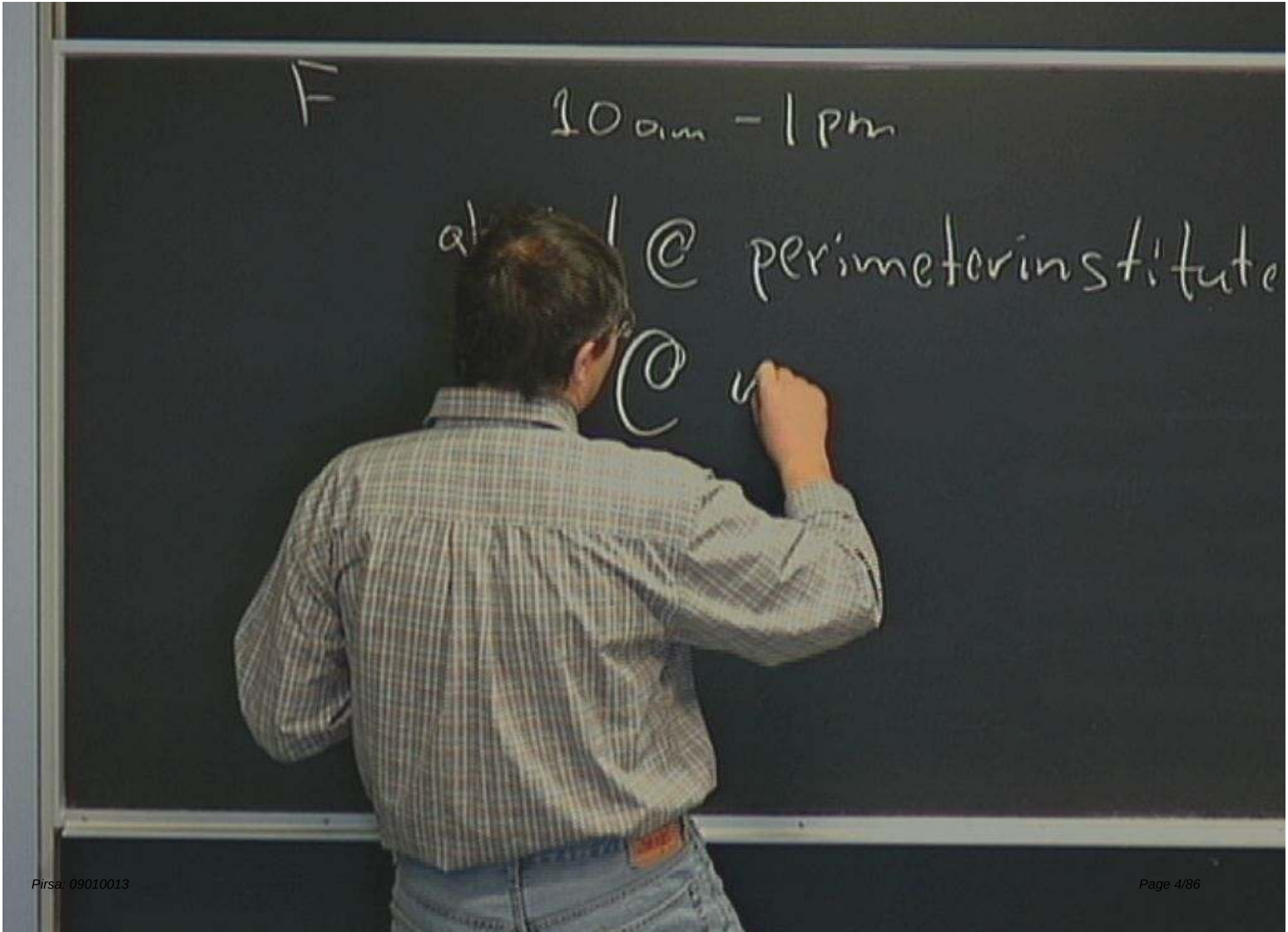
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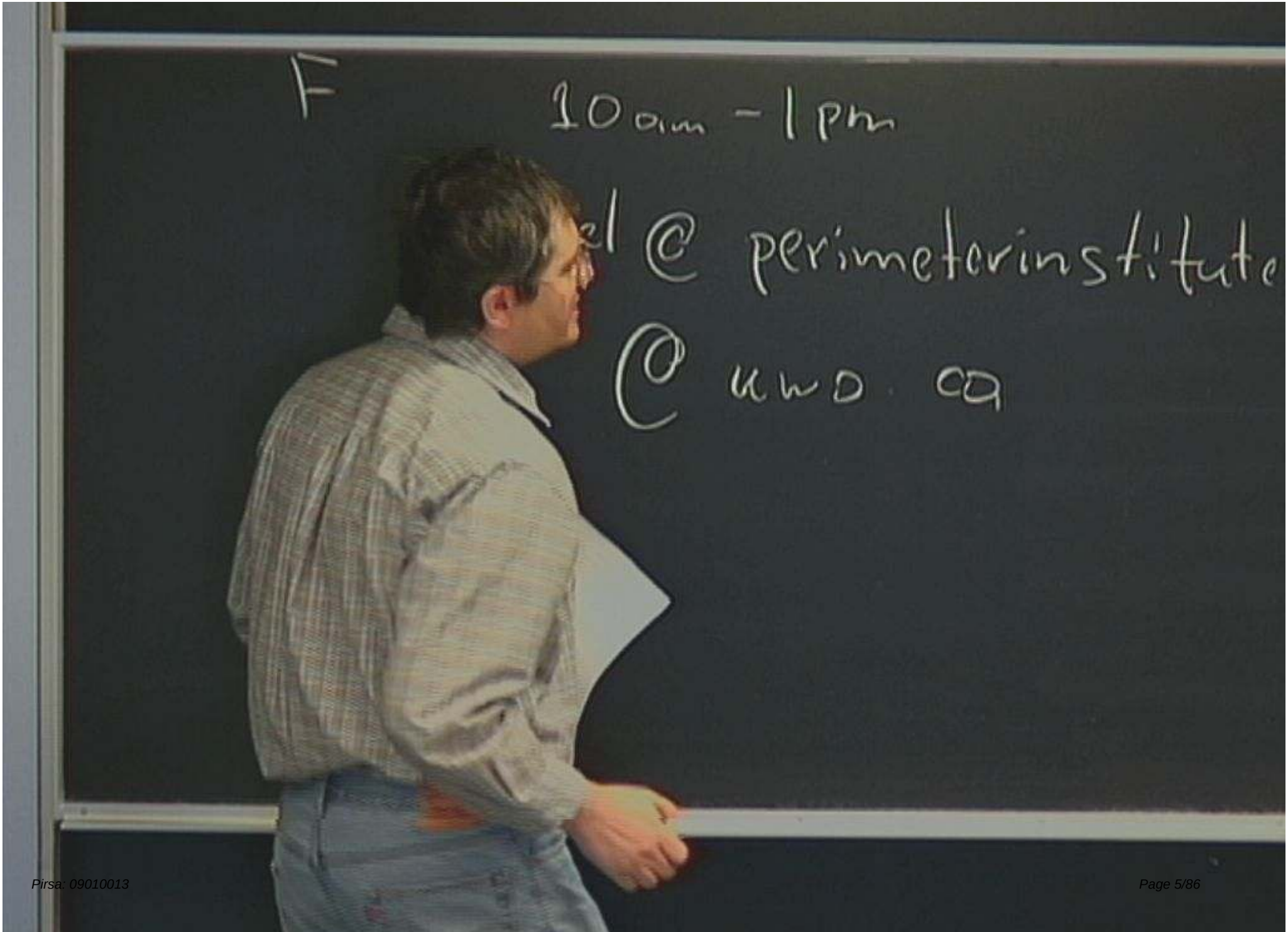
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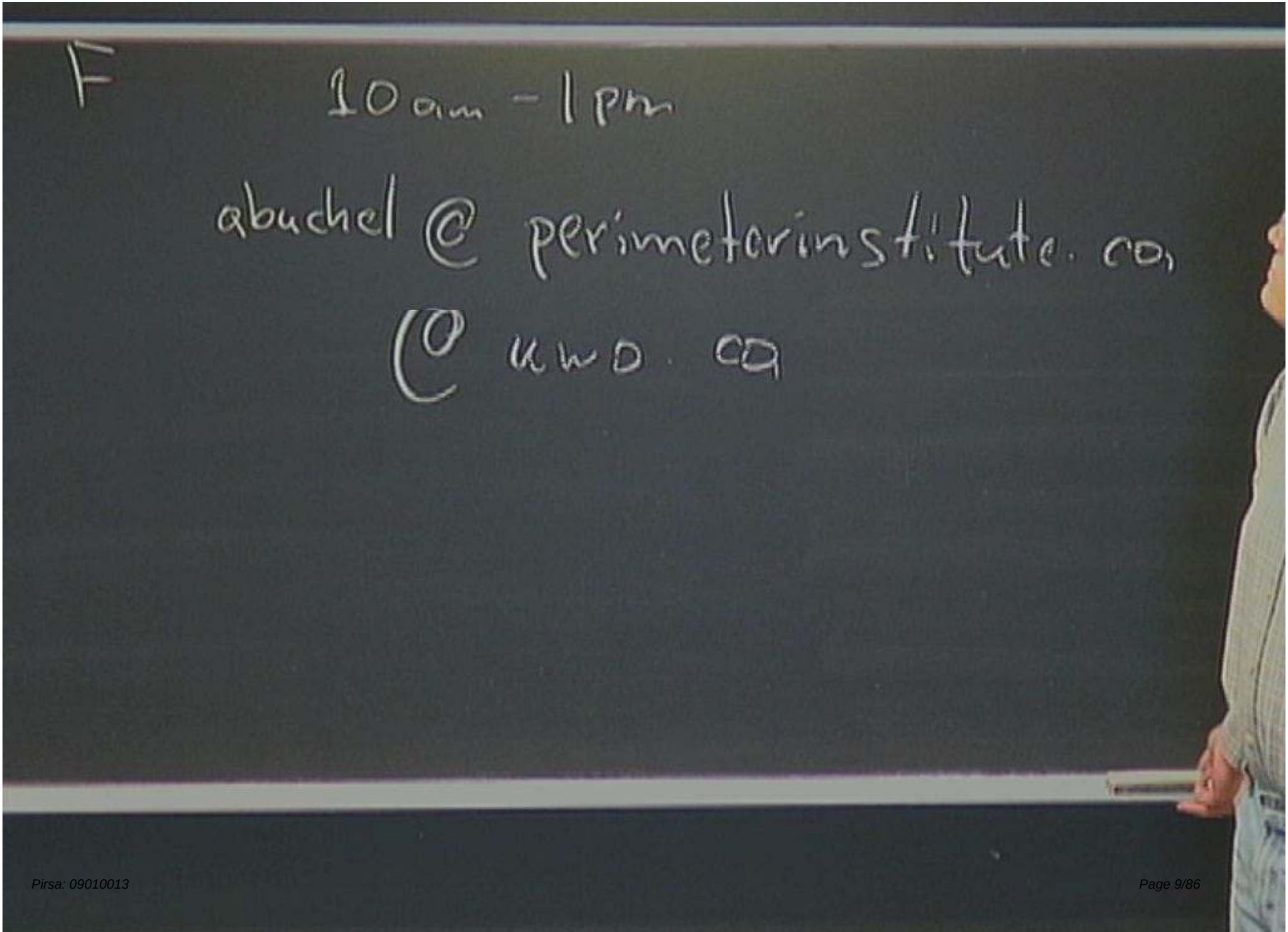
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10 am - 1 pm
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(I) Action principle & light-cone quantization of p.p. and string

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⇒ critical dim of bosonic string.

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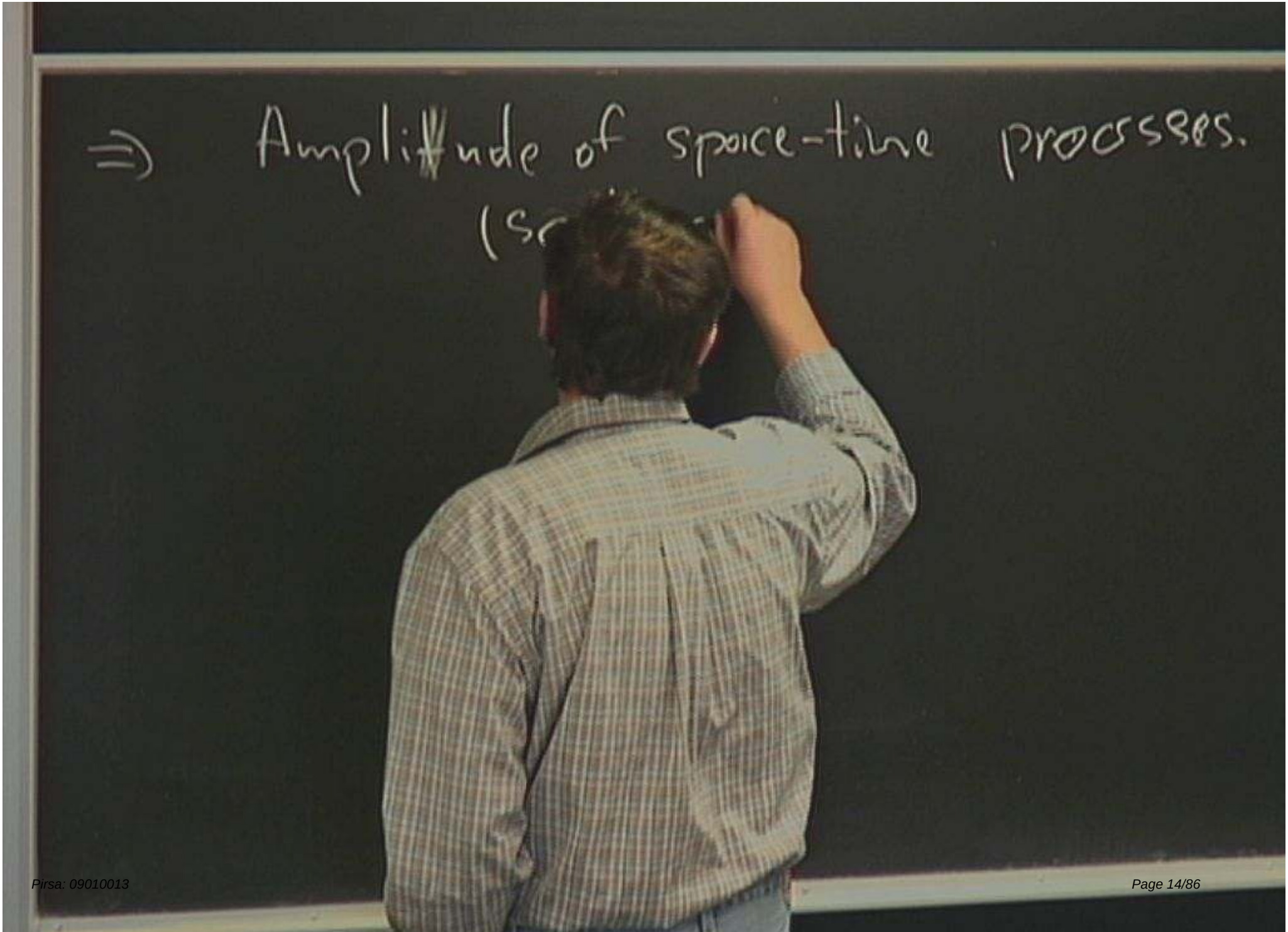
(II) CFT

(I) Action principle & light-cone quantization of P.P. and string
⇒ critical dim of bosonic string.

(II) CFT

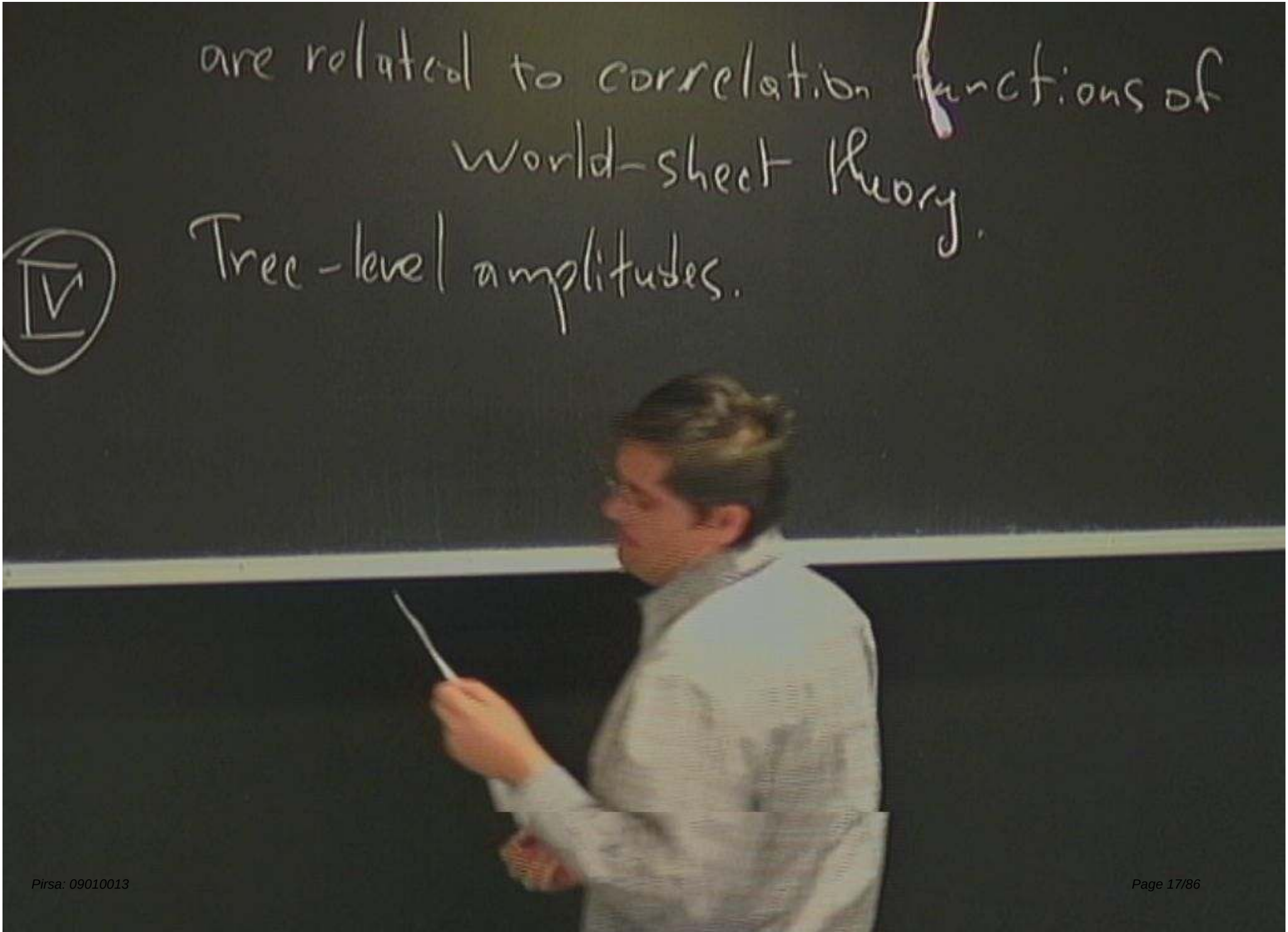
(III) Polyakov path integral.


⇒ Amplitude of space-time processes.

⇒ Amplitude of space-time processes. (scattering) in $D = 26$

$\Rightarrow$ Amplitude of space-time processes. (scattering) in $D = 26$

are related to correlation functions of World-sheet theory.


are related to correlation functions of
World-sheet theory.
IV
Tree-level amplitudes.

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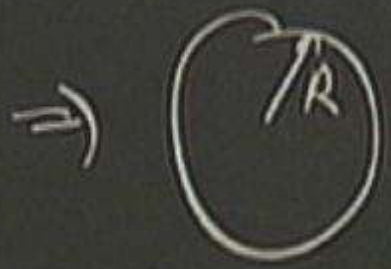
IV Tree-level amplitudes.

V

Toroidal compactifications and T-duality

V

Toroidal compactifications and T-duality



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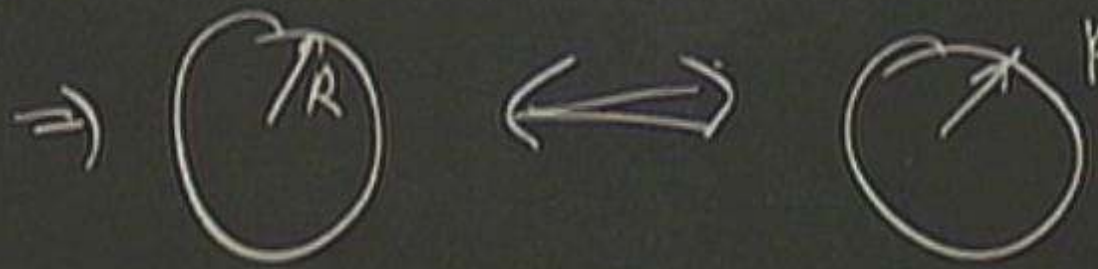
Toroidal compactifications and T-duality



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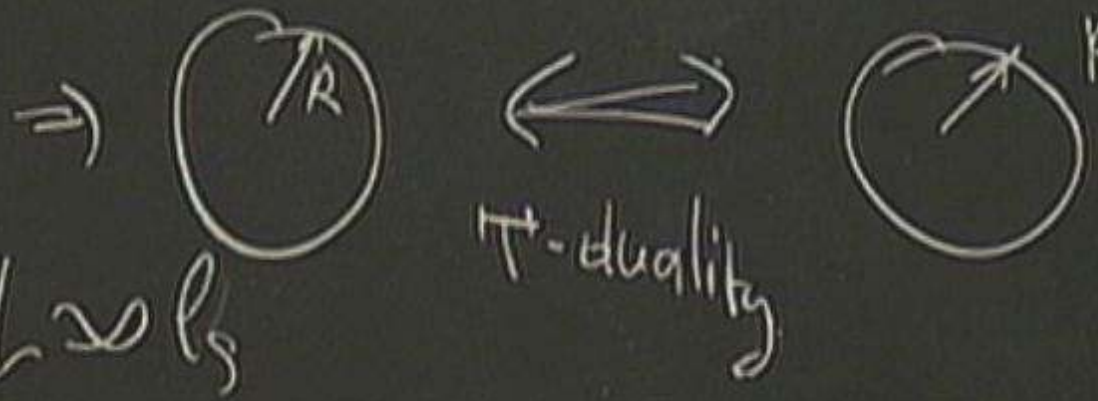
Toroidal compactifications and T-duality



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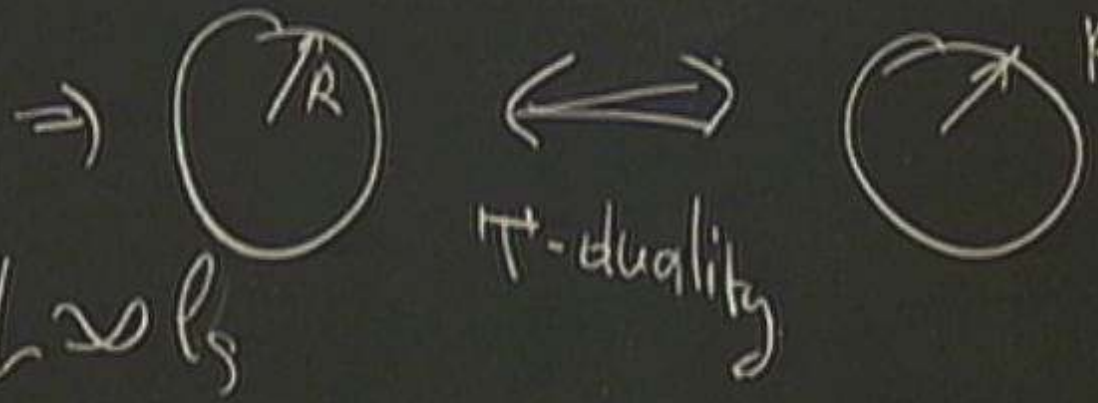
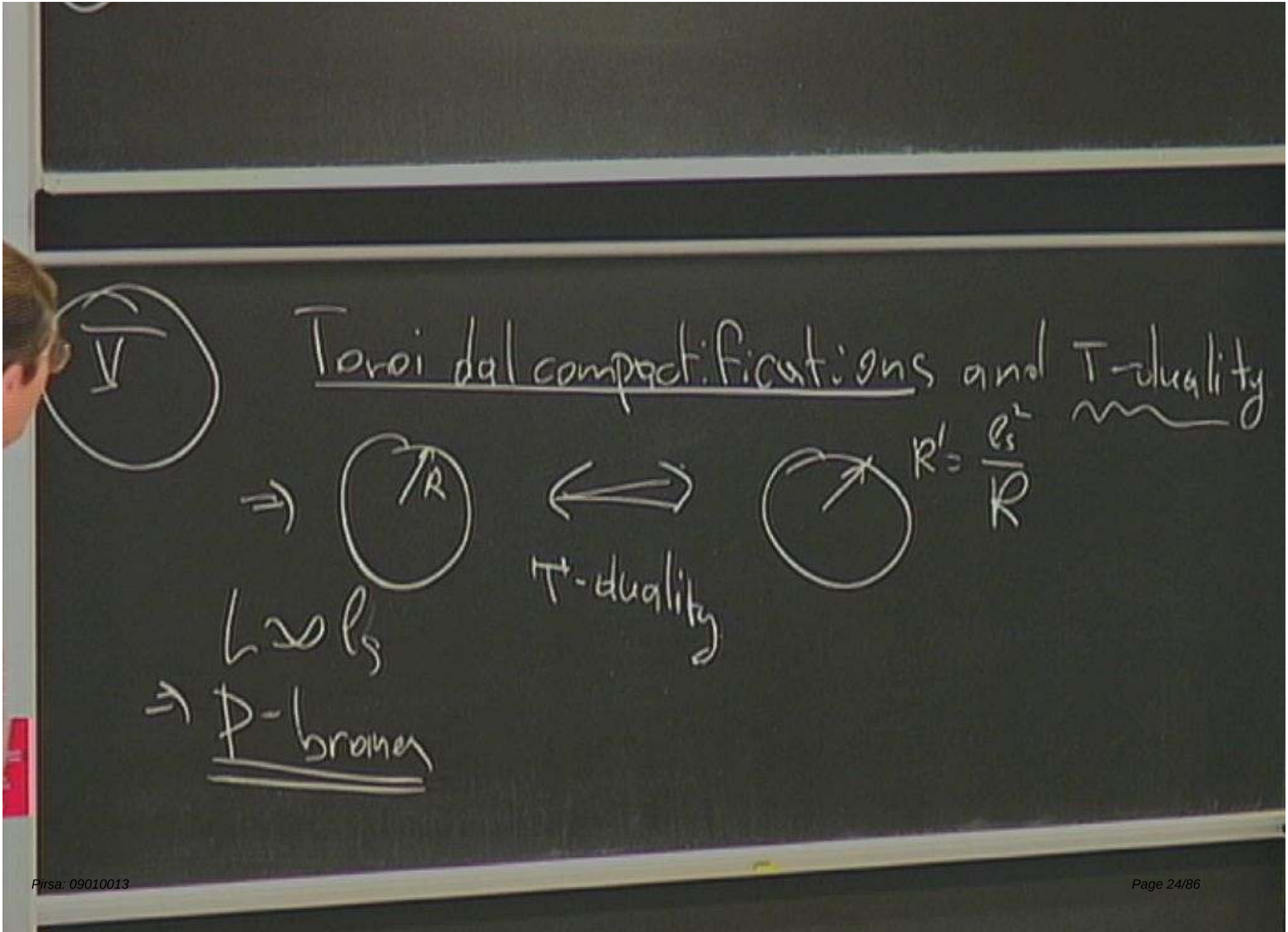
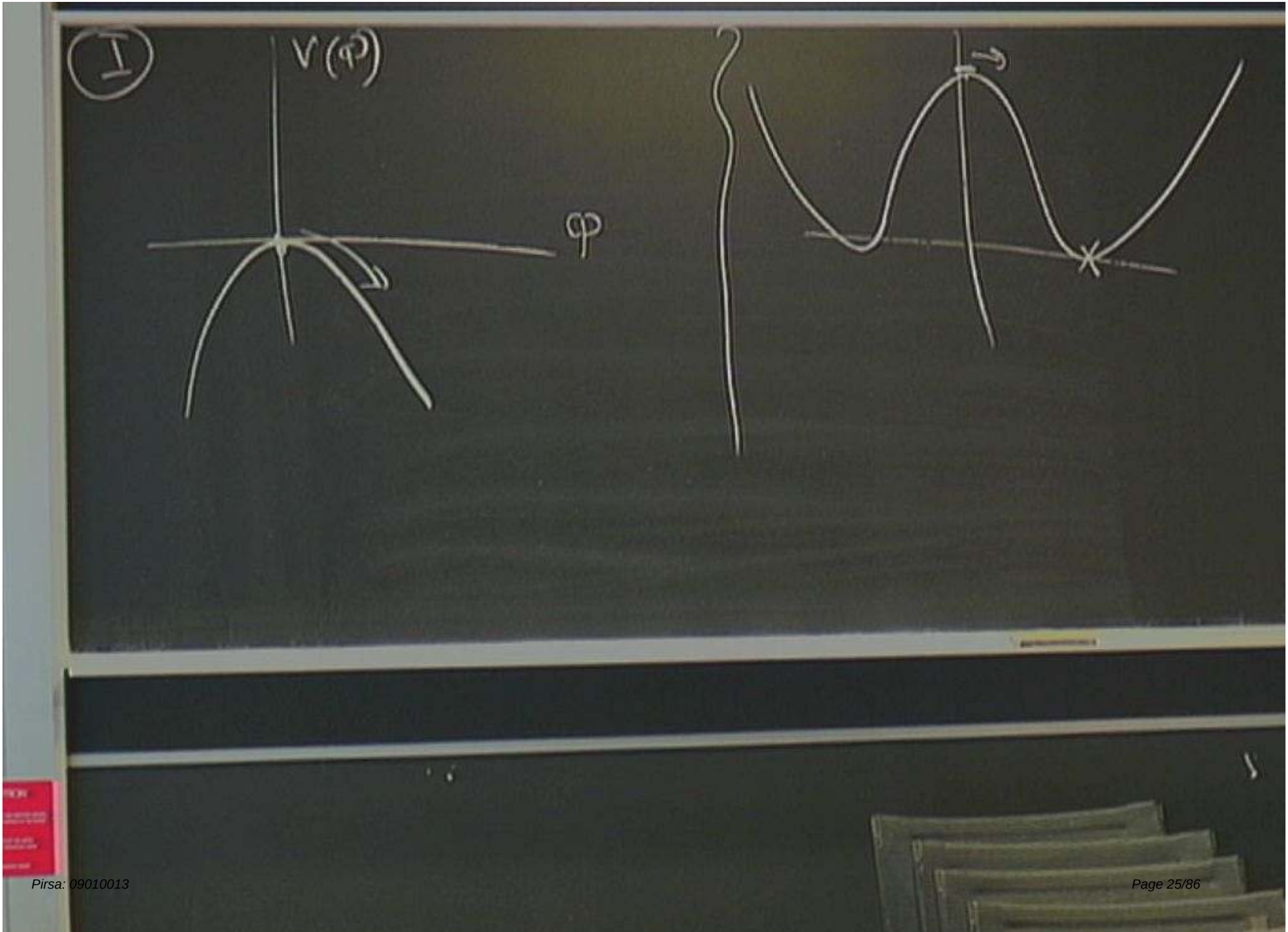
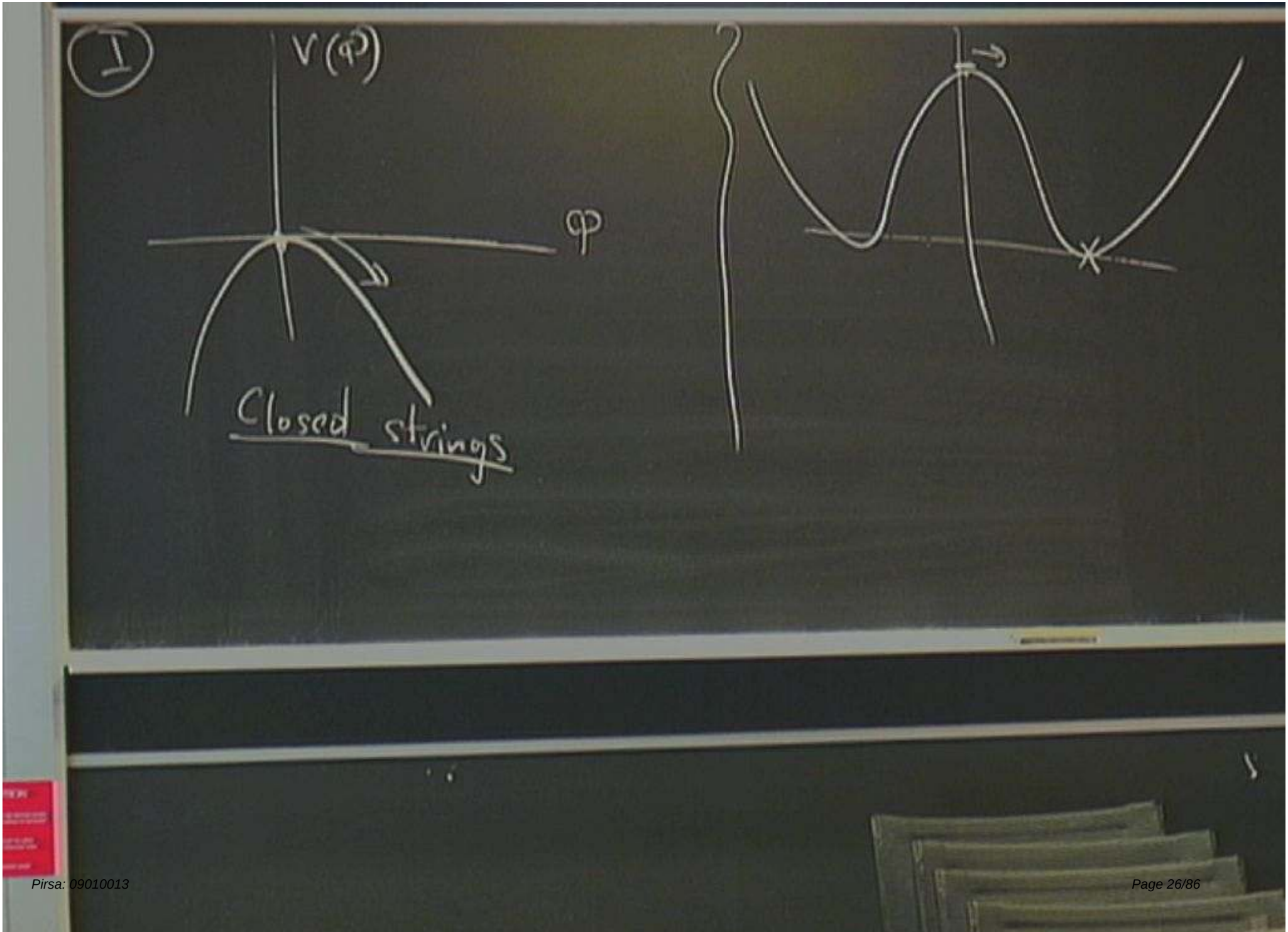
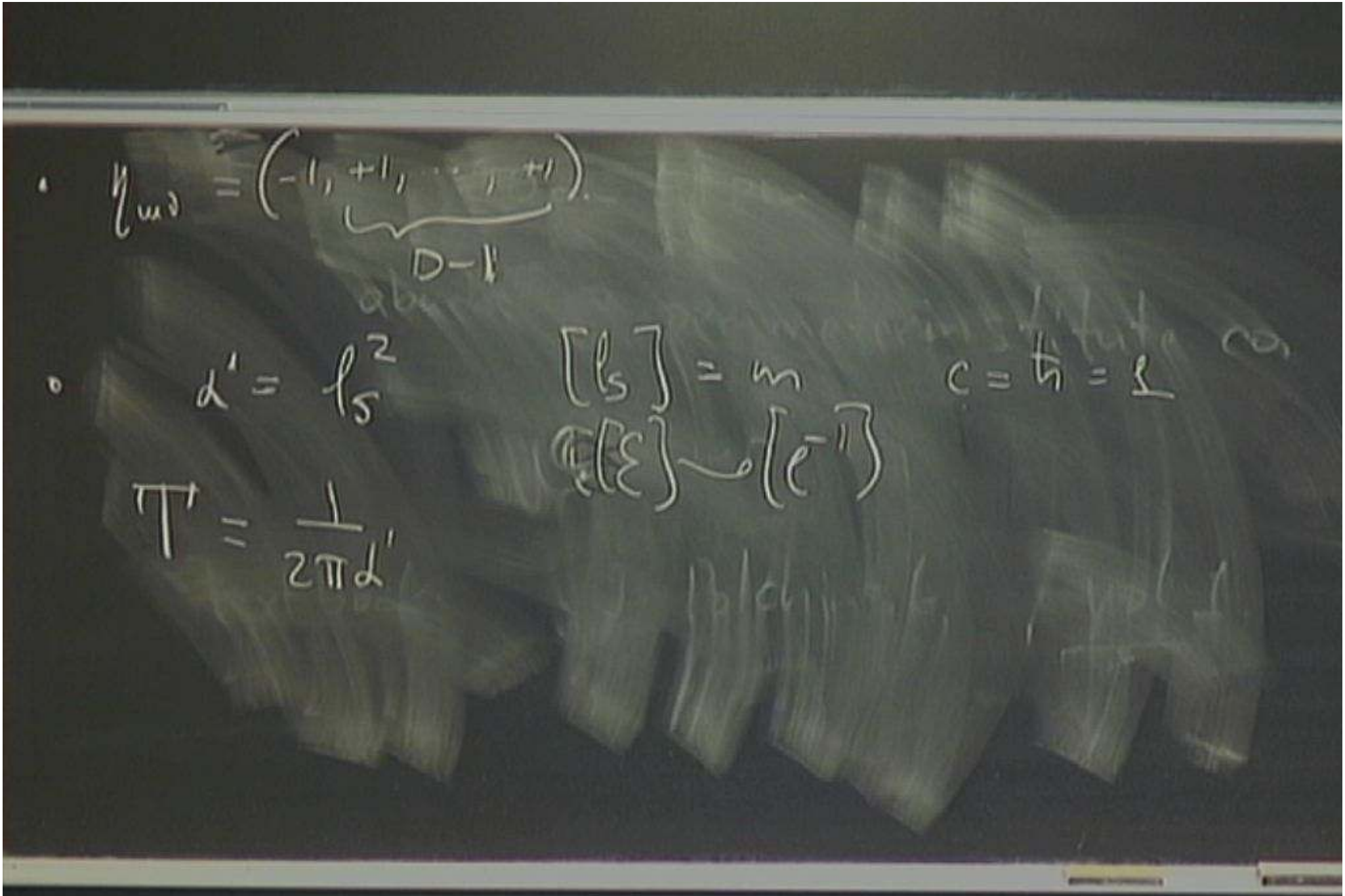
Toroidal compactifications and T-duality

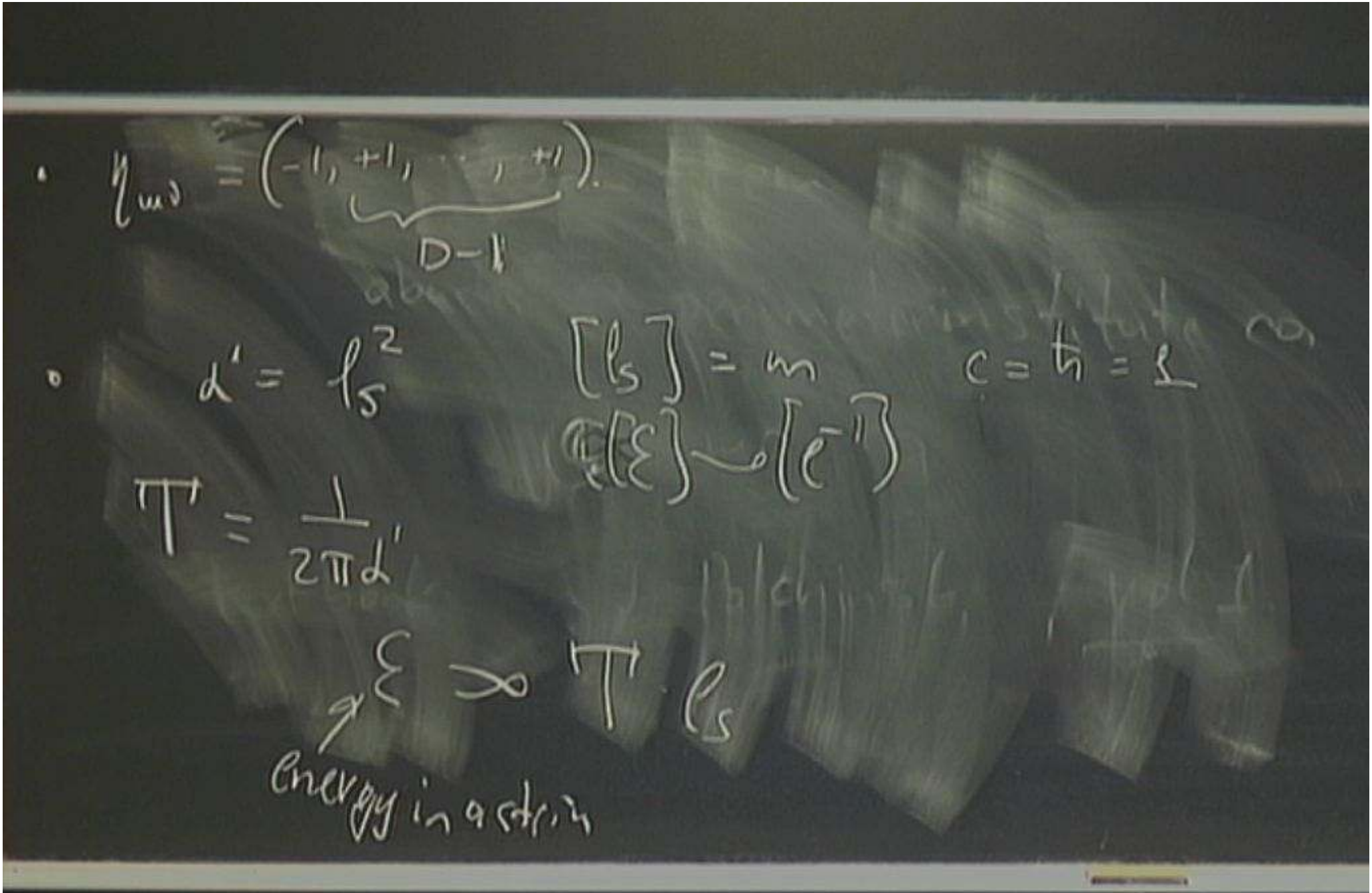


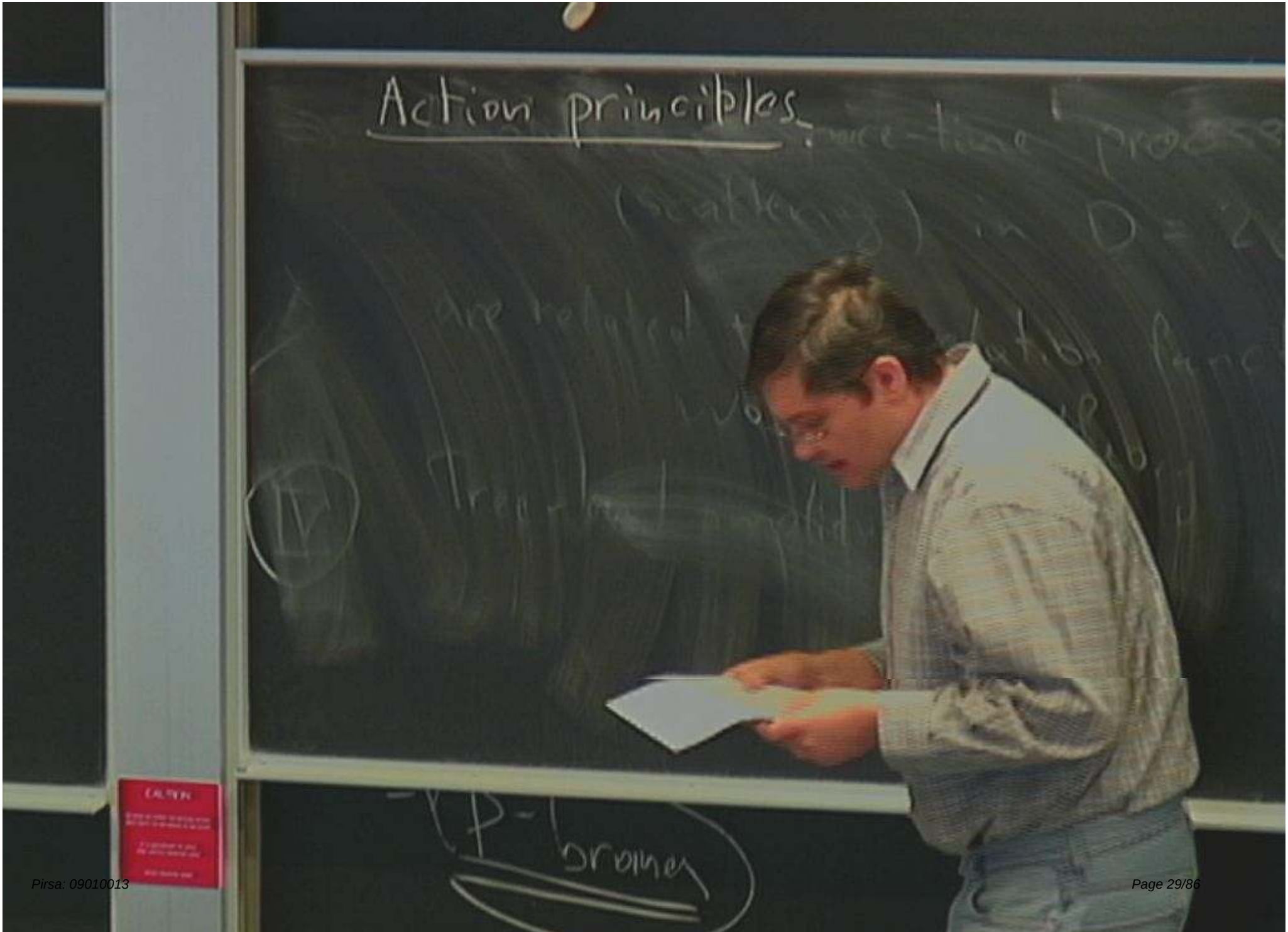

Toroidal compactifications and T-duality
R
R' = $\frac{\ell_s^2}{R}$
T-duality
$L \gg \ell_s$
⇒ D-branes


I
V(φ)
φ


I
V(φ)
φ
Closed strings

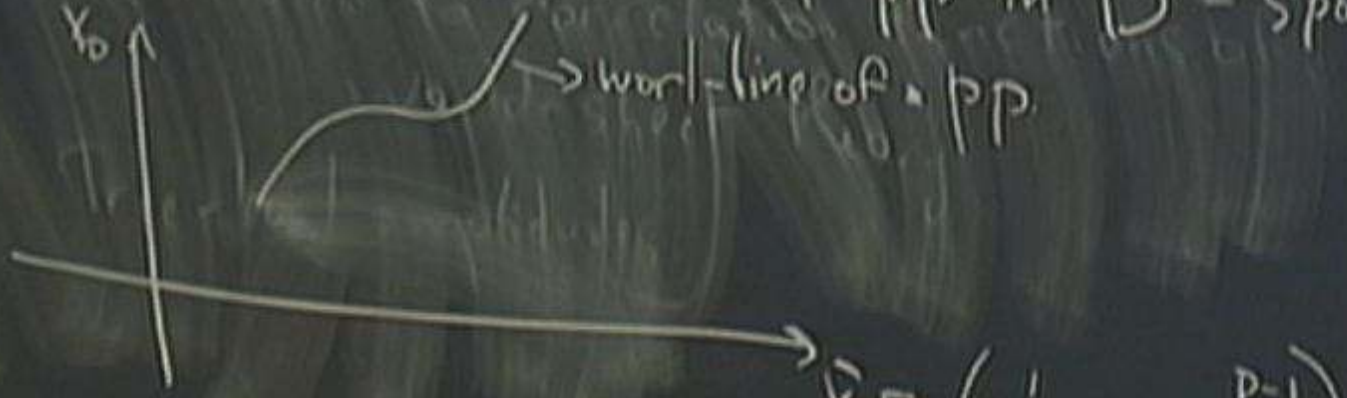

$\eta_{\mu\nu} = (-1, +1, \cdots, +1)$
$D-1$
$\alpha' = \ell_s^2$
$[\ell_s] = m$
$c = \hbar = 1$
$T = \frac{1}{2\pi\alpha'}$

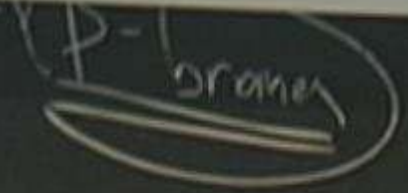

$\eta_{\mu\nu} = (-1, +1, \ldots, +1)$
D-1
$\alpha' = \ell_s^2$
$[\ell_s] = m$
$c = \hbar = 1$
$\mathcal{E}[E] \sim [e^{-1}]$
$T = \frac{1}{2\pi\alpha'}$
$\mathcal{E} \sim T \ell_s$
Energy in a string

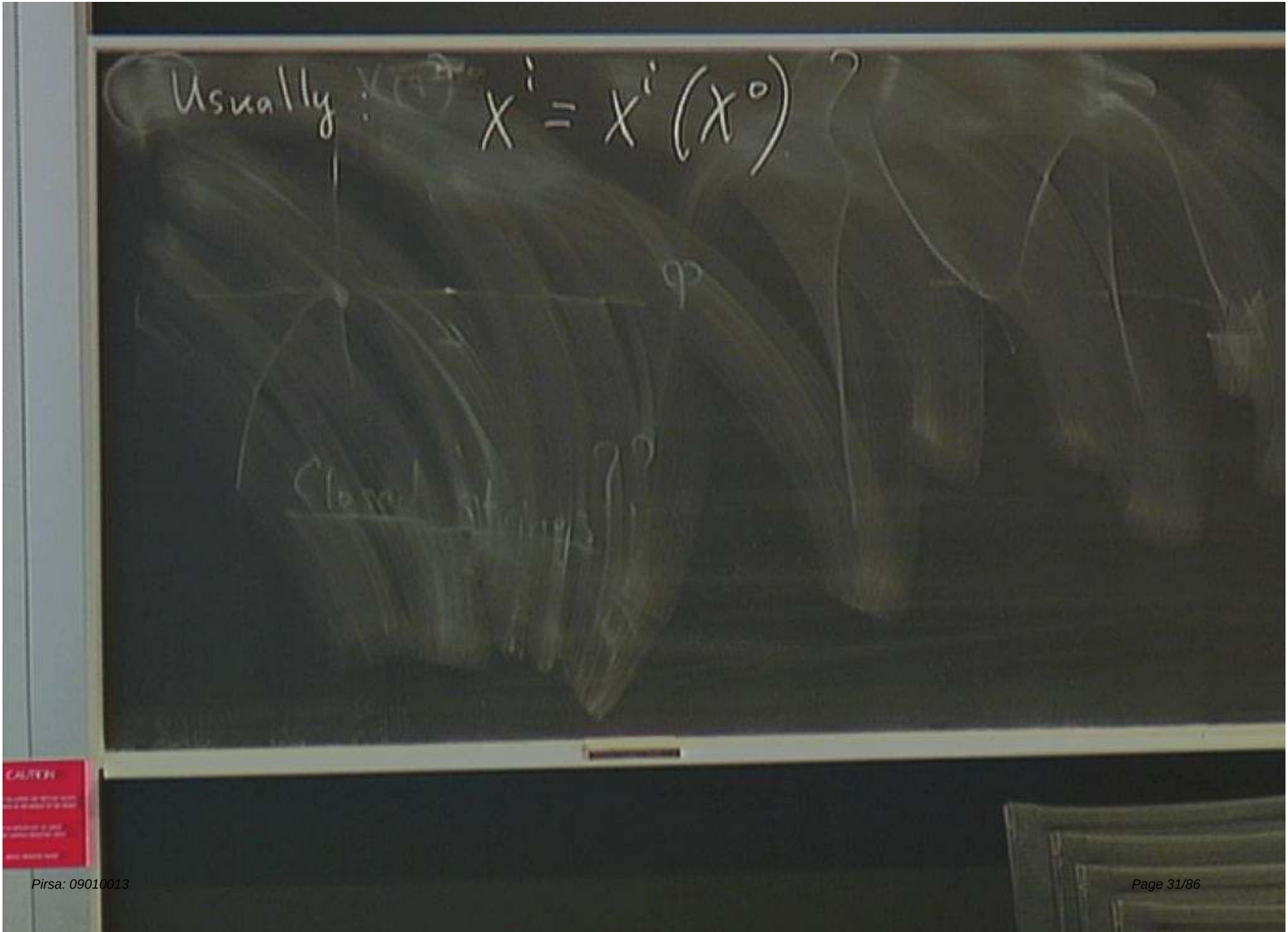

Action principles

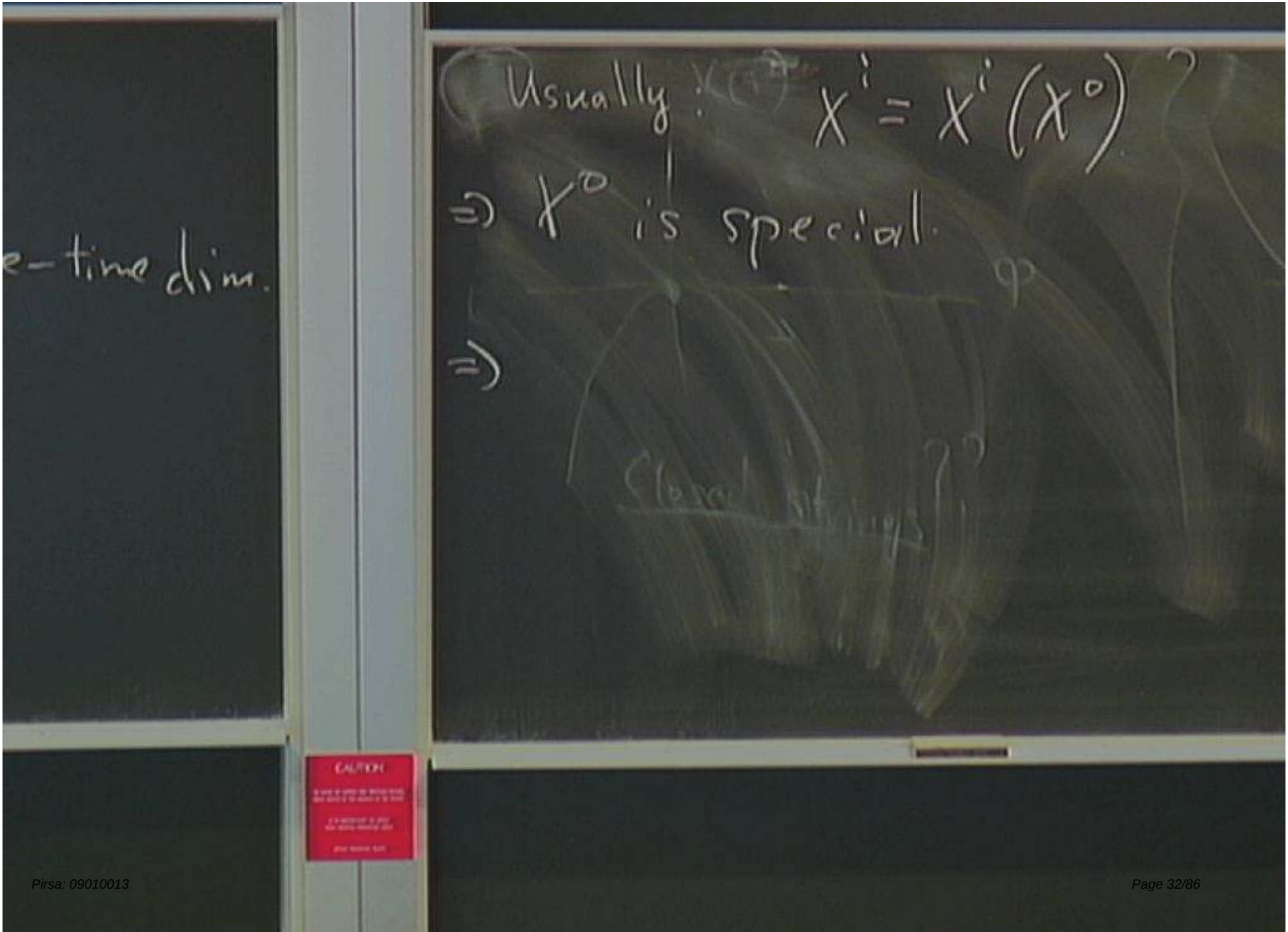
Action principles

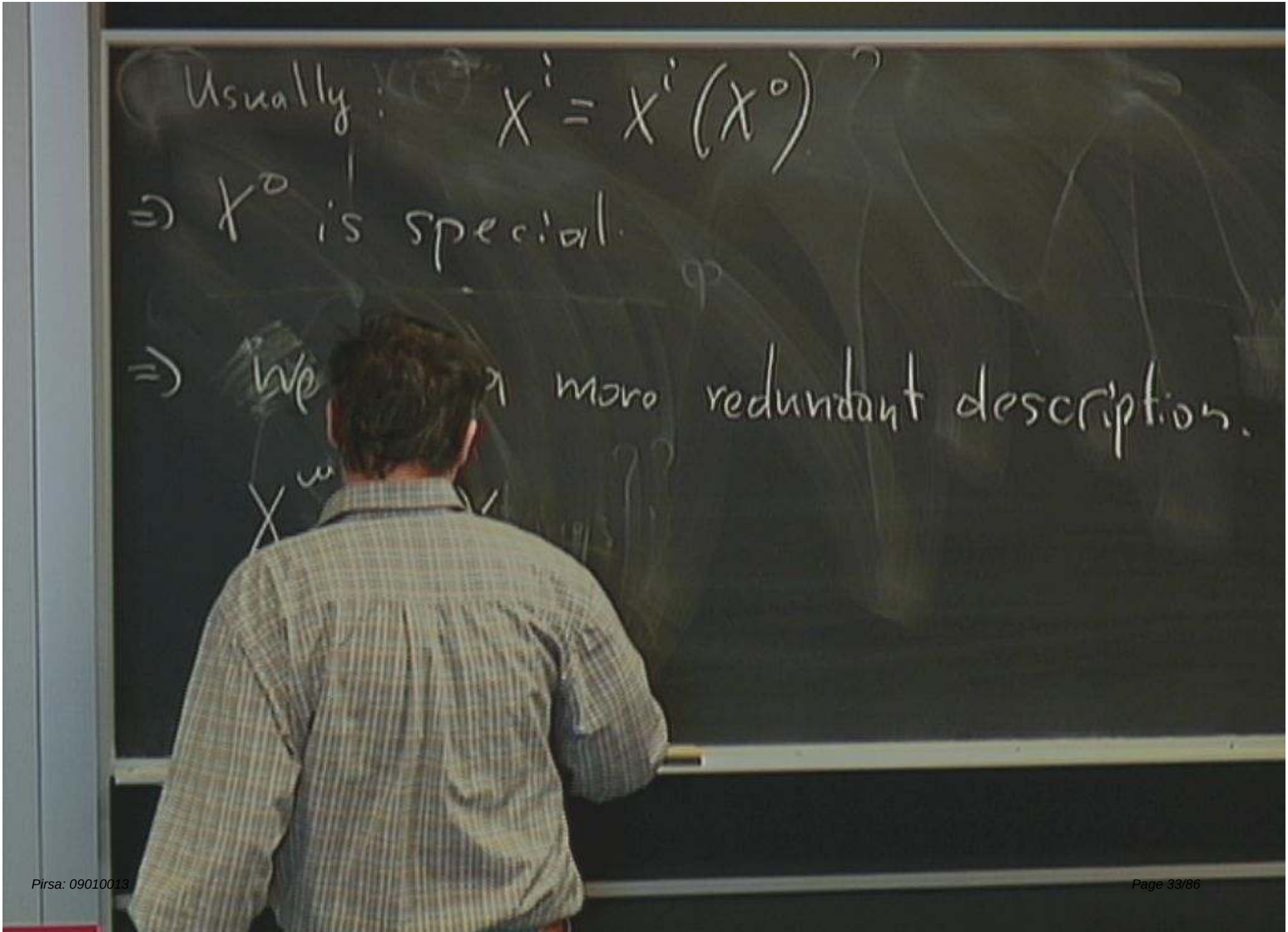
$\Rightarrow$ classical mechanics of a PP in D - spate-time dim.






Usually: $x^i = x^i(x^0)$
φ


e-time dim.
Usually: $x^i = x^i(x^0)$?
$\Rightarrow$ x^0 is special.
$\Rightarrow$
CAUTION


Usually: $x^i = x^i(x^0)$
$\Rightarrow x^0$ is special.
$\Rightarrow$ more redundant description.

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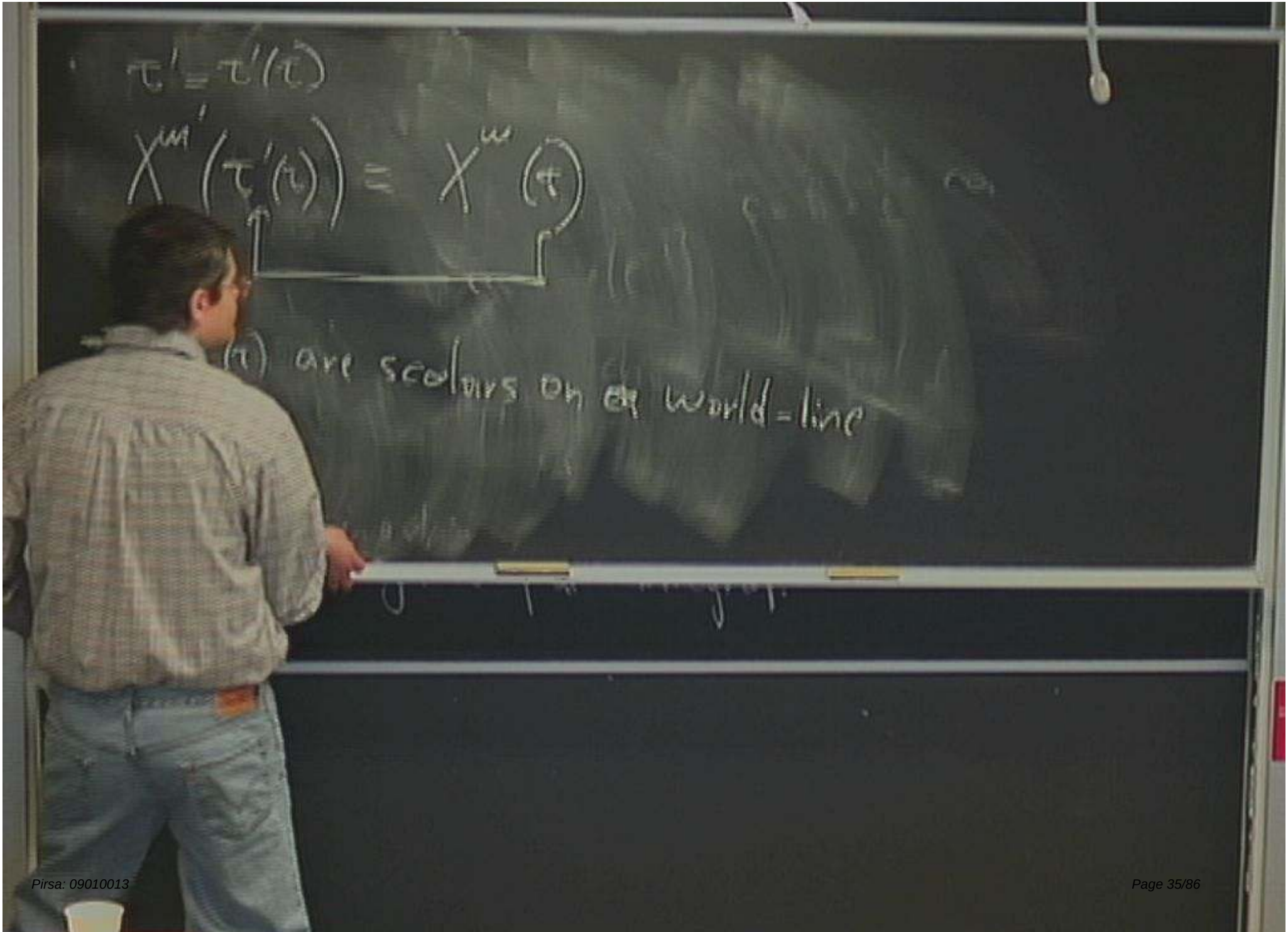
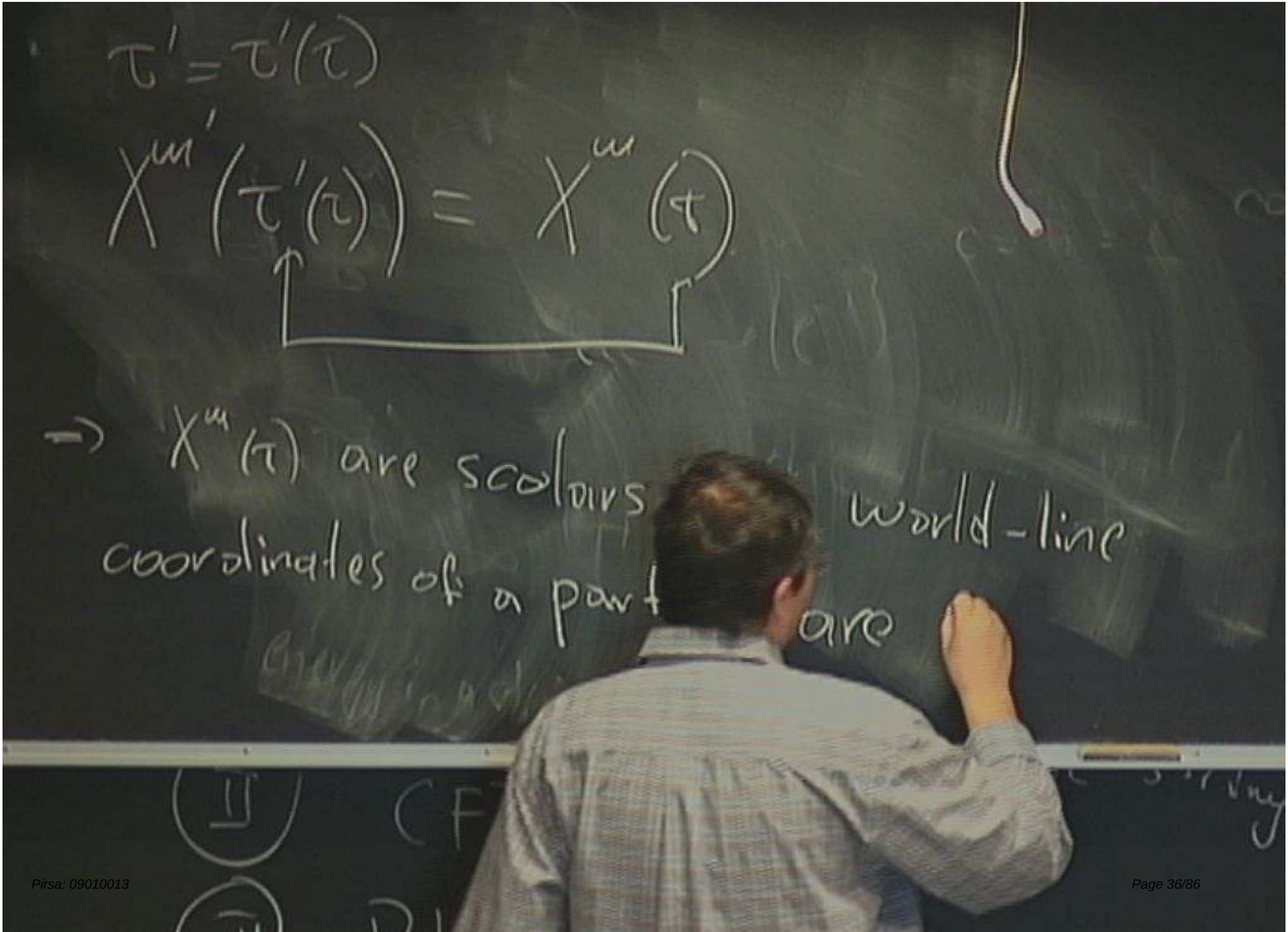
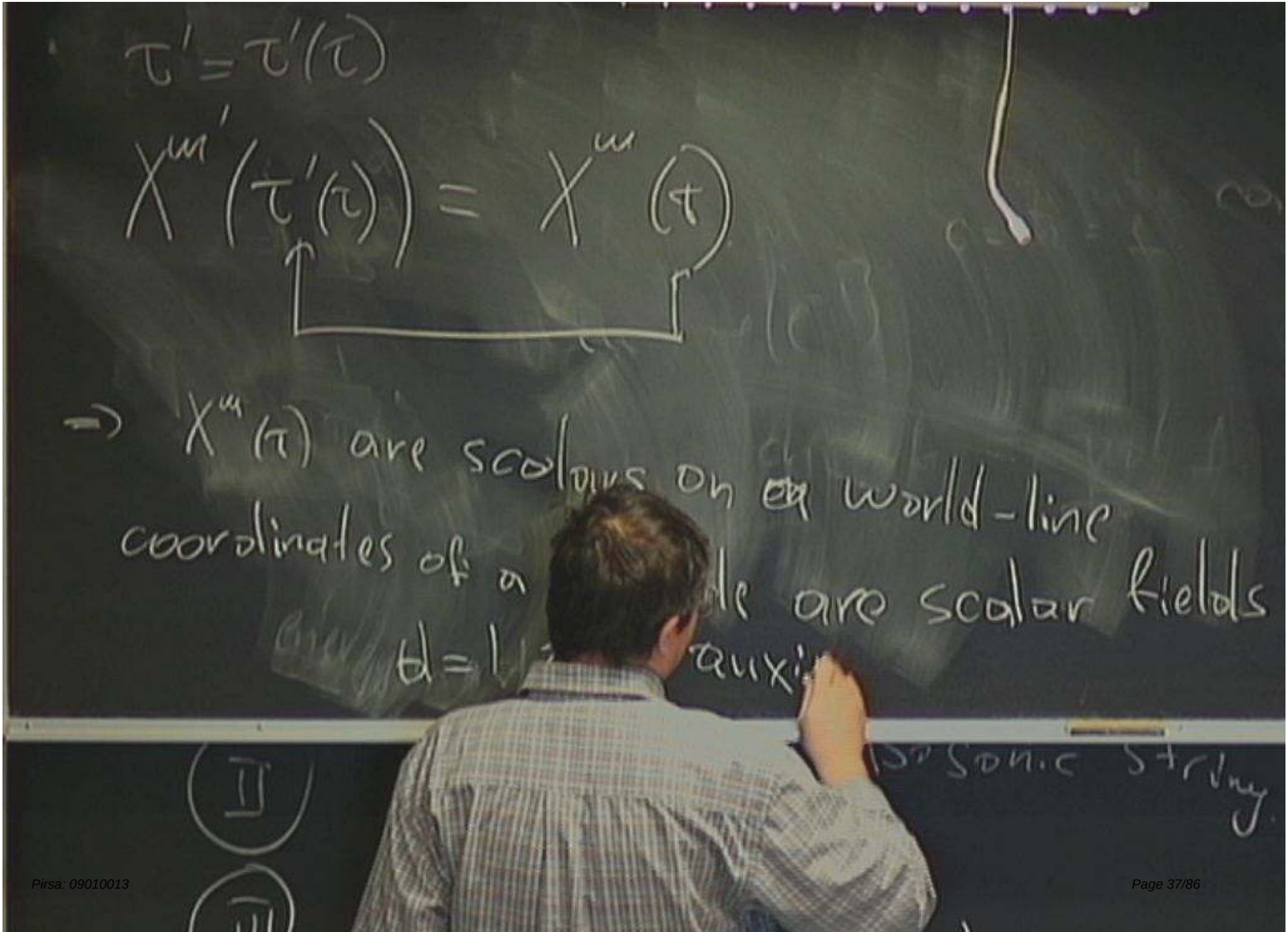
$\Rightarrow x^0$ is special.

$\Rightarrow$ We do a more redundant description.

$X^\mu = X^\mu(\tau)$ → arbitrary parameter on a world-line


$\tau' = \tau'(\tau)$
$X^{\mu'}(\tau'(\tau)) = X^{\mu}(\tau)$
(τ) are scalars on the world-line


$\tau' = \tau'(\tau)$
$X^{\mu'}(\tau'(\tau)) \equiv X^{\mu}(\tau)$
$\Rightarrow$ $X^{\mu}(\tau)$ are scalars
coordinates of a part
world-line
are


$\tau' = \tau'(\tau)$
$X^{\mu'}(\tau'(\tau)) = X^{\mu}(\tau)$
$\Rightarrow$ $X^{\mu}(\tau)$ are scalars on a world-line
coordinates of a
are scalar fields
$d=1$
sonic string

→ $X^\mu(\tau)$ are scalars on the world-line

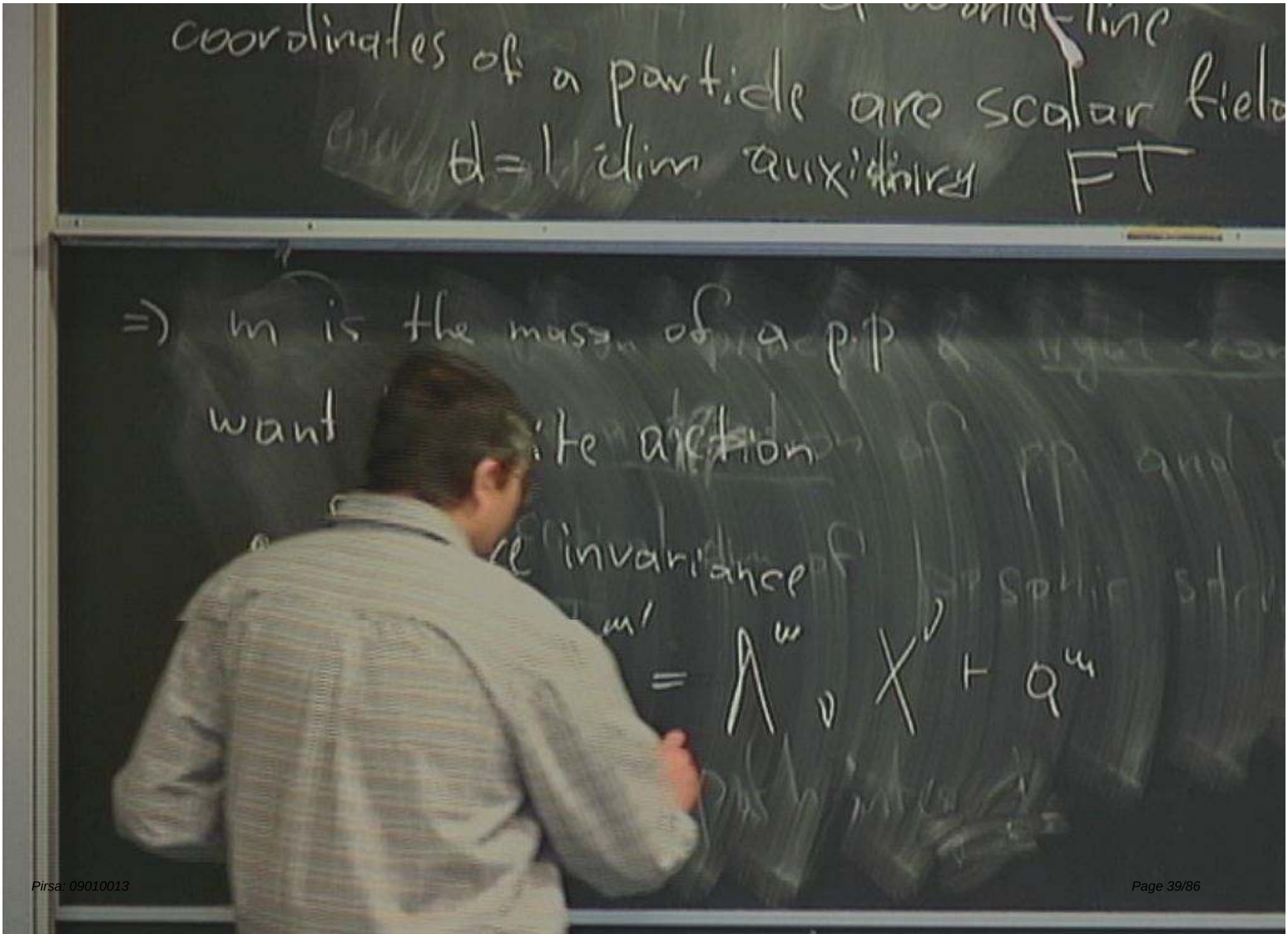
coordinates of a particle are scalar fields

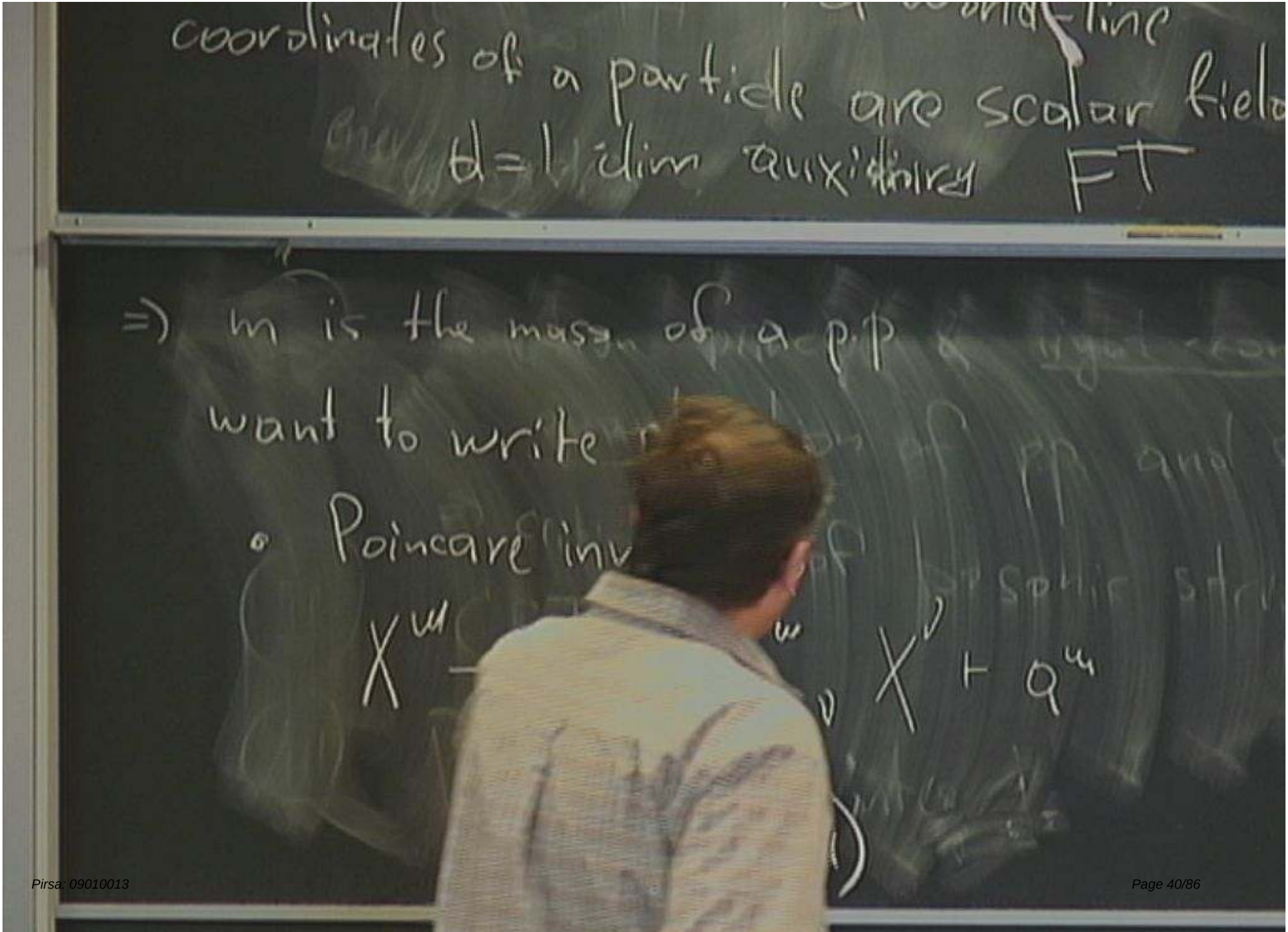
$d=1$ dim auxiliary FT

(I) Action principle & light-cone quantization of p.p. and str

⇒ critical dim of bosonic string.

(II) CFT


coordinates of a particle are scalar fiel
d=1 dim auxiliary FT
⇒ m is the mass of a P.P
want
invariance
$= \Lambda^{\mu}{}_{\nu} X^{\nu} + a^{\mu}$


coordinates of a particle are scalar
d=1, dim auxiliary FT
⇒ m is the mass of a p.p
want to write
• Poincare inv

coordinates of a particle are scalar field

$d=1$, 1dim auxiliary FT

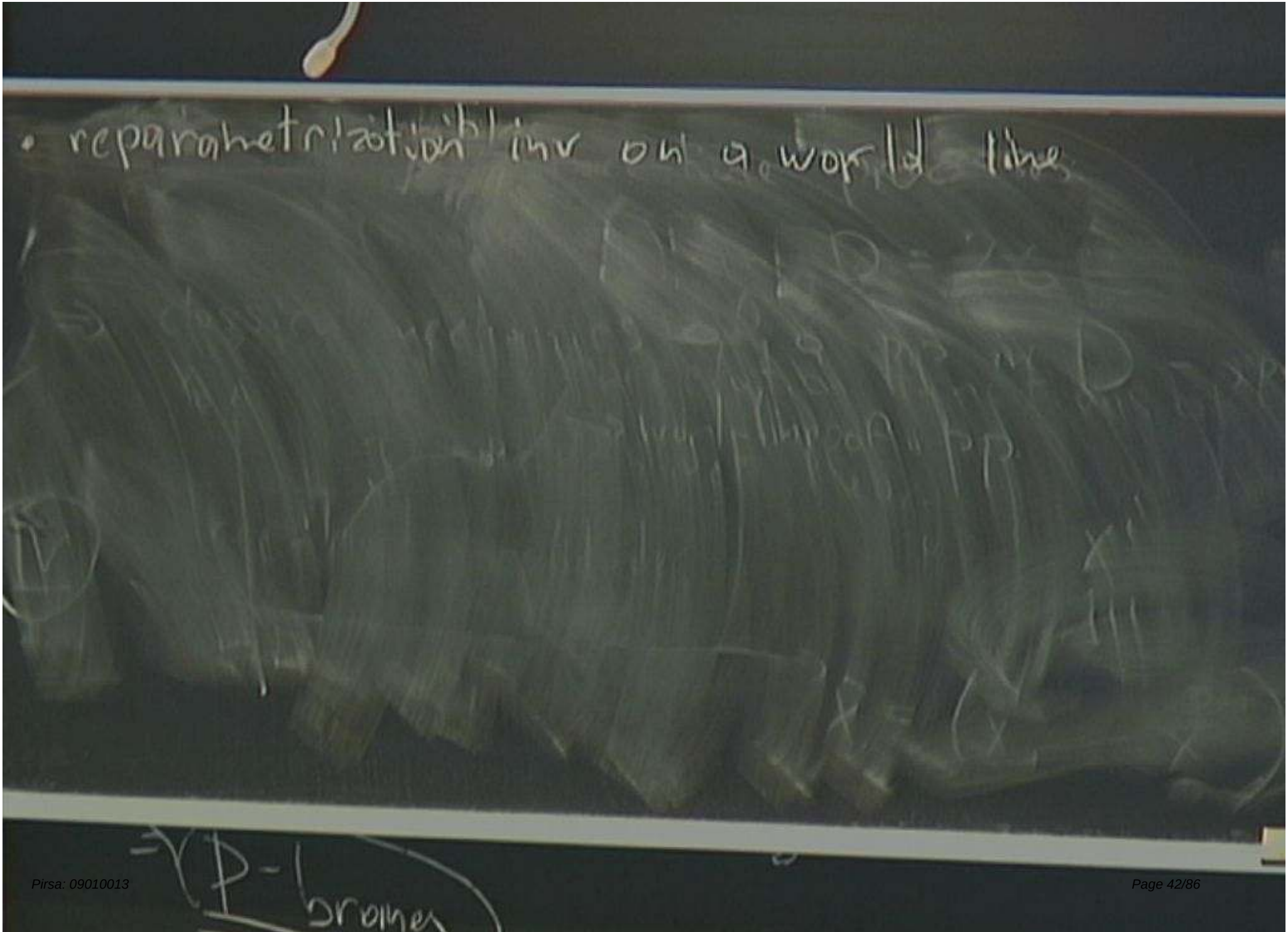
$\Rightarrow$ m is the mass of a p.p

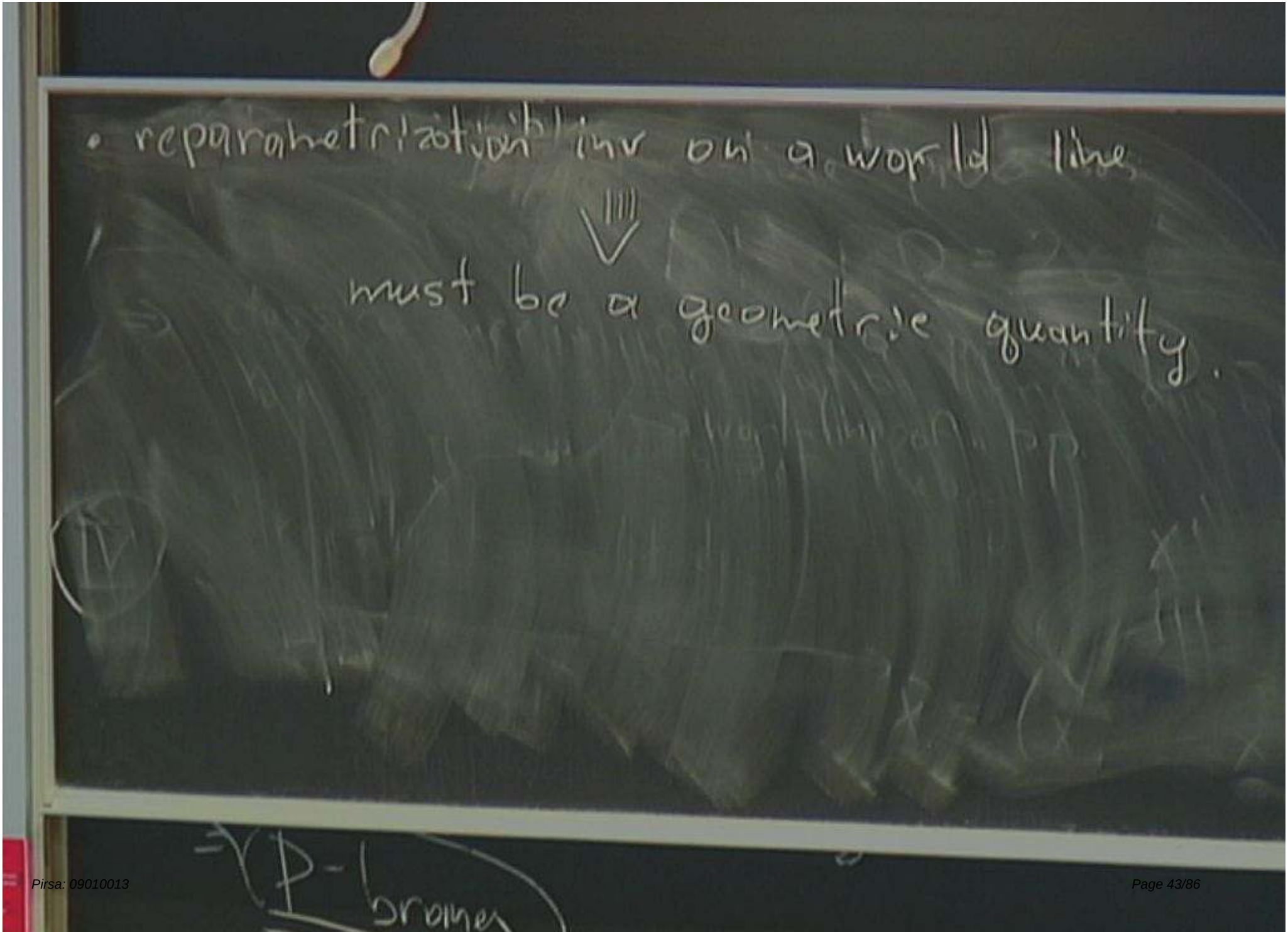
want to write action

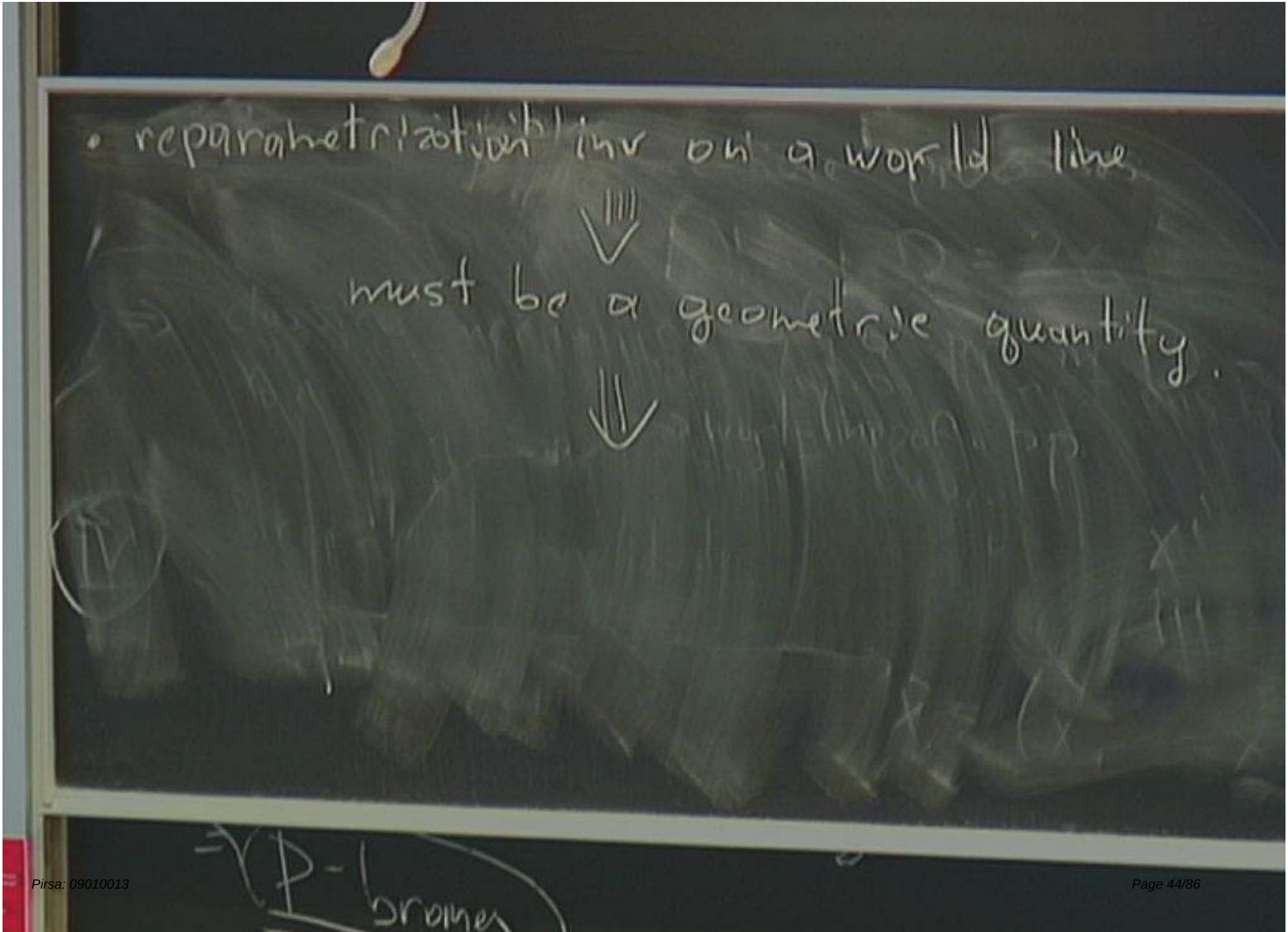
- Poincaré invariance

$$X^\mu \to X^{\mu\prime} = \Lambda^\mu{}_\nu X^\nu + a^\mu$$

$$SO(D-1,1)$$


• reparametrization inv on a world line


• reparametrization inv on a world line
must be a geometric quantity.


reparametrization inv on a world line
must be a geometric quantity.

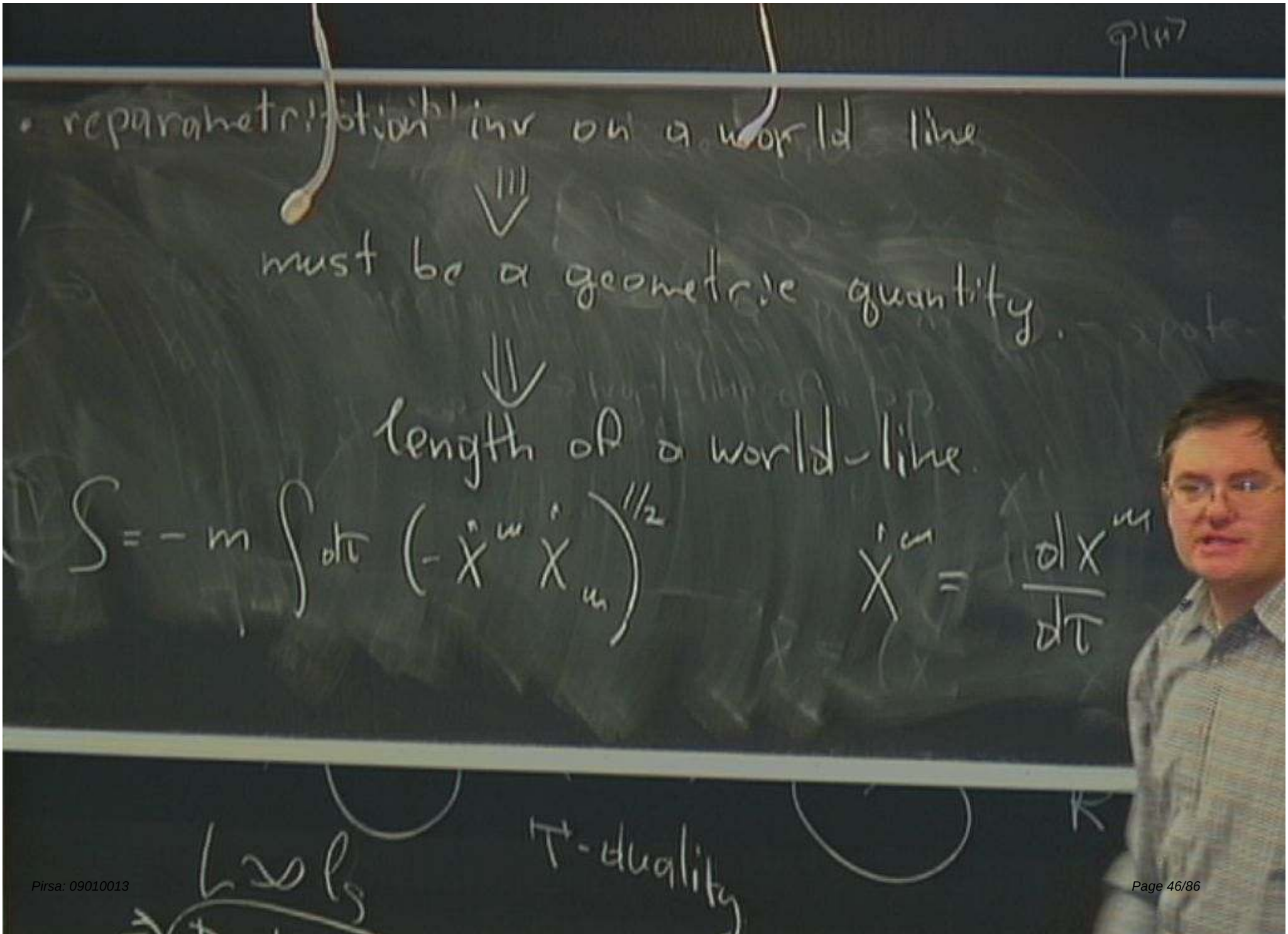
must be a geometric quantity.

length of a world-line.

$L \propto l_s$

p-branes

T-duality


• reparametrization inv on a world line
must be a geometric quantity.
length of a world-line
$S = -m \int d\tau \left(-\dot{x}^\mu \dot{x}_\mu\right)^{1/2}$
$\dot{x}^\mu = \frac{dx^\mu}{d\tau}$
T-duality

- reparametrization inv on a world line

⇓

must be a geometric quantity.

⇓

length of a world-line.

$$S = -m \int d\tau \left(-\dot{X}^\mu \dot{X}_\mu \right)^{1/2} \qquad \dot{X}^\mu = \frac{dX^\mu}{d\tau}$$

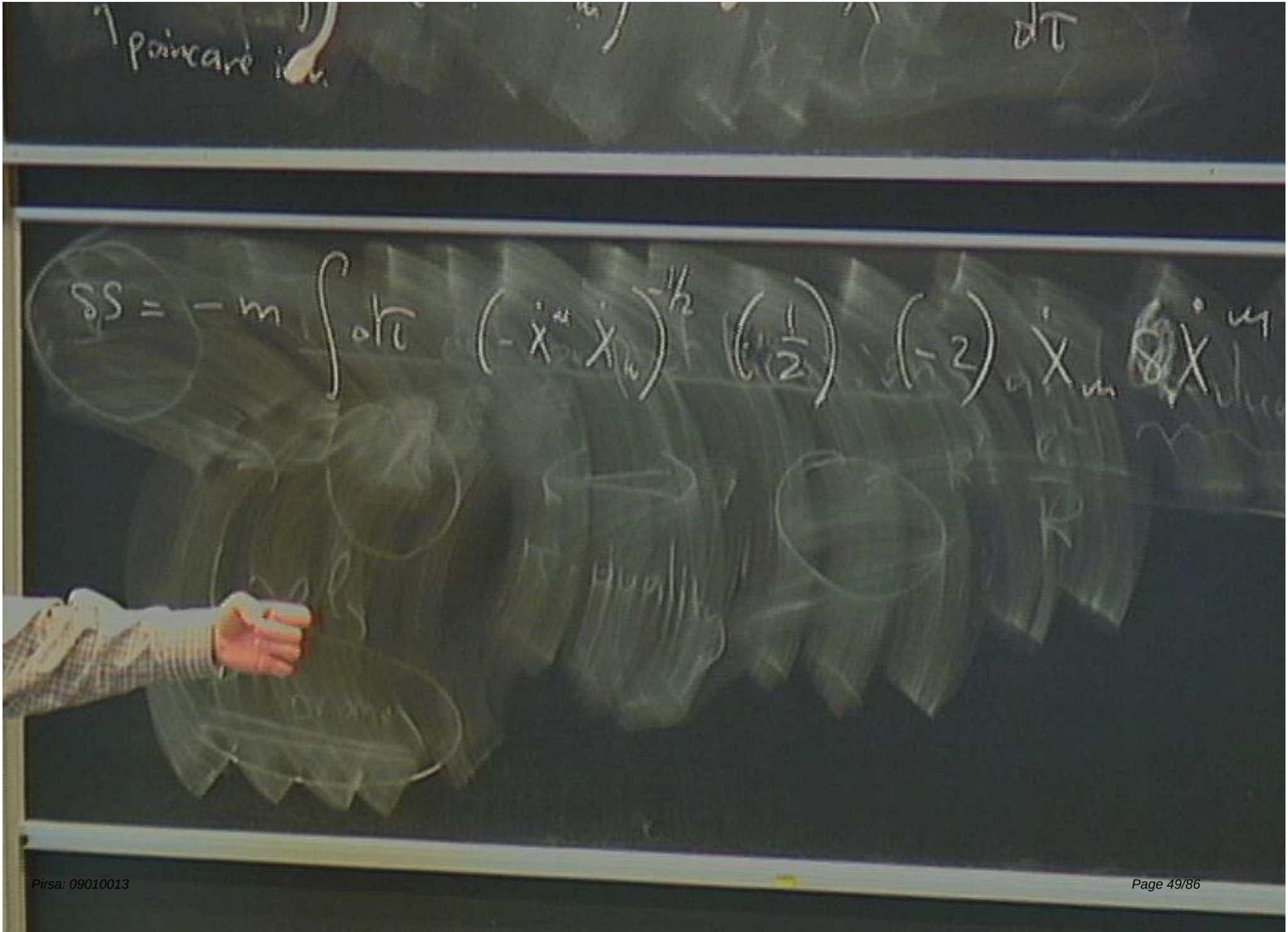
T-duality

Loops

R

Poincaré inv.

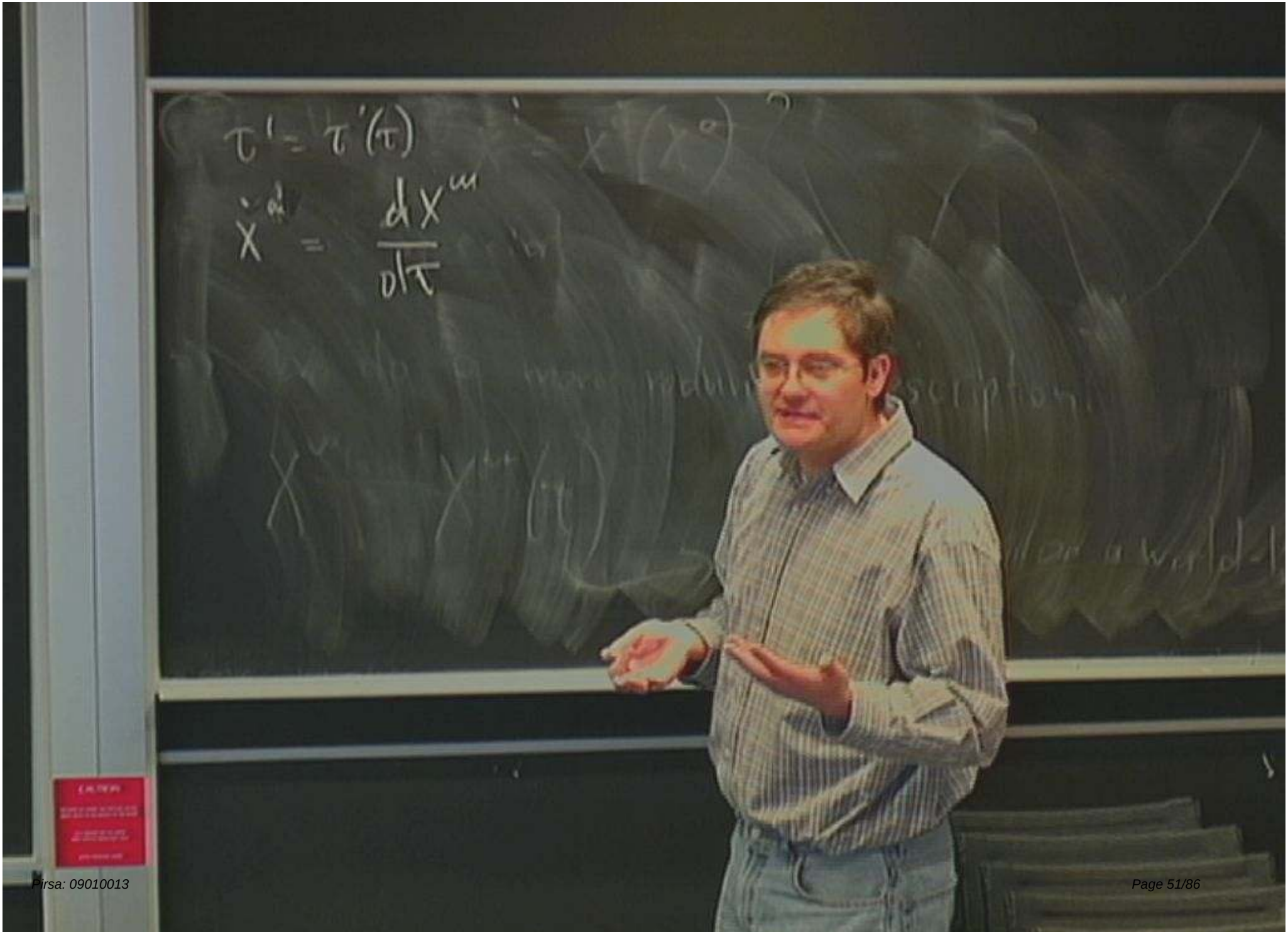
$$\delta S = -m \int d\tau \, \left(-\dot{x}^\mu \dot{x}_\mu\right)^{-1/2} \left(-\frac{1}{2}\right) \left(-2\right)$$

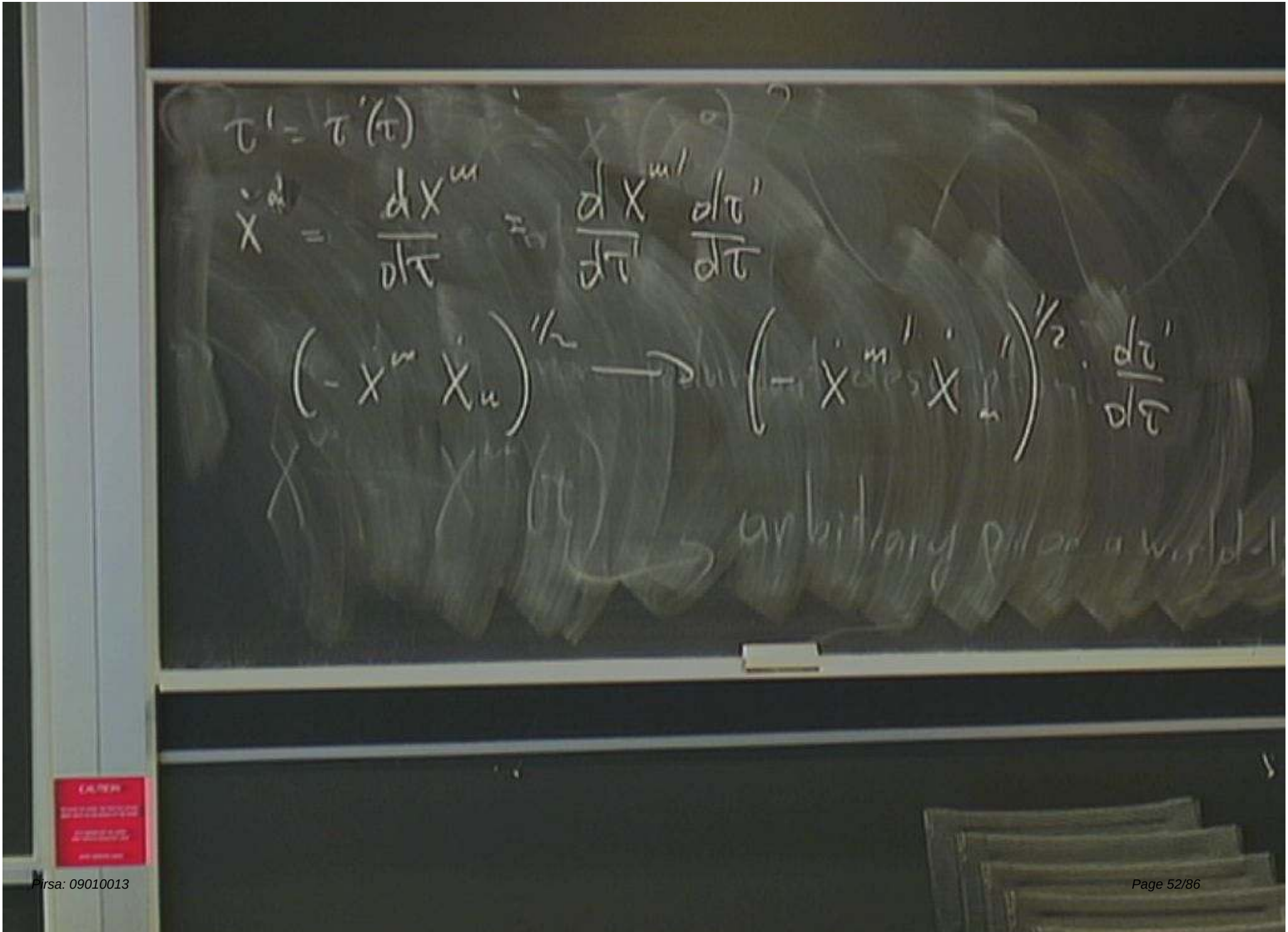

Poincaré
dτ
$\delta S = -m \int_0 d\tau \; (-\dot{x}^\mu \dot{x}_\mu)^{-\frac{1}{2}} \left(-\frac{1}{2}\right)(-2)\, \dot{x}_\mu \delta \dot{x}^\mu$

poincaré i...

$d\tau$

$$\delta S = -m \int d\tau \, (-\dot{x}^\mu \dot{x}_\mu)^{-1/2} \left(\frac{1}{2}\right) (-2) \dot{x}_\mu \delta \dot{x}^\mu$$

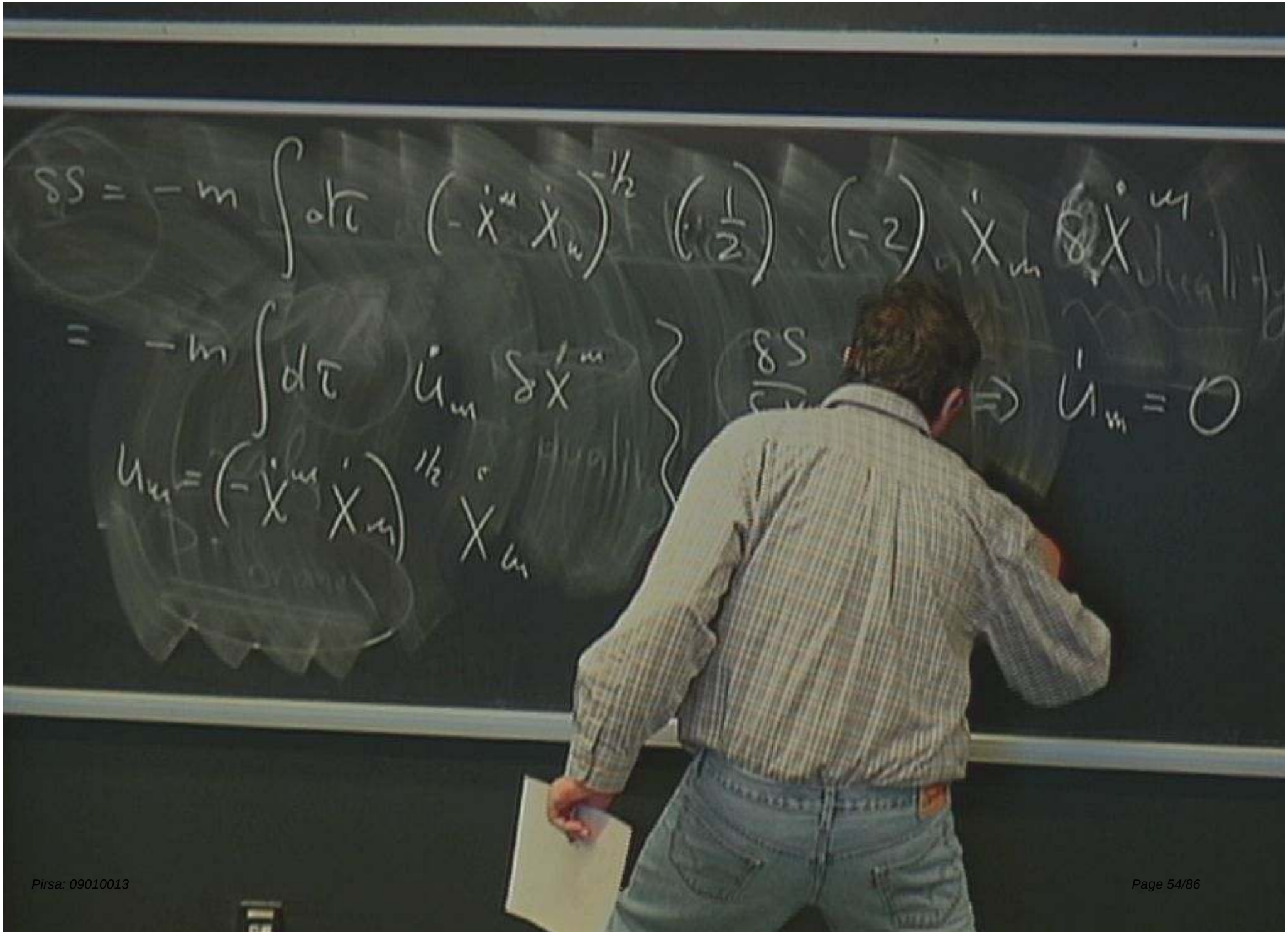

$\tau' = \tau'(\tau)$
$\dot{X}^{\mu} = \frac{dX^{\mu}}{d\tau}$


$\tau' = \tau'(\tau)$
$\dot{X}^{\mu} = \frac{dX^{\mu}}{d\tau} = \frac{dX^{\mu\prime}}{d\tau'}\frac{d\tau'}{d\tau}$
$\left(-\dot{X}^{\mu}\dot{X}_{\mu}\right)^{1/2} \longrightarrow \left(-\dot{X}^{\mu\prime}\dot{X}'_{\mu}\right)^{1/2}\cdot\frac{d\tau'}{d\tau}$
arbitrary

$$\delta S = -m \int d\tau \, (-\dot{x}^\mu \dot{x}_\mu)^{-1/2} \left(-\frac{1}{2}\right) (-2)\, \dot{x}_\mu \delta \dot{x}^\mu$$

$$= -m \int d\tau \, u_\mu \, \delta \dot{x}^\mu$$

$$u_\mu = (-\dot{x}^\nu \dot{x}_\nu)^{-1/2} \, \dot{x}_\mu$$


$\delta S = -m \int d\tau \left(-\dot{x}^\mu \dot{x}_\mu\right)^{-1/2} \left(\frac{1}{2}\right)(-2)\, \dot{x}_\mu \delta \dot{x}^\mu$
$= -m \int d\tau\, u_\mu\, \delta x'^\mu$
$\frac{\delta S}{\delta x^\mu} \Rightarrow \dot{u}_\mu = 0$
$u_\mu = \left(-\dot{x}^\mu \dot{x}_\mu\right)^{-1/2} \dot{x}_\mu$

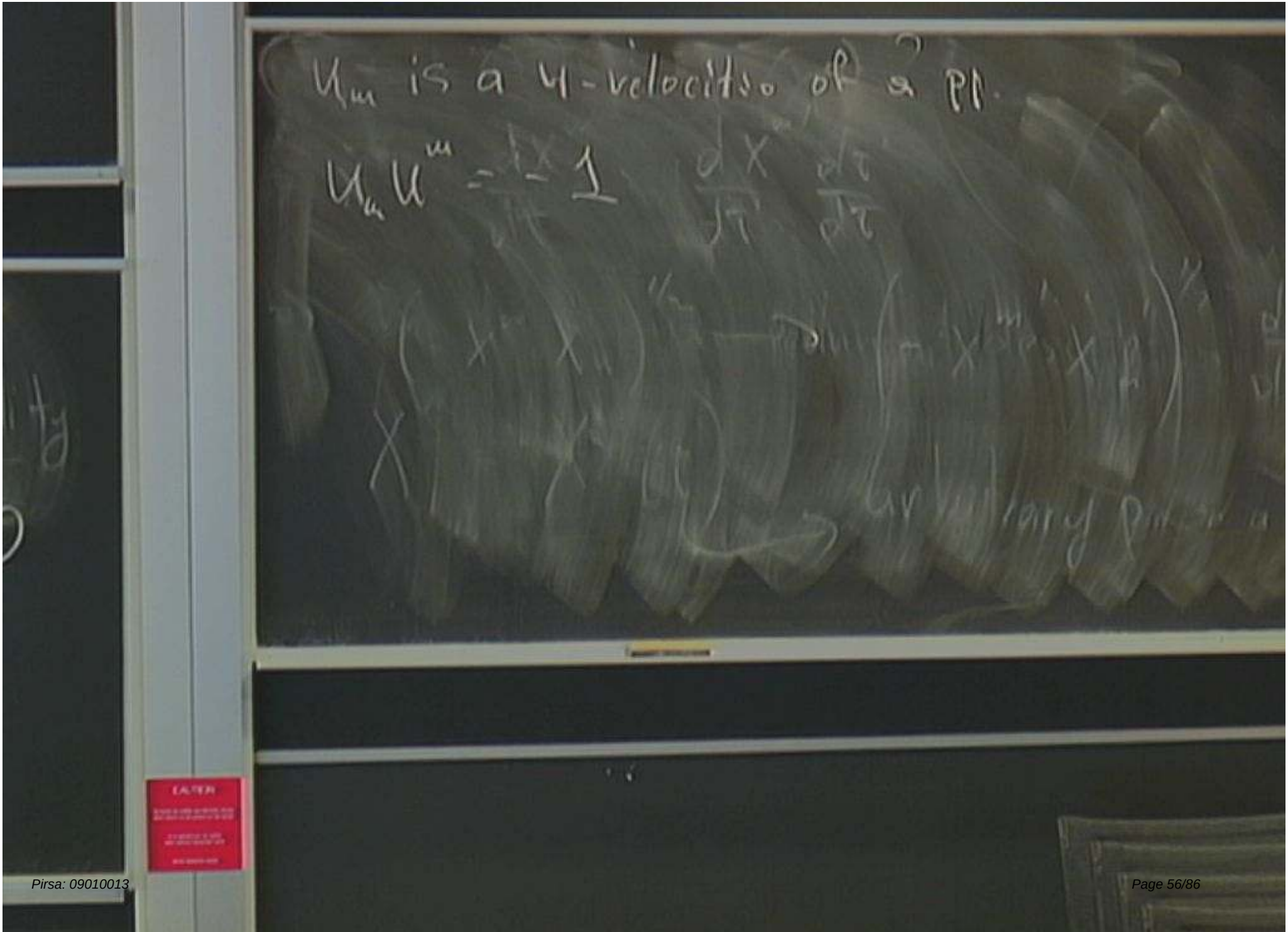
$$\delta S = -m \int d\tau \; (-\dot{X}^\mu \dot{X}_\mu)^{-1/2} \left(\tfrac{1}{2}\right)(-2)\, \dot{X}_\mu \delta \dot{X}^\mu$$

$$= -m \int d\tau \; \dot{u}_\mu \delta X^\mu$$

$$u_\mu = (-\dot{X}^\mu \dot{X}_\mu)^{-1/2} \dot{X}_\mu$$

$$\frac{\delta S}{\delta X^\mu} = 0 \Rightarrow \dot{u}_\mu = 0$$

$$u_\mu = \text{const.}$$


U_μ is a 4-velocity of a PT.
$U_\mu U^\mu = -1$
$\frac{dx^\mu}{d\tau} \frac{dx^\mu}{d\tau}$

u_m is a 4-velocity of a PT.

$u_m u^m = -1$ $\quad \frac{dx^m}{d\tau} \frac{dx^m}{d\tau}$

$\Rightarrow$ problem with $m \to 0$

U_m is a 4-velocity of a PP.

$$U_m U^m = -1 \qquad \frac{dx}{d\tau}\,\frac{dx}{d\tau}$$

⇒ problem with $m \to 0$

⇓

make description more redundant

by introducing arbitrary metric of a worldline.

GM

$\int dx_1 \dots dx_D = \int d\xi_1 \dots d\xi_D \sqrt{-g}$

↳ 1-dim $\eta(\tau)$ → additional field on a worldline

$d\tau\, \eta(\tau) = d\tau'\, \eta'(\tau') \Rightarrow \eta'' \to \eta' = \eta \frac{d\tau}{d\tau'}$

$$S'_{PP} = \frac{1}{2}\int d\tau \left[\eta^{-1} \dot{X}'^{m} \dot{X}'_{m} - \eta\, m'^{2} \right]$$

$\Updownarrow$

must be equivalent (at least classically) to S_{PP}

$$\dot{X}'^{m} = \frac{dX'^{m}}{d\tau}$$

$$\Rightarrow S'_{PP} = \frac{1}{2}\int d\tau \left[\dot{X}^m \dot{X}_m - \eta^2 m^2 \right]$$

$\Downarrow$

must be equivalent (at least classically) to S_{PP}

- Poincare invariance.

$$\dot{X} = \frac{d\tau}{dt}\frac{d\tau'}{d\tau}\dot{X}' \qquad \eta = \eta' \frac{d\tau}{d\tau'}$$

$$\left[\eta^{-1}\dot{X}^{\mu}\dot{X}_{\mu} - \eta m^2\right] = \left[\eta'^{-1}\frac{d\tau'}{d\tau}\dot{X}'^{\mu}\dot{X}'_{\mu}\left(\frac{dt'}{d\tau}\right)^2\right]$$

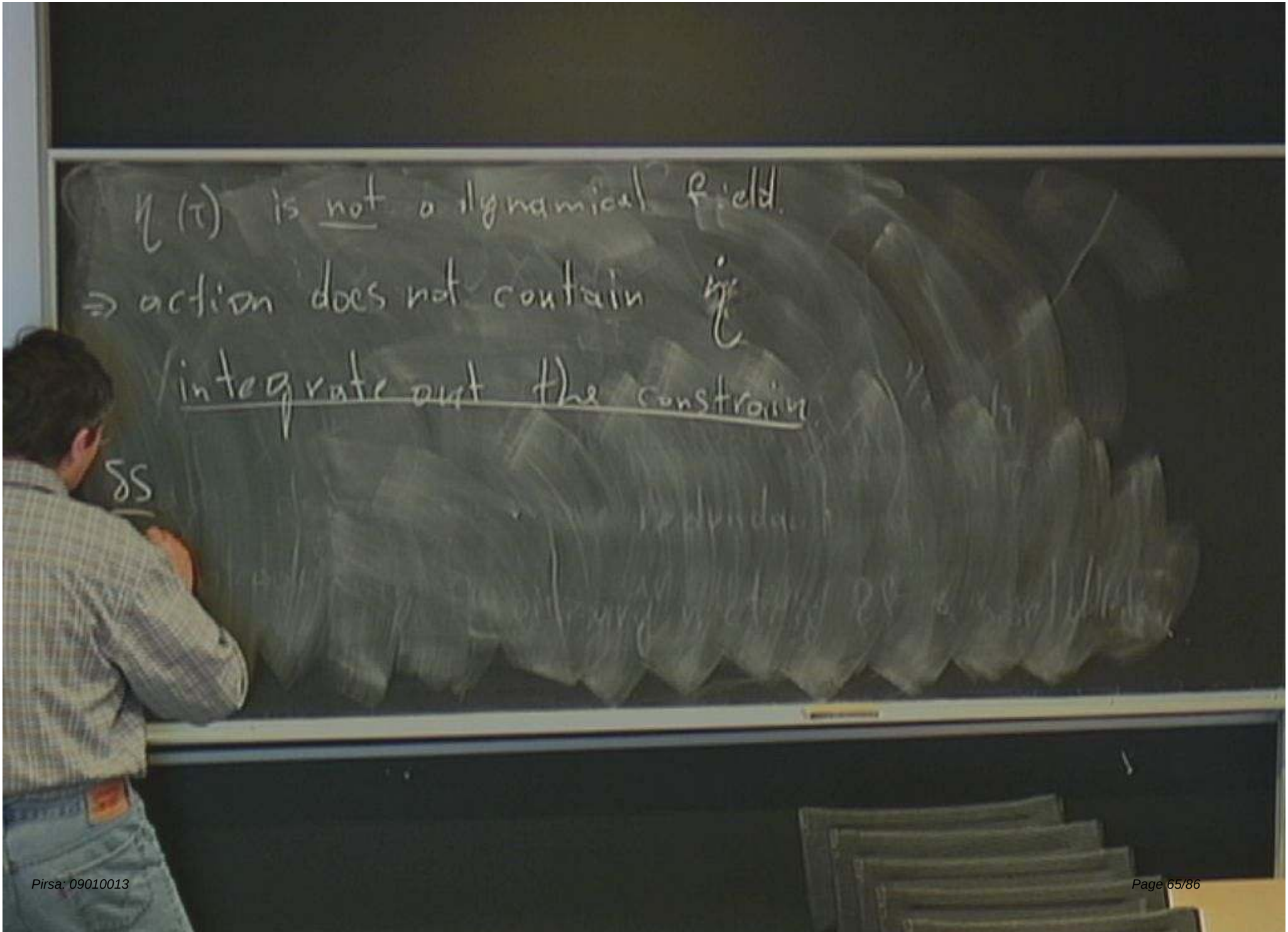
$$\dot{X}^\mu = \frac{d X^\mu}{dt} = \frac{dX}{d\tau'} \cdot \frac{d\tau'}{d\tau} \qquad \eta = \eta' \cdot \frac{d\tau'}{d\tau}$$

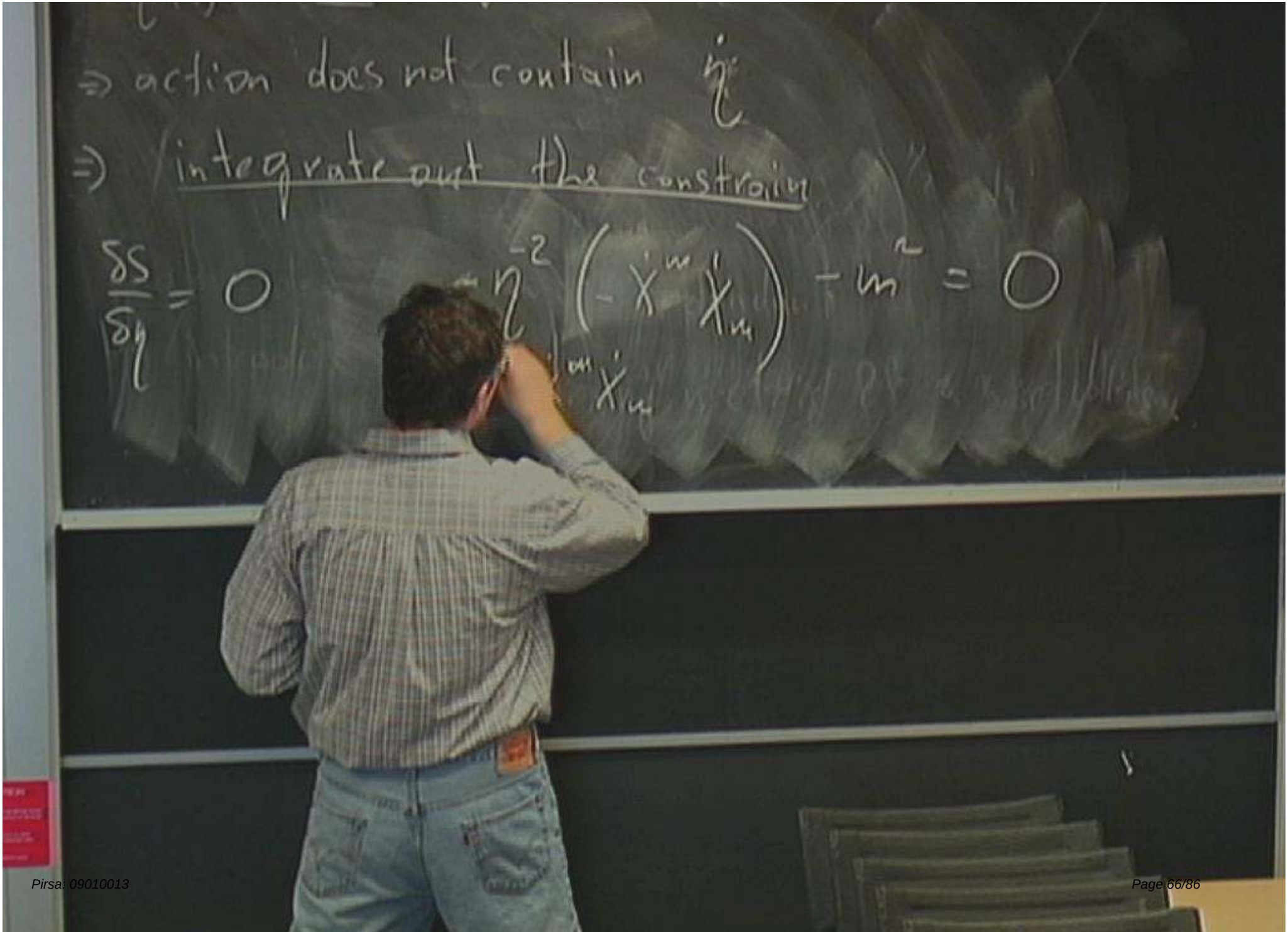
$$\left[\eta^{-1} \dot{X}^\mu \dot{X}_\mu - \eta m^2 \right] = \left[\eta'^{-1} \frac{d\tau'}{d\tau} \dot{X}'^\mu \dot{X}'_\mu \left(\frac{d\tau'}{d\tau} \right)^2 - m^2 \eta' \frac{d\tau'}{d\tau} \right]$$

$$S'_{PP}[\tau'(\tau)] = S'_{PP}[\tau] \left[\quad \right]' \frac{d\tau'}{d\tau}$$

$\eta(\tau)$ is not a dynamical field.

$\Rightarrow$ action does not contain $\dot{\eta}$


η(τ) is not a dynamical field.
⇒ action does not contain it
integrate out the constraint
δS


⇒ action does not contain $\dot{\eta}$
⇒ integrate out the constraint
$\frac{\delta S}{\delta \eta} = 0$
$-\eta^{-2}\left(-\dot{X}^m X'_m\right) + m^2 = 0$
X'_m

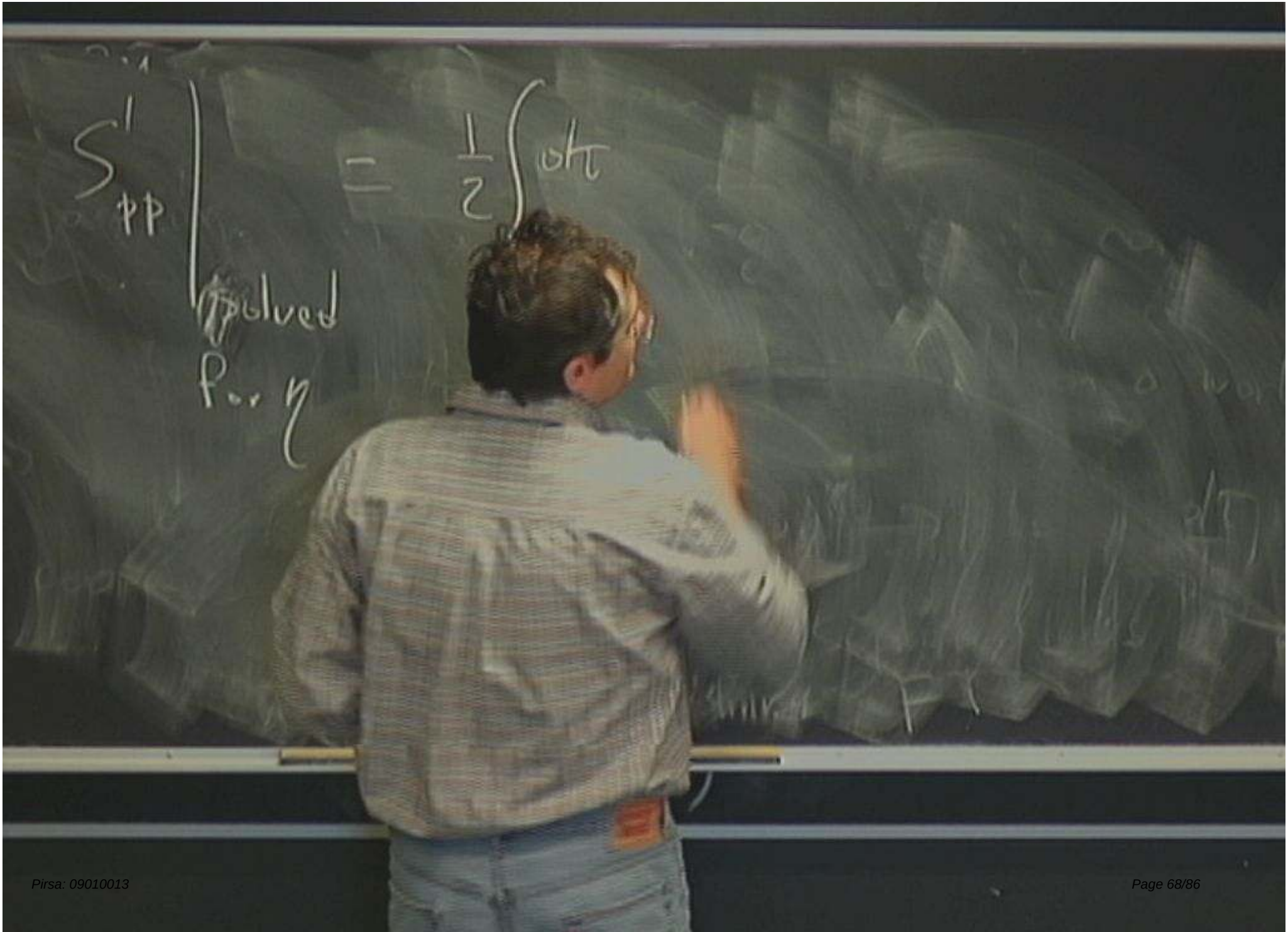
$\eta(\tau)$ is not a dynamical field

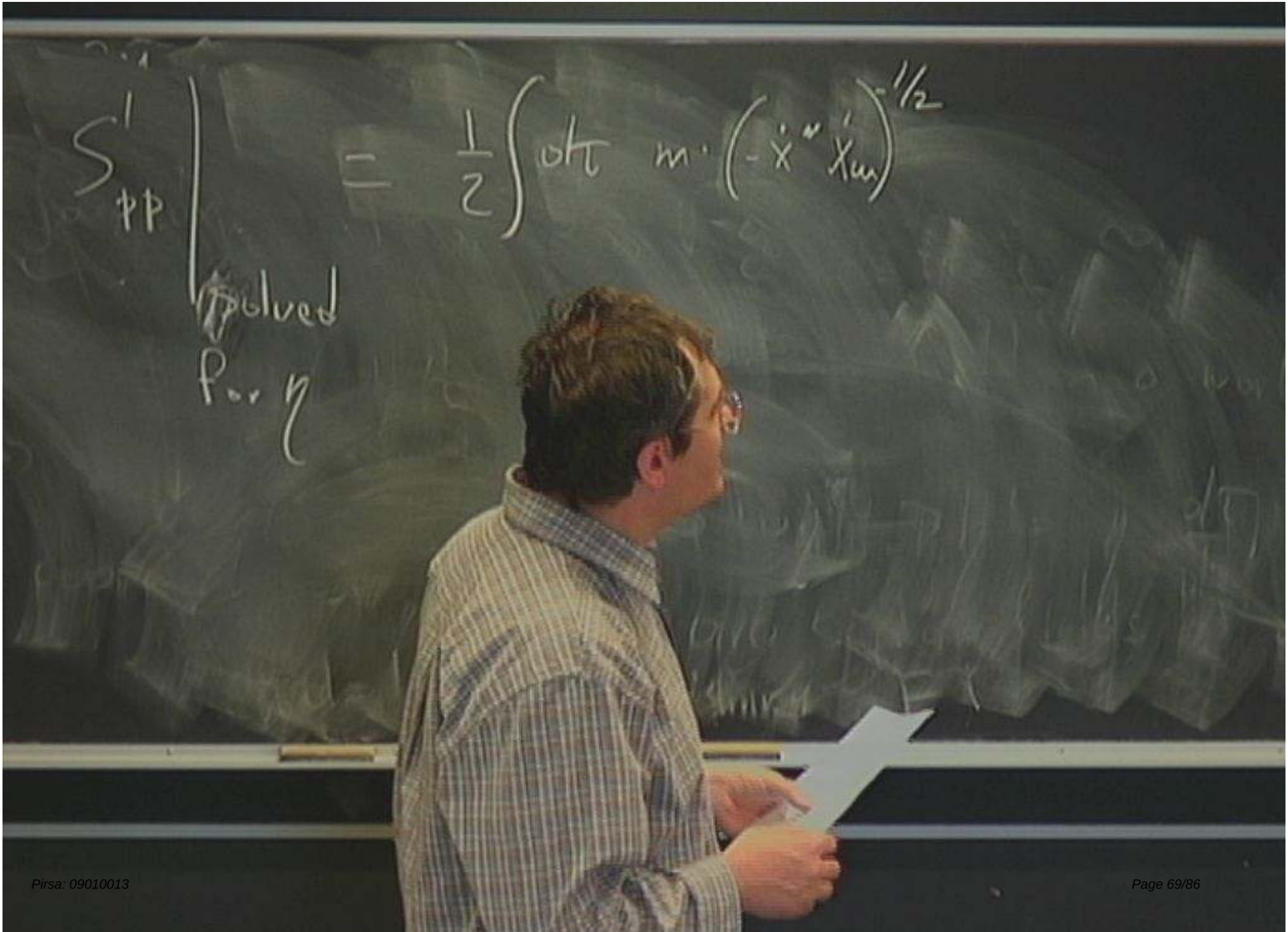
$\Rightarrow$ action does not contain $\dot{\eta}$

$\Rightarrow$ integrate out the constraint

$$\frac{\delta S}{\delta \eta} = 0 \qquad -\eta^{-2}\left(-\dot{X}^{\mu}\dot{X}_{\mu}\right) + m^2 = 0$$

$$\eta = \frac{\left(-\dot{X}^{\mu}\dot{X}_{\mu}\right)^{1/2}}{m}$$


$S'_{pp}\Big|_{\text{solved for } \eta} = \frac{1}{2}\int d\tau$


$S'_{pp}\Big|_{\text{solved for } \eta} = \frac{1}{2}\int dt\ m\cdot\left(-\dot{x}^{\mu}\dot{x}_{\mu}\right)^{-1/2}$

$$\Rightarrow S'_{pp} = \frac{1}{2}\int d\tau \left[\eta^{-1} \dot{X}'^{\mu} \dot{X}'_{\mu} - \eta m^2 \right]$$

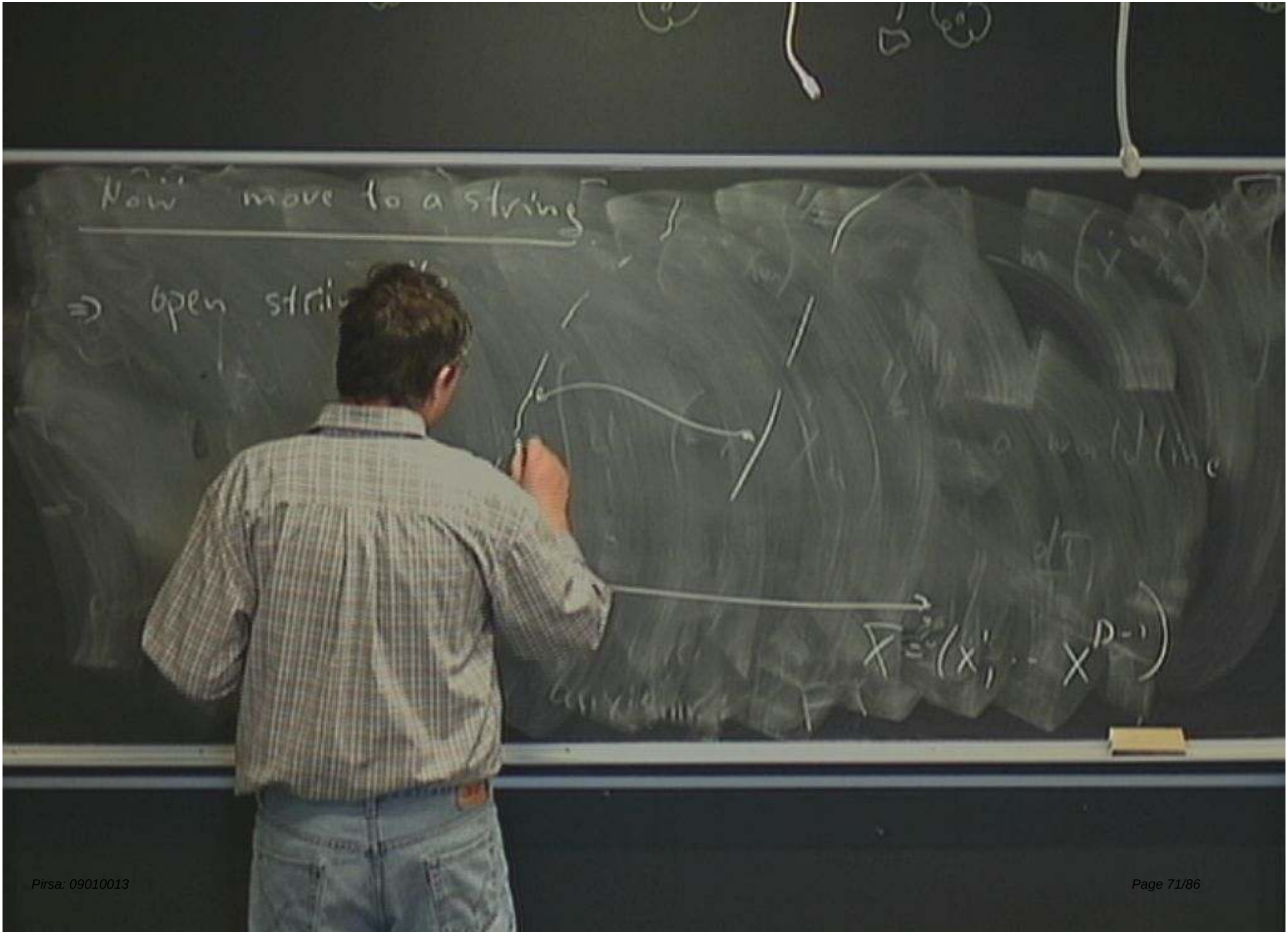
$\Updownarrow$

must be equivalent (at least classically) to S_{pp}

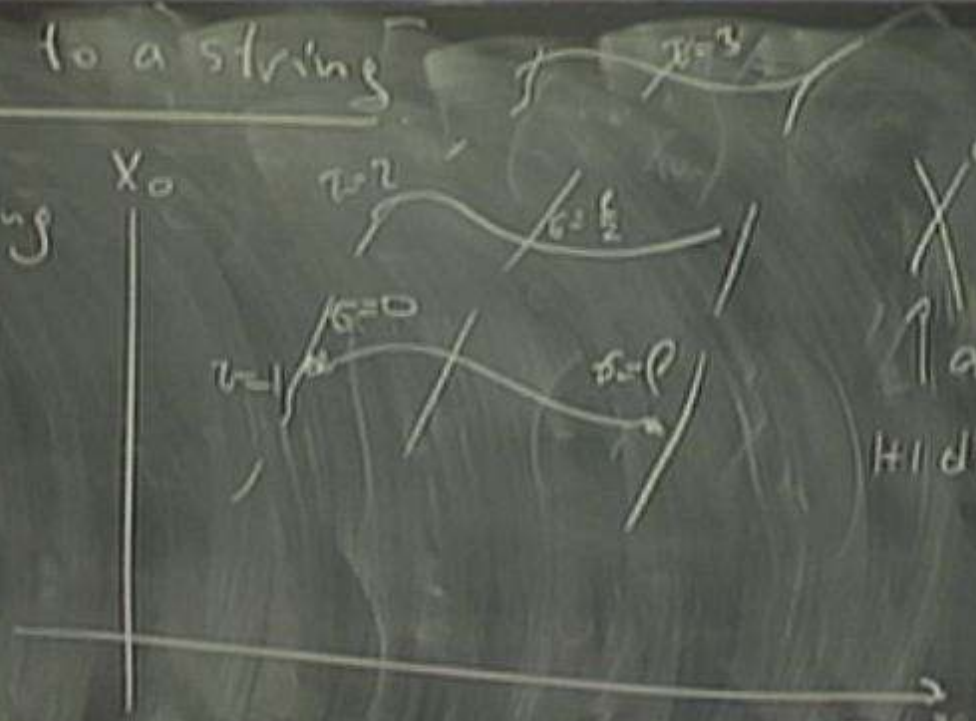
- Poincare invariance

- $\tau \to \tau' = \delta'(\tau)$

$$\dot{X}'^{\mu} = \frac{dX'^{\mu}}{d\tau} = \frac{dX'^{\mu}}{d\tau'} \cdot \frac{d\tau'}{d\tau} \qquad \eta = \eta' \cdot \frac{d\tau'}{d\tau}$$


Now move to a string
open strin

Now move to a string



⇒ open string

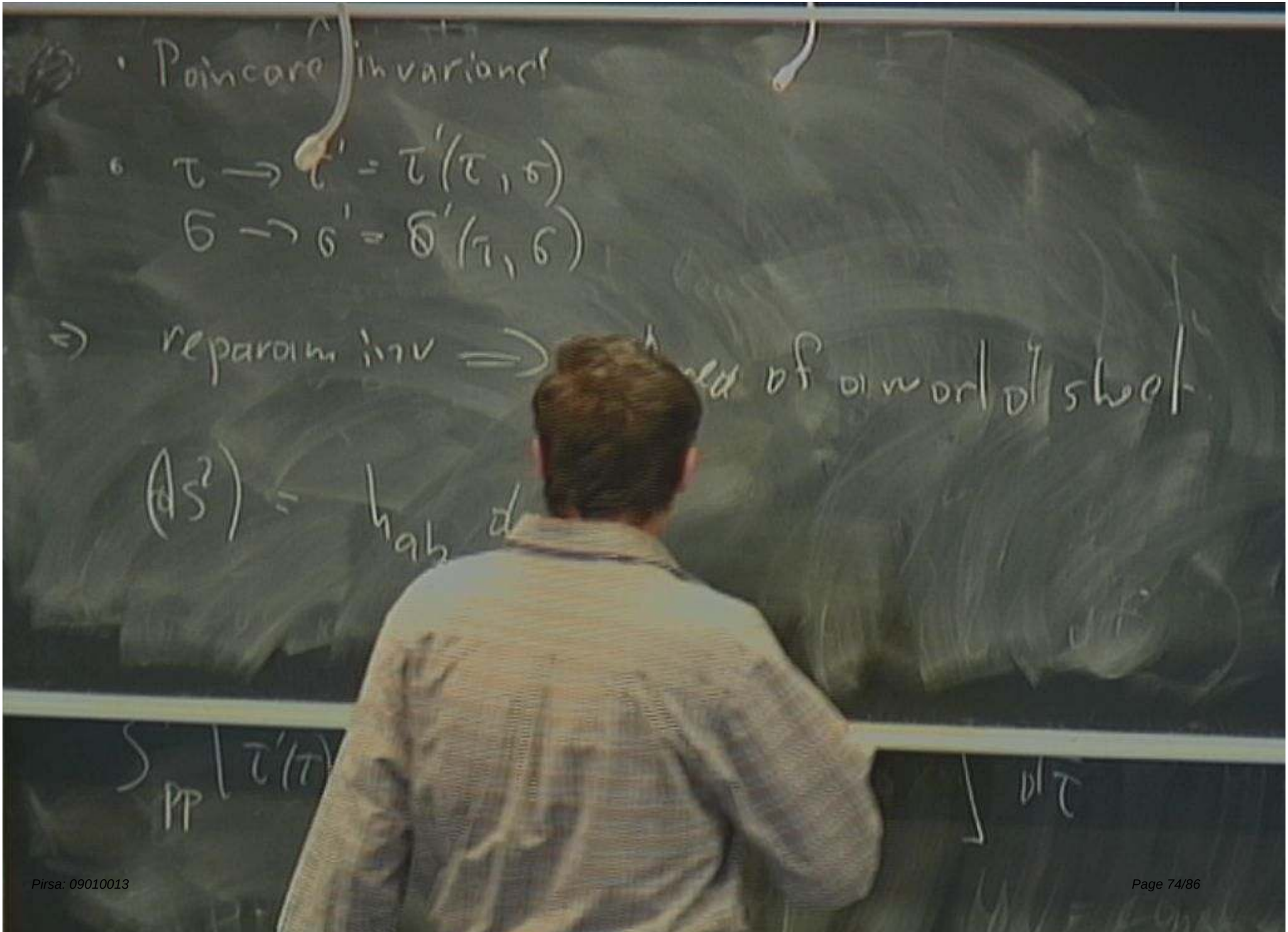
$X^\mu = X^\mu(\tau, \sigma)$

↑ a field of
11 dim world-sheet FT

$\vec{x} = (x^1, \ldots x^{D-1})$

• Poincare invariance

$$\tau \to \tau' = \tau'(\tau, \sigma)$$
$$\sigma \to \sigma' = \sigma'(\tau, \sigma)$$


· Poincare invariant
$\tau \to \tau' = \tau'(\tau, \sigma)$
$\sigma \to \sigma' = \sigma'(\tau, \sigma)$
$(ds^2) = h_{ab}\, d$

• Poincare invariance

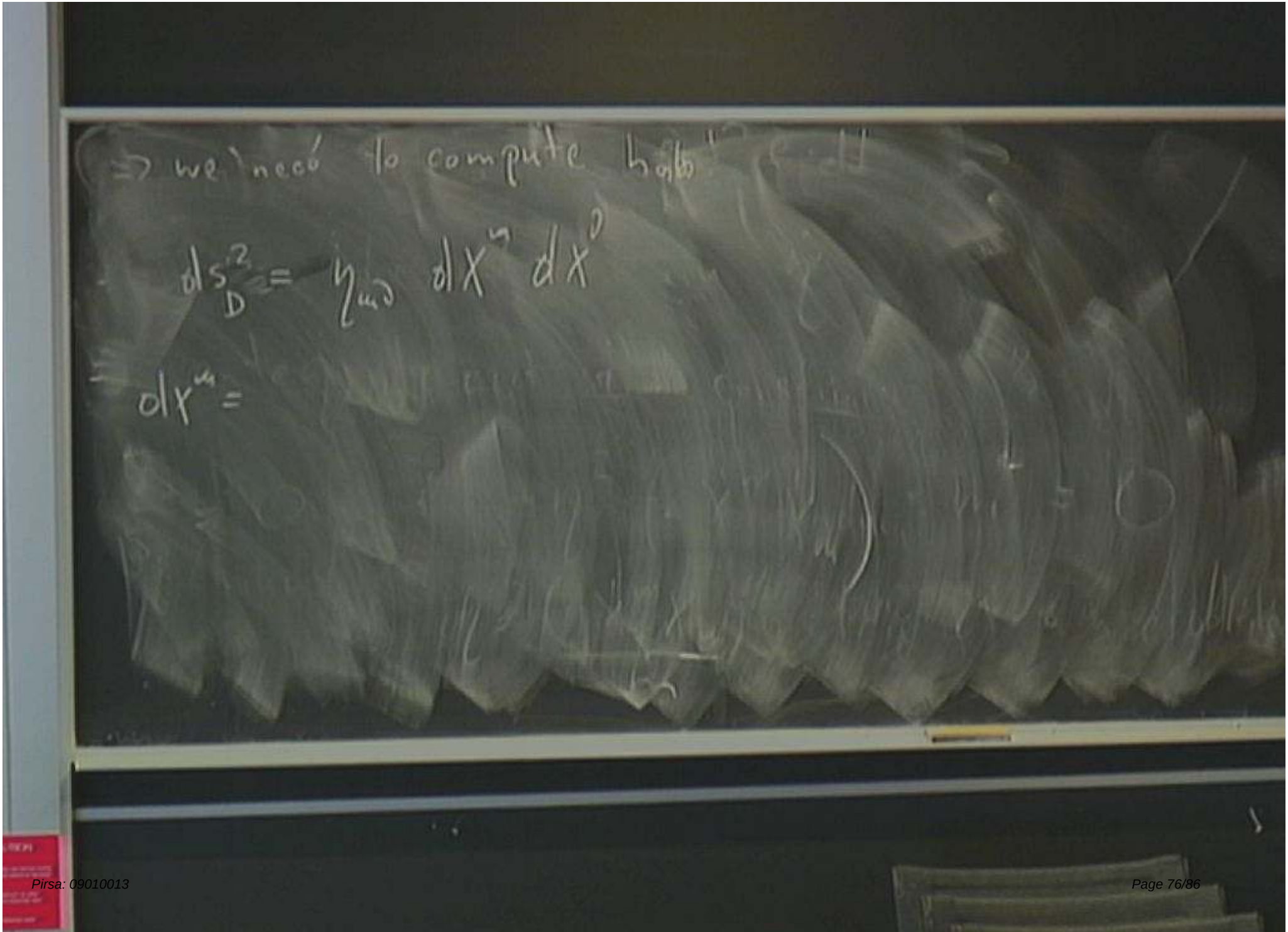
• $\tau \to \tau' = \tau'(\tau, \sigma)$
$\sigma \to \sigma' = \sigma'(\tau, \sigma)$

⇒ reparam inv ⟹ Area of world sheet

$(ds^2) = h_{ab}\, dy^a\, dy^b$ $\qquad A = \int d\tau d\sigma \sqrt{-h}$

$h = \det(h_{ab})$

$S_{PP}[\tau'(\tau)] = S_{PP}[\tau]$ $\int d\tau$



$\Rightarrow$ we need to compute h_{ab}

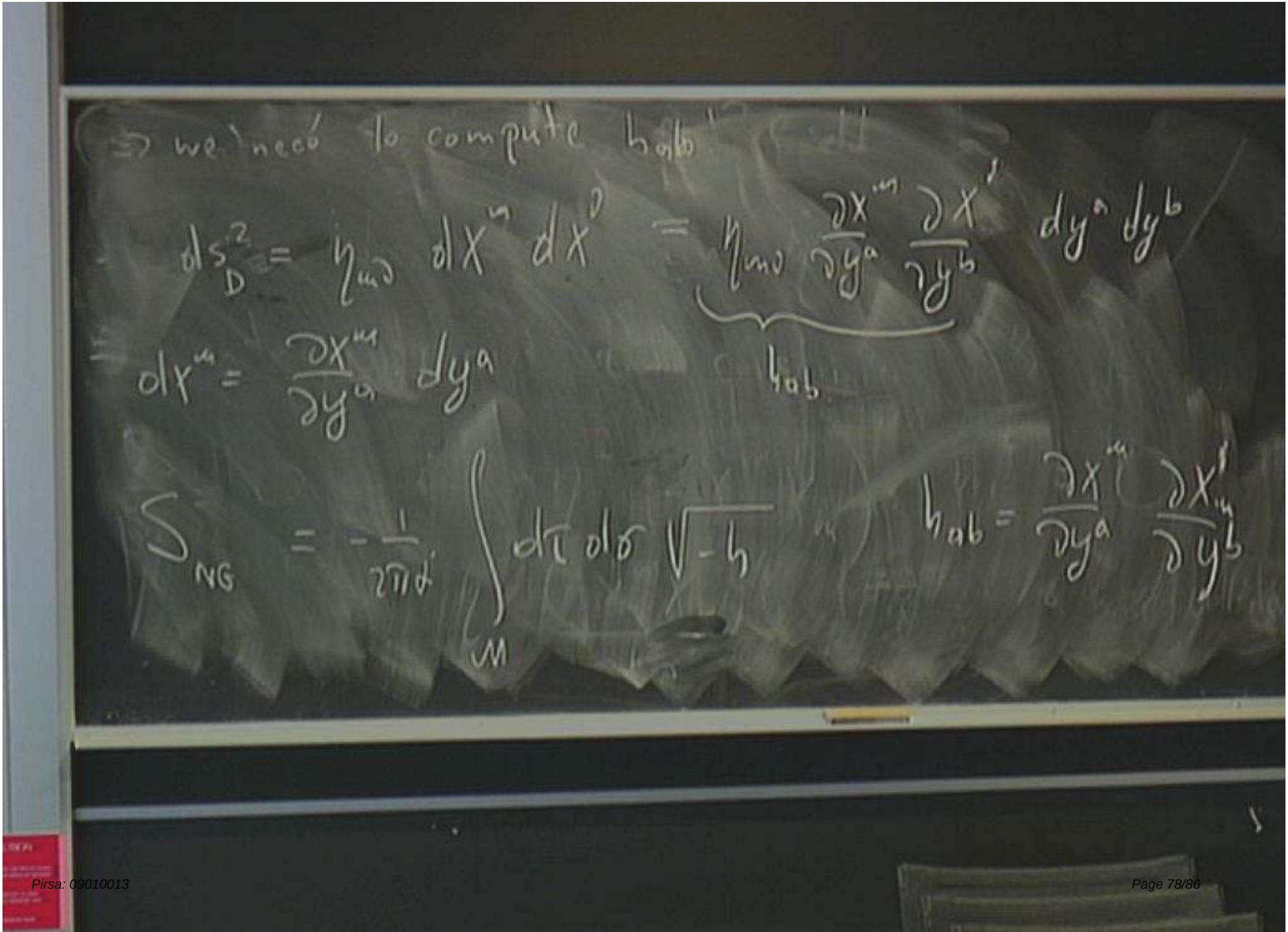
$$ds_D^2 = \eta_{\mu\nu}\, dx^\mu\, dx^\nu$$

$$dx^\mu =$$

⇒ we need to compute h_{ab}

$$ds^2_D = \eta_{\mu\nu}\, dX^\mu dX^\nu = \underbrace{\eta_{\mu\nu} \frac{\partial X^\mu}{\partial y^a} \frac{\partial X^\nu}{\partial y^b}}_{h_{ab}} dy^a dy^b$$

$$dX^\mu = \frac{\partial X^\mu}{\partial y^a} dy^a$$


⇒ we need to compute h_{ab}
$ds^2_D = \eta_{\mu\nu}\, dX^\mu dX^\nu = \eta_{\mu\nu} \frac{\partial X^\mu}{\partial y^a} \frac{\partial X^\nu}{\partial y^b}\, dy^a dy^b$
h_{ab}
$dX^\mu = \frac{\partial X^\mu}{\partial y^a}\, dy^a$
$S_{NG} = -\frac{1}{2\pi\alpha'} \int_M d\tau\, d\sigma\, \sqrt{-h}$
$h_{ab} = \frac{\partial X^\mu}{\partial y^a} \frac{\partial X^\nu}{\partial y^b}$

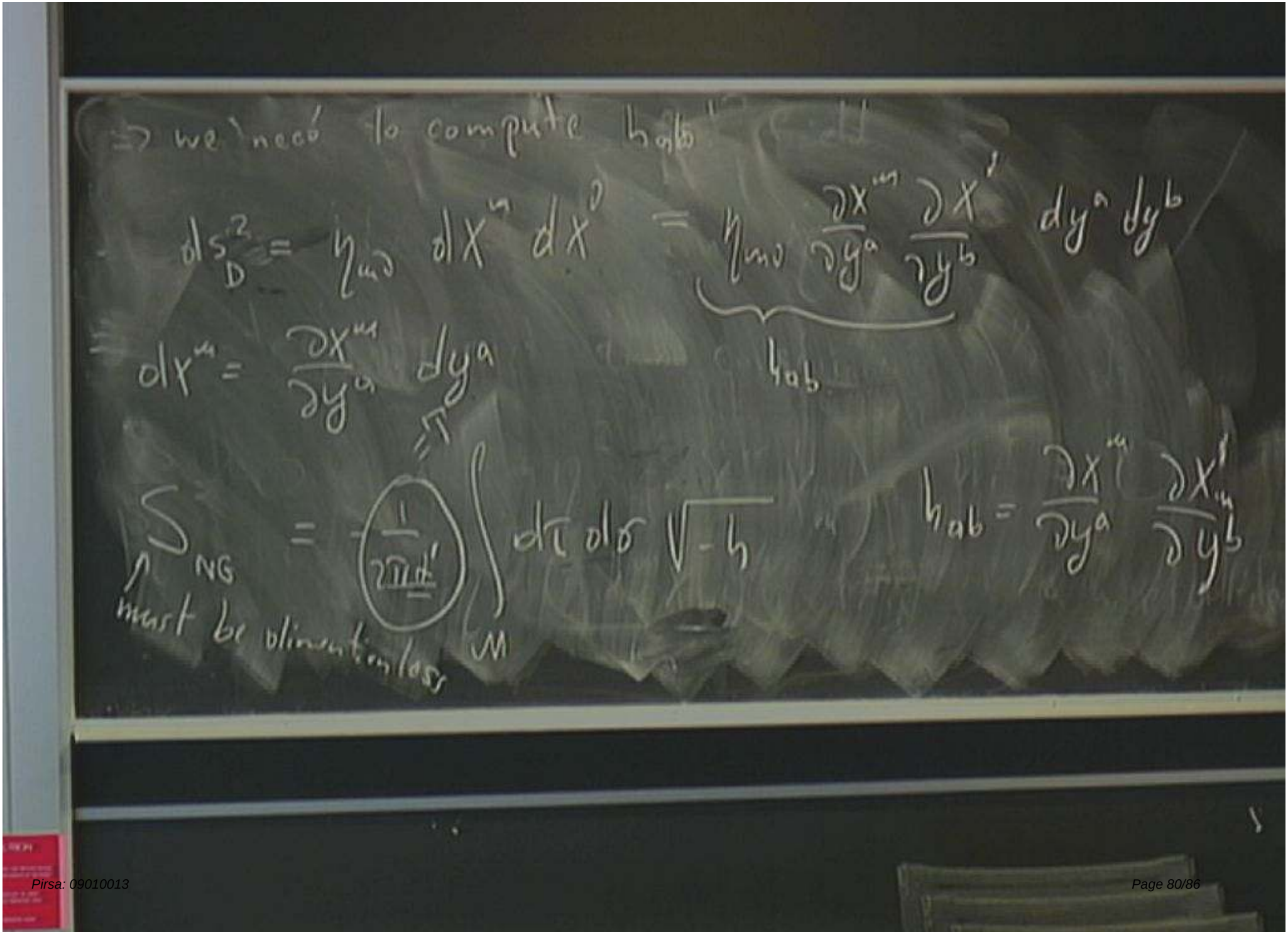
⇒ we need to compute h_{ab}

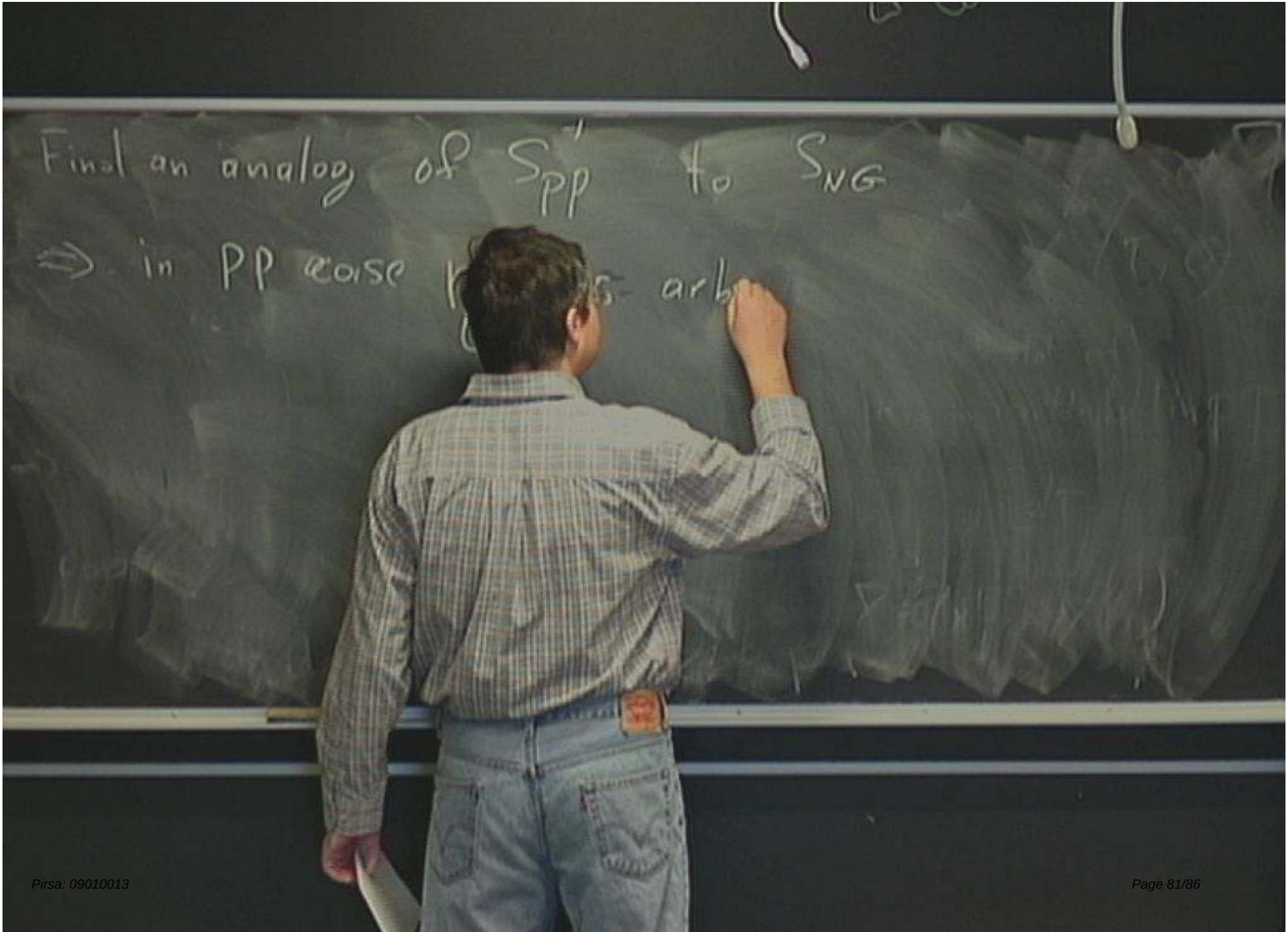
$$ds^2_D = \eta_{\mu\nu}\, dX^\mu dX^\nu = \underbrace{\eta_{\mu\nu}\frac{\partial X^\mu}{\partial y^a}\frac{\partial X^\nu}{\partial y^b}}_{h_{ab}}\, dy^a dy^b$$

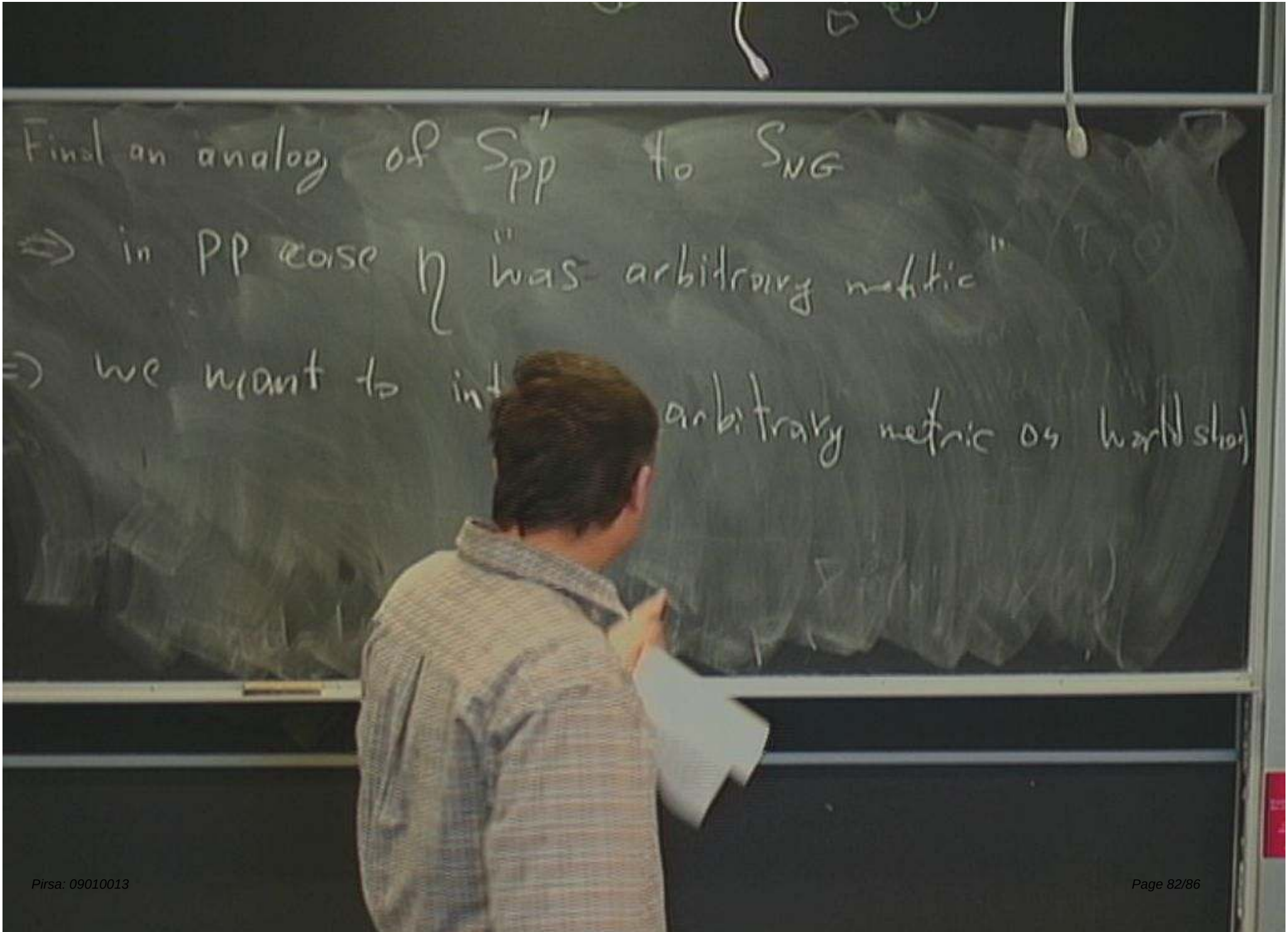
$$dX^\mu = \frac{\partial X^\mu}{\partial y^a}\, dy^a$$

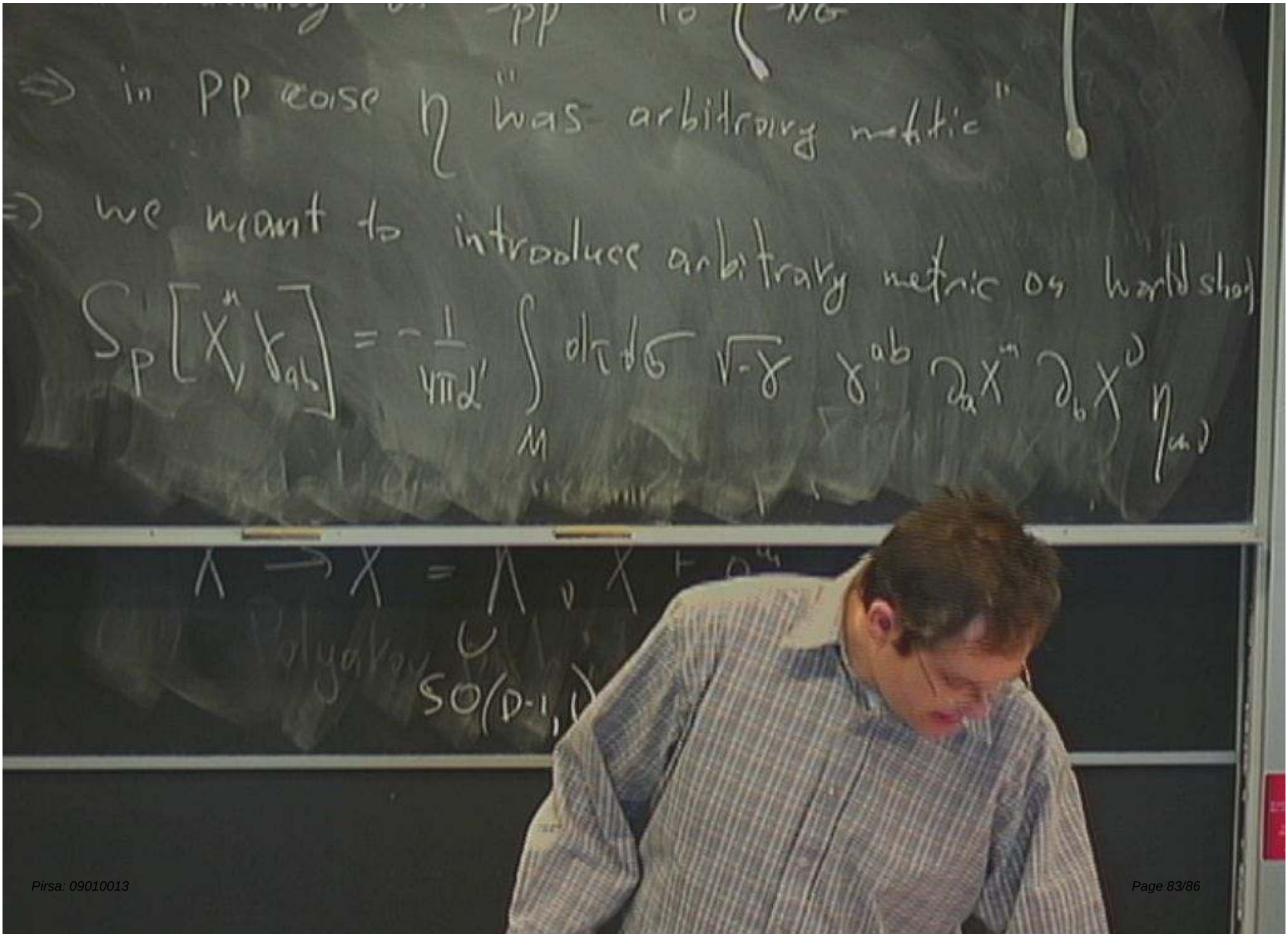
$$S_{NG} = -\frac{1}{2\pi\alpha'}\int_M d\tau\, d\sigma\, \sqrt{-h} \qquad h_{ab} = \frac{\partial X^\mu}{\partial y^a}\frac{\partial X^\nu}{\partial y^b}\eta_{\mu\nu}$$

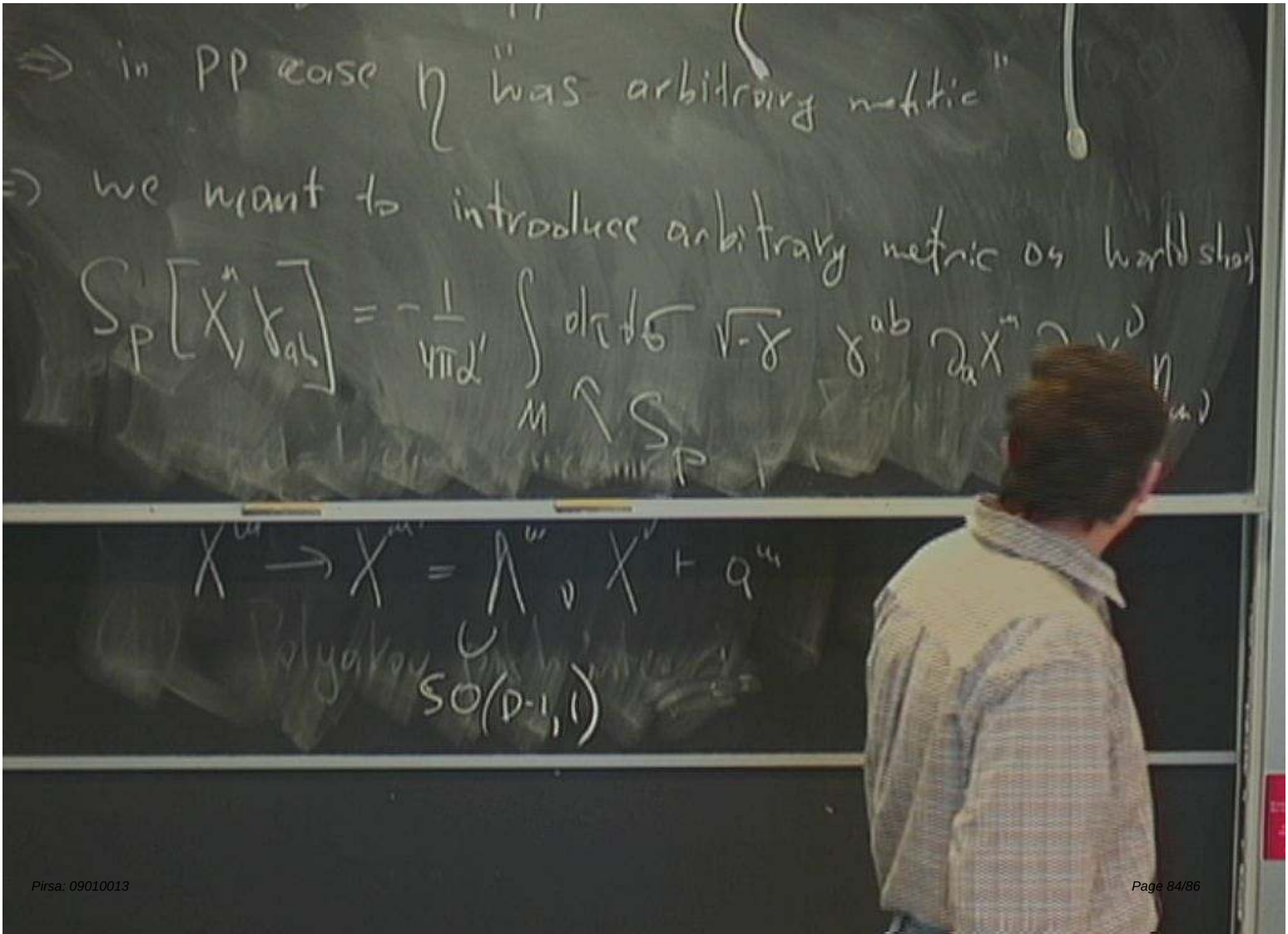
must be dimensionless


⇒ we need to compute h_{ab}
$ds^2_D = \eta_{\mu\nu}\, dX^\mu dX^\nu = \eta_{\mu\nu} \frac{\partial X^\mu}{\partial y^a} \frac{\partial X^\nu}{\partial y^b}\, dy^a dy^b$
$dX^\mu = \frac{\partial X^\mu}{\partial y^a} dy^a$
h_{ab}
$S_{NG} = -\frac{1}{2\pi\alpha'} \int_M d\tau\, d\sigma \sqrt{-h}$
$h_{ab} = \frac{\partial X^\mu}{\partial y^a} \frac{\partial X^\nu}{\partial y^b}$
must be dimensionless


Find an analog of S_{PP}^{-1} to S_{NG}
⇒ in PP case


Find an analog of S_{PP}' to S_{NG}
⇒ in PP case η "was arbitrary metric"
⇒ we want to int
arbitrary metric on world sho


⇒ in PP case η was arbitrary metric
⇒ we want to introduce arbitrary metric on world sheet
$S_P[X^\mu, \gamma_{ab}] = -\frac{1}{4\pi\alpha'} \int_M d\tau d\sigma \sqrt{-\gamma}\, \gamma^{ab} \partial_a X^\mu \partial_b X^\nu \eta_{\mu\nu}$
$X \to \Lambda X = \Lambda_\nu X + a^\mu$
SO(D-1,1)


⇒ in PP case η was arbitrary metric
⇒ we want to introduce arbitrary metric on world sheet
$S_P[X^\mu, \gamma_{ab}] = -\frac{1}{4\pi\alpha'} \int_M d\tau d\sigma \sqrt{-\gamma}\, \gamma^{ab} \partial_a X^\mu \partial_b X^\nu \eta_{\mu\nu}$
$X^\mu \to X'^\mu = \Lambda^\mu{}_\nu X^\nu + a^\mu$
Poincaré
$SO(D-1,1)$

Find an analog of S_{PP} to S_{NG}

$\Rightarrow$ in PP case η "was arbitrary metric"

$\Rightarrow$ we want to introduce arbitrary metric on world sheet

$$S_P[X^\mu, \gamma_{ab}] = -\frac{1}{4\pi\alpha'} \int_M d\tau d\sigma \sqrt{-\gamma}\, \gamma^{ab} \partial_a X^\mu \partial_b X^\nu \eta_{\mu\nu}$$

S_P

$$X^\mu \to X'^\mu = \Lambda^\mu{}_\nu X^\nu + a^\mu$$

Poincaré

$SO(D-1,1)$

find an analog of S_{PP} to S_{NG}

⇒ in PP case η "was arbitrary metric"

we want to introduce arbitrary metric on world sh

$$S_P[X^\mu, \gamma_{ab}] = -\frac{1}{4\pi\alpha'} \int_M \underbrace{d\tau\, d\sigma\, \sqrt{-\gamma}}_{\text{inv.}}\, \gamma^{ab}\, \partial_a X^\mu\, \partial_b X^\nu\, \eta_{\mu\nu}$$

S_P

$$X^\mu \to X^{\mu'} = \Lambda^\mu{}_\nu\, X^\nu$$