Title: New Techniques for One-loop Gauge Theory Amplitudes

Date: Jan 20, 2009 11:00 AM

URL: http://pirsa.org/09010012

Abstract: In the past couple of years many new developments have been made in the techniques used for computing one-loop gauge theory amplitudes. These developments have mainly involved exploiting generalized unitarity techniques to construct the coefficients of the basis integral functions which make up a one-loop amplitude. I will outline these new developments along with their application to both QCD and N=8 supergravity amplitudes.

Darren Forde

(SLAC & UCLA)

NEW TECHNIQUES FOR ONE-LOOP GAUGE THEORY AMPLITUDES

arXiv:arXiv:0808.0941 [hep-ph], arXiv:0803.4180 [hep-ph], arXiv:0707.1035 [hep-th], arXiv:0704.1835 [hep-ph]

In collaboration with C. Berger, Z. Bern, L. Dixon, F. Febres Cordero, D. Maitre, H. Ita & D. Kosower.

OVERVIEW

Why do we want to compute one-loop amplitudes?

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Applications to QCD and $\mathcal{N}=8$ Supergravity

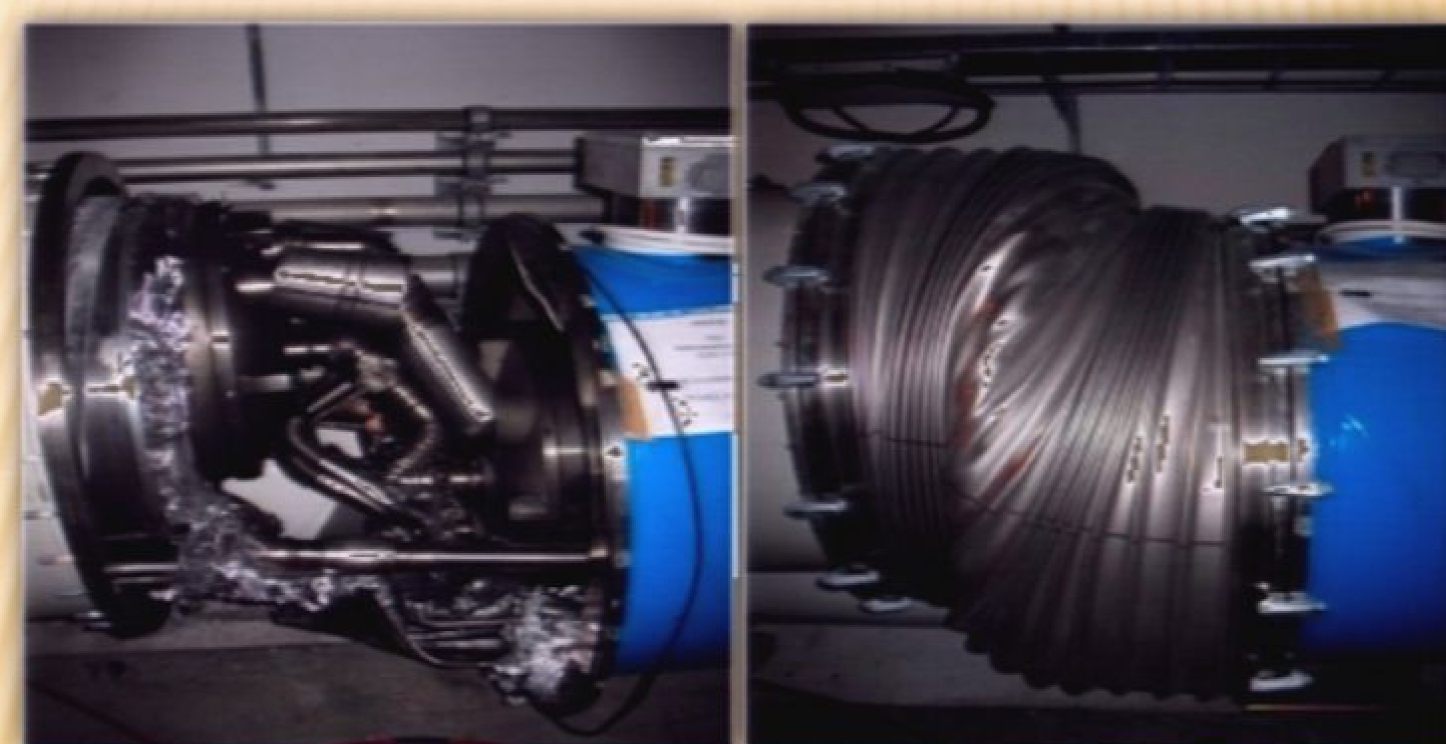
- Scaling behaviour of one-loop amplitudes.
- Automatic computation of QCD amplitudes.

WHAT'S THE PROBLEM?

- The LHC

STARTED WITH A BANG

- Unfortunate incident on the 19th Sep during commissioning.



- Delayed until June 2009.

NEW PHYSICS

- Probe beyond the Standard Model. hope to see new physics.
  - Search for events with missing E_t and jets.
  - Standard Model processes can produce similar events $\Rightarrow$ Backgrounds to new physics searches.

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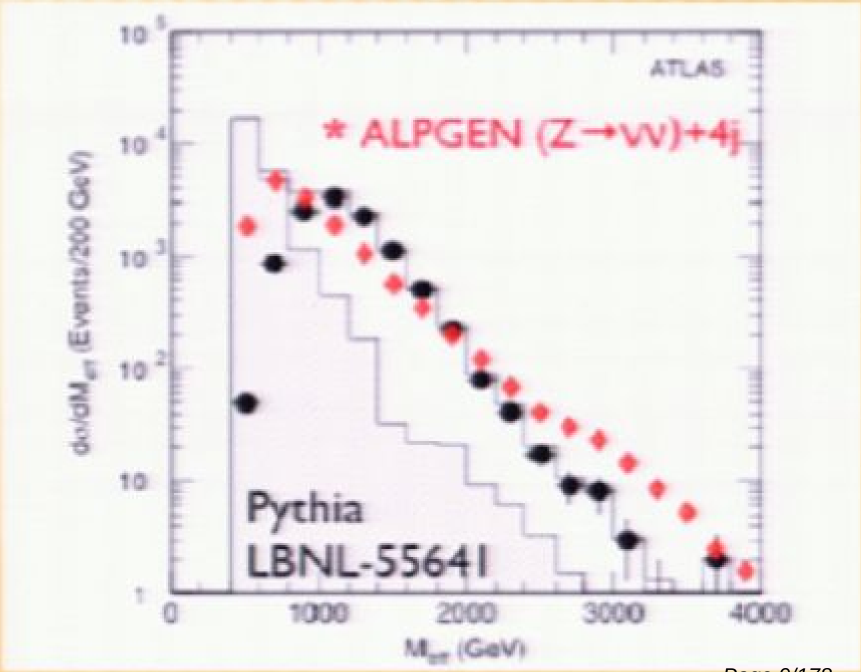

[Gianotti, Mangano]

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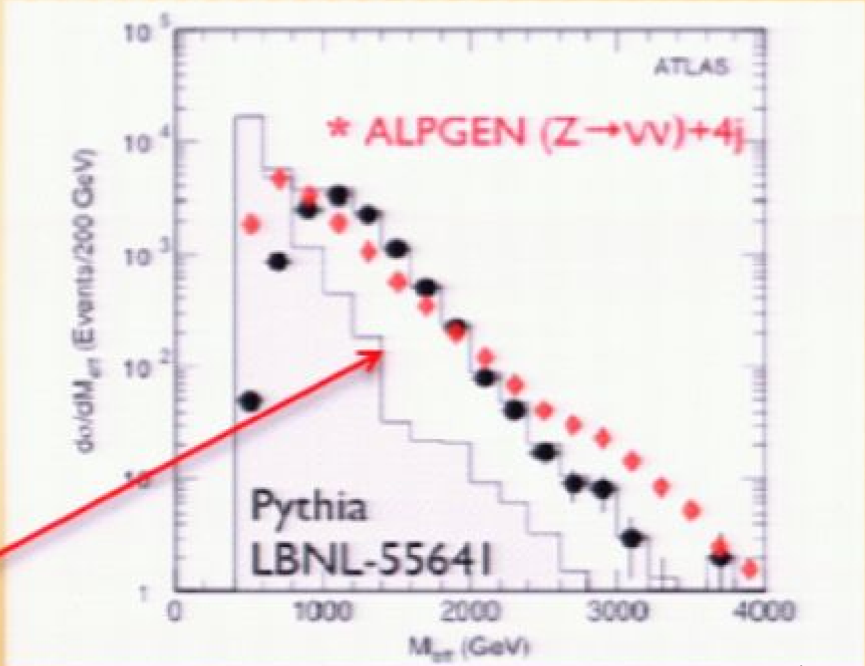

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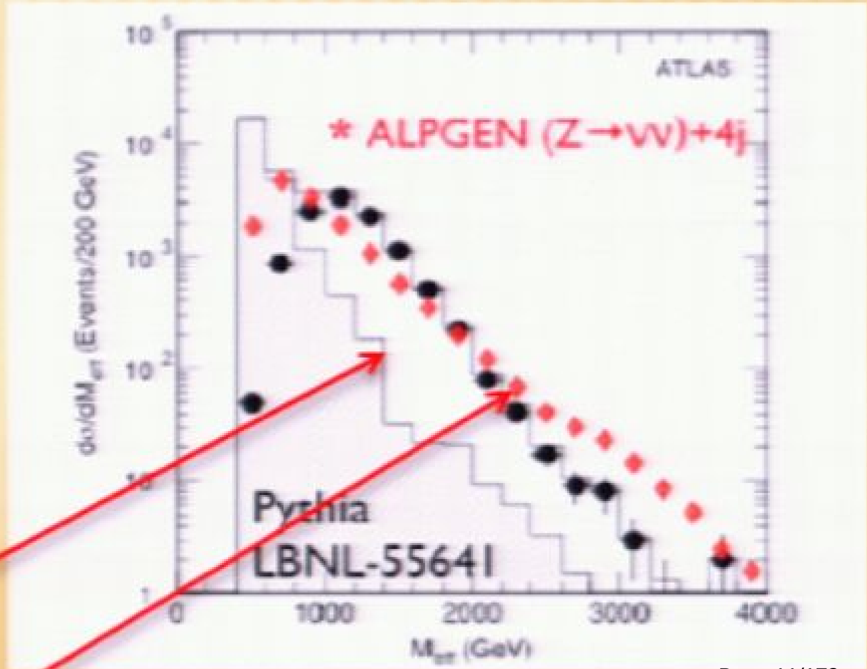

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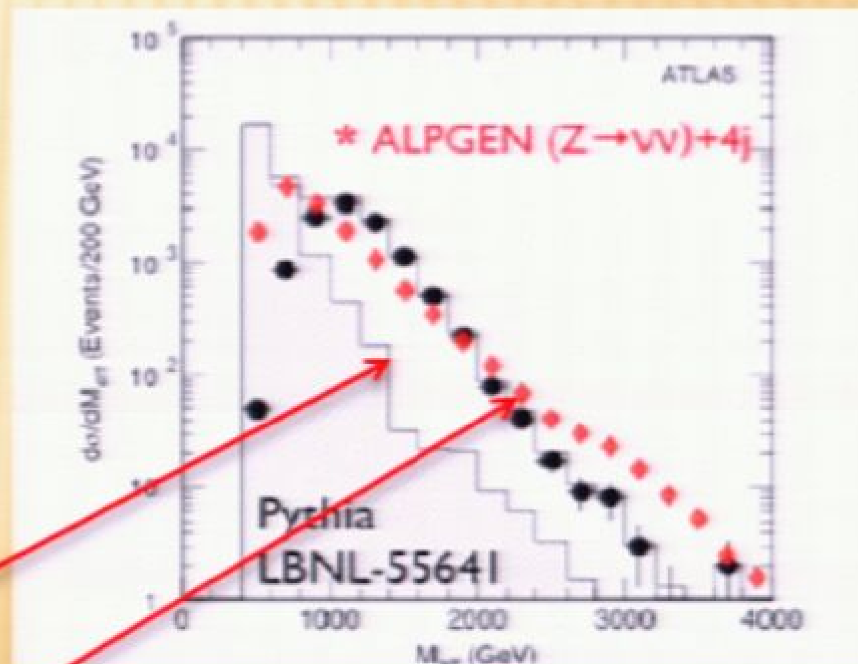
- Need NLO (1-loop) amplitudes to help identify signal.



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UNDERSTANDING $\mathcal{N}=8$ SUPERGRAVITY

- Recent 3 loop 4-point $\mathcal{N}=8$ supergravity perturbative computations suggest: [Bern, Dixon, Roiban] [Bern, Carrasco, Dixon, Johansson, Kosower, Roiban]
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- At one-loop this corresponds to the "No-triangle hypothesis". recently proved [Bjerrum-Bohr, Vanhove], [Arkani-Hamed, Cachazo, Kaplan]
  - One-loop amplitude is made up entirely of scalar boxes.
  - Can we see understand why this is? Examining one-loop amplitudes can tell us something.

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- Numerical approaches (challenge to preserve numerical stability with large cancellations between terms) [von Hameren, Vollinga, Weinzierl], [Giele, Glover], [Giele, Glover, Zanderighi], [Ellis, Giele, Zanderighi], [Binoth, Guillet, Heinrich, Pilon, Schubert]

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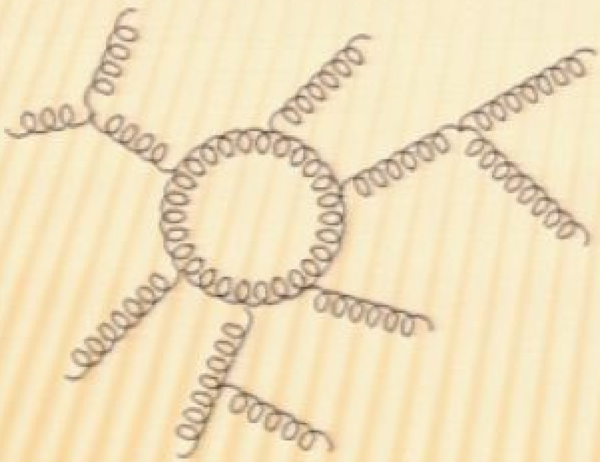

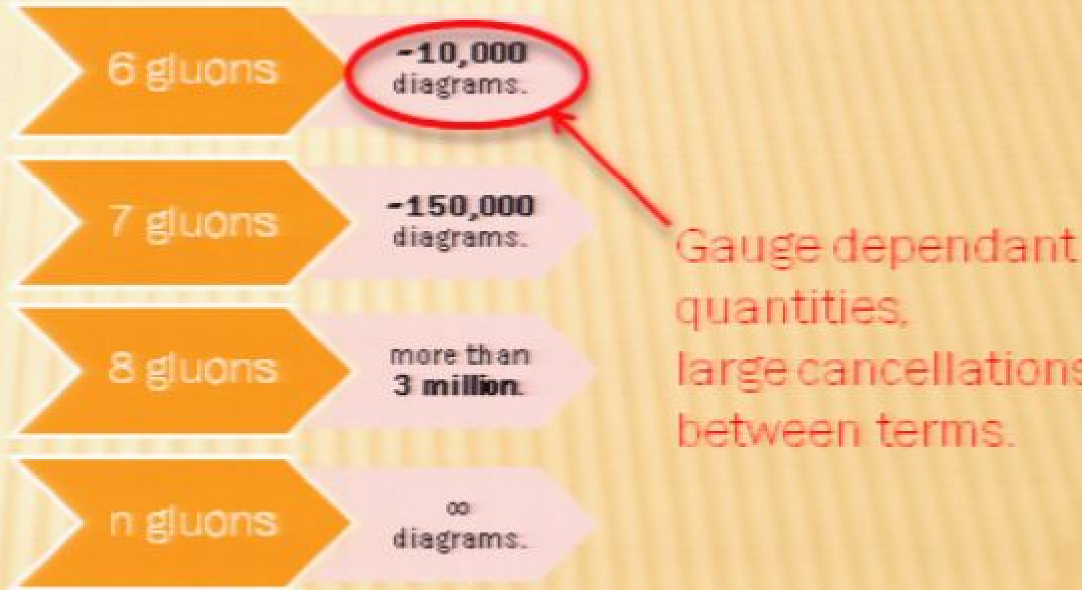

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- Calculating using Feynman diagrams is **Hard!**

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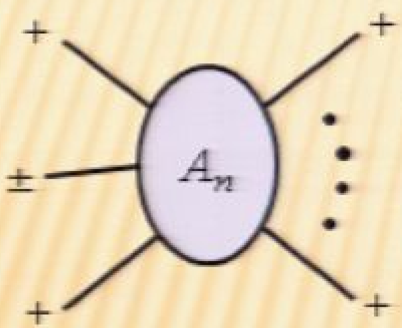

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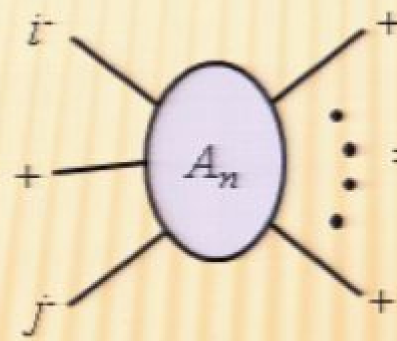

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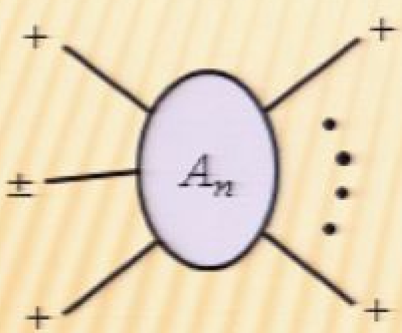

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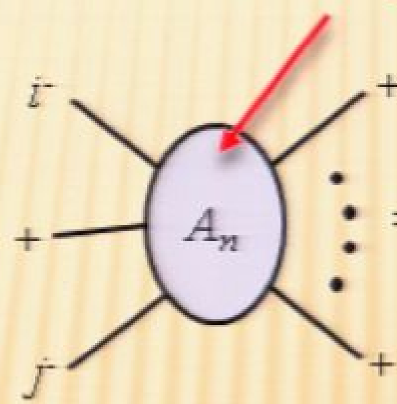

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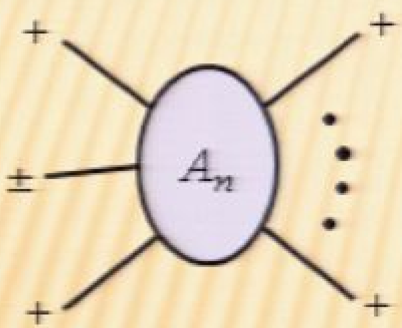

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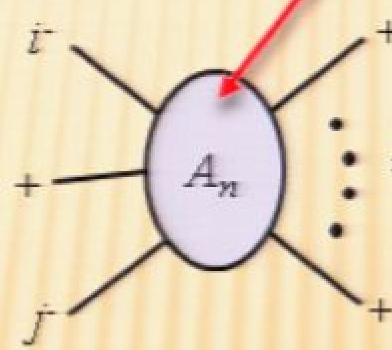
MHV Amplitude

Spinor helicity notation



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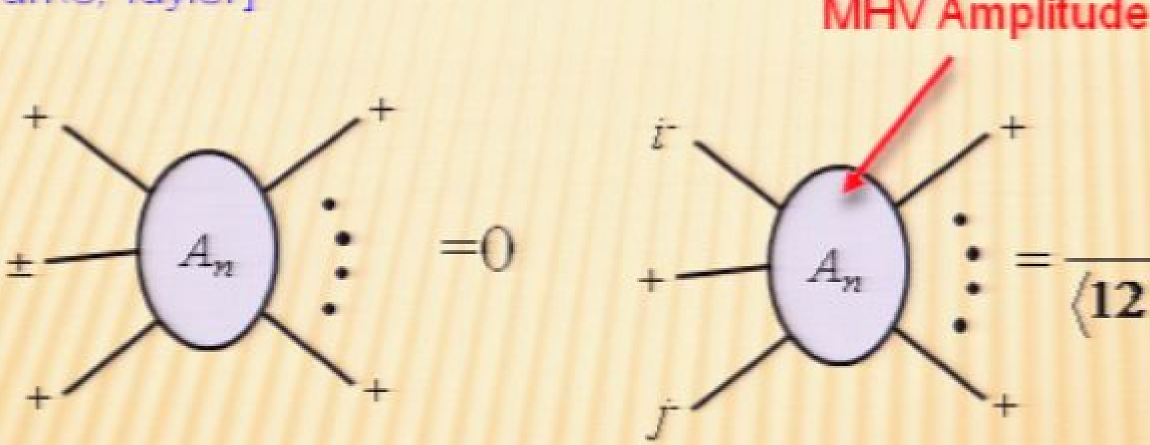

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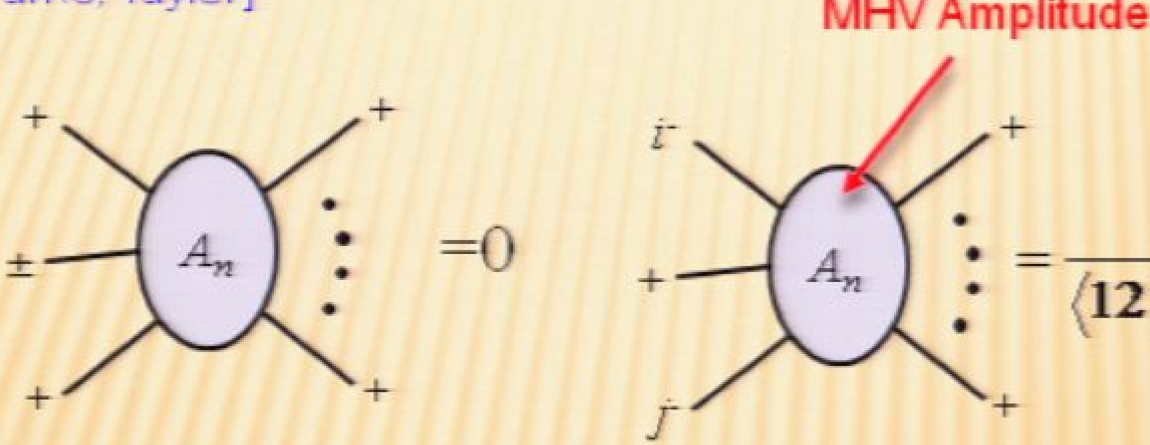

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- Want to use on-shell quantities only.

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- Can we expose this simplicity when computing tree-level amplitudes more directly?

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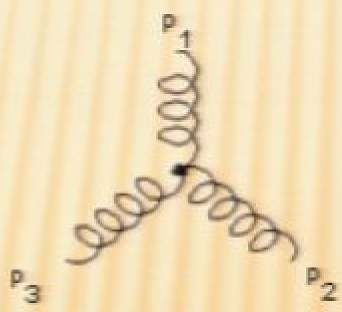

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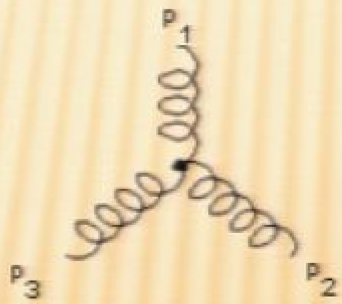

- Take better advantage of the analytic structure of amplitudes.

AMPLITUDES AND THE COMPLEX PLANE

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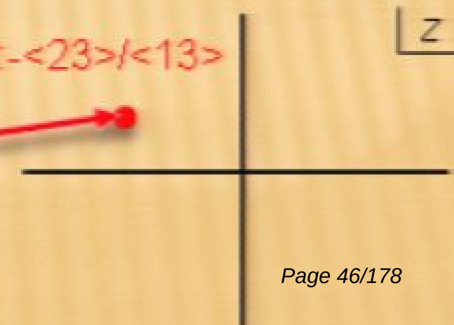

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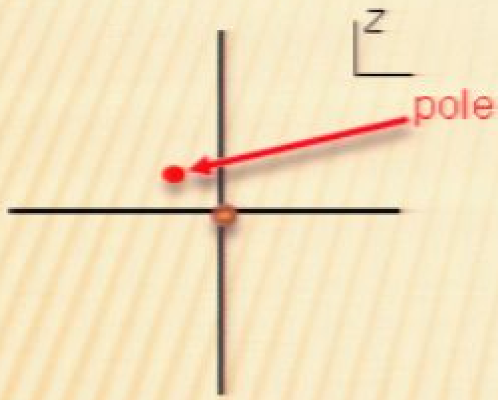

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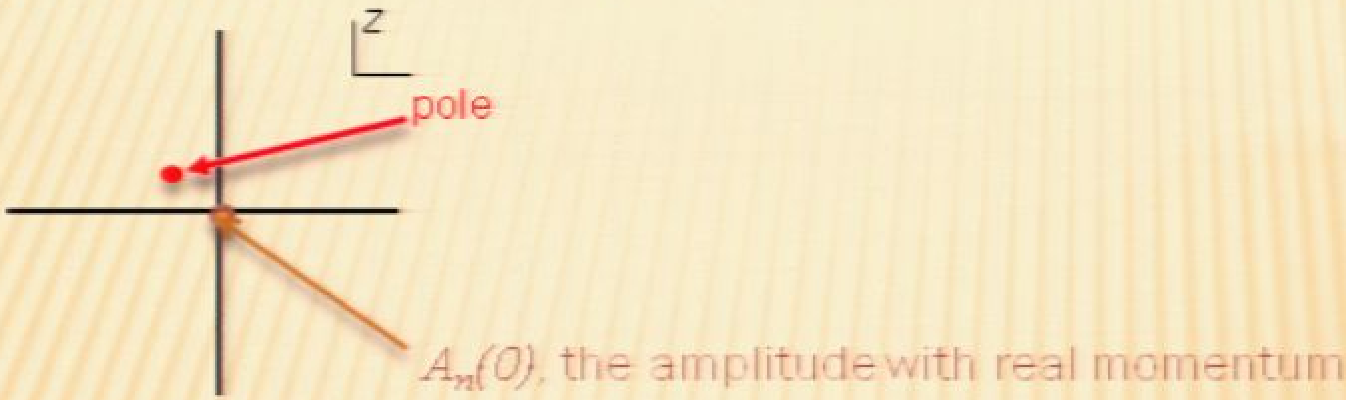

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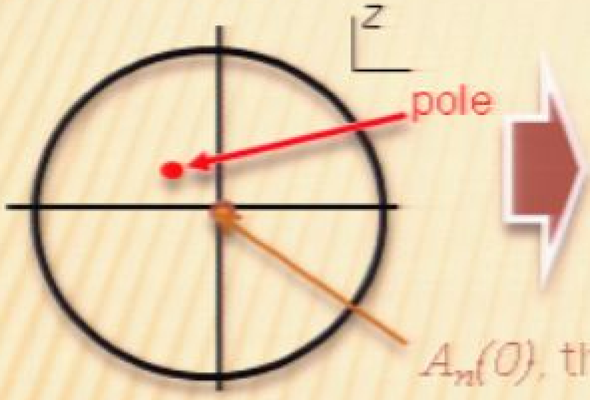

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$A_n(0)$, the amplitude with real momentum

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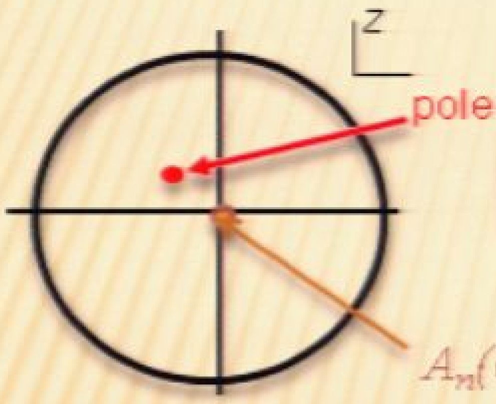

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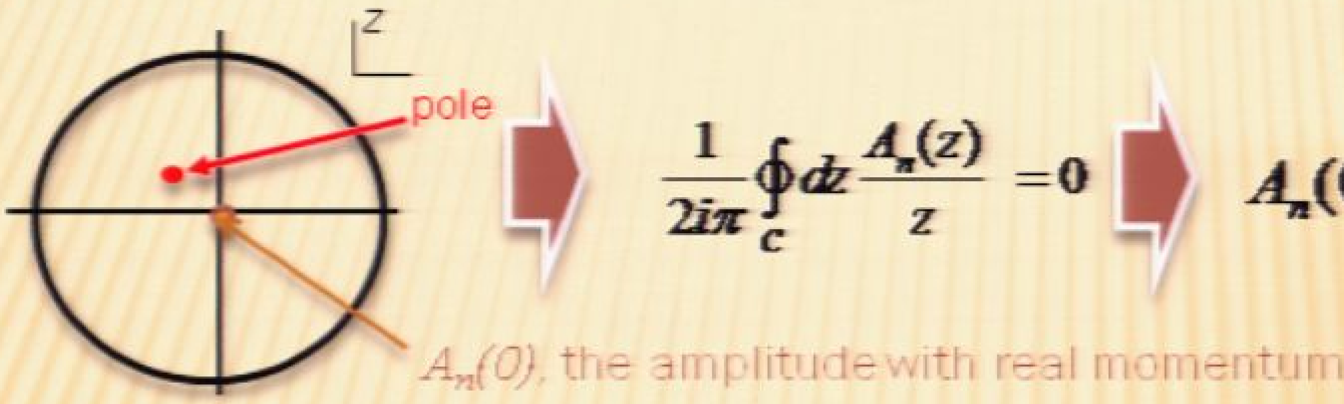

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$$A_n \overset{P^2 \to 0}{=} \sum_{i\in L, j\in R} A_L(\ldots, i, \ldots, P) \frac{1}{P^2} A_R(\ldots, j, \ldots, P)$$

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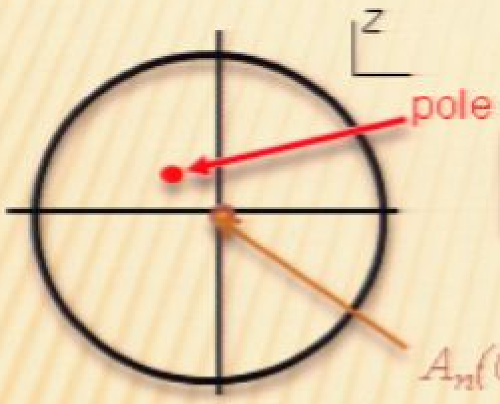

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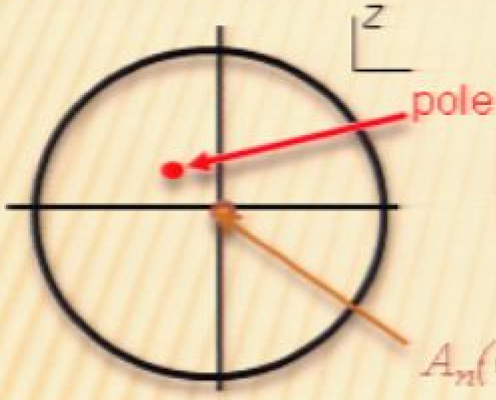

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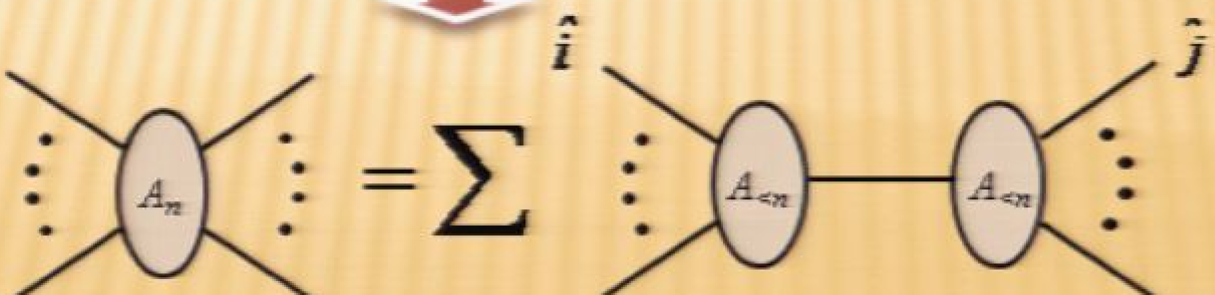
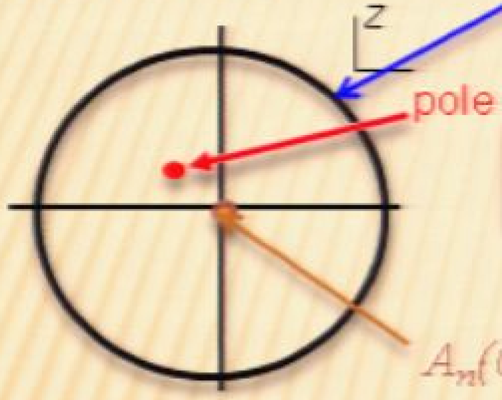


A SIMPLE IDEA

Require that the circle at infinity vanishes.
Vanishing depends upon the theory in question.

- Function of a complex variable containing only simple poles



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On-shell recursion

STRUCTURE OF A 1-LOOP AMPLITUDE

- Analytic form of a 1-loop amplitude

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e.g. $t_{34}\,log(s_{12})$

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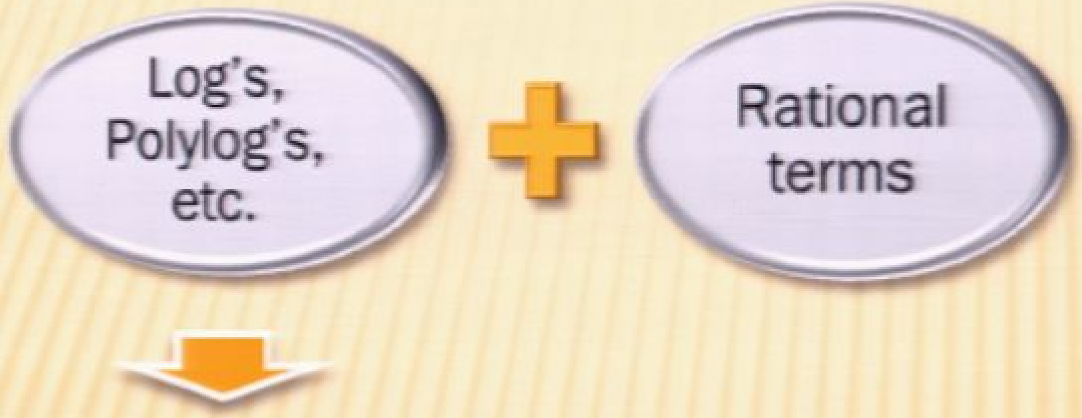

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e.g. $\frac{s_{34}}{s_{12}}$

STRUCTURE OF A 1-LOOP AMPLITUDE

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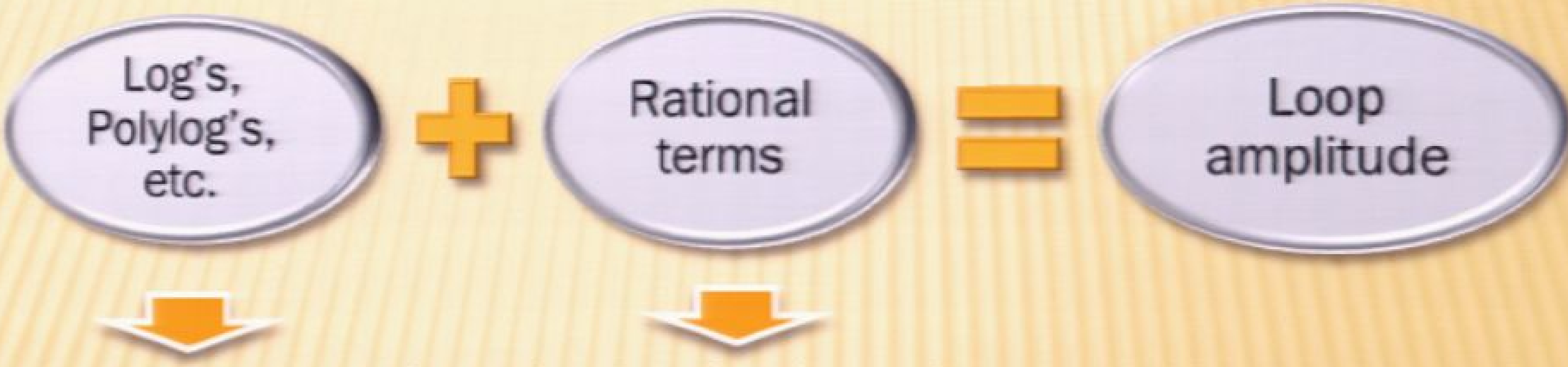

Branch Cuts
e.g. $t_{34}\, log(s_{12})$

Invariants of external momenta
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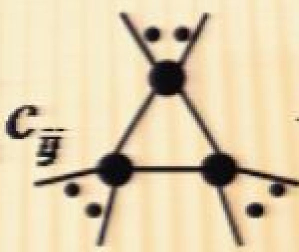
- Want to efficiently compute both pieces.

ONE-LOOP INTEGRAL BASIS

- Cut pieces described by a basis of one-loop integrals

$$R_n + r_\Gamma \frac{\left(\mu^2\right)^{\varepsilon}}{\left(4\pi\right)^{2-\varepsilon}} \left(\sum_i b_{\bar{i}}$$

$$+ \sum_{\bar{i}\bar{j}} c_{\bar{i}\bar{j}}$$

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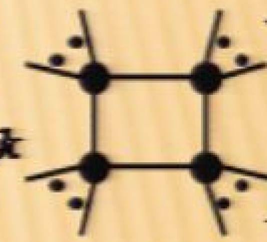
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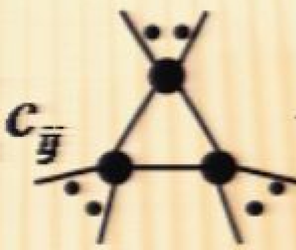
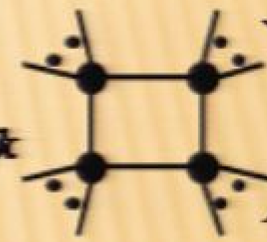
$R_n + r_\Gamma \frac{(\mu^2)^\varepsilon}{(4\pi)^{2-\varepsilon}} \Big(\sum_i b_i$  $+ \sum_{ij} c_{ij}$  $+ \sum_{ijk} d_{ijk}$  $\Big)$

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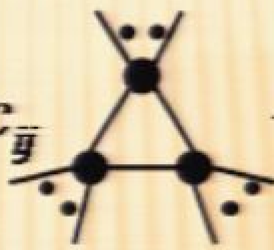

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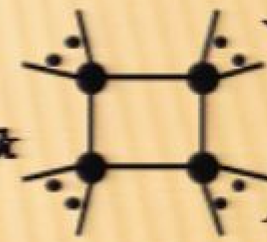

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1-loop scalar integral [Ellis, Zanderighi], [Denner, U. Nierste and R. Scharf], [van Oldenborgh, Vermaseren] + many others.

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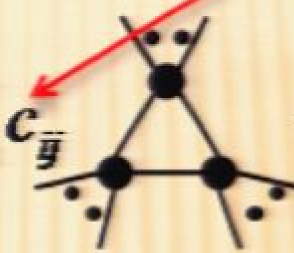
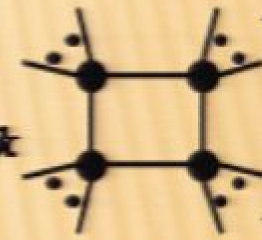
$R_n + r_\Gamma \frac{(\mu^2)^\varepsilon}{(4\pi)^{2-\varepsilon}} \Big(\sum_i b_i$  $+ \sum_{ij} c_{ij}$  $+ \sum_{ijk} d_{ijk}$  $\Big)$

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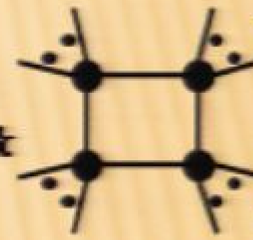
$R_n + r_\Gamma \frac{(\mu^2)^\varepsilon}{(4\pi)^{2-\varepsilon}} \Big(\sum_i b_i$  $+ \sum_{ij} c_{ij}$  $+ \sum_{ijk} d_{ijk}$  $\Big)$

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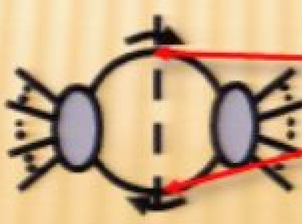

Glue together tree amplitudes

UNITARITY CUTTING TECHNIQUES

- Basic idea. glue together tree amplitudes to form a loop.
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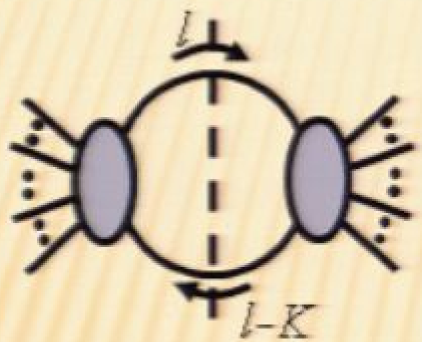

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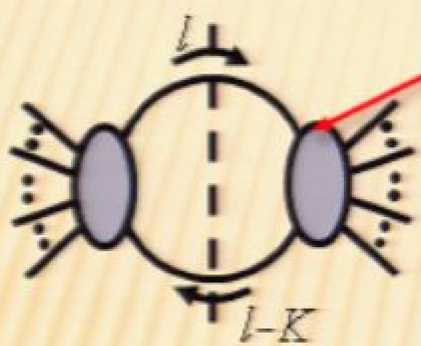

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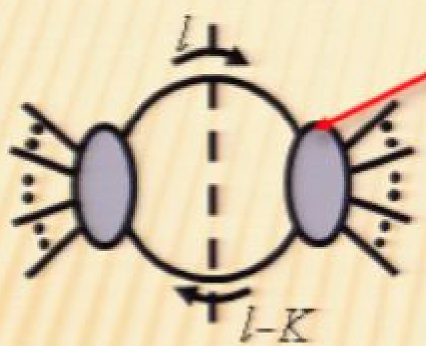

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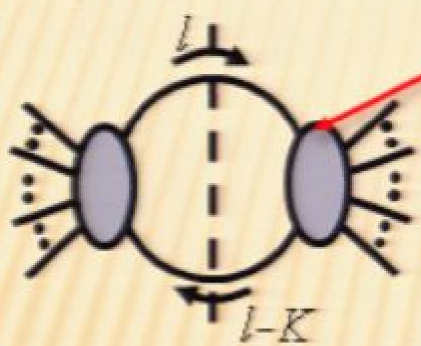

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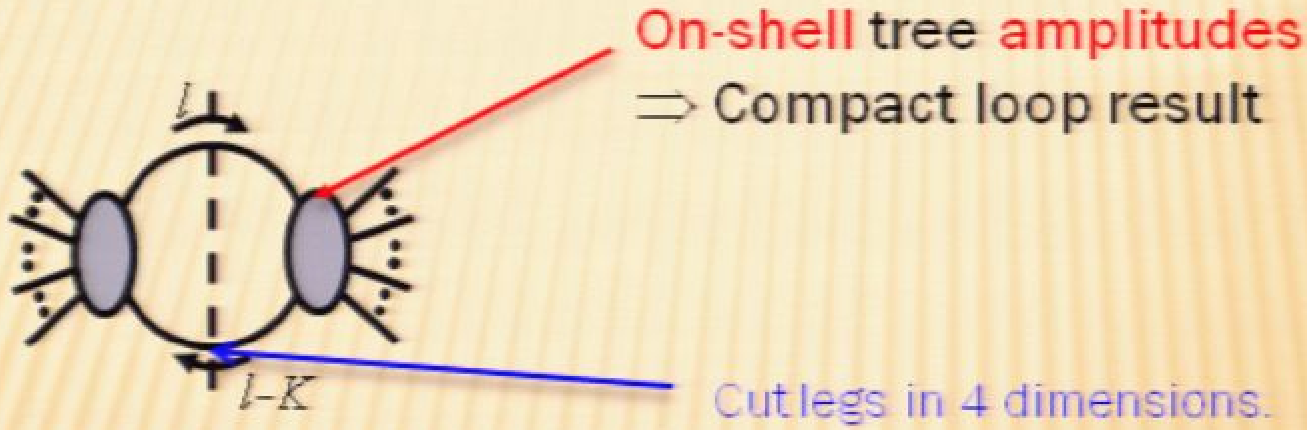

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  + 4 dimensional cuts will miss rational terms.

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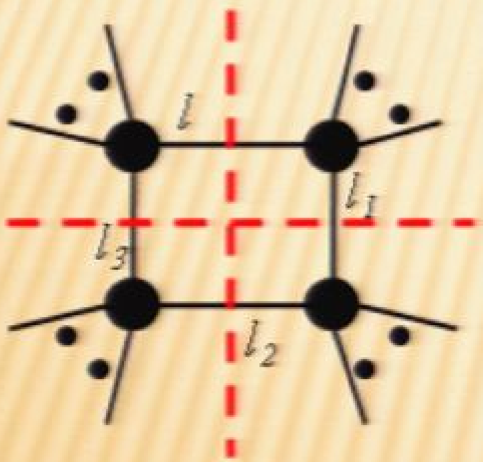

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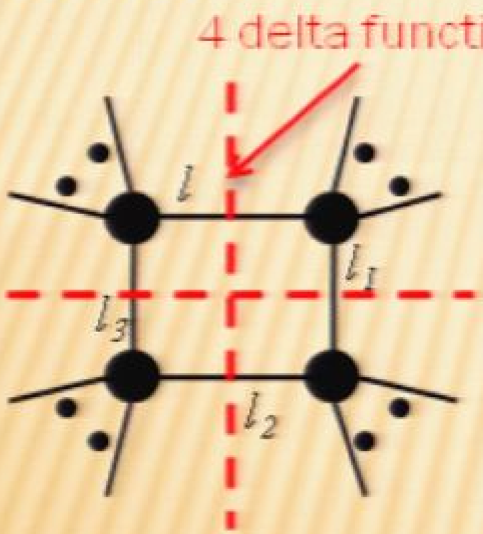

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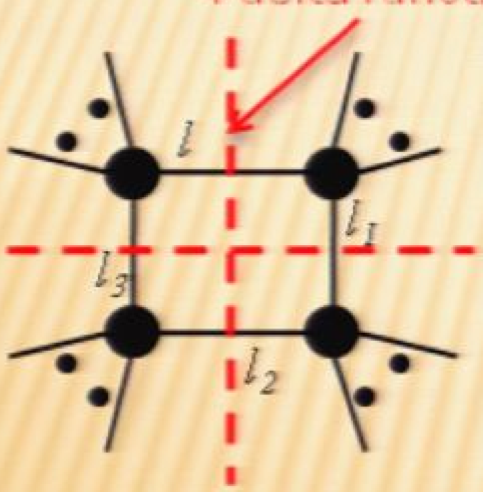

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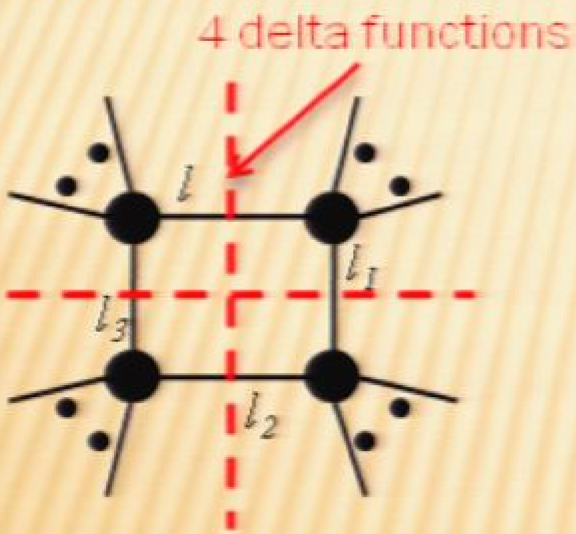

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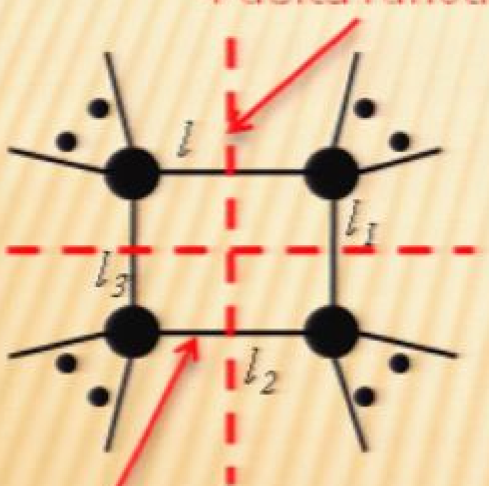
4 delta functions



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In general only solve all constraints with complex l^μ

TWO-PARTICLE AND TRIPLE CUTS

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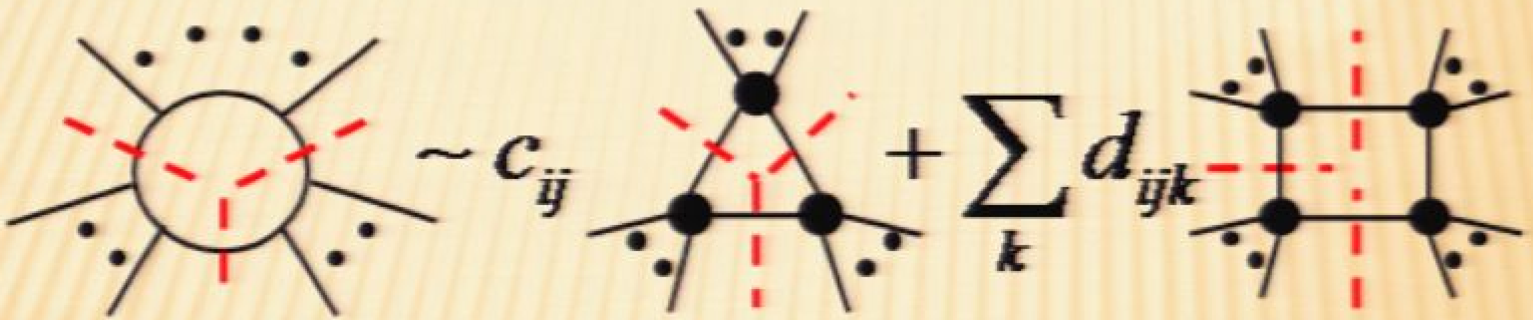

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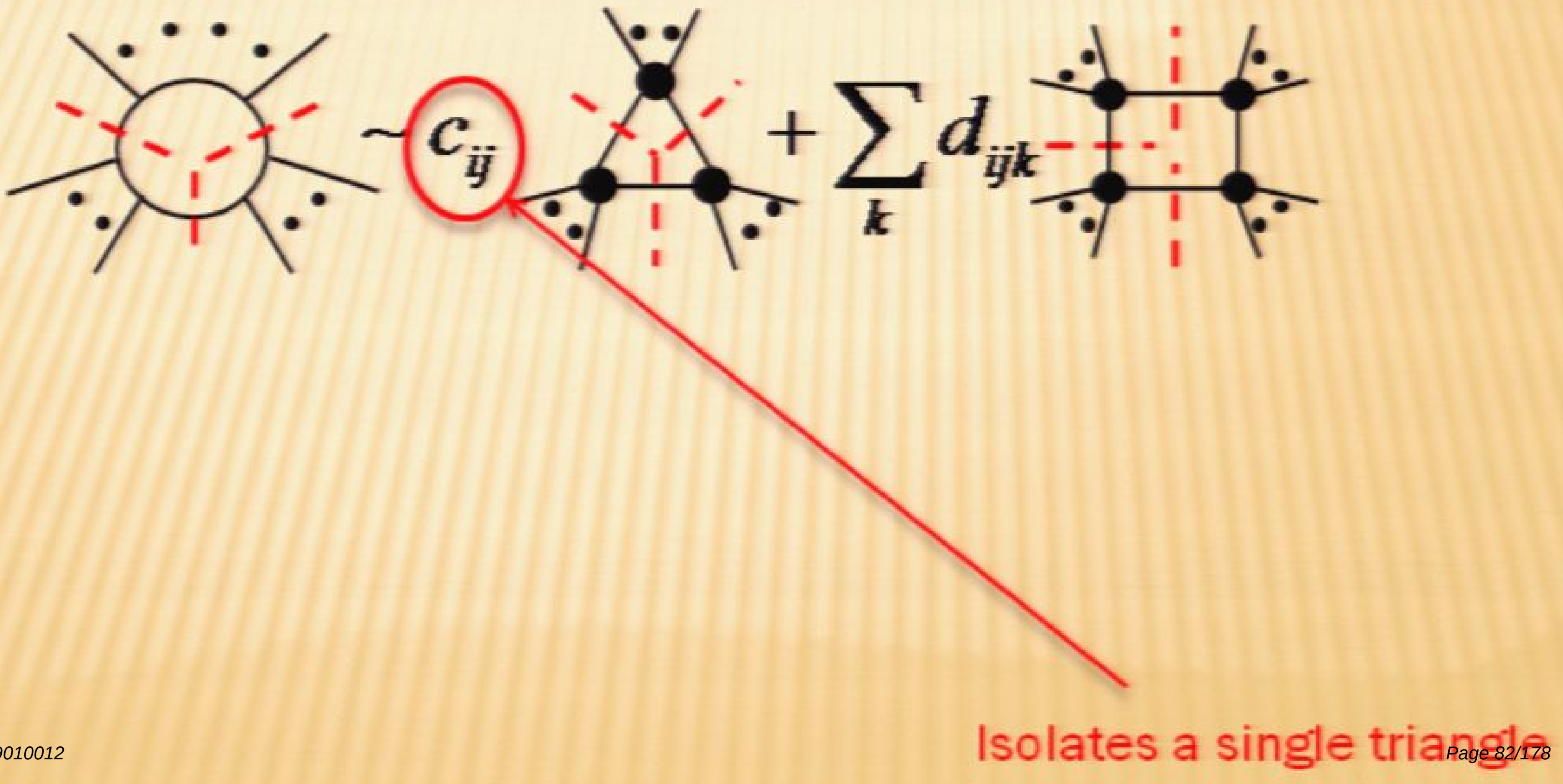

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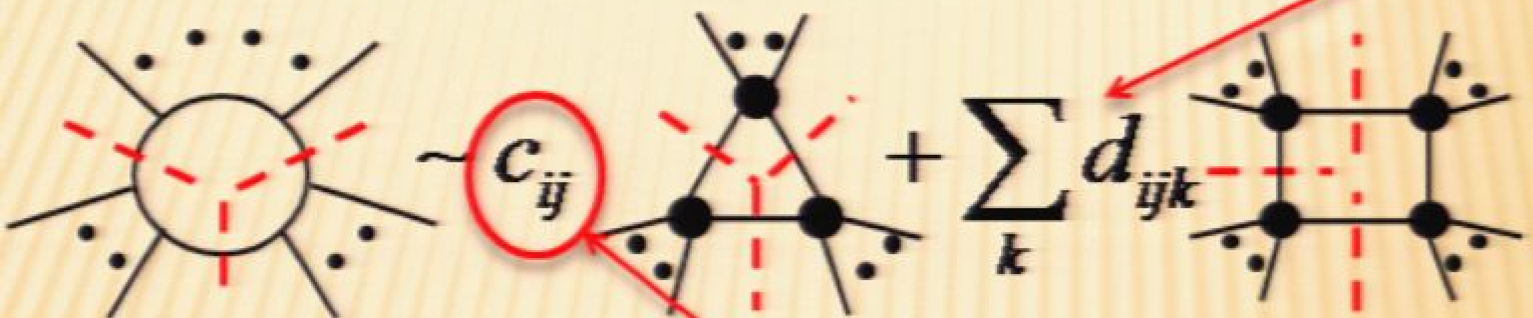

Isolates a single triangle

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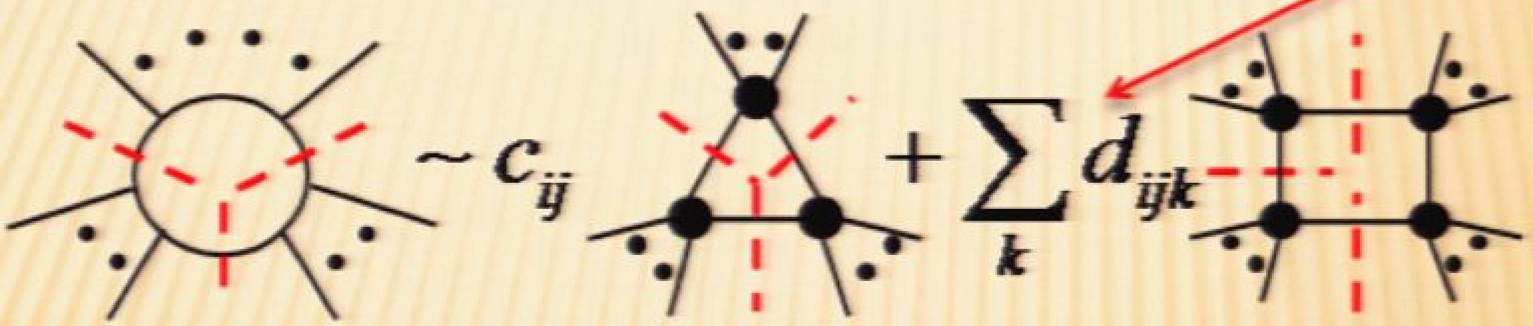

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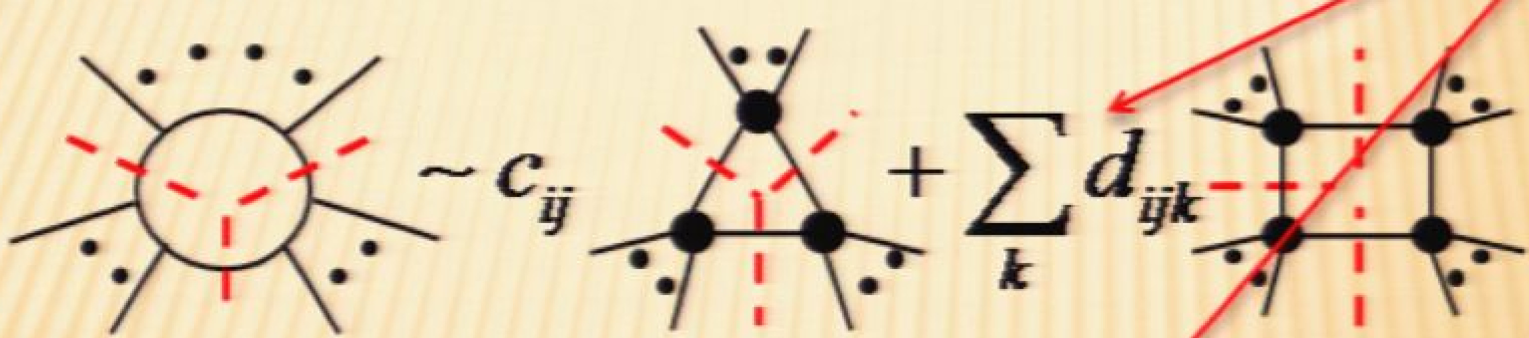

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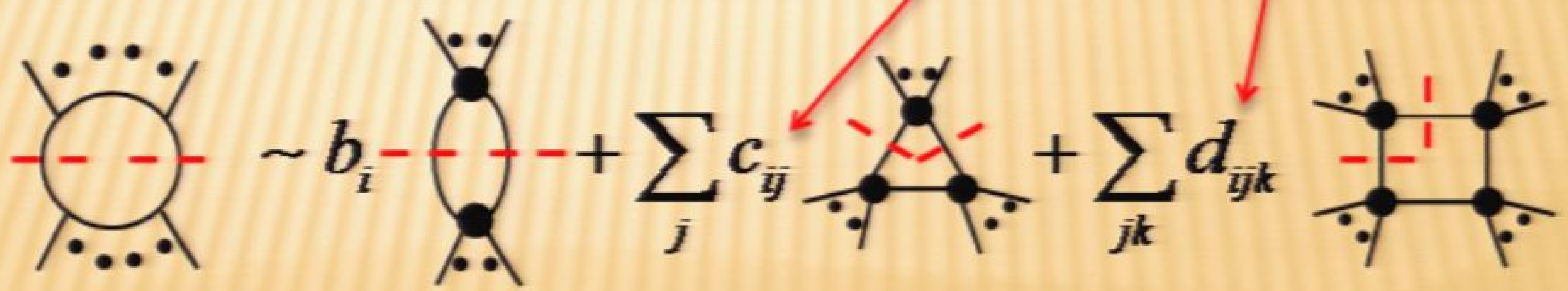

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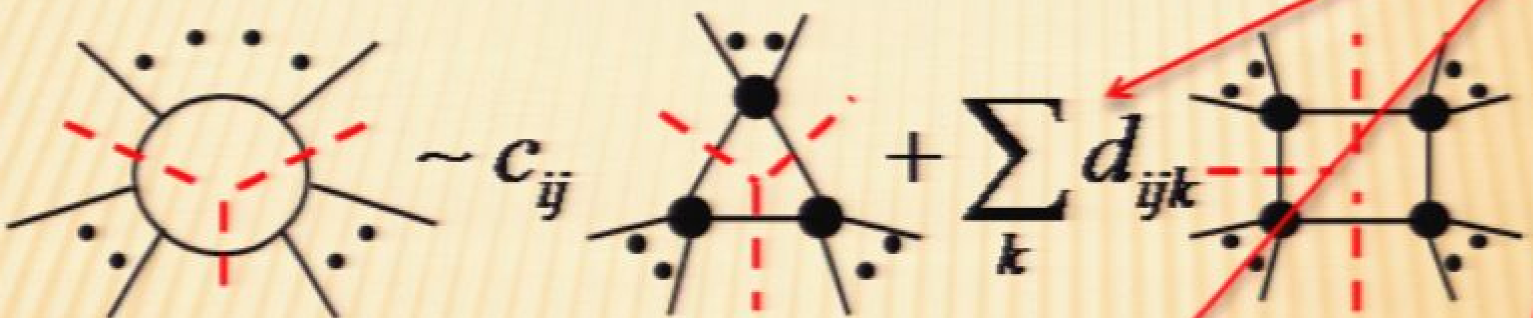
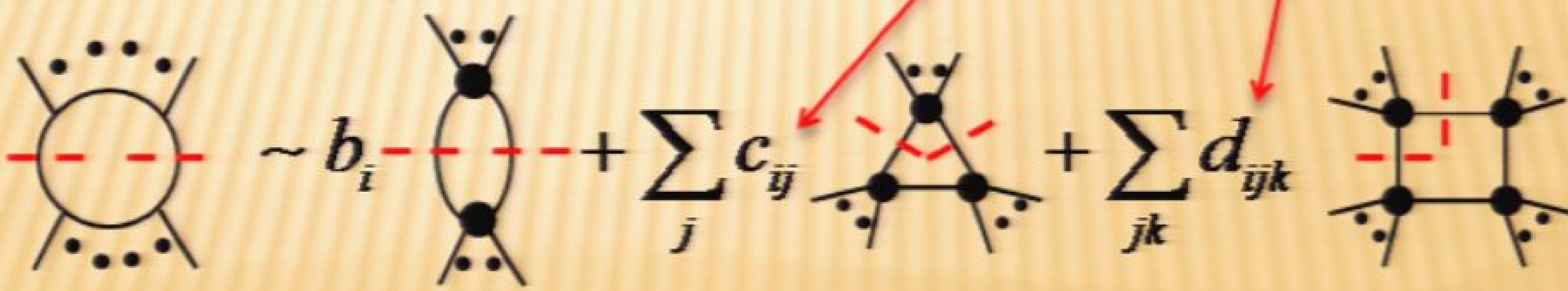


$$\sim c_{ij} \,[\text{triangle}] + \sum_k d_{ijk} \,[\text{box}]$$

- Two-particle cut $\Rightarrow$ Scalar bubble coefficients?



$$\sim b_i \,[\text{bubble}] + \sum_j c_{ij} \,[\text{triangle}] + \sum_{jk} d_{ijk} \,[\text{box}]$$

- Disentangle these coefficients.

DISENTANGLING COEFFICIENTS

- Many approaches.
  - Unitarity technique. [Bern, Dixon, Dunbar, Kosower]
  - MHV vertex techniques. [Bedford, Brandhuber, Spence, Traviglini], [Quigley, Rozali]
  - Unitarity cuts & integration of spinors. [Britto, Cachazo, Feng] + [Mastrolia] + [Anastasiou, Kunszt]
  - Recursion relations. [Bern, Bjerrum-Bohr, Dunbar, Ita]
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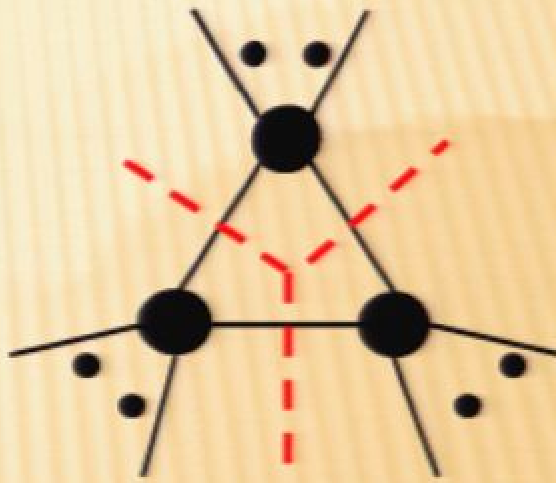
- Large parameter behaviour of the integrand. [DF]

LARGE PARAMETER BEHAVIOUR

- Apply a triple cut to an amplitude



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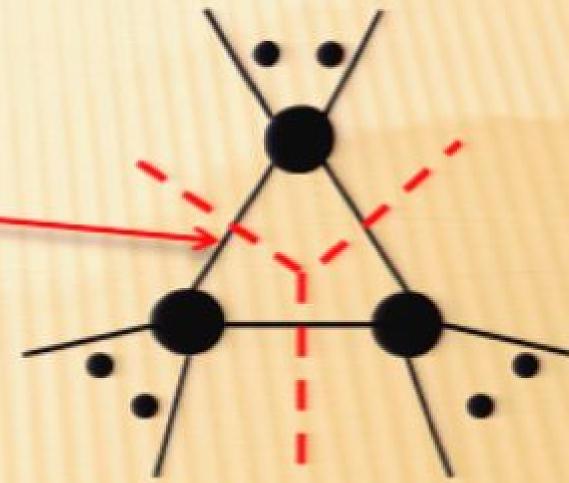
Cut momentum parameterisation

[DF], [del Aguila, Ossola, Papadopoulos, Pittau]

$$l^\mu = K_1^{\flat\mu} + K_2^{\flat\mu} + \frac{t}{2}\langle K_1^{\flat-}|\gamma^\mu|K_2^{\flat-}\rangle + \frac{1}{2t}\langle K_1^{\flat+}|\gamma^\mu|K_2^{\flat+}\rangle$$



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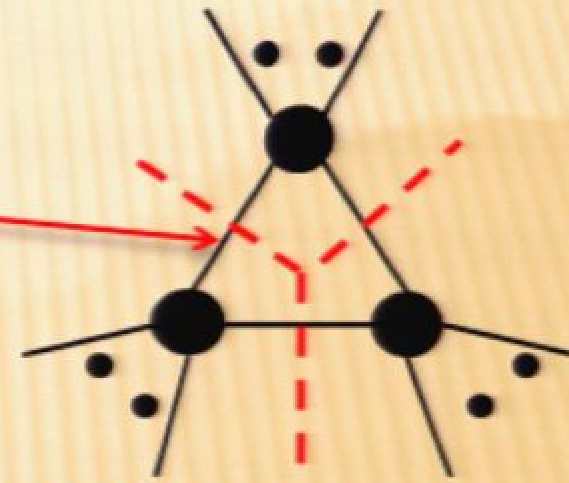

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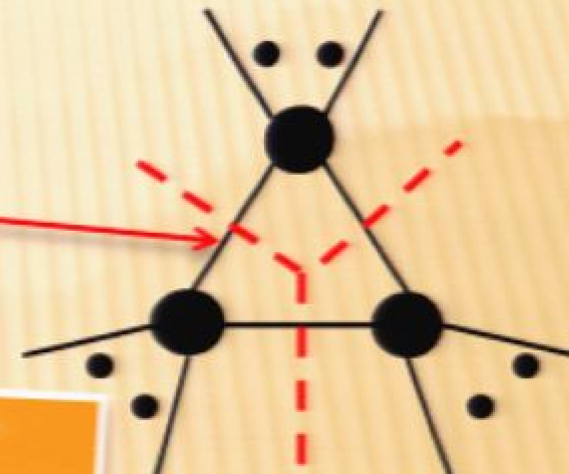

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- Extract large parameter behaviour by series expanding around $t \longrightarrow \infty$.

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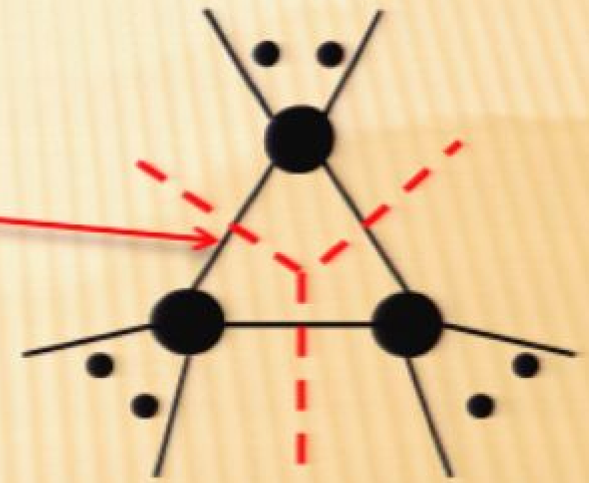

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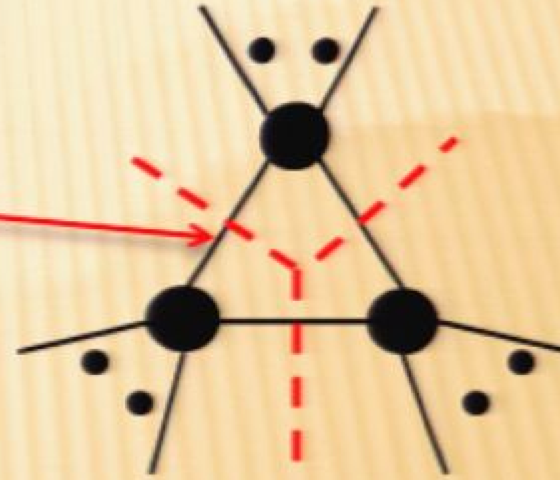

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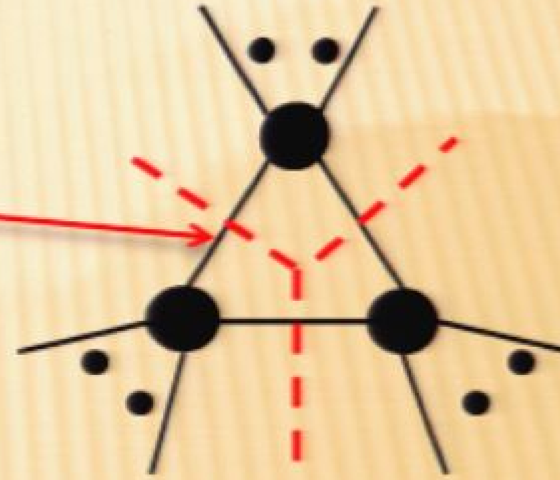

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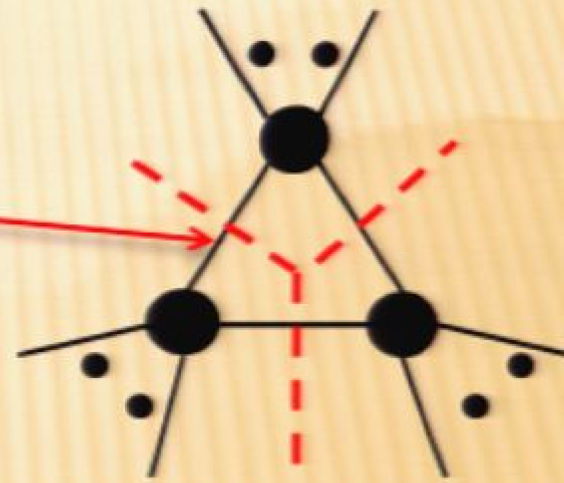

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Triangle Coeff

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TRIANGLE COEFFICIENTS

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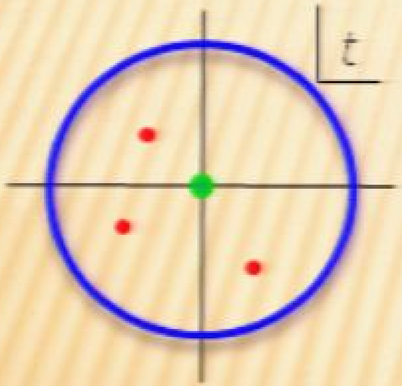

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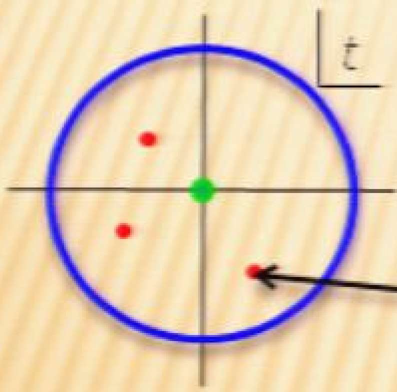

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Pole ⇒ Additional propagator ⇒ Box

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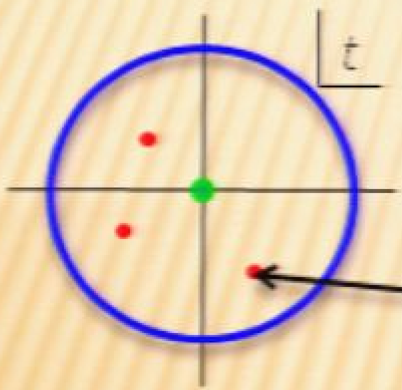

Previously computed from quadruple cut

$$\oint T_3(t) = \oint \frac{C_{-3}}{t^3} + \frac{C_{-2}}{t^2} + \frac{C_{-1}}{t} + C_0 + tC_1 + t^2C_2 + t^3C_3 + \sum_{i,\sigma=\pm} \frac{d_i^\sigma}{\xi_i^\sigma (t - t_i^\sigma)}$$

Pole $\Rightarrow$ Additional propagator $\Rightarrow$ Box

- Subtract box terms. [Berger, Bern, Dixon, Febres Cordero, DF, Ita, Kosower, Maître]

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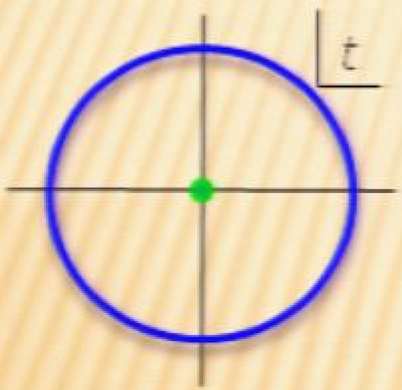

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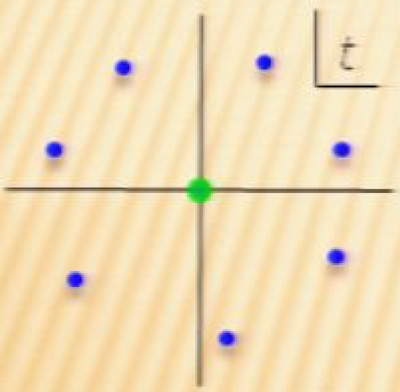

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$$C_0 = \frac{1}{2p+1} \sum_{j=-p}^{p} T_3\left(t_0 e^{2\pi i j/(2p+1)}\right)$$

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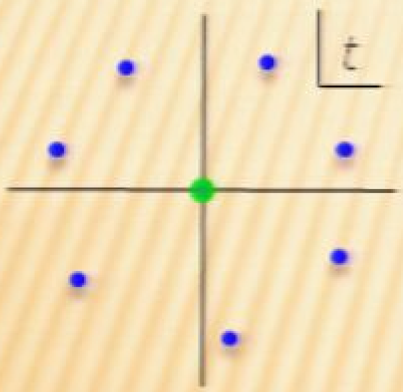

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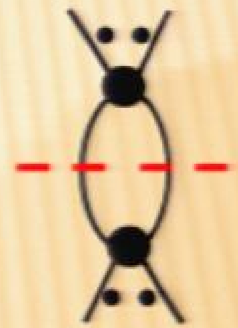
- Construct a two-particle cut.



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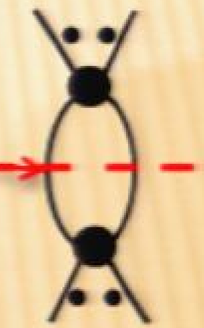

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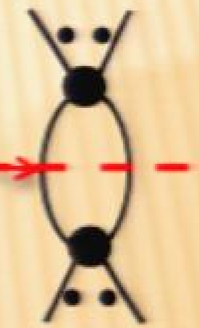

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- Integrals over t vanish.

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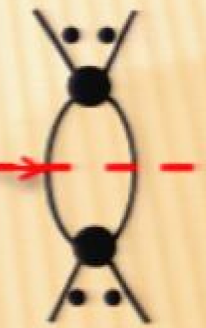

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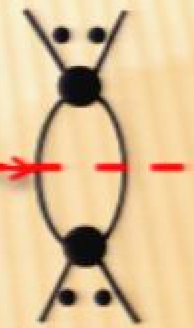

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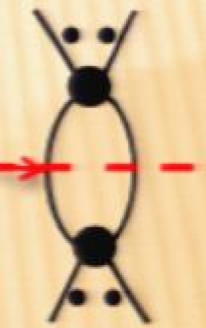

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The bubble coefficient?

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$$\int B_2(y,t) = \int b_0 + \frac{b_1}{2} + \frac{b_2}{3}$$

The bubble coefficient?

$$+ \sum_i \frac{A_1(y,t)A_2(y,t)A_3(y,t)}{\left(l(y,t)-k_i\right)^2}$$

Does not vanish when t and y are taken to ∞

- Integrals over t vanish.
- Integrals over y do not vanish. can show. $\int dy\, y^i = \frac{1}{i+1}\int dy$

"TRIPLE CUT CONTRIBUTIONS"

- Reuse the previously computed triangle and box information

$$\frac{A_1(y,t)A_2(y,t)A_3(y,t)}{\left(l(y,t)-K\right)^2}=\frac{C_{-3}t_{IP}^3+C_{-2}t_{IP}^2+C_{-1}t_{IP}+C_0+C_1t_P+C_2t_P^2+C_3t_P^3}{\left(l(y,t)-K\right)^2}$$

"TRIPLE CUT CONTRIBUTIONS"

- Reuse the previously computed triangle and box information

$$\frac{A_1(y,t)A_2(y,t)A_3(y,t)}{\left(l(y,t)-K\right)^2} = \frac{C_{-3}t_{IP}^3 + C_{-2}t_{IP}^2 + C_{-1}t_{IP} + C_0 + C_1 t_P + C_2 t_P^2 + C_3 t_P^3}{\left(l(y,t)-K\right)^2}$$

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Previously computed triangle coefficients

Change variables from *t* of the triangle parameterisation to that in the bubble.

$$t_P=\frac{1}{\gamma}\left(y\left\langle K_2^{\flat-}\middle|\bar{K}_1\middle|K_1^{\flat-}\right\rangle+(1-y)\left\langle K_2^{\flat-}\middle|\chi\middle|K_1^{\flat-}\right\rangle+\frac{t}{2}\left\langle\bar{K}_1K_2^{\flat}\right\rangle\left[K_1^{\flat}\chi\right]+\frac{y(1-y)}{2t}\left\langle\chi K_2^{\flat}\right\rangle\left[K_1^{\flat}\bar{K}_1\right]\right)$$

$$t_{\bar{P}}=\frac{1}{\gamma}\left(y\left\langle K_1^{\flat-}\middle|\bar{K}_1\middle|K_2^{\flat-}\right\rangle+(1-y)\left\langle K_1^{\flat-}\middle|\chi\middle|K_2^{\flat-}\right\rangle+\frac{t}{2}\left\langle\bar{K}_1K_1^{\flat}\right\rangle\left[K_2^{\flat}\chi\right]+\frac{y(1-y)}{2t}\left\langle\chi K_1^{\flat}\right\rangle\left[K_2^{\flat}\bar{K}_1\right]\right)$$

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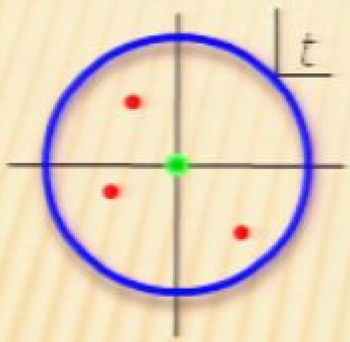
- Compute general form for the ∞ limit
  - Combine with two-particle cut terms for the complete result.

BUBBLE COEFFICIENTS NUMERICALLY

- Two free parameters y and t in the integrand B_2.
  - Contour integral in terms of t (with $y \in [0,1]$) now contains poles from triangle & box propagators.



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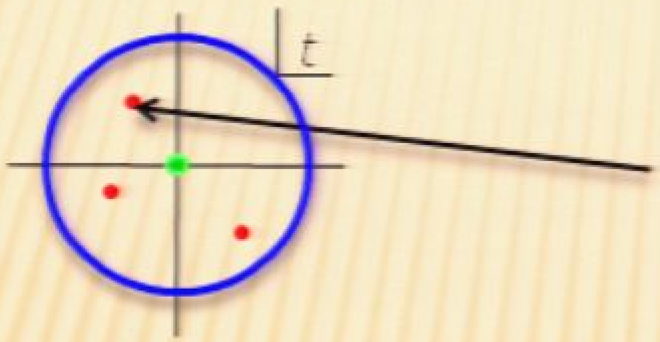

Pole $\Rightarrow$ Additional propagator
$\Rightarrow$ Triangle/Box

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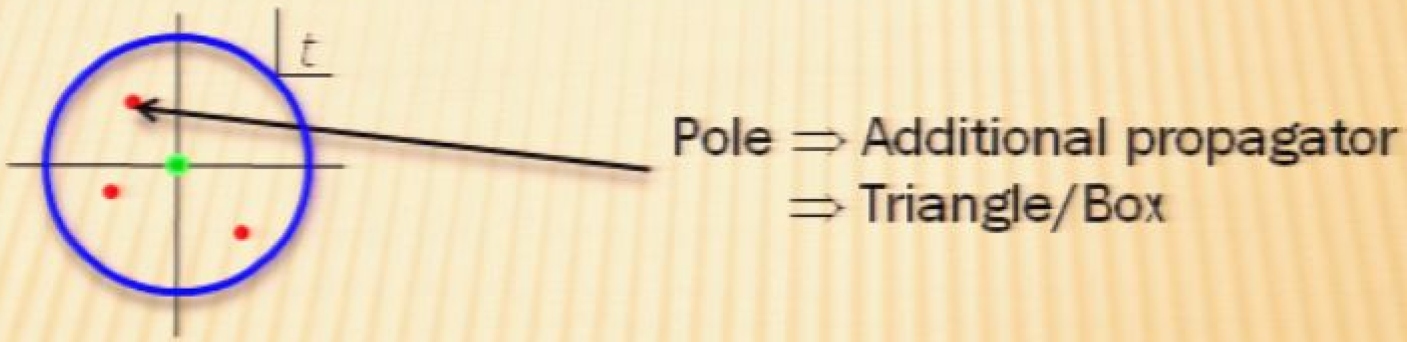

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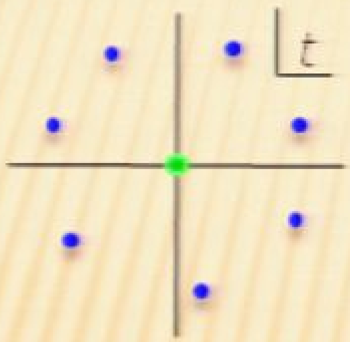

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$$B_0 = \frac{1}{20}\sum_{j=0}^{4}\left[B_2(0, t_0 e^{2\pi i j/5}) + 3B_2(2/3, t_0 e^{2\pi i j/5})\right]$$

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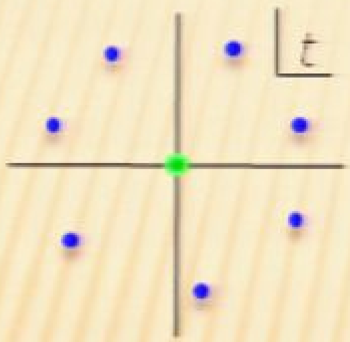

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Only need to evaluate at 2 points in y

SCALING OF TREES IN GRAVITY

- From standard power counting would naively expect triangle and bubble power counting to scale as the number of external legs
  - e.g. for a triangle $$\int \frac{d^{4-2\epsilon} l}{(2\pi)^{4-\epsilon}} \frac{\left(l^\mu l^\nu\right)^{n+3}}{l^2 (l-p_1)^2 (l-p_2)^2}$$

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- Related to the scaling of t and y in tree level amplitudes.
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- Maximum scaling of triangle is only $t^6=(l^\mu)^6$, much better than expected.

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- Complete "no triangle" hypothesis proven using a related technique. [Arkani-Hamed, Cachazo, Kaplan] (first proved using an unrelated technique by [Bjerrum-Bohr, Vanhove])

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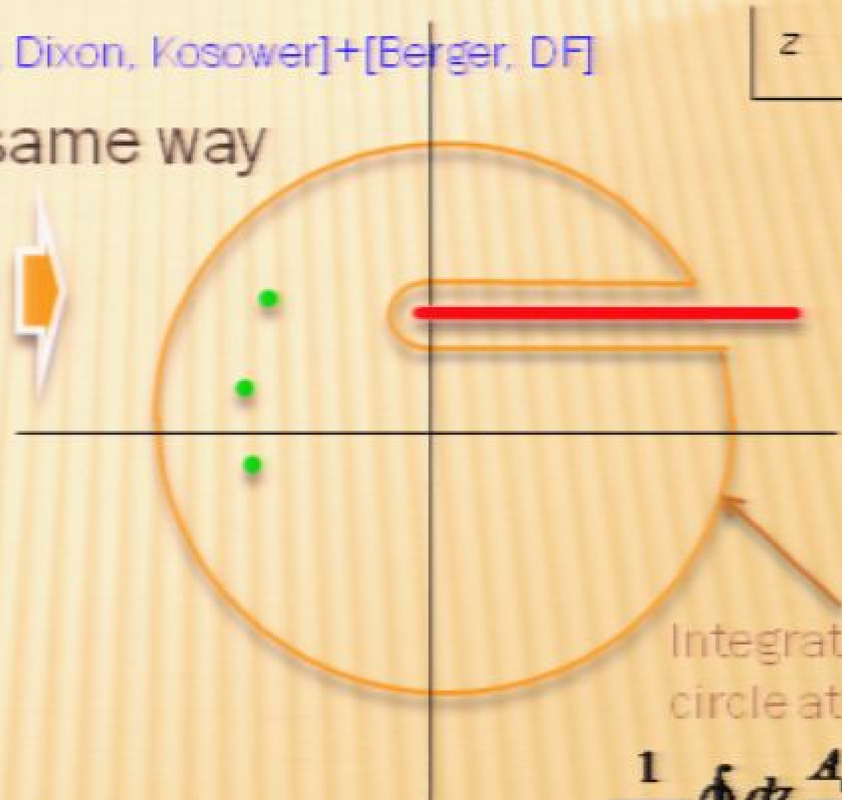

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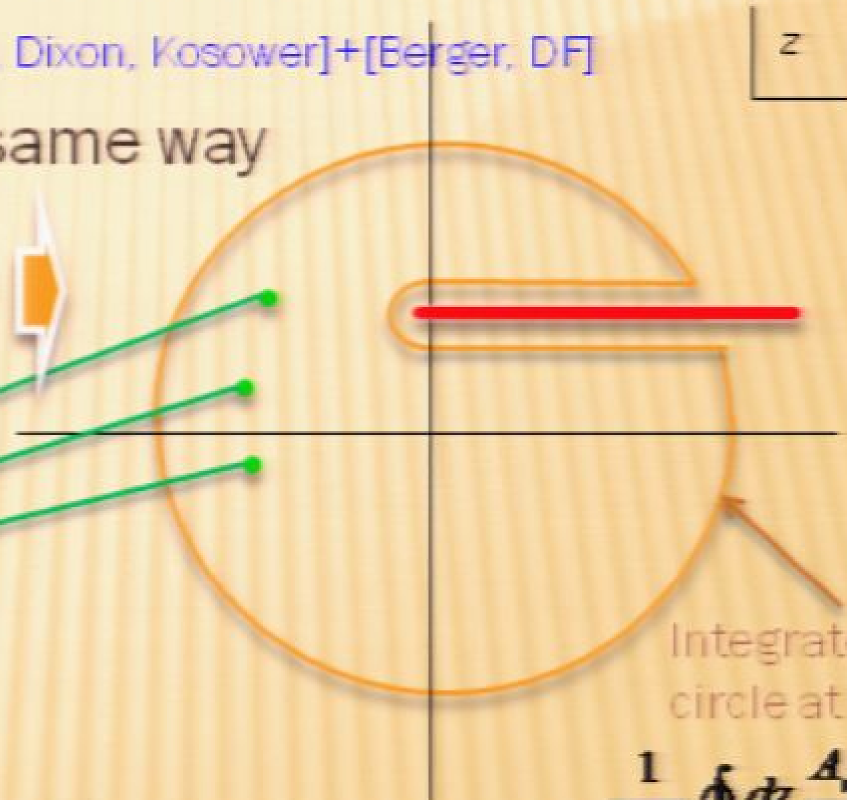

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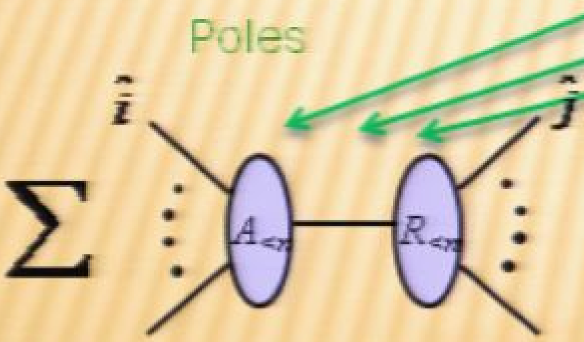

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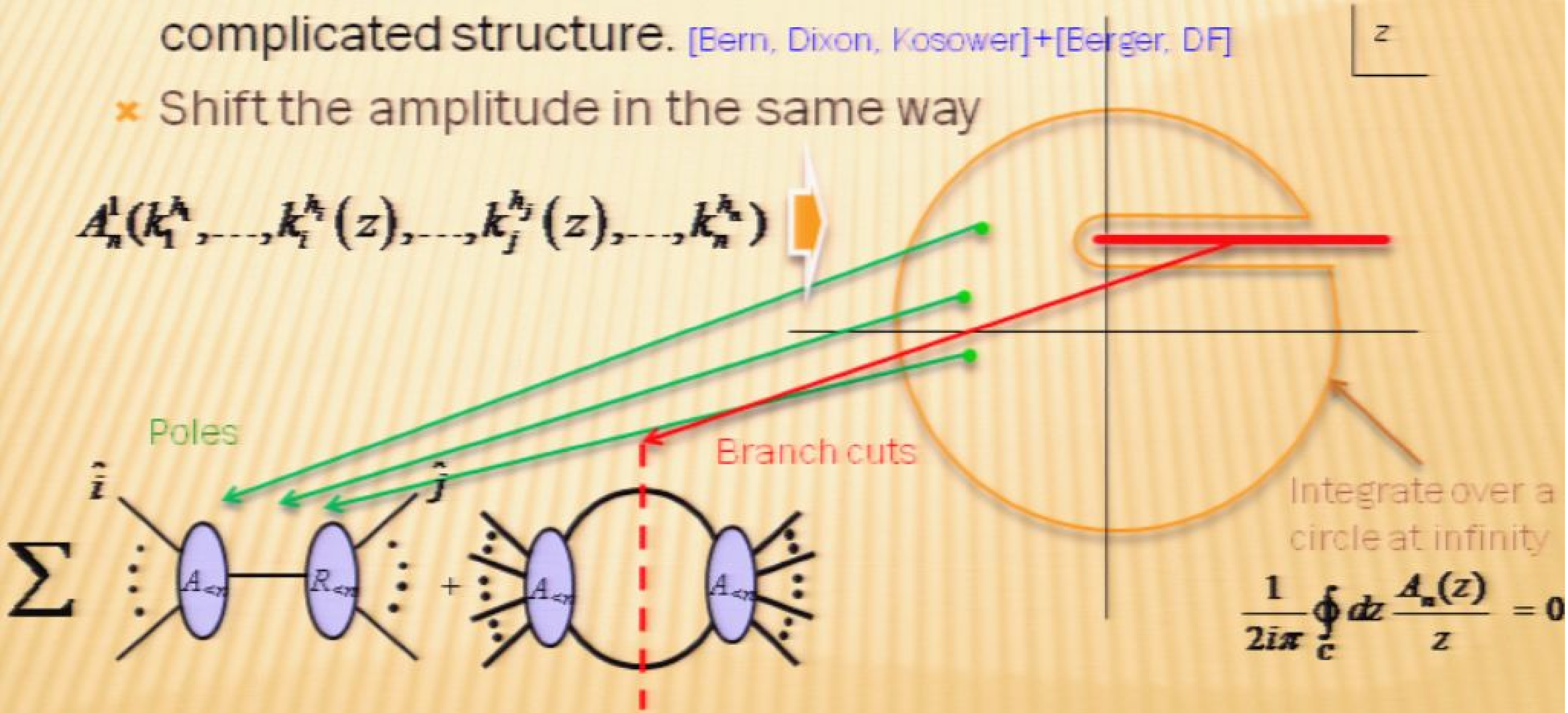

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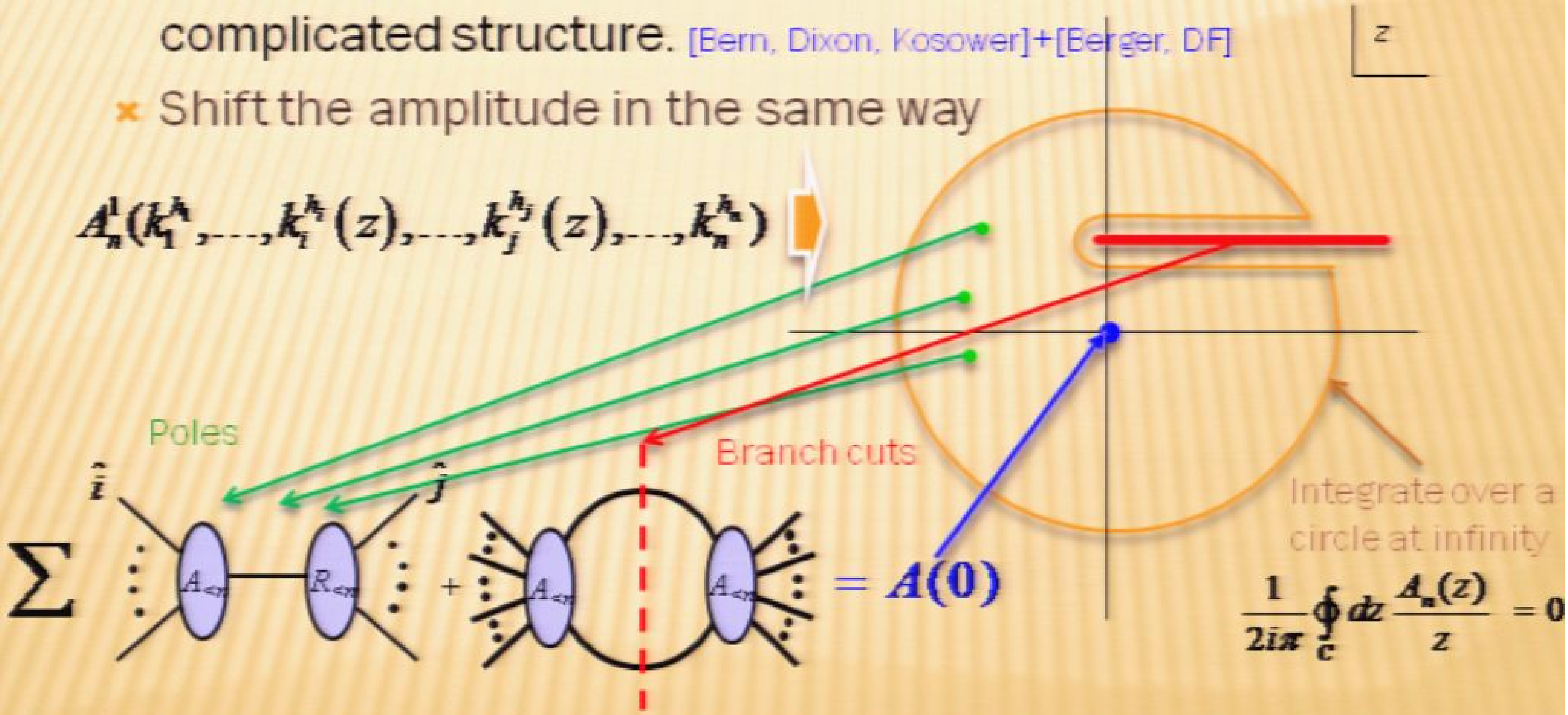

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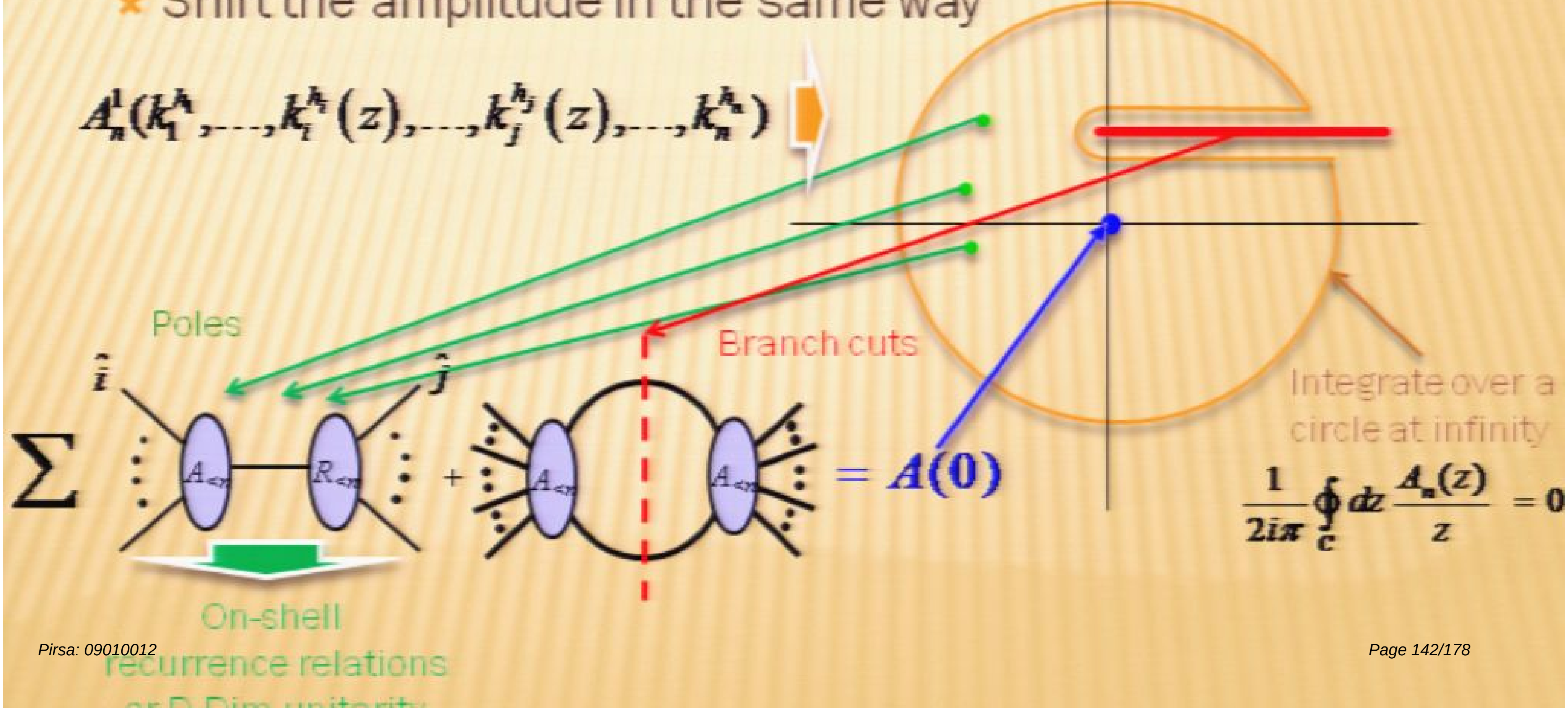

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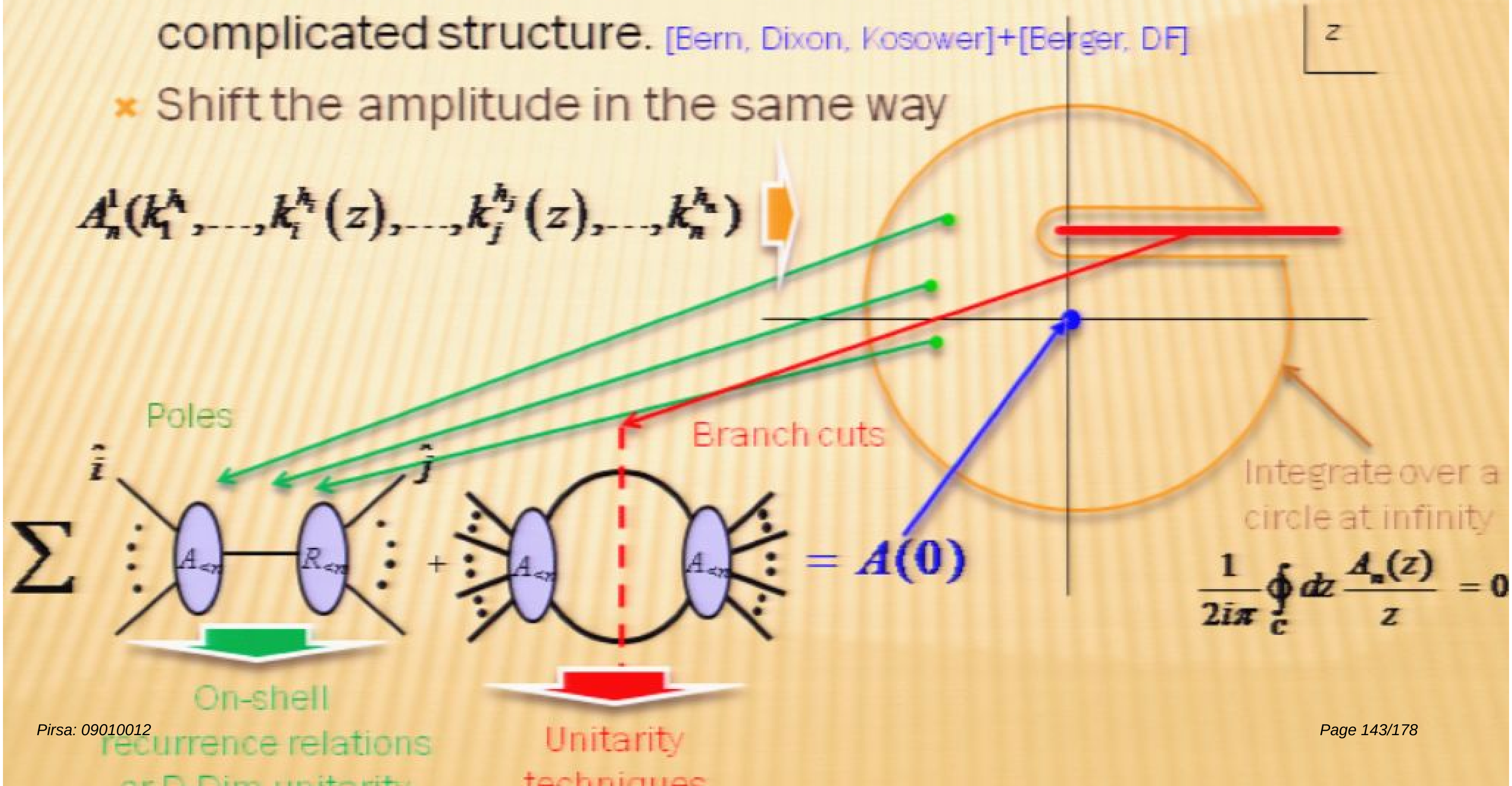

$$\sum \ldots + \ldots = A(0)$$

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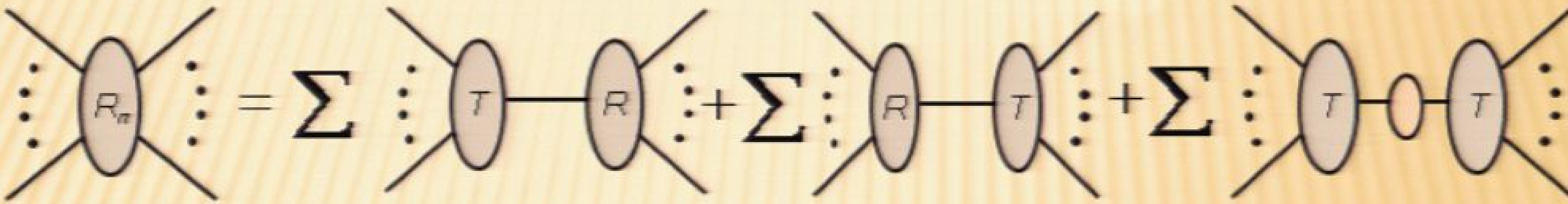
LOOP ON-SHELL RECURSION RELATIONS

- At one-loop recursion using on-shell tree amplitudes, T, and rational pieces of one-loop amplitudes, R.



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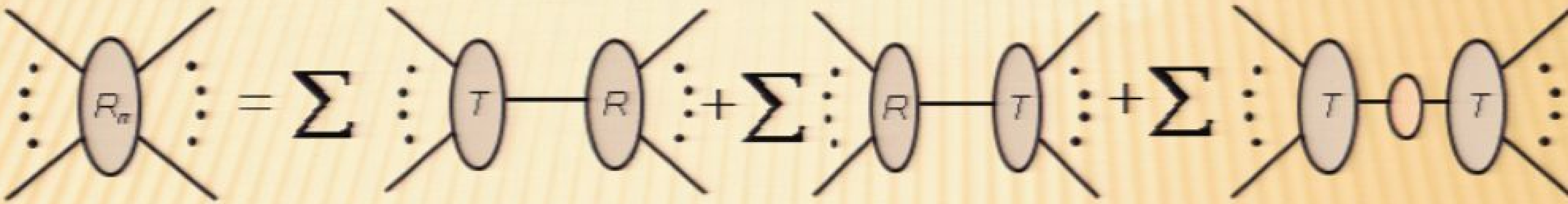

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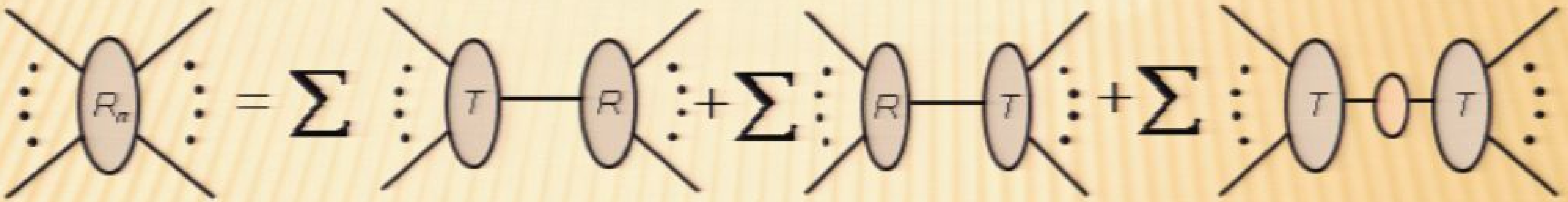

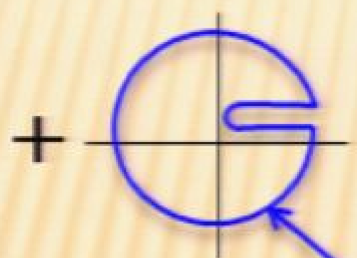

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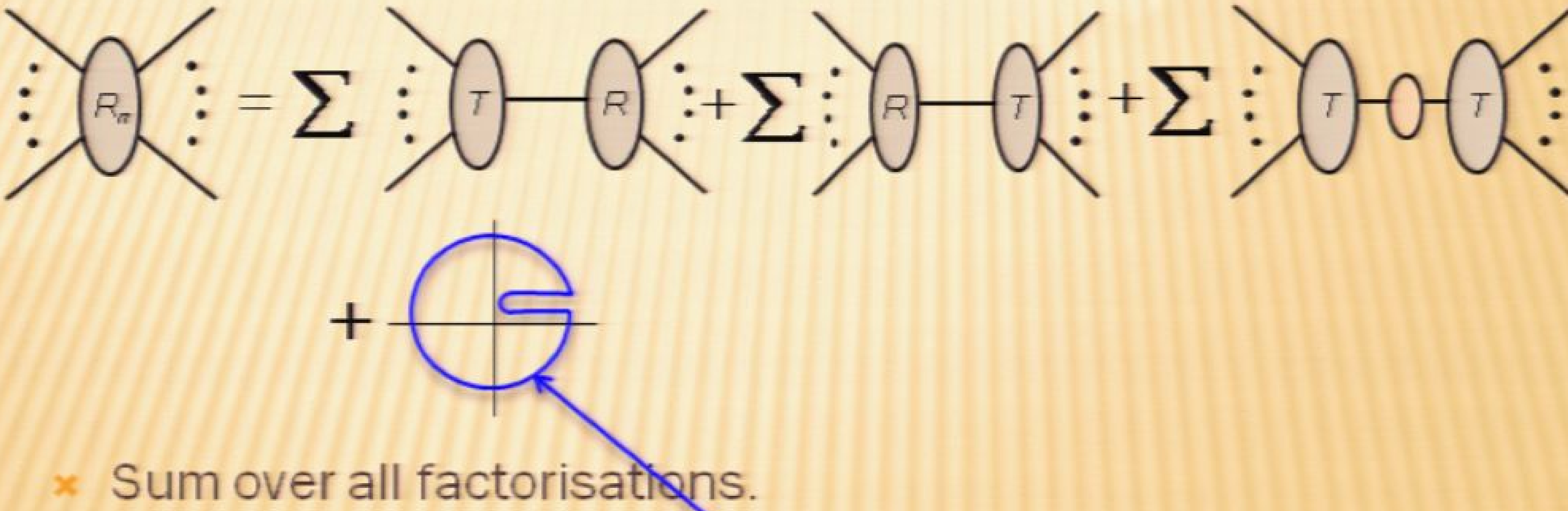
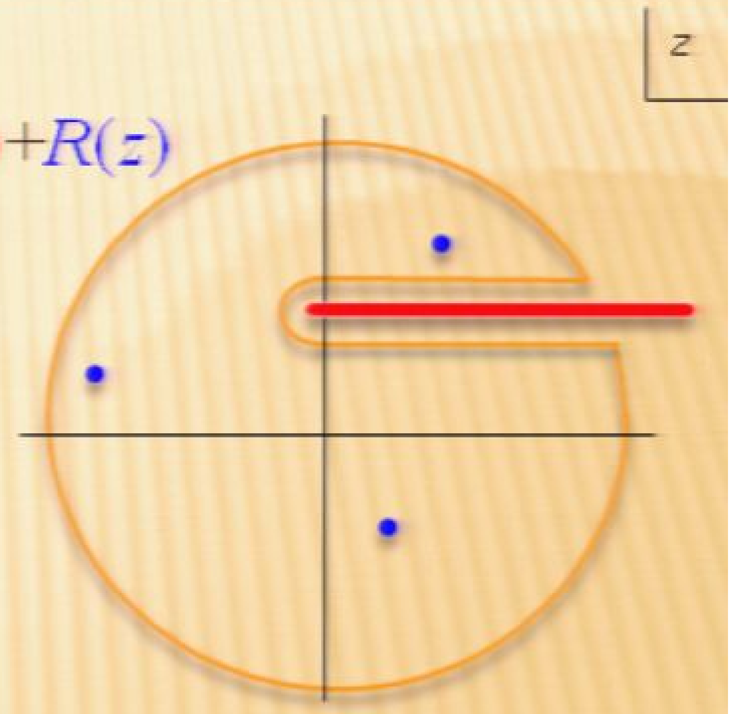


- Sum over all factorisations.
- Contribution from the boundary, compute using a second recursion (it is purely rational).
- Not the complete rational result, missing "Spurious" poles.

SPURIOUS POLES

- Shifting the amplitude by $z \Rightarrow A(z) = C(z) + R(z)$



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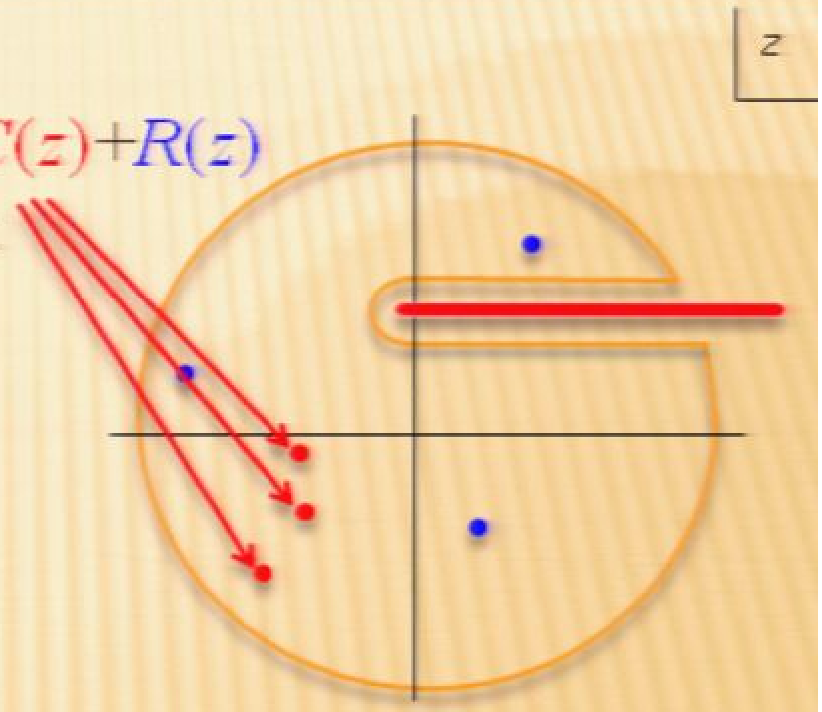
$$bI_2 = \frac{\tilde{b}}{K_1^2 - K_2^2}\ln(-K_1^2) \rightarrow \frac{\tilde{b}}{K_1^2 - K_2^2 - zY}\ln(-K_1^2 - z\tilde{Y})$$



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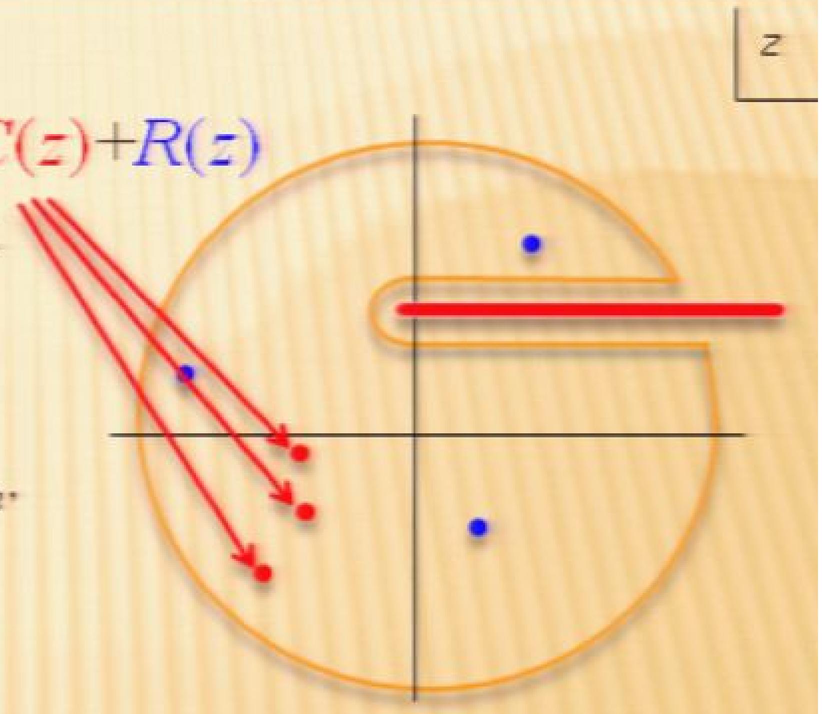

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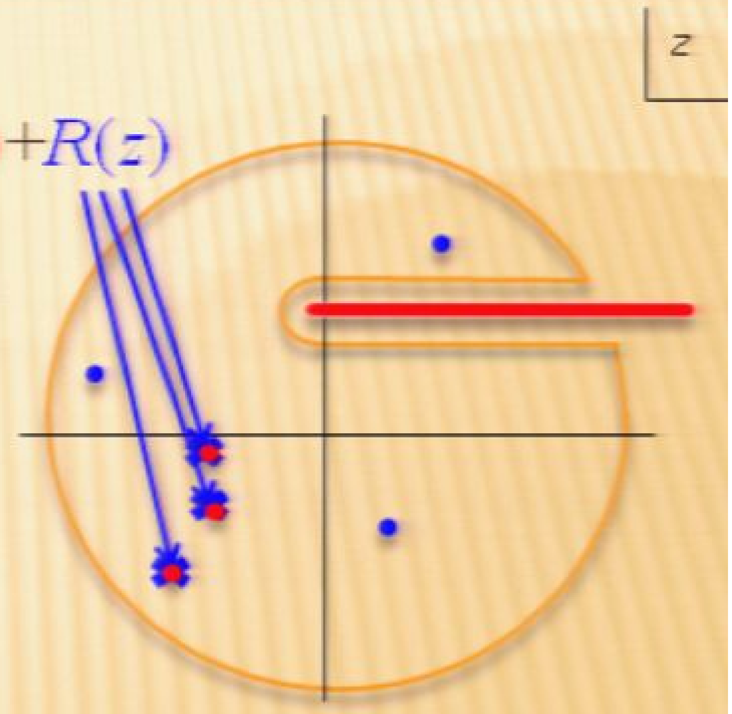

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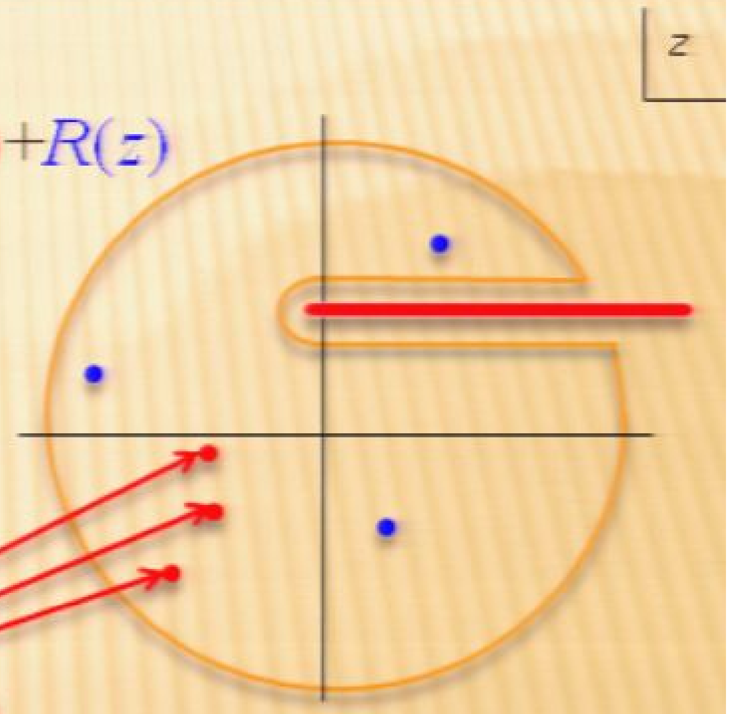

$$-\sum \mathrm{Res}_{z=z_s} \frac{C(z)}{z}$$

- Cancel against poles in the rational part.
- Compute spurious poles from residues of the cut terms.
  - Poles located at the vanishing of shifted Gram determinants of boxes and triangles

D-DIMENSIONAL UNITARITY

- l^μ in dimensionally regulated integral in $D=4-2\varepsilon$ dimensions.
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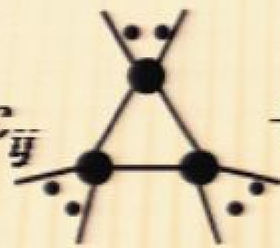
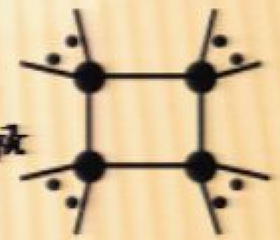
$R_n + \sum_i b_i$  $+ \sum_{ij} c_{ij}$  $+ \sum_{ijk} d_{ijk}$ 

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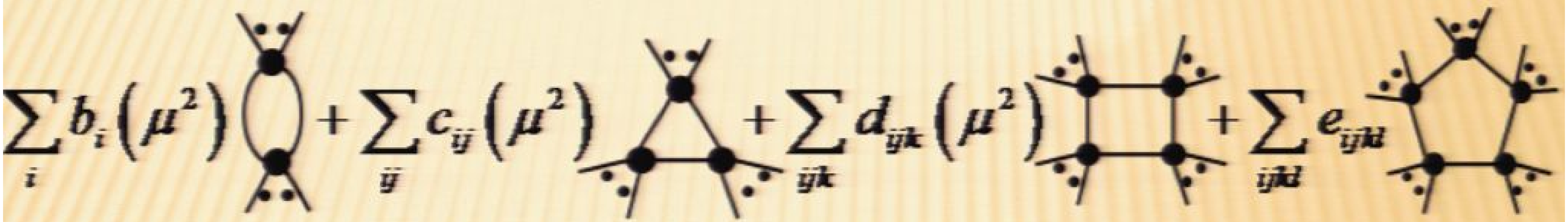

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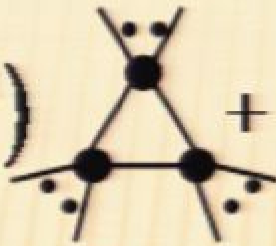
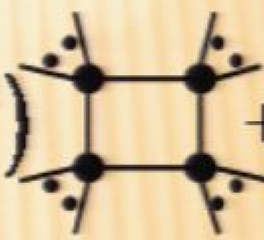
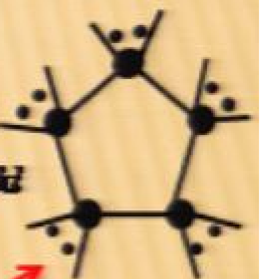
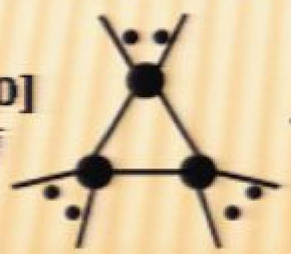
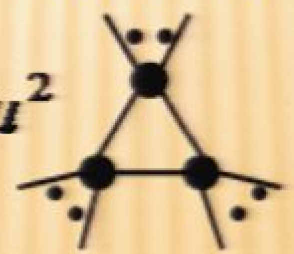
$\sum_i b_i(\mu^2)$  $+\sum_{ij} c_{ij}(\mu^2)$  $+\sum_{ijk} d_{ijk}(\mu^2)$  $+\sum_{ijkl} e_{ijkl}$ 

Pentagons in more than 4 dimensions

- Relate the μ^2 dependant parts of the coefficients to rational terms

[Giele, Kunszt, Melnikov][Ossola, Papadopoulos, Pittau] [Badger]

$c_{ij}(\mu^2)$  $= c_{ij}^{[0]}$  $+ c_{ij}^{[1]}\mu^2$ 

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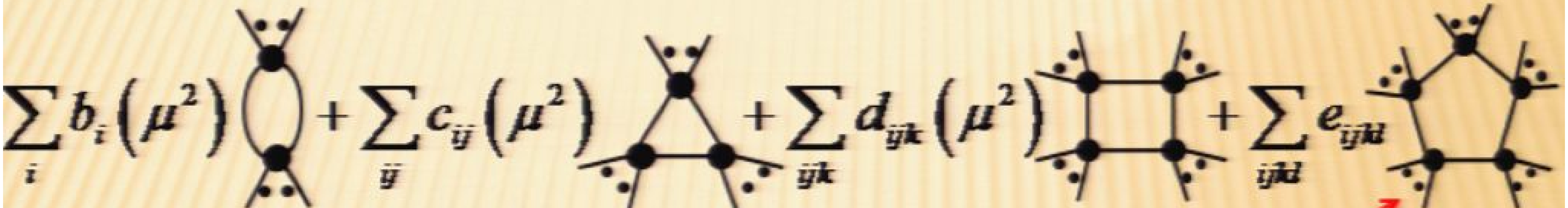

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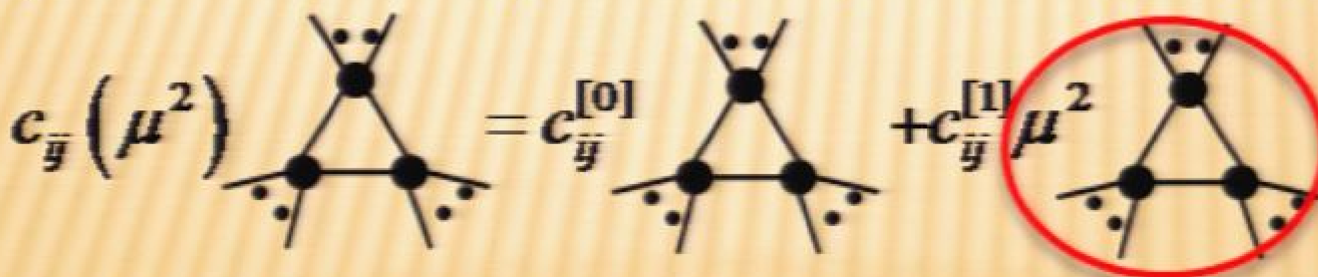

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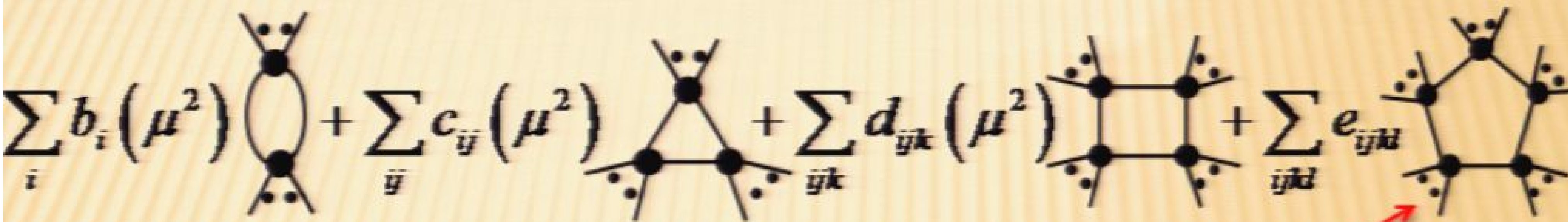

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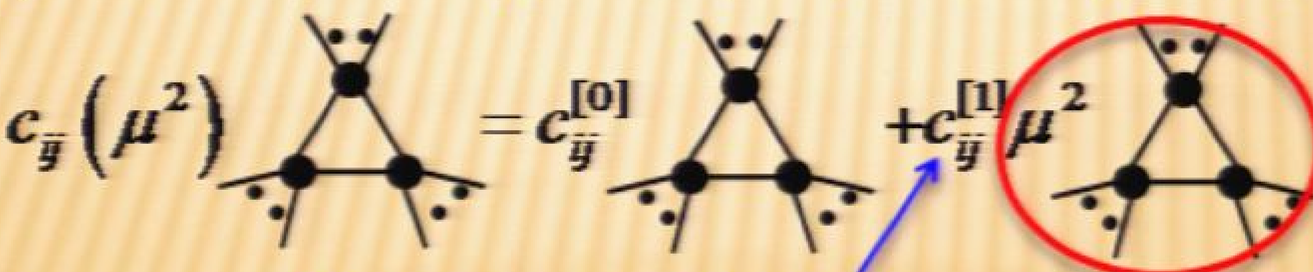

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- Extract additional coefficients proportional to μ^2. in the same way as the cut terms.

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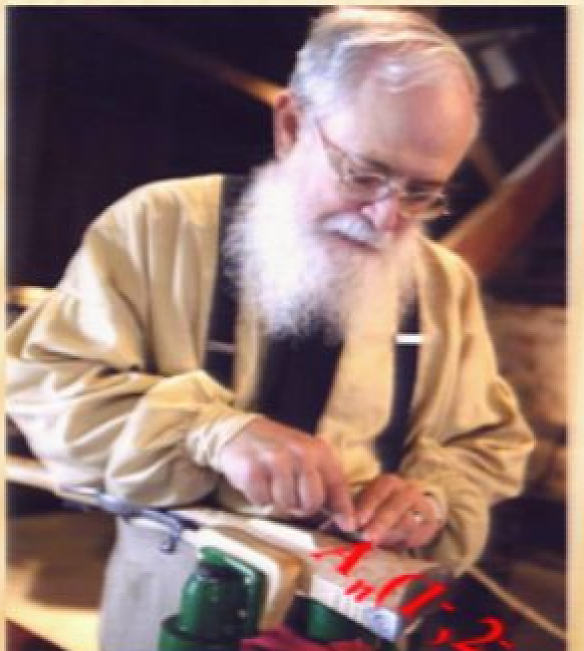

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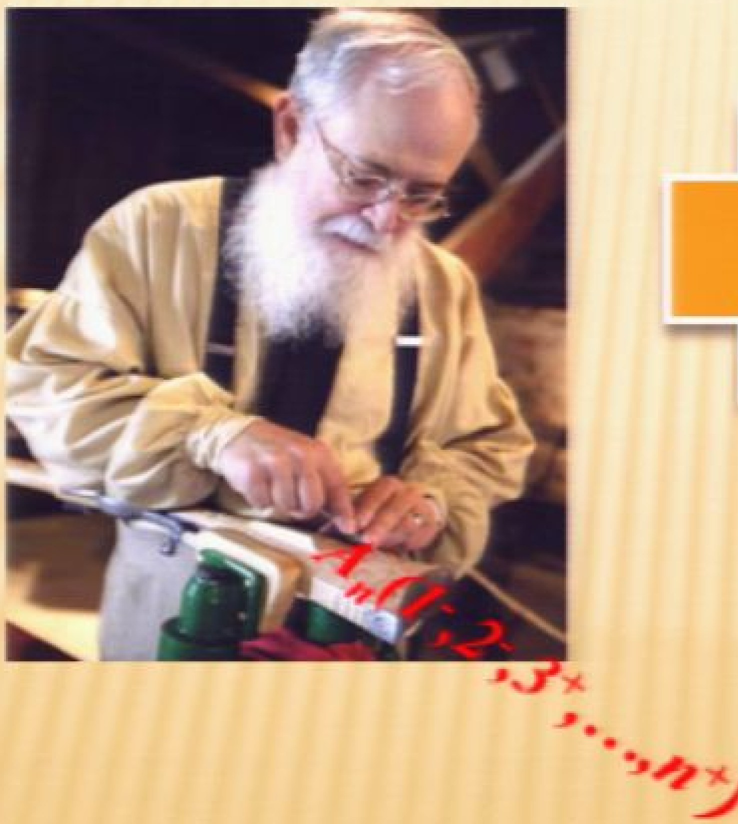

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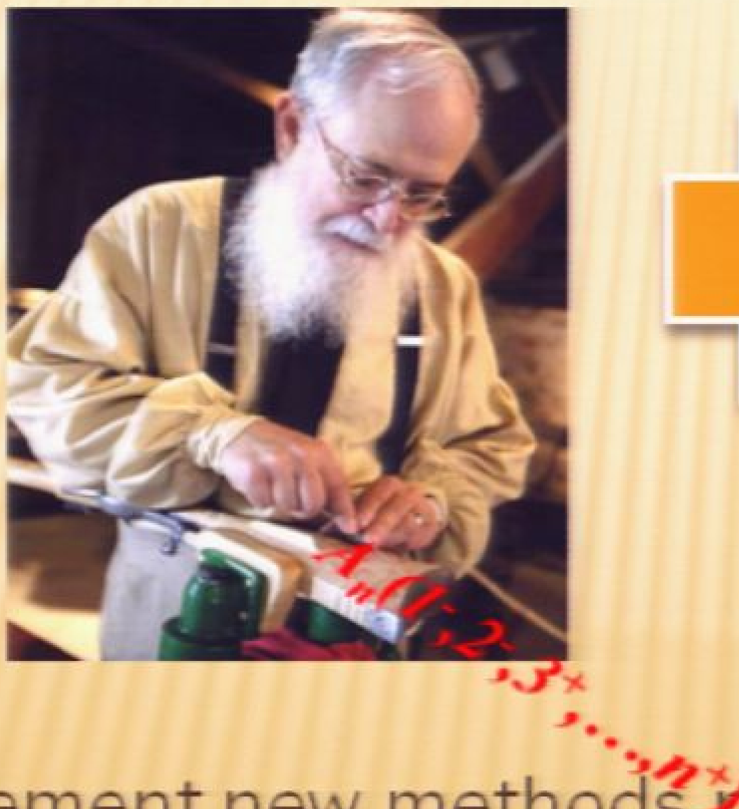

- Implement new methods numerically.

[Berger, Bern, Dixon, Febres Cordero, DF, Ita, Kosower, Maître]

BlackHat

- Numerical implementation of the unitarity bootstrap approach in `c++`.

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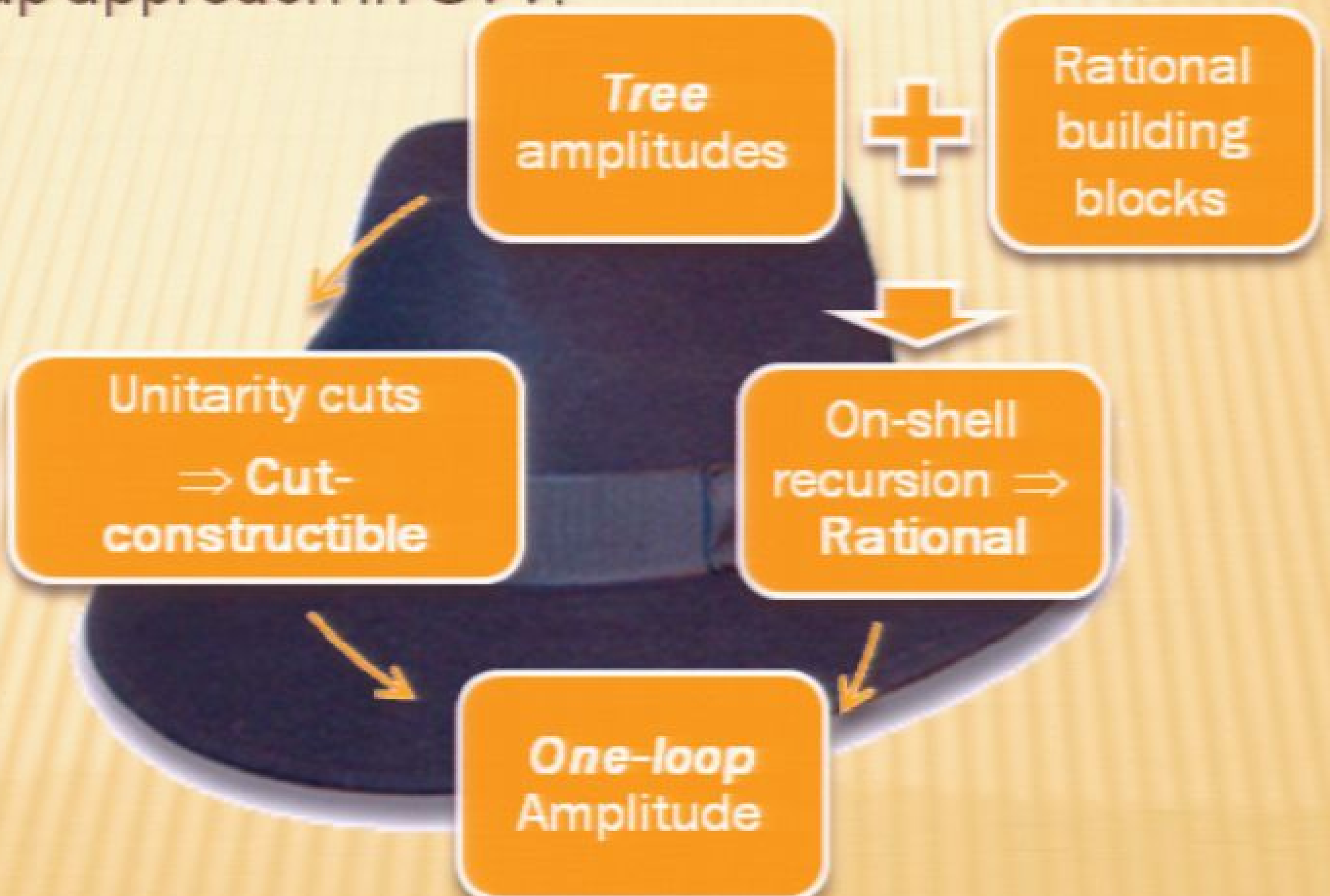

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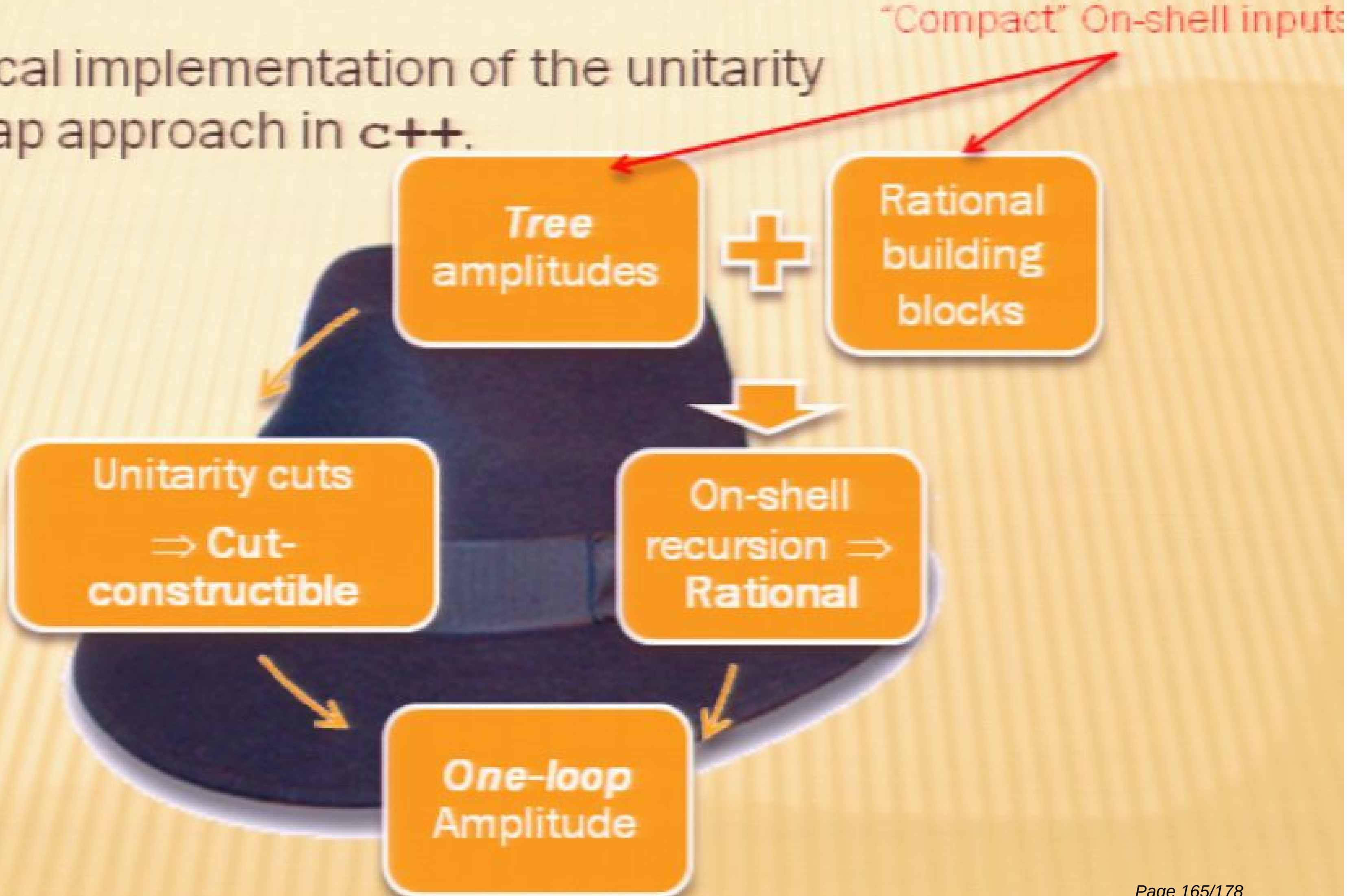

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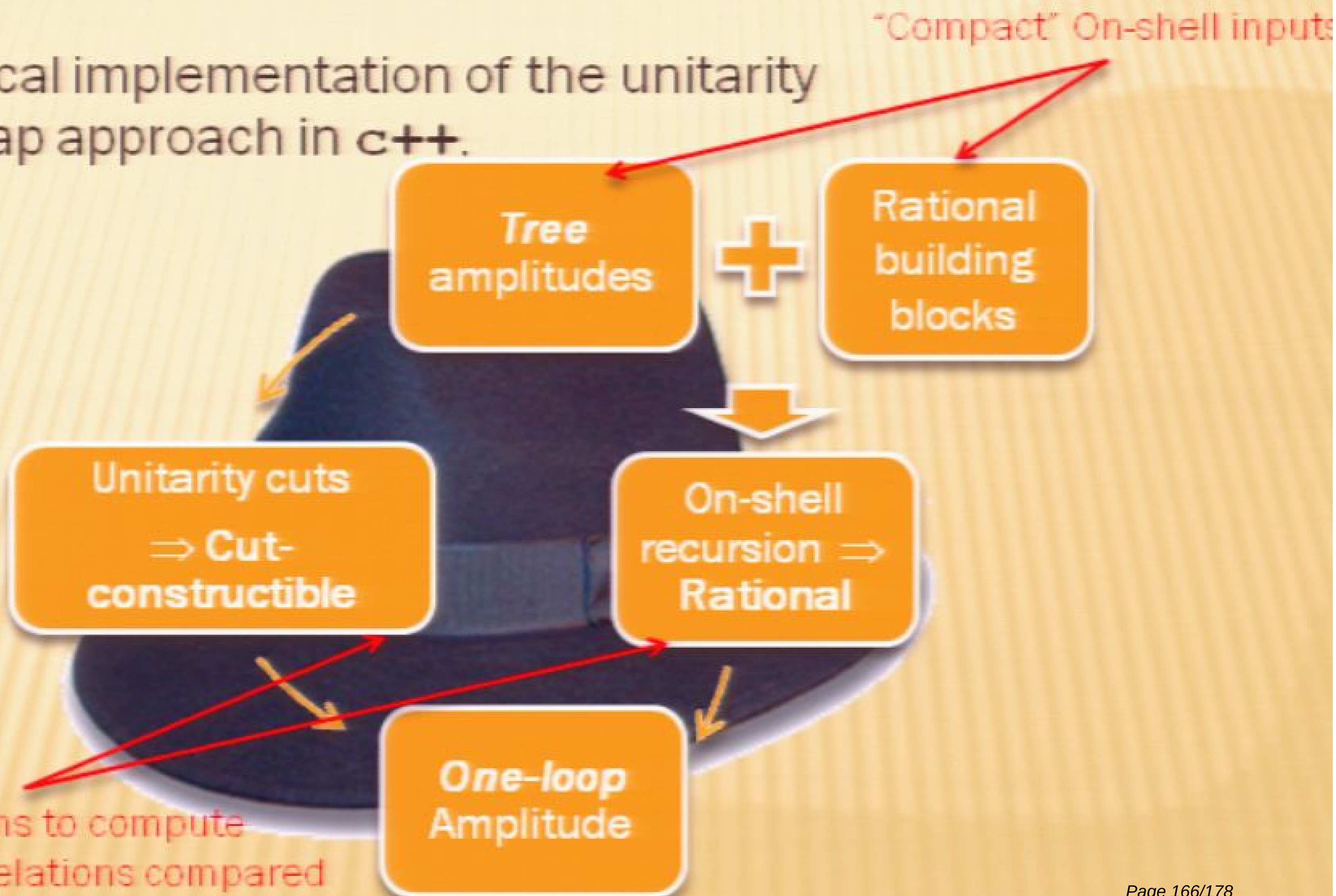

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- Other numerical tools "Rocket" [Giele, Zanderighi] [Ellis, Giele, Kunszt, Melnikov, Zanderighi]. "CutTools" [Ossola, Papadopoulos, Pittau] [Mastrolia, Ossola, Papadopoulos, Pittau]. "Golem" (Golem95) [Binoth, Guillett, Heinrich, Pilon, Reiter]+[Guffanti, Karg, Kauer]

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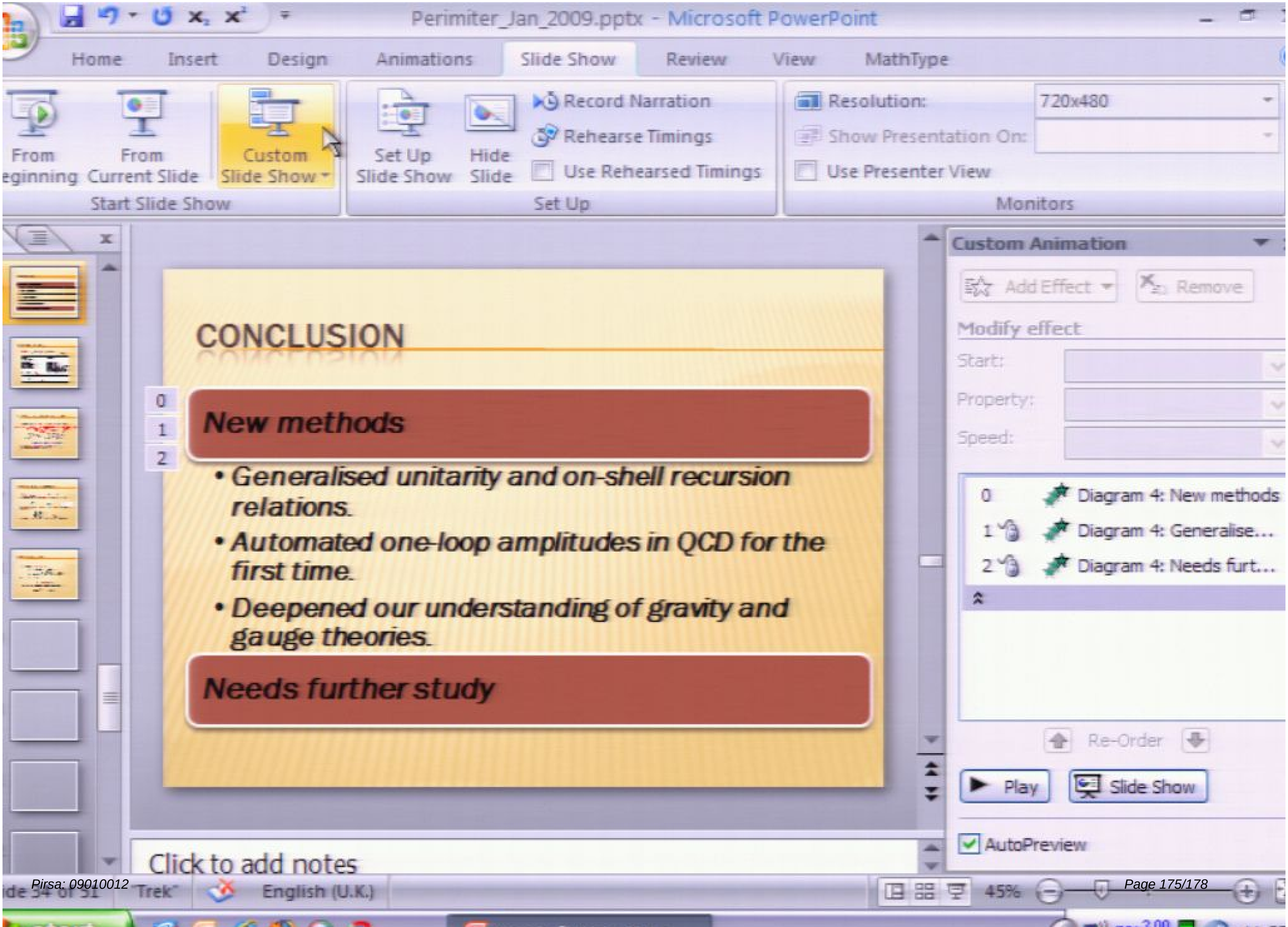
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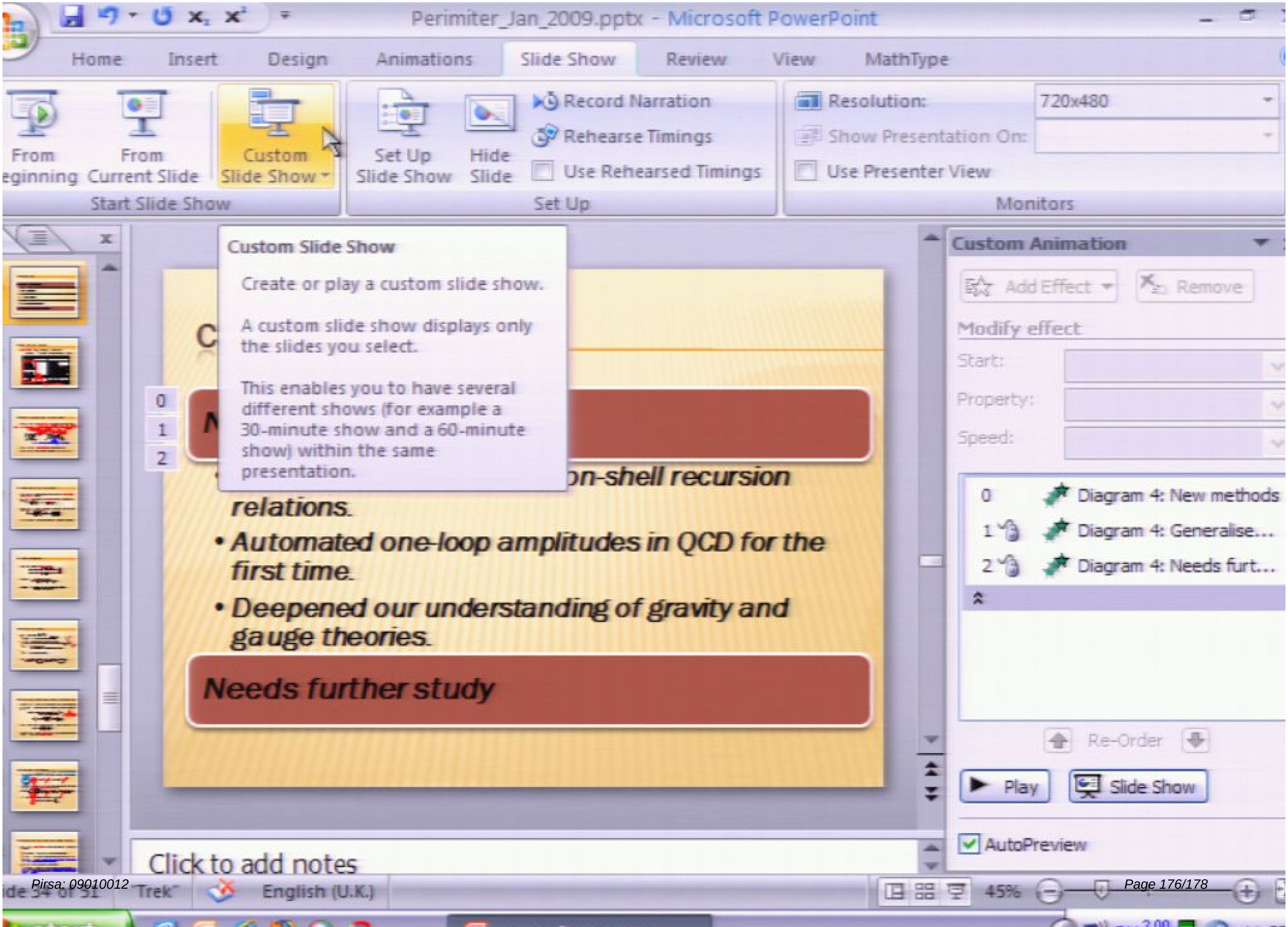
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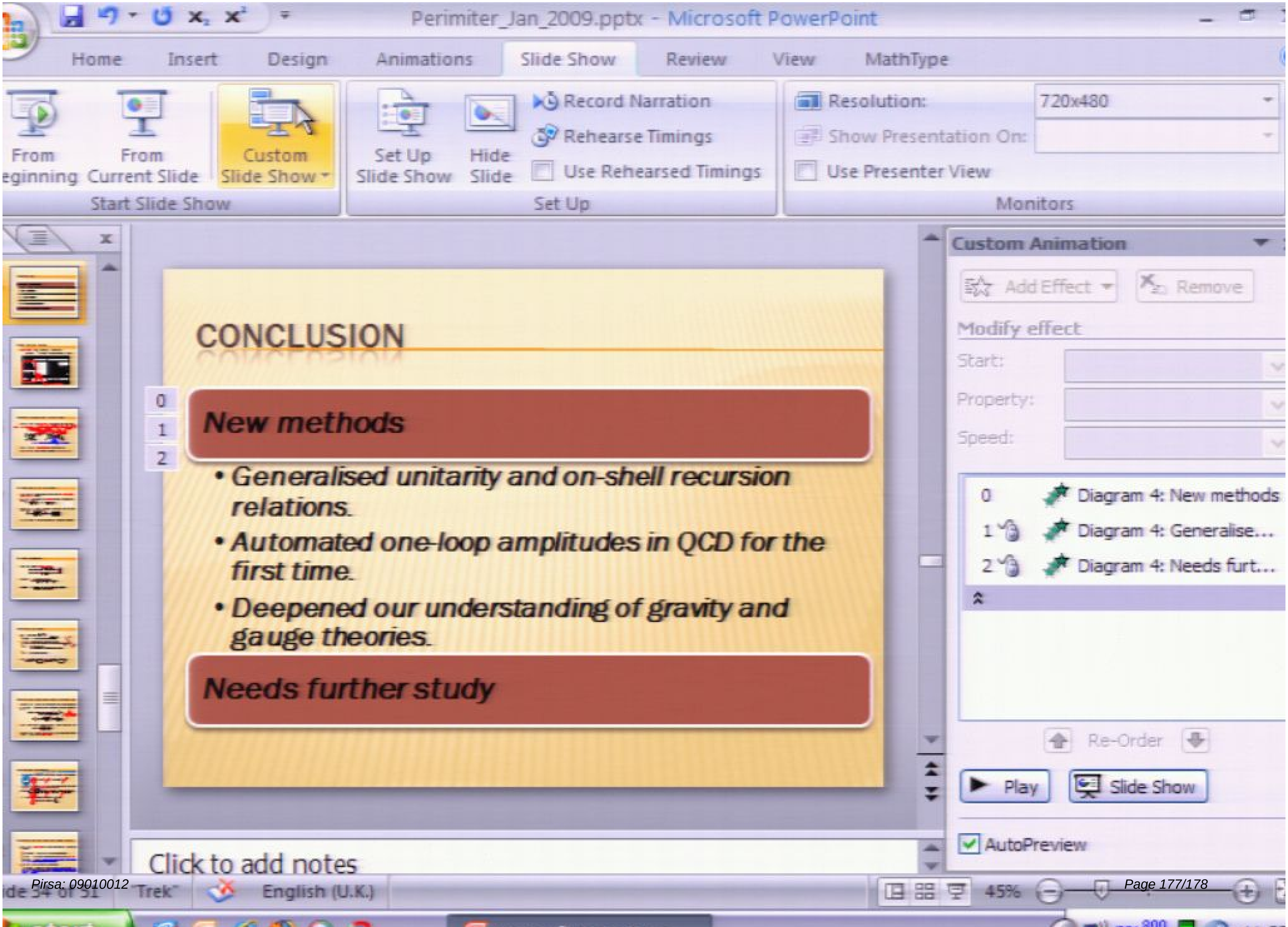

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WHAT DO WE NEED?

- One-loop high multiplicity processes,

  Newest "Famous" Les Houches list, (2007)

Process ($V \in \{Z, W, \gamma\}$)	Comments
4. $pp \to t\bar{t}\, b\bar{b}$	relevant for $t\bar{t}H$
5. $pp \to t\bar{t}$+2jets	relevant for $t\bar{t}H$
6. $pp \to VV\, b\bar{b}$,	relevant for VBF $\to H \to VV$, $t\bar{t}H$
7. $pp \to VV$+2jets	relevant for VBF $\to H \to VV$ VBF contributions calculated by (Bozzi/)Jäger/Oleari/Zeppenfeld.
8. $pp \to V$+3jets	various new physics signatures
NLO calculations added to list in 2007	
9. $pp \to b\bar{b}b\bar{b}$	Higgs and new physics signatures