Title: The Effect of Dark Energy Perturbations on the Growth of Structures

Date: Jan 13, 2009 02:00 PM

URL: http://pirsa.org/09010011

Abstract: The growth of matter perturbations in the presence of dark energy with small fluctuations depends on the speed of sound of these fluctuations and the comoving scale. The growth index can differ from the value that it takes in the limit of no dark energy perturbations by an amount comparable to the accuracy of future observations. This may contribute to a better characterization of the dark energy properties.

The effect of dark energy perturbations on the growth of structures

Guillermo Ballesteros

[Work with Antonio Riotto]

Instituto de Física Teórica
Universidad Autónoma de Madrid

Perimeter Institute
January 13, 2009

Overview

- Dark energy perturbations and the growth of structures
  Phys.Lett.B668:171-176,2008 arXiv:0807.3343
  - Introduction
    - The acceleration of the Universe
    - Dark energy vs modified gravity and the nature of dark energy
  - Expansion history and Growth history
  - The growth index
  - Entropy perturbations and the sound speed
  - Results
  - Conclusions

a BIG problem

SN Type Ia + Cosmological Principle $\Longrightarrow$ Accelerated expansion

a BIG problem

SN Type Ia + Cosmological Principle $\Longrightarrow$ Accelerated expansion

The Cosmological Principle:

Spatial homogeneity and isotropy on very large scales

FLRW metric:

$$\mathrm{d}s^2 = a^2 \left[-\mathrm{d}\tau^2 + \delta_{ij}\mathrm{d}x^i\mathrm{d}x^j + \kappa \frac{\left(\delta_{ij}x^i\mathrm{d}x^j\right)^2}{1-\kappa\,\delta_{ij}x^i x^j} \right], \quad \kappa = \{-1, 0, 1\}.$$

Einstein Equations (General Relativity), perfect fluid.

$$G_{\mu\nu} = 8\pi G\, T_{\mu\nu}\,,$$

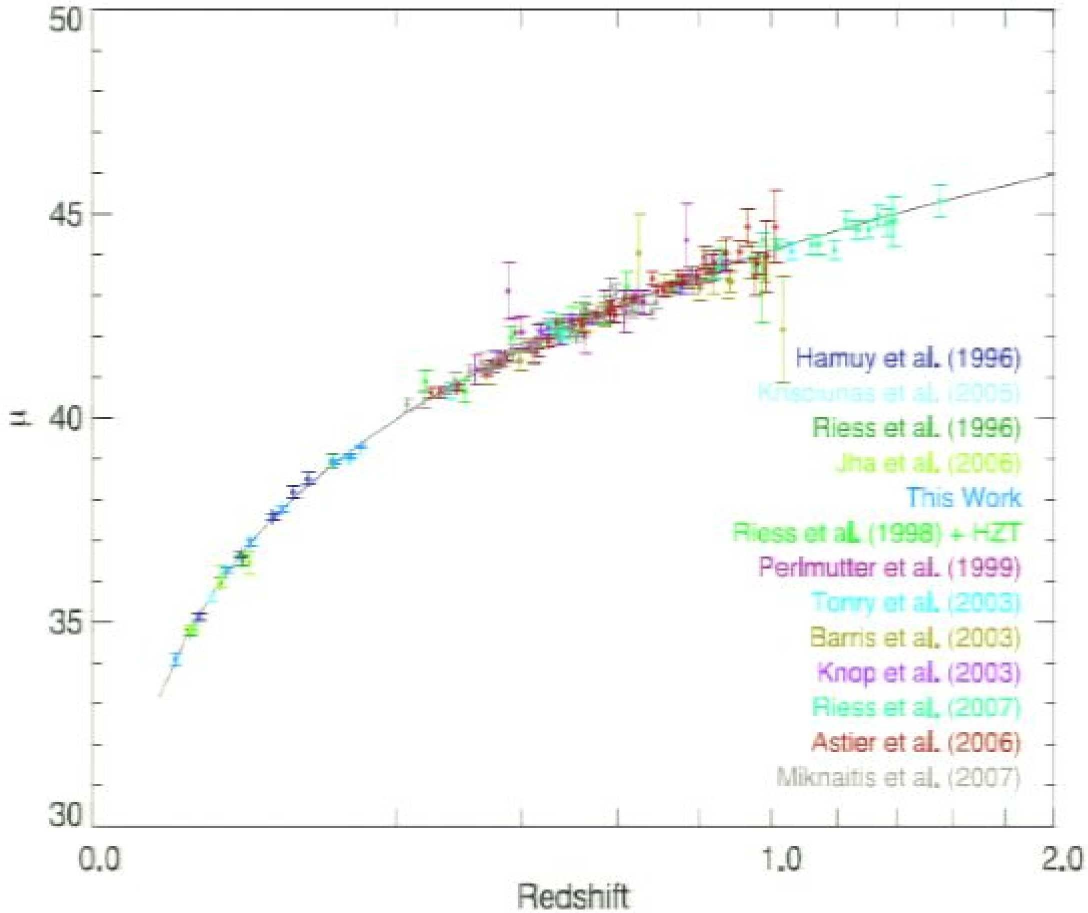
$$\mathcal{H}^2 = \frac{8\pi G}{3} a^2 \rho - \kappa\,,$$

$$-\mathcal{H}' = \frac{4\pi G}{3} a^2 \left(\rho + 3P\right),$$

a BIG problem

SN Type Ia + Cosmological Principle $\Longrightarrow$ Accelerated expansion

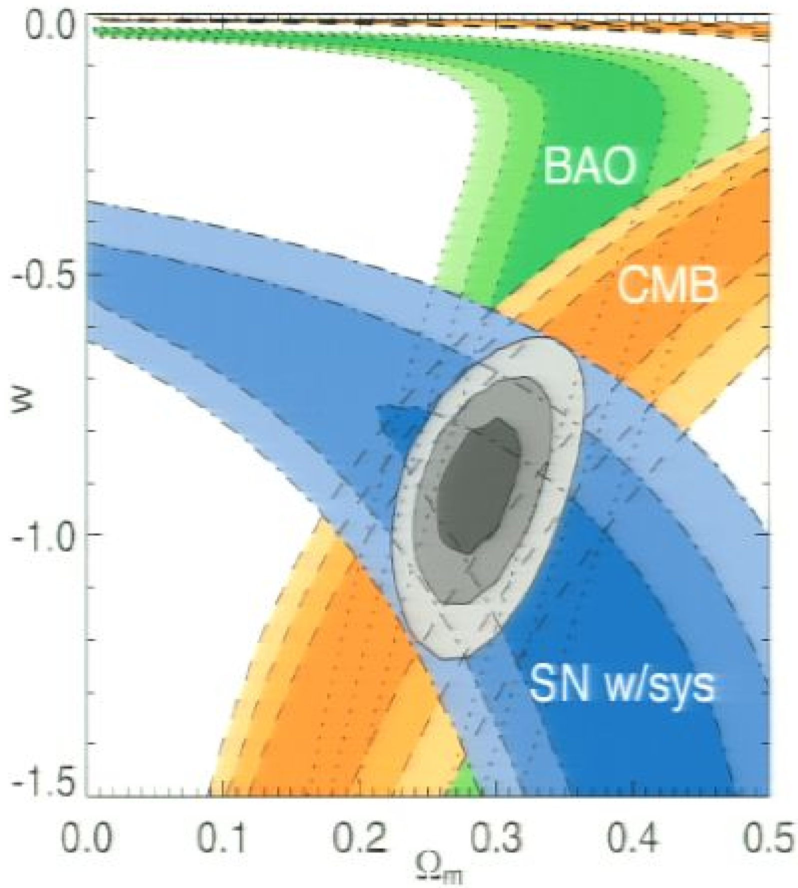
Supernova Type Ia: standard candles



Kowalski et al. 08

a BIG problem

SN Type Ia + Cosmological Principle $\Longrightarrow$ Accelerated expansion



Rubin et al. 08

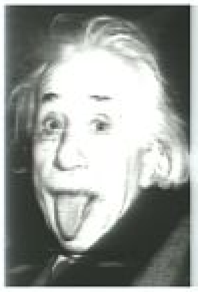
a BIG problem

SN Type Ia + Cosmological Principle $\Longrightarrow$ Accelerated expansion

(some) Possible explanations/descriptions

- Cosmological constant :

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi G T_{\mu\nu}^{(matter)} - \Lambda g_{\mu\nu}$$
$$P = -\rho$$



A puzzling result

* $\Omega_\Lambda^0 \sim 0.75$ Three quarters of the energy density of the universe.
* Smallness of Λ. Cosmological constant problem.
* Coincidence 'problem'. $\Omega_{matter}^0 \sim \Omega_\Lambda^0$ (order of magnitude)

a BIG problem

SN Type Ia + Cosmological Principle $\Longrightarrow$ Accelerated expansion

(some) Possible explanations/descriptions

- Dark Energy (DE):

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}^{(matter)} + 8\pi G T_{\mu\nu}^{(\mathrm{DE})}$$

$$P = w\rho, \quad w < -1/3 \text{ (required)}$$

a BIG problem

SN Type Ia + Cosmological Principle $\Longrightarrow$ Accelerated expansion

(some) Possible explanations/descriptions

- Dark Energy (DE):

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}^{(matter)} + 8\pi G T_{\mu\nu}^{(\mathrm{DE})}$$

$$P = w\rho, \quad w < -1/3 \text{ (required)}$$

- Modified Gravity (MG):

$$\cancel{R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}}$$

$$= 8\pi G T_{\mu\nu}^{(matter)}$$

Two questions

i) Dark energy (DE) or modified gravity (MG)?

ii) What is the nature of DE?

Two questions

i) Dark energy (DE) or modified gravity (MG)?

ii) What is the nature of DE?

The expansion history of the universe, $\mathcal{H}(z)$, is not enough to distinguish between DE and MG.

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}^{(matter)} + 8\pi G T_{\mu\nu}^{(\mathrm{DE})}$$

Two questions

i) Dark energy (DE) or modified gravity (MG)?

ii) What is the nature of DE?

The expansion history of the universe, $\mathcal{H}(z)$, is not enough to distinguish between DE and MG.

$$-8\pi G T_{\mu\nu}^{(\mathrm{DE})} + R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}^{(matter)}$$

Any expansion history can be reproduced in GR choosing an appropiate time dependent w

$$w(z) = \frac{3}{2}\left(\frac{2/3H'(z) + H^2(z)}{H^2(z) - H_0^2\Omega_c^0(1+z)^3}\right)$$

Two questions

i) Dark energy (DE) or modified gravity (MG)?

ii) What is the nature of DE?

Growth history of the Universe

$$G_{\mu\nu} + \delta G_{\mu\nu} = 8\pi G T_{\mu\nu} + 8\pi G \delta T_{\mu\nu}$$

(Dark) matter perturbations $\delta_c = \delta\rho_c/\rho_c$

$$\delta_c'' + \mathcal{H}\delta_c' - \frac{3}{2}\mathcal{H}^2\Omega_c\delta_c = 0$$

Two questions

i) Dark energy (DE) or modified gravity (MG)?

ii) What is the nature of DE?

The expansion history of the universe, $\mathcal{H}(z)$, is not enough to distinguish between DE and MG.

$$-8\pi G T_{\mu\nu}^{(\mathrm{DE})} + R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}^{(matter)}$$

Any expansion history can be reproduced in GR choosing an appropiate time dependent w

$$w(z) = \frac{3}{2}\left(\frac{2/3H'(z) + H^2(z)}{H^2(z) - H_0^2\Omega_c^0(1+z)^3}\right)$$

Two questions

i) Dark energy (DE) or modified gravity (MG)?

ii) What is the nature of DE?

Growth history of the Universe

$$G_{\mu\nu} + \delta G_{\mu\nu} = 8\pi G T_{\mu\nu} + 8\pi G \delta T_{\mu\nu}$$

(Dark) matter perturbations $\delta_c = \delta\rho_c/\rho_c$

$$\delta_c'' + \mathcal{H}\delta_c' - \frac{3}{2}\mathcal{H}^2\Omega_c\delta_c = 0$$

Two questions

i) Dark energy (DE) or modified gravity (MG)?

ii) What is the nature of DE?

Growth history of the Universe

$$G_{\mu\nu} + \delta G_{\mu\nu} = 8\pi G T_{\mu\nu} + 8\pi G \delta T_{\mu\nu}$$

(Dark) matter perturbations $\delta_c = \delta\rho_c/\rho_c$

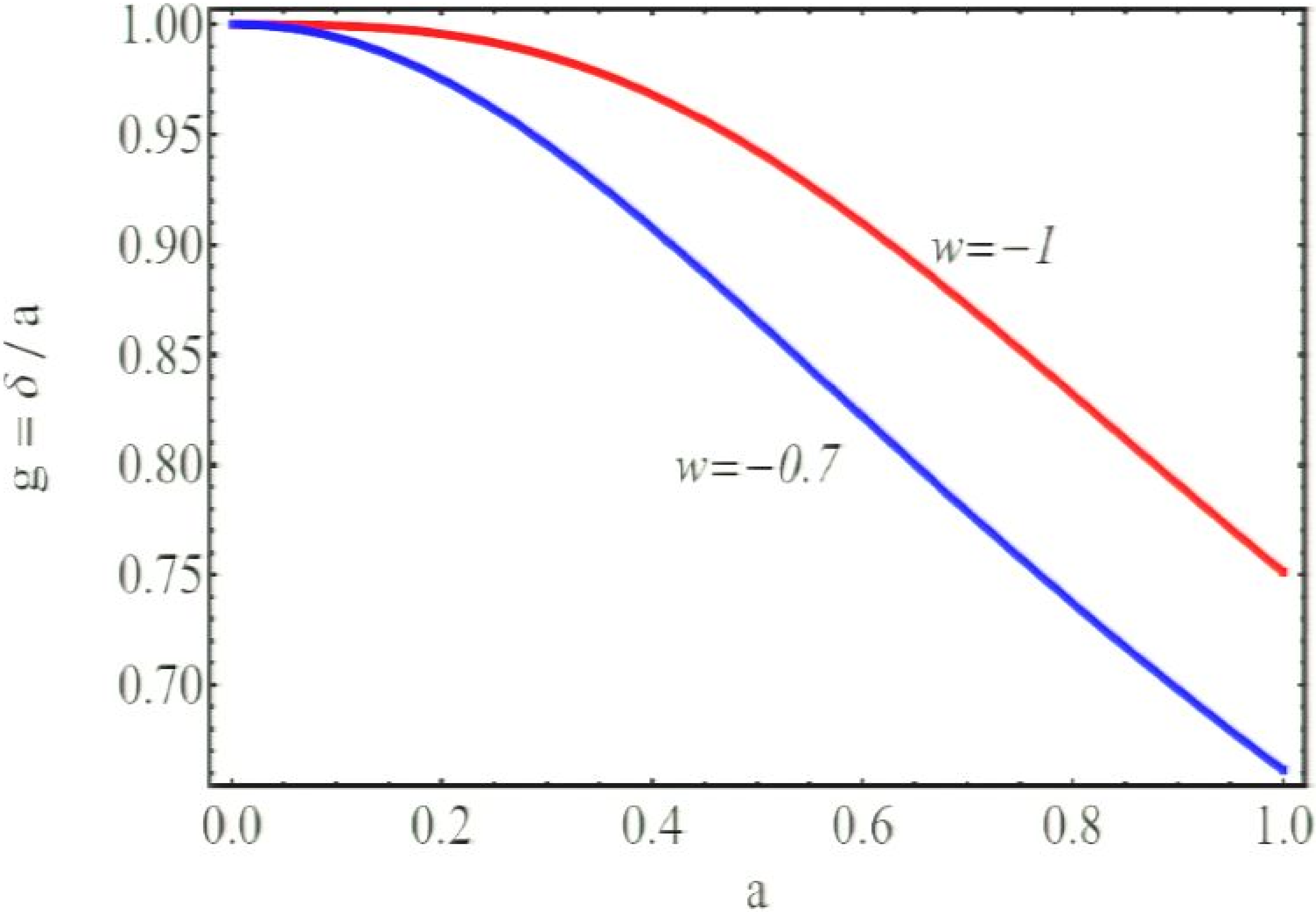
$$\delta_c'' + \mathcal{H}\delta_c' - \frac{3}{2}\mathcal{H}^2\Omega_c\delta_c = 0$$

Growth factor

$$g \equiv \delta_c/a$$

If matter dominated ($\Omega_c \to 1$): $\mathcal{H} \sim \tau^{-1}$, $\delta_c \sim \tau^2 \sim a \implies g' \sim 0$

Growth factor in General Relativity



The growth index γ

$$g(a) = g(a_i) \exp \int_{a_i}^{a} \left([\Omega_m(\hat{a})]^{\gamma} - 1 \right) \frac{\mathrm{d}\hat{a}}{\hat{a}}$$

Linder 05

Ω_m characterizes the expansion history and γ encodes the growth history but clearly growth and expansion are not unrelated.

- In General Relativity:

$$\delta_c'' + \mathcal{H}\delta_c' - \frac{3}{2}\mathcal{H}^2\Omega_c\delta_c = 0 \implies \gamma(w) = 0.55 + 0.05\,[1 + w(z=1)]$$

0.2% fitting accuracy

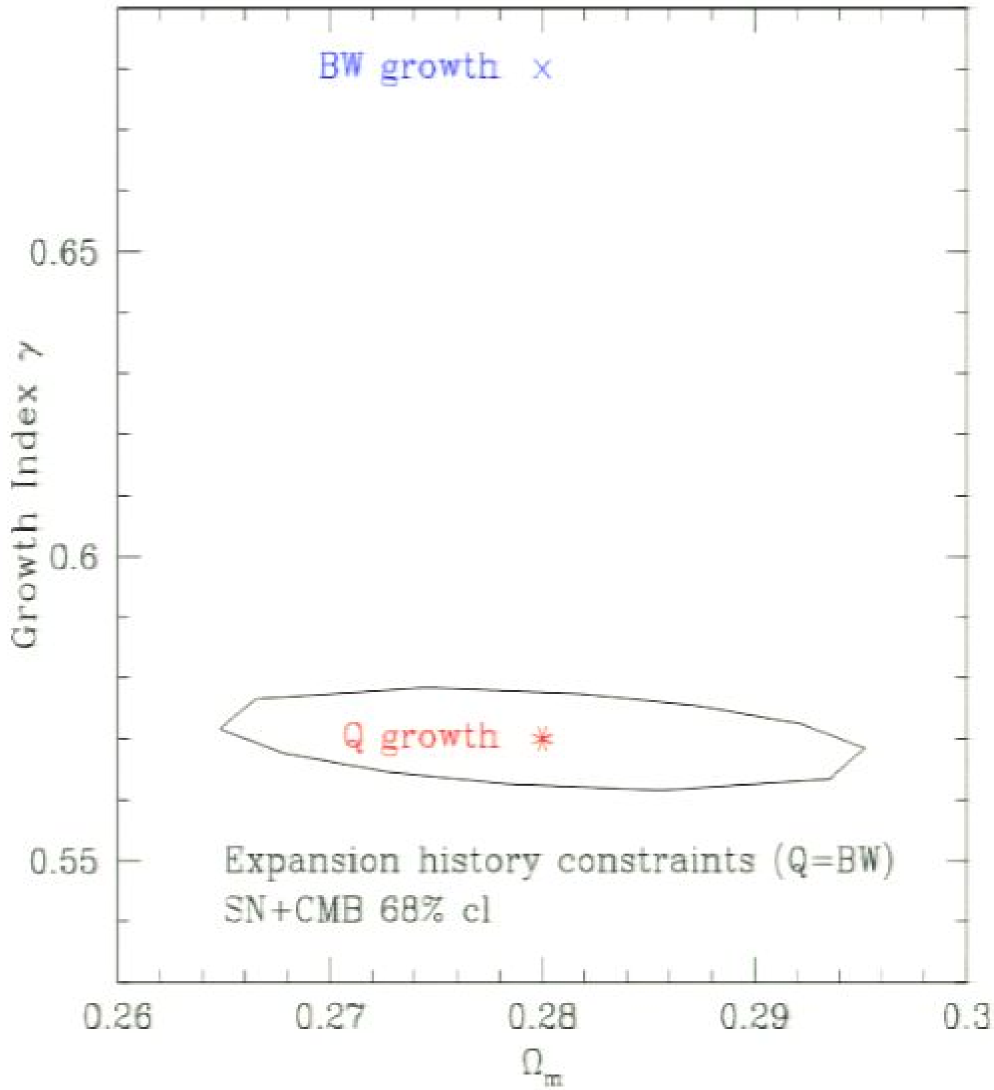
- For DGP:

$$\gamma \simeq 0.68$$

- For several $f(R)$ models:

$$\gamma \sim 0.4$$

Gannouji, Moraes and Polarksi 08



BW growth ×
Q growth *
Expansion history constraints (Q=BW)
SN+CMB 68% cl
Growth Index γ
0.65
0.6
0.55
0.26
0.27
0.28
0.29
0.3
Ω_m

Linder 05

The growth index γ

$$g(a) = g(a_i)\exp\int_{a_i}^{a}\left([\Omega_m(\hat{a})]^{\gamma} - 1\right)\frac{\mathrm{d}\hat{a}}{\hat{a}}$$

Linder 05

Ω_m characterizes the expansion history and γ encodes the growth history but clearly growth and expansion are not unrelated.

- In General Relativity:

$$\delta_c'' + \mathcal{H}\delta_c' - \frac{3}{2}\mathcal{H}^2\Omega_c\delta_c = 0 \implies \gamma(w) = 0.55 + 0.05\,[1 + w(z=1)]$$

0.2% fitting accuracy

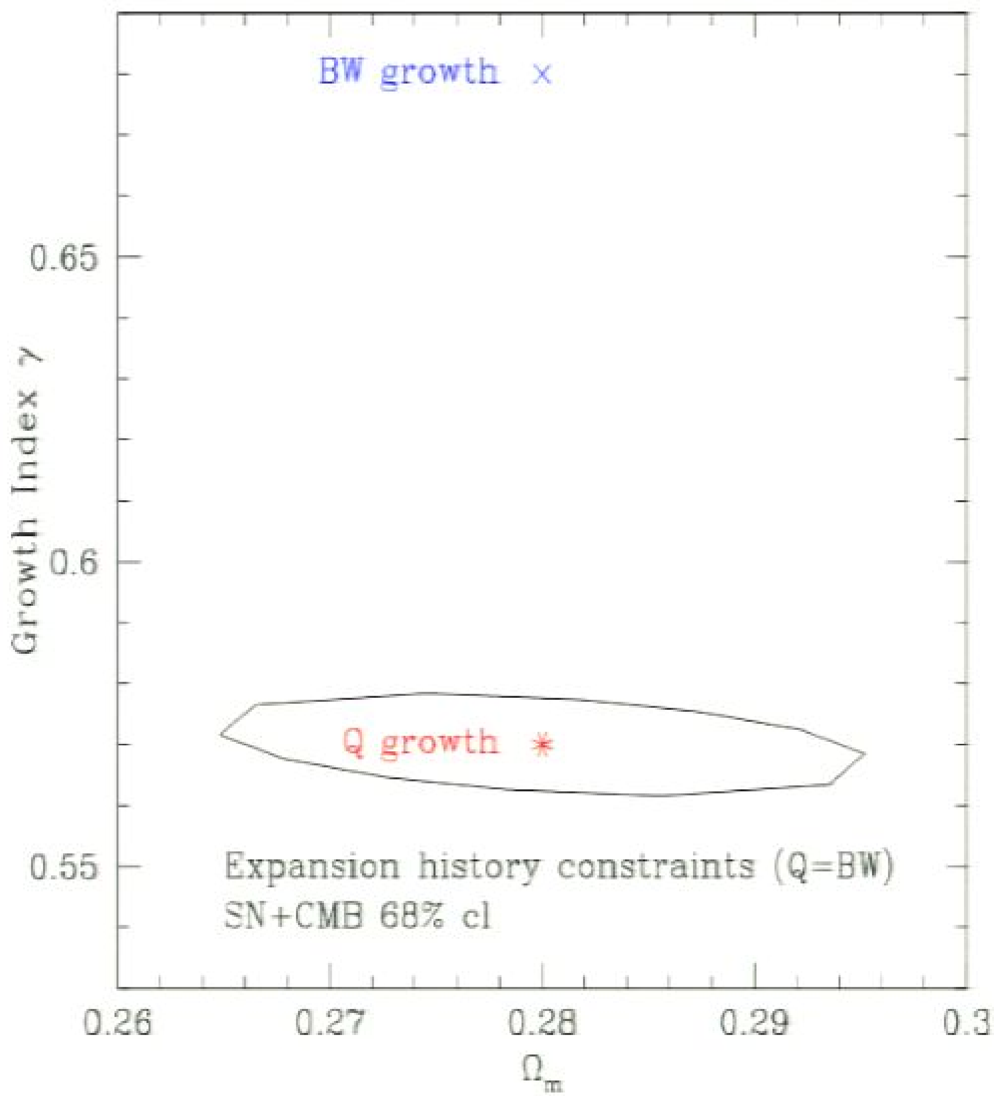
- For DGP:

$$\gamma \simeq 0.68$$

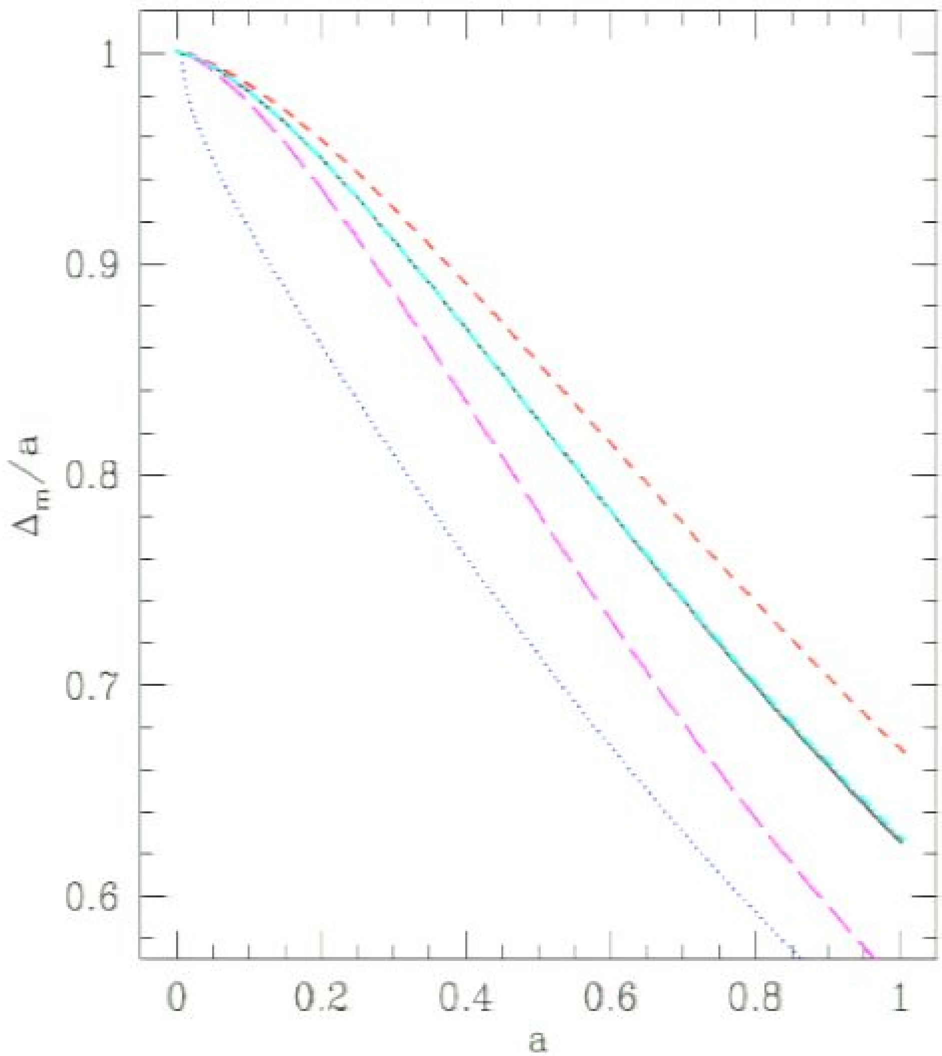
- For several $f(R)$ models:

$$\gamma \sim 0.4$$

Gannouji, Moraes and Polarksi 08


BW growth ×
Q growth *
Expansion history constraints (Q=BW)
SN+CMB 68% cl
Growth Index γ
0.65
0.6
0.55
0.26
0.27
0.28
0.29
0.3
Ωm

Linder 05

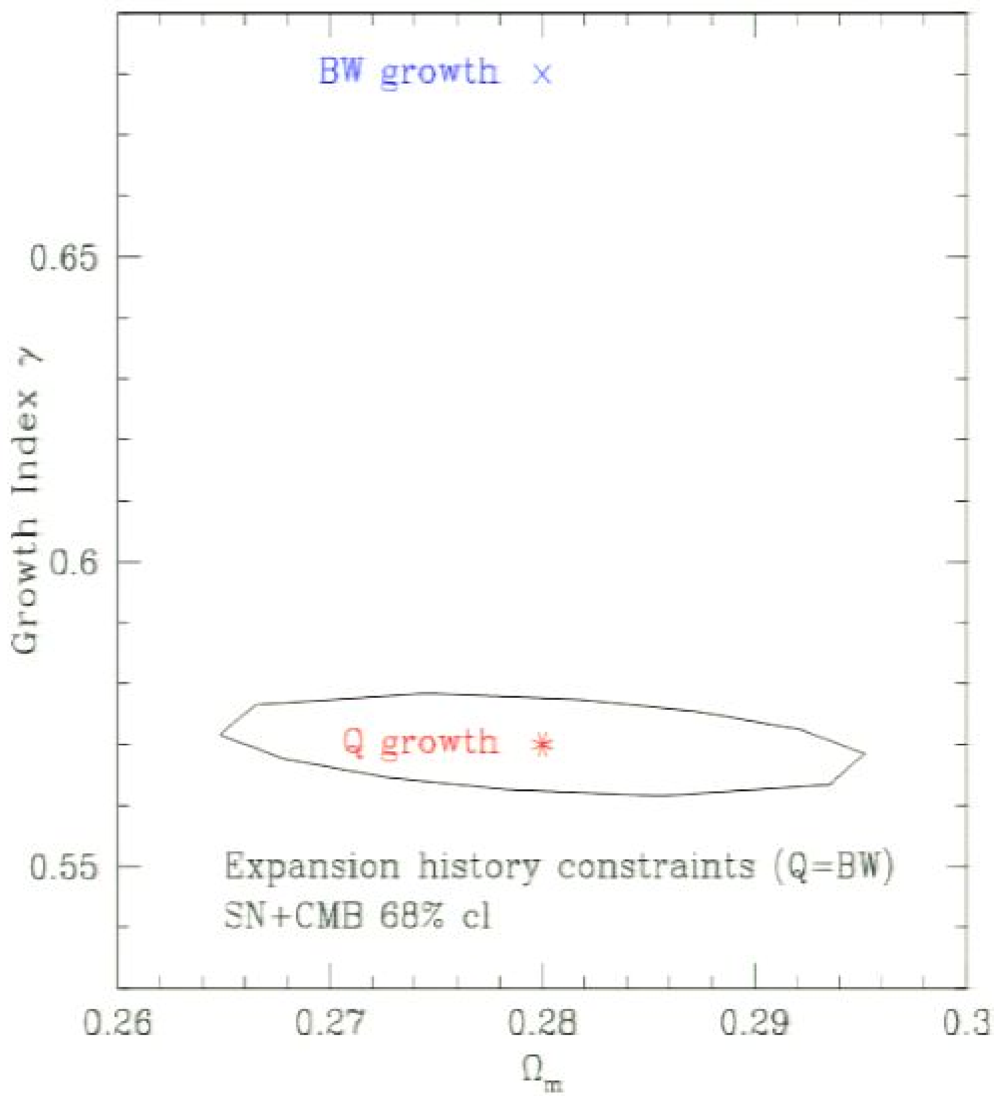


Dashed red on top: DE in GR

Black dotted line: DGP

Dashed cyan: anisotropic DE in GR

Kunz and Sapone 06


BW growth ×
Q growth *
Expansion history constraints (Q=BW)
SN+CMB 68% cl
Growth Index γ
0.65
0.6
0.55
0.26
0.27
0.28
0.29
0.3
Ω_m

Linder 05

The growth index γ

$$g(a) = g(a_i)\exp\int_{a_i}^{a}\left([\Omega_m(\hat{a})]^{\gamma} - 1\right)\frac{\mathrm{d}\hat{a}}{\hat{a}}$$

Linder 05

Ω_m characterizes the expansion history and γ encodes the growth history but clearly growth and expansion are not unrelated.

- In General Relativity:

$$\delta_c'' + \mathcal{H}\delta_c' - \frac{3}{2}\mathcal{H}^2\Omega_c\delta_c = 0 \implies \gamma(w) = 0.55 + 0.05\,[1 + w(z=1)]$$

0.2% fitting accuracy

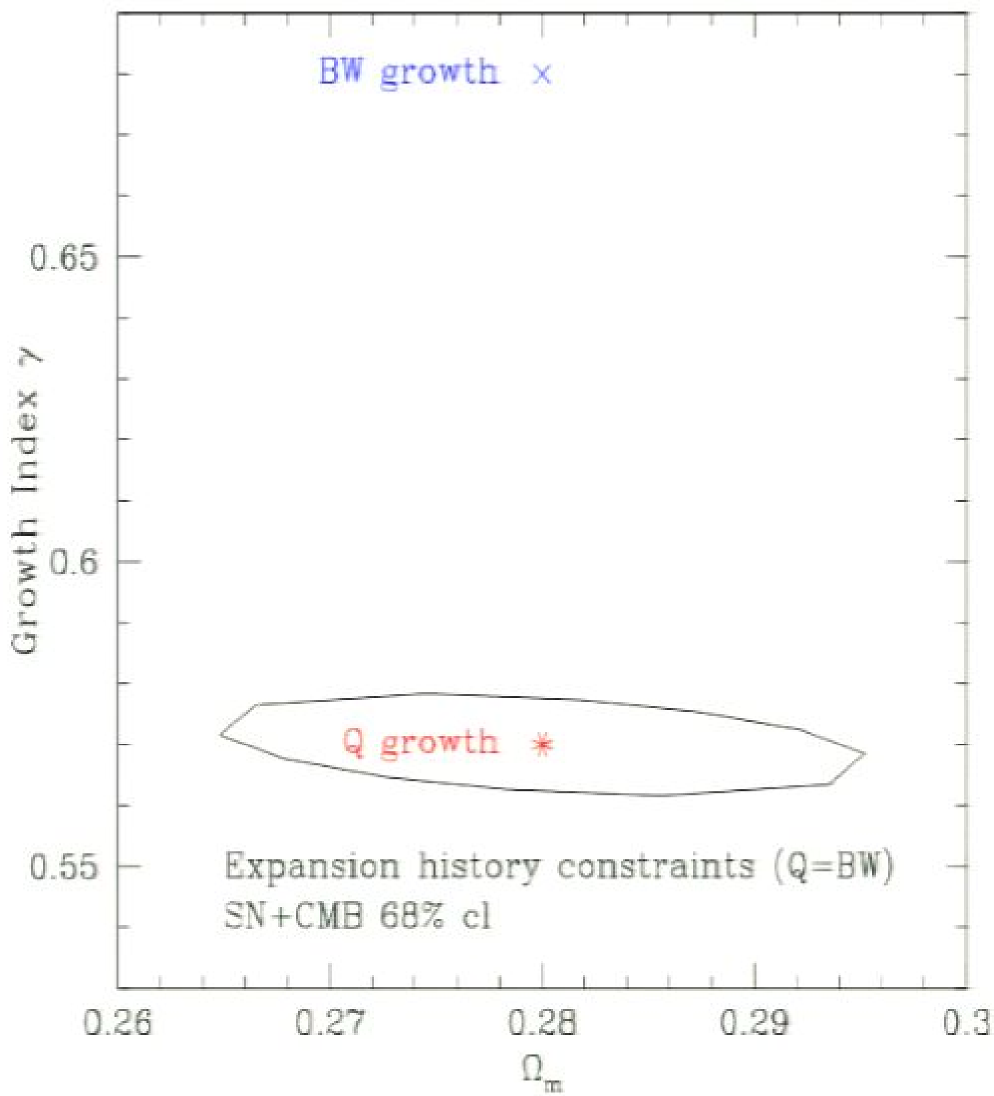
- For DGP:

$$\gamma \simeq 0.68$$

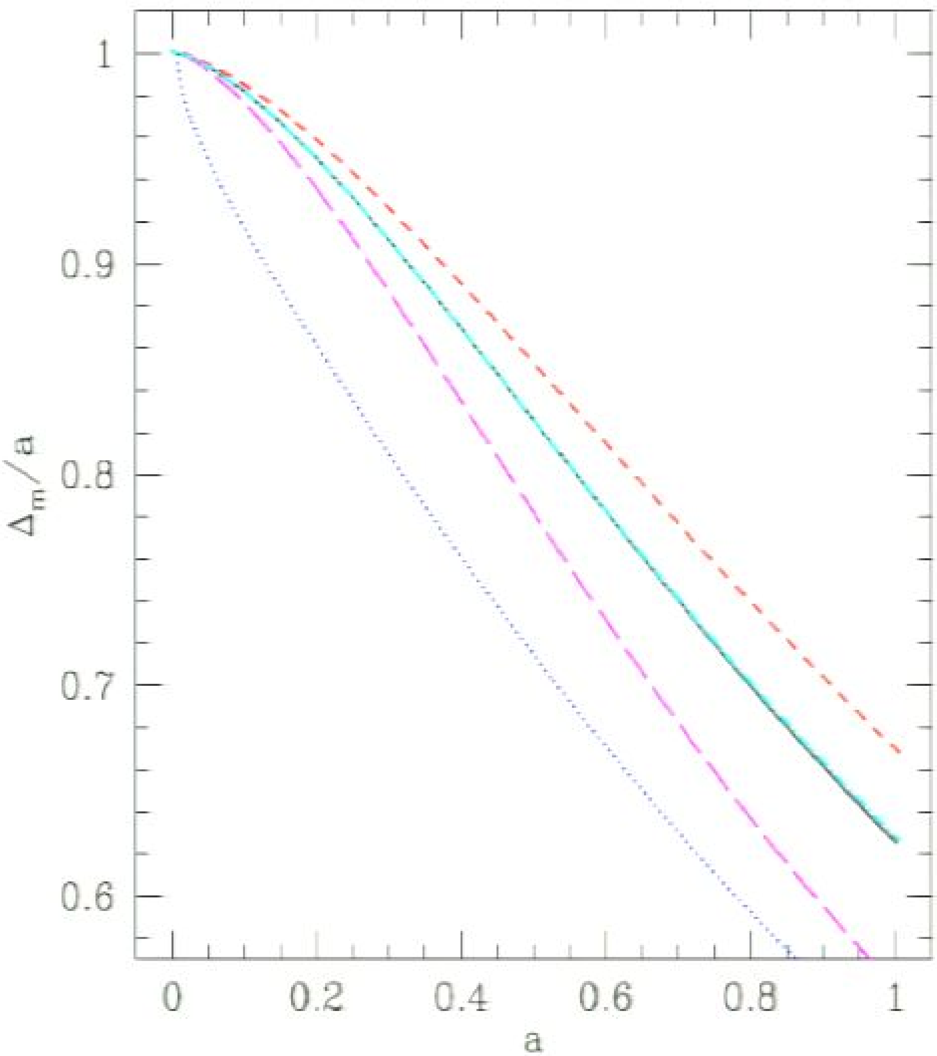
- For several $f(R)$ models:

$$\gamma \sim 0.4$$

Gannouji, Moraes and Polarksi 08


BW growth ×
0.65
0.6
0.55
Growth Index γ
Q growth *
Expansion history constraints (Q=BW)
SN+CMB 68% cl
0.26
0.27
0.28
0.29
0.3
Ωm

Linder 05

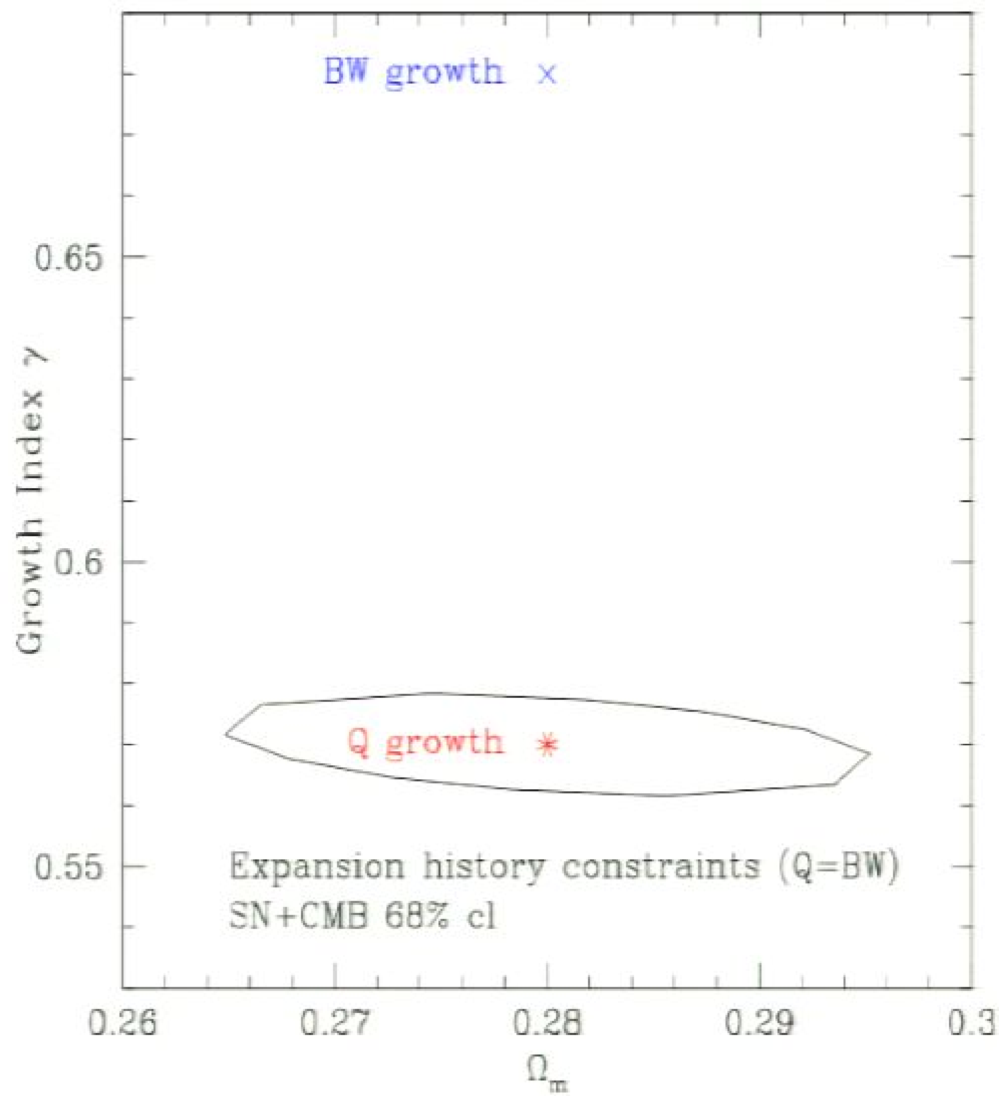


Dashed red on top: DE in GR

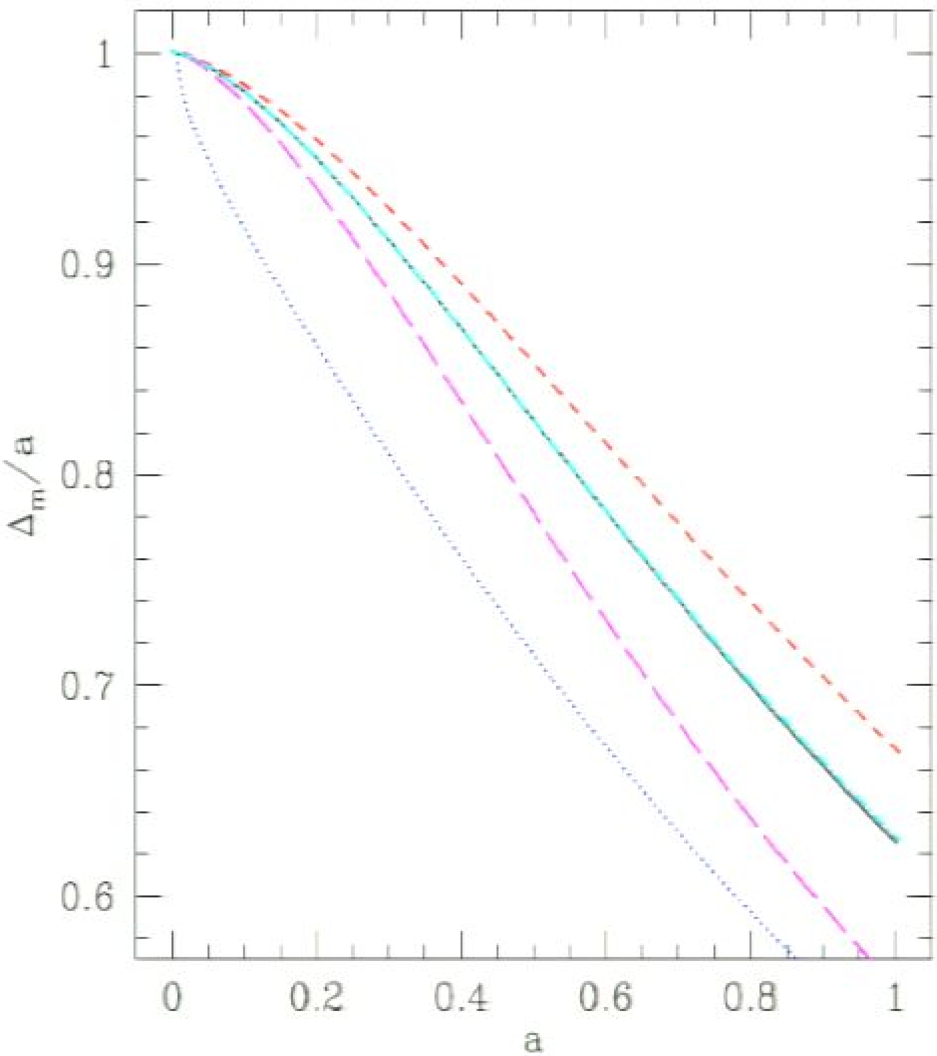
Black dotted line: DGP

Dashed cyan: anisotropic DE in GR

Kunz and Sapone 06


BW growth ×
Q growth *
Expansion history constraints (Q=BW)
SN+CMB 68% cl
Growth Index γ
0.65
0.6
0.55
0.26
0.27
0.28
0.29
0.3
Ωm

Linder 05



Dashed red on top: DE in GR

Black dotted line: DGP

Dashed cyan: anisotropic DE in GR

Kunz and Sapone 06

Dark Energy or Modified Gravity?

If the DE component is sufficiently general:

- shear stress perturbations
- entropy perturbations

in general one can not tell apart DE and MG.

Bertschinger and Zukin 08

Kunz and Sapone 06

...

$$\left\{ \begin{array}{r} \text{Weak lensing} \\ + \\ \text{LSS observations} \end{array} \right. \rightarrow \left\{ \begin{array}{r} \text{Gravitational potentials} \\ \Psi \quad and \quad \Phi \end{array} \right. \rightarrow \left\{ \begin{array}{r} \text{DE } (\sigma, \hat{c}_s^2) \\ or \\ \text{MG } (\beta, G_{eff}) \end{array} \right.$$

Dark Energy or Modified Gravity?

If the DE component is sufficiently general:

- shear stress perturbations
- entropy perturbations

in general one can not tell apart DE and MG.

Bertschinger and Zukin 08

Kunz and Sapone 06

...

$$\left\{ \begin{array}{r} \text{Weak lensing} \\ + \\ \text{LSS observations} \end{array} \right. \rightarrow \left\{ \begin{array}{r} \text{Gravitational potentials} \\ \Psi \quad and \quad \Phi \end{array} \right. \rightarrow \left\{ \begin{array}{r} \text{DE } (\sigma, \hat{c}_s^2) \\ or \\ \text{MG } (\beta, G_{eff}) \end{array} \right.$$

- However, the perturbations can be useful if we assume a description of the acceleration; in particular DE.

The speed of sound

$$c_s^2 \equiv \frac{\delta P}{\delta \rho}$$

For a homogeneous perfect fluid (adiabatic):

$$c_a^2 \equiv \frac{P'}{\rho'} = w - \frac{w'}{3\mathcal{H}(1+w)}$$

Imperfect fluid (entropy perturbation):

$$w\Gamma \equiv \left(c_s^2 - c_a^2\right)\delta = \frac{P'}{\rho}\left(\frac{\delta P}{P'} - \frac{\delta\rho}{\rho'}\right)$$

In general, c_s^2 is gauge dependent but c_a^2 and Γ are gauge invariant

$$\hat{\delta} = \delta + 3\mathcal{H}(1+w)\frac{\theta}{k^2} \implies \delta P = \hat{c}_s^2\,\delta\rho + 3\mathcal{H}\,(1+w)\left(\hat{c}_s^2 - w\right)\rho\frac{\theta}{k^2}$$

δ is invariant in the rest frame $\longrightarrow \hat{\delta}$

The speed of sound

$$\hat{c}_s^2 \equiv \frac{\delta P}{\delta \rho}$$

- Matter: $\hat{c}_s^2 = 0$
- Radiation: $\hat{c}_s^2 = 1/3$
- Quintessence (scalar field DE) with a canonical kinetic term: $\hat{c}_s^2 = 1$
- K-essence (non-canonical kinetic terms): $\hat{c}_s^2 \neq 1$ (even $\rightarrow \infty$)
- Two scalar fields: $\hat{c}_s^2 \neq 1$
- String gas: $0 < \hat{c}_s^2 << 1$
- Chaplygin gas ($P \propto 1/\rho$): $\hat{c}_s^2 \rightarrow 0$
- ...

If $\hat{c}_s^2 \neq 1$ DE perturbations can be important.

The speed of sound

$$c_s^2 \equiv \frac{\delta P}{\delta \rho}$$

For a homogeneous perfect fluid (adiabatic):

$$c_a^2 \equiv \frac{P'}{\rho'} = w - \frac{w'}{3\mathcal{H}(1+w)}$$

Imperfect fluid (entropy perturbation):

$$w\Gamma \equiv \left(c_s^2 - c_a^2\right)\delta = \frac{P'}{\rho}\left(\frac{\delta P}{P'} - \frac{\delta\rho}{\rho'}\right)$$

In general, c_s^2 is gauge dependent but c_a^2 and Γ are gauge invariant

$$\hat{\delta} = \delta + 3\mathcal{H}(1+w)\frac{\theta}{k^2} \implies \delta P = \hat{c}_s^2\,\delta\rho + 3\mathcal{H}\,(1+w)\left(\hat{c}_s^2 - w\right)\rho\frac{\theta}{k^2}$$

δ is invariant in the rest frame $\longrightarrow \hat{\delta}$

The speed of sound

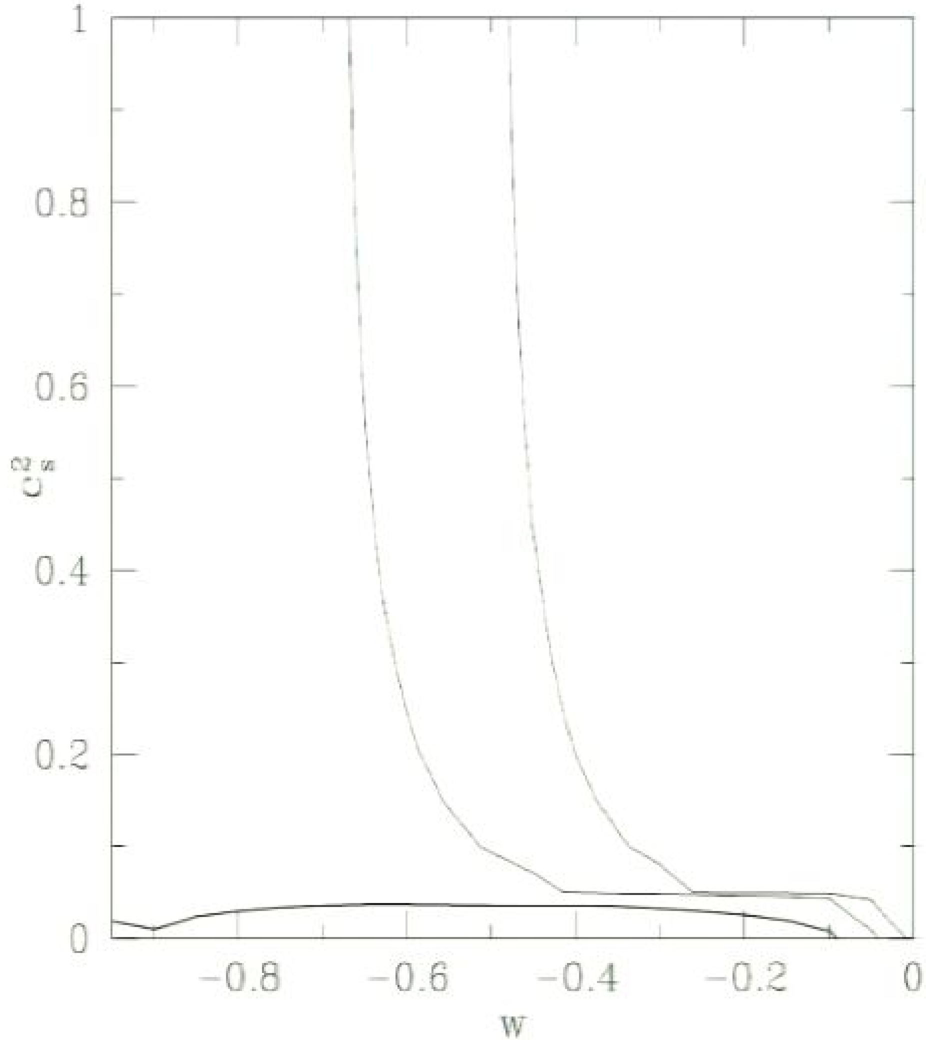
$$\hat{c}_s^2 \equiv \frac{\delta P}{\delta \rho}$$

- Matter: $\hat{c}_s^2 = 0$
- Radiation: $\hat{c}_s^2 = 1/3$
- Quintessence (scalar field DE) with a canonical kinetic term: $\hat{c}_s^2 = 1$
- K-essence (non-canonical kinetic terms): $\hat{c}_s^2 \neq 1$ (even $\rightarrow \infty$)
- Two scalar fields: $\hat{c}_s^2 \neq 1$
- String gas: $0 < \hat{c}_s^2 << 1$
- Chaplygin gas ($P \propto 1/\rho$): $\hat{c}_s^2 \rightarrow 0$
- ...

If $\hat{c}_s^2 \neq 1$ DE perturbations can be important.

The speed of sound of dark energy

$\hat{c}_s^2$ of dark energy is unconstrained by present data (CMB, LSS, ...)

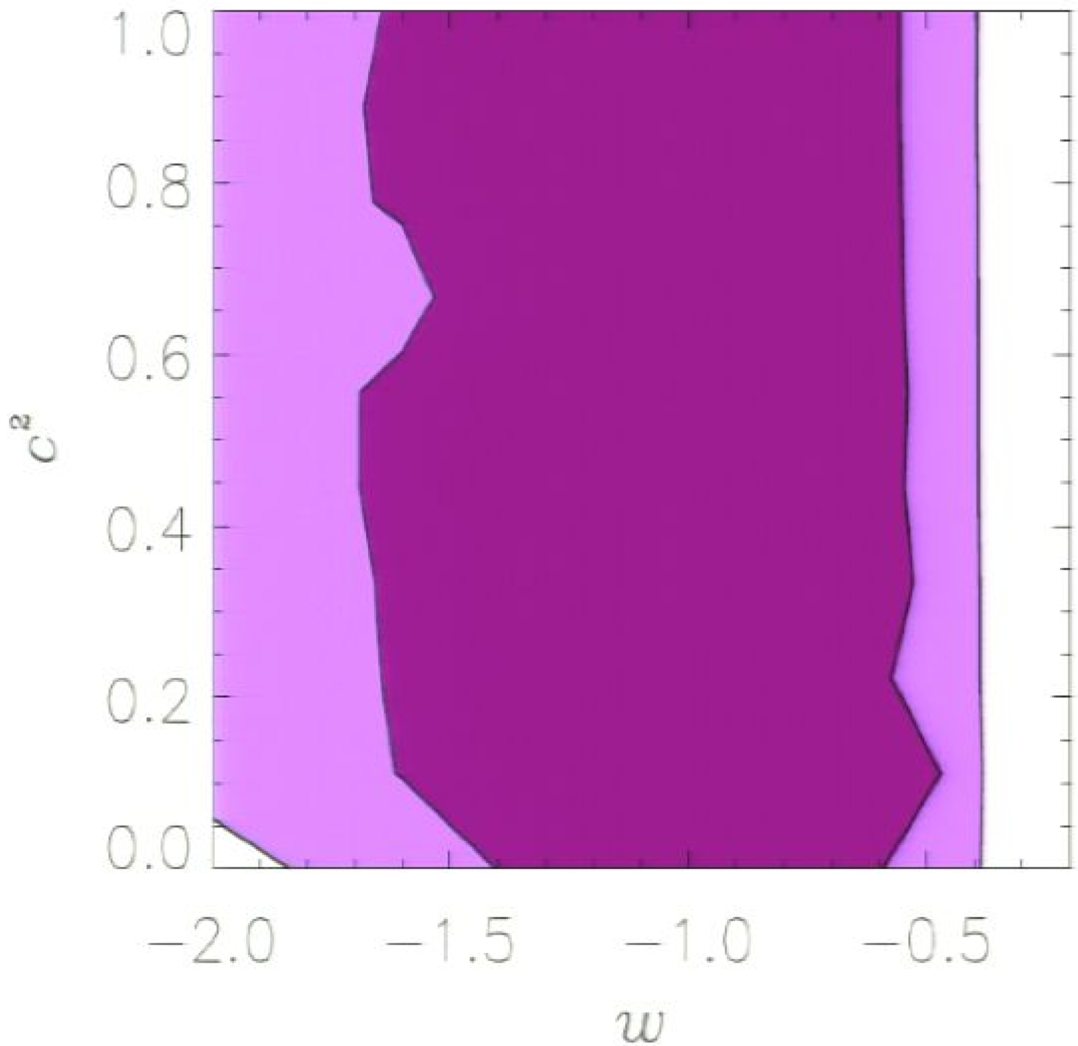


From CMB: $\hat{c}_s^2 < 0.04$ at 1σ level

Bean and Doré 03

The speed of sound of dark energy

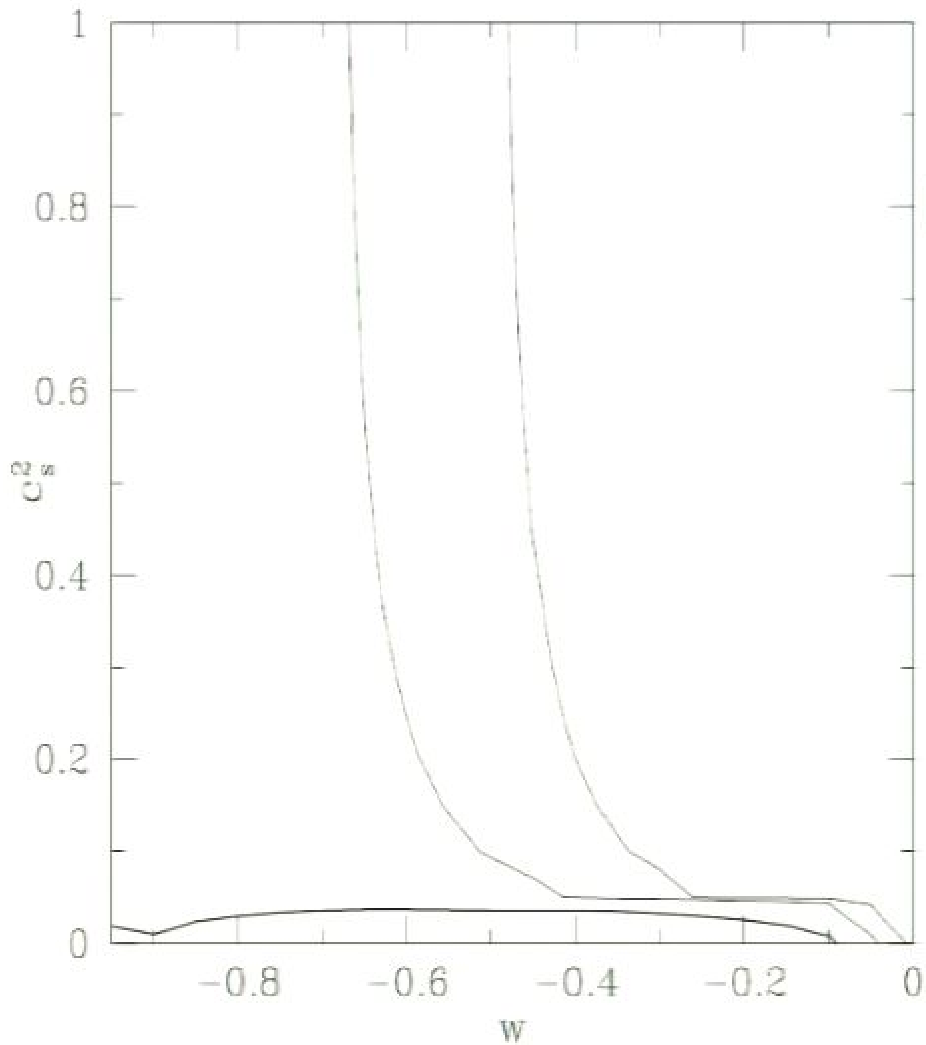
$\hat{c}_s^2$ of dark energy is unconstrained by present data (CMB, LSS, ...)



Hannestad 05

The speed of sound of dark energy

$\hat{c}_s^2$ of dark energy is unconstrained by present data (CMB, LSS, ...)



From CMB: $\hat{c}_s^2 < 0.04$ at 1σ level

Bean and Doré 03

The speed of sound

$$\hat{c}_s^2 \equiv \frac{\delta P}{\delta \rho}$$

- Matter: $\hat{c}_s^2 = 0$
- Radiation: $\hat{c}_s^2 = 1/3$
- Quintessence (scalar field DE) with a canonical kinetic term: $\hat{c}_s^2 = 1$
- K-essence (non-canonical kinetic terms): $\hat{c}_s^2 \neq 1$ (even $\to \infty$)
- Two scalar fields: $\hat{c}_s^2 \neq 1$
- String gas: $0 < \hat{c}_s^2 << 1$
- Chaplygin gas ($P \propto 1/\rho$): $\hat{c}_s^2 \to 0$
- ...

If $\hat{c}_s^2 \neq 1$ DE perturbations can be important.

The speed of sound

$$c_s^2 \equiv \frac{\delta P}{\delta \rho}$$

For a homogeneous perfect fluid (adiabatic):

$$c_a^2 \equiv \frac{P'}{\rho'} = w - \frac{w'}{3\mathcal{H}(1+w)}$$

Imperfect fluid (entropy perturbation):

$$w\Gamma \equiv \left(c_s^2 - c_a^2\right)\delta = \frac{P'}{\rho}\left(\frac{\delta P}{P'} - \frac{\delta\rho}{\rho'}\right)$$

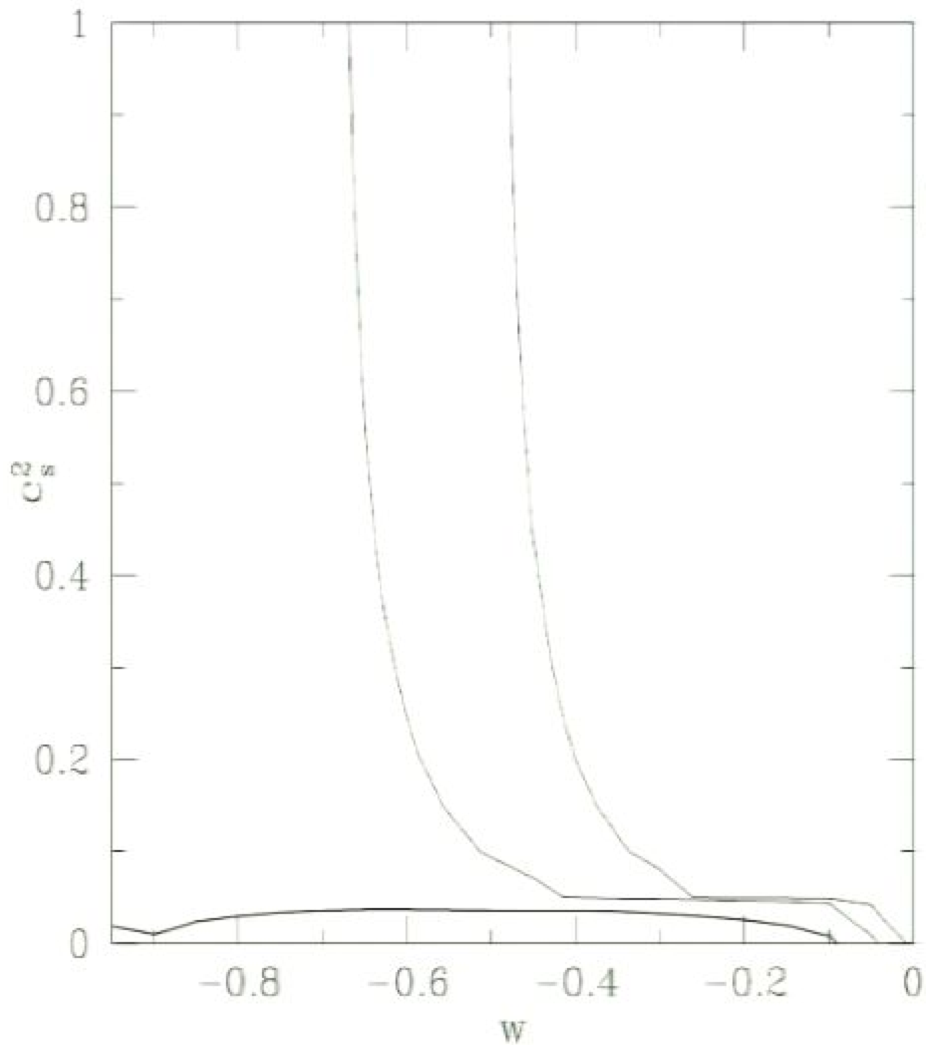
In general, c_s^2 is gauge dependent but c_a^2 and Γ are gauge invariant

$$\hat{\delta} = \delta + 3\mathcal{H}(1+w)\frac{\theta}{k^2} \implies \delta P = \hat{c}_s^2\,\delta\rho + 3\mathcal{H}\left(1+w\right)\left(\hat{c}_s^2 - w\right)\rho\frac{\theta}{k^2}$$

δ is invariant in the rest frame $\longrightarrow \hat{\delta}$

The speed of sound of dark energy

$\hat{c}_s^2$ of dark energy is unconstrained by present data (CMB, LSS, ...)

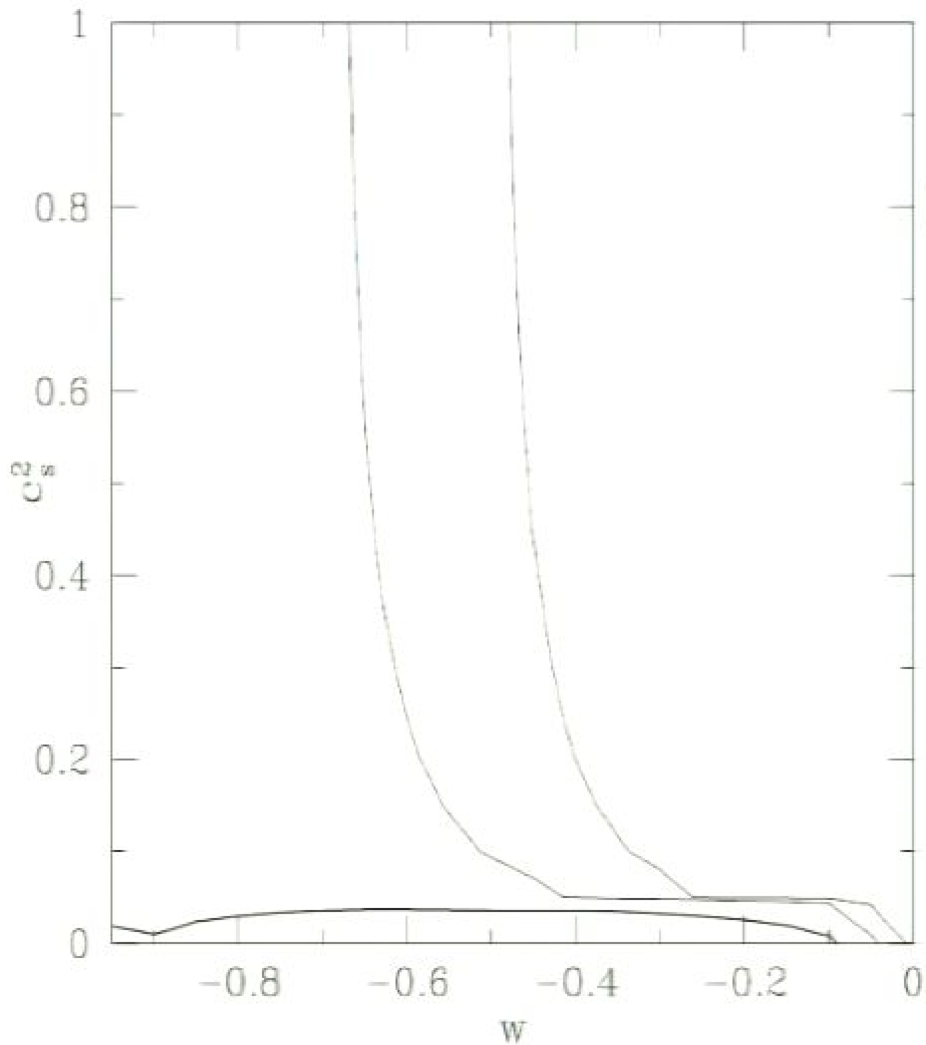


From CMB: $\hat{c}_s^2 < 0.04$ at 1σ level

Bean and Doré 03

The speed of sound of dark energy

$\hat{c}_s^2$ of dark energy is unconstrained by present data (CMB, LSS, ...)



From CMB: $\hat{c}_s^2 < 0.04$ at 1σ level

Bean and Doré 03

The speed of sound

$$c_s^2 \equiv \frac{\delta P}{\delta \rho}$$

For a homogeneous perfect fluid (adiabatic):

$$c_a^2 \equiv \frac{P'}{\rho'} = w - \frac{w'}{3\mathcal{H}(1+w)}$$

Imperfect fluid (entropy perturbation):

$$w\Gamma \equiv \left(c_s^2 - c_a^2\right)\delta = \frac{P'}{\rho}\left(\frac{\delta P}{P'} - \frac{\delta \rho}{\rho'}\right)$$

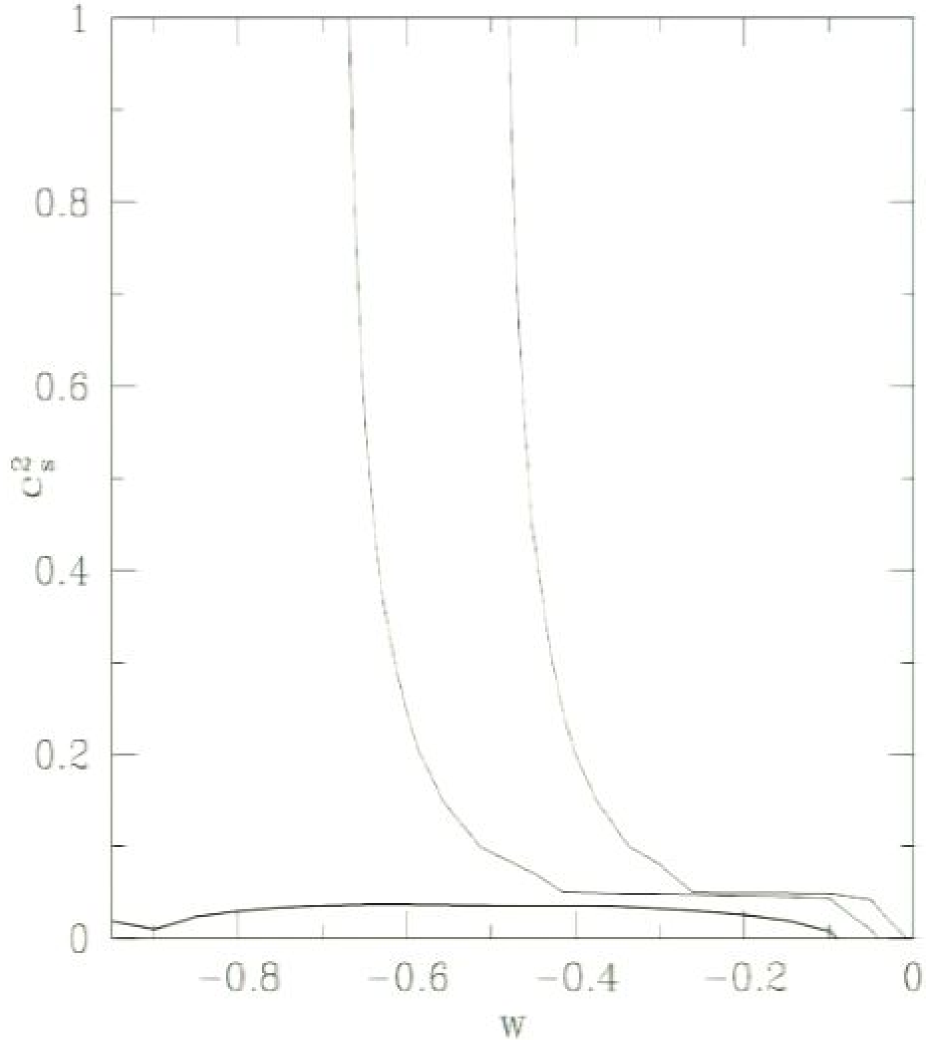
In general, c_s^2 is gauge dependent but c_a^2 and Γ are gauge invariant

$$\hat{\delta} = \delta + 3\mathcal{H}(1+w)\frac{\theta}{k^2} \implies \delta P = \hat{c}_s^2\,\delta\rho + 3\mathcal{H}\,(1+w)\left(\hat{c}_s^2 - w\right)\rho\frac{\theta}{k^2}$$

δ is invariant in the rest frame $\longrightarrow \hat{\delta}$

The speed of sound of dark energy

$\hat{c}_s^2$ of dark energy is unconstrained by present data (CMB, LSS, …)

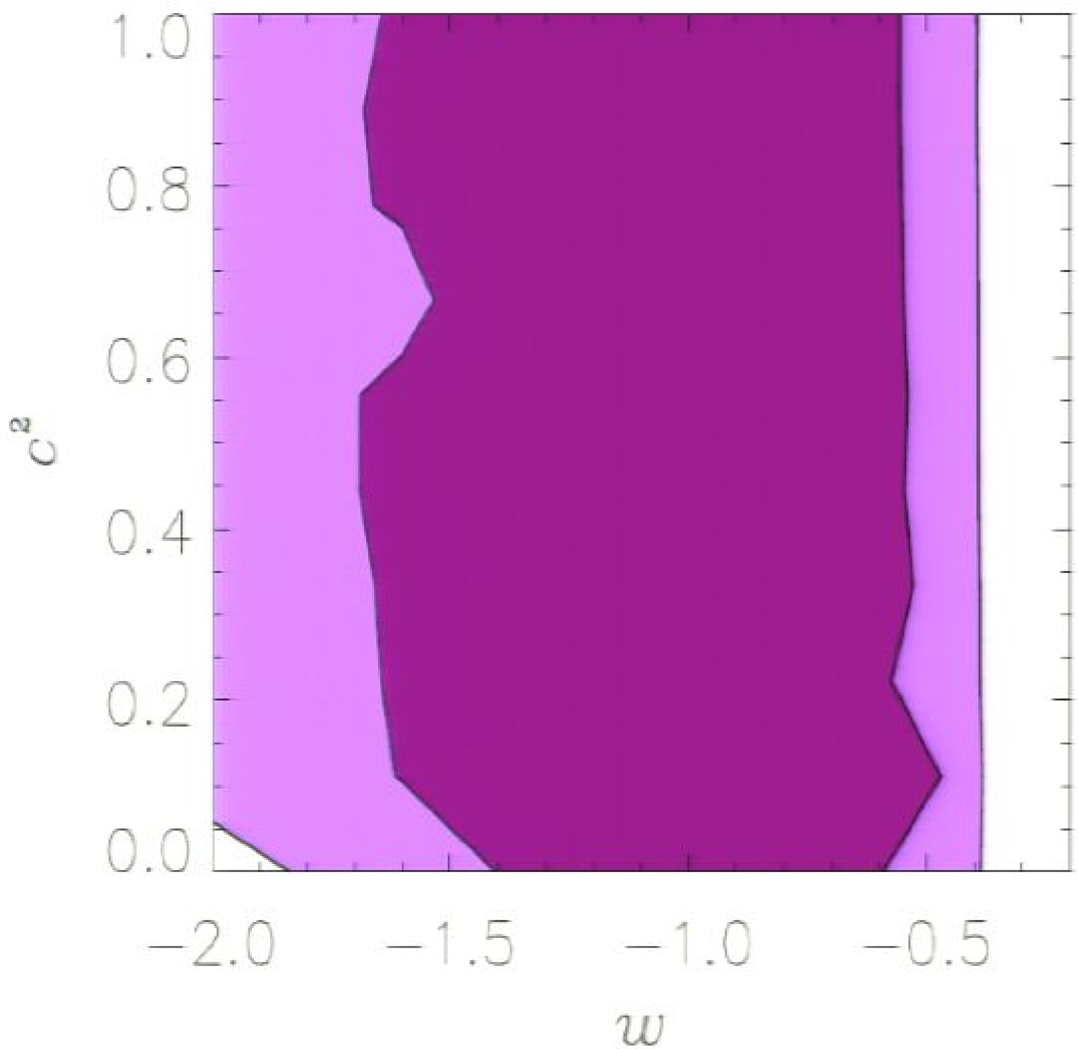


From CMB: $\hat{c}_s^2 < 0.04$ at 1σ level

Bean and Doré 03

The speed of sound of dark energy

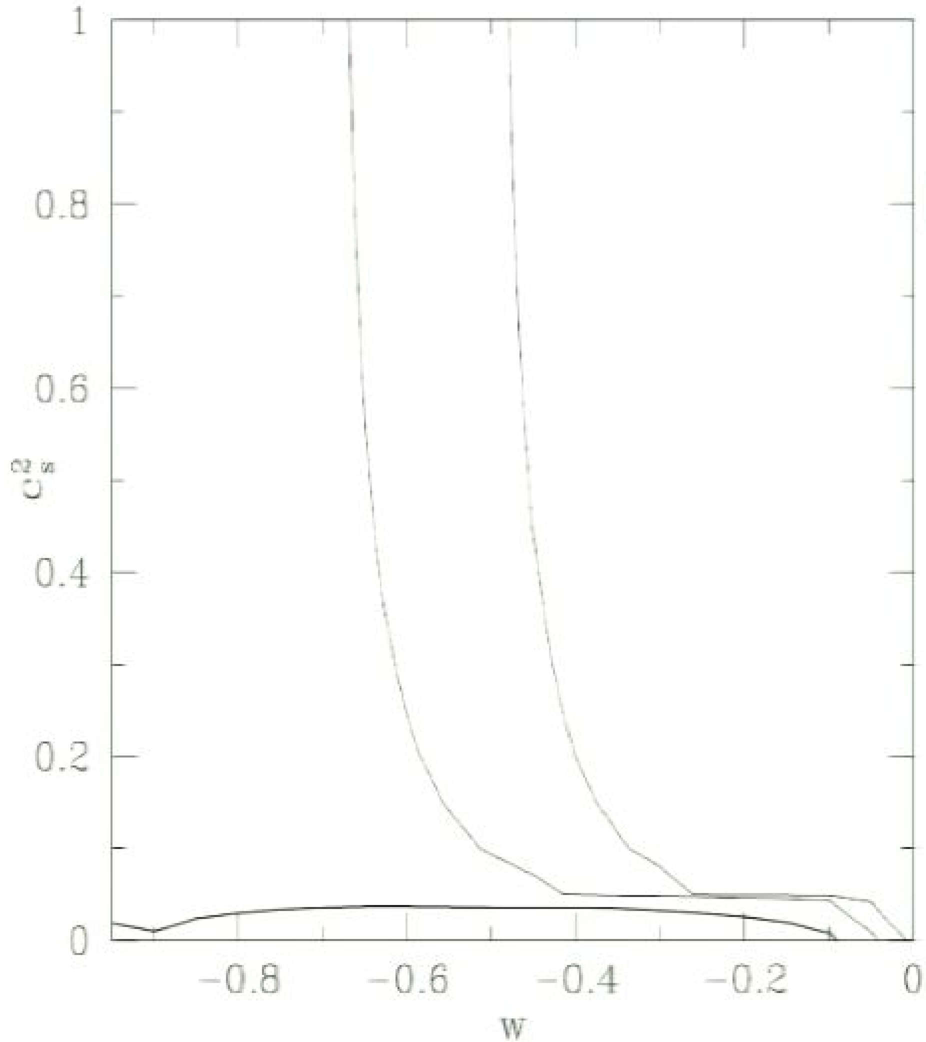
$\hat{c}_s^2$ of dark energy is unconstrained by present data (CMB, LSS, ...)



Hannestad 05

The speed of sound of dark energy

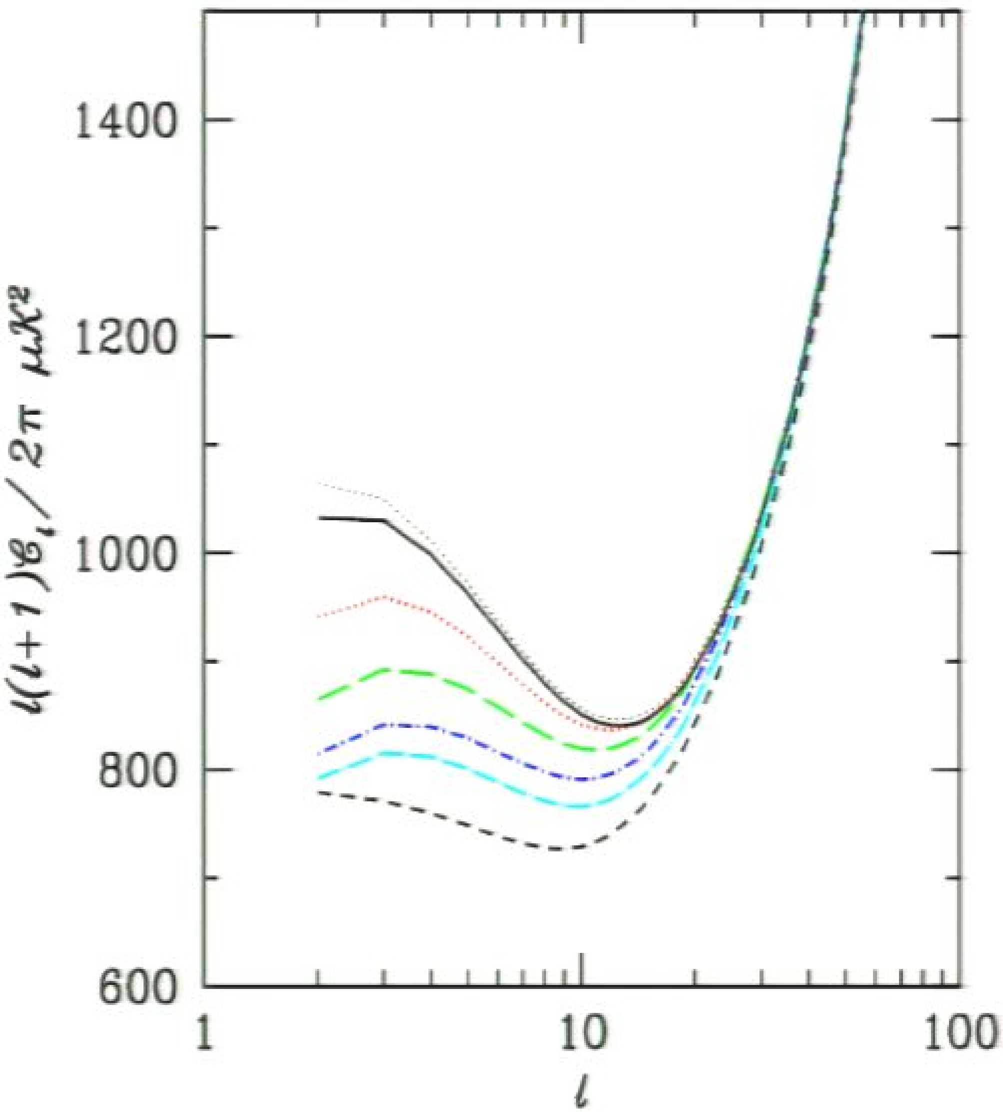
$\hat{c}_s^2$ of dark energy is unconstrained by present data (CMB, LSS, ...)



From CMB: $\hat{c}_s^2 < 0.04$ at 1σ level

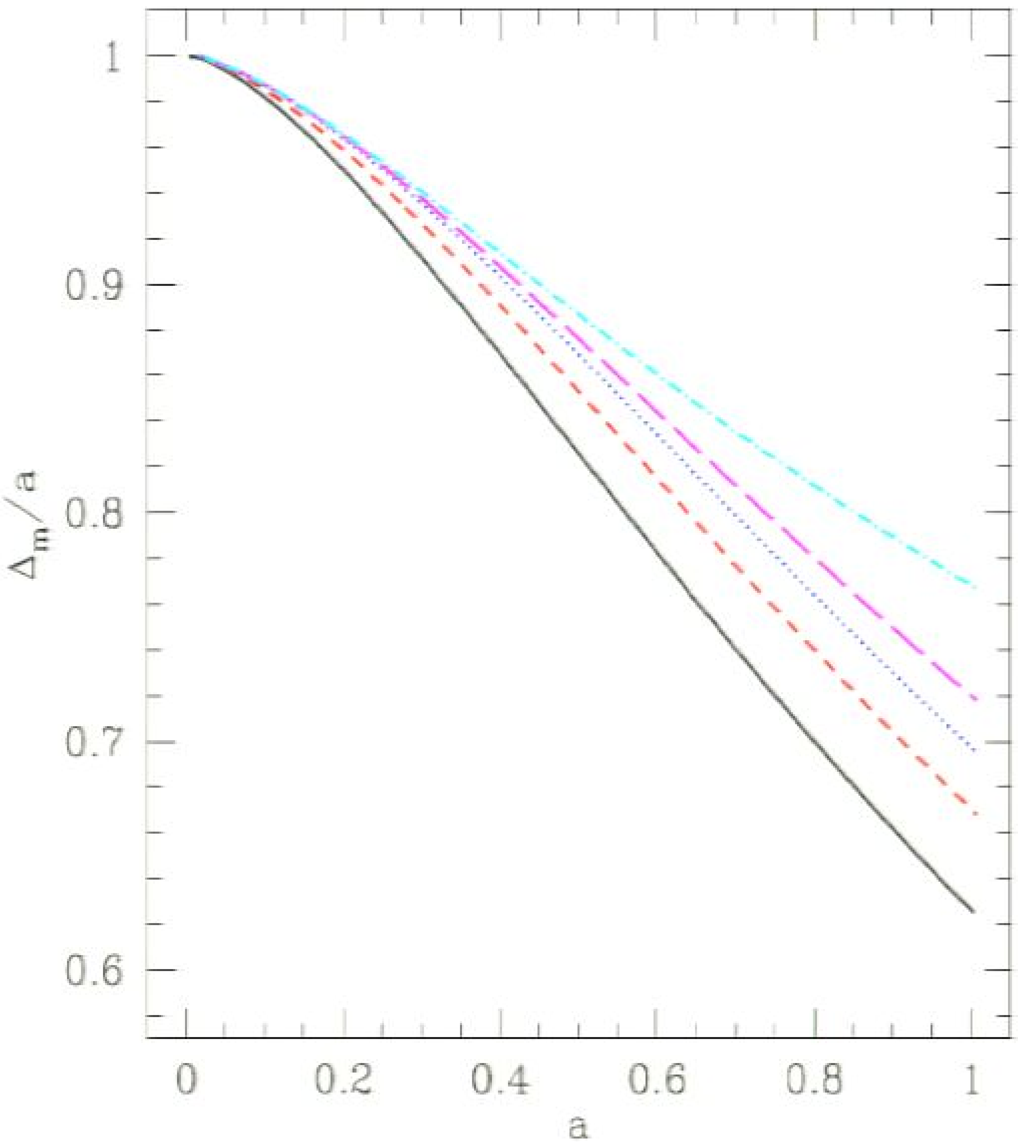
Bean and Doré 03

CMB and the DE speed of sound



Lewis and Weller 06

Growth factor and the speed of sound of dark energy



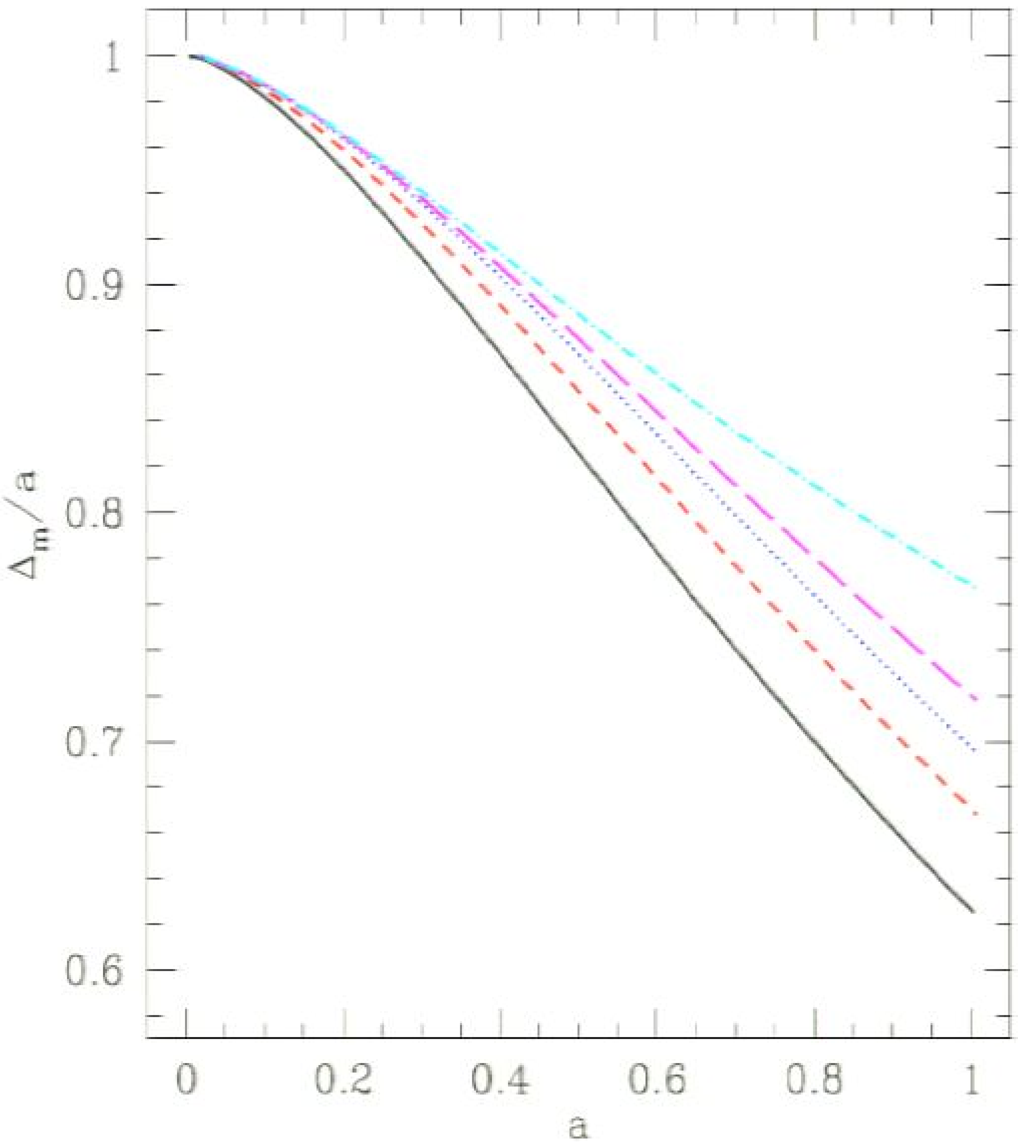
Kunz and Sapone 06

Dark energy perturbations

$$\delta_c'' + \mathcal{H}\delta_c' - \frac{3}{2}\mathcal{H}^2\Omega_c\delta_c = \frac{3}{2}\mathcal{H}^2\Omega_x\left[\left(1+3\hat{c}_s^2\right)\delta_x + 9\left(1+w\right)\mathcal{H}\left(\hat{c}_s^2 - w\right)\frac{\theta_x}{k^2}\right]$$

Bean and Doré 03

Growth factor and the speed of sound of dark energy



Kunz and Sapone 06

Dark energy perturbations

$$\delta_c'' + \mathcal{H}\delta_c' - \frac{3}{2}\mathcal{H}^2\Omega_c\delta_c = \frac{3}{2}\mathcal{H}^2\Omega_x\left[\left(1+3\hat{c}_s^2\right)\delta_x + 9\left(1+w\right)\mathcal{H}\left(\hat{c}_s^2 - w\right)\frac{\theta_x}{k^2}\right]$$

Bean and Doré 03

Dark energy perturbations

$$\delta_c'' + \mathcal{H}\delta_c' - \frac{3}{2}\mathcal{H}^2\Omega_c\delta_c = \frac{3}{2}\mathcal{H}^2\Omega_x\left[\left(1+3\hat{c}_s^2\right)\delta_x + 9\left(1+w\right)\mathcal{H}\left(\hat{c}_s^2 - w\right)\frac{\theta_x}{k^2}\right]$$

$$\begin{aligned}\delta_x'' &+ \left[3\left(\hat{c}_s^2 - w\right)\mathcal{H} - \mathcal{F}\right]\delta_x' \\ &+ \left\{\hat{c}_s^2 k^2 - \frac{3}{2}\left(\hat{c}_s^2 - w\right)\mathcal{H}\left[\left(1 + 3w\Omega_x - 6\hat{c}_s^2\right)\mathcal{H} + 2\mathcal{F}\right]\right\}\delta_x \\ &= (1+w)\delta_c'' - (1+w)\mathcal{F}\delta_c' \,.\end{aligned}$$

$$\frac{\theta_x'}{k^2} = -\left(1 - 3\hat{c}_s^2\right)\mathcal{H}\frac{\theta_x}{k^2} + \frac{\hat{c}_s^2}{1+w}\delta_x$$

$$\mathcal{F} \equiv -9\left(1 + 3w\Omega_x\right)\frac{\hat{c}_s^2 - w}{k^2 + 9\left(\hat{c}_s^2 - w\right)\mathcal{H}^2}\mathcal{H}^3 - (1 - 3\hat{c}_s^2)\mathcal{H} \,.$$

Dark energy perturbations

$$\delta_c'' + \mathcal{H}\delta_c' - \frac{3}{2}\mathcal{H}^2\Omega_c\delta_c = \frac{3}{2}\mathcal{H}^2\Omega_x\left[\left(1+3\hat{c}_s^2\right)\delta_x + 9\left(1+w\right)\mathcal{H}\left(\hat{c}_s^2 - w\right)\frac{\theta_x}{k^2}\right]$$

$$\delta_x'' = \delta_x''\left(\delta_x', \delta_x, \delta_c', \delta_c\right), \quad \theta_x' = \theta_x'\left(\delta_x, \theta_x\right)$$

Dark energy perturbations

$$\delta_c'' + \mathcal{H}\delta_c' - \frac{3}{2}\mathcal{H}^2\Omega_c\delta_c = \frac{3}{2}\mathcal{H}^2\Omega_x\left[\left(1+3\hat{c}_s^2\right)\delta_x + 9\left(1+w\right)\mathcal{H}\left(\hat{c}_s^2 - w\right)\frac{\theta_x}{k^2}\right]$$

$$\delta_x'' = \delta_x''\left(\delta_x', \delta_x, \delta_c', \delta_c\right), \quad \theta_x' = \theta_x'\left(\delta_x, \theta_x\right)$$

- Gravitational potentials (metric pert. d.o.f.) not present (synchronous gauge) $\mathrm{d}s^2 = a^2(\tau)\left[-\mathrm{d}\tau^2 + (\delta_{ij} + h_{ij})\,\mathrm{d}x^i\mathrm{d}x^j\right]$, zero curvature (dark matter velocity perturbation $\rightarrow 0$, fixing the gauge)
- DE perturbations are necessary for consistency (generated at later times even if set to zero initially)

Dark energy perturbations

$$\delta_c'' + \mathcal{H}\delta_c' - \frac{3}{2}\mathcal{H}^2\Omega_c\delta_c = \frac{3}{2}\mathcal{H}^2\Omega_x\left[\left(1+3\hat{c}_s^2\right)\delta_x + 9\left(1+w\right)\mathcal{H}\left(\hat{c}_s^2 - w\right)\frac{\theta_x}{k^2}\right]$$

$$\begin{aligned}\delta_x'' &+ \left[3\left(\hat{c}_s^2 - w\right)\mathcal{H} - \mathcal{F}\right]\delta_x' \\ &+ \left\{\hat{c}_s^2 k^2 - \frac{3}{2}\left(\hat{c}_s^2 - w\right)\mathcal{H}\left[\left(1 + 3w\Omega_x - 6\hat{c}_s^2\right)\mathcal{H} + 2\mathcal{F}\right]\right\}\delta_x \\ &= (1+w)\delta_c'' - (1+w)\mathcal{F}\delta_c' \,.\end{aligned}$$

$$\frac{\theta_x'}{k^2} = -\left(1 - 3\hat{c}_s^2\right)\mathcal{H}\frac{\theta_x}{k^2} + \frac{\hat{c}_s^2}{1+w}\delta_x$$

$$\mathcal{F} \equiv -9\left(1 + 3w\Omega_x\right)\frac{\hat{c}_s^2 - w}{k^2 + 9\left(\hat{c}_s^2 - w\right)\mathcal{H}^2}\mathcal{H}^3 - (1 - 3\hat{c}_s^2)\mathcal{H} \,.$$

Dark energy perturbations

$$\delta_c'' + \mathcal{H}\delta_c' - \frac{3}{2}\mathcal{H}^2\Omega_c\delta_c = \frac{3}{2}\mathcal{H}^2\Omega_x\left[\left(1+3\hat{c}_s^2\right)\delta_x + 9\left(1+w\right)\mathcal{H}\left(\hat{c}_s^2 - w\right)\frac{\theta_x}{k^2}\right]$$

$$\delta_x'' = \delta_x''\left(\delta_x', \delta_x, \delta_c', \delta_c\right), \quad \theta_x' = \theta_x'\left(\delta_x, \theta_x\right)$$

Dark energy perturbations

$$\delta_c'' + \mathcal{H}\delta_c' - \frac{3}{2}\mathcal{H}^2\Omega_c\delta_c = \frac{3}{2}\mathcal{H}^2\Omega_x\left[\left(1+3\hat{c}_s^2\right)\delta_x + 9\left(1+w\right)\mathcal{H}\left(\hat{c}_s^2 - w\right)\frac{\theta_x}{k^2}\right]$$

$$\delta_x'' = \delta_x''\left(\delta_x', \delta_x, \delta_c', \delta_c\right), \quad \theta_x' = \theta_x'\left(\delta_x, \theta_x\right)$$

- Gravitational potentials (metric pert. d.o.f.) not present (synchronous gauge) $\mathrm{d}s^2 = a^2(\tau)\left[-\mathrm{d}\tau^2 + \left(\delta_{ij} + h_{ij}\right)\mathrm{d}x^i\mathrm{d}x^j\right]$, zero curvature (dark matter velocity perturbation $\to 0$, fixing the gauge)
- DE perturbations are necessary for consistency (generated at later times even if set to zero initially)

Dark energy perturbations

$$\delta_c'' + \mathcal{H}\delta_c' - \frac{3}{2}\mathcal{H}^2\Omega_c\delta_c = \frac{3}{2}\mathcal{H}^2\Omega_x\left[\left(1+3\hat{c}_s^2\right)\delta_x + 9\left(1+w\right)\mathcal{H}\left(\hat{c}_s^2 - w\right)\frac{\theta_x}{k^2}\right]$$

$$\delta_x'' = \delta_x''\left(\delta_x', \delta_x, \delta_c', \delta_c\right), \quad \theta_x' = \theta_x'\left(\delta_x, \theta_x\right)$$

- Gravitational potentials (metric pert. d.o.f.) not present (synchronous gauge)
  $\mathrm{d}s^2 = a^2(\tau)\left[-\mathrm{d}\tau^2 + \left(\delta_{ij} + h_{ij}\right)\mathrm{d}x^i\mathrm{d}x^j\right]$, zero curvature
  (dark matter velocity perturbation $\to 0$, fixing the gauge)
- DE perturbations are necessary for consistency
  (generated at later times even if set to zero initially)

$$g(a) = g(a_i)\exp\int_{a_i}^{a}\left(\left[\Omega_m(\hat{a})\right]^{\gamma} - 1\right)\frac{\mathrm{d}\hat{a}}{\hat{a}}$$

Dark energy perturbations

$$\delta_c'' + \mathcal{H}\delta_c' - \frac{3}{2}\mathcal{H}^2\Omega_c\delta_c = \frac{3}{2}\mathcal{H}^2\Omega_x\left[\left(1+3\hat{c}_s^2\right)\delta_x + 9\left(1+w\right)\mathcal{H}\left(\hat{c}_s^2 - w\right)\frac{\theta_x}{k^2}\right]$$

$$\begin{aligned}
\delta_x'' &+ \left[3\left(\hat{c}_s^2 - w\right)\mathcal{H} - \mathcal{F}\right]\delta_x' \\
&+ \left\{\hat{c}_s^2 k^2 - \frac{3}{2}\left(\hat{c}_s^2 - w\right)\mathcal{H}\left[\left(1 + 3w\Omega_x - 6\hat{c}_s^2\right)\mathcal{H} + 2\mathcal{F}\right]\right\}\delta_x \\
&= (1+w)\delta_c'' - (1+w)\mathcal{F}\delta_c' \, .
\end{aligned}$$

$$\frac{\theta_x'}{k^2} = -\left(1 - 3\hat{c}_s^2\right)\mathcal{H}\frac{\theta_x}{k^2} + \frac{\hat{c}_s^2}{1+w}\delta_x$$

$$\mathcal{F} \equiv -9\left(1 + 3w\Omega_x\right)\frac{\hat{c}_s^2 - w}{k^2 + 9\left(\hat{c}_s^2 - w\right)\mathcal{H}^2}\mathcal{H}^3 - (1 - 3\hat{c}_s^2)\mathcal{H} \, .$$

Dark energy perturbations

$$\delta_c'' + \mathcal{H}\delta_c' - \frac{3}{2}\mathcal{H}^2\Omega_c\delta_c = \frac{3}{2}\mathcal{H}^2\Omega_x\left[\left(1+3\hat{c}_s^2\right)\delta_x + 9\left(1+w\right)\mathcal{H}\left(\hat{c}_s^2 - w\right)\frac{\theta_x}{k^2}\right]$$

$$\delta_x'' = \delta_x''\left(\delta_x', \delta_x, \delta_c', \delta_c\right), \quad \theta_x' = \theta_x'\left(\delta_x, \theta_x\right)$$

- Gravitational potentials (metric pert. d.o.f.) not present (synchronous gauge)
  $ds^2 = a^2(\tau)\left[-d\tau^2 + \left(\delta_{ij} + h_{ij}\right)dx^i dx^j\right]$, zero curvature
  (dark matter velocity perturbation $\rightarrow 0$, fixing the gauge)
- DE perturbations are necessary for consistency
  (generated at later times even if set to zero initially)

Dark energy perturbations

$$\delta_c'' + \mathcal{H}\delta_c' - \frac{3}{2}\mathcal{H}^2\Omega_c\delta_c = \frac{3}{2}\mathcal{H}^2\Omega_x\left[\left(1+3\hat{c}_s^2\right)\delta_x + 9\left(1+w\right)\mathcal{H}\left(\hat{c}_s^2 - w\right)\frac{\theta_x}{k^2}\right]$$

$$\delta_x'' = \delta_x''\left(\delta_x', \delta_x, \delta_c', \delta_c\right), \quad \theta_x' = \theta_x'\left(\delta_x, \theta_x\right)$$

Dark energy perturbations

$$\delta_c'' + \mathcal{H}\delta_c' - \frac{3}{2}\mathcal{H}^2\Omega_c\delta_c = \frac{3}{2}\mathcal{H}^2\Omega_x\left[\left(1+3\hat{c}_s^2\right)\delta_x + 9\left(1+w\right)\mathcal{H}\left(\hat{c}_s^2 - w\right)\frac{\theta_x}{k^2}\right]$$

$$\begin{aligned}\delta_x'' &+ \left[3\left(\hat{c}_s^2 - w\right)\mathcal{H} - \mathcal{F}\right]\delta_x' \\ &+ \left\{\hat{c}_s^2 k^2 - \frac{3}{2}\left(\hat{c}_s^2 - w\right)\mathcal{H}\left[\left(1+3w\Omega_x - 6\hat{c}_s^2\right)\mathcal{H} + 2\mathcal{F}\right]\right\}\delta_x \\ &= (1+w)\delta_c'' - (1+w)\mathcal{F}\delta_c' \,.\end{aligned}$$

$$\frac{\theta_x'}{k^2} = -\left(1-3\hat{c}_s^2\right)\mathcal{H}\frac{\theta_x}{k^2} + \frac{\hat{c}_s^2}{1+w}\delta_x$$

$$\mathcal{F} \equiv -9\left(1+3w\Omega_x\right)\frac{\hat{c}_s^2 - w}{k^2 + 9\left(\hat{c}_s^2 - w\right)\mathcal{H}^2}\mathcal{H}^3 - (1-3\hat{c}_s^2)\mathcal{H} \,.$$

Dark energy perturbations

$$\delta_c'' + \mathcal{H}\delta_c' - \frac{3}{2}\mathcal{H}^2\Omega_c\delta_c = \frac{3}{2}\mathcal{H}^2\Omega_x\left[\left(1+3\hat{c}_s^2\right)\delta_x + 9\left(1+w\right)\mathcal{H}\left(\hat{c}_s^2 - w\right)\frac{\theta_x}{k^2}\right]$$

$$\delta_x'' = \delta_x''\left(\delta_x', \delta_x, \delta_c', \delta_c\right), \quad \theta_x' = \theta_x'\left(\delta_x, \theta_x\right)$$

Dark energy perturbations

$$\delta_c'' + \mathcal{H}\delta_c' - \frac{3}{2}\mathcal{H}^2\Omega_c\delta_c = \frac{3}{2}\mathcal{H}^2\Omega_x\left[\left(1+3\hat{c}_s^2\right)\delta_x + 9\left(1+w\right)\mathcal{H}\left(\hat{c}_s^2 - w\right)\frac{\theta_x}{k^2}\right]$$

$$\delta_x'' = \delta_x''\left(\delta_x', \delta_x, \delta_c', \delta_c\right), \quad \theta_x' = \theta_x'\left(\delta_x, \theta_x\right)$$

- Gravitational potentials (metric pert. d.o.f.) not present (synchronous gauge) $\mathrm{d}s^2 = a^2(\tau)\left[-\mathrm{d}\tau^2 + \left(\delta_{ij} + h_{ij}\right)\mathrm{d}x^i\mathrm{d}x^j\right]$, zero curvature (dark matter velocity perturbation $\rightarrow 0$, fixing the gauge)
- DE perturbations are necessary for consistency (generated at later times even if set to zero initially)

Dark energy perturbations

$$\delta_c'' + \mathcal{H}\delta_c' - \frac{3}{2}\mathcal{H}^2\Omega_c\delta_c = \frac{3}{2}\mathcal{H}^2\Omega_x\left[\left(1+3\hat{c}_s^2\right)\delta_x + 9\left(1+w\right)\mathcal{H}\left(\hat{c}_s^2 - w\right)\frac{\theta_x}{k^2}\right]$$

$$\delta_x'' = \delta_x''\left(\delta_x', \delta_x, \delta_c', \delta_c\right), \quad \theta_x' = \theta_x'\left(\delta_x, \theta_x\right)$$

- Gravitational potentials (metric pert. d.o.f.) not present (synchronous gauge)
  $\mathrm{d}s^2 = a^2(\tau)\left[-\mathrm{d}\tau^2 + (\delta_{ij} + h_{ij})\,\mathrm{d}x^i\mathrm{d}x^j\right]$, zero curvature
  (dark matter velocity perturbation $\to 0$, fixing the gauge)
- DE perturbations are necessary for consistency (generated at later times even if set to zero initially)

$$g(a) = g(a_i)\exp\int_{a_i}^{a}\left(\left[\Omega_m(\hat{a})\right]^{\gamma} - 1\right)\frac{\mathrm{d}\hat{a}}{\hat{a}}$$

Dark energy perturbations

$$\delta_c'' + \mathcal{H}\delta_c' - \frac{3}{2}\mathcal{H}^2\Omega_c\delta_c = \frac{3}{2}\mathcal{H}^2\Omega_x\left[\left(1+3\hat{c}_s^2\right)\delta_x + 9\left(1+w\right)\mathcal{H}\left(\hat{c}_s^2 - w\right)\frac{\theta_x}{k^2}\right]$$

$$\delta_x'' = \delta_x''\left(\delta_x', \delta_x, \delta_c', \delta_c\right), \quad \theta_x' = \theta_x'\left(\delta_x, \theta_x\right)$$

- Gravitational potentials (metric pert. d.o.f.) not present (synchronous gauge)
  $\mathrm{d}s^2 = a^2(\tau)\left[-\mathrm{d}\tau^2 + \left(\delta_{ij} + h_{ij}\right)\mathrm{d}x^i\mathrm{d}x^j\right]$, zero curvature
  (dark matter velocity perturbation $\to 0$, fixing the gauge)
- DE perturbations are necessary for consistency
  (generated at later times even if set to zero initially)

$$\gamma = \left(\ln\Omega_m\right)^{-1}\ln\left(\frac{a}{\delta_c}\frac{\mathrm{d}\delta_c}{\mathrm{d}a}\right)$$

$$\gamma = \gamma\left(\Omega_m^0, \hat{c}_s, k, w, a\right)$$

Dark energy perturbations

$$\delta_c'' + \mathcal{H}\delta_c' - \frac{3}{2}\mathcal{H}^2\Omega_c\delta_c = \frac{3}{2}\mathcal{H}^2\Omega_x\left[\left(1+3\hat{c}_s^2\right)\delta_x + 9\left(1+w\right)\mathcal{H}\left(\hat{c}_s^2 - w\right)\frac{\theta_x}{k^2}\right]$$

$$\delta_x'' = \delta_x''\left(\delta_x', \delta_x, \delta_c', \delta_c\right), \quad \theta_x' = \theta_x'\left(\delta_x, \theta_x\right)$$

- Gravitational potentials (metric pert. d.o.f.) not present (synchronous gauge)
  $\mathrm{d}s^2 = a^2(\tau)\left[-\mathrm{d}\tau^2 + (\delta_{ij} + h_{ij})\,\mathrm{d}x^i\mathrm{d}x^j\right]$, zero curvature
  (dark matter velocity perturbation $\rightarrow 0$, fixing the gauge)
- DE perturbations are necessary for consistency
  (generated at later times even if set to zero initially)

$$g(a) = g(a_i)\exp\int_{a_i}^{a}\left(\left[\Omega_m(\hat{a})\right]^{\gamma} - 1\right)\frac{\mathrm{d}\hat{a}}{\hat{a}}$$

Dark energy perturbations

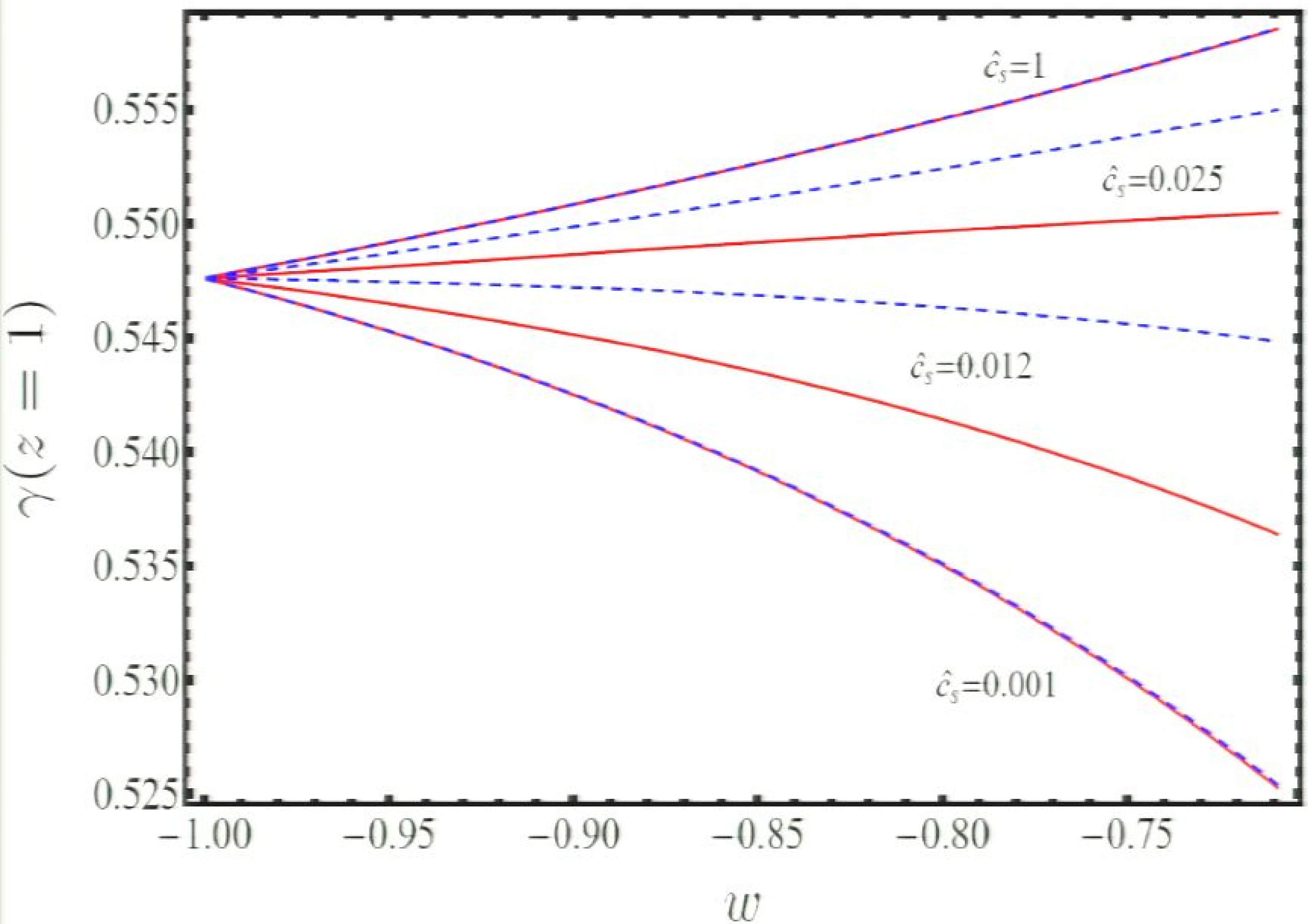
$$\delta_c'' + \mathcal{H}\delta_c' - \frac{3}{2}\mathcal{H}^2\Omega_c\delta_c = \frac{3}{2}\mathcal{H}^2\Omega_x\left[\left(1+3\hat{c}_s^2\right)\delta_x + 9\left(1+w\right)\mathcal{H}\left(\hat{c}_s^2 - w\right)\frac{\theta_x}{k^2}\right]$$

$$\delta_x'' = \delta_x''\left(\delta_x', \delta_x, \delta_c', \delta_c\right), \quad \theta_x' = \theta_x'\left(\delta_x, \theta_x\right)$$

- Gravitational potentials (metric pert. d.o.f.) not present (synchronous gauge) $\mathrm{d}s^2 = a^2(\tau)\left[-\mathrm{d}\tau^2 + \left(\delta_{ij} + h_{ij}\right)\mathrm{d}x^i\mathrm{d}x^j\right]$, zero curvature (dark matter velocity perturbation $\rightarrow 0$, fixing the gauge)
- DE perturbations are necessary for consistency (generated at later times even if set to zero initially)

$$\gamma = \left(\ln\Omega_m\right)^{-1}\ln\left(\frac{a}{\delta_c}\frac{\mathrm{d}\delta_c}{\mathrm{d}a}\right)$$

$$\gamma = \gamma\left(\Omega_m^0, \hat{c}_s, k, w, a\right)$$



Red: $k = 0.050\, h\mathrm{Mpc}^{-1}$ and Blue dashed: $k = 0.078\, h\mathrm{Mpc}^{-1}$

Dark energy perturbations

$$\delta_c'' + \mathcal{H}\delta_c' - \frac{3}{2}\mathcal{H}^2\Omega_c\delta_c = \frac{3}{2}\mathcal{H}^2\Omega_x\left[\left(1+3\hat{c}_s^2\right)\delta_x + 9\left(1+w\right)\mathcal{H}\left(\hat{c}_s^2 - w\right)\frac{\theta_x}{k^2}\right]$$

$$\delta_x'' = \delta_x''\left(\delta_x', \delta_x, \delta_c', \delta_c\right), \quad \theta_x' = \theta_x'\left(\delta_x, \theta_x\right)$$

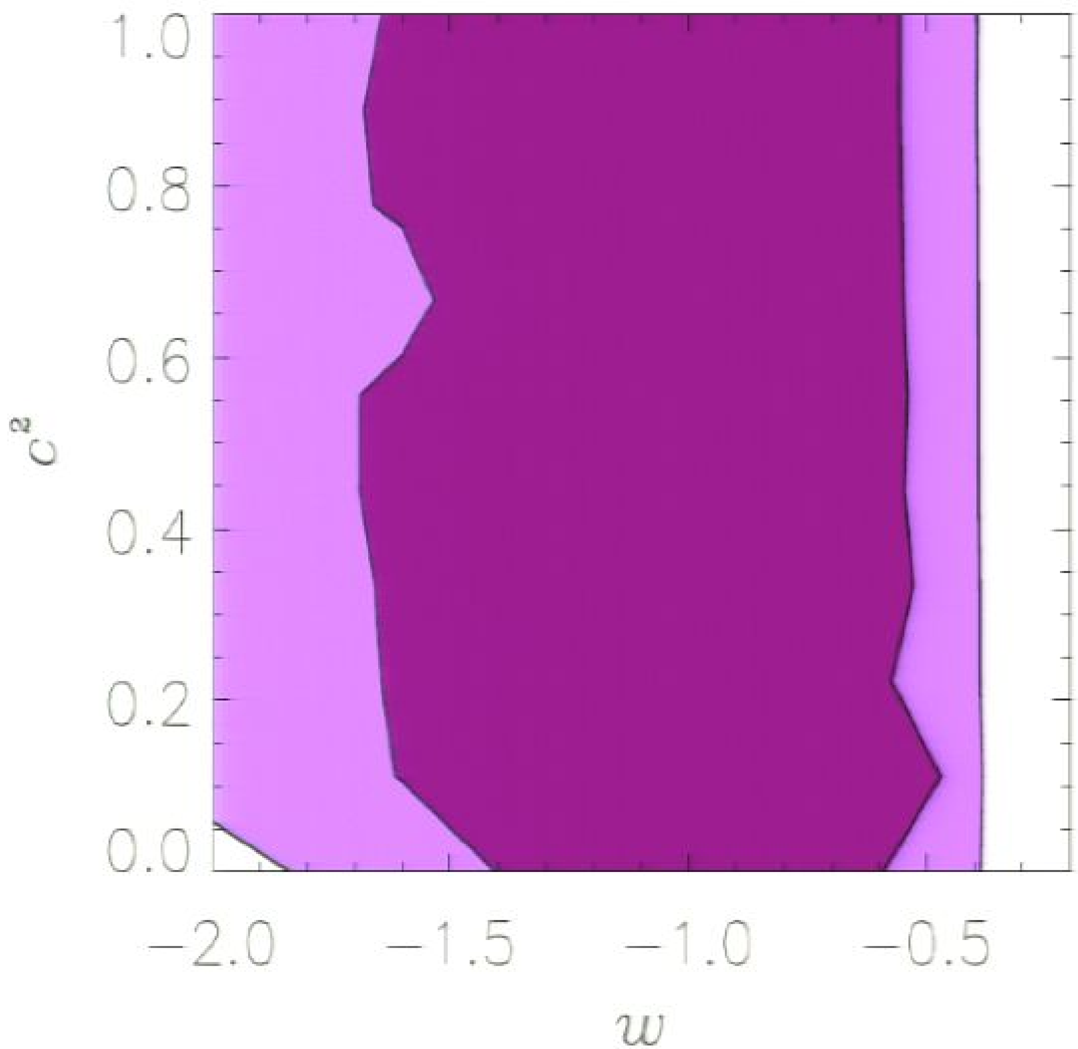
- Gravitational potentials (metric pert. d.o.f.) not present (synchronous gauge) $\mathrm{d}s^2 = a^2(\tau)\left[-\mathrm{d}\tau^2 + \left(\delta_{ij} + h_{ij}\right)\mathrm{d}x^i\mathrm{d}x^j\right]$, zero curvature (dark matter velocity perturbation $\rightarrow 0$, fixing the gauge)
- DE perturbations are necessary for consistency (generated at later times even if set to zero initially)

$$\gamma = \left(\ln\Omega_m\right)^{-1}\ln\left(\frac{a}{\delta_c}\frac{\mathrm{d}\delta_c}{\mathrm{d}a}\right)$$

$$\gamma = \gamma\left(\Omega_m^0, \hat{c}_s, k, w, a\right)$$

The speed of sound of dark energy

$\hat{c}_s^2$ of dark energy is unconstrained by present data (CMB, LSS, ...)



Hannestad 05

The speed of sound

$$c_s^2 \equiv \frac{\delta P}{\delta \rho}$$

For a homogeneous perfect fluid (adiabatic):

$$c_a^2 \equiv \frac{P'}{\rho'} = w - \frac{w'}{3\mathcal{H}(1+w)}$$

Imperfect fluid (entropy perturbation):

$$w\Gamma \equiv \left(c_s^2 - c_a^2\right)\delta = \frac{P'}{\rho}\left(\frac{\delta P}{P'} - \frac{\delta\rho}{\rho'}\right)$$

In general, c_s^2 is gauge dependent but c_a^2 and Γ are gauge invariant

$$\hat{\delta} = \delta + 3\mathcal{H}(1+w)\frac{\theta}{k^2} \implies \delta P = \hat{c}_s^2\,\delta\rho + 3\mathcal{H}\,(1+w)\left(\hat{c}_s^2 - w\right)\rho\frac{\theta}{k^2}$$

δ is invariant in the rest frame $\longrightarrow \hat{\delta}$

Dark energy perturbations

$$\delta_c'' + \mathcal{H}\delta_c' - \frac{3}{2}\mathcal{H}^2\Omega_c\delta_c = \frac{3}{2}\mathcal{H}^2\Omega_x\left[\left(1+3\hat{c}_s^2\right)\delta_x + 9\left(1+w\right)\mathcal{H}\left(\hat{c}_s^2 - w\right)\frac{\theta_x}{k^2}\right]$$

$$\begin{aligned}\delta_x'' &+ \left[3\left(\hat{c}_s^2 - w\right)\mathcal{H} - \mathcal{F}\right]\delta_x' \\ &+ \left\{\hat{c}_s^2 k^2 - \frac{3}{2}\left(\hat{c}_s^2 - w\right)\mathcal{H}\left[\left(1+3w\Omega_x - 6\hat{c}_s^2\right)\mathcal{H} + 2\mathcal{F}\right]\right\}\delta_x \\ &= (1+w)\delta_c'' - (1+w)\mathcal{F}\delta_c' \, .\end{aligned}$$

$$\frac{\theta_x'}{k^2} = -\left(1-3\hat{c}_s^2\right)\mathcal{H}\frac{\theta_x}{k^2} + \frac{\hat{c}_s^2}{1+w}\delta_x$$

$$\mathcal{F} \equiv -9\left(1+3w\Omega_x\right)\frac{\hat{c}_s^2 - w}{k^2 + 9\left(\hat{c}_s^2 - w\right)\mathcal{H}^2}\mathcal{H}^3 - (1-3\hat{c}_s^2)\mathcal{H} \, .$$

Dark energy perturbations

$$\delta_c'' + \mathcal{H}\delta_c' - \frac{3}{2}\mathcal{H}^2\Omega_c\delta_c = \frac{3}{2}\mathcal{H}^2\Omega_x\left[\left(1+3\hat{c}_s^2\right)\delta_x + 9\left(1+w\right)\mathcal{H}\left(\hat{c}_s^2 - w\right)\frac{\theta_x}{k^2}\right]$$

$$\delta_x'' = \delta_x''\left(\delta_x', \delta_x, \delta_c', \delta_c\right), \quad \theta_x' = \theta_x'\left(\delta_x, \theta_x\right)$$

- Gravitational potentials (metric pert. d.o.f.) not present (synchronous gauge)
  $\mathrm{d}s^2 = a^2(\tau)\left[-\mathrm{d}\tau^2 + \left(\delta_{ij} + h_{ij}\right)\mathrm{d}x^i\mathrm{d}x^j\right]$, zero curvature
  (dark matter velocity perturbation $\to 0$, fixing the gauge)
- DE perturbations are necessary for consistency
  (generated at later times even if set to zero initially)

Dark energy perturbations

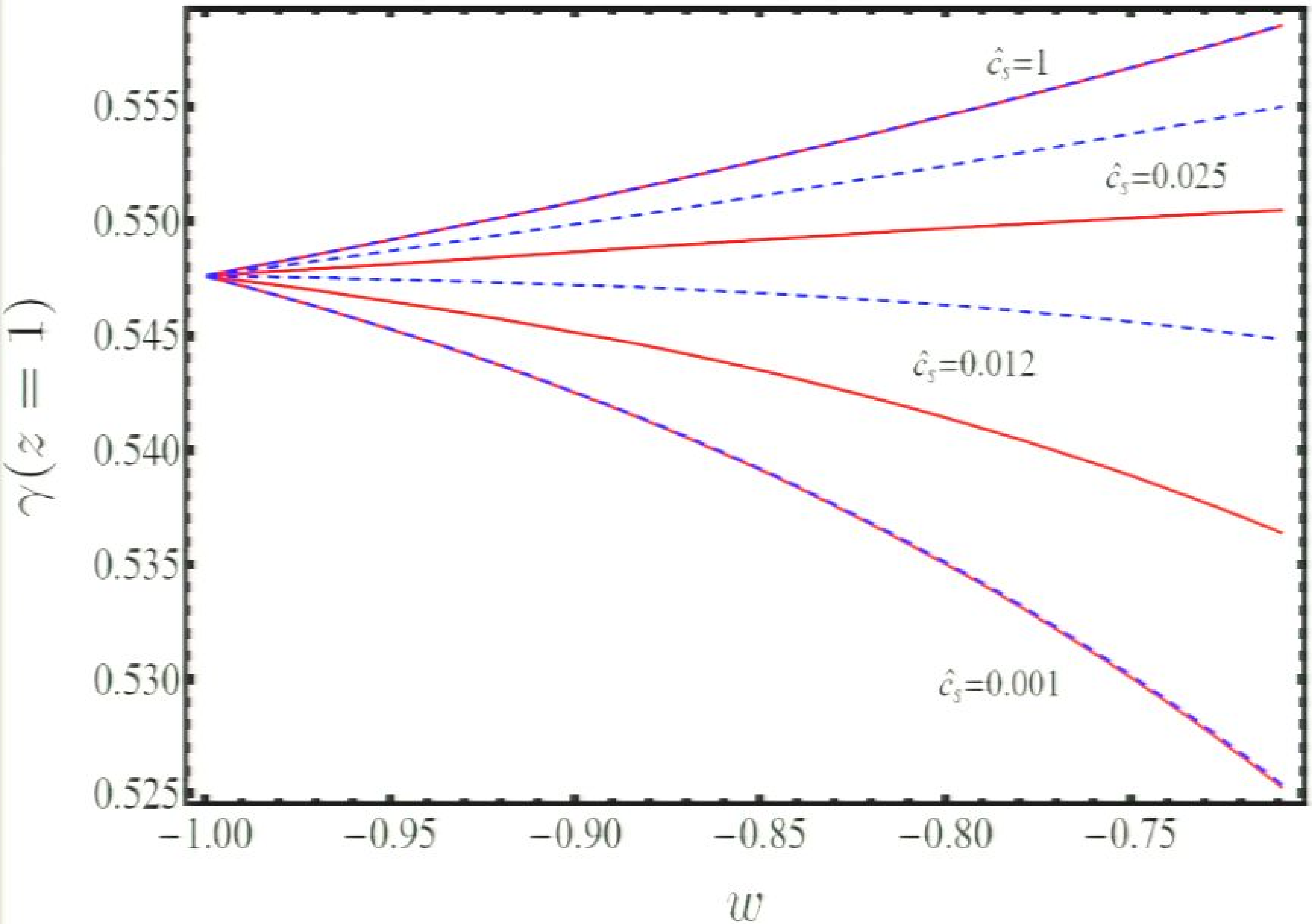
$$\delta_c'' + \mathcal{H}\delta_c' - \frac{3}{2}\mathcal{H}^2\Omega_c\delta_c = \frac{3}{2}\mathcal{H}^2\Omega_x\left[\left(1+3\hat{c}_s^2\right)\delta_x + 9\left(1+w\right)\mathcal{H}\left(\hat{c}_s^2 - w\right)\frac{\theta_x}{k^2}\right]$$

$$\delta_x'' = \delta_x''\left(\delta_x', \delta_x, \delta_c', \delta_c\right), \quad \theta_x' = \theta_x'\left(\delta_x, \theta_x\right)$$

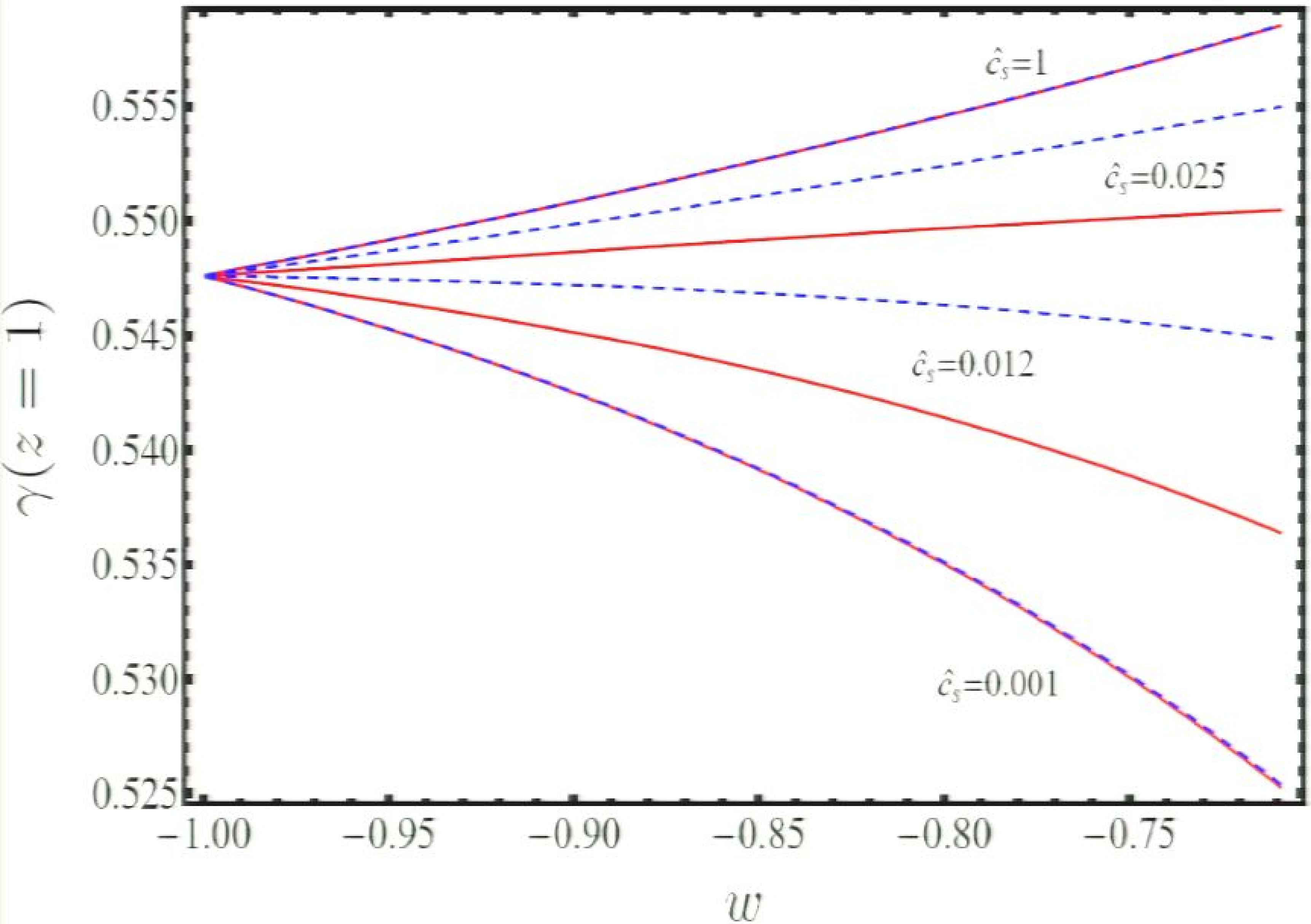
- Gravitational potentials (metric pert. d.o.f.) not present (synchronous gauge) $\mathrm{d}s^2 = a^2(\tau)\left[-\mathrm{d}\tau^2 + \left(\delta_{ij} + h_{ij}\right)\mathrm{d}x^i\mathrm{d}x^j\right]$, zero curvature (dark matter velocity perturbation $\rightarrow 0$, fixing the gauge)
- DE perturbations are necessary for consistency (generated at later times even if set to zero initially)

$$\gamma = \left(\ln\Omega_m\right)^{-1}\ln\left(\frac{a}{\delta_c}\frac{\mathrm{d}\delta_c}{\mathrm{d}a}\right)$$

$$\gamma = \gamma\left(\Omega_m^0, \hat{c}_s, k, w, a\right)$$



Red: $k = 0.050\,h\mathrm{Mpc}^{-1}$ and Blue dashed: $k = 0.078\,h\mathrm{Mpc}^{-1}$



Red: $k = 0.050\, h\mathrm{Mpc}^{-1}$ and Blue dashed: $k = 0.078\, h\mathrm{Mpc}^{-1}$

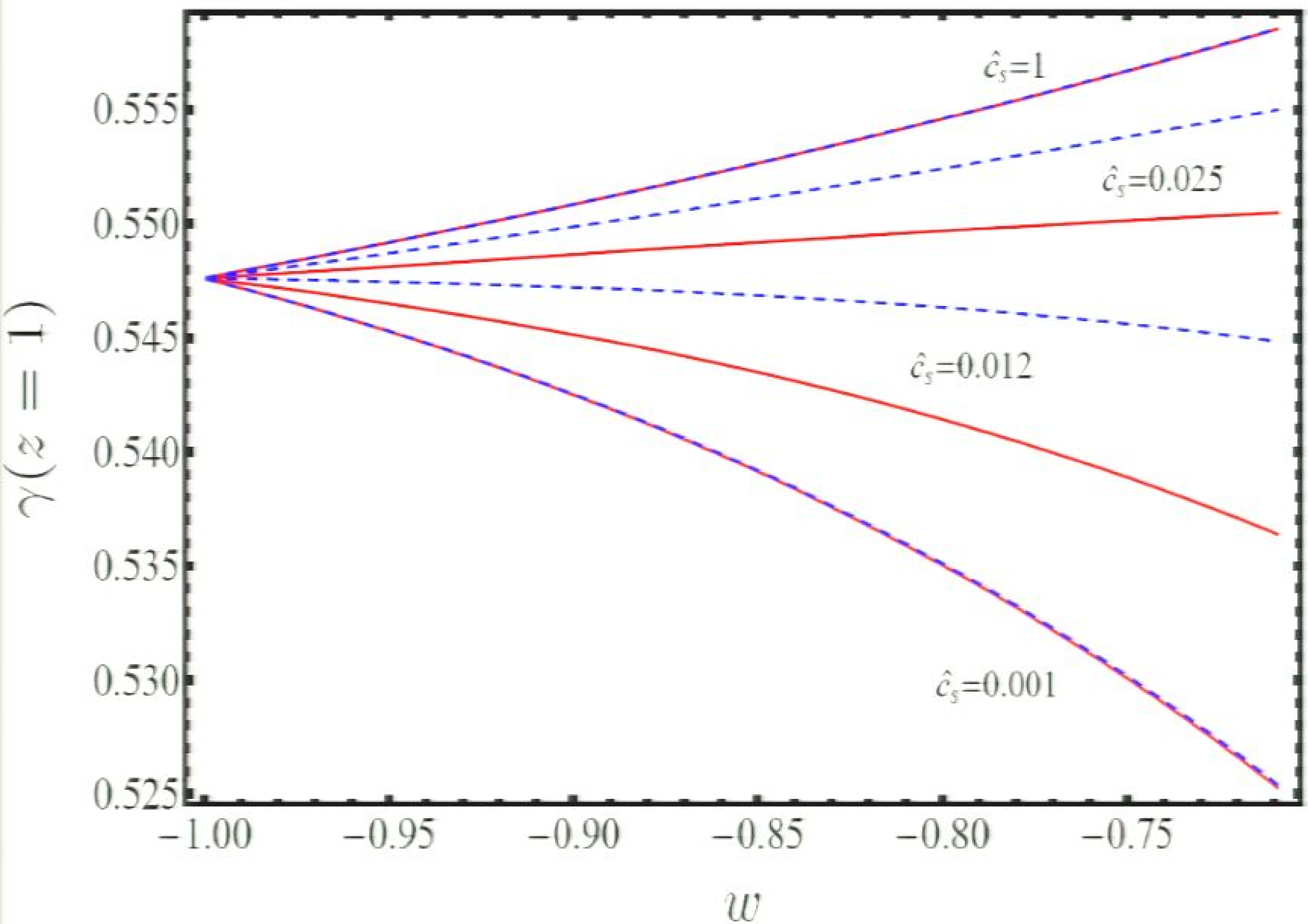
Status of γ

Theory

- Different theories ($f(R)$, DGP, GR...) predict different growths
- But the results can be mimicked with a general DE fluid
- A parameterization of γ can help to determine DE properties

Experiment

- Current data (BAO, WL, LSS, X-ray clust., Lyman–α, ISW) still poor
- Forecasted accuracy of future experiments $\Delta\gamma \simeq 0.04 \to 7\%$...
- Effect of DE perturbations also can be as large as $\Delta\gamma$



Red: $k = 0.050\,h\mathrm{Mpc}^{-1}$ and Blue dashed: $k = 0.078\,h\mathrm{Mpc}^{-1}$

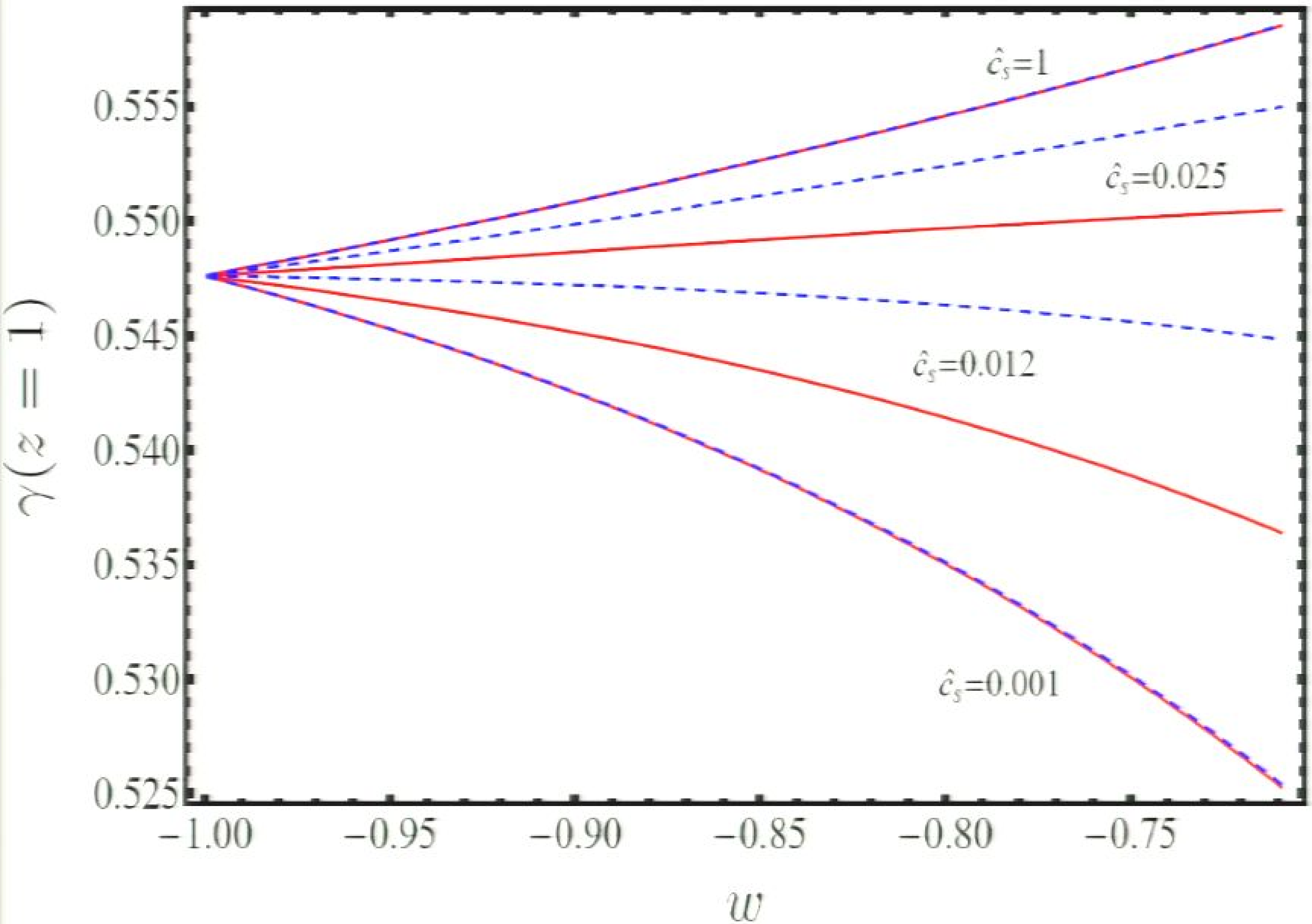
Status of γ

Theory

- Different theories ($f(R)$, DGP, GR...) predict different growths
- But the results can be mimicked with a general DE fluid
- A parameterization of γ can help to determine DE properties

Experiment

- Current data (BAO, WL, LSS, X-ray clust., Lyman–α, ISW) still poor
- Forecasted accuracy of future experiments $\Delta\gamma \simeq 0.04 \to 7\%\ldots$
- Effect of DE perturbations also can be as large as $\Delta\gamma$



Red: $k = 0.050\, h\mathrm{Mpc}^{-1}$ and Blue dashed: $k = 0.078\, h\mathrm{Mpc}^{-1}$

Status of γ

Theory

- Different theories ($f(R)$, DGP, GR...) predict different growths
- But the results can be mimicked with a general DE fluid
- A parameterization of γ can help to determine DE properties

Experiment

- Current data (BAO, WL, LSS, X-ray clust., Lyman–α, ISW) still poor
- Forecasted accuracy of future experiments $\Delta\gamma \simeq 0.04 \to 7\%\ldots$
- Effect of DE perturbations also can be as large as $\Delta\gamma$

The fitting formula for the growth index γ

Working assumptions

- Just non–interacting DE and DM $\quad \Omega_m^0 \in [0.25, 0.30]$
- Constant $w \in [-1, -0.7]$ and $\hat{c}_s^2 \in [0, 1]$
- $k \in [0.01h\,, 0.2h]\,\mathrm{Mpc}^{-1}$: (LSS data on the matter spectrum)
- No shear stress perturbations
- Scale factor from $a_{eq}|\,\Omega_m = \Omega_x = 1/2$ up to now: $z \in\sim (0, 0.3)$
- Initial Conditions: adiabatic at matt–rad eq.: $\delta\rho_c/\bar{\rho_c}' = \delta\rho_x/\bar{\rho_x}'$
  …and zero initial time derivatives of δ_c and δ_x
  reminder: in matter domination $\delta_x \propto (1+w)\delta_c \propto \tau^2$ (constant g)

$$\gamma = \gamma\left(\Omega_m^0, \hat{c}_s, k, w, a\right)$$

Dark energy perturbations

$$\delta_c'' + \mathcal{H}\delta_c' - \frac{3}{2}\mathcal{H}^2\Omega_c\delta_c = \frac{3}{2}\mathcal{H}^2\Omega_x\left[\left(1+3\hat{c}_s^2\right)\delta_x + 9\left(1+w\right)\mathcal{H}\left(\hat{c}_s^2 - w\right)\frac{\theta_x}{k^2}\right]$$

$$\delta_x'' = \delta_x''\left(\delta_x', \delta_x, \delta_c', \delta_c\right), \quad \theta_x' = \theta_x'\left(\delta_x, \theta_x\right)$$

- Gravitational potentials (metric pert. d.o.f.) not present (synchronous gauge)
  $\mathrm{d}s^2 = a^2(\tau)\left[-\mathrm{d}\tau^2 + (\delta_{ij} + h_{ij})\,\mathrm{d}x^i\mathrm{d}x^j\right]$, zero curvature
  (dark matter velocity perturbation $\to 0$, fixing the gauge)
- DE perturbations are necessary for consistency
  (generated at later times even if set to zero initially)

$$\gamma = (\ln\Omega_m)^{-1}\ln\left(\frac{a}{\delta_c}\frac{\mathrm{d}\delta_c}{\mathrm{d}a}\right)$$

$$\gamma = \gamma\left(\Omega_m^0, \hat{c}_s, k, w, a\right)$$

The fitting formula for the growth index γ

Working assumptions

- Just non–interacting DE and DM $\quad \Omega_m^0 \in [0.25, 0.30]$
- Constant $w \in [-1, -0.7]$ and $\hat{c}_s^2 \in [0, 1]$
- $k \in [0.01h\,, 0.2h]\,\mathrm{Mpc}^{-1}$: (LSS data on the matter spectrum)
- No shear stress perturbations
- Scale factor from $a_{eq}|\,\Omega_m = \Omega_x = 1/2$ up to now: $z \in\sim (0, 0.3)$
- Initial Conditions: adiabatic at matt–rad eq.: $\delta\rho_c/\bar{\rho_c}' = \delta\rho_x/\bar{\rho_x}'$
  ...and zero initial time derivatives of δ_c and δ_x
  reminder: in matter domination $\delta_x \propto (1+w)\delta_c \propto \tau^2$ (constant g)

$$\gamma = \gamma\left(\Omega_m^0, \hat{c}_s, k, w, a\right)$$

The fitting formula for the growth index γ

Main results

- Estimated effect of baryons on γ is $< 0.2\%$
- Affine dependence on a: $\gamma \sim \gamma_0 + \beta_0 \, z^{-1}$
- $\gamma \sim \tanh \log_{10} \hat{c}_s^2$
- robust changing the initial conditions

$$\gamma\left(\Omega_m^0, \hat{c}_s, k, w, a\right) = \gamma_{eq}\left(\hat{c}_s, k, w\right) + \left[\zeta\left(\Omega_m^0, \hat{c}_s, k, w\right) + \eta\left(\Omega_m^0\right)\right]\left[a - a_{eq}\left(\Omega_m^0, w\right)\right] .$$

$$\gamma_j\left(\hat{c}_s, k, w\right) = h_j(w) \tanh\left[\left(\log_{10} \hat{c}_s - g_j(k)\right) \frac{r_j(w)}{h_j(w)}\right] + f_j(w) \ , \ j = \{eq, 0\} .$$

$$\zeta\left(\Omega_m^0, \hat{c}_s, k, w\right) = \frac{\gamma_0 - \gamma_{eq}}{1 - a_{eq}} .$$

h_j, f_j, g_j, r_j and η are polynomials on their variables.

The fitting formula for the growth index γ

Working assumptions

- Just non–interacting DE and DM $\quad \Omega_m^0 \in [0.25, 0.30]$
- Constant $w \in [-1, -0.7]$ and $\hat{c}_s^2 \in [0, 1]$
- $k \in [0.01h\,, 0.2h]\,\mathrm{Mpc}^{-1}$: (LSS data on the matter spectrum)
- No shear stress perturbations
- Scale factor from $a_{eq}|\,\Omega_m = \Omega_x = 1/2$ up to now: $z \in\sim (0, 0.3)$
- Initial Conditions: adiabatic at matt–rad eq.: $\delta\rho_c/\bar{\rho_c}' = \delta\rho_x/\bar{\rho_x}'$
  …and zero initial time derivatives of δ_c and δ_x
  reminder: in matter domination $\delta_x \propto (1+w)\delta_c \propto \tau^2$ (constant g)

$$\gamma = \gamma\left(\Omega_m^0, \hat{c}_s, k, w, a\right)$$

The fitting formula for the growth index γ

Main results

- Estimated effect of baryons on γ is $< 0.2\%$
- Affine dependence on a: $\gamma \sim \gamma_0 + \beta_0 \, z^{-1}$
- $\gamma \sim \tanh \log_{10} \hat{c}_s^2$
- robust changing the initial conditions

$$\begin{aligned} \gamma\left(\Omega_m^0, \hat{c}_s, k, w, a\right) &= \gamma_{eq}\left(\hat{c}_s, k, w\right) \\ &+ \left[\zeta\left(\Omega_m^0, \hat{c}_s, k, w\right) + \eta\left(\Omega_m^0\right)\right]\left[a - a_{eq}\left(\Omega_m^0, w\right)\right] . \end{aligned}$$

$$\gamma_j\left(\hat{c}_s, k, w\right) = h_j(w) \tanh\left[\left(\log_{10} \hat{c}_s - g_j(k)\right) \frac{r_j(w)}{h_j(w)}\right] + f_j(w) \; , \; j = \{eq, 0\} .$$

$$\zeta\left(\Omega_m^0, \hat{c}_s, k, w\right) = \frac{\gamma_0 - \gamma_{eq}}{1 - a_{eq}} .$$

$h_j \, , f_j \, , g_j \, , r_j$ and η are polynomials on their variables.

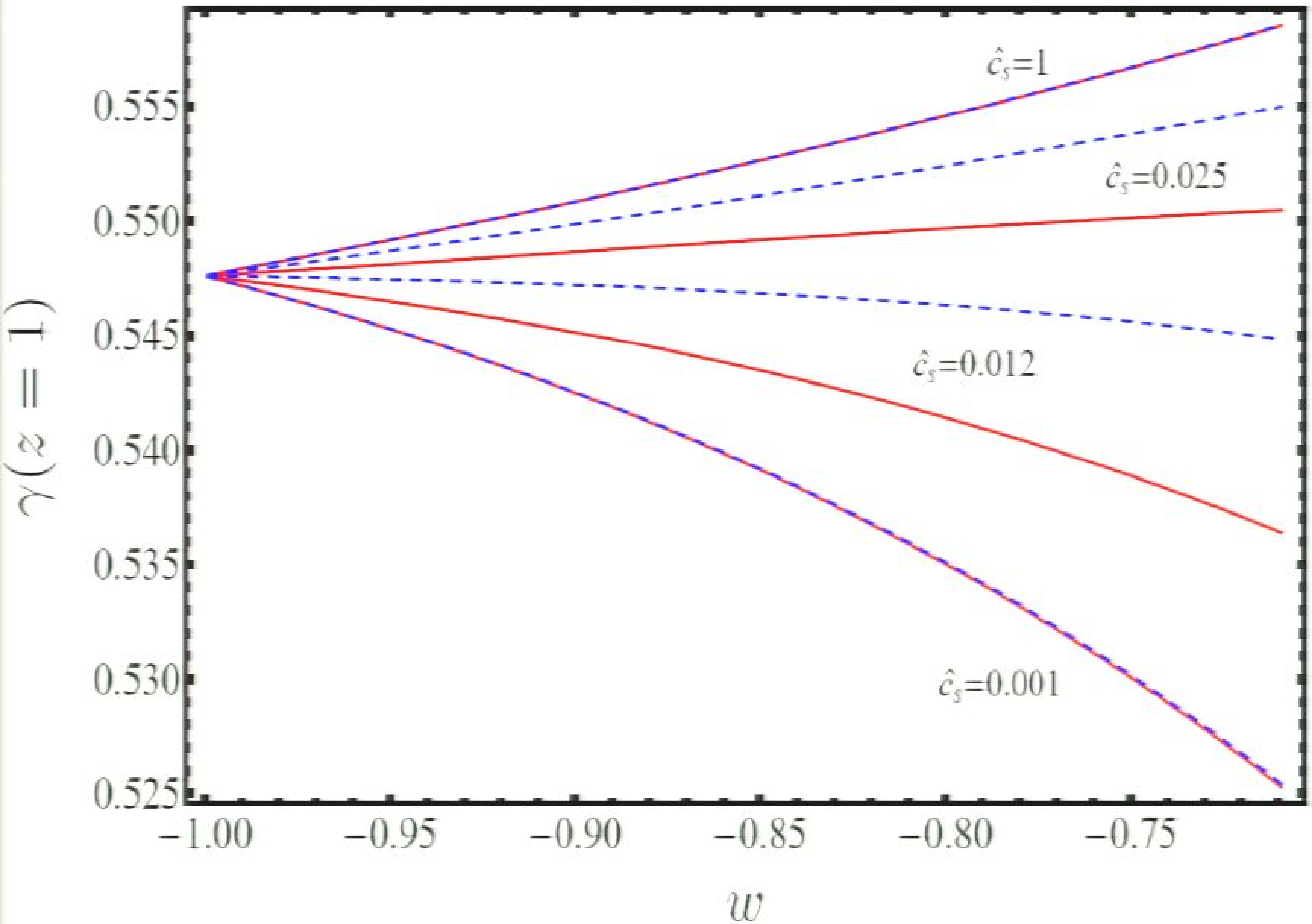
Dark energy perturbations

$$\delta_c'' + \mathcal{H}\delta_c' - \frac{3}{2}\mathcal{H}^2\Omega_c\delta_c = \frac{3}{2}\mathcal{H}^2\Omega_x\left[\left(1+3\hat{c}_s^2\right)\delta_x + 9\left(1+w\right)\mathcal{H}\left(\hat{c}_s^2 - w\right)\frac{\theta_x}{k^2}\right]$$

$$\begin{aligned}\delta_x'' &+ \left[3\left(\hat{c}_s^2 - w\right)\mathcal{H} - \mathcal{F}\right]\delta_x' \\ &+ \left\{\hat{c}_s^2 k^2 - \frac{3}{2}\left(\hat{c}_s^2 - w\right)\mathcal{H}\left[\left(1+3w\Omega_x - 6\hat{c}_s^2\right)\mathcal{H} + 2\mathcal{F}\right]\right\}\delta_x \\ &= (1+w)\delta_c'' - (1+w)\mathcal{F}\delta_c'\,.\end{aligned}$$

$$\frac{\theta_x'}{k^2} = -\left(1-3\hat{c}_s^2\right)\mathcal{H}\frac{\theta_x}{k^2} + \frac{\hat{c}_s^2}{1+w}\delta_x$$

$$\mathcal{F} \equiv -9\left(1+3w\Omega_x\right)\frac{\hat{c}_s^2 - w}{k^2 + 9\left(\hat{c}_s^2 - w\right)\mathcal{H}^2}\mathcal{H}^3 - (1-3\hat{c}_s^2)\mathcal{H}\,.$$



Red: $k = 0.050\, h\mathrm{Mpc}^{-1}$ and Blue dashed: $k = 0.078\, h\mathrm{Mpc}^{-1}$

Status of γ

Theory

- Different theories ($f(R)$, DGP, GR...) predict different growths
- But the results can be mimicked with a general DE fluid
- A parameterization of γ can help to determine DE properties

Experiment

- Current data (BAO, WL, LSS, X-ray clust., Lyman–α, ISW) still poor
- Forecasted accuracy of future experiments $\Delta\gamma \simeq 0.04 \to 7\%$...
- Effect of DE perturbations also can be as large as $\Delta\gamma$

The fitting formula for the growth index γ

Main results

- Estimated effect of baryons on γ is $< 0.2\%$
- Affine dependence on a: $\gamma \sim \gamma_0 + \beta_0 \, z^{-1}$
- $\gamma \sim \tanh \log_{10} \hat{c}_s^2$
- robust changing the initial conditions

For any combination of the variables

Error on $\gamma < 0.2(fittingaccuracy)\% << $ forecasted $\Delta\gamma \simeq 7\%$

(and typically much smaller than 0.2%)

The fitting formula for the growth index γ

Main results

- Estimated effect of baryons on γ is $< 0.2\%$
- Affine dependence on a: $\gamma \sim \gamma_0 + \beta_0\, z^{-1}$
- $\gamma \sim \tanh \log_{10} \hat{c}_s^2$
- robust changing the initial conditions

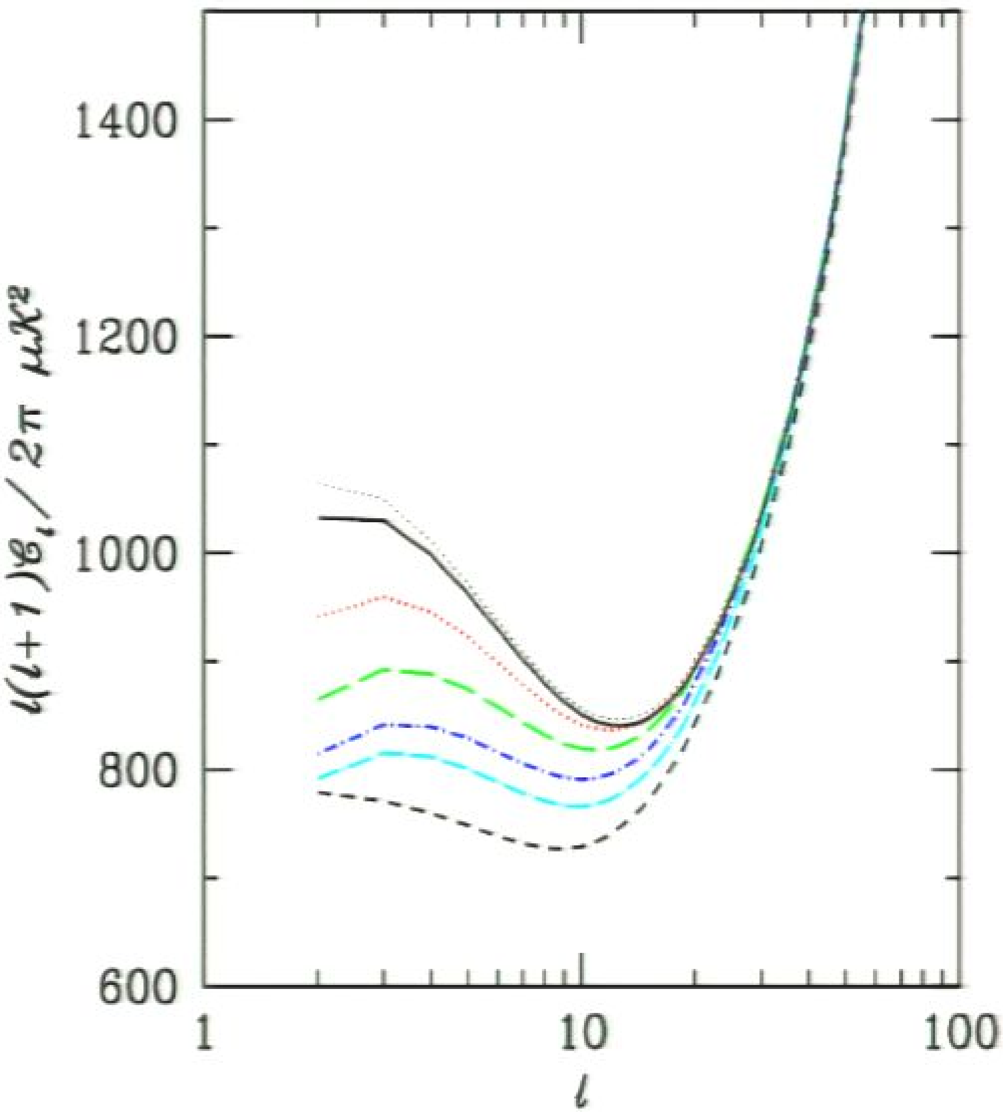
$$\begin{aligned}\gamma\left(\Omega_m^0, \hat{c}_s, k, w, a\right) &= \gamma_{eq}\left(\hat{c}_s, k, w\right) \\ &+ \left[\zeta\left(\Omega_m^0, \hat{c}_s, k, w\right) + \eta\left(\Omega_m^0\right)\right]\left[a - a_{eq}\left(\Omega_m^0, w\right)\right] .\end{aligned}$$

$$\gamma_j\left(\hat{c}_s, k, w\right) = h_j(w) \tanh\left[\left(\log_{10}\hat{c}_s - g_j(k)\right)\frac{r_j(w)}{h_j(w)}\right] + f_j(w)\ , \ j = \{eq, 0\} .$$

$$\zeta\left(\Omega_m^0, \hat{c}_s, k, w\right) = \frac{\gamma_0 - \gamma_{eq}}{1 - a_{eq}} .$$

$h_j\, , f_j\, , g_j\, , r_j$ and η are polynomials on their variables.

CMB and the DE speed of sound



Lewis and Weller 06

The speed of sound

$$\hat{c}_s^2 \equiv \frac{\delta P}{\delta \rho}$$

- Matter: $\hat{c}_s^2 = 0$
- Radiation: $\hat{c}_s^2 = 1/3$
- Quintessence (scalar field DE) with a canonical kinetic term: $\hat{c}_s^2 = 1$
- K-essence (non-canonical kinetic terms): $\hat{c}_s^2 \neq 1$ (even $\to \infty$)
- Two scalar fields: $\hat{c}_s^2 \neq 1$
- String gas: $0 < \hat{c}_s^2 << 1$
- Chaplygin gas ($P \propto 1/\rho$): $\hat{c}_s^2 \to 0$
- ...

If $\hat{c}_s^2 \neq 1$ DE perturbations can be important.

Two questions

i) Dark energy (DE) or modified gravity (MG)?

ii) What is the nature of DE?

Growth history of the Universe

$$G_{\mu\nu} + \delta G_{\mu\nu} = 8\pi G T_{\mu\nu} + 8\pi G \delta T_{\mu\nu}$$

(Dark) matter perturbations $\delta_c = \delta\rho_c/\rho_c$

$$\delta_c'' + \mathcal{H}\delta_c' - \frac{3}{2}\mathcal{H}^2\Omega_c\delta_c = 0$$

Growth factor

$$g \equiv \delta_c/a$$

If matter dominated ($\Omega_c \to 1$): $\mathcal{H} \sim \tau^{-1}$, $\delta_c \sim \tau^2 \sim a \implies g' \sim 0$

The growth index γ

$$g(a) = g(a_i)\exp\int_{a_i}^{a}\left([\Omega_m(\hat{a})]^{\gamma} - 1\right)\frac{\mathrm{d}\hat{a}}{\hat{a}}$$

Linder 05

Ω_m characterizes the expansion history and γ encodes the growth history

but clearly growth and expansion are not unrelated.

- In General Relativity:

$$\delta_c'' + \mathcal{H}\delta_c' - \frac{3}{2}\mathcal{H}^2\Omega_c\delta_c = 0 \implies \gamma(w) = 0.55 + 0.05\left[1 + w(z=1)\right]$$

0.2% fitting accuracy

- For DGP:

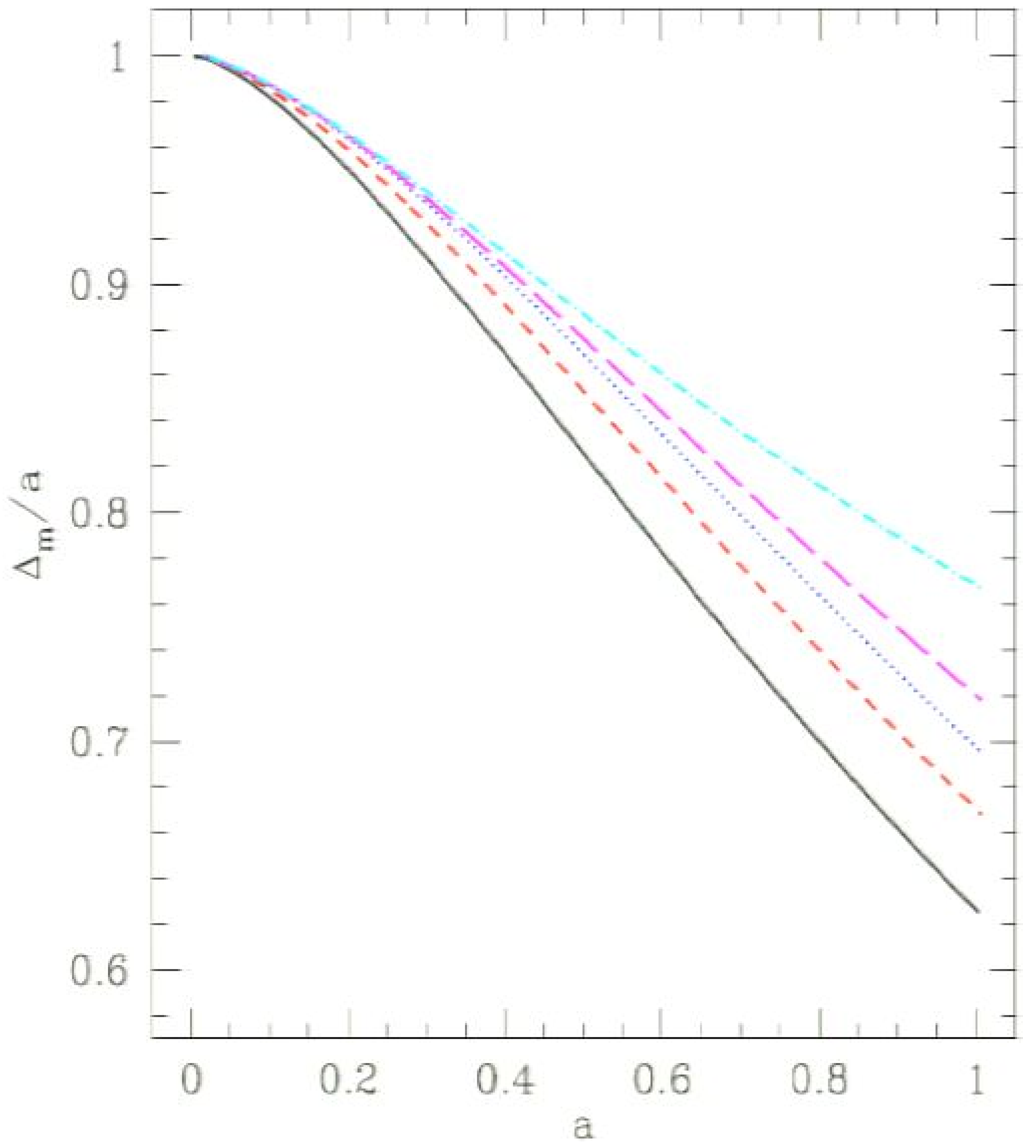
$$\gamma \simeq 0.68$$

- For several $f(R)$ models:

$$\gamma \sim 0.4$$

Gannouji, Moraes and Polarksi 08

Growth factor and the speed of sound of dark energy



Kunz and Sapone 06

Dark energy perturbations

$$\delta_c'' + \mathcal{H}\delta_c' - \frac{3}{2}\mathcal{H}^2\Omega_c\delta_c = \frac{3}{2}\mathcal{H}^2\Omega_x\left[\left(1+3\hat{c}_s^2\right)\delta_x + 9\left(1+w\right)\mathcal{H}\left(\hat{c}_s^2 - w\right)\frac{\theta_x}{k^2}\right]$$

$$\delta_x'' = \delta_x''\left(\delta_x', \delta_x, \delta_c', \delta_c\right), \quad \theta_x' = \theta_x'\left(\delta_x, \theta_x\right)$$

The speed of sound

$$\hat{c}_s^2 \equiv \frac{\delta P}{\delta \rho}$$

- Matter: $\hat{c}_s^2 = 0$
- Radiation: $\hat{c}_s^2 = 1/3$
- Quintessence (scalar field DE) with a canonical kinetic term: $\hat{c}_s^2 = 1$
- K-essence (non-canonical kinetic terms): $\hat{c}_s^2 \neq 1$ (even $\rightarrow \infty$)
- Two scalar fields: $\hat{c}_s^2 \neq 1$
- String gas: $0 < \hat{c}_s^2 << 1$
- Chaplygin gas ($P \propto 1/\rho$): $\hat{c}_s^2 \rightarrow 0$
- ...

If $\hat{c}_s^2 \neq 1$ DE perturbations can be important.

The speed of sound

$$c_s^2 \equiv \frac{\delta P}{\delta \rho}$$

For a homogeneous perfect fluid (adiabatic):

$$c_a^2 \equiv \frac{P'}{\rho'} = w - \frac{w'}{3\mathcal{H}(1+w)}$$

Imperfect fluid (entropy perturbation):

$$w\Gamma \equiv \left(c_s^2 - c_a^2\right)\delta = \frac{P'}{\rho}\left(\frac{\delta P}{P'} - \frac{\delta\rho}{\rho'}\right)$$

In general, c_s^2 is gauge dependent but c_a^2 and Γ are gauge invariant

$$\hat{\delta} = \delta + 3\mathcal{H}(1+w)\frac{\theta}{k^2} \implies \delta P = \hat{c}_s^2\,\delta\rho + 3\mathcal{H}\,(1+w)\left(\hat{c}_s^2 - w\right)\rho\frac{\theta}{k^2}$$

δ is invariant in the rest frame $\longrightarrow \hat{\delta}$

Dark energy perturbations

$$\delta_c'' + \mathcal{H}\delta_c' - \frac{3}{2}\mathcal{H}^2\Omega_c\delta_c = \frac{3}{2}\mathcal{H}^2\Omega_x\left[\left(1+3\hat{c}_s^2\right)\delta_x + 9\left(1+w\right)\mathcal{H}\left(\hat{c}_s^2 - w\right)\frac{\theta_x}{k^2}\right]$$

$$\delta_x'' = \delta_x''\left(\delta_x', \delta_x, \delta_c', \delta_c\right), \quad \theta_x' = \theta_x'\left(\delta_x, \theta_x\right)$$

- Gravitational potentials (metric pert. d.o.f.) not present (synchronous gauge) $\mathrm{d}s^2 = a^2(\tau)\left[-\mathrm{d}\tau^2 + (\delta_{ij} + h_{ij})\,\mathrm{d}x^i\mathrm{d}x^j\right]$, zero curvature (dark matter velocity perturbation $\rightarrow 0$, fixing the gauge)
- DE perturbations are necessary for consistency (generated at later times even if set to zero initially)

Dark energy perturbations

$$\delta_c'' + \mathcal{H}\delta_c' - \frac{3}{2}\mathcal{H}^2\Omega_c\delta_c = \frac{3}{2}\mathcal{H}^2\Omega_x\left[\left(1+3\hat{c}_s^2\right)\delta_x + 9\left(1+w\right)\mathcal{H}\left(\hat{c}_s^2 - w\right)\frac{\theta_x}{k^2}\right]$$

$$\delta_x'' = \delta_x''\left(\delta_x', \delta_x, \delta_c', \delta_c\right), \quad \theta_x' = \theta_x'\left(\delta_x, \theta_x\right)$$

- Gravitational potentials (metric pert. d.o.f.) not present (synchronous gauge) $\mathrm{d}s^2 = a^2(\tau)\left[-\mathrm{d}\tau^2 + \left(\delta_{ij} + h_{ij}\right)\mathrm{d}x^i\mathrm{d}x^j\right]$, zero curvature (dark matter velocity perturbation $\rightarrow 0$, fixing the gauge)
- DE perturbations are necessary for consistency (generated at later times even if set to zero initially)

$$\gamma = \left(\ln\Omega_m\right)^{-1}\ln\left(\frac{a}{\delta_c}\frac{\mathrm{d}\delta_c}{\mathrm{d}a}\right)$$

$$\gamma = \gamma\left(\Omega_m^0, \hat{c}_s, k, w, a\right)$$

Dark energy perturbations

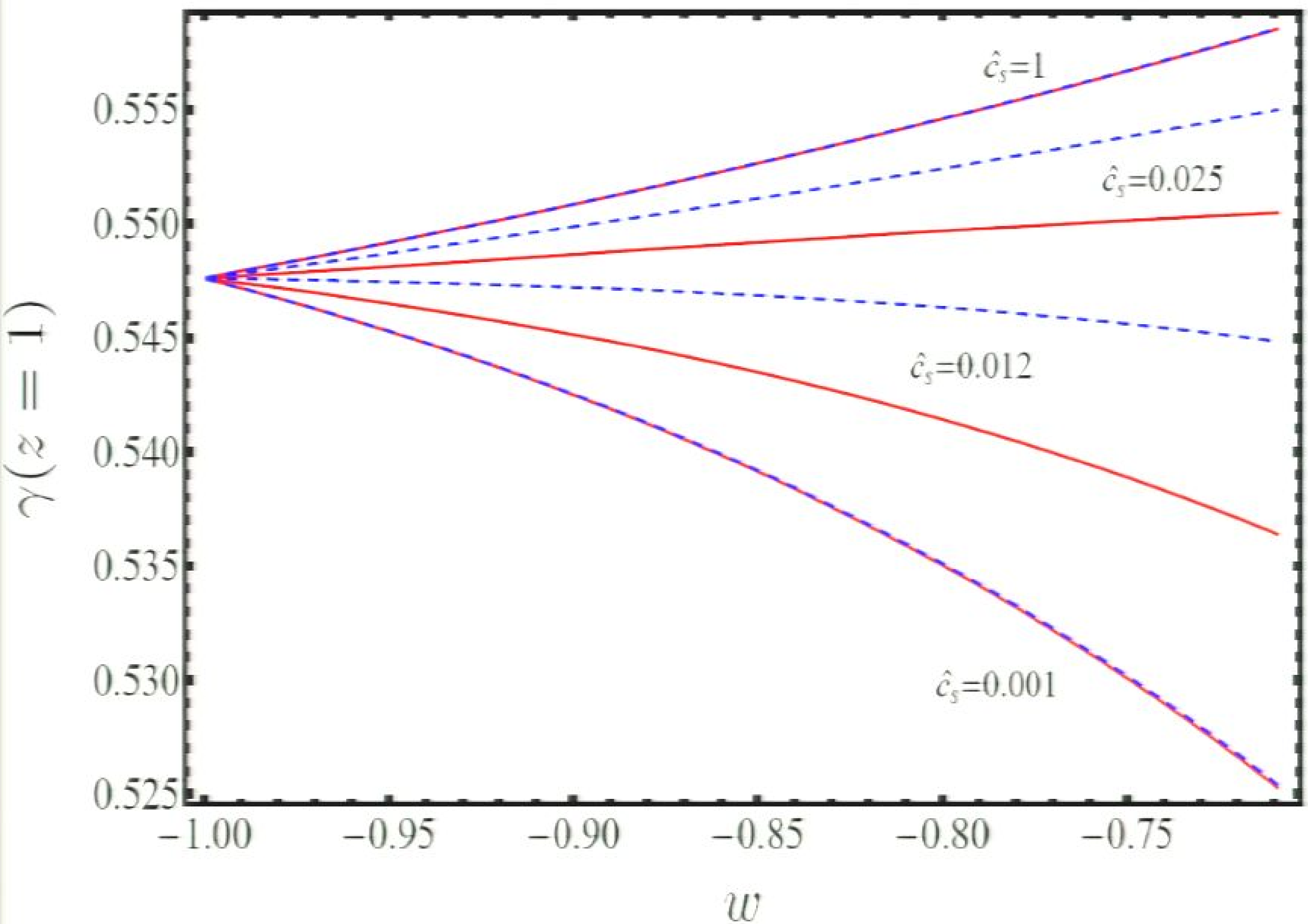
$$\delta_c'' + \mathcal{H}\delta_c' - \frac{3}{2}\mathcal{H}^2\Omega_c\delta_c = \frac{3}{2}\mathcal{H}^2\Omega_x\left[\left(1+3\hat{c}_s^2\right)\delta_x + 9\left(1+w\right)\mathcal{H}\left(\hat{c}_s^2 - w\right)\frac{\theta_x}{k^2}\right]$$

$$\delta_x'' = \delta_x''\left(\delta_x', \delta_x, \delta_c', \delta_c\right), \quad \theta_x' = \theta_x'\left(\delta_x, \theta_x\right)$$

- Gravitational potentials (metric pert. d.o.f.) not present (synchronous gauge) $\mathrm{d}s^2 = a^2(\tau)\left[-\mathrm{d}\tau^2 + \left(\delta_{ij} + h_{ij}\right)\mathrm{d}x^i\mathrm{d}x^j\right]$, zero curvature (dark matter velocity perturbation $\rightarrow 0$, fixing the gauge)
- DE perturbations are necessary for consistency (generated at later times even if set to zero initially)

$$\gamma = \left(\ln\Omega_m\right)^{-1}\ln\left(\frac{a}{\delta_c}\frac{\mathrm{d}\delta_c}{\mathrm{d}a}\right)$$

$$\gamma = \gamma\left(\Omega_m^0, \hat{c}_s, k, w, a\right)$$



Red: $k = 0.050\,h\mathrm{Mpc}^{-1}$ and Blue dashed: $k = 0.078\,h\mathrm{Mpc}^{-1}$

Status of γ

Theory

- Different theories ($f(R)$, DGP, GR...) predict different growths
- But the results can be mimicked with a general DE fluid
- A parameterization of γ can help to determine DE properties

Experiment

- Current data (BAO, WL, LSS, X-ray clust., Lyman–α, ISW) still poor
- Forecasted accuracy of future experiments $\Delta\gamma \simeq 0.04 \to 7\%\dots$
- Effect of DE perturbations also can be as large as $\Delta\gamma$

The fitting formula for the growth index γ

Working assumptions

- Just non–interacting DE and DM $\Omega_m^0 \in [0.25, 0.30]$
- Constant $w \in [-1, -0.7]$ and $\hat{c}_s^2 \in [0, 1]$
- $k \in [0.01h\,, 0.2h]\,\mathrm{Mpc}^{-1}$: (LSS data on the matter spectrum)
- No shear stress perturbations
- Scale factor from $a_{eq}|\,\Omega_m = \Omega_x = 1/2$ up to now: $z \in\sim (0, 0.3)$
- Initial Conditions: adiabatic at matt–rad eq.: $\delta\rho_c/\bar{\rho_c}' = \delta\rho_x/\bar{\rho_x}'$
  ...and zero initial time derivatives of δ_c and δ_x
  reminder: in matter domination $\delta_x \propto (1+w)\delta_c \propto \tau^2$ (constant g)

$$\gamma = \gamma\left(\Omega_m^0, \hat{c}_s, k, w, a\right)$$

The fitting formula for the growth index γ

Main results

- Estimated effect of baryons on γ is $< 0.2\%$
- Affine dependence on a: $\gamma \sim \gamma_0 + \beta_0 z^{-1}$
- $\gamma \sim \tanh \log_{10} \hat{c}_s^2$
- robust changing the initial conditions

$$\gamma\left(\Omega_m^0, \hat{c}_s, k, w, a\right) = \gamma_{eq}\left(\hat{c}_s, k, w\right) + \left[\zeta\left(\Omega_m^0, \hat{c}_s, k, w\right) + \eta\left(\Omega_m^0\right)\right]\left[a - a_{eq}\left(\Omega_m^0, w\right)\right] .$$

$$\gamma_j\left(\hat{c}_s, k, w\right) = h_j(w) \tanh\left[\left(\log_{10} \hat{c}_s - g_j(k)\right) \frac{r_j(w)}{h_j(w)}\right] + f_j(w) \ , \ j = \{eq, 0\} .$$

$$\zeta\left(\Omega_m^0, \hat{c}_s, k, w\right) = \frac{\gamma_0 - \gamma_{eq}}{1 - a_{eq}} .$$

h_j , f_j , g_j , r_j and η are polynomials on their variables.

The fitting formula for the growth index γ

Main results

- Estimated effect of baryons on γ is $< 0.2\%$
- Affine dependence on a: $\gamma \sim \gamma_0 + \beta_0\, z^{-1}$
- $\gamma \sim \tanh \log_{10} \hat{c}_s^2$
- robust changing the initial conditions

For any combination of the variables

Error on $\gamma < 0.2(fittingaccuracy)\% << $ forecasted $\Delta\gamma \simeq 7\%$

(and typically much smaller than 0.2%)

The fitting formula for the growth index γ

Main results

- Estimated effect of baryons on γ is $< 0.2\%$
- Affine dependence on a: $\gamma \sim \gamma_0 + \beta_0 \, z^{-1}$
- $\gamma \sim \tanh \log_{10} \hat{c}_s^2$
- robust changing the initial conditions

$$\begin{aligned}\gamma\left(\Omega_m^0, \hat{c}_s, k, w, a\right) &= \gamma_{eq}\left(\hat{c}_s, k, w\right) \\ &\quad + \left[\zeta\left(\Omega_m^0, \hat{c}_s, k, w\right) + \eta\left(\Omega_m^0\right)\right]\left[a - a_{eq}\left(\Omega_m^0, w\right)\right] .\end{aligned}$$

$$\gamma_j\left(\hat{c}_s, k, w\right) = h_j(w) \tanh\left[\left(\log_{10}\hat{c}_s - g_j(k)\right)\frac{r_j(w)}{h_j(w)}\right] + f_j(w) \ , \ j = \{eq, 0\} .$$

$$\zeta\left(\Omega_m^0, \hat{c}_s, k, w\right) = \frac{\gamma_0 - \gamma_{eq}}{1 - a_{eq}} .$$

h_j , f_j , g_j , r_j and η are polynomials on their variables.

The fitting formula for the growth index γ

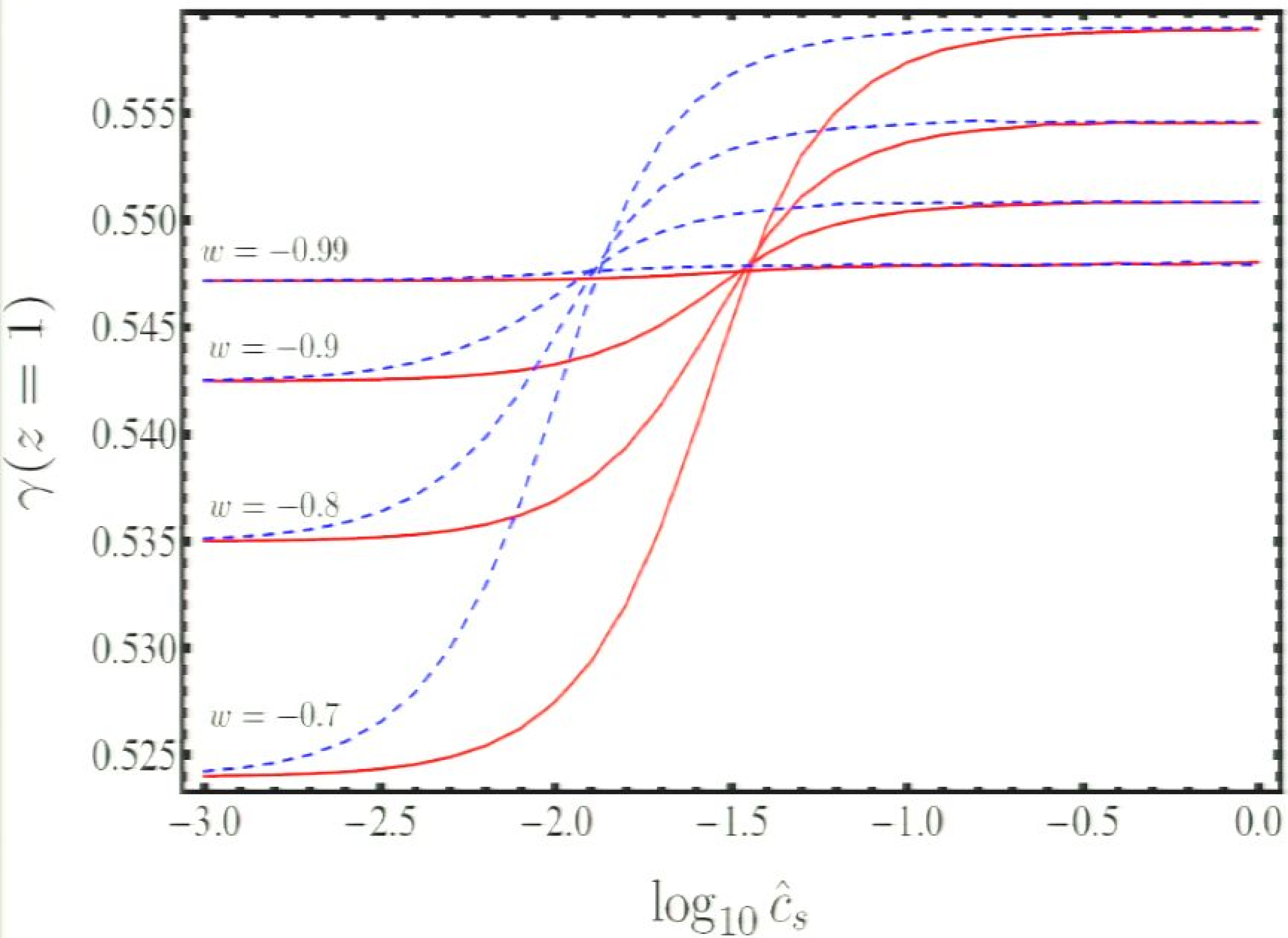
Main results

- Estimated effect of baryons on γ is $< 0.2\%$
- Affine dependence on a: $\gamma \sim \gamma_0 + \beta_0\, z^{-1}$
- $\gamma \sim \tanh \log_{10} \hat{c}_s^2$
- robust changing the initial conditions

For any combination of the variables

Error on $\gamma < 0.2(fitting accuracy)\% << $ forecasted $\Delta\gamma \simeq 7\%$

(and typically much smaller than 0.2%)



Red $k = 0.03\, h\, \mathrm{Mpc}^{-1}$ Blue dashed $k = 0.08\, h\, \mathrm{Mpc}^{-1}$

The fitting formula for the growth index γ

Main results

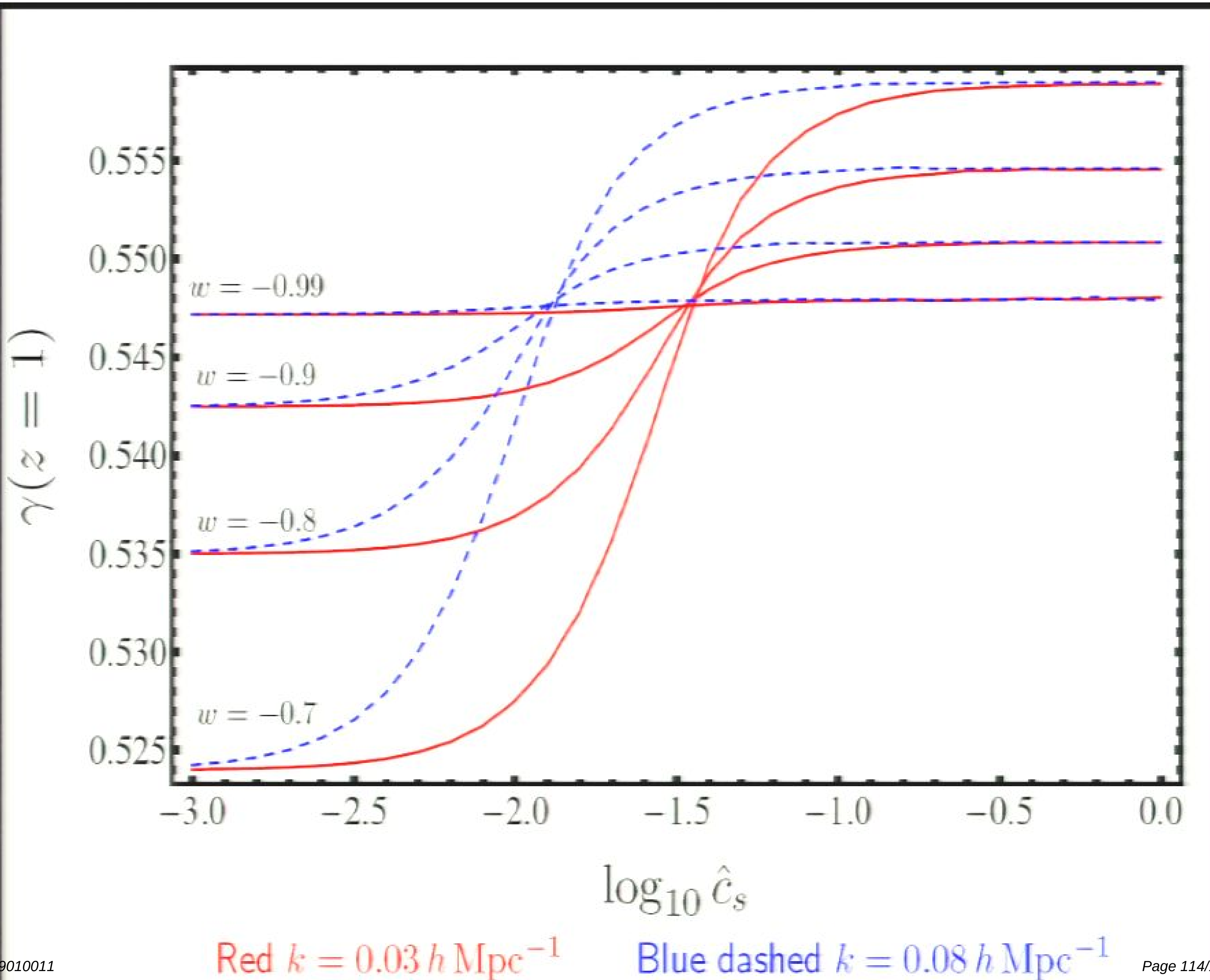
- Estimated effect of baryons on γ is $< 0.2\%$
- Affine dependence on a: $\gamma \sim \gamma_0 + \beta_0 z^{-1}$
- $\gamma \sim \tanh \log_{10} \hat{c}_s^2$
- robust changing the initial conditions

$$\gamma\left(\Omega_m^0, \hat{c}_s, k, w, a\right) = \gamma_{eq}\left(\hat{c}_s, k, w\right) + \left[\zeta\left(\Omega_m^0, \hat{c}_s, k, w\right) + \eta\left(\Omega_m^0\right)\right]\left[a - a_{eq}\left(\Omega_m^0, w\right)\right] .$$

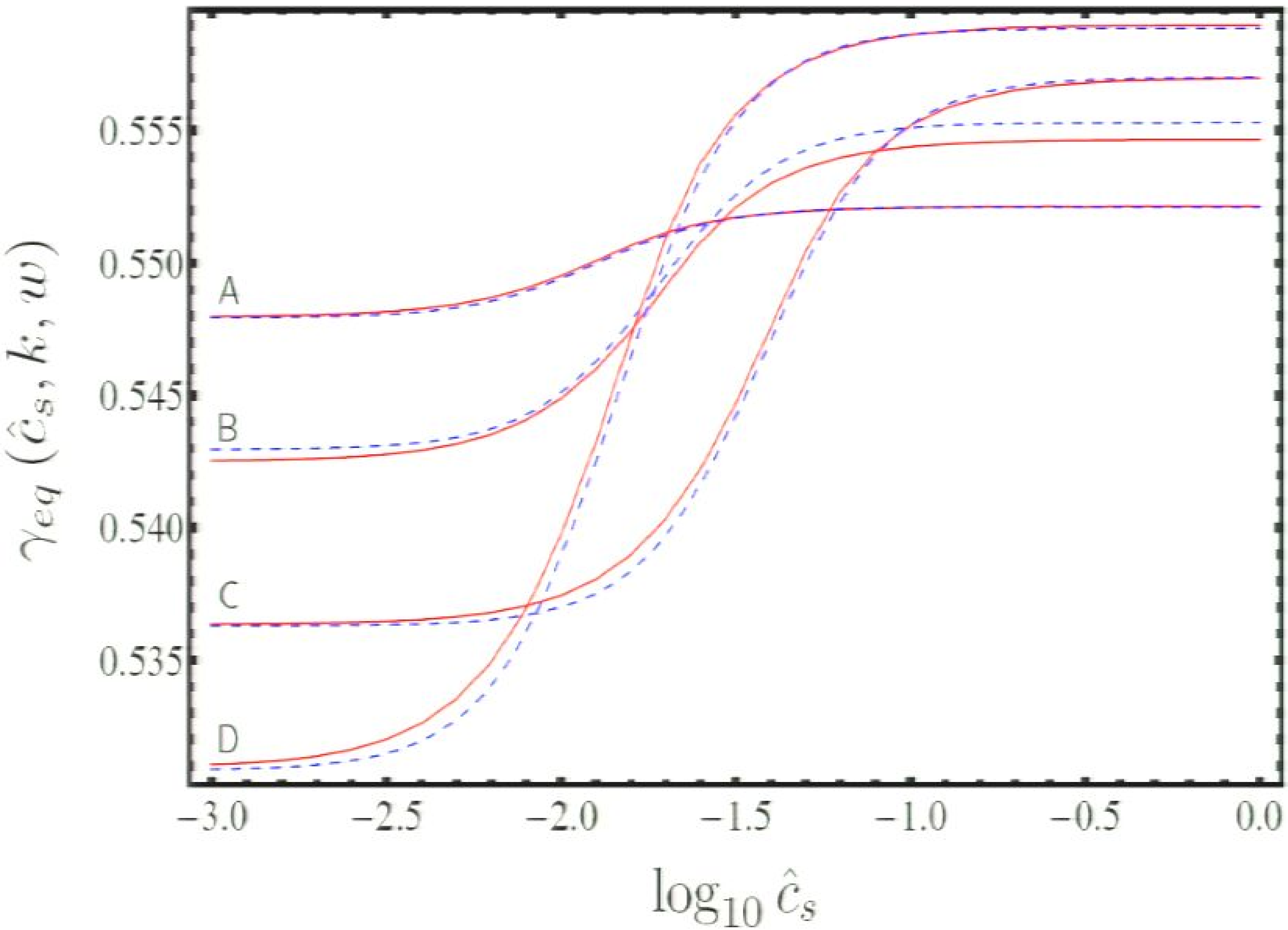
$$\gamma_j\left(\hat{c}_s, k, w\right) = h_j(w) \tanh\left[\left(\log_{10} \hat{c}_s - g_j(k)\right) \frac{r_j(w)}{h_j(w)}\right] + f_j(w) \ , \ j = \{eq, 0\} .$$

$$\zeta\left(\Omega_m^0, \hat{c}_s, k, w\right) = \frac{\gamma_0 - \gamma_{eq}}{1 - a_{eq}} .$$

h_j , f_j , g_j , r_j and η are polynomials on their variables.



0.555
0.550
0.545
0.540
0.535
0.530
0.525
$\gamma(z=1)$
$w=-0.99$
$w=-0.9$
$w=-0.8$
$w=-0.7$
−3.0
−2.5
−2.0
−1.5
−1.0
−0.5
0.0
$\log_{10} \hat{c}_s$
Red $k = 0.03\,h\,\mathrm{Mpc}^{-1}$
Blue dashed $k = 0.08\,h\,\mathrm{Mpc}^{-1}$



A = {0.08 h Mpc^{-1}, −0.95} , B = {0.02 h Mpc^{-1}, −0.7} , C = {0.04 h Mpc^{-1}, −0.87}

D = {0.06 h Mpc^{-1}, −0.75} . Red: exact numerical Blue dashed: fits

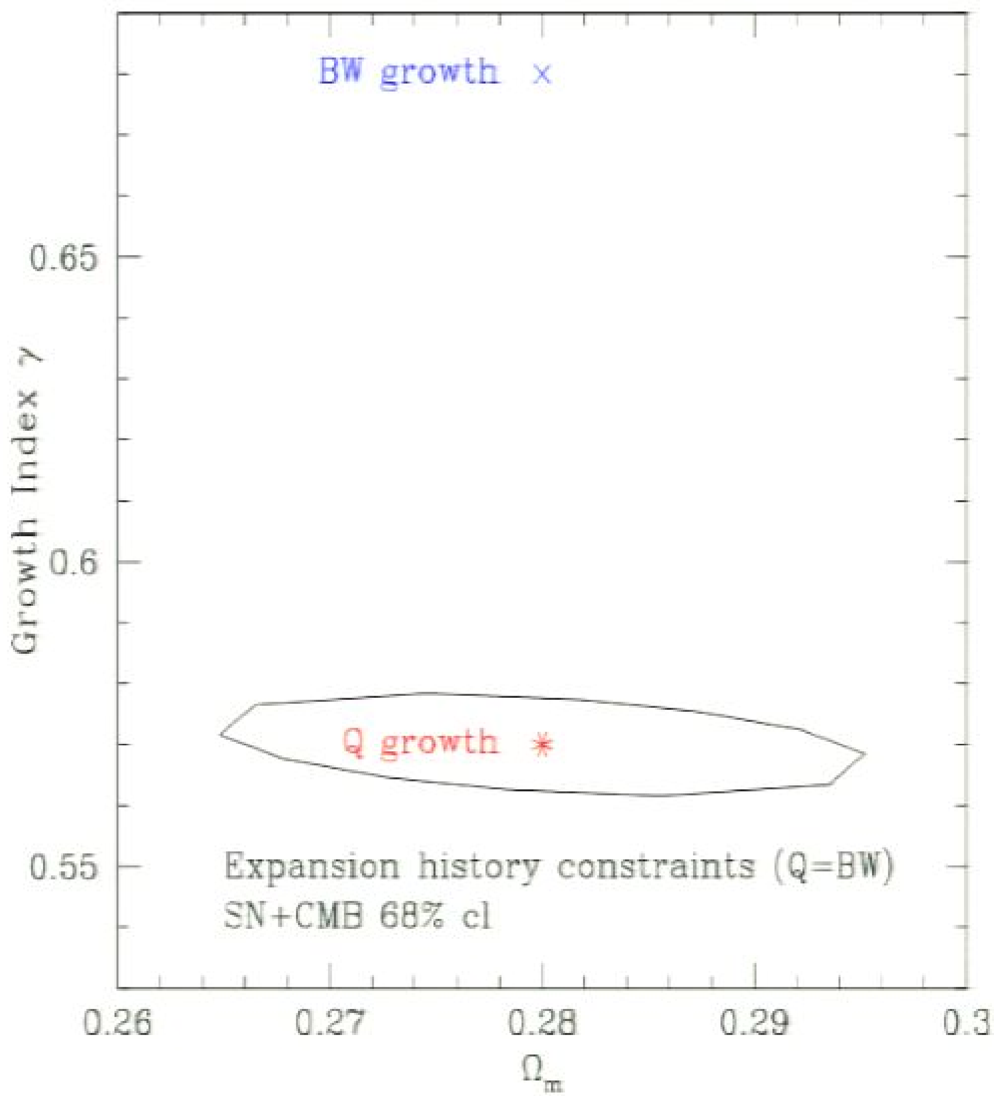
Conclusions

1. Dark energy perturbations are needed for consistency
2. The growth index γ depends on
   - equation of state of DE, w
   - comoving scale, k (from Fourier space)
   - speed of sound of DE, c_s^2
   - redshift (or scale factor)
   - amount of dark matter, Ω_c^0
3. Including DE perturbations, the variation in γ can be comparable with the accuracy forecasts for the next generation of observations $\sim 7\%$

Dark energy perturbations

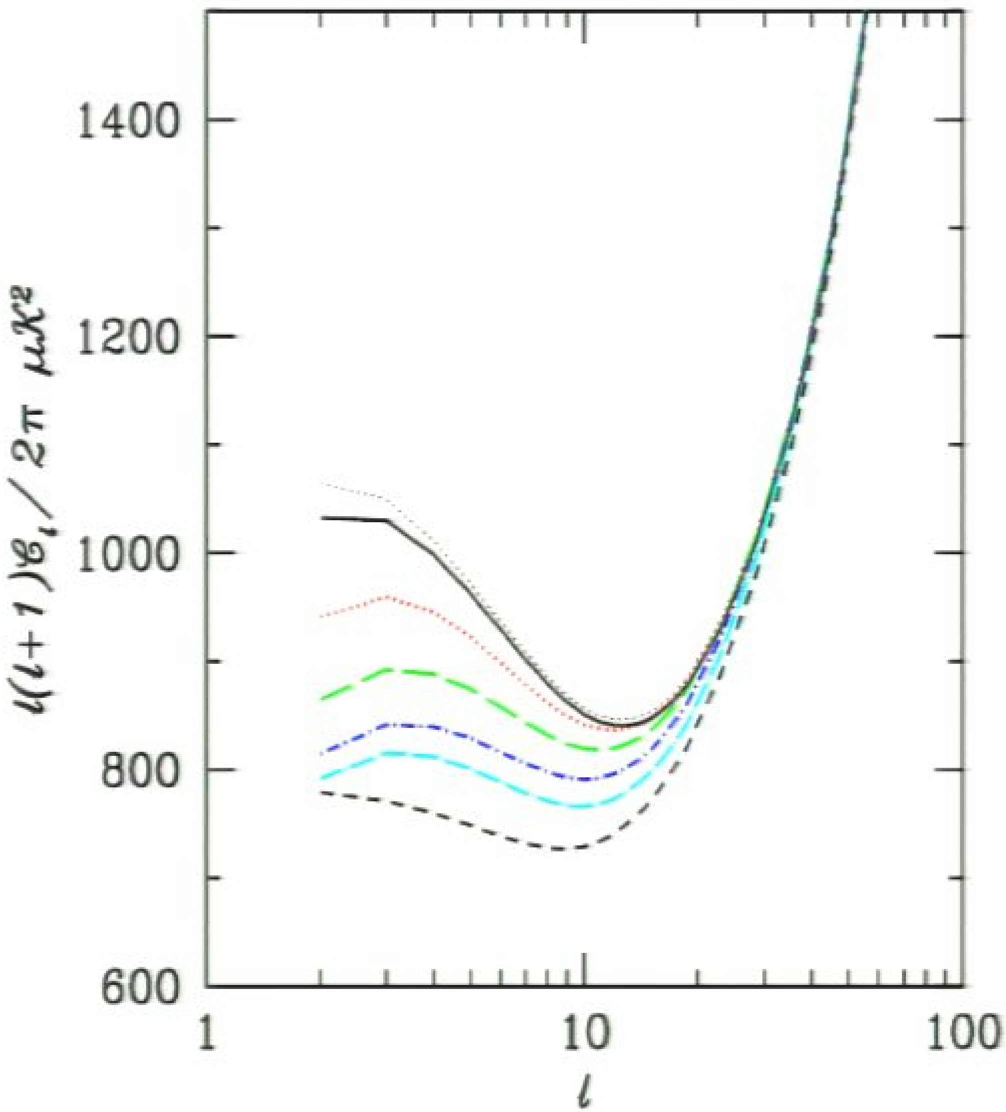
$$\delta_c'' + \mathcal{H}\delta_c' - \frac{3}{2}\mathcal{H}^2\Omega_c\delta_c = \frac{3}{2}\mathcal{H}^2\Omega_x\left[\left(1+3\hat{c}_s^2\right)\delta_x + 9\left(1+w\right)\mathcal{H}\left(\hat{c}_s^2 - w\right)\frac{\theta_x}{k^2}\right]$$

Bean and Doré 03


BW growth ×
Q growth *
Expansion history constraints (Q=BW)
SN+CMB 68% cl
Growth Index γ
0.65
0.6
0.55
0.26
0.27
0.28
0.29
0.3
Ω_m

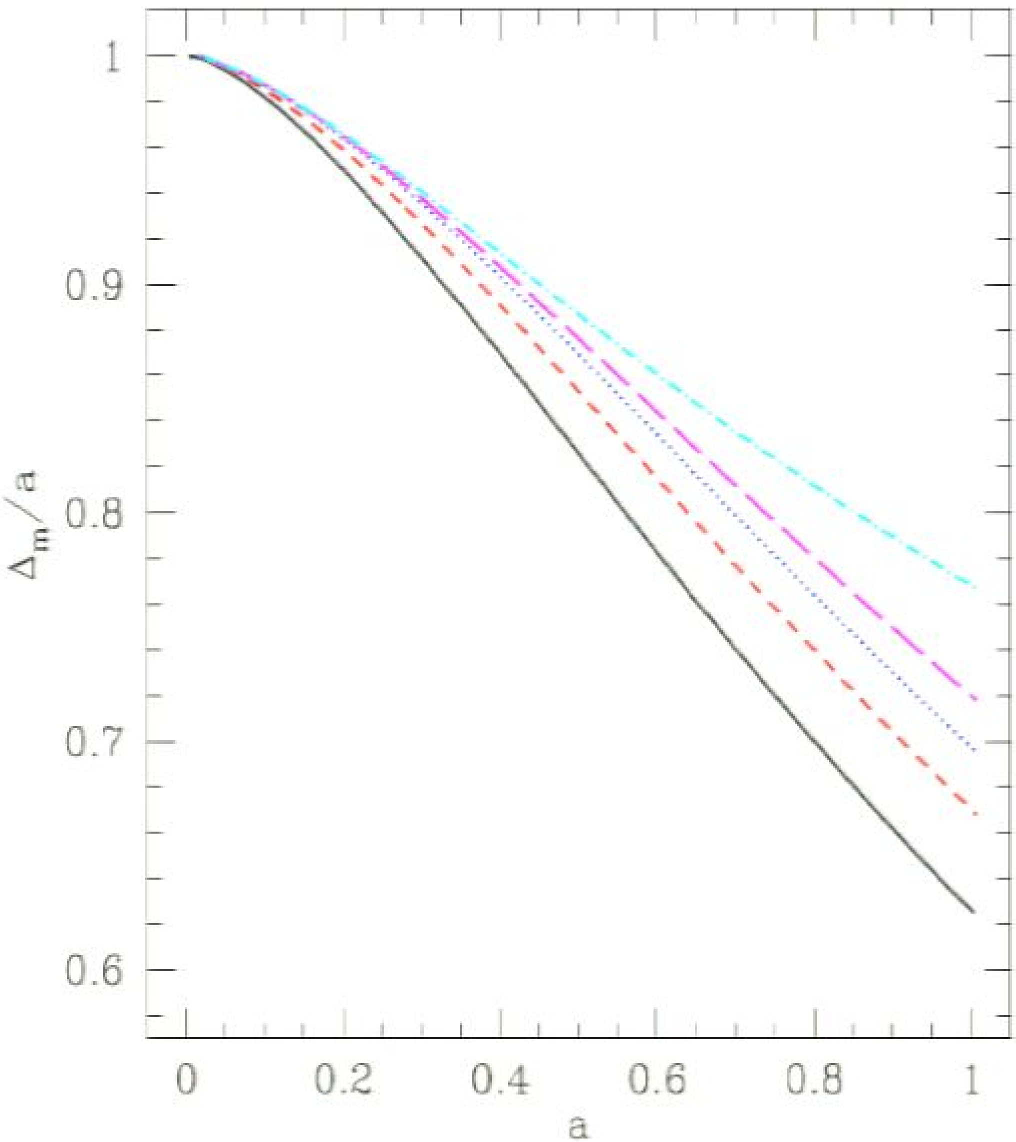
Linder 05

CMB and the DE speed of sound

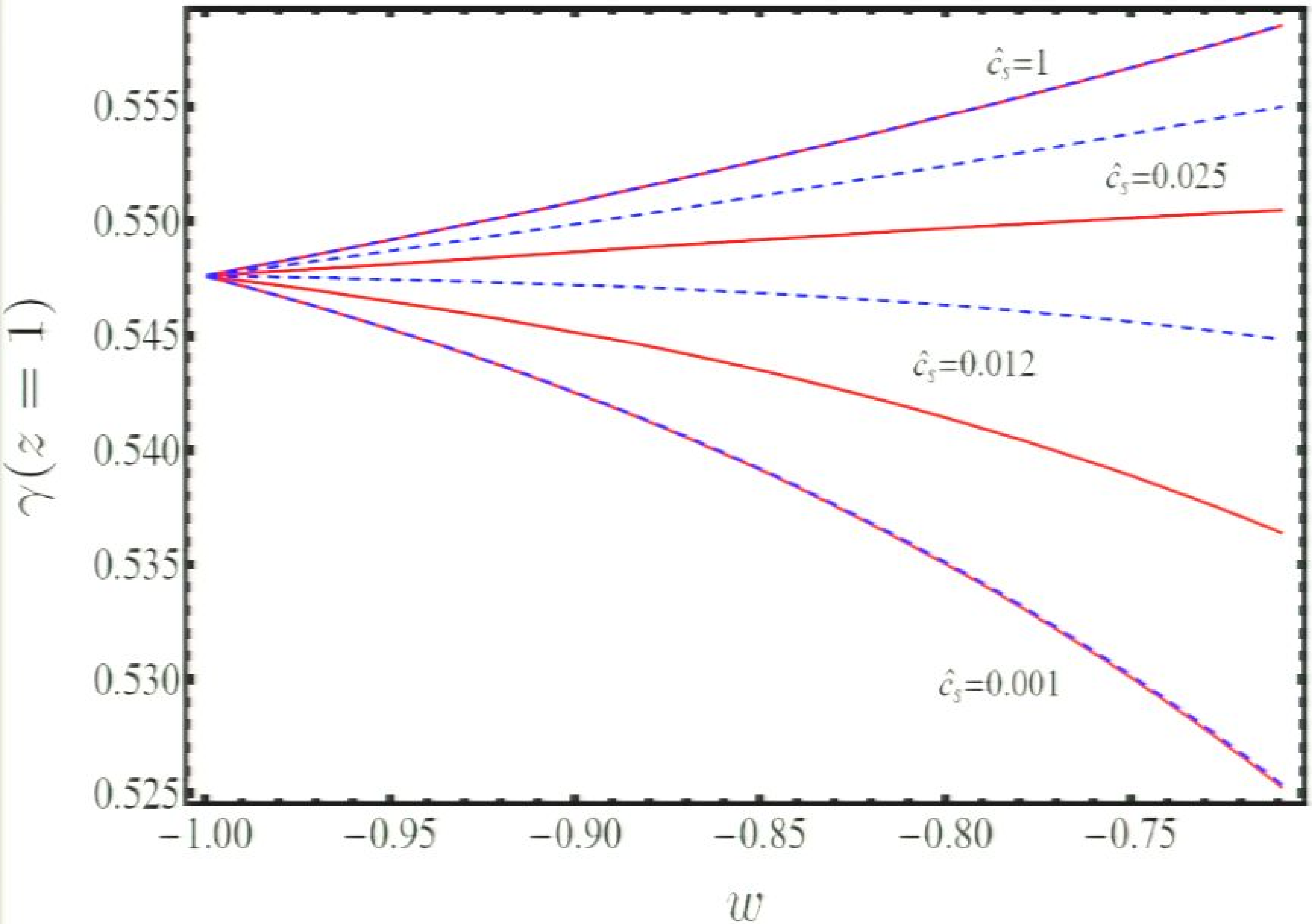


Lewis and Weller 06

Growth factor and the speed of sound of dark energy



Kunz and Sapone 06



Red: $k = 0.050\,h\mathrm{Mpc}^{-1}$ and Blue dashed: $k = 0.078\,h\mathrm{Mpc}^{-1}$

The fitting formula for the growth index γ

Working assumptions

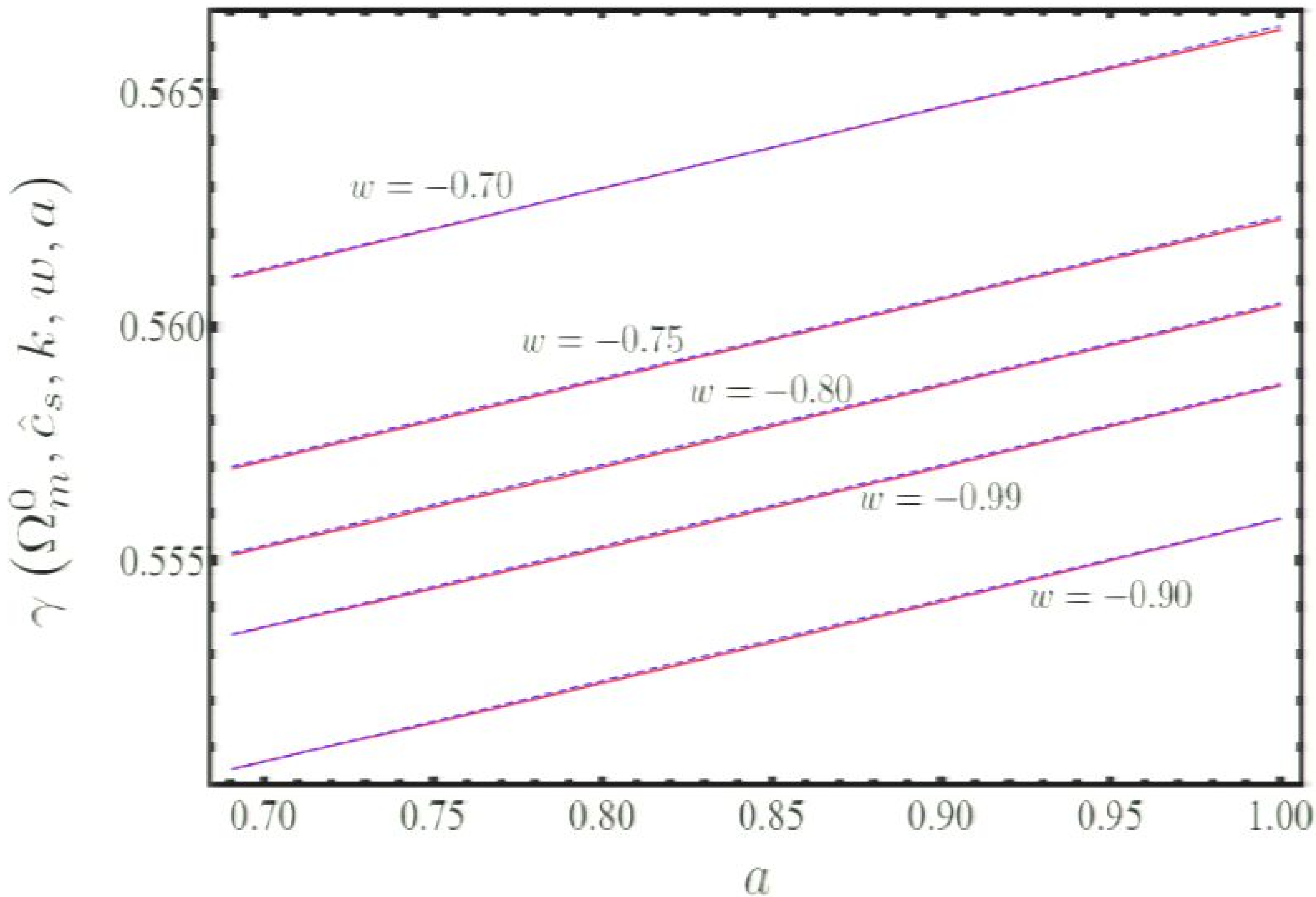
- Just non–interacting DE and DM $\Omega_m^0 \in [0.25, 0.30]$
- Constant $w \in [-1, -0.7]$ and $\hat{c}_s^2 \in [0, 1]$
- $k \in [0.01h\,, 0.2h]\,\mathrm{Mpc}^{-1}$: (LSS data on the matter spectrum)
- No shear stress perturbations
- Scale factor from $a_{eq}|\,\Omega_m = \Omega_x = 1/2$ up to now: $z \in\sim (0, 0.3)$
- Initial Conditions: adiabatic at matt–rad eq.: $\delta\rho_c/\bar{\rho_c}' = \delta\rho_x/\bar{\rho_x}'$
  ...and zero initial time derivatives of δ_c and δ_x
  reminder: in matter domination $\delta_x \propto (1+w)\delta_c \propto \tau^2$ (constant g)

$$\gamma = \gamma\left(\Omega_m^0, \hat{c}_s, k, w, a\right)$$

Conclusions

1. Dark energy perturbations are needed for consistency
2. The growth index γ depends on
   - equation of state of DE, w
   - comoving scale, k (from Fourier space)
   - speed of sound of DE, c_s^2
   - redshift (or scale factor)
   - amount of dark matter, Ω_c^0
3. Including DE perturbations, the variation in γ can be comparable with the accuracy forecasts for the next generation of observations $\sim 7\%$

- Will we 'measure' DE fluctuations ?
- How to distinguish between MG and GR (dark energy)?



$k = 0.033\, h\, \mathrm{Mpc}^{-1}$, $\Omega_m^0 = 0.27$ $\hat{c}_s^2 = 0.01$

Red: exact numerical Blue dashed: fits

Status of γ

Theory

- Different theories ($f(R)$, DGP, GR...) predict different growths
- But the results can be mimicked with a general DE fluid
- A parameterization of γ can help to determine DE properties

Experiment

- Current data (BAO, WL, LSS, X-ray clust., Lyman–α, ISW) still poor
- Forecasted accuracy of future experiments $\Delta\gamma \simeq 0.04 \to 7\%\ldots$
- Effect of DE perturbations also can be as large as $\Delta\gamma$