Title: The S-matrix reloaded

Date: Jan 21, 2009 02:00 PM

URL: http://pirsa.org/09010009

Abstract: In the 60â€™s, the analytic S-matrix program was developed in an attempt to describe the strong interactions â€“ at the time, this was a theory of massive particles like pions. The S-matrix is an object that encodes the information of the probability of producing a certain set of final particles from a given set of initial particles. Eventually, the S-matrix program was replaced by Quantum Field Theory and in particular by Quantum Chromo Dynamics as the description of the strong interactions. In recent years there has been a resurrection of the S-matrix paradigm. The current view is that S-matrix techniques are most natural and powerful in theories of massless particles! Moreover, from this new perspective, the simplest quantum field theory to consider is now believed to be the maximally supersymmetric gravity theory. If the expectation is correct then N=8 supergravity will turn out to be a finite theory of gravity in perturbation theory.

Aristocrats vs. Democrats.

Back in time 40's.

Heisenberg.

Aristocrats vs. Democrats.

Back in time. 40's.
Heisenberg. 1942.

- Energy, Momentum, spin, mass, ..

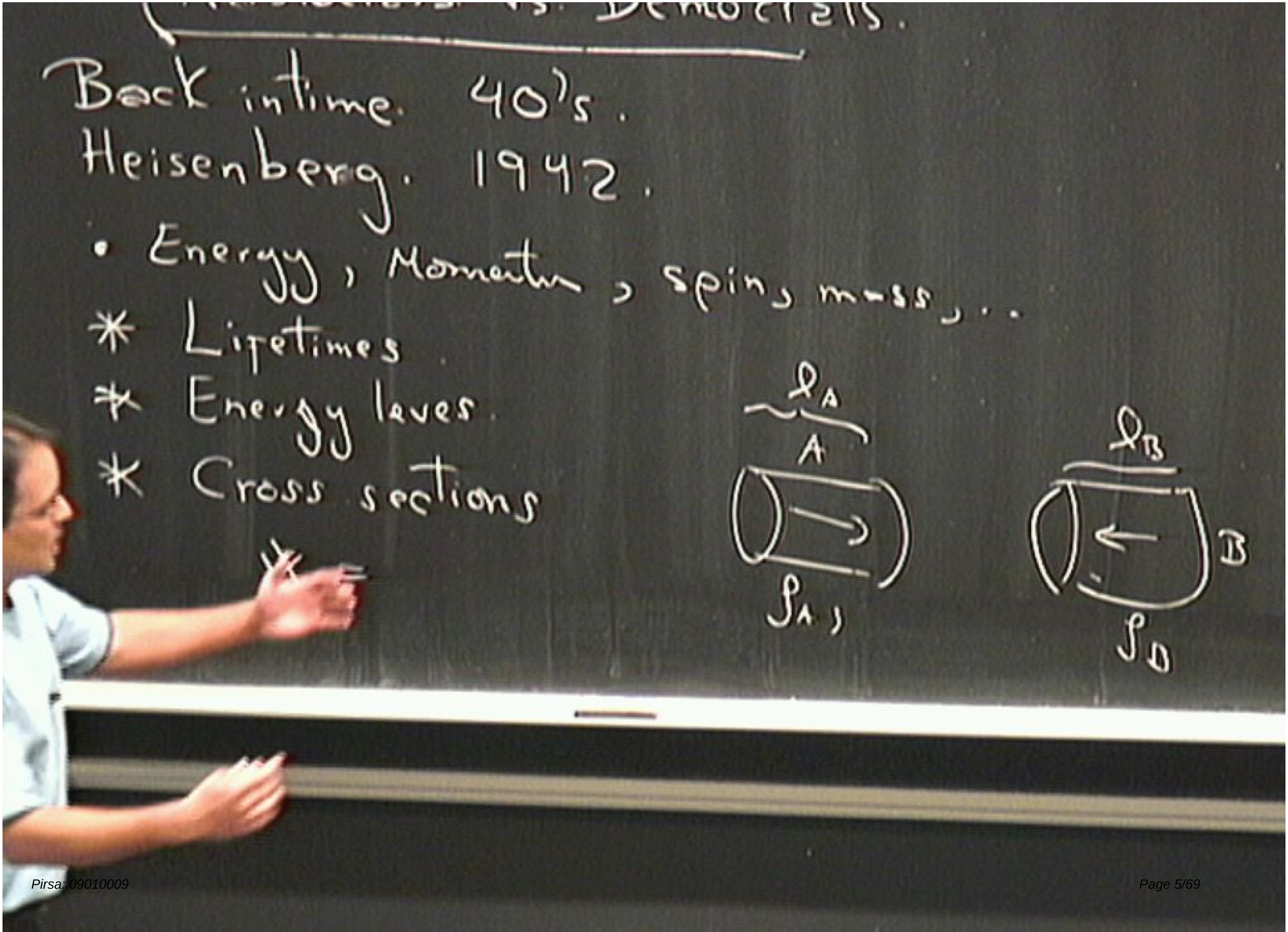
Aristocrats vs. Democrats.

Back in time 40's.

Heisenberg. 1942.

- Energy, Momentum, spin, mass, ...

* Lifetimes
* Energy leves
* Cross sections


Back in time 40's.
Heisenberg. 1942.
• Energy, Momentum, spin, mass, ..
* Lifetimes
* Energy leves
* Cross sections
A
B

... vs. Democrats.

Back in time 40's.

Heisenberg. 1942.

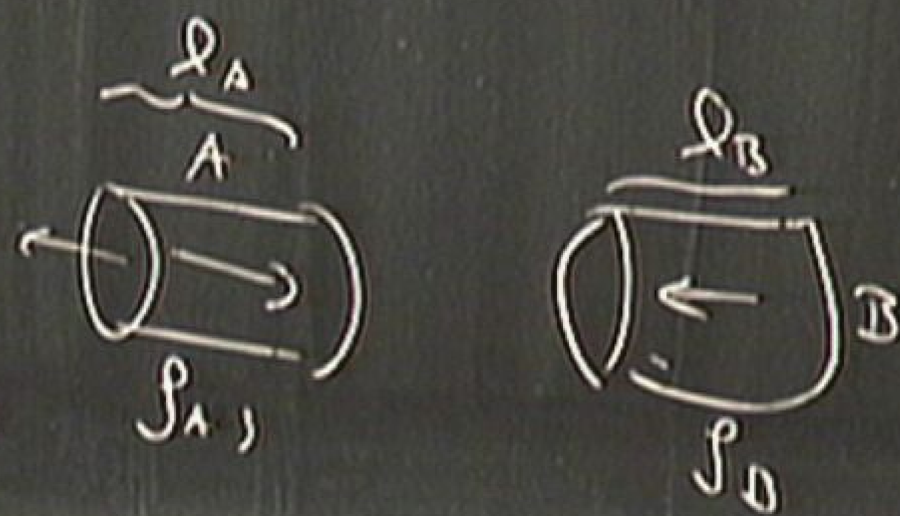
- Energy, Momentum, spin, mass, ..

* Lifetimes

* Energy leves.

* Cross sections σ

* $\sigma\, \ell_A \ell_B \rho_A \rho_B$



Back in time. 40's.

Heisenberg. 1942.

- Energy, Momentum, spin, mass, ...

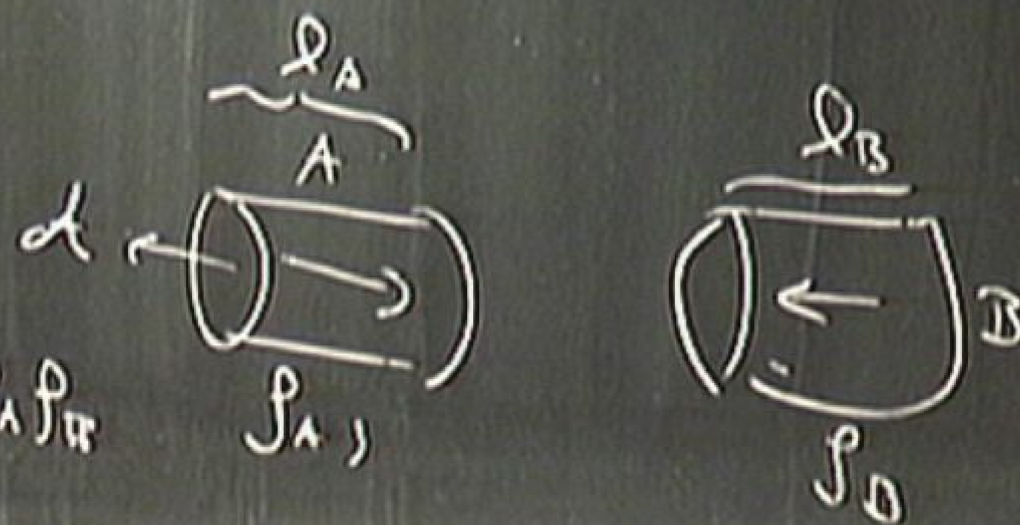
* Lifetimes.

* Energy leves.

* Cross sections

$\sigma \; \ell_A \ell_B \rho_A \rho_B$

Area



Scattering Matrix.

Classical Theory.

ω_i

$$\omega_f = \sum_f \omega_{fi}\, \omega_i$$

$\omega_{fi} \to$ Conditional Probability.

Scattering Matrix.

Classical Theory.

ω_i $\qquad$ $\omega_f = \sum_f \omega_{fi}\, \omega_i$

ω_{fi} → Condition Probability.

Quantum Theory. $\omega_{fi} = |S_{fi}|^2.$

S_{fi} → cplx number.

Scattering Matrix.

Classical Theory.

ω_i

$$\omega_f = \sum_f \omega_{fi}\,\omega_i$$

Conditional Probability

Quantum Theory.

$\omega_i = |a_i|^2$

$$\omega_{fi} = |S_{fi}|^2.$$

Cplx number.

$$a_f = \sum S_{fi}\,a_i$$

Back to the Future. 50's & 60's.

+ Exp. Plethora.

Back to the Future. 50's & 60's.

* Exp. Plethora of new part. → Hadrons.

↳ String Int.

Massive.

Back to the Future. 50's & 60's.

* Exp. Plethora of new part. → Hadrons.

↳ String Int.

Massive.

T.P → Fundam. Particles (Aristocrats)

→

Back to the Future. 50's & 60's.

* Exp. Plethora of new part. → Hadrons.

↳ String Int.

Massive.

T.P → Fundam. Particles (Aristocrats)

T.P → Democratic way.

Postulates:

1) Initial & Final $\longrightarrow$ Irreps. of the Poincaré group

$\downarrow$

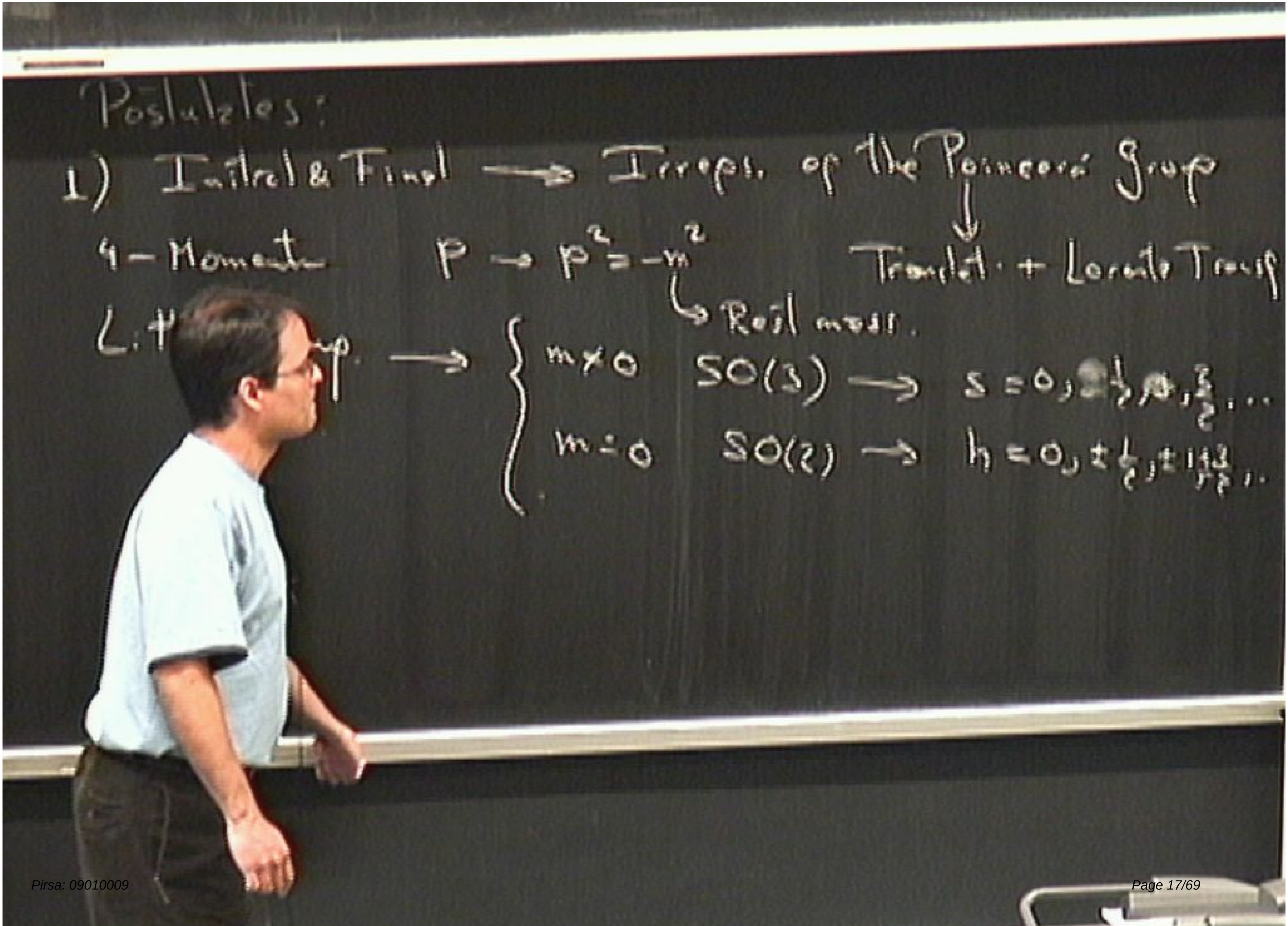
Translat. + Lorentz Trans

Postulates:

1) Initial & Final $\longrightarrow$ Irreps. of the Poincaré group

4 - Momentum $\quad p \to p^2 = -m^2$ $\quad$ Translat. + Lorentz Transf.

$\hookrightarrow$ Real mass.


Postulates:
1) Initial & Final → Irreps. of the Poincaré Group
4-Momentum P → p² = -m²
Translat. + Lorentz Transf
Rest mass.
m≠0 SO(3) → s = 0, 3/2, ...
m=0 SO(2) → h = 0, ±1/2, ...

1) Initial & Final $\longrightarrow$ Irreps. of the Poincaré group

$\downarrow$ Translat. + Lorentz Transf

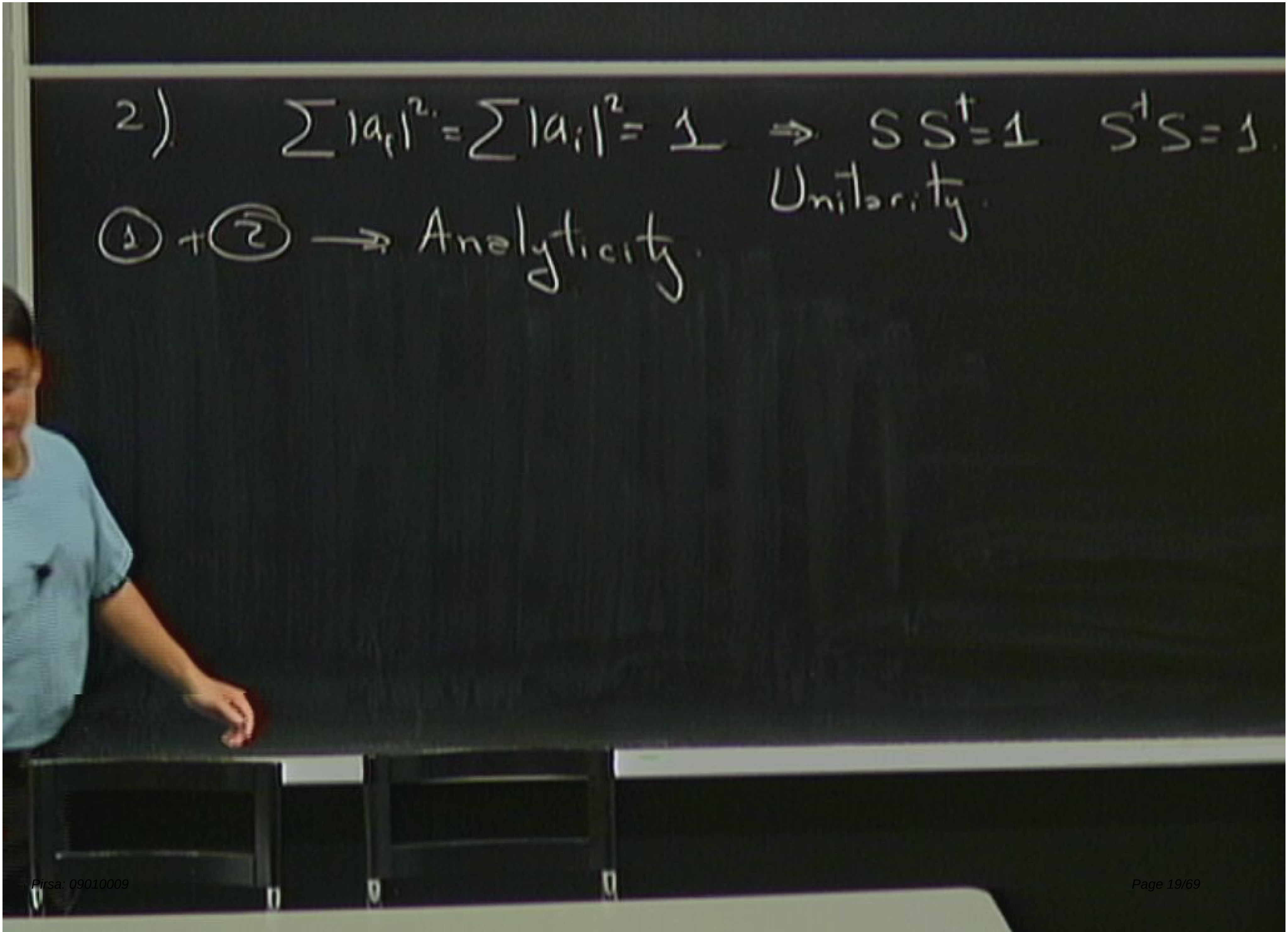
4-Momentum $\quad p \to p^2 = -m^2$

$\hookrightarrow$ Real mass.

Little group $\longrightarrow$

$$\begin{cases} m \neq 0 & SO(3) \longrightarrow s = 0, \frac{1}{2}, 1, \frac{3}{2}, \ldots \\ m = 0 & SO(2) \longrightarrow h = 0, \pm\frac{1}{2}, \pm 1, \pm\frac{3}{2}, \ldots \end{cases}$$

h: helicity.


2). $\sum |a_i|^2 = \sum |a_i|^2 = 1 \Rightarrow S S^\dagger = 1 \quad S^\dagger S = 1$
Unitarity.
1 + 2 → Analyticity.

2) $\sum |a_f|^2 = \sum |a_i|^2 = 1 \Rightarrow S S^\dagger = 1 \quad S^\dagger S = 1$

Unitarity.

1 + 2 $\longrightarrow$ Analyticity.

* Preliminary. a

a) $S(P_i, P_f) = e^{i(P_i - P_f)\cdot a}\, S(P_i, P_f)$

2) $\sum |a_f|^2 = \sum |a_i|^2 = 1 \Rightarrow S^\dagger S = 1 \quad S S^\dagger = 1$

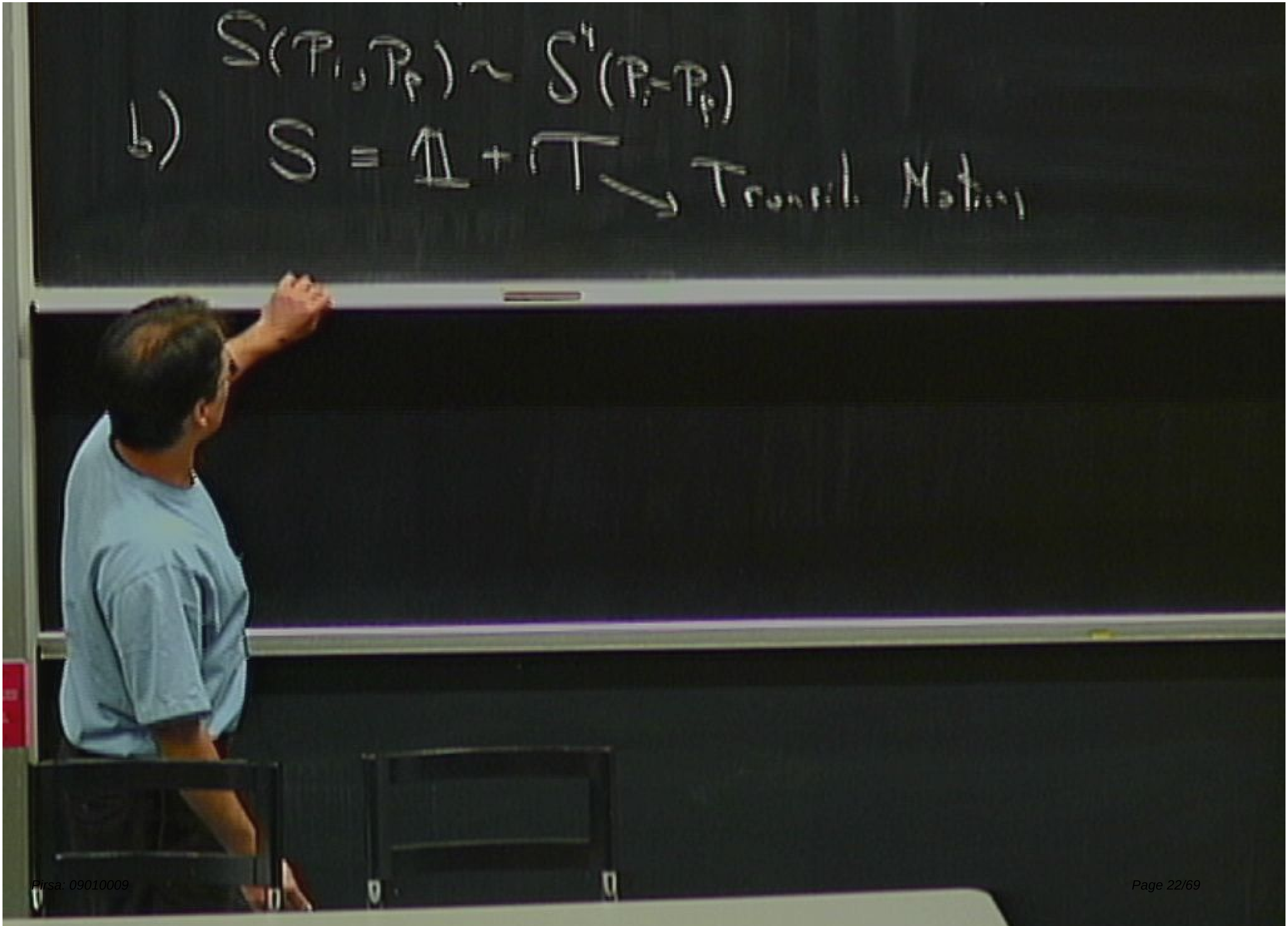
Unitarity.

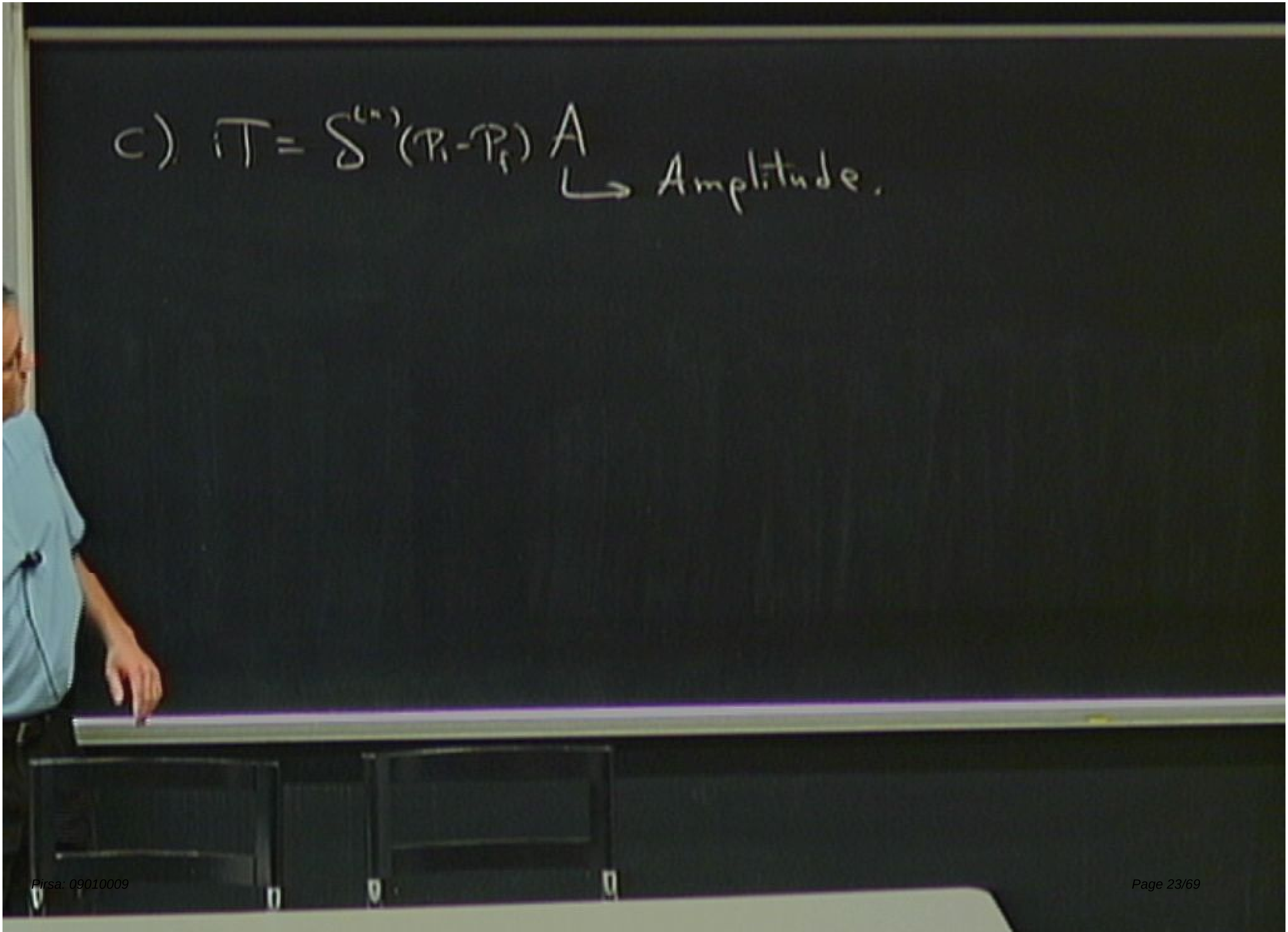
① + ② $\Longrightarrow$ Analyticity.

* Preliminary.

a) $S(P_i, P_f) = e^{i(P_i - P_f)\cdot a} S(P_i, P_f)$

$S(P_i, P_f) \sim \delta^4(P_i - P_f)$

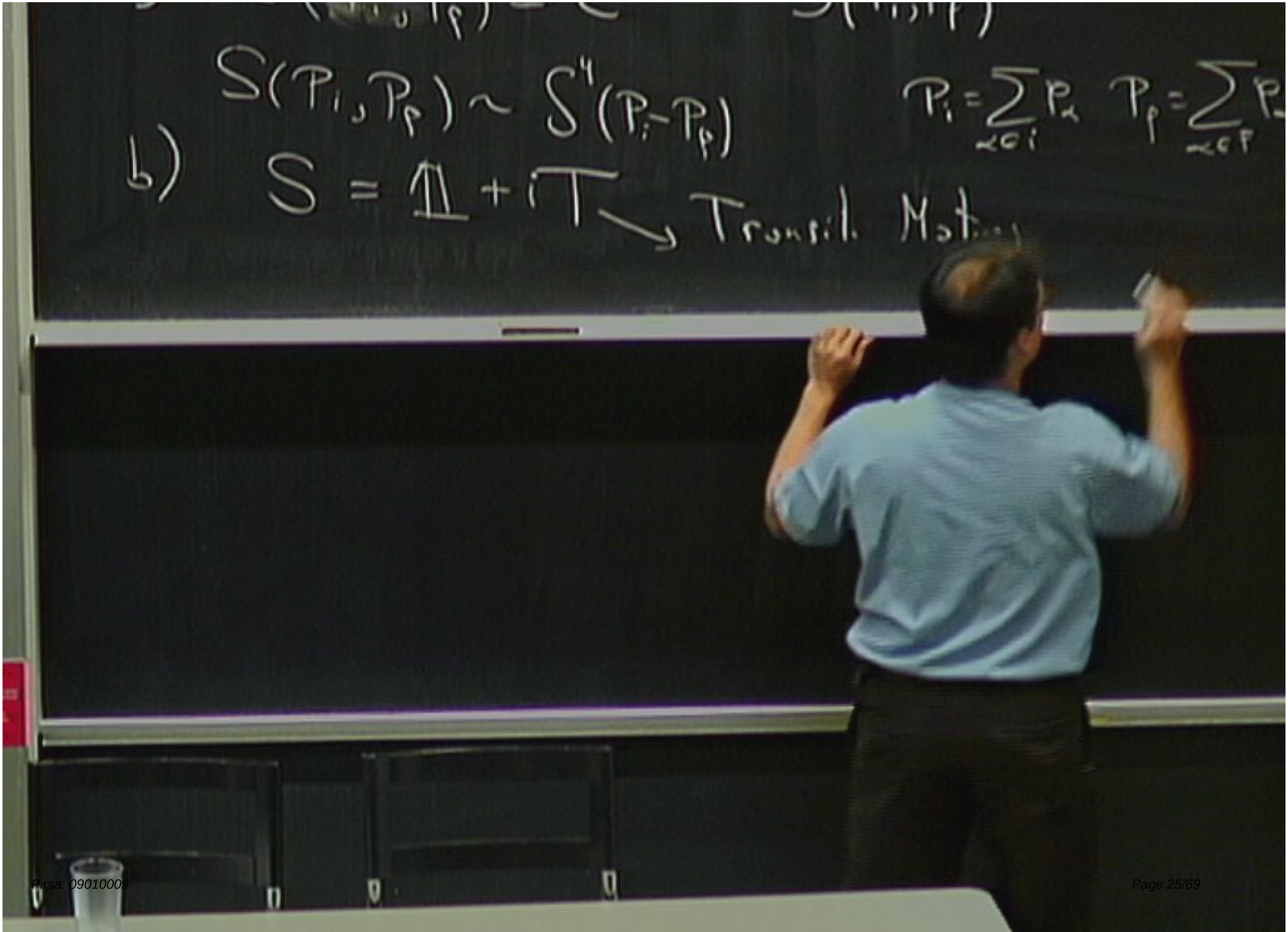

$S(P_i, P_f) \sim \delta^4(P_i - P_f)$
b) $S = 1\!\!1 + iT$
Transit. Matrix


c) $iT = \delta^{(4)}(P_i - P_f)\, A$
↳ Amplitude.

c) $iT = \delta^{(4)}(P_i - P_f)\, A$

$\hookrightarrow$ Amplitude.

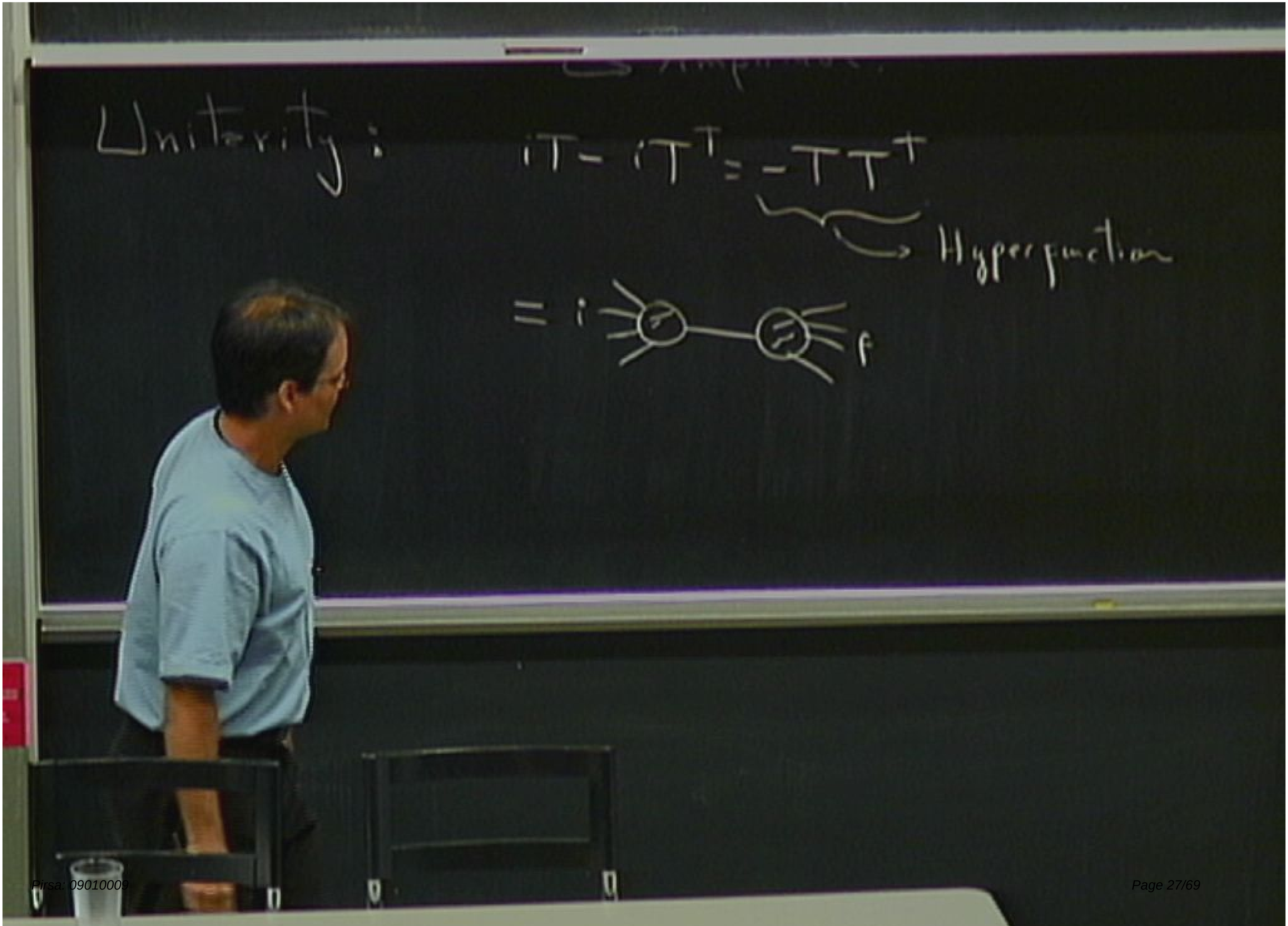
Unitarity:

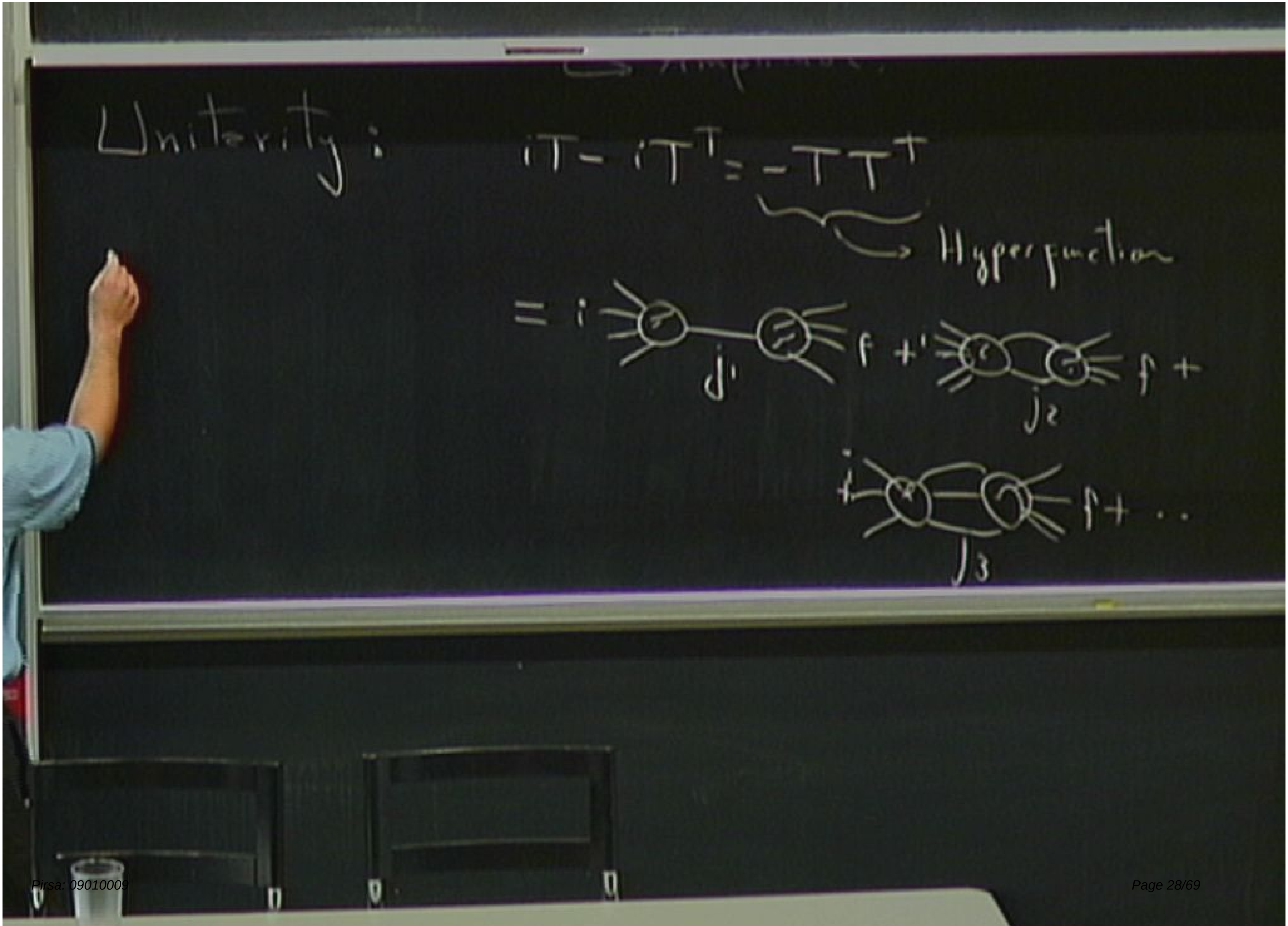

$S(P_i, P_f) \sim \delta^4(P_i - P_f)$
$P_i = \sum_{\alpha \in i} P_\alpha$
$P_f = \sum_{\alpha \in f}$
b)
$S = \mathbb{1} + iT$
Transit. Mat

c). $iT = \delta^{(4)}(P_i - P_f)\, A$

$\hookrightarrow$ Amplitude.

Unitarity: $\quad iT - iT^\dagger = -TT^\dagger$

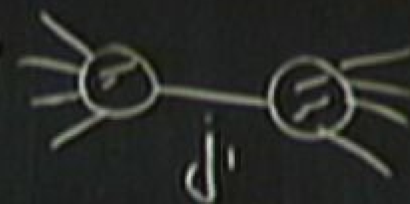
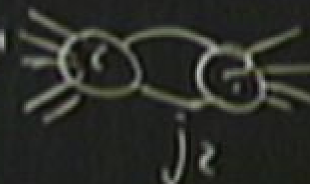

Unitarity:
$iT - iT^\dagger = -TT^\dagger$
Hyperfunction
$= i$
f

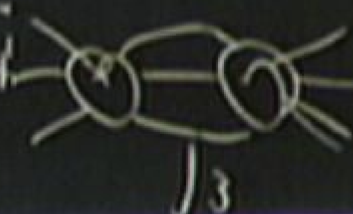

Unitarity:
iT − iT† = −TT†
Hyperfunction
= i
j1
f +i
j2
f +
i
j3
f + . .

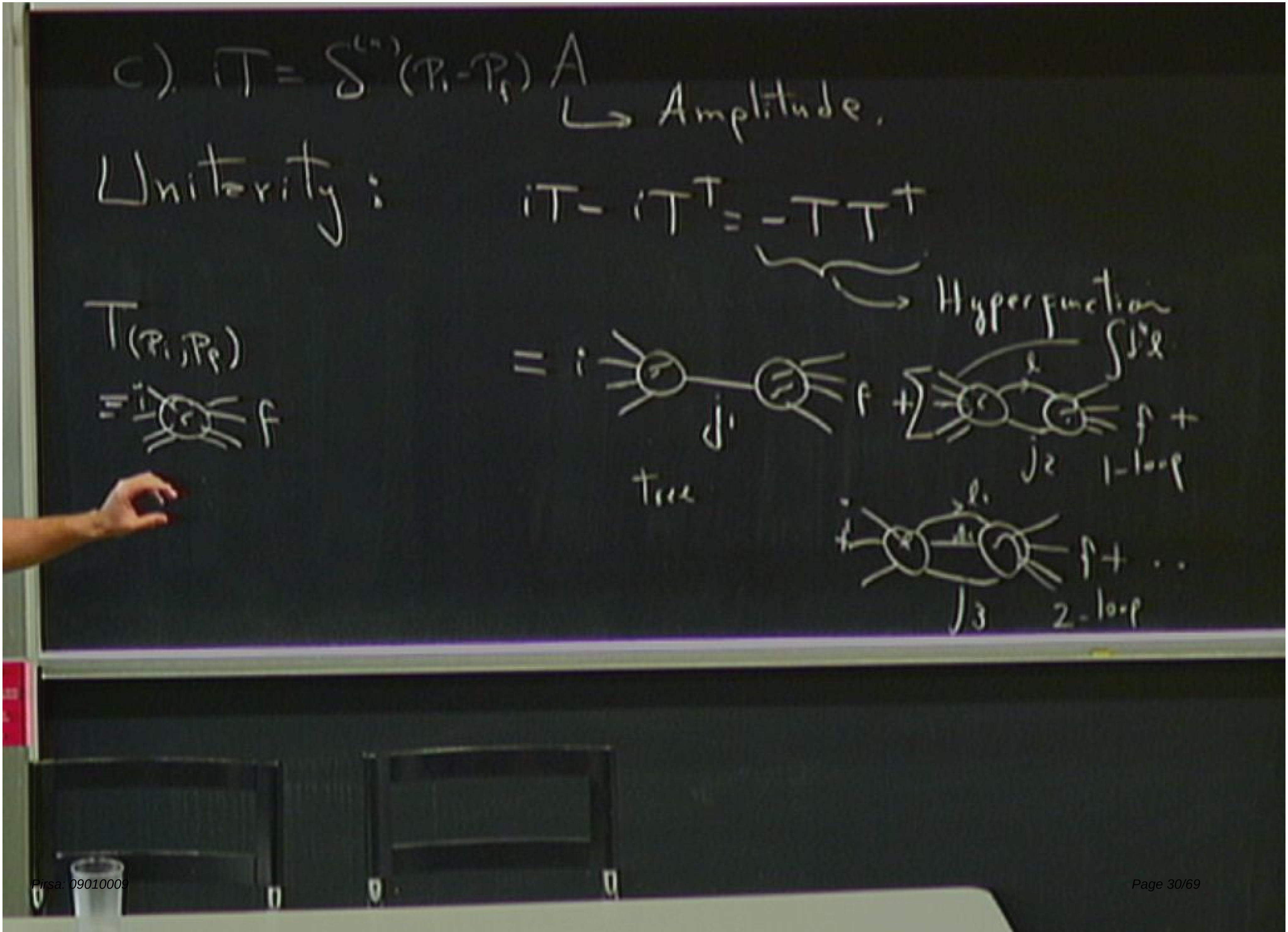
Unitarity: $iT - iT^\dagger = \underbrace{-TT^\dagger}$ → Hyperfunction

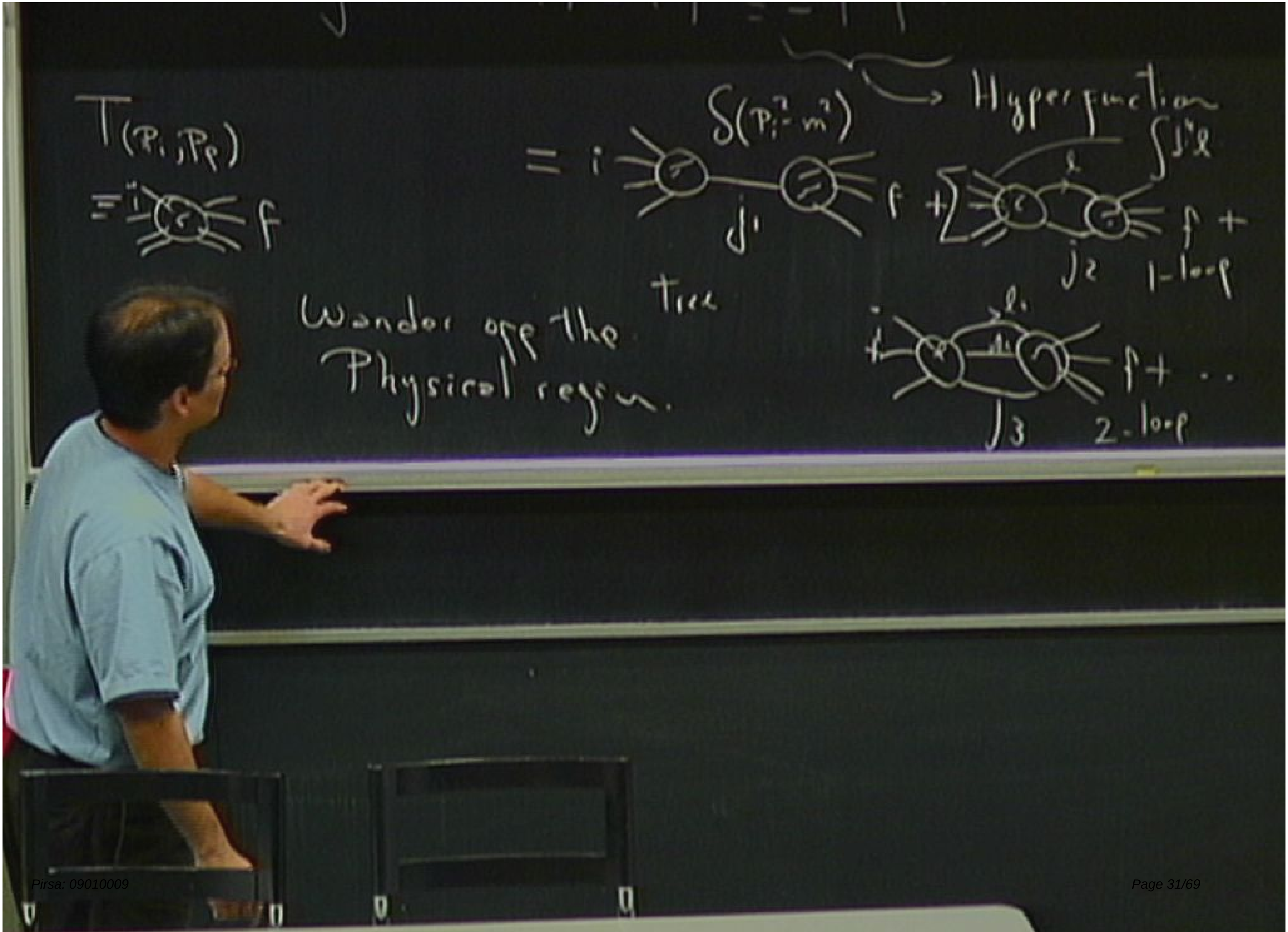
$T(p_i, p_f)$

$= i$  f

$= i$  f $+ i$  f $+$

i  $f + \ldots$


c). iT = δ(4)(Pi - Pf) A
Amplitude.
Unitarity:
iT - iT† = -TT†
Hyperfunction
T(Pi, Pf)
= i f
= i f + Σ f + f + . .
tree
∫d4l
1-loop
2-loop


Hyperfunction
tree
1-loop
2-loop
Wander off the
Physical region.

$$\lim_{\epsilon \to 0} \left(\frac{1}{(x-a)+i\epsilon} - \frac{1}{(x-a)-i\epsilon} \right) = \delta(x-a)$$

$$\underbrace{\lim_{\epsilon \to 0} \left(\frac{1}{(x-a)+i\epsilon} - \frac{1}{(x-a)-i\epsilon} \right)}_{\mathrm{Disc}\left(\frac{1}{(x-a)}\right)} = \delta(x-a)$$

$$\mathrm{Disc}(T) =$$

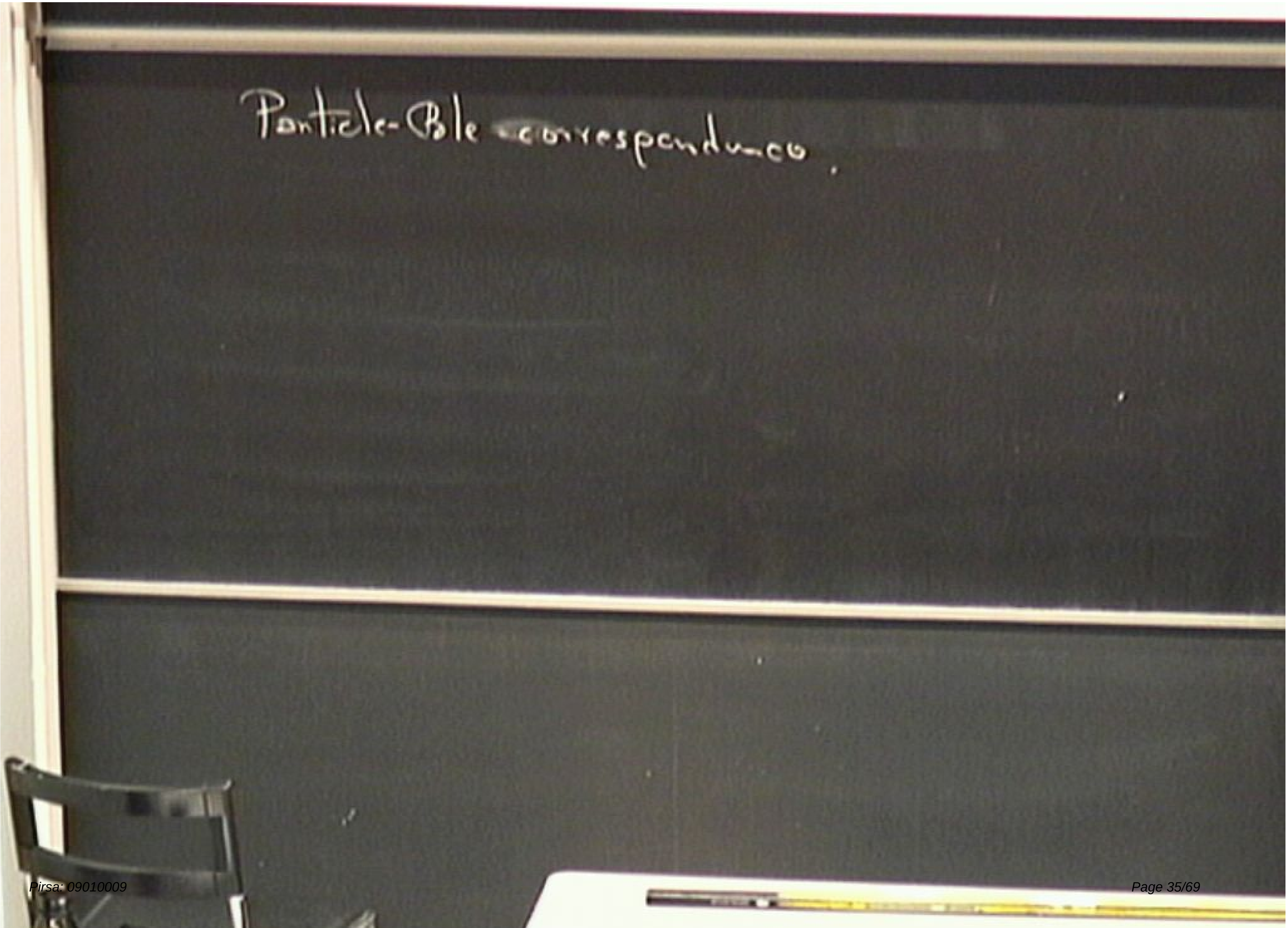
$$+ \cdots$$

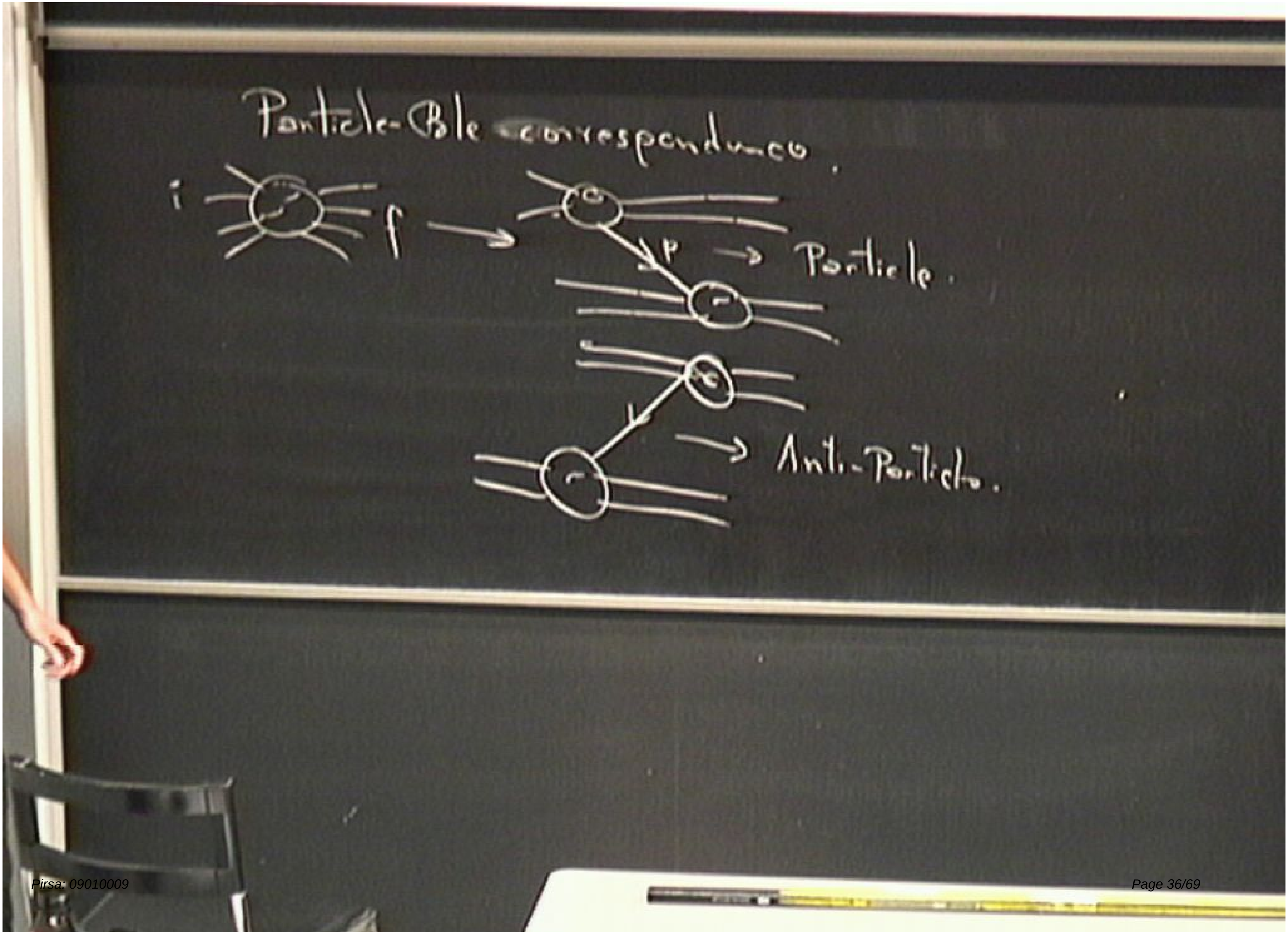
$$\underbrace{\lim_{\epsilon\to 0}\left(\frac{1}{(x-a)+i\epsilon}-\frac{1}{(x-a)+i\epsilon}\right)}_{=\ \mathrm{Disc}\left(\frac{1}{(x-a)}\right)}=\delta(x-a)$$

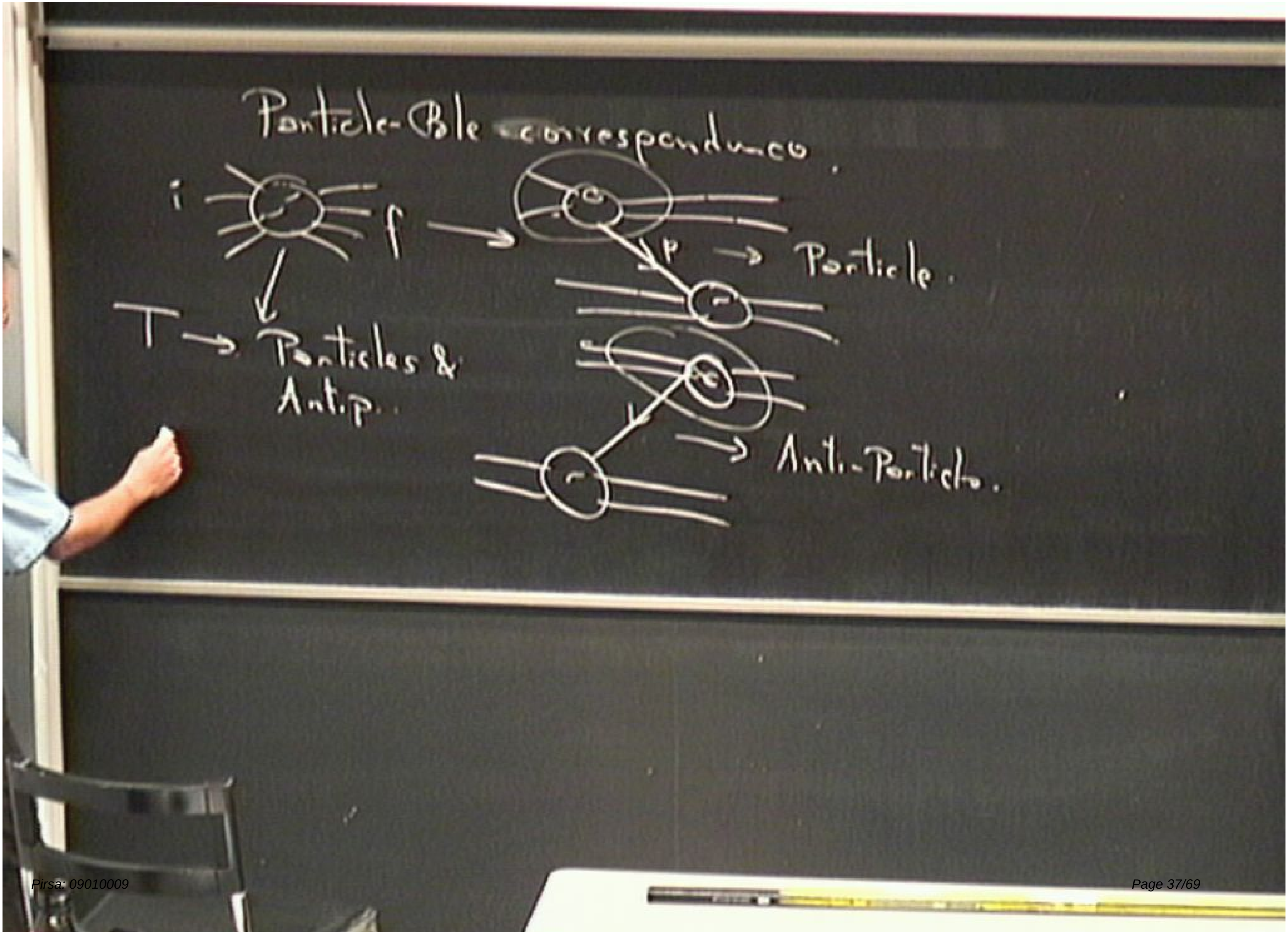
$$\mathrm{Disc}(T) =$$

$$+ \cdots$$

$$T \sim \frac{1}{(P_1^2-m^2)}$$


Particle-Hole correspondence.


Particle-Hole correspondence.
i
f
p
Particle.
Anti-Particle.


Particle-Hole correspondence.
i
f
T → Particles & Antip.
p → Particle.
Anti-Particle.

Analytic S-matrix :

Analytic S-matrix

- Death of the Program.

Analytic S-matrix

- Death of the Program ⟷ Very hard mathematical Problem

QFT AF

QCD

70's → ...

Analytic S-matrix

- Death of the Program $\longleftrightarrow$ Very hard mathematical Problems

QFT AF

QCD

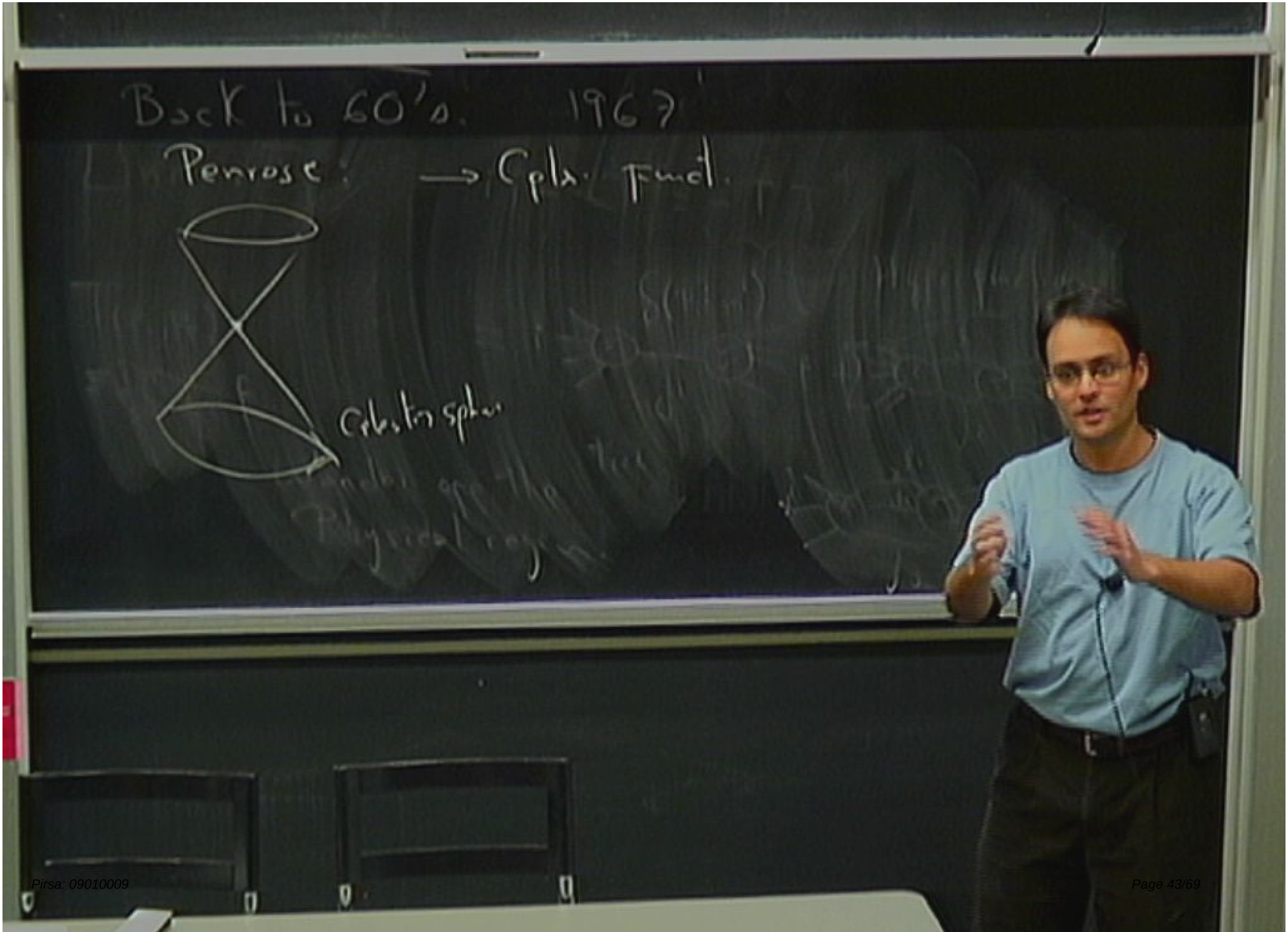
70's $\longrightarrow$ ...

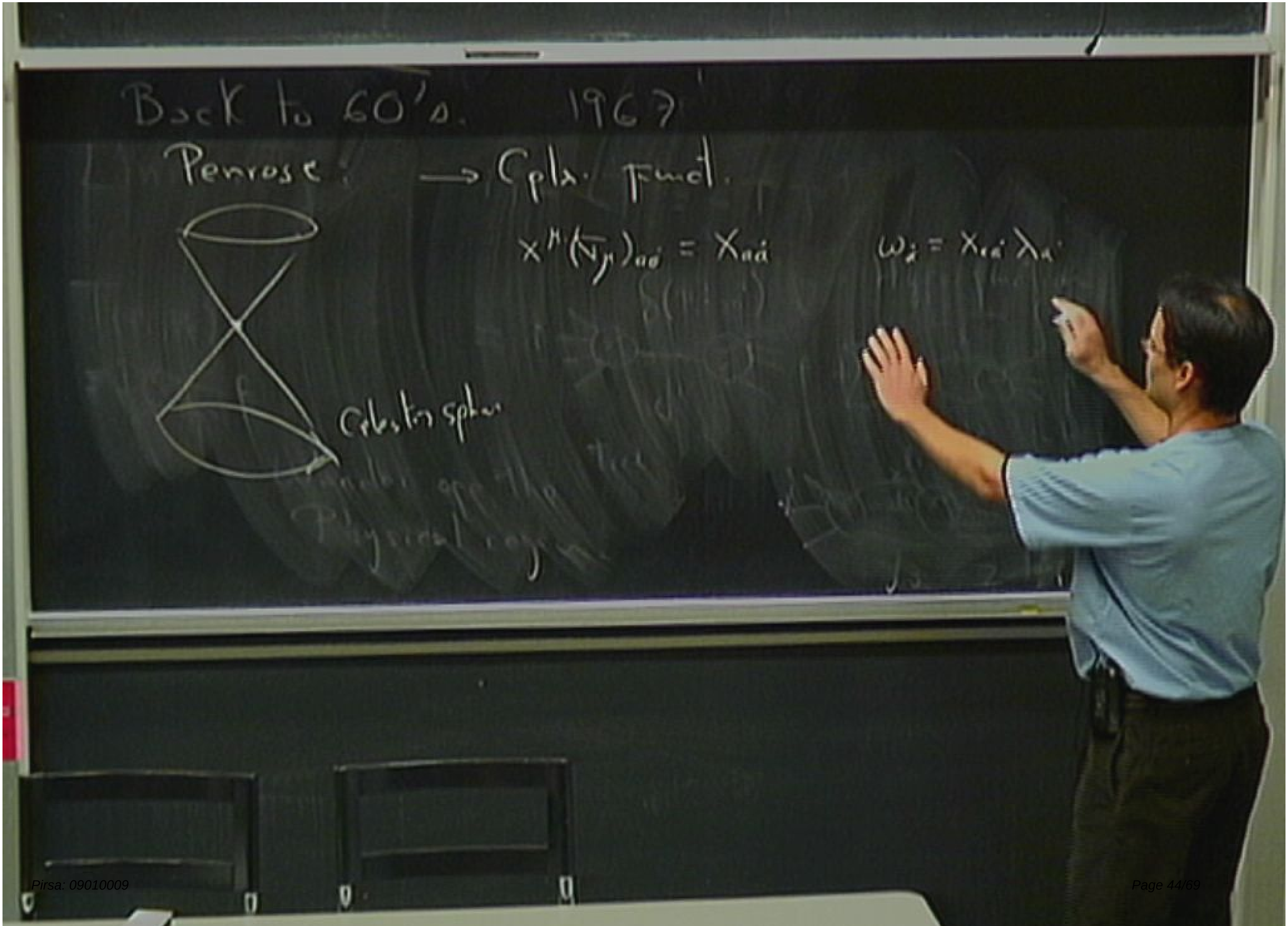
82 Z, W

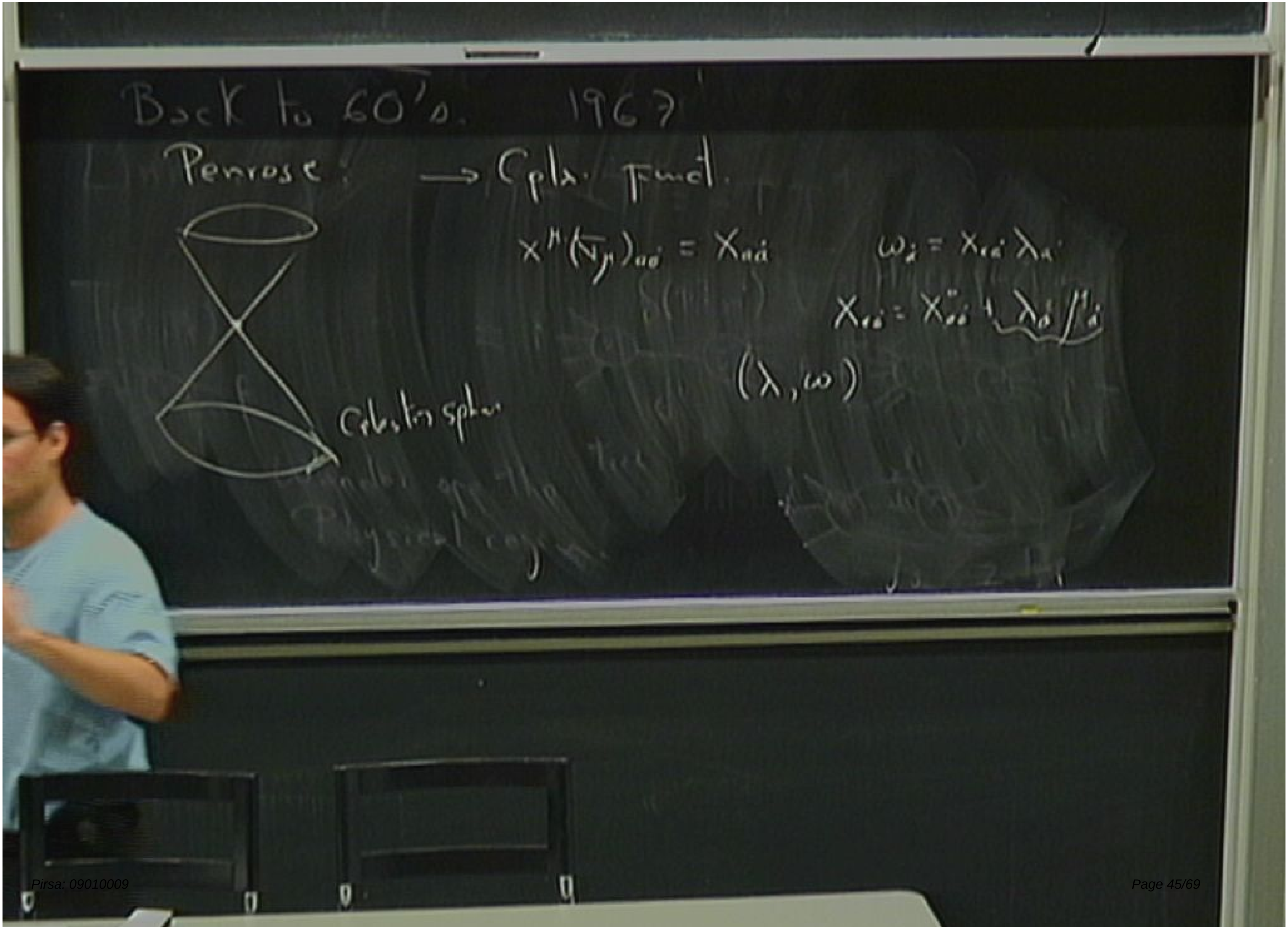
Aside

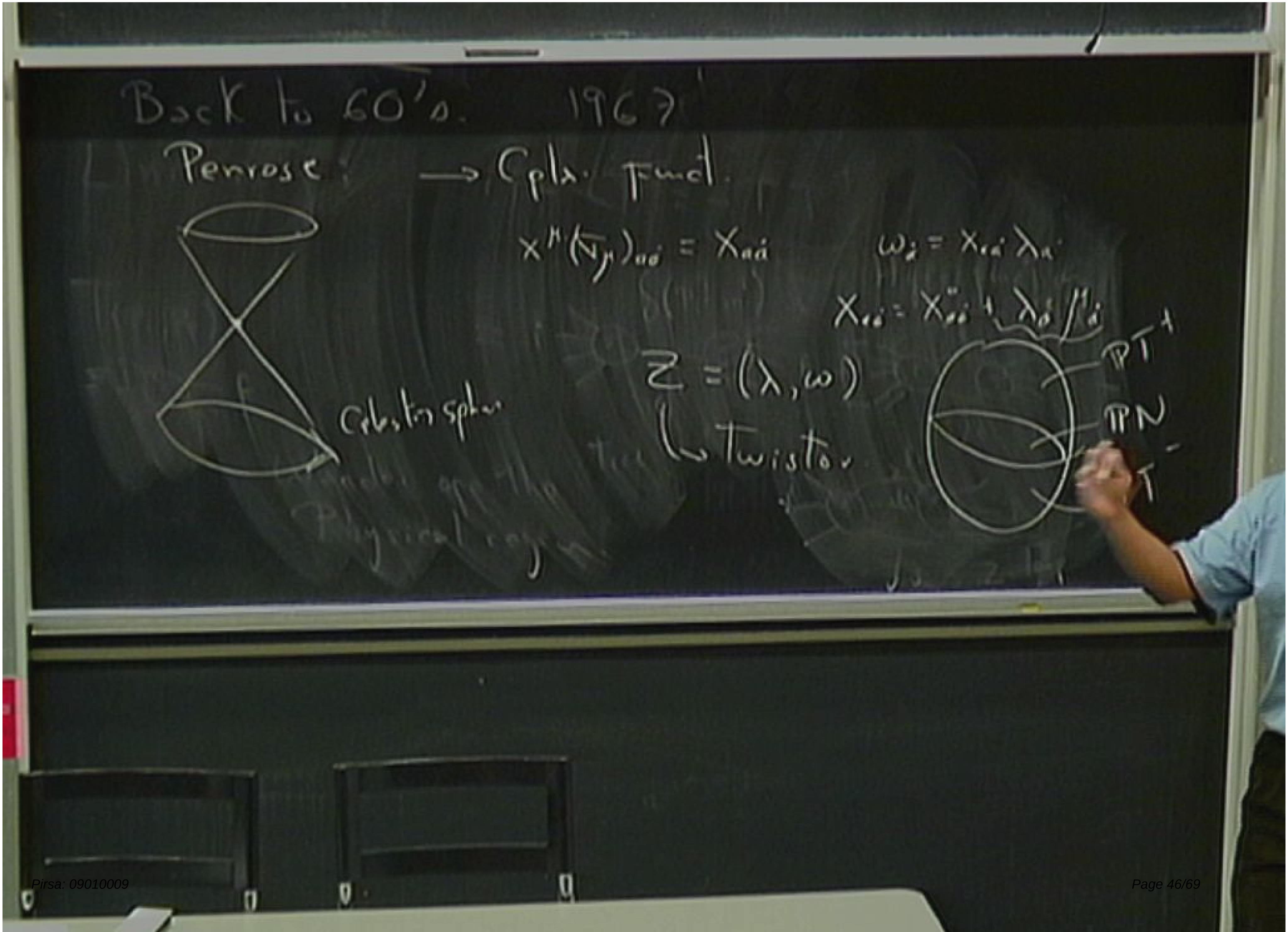
Back to 60's. 1967

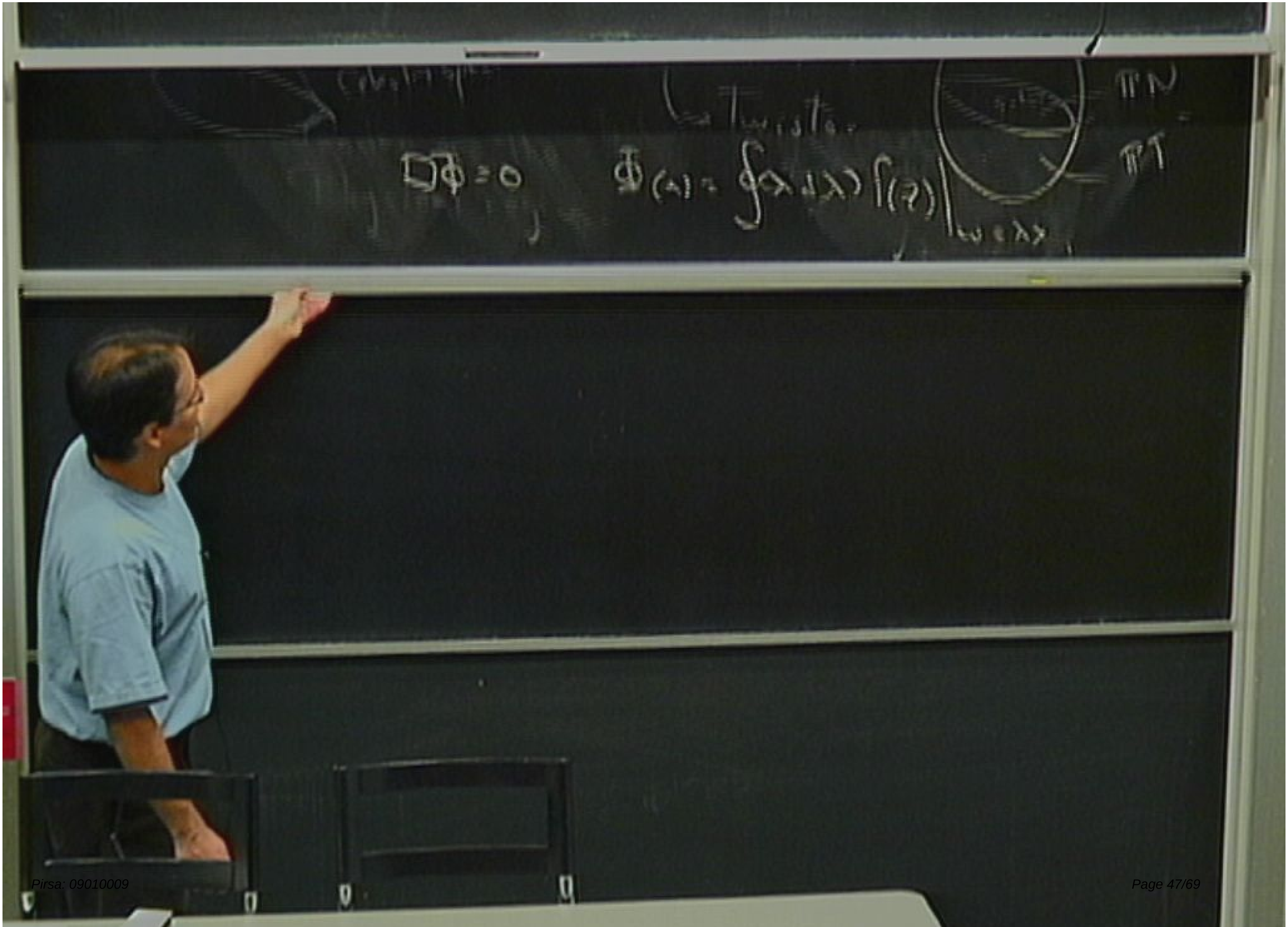
Penrose:


Back to 60's. 1967
Penrose: → Cplx. fund.


Back to 60's. 1967
Penrose
$x^\mu(\bar{\sigma}_\mu)_{\alpha\dot{\alpha}} = X_{\alpha\dot{\alpha}}$
$\omega_\alpha = X_{\alpha\dot{\alpha}}\lambda_{\dot{\alpha}}$

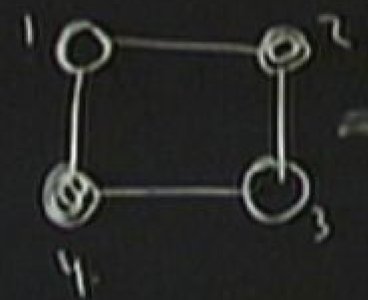

Back to 60's. 1967
Penrose: → Cplx. fund.
$x^\mu(\bar\sigma_\mu)_{a\dot a} = X_{a\dot a}$
$\omega_{\dot a} = X_{a\dot a}\lambda_{a}$
(λ, ω)
Celestial sphere

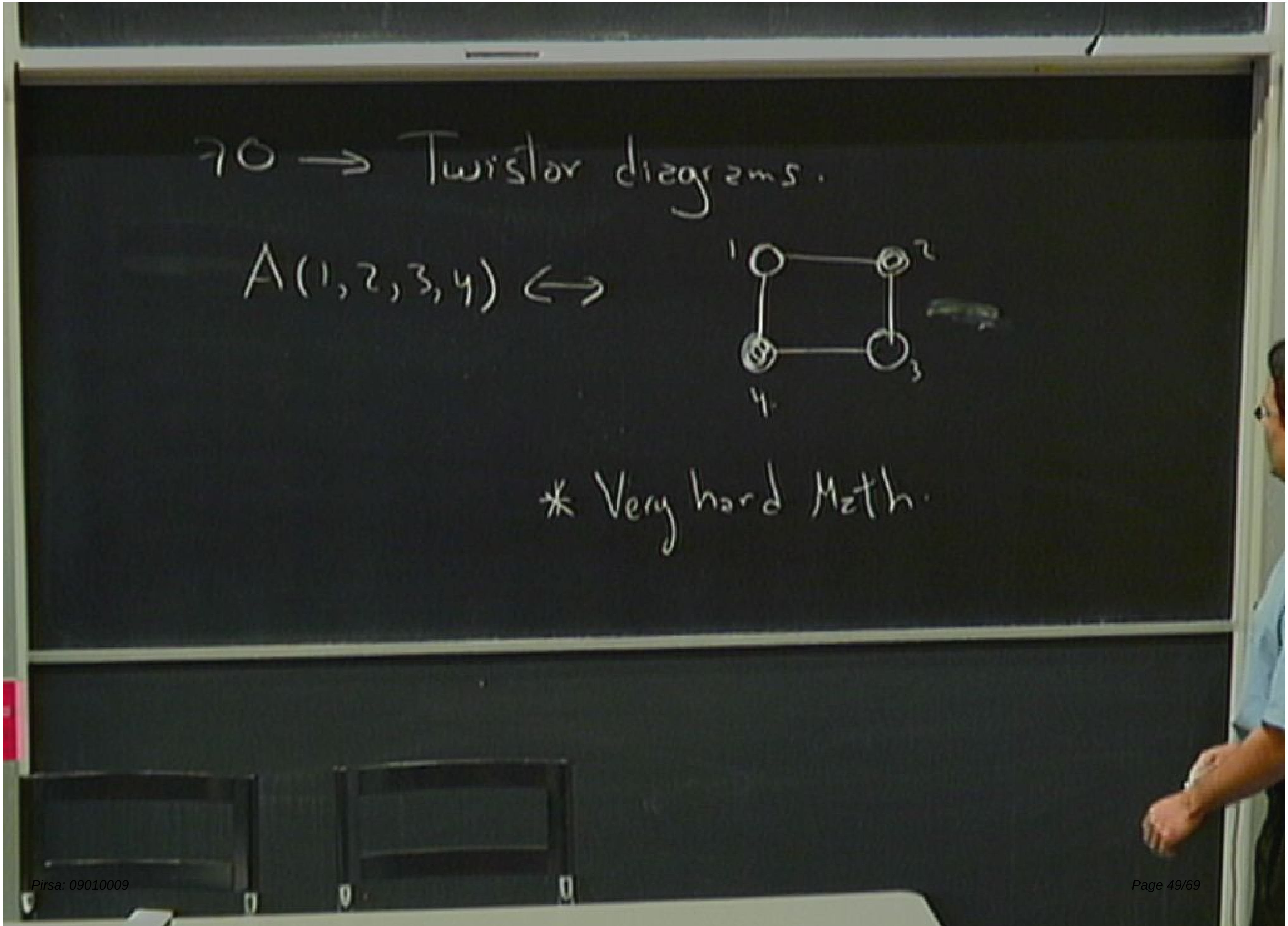

Back to 60's. 1967
Penrose: → Cplx. Fund.
Celestial sphere
Z = (λ, ω)
Twistor
PT+
PN
PT-

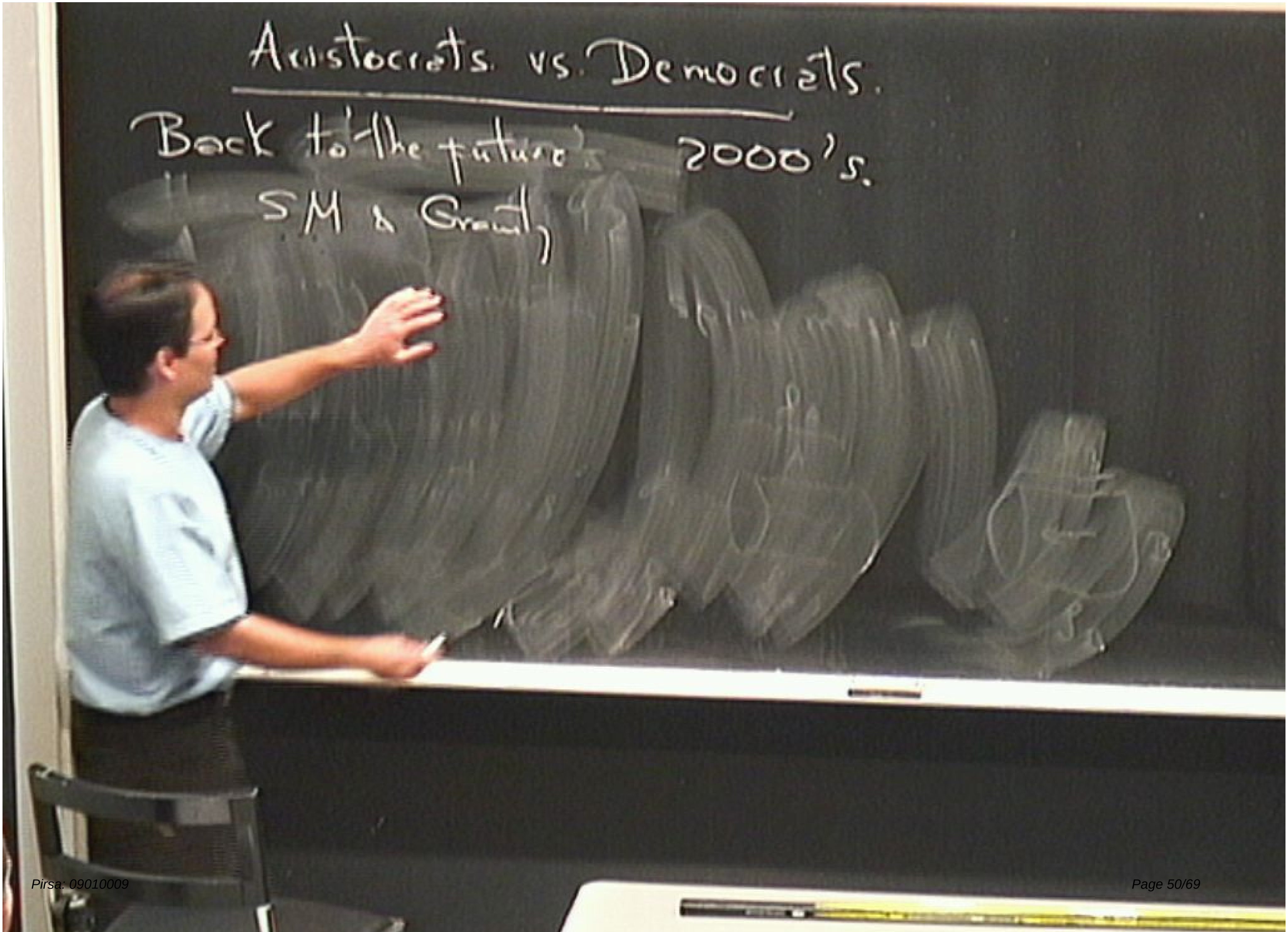

Twistor
□Φ = 0

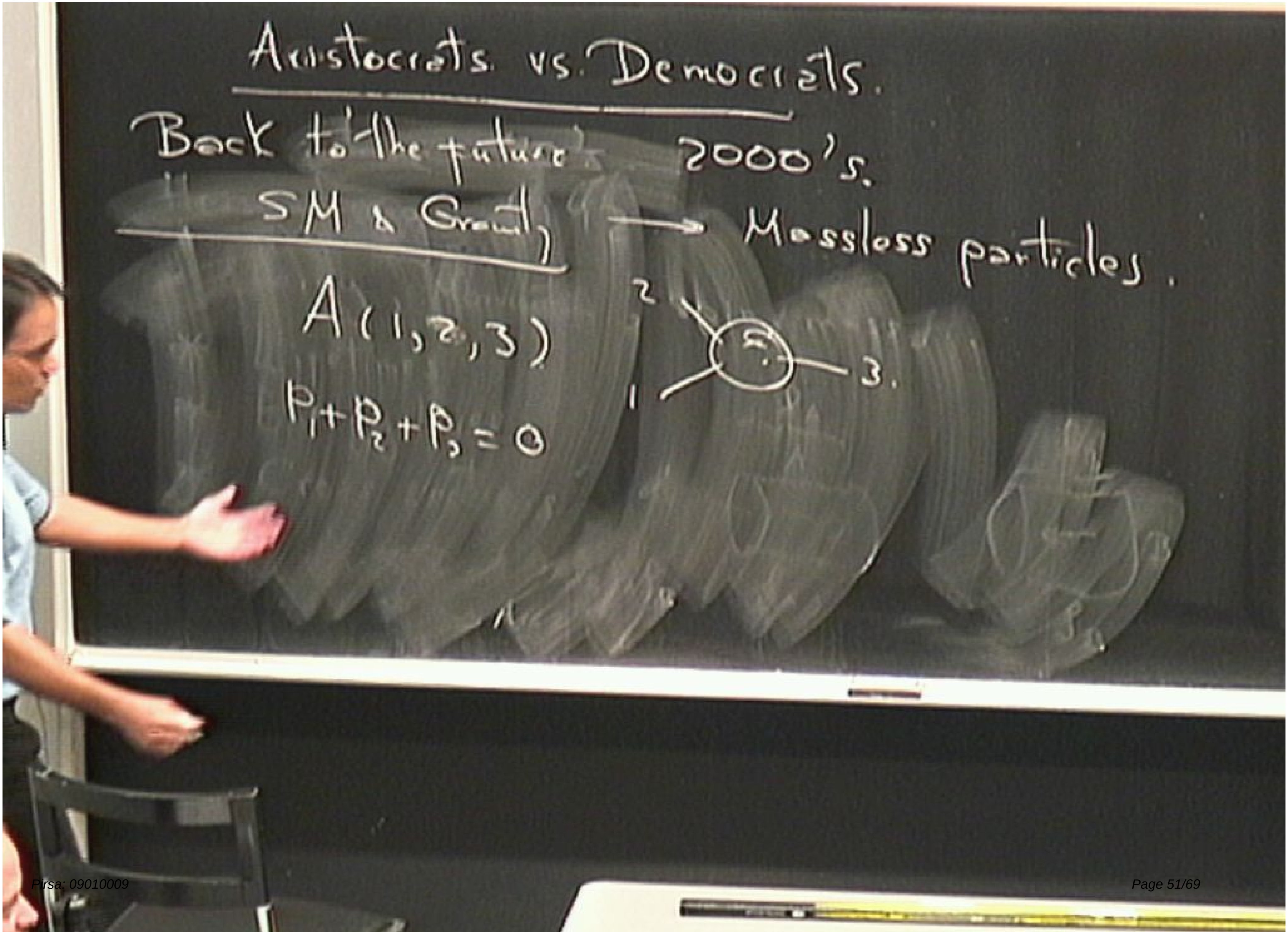
70 $\longrightarrow$ Twistor diagrams.

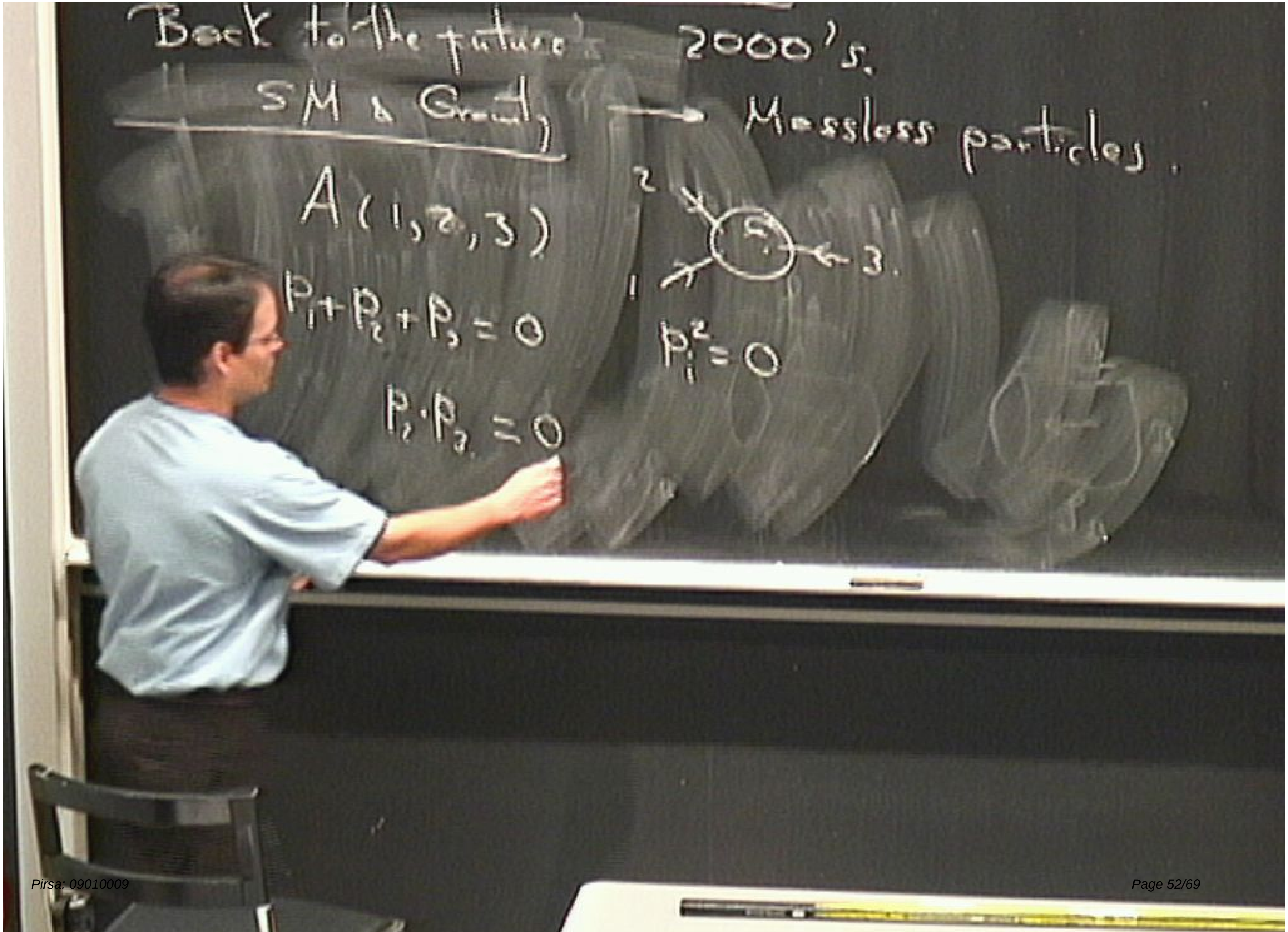
$A(1,2,3,4) \longleftrightarrow$




70 → Twistor diagrams.
A(1,2,3,4) ⟷
1
2
4
3
* Very hard Math.


Aristocrats vs. Democrats.
Back to the future: 2000's.
SM & Gravity,

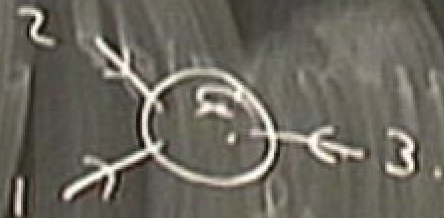

Aristocrats vs. Democrats.
Back to the future: 2000's.
SM & Gravity → Massless particles.
A(1,2,3)
p1+p2+p3 = 0
2
1
3.


Back to the future: 2000's.
SM & Gravity
Massless particles.
A(1,2,3)
P1+P2+P3 = 0
P1·P2 = 0
Pi² = 0
1
2
3.

Back to the future 2000's.

SM & Gravity $\longrightarrow$ Massless particles.

$A(1,2,3)$

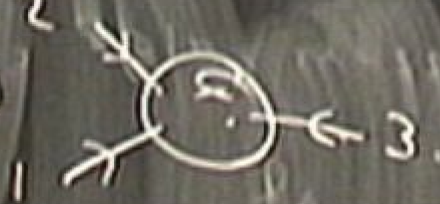


$p_1+p_2+p_3=0$ $\quad$ $p_i^2=0$

$p_2\cdot p_3=0 \Rightarrow p_1\cdot p_2=0 \quad p_1\cdot p_3=0$

SM & Gravity $\longrightarrow$ Massless particles.

$$A(1,2,3)=0$$



$$p_1+p_2+p_3=0 \qquad p_1^2=0 \qquad z_{12}\bar{z}_{12}=0 \qquad z_{13}\bar{z}_{13}=0$$

$$p_2\cdot p_3=0 \quad\Rightarrow\quad p_1\cdot p_2=0 \qquad p_1\cdot p_3=0$$

$$z_{23}\,\bar{z}_{23}$$

SM & Gravity $\longrightarrow$ Massless particles.

$$A(1,2,3)=0$$

$$p_1+p_2+p_3=0 \qquad p_i^2=0$$

$$p_2\cdot p_3=0 \Rightarrow p_1\cdot p_2=0 \rightarrow z_{12}\bar{z}_{12}=0 \rightarrow z_{12}\bar{W}_{12}=0$$

$$z_{23}\bar{z}_{23}$$

$$p_1\cdot p_3=0 \rightarrow z_{13}\bar{z}_{13}=0$$

$$A(1,2,3) = f(z_{12}, z_{23}, z_{31}) + g(\bar{w}_{12}, \bar{w}_{23}, \bar{w}_{31})$$

agram. $\longleftrightarrow$ Very hard mathematical Problem

8 2   z, w

70's $\longrightarrow$ . . .

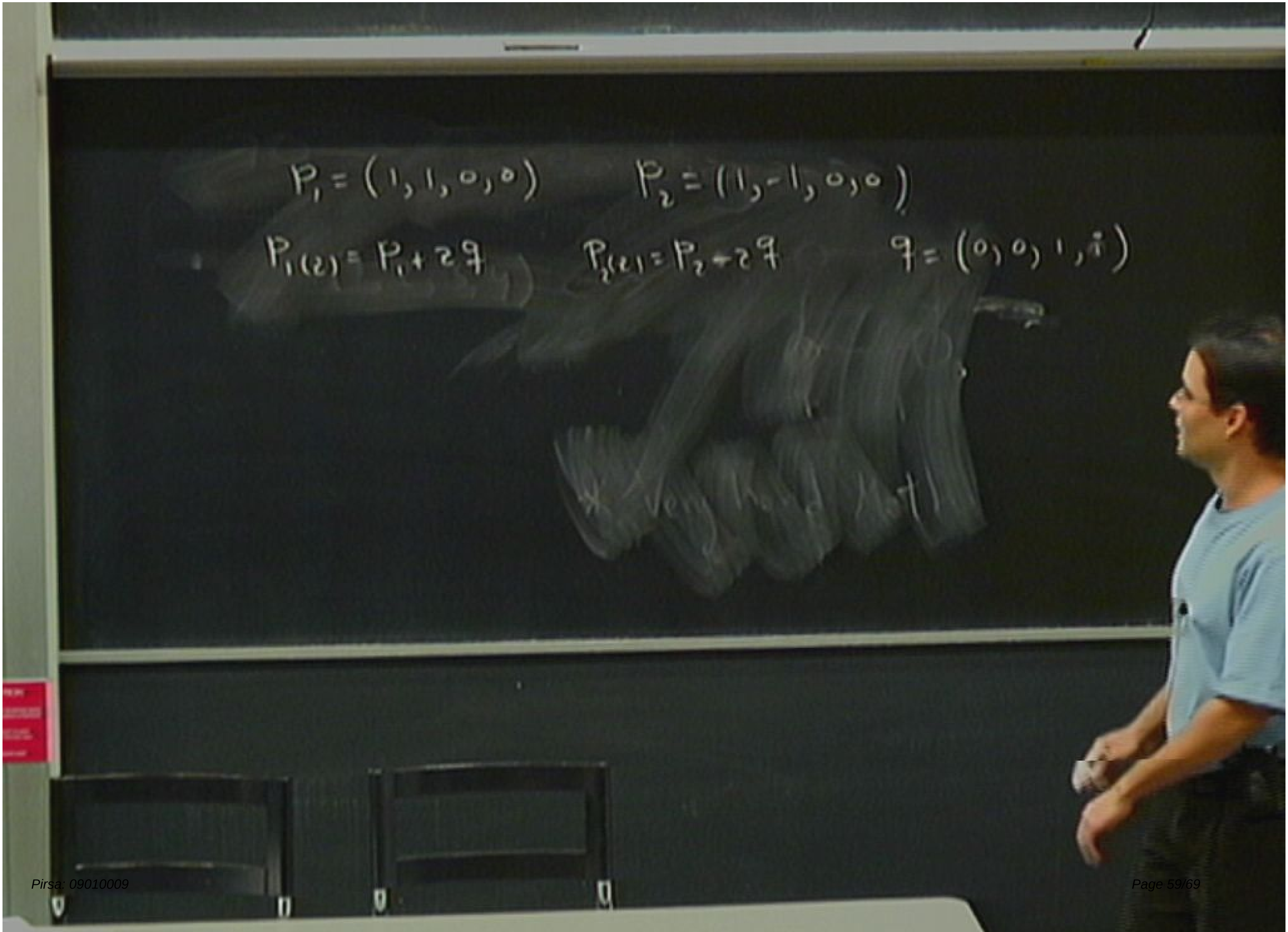
$$A(1,2,3) = f(z_{12}, z_{23}, z_{31}) + g(\bar{w}_{12}, \bar{w}_{23}, \bar{w}_{31})$$

$$A(1,2,3) = f(z_{12}, z_{23}, z_{31}) + g(\bar{w}_{12}, \bar{w}_{23}, \bar{w}_{31})$$

$z_{12} \to 0$ / $\bar{w}_{12} \to 0$: either f or $g \to \infty$, the other $\to 0$

$$A(1^-, 2^-, 3^+) = g \left(\frac{z_{12}^3}{z_{23} z_{31}} \right)^{\Delta}$$

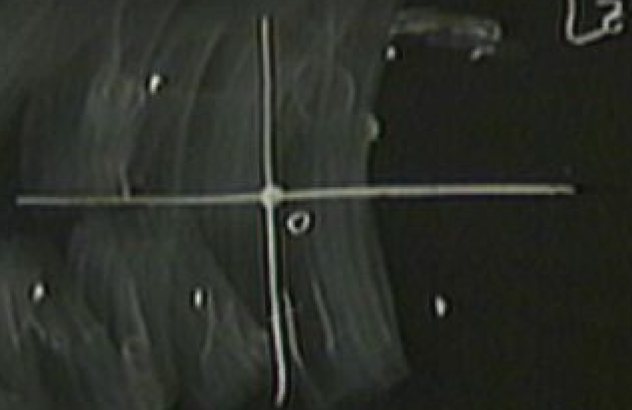
$h = \pm \Delta$

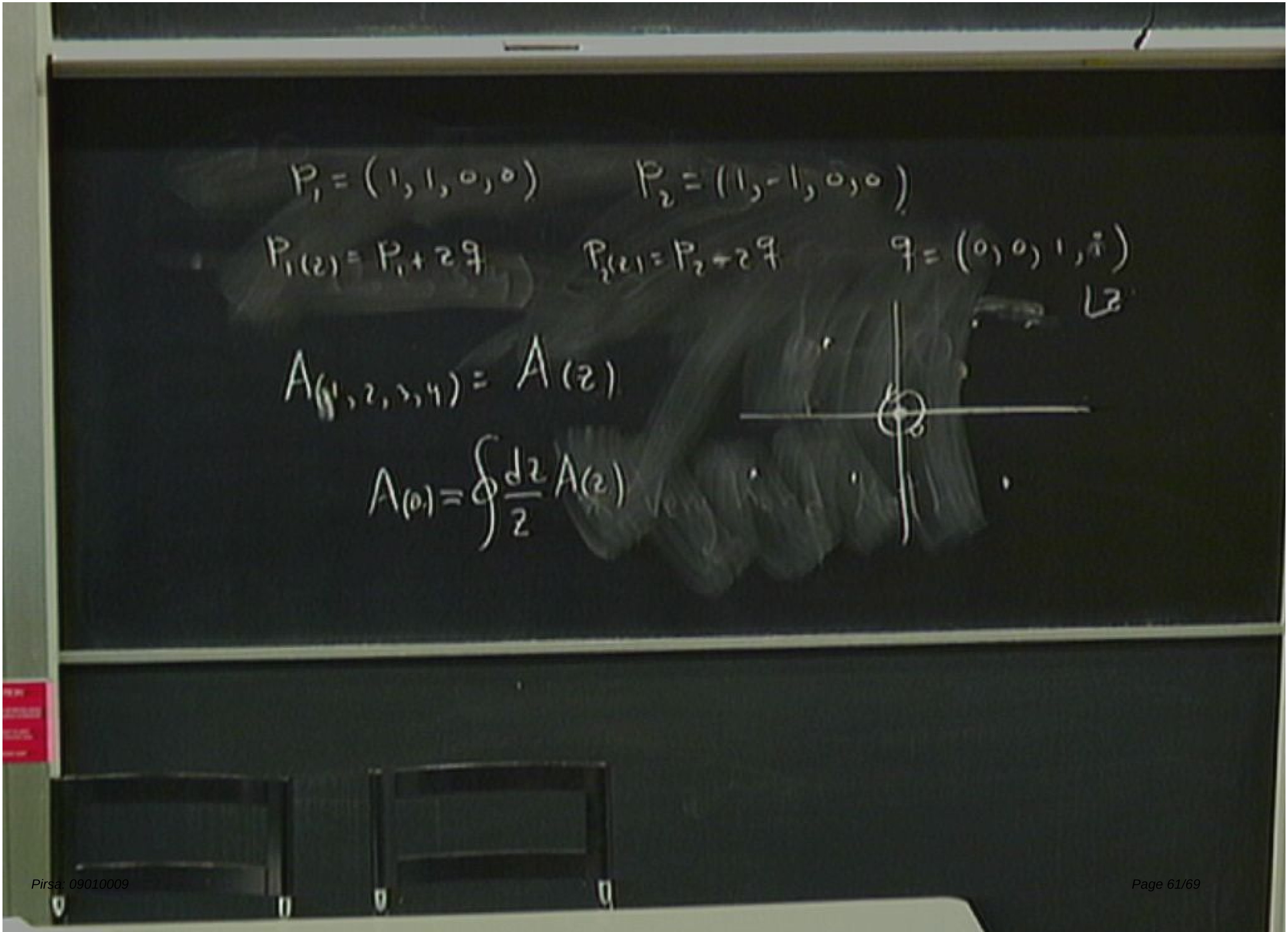

$p_1 = (1, 1, 0, 0)$
$p_2 = (1, -1, 0, 0)$
$p_{1(2)} = p_1 + 2q$
$p_{2(2)} = p_2 + 2q$
$q = (0, 0, 1, i)$

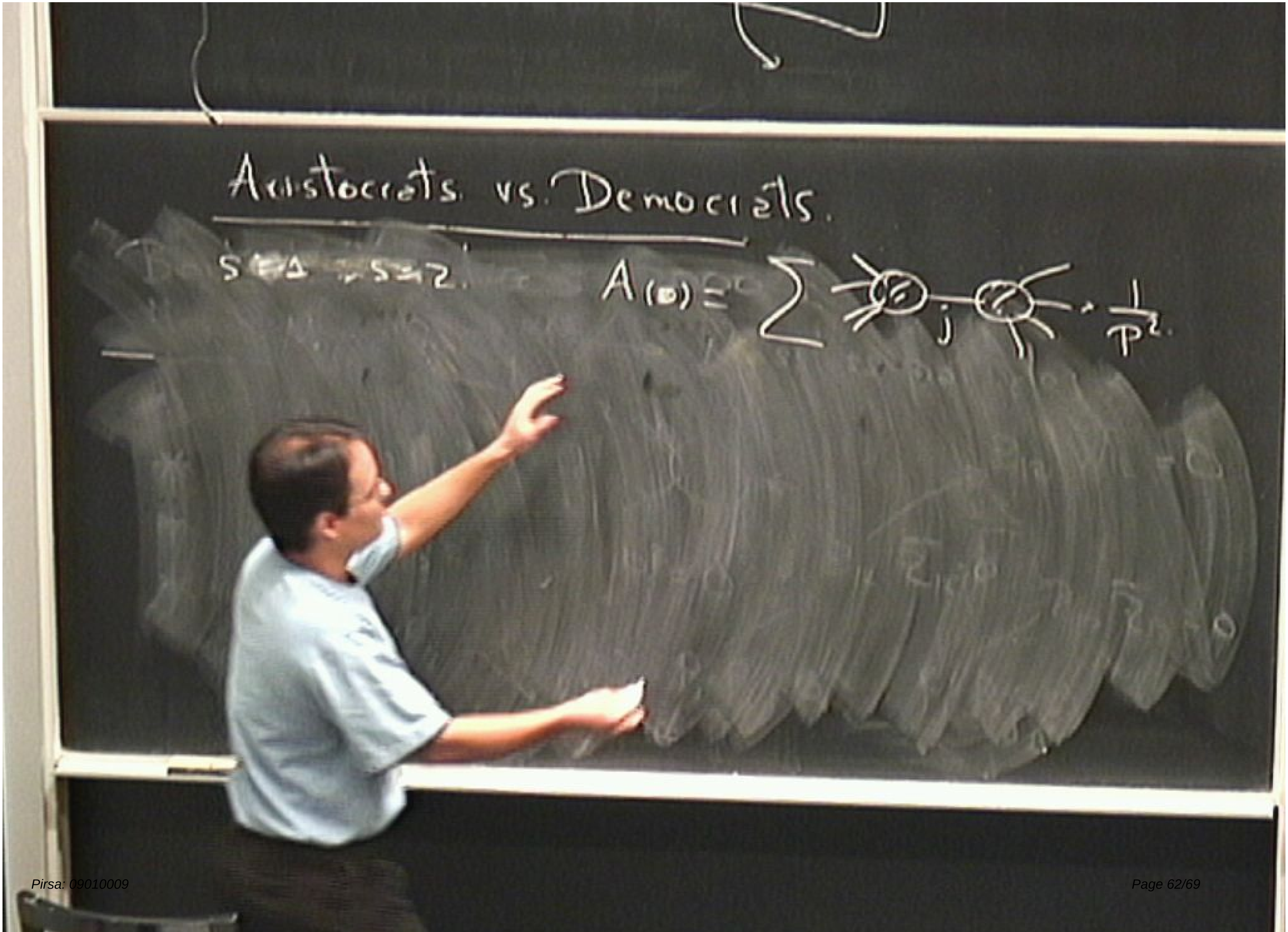
$P_1 = (1, 1, 0, 0)$ $\qquad$ $P_2 = (1, -1, 0, 0)$

$P_1(z) = P_1 + z q$ $\qquad$ $P_2(z) = P_2 - z q$ $\qquad$ $q = (0, 0, 1, i)$

$A(1, 2, 3, 4) = A(z)$

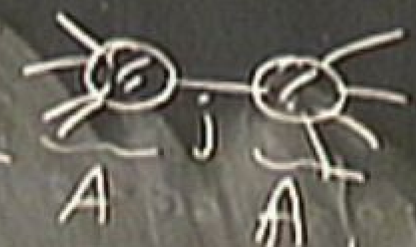



$P_1 = (1,1,0,0)$
$P_2 = (1,-1,0,0)$
$P_1(z) = P_1 + z q$
$P_2(z) = P_2 + z q$
$q = (0,0,1,i)$
$A(1,2,3,4) = A(z)$
$A(0) = \oint \frac{dz}{z} A(z)$


Aristocrats vs. Democrats.

Aristocrats vs. Democrats.

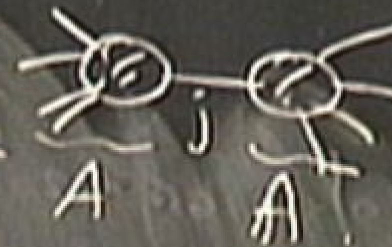
s ... s = 2.

$A(\phi) = \sum$  $\cdot \frac{1}{p^2}$

Recurs. Relat.

Aristocrats vs. Democrats.

s ... s=2.

$A(\infty) = \sum$  $\frac{1}{P^2}$

Recurs. Relat.

The whole tree level S-matrix is completely determined in terms of Poincaré invar. & Physical singu..

Aristocrats vs. Democrats.

YM ←

↳ Gravity

Recurs. Relat.

$$A(\cdot) = \sum A \; j \; A \cdot \frac{1}{P^2}$$

The whole tree level S-matrix is completely determined in terms of Poincaré invar. & Physical singu.

$\Downarrow$

S-matrix

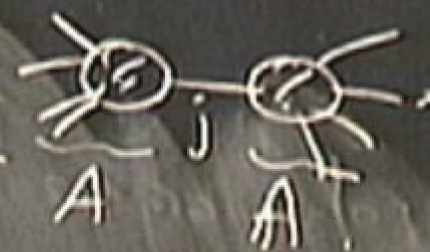
Aristocrats vs. Democrats.

S ... S ≠ 2.

YM. ↙ ↳ Gravity

Recurs. Relat.

$A(\cdot) = \sum_j$



$\cdot \frac{1}{P^2}$

The whole tree level S-matrix is completely determined in terms of Poincaré invar. & Physical singu..

S-matrix

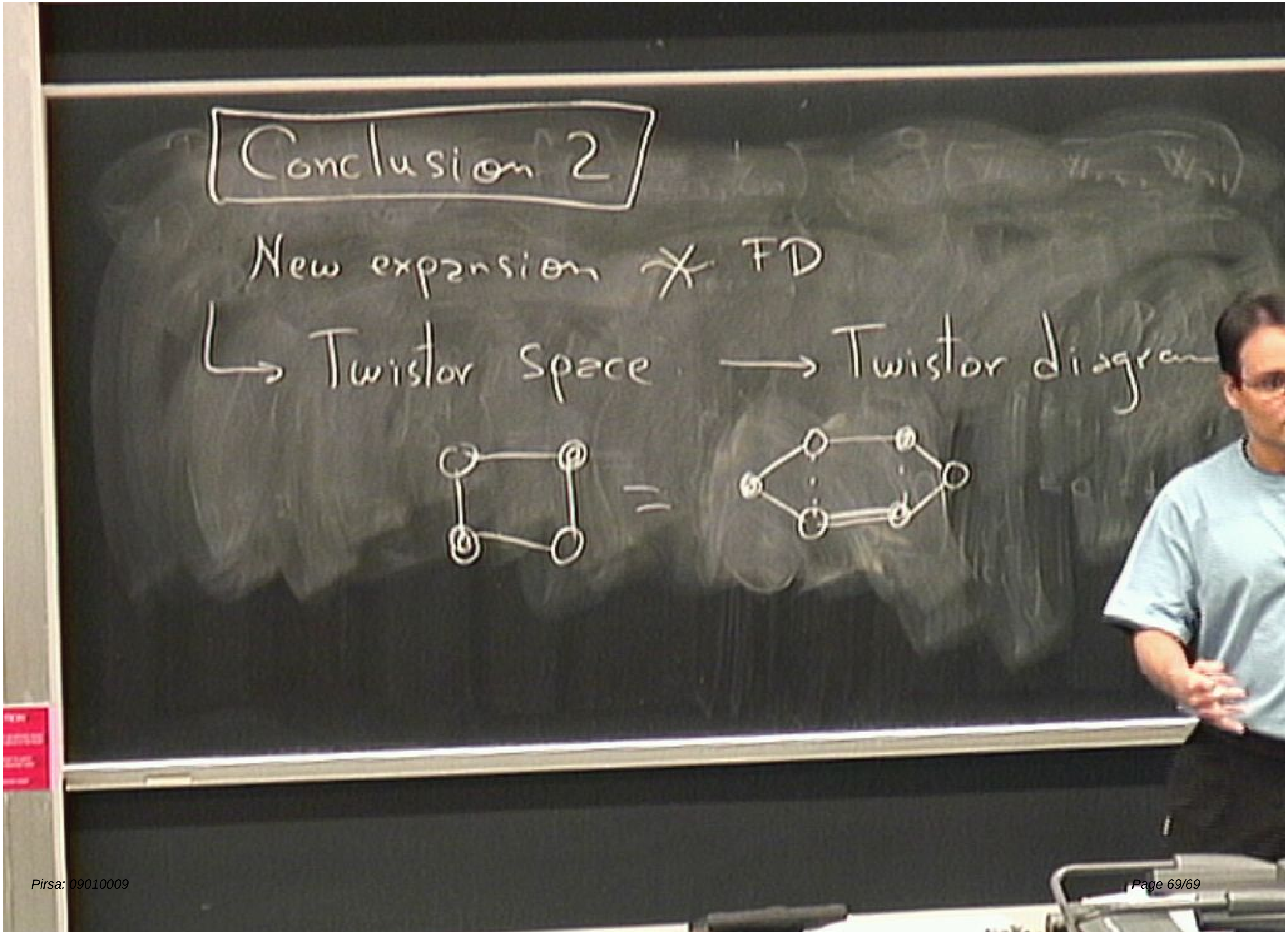
$\boxed{\text{Conclusion 1}}$

$\boxed{\text{Conclusion 2}}$

Conclusion 2

New expansion $\neq$ FD

↳ Twistor Space ⟶ Twistor diagram


Conclusion 2
New expansion ≁ FD
↳ Twistor Space ⟶ Twistor diagra
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