Title: Against commutators

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Abstract: The essential ingredients of a quantum theory are usually a Hilbert space of states and an algebra of operators encoding observables. The mathematical operations available with these structures translate fairly well into physical operations (preparation, measurement etc.) in a non-relativistic world. This correspondence weakens in quantum field theory, where the direct operational meaning of the observable algebra structure (encoded usually through commutators) is lost. The situation becomes even worse when we want to give a more dynamical role to spacetime as for example in attempts to formulate a quantum theory of gravity. I argue that a revision of the structures that we think of as fundamental in a quantum theory is in order. I go on to outline a proposal in this direction, based on the so called 'general boundary formulation', emphasizing the operational meaning of the ingredients. If time permits I will also comment on the relation to the framework of algebraic quantum field theory.

Against Commutators

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20 January 2009

Abstract

The fundamental ingredients for the description of a quantum system are usually taken to be a Hilbert space of states and an operator algebra of observables. In this talk I want to argue against this. In moving from a non-relativistic via a special relativistic to a general relativistic world, the standard ingredients of a quantum theory become increasingly inadequate in their operational relation with reality. I will outline a proposal for more adequate foundations and discuss how the usual structures are recovered.

Outline

1. The standard framework and its problems
2. The need for new foundations
   - A lesson from Quantum Field Theory
3. The general boundary formulation
   - Overview
   - Probability interpretation
   - Observables
4. Where are the commutators?
5. Towards a new correspondence principle

Ingredients of standard Quantum Theory

Quantum theory is modeled after non-relativistic classical mechanics.

	classical mechanics	quantum theory
states	phase space (manifold) P	Hilbert space $\mathcal{H}$
infinitesimal dynamics	Hamiltonian vector field $H \in \Gamma(TP)$	Hamiltonian operator $H : \mathcal{H} \to \mathcal{H}$
finite dynamics	symplectic transformation $U_{[t_1,t_2]} : P \to P$	time-evolution operator $U_{[t_1,t_2]} : \mathcal{H} \to \mathcal{H}$
instantaneous observables	form an algebra of functions $A : P \to \mathbb{R}$	form an algebra $\mathcal{A}$ of operators $A : \mathcal{H} \to \mathcal{H}$

- The operational role of the quantum mechanical structures is quite different from that of their classical counterparts.
- Quantum theory is tied much more strongly to a non-relativistic setting than is classical mechanics.

Operational meaning tied to background time

The physical role of key ingredients of Quantum Theory…

- A Hilbert space $\mathcal{H}$ of states.
  - A state encodes information about the system between measurements.
  - The inner product allows to extract probabilities.
- An algebra of observables $\mathcal{A}$.
  - An observable encodes a possible measurement on the system.
  - A measurement changes a state to a new state.
  - The product of $\mathcal{A}$ encodes temporal composition of measurements.
- Certain unitary operators describe evolution of the system in time.
  - Probability is conserved in time.

…makes reference to an *external notion of time*, i.e., a notion of time independent of a state.

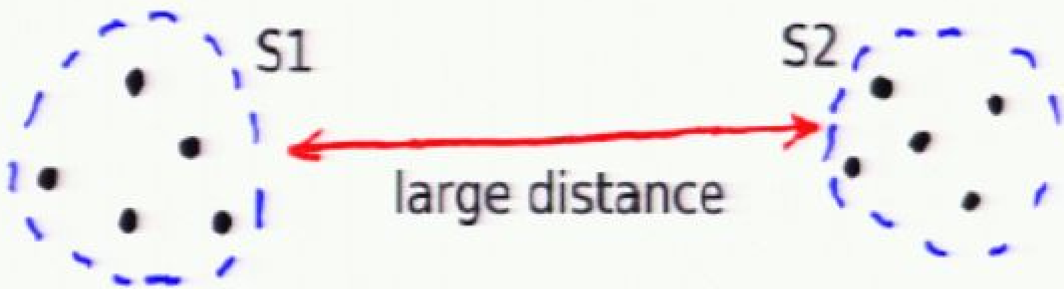
The background time problem

The operational meaning depends on a background time, but...

- in special relativistic physics there is no preferred frame and hence no preferred background time. (This problem can be fixed.)
- in general relativistic physics there is no fixed metric and hence no background time at all.

The quantum cosmology problem

- In a fundamental quantum theory a state is a priori a state of the universe. But, in quantum theory the observer must be outside the observed system. Also, we cannot hope to be able to describe the universe in all its details.
- In quantum field theory distant systems (with respect to the background metric) are independent. Cluster decomposition means that the S-matrix factorises, $S = S_1 S_2$:



  We can thus successfully describe a local system as if it was alone in a Minkowski universe.
- In the absence of a background metric there is no known solution to this problem.

The need for new foundations

Conclusion

The standard ingredients (state space plus observable algebra) are unsuitable as a foundation for quantum theory in general.

- Because of the background time problem we need an interpretation that does not refer explicitly to a background (space)time.
- Because of the quantum cosmology problem we need structures that can describe physics in a manifestly local way.

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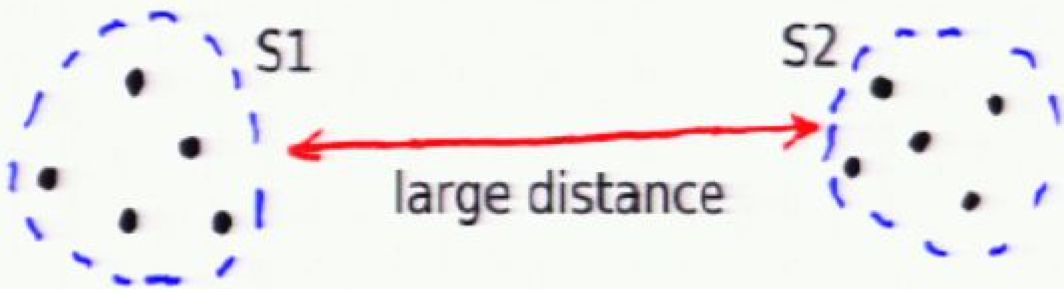

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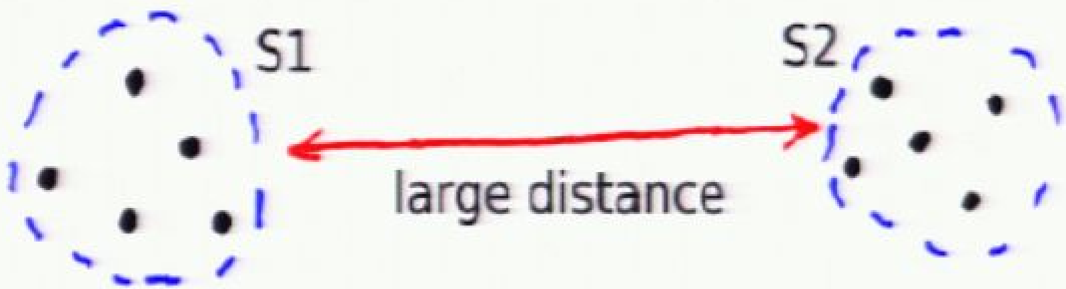

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A lesson from Quantum Field Theory

- Standard observables of QFT are values of fields and their derivatives at spacetime points.
- These observables carry a label specifying when (and where) they are applied.
- There is only one operationally meaningful composition of two such observables, given by the commutative time-ordered product.
- In QFT all physically measurable quantities are constructed via the time-ordered product. The noncommutative operator product is never used.
- The equal-time commutation relations can be recovered: $[A(t,x),B(t,y)] = \lim_{\epsilon\to 0} TA(t+\epsilon,x)B(t-\epsilon,y) - TB(t+\epsilon,y)A(t-\epsilon,x)$
- To ensure consistency under change of reference frame the operator product must satisfy $[A(p),B(q)] = 0$ if p and q are not causally related.

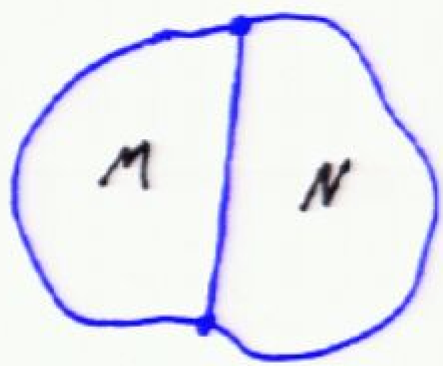
Another ingredient: Locality

Consider classical field theory.

- The physics in a region M of spacetime is described by the space L_M of solutions of the equations of motion in M.
- The observables in a region M form an algebra $C(L_M)$ of functions $L_M \to \mathbb{R}$.

Suppose we have regions $N \subset M$. Then, $L_M \to L_N$ by restriction. This induces $C(N) \to C(M)$.

Suppose we have adjacent regions M, N. Then, $L_{M \cup N} \to L_M \times L_N$ and $C(L_M) \otimes C(L_N) \to C(L_{M \cup N})$.



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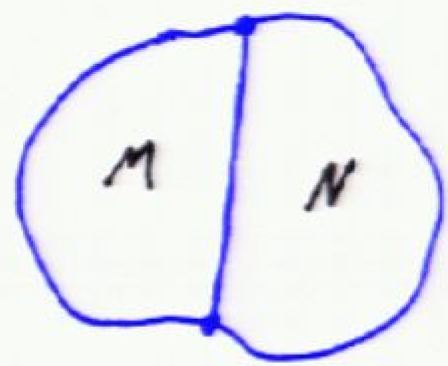

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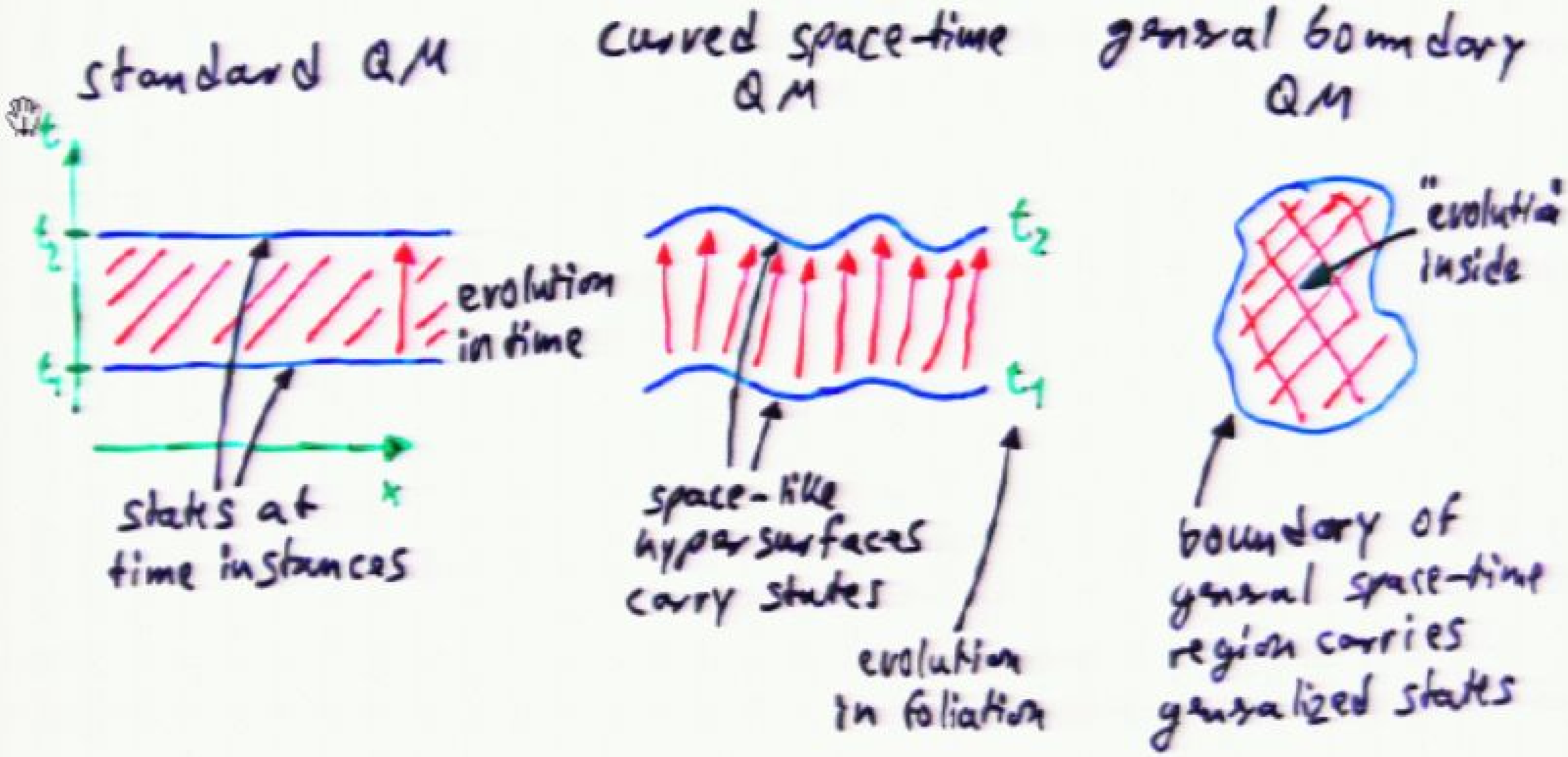

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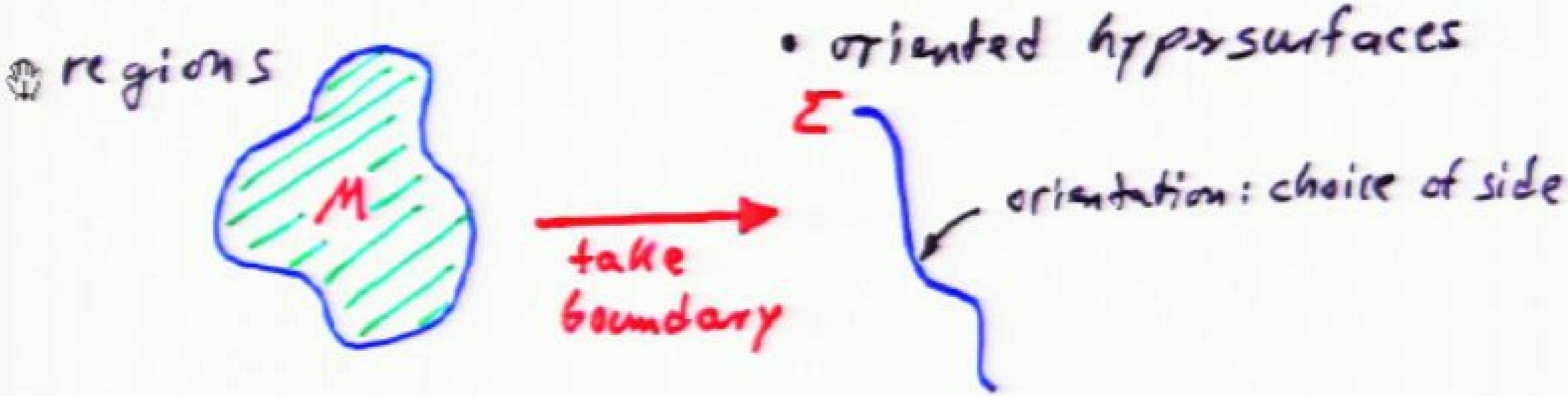

Basic algebraic structures:

- To each hypersurface Σ associate a Hilbert space $\mathcal{H}_\Sigma$ of states.
- To each region M with boundary Σ associate a linear amplitude map $\rho_M : \mathcal{H}_\Sigma \to \mathbb{C}$.

Main axioms

The structures are subject to a number of axioms. The most important are:

- $\overline{\Sigma}$ is Σ with opposite orientation. Then $\mathcal{H}_{\overline{\Sigma}} = \mathcal{H}_{\Sigma}^*$.
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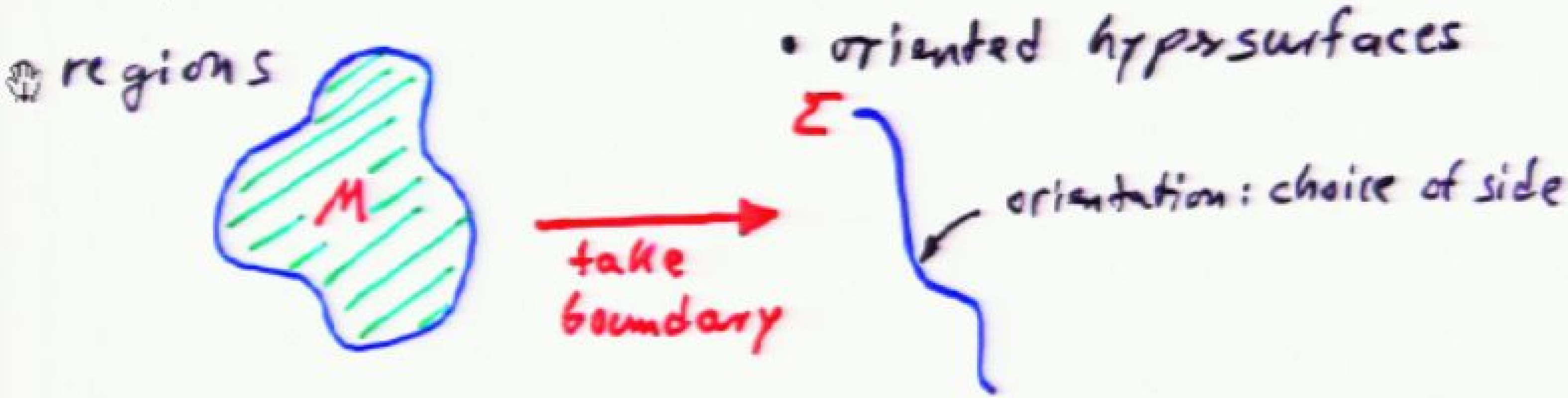

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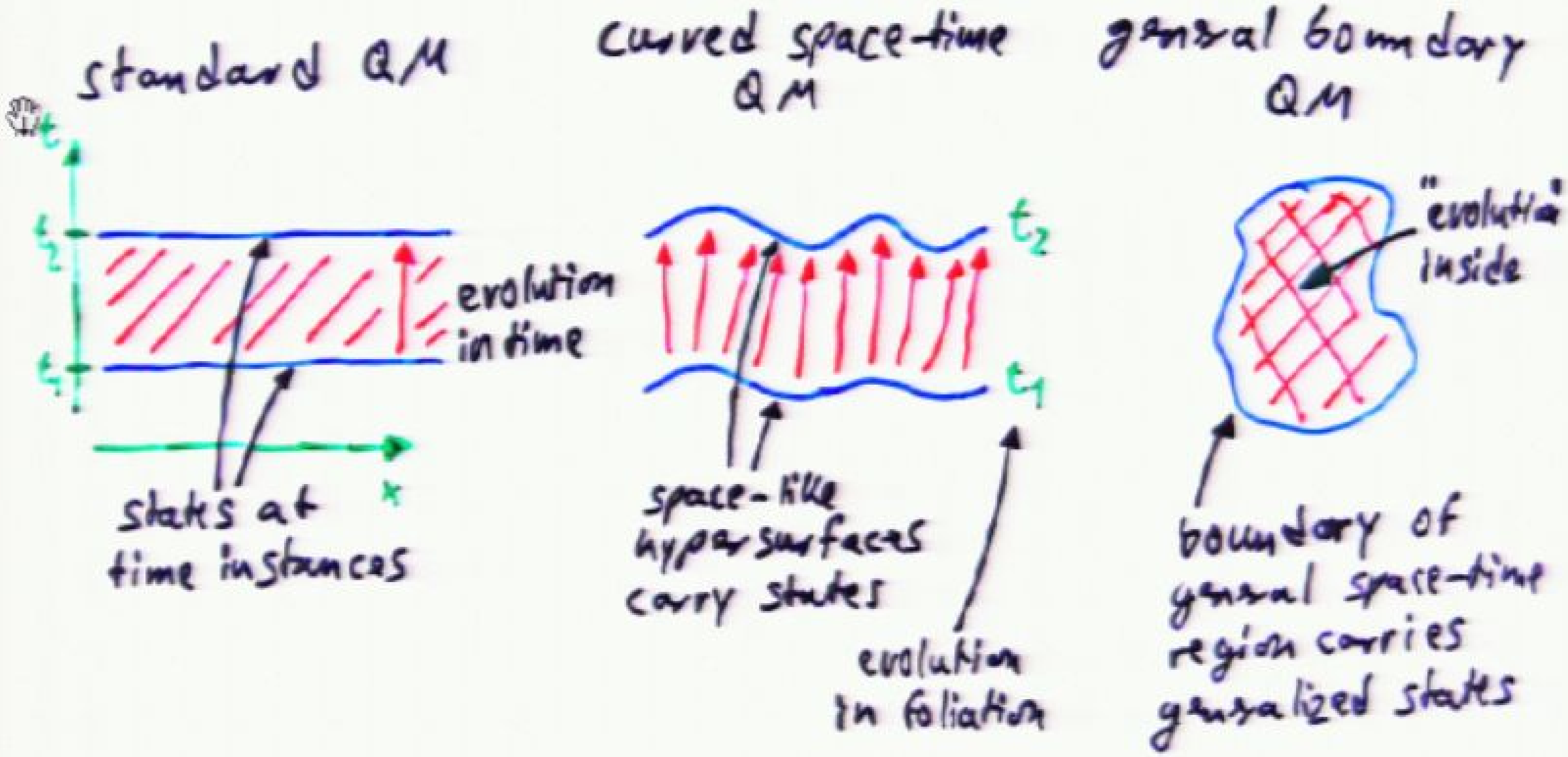

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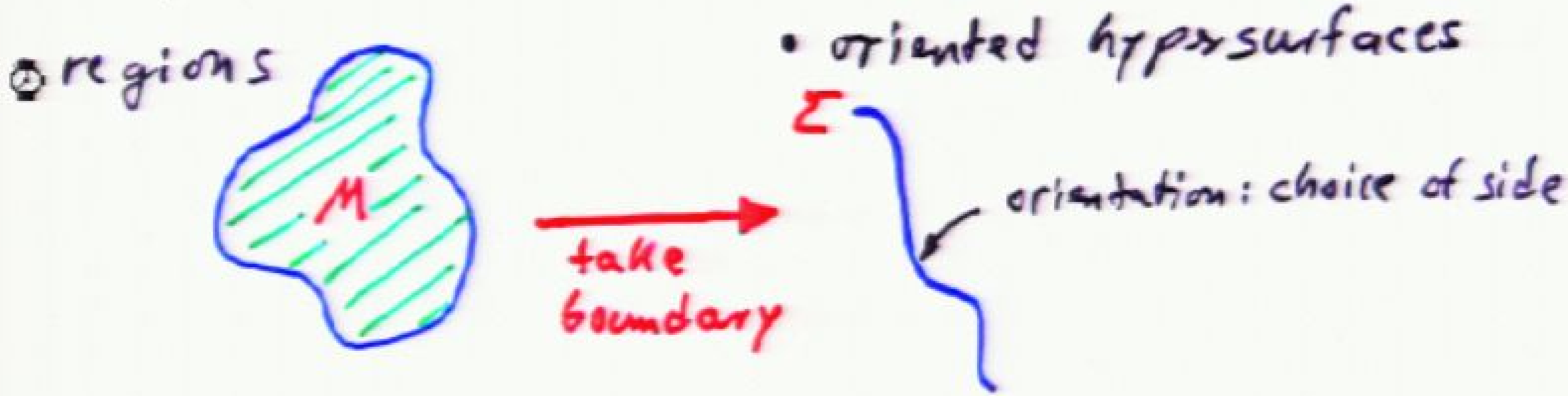

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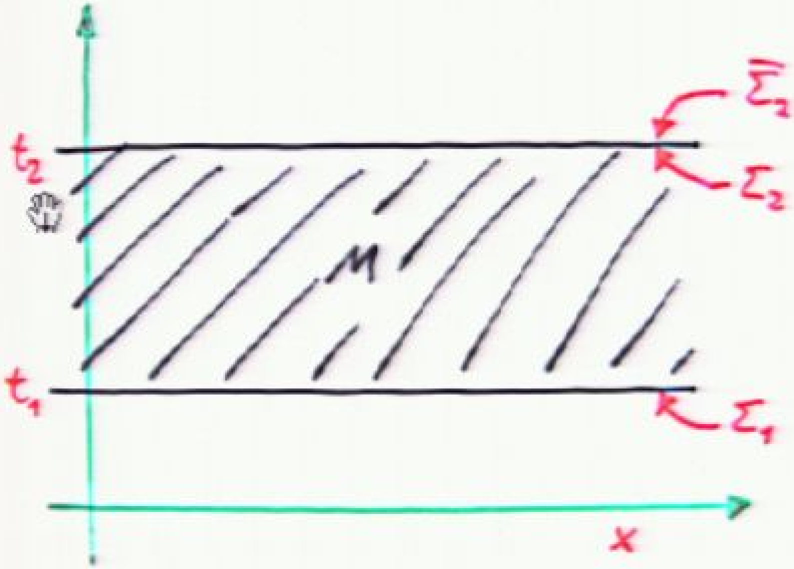
Recovering standard transition amplitudes

Consider the geometry of a standard transition.



- region: $M = [t_1, t_2] \times \mathbb{R}^3$
- boundary: $\partial M = \Sigma_1 \cup \overline{\Sigma}_2$
- state space: $\mathcal{H}_{\partial M} = \mathcal{H}_{\Sigma_1} \otimes \mathcal{H}_{\overline{\Sigma}_2} = \mathcal{H}_{\Sigma_1} \otimes \mathcal{H}^*_{\Sigma_2}$

- Via time-translation symmetry identify $\mathcal{H}_{\Sigma_1} \cong \mathcal{H}_{\Sigma_2} \cong \mathcal{H}$, where $\mathcal{H}$ is the state space of standard quantum mechanics.
- Write the amplitude map as $\rho_M : \mathcal{H} \otimes \mathcal{H}^* \to \mathbb{C}$.
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$$\rho_M(\psi \otimes \eta) = \langle \eta | U(t_2 - t_1) | \psi \rangle$$

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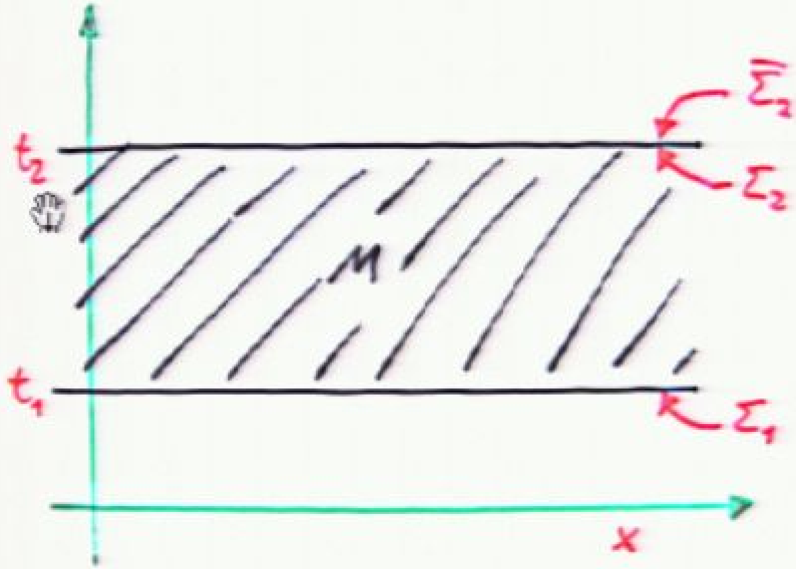

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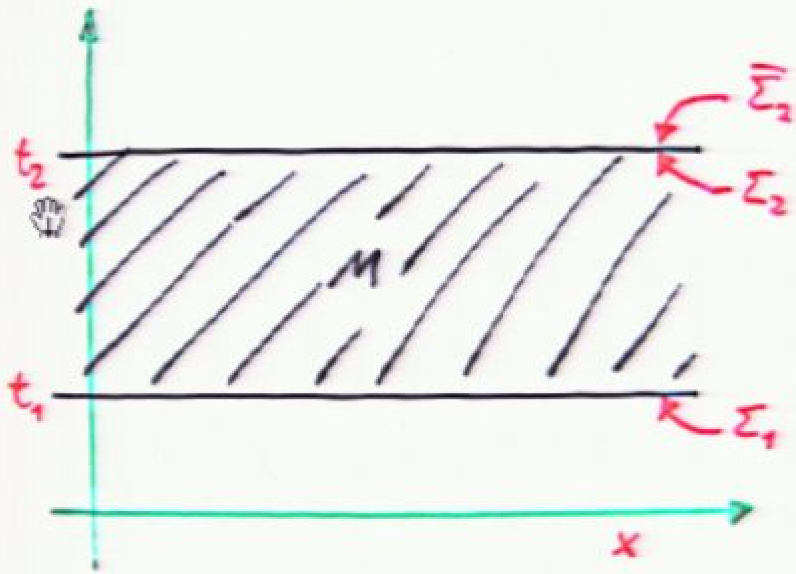

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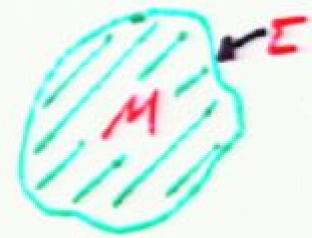
Generalized probability interpretation

Consider the context of a general spacetime region M with boundary Σ.



Probabilities in quantum theory are generally conditional probabilities. They depend on two pieces of information. Here these are:

- $\mathcal{S} \subset \mathcal{H}_\Sigma$ representing preparation or knowledge
- $\mathcal{A} \subset \mathcal{H}_\Sigma$ representing observation or the question

The probability that the system is described by $\mathcal{A}$ given that it is described by $\mathcal{S}$ is:

$$P(\mathcal{A}|\mathcal{S}) = \frac{|\rho_M \circ P_\mathcal{S} \circ P_\mathcal{A}|^2}{|\rho_M \circ P_\mathcal{S}|^2}$$

- $P_\mathcal{S}$ and $P_\mathcal{A}$ are the orthogonal projectors onto the subspaces.

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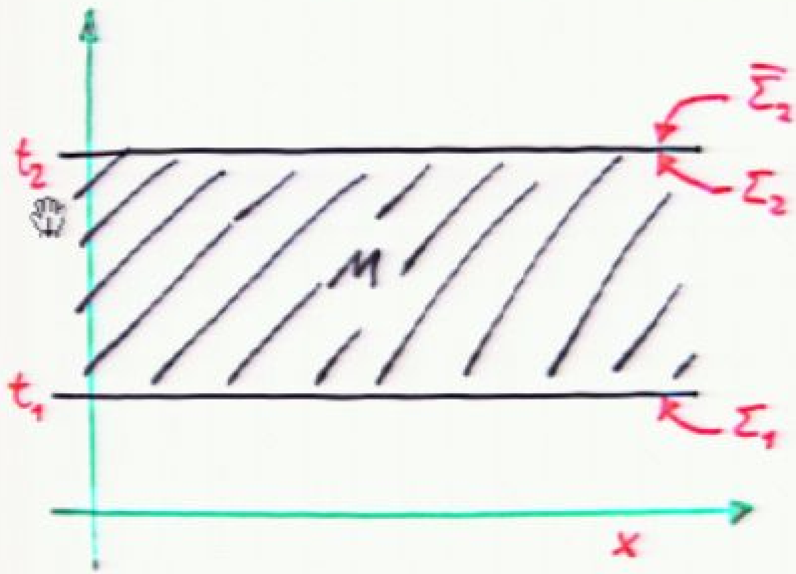

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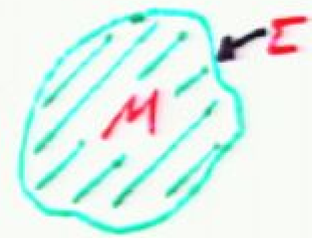

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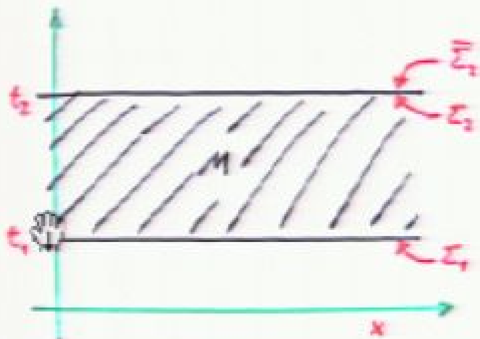


Recall the geometry for standard transition amplitudes with $\mathcal{H}_{\partial M} = \mathcal{H} \otimes \mathcal{H}^*$ and $\rho_M(\psi \otimes \eta) = \langle \eta | U(t_2 - t_1) | \psi \rangle$.

We want to compute the probability of measuring η at t_2 given that we prepared ψ at t_1. This is encoded via

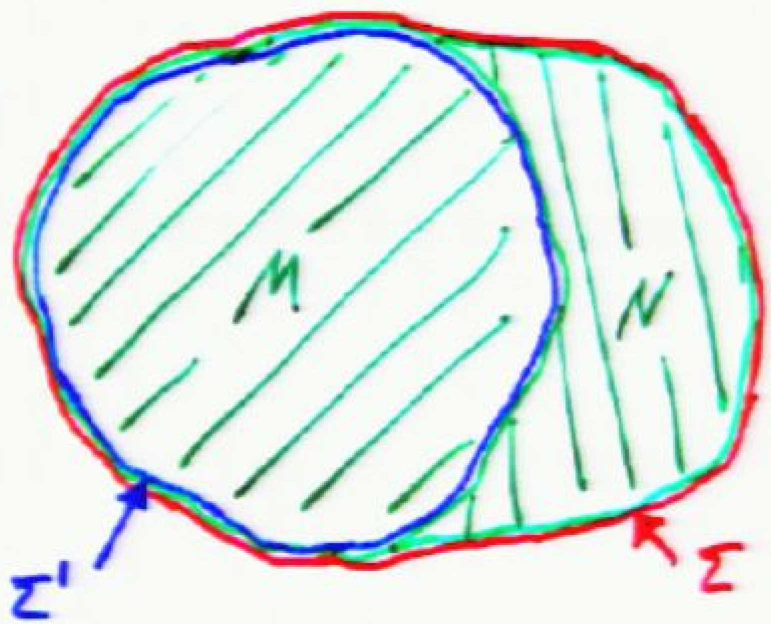
$$\mathcal{S} = \psi \otimes \mathcal{H}^*, \quad \mathcal{A} = \mathcal{H} \otimes \eta.$$

The resulting expression yields correctly

$$P(\mathcal{A}|\mathcal{S}) = \frac{|\rho_M \circ P_\mathcal{S} \circ P_\mathcal{A}|^2}{|\rho_M \circ P_\mathcal{S}|^2} = \frac{|\rho_M(\psi \otimes \eta)|^2}{1} = |\langle \eta | U(t_2 - t_1) | \psi \rangle|^2.$$

Probability conservation

Probability conservation in time is generalized to probability conservation in spacetime.



Consider a region M and a region N "deforming" it. Call Σ the boundary of $M \cup N$ and Σ' the boundary of M.

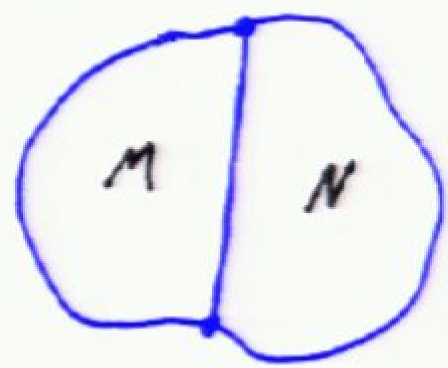
- The amplitude map for N induces a unitary map $\tilde{\rho} : \mathcal{H}_\Sigma \to \mathcal{H}_{\Sigma'}$.
- Let $\mathcal{S} \subset \mathcal{H}_\Sigma$ and $\mathcal{A} \subset \mathcal{H}_\Sigma$. Define $\mathcal{S}' := \tilde{\rho}(\mathcal{S})$ and $\mathcal{A}' := \tilde{\rho}(\mathcal{A})$.
- Then, probability is conserved, $P(\mathcal{A}|\mathcal{S}) = P(\mathcal{A}'|\mathcal{S}')$.

Observables

- Observables are associated to spacetime regions.
- For a region M an observable f is encoded in a *modified amplitude map* $\rho_M^f : \mathcal{H}_{\partial M} \to \mathbb{C}$.

Suppose we have regions $N \subset M$. An observable in N gives rise to an observable in M, $\rho_{M \cup N}^f = \rho_M^f \diamond \rho_{M \setminus N}$.

Suppose M and N are adjacent regions with observables f in M and g in N. Then we can form a composite observable in $M \cup N$ given by $\rho_{M \cup N}^{f \star g} = \rho_M^f \diamond \rho_N^g$.



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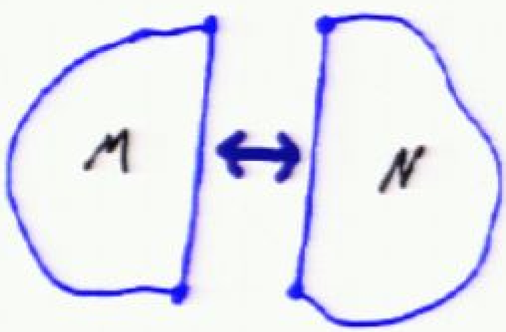
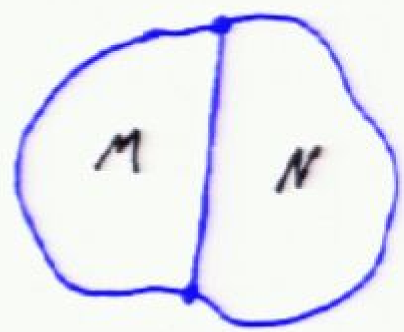
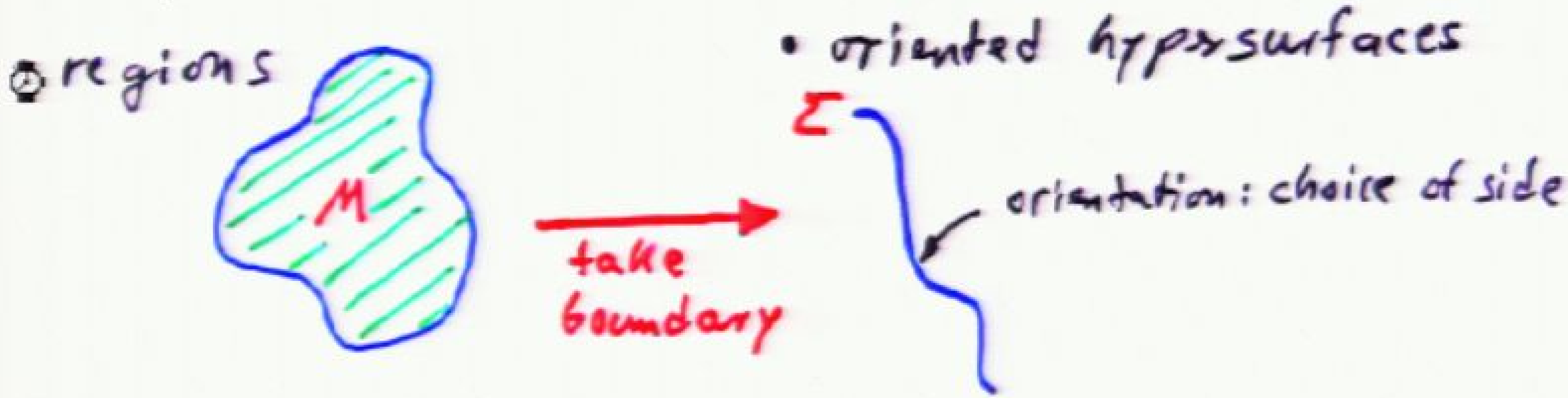




Basic structures

Basic spacetime structures:



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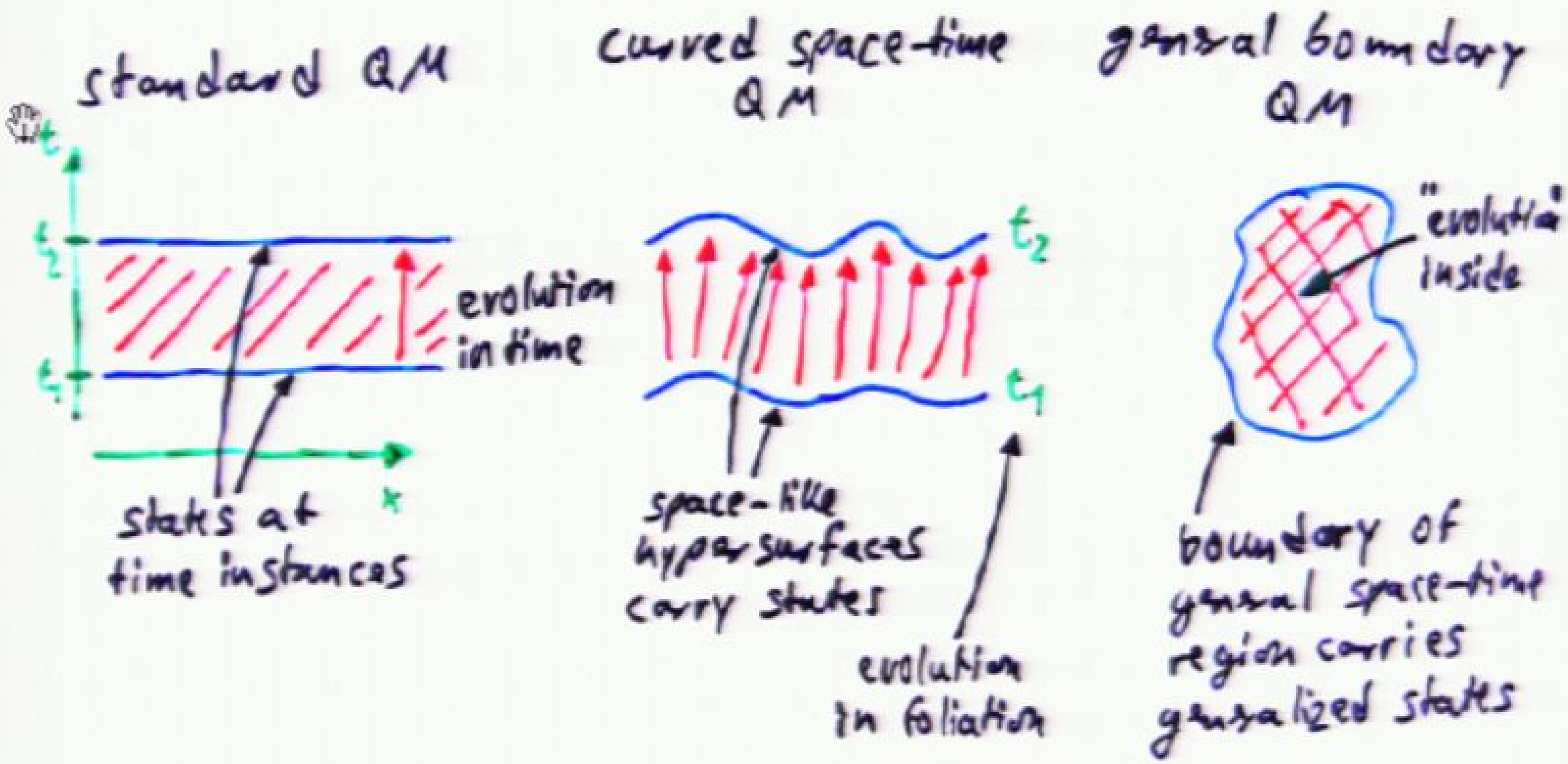

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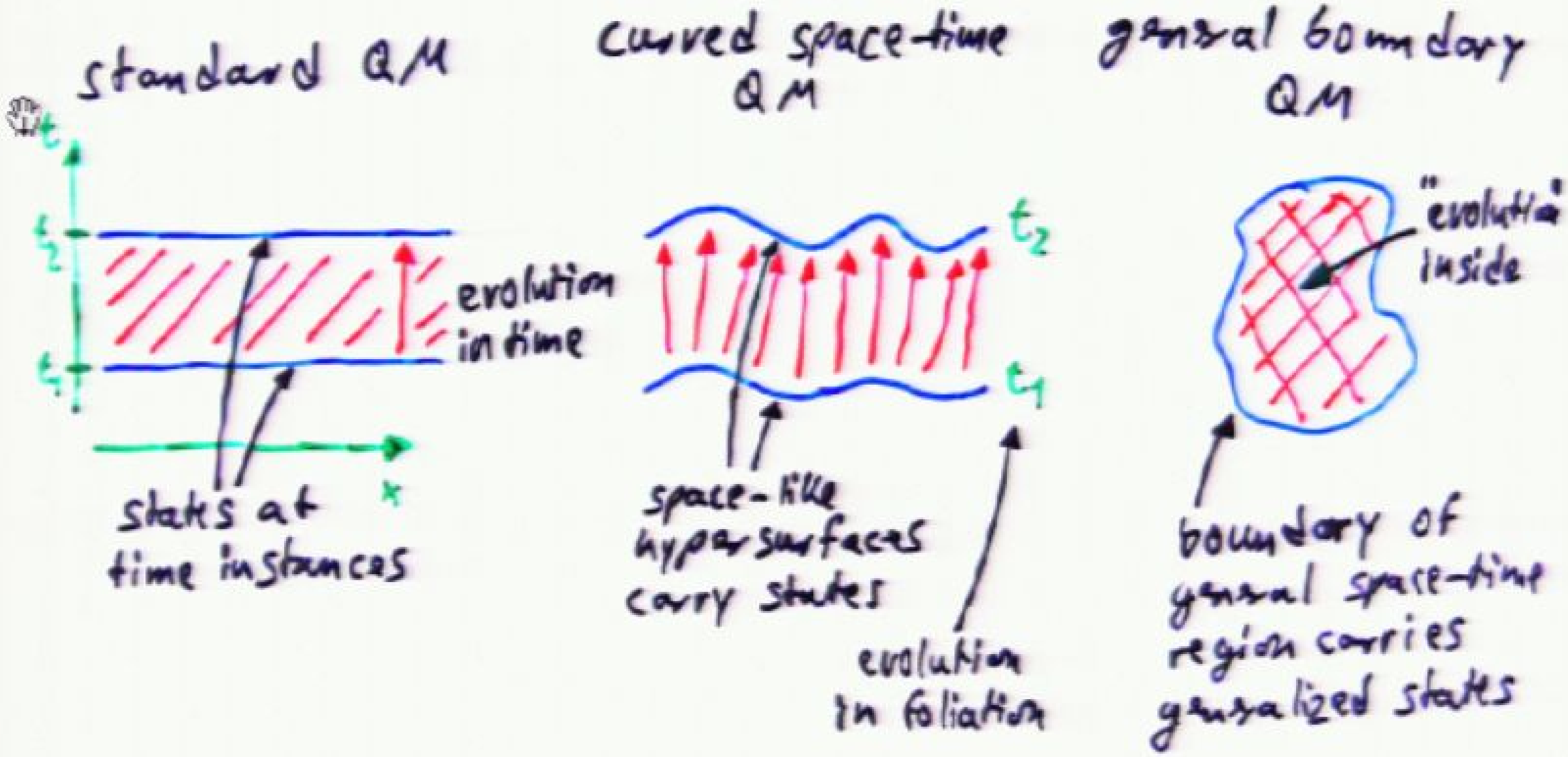

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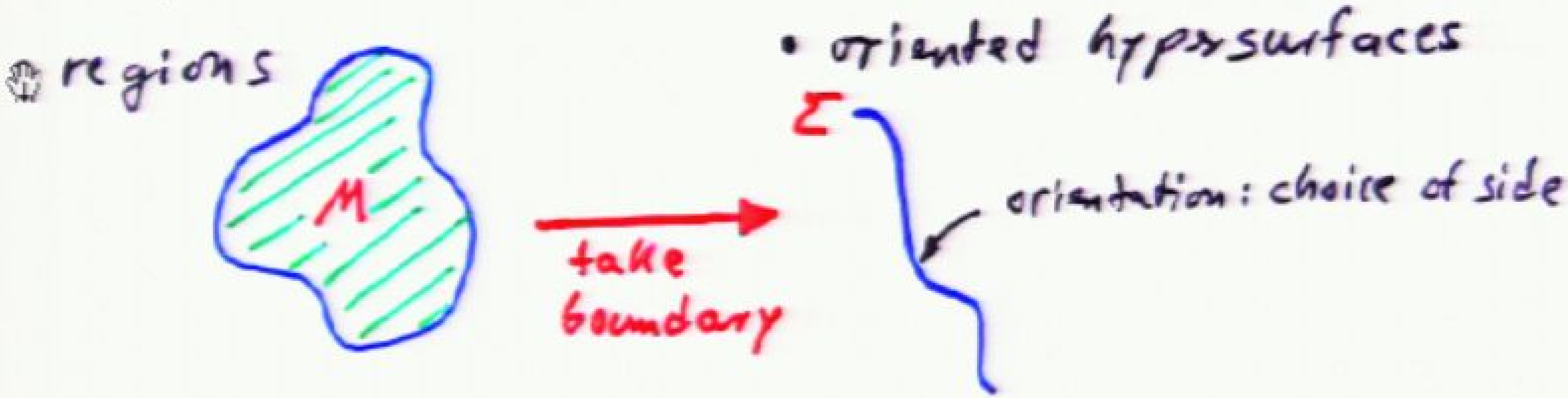

Basic algebraic structures:

- To each hypersurface Σ associate a Hilbert space $\mathcal{H}_\Sigma$ of states.
- To each region M with boundary Σ associate a linear amplitude map $\rho_M : \mathcal{H}_\Sigma \to \mathbb{C}$.

Another ingredient: Locality

Consider classical field theory.

- The physics in a region M of spacetime is described by the space L_M of solutions of the equations of motion in M.
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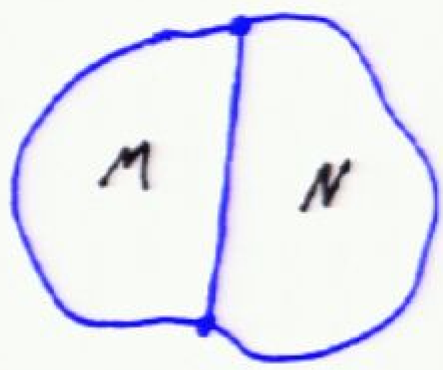
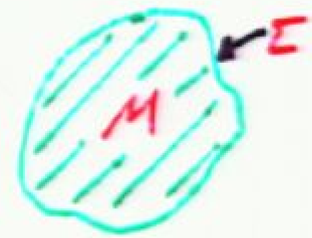


Generalized probability interpretation

Consider the context of a general spacetime region M with boundary Σ.



Probabilities in quantum theory are generally conditional probabilities. They depend on two pieces of information. Here these are:

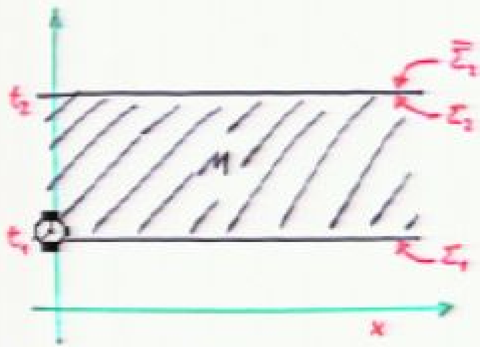
- $\mathcal{S} \subset \mathcal{H}_\Sigma$ representing preparation or knowledge
- $\mathcal{A} \subset \mathcal{H}_\Sigma$ representing observation or the question

The probability that the system is described by $\mathcal{A}$ given that it is described by $\mathcal{S}$ is:

$$P(\mathcal{A}|\mathcal{S}) = \frac{|\rho_M \circ P_\mathcal{S} \circ P_\mathcal{A}|^2}{|\rho_M \circ P_\mathcal{S}|^2}$$

- $P_\mathcal{S}$ and $P_\mathcal{A}$ are the orthogonal projectors onto the subspaces.

Recovering standard probabilities



Recall the geometry for standard transition amplitudes with $\mathcal{H}_{\partial M} = \mathcal{H} \otimes \mathcal{H}^*$ and $\rho_M(\psi \otimes \eta) = \langle \eta | U(t_2 - t_1) | \psi \rangle$.

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The resulting expression yields correctly

$$P(\mathcal{A}|\mathcal{S}) = \frac{|\rho_M \circ P_{\mathcal{S}} \circ P_{\mathcal{A}}|^2}{|\rho_M \circ P_{\mathcal{S}}|^2} = \frac{|\rho_M(\psi \otimes \eta)|^2}{1} = |\langle \eta | U(t_2 - t_1) | \psi \rangle|^2.$$

Observables

- Observables are associated to spacetime regions.
- For a region M an observable f is encoded in a *modified amplitude map* $\rho_M^f : \mathcal{H}_{\partial M} \to \mathbb{C}$.

Suppose we have regions $N \subset M$. An observable in N gives rise to an observable in M, $\rho_{M \cup N}^f = \rho_M^f \diamond \rho_{M \setminus N}$.

Suppose M and N are adjacent regions with observables f in M and g in N. Then we can form a composite observable in $M \cup N$ given by $\rho_{M \cup N}^{f \star g} = \rho_M^f \diamond \rho_N^g$.

Expectation values

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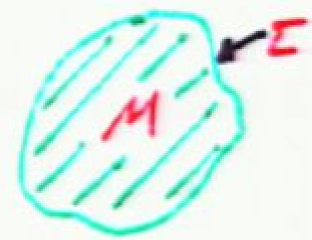


The expectation value of the observable f conditional on the system being prepared in the subspace $\mathcal{S} \subset \mathcal{H}_\Sigma$ can be represented as follows:

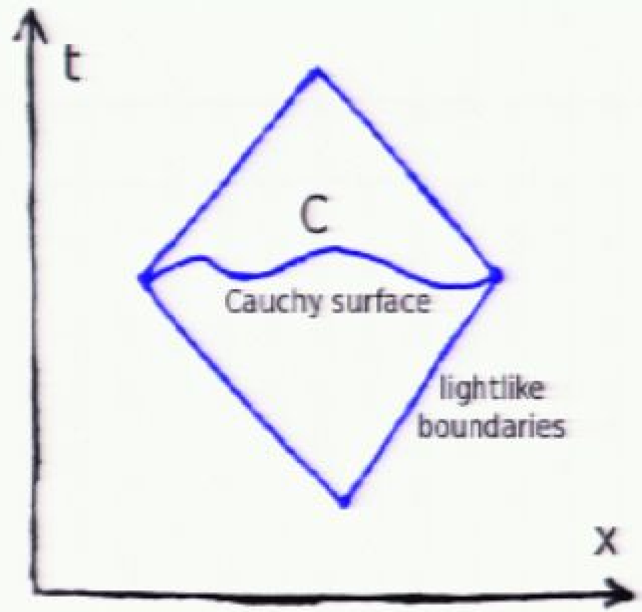
$$\langle f \rangle_{\mathcal{S}} = \frac{\langle \rho_M^{\mathcal{S}}, \rho_M^f \rangle}{|\rho_M^{\mathcal{S}}|^2}$$

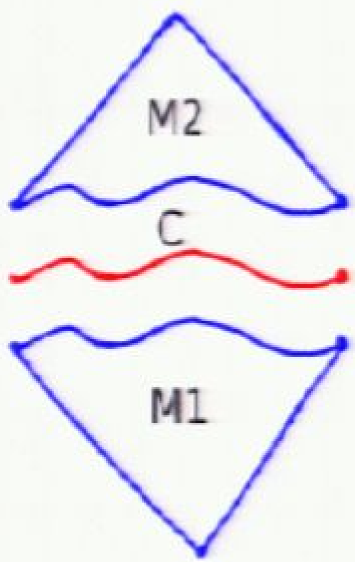
Here we write $\rho_M^{\mathcal{S}} := \rho_M \circ P_{\mathcal{S}}$.

(We also use a certain simplifying condition which in the standard formalism is always satisfied.)

Where are the commutators?

Given a metric background structure we can recover the usual operator algebras (and commutators). Consider a spacetime region M containing a Cauchy hypersurface C, say a causal diamond.





Decompose the region into three pieces, $M = M_1 \cup C \cup M_2$. We think of C as an "infinitely thin" region. For an observable f in M there is a unique ρ^f_C such that $\rho^f_M = \rho_{M_1} \diamond \rho^f_C \diamond \rho_{M_2}$. We can then interpret ρ^f_C as an operator on $\mathcal{H}_C$.

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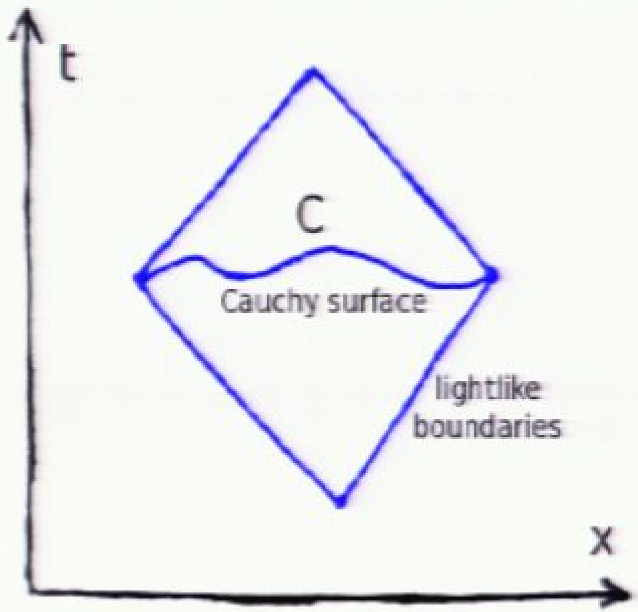

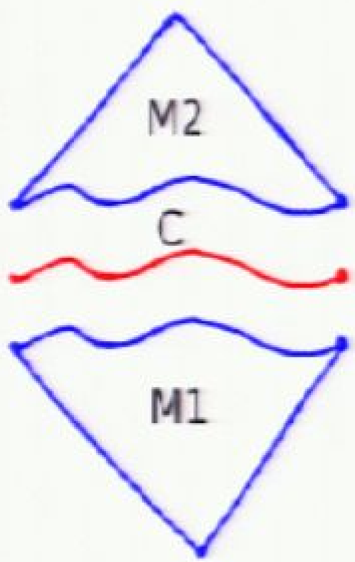

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Towards a new correspondence principle

Suppose we have a map that associates to every classical observable $A \in C(L_M)$ a quantum observable $\rho_M^A : \mathcal{H}_{\partial M} \to \mathbb{C}$. For classical observables A, B in adjacent regions M and N we can consider the product observable $A \cdot B$ in the joint region $M \cup N$. We can then demand:

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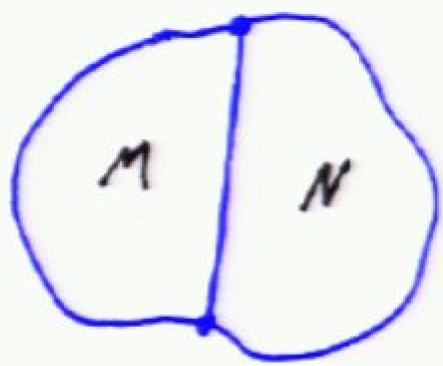

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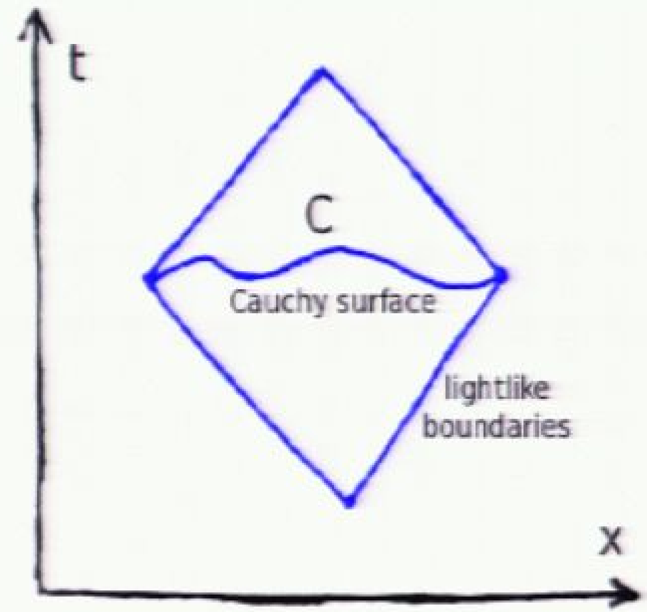

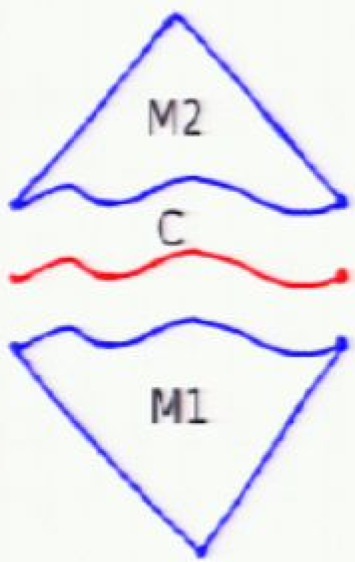

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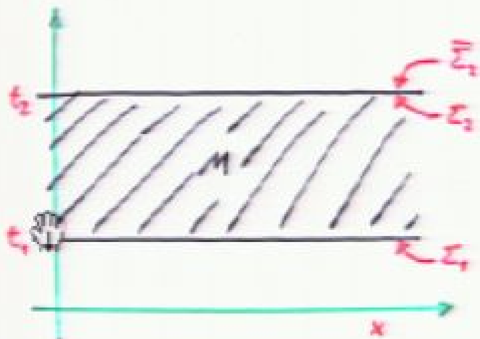

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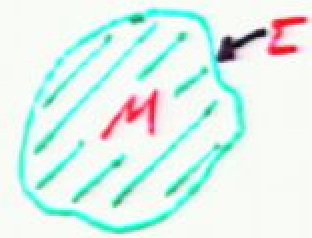

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A lesson from Quantum Field Theory

- Standard observables of QFT are values of fields and their derivatives at spacetime points.
- These observables carry a label specifying when (and where) they are applied.
- There is only one operationally meaningful composition of two such observables, given by the commutative time-ordered product.
- In QFT all physically measurable quantities are constructed via the time-ordered product. The noncommutative operator product is never used.
- The equal-time commutation relations can be recovered: $[A(t,x), B(t,y)] = \lim_{\epsilon \to 0} TA(t+\epsilon, x)B(t-\epsilon, y) - TB(t+\epsilon, y)A(t-\epsilon, x)$
- To ensure consistency under change of reference frame the operator product must satisfy $[A(p), B(q)] = 0$ if p and q are not causally related.

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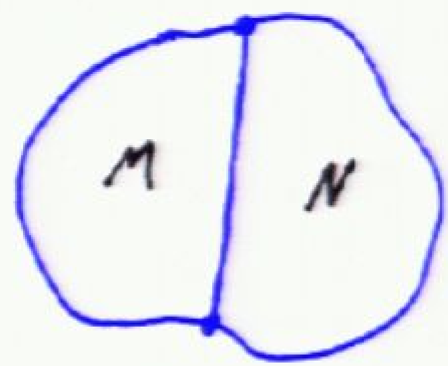

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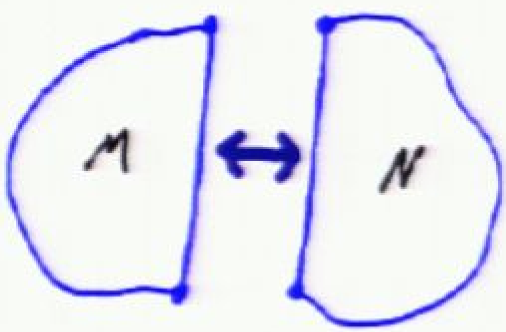
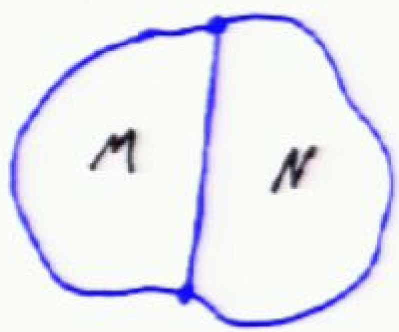
Main axioms

The structures are subject to a number of axioms. The most important are:

- $\overline{\Sigma}$ is Σ with opposite orientation. Then $\mathcal{H}_{\overline{\Sigma}} = \mathcal{H}_{\Sigma}^*$.
- $\Sigma = \Sigma_1 \cup \Sigma_2$ is a disjoint union of hypersurfaces. Then $\mathcal{H}_{\Sigma} = \mathcal{H}_{\Sigma_1} \otimes \mathcal{H}_{\Sigma_2}$.
- If M and N are adjacent regions, then $\rho_{M \cup N} = \rho_M \diamond \rho_N$. The composition $\diamond$ involves a sum over a complete basis on the boundary shared by M and N.





Basic structures

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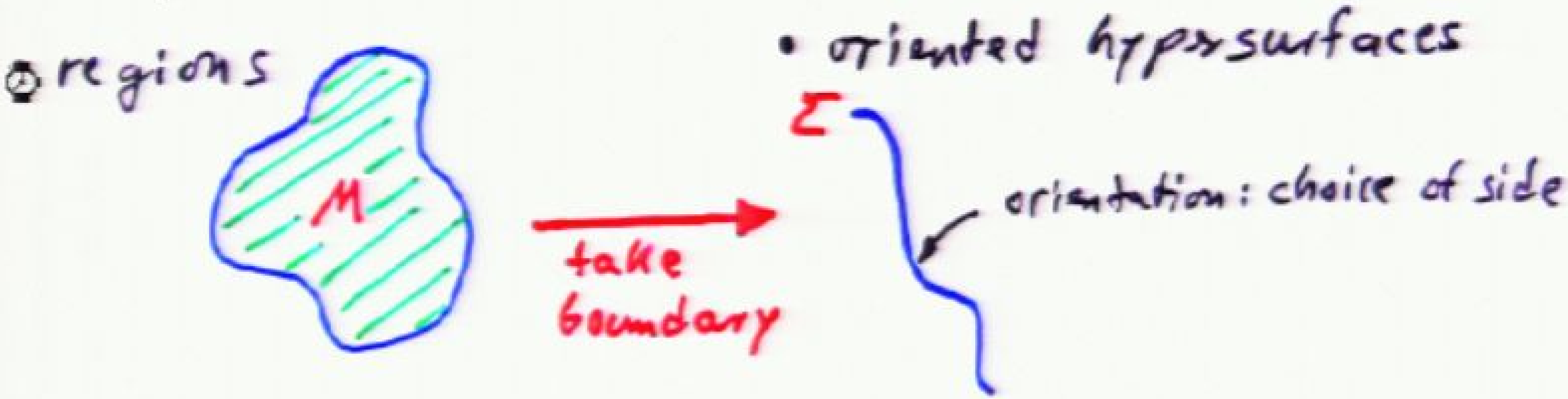

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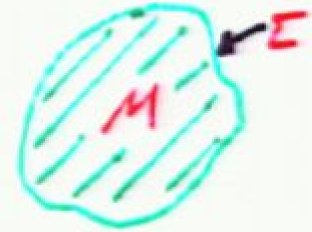

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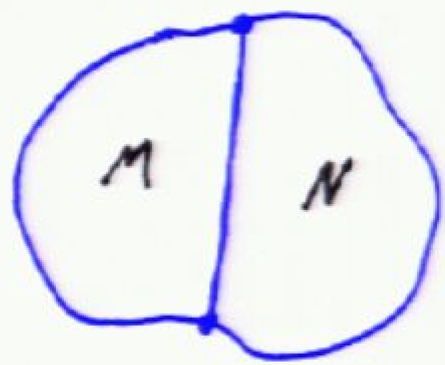

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