Title: GR/EFT and Black Hole dynamics

Date: Jan 28, 2009 04:00 PM

URL: http://pirsa.org/09010001

Abstract: In this talk I will review how ideas borrowed from perturbative Quantum Gravity and Effective Field Theory (EFT) in Particle Physics can be applied to problems in General Relativity (GR), such as calculating gravitational wave emission by inspiralling spinning binary systems, including finite size effects and absorption. I will discuss in somewhat more detail how to account for dissipative effects, where the GR/EFT duality is used to predict the power loss due to absorption in the dynamics of binary spinning Black Holes.

(100%) Wed 4:02 PM

GR/EFT and Black Hole dynamics

Rafael A. Porto (UCSB)

In collaboration with: I. Rothstein (CMU) and W. Goldberger (Yale)

Wed 4:03 PM

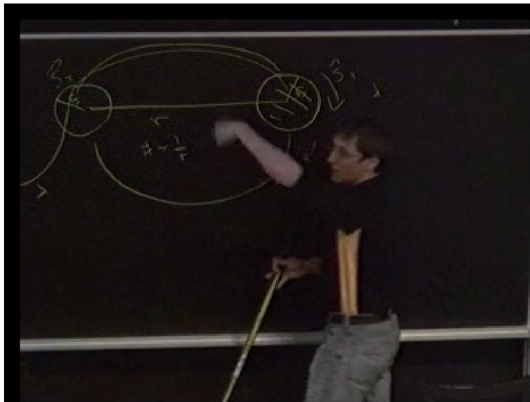
EFT meets GR:

Solving *classical* BH dynamics with Feynman diagrams.

Some Literature

- Motion and gravitational radiation for binary BHs or NSs. non-spinning: Goldberger & Rothstein PRD73 104029 (06'); spinning: RAP, PRD73 104031 (06').
- New results (including finite size effects) at NLO order for spinning BHs or NSs. RAP & Rothstein, PRL97 021101 (06'), PRD78 044012 (08'), PRD78 044013 (08').
- NNLO for non-spinning BHs or NSs. Gilmore & Ross. arXiv:0810.1328.
- Dissipation/Absorption for binary BHs or NSs. non-spinning: Goldberger & Rothstein PRD73 104030 (06'). spinning: RAP, PRD77 064026 (08').
- EMRI. Self-force. Galley & Hu. ArXiv:0801.0900.
- Reviews (Les Houches): Goldberger, hepph/0701129; RAP & Sturani grqc/0701105.

PERIMETER INSTITUTE



(100%) Wed 4:06 PM

The problem of motion:

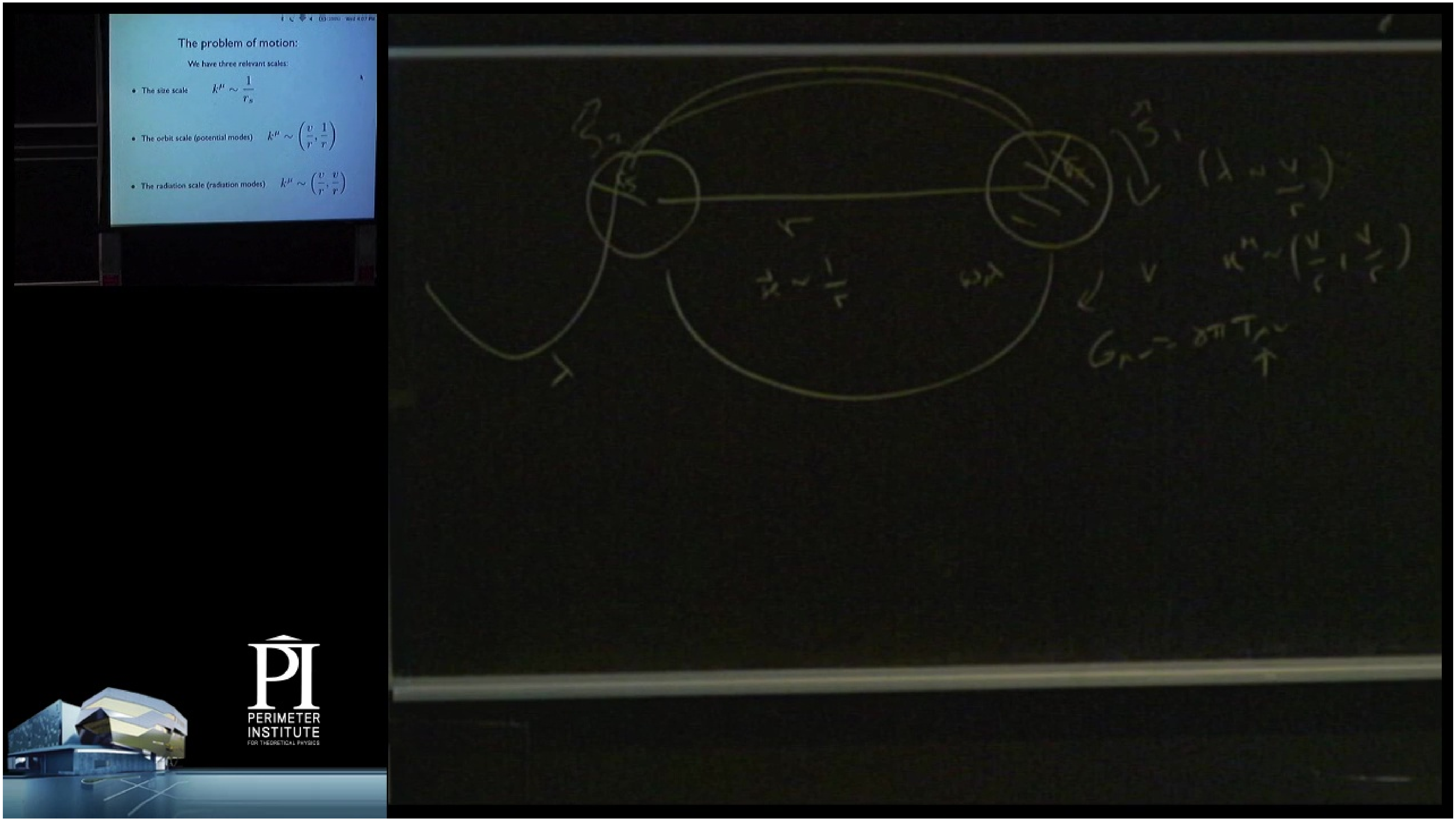
We have three relevant scales:

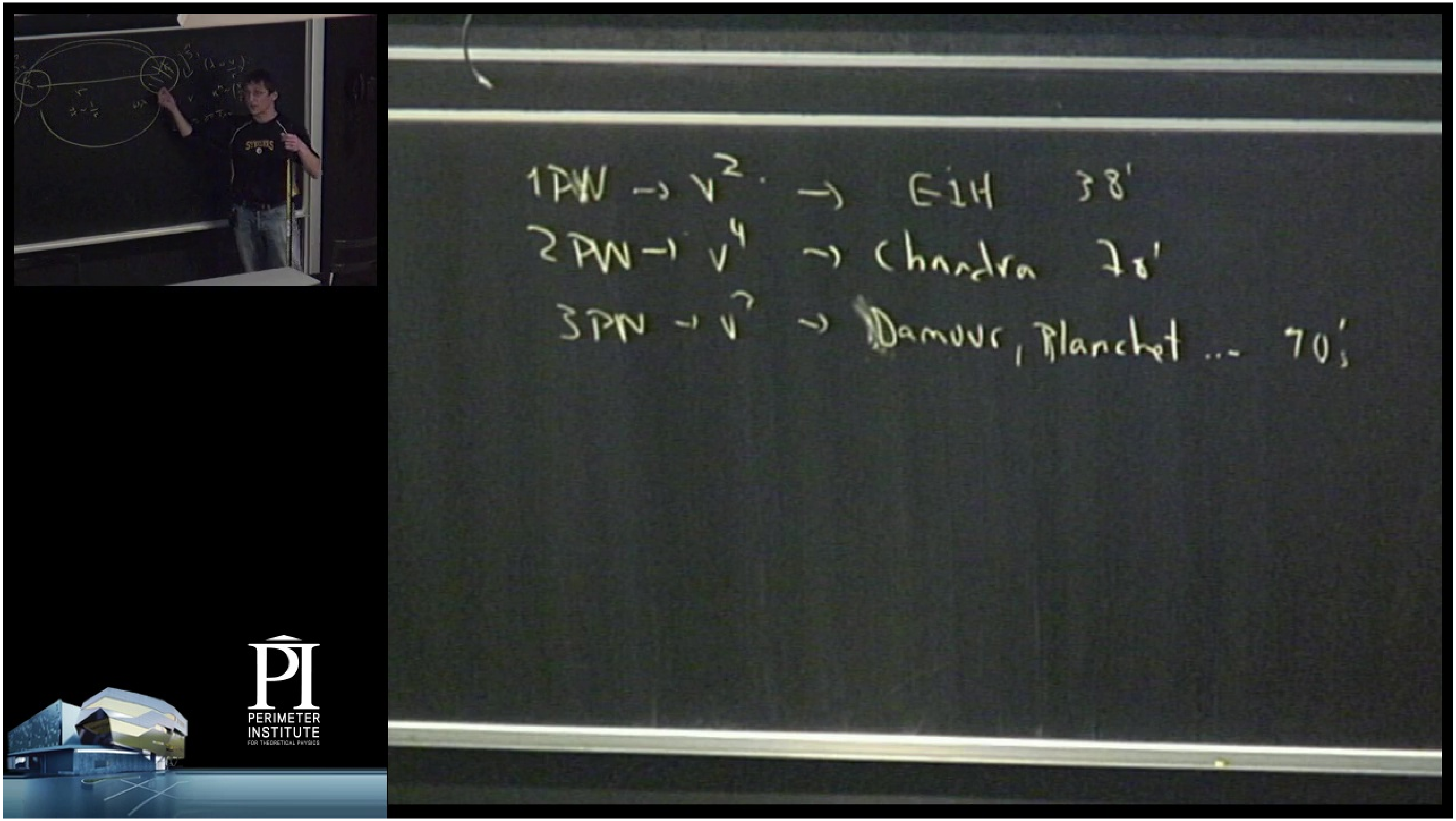
- The size scale $k^\mu \sim \dfrac{1}{r_s}$

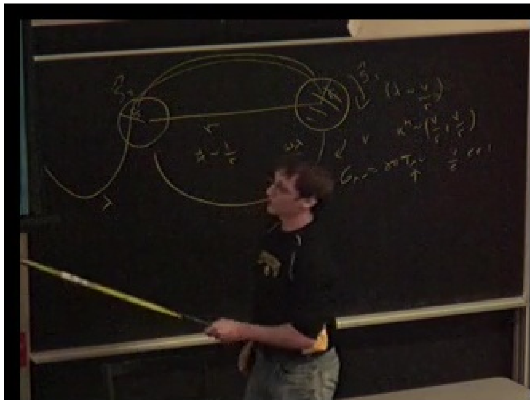
- The orbit scale (potential modes) $k^\mu \sim \left(\dfrac{v}{r}, \dfrac{1}{r}\right)$

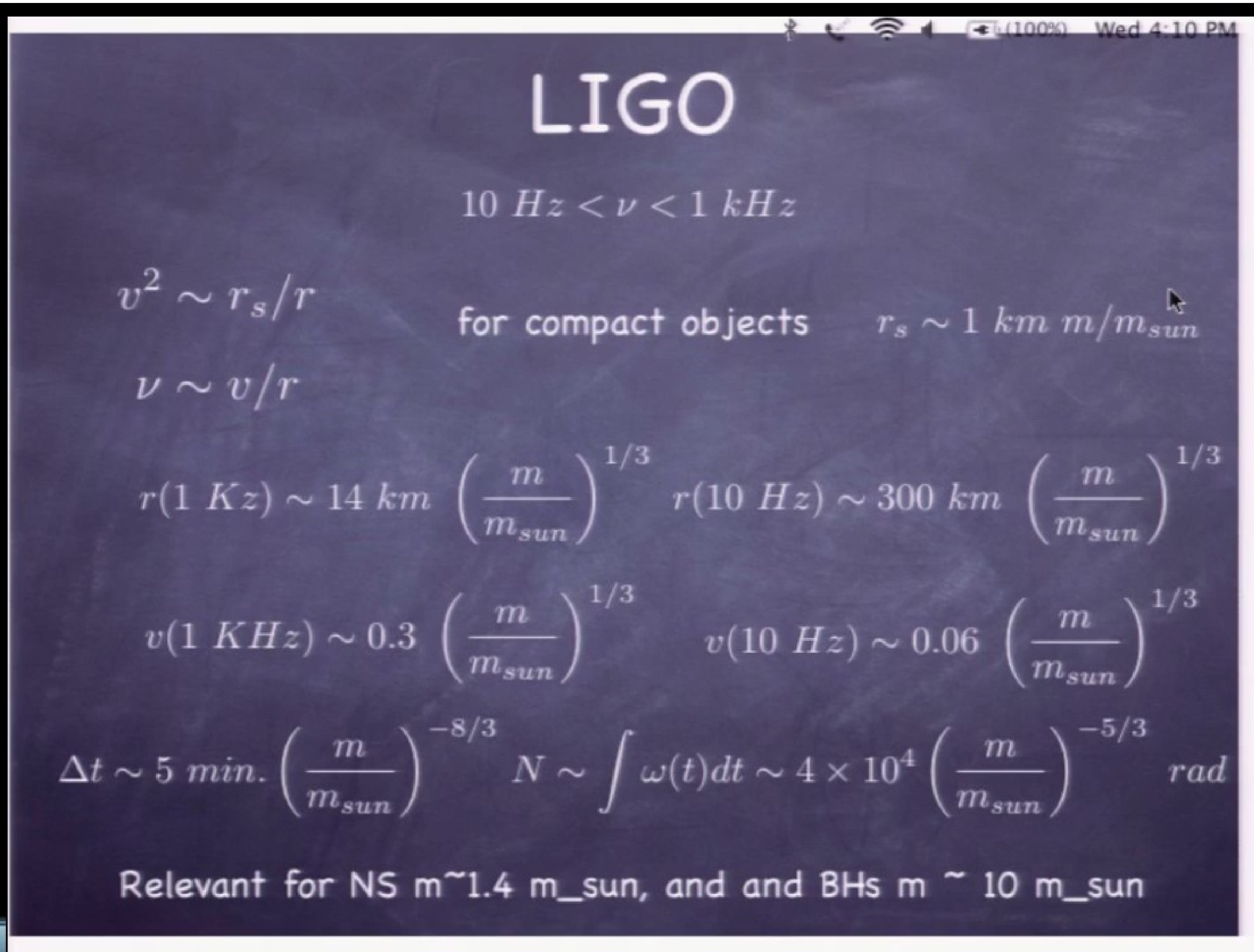
- The radiation scale (radiation modes) $k^\mu \sim \left(\dfrac{v}{r}, \dfrac{v}{r}\right)$

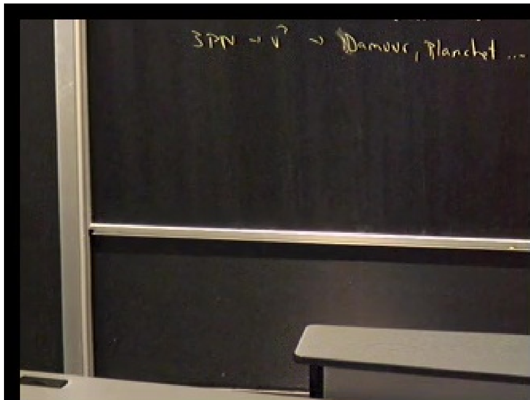
PERIMETER INSTITUTE FOR THEORETICAL PHYSICS


The problem of motion:
We have three relevant scales:
The size scale $k^\mu \sim \frac{1}{r_s}$
The orbit scale (potential modes) $k^\mu \sim \left(\frac{v}{r}, \frac{1}{r}\right)$
The radiation scale (radiation modes) $k^\mu \sim \left(\frac{v}{r}, \frac{v}{r}\right)$
PERIMETER
INSTITUTE
FOR THEORETICAL PHYSICS


1PN → v2 → EIH 38'
2PN → v4 → Chandra 70'
3PN → v6 → Damour, Blanchet ... 90's
PERIMETER INSTITUTE
FOR THEORETICAL PHYSICS




Wed 4:10 PM
LIGO
$10\ Hz < \nu < 1\ kHz$
$v^2 \sim r_s/r$
for compact objects
$r_s \sim 1\ km\ m/m_{sun}$
$\nu \sim v/r$
$r(1\ Kz) \sim 14\ km \left(\frac{m}{m_{sun}}\right)^{1/3}$
$r(10\ Hz) \sim 300\ km \left(\frac{m}{m_{sun}}\right)^{1/3}$
$v(1\ KHz) \sim 0.3 \left(\frac{m}{m_{sun}}\right)^{1/3}$
$v(10\ Hz) \sim 0.06 \left(\frac{m}{m_{sun}}\right)^{1/3}$
$\Delta t \sim 5\ min. \left(\frac{m}{m_{sun}}\right)^{-8/3}$
$N \sim \int \omega(t) dt \sim 4 \times 10^4 \left(\frac{m}{m_{sun}}\right)^{-5/3}\ rad$
Relevant for NS m~1.4 m_sun, and and BHs m ~ 10 m_sun



(100%) Wed 4:11 PM

EFT approach:

Do the *classical* path integral (saddle point) one scale at the time

$$\exp[iS_{eff}(x_a)] = \int \mathcal{D}\phi\mathcal{D}H \exp[iS(g+H,\phi,x_a) + iS_{GF}]$$

Shrinking the BHs in the long wavelength limit

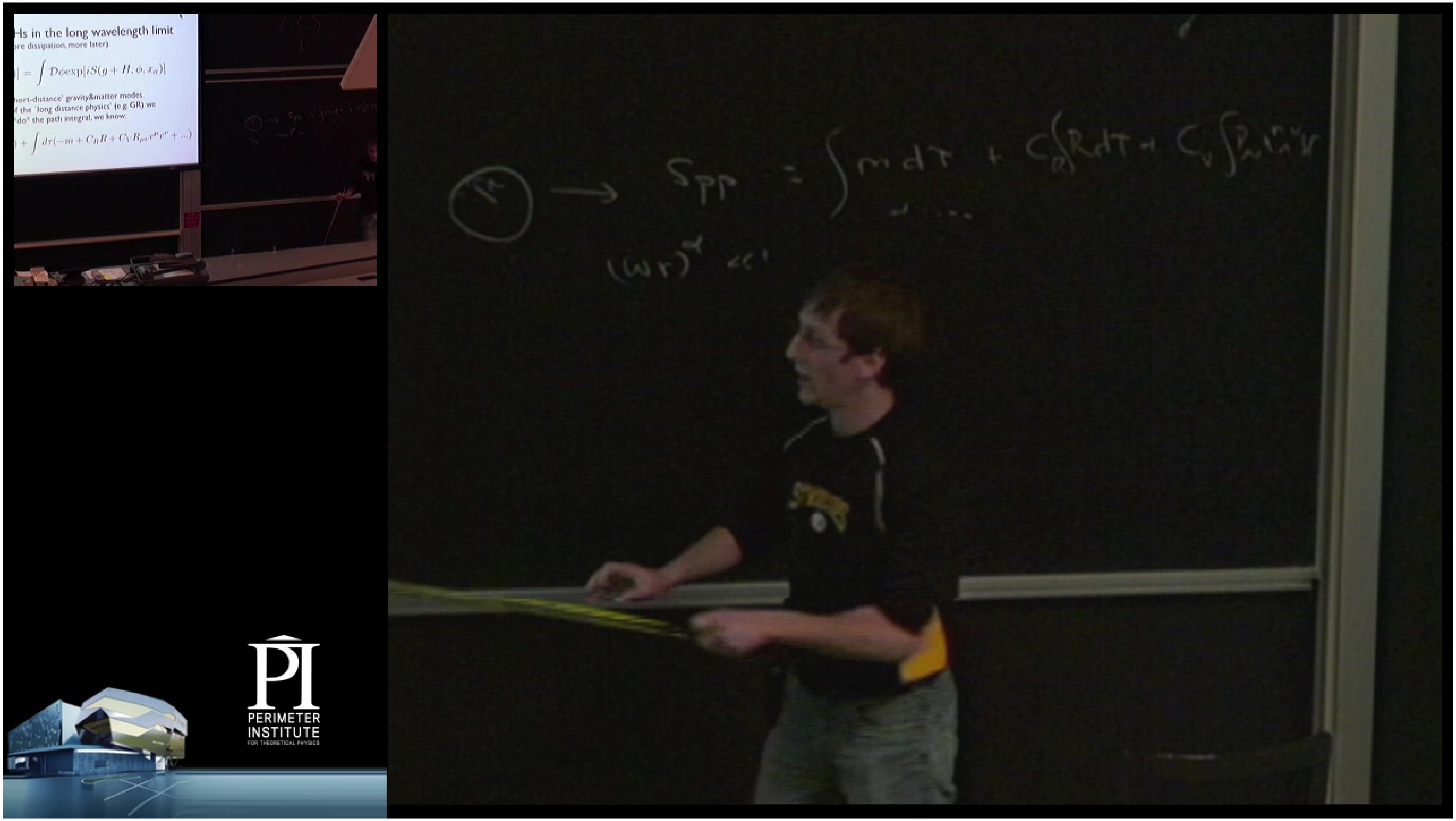
(Ignore dissipation, more later):

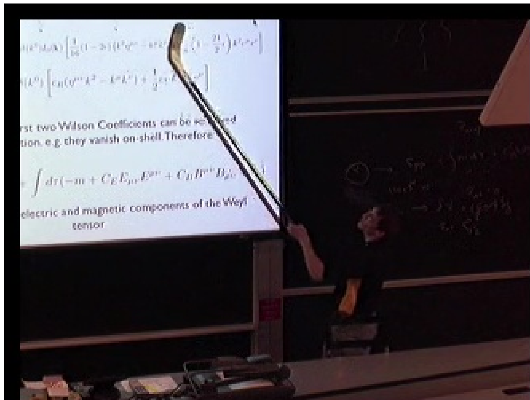
$$\exp[iS_{pp}(x_a,g)] = \int \mathcal{D}\phi \exp[iS(g+H,\phi,x_a)]$$

where ϕ includes 'short-distance' gravity&matter modes.
From the symmetries of the `long distance physics' (e.g. GR) we don't need to *do* the path integral, we know:

$$S_{pp}(g,x_a) = S_{EH}(g) + \int d\tau(-m + C_R R + C_V R_{\mu\nu}v^\mu v^\nu + ...)$$

PERIMETER INSTITUTE FOR THEORETICAL PHYSICS


Hs in the long wavelength limit
re dissipation, more later)
hort-distance' gravity&matter modes.
f the 'long distance physics' (e.g. GR) we
"do" the path integral, we know:
PERIMETER
INSTITUTE



(100%) Wed 4:22 PM

All divergences of the pp approximation are absorbed into the Wilson coefficients to all orders.

$$T^{\mu\nu}_{(3)}(k) = \frac{1}{3!}\left(-\frac{im}{2m_{Pl}}\right)^3 \frac{2}{m_{Pl}}(2\pi)\delta(k^0)I_0(\mathbf{k})\left[\frac{1}{16}(1-2\epsilon)\left(k^2\eta^{\mu\nu}-k^\mu k^\nu\right)-\frac{1}{8}\left(1-\frac{21}{2}\epsilon\right)k^2 v^\mu v^\nu\right]$$

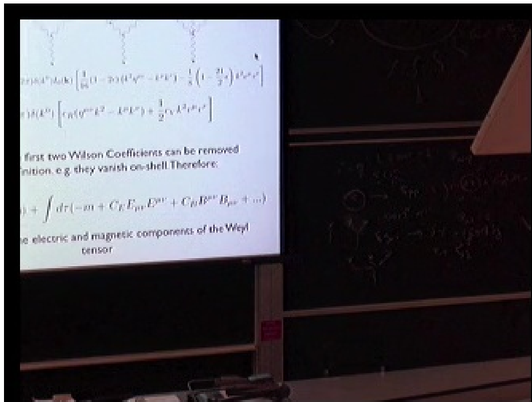
$$T^{\mu\nu}_{ct}(k) = (2\pi)\delta(k^0)\left[c_R(\eta^{\mu\nu}k^2 - k^\mu k^\nu) + \frac{1}{2}c_V k^2 v^\mu v^\nu\right]$$

Unfortunately the first two Wilson Coefficients can be removed by field redefinition. e.g. they vanish on-shell. Therefore:

$$S_{pp}(g, x_a) = S_{EH}(g) + \int d\tau(-m + C_E E_{\mu\nu}E^{\mu\nu} + C_B B^{\mu\nu}B_{\mu\nu} + ...)$$

Where E,B are the electric and magnetic components of the Weyl tensor

PERIMETER INSTITUTE FOR THEORETICAL PHYSICS



(100%) Wed 4:29 PM

Matching in the low frequency regime:

We compute graviton scattering in the full theory and in the EFT and 'match'

$$\sigma_{full}(\omega) = r_s^2 F(\omega r_s) \sim r_s^2(... + \alpha r_s^8 \omega^8 + ...)$$

$$\sigma_{EFT}(\omega) = ... + \frac{C_{E,B}^2}{m_{pl}^4}\omega^8 + ...$$

$$C_{E,B} \sim r_s^5 m_{pl}^2$$

One can show finite size effects start out at v^10 for spinless bodies

PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

Matching in the low frequency regime:

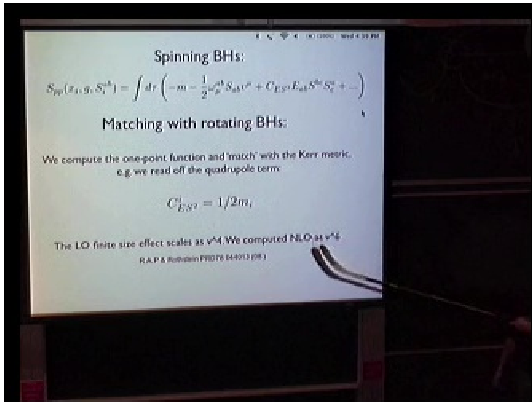
We compute graviton scattering in the full theory and in the EFT and 'match'

$$\sigma_{full}(\omega) = r_s^2 F(\omega r_s) \sim r_s^2(... + \alpha r_s^8 \omega^8 + ...)$$

$$\sigma_{EFT}(\omega) = ... + \frac{C_{E,B}^2}{m_{pl}^4}\omega^8 + ...$$

$$C_{E,B} \sim r_s^5 m_{pl}^2$$

One can show finite size effects start out at v^10 for spinless bodies



Wed 4:38 PM

Spinning BHs:

$$S_{pp}(x_i, g, S_i^{ab}) = \int d\tau \left(-m - \frac{1}{2}\omega_\mu^{ab} S_{ab} v^\mu + C_{ES^2} E_{ab} S^{bc} S_c^a + ... \right)$$

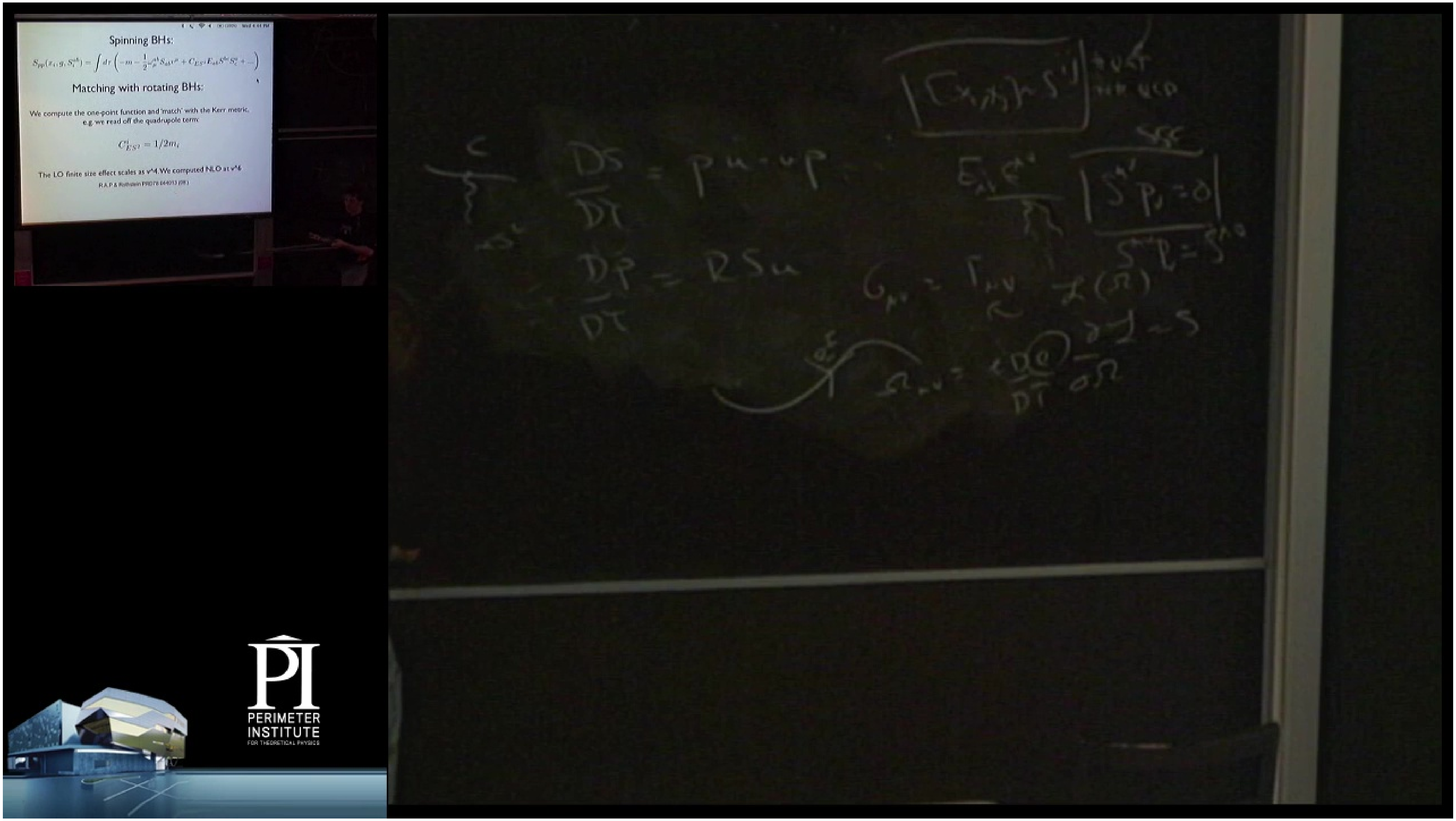
Matching with rotating BHs:

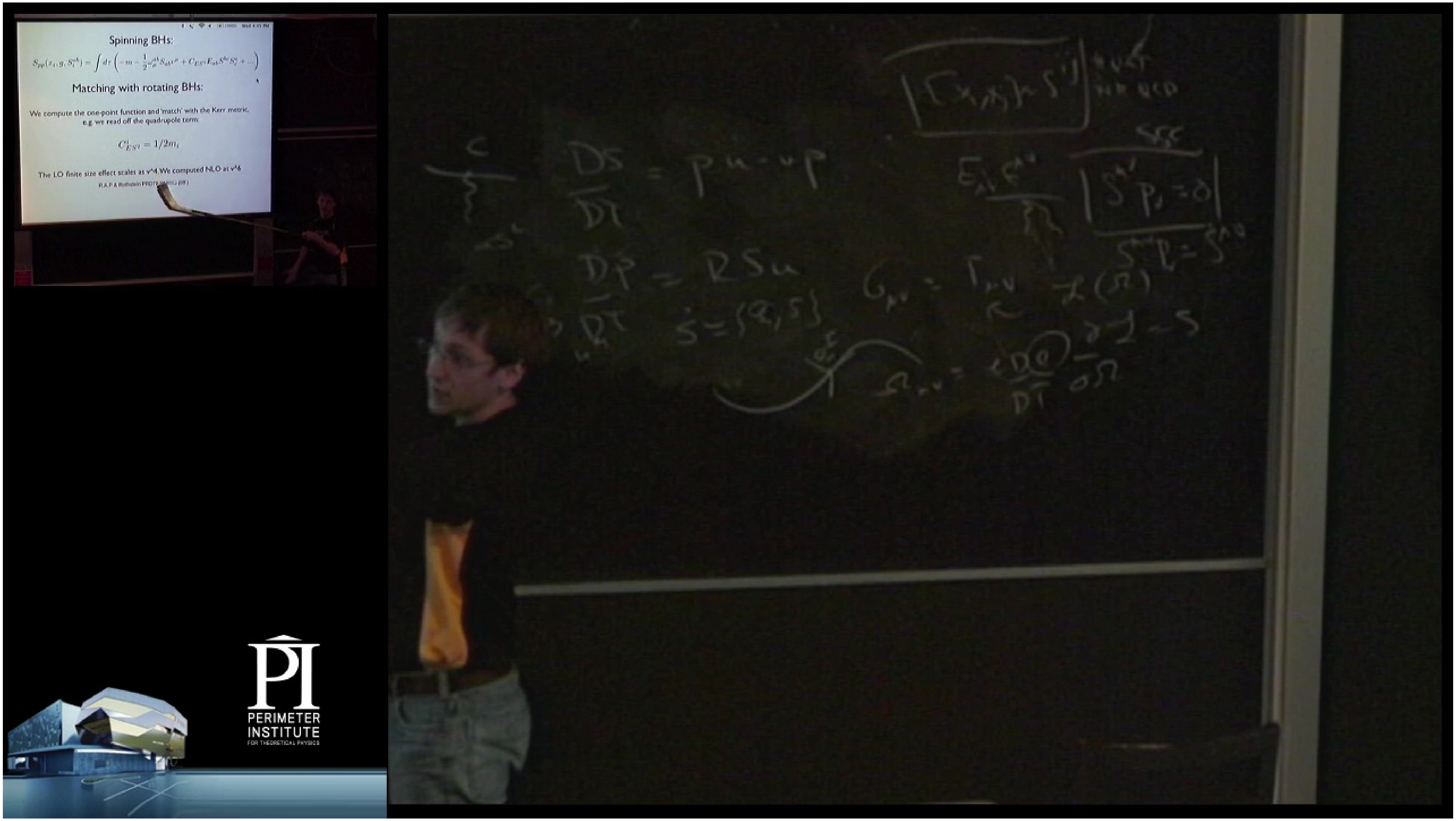
We compute the one-point function and 'match' with the Kerr metric, e.g. we read off the quadrupole term:

$$C^i_{ES^2} = 1/2m_i$$

The LO finite size effect scales as v^4. We computed NLO at v^6

R.A.P & Rothstein PRD78 044013 (08')


Spinning BHs:
Matching with rotating BHs:
We compute the one-point function and 'match' with the Kerr metric, e.g. we read off the quadrupole term:
$C^i_{ES^2} = 1/2m_i$
The LO finite size effect scales as v^4. We computed NLO at v^6
PERIMETER INSTITUTE
FOR THEORETICAL PHYSICS


Spinning BHs:
Matching with rotating BHs:
We compute the one-point function and 'match' with the Kerr metric, e.g. we read off the quadrupole term:
The LO finite size effect scales as v^4. We computed NLO at v^6
PERIMETER INSTITUTE
FOR THEORETICAL PHYSICS

(100%) Wed 4:46 PM

Shrinking the binary system in the PN approximation

We have two more modes to take care of: off-shell potential (v/r, 1/r), and on-shell radiation modes (v/r,v/r). We integrate out the potential modes first.

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \qquad\qquad h_{\mu\nu} = \bar{h}_{\mu\nu} + H_{\mu\nu}$$

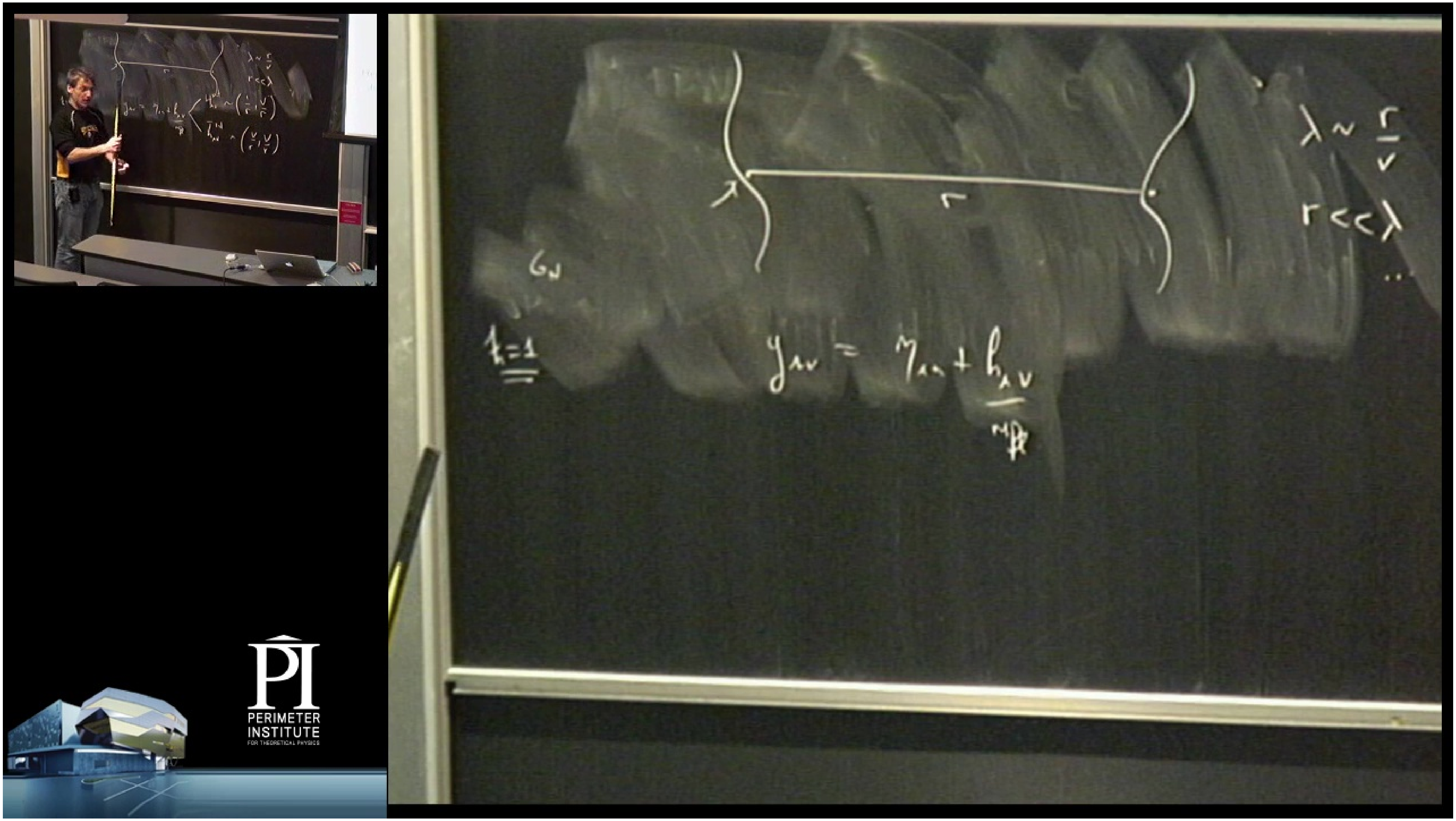
$$\exp[iS_{rad}(x_i, S_i, C_i, \bar{h})] = \int \mathcal{D}H \exp[iS(g = \eta + \bar{h} + H, x, S, C) + iS_{GF}]$$

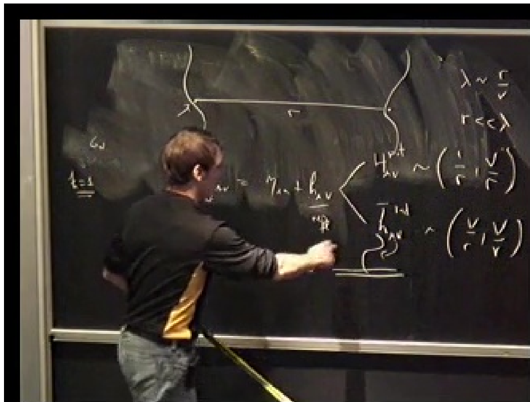
We work in the background field:

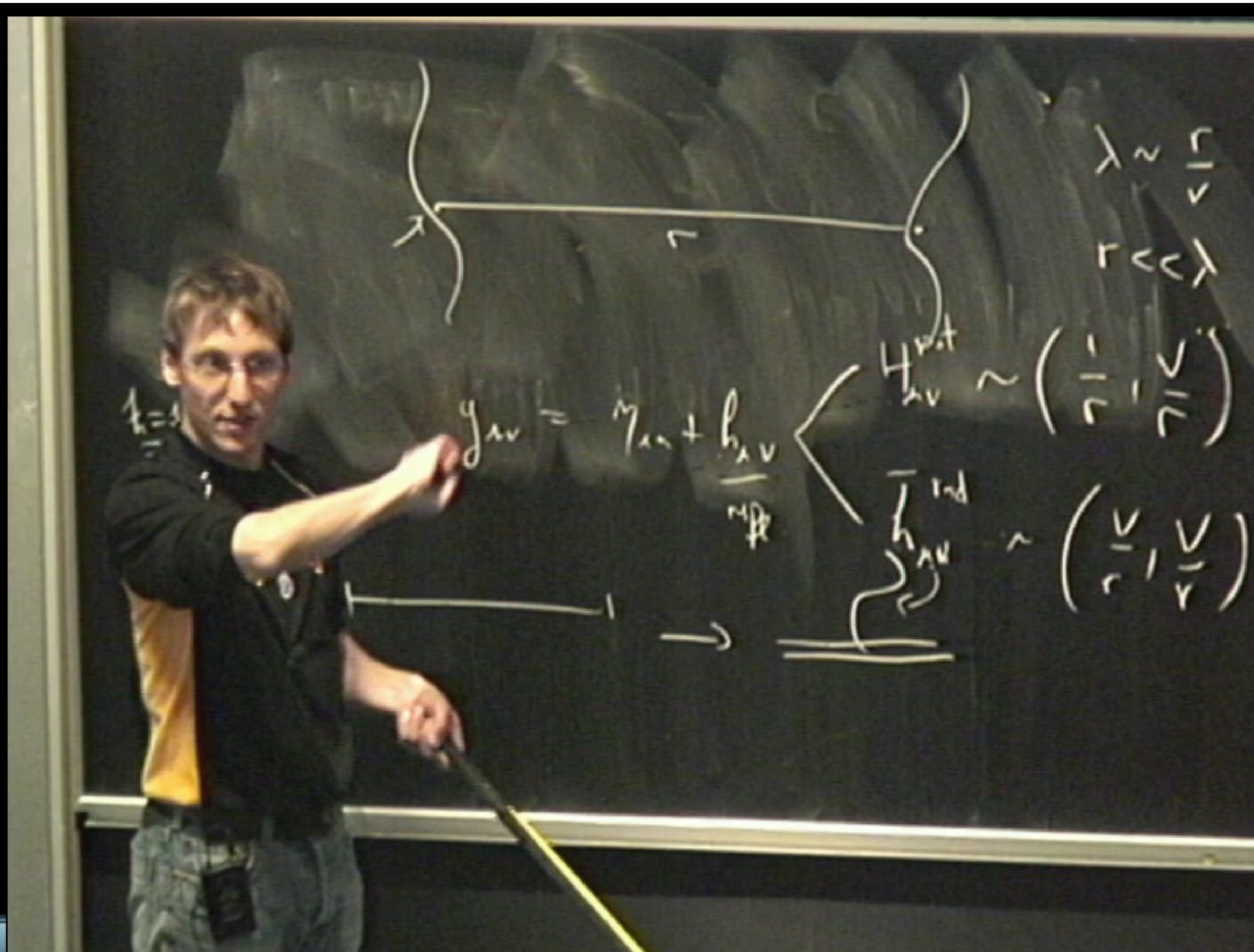
$$S_{GF} = \int \sqrt{\bar{g}}\Gamma^2 \quad \Gamma_\mu = \bar{D}_\alpha H^\alpha_\mu - \frac{1}{2}\bar{D}_\mu H^\alpha_\alpha$$

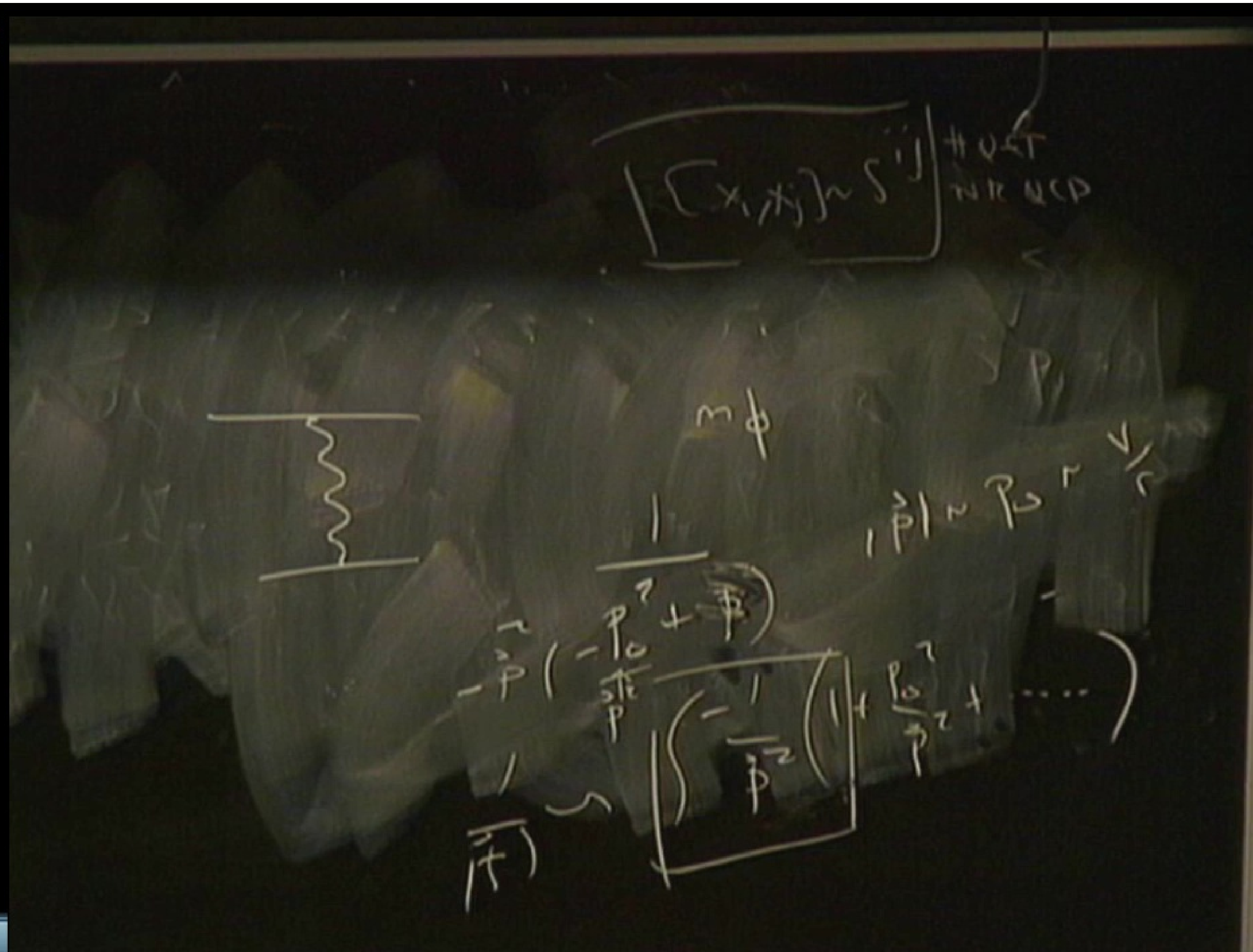
Here we know the full theory and do the matching explicitly. From the off-shell modes we obtain the real part of the effective action and hence the potential energy (ignoring self-force effects).

PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

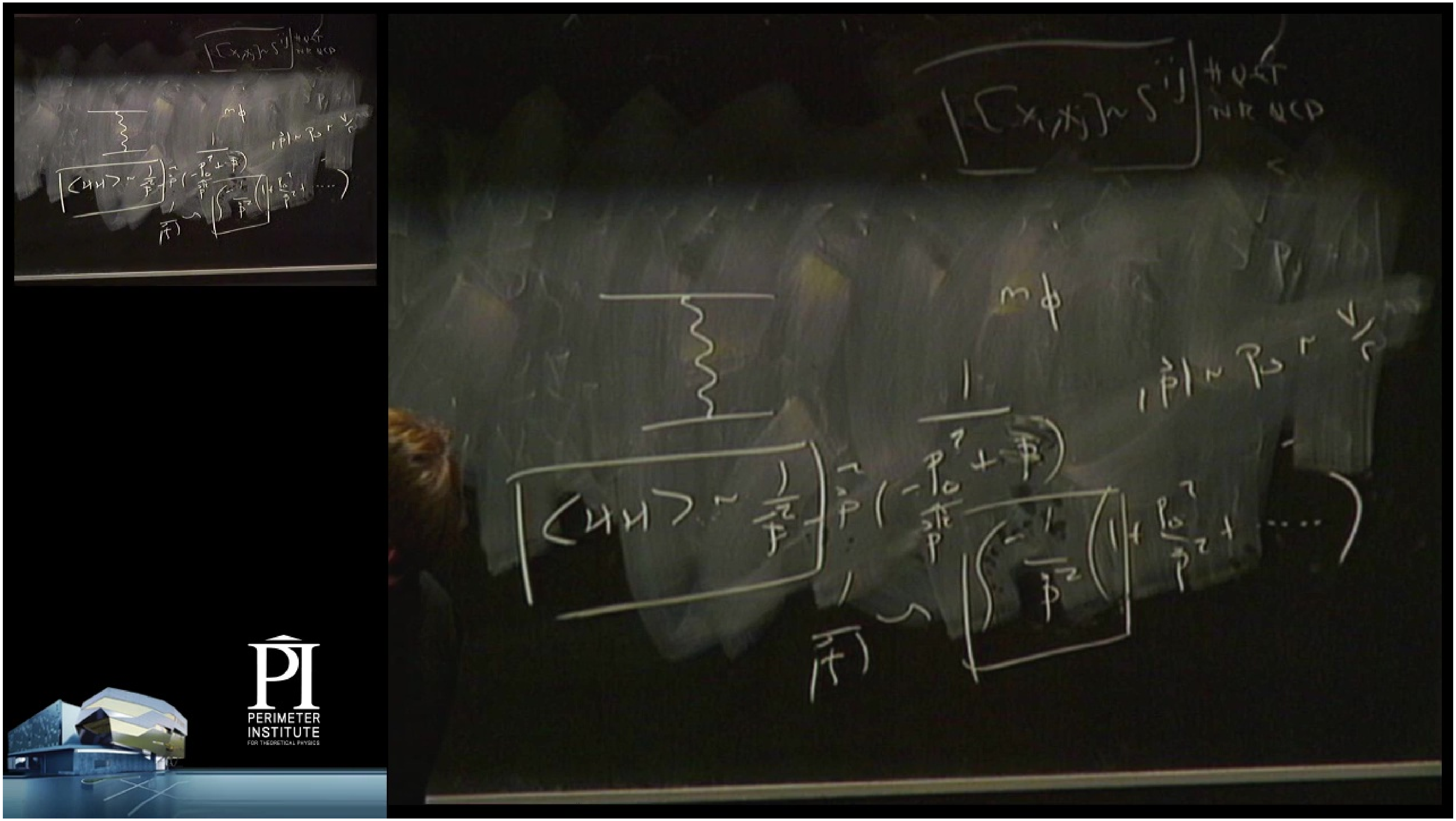

λ ~ r/v
r << λ
r
g_μν = η_μν + h_μν

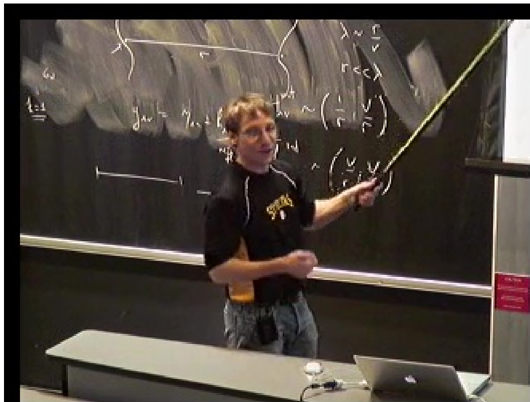



λ ~ r/v
r<<λ

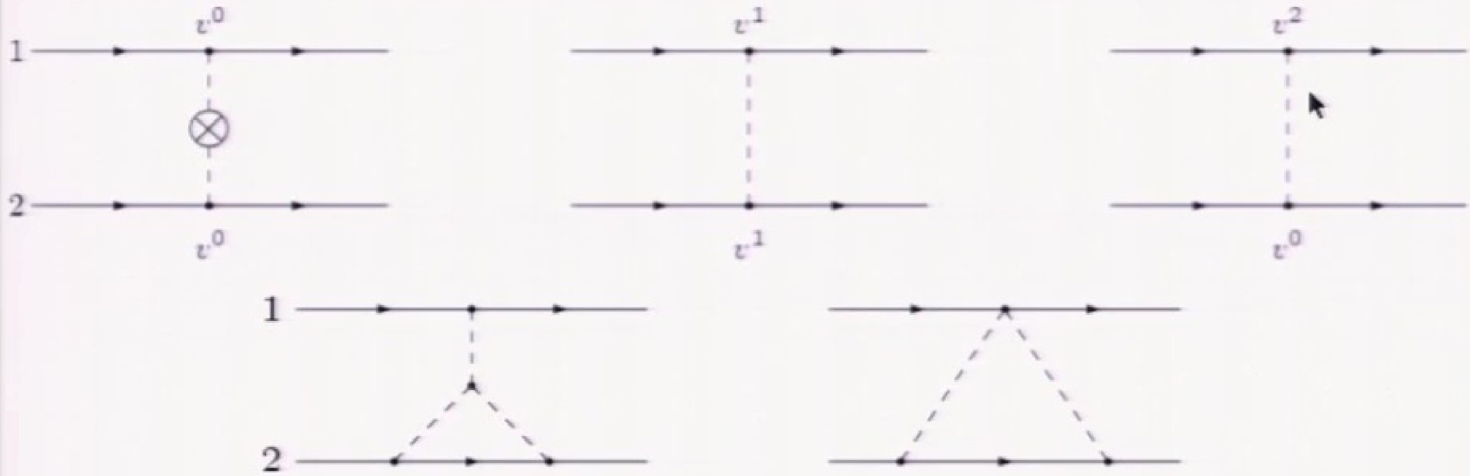


NR QCD


PERIMETER
INSTITUTE
FOR THEORETICAL PHYSICS

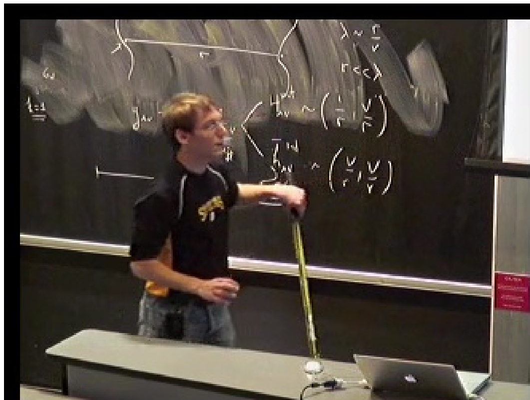


Einstein-Infeld-Hoffmann



$$G(p) \propto \frac{1}{p_0^2 - \vec{p}^2} \approx -\frac{1}{\vec{p}^2} - \frac{p_0^2}{\vec{p}^4} + \dots$$

$$L_{EIH} = \frac{1}{8}\sum_a m_a \mathbf{v}_a^4 + \frac{G_N m_1 m_2}{2|\mathbf{x}_1 - \mathbf{x}_2|}\left[3(\mathbf{v}_1^2 + \mathbf{v}_2^2) - 7(\mathbf{v}_1 \cdot \mathbf{v}_2) - \frac{(\mathbf{v}_1 \cdot \mathbf{x}_{12})(\mathbf{v}_2 \cdot \mathbf{x}_{12})}{|\mathbf{x}_1 - \mathbf{x}_2|^2}\right] - \frac{G_N^2 m_1 m_2 (m_1 + m_2)}{2|\mathbf{x}_1 - \mathbf{x}_2|^2}$$



S1S2 spin Hamiltonian to 3PN order

RAP & Rothstein, PRL97 021101 (06'), PRD78 044012 (08')

$$
\begin{aligned}
H^{spin}_{NW} = {} & \frac{G_N}{2m_1m_2r^3}\Big[\frac{3}{2}(\mathcal{P}_1\times\mathbf{S}_1)\cdot\mathbf{n}(\mathcal{P}_2\times\mathbf{S}_2)\cdot\mathbf{n}+6(\mathcal{P}_2\times\mathbf{S}_1)\cdot\mathbf{n}(\mathcal{P}_1\times\mathbf{S}_2)\cdot\mathbf{n} \\
& -15(\mathcal{P}_1\cdot\mathbf{n})(\mathcal{P}_2\cdot\mathbf{n})(\mathbf{S}_1\cdot\mathbf{n})(\mathbf{S}_2\cdot\mathbf{n})+\frac{3}{2}(\mathcal{P}_2\cdot\mathbf{S}_1)(\mathcal{P}_1\cdot\mathbf{S}_2)-\frac{3}{2}(\mathcal{P}_2\cdot\mathcal{P}_1)(\mathbf{S}_1\cdot\mathbf{S}_2) \\
& -3(\mathcal{P}_1\cdot\mathcal{P}_2)(\mathbf{S}_1\cdot\mathbf{n})(\mathbf{S}_2\cdot\mathbf{n})+3(\mathcal{P}_1\cdot\mathbf{S}_1)(\mathbf{n}\cdot\mathcal{P}_2)(\mathbf{S}_2\cdot\mathbf{n})+3(\mathcal{P}_2\cdot\mathbf{S}_2)(\mathbf{n}\cdot\mathcal{P}_1)(\mathbf{S}_1\cdot\mathbf{n}) \\
& +3(\mathcal{P}_2\cdot\mathbf{n})(\mathbf{n}\cdot\mathcal{P}_1)(\mathbf{S}_1\cdot\mathbf{S}_2)+(\mathcal{P}_2\cdot\mathbf{S}_2)(\mathcal{P}_1\cdot\mathbf{S}_1)\Big] \\
& +\frac{G_N}{2m_1^2r^3}\left[\mathcal{P}_1^2(\mathbf{S}_1\cdot\mathbf{S}_2)-3(\mathcal{P}_1\times\mathbf{S}_1)\cdot\mathbf{n}(\mathcal{P}_1\times\mathbf{S}_2)\cdot\mathbf{n}-(\mathcal{P}_1\cdot\mathbf{S}_2)(\mathcal{P}_1\cdot\mathbf{S}_1)\right] \\
& +\frac{G_N}{2m_2^2r^3}\left[\mathcal{P}_2^2(\mathbf{S}_1\cdot\mathbf{S}_2)-3(\mathcal{P}_2\times\mathbf{S}_1)\cdot\mathbf{n}(\mathcal{P}_2\times\mathbf{S}_2)\cdot\mathbf{n}-(\mathcal{P}_2\cdot\mathbf{S}_2)(\mathcal{P}_2\cdot\mathbf{S}_1)\right] \\
& +\frac{G_N^2(m_1+m_2)}{2r^4}(11\mathbf{S}_1\cdot\mathbf{S}_2-23(\mathbf{S}_1\cdot\mathbf{n})(\mathbf{S}_2\cdot\mathbf{n}))-\frac{G_N}{r^3}(\mathbf{S}_1\cdot\mathbf{S}_2-3(\mathbf{S}_1\cdot\mathbf{n})(\mathbf{S}_2\cdot\mathbf{n})) \\
& +\frac{G_N}{r^2}\left[\frac{3m_2}{2m_1}(\mathbf{n}\times\mathcal{P}_1)\cdot\mathbf{S}_1-2(\mathbf{n}\times\mathcal{P}_2)\cdot\mathbf{S}_1+2(\mathbf{n}\times\mathcal{P}_1)\cdot\mathbf{S}_2-\frac{3m_1}{2m_2}(\mathbf{n}\times\mathcal{P}_2)\cdot\mathbf{S}_2\right],
\end{aligned}
\tag{59}
$$

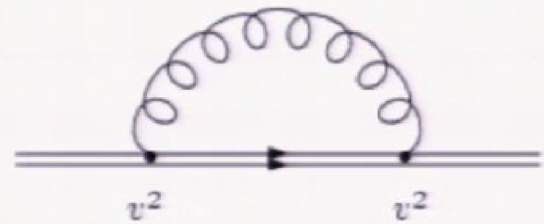
To retain manifest power counting we need to multiple expand the radiation field whenever it couples to worldlines or potential modes.
Then we obtain (non-spinning):

$$_{rad} \sim \ldots + \frac{1}{2}E_{ab}(X,x^0)Q^{ab} \qquad Q_{ab} = \sum_i \mathbf{x}_i^a\mathbf{x}_i^b - \frac{1}{3}\delta^{ab}\mathbf{x}^2$$

Since we work in background field we knew from gauge invariance this was it.

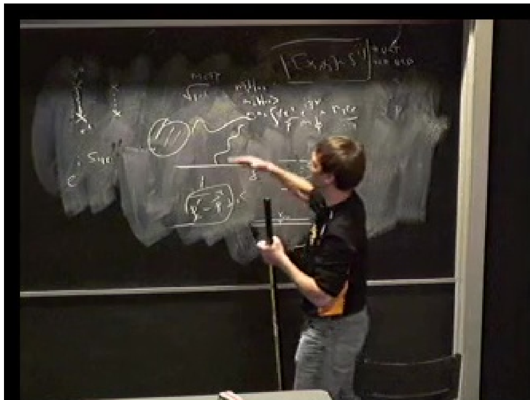
Last step, we need to integrate our h_rad, from the optical theorem we know

$$\frac{1}{T}\mathrm{Im}S_{eff} = \frac{1}{2}\int dEd\Omega\frac{d^2\Gamma}{dEd\Omega}$$



$$\mathrm{Im}S_{eff} = -\frac{1}{80m_{pl}^2}\int_{\mathbf{k}}\frac{1}{2|\mathbf{k}|}\mathbf{k}^4|Q_{ij}|^2 \rightarrow \frac{dE}{dt} = -\frac{G_N}{5}\langle\frac{d^3}{dt^3}Q_{ij}\frac{d^3}{dt^3}Q^{ij}\rangle$$

Comment: Re S_eff accounts for the self-force (see diagram)



Dissipation/Absorption: Membrane Paradigm

In the previous formalism we ignored absorption. We can add complex Wilson coefficients or realize we need to keep the BH modes in the horizon.

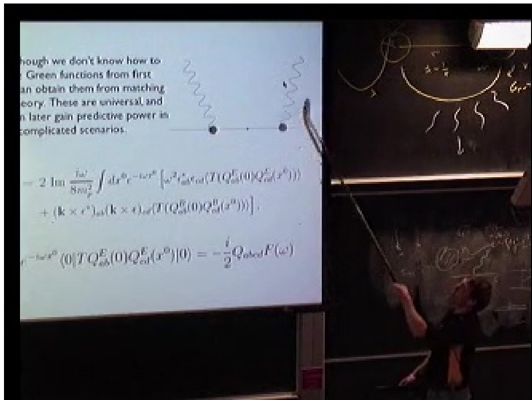
3+1 BH dynamics / 0+1 EFT Duality

We assume that in the long wavelength approximation the energy loss thru the BH horizon can be described by a worldline theory EFT where bulk fields couple to localized degrees of freedom. The coupling is determined by the symmetries. Isometries => Global Symmetries (a=1,2,3)

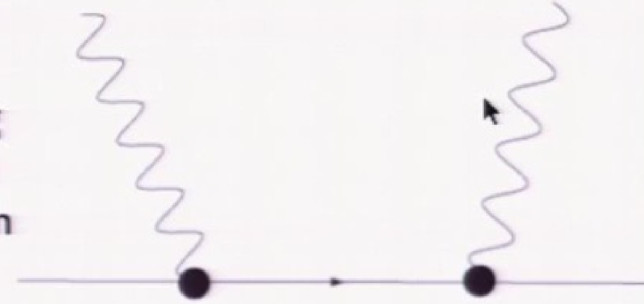
$$S = -\int d\tau Q^E_{ab} E^{ab} + Q^B_{ab} B^{ab} + \ldots$$

Imagine we have the full Lagrangian and 'integrate out' Qab then we generate:

$$S_{pp}(g, x_a) = S_{EH}(g) + \int d\tau(-m + C_E E_{\mu\nu} E^{\mu\nu} + C_B B^{\mu\nu} B_{\mu\nu} + \ldots)$$

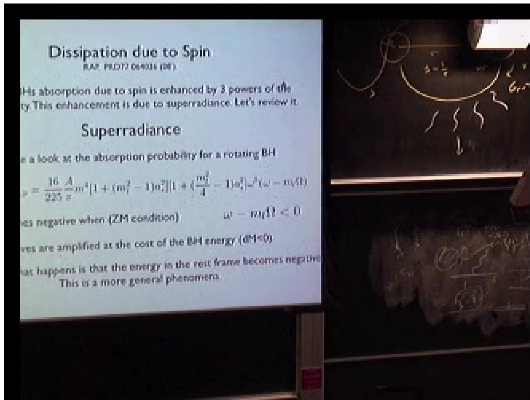


Lesson: Even though we don't know how to compute the Green functions from first principles, we can obtain them from matching with the full theory. These are universal, and we can use them later gain predictive power in more complicated scenarios.



$$\sigma^{eft}_{abs}(\omega) = 2\,\mathrm{Im}\,\frac{i\omega}{8m_p^2}\int dx^0 e^{-i\omega x^0}\Big[\omega^2\epsilon^*_{ab}\epsilon_{cd}\langle T(Q^E_{ab}(0)Q^E_{cd}(x^0))\rangle + (\mathbf{k}\times\epsilon^*)_{ab}(\mathbf{k}\times\epsilon)_{cd}\langle T(Q^B_{ab}(0)Q^B_{cd}(x^0))\rangle\Big],$$

$$\int dx^0 e^{-i\omega x^0}\langle 0|TQ^E_{ab}(0)Q^E_{cd}(x^0)|0\rangle = -\frac{i}{2}Q_{abcd}F(\omega)$$



Dissipation due to Spin

RAP, PRD77 064026 (08').

For binary BHs absorption due to spin is enhanced by 3 powers of the relative velocity. This enhancement is due to superradiance. Let's review it.

Superradiance

Let's take a look at the absorption probability for a rotating BH

$$\Gamma_{s=2,\omega,l=s,m,p} = \frac{16}{225}\frac{A}{\pi}m^4[1+(m_l^2-1)a_*^2][1+(\frac{m_l^2}{4}-1)a_*^2]\omega^5(\omega - m_l\Omega)$$

It becomes negative when (ZM condition) $\qquad \omega - m_l\Omega < 0$

The waves are amplified at the cost of the BH energy (dM<0).

Heuristically what happens is that the energy in the rest frame becomes negative. This is a more general phenomena.

Universality

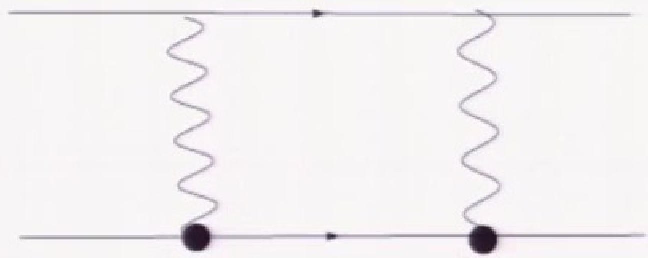
Let's use our dual EFT to predict the power of absorption in binary systems

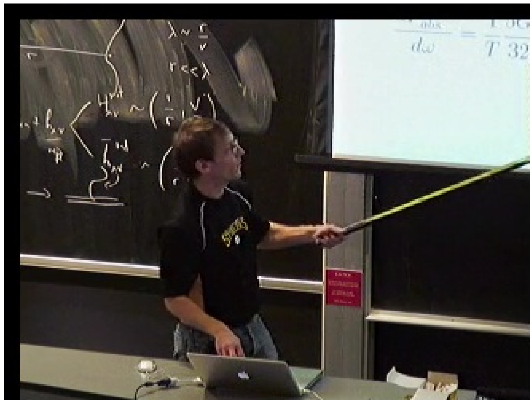
In the NR limit we have w~v/r. Therefore we can power count Q^E(B)

$$\left(Q_{ab}^{E(B)}\right)_{spin} \sim \sqrt{a_*(1+3a_*^2)}Lv^{5/2}/m_p$$

$$\int d\tau \left(Q_{ab}^{E}\right)_{spin} E^{ab}[H] \sim \sqrt{a_*(1+3a_*^2)}v^5$$

This diagram scales as v^10, contrary to v^13 in the spinless case. This represents an enhancement of v^3!





Neutron Stars

Same trick. Assume we know the abs probabilities and assume they show superradiance (very compact ones). We expand at leading order:

$$\Gamma^{NS}_{l,m_l}[\omega,\Omega] = \tilde{\Gamma}^{NS}_{l,m_l}[\omega - m_l\Omega, \omega](\omega - m_l\Omega),$$

$$\Gamma^{NS}_{l,m_l}[\omega,\Omega] \sim -\gamma^{NS}_{m_l,l=2}[a_*,\omega]m_l\Omega$$

$$\frac{dP^{spin}_{abs}}{d\omega} = \frac{1}{T}\frac{5G_N}{32\pi}\left\langle \sum_{a\neq b} \frac{\tilde{\gamma}^b_{-2}[\omega]a_*}{\omega^4} m_a^2 q^b_{ij}(\omega) q^{b^*}_{il}(\omega) s^b_{jl}(\omega)\right\rangle + \ldots.$$

Test spinning particle in a time dependent curved background

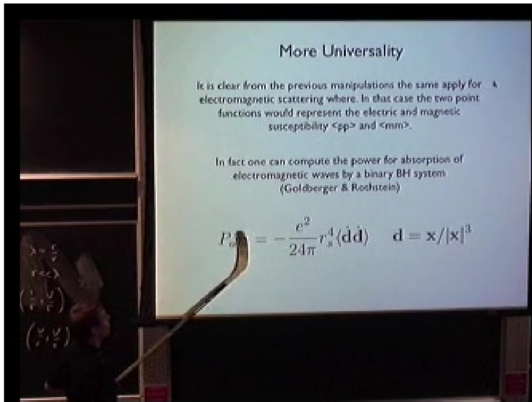
Let's assume a small BH rs<<R. With R the curvature scale of the spacetime environment. Then we treat the E,B fields as external and time dependent.

The LO dissipation comes from two insertions of the quadrupole operators. Therefore we have

$$2\,\mathrm{Im} S_{eff}[x] = \mathrm{Im}\, i \int d\tau d\bar{\tau} \langle T Q^E_{ab}(\tau) Q^E_{cd}(\bar{\tau})\rangle \left[E^{ab}(\tau)E^{cd}(\bar{\tau}) + B^{ab}(\tau)B^{cd}(\bar{\tau})\right] + \ldots$$

$$P_{abs} = \frac{8}{45} a_*(1+3a_*^2) G_N^4 m^5 \left\langle \left(\dot{E}_{ij}E_{il} + \dot{B}_{ij}B_{jl}\right) s_{jl} \right\rangle$$

$$\dot{E}^{ij} = e^i_\mu e^j_\nu (v\cdot D) E^{\mu\nu}$$



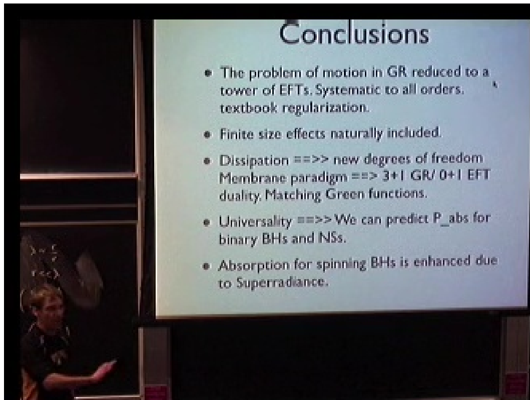
More Universality

It is clear from the previous manipulations the same apply for electromagnetic scattering where. In that case the two point functions would represent the electric and magnetic susceptibility <pp> and <mm>.

In fact one can compute the power for absorption of electromagnetic waves by a binary BH system (Goldberger & Rothstein)

$$P_{abs} = -\frac{e^2}{24\pi} r_s^4 \langle \dot{\mathbf{d}} \dot{\mathbf{d}} \rangle \qquad \mathbf{d} = \mathbf{x}/|\mathbf{x}|^3$$

PERIMETER INSTITUTE FOR THEORETICAL PHYSICS



Conclusions

- The problem of motion in GR reduced to a tower of EFTs. Systematic to all orders. textbook regularization.
- Finite size effects naturally included.
- Dissipation ==>> new degrees of freedom Membrane paradigm ==> 3+1 GR/ 0+1 EFT duality. Matching Green functions.
- Universality ==>> We can predict P_abs for binary BHs and NSs.
- Absorption for spinning BHs is enhanced due to Superradiance.