Title: Story of a & c

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Abstract: In two dimensional CFTs the Zamolodchikov\'s c-theorem is fundamental in that it shows that the number of degrees of freedom decreases along the renormalization group flow. I will give a short history of and discuss recent developments in the quest to find its four-dimensional analogue using the central charges a & c.

Story of a & c :
a counterexample to the a-theorem

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with Alfred D. Shapere (Kentucky)
arXiv:0804.1957 and 0809.3238

Dec, 2008

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2. Argyres-Douglas points

3. Summary

Contents

Central charge c in 2d

$$T(z)T(0) \sim \frac{c}{2z^4} + \cdots$$

$$\langle T \rangle = -\frac{cR}{12}$$

- Captures the asymptotic growth of states.
- (Need to use modular transformation)
- Additive.

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Zamolodchikov's c-theorem

Let

$$
\begin{aligned}
F(r) &= \quad z^4 \langle T_{zz}(z,\bar{z}) T_{zz}(0,0)\rangle, \\
G(r) &= z^3\bar{z}\ \langle T_{zz}(z,\bar{z}) T_{z\bar{z}}(0,0)\rangle, \\
H(r) &= z^2\bar{z}^2 \langle T_{z\bar{z}}(z,\bar{z}) T_{z\bar{z}}(0,0)\rangle,
\end{aligned}
$$

and define

$$C(r) = 2F(r) - 4G(r) - 6H(r).$$

- $C(r)$ is the central charge c for CFTs.
- $r\frac{\partial}{\partial r}C(r) = -\frac{3}{2}H \leq 0.$
- # DOF decreases along the RG flow

4d central charges: a and c

$$\langle T^\mu_\mu \rangle = \frac{c}{16\pi^2}\text{Weyl}^2 - \frac{a}{16\pi^2}\text{Euler}$$

- Only c appears in 2pt functions.
- Additive.
- No modular tr. ⟶ no immediate relation to #DOF.

	a	c
$\mathcal{N}=1$ chiral mult.	$1/48$	$1/24$
$\mathcal{N}=1$ vector mult.	$3/16$	$1/8$

Conjectural 'a-theorem'

[Cardy 1988]

In 2d, it was $c \propto \int_{S^2} \langle T \rangle$.

Why don't we choose $a \propto \int_{S^4} \langle T \rangle$ in 4d?

- He somehow chose S^4, which is conformally flat.
- Weyl^2 happened to dropped out.
- Anyway his proposal stood the test of time...

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Test 1 [Cardy 1988]

Consider massless QCD with N_c colors and N_f flavors.

UV:	quarks & gluons	$a \sim N_c^2 + N_c N_f$
IR:	pions	$a \sim N_f^2$

- a-theorem violated if $\dfrac{N_f}{N_c} \gtrsim \mathbf{15.1}$
- loses asymptotic freedom if $\dfrac{N_f}{N_c} \gtrsim \mathbf{5.5}$
- (Chiral symmetry ceases to break at much lower N_f/N_c)
- [Ball-Damgaard 2001] checked other G and matter contents

Test 2 [Anselmi, Freedman et al. 1998]

- Supersymmetry relates a, c to 't Hooft anomalies of R symmetry:

$$a = \frac{3}{32}\left[3\,\mathrm{tr}\,R^3 - \mathrm{tr}\,R\right],$$
$$c = \frac{1}{32}\left[9\,\mathrm{tr}\,R^3 - 5\,\mathrm{tr}\,R\right]$$

- R symmetry must be non-anomalous,
- This condition alone sometimes fixes it
- e.g. SQCD in Seiberg's conformal window
- $a_{IR} < a_{UV}$, but other combinations $a + kc$ don't work

Test 3 [Intriligator, Wecht 2003]

a-maximization

Let the trial a function be

$$a(R) = \frac{3}{32}\left[3\,\mathrm{tr}\,R^3 - \mathrm{tr}\,R\right],$$

a function of trial R symmetry,

$$R = R_0 + t_1 F_1 + \cdots.$$

The right R maximizes $a(R)$.

- Q should have charge **1** under R.
- Marginal terms in the superpotential W have charge **2** under R.
- UV: some terms in W irrelevant → less condition on R
- IR: some terms in W marginal → more condition on R
- Maximization in a smaller subset gives smaller number.

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Test 4 [Freedman, Gubser et al. 1999]

- Use AdS/CFT correspondence:

$$a \sim c \sim \Lambda^{-3/2}$$

- RG flow $\sim$ flow of scalars changing vacuum energy $\Lambda = V(\phi)$
- Null energy condition

$$T_{\mu\nu} n^\mu n^\nu \geq 0$$

guarantees monotony of $V(\phi)$ along the flow.

Test 5 [Gauntlett-Martelli-Sparks-Yau, 2006]

- Use AdS/CFT correspondence:

$$a \sim c \sim \frac{\pi^3 N^2}{4\,\mathrm{vol}\,X_5}$$

for N D3-branes on the cone over X_5,

- X_5 normalized to have $R_{ij} = 4g_{ij}$.

- Bishop's theorem: $\mathrm{vol}\,X_5 \leq \mathrm{vol}\,S^5 \longrightarrow a(X_5) \geq a(S^5)$.
- Flow associated to 'Higgsing'
which moves all D3-branes away from the tip.

a-theorem?

So far so good.

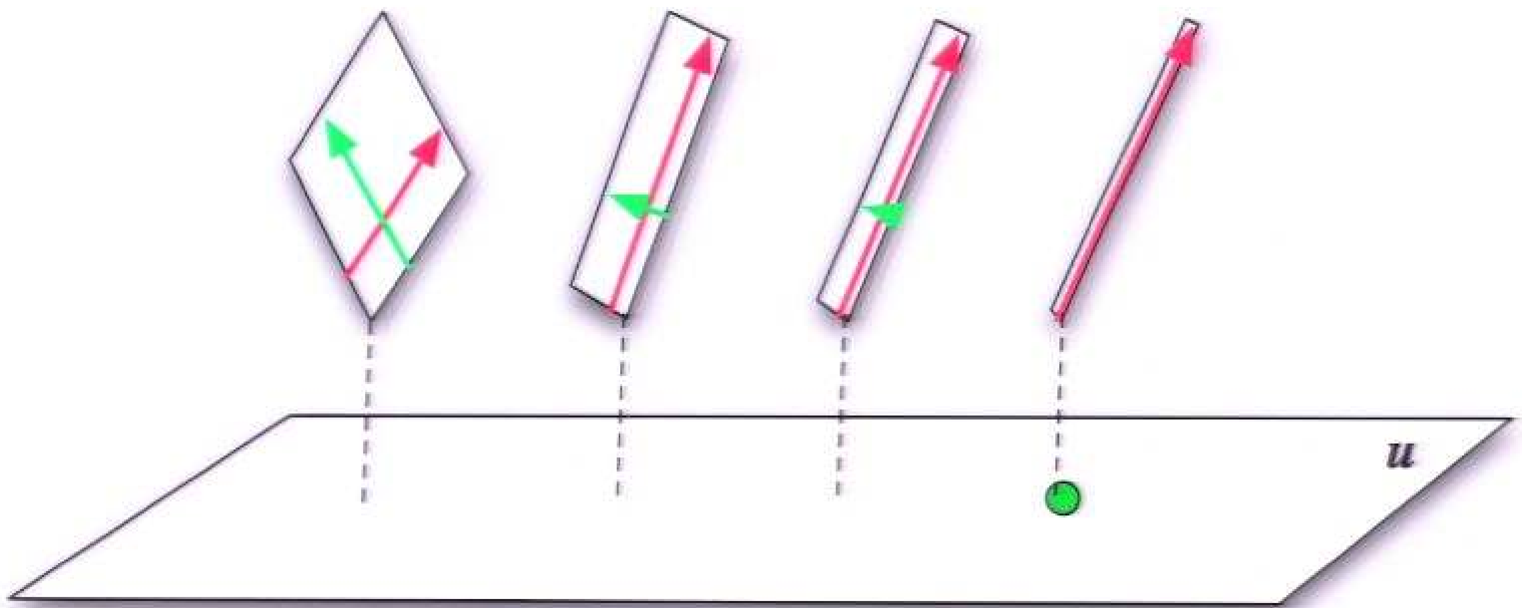
a-theorem?

So far so good.

But there is another class of SCFTs which is far less understood ...

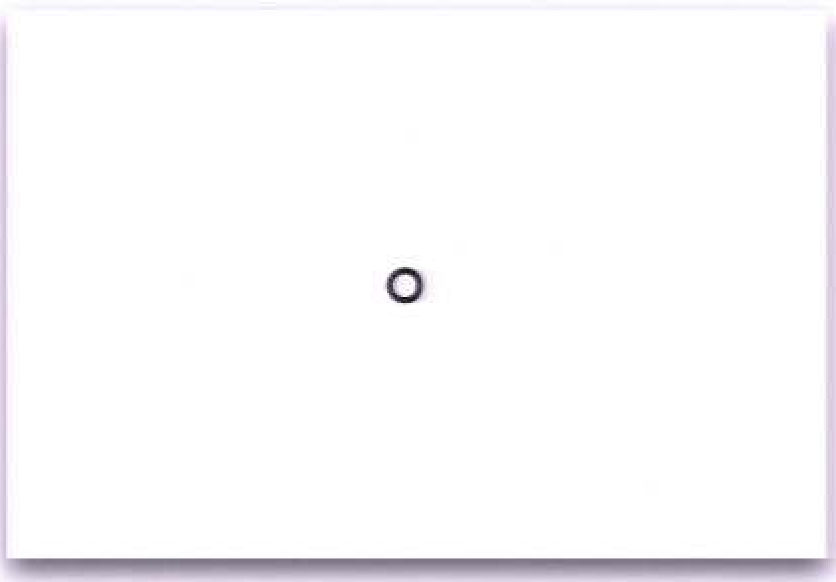
Contents

Seiberg-Witten theory



- **SU(2)** gauge theory, an adjoint complex scalar ϕ
- SW curve parametrized by the vev $u = \langle \mathbf{tr}\, \phi^2 \rangle$
- Electron mass = $\int_A \lambda_{SW}$, Monopole mass = $\int_B \lambda_{SW}$

pure $\mathcal{N} = 2$ $SU(2)$: classical and quantum



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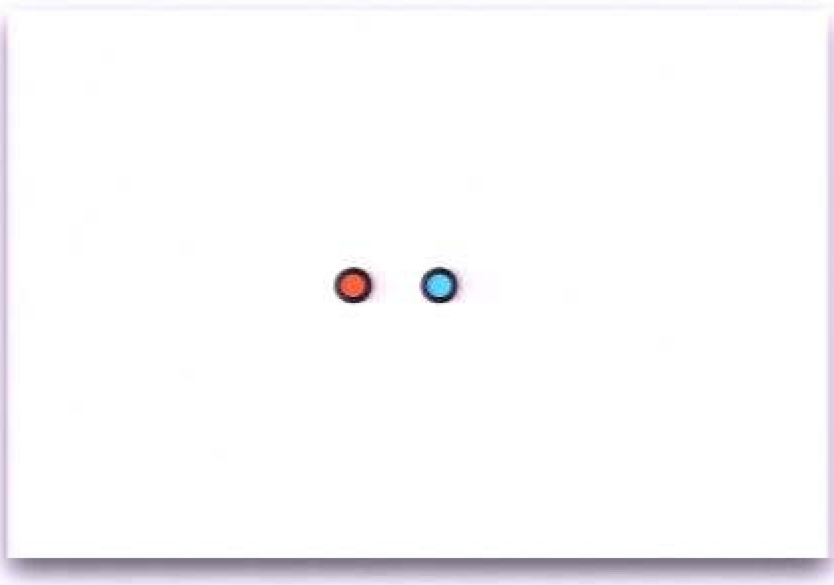

- Enhanced $SU(2)$ symmetry at the origin $u = 0$ →
- Monopole point $u = \Lambda^2$
- Dyon point $u = -\Lambda^2$

$N_f = 1$

- $m \gg \Lambda \longrightarrow u \sim m^2/4, \quad u \sim \pm 2(m\Lambda^3)^{1/2}$

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- $m = 3\Lambda \longrightarrow u = 3\Lambda^2$ (double), $\qquad u = -15\Lambda^2/4$

Strongly coupled $\mathcal{N} = 2$ SCFT

- Electron & Monopole both massless at $u = 0$

  → likely to be a conformal theory

  [Argyres-Douglas, Argyres-Plesser-Seiberg-Witten, Eguchi-Hori-Ito-Yang]
- Suppose in a conformal theory there is an operator $F_{\mu\nu}$
- Suppose it has non-zero anomalous dimension.

$$\longrightarrow \partial_\mu F^{\mu\nu} \neq 0, \qquad \partial_\mu \tilde{F}^{\mu\nu} \neq 0$$

→ There should be both electric & magnetic sources.

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Argyres-Douglas point

- SW curve close to the AD point:

$$y^2 = \tilde{x}^3 + \tilde{m}\tilde{x} - \tilde{u}$$

- $D(\tilde{x}) : D(\tilde{y}) : D(\tilde{m}) : D(\tilde{u}) = 2 : 3 : 4 : 6$
- $D(\int_A \lambda_{SW}) = 1$, $\lambda_{SW} = \dfrac{\tilde{u}d\tilde{x}}{y}$

  $\longrightarrow D(\tilde{u}) = 6/5, D(\tilde{m}) = 4/5.$
- Recall $u = \mathrm{tr}\,\phi^2$, so $D_{UV}(u) = 2$. Strongly coupled !
- That's all what was known about $\mathcal{N} = 2$ SCFT before Nov 2007.

a and c of AD points

- a and c measure the response of the CFT to the external gravity

$$\langle T^{\mu}_{\mu} \rangle = a \times \text{Euler} + c \times \text{Weyl}^2$$

- the best way to couple $\mathcal{N} = 2$ supersymmetric theory to gravity = topological twisting.
- Are a and c encoded in the topological theory ?
  Yes ! in the so-called $A^{\chi} B^{\sigma}$ term which is known for 10 yrs by [Witten, Moore, Mariño, Losev, Nekrasov, Shatashvili]

$A^{\chi} B^{\sigma}$ term

Just as nontrivial $\tau(u) F \wedge F$ is generated, on a curved manifold

$$S_{\text{curved}} = [\log A(u)] \epsilon_{abcd} R^{ab} \wedge R^{cd} + [\log B(u)] R^{a}{}_{b} \wedge R^{b}{}_{a} + \cdots$$

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$\tau(u)$ was determined from

- Holomorphy
- Semiclassical behavior
- Behavior around the singular point in the moduli

The same works for $A^{\chi}B^{\sigma}$.

Determination of $A^\chi B^\sigma$

$$A(u)^2 = \det \frac{\partial u_i}{\partial a_I} \qquad B(u)^8 = \Delta$$

- $u_i = \mathrm{tr}\,\phi^i$: gauge-invariant coordinates
- a^I: special coordinates i.e. masses of BPS particles
- Δ: the discriminant of the SW curve

[Witten,Moore,Mariño,Nekrasov,Losev,Shatashivili]

$A^{\chi}B^{\sigma}$ and a, c

Al and I showed

$$a = \frac{1}{4}R(A) + \frac{1}{6}R(B) + \frac{5}{24}r, \qquad c = \frac{1}{3}R(B) + \frac{1}{6}r.$$

where

- r is the number of free vector multiplet
- $R(A)$ is the R-charge of A, etc.

A and B have been calculated .

→ taking their R-charges, we get a and c.

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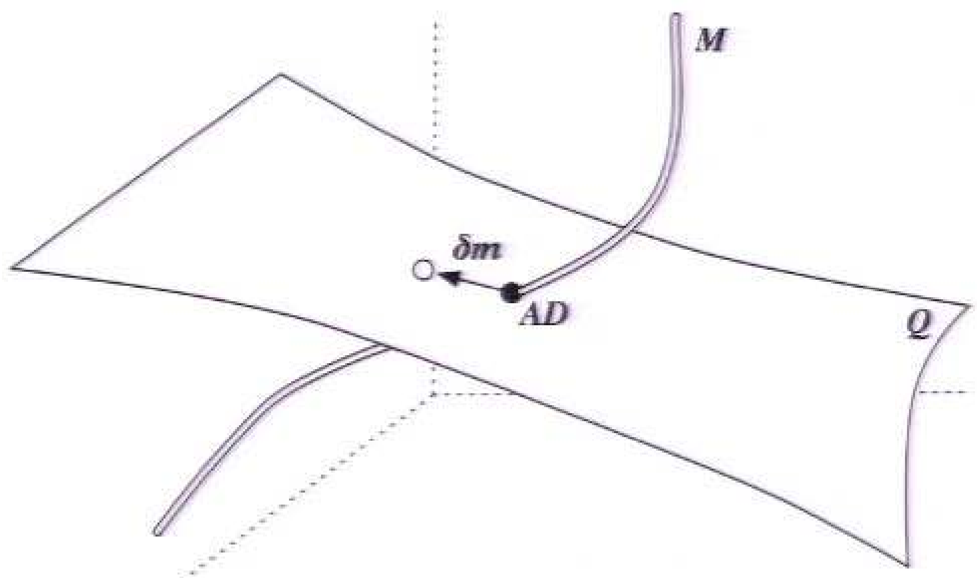
Argyres-Douglas points

- Consider $\mathbf{U}(N_c)$ with N_f quarks of bare mass m
- There's a subspace where $\mathbf{U}(N_c - 1)$ with N_f massless quarks is realized semiclassically: $\phi = \mathbf{diag}(m, 0, 0, \ldots, 0)$
- A monopole locus intersects at the strongly-coupled region



Argyres-Douglas points

$\mathbf{U}(N+1)$ gauge theory with $N_f = 2N$ flavors :

Curve: $y^2 = x^{2N+1}(x+2\Lambda)$ with $\lambda_{SW} \sim x d(y/P) \sim x^{1-N} dy.$

$$\longrightarrow \qquad D(x) = \frac{2}{3}, \qquad D(u_j) = jD(x).$$

Therefore
$$\begin{cases} A^2 = \det \dfrac{\partial u_i}{\partial a^I} & \sim x^{N^2/2}, \\ B^8 = \Delta = \prod (e_i - e_j)^2 & \sim x^{2N(2N+1)}. \end{cases}$$

Use our formula

$$a = \frac{1}{4}R(A) + \frac{1}{6}R(B) + \frac{5}{24}r, \qquad c = \frac{1}{3}R(B) + \frac{1}{6}r.$$

and get

$$a = \frac{14N^2 + 19N}{72}, \qquad c = \frac{4N^2 + 5N}{18}.$$

A counterexample to the a-theorem

Flow associated to Higgsing:

$$\textbf{SU}(N+1) \text{ with } \mathbf{2}N \text{ flavors at UV:} \quad a \sim \tfrac{7}{24} N^2$$

$$\textbf{SU}(N+1) \text{ with } \mathbf{2}N \text{ flavors at IR:} \quad a \sim \tfrac{7}{36} N^2$$

$$\textbf{SU}(N) \text{ with } \mathbf{2}N \text{ flavors:} \quad a \sim \tfrac{7}{24} N^2$$

One way to understand the factor **2/3** : our formula implies

$$4(2a - c) = \sum_j [2D(u_j) - 1].$$

o..iginally conjectured by Aharony and Argyres.

$$D_{AD}(u_j) = jD_{AD}(x) = \frac{2}{3}j, \qquad D_{IR}(u_j) = jD_{IR}(x) = j.$$

Compatibility to the previous results

a-maximization-based proofs

Our R symmetry is completely accidental,
't Hooft anomaly matching unusable

AdS/CFT-based proofs

Our theories can not have weakly-curved AdS duals,
because $a/c \to 7/8$ in the large N limit.

Possible objections...

- Is our method correct? → Yes I believe so.
- **USp(2N)** + N_f flavors + 1 antisymmetric can also be analyzed holographically. [Aharony-YT]
- This gives the same result, including $1/N$ corrections.

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- Is the direction of the flow correct?
  → Yes I believe so. More in our paper...

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- I would be happier if I can calculate η/s and s/T^3 of this thing.

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