Title: Asymptotic Expansion of the N=4 Dyon Degeneracy

Date: Dec 11, 2008 04:40 PM

URL: http://pirsa.org/08120053

Abstract: We study various aspects of power suppressed as well as exponentially suppressed corrections in the asymptotic expansion of the degeneracy of quarter BPS dyons in N=4 supersymmetric string theories. In particular we explicitly calculate the power suppressed corrections up to second order and the first exponentially suppressed corrections. We also propose a macroscopic origin of the exponentially suppressed corrections using the quantum entropy function formalism. This suggests a universal pattern of exponentially suppressed corrections to all four dimensional extremal black hole entropies in string theory.

Asymptotic Expansion of $\mathcal{N} = 4$ Dyon Degeneracy

Nabamita Banerjee

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References:
(1) arXiv:0807.1314 [hep-th] [NB]
(2) arXiv:0810.3472 [hep-th] [NB, D. Jatkar, A. Sen]

YRC, Perimeter Institute

Plan of the talk

1. Introduction
2. Setup
3. Macroscopic Understanding
4. Result

Motivation

- Black Holes are solutions of Einstein-Maxwell theory (low energy limit of string theory). They carry certain charges and quantum mechanically behave as thermodynamic objects.
- They can also be described in terms of specific configuration of states in the full string theory, carrying similar set of charges.

- We want to understand the statistical origin of Black Hole entropy , as the logarithm of the degeneracy of these states.

- Major success: for extremal Black Holes,

$$S_{BH} = S_{stat} \equiv \ln d(\vec{Q}), \quad \textit{in large charge}(\vec{Q}) \textit{ limit}.$$

- Can we go beyond large charge limit?

$$i.e. \qquad S_{BH} \stackrel{?}{=} \ln d(\vec{Q}),$$

for large but finite charges.

To do this, we need to take two way approach to the problem.

Macroscopic Side

- On gravity side, we need to consider all α' corrections and stringloop corrections.
  - α' corrections: Entropy function technology can be used.
  - stringloop corrections: Quantum entropy function can be used.
- Entropy corrections come as an expansion in inverse power of charges.

Microscopic Side

- One needs to compute the degeneracy of states more accurately.

GOAL

To understand these corrections to the entropy in the statistical side by doing systematic asymptotic expansion of the degeneracy function.

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Setup

Theory

- We consider $\mathcal{N} = 4$ superstring theory with rank r gauge group.
- At a generic point in the moduli space,the unbroken gauge group is $U(1)^r$.
- The low energy SUGRA theory has a continuous $SO(6, r-6) \times SL(2, R)$ symmetry.
- We denote the $SO(6, r-6)$ invariant metric by L. All inner products are defined with respect to L.

Two Descriptions

First Description

- Type IIB string theory on $K3 \times S^1 \times \tilde{S}^1 / \mathbb{Z}_N$.
- A dyonic state in this theory is a particular brane configuration.

Second Description

- Equivalently heterotic string theory on $T^4 \times S^1 \times \hat{S}^1 / \mathbb{Z}_N$
- A dyonic state in this theory is described by a state carrying some electric and magnetic charges.

Duality

- The two descriptions of the theory are related by a chain of duality transformations as follows:

$$\begin{pmatrix} \text{IIB} \\ S^1 \times S^1 \end{pmatrix} \overset{S}{=} \begin{pmatrix} \text{IIB} \\ S^1 \times S^1 \end{pmatrix} \overset{T}{=} \begin{pmatrix} \text{IIA} \\ S^1 \times S^1 \end{pmatrix} \overset{st-st}{=} \begin{pmatrix} \text{Heterotic} \\ T^6 \end{pmatrix}$$

Charge Vectors

- In general, any given state is characterized by r dimensional electric and magnetic charge vectors, $\vec{Q}$ and $\vec{P}$.
- The T-duality invariants are,

$$Q^2 = Q^T L Q \quad P^2 = P^T L P \quad Q.P = Q^T L P,$$

- The discreate T-duality invariant (gcd of $(Q \wedge P)$) is set to 1.

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Macroscopic Side

- We will consider quarter BPS dyonic Black Holes in the Heterotic theory.
- Restricting to Supergravity approximation, we can find the leading entropy carried by these Black Holes .

Microscopic Side

- By duality, we can also regard the states associated with this Black Holes as states of some particular quarter BPS D-brane configuration in the type IIB theory.
- Considering the dynamics of various fields in the D-brane configuration, the complete degeneracy function has been evaluated (J. David, A. Sen; NB, D. Jatkar, A. Sen).

Degeneracy Formula

- The microscopic degeneracy is,

$$d(\vec{Q},\vec{P}) = (-1)^{Q\cdot P+1} A \int_{\mathcal{C}} d\rho\, d\sigma\, dv\, \frac{e^{-\pi i(\rho Q^2+\sigma P^2+2vQ\cdot P)}}{\Phi(\rho,\sigma,v)}$$

- Contour $\mathcal{C}$ is a three real dimensional subspace of the complex dimensional space labeled by (ρ, σ, v).

- For $N = 1$ theory, the function $\Phi(\rho, \sigma, v)$ is a modular form of weight 10.
- The analogous modular forms are also known for many other models .

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Asymptotic Expansion

A .

- For a given set of charges, there are single centered and multi centered Black Hole solutions.
- We are interested in single centered Black Hole entropy.
- We organize the integral such that the result can pick up the contribution from single centered Black holes. This is done by choosing the integration contour $\mathcal{C}$ in a specific way.
- In particular, we need to set the asymptotic values of the moduli fields equal to their attractor values.

B .

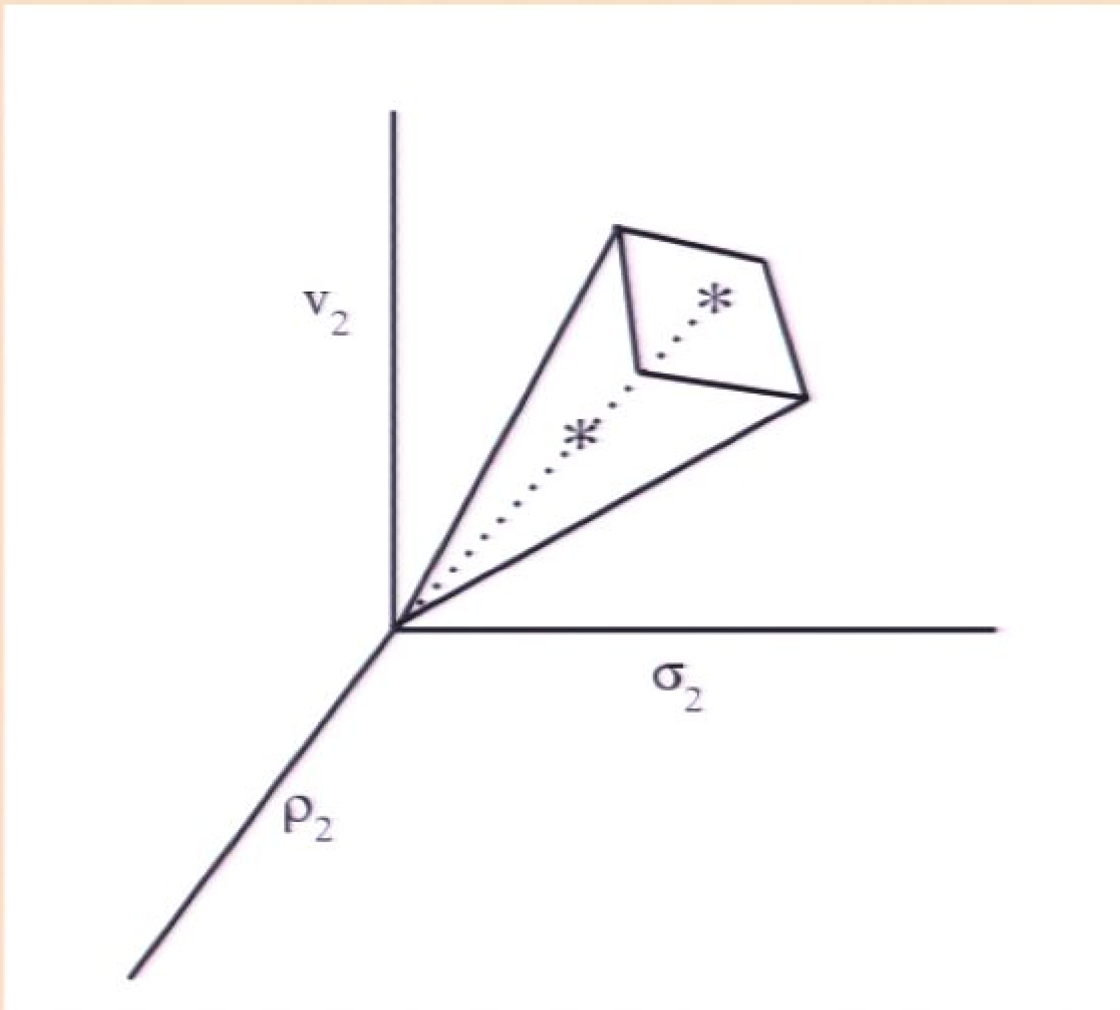
- We have to do three integrals, over (ρ, σ, v). For this,we need the pole of the integrand.
- The function $\Phi(\rho, \sigma, v)$ has a second order zero at,

$$n_2(\sigma\rho - v^2) + jv + n_1\sigma - m_1\rho + m_2 = 0$$

for
$m_1, n_1, m_2, n_2 \in \mathbb{Z}, j \in 2\,\mathbb{Z} + 1, \quad m_1 n_1 + m_2 n_2 + \frac{j^2}{4} = \frac{1}{4}$

- We consider cases with $n_2 \geq 1$.

Pole structure



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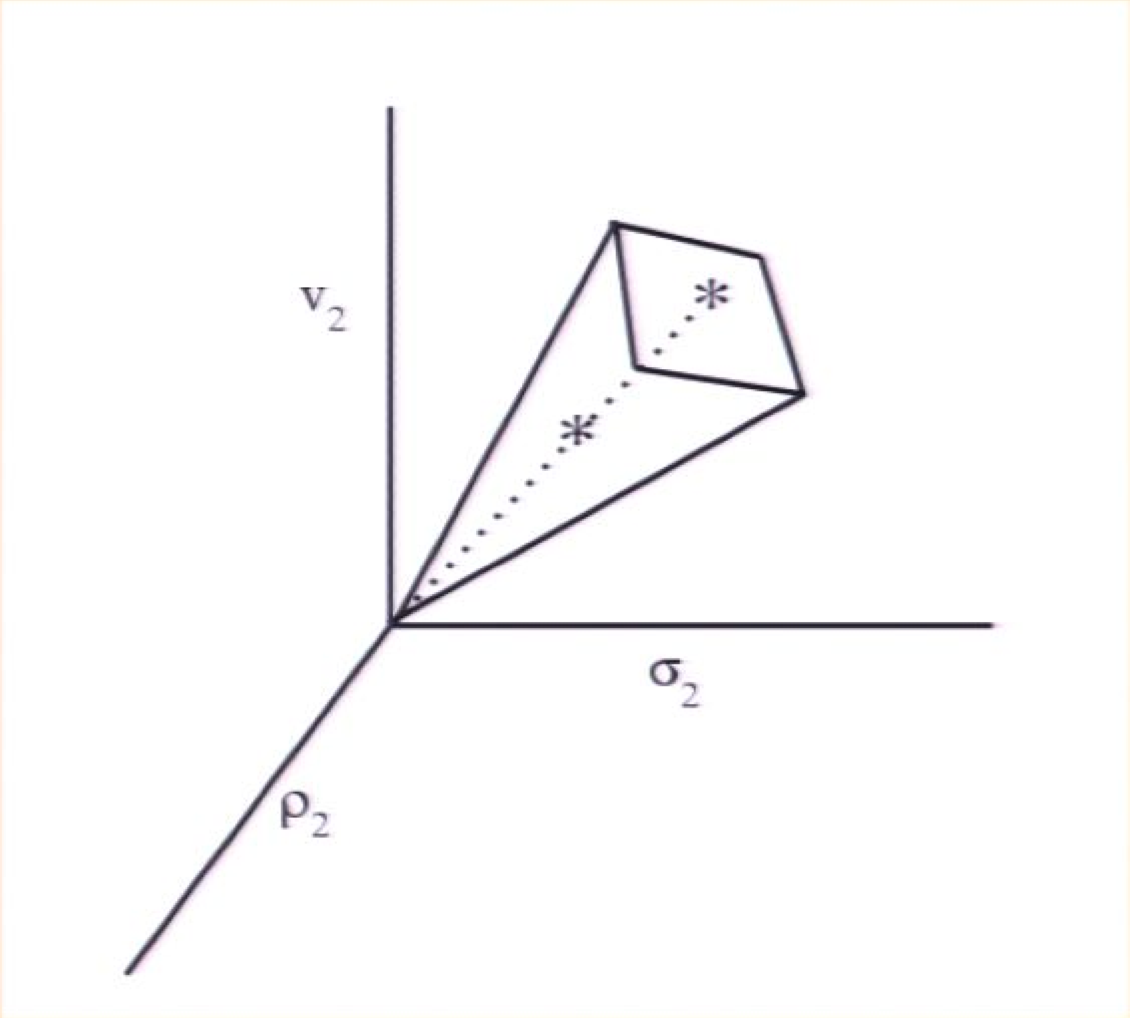

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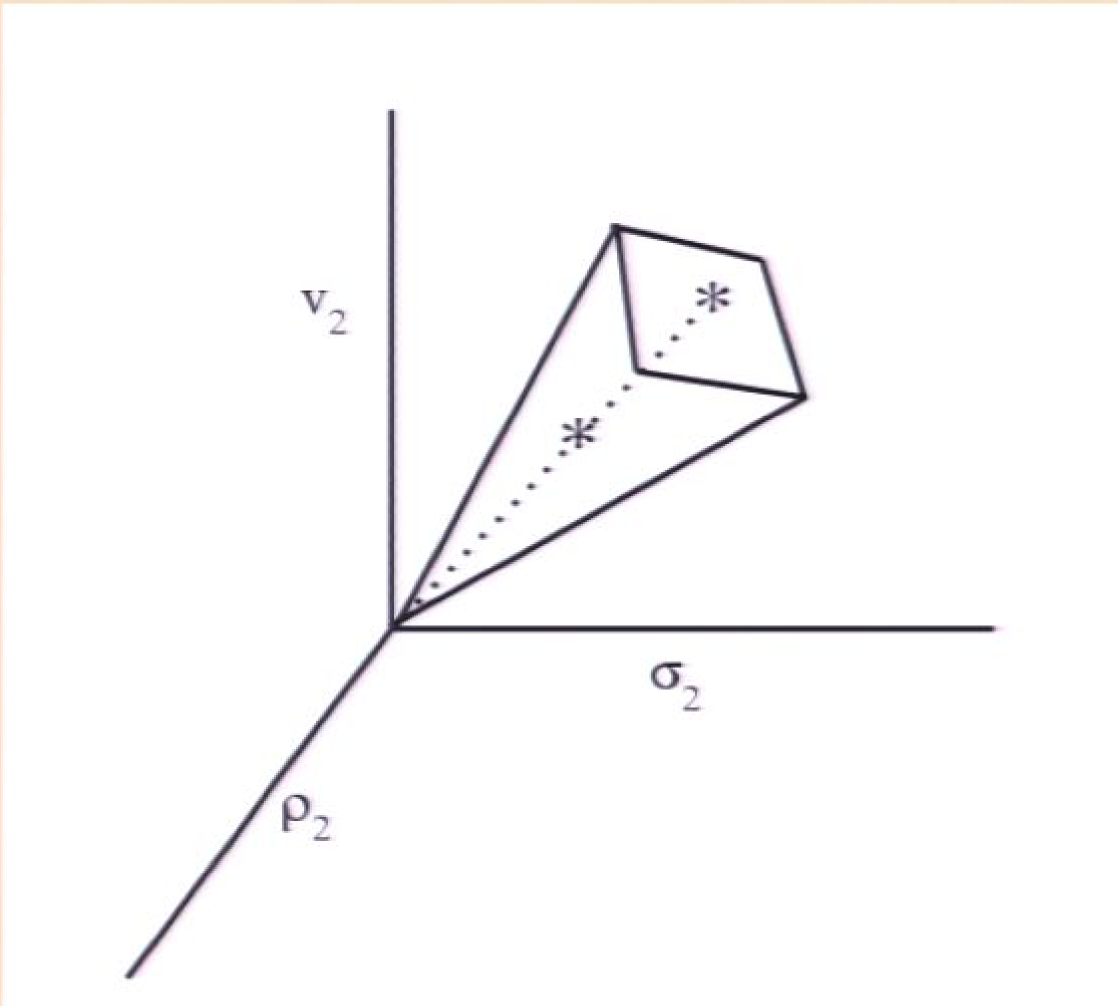

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B.1

- In saddle point approximation of the integral, the degeneracy is,

$$d(\vec{Q}, \vec{P}) = e^{\pi\sqrt{Q^2P^2-(Q.P)^2}/n_2}$$

- For $n_2 = 1$, we get the maximum contribution to the degeneracy.

- For $n_2 \geq 2$, the degeneracy is exponentially suppressed compared to the leading one. Hence, to compute exponentially suppressed contribution, we need to look at these sub-leading poles.

B.2

- For the leading pole, one integral can be done by residue method. The other two integral are done by saddle point analysis.
- The v integral is done by residue method. Near the pole, the function Φ behaves as.

$$\Phi(\rho, \sigma, v) \rightarrow v^2 g(\rho) g(\sigma) + O(v^4)$$

- The (ρ, σ) integral takes the form,

$$e^{S_{stat}(\vec{Q},\vec{P})} = d(\vec{Q},\vec{P}) \sim \int \frac{d^2\tau}{\tau_2^2} e^{F(\vec{\tau})}$$

where, $\rho = \tau_1 + i\tau_2$ and $\sigma = -\tau_1 + i\tau_2$

- The function $F(\tau)$ can be easily computed after doing the v integral.

- This can be regarded as a zero dimensional field theory with fields (τ_1, τ_2) with action $F(\vec{\tau}) - 2\ln\tau_2$.

- The result for statistical entropy S_{stat} can be obtained by computing the possible diagrams of this field theory up to any desired order in charges.

- This will produce all sub-leading correction which are power suppressed in charges.

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Up to $1/charge^2$ corrections

Statistical Entropy Function

- Leading result:

$$\Gamma_0(\vec{\tau}_B) = -\frac{\pi}{2\tau_{B_2}}|Q - \tau_B P|^2$$

- The $charge^0$ correction:

$$\Gamma_1(\vec{\tau}_B) = \ln g(\tau_B) + \ln g(-\bar{\tau}_B) + (k+2)\ln(2\tau_{B_2})$$

The $1/charge^2$ correction :

$$\Gamma_2(\vec{\tau}_B) = \ln d_2(\vec{Q}, \vec{P}) = -\frac{\tau_{2B}}{\pi|Q - \tau_B P|^2}(k+2)$$

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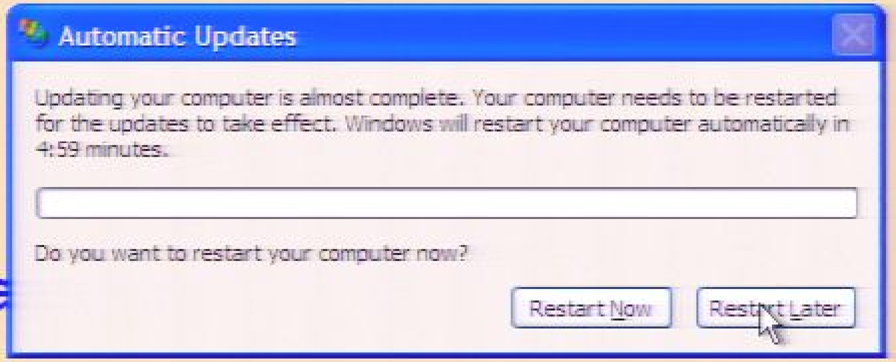

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- Leading entropy: $S^{(0)} = \pi\sqrt{Q^2P^2 - (Q.P)^2}$
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$$S^{(1)} = -\ln g(\tau_{(0)}) - \ln g(-\bar{\tau}_{(0)}) - (k+2)\ln(2\tau_{(0)_2})$$

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$$S^{(2)} = \frac{2+k}{2\pi\sqrt{Q^2P^2 - (Q\cdot P)^2}} + \frac{4\tau_{(0)2}^3}{\pi|Q - \tau_{(0)}P|^2}$$

$$\left[\left(\frac{g'(\tau_{(0)})}{g(\tau_{(0)})} + \frac{k+2}{\tau_{(0)} - \bar{\tau}_{(0)}}\right)\left(\frac{g'(-\bar{\tau}_{(0)})}{g(-\bar{\tau}_{(0)})} + \frac{k+2}{\tau_{(0)} - \bar{\tau}_{(0)}}\right)\right]$$

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BMPV Black Holes

Microscopic Configuration

- It involves a D1-D5 system carrying crtain momenta and angular momenta in flat space-time.

Results [NB]

- We derive the higher derivative corrections to classical entropy.
- the results are valid even out of Farey tail limit.
- The leading walls of marginal stability is absent in this case.

Up to $1/charge^2$ corrections

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Exponential Suppressed Correction

- The zeros of the function Φ,

$$n_2(\sigma\rho - v^2) + jv + n_1\sigma - m_1\rho + m_2 = 0$$

- We get such corrections from the sub-leading poles, $n_2 \geq 2$.

- For this we define

$$\Omega = \begin{bmatrix} \rho & v \\ v & \sigma \end{bmatrix}$$

- We look for a symplectic transformation of the form:

$$\begin{bmatrix} \hat{\rho} & \hat{v} \\ \hat{v} & \hat{\sigma} \end{bmatrix} \equiv \hat{\Omega} = (A\,\Omega + B)(C\,\Omega + D)^{-1},$$

such that

$$\hat{v} = \frac{n_2(\sigma\rho - v^2) + jv + n_1\sigma - m_1\rho + m_2}{\det(C\Omega + D)}$$

- The behavior of Φ_k near the zero is,

$$\Phi_k(\rho, \sigma, v) = -\{\det(C\,\Omega + D)\}^{-k}\, 4\pi^2\, \hat{v}^2\, g(\hat{\rho})\, g(\hat{\sigma}) + O(v^4)$$

Degeneracy Formula

- The degeneracy formula for any sub-leading pole is,

$$\frac{\exp\left(\pi\sqrt{Q^2P^2-(Q\cdot P)^2}/n_2\right)}{n_2}\left[\det(C\Omega+D)^{k+2}\, g(\rho)^{-1}\, g(\sigma)^{-1}\right]_{saddle}$$
$$(-1)^{Q\cdot P}\exp\left[i\pi(n_1P^2-m_1Q^2+jQ\cdot P)/n_2\right]$$

- To evaluate $\det(C\Omega+D)^{k+2}g(\rho)^{-1}\,g(\sigma)^{-1}$, we actually need the transformation matrix A, B, C, D.

- We will now compare the entropy with the leading pole results.

Comparison

Q^2, P^2	$Q.P$	$d(Q,P)$	S_{stat}	$S_{stat}^{(0)}$	$+S_{stat}^{(1)}$	$+S_{stat}^{(2)}$	Δd
2	0	5×10^4	10.82	6.28	10.62	11.58	34.6
4	0	3×10^7	17.31	12.57	16.90	17.38	480.6
6	0	1×10^{10}	23.51	18.85	23.19	23.51	18573
6	3	9×10^8	20.64	16.32	20.41	20.77	0
6	-3	2×10^9	21.78	16.32	20.41	20.77	-

Comparison

Q^2, P^2	$Q.P$	[illegible]	[illegible]	[illegible]	$S^{(1)}_{stat}$	$+S^{(2)}_{stat}$	Δd
2	0	[illegible]	[illegible]	[illegible]	[illegible]62	11.58	34.6
4	0	[illegible]	[illegible]	[illegible]	[illegible]90	17.38	480.6
6	0	[illegible]	[illegible]	[illegible]	[illegible]19	23.51	18573
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Comparison

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Send Error Report | Don't Send

Wireless Network Connection is now connected

Connected to: GuestPass
Signal Strength: Excellent

Comparison

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4	0	3×10^7	17.31	12.57	16.90	17.38	480.6
6	0	1×10^{10}	23.51	18.85	23.19	23.51	18573
6	3	9×10^8	20.64	16.32	20.41	20.77	0
6	-3	2×10^9	21.78	16.32	20.41	20.77	-

Comparison

Q^2, P^2	$Q.P$	$d(Q,P)$	S_{stat}	$S^{(0)}_{stat}$	$+S^{(1)}_{stat}$	$+S^{(2)}_{stat}$	Δd
2	0	[illegible]	[illegible]	[illegible]	[illegible]	11.58	34.6
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Macroscopic Understanding

Power suppressed correction

- Power suppressed corrections are identified to the α' / g_{string} correction to Black Hole macroscopic entropy.
- This Entropy function is just the value of the corresponding six derivative term in the Black Hole action computed on the $AdS_2 \times S^2$ background.
- We do not know a candidate for this term in the action. Our Analysis tells us that the term has to be duality invariant and puts a strong constraint to the possible terms.
- We have also been able to eliminate terms like R^3 as they gives zero result.

Exponentially Suppressed Correction

- These corrections naturally come from Quantum Entropy Function.

Quantum Entropy Function

- This is a proposal for computing the exact degeneracy of states of an extremal Black Holes (A. Sen).
- These Black Holes have the following near horizon geometry,

$$ds^2 = v\left((r^2-1)d\theta^2 + \frac{dr^2}{r^2-1}\right), \quad F_{rt}^{(i)} = -ie_i, \quad \cdots$$

v, e_i are constants, ... denotes near horizon values for other fields.

- The degeneracy is given as,

$$d(\vec{q}) = \left\langle \exp[-iq_i \oint d\theta \, A_\theta^{(i)}] \right\rangle_{AdS_2}^{finite}$$

  - where $\langle \ \rangle_{AdS_2}$ denotes the unnormalized path integral over various fields of string theory on euclidean global AdS_2.
  - The superscript '*finite*' refers to the finite part of the amplitude. To get this, we need to put a cut of to regularize the AdS volume.

- Explicit computation shows that this proposal reproduces the right classical degeneracy of states for quarter BPS Black Holes in $\mathcal{N} = 4$ theories as,

$$d(q) \simeq \exp\left(\pi\sqrt{Q^2P^2 - (Q\cdot P)^2}\right).$$

Possible Quantum Corrections

- There are two sources of quantum corrections.
  - From fluctuations of the string fields around AdS_2 background.
  - There can be different classical solutions with similar asymptotic configuration.

Fluctuation of the Background

- The degeneracy is actually given as the finite part of the amplitude in the AdS_2 background, and hence can only get power law corrections from fluctuation modes.

Different Solutions

- The different solution can come with a different action, and hence we can get a different exponential factor.
- Q. Can we identify such a different solution?

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Solution

- We consider a $\mathbb{Z}_N$ quotient of the previous background by,

$$\theta \to \theta + \frac{2\pi}{N}, \quad \phi \to \phi - \frac{2\pi}{N}$$

- The new solution looks like,

$$ds^2 = v\left((\tilde{r}^2 - 1)d\tilde{\theta}^2 + \frac{d\tilde{r}^2}{\tilde{r}^2 - 1}\right), \quad F^{(i)}_{\tilde{r}\tilde{\theta}} = -i\, e_i, \quad \cdots,$$

$$\tilde{\theta} \equiv \tilde{\theta} + \frac{2\pi}{N}$$

- In a new coordinate $r = \tilde{r}/N, \quad \theta = N\tilde{\theta}$, the solution looks as,

$$ds^2 = v\left((r^2 - N^{-2})d\theta^2 + \frac{dr^2}{r^2 - N^{-2}}\right), \quad F^{(i)}_{r\theta} = -i\, e_i, \quad \cdots$$

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- This has the same asymptotic behavior as the original solution.

- The finite contribution to the quantum entropy function is,

$$d(\vec{Q}, \vec{P}) = \exp\left(\pi\sqrt{Q^2P^2 - (Q\cdot P)^2}/N\right)$$

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Automatic Updates

Updating your computer is almost complete. Your computer needs to be restarted for the updates to take effect. Windows will restart your computer automatically in 4:58 minutes.

Do you want to restart your computer now?

Restart Now | Restart Later

- This has th... the original solution.

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Results

- We have shown that the degeneracy formula is valid for a generic quarter BPS dyonic Black Hole state in $\mathcal{N} = 4$ theory.
- We have explored possible power suppressed and exponentially suppressed corrections to the microscopic degeneracy formula.
- We have also identified the roots of these corrections in the Black Hole macroscopic entropy.
- The exponentially suppressed corrections to the black hole entropy is universal, does not depend on the particular kind of extremal Black Holes.

THANK YOU