Title: Constraint free canonical GR using characteristic data

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Abstract: The handling of the constraints on initial data is a major issue in most canonical formulations of general relativity. Since the 1960s unconstrained initial data for GR that living on null hypersurfaces has been known, but no canoncial formulation based on these data was developed due to conceptual and technical difficulties. I will explain how these dificulties have been overcome and outline the resulting canonical framework. I will also explain how this might be the ideal setting to attempt a proof of the Bousso entropy bound, or to incorporate the associated holographic principle in a quantization of gravity.

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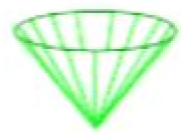
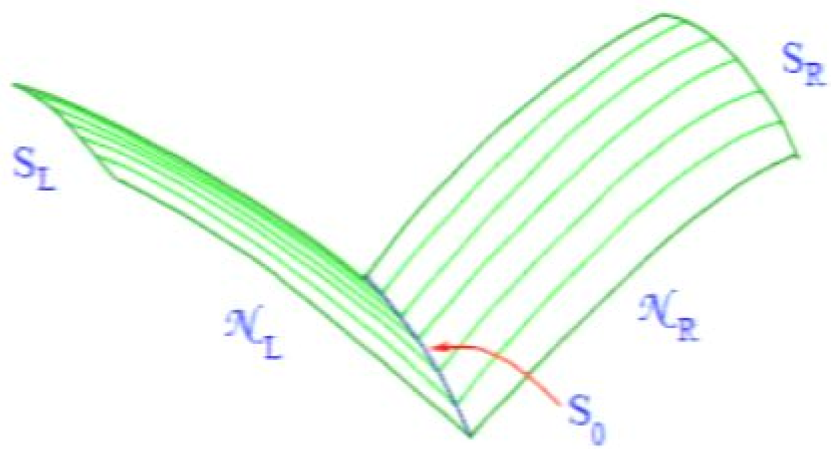
PLAN OF TALK

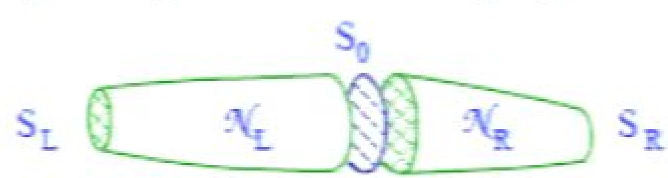
- What is it?
- Why study this?
- Why it was not done sooner
- The free characteristic initial data
- The Poisson brackets
- Some comments on quantization

What is canonical GR using characteristic data?

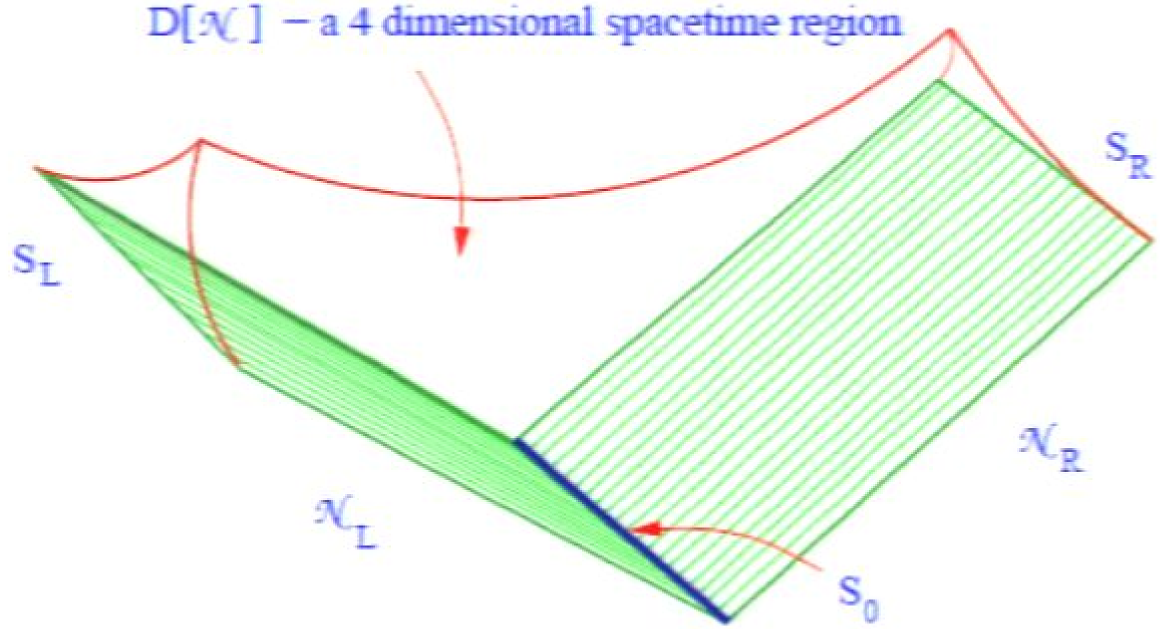
- It is canonical GR using initial data on a piecewise null hypersurface
  - could be a future light cone 
  - we will use a pair of intersecting null hypersurfaces (or "lightfronts") - like an open book in spacetime.



  - $\mathcal{N}_R, \mathcal{N}_L$ are 3-surfaces swept out by null geodesics emerging normally from the two sides of 2-disk S_0.

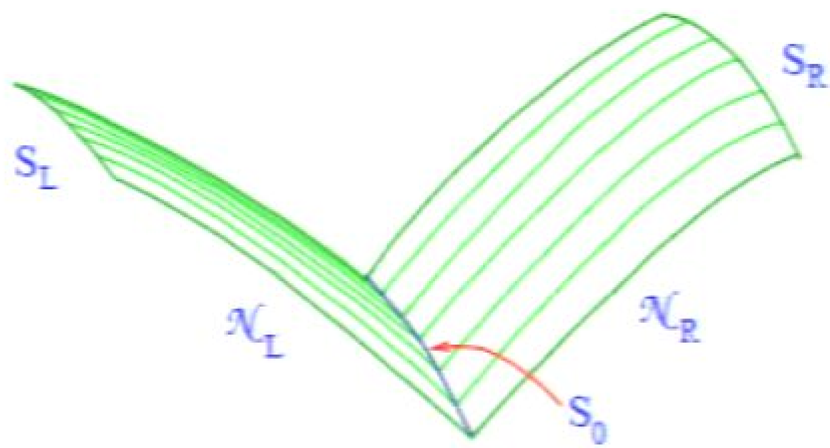


- initial data on $\mathcal{N} = \mathcal{N}_L \cup \mathcal{N}_R$ specifies solution in domain of dependence $D[\mathcal{N}]$



WHY STUDY THIS?

- No constraints -can identify free, complete data ($\sim$ 1962 Sachs, Bondi, van der Burg, Metzner, Penrose, Dautcourt)
- Lorentzian
- Observables - main free initial data has direct interpretation in terms of test lightrays $\rightarrow$ allow formulation of observables
- Time evolution conceptually straightforward



- Holography Beckenstein - 't Hooft Susskind - Bousso bound: If generators of a branch ($\mathcal{N}_R$ say) are non-expanding at S_0 then they argue

$$\text{Entropy on } \mathcal{N}_R \leq \frac{\text{Area}[S_0]}{4\text{A}_{Planck}}$$

with saturation possible.

- Normally the highest entropy thermodynamic macrostate of a system has essentially *all* microstates. This suggests

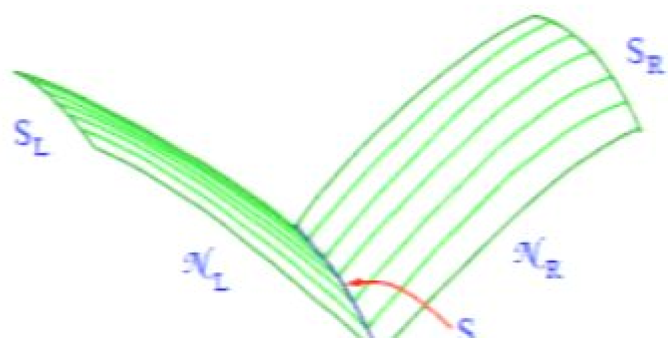
$$dim H_{\mathcal{N}_R} = e^{\frac{A[S_0]}{4A_{Planck}}}$$

or

$$dim H_{\mathcal{N}} = e^{\frac{A[S_0]}{2A_{Planck}}}$$

with $H_{\mathcal{N}}$ the Hilbert space of vacuum GR in $D[\mathcal{N}]$.

- Canonical GR on $\mathcal{N}$ seems ideal framework to check this.



Microsoft Windows xp Professional

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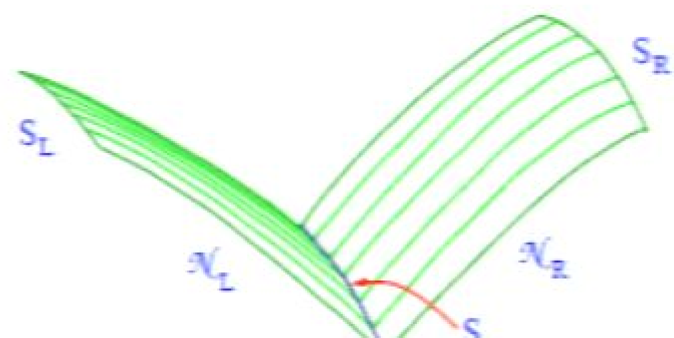
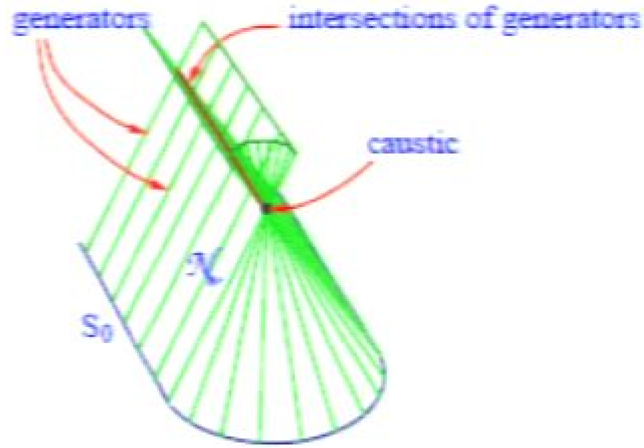


Why it was not done sooner

- Focusing of generators: Once generators cross they enter $D[\mathcal{N}]$ so data on them not independent of other data. Was not clear how to control crossings by conditions on data. And such conditions would also spoil simplicity of free data.



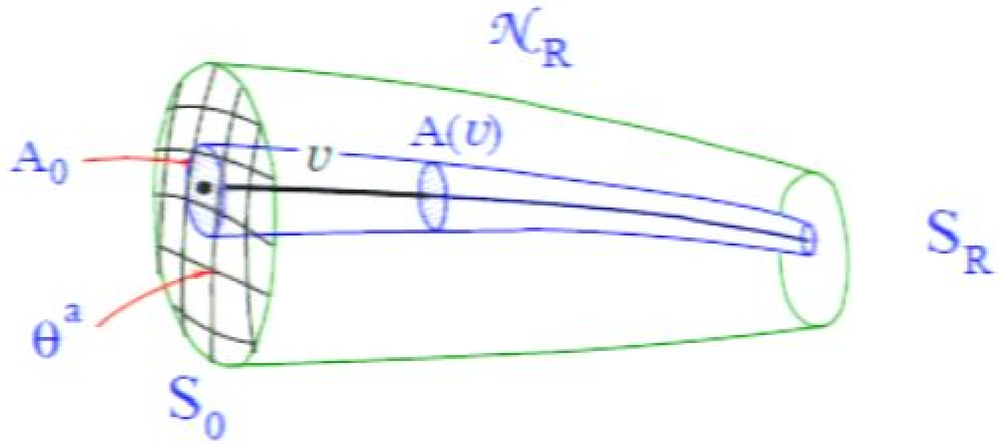
  - Solution: Caustics easily excluded from data. Once caustics gone can "unidentify" other crossings - Pull back metric to normal bundle of S_0. Locally isometric spacetime with no generator crossings. Then eliminating crossings requires no restrictions on data - All data correspond to a soln of Einstein's eqns.

- Finding Poisson brackets of data hard work.

  - Solution: Work hard!

THE FREE INITIAL DATA

- Coordinates adapted to $\mathcal{N}$



- θ^1, θ^2 coordinates on S_0. Held constant on generators.

- v parameter along each generator.

  Definition: Cross sectional area of infinitesimal bundle of neighboring generators

$$A(v) = A_0 v^2$$

A_0 is the cross sectional area at S_0.

- "Bulk" data lives on the 3-manifolds $\mathcal{N}_L$ and $\mathcal{N}_R$. Additional data lives on S_0.

- Bulk data = conformal 2-metric $e_{ab}(\theta^1, \theta^2, v)$

  - Metric on $\mathcal{N}_R$ ($\mathcal{N}_L$) degenerate because $\mathcal{N}_R$ ($\mathcal{N}_L$) is null, so

$$ds^2 = h_{ab} d\theta^a d\theta^b$$

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  - Definition:

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- Data on S_0:
  - $\rho_0 = \sqrt{det\, h_{ab}}$ = area density on S_0.
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The Poisson brackets

Brackets of e_{ab}:[2]

Parametrize e_{ab} by a complex scalar μ

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\begin{aligned}
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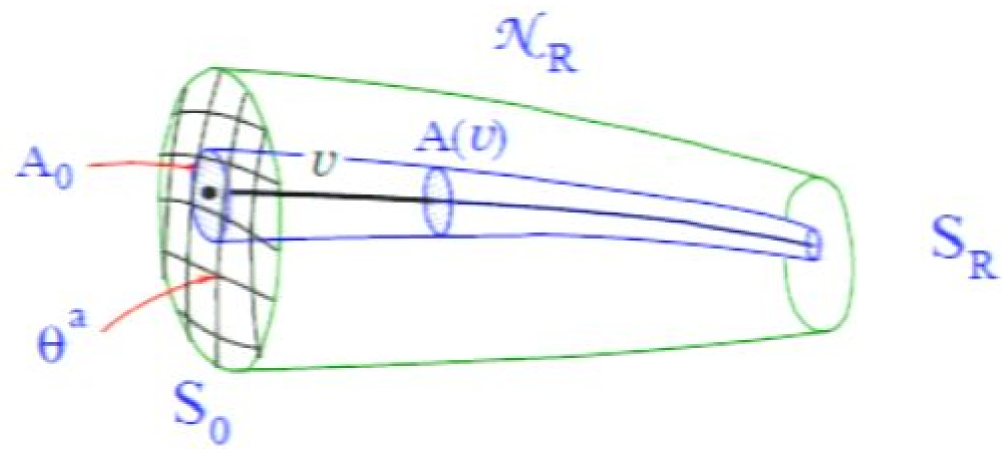

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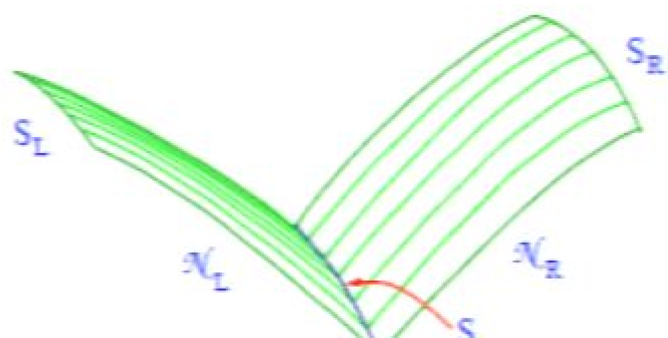

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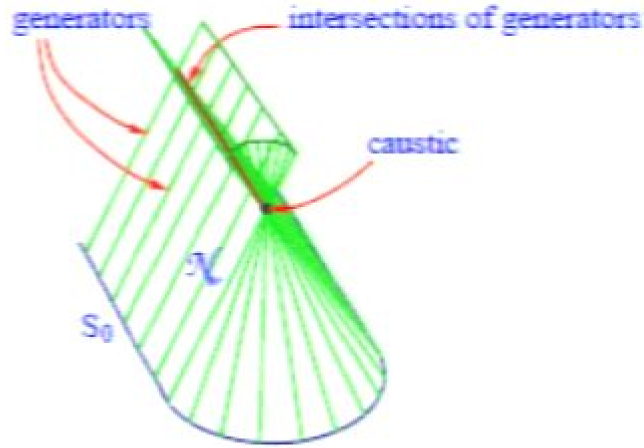

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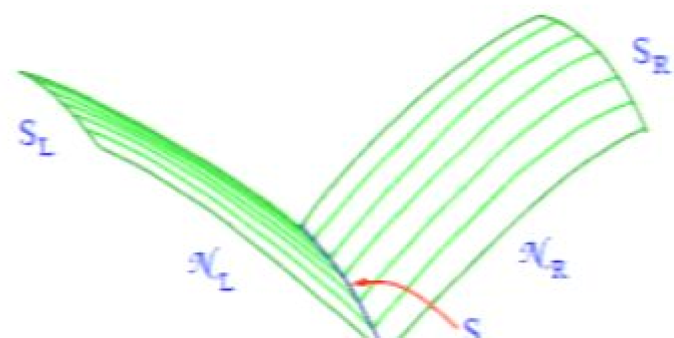

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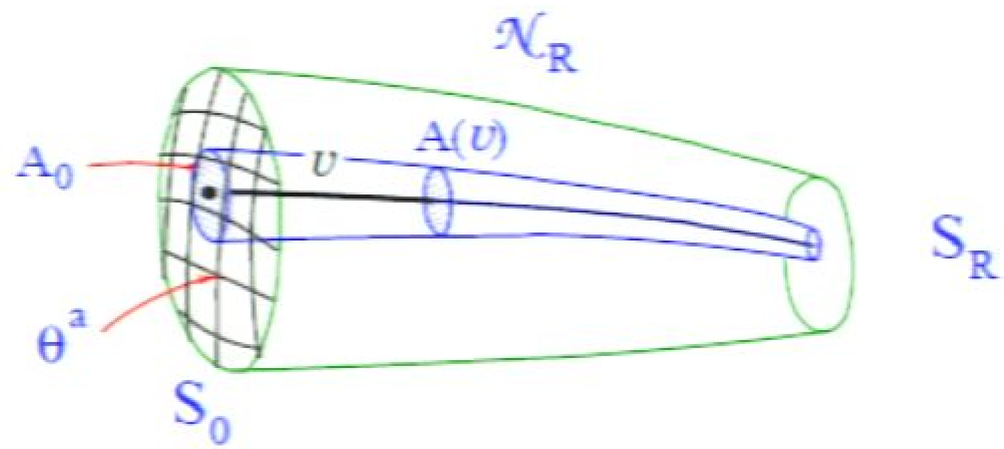

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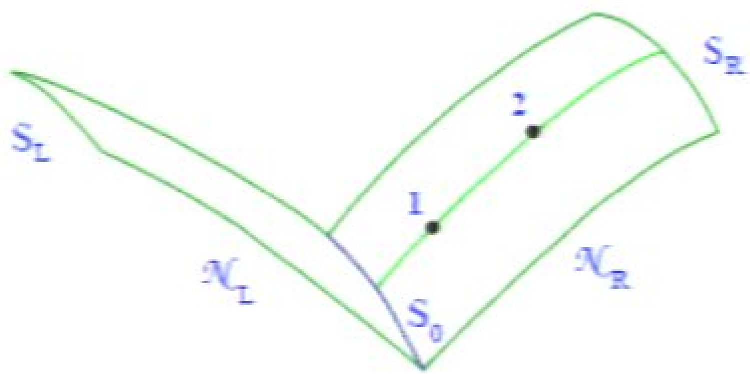
Then

$$\{\mu(\mathbf{1}),\bar{\mu}(\mathbf{2})\}_{\bullet} = 4\pi G\frac{1}{\rho_0 v_1 v_2}\delta^2(\theta_2-\theta_1)\,H(\mathbf{1},\mathbf{2})$$
$$\times[1-\mu\bar{\mu}]_{\mathbf{1}}[1-\mu\bar{\mu}]_{\mathbf{2}}\,e^{\int_{\mathbf{1}}^{\mathbf{2}}(\bar{\mu}d\mu-\mu d\bar{\mu})/(1-\mu\bar{\mu})}.$$

$H(\mathbf{1},\mathbf{2})$ is a step function = 1 if **2** follows **1** along the generator, and 0 otherwise.



- Only data on same generator have non-zero bracket. From causality since points on distinct generators are spacelike separated.
- First line is bracket for Klein-Gordon scalar in Minkowski space.
- Bracket covariant under change of θ chart.

A LITTLE QUANTIZATION

- A polarization
  - $\{\mu(\mathbf{1}), \mu(\mathbf{2})\}_\bullet = 0$. Indeed μ, ρ_0, ϕ form a maximal commuting set of data.
    $\Longrightarrow$ Can be used as configuration variables.
  - can try to quantize using wavefunctions $\Psi[\mu, \rho_0, \phi]$ analytic in μ.

- Quantizing a simple part of Poisson brackets:
  - Let $A \equiv \text{Area}[S_0] = \int_{S_0} |\rho_0| d^2\theta$
  - In order that
$$e^{\frac{A}{2A_{Planck}}} = dim\, H_N$$
    it must be an integer
  - $\{\rho_0(\theta_1), \lambda(\theta_2)\}_\bullet = 8\pi G \delta^2(\theta_2 - \theta_1)$
  - In "loop quantization" of scalar λ and density ρ_0 $\exists$ $\widehat{e^{-i\kappa\lambda(p)}}$ unitary which adds $8\pi\kappa A_{Planck}\delta^2(p, \cdot)$ to ρ_0.
  - Spectrum of A is therefore $8\pi\kappa A_{Planck} n$, $n \geq 0$, integer.
  - Set $\kappa = \frac{\ln 2}{4\pi}$ then . $e^{\frac{A}{2A_{Planck}}}$ has eigenvalues 2^n. - integers!

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