Title: IR modification of gravity and the forbidden mass range of spin-2 particles in De Sitter space

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Abstract: One of the most challenging problems in theoretical physics today is the so called cosmological constant problem. While current observations are consistent with the prediction of GR with an unexplainable tiny cosmological constant, it remains possible that it\'s the deviation of the law of gravity at large distance from Einstein\'s theory that resolves the puzzle. In this talk, I will briefly review some of the theoretical attempts we made along this line, in particular, the so called \'classically constrained gravity\' and its implications in quantum cosmology. I will also present some most recent study on massive spin-2 particles in De Sitter space, and describe a model, initially motivated by DGP theory, which allows one to explore the Higuchi forbidden mass range of the graviton on the De Sitter background.

IR modification of Gravity and the forbidden mass range of graviton in de Sitter space

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Department of Physics
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Young Researchers Conference 2008
Perimeter Institute

Outline

1. Motivation
2. A few words about massive gravity
3. DGP theory and the Higuchi ghost in de Sitter space
   - Higuchi ghost
   - The connection to DGP theory
   - Attempts to cure the theory
   - More general study for $m^2 \neq 2H^2$
   - Outlook

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Motivation

- Cosmological constant problem
- Observed cosmic expansion
- Idea: modification of gravity at large scale may help to solve all the puzzles.
  Most trivially, FP-massive gravity $\Rightarrow$

$$G^L_{\mu\nu} - \frac{1}{2}m^2(h_{\mu\nu} - h\eta_{\mu\nu}) = \Lambda\eta_{\mu\nu}$$

  $h_{\mu\nu} = \frac{2\Lambda}{3m^2}\eta_{\mu\nu}$ is a flat solution.
- Degravitation of the dark energy, G. Dvali, S. Hofmann, J. Khoury, A. Tolley, C. de Rham and etc.

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A few more words about FP theory

- Most trivially, introduce a mass term in the Linearized EH action:

$$\mathcal{L}_{\rm FP} = \frac{1}{4} m^2 (h_{\mu\nu} h^{\mu\nu} - \alpha h^2),$$

$\alpha = -\frac{1}{2} \Rightarrow$ cosmological constant term. Any other values of $\alpha \neq 1$ introduces a ghost into the theory.

- $h_{\mu\nu} = h^{\rm TT}_{\mu\nu} + \nabla_\mu A^{\rm T}_\nu + \nabla_\mu A^{\rm T}_\nu + \gamma_{\mu\nu}\sigma + \nabla_\mu \nabla_\nu \tau \Rightarrow$

$$h^2_{\mu\nu} = h^{\rm TT\,2}_{\mu\nu} - 2A^{\rm T}_\nu \Box A^{\rm T\nu} - 6H^2 A^{\rm T}_\nu A^{\rm T\nu} + 4\sigma^2 + \tau\Box^2\tau + 3H^2\tau\Box\tau + 2\sigma\Box\tau$$

$$h^2 = 16\sigma^2 + 8\sigma\Box\tau + \tau\Box^2\tau$$

The only way to avoid introducing $\tau\Box^2\tau$ in the mass term is by choosing $\alpha = 1$. True in any spacetime of constant curvature.

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Problems: vDVZ discontinuity

- vDVZ discontinuity: Graviton propagator is given by

$$h_{\mu\nu} = -\frac{T_{\mu\nu} - \frac{1}{3}T\eta_{\mu\nu}}{\Box + m^2}$$

Instead of that in the pure EH gravity:

$$h_{\mu\nu} = -\frac{T_{\mu\nu} - \frac{1}{2}T\eta_{\mu\nu}}{\Box}$$

The discrepancy can be easily observed by comparing the Newton's constant measured by light bending and planetary orbits.
- Nonlinear completion of the theory, if exists, should cure the problem.

Problems: no consistent non-linear completion in 4-D

No known consistent nonlinear completion without extra dimensions (codimension-2 DGP models)

- Naively FP-like theories propagate 6 degrees of freedom, but incidentally eom+Bianchi demands

$$\frac{3}{2}m^2 h = T$$

  eliminating one additional degree of freedom

- Alternatively,

$$\mathcal{L}_{\rm PF} = 3m^2\sigma^2 + \frac{3}{2}m^2\sigma\Box\tau + \dots$$

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Why de Sitter space

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  What happens during inflation if gravity is “masive”?
- DGP-theory: low energy limit of the self-accelerating branch is a theory of massive spin-2 on de Sitter background.
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When $H^2 > 0$, there is a ghost if $m^2 < 2H^2$. Higuchi, Deser, Waldron ...

- Focus on the extra spin-0 part: $\frac{m^2}{6(m^2-2H^2)} T g_{\mu\nu}$
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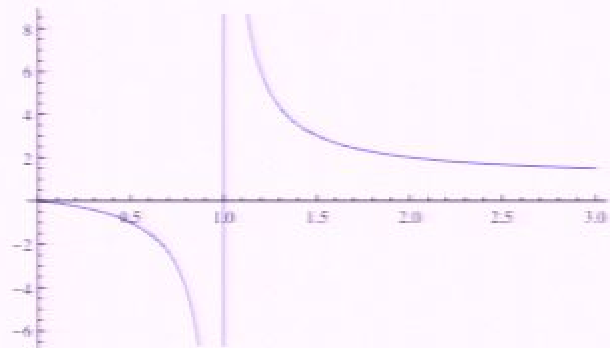

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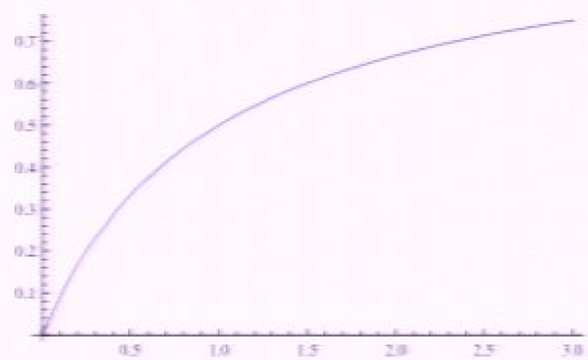

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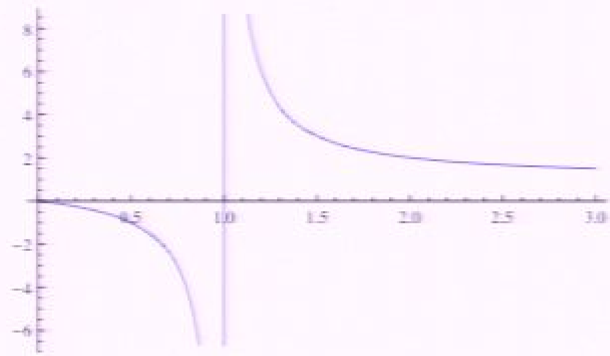

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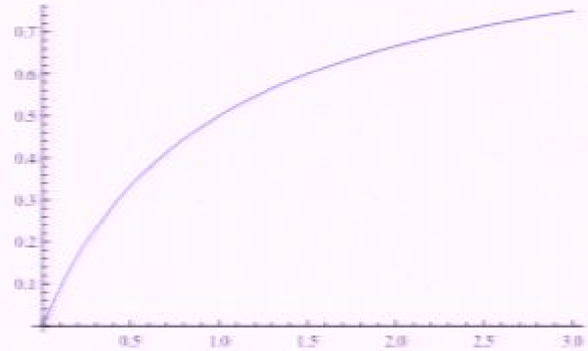

A bit more rigorously . . .

The action:

- $$\mathcal{L} = \mathcal{L}_{\text{EH}}^{(2)}(h_{\mu\nu}) - \frac{1}{4}m^2(h_{\mu\nu}^2 - h^2)$$

  where $\mathcal{L}_{\text{EH}}^{(2)}$ is the second order EH action around de Sitter background with cosmological constant $\Lambda = 3H^2$:

  $$\mathcal{L}_{\text{EH}}^{(2)} = \frac{1}{4}h_{\mu\nu}\Box h^{\mu\nu} + \frac{1}{2}(\nabla_\mu h^{\mu\nu})^2 + \frac{1}{2}h_{\mu\nu}\nabla^\mu\nabla^\nu h - \frac{1}{4}h\Box h - \frac{1}{2}H^2\left(h_{\mu\nu}^2 + \frac{1}{2}h^2\right)$$

- This theory is consistent when $H = 0$.

Decomposition of the modes

When $H^2 \neq 0$, there is a ghost if $m^2 < 2H^2$.
Higuchi, Deser, Waldron ...

- If we decompose $h_{\mu\nu}$ as:

$$h_{\mu\nu} = h^{\mathrm{TT}}_{\mu\nu} + \nabla_\mu A^{\mathrm{T}}_\nu + \nabla_\mu A^{\mathrm{T}}_\nu + \gamma_{\mu\nu}\sigma + \nabla_\mu \nabla_\nu \tau$$

- we find

$$\begin{aligned}\mathcal{L}^{(2)}_{\mathrm{EH}} + \mathcal{L}_{\mathrm{FP}} =& \frac{1}{4} h^{\mathrm{TT}}_{\mu\nu}(\Box - 2H^2) h^{\mathrm{TT}\mu\nu} - \frac{3}{2}\sigma(\Box + 4H^2)\sigma \\ & - \frac{1}{4} m^2 h^{\mathrm{TT}\,2}_{\mu\nu} + 3m^2\sigma^2 - \frac{3}{4} H^2 m^2 \tau\Box\tau + \frac{3}{2} m^2 \sigma \Box \tau \\ & + \frac{1}{2} m^2 A^{\mathrm{T}}_\nu \Box A^{\mathrm{T}\nu} + \frac{3}{2} H^2 m^2 A^{\mathrm{T}}_\nu A^{\mathrm{T}\nu}\end{aligned}$$

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Effective action

- Integrate out the non-dynamical fields τ and A^{T}_{μ}:

$$\mathcal{L}^{(2)}_{\mathrm{EH+PF}} = \frac{1}{4} h^{\mathrm{TT}}_{\mu\nu} \Box h^{\mathrm{TT}\mu\nu} - \frac{1}{4}(m^2 + 2H^2) h^{\mathrm{TT}\,2}_{\mu\nu} + \frac{3(m^2 - 2H^2)}{4H^2} \left(\sigma \Box \sigma + 4H^2 \sigma^2\right)$$

- σ is a ghost when $m^2 < 2H^2$.
- At the special point when $m^2 = 2H^2$, σ disappears in the action and the theory contains extra local symmetry:

$$h_{\mu\nu}(x) \to h_{\mu\nu}(x) + (\nabla_\mu \nabla_\nu + H^2 \gamma_{\mu\nu}) \rho(x)$$

- Equations of motion demand

$$(m^2 - 2H^2) h = T$$

when $m^2 = 2H^2$, the theory can not couple to the trace of $T_{\mu\nu}$.

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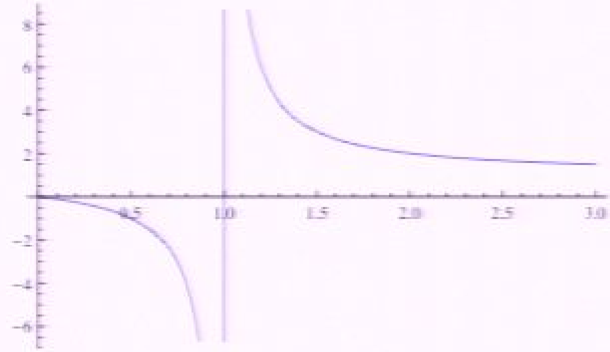

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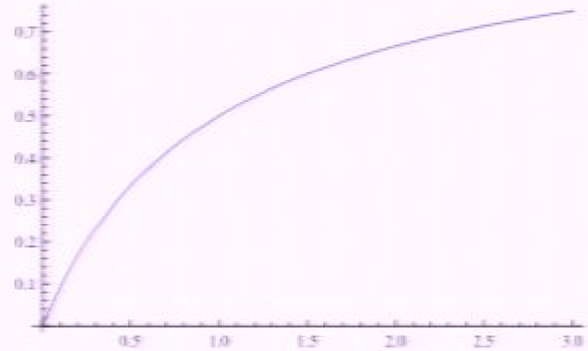

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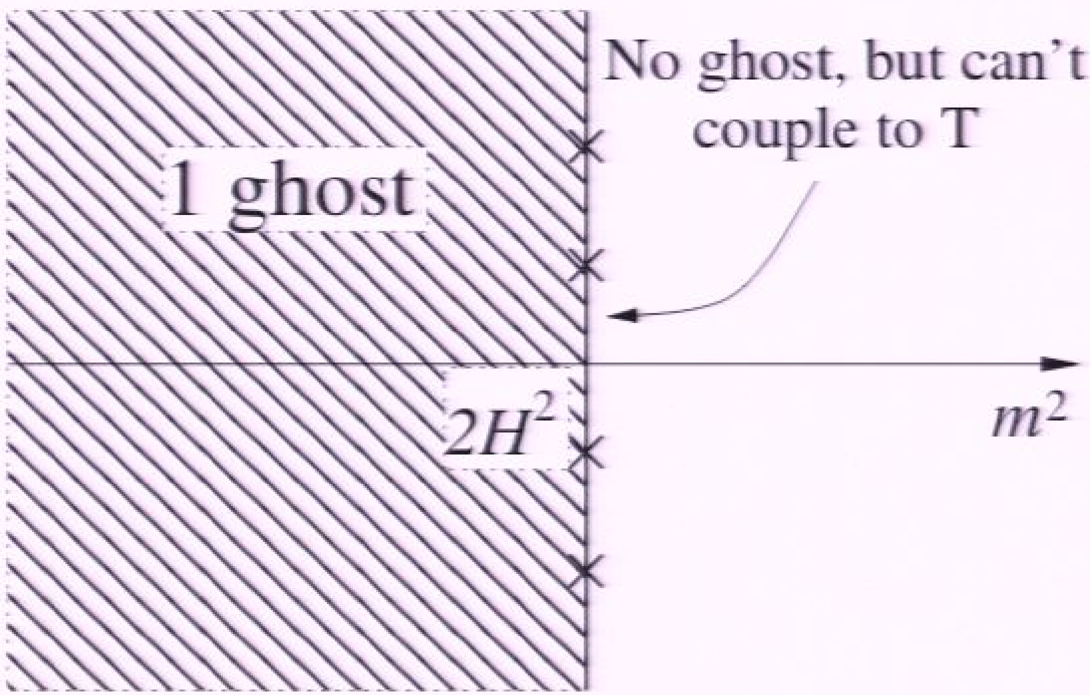
A summary of massive spin-2 on de Sitter background:

- when $m^2 < 2H^2$ there's a ghost
- when $m^2 > 2H^2$ no ghost.
- when $m^2 = 2H^2$, propagating only the spin-2 modes, no ghost or tachyon, but enhanced local symmetry forbids the coupling to T



The Dvali-Gabadadze-Porrati theory

- The codimensional-1 model in 5-d spacetime:

$$S = M_5^3 \int \mathrm{d}^5x\sqrt{-g}R + M_4^2 \int_{\text{brane}} \mathrm{d}^4x\sqrt{-g^*}(R^* + \sigma)$$

- The Freedman equation reads

$$H^2 + r_c H = 0$$

$$r_c \equiv \frac{M_4^2}{2M_5^2}$$

$H = 1/r_c$ is the self-accelerating branch

- From the 4-D point of the view, the self-accelerating branch contains a normalizable spin-2 mode of mass $m_*^2 = 2H^2$ and a continuous tower starting from $m^2 = (9/4)H^2$. K. Koyama, D. Gorbunov, S. Sibiryakov and et al.
- Careful analysis showed that this branch has an unstable mode.

...where this study was initiated

G. Gabadadze and A. Iglesias, JCAP **0802**, 014 (2008) [arXiv:0801.2165 [hep-th]].

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- The presence of additional scalar:

$$\mathcal{L} = \mathcal{L}_{\text{EH+FP}} - \phi\mathcal{O}^{\mu\nu}h_{\mu\nu} + h_{\mu\nu}T^{\mu\nu}$$

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- Despite its origin from the DGP model, operator $\mathcal{O}$ can be motivated by the Bianchi identities:

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DGP 4-D effective theory ...

- Equations of motion are

$$\mathcal{O}^{\mu\nu} h_{\mu\nu} = 0$$

$$G^{\mathrm{dS}}_{\mu\nu} - H^2(h_{\mu\nu} - \gamma_{\mu\nu} h) - \mathcal{O}_{\mu\nu}\phi = -T_{\mu\nu}$$

- Using Bianchi's+trace as usual, we find

$$\frac{1}{2}h_{\mu\nu} = \frac{1}{\Delta_L - 4H^2} T^{(1/3)}_{\mu\nu} - \frac{\gamma_{\mu\nu}}{12}\frac{T}{\Box + 4H^2} + \frac{1}{3}\left(\nabla_\mu\nabla_\nu - \frac{1}{4}\gamma_{\mu\nu}\Box\right)\frac{T}{(\Box + 4H^2)^2}$$

where Δ_L is the Lichnerowicz operator:

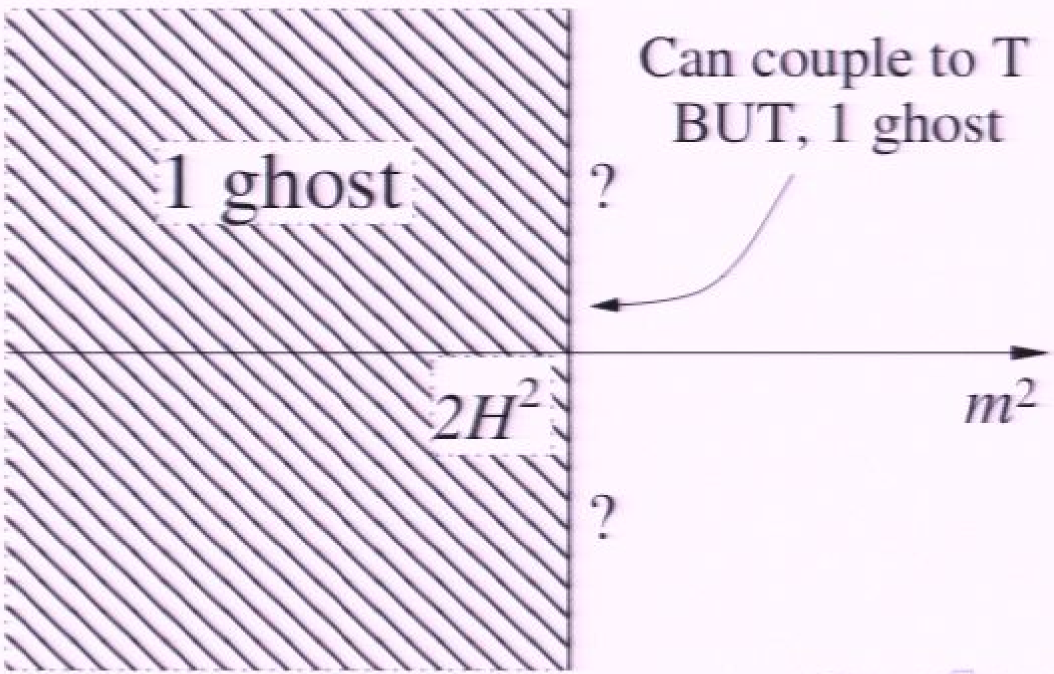
$$\Delta_L h_{\mu\nu} = -\Box h_{\mu\nu} + 2R^\rho{}_{\mu\nu\sigma} h^\sigma{}_\rho + R_{\rho\mu} h^\rho{}_\nu + R_{\rho\nu} h^\rho{}_\mu$$

A bit more careful analysis

- The amplitude

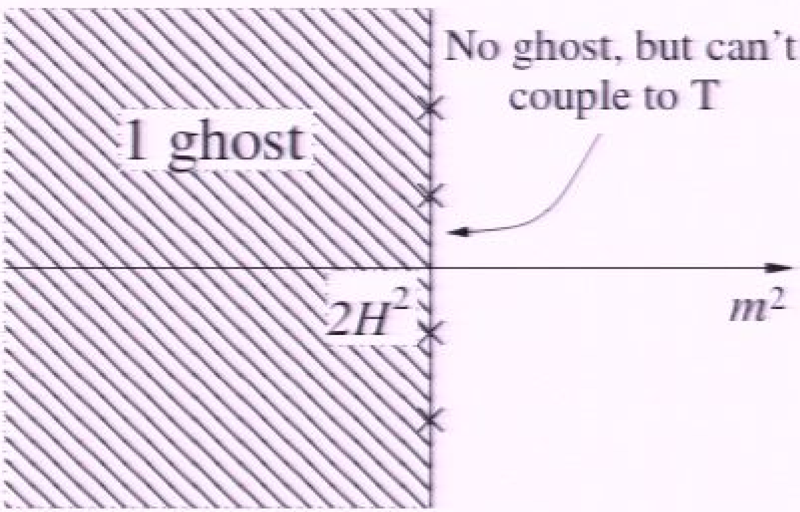
$$\frac{1}{2}h_{\mu\nu} = \frac{1}{\Delta_L - 4H^2}T^{(1/3)}_{\mu\nu} - \frac{1}{3}\left(\nabla_\mu\nabla_\nu + \gamma_{\mu\nu}H^2\right)\frac{T}{(\Box + 4H^2)^2}$$

The double pole can be decomposed into a sum of single poles giving rise to a tachyon and ghost mode. (Strongly coupled, problematic.)

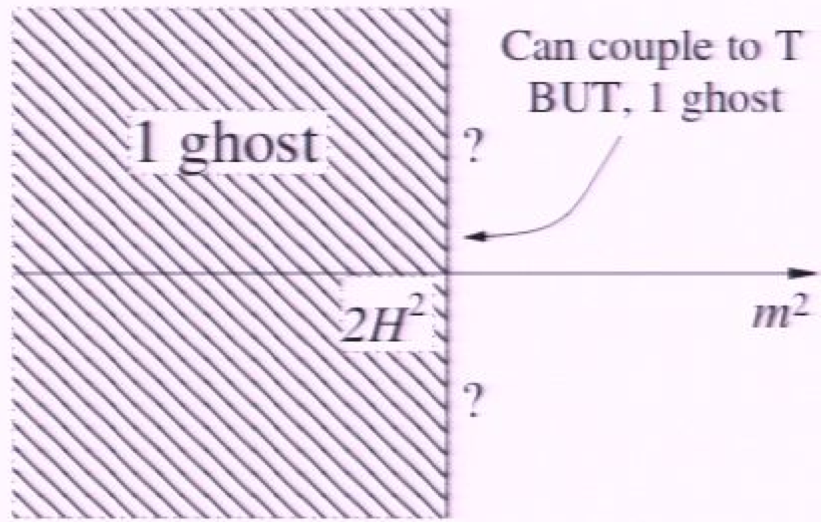


Recap

Simple massive spin-2, when $m^2 = 2H^2$, no ghost, but can not couple to T.



In DGP, self-accelerating branch, always $m^2 = 2H^2$, can couple to T, but there's ghost and strong coupling.



- **Question:** can we improve the DGP theory?

Decouple the ghost

- The amplitude:

$$\frac{1}{2}h_{\mu\nu} = \frac{1}{\Delta_L - 4H^2}T^{(1/3)}_{\mu\nu} + \frac{1}{3}(\nabla_\mu\nabla_\nu + \gamma_{\mu\nu}H^2)\frac{T}{(\Box + 4H^2)^2}$$

- To decouple the ghost, let's introduce a kinetic term for the scalar ϕ

$$\mathcal{L} = \mathcal{L}_{\text{EH+FP}} - \phi\mathcal{O}^{\mu\nu}h_{\mu\nu} + \phi\mathcal{K}\phi + h_{\mu\nu}T^{\mu\nu} + q\phi T$$

where $\mathcal{K}$ is a scalar derivative operator:

$$\mathcal{K} = 3aQ + 3H^2$$

a is unknown, and

$$Q \equiv -\Box - 4H^2$$

- Equations of motion are

$$G^{\rm dS}_{\mu\nu} - H^2(h_{\mu\nu} - \gamma_{\mu\nu}h) - \mathcal{O}_{\mu\nu}\phi = -T_{\mu\nu}$$
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$$\phi = \frac{1}{3Q}T \quad h = -\frac{1}{3H^2}\left(q + \frac{2\mathcal{K}}{3Q}\right)T$$

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- We'll achieve our goal if we choose

$$q - a = 0$$

No ghost or tachyons couple to the source $T_{\mu\nu}$.

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So what's going on?

- Generally the system still contains a ghost, but not coupled to $T_{\mu\nu}$ at the linear level,
  or, is there?
- Any way to improve it further? What about $m^2 \neq 2H^2$?
  Can we follow the same strategy and construct a theory that removes the ghost?
- and maybe even in the forbidden region $m^2 < 2H^2$?
  G. Gabadadze, A. Iglesias and Y. Shang, arXiv:0809.2996 [hep-th].

So what about $m^2 \neq 2H^2$?

- Consider the action for a graviton of mass m and a scalar field ϕ, and as before, with kinetic mixing

$$\mathcal{L}_{\rm eff} = \mathcal{L}^{(2)}_{\rm EH}(h_{\mu\nu}) - \frac{1}{4}m^2(h^2_{\mu\nu} - h^2) - \phi\mathcal{O}^{\mu\nu}h_{\mu\nu} + \phi\mathcal{K}\phi + h_{\mu\nu}T^{\mu\nu} + q\phi T$$

  We keep the operator $\mathcal{O}$

$$\mathcal{O}_{\mu\nu} = \nabla_\mu\nabla_\nu - \gamma_{\mu\nu}\Box - 3H^2\gamma_{\mu\nu}$$

- Again we have left the kinetic term of ϕ undetermined, but simply assume that

$$\mathcal{K} = A\Box + B$$

  Just to remind you:

$$Q \equiv -\Box - 4H^2$$

  while A and B are assumed to be constants and $\mathcal{K}$ commutes with Q.

- Equations of motion are just a bit messier:

$$G^{\rm ds}_{\mu\nu} - \frac{m^2}{2}(h_{\mu\nu} - \gamma_{\mu\nu}h) - \mathcal{O}_{\mu\nu}\phi = -T_{\mu\nu}$$
$$\mathcal{O}^{\mu\nu}h_{\mu\nu} - 2\mathcal{K}\phi = qT$$

Bianchi $\Rightarrow$

$$\nabla^\mu h_{\mu\nu} = \nabla_\nu h$$

which can be used to reduce to:

$$\frac{1}{2}\left[\Box h_{\mu\nu} - (2H^2 + m^2)h_{\mu\nu} - \gamma_{\mu\nu}(H^2 - m^2)h - \nabla_\mu\nabla_\nu h\right] = -T_{\mu\nu} + \mathcal{O}_{\mu\nu}\phi$$

and the trace of this equation:

$$\left(3H^2 - \frac{3}{2}m^2\right)h + 3Q\phi = T$$

- EOM of ϕ implies that

$$-3H^2 h - 2\mathcal{K}\phi = qT$$

All together, we find:

$$\phi = \frac{(1+q)H^2 - \frac{1}{2}qm^2}{3H^2 Q + (m^2 - 2H^2)\mathcal{K}} T$$

$$h = \frac{qQ + \frac{2}{3}\mathcal{K}}{\frac{1}{2}qm^2 - (1+q)H^2}\phi$$

- and the amplitude:

$$\frac{1}{2}(\Delta_L - 6H^2 + m^2)h_{\mu\nu} = T_{\mu\nu} - \mathcal{O}_{\mu\nu}\phi - \frac{1}{2}\mathcal{M}_{\mu\nu}h$$

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The physical metric perturbation

- Again, let's focus on the physical metric perturbation

$$h^{\rm phy}_{\mu\nu} = h_{\mu\nu} + q\gamma_{\mu\nu}\phi \; ,$$

which couples to $T_{\mu\nu}$

- We find the amplitude:

$$\begin{aligned}\frac{1}{2}h^{\rm phy}_{\mu\nu} =& \frac{1}{\Delta_L - 6H^2 + m^2}\left\{T_{\mu\nu} - \mathcal{O}_{\mu\nu}\phi - \frac{1}{2}\mathcal{M}_{\mu\nu}h\right\} + \frac{1}{2}\gamma_{\mu\nu}q\phi \\ =& \frac{1}{\Delta_L - 6H^2 + m^2}\left\{T_{\mu\nu} - \mathcal{O}_{\mu\nu}\phi - \frac{1}{2}\mathcal{M}_{\mu\nu}h \right. \\ & \left. + \frac{q(Q - 2H^2 + m^2)}{2}\gamma_{\mu\nu}\phi\right\} \; ,\end{aligned}$$

- q and $\mathcal{K}$ are still to be determined.
- Generically ϕ and h contain single poles.

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The main trick toward a solution

- Notice that there's nothing we can do about the operator $\mathcal{M}_{\mu\nu}$ in front of h. For any cancellation of the single poles to be possible, the term $\mathcal{O}_{\mu\nu}\phi$ must contain a term of the form of $\mathcal{M}_{\mu\nu}h$, or $\mathcal{M}_{\mu\nu}\phi$ since ϕ and h are related by

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$$\mathcal{O}_{\mu\nu}\phi = \frac{1}{3}\gamma_{\mu\nu}T + \mathcal{M}_{\mu\nu}\phi$$

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The master equation for $\mathcal{K}$

- This leads to the magic equation that determines $\mathcal{K}$:

$$(2H^2 - m^2)\mathcal{K} = -\frac{3q}{2}\left(2H^2 - m^2\right) Q + 3\left(2H^2 - m^2\right)\left[(1+q)H^2 - \frac{1}{2}qm^2\right]$$

- Notice that when $m^2 = 2H^2$, the above equation is automatically satisfied and $\mathcal{K}$ remains completely arbitrary. That is why for this special case one finds additional freedom as discussed earlier.
- Things are very different when $m^2 \neq 2H^2$. Here $\mathcal{K}$ is fixed to be

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Such a choice of $\mathcal{K}$ gives rise to a series of surprising simplifications of the solutions.

h is ϕ

- An immediate consequence of the given choice of $\mathcal{K}$ is that h is directly proportional to ϕ:

$$h = -2\phi$$

removing of of the degree of freedom among the two.

- Furthermore, we find

$$\phi = -\frac{1}{3(\Box + 6H^2 - m^2)}T$$

- The single poles inside the curly brackets, besides the $q(Q - 2H^2 + m^2)\phi$ term, cancel exactly, and the final result:

$$\frac{1}{2}h^{\rm phy}_{\mu\nu} = \frac{1}{\Delta_L - 6H^2 + m^2}\left(T^{(1/2)}_{\mu\nu} + \frac{1+q}{6}\gamma_{\mu\nu}T\right)$$

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Are there ghosts?

- Recall the amplitude:

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  No ghost if $1 + q > 0 \Rightarrow q > -1$.
- How about the extra degree of freedom not sourced by $T_{\mu\nu}$? To determine that, we use the following trick: temporarily set to zero $T_{\mu\nu}$ and add a putative source J via the term $+J\phi$ in the action.
- J should excite a different combinations of the helicity-0 mode and ϕ; if there is a ghost not sourced by T, it should be sourced by J.
- Performing this analysis in a similar way, we find that for

$$q > \frac{2H^2}{m^2 - 2H^2}$$

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Unfortunately we can't get rid of the ghost all together

- Quadratic action can do majic?
- No matter what it is, it has to be $\mathcal{L}_{\text{EH+FP}} + \mathcal{L}_{\text{scalar}}$, except maybe some special points?
- The theory can be diagonalized:

$$h_{\mu\nu} = \tilde{h}_{\mu\nu} + \frac{2}{2H^2 - m^2}\left(H^2\gamma_{\mu\nu}\phi + \nabla_\mu\nabla_\nu\phi\right)$$

The action becomes

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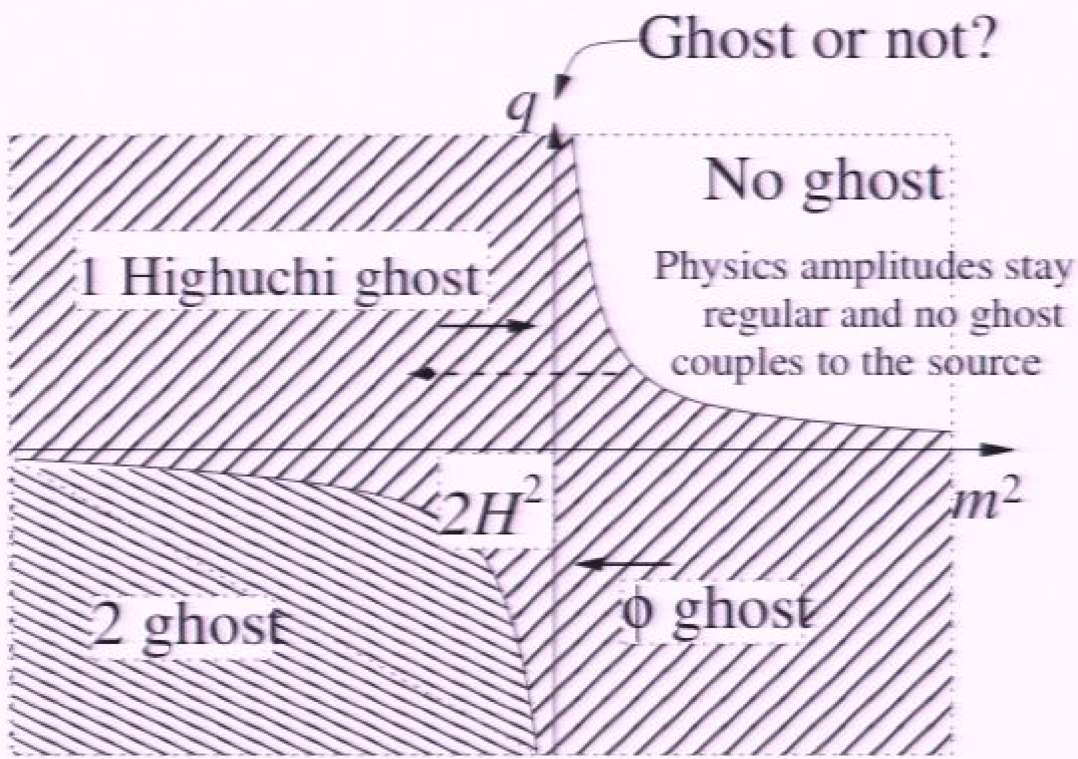
The phase diagram of our contrived theory

- when $q > \frac{2H^2}{m^2-2H^2}$ the scalar ϕ is not a ghost.



- What about the boundary $m^2 = 2H^2$ now? No ghost?

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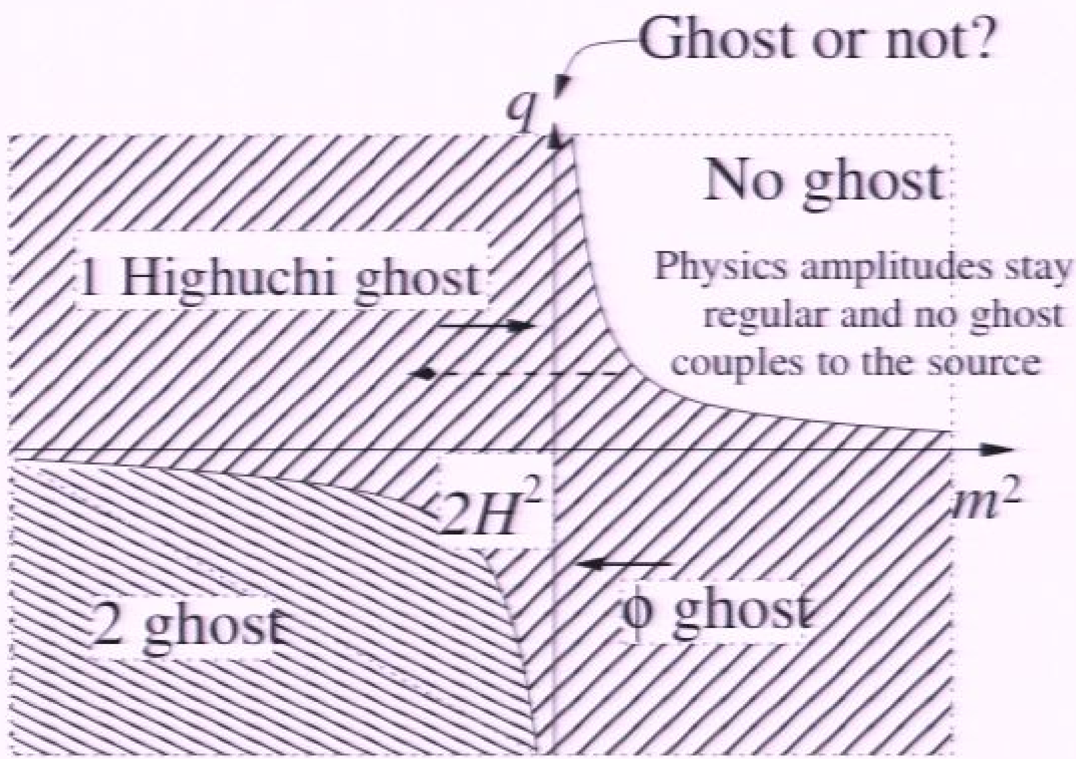
$$\mathcal{L} = \mathcal{L}_{\text{EH+FP}}(\tilde{h}_{\mu\nu}) + \frac{3}{2}\left(q + \frac{2H^2}{2H^2 - m^2}\right)\phi\Box\phi + \text{mass terms for } \phi$$
$$+ \text{complicated couplings to } T_{\mu\nu}$$

- Notice, however, the diagonalization becomes singular when $m^2 = 2H^2$.

The phase diagram of our contrived theory

- when $q > \frac{2H^2}{m^2 - 2H^2}$ the scalar ϕ is not a ghost.



- What about the boundary $m^2 = 2H^2$ now? No ghost?

ı is ϕ

- An immediate consequence of the given choice of $\mathcal{K}$ is that h is directly proportional to ϕ:

$$h = -2\phi$$

  removing of of the degree of freedom among the two.

- Furthermore, we find

$$\phi = -\frac{1}{3(\Box + 6H^2 - m^2)}T$$

- The single poles inside the curly brackets, besides the $q(Q - 2H^2 + m^2)\phi$ term, cancel exactly. and the final result:

$$\frac{1}{2}h^{\text{phy}}_{\mu\nu} = \frac{1}{\Delta_L - 6H^2 + m^2}\left(T^{(1/2)}_{\mu\nu} + \frac{1+q}{6}\gamma_{\mu\nu}T\right)$$

Are there ghosts?

- Recall the amplitude:

$$\frac{1}{2}h^{\rm phy}_{\mu\nu} = \frac{1}{\Delta_L - 6H^2 + m^2}\left(T^{(1/2)}_{\mu\nu} + \frac{1+q}{6}\gamma_{\mu\nu}T\right)$$

No ghost if $1 + q > 0 \Rightarrow q > -1$.

- How about the extra degree of freedom not sourced by $T_{\mu\nu}$? To determine that, we use the following trick: temporarily set to zero $T_{\mu\nu}$ and add a putative source J via the term $+J\phi$ in the action.
- J should excite a different combinations of the helicity-0 mode and ϕ; if there is a ghost not sourced by T, it should be sourced by J.
- Performing this analysis in a similar way, we find that for

$$q > \frac{2H^2}{m^2 - 2H^2}$$

no ghosts are excited by J either.

- If $m^2 < 2H^2$, $\frac{2H^2}{m^2-2H^2} < -1$.

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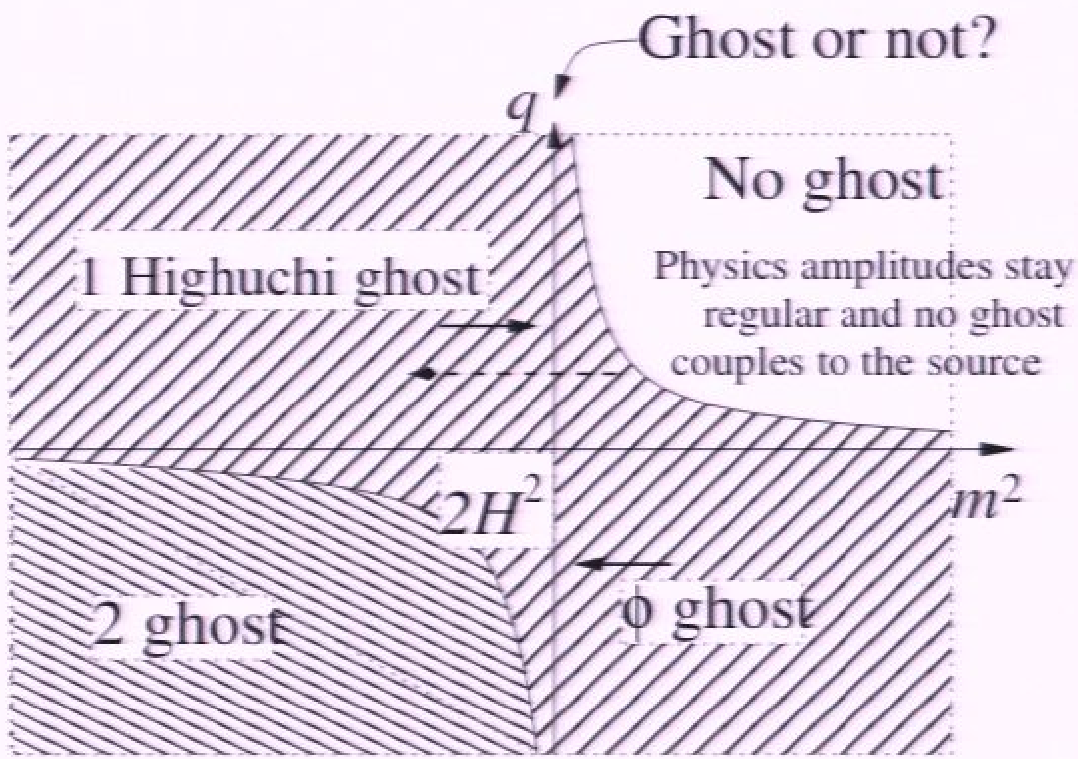
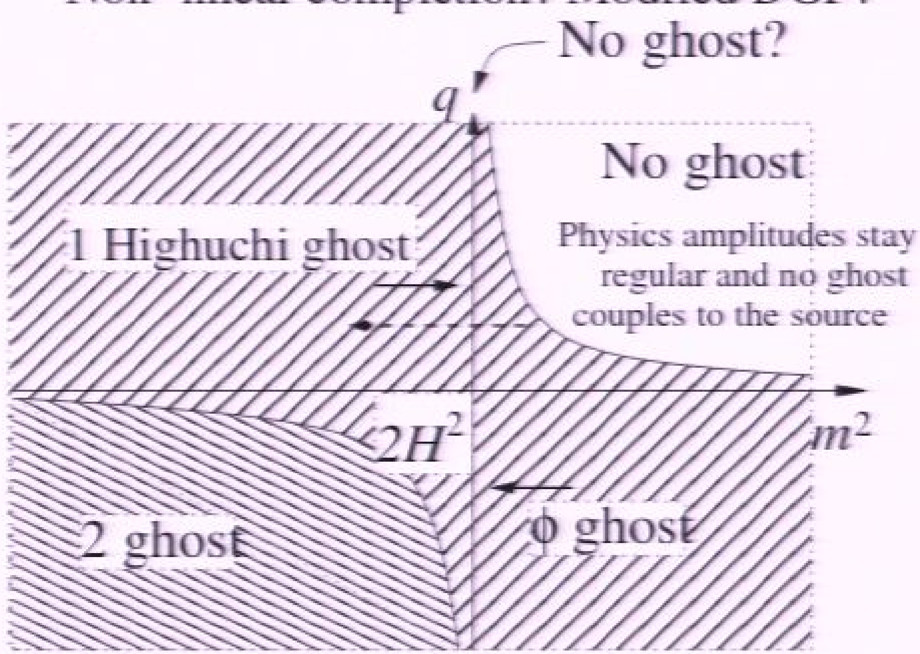


- What about the boundary $m^2 = 2H^2$ now? No ghost?

Still working on it . . .

- the diagonalization becomes singular when $m^2 = 2H^2$. Need more careful analysis.
- At least no ghost coupled to the physical $T_{\mu\nu}$ for the entire region $q > -1$. All amplitudes are regular function of q. Brings us hope to truncate the theory or find a healthy theory on the boundary.

Non–linear completion? Modfied DGP?



Unfortunately we can't get rid of the ghost all together

- Quadratic action can do majic?
- No matter what it is, it has to be $\mathcal{L}_{\text{EH+FP}} + \mathcal{L}_{\text{scalar}}$, except maybe some special points?
- The theory can be diagonalized:

$$h_{\mu\nu} = \tilde{h}_{\mu\nu} + \frac{2}{2H^2 - m^2}(H^2\gamma_{\mu\nu}\phi + \nabla_\mu\nabla_\nu\phi)$$

The action becomes

$$\mathcal{L} = \mathcal{L}_{\text{EH+FP}}(\tilde{h}_{\mu\nu}) + \frac{3}{2}\left(q + \frac{2H^2}{2H^2 - m^2}\right)\phi\Box\phi + \text{mass terms for } \phi$$
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Why de Sitter space

- de Sitter background might be relevant to both late time cosmology and inflationary epoch.
  What happens during inflation if gravity is "masive"?
- DGP-theory: low energy limit of the self-accelerating branch is a theory of massive spin-2 on de Sitter background.
- No vDVZ discontinuity in (A)dS space, M. Porratti.

$$h_{\mu\nu} = -\frac{1}{\Box + m^2}\left[T^{(1/2)}_{\mu\nu} + \frac{m^2}{6(m^2 - 2H^2)} T g_{\mu\nu}\right]$$

$$T^{(1/2)}_{\mu\nu} \equiv T_{\mu\nu} - \frac{1}{2} T g_{\mu\nu}$$

$$-\frac{1}{2} + \frac{m^2}{6(m^2 - 2H^2)} = \begin{cases} -\frac{1}{2} & m^2 \to 0, H^2 \neq 0 \\ -\frac{1}{3} & m^2 \neq 0, H^2 \to 0 \end{cases}$$