Title: Black hole entropy in Loop Quantum Gravity

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Abstract: I will review the present status of the black hole entropy computation in Loop Quantum Gravity within the isolated horizon framework. Starting from the recently discovered discretization effect, I will give an overview of the subsequent developments that have been obtained motivated by it. Through this further analysis of the problem I will present some new related results and the promising new open windows that they give rise to.

Young Researchers Conference. Perimeter Institute. December 8, 2008

Black Hole entropy in LQG: Isolated horizon framework

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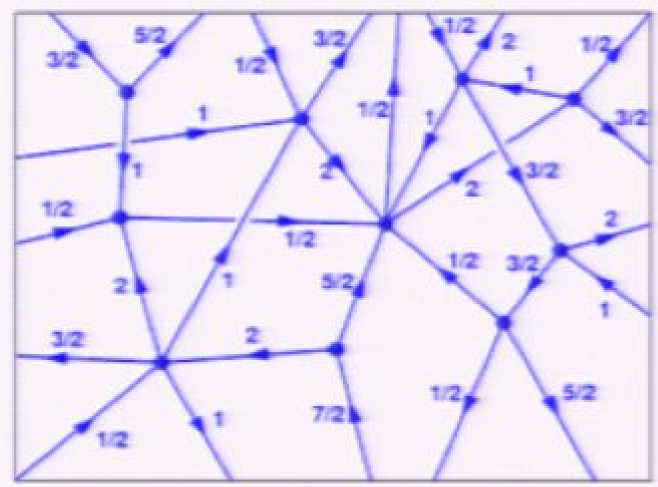
Plan of the talk

- Brief and incomplete introduction to LQG
- From isolated horizons to combinatorics
- Discretization by brute force
- Playing with numbers
- Beyond the stair
- Further discussions

Brief and incomplete introduction to LQG

- LQG is an attemp to make a canonical quantization of GR.
  (Thiemann's book, and references therein)
- Key Point: Make use of the Ashtekar variables (SU(2)-connection) .
  - metric → connection
  - You can see GR as a kind of YM-theory with the proper additional constraints.
- At the quantum level the gravitational degrees of fredoom are described by the spin networks.
- To define a spin network you need:
  - An oriented graph
  - Graph = set of edges and vertices
  - Each edge carries an SU(2)-irrep labelled by j
  - Intertwiners at vertices ensure gauge invariance



Nodes + Lines+ Arrows +Labels = Spin network

From Isolated Horizons to combinatorics

htekar, Baez, Corichi, Krasnov **Phys.Rev.Lett.80:904-907,1998** Ashtekar, Corichi, Krasnov **Adv.Theor.Math.Phys.3:419-478,2000**

htekar, Baez, Krasnov **Adv.Theor.Math.Phys.4:1-94,2000**.

- A black hole can be described in causal terms. We can indentify such an object through the identification of an event horizon.
- In order to identify such a horizon we must know the complete history of your spacetime. An event horizon is a teleological concept.
- The necessity of a more local definition of horizon leads to isolated horizons. The definition of this kind of horizons is purely geometrical in contrast with the later.
- The mathematical definition of isolated horizons is cumbersome. But, for our purposes we only need to take into account the following points:
  - The area of the horizon surface is constant.
  - We are not allowing matter or radiation infalling through the horizon. (Isolation in the usual thermodynamical sense)
  - We are interested in horizons with spherical topology.

The isolated horizons satisfy the laws of black hole mechanics.

So, thermodynamical properties are well defined.

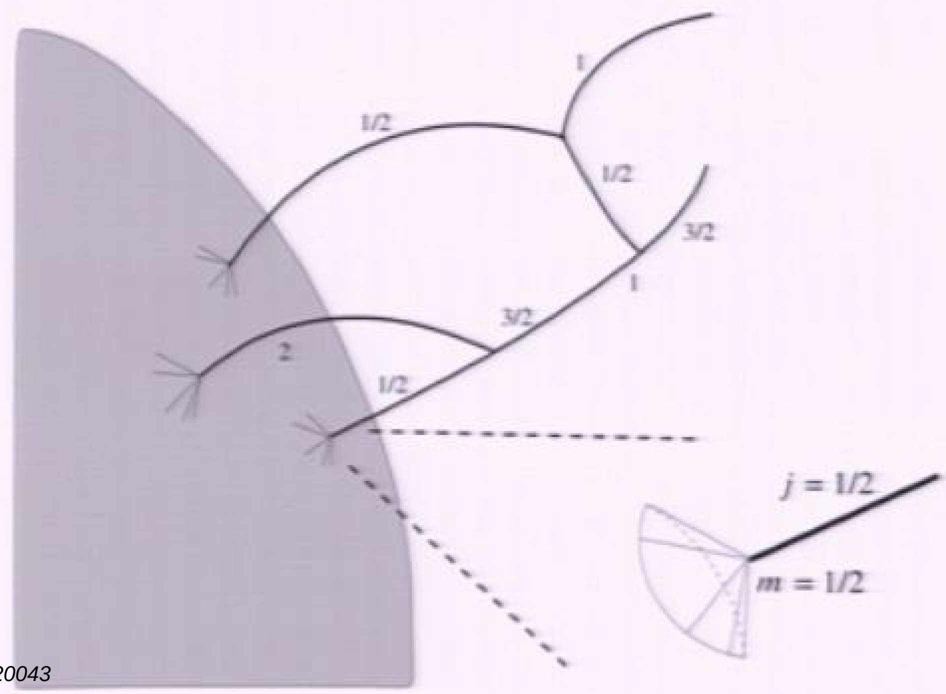
From Isolated Horizons to combinatorics

- At the classical level, the gravitational action (written with Ashtekar variables) acquires an additional term due to the presence of an isolated horizon.
- This term coincides with a U(1) Chern-Simons theory.
- The origin of this U(1)-CS theory is on the fact that the pull-back of Ashtekar SU(2)-connection (in the bulk) to the surface horizon is completly determined by a U(1)-connection.
- So, in presence of an isolated horizon we have the usual gravitational action in the bulk plus an action of an U(1)-CS theory in the surface horizon.

From Isolated Horizons to combinatorics

Quantum level:

- In the bulk we have the spin-networks.
- These intersect the surface horizon in a finite set of points (punctures).
- This punctures make the U(1)-CS theory on the horizon non-trivial. This new degrees of freedom will be the origin of the black hole entropy.



- At the punctures, edges acquire a new label (m).
- This can be understood as the projection of the spin (j).
- The CS-states are characterized by a label (a) quantifying the angle defficits (holonomies) around punctures.

From Isolated Horizons to combinatorics

- The isolated horizon boundary conditions establish:
  - Fixed area A
  - $2m_i = a_i$
- Spherical topology $\rightarrow \sum a_i = 0 \rightarrow \sum m_i = 0$ (Projection constraint)

Entropy Counting

- Entropy of a black hole given by $S(A) = \log N(A)$, $N(A)$ being the number of horizon states labelled by $\{a_i\}$ lists for a given value of area A $\rightarrow$ combinatorial problem
- Area of the horizon given by $A = 8\pi\gamma\ell_P^2 \sum_i \sqrt{j_i(j_i+1)}$

From Isolate Horizons to combinatorics

- Given a value A for the horizon area, the combinatorial problem can be formulated as:
  (Domagała, Lewandowski)

N(A) is given by the number of all the finite, arbitrarily long sequences $m=(m_1,\ldots,m_n)$ *of non-zero half-integers, such that:*

$$\sum_{i=1}^{n} m_i = 0, \qquad \sum_{i=1}^{n} \sqrt{|m_i|(|m_i|+1)} \leq \frac{A}{8\pi\gamma\ell_P^2}$$

Lewandowski, Domagala Class.Quant.Grav.21:5233-5244,2004.

Discretization by brute force

- Explicit enumeration combinatorial algorithm

Corichi, Diaz-Polo, FB Class.Quant.Grav.24:243-251,2007

- Compatible with previous results (B-H, Log corr.)
- But we cannot reach more than a few hundred Planck areas. (So physically uninteresting)

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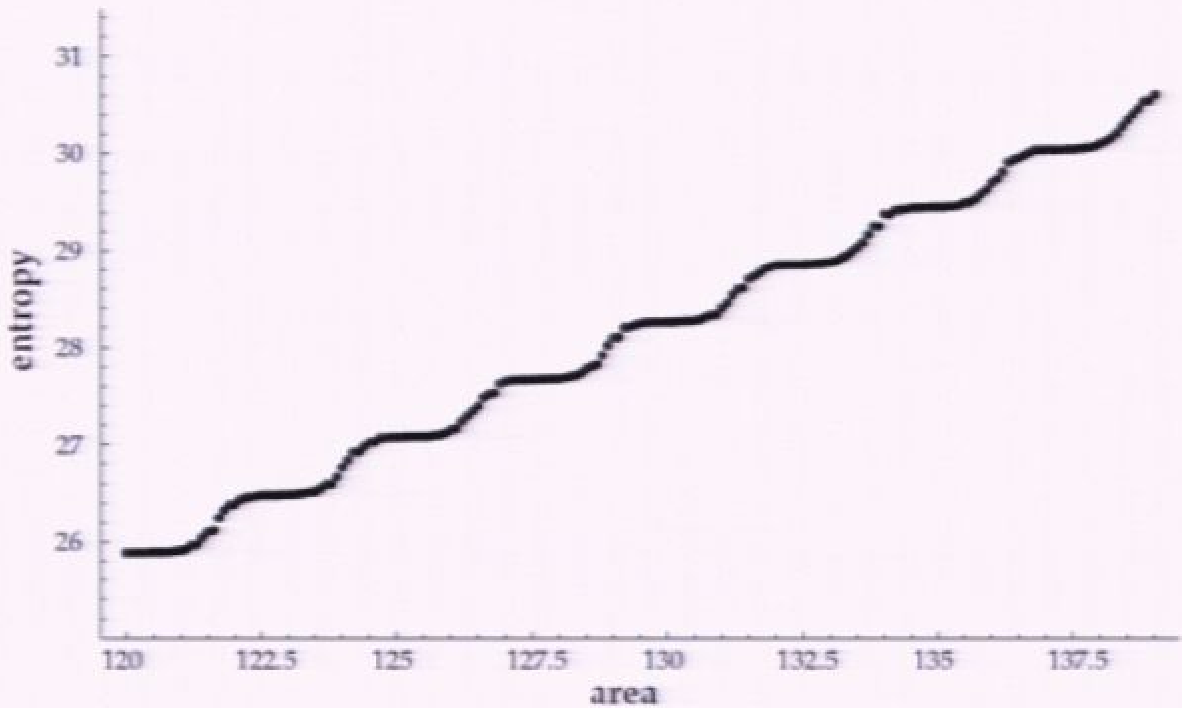

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Discretization by brute force

- Looking at the degeneracy of each area eigenvalue.

$$\Delta A = \gamma\,\chi \;\; \rightarrow \;\; \chi \approx 8\ln 3 \text{ (at least up to the fith decimal figure)}$$

Corichi, Diaz-Polo, F.B. **Phys.Rev.Lett.98:181301,2007.**

Diaz-Polo, F.B. **Class.Quant.Grav.25:105007,2008**

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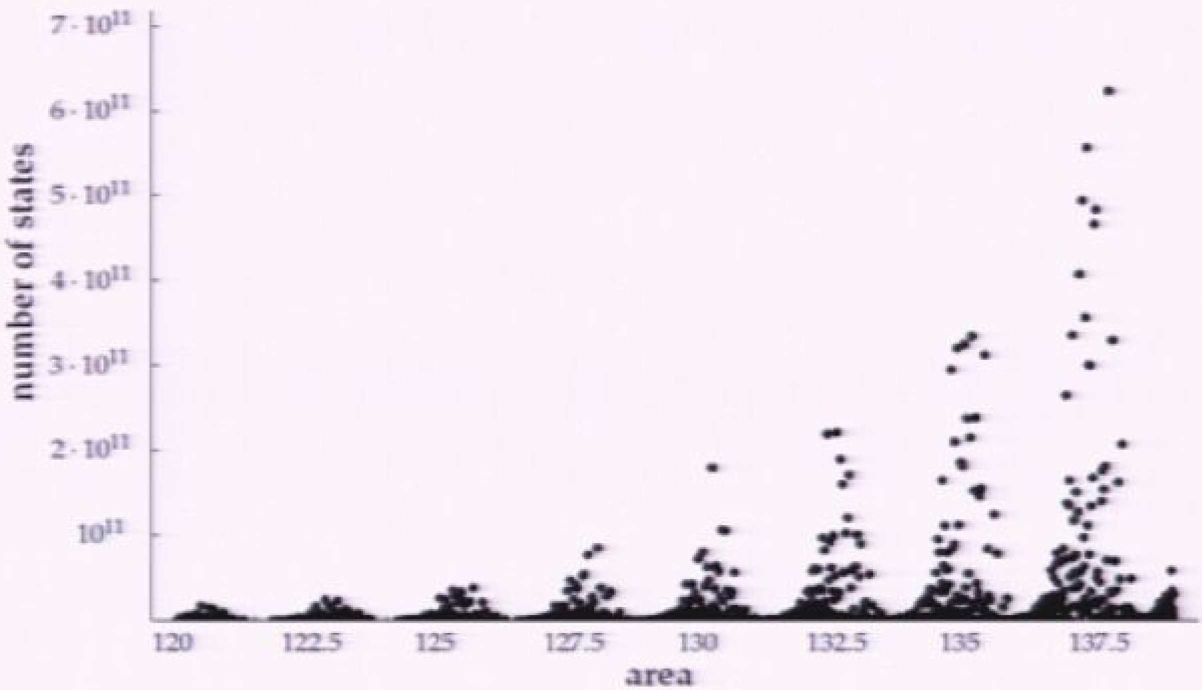

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Playing with numbers

- In order to quantize the CS theory we need to give an ordering to the punctures. But you can choose a given ordering to compute the number of states.
- The Hilbert space for two different orderings are related by an unitary transformation (diffeomorphism).
- So, you must only consider reorderings of labels over punctures.
- Let n_k be the number of punctures with a given k label, where $k = 2|m|$.
- A sequence of n_k characterizes a state up to reorderings and sign assignments.

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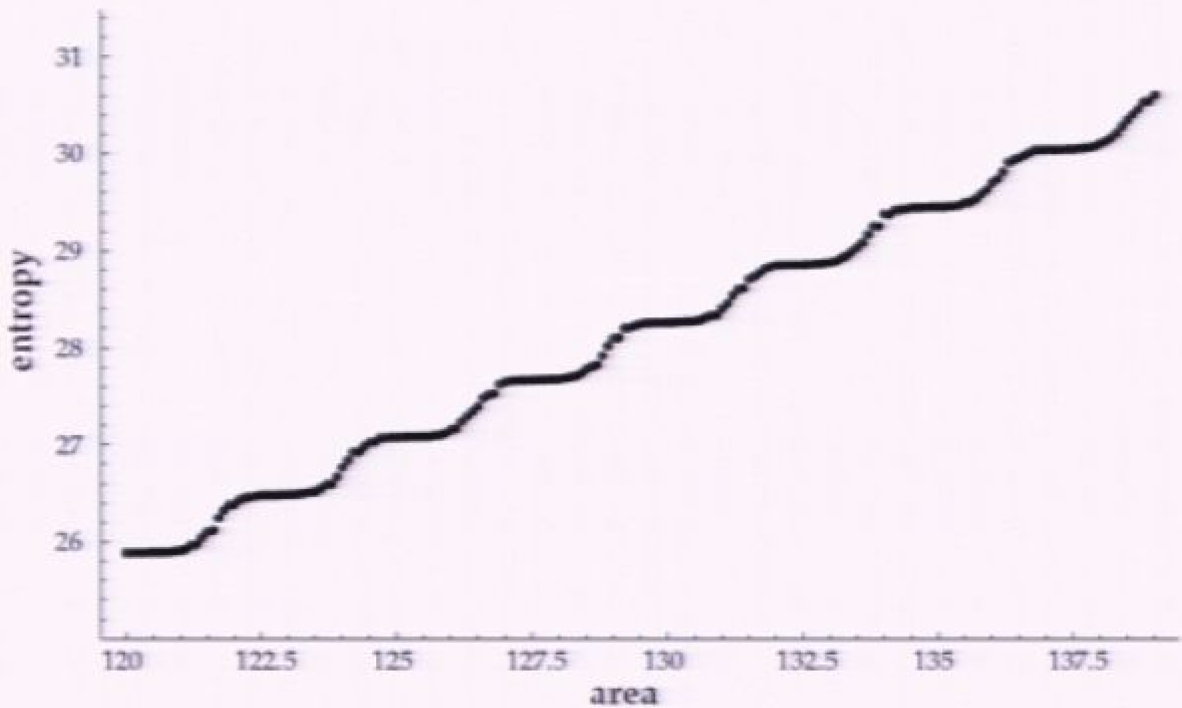

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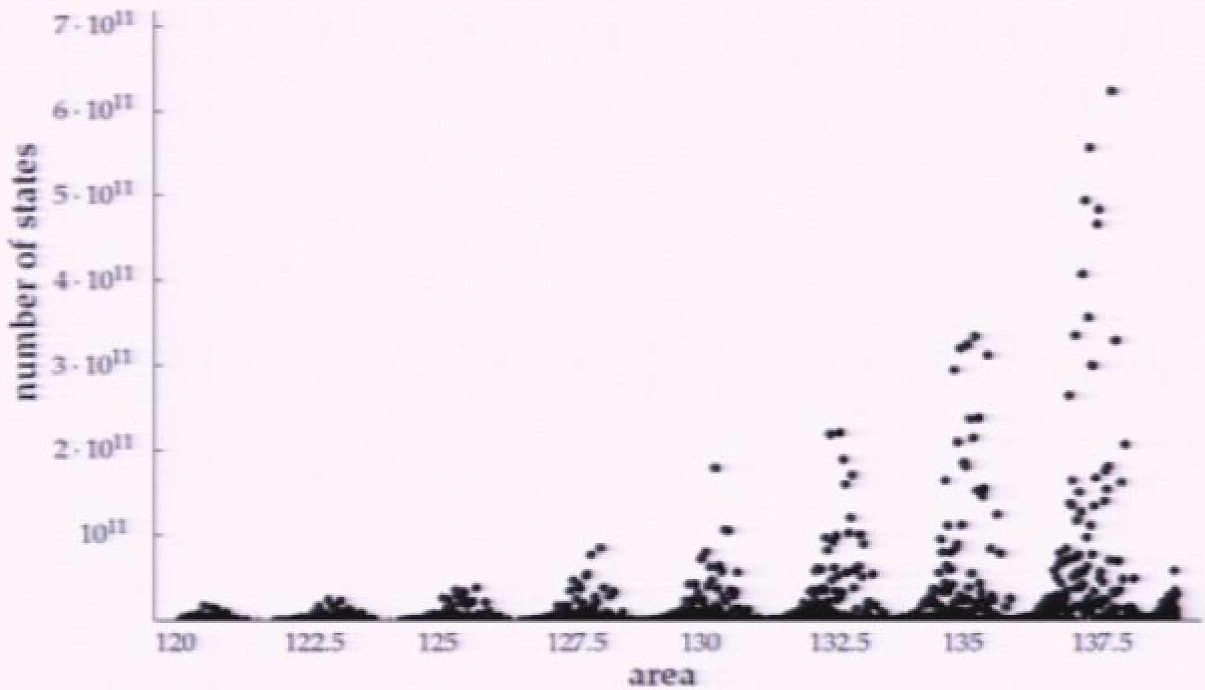

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Playing with numbers

The question:

For a given area, what are the sequences of n_k compatible with it?

The answer:

Pell and diophantine equations.

- The corresponding value of area can be written as

$$A = \sum_k n_k \sqrt{k(k+2)} = \sum_k n_k \sqrt{(k+1)^2 - 1}$$

- Any eigenvalue of the area spectrum can be written in terms of a linear combination
  - $q_i \rightarrow$ positive integers
  - $p_i \rightarrow$ square-free numbers

$$\sum_i q_i \sqrt{p_i}$$

Playing with numbers

Agullo, Barbero, Diaz-Polo, F.B., Villasenor **Phys.Rev.Lett.100:211301,2008**

- In order to characterize the area spectrum we need to solve the equation $\sum_k n_k \sqrt{(k+1)^2-1} = \sum_i q_i \sqrt{p_i}$
- Consider a single $p_i \rightarrow \sqrt{(k+1)^2-1} = y\sqrt{p_i}$
- Known equation in number theory: Pell equation $x^2 - p_i y^2 = 1$
  - For every square-free number p_i one can always find an (infinite) set of solutions $\{(k^i_m, y^i_m)\}$ using the relation $k^i_m + 1 + y^i_m\sqrt{p_i} = \left(k^i_1 + 1 + y^i_1\sqrt{p_i}\right)^m$
- We can rewrite $\sum_i \sum_m n_{k_m} y^i_m \sqrt{p_i} = \sum_i q_i \sqrt{p_i}$
- Square-free square roots are linearly independent with integer (rational) coefficients → we end up with a set of decoupled diophantine equations $\sum_m n_{k^i_m} y^i_m = q_i$
- For each area $\{q_i\}$ we obtain all compatible sets $\{n_k\}$

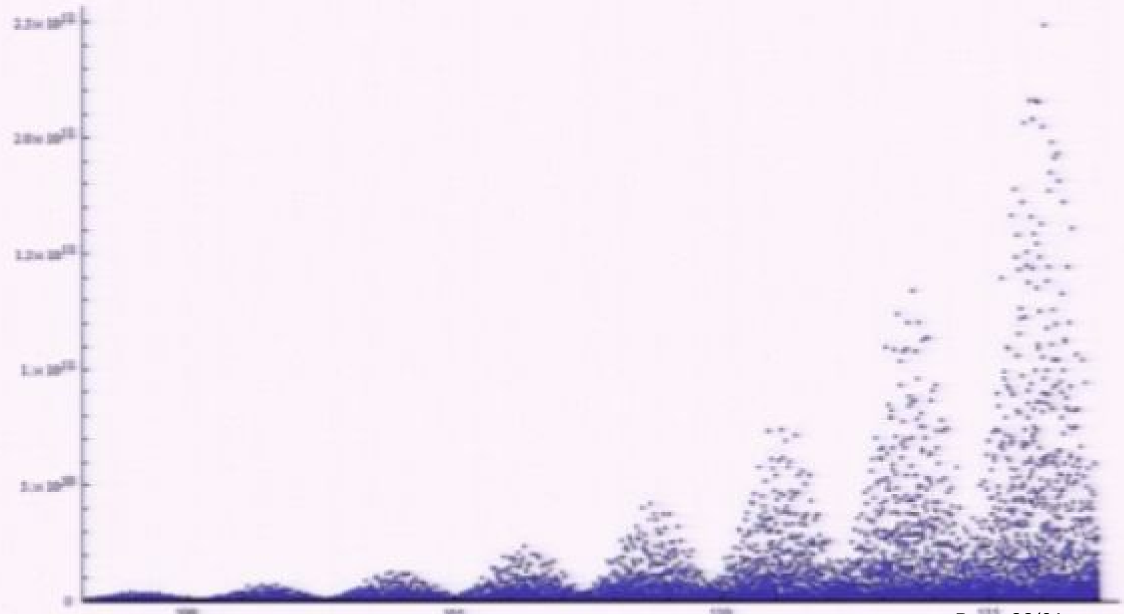
Playing with numbers

- Sign assingnments → Equivalent to a 'partition problem' in number theory
- Solution, given a $\{k_i\}$ list:

$$P(\{k_i\}) = \frac{2^n}{M} \sum_{s=0}^{M-1} \prod_{i=1}^{n} \cos(2\pi s k_i / M), \qquad M = 1 + \sum_i k_i$$

- Reorderings for a $\{n_k\}$ list:

$$R(\{n_k\}) = \frac{(\sum_k n_k)!}{\prod_k n_k!}$$



Playing with numbers

Barbero, Villasenor **Phys.Rev.D77:121502,2008,** and arXiv:0810.1599 [gr-qc]

Agullo, Barbero, Diaz-Polo, F.B., Villasenor work in progress

- We can compute the whole degeneracy and also each single contribution making use of generating functions.
- Very convenient way of codifying **all** the information about the combinatorial problem
- Allow to obtain closed analytical expressions for the solution
- For instance, the solution to the sign assignments is given by the expression:

$$\frac{1}{2\pi}\int_{0}^{2\pi} d\theta \prod_{i} 2\cos(k_i\theta)$$

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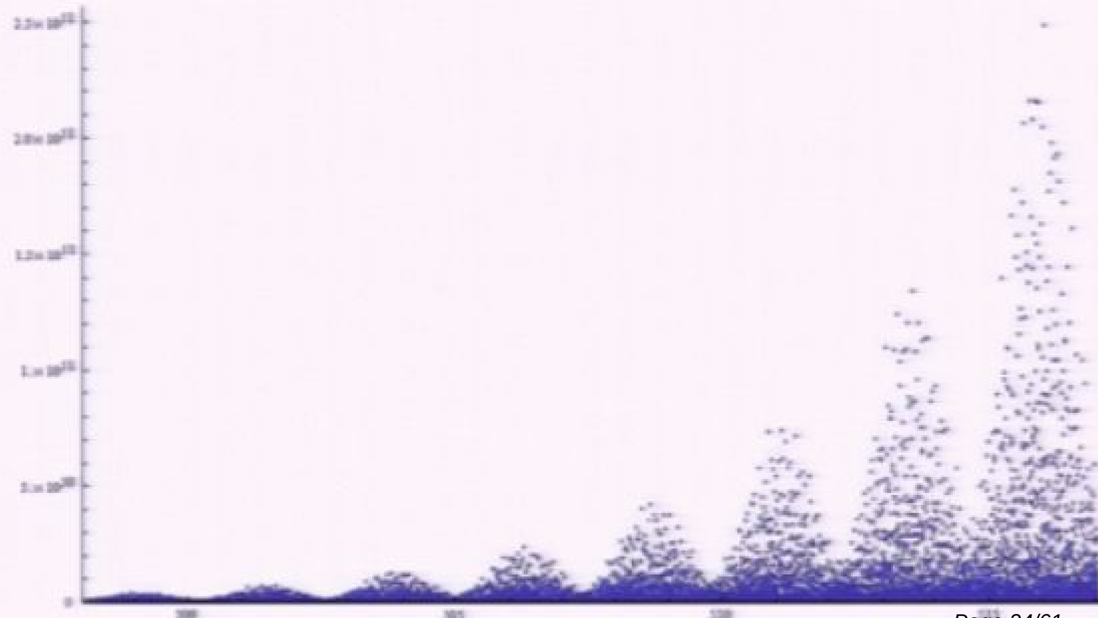

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Beyond the stair

- Once the problem has been set up in an analytical form, we can search for the implications of it.
- Nowdays, we have three interesting windows to look at.
  - Can this setting be related with conformal field theory?
  - Is the discrete entropy present for macroscopical black holes?
  - What are the implications of this effect in the black hole radiation spectrum?

Beyond the stair

Conformal tales: Witten Commun.Math.Phys.121:351,1989, Carlip, Gen.Rel.Grav.39:1519-1523,2007

- Motivation:
  - Carlip's idea of an underlying conformal symmetry driving the linear behavior of black hole entropy.
  - Witten's analogy between Chern-Simons theory and WZW models.
- Pevious work:

  Kaul, Majumdar Phys.Lett.B439:267-270,1998.
  - SU(2) Chern-Simons theory on the horizon
  - SU(2) WZW model on a 2-sphere (boundary)
  - Compute the dimension of the space of conformal blocks
    - Fusion rules for the product of representations

$$[\phi_i] \otimes [\phi_j] = \sum_r N_{ij}^r [\phi_r]$$

  - Problem: In LQG the horizon is described by U(1) Chern-Simons

Beyond the stair

- How to account for the U(1) degrees of freedom?

Beyond the stair

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U(1)
Chern-Simons

Beyond the stair

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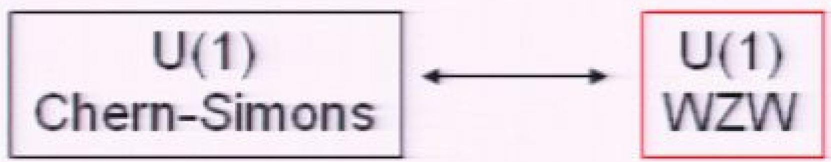

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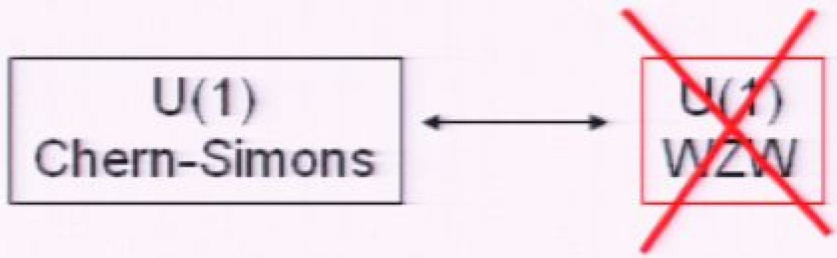

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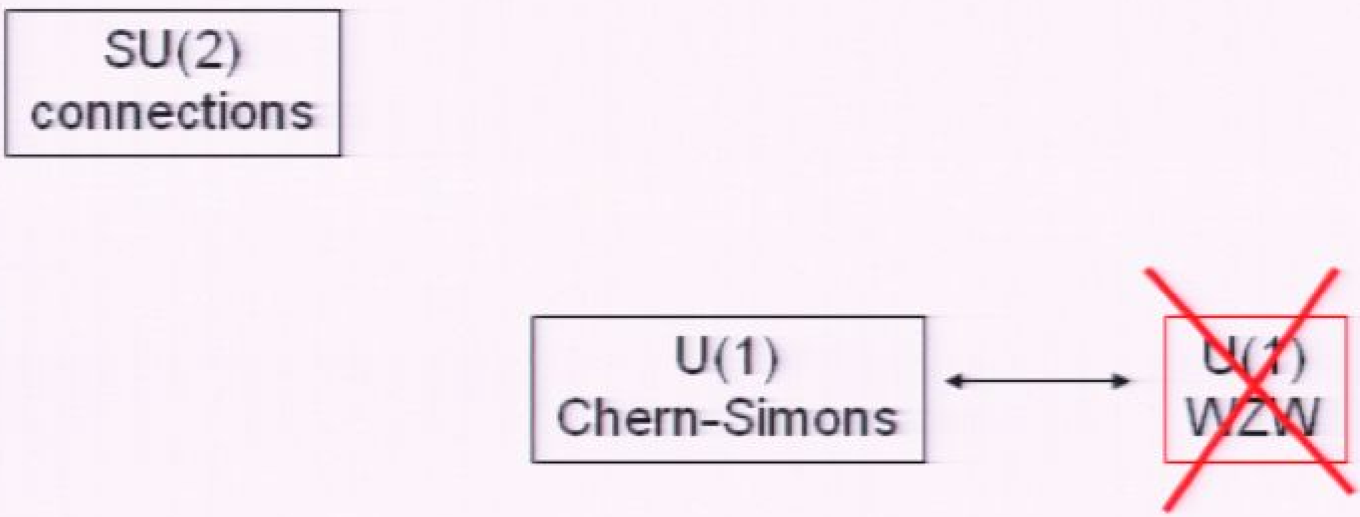

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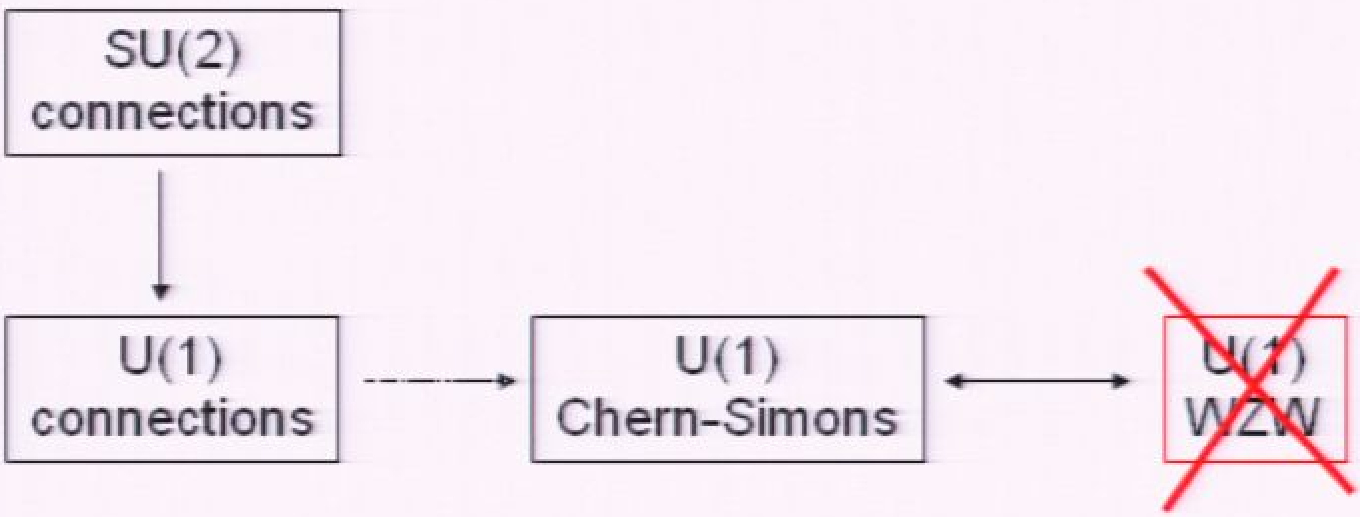

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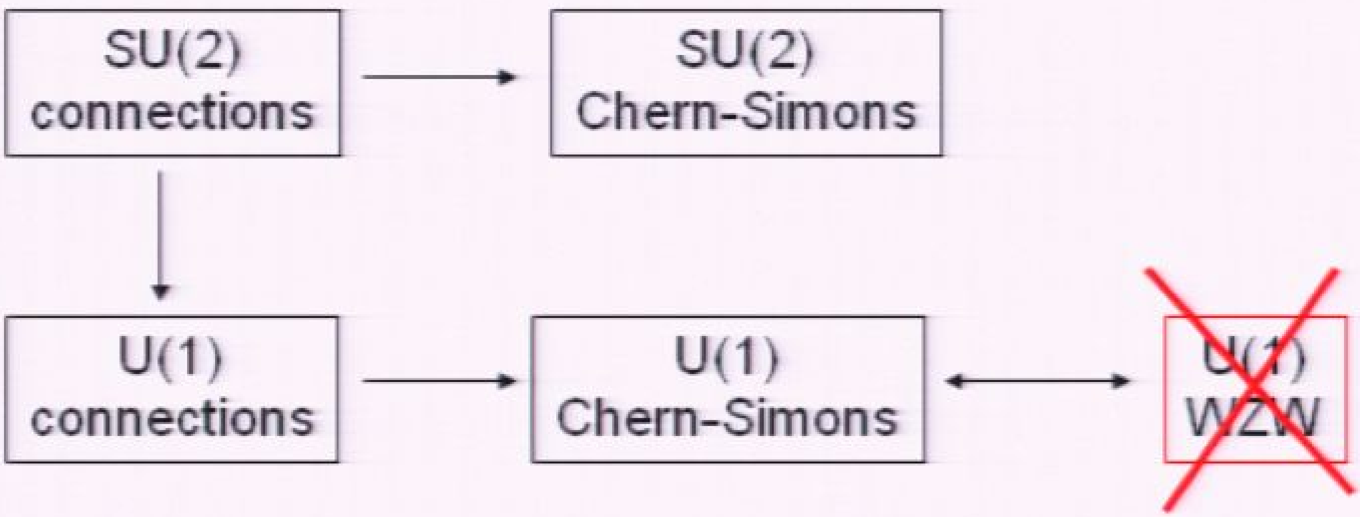

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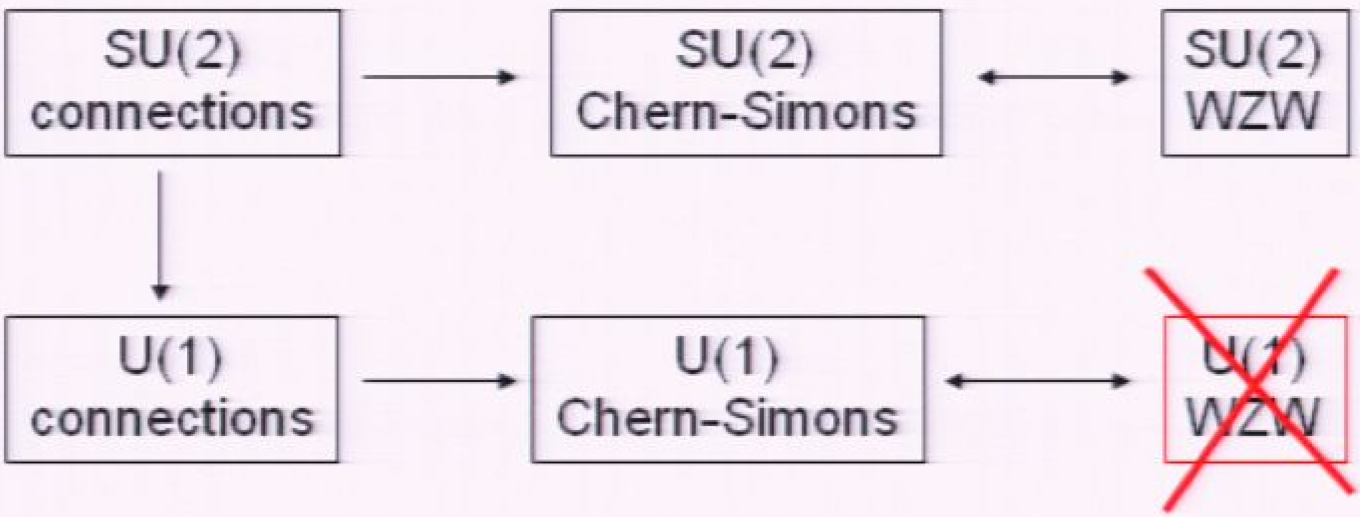

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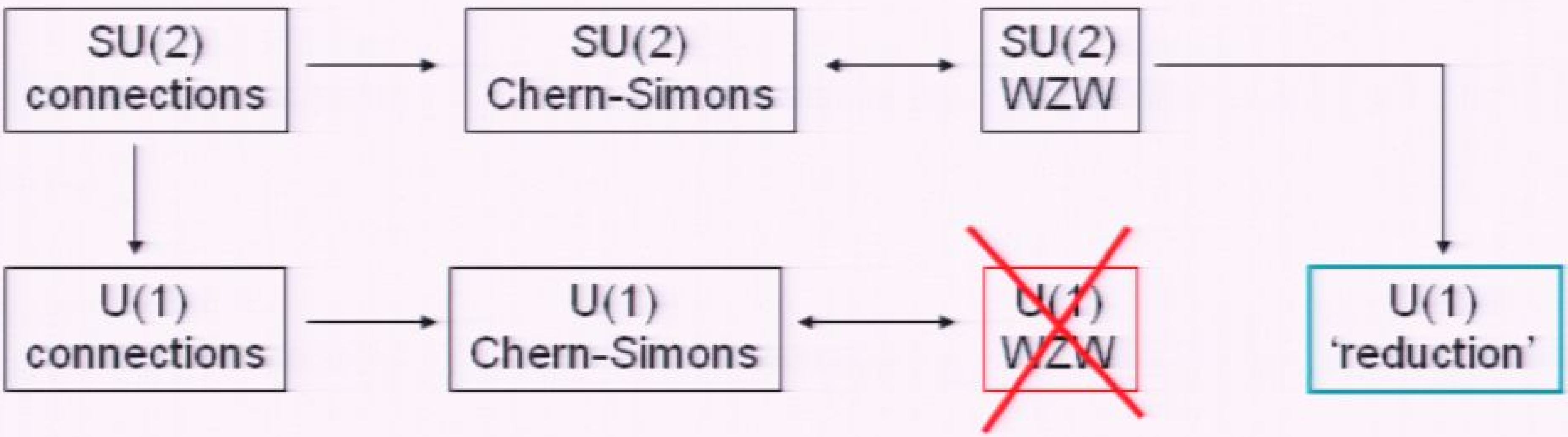

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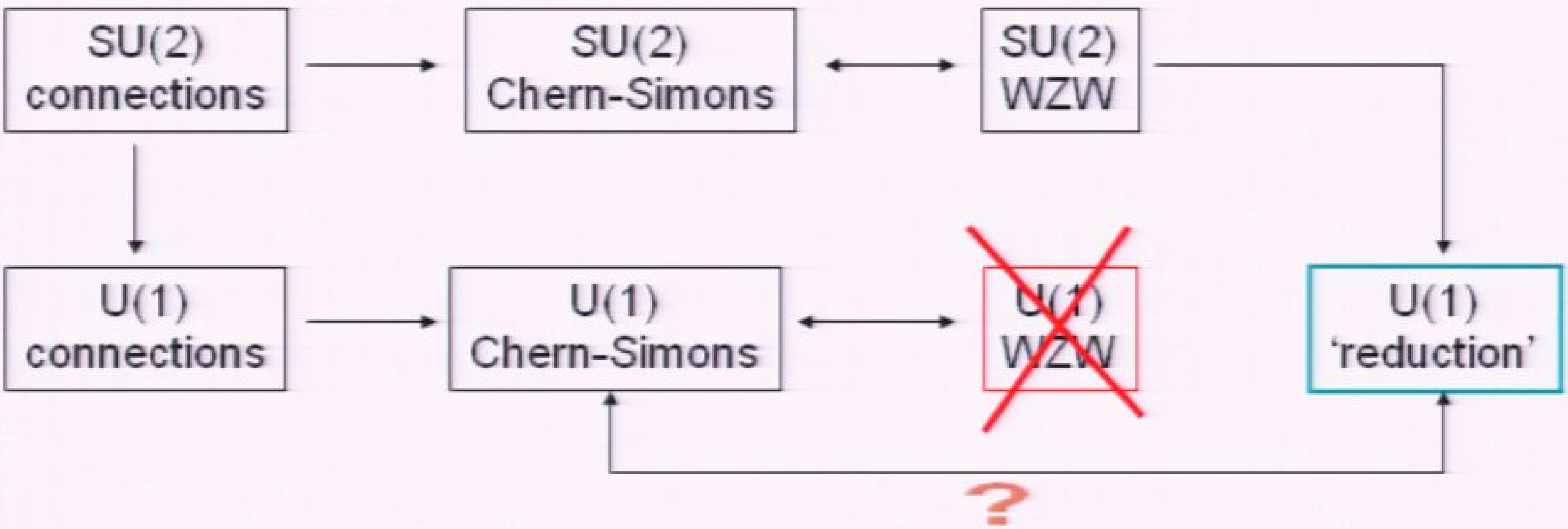

Beyond the stair

- We start with a SU(2) Chern-Simons theory (over a punctured sphere) on the (2+1) horizon
- Witten → Hilbert space corresponds to a (1+1) WZW model
- Compute the dimension of the space of 'U(1) reduced' conformal blocks
- Geometrical symmetry breaking
  - The homomorphism H → G induces the bundle reduction P(G, Σ) → Q(H, Σ)
- $U(1) \cong T(SU(2)) = \{diag(z, z^{-1}) \mid z \in U(1)\}$
- All homomorphisms in Hom(U(1), T(SU(2))) are given by

$$\lambda_k : z \mapsto diag(z^k, z^{-k})$$

  - k and -k conjugated by the Weil group action → $k \in N_0$

Beyond the stair

- We want to make this symmetry reduction locally at each puncture
- Each SU(2) irrep j is the direct sum of 2j+1 U(1) irreps
- To be consistent with the embeddings $U(1) \rightarrow SU(2)$ we cannot differenciate signs
- At each puncture we take the direct sum of two U(1) irreps ($\pm m$) which gives us a (reducible) U(1) representation
- This is precisely using the homomorphisms λ_k as U(1) representations
- We consider these representations instead of the irreducible ones as a consequence of the reduction from SU(2)

Beyond the stairs

Agullo, Diaz-Polo, F.B. work in progress

- With this, we can project the product of characters over the character of the trivial U(1) irreducible representation, to require the gauge invariance
- This can be seen as an elegant way of imposing the projection constraint
- The characters of the representations we are using are

  $\chi_k = z^k + z^{-k} = 2\cos(k\theta)$ provided $z = e^{i\theta} \in U(1)$
- For each $\{k_i\}$ list, compute the product of characters

$$\chi_{k_1}\chi_{k_2}\cdots\chi_{k_n} = \prod_{i=1}^{n} 2\cos(k_i\theta)$$

- Projecting over the character $\chi_0 = 1$ of the trivial U(1) irrep

$$\boxed{\left\langle \chi_{k_1}\chi_{k_2}\cdots\chi_{k_n} \middle| \chi_0 \right\rangle = \frac{1}{2\pi}\int_0^{2\pi} d\theta \prod_{i=1}^{n} 2\cos(k_i\theta)}$$

Beyond the stairs

Asymptotic expansions…

- In order to know if the black hole entropy discretization is present in macroscopic black holes we need to study the asymptotic behavior of the number of states with area.
- This is a very interesting and technically difficult problem.
- At present we have some indications that this effect is mantained in the asymptotic regime, but we have not a definte proof.

Beyond the stair

Meissner Class.Quant.Grav.21:5245-5252,2004

In a well known paper of Meissner we can found a disturbing answer:

Meissner computed the Laplace transform of the number of states N(a):

$$P(s) := \int_0^\infty \mathrm{d}a\, N(a)\, e^{-sa}$$

$$P(s) = \frac{2\sum_{k=1}^{\infty} e^{-s\sqrt{k(k+2)/4}}}{s\left(1 - 2\sum_{k=1}^{\infty} e^{-s\sqrt{k(k+2)/4}}\right)}$$

Beyond the stair

Then, the inverse Laplace transform gives us the N(a). We can study the poles of it and establish:

$$N(a) = \sum_{s_i, \mathrm{Re}(s_i)>0} \mathrm{res}_{s_i} e^{s_i a} + O(a^n)$$

The leading order is given by the real pole.

Seemingly, the discrete entropy is lost.

$$N(a) = C_M e^{\tilde{\gamma}_M a}$$

Beyond the stair

Fortunately, this is not the case: (Barbero, Villasenor arXiv:0810.1599v1 [gr-qc])

One can only express the inverse Laplace transform as a sum over residues when one has a finite number of poles, or countably infinity without and accumulation point.

But the P(s) has poles whose real part accumulate to the real pole. With this, one cannot discard the possibility of finding the discretization effect in the asymptotical regime.

Work in progress: We are trying make use of differents kind of transform, like the Mellin transform in order to simplify the pole structure of the P(s) and be able reach the asymptotic regime. Partial results indicate that these transforms give us generalized zeta functions, with known pole structure. We hope to have a final result very soon. (Agullo, Barbero, Diaz-Polo, FB, Villasenor)

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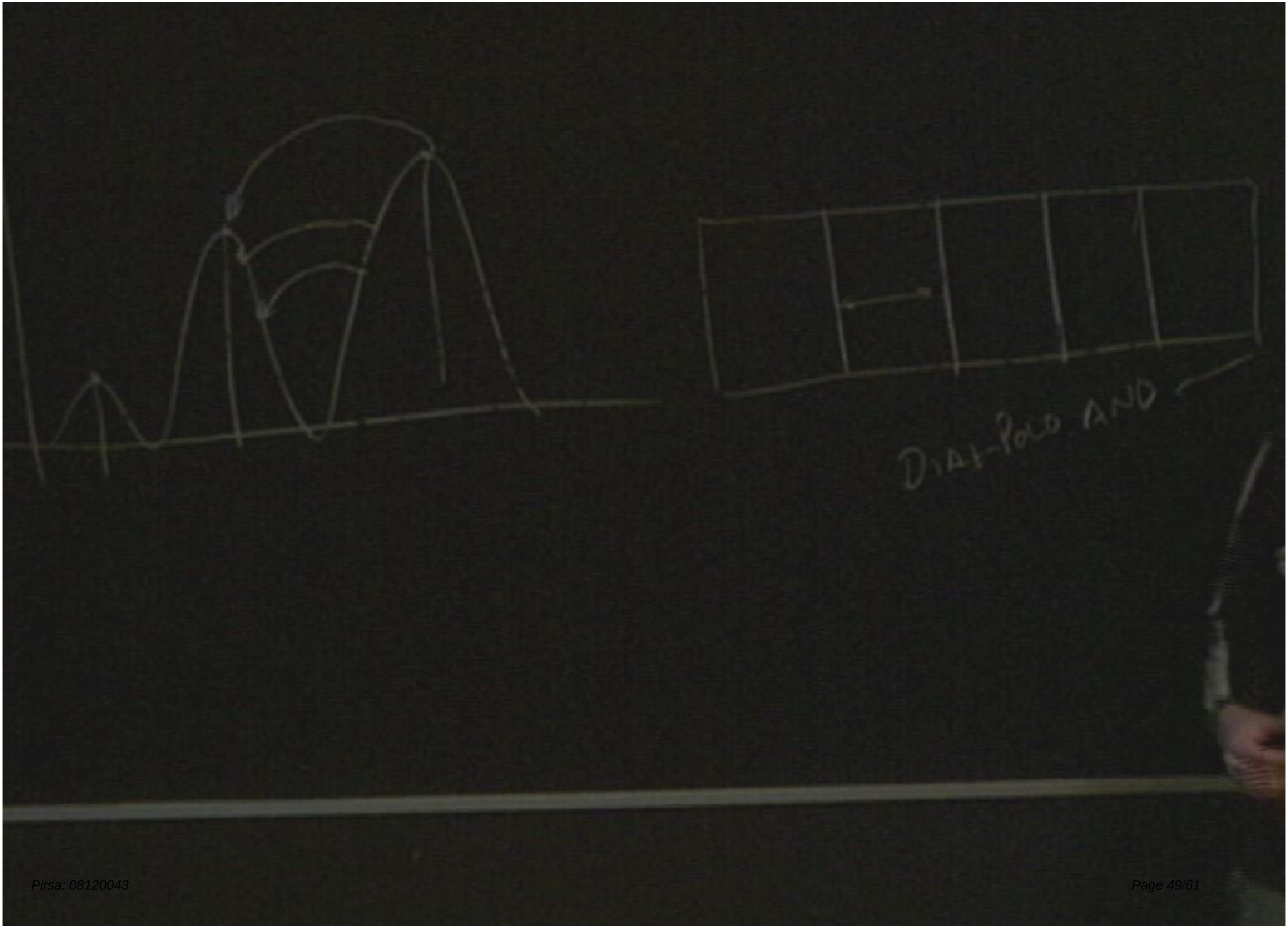
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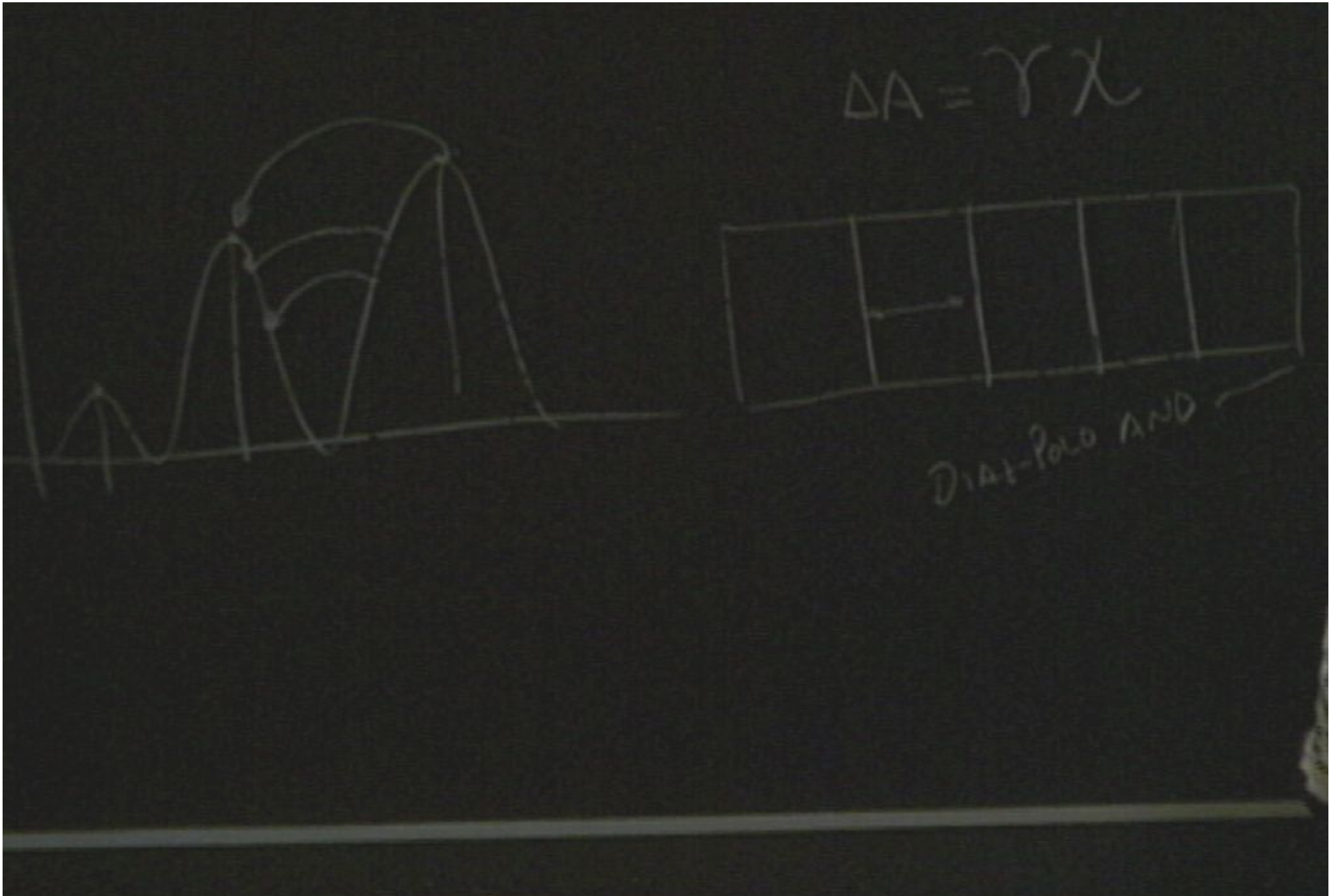
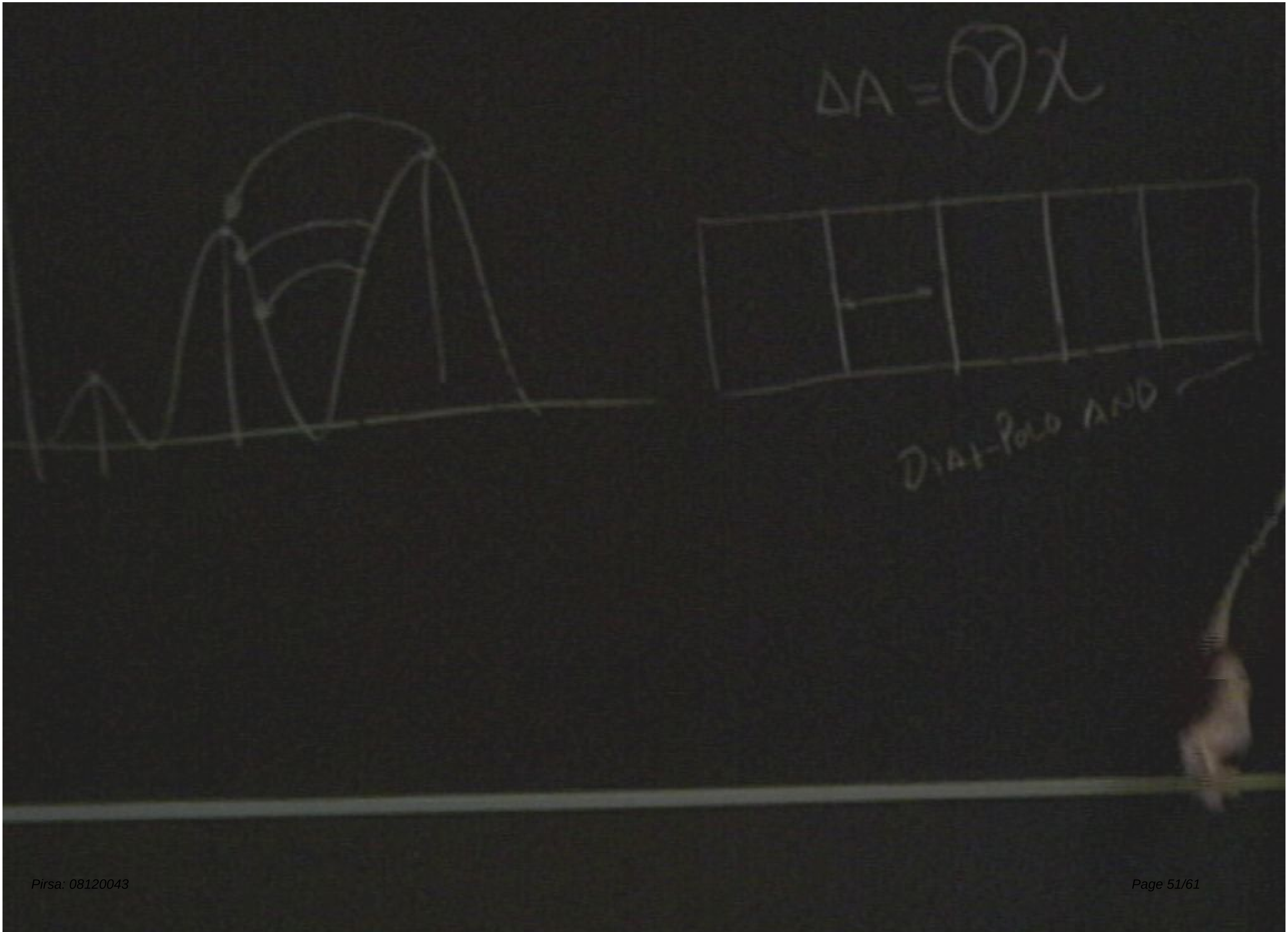

ΔA = γχ
DIAL-POLO AND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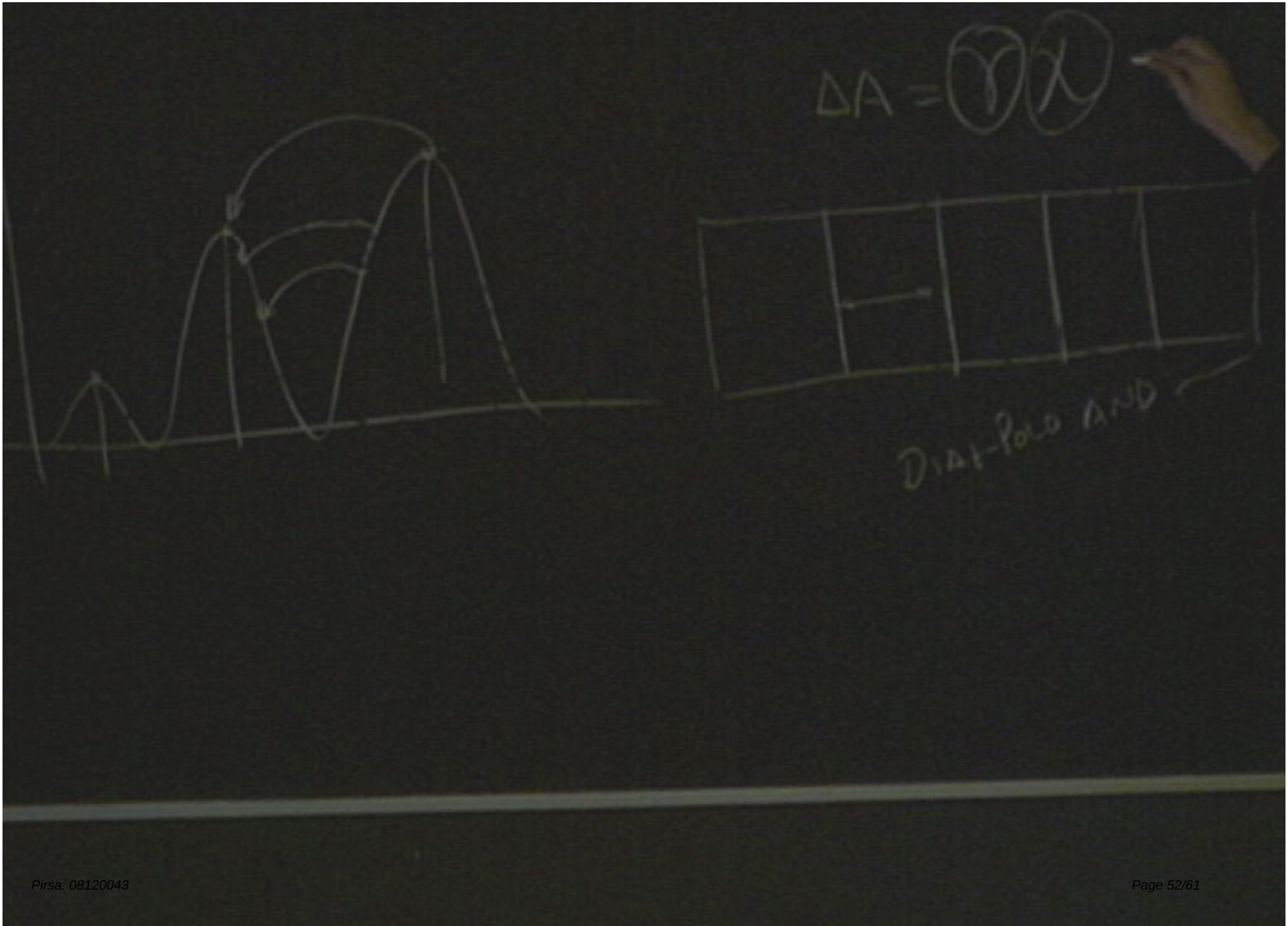
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$$\left(P\ SU(2) \right)$$

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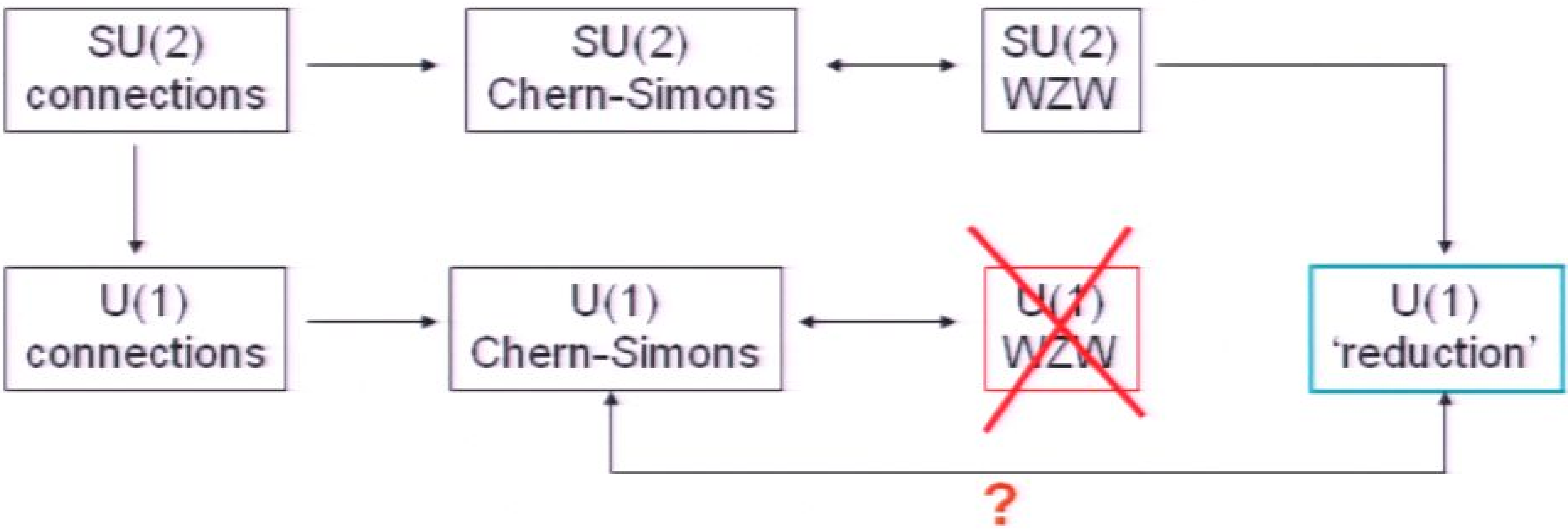
$$\chi_k = 2\cos(k\theta)$$

Beyond the stair

- How to account for the U(1) degrees of freedom?



Beyond the stairs

Agullo, Diaz-Polo, F.B. work in progress

- With this, we can project the product of characters over the character of the trivial U(1) irreducible representation, to require the gauge invariance
- This can be seen as an elegant way of imposing the projection constraint
- The characters of the representations we are using are

  $\chi_k = z^k + z^{-k} = 2\cos(k\theta)$ provided $z = e^{i\theta} \in U(1)$
- For each $\{k_i\}$ list, compute the product of characters

$$\chi_{k_1}\chi_{k_2}\cdots\chi_{k_n} = \prod_{i=1}^{n} 2\cos(k_i\theta)$$

- Projecting over the character $\chi_0 = 1$ of the trivial U(1) irrep

$$\left\langle \chi_{k_1}\chi_{k_2}\cdots\chi_{k_n} \middle| \chi_0 \right\rangle = \frac{1}{2\pi}\int_0^{2\pi} d\theta \prod_{i=1}^{n} 2\cos(k_i\theta)$$

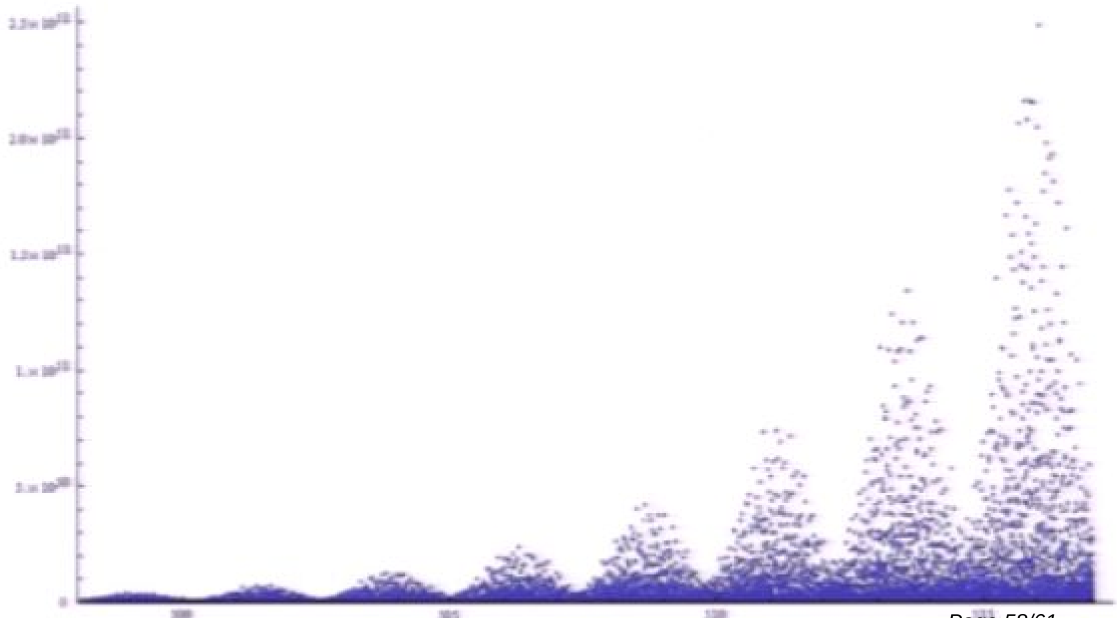
Playing with numbers

- Sign assingnments → Equivalent to a 'partition problem' in number theory
- Solution, given a $\{k_i\}$ list:

$$P(\{k_i\}) = \frac{2^n}{M}\sum_{s=0}^{M-1}\prod_{i=1}^{n}\cos(2\pi s k_i/M), \qquad M = 1+\sum_i k_i$$

- Reorderings for a $\{n_k\}$ list:

$$R(\{n_k\}) = \frac{(\sum_k n_k)!}{\prod_k n_k!}$$



Playing with numbers

Barbero, Villasenor **Phys.Rev.D77:121502,2008,** and arXiv:0810.1599 [gr-qc]

Agullo, Barbero, Diaz-Polo, F.B., Villasenor work in progress

- We can compute the whole degeneracy and also each single contribution making use of generating functions.
- Very convenient way of codifying **all** the information about the combinatorial problem
- Allow to obtain closed analytical expressions for the solution
- For instance, the solution to the sign assignments is given by the expression:

$$\frac{1}{2\pi}\int_{0}^{2\pi} d\theta \prod_{i} 2\cos(k_i\theta)$$

- These kind of expressions make the computations even faster.

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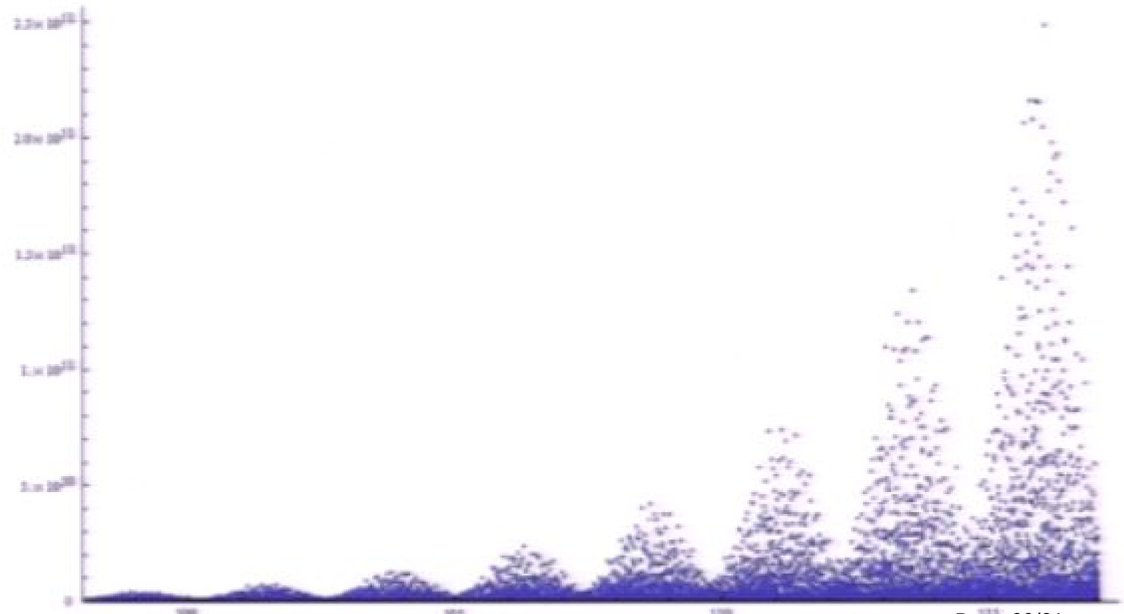

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