Title: The impossibility of partially local hidden variable models for quantum theory and the relation to cryptography

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Abstract: More than 40 years ago, Bell ruled out completely local hidden variable models as an explanation for quantum correlations. However, a new type of hidden variable model has recently been brought to light by the work of Leggett. Such a model has both local and non-local parts. Roughly speaking, having a local part means that the measurement outcomes can be guessed with better than 50% success. In this talk, I will explain that there exist quantum correlations for which any hidden variable model must have a trivial local part. I will then discuss how an extension of the original theorem implies that these correlations can be used to enhance the quality of a private random string.

The impossibility of partially local hidden variable models for quantum theory and the relation to cryptography

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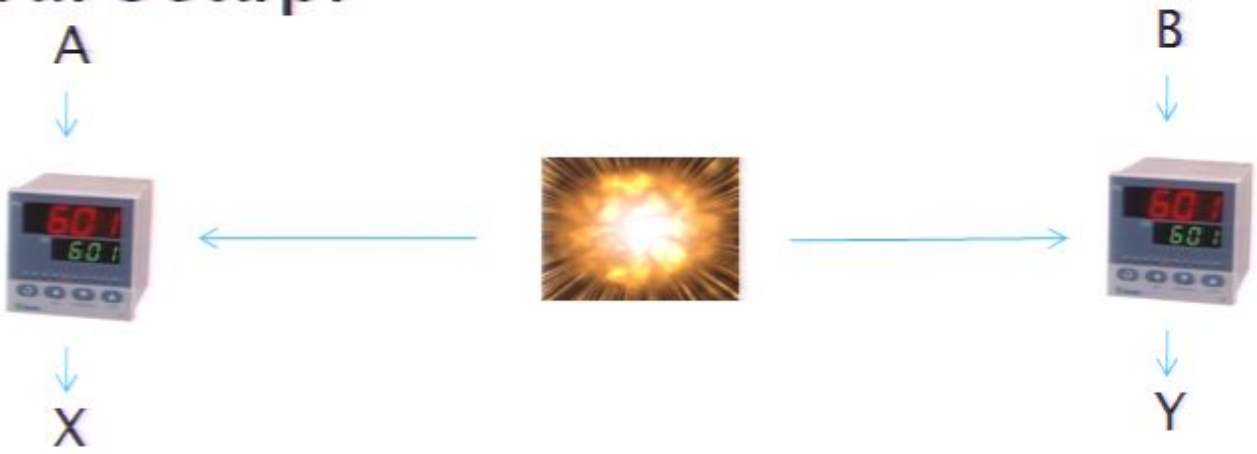
(based on work with Renato Renner
PRL 101 050403)

Hidden Variable Models

- General Setup:



- Scenario described by distribution P(X,Y|A,B).
- In quantum theory, X and Y are not determined until measured.
- Can the correlations be explained with a hidden variable model?

Hidden Variable Models

- We introduce U, V and W as hidden variables:

- These variables (if known) would determine the outcomes completely:

  i.e., X=f(A,B,U,V,W) and Y=g(A,B,U,V,W).

Hidden Variable Models

- In the Bell model (completely local), X=f(A,U), Y=g(B,V). This is incompatible with QM.
- Conversely, a completely non-local theory is compatible with all quantum correlations.
- What about in-between models with a local and non-local part?

Mixed models

- We have X=f(A,B,U,V,W) and Y=g(A,B,U,V,W).
- To give the model a local part, we restrict P(X|ABUV)=P(X|AU) (and similarly for Y).
- A local part essentially means that given knowledge of local parameters only, we have some knowledge about X.

No local part

- For a particular set of quantum correlations, it can be shown that P(X|AU) is uniform (and therefore independent of A and U).
- Hence, hidden variable models for quantum mechanics have no local part.
- In other words, even given access to all the local parameters, the outcomes of the measurement devices are completely unpredictable.

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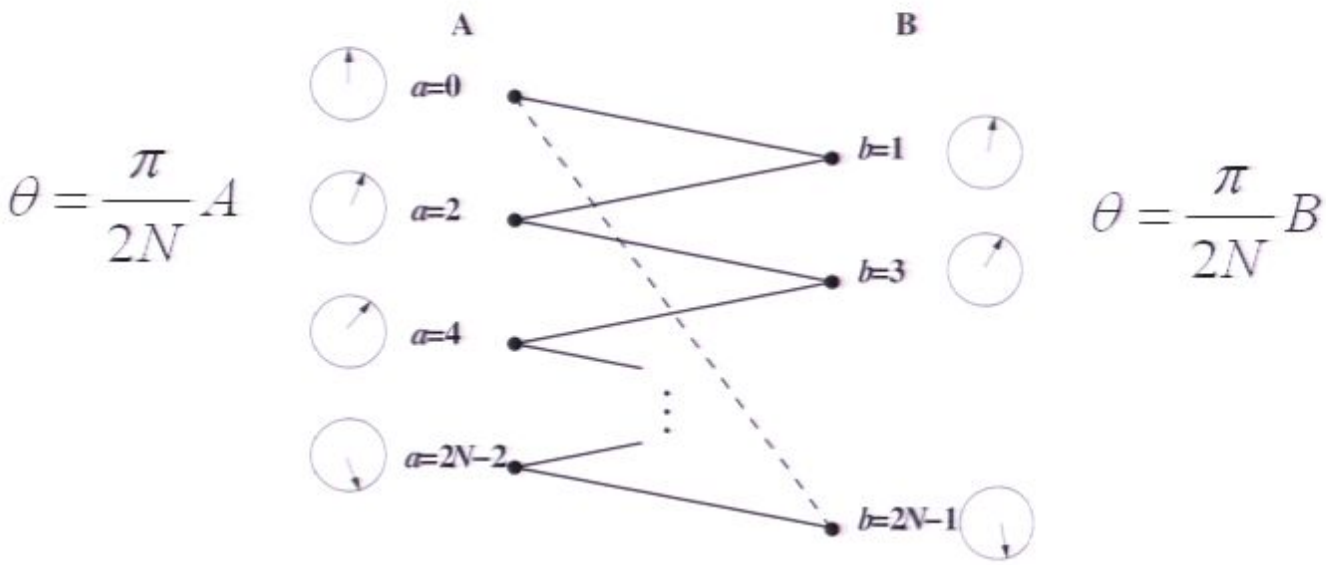
Chained Bell Inequalities

- In order to show our result, we use chained Bell inequalities:

$$I_N := P(X = Y \mid 0, 2N-1) + \sum_{|A-B|=1} P(X \neq Y \mid A, B) \geq 1$$



Main Theorem

- In the original version of the theorem, we show that, for any non-signalling distribution and for A and B independent of the hidden variables, that, for all a, b

$$D(P_{XU|a}, P_{\overline{X}} \times P_U) \leq \frac{I_N}{2},$$

$$D(P_{YV|v}, P_{\overline{Y}} \times P_V) \leq \frac{I_N}{2}.$$

$P_{\overline{X}}$ denotes the uniform distribution on X and D is the variational distance.

Significance of D

- D is a measure of the distance between two distributions; the smaller D is, the closer the two distributions.
- D directly determines the maximum probability of correctly distinguishing the two distributions. $P_{guess} = \frac{1}{2}(1+D)$.
- If two distributions have distance D, they behave identically in all situations, except with probability at most D.

No local part

- Quantum mechanics allows us to obtain $I_N \approx \pi^2 / 8N \xrightarrow{N\to\infty} 0$. Hence, $D(P_{XU|a}, P_{\bar{X}} \times P_U)$ can be bounded by an arbitrarily small number.

- In the limit, we have $P_{XU|a} = P_{\bar{X}} \times P_U$, i.e. X is uniformly distributed, and independent of U.

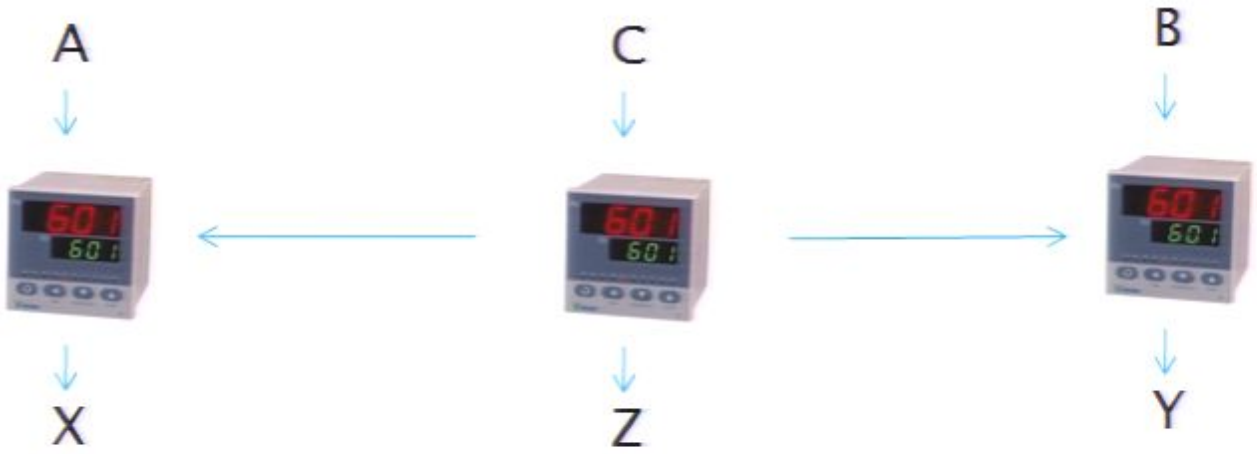
- In other words, there is no local part.

Cryptographic Setting

- Now consider the use of these correlations for cryptography, i.e. Alice and Bob use them to establish a shared private key.

Cryptographic Setting

- We replace the source with a system held by an Eavesdropper, Eve.



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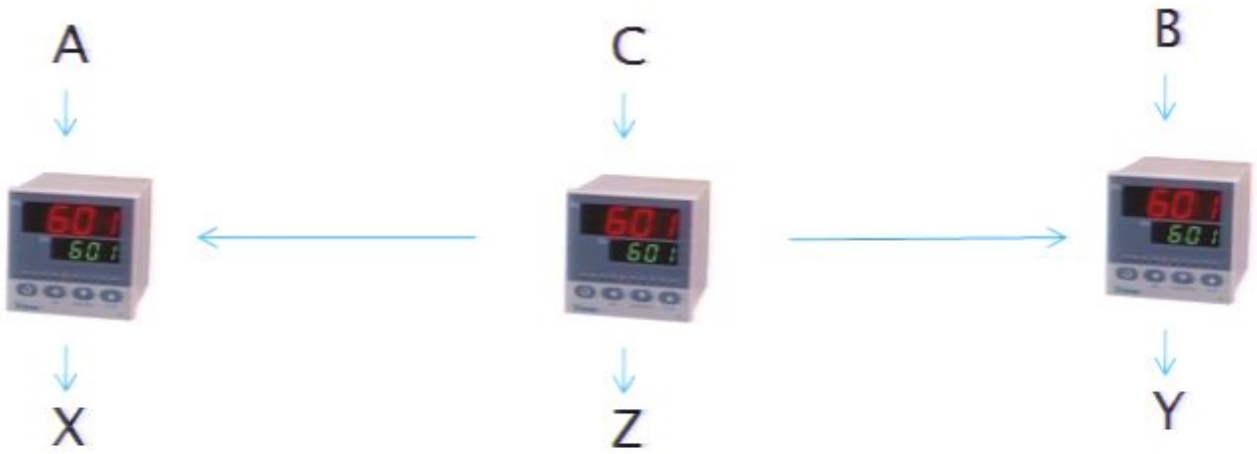

- We assume that the entire setup is created by Eve, and demand security even if so.
- If Alice and Bob can verify that I_N is small, then they are assured of the key's security.

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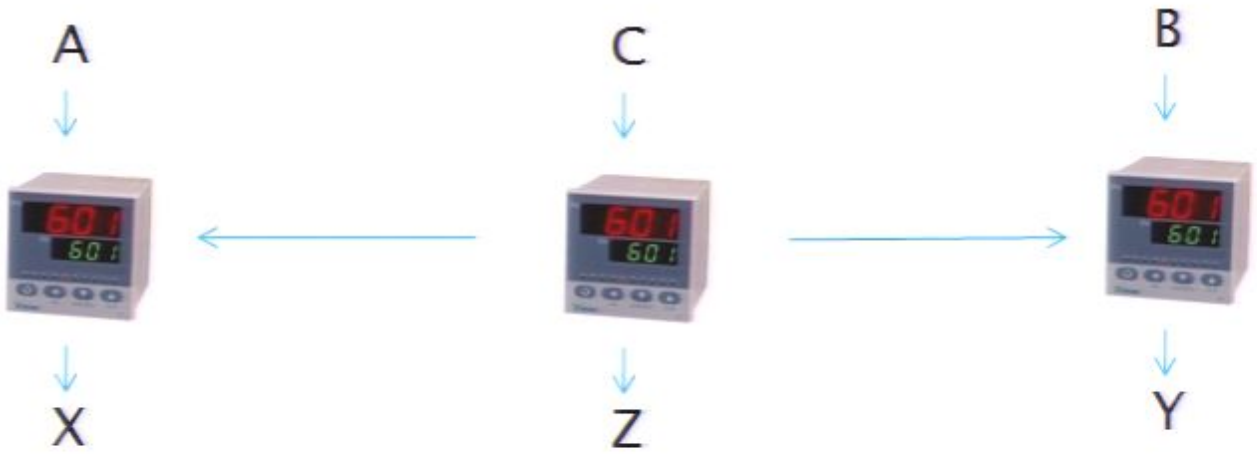

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Stronger Theorem

- In the original theorem, we bounded $D(P_{XU|a}, P_{\overline{X}} \times P_U)$ assuming that the choice of measurement, A, is independent of the local hidden variables.
- If the hidden variables could be chosen knowing A and B, then a completely local hidden variable model is possible.
- Amazingly, we can show that even if U and V are almost completely dependent on A and B, then the output is close to uniform.

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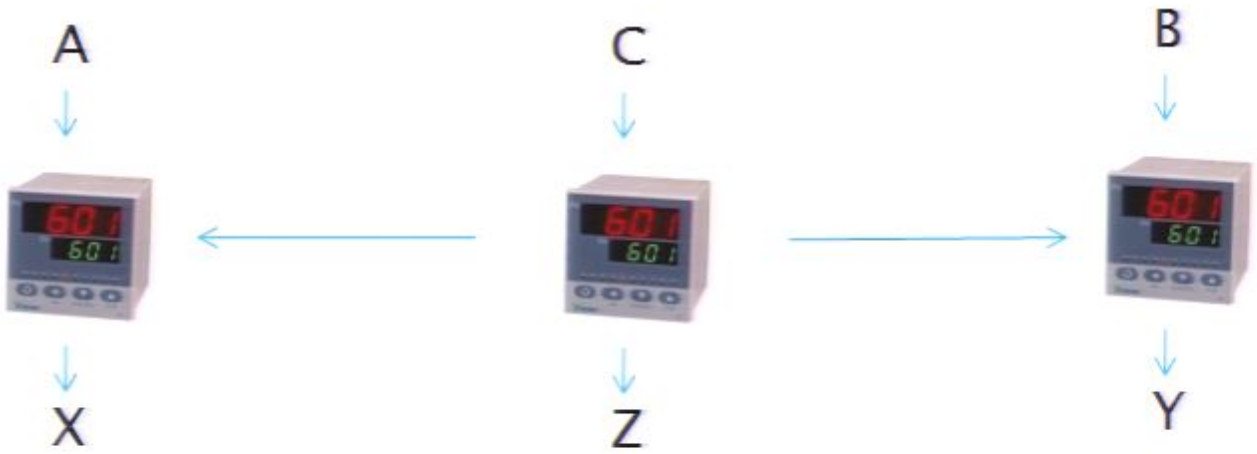

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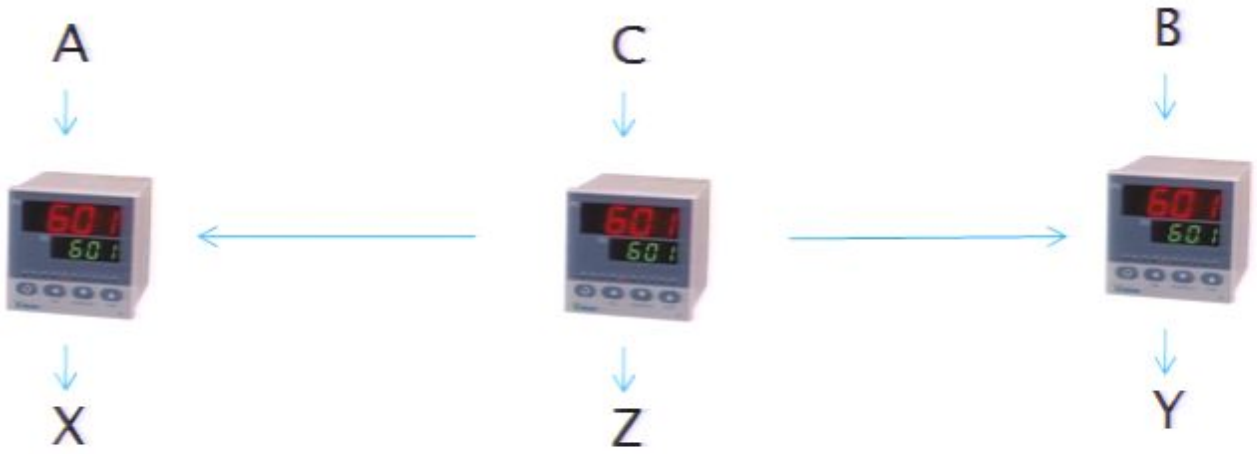

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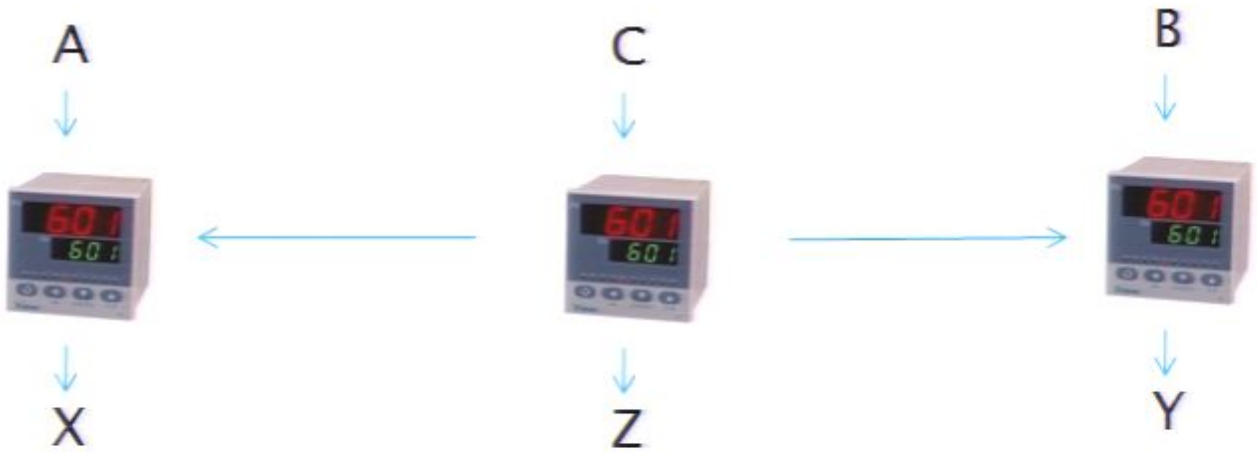

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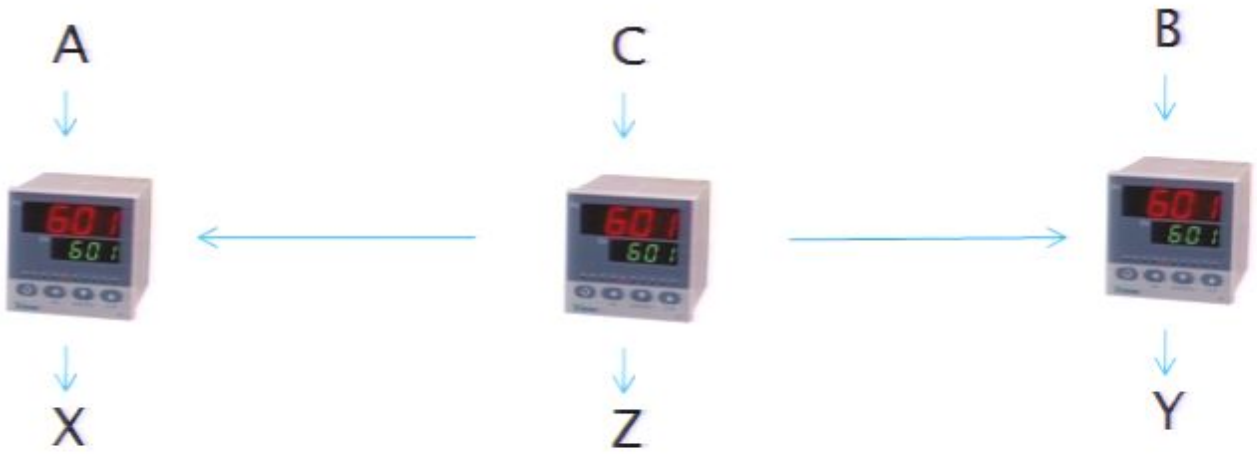

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- For any distribution P(XYZ|ABC) that is non-signalling between Alice and Bob, we have

$$D(P_{XZC|ab}, P_{\overline{X}} \times P_{ZC|ab}) \leq \frac{I_N}{2}.$$

- In other words, even if C and Z can depend on A and B, if I_N is small, then X is close to uniform.
- This shows that if Alice and Bob have strings of low privacy, they can enhance this privacy.

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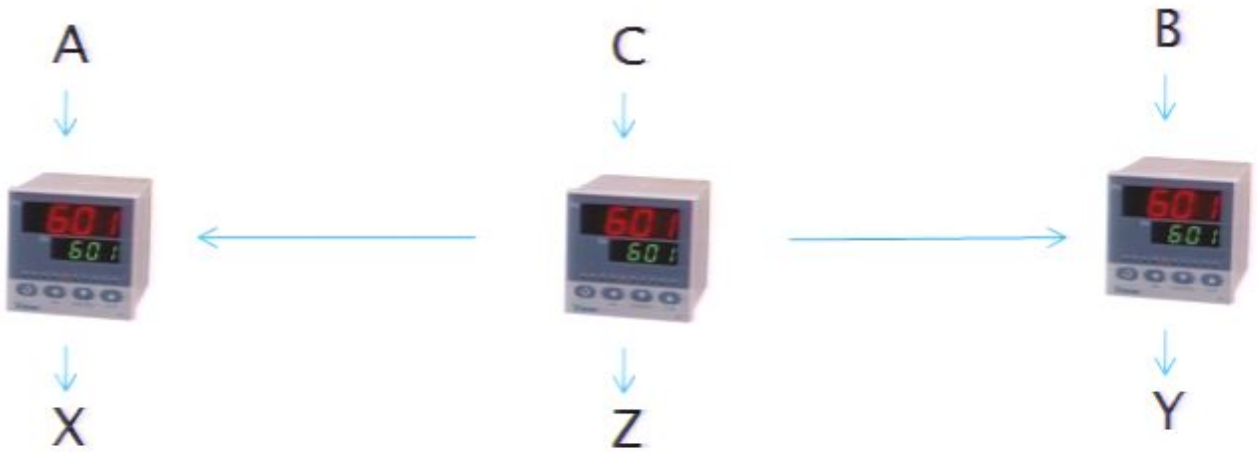

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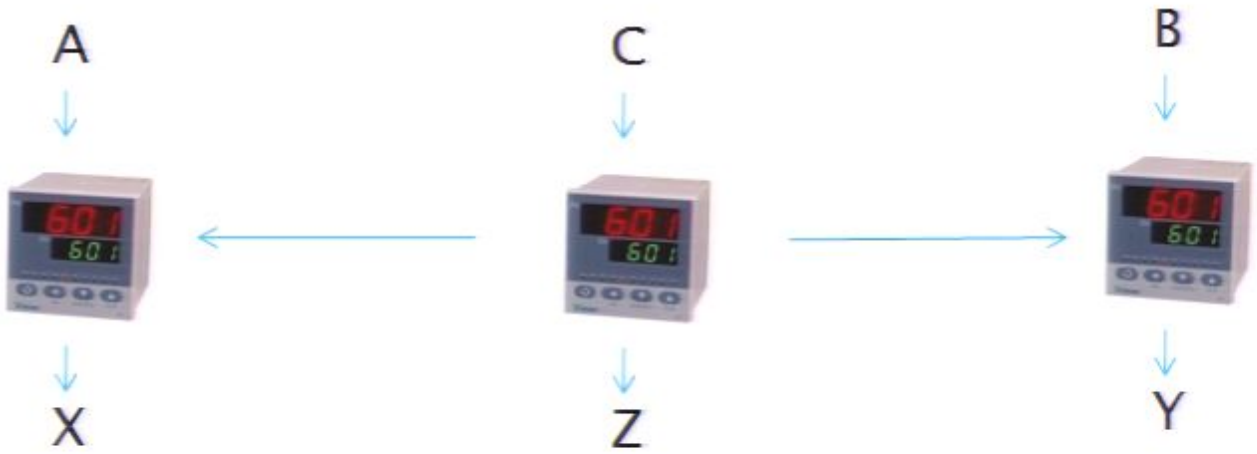

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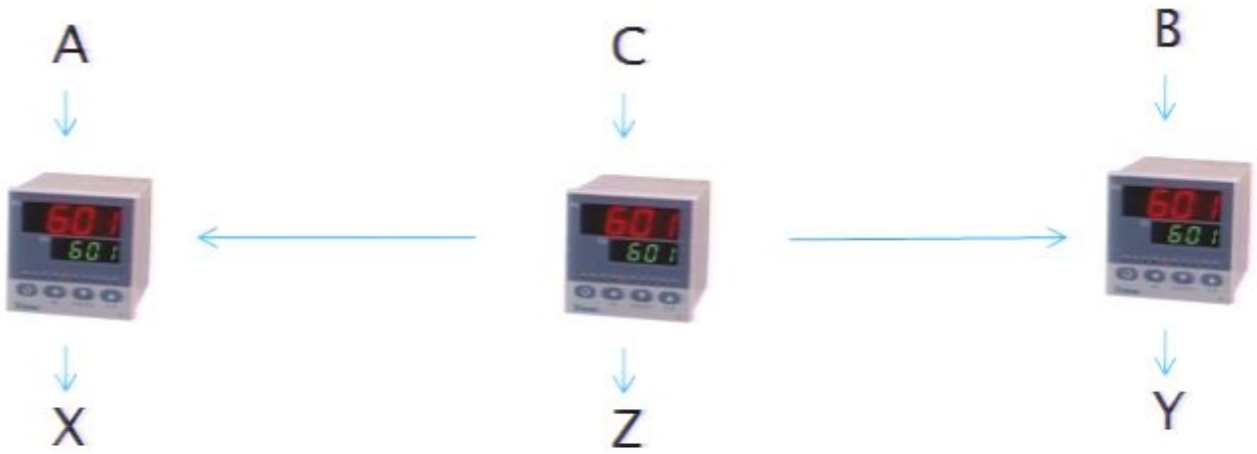

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- The local part is related to the knowledge an eavesdropper could have on the privacy of a string. Correlations with no local part are completely private.
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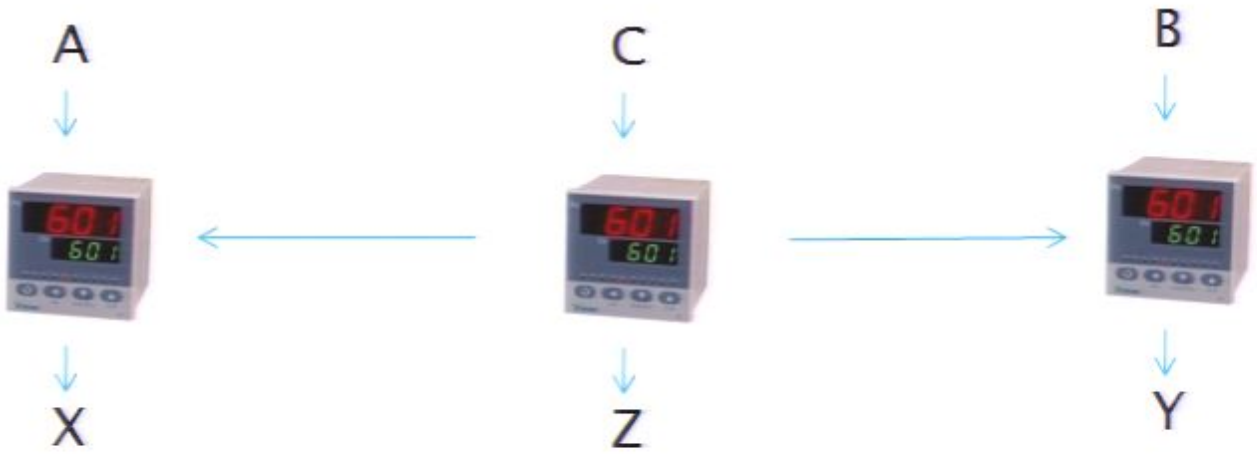

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