

Title: Structure Formation in Modified Gravity

Date: Dec 09, 2008 03:00 PM

URL: <http://pirsa.org/08120030>

Abstract: Instead of adding another dark component to the energy budget of the Universe in trying to explain the accelerated expansion, one can ask whether the cause is in fact the laws of gravity itself on the largest scales. In this talk, I will consider a sub-class of so-called  $f(R)$  gravity theories which closely follow the  $\Lambda$ CDM expansion history, while at the same time evading tight Solar System constraints on gravity. I will present new results from cosmological N-body simulations which consistently solve for the modified gravitational force. In particular, I will discuss the effects of modified gravity on structure formation, dark matter halo properties, and cosmological observables.

# The Universe is Accelerating

- 3 ingredients of standard cosmology:

Homogeneity & Isotropy  
- FRW metric

General Relativity (GR)  
- Friedmann Equation

Stress-Energy Content  
- Matter & Radiation

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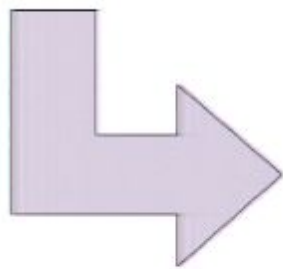
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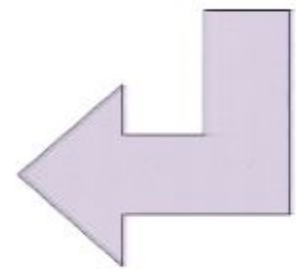
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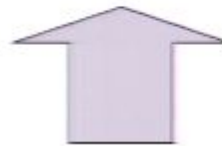
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**Decelerating Universe**



What is wrong ?  
Or missing ?



Stress-Energy Content  
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# Physics behind Acceleration

- **Modify any of the ingredients:**

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**Accelerating Universe**

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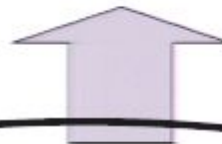
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**Accelerating Universe**



Stress-Energy Content  
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and  
Dark Energy ?

# Physics behind Acceleration

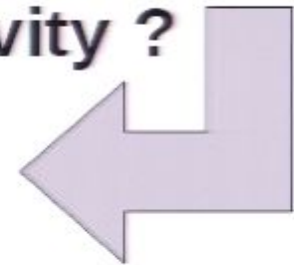
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Modified Gravity ?

**Accelerating Universe**



*(focus of this talk)*

Stress-Energy Content  
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# Physics behind Acceleration

- Minimal solution: cosmological constant  $\Lambda$

Homogeneity & Isotropy  
- FRW metric

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Accelerating Universe

Stress-Energy Content  
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# Modified Gravity

- **Theoretical Challenge:** consistent gravity theory with accelerating cosmology
  - Has to satisfy all constraints on gravity
- **Observational Challenge:** distinguish between Modified Gravity and Dark Energy
  - i.e. *smooth, uncoupled Dark Energy*
  - No clean division in general case

# Modified Gravity: Challenges

## *Theoretical Challenge:*

- **Gravity constrained on wide range of scales:**
  - Early Universe: BBN, CMB
  - Growth of structure
  - Solar System
- **Idea: reduce to GR in high-curvature regime**
  - Applies to Early Universe as well as high-density regions today

# Modified Gravity vs Dark Energy

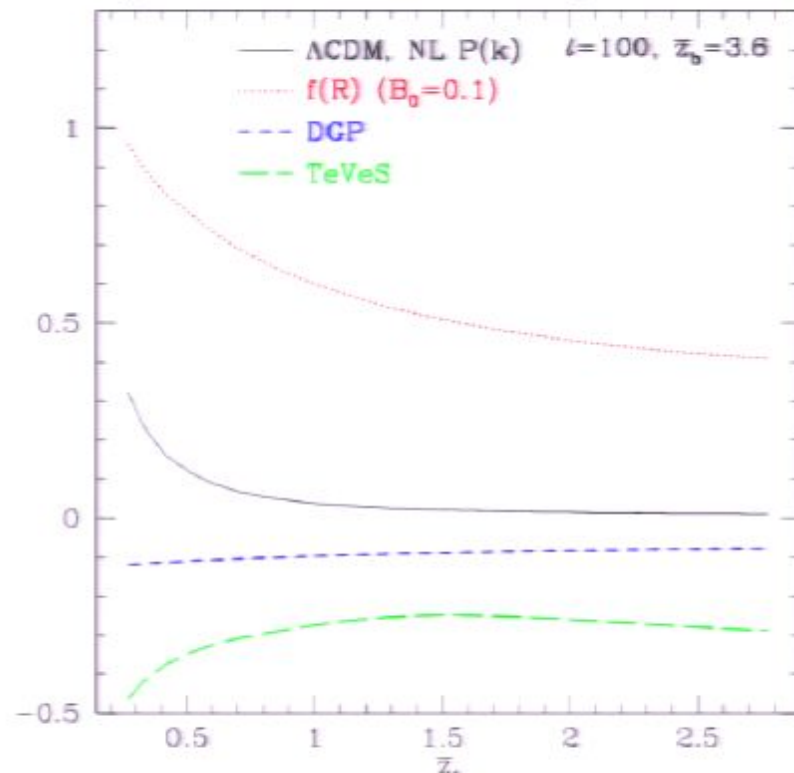
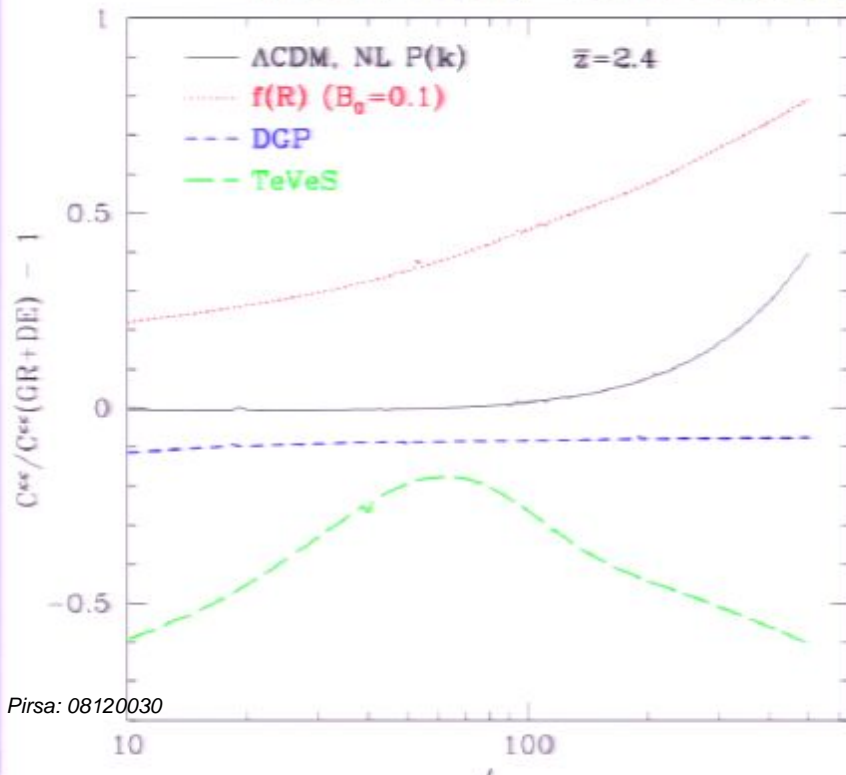
## *Observational Challenge:*

- How can we distinguish **Modified Gravity** from **GR + Dark Energy** ?
  - (Almost) any expansion possible with Dark Energy
- **Beyond background: growth of structure**
  - Predictions worked out in the *linear regime*
  - Compare predictions with GR+DE model with identical expansion history

# Weak Lensing as Probe Of Gravity

F.S., arXiv:0805.4811

- Shear correlations sensitive to growth of structure:
  - Redshift- and scale-dependence of growth



$C^{\kappa\kappa}(l)$

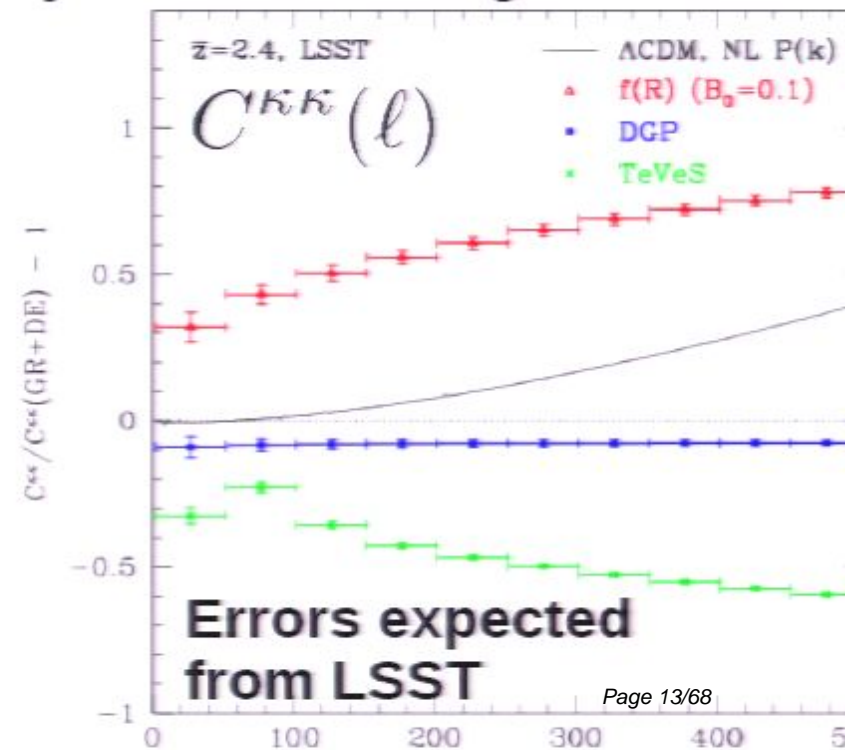
for mod. gravity rel. to GR+DE

restricted to linear scales

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F.S., arXiv:0805.4811

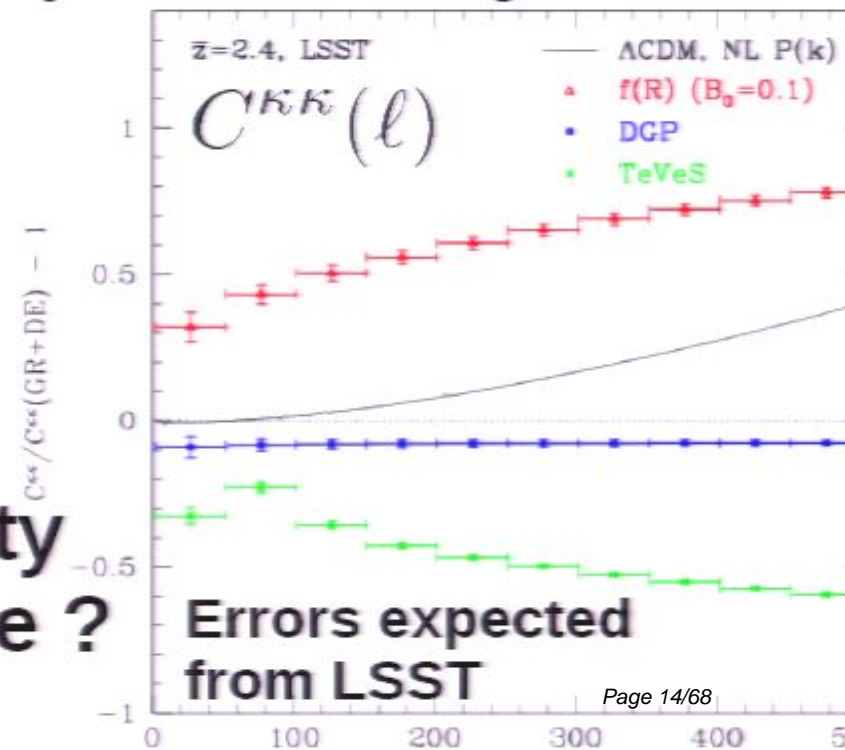
- Future weak lensing surveys will **constrain growth at percent level**
  - Most sensitive probe of gravity on cosmological scales



# Weak Lensing as Probe Of Gravity

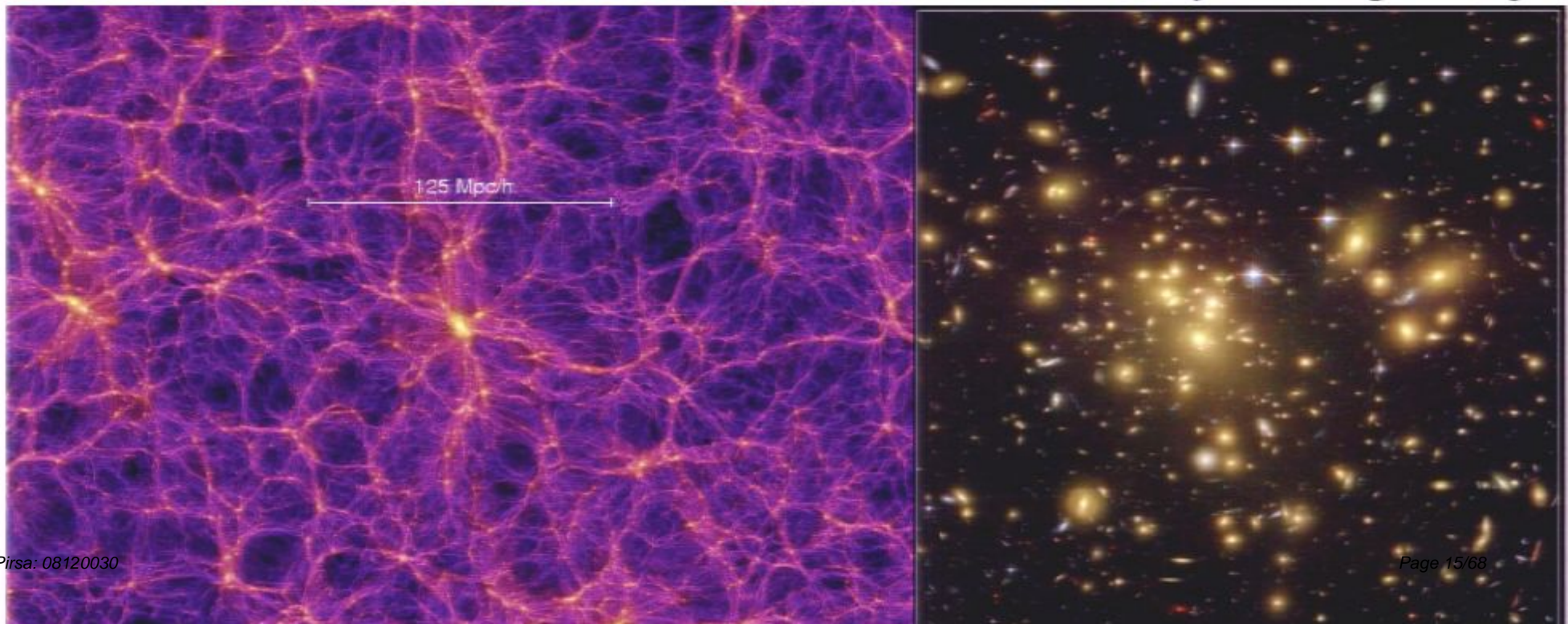
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- Future weak lensing surveys will **constrain growth at percent level**
  - Most sensitive probe of gravity on cosmological scales
- However: bulk of constraining power **in non-linear regime**
- How does modified gravity affect non-linear structure ?



# Probing Gravity with Cosmic Structure

- **Wide range of observables**
  - almost all in the non-linear regime
  - Can we unlock this vast information to probe gravity?



# f(R) Gravity

Hu & Sawicki, arXiv:0705.115

- **Simplest workable** modified gravity model
- **Generalize Lagrangian of General Relativity:**

$$\mathcal{L}_g = \frac{1}{16\pi G}(R - 2\Lambda) \longrightarrow \frac{1}{16\pi G}(R + f(R))$$

- **Choose function which (when close to GR) becomes:**

$$f(R) \approx -2\Lambda - f_{R0} \frac{R_0^2}{R},$$

- **Reduces to GR in high-curvature regime**



# f(R) Gravity

Hu & Sawicki, arXiv:0705.115

- **f(R) model produces  $\Lambda$ CDM expansion history **without true  $\Lambda$****
- Some fine-tuning involved...
- Departures from  $\Lambda$ CDM expansion of order  $f_{R0} \ll 1$
- **Equivalent to scalar-tensor theory**
- Scalar field  $f_R \equiv \frac{df}{dR} \rightarrow 5^{\text{th}}$  force
- Grav. force enhanced by 4/3 within  $\lambda_C = \sqrt{3f_{RR}}$

$$\lambda_C \approx 23 \text{ Mpc}/h \sqrt{\frac{|f_{R0}|}{10^{-4}}} \text{ in the background today}$$

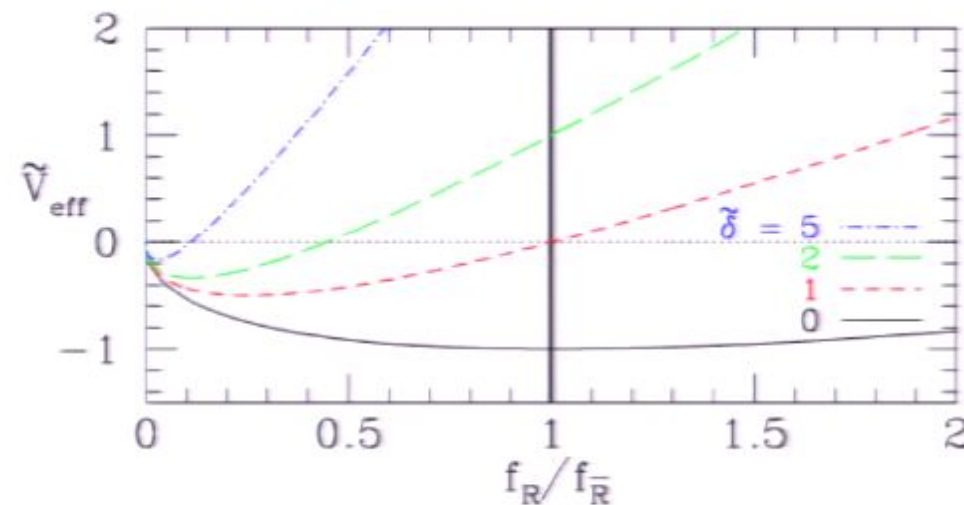
# Evading Solar System Tests: Chameleon Mechanism in $f(R)$

- **Scalar field  $f_R$  with density-dependent potential:**

$$\nabla^2 f_R = \frac{\partial \tilde{V}_{\text{eff}}(f_R; \rho_m)}{\partial f_R}$$

Khoury & Weltman, PRD, 2004

Hu & Sawicki, arXiv:0705.1158



- **GR restored in high-density environments**

– Chameleon operates when background field small enough:

$$\overline{f_R} \sim \Psi \lesssim 10^{-5}$$

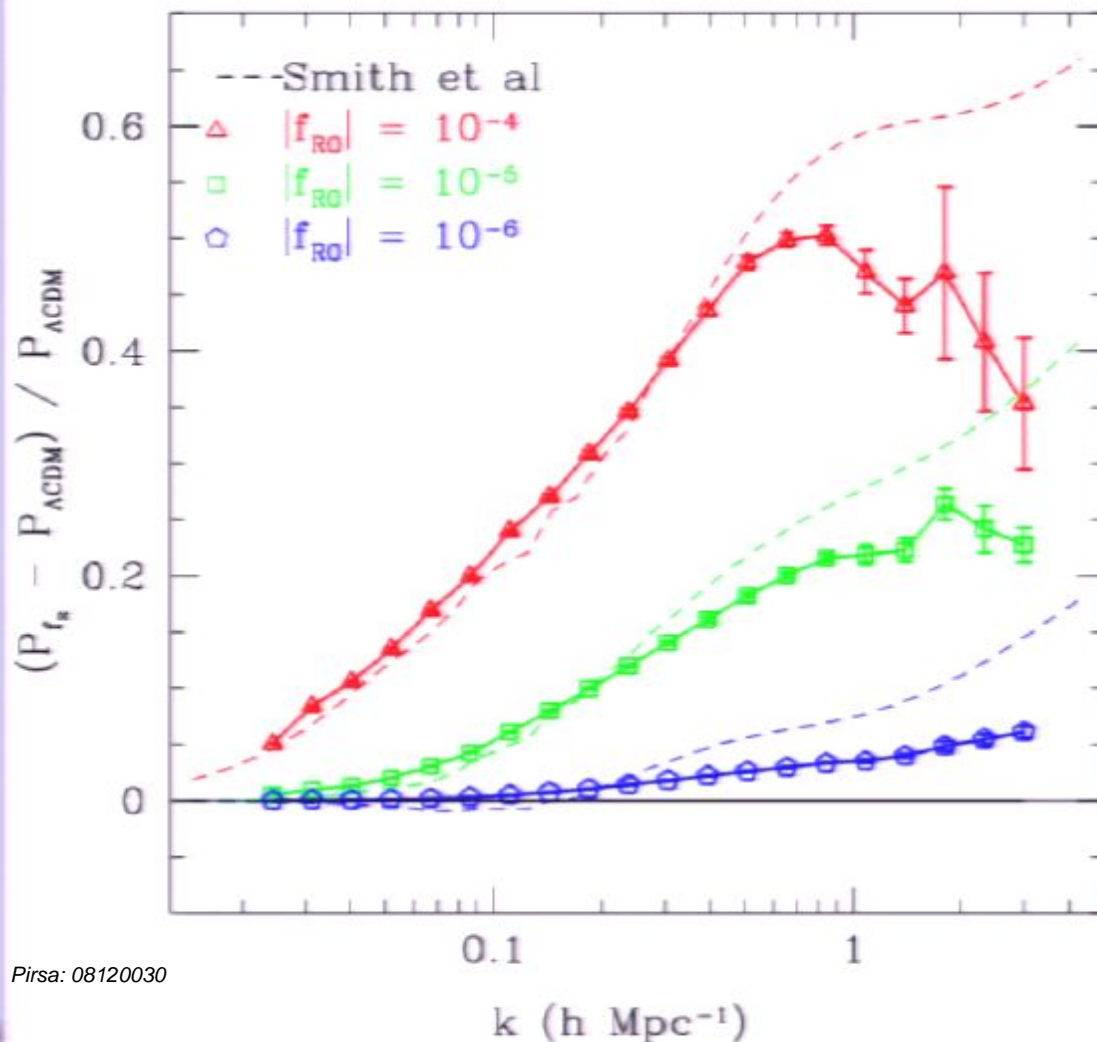
# **f(R) Gravity in the Non-linear Regime**

- **Modified forces exist throughout in f(R)**
  - $\lambda_C$  adds additional scale to the problem
  - Modifications to non-linear structure formation
- **Chameleon is non-linear effect**
  - Environment-dependent
- **Can't trust fitting formulas based on GR**

**Full N-body simulation of f(R) model needed**

# Matter Power Spectrum in $f(R)$

Oyaizu et al., arXiv:0807.2466



- **GR fitting formulas using linear  $P(k)$** 
  - **Fail to capture trend** at high  $k$
  - Enhancement overestimated
- **Can we model enhanced non-linear  $P(k)$  in  $f(R)$  ?**

# Dark Matter Halos in $f(R)$

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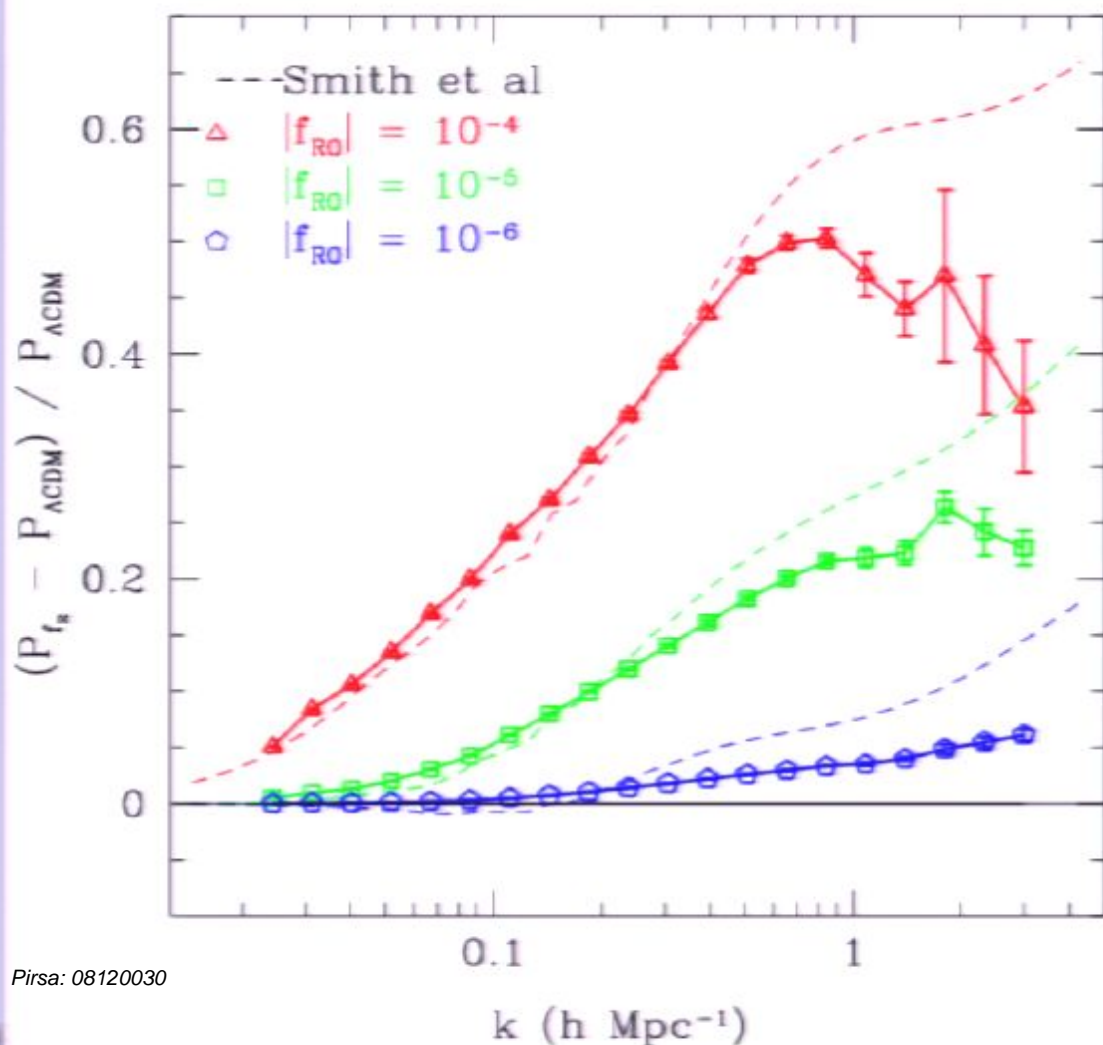
- “building blocks” of non-linear structure

## Halo properties:

1. Mass function
2. Halo bias
3. Halo profiles

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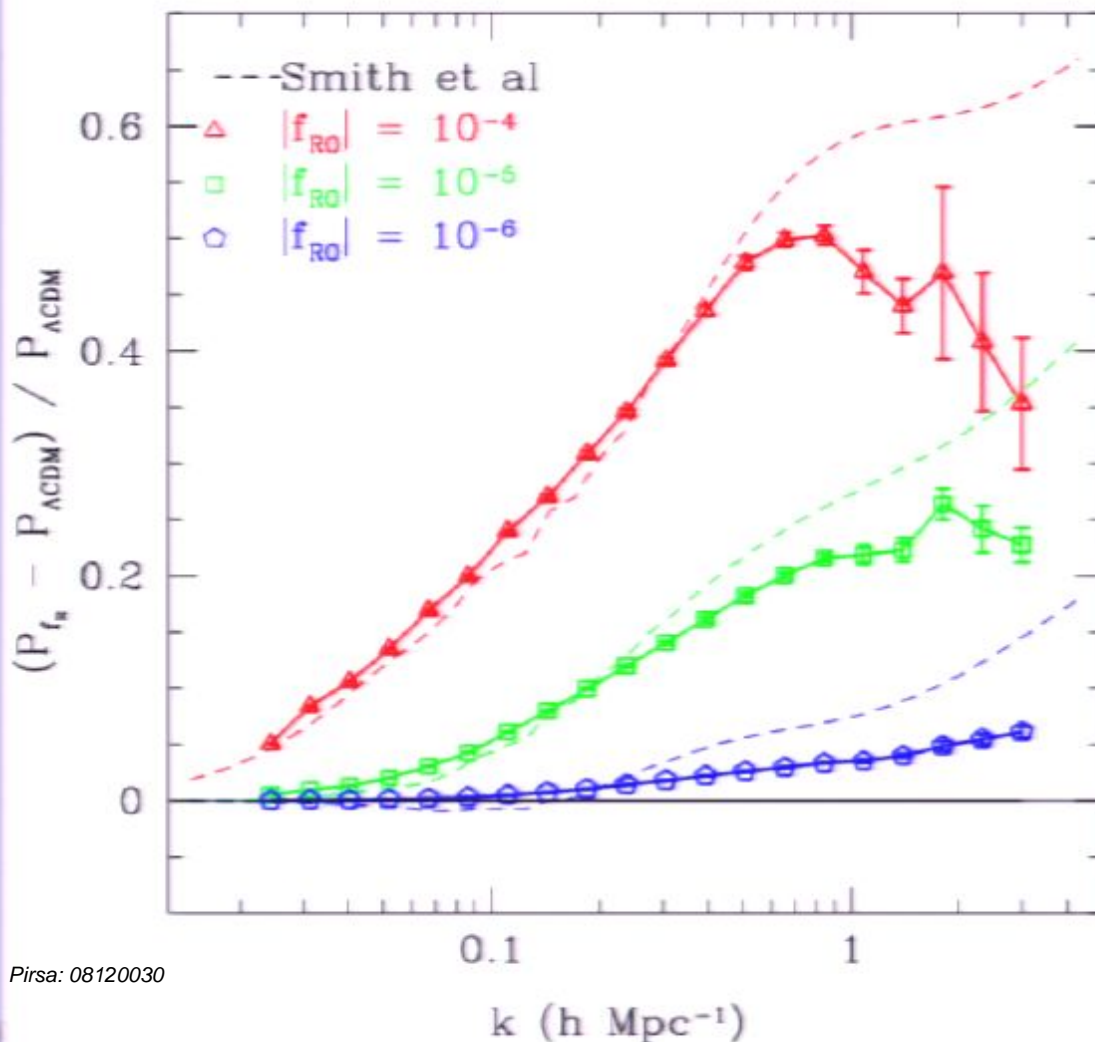
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Spherical Collapse &  
Press-Schechter Theory  
(Sheth-Tormen)

NFW profile  
+ concentration relation



# Dark Matter Halos in $f(R)$

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*Scale-dependent modification of gravitational force*

} - enhanced  $P_L(k)$   
- spherical collapse modified

Halo profiles modified ?

# Modified Spherical Collapse

- **Exact spherical collapse solution complicated: not scale-free** due to  $\lambda_C$
- **Simplest limiting cases** of  $f(R)$  model:
  - Unmodified forces  $\rightarrow$  GR
  - Forces enhanced by  $4/3$  throughout
- **For enhanced forces:**
  - Collapse threshold  $\delta_c$  increases
  - Virial overdensity  $\Delta_{\text{vir}}$  decreases

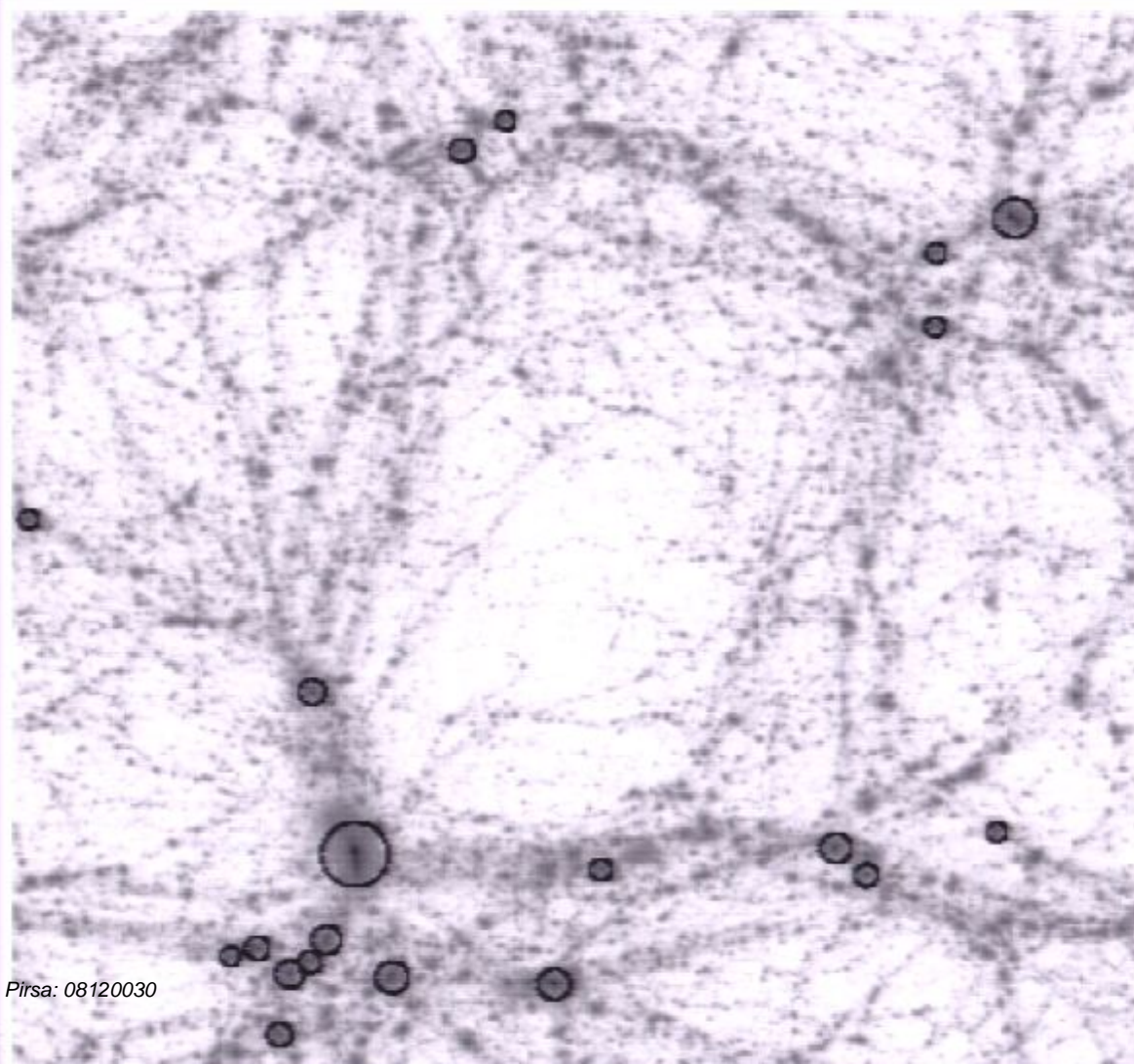
# Simulating $f(R)$ Gravity

H. Oyaizu, arXiv:0807.244

## Self-consistent solution of field & particles

- Particle-mesh code
- Given density field, solve **non-linear field equation** for  $f_R$  (*in quasi-static approx.*)
  - Multigrid relaxation scheme speeds up convergence
- Given  $f_R$ , calculate Newtonian potential
  - Propagate particles as in GR
- Compare with **“no-chameleon” simulations**
  - Using linearized field equation

# Results: Structure Formation

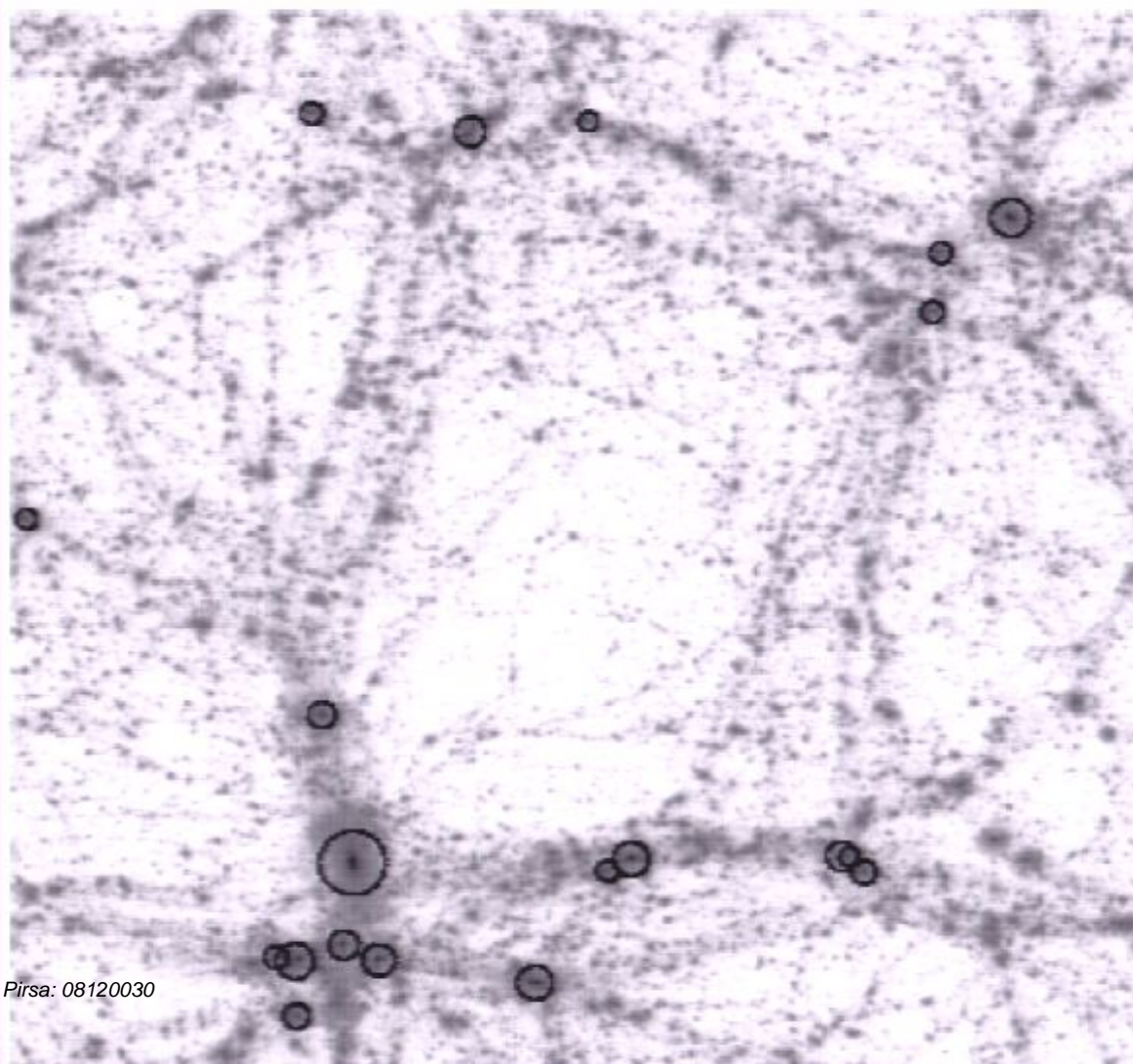


Slice through simulation  
at  $z=0$ , size: 64 Mpc/h

**GR -  $\Lambda$ CDM**

Circles: 20 most massive  
halos

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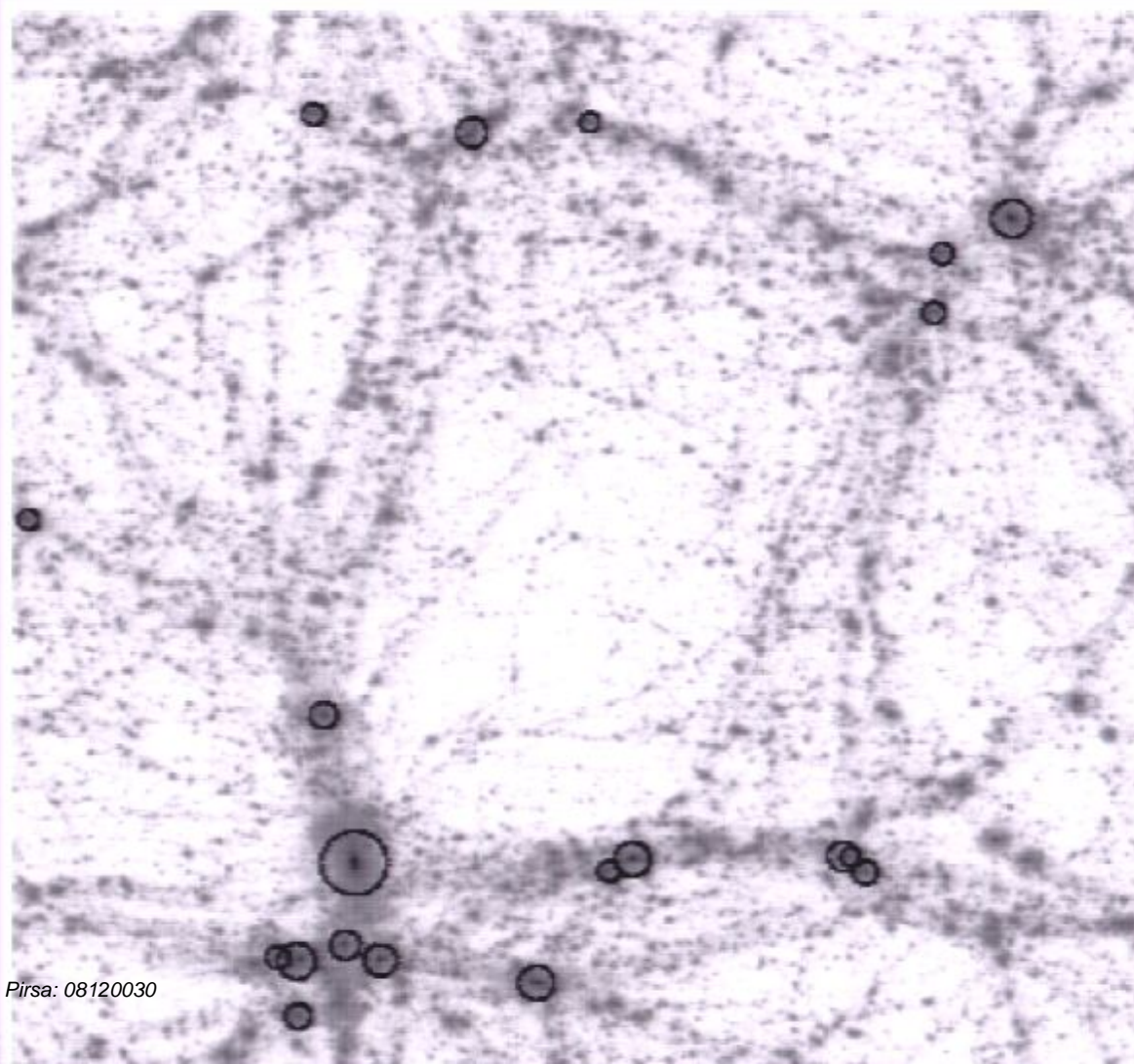


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$f(R)$  with  $f_{R0} = 10^{-4}$

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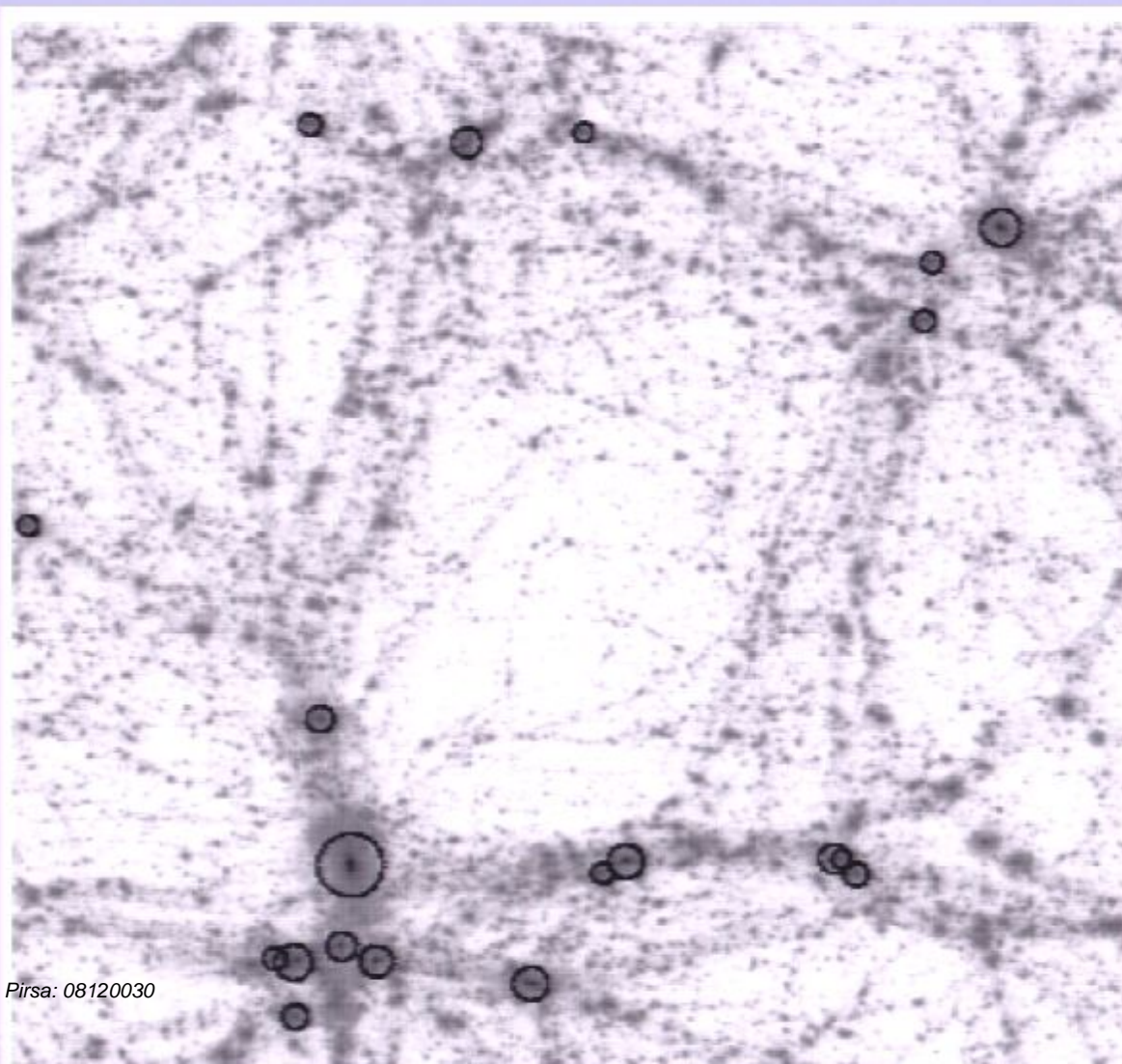
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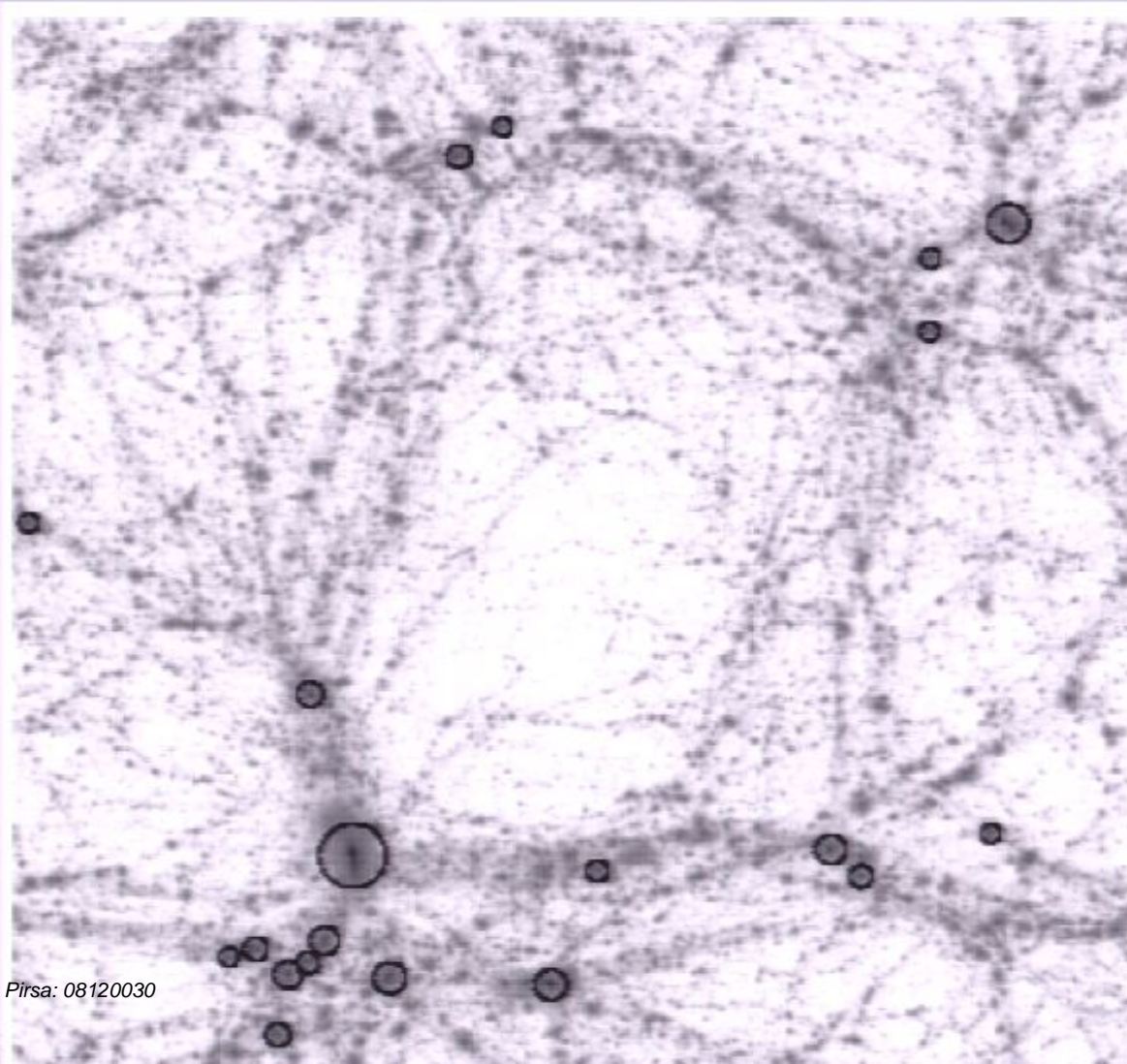


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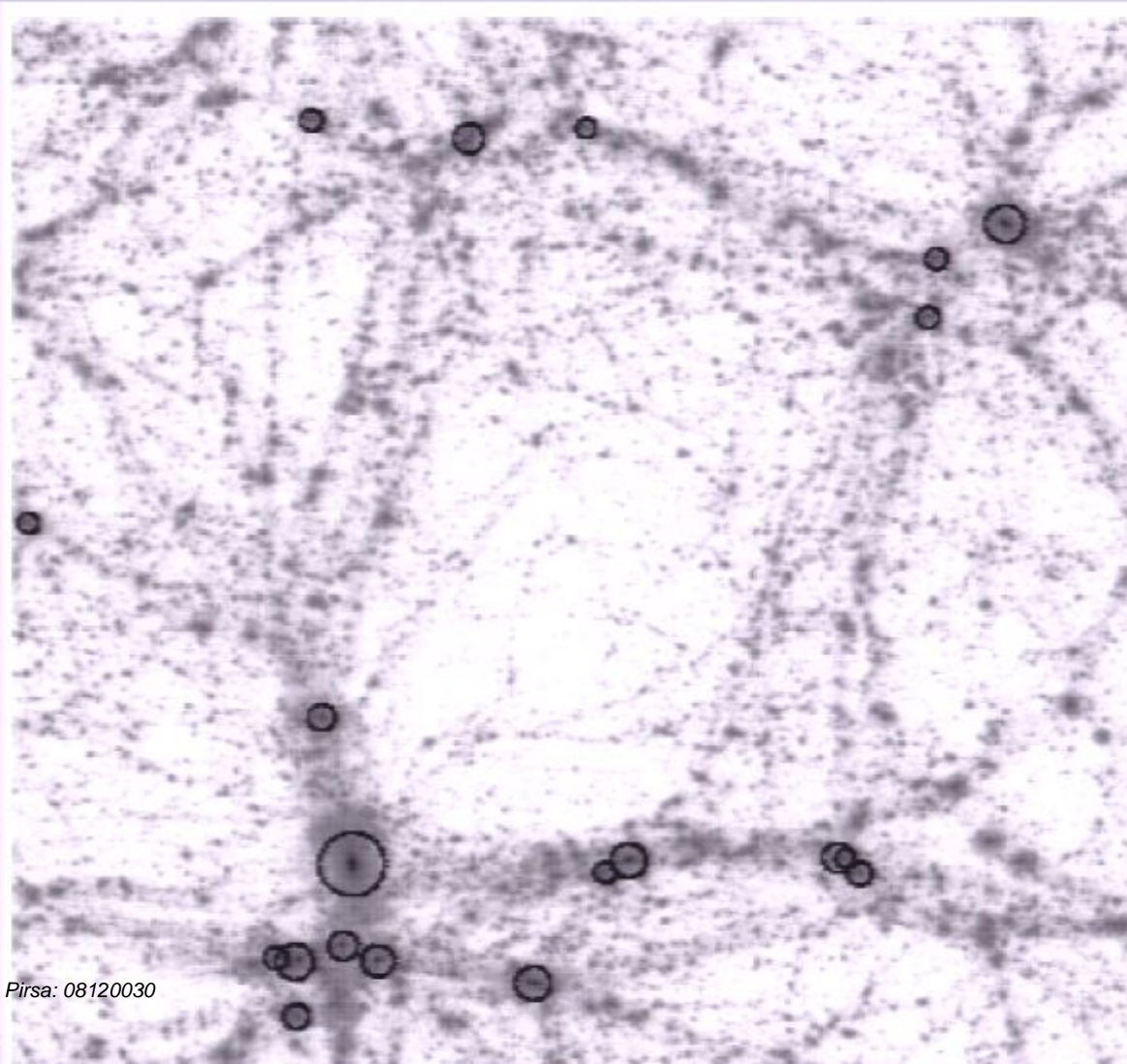


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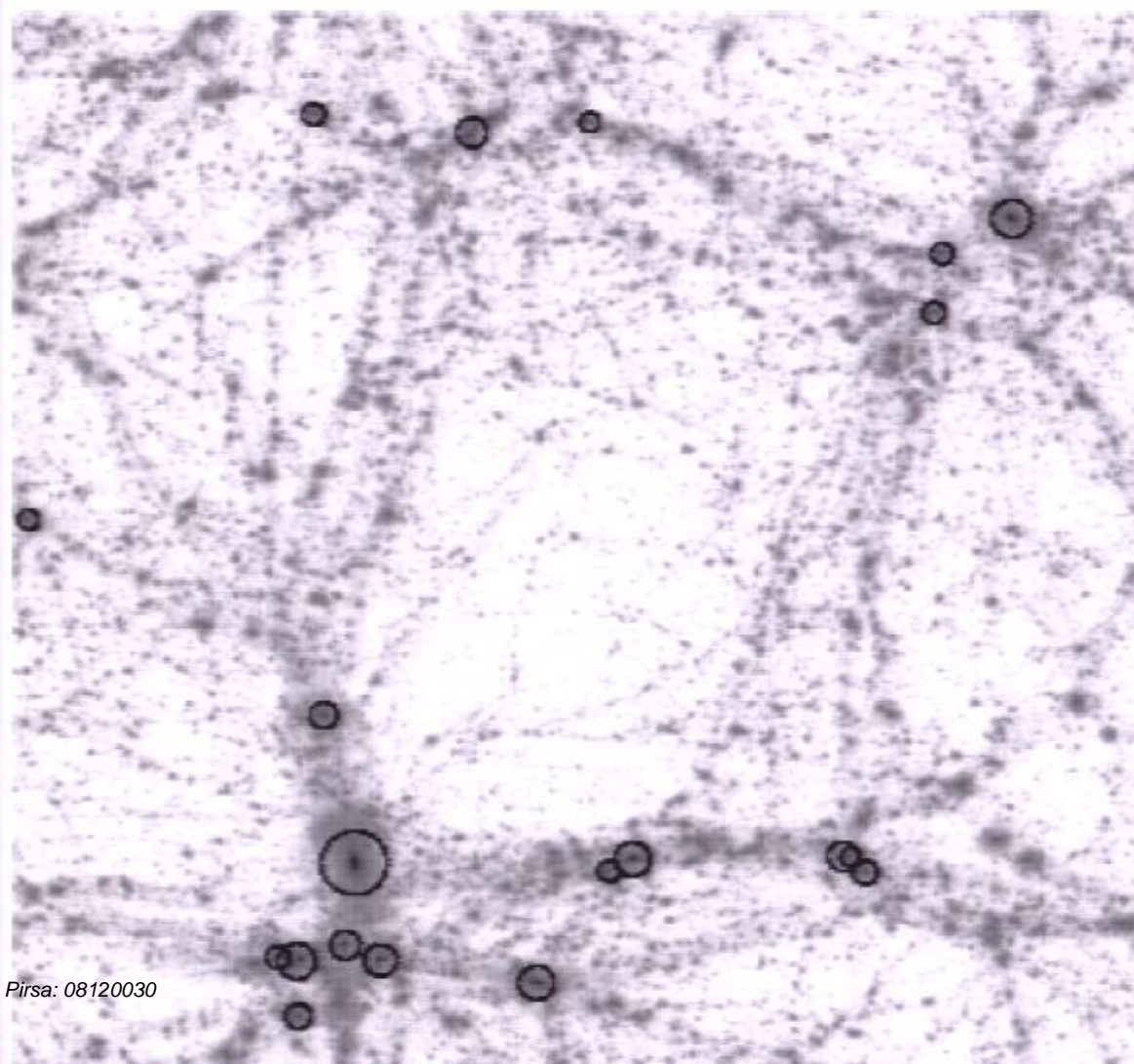


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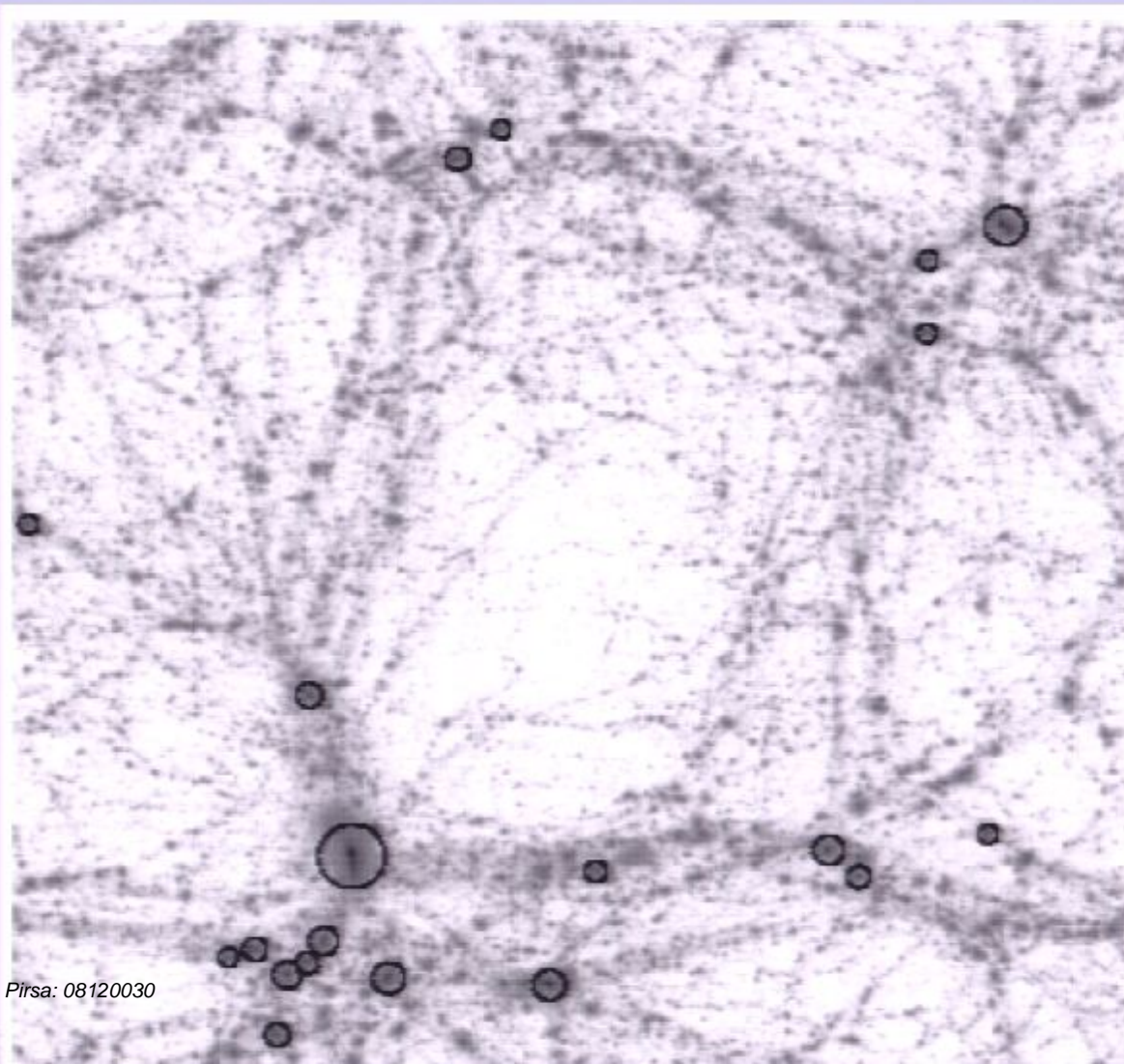


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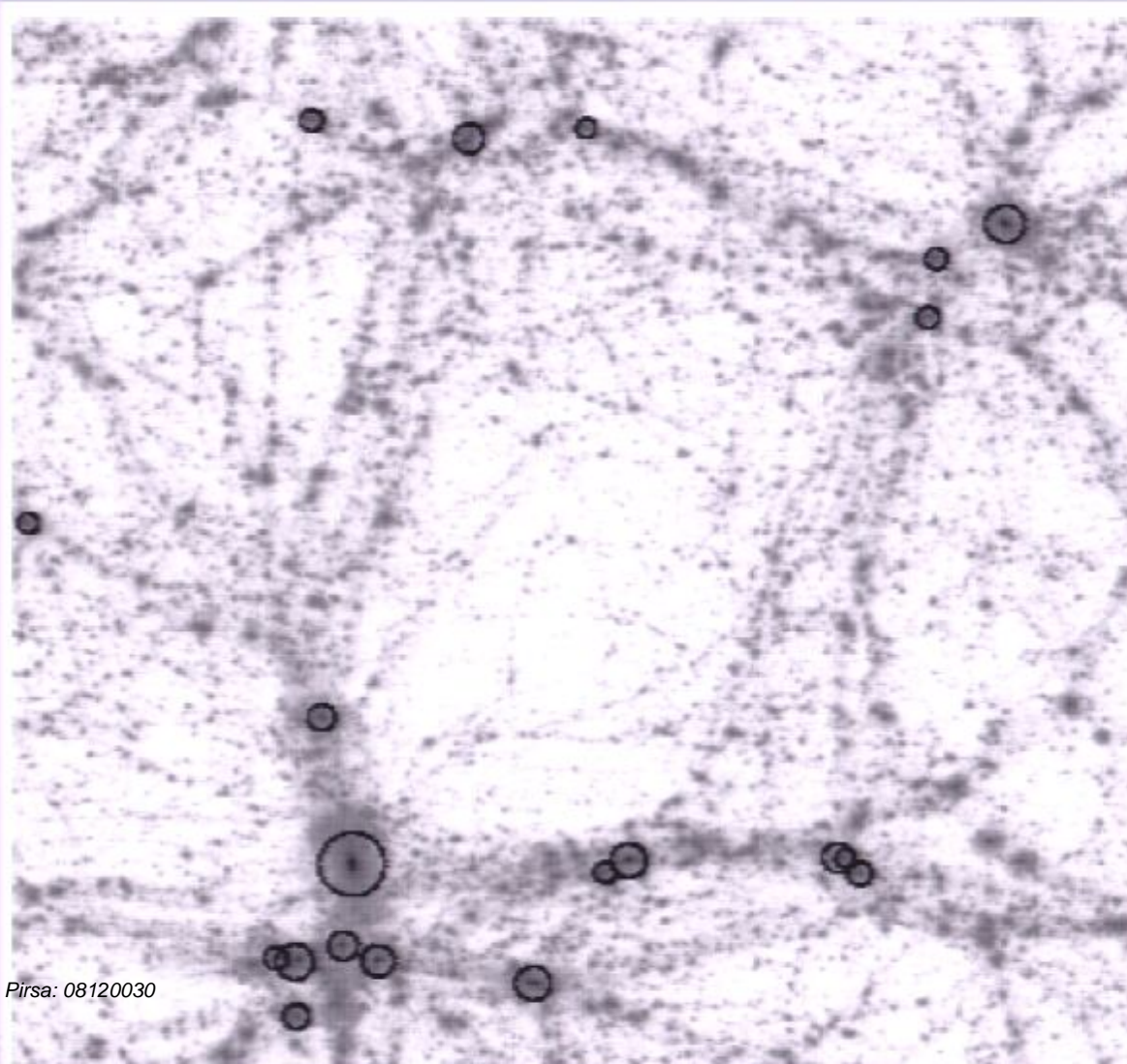


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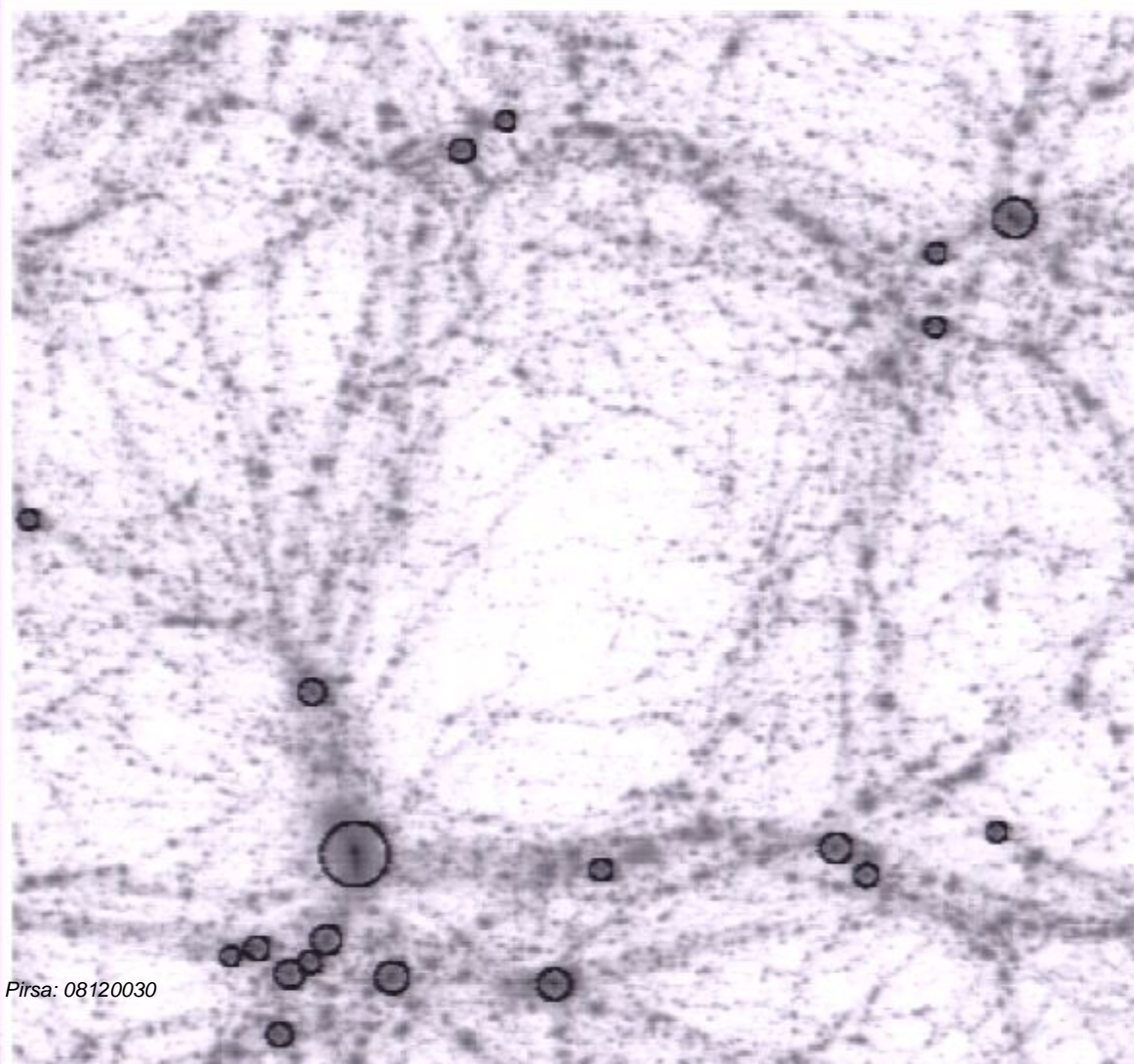


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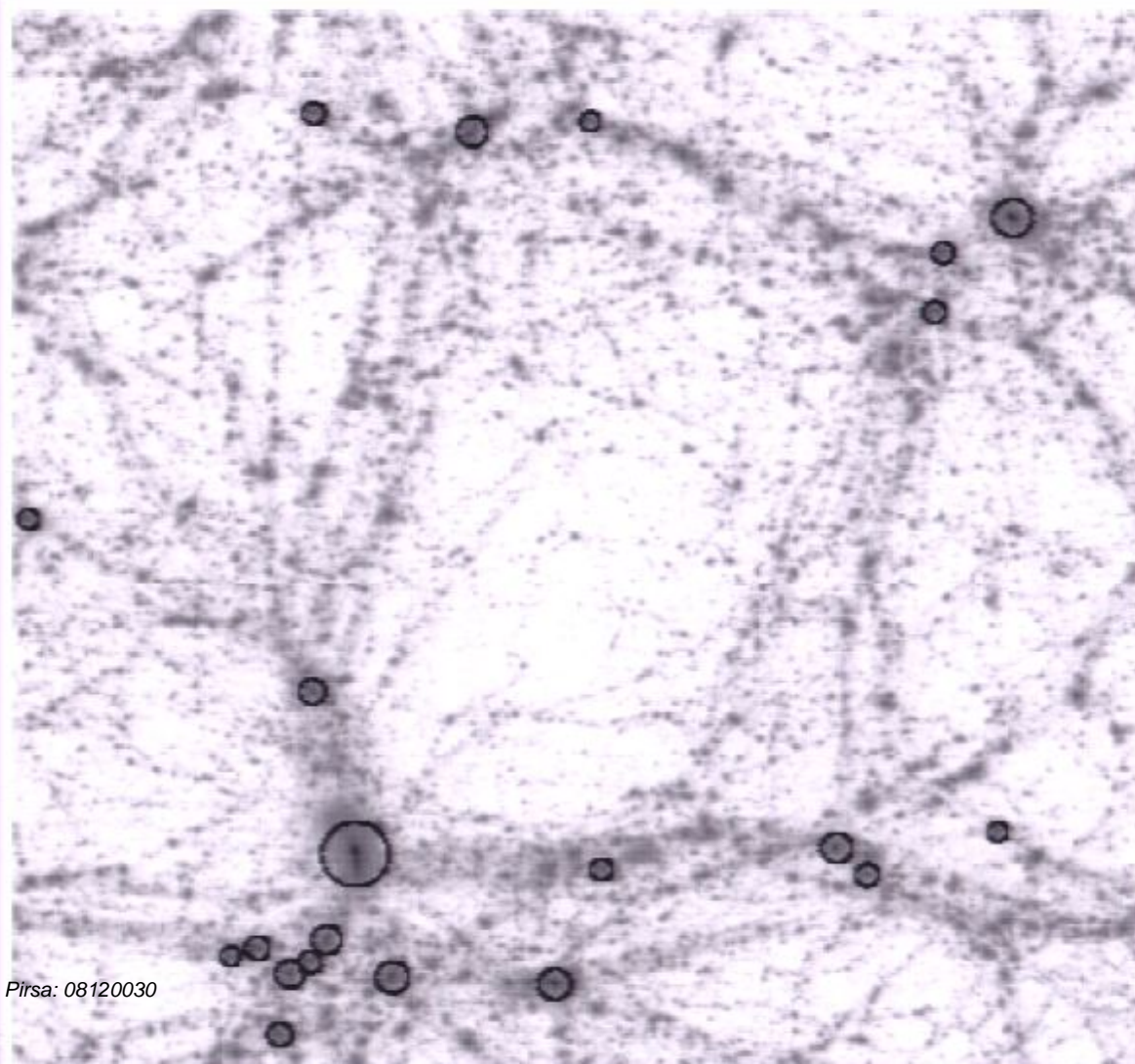


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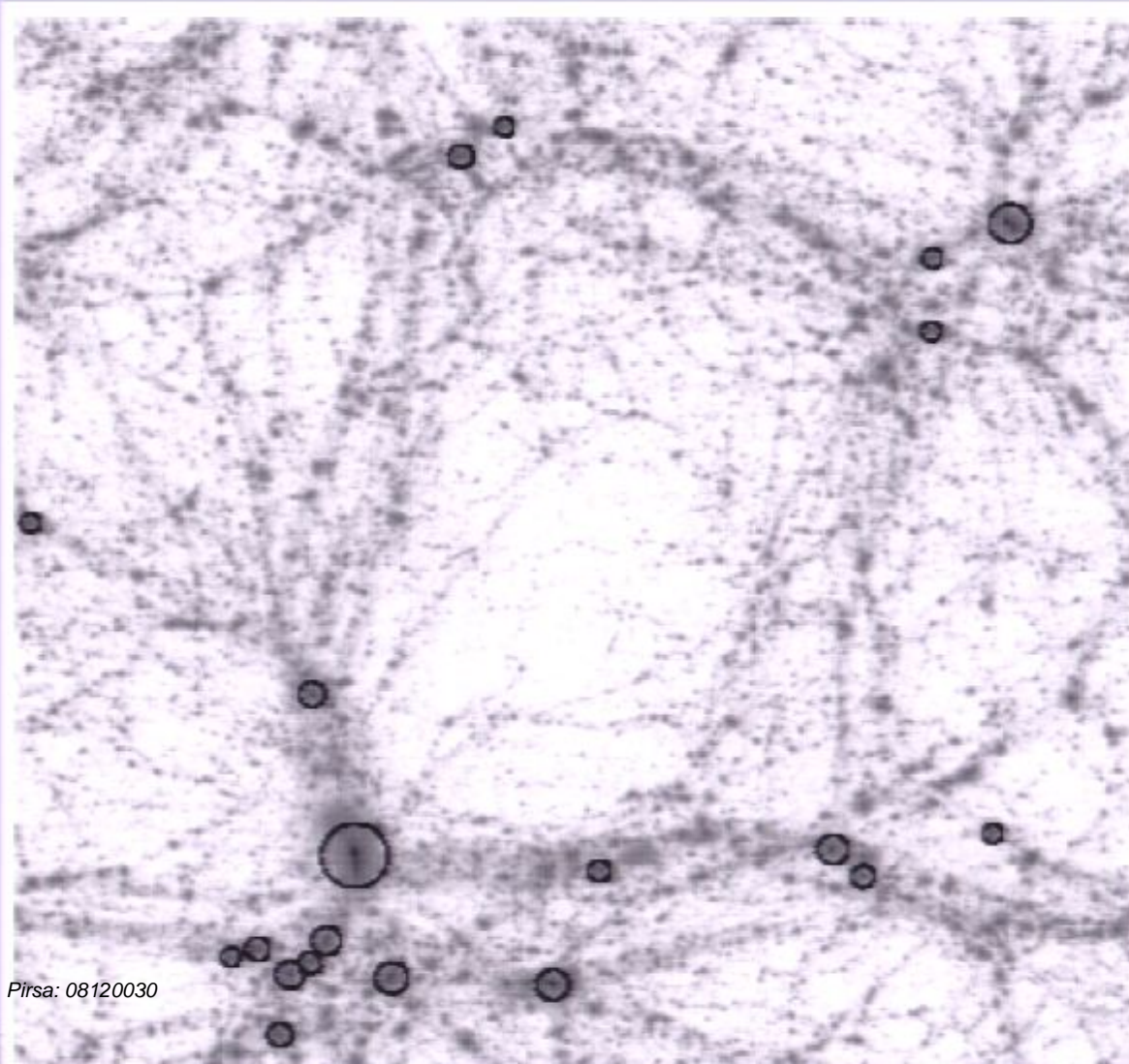
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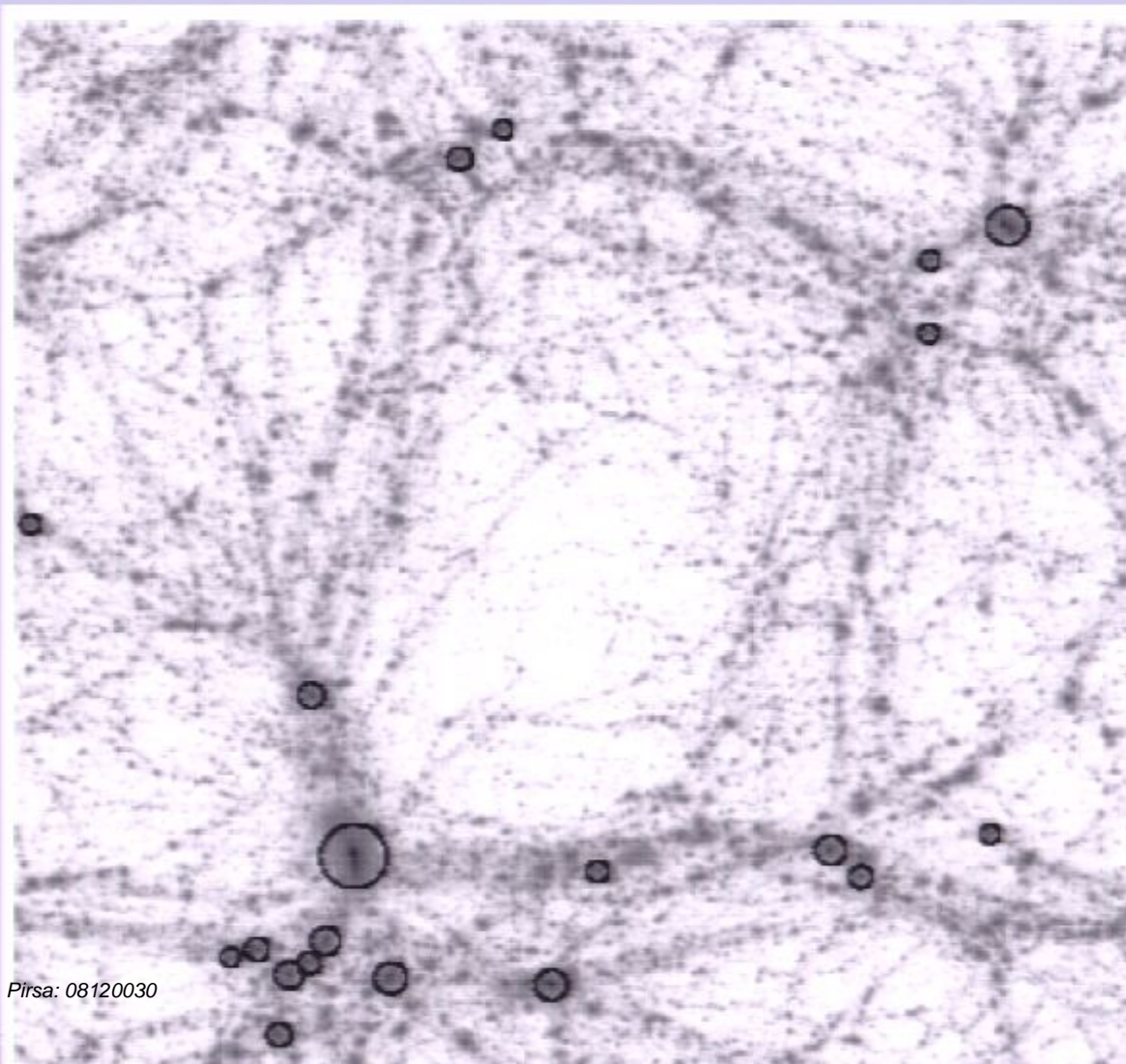


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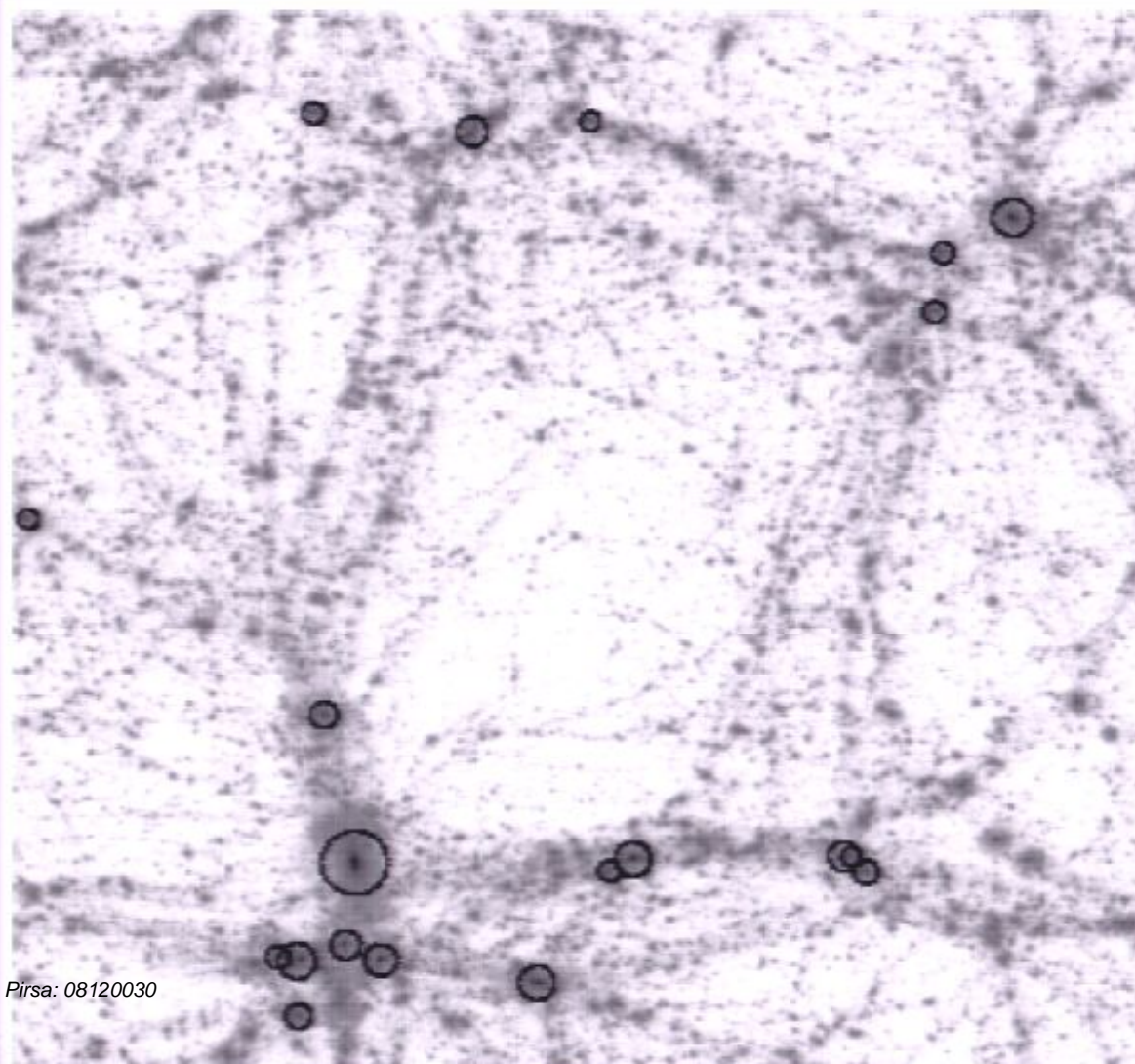


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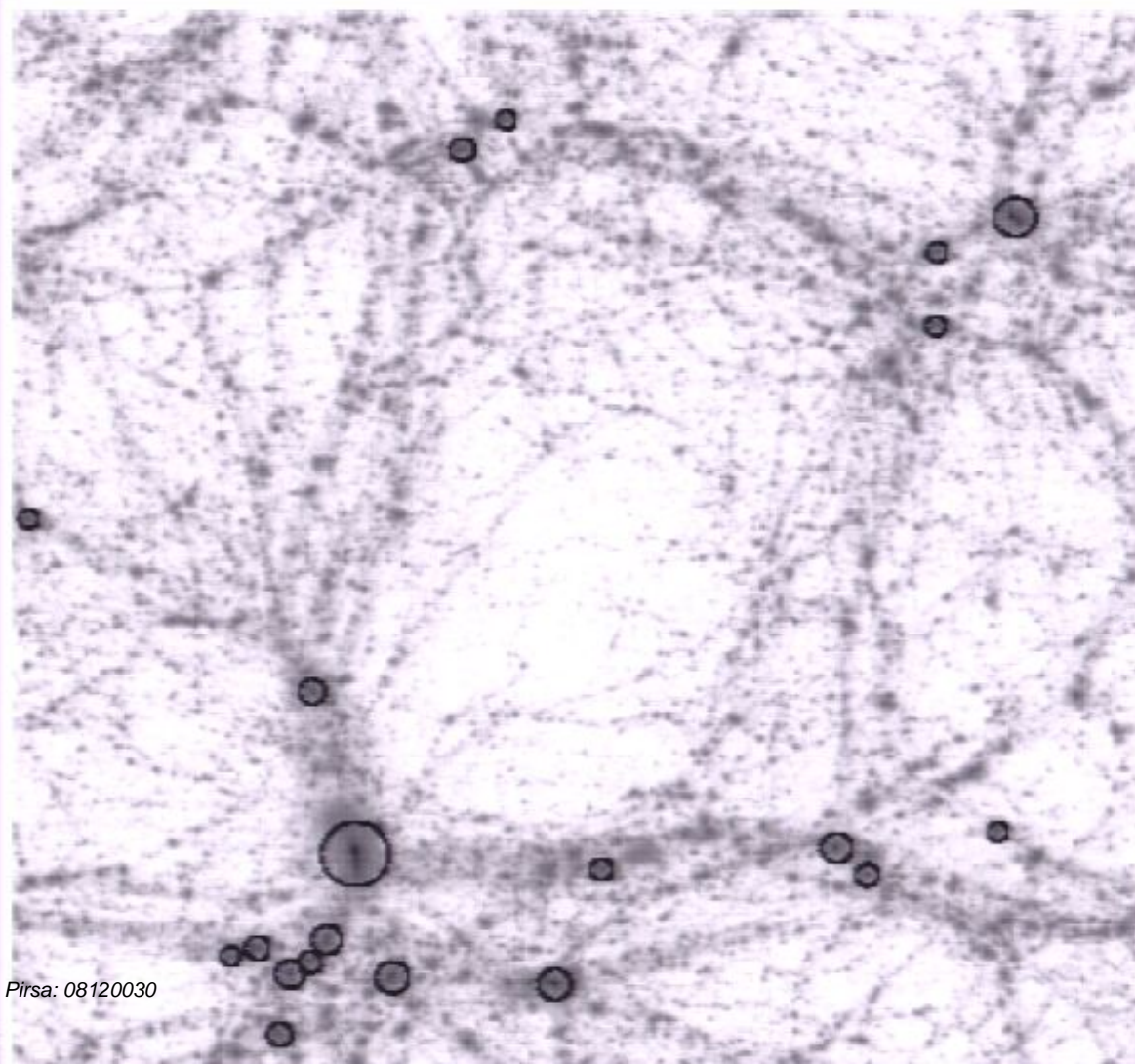


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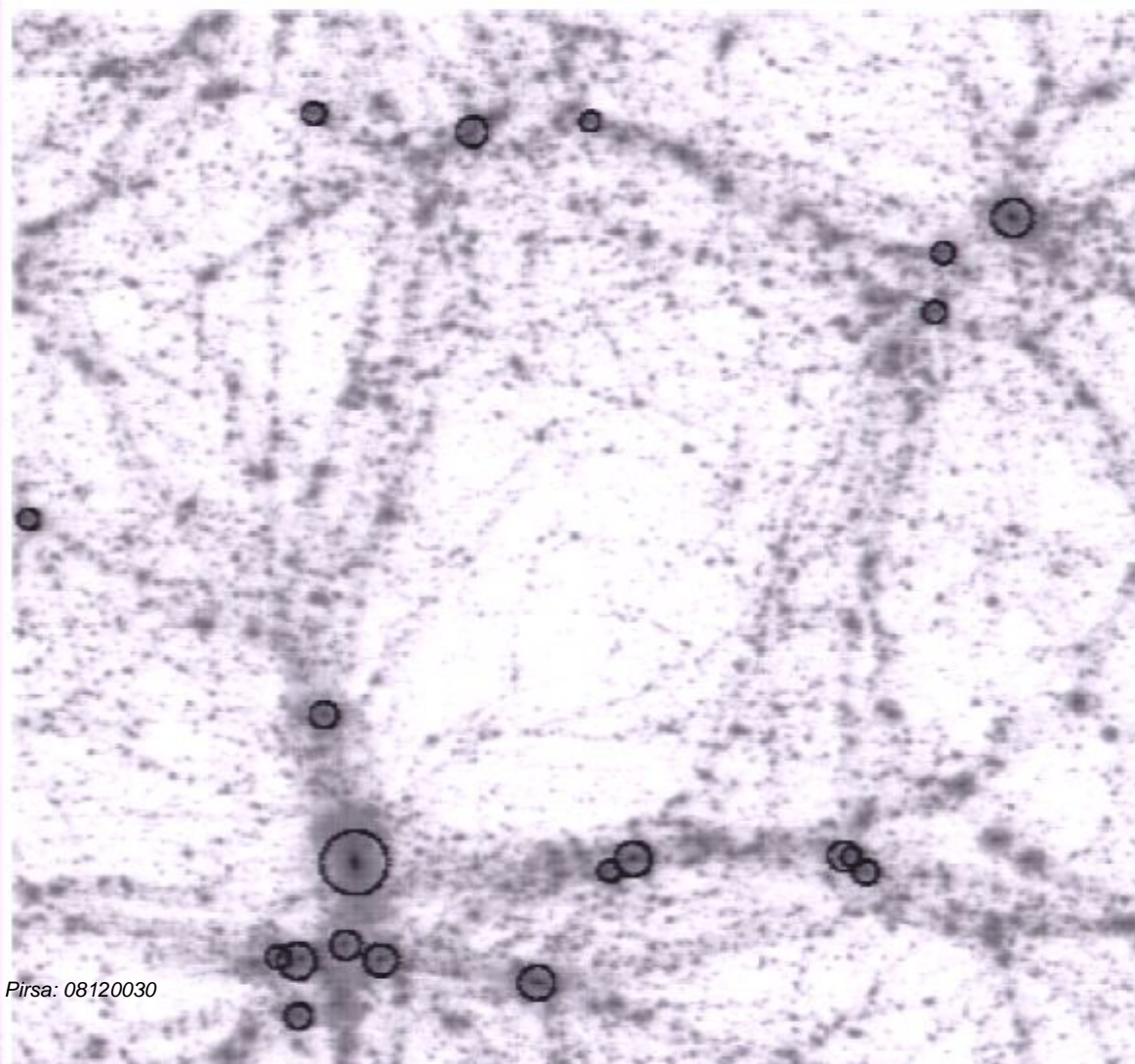


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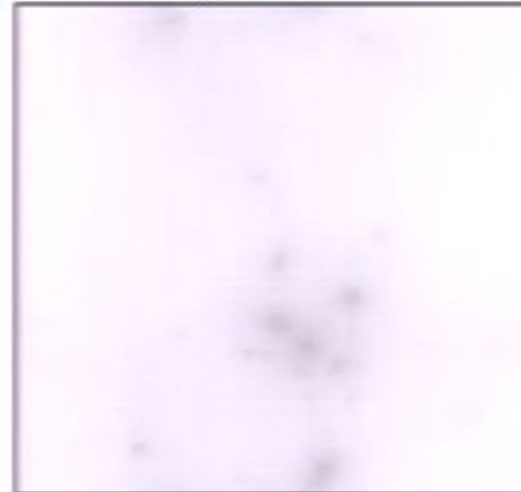
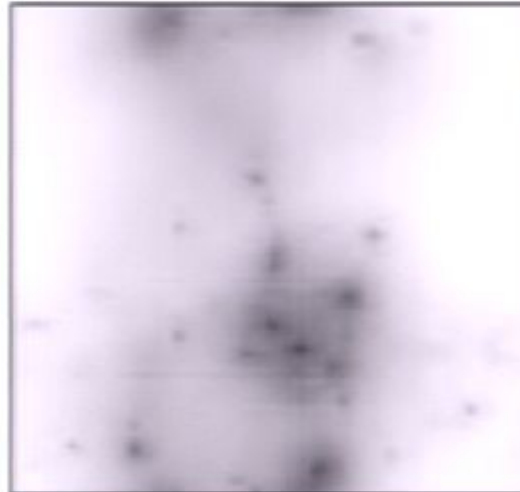
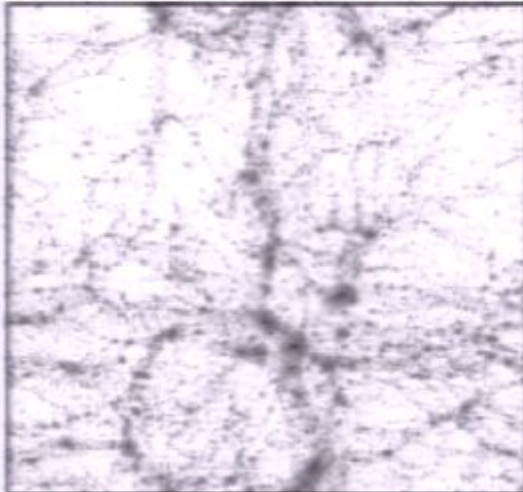
Oyaizu et al., arXiv:0807.2463

Density

Potential

$f_R$  field

At  $z=0$

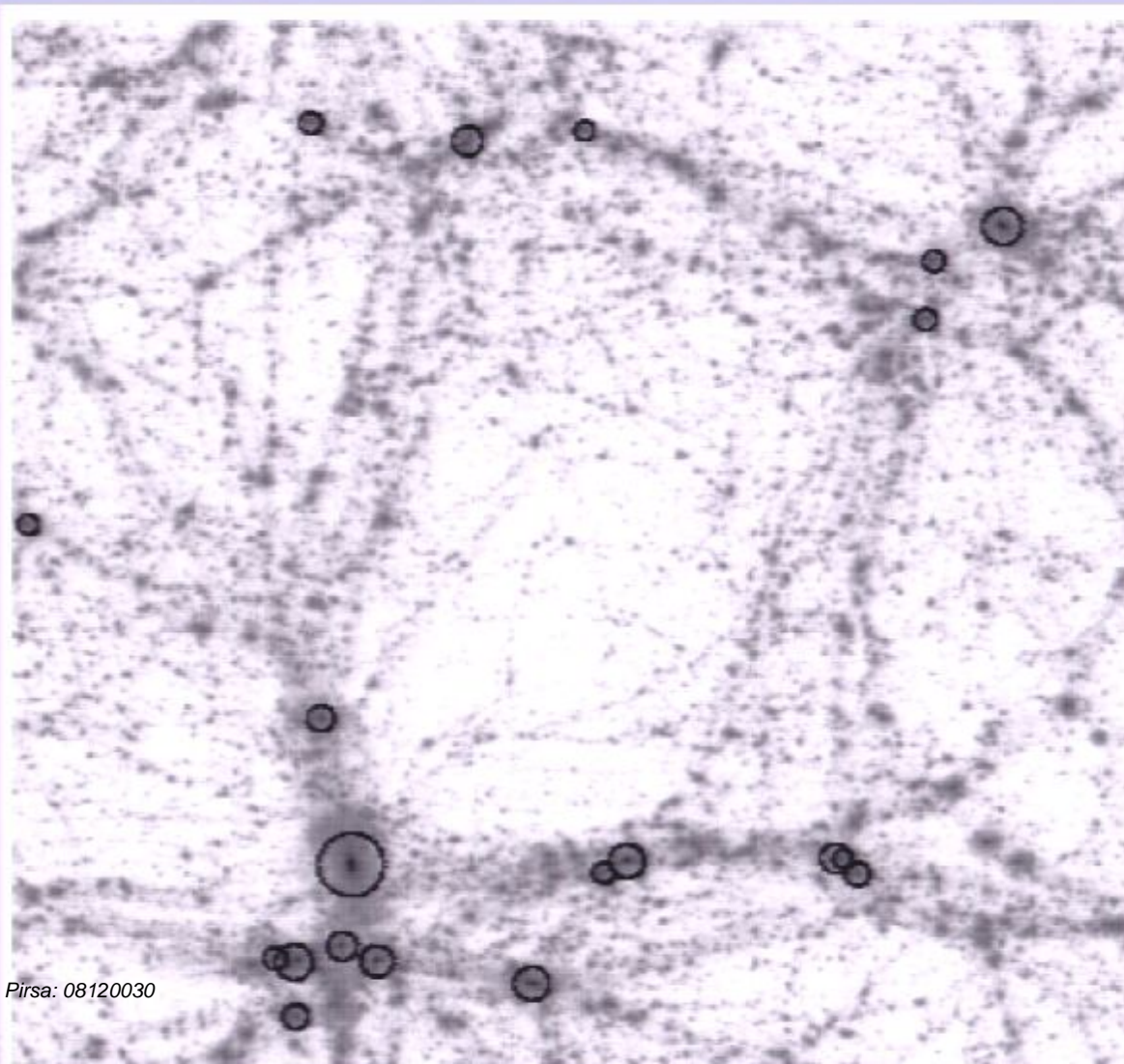


**Strong field**

$$f_{R0} = 10^{-4}$$

$f_R$  field at bg.  
value

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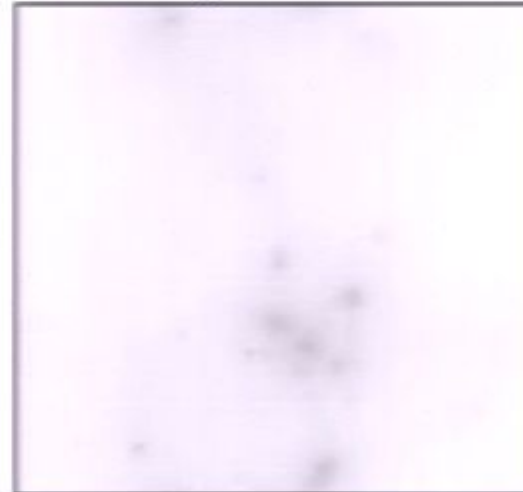
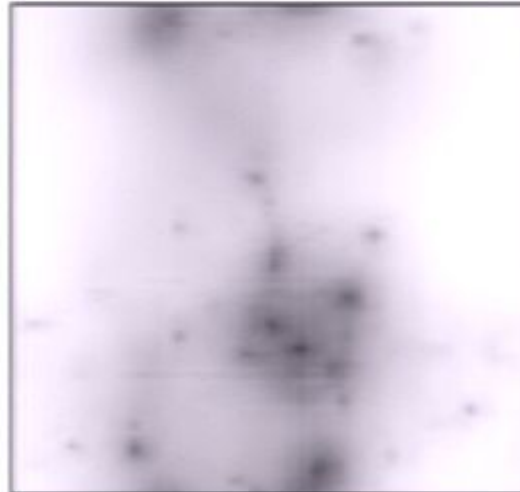
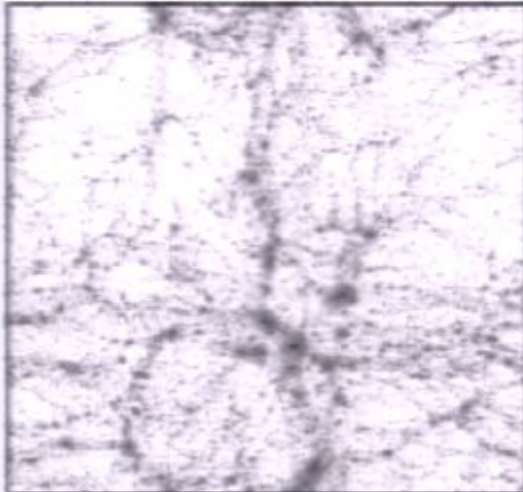
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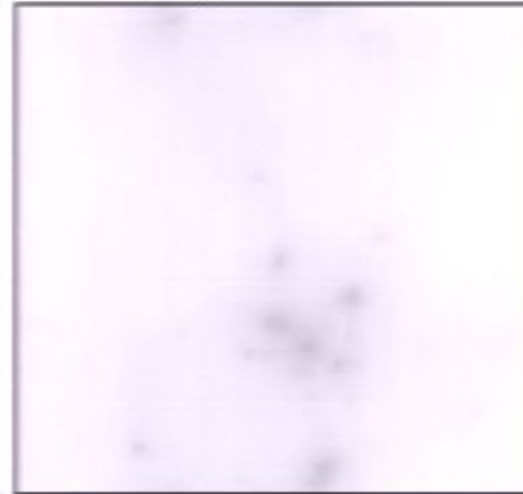
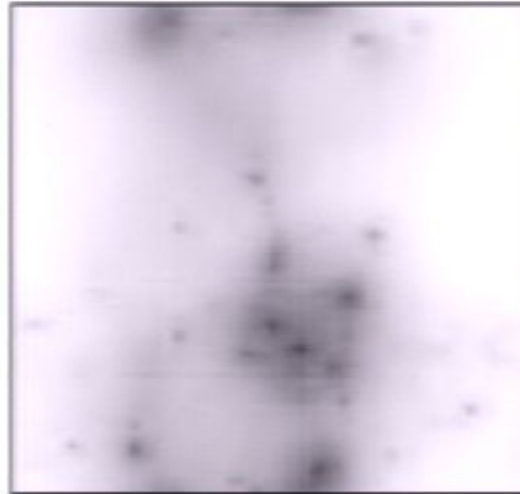
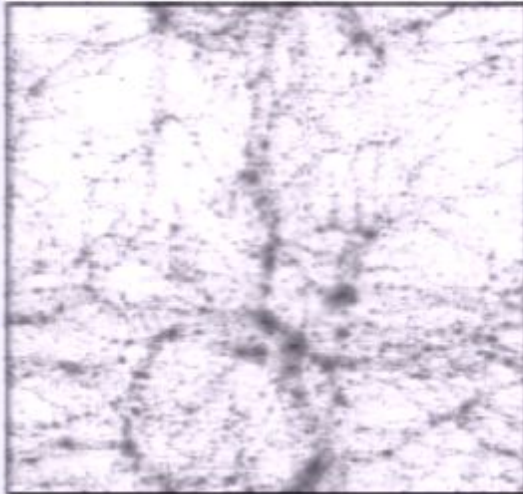
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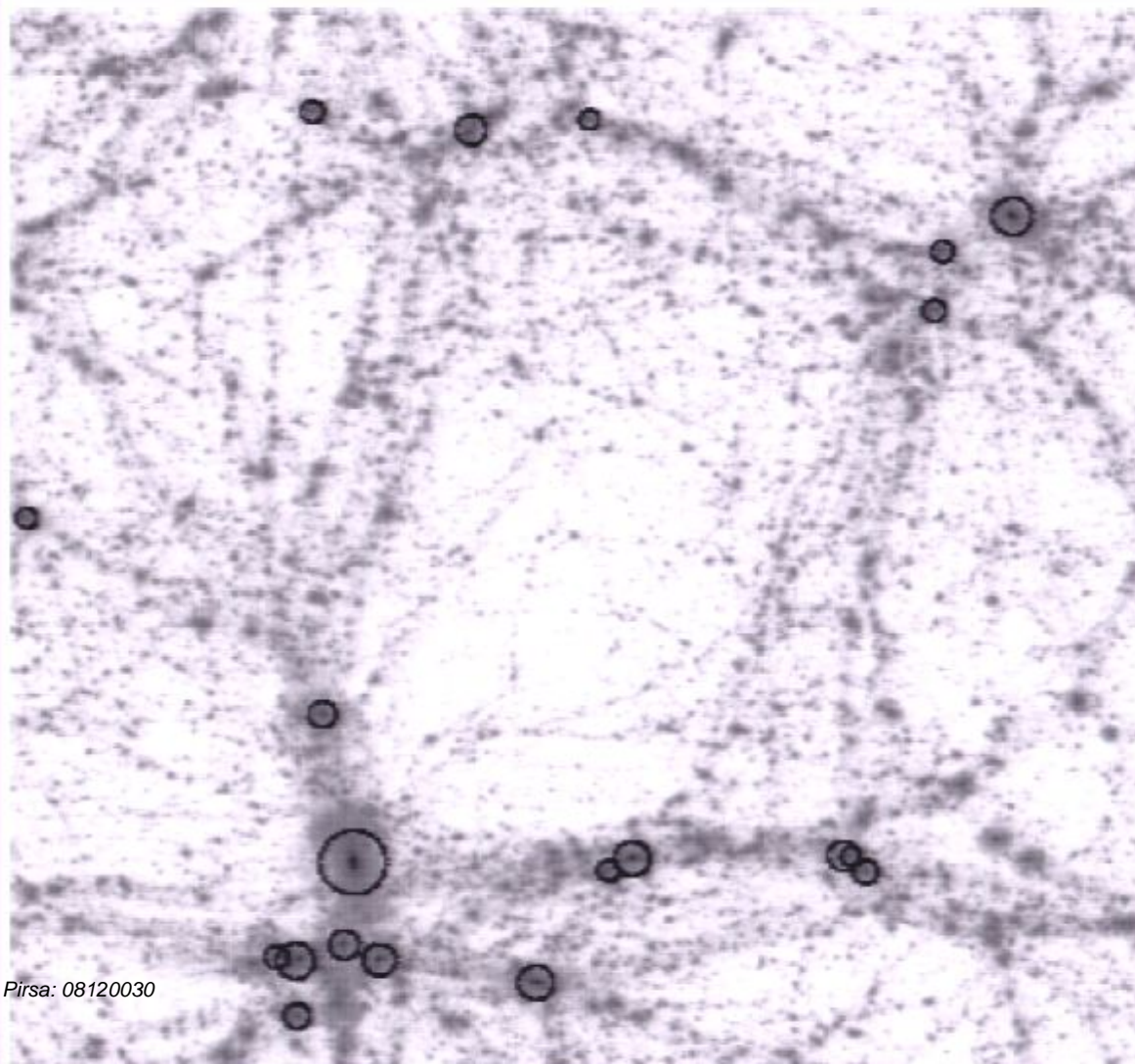


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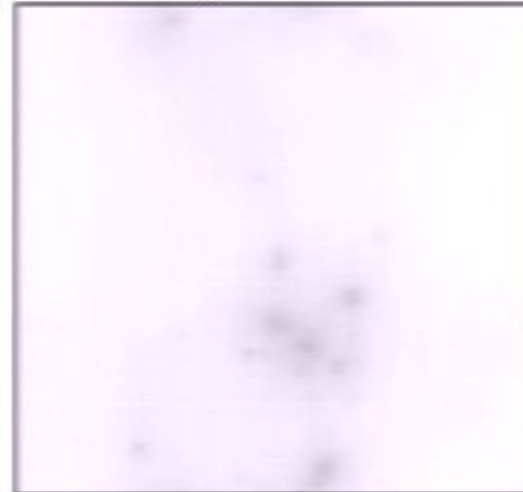
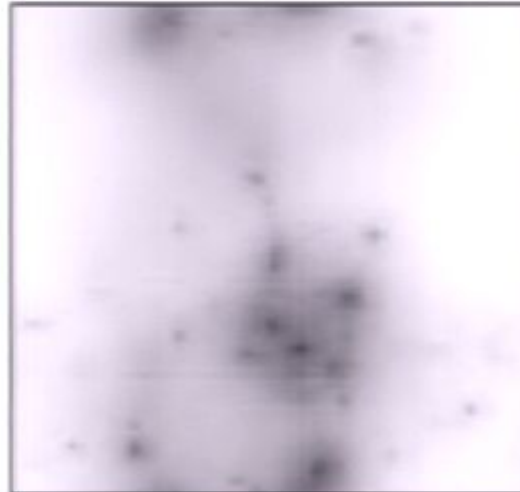
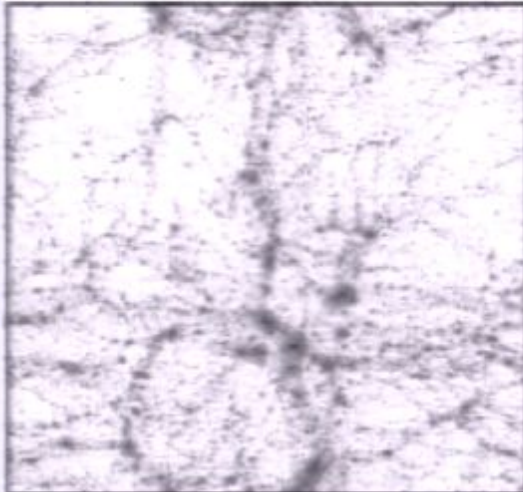
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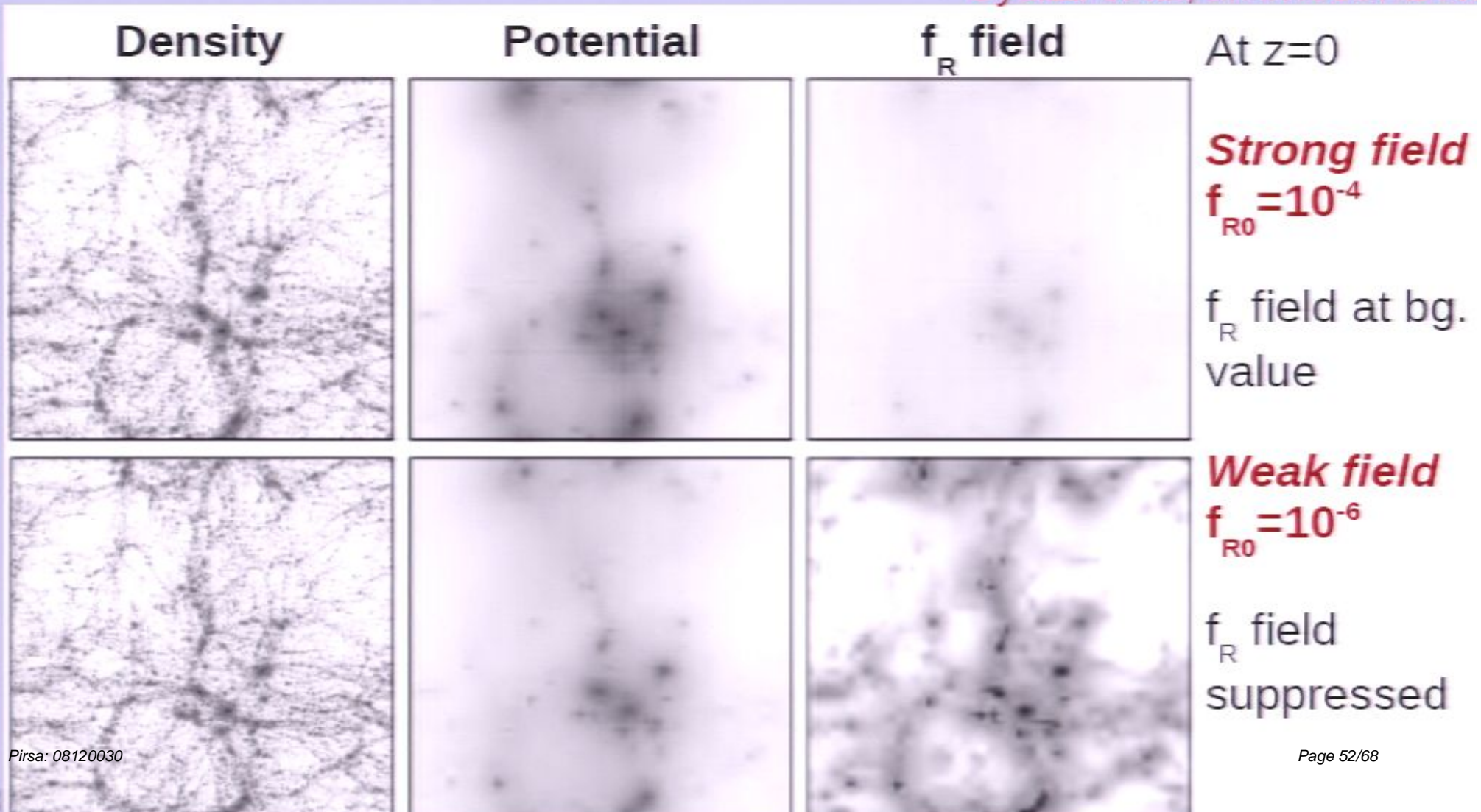
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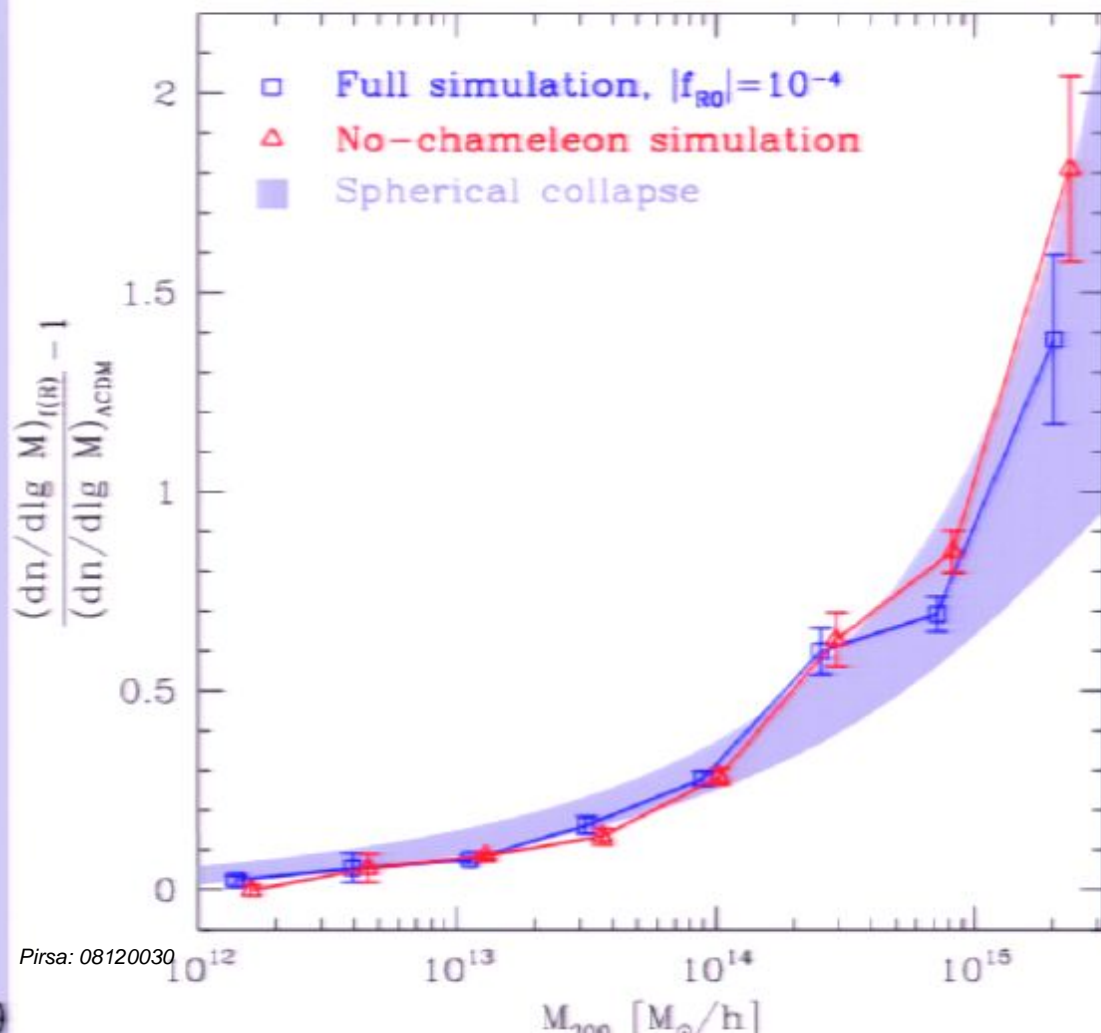
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# Halo Mass Function

F.S. et al., arXiv:0812.0549

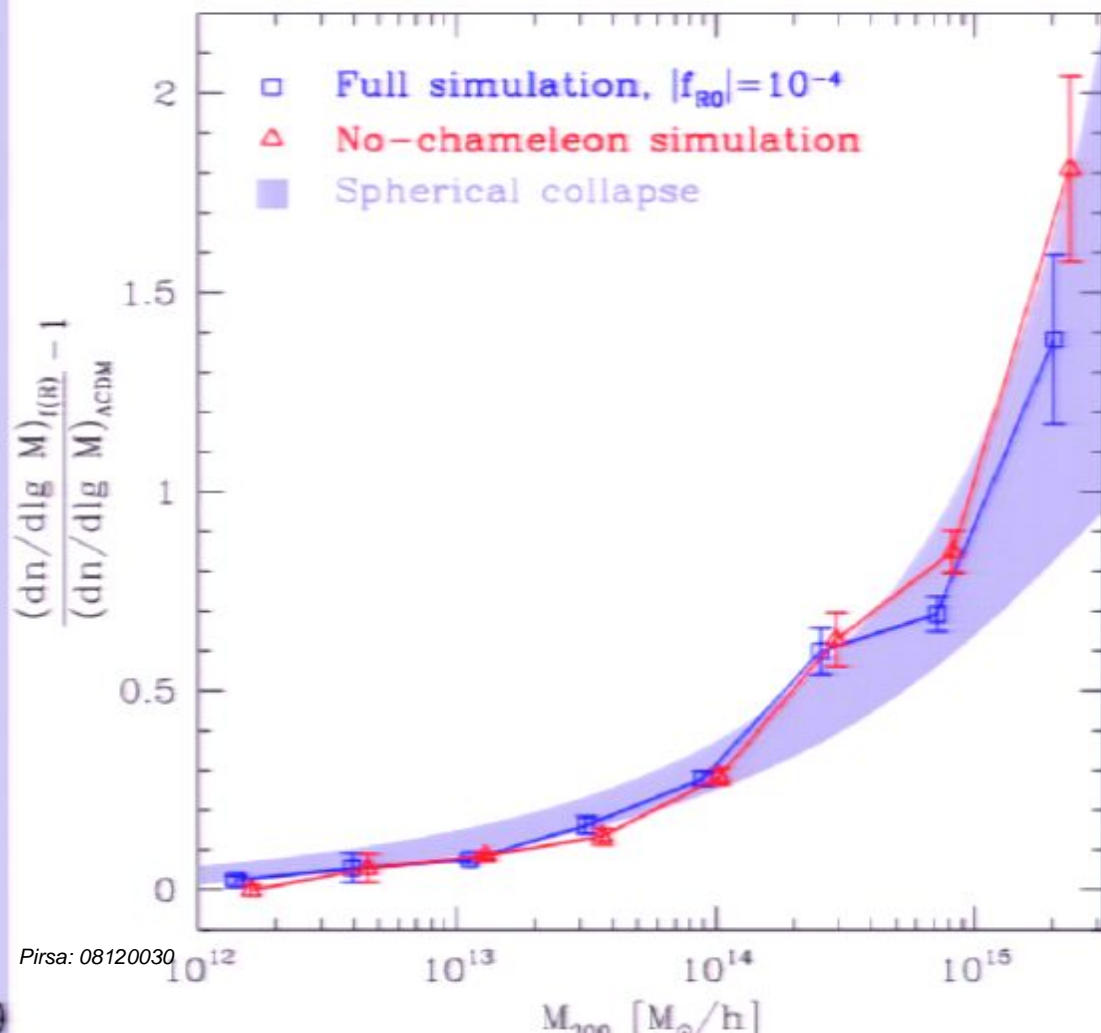


## Effect on $dn/d\ln M_{300}$

- **Strong field ( $10^{-4}$ ):**
  - Abundance **increased by  $\sim 100\%$**  for high masses
    - *Exponential* dependence on  $\sigma(M)$
  - Spherical collapse prediction OK

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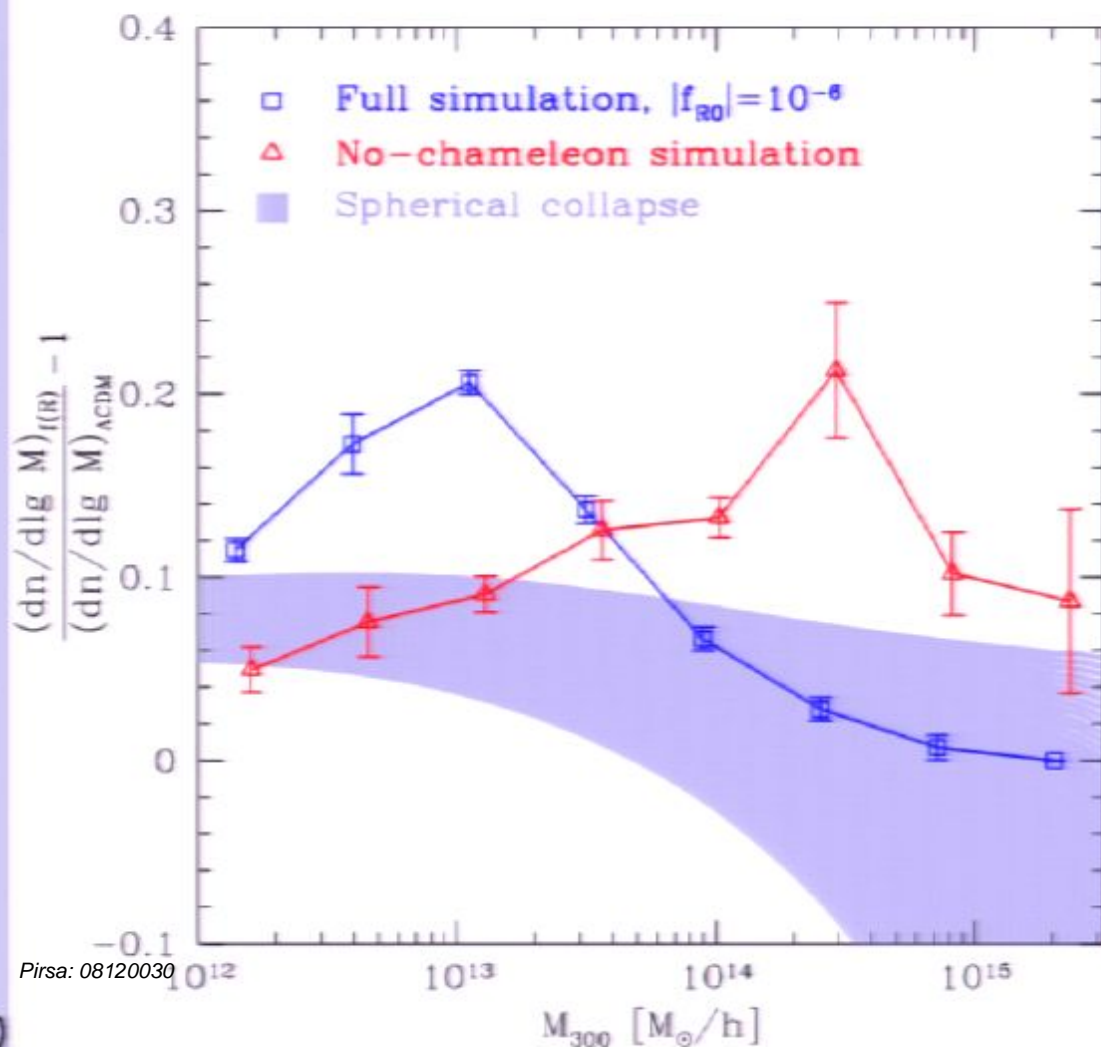


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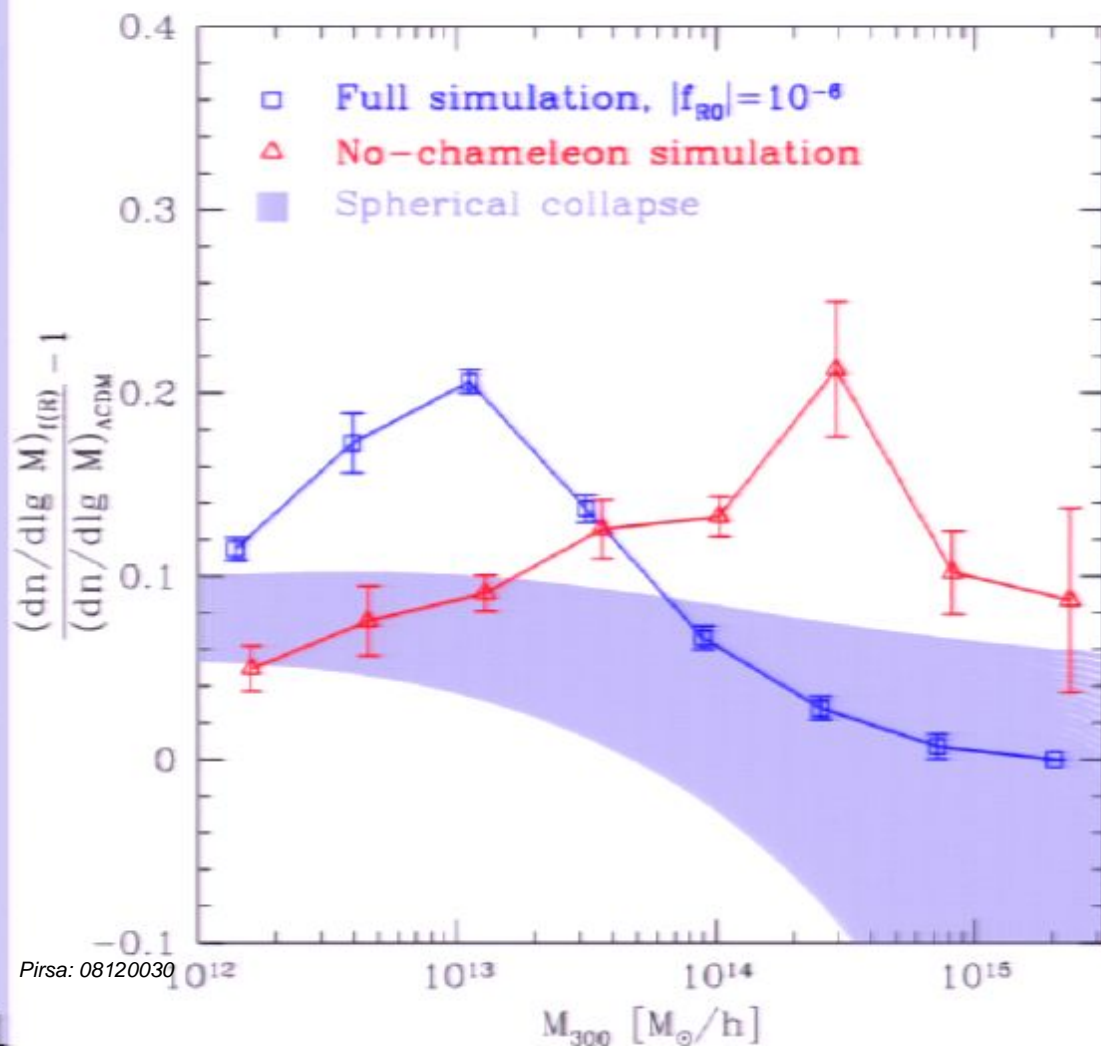


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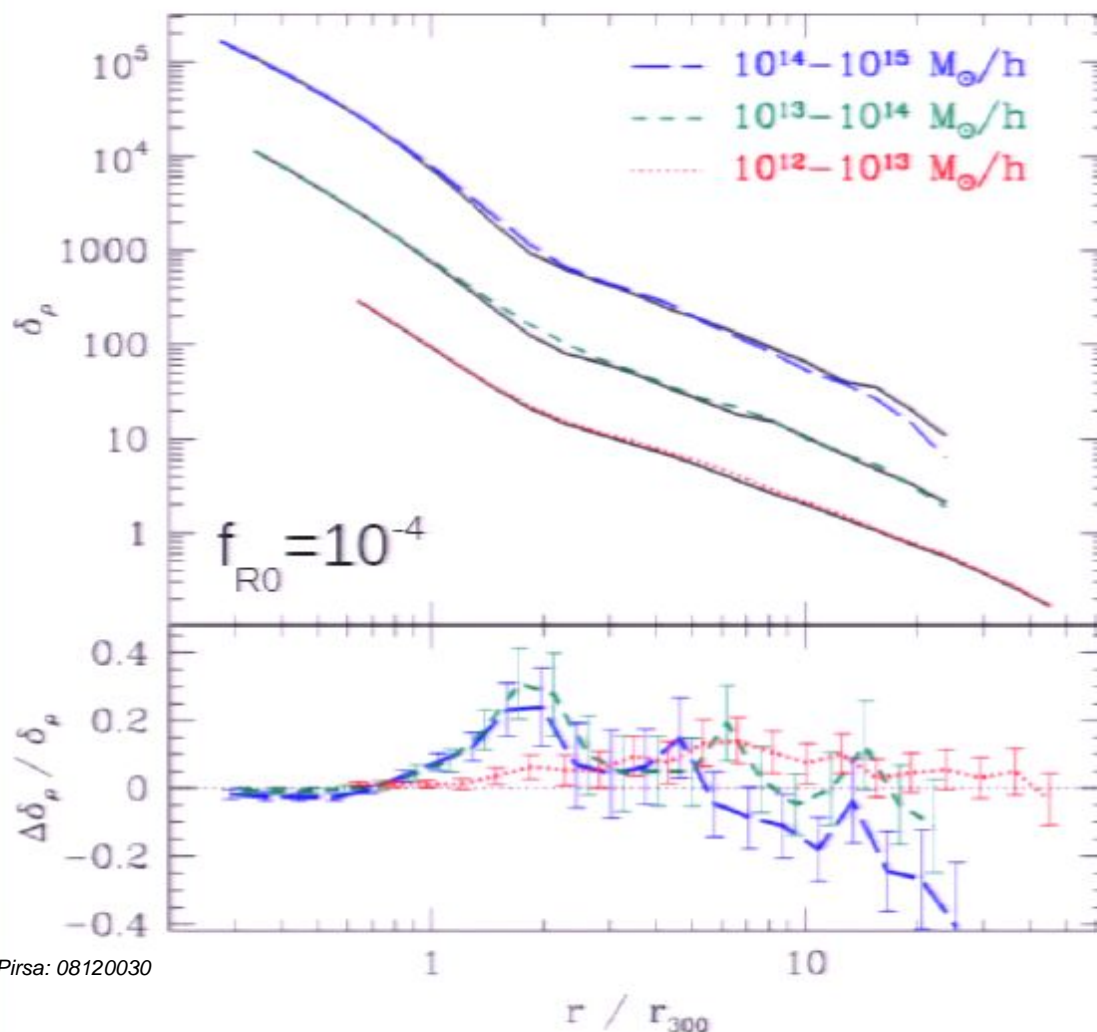
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- **Halo bias less affected by  $f(R)$** 
  - Few – 10% effect



# Halo Profiles and Environment

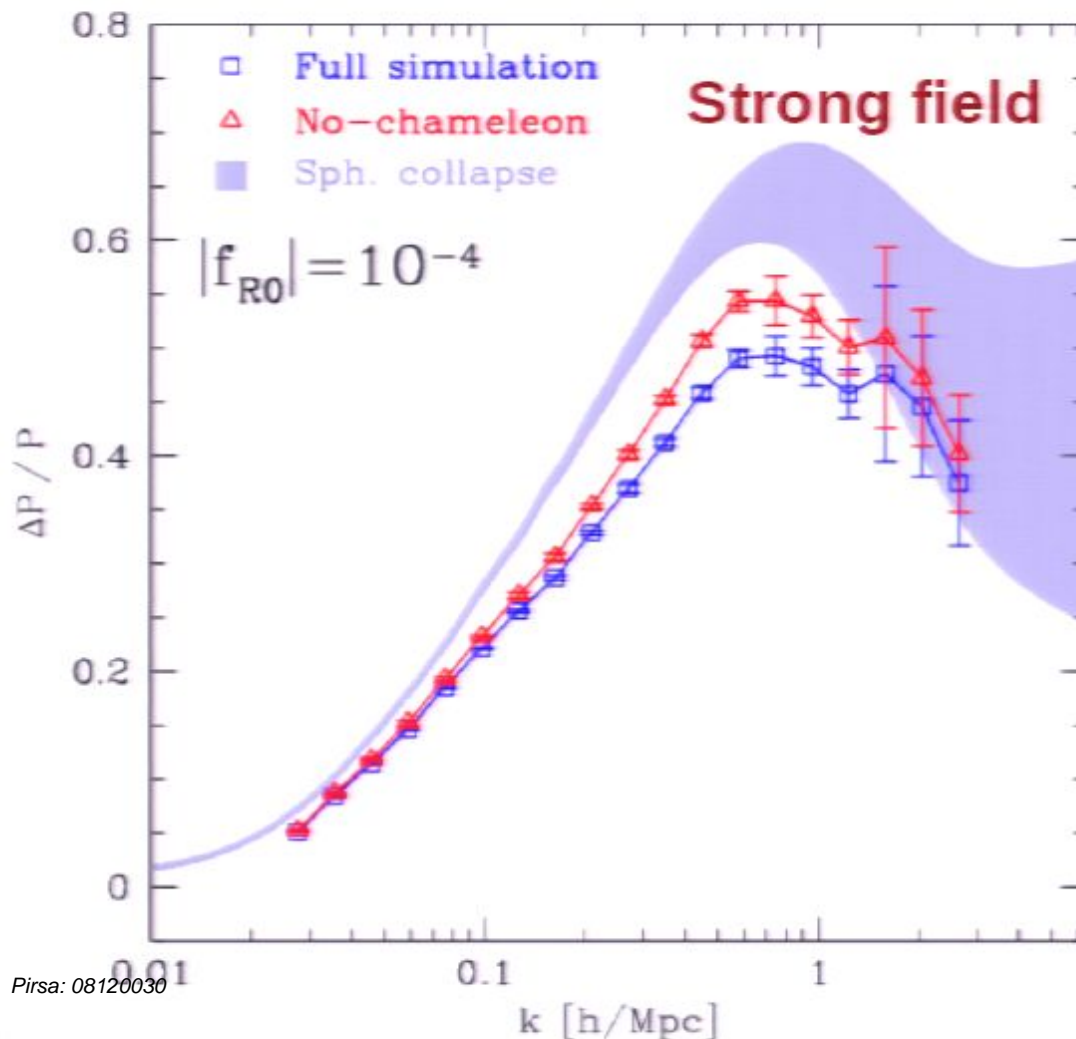
F.S. et al., arXiv:0812.054



- **Stacked halo profiles for  $\Lambda$ CDM and strong field**
  - Scaled to same overdensity radius
  - Inner halo profiles unchanged
  - Some effects on surrounding mass

# Non-linear Matter Power Spectrum

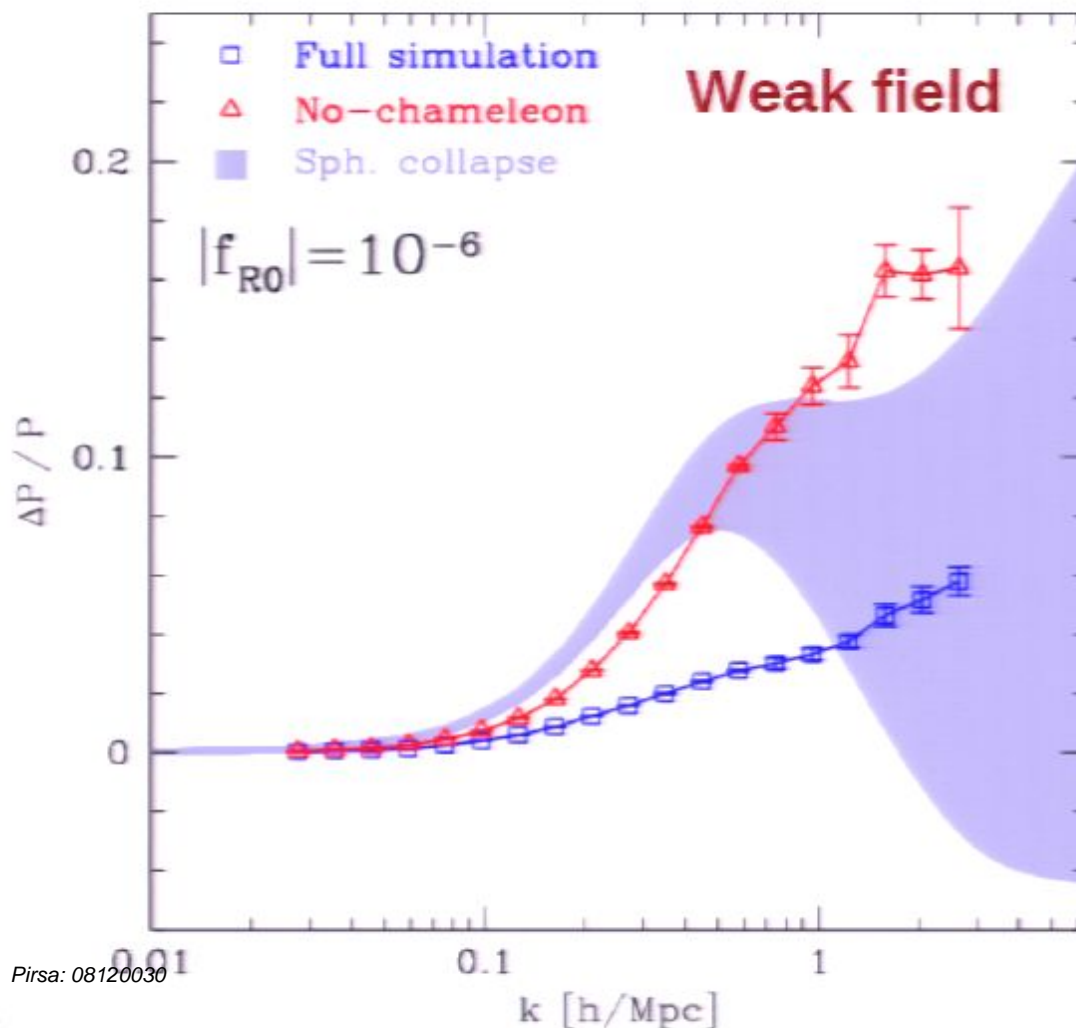
F.S. et al., arXiv:0812.054



- **Power spectrum enhancement relative to  $\Lambda$ CDM**
  - Scales within  $\lambda_C$  experience enhanced growth
- **Spherical collapse prediction:**
  - Performs better than fitting formulas

# Non-linear Matter Power Spectrum

F.S. et al., arXiv:0812.054



- **Chameleon effect changes behavior qualitatively**
  - Simple spherical collapse fails
  - More detailed modeling necessary

# Constraining Modified Gravity

*Just two examples:*

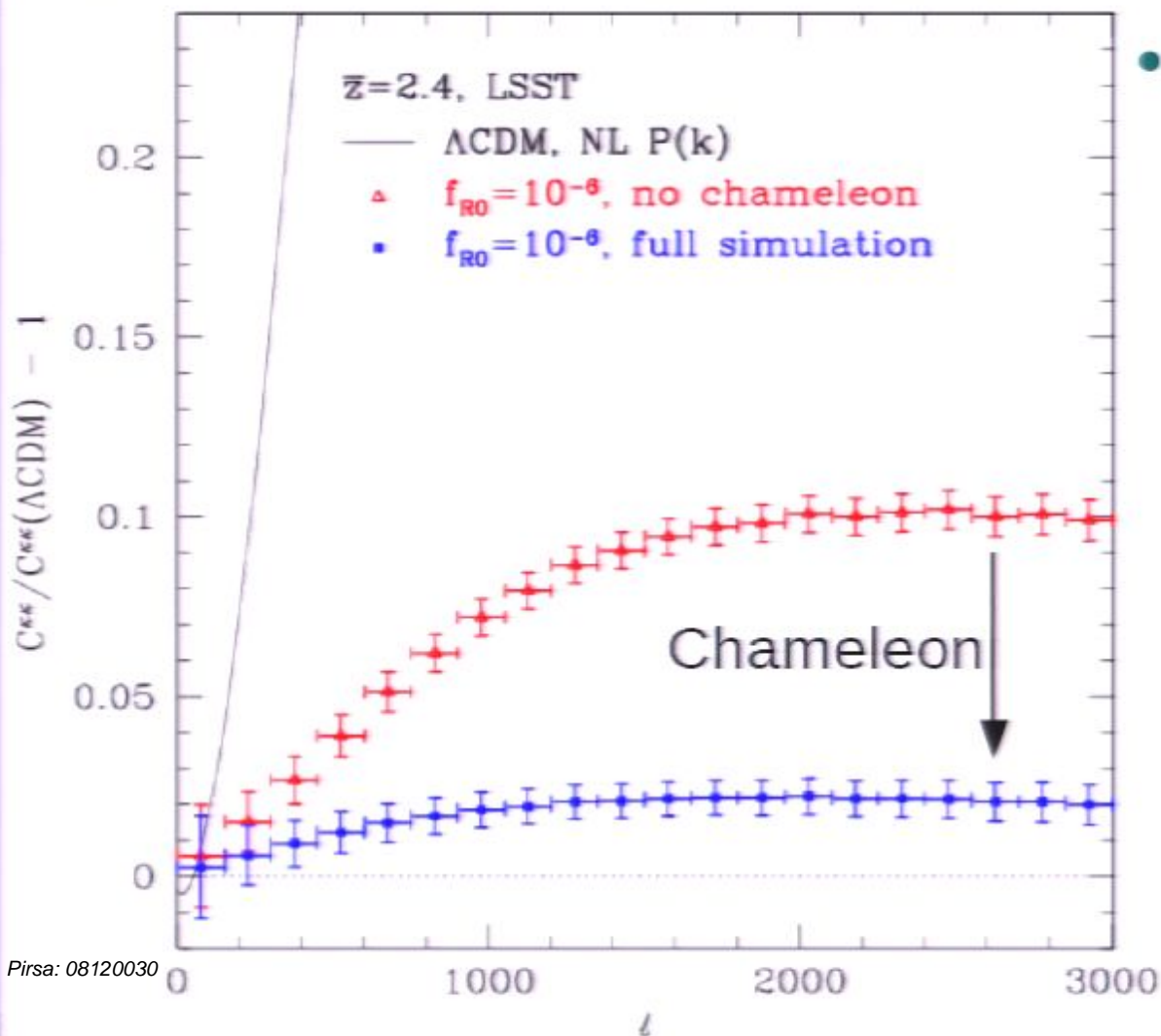
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- **Halo mass function**
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  - Constraints of this order within reach
- **Weak lensing power spectrum**
  - **Constraints  $< 10^{-6}$**  expected from future surveys

# Future Weak Lensing Constraints

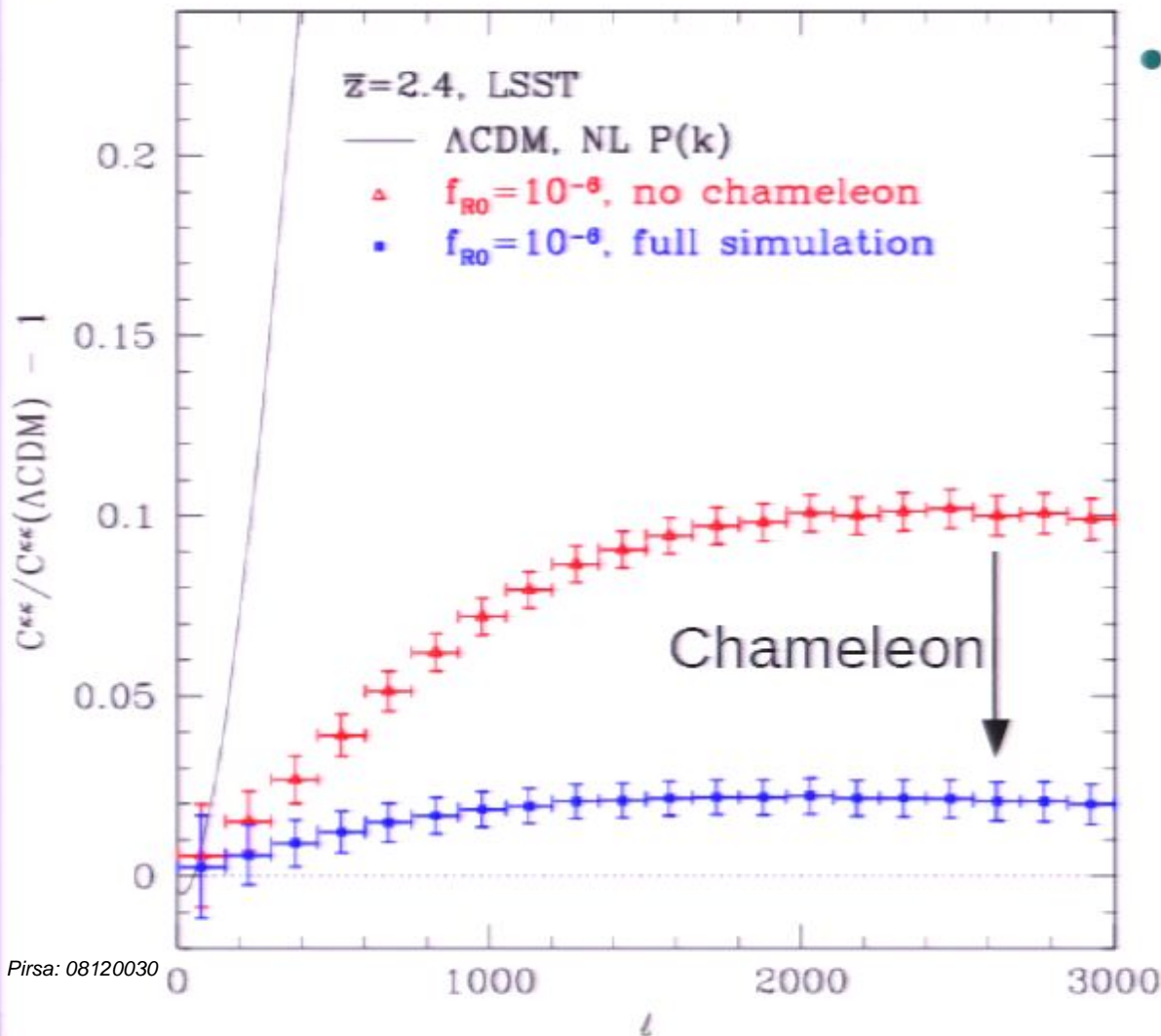


- Shear correlations on **non-linear** scales for  $f_{R0} = 10^{-6}$ 
  - Extremely tight constraints expected
  - Sensitive to **deeply non-linear** regime
  - **Careful modeling essential** for constraints

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# Conclusions

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- **Opening the door to probing gravity on Mpc scales**