Title: A Toy Model of Unparticle Physics

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Abstract: I will discuss a simple two-dimensional theory, whose unparticle sector is a modification of the Schwinger model, that gives new insights into the qualitative features of unparticle physics. I will analyze the transition between the short-distance perturbative physics and large-distance unparticle behavior. Then I will show how to compute processes that involve unparticle self-interactions, for which nontrivial higher n-point functions of the conformal theory are essential.

A Toy Model of Unparticle Physics

Yevgeny Kats

Harvard University

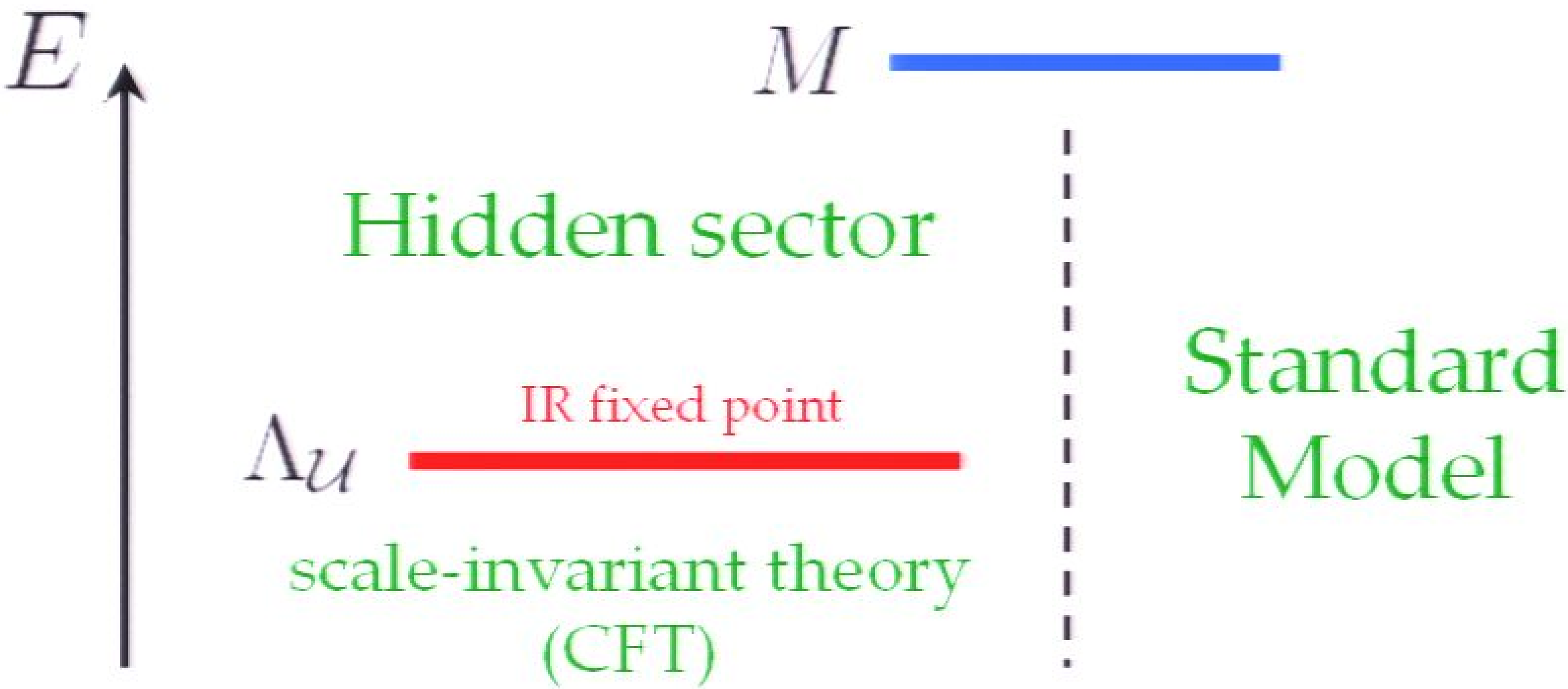
Unparticle example in 2D

H. Georgi and YK, PRL 101, 131603 (2008) arXiv:0805.3953 [hep-ph]

Nontrivial unparticle interactions in 2D

H. Georgi and YK, in preparation

What is unparticle physics?



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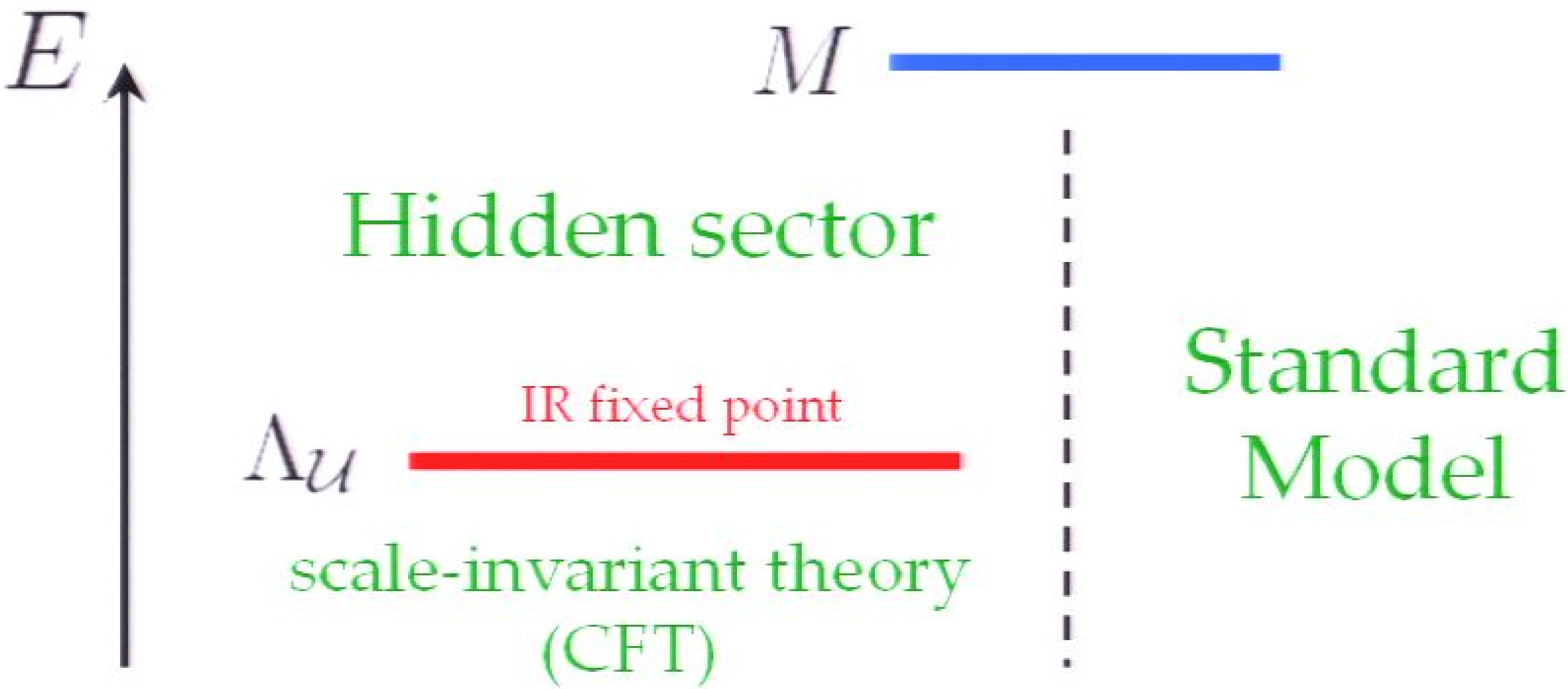

Motivations

- Interesting physics: CFT is not made of particles!

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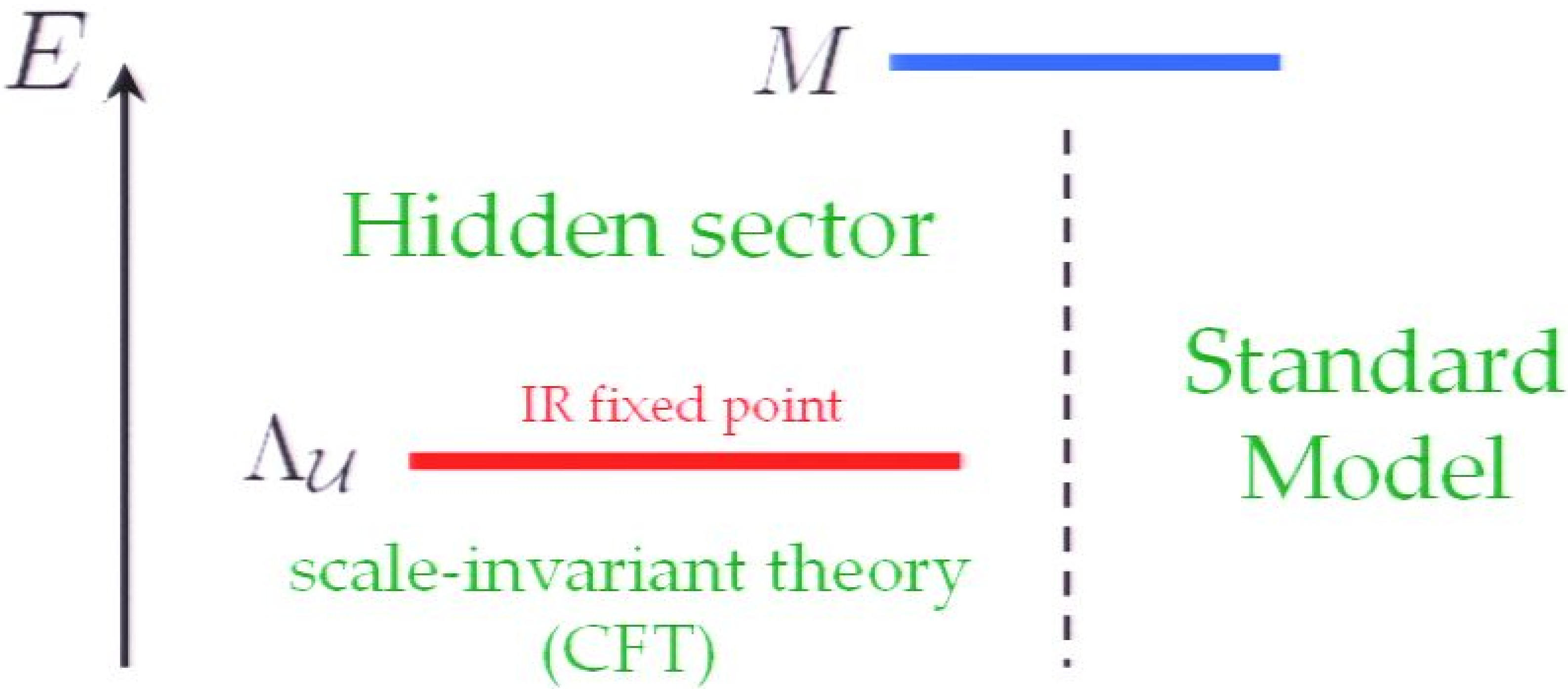

Motivations

- Interesting physics: CFT is not made of particles!
- Hidden sectors appear generically in physics beyond SM and they can generically have IR fixed points.
  How will this physics manifest itself in experiment?

Simplest processes

$$\mathcal{L}_{\rm int} \sim \frac{1}{M^n}\,\phi_1^{\rm SM}\,\phi_2^{\rm SM}\,\mathcal{O}_{\mathcal{U}}$$

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Disappearance process

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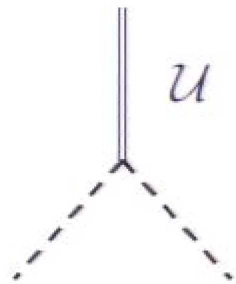

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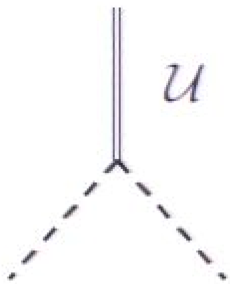

Interference with a SM process

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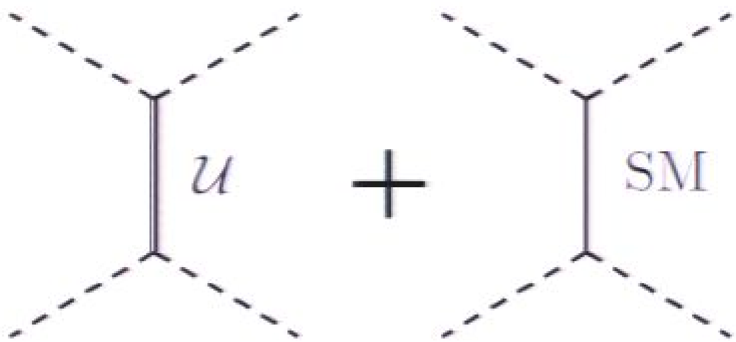

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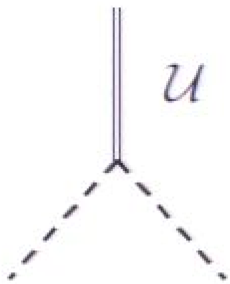
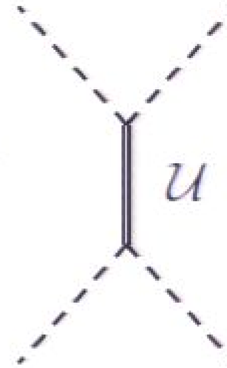


Cross section via optical theorem



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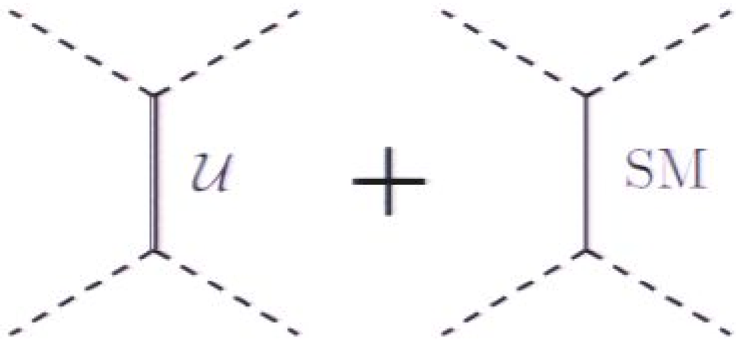

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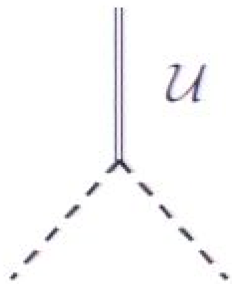

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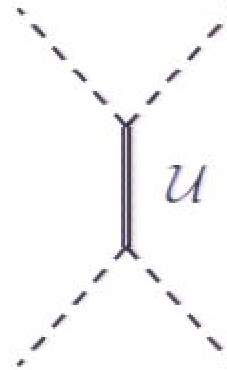

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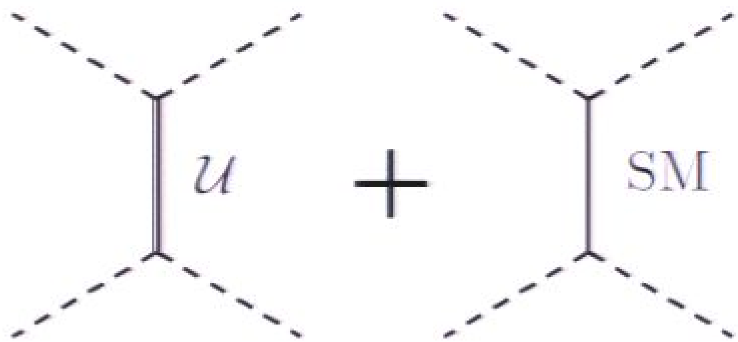

Unparticle 2-point function is fixed by scale invariance

$$i\Delta_{\mathcal{U}}(x) \equiv \langle 0|\mathrm{T}\mathcal{O}_{\mathcal{U}}(x)\mathcal{O}_{\mathcal{U}}^{\dagger}(0)|0\rangle \propto \frac{1}{(-x^2+i\epsilon)^{d_{\mathcal{U}}}}$$

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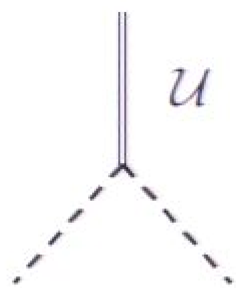

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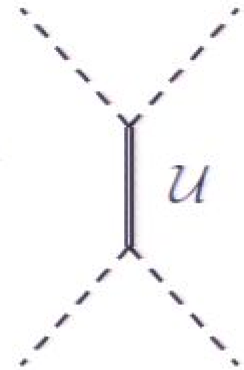

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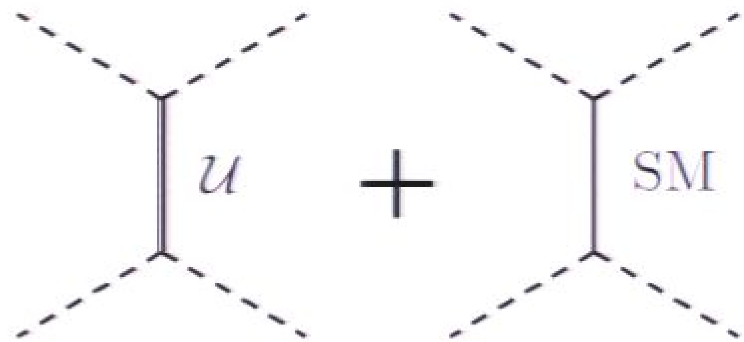

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$$i\Delta_{\mathcal{U}}(p) \propto \frac{i}{(-p^2-i\epsilon)^{2-d_{\mathcal{U}}}} \propto \int_0^\infty \frac{dM^2}{(M^2)^{2-d_{\mathcal{U}}}}\,\frac{i}{p^2-M^2+i\epsilon}$$

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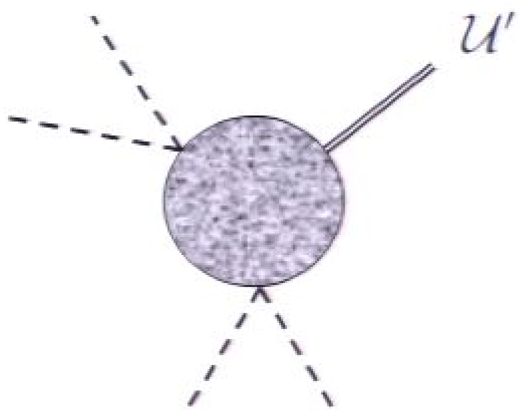

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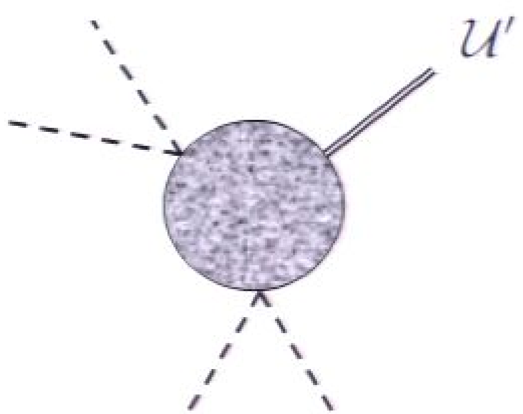

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- What information about the CFT is needed?
- How to compute?

2D toy model of unparticle physics

Unparticle sector: Sommerfield model

$$\mathcal{L} = \overline{\psi}\,(i\,\partial\!\!\!/ - e\,A\!\!\!/)\,\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{m_0^2}{2}A^{\mu}A_{\mu}$$

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- Fractional anomalous dimensions
- Exactly solvable

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The interaction: $$\mathcal{L}_{\text{int}} = \frac{h}{2}\left(\mathcal{O}\,\phi^{*2} + \mathcal{O}^*\phi^2\right)$$

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Conserved U(1) charge: $\phi \to e^{i\alpha}\phi\,, \quad \psi \to e^{i\alpha\gamma^5}\psi$

Solution of the Sommerfield model

$$\mathcal{L} = \bar{\psi}\,(i\,\partial\!\!\!/ - e\,A\!\!\!/)\,\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{\mu_0^2}{2}A^\mu A_\mu$$

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$\xi = e^{\gamma_E}/2$

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The unparticle limit

$$i\Delta_{\mathcal{O}}(x) \;\rightarrow\; i\Delta_{\mathcal{U}}(x) = \frac{1}{4\pi^2(\xi m)^{2a}\left(-x^2+i\epsilon\right)^{1+a}}$$

$$a \equiv -\frac{e^2}{\pi m^2} = -\frac{1}{1+\pi m_0^2/e^2}$$

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$$i\Delta_{\mathcal{U}}(p) = \frac{iA(a)}{2(\xi m)^{2a}\sin(\pi a)}(-p^2-i\epsilon)^a$$

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$$= \frac{A(a)}{2\pi(\xi m)^{2a}}\int_0^\infty dM^2\,(M^2)^a\,\frac{i}{p^2-M^2+i\epsilon}$$

$$A(a) \equiv -\frac{\sin(\pi a)\,\Gamma(-a)}{2^{1+2a}\,\pi\Gamma(1+a)}$$

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$$i\Delta_{\mathcal{U}}(p) = \frac{iA(a)}{2(\xi m)^{2a}\sin(\pi a)}(-p^2 - i\epsilon)^a$$

anomalous dimension $-1 < a < 0$

$$= \frac{A(a)}{2\pi(\xi m)^{2a}}\int_0^\infty dM^2\,(M^2)^a\,\frac{i}{p^2 - M^2 + i\epsilon}$$

$$A(a) \equiv -\frac{\sin(\pi a)\,\Gamma(-a)}{2^{1+2a}\,\pi\Gamma(1+a)}$$

phase space $\Phi_{\mathcal{U}}(p) = \frac{A(a)}{(\xi m)^{2a}}\,\theta(p^0)\,\theta(p^2)\,(p^2)^a$

The unparticle limit

$$i\Delta_{\mathcal{O}}(x) \;\rightarrow\; i\Delta_{\mathcal{U}}(x) = \frac{1}{4\pi^2(\xi m)^{2a}\left(-x^2 + i\epsilon\right)^{1+a}}$$

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Schwinger model: $A(-1) = 0$
No unparticle stuff!

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unparticle limit $\quad s \ll m^2$

$$\phi + \phi \to \mathcal{U}$$

$$\sigma = \frac{A(a)}{2}\frac{h^2}{(\xi m)^{2a}}\frac{1}{s^{1-a}}$$

The unparticle limit

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unparticle limit $s \ll m^2$

$$\phi + \phi \to \mathcal{U}$$

$$\sigma = \frac{A(a)}{2}\frac{h^2}{(\xi m)^{2a}}\frac{1}{s^{1-a}}$$

free-fermion limit $s \gg m^2$

$$\phi + \phi \to \psi_2 + \overline{\psi}_1$$

$$\sigma = \frac{h^2}{4}\frac{1}{s}$$

Beyond the unparticle limit

$$\Delta_{\mathcal{O}}(x) = \frac{C(x)^4}{4\pi^2\left(-x^2+i\epsilon\right)} \qquad C(x) = \exp\left[i\frac{e^2}{m^2}\left[(\Delta(x)-\Delta(0)) - (D(x)-D(0))\right]\right]$$

$$= \exp\left[\frac{e^2}{2\pi m^2}\left[K_0\left(m\sqrt{-x^2+i\epsilon}\right) + \ln\left(\xi m\sqrt{-x^2+i\epsilon}\right)\right]\right]$$

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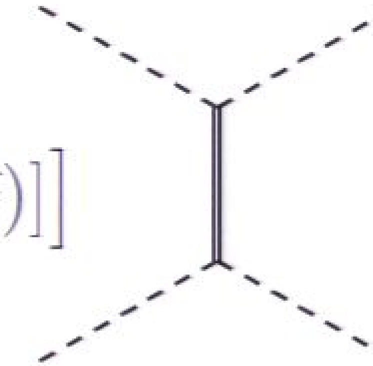

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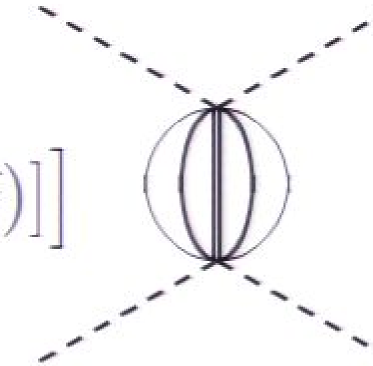
Beyond the unparticle limit



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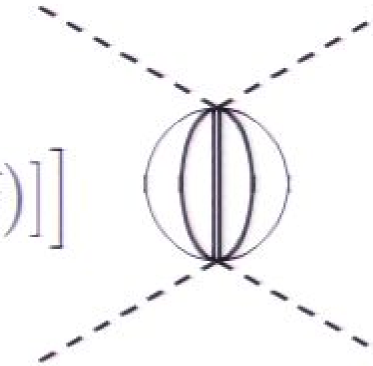

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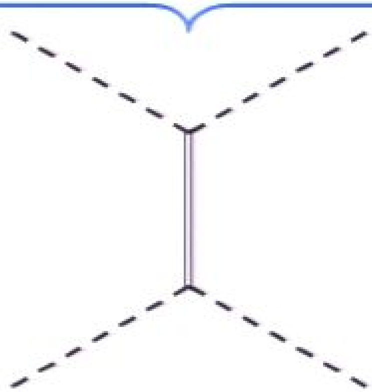

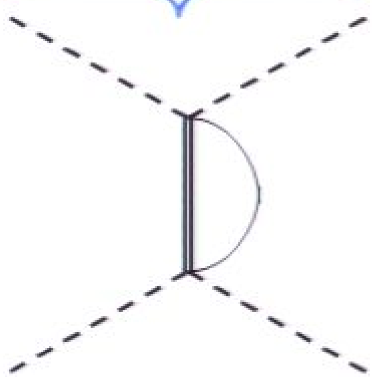

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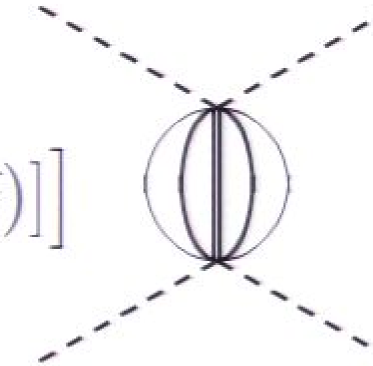

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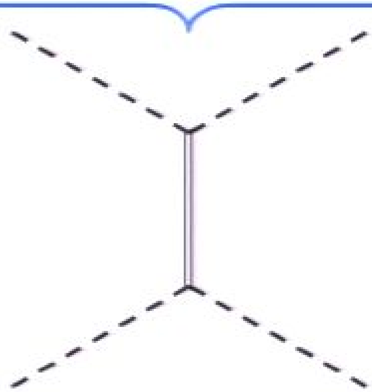

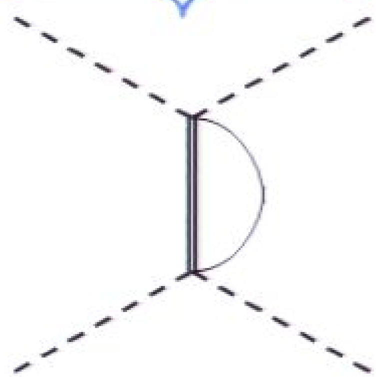

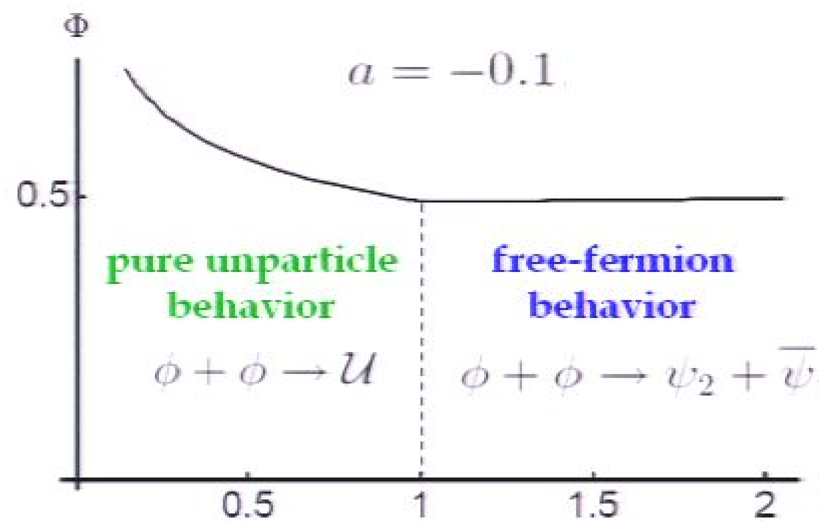

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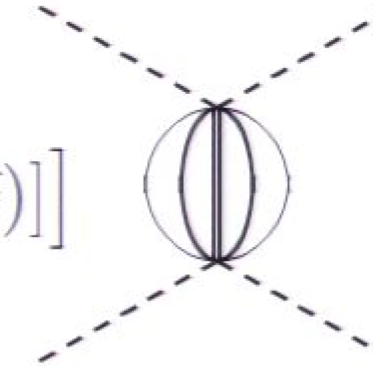

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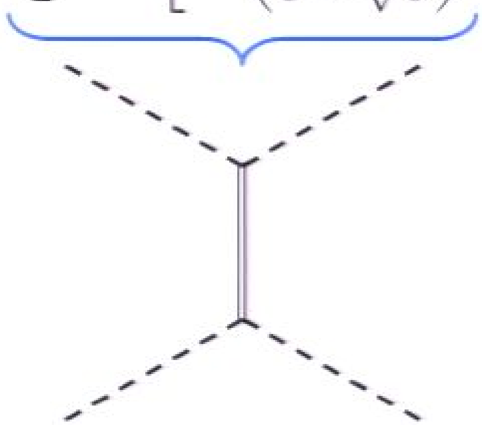

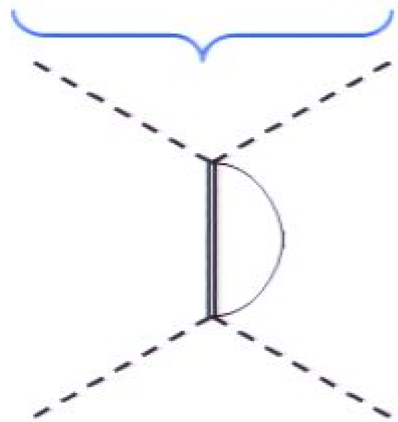

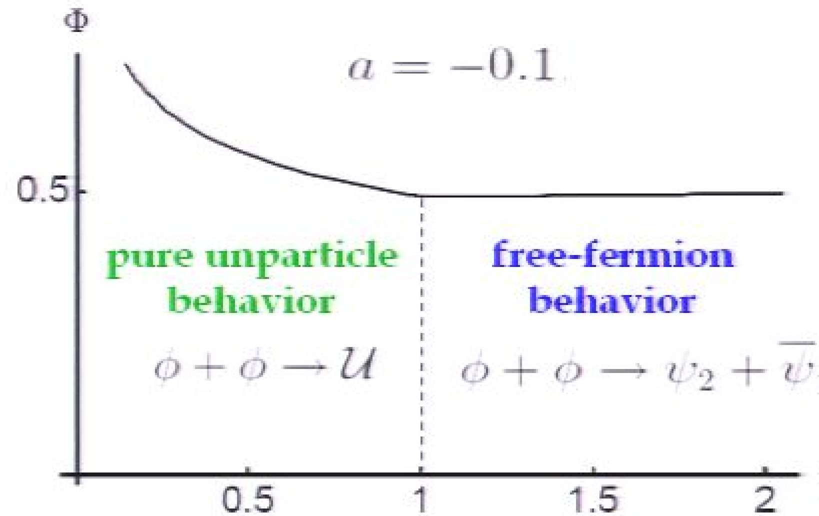

Sommerfield model: free fermions = unparticle stuff + bosons $m^2 = m_0^2 + e^2/\pi$

Beyond the unparticle limit

$$\Delta_{\mathcal{O}}(x) = \frac{C(x)^4}{4\pi^2(-x^2+i\epsilon)} \qquad C(x) = \exp\left[i\frac{e^2}{m^2}\left[(\Delta(x)-\Delta(0)) - (D(x)-D(0))\right]\right]$$

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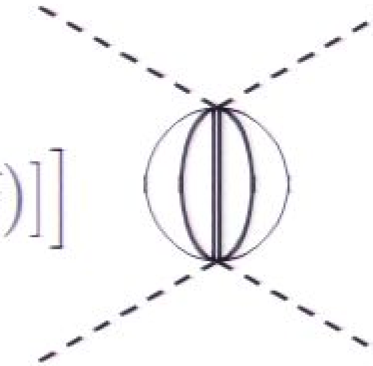

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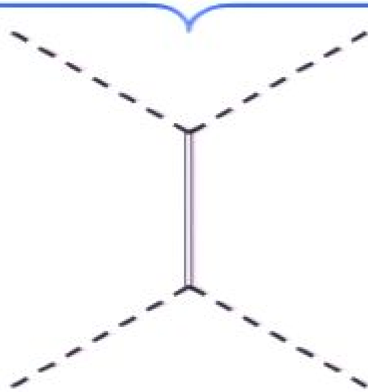
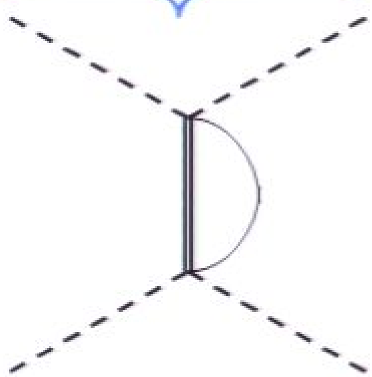
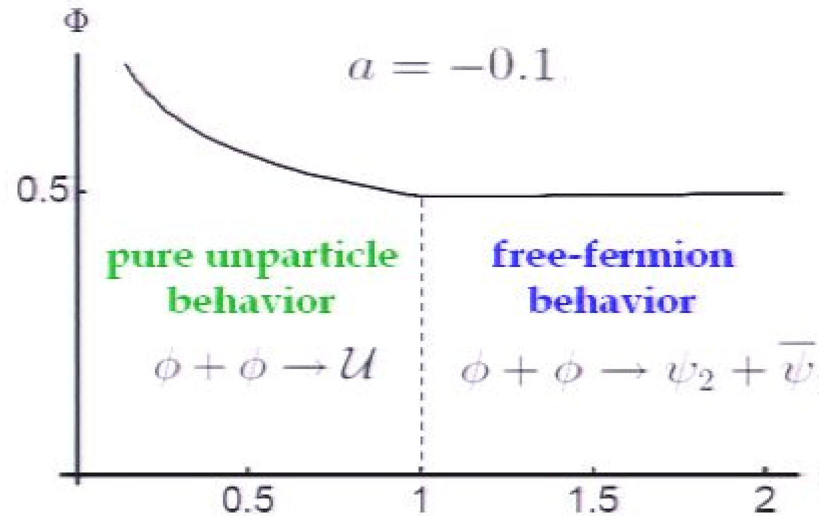




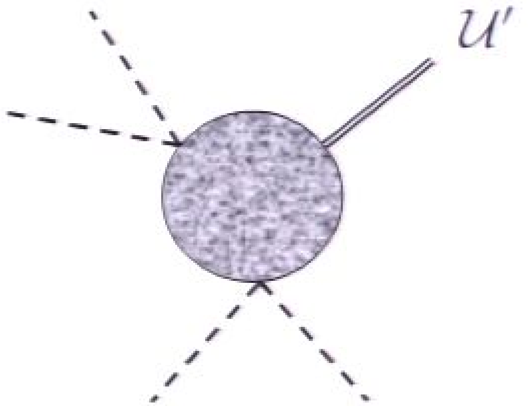


Sommerfield model: free fermions = unparticle stuff + bosons $m^2 = m_0^2 + e^2/\pi$

Schwinger model: free fermions = bosons $m^2 = e^2/\pi$ Casher, Kogut, Susskind 1973

Unparticle self-interactions

$$\mathcal{L}_{\text{int}} \sim \frac{1}{M^n}\, \phi_1^{\text{SM}}\, \phi_2^{\text{SM}}\, \mathcal{O}_{\mathcal{U}}$$



$$\phi_1^{\text{SM}} + \phi_2^{\text{SM}} \to \phi_1^{\text{SM}} + \phi_2^{\text{SM}} + \mathcal{U}'$$

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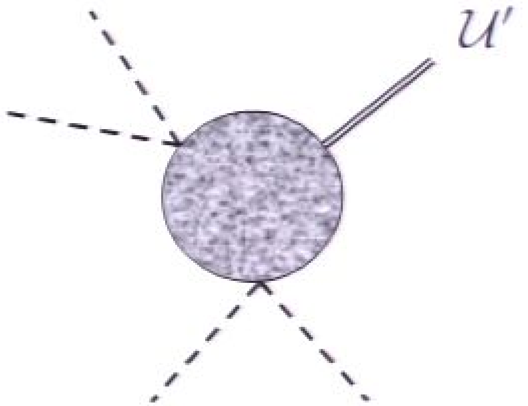

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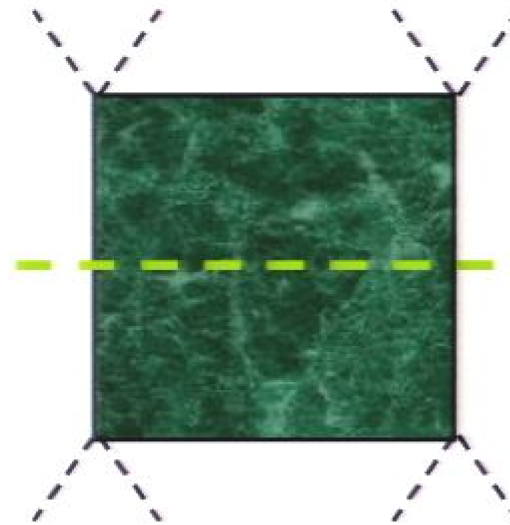
Use cutting rules on 4-point function of $\mathcal{O}_{\mathcal{U}}$



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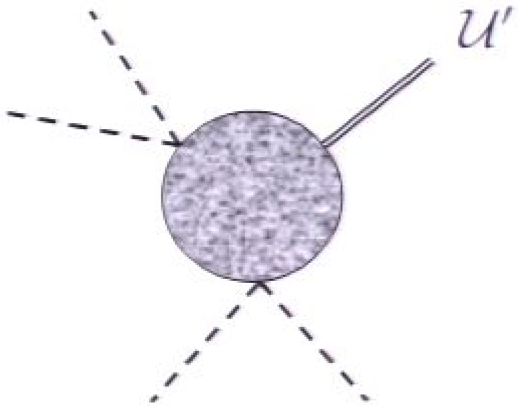

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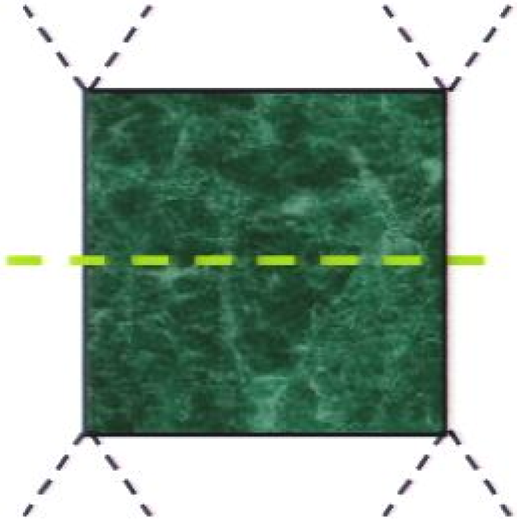

By conformal invariance (in any number of dimensions):

$$\langle 0|\mathrm{T}\mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\mathcal{O}_4(x_4)|0\rangle = \frac{f(u,v)}{(x_{13}^2)^d (x_{24}^2)^d}$$

where

$$u = \frac{x_{12}^2\,x_{34}^2}{x_{13}^2\,x_{24}^2} \qquad v = \frac{x_{12}^2\,x_{34}^2}{x_{14}^2\,x_{23}^2} \qquad x_{ij} = x_i - x_j$$

Unparticle self-interactions

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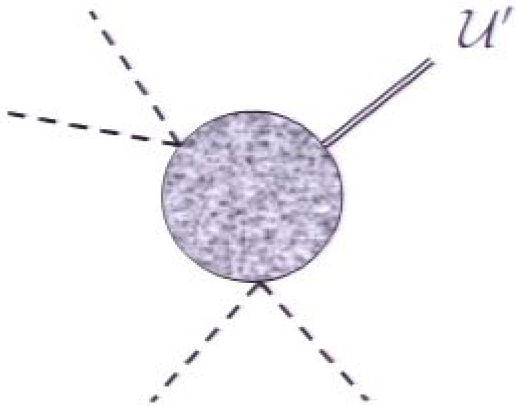

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Use cutting rules
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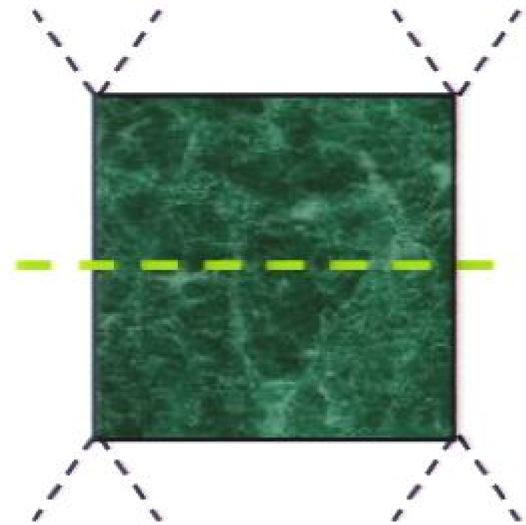

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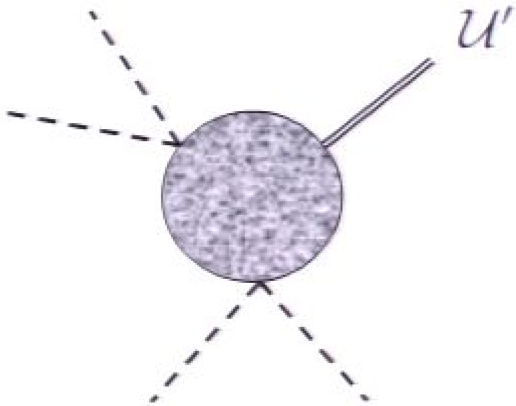
In the Sommerfield model:

$$\langle 0|\mathrm{T}\mathcal{O}(x_1)\mathcal{O}(x_2)\,\mathcal{O}^*(y_1)\mathcal{O}^*(y_2)|0\rangle = \frac{i\Delta_{\mathcal{O}}(x_1-y_1)\, i\Delta_{\mathcal{O}}(x_1-y_2)\, i\Delta_{\mathcal{O}}(x_2-y_1)\, i\Delta_{\mathcal{O}}(x_2-y_2)}{i\Delta_{\mathcal{O}}(x_1-x_2)\, i\Delta_{\mathcal{O}}(y_1-y_2)}$$

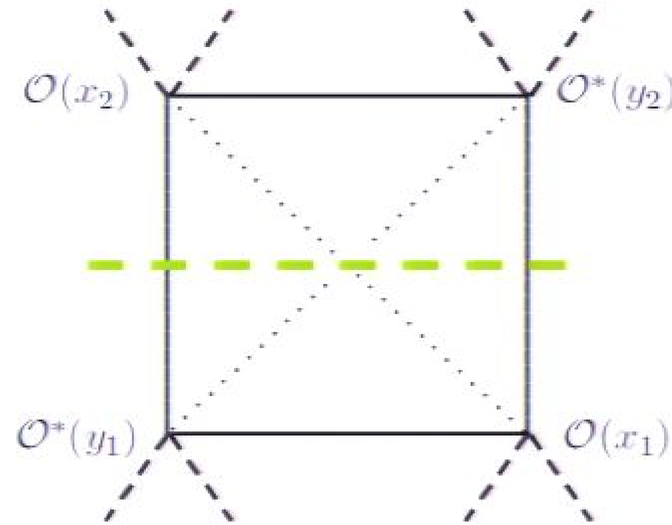
Unparticle self-interactions

$$\mathcal{L}_{\rm int} \sim \frac{1}{M^n}\,\phi_1^{\rm SM}\,\phi_2^{\rm SM}\,\mathcal{O}_\mathcal{U}$$



$$\phi_1^{\rm SM} + \phi_2^{\rm SM} \to \phi_1^{\rm SM} + \phi_2^{\rm SM} + \mathcal{U}'$$

Use cutting rules
on 4-point function of $\mathcal{O}_\mathcal{U}$



$$\phi + \phi \to \phi + \phi + \mathcal{U}$$

By conformal invariance (in any number of dimensions):

$$\langle 0|\mathrm{T}\mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\mathcal{O}_4(x_4)|0\rangle = \frac{f(u,v)}{(x_{13}^2)^d(x_{24}^2)^d}$$

where

$$u = \frac{x_{12}^2\,x_{34}^2}{x_{13}^2\,x_{24}^2} \qquad v = \frac{x_{12}^2\,x_{34}^2}{x_{14}^2\,x_{23}^2} \qquad x_{ij} = x_i - x_j$$

In the Sommerfield model:

$$\langle 0|\mathrm{T}\mathcal{O}(x_1)\mathcal{O}(x_2)\,\mathcal{O}^*(y_1)\mathcal{O}^*(y_2)|0\rangle = \frac{i\Delta_\mathcal{O}(x_1-y_1)\,i\Delta_\mathcal{O}(x_1-y_2)\,i\Delta_\mathcal{O}(x_2-y_1)\,i\Delta_\mathcal{O}(x_2-y_2)}{i\Delta_\mathcal{O}(x_1-x_2)\,i\Delta_\mathcal{O}(y_1-y_2)}$$

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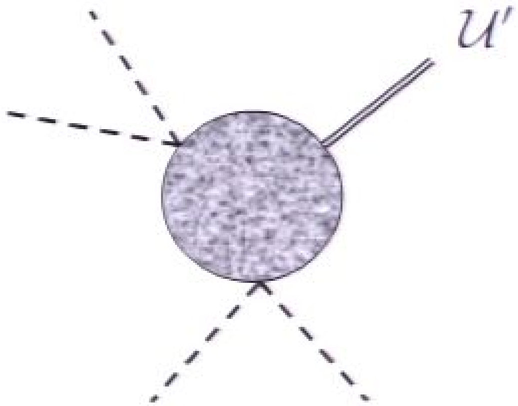

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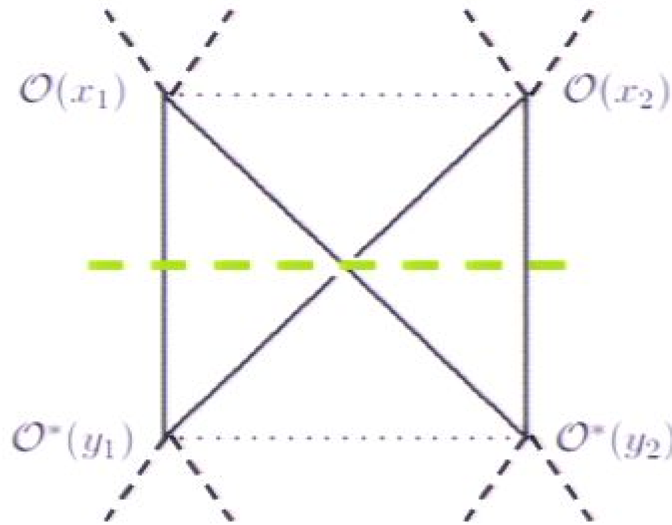

$$\phi + \phi \rightarrow \overline{\phi} + \overline{\phi} + \mathcal{U}$$

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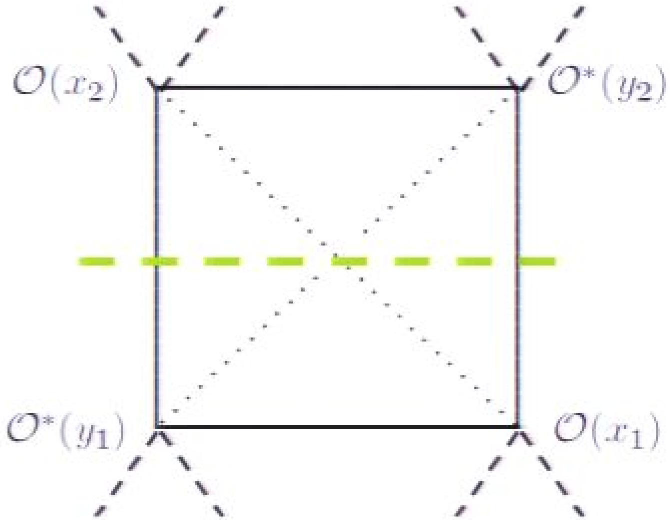
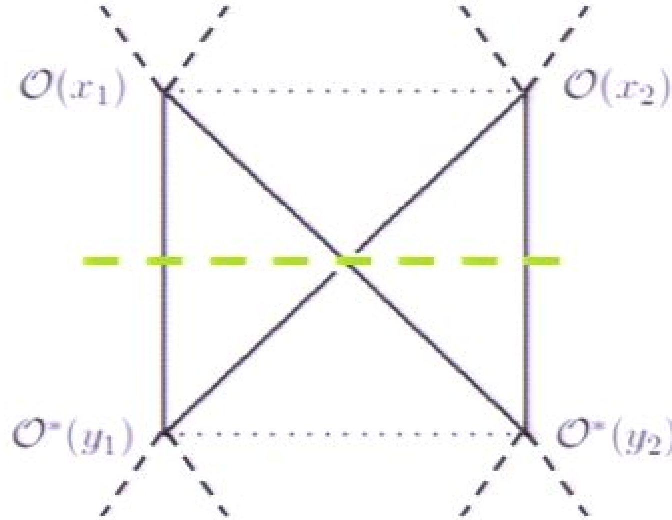
Unparticle self-interactions (cont'd)



$\phi + \phi \to \phi + \phi + \mathcal{U}$



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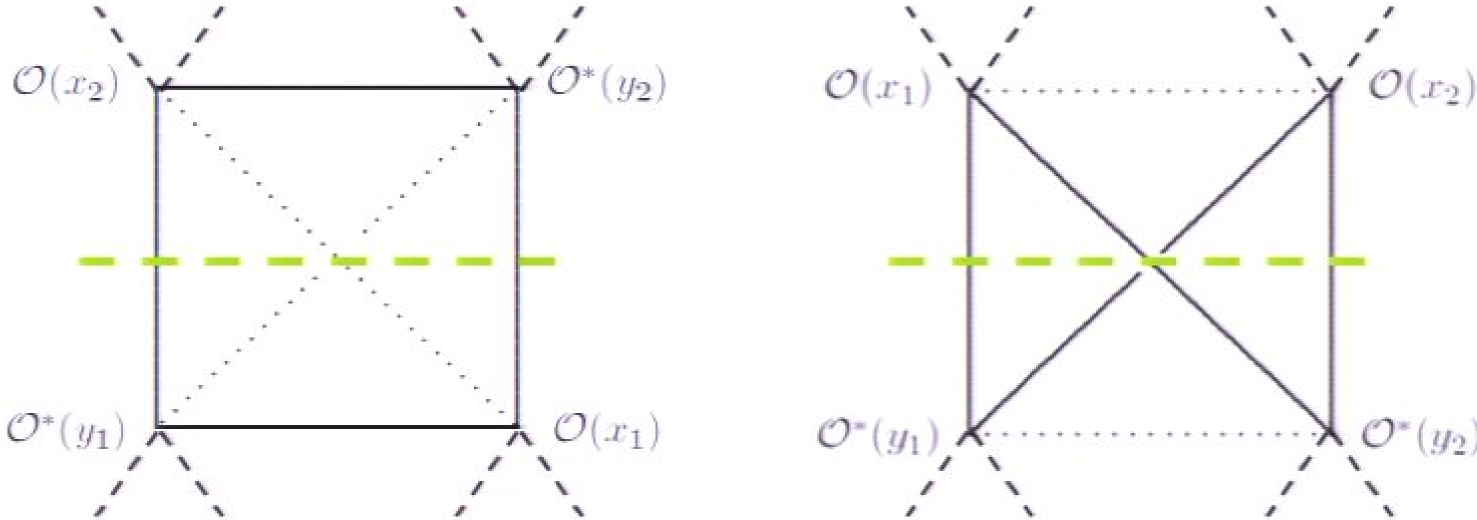
Unparticle self-interactions (cont′d)



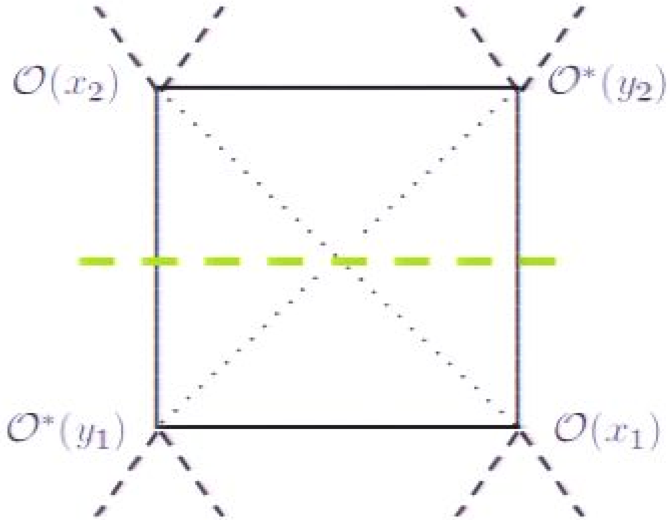
$\phi + \phi \to \phi + \phi + \mathcal{U}$ $\qquad$ $\phi + \phi \to \bar{\phi} + \bar{\phi} + \mathcal{U}$

n momentum space:

—— $$i\Delta_{\mathcal{O}}(k) = \int \frac{d^2 p_{\mathcal{U}}}{(2\pi)^2}\, i\Delta_{\mathcal{U}}(p_{\mathcal{U}}) \sum_{n=0}^{\infty} \frac{(-4\pi a)^n}{n!} \left[\prod_{i=1}^{n} \frac{d^2 p_i}{(2\pi)^2} i\Delta(p_i)\right] (2\pi)^2 \delta^2 \left(k - p_{\mathcal{U}} - \sum_{j=1}^{n} p_j\right)$$

········ $$i\tilde{\Delta}_{\mathcal{O}}(k) = -i\frac{8\pi^4 A(-2-a)(\xi m)^{2a}}{\sin(\pi a)} \int \frac{d^2 p}{(2\pi)^2} \frac{1}{(-p^2 - i\epsilon)^{2+a}} \sum_{n=0}^{\infty} \frac{(4\pi a)^n}{n!} \left[\prod_{i=1}^{n} \frac{d^2 p_i}{(2\pi)^2} i\Delta(p_i)\right] (2\pi)^2 \delta^2 \left(k - p - \sum_{j=1}^{n} p_j\right.$$

Unparticle self-interactions (cont'd)



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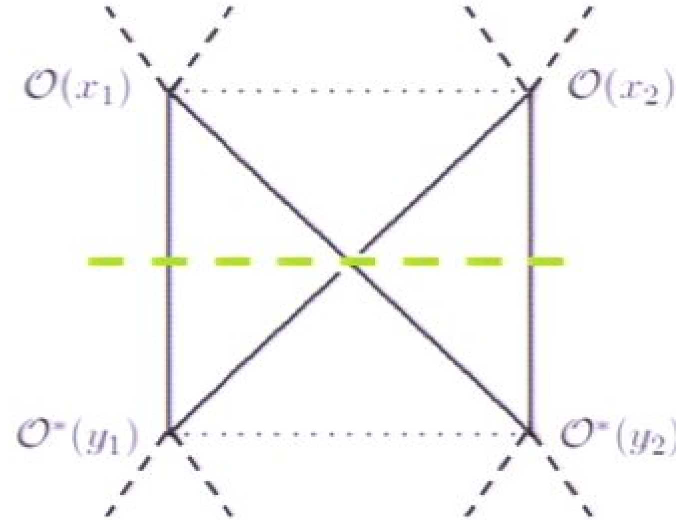

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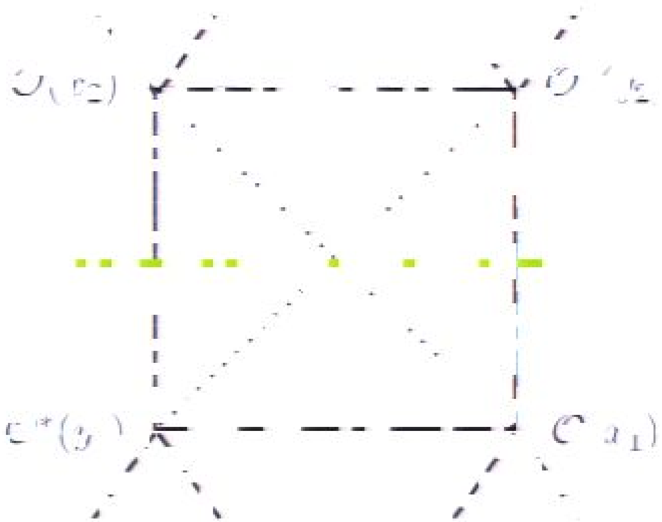
We can compute the cross-sections for these processes.

But what do they correspond to physically?

Unparticle self-interactions (cont′d)

Limit of small unparticle momentum



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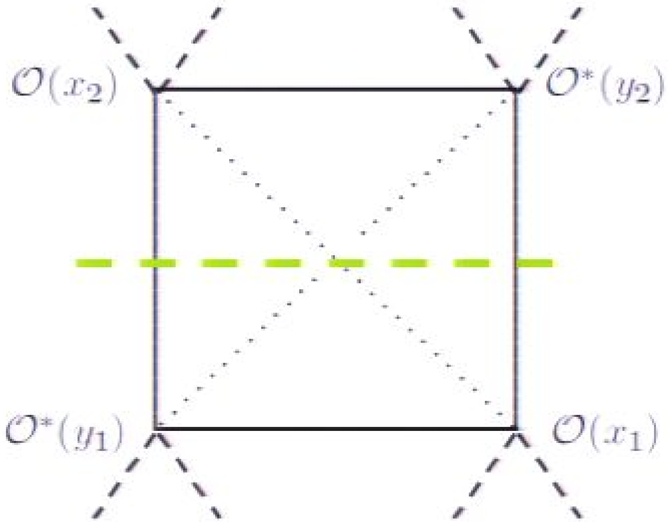
Unparticle self-interactions (cont′d)

Limit of small unparticle momentum



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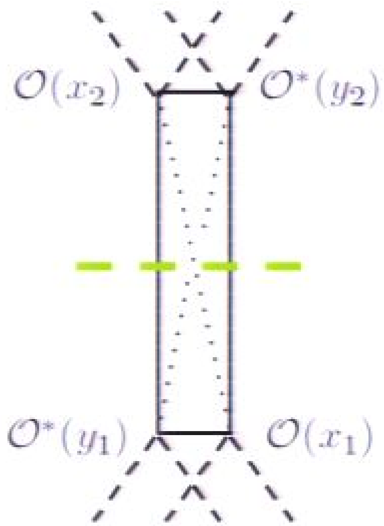
Unparticle self-interactions (cont'd)

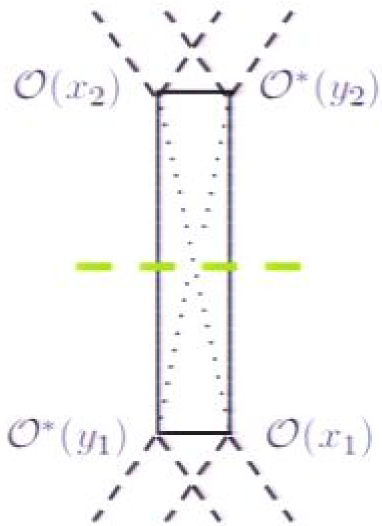
Limit of small unparticle momentum



$$\underbrace{\phi + \phi}_{P} \to \underbrace{\phi + \phi}_{\simeq P} + \underbrace{\mathcal{U}}_{Q \ll P}$$

Unparticle self-interactions (cont'd)

Limit of small unparticle momentum



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Operator product expansion

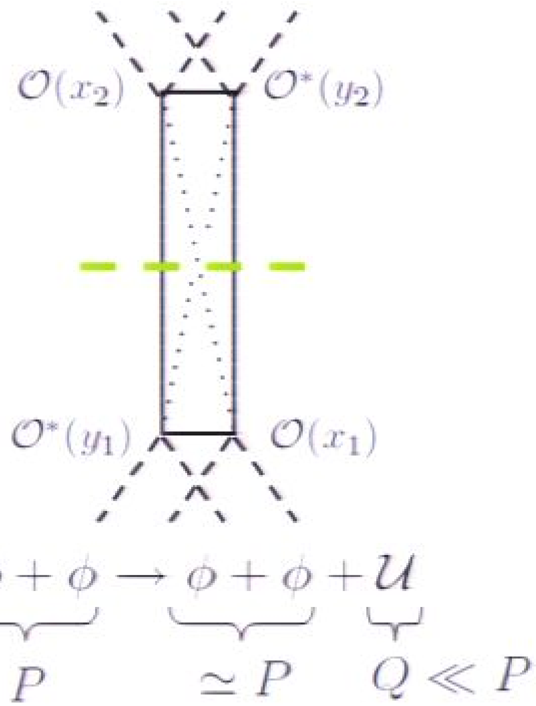
$$\mathrm{T}\,\mathcal{O}(x_i)\,\mathcal{O}^*(y_i) = i\Delta_{\mathcal{U}}(\zeta_i) + \sum_k c_k(\zeta_i)\,\mathcal{O}_k(x_i)$$

$$\zeta_i \equiv x_i - y_i$$

$$X \equiv x_2 - x_1$$

Unparticle self-interactions (cont′d)

Limit of small unparticle momentum



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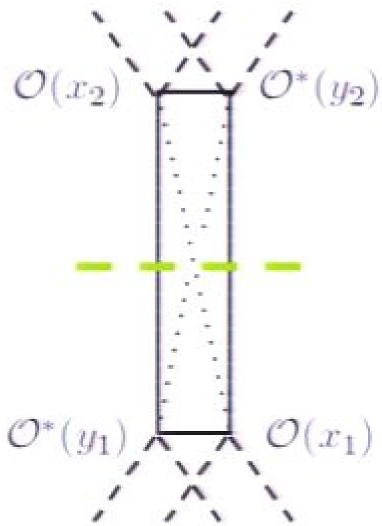
Unparticle self-interactions (cont'd)

Limit of small unparticle momentum



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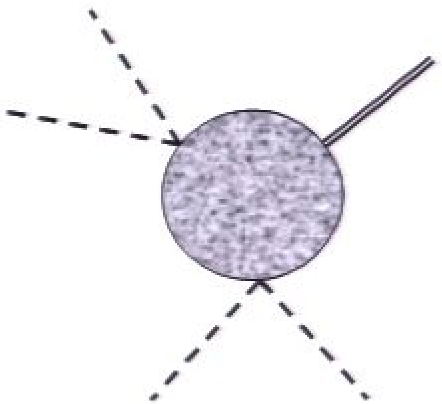
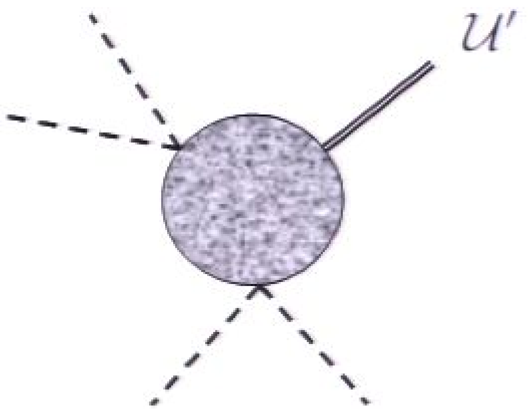
This process produces unparticle stuff
that corresponds to other operators of the CFT.

Unparticle self-interactions (cont′d)



Is it possible to compute the amplitude $\mathcal{M}$ and then $\mathcal{M}^*\mathcal{M}$, rather than directly the inclusive cross-section?

Unparticle self-interactions (cont'd)



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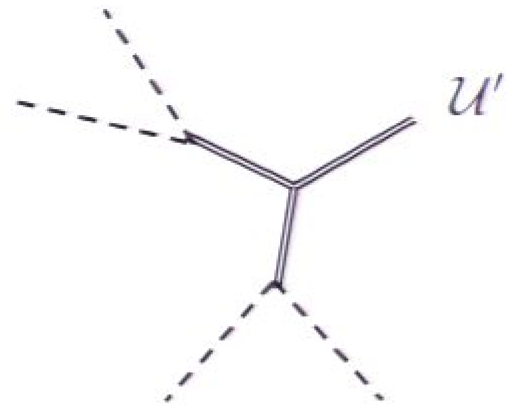

Define the amputated 3-point function $Q_i(z|x,y)$

$$\int d^2z \, \langle 0|\mathcal{O}_i^*(z')\mathcal{O}_i(z)|0\rangle \, Q_i(z|x,y) = \langle 0|\mathcal{O}_i^*(z')\mathcal{O}(x)\mathcal{O}^*(y)|0\rangle$$

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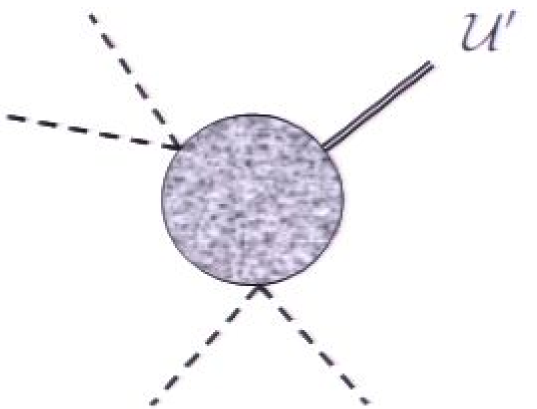

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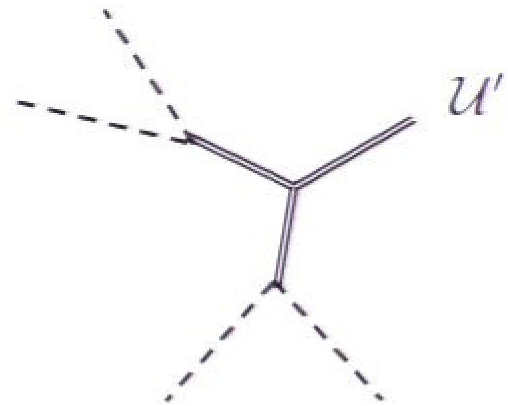

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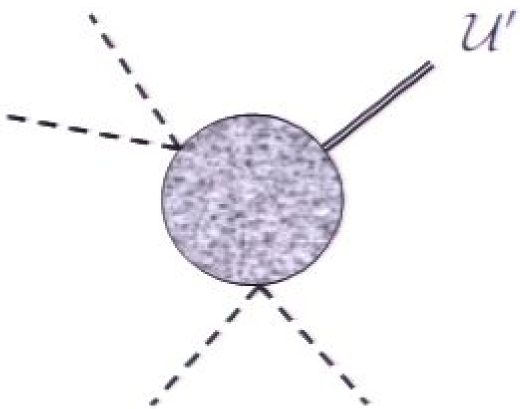

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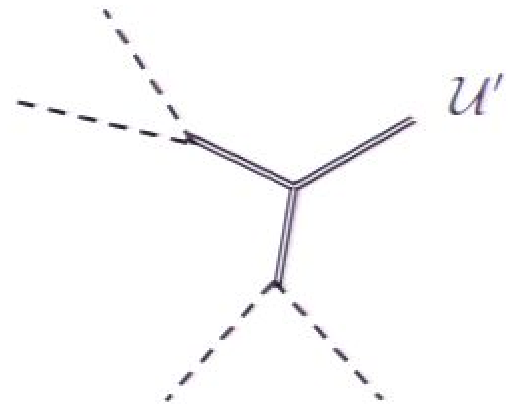

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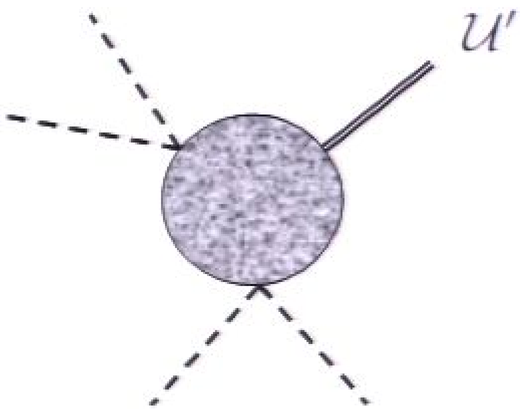
Ferrara, Gatto, Grillo, Parisi 1972

Unparticle self-interactions (cont'd)



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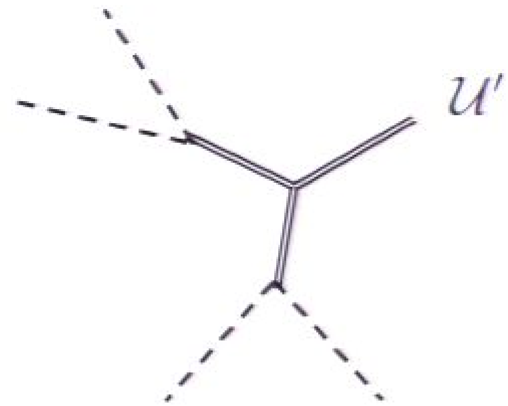

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$$\langle 0|\mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}^*(y_1)\mathcal{O}^*(y_2)|0\rangle = \sum \int d^2z\, d^2z'\, Q_i^*(z'|y_2, x_2)\, \langle 0|\mathcal{O}_i^*(z')\mathcal{O}_i(z)|0\rangle\, Q_i(z|x_1, y_1)$$

Summary and Conclusions

$$\text{Schwinger model} \leftrightarrow \text{QCD}$$

$$\text{Sommerfield model} \leftrightarrow \text{unparticle sector}$$

- (At least) in some models, expect to have "hardons" of mass $\Lambda_{\mathcal{U}}$ in addition to unparticle stuff.

Summary and Conclusions

Schwinger model $\leftrightarrow$ QCD

Sommerfield model $\leftrightarrow$ unparticle sector

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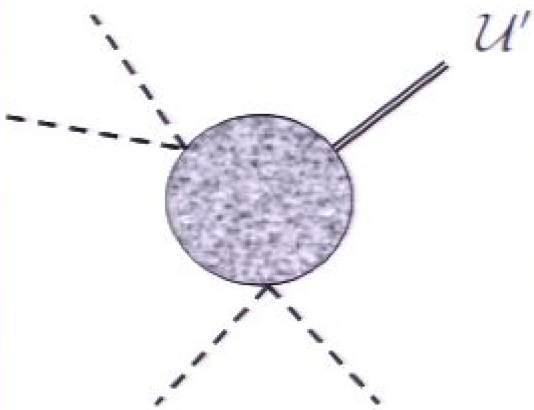

Higher n-point functions of the CFT are important for many aspects of unparticle physics.

- Production of unparticle stuff corresponding to operators to which the standard model does not couple directly.
- We know how to compute these processes, and in particular how to deal with IR divergences.

THE END

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