Title: Bell\'s theorem and monogamy

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URL: http://pirsa.org/08120020

Abstract: Quantum entanglement has two remarkable properties. First, according to Bell\'s theorem, the statistical correlations between entangled quantum systems are inconsistent with any theory of local hidden variables. Second, entanglement is monogamous -- that is, to the degree that A and B are entangled with each other, they cannot be entangled with any other systems. It turns out that these properties are intimately related.

EPR

QM

EPR

QM +

EPR

QM + locality →

"criterion of reality"

EPR

QM + locality $\longrightarrow$ hidden variables

"criterion of reality"

EPR

QM + locality $\longrightarrow$ hidden variables

"criterion of reality"

Bell

EPR

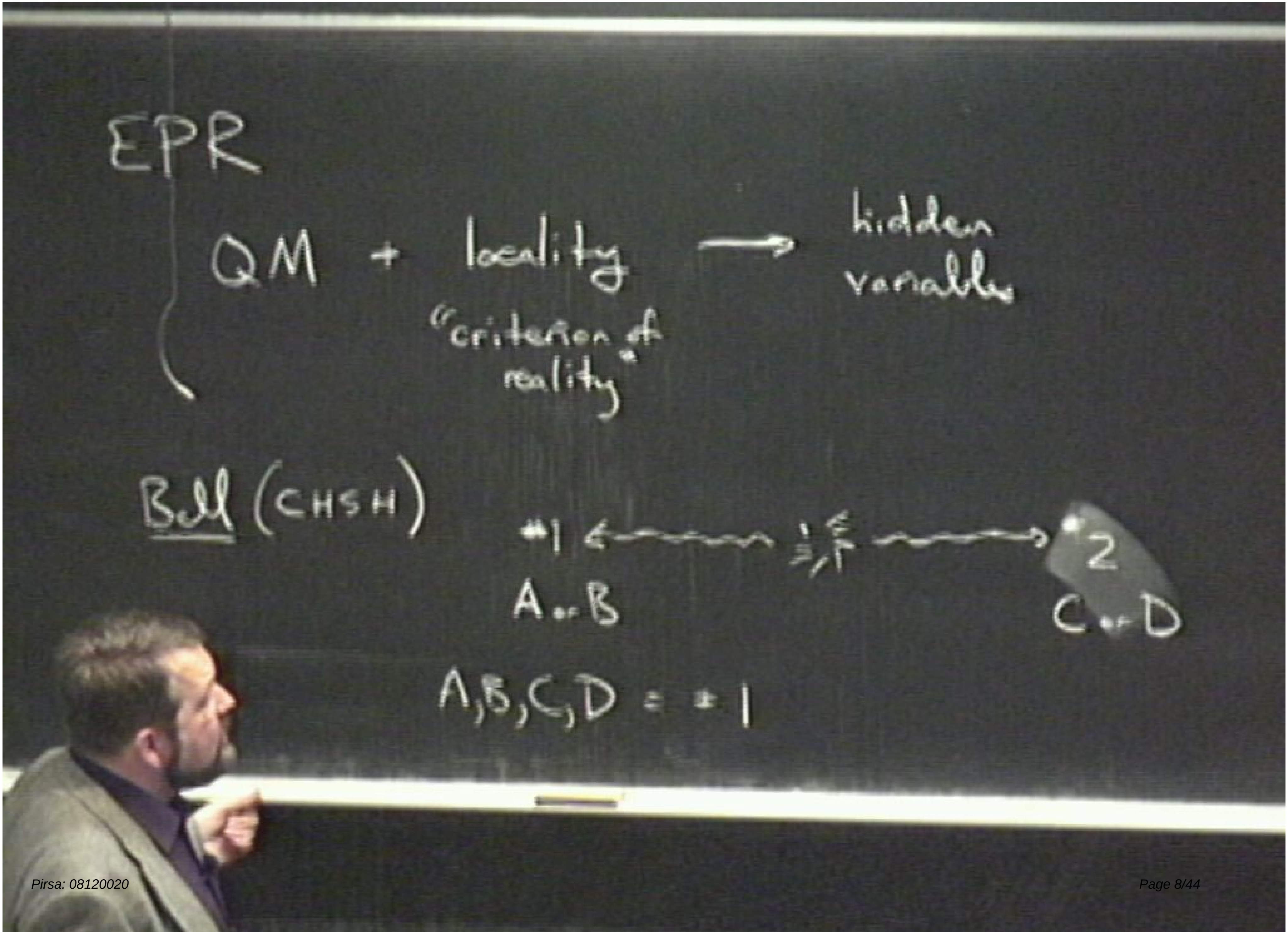
QM + locality → hidden variables

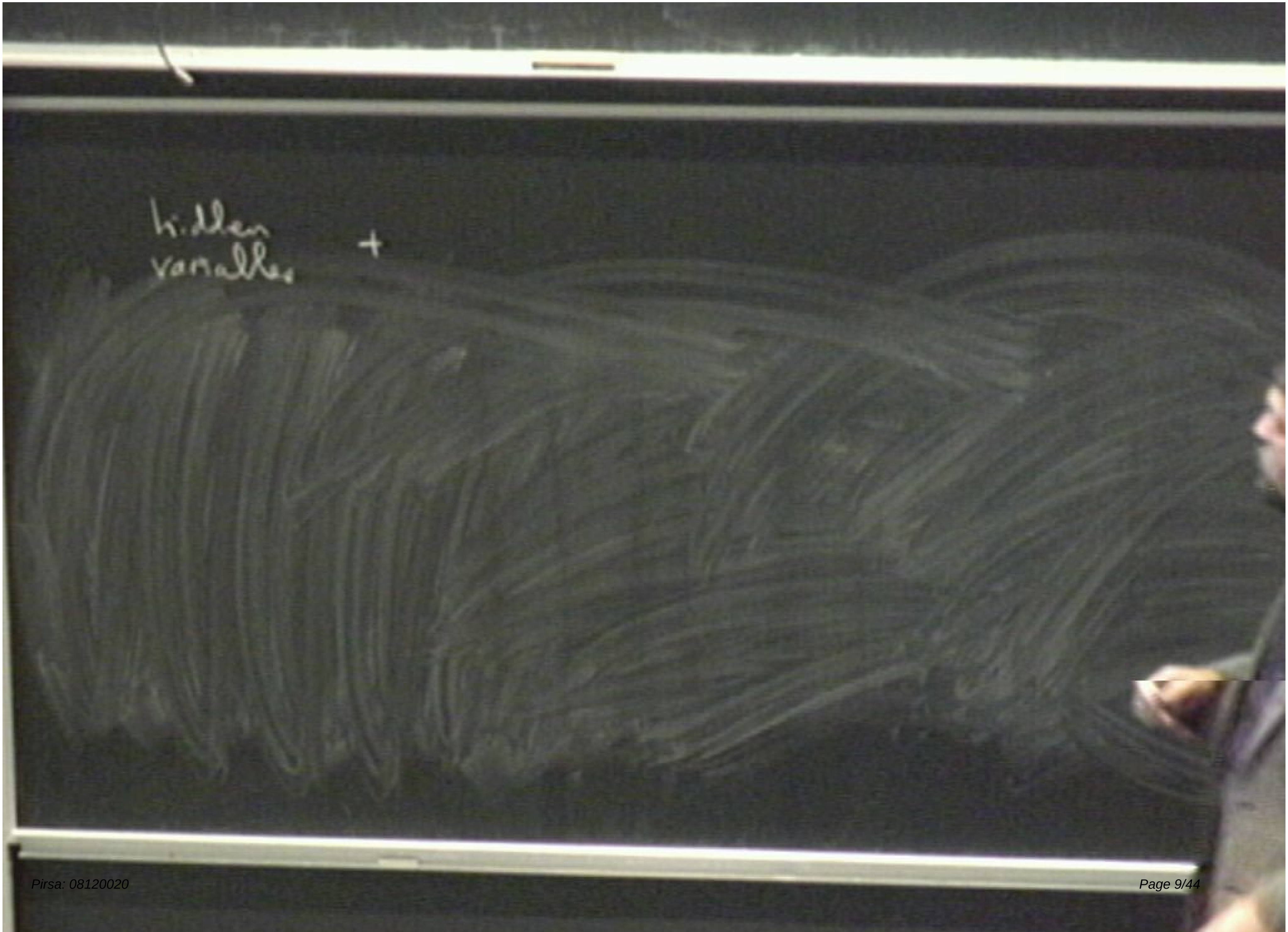
"criterion of reality"

Bell (CHSH)

#1 ← ~~~ ※ ~~~ → #2

A or B C or D


EPR
QM + locality → hidden variables
"criterion of reality"
Bell (CHSH)
#1
#2
A or B
C or D
A,B,C,D = ±1


hidden
variables
+

hidden variables + locality

hidden variables + locality $\Rightarrow$?

Consider $A(C-D) + B(C+D) = \pm 2$

hidden variables + locality $\Rightarrow$?

Consider $$A(C-D) + B(C+D) = \pm 2$$

$$\left| \langle AC \rangle - \langle AD \rangle + \langle BC \rangle + \langle BD \rangle \right| \leq 2$$

hidden variables + locality $\Rightarrow$?

Bell

Consider $A(C-D) + B(C+D) = \pm 2$

$$\left| \langle AC \rangle - \langle AD \rangle + \langle BC \rangle + \langle BD \rangle \right| \leq 2$$

QM: Two spins in $|\Psi^-\rangle$

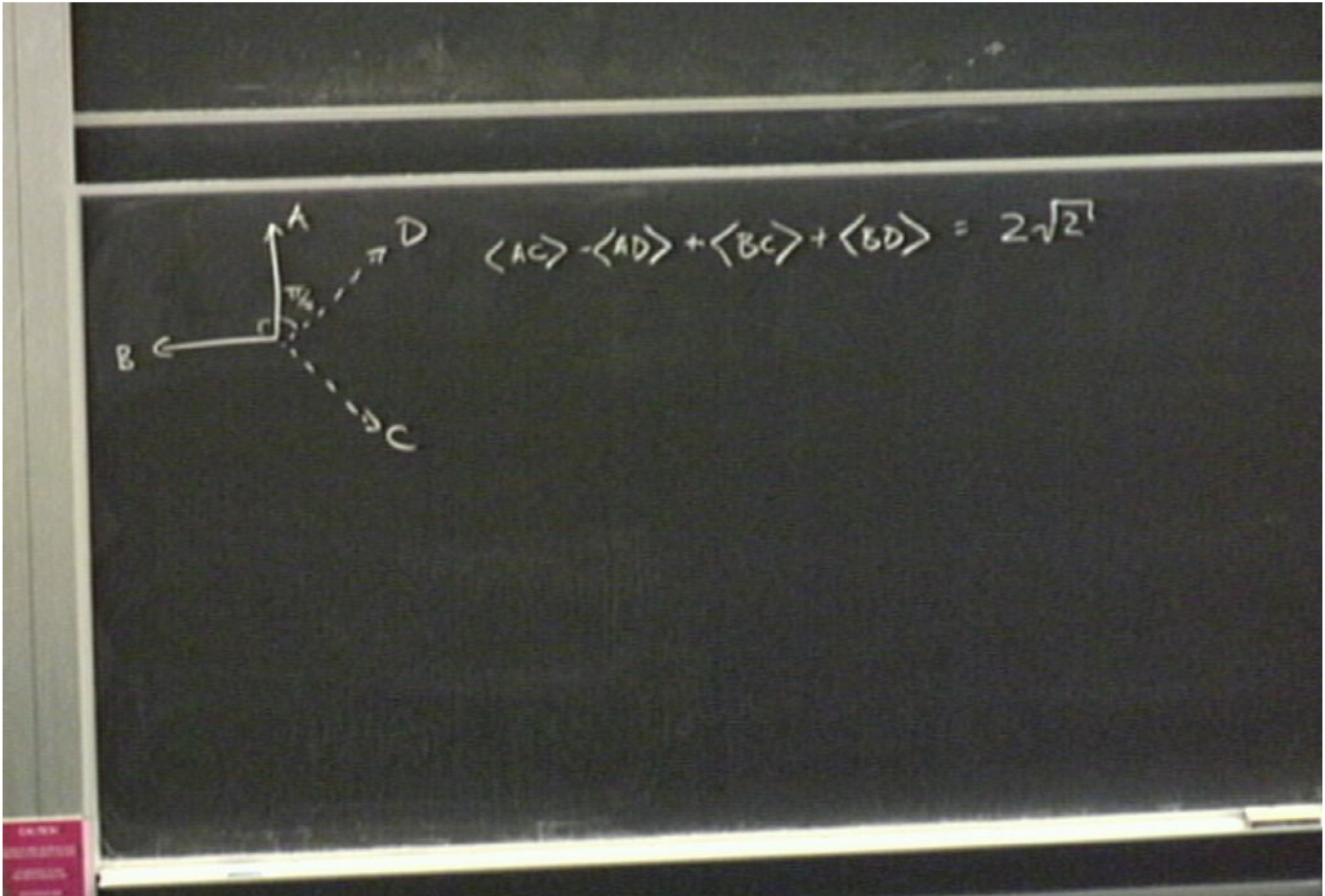
hidden variables + locality $\Rightarrow$?

Consider $A(C-D) + B(C+D) = \pm 2$ Bell

$$\left| \langle AC \rangle - \langle AD \rangle + \langle BC \rangle + \langle BD \rangle \right| \leq 2$$

QM: Two spins in $|\Psi^-\rangle$

$\langle AC \rangle = -\cos\theta$


A
D
B
C
π/4
⟨AC⟩ − ⟨AD⟩ + ⟨BC⟩ + ⟨BD⟩ = 2√2

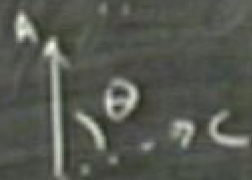
$$\begin{pmatrix}\text{hidden}\\ \text{variables}\end{pmatrix} + (\text{locality}) \Rightarrow \underline{\underline{\text{NOT}}}\ \text{QM}$$

Bell ≠

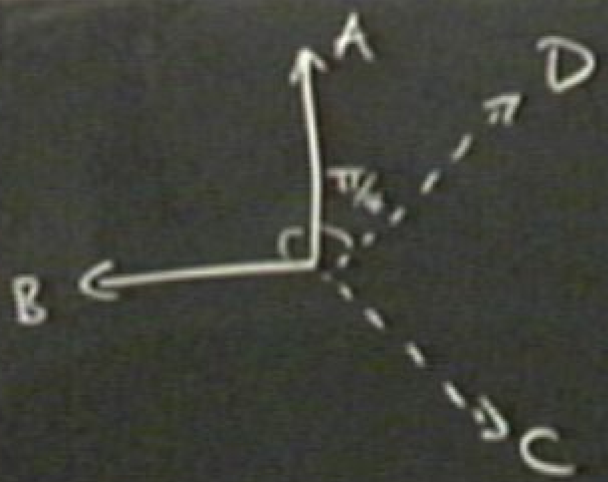
Consider $A(C-D) + B(C+D) = \pm 2$

$$\left|\langle AC\rangle - \langle AD\rangle + \langle BC\rangle + \langle BD\rangle\right| \leq 2$$

QM: Two spins in $|\Psi^-\rangle$



$\langle AC\rangle = -\cos\theta$



$$\langle AC\rangle - \langle AD\rangle + \langle BC\rangle + \langle BD\rangle = 2\sqrt{2}$$

It suffices to have

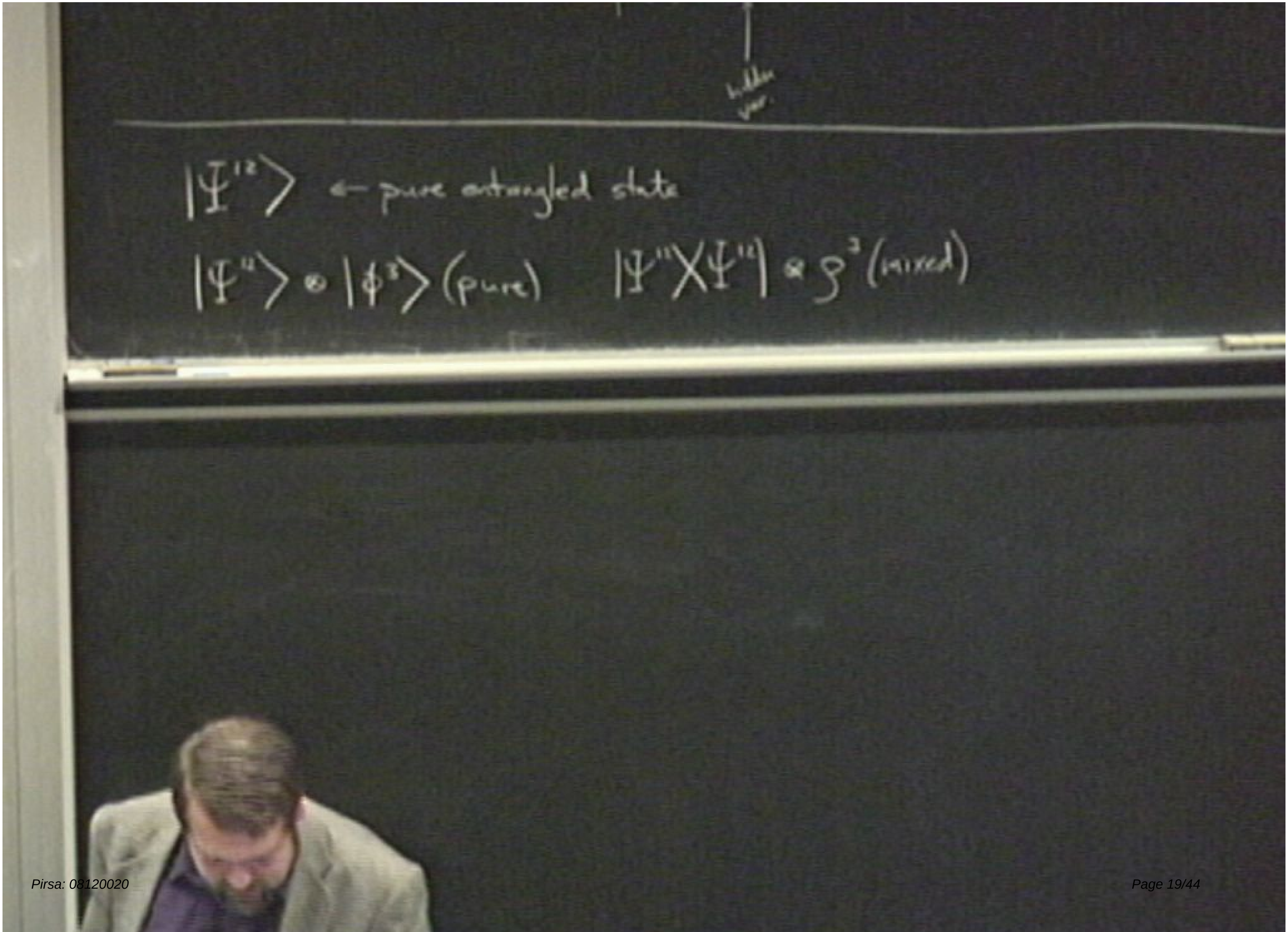
$$p(a,c|\lambda) = p(a|\lambda)\, p(c|\lambda)$$

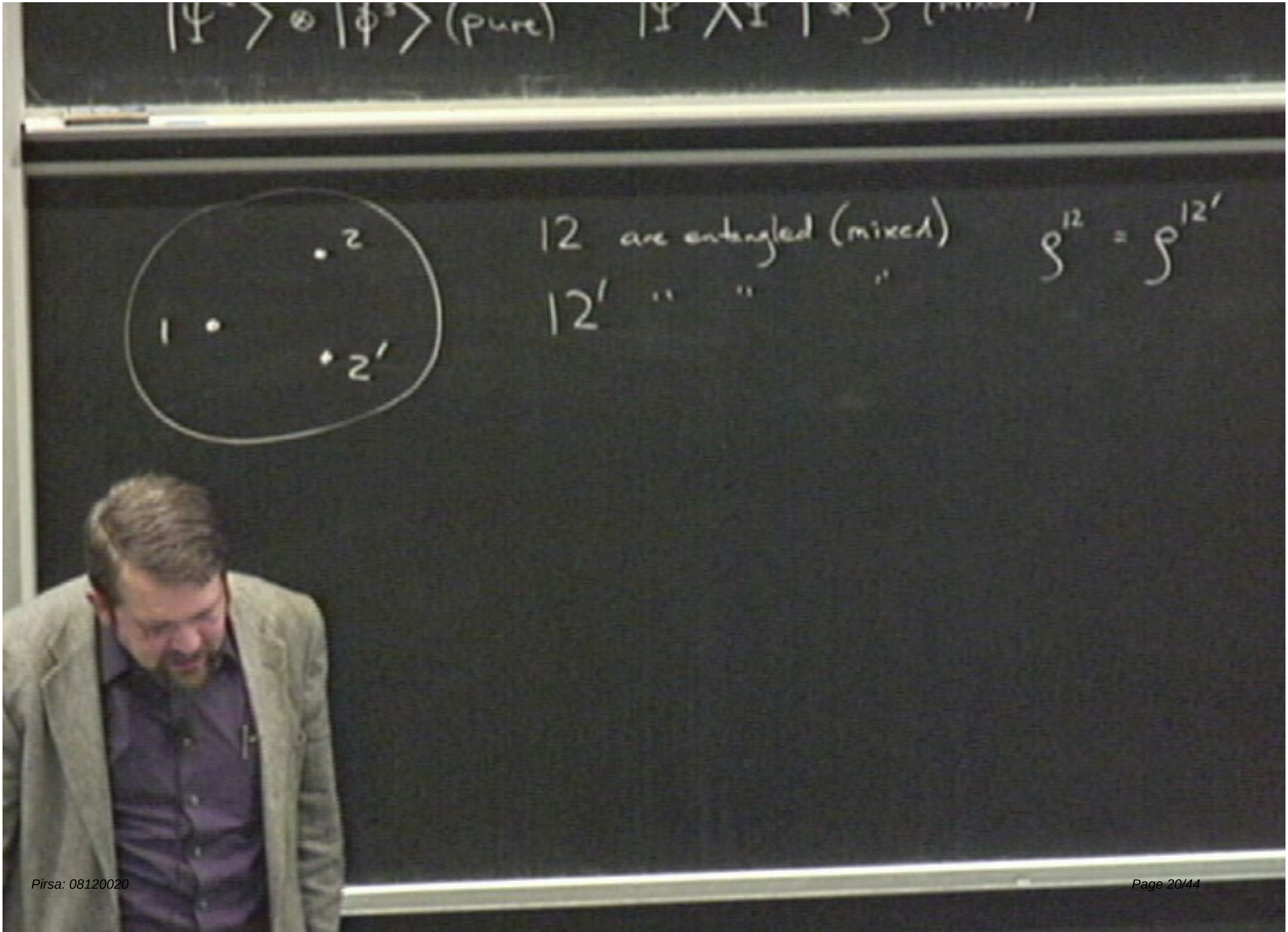
hidden var.

$$p(a,c|\lambda) = p(a|\lambda)\,p(c|\lambda)$$

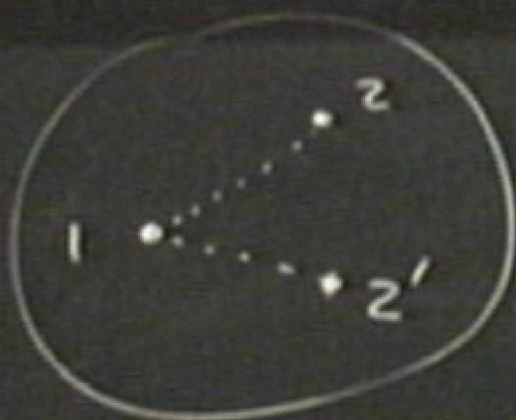
hidden var.

$|\Psi^{12}\rangle \leftarrow$ pure entangled state


hidden var.
$|\Psi^{12}\rangle$ ← pure entangled state
$|\Psi^{12}\rangle \otimes |\phi^{3}\rangle$ (pure) $|\Psi^{12}\rangle\langle\Psi^{12}| \otimes \rho^{3}$ (mixed)


(pure)
12 are entangled (mixed)
12' " " "
ρ^12 = ρ^12'
1
2
2'

$|\Psi\rangle \otimes |\phi^3\rangle$ (pure) $\quad |\Psi\rangle\langle\Psi| \otimes \rho$ (mixed)



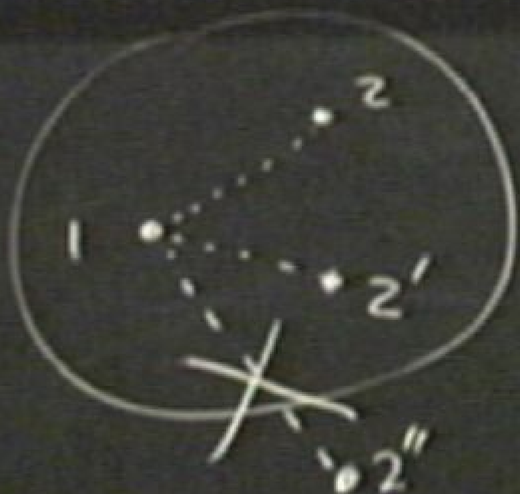
12 are entangled (mixed) $\quad \rho^{12} = \rho^{12'}$

12' " " "

But QM forbids existence of 2'' with

$$\rho^{12''} = \rho^{12} = \rho^{12'}$$

$|\Psi\rangle \otimes |\phi\rangle$ (pure) $|\Psi\rangle\langle\Psi| \otimes \rho$



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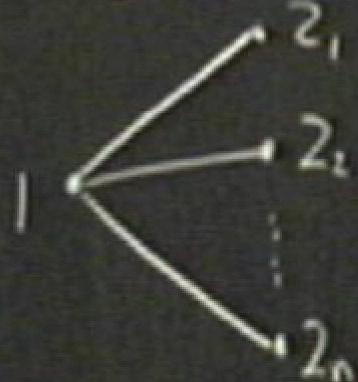
12′ " " "

But QM forbids existence of 2″ with

$$\rho^{12''} = \rho^{12} = \rho^{12'}$$

Each line is a term $\vec{S}_1 \cdot \vec{S}_{2_k}$



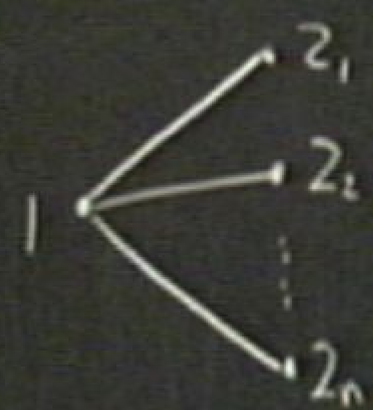
$|\Psi^{\cdot}\rangle \otimes |\phi^3\rangle$ (pure) $\quad |\Psi\rangle\langle\Psi| \otimes \rho$ (mixed)

12 are entangled (mixed) $\quad \rho^{12} = \rho^{12'}$

12' " " "

But QM forbids existence of $2''$ with

$$\rho^{12''} = \rho^{12} = \rho^{12'}$$



Each line is a term $\vec{S}_1 \cdot \vec{S}_{2_i}$ in H

Find ground state

① ρ^{12_i} same entangled

②

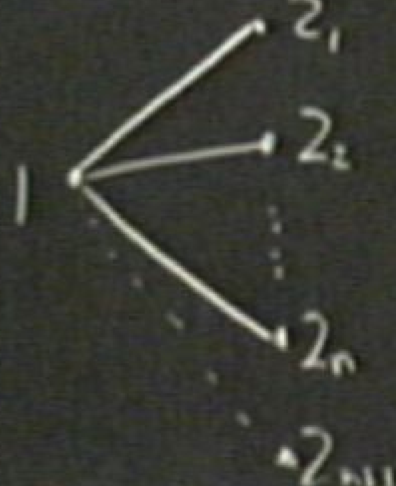
12 are entangled (mixed) $\rho^{12} = \rho^{12'}$

12' " " "

But QM forbids existence of 2'' with

$$\rho^{12''} = \rho^{12} = \rho^{12'}$$

Each line is a term $\vec{S}_1 \cdot \vec{S}_{2_k}$ in H



Find ground state

① ρ^{12_k} same entangled

② QM forbids 2_{n+1} with same ρ^{12}

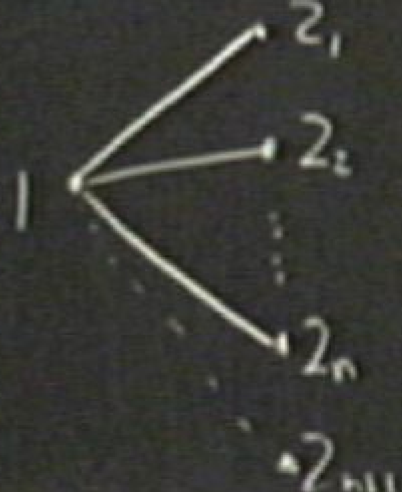
12 are entangled (mixed) $\qquad \rho^{12} = \rho^{12'}$

12' " " "

But QM forbids existence of 2'' with

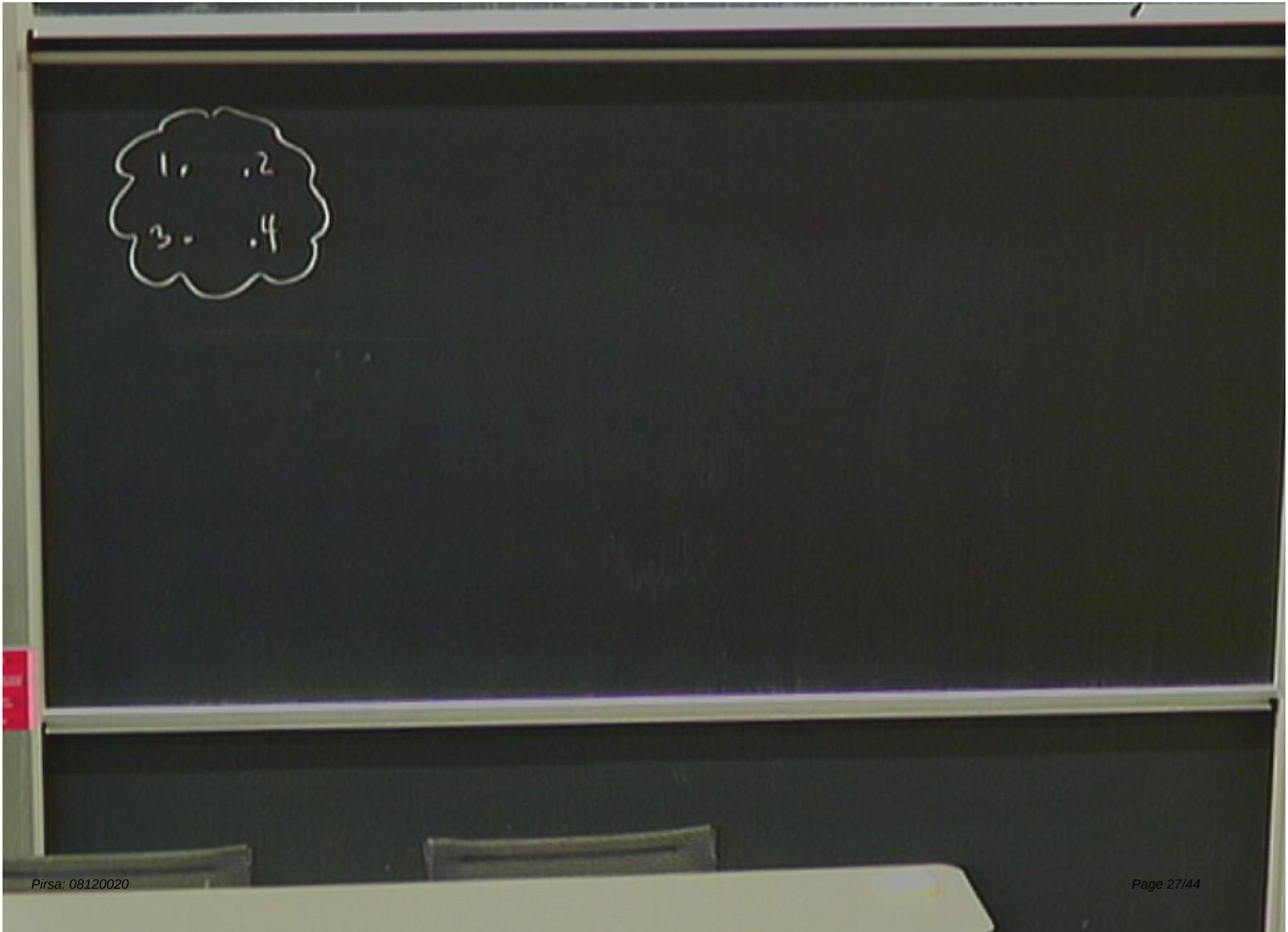
$$\rho^{12''} = \rho^{12} = \rho^{12'}$$

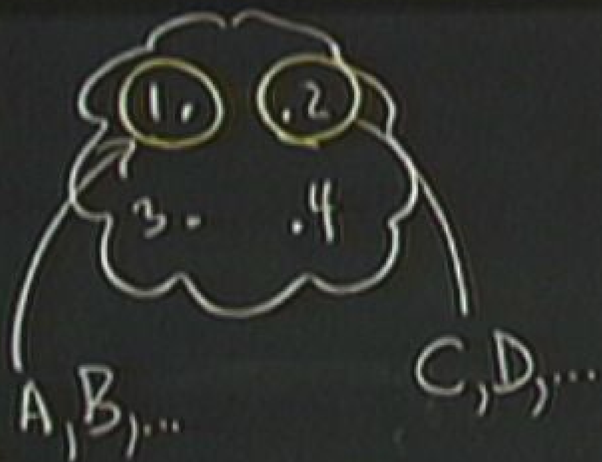
Each line is a term $\vec{S}_1 \cdot \vec{S}_{2_k}$ in H



Find ground state

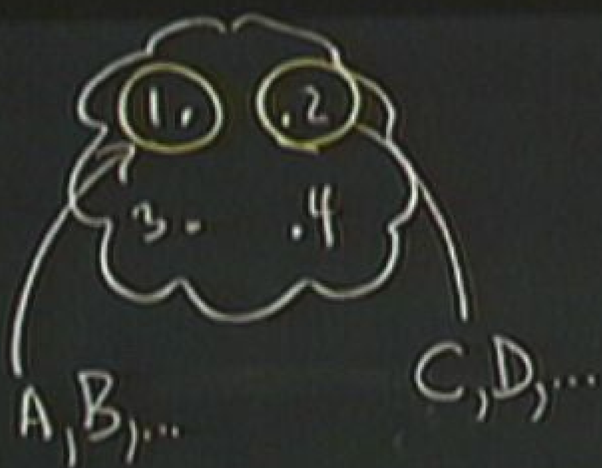
1. ρ^{12_k} some entangled state
2. QM forbids 2_{n+1} with same $\rho^{12_{n+1}}$


1. .2
3. .4



Statistical relation $\mathbb{R}(1,2)$

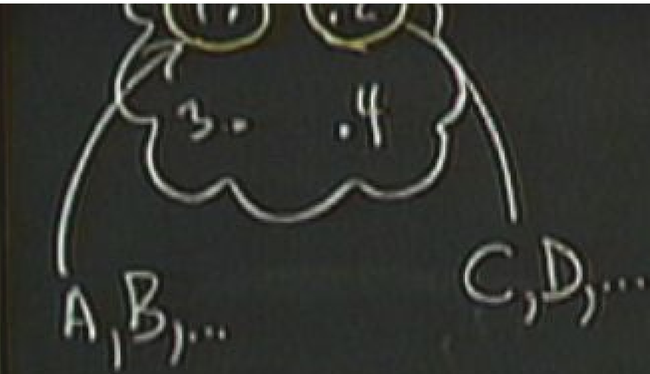
$$p(a,c \mid A,C)$$



Statistical relation $\mathbb{R}(1,2)$

$p(a,c \mid A,C)$ for all A, C choices

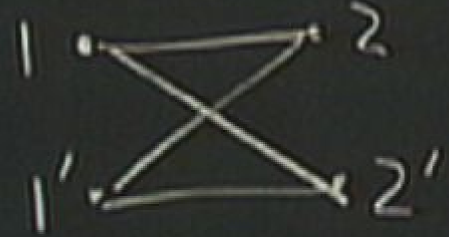
outcomes (a,c) choices (A,C)



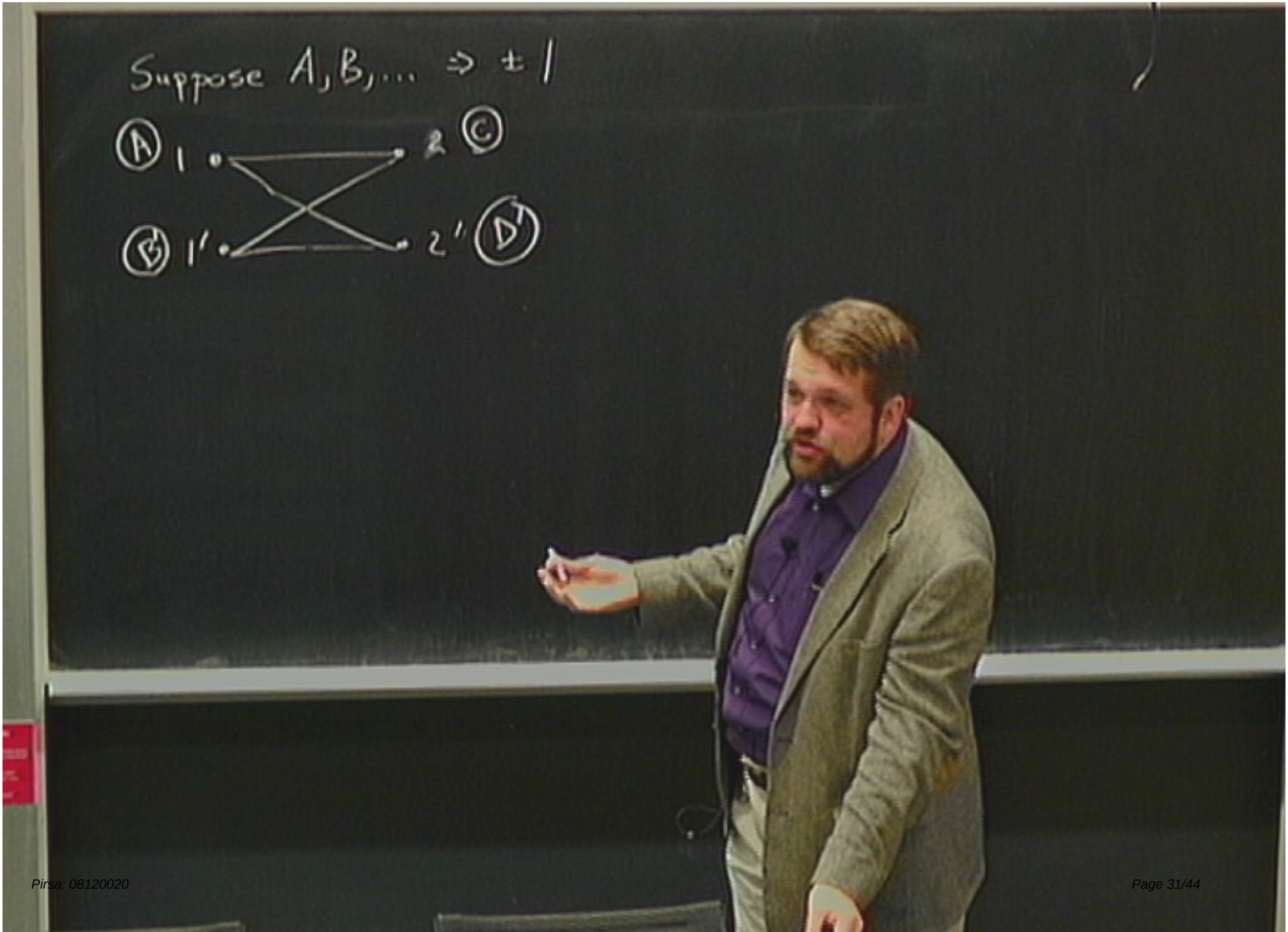
$$\{ p(a,c \mid A,C) \} \text{ for all } A,C \text{ choice}$$

$\uparrow$ outcomes $\quad$ $\uparrow$ choices

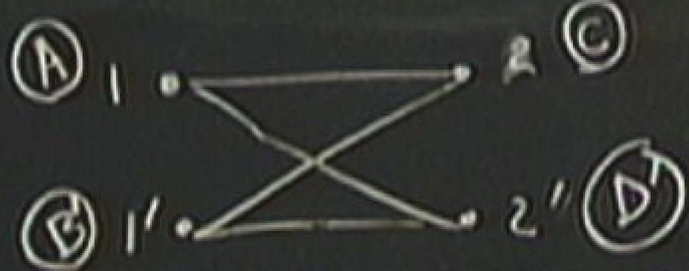
Could there be $1'$, $2'$



$$\begin{array}{ccc} R(1,2) & = & R(1,2') \\ \| & & \| \\ R(1',2) & = & R(1',2') \end{array}$$

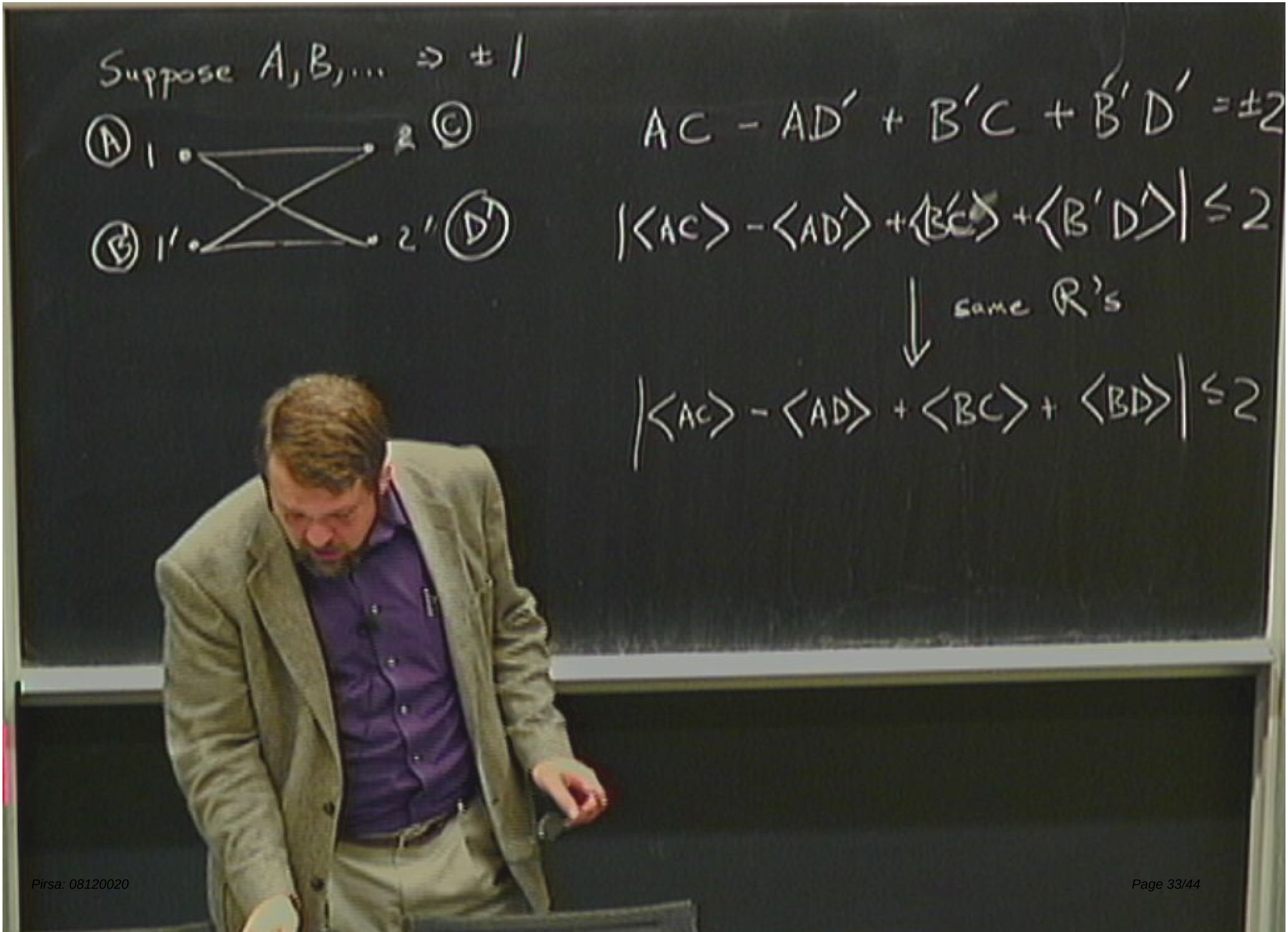

Suppose A, B, ... ⇒ ±1
A 1
2 C
B 1'
2' D^T

Suppose $A, B, \ldots \Rightarrow \pm 1$

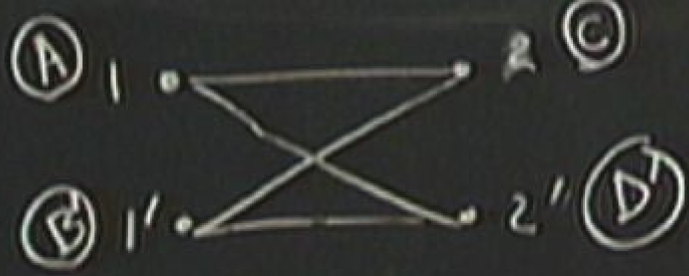


$$AC - AD' + B'C + B'D' = \pm 2$$

$$|\langle AC\rangle - \langle AD'\rangle + \langle B'C\rangle + \langle B'D'\rangle| \leq 2$$


Suppose $A, B, \ldots \Rightarrow \pm 1$
A 1 2 C
B 1' 2'' D'
$AC - AD' + B'C + B'D' = \pm 2$
$|\langle AC\rangle - \langle AD'\rangle + \langle B'C\rangle + \langle B'D'\rangle| \leq 2$
same R's
$|\langle AC\rangle - \langle AD\rangle + \langle BC\rangle + \langle BD\rangle| \leq 2$

Suppose $A, B, \ldots \Rightarrow \pm 1$



$$AC - AD' + B'C + B'D' = \pm 2$$

$$\left| \langle AC \rangle - \langle AD' \rangle + \langle B'C \rangle + \langle B'D' \rangle \right| \leq 2$$

$$\Big\downarrow \text{ same } R\text{'s}$$

$$\left| \langle AC \rangle - \langle AD \rangle + \langle BC \rangle + \langle BD \rangle \right| \leq 2$$

Violated by QM!

$$p(a,c) = \sum_{\lambda} p(a|\lambda)\, p(c|\lambda)\, p(\lambda)$$

Given $p(a|\lambda)$

$$p(a,c) = \sum_{\lambda} p(a|\lambda)\, p(c|\lambda)\, p(\lambda)$$

Given $p(a|\lambda)$

Can find a' with <u>same</u>

$p(a'|\lambda)$

$$p(a,c) = \sum_\lambda p(a|\lambda)\, p(c|\lambda)\, p(\lambda)$$

$$= \sum_\lambda p(a'|\lambda)\, p(c|\lambda)\, p(\lambda)$$

$$= p(a',c)$$

Given $p(a|\lambda)$

Can find a' with <u>same</u> $p(a'|\lambda)$

$$p(a,c) = \sum_\lambda p(a|\lambda)\, p(c|\lambda)\, p(\lambda)$$

$$= \sum_\lambda p(a'|\lambda)\, p(c|\lambda)\, p(\lambda)$$

$$= p(a',c)$$

Given $p(a|\lambda)$

Can find a' with <u>same</u> $p(a'|\lambda)$

local hidden variable $\Longrightarrow$ $\mathcal{R}(1,2)$ is sharable

$$p(a,c) = \sum_\lambda p(a|\lambda)\, p(c|\lambda)\, p(\lambda)$$

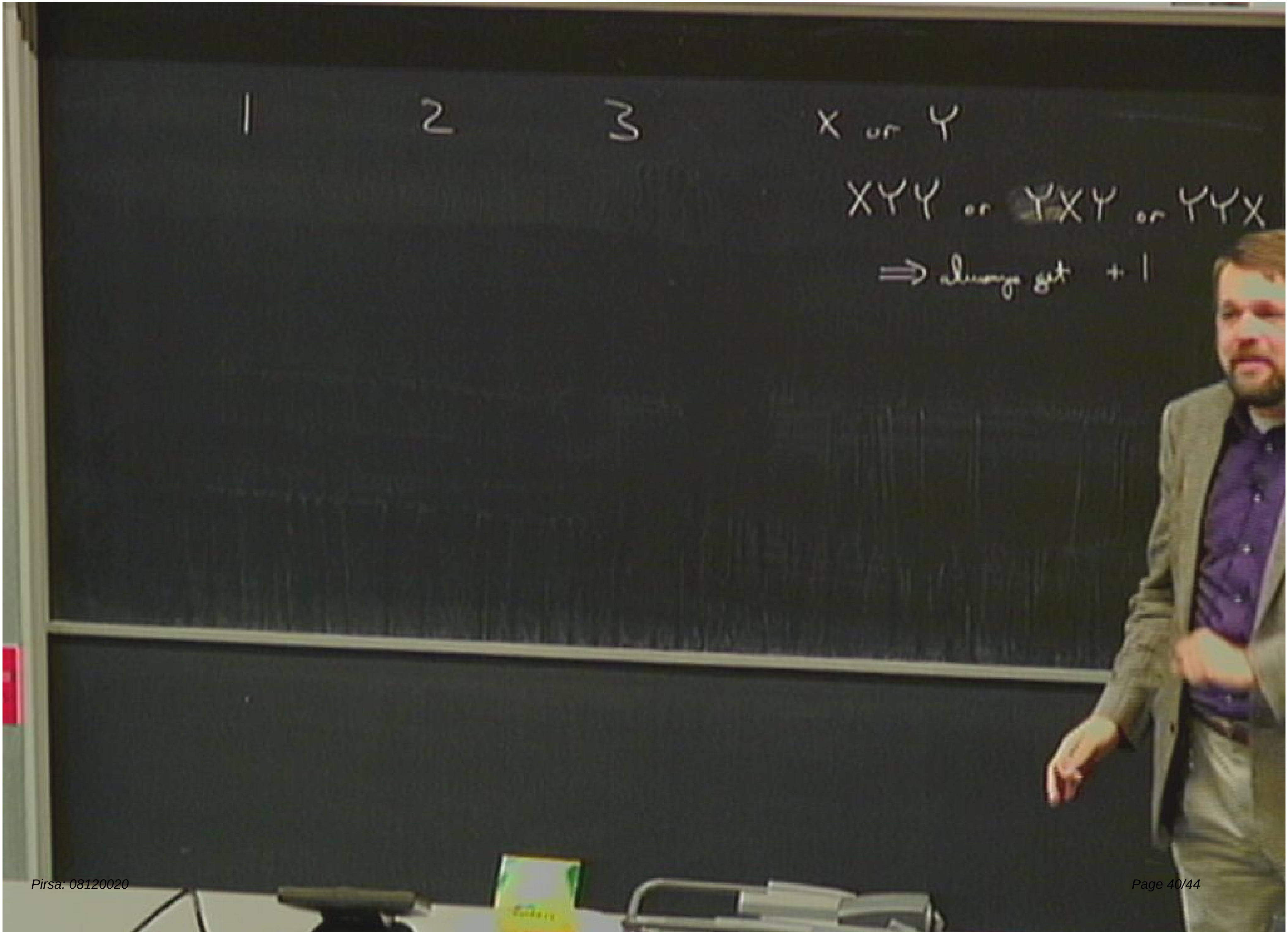
$$= \sum_\lambda p(a'|\lambda)\, p(c|\lambda)\, p(\lambda)$$

$$= p(a',c)$$

Given $p(a|\lambda)$

Can find a's with <u>same</u> $p(a'|\lambda)$

local hidden variable $\Longrightarrow$ $\mathcal{R}(1,2)$ is sharable $\Longrightarrow$ Bell $\neq$


1 2 3
X or Y
XYY or YXY or YYX
⇒ always get +1

1 2 3 X or Y

1' 2' 3' Obs. XYY or YXY or YYX

$\Longrightarrow$ always get +1

1 2 3 X or Y

1' 2' 3' Obs. XYY or YXY or YYX

$\Longrightarrow$ always get +1

Assume 1', 2', 3' exist

all R(1,2,3) same

X 1 $\quad$ X 2 $\quad$ X 3

Y' 1' $\quad$ Y' 2' $\quad$ Y' 3'

$(X Y' Y')$

$+1$

X or Y

Obs. XYY or YXY or YYX

$\Longrightarrow$ always get $+1$

Assume 1', 2', 3' exist (maybe)

all $R(1,2,3)$ same

$^{X}1$ $\quad$ $^{X}2$ $\quad$ $^{X}3$

$Y'\,1'$ $\quad$ $Y'\,2'$ $\quad$ $Y'\,3'$

X or Y

Obs. XYY or YXY or YYX

$\Rightarrow$ always get +1

Assume 1', 2', 3' exist (maybe)

all R(1,2,3) same

$$(XY'Y')(Y'XY')(Y'Y'X)$$

$$+1 \qquad +1 \qquad +1$$

$$\Rightarrow XXX = +1$$

$$\text{GHZ} \Rightarrow XXX = -1$$