Title: Quantum Field Theory 1 - Lecture 14B

Date: Dec 10, 2008 03:30 PM

URL: http://pirsa.org/08120012

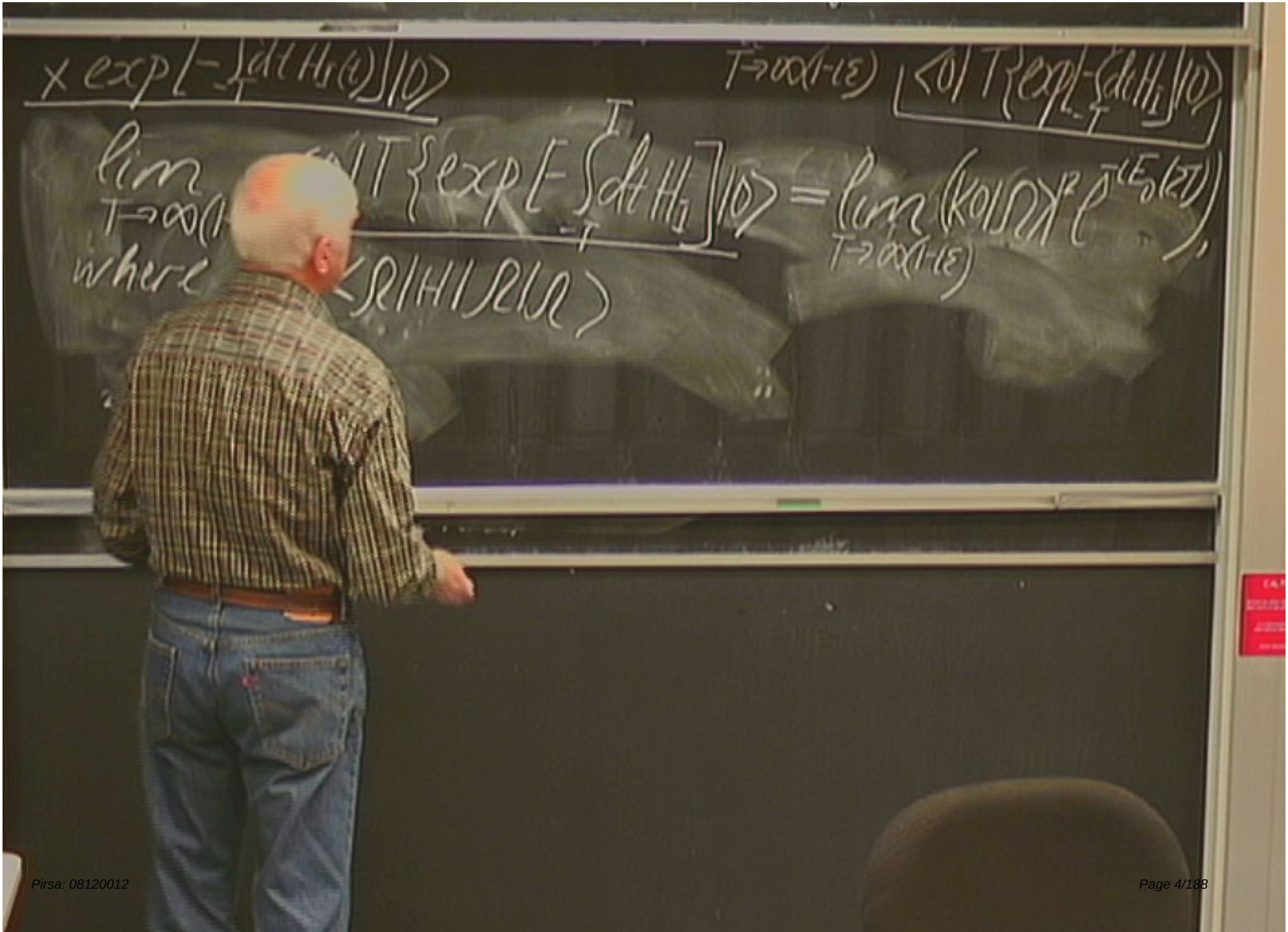
Abstract: Quantum Field Theory I course taught by Volodya Miransky of the University of Western Ontario

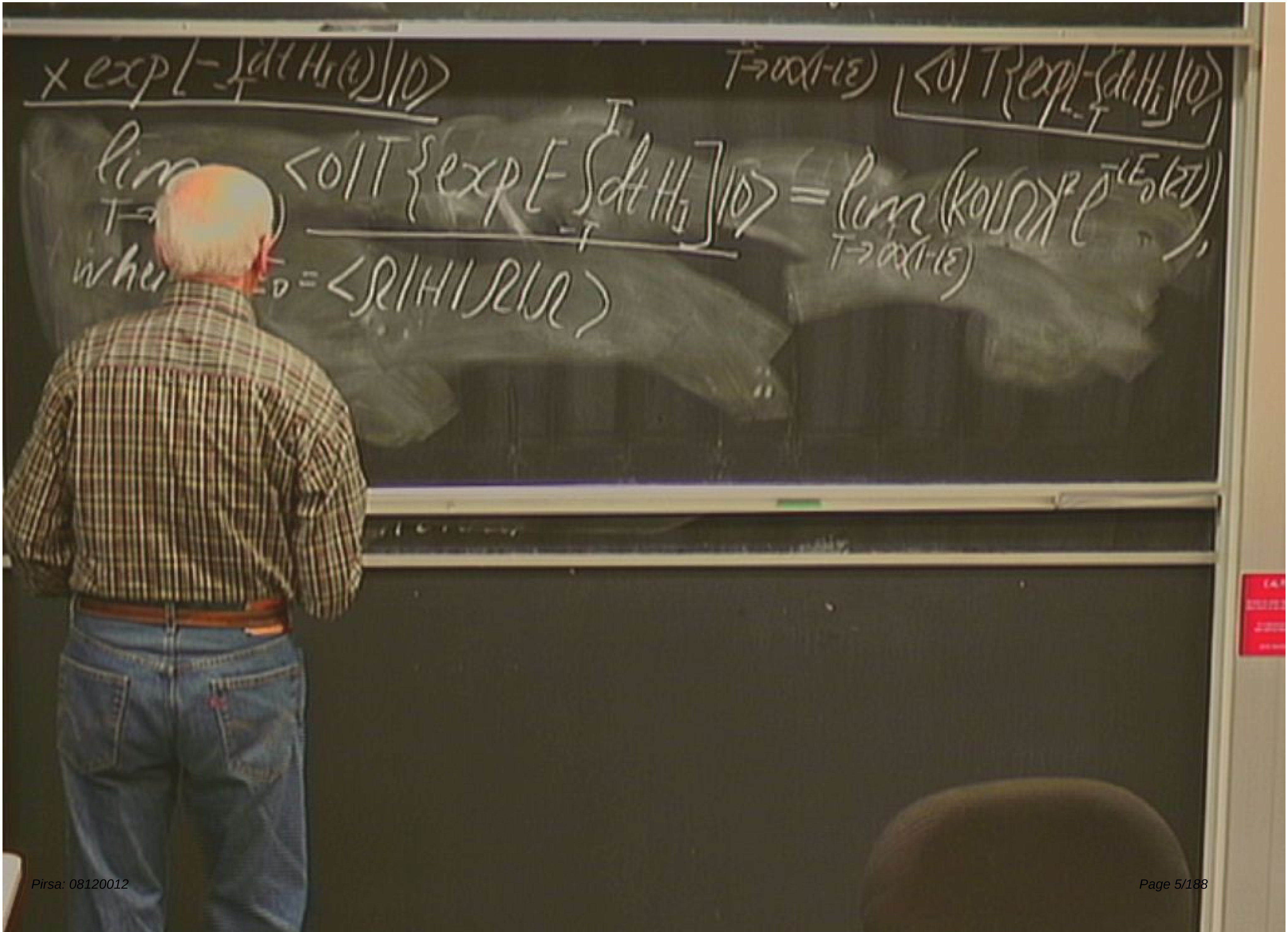
$$\langle \Omega | T\{\varphi(x_1)\varphi(x_2)\cdots\varphi(x_n)\}|\Omega\rangle = \lim_{T\to\infty(1-i\varepsilon)} \frac{\langle 0|T\{\varphi_I(x_1)\cdots\varphi_I(x_n) \times \exp[-i\int_{-T}^{T} dt\, H_I(t)]\}|0\rangle}{\langle 0|T\{\exp[-i\int_{-T}^{T} dt\, H_I]\}|0\rangle}$$

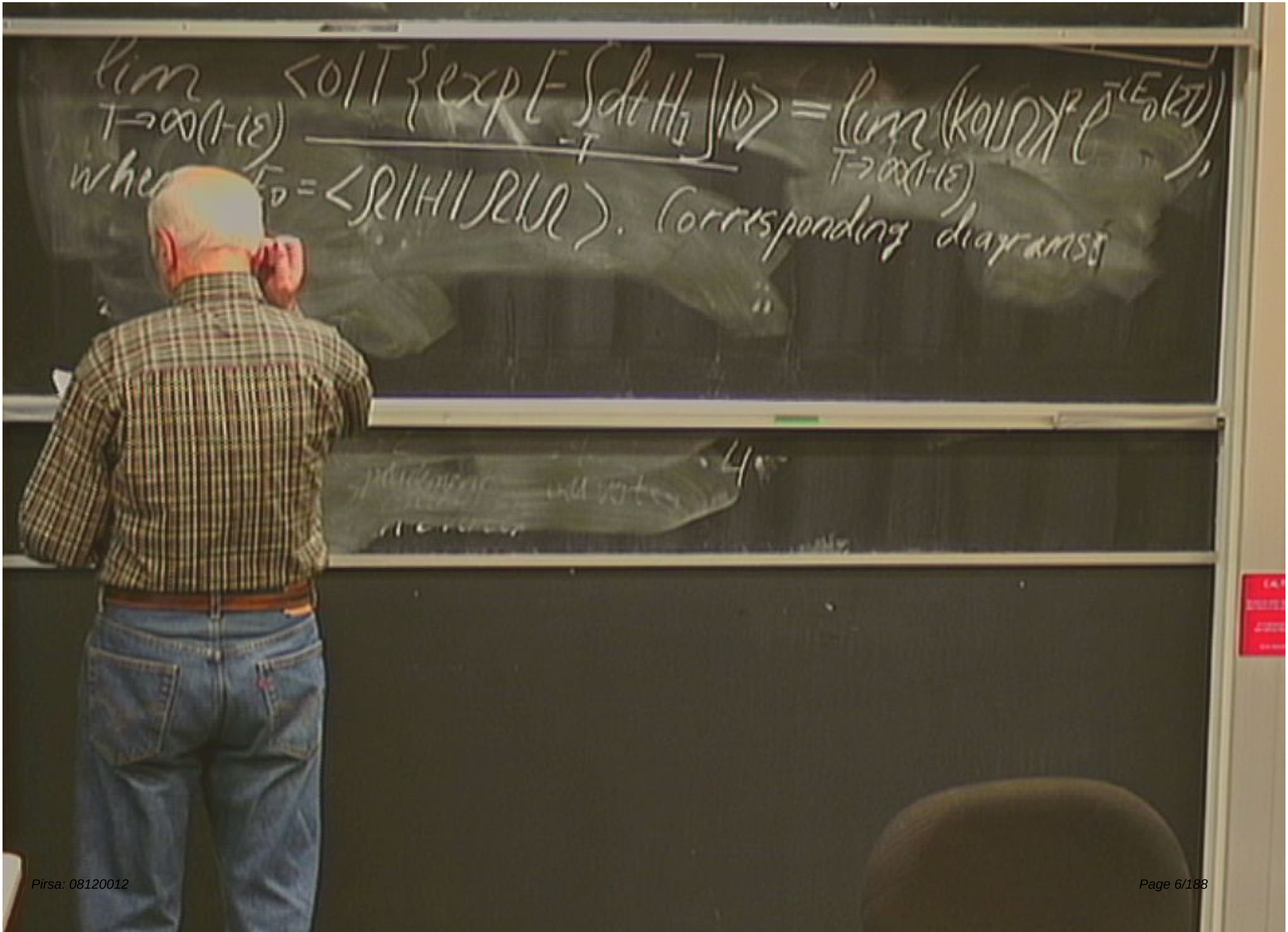
$$\langle\Omega|\,T_I\{\varphi(x_1)\varphi(x_2)\cdots\cdots\varphi(x_n)\}|\Omega\rangle = \lim_{T\to\infty(1-i\varepsilon)} \frac{\langle 0|T\{\varphi_I(x_1)\cdots\varphi_I(x_n)\times\exp[-i\int_{-T}^{T}dt\,H_I(t)]\}|0\rangle}{\langle 0|T\{\exp[-i\int_{-T}^{T}dt\,H_I]\}|0\rangle}$$

$$\lim_{T\to\infty(1-i\varepsilon)} \langle 0|T\{\exp[-i\int_{-T}^{T}dt\,H_I]\}|0\rangle = \lim_{T\to\infty(1-i\varepsilon)} \left(|\langle 0|\Omega\rangle|^2 e^{-iE_0(2T)}\right)$$

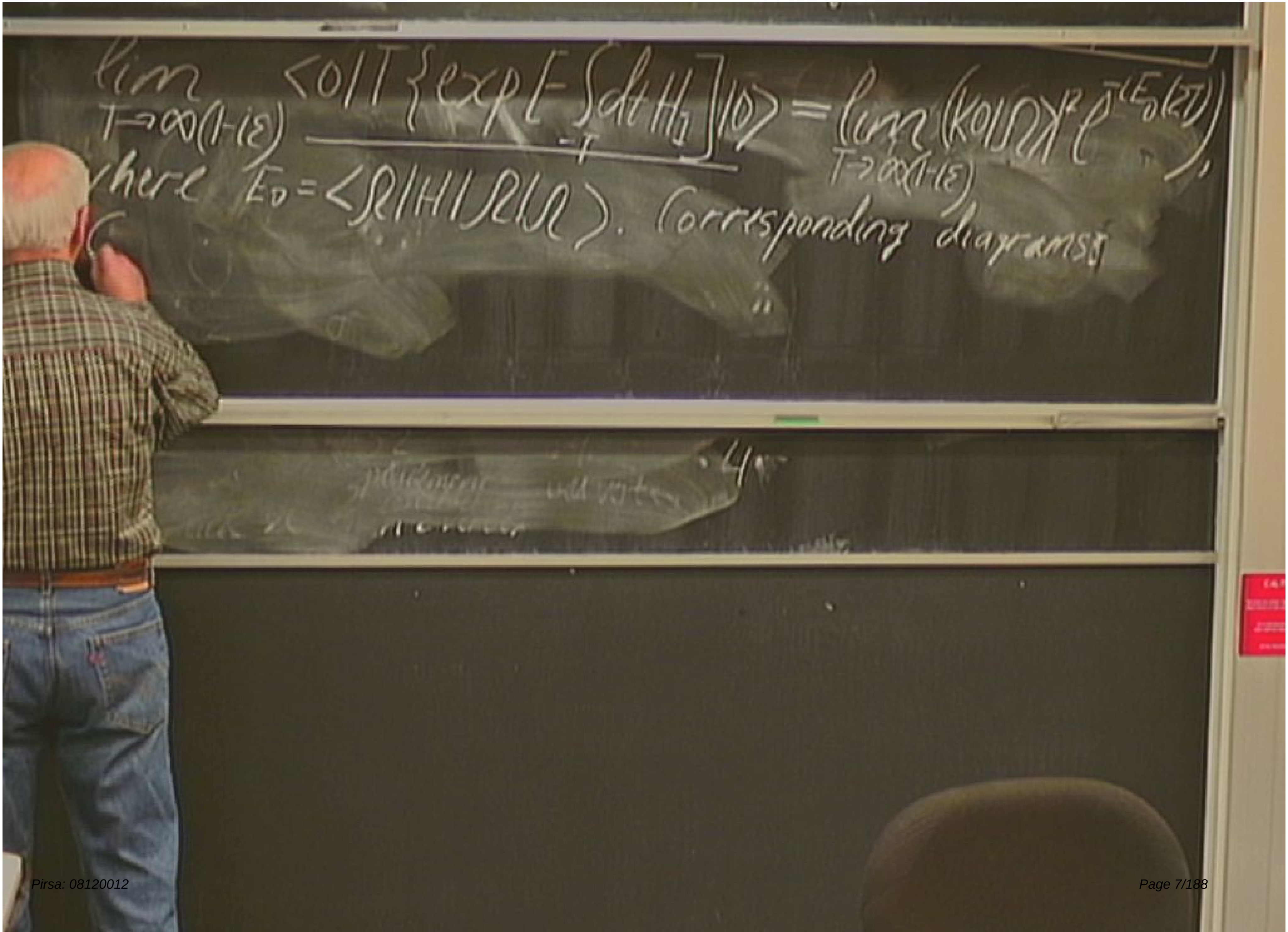
where $E_0 = \langle\Omega|H|\Omega\rangle$

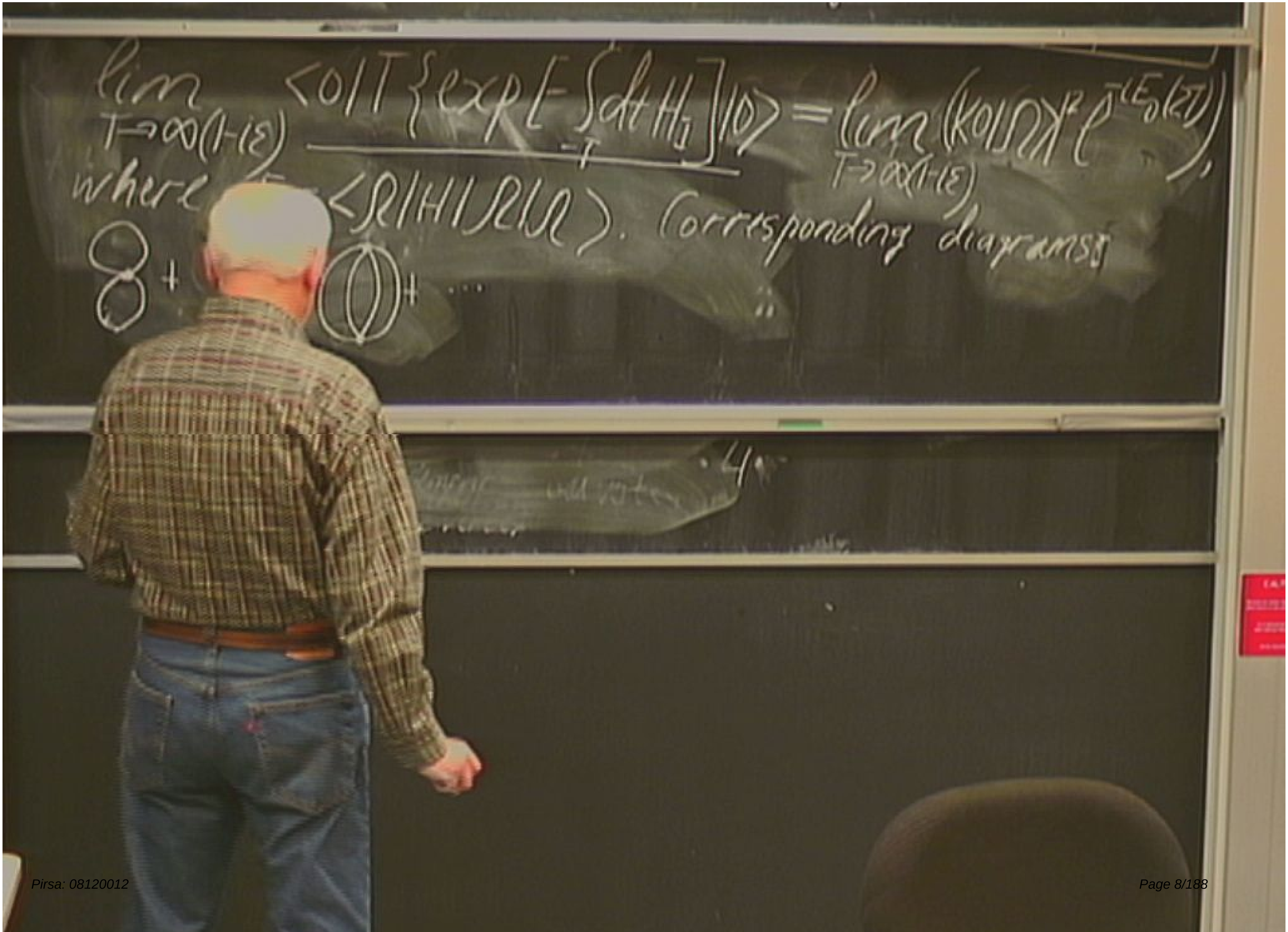

x exp[-∫ dt H_I(t)]|0⟩
T→∞(1-iε) ⟨0|T{exp[-∫ dt H_I]|0⟩
lim ⟨0|T{exp[-∫ dt H_I]|0⟩ = lim (k⟨0|Ω⟩|² e^{-iE_0(2T)}),
T→∞(1-iε)
where ⟨Ω|H|Ω⟩

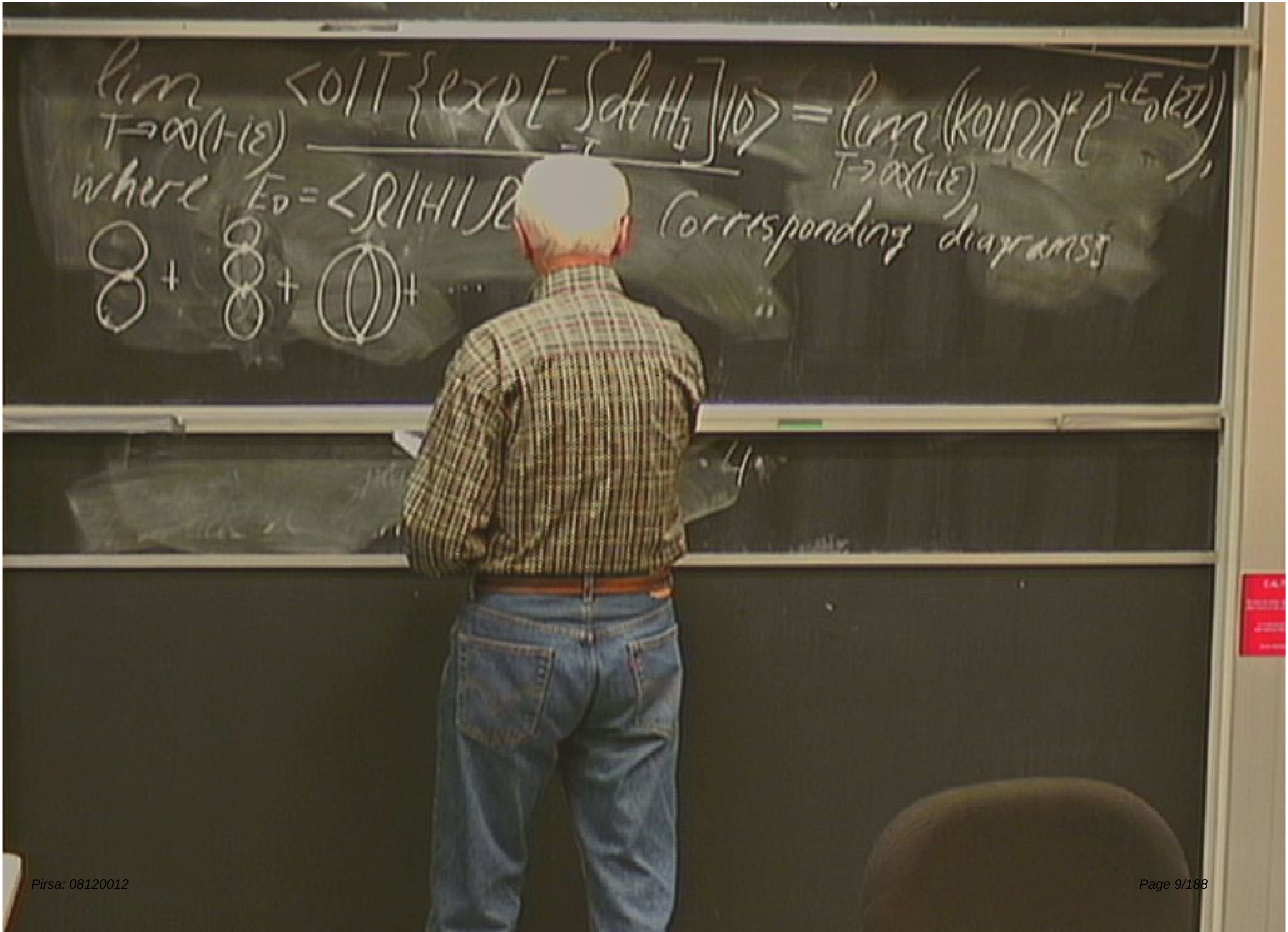

$\times \exp[-\int_{-T}^{T} dt\, H_I(t)]|0\rangle$
$T\to\infty(1-i\epsilon)$
$\langle 0|T\{\exp[-\int_{-T}^{T} dt\, H_I]|0\rangle$
$\lim \langle 0|T\{\exp[-\int_{-T}^{T} dt\, H_I]|0\rangle = \lim_{T\to\infty(1-i\epsilon)} (|\langle 0|\Omega\rangle|^2 e^{-iE_0(2T)}),$
$= \langle \Omega|H|\Omega\rangle$

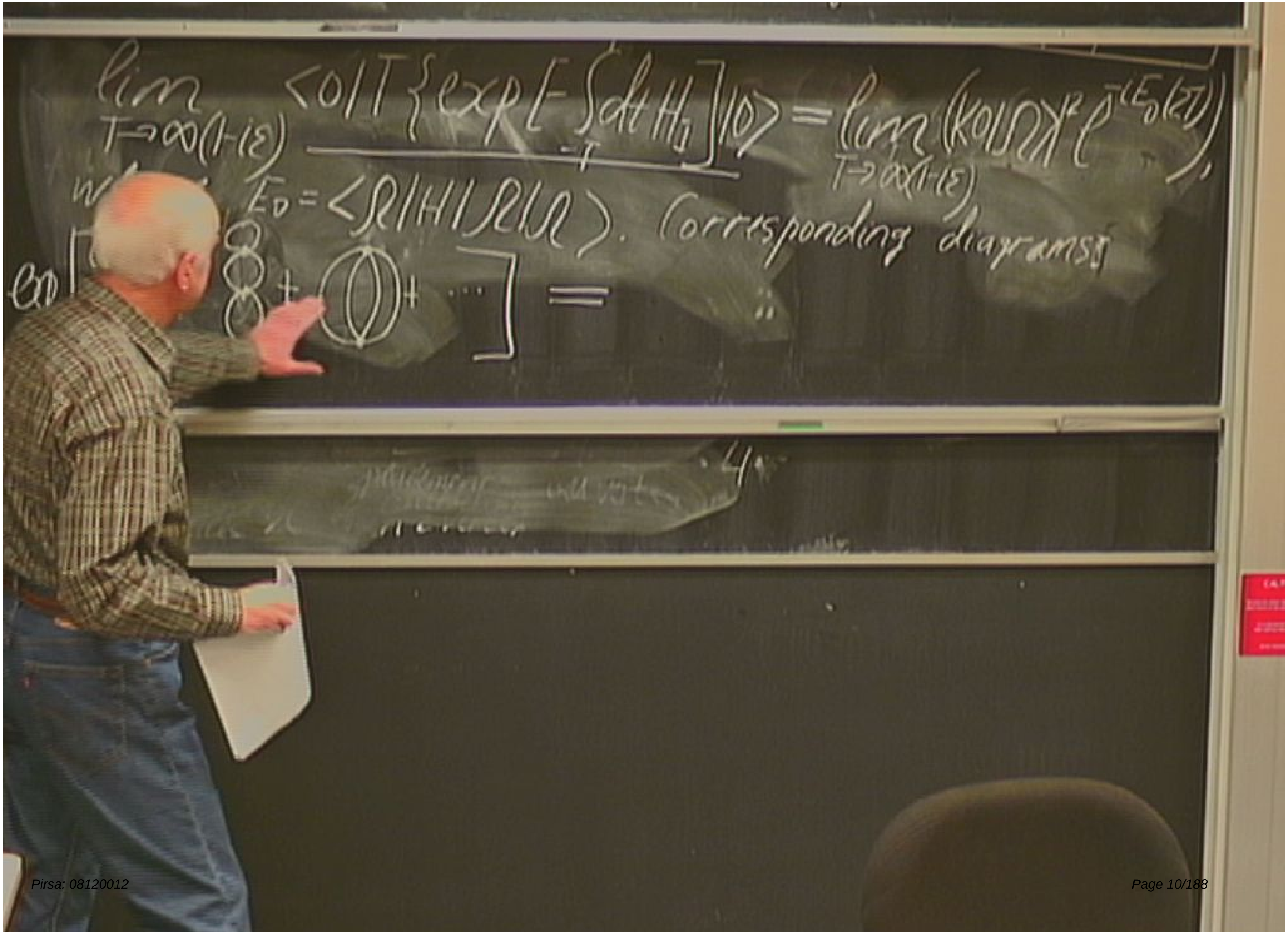


$\lim_{T\to\infty(1-i\varepsilon)} \langle 0|T\{\exp[-i\int_{-T}^{T} dt\, H_I]\}|0\rangle = \lim_{T\to\infty(1-i\varepsilon)} (\langle 0|\Omega\rangle)^2 e^{-iE_0(2T)},$
where $E_0 = \langle\Omega|H|\Omega\rangle$. Corresponding diagrams:


$\lim_{T\to\infty(1-i\varepsilon)} \langle 0|T\{\exp[-i\int_{-T}^{T} dt\, H_I]\}|0\rangle = \lim_{T\to\infty(1-i\varepsilon)} \left(|\langle 0|\Omega\rangle|^2 e^{-iE_0(2T)}\right),$
where $E_0 = \langle\Omega|H|\Omega\rangle$. Corresponding diagrams:


$\lim_{T\to\infty(1-i\varepsilon)} \langle 0|T\{\exp[-i\int_{-T}^{T} dt\, H_I]\}|0\rangle = \lim_{T\to\infty(1-i\varepsilon)} \left(|\langle 0|\Omega\rangle|^2 e^{-iE_0(2T)}\right),$
where $E_0 = \langle\Omega|H|\Omega\rangle$. Corresponding diagrams:

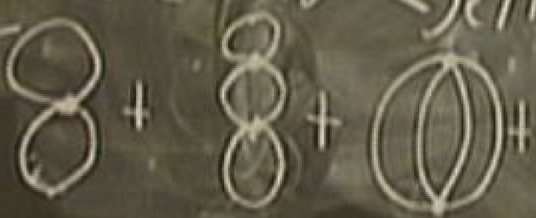

where
Corresponding diagrams:



lim <0|T{exp[-i∫dt H_I]}|0> = lim (<0|Ω>)² e^{-iE_0(2T)},
T→∞(1-iε)
T→∞(1-iε)
E_0 = <Ω|H|Ω>. Corresponding diagrams

$$\lim_{T\to\infty(1-i\varepsilon)} \langle 0|T\left\{\exp\left[-i\int_{-T}^{T} dt\, H_I\right]\right\}|0\rangle = \lim_{T\to\infty(1-i\varepsilon)} \left(|\langle 0|\Omega\rangle|^2 e^{-iE_0(2T)}\right),$$

where $E_0 = \langle \Omega | H | \Omega \rangle$. Corresponding diagrams:

$\exp\Big[$  $+ \cdots \Big] =$

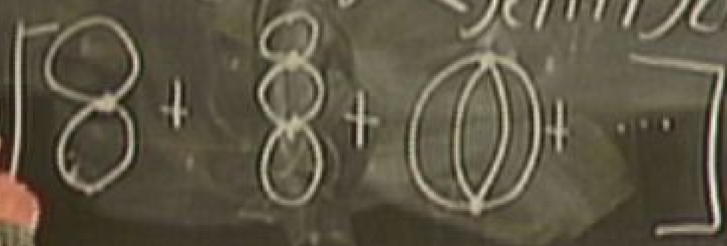
$$\lim_{T\to\infty(1-i\epsilon)} \langle 0|T\left\{\exp\left[-i\int_{-T}^{T} dt\, H_I\right]\right\}|0\rangle = \lim_{T\to\infty(1-i\epsilon)} \left(|\langle 0|\Omega\rangle|^2 e^{-iE_0(2T)}\right),$$

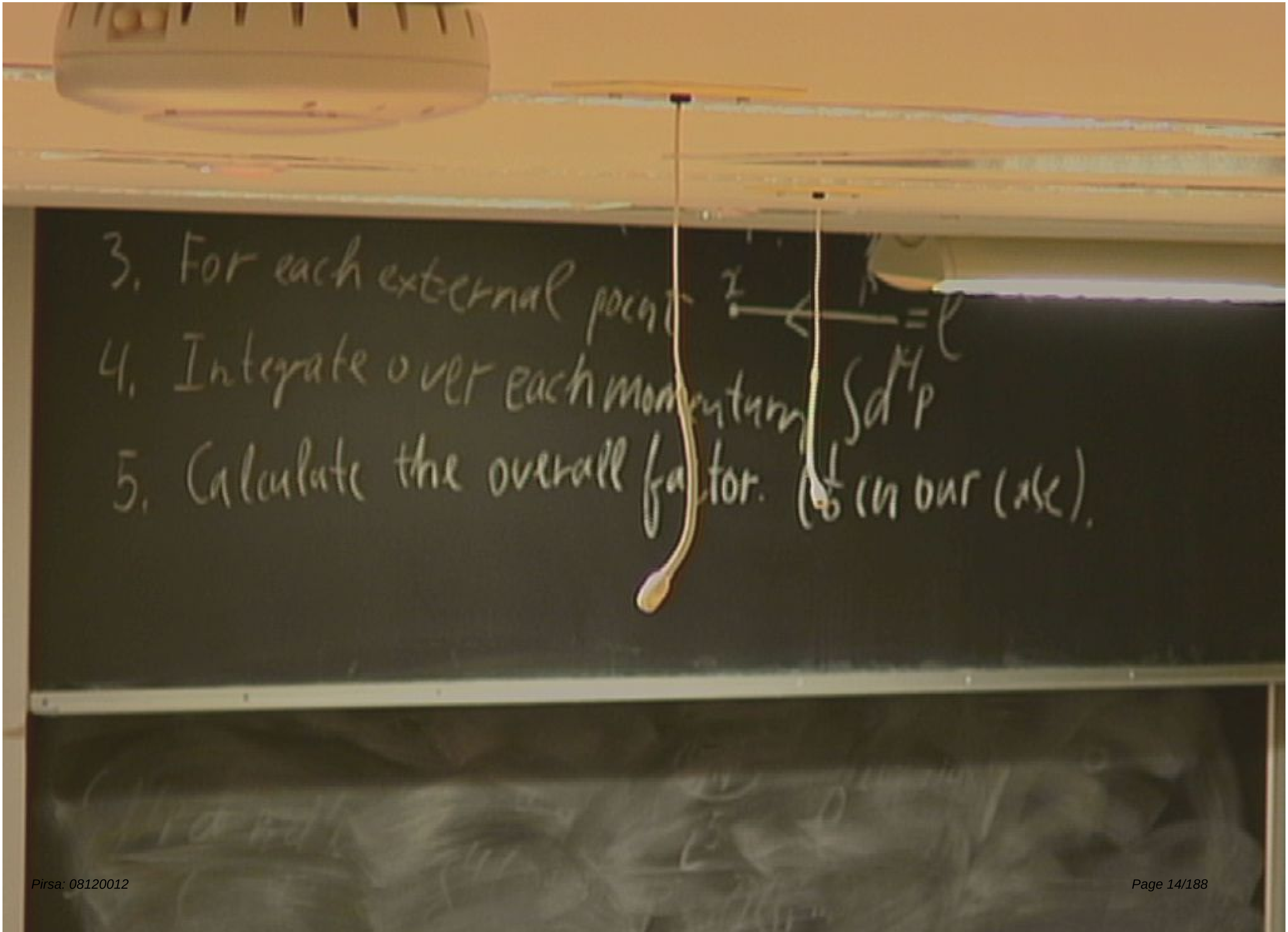
where $E_0 = \langle\Omega|H|\Omega\rangle$. Corresponding diagrams:

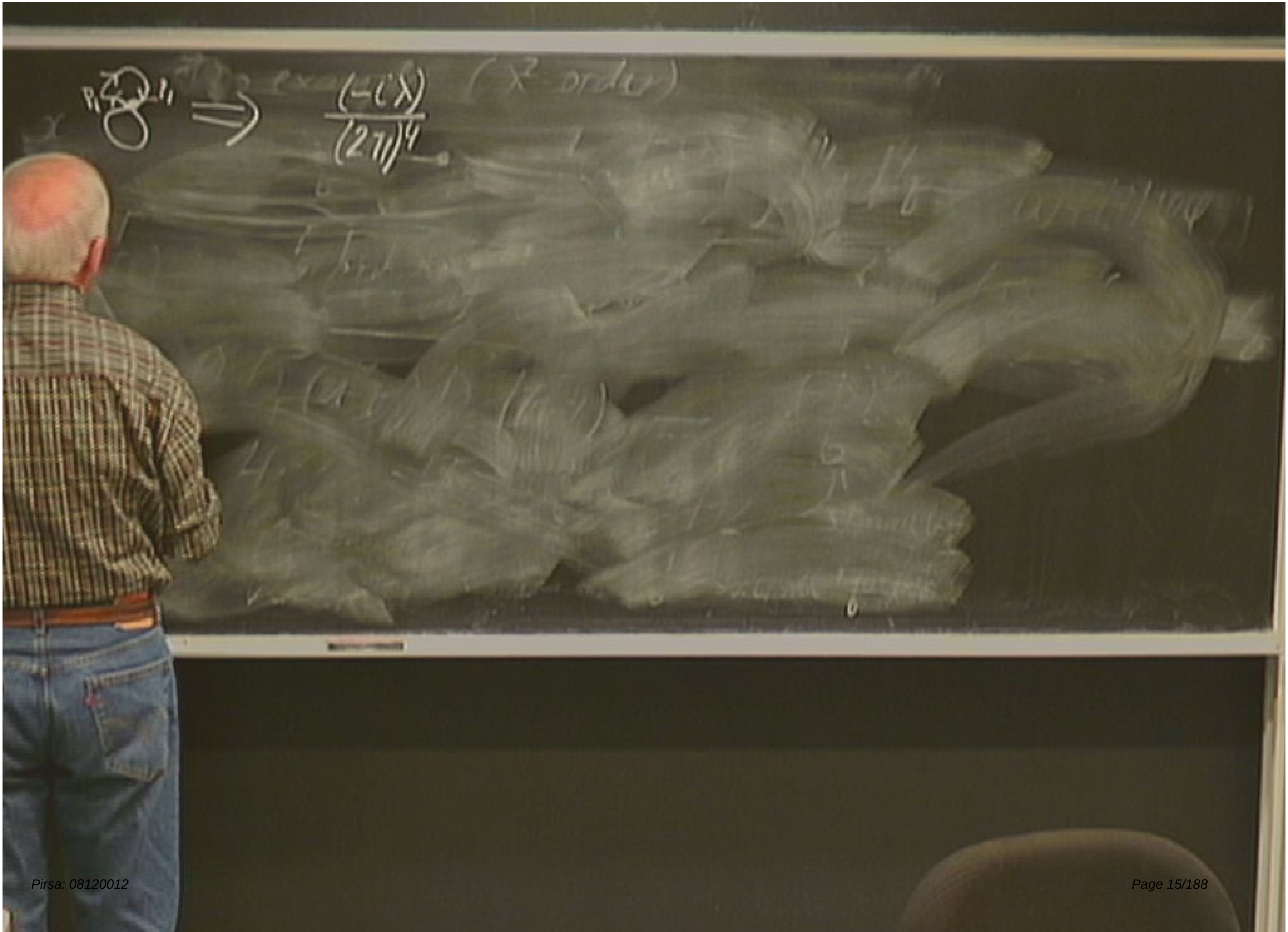
$\exp\Big[$  $+ \cdots \Big] =$

$$\lim_{T\to\infty(1-i\epsilon)} \langle 0|T\{\exp[-i\int_{-T}^{T} dt\, H_I]\}|0\rangle = \lim_{T\to\infty(1-i\epsilon)} \left(|\langle 0|\Omega\rangle|^2 e^{-iE_0(2T)}\right),$$

where $E_0 = \langle\Omega|H|\Omega\rangle$. Corresponding diagrams:

exp  (vacuum bubbles)

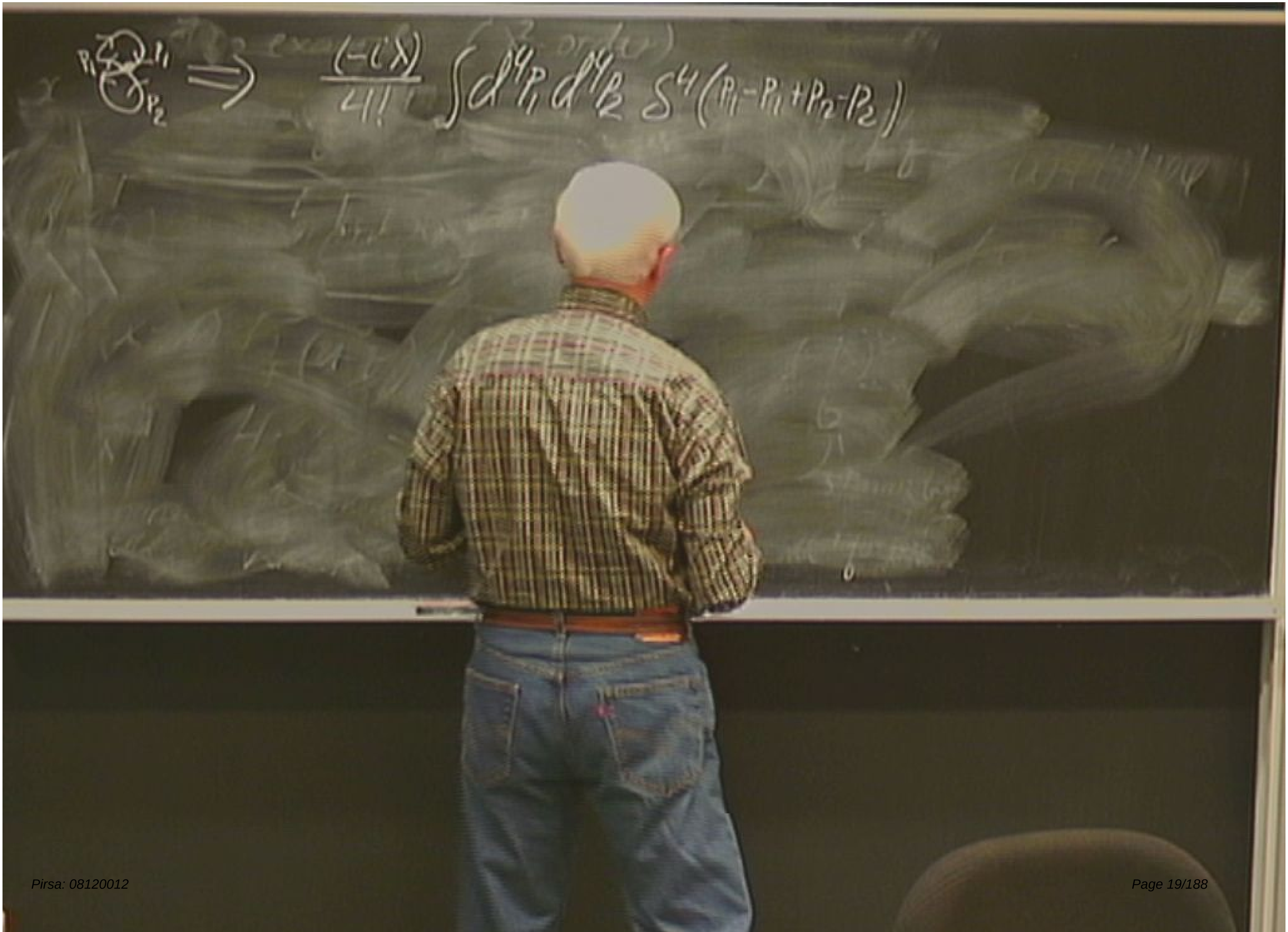

3. For each external point = 1
4. Integrate over each momentum $\int d^4p$
5. Calculate the overall factor. (1 in our case).

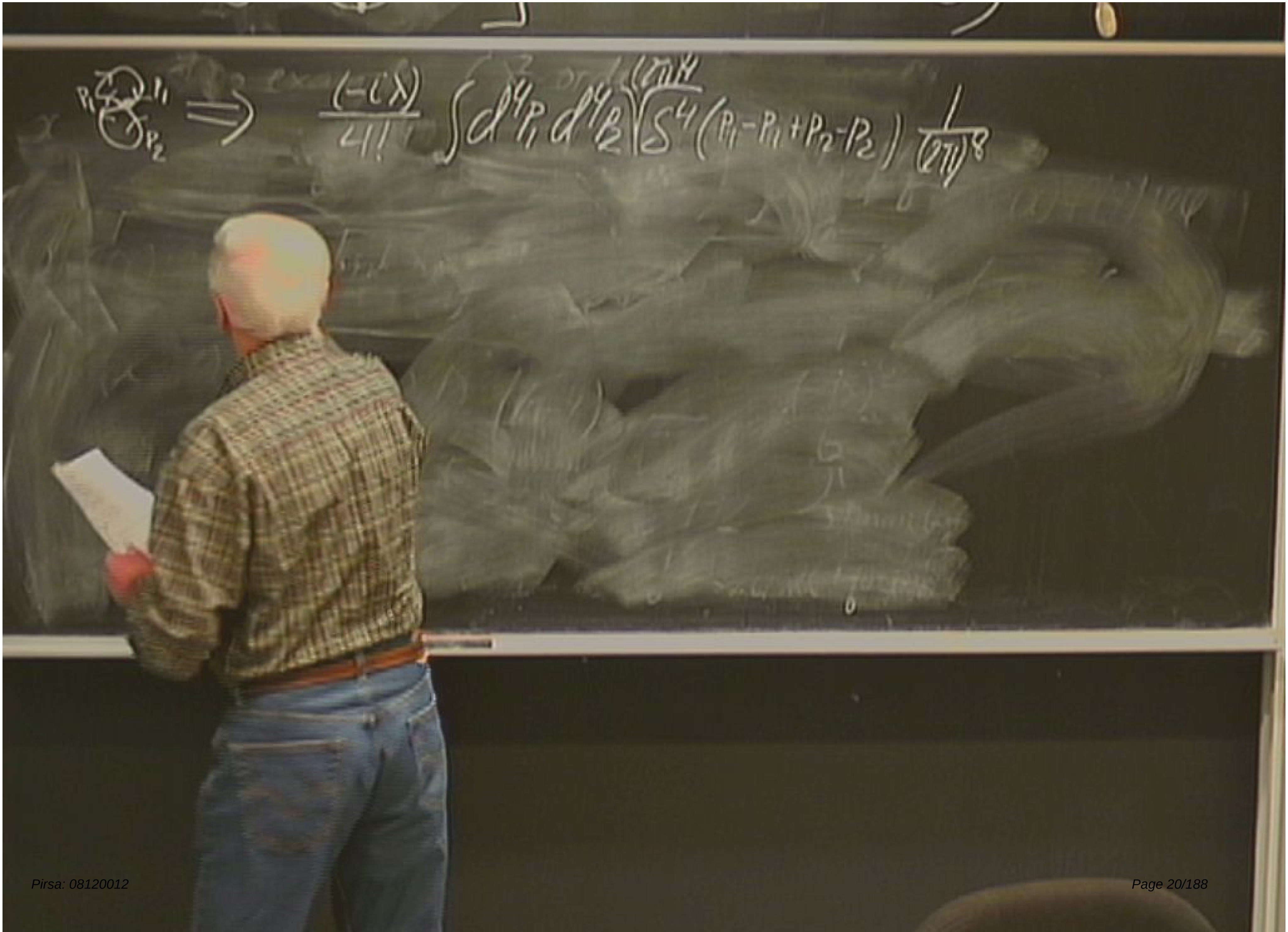

(-iCλ)
(2π)^4
(λ² order)

$$\Longrightarrow \frac{(-i\lambda)}{(2\pi)^4} \int d^4p_1\, d^4p_2$$

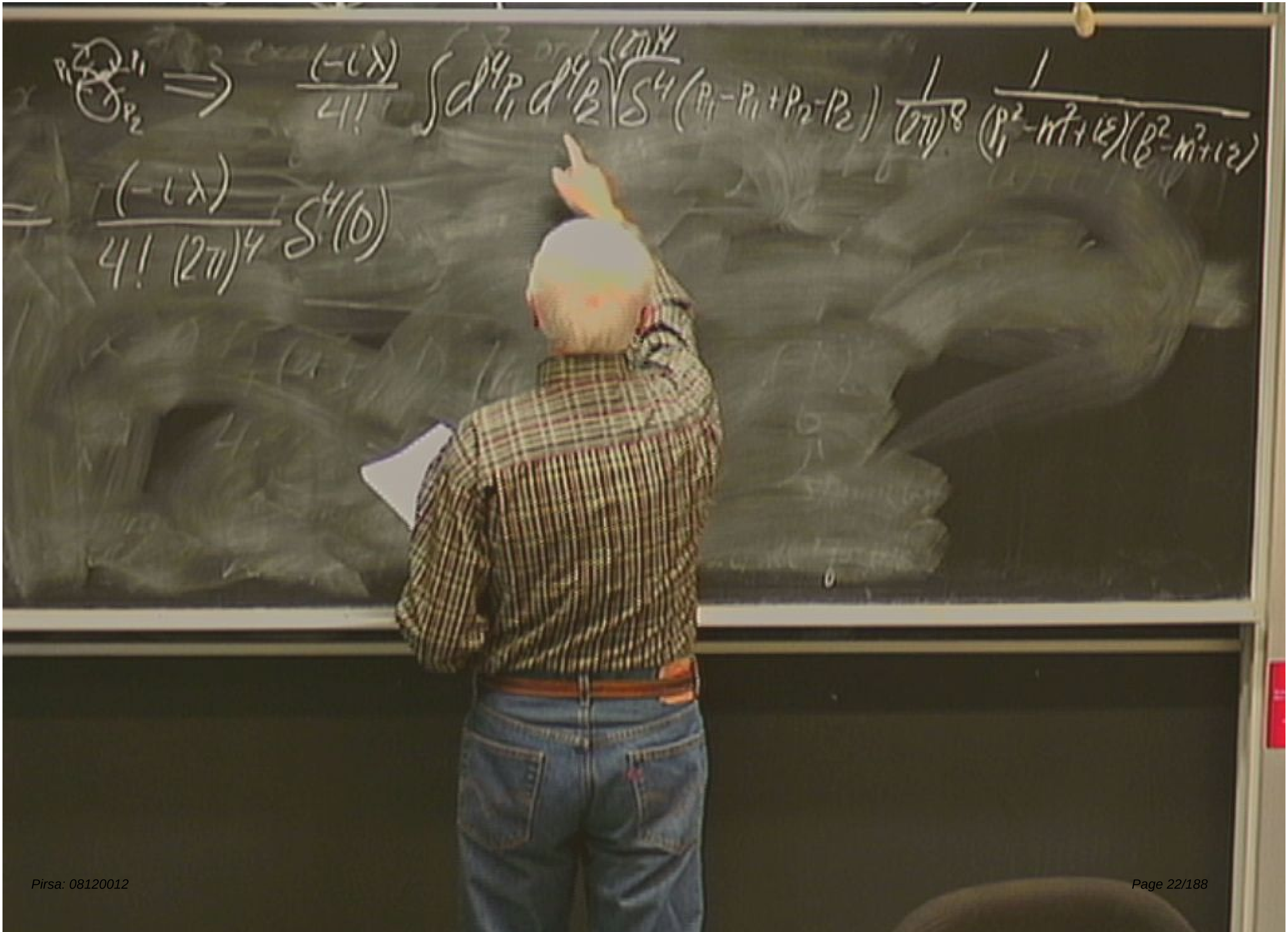
$$\Longrightarrow \quad \frac{(-i\lambda)}{(2\pi)^4}\int d^4p_1\, d^4p_2$$

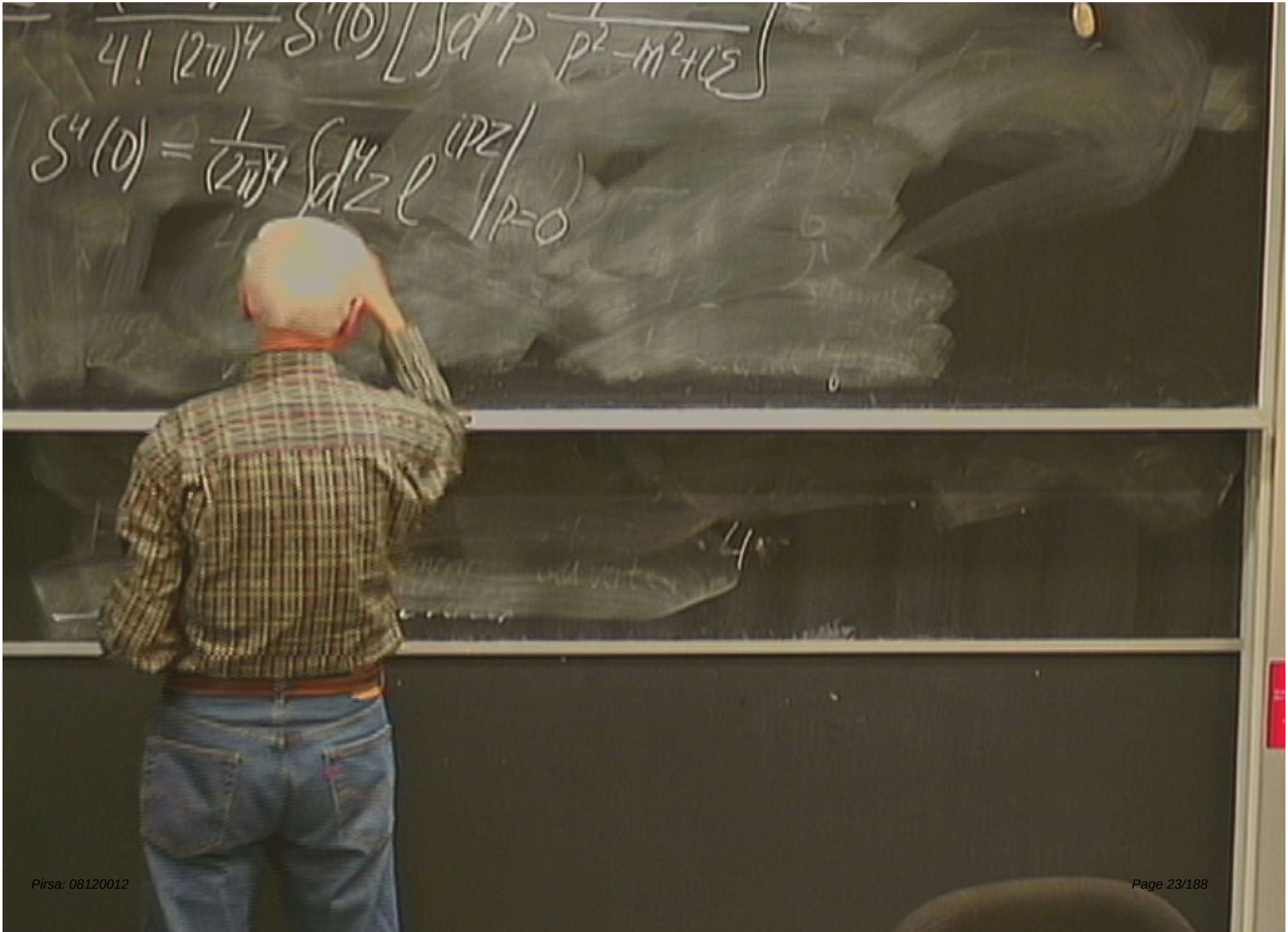
$$\Rightarrow \frac{(-i\lambda)}{4!}\int d^4p_1\, d^4p_2$$


$\frac{(-i\lambda)}{4!} \int d^4p_1\, d^4p_2\, \delta^4(p_1-p_1+p_2-p_2)$

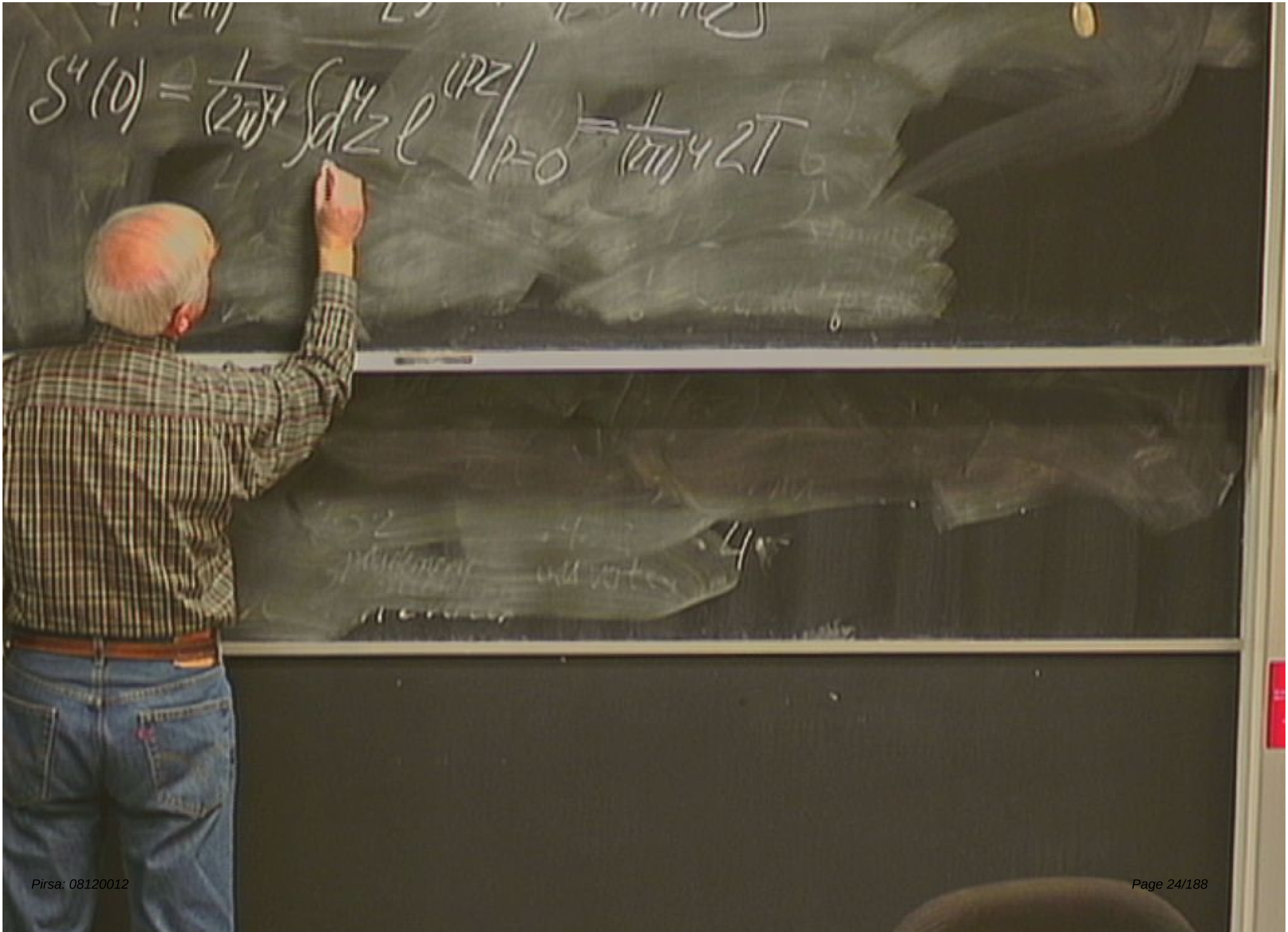

$\Rightarrow \frac{(-i\lambda)}{4!}\int d^4p_1\, d^4p_2\, (2\pi)^4\delta^4(p_1-p_1+p_2-p_2)\frac{1}{(2\pi)^8}$

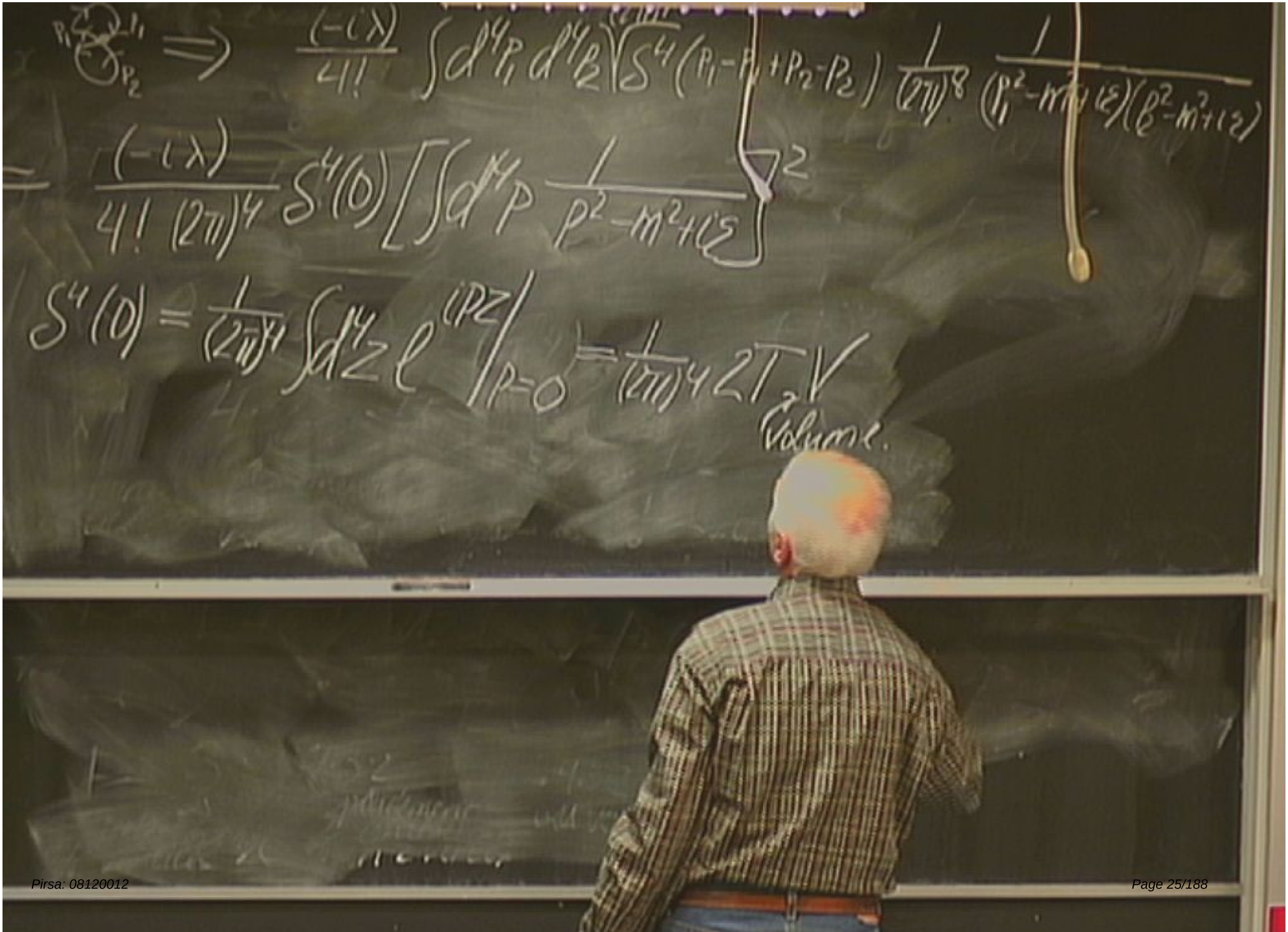
$$\Longrightarrow \frac{(-i\lambda)}{4!}\int d^4P_1\, d^4P_2\, (2\pi)^4\,\delta^4(P_1-P_1+P_2-P_2)\,\frac{1}{(2\pi)^8}\,\frac{1}{(P_1^2-m^2+i\epsilon)(P_2^2-m^2+i\epsilon)}$$

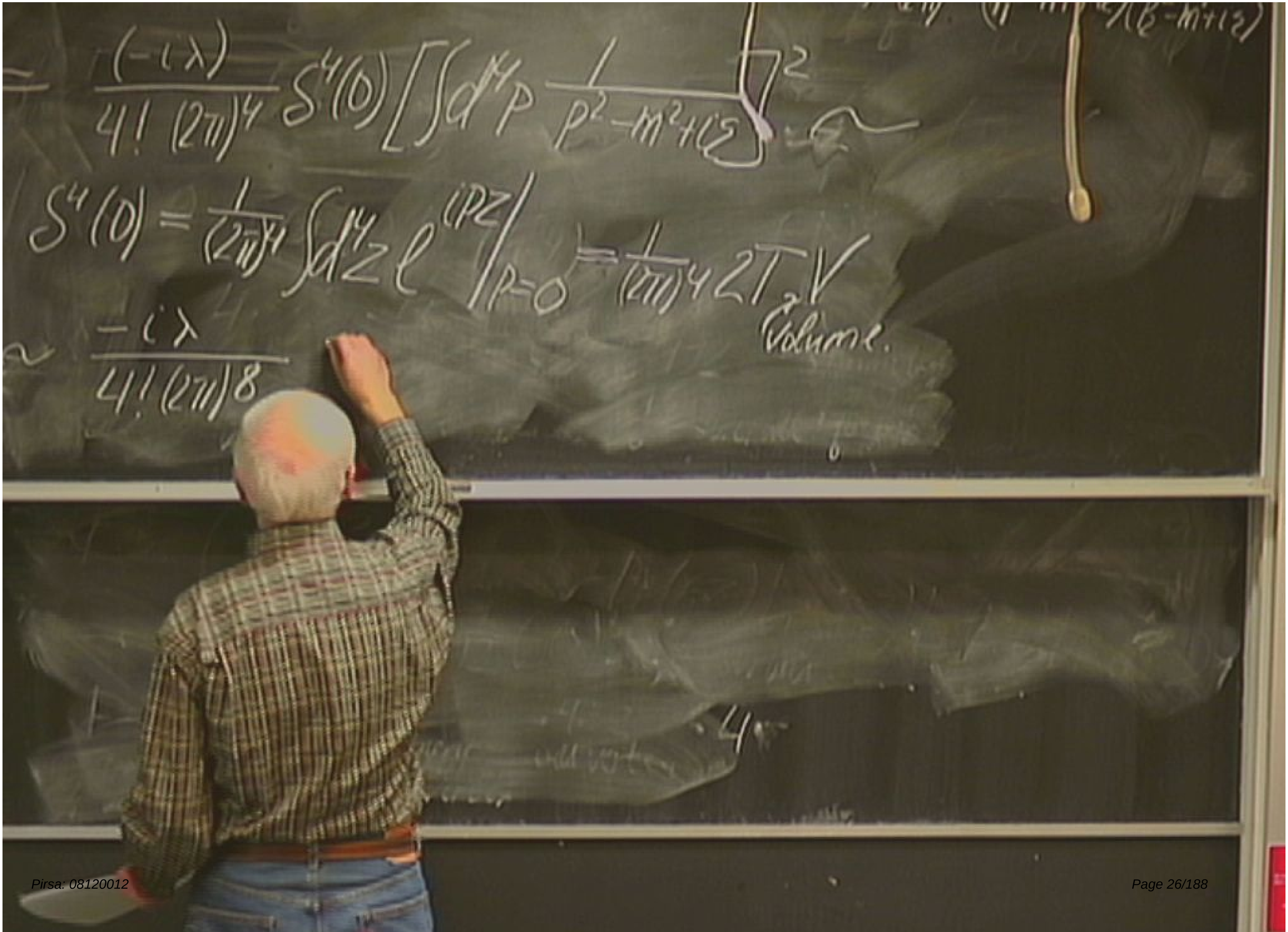

$\frac{(-i\lambda)}{4!}\int d^4p_1\, d^4p_2\, (2\pi)^4\, \delta^4(p_1-p_1+p_2-p_2)\, \frac{1}{(2\pi)^8}\, \frac{1}{(p_1^2-m^2+i\epsilon)(p_2^2-m^2+i\epsilon)}$
$\frac{(-i\lambda)}{4!\,(2\pi)^4}\,\delta^4(0)$

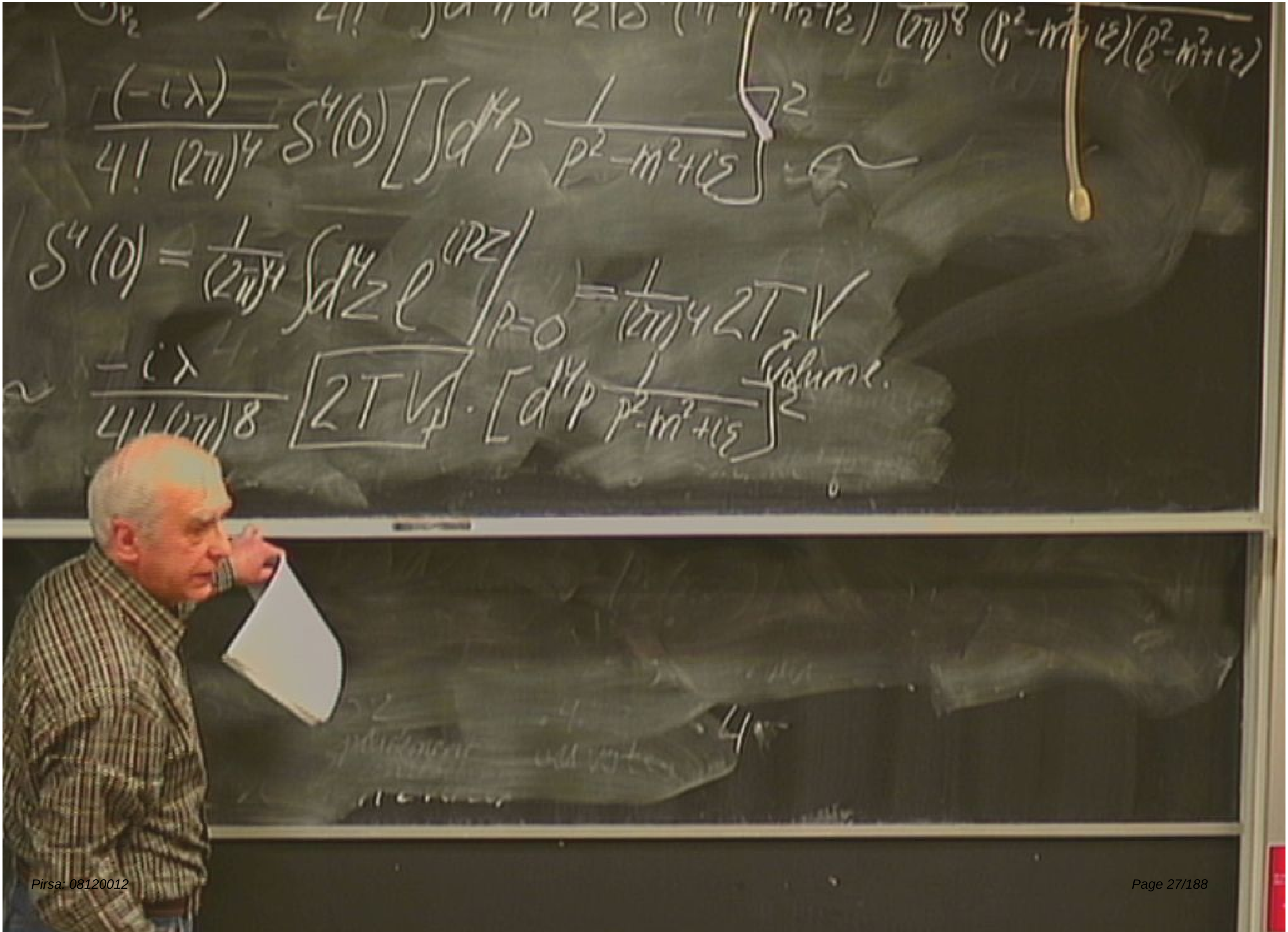


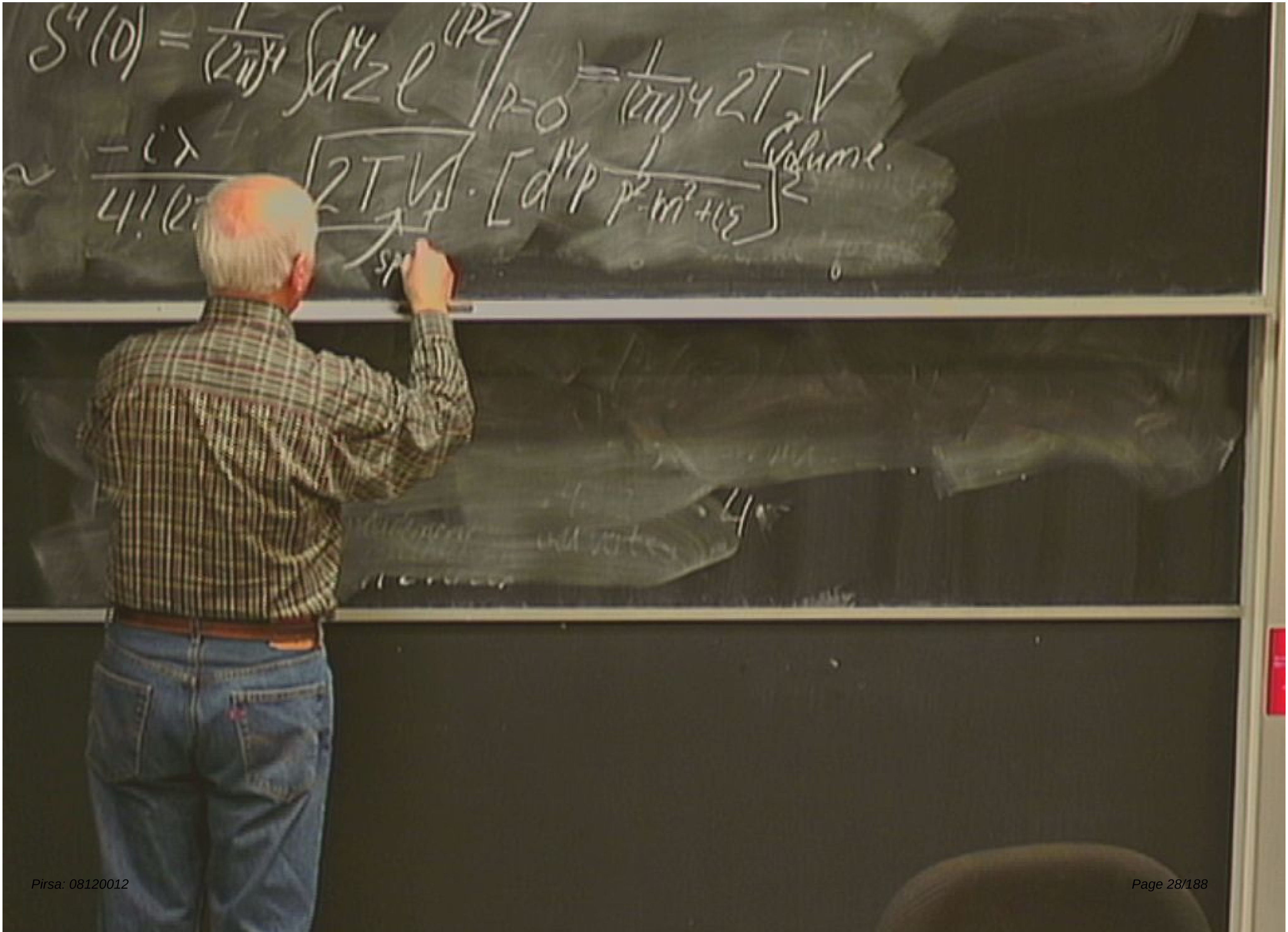
$\frac{}{4!\,(2\pi)^4}\,\delta^4(0)\left[\int d^4p\,\frac{1}{p^2-m^2+i\varepsilon}\right]$
$\delta^4(0)=\frac{1}{(2\pi)^4}\int d^4z\,e^{ipz}\Big|_{p=0}$

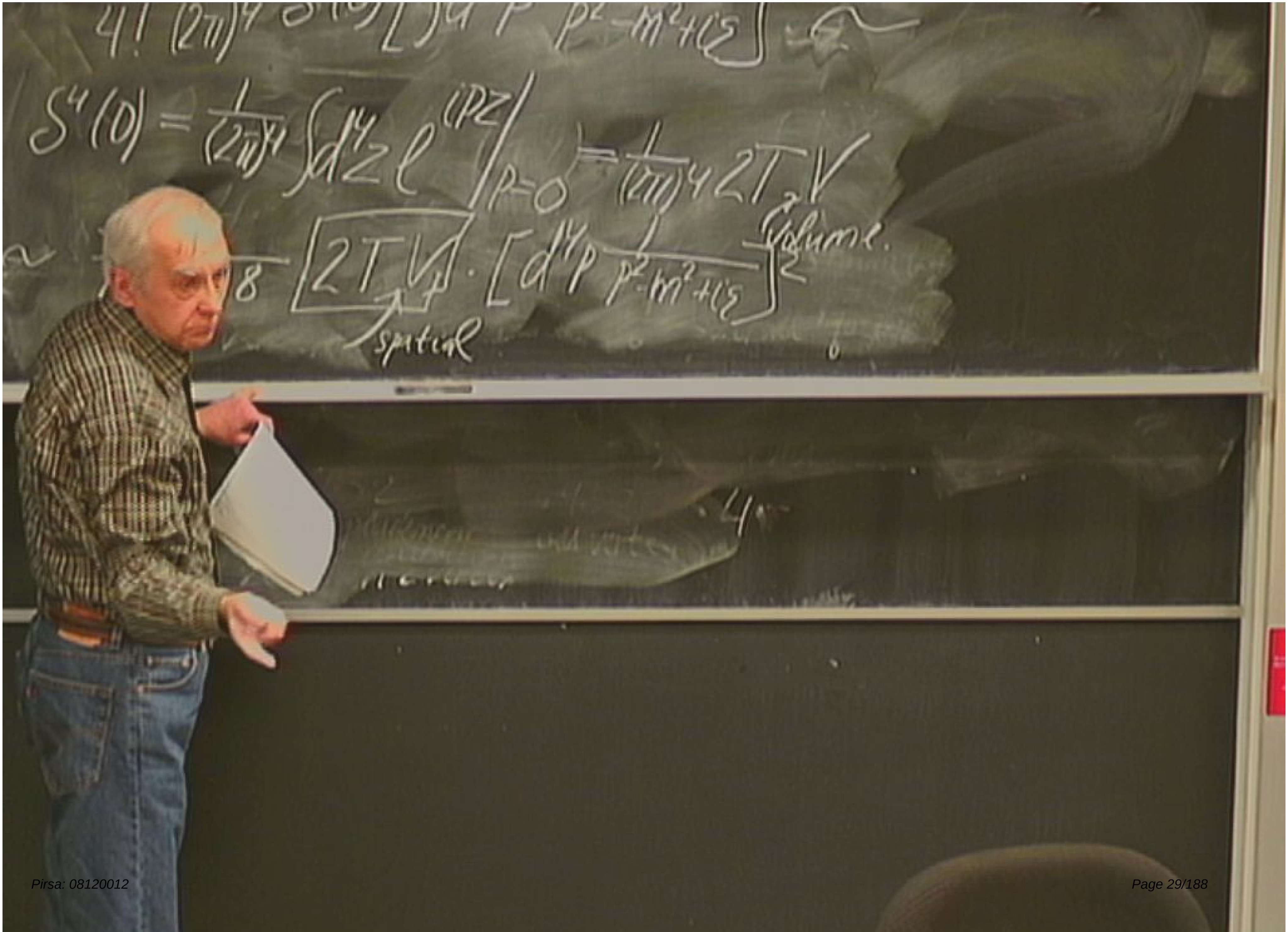

$S^4(0) = \frac{1}{(2\pi)^4}\int d^4z\, e^{ipz}\big|_{p=0} = \frac{1}{(2\pi)^4} 2T$

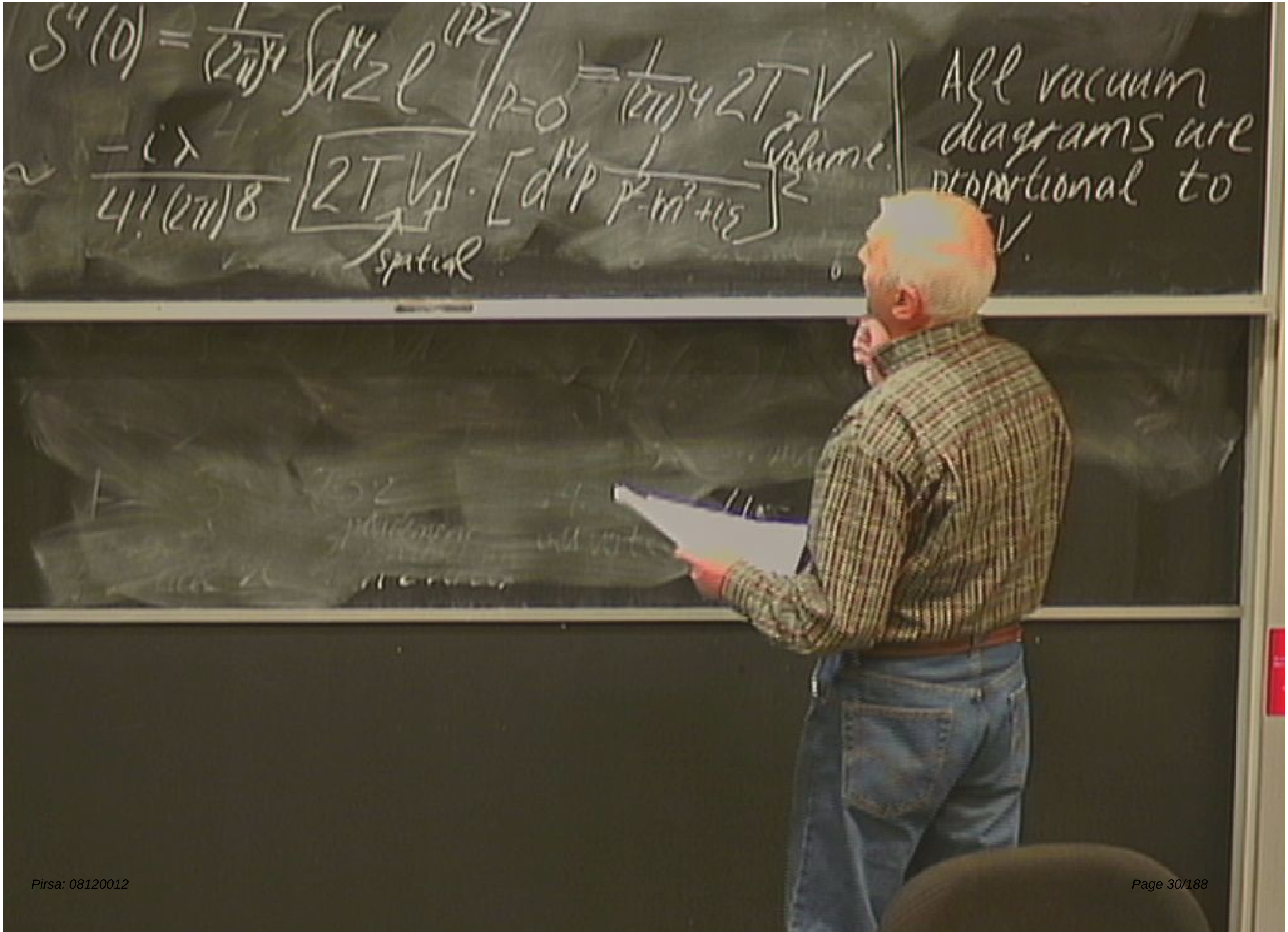

(-iλ)/4! ∫d^4p_1 d^4p_2 δ^4(p_1-p_1+p_2-p_2) 1/(2π)^8 1/((p_1^2-m^2+iε)(p_2^2-m^2+iε))
= (-iλ)/(4! (2π)^4) δ^4(0) [∫d^4p 1/(p^2-m^2+iε)]^2
δ^4(0) = 1/(2π)^4 ∫d^4z e^{ipz}|_{p=0} = 1/(2π)^4 2T_0 V
Volume.

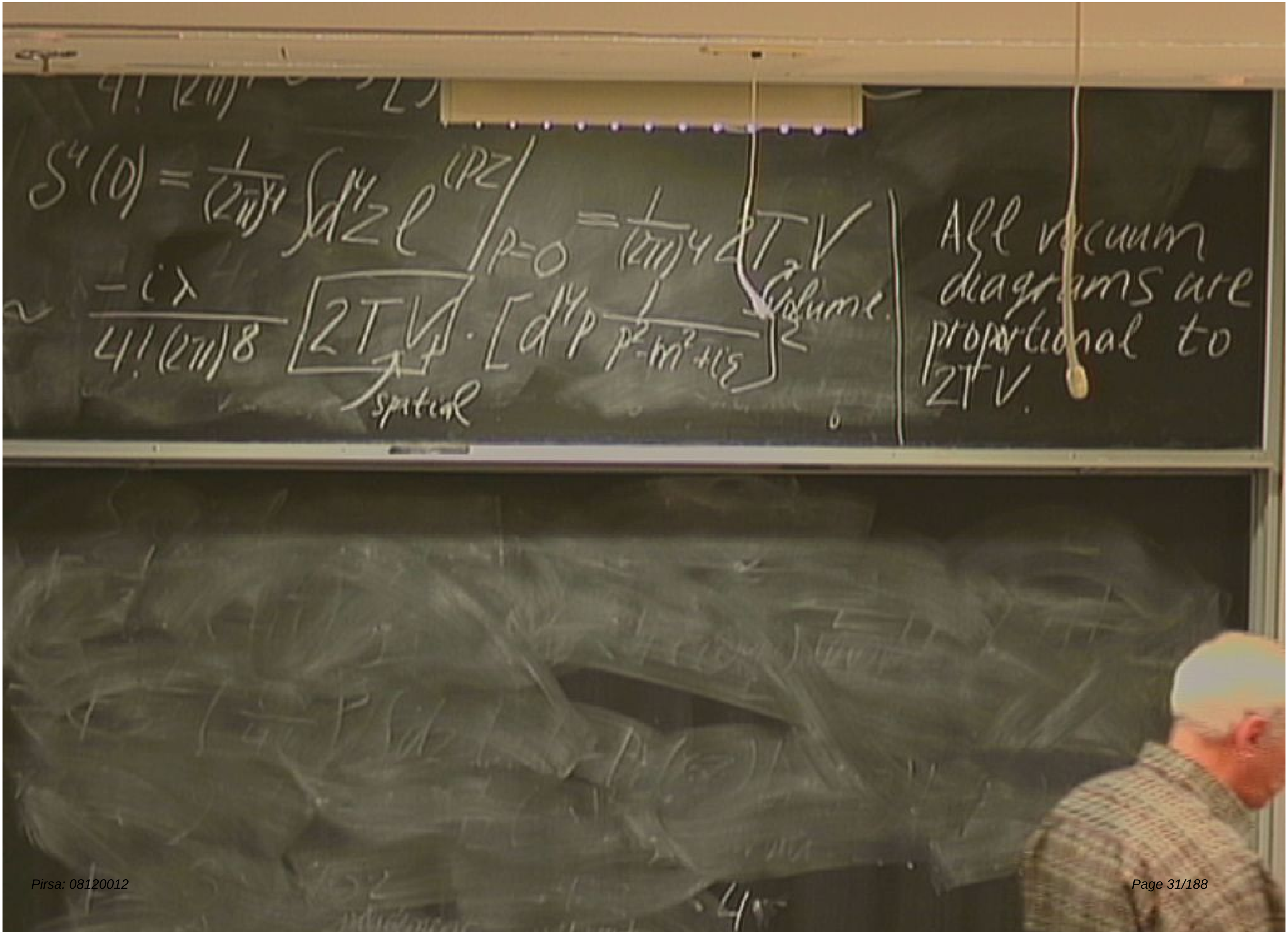

$\frac{(-i\lambda)}{4!(2\pi)^4}\delta^4(0)\left[\int d^4p \frac{1}{p^2-m^2+i\varepsilon}\right]^2$
$\delta^4(0) = \frac{1}{(2\pi)^4}\int d^4z\, e^{ipz}\Big|_{p=0} = \frac{1}{(2\pi)^4}2T_0V$
Volume.
$\frac{-i\lambda}{4!(2\pi)^8}$

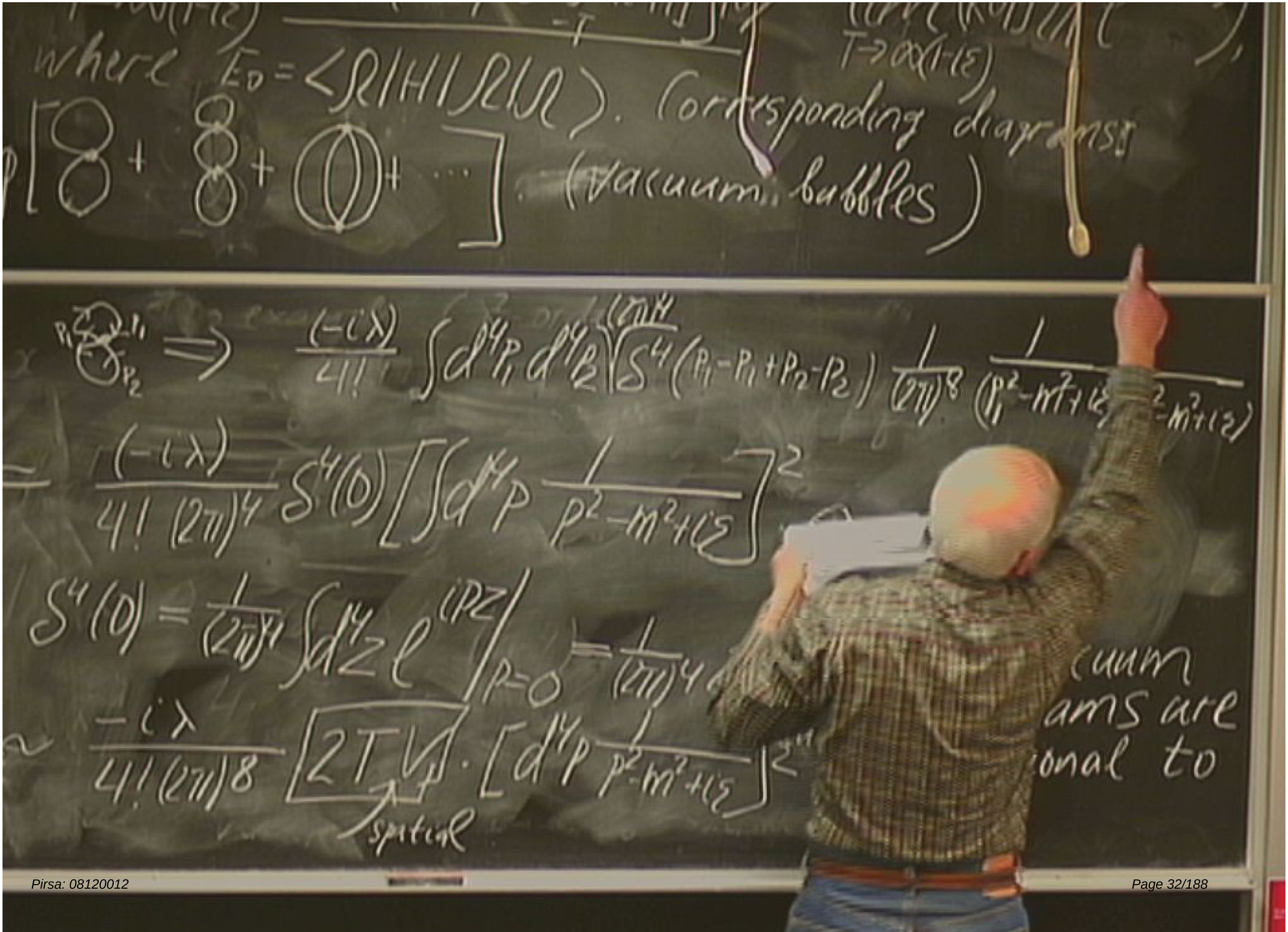

$\frac{(-i\lambda)}{4!\,(2\pi)^4}\,\delta^4(0)\left[\int d^4p\,\frac{1}{p^2-m^2+i\varepsilon}\right]^2$
$\delta^4(0) = \frac{1}{(2\pi)^4}\int d^4z\, e^{ipz}\Big|_{p=0} = \frac{1}{(2\pi)^4}\,2TV$
volume
$\frac{-i\lambda}{4!\,(2\pi)^8}\,\boxed{2TV}\cdot\left[\int d^4p\,\frac{1}{p^2-m^2+i\varepsilon}\right]^2$


Volume


spatial
volume.


$S^4(0) = \frac{1}{(2\pi)^4} \int d^4z\, e^{ipz} \Big|_{p=0} = \frac{1}{(2\pi)^4} 2T\,V$
volume
$\sim \frac{-i\lambda}{4!(2\pi)^8} \boxed{2T\,V_s} \cdot \left[\int d^4p \frac{1}{p^2-m^2+i\epsilon}\right]^2$
spatial
All vacuum diagrams are proportional to V.


δ⁴(0) = 1/(2π)⁴ ∫d⁴z e^(ipz) |p=0 = 1/(2π)⁴ 2T V
volume
−iλ / 4!(2π)8 [2T V] · [∫d⁴p 1/(p²−m²+iε)]²
spatial
All vacuum diagrams are proportional to 2TV.


where $E_0 = \langle \Omega | H | \Omega \rangle$. Corresponding diagrams:
(vacuum bubbles)
$\frac{(-i\lambda)}{4!} \int d^4p_1 d^4p_2 (2\pi)^4 \delta^4(p_1 - p_1 + p_2 - p_2) \frac{1}{(2\pi)^8}$
$\frac{(-i\lambda)}{4!\,(2\pi)^4} \delta^4(0) \left[\int d^4p \frac{1}{p^2 - m^2 + i\varepsilon} \right]^2$
$\delta^4(0) = \frac{1}{(2\pi)^4} \int d^4z\, e^{ipz} \Big|_{p=0} = \frac{1}{(2\pi)^4}$
$\frac{-i\lambda}{4!\,(2\pi)^8} \boxed{2TV} \cdot \left[\int d^4p \frac{1}{p^2 - m^2 + i\varepsilon} \right]^2$
spatial
cuum
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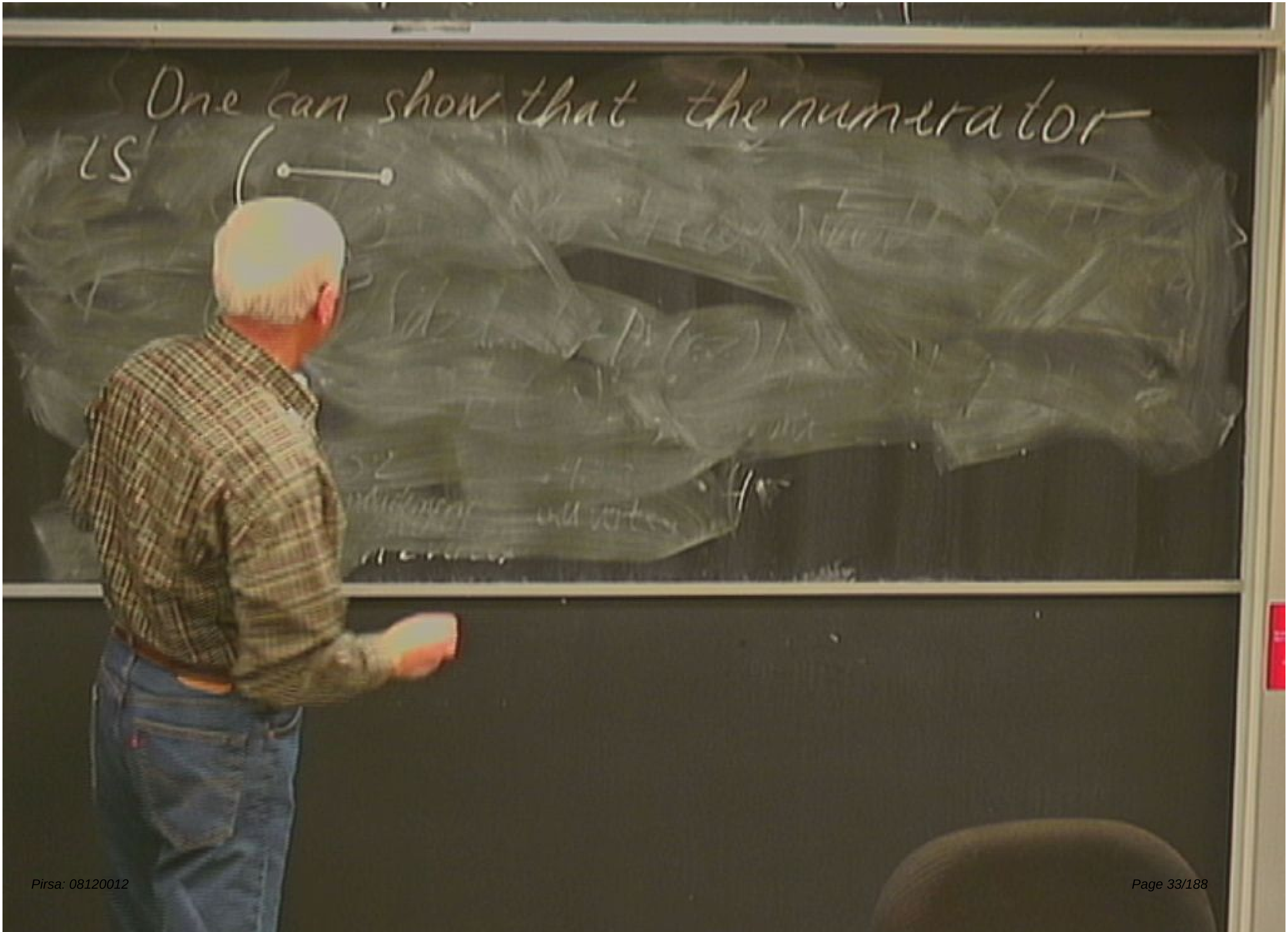
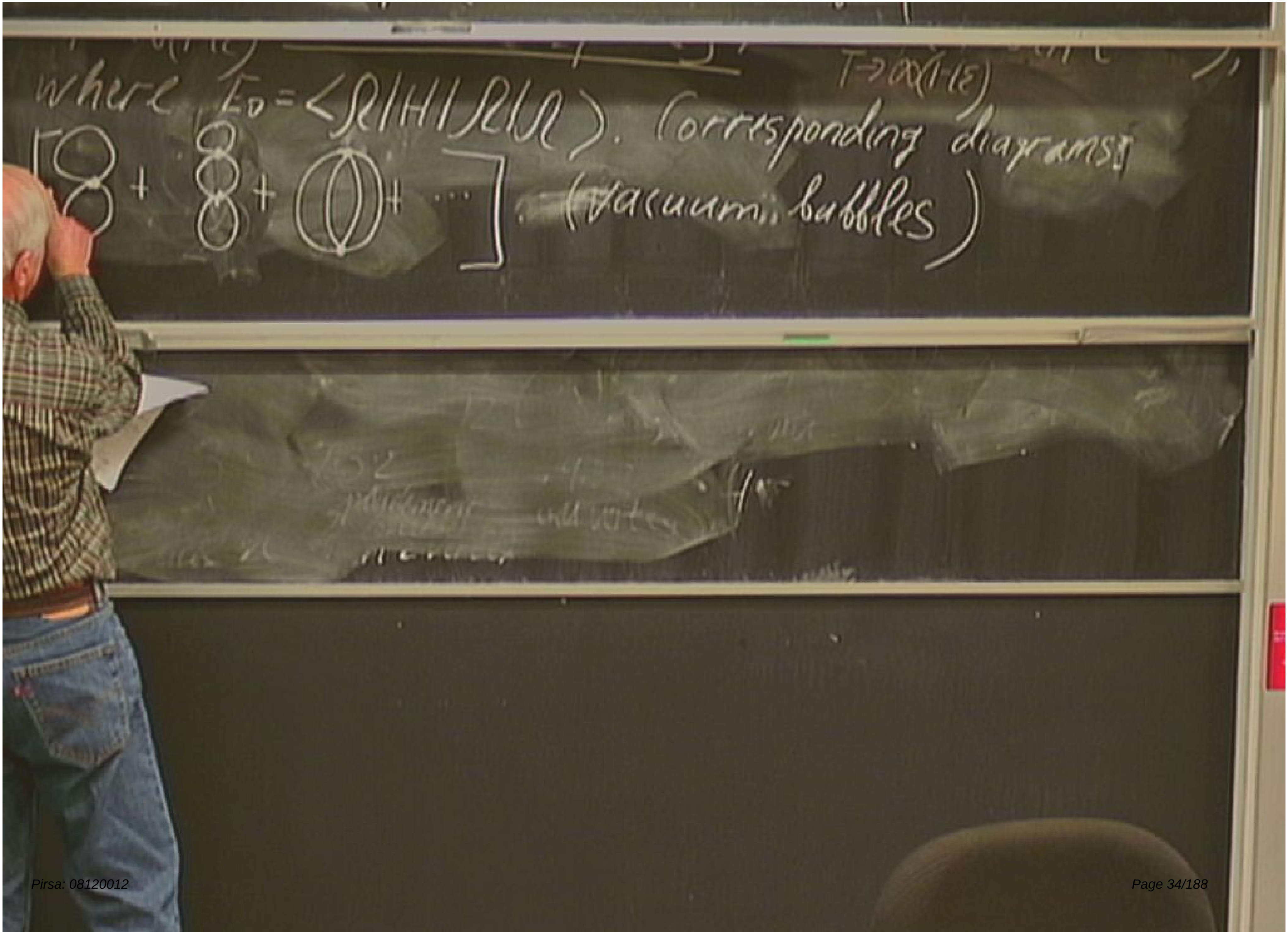
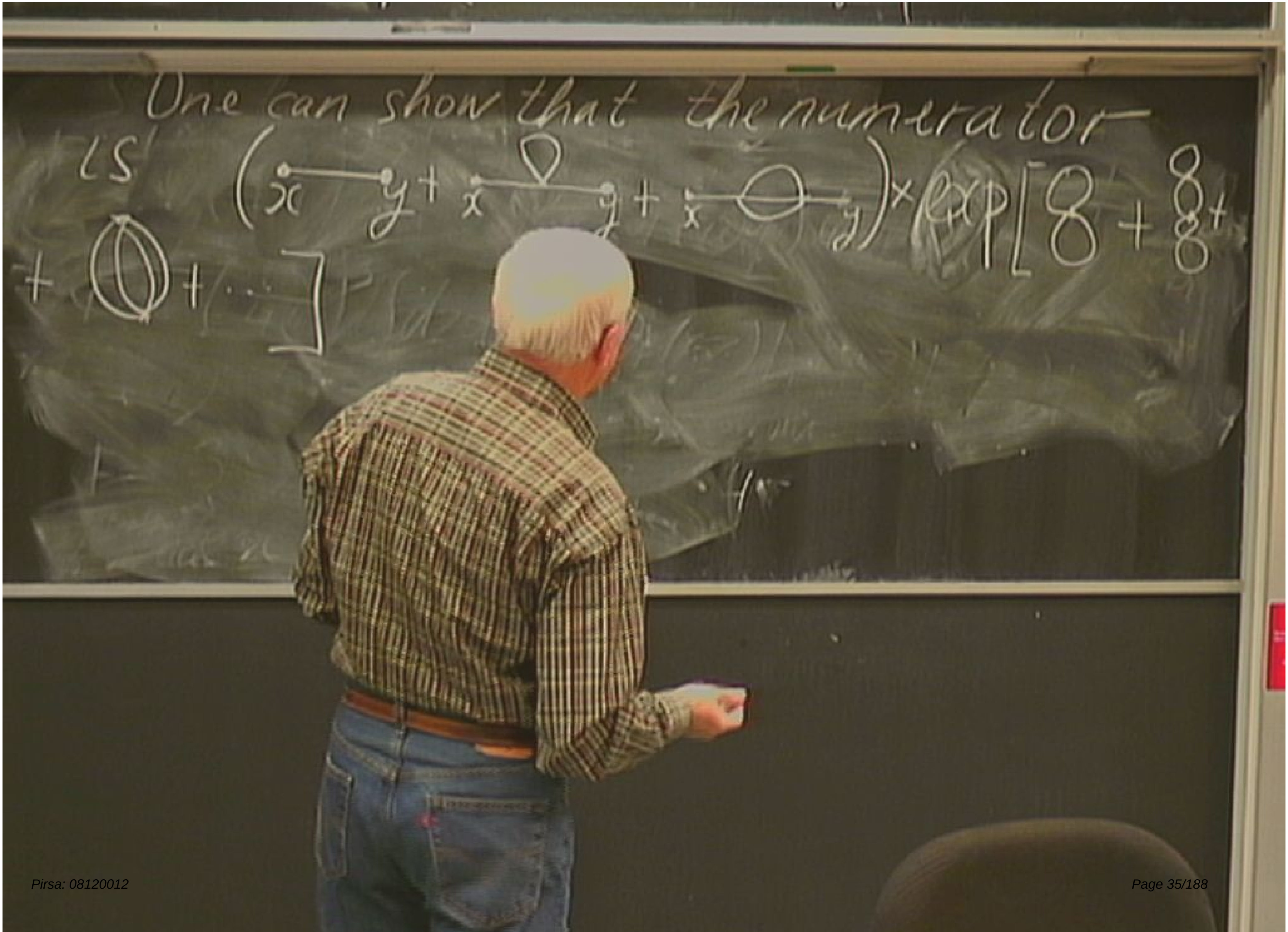
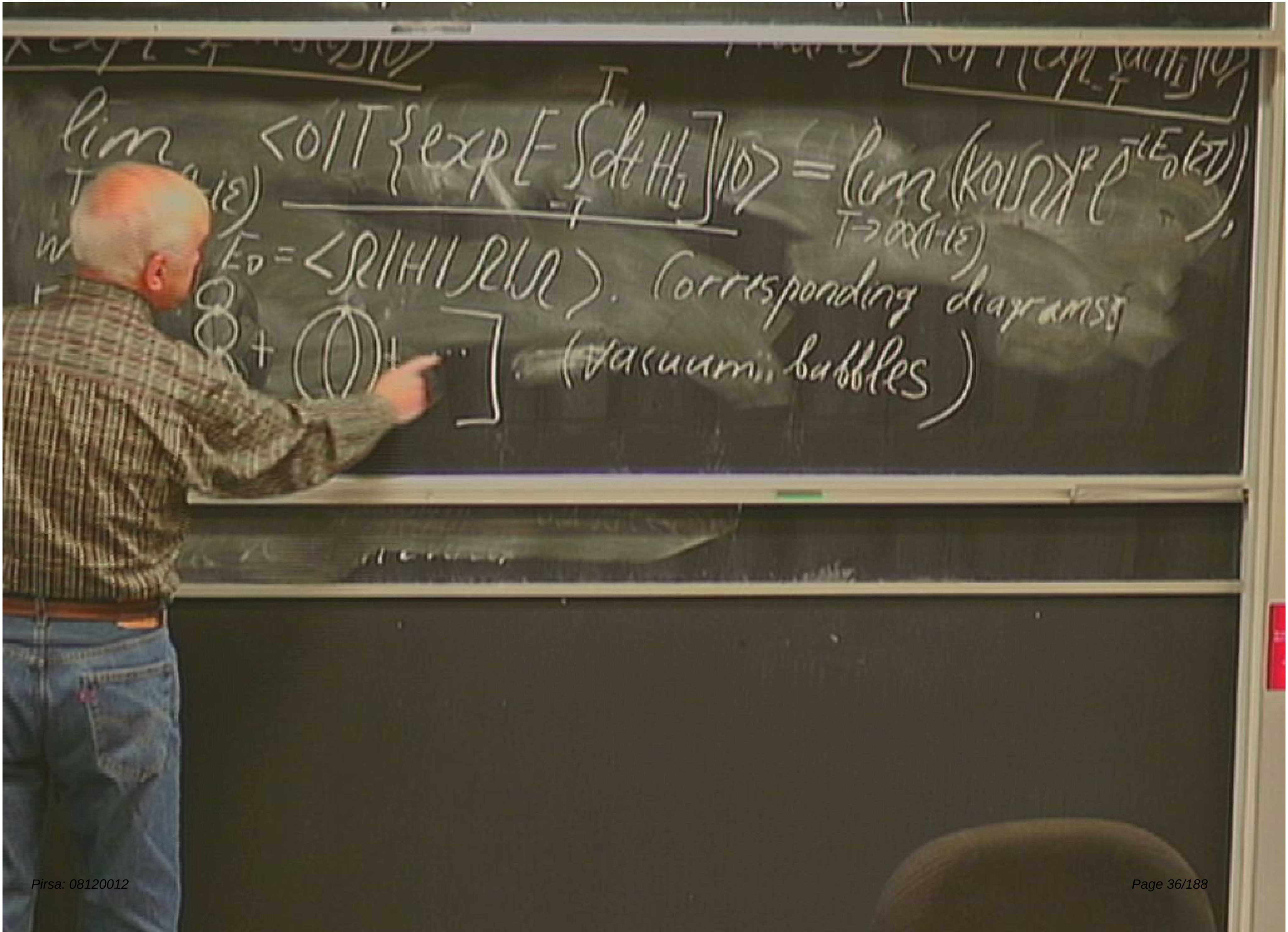
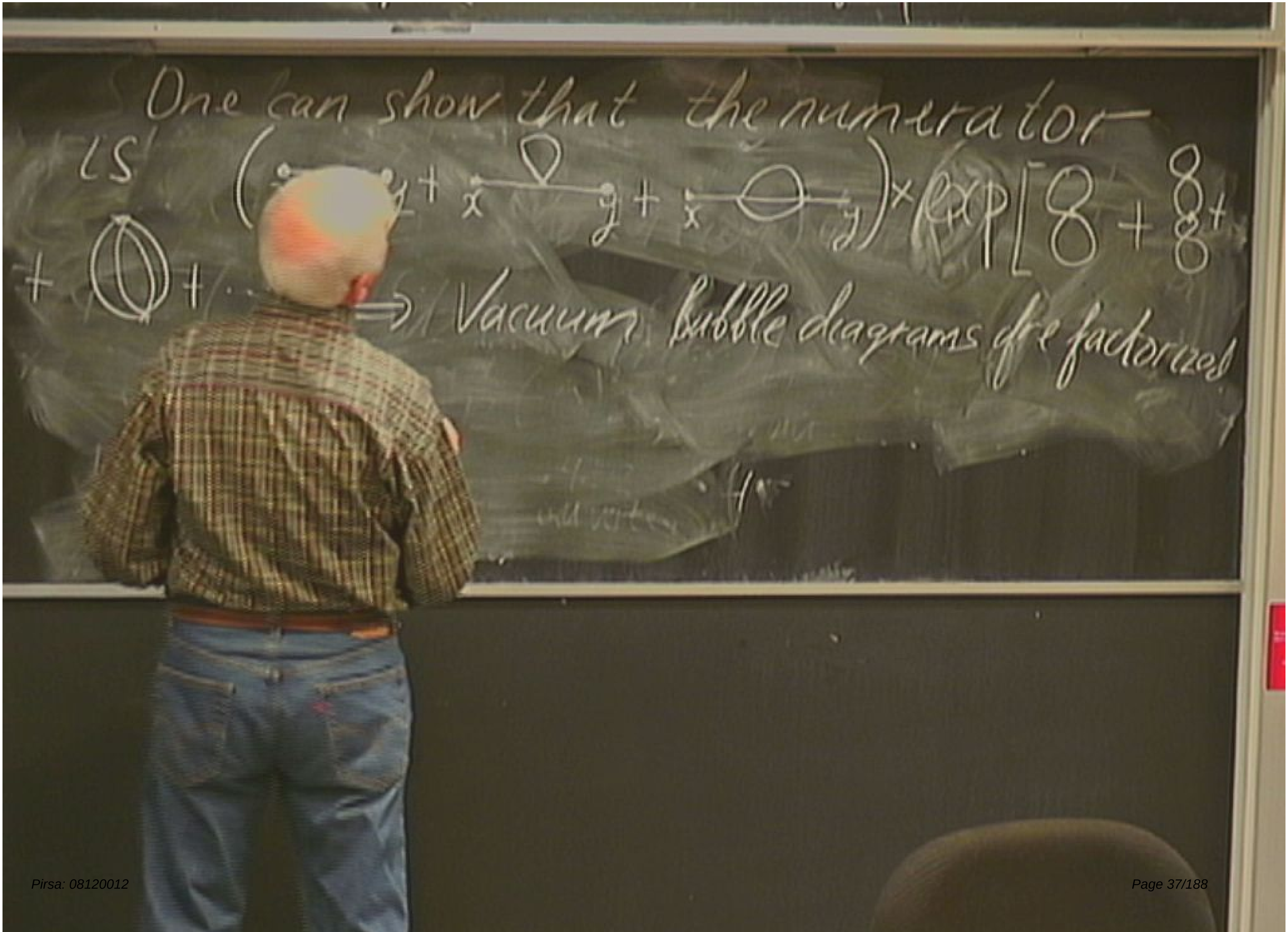
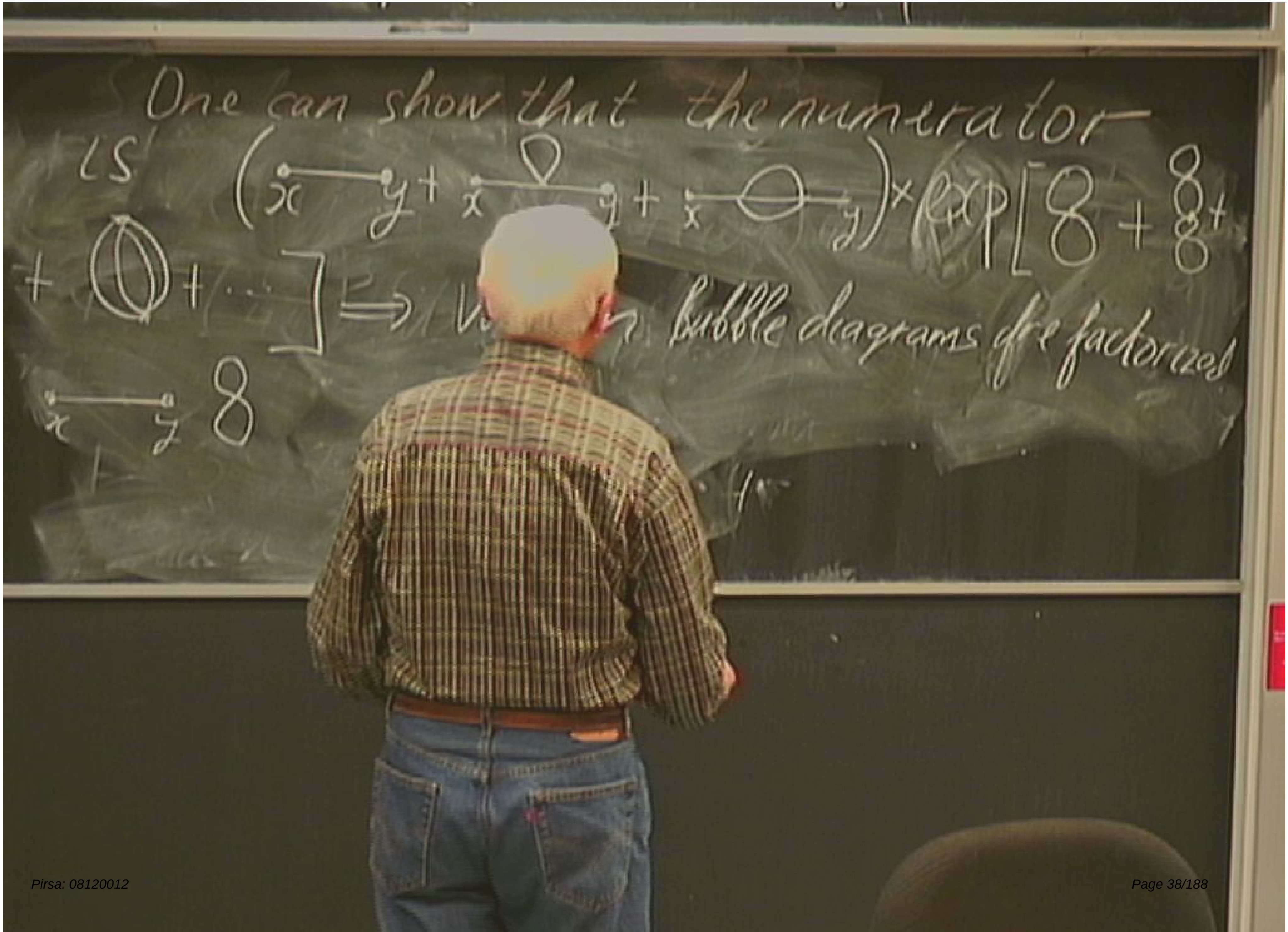

One can show that the numerator
is

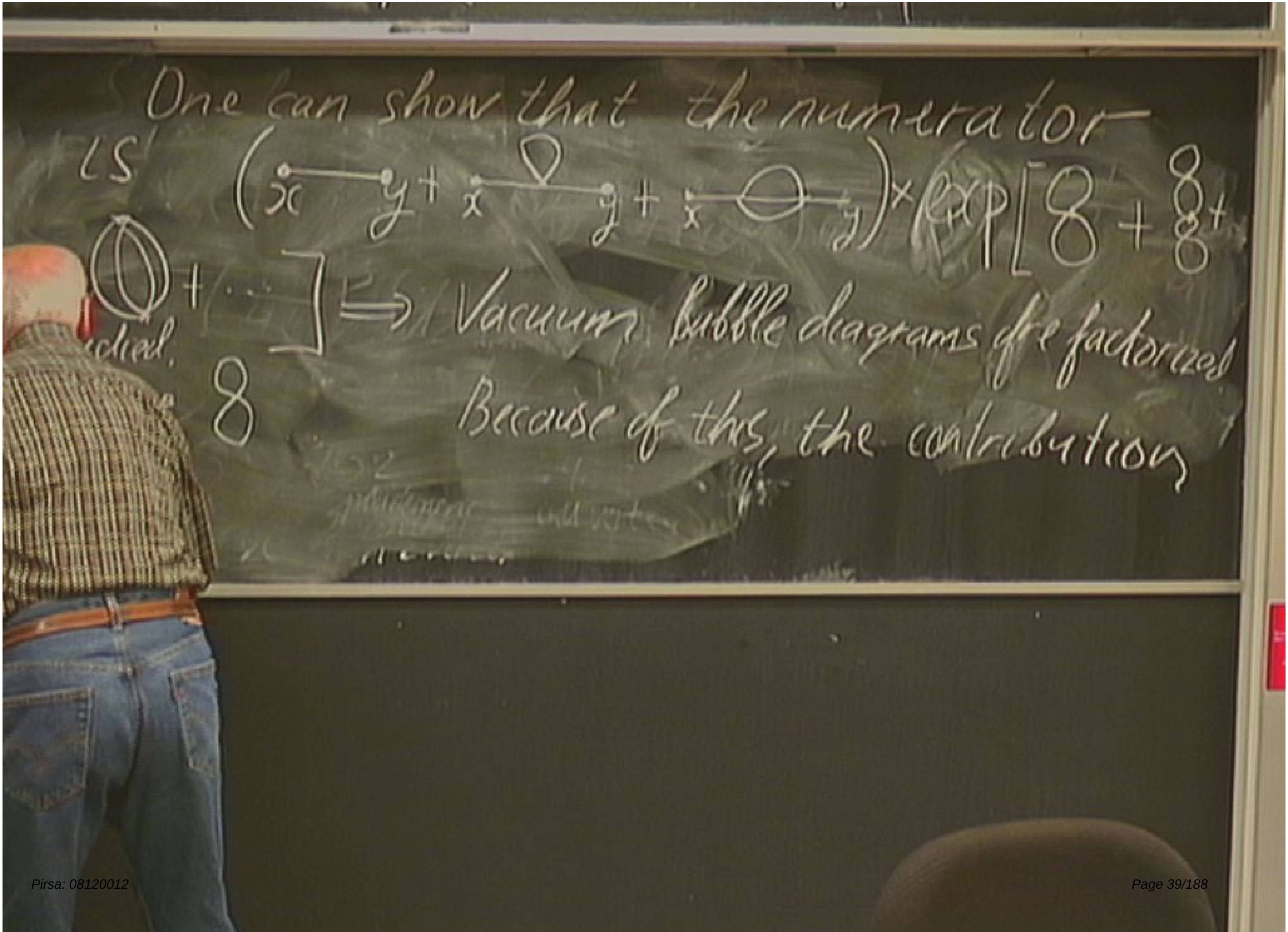

T→∞(1-iε)
where E₀=<Ω|H|Ω>. Corresponding diagrams:
(vacuum bubbles)


One can show that the numerator
is

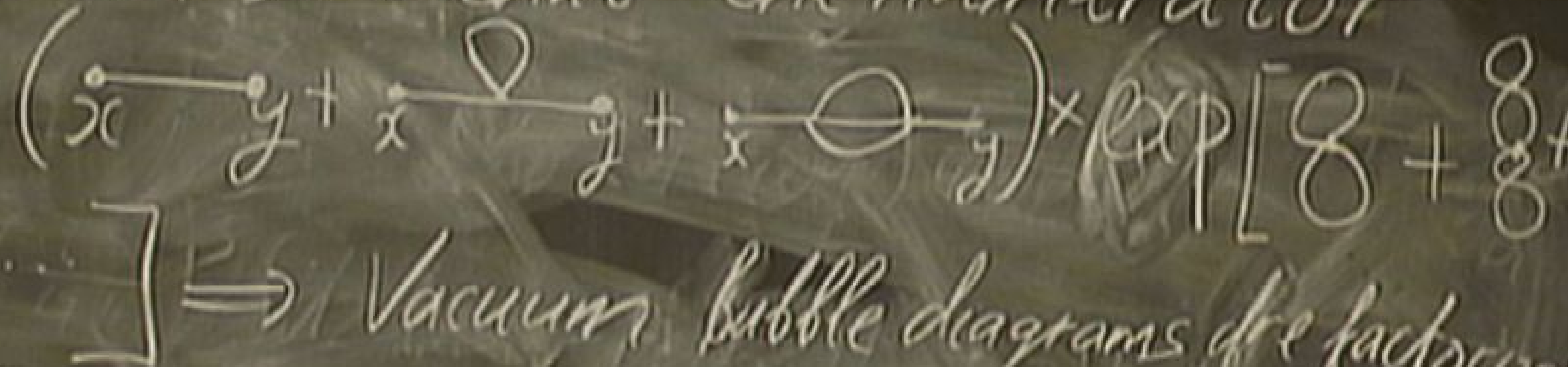

Corresponding diagrams:
(vacuum bubbles)


One can show that the numerator
is
exp
Vacuum bubble diagrams are factorized


One can show that the numerator
is
bubble diagrams are factorized

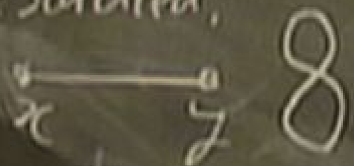

One can show that the numerator
is
Vacuum bubble diagrams are factorized
Because of this, the contribution

One can show that the numerator is

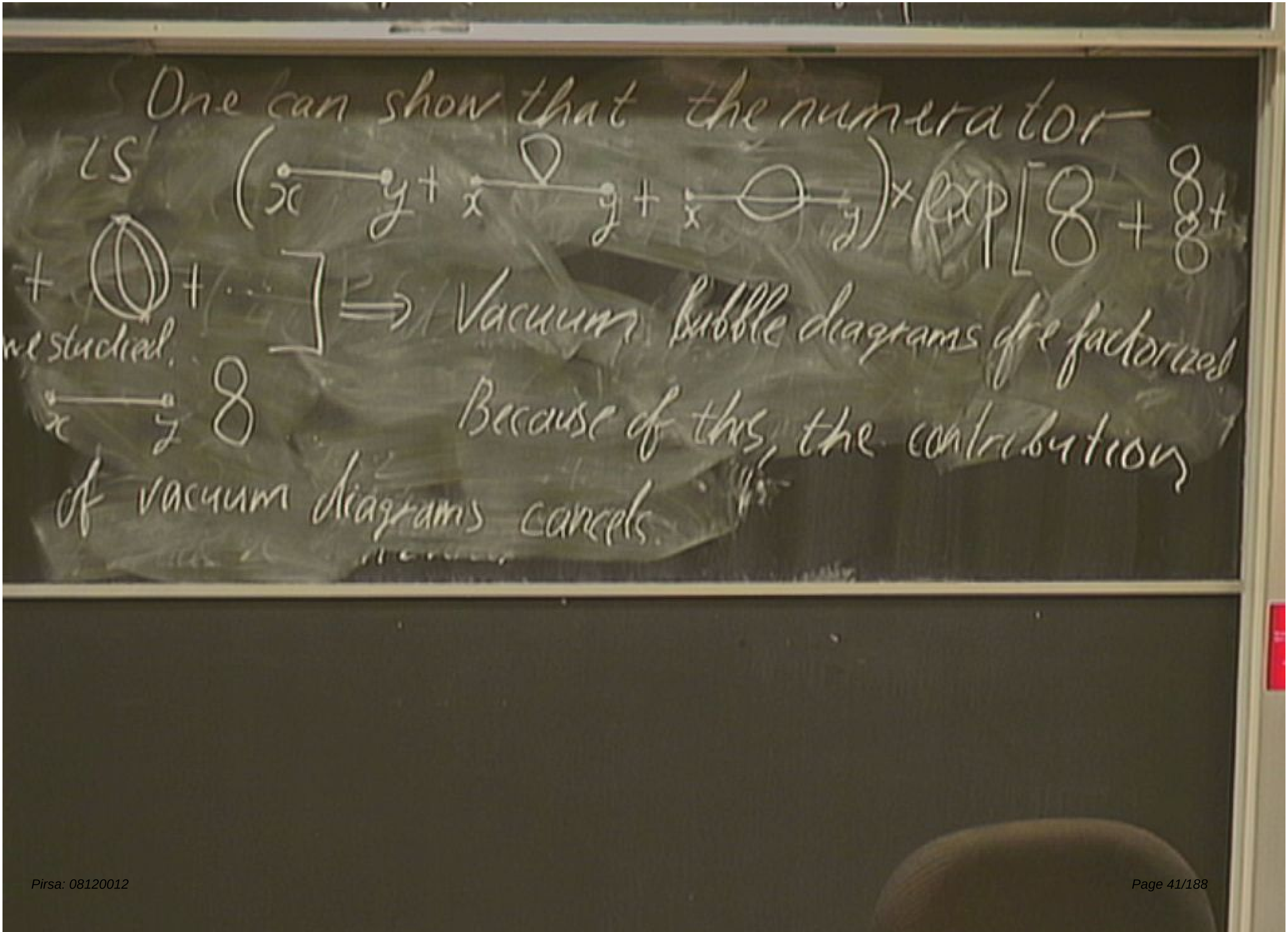


$\Rightarrow$ Vacuum bubble diagrams are factorized

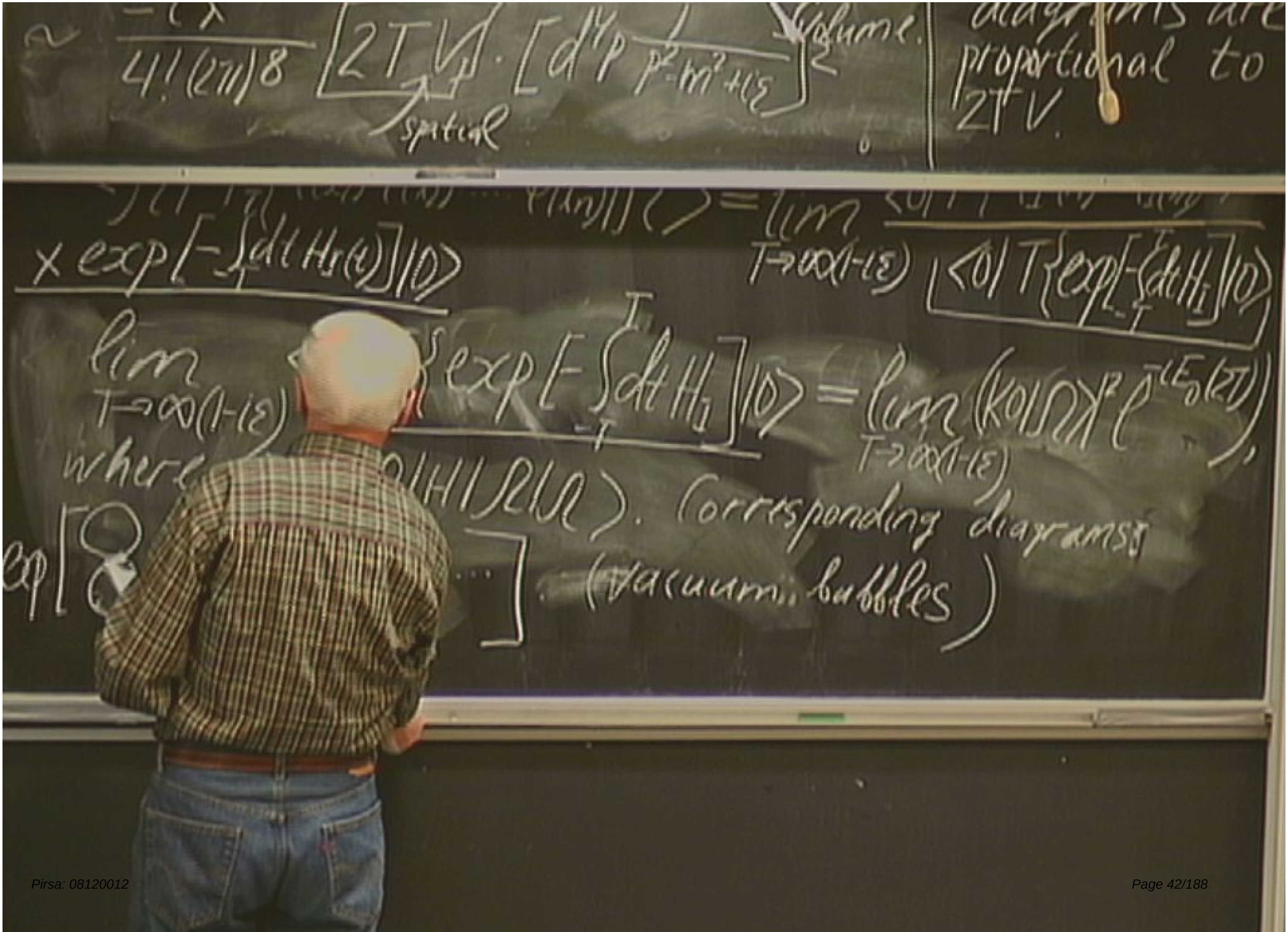
we studied.

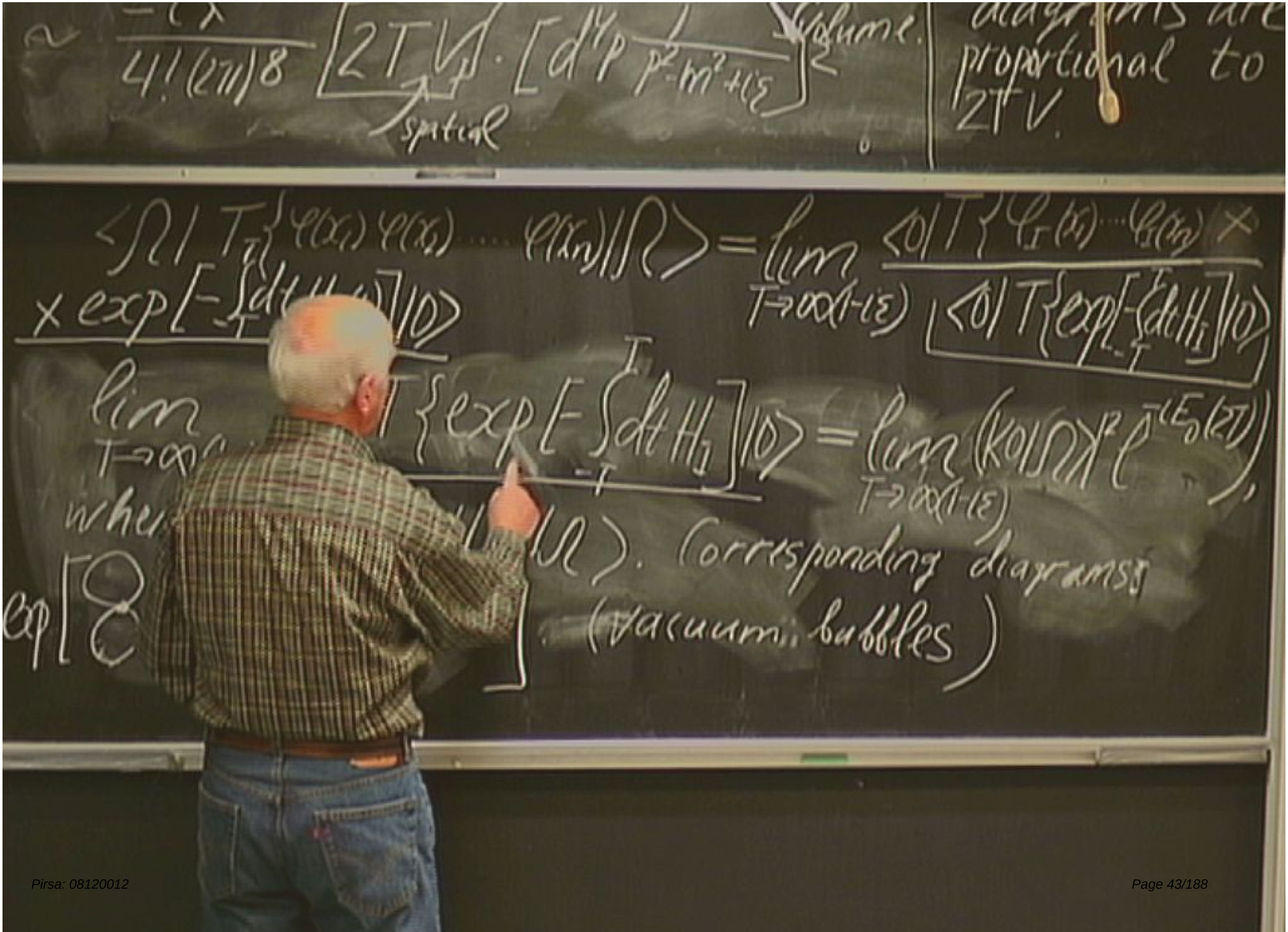


Because of this, the contribution of vacuum diagrams cancels.



One can show that the numerator
is
x
y
exp
Vacuum bubble diagrams are factorized
we studied.
Because of this, the contribution
of vacuum diagrams cancels.


spatial
volume.
diagrams are proportional to 2TV.
lim T→∞(1−iε)
× exp[−i∫dt H_I(t)]|0⟩
⟨0|T{exp[−i∫dt H_I]}|0⟩
where
Corresponding diagrams
(vacuum bubbles)



spatial
volume.
proportional to 2TV.
Corresponding diagrams
(vacuum bubbles)

$$\sim \frac{-i\lambda}{4!\,(2\pi)^8}\,\left[2T V_3\right]\cdot\left[\int d^4p\,\frac{i}{p^2-m^2+i\varepsilon}\right]^2$$

volume.

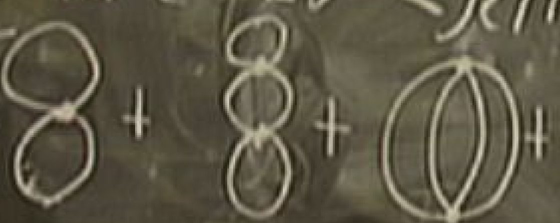
spatial

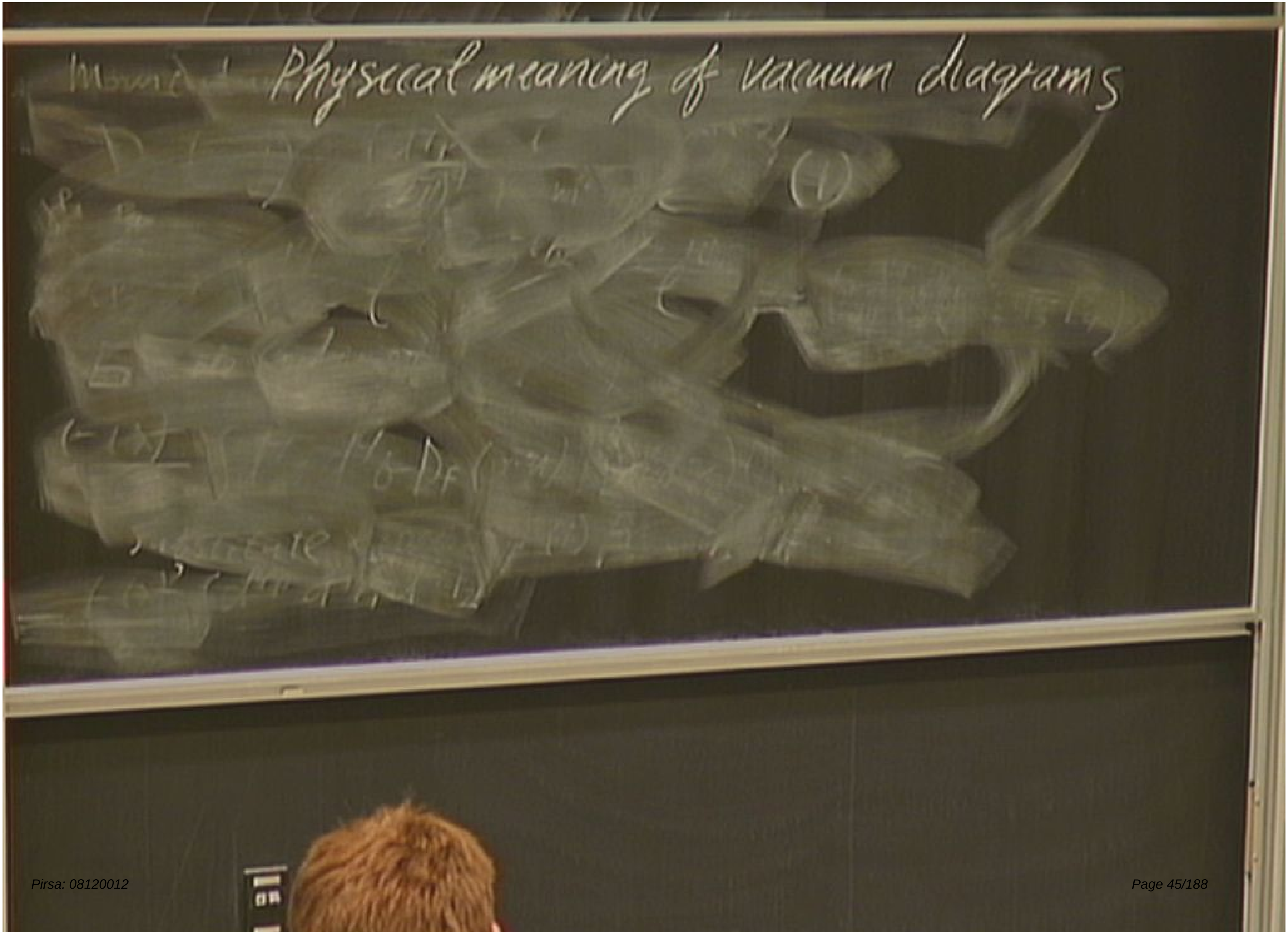
diagrams are proportional to $2TV$.

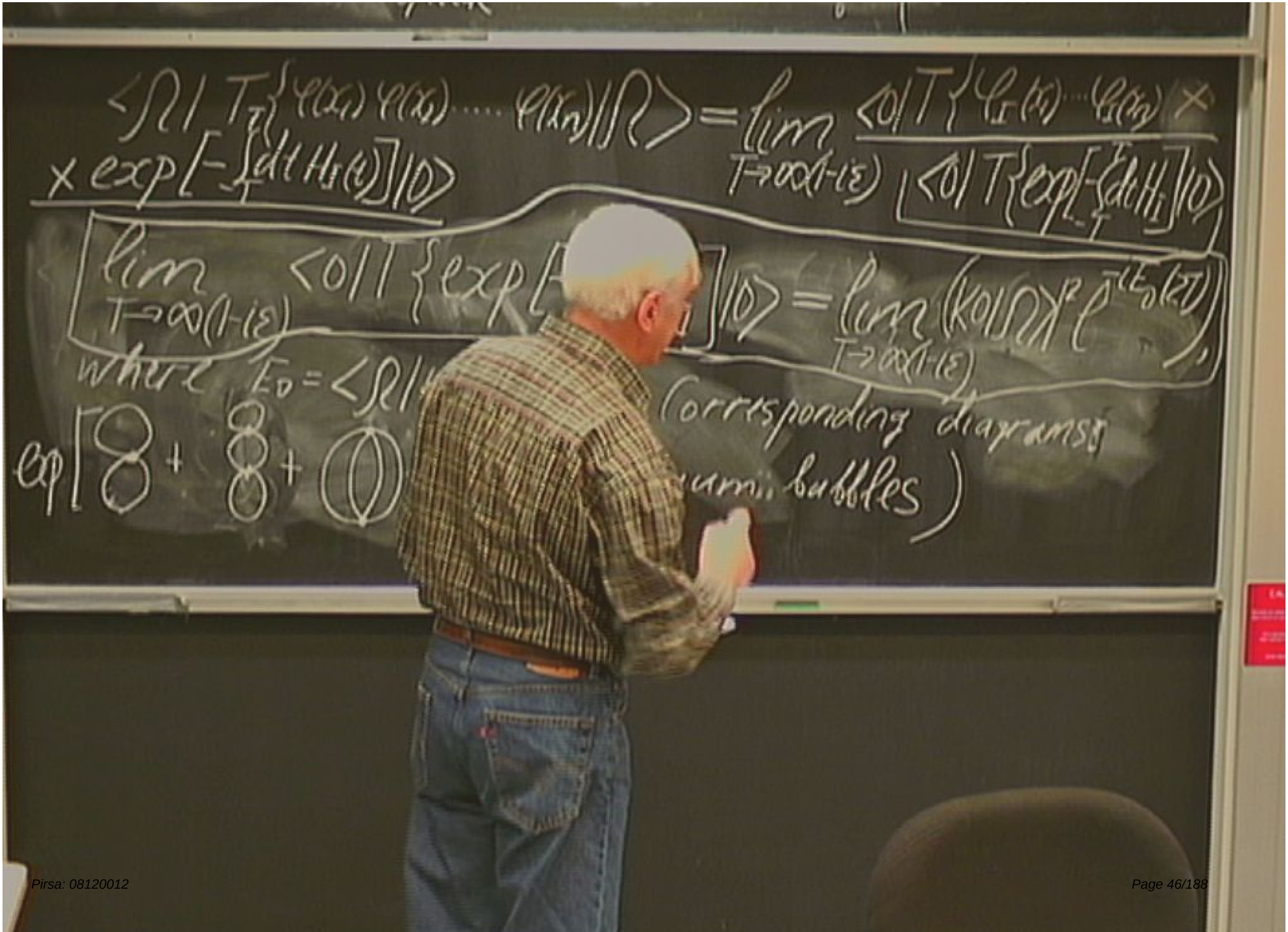
$$\langle\Omega|\,T_I\{\varphi(x_1)\varphi(x_2)\cdots\varphi(x_n)\}|\Omega\rangle = \lim_{T\to\infty(1-i\varepsilon)} \frac{\langle 0|T\{\varphi_I(x_1)\cdots\varphi_I(x_n)\times\exp[-i\int_{-T}^{T}dt\,H_I(t)]\}|0\rangle}{\langle 0|T\{\exp[-i\int_{-T}^{T}dt\,H_I]\}|0\rangle}$$

$$\lim_{T\to\infty(1-i\varepsilon)} \langle 0|T\{\exp[-i\int_{-T}^{T}dt\,H_I]\}|0\rangle = \lim_{T\to\infty(1-i\varepsilon)} \left(|\langle 0|\Omega\rangle|^2 e^{-iE_0(2T)}\right),$$

where $E_0=\langle\Omega|H|\Omega\rangle$. Corresponding diagrams:

exp[ + ⋯] (vacuum bubbles)


Physical meaning of vacuum diagrams

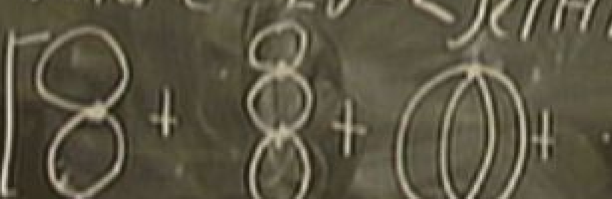


$\langle \Omega | T\{\varphi(x_1)\varphi(x_2)\cdots\varphi(x_n)\} | \Omega \rangle = \lim_{T\to\infty(1-i\varepsilon)} \frac{\langle 0 | T\{\varphi_I(x_1)\cdots\varphi_I(x_n) \times \exp[-i\int_{-T}^{T} dt\, H_I(t)]\} | 0 \rangle}{\langle 0 | T\{\exp[-i\int_{-T}^{T} dt\, H_I]\} | 0 \rangle}$
$\lim_{T\to\infty(1-i\varepsilon)} \langle 0 | T\{\exp[\ldots]\} | 0 \rangle = \lim_{T\to\infty(1-i\varepsilon)} |\langle 0 | \Omega \rangle|^2 e^{-iE_0(2T)}$
where $E_0 = \langle \Omega | H$
(corresponding diagrams: vacuum bubbles)
$\exp[$

$$\langle \Omega | T_\Omega\{\varphi(x_1)\varphi(x_2)\cdots\varphi(x_n)\}|\Omega\rangle = \lim_{T\to\infty(1-i\varepsilon)} \frac{\langle 0|T\{\varphi_I(x_1)\cdots\varphi_I(x_n)\times \exp[-i\int_{-T}^{T} dt\, H_I(t)]\}|0\rangle}{\langle 0|T\{\exp[-i\int_{-T}^{T} dt\, H_I]\}|0\rangle}$$

$$\boxed{\lim_{T\to\infty(1-i\varepsilon)} \langle 0|T\{\exp[-i\int_{-T}^{T} dt\, H_I]\}|0\rangle = \lim_{T\to\infty(1-i\varepsilon)} \left(|\langle 0|\Omega\rangle|^2 e^{-iE_0(2T)}\right)}$$

where $E_0 = \langle \Omega | H | \Omega \rangle$. Corresponding diagrams:

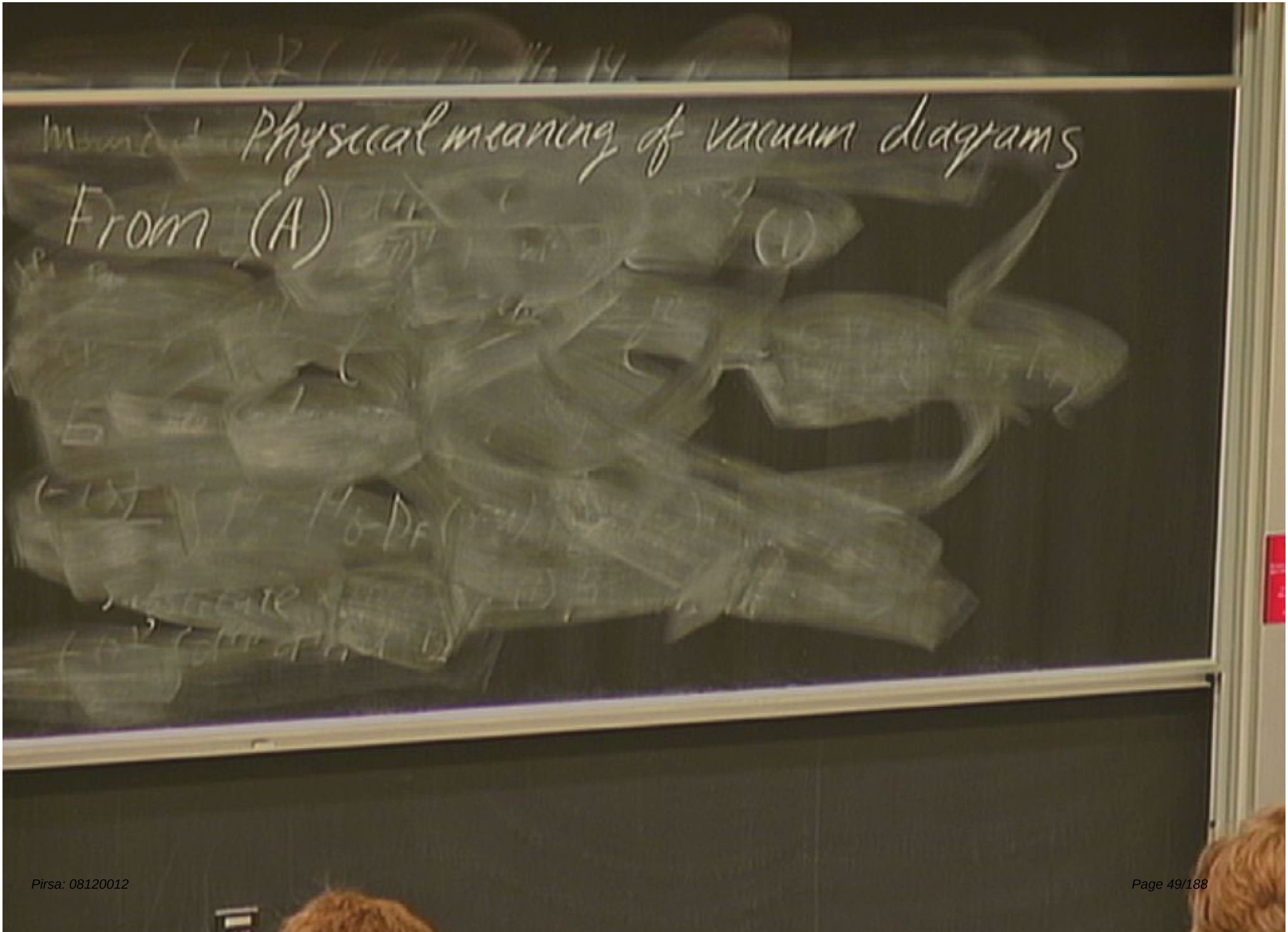
exp[ + ...] (vacuum bubbles)

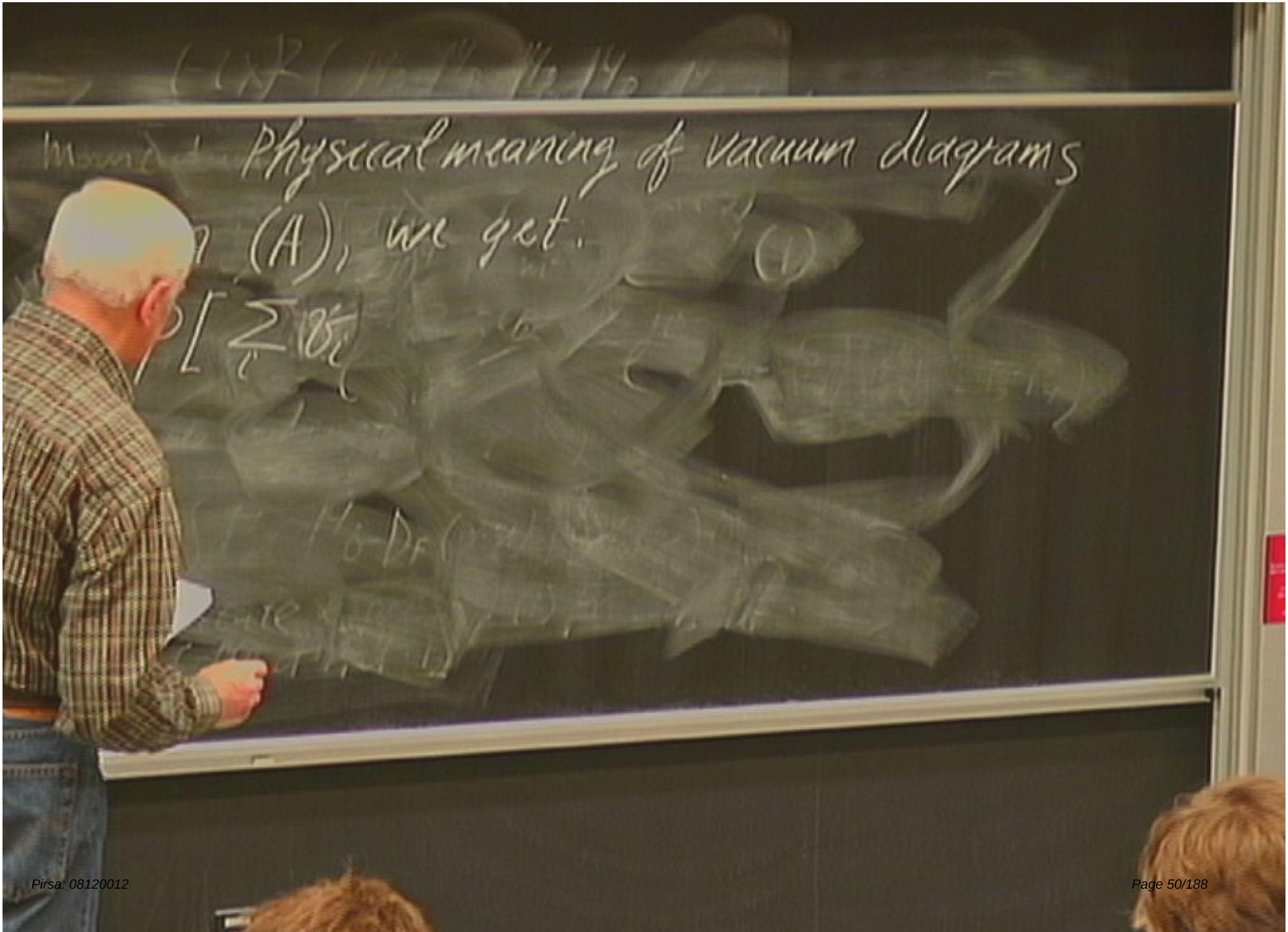
$$\langle\Omega| T_I\{\varphi(x_1)\varphi(x_2)\cdots\varphi(x_n)\}|\Omega\rangle = \lim_{T\to\infty(1-i\varepsilon)} \frac{\langle 0| T\{\varphi_I(x_1)\cdots\varphi_I(x_n)\times \exp[-\int_{-T}^{T} dt\, H_I(t)]\}|0\rangle}{\langle 0| T\{\exp[-\int_{-T}^{T} dt\, H_I]\}|0\rangle}$$

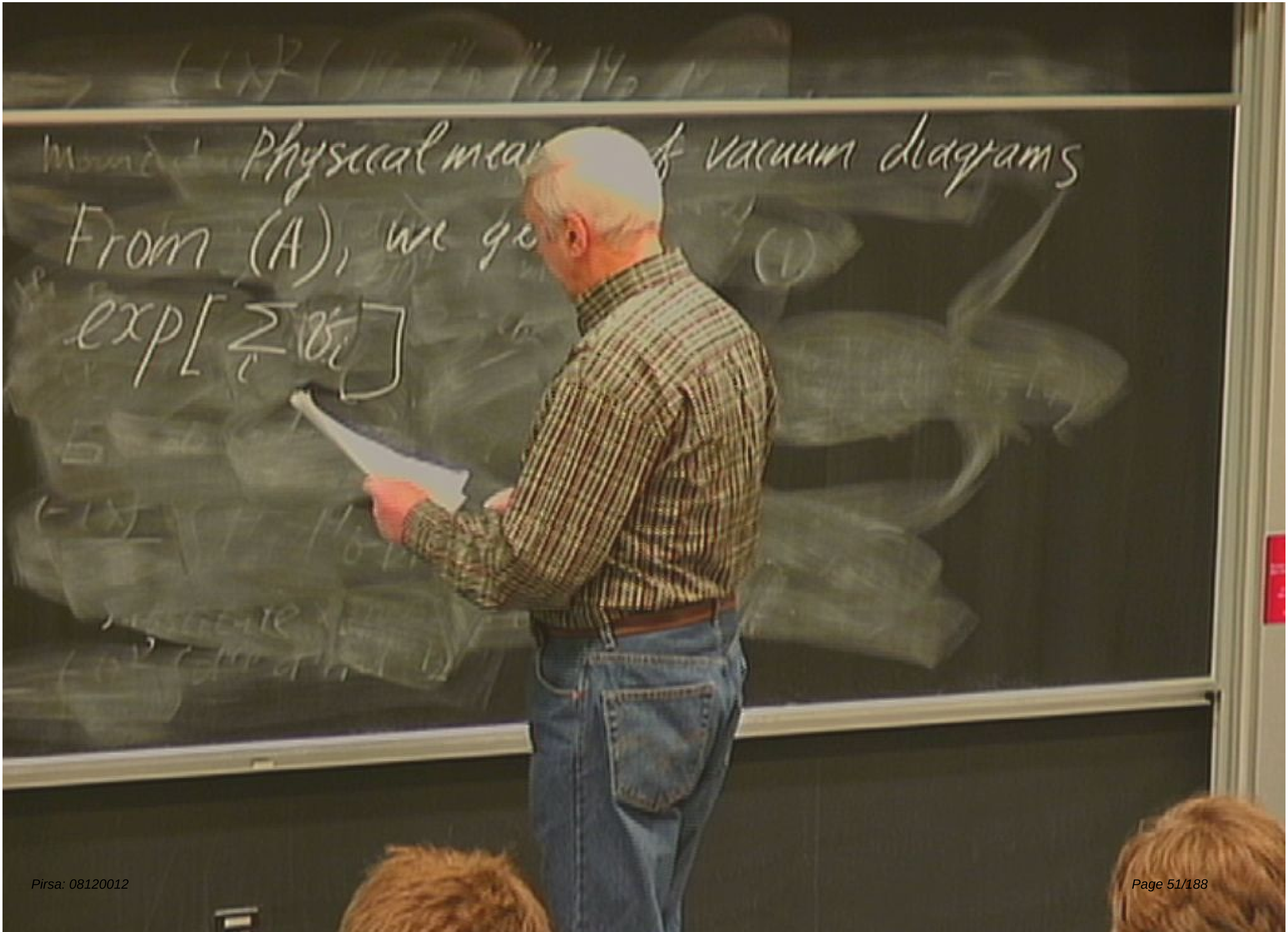
$$\boxed{\lim_{T\to\infty(1-i\varepsilon)} \langle 0| T\{\exp[-\int_{-T}^{T} dt\, H_I]\}|0\rangle = \lim_{T\to\infty(1-i\varepsilon)} |\langle 0|\Omega\rangle|^2 e^{-iE_0(2T)}}$$

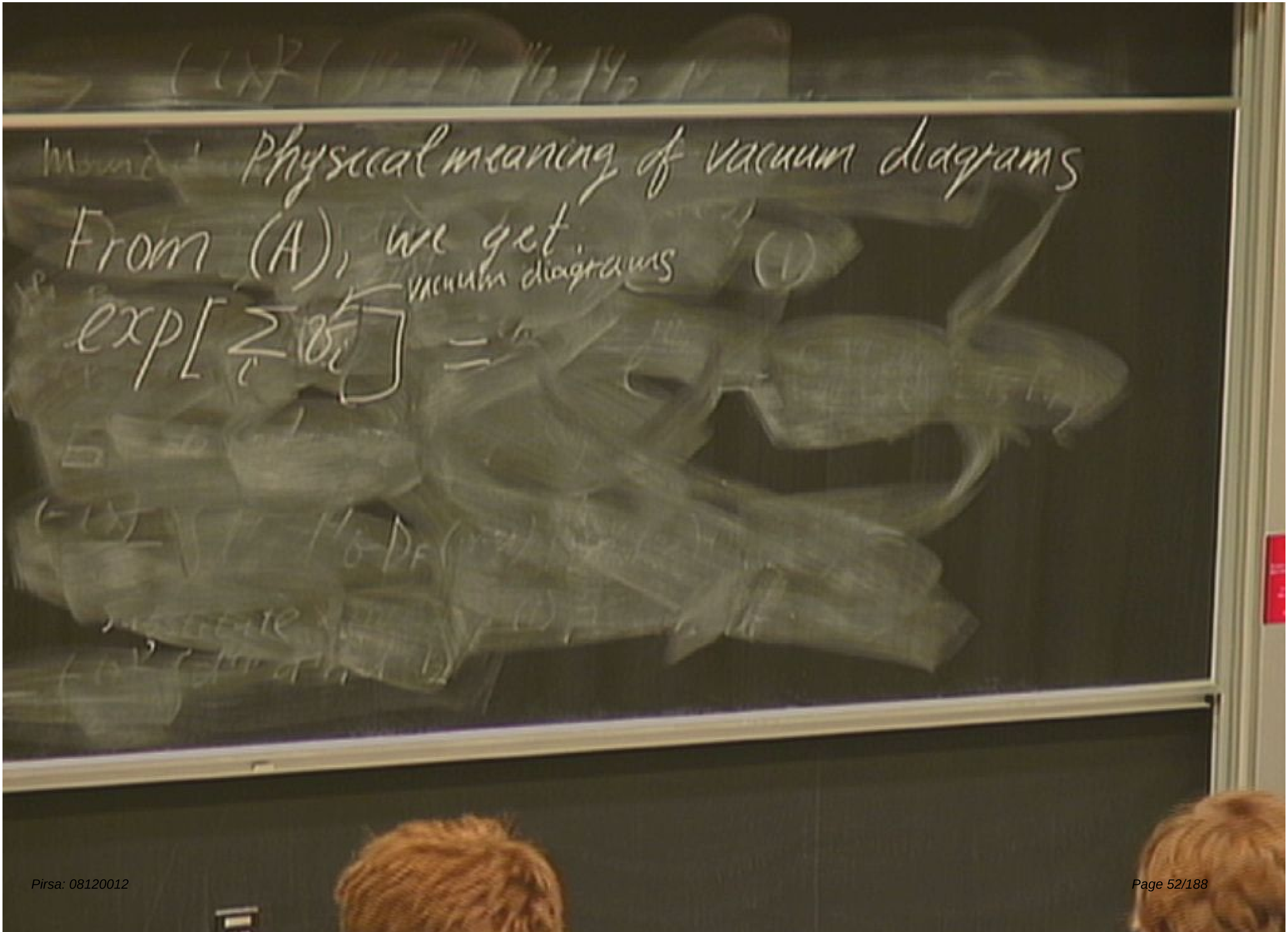
where $E_0 = \langle\Omega|H|\Omega\rangle$. Corresponding diagrams:

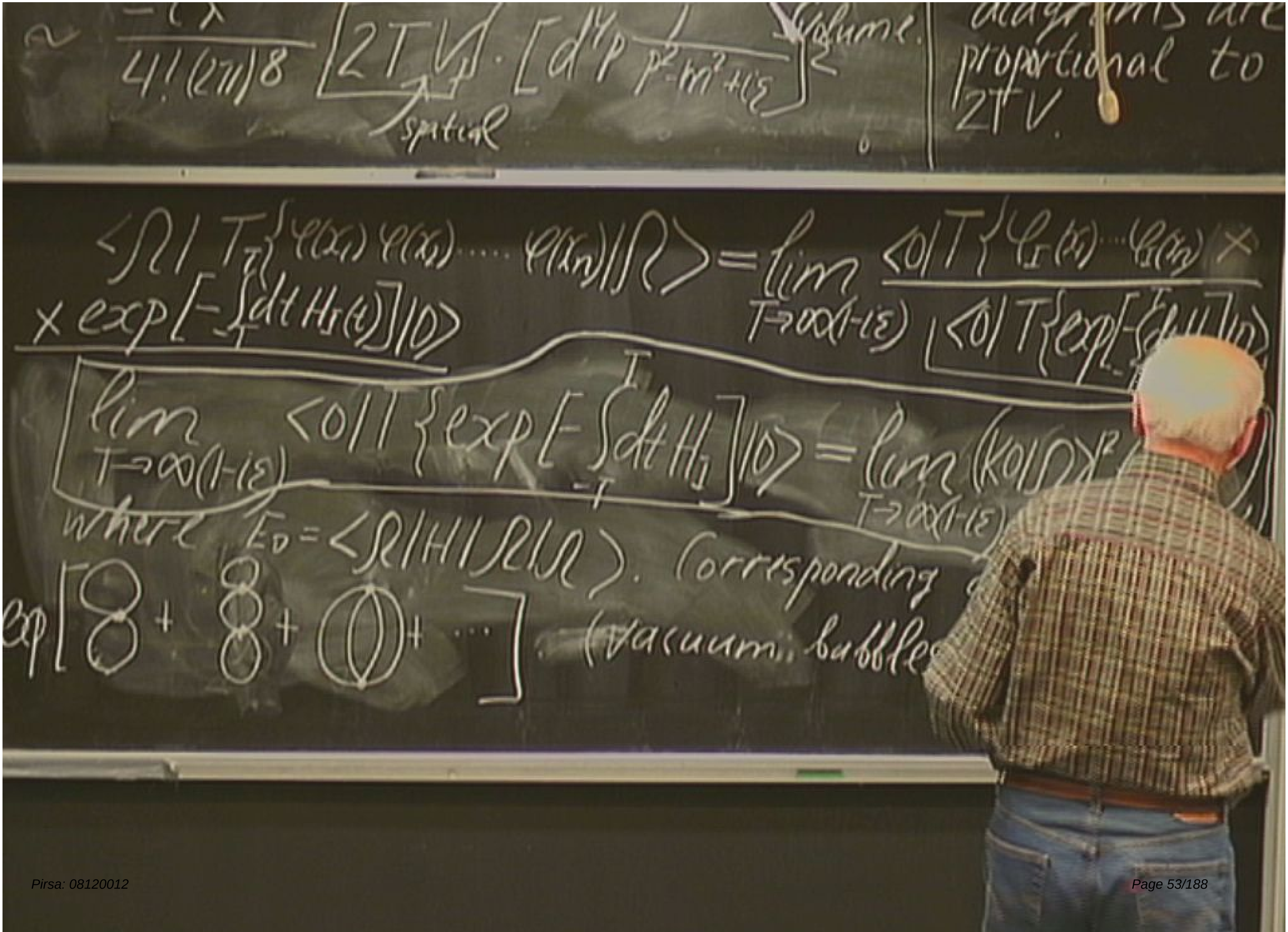
$\exp[\ 8 + 8 + \ldots + \cdots\ ]$ (vacuum bubbles)


Physical meaning of vacuum diagrams
From (A)


Physical meaning of vacuum diagrams
(A), we get.


Physical mea f vacuum diagrams
From (A), we g
exp[Σ


Physical meaning of vacuum diagrams
From (A), we get
vacuum diagrams
(1)
exp[Σ V_i] =



volume
diagrams are proportional to 2TV.
spatial
⟨Ω| T{φ(x₁) φ(x₂) ..... φ(xₙ)}|Ω⟩ = lim ⟨0|T{φ_I(x₁)···φ_I(xₙ) × exp[-i∫dt H_I(t)]}|0⟩ / ⟨0|T{exp[-i∫dt H_I]}|0⟩
T→∞(1-iε)
lim ⟨0|T{exp[-i∫dt H_I]}|0⟩ = lim (|⟨0|Ω⟩|²
T→∞(1-iε)
where E₀ = ⟨Ω|H|Ω⟩. Corresponding
exp[+ + + ···] (vacuum bubbles

$$\sim \frac{-i\lambda}{4!(2\pi)^8}\,[2T\,V_s]\cdot\left[\int d^4p\,\frac{i}{p^2-m^2+i\varepsilon}\right]^2$$

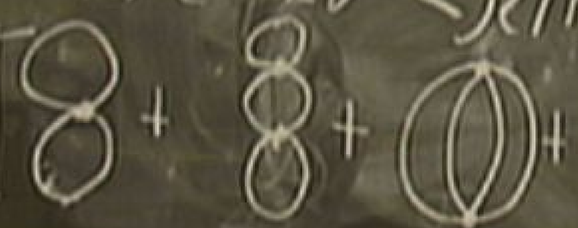
spatial volume.

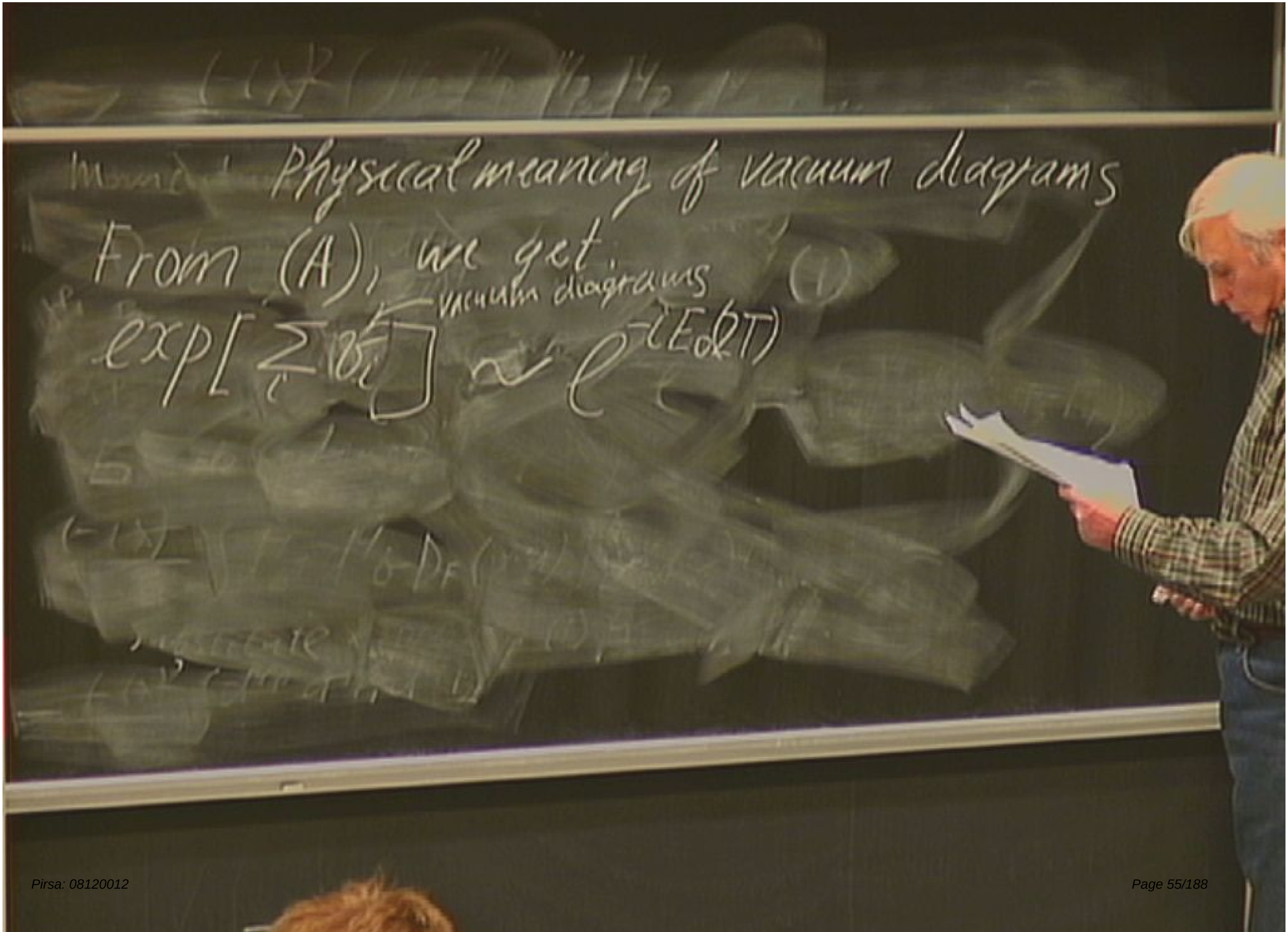
diagrams are proportional to $2TV$.

$$\langle\Omega|\,T\{\varphi(x_1)\varphi(x_2)\cdots\varphi(x_n)\}\,|\Omega\rangle = \lim_{T\to\infty(1-i\varepsilon)} \frac{\langle 0|\,T\{\varphi_I(x_1)\cdots\varphi_I(x_n) \times \exp[-i\int_{-T}^{T} dt\, H_I(t)]\}\,|0\rangle}{\langle 0|\,T\{\exp[-i\int_{-T}^{T} dt\, H_I]\}\,|0\rangle}$$

$$\boxed{\lim_{T\to\infty(1-i\varepsilon)} \langle 0|\,T\{\exp[-i\int_{-T}^{T} dt\, H_I]\}\,|0\rangle = \lim_{T\to\infty(1-i\varepsilon)} |\langle 0|\Omega\rangle|^2\, e^{-iE_0(2T)}}$$

where $E_0 = \langle\Omega|H|\Omega\rangle$. Corresponding diagrams:

exp[ + ...] (vacuum bubbles)


Physical meaning of vacuum diagrams
From (A), we get
vacuum diagrams
$\exp[\sum_i V_i] \sim e^{-iE_0(2T)}$

Physical meaning of vacuum diagrams

From (A), we get

$$\exp\Big[\sum_i \underbrace{V_i}_{\text{vacuum diagrams}}\Big] \sim e^{-iE_0(2T)} \Rightarrow E_0 2T = i\sum_i V_i$$

Physical meaning of vacuum diagrams

From (A), we get

$$\exp\Big[\sum_i \overbrace{v_i}^{\text{vacuum diagrams}}\Big] \sim e^{-iE_0(2T)} \Rightarrow E_0 2T = i\sum_i v_i$$

Physical meaning of vacuum diagrams

From (A), we get

$$\exp\Big[\overbrace{\sum_i V_i}^{\text{vacuum diagrams}}\Big] \sim e^{-iE_0(2T)} \Rightarrow E_0 2T = i\sum_i V_i$$

$$\frac{(-i\lambda)}{4!\,(2\pi)^4}\,\delta^4(0)\left[\int d^4p\,\frac{1}{p^2-m^2+i\varepsilon}\right]^2$$

$$\delta^4(0)=\frac{1}{(2\pi)^4}\int d^4z\,e^{ipz}\Big|_{p=0}=\frac{1}{(2\pi)^4}\,2T\,V$$

volume.

$$\frac{-i\lambda}{4!\,(2\pi)^8}\,\boxed{2TV}\cdot\left[\int d^4p\,\frac{1}{p^2-m^2+i\varepsilon}\right]^2$$

spatial

All vacuum diagrams a[re] proportional t[o] 2TV.

$\ldots\exp[-i\int dt\, H_I(t)]|0\rangle \quad T\to\infty(1-i\varepsilon) \quad \langle 0|T\{\exp[-i\int dt\, H_I \ldots$

$$\lim_{T\to\infty(1-i\varepsilon)}\langle 0|T\left\{\exp\left[-i\int_{-T}^{T}dt\,H_I\right]\right\}|0\rangle=\lim_{T\to\infty(1-i\varepsilon)}\left(|\langle 0|\Omega\rangle|^2 e^{-iE_0\ldots}\right) \quad (A)$$

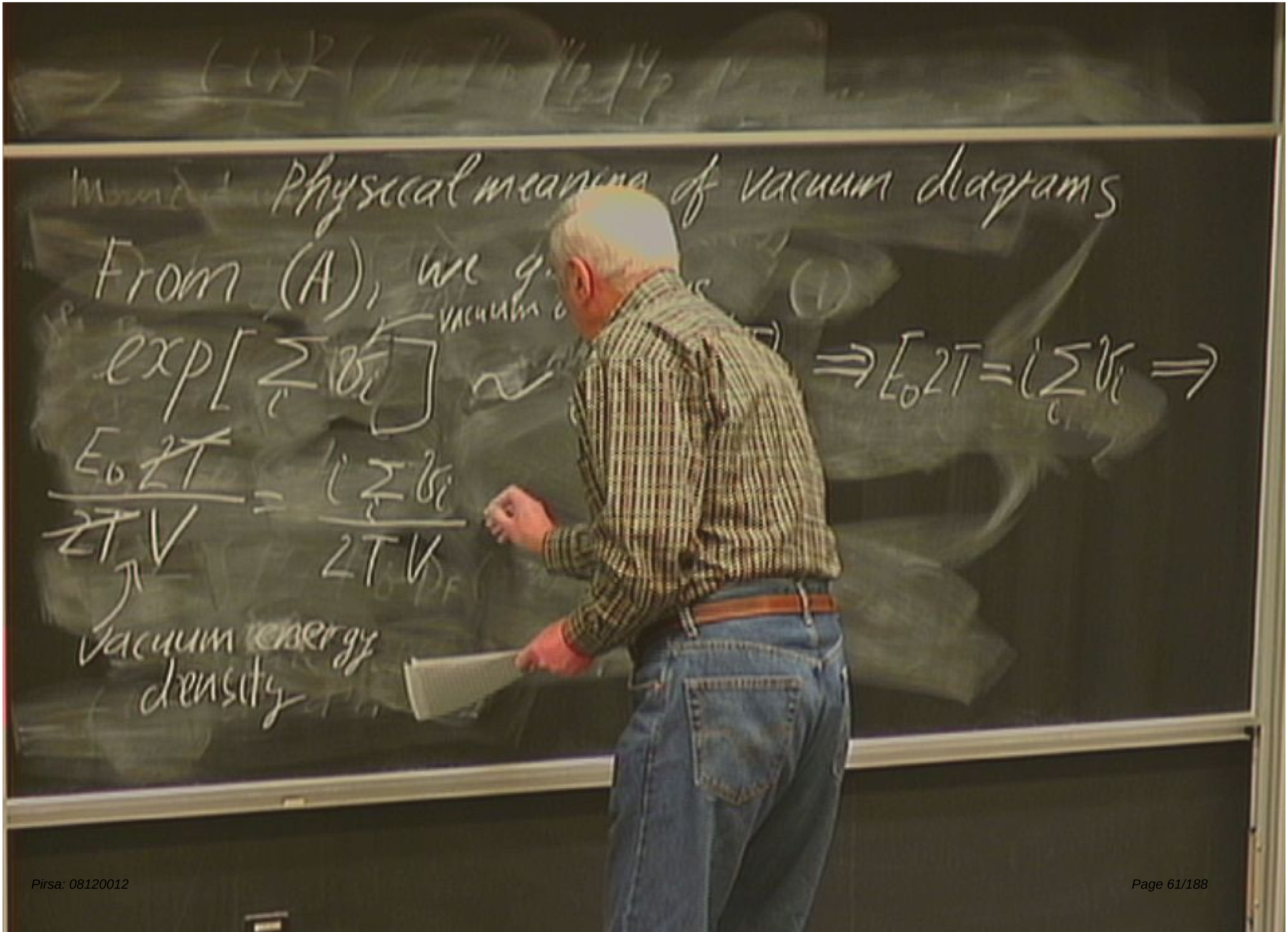
where $E_0=\langle\Omega|H|\Omega\rangle$. Corresponding diagrams

Physical meaning of vacuum diagrams

From (A), we get

$$\exp\Big[\underbrace{\sum_i v_i}_{\text{vacuum diagrams}}\Big] \sim e^{-iE_0(2T)} \Rightarrow E_0 2T = i\sum_i v_i \Rightarrow$$

$$\frac{E_0 \cancel{2T}}{\cancel{2T}\,V} = \frac{i\sum_i v_i}{2T\,V}$$


Physical meaning of vacuum diagrams
From (A), we g
vacuum
$\exp[\sum_i \mathcal{V}_i] \sim$
$\Rightarrow E_0 2T = i\sum_i \mathcal{V}_i \Rightarrow$
$\frac{E_0 \cancel{2T}}{\cancel{2T} V} = \frac{i\sum_i \mathcal{V}_i}{2TV}$
Vacuum energy density

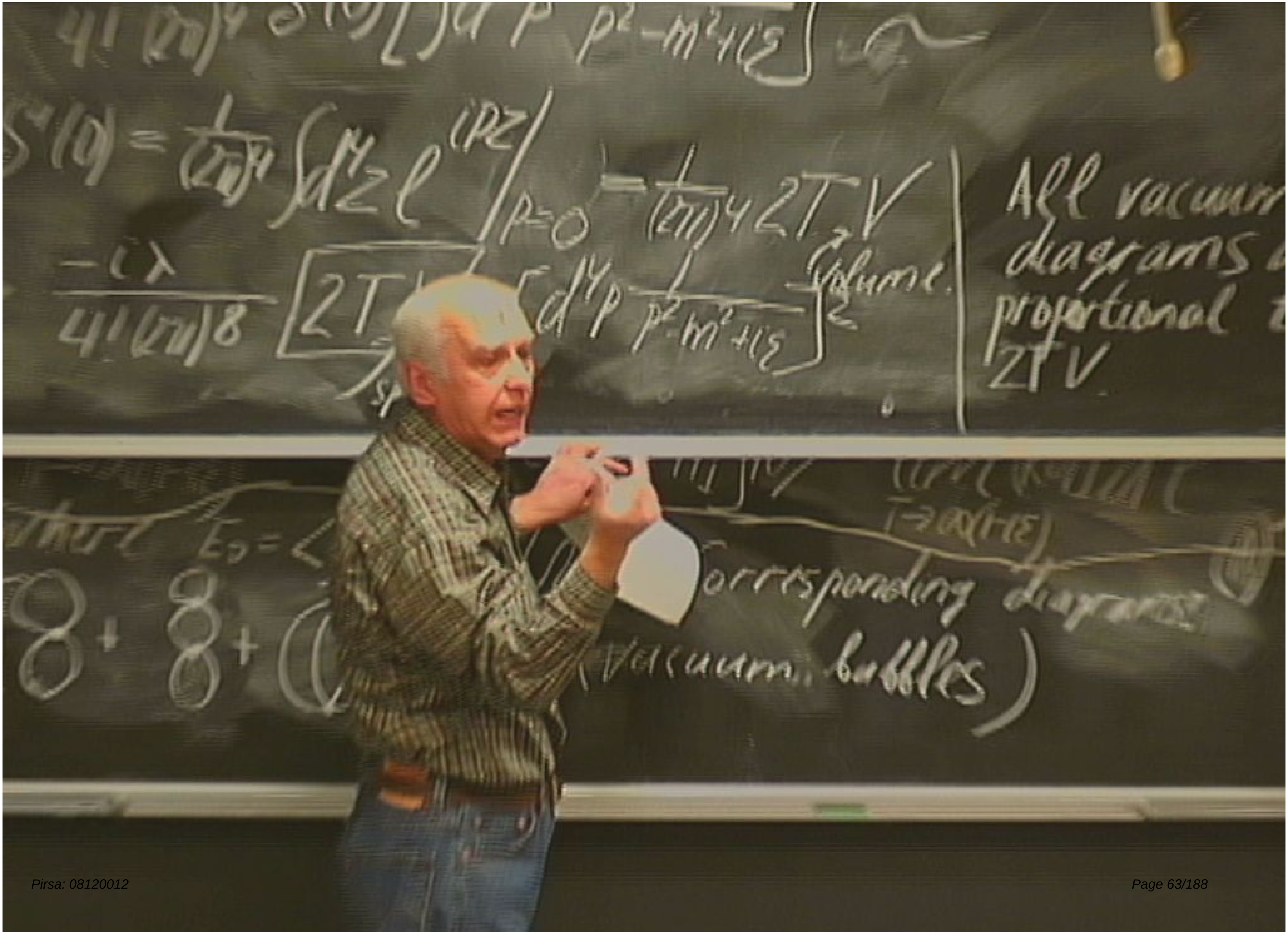
Physical meaning of vacuum diagrams

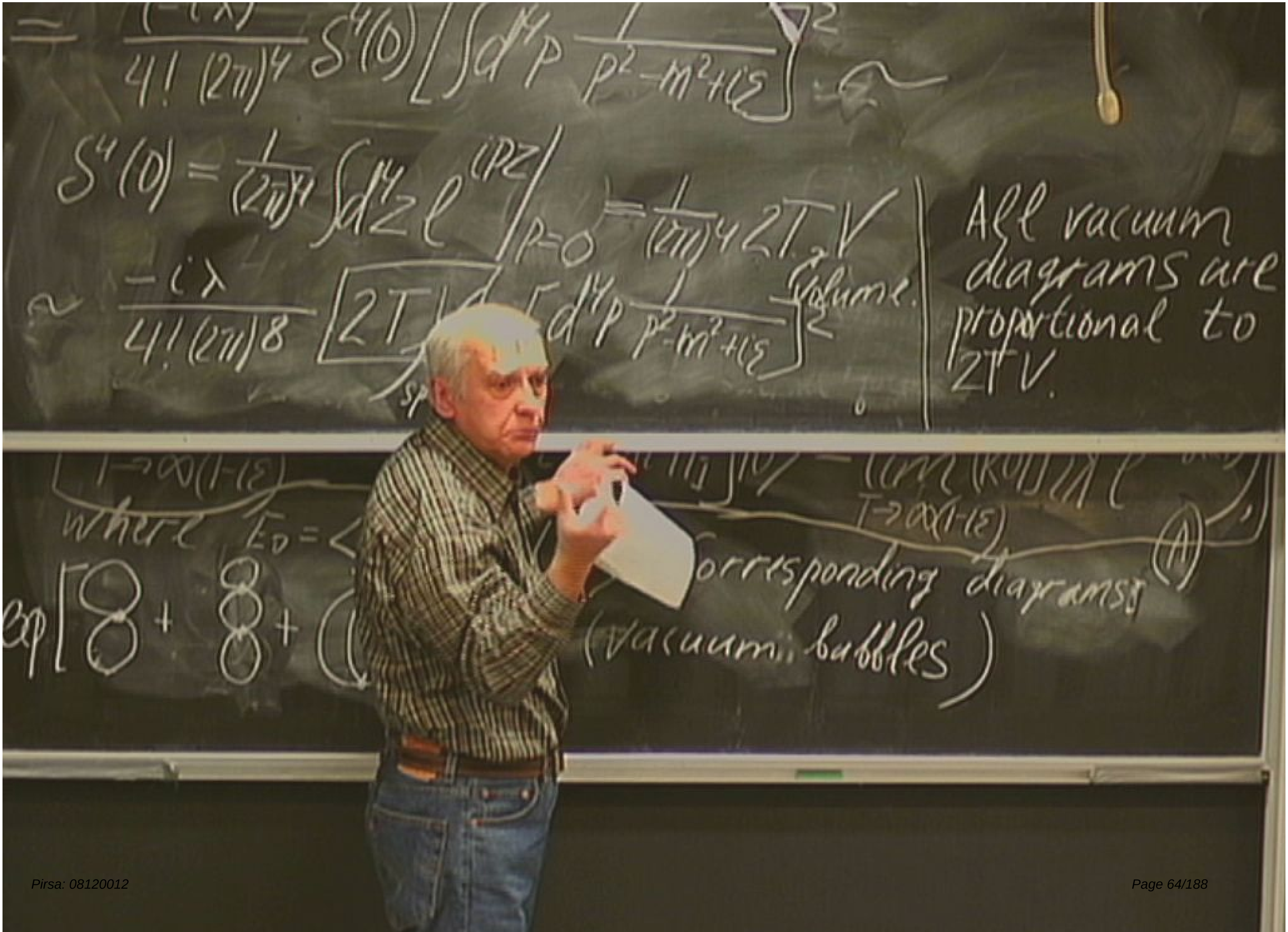
From (A), we get

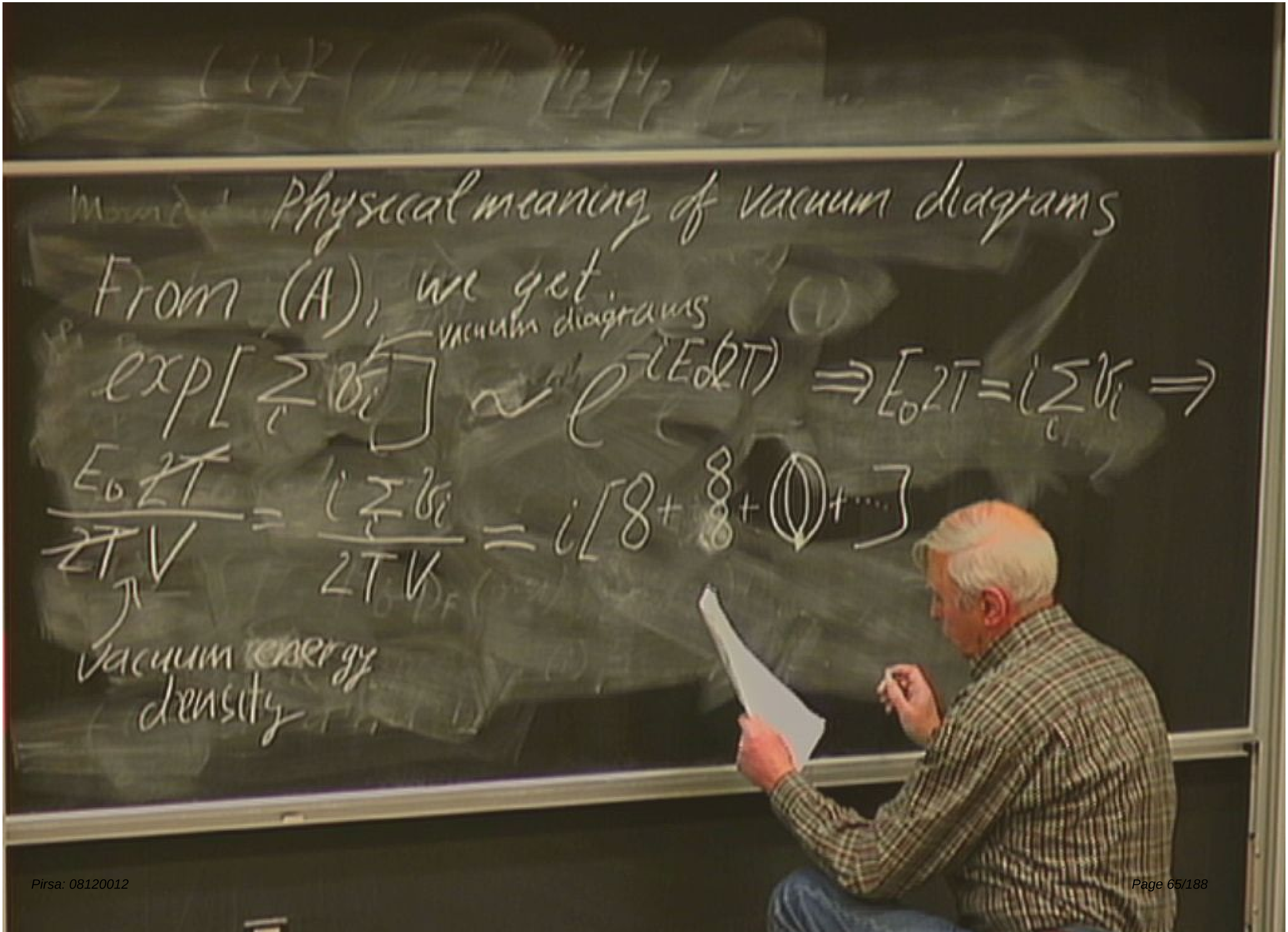
$$\exp\Big[\sum_i \underbrace{v_i}_{\text{vacuum diagrams}}\Big] \sim e^{-iE_0(2T)} \Rightarrow E_0 2T = i\sum_i v_i \Rightarrow$$

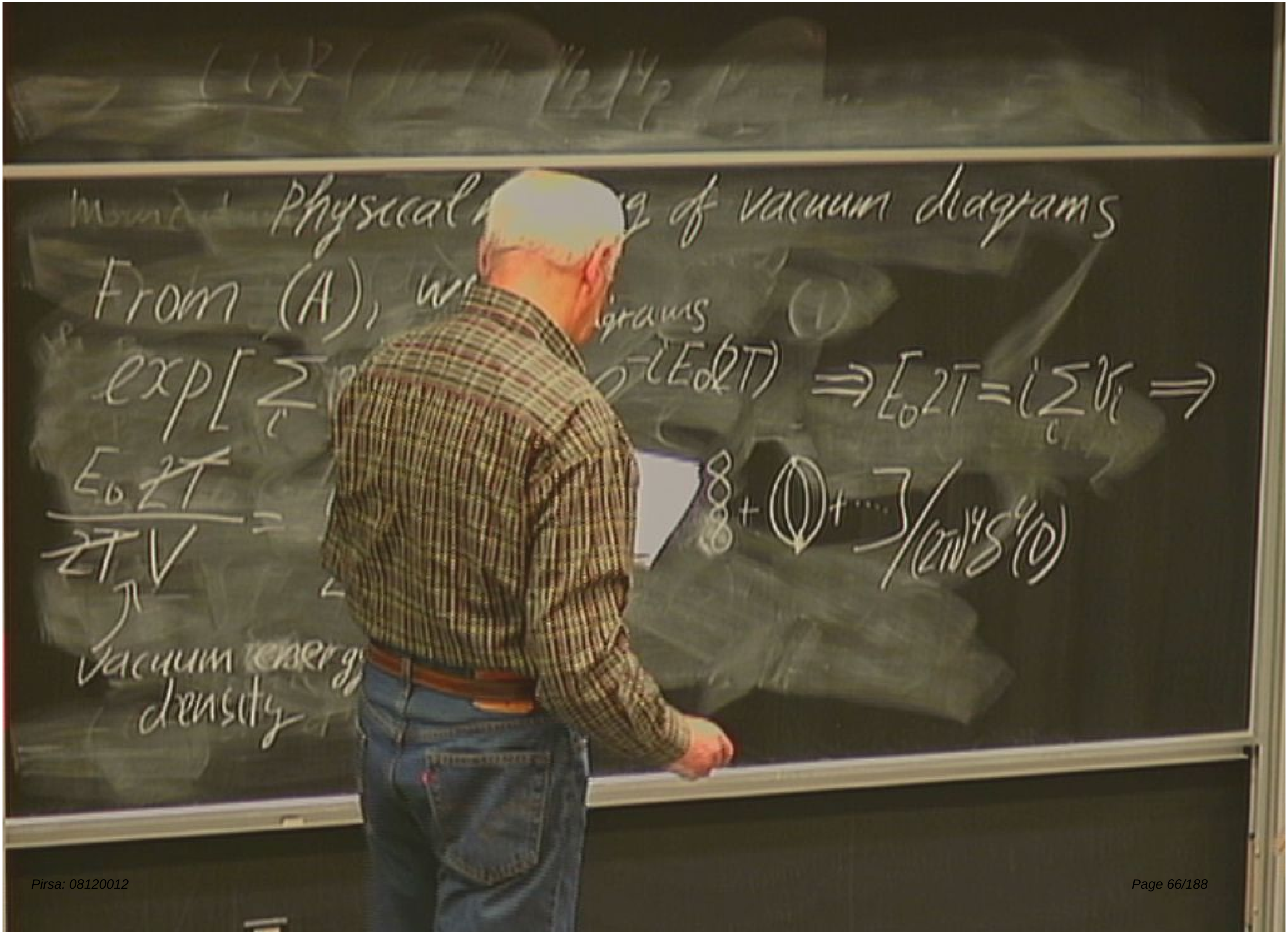
$$\frac{E_0 \cancel{2T}}{\cancel{2T}\,V} = \frac{i\sum_i v_i}{2TV} =$$

vacuum energy density


All vacuum
diagrams
proportional
2TV
volume.
corresponding diagrams
vacuum bubbles


All vacuum diagrams are proportional to 2TV.
volume
where
corresponding diagrams
(vacuum bubbles)


Physical meaning of vacuum diagrams
From (A), we get
vacuum diagrams
$\exp\left[\sum_i V_i\right] \sim e^{-iE_0(2T)} \Rightarrow E_0 2T = i\sum_i V_i \Rightarrow$
$\frac{E_0 \cancel{2T}}{\cancel{2T} V} = \frac{i\sum_i V_i}{2TV} = i[8 + 8 + \ldots]$
vacuum energy density



Physical
of vacuum diagrams
From (A),
vacuum
density

Physical meaning of vacuum diagrams

From (A), we get

$$\exp\Big[\sum_i \underbrace{\boxed{V_i}}_{\text{vacuum diagrams}}\Big] \sim e^{-iE_0(2T)} \Rightarrow E_0 2T = i\sum_i V_i \Rightarrow$$

$$\frac{E_0 \cancel{2T}}{\cancel{2T}\, V} = \frac{i\sum_i V_i}{2TV} = i\left[\ + \ + \ + \cdots\right]/(2\pi)^4\delta^4(0)$$

vacuum energy density

Physical meaning of vacuum diagrams

From (A), we get

$$\exp\Big[\sum_i \overbrace{V_i}^{\text{vacuum diagrams}}\Big] \sim e^{-iE_0(2T)} \Rightarrow E_0 2T = i\sum_i V_i \Rightarrow$$

$$\frac{E_0 \cancel{2T}}{\cancel{2T}\, V} = \frac{i\sum_i V_i}{2TV} = i\Big[$$

$$\Big]\Big/(2\pi)^4\delta^4(0)$$

$\frac{E_0}{V}$: vacuum energy density

$$\frac{}{4!\,(2\pi)^4}\,\delta^4(0)\left[\int d^4p\,\frac{1}{p^2-m^2+i\varepsilon}\right]$$

$$\delta^4(0)=\frac{1}{(2\pi)^4}\int d^4z\,e^{ipz}\Big|_{p=0}=\frac{1}{(2\pi)^4}2T\,V$$

$$\sim\frac{-i\lambda}{4!\,(2\pi)^8}\boxed{2TV_{\uparrow}}\cdot\left[\int d^4p\,\frac{1}{p^2-m^2+i\varepsilon}\right]^2$$

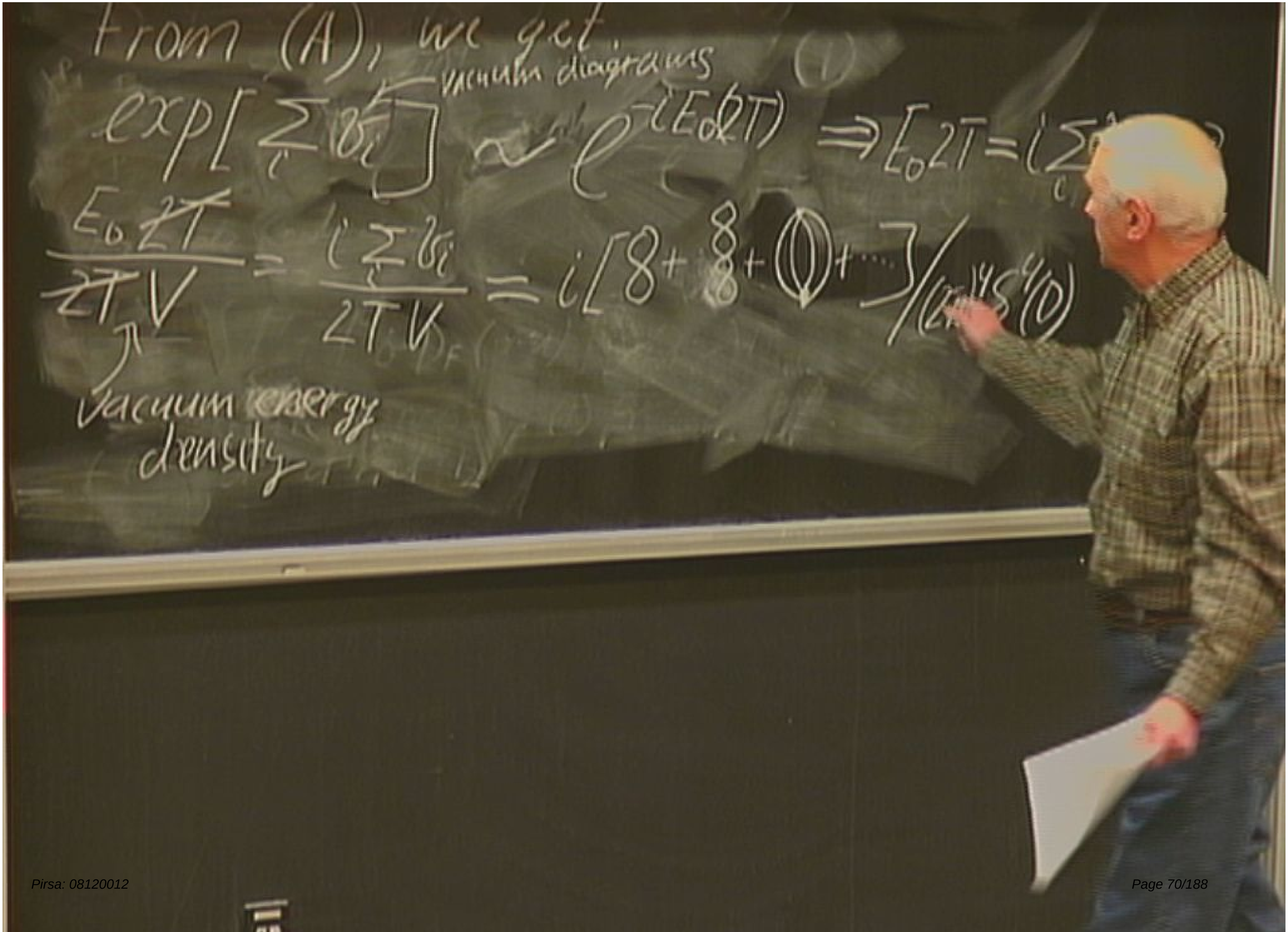
spatial volume.

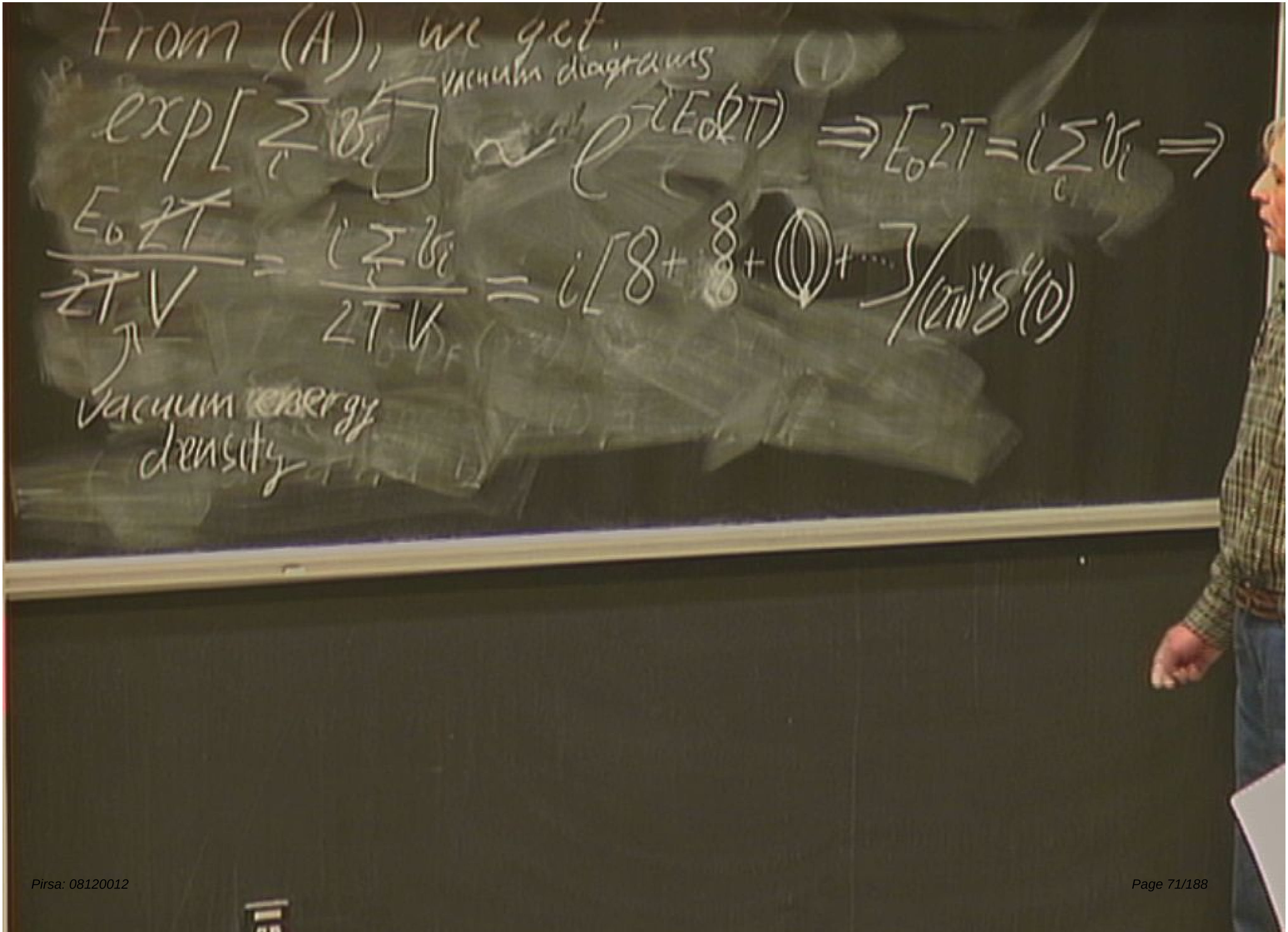
All vacuum diagrams are proportional to $2TV$.

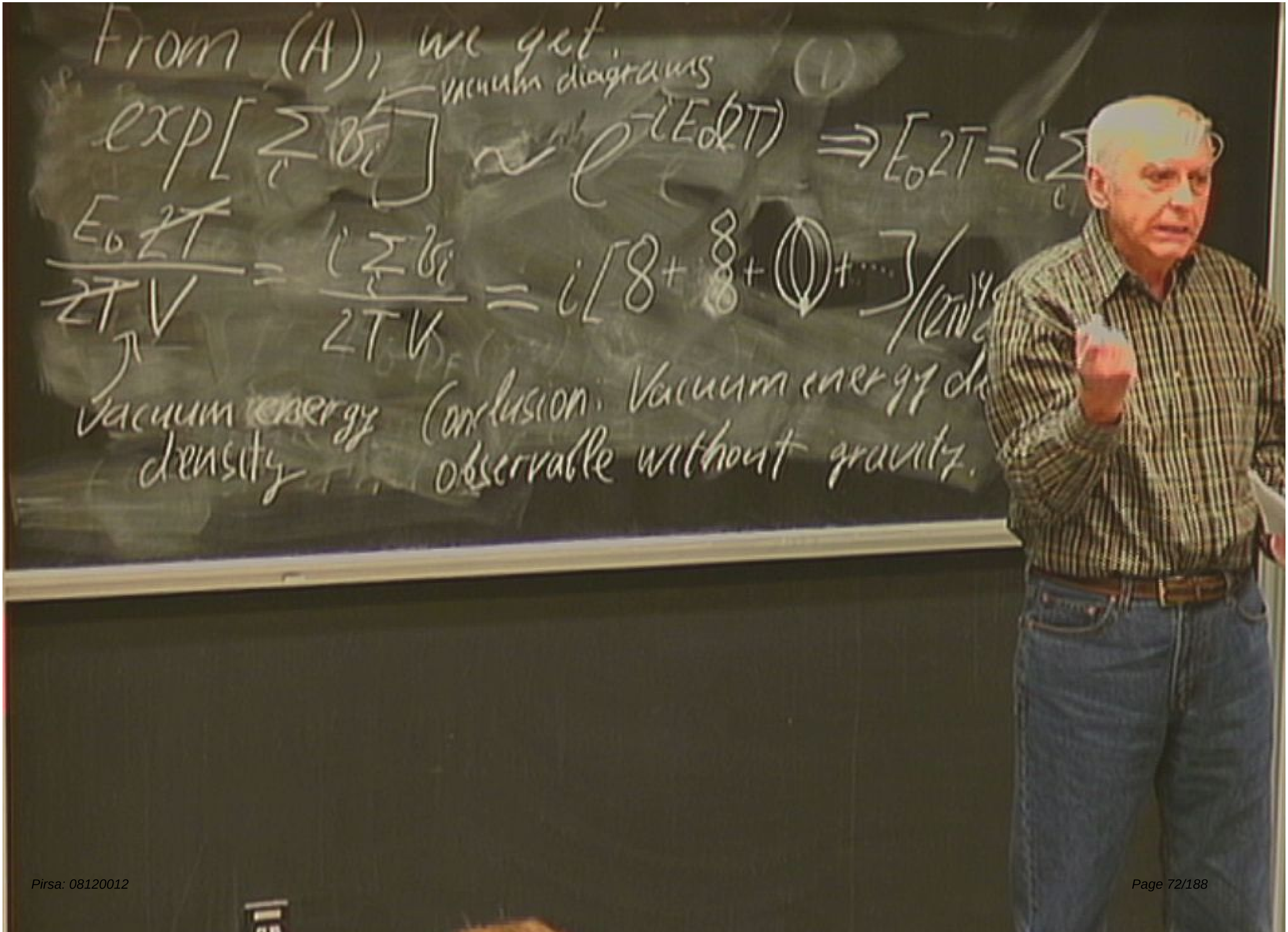
$$\lim_{T\to\infty(1-i\varepsilon)}\ldots\left(\ldots\int_{-T}dt\,H_I\right)|0\rangle=\lim_{T\to\infty(1-i\varepsilon)}\left(\langle 0|\Omega\rangle\right)^2 e^{iE_0(2T)}$$

where $E_0=\langle\Omega|H|\Omega\rangle$. Corresponding diagrams:

exp [8 + 8 + ⦶ + ...] (vacuum bubbles)


From (A), we get.
vacuum diagrams
vacuum energy
density


From (A), we get.
vacuum diagrams
$\exp[\sum_i V_i] \sim e^{-iE_0(2T)} \Rightarrow E_0 2T = i\sum_i V_i \Rightarrow$
$\frac{E_0 \cancel{2T}}{\cancel{2T} V} = \frac{i\sum_i V_i}{2TV} = i[8 + 8 + \cdots]/(2\pi)^4\delta^4(0)$
Vacuum energy density


From (A), we get.
vacuum diagrams
Vacuum energy density
Conclusion: Vacuum energy d
observable without gravity.

From (A), we get:

$$\exp\Big[\underbrace{\sum_i V_i}_{\text{vacuum diagrams}}\Big] \sim e^{-iE_0(2T)} \Rightarrow E_0 2T = i\sum_i V_i \Rightarrow$$

$$\frac{E_0 \cancel{2T}}{\cancel{2T}\, V} = \frac{i\sum_i V_i}{2TV} = i\left[\ \ + \ \ + \ \ + \cdots\right]\Big/(2\pi)^4\delta^4(0)$$

Vacuum energy density

Conclusion: Vacuum energy density is not observable without gravity.

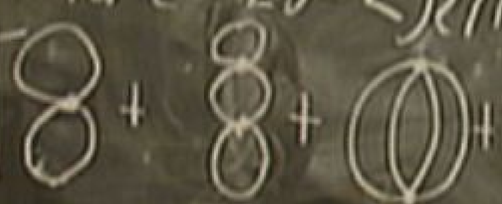
$$\frac{\cdots}{4!(2\pi)^8}\left[2T\underbrace{V}_{\text{spatial}}\right]\cdot\left[\int d^4p\,\frac{\cdots}{p^2-m^2+i\epsilon}\right]^2$$

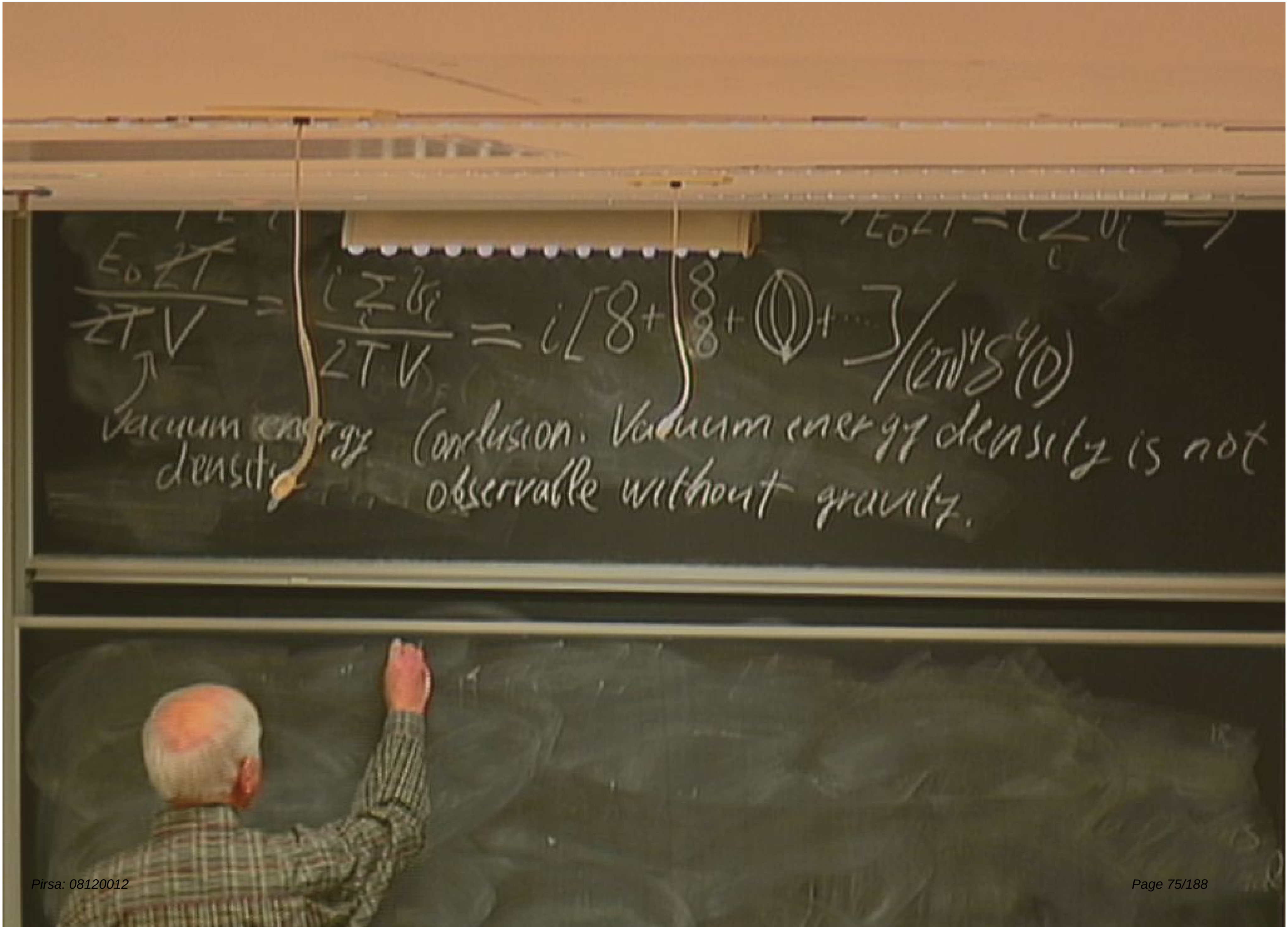
proportional to $2TV$.

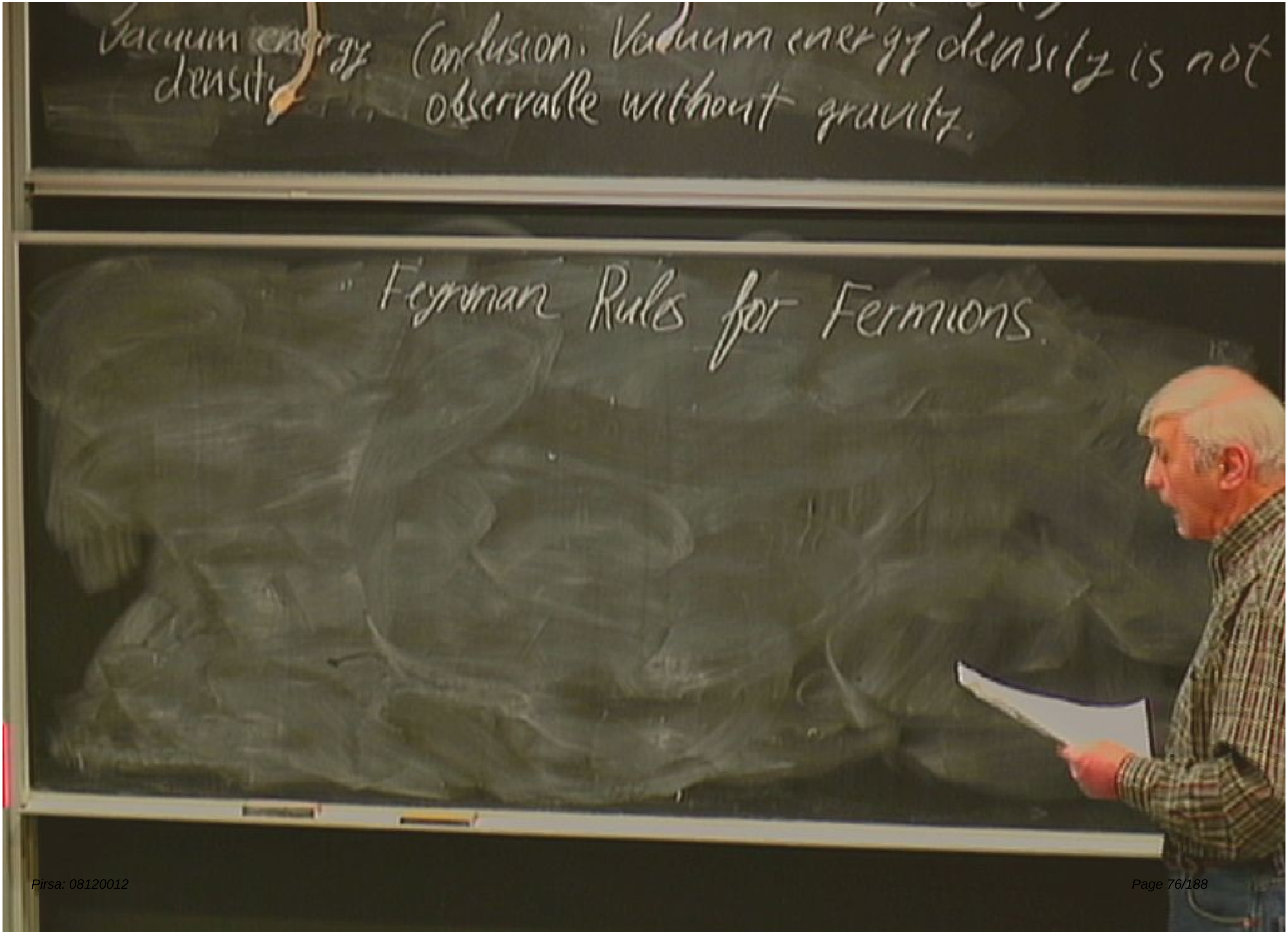
$$\langle\Omega|T_I\{\varphi(x_1)\varphi(x_2)\cdots\varphi(x_n)\}|\Omega\rangle=\lim_{T\to\infty(1-i\epsilon)}\frac{\langle 0|T\{\varphi_I(x_1)\cdots\varphi_I(x_n)\times\exp[-i\int_{-T}^{T}dt\,H_I(t)]\}|0\rangle}{\langle 0|T\{\exp[-i\int_{-T}^{T}dt\,H_I]\}|0\rangle}$$

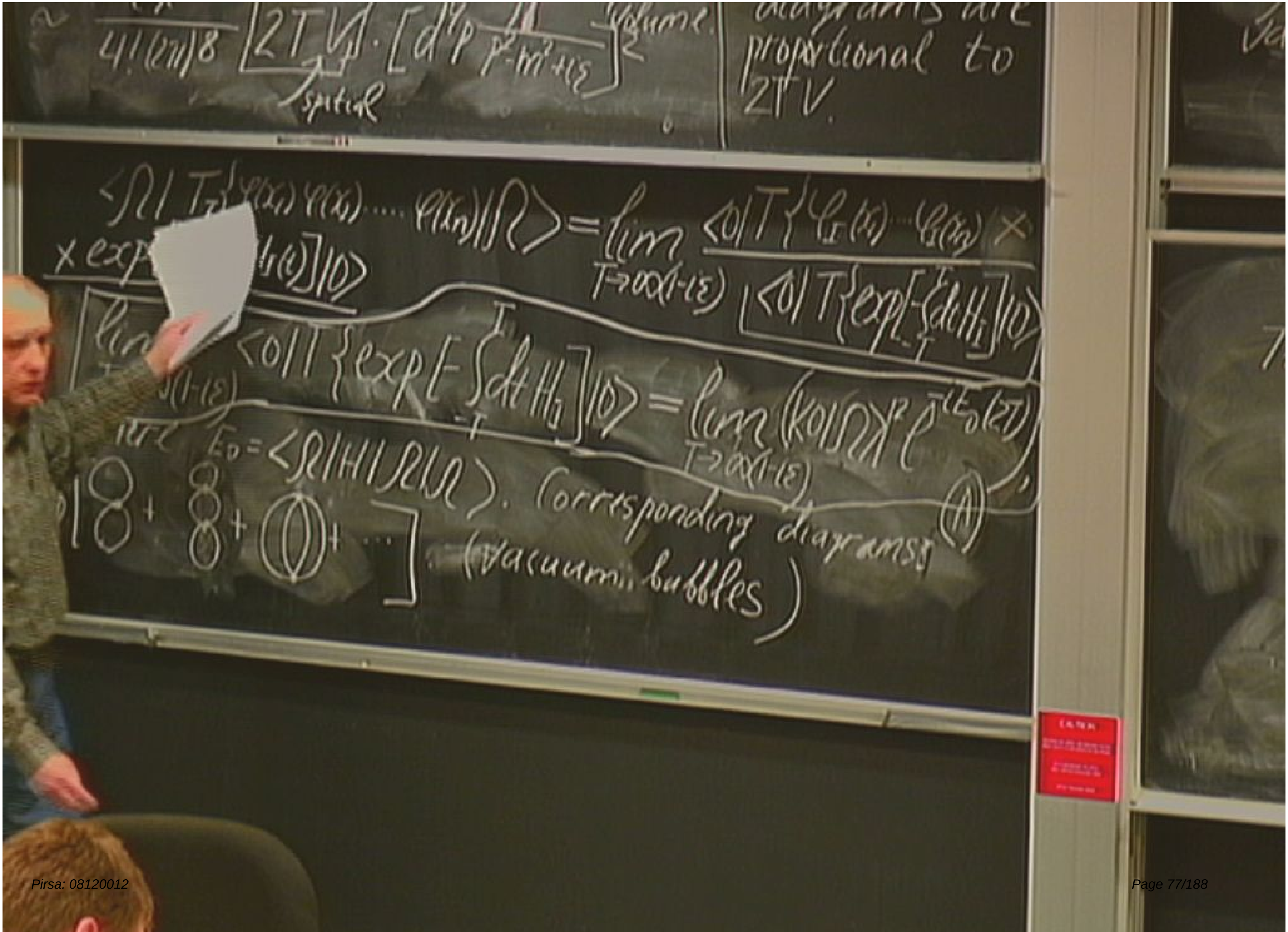
$$\boxed{\lim_{T\to\infty(1-i\epsilon)}\langle 0|T\{\exp[-i\int_{-T}^{T}dt\,H_I]\}|0\rangle=\lim_{T\to\infty(1-i\epsilon)}|\langle 0|\Omega\rangle|^2 e^{-iE_0(2T)}}\quad (A)$$

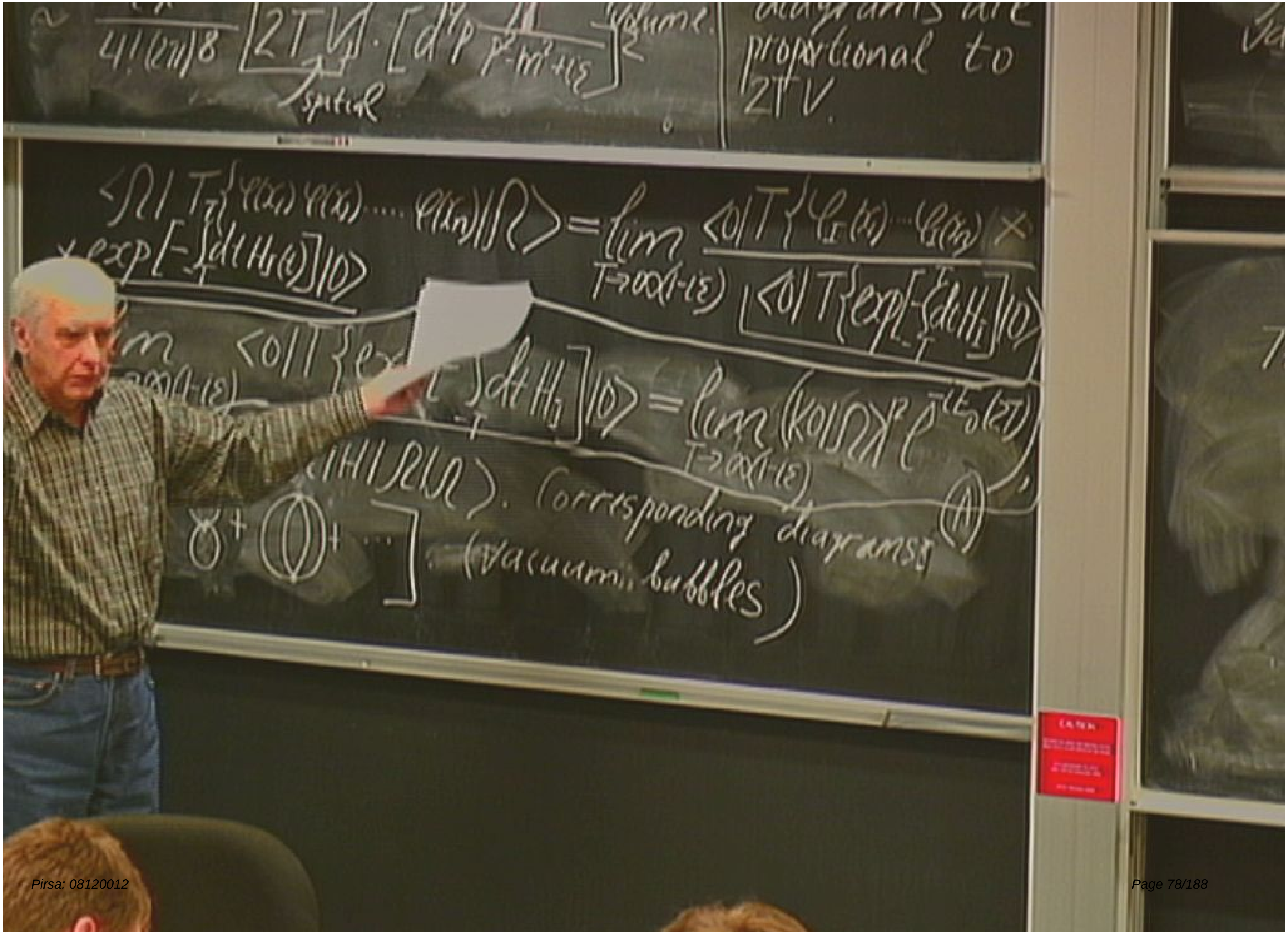
where $E_0=\langle\Omega|H|\Omega\rangle$. Corresponding diagrams:

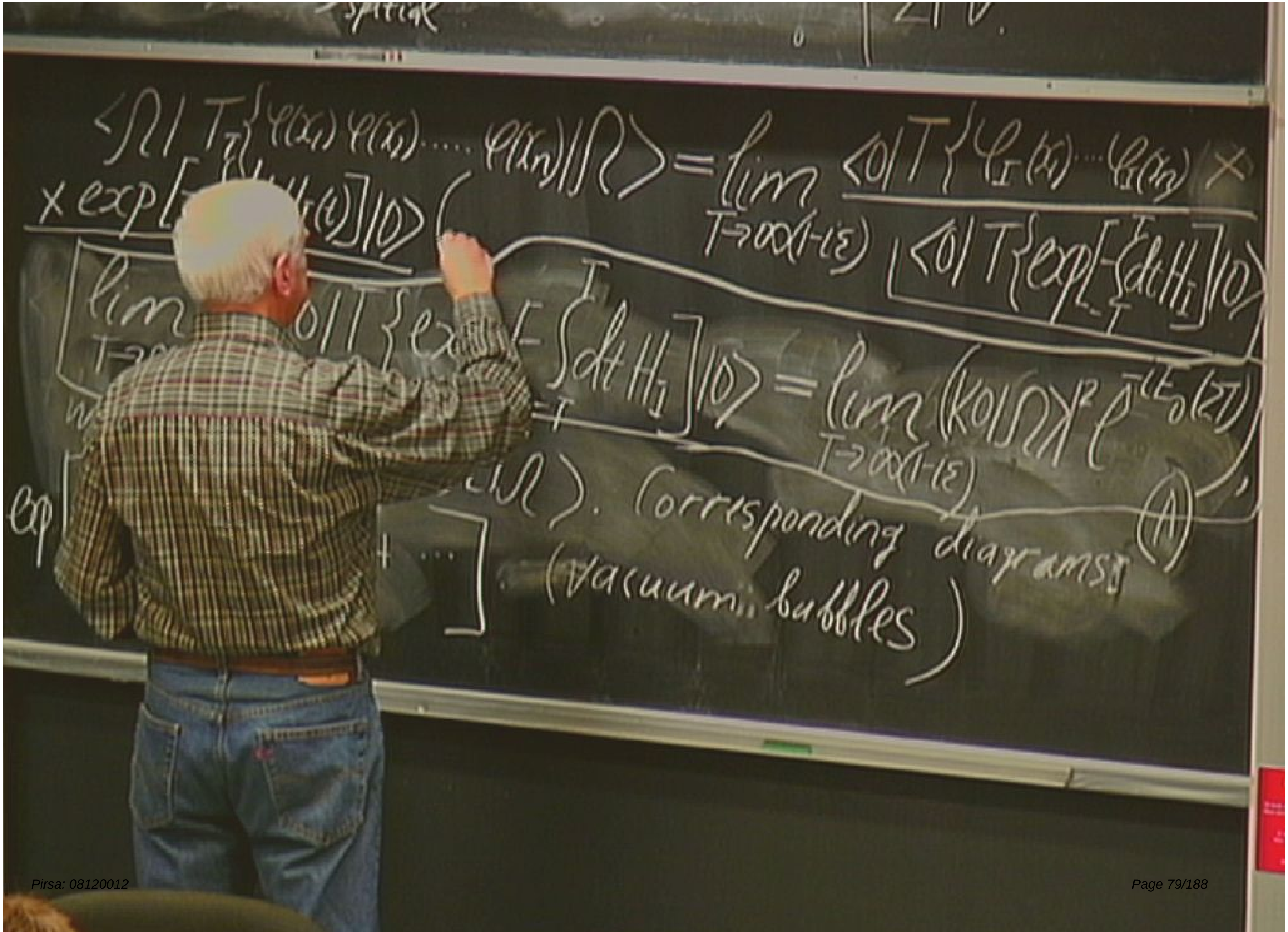
exp [ + ...] (vacuum bubbles)


Vacuum energy density
Conclusion: Vacuum energy density is not observable without gravity.


Vacuum energy density
Conclusion. Vacuum energy density is not observable without gravity.
"Feynman Rules for Fermions."


4!(2π)8
spatial
volume
diagrams are
proportional to
2TV.
Corresponding diagrams
(vacuum bubbles)

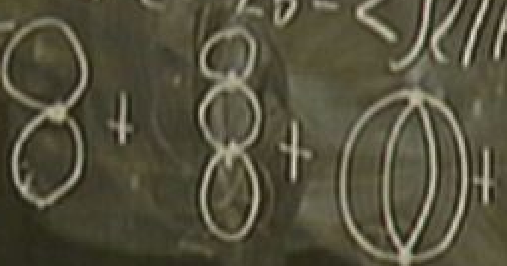

spatial
volume
proportional to
2TV.
Corresponding diagrams
(vacuum bubbles)

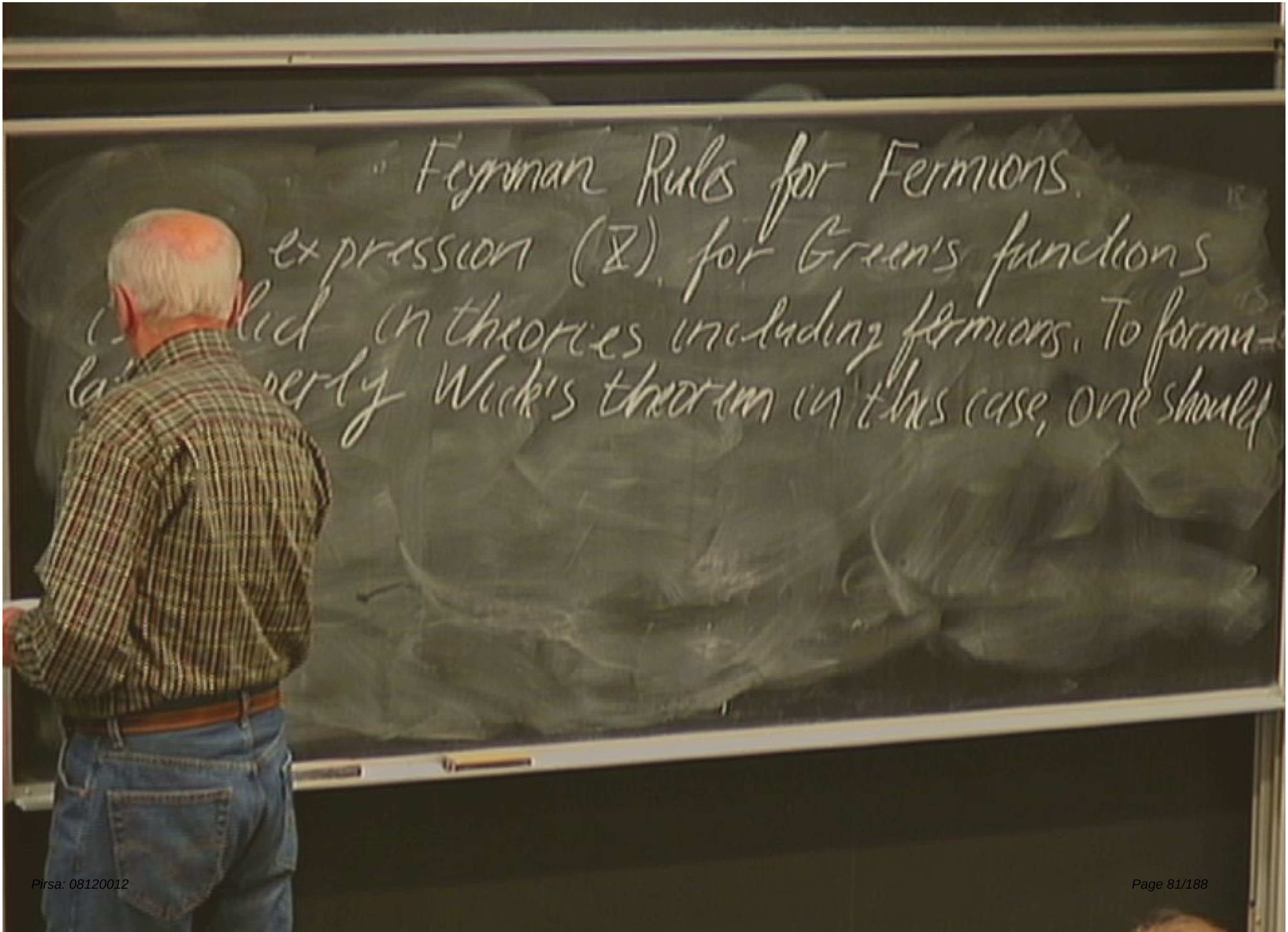

Corresponding diagrams
(vacuum bubbles)

$$\langle\Omega| T\{\varphi(x_1)\varphi(x_2)\cdots\varphi(x_n)\}|\Omega\rangle = \lim_{T\to\infty(1-i\epsilon)} \frac{\langle 0|T\{\varphi_I(x_1)\cdots\varphi_I(x_n) \times \exp[-i\int_{-T}^{T} dt\, H_I(t)]\}|0\rangle}{\langle 0|T\{\exp[-i\int_{-T}^{T} dt\, H_I]\}|0\rangle} \quad (X)$$

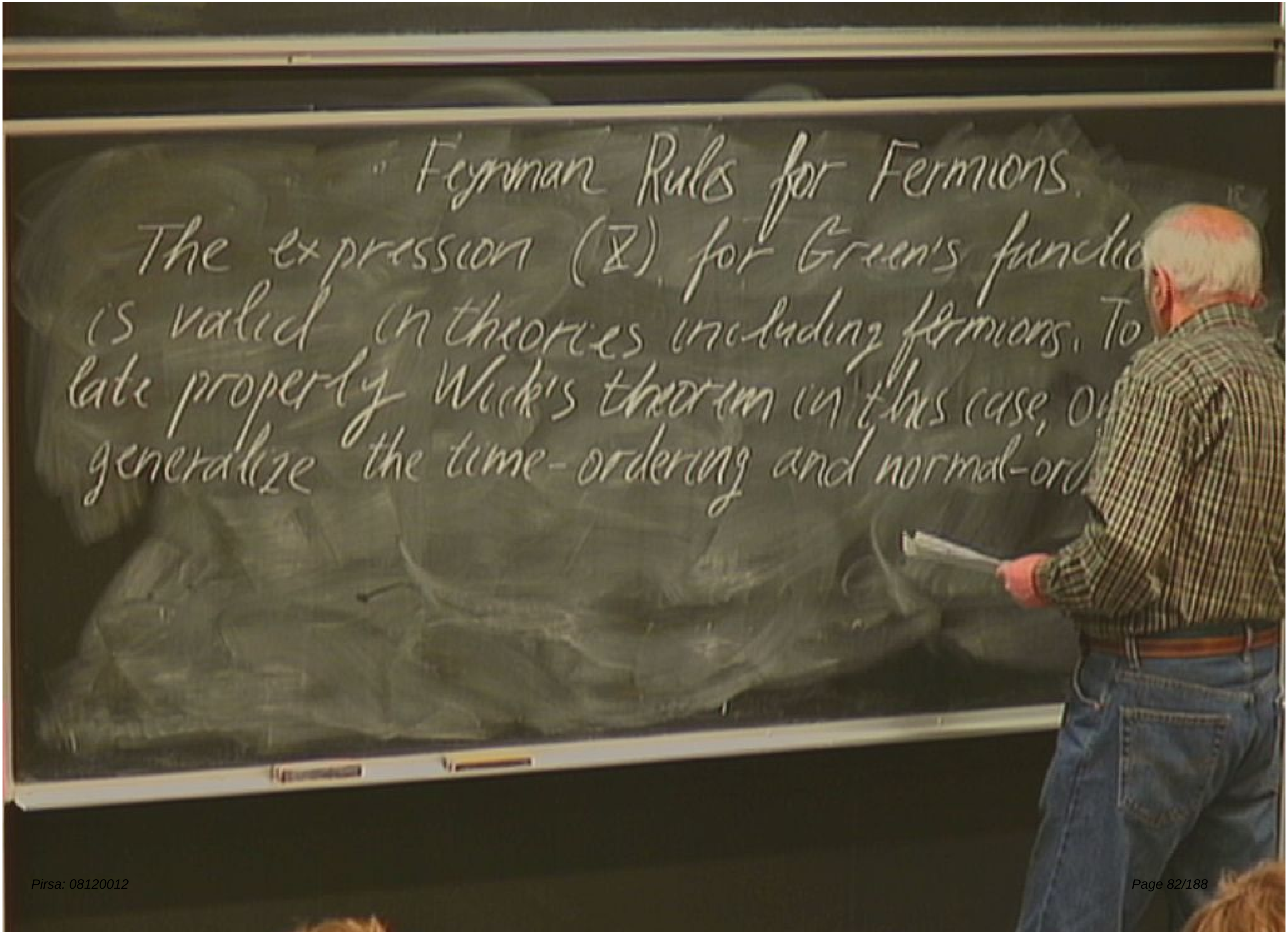
$$\lim_{T\to\infty(1-i\epsilon)} \langle 0|T\{\exp[-i\int_{-T}^{T} dt\, H_I]\}|0\rangle = \lim_{T\to\infty(1-i\epsilon)} \left(|\langle 0|\Omega\rangle|^2 e^{-iE_0(2T)}\right) \quad (A)$$

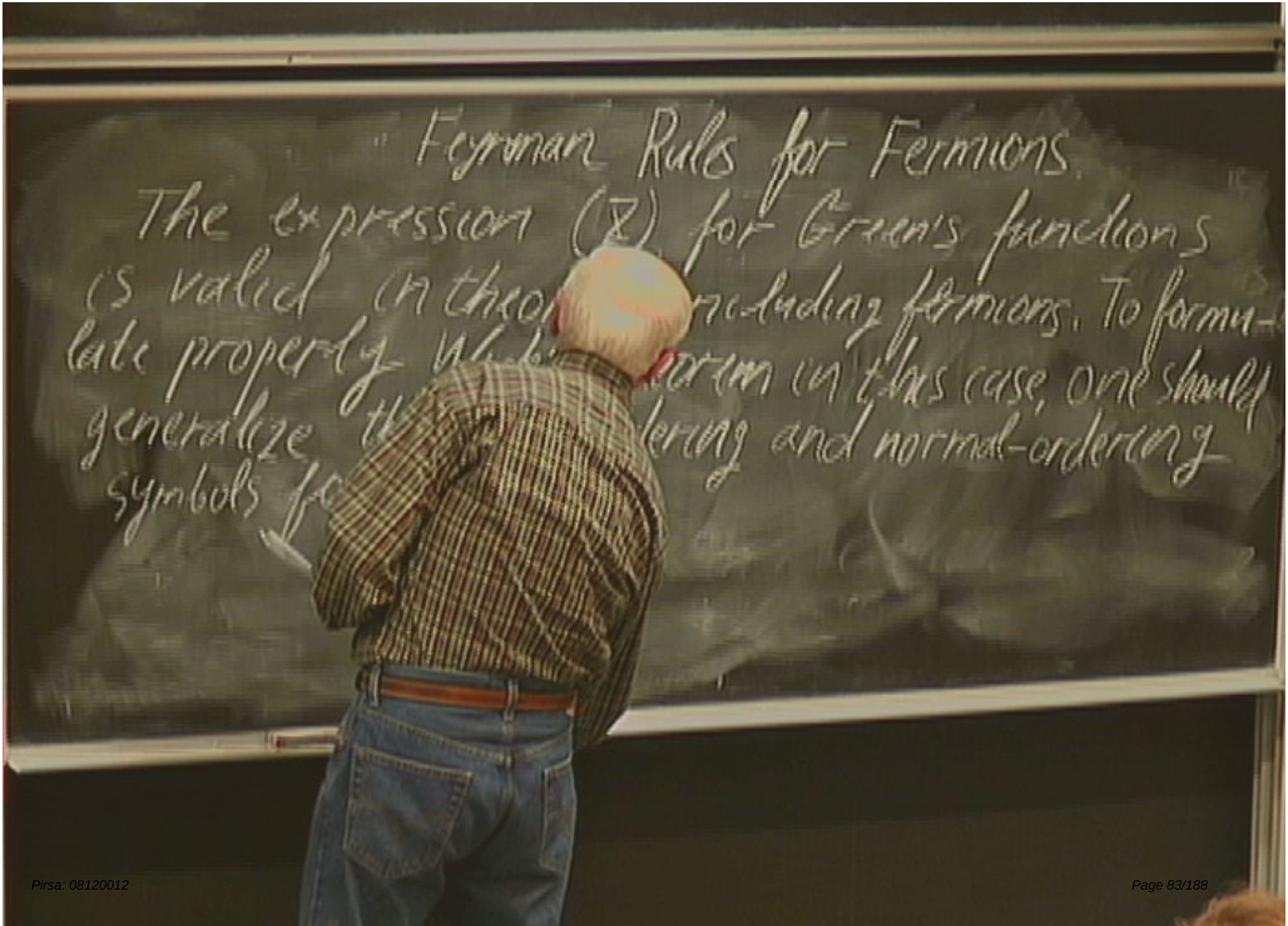
where $E_0 = \langle\Omega|H|\Omega\rangle$. Corresponding diagrams:

exp[ + ...] (vacuum bubbles)



"Feynman Rules for Fermions.
expression (X) for Green's functions
lied in theories including fermions. To formu-
perly Wick's theorem in this case, one should


" Feynman Rules for Fermions.
The expression (X) for Green's functio
is valid in theories including fermions. To
late properly Wick's theorem in this case, o
generalize the time-ordering and normal-ord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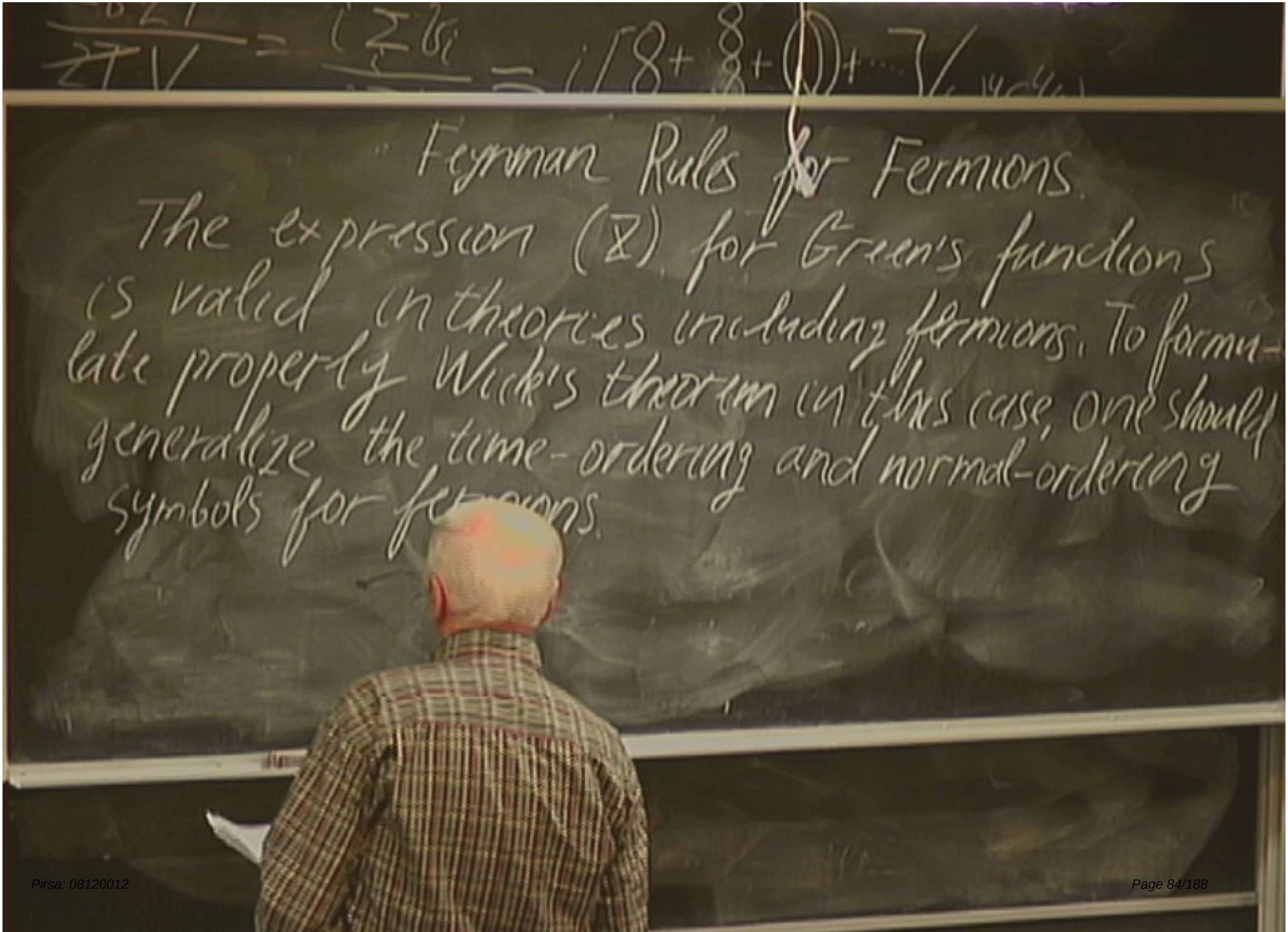
Feynman Rules for Fermions.
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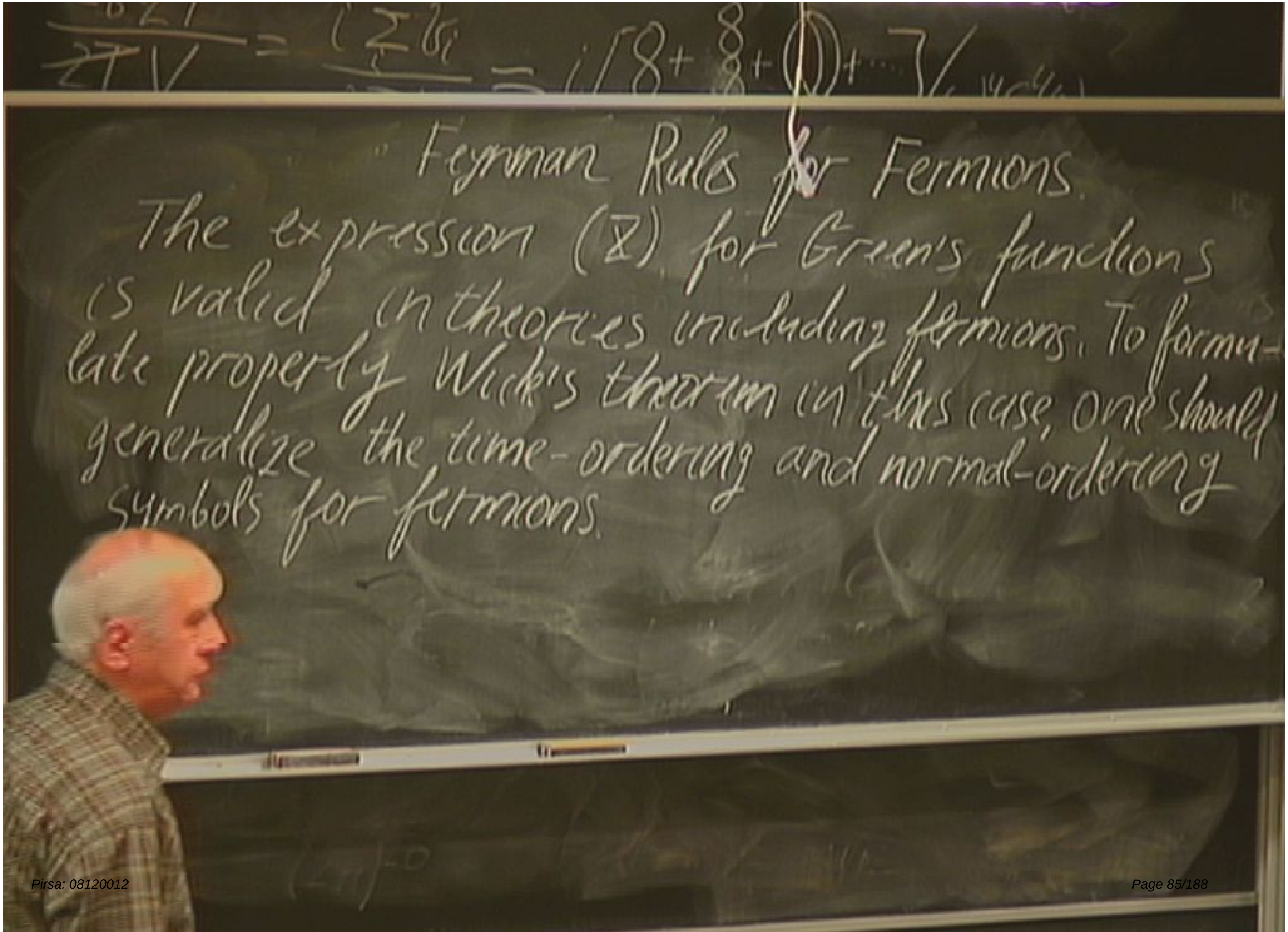
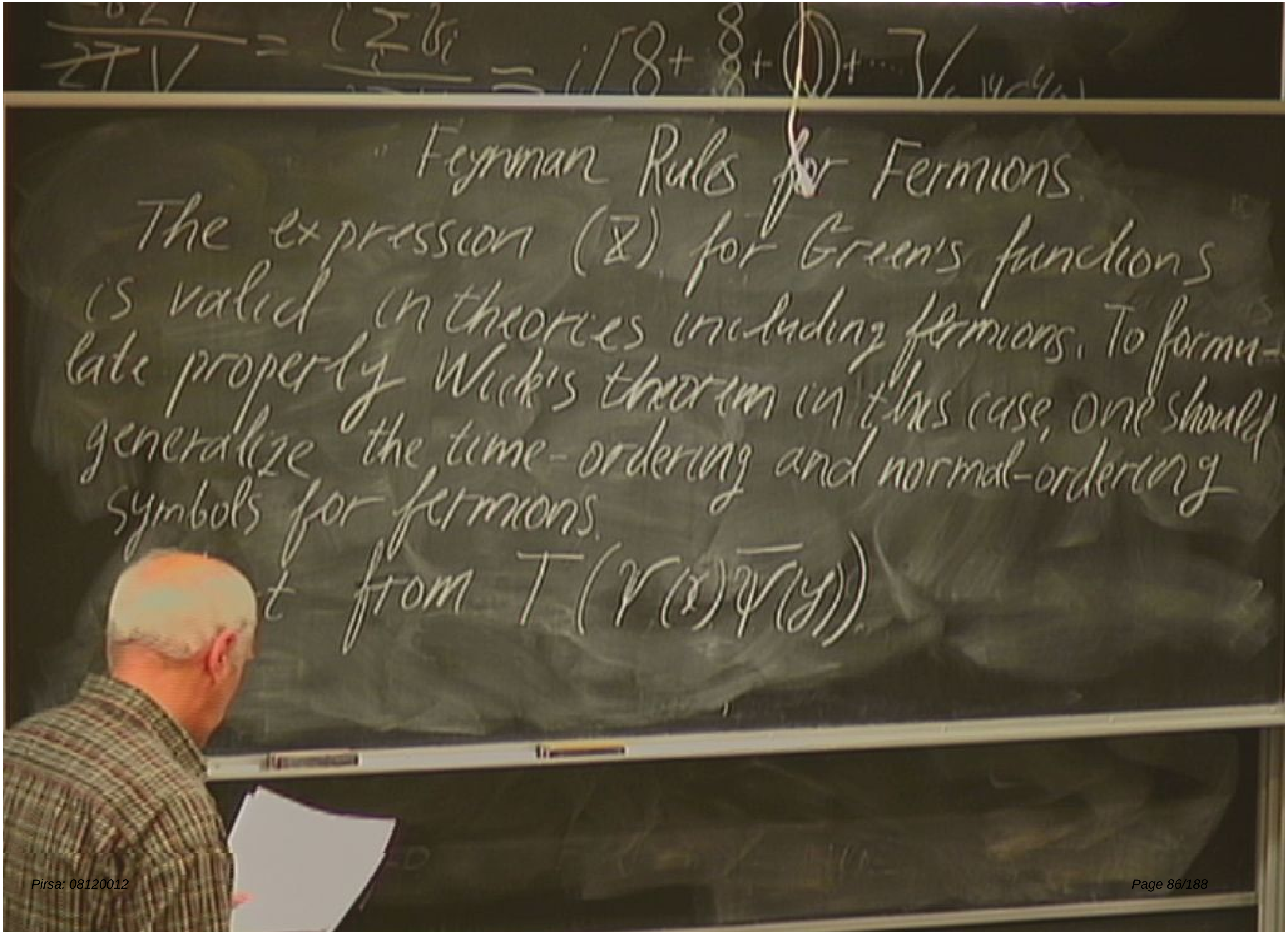
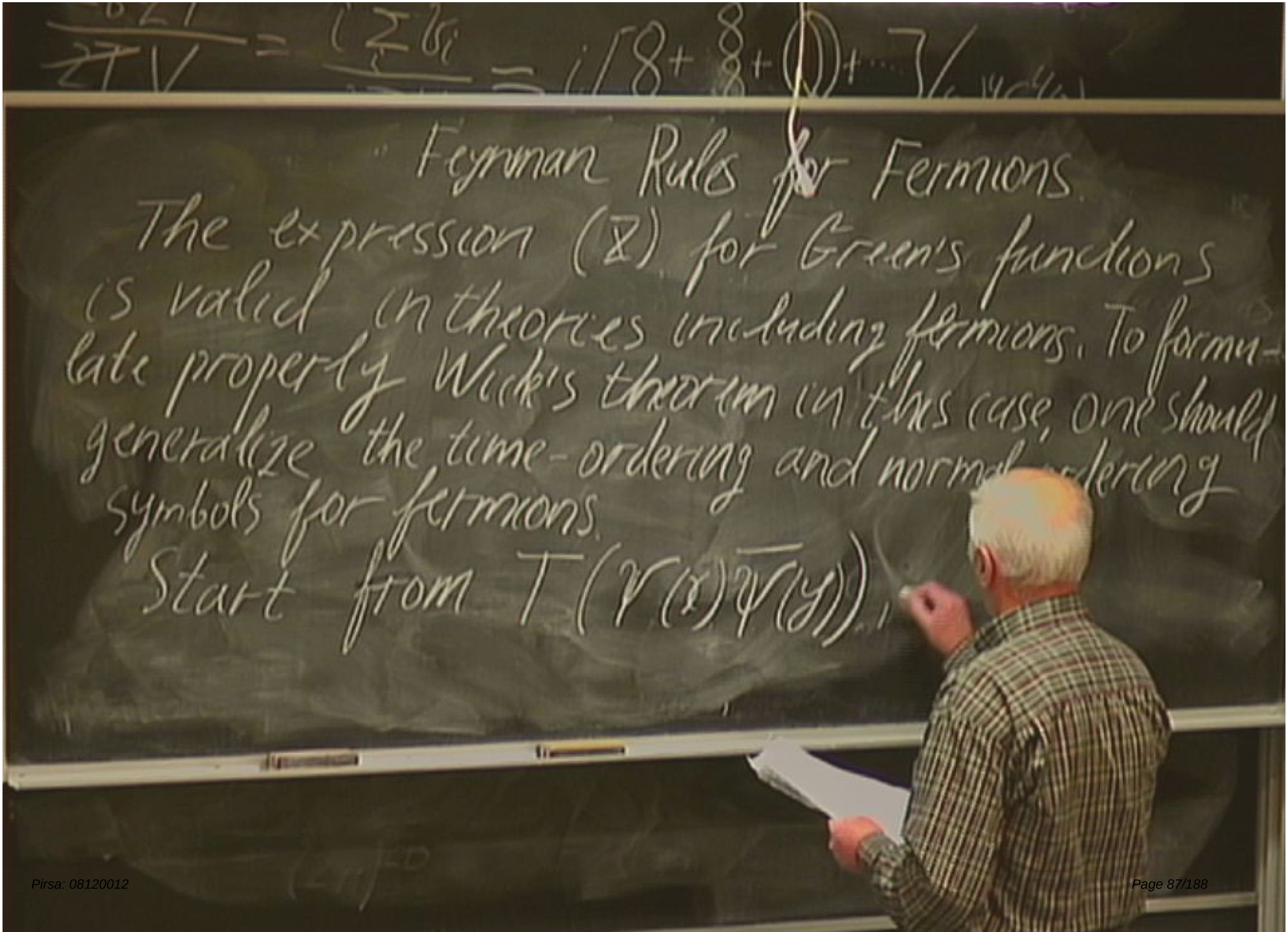

"Feynman Rules for Fermions.
The expression (X) for Green's functions is valid in theories including fermions. To formulate properly Wick's theorem in this case, one should generalize the time-ordering and normal-ordering symbols for fermions.



Feynman Rules for Fermions.
The expression (VIII) for Green's functions is valid in theories including fermions. To formulate properly Wick's theorem in this case, one should generalize the time-ordering and normal-ordering symbols for fermions.
t from $T(\psi(x)\bar{\psi}(y))$


Feynman Rules for Fermions.
The expression (X) for Green's functions
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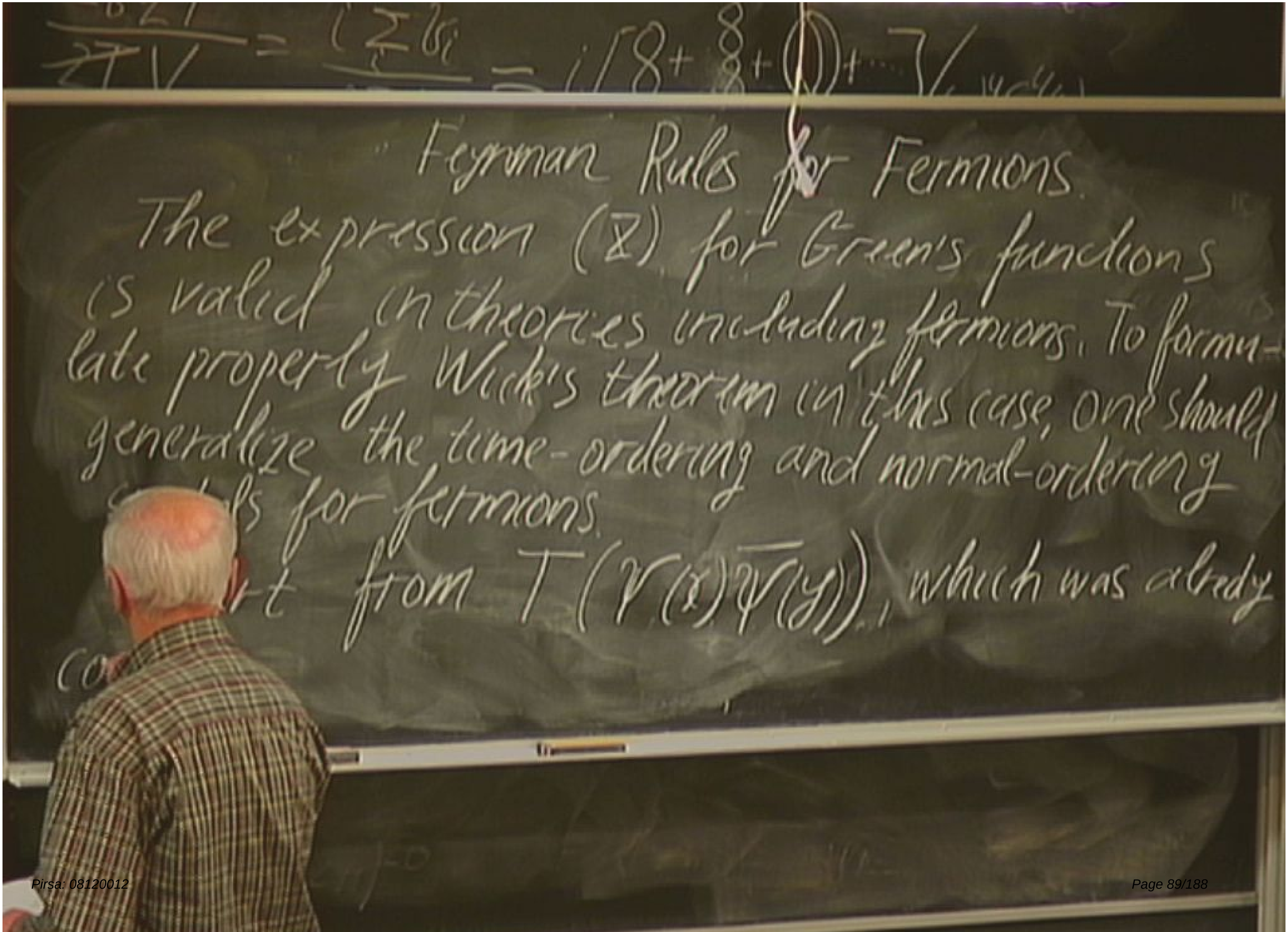
late properly Wick's theorem in this case, one should
generalize the time-ordering and normal ordering
symbols for fermions.
Start from $T(\psi(x)\bar{\psi}(y))$.

"Feynman Rules for Fermions"

The expression (X) for Green's functions is valid in theories including fermions. To formulate properly Wick's theorem in this case, one should generalize the time-ordering and normal-ordering symbols for fermions.

Start from $T(\psi(x)\bar{\psi}(y))$, which was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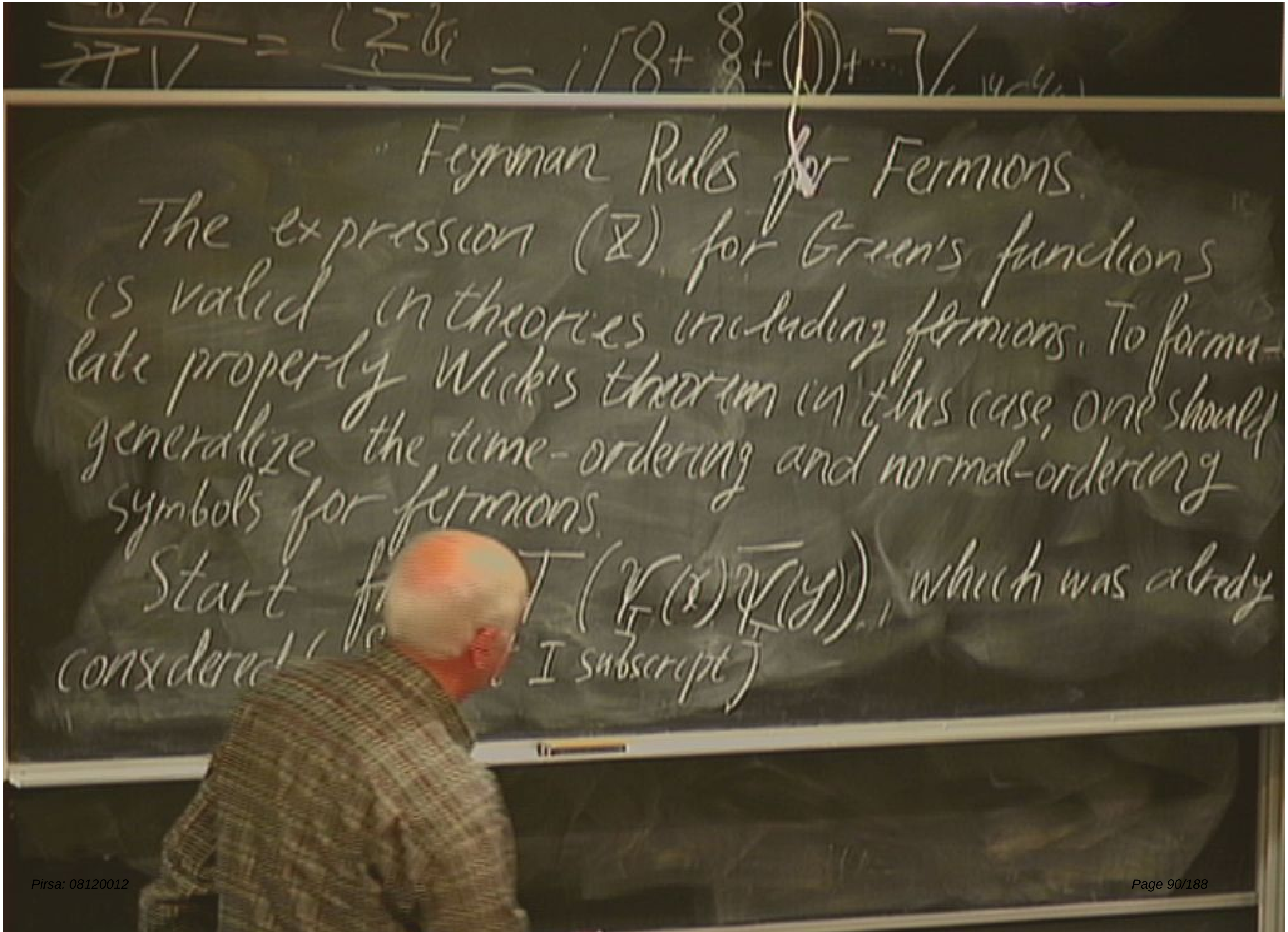
symbols for fermions.
from T(ψ(x)ψ̄(y)), which was already


Feynman Rules for Fermions.
The expression (X) for Green's functions
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late properly Wick's theorem in this case, one should
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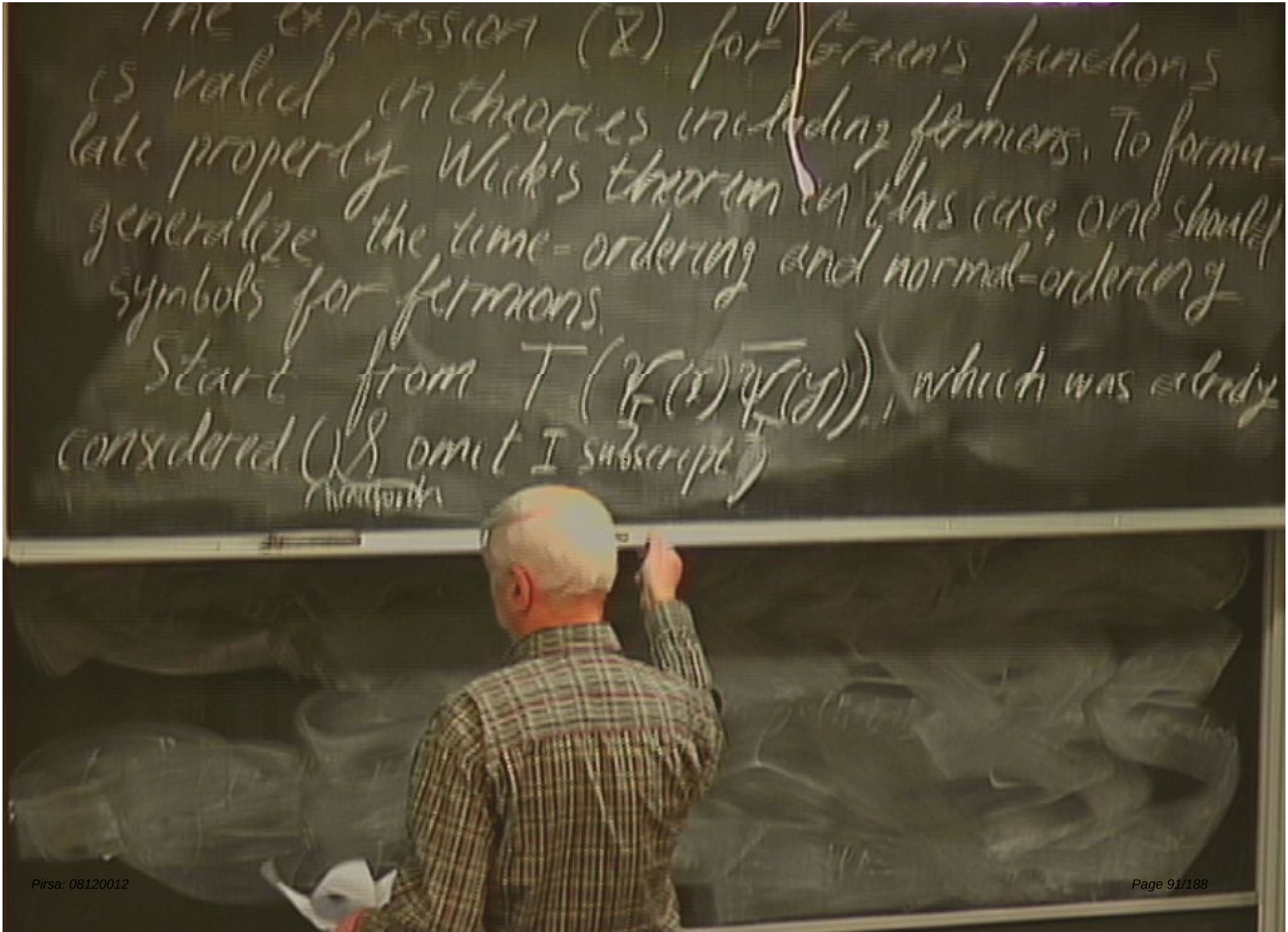
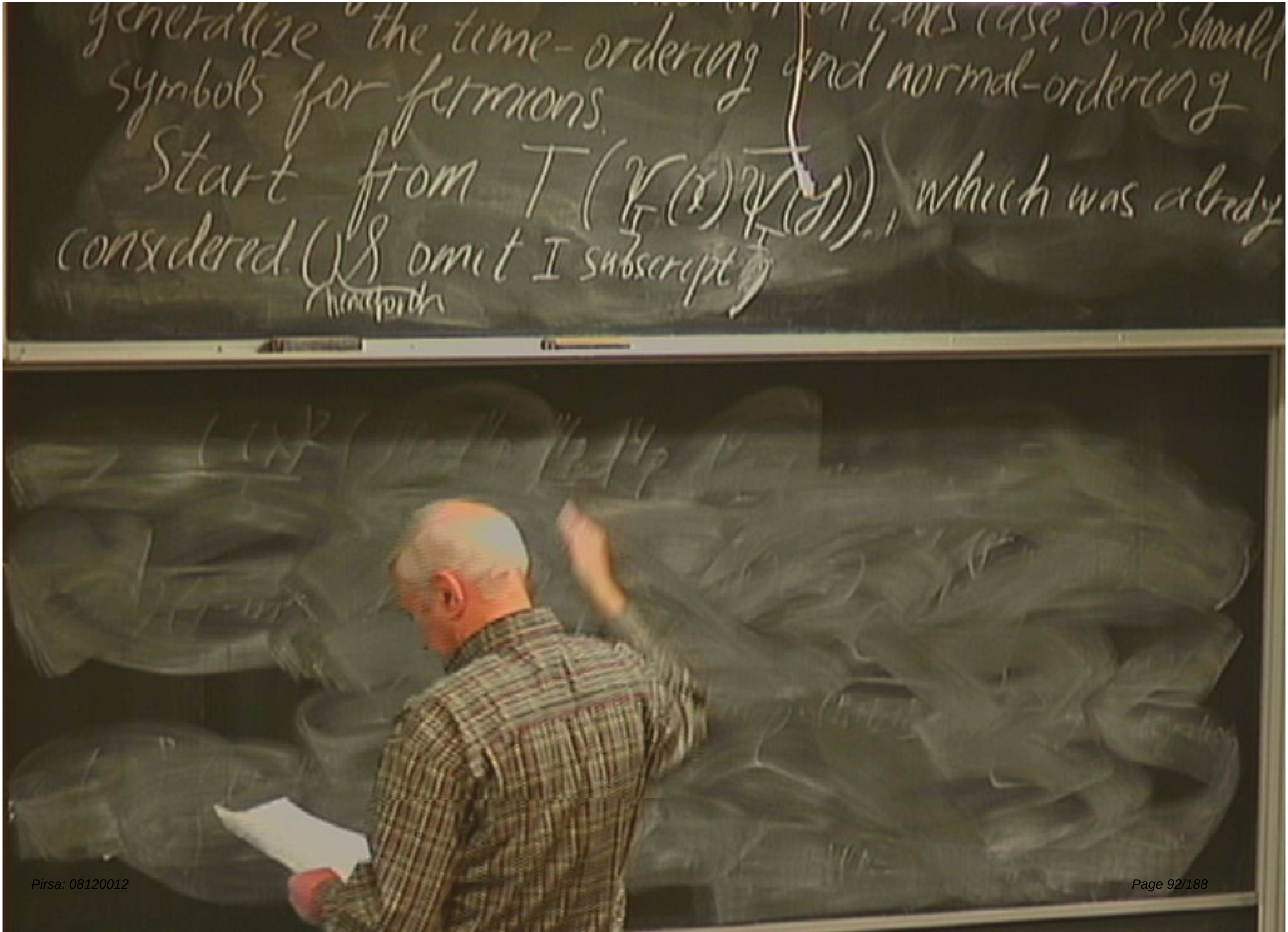
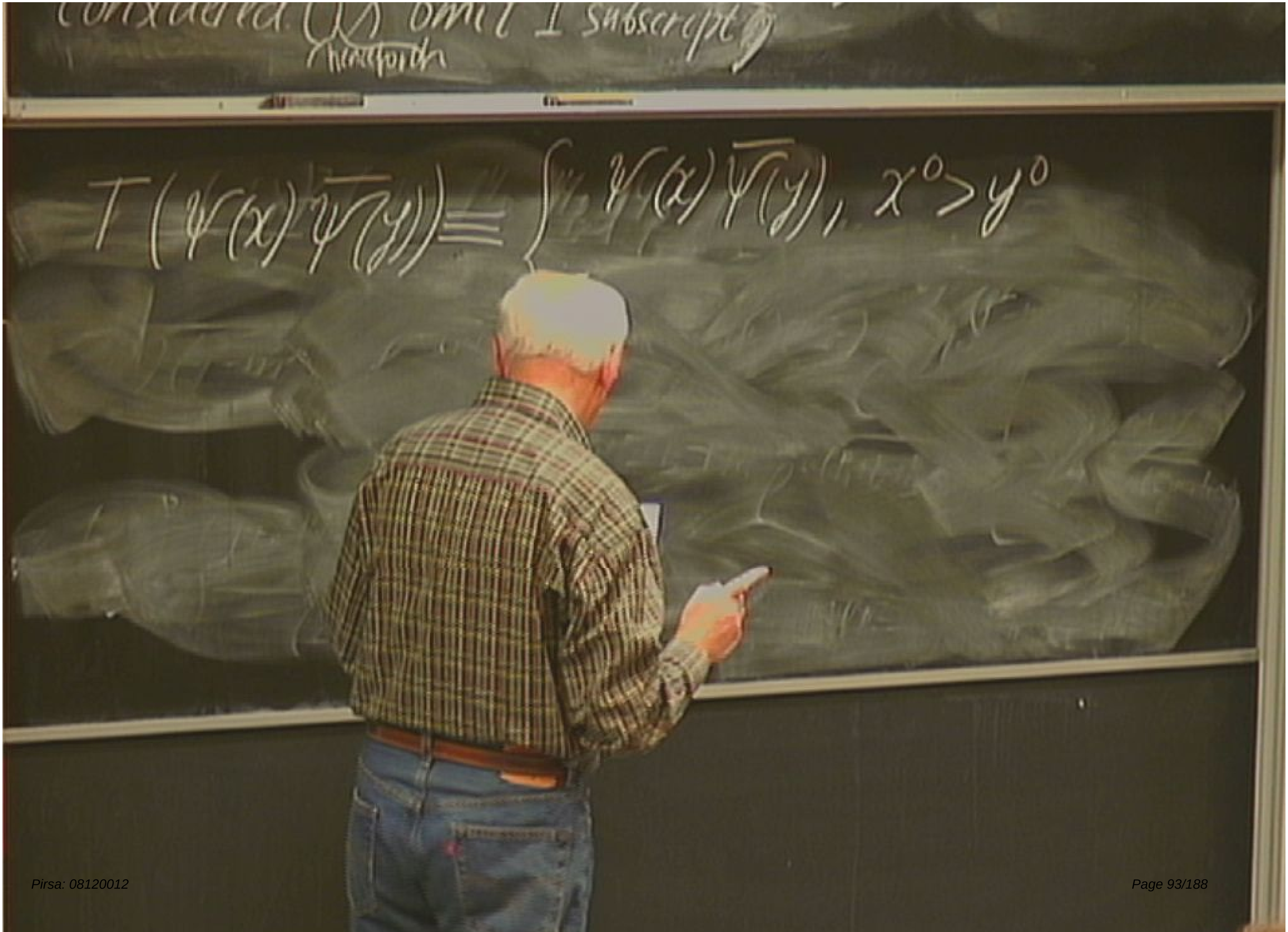
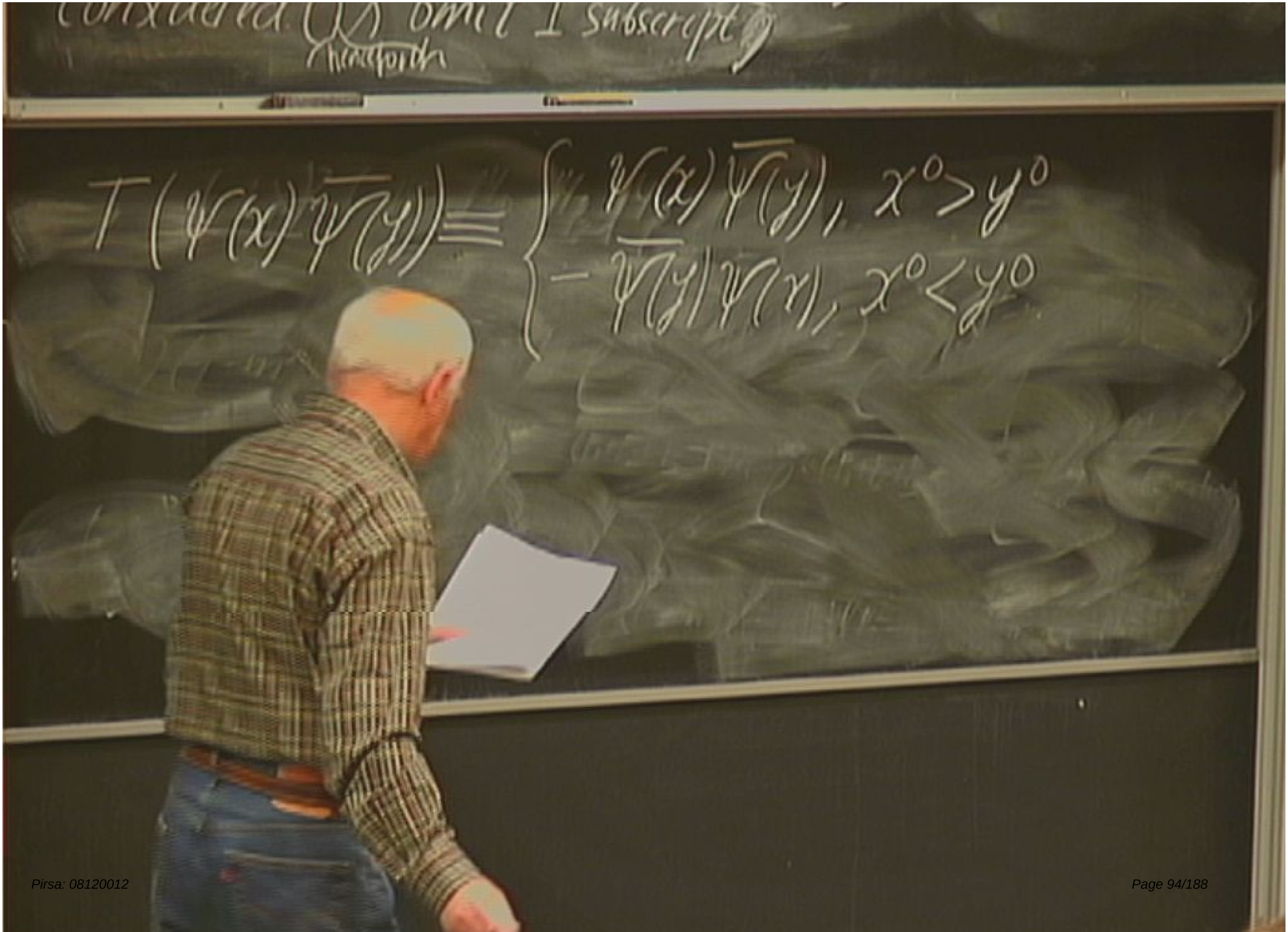
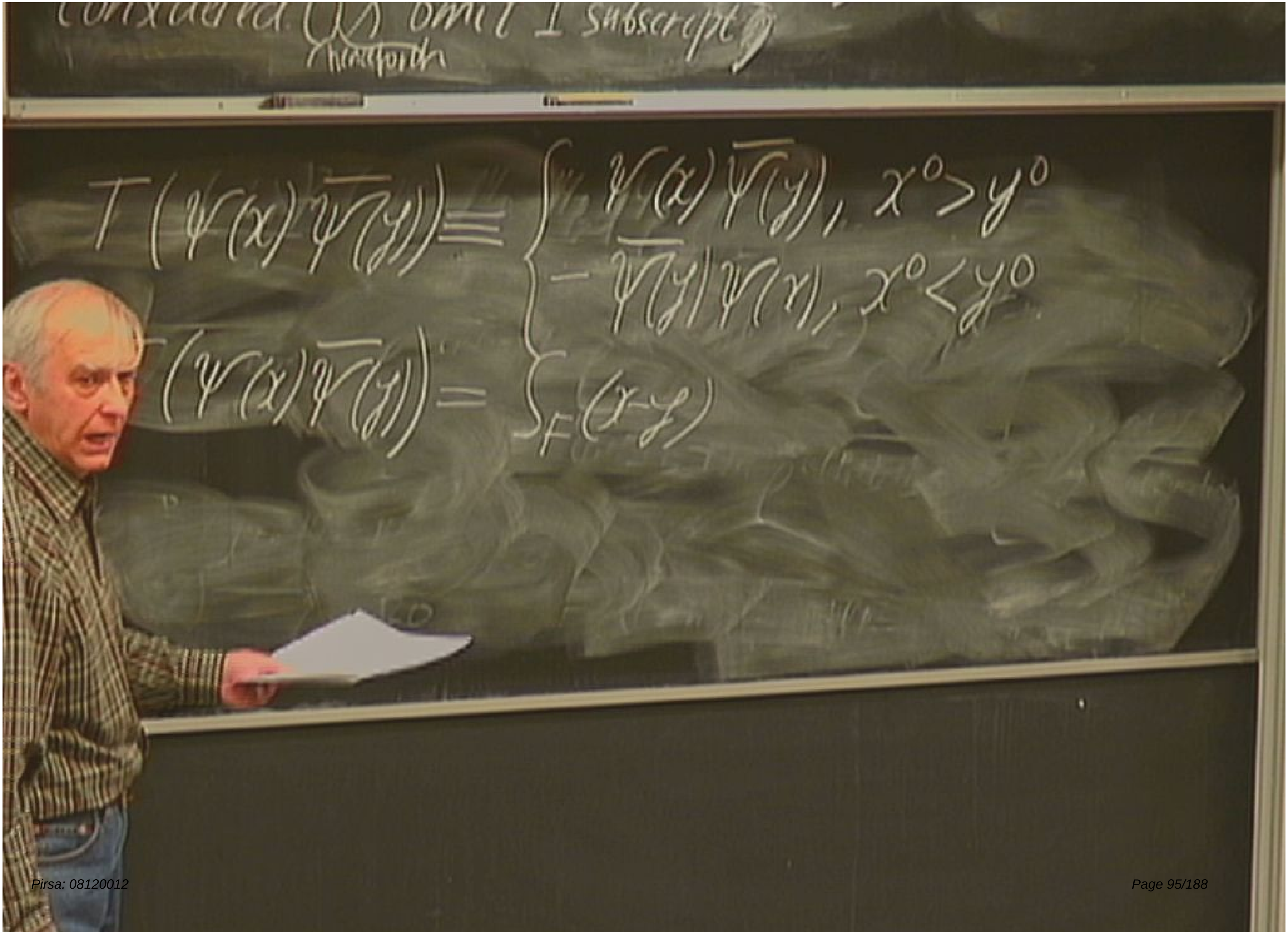
symbols for fermions.
Start
$T(\psi_I(x)\bar{\psi}_I(y))$, which was already
considered
I subscript)

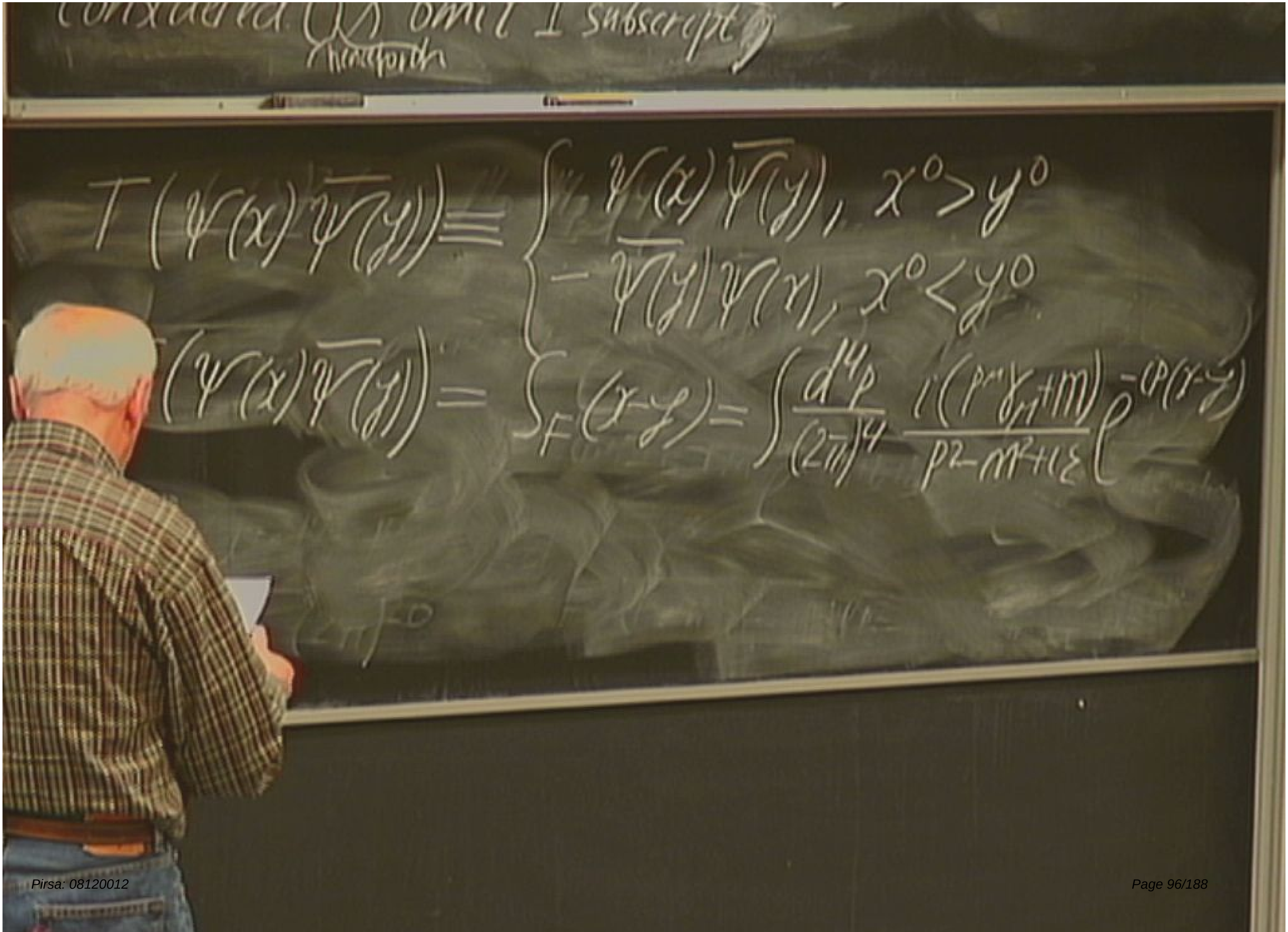

The expression (8) for Green's functions
is valid in theories including fermions. To formu-
late properly Wick's theorem in this case, one should
generalize the time-ordering and normal-ordering
symbols for fermions.
Start from $T(\psi_I(x)\bar{\psi}_I(y))$, which was already
considered (§8 omit I subscript)

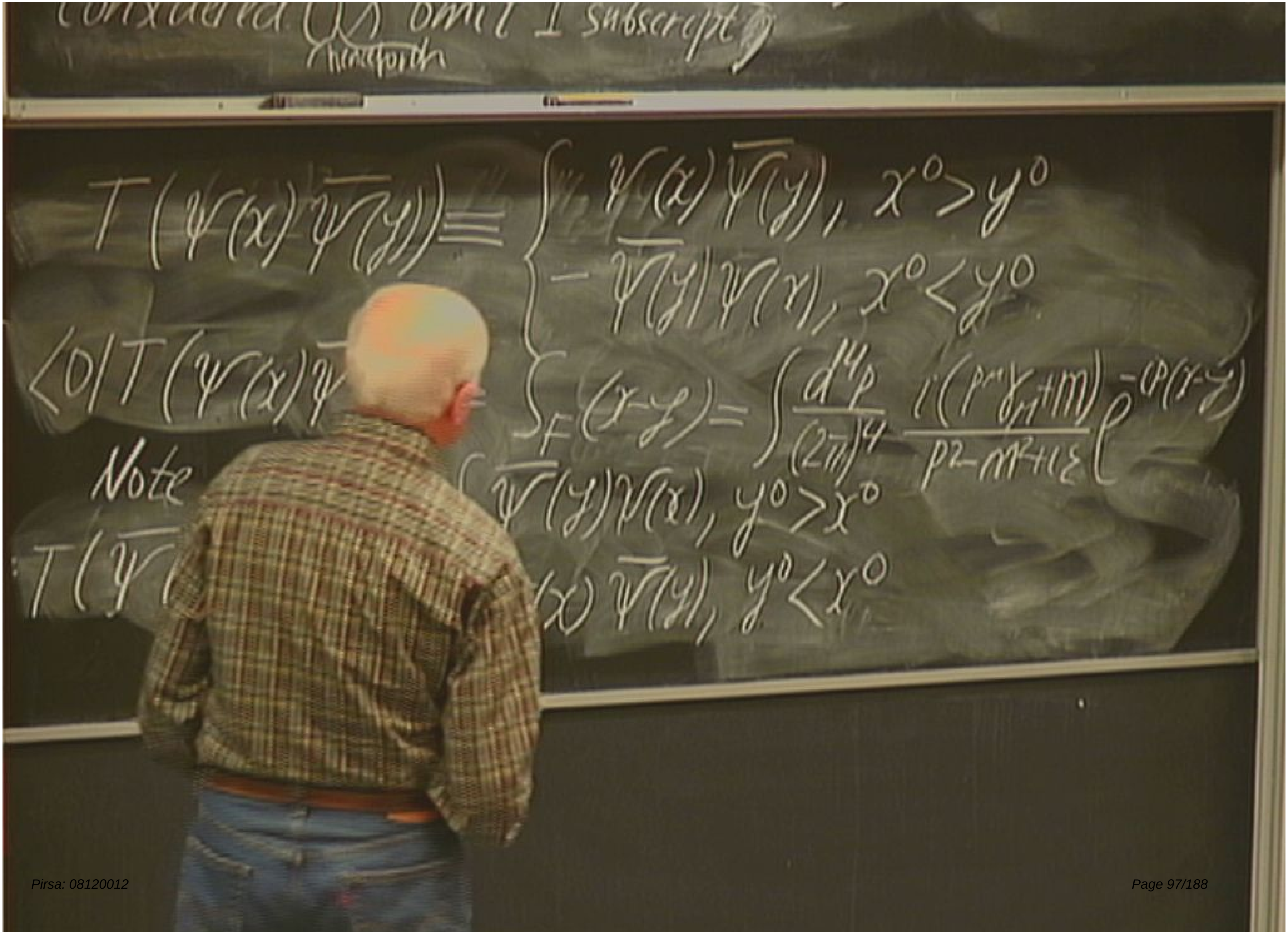

this case, one should
generalize the time-ordering and normal-ordering
symbols for fermions.
Start from T(ψ_I(x) ψ̄_I(y)), which was already
considered. (Henceforth omit I subscript)

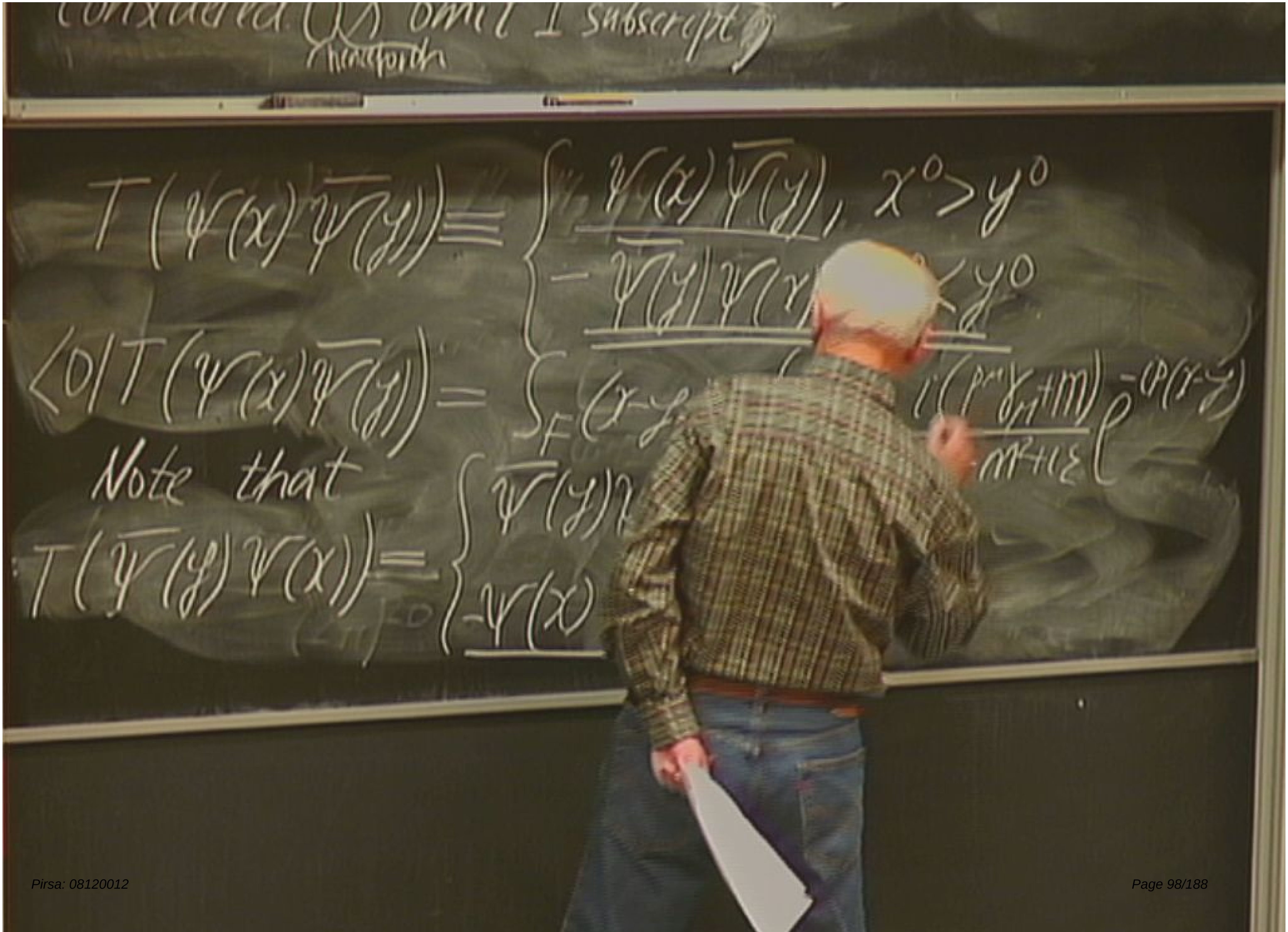

omit 1 subscript
henceforth
$T(\psi(x)\bar{\psi}(y)) \equiv \{ \psi(x)\bar{\psi}(y), \; x^0 > y^0$

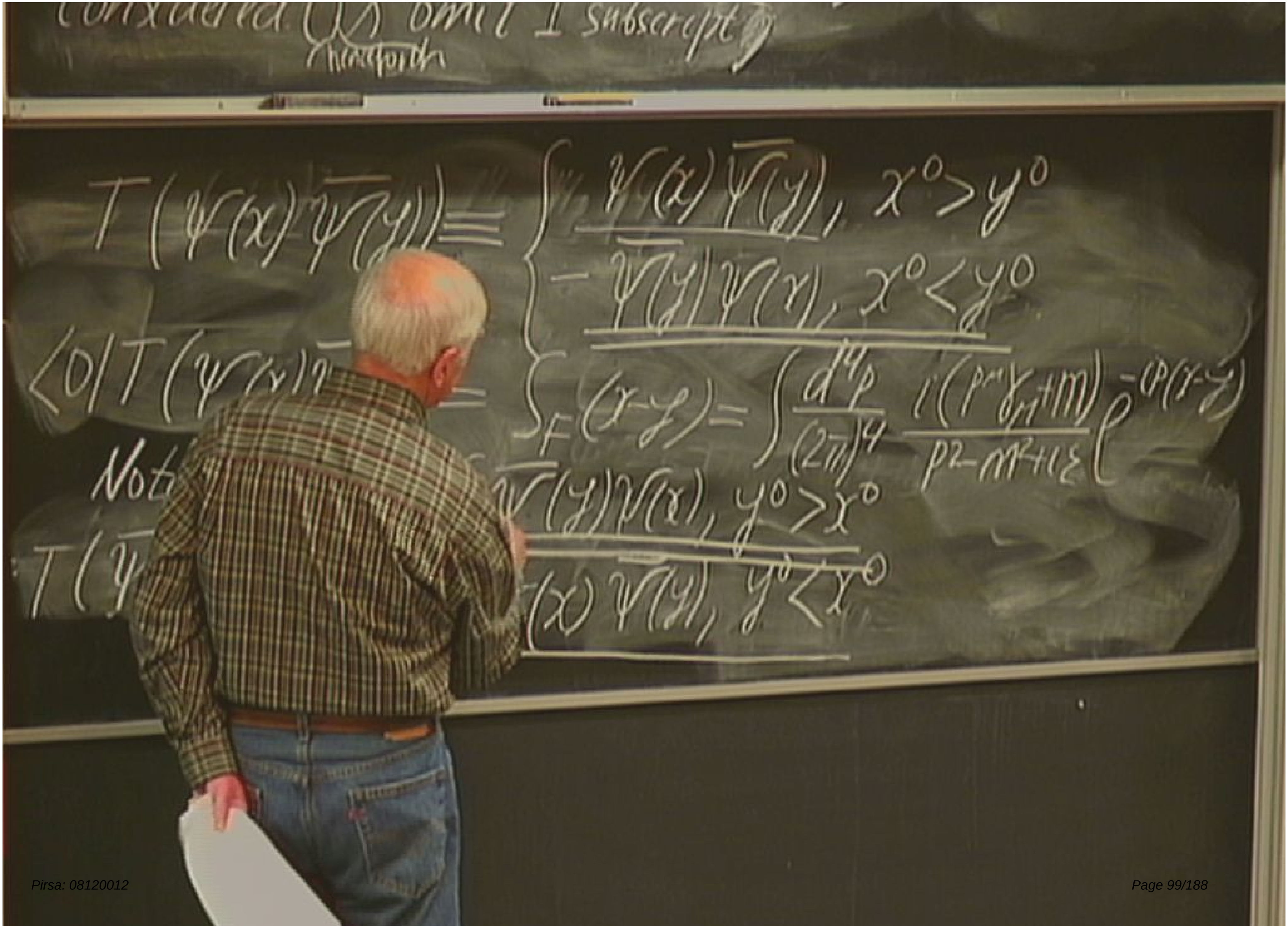

omit 1 subscript
henceforth
$T(\psi(x)\bar{\psi}(y)) \equiv \begin{cases} \psi(x)\bar{\psi}(y), & x^0 > y^0 \\ -\bar{\psi}(y)\psi(x), & x^0 < y^0 \end{cases}$


considered (1) omit I subscript
henceforth
$T(\psi(x)\bar{\psi}(y)) \equiv \begin{cases} \psi(x)\bar{\psi}(y), & x^0 > y^0 \\ -\bar{\psi}(y)\psi(x), & x^0 < y^0 \end{cases}$
$T(\psi(x)\bar{\psi}(y)) = S_F(x-y)$


considered (I) omit I subscript
henceforth
$T(\psi(x)\bar{\psi}(y)) \equiv \begin{cases} \psi(x)\bar{\psi}(y), & x^0 > y^0 \\ -\bar{\psi}(y)\psi(x), & x^0 < y^0 \end{cases}$
$T(\psi(x)\bar{\psi}(y)) = S_F(x-y) = \int \frac{d^4p}{(2\pi)^4} \frac{i(p\cdot\gamma+m)}{p^2-m^2+i\epsilon} e^{-ip(x-y)}$


omit I subscript
henceforth
T(ψ(x) ψ̄(y)) ≡ { ψ(x) ψ̄(y), x⁰ > y⁰ ; − ψ̄(y) ψ(x), x⁰ < y⁰ }
⟨0|T(ψ(x) ψ̄
S_F(x−y) = ∫ d⁴p/(2π)⁴ i(p·γ+m)/(p²−m²+iε) e^{−ip(x−y)}
Note
T(ψ̄
ψ̄(y)ψ(x), y⁰ > x⁰
(x) ψ̄(y), y⁰ < x⁰


omit 1 subscript
henceforth
T(ψ(x) ψ̄(y)) ≡ ψ(x) ψ̄(y), x⁰ > y⁰
− ψ̄(y) ψ(x
⟨0| T(ψ(x) ψ̄(y)) =
Note that
T(ψ̄(y) ψ(x)) ≡
−ψ(x)



subscript
$T(\psi(x)\bar{\psi}(y)) \equiv \begin{cases} \psi(x)\bar{\psi}(y), & x^0 > y^0 \\ -\bar{\psi}(y)\psi(x), & x^0 < y^0 \end{cases}$
$S_F(x-y) = \int \frac{d^4p}{(2\pi)^4} \frac{i(p\cdot\gamma+m)}{p^2-m^2+i\epsilon} e^{-ip(x-y)}$
$\psi(y)\psi(x), \ y^0 > x^0$
$\psi(x)\psi(y), \ y^0 < x^0$

considered (...) omit 1 subscript henceforth

$$T\left(\psi(x)\bar{\psi}(y)\right) \equiv \begin{cases} \psi(x)\bar{\psi}(y), & x^0 > y^0 \\ -\bar{\psi}(y)\psi(x), & x^0 < y^0 \end{cases}$$

$$\langle 0|T\left(\psi(x)\bar{\psi}(y)\right)|0\rangle = S_F(x-y) = \int \frac{d^4p}{(2\pi)^4}\, \frac{i(p^\mu\gamma_\mu + m)}{p^2 - m^2 + i\epsilon}\, e^{-ip(x-y)}$$

Note that

$$T\left(\bar{\psi}(y)\psi(x)\right) \equiv \begin{cases} \bar{\psi}(y)\psi(x), & y^0 > x^0 \\ -\psi(x)\bar{\psi}(y), & y^0 < x^0 \end{cases}$$

considered (...) omit 1 subscript henceforth

$$T\left(\psi(x)\bar{\psi}(y)\right) \equiv \begin{cases} \psi(x)\bar{\psi}(y), & x^0 > y^0 \\ -\bar{\psi}(y)\psi(x), & x^0 < y^0 \end{cases}$$

$$\langle 0|T\left(\psi(x)\bar{\psi}(y)\right)|0\rangle = S_F(x-y) = \int \frac{d^4p}{(2\pi)^4}\,\frac{i(p^\mu\gamma_\mu+m)}{p^2-m^2+i\varepsilon}\,e^{-ip(x-y)}$$

Note that

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considered (I) omit I subscript
henceforth

$$T(\psi(x)\bar{\psi}(y)) \equiv \begin{cases} \psi(x)\bar{\psi}(y), & x^0 > y^0 \\ -\bar{\psi}(y)\psi(x), & x^0 < y^0 \end{cases}$$

$$\langle 0|T(\psi(x)\bar{\psi}(y))| = S_F(x-y) = \int \frac{d^4p}{(2\pi)^4} \frac{i(p^\mu\gamma_\mu + m)}{p^2 - m^2 + i\varepsilon} e^{-ip(x-y)}$$

Note that

$$T(\bar{\psi}(y)\psi(x)) = \begin{cases} \bar{\psi}(y)\psi(x), & y^0 > x^0 \\ -\psi(x)\bar{\psi}(y), & y^0 < x^0 \end{cases}$$

considered. (X omit I subscript)

$$T\left(\psi(x)\bar{\psi}(y)\right) \equiv \begin{cases} \psi(x)\bar{\psi}(y), & x^0 > y^0 \\ -\bar{\psi}(y)\psi(x), & x^0 < y^0 \end{cases}$$

$$\langle 0|T\left(\psi(x)\bar{\psi}(y)\right) = S_F(x-y) = \int \frac{d^4p}{(2\pi)^4} \frac{i(p^\mu\gamma_\mu + m)}{p^2 - m^2 + i\varepsilon} e^{-ip(x-y)}$$

Note that

$$T\left(\bar{\psi}(y)\psi(x)\right) = \begin{cases} \bar{\psi}(y)\psi(x), & y^0 > x^0 \\ -\psi(x)\bar{\psi}(y), & y^0 < x^0 \end{cases}$$

Then

$$\Rightarrow T\left(\psi(x)\bar{\psi}(y)\right) = -T\left(\bar{\psi}(y) \ldots\right)$$

considered ... omit I subscript

$$T(\psi(x)\bar{\psi}(y)) \equiv \begin{cases} \psi(x)\bar{\psi}(y), & x^0 > y^0 \\ -\bar{\psi}(y)\psi(x), & x^0 < y^0 \end{cases}$$

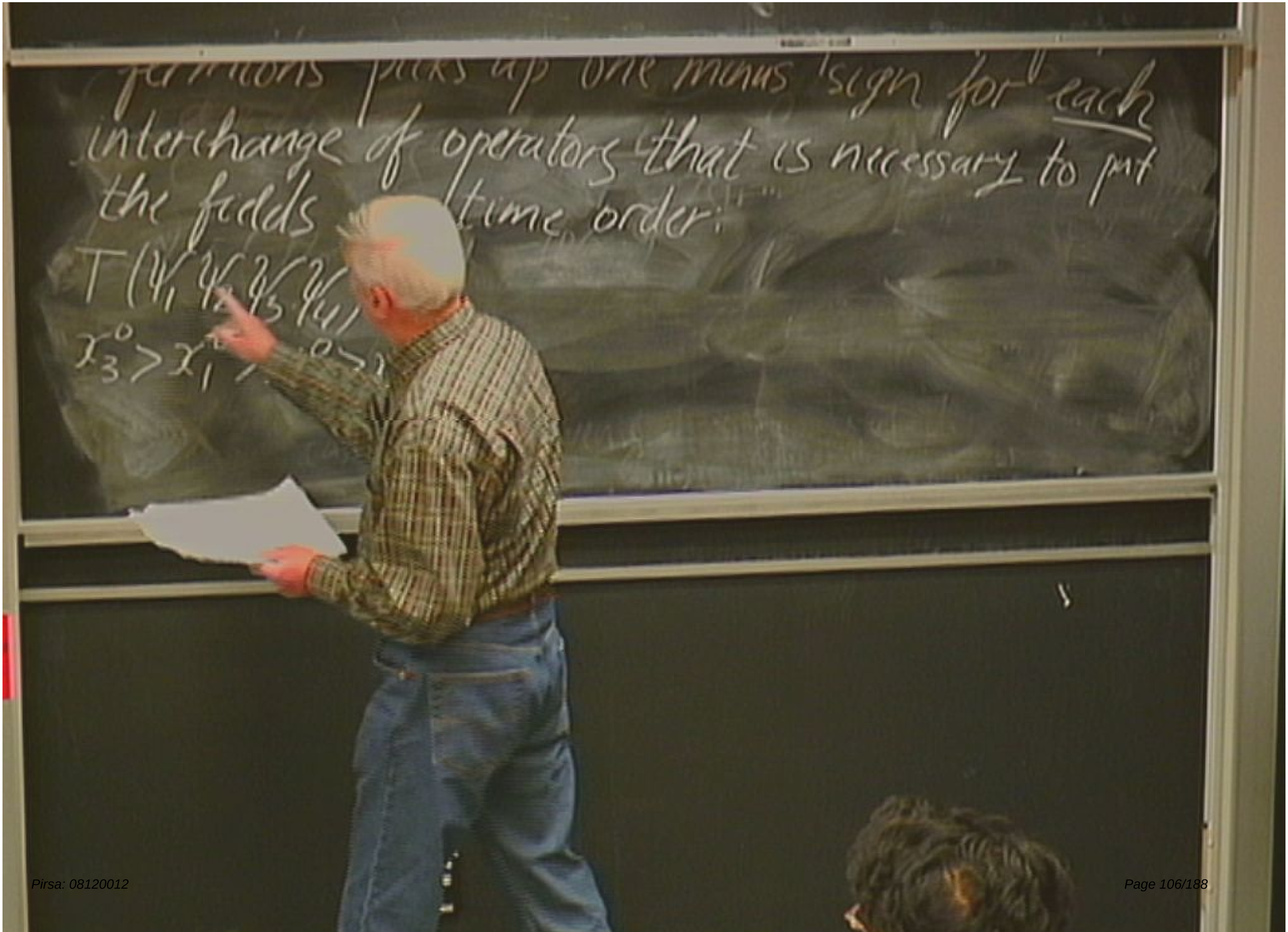
$$\langle 0|T(\psi(x)\bar{\psi}(y)) = S_F(x-y) = \int \frac{d^4p}{(2\pi)^4} \frac{i(p^\mu\gamma_\mu + m)}{p^2 - m^2 + i\epsilon} e^{-ip(x-y)}$$

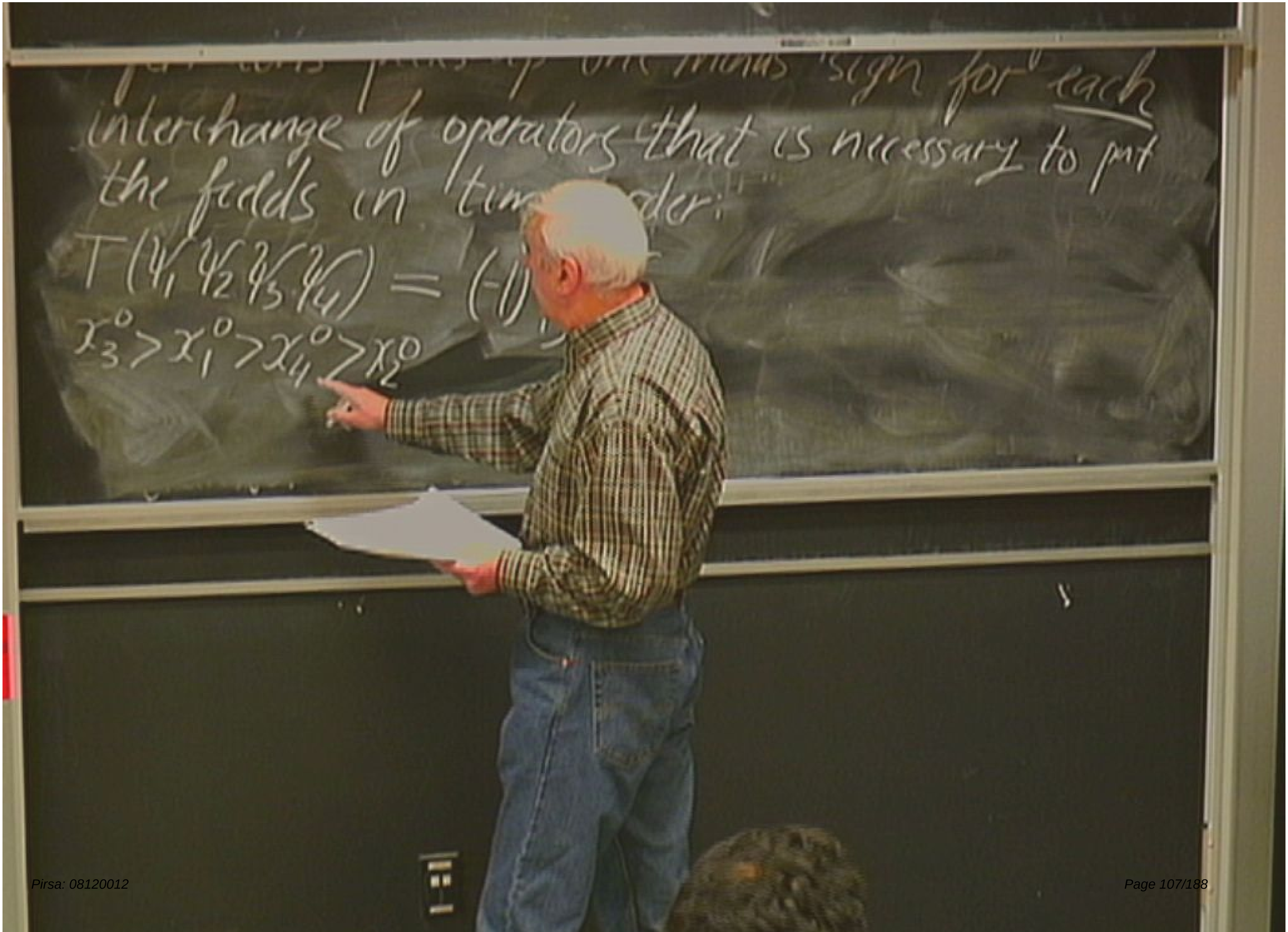
Note that

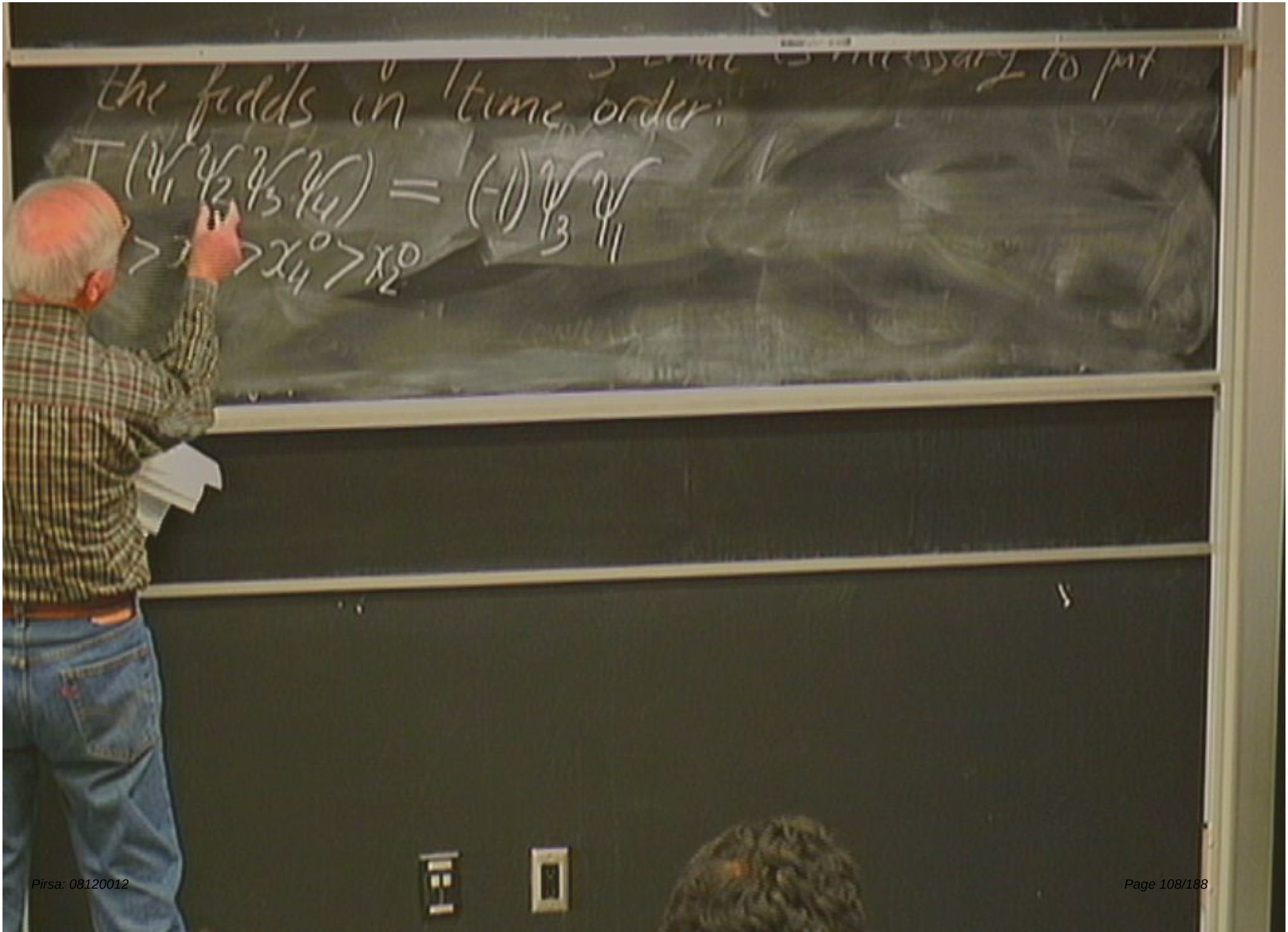
$$T(\bar{\psi}(y)\psi(x)) = \begin{cases} \bar{\psi}(y)\psi(x), & y^0 > x^0 \\ -\psi(x)\bar{\psi}(y), & y^0 < x^0 \end{cases}$$

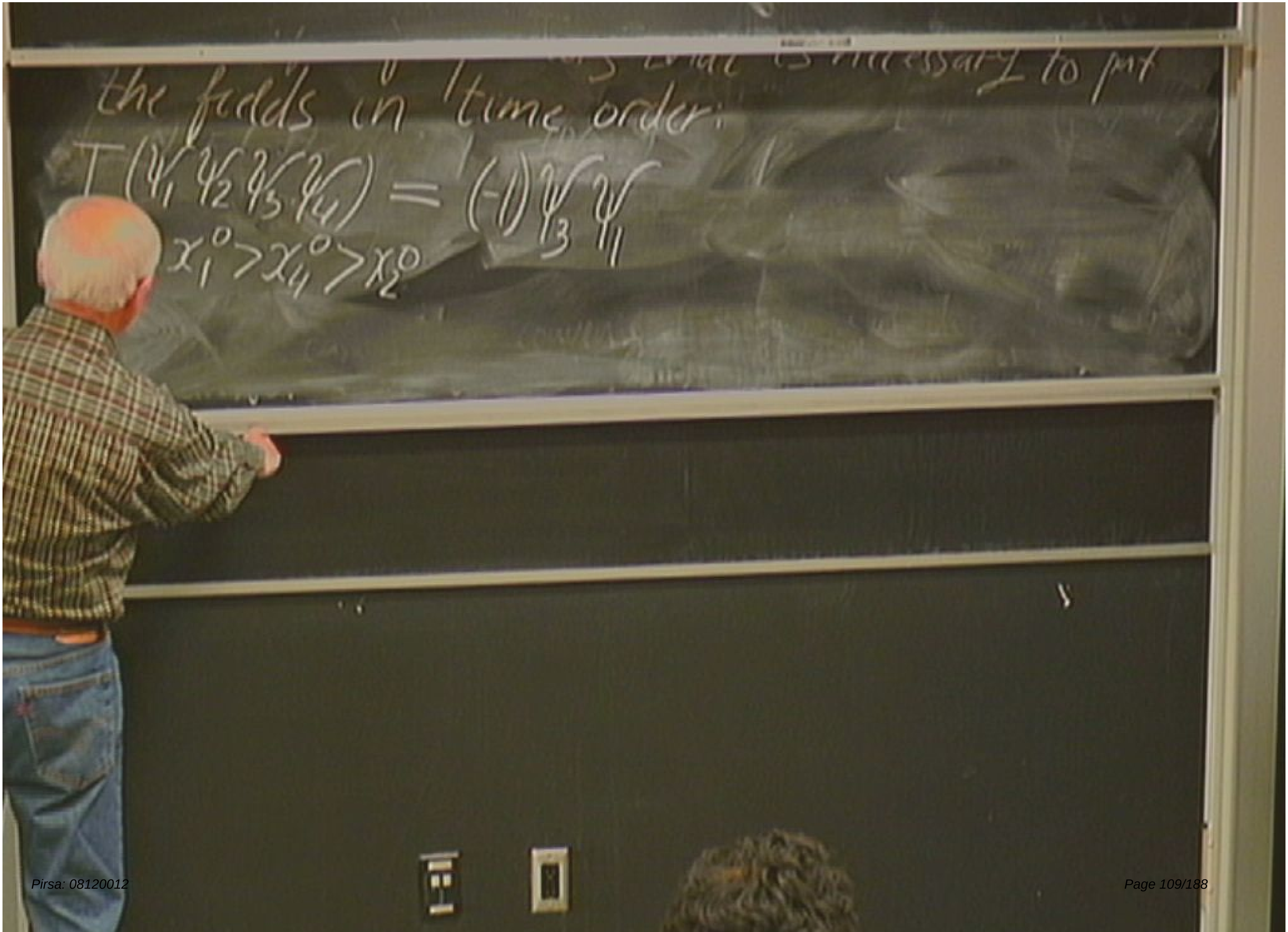
Then $\Rightarrow T(\psi(x)\bar{\psi}(y)) = -T(\bar{\psi}(y)\psi(x))$

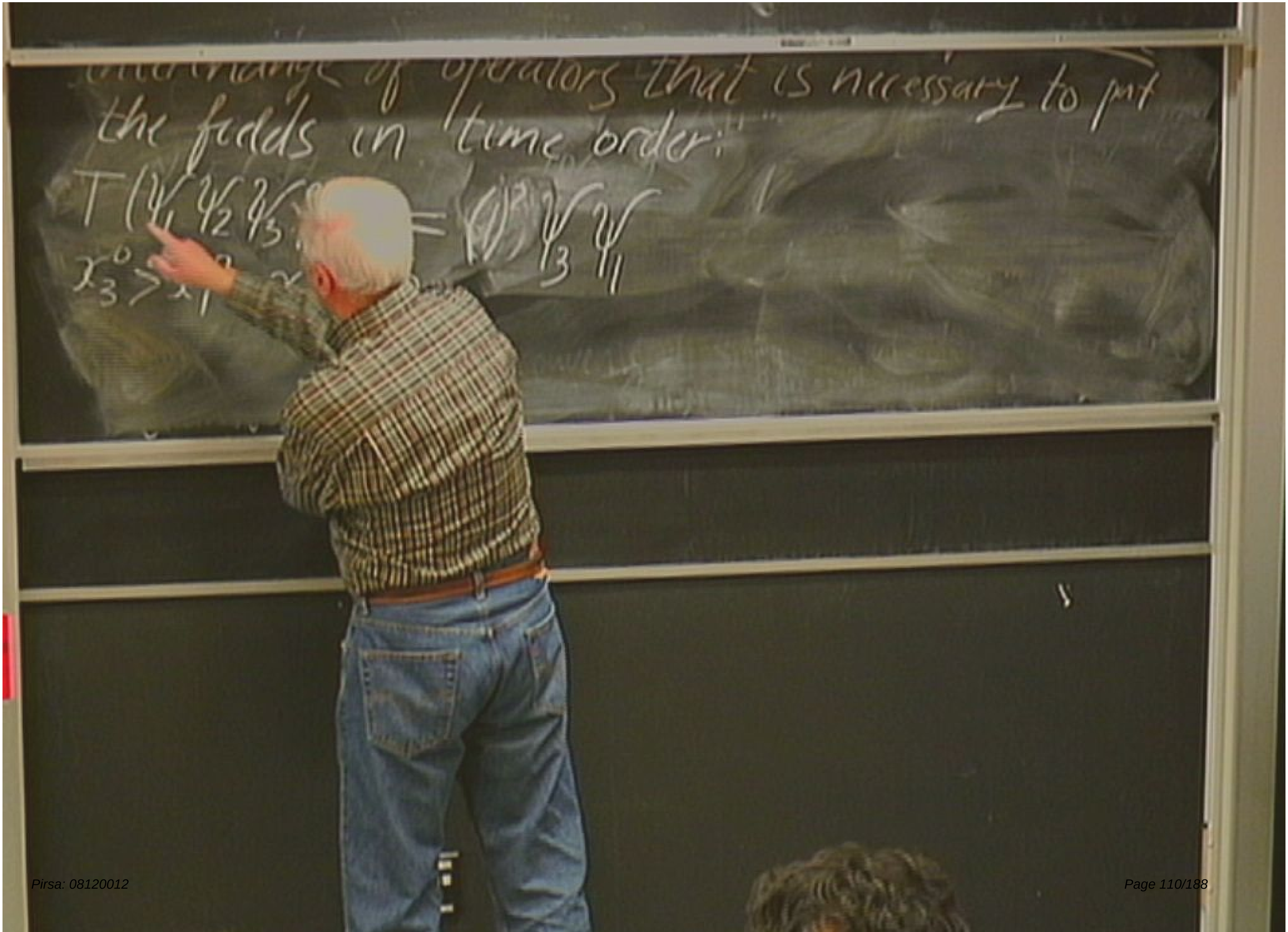
Generalization: Time ordered product for
ermions picks up one minus sign for <u>each</u>
terchange of operators that is necessary to put

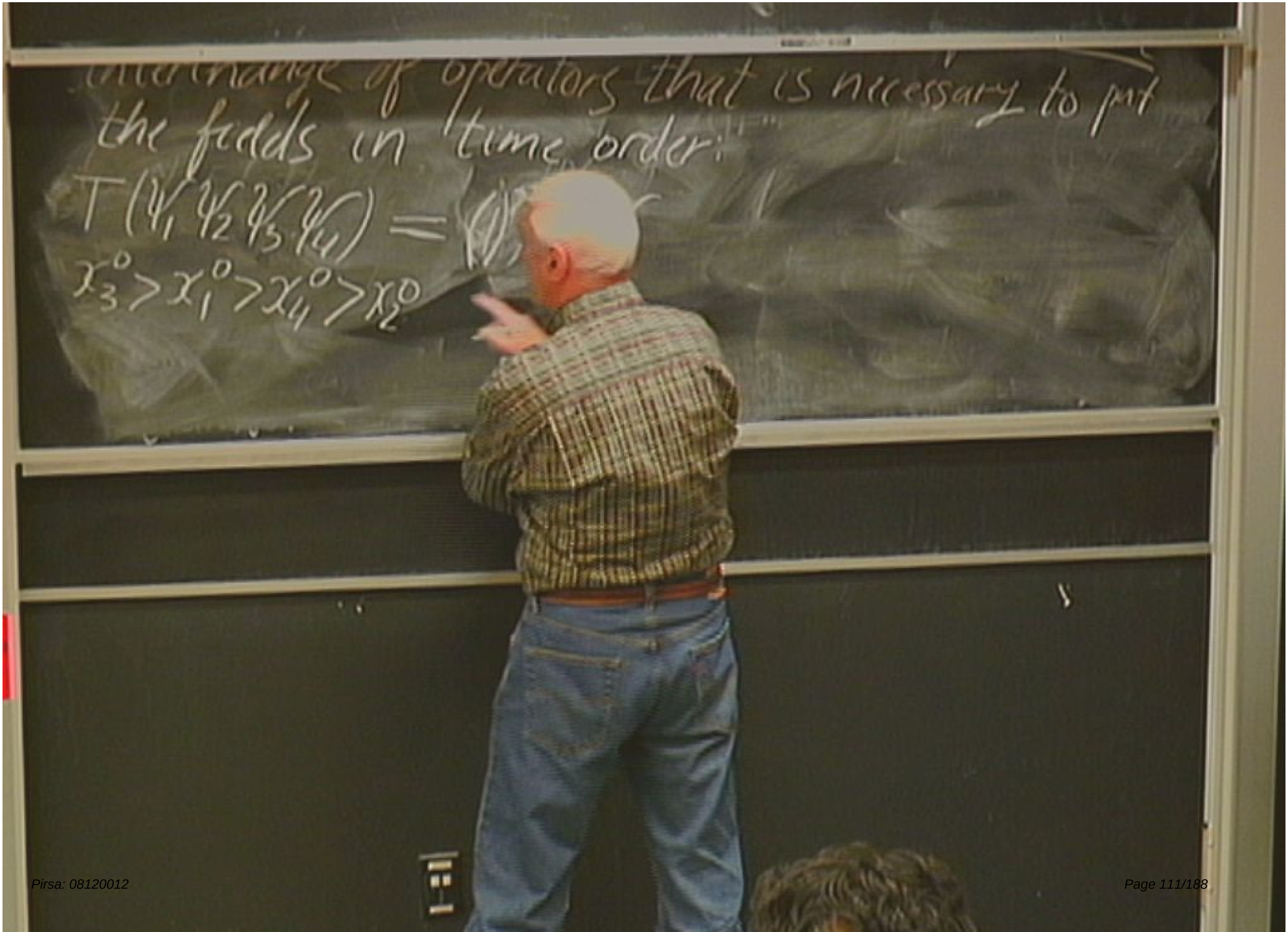

fermions picks up one minus sign for each
interchange of operators that is necessary to put
the fields time order."
T(ψ1 ψ2 ψ3 ψ4)

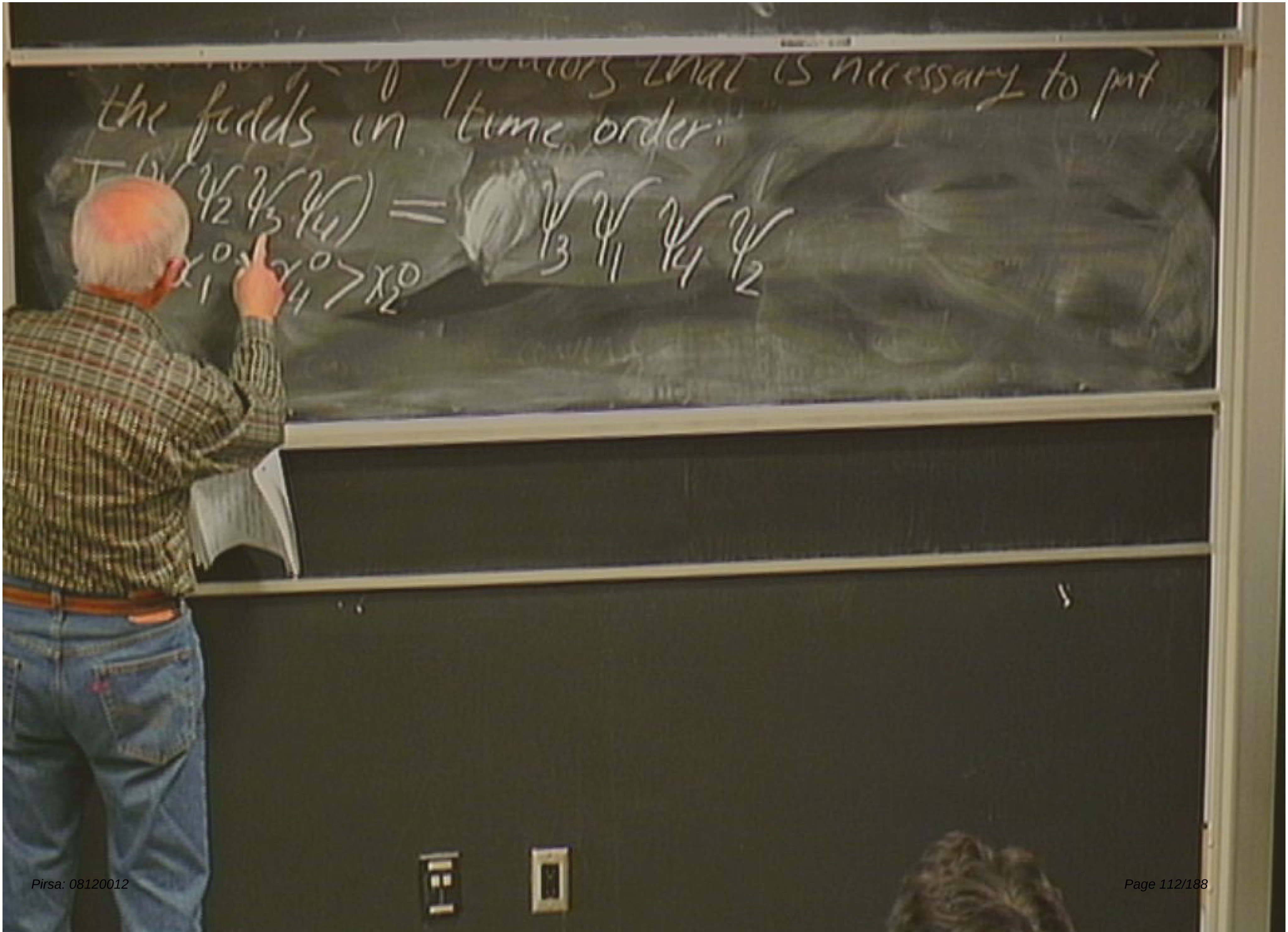

sign for each
interchange of operators that is necessary to put
the fields in
$T(\psi_1 \psi_2 \psi_3 \psi_4) = (-1)$
$x_3^0 > x_1^0 > x_4^0 > x_2^0$

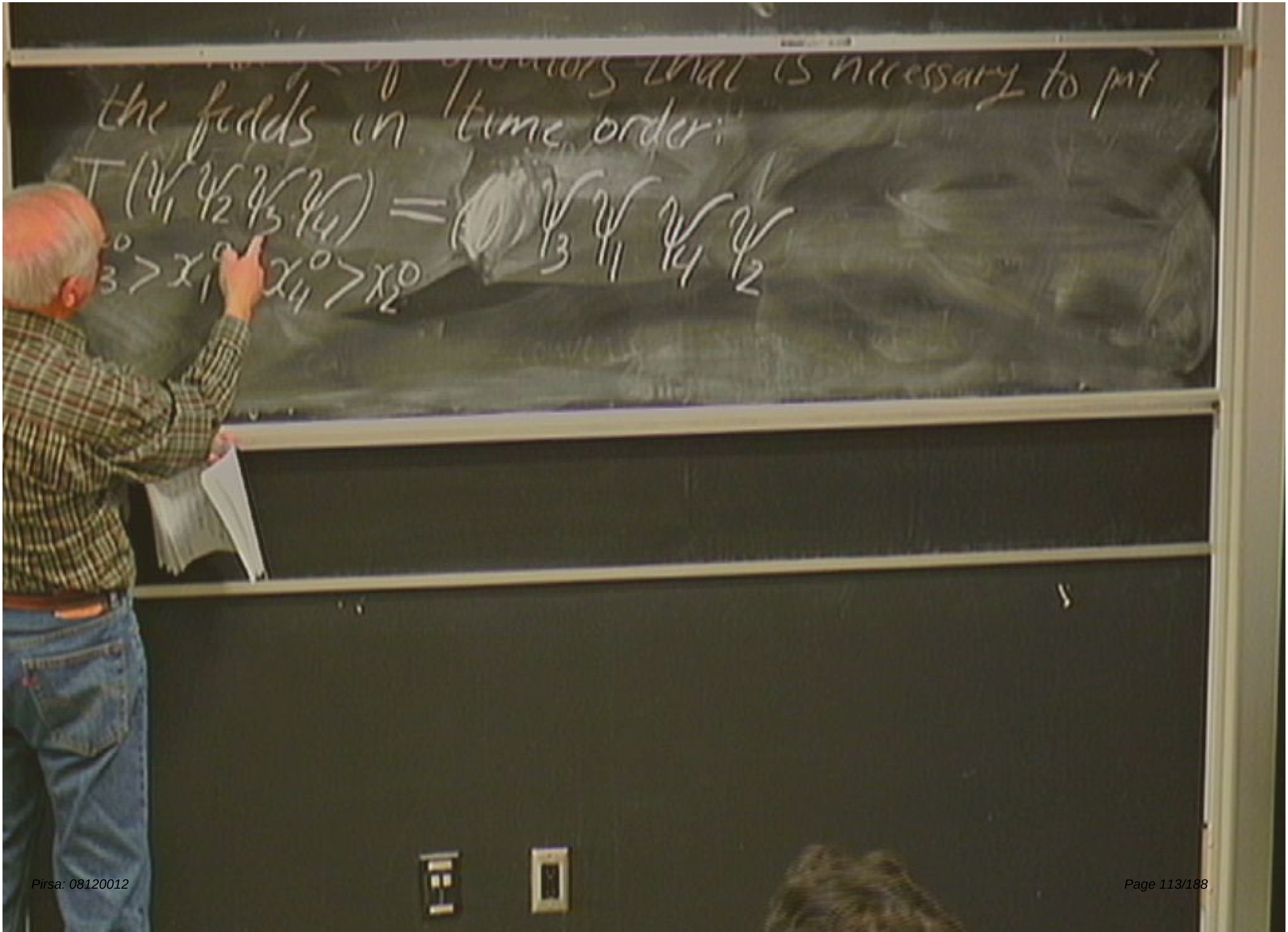

necessary to put
the fields in time order:
T(ψ1 ψ2 ψ3 ψ4) = (-1) ψ3 ψ1
> x4° > x2°

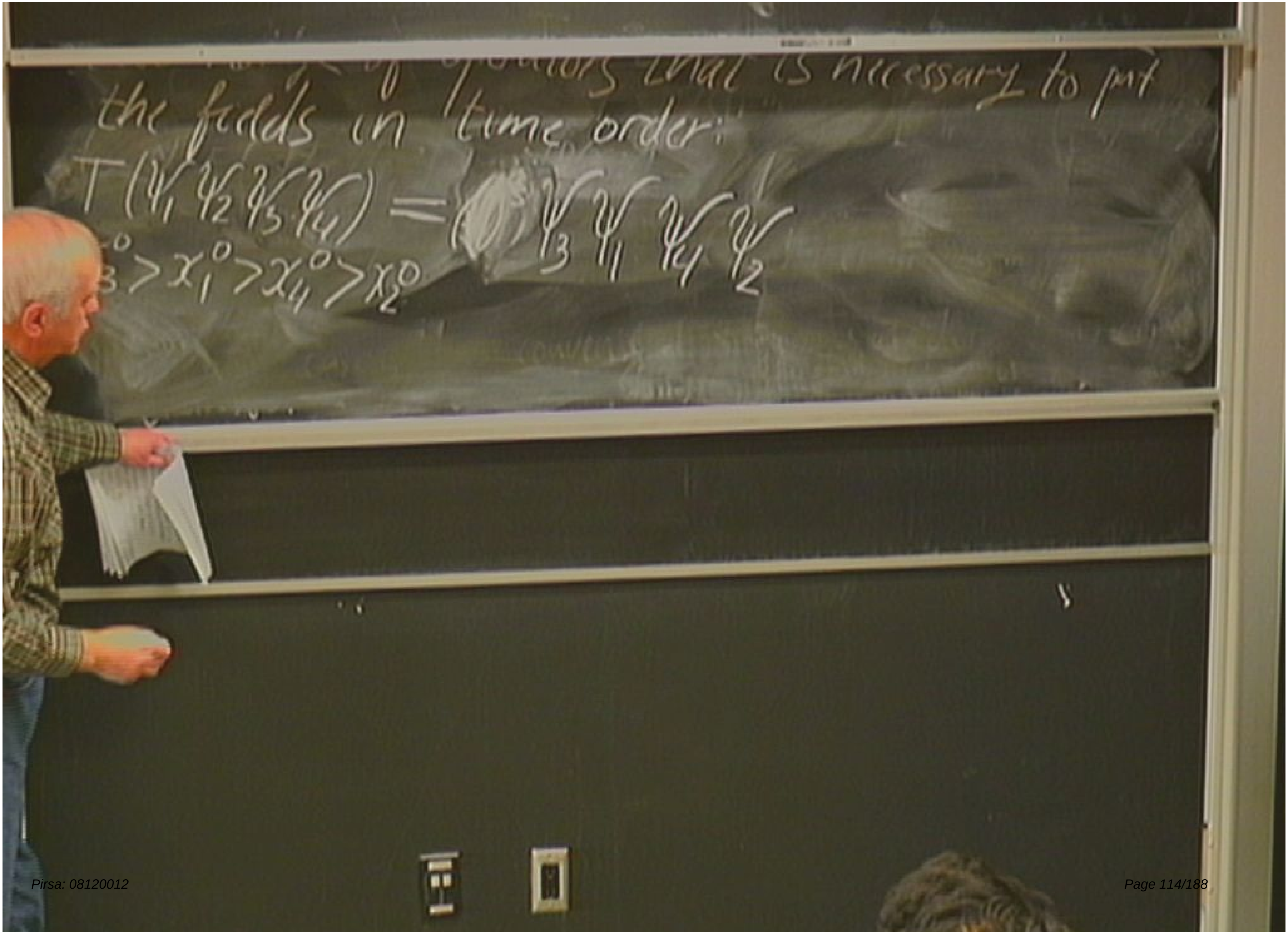

necessary to put
the fields in time order:
T(ψ1 ψ2 ψ3 ψ4) = (-1) ψ3 ψ1
x1^0 > x4^0 > x2^0

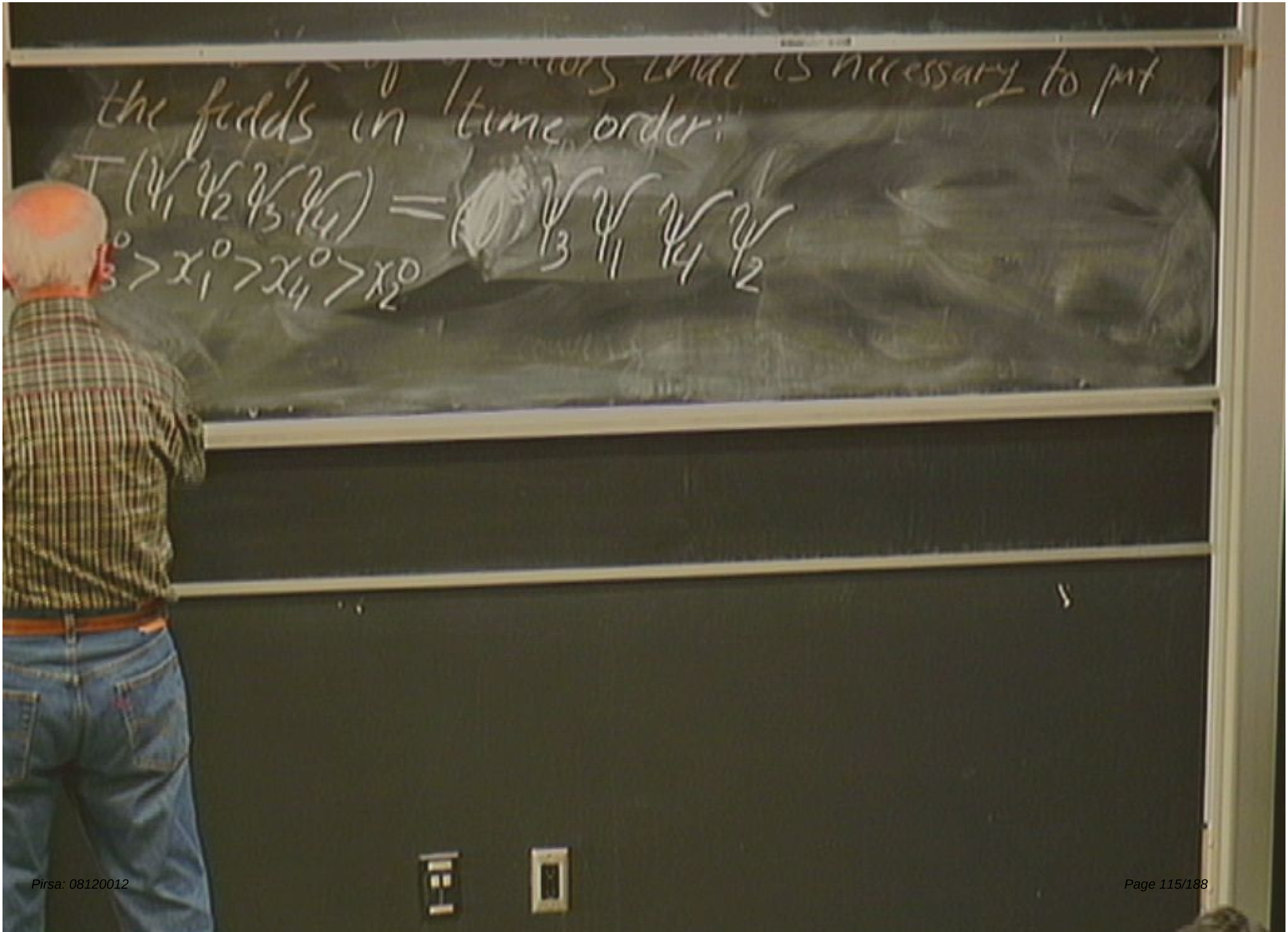

of operators that is necessary to put
the fields in 'time order':"

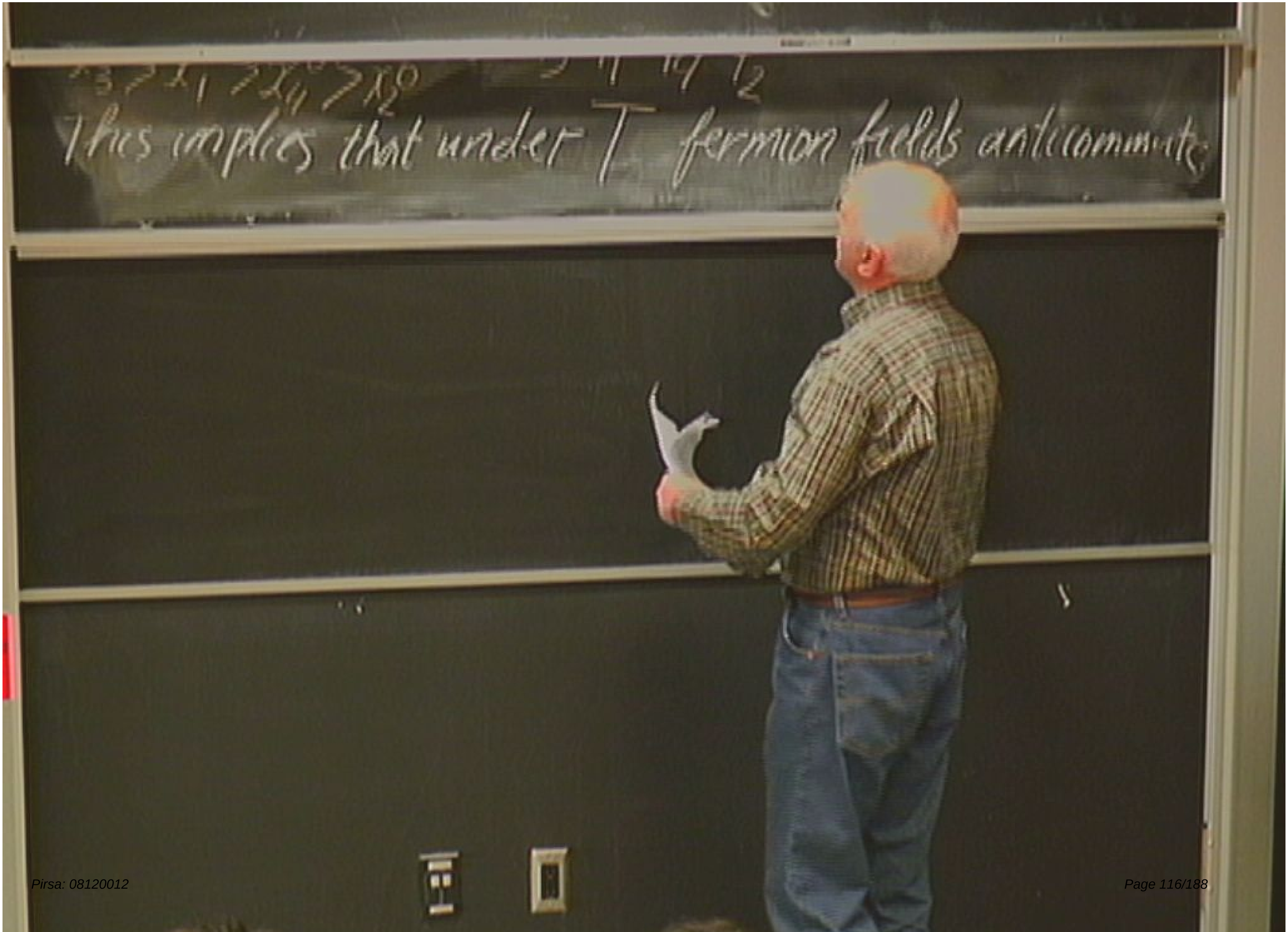

interchange of operators that is necessary to put
the fields in time order:
$T(\psi_1 \psi_2 \psi_3 \psi_4) =$
$x_3^0 > x_1^0 > x_4^0 > x_2^0$


that is necessary to put
the fields in time order:
$\psi_3 \psi_1 \psi_4 \psi_2$
x_2^0


that is necessary to put
the fields in time order:
T(ψ1 ψ2 ψ3 ψ4) = ψ3 ψ1 ψ4 ψ2
> x1 > x4 > x2


operators that is necessary to put
the fields in time order:
T(ψ1 ψ2 ψ3 ψ4) = ψ3 ψ1 ψ4 ψ2
x3^0 > x1^0 > x4^0 > x2^0


that is necessary to put
the fields in time order:
$T(\psi_1 \psi_2 \psi_3 \psi_4) = (\ ) \psi_3 \psi_1 \psi_4 \psi_2$
$> x_1^0 > x_4^0 > x_2^0$

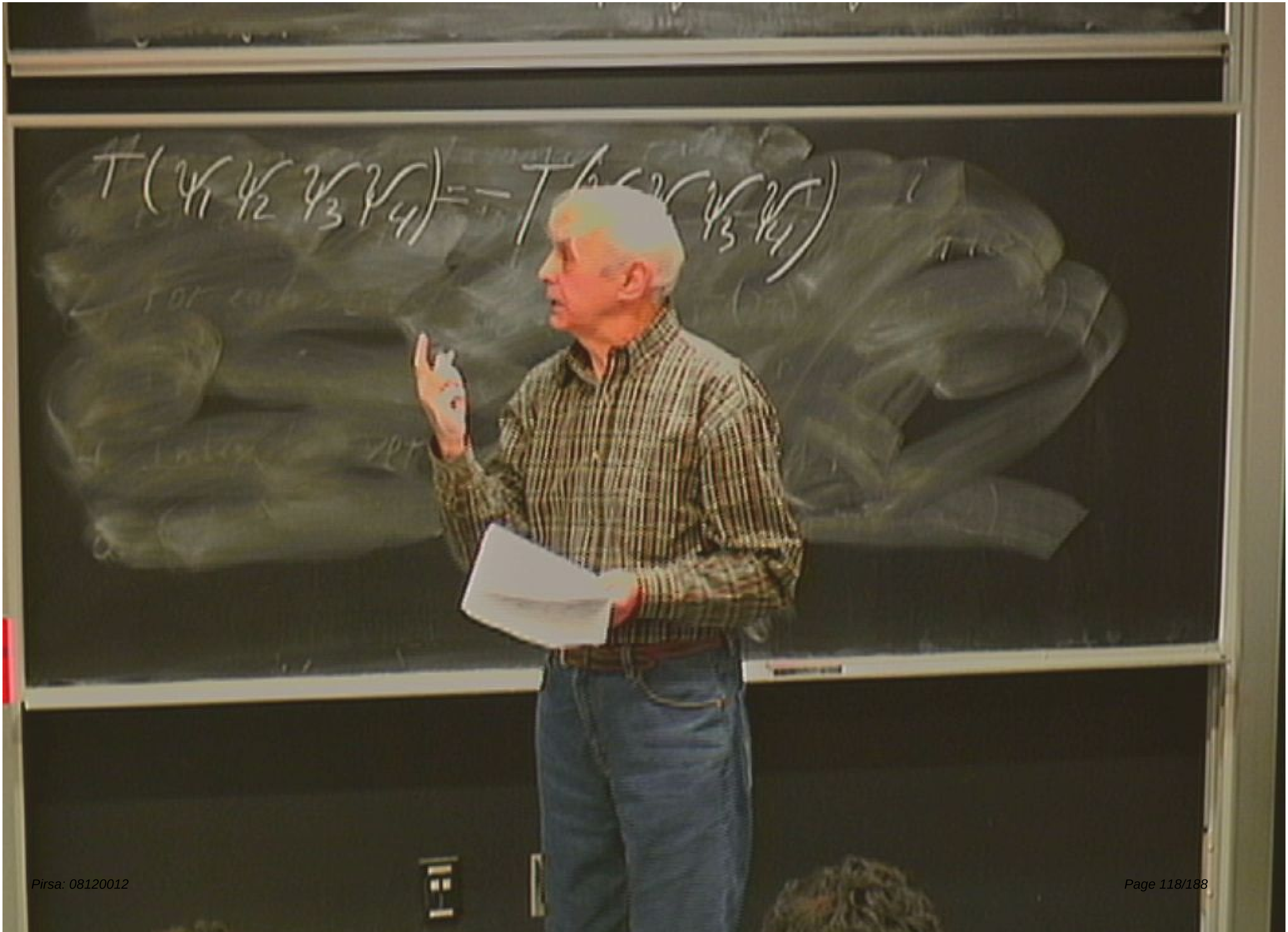

This implies that under T fermion fields anticommute

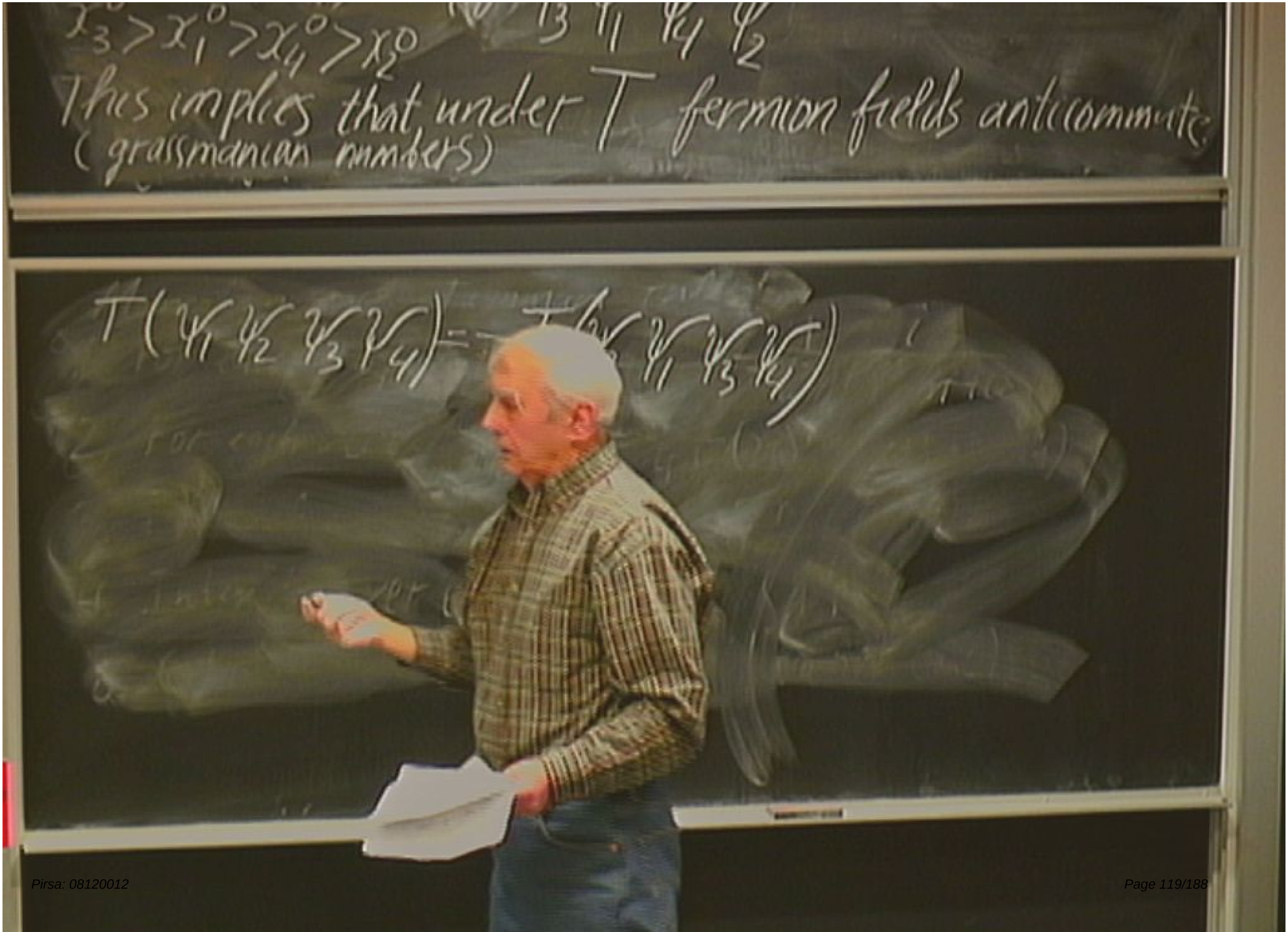
the fields in time order:

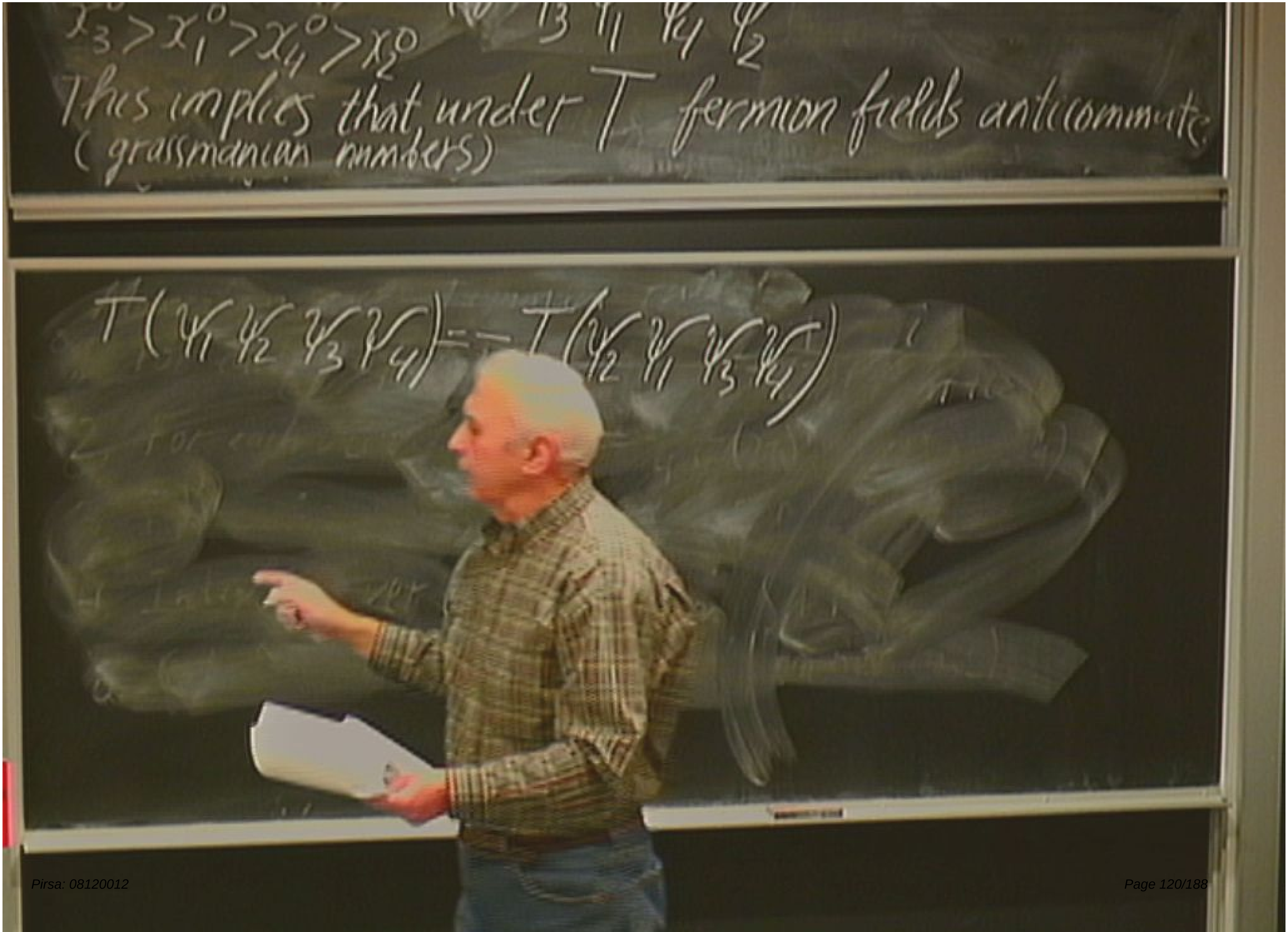
$$T(\psi_1\psi_2\psi_3\psi_4) = (-1)^3\,\psi_3\psi_1\psi_4\psi_2$$

$$x_3^0 > x_1^0 > x_4^0 > x_2^0$$

This implies that under T fermion fields anticommute.




$x_3^0 > x_1^0 > x_4^0 > x_2^0$
$\psi_3 \; \psi_1 \; \psi_4 \; \psi_2$
This implies that under T fermion fields anticommute
(grassmanian numbers)
$T(\psi_1 \psi_2 \psi_3 \psi_4) = -T(\psi_2 \psi_1 \psi_3 \psi_4)$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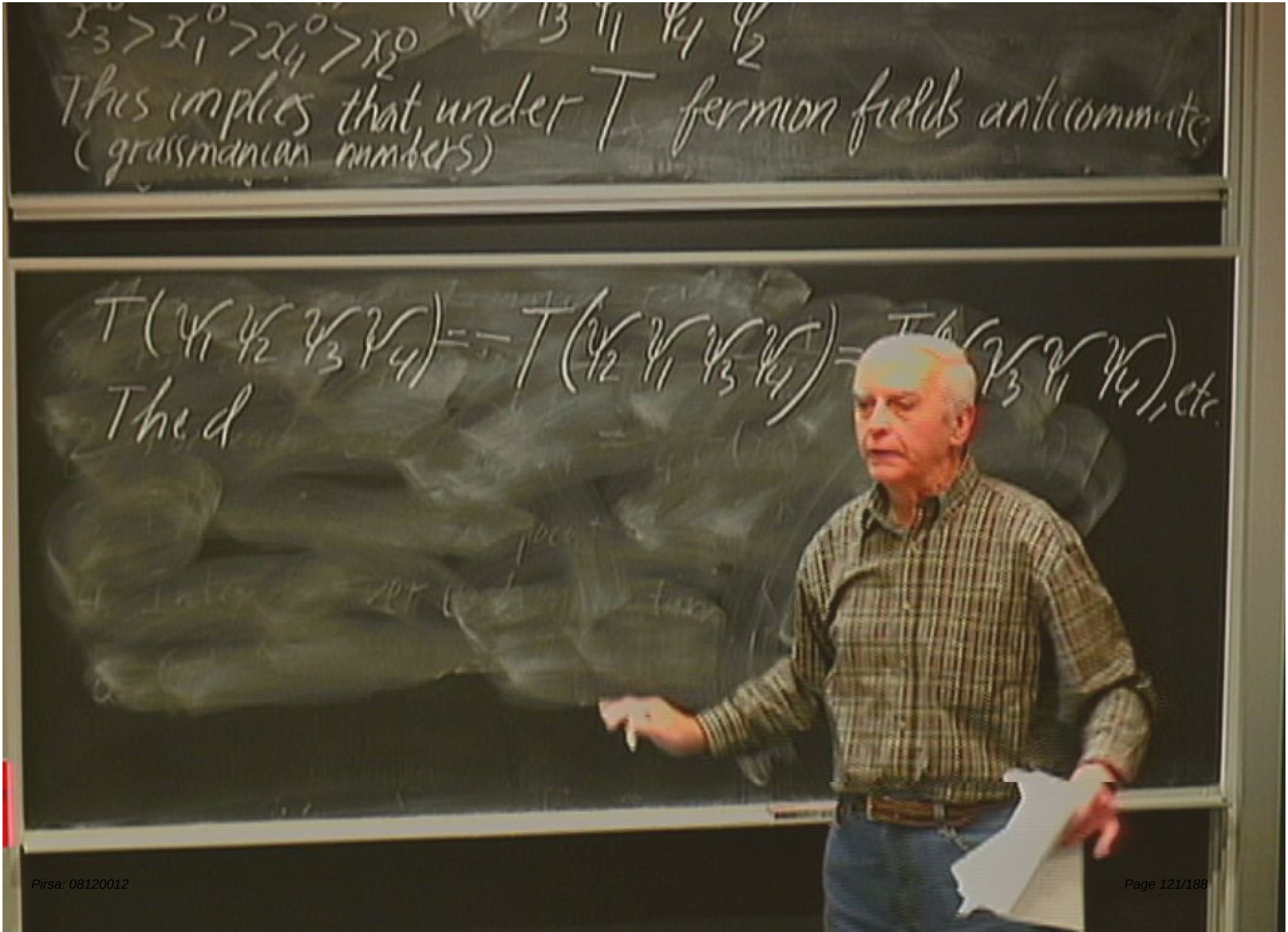
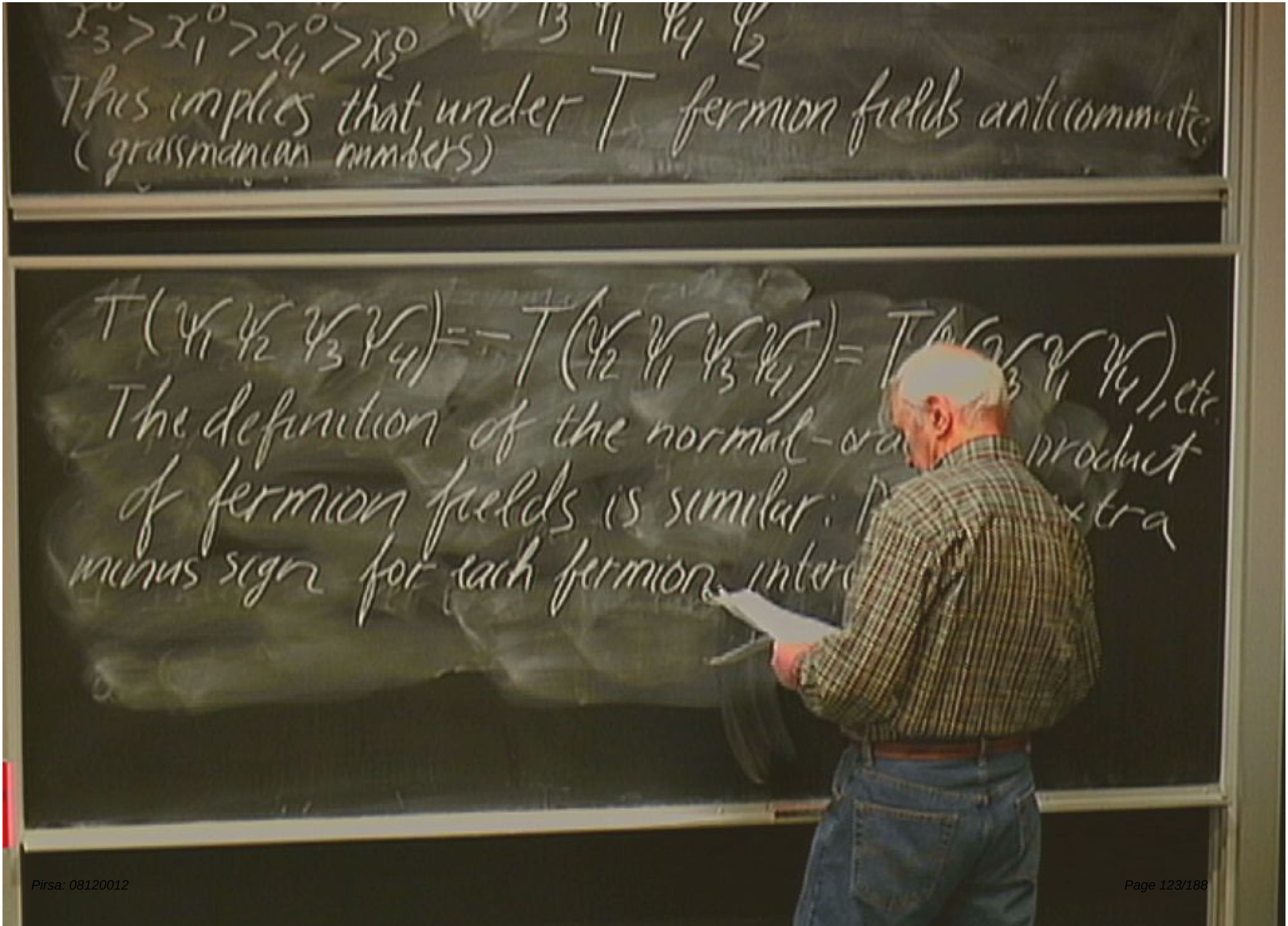

$x_3 > x_1^0 > x_4^0 > x_2^0$
This implies that under T fermion fields anticommute
(grassmanian numbers)
$T(\psi_1 \psi_2 \psi_3 \psi_4) = -T(\psi_2 \psi_1 \psi_3 \psi_4) =$ etc.

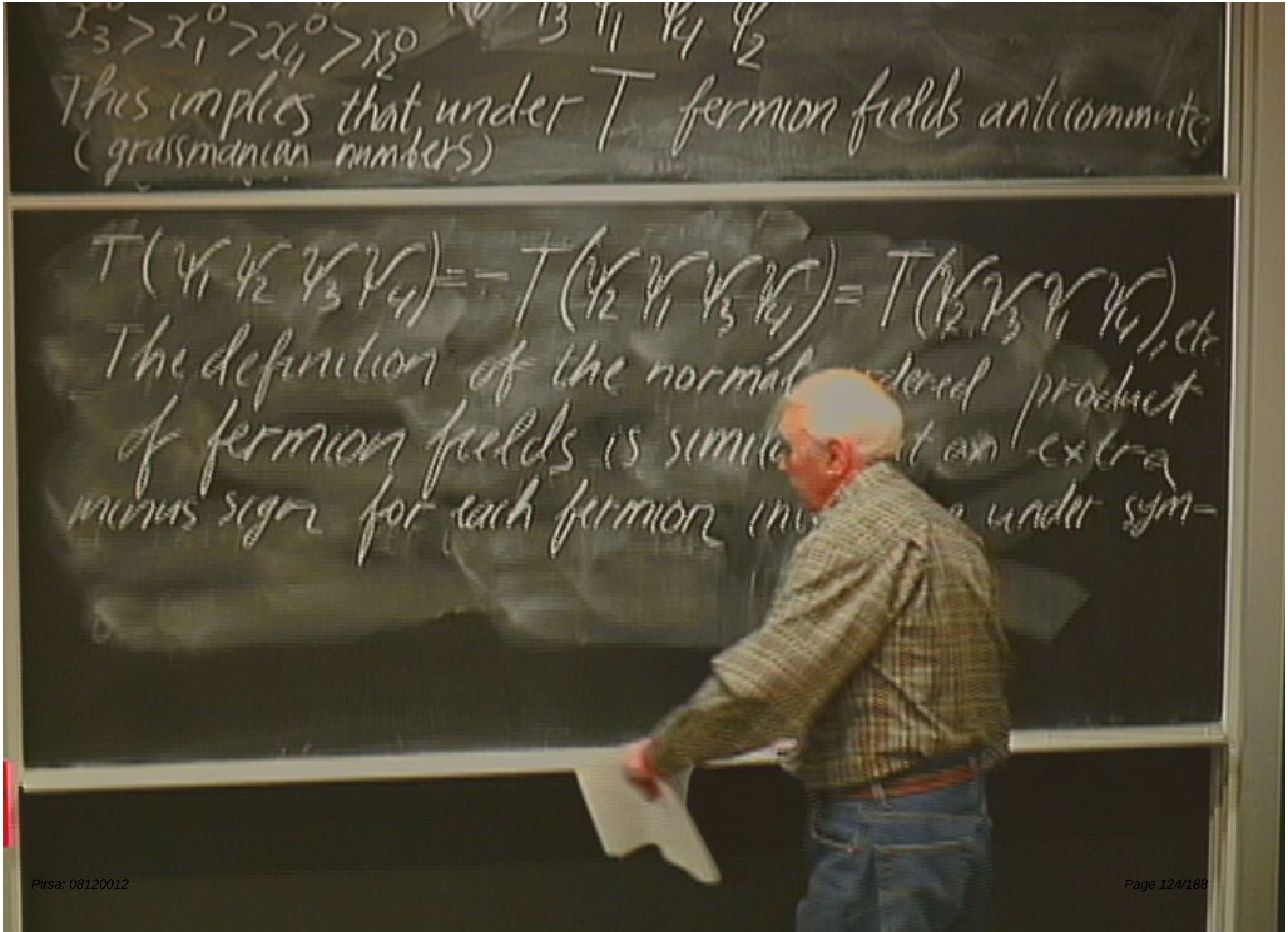
$x_3^0 > x_1^0 > x_4^0 > x_2^0$ $\quad \psi_3\ \psi_1\ \psi_4\ \psi_2$

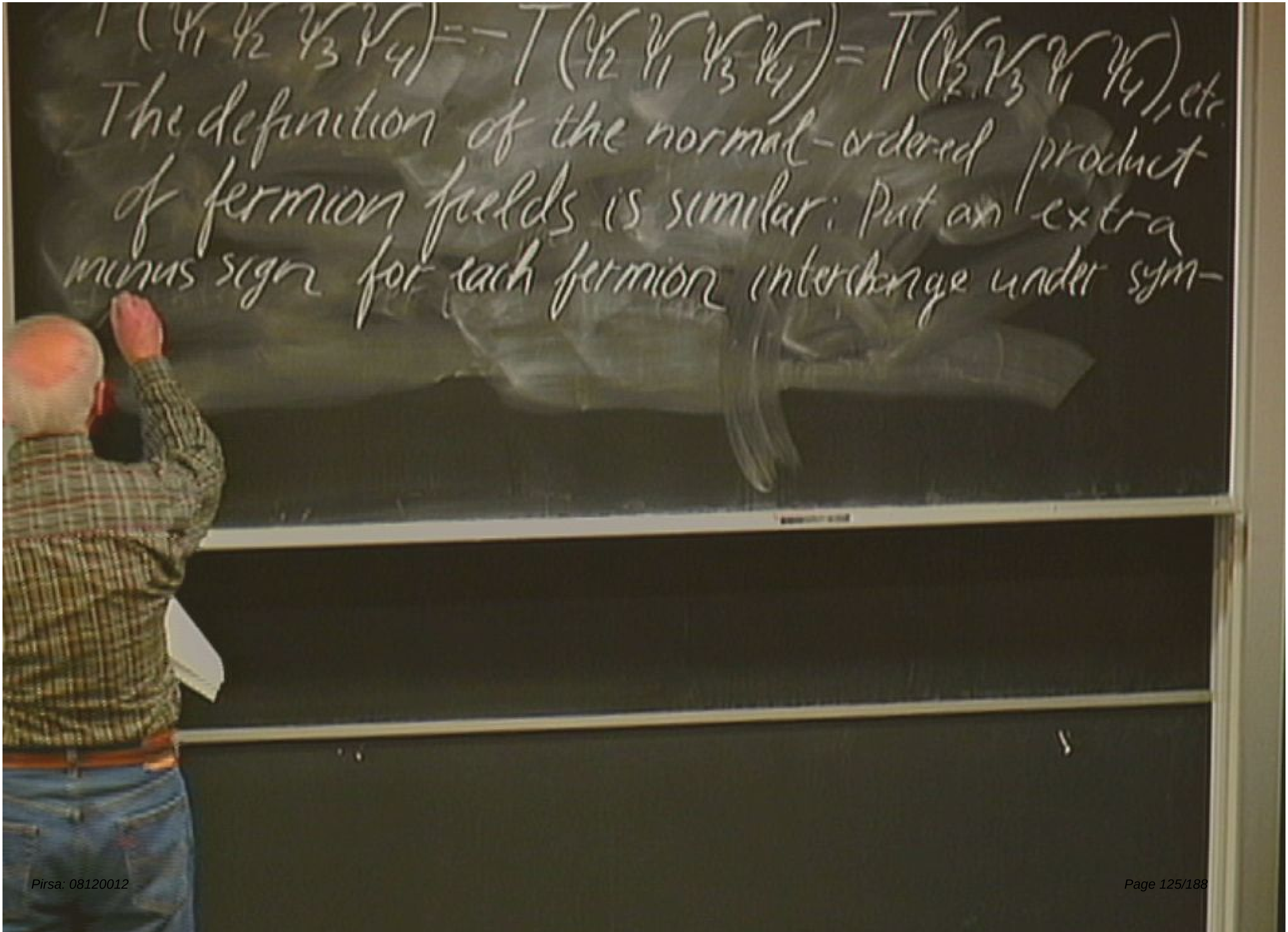
This implies that under T fermion fields anticommute (grassmanian numbers)

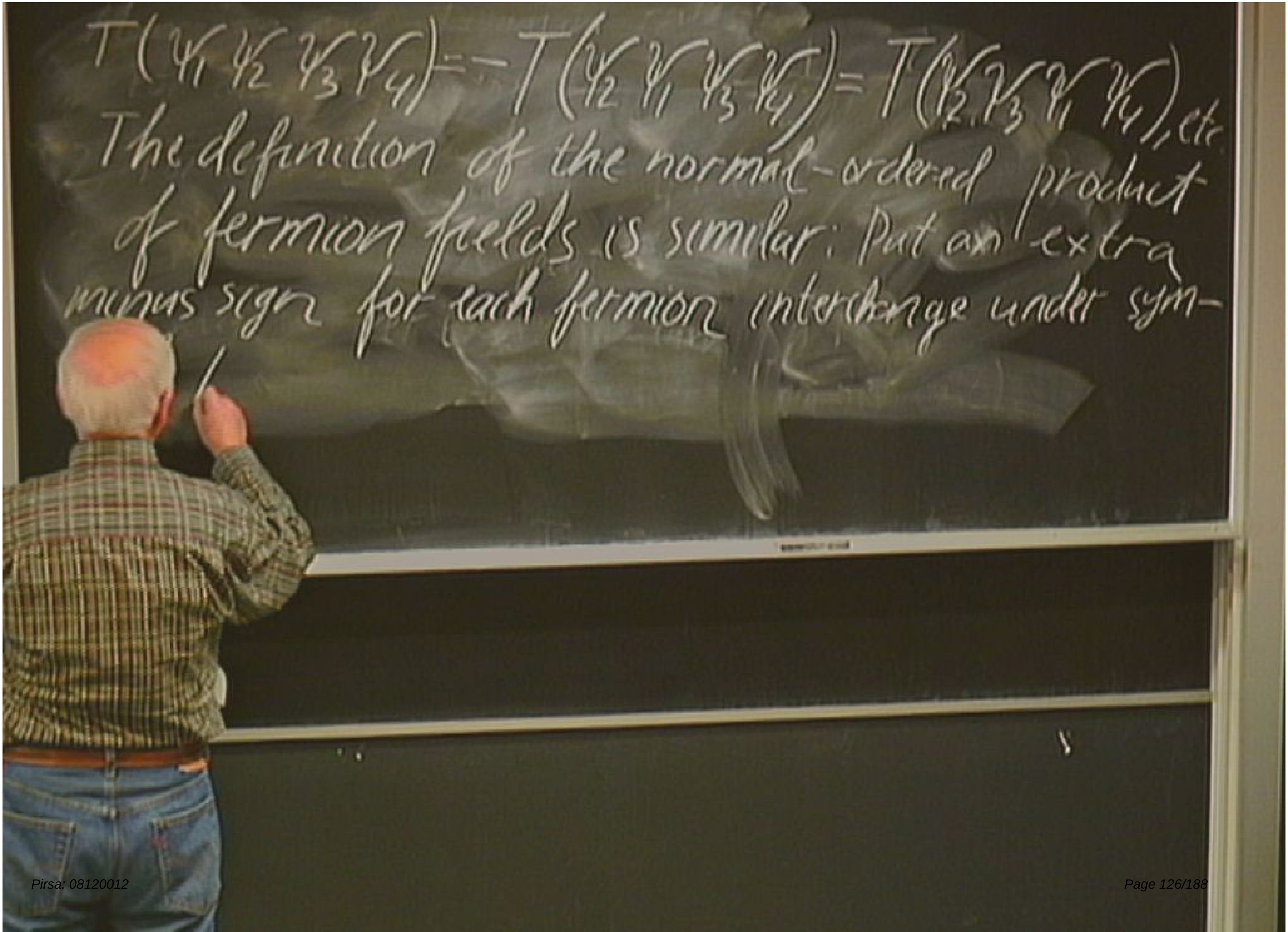
$$T(\psi_1\psi_2\psi_3\psi_4) = -T(\psi_2\psi_1\psi_3\psi_4) = T(\psi_2\psi_3\psi_1\psi_4), \text{etc.}$$

The d


$x_3^0 > x_1^0 > x_4^0 > x_2^0$
$\psi_3 \psi_1 \psi_4 \psi_2$
This implies that under T fermion fields anticommute
(grassmanian numbers)
$T(\psi_1 \psi_2 \psi_3 \psi_4) = -T(\psi_2 \psi_1 \psi_3 \psi_4) = T$
$\psi_3 \psi_1 \psi_4)$, etc
The definition of the normal-
product
of fermion fields is similar:
xtra
minus sign for each fermion inter


$x_3^0 > x_1^0 > x_4^0 > x_2^0$
$\psi_3 \; \psi_1 \; \psi_4 \; \psi_2$
This implies that under T fermion fields anticommute
(grassmanian numbers)
$T(\psi_1 \psi_2 \psi_3 \psi_4) = -T(\psi_2 \psi_1 \psi_3 \psi_4) = T(\psi_2 \psi_3 \psi_1 \psi_4)$, etc
The definition of the normal ordered product
of fermion fields is simil t an extra
minus sign for each fermion under sym-


$T(\psi_1\psi_2\psi_3\psi_4) = -T(\psi_2\psi_1\psi_3\psi_4) = T(\psi_2\psi_3\psi_1\psi_4)$, etc.
The definition of the normal-ordered product
of fermion fields is similar: Put an extra
minus sign for each fermion interchange under sym-


$T(\psi_1\psi_2\psi_3\psi_4) = -T(\psi_2\psi_1\psi_3\psi_4) = T(\psi_2\psi_3\psi_1\psi_4)$, etc
The definition of the normal-ordered product
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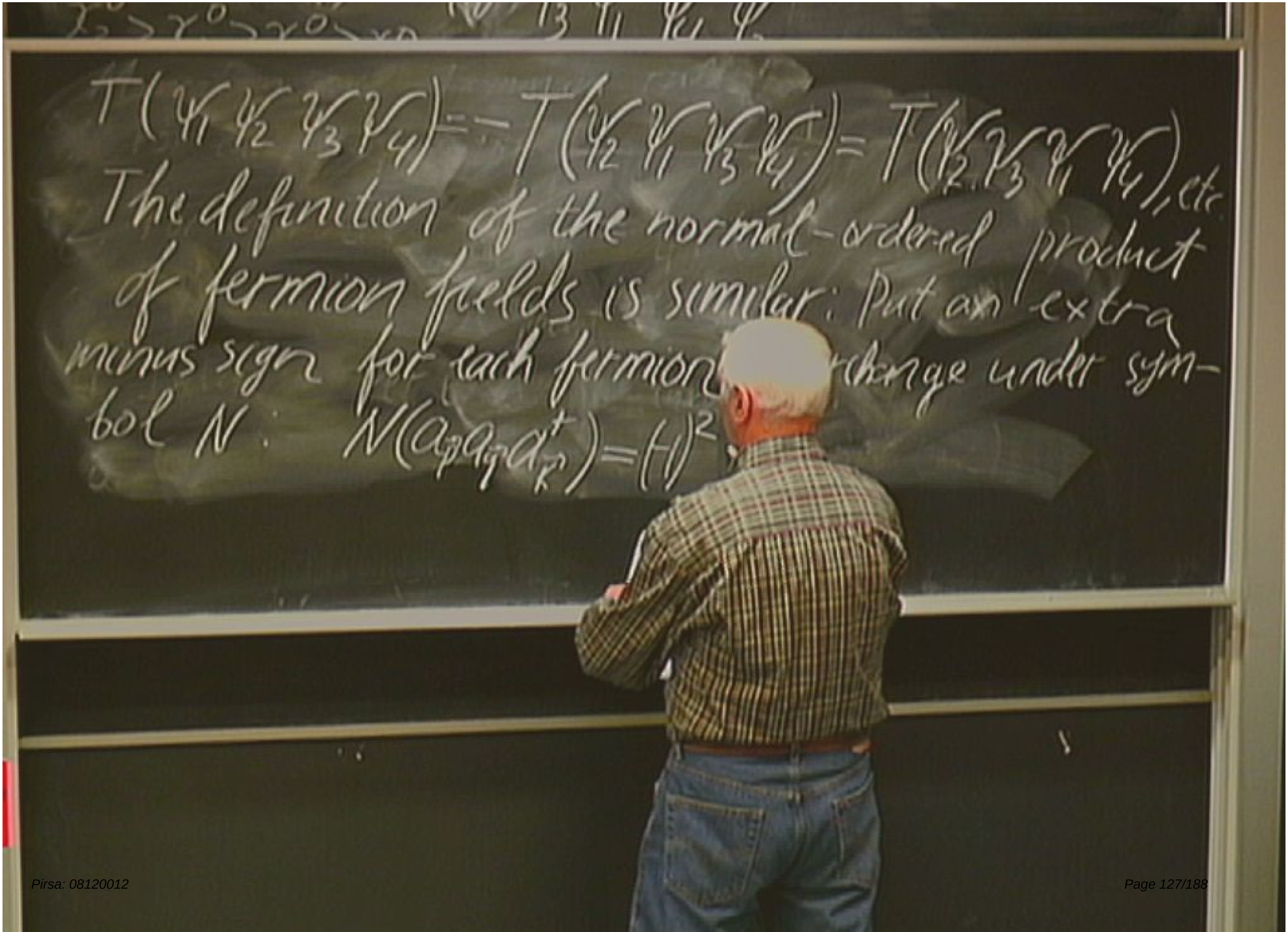
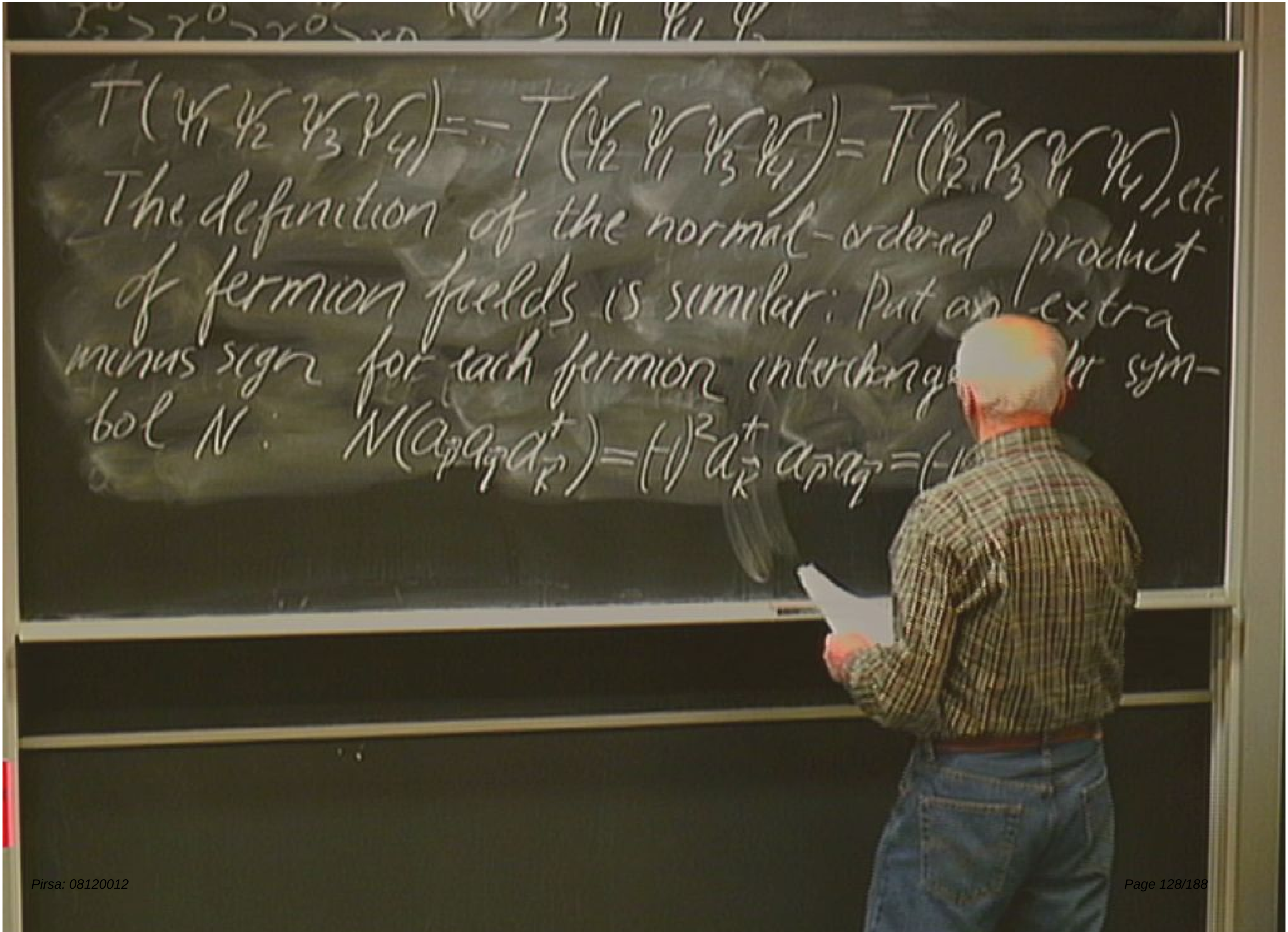
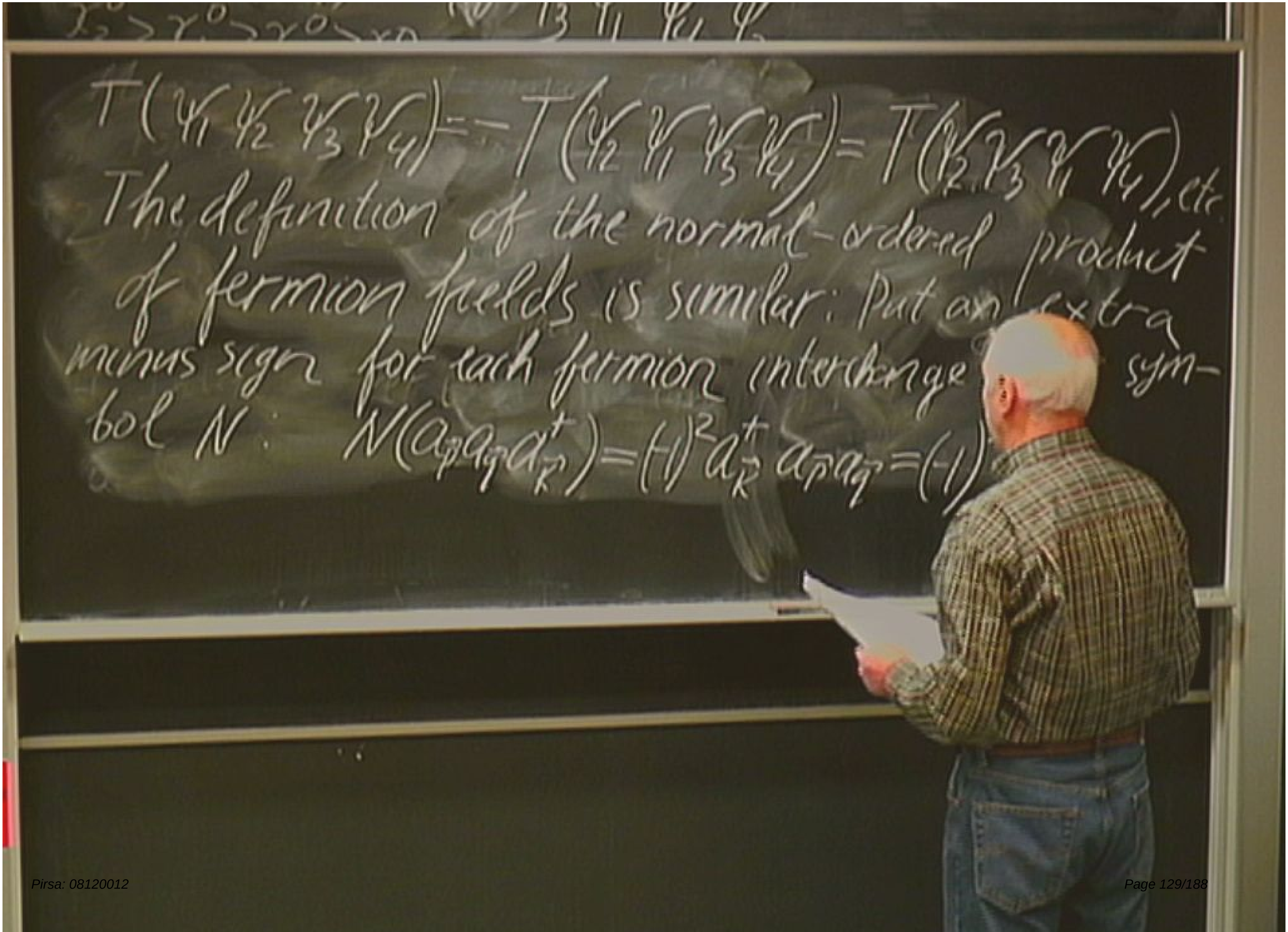

$T(\psi_1\psi_2\psi_3\psi_4) = -T(\psi_2\psi_1\psi_3\psi_4) = T(\psi_2\psi_3\psi_1\psi_4)$, etc
The definition of the normal-ordered product
of fermion fields is similar: Put an extra
minus sign for each fermion interchange under sym-
bol N: $N(a_p a_q a_k^\dagger) = (-)^2$

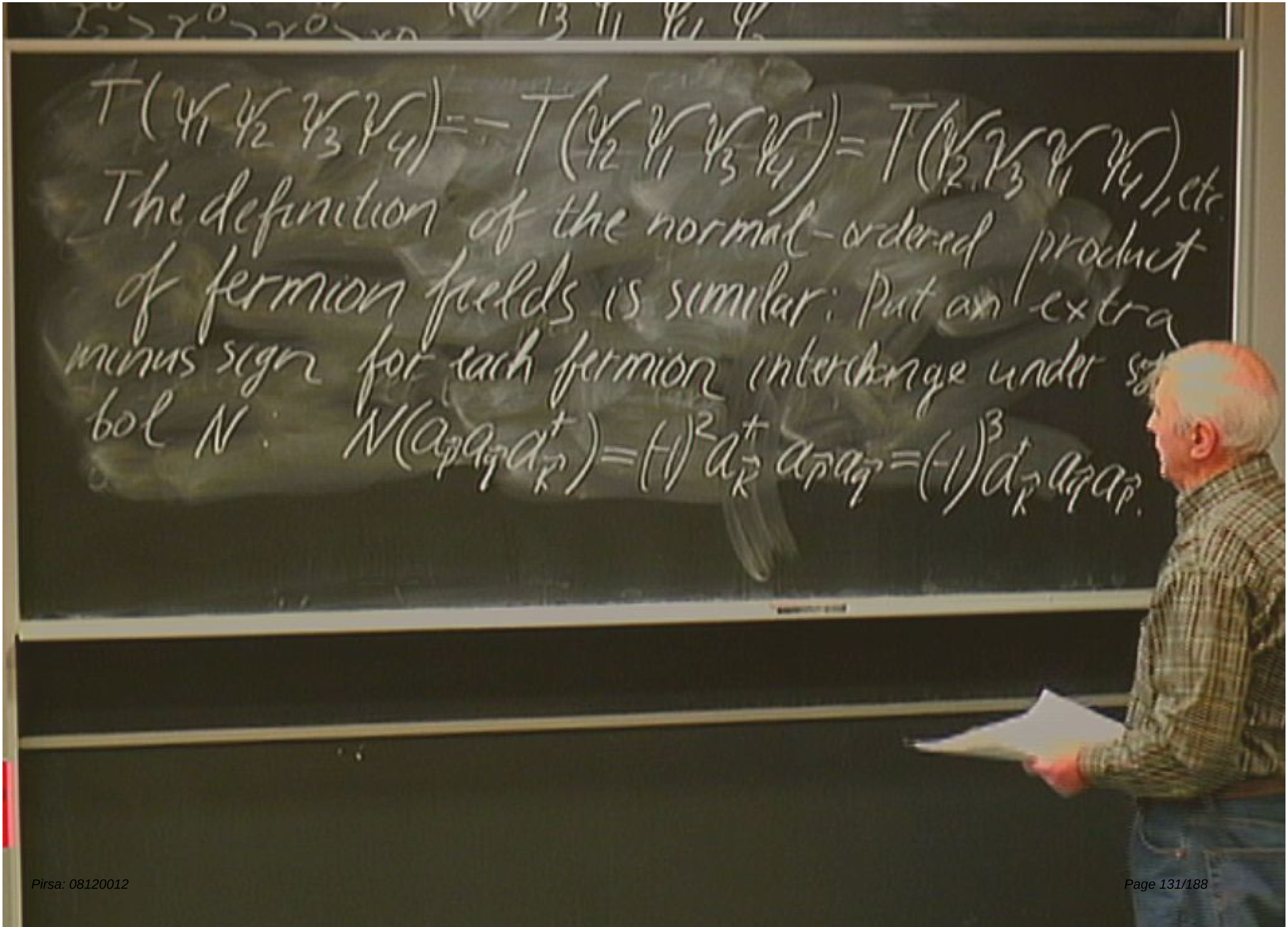

$T(\psi_1\psi_2\psi_3\psi_4) = -T(\psi_2\psi_1\psi_3\psi_4) = T(\psi_2\psi_3\psi_1\psi_4)$, etc
The definition of the normal-ordered product
of fermion fields is similar: Put an extra
minus sign for each fermion interchange
symbol N: $N(a_{\vec{p}}a_{\vec{q}}a^\dagger_{\vec{k}}) = (-1)^2 a^\dagger_{\vec{k}} a_{\vec{p}} a_{\vec{q}} =$

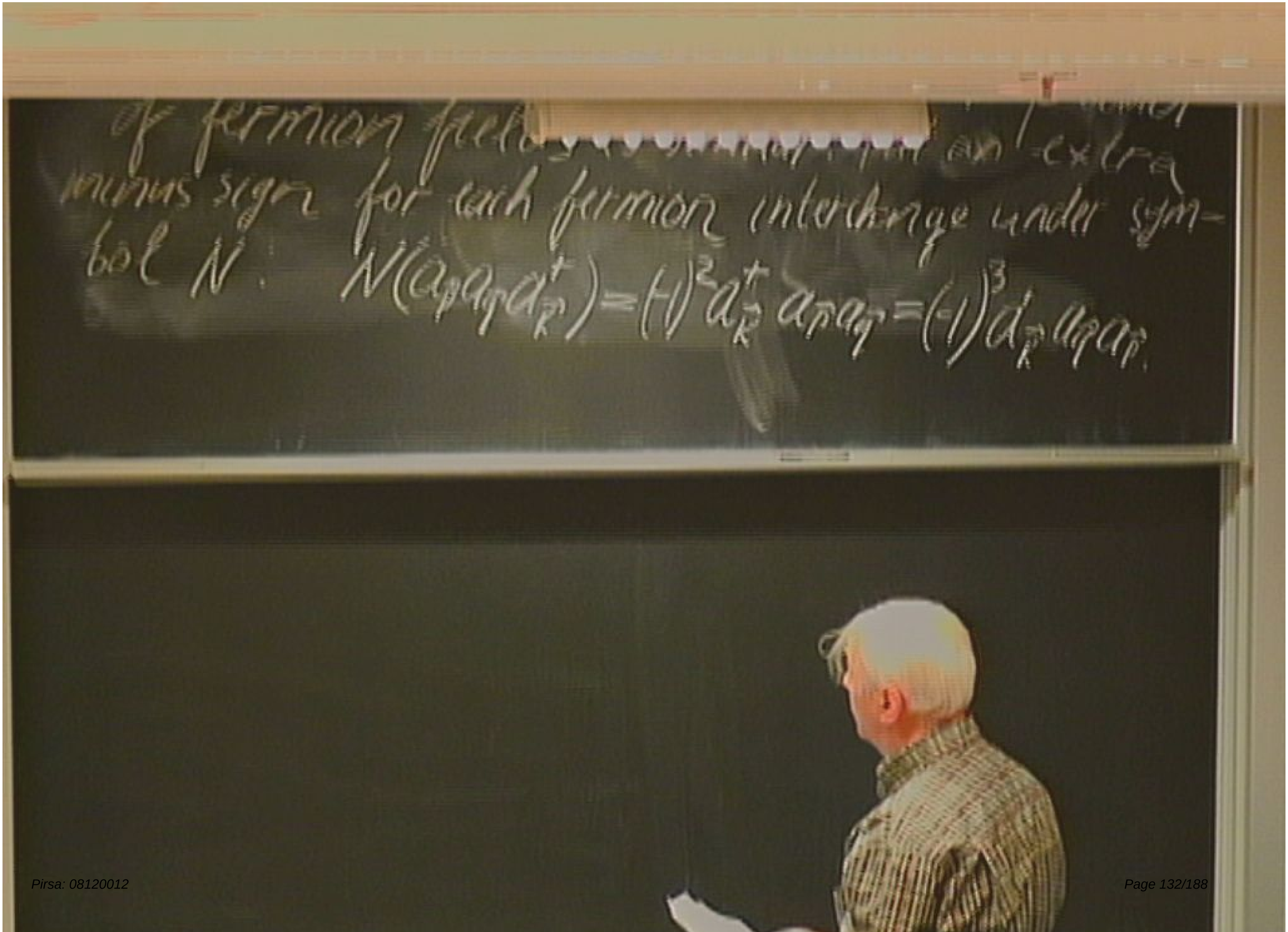


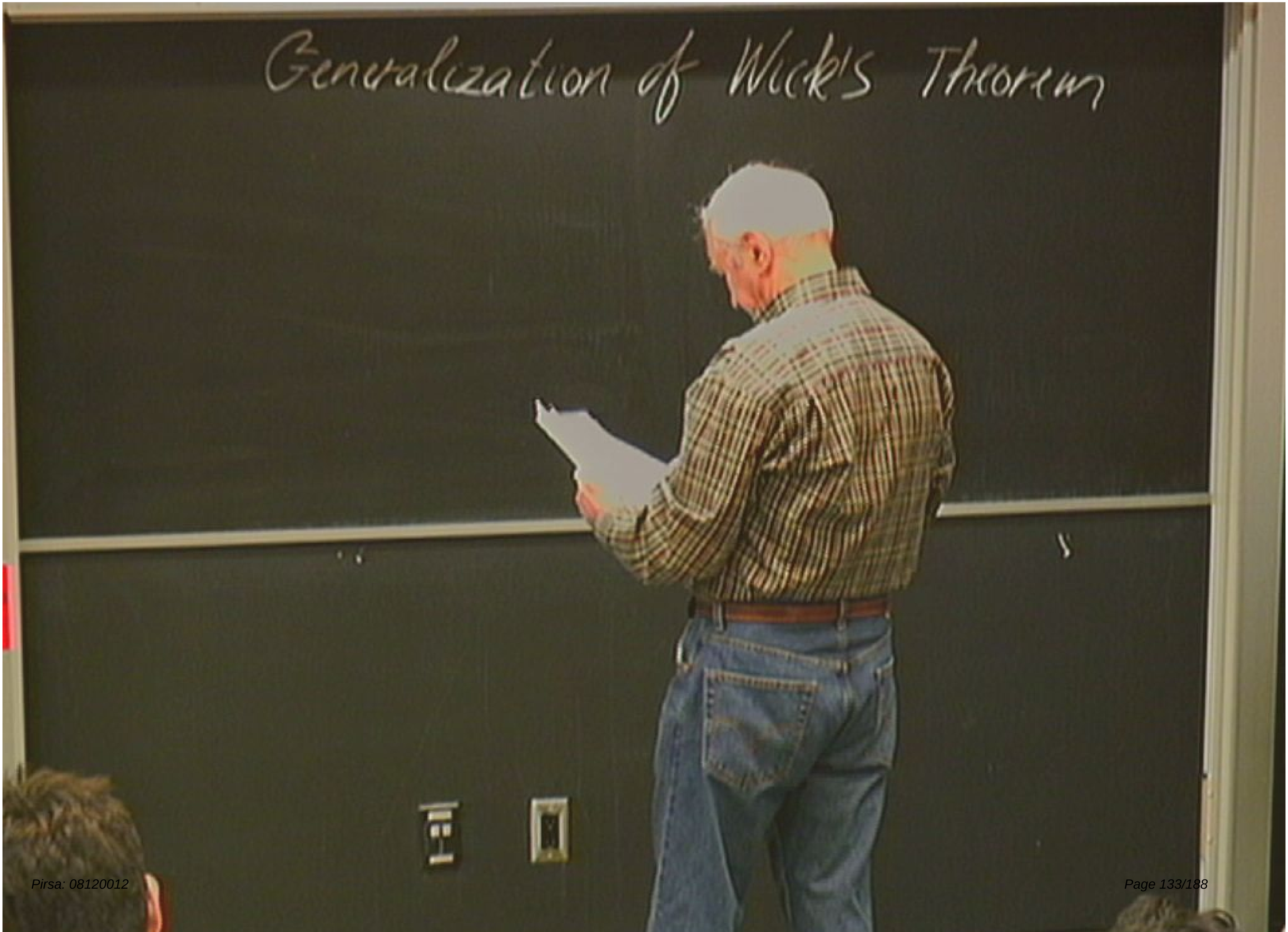
$T(\psi_1\psi_2\psi_3\psi_4) = -T(\psi_2\psi_1\psi_3\psi_4) = T(\psi_2\psi_3\psi_1\psi_4)$, etc
The definition of the normal-ordered product
of fermion fields is similar: Put an extra
minus sign for each fermion interchange sym-
bol N: $N(a_{\vec{p}}a_{\vec{q}}a^\dagger_{\vec{k}}) = (-1)^2 a^\dagger_{\vec{k}} a_{\vec{p}} a_{\vec{q}} = (-1)$

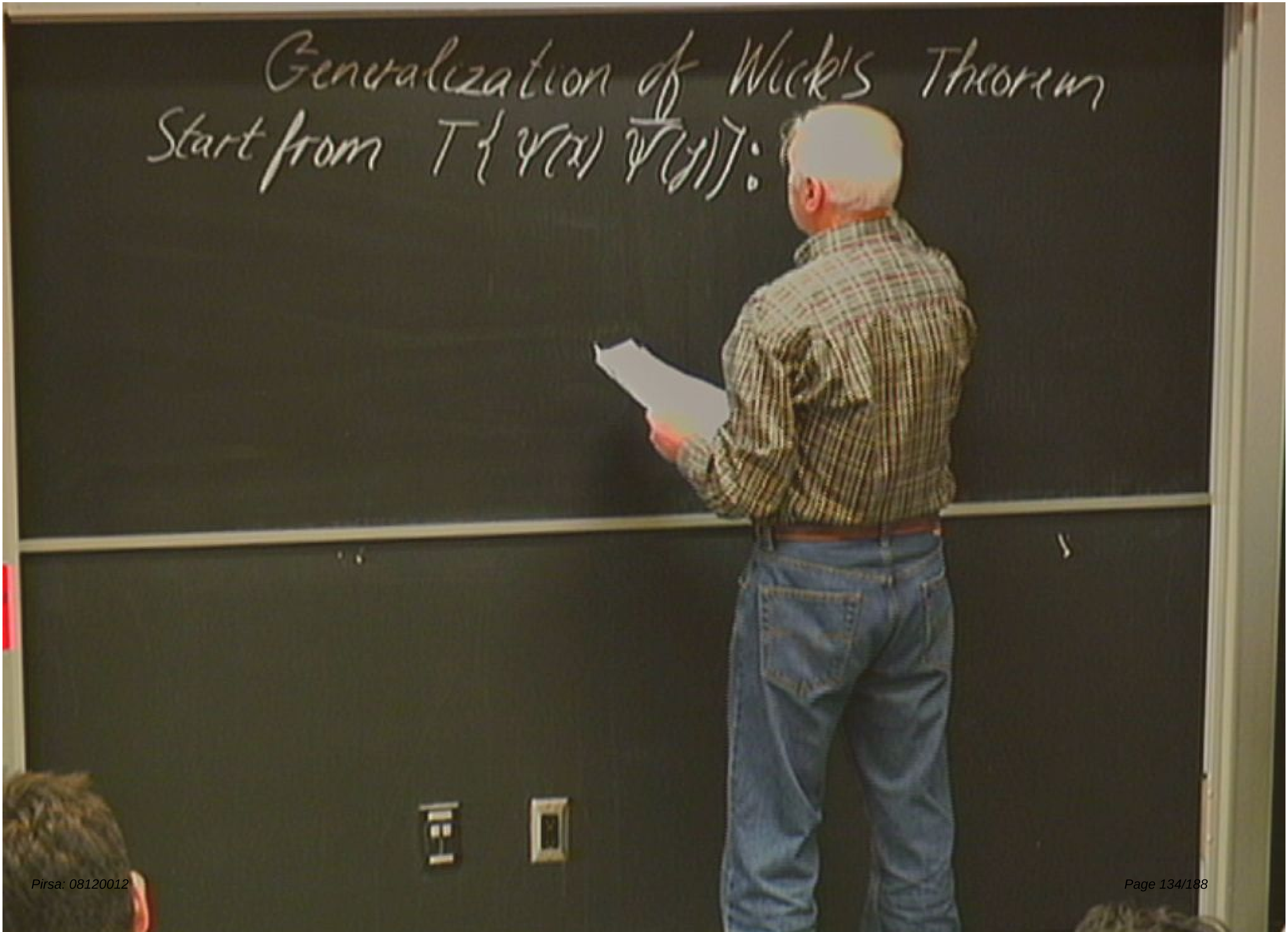
$$T(\psi_1\psi_2\psi_3\psi_4) = -T(\psi_2\psi_1\psi_3\psi_4) = T(\psi_2\psi_3\psi_1\psi_4), \text{ etc}$$

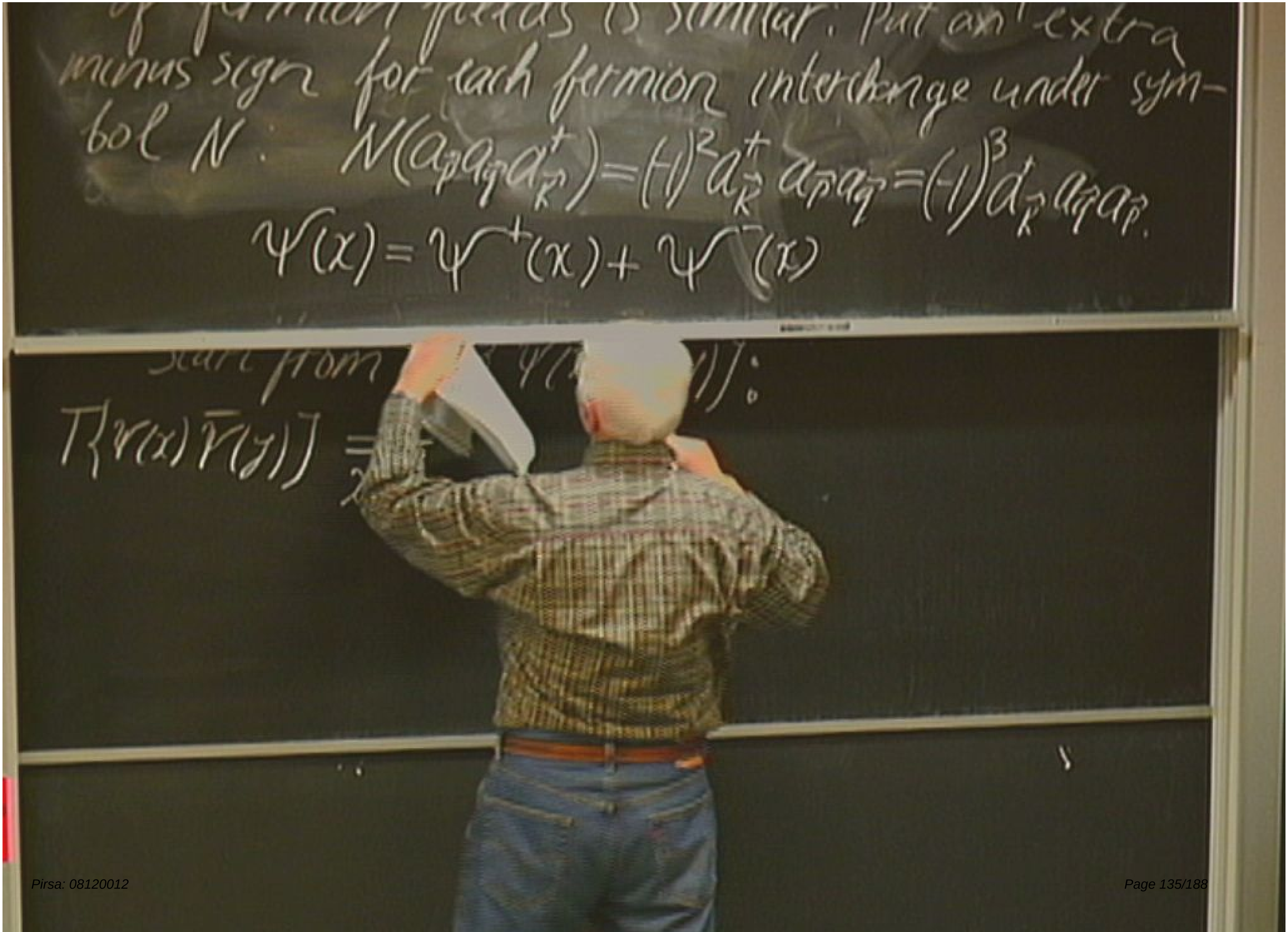
The definition of the normal-ordered product of fermion fields is similar: Put an extra minus sign for each fermion interchange under symbol N: $N(a_{\vec p}\, a_{\vec q}\, a^{\dagger}_{\vec k}) = (-1)^2 a^{\dagger}_{\vec k}\, a_{\vec p}\, a_{\vec q} = (-1)^3$

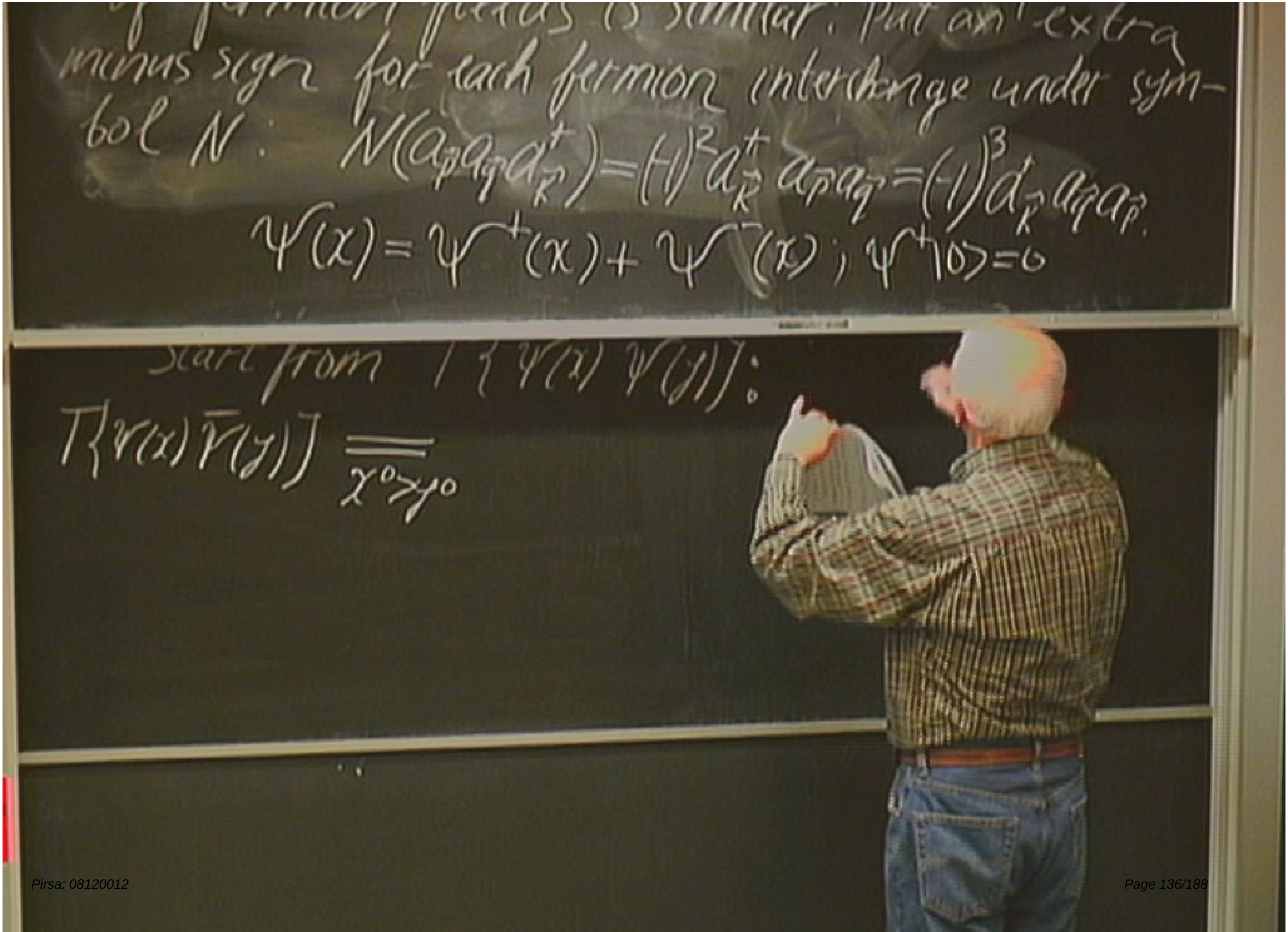

$T(\psi_1\psi_2\psi_3\psi_4) = -T(\psi_2\psi_1\psi_3\psi_4) = T(\psi_2\psi_3\psi_1\psi_4)$, etc.
The definition of the normal-ordered product of fermion fields is similar: Put an extra minus sign for each fermion interchange under symbol N:
$N(a_{\vec p}a_{\vec q}a^\dagger_{\vec k}) = (-1)^2 a^\dagger_{\vec k} a_{\vec p} a_{\vec q} = (-1)^3 a^\dagger_{\vec k} a_{\vec q} a_{\vec p}$.


of fermion fields
an extra
minus sign for each fermion interchange under sym-
bol N. N(a_p a_q a_k^†) = (-1)^2 a_k^† a_p a_q = (-1)^3 a_k a_q a_p.


Generalization of Wick's Theorem


Generalization of Wick's Theorem
Start from $T\{\psi(x)\,\bar{\psi}(y)\}$:


of fermion fields is similar: Put an extra
minus sign for each fermion interchange under sym-
bol N: $N(a_{\vec p}a_{\vec q}a^+_{\vec k}) = (-1)^2 a^+_{\vec k}a_{\vec p}a_{\vec q} = (-1)^3 a^+_{\vec k}a_{\vec q}a_{\vec p}$.
$\psi(x) = \psi^+(x) + \psi^-(x)$
Start from
$T\{\psi(x)\bar{\psi}(y)] =$

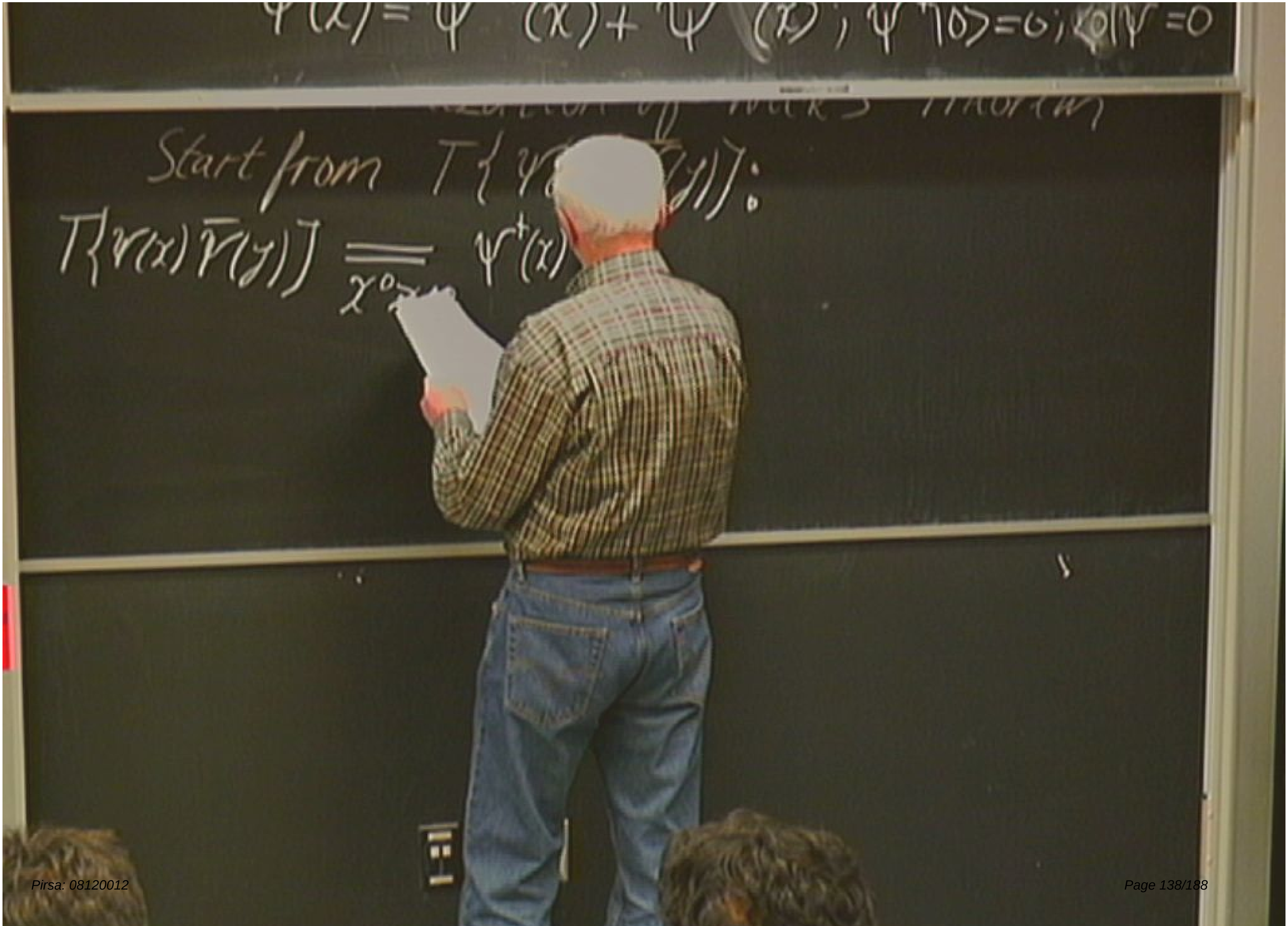

of fermion fields is similar: put an extra
minus sign for each fermion interchange under sym-
bol N: N(a_p a_q a_k^†) = (-1)^2 a_k^† a_p a_q = (-1)^3 a_k^† a_q a_p.
ψ(x) = ψ^+(x) + ψ^-(x); ψ^+|0> = 0
Start from T{ψ(x) ψ̄(y)}:
T{ψ(x) ψ̄(y)} =
x^0 > y^0

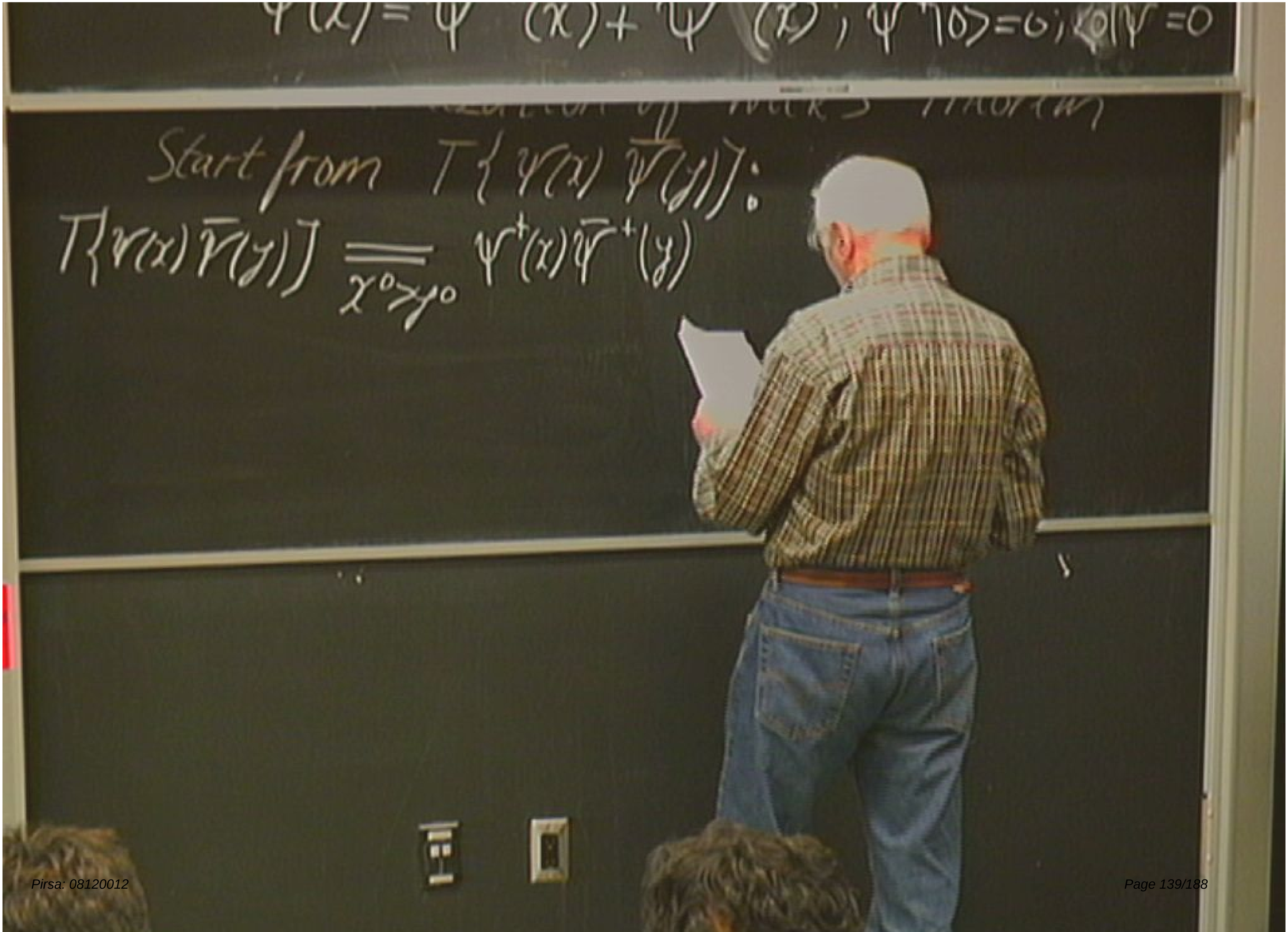
of fermion fields is similar: Put an extra minus sign for each fermion interchange under symbol N: $N(a_{\vec{p}}a_{\vec{q}}a^{\dagger}_{\vec{k}}) = (-1)^2 a^{\dagger}_{\vec{k}} a_{\vec{p}} a_{\vec{q}} = (-1)^3 a^{\dagger}_{\vec{k}} a_{\vec{q}} a_{\vec{p}}$

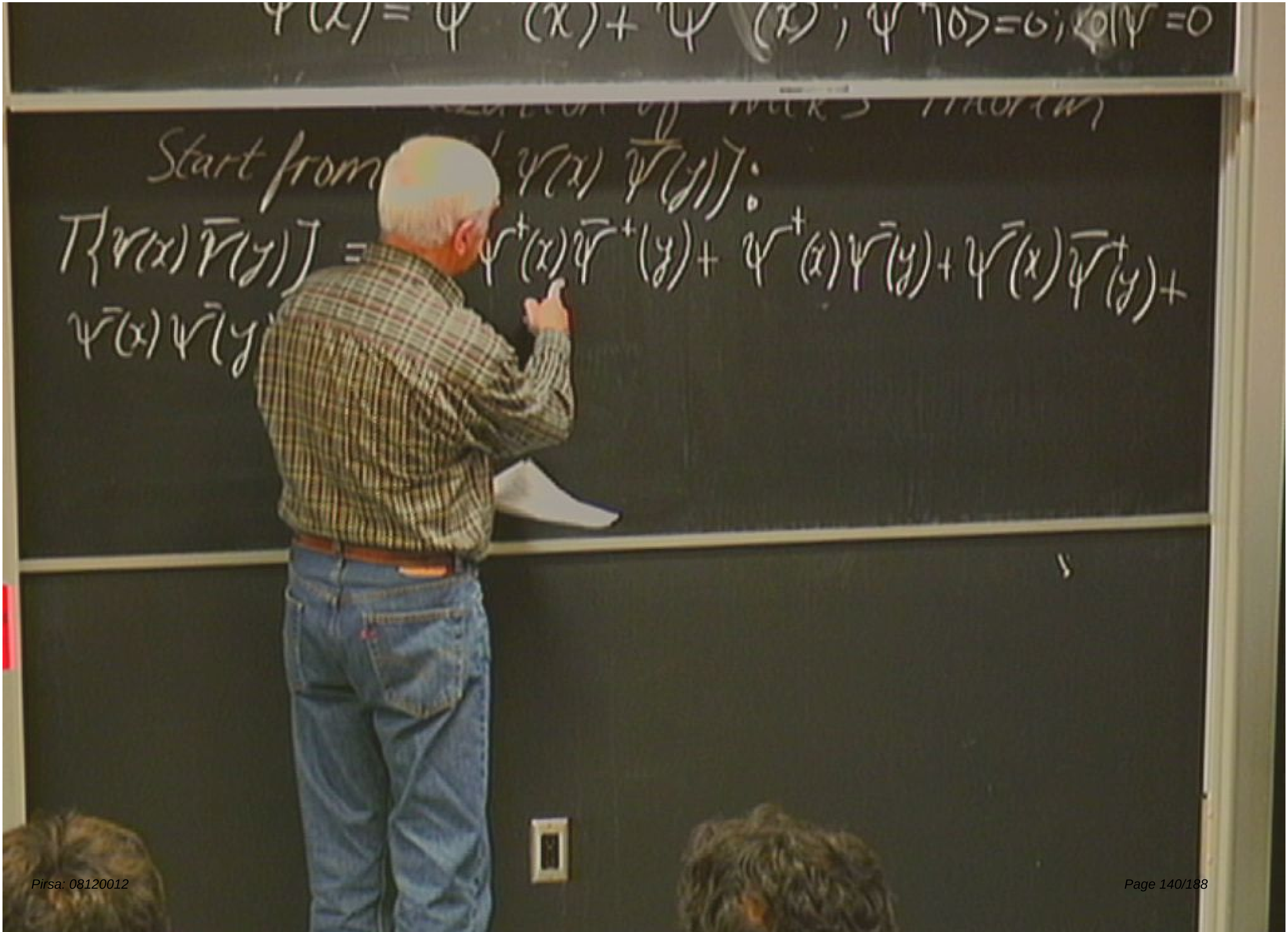
$$\psi(x) = \psi^{+}(x) + \psi^{-}(x)\,;\ \psi^{+}|0\rangle = 0\,;\ \langle 0|\bar{\psi}^{-} = 0$$

Start from $T\{\psi(x)\bar{\psi}(y)\}$:

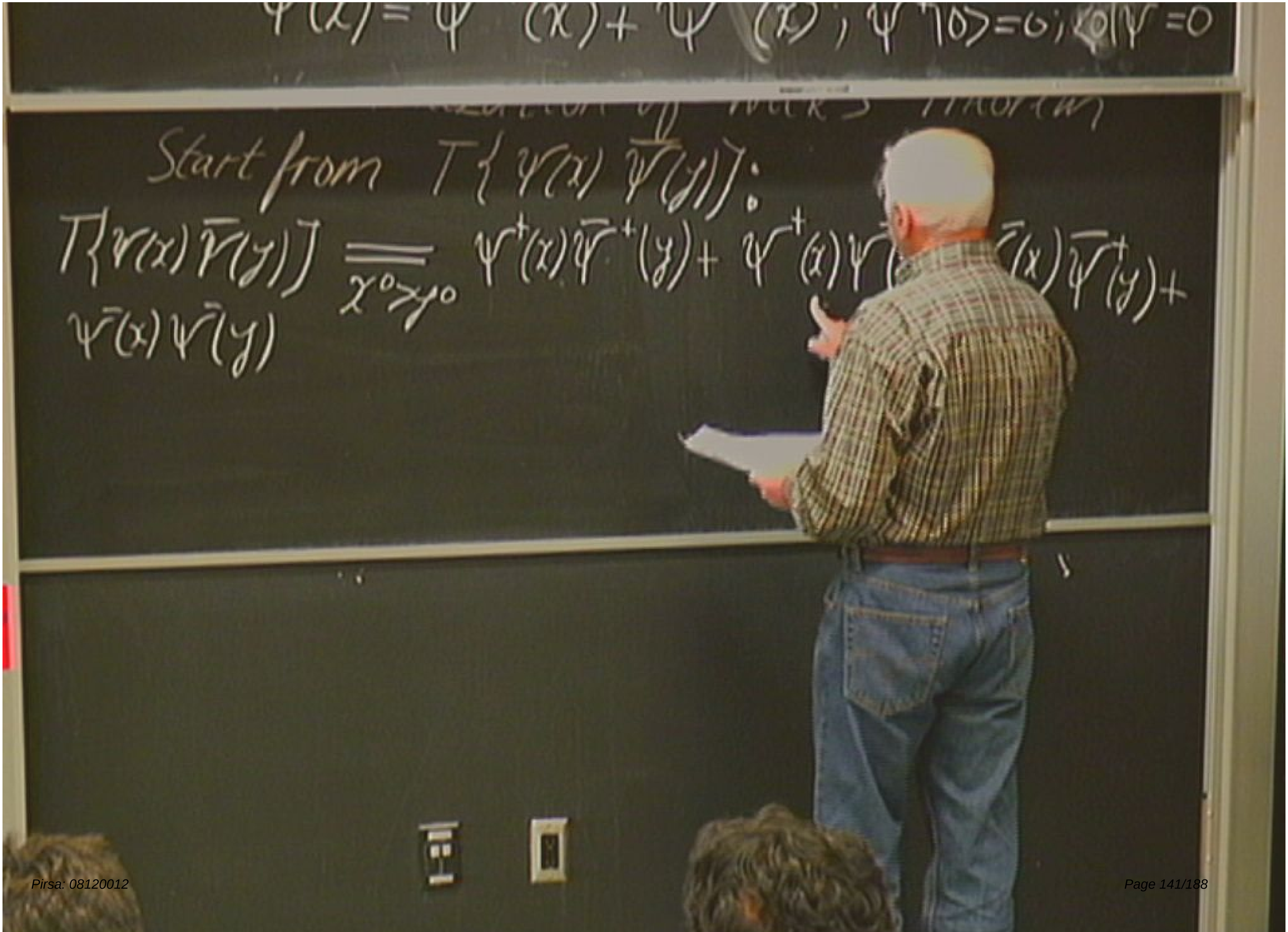
$$T\{\psi(x)\bar{\psi}(y)\} \underset{x^0>y^0}{=}$$

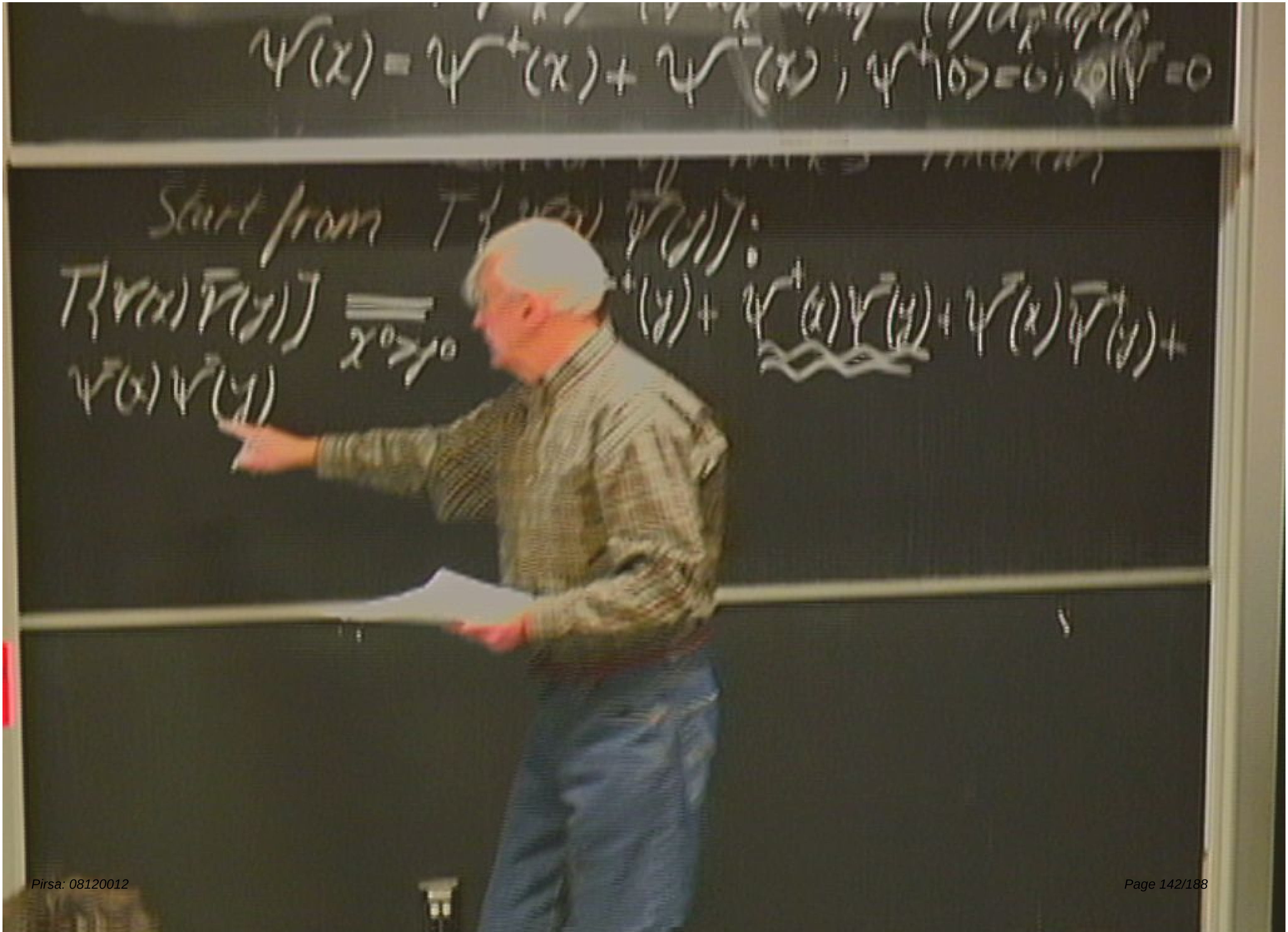

Start from


Start from T{ψ(x) ψ̄(y)}:
T{ψ(x) ψ̄(y)} = ψ⁺(x) ψ̄⁺(y)
x⁰>y⁰

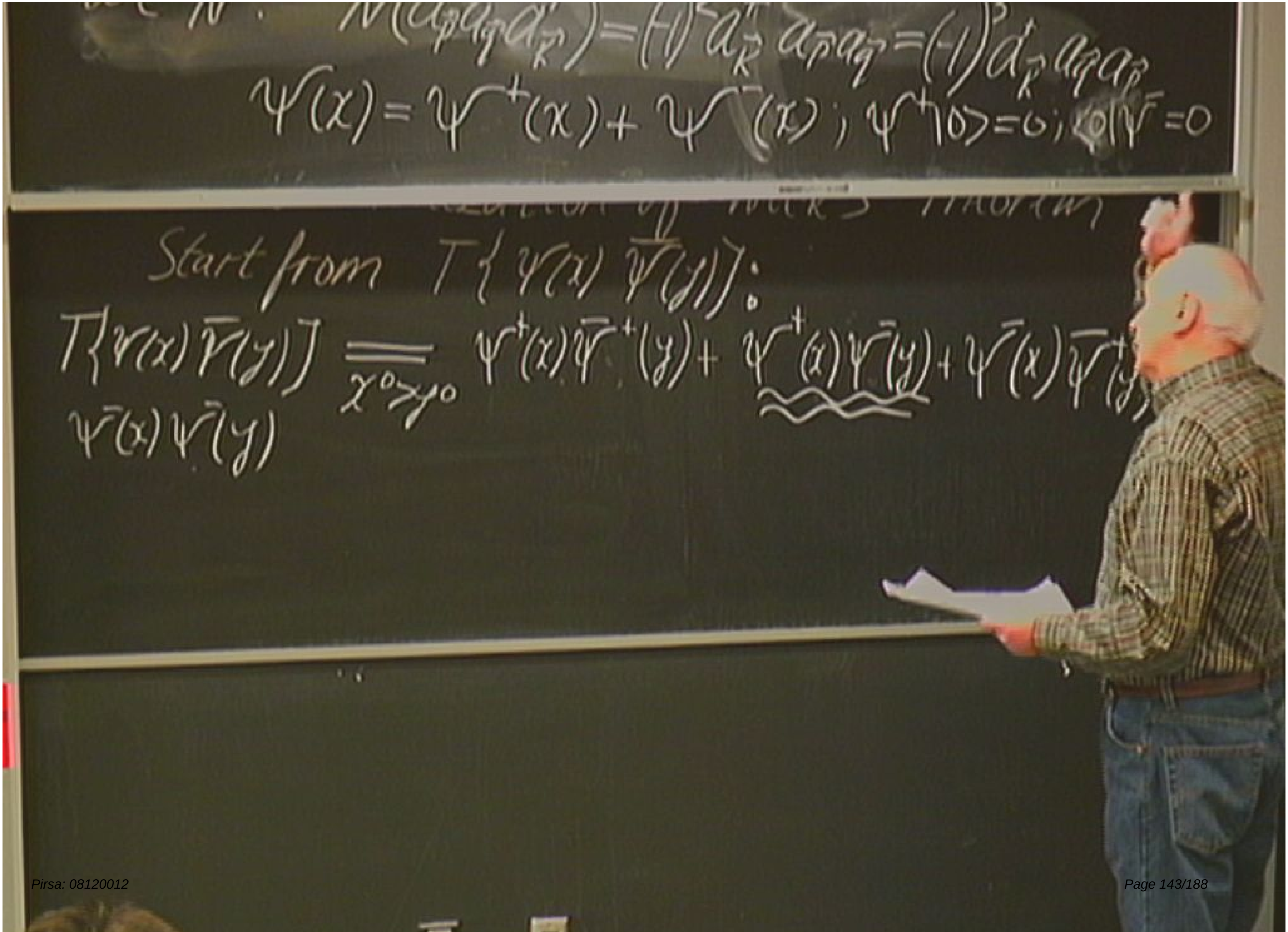


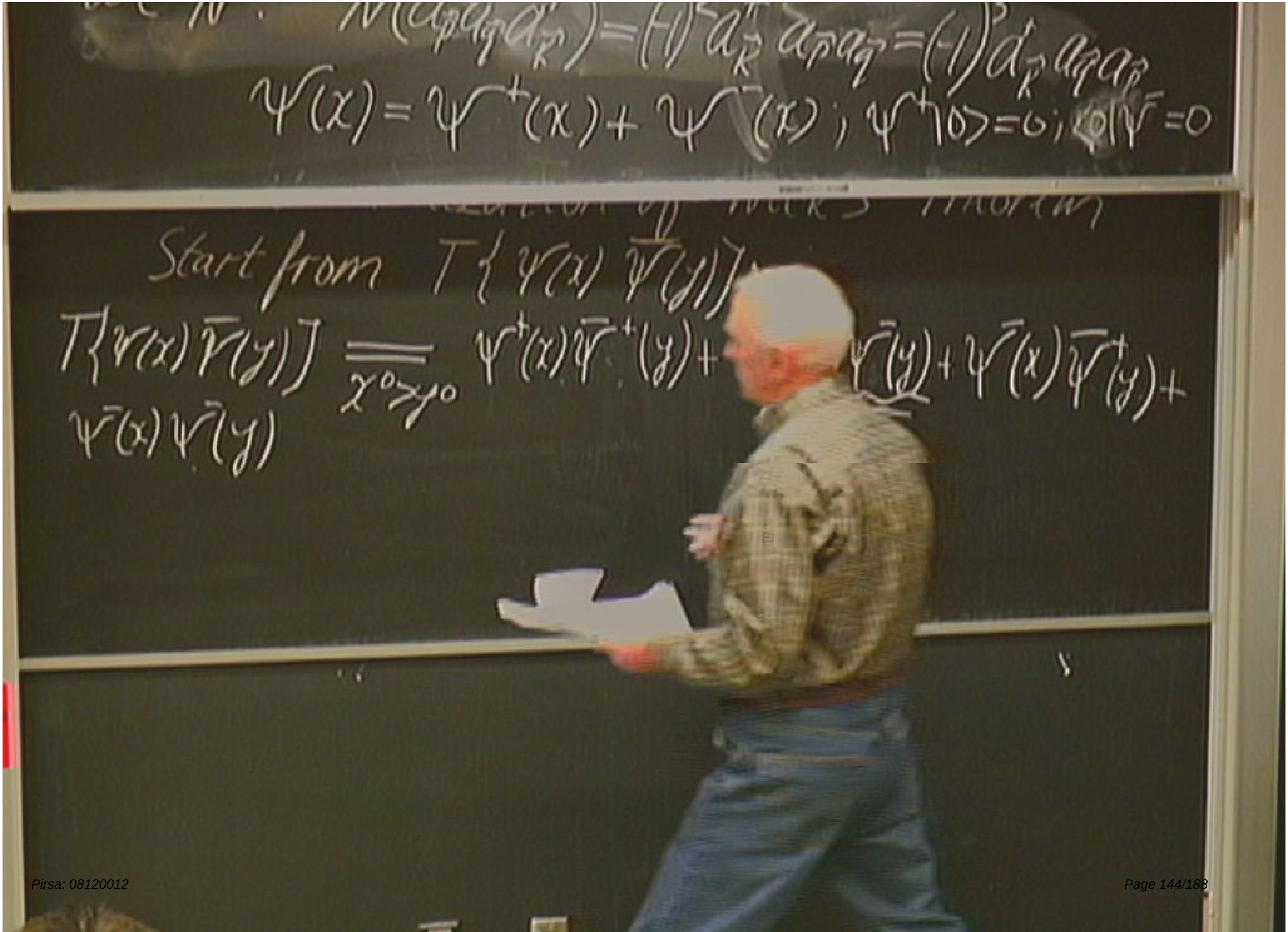
$\psi(x) = \psi^{+}(x) + \psi^{-}(x)$; $\psi^{+}|0\rangle = 0$; $\langle 0|\psi^{-} = 0$
Start from
$T\{\psi(x)\bar{\psi}(y)\} = \psi^{+}(x)\bar{\psi}^{+}(y) + \psi^{+}(x)\bar{\psi}^{-}(y) + \psi^{-}(x)\bar{\psi}^{+}(y) +$
$\psi^{-}(x)\bar{\psi}^{-}(y)$

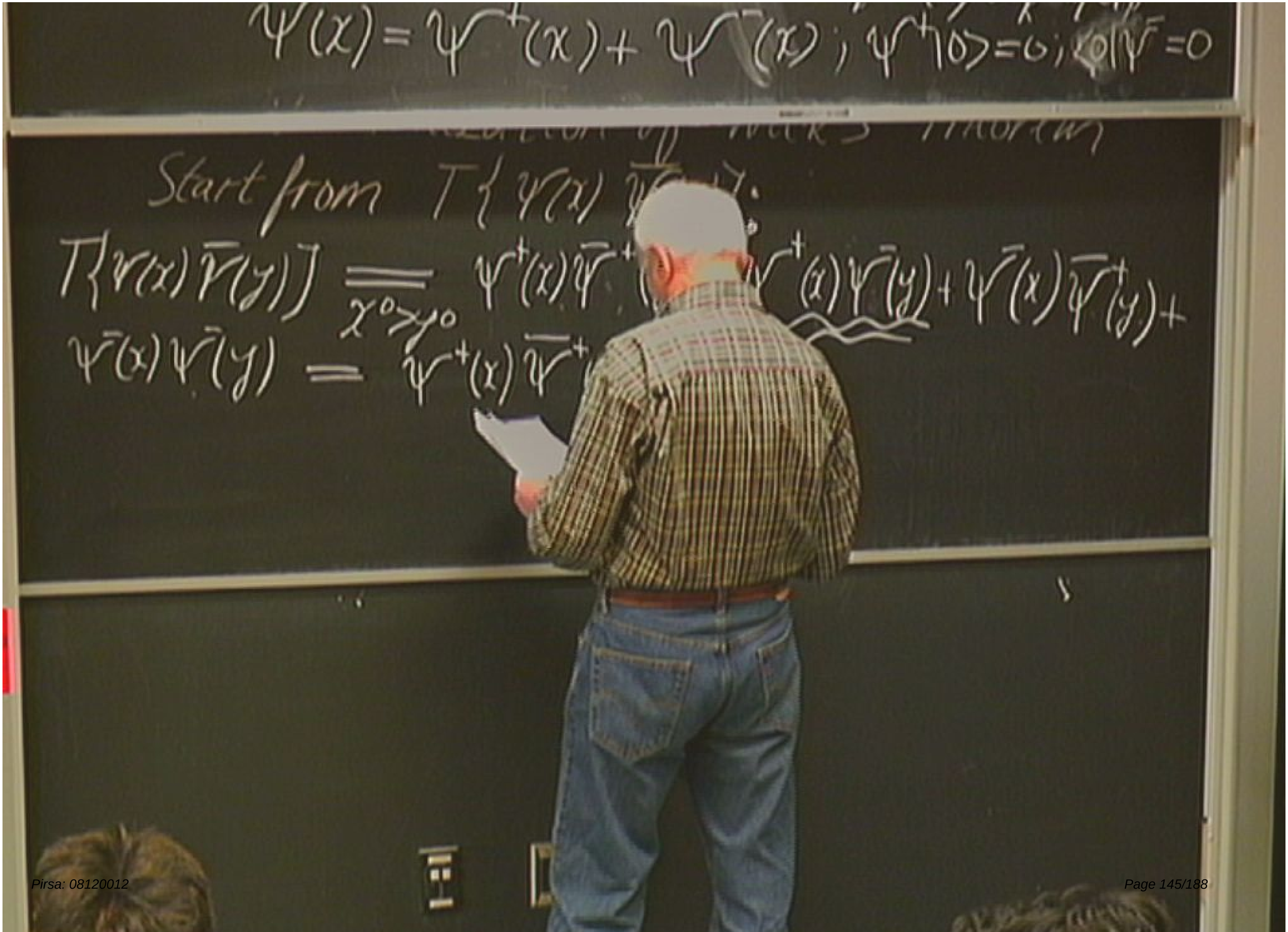

Start from T{ψ(x) ψ̄(y)}:
ψ⁻(x) ψ̄⁻(y)

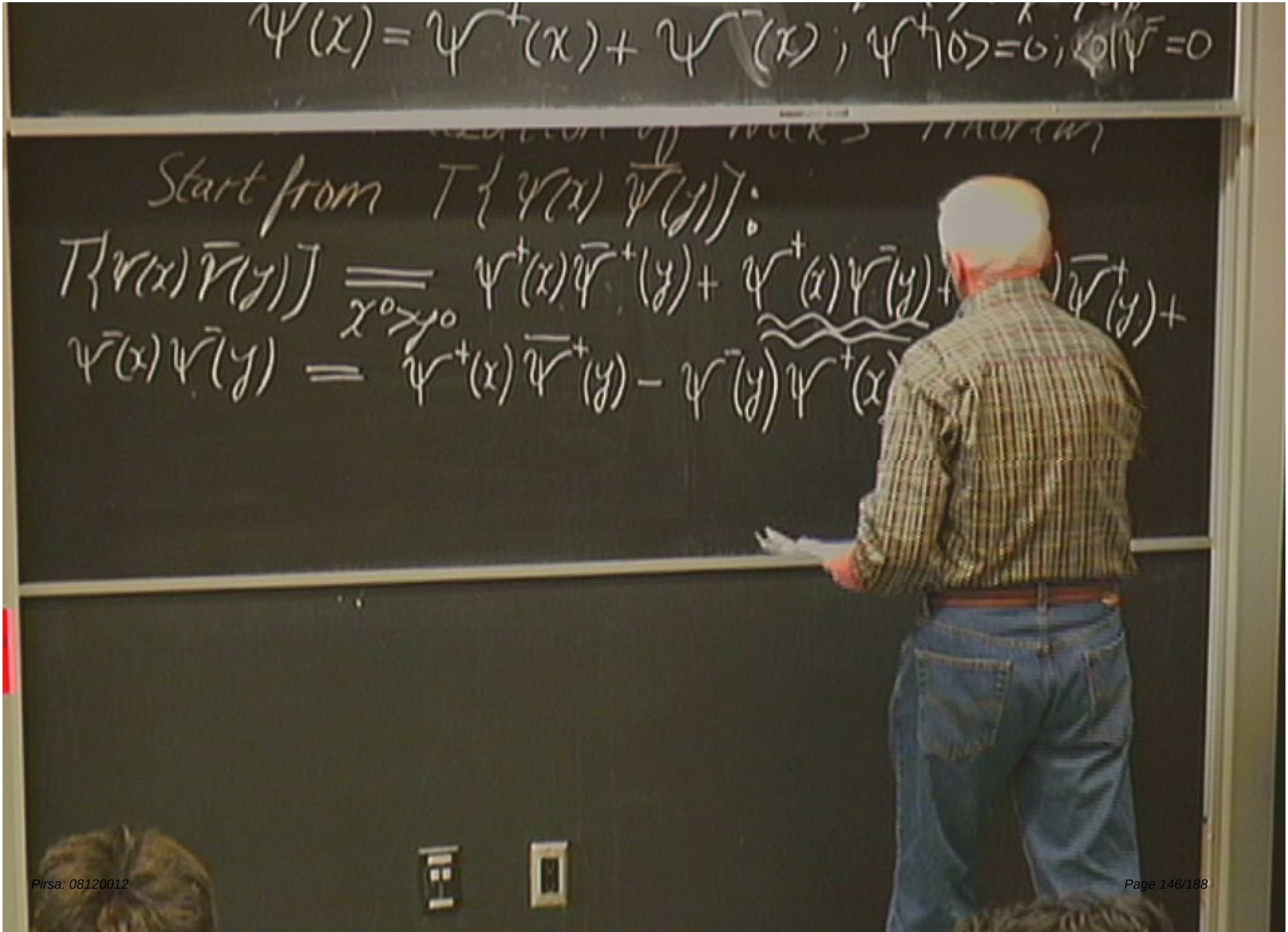


ψ(x) = ψ⁺(x) + ψ⁻(x)
Start from


$\psi(x) = \psi^{+}(x) + \psi^{-}(x) \; ; \; \psi^{+}|0\rangle = 0 \; ; \; \langle 0|\bar{\psi}^{-} = 0$
Start from $T\{\psi(x)\bar{\psi}(y)\}$:
$T\{\psi(x)\bar{\psi}(y)\} \underset{x^0>y^0}{=} \psi^{+}(x)\bar{\psi}^{+}(y) + \psi^{+}(x)\bar{\psi}^{-}(y) + \psi^{-}(x)\bar{\psi}^{+}(y) + \psi^{-}(x)\bar{\psi}^{-}(y)$


$\psi(x) = \psi^{+}(x) + \psi^{-}(x)$; $\psi^{+}|0\rangle = 0$; $\langle 0|\bar\psi^{-} = 0$
Start from $T\{\psi(x)\,\bar\psi(y)\}$
$T\{\psi(x)\bar\psi(y)\} \underset{x^0>y^0}{=} \psi^{+}(x)\bar\psi^{+}(y) +$
$\bar\psi^{-}(y) + \psi^{-}(x)\bar\psi^{+}(y) + \psi^{-}(x)\bar\psi^{-}(y)$


ψ(x) = ψ⁺(x) + ψ⁻(x) ; ψ⁺|0⟩ = 0 ; ⟨0|ψ̄⁻ = 0
Start from


$\psi(x) = \psi^{+}(x) + \psi^{-}(x)\ ;\ \psi^{+}|0\rangle = 0\ ;\ \langle 0|\bar{\psi}^{-} = 0$
Start from $T\{\psi(x)\bar{\psi}(y)\}$:
$T\{\psi(x)\bar{\psi}(y)\} \underset{x^0>y^0}{=} \psi^{+}(x)\bar{\psi}^{+}(y) + \psi^{+}(x)\bar{\psi}^{-}(y) + \ldots \bar{\psi}^{+}(y) +$
$\psi^{-}(x)\bar{\psi}^{-}(y) = \psi^{+}(x)\bar{\psi}^{+}(y) - \bar{\psi}^{-}(y)\psi^{+}(x)$

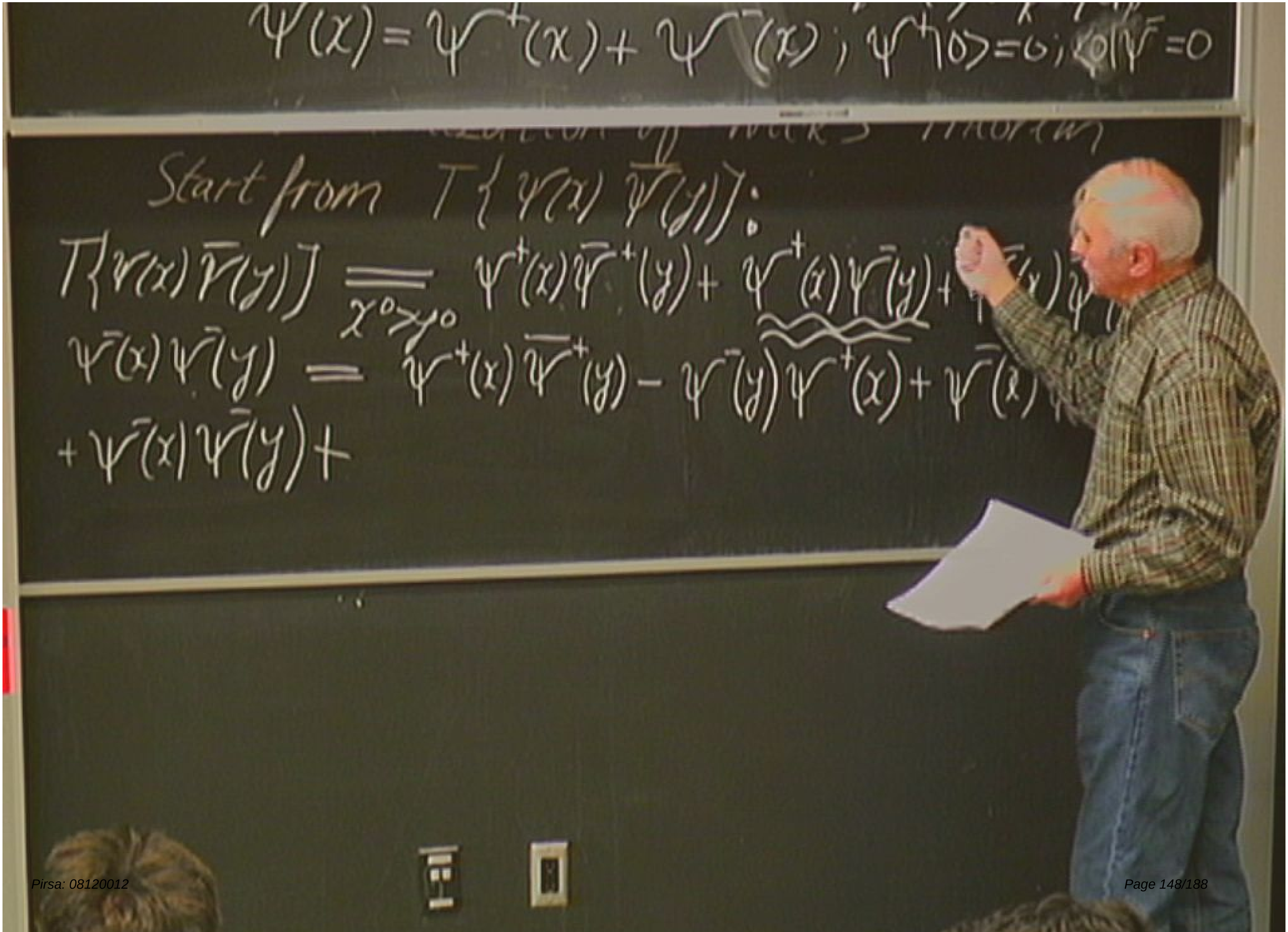
$$\psi(x) = \psi^+(x) + \psi^-(x)\,;\quad \psi^+|0\rangle = 0\,;\quad \langle 0|\bar\psi^- = 0$$

... of Wick's Theorem

Start from $T\{\psi(x)\bar\psi(y)\}$:

$$T\{\psi(x)\bar\psi(y)\} \underset{x^0>y^0}{=} \psi^+(x)\bar\psi^+(y) + \psi^+(x)\bar\psi^-(y) + \psi^-(x)\bar\psi^+(y) +$$

$$\ldots\bar\psi^-(y) = \psi^+(x)\bar\psi^+(y) - \bar\psi^-(y)\psi^+(x) + \psi^-(x)\bar\psi^+(y) +$$


$\psi(x) = \psi^+(x) + \psi^-(x)\,;\ \psi^+|0\rangle = 0\,;\ \langle 0|\bar\psi^- = 0$
Start from $T\{\psi(x)\,\bar\psi(y)\}$:
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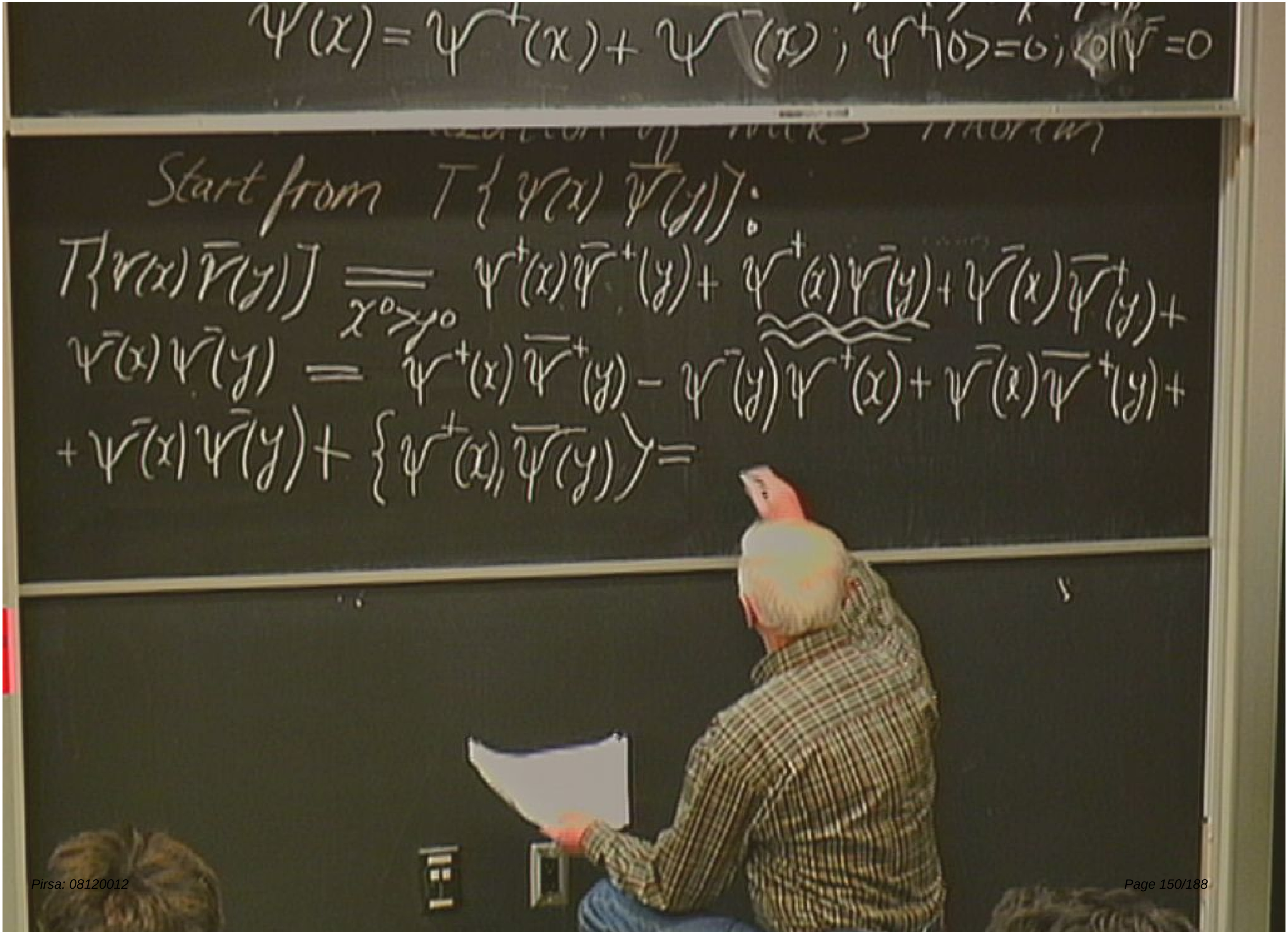
$\psi^-(x)\bar\psi^-(y) = \psi^+(x)\bar\psi^+(y) - \bar\psi^-(y)\psi^+(x) + \psi^-(x)$
$+\ \psi^-(x)\bar\psi^-(y) +$

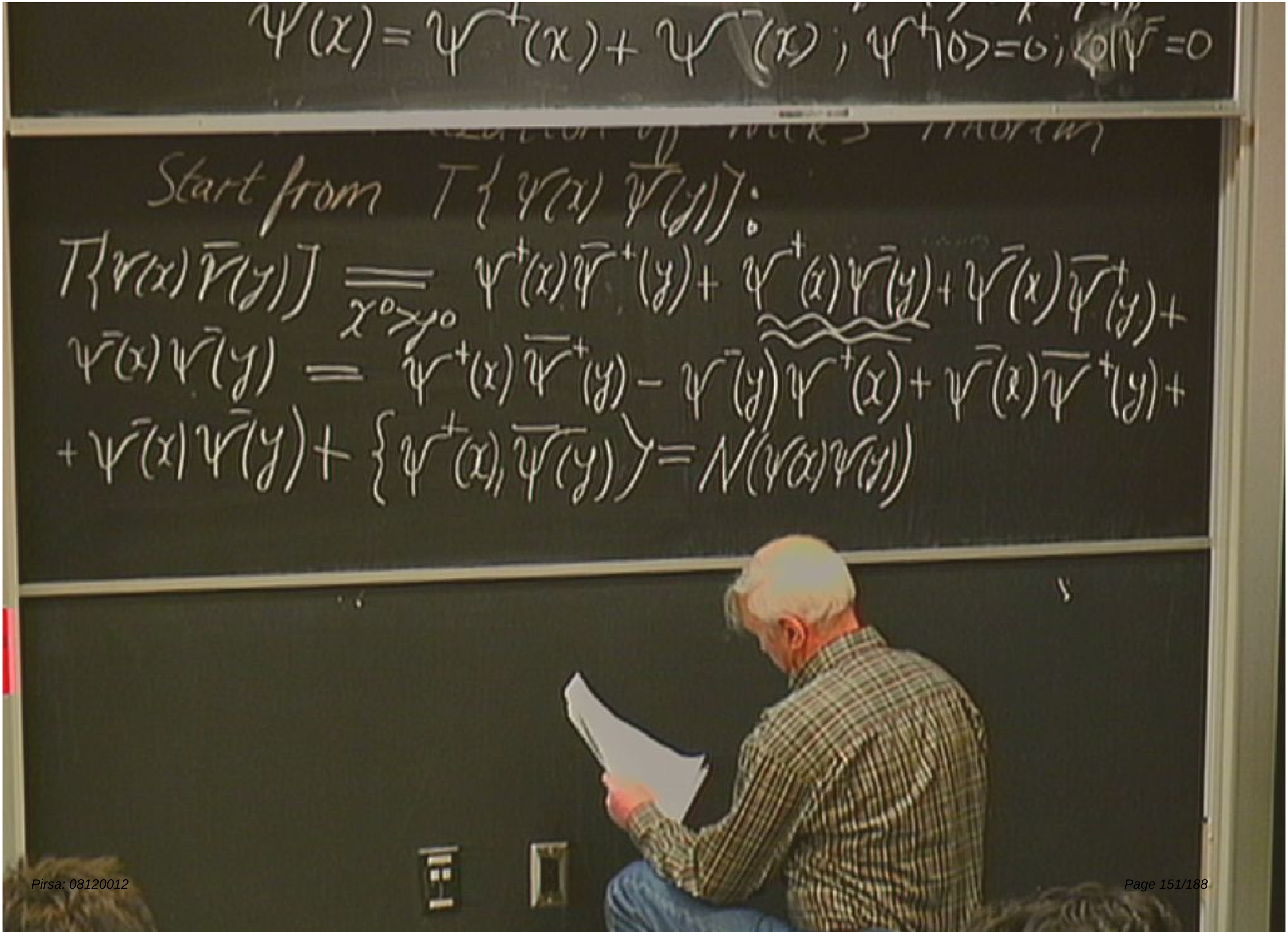
$$\psi(x) = \psi^+(x) + \psi^-(x)\ ;\ \psi^+|0\rangle = 0\ ;\ \langle 0|\bar\psi^- = 0$$

... of Wick's Theorem

Start from $T\{\psi(x)\bar\psi(y)\}$:

$$T\{\psi(x)\bar\psi(y)\} \underset{x^0>y^0}{=} \psi^+(x)\bar\psi^+(y) + \psi^+(x)\bar\psi^-(y) + \psi^-(x)\bar\psi^+(y) + \psi^-(x)\bar\psi^-(y) = \psi^+(x)\bar\psi^+(y) - \bar\psi^-(y)\psi^+(x) + \psi^-(x)\bar\psi^+(y) + \psi^-(x)\bar\psi^-(y) + \{$$


$\psi(x) = \psi^{+}(x) + \psi^{-}(x)\,;\ \psi^{+}|0\rangle = 0\,;\ \langle 0|\bar\psi^{-} = 0$
Start from $T\{\psi(x)\bar\psi(y)\}$:
$T\{\psi(x)\bar\psi(y)\} \underset{x^0>y^0}{=} \psi^{+}(x)\bar\psi^{+}(y) + \psi^{+}(x)\bar\psi^{-}(y) + \psi^{-}(x)\bar\psi^{+}(y) +$
$\psi^{-}(x)\bar\psi^{-}(y) = \psi^{+}(x)\bar\psi^{+}(y) - \bar\psi^{-}(y)\psi^{+}(x) + \psi^{-}(x)\bar\psi^{+}(y) +$
$+\psi^{-}(x)\bar\psi^{-}(y) + \{\psi^{+}(x), \bar\psi^{-}(y)\} =$


$\psi(x) = \psi^+(x) + \psi^-(x)\,;\ \psi^+|0\rangle = 0\,;\ \langle 0|\bar\psi^- = 0$
Start from $T\{\psi(x)\,\bar\psi(y)\}$:
$T\{\psi(x)\bar\psi(y)\} \underset{x^0>y^0}{=\!=} \psi^+(x)\bar\psi^+(y) + \psi^+(x)\bar\psi^-(y) + \psi^-(x)\bar\psi^+(y) +$
$\psi^-(x)\bar\psi^-(y) = \psi^+(x)\bar\psi^+(y) - \bar\psi^-(y)\psi^+(x) + \psi^-(x)\bar\psi^+(y) +$
$+ \psi^-(x)\bar\psi^-(y) + \{\psi^+(x), \bar\psi^-(y)\} = N(\psi(x)\bar\psi(y))$

$$\psi(x) = \psi^+(x) + \psi^-(x)\,;\quad \psi^+|0\rangle = 0\,;\quad \langle 0|\bar\psi^- = 0$$

...ation of Wick's Theorem

Start from $T\{\psi(x)\bar\psi(y)\}$:

$$T\{\psi(x)\bar\psi(y)\} \underset{x^0>y^0}{=} \psi^+(x)\bar\psi^+(y) + \psi^+(x)\bar\psi^-(y) + \psi^-(x)\bar\psi^+(y) + \psi^-(x)\bar\psi^-(y)$$

$$= \psi^+(x)\bar\psi^+(y) - \bar\psi^-(y)\psi^+(x) + \psi^-(x)\bar\psi^+(y) + \psi^-(x)\bar\psi^-(y) + \{\psi^+(x),\bar\psi^-(y)\} = N(\psi(x)\bar\psi(y)) + \{\psi^+(x),\bar\psi^-(y)\}$$

$$\psi(x) = \psi^+(x) + \psi^-(x)\,;\quad \psi^+|0\rangle = 0\,;\quad \langle 0|\bar\psi^- = 0$$

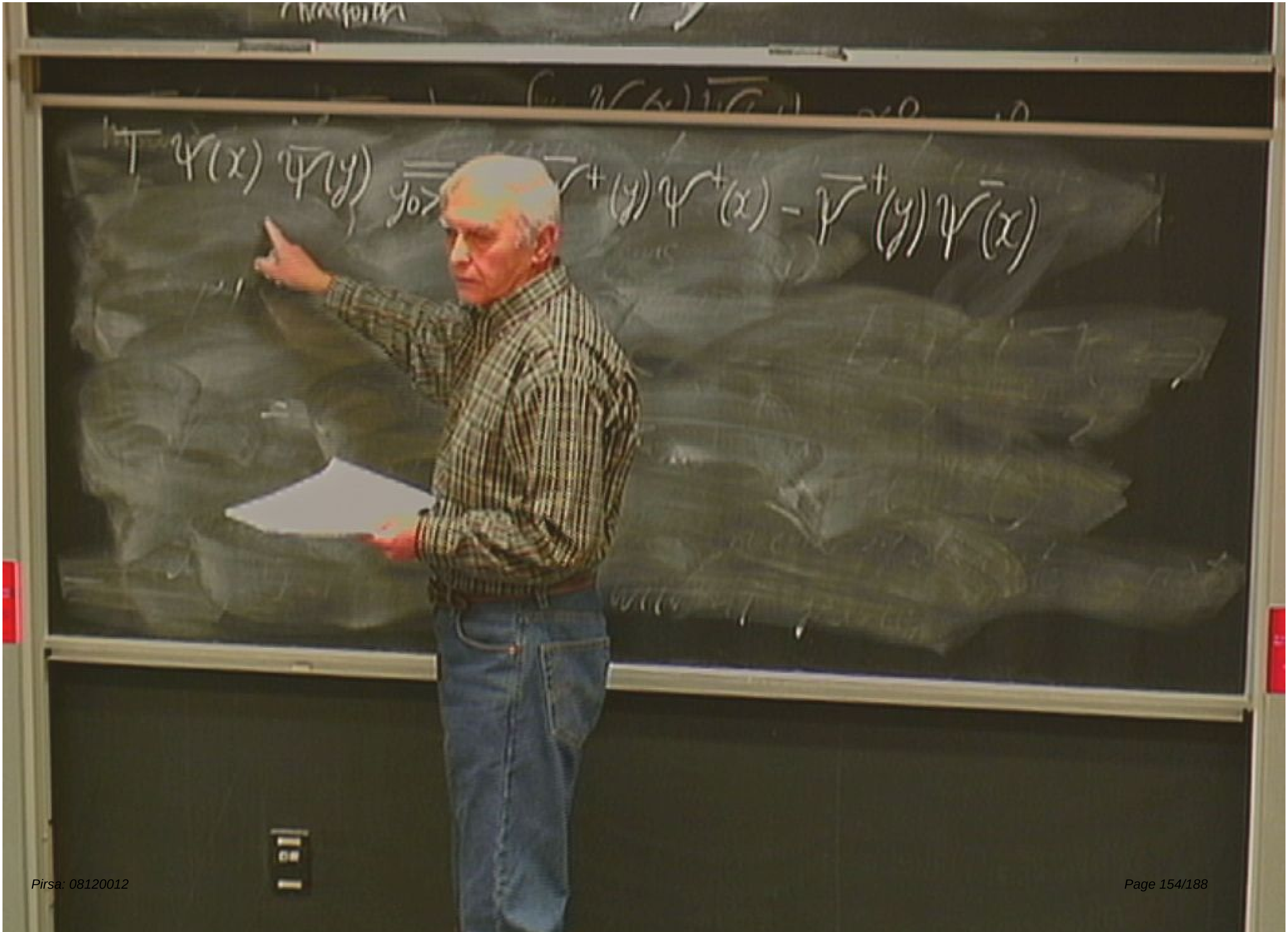
...zation of Wick's Theorem

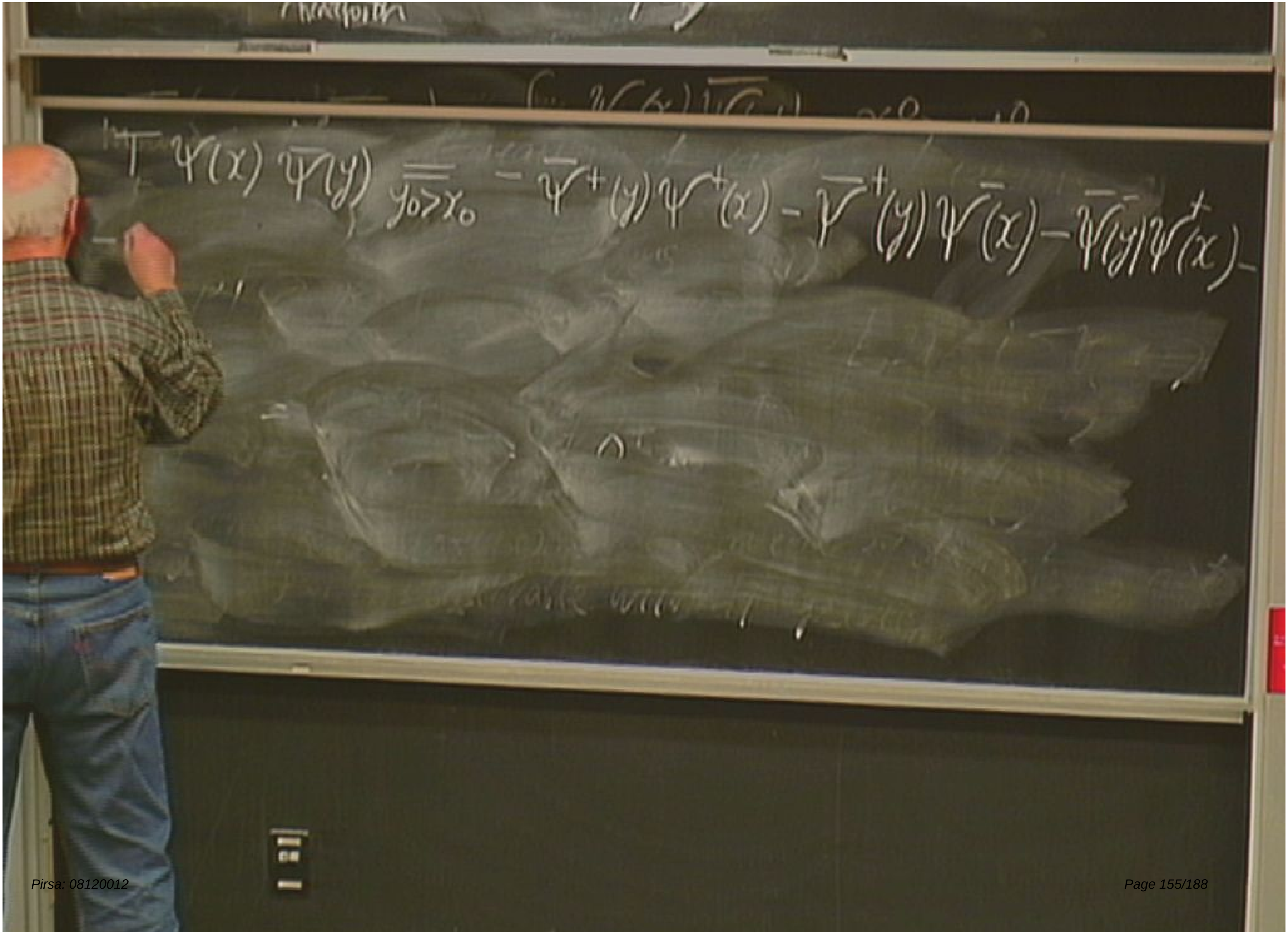
Start from $T\{\psi(x)\bar\psi(y)\}$:

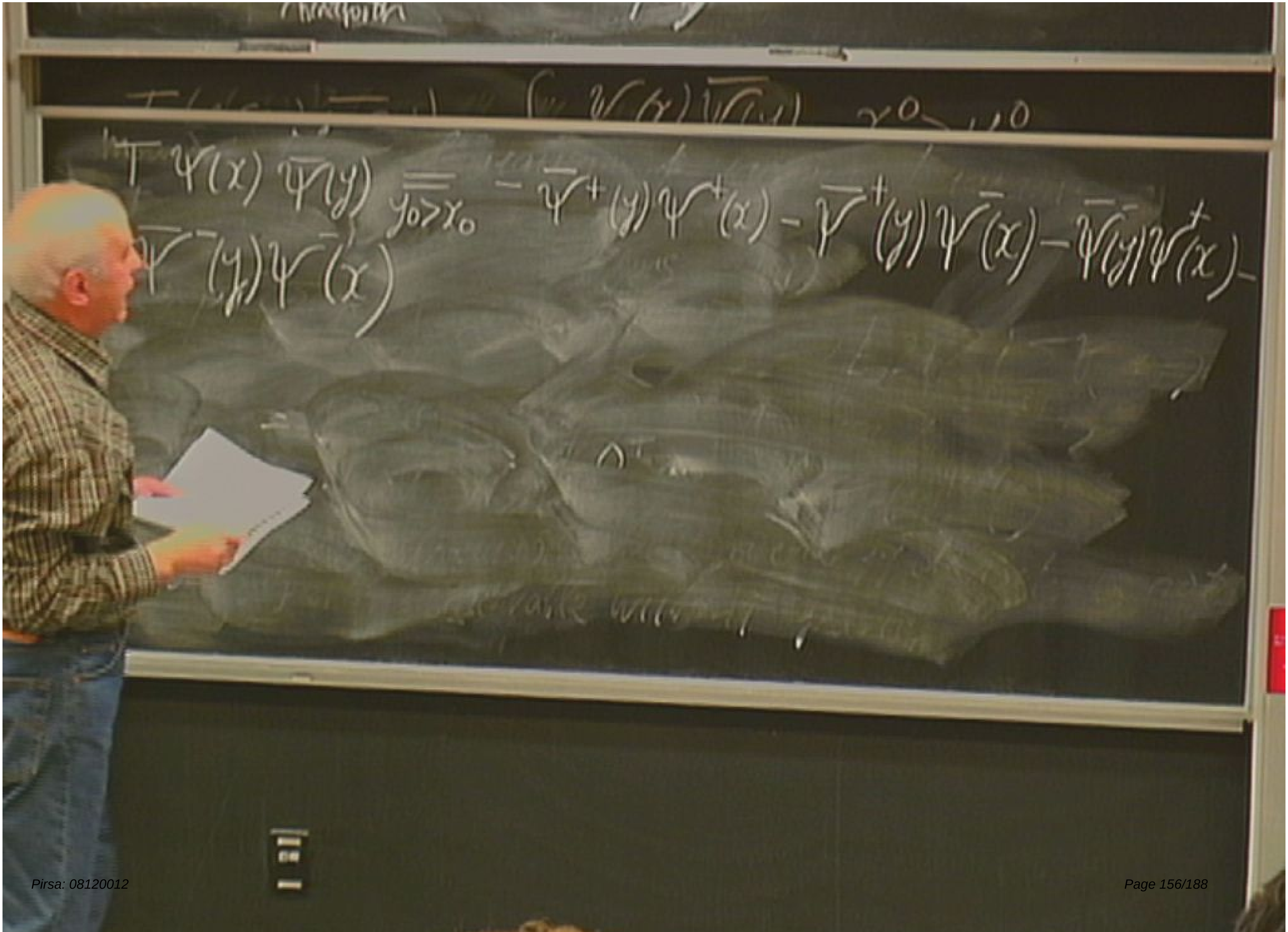
$$T\{\psi(x)\bar\psi(y)\} \underset{x^0>y^0}{=} \psi^+(x)\bar\psi^+(y) + \psi^+(x)\bar\psi^-(y) + \psi^-(x)\bar\psi^+(y) +$$

$$\psi^-(x)\bar\psi^-(y) = \psi^+(x)\bar\psi^+(y) - \bar\psi^-(y)\psi^+(x) + \psi^-(x)\bar\psi^+(y) +$$

$$+\psi^-(x)\bar\psi^-(y) + \{\psi^+(x),\bar\psi^-(y)\} = N(\psi(x)\bar\psi(y)) + \{\psi^+(x),\bar\psi^-(y)\}$$

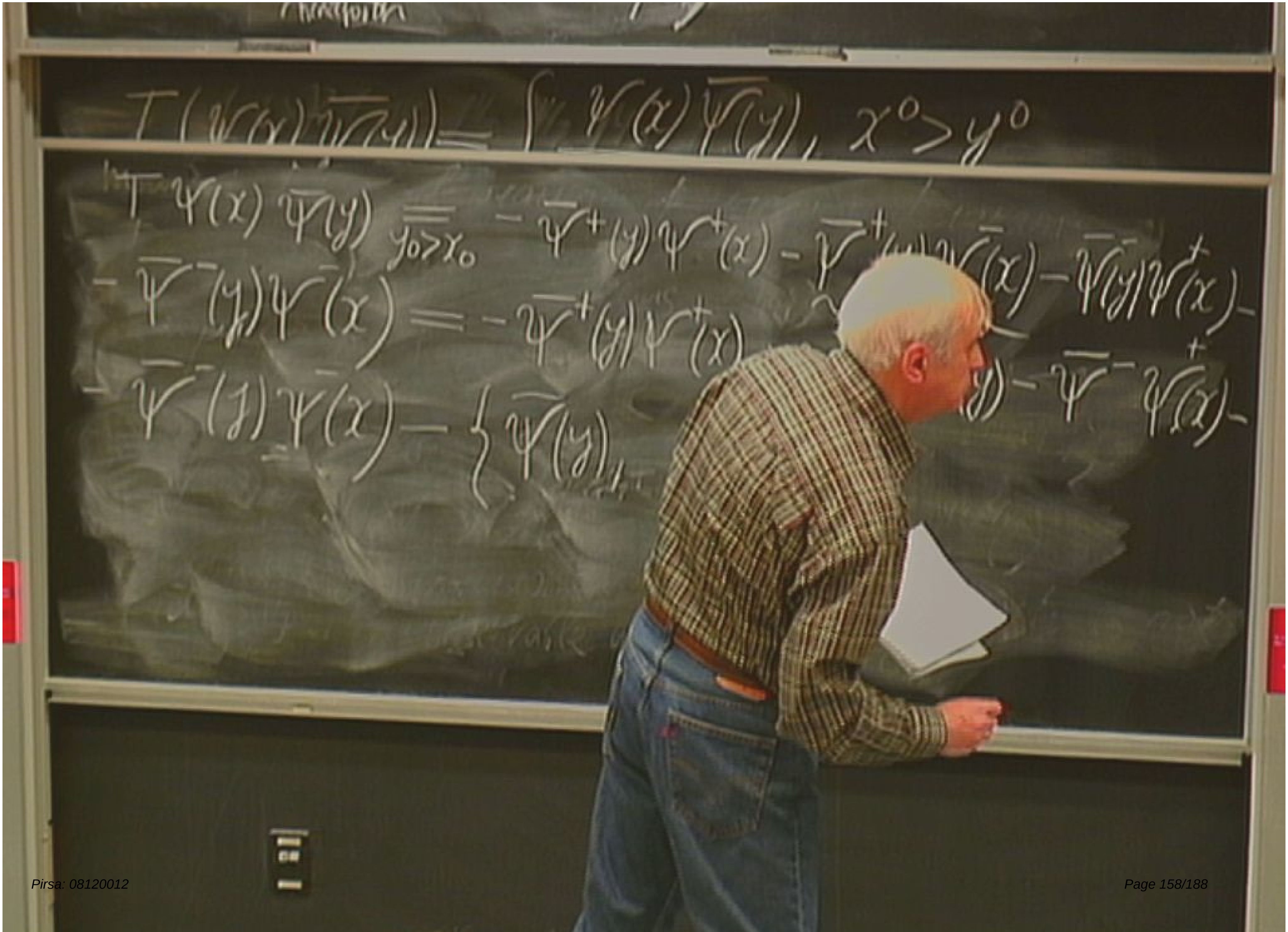



$T\,\psi(x)\,\bar\psi(y) \underset{y_0>x_0}{=} -\bar\psi^+(y)\,\psi^+(x) - \bar\psi^+(y)\,\psi^-(x) - \bar\psi^-(y)\,\psi^+(x) -$


$T\,\psi(x)\,\bar\psi(y) \underset{y_0>x_0}{=} -\bar\psi^{+}(y)\psi^{+}(x) - \bar\psi^{+}(y)\psi^{-}(x) - \bar\psi^{-}(y)\psi^{+}(x) -$
$\bar\psi^{-}(y)\psi^{-}(x)$

$$T(\psi(x)\bar{\psi}(y)) \quad \int \ldots \psi(x)\bar{\psi}(y), \quad x^0 > y^0$$

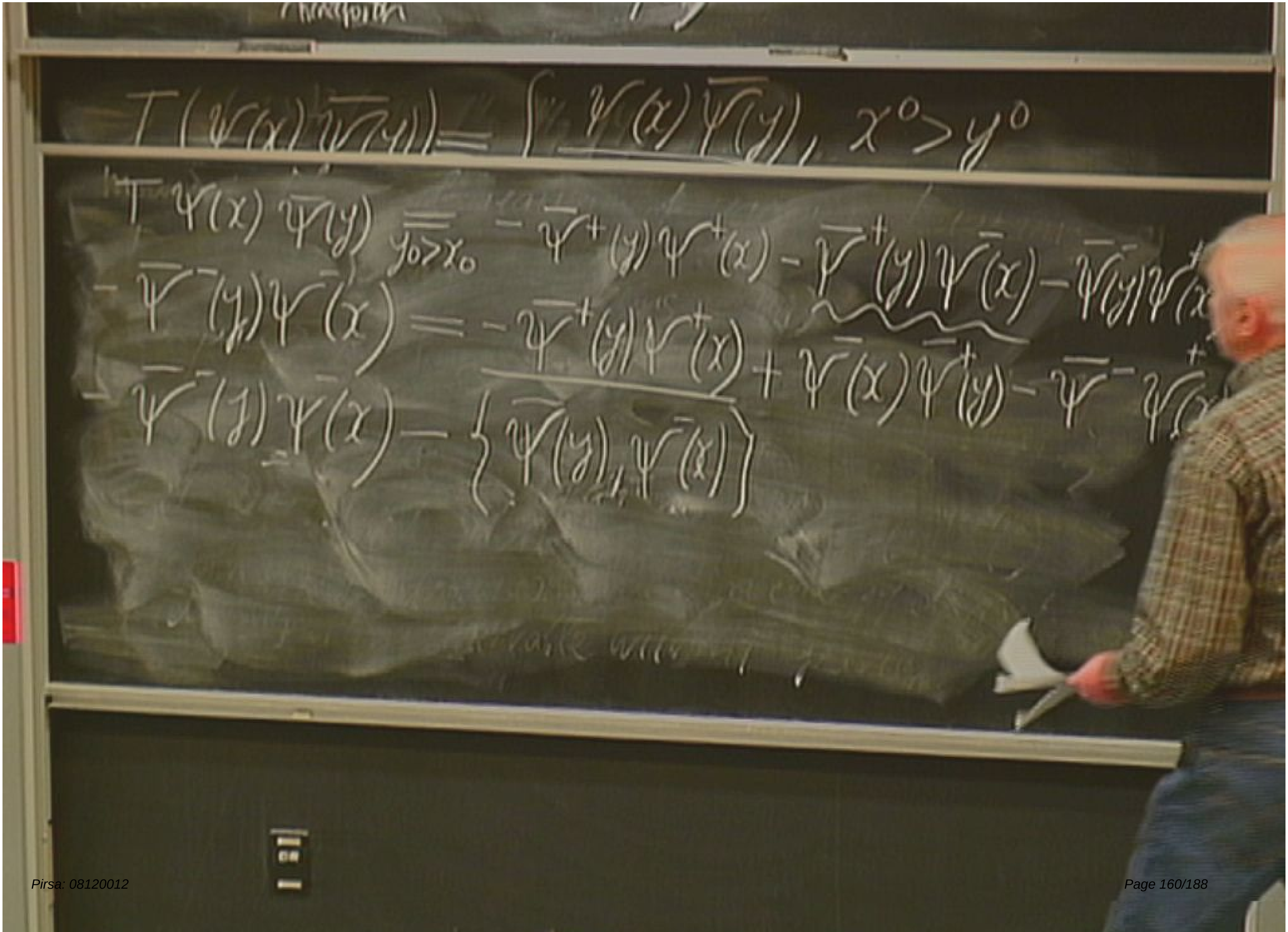
$$T\,\psi(x)\bar{\psi}(y) \underset{y_0 > x_0}{=} -\bar{\psi}^+(y)\psi^+(x) - \bar{\psi}^+(y)\psi^-(x) - \bar{\psi}^-(y)\psi^+(x) - \bar{\psi}^-(y)\psi^-(x) = -\bar{\psi}^+(y)\psi^+(x) + \psi^-(x)\bar{\psi}^+(y) - \bar{\psi}^-\psi^+(x) -$$



T(ψ(x)ψ̄(y)) = { ψ(x)ψ̄(y), x⁰ > y⁰

$$T(\psi(x)\bar{\psi}(y)) = \{\, \psi(x)\bar{\psi}(y), \quad x^0 > y^0$$

$$T\psi(x)\bar{\psi}(y) \underset{y_0 > x_0}{=} -\bar{\psi}^+(y)\psi^+(x) - \bar{\psi}^+(y)\psi^-(x) - \bar{\psi}^-(y)\psi^+(x) - \bar{\psi}^-(y)\psi^-(x) = -\bar{\psi}^+(y)\psi^+(x) + \psi^-(x)\bar{\psi}^+(y) - \bar{\psi}^-\psi^+(x) - \bar{\psi}^-(y)\psi^-(x) - \{\bar{\psi}^+(y), \psi^-(x)\}$$



$T(\psi(x)\bar{\psi}(y)) = \{ \psi(x)\bar{\psi}(y), \; x^0 > y^0$
$T\,\psi(x)\,\bar{\psi}(y) \underset{y_0 > x_0}{=} -\bar{\psi}^+(y)\psi^+(x) - \bar{\psi}^+(y)\psi^-(x) - \bar{\psi}^-(y)\psi^+(x)$
$-\bar{\psi}^-(y)\psi^-(x) = -\bar{\psi}^+(y)\psi^+(x) + \psi^-(x)\bar{\psi}^+(y) - \bar{\psi}^-\psi^+(x)$
$-\bar{\psi}^-(y)\psi^-(x) - \{\bar{\psi}^+(y), \psi^-(x)\}$

$$T(\psi(x)\bar\psi(y)) = \psi(x)\bar\psi(y), \quad x^0 > y^0$$

$$T\,\psi(x)\,\bar\psi(y) \underset{y_0 > x_0}{=} -\bar\psi^+(y)\psi^+(x) - \bar\psi^+(y)\psi^-(x) - \bar\psi^-(y)\psi^+(x) - \bar\psi^-(y)\psi^-(x) = -\bar\psi^+(y)\psi^+(x) + \psi^-(x)\bar\psi^+(y) - \bar\psi^-\psi^+(x) - \bar\psi^-(y)\psi^-(x) - \{\bar\psi^+(y), \psi^-(x)\}$$

$$T(\psi(x)\bar{\psi}(y)) = \{\ \psi(x)\bar{\psi}(y),\quad x^0 > y^0$$

$$T\,\psi(x)\,\bar{\psi}(y) \underset{y_0 > x_0}{=} -\bar{\psi}^+(y)\psi^+(x) - \bar{\psi}^+(y)\psi^-(x) - \bar{\psi}^-(y)\psi^+(x) -$$

$$-\bar{\psi}^-(y)\psi^-(x) = -\bar{\psi}^+(y)\psi^+(x) + \psi^-(x)\bar{\psi}^+(y) - \bar{\psi}^-\psi^+(x) -$$

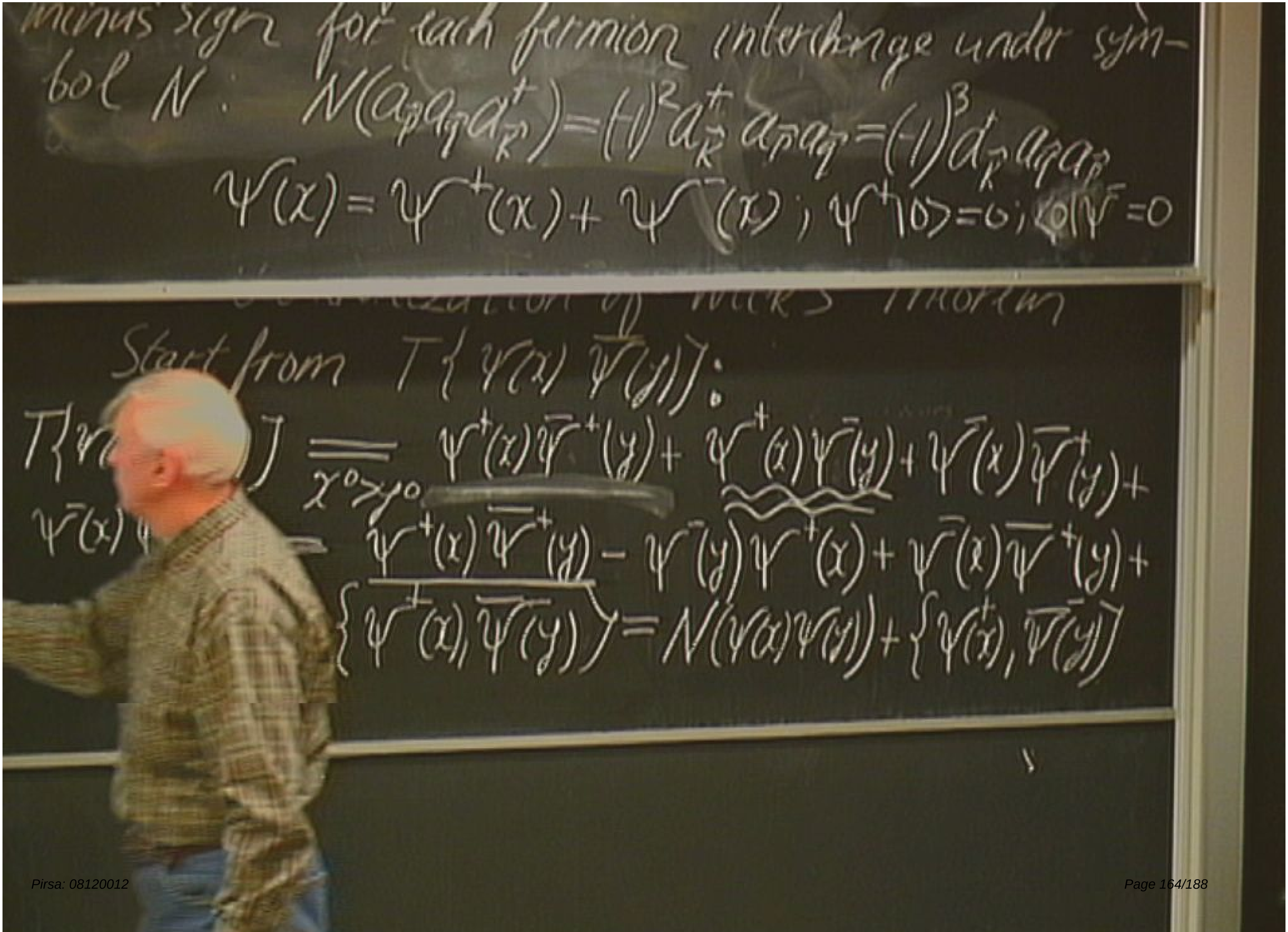
$$-\bar{\psi}^-(y)\psi^-(x) - \{\bar{\psi}^-(y), \psi^-(x)\}$$

$$T\left(\psi(x)\bar{\psi}(y)\right) = \begin{cases} \psi(x)\bar{\psi}(y), & x^0 > y^0 \end{cases}$$

$$T\,\psi(x)\,\bar{\psi}(y) \underset{y_0>x_0}{=} -\bar{\psi}^+(y)\psi^+(x) - \bar{\psi}^+(y)\psi^-(x) - \bar{\psi}^-(y)\psi^+(x)$$

$$- \bar{\psi}^-(y)\psi^-(x) = -\bar{\psi}^+(y)\psi^+(x) + \psi^-(x)\bar{\psi}^+(y) - \bar{\psi}^-(y)\psi^+(x)$$

$$- \bar{\psi}^-(y)\psi^-(x) - \left\{\bar{\psi}^+(y), \psi^-(x)\right\}$$


minus sign for each fermion interchange under sym-
bol N: $N(a_{\vec p} a_{\vec q} a^\dagger_{\vec k}) = (-1)^2 a^\dagger_{\vec k} a_{\vec p} a_{\vec q} = (-1)^3 a^\dagger_{\vec k} a_{\vec q} a_{\vec p}$
$\psi(x) = \psi^+(x) + \psi^-(x)$; $\psi^+|0\rangle = 0$; $\langle 0|\bar\psi^- = 0$
Wick's Theorem
Start from $T\{\psi(x)\bar\psi(y)\}$:
$\underset{x^0>y^0}{=} \psi^+(x)\bar\psi^+(y) + \psi^+(x)\bar\psi^-(y) + \psi^-(x)\bar\psi^+(y) +$
$= \psi^+(x)\bar\psi^+(y) - \bar\psi^-(y)\psi^+(x) + \psi^-(x)\bar\psi^+(y) +$
$\{\psi^+(x), \bar\psi^-(y)\} = N(\psi(x)\bar\psi(y)) + \{\psi^+(x), \bar\psi^-(y)\}$

minus sign for each fermion interchange under symbol N: $N(a_{\vec p}a_{\vec q}a^{\dagger}_{\vec k}) = (-1)^2 a^{\dagger}_{\vec k}a_{\vec p}a_{\vec q} = (-1)^3 a^{\dagger}_{\vec k}a_{\vec q}a_{\vec p}$

$\psi(x) = \psi^{+}(x) + \psi^{-}(x)$; $\psi^{+}|0\rangle = 0$; $\langle 0|\bar\psi^{-} = 0$

Generalization of Wick's Theorem

Start from $T\{\psi(x)\bar\psi(y)\}$:

$$T\{\psi(x)\bar\psi(y)\} \underset{x^0>y^0}{=} \psi^{+}(x)\bar\psi^{+}(y) + \psi^{+}(x)\bar\psi^{-}(y) + \psi^{-}(x)\bar\psi^{+}(y) + \psi^{-}(x)\bar\psi^{-}(y) = \psi^{+}(x)\bar\psi^{+}(y) - \bar\psi^{-}(y)\psi^{+}(x) + \psi^{-}(x)\bar\psi^{+}(y) + \psi^{-}(x)\bar\psi^{-}(y) + \{\psi^{+}(x),\bar\psi^{-}(y)\} = N(\psi(x)\bar\psi(y)) + \{\psi^{+}(x),\bar\psi^{-}(y)\}$$

minus sign for each fermion interchange under symbol N: $N(a_{\vec p} a_{\vec q} a^{\dagger}_{\vec k}) = (-1)^2 a^{\dagger}_{\vec k} a_{\vec p} a_{\vec q} = (-1)^3 a^{\dagger}_{\vec k} a_{\vec q} a_{\vec p}$

$$\psi(x) = \psi^{+}(x) + \psi^{-}(x);\ \psi^{+}|0\rangle = 0;\ \langle 0|\bar\psi^{-} = 0$$

Generalization of Wick's Theorem

Start from $T\{\psi(x)\bar\psi(y)\}$:

$$T\{\psi(x)\bar\psi(y)\} \underset{x^0>y^0}{=\!=} \psi^{+}(x)\bar\psi^{+}(y) + \psi^{+}(x)\bar\psi^{-}(y) + \psi^{-}(x)\bar\psi^{+}(y) +$$

$$\psi^{-}(x)\bar\psi^{-}(y) = \psi^{+}(x)\bar\psi^{+}(y) - \bar\psi^{-}(y)\psi^{+}(x) + \psi^{-}(x)\bar\psi^{+}(y) +$$

$$+ \psi^{-}(x)\bar\psi^{-}(y) + \{\psi^{+}(x), \bar\psi^{-}(y)\} = N(\psi(x)\bar\psi(y)) + \{\psi^{+}(x), \bar\psi^{-}(y)\}$$

$$T(\psi(x)\bar{\psi}(y)) = \{ \psi(x)\bar{\psi}(y), \quad x^0 > y^0$$

$$T\,\psi(x)\,\bar{\psi}(y) \underset{y_0>x_0}{=} -\bar{\psi}^+(y)\psi^+(x) - \bar{\psi}^+(y)\psi^-(x) - \bar{\psi}^-(y)\psi^+(x) -$$

$$-\bar{\psi}^-(y)\psi^-(x) = -\bar{\psi}^+(y)\psi^+(x) + \psi^-(x)\bar{\psi}^+(y) - \bar{\psi}^-\psi^+(x) -$$

$$-\bar{\psi}^-(y)\psi^-(x) - \{\bar{\psi}^-(y), \psi^-(x)\}$$

$$T\,\psi(x)\,\bar\psi(y) \underset{y_0>x_0}{=} -\bar\psi^{+}(y)\psi^{+}(x) - \bar\psi^{+}(y)\psi^{-}(x) - \bar\psi^{-}(y)\psi^{+}(x) - \bar\psi^{-}(y)\psi^{-}(x) = -\bar\psi^{+}(y)\psi^{+}(x) + \psi^{-}(x)\bar\psi^{+}(y) - \bar\psi^{-}\psi^{+}(x) - \bar\psi^{-}(y)\psi^{-}(x) - \{\bar\psi^{-}(y), \psi^{-}(x)\}$$

$$\psi(x) = \psi^+(x) + \psi^-(x)\ ;\ \psi^+|0\rangle = 0\ ;\ \langle 0|\bar\psi^- = 0$$

Generalization of Wick's Theorem

Start from $T\{\psi(x)\bar\psi(y)\}$:

$$T\{\psi(x)\bar\psi(y)\} \underset{x^0>y^0}{=} \psi^+(x)\bar\psi^+(y) + \psi^+(x)\bar\psi^-(y) + \psi^-(x)\bar\psi^+(y) + \psi^-(x)\bar\psi^-(y) = \psi^+(x)\bar\psi^+(y) - \bar\psi^-(y)\psi^+(x) + \psi^-(x)\bar\psi^+(y) + \psi^-(x)\bar\psi^-(y) + \{\psi^+(x), \bar\psi^-(y)\} = N(\psi(x)\bar\psi(y)) + \{\psi^+(x), \bar\psi^-(y)\}$$

$$T(\psi(x)\bar\psi(y)) = \{\, \psi(x)\bar\psi(y), \quad x^0 > y^0$$

$$T\,\psi(x)\bar\psi(y) \underset{y_0>x_0}{=} -\bar\psi^+(y)\psi^+(x) - \bar\psi^+(y)\psi^-(x) - \bar\psi^-(y)\psi^+(x) -$$

$$-\bar\psi^-(y)\psi^-(x) = -\bar\psi^+(y)\psi^+(x) + \psi^-(x)\bar\psi^+(y) - \bar\psi^-\psi^+$$

$$-\bar\psi^-(y)\psi^-(x) - \{\bar\psi(y), \psi(x)\}$$

$$T(\psi(x)\bar{\psi}(y)) = \begin{cases} \psi(x)\bar{\psi}(y), & x^0 > y^0 \end{cases}$$

$$T\,\psi(x)\,\bar{\psi}(y) \underset{y_0 > x_0}{=} -\bar{\psi}^{+}(y)\psi^{+}(x) - \bar{\psi}^{+}(y)\psi^{-}(x) - \bar{\psi}^{-}(y)\psi^{+}(x) - \bar{\psi}^{-}(y)\psi^{-}(x) = -\bar{\psi}^{+}(y)\psi^{+}(x) + \psi^{-}(x)\bar{\psi}^{+}(y) - \bar{\psi}^{-}\psi^{+}(x) - \bar{\psi}^{-}(y)\psi^{-}(x) - \{\bar{\psi}^{+}(y),\psi^{-}(x)\}$$

$$\psi(x) = \psi^+(x) + \psi^-(x)\ ; \quad \psi^+|0\rangle = 0\ ; \quad \langle 0|\bar\psi^- = 0$$

Generalization of Wick's Theorem

Start from $T\{\psi(x)\bar\psi(y)\}$:

$$T\{\psi(x)\bar\psi(y)\} \underset{x^0>y^0}{=} \psi^+(x)\bar\psi^+(y) + \psi^+(x)\bar\psi^-(y) + \psi^-(x)\bar\psi^+(y) + \psi^-(x)\bar\psi^-(y) = \psi^+(x)\bar\psi^+(y) - \bar\psi^-(y)\psi^+(x) + \psi^-(x)\bar\psi^+(y) + \psi^-(x)\bar\psi^-(y) + \{\psi^+(x),\bar\psi^-(y)\} = N(\psi(x)\bar\psi(y)) + \{\psi^+(x),\bar\psi^-(y)\}$$

$$T(\psi(x)\bar{\psi}(y)) = \psi(x)\bar{\psi}(y), \quad x^0 > y^0$$

$$T\,\psi(x)\bar{\psi}(y) \underset{y_0 > x_0}{=} -\bar{\psi}^+(y)\psi^+(x) - \bar{\psi}^+(y)\psi^-(x) - \bar{\psi}^-(y)\psi^+(x) - \bar{\psi}^-(y)\psi^-(x) = -\bar{\psi}^+(y)\psi^+(x) + \psi^-(x)\bar{\psi}^+(y) - \bar{\psi}^-(y)\psi^+(x) - \bar{\psi}^-(y)\psi^-(x) - \{\bar{\psi}^+(y), \psi^-(x)\}$$

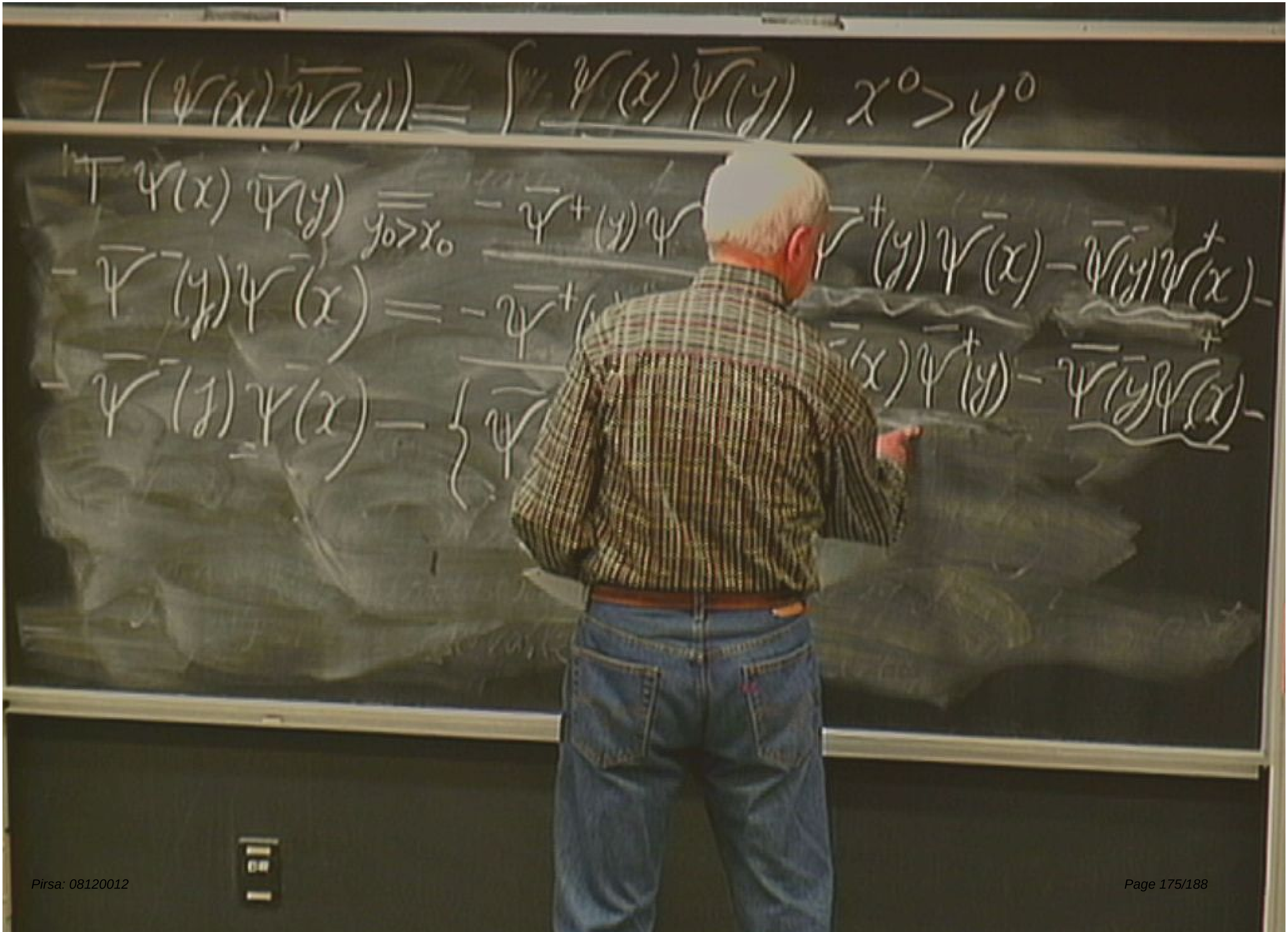
Generalization of Wick's Theorem

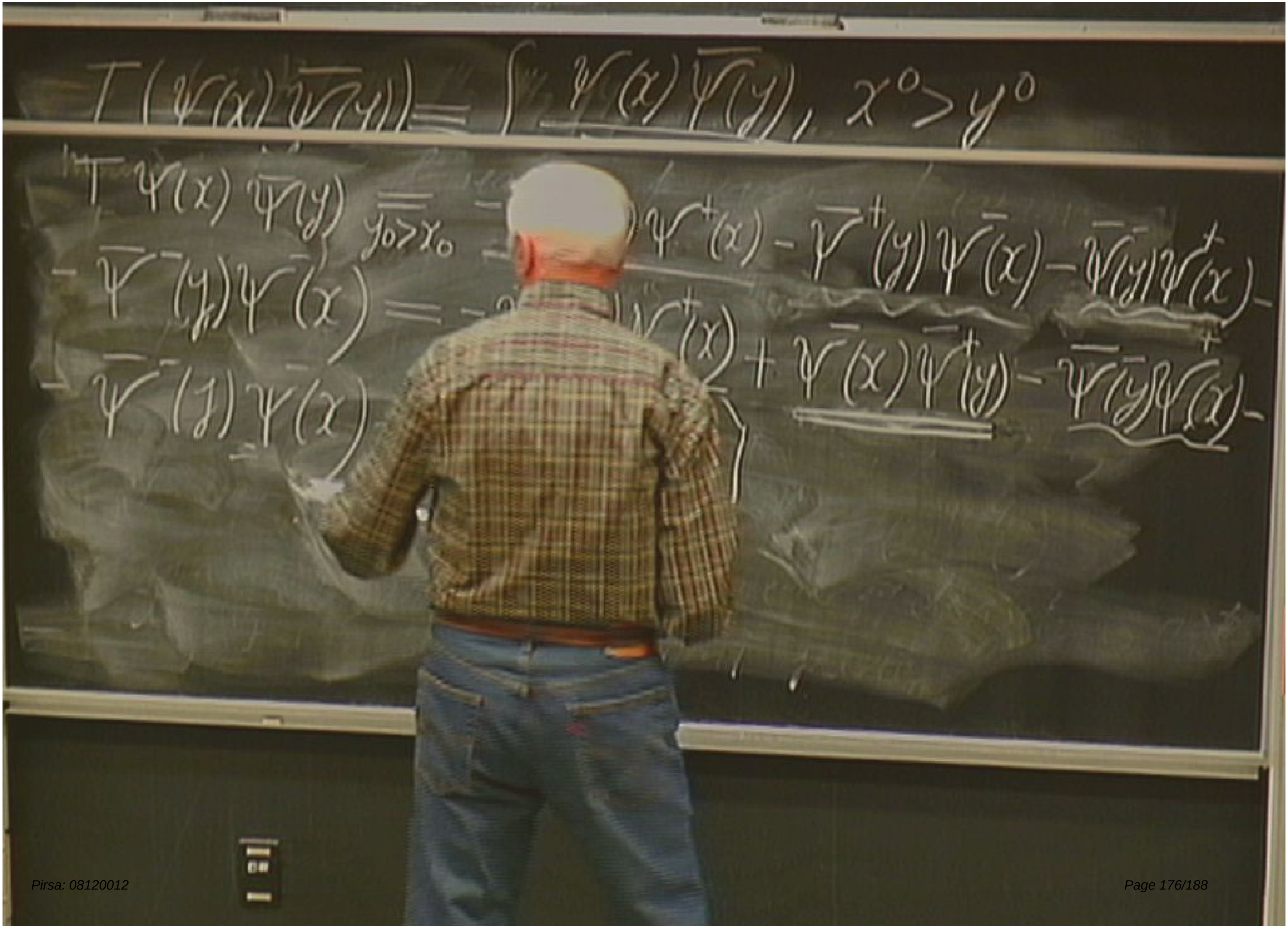
Start from $T\{\psi(x)\bar{\psi}(y)\}$:

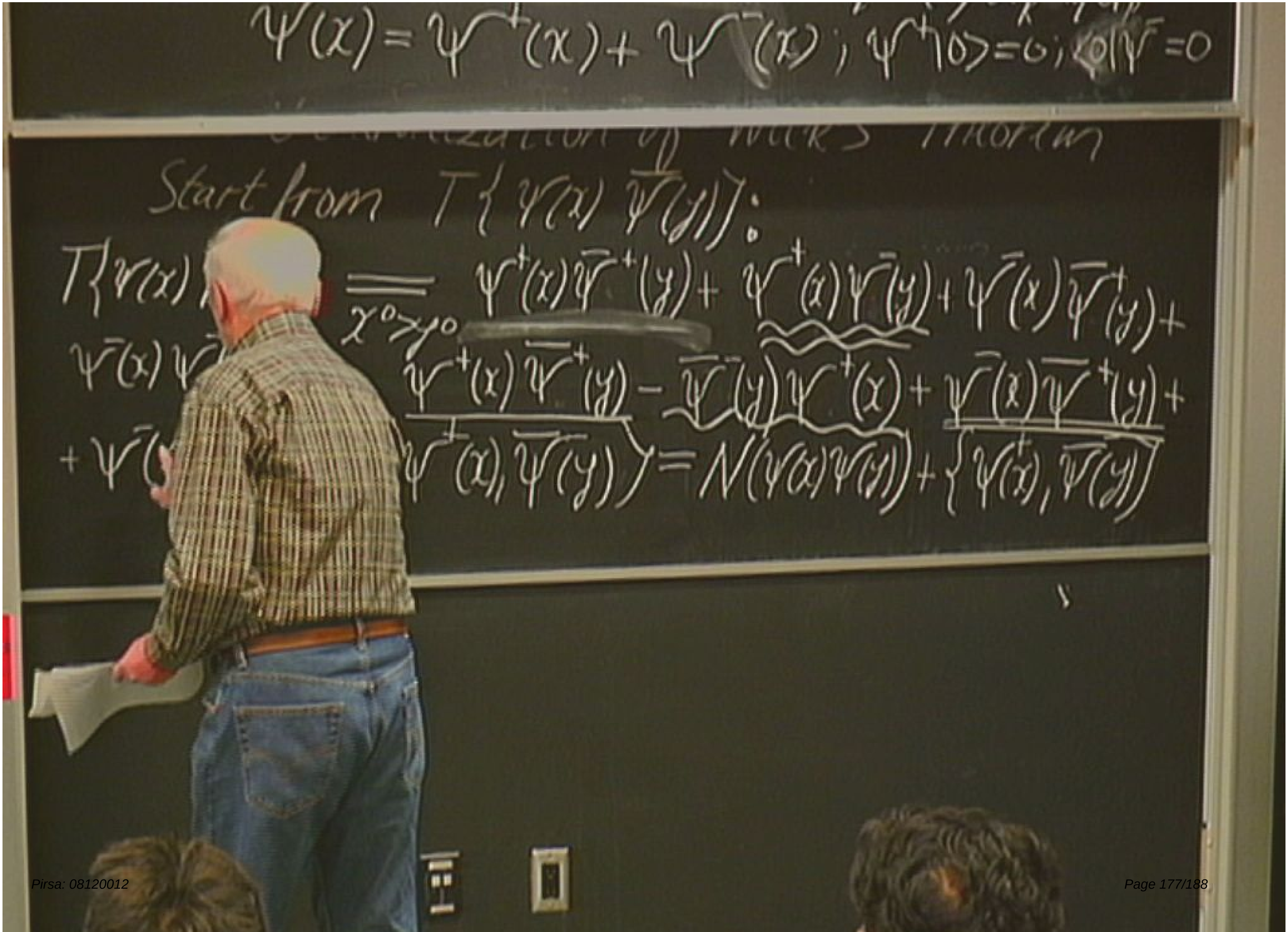
$$T\{\psi(x)\bar{\psi}(y)\} \underset{x^0>y^0}{=} \psi^+(x)\bar{\psi}^+(y) + \psi^+(x)\bar{\psi}^-(y) + \psi^-(x)\bar{\psi}^+(y) +$$

$$\psi^-(x)\bar{\psi}^-(y) = \psi^+(x)\bar{\psi}^+(y) - \bar{\psi}^-(y)\psi^+(x) + \psi^-(x)\bar{\psi}^+(y) +$$

$$+ \psi^-(x)\bar{\psi}^-(y) + \{\psi^+(x), \bar{\psi}^-(y)\} = N(\psi(x)\bar{\psi}(y)) + \{\psi^+(x), \bar{\psi}^-(y)\}$$




T(ψ(x) ψ̄(y)) = ∫ ψ(x) ψ̄(y), x⁰ > y⁰
y₀ > x₀



$\psi(x) = \psi^{+}(x) + \psi^{-}(x)\ ;\ \psi^{+}|0\rangle = 0\ ;\ \langle 0|\bar{\psi}^{-} = 0$
Start from $T\{\psi(x)\,\bar{\psi}(y)\}$:
$= N(\psi(x)\bar{\psi}(y)) + \{\psi^{+}(x), \bar{\psi}^{-}(y)\}$

$$\psi(x) = \psi^{+}(x) + \psi^{-}(x)\,;\quad \psi^{+}|0\rangle = 0\,;\quad \langle 0|\bar{\psi}^{-} = 0$$

Generalization of Wick's Theorem

Start from $T\{\psi(x)\,\bar{\psi}(y)\}$:

$$T\{\psi(x)\bar{\psi}(y)\} \underset{x^0>y^0}{=\!=} \psi^{+}(x)\bar{\psi}^{+}(y) + \psi^{+}(x)\bar{\psi}^{-}(y) + \psi^{-}(x)\bar{\psi}^{+}(y) + \psi^{-}(x)\bar{\psi}^{-}(y) = \underline{\psi^{+}(x)\bar{\psi}^{+}(y)} - \bar{\psi}^{-}(y)\psi^{+}(x) + \underline{\psi^{-}(x)\bar{\psi}^{+}(y)} + \psi^{-}(x)\bar{\psi}^{-}(y) + \{\psi^{+}(x), \bar{\psi}^{-}(y)\} = N(\psi(x)\bar{\psi}(y)) + \{\psi^{+}(x), \bar{\psi}^{-}(y)\}$$

$$\psi(x) = \psi^+(x) + \psi^-(x)\,;\quad \psi^+|0\rangle = 0\,;\quad \langle 0|\bar\psi^- = 0$$

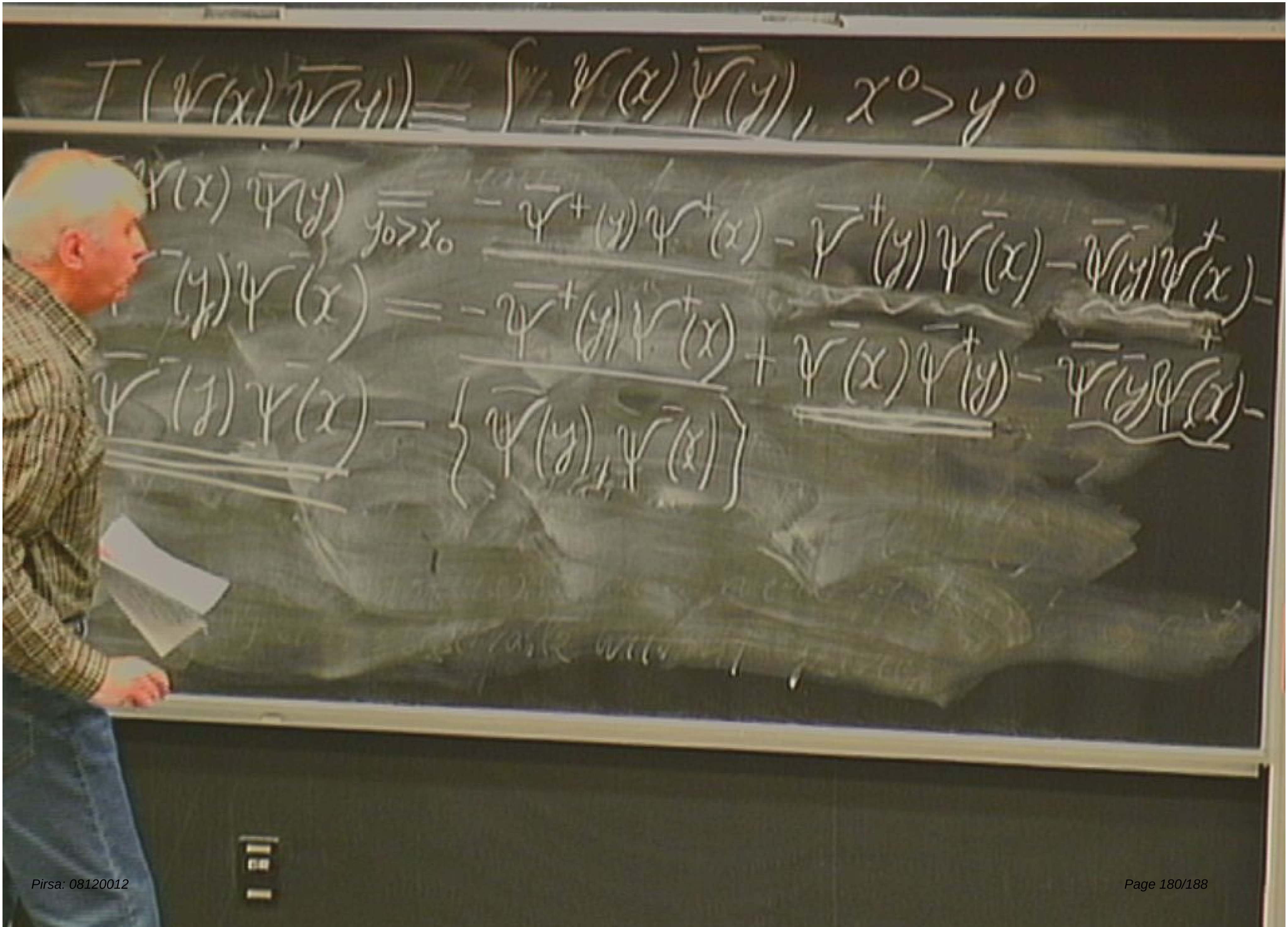
Generalization of Wick's Theorem

Start from $T\{\psi(x)\,\bar\psi(y)\}$:

$$T\{\psi(x)\bar\psi(y)\} \underset{x^0>y^0}{=} \psi^+(x)\bar\psi^+(y) + \psi^+(x)\bar\psi^-(y) + \psi^-(x)\bar\psi^+(y) +$$

$$\psi^-(x)\bar\psi^-(y) = \psi^+(x)\bar\psi^+(y) - \bar\psi^-(y)\psi^+(x) + \psi^-(x)\bar\psi^+(y) +$$

$$+\,\psi^-(x)\bar\psi^-(y) + \{\psi^+(x), \bar\psi^-(y)\} = N(\psi(x)\bar\psi(y)) + \{\psi^+(x), \bar\psi^-(y)\}$$

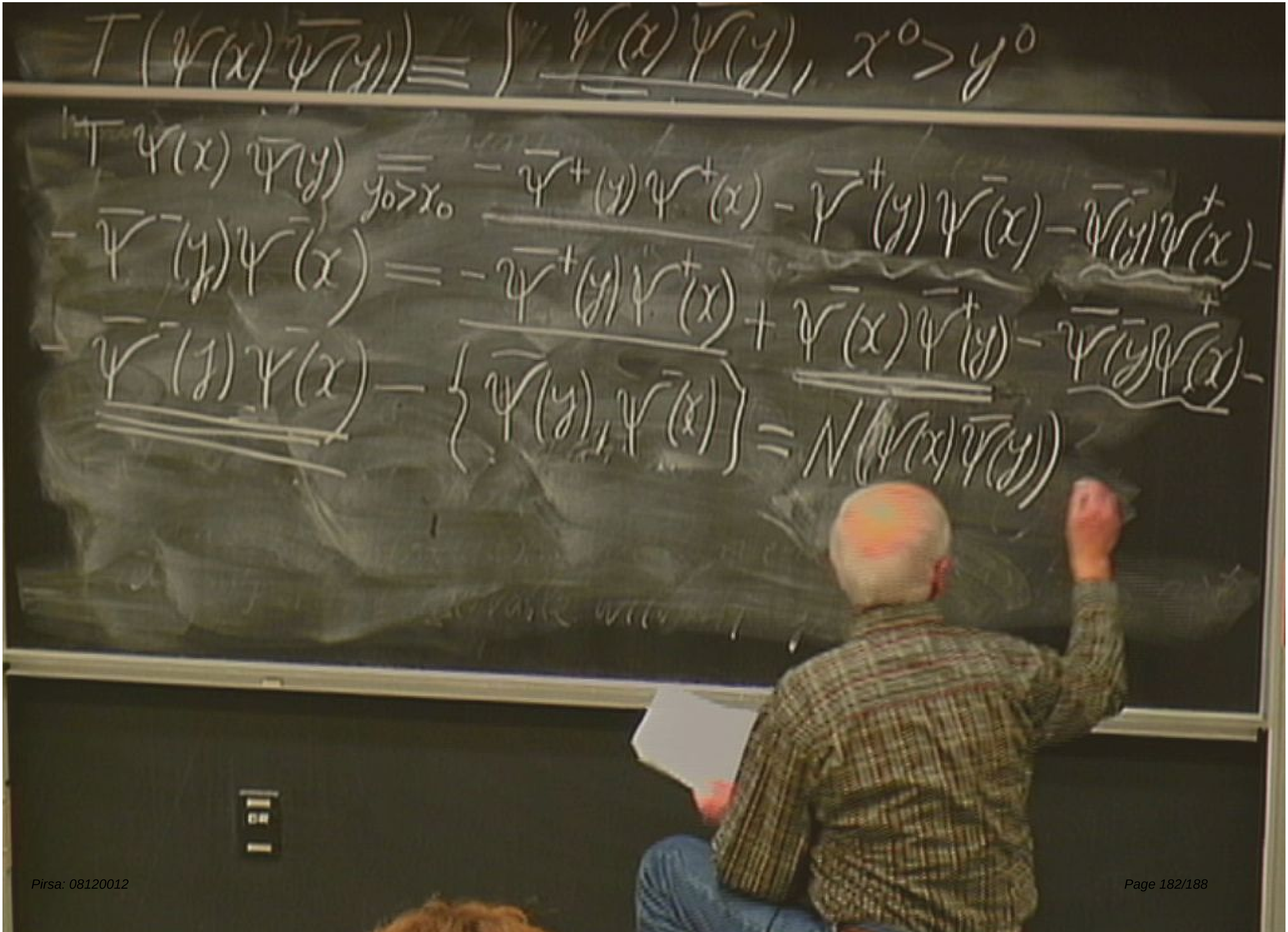

T(ψ(x)ψ̄(y)) = { ψ(x)ψ̄(y), x⁰ > y⁰

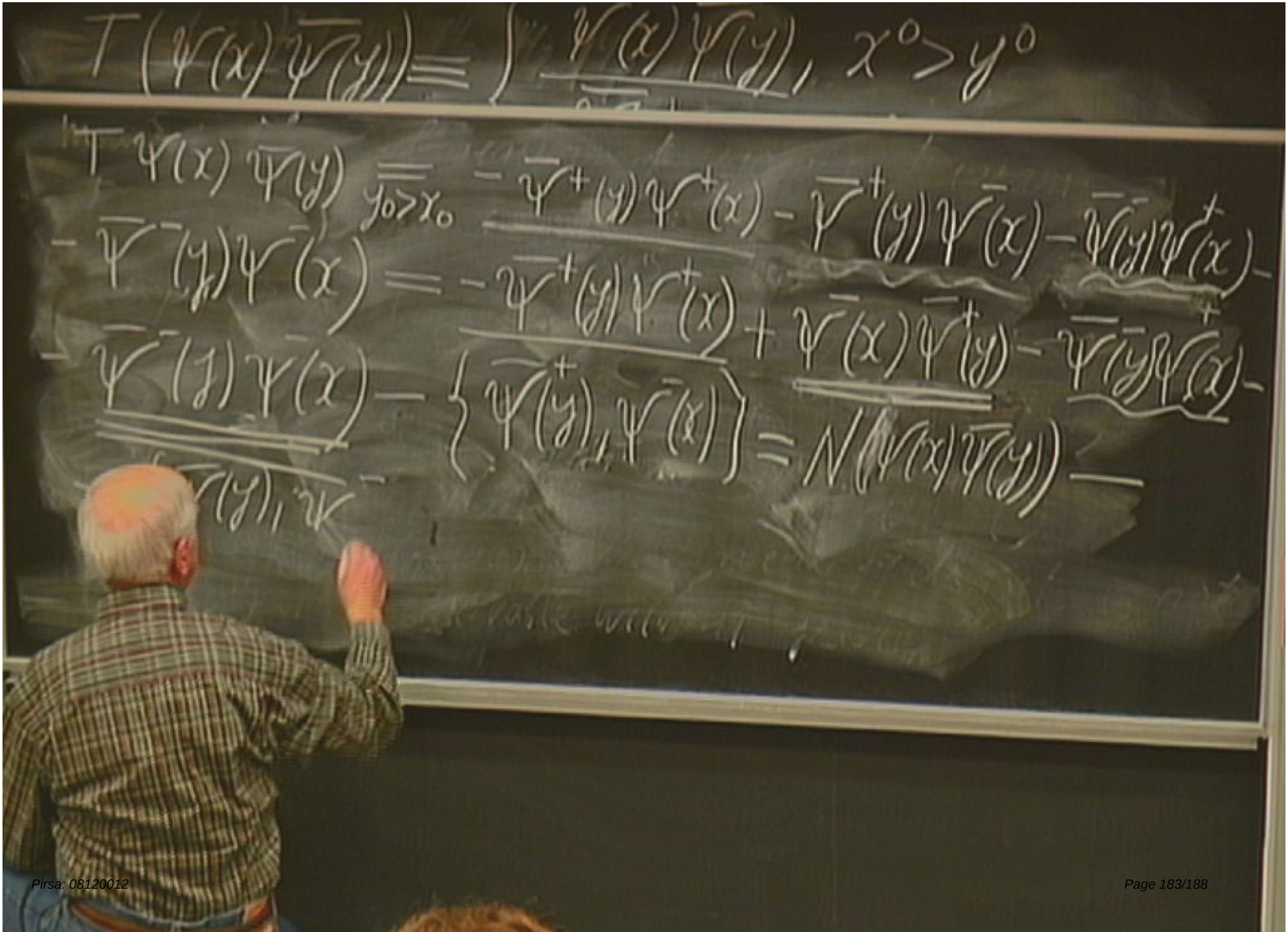
$\psi(x) = \psi^+(x) + \psi^-(x)\,;\ \psi^+|0\rangle = 0\,;\ \langle 0|\bar\psi^- = 0$

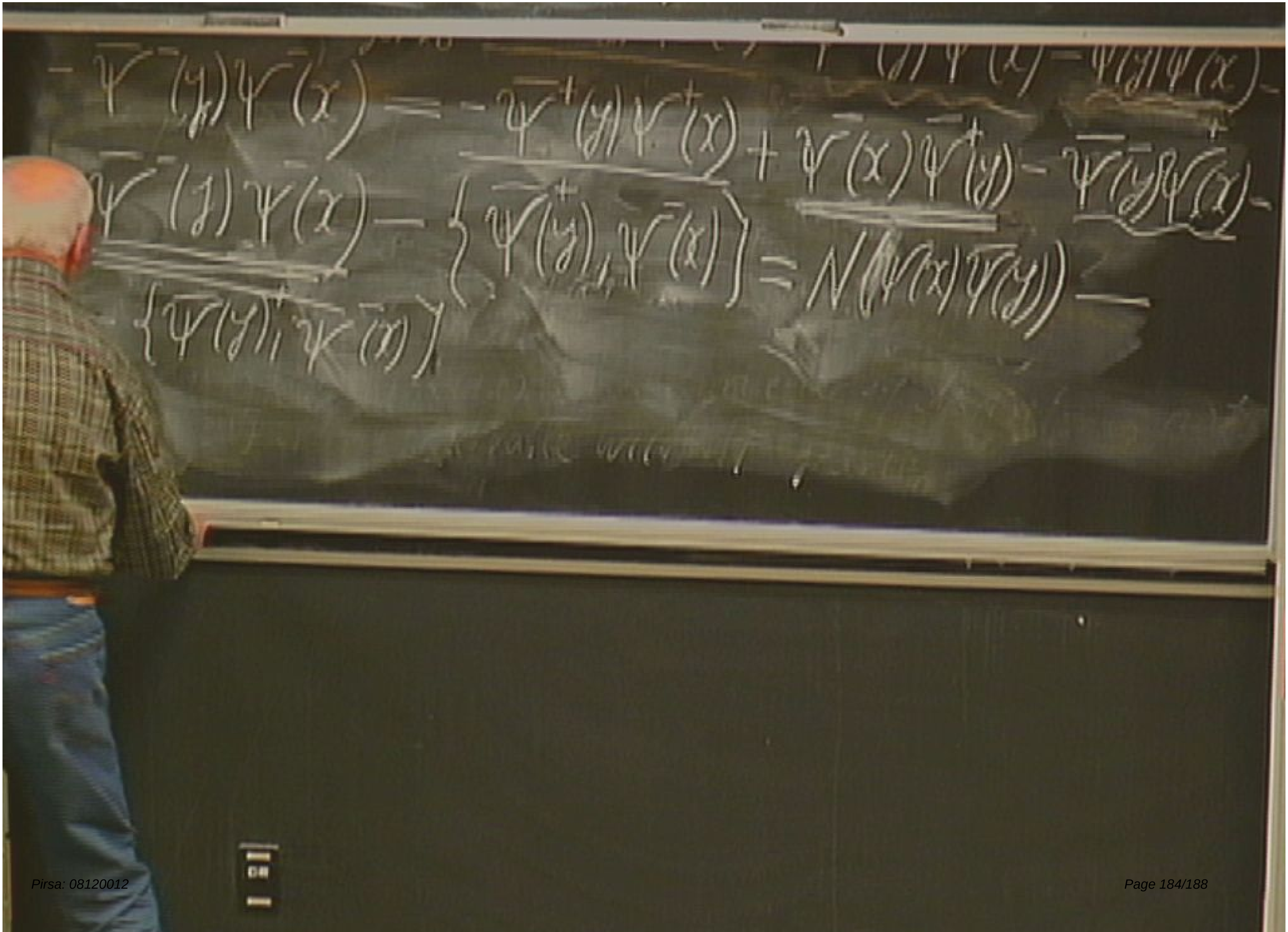
Generalization of Wick's Theorem

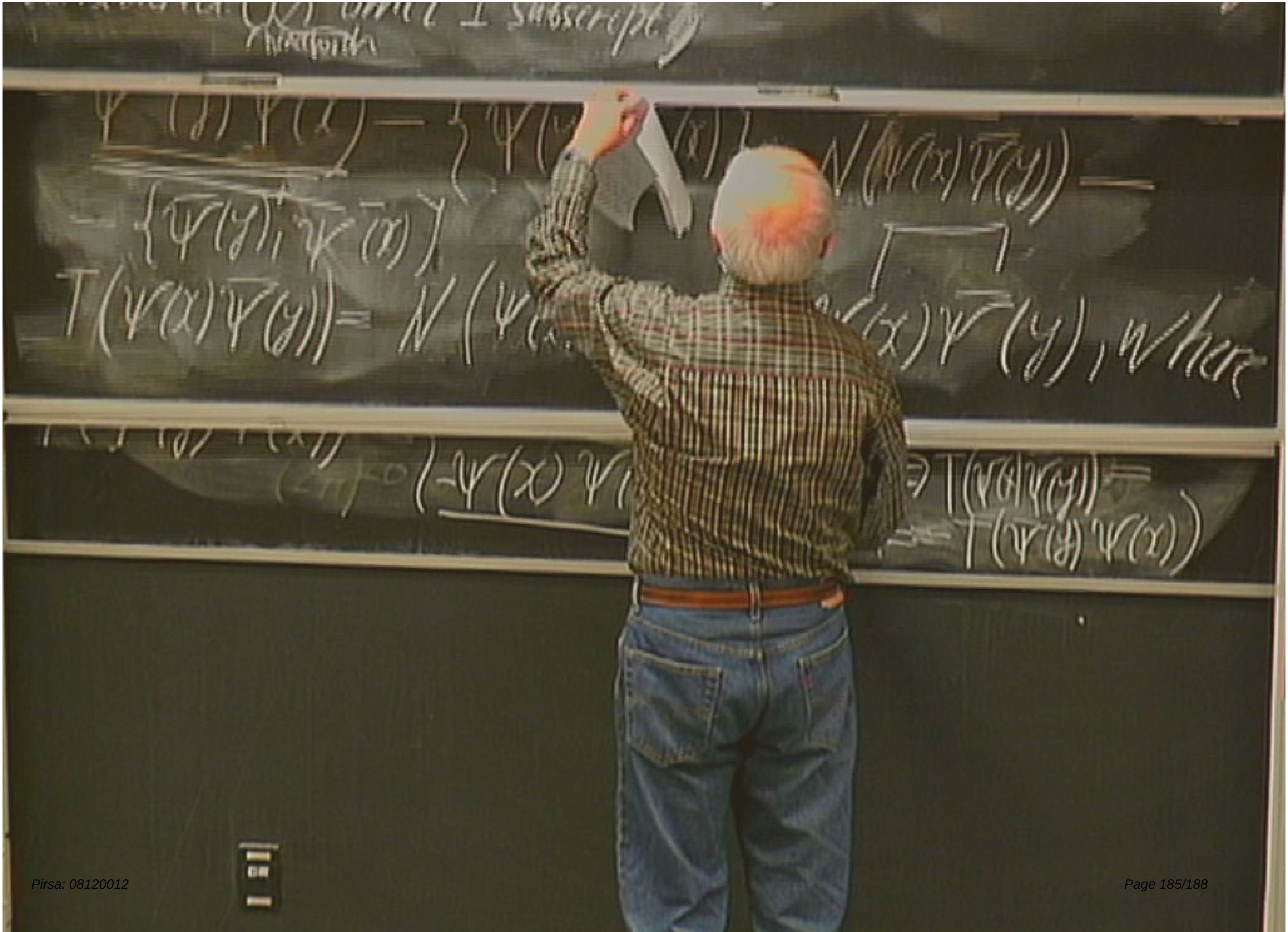
Start from $T\{\psi(x)\bar\psi(y)\}$:

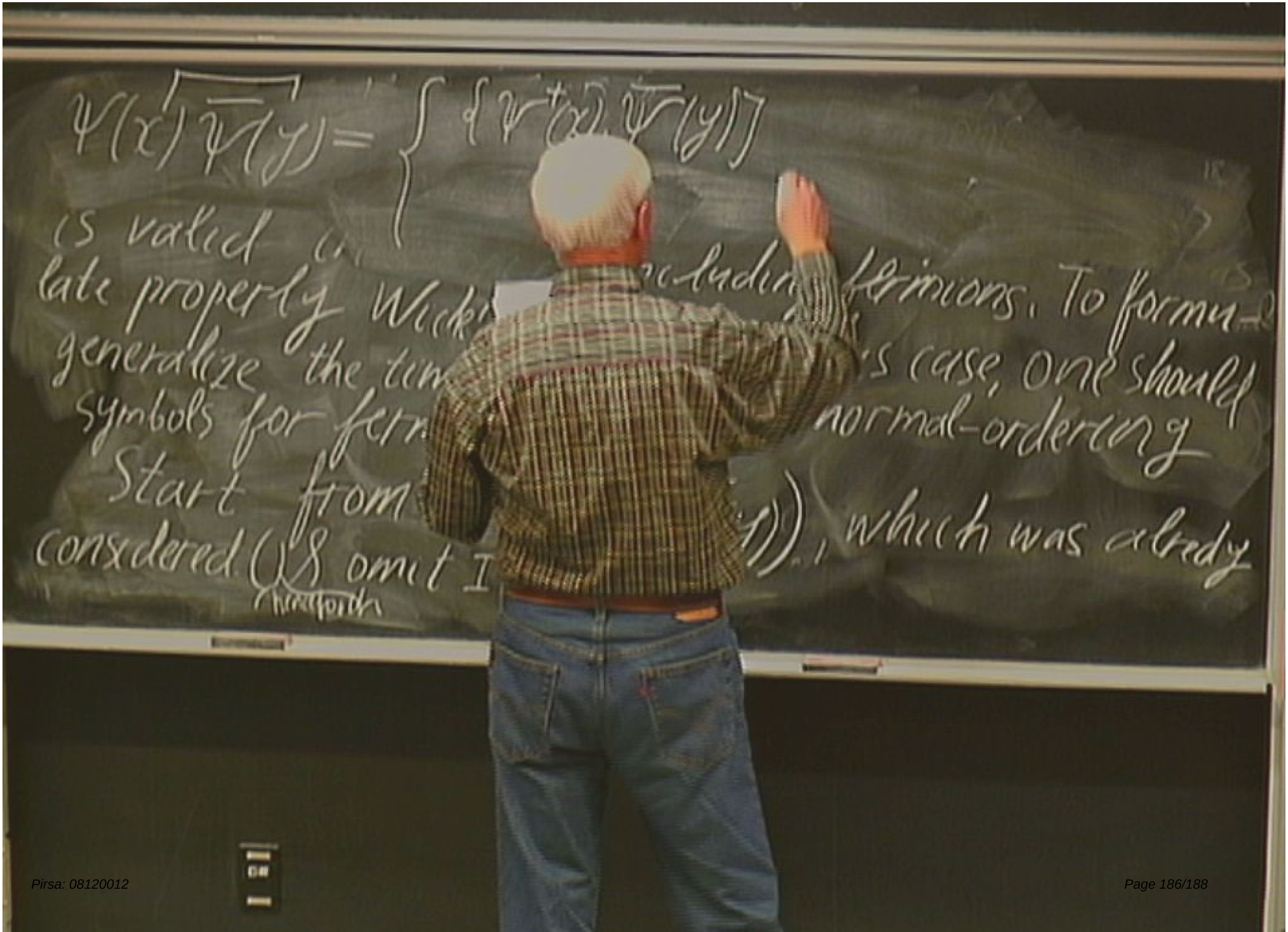
$$T\{\psi(x)\bar\psi(y)\} \underset{x^0>y^0}{=\!=} \psi^+(x)\bar\psi^+(y) + \psi^+(x)\bar\psi^-(y) + \psi^-(x)\bar\psi^+(y) + \psi^-(x)\bar\psi^-(y) = \psi^+(x)\bar\psi^+(y) - \bar\psi^-(y)\psi^+(x) + \psi^-(x)\bar\psi^+(y) + \psi^-(x)\bar\psi^-(y) + \{\psi^+(x), \bar\psi^-(y)\} = N(\psi(x)\bar\psi(y)) + \{\psi^+(x), \bar\psi(y)\}$$

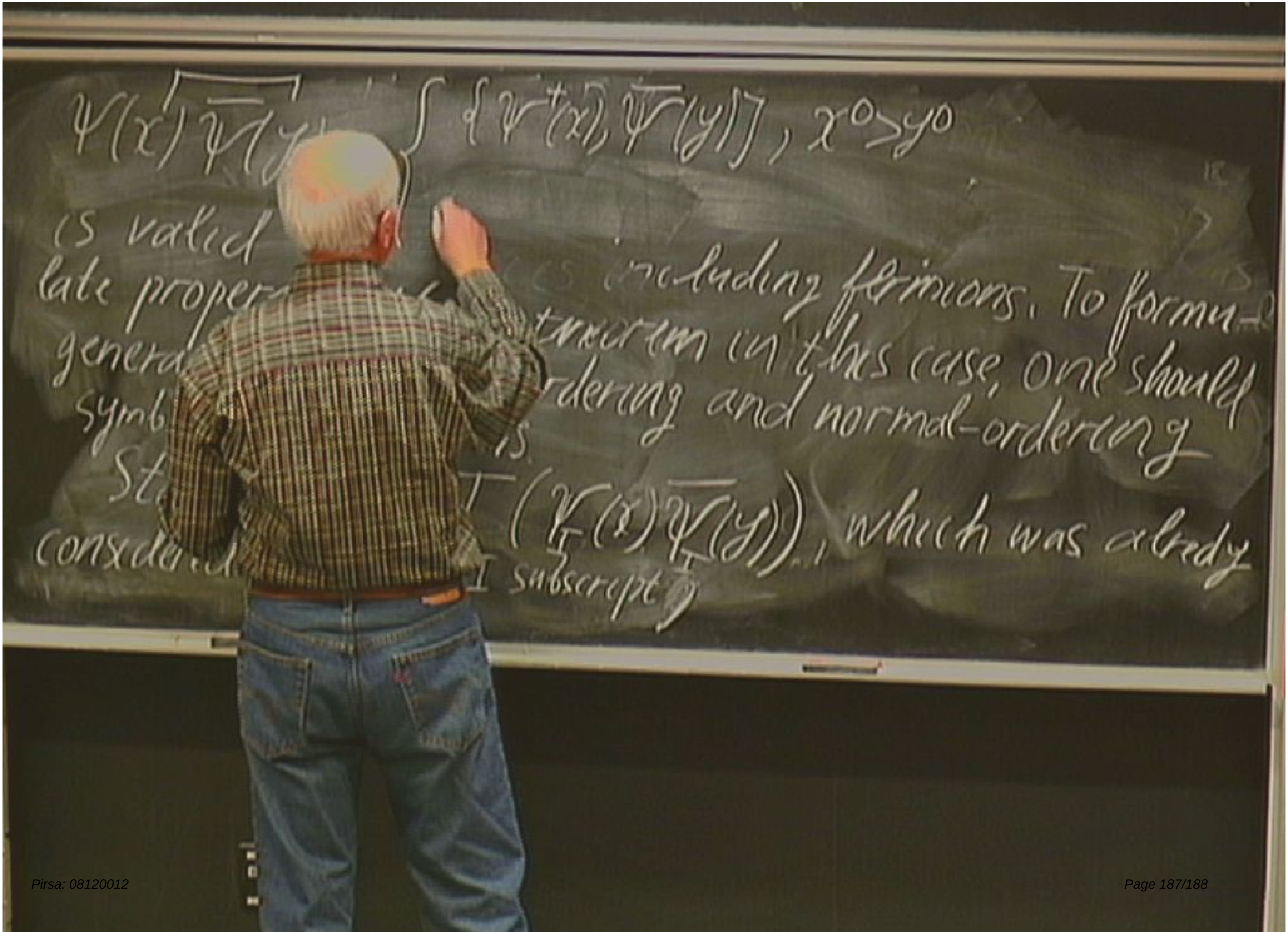

T(ψ(x) ψ̄(y)) ≡ ψ(x)ψ̄(y), x⁰ > y⁰
T ψ(x) ψ̄(y) ≡ (y₀ > x₀)
= N(ψ(x) ψ̄(y))


$T(\psi(x)\bar\psi(y)) \equiv \{\psi(x)\bar\psi(y),\ x^0>y^0$
$T\,\psi(x)\bar\psi(y) \underset{y_0>x_0}{=} -\bar\psi^+(y)\psi^+(x) - \bar\psi^+(y)\psi^-(x) - \bar\psi^-(y)\psi^+(x) -$
$-\bar\psi^-(y)\psi^-(x) = -\bar\psi^+(y)\psi^+(x) + \psi^-(x)\bar\psi^+(y) - \bar\psi^-(y)\psi^+(x) -$
$-\bar\psi^-(y)\psi^-(x) - \{\bar\psi^+(y),\psi^-(x)\} = N(\psi(x)\bar\psi(y)) -$






is valid
including fermions. To formu-
late properly Wick
generalize the
case, one should
symbols for
normal-ordering
Start from
which was already
considered
omit


x^0>y^0
is valid
including fermions. To formu-
late proper
theorem in this case, one should
ordering and normal-ordering
which was already
subscript

$$T(\psi(x)\overline{\psi}(y)) = N(\psi(x)\overline{\psi}(y)) + \psi(x)\overline{\psi}(y), \text{ where}$$

$$\psi(x)\overline{\psi}(y) = \begin{cases} \{\psi^+(x), \overline{\psi}^-(y)\}, & x^0 > y^0 \\ -\{\overline{\psi}^+(y), \psi^-(x)\}, & y^0 > x^0 \end{cases} = S_F(x-y).$$

is valid ... including fermions. To formulate properly Wick's theorem in this case, one should generalize the time-ordering and normal-ordering symbols for fermions.

Start from $T(\psi_I(x)\overline{\psi}_I(y))$, which was already considered (I'll omit I subscript)

Chistopher