

Title: Quantum Field Theory 1 - Lecture 14B

Date: Dec 10, 2008 03:30 PM

URL: <http://pirsa.org/08120012>

Abstract: Quantum Field Theory I course taught by Volodya Miransky of the University of Western Ontario

$$\langle \mathcal{R} | T_{\mathcal{R}} \{ \psi(x_1) \psi(x_2) \dots \psi(x_n) \} | \mathcal{R} \rangle = \lim_{T \rightarrow \infty (1-i\epsilon)} \frac{\langle 0 | T \{ \psi_{\mathcal{R}}(x_1) \dots \psi_{\mathcal{R}}(x_n) \} \times \exp[-\int_T dt H_{\mathcal{R}}(t)] | 0 \rangle}{\langle 0 | T \{ \exp[-\int_T dt H_{\mathcal{R}}(t)] \} | 0 \rangle}$$

$$\frac{\langle \Omega | T \{ \psi(x_1) \psi(x_2) \dots \psi(x_n) \} | \Omega \rangle}{\times \exp[-\int_{-T}^T dt H_1(t)] | 0 \rangle} = \lim_{T \rightarrow \infty(1-i\epsilon)} \frac{\langle 0 | T \{ \psi_I(x_1) \dots \psi_I(x_n) \} \times \exp[-\int_{-T}^T dt H_1(t)] | 0 \rangle}{\langle 0 | T \{ \exp[-\int_{-T}^T dt H_1(t)] | 0 \rangle}$$

$$\lim_{T \rightarrow \infty(1-i\epsilon)} \langle 0 | T \{ \exp[-\int_{-T}^T dt H_1(t)] | 0 \rangle = \lim_{T \rightarrow \infty(1-i\epsilon)} (\langle \Omega | \Omega \rangle)^2 e^{-iE_0(2T)}$$

where $E_0 = \langle \Omega | H | \Omega \rangle$

$$\begin{aligned}
 & \times \exp\left[-\int_{-T}^0 dt H_1(t)\right] |0\rangle \\
 & \lim_{T \rightarrow \infty(1-\epsilon)} \langle 0| T \left\{ \exp\left[-\int_{-T}^T dt H_1(t)\right] \right\} |0\rangle = \lim_{T \rightarrow \infty(1-\epsilon)} \left(\langle 0| \Omega \Omega^\dagger e^{-iE_0(2T)} \right) \\
 & \text{where } \langle R|H|R\rangle
 \end{aligned}$$

$$\frac{\langle 0 | \exp[-\int_{-T}^T dt H_0] | 0 \rangle}{\langle 0 | \exp[-\int_{-T}^T dt H_0] | 0 \rangle}$$

$$T \rightarrow \infty(1-\epsilon) \quad \langle 0 | T \exp[-\int_{-T}^T dt H_0] | 0 \rangle$$

$$\lim_{T \rightarrow \infty} \frac{\langle 0 | T \left\{ \exp \left[-\int_{-T}^T dt H_0 \right] \right\} | 0 \rangle}{\langle 0 | \exp[-\int_{-T}^T dt H_0] | 0 \rangle} = \lim_{T \rightarrow \infty(1-\epsilon)} \left(\frac{\langle 0 | \Omega \exp[-E_0(2T)] \Omega^\dagger | 0 \rangle}{\langle 0 | \exp[-E_0(2T)] | 0 \rangle} \right)$$

where $E_0 = \langle 0 | H_0 | 0 \rangle$



$$\lim_{T \rightarrow \infty (1-i\epsilon)} \frac{\langle 0 | T \{ \exp [- \int dt H_0] | 0 \rangle}{-T} = \lim_{T \rightarrow \infty (1-i\epsilon)} (k_0 | \Omega \rangle^2 e^{-iE_0(2T)}$$

where $E_0 = \langle \Omega | H | \Omega \rangle$. (Corresponding diagrams)

plückerung und verteilung
 4

$$\lim_{T \rightarrow \infty (1-i\epsilon)} \frac{\langle 0 | T \{ \exp [- \int dt H_0] | 0 \rangle}{-T} = \lim_{T \rightarrow \infty (1-i\epsilon)} (K_0)^{-1} e^{-iE_0(2T)}$$

where $E_0 = \langle 0 | H | 0 \rangle$. (Corresponding diagrams)



plückerung und verteilung
 4

$$\lim_{T \rightarrow \infty (1-i\epsilon)} \frac{\langle 0 | T \{ \exp[-\int dt H_0] | 0 \rangle}{T} = \lim_{T \rightarrow \infty (1-i\epsilon)} (K_0 | \Omega \rangle^2 e^{-iE_0(2T)}$$

where $\langle R | H | R \rangle$. (Corresponding diagrams)



... 4 ...

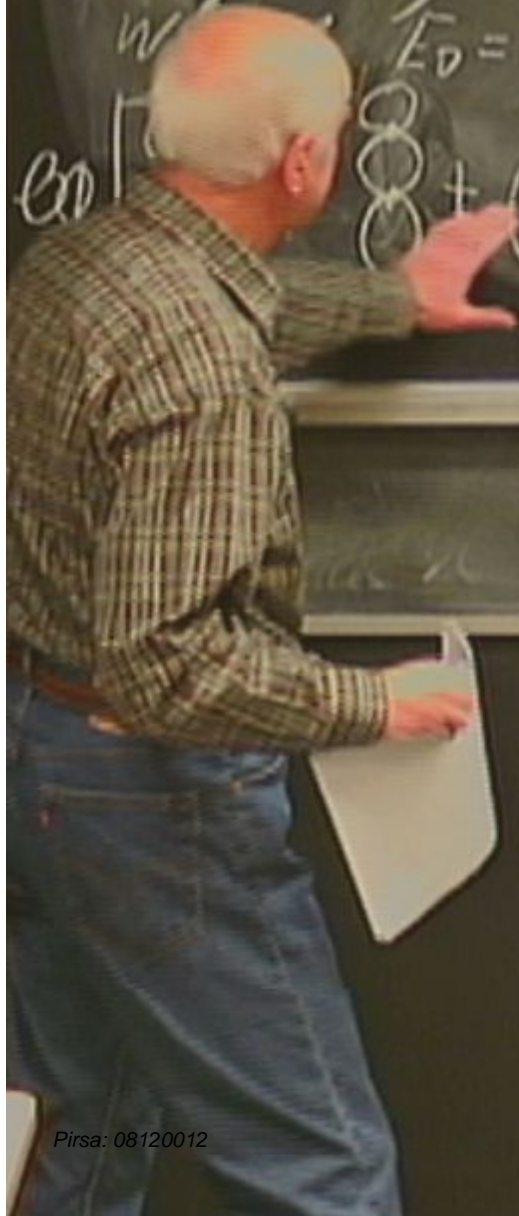
$$\lim_{T \rightarrow \infty (1-i\epsilon)} \frac{\langle 0 | T \{ \exp[-\int dt H_0] | 0 \rangle}{T} = \lim_{T \rightarrow \infty (1-i\epsilon)} (K_0 | \Omega \rangle^2 e^{-iE_0(zT)})$$

where $E_0 = \langle R | H | R \rangle$ (Corresponding diagrams)



$$\lim_{T \rightarrow \infty (1-i\epsilon)} \frac{\langle 0 | T \{ \exp[-i \int dt H_0] | 0 \rangle}{-i} = \lim_{T \rightarrow \infty (1-i\epsilon)} (K_0 | \Omega \rangle e^{-i E_0 (2T)}$$

with $E_0 = \langle \Omega | H | \Omega \rangle$. (Corresponding diagrams)



[Faded handwritten notes on the lower chalkboard, including the number 4.]

$$\lim_{T \rightarrow \infty (1-i\epsilon)} \frac{\langle 0 | T \{ \exp[-\int dt H_0] | 0 \rangle}{-T} = \lim_{T \rightarrow \infty (1-i\epsilon)} (K_0 | \Omega \rangle e^{-E_0(2T)}$$

where $E_0 = \langle \Omega | H | \Omega \rangle$. (Corresponding diagrams)

$$\exp[\text{diagram 1} + \text{diagram 2} + \text{diagram 3} + \dots] =$$

[Faint handwritten notes, possibly including the number 4]

$$\lim_{T \rightarrow \infty (1-i\epsilon)} \frac{\langle 0 | T \{ \exp[-\int dt H_0] | 0 \rangle}{-T} = \lim_{T \rightarrow \infty (1-i\epsilon)} (K_0 | \Omega \rangle e^{-E_0(2T)}$$

where $E_0 = \langle R | H | R \rangle$. (Corresponding diagrams)

$$\exp[\text{diagram 1} + \text{diagram 2} + \text{diagram 3} + \dots] = \dots$$

plückerung und verteilung
 4

$$\lim_{T \rightarrow \infty (1-i\epsilon)} \frac{\langle 0 | T \{ \exp [- \int dt H_0] | 0 \rangle}{-T} = \lim_{T \rightarrow \infty (1-i\epsilon)} (K_0 | \Omega \rangle^2 e^{-i E_0 (2T)}$$

where $E_0 = \langle R | H | R \rangle$. (Corresponding diagrams)

ex. $[\text{diagram 1} + \text{diagram 2} + \text{diagram 3} + \dots]$. (vacuum bubbles)

[Faint, mostly illegible handwritten notes on the lower chalkboard panel]

3. For each external point \vec{x}
4. Integrate over each momentum $\int d^4 p$
5. Calculate the overall factor. ($\frac{1}{i}$ in our case).

$\mathbb{R}[x]$ \Rightarrow $\frac{(L(x))}{(271)^4}$ (x^2 -order)



$\mathcal{D}^2 \Rightarrow \frac{(-i)^4}{(2\pi)^4} \int d^4 p_1 d^4 p_2$

(2 order)

$$\begin{array}{c}
 \text{P}_1 \\
 \text{P}_2
 \end{array}
 \Rightarrow \frac{(-i\pi)^2}{(2\pi)^4} \int d^4 p_1 d^4 p_2 \quad (\text{2 order})$$

$$\int_{\mathcal{P}_1} \int_{\mathcal{P}_2} \Rightarrow \frac{(-LX)}{4!!} \int d^4 p_1 d^4 p_2$$

(2 order)

$$P_1 \otimes P_2 \Rightarrow \frac{(-i)^4}{4!} \int d^4 P_1 d^4 P_2 S^4(P_1 - P_1 + P_2 - P_2)$$



$$\begin{aligned}
 & \int_{P_1}^{\infty} \int_{P_2}^{\infty} \Rightarrow \frac{(-LX)}{4!!} \int d^4 P_1 d^4 P_2 \sqrt{S^4 (P_1 - P_1 + P_2 - P_2)} \frac{1}{(2\pi)^8}
 \end{aligned}$$

$$\begin{aligned}
 & \text{Diagram: } P_1 \text{ and } P_2 \text{ with arrows indicating interaction} \Rightarrow \\
 & \frac{(-i)^4}{4!} \int d^4 P_1 d^4 P_2 \sqrt{S^4 (P_1 - P_1 + P_2 - P_2)} \frac{1}{(2\pi)^8} \frac{1}{(P_1^2 - m^2 + i\epsilon)(P_2^2 - m^2 + i\epsilon)}
 \end{aligned}$$

$$\int_{\mathbb{R}^2} \delta^4(p_1 - p_2) \Rightarrow \frac{(-i\lambda)}{4!} \int d^4 p_1 d^4 p_2 \delta^4(p_1 - p_2) \frac{1}{(2\pi)^8} \frac{1}{(p_1^2 - m^2 + i\epsilon)(p_2^2 - m^2 + i\epsilon)}$$

$$= \frac{(-i\lambda)}{4! (2\pi)^4} \delta^4(0)$$

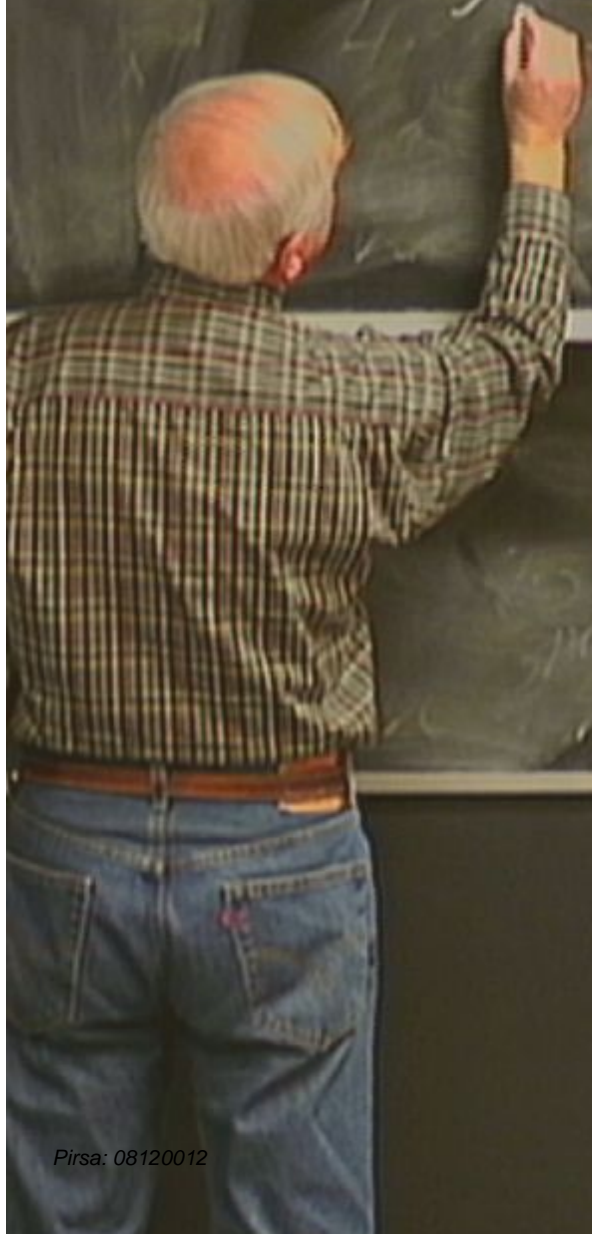


$$\frac{4!}{(2\pi)^4} S'(0) \left[\int d^4p \frac{1}{p^2 - m^2 + i\epsilon} \right]$$

$$S^4(0) = \frac{1}{(2\pi)^4} \int d^4z \ell \left. \frac{1}{p=0} \right|$$



$$S^4(0) = \frac{1}{(2\pi)^4} \int_{\mathbb{R}^4} dz \ell^{\text{IPZ}} \Big|_{P=0} = \frac{1}{(2\pi)^4} 42T$$



$$P_1^2 P_2^2 \Rightarrow \frac{(-1)^\lambda}{4!} \int d^4 P_1 d^4 P_2 \sqrt{S^4(P_1 - P_1 + P_2 - P_2)} \frac{1}{(2\pi)^8} \frac{1}{(P_1^2 - m^2 + i\epsilon)(P_2^2 - m^2 + i\epsilon)}$$

$$= \frac{(-1)^\lambda}{4! (2\pi)^4} S^4(0) \left[\int d^4 P \frac{1}{P^2 - m^2 + i\epsilon} \right]^2$$

$$S^4(0) = \frac{1}{(2\pi)^4} \int d^4 z e^{iPz} \Big|_{P=0} = \frac{1}{(2\pi)^4} 2T_2 V$$

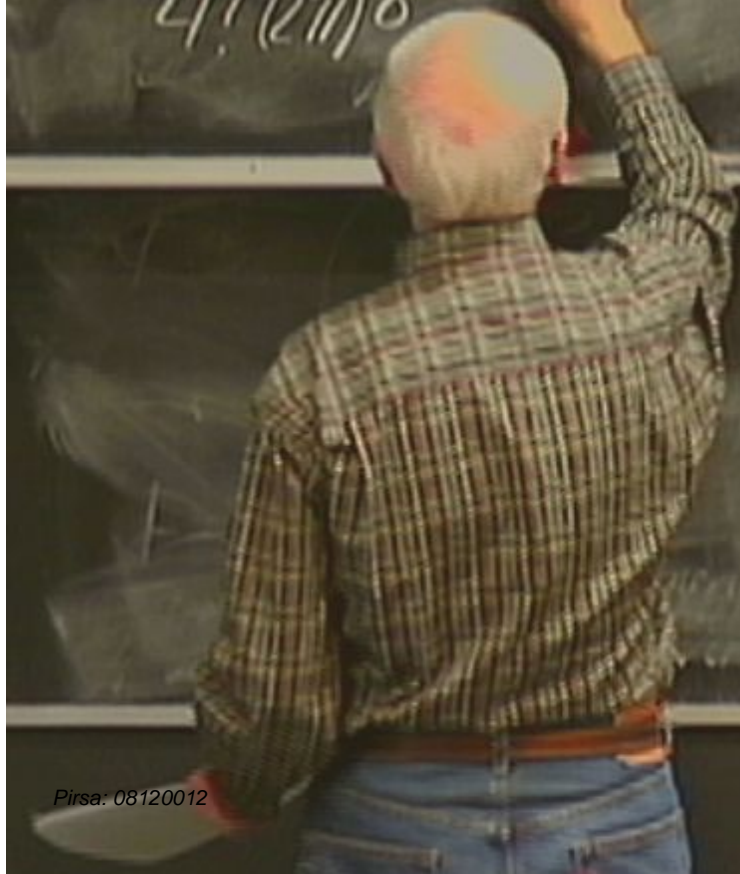
Volume.

$$= \frac{(-i\lambda)}{4! (2\pi)^4} S^4(0) \left[\int d^4p \frac{1}{p^2 - m^2 + i\epsilon} \right]^2$$

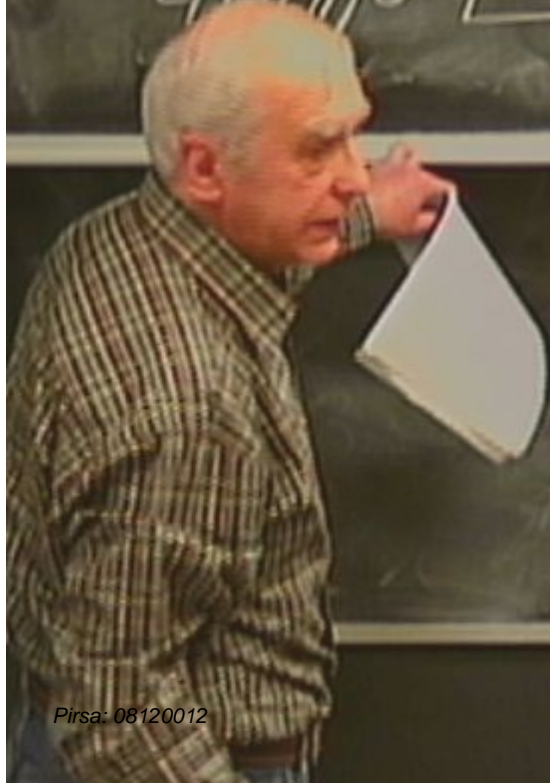
$$S^4(0) = \frac{1}{(2\pi)^4} \int d^4z e^{i(pz)} \Big|_{p=0} = \frac{1}{(2\pi)^4} 2T_2 V$$

Volume.

$$\sim \frac{-i\lambda}{4! (2\pi)^8}$$



$$\begin{aligned}
 & \frac{(-i\lambda)}{4! (2\pi)^4} S^4(0) \left[\int d^4 p \frac{1}{p^2 - m^2 + i\epsilon} \right]^2 \\
 S^4(0) &= \frac{1}{(2\pi)^4} \int d^4 z e^{i p z} \Big|_{p=0} = \frac{1}{(2\pi)^4} 2T_2 V \\
 & \frac{-i\lambda}{4! (2\pi)^8} \boxed{2T_2 V} \cdot \left[\int d^4 p \frac{1}{p^2 - m^2 + i\epsilon} \right]^2 \quad \text{Volume.}
 \end{aligned}$$



$$S^4(0) = \frac{1}{(2\pi)^4} \int d^4z \ell^{\text{LPZ}} \Big|_{P=0} = \frac{1}{(2\pi)^4} 2T_2 V$$

$$\sim \frac{-c_2}{4! (2\pi)^4} \sqrt{2T_2 V} \cdot \left[d^4P \frac{1}{P^2 - m^2 + i\epsilon} \right]^2 \text{Volume.}$$

↑
SP

$$4! (2\pi)^4 \delta^{(4)}(p) [p^2 - m^2 + i\epsilon]^{-2}$$

$$S^4(0) = \frac{1}{(2\pi)^4} \int d^4z \ell^{\mu\nu} \Big|_{p=0} = \frac{1}{(2\pi)^4} 2T_2 V$$

$$\frac{1}{8} \left[\underset{\substack{\uparrow \\ \text{spatial}}}{2T_2 V} \cdot \left[d^4p \frac{1}{p^2 - m^2 + i\epsilon} \right]^2 \right] \text{Volume.}$$



$$S^4(0) = \frac{1}{(2\pi)^4} \int d^4z \ell^{\text{LPZ}} \Big|_{P=0} = \frac{1}{(2\pi)^4} 4 \cdot 2T \cdot V$$

All vacuum diagrams are proportional to V .

$$\sim \frac{-i\lambda}{4!(2\pi)^8} \underbrace{[2TV]}_{\text{spatial}} \cdot \underbrace{[d^4P \frac{1}{P^2 - m^2 + i\epsilon}]}_{\text{Volume}}$$

[Faded handwritten notes on the lower chalkboard panel, including the word "spatial" and "Volume" written vertically.]



$$S^4(0) = \frac{1}{(2\pi)^4} \int d^4z \ell^{\text{LPZ}} \Big|_{P=0} = \frac{1}{(2\pi)^4} 2TV$$

$$\sim \frac{-i\gamma}{4!(2\pi)^8} \boxed{2TV} \cdot \left[d^4P \frac{1}{P^2 - m^2 + i\epsilon} \right]$$

↑ spatial

All vacuum diagrams are proportional to $2TV$.

where $E_0 = \langle R|H|R \rangle$. (Corresponding diagrams)

$[\text{diagram 1} + \text{diagram 2} + \text{diagram 3} + \dots]$ (vacuum bubbles)

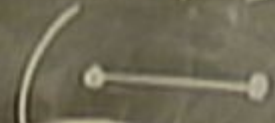
$T \rightarrow \infty (1-\epsilon)$

$$\begin{aligned}
 & \Rightarrow \frac{(-i\lambda)}{4!} \int d^4 p_1 d^4 p_2 S^4(p_1 - p_1 + p_2 - p_2) \frac{1}{(2\pi)^8} \frac{1}{(p_1^2 - m^2 + i\epsilon)} \frac{1}{(p_2^2 - m^2 + i\epsilon)} \\
 & = \frac{(-i\lambda)}{4! (2\pi)^4} S^4(0) \left[\int d^4 p \frac{1}{p^2 - m^2 + i\epsilon} \right]^2 \\
 & S^4(0) = \frac{1}{(2\pi)^4} \int d^4 z e^{i p z} \Big|_{p=0} = \frac{1}{(2\pi)^4} \int d^4 z \\
 & \sim \frac{-i\lambda}{4! (2\pi)^8} \boxed{2TV} \cdot \left[\int d^4 p \frac{1}{p^2 - m^2 + i\epsilon} \right]^2
 \end{aligned}$$

(num
ams are
onal to

One can show that the numerator

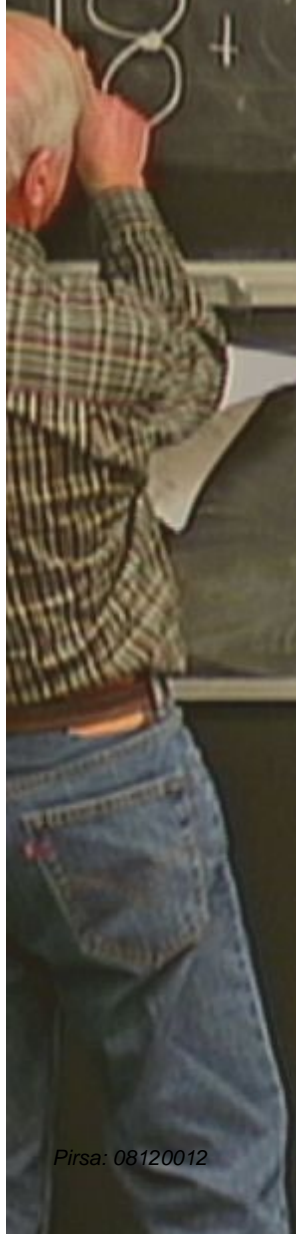
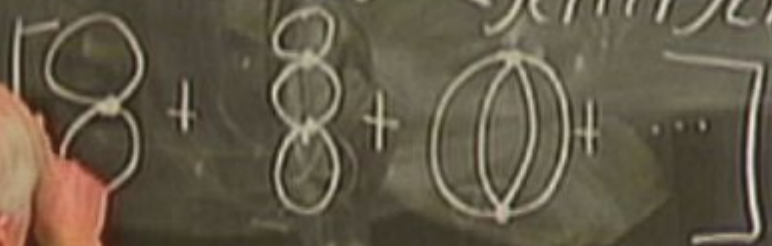
LS



where $E_0 = \langle R|H|R \rangle$. (Corresponding diagrams)

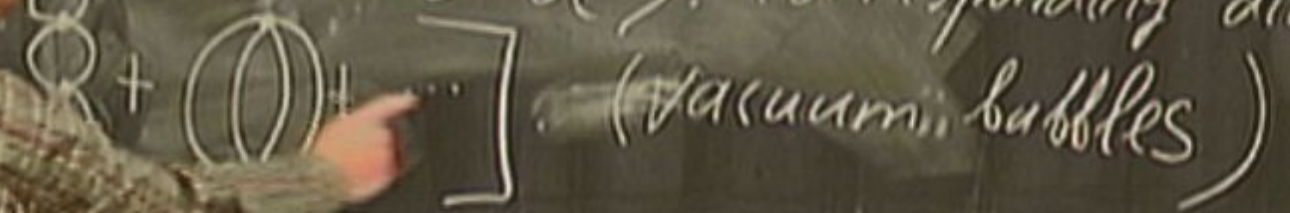
$T \rightarrow \alpha(1-\epsilon)$

$[\text{diagram 1} + \text{diagram 2} + \text{diagram 3} + \dots]$ (vacuum bubbles)



$$\lim_{T \rightarrow \infty(1-\epsilon)} \frac{\langle 0|T \left\{ \exp \left[- \int_{-T}^T dt H_1 \right] \right\} |0\rangle}{\langle 0|0\rangle} = \lim_{T \rightarrow \infty(1-\epsilon)} \langle 0| \Omega T^2 e^{-iE_0(2T)} \rangle$$

$E_0 = \langle 0|H|0\rangle$. (Corresponding diagrams)



One can show that the numerator

$$\begin{aligned}
 & \text{LS} \left(\text{diagram 1} + \text{diagram 2} \right) \times \exp \left[\text{diagram 3} + \text{diagram 4} \right] \\
 & + \text{diagram 5} + \dots \rightarrow \text{Vacuum bubble diagrams are factorized}
 \end{aligned}$$

One can show that the numerator

$$\begin{aligned}
 & \text{LS} \left[\left(\overset{\circ}{x} \xrightarrow{\quad} \overset{\circ}{y} + \overset{\circ}{x} \xrightarrow{\Delta} \overset{\circ}{y} + \overset{\circ}{x} \xrightarrow{\ominus} \overset{\circ}{y} \right) \times \exp \left[\textcircled{8} + \textcircled{8} + \right. \right. \\
 & \left. \left. + \textcircled{8} + \dots \right] \right] \Rightarrow \text{where bubble diagrams are factorized} \\
 & \overset{\circ}{x} \xrightarrow{\quad} \overset{\circ}{y} \textcircled{8}
 \end{aligned}$$



One can show that the numerator

$$LS \left(\begin{array}{c} \text{---} \bullet \text{---} \bullet \text{---} \\ \text{---} \bullet \text{---} \bullet \text{---} \\ \text{---} \bullet \text{---} \bullet \text{---} \end{array} \right) \times \exp \left[\text{---} \bullet \text{---} \bullet \text{---} + \text{---} \bullet \text{---} \bullet \text{---} \right]$$



vac. \mathcal{O}

\mathcal{O}

\Rightarrow Vacuum bubble diagrams are factorized

Because of this, the contribution


One can show that the numerator

$$LS \left(\begin{array}{c} \text{---} \circ \text{---} \circ \text{---} \\ \text{---} \circ \text{---} \circ \text{---} \\ \text{---} \circ \text{---} \circ \text{---} \end{array} + \text{---} \circ \text{---} \circ \text{---} + \text{---} \circ \text{---} \circ \text{---} \right) \times \exp \left[\text{---} \circ \text{---} \circ \text{---} + \text{---} \circ \text{---} \circ \text{---} + \dots \right] \Rightarrow \text{Vacuum bubble diagrams are factorized}$$

Because of this, the contribution of vacuum diagrams cancels.

One can show that the numerator

$$LS \left(\begin{array}{c} \text{---} \text{---} \text{---} \\ \text{---} \text{---} \text{---} \end{array} + \begin{array}{c} \text{---} \text{---} \text{---} \\ \text{---} \text{---} \text{---} \end{array} + \begin{array}{c} \text{---} \text{---} \text{---} \\ \text{---} \text{---} \text{---} \end{array} \right) \times \exp \left[\begin{array}{c} \text{---} \text{---} \text{---} \\ \text{---} \text{---} \text{---} \end{array} + \begin{array}{c} \text{---} \text{---} \text{---} \\ \text{---} \text{---} \text{---} \end{array} + \dots \right] \Rightarrow \text{Vacuum bubble diagrams are factorized}$$

+  + ...
we studied.

Because of this, the contribution of vacuum diagrams cancels.

$$\sim \frac{-(\lambda)}{4!(2\pi)^8} [2TV] \cdot \left[\frac{d^4 p}{p^2 - m^2 + i\epsilon} \right] \text{Volume.}$$

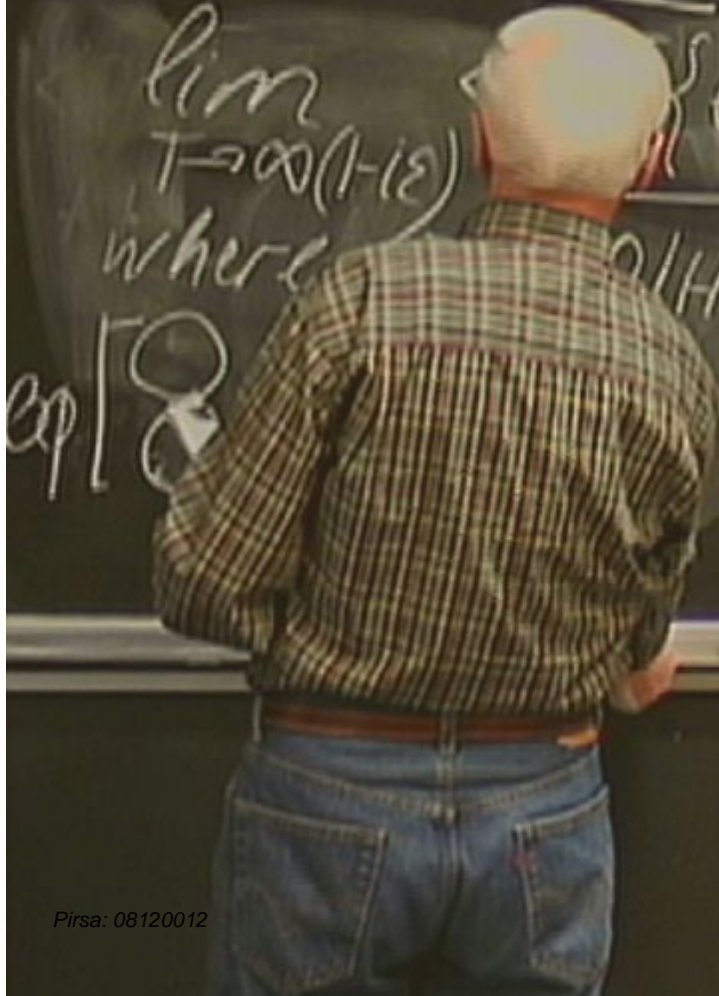
spiral

diagrams are proportional to $2TV$.

$$\lim_{T \rightarrow \infty(1-i\epsilon)} \langle 0 | T \exp \left[- \int_T dt H_0(t) \right] | 0 \rangle = \lim_{T \rightarrow \infty(1-i\epsilon)} \langle 0 | T \exp \left[- \int_T dt H_0(t) \right] | 0 \rangle$$

$$\lim_{T \rightarrow \infty(1-i\epsilon)} \langle \exp \left[- \int_T dt H_0(t) \right] | 0 \rangle = \lim_{T \rightarrow \infty(1-i\epsilon)} (k_0 | \Omega \rangle e^{-iE_0(2T)},$$

where $\langle \Omega | H_0 | \Omega \rangle = E_0$. (Corresponding diagrams (vacuum bubbles))



$$\sim \frac{-(c)}{4!(2\pi)^8} [2TV] \cdot [d^4p \frac{1}{p^2 - m^2 + i\epsilon}] \quad \begin{matrix} \text{Volume} \\ \text{spatial} \end{matrix}$$

diagrams are proportional to $2TV$.

$$\langle \Omega | T \{ \psi(x_1) \psi(x_2) \dots \psi(x_n) \} | \Omega \rangle = \lim_{T \rightarrow \infty(1-i\epsilon)} \frac{\langle 0 | T \{ \psi_I(x_1) \dots \psi_I(x_n) \} \times \exp[-\int_T dt H_I] | 0 \rangle}{\langle 0 | T \{ \exp[-\int_T dt H_I] | 0 \rangle}$$

$$\lim_{T \rightarrow \infty} \frac{\langle 0 | T \{ \exp[-\int_T dt H_I] | 0 \rangle}{\langle 0 | T \{ \exp[-\int_T dt H_I] | 0 \rangle} = \lim_{T \rightarrow \infty(1-i\epsilon)} (K_0 | \Omega \rangle e^{-iE_0(2T)})$$

where $| \Omega \rangle = \lim_{T \rightarrow \infty} \frac{1}{\sqrt{Z}} \int \mathcal{D}\phi \exp[-\int_T dt H_I] | 0 \rangle$. (Corresponding diagrams) (vacuum bubbles)

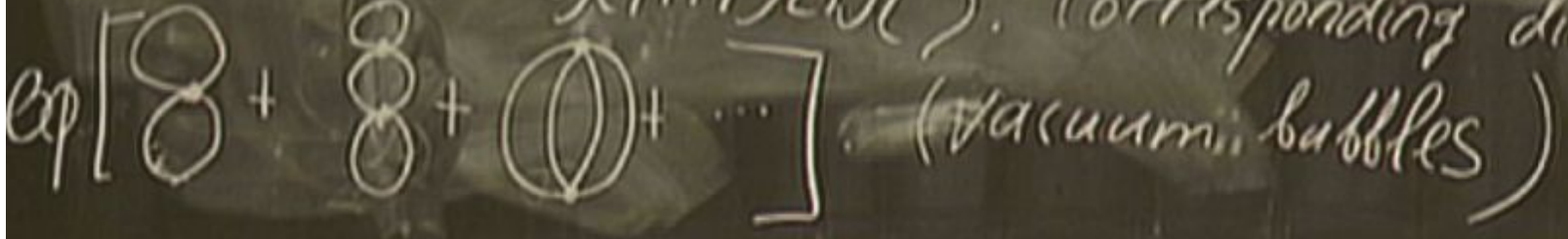
$$\sim \frac{-cT}{4!(2\pi)^8} [2TV] \cdot [d^4p \frac{1}{p^2 - m^2 + i\epsilon}] \quad \begin{matrix} \text{Volume} \\ \text{spatial} \end{matrix}$$

diagrams are proportional to $2TV$.

$$\frac{\langle \Omega | T_{\tau_1} \psi(x_1) \psi(x_2) \dots \psi(x_n) | \Omega \rangle}{\langle 0 | T \exp[-\int_T dt H_I(t)] | 0 \rangle} = \lim_{T \rightarrow \infty(1-i\epsilon)} \frac{\langle 0 | T \psi_I(x_1) \dots \psi_I(x_n) \times \exp[-\int_T dt H_I(t)] | 0 \rangle}{\langle 0 | T \exp[-\int_T dt H_I(t)] | 0 \rangle}$$

$$\lim_{T \rightarrow \infty(1-i\epsilon)} \frac{\langle 0 | T \exp[-\int_T dt H_I(t)] | 0 \rangle}{\langle 0 | T \exp[-\int_T dt H_I(t)] | 0 \rangle} = \lim_{T \rightarrow \infty(1-i\epsilon)} (K_0 | \Omega \rangle)^2 e^{-iE_0(2T)}$$

where $E_0 = \langle \Omega | H | \Omega \rangle$. (corresponding diagrams)



Physical meaning of vacuum diagrams

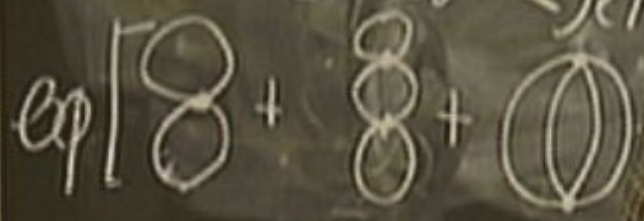


$$\frac{\langle \mathcal{R} | T_{\mathcal{R}} \{ \psi(x_1) \psi(x_2) \dots \psi(x_n) \} | \mathcal{R} \rangle}{\times \exp[-\int dt H_1(t)] | 0 \rangle} = \lim_{T \rightarrow \infty (1-i\epsilon)} \frac{\langle 0 | T \{ \psi(x_1) \dots \psi(x_n) \} \times \exp[-\int dt H_1(t)] | 0 \rangle}{\langle 0 | T \{ \exp[-\int dt H_1(t)] \} | 0 \rangle}$$

$$\lim_{T \rightarrow \infty (1-i\epsilon)} \langle 0 | T \{ \exp[-\int dt H_1(t)] \} | 0 \rangle = \lim_{T \rightarrow \infty (1-i\epsilon)} (K_0 | \mathcal{R} \rangle e^{-iE_0 T}$$

where $E_0 = \langle \mathcal{R} | H_0 | \mathcal{R} \rangle$

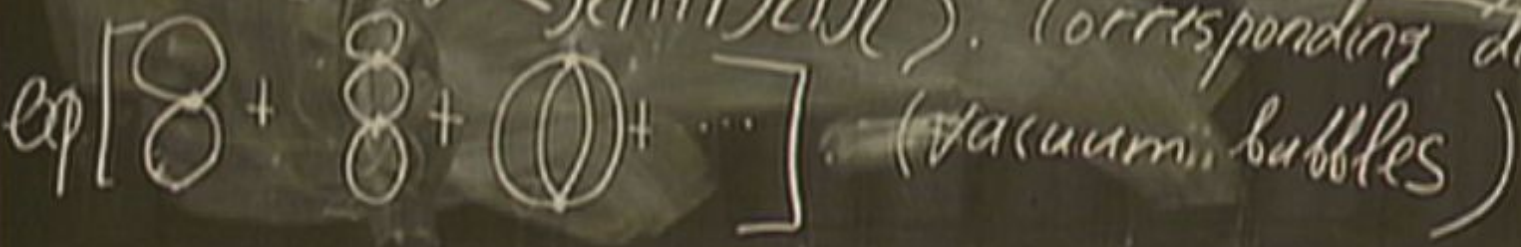
(corresponding diagrams
sum bubbles)



$$\frac{\langle \Omega | T_T \{ \psi(x_1) \psi(x_2) \dots \psi(x_n) \} | \Omega \rangle}{\times \exp[-\int_T dt H_I(t)] | 0 \rangle} = \lim_{T \rightarrow \infty(1-i\epsilon)} \frac{\langle 0 | T \{ \psi(x_1) \dots \psi(x_n) \} \times \exp[-\int_T dt H_I(t)] | 0 \rangle}{\langle 0 | T \{ \exp[-\int_T dt H_I(t)] | 0 \rangle}$$

$$\lim_{T \rightarrow \infty(1-i\epsilon)} \langle 0 | T \{ \exp[-\int_T dt H_I(t)] | 0 \rangle = \lim_{T \rightarrow \infty(1-i\epsilon)} (K_0 | \Omega \rangle)^2 e^{-iE_0(2T)}$$

where $E_0 = \langle \Omega | H | \Omega \rangle$. (Corresponding diagrams)

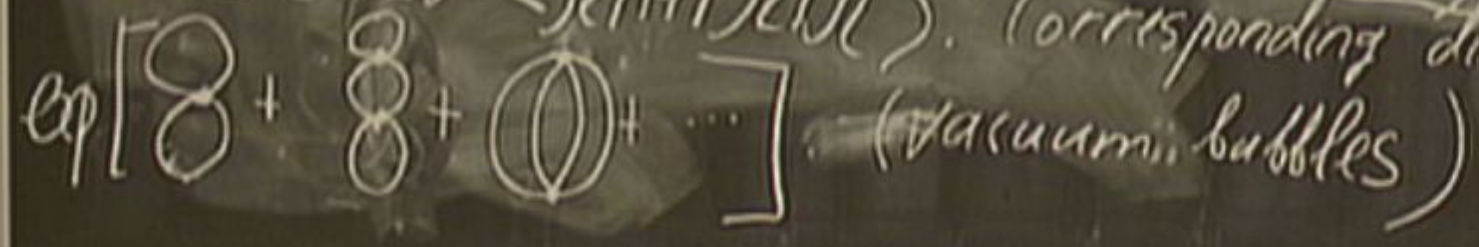


spiral

$$\langle \Omega | T \{ \psi(x_1) \psi(x_2) \dots \psi(x_n) \} | \Omega \rangle = \lim_{T \rightarrow \infty(1-i\epsilon)} \frac{\langle 0 | T \{ \psi(x_1) \dots \psi(x_n) \} \times \exp[-\int_{-T}^T dt H_1(t)] | 0 \rangle}{\langle 0 | T \{ \exp[-\int_{-T}^T dt H_1(t)] | 0 \rangle}$$

$$\lim_{T \rightarrow \infty(1-i\epsilon)} \langle 0 | T \{ \exp[-\int_{-T}^T dt H_1(t)] | 0 \rangle = \lim_{T \rightarrow \infty(1-i\epsilon)} (K_0 | \Omega \rangle^2 e^{-E_0(2T)}$$

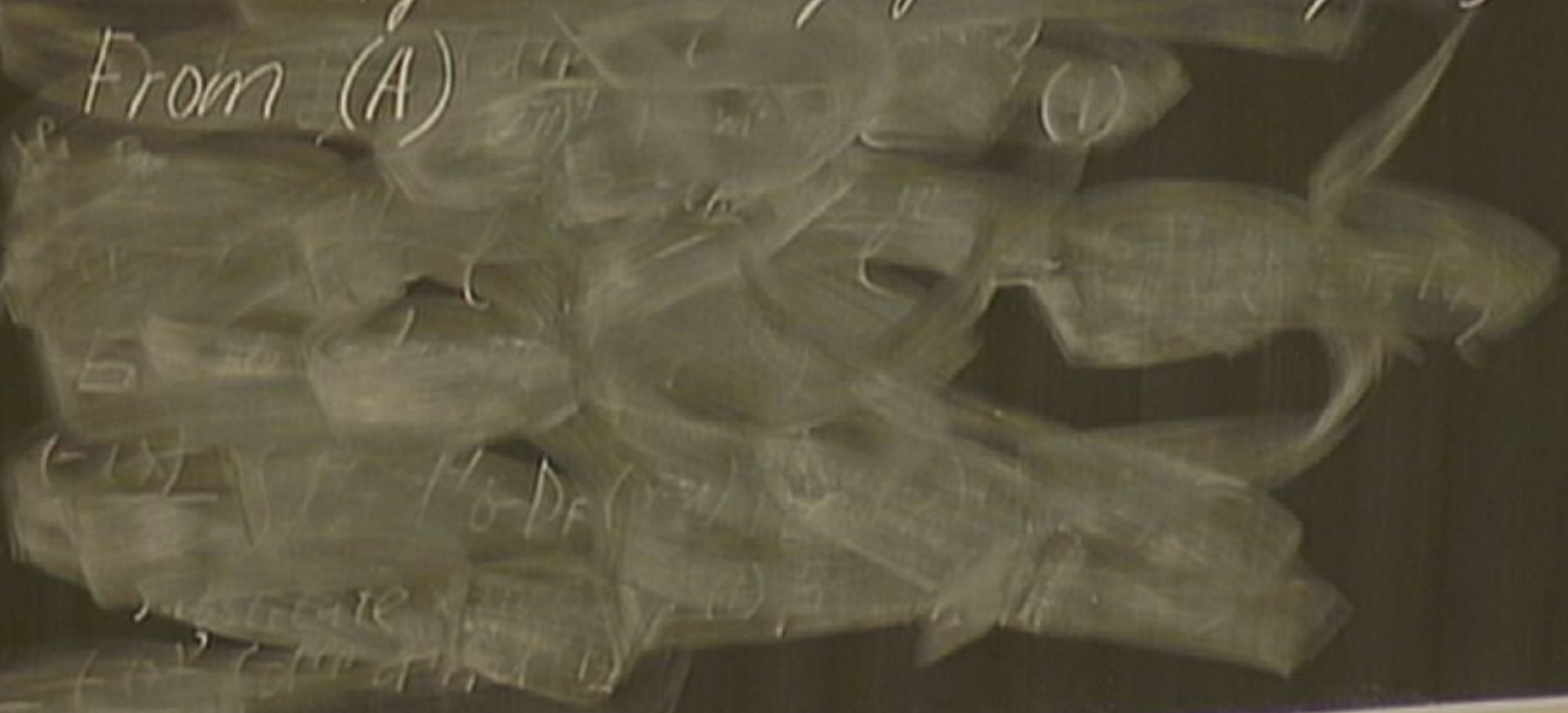
where $E_0 = \langle \Omega | H | \Omega \rangle$. (Corresponding diagrams)



6/27/14

Physical meaning of vacuum diagrams

From (A)

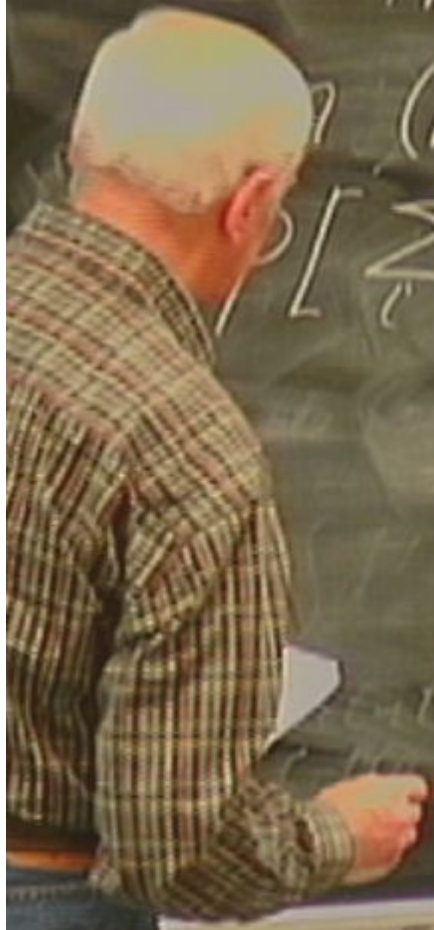


6-147 (1/10/14, 1/12/14, 1/14/14)

Physical meaning of vacuum diagrams

(A), we get.

$$\left[\sum_i \delta_i \right]$$



$$\langle \text{LX} \rangle = \langle \text{LX} \rangle_{\text{vac}} + \langle \text{LX} \rangle_{\text{non-vac}}$$

Physical meaning of vacuum diagrams

From (A), we get

$$\exp\left[\sum_i \delta_i\right]$$

$$\langle \text{LX} \rangle = \langle \text{LX} \rangle_{\text{vac}} + \langle \text{LX} \rangle_{\text{non-vac}}$$

(LX) (11/11/14)

Physical meaning of vacuum diagrams

From (A), we get
vacuum diagrams

$$\exp\left[\sum_i \delta_i\right] = \dots$$

(1)

(LX) (11/11/14)

vacuum diagrams

$$\sim \frac{-(cT)}{4!(2\pi)^8} \left[2TV \int_{\text{spiral}} [d^4p \frac{1}{p^2 - m^2 + i\epsilon}] \right]^{\text{Volume}}$$

diagrams are proportional to $2TV$.

$$\langle \mathcal{R} | T_{\tau_1}^{\tau_2} \psi(x_1) \psi(x_2) \dots \psi(x_n) | \mathcal{R} \rangle = \lim_{T \rightarrow \infty(1-i\epsilon)} \frac{\langle 0 | T \{ \psi(x_1) \dots \psi(x_n) \} \times \exp[-\int_T dt H_I(t)] | 0 \rangle}{\langle 0 | T \exp[-\int_T dt H_I(t)] | 0 \rangle}$$

$$\lim_{T \rightarrow \infty(1-i\epsilon)} \langle 0 | T \{ \exp[-\int_T dt H_I] | 0 \rangle = \lim_{T \rightarrow \infty(1-i\epsilon)} (K | \Omega \rangle \langle \Omega | K^\dagger)$$

where $E_0 = \langle \mathcal{R} | H | \mathcal{R} \rangle$. (Corresponding to

$\exp[\text{diagram} + \text{diagram} + \text{diagram} + \dots]$ (vacuum bubbles)

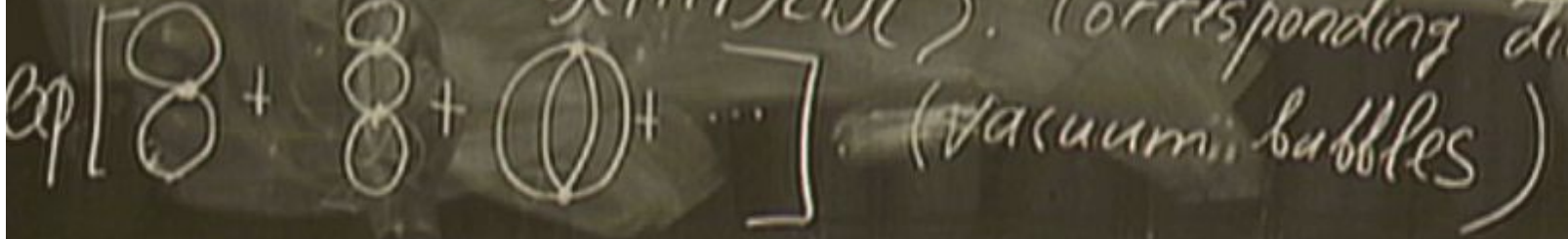
$$\sim \frac{-cT}{4!(2\pi)^8} [2TV] \cdot \left[\frac{d^4 p}{p^2 + i\epsilon} \right]^{\text{Volume}}$$

diagrams are proportional to $2TV$.

$$\langle \mathcal{R} | T_{\mathcal{R}} \{ \psi(x_1) \psi(x_2) \dots \psi(x_n) \} | \mathcal{R} \rangle = \lim_{T \rightarrow \infty(1-i\epsilon)} \frac{\langle 0 | T \{ \psi(x_1) \dots \psi(x_n) \} \times \exp[-\int_T dt H_I(t)] | 0 \rangle}{\langle 0 | T \{ \exp[-\int_T dt H_I]] | 0 \rangle}$$

$$\lim_{T \rightarrow \infty(1-i\epsilon)} \langle 0 | T \{ \exp[-\int_T dt H_I]] | 0 \rangle = \lim_{T \rightarrow \infty(1-i\epsilon)} (K_0 | \mathcal{R} \rangle e^{-iE_0(2T)}$$

where $E_0 = \langle \mathcal{R} | H | \mathcal{R} \rangle$. (Corresponding diagrams)



Moment Physical meaning of vacuum diagrams

From (A), we get.

$$\exp\left[\sum_i \delta_i\right] \sim e^{-iE_0(T)}$$

vacuum diagrams

$(L\mathcal{H}) / (1 + \mathcal{H}^2)$

Physical meaning of vacuum diagrams

from (A), we get

$$\exp\left[\sum_i \mathcal{V}_i\right] \sim e^{-iE_0(T)} \Rightarrow E_0(T) = i \sum_i \mathcal{V}_i$$

vacuum diagrams

Physical meaning of vacuum diagrams

From (A), we get.

$$\exp\left[\sum_i \delta_i\right] \sim e^{-iE_0(T)} \Rightarrow E_0(T) = i \sum_i \delta_i$$

Physical meaning of vacuum diagrams

From (A), we get.

$\exp\left[\sum_i \delta_i\right] \sim e^{-iE_0(T)} \Rightarrow E_0(T) = i \sum_i \delta_i$

vacuum diagrams

$$\frac{(-i\lambda)}{4! (2\pi)^4} S'(0) \left[\int d^4 p \frac{1}{p^2 - m^2 + i\epsilon} \right]^2$$

$$(0) = \frac{1}{(2\pi)^4} \int d^4 z \ell^{(pZ)} \Big|_{p=0} = \frac{1}{(2\pi)^4} 2T V$$

Volume

$$\frac{-i\lambda}{4! (2\pi)^8} \boxed{2T V} \cdot \left[\int d^4 p \frac{1}{p^2 - m^2 + i\epsilon} \right]^2$$

spatial

All vacuum diagrams are proportional to $2T V$.

$$\lim_{T \rightarrow \infty (1-i\epsilon)} \langle 0 | T \{ \exp [- \int dt H_I] \} | 0 \rangle = \lim_{T \rightarrow \infty (1-i\epsilon)} (K_0 | \Omega \rangle \langle \Omega | K_0^\dagger)^{-1} e^{-iE_0 T}$$

here $E_0 = \langle \Omega | H | \Omega \rangle$. (Corresponding diagrams)

QFT (11/10/13) 14

Physical meaning of vacuum diagrams

From (A), we get

$$\exp\left[\sum_i \text{vacuum diagrams}\right] \sim e^{-iE_0(2T)} \Rightarrow E_0(2T) = i \sum_i \text{vacuum diagrams} \Rightarrow$$

$$\frac{E_0(2T)}{2TV} = \frac{i \sum_i \text{vacuum diagrams}}{2TV}$$

Physical meaning of vacuum diagrams

From (A), we get

$$\exp\left[\sum_i \delta_i\right] \sim \Rightarrow E_0 ZT = i \sum_i \delta_i \Rightarrow$$

$$\frac{E_0 ZT}{ZT V} = \frac{i \sum_i \delta_i}{ZT V}$$

Vacuum energy density

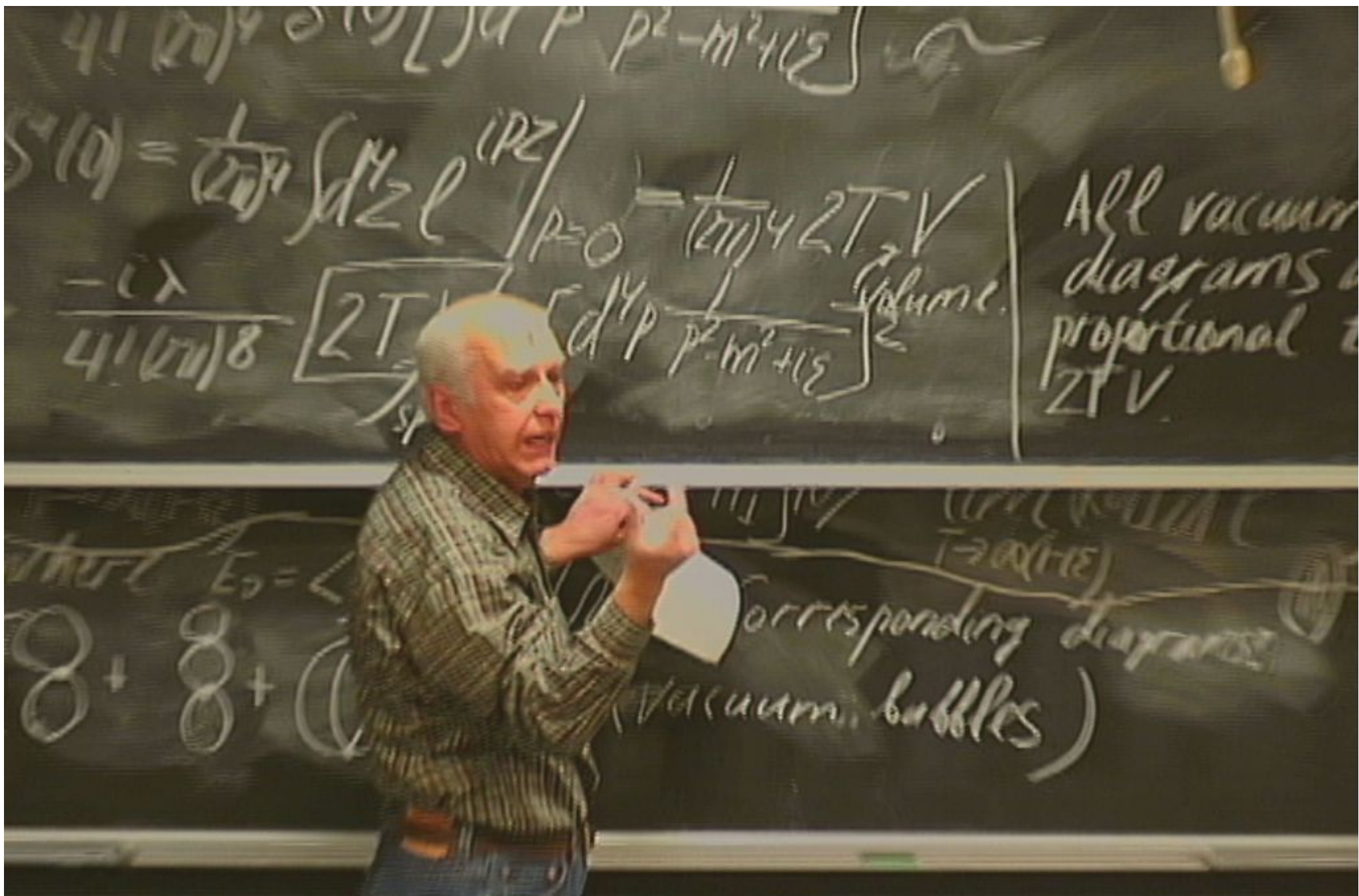
Physical meaning of vacuum diagrams

From (A), we get

$$\exp\left[\sum_i \text{vacuum diagrams}_i\right] \sim e^{-iE_0(2T)} \Rightarrow E_0(2T) = i \sum_i \text{vacuum diagrams}_i \Rightarrow$$

$$\frac{E_0(2T)}{2TV} = \frac{i \sum_i \text{vacuum diagrams}_i}{2TV} =$$

vacuum energy density



$$S^2(0) = \frac{1}{(2\pi)^4} \int d^4z \ell \left[\frac{1}{p^2 - m^2 + i\epsilon} \right]_{p=0} = \frac{1}{(2\pi)^4} 2T_2 V$$

$$\frac{-i\lambda}{4! (2\pi)^8} \left[2T_2 V \int d^4p \frac{1}{p^2 - m^2 + i\epsilon} \right]^2$$

All vacuum diagrams are proportional to ZTV

corresponding diagrams (vacuum bubbles)


$$= \frac{(-i)^4}{4! (2\pi)^4} S^4(0) \left[\int d^4 p \frac{1}{p^2 - m^2 + i\epsilon} \right]^2$$

$$S^4(0) = \frac{1}{(2\pi)^4} \int d^4 z e^{i p z} \Big|_{p=0} = \frac{1}{(2\pi)^4} 2T_2 V$$

$$\sim \frac{(-i)^4}{4! (2\pi)^8} [2T_2 V] \left[\int d^4 p \frac{1}{p^2 - m^2 + i\epsilon} \right]^2$$

All vacuum diagrams are proportional to $2T_2 V$.

where $E_0 = \dots$

exp []

corresponding diagrams (vacuum bubbles)

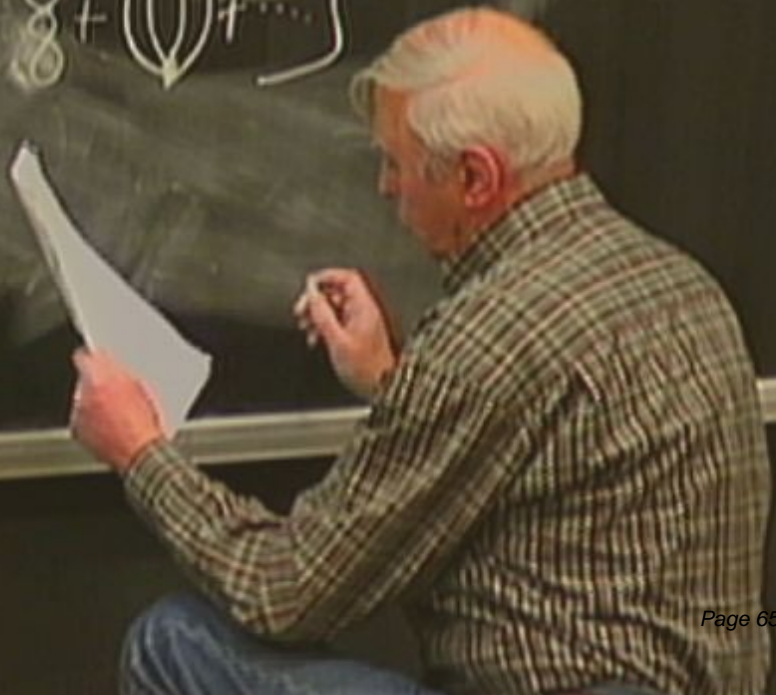
Physical meaning of vacuum diagrams

From (A), we get

$$\exp\left[\sum_i \mathcal{V}_i\right] \sim e^{-iE_0(2T)} \Rightarrow E_0(2T) = i\sum_i \mathcal{V}_i \Rightarrow$$

$$\frac{E_0(2T)}{2TV} = \frac{i\sum_i \mathcal{V}_i}{2TV} = i[\text{8} + \text{8} + \text{0} + \dots]$$

vacuum energy density



(Faint handwritten text at the top of the board)

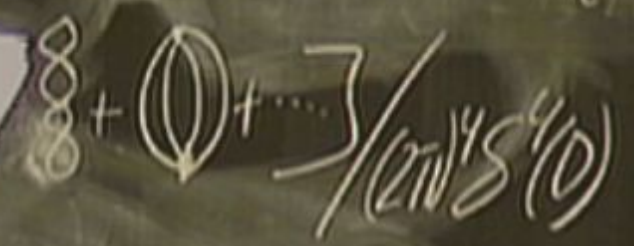
Physical interpretation of vacuum diagrams

From (A), we have vacuum diagrams

$$\exp\left[\sum_i \dots -iE_0(2T)\right] \Rightarrow E_0(2T) = i \sum_i \dots \Rightarrow$$

$$\frac{E_0(2T)}{2T V} =$$

vacuum energy density



Physical meaning of vacuum diagrams

From (A), we get.

$$\exp\left[\sum_i \mathcal{V}_i\right] \sim e^{-iE_0(2T)} \Rightarrow E_0(2T) = i \sum_i \mathcal{V}_i \Rightarrow$$

$$\frac{E_0(2T)}{2TV} = \frac{i \sum_i \mathcal{V}_i}{2TV} = i \left[\text{8} + \text{8} + \text{0} + \dots \right] / (2\pi)^4 S^4(0)$$

vacuum energy density

Physical meaning of vacuum diagrams

From (A), we get

$$\exp\left[\sum_i \mathcal{V}_i\right] \sim e^{-iE_0(2T)} \Rightarrow E_0(2T) = i \sum_i \mathcal{V}_i \Rightarrow$$

$$\frac{E_0(2T)}{2T V} = \frac{i \sum_i \mathcal{V}_i}{2T V} = i \left[\text{8} + \text{8} + \text{0} + \dots \right] / (2\pi)^4 S^4(0)$$

vacuum energy density

$$\frac{4! (2\pi)^4 S'(0)}{[d^4 p \frac{1}{p^2 - m^2 + i\epsilon}]}$$

$$S^4(0) = \frac{1}{(2\pi)^4} \int d^4 z \mathcal{L} \Big|_{p=0} = \frac{1}{(2\pi)^4} 2T_+ V$$

$$\sim \frac{-i\lambda}{4! (2\pi)^8} \underbrace{[2T_+ V]}_{\text{spatial}} \cdot \underbrace{[d^4 p \frac{1}{p^2 - m^2 + i\epsilon}]^2}_{\text{Volume}}$$

All vacuum diagrams are proportional to $2T_+ V$.

$$\langle T \rightarrow \infty (1-i\epsilon) \int d^4 x H_0 | 0 \rangle = \lim_{T \rightarrow \infty (1-i\epsilon)} (\text{vacuum diagrams})$$

where $E_0 = \langle H | H | H \rangle$. (Corresponding diagrams)



from (A), we get.

vacuum diagrams

$$\exp\left[\sum_i \delta_i\right] \sim e^{-iE_0(2T)} \Rightarrow E_0(2T) = i \sum_i \delta_i$$

$$\frac{E_0(2T)}{2TV} = \frac{i \sum_i \delta_i}{2TV} = i \left[\delta + \delta + \text{loop} + \dots \right] / (2\pi)^4 \delta^4(0)$$

vacuum energy density

from (A), we get.

vacuum diagrams

$$\exp\left[\sum_i \delta_i\right] \sim e^{-iE_0(2T)} \Rightarrow E_0(2T) = i \sum_i \delta_i \Rightarrow$$

$$\frac{E_0(2T)}{2TV} = \frac{i \sum_i \delta_i}{2TV} = i \left[\delta + \delta + \text{O} + \dots \right] / (2V^4 S^4(0))$$

vacuum energy density

From (A), we get
 vacuum diagrams (1)

$$\exp\left[\sum_i \delta_i\right] \sim e^{-iE_0 \Delta T} \Rightarrow E_0 \Delta T = i \sum_i \delta_i$$

$$\frac{E_0 \Delta T}{\Delta T V} = \frac{i \sum_i \delta_i}{\Delta T V} = i \left[\text{8} + \text{8} + \text{0} + \dots \right] / (2\pi)^4$$

vacuum energy density

Conclusion: Vacuum energy density is not observable without gravity.



From (A), we get.

vacuum diagrams

(1)

$$\exp\left[\sum_i \delta_i\right] \sim e^{-iE_0 \mathcal{Z}T} \Rightarrow E_0 \mathcal{Z}T = i \sum_i \delta_i \Rightarrow$$

$$\frac{E_0 \mathcal{Z}T}{\mathcal{Z}T V} = \frac{i \sum_i \delta_i}{\mathcal{Z}T V} = i \left[\delta + \delta + \text{loop} + \dots \right] / (2\pi)^4 \delta^4(0)$$

vacuum energy density (conclusion: Vacuum energy density is not observable without gravity.)

$$4! (2\pi)^8 \int \frac{d^4 p}{(2\pi)^4} \cdot \left[\frac{d^4 p}{p^2 - m^2 + i\epsilon} \right]^2$$

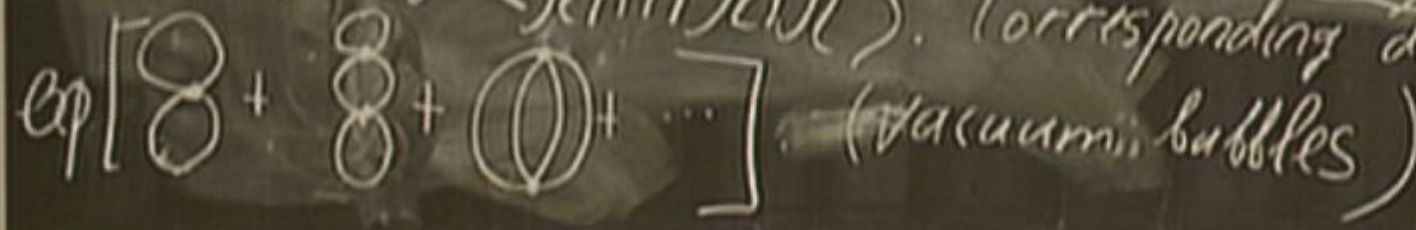
spiral

proportional to $2T V$.

$$\langle \Omega | T \{ \psi(x_1) \bar{\psi}(x_2) \dots \psi(x_n) \} | \Omega \rangle = \lim_{T \rightarrow \infty(1-i\epsilon)} \frac{\langle 0 | T \{ \psi(x_1) \bar{\psi}(x_2) \dots \psi(x_n) \} \times \exp[-\int_T dt H_I(t)] | 0 \rangle}{\langle 0 | T \{ \exp[-\int_T dt H_I(t)] | 0 \rangle}$$

$$\lim_{T \rightarrow \infty(1-i\epsilon)} \langle 0 | T \{ \exp[-\int_T dt H_I(t)] | 0 \rangle = \lim_{T \rightarrow \infty(1-i\epsilon)} (K_0 | \Omega \rangle \langle \Omega |) e^{-iE_0(2T)}$$

where $E_0 = \langle \Omega | H | \Omega \rangle$. (Corresponding diagrams)



$$\frac{E_0 \cancel{2T}}{\cancel{2T} V} = \frac{i \sum b_i}{\cancel{2T} V} = i \left[8 + \left(\frac{8}{8} + \frac{0}{0} + \dots \right) \right] / (2\pi)^4 5^4 / 10$$

vacuum energy density

Conclusion. Vacuum energy density is not observable without gravity.

Vacuum energy density Conclusion. Vacuum energy density is not observable without gravity.

"Feynman Rules for Fermions"


$$\frac{1}{4! (2\pi)^8} [2TV] \cdot \left[d^4 p \frac{1}{p^2 - m^2 + i\epsilon} \right]^{\text{volume}}$$

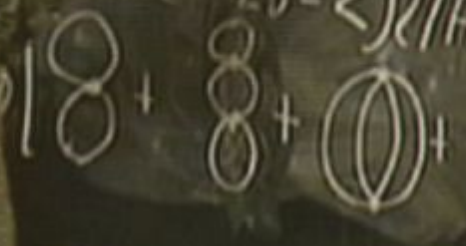
↑
spatial

diagrams are proportional to $2TV$.

$$\langle \Omega | T \{ \psi(x_1) \bar{\psi}(x_2) \dots \psi(x_n) \} | \Omega \rangle = \lim_{T \rightarrow \infty(1-i\epsilon)} \frac{\langle 0 | T \{ \psi(x_1) \bar{\psi}(x_2) \dots \psi(x_n) \} \times \exp[-i \int_{-T}^T dt H_0] | 0 \rangle}{\langle 0 | T \{ \exp[-i \int_{-T}^T dt H_0] | 0 \rangle}$$

$$\lim_{T \rightarrow \infty(1-i\epsilon)} \frac{\langle 0 | T \{ \exp[-i \int_{-T}^T dt H_0] | 0 \rangle}{\langle 0 | T \{ \exp[-i \int_{-T}^T dt H_0] | 0 \rangle} = \lim_{T \rightarrow \infty(1-i\epsilon)} (K_0 | \Omega \rangle \langle \Omega | K_0^\dagger) e^{-iE_0(2T)}$$

$E_0 = \langle \Omega | H | \Omega \rangle$. (Corresponding diagrams: )
(vacuum bubbles)



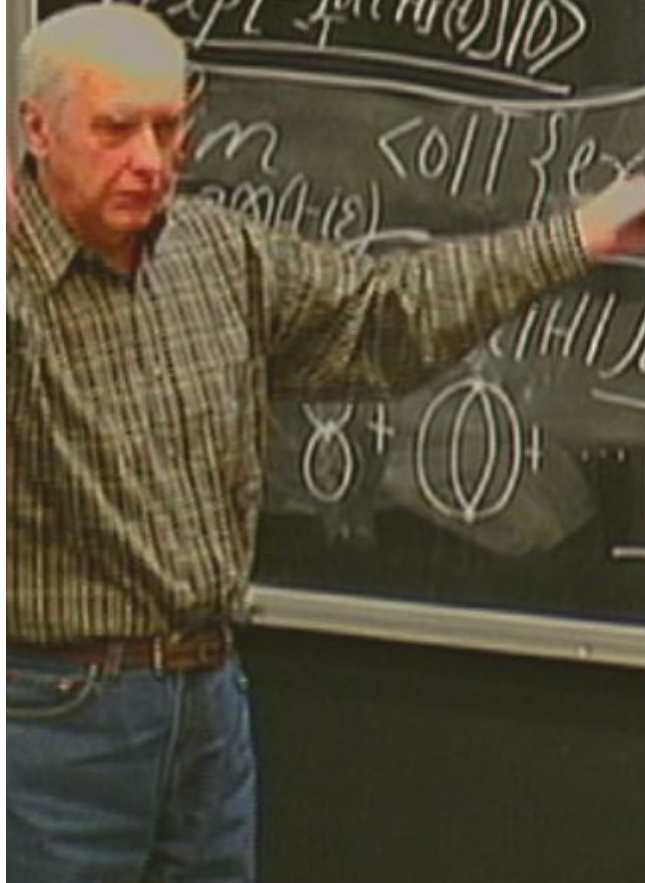
CAUTION
DO NOT TOUCH THE BOARD
OR THE EQUIPMENT ON IT

$$\frac{1}{4! (2\pi)^8} [2TV] \cdot \left[d^4 p \frac{1}{p^2 - m^2 + i\epsilon} \right]$$

spatial

diagrams are proportional to $2TV$.

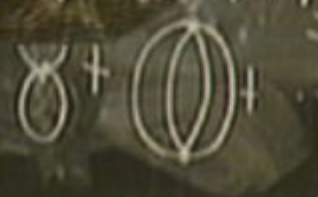
$$\langle \Omega | T_n \{ \psi(x_1) \psi(x_2) \dots \psi(x_n) \} | \Omega \rangle = \lim_{T \rightarrow \infty(1-i\epsilon)} \frac{\langle 0 | T \{ \psi(x_1) \dots \psi(x_n) \} \times \exp[-\int_T dt H_0(t)] | 0 \rangle}{\langle 0 | T \{ \exp[-\int_T dt H_0(t)] | 0 \rangle}$$



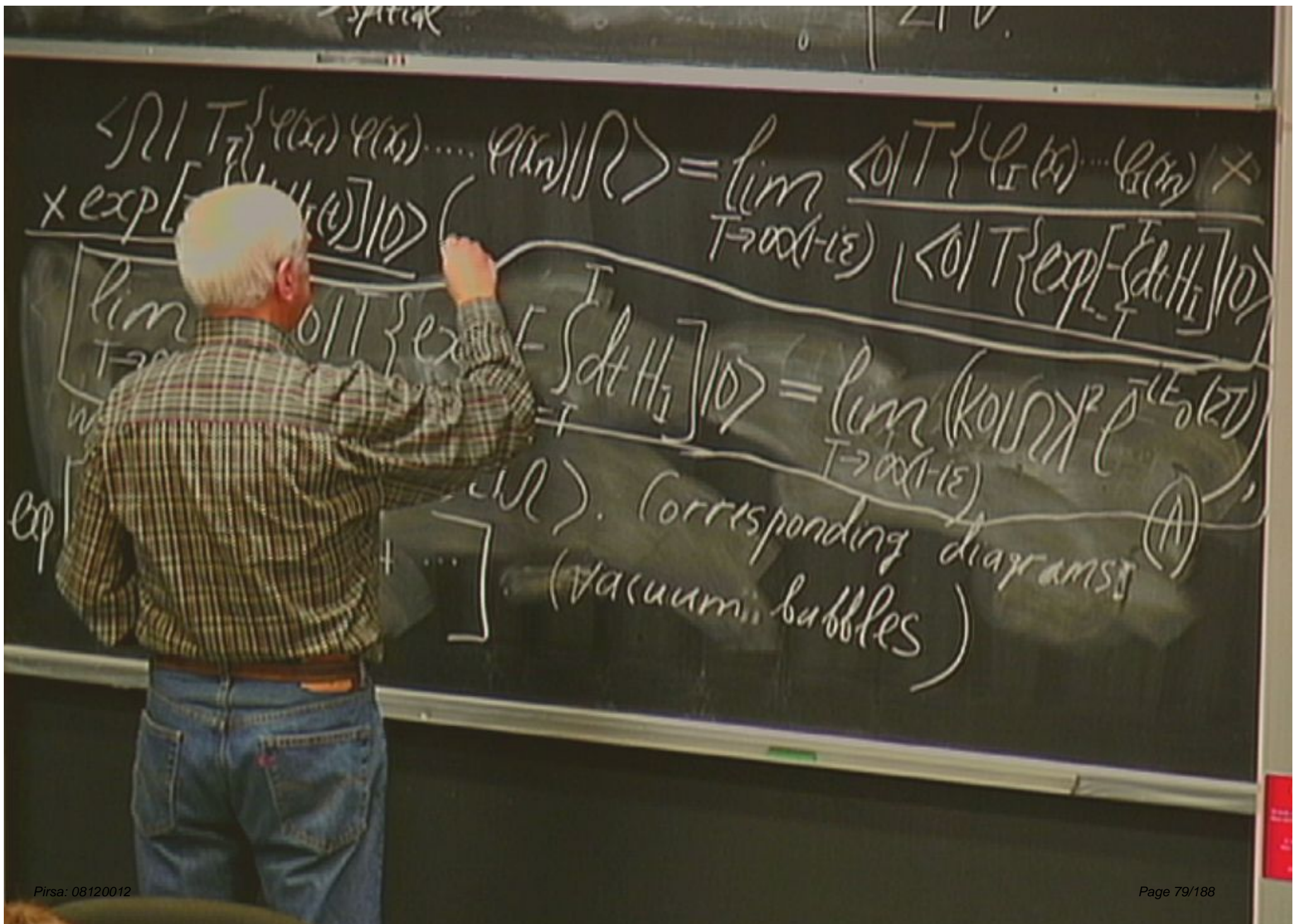
$$\lim_{T \rightarrow \infty(1-i\epsilon)} \langle 0 | T \{ \exp[-\int_T dt H_0(t)] | 0 \rangle = \lim_{T \rightarrow \infty(1-i\epsilon)} (K_0 | \Omega \rangle \langle \Omega | K_0^\dagger e^{-iE_0(2T)})$$

(corresponding diagrams) \textcircled{A}

(vacuum bubbles)



CAUTION



$$\langle \Omega | T \{ \psi(x_1) \psi(x_2) \dots \psi(x_n) \} | \Omega \rangle = \lim_{T \rightarrow \infty (1-i\epsilon)} \frac{\langle 0 | T \{ \psi_I(x_1) \dots \psi_I(x_n) \times \exp[-\int_{-T}^T dt H_I] | 0 \rangle}{\langle 0 | T \{ \exp[-\int_{-T}^T dt H_I] | 0 \rangle}$$

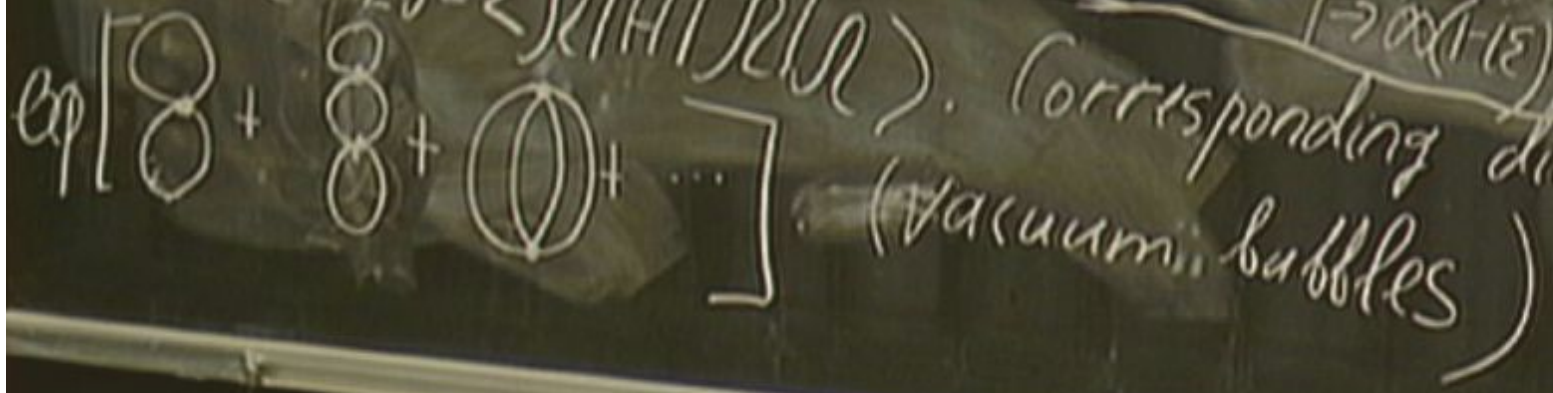
$$\lim_{T \rightarrow \infty} \langle 0 | T \{ \exp[-\int_{-T}^T dt H_I] | 0 \rangle = \lim_{T \rightarrow \infty (1-i\epsilon)} \langle 0 | \Omega \rangle^2 e^{-iE_0(2T)}$$

exp [...] (corresponding diagrams) (A)
 (vacuum bubbles)

$$\langle \Omega | T_{\tau_1} \psi(x_1) \psi(x_2) \dots \psi(x_n) | \Omega \rangle = \lim_{T \rightarrow \infty(1-i\epsilon)} \frac{\langle 0 | T \{ \psi(x_1) \dots \psi(x_n) \} \times \exp[-\int_T dt H_I(t)] | 0 \rangle}{\langle 0 | T \exp[-\int_T dt H_I] | 0 \rangle} \quad (\text{X})$$

$$\lim_{T \rightarrow \infty(1-i\epsilon)} \langle 0 | T \{ \exp[-\int_T dt H_I] | 0 \rangle = \lim_{T \rightarrow \infty(1-i\epsilon)} (\langle 0 | \Omega \rangle)^2 e^{-iE_0(2T)} \quad (\text{A})$$

where $E_0 = \langle \Omega | H | \Omega \rangle$. (Corresponding diagrams)



Feynman Rules for Fermions

expression (8) for Green's functions
in theories including fermions. To formulat
properly Wick's theorem in this case, one should

Feynman Rules for Fermions

The expression (8) for Green's function is valid in theories including fermions. To take properly Wick's theorem in this case, one generalizes the time-ordering and normal-ordering

Feynman Rules for Fermions

The expression (8) for Green's functions is valid in theories including fermions. To formulate properly Wick's theorem in this case, one should generalize the ordering and normal-ordering symbols for fermions.

$$\frac{\delta Z}{\delta V} = \frac{(\sum b_i)}{Z} = i(\sqrt{8} + \dots) + \dots$$

Feynman Rules for Fermions

The expression (8) for Green's functions is valid in theories including fermions. To formulate properly Wick's theorem in this case, one should generalize the time-ordering and normal-ordering symbols for fermions.

$$\frac{\delta Z}{\delta V} = \frac{(\sum b_i)}{Z} = i(\gamma + \delta + \dots) \sqrt{V}$$

Feynman Rules for Fermions.

The expression (8) for Green's functions is valid in theories including fermions. To formulate properly Wick's theorem in this case, one should generalize the time-ordering and normal-ordering symbols for fermions.

$$\frac{\delta Z}{\delta V} = \frac{i \sum b_i}{Z} = i(8 + 8 + 0 + \dots) \sqrt{4c^4}$$

Feynman Rules for Fermions

The expression (8) for Green's functions is valid in theories including fermions. To formulate properly Wick's theorem in this case, one should generalize the time-ordering and normal-ordering symbols for fermions.

from $T(\psi(x)\bar{\psi}(y))$

$$\frac{\delta Z}{\delta V} = \frac{(\sum b_i)}{Z} = i(\gamma_8 + \gamma_8 + \dots) + \dots$$

Feynman Rules for Fermions

The expression (8) for Green's functions is valid in theories including fermions. To formulate properly Wick's theorem in this case, one should generalize the time-ordering and normal ordering symbols for fermions.

Start from $T(\psi(x)\bar{\psi}(y))$.

$$\frac{\delta Z}{\delta V} = \frac{(\sum b_i)}{Z} = i(\gamma_8 + \gamma_8 + \dots) + \dots$$

Feynman Rules for Fermions

The expression (8) for Green's functions is valid in theories including fermions. To formulate properly Wick's theorem in this case, one should generalize the time-ordering and normal-ordering symbols for fermions.

Start from $T(\psi(x)\bar{\psi}(y))$, which was

$$\frac{\delta Z}{\delta V} = \frac{(\sum \psi_i)}{Z} = i(\psi + \psi + \dots) + \dots$$

Feynman Rules for Fermions

The expression (8) for Green's functions is valid in theories including fermions. To formulate properly Wick's theorem in this case, one should generalize the time-ordering and normal-ordering operators for fermions.

Start from $T(\psi(x)\bar{\psi}(y))$, which was already

$$\frac{\delta Z}{\delta V} = \frac{(\sum b_i)}{Z} = i(8 + 8 + 0) + \dots$$

Feynman Rules for Fermions.

The expression (8) for Green's functions is valid in theories including fermions. To formulate properly Wick's theorem in this case, one should generalize the time-ordering and normal-ordering symbols for fermions.

Start from $T(\psi_I(x) \bar{\psi}_I(y))$, which was already considered (I subscript)

The expression (8) for Green's functions is valid in theories including fermions. To formulate properly Wick's theorem in this case, one should generalize the time-ordering and normal-ordering symbols for fermions.

Start from $T(\psi(x)\bar{\psi}(y))$, which was already considered (omit I subscript)

generalize the time-ordering and normal-ordering symbols for fermions.

Start from $T(\psi_I(x)\psi_I(y))$, which was already considered. (I omit I subscript)
henceforth

considered (omit subscript)
henceforth

$$T(\psi(x)\overline{\psi(y)}) \equiv \int_{x^0 > y^0} \psi(x)\overline{\psi(y)}$$

considered (omit I subscript)
henceforth

$$T(\psi(x)\overline{\psi(y)}) \equiv \begin{cases} \psi(x)\overline{\psi(y)}, & x^0 > y^0 \\ -\overline{\psi(y)}\psi(x), & x^0 < y^0 \end{cases}$$

considered (omit I subscript)
henceforth

$$T(\psi(x)\overline{\psi(y)}) \equiv \begin{cases} \psi(x)\overline{\psi(y)}, & x^0 > y^0 \\ -\overline{\psi(y)}\psi(x), & x^0 < y^0 \end{cases}$$
$$T(\psi(x)\overline{\psi(y)}) = \int_F(x-y)$$

(considered as omit 1 subscript)
 henceforth

$$T(\psi(x)\bar{\psi}(y)) \equiv \begin{cases} \psi(x)\bar{\psi}(y), & x^0 > y^0 \\ -\bar{\psi}(y)\psi(x), & x^0 < y^0 \end{cases}$$

$$(\psi(x)\bar{\psi}(y)) = \int_F (x-y) = \int \frac{d^4 p}{(2\pi)^4} \frac{i(p\gamma_{\mu} + m)}{p^2 - m^2 + i\epsilon} e^{-i p(x-y)}$$

considered (omit subscript)
 henceforth

$$T(\psi(x)\bar{\psi}(y)) \equiv \begin{cases} \psi(x)\bar{\psi}(y), & x^0 > y^0 \\ -\bar{\psi}(y)\psi(x), & x^0 < y^0 \end{cases}$$

$$\langle 0 | T(\psi(x)\bar{\psi}(y)) | 0 \rangle$$

$$S_F(x-y) = \int \frac{d^4 p}{(2\pi)^4} \frac{i(\not{p} \gamma_5 + m) e^{-ip(x-y)}}{p^2 - m^2 + i\epsilon}$$

Note

$$T(\bar{\psi}(x)\psi(y))$$

$$\begin{cases} \bar{\psi}(y)\psi(x), & y^0 > x^0 \\ \psi(x)\bar{\psi}(y), & y^0 < x^0 \end{cases}$$

(considered (1) omit 1 subscript)
 henceforth

$$T(\psi(x)\bar{\psi}(y)) = \begin{cases} \psi(x)\bar{\psi}(y), & x^0 > y^0 \\ -\bar{\psi}(y)\psi(x), & x^0 < y^0 \end{cases}$$

$$\langle 0 | T(\psi(x)\bar{\psi}(y)) | 0 \rangle = \int_{\mathcal{F}} \psi(x) \bar{\psi}(y) \dots$$

Note that

$$T(\bar{\psi}(y)\psi(x)) = \begin{cases} \bar{\psi}(y)\psi(x) \\ \psi(x)\bar{\psi}(y) \end{cases}$$

(considered as omit 1 subscript)
 henceforth

$$T(\psi(x)\bar{\psi}(y)) = \begin{cases} \psi(x)\bar{\psi}(y), & x^0 > y^0 \\ -\bar{\psi}(y)\psi(x), & x^0 < y^0 \end{cases}$$

$$\langle 0|T(\psi(x)\bar{\psi}(y))|0\rangle = S_F(x-y) = \int \frac{d^4 p}{(2\pi)^4} \frac{i(\not{p} \gamma_4 + m)}{p^2 - m^2 + i\epsilon} e^{-i p(x-y)}$$

Note

$$T(\bar{\psi}(y)\psi(x)) = \begin{cases} \bar{\psi}(y)\psi(x), & y^0 > x^0 \\ \psi(x)\bar{\psi}(y), & y^0 < x^0 \end{cases}$$

(considered as omit 1 subscript)
 henceforth

$$T(\psi(x)\bar{\psi}(y)) = \begin{cases} \psi(x)\bar{\psi}(y), & x^0 > y^0 \\ -\bar{\psi}(y)\psi(x), & x^0 < y^0 \end{cases}$$

$$\langle 0 | T(\psi(x)\bar{\psi}(y)) | 0 \rangle = \int_{\mathcal{F}} (x-y) = \int \frac{d^4 p}{(2\pi)^4} \frac{i(p \cdot \gamma + m)}{p^2 - m^2 + i\epsilon} e^{-i p \cdot (x-y)}$$

Note that

$$T(\bar{\psi}(y)\psi(x)) = \begin{cases} \bar{\psi}(y)\psi(x), & y^0 > x^0 \\ \psi(x)\bar{\psi}(y), & x^0 > y^0 \end{cases}$$

(considered as omit 1 subscript)
 henceforth

$$T(\psi(x)\bar{\psi}(y)) \equiv \begin{cases} \psi(x)\bar{\psi}(y), & x^0 > y^0 \\ -\bar{\psi}(y)\psi(x), & x^0 < y^0 \end{cases}$$

$$i\partial/T(\psi(x)\bar{\psi}(y)) = S_F(x-y) = \int \frac{d^4 p}{(2\pi)^4} \frac{i(p\gamma + m)}{p^2 - m^2 + i\epsilon} e^{-i p(x-y)}$$

Note that

$$(\bar{\psi}(y)\psi(x)) \equiv \begin{cases} \bar{\psi}(y)\psi(x), & y^0 > x^0 \\ \psi(x)\bar{\psi}(y), & x^0 > y^0 \end{cases}$$

(considered as omit 1 subscript)
 henceforth

$$T(\psi(x)\bar{\psi}(y)) = \begin{cases} \psi(x)\bar{\psi}(y), & x^0 > y^0 \\ -\bar{\psi}(y)\psi(x), & x^0 < y^0 \end{cases}$$

$$\langle 0 | T(\psi(x)\bar{\psi}(y)) | 0 \rangle = \int_{\mathcal{F}} (x-y) = \int \frac{d^4 p}{(2\pi)^4} \frac{i(p \cdot \gamma + m)}{p^2 - m^2 + i\epsilon} e^{-i p(x-y)}$$

Note that

$$T(\bar{\psi}(y)\psi(x)) = \begin{cases} \bar{\psi}(y)\psi(x), & y^0 > x^0 \\ \psi(x)\bar{\psi}(y), & y^0 < x^0 \end{cases}$$

considered \Rightarrow omit I subscript
 henceforth

$$T(\psi(x)\bar{\psi}(y)) = \begin{cases} \psi(x)\bar{\psi}(y), & x^0 > y^0 \\ -\bar{\psi}(y)\psi(x), & x^0 < y^0 \end{cases}$$

$$\langle 0|T(\psi(x)\bar{\psi}(y))|0\rangle = S_F(x-y) = \int \frac{d^4p}{(2\pi)^4} \frac{i(\not{p} + m)}{p^2 - m^2 + i\epsilon} e^{-ip(x-y)}$$

Note that

$$T(\bar{\psi}(y)\psi(x)) = \begin{cases} \bar{\psi}(y)\psi(x), & y^0 > x^0 \\ \psi(x)\bar{\psi}(y), & y^0 < x^0 \end{cases} \Rightarrow T(\bar{\psi}(y)\psi(x)) = T(\psi(x)\bar{\psi}(y))$$

Then
 $\Rightarrow T(\bar{\psi}(y)\psi(x)) = T(\psi(x)\bar{\psi}(y))$

considered ψ omit I subscript
 henceforth

$$T(\psi(x)\bar{\psi}(y)) = \begin{cases} \psi(x)\bar{\psi}(y), & x^0 > y^0 \\ -\bar{\psi}(y)\psi(x), & x^0 < y^0 \end{cases}$$

$$\langle 0|T(\psi(x)\bar{\psi}(y))|0\rangle = S_F(x-y) = \int \frac{d^4 p}{(2\pi)^4} \frac{i(\not{p}\gamma_4 + m)}{p^2 - m^2 + i\epsilon} e^{-i p(x-y)}$$

Note that

$$T(\bar{\psi}(y)\psi(x)) = \begin{cases} \bar{\psi}(y)\psi(x), & y^0 > x^0 \\ \psi(x)\bar{\psi}(y), & y^0 < x^0 \end{cases}$$

Then
 $\Rightarrow T(\bar{\psi}(y)\psi(x)) = -T(\psi(x)\bar{\psi}(y))$
 $\Rightarrow T(\psi(x)\bar{\psi}(y)) = -T(\bar{\psi}(y)\psi(x))$

Generalization: Time ordered product for fermions picks up one minus sign for each interchange of operators that is necessary to put

fermions picks up one minus sign for each interchange of operators that is necessary to put the fields in time order:

$$T(\psi_1 \psi_2 \psi_3 \psi_4)$$
$$x_3^0 > x_1^0, \quad 0 \rightarrow 2$$

performs pairs of one minus 'sign for' each
interchange of operators that is necessary to put
the fields in 'time order'.

$$T(\psi_1 \psi_2 \psi_3 \psi_4) = (-1)^{N}$$

$$x_3^0 > x_1^0 > x_4^0 > x_2^0$$

the fields in time order: ... necessary to put

$$T(\psi_1 \psi_2 \psi_3 \psi_4) = (-1)^{\sum_{i < j} \epsilon_i \epsilon_j} \psi_3 \psi_1$$

$x_1^0 > x_4^0 > x_2^0$

... is necessary to put
the fields in time order:

$$T(\psi_1 \psi_2 \psi_3 \psi_4) = (-1)^{\psi_3 \psi_4}$$

$x_1^0 > x_4^0 > x_2^0$

...range of operators that is necessary to put
the fields in time order:

$$T(\psi_1 \psi_2 \psi_3) = \psi_3 \psi_2 \psi_1$$

$x_3^0 > x_1^0$

interchange of operators that is necessary to put
the fields in time order:

$$T(\psi_1 \psi_2 \psi_3 \psi_4) = \psi_3 \psi_1 \psi_4 \psi_2$$

$x_3^0 > x_1^0 > x_4^0 > x_2^0$

... of operators that is necessary to put
the fields in time order:

$$T(\psi_2(x_2^0) \psi_3(x_3^0) \psi_4(x_4^0)) = \psi_3 \psi_1 \psi_4 \psi_2$$



... of operators that is necessary to put
the fields in time order:

$$T(\psi_1 \psi_2 \psi_3 \psi_4) = \psi_3 \psi_1 \psi_4 \psi_2$$

$x_3^0 > x_1^0$ $x_4^0 > x_2^0$

... of operators that is necessary to put
the fields in time order:

$$T(\psi_1 \psi_2 \psi_3 \psi_4) = \psi_3 \psi_1 \psi_4 \psi_2$$

$x_3^0 > x_1^0 > x_4^0 > x_2^0$



of operators that is necessary to put
the fields in time order:

$$T(\psi_1 \psi_2 \psi_3 \psi_4) = \psi_3 \psi_1 \psi_4 \psi_2$$

$x_3^0 > x_1^0 > x_4^0 > x_2^0$

$3 \times 2 \times 1 = 6$
 $4 \times 3 \times 2 \times 1 = 24$
 $5 \times 4 \times 3 \times 2 \times 1 = 120$

$3 \times 1 \times 1/2$

This implies that under T fermion fields anticommute.



page 10/11

the fields in 'time order:

$$T(\psi_1 \psi_2 \psi_3 \psi_4) = (-1)^3 \psi_3 \psi_1 \psi_4 \psi_2$$

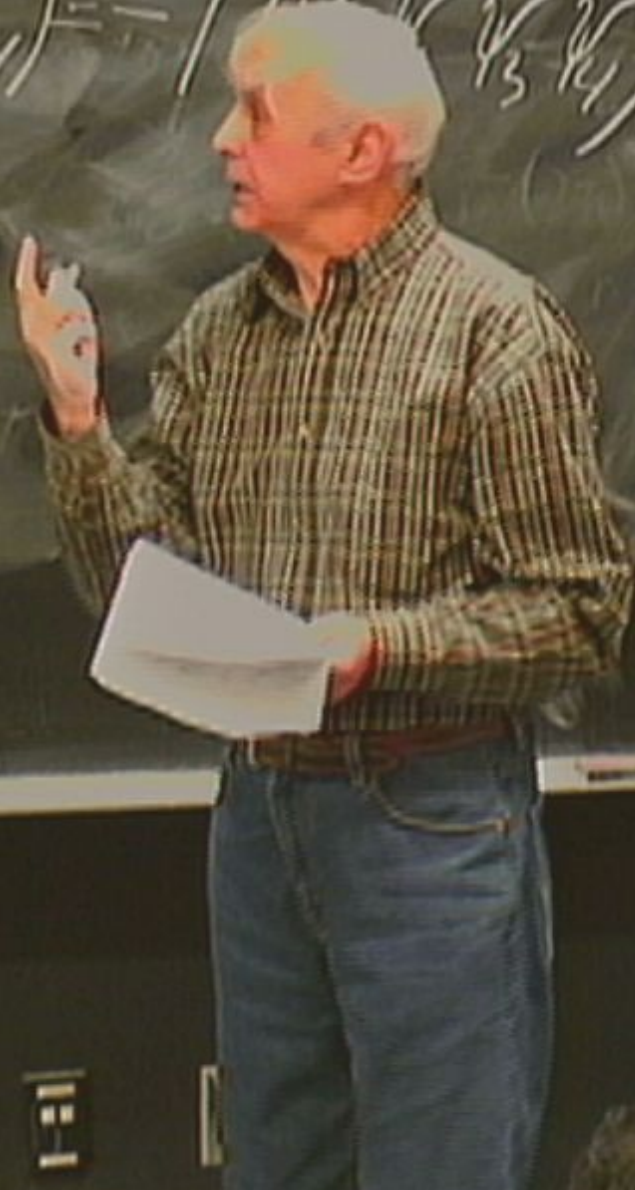
$$x_3^0 > x_1^0 > x_4^0 > x_2^0$$

This implies that under T fermion fields anticommute.



[The lower chalkboard contains very faint and mostly illegible handwritten notes, possibly including the word 'fermion' and some mathematical symbols.]

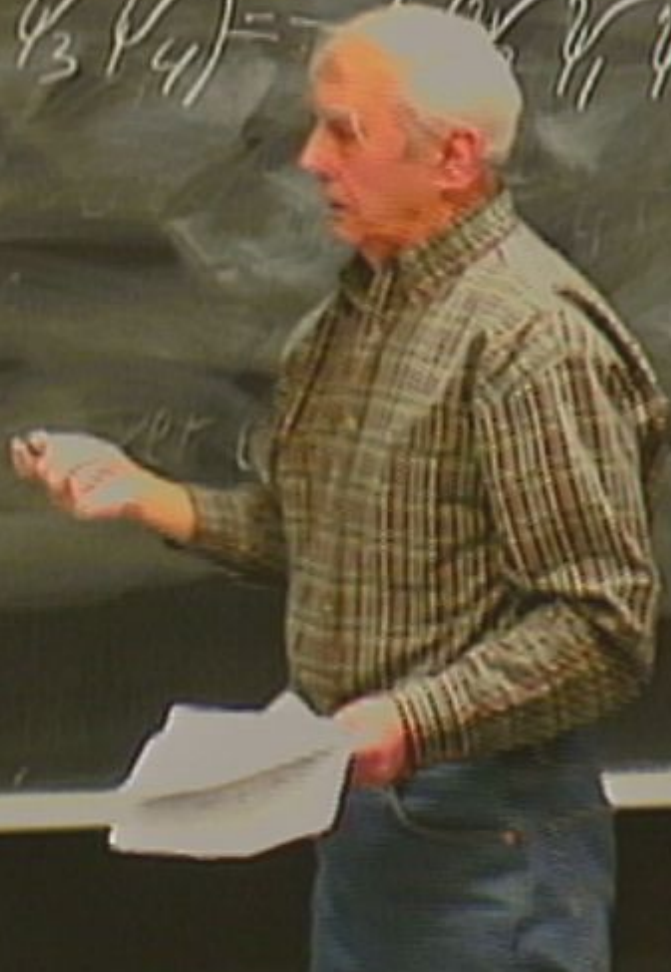
$$T(\psi_1 \psi_2 \psi_3 \psi_4) = -T(\psi_3 \psi_4 \psi_1 \psi_2)$$



$$\chi_3 > \chi_1 > \chi_4 > \chi_2$$

This implies that under T fermion fields anticommute
(grassmannian numbers)

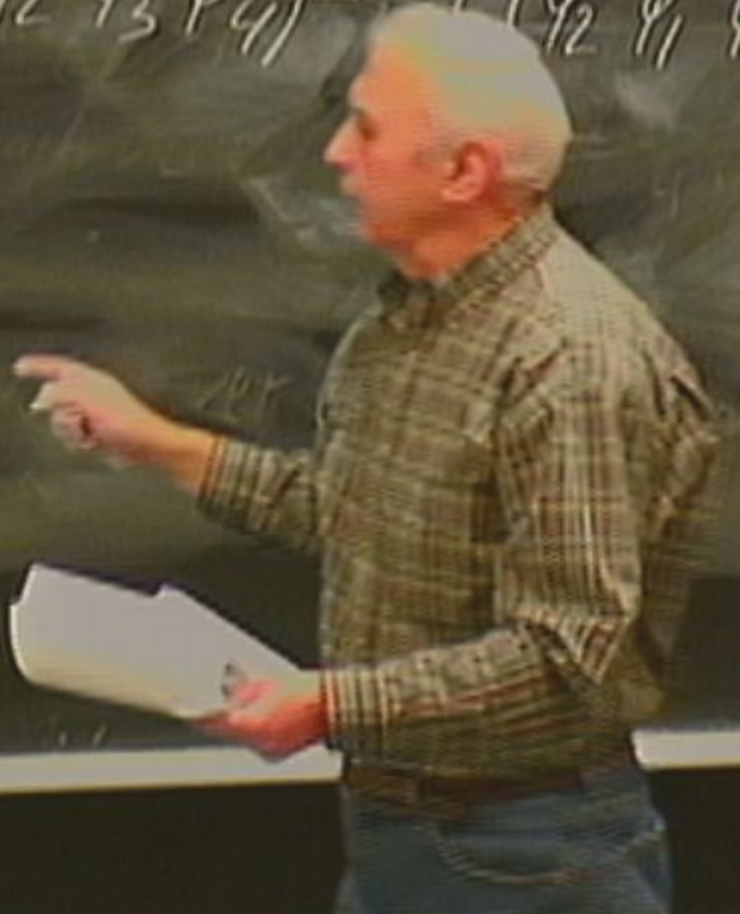
$$T(\psi_1 \psi_2 \psi_3 \psi_4) = -T(\psi_1 \psi_3 \psi_2 \psi_4)$$



$$\chi_3 > \chi_1 > \chi_4 > \chi_2 \quad \psi_3 \quad \psi_1 \quad \psi_4 \quad \psi_2$$

This implies that under T fermion fields anticommute
(grassmannian numbers)

$$T(\psi_1 \psi_2 \psi_3 \psi_4) = -T(\psi_2 \psi_1 \psi_3 \psi_4)$$

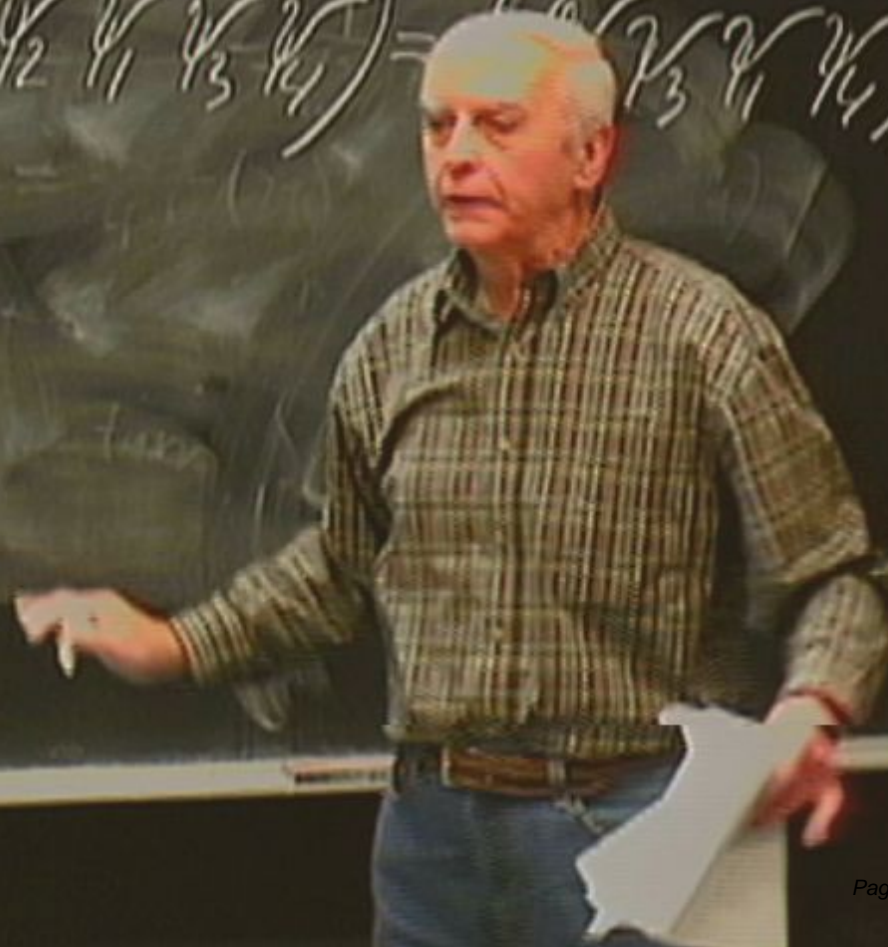


$$\chi_3^0 > \chi_1^0 > \chi_4^0 > \chi_2^0 \quad \psi_3 \quad \psi_1 \quad \psi_4 \quad \psi_2$$

This implies that under T fermion fields anticommute (grassmannian numbers)

$$T(\psi_1 \psi_2 \psi_3 \psi_4) = -T(\psi_2 \psi_1 \psi_3 \psi_4) = T(\psi_3 \psi_1 \psi_4), \text{ etc.}$$

The d



$$\chi_3^0 > \chi_1^0 > \chi_4^0 > \chi_2^0 \quad \psi_3 \quad \psi_1 \quad \psi_4 \quad \psi_2$$

This implies that under T fermion fields anticommute (grassmannian numbers)

$$T(\psi_1 \psi_2 \psi_3 \psi_4) = -T(\psi_2 \psi_1 \psi_3 \psi_4) = T(\psi_2 \psi_3 \psi_1 \psi_4), \text{ etc.}$$

The d

$$x_3 > x_1 > x_4 > x_2 \quad \psi_3 \psi_1 \psi_4 \psi_2$$

This implies that under T fermion fields anticommute (grassmannian numbers)

$$T(\psi_1 \psi_2 \psi_3 \psi_4) = -T(\psi_2 \psi_1 \psi_3 \psi_4) = T(\psi_3 \psi_1 \psi_2 \psi_4), \text{ etc.}$$

The definition of the normal-ordered product of fermion fields is similar. For each fermion, insert a minus sign for each fermion interchange.

$$x_3 > x_1 > x_4 > x_2$$

This implies that under T fermion fields anticommute (grassmannian numbers)

$$T(\psi_1 \psi_2 \psi_3 \psi_4) = -T(\psi_2 \psi_1 \psi_3 \psi_4) = T(\psi_2 \psi_3 \psi_1 \psi_4), \text{ etc}$$

The definition of the normal ordered product of fermion fields is similar but an extra minus sign for each fermion involved under sym-

$$T(\psi_1 \psi_2 \psi_3 \psi_4) = -T(\psi_2 \psi_1 \psi_3 \psi_4) = T(\psi_2 \psi_3 \psi_1 \psi_4), \text{ etc.}$$

The definition of the normal-ordered product of fermion fields is similar: Put an extra minus sign for each fermion interchange under sym-

$$T(\psi_1 \psi_2 \psi_3 \psi_4) = -T(\psi_2 \psi_1 \psi_3 \psi_4) = T(\psi_2 \psi_3 \psi_1 \psi_4), \text{ etc.}$$

The definition of the normal-ordered product of fermion fields is similar: Put an extra minus sign for each fermion interchange under sym-

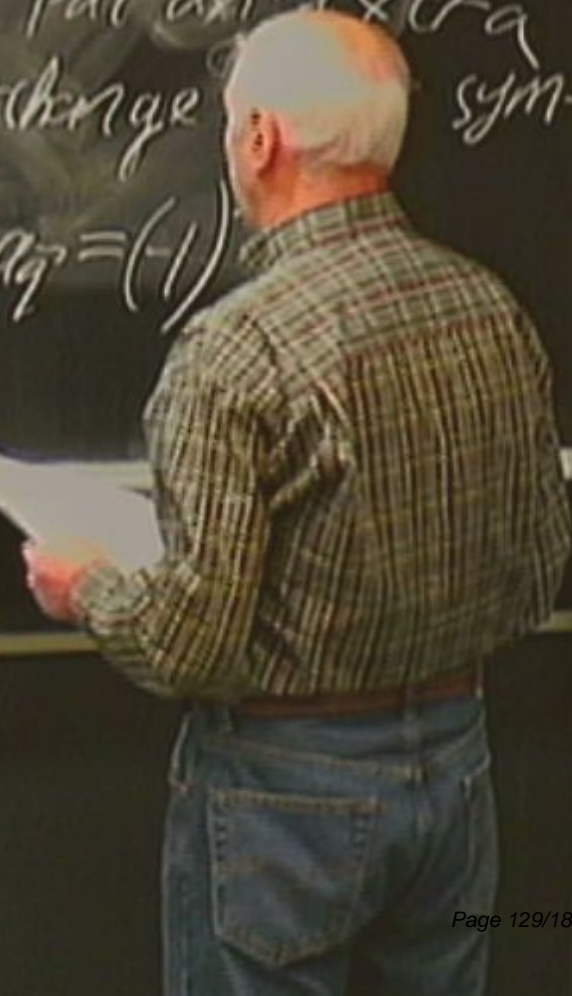
$$T(\psi_1 \psi_2 \psi_3 \psi_4) = -T(\psi_2 \psi_1 \psi_3 \psi_4) = T(\psi_2 \psi_3 \psi_1 \psi_4), \text{ etc.}$$

The definition of the normal-ordered product of fermion fields is similar: Put an extra minus sign for each fermion exchange under symbol N . $N(a_p a_q a_r^\dagger) = (-1)^2$

$\psi_2 > \psi_0 > \psi_0 > \psi_0$ $\psi_3 \psi_1 \psi_4 \psi_2$

$$T(\psi_1 \psi_2 \psi_3 \psi_4) = -T(\psi_2 \psi_1 \psi_3 \psi_4) = T(\psi_2 \psi_3 \psi_1 \psi_4), \text{ etc.}$$

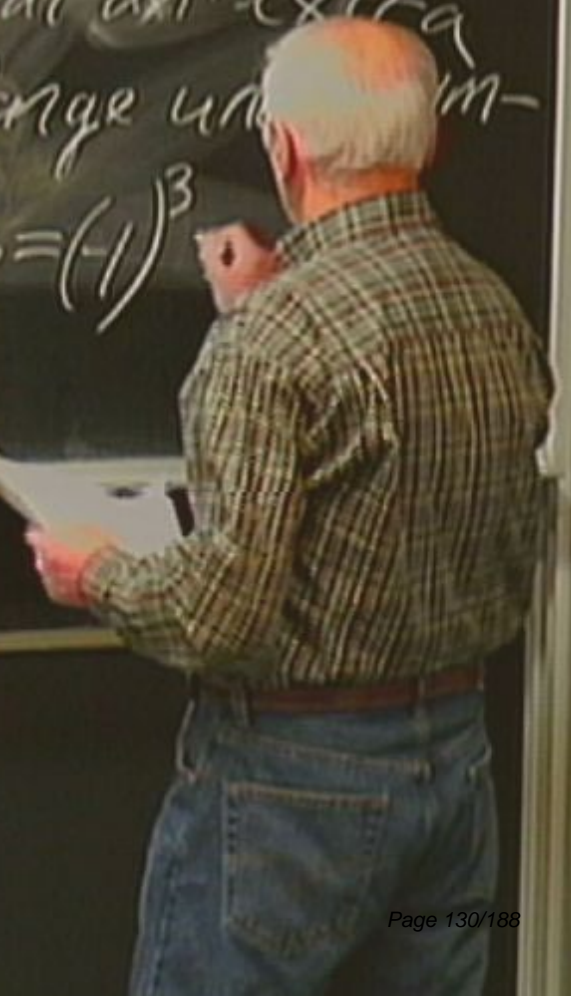
The definition of the normal-ordered product of fermion fields is similar: Put an extra minus sign for each fermion interchange symbol N . $N(a_p a_q a_r^\dagger) = (-1)^2 a_r^\dagger a_p a_q = (-1)$



$$T(\psi_1 \psi_2 \psi_3 \psi_4) = -T(\psi_2 \psi_1 \psi_3 \psi_4) = T(\psi_2 \psi_3 \psi_1 \psi_4), \text{ etc.}$$

The definition of the normal-ordered product of fermion fields is similar: Put an extra minus sign for each fermion interchange under

$$\text{bol } N: \quad N(a_p a_q a_r^\dagger) = (-1)^2 a_r^\dagger a_p a_q = (-1)^3$$



$$T(\psi_1 \psi_2 \psi_3 \psi_4) = -T(\psi_2 \psi_1 \psi_3 \psi_4) = T(\psi_2 \psi_3 \psi_1 \psi_4), \text{ etc.}$$

The definition of the normal-ordered product of fermion fields is similar: Put an extra minus sign for each fermion interchange under symbol N . $N(a_p a_q a_r^\dagger) = (-1)^2 a_r^\dagger a_p a_q = (-1)^3 a_r^\dagger a_q a_p$.

of fermion fields. ... an extra
minus sign for each fermion interchange under sym-
bol N . $N(a_p a_q a_r^\dagger) = (-1)^2 a_r^\dagger a_p a_q = (-1)^3 a_r^\dagger a_q a_p$.

Generalization of Wick's Theorem



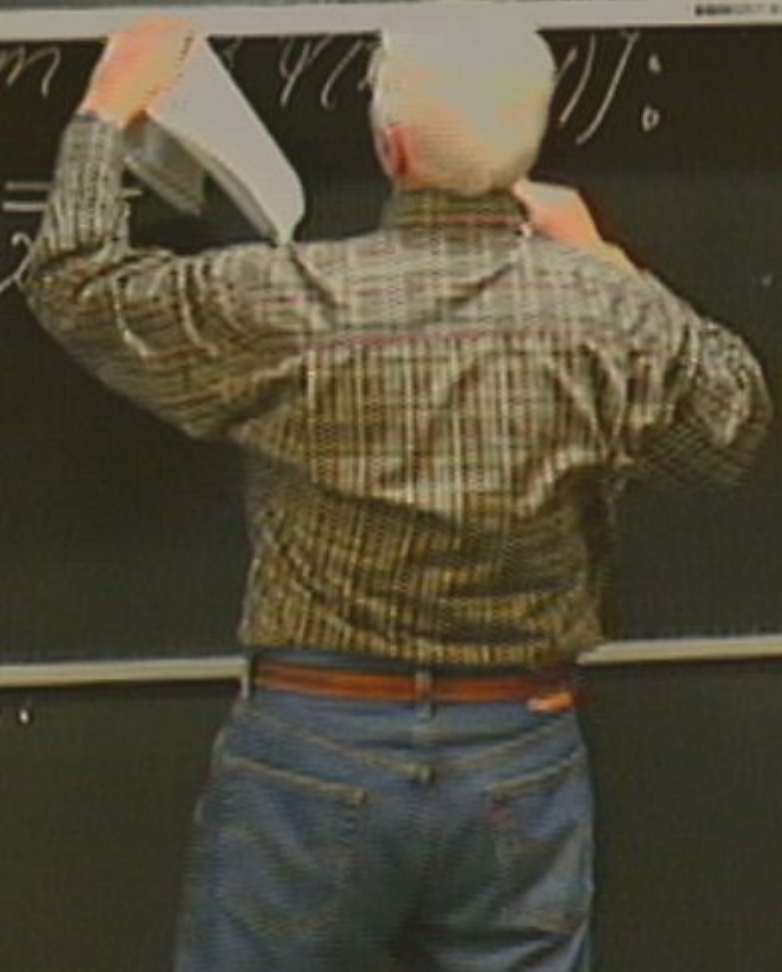
Generalization of Wick's Theorem
Start from $T\{\psi(x)\bar{\psi}(y)\}$:

of fermion fields is similar. Put an extra minus sign for each fermion interchange under symbol N . $N(a_p a_q a_r^\dagger) = (-1)^2 a_r^\dagger a_p a_q = (-1)^3 a_r^\dagger a_q a_p$.

$$\psi(x) = \psi^+(x) + \psi^-(x)$$

start from $\psi(x) = \int d^3p \dots$

$$T[\psi(x) \bar{\psi}(y)] =$$



of fermion fields is similar. Put an extra minus sign for each fermion interchange under symbol N :

$$N(a_p a_q a_r^\dagger) = (-1)^2 a_r^\dagger a_p a_q = (-1)^3 a_r^\dagger a_q a_p.$$

$$\psi(x) = \psi^+(x) + \psi^-(x); \quad \psi^+ |0\rangle = 0$$

start from $T\{\psi(x)\psi(y)\}$:

$$T\{\psi(x)\bar{\psi}(y)\} \stackrel{\text{if } x^0 > y^0}{=} \bar{\psi}(y)\psi(x)$$



of fermion fields is similar. Put an extra minus sign for each fermion interchange under symbol N .

$$N(a_p a_q a_r^\dagger) = (-1)^2 a_r^\dagger a_p a_q = (-1)^3 a_r^\dagger a_q a_p$$

$$\psi(x) = \psi^+(x) + \psi^-(x); \quad \psi^+|0\rangle = 0; \quad \langle 0|\psi^- = 0$$

... from $(\psi(x), \psi(y))$:

$$T[\psi(x)\bar{\psi}(y)] \stackrel{=}{=} \bar{\psi}(y)\psi(x)$$



$$\psi(x) = \psi_+(x) + \psi_-(x); \quad \psi_+|0\rangle = 0; \quad \langle 0|\psi_- = 0$$

PROOF OF MERK'S THEOREM

Start from $T\{\psi_+(x)\psi_-(y)\}$:

$$T\{\psi_+(x)\bar{\psi}_-(y)\} \stackrel{x \rightarrow y}{=} \psi_+^+(x)$$

$$\psi(x) = \psi_1(x) + \psi_2(x); \quad \psi^* |0\rangle = 0; \quad \langle 0 | \psi = 0$$

Start from $T\{\psi(x)\bar{\psi}(y)\}$:

$$T\{\psi(x)\bar{\psi}(y)\} \stackrel{\text{ok } x_0 > y_0}{=} \psi^+(x)\bar{\psi}^+(y)$$

$$\psi(x) = \psi^+(x) + \psi^-(x); \quad \psi^+ |0\rangle = 0; \quad \langle 0 | \psi^- = 0$$

Start from $T[\psi(x)\bar{\psi}(y)]$:

$$T[\psi(x)\bar{\psi}(y)] = \psi^+(x)\bar{\psi}^+(y) + \psi^+(x)\bar{\psi}^-(y) + \psi^-(x)\bar{\psi}^+(y) + \psi^-(x)\bar{\psi}^-(y)$$

$$\psi(x) = \psi^+(x) + \psi^-(x); \quad \psi^+|0\rangle = 0; \quad \langle 0|\psi^- = 0$$

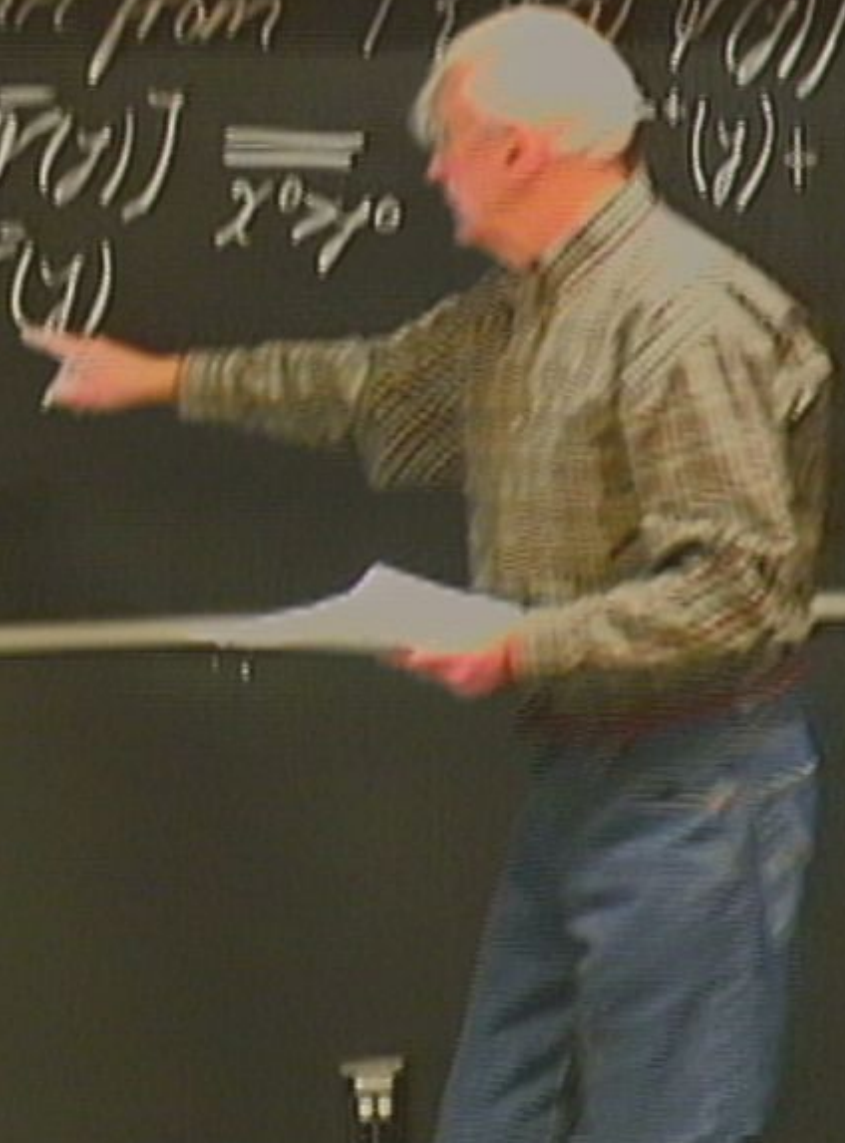
Start from $T\{\psi(x)\bar{\psi}(y)\}$:

$$T\{\psi(x)\bar{\psi}(y)\} \stackrel{x^0 > y^0}{=} \psi^+(x)\bar{\psi}^+(y) + \psi^-(x)\bar{\psi}^-(y) + \psi^-(x)\bar{\psi}^+(y) + \psi^+(x)\bar{\psi}^-(y)$$

$$\psi(x) = \psi^+(x) + \psi^-(x), \quad \psi^+|0\rangle = 0, \quad \psi^-|0\rangle = 0$$

Start from $T\{\psi(x)\bar{\psi}(y)\}$:

$$T\{\psi(x)\bar{\psi}(y)\} \stackrel{x^0 > y^0}{=} \psi^+(x)\bar{\psi}^-(y) + \psi^-(x)\bar{\psi}^+(y) + \psi(x)\bar{\psi}(y)$$



$$N(a_p a_q a_r) = (-1)^p a_r^+ a_p a_q = (-1)^p a_r^+ a_q a_p$$

$$\psi(x) = \psi^+(x) + \psi^-(x); \quad \psi^+ |0\rangle = 0; \quad \langle 0 | \psi^- = 0$$

Start from $T\{\psi(x)\bar{\psi}(y)\}$:

$$T\{\psi(x)\bar{\psi}(y)\} \stackrel{x^0 > y^0}{=} \psi^+(x)\bar{\psi}^+(y) + \underbrace{\psi^-(x)\bar{\psi}(y)} + \psi^-(x)\bar{\psi}^+(y)$$

$$\psi^-(x)\psi^-(y)$$

$$N(a_p a_q a_r) = (-1)^p a_r a_p a_q = (-1)^p a_r a_q a_p$$

$$\psi(x) = \psi^+(x) + \psi^-(x); \quad \psi^+|0\rangle = 0; \quad \psi^-|0\rangle = 0$$

Start from $T\{\psi(x)\bar{\psi}(y)\}$

$$T\{\psi(x)\bar{\psi}(y)\} \stackrel{x^0 > y^0}{=} \psi^+(x)\bar{\psi}^+(y) + \psi^-(y)\bar{\psi}^-(x) + \psi^-(x)\bar{\psi}^+(y) + \psi^+(y)\bar{\psi}^-(x)$$

$$\psi(x) = \psi^+(x) + \psi^-(x); \quad \psi^+|0\rangle = 0; \quad \langle 0|\psi^- = 0$$

Start from $T\{\psi(x)\bar{\psi}(y)\}$.

$$T\{\psi(x)\bar{\psi}(y)\} \stackrel{x^0 > y^0}{=} \psi^+(x)\bar{\psi}^+(y) + \psi^+(x)\bar{\psi}^-(y) + \psi^-(x)\bar{\psi}^+(y) + \psi^-(x)\bar{\psi}^-(y)$$

$$\psi^-(x)\bar{\psi}^-(y) = \psi^-(x)\bar{\psi}^-(y)$$

$$\psi(x) = \psi^+(x) + \psi^-(x); \quad \psi^+|0\rangle = 0; \quad \langle 0|\psi^- = 0$$

Start from $T\{\psi(x)\bar{\psi}(y)\}$:

$$T\{\psi(x)\bar{\psi}(y)\} \stackrel{x^0 > y^0}{=} \psi^+(x)\bar{\psi}^+(y) + \psi^+(x)\bar{\psi}^-(y) + \psi^-(x)\bar{\psi}^+(y) + \psi^-(x)\bar{\psi}^-(y)$$

$$\psi^-(x)\bar{\psi}^-(y) = \psi^+(x)\bar{\psi}^+(y) - \psi^-(y)\bar{\psi}^+(x)$$

$$\psi(x) = \psi^+(x) + \psi^-(x); \quad \psi^+|0\rangle = 0; \quad \langle 0|\psi^- = 0$$

Start from $T\{\psi(x)\bar{\psi}(y)\}$:

$$T\{\psi(x)\bar{\psi}(y)\} \stackrel{x_0 > y_0}{=} \psi^+(x)\bar{\psi}^+(y) + \psi^+(x)\bar{\psi}^-(y) + \psi^-(x)\bar{\psi}^+(y) + \psi^-(x)\bar{\psi}^-(y)$$

$$= \psi^+(x)\bar{\psi}^+(y) - \psi^-(y)\bar{\psi}^+(x) + \psi^-(x)\bar{\psi}^+(y) + \psi^-(x)\bar{\psi}^-(y)$$

$$\psi(x) = \psi^+(x) + \psi^-(x); \quad \psi^+|0\rangle = 0; \quad \langle 0|\psi^- = 0$$

Start from $T\{\psi(x)\bar{\psi}(y)\}$:

$$T\{\psi(x)\bar{\psi}(y)\} \stackrel{x^0 > y^0}{=} \psi^+(x)\bar{\psi}^+(y) + \underbrace{\psi^+(x)\bar{\psi}^-(y)} + \psi^-(x)\bar{\psi}^-(y)$$

$$\psi^-(x)\bar{\psi}^-(y) = \psi^+(x)\bar{\psi}^+(y) - \psi^-(y)\bar{\psi}^+(x) + \psi^-(x)\bar{\psi}^-(y) +$$

$$\psi(x) = \psi^+(x) + \psi^-(x); \quad \psi^+|0\rangle = 0; \quad \langle 0|\psi^- = 0$$

Start from $T\{\psi(x)\bar{\psi}(y)\}$:

$$T\{\psi(x)\bar{\psi}(y)\} \stackrel{x^0 > y^0}{=} \psi^+(x)\bar{\psi}^+(y) + \psi^+(x)\bar{\psi}^-(y) + \psi^-(x)\bar{\psi}^+(y) + \psi^-(x)\bar{\psi}^-(y)$$

$$= \psi^+(x)\bar{\psi}^+(y) - \psi^-(y)\bar{\psi}^+(x) + \psi^-(x)\bar{\psi}^+(y) + \psi^-(x)\bar{\psi}^-(y) + \{$$

$$\psi(x) = \psi^+(x) + \psi^-(x); \quad \psi^+(\infty) = 0; \quad \psi^-(\infty) = 0$$

Start from $T\{\psi(x)\bar{\psi}(y)\}$:

$$T\{\psi(x)\bar{\psi}(y)\} \stackrel{x^0 > y^0}{=} \psi^+(x)\bar{\psi}^+(y) + \psi^+(x)\bar{\psi}^-(y) + \psi^-(x)\bar{\psi}^+(y) + \psi^-(x)\bar{\psi}^-(y)$$

$$= \psi^+(x)\bar{\psi}^+(y) - \psi^-(y)\bar{\psi}^+(x) + \psi^-(x)\bar{\psi}^+(y) + \psi^-(x)\bar{\psi}^-(y) + \{\psi^+(x), \bar{\psi}^-(y)\} =$$

$$\psi(x) = \psi^+(x) + \psi^-(x); \quad \psi^+(\infty) = 0; \quad \psi^-(\infty) = 0$$

Start from $T\{\psi(x)\bar{\psi}(y)\}$:

$$T\{\psi(x)\bar{\psi}(y)\} \stackrel{x^0 > y^0}{=} \psi^+(x)\bar{\psi}^+(y) + \psi^+(x)\bar{\psi}^-(y) + \psi^-(x)\bar{\psi}^+(y) + \psi^-(x)\bar{\psi}^-(y)$$

$$= \psi^+(x)\bar{\psi}^+(y) - \psi^-(y)\bar{\psi}^+(x) + \psi^-(x)\bar{\psi}^+(y) + \psi^-(x)\bar{\psi}^-(y) + \{\psi^+(x), \bar{\psi}^-(y)\} = N(\psi(x)\bar{\psi}(y))$$

$$\psi(x) = \psi^+(x) + \psi^-(x); \quad \psi^+|0\rangle = 0; \quad \psi^-|0\rangle = 0$$

Start from $T\{\psi(x)\bar{\psi}(y)\}$:

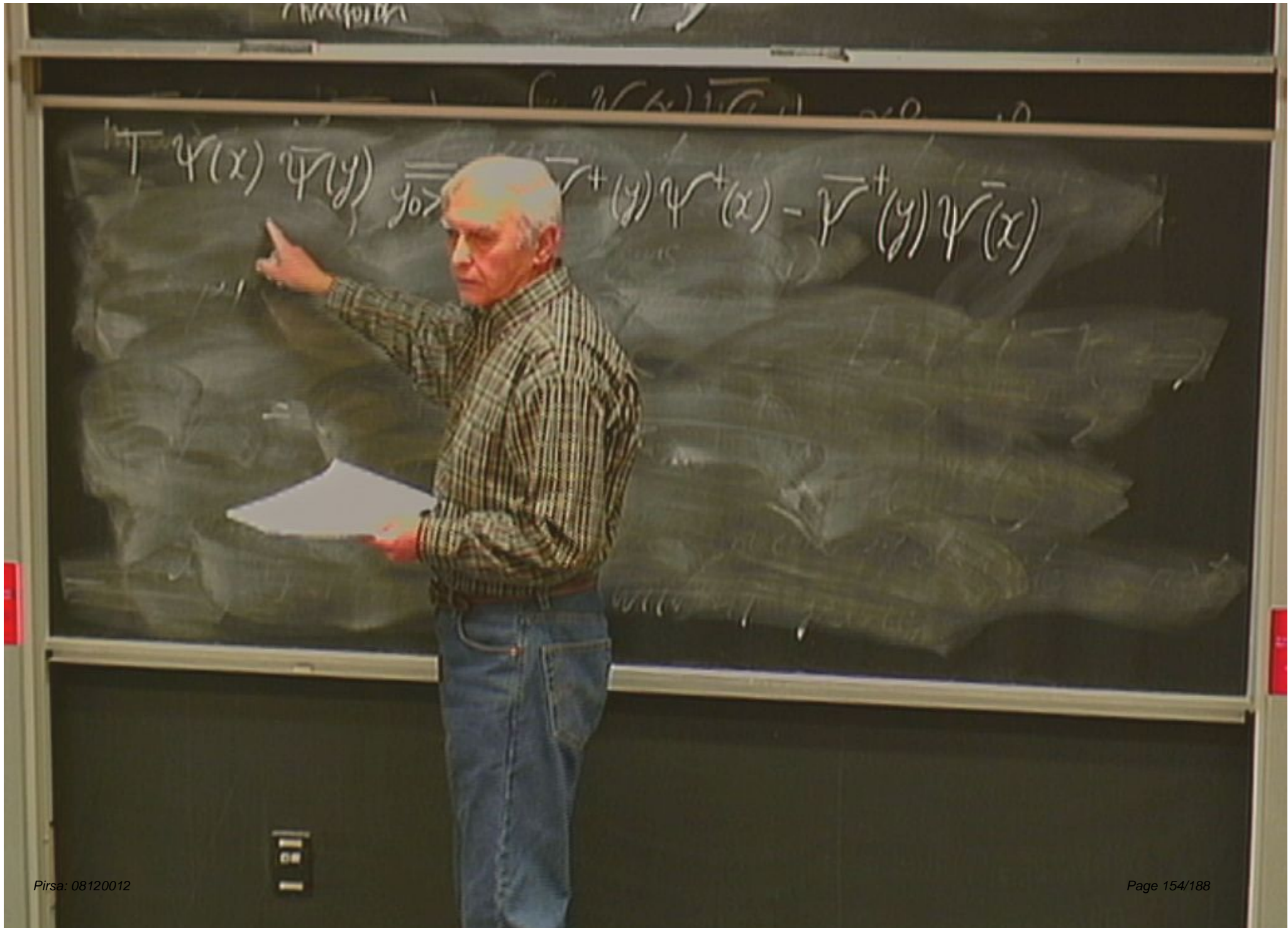
$$T\{\psi(x)\bar{\psi}(y)\} \stackrel{x^0 > y^0}{=} \psi^+(x)\bar{\psi}^+(y) + \psi^+(x)\bar{\psi}^-(y) + \psi^-(x)\bar{\psi}^+(y) + \psi^-(x)\bar{\psi}^-(y)$$

$$= \psi^+(x)\bar{\psi}^+(y) - \psi^-(y)\bar{\psi}^+(x) + \psi^-(x)\bar{\psi}^+(y) + \psi^-(x)\bar{\psi}^-(y) + \{\psi^+(x), \bar{\psi}^-(y)\} = N(\psi(x)\bar{\psi}(y)) + \{\psi^+(x), \bar{\psi}^-(y)\}$$

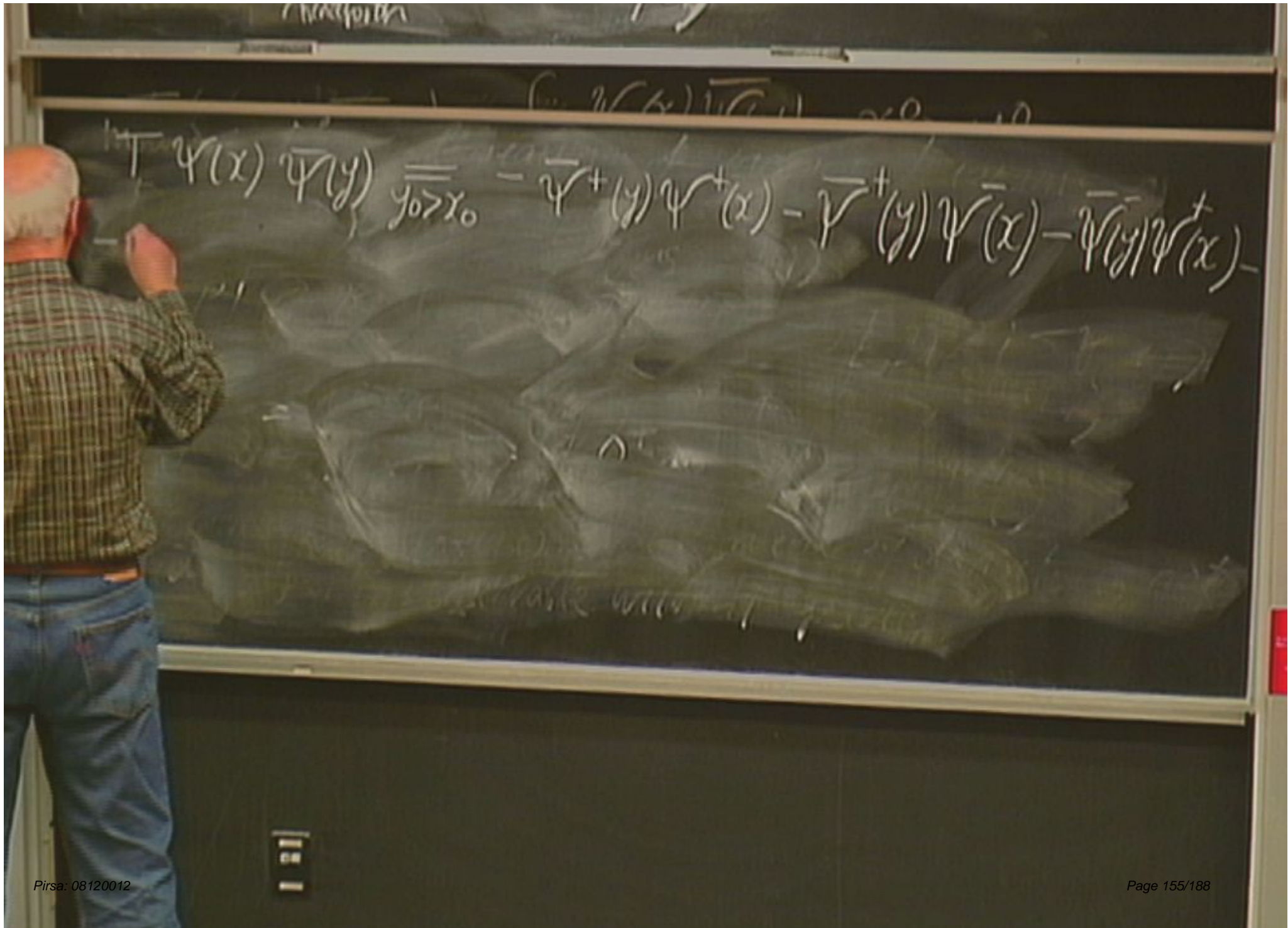
$$\psi(x) = \psi^+(x) + \psi^-(x); \quad \psi^+|0\rangle = 0; \quad \langle 0|\psi^- = 0$$

Start from $T\{\psi(x)\bar{\psi}(y)\}$:

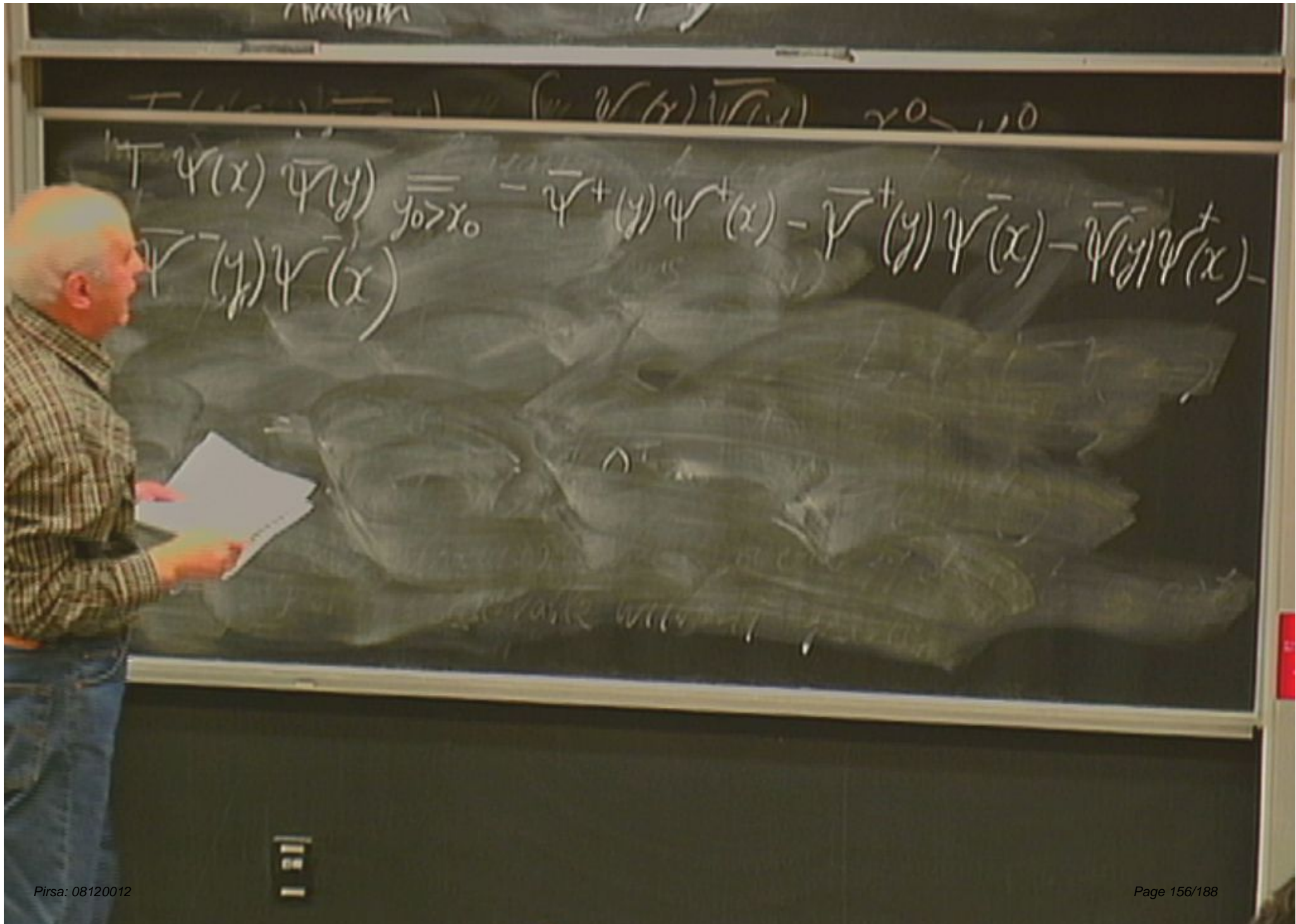
$$\begin{aligned}
 T\{\psi(x)\bar{\psi}(y)\} &\stackrel{x^0 > y^0}{=} \psi^+(x)\bar{\psi}^+(y) + \underbrace{\psi^+(x)\bar{\psi}^-(y)} + \psi^-(x)\bar{\psi}^+(y) + \psi^-(x)\bar{\psi}^-(y) \\
 &= \psi^+(x)\bar{\psi}^+(y) - \psi^-(y)\bar{\psi}^+(x) + \psi^-(x)\bar{\psi}^+(y) + \psi^-(x)\bar{\psi}^-(y) + \{\psi^+(x), \bar{\psi}^-(y)\} = N(\psi(x)\bar{\psi}(y)) + \{\psi^+(x), \bar{\psi}^-(y)\}
 \end{aligned}$$



$$T \psi(x) \bar{\psi}(y) \overleftrightarrow{\partial}_0 \psi^+(y) \psi^+(x) - \bar{\psi}^+(y) \psi^-(x)$$



$$\int \psi(x) \bar{\psi}(y) \overline{y_0 > x_0} - \bar{\psi}^+(y) \psi^+(x) - \bar{\psi}^+(y) \psi^-(x) - \bar{\psi}^-(y) \psi^+(x)$$



Handwritten text at the top of the chalkboard, possibly a name or title.

$$T(\psi(x), \bar{\psi}(y))_{y_0 > x_0} = \bar{\psi}^+(y) \psi^+(x) - \bar{\psi}^+(y) \psi^-(x) - \bar{\psi}^-(y) \psi^+(x) - \bar{\psi}^-(y) \psi^-(x)$$

$$T(\psi(x)\bar{\psi}(y)) \Big|_{x^0 > y^0}$$

$$T(\psi(x)\bar{\psi}(y)) \Big|_{y_0 > x_0} = \bar{\psi}^+(y)\psi^+(x) - \bar{\psi}^+(y)\psi^-(x) - \bar{\psi}^-(y)\psi^-(x) - \bar{\psi}^-(y)\psi^+(x) - \bar{\psi}^-(y)\psi^+(x) - \bar{\psi}^-(y)\psi^-(x)$$

$$T(\psi(x)\overline{\psi(y)}) = \int \psi(x)\overline{\psi(y)}, x^0 > y^0$$

$$T(\psi(x)\overline{\psi(y)})_{y_0 > x_0} = \overline{\psi^+(y)}\psi^+(x) - \overline{\psi^-(y)}\psi^-(x) - \overline{\psi^-(y)}\psi^+(x) - \overline{\psi^+(y)}\psi^-(x)$$

$$- \overline{\psi^-(y)}\psi^-(x) = -\overline{\psi^+(y)}\psi^+(x)$$

$$- \overline{\psi^-(y)}\psi^-(x) = \overline{\psi^-(y)}\psi^-(x)$$

$$T(\psi(x)\overline{\psi(y)}) = \int \psi(x)\overline{\psi(y)}, x^0 > y^0$$

$$\begin{aligned}
 T(\psi(x)\overline{\psi(y)})_{y_0 > x_0} &= \overline{\psi^+(y)}\psi^+(x) - \overline{\psi^-(y)}\psi^-(x) - \overline{\psi^+(y)}\psi^-(x) - \overline{\psi^-(y)}\psi^+(x) \\
 - \overline{\psi^-(y)}\psi^-(x) &= -\overline{\psi^+(y)}\psi^+(x) + \overline{\psi^-(y)}\psi^-(x) + \overline{\psi^-(y)}\psi^+(x) - \overline{\psi^+(y)}\psi^-(x) \\
 - \overline{\psi^-(y)}\psi^-(x) &= \{\overline{\psi^-(y)}\psi^-(x)\}
 \end{aligned}$$

$$T(\psi(x)\bar{\psi}(y)) = \int \psi(x)\bar{\psi}(y), x^0 > y^0$$

$$T(\psi(x)\bar{\psi}(y))_{y^0 > x^0} = \bar{\psi}^+(y)\psi^+(x) - \bar{\psi}^+(y)\psi^-(x) - \bar{\psi}^-(y)\psi^-(x) - \bar{\psi}^-(y)\psi^+(x)$$

$$- \bar{\psi}^-(y)\psi^-(x) = -\bar{\psi}^+(y)\psi^+(x) + \bar{\psi}^-(y)\psi^-(x) - \bar{\psi}^-(y)\psi^+(x) - \bar{\psi}^+(y)\psi^-(x)$$

$$- \bar{\psi}^-(y)\psi^-(x) = \left\{ \bar{\psi}^-(y)\psi^-(x) \right\}$$

$$T(\psi(x)\bar{\psi}(y)) = \int \psi(x)\bar{\psi}(y), x^0 > y^0$$

$$T(\psi(x)\bar{\psi}(y))_{y_0 > x_0} = \overline{\psi^+(y)\psi^+(x)} - \overline{\psi^+(y)\psi^-(x)} - \overline{\psi^-(y)\psi^+(x)} - \overline{\psi^-(y)\psi^-(x)}$$

$$- \overline{\psi^-(y)\psi^-(x)} = -\overline{\psi^+(y)\psi^+(x)} + \overline{\psi^-(x)\psi^+(y)} - \overline{\psi^-(x)\psi^-(y)}$$

$$- \overline{\psi^-(y)\psi^-(x)} = \left\{ \overline{\psi^-(y)\psi^-(x)} \right\}$$

$$T(\psi(x)\bar{\psi}(y)) = \int \psi(x)\bar{\psi}(y), x^0 > y^0$$

$$T\psi(x)\bar{\psi}(y) \Big|_{y_0 > x_0} = \overline{\psi^+(y)\psi^+(x)} - \overline{\psi^+(y)\psi^-(x)} - \overline{\psi^-(y)\psi^+(x)} - \overline{\psi^-(y)\psi^-(x)}$$

$$- \overline{\psi^-(y)\psi^-(x)} = -\overline{\psi^+(y)\psi^+(x)} + \overline{\psi^-(x)\psi^+(y)} - \overline{\psi^-(x)\psi^-(y)}$$

$$- \overline{\psi^-(y)\psi^-(x)} = \left\{ \overline{\psi^-(y)\psi^-(x)} \right\}$$

$$T(\psi(x)\bar{\psi}(y)) = \int \psi(x)\bar{\psi}(y), x^0 > y^0$$

$$T\psi(x)\bar{\psi}(y) \Big|_{y_0 > x_0} = \frac{\bar{\psi}^+(y)\psi^+(x) - \bar{\psi}^+(y)\psi^-(x) - \bar{\psi}^-(y)\psi^+(x) - \bar{\psi}^-(y)\psi^-(x)}{\hbar} - \frac{\bar{\psi}^-(y)\psi^-(x) - \bar{\psi}^-(y)\psi^+(x)}{\hbar} + \frac{\bar{\psi}^+(y)\psi^+(x) + \bar{\psi}^-(y)\psi^-(x) - \bar{\psi}^-(y)\psi^+(x) - \bar{\psi}^+(y)\psi^-(x)}{\hbar} - \left\{ \bar{\psi}(y), \psi(x) \right\}$$

minus sign for each fermion interchange under symbol N . $N(a_p a_q a_r^\dagger) = (-1)^2 a_r^\dagger a_p a_q = (-1)^3 a_r^\dagger a_q a_p$

$\psi(x) = \psi^+(x) + \psi^-(x)$; $\psi^+|0\rangle = 0$; $\langle 0|\psi^- = 0$

Useful realization of Wick's theorem

Start from $T\{\psi(x)\bar{\psi}(y)\}$:

$$T\{\psi(x)\bar{\psi}(y)\} \stackrel{x^0 > y^0}{=} \psi^+(x)\bar{\psi}^+(y) + \psi^+(x)\bar{\psi}^-(y) + \psi^-(x)\bar{\psi}^+(y) + \psi^-(x)\bar{\psi}^-(y)$$

$$\stackrel{x^0 < y^0}{=} \psi^+(x)\bar{\psi}^-(y) - \psi^-(y)\bar{\psi}^+(x) + \psi^-(x)\bar{\psi}^+(y) + \psi^-(x)\bar{\psi}^-(y)$$

$$\{\psi^+(x), \bar{\psi}^-(y)\} = N(\psi(x)\bar{\psi}(y)) + \{\psi^+(x), \bar{\psi}^-(y)\}$$

minus sign for each fermion interchange under symbol N : $N(a_p a_q a_r^\dagger) = (-1)^2 a_r^\dagger a_p a_q = (-1)^3 a_r^\dagger a_q a_p$

$\psi(x) = \psi^+(x) + \psi^-(x)$; $\psi^+|0\rangle = 0$; $\langle 0|\psi^- = 0$

Quantization of Wick's Theorem

Start from $T\{\psi(x)\bar{\psi}(y)\}$:

$$T\{\psi(x)\bar{\psi}(y)\} \stackrel{x^0 > y^0}{=} \psi^+(x)\bar{\psi}^+(y) + \psi^+(x)\bar{\psi}^-(y) + \psi^-(x)\bar{\psi}^+(y) + \psi^-(x)\bar{\psi}^-(y)$$

$$T\{\psi(x)\bar{\psi}(y)\} \stackrel{x^0 < y^0}{=} \psi^-(y)\bar{\psi}^-(x) + \psi^-(y)\bar{\psi}^+(x) + \psi^+(y)\bar{\psi}^-(x) + \psi^+(y)\bar{\psi}^+(x)$$

$$T\{\psi(x)\bar{\psi}(y)\} = N(\psi(x)\bar{\psi}(y)) + \{\psi(x), \bar{\psi}(y)\}$$

minus sign for each fermion interchange under symbol N . $N(a_p a_q a_r^+) = (-1)^2 a_r^+ a_p a_q = (-1)^3 a_r^+ a_q a_p$
 $\psi(x) = \psi^+(x) + \psi^-(x); \psi^+|0\rangle = 0; \langle 0|\psi^- = 0$

Quantization of Wick's theorem

Start from $T\{\psi(x)\bar{\psi}(y)\}$:

$$T\{\psi(x)\bar{\psi}(y)\} \stackrel{x^0 > y^0}{=} \psi^+(x)\bar{\psi}^+(y) + \psi^+(x)\bar{\psi}^-(y) + \psi^-(x)\bar{\psi}^+(y) + \psi^-(x)\bar{\psi}^-(y)$$

$$= \psi^+(x)\bar{\psi}^+(y) - \psi^-(y)\bar{\psi}^+(x) + \psi^-(x)\bar{\psi}^+(y) + \psi^-(x)\bar{\psi}^-(y) + \{\psi^+(x), \bar{\psi}^-(y)\} = N(\psi(x)\bar{\psi}(y)) + \{\psi^+(x), \bar{\psi}^-(y)\}$$

$$T(\psi(x)\overline{\psi(y)}) = \int \psi(x)\overline{\psi(y)}, x^0 > y^0$$

$$T(\psi(x)\overline{\psi(y)})_{y_0 > x_0} = \overline{\psi^+(y)\psi^+(x)} - \overline{\psi^+(y)\psi^-(x)} - \overline{\psi^-(y)\psi^+(x)} - \overline{\psi^-(y)\psi^-(x)}$$

$$- \overline{\psi^-(y)\psi^-(x)} = -\overline{\psi^+(y)\psi^+(x)} + \overline{\psi^-(x)\psi^+(y)} - \overline{\psi^-(x)\psi^-(y)}$$

$$- \overline{\psi^-(y)\psi^-(x)} = \{ \overline{\psi^-(y)\psi^-(x)} \}$$

$$|\Psi(x)\Psi(y)| = |\Psi(y)\Psi(x)|, \quad x > y^0$$

$$\begin{aligned}
 \Psi(x)\Psi(y) &= \frac{\Psi^+(y)\Psi^+(x) - \Psi^+(y)\Psi^-(x) - \Psi^-(y)\Psi^+(x) - \Psi^-(y)\Psi^-(x)}{2} \\
 &= \frac{\Psi^+(y)\Psi^+(x) + \Psi^-(x)\Psi^+(y) - \Psi^-(x)\Psi^-(y) - \Psi^-(y)\Psi^-(x)}{2} \\
 &= \frac{1}{2} \{ \Psi^+(y), \Psi^-(x) \}
 \end{aligned}$$

$$\psi(x) = \psi^+(x) + \psi^-(x); \quad \psi^+|0\rangle = 0; \quad \langle 0|\psi^- = 0$$

Start from $T\{\psi(x)\bar{\psi}(y)\}$:

$$\begin{aligned}
 T\{\psi(x)\bar{\psi}(y)\} & \stackrel{x_0 > y_0}{=} \psi^+(x)\bar{\psi}^+(y) + \psi^+(x)\bar{\psi}^-(y) + \psi^-(x)\bar{\psi}^+(y) + \\
 & \psi^-(x)\bar{\psi}^-(y) = \psi^+(x)\bar{\psi}^+(y) - \psi^-(y)\bar{\psi}^+(x) + \psi^-(x)\bar{\psi}^+(y) + \\
 & + \psi^-(x)\bar{\psi}^-(y) + \{\psi^+(x), \bar{\psi}^-(y)\} = N(\psi(x)\bar{\psi}(y)) + \{\psi^+(x), \bar{\psi}^-(y)\}
 \end{aligned}$$

$$T(\psi(x)\bar{\psi}(y)) = \int \psi(x)\bar{\psi}(y), x^0 > y^0$$

$$T(\psi(x)\bar{\psi}(y))_{y^0 > x^0} = \bar{\psi}^+(y)\psi^+(x) - \bar{\psi}^+(y)\psi^-(x) - \bar{\psi}^-(y)\psi^+(x) - \bar{\psi}^-(y)\psi^-(x) - \bar{\psi}^-(y)\psi^+(x) - \bar{\psi}^-(y)\psi^-(x)$$

$$= -\bar{\psi}^+(y)\psi^+(x) + \bar{\psi}^-(y)\psi^+(x) - \bar{\psi}^-(y)\psi^-(x)$$

$$= \{\bar{\psi}^-(y), \psi^-(x)\}$$

$$T(\psi(x)\bar{\psi}(y)) = \int \psi(x)\bar{\psi}(y), x^0 > y^0$$

$$T(\psi(x)\bar{\psi}(y))_{y^0 > x^0} = \bar{\psi}^+(y)\psi^+(x) - \bar{\psi}^+(y)\psi^-(x) - \bar{\psi}^-(y)\psi^+(x) - \bar{\psi}^-(y)\psi^-(x)$$

$$- \bar{\psi}^-(y)\psi^-(x) = -\bar{\psi}^+(y)\psi^+(x) + \bar{\psi}^-(x)\psi^+(y) - \bar{\psi}^-(x)\psi^-(y) - \bar{\psi}^-(x)\psi^-(y)$$

$$- \bar{\psi}^-(x)\psi^-(y) = \{ \bar{\psi}^-(x)\psi^-(y), \psi^+(x) \}$$

$$\psi(x) = \psi^+(x) + \psi^-(x); \quad \psi^+|0\rangle = 0; \quad \langle 0|\psi^- = 0$$

Generalization of Wick's theorem

Start from $T\{\psi(x)\bar{\psi}(y)\}$:

$$\begin{aligned}
 T\{\psi(x)\bar{\psi}(y)\} &\stackrel{x^0 > y^0}{=} \psi^+(x)\bar{\psi}^+(y) + \psi^+(x)\bar{\psi}^-(y) + \psi^-(x)\bar{\psi}^+(y) + \\
 &\psi^-(x)\bar{\psi}^-(y) = \psi^+(x)\bar{\psi}^+(y) - \bar{\psi}^-(y)\psi^+(x) + \psi^-(x)\bar{\psi}^+(y) + \\
 &+ \psi^-(x)\bar{\psi}^-(y) + \{\psi^+(x), \bar{\psi}^-(y)\} = N(\psi(x)\bar{\psi}(y)) + \{\psi^+(x), \bar{\psi}^-(y)\}
 \end{aligned}$$

$$T(\psi(x)\bar{\psi}(y)) = \int \psi(x)\bar{\psi}(y), x^0 > y^0$$

$$T(\psi(x)\bar{\psi}(y))_{y_0 > x_0} = \bar{\psi}^+(y)\psi^+(x) - \bar{\psi}^+(y)\psi^-(x) - \bar{\psi}^-(y)\psi^+(x) - \bar{\psi}^-(y)\psi^-(x)$$

$$- \bar{\psi}^-(y)\psi^-(x) = -\bar{\psi}^+(y)\psi^+(x) + \bar{\psi}^-(y)\psi^-(x) - \bar{\psi}^-(y)\psi^+(x) - \bar{\psi}^+(y)\psi^-(x)$$

$$- \bar{\psi}^-(y)\psi^-(x) = \left\{ \bar{\psi}^-(y), \psi^-(x) \right\}$$

Start from $T\{\psi(x)\bar{\psi}(y)\}$:

$$\begin{aligned}
 T\{\psi(x)\bar{\psi}(y)\} &= \underbrace{\psi^+(x)\bar{\psi}^+(y)}_{x^0 > y^0} + \underbrace{\psi^+(x)\bar{\psi}(y)} + \underbrace{\psi^-(x)\bar{\psi}^+(y)} + \underbrace{\psi^-(x)\bar{\psi}(y)} \\
 \psi^-(x)\bar{\psi}(y) &= \underbrace{\psi^+(x)\bar{\psi}^+(y)} - \underbrace{\bar{\psi}^-(y)\psi^+(x)} + \underbrace{\psi^-(x)\bar{\psi}^+(y)} + \underbrace{\bar{\psi}^-(y)\psi^-(x)} \\
 + \psi^-(x)\bar{\psi}(y) + \{\psi^+(x), \bar{\psi}^+(y)\} &= N(\psi(x)\bar{\psi}(y)) + \{\psi^+(x), \bar{\psi}^+(y)\}
 \end{aligned}$$

$$T(\psi(x)\overline{\psi(y)}) = \int \psi(x)\overline{\psi(y)}, x^0 > y^0$$

$$T(\psi(x)\overline{\psi(y)})_{y_0 > x_0} = \overline{\psi^+(y)}\psi^-(x) - \psi^+(y)\overline{\psi^-(x)} - \overline{\psi^-(y)}\psi^+(x) - \psi^-(y)\overline{\psi^+(x)}$$

$$= -\overline{\psi^+(y)}\psi^-(x) - \psi^+(y)\overline{\psi^-(x)} - \overline{\psi^-(y)}\psi^+(x) - \psi^-(y)\overline{\psi^+(x)}$$



$$T(\psi(x)\bar{\psi}(y)) = \int \psi(x)\bar{\psi}(y), x^0 > y^0$$

$$T\psi(x)\bar{\psi}(y) \stackrel{y_0 > x_0}{=} -\psi^+(x) - \bar{\psi}^+(y)\psi^-(x) - \bar{\psi}(y)\psi^+(x) - \bar{\psi}^-(y)\psi^-(x)$$

$$- \bar{\psi}^-(y)\psi^-(x) = -\psi^+(x) + \bar{\psi}^-(x)\bar{\psi}^+(y) - \bar{\psi}^-(y)\psi^+(x) - \bar{\psi}^-(y)\psi^-(x)$$

$$\psi(x) = \psi^+(x) + \psi^-(x); \quad \psi^+|0\rangle = 0; \quad \langle 0|\psi^- = 0$$

Generalization of Wick's theorem

Start from $T\{\psi(x)\bar{\psi}(y)\}$:

$$\begin{aligned}
 T\{\psi(x)\bar{\psi}(y)\} &= \psi^+(x)\bar{\psi}^+(y) + \psi^+(x)\bar{\psi}^-(y) + \psi^-(x)\bar{\psi}^+(y) + \\
 &\quad \psi^-(x)\bar{\psi}^-(y) \\
 &\stackrel{x^0 > y^0}{=} \psi^+(x)\bar{\psi}^+(y) - \bar{\psi}^-(y)\psi^+(x) + \psi^-(x)\bar{\psi}^+(y) + \\
 &\quad \psi^-(x)\bar{\psi}^-(y) \\
 &= N(\psi(x)\bar{\psi}(y)) + \langle \psi(x), \bar{\psi}(y) \rangle
 \end{aligned}$$

$$\psi(x) = \psi^+(x) + \psi^-(x); \quad \psi^+|0\rangle = 0; \quad \langle 0|\psi^- = 0$$

Generalization of Wick's theorem

Start from $T\{\psi(x)\bar{\psi}(y)\}$:

$$\begin{aligned}
 T\{\psi(x)\bar{\psi}(y)\} &\stackrel{x^0 > y^0}{=} \psi^+(x)\bar{\psi}^+(y) + \psi^+(x)\bar{\psi}^-(y) + \psi^-(x)\bar{\psi}^+(y) + \\
 &\psi^-(x)\bar{\psi}^-(y) = \psi^+(x)\bar{\psi}^+(y) - \bar{\psi}^-(y)\psi^+(x) + \psi^-(x)\bar{\psi}^+(y) + \\
 &+ \psi^-(x)\bar{\psi}^-(y) + \{\psi^+(x), \bar{\psi}^-(y)\} = N(\psi(x)\bar{\psi}(y)) + \{\psi^+(x), \bar{\psi}^-(y)\}
 \end{aligned}$$

$$\psi(x) = \psi^+(x) + \psi^-(x); \quad \psi^+|0\rangle = 0; \quad \langle 0|\psi^- = 0$$

Quantization of Wick's theorem

Start from $T\{\psi(x)\bar{\psi}(y)\}$:

$$\begin{aligned}
 T\{\psi(x)\bar{\psi}(y)\} &\stackrel{x^0 > y^0}{=} \psi^+(x)\bar{\psi}^+(y) + \psi^+(x)\bar{\psi}^-(y) + \psi^-(x)\bar{\psi}^+(y) + \\
 &\psi^-(x)\bar{\psi}^-(y) = \psi^+(x)\bar{\psi}^+(y) - \bar{\psi}^-(y)\psi^+(x) + \psi^-(x)\bar{\psi}^+(y) + \\
 &+ \psi^-(x)\bar{\psi}^-(y) + \{\psi^+(x), \bar{\psi}^-(y)\} = N(\psi(x)\bar{\psi}(y)) + \{\psi^+(x), \bar{\psi}^-(y)\}
 \end{aligned}$$

$$T(\psi(x)\bar{\psi}(y)) = \int \psi(x)\bar{\psi}(y), x^0 > y^0$$

$$\begin{aligned} \psi(x)\bar{\psi}(y) \Big|_{y_0 > x_0} &= \bar{\psi}^+(y)\psi^+(x) - \bar{\psi}^-(y)\psi^-(x) - \bar{\psi}(y)\psi^+(x) - \bar{\psi}^+(y)\psi^-(x) \\ &= -\bar{\psi}^+(y)\psi^+(x) + \bar{\psi}^-(y)\psi^-(x) + \bar{\psi}^-(y)\psi^+(x) - \bar{\psi}^+(y)\psi^-(x) \\ &= \psi^-(x)\bar{\psi}^-(y) - \psi^+(x)\bar{\psi}^+(y) \end{aligned}$$



$$\psi(x) = \psi^+(x) + \psi^-(x); \quad \psi^+|0\rangle = 0; \quad \langle 0|\psi^- = 0$$

Generalization of Wick's theorem

Start from $T\{\psi(x)\bar{\psi}(y)\}$:

$$\begin{aligned}
 T\{\psi(x)\bar{\psi}(y)\} & \stackrel{x^0 > y^0}{=} \psi^+(x)\bar{\psi}^+(y) + \psi^+(x)\bar{\psi}^-(y) + \psi^-(x)\bar{\psi}^+(y) + \\
 & \psi^-(x)\bar{\psi}^-(y) = \psi^+(x)\bar{\psi}^+(y) - \bar{\psi}^-(y)\psi^+(x) + \psi^-(x)\bar{\psi}^+(y) + \\
 & \psi^-(x)\bar{\psi}^-(y) + \{\psi^+(x), \bar{\psi}^-(y)\} = \underbrace{N(\psi(x)\bar{\psi}(y))}_{\text{normal ordered}} + \underbrace{\{\psi^+(x), \bar{\psi}^-(y)\}}_{\text{commutator}}
 \end{aligned}$$

$$T(\psi(x)\bar{\psi}(y)) \equiv \underline{\psi(x)\bar{\psi}(y)}, \quad x^0 > y^0$$

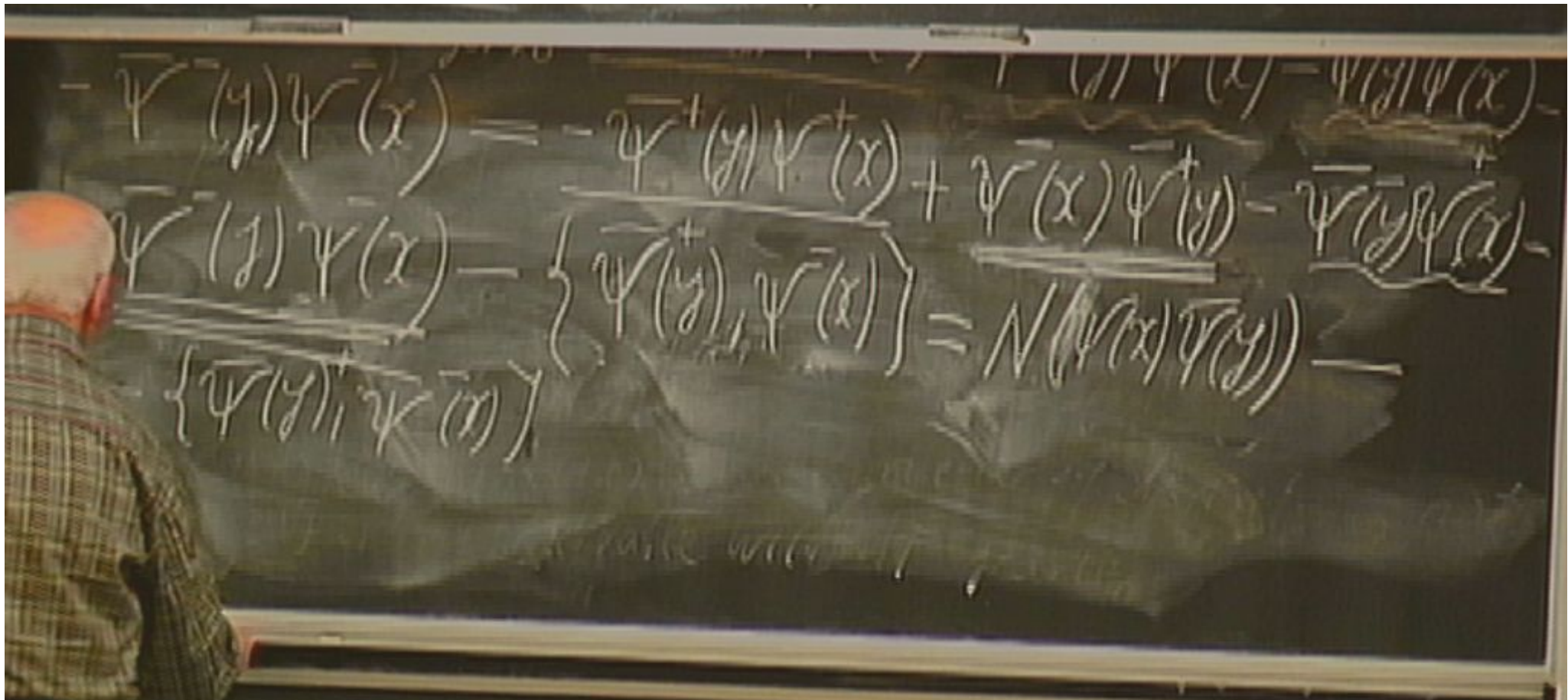
$$\begin{aligned}
 T(\psi(x)\bar{\psi}(y)) &\stackrel{y_0 > x_0}{=} -\bar{\psi}^+(y)\psi^+(x) - \bar{\psi}^+(y)\bar{\psi}(x) - \bar{\psi}(y)\psi^+(x) - \\
 &\quad - \bar{\psi}(y)\psi(x) = -\bar{\psi}^+(y)\psi^+(x) + \bar{\psi}(x)\bar{\psi}^+(y) - \bar{\psi}(y)\psi(x) - \\
 &\quad - \bar{\psi}(y)\psi(x) = \underline{\underline{\bar{\psi}(y)\psi(x)}} = N(\psi(x)\bar{\psi}(y))
 \end{aligned}$$

$$T(\psi(x)\bar{\psi}(y)) \equiv \frac{\psi(x)\bar{\psi}(y)}{\bar{\psi}(x)\psi(y)}, x^0 > y^0$$

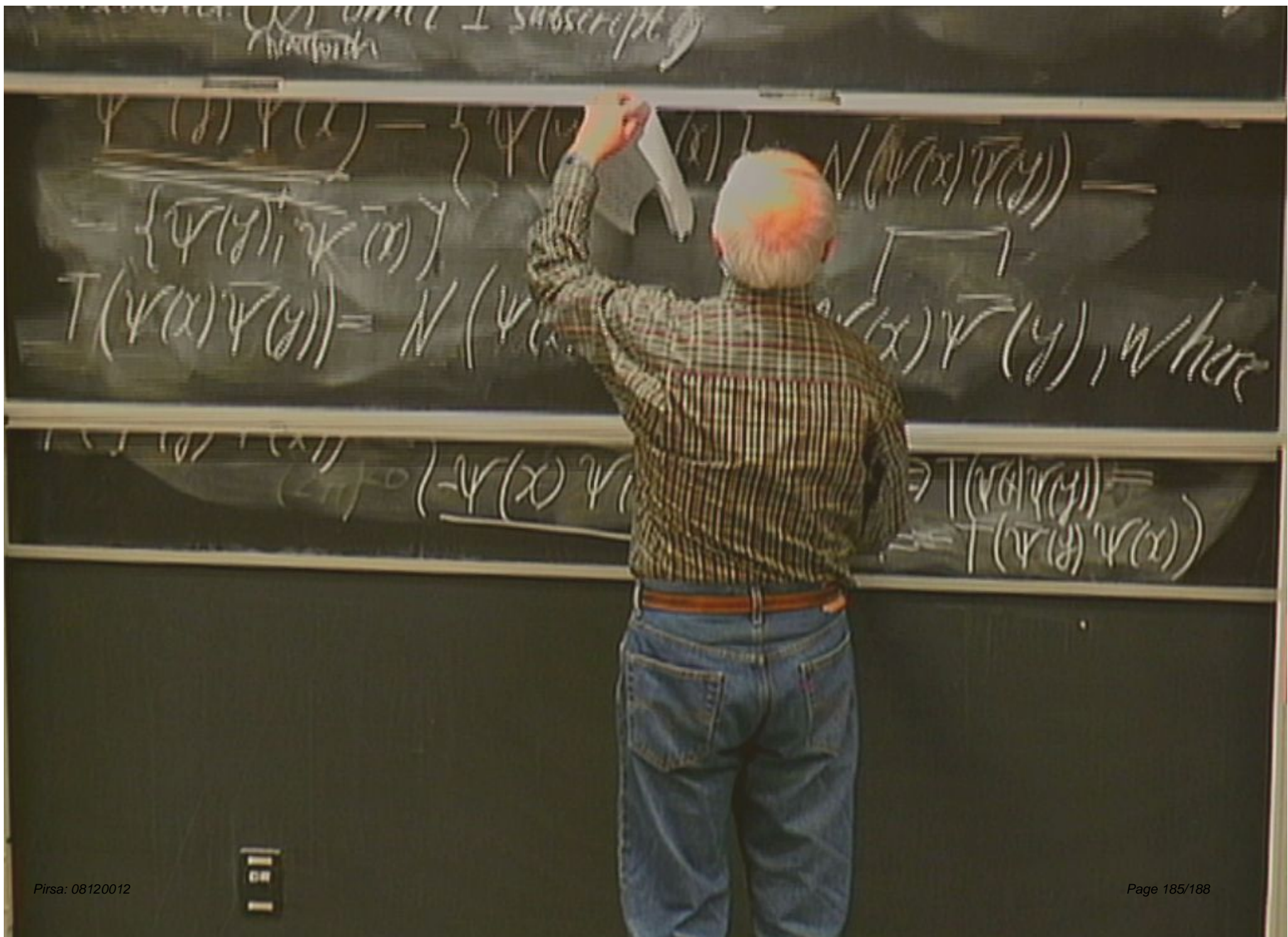
$$T\psi(x)\bar{\psi}(y) \stackrel{y_0 > x_0}{=} \bar{\psi}^+(y)\psi^+(x) - \bar{\psi}^+(y)\psi^-(x) - \bar{\psi}^-(y)\psi^-(x) - \bar{\psi}^-(y)\psi^+(x)$$

$$- \bar{\psi}^-(y)\psi^-(x) = -\bar{\psi}^+(y)\psi^+(x) + \bar{\psi}^-(x)\bar{\psi}^+(y) - \bar{\psi}^-(y)\psi^-(x) - \bar{\psi}^-(y)\psi^+(x)$$

$$- \bar{\psi}^-(y)\psi^-(x) = \{ \bar{\psi}^+(y), \psi^-(x) \} = N(\bar{\psi}(x)\bar{\psi}(y))$$



$$\begin{aligned}
 & \psi^-(y)\psi^-(x) = -\psi^+(y)\psi^+(x) + \psi^-(x)\psi^+(y) - \psi^+(x)\psi^-(y) \\
 & \psi^-(y)\psi^-(x) - \psi^+(y)\psi^+(x) = \psi^-(x)\psi^+(y) - \psi^+(x)\psi^-(y) \\
 & \psi^-(y)\psi^-(x) - \psi^+(y)\psi^+(x) = N(\psi^-(x)\psi^+(y) - \psi^+(x)\psi^-(y)) \\
 & \psi^-(y)\psi^-(x) = N(\psi^-(x)\psi^+(y) - \psi^+(x)\psi^-(y))
 \end{aligned}$$



$$\psi(x)\bar{\psi}(y) = \left\{ \psi^+(x)\bar{\psi}(y) \right\}$$

is valid in including fermions. To formulate properly Wick's theorem's case, one should generalize the terminology symbols for fermions normal-ordering Start from $\psi(x)\bar{\psi}(y)$, which was already considered (I omit I handwritten)

$$\psi(x) \bar{\psi}(y) \Big|_{x^0 > y^0} = \{ \psi(x), \bar{\psi}(y) \}, x^0 > y^0$$

is valid late proper general symb including fermions. To formulate in this case, one should ordering and normal-ordering

is the same as the one considered in the previous section. The time-ordered product is $T(\psi(x) \bar{\psi}(y))$, which was already considered in the previous section (I subscript)

$$T(\psi(x)\bar{\psi}(y)) = N(\psi(x)\bar{\psi}(y)) + \psi(x)\bar{\psi}(y), \text{ where}$$

$$\psi(x)\bar{\psi}(y) = \begin{cases} [\psi(x), \bar{\psi}(y)], & x^0 > y^0 \\ -[\bar{\psi}(y), \psi(x)], & y^0 > x^0 \end{cases} = S_F(x-y)$$

is valid including fermions. To formulate properly Wick's theorem in this case, one should generalize the time-ordering and normal-ordering symbols for fermions.

Start from $T(\psi_I(x)\bar{\psi}_I(y))$, which was already considered (omit I subscript)