

Title: Quantum Field Theory 1 - Lecture 13B

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Abstract: Quantum Field Theory I course taught by Volodya Miransky of the University of Western Ontario

WICK'S Theorem

Question: What is an effective method to calculate  $\langle 0|T \psi_I(x_1) \dots \psi_I(x_n)|0\rangle$ ?

contraction of two fields.

*[The chalkboard contains several lines of very faint, mostly illegible handwritten mathematical notes and equations.]*



Wick's Theorem

Question: what is an effective method to calculate  $\langle 0|T \psi_I(x) \dots \psi_I(y)|0\rangle$ ?

contraction of two fields:

$$\overline{\psi_I(x)\psi_I(y)} \equiv \begin{cases} [\psi_I^+(x), \bar{\psi}_I(y)], & x^0 > y^0 \\ [\psi_I^+(y), \bar{\psi}_I(x)], & y^0 > x^0 \end{cases} = D_F(x-y)$$

Normal order.  $N(\psi_I(x)\psi_I(y)) = \psi_I^+(x)\psi_I^+(y) + \bar{\psi}_I^-(y)\psi_I^-(x)$   
 $+ \psi_I^-(x)\psi_I^+(y) + \bar{\psi}_I^-(x)\psi_I^-(y)$

$$\langle 0|N(\psi_I(x)\psi_I(y))|0\rangle = 0$$

# WICK'S Theorem

Question: what is an effective method to calculate  $\langle 0 | T \psi_I(x) \dots \psi_I(y) | 0 \rangle$ ?

contraction of two fields:

$$\overline{\psi_I(x) \psi_I(y)} \equiv \begin{cases} [\psi_I^+(x), \bar{\psi}_I(y)], & x^0 > y^0 \\ [\psi_I^+(y), \bar{\psi}_I(x)], & y^0 > x^0 \end{cases} = D_F(x-y)$$

Normal order:  $N(\psi_I(x) \psi_I(y)) = \psi_I^+(x) \psi_I^+(y) + \bar{\psi}_I(y) \psi_I^+(x) + \psi_I^-(x) \psi_I^-(y) + \bar{\psi}_I(x) \psi_I^-(y)$

$$\langle 0 | N(\psi_I(x) \psi_I(y)) | 0 \rangle = 0$$

Normal order is that when all  $a_p$  terms are to the right of  $a_p^+$  terms

# WICK'S Theorem

Question: what is an effective method to calculate

Normal order is that when all  $a_p$  terms are to the right of  $a_p^+$  terms

$$N(\psi^+(x) \bar{\psi}(y) \psi^+(u) \psi(v)) =$$

# WICK'S Theorem

Question: what is an effective method to calculate

Normal order is that when all  $a_p$  terms are to the right of  $a_p^+$  terms

$$N(\psi^+(x) \bar{\psi}(y) \psi^+(u) \psi^+(v)) = \bar{\psi}(y) \bar{\psi}(v) \psi^+(x) \psi^+(u)$$

WICK'S Theorem

Question: what is an effective method to calculate  $\langle 0|T \psi_I(x_1) \dots \psi_I(x_n)|0\rangle$ ?

contraction of two fields:

$$\overline{\psi_I(x)\psi_I(y)} \equiv \begin{cases} [\psi_I^+(x), \bar{\psi}_I(y)], & x^0 > y^0 \\ [\psi_I^+(y), \bar{\psi}_I(x)], & y^0 > x^0 \end{cases} D_F(x-y)$$

Normal order.  $N(\psi_I(x)\psi_I(y)) = \psi_I^+(x)\bar{\psi}_I(y)\psi_I^-(x) + \bar{\psi}_I(x)\psi_I^-(y) + \dots$

Normal order is the order where all annihilation operators are to the right of all creation operators.

$$N(\psi^+(x)\bar{\psi}(y)\psi^+(u)\bar{\psi}(v)) = \bar{\psi}(v)\psi^+(u)\bar{\psi}(y)\psi^+(x)$$

$$\langle 0|N(\text{any operator})|0\rangle = 0.$$



# WICK'S Theorem

Question: what is an effective method to calculate  $\langle 0 | T \psi_I(x) \dots \psi_I(z) | 0 \rangle$ ?

contraction of two fields:

$$\overline{\psi_I(x) \psi_I(y)} \equiv \begin{cases} [\psi_I^+(x), \bar{\psi}_I(y)], & x^0 > y^0 \\ [\psi_I^+(y), \bar{\psi}_I(x)], & y^0 > x^0 \end{cases} = D_F(x-y)$$

Normal order.  $N(\psi_I(x) \psi_I(y)) = \psi_I^+(x) \psi_I^+(y) + \bar{\psi}_I(y) \psi_I^+(x) + \psi_I^-(x) \psi_I^-(y) + \bar{\psi}_I(x) \psi_I^-(y)$

$$\langle 0 | N(\psi_I(x) \psi_I(y)) | 0 \rangle = 0.$$

Normal order is that when all  $a_p$  terms are to the right of  $a_p^\dagger$  terms

$$N(\psi^+(x) \bar{\psi}(y) \psi^+(u) \bar{\psi}(z)) = \bar{\psi}(y) \psi^+(x) \bar{\psi}(z) \psi^+(u)$$

$$\langle 0 | N(\text{any operator}) | 0 \rangle = 0.$$

$$N(\psi(x)\psi(y)\psi(z)\psi(w)) = \psi(y)\psi(z)\psi(w)\psi(x)$$

$$\langle 0|N(\text{any operator})|0\rangle = 0.$$

Normal order.  $N(\psi_I^-(x)\psi_I^+(y) + \psi_I^-(y)\psi_I^+(x) + \psi_I^-(x)\psi_I^-(y) + \psi_I^-(y)\psi_I^-(x) + \psi_I^+(x)\psi_I^+(y) + \psi_I^+(y)\psi_I^+(x))|0\rangle = 0.$

$$N \rightarrow : : \quad N(\psi(x)\psi(y)) \equiv : \psi(x)\psi(y) :$$

contraction of two fields:

$$\overline{\psi_I(x)\psi_I(y)} \equiv \begin{cases} [\psi_I^+(x), \psi_I^-(y)] & x^0 > y^0 \\ [\psi_I^+(y), \psi_I^-(x)] & y^0 > x^0 \end{cases} = D_F(x-y)$$

Normal order.

$$N(\psi_I(x)\psi_I(y)) + \psi_I^-(x)\psi_I^+(y) + \psi_I^-(y)\psi_I^+(x) + \psi_I^-(x)\psi_I^-(y) + \psi_I^+(x)\psi_I^+(y) \langle 0 | \dots | 0 \rangle = 0.$$

Wick's theorem:  
 $T \{ \psi(x_1) \psi(x_2) \dots \psi(x_n) \} = N \{ \psi(x_1) \psi(x_2) \dots \}$



Wick's theorem:

$$\langle \psi(x_1) \psi(x_2) \dots \psi(x_m) \rangle = N \{ \psi(x_1) \psi(x_2) \dots \psi(x_m) + \text{all} \}$$

Wick's theorem:

$$T \{ \varphi(x_1) \varphi(x_2) \dots \varphi(x_m) \} = N \{ \varphi(x_1) \varphi(x_2) \dots \varphi(x_m) \} + \text{all possible contractions}$$

I.

$$U(t) = \mathcal{P} \exp \left[ -i \int_{t_0}^t H(t') dt' \right]$$

Wick's theorem:

$$T\{\varphi(x_1)\varphi(x_2)\dots\varphi(x_m)\} = N\{\varphi(x_1)\varphi(x_2)\dots\varphi(x_m)\} + \text{all possible contractions}$$

Illustration:  $T(\varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4)) =$



Wick's theorem:

$$T\{\varphi(x_1)\varphi(x_2)\dots\varphi(x_m)\} = N\{\varphi(x_1)\varphi(x_2)\dots\varphi(x_m)\} + \text{all possible contractions}$$

Illustration:  $T\{\varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4)\} = N\{\varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4)\} + \dots$

*[Faded handwritten notes and scribbles]*



Wick's theorem:

$$T \{ \varphi(x_1) \varphi(x_2) \dots \varphi(x_m) \} = N \{ \varphi(x_1) \varphi(x_2) \dots \varphi(x_m) \} + \text{all possible contractions}$$

Illustration:  $T(\varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4)) = N \{ \overbrace{\varphi(x_1)\varphi(x_2)} \varphi(x_3)\varphi(x_4) + \overbrace{\varphi(x_1)\varphi(x_3)} \varphi(x_2)\varphi(x_4) + \overbrace{\varphi(x_1)\varphi(x_4)} \varphi(x_2)\varphi(x_3) + \overbrace{\varphi(x_2)\varphi(x_3)} \varphi(x_1)\varphi(x_4) + \overbrace{\varphi(x_2)\varphi(x_4)} \varphi(x_1)\varphi(x_3) + \overbrace{\varphi(x_3)\varphi(x_4)} \varphi(x_1)\varphi(x_2) \}$



possible contractions

Illustration:  $T(\varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4)) = N \left( \varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4) + \varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4) + \varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4) + \varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4) + \varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4) + \varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4) + \varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4) + \varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4) \right)$

$\varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4)$   
 $\varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4)$   
 $\varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4)$   
 $\varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4)$   
 $\varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4)$   
 $\varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4)$   
 $\varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4)$   
 $\varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4)$



$$\begin{aligned}
 & + \varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4) + \varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4) + \varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4) = N \varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4) \\
 & + \varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4) + \varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4) + \varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4) + \varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4) \\
 & + \varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4) + \varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4) + \varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4) + \varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4)
 \end{aligned}$$



$$\begin{aligned}
 & \varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4) + \varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4) + \varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4) + \varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4) \\
 & + \varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4) + \varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4) + \varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4) + \varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4) \\
 & \varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4) + \varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4) + \varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4) + \varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4)
 \end{aligned}$$



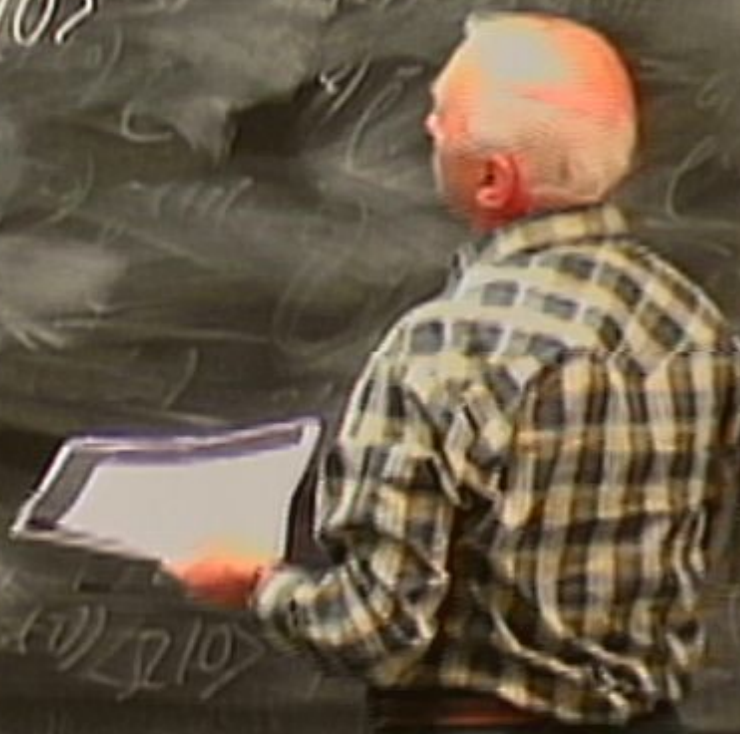
$$\langle 0 | T \{ \psi(x_1) \psi(x_2) \psi(x_3) \psi(x_4) \} | 0 \rangle$$





$$\begin{aligned}
 & + \varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4) + \varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4) = N \{ \varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4) \\
 & + \varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4) + \varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4) + \varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4) + \\
 & \varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4) + \varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4) + \varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4) + \\
 & \varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4) + \varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4) + \varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4) + \varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4)
 \end{aligned}$$

$$\langle 0 | T \{ \varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4) \} | 0 \rangle$$



$$\begin{aligned}
 & + \psi(x_1)\psi(x_2)\psi(x_3)\psi(x_4) + \psi(x_1)\psi(x_2)\psi(x_3)\psi(x_4) + \psi(x_1)\psi(x_2)\psi(x_3)\psi(x_4) + \\
 & \psi(x_1)\psi(x_2)\psi(x_3)\psi(x_4) + \psi(x_1)\psi(x_2)\psi(x_3)\psi(x_4) + \psi(x_1)\psi(x_2)\psi(x_3)\psi(x_4) + \psi(x_1)\psi(x_2)\psi(x_3)\psi(x_4)
 \end{aligned}$$

$$\langle 0 | T (\psi(x_1)\psi(x_2)\psi(x_3)\psi(x_4)) | 0 \rangle = D_F(x_1-x_2)D_F(x_3-x_4) + D_F(x_1-x_3) \cdot$$





$$\begin{aligned}
 & + \psi(x_1) \psi(x_2) \psi(x_3) \psi(x_4) + \psi(x_1) \psi(x_2) \psi(x_3) \psi(x_4) + \psi(x_1) \psi(x_2) \psi(x_3) \psi(x_4) + \psi(x_1) \psi(x_2) \psi(x_3) \psi(x_4) \\
 & \psi(x_1) \psi(x_2) \psi(x_3) \psi(x_4) + \psi(x_1) \psi(x_2) \psi(x_3) \psi(x_4) + \psi(x_1) \psi(x_2) \psi(x_3) \psi(x_4) + \psi(x_1) \psi(x_2) \psi(x_3) \psi(x_4)
 \end{aligned}$$

$$\langle 0 | T (\psi(x_1) \psi(x_2) \psi(x_3) \psi(x_4)) | 0 \rangle = D_F(x_1, x_2) D_F(x_3, x_4) + D_F(x_1, x_3) \cdot D_F(x_2, x_4) + D_F(x_1, x_4) D_F(x_2, x_3)$$



$$\begin{aligned}
 & + \psi(x_1) \psi(x_2) \psi(x_3) \psi(x_4) + \psi(x_1) \psi(x_2) \psi(x_3) \psi(x_4) + \psi(x_1) \psi(x_2) \psi(x_3) \psi(x_4) + \psi(x_1) \psi(x_2) \psi(x_3) \psi(x_4) \\
 & + \psi(x_1) \psi(x_2) \psi(x_3) \psi(x_4) + \psi(x_1) \psi(x_2) \psi(x_3) \psi(x_4) + \psi(x_1) \psi(x_2) \psi(x_3) \psi(x_4) + \psi(x_1) \psi(x_2) \psi(x_3) \psi(x_4) \\
 & + \psi(x_1) \psi(x_2) \psi(x_3) \psi(x_4) + \psi(x_1) \psi(x_2) \psi(x_3) \psi(x_4) + \psi(x_1) \psi(x_2) \psi(x_3) \psi(x_4) + \psi(x_1) \psi(x_2) \psi(x_3) \psi(x_4)
 \end{aligned}$$

$$\langle 0 | T \{ \psi(x_1) \psi(x_2) \psi(x_3) \psi(x_4) \} | 0 \rangle = D_F(x_1-x_2) D_F(x_3-x_4) + D_F(x_1-x_3) \cdot D_F(x_2-x_4) + D_F(x_1-x_4) D_F(x_2-x_3)$$

Normal order is that when all  $a_p$  terms are to the right of  $a_p^\dagger$  terms

$$N(\psi^\dagger(x) \bar{\psi}(y) \psi^\dagger(u) \bar{\psi}(v)) = \bar{\psi}(y) \bar{\psi}(v) \psi^\dagger(u) \psi^\dagger(x)$$

$$\langle 0 | N(\text{any operator}) | 0 \rangle = 0.$$

$$a_p | 0 \rangle = 0, \text{ so } a_p^\dagger | 0 \rangle = 0$$

$$N \rightarrow : : \text{ ; } N(\psi(x) \bar{\psi}(y)) \equiv : \psi(x) \bar{\psi}(y) :$$

QUESTION: what is an effective method to calculate  $\langle 0 | T \psi_1(x) \dots \bar{\psi}_1(x) | 0 \rangle$ ?

$$N \rightarrow \dots, N(\psi(x)\psi(y)) \equiv \psi(x)\psi(y)$$

contraction of two fields:

$$\overline{\psi_I(x)\psi_I(y)} \equiv \begin{cases} [\psi_I^+(x), \bar{\psi}_I(y)], & x^0 > y^0 \\ [\psi_I^+(y), \bar{\psi}_I(x)], & y^0 > x^0 \end{cases} = \underline{D_F(x-y)}$$

Normal order:

$$N(\psi_I(x)\psi_I(y)) = \psi_I^+(x)\psi_I^+(y) + \bar{\psi}_I^-(y)\psi_I^+(x) + \psi_I^-(y)\bar{\psi}_I^-(x)$$

$$\langle 0 | N(\psi_I(x)\psi_I(y)) | 0 \rangle = 0$$

$$N \rightarrow \dots, N(\varphi(x)\varphi(y)) \equiv \varphi(x)\varphi(y)$$

contraction of two fields:

$$\overline{\varphi_I(x)\varphi_I(y)} \equiv \begin{cases} [\varphi_I^+(x), \varphi_I^-(y)], & x^0 > y^0 \\ [\varphi_I^+(y), \varphi_I^-(x)], & y^0 > x^0 \end{cases} = \underline{D_F(x-y)}$$

Normal order:  $N(\varphi_I(x)\varphi_I(y)) = \varphi_I^+(x)\varphi_I^+(y) + \varphi_I^-(y)\varphi_I^+(x) + \varphi_I^-(x)\varphi_I^+(y) + \varphi_I^-(x)\varphi_I^-(y)$

$$\langle 0 | N(\varphi_I(x)\varphi_I(y)) | 0 \rangle = 0$$

VII.1.3 THEOREM  
 $T(\varphi(x_1)\varphi(x_2)\dots\varphi(x_m)) \equiv N\{\varphi(x_1)\varphi(x_2)\dots\varphi(x_m)\} + \text{all possible contractions}$

Illustration:  $T(\varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4)) = N\{\varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4)\} + \varphi(x_1)\varphi(x_2)\varphi(x_3) + \varphi(x_1)\varphi(x_2)\varphi(x_4) + \varphi(x_1)\varphi(x_3)\varphi(x_4) + \varphi(x_2)\varphi(x_3)\varphi(x_4) + \varphi(x_1)\varphi(x_2)\varphi(x_3) + \varphi(x_1)\varphi(x_2)\varphi(x_4) + \varphi(x_1)\varphi(x_3)\varphi(x_4) + \varphi(x_2)\varphi(x_3)\varphi(x_4)$

Generalization for arbitrary Green's function:

$$\langle \Omega | T(\psi(x_1)\psi(x_2)\dots\psi(x_n)) | \Omega \rangle = \lim_{T \rightarrow \infty(1-i\epsilon)} \frac{\langle 0 | T\{\psi_I(x_1)\psi_I(x_2)\dots\psi_I(x_n) \exp[-i \int_{-T}^T dt H_I(t)] \} | 0 \rangle}{\langle 0 | T\{\exp[-i \int_{-T}^T dt H_I(t)] \} | 0 \rangle}$$

Wick's Theorem

Question: What is an effective method to calculate  $\langle 0 | T\{\psi_I(x_1)\dots\psi_I(x_n)\} | 0 \rangle$ ?

$$\langle 0|T(\psi(x_1)\psi(x_2)\psi(x_3)\psi(x_4))|0\rangle = \frac{D_F(x_1-x_2)D_F(x_3-x_4) + D_F(x_1-x_3) \cdot D_F(x_2-x_4) + D_F(x_1-x_4)D_F(x_2-x_3)}{2}$$

### Feynman Diagrams

$$\langle 0|T(\psi_1\psi_2\psi_3\psi_4)|0\rangle$$





$$D_F(x_2-x_4) + D_F(x_1-x_4)D_F(x_2-x_3) \quad D_F(x_1-x_2)D_F(x_3-x_4) + D_F(x_1-x_3)$$

### Feynman Diagrams

$$\langle 0|T(\psi_1\psi_2\psi_3\psi_4)|0\rangle$$



# Feynman Diagrams

$$\langle 0 | T(\psi_1 \psi_2 \psi_3 \psi_4) | 0 \rangle : D_F(x-y) \Rightarrow \begin{array}{c} \text{---} \\ \text{---} \end{array}$$

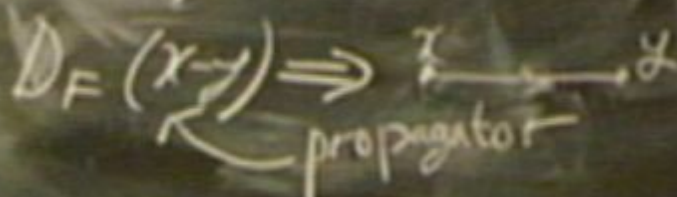
$$\begin{aligned} & \psi(x_1) \psi(x_2) \psi(x_3) \psi(x_4) + \psi(x_1) \psi(x_2) \psi(x_3) \psi(x_4) + \psi(x_1) \psi(x_2) \psi(x_3) \psi(x_4) \\ & \psi(x_1) \psi(x_2) \psi(x_3) \psi(x_4) + \psi(x_1) \psi(x_2) \psi(x_3) \psi(x_4) + \psi(x_1) \psi(x_2) \psi(x_3) \psi(x_4) \end{aligned}$$

$$\langle 0 | T(\psi_1 \psi_2 \psi_3 \psi_4) | 0 \rangle = \dots$$

$D_F(x-y) \Rightarrow \begin{array}{c} x \longrightarrow y \\ \text{propagator} \end{array}$

$$\begin{aligned}
 & + \psi(x_1) \psi(x_2) \psi(x_3) \psi(x_4) + \dots \\
 & \psi(x_1) \psi(x_2) \psi(x_3) \psi(x_4) + \dots \\
 & \psi(x_1) \psi(x_2) \psi(x_3) \psi(x_4) + \dots
 \end{aligned}$$

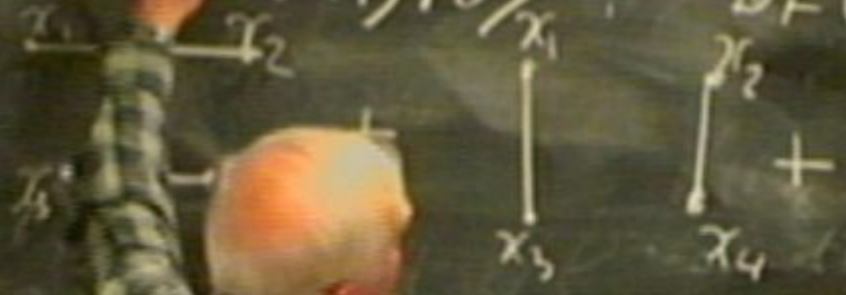
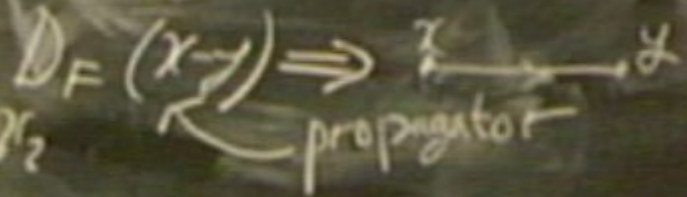
$$\langle 0 | T(\psi_1 \psi_2 \psi_3 \psi_4) | 0 \rangle =$$



$$\begin{aligned}
 & \psi(x_1) \psi(x_2) \psi(x_3) \psi(x_4) + \psi(x_1) \psi(x_2) \psi(x_4) \psi(x_3) + \psi(x_1) \psi(x_3) \psi(x_2) \psi(x_4) + \psi(x_1) \psi(x_3) \psi(x_4) \psi(x_2) + \\
 & \psi(x_1) \psi(x_4) \psi(x_2) \psi(x_3) + \psi(x_1) \psi(x_4) \psi(x_3) \psi(x_2) + \psi(x_2) \psi(x_1) \psi(x_3) \psi(x_4) + \psi(x_2) \psi(x_1) \psi(x_4) \psi(x_3) + \\
 & \psi(x_2) \psi(x_3) \psi(x_1) \psi(x_4) + \psi(x_2) \psi(x_3) \psi(x_4) \psi(x_1) + \psi(x_3) \psi(x_1) \psi(x_2) \psi(x_4) + \psi(x_3) \psi(x_1) \psi(x_4) \psi(x_2) + \\
 & \psi(x_3) \psi(x_2) \psi(x_1) \psi(x_4) + \psi(x_3) \psi(x_2) \psi(x_4) \psi(x_1) + \psi(x_4) \psi(x_1) \psi(x_2) \psi(x_3) + \psi(x_4) \psi(x_1) \psi(x_3) \psi(x_2) + \\
 & \psi(x_4) \psi(x_2) \psi(x_1) \psi(x_3) + \psi(x_4) \psi(x_2) \psi(x_3) \psi(x_1)
 \end{aligned}$$

# Feynman Diagrams

$$\langle 0 | T(\psi(x_1) \psi(x_2) \psi(x_3) \psi(x_4)) | 0 \rangle =$$



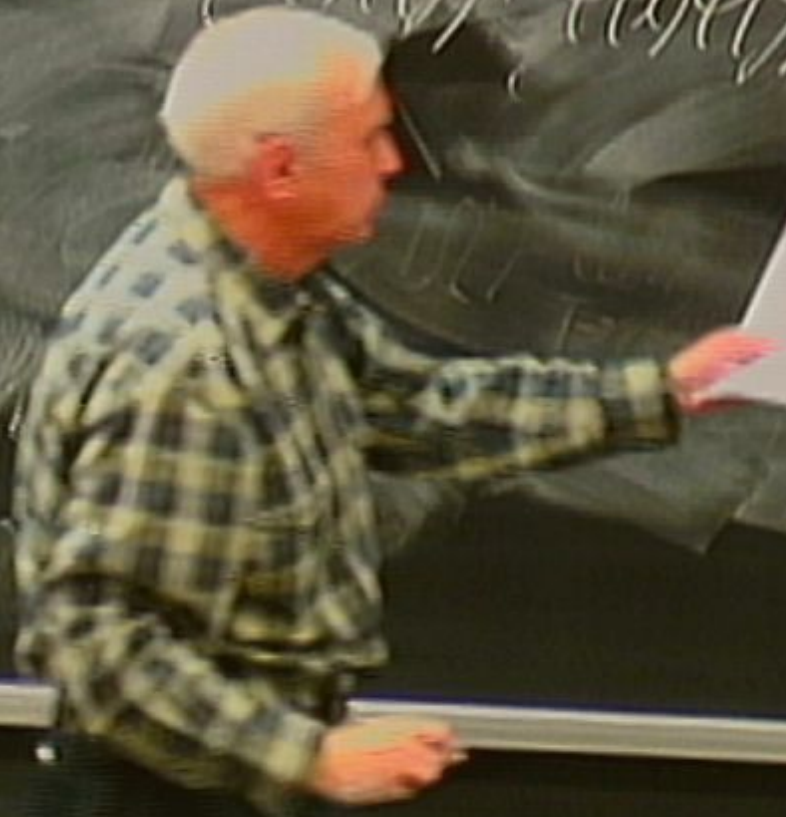
$$= \int d^4x_1 d^4x_2 d^4x_3 d^4x_4 \left[ \psi(x_1) \psi(x_2) \psi(x_3) \psi(x_4) + \psi(x_1) \psi(x_3) \psi(x_2) \psi(x_4) + \psi(x_1) \psi(x_4) \psi(x_2) \psi(x_3) + \psi(x_1) \psi(x_2) \psi(x_4) \psi(x_3) + \psi(x_3) \psi(x_1) \psi(x_2) \psi(x_4) + \psi(x_3) \psi(x_4) \psi(x_1) \psi(x_2) \right]$$

$$\langle 0|T(\psi_1 \psi_2 \psi_3 \psi_4)|0\rangle = D_F(x_1 \dots x_4) \Rightarrow \text{propagator}$$

$$\begin{aligned} & \psi(x_1)\psi(x_2)\psi(x_3)\psi(x_4) + \psi(x_1)\psi(x_2)\psi(x_3)\psi(x_4) + \psi(x_1)\psi(x_2)\psi(x_3)\psi(x_4) + \psi(x_1)\psi(x_2)\psi(x_3)\psi(x_4) + \\ & \psi(x_1)\psi(x_2)\psi(x_3)\psi(x_4) + \psi(x_1)\psi(x_2)\psi(x_3)\psi(x_4) + \psi(x_1)\psi(x_2)\psi(x_3)\psi(x_4) + \psi(x_1)\psi(x_2)\psi(x_3)\psi(x_4) + \\ & \psi(x_1)\psi(x_2)\psi(x_3)\psi(x_4) + \psi(x_1)\psi(x_2)\psi(x_3)\psi(x_4) + \psi(x_1)\psi(x_2)\psi(x_3)\psi(x_4) + \psi(x_1)\psi(x_2)\psi(x_3)\psi(x_4) + \end{aligned}$$

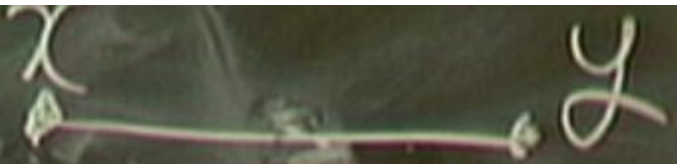
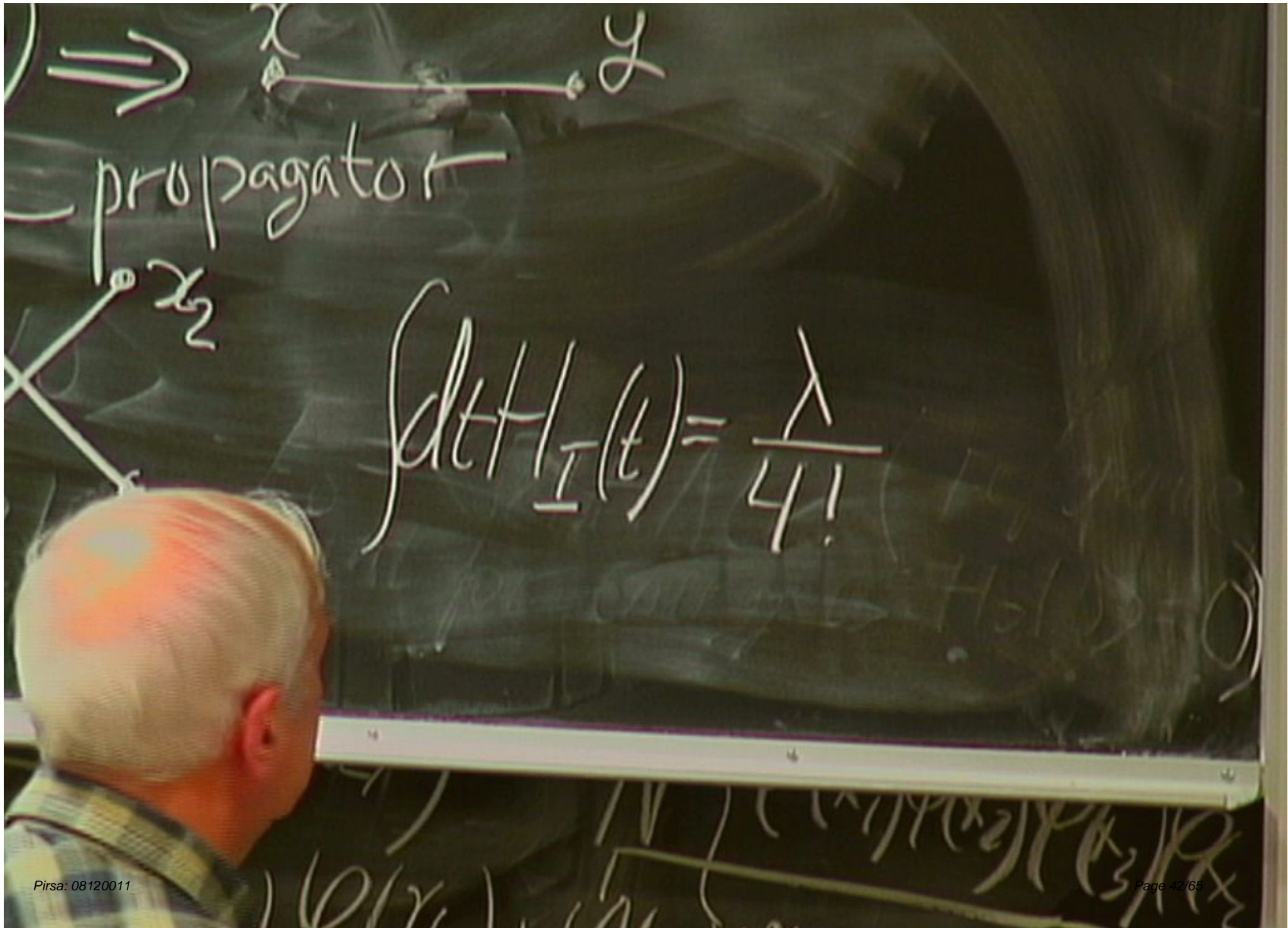
Two point Green's function:  $\langle 0|T\{\dots\}$

Two point Green's function:  $\langle 0|T\{\psi(x)\psi(y) + \psi(x)\psi(y)\} \times$   
 $\times [-i \int dt H_i(t)]$

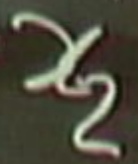




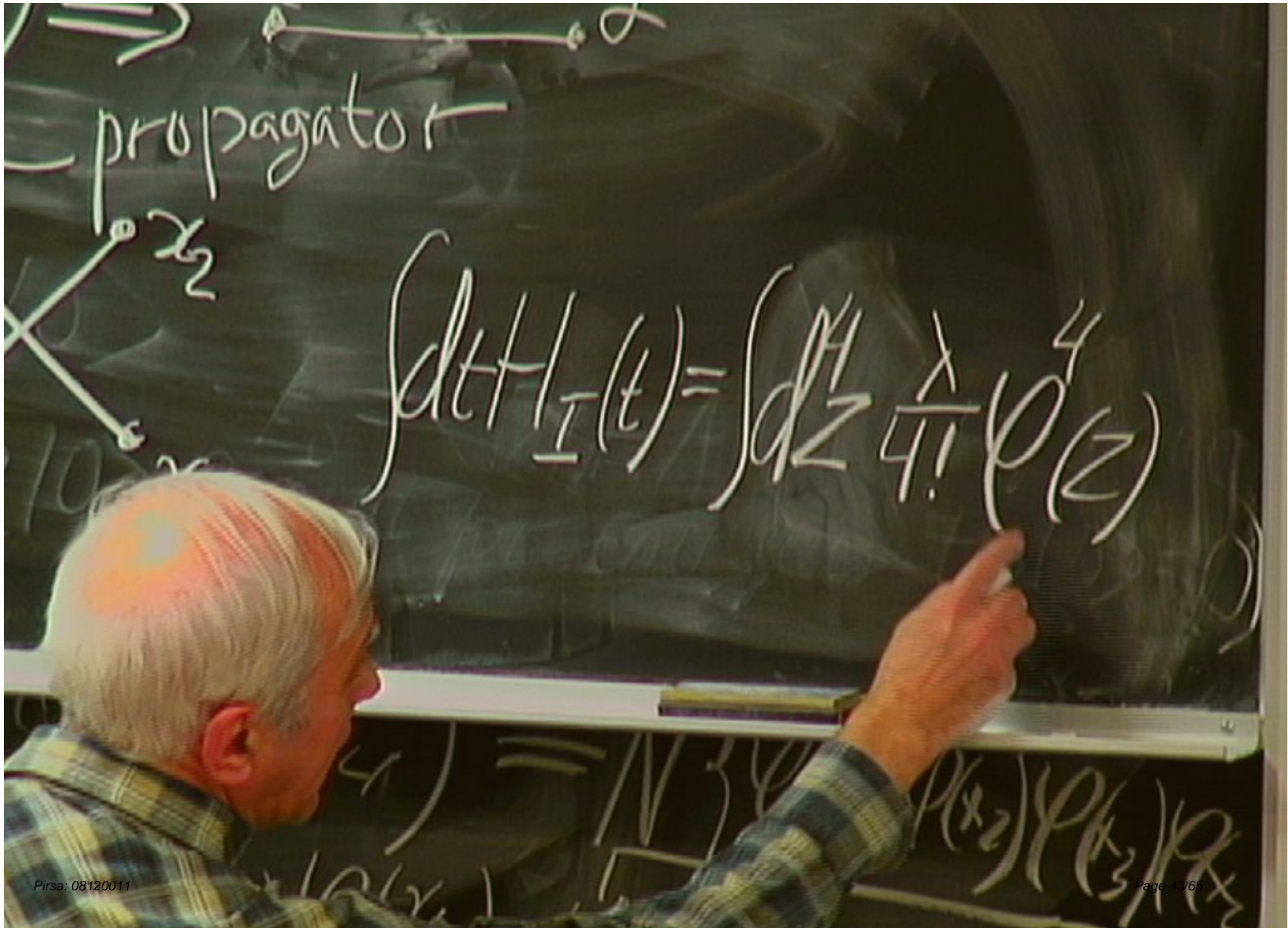
Two point Green's function:  $\langle 0|T\{\underbrace{\varphi(x)\varphi(y)} + \varphi(x)\varphi(y) \times$   
 $\times [-i\int dt H_I(t)] + \dots\} = D_F(x-y) + \langle 0|\varphi(x)\varphi(y)$



propagator



$$\int dt H_I(t) = \frac{\Delta}{4!}$$



propagator

$$\int dt \text{Tr} U_T(t) = \int dZ \frac{\lambda}{4!} \rho^4(Z)$$

# Two point Green's function

$$\times \left[ -i \int dt H_I(t) \right] + \dots \Bigg\} = \dots$$

$$\int d^4z \phi^4(z) |0\rangle$$

CAUTION

BE CAREFUL IN USING THE SCISSORS TO CUT THE TAPE FROM THE BACK OF THE BOARD. DO NOT CUT THE TAPE FROM THE FRONT OF THE BOARD.

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Two point Green's function:  $\langle 0 | T \{ \underbrace{\varphi(x)\varphi(y)} + \varphi(x)\varphi(y) \} \times$   
 $[-i \int dt H_I(t)] + \dots \rangle = D_F(x-y) + \langle 0 | T \{ \varphi(x)\varphi(y) \} \left( \frac{-i\lambda}{4!} \right)$   
 $\langle d^4 z \varphi^4(z) \rangle | 0 \rangle$

$$\frac{\psi(x_1)\psi(x_2)\psi(x_3)\psi(x_4)}{\psi(x_2)\psi(x_3)\psi(x_4)} + \dots$$

Two point Green's function

$$\times \left[ -i \int dt H_I(t) \right] + \dots$$

$$\cdot \left( \int d^4z \psi^4(z) \right) / \langle 0 | 0 \rangle$$

$$C_0^2$$

CAUTION  
 DO NOT TOUCH THE BOARD SURFACE  
 OR THE BOARD SURFACE WILL BE DAMAGED

Two point Green's function:

$$\times \left[ -i \int dt H_I(t) + \dots \right] = D_F(\dots)$$

$$\int d^4z \varphi^4(z) |10\rangle$$

$$C_6^2 = \frac{6 \cdot 5}{2} = 15 \text{ combinations of con}$$

$$\int d^4z \varphi^4(z) |0\rangle = D_F(x-y) + \langle 0 | \varphi(x) \varphi(y) | 0 \rangle$$

$C_6^2 = \frac{6 \cdot 5}{2} = 15$  combinations of contractions. Two types  
 1)  $\varphi(x) \varphi(y)$



$$\langle 0 | \int d^4z \varphi^4(z) | 0 \rangle = D_F(x-y) + \langle 0 | \int d^4z \varphi^4(z) | 0 \rangle$$

$C_6^2 = \frac{6 \cdot 5}{2} = 15$  combinations of contractions.

Two types

1)  $\varphi(x)\varphi(y) : \left(\frac{-i\lambda}{4!}\right) D_F(x-y) \langle 0 | \int d^4z \varphi^4(z) | 0 \rangle$

$$\varphi(x_4) | 0 \rangle = \frac{D_F(x_1 - x_2) D_F(x_3 - x_4) + D_F(x_1 - x_3) D_F(x_2 - x_3)}{D_F(x_2 - x_3)}$$

man Diagrams



$$\langle 0 | T \{ \psi(x_1) \psi(x_2) \psi(x_3) \psi(x_4) \} | 0 \rangle =$$

$$\underline{D_F(x_2 - x_4)} + \underline{D_F(x_1 - x_4)} D_F(x_2 - x_3)$$

Feynman Diag

$$\langle 0 | T (\psi_1 \psi_2 \psi_3 \psi_4) | 0 \rangle =$$

The diagrams illustrate the Wick contractions for the four-point function. The first diagram shows a contraction between  $\psi_1$  and  $\psi_2$  (vertical line) and another between  $\psi_3$  and  $\psi_4$  (vertical line). The second diagram shows a contraction between  $\psi_1$  and  $\psi_4$  (vertical line) and another between  $\psi_2$  and  $\psi_3$  (vertical line). Both diagrams are separated by a plus sign.

Combinations of contractions.

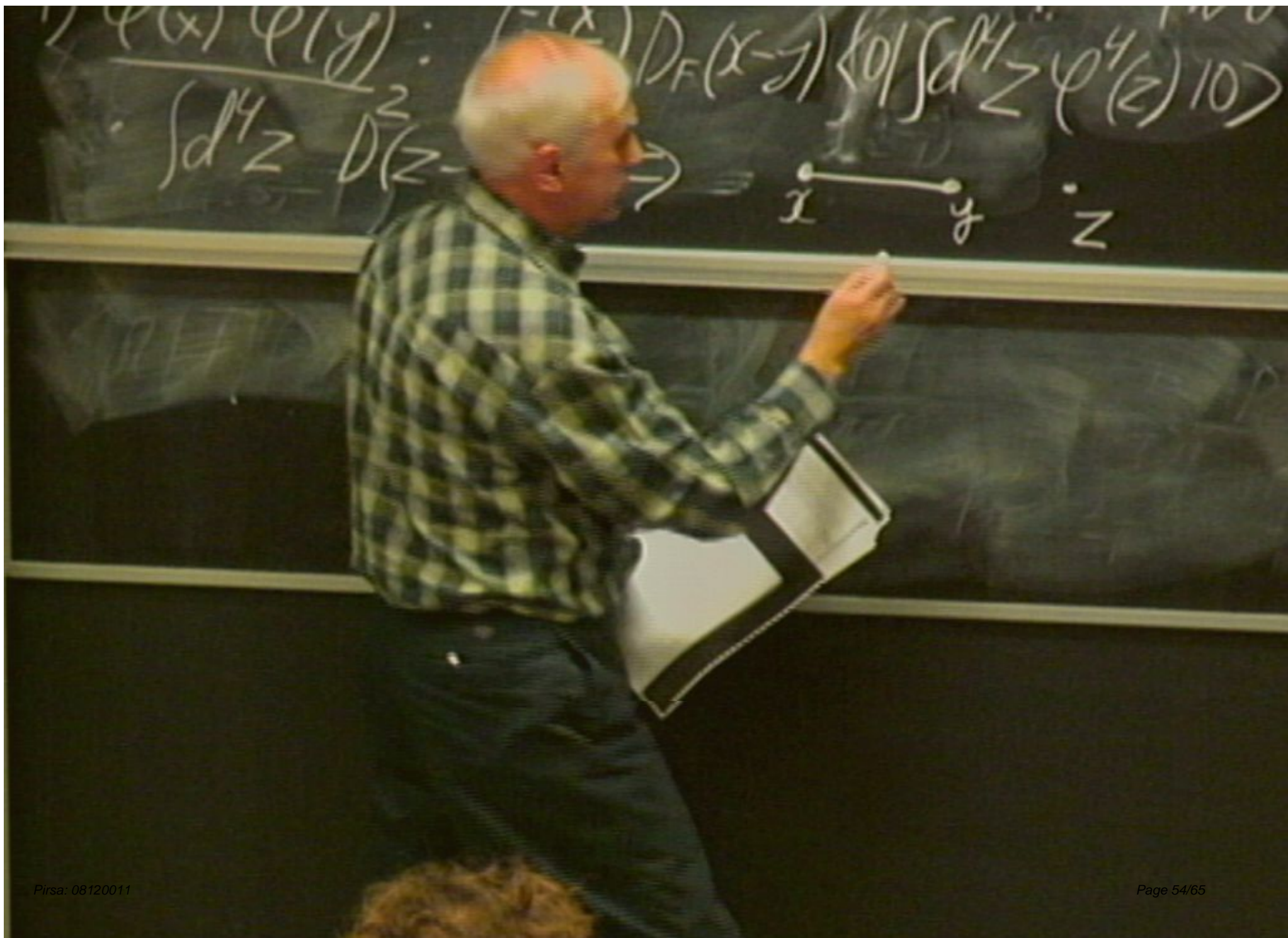
Two types:

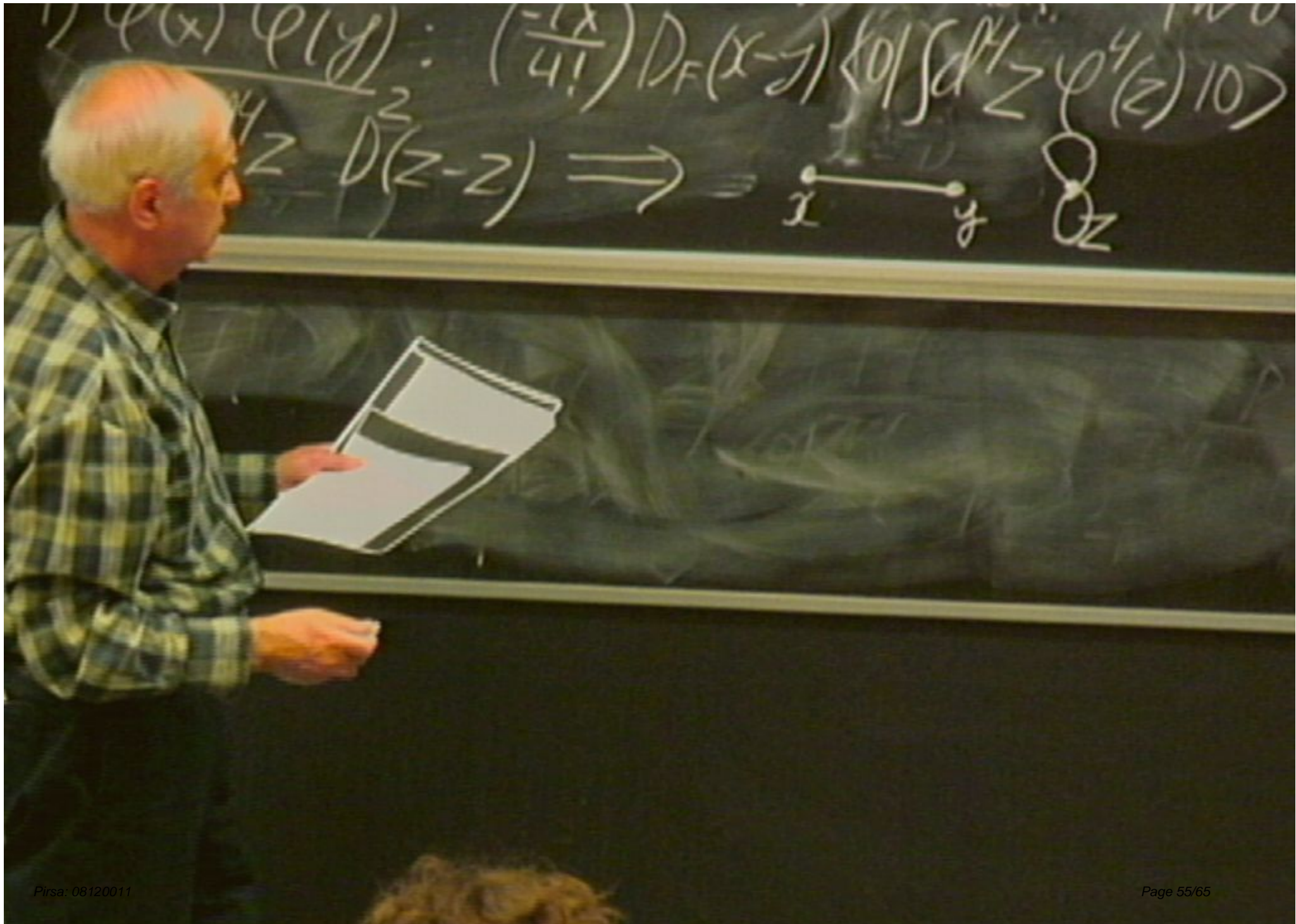
$$\frac{\lambda}{4!} D_F(x-y) \langle 0 | \int d^4z \varphi^4(z) | 0 \rangle = 3 \left( \frac{\lambda}{4!} \right) D_F(x-y).$$

$$C_6^2 = \frac{6 \cdot 5}{2} = 15 \text{ combinations of contractions}$$

$$1) \frac{\varphi(x) \varphi(y)}{\int d^4z D(z-x) D(z-y)} : \left( \frac{-i\lambda}{4!} \right) D_F(x-y) \langle 0 | \int d^4z$$

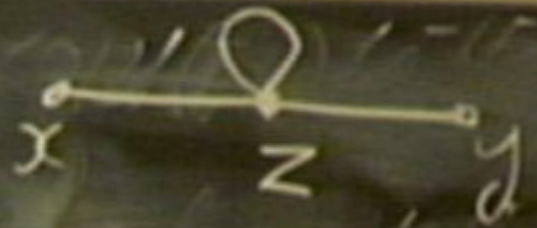
CAUTION





$$\frac{\varphi(x)\varphi(y)}{z^4} : \left(\frac{-1}{4!}\right) D_F(x-z) \langle 0 | \int d^4z \varphi^4(z) | 0 \rangle$$
$$D(z-z) \Rightarrow \begin{matrix} x & \text{---} & y \\ & & \text{---} & \text{---} & z \end{matrix}$$

2)  $\varphi(x)$  and  $\varphi(y)$  are contracted  
with  $\varphi(z)$  operators






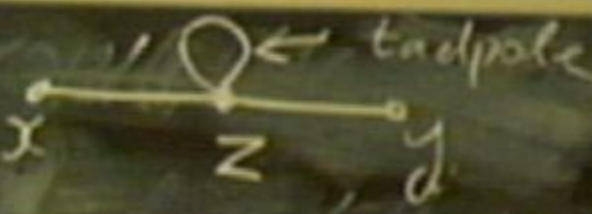
$$\left[ \dots \right] = D_F(x-y) + \langle 0 | \overline{\psi}(x) \psi(y) \left( \frac{-i\lambda}{4!} \right)$$

$\int d^4z \psi^4(z) |0\rangle$   
 $\binom{6}{2} = \frac{6 \cdot 5}{2} = 15$  combinations of contractions. Two types:

1)  $\psi(x) \psi(y)$ :  $\left( \frac{-i\lambda}{4!} \right) D_F(x-y) \langle 0 | \int d^4z \psi^4(z) |0\rangle = 3 \left( \frac{-i\lambda}{4!} \right) D_F(x-y)$



2)  $d\psi(y)$  are contracted (2) operators

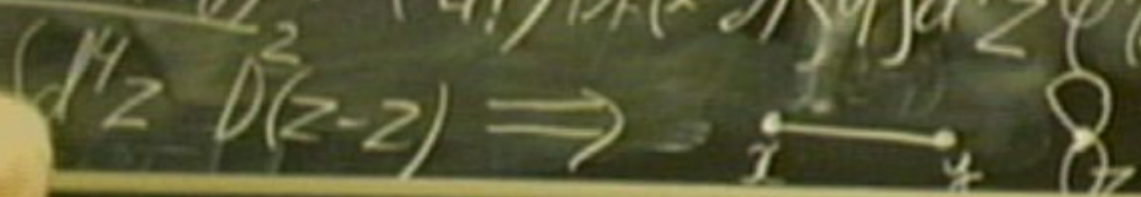


$$\left[ \frac{(-1)^j}{j!} \frac{d^j H_1(\varphi)}{dz^j} \right] + \dots = D_F(x-y) + \langle 0 | \overline{\varphi(x) \varphi(y)} \left( \frac{-i\lambda}{4!} \right)$$

$$\langle d^4 z \varphi^4(z) | 0 \rangle$$

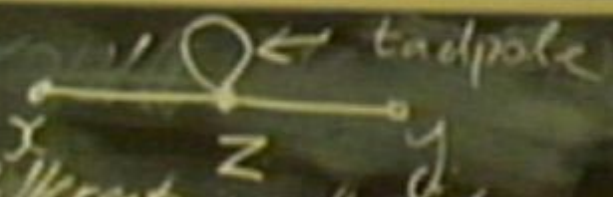
$C_6^2 = \frac{6 \cdot 5}{2} = 15$  combinations of contractions. Two types:

1)  $\overbrace{\varphi(x) \varphi(y)}_2$ :  $\left( \frac{-i\lambda}{4!} \right) D_F(x-y) \langle 0 | \int d^4 z \varphi^4(z) | 0 \rangle = 3 \left( \frac{-i\lambda}{4!} \right) D_F(x-y)$



$\varphi(x)$  and  $\varphi(y)$  are contracted

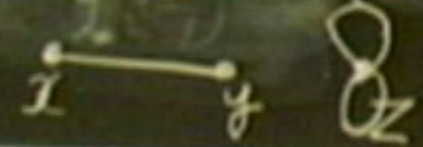
with  $\varphi(z)$  operators. There 12 different contractions for  $\varphi^4(z)$  with  $\varphi^4(z)$

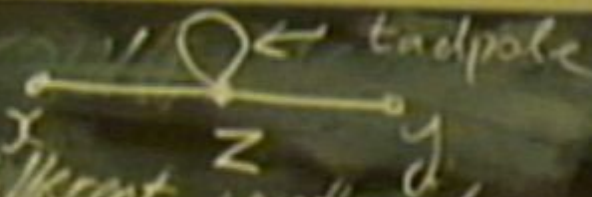


Two point Green's function  $\langle 0|T\{\underbrace{\varphi(x)\varphi(y)}_{\varphi(x)\varphi(y)} + \varphi(x)\varphi(y) \times$   
 $\times [-i \int dt H_I(t)] + \dots \rangle = D_F(x-y) + \langle 0|T\{\underbrace{\varphi(x)\varphi(y)}_{\varphi(x)\varphi(y)} \underbrace{(-i \int dt H_I(t))}_{\varphi\varphi\varphi\varphi(z)}\rangle$   
 $\langle d^4z \varphi^4(z) | 0 \rangle$

$C_6^2 = 6 \cdot 5 = 15$  combinations of contractions. Two types:

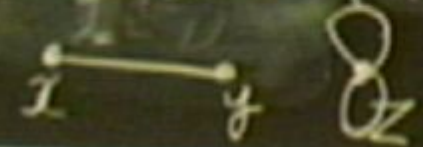
1)  $\langle \varphi(x)\varphi(y) \underbrace{(-i \int dt H_I(t))}_{\varphi\varphi\varphi\varphi(z)} \rangle = 3 \underbrace{(-i \int dt H_I(t))}_{\varphi\varphi\varphi\varphi(z)} D_F(x-y)$   
 $\langle d^4z \varphi^4(z) | 0 \rangle = 3 \underbrace{(-i \int dt H_I(t))}_{\varphi\varphi\varphi\varphi(z)} D_F(x-y)$



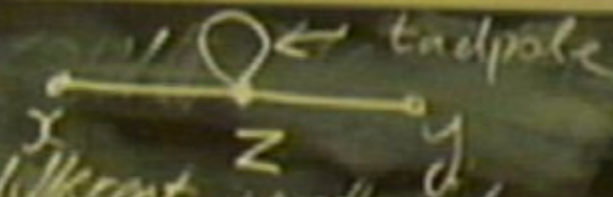
2)  $\varphi(x)\varphi(y)$  are contracted:  tadpole  
 $\varphi(x)\varphi(y)$  operators: There 12 different contractions for

Two point Green's function  $\langle 0|T\{\underbrace{\varphi(x)\varphi(y)} + \varphi(x)\varphi(y)\} \times$   
 $\times [-i\int dt H_I(t)] + \dots \rangle = D_F(x-y) + \langle 0|T\{\underbrace{\varphi(x)\varphi(y)} \underbrace{\varphi\varphi\varphi\varphi(z)}\} \left(\frac{-i\lambda}{4!}\right)$   
 $\langle d^4z \varphi^4(z) | 0 \rangle$

$C_6^2 = \frac{6 \cdot 5}{2} = 15$  combinations of contractions. Two types:

$\varphi(x)\varphi(y) : \left(\frac{-i\lambda}{4!}\right) D_F(x-y) \langle 0| \int d^4z \varphi^4(z) | 0 \rangle = 3 \left(\frac{-i\lambda}{4!}\right) D_F(x-y)$   
 $D(z-z) \Rightarrow$  


$\varphi(x)$  and  $\varphi(y)$  are contracted with  $\varphi(z)$  operators. There 12 different contractions for  $\varphi(x)\varphi(y)$  with  $\varphi^4(z)$



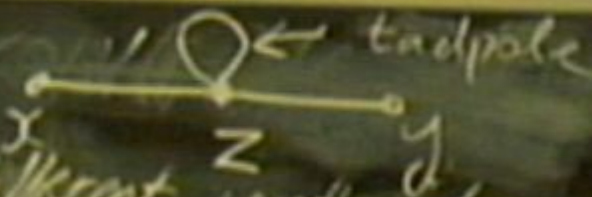
Two point Green's function  $\langle 0|T\{\underbrace{\varphi(x)\varphi(y)} + \varphi(x)\varphi(y) \times$   
 $\times [-i \int dt H_I(t)] + \dots \rangle = D_F(x-y) + \langle 0|T\{\underbrace{\varphi(x)\varphi(y)} \underbrace{\varphi\varphi\varphi\varphi(z)} \frac{(-i\lambda)}{4!} \rangle$   
 $\cdot \int d^4z \varphi^4(z) |0\rangle$

$C_6^2 = \frac{6 \cdot 5}{2} = 15$  combinations of contractions. Two types:

1)  $\frac{\varphi(x)\varphi(y)}{\int d^4z D^2(z-z)} : \left(\frac{-i\lambda}{4!}\right) D_F(x-y) \langle 0| \int d^4z \varphi^4(z) |0\rangle = 3 \left(\frac{-i\lambda}{4!}\right) D_F(x-y)$

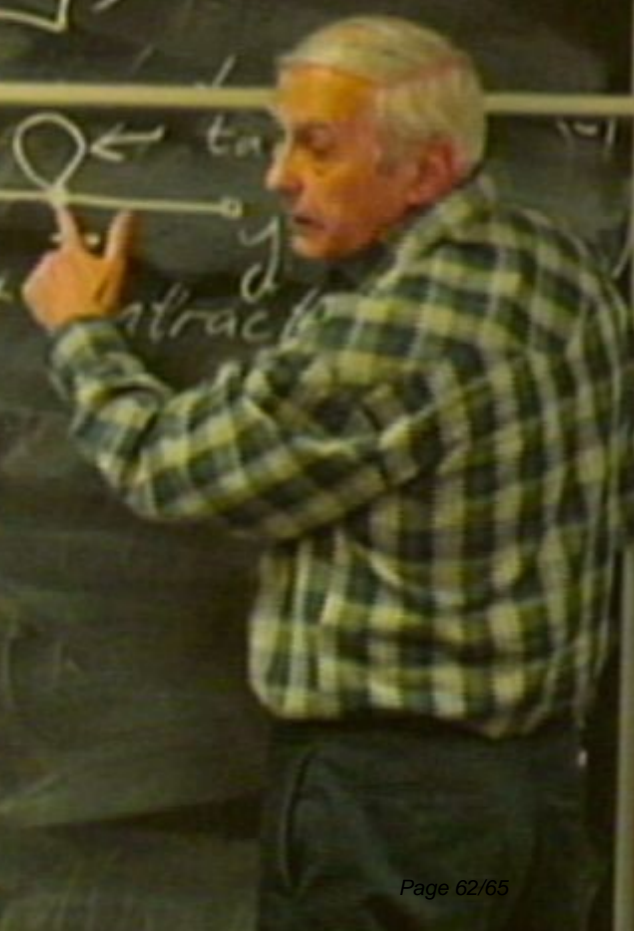
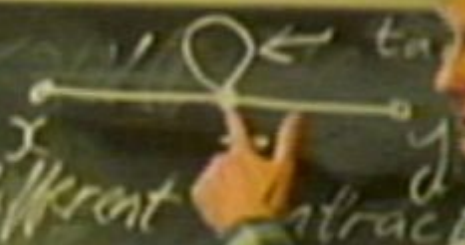


2)  $\varphi(x)$  and  $\varphi(y)$  are contracted with  $\varphi(z)$  operators. There are 12 different contractions for  $\varphi(x), \varphi(y)$  with  $\varphi^4(z)$



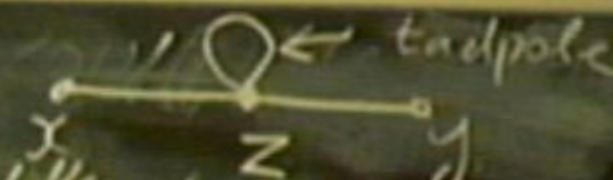
Two point Green's function  $\langle 0|T\{\underbrace{\varphi(x)\varphi(y)} + \varphi(x)\varphi(y) \times$   
 $\times [-i\int dt H_I(t)] + \dots \rangle = D_F(x-y) + \langle 0|T\{\underbrace{\varphi(x)\varphi(y)} \underbrace{\varphi\varphi\varphi\varphi(z)}\} \left(\frac{-i\lambda}{4!}\right)$   
 $\langle d^4z \varphi^4(z) \rangle |0\rangle$   
 $C_6^2 = \frac{6 \cdot 5}{2} = 15$  combinations of contraction

2)  $\varphi(x)$  and  $\varphi(y)$  are contracted with  $\varphi(z)$  operators. There 12 different  $\varphi(x), \varphi(y)$  with  $\varphi^4(z)$ :  $12 \cdot \left(\frac{-i\lambda}{4!}\right)$



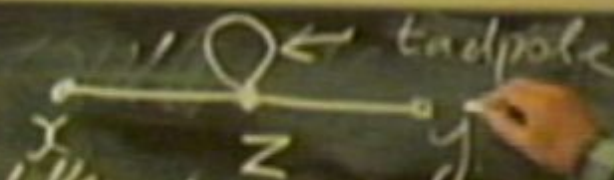
Two point Green's function  $\langle 0|T\{\underbrace{\varphi(x)\varphi(y)} + \varphi(x)\varphi(y) + \dots\} \times$   
 $\times [-i\int dt A_i(t)] + \dots \rangle = D_F(x-y) + \langle 0|T\{\underbrace{\varphi(x)\varphi(y)} \underbrace{\varphi\varphi\varphi\varphi(z)}\} \left(\frac{-i\lambda}{4!}\right)$   
 $\langle d^4z \varphi^4(z) \rangle |0\rangle$   
 $C_6^2 = \frac{6 \cdot 5}{2} = 15$  combinations of contractions

2)  $\varphi(x)$  and  $\varphi(y)$  are contracted with  $\varphi(z)$  operators: There 12 different contractions for  $\varphi(x), \varphi(y)$  with  $\varphi^4(z)$ :  $12 \cdot \left(\frac{-i\lambda}{4!}\right) \int d^4z D(x-z) D(y-z)$



Two point Green's function  $\langle 0|T\{\underbrace{\varphi(x)\varphi(y)} + \varphi(x)\varphi(y) + \dots\} \times$   
 $\times [-i\{dt A_I(t)\} + \dots] = D_F(x-y) + \langle 0|T\{\underbrace{\varphi(x)\varphi(y)} \underbrace{\varphi\varphi\varphi\varphi(z)}\} \left(\frac{-i\lambda}{4!}\right)$   
 $\int d^4z \varphi^4(z) |0\rangle$   
 $C_6^2 = \frac{6 \cdot 5}{2} = 15$  (combinations of contraction)

2)  $\varphi(x)$  and  $\varphi(y)$  are contracted with  $\varphi(z)$  operators. There 12 different contraction  $\varphi(x), \varphi(y)$  with  $\varphi^4(z)$ :  $12 \cdot \left(\frac{-i\lambda}{4!}\right) \int d^4z D(x-z) D(y-z) D(z-z)$

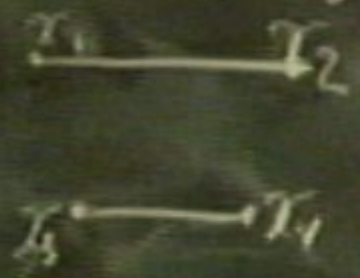




$$\langle 0 | T(\psi(x_1)\psi(x_2)\psi(x_3)\psi(x_4)) | 0 \rangle = D_F(x_1-x_2)D_F(x_3-x_4) + D_F(x_1-x_3)D_F(x_2-x_4) + D_F(x_1-x_4)D_F(x_2-x_3)$$

## Feynman Diagrams

$$\langle 0 | T(\psi_1\psi_2\psi_3\psi_4) | 0 \rangle =$$



+



+



$$\int d^4x \mathcal{H}_I(x) = \int d^4z \frac{1}{4!} \psi^4$$

