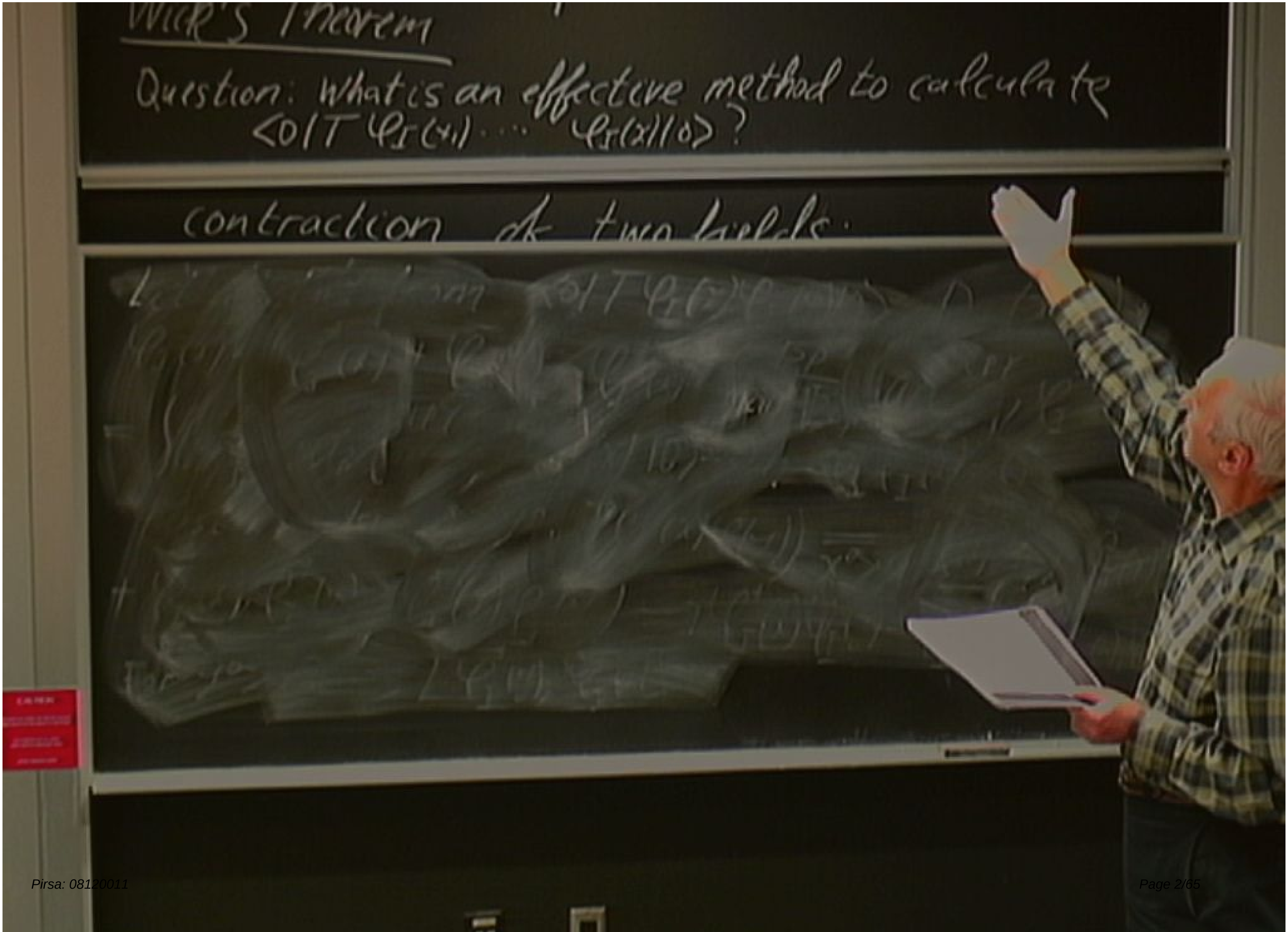
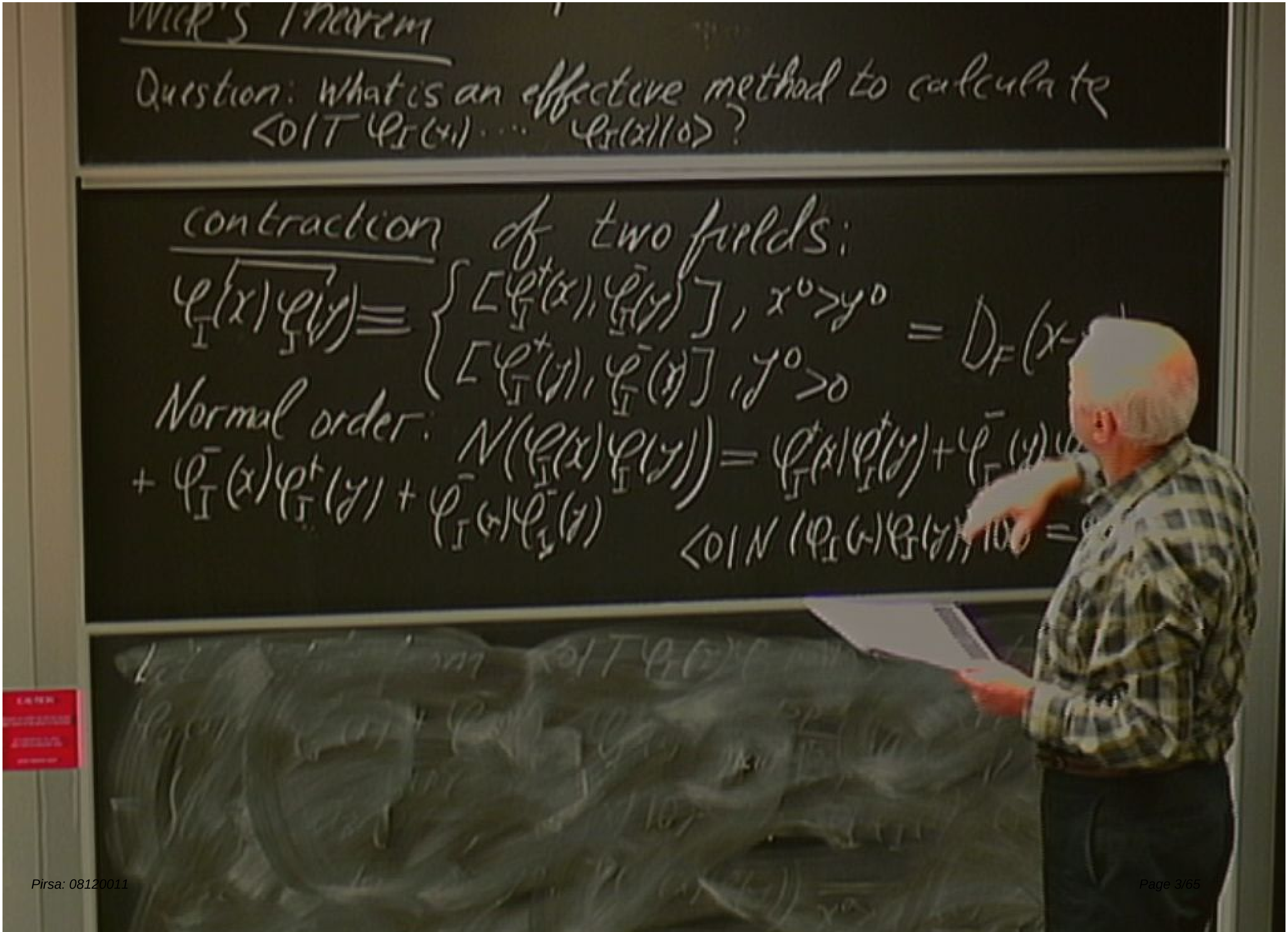
Title: Quantum Field Theory 1 - Lecture 13B

Date: Dec 03, 2008 03:30 PM

URL: http://pirsa.org/08120011

Abstract: Quantum Field Theory I course taught by Volodya Miransky of the University of Western Ontario


Wick's Theorem
Question: What is an effective method to calculate
<0|T φ_I(x_1) ··· φ_I(x_n)|0> ?
contraction of two fields:


Wick's Theorem
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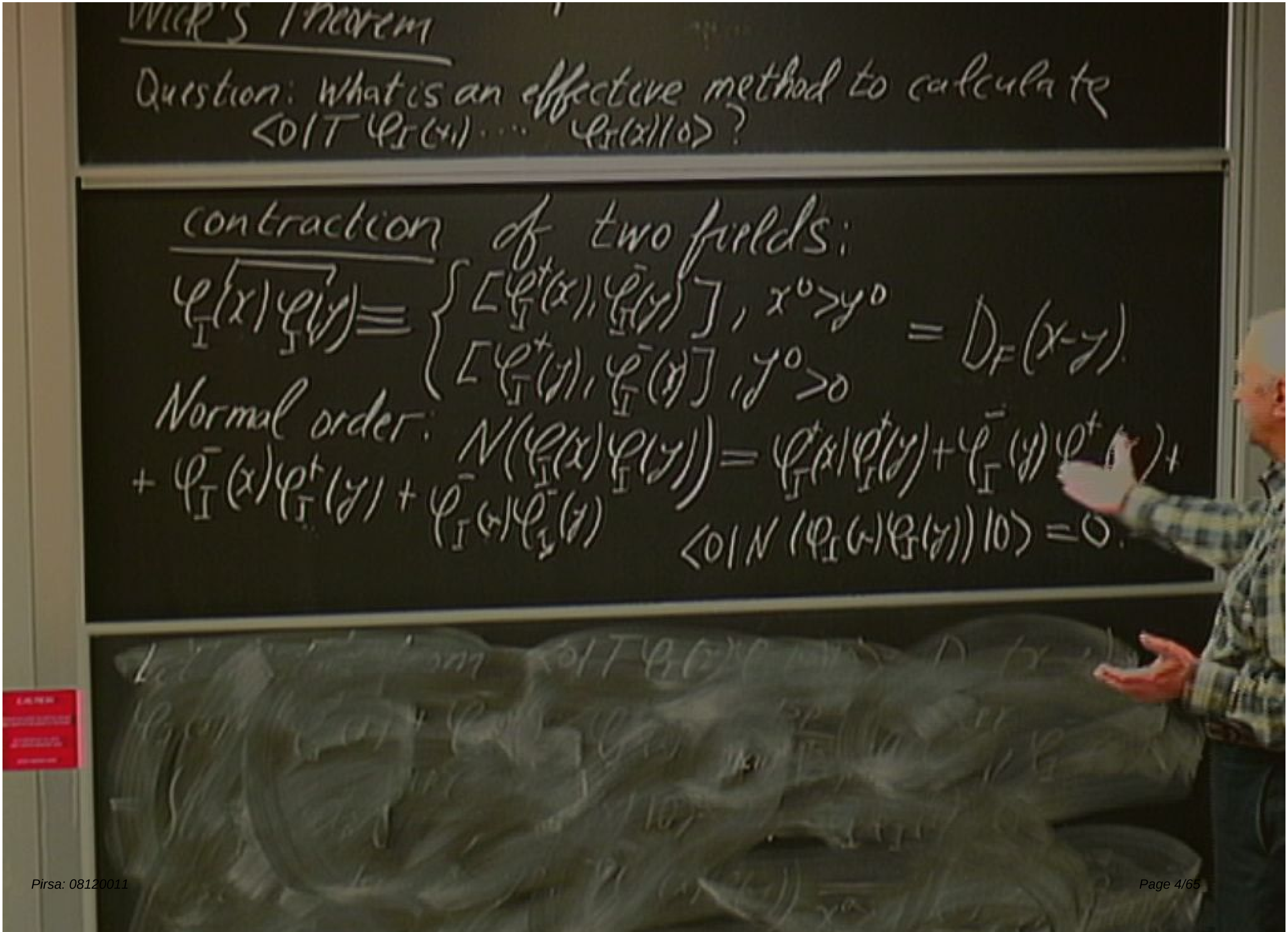
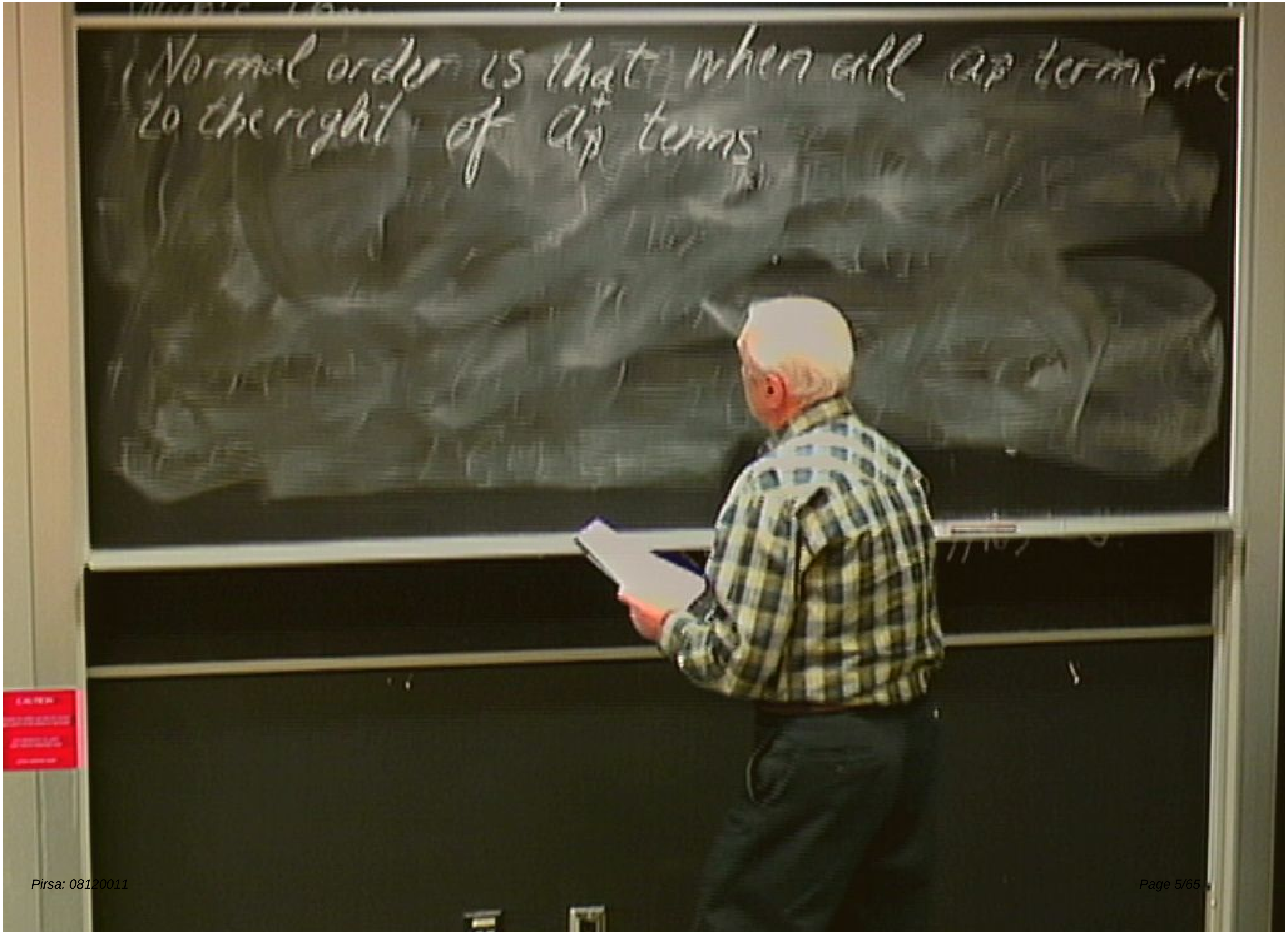
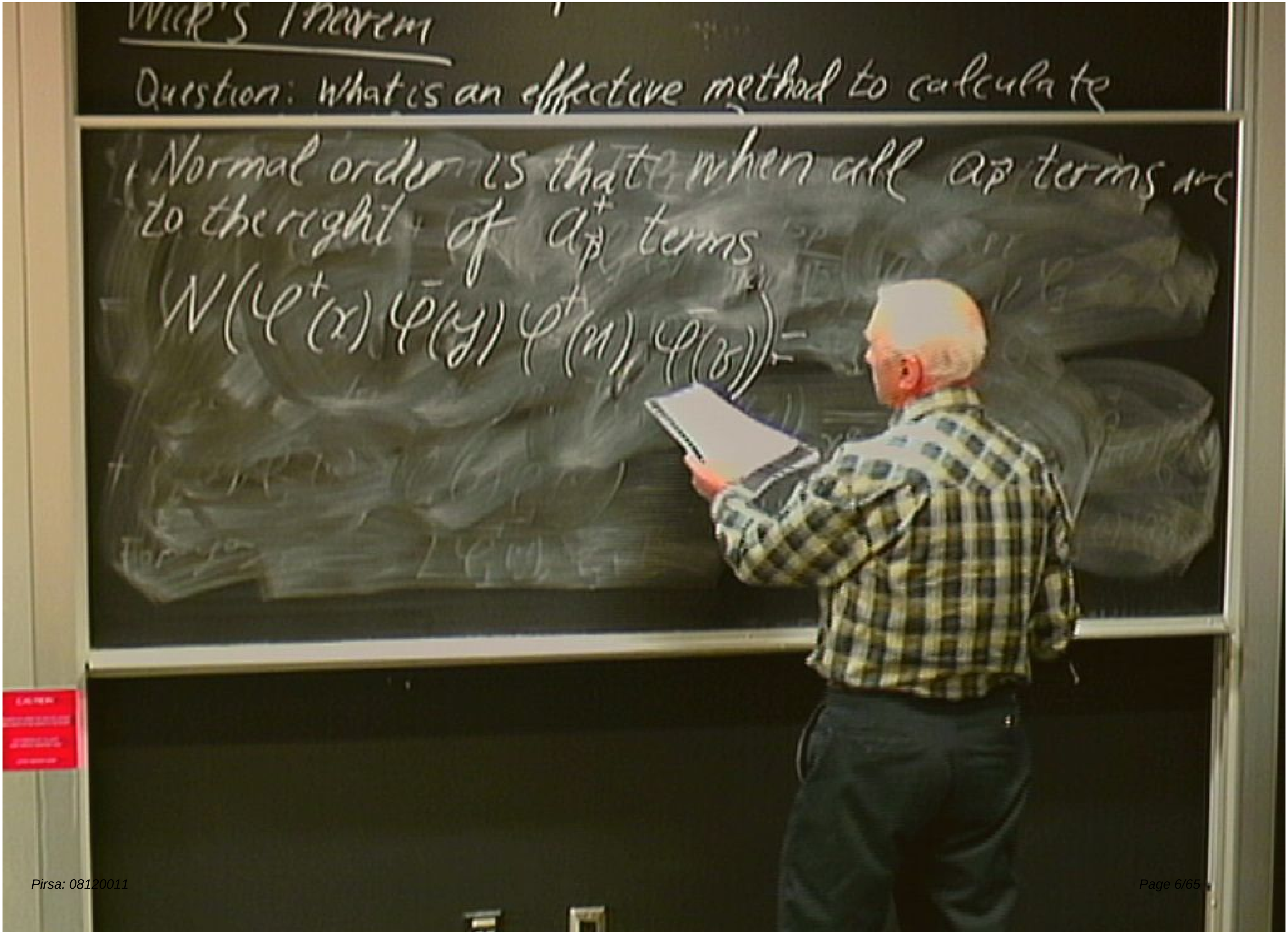
Normal order:


Wick's Theorem
Question: What is an effective method to calculate
<0|T φ_I(x_1) ⋯ φ_I(x_n)|0> ?
contraction of two fields:
φ_I(x) φ_I(y) ≡ { [φ_I^+(x), φ_I^-(y)], x^0 > y^0 ; [φ_I^+(y), φ_I^-(x)], y^0 > x^0 } = D_F(x-y).
Normal order: N(φ_I(x) φ_I(y)) = φ_I^+(x) φ_I^+(y) + φ_I^-(y) φ_I^+(x) +
+ φ_I^-(x) φ_I^+(y) + φ_I^-(x) φ_I^-(y)
<0| N (φ_I(x) φ_I(y)) |0> = 0.


Normal order is that when all a_p terms are
to the right of $a_p^\dagger$ terms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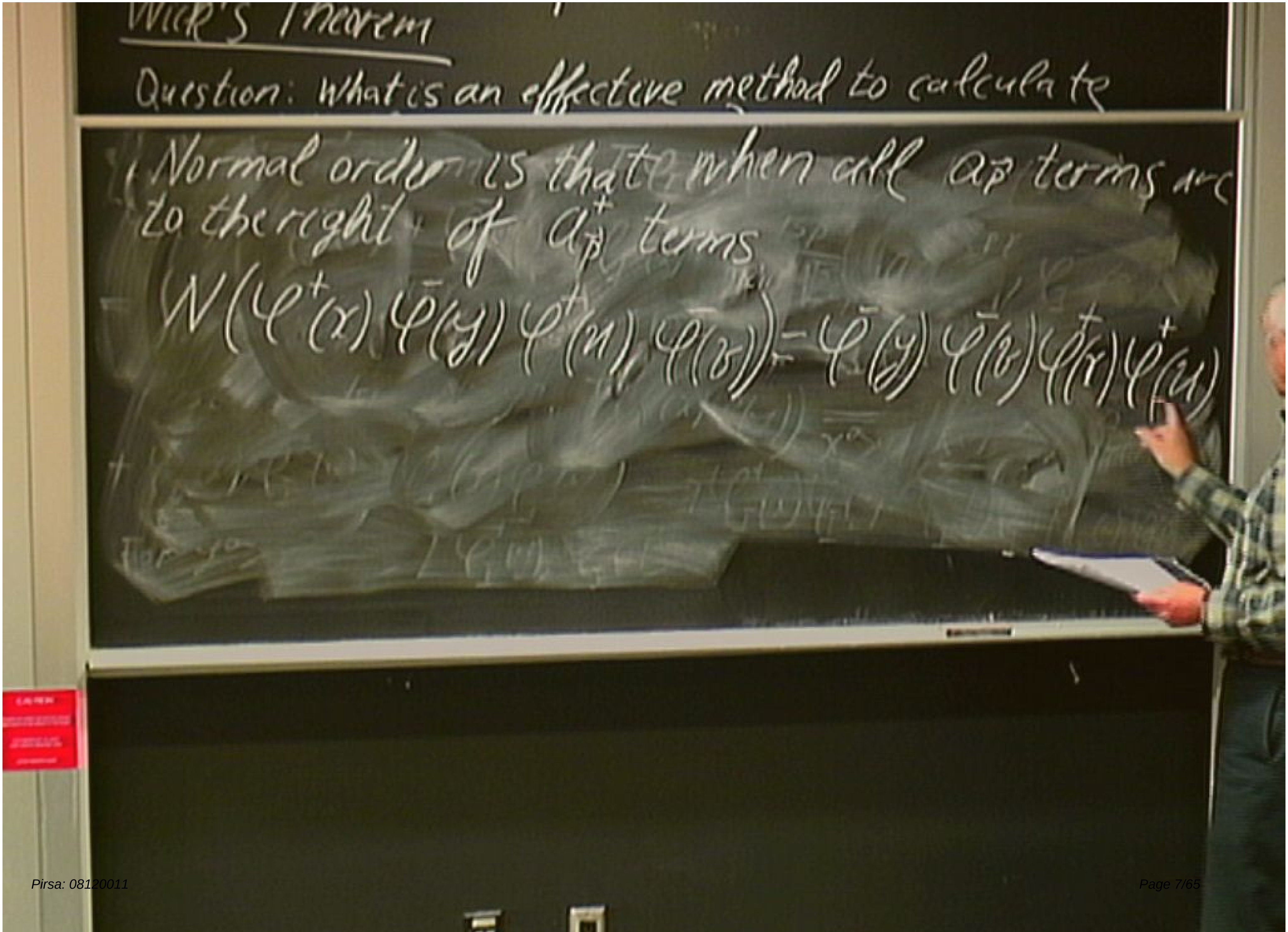
N(φ+(x) φ-(y) φ+(u) φ-(v))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Question: what is an effective method to calculate
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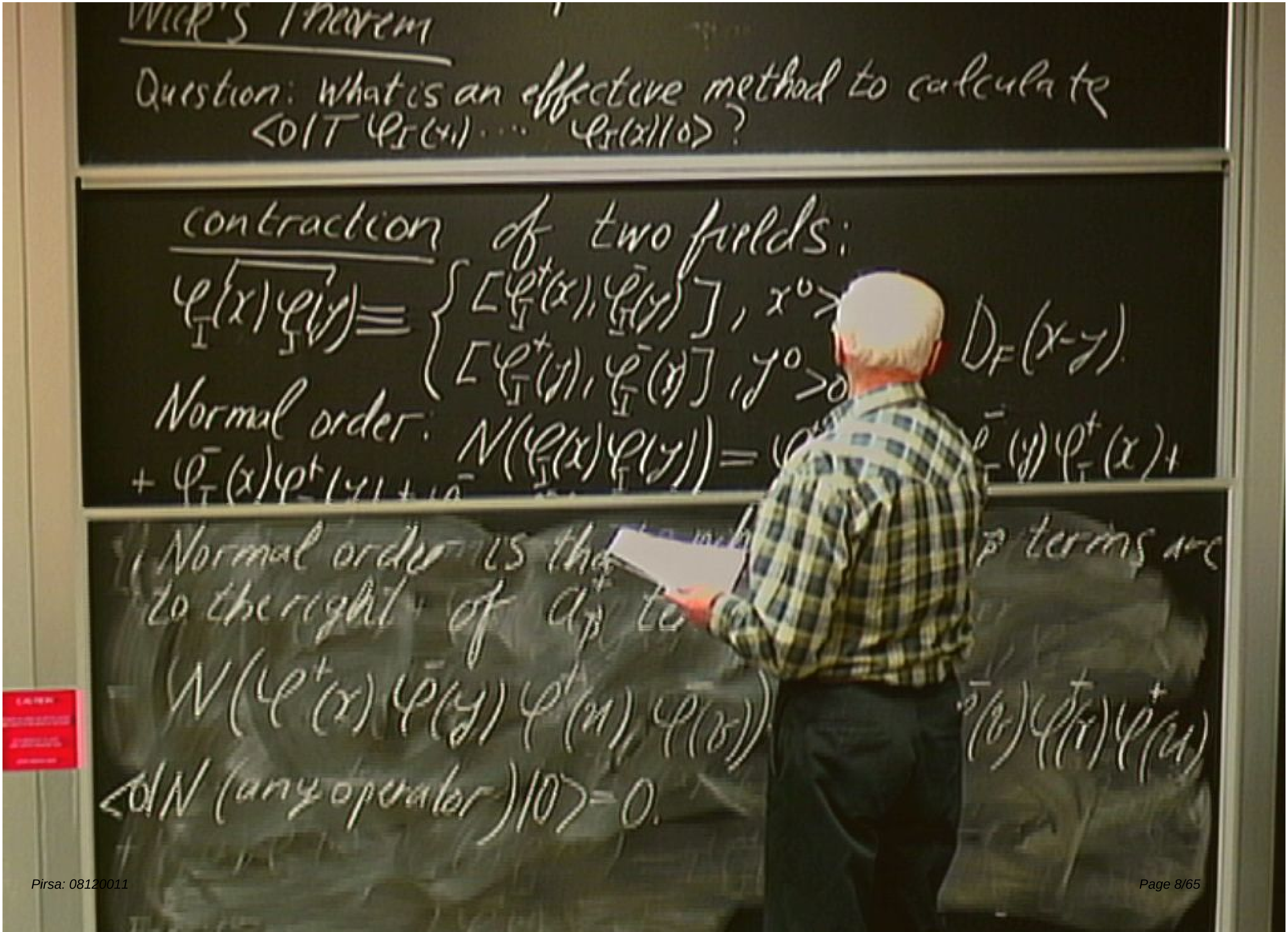
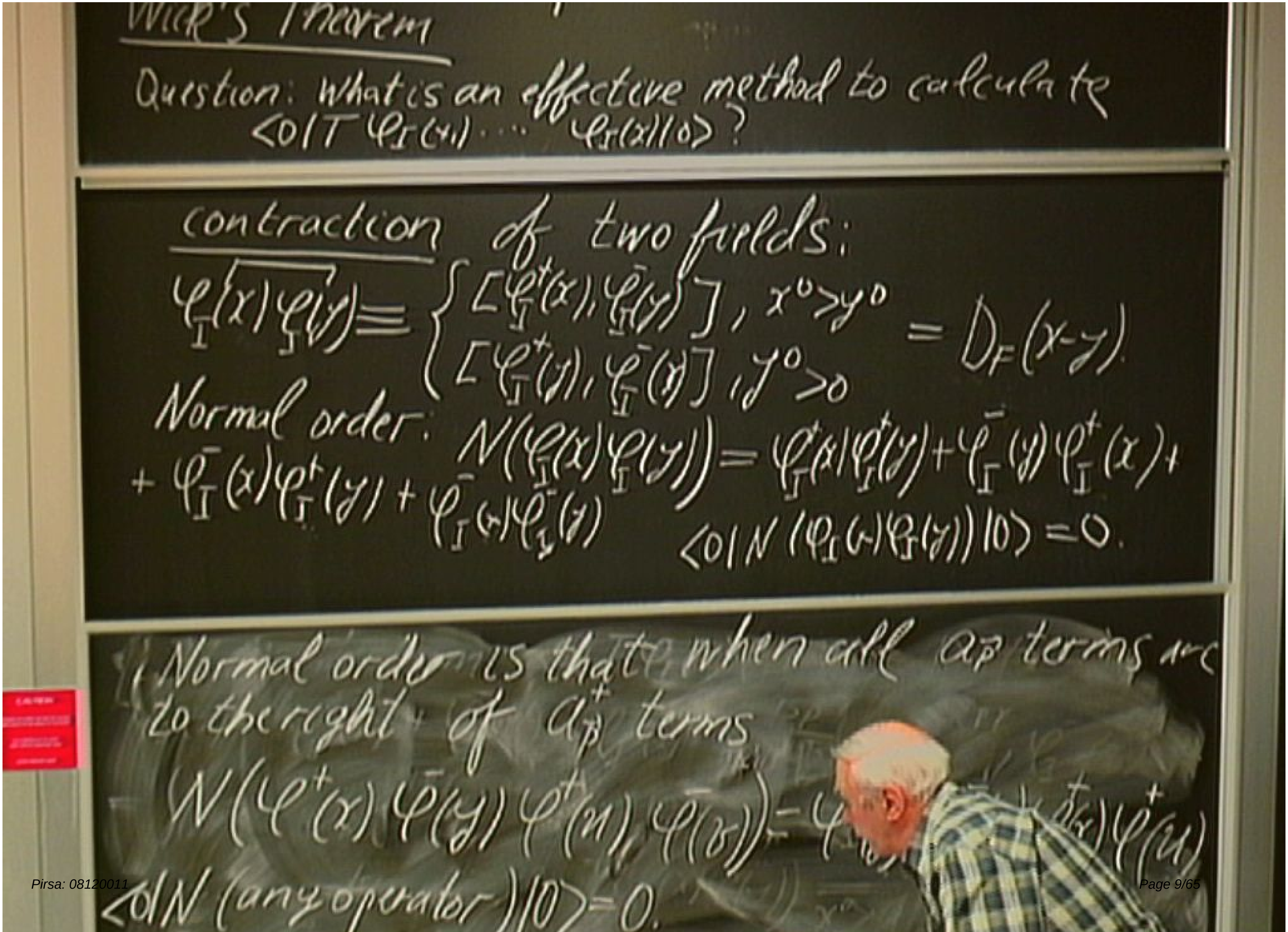
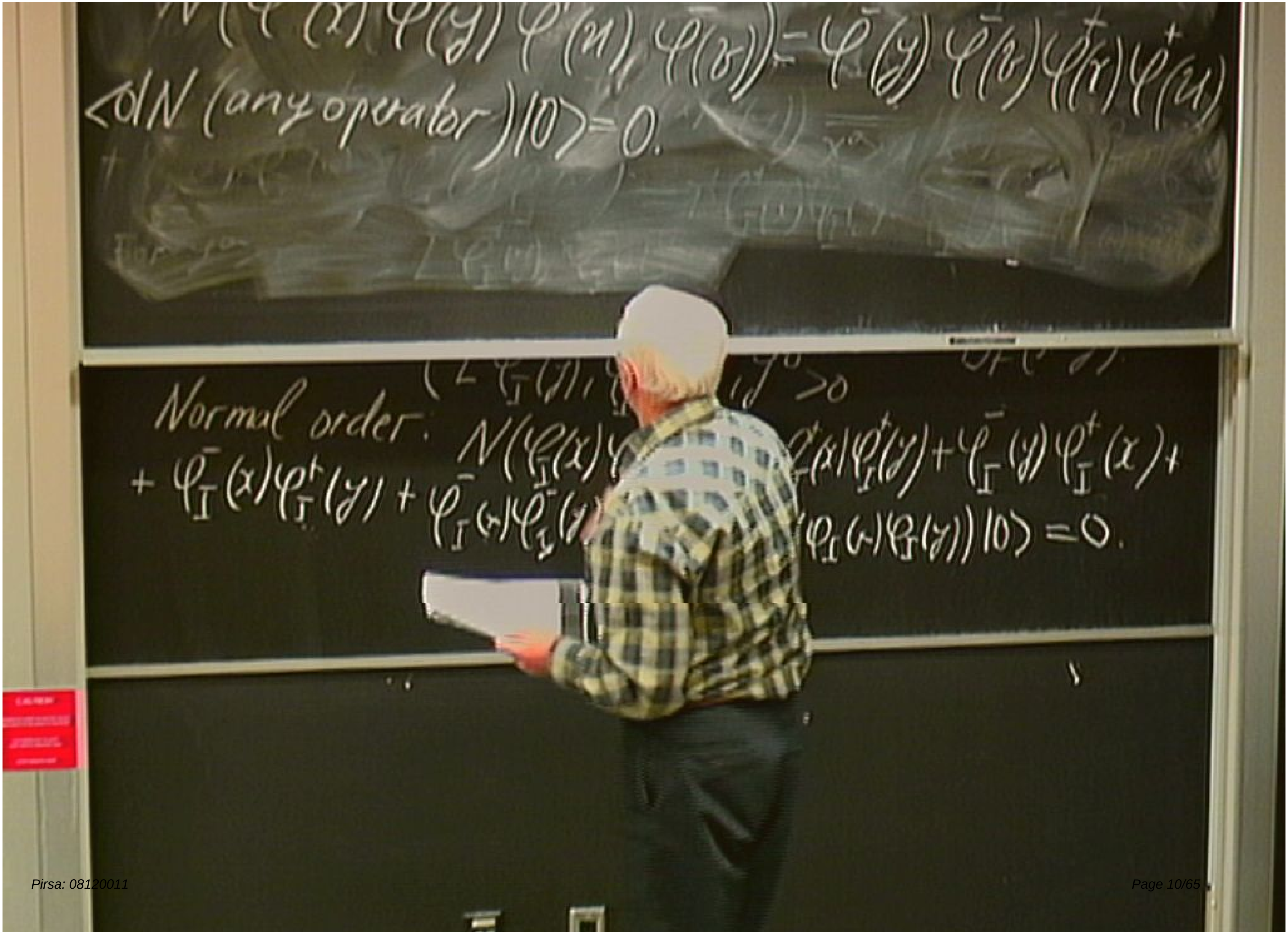
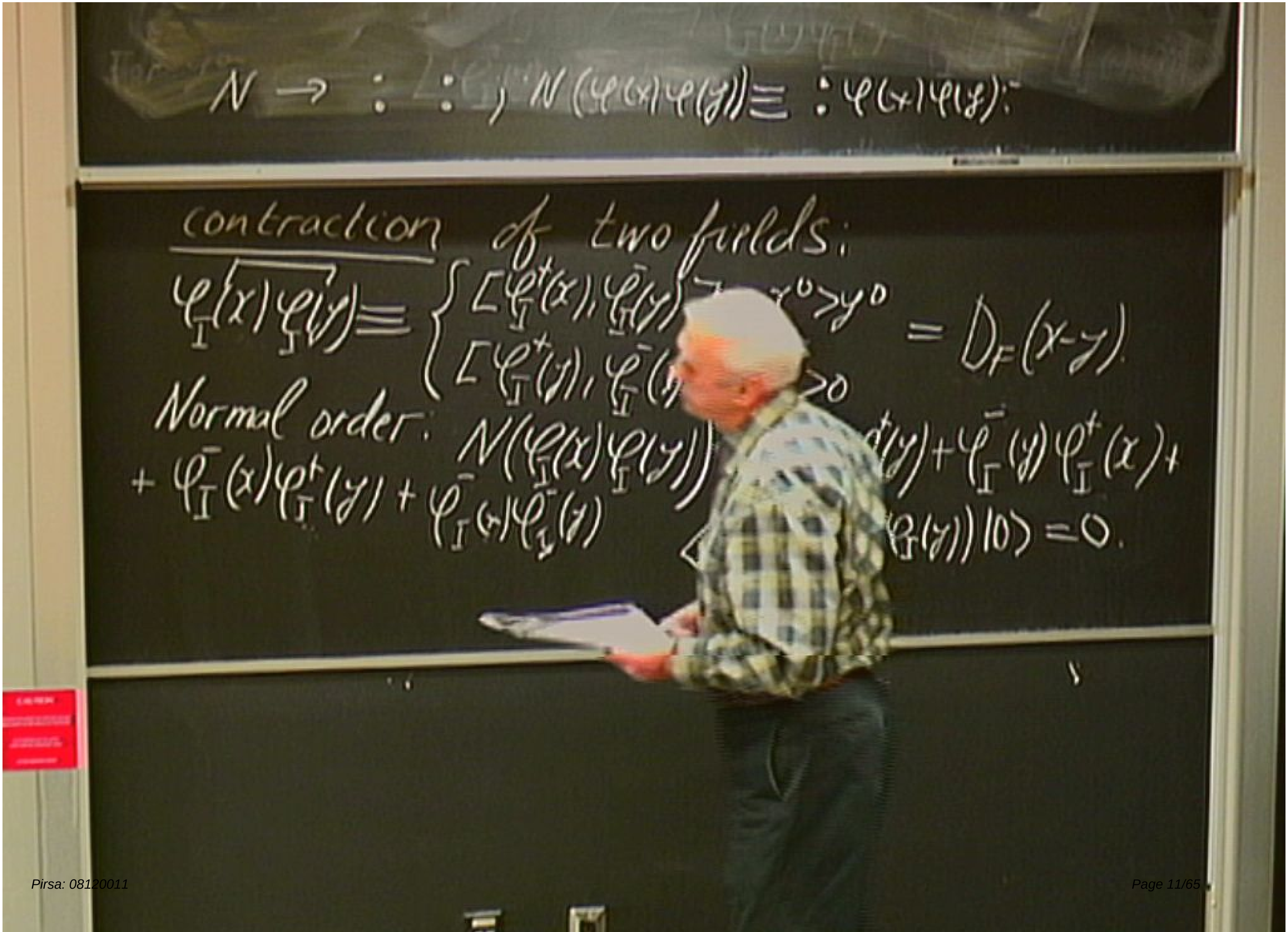

Wick's Theorem
Question: What is an effective method to calculate
contraction of two fields:
Normal order:
Normal order is the
to the right of
<0|N (any operator)|0> = 0.


Wick's Theorem
Question: what is an effective method to calculate
$\langle 0 | T \varphi_I(x_1) \cdots \varphi_I(x_n) | 0 \rangle$?
contraction of two fields:
$\varphi_I(x)\varphi_I(y) \equiv \begin{cases} [\varphi_I^+(x), \varphi_I^-(y)], & x^0 > y^0 \\ [\varphi_I^+(y), \varphi_I^-(x)], & y^0 > x^0 \end{cases} = D_F(x-y).$
Normal order: $N(\varphi_I(x)\varphi_I(y)) = \varphi_I^+(x)\varphi_I^+(y) + \varphi_I^-(y)\varphi_I^+(x) + \varphi_I^-(x)\varphi_I^+(y) + \varphi_I^-(x)\varphi_I^-(y)$
$\langle 0 | N(\varphi_I(x)\varphi_I(y)) | 0 \rangle = 0.$
Normal order is that when all $a_{\vec p}$ terms are to the right of $a_{\vec p}^+$ terms
$N(\varphi^+(x)\varphi^-(y)\varphi^+(u)\varphi^-(v)) = \varphi^- \cdots \varphi^+(x)\varphi^+(u)$
$\langle 0 | N(\text{any operator}) | 0 \rangle = 0.$



<0|N (any operator)|0> = 0.
Normal order:


N → : : , N(φ(x)φ(y)) ≡ :φ(x)φ(y):
contraction of two fields:
= D_F(x-y).
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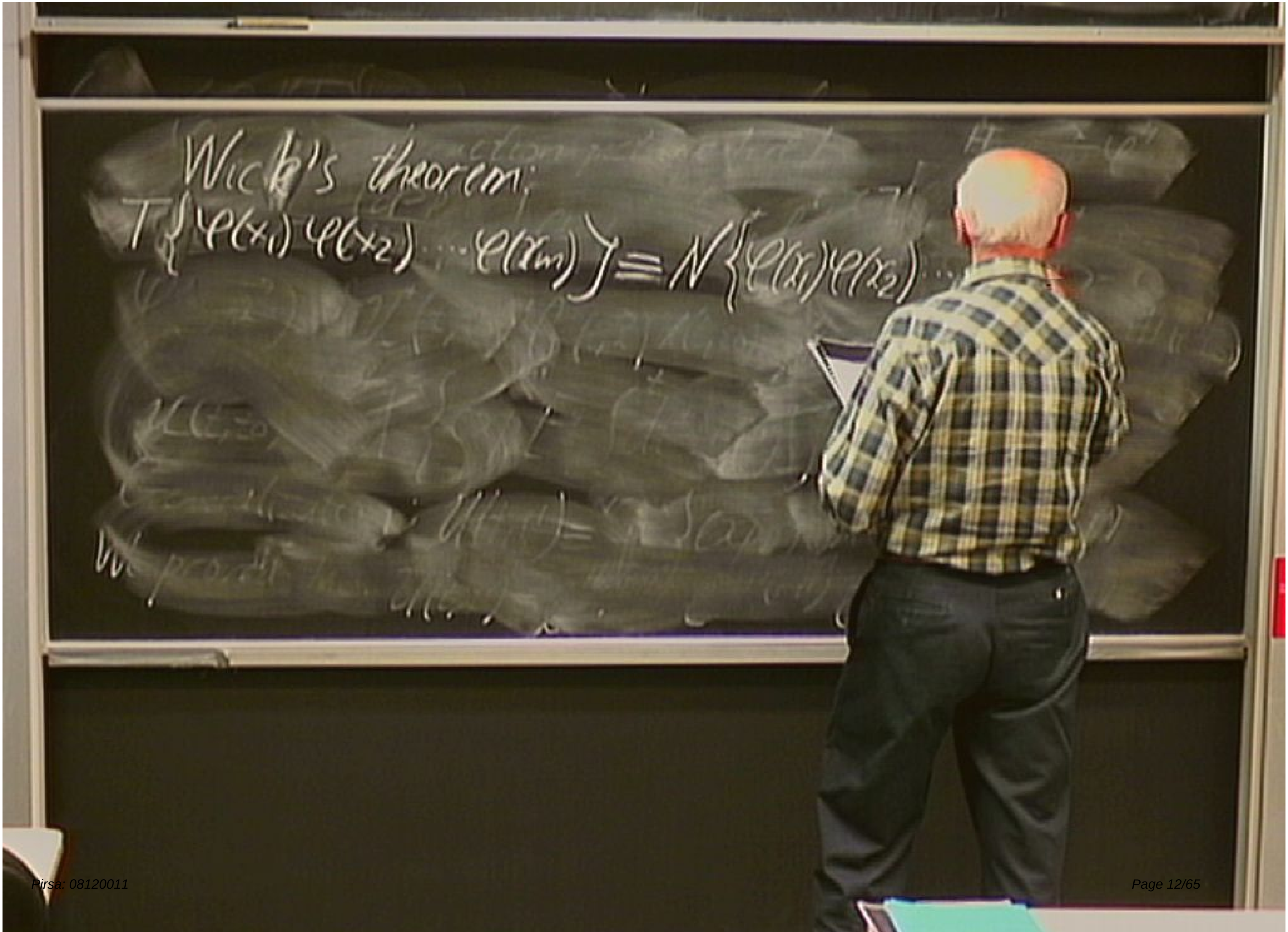
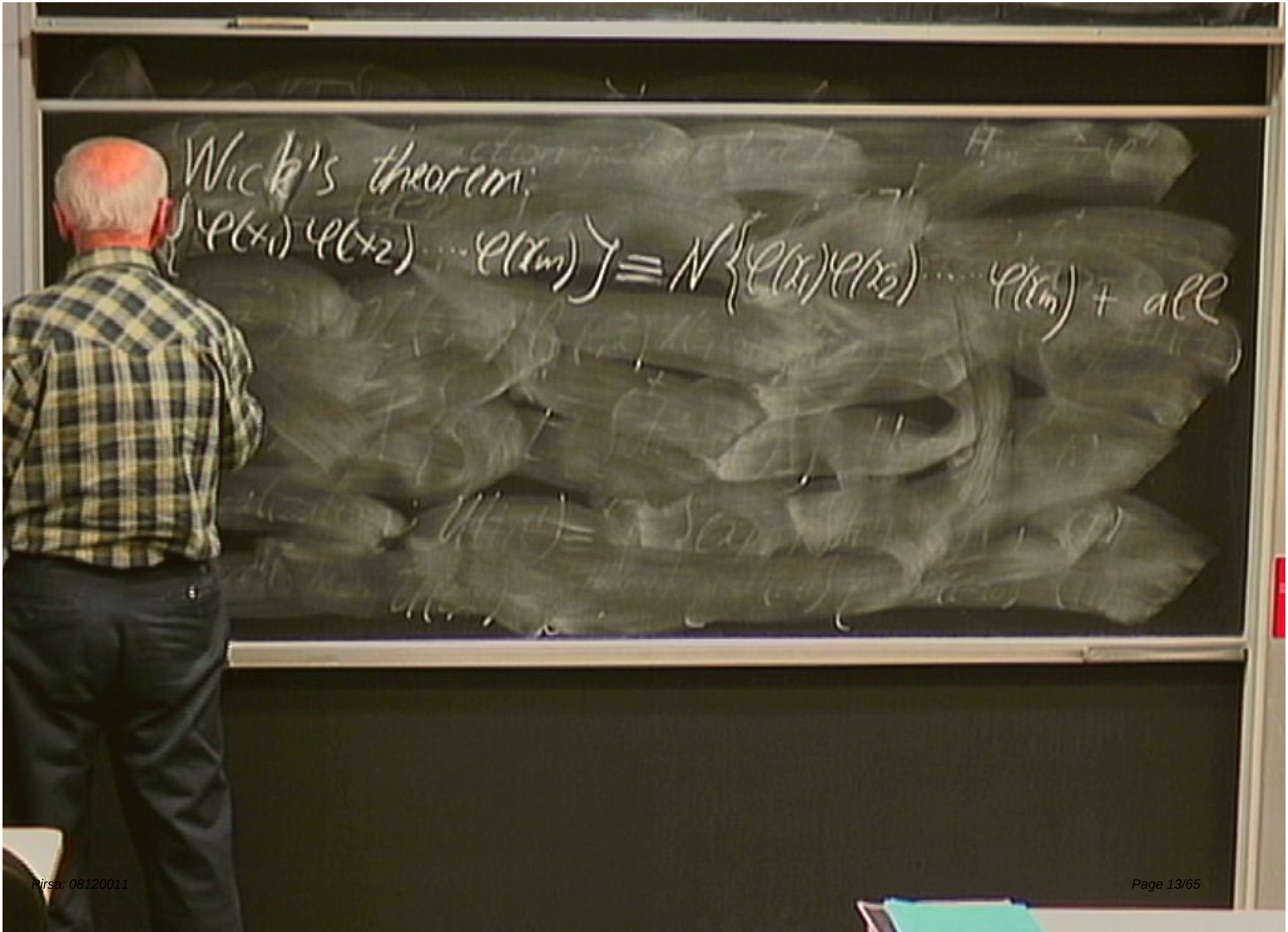
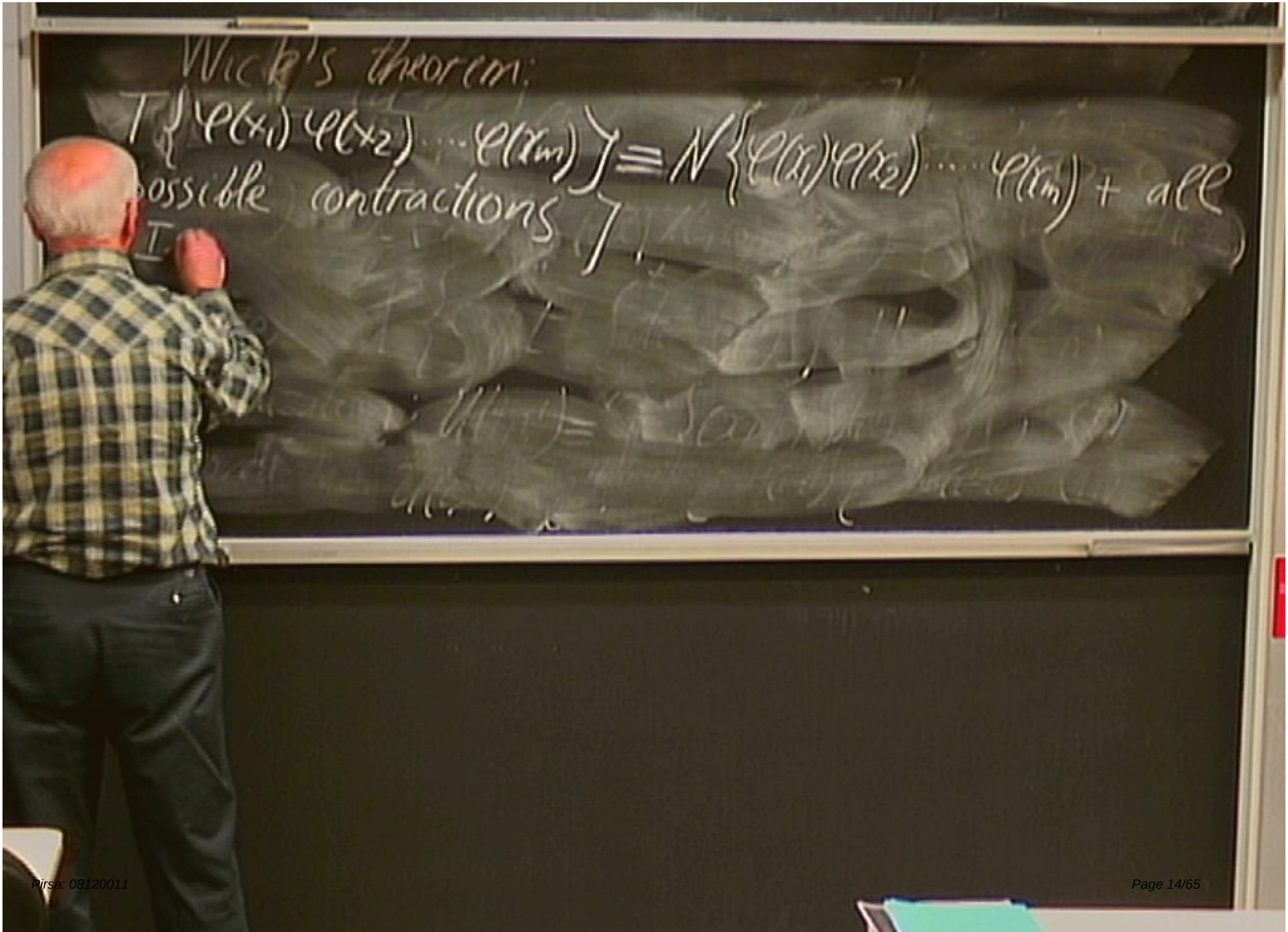
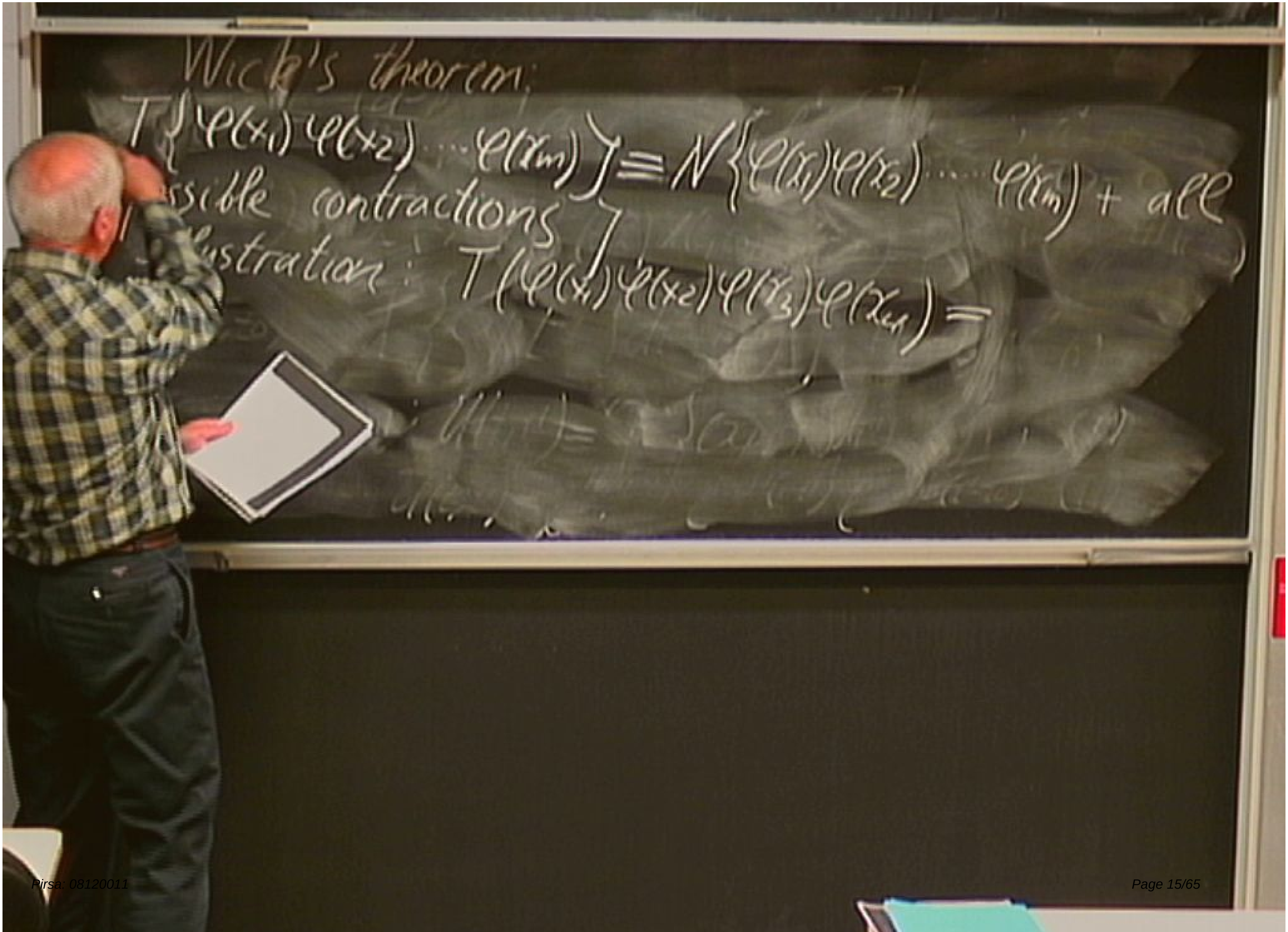

Wick's theorem:
T{φ(x1) φ(x2) ··· φ(xn)} = N{φ(x1)φ(x2) ···


Wick's theorem:
T{φ(x1) φ(x2) ...... φ(xm)} = N{φ(x1)φ(x2) ...... φ(xm) + all


Wick's theorem:
T{φ(x1)φ(x2)....φ(xm)} = N{φ(x1)φ(x2).....φ(xm) + all
ossible contractions}.


Wick's theorem:
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ssible contractions }
stration: T(φ(x1)φ(x2)φ(x3)φ(x4)) =

Wick's theorem:

$$T\{\varphi(x_1)\varphi(x_2)\cdots\varphi(x_m)\} = N\{\varphi(x_1)\varphi(x_2)\cdots\varphi(x_m) + \text{all possible contractions}\}$$

Illustration: $T(\varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4)) = N\{\varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4)$

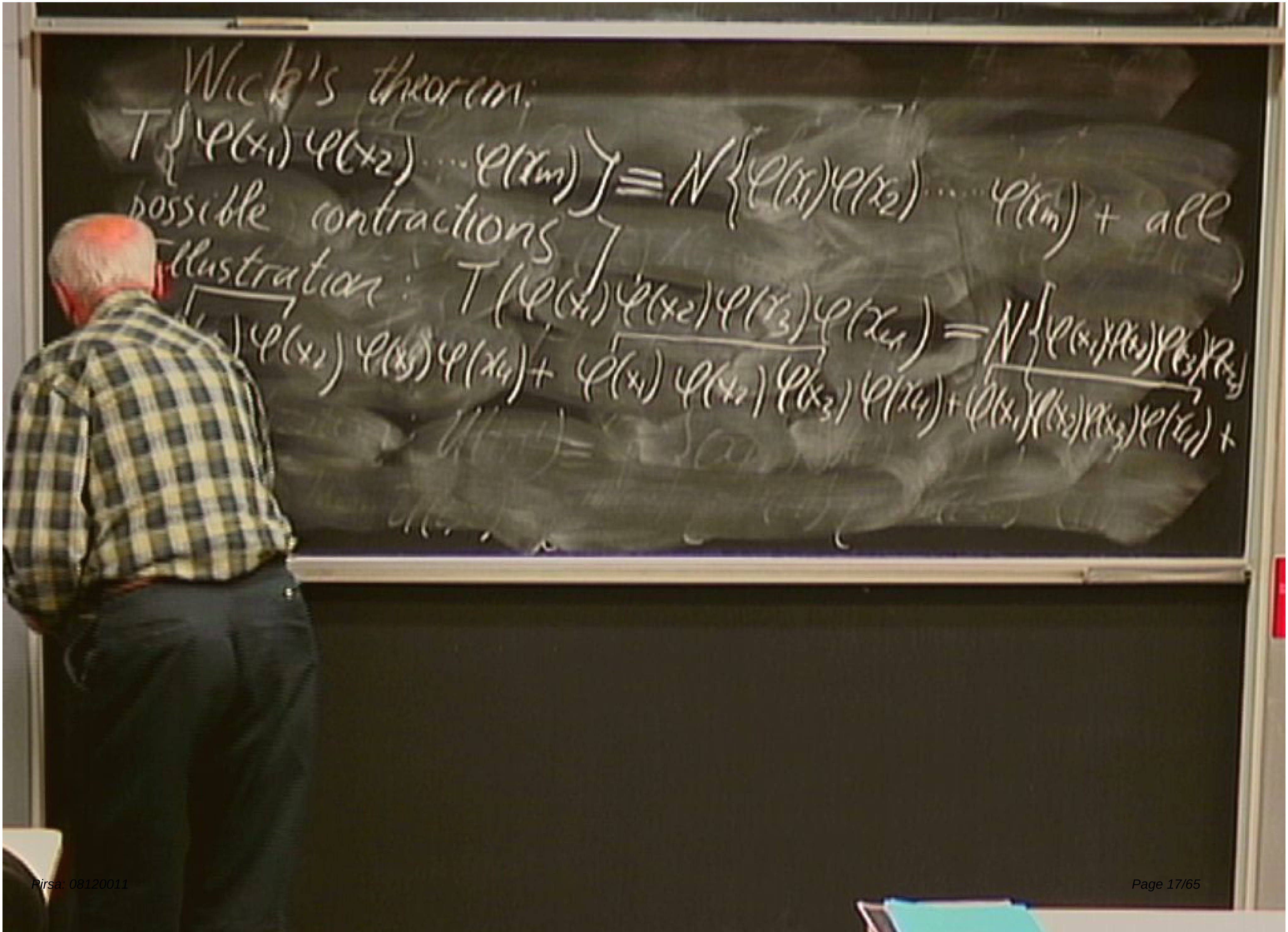
Wick's theorem:
T{φ(x1) φ(x2) ····φ(xm)} = N{φ(x1)φ(x2)······φ(xm) + all
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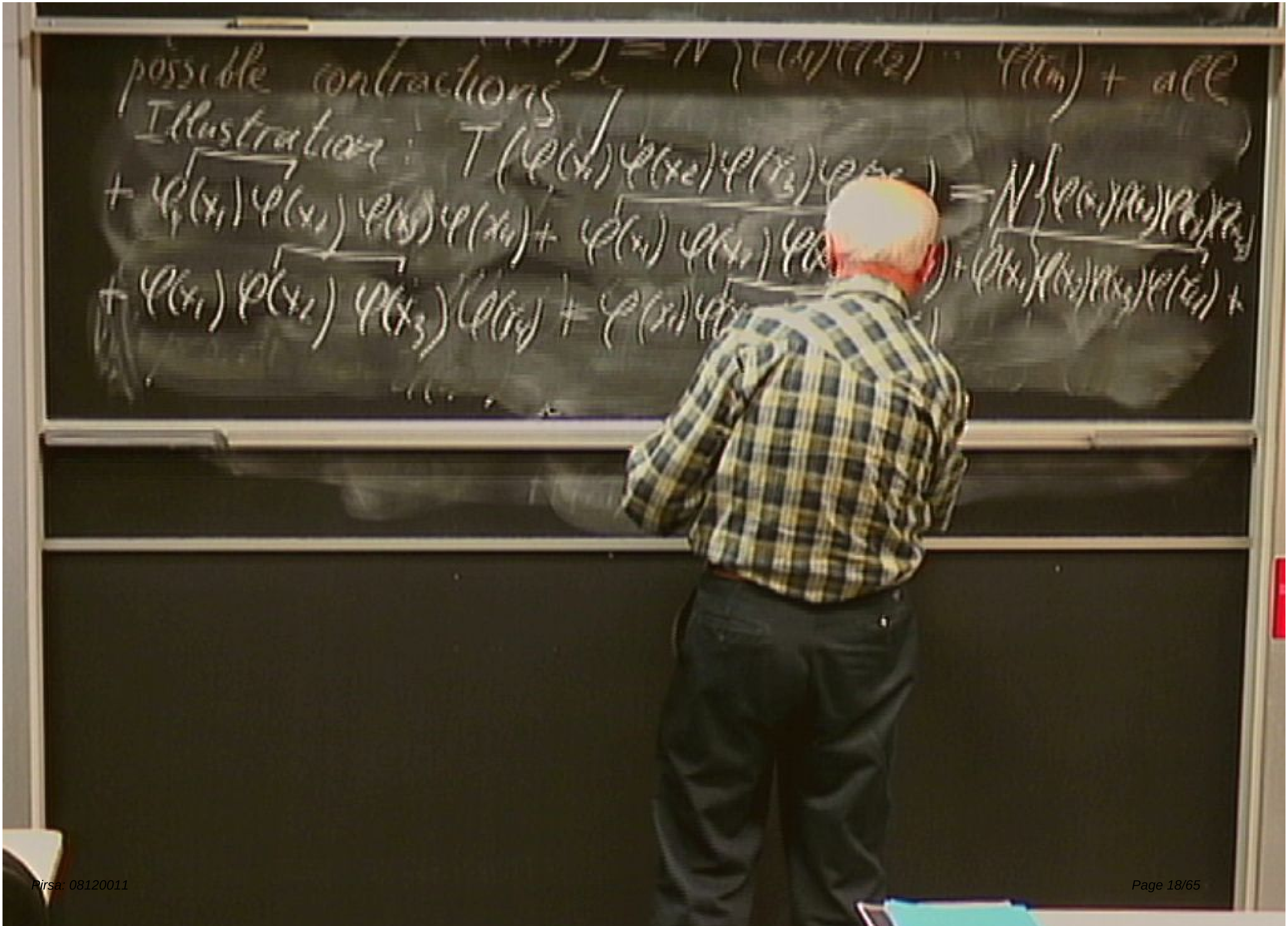
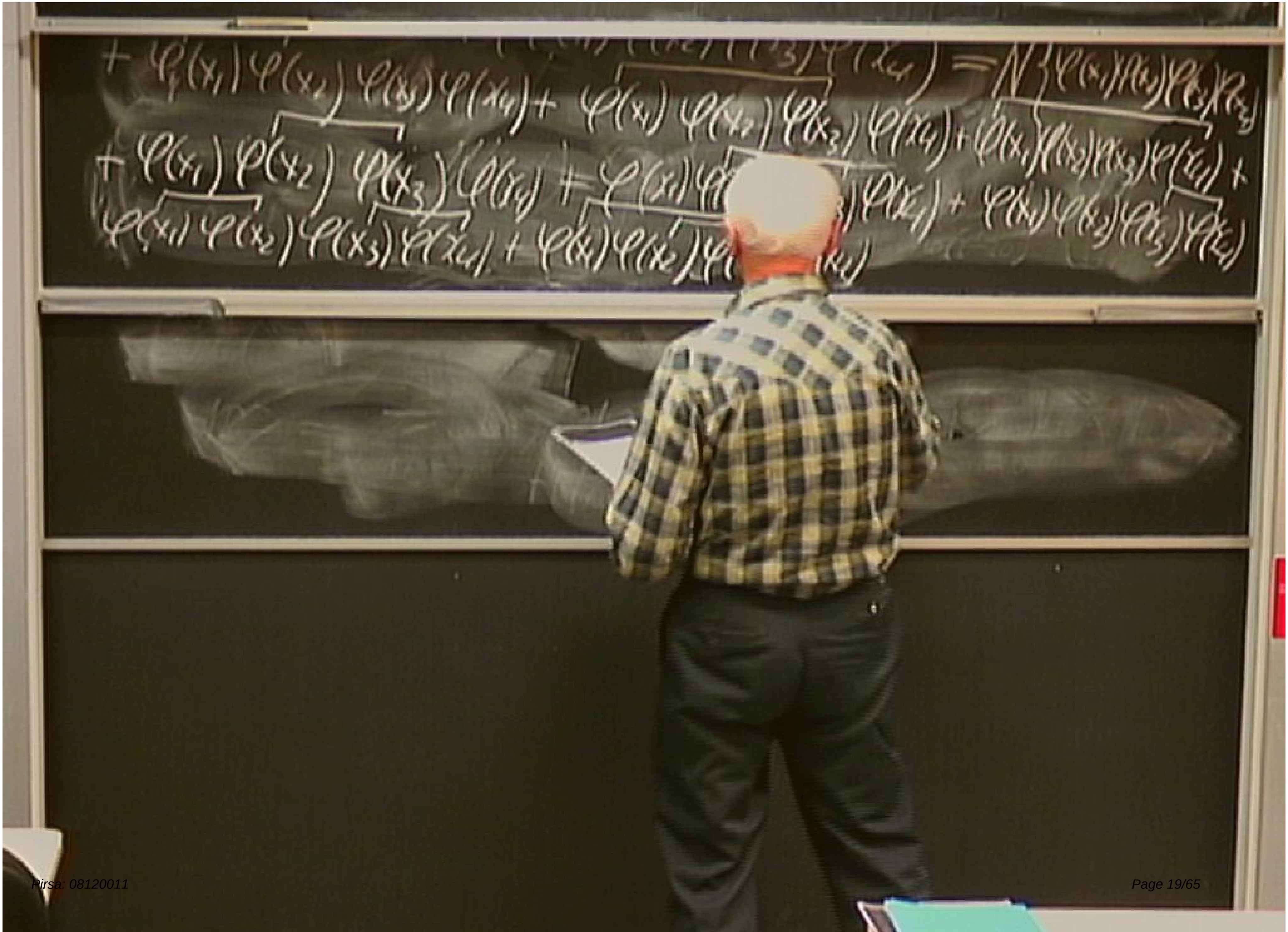
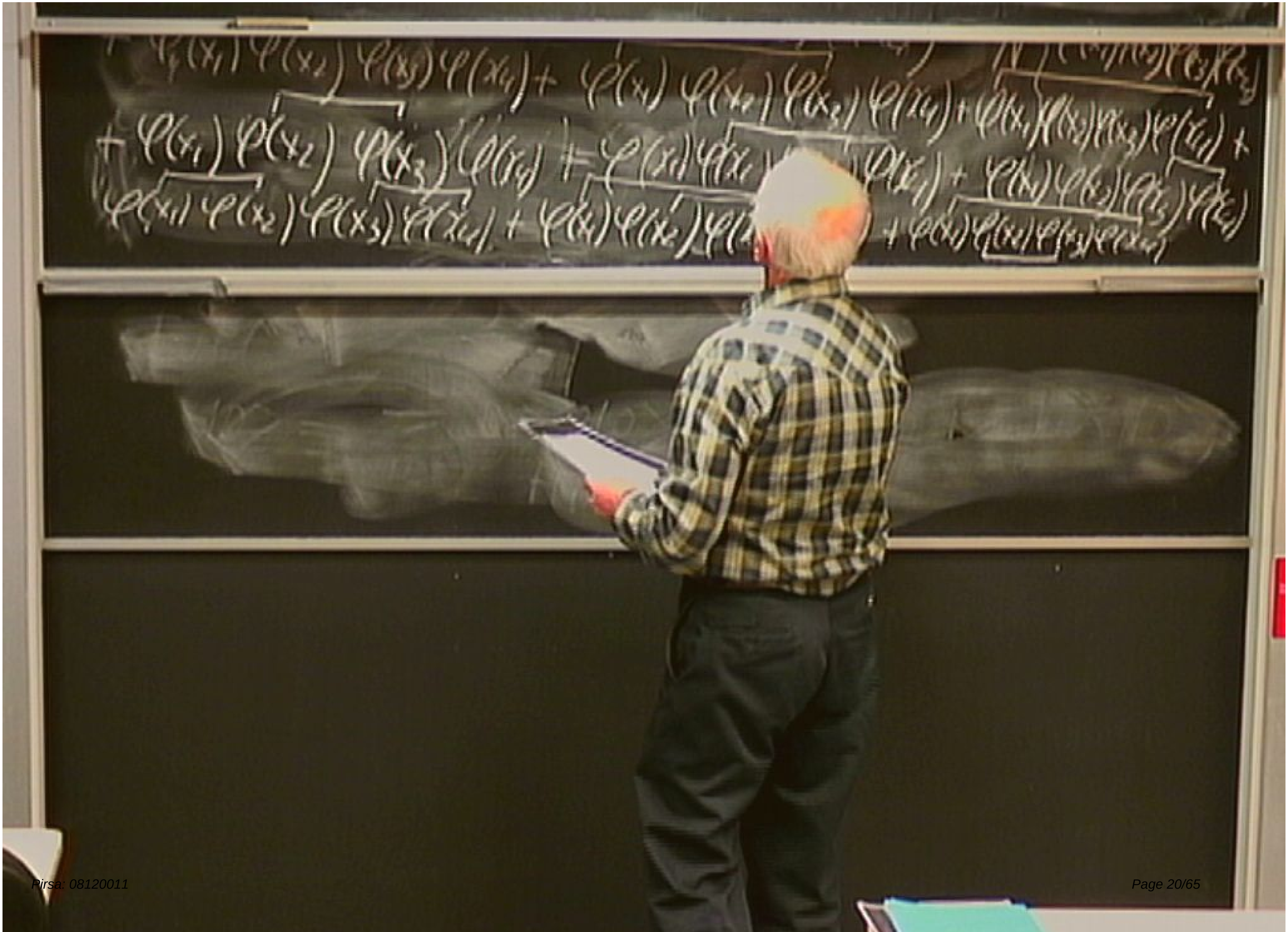
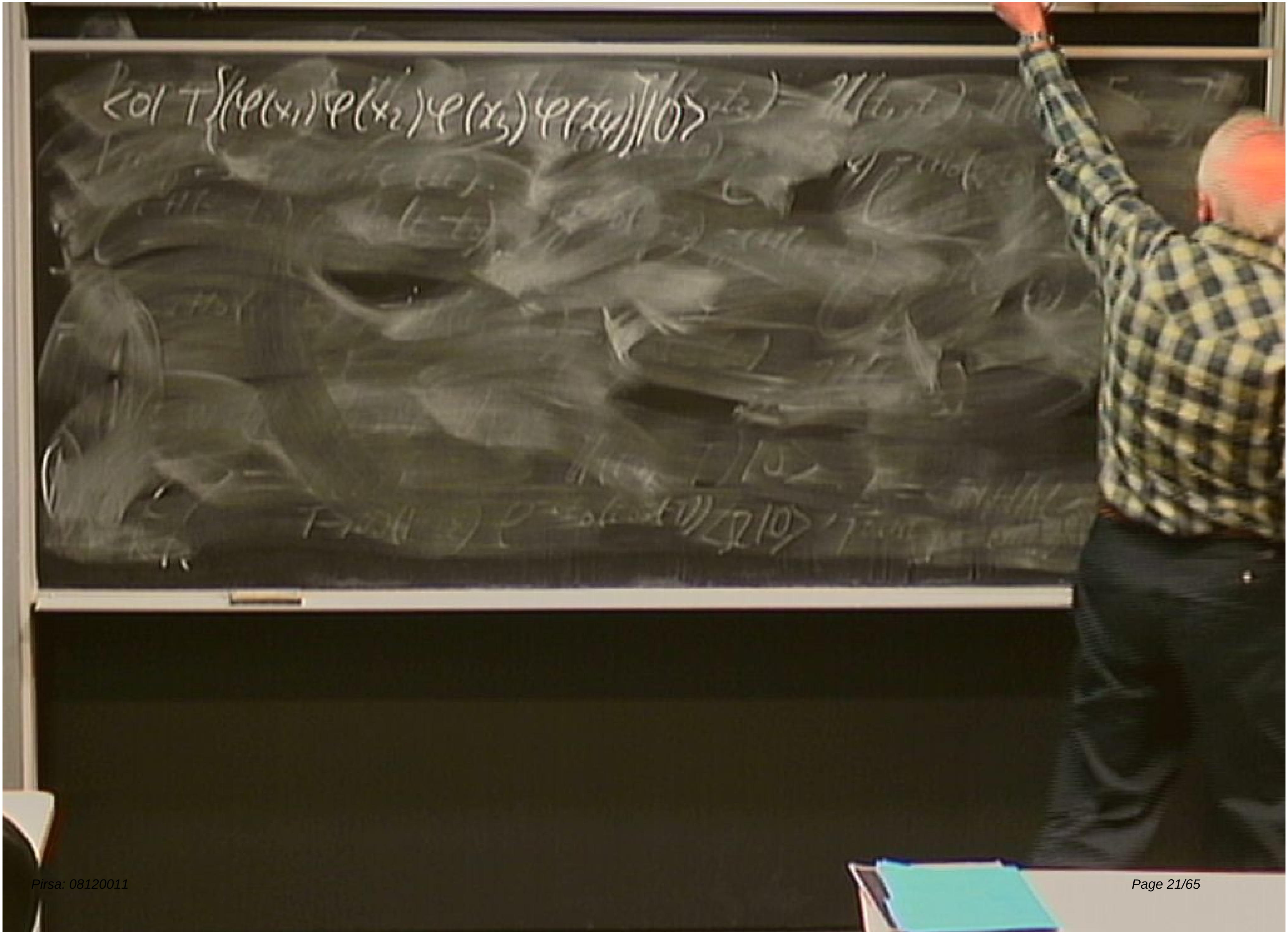
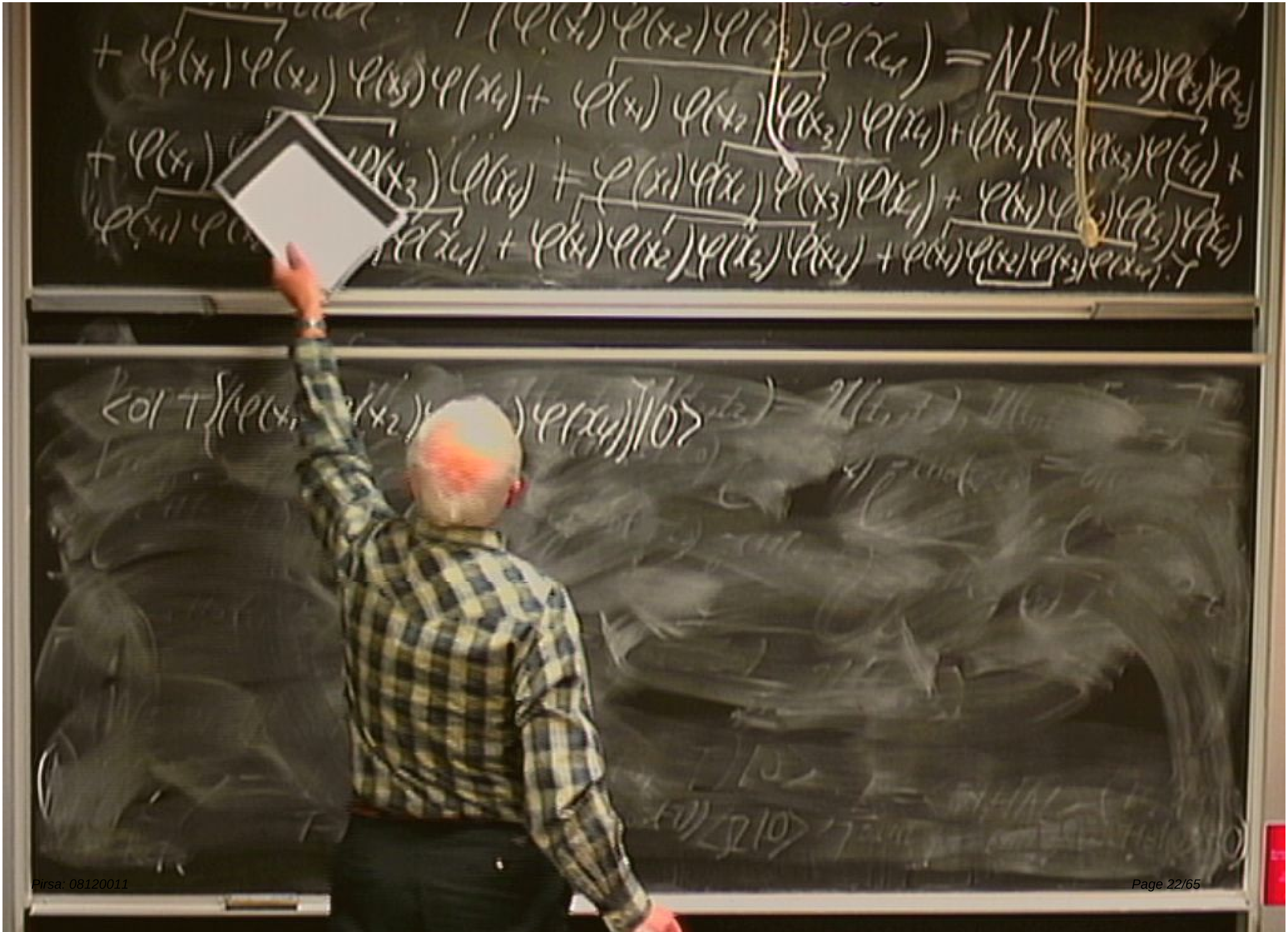
Illustration: T(φ(x1)φ(x2)φ(x3)φ(x4)) = N{φ(x1)φ(x2)φ(x3)φ(x4)
) φ(x2) φ(x3)φ(x4) + φ(x1) φ(x2) φ(x3) φ(x4) + φ(x1)φ(x2)φ(x3)φ(x4) +

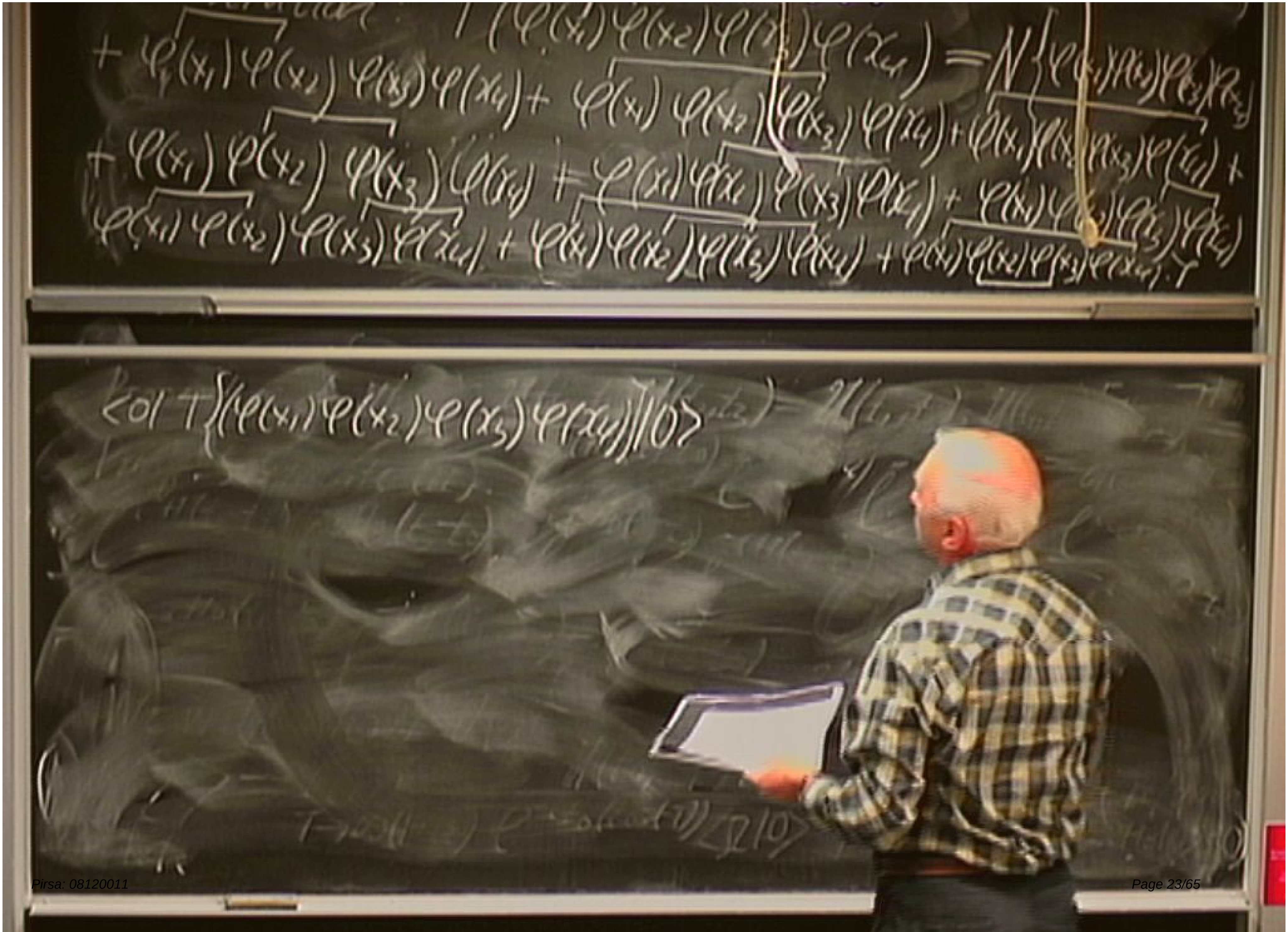

possible contractions
Illustration

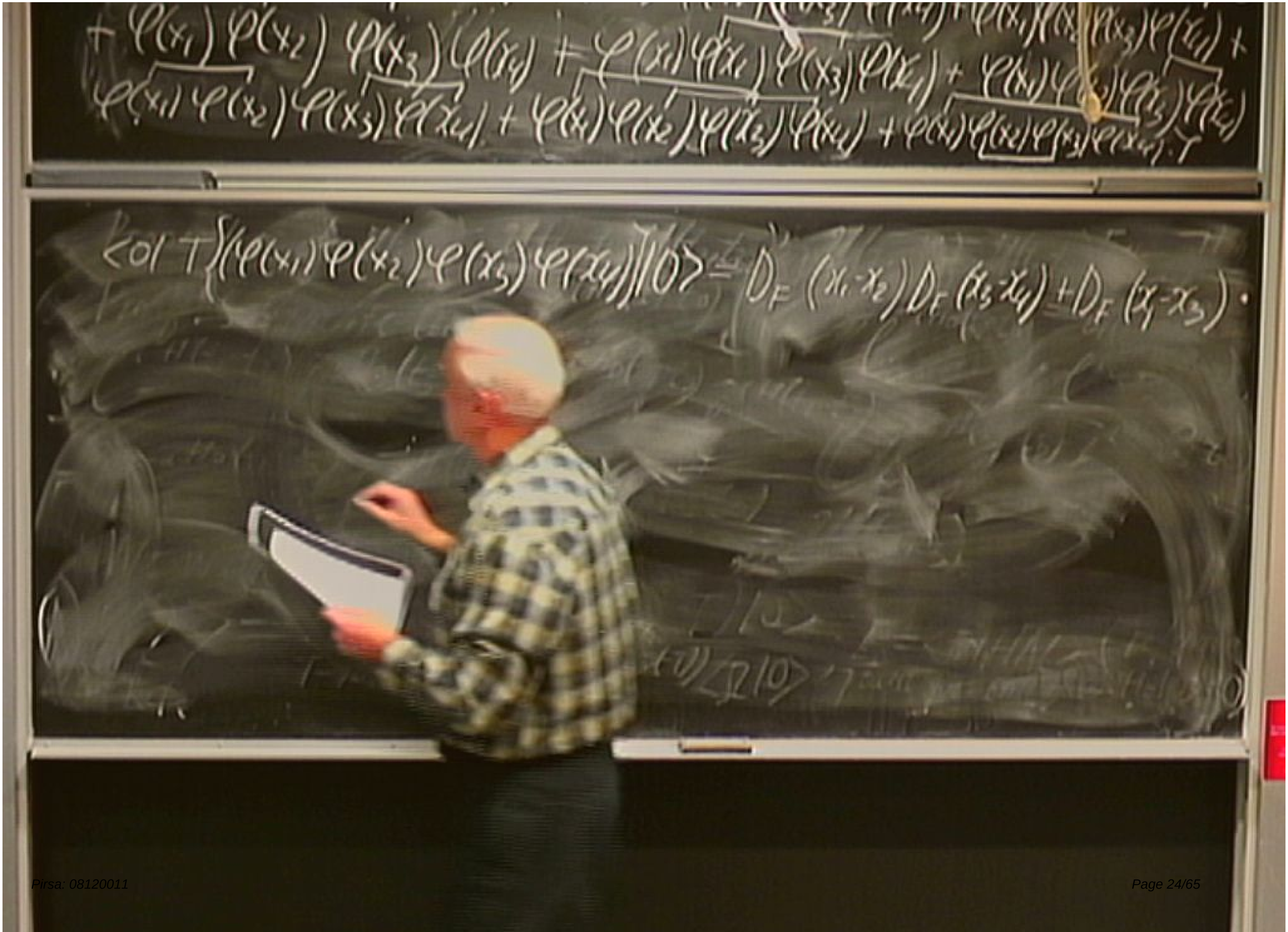


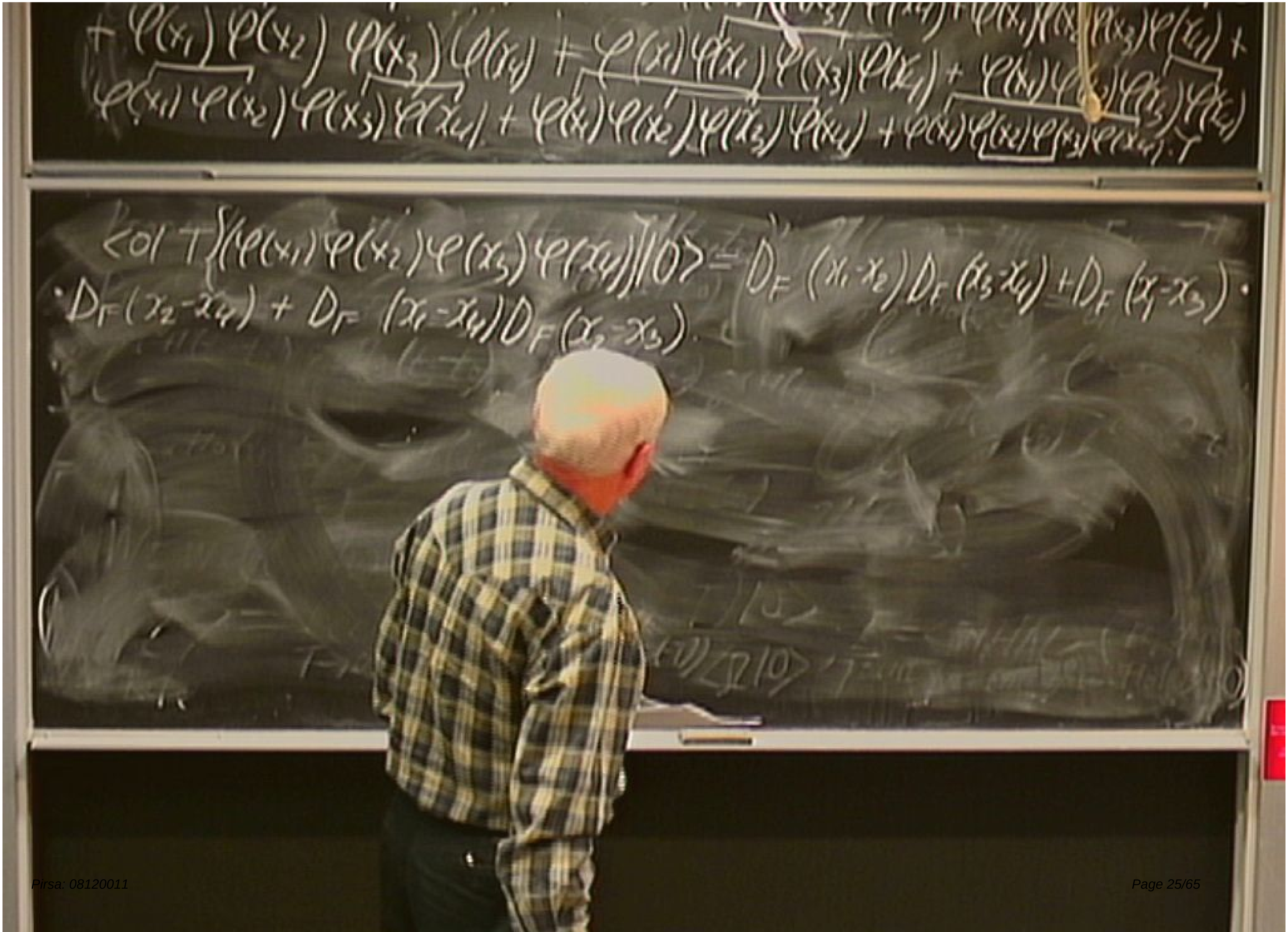



$\langle 0| T\{(\varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4))\}|0\rangle$




<0| T{(φ(x1)φ(x2)φ(x3)φ(x4))}|0>


<0| T{φ(x1)φ(x2)φ(x3)φ(x4)}|0> = DF(x1-x2) DF(x3-x4) + DF(x1-x3) ·

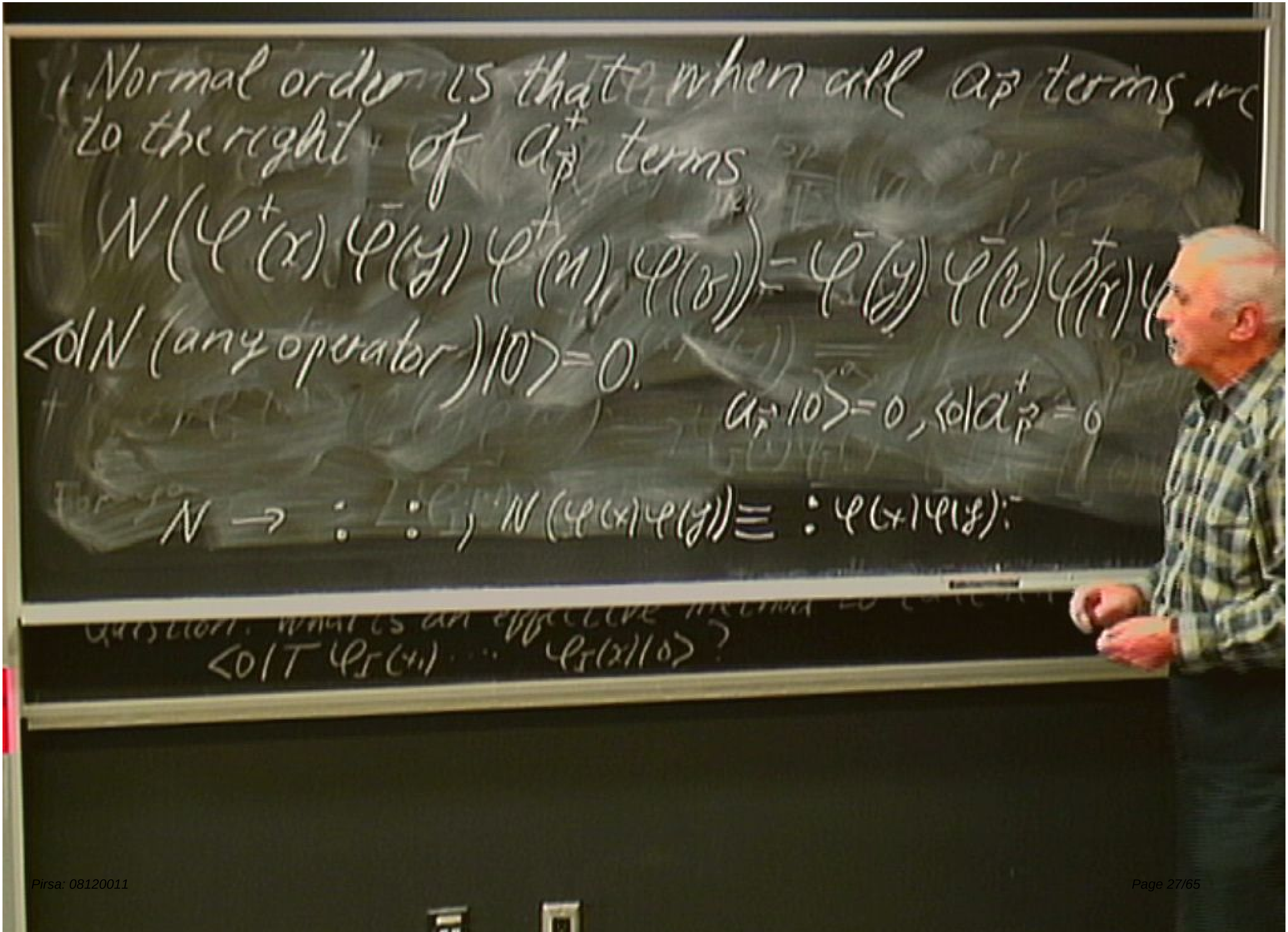

⟨0| T{φ(x1) φ(x2) φ(x3) φ(x4)} |0⟩ = DF (x1 - x2) DF (x3 - x4) + DF (x1 - x3) ·
DF (x2 - x4) + DF (x1 - x4) DF (x2 - x3).

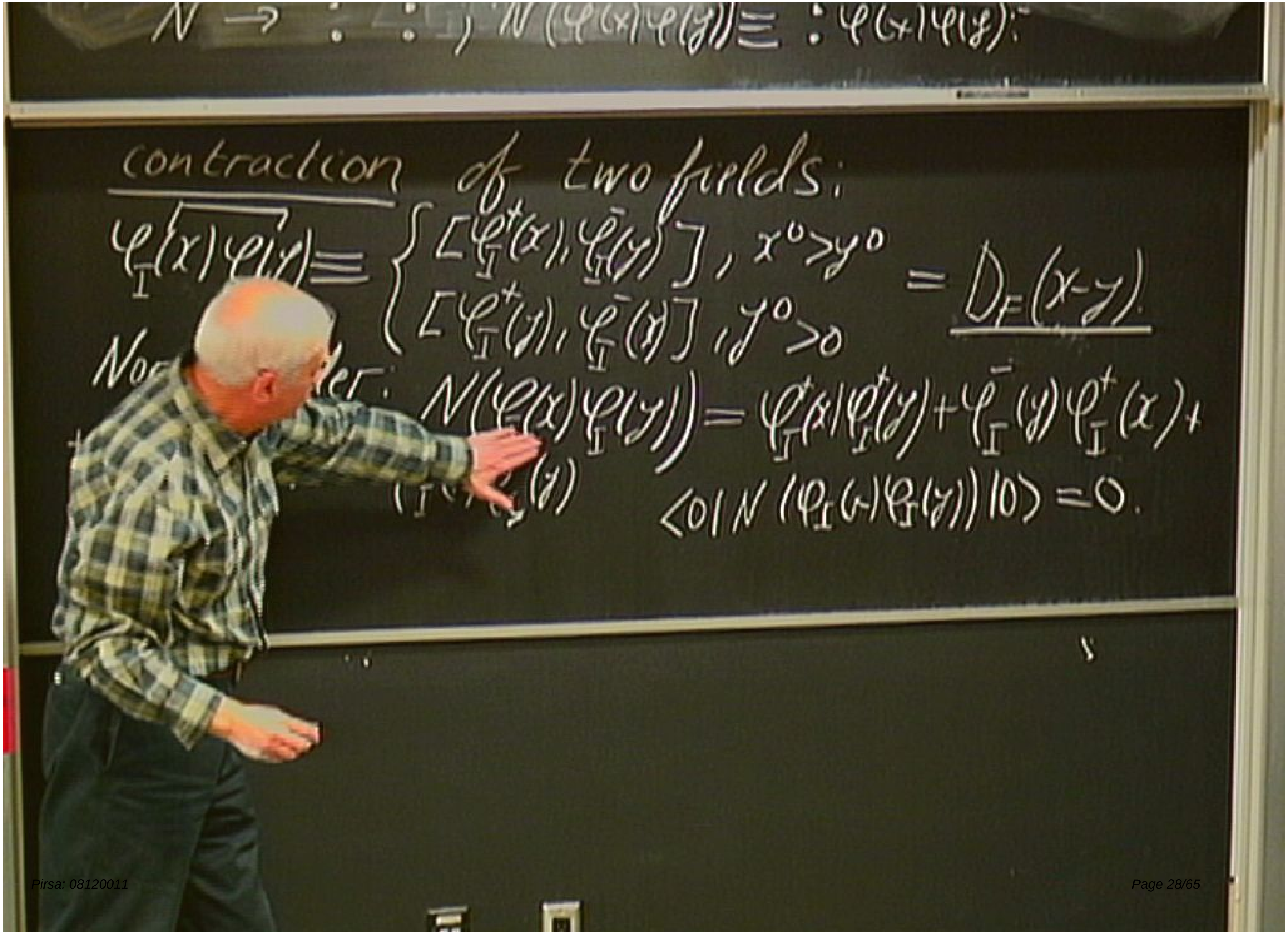
$$\ldots \varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4) + \varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4) +$$

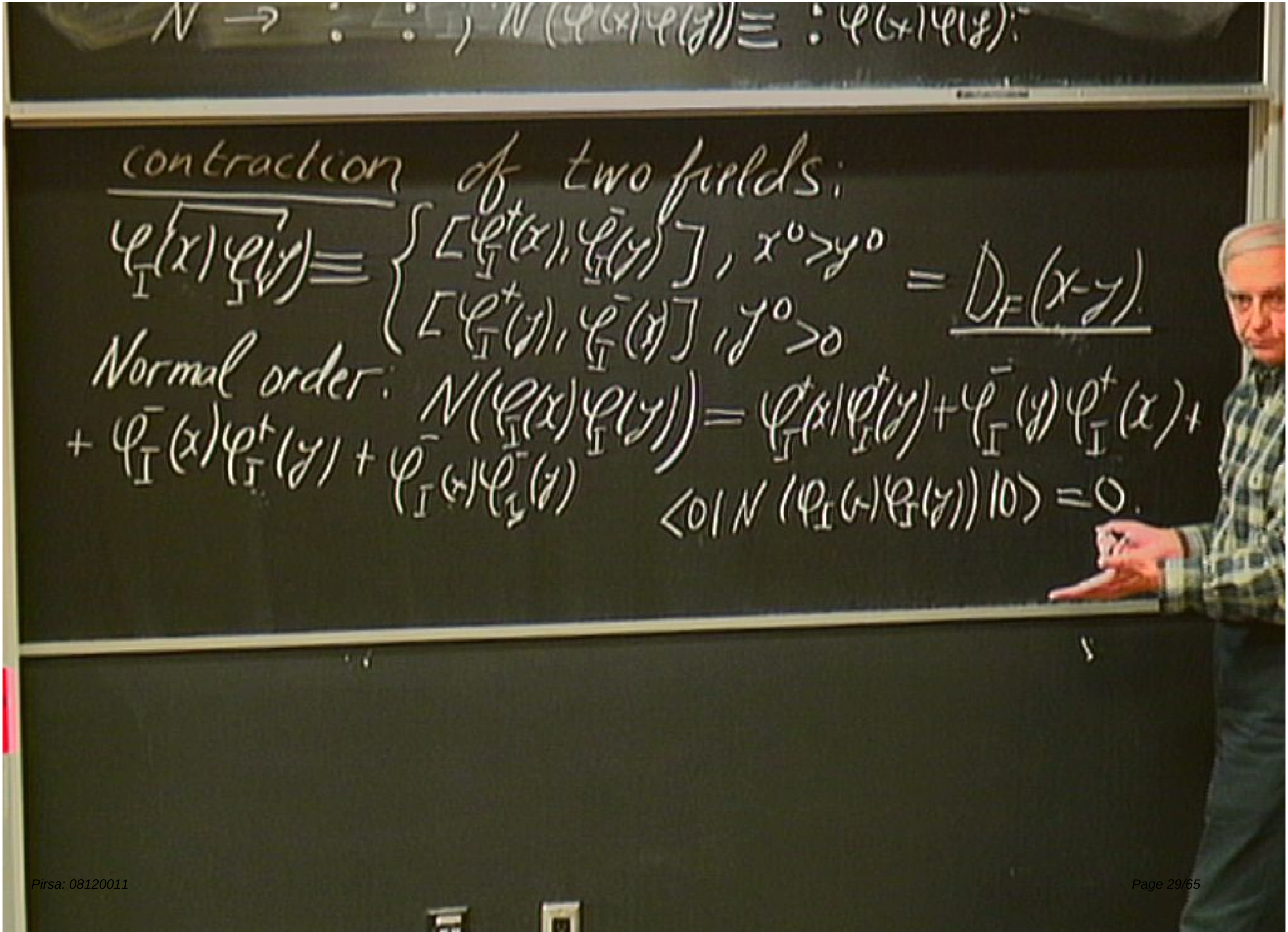
$$+ \varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4) + \varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4) + \varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4) +$$

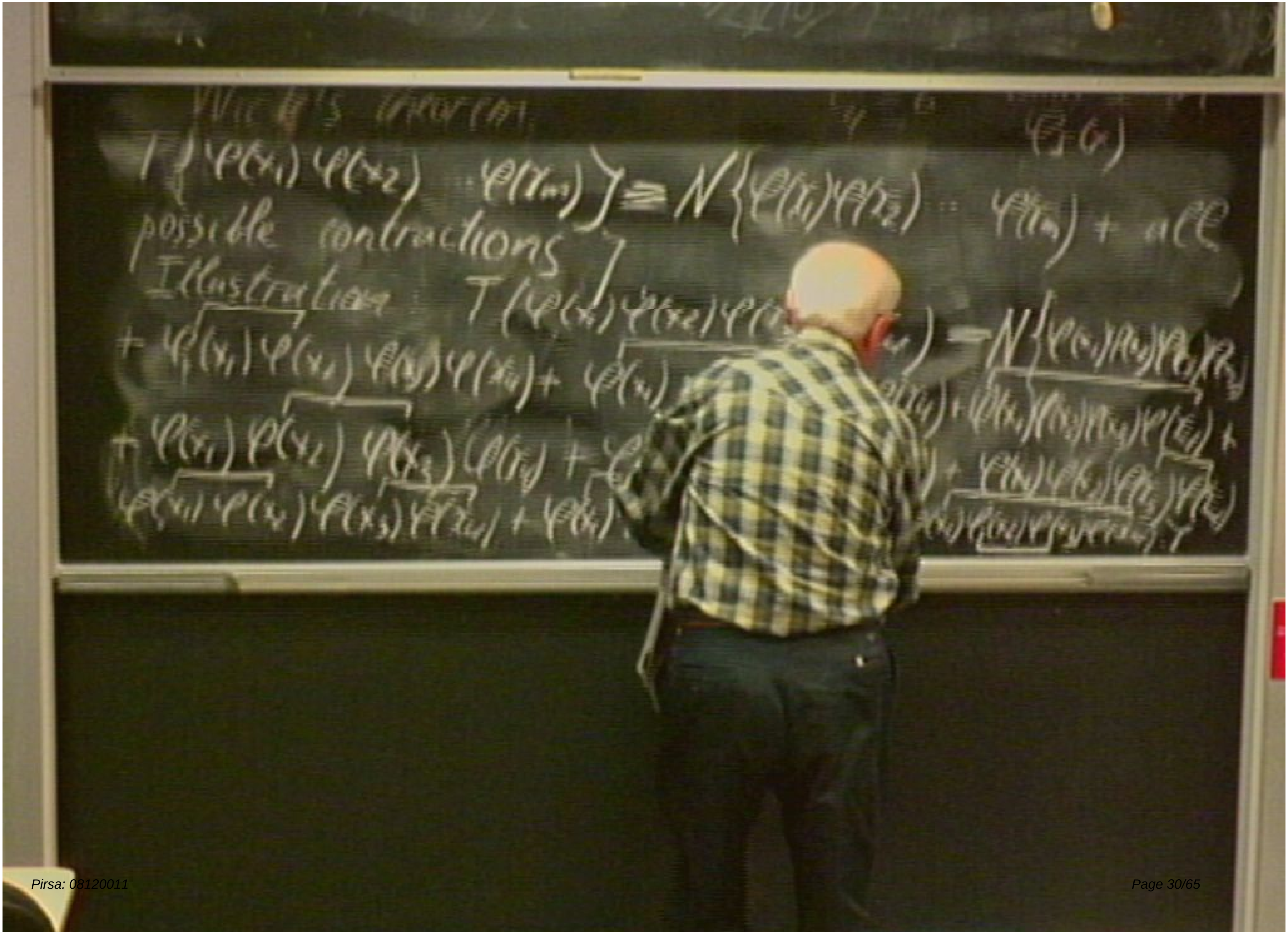
$$\varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4) + \varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4) + \varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4) :]$$

$$\langle 0| T\{\varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4)\}|0\rangle = D_F(x_1-x_2)D_F(x_3-x_4) + D_F(x_1-x_3)\cdot D_F(x_2-x_4) + D_F(x_1-x_4)D_F(x_2-x_3).$$


Normal order is that when all a_p terms are
to the right of a_p^+ terms
<0|N (any operator)|0> = 0.
a_p|0> = 0, <0|a_p^+ = 0
N → : : , N(φ(x)φ(y)) ≡ :φ(x)φ(y):
<0|T φ_I(x_1) ... φ_I(x_n)|0> ?


contraction of two fields:
= D_F(x-y)
⟨0| N (φ_I(x) φ_I(y)) |0⟩ = 0.


$N \rightarrow : \; :$, $N(\varphi(x)\varphi(y)) \equiv :\varphi(x)\varphi(y):$
contraction of two fields:
$\varphi_I(x)\varphi_I(y) \equiv \begin{cases} [\varphi_I^+(x), \varphi_I^-(y)], & x^0>y^0 \\ [\varphi_I^+(y), \varphi_I^-(x)], & y^0>0 \end{cases} = D_F(x-y)$
Normal order: $N(\varphi_I(x)\varphi_I(y)) = \varphi_I^+(x)\varphi_I^+(y) + \varphi_I^-(y)\varphi_I^+(x) +$
$+ \varphi_I^-(x)\varphi_I^+(y) + \varphi_I^-(x)\varphi_I^-(y)$
$\langle 0 | N(\varphi_I(x)\varphi_I(y)) | 0 \rangle = 0.$

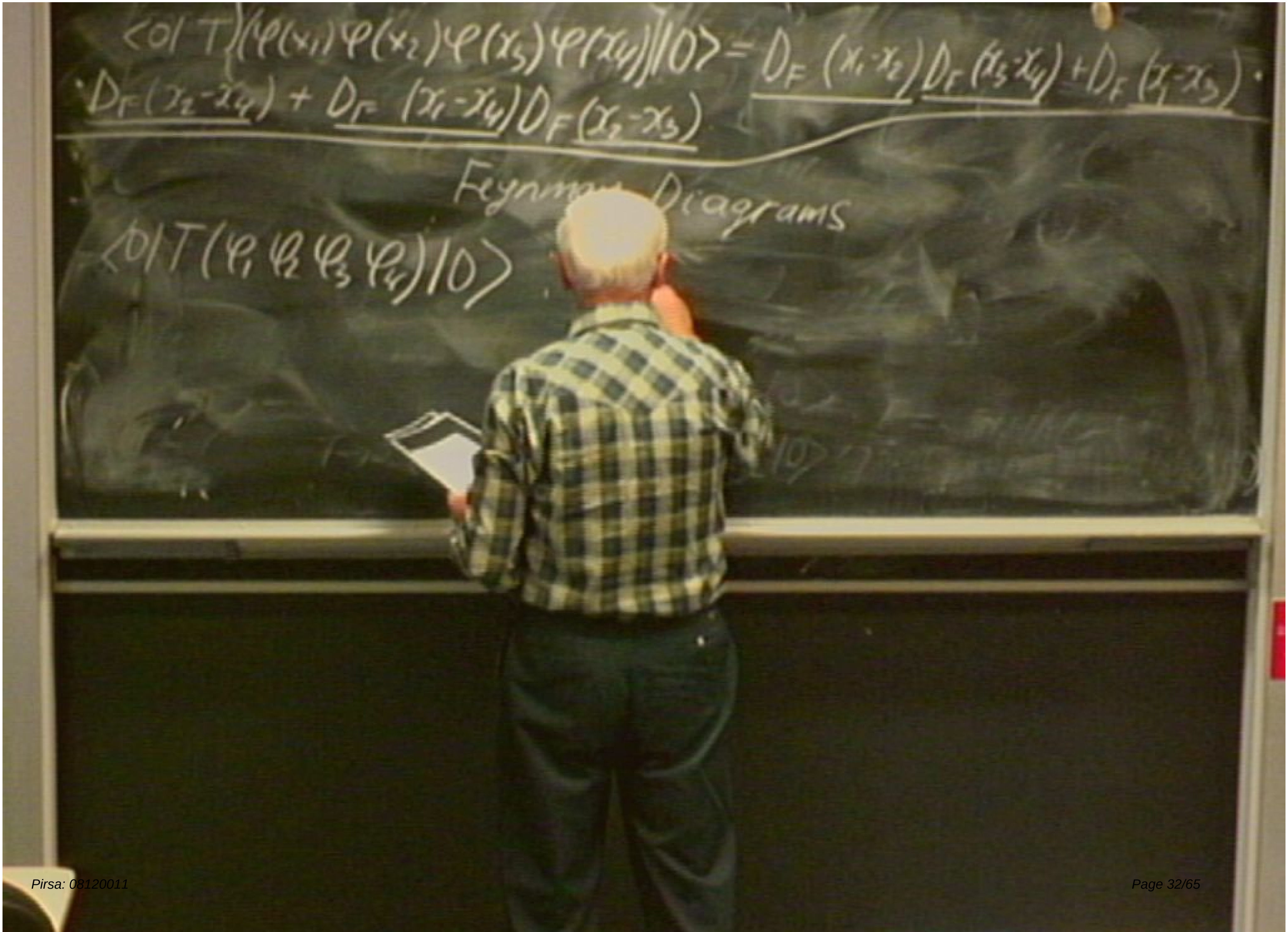

possible contractions
Illustration

Generalization for arbitrary Green's function:

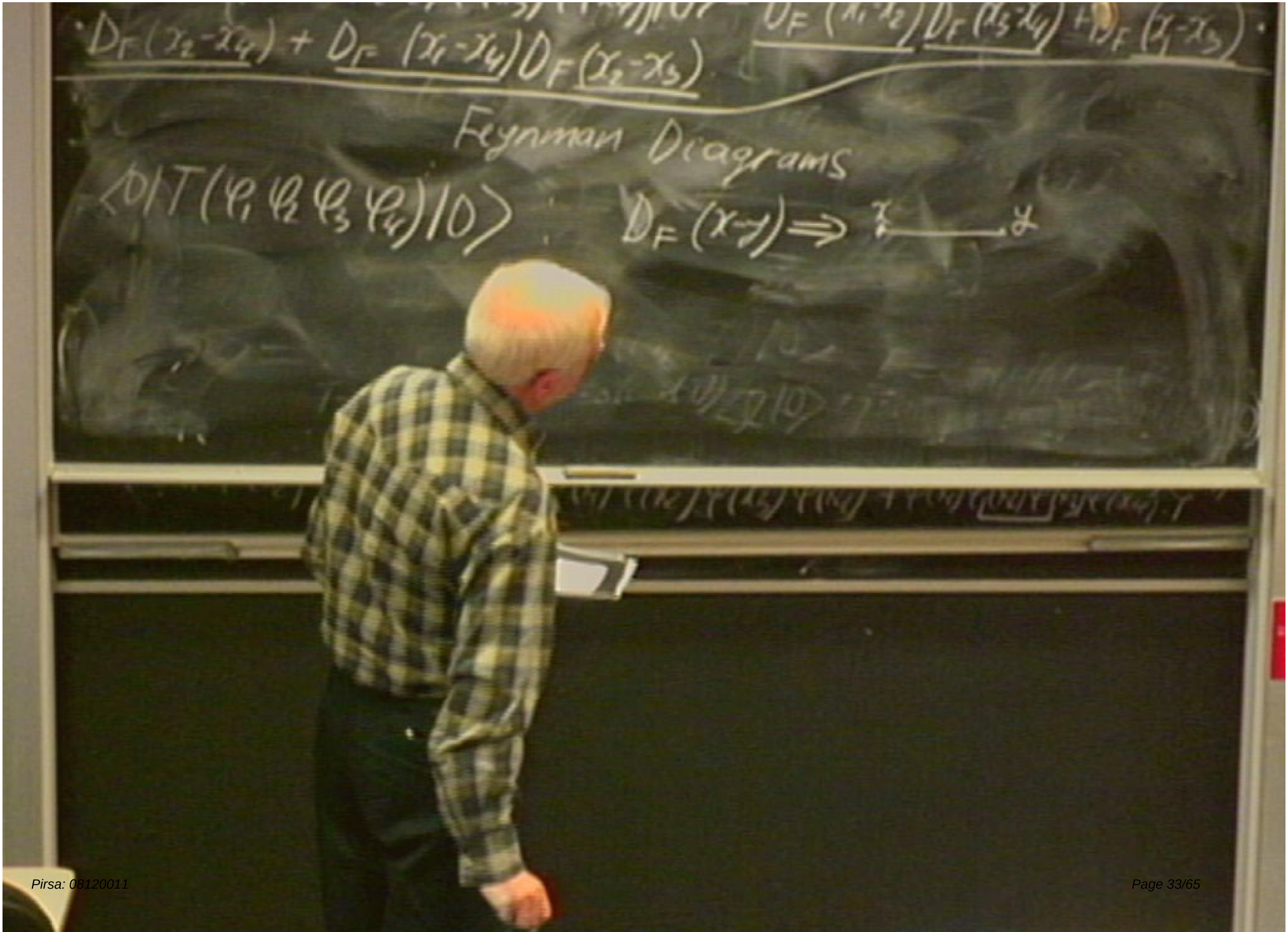
$$\langle\Omega| T(\varphi(x_1)\varphi(x_2)\cdots\varphi(x_n))|\Omega\rangle = \lim_{T\to\infty(1-i\varepsilon)} \frac{\langle 0| T\{\varphi_I(x_1)\varphi_I(x_2)\cdots\varphi_I(x_n)\exp[-i\int_{-T}^{T}dt\, H_I(t)]\}|0\rangle}{\langle 0| T\{\exp[-i\int_{-T}^{T}dt\, H_I(t)]\}|0\rangle}.$$

Wick's Theorem

Question: what is an effective method to calculate $\langle 0|T\varphi_I(x_1)\cdots\varphi_I(x_n)|0\rangle$?



<0| T{φ(x1)φ(x2)φ(x3)φ(x4)}|0> = DF (x1-x2) DF (x3-x4) + DF (x1-x3) ·
DF (x2-x4) + DF (x1-x4) DF (x2-x3)
Diagrams
<0|T(φ1 φ2 φ3 φ4)|0>



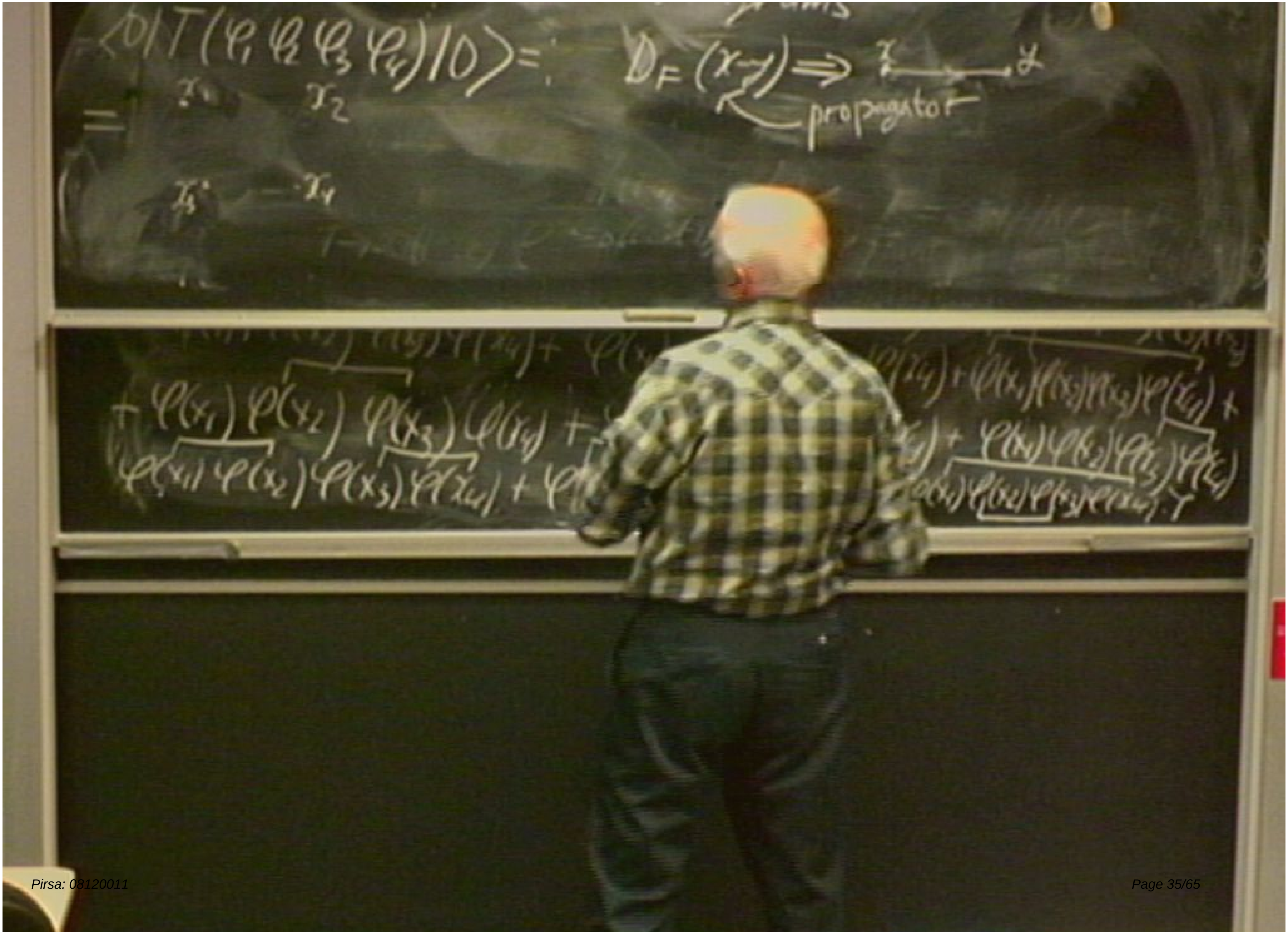
Feynman Diagrams
⟨0|T(φ1 φ2 φ3 φ4)|0⟩
DF(x−y) ⇒ x —— y

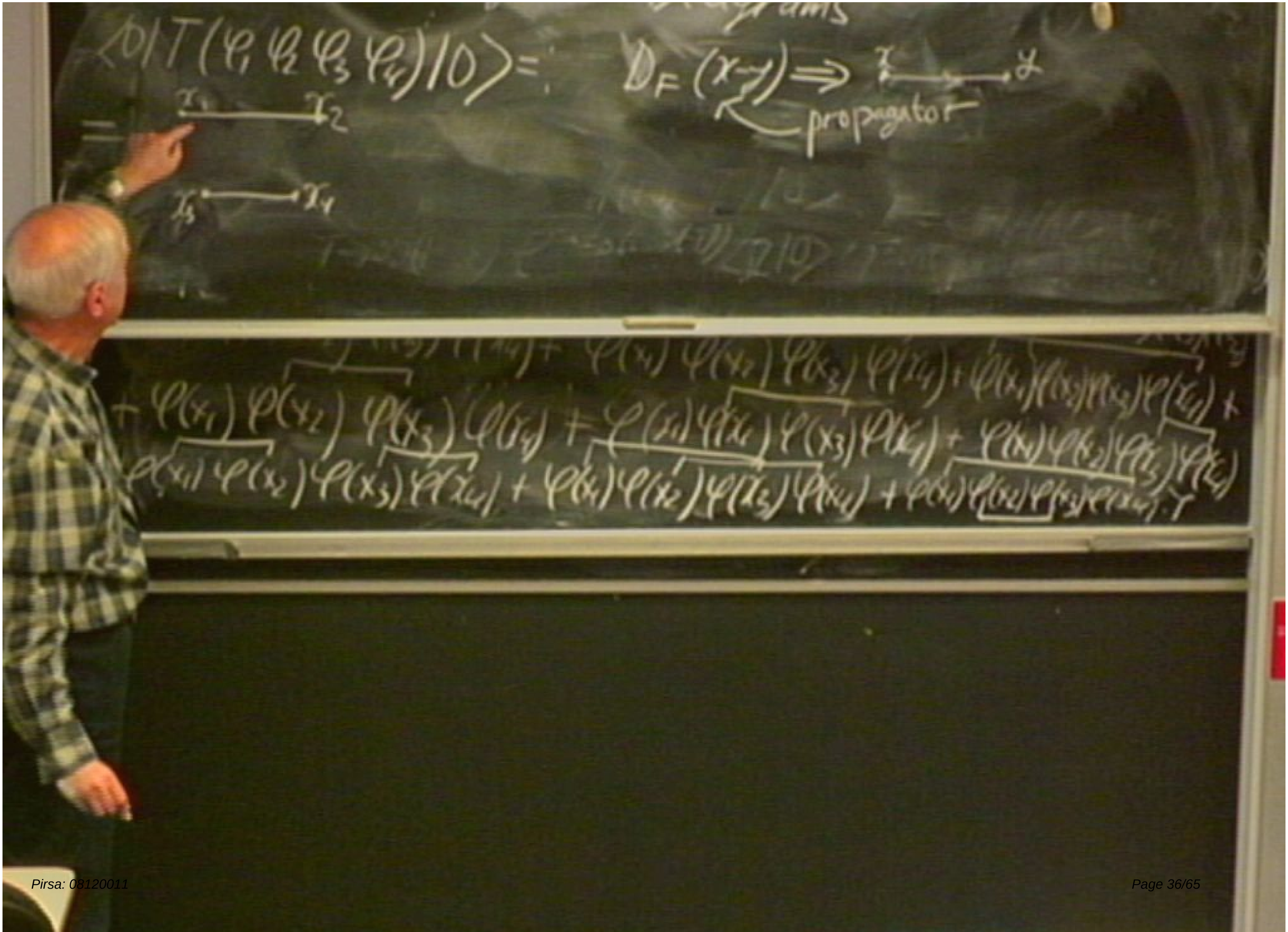
Feynman Diagrams

$\langle 0|T(\varphi_1\varphi_2\varphi_3\varphi_4)|0\rangle$: $D_F(x-y) \Rightarrow$ x ——— y

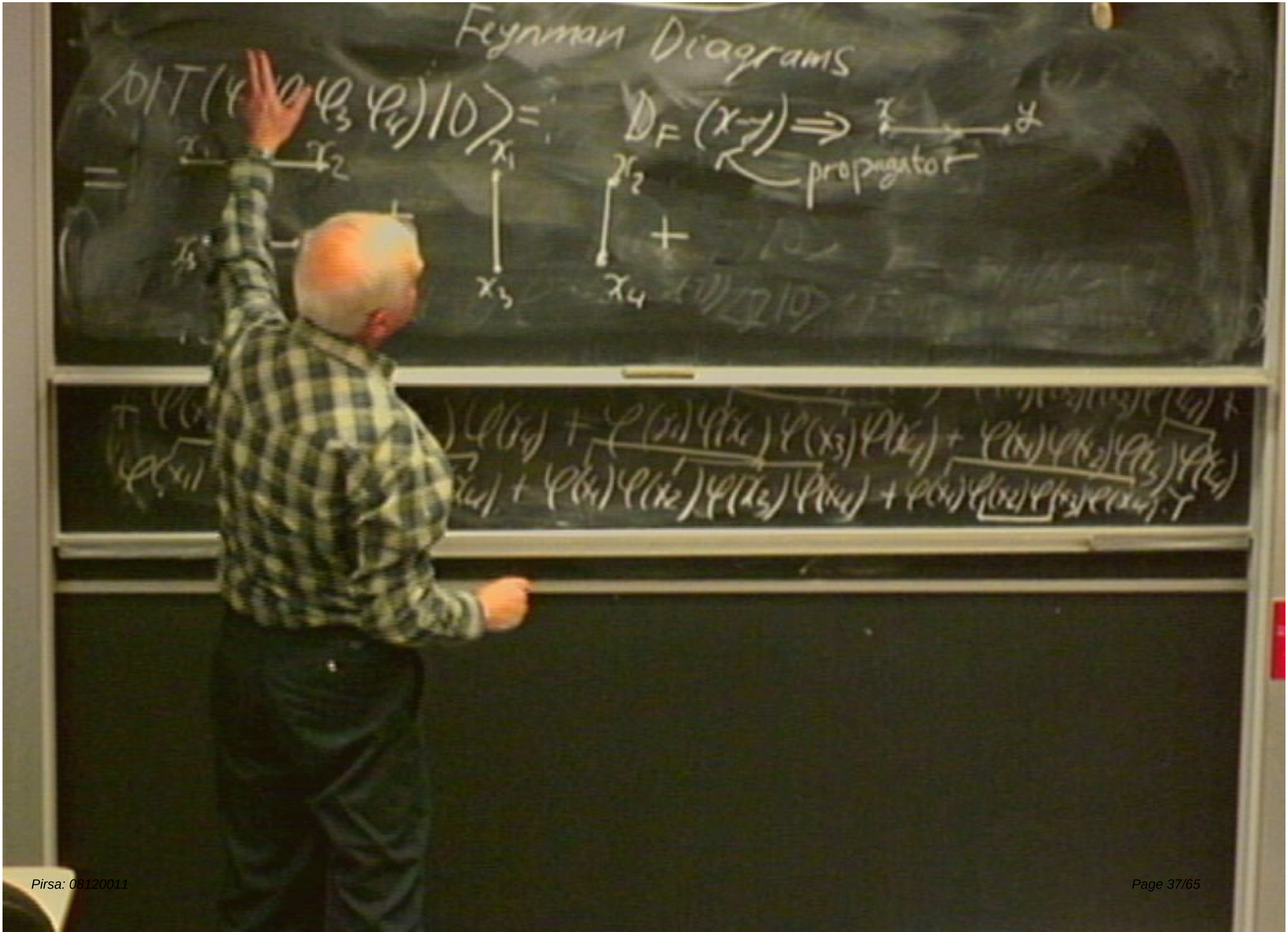
$+ \varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4) + \varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4) + \varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4)$

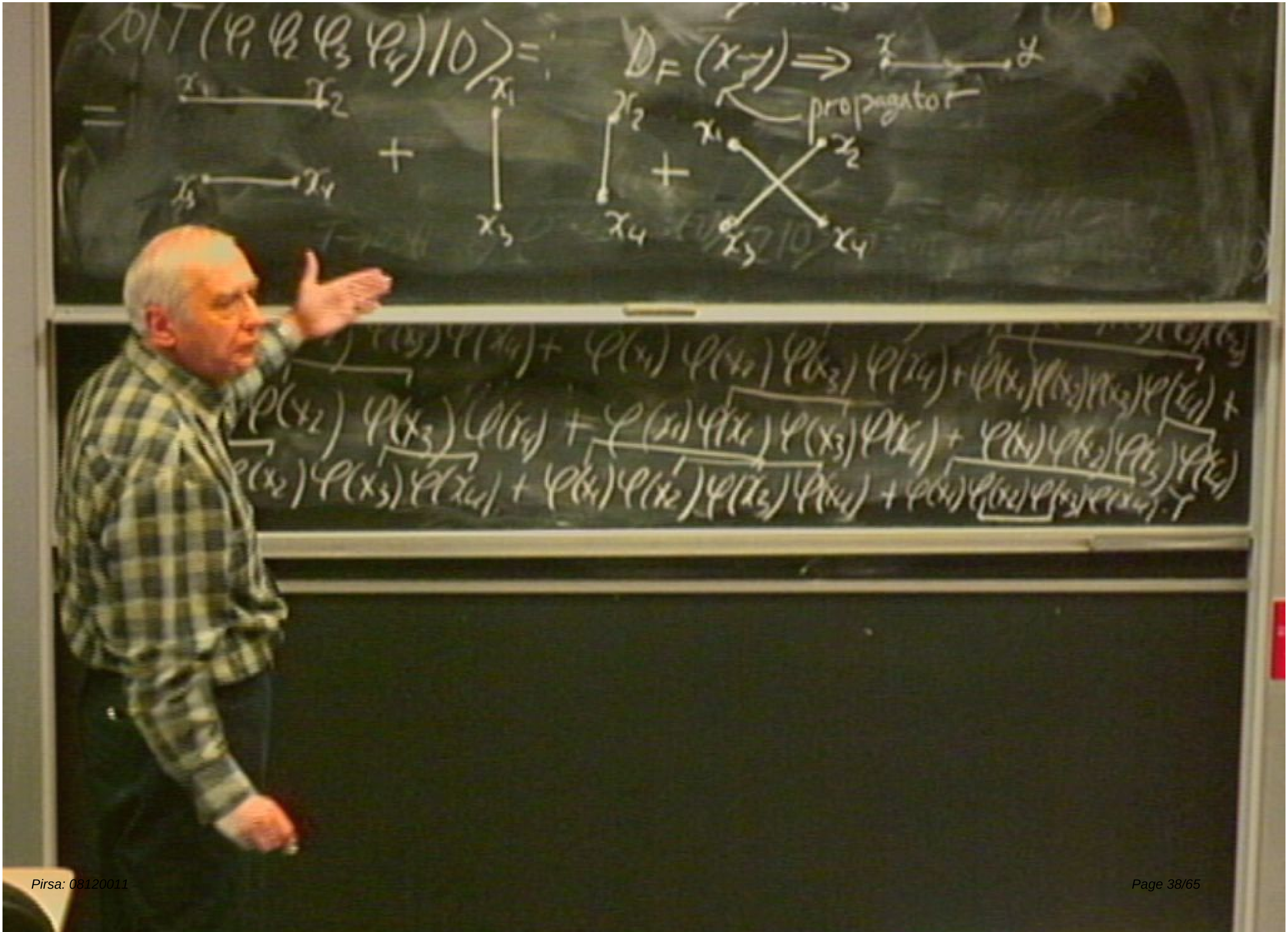
$\varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4) + \varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4) + \varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4) :$

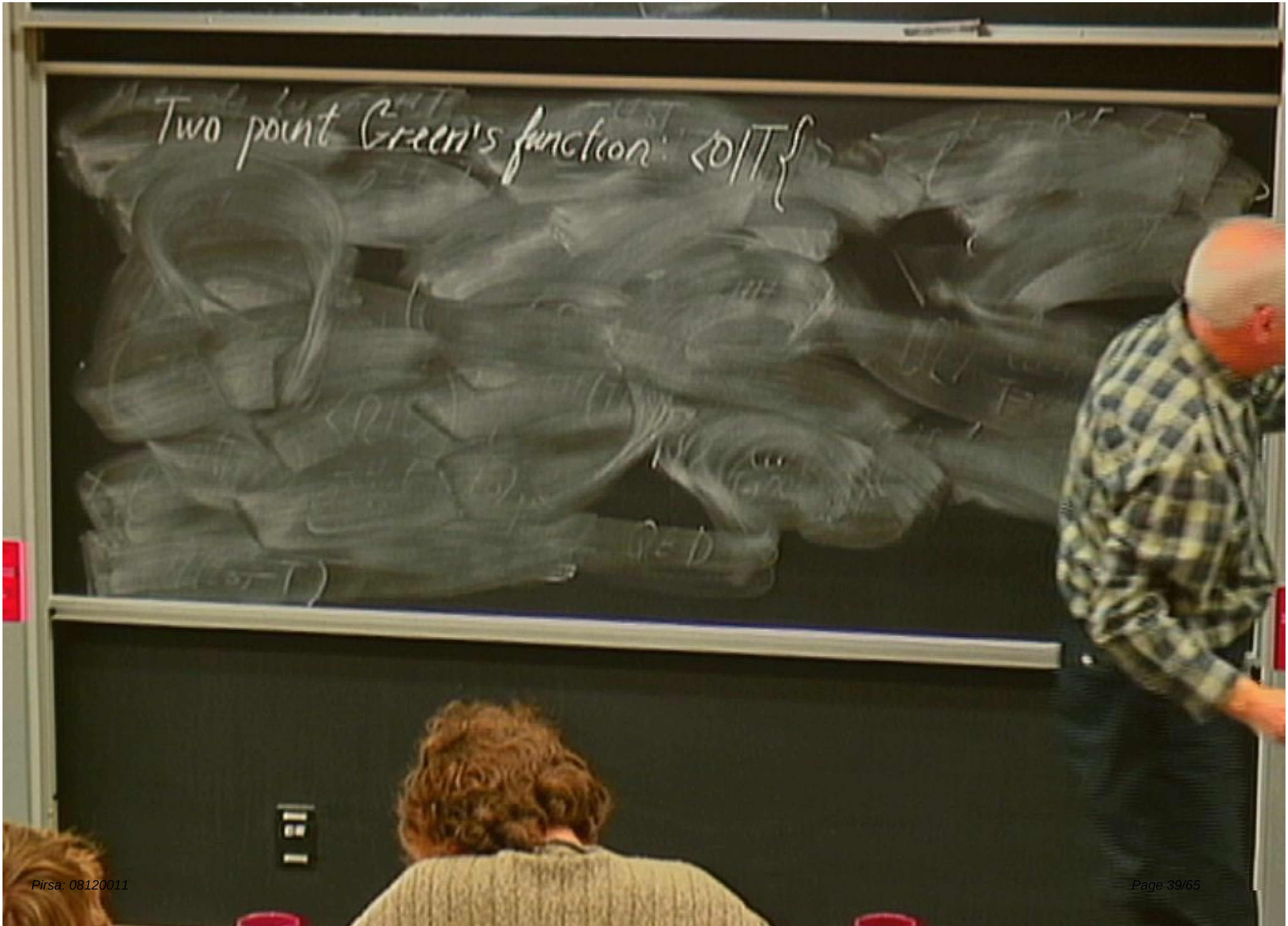

⟨0|T(φ1 φ2 φ3 φ4)|0⟩ =
D_F(x−y) ⇒
propagator

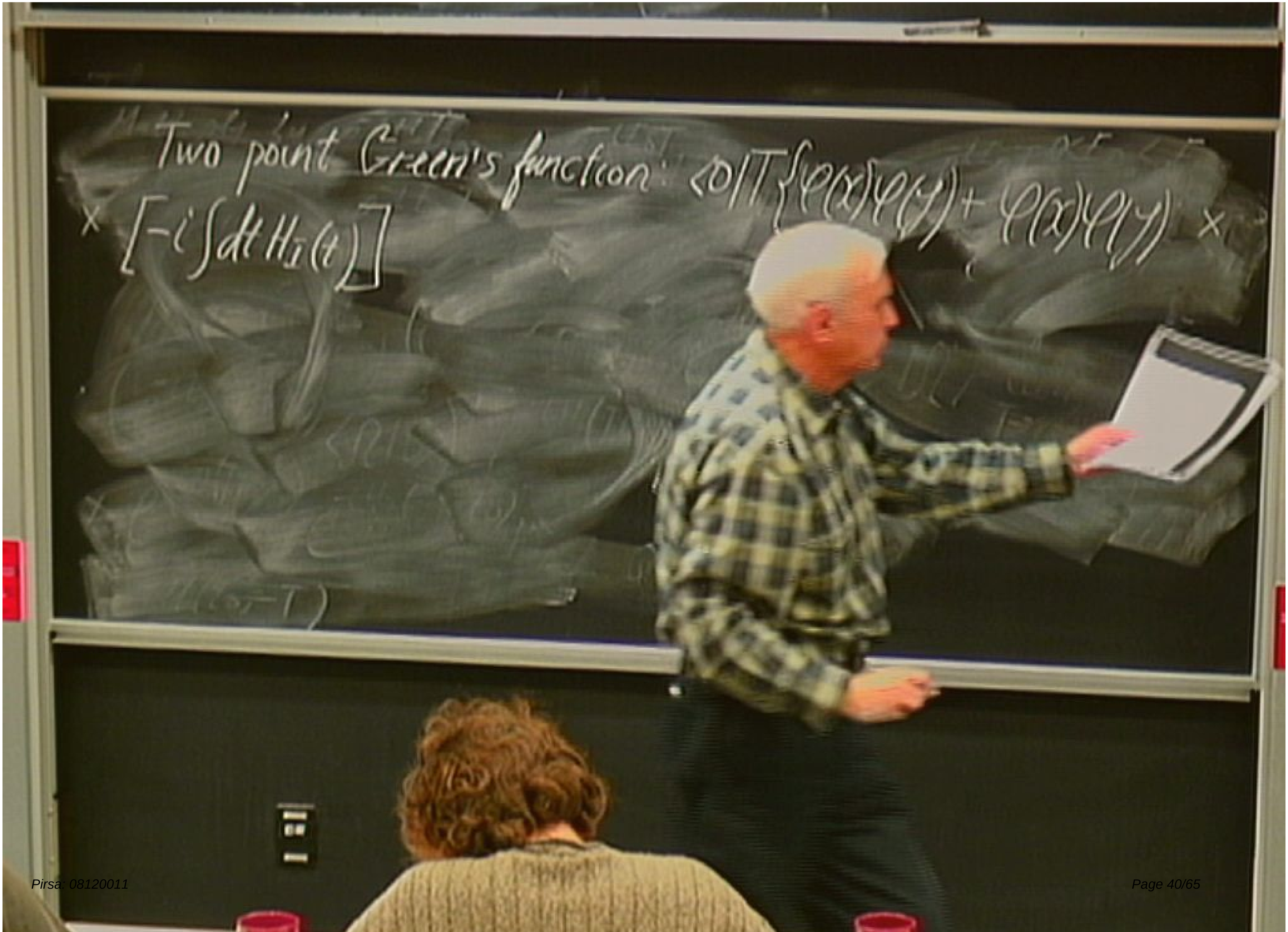


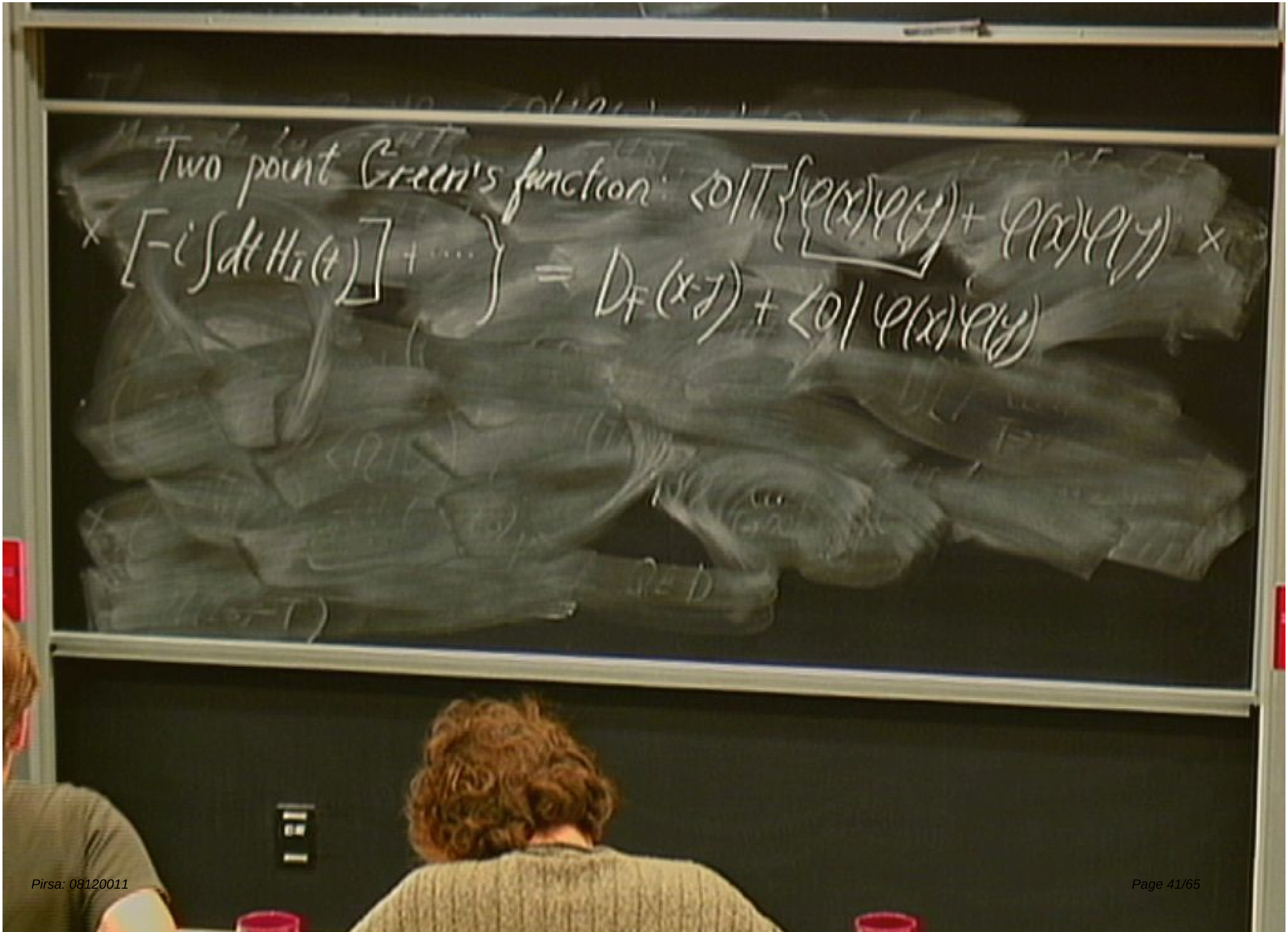
⟨0|T(φ1 φ2 φ3 φ4)|0⟩ =
D_F (x−y) ⇒
x
y
propagator
x1
x2
x3
x4

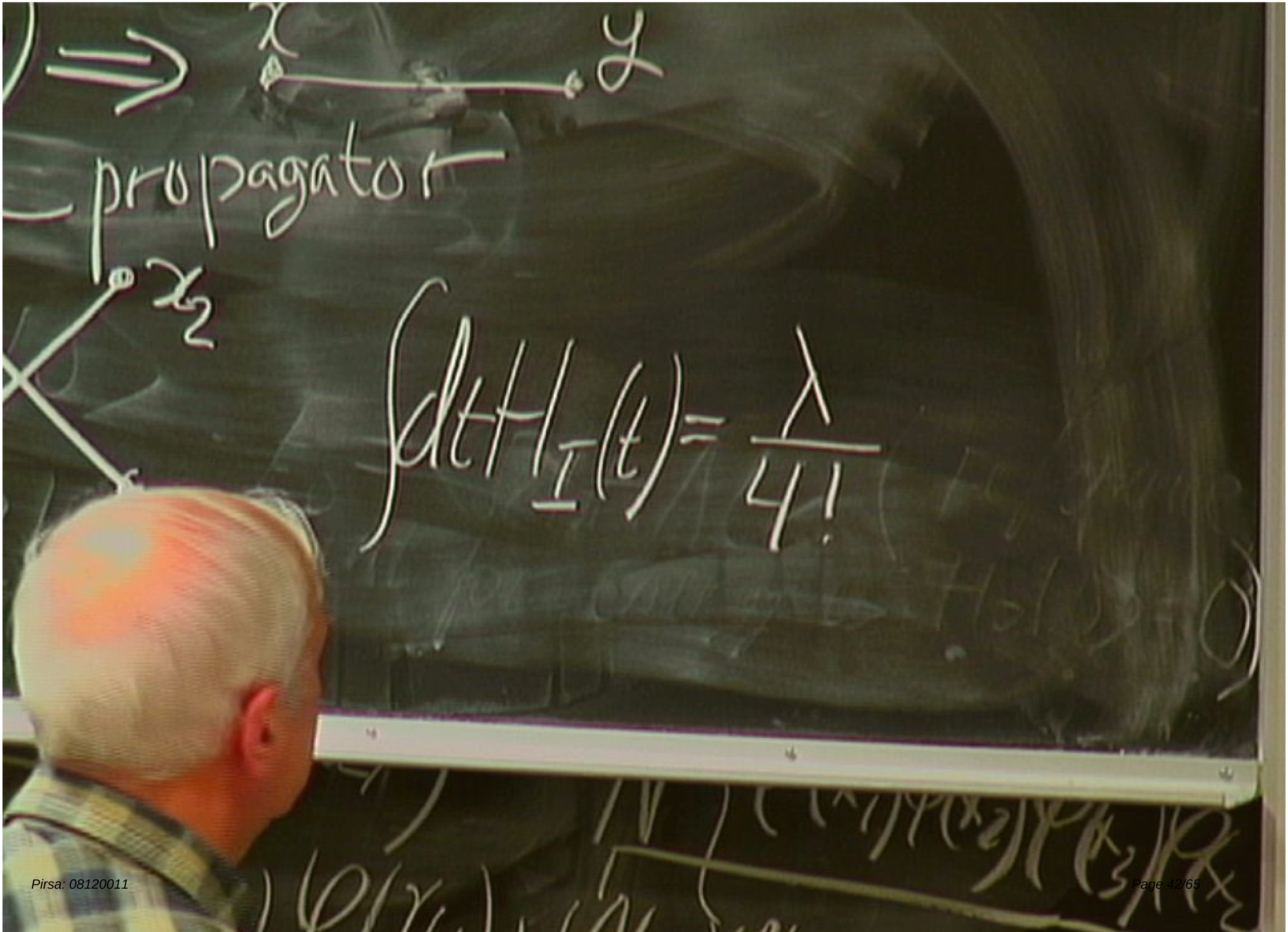

Feynman Diagrams
propagator

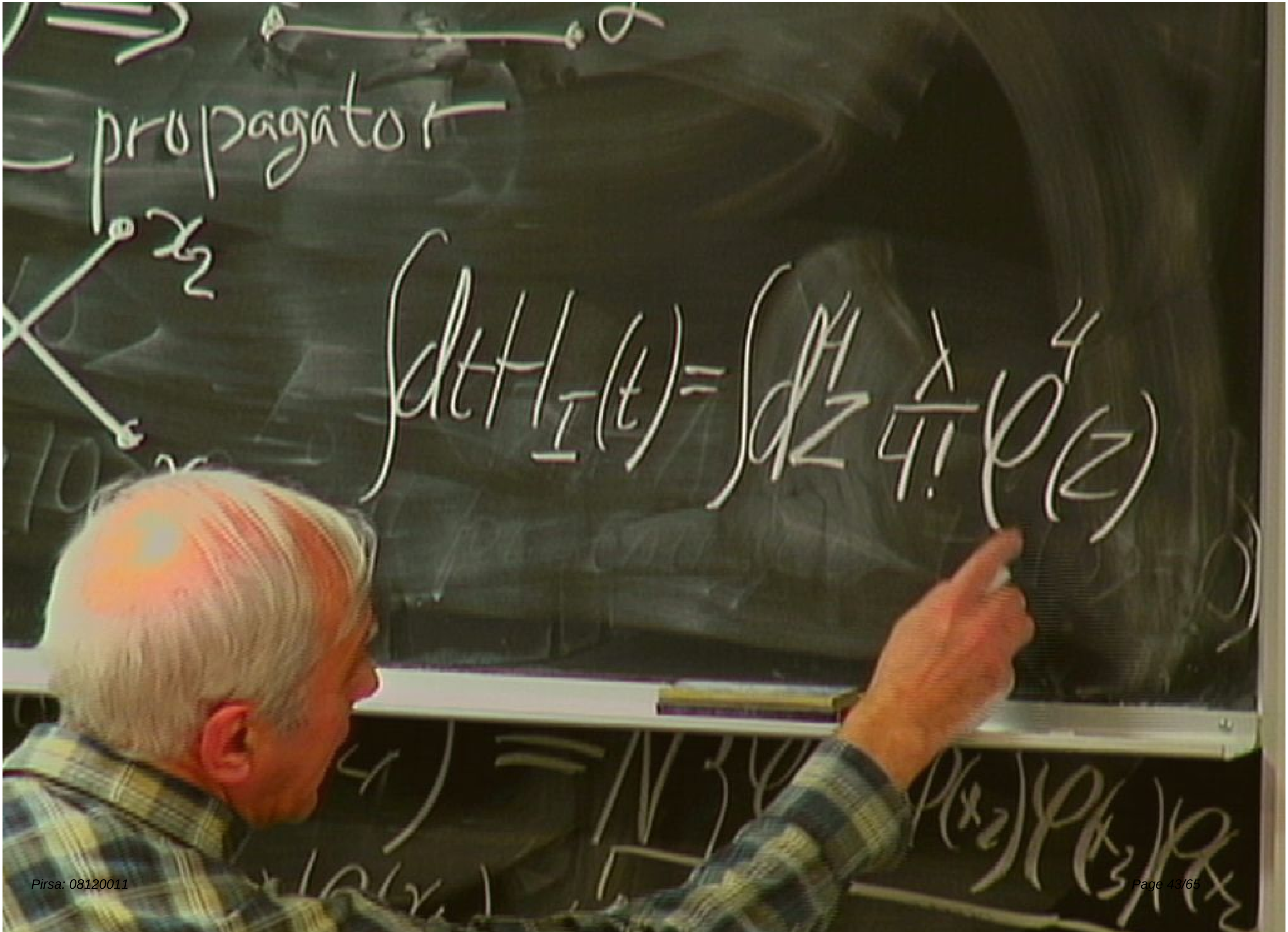

propagator

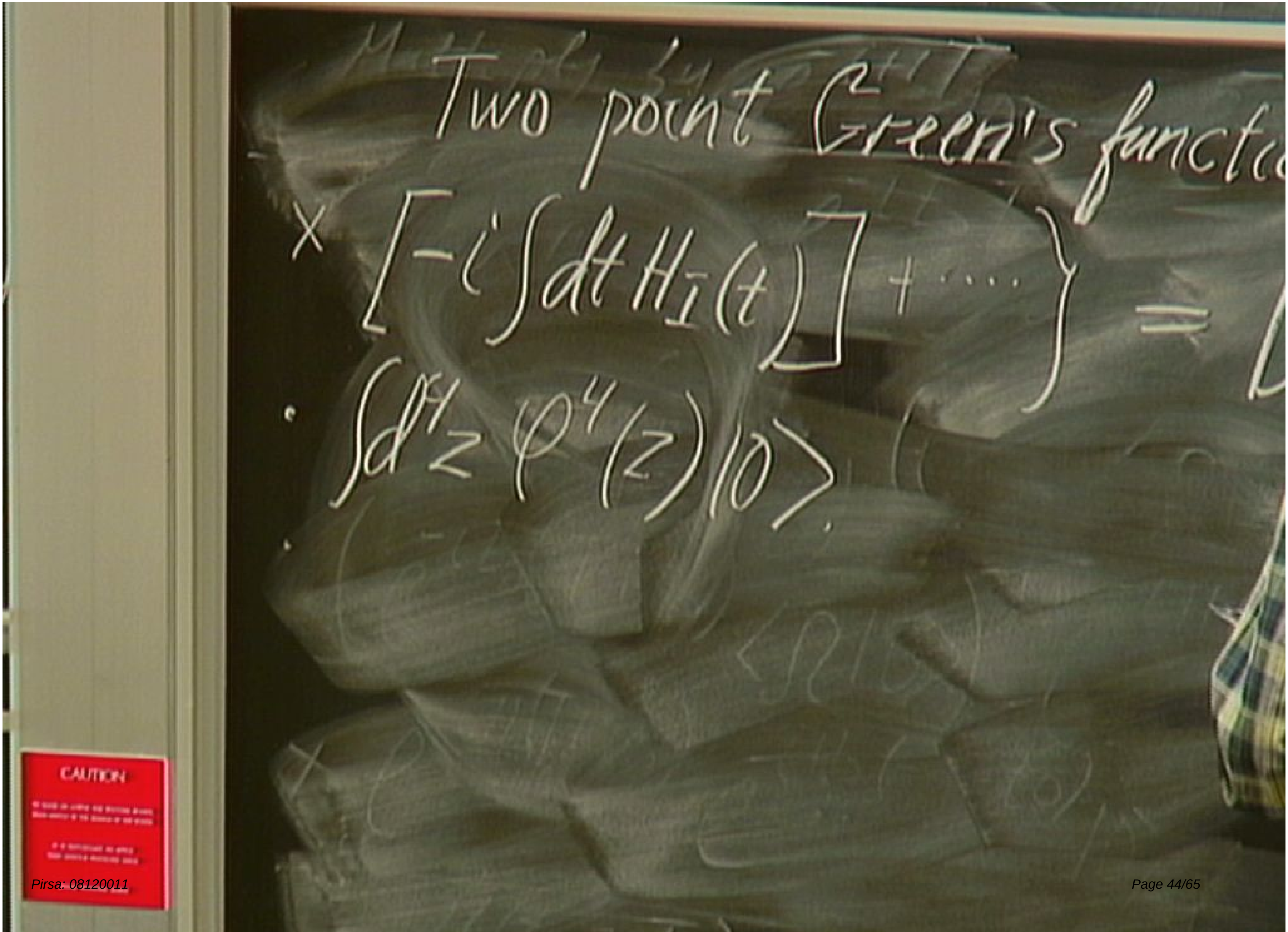

Two point Green's function: <0|T{

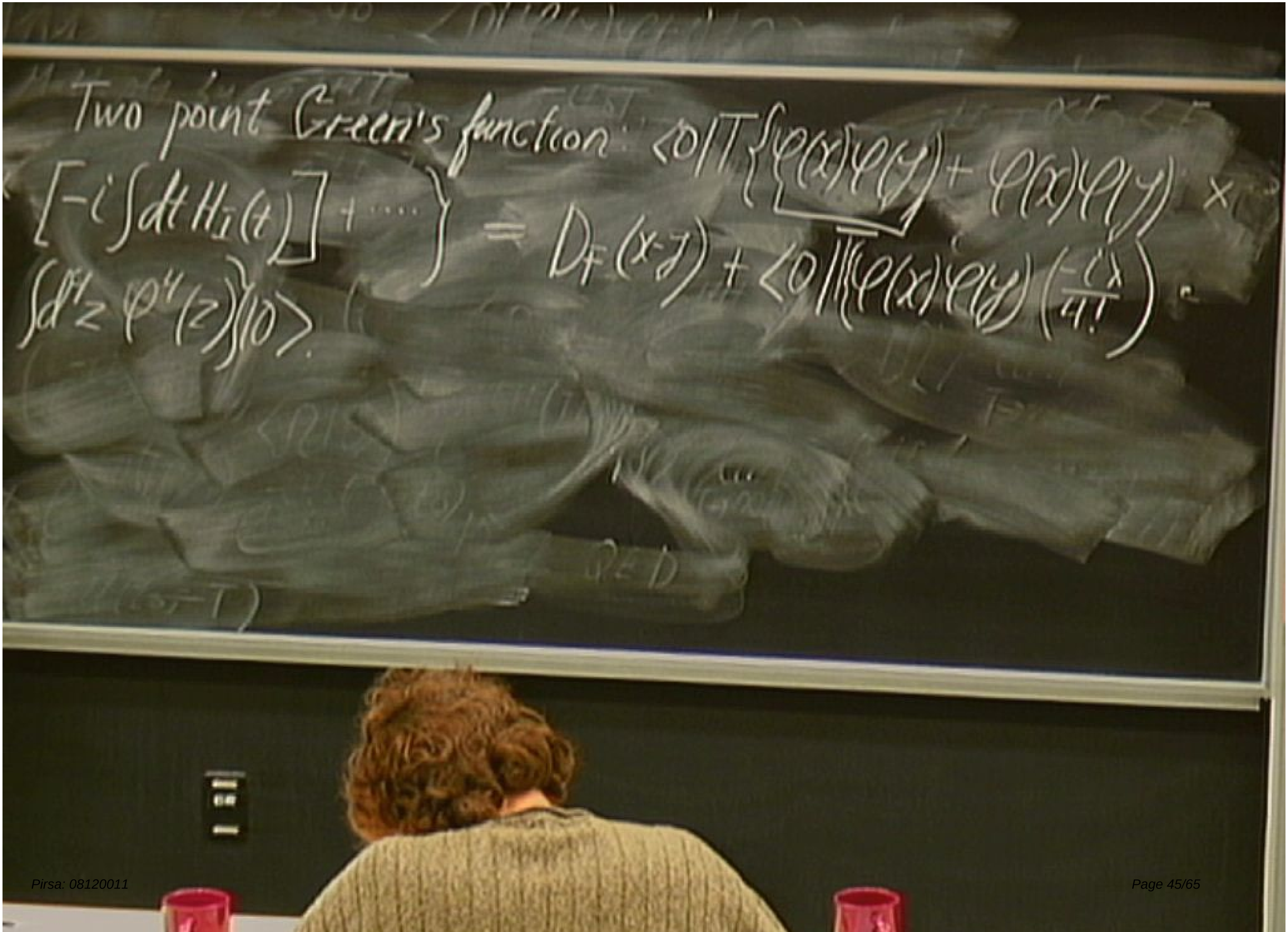

Two point Green's function: <0|T{φ(x)φ(y) + φ(x)φ(y) ×
× [−i∫dt H_I(t)]

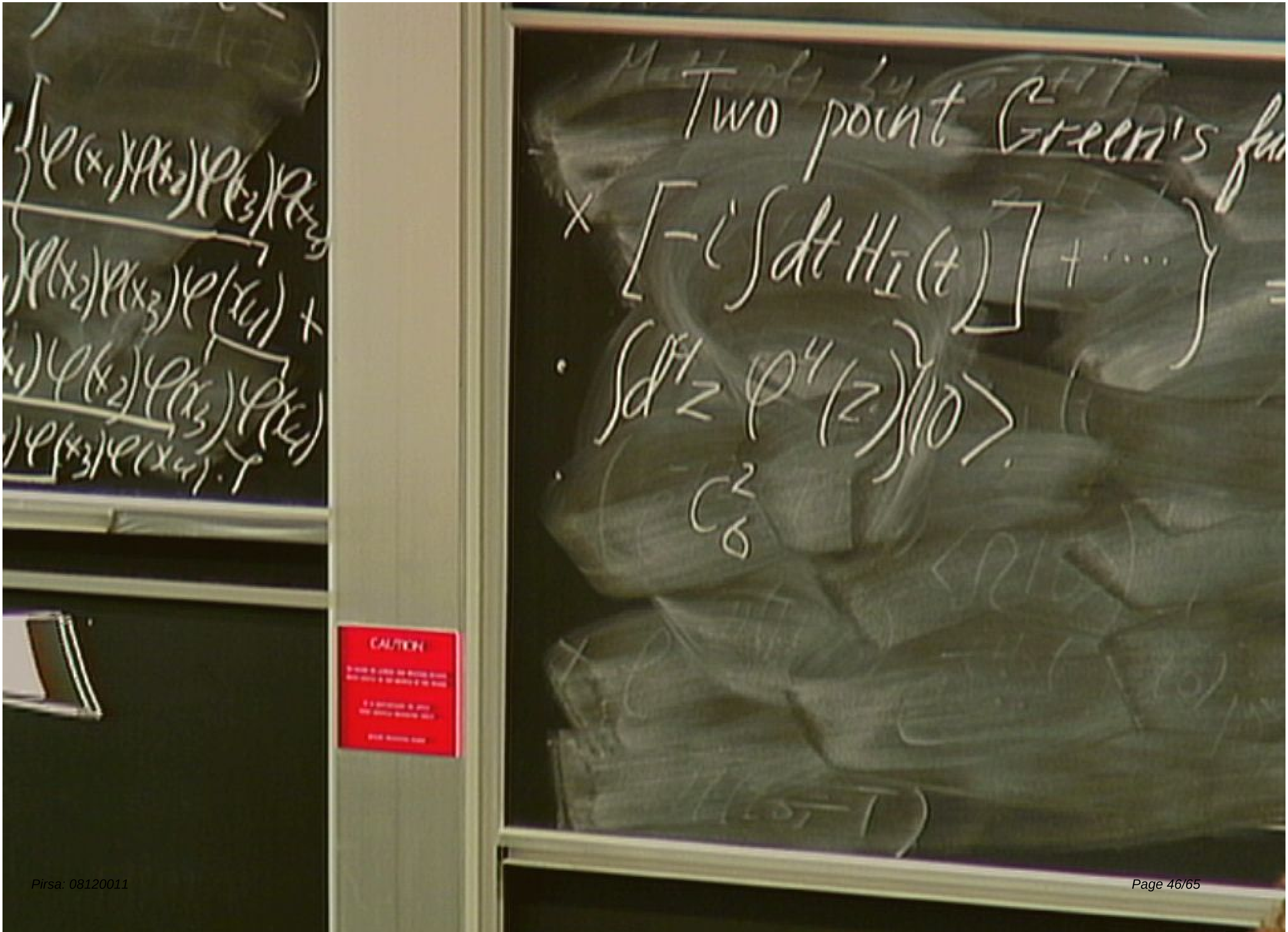

Two point Green's function: ⟨0|T{φ(x)φ(y) + φ(x)φ(y) ×
× [−i∫dt H_I(t)] + ... } = D_F(x−y) + ⟨0| φ(x)φ(y)


x
y
propagator
x2
∫dt H_I(t) = λ/4!


propagator
∫dt H_I(t) = ∫d⁴z λ/4! φ⁴(z)


Two point Green's functi
× [−i ∫dt H_I(t)] + ... } =
∫d^4z φ^4(z)|0⟩.
CAUTION

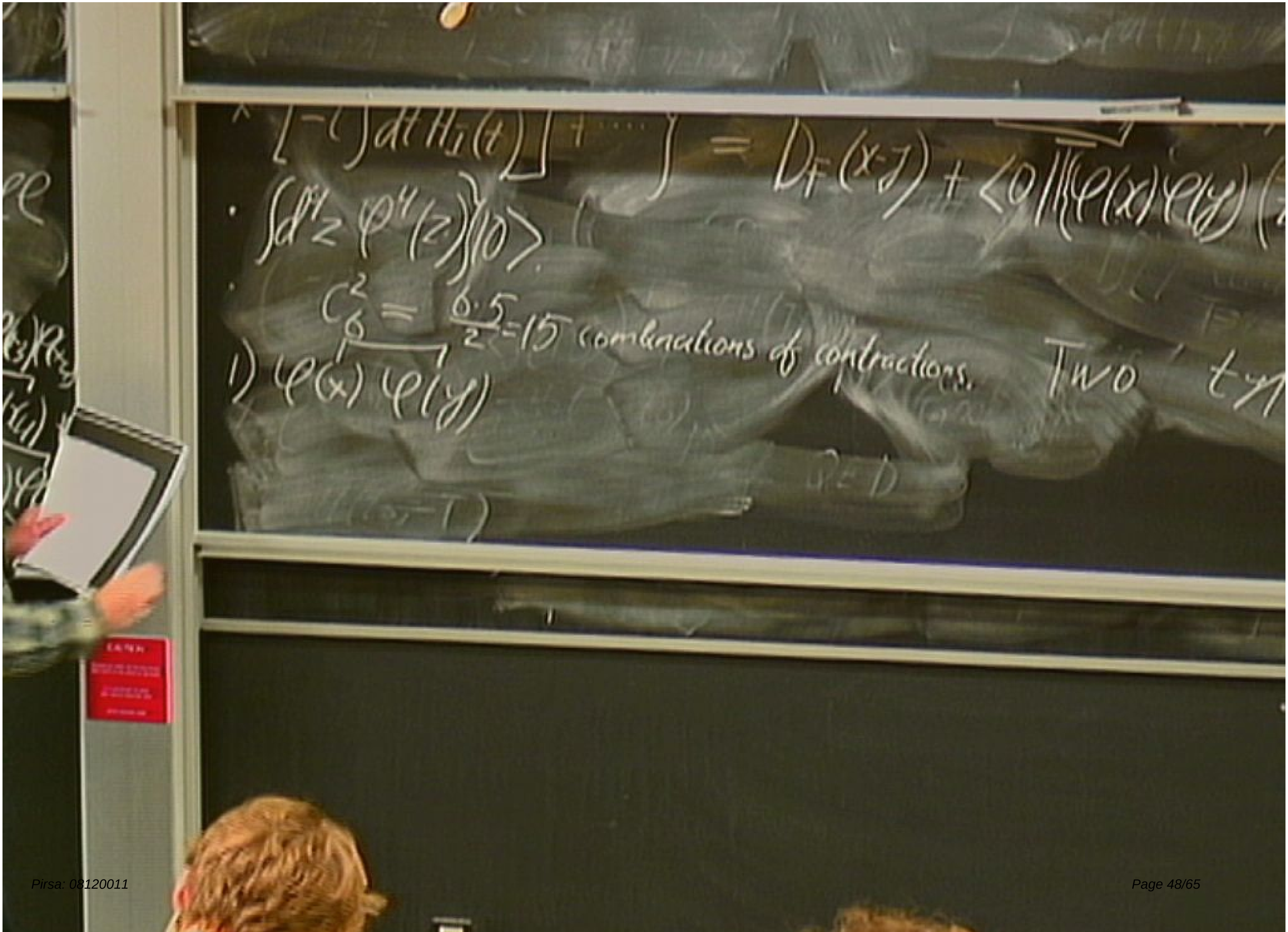

Two point Green's function: <0|T{φ(x)φ(y) + φ(x)φ(y) ×
[-i∫dt H_I(t)] + ... } = D_F(x-y) + <0|T φ(x)φ(y) (-iλ/4!)
∫d^4z φ^4(z)}|0>.

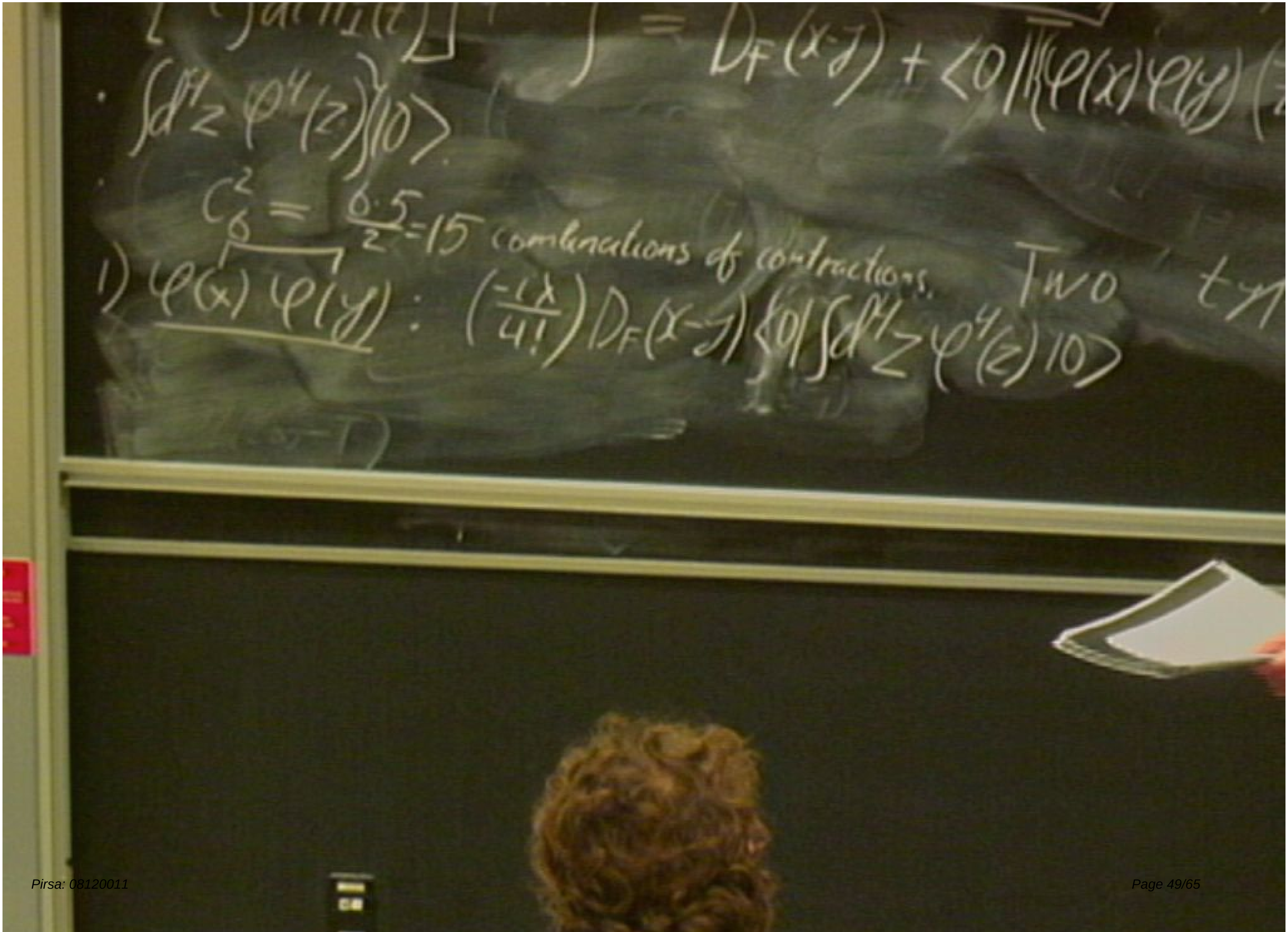

Two point Green's
CAUTION

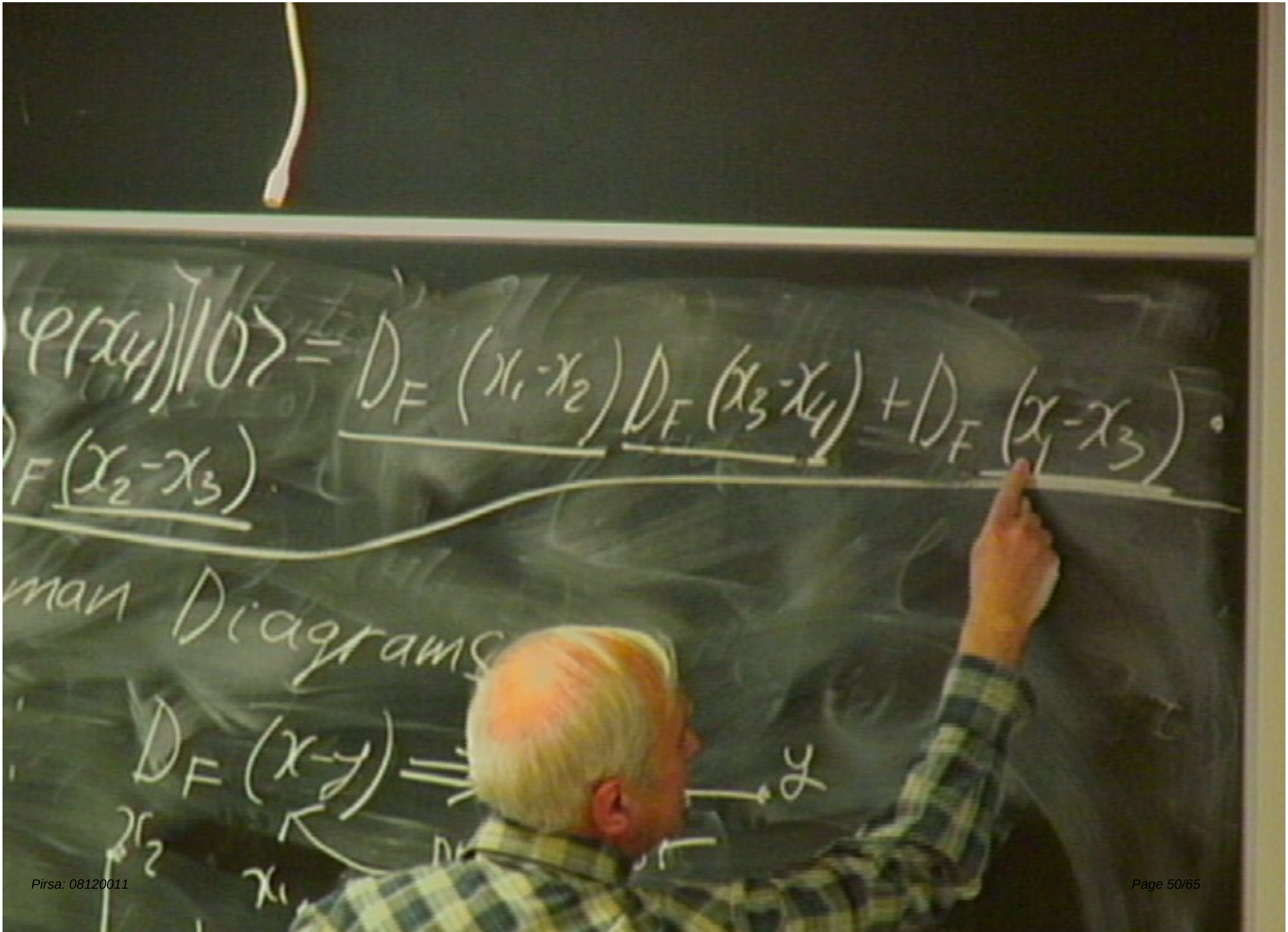
Two point Green's function:

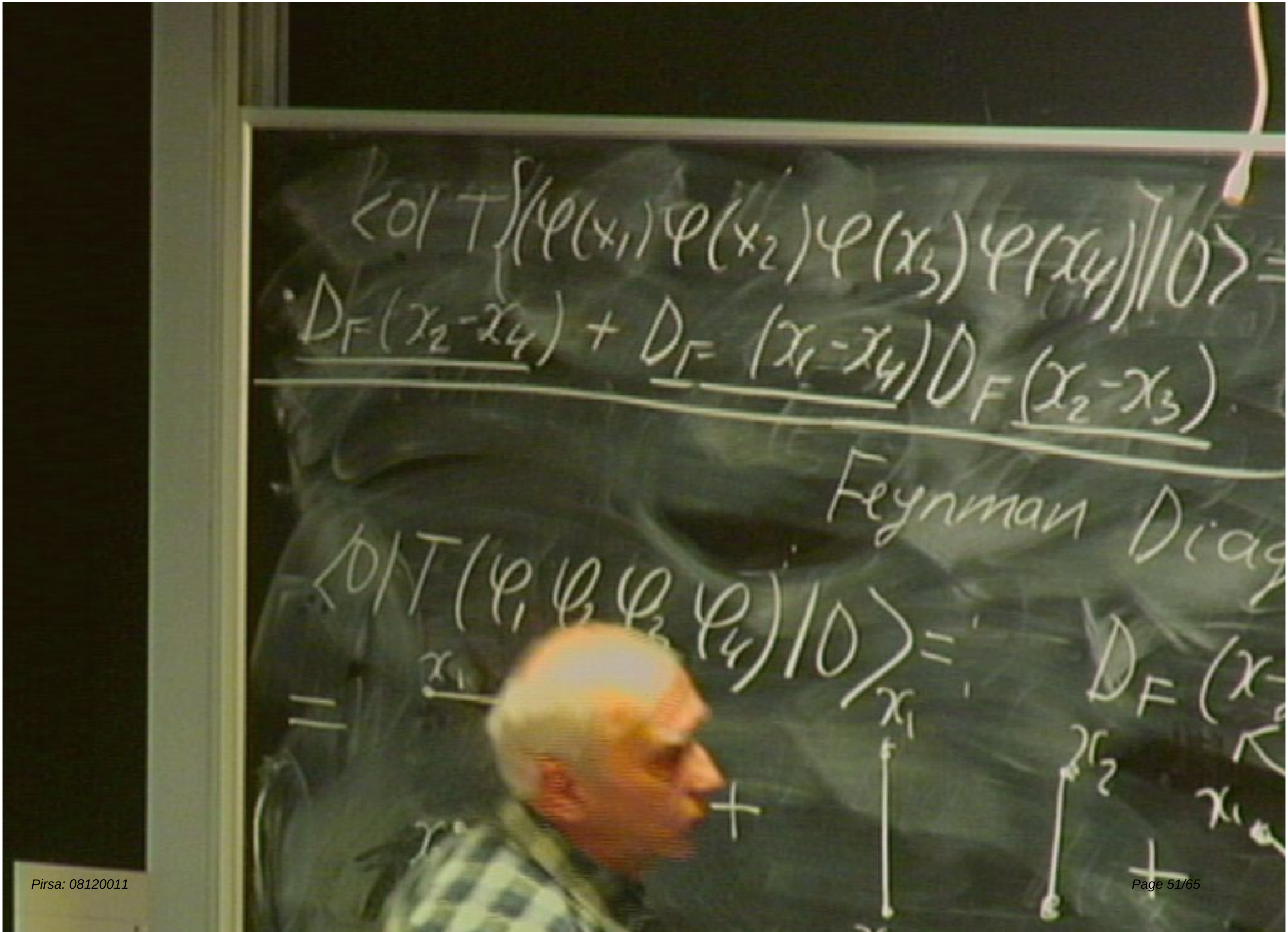
$$\times \left[-i \int dt\, H_I(t) \right] + \cdots \Big\} = D_F($$

- $\int d^4 z\, \varphi^4(z) \Big\} |0\rangle.$
- $C_6^2 = \frac{6 \cdot 5}{2} = 15$ combinations of con


$\int d^4z\, \varphi^4(z)\}|0\rangle$
$= D_F(x-y) + \langle 0|T\{\varphi(x)\varphi(y)$
$C_6^2 = \frac{6 \cdot 5}{2} = 15$ combinations of contractions.
Two
1) $\varphi(x)\, \varphi(y)$

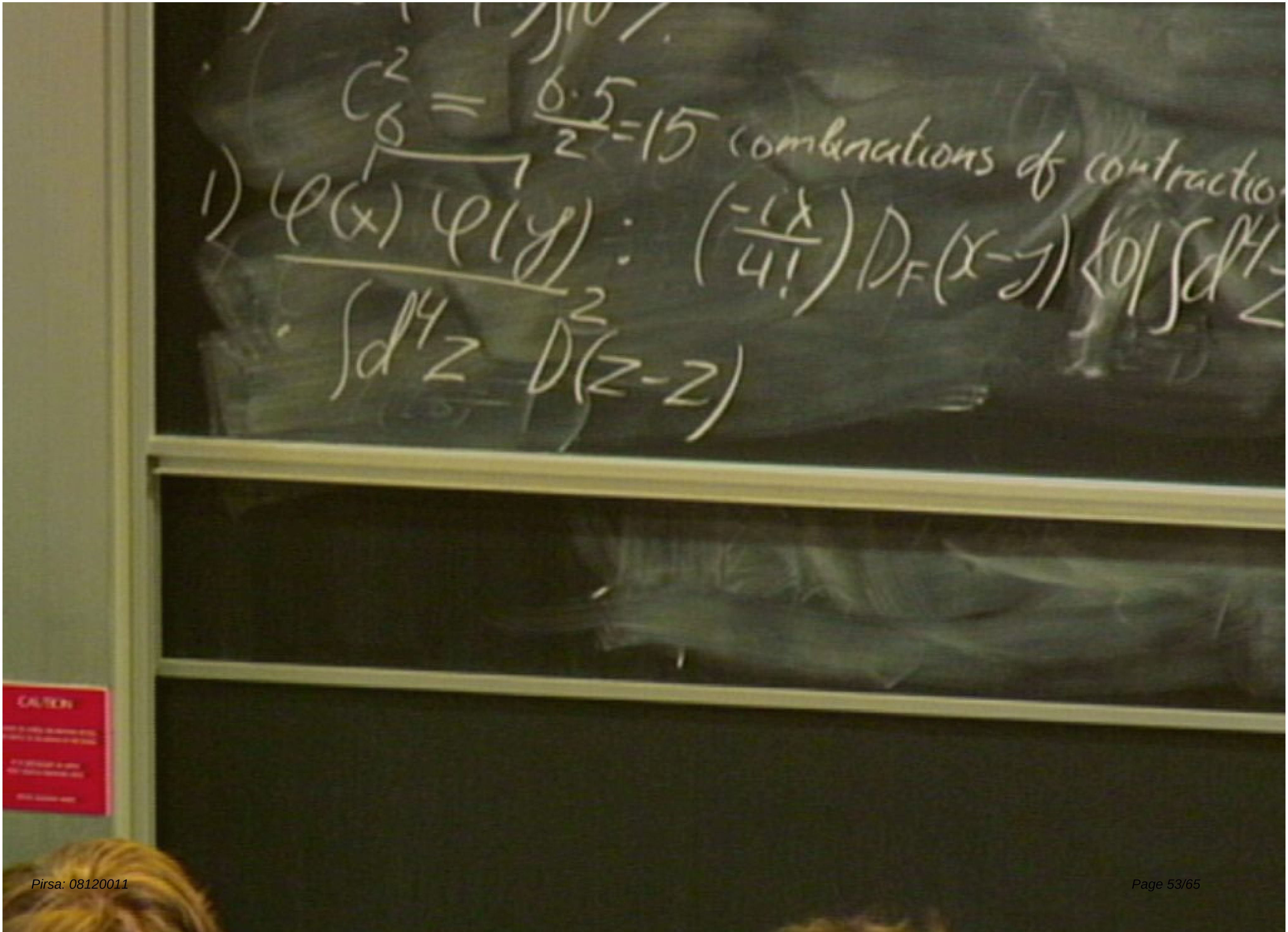

= D_F(x-y) + <0|T{φ(x)φ(y)
∫d^4z φ^4(z)}|0>
C^2_6 = 6·5/2 = 15 combinations of contractions. Two
1) φ(x) φ(y) : (-iλ/4!) D_F(x-y) <0| ∫d^4z φ^4(z) |0>

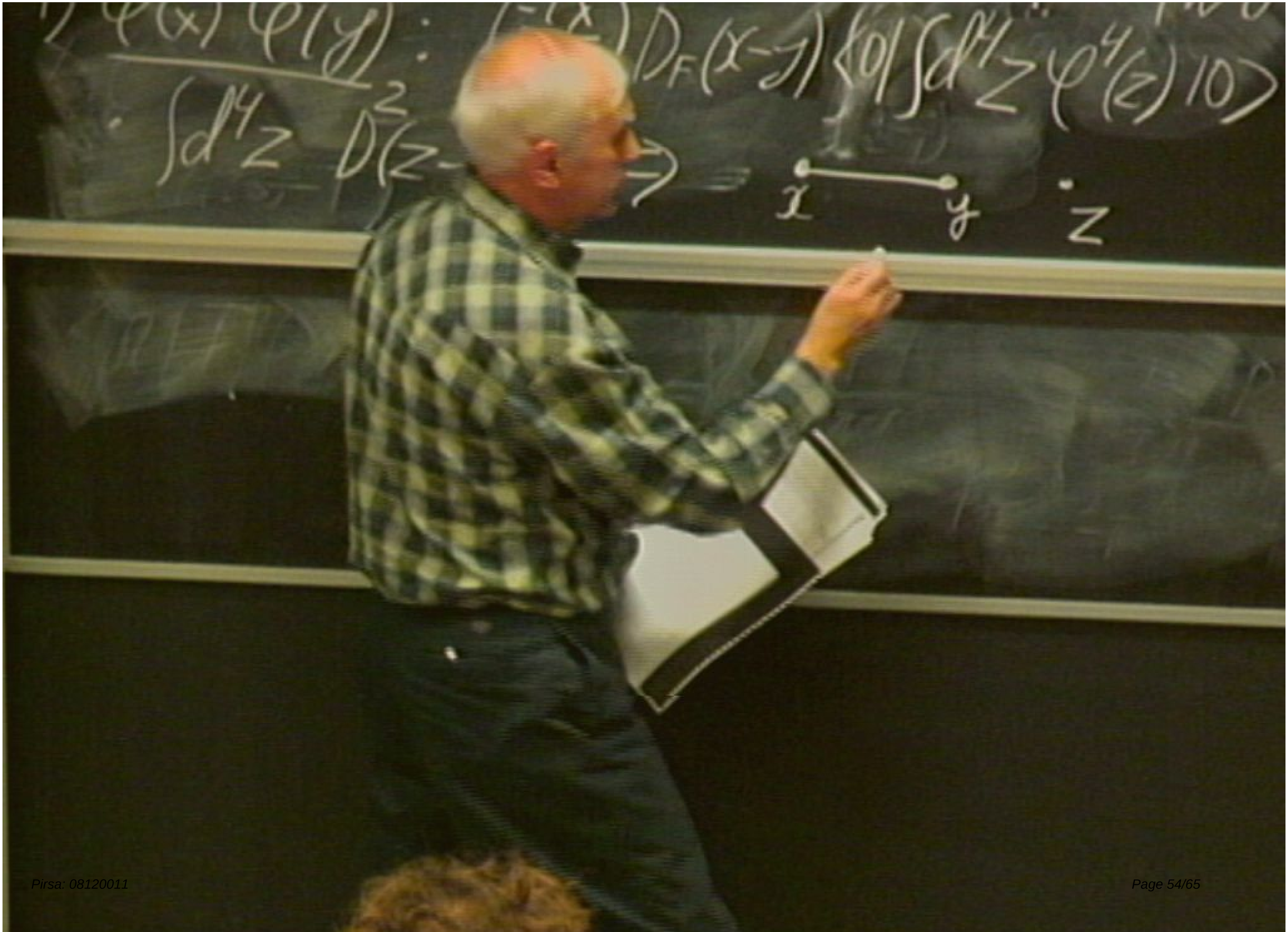

φ(x4)]|0⟩ = DF (x1-x2) DF (x3-x4) + DF (x1-x3) ·
DF (x2-x3).
man Diagrams
DF (x-y) ⟹ y
x2
x1

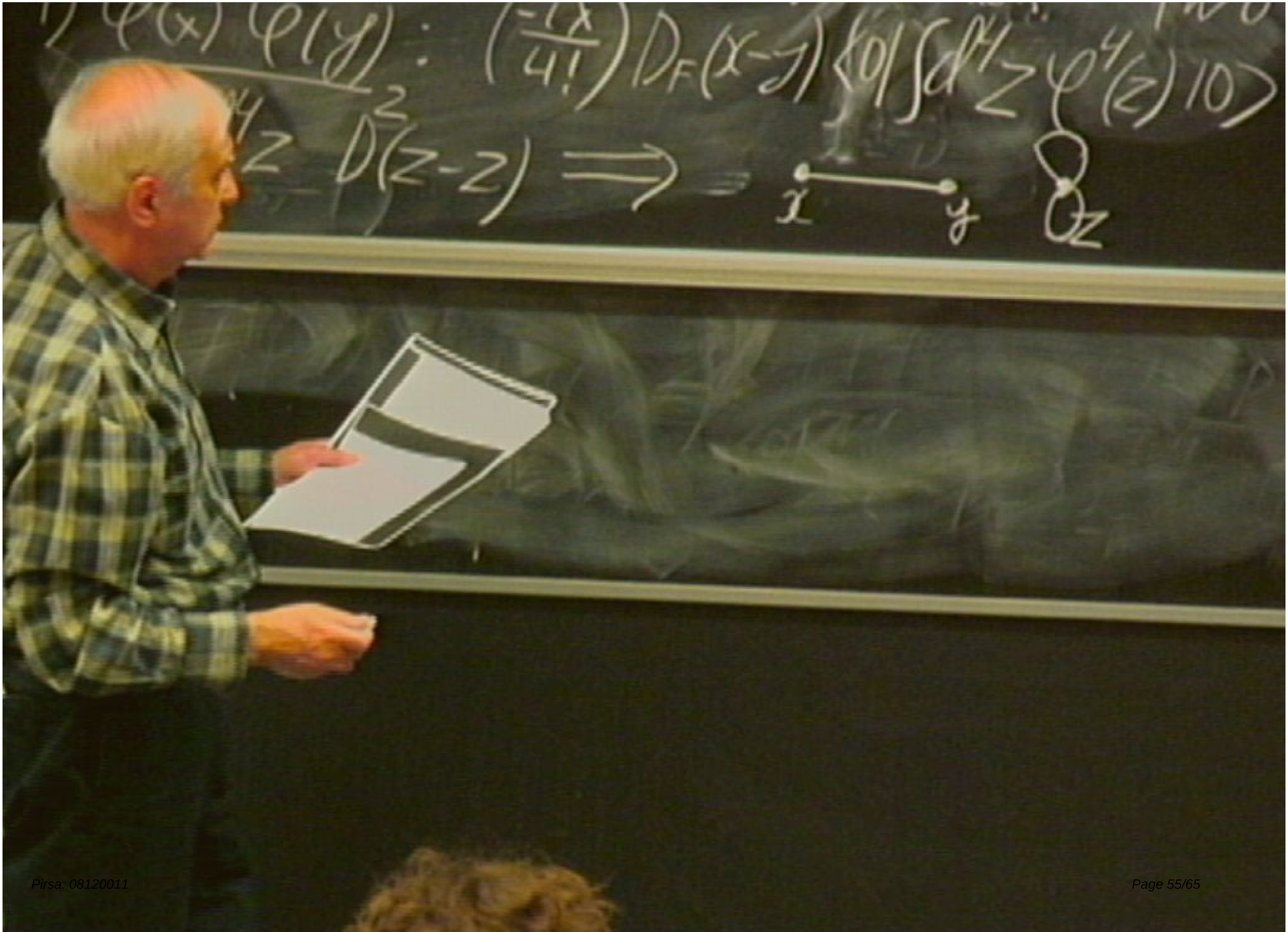

⟨0| T{φ(x1) φ(x2) φ(x3) φ(x4)}|0⟩ =
DF(x2 - x4) + DF (x1 - x4) DF (x2 - x3)
Feynman Diag
⟨0| T(φ1 φ2 φ3 φ4)|0⟩ =
DF (x
x1
x2

...mbinations of contractions. Two types:

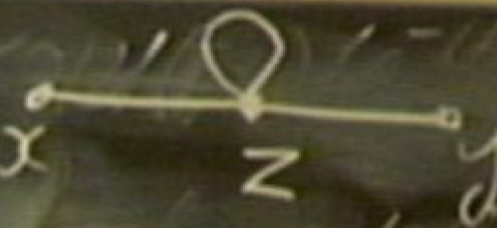
$$\ldots) D_F(x-y) \langle 0 | \int d^4z \, \varphi^4(z) | 0 \rangle = 3 \left(\frac{-i\lambda}{4!}\right) \cdot D_F(x-y).$$

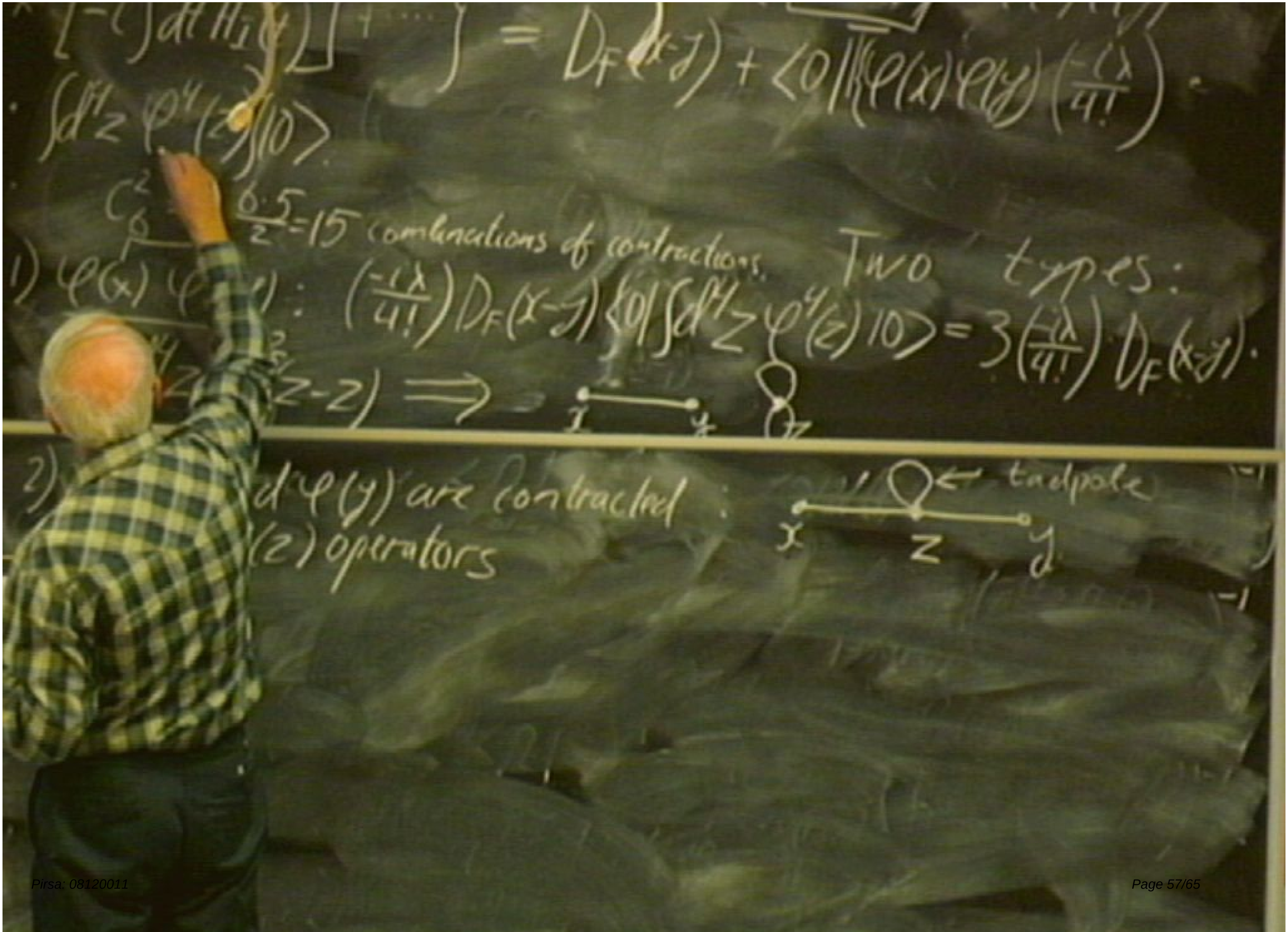

$C_6^2 = \frac{6 \cdot 5}{2} = 15$ combinations of contractio
1) $\varphi(x)\,\varphi(y)$: $\left(\frac{-i\lambda}{4!}\right) D_F(x-y) \langle 0| \int d^4z$
$\cdot \int d^4z \; D(z-z)$
CAUTION

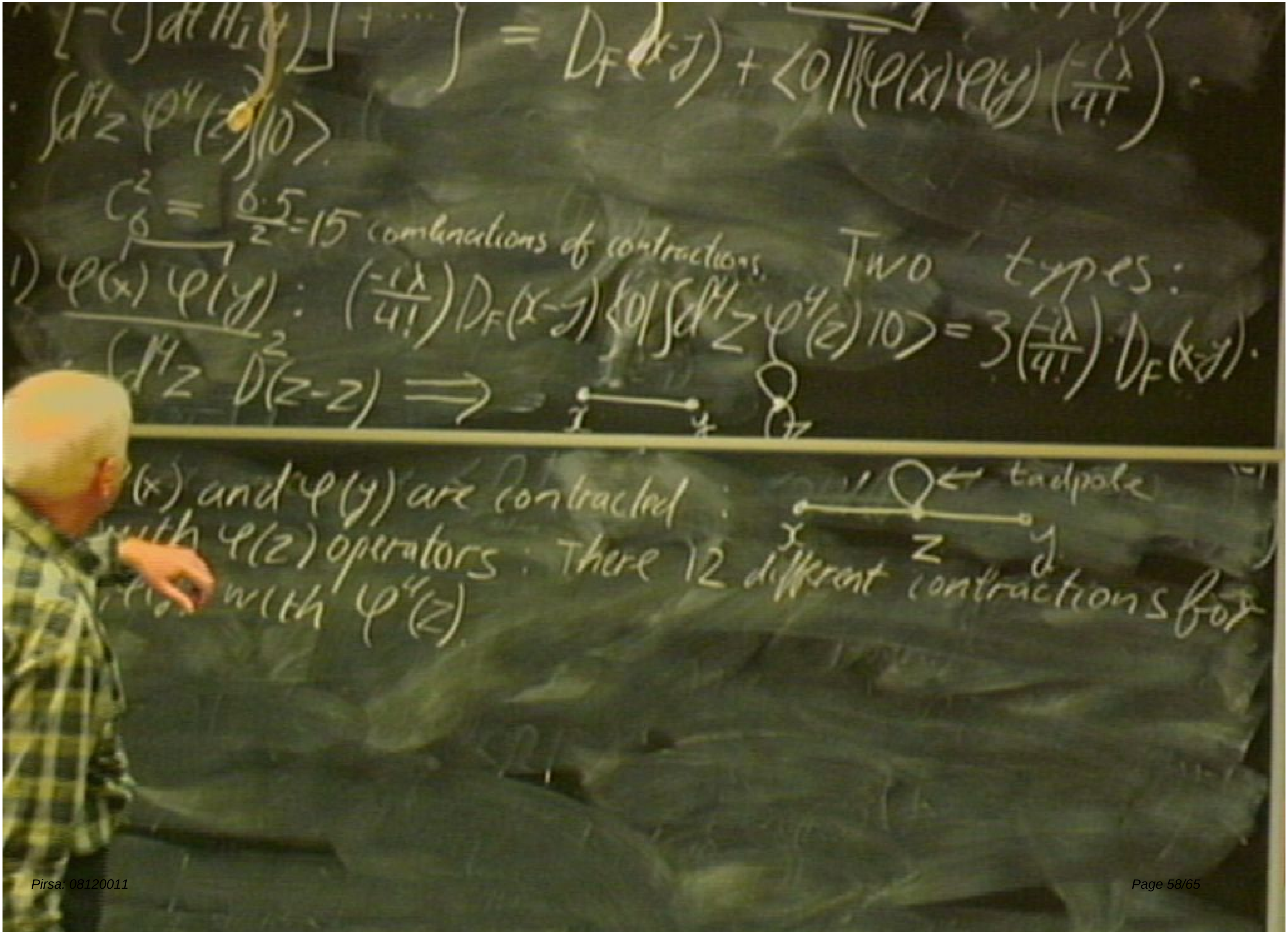




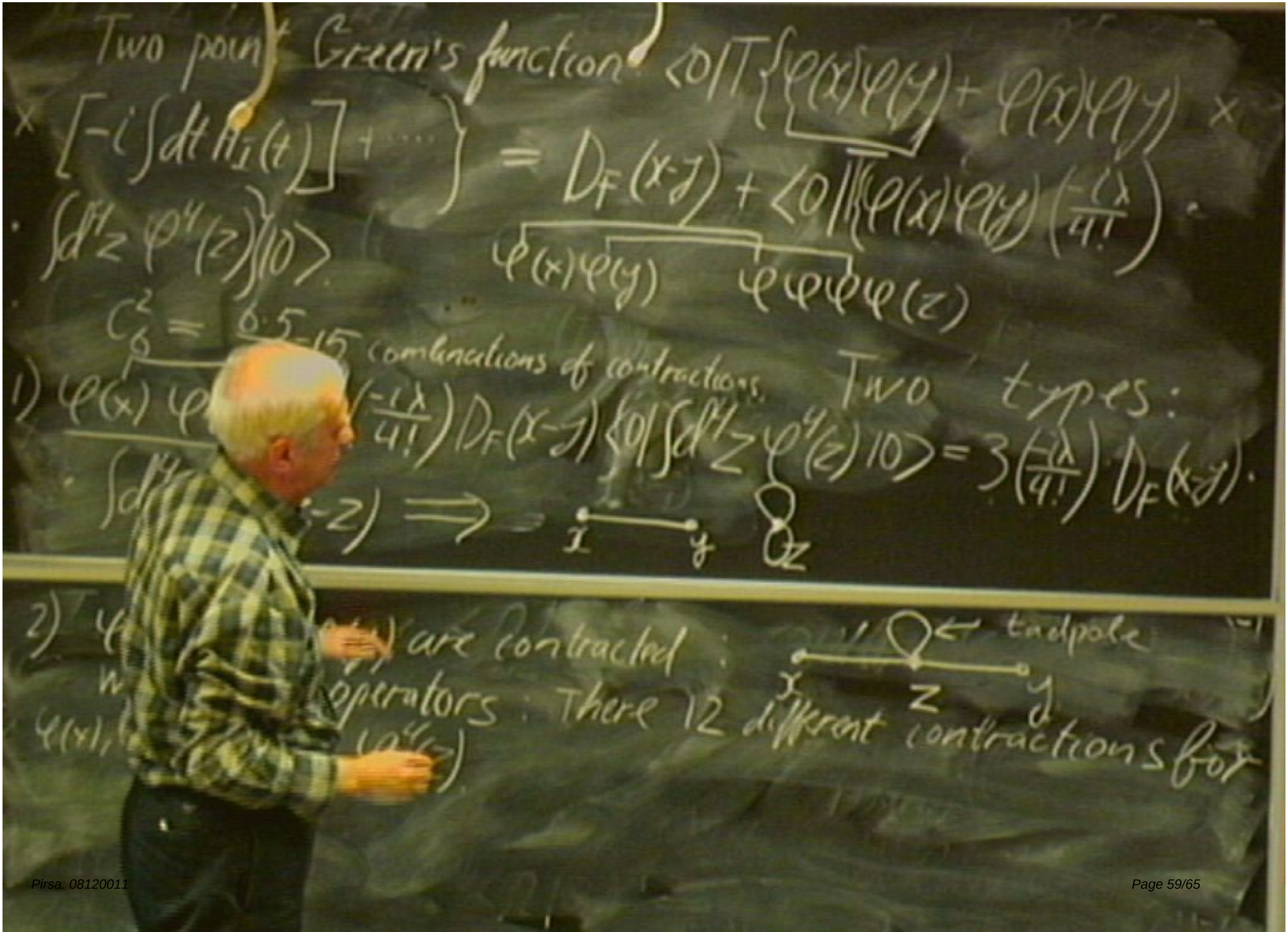
2) $\varphi(x)$ and $\varphi(y)$ are contracted with $\varphi(z)$ operators :

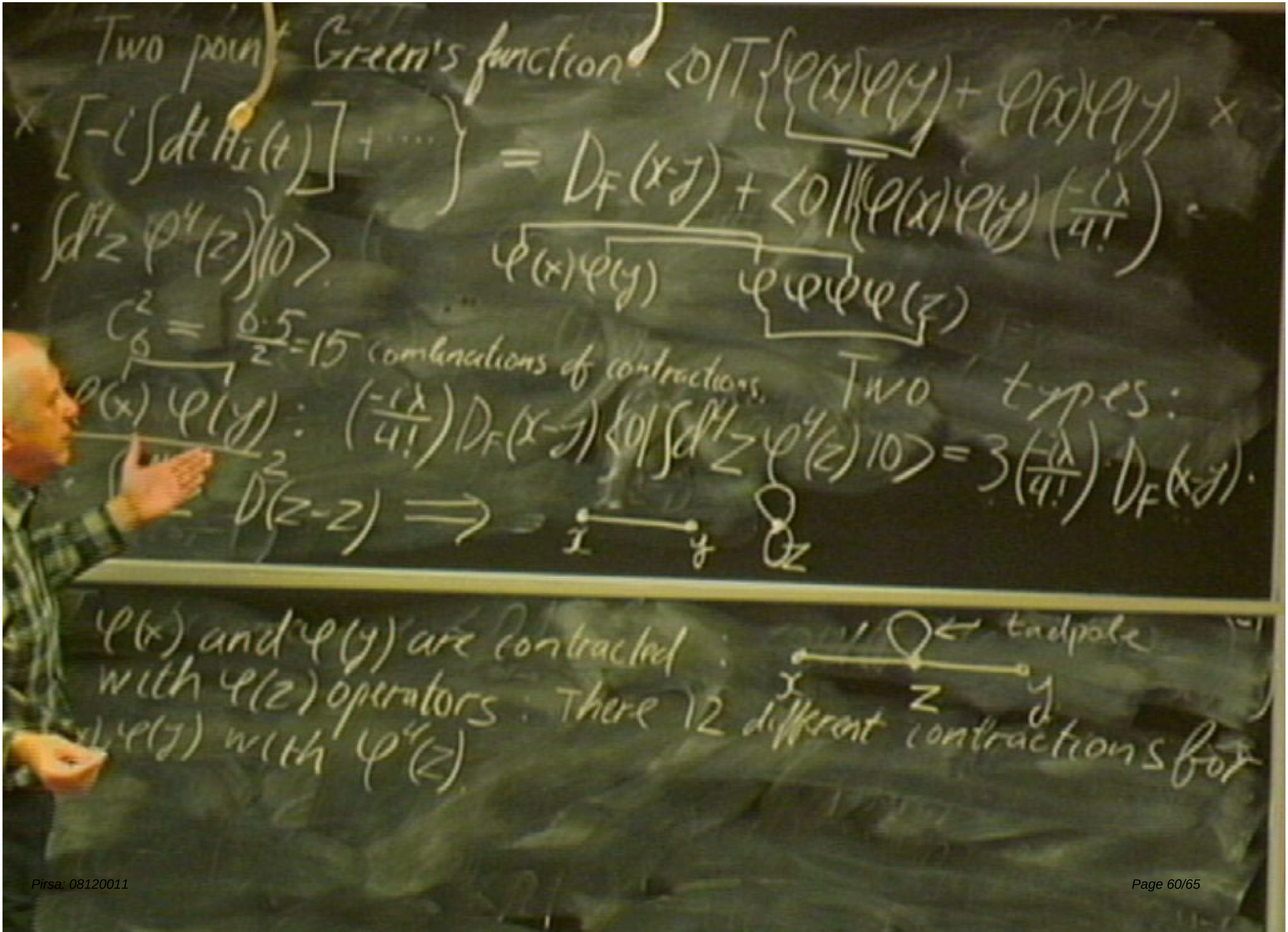



=15 combinations of contractions
Two types:
are contracted
operators
tadpole

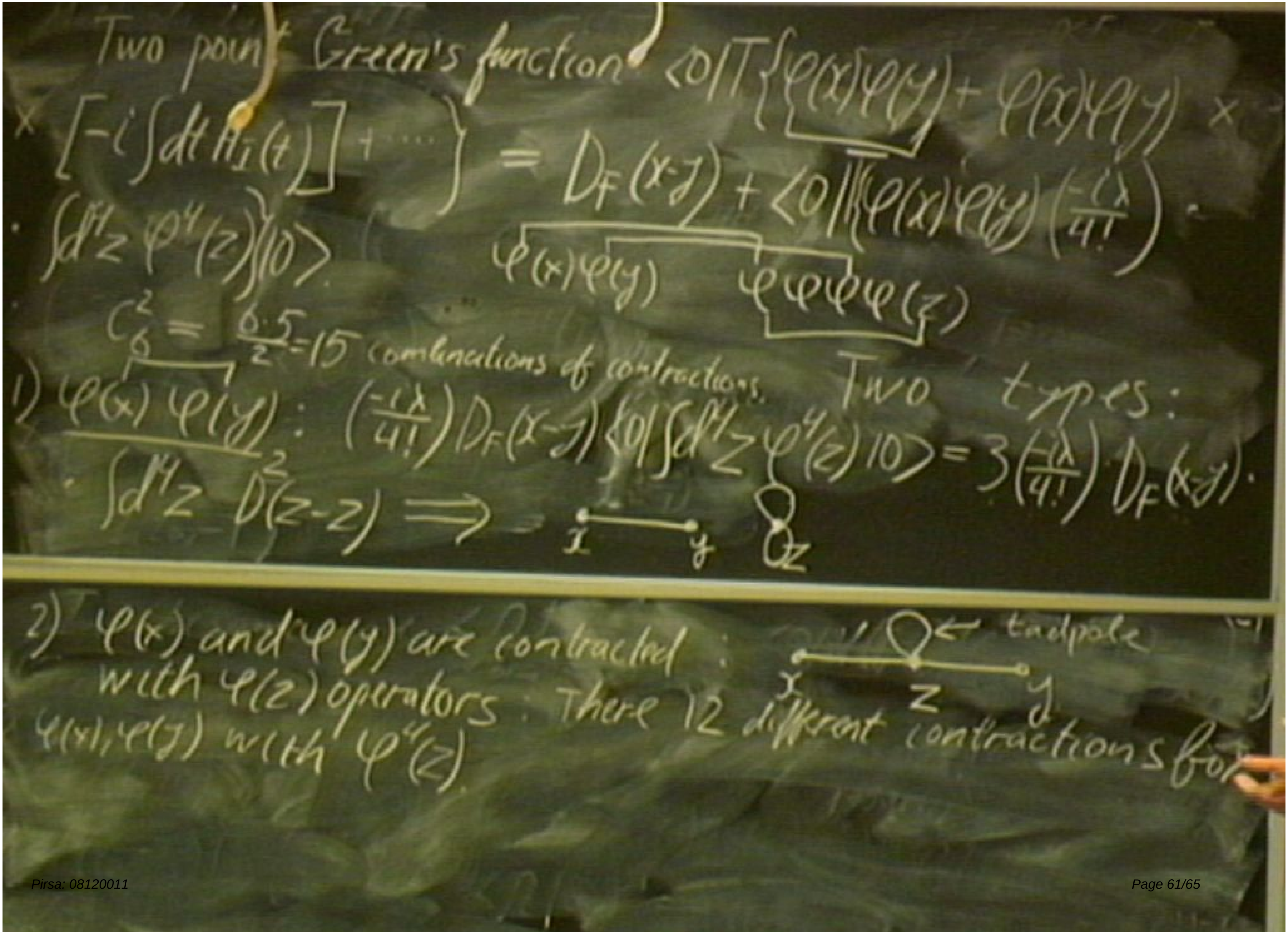


= D_F(x-y) + <0|T φ(x) φ(y) (-iλ/4!)
∫d^4z φ^4(z)|0>.
C_6^2 = 6·5/2 = 15 combinations of contractions. Two types:
1) φ(x) φ(y) : (-iλ/4!) D_F(x-y) <0|∫d^4z φ^4(z)|0> = 3(-iλ/4!) D_F(x-y) ·
∫d^4z D(z-z) ⟹
x
y
z
(x) and φ(y) are contracted :
tadpole
x
z
y
with φ(z) operators : There 12 different contractions for
with φ^4(z).

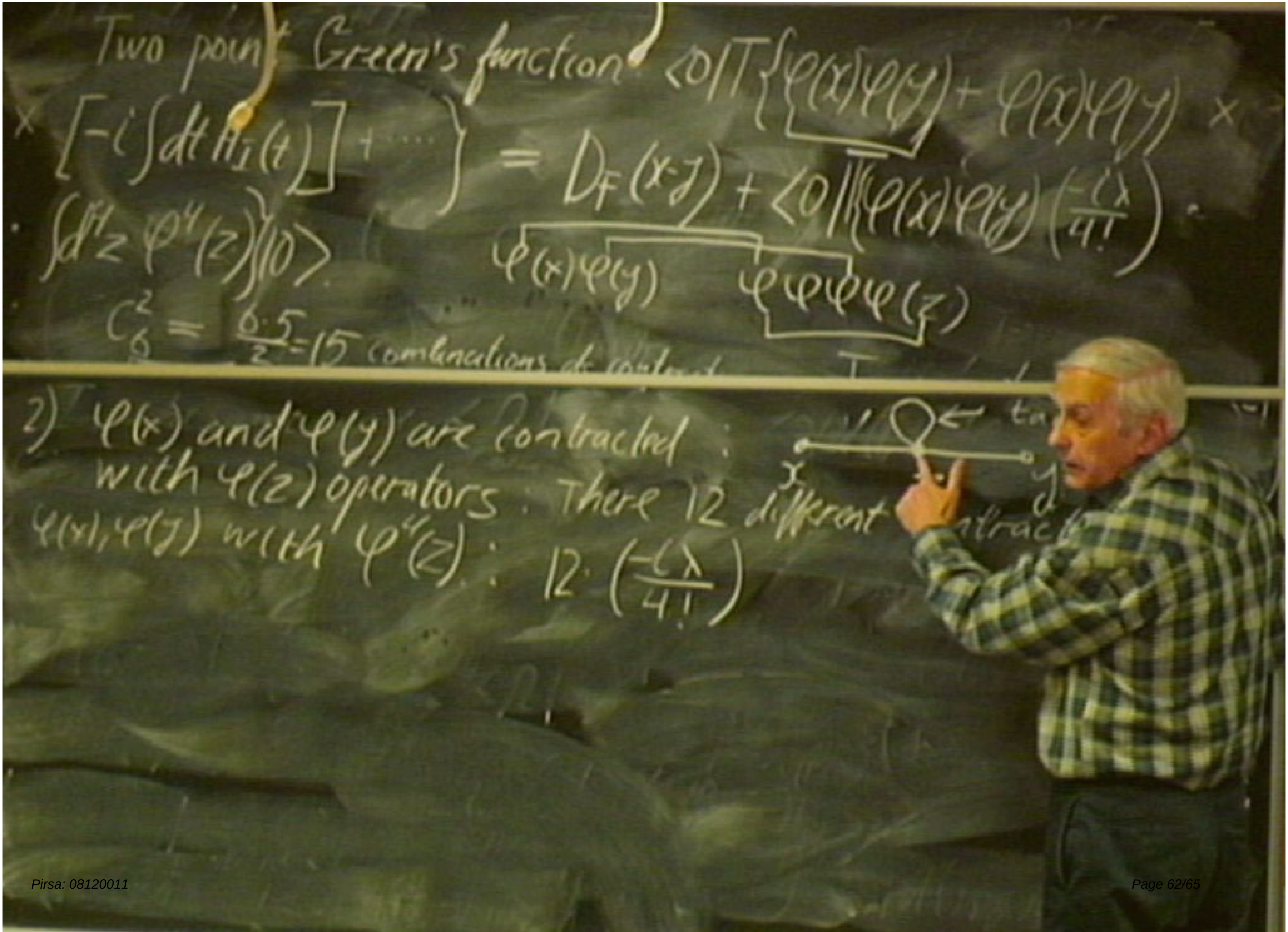

Two point Green's function: <0|T{φ(x)φ(y) + φ(x)φ(y) ×
[−i ∫dt H_I(t)] + ...} = D_F(x−y) + <0|T{φ(x)φ(y) (−iλ/4!) ·
∫d⁴z φ⁴(z)}|0>
φ(x)φ(y) φφφφ(z)
C²₆ = 6·5/2 = 15 combinations of contractions. Two types:
1) φ(x) φ(y) (−iλ/4!) D_F(x−y) <0| ∫d⁴z φ⁴(z) |0> = 3 (−iλ/4!) D_F(x−y) ·
∫d⁴z D_F(z−z) ⟹
x y z
2) are contracted
operators. There 12 different contractions for
φ(x), (φ⁴(z))
tadpole
x z y

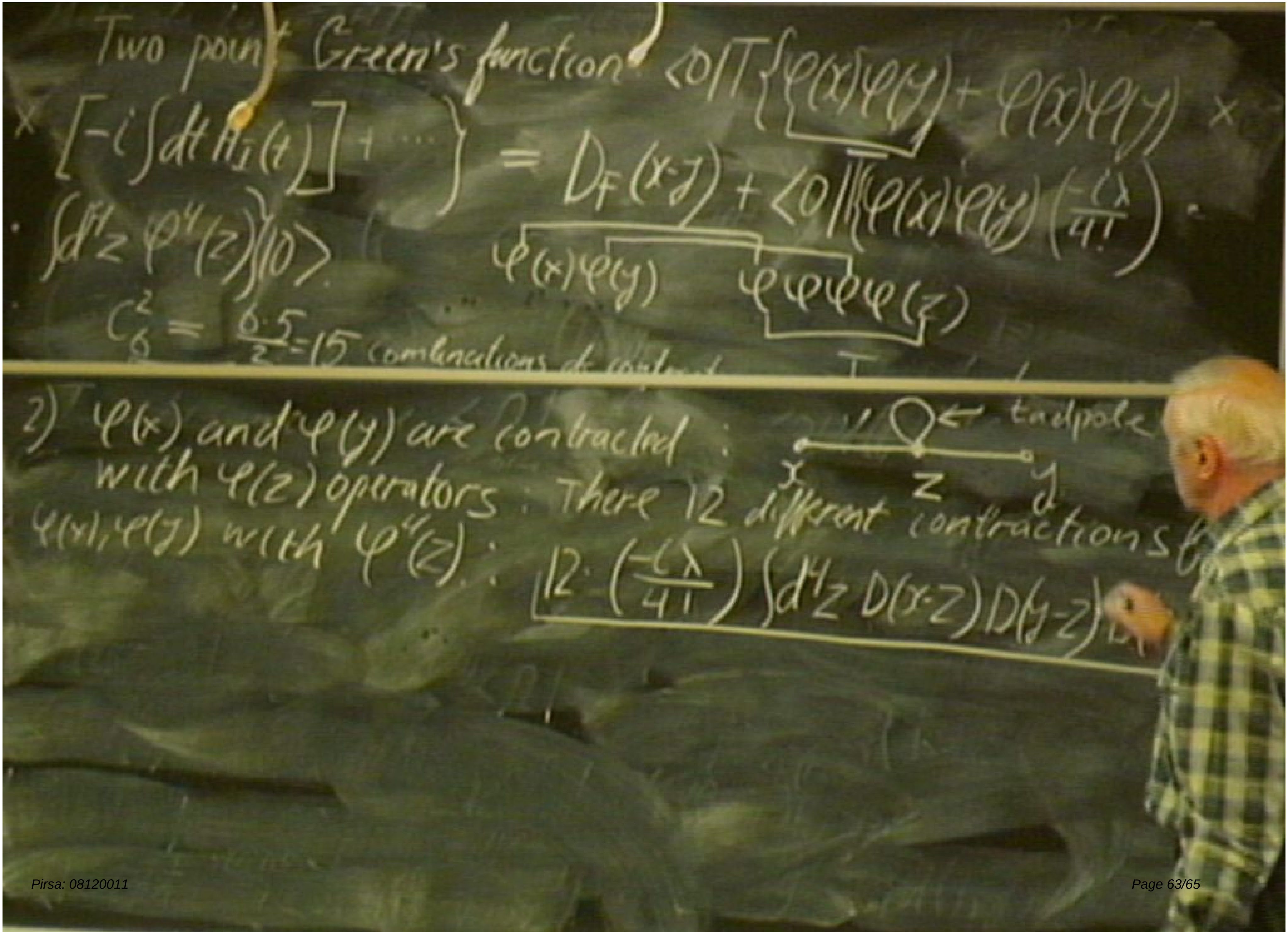


Two point Green's function: $\langle 0|T\{\varphi(x)\varphi(y) + \varphi(x)\varphi(y)\ \times$
$\times\ [-i\int dt\, H_I(t)] + \cdots\} = D_F(x-y) + \langle 0|T\{\varphi(x)\varphi(y)\left(\frac{-i\lambda}{4!}\right)\cdot$
$\int d^4z\, \varphi^4(z)\}|0\rangle.$
$\varphi(x)\varphi(y)\quad \varphi\varphi\varphi\varphi(z)$
$C_6^2 = \frac{6\cdot 5}{2} = 15$ combinations of contractions. Two types:
$\varphi(x)\varphi(y)$: $\left(\frac{-i\lambda}{4!}\right) D_F(x-y)\langle 0|\int d^4z\, \varphi^4(z)|0\rangle = 3\left(\frac{-i\lambda}{4!}\right)\cdot D_F(x-y)\cdot$
$D(z-z) \Longrightarrow$
x
y
z
$\varphi(x)$ and $\varphi(y)$ are contracted with $\varphi(z)$ operators: There 12 different contractions for
$\varphi(y)$ with $\varphi^4(z)$.
tadpole
x
z
y



Two point Green's function ⟨0|T{φ(x)φ(y) + φ(x)φ(y) ×
[−i∫dt H_I(t)] + ……} = D_F(x−y) + ⟨0|T{φ(x)φ(y) (−iλ/4!) ·
∫d⁴z φ⁴(z)}|0⟩.
φ(x)φ(y) φφφφ(z)
C²₆ = 6·5/2 = 15 combinations of contractions. Two types:
1) φ(x) φ(y) : (−iλ/4!) D_F(x−y) ⟨0|∫d⁴z φ⁴(z)|0⟩ = 3(−iλ/4!) D_F(x−y) ·
∫d⁴z D(z−z) ⟹
x y z
2) φ(x) and φ(y) are contracted : tadpole
with φ(z) operators. There 12 different contractions for
φ(x), φ(y) with φ⁴(z).
x z y


Two point Green's function
2) φ(x) and φ(y) are contracted
with φ(z) operators. There 12 different
φ(x), φ(y) with φ⁴(z):
15 combinations

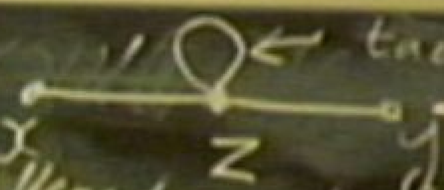

Two point Green's function: ⟨0|T{φ(x)φ(y) + φ(x)φ(y) ×
[−i∫dt H_I(t)] + ...} = D_F(x−y) + ⟨0|T{φ(x)φ(y) (−iλ/4!)
∫d⁴z φ⁴(z)}|0⟩.
φ(x)φ(y) φφφφ(z)
C²₆ = 6·5/2 = 15 combinations of contract
2) φ(x) and φ(y) are contracted
with φ(z) operators. There 12 different contractions
φ(x), φ(y) with φ⁴(z):
12·(−iλ/4!) ∫d⁴z D(x−z) D(y−z)
tadpole
x
z
y

Two point Green's function: $\langle 0 | T\{ \varphi(x)\varphi(y) + \varphi(x)\varphi(y) \times \left[-i\int dt\, H_I(t)\right] + \dots \} = D_F(x-y) + \langle 0 | T \varphi(x)\varphi(y) \left(\frac{-i\lambda}{4!}\right) \cdot \int d^4z\, \varphi^4(z)\} |0\rangle$.

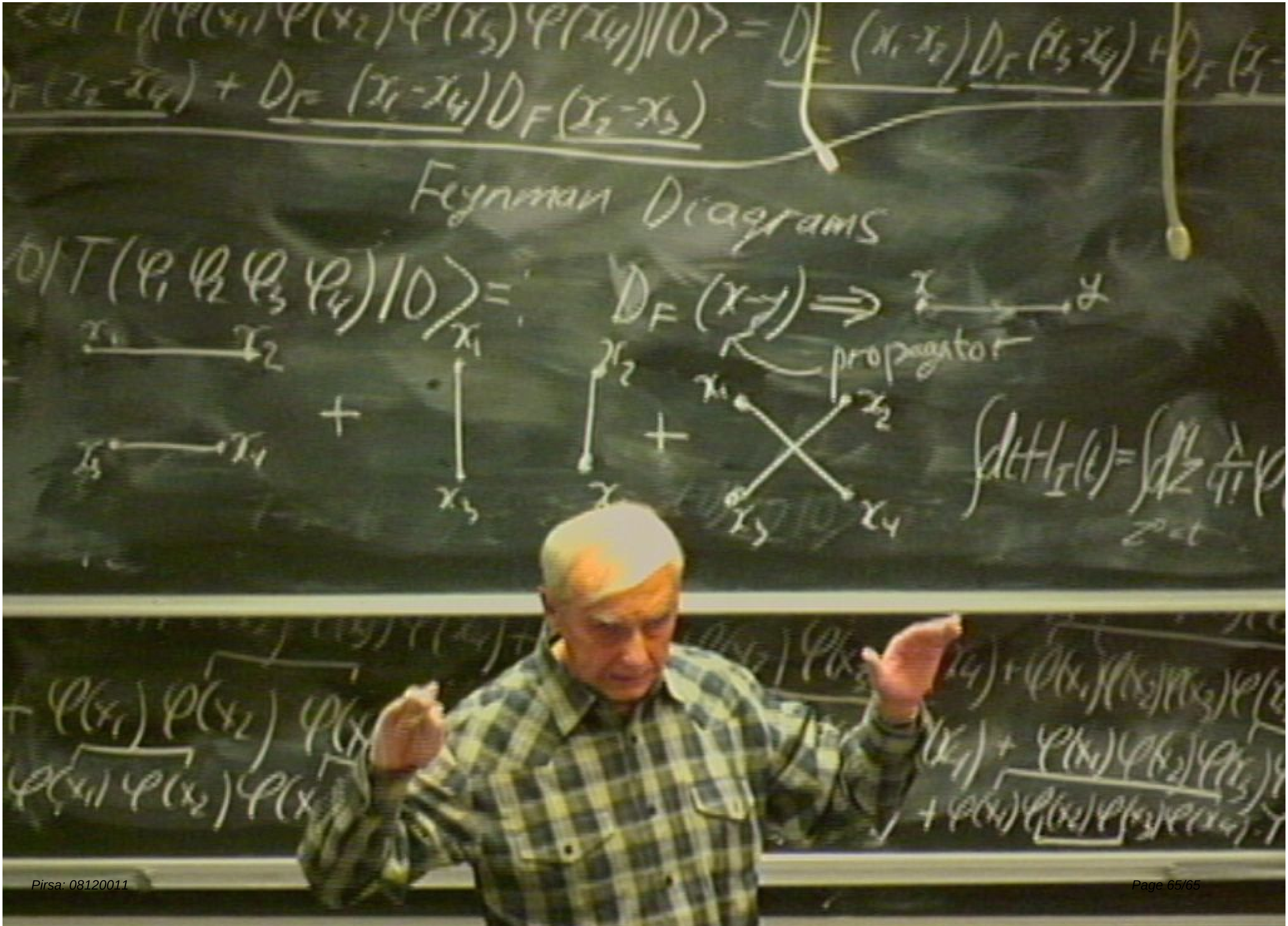
$\varphi(x)\varphi(y) \quad \varphi\varphi\varphi\varphi(z)$

$C_6^2 = \frac{6\cdot 5}{2} = 15$ combinations of contract...

2) $\varphi(x)$ and $\varphi(y)$ are contracted with $\varphi(z)$ operators: There 12 different contractions of $\varphi(x), \varphi(y)$ with $\varphi^4(z)$:



$$12 \cdot \left(\frac{-i\lambda}{4!}\right) \int d^4z\, D(x-z) D(y-z) D(z-z)$$


Feynman Diagrams
propagator