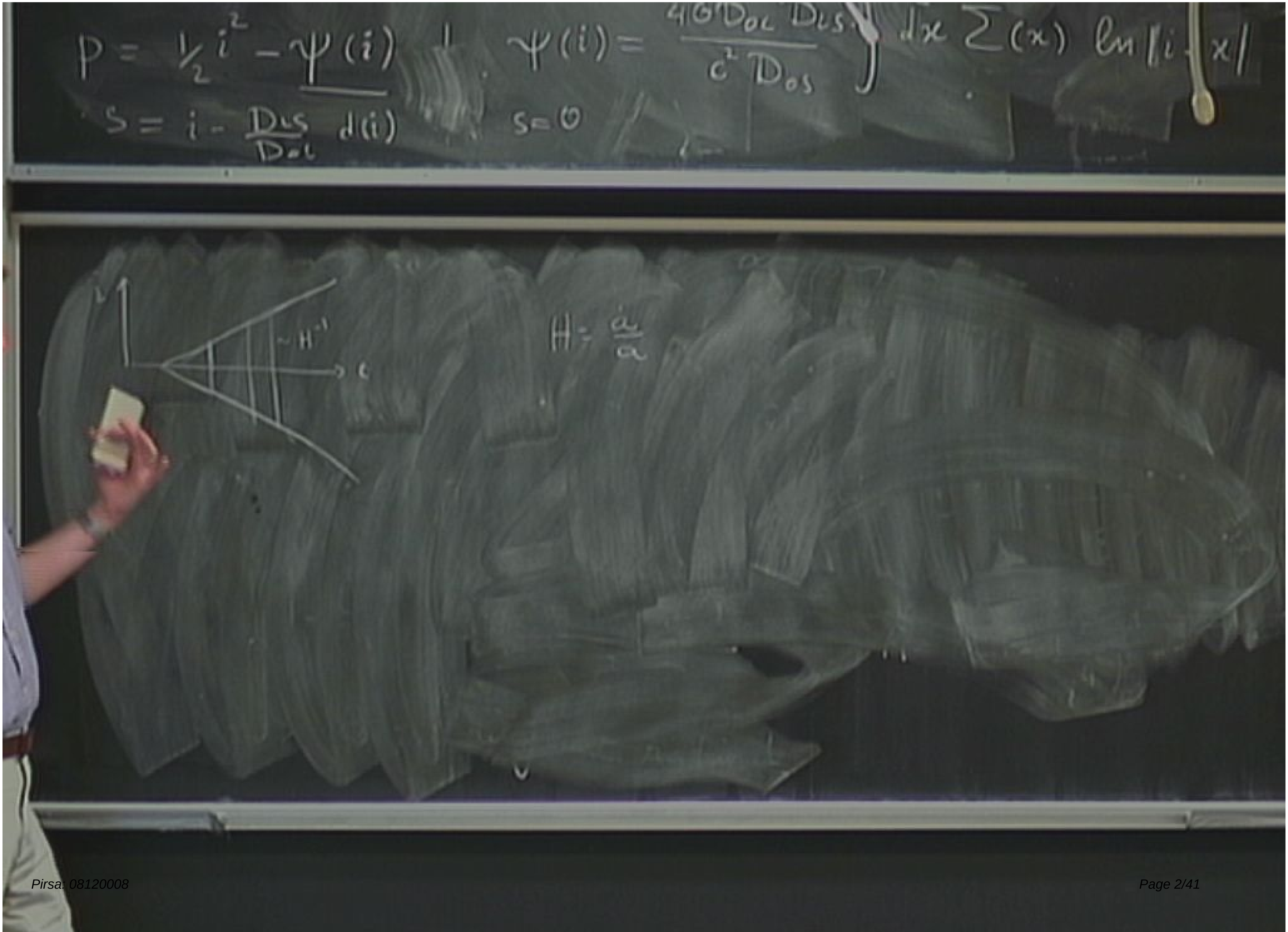
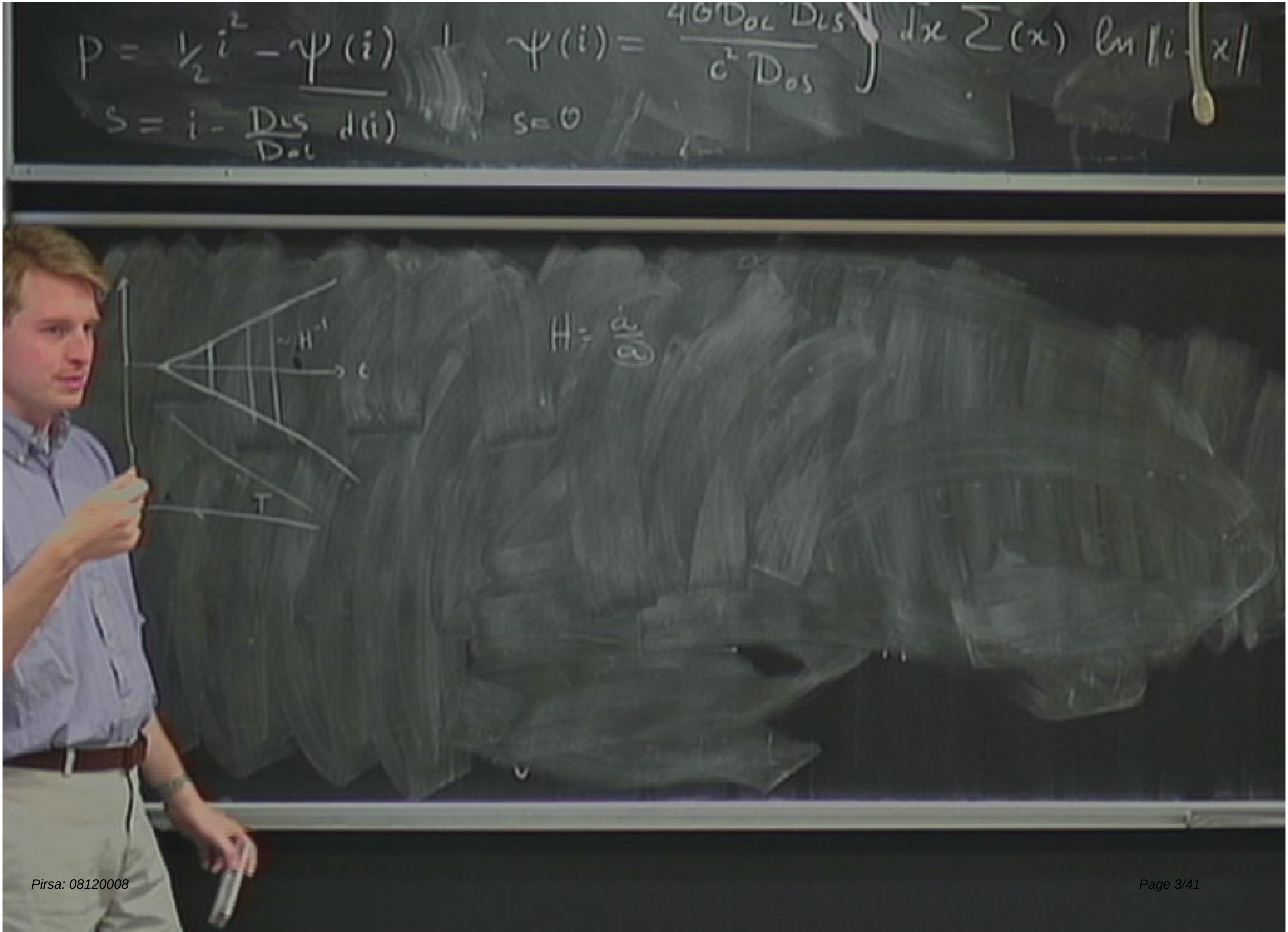
Title: Lecture 14B

Date: Dec 04, 2008 12:30 PM

URL: http://pirsa.org/08120008

Abstract: This course is aimed at advanced undergraduate and beginning graduate students, and is inspired by a book by the same title, written by Padmanabhan. Each session consists of solving one or two pre-determined problems, which is done by a randomly picked student. While the problems introduce various subjects in Astrophysics and Cosmology, they do not serve as replacement for standard courses in these subjects, and are rather aimed at educating students with hands-on analytic/numerical skills to attack new problems.


$P = \frac{1}{2} i^2 - \psi(i)$
$\psi(i) = \frac{4G D_{ol} D_{ls}}{c^2 D_{os}} \int dx\, \Sigma(x) \ln|i - x|$
$S = i - \frac{D_{ls}}{D_{ol}} d(i)$
$s = 0$
$\sim H^{-1}$
$H = \frac{\dot{a}}{a}$



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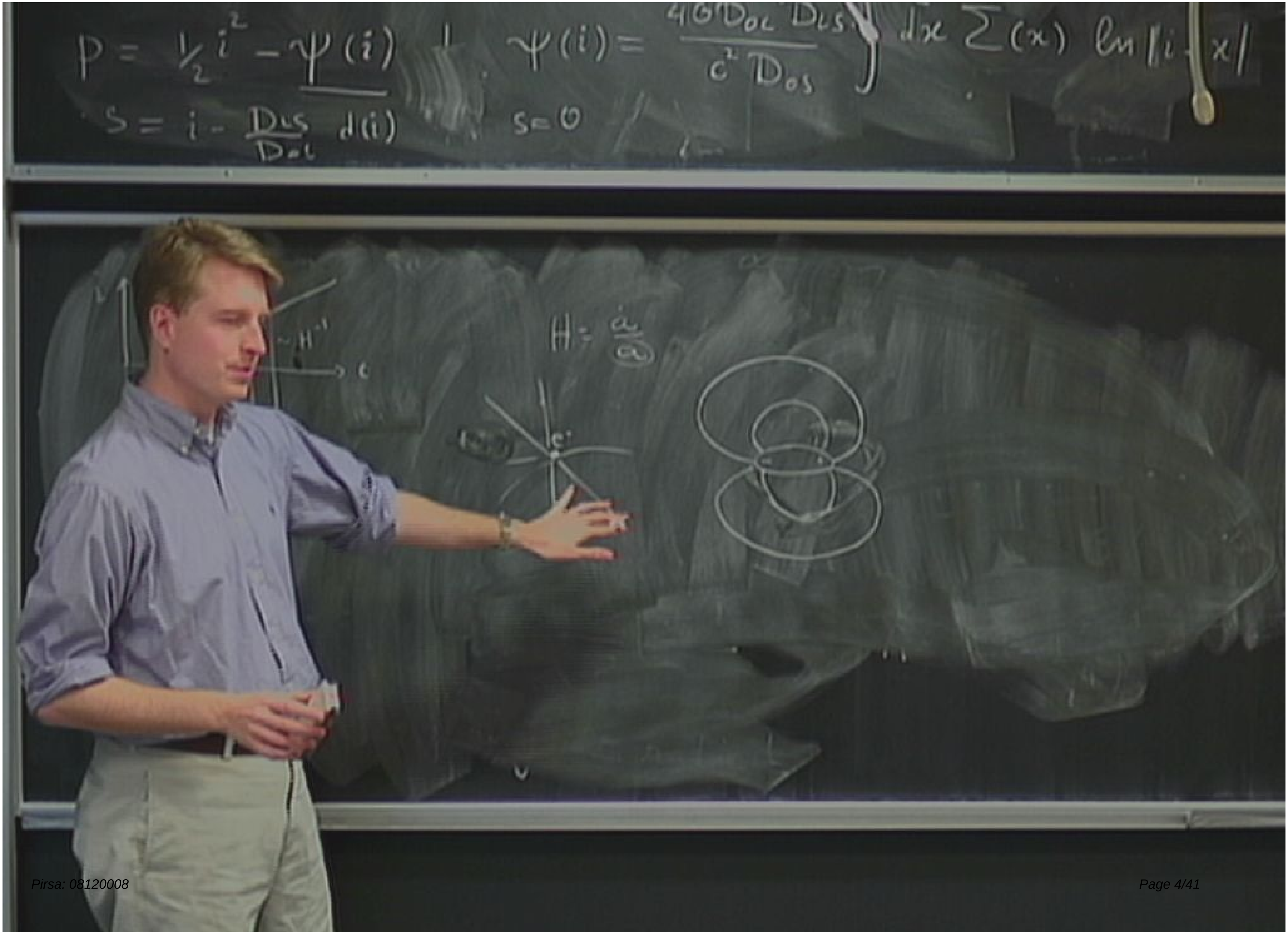
$S = i - \frac{D_{LS}}{D_{OL}} \alpha(i)$
$s = 0$
$H = \frac{\dot{a}}{a}$
$\sim H^{-1}$
t
T



$P = \frac{1}{2} i^2 - \psi(i)$
$\psi(i) = \frac{4 G D_{OL} D_{LS}}{c^2 D_{OS}} \int dx\, \Sigma(x) \ln|i - x|$
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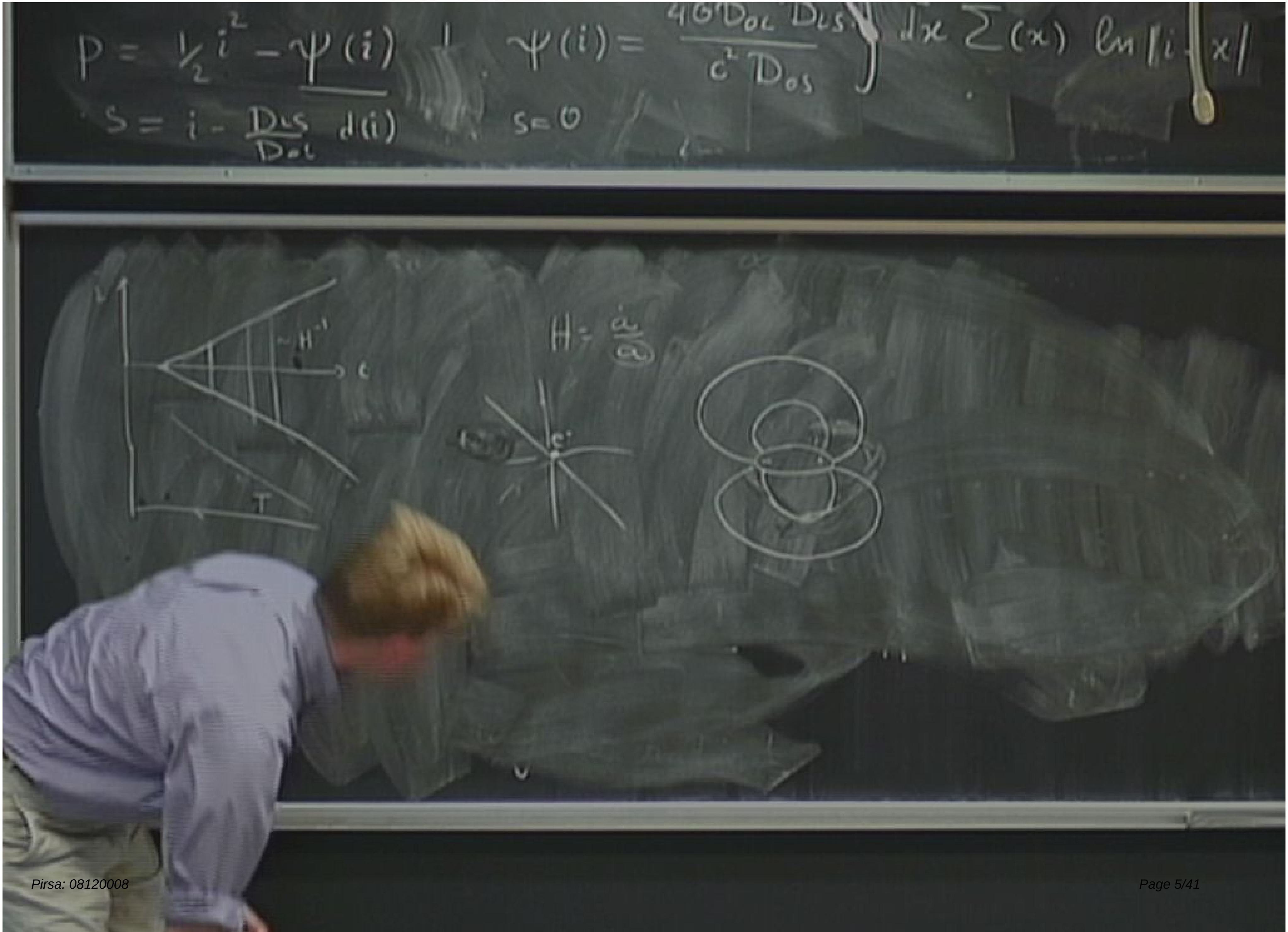
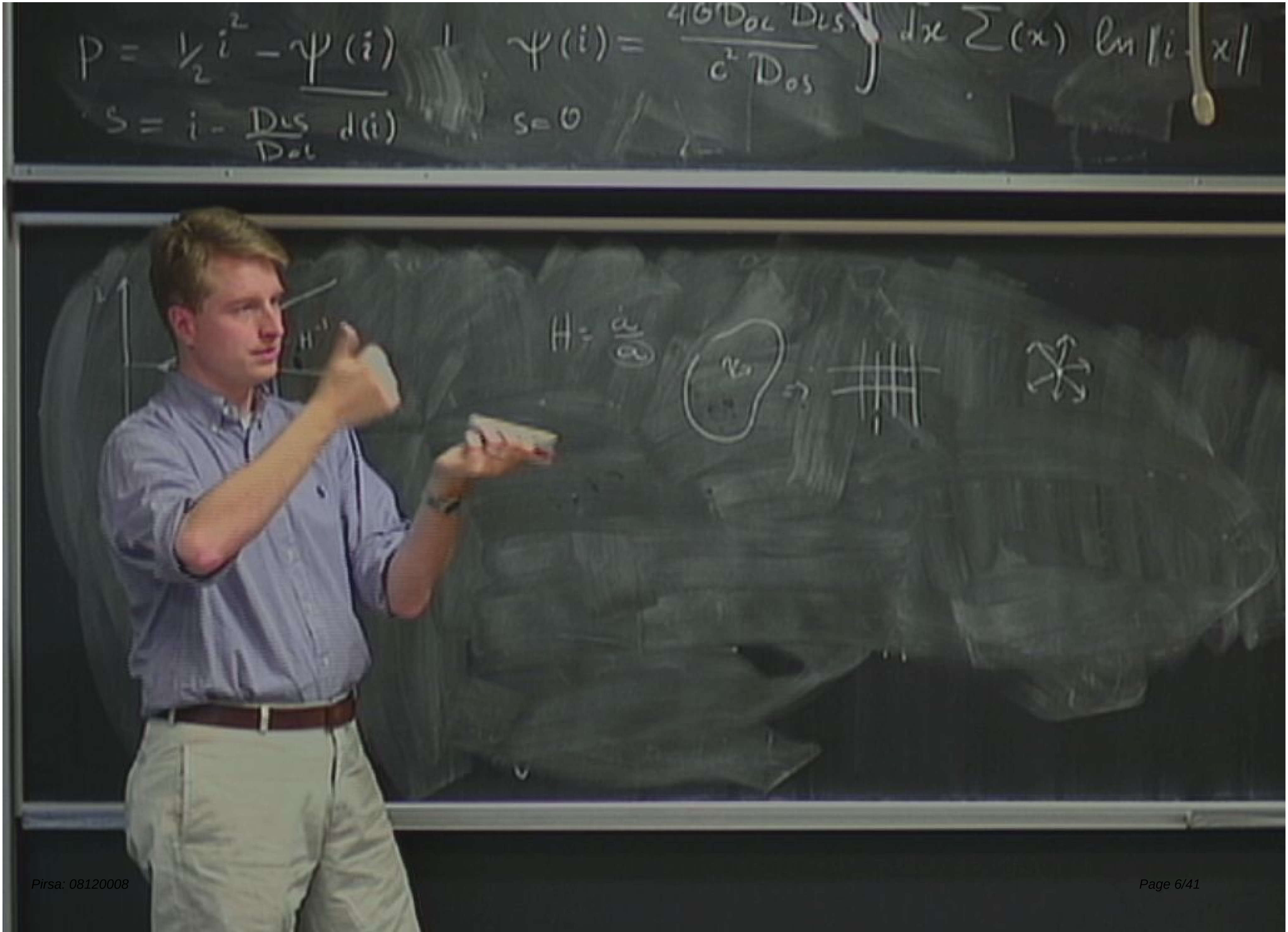
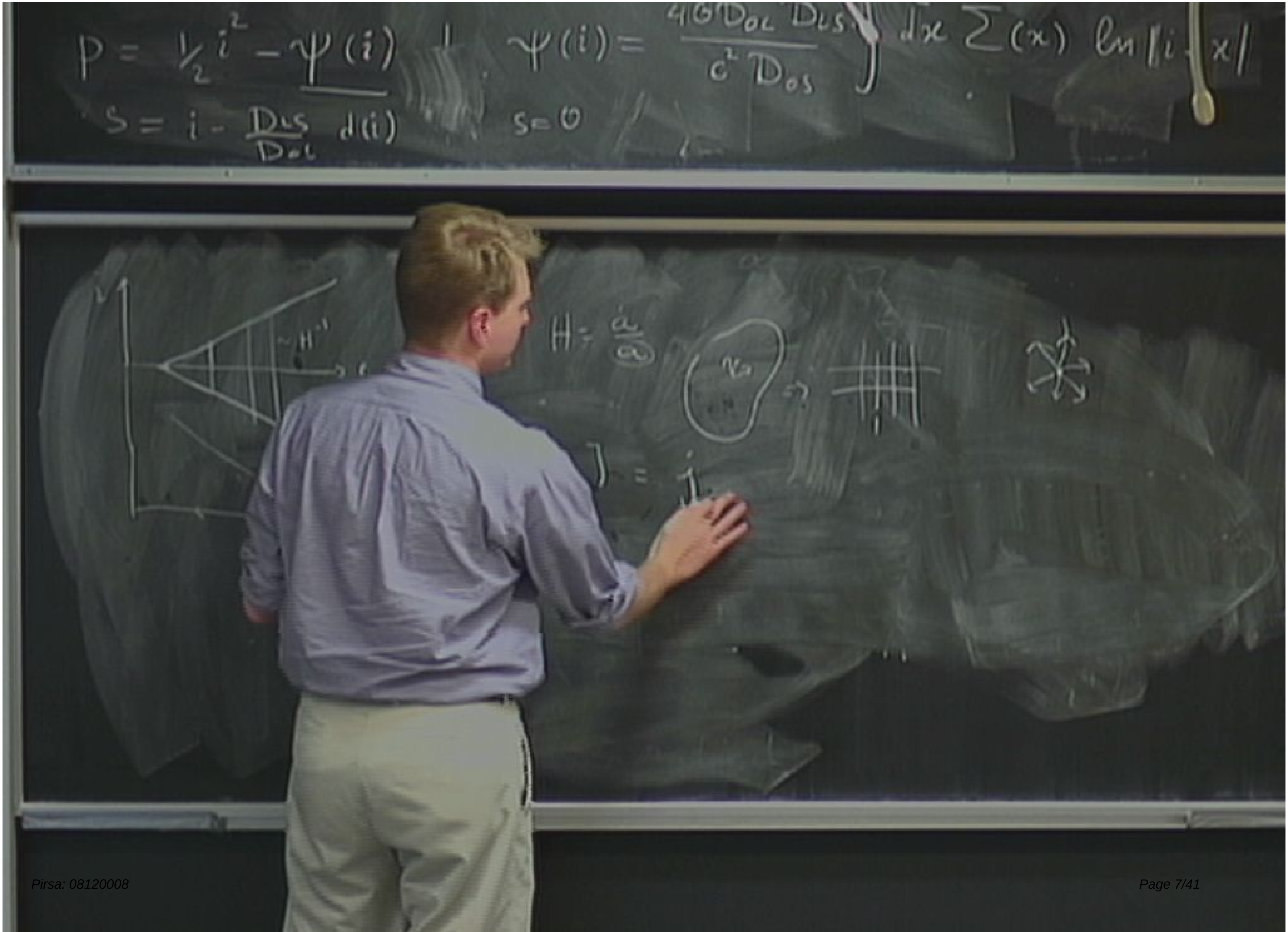

P = ½ i² − ψ(i)
ψ(i) = 4G D_ol D_ls / c² D_os ∫ dx Σ(x) ln|i − x|
S = i − D_ls/D_ol d(i)
s = 0
H = ȧ/a
~H⁻¹
T


P = ½ i² − ψ(i)
ψ(i) = 4G D_ol D_ls / c² D_os ∫ dx Σ(x) ln|i − x|
S = i − (D_ls / D_ol) d(i)
s = 0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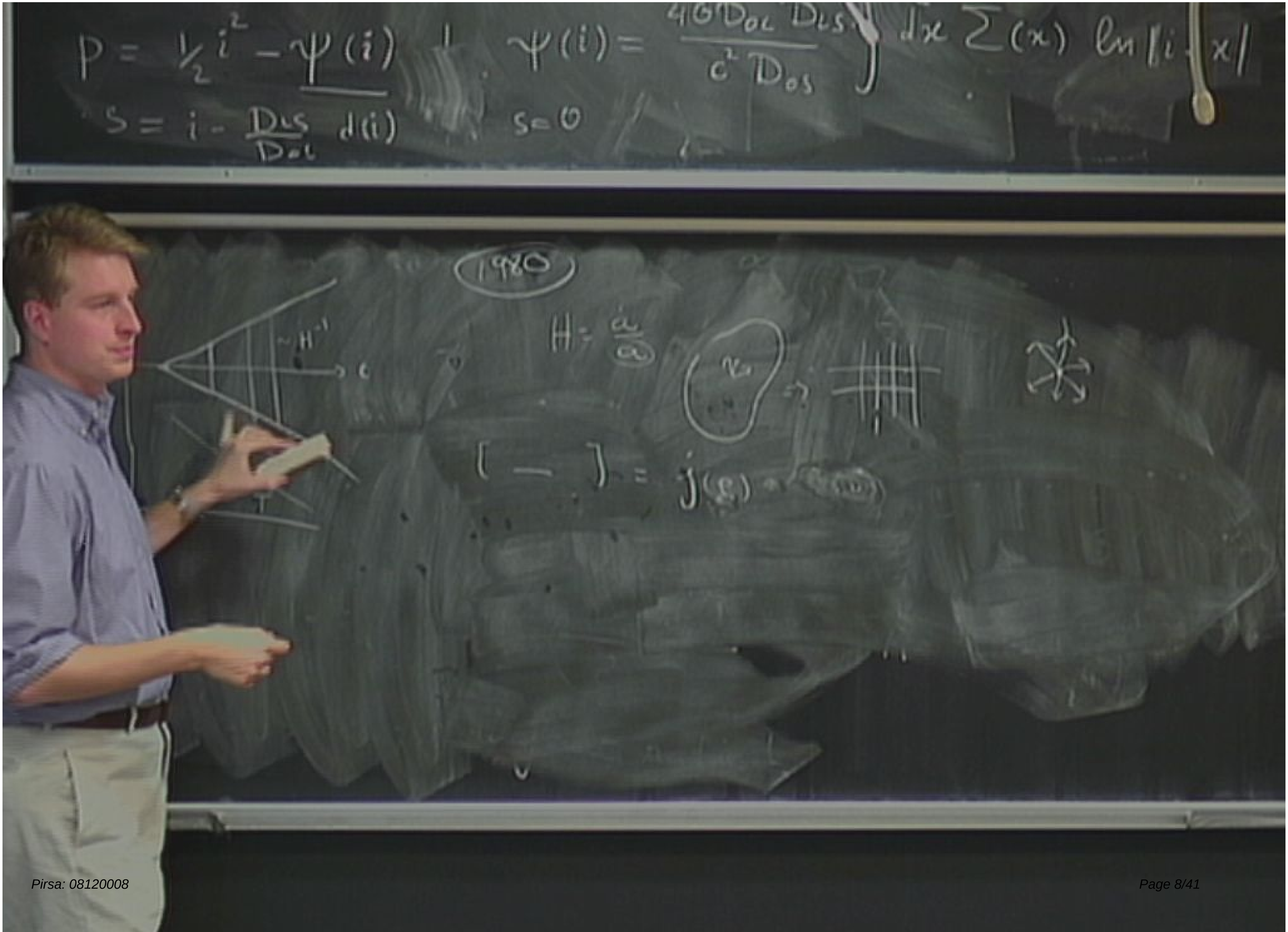
$S = i - \frac{D_{LS}}{D_{OL}} d(i)$
$s = 0$
$H = \frac{\dot{a}}{a}$


P = ½ i² − ψ(i)
ψ(i) = 4G D_ol D_ls / c² D_os ∫ dx Σ(x) ln |i − x|
S = i − D_ls/D_ol d(i)
s = 0
1980
H = ȧ/a

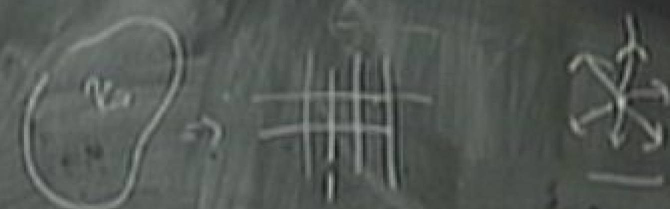
$P = \frac{1}{2} i^2 - \psi(i)$ ; $\psi(i) = \frac{4 G D_{OL} D_{LS}}{c^2 D_{OS}} \int dx \, \Sigma(x) \ln|i - x|$

$S = i - \frac{D_{LS}}{D_{OL}} \alpha(i)$ $\quad S = 0$



1980

$H = \frac{\dot a}{a}$



$[\ -\ ] = j(g) + \ldots$

$$H^2 = \frac{8\pi G \rho}{3} - \frac{1}{R^2 a^2}$$

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$$= H_0^2 \left( \frac{\Omega_m}{a^3} + \frac{\Omega_{Rad}}{a^4} + \Omega_\Lambda + \frac{(1-\Omega_{sum})}{a^2} \right)$$

$\uparrow$ today's $H$

$$H^2 = \frac{8\pi G\rho}{3} - \frac{1}{R^2 a^2}$$

$$= H_0^2 \left( \frac{\Omega_m^0}{a^3} + \frac{\Omega_{Rad}^0}{a^4} + \Omega_\Lambda^0 + \frac{(1-\Omega_{sum})}{a^2} \right)$$

$\uparrow$ today's $H$

$$H^2 = \frac{8\pi G \rho}{3} - \frac{1}{R^2 a^2}$$

$R^2 \to R_0$

$$\rho_0 = \frac{3H^2}{8\pi G}$$

$$= H_0^2 \left( \frac{\Omega_m}{a^3} + \frac{\Omega_{Rad}}{a^4} + \Omega_\Lambda + \frac{(1-\Omega_{sum})}{a^2} \right)$$

today, H

$$H^2 = \frac{8\pi G \rho}{3} - \frac{1}{R^2 a^2}$$

$R^2 \to R_0$

$$\rho_0 = \frac{3H^2}{8\pi G}$$

$$= H_0^2 \left( \frac{\Omega_m}{a^3} + \frac{\Omega_{Rad}}{a^4} + \Omega_\Lambda + \frac{(1 - \Omega_{sum})}{a^2} \right)$$

$\Omega_m = \frac{\rho_m}{\rho_0}$, $\Omega_{Rad} = \frac{\rho_{Rad}}{\rho_0}$

today's H

$$H^2 = \frac{8\pi G\rho}{3} - \frac{1}{R^2 a^2} \qquad R^2 \to \infty \qquad \rho_0 = \frac{3H^2}{8\pi G}$$

$$= \underbrace{H_0^2}_{\text{today's } H}\left(\frac{\overset{\rho_m/\rho_0}{\Omega_m}}{a^3} + \frac{\overset{\rho_{Rad}/\rho_0}{\Omega_{Rad}}}{a^4} + \Omega_\Lambda + \frac{(1-\Omega_{sum})}{a^2}\right)$$

$$H^2 = \frac{8\pi G \rho}{3} - \frac{1}{R^2 a^2}$$

$R^2 \to \infty$

$$\rho_c = \frac{3H^2}{8\pi G}$$

$$= H_0^2 \left( \frac{\Omega_m}{a^3} + \frac{\Omega_{Rad}}{a^4} + \Omega_\Lambda + \frac{(1-\Omega_{sum})}{a^2} \right)$$

$\Omega_m = \frac{\rho_m}{\rho_0}$, $\Omega_{Rad} = \frac{\rho_{Rad}}{\rho_0}$

$H_0 \uparrow$ today's $H$

$$H^2 = \frac{8\pi G \rho}{3} - \frac{1}{R^2 a^2} \qquad R^2 \to R_0 \qquad \rho_0 = \frac{3H^2}{8\pi G}$$

$$= H_0^2 \left( \frac{\Omega_m}{a^3} + \frac{\Omega_{Rad}}{a^4} + \Omega_\Lambda + \frac{(1-\Omega_{sum})}{a^2} \right)$$

$\Omega_m = \frac{\rho_m}{\rho_0}$, $\Omega_{Rad} = \frac{\rho_{Rad}}{\rho_0}$

$H_0$: today's H

$$H^2 = \frac{8\pi G \rho}{3} - \frac{1}{R^2 a^2} \qquad R^2 \to R_0 \qquad \rho_0 = \frac{3H^2}{8\pi G}$$

$$= H_0^2 \left( \frac{\overset{\rho_m/\rho_0}{\Omega_m}}{a^3} + \frac{\overset{\rho_{Rad}/\rho_0}{\Omega_{Rad}}}{a^4} + \Omega_\Lambda + \frac{(1 - \Omega_{sum})}{a^2} \right)$$

today's H

$$\simeq H_0^2 \left( \frac{\Omega_{Rad}}{a^4} + \frac{(1 - \Omega_{rad})}{a^2} \right)$$

$$H^2 = \frac{8\pi G \rho}{3} - \frac{1}{R^2 a^2}$$

$R \to R_0$

$$\rho_0 = \frac{3H^2}{8\pi G}$$

$$= H_0^2 \left( \frac{\Omega_m}{a^3} + \frac{\Omega_{Rad}}{a^4} + \Omega_\Lambda + \frac{(1-\Omega_{sum})}{a^2} \right)$$

$\Omega_m = \frac{\rho_m}{\rho_0}$, $\Omega_{Rad} = \frac{\rho_{Rad}}{\rho_0}$

$H_0$: today's H

$$\simeq H_0^2 \left( \frac{\Omega_{Rad}}{a^4} + \frac{(1-\Omega_{Rad})}{a^2} \right)$$

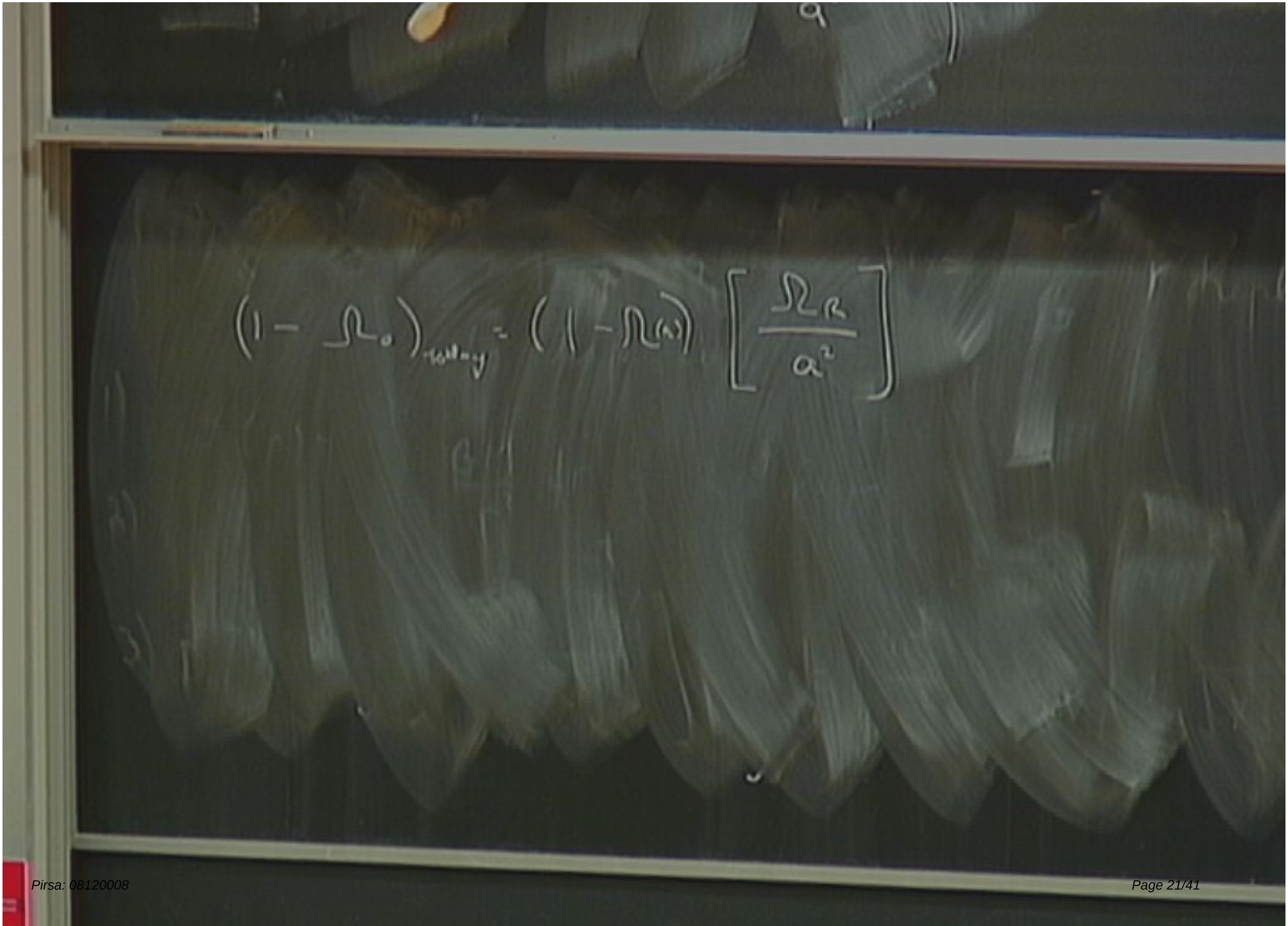
$$H^2 = \frac{8\pi G \rho}{3} - \frac{1}{R^2 a^2} \qquad R^2 \to R_0 \qquad \rho_0 = \frac{3H^2}{8\pi G}$$

$$\frac{\rho_m}{\rho_0} \qquad \frac{\rho_{Rad}}{\rho_0}$$

$$= H_0^2 \left( \frac{\Omega_m}{a^3} + \frac{\Omega_{Rad}}{a^4} + \Omega_\Lambda + \frac{(1-\Omega_{sum})}{a^2} \right)$$

today's $H$

$$\frac{H^2}{H_0^2} \simeq \left( \frac{\Omega_{Rad}}{a^4} + \frac{(1-\Omega_{Rad})}{a^2} \right)$$

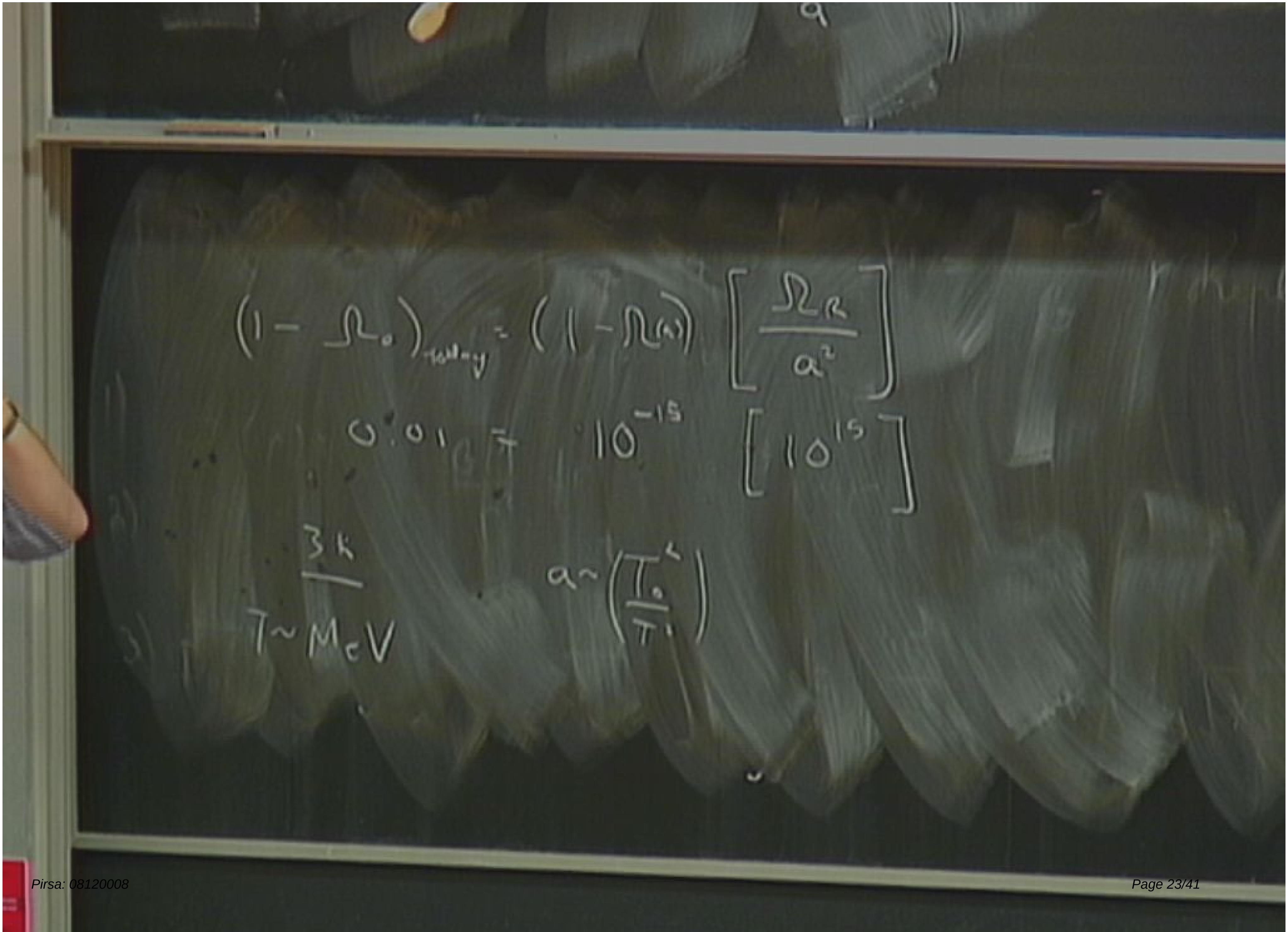

$(1-\Omega_0)_{\text{today}} = (1-\Omega(a))\left[\frac{\Omega_R}{a^2}\right]$

$$(1-\Omega_0)_{today} = (1-\Omega(a)) \left[ \frac{\Omega_R}{a^2} \right]$$

$$0.01 =$$

$3k$

$$a \sim \left( \frac{T_0}{T_i} \right)^2$$

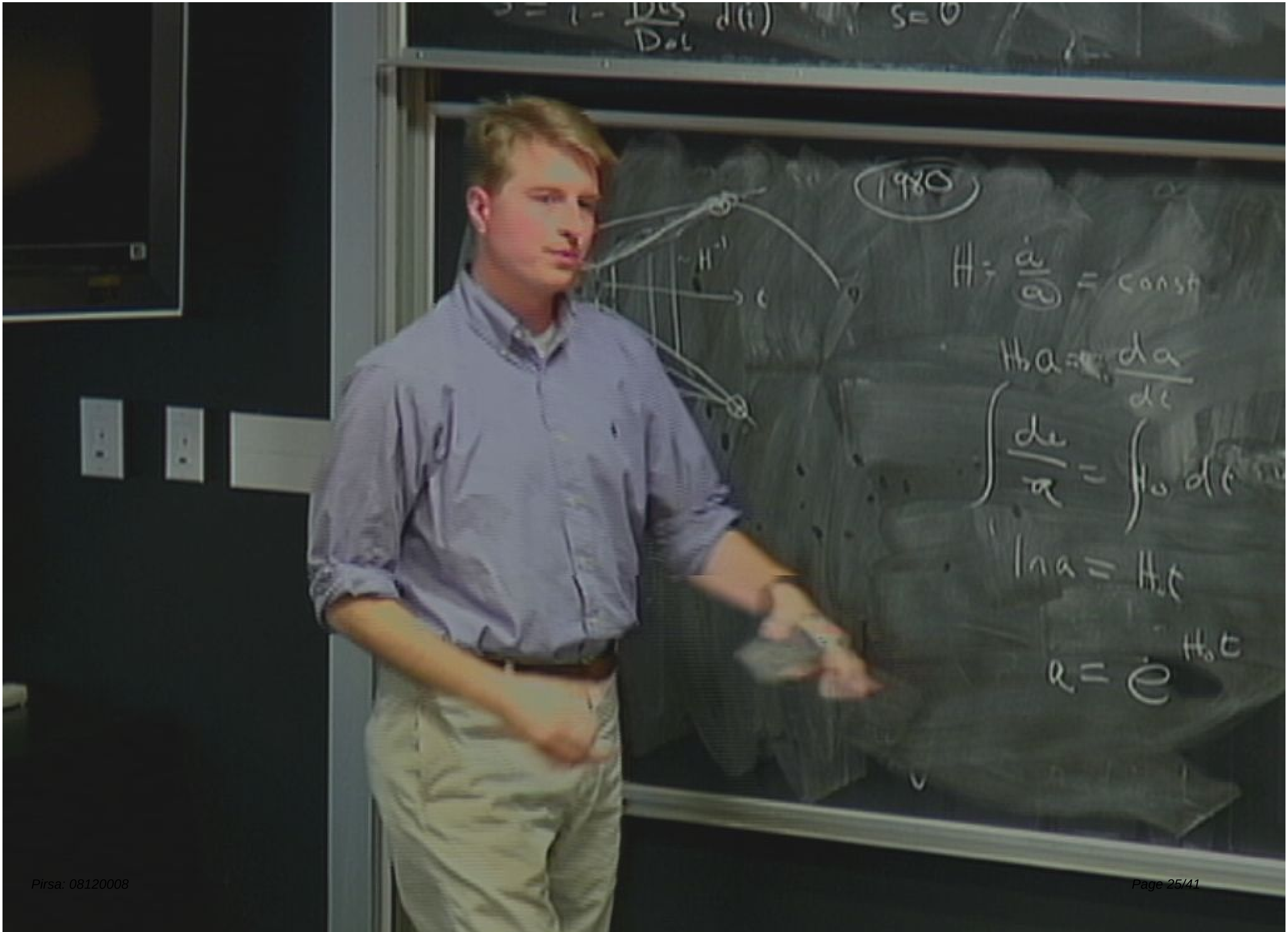

$(1-\Omega_0)_{\text{today}} = (1-\Omega(a)) \left[\frac{\Omega_R}{a^2}\right]$
$0.01 = 10^{-15} \left[10^{15}\right]$
$3K$
$T \sim MeV$
$a \sim \left(\frac{T_0}{T_1}\right)$

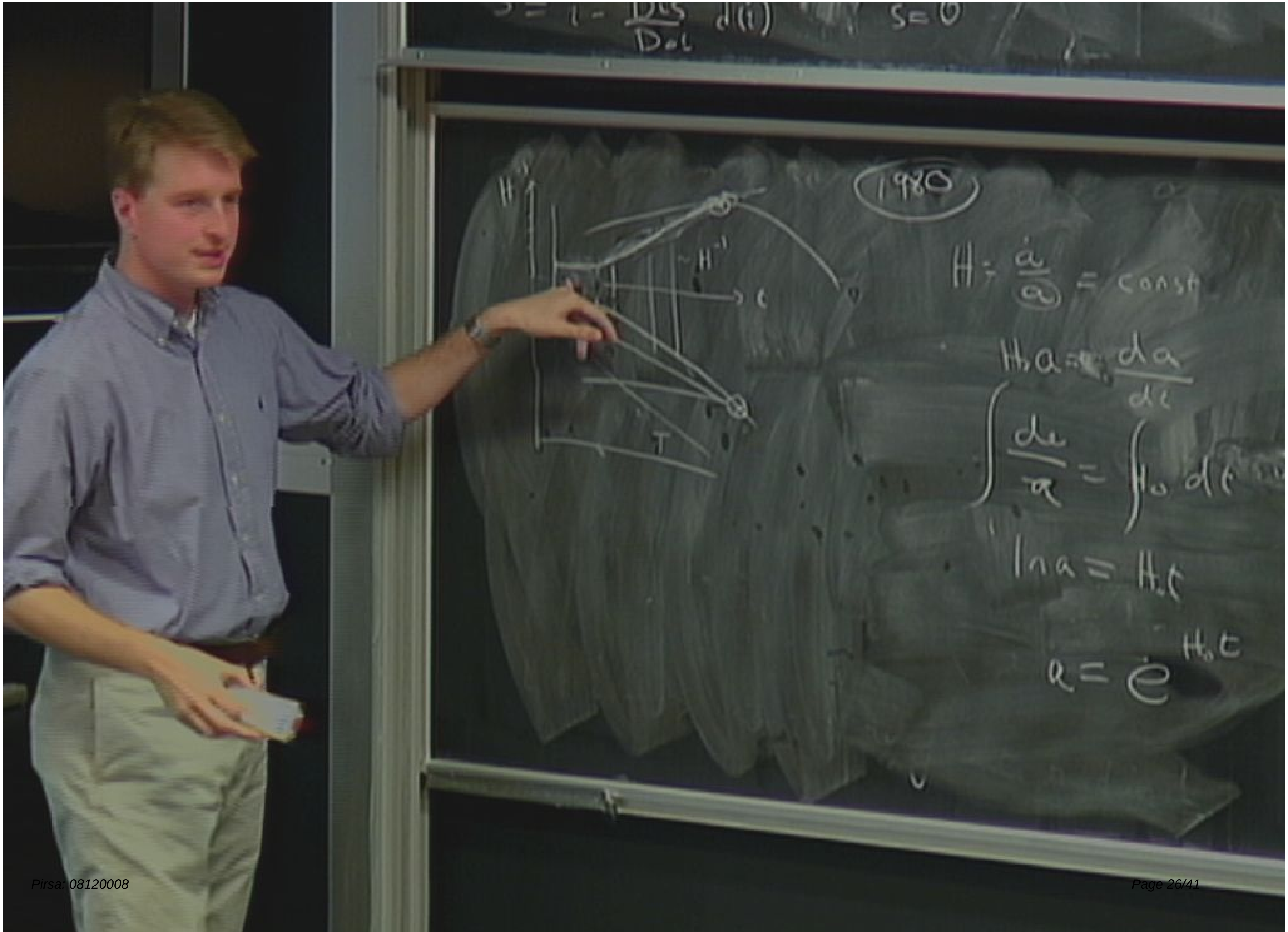
$$(1-\Omega_0)_{\text{today}} = (1-\Omega(a))\left[\frac{\Omega_R}{a^2}\right]$$

$$0.1 \approx 10^{-15}\left[10^{15}\right]$$

$$\frac{3k}{\ }$$

$$T \sim \text{MeV} \qquad a \sim \left(\frac{T_0}{T_i}\right)$$


1980
H = ȧ/a = const
H₀a = da/dt
∫ da/a = ∫ H₀ dt
ln a = H₀t
a = e^{H₀t}


1980
H = ȧ/a = const
H₀a = da/dt
∫ da/a = ∫ H₀ dt
ln a = H₀t
a = e^{H₀t}

$$(1-\Omega_0)_{today} = (1-\Omega(a))\left[\frac{\Omega_R}{a^2}\right]$$

$$0.\ \ = 10^{-15}\left[10^{15}\right]$$

$$e^{N} \sim 10^{30}$$

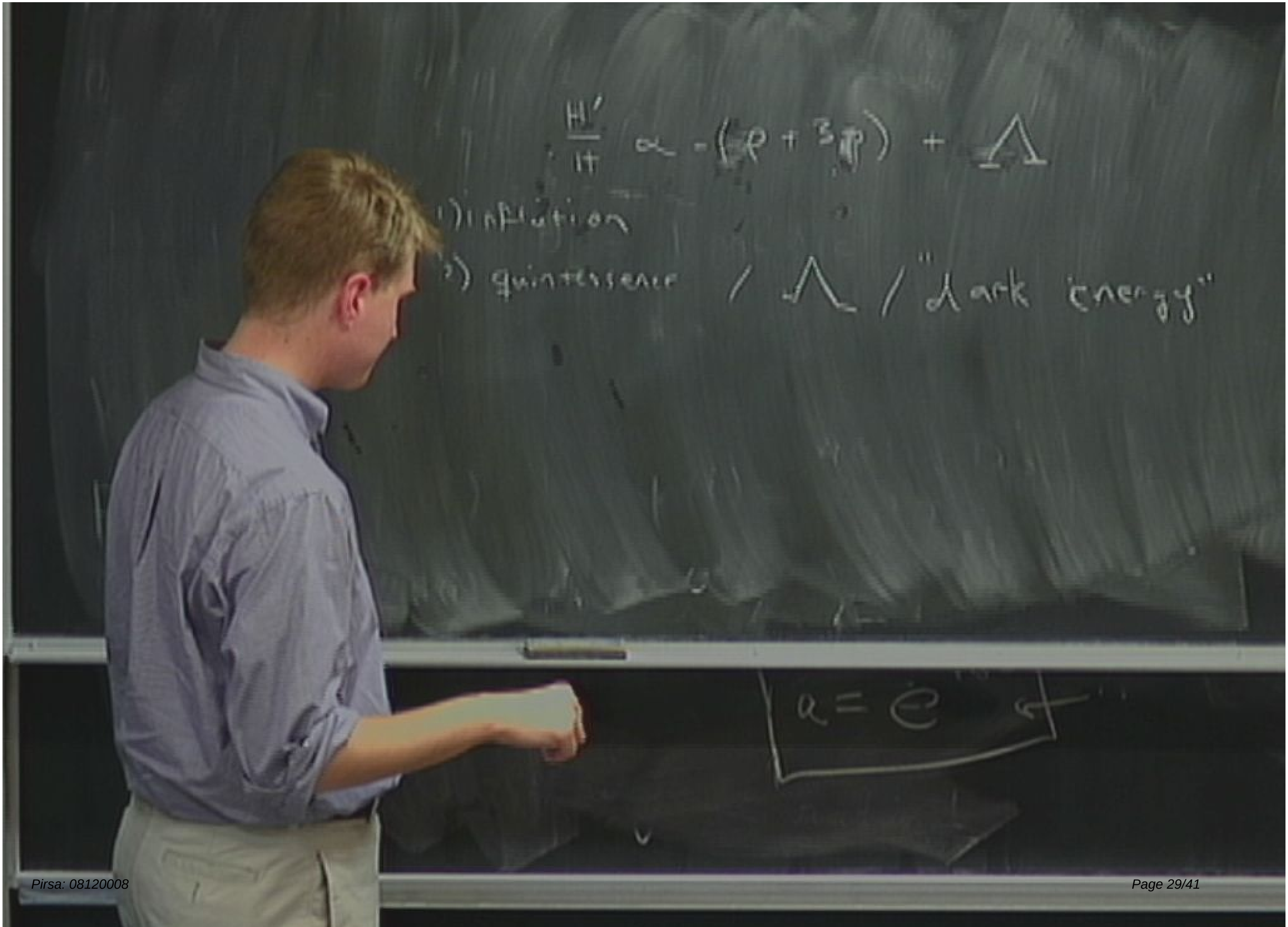
$$N \sim 60$$

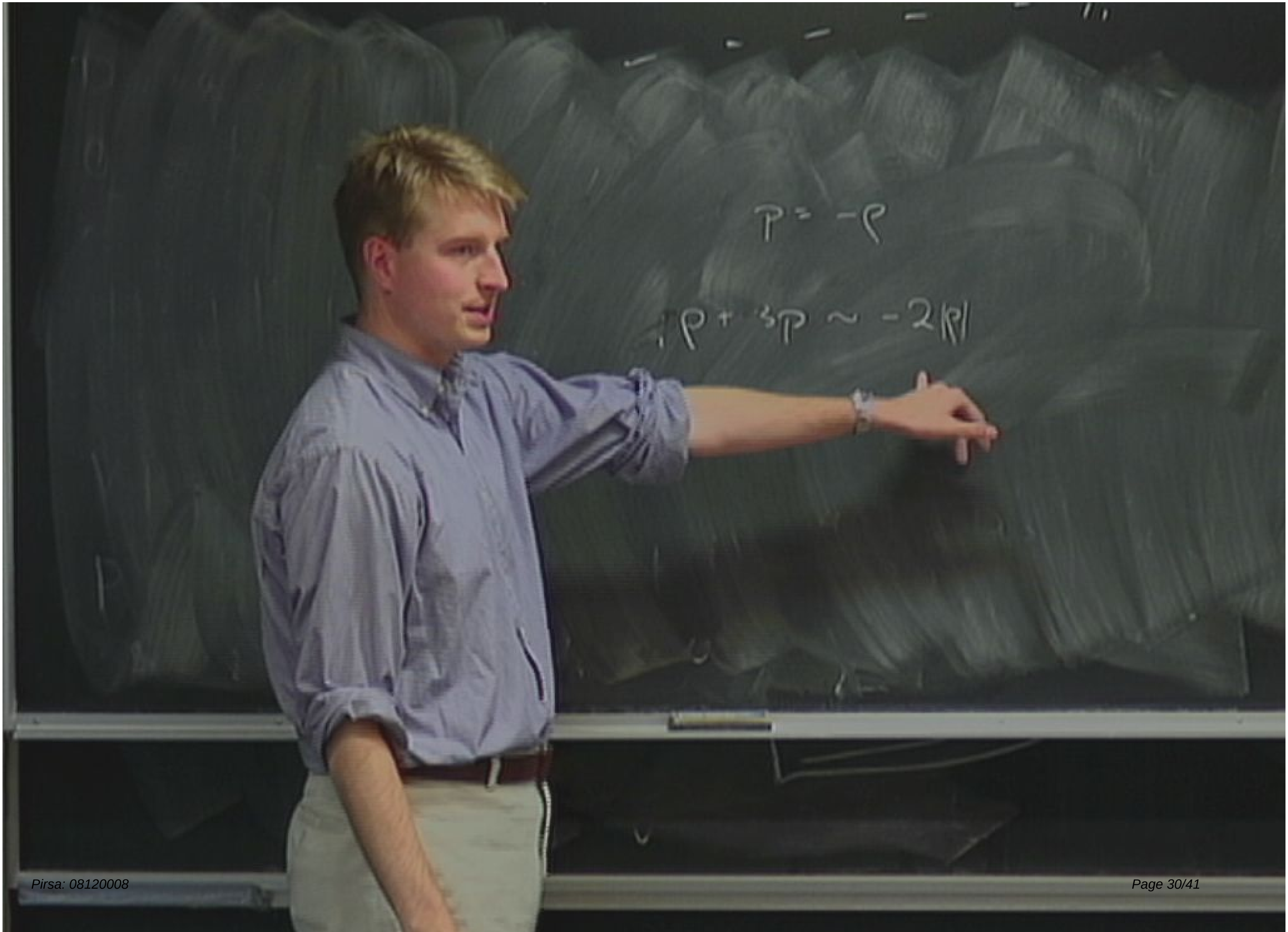
$3\,K$

$T \sim MeV$

$$a \sim \left(\frac{T_0}{T_i}\right)$$

$$\frac{H'}{H} \propto -(\rho + 3p) + \Lambda$$


$\frac{H'}{H} \propto -(\rho + 3p) + \Lambda$
1) inflation
2) quintessence / $\Lambda$ / "dark energy"
$a = e$


p = -ρ

$p = -\rho$

$\rho + 3p \sim -2|\rho|$

Universe dominated by scalar field

$$H^2 = \frac{8\pi G}{3} \rho_\phi$$

$$= \frac{8\pi G}{3} \left( \frac{1}{2}\dot{\phi}^2 + V(\phi) \right)$$

$p = -\rho$

$\rho + 3p \sim -2|\rho|$

Universe dominated by scalar field

$$H^2 = \frac{8\pi G}{3}\rho_\phi$$

$$= \frac{8\pi G}{3}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right)$$

$$p = -\rho$$

$$\rho + 3p \sim -2|\rho|$$

$$\mathcal{L} = (\partial_\mu\phi)(\partial^\mu\phi) - V(\phi)$$

Universe dominated by scalar field

$$H^2 = \frac{8\pi G}{3}\rho_\phi$$

$$= \frac{8\pi G}{3}\left(\tfrac{1}{2}\dot{\phi}^2 + V(\phi)\right)$$

Universe dominated by scalar field

$$\boxed{p = -\rho}$$

$$\rho + 3p \sim -2|\rho|$$

$$\mathcal{L} = (\partial_\mu \phi)(\partial^\mu \phi) - V(\phi)$$

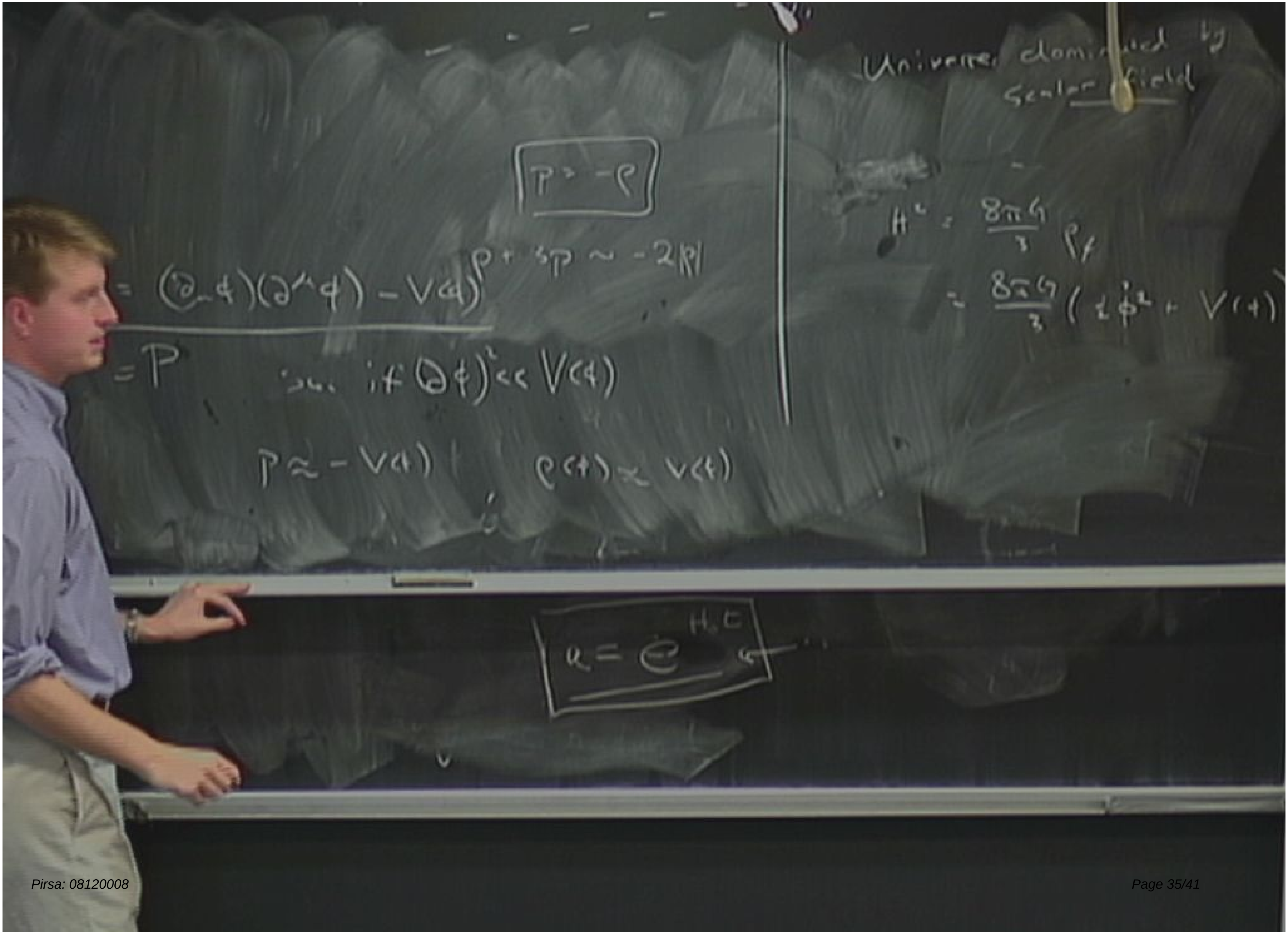
$$= P \quad \text{if } (\partial\phi)^2 \ll V(\phi)$$

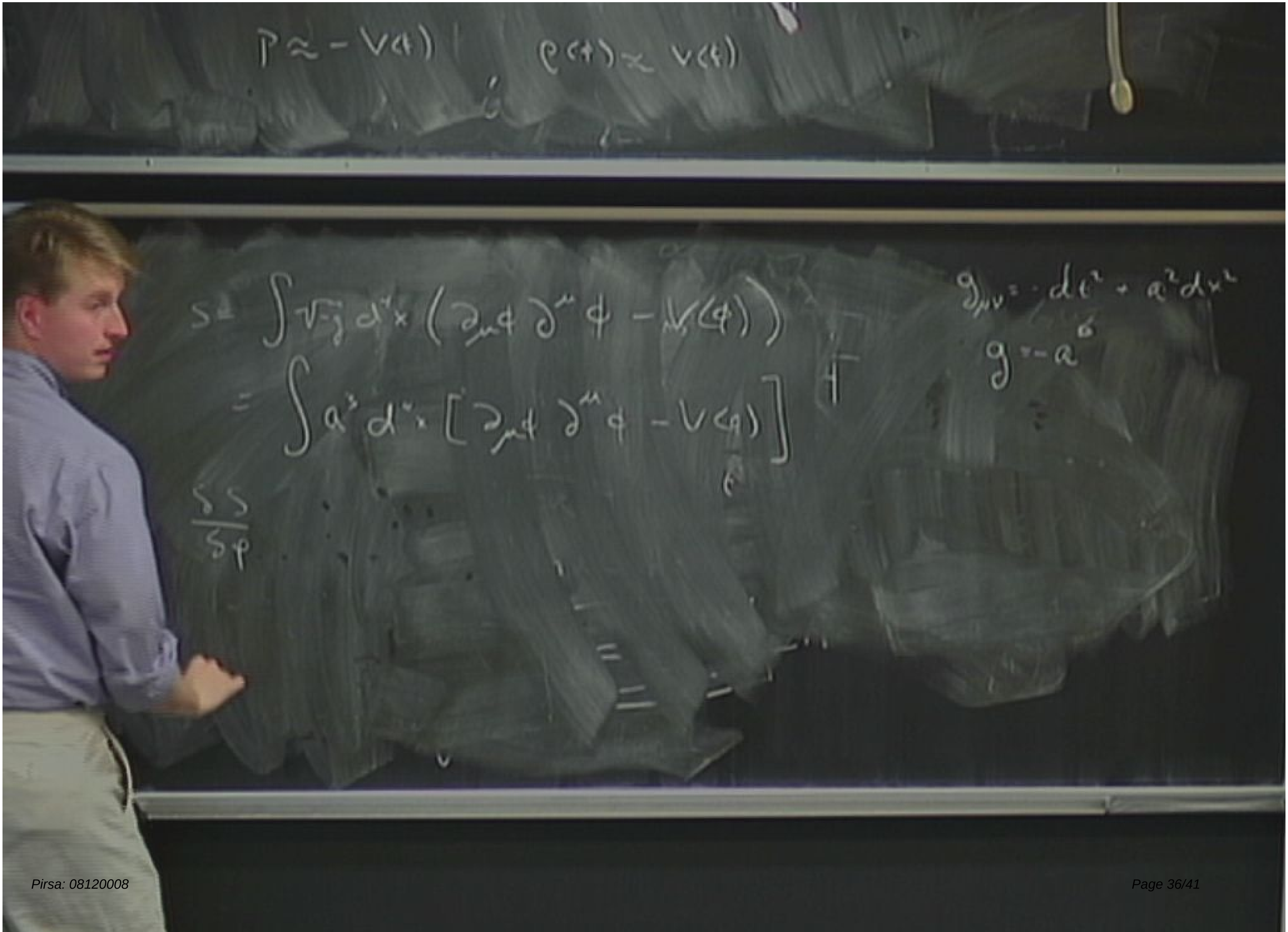
$$\simeq -V(\phi) \qquad \rho(\phi) \simeq V(\phi)$$

$$H^2 = \frac{8\pi G}{3}\rho_\phi$$

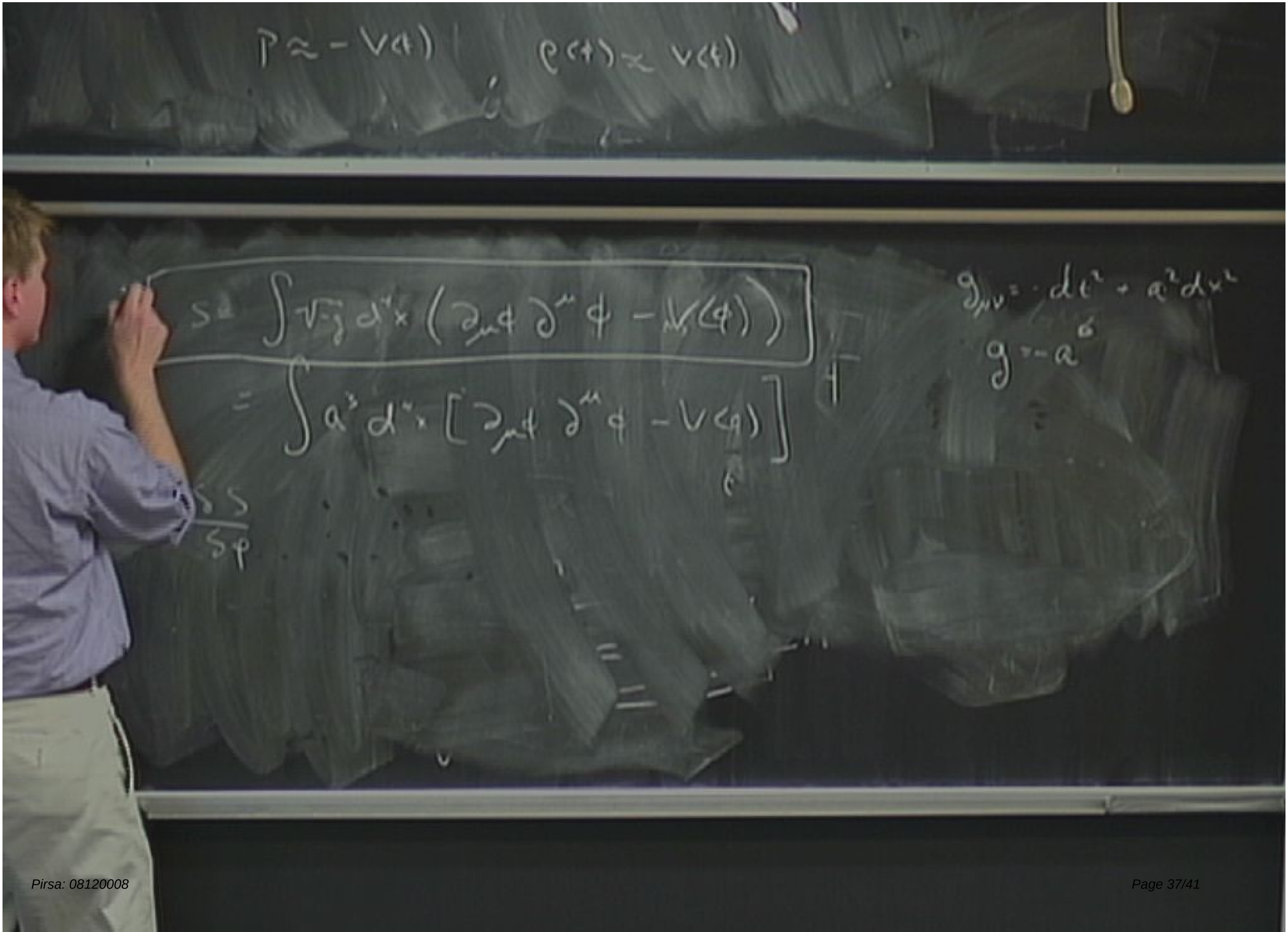
$$= \frac{8\pi G}{3}\left(\frac{1}{2}\dot\phi^2 + V(\phi)\right)$$

$$\boxed{a = e^{H_0 t}}$$


Universe dominated by scalar field
p = -ρ
= (∂_μ φ)(∂^μ φ) - V(φ)
ρ + 3p ~ -2|ρ|
= P
if (∂φ)² << V(φ)
P ≈ -V(φ)
ρ(φ) ≈ V(φ)
H² = 8πG/3 ρ_φ
= 8πG/3 (½ φ̇² + V(φ))
a = e^{H₀t}



$P \approx -V(\phi)$
$\rho(\phi) \approx V(\phi)$
$S = \int \sqrt{-g}\, d^4x \left(\partial_\mu \phi\, \partial^\mu \phi - V(\phi)\right)$
$= \int a^3 d^4x \left[\partial_\mu \phi\, \partial^\mu \phi - V(\phi)\right]$
$\frac{\delta S}{\delta \phi}$
$g_{\mu\nu} = -dt^2 + a^2 dx^2$
$g = -a^6$


$P \approx -V(\phi)$
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$= \int a^3 d^4x \left[ \partial_\mu \phi \, \partial^\mu \phi - V(\phi) \right]$
$\frac{\delta S}{\delta \phi}$
$g_{\mu\nu} = -dt^2 + a^2 dx^2$
$g = -a^6$

$$p \approx -V(\phi) \qquad \rho(\phi) \approx V(\phi)$$

Def. scalar energy field

$\phi$: spin 0

$$\boxed{S = \int \sqrt{-g}\, d^4x \left( \partial_\mu \phi\, \partial^\mu \phi - V(\phi) \right)}$$

$$= \int a^3 d^4x \left[ \partial_\mu \phi\, \partial^\mu \phi - V(\phi) \right]$$

$$g_{\mu\nu} = -dt^2 + a^2 dx^2$$

$$g = -a^6$$

$$\frac{\delta S}{\delta \phi}$$

$P \approx -V(\phi)$ $\qquad \rho(\phi) \approx V(\phi)$

Def. scalar energy field

$\phi$: spin 0

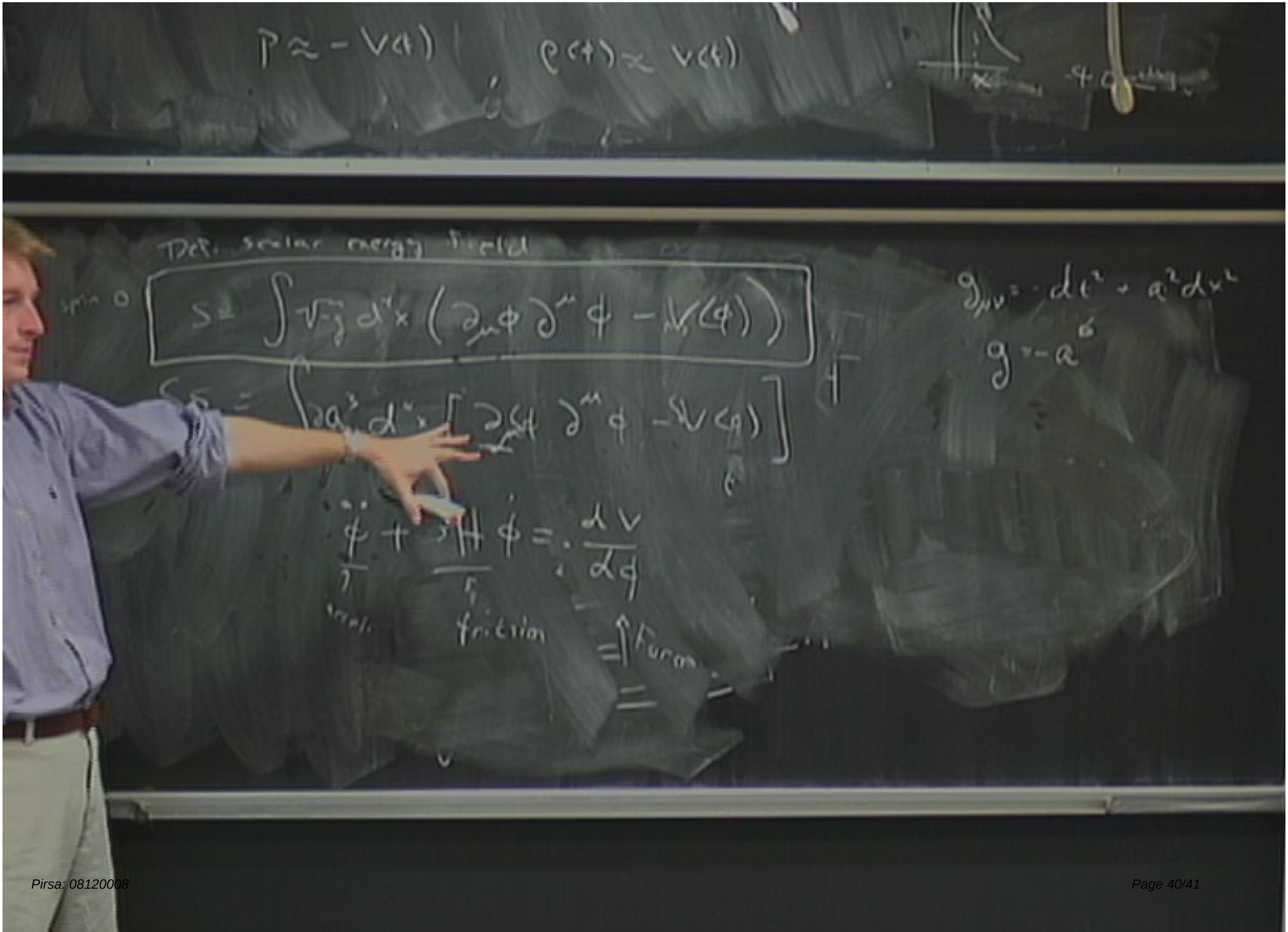
$$\boxed{S = \int \sqrt{-g}\, d^4x \left( \partial_\mu \phi\, \partial^\mu \phi - V(\phi) \right)}$$

$$= \int a^3 d^4x \left[ \partial_\mu \phi\, \partial^\mu \phi - V(\phi) \right]$$

$$g_{\mu\nu} = -dt^2 + a^2 dx^2$$

$$g = -a^6$$

$$\frac{\delta S}{\delta \phi}$$


P ≈ - V(φ)
ρ(φ) ≈ V(φ)
Def. scalar energy field
spin 0
S = ∫ √-g d⁴x ( ∂_μ φ ∂^μ φ - V(φ) )
g_μν = -dt² + a²dx²
g = -a⁶
friction