Title: Quantum Field Theory 1 - Lecture 15A

Date: Dec 17, 2008 10:30 AM

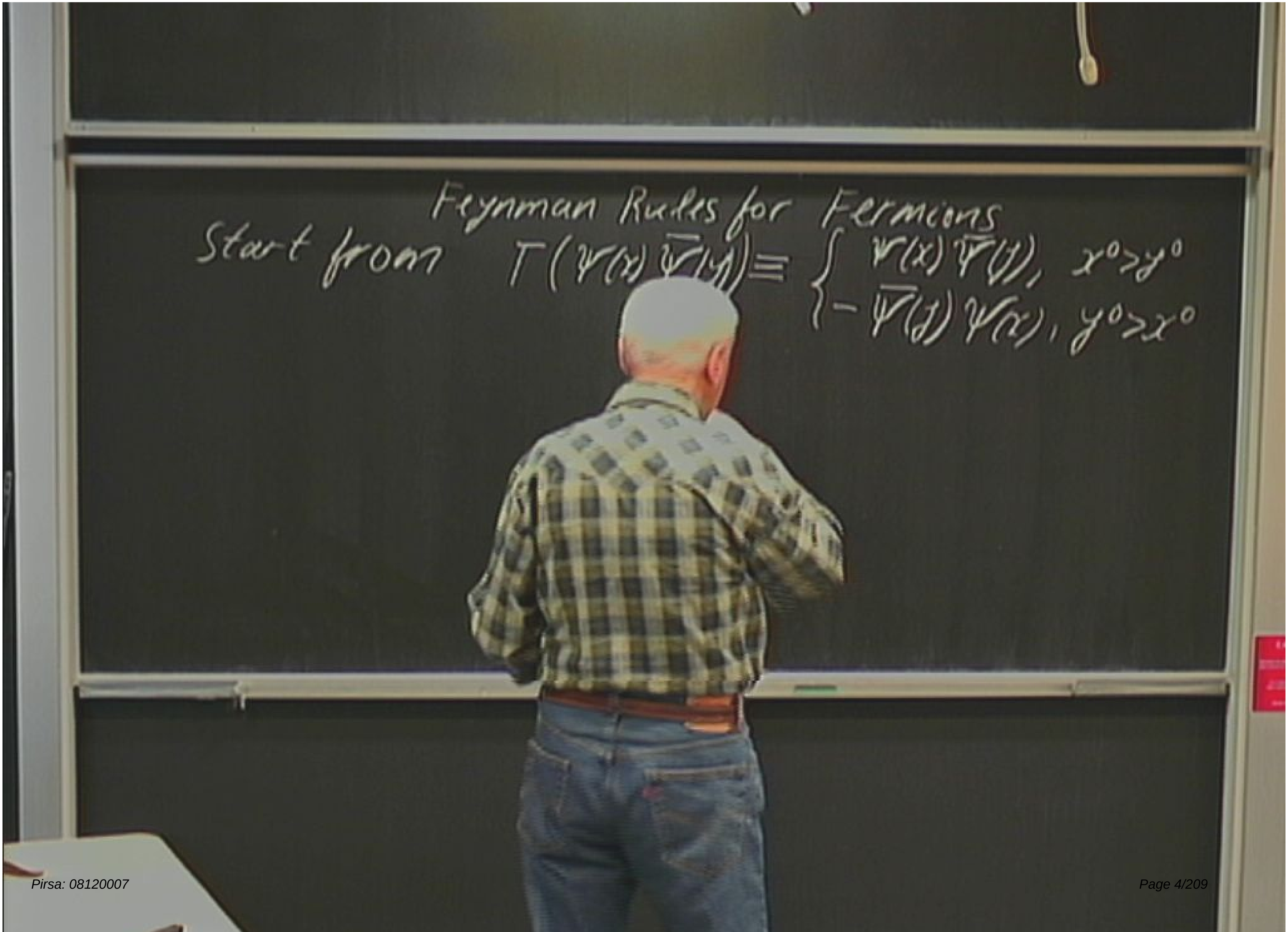
URL: http://pirsa.org/08120007

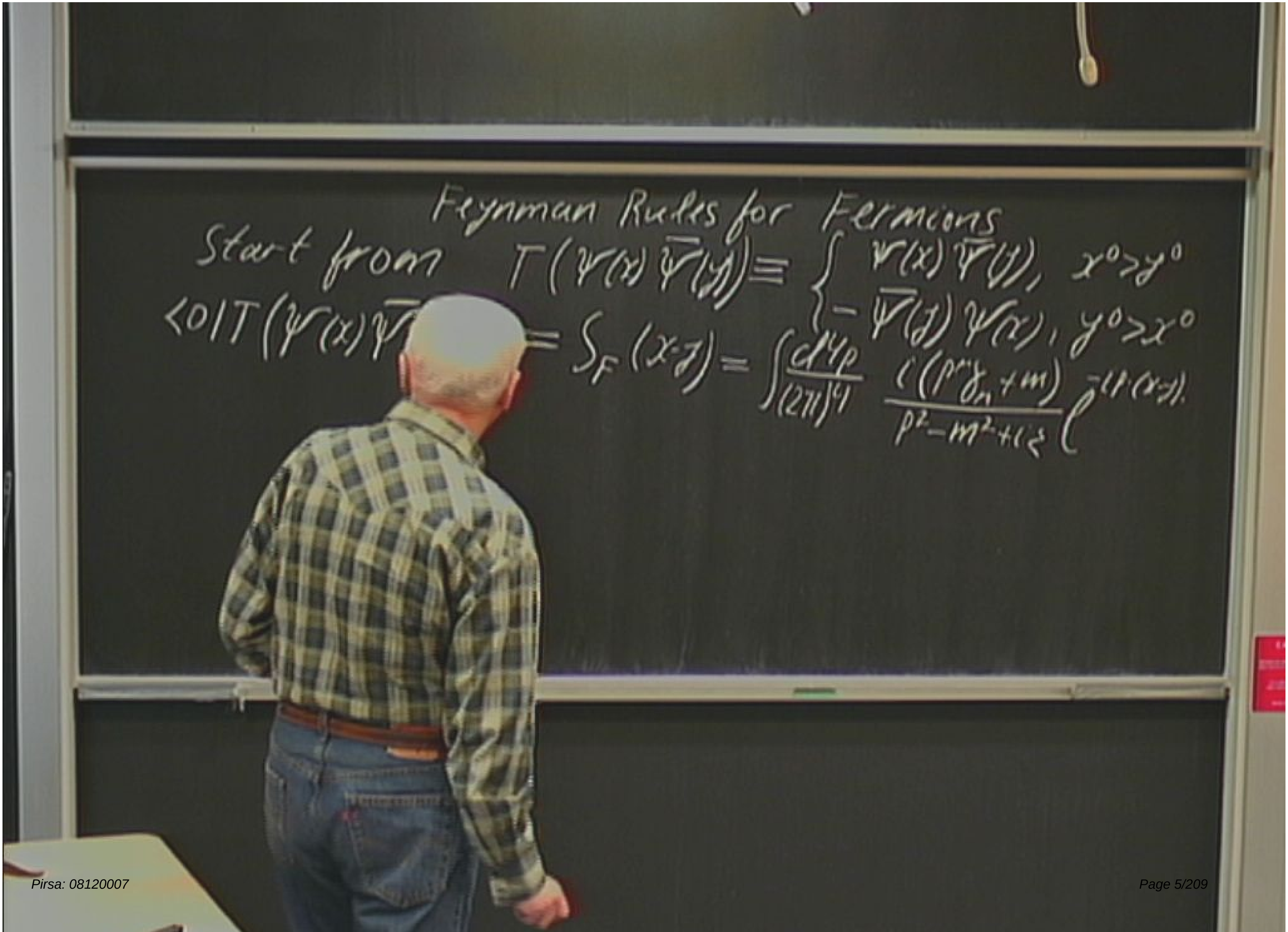
Abstract: Quantum Field Theory I course taught by Volodya Miransky of the University of Western Ontario

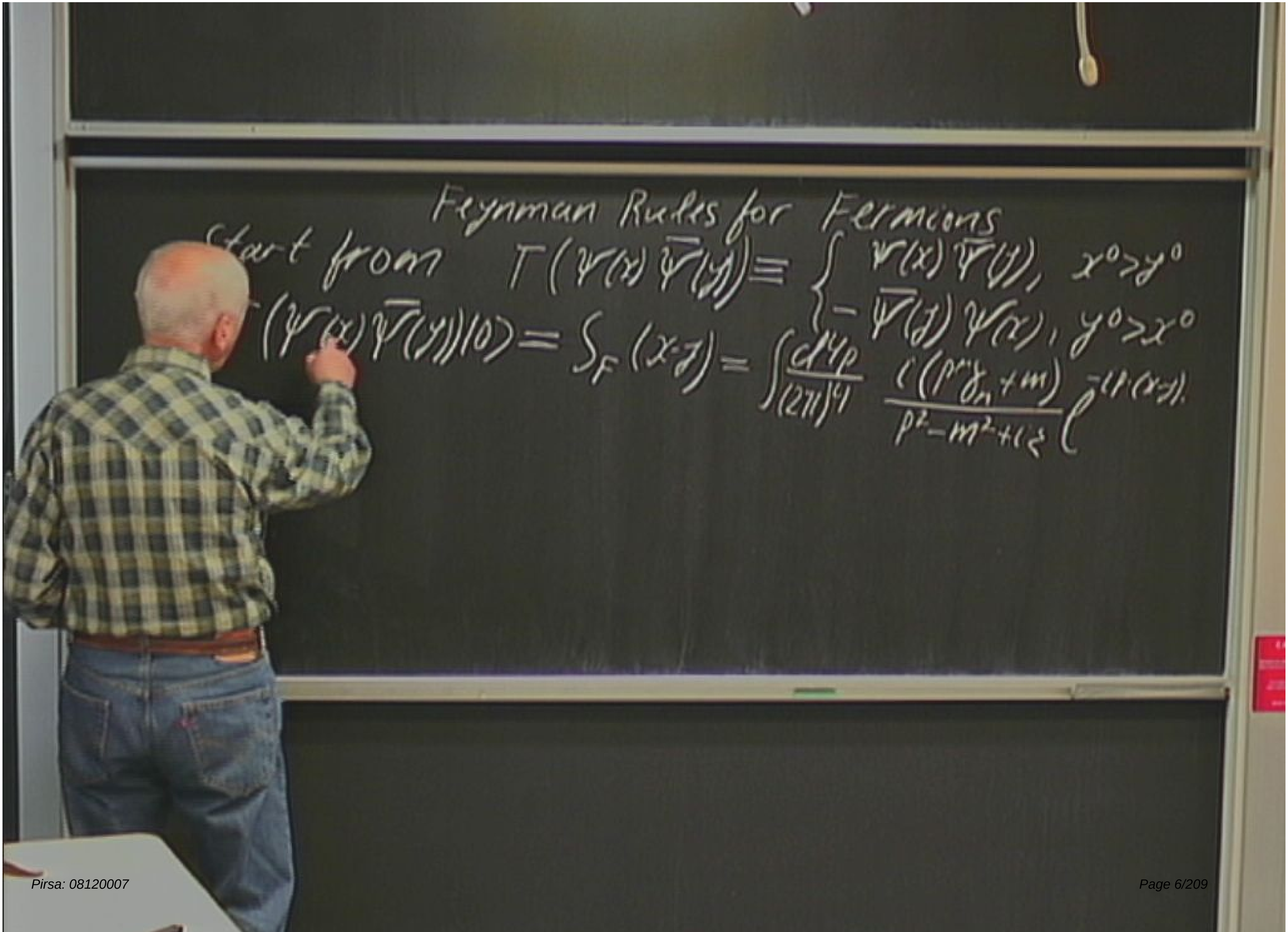
Feynman Rules for Fermions

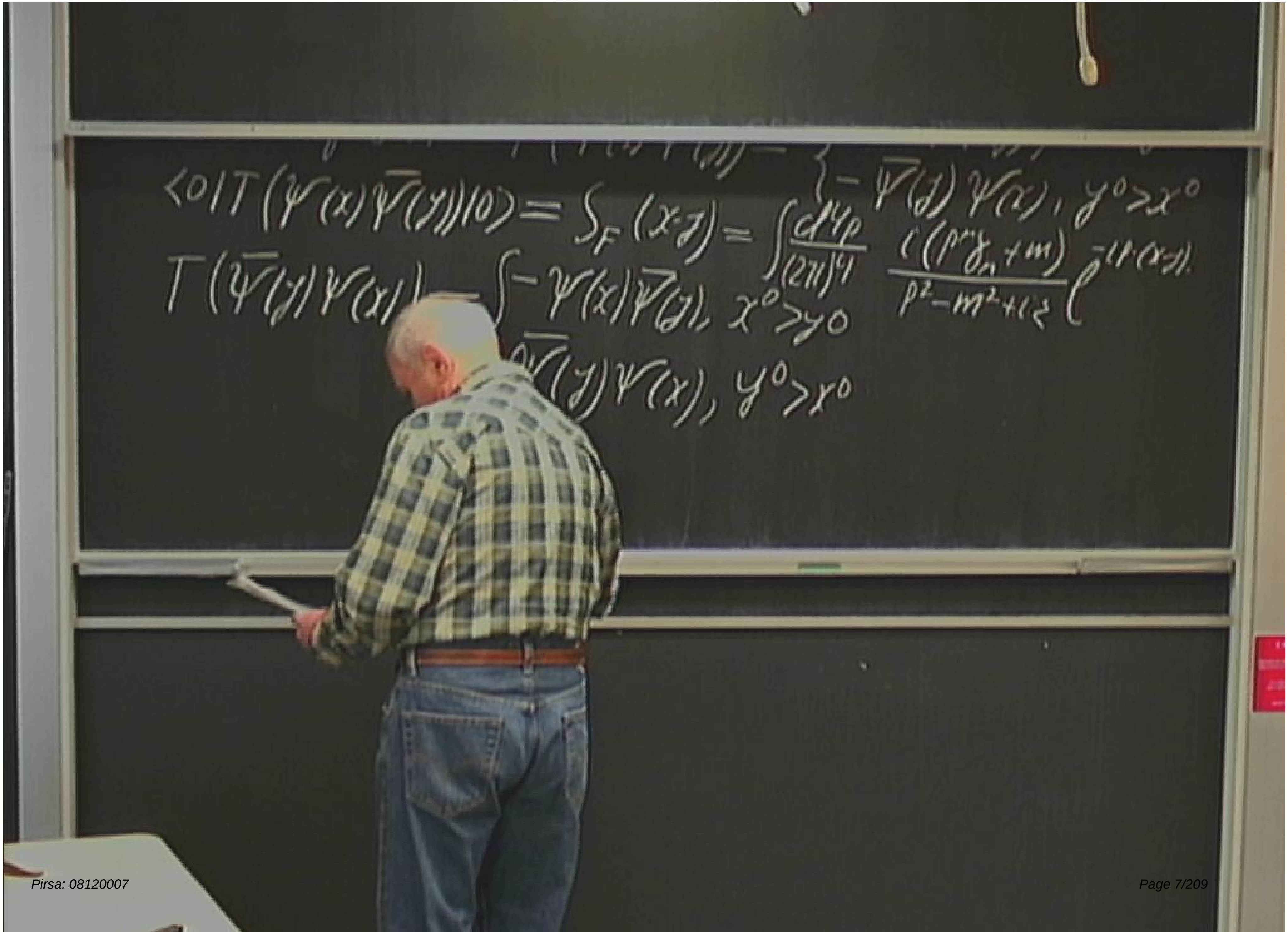
Feynman Rules for Fermions

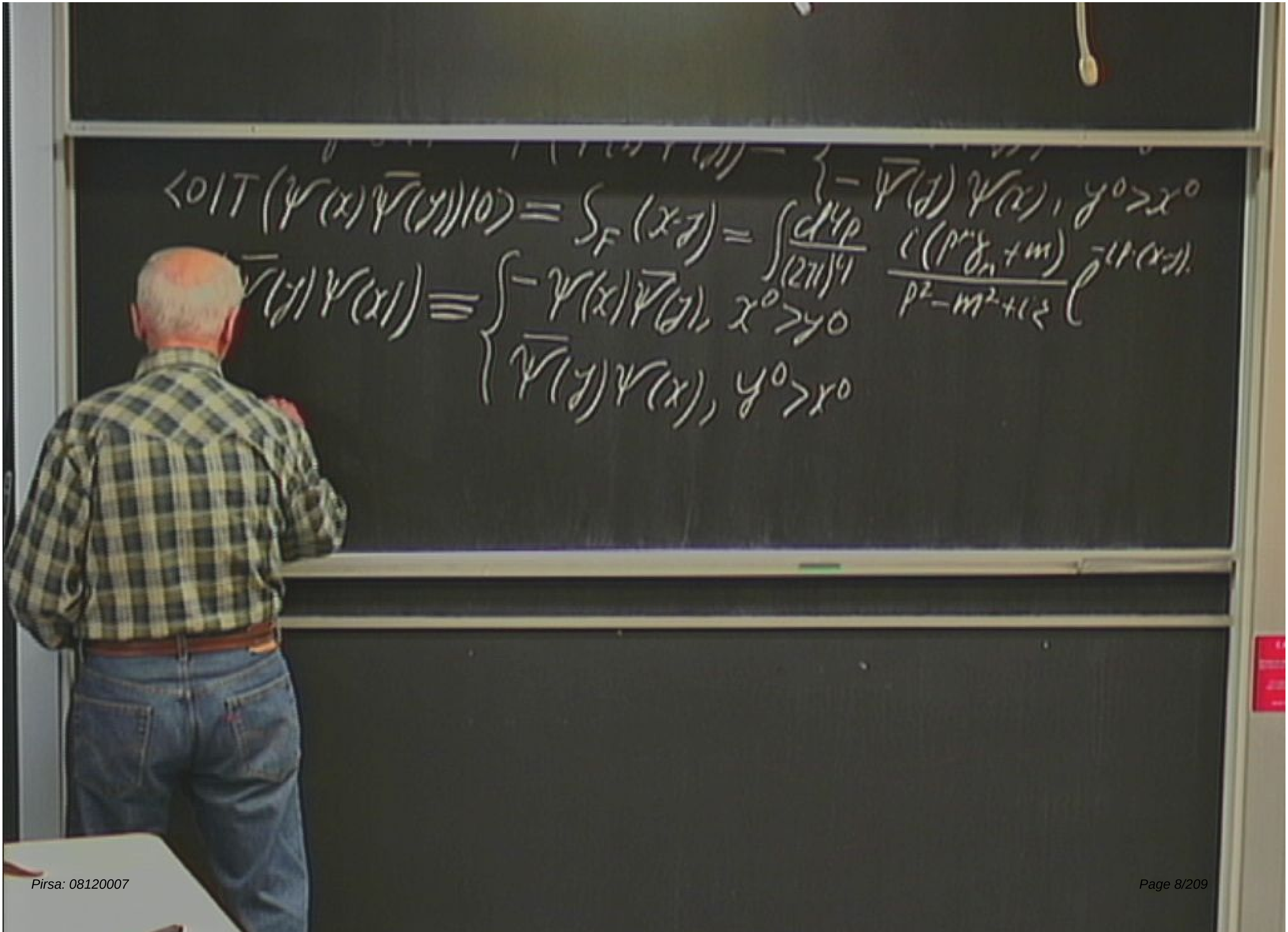
Start from $T(\psi(x)\bar{\psi}(y)) \equiv \{$

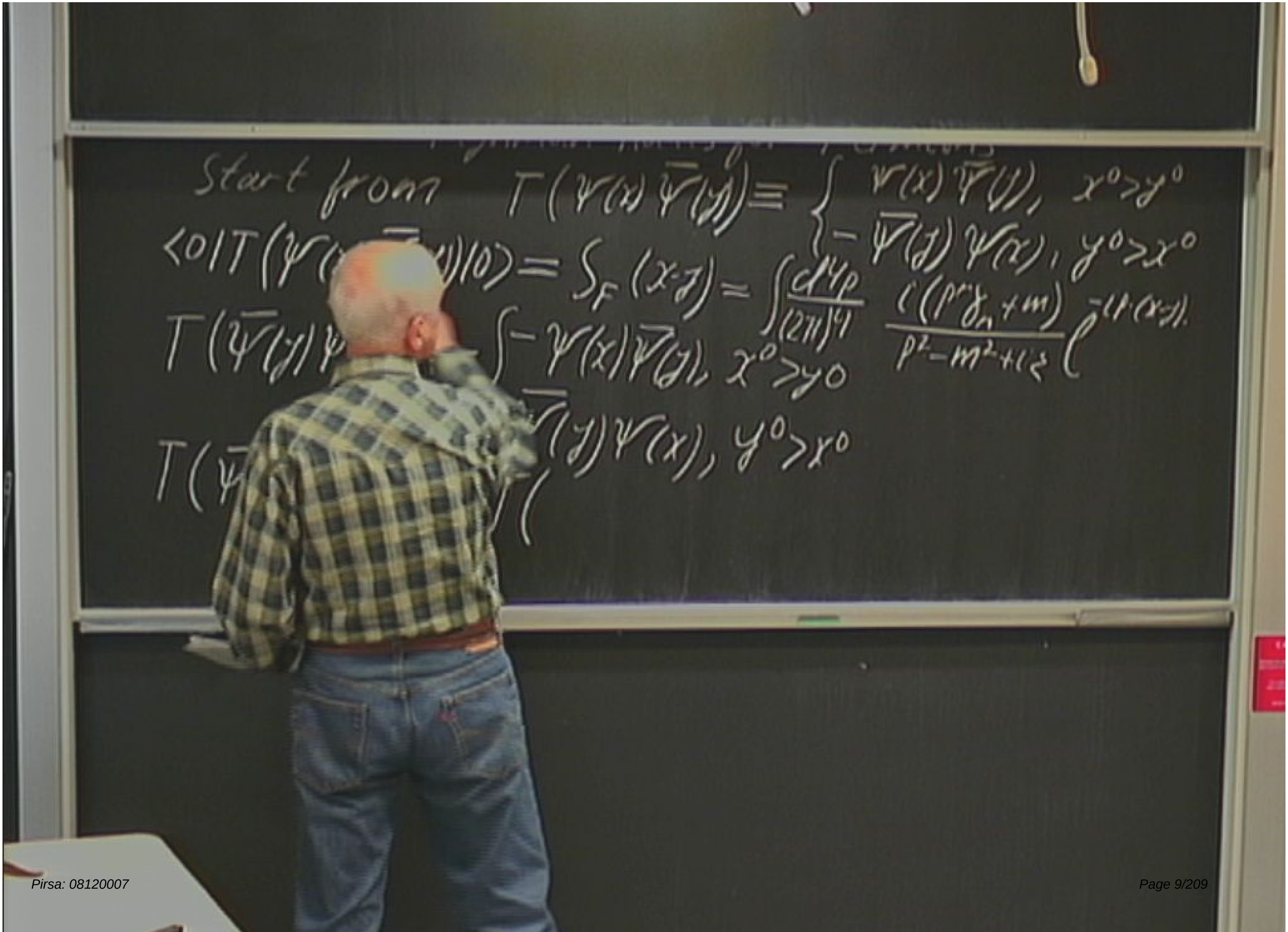

Feynman Rules for Fermions
Start from $T(\psi(x)\bar{\psi}(y)) \equiv \begin{cases} \psi(x)\bar{\psi}(y), & x^0 > y^0 \\ -\bar{\psi}(y)\psi(x), & y^0 > x^0 \end{cases}$

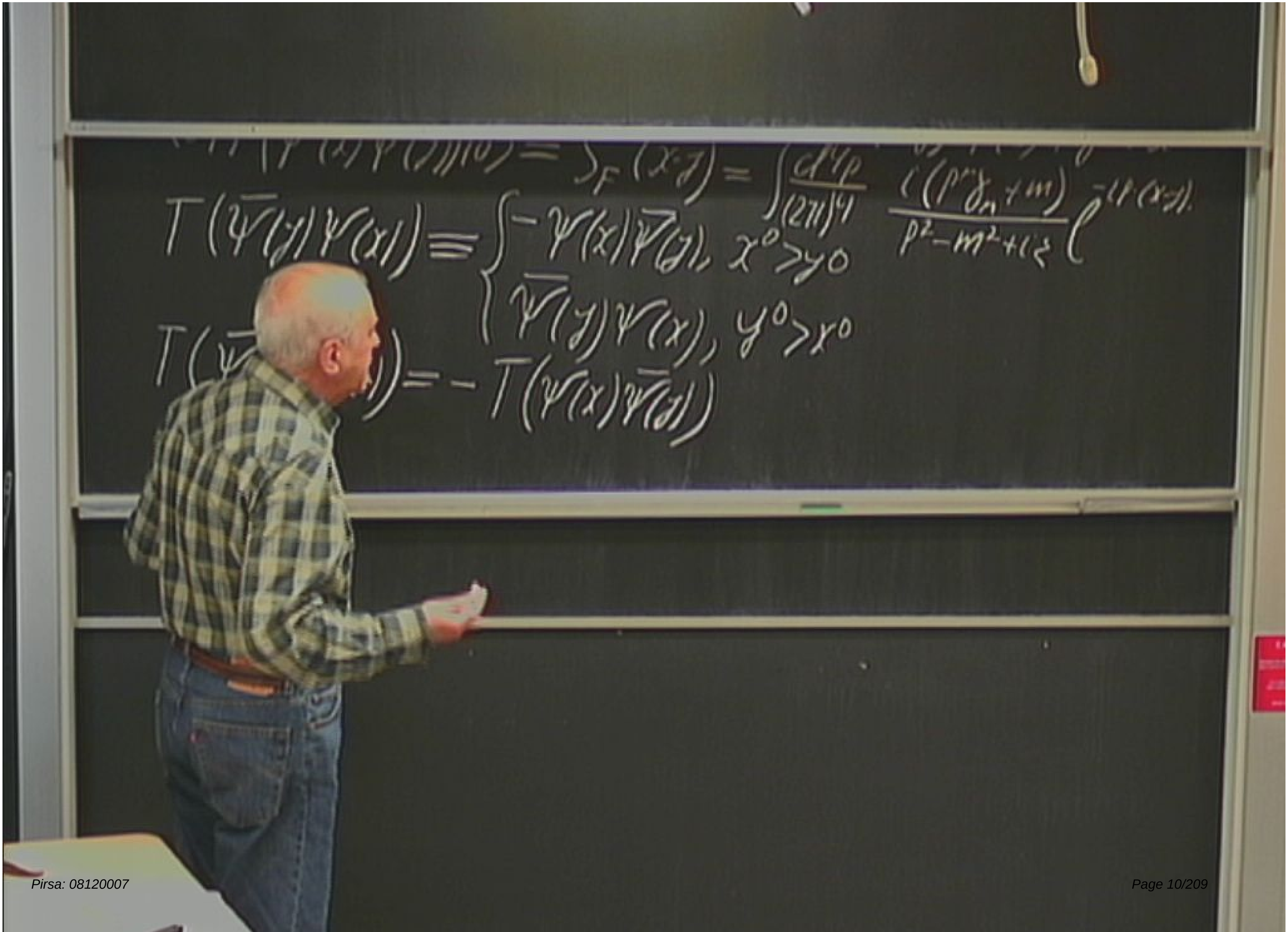

Feynman Rules for Fermions
Start from $T(\psi(x)\bar{\psi}(y)) \equiv \begin{cases} \psi(x)\bar{\psi}(y), & x^0>y^0 \\ -\bar{\psi}(y)\psi(x), & y^0>x^0 \end{cases}$
$\langle 0|T(\psi(x)\bar{\psi}$ $= S_F(x-y) = \int \frac{d^4p}{(2\pi)^4} \frac{i(p^\mu\gamma_\mu+m)}{p^2-m^2+i\epsilon} e^{-ip\cdot(x-y)}$


Feynman Rules for Fermions
Start from $T(\psi(x)\bar{\psi}(y)) \equiv \begin{cases} \psi(x)\bar{\psi}(y), & x^0 > y^0 \\ -\bar{\psi}(y)\psi(x), & y^0 > x^0 \end{cases}$
$\langle 0|T(\psi(x)\bar{\psi}(y))|0\rangle = S_F(x-y) = \int \frac{d^4p}{(2\pi)^4} \frac{i(p^\mu\gamma_\mu + m)}{p^2 - m^2 + i\epsilon} e^{-ip\cdot(x-y)}$


$\{-\bar\psi(y)\psi(x),\ y^0>x^0$
$\langle 0|T(\psi(x)\bar\psi(y))|0\rangle = S_F(x-y) = \int \frac{d^4p}{(2\pi)^4} \frac{i(p^\mu\gamma_\mu+m)}{p^2-m^2+i\epsilon} e^{-ip\cdot(x-y)}$
$T(\bar\psi(y)\psi(x)) = \{-\psi(x)\bar\psi(y),\ x^0>y^0$
$\bar\psi(y)\psi(x),\ y^0>x^0$


$\{-\bar{\psi}(y)\psi(x),\ y^0 > x^0$
$\langle 0|T(\psi(x)\bar{\psi}(y))|0\rangle = S_F(x-y) = \int \frac{d^4p}{(2\pi)^4} \frac{i(p^\mu\gamma_\mu + m)}{p^2 - m^2 + i\epsilon} e^{-ip\cdot(x-y)}$
$T(\bar{\psi}(y)\psi(x)) \equiv \begin{cases} -\psi(x)\bar{\psi}(y), & x^0 > y^0 \\ \bar{\psi}(y)\psi(x), & y^0 > x^0 \end{cases}$

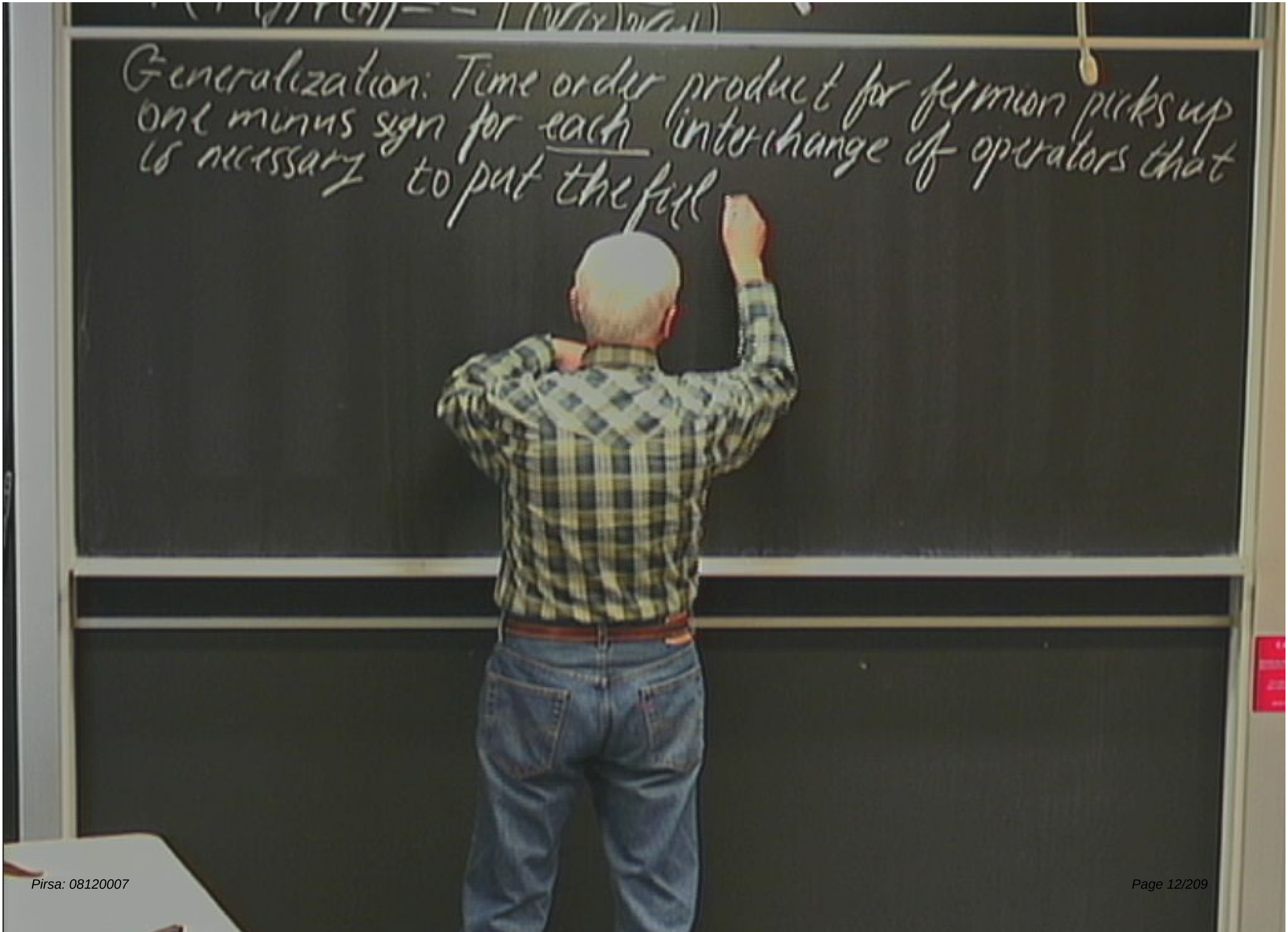

Start from $T(\psi(x)\bar{\psi}(y)) \equiv \begin{cases} \psi(x)\bar{\psi}(y), & x^0 > y^0 \\ -\bar{\psi}(y)\psi(x), & y^0 > x^0 \end{cases}$
$= S_F(x-y) = \int \frac{d^4p}{(2\pi)^4} \frac{i(p^\mu\gamma_\mu + m)}{p^2 - m^2 + i\epsilon} e^{-ip\cdot(x-y)}$
$-\psi(x)\bar{\psi}(y), \; x^0 > y^0$
$\bar{\psi}(y)\psi(x)), \; y^0 > x^0$

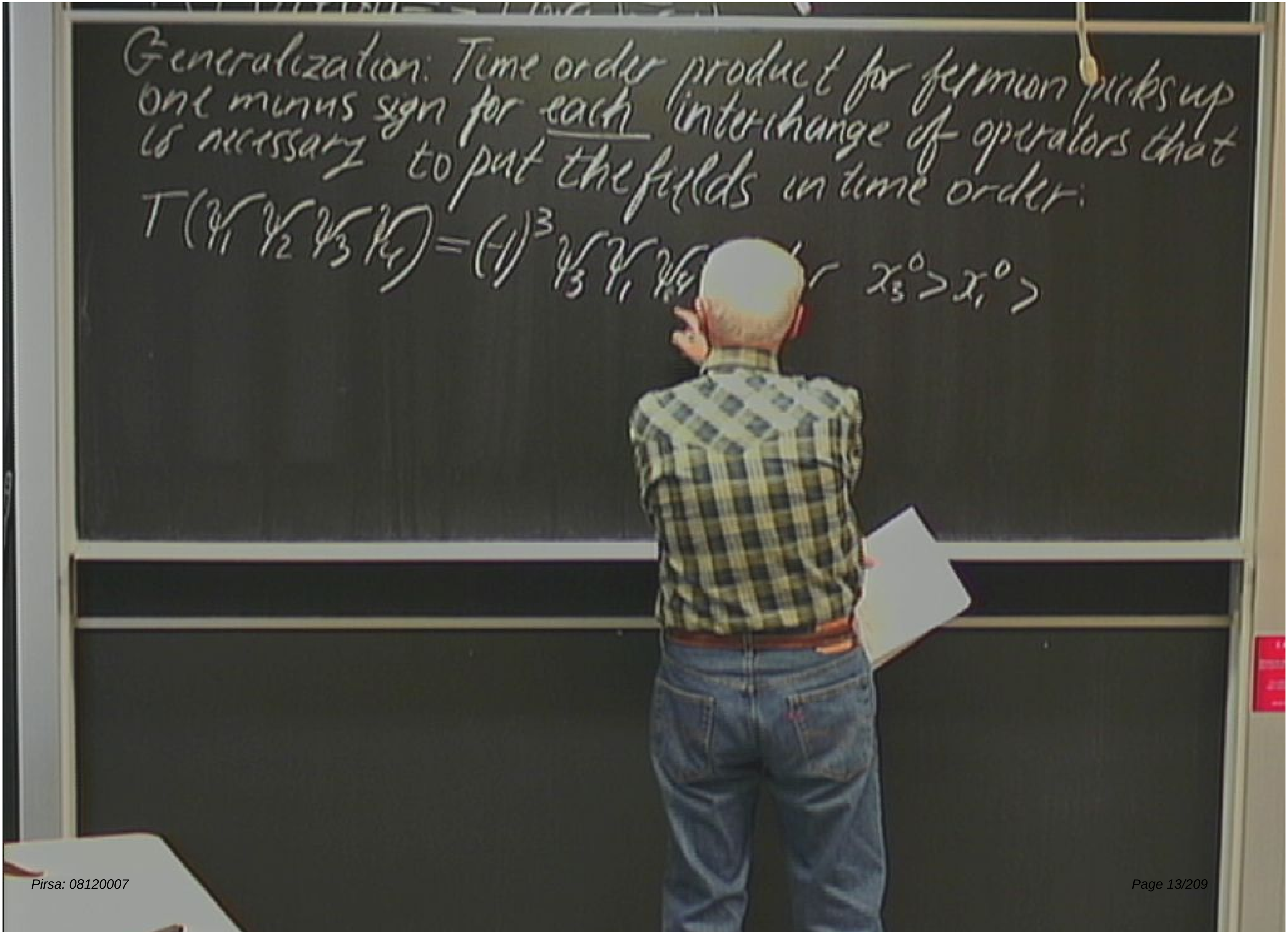

$\langle 0|T(\psi(x)\bar\psi(y))|0\rangle = S_F(x-y) = \int \frac{d^4p}{(2\pi)^4} \frac{i(p^\mu\gamma_\mu+m)}{p^2-m^2+i\epsilon} e^{-ip(x-y)}$
$T(\bar\psi(y)\psi(x)) \equiv \begin{cases} -\psi(x)\bar\psi(y), & x^0>y^0 \\ \bar\psi(y)\psi(x), & y^0>x^0 \end{cases}$
$T(\bar\psi(y)\psi(x)) = -T(\psi(x)\bar\psi(y))$

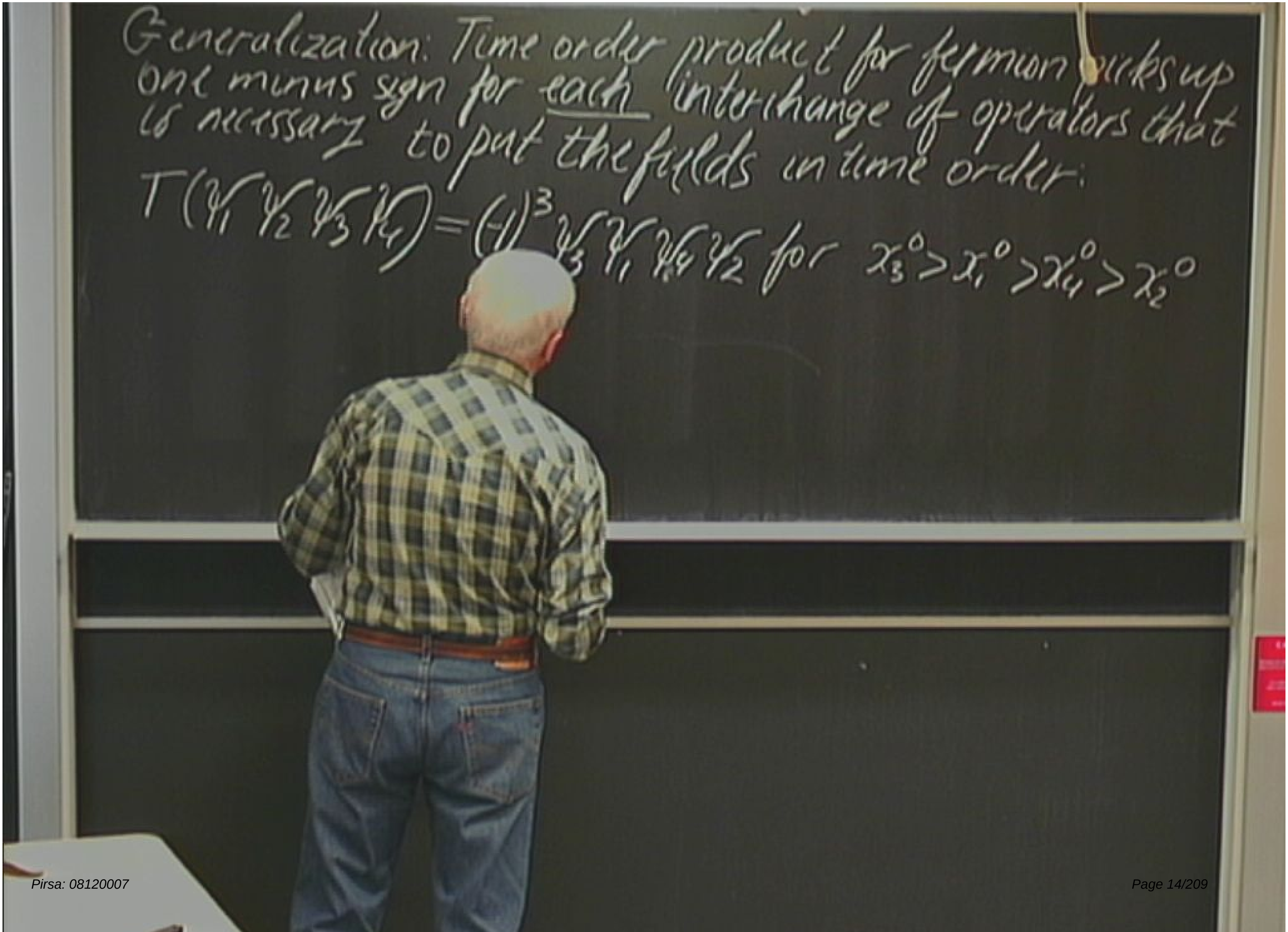
$$\langle 0|T(\psi(x)\bar{\psi}(y))|0\rangle = S_F(x-y) = \int \frac{d^4p}{(2\pi)^4}\, \frac{i(p^\mu\gamma_\mu + m)}{p^2 - m^2 + i\epsilon}\, e^{-ip\cdot(x-y)}$$

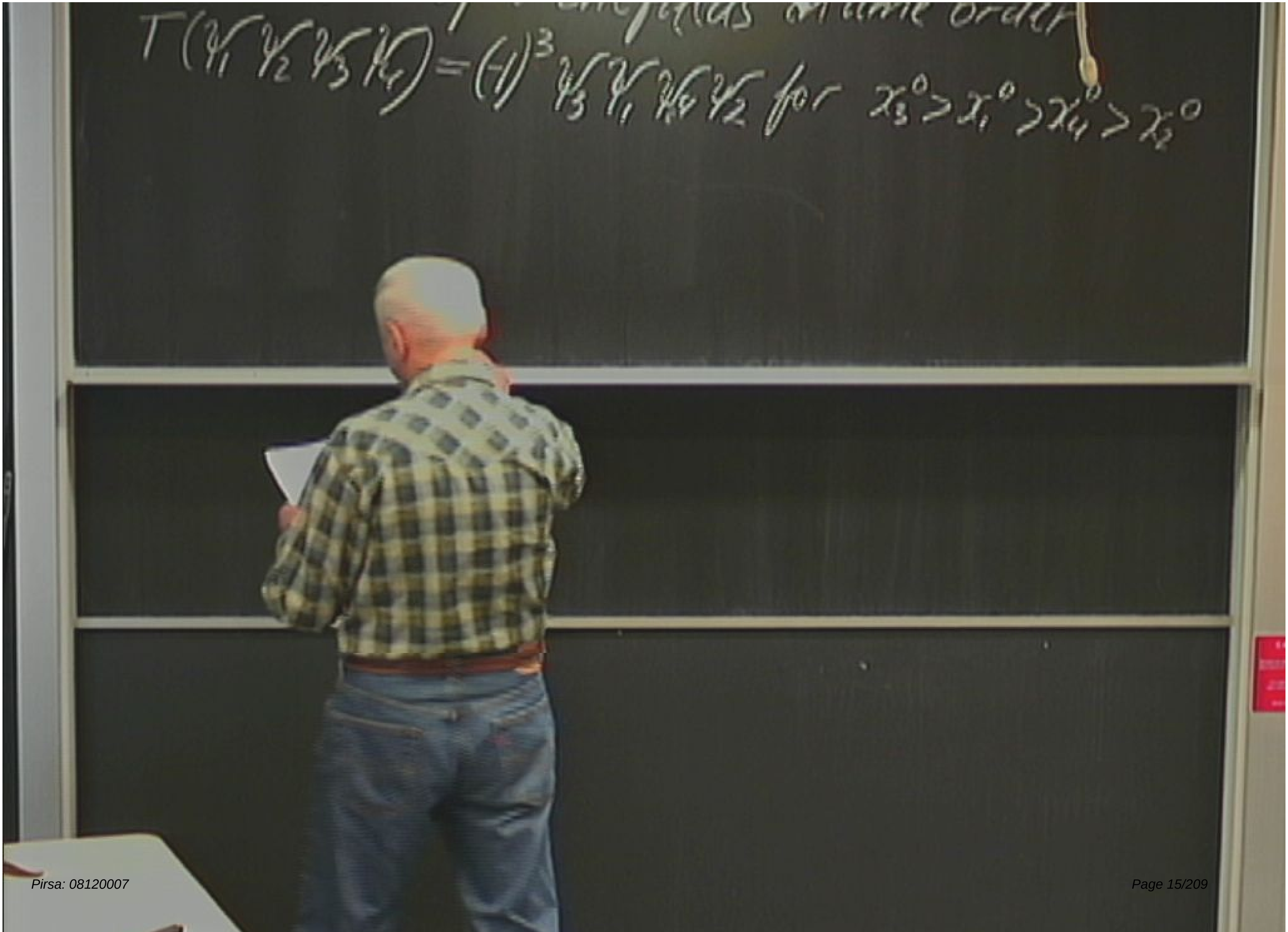
$$T(\bar{\psi}(y)\psi(x)) \equiv \begin{cases} -\psi(x)\bar{\psi}(y), & x^0 > y^0 \\ \bar{\psi}(y)\psi(x), & y^0 > x^0 \end{cases}$$

$$T(\bar{\psi}(y)\psi(x)) = -T(\psi(x)\bar{\psi}(y))$$


Generalization: Time order product for fermion picks up
one minus sign for each interchange of operators that
is necessary to put the fiel


Generalization: Time order product for fermion picks up
one minus sign for each interchange of operators that
is necessary to put the fields in time order:
$T(\psi_1 \psi_2 \psi_3 \psi_4) = (-1)^3 \psi_3 \psi_1 \psi_4$
$x_3^0 > x_1^0 >$

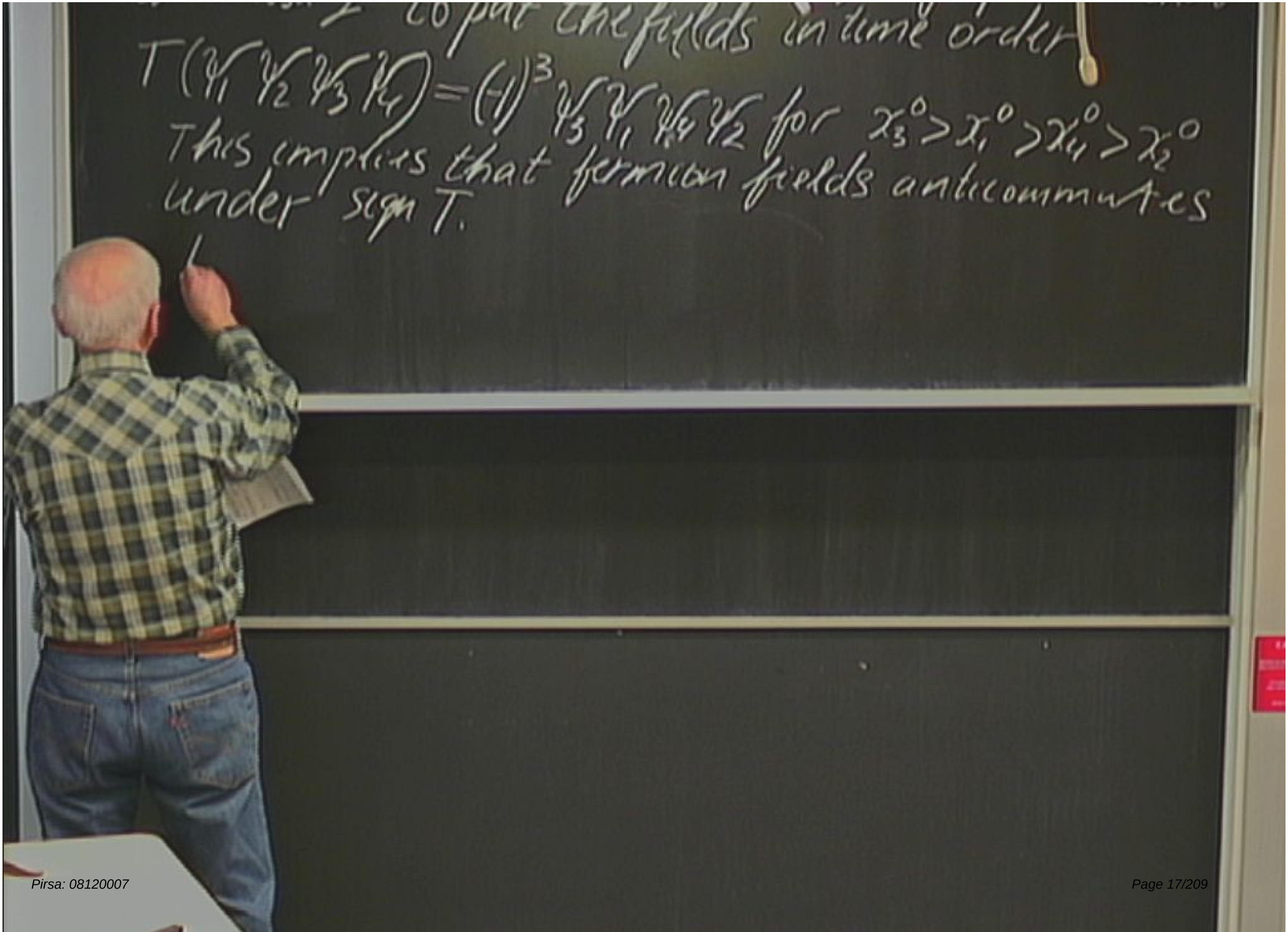

Generalization: Time order product for fermion picks up one minus sign for each interchange of operators that is necessary to put the fields in time order:
$T(\psi_1 \psi_2 \psi_3 \psi_4) = (-1)^3 \psi_3 \psi_1 \psi_4 \psi_2$ for $x_3^0 > x_1^0 > x_4^0 > x_2^0$

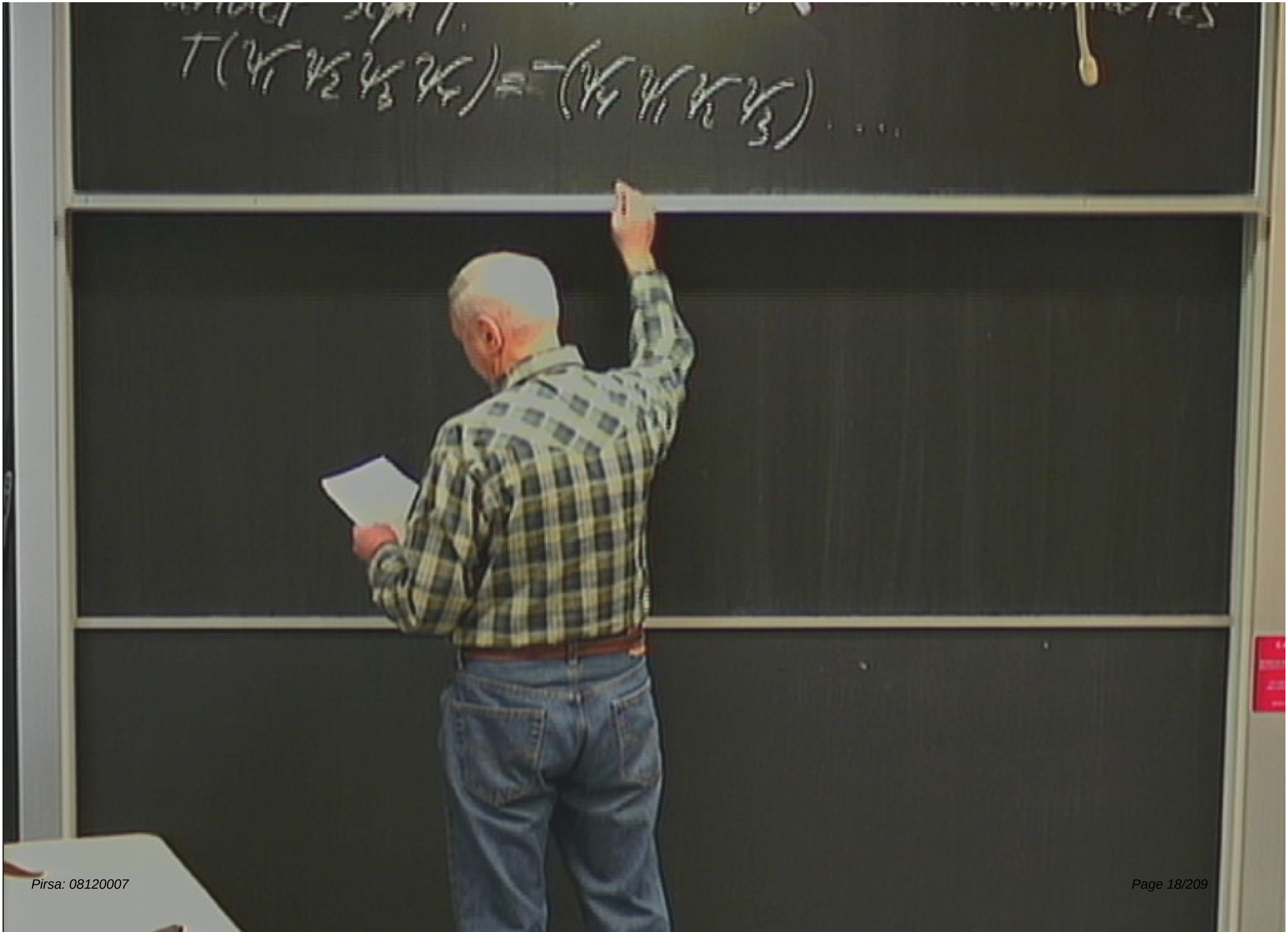

$T(\psi_1 \psi_2 \psi_3 \psi_4) = (-1)^3 \psi_3 \psi_1 \psi_4 \psi_2$ for $x_3^0 > x_1^0 > x_4^0 > x_2^0$

one minus sign for each interchange of operators that is necessary to put the fields in time order:

$$T(\psi_1\psi_2\psi_3\psi_4) = (-1)^3\,\psi_3\psi_1\psi_4\psi_2 \quad \text{for } x_3^0 > x_1^0 > x_4^0 > x_2^0$$

This implies that fermion fields anticommute

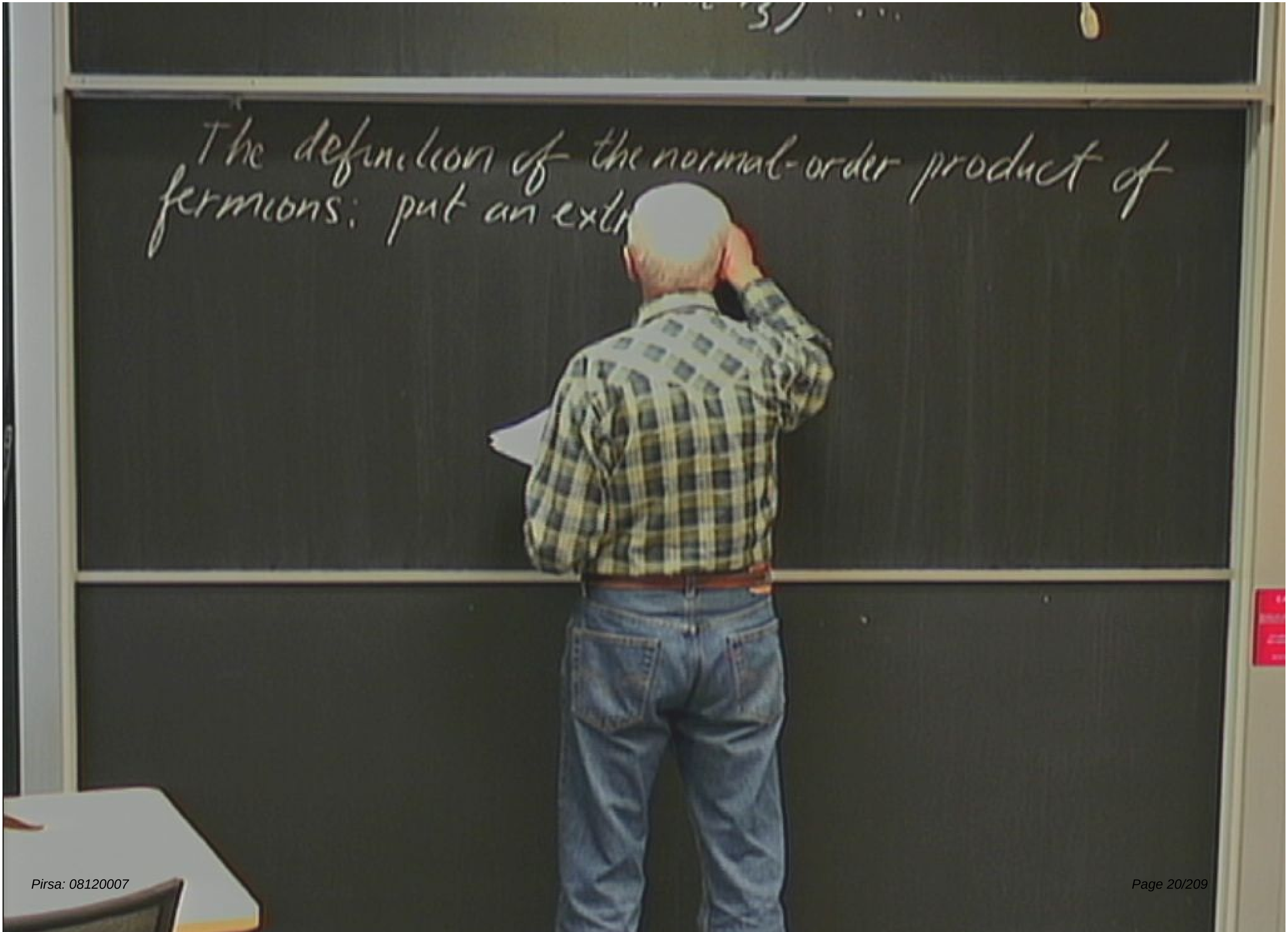

to put the fields in time order
$T(\psi_1 \psi_2 \psi_3 \psi_4) = (-1)^3 \psi_3 \psi_1 \psi_4 \psi_2$ for $x_3^0 > x_1^0 > x_4^0 > x_2^0$
This implies that fermion fields anticommutes
under sign T.

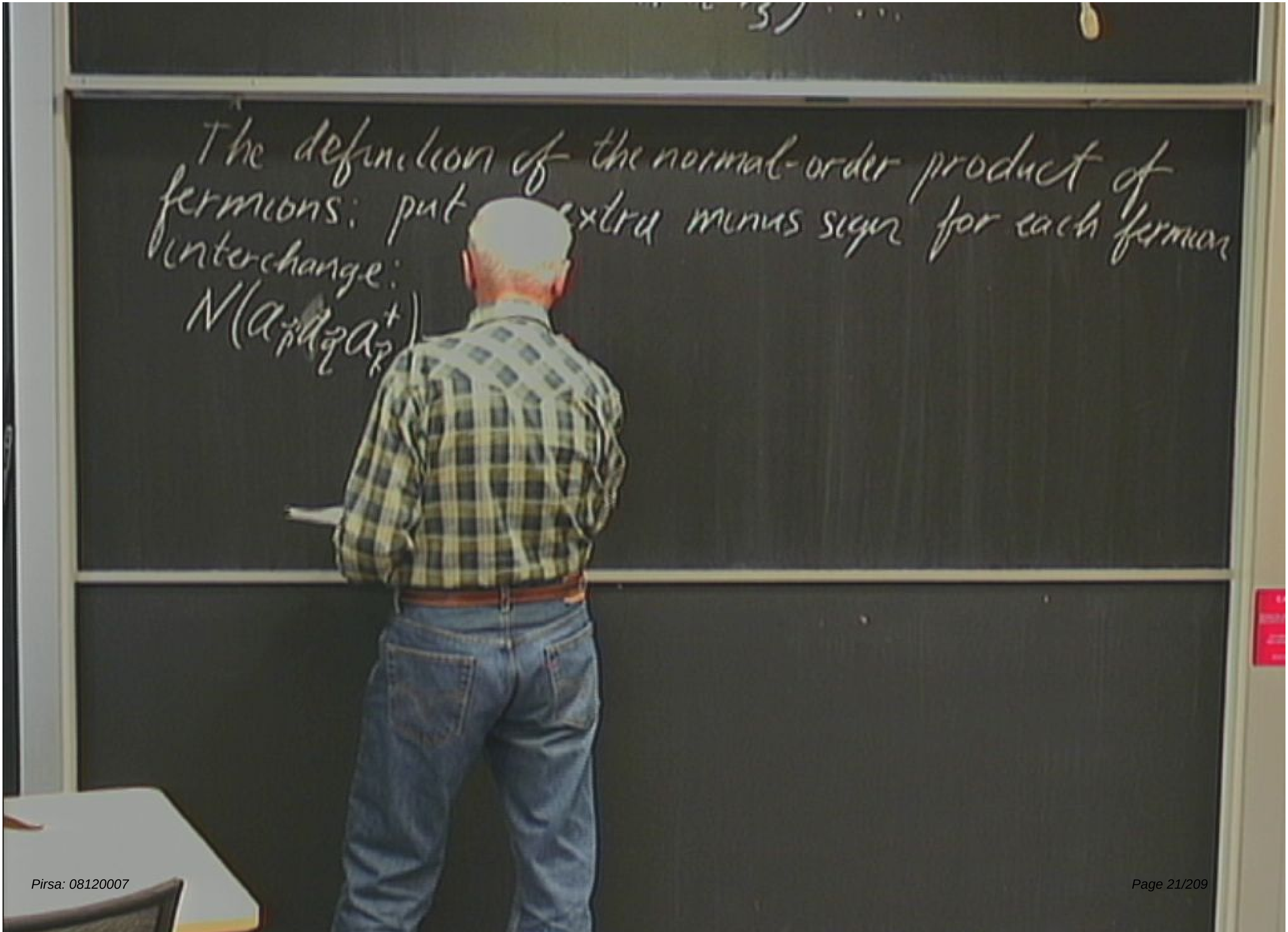

$T(\psi_1 \psi_2 \psi_3 \psi_4) = -(\psi_4 \psi_1 \psi_2 \psi_3) \dots$

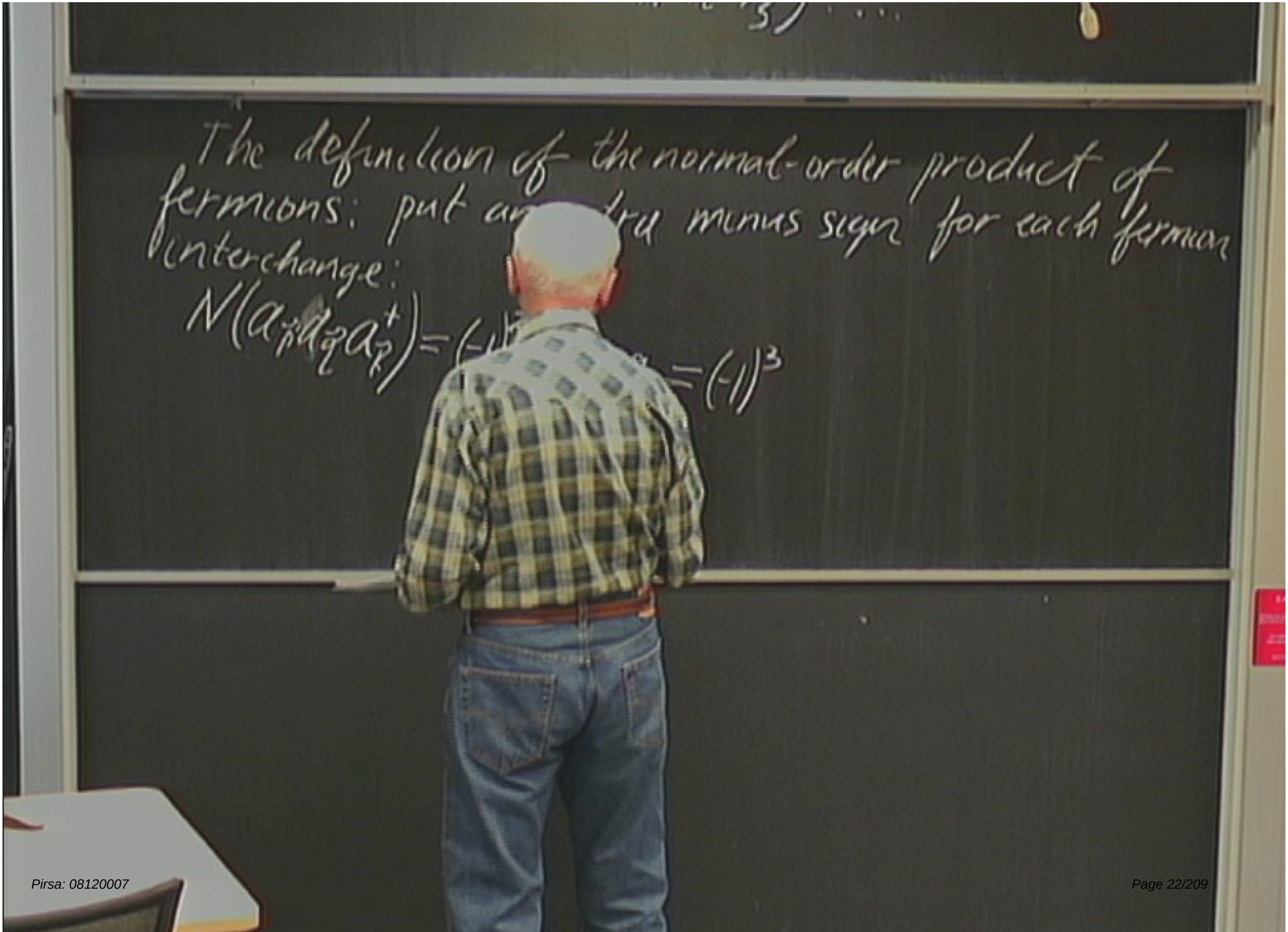
$T(\psi_1\,\psi_2\,\psi_3\,\psi_4) = (\ldots)\ \psi_3\,\psi_1\,\psi_4\,\psi_2$ for $x_3^0 > x_1^0 > x_4^0 > x_2^0$

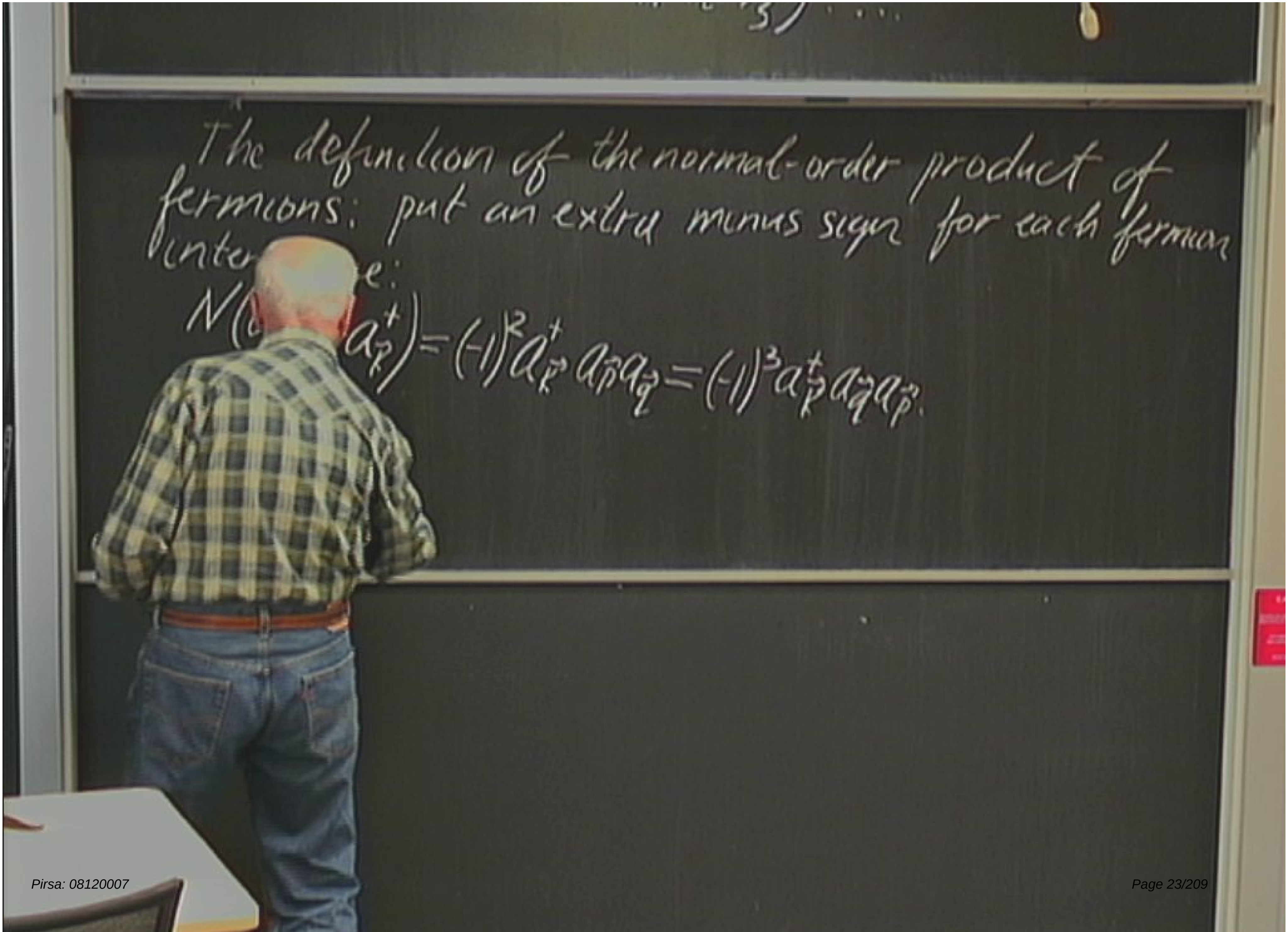
This implies that fermion fields anticommutes under sign T.

$T(\psi_1\,\psi_2\,\psi_3\,\psi_4) = -(\psi_4\,\psi_1\,\psi_2\,\psi_3)\ \ldots$


The definition of the normal-order product of
fermions: put an extr


The definition of the normal-order product of
fermions: put extra minus sign for each fermion
interchange:
N(apaqak+)


The definition of the normal-order product of fermions: put an extra minus sign for each fermion interchange:
$N(a_p a_q a_r^+) = (-1)$
$= (-1)^3$


The definition of the normal-order product of fermions: put an extra minus sign for each fermion
inter
e:
$=(-1)^2 a^+_{\vec k} a_{\vec p} a_{\vec q} = (-1)^3 a^+_{\vec k} a_{\vec q} a_{\vec p}.$

The definition of the normal-order product of fermions: put an extra minus sign for each fermion interchange:

$$N(a_{\vec{p}}a_{\vec{q}}a_{\vec{k}}^{+}) = (-1)^2 a_{\vec{k}}^{+} a_{\vec{p}} a_{\vec{q}} = (-1)^3 a_{\vec{k}}^{+} a_{\vec{q}} a_{\vec{p}}.$$

Generalization of Wick's Theorem.

Start from $\Psi\left(\psi(x)\overline{\psi}(y)\right)$; $\psi(x) = \psi^{+}(x) + \psi^{-}(x)$;

$\psi^{+}(x) = \int \frac{d^3p}{(2\pi)^3 \sqrt{2E_p}}$

... $\psi_1\psi_2$ for $x_3^0 > x_1^0 > x_4^0 > x_2^0$

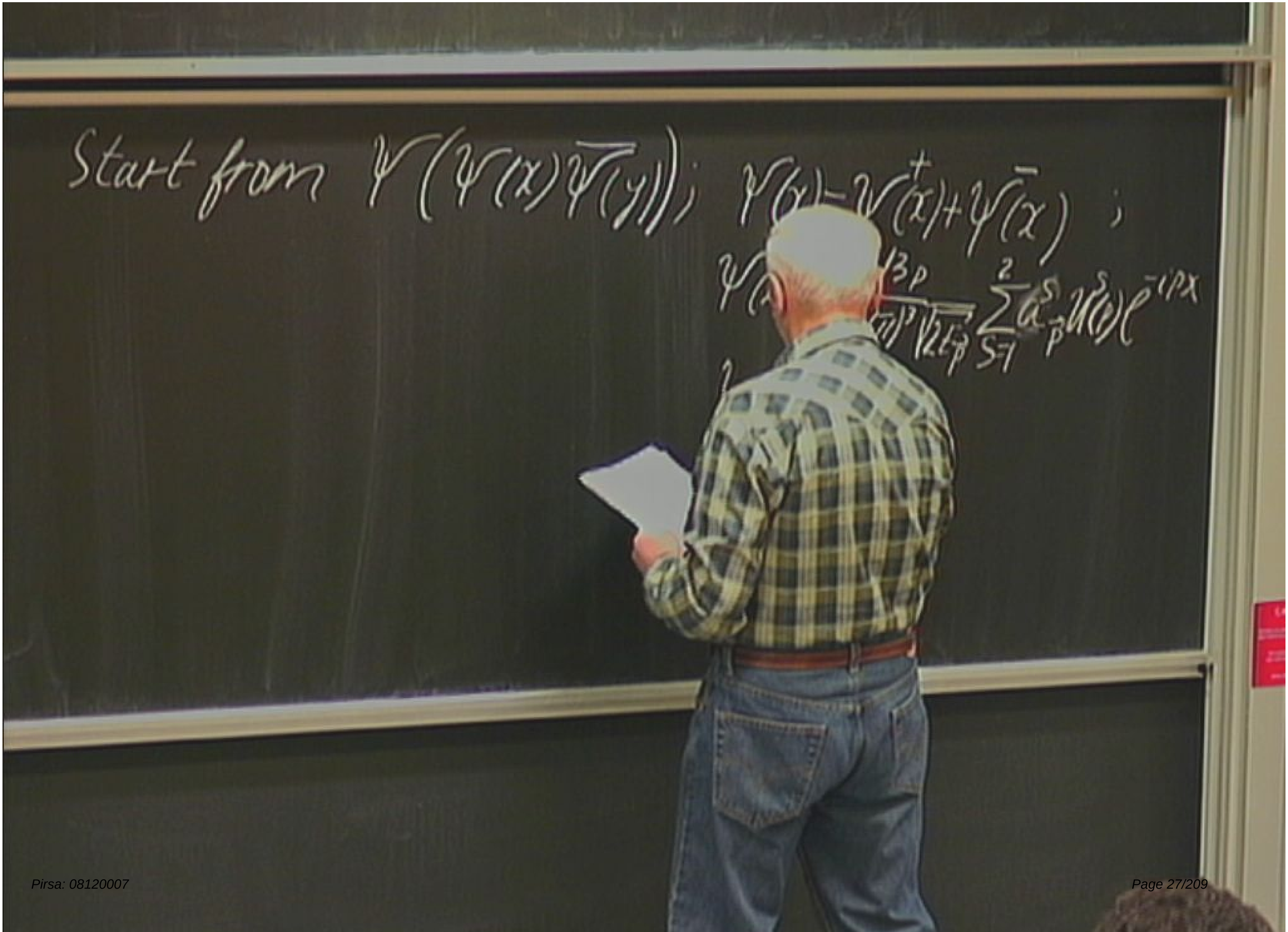
This implies that fermion fields anticommute under sign T.

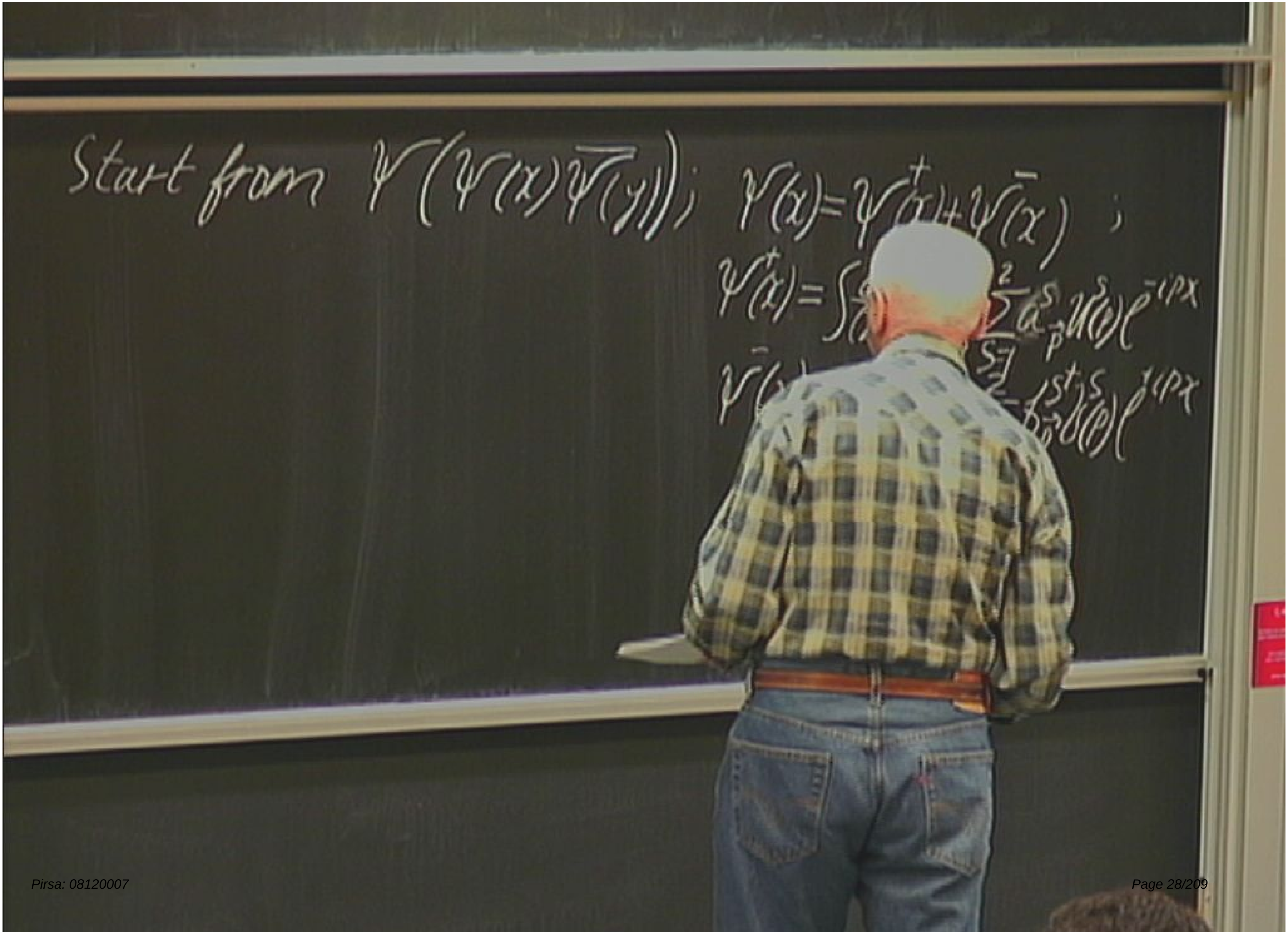
$$T(\psi_1\psi_2\psi_3\psi_4) = -(\psi_4\psi_1\psi_2\psi_3) \ldots$$

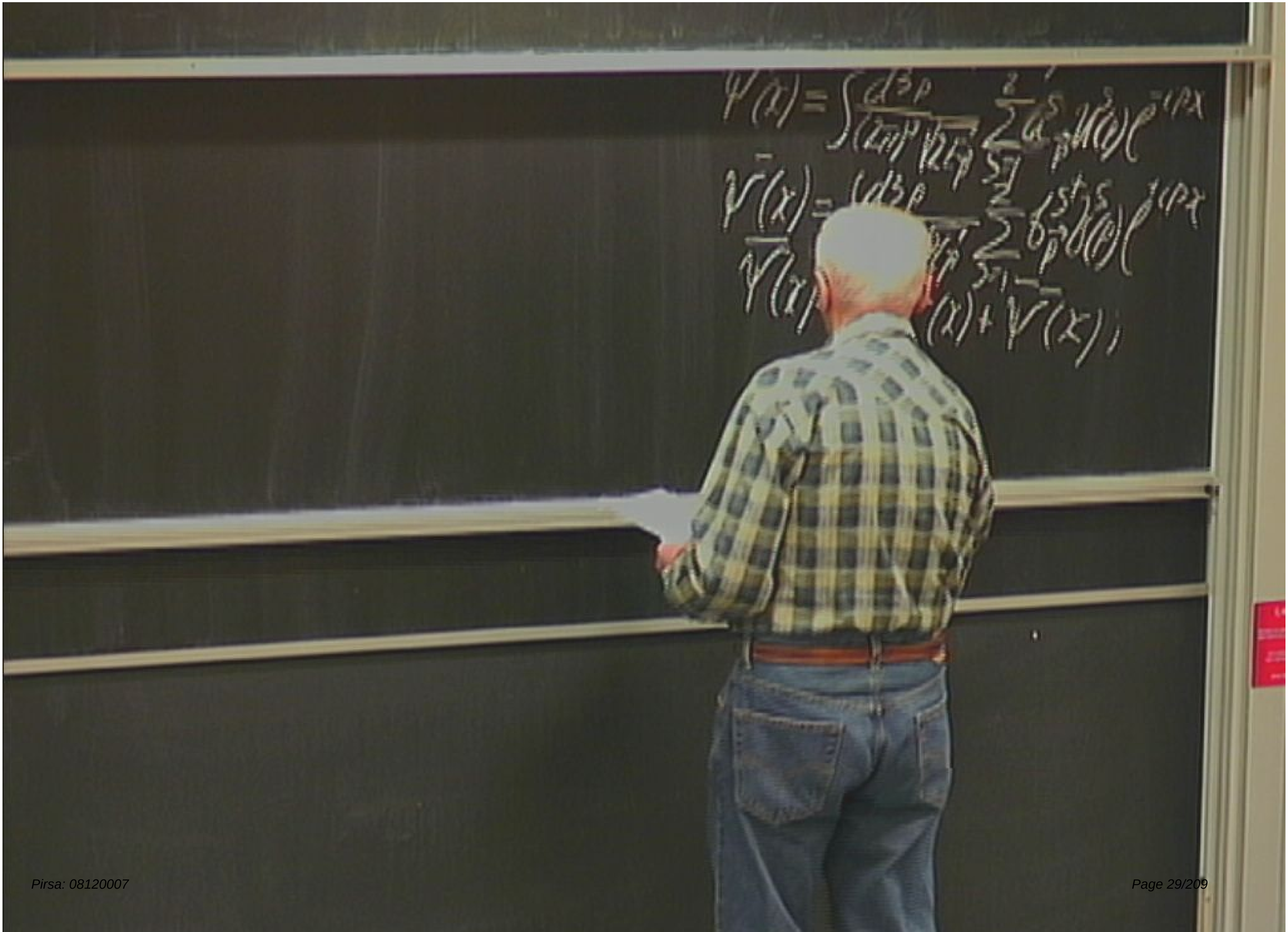
The definition of the normal-[illegible] product of fermions: put an extra minus sign for each fermion interchange:

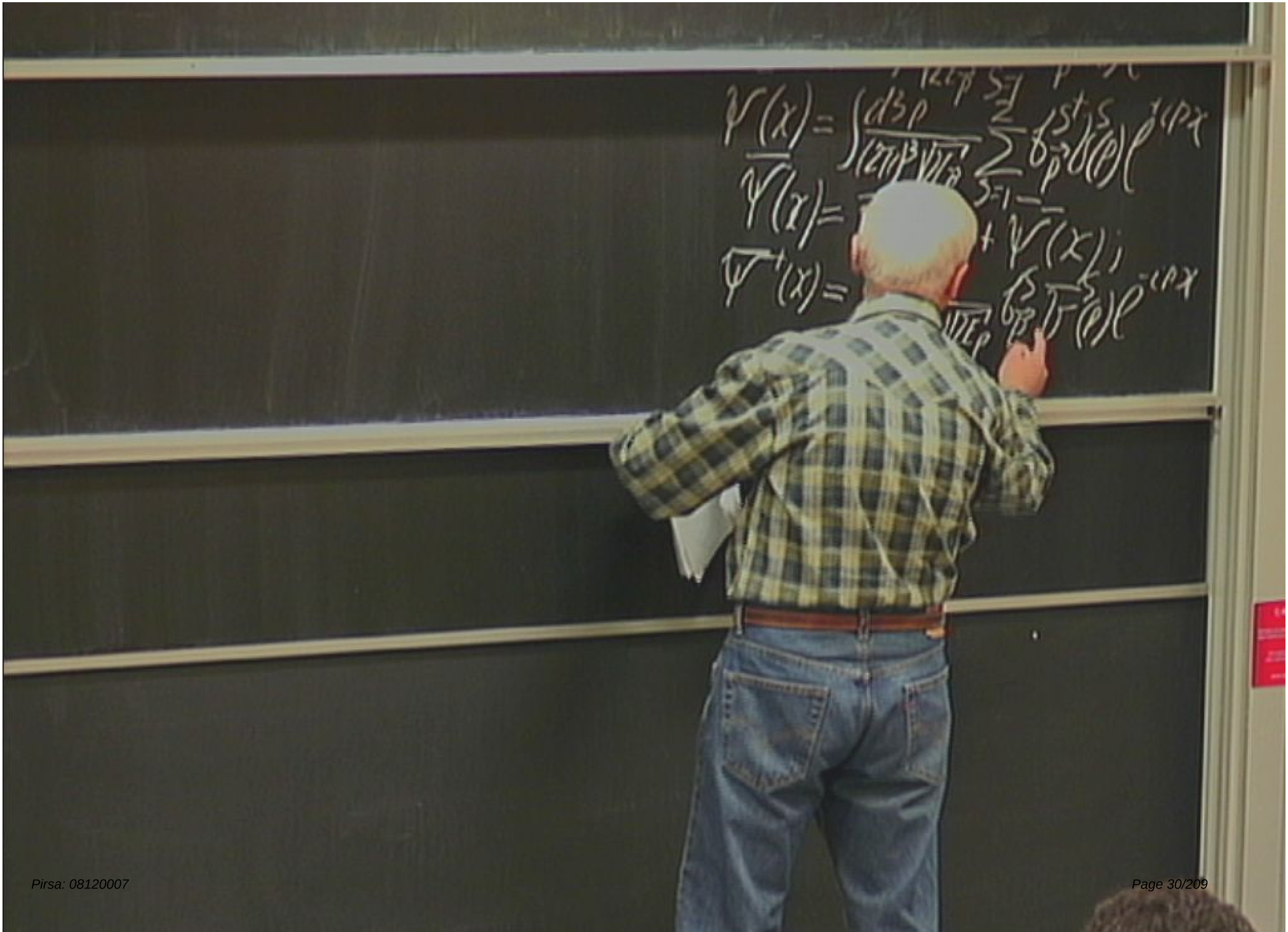
$$N(a_{\vec{p}}\, a_{\vec{q}}\, a^+_{\vec{k}}) = (-1)^2 a^+_{\vec{k}}\, a_{\vec{p}}\, a_{\vec{q}} = (-1)^3 \ldots$$

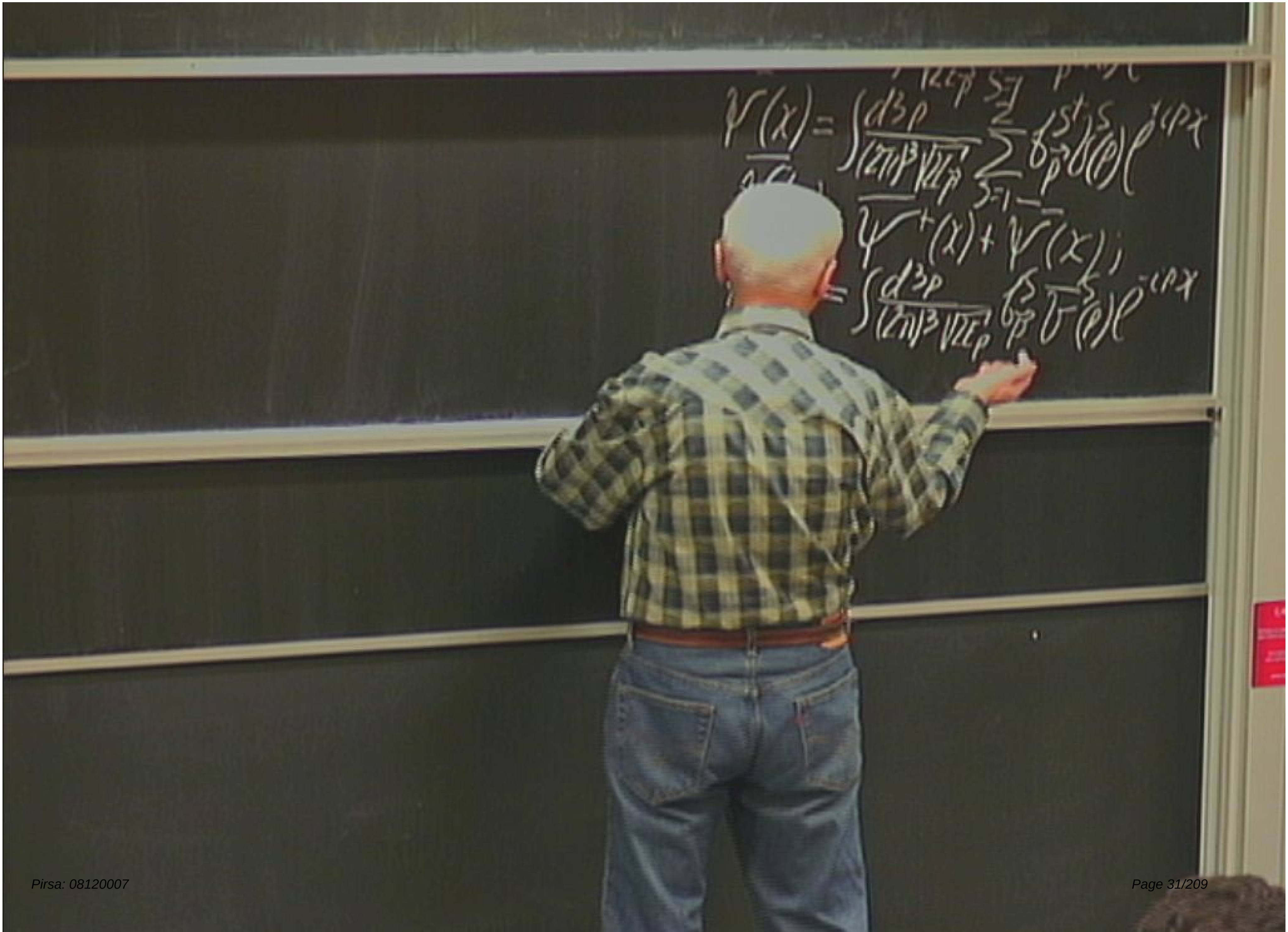
Generalization of Wick's Th[illegible]

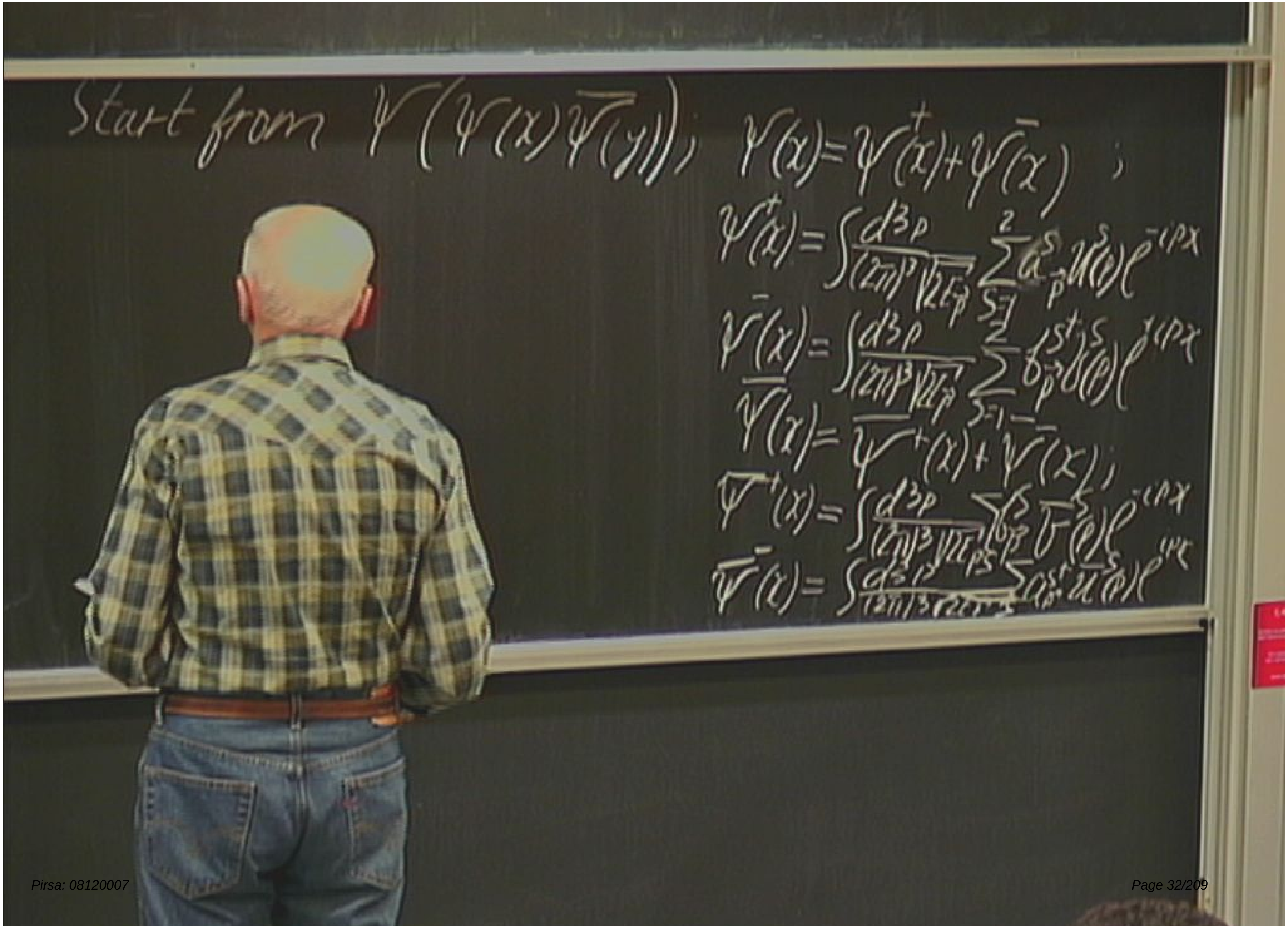

Start from

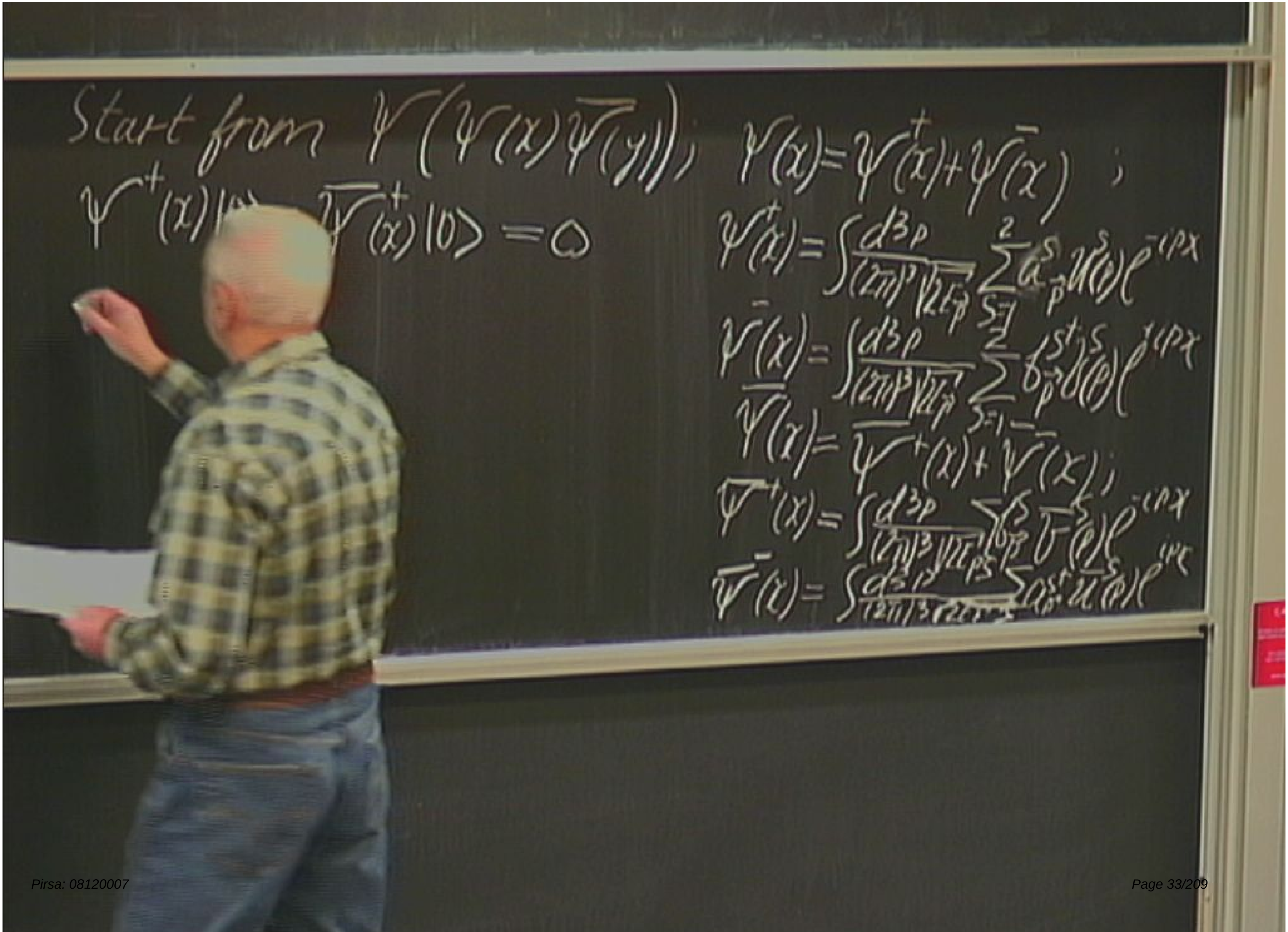

Start from $T(\psi(x)\bar{\psi}(y))$; $\psi(x)=\psi^{+}(x)+\psi^{-}(x)$;



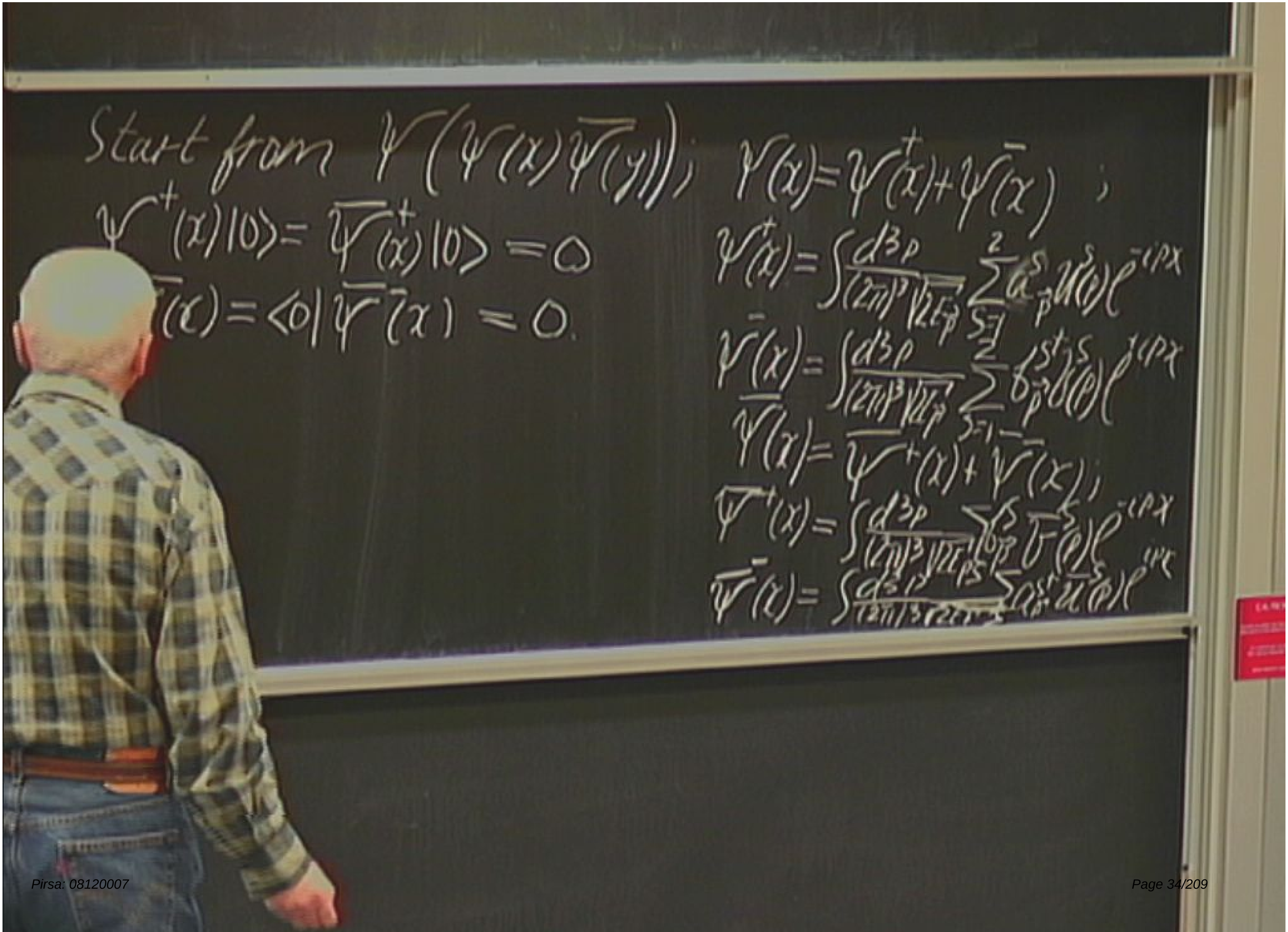


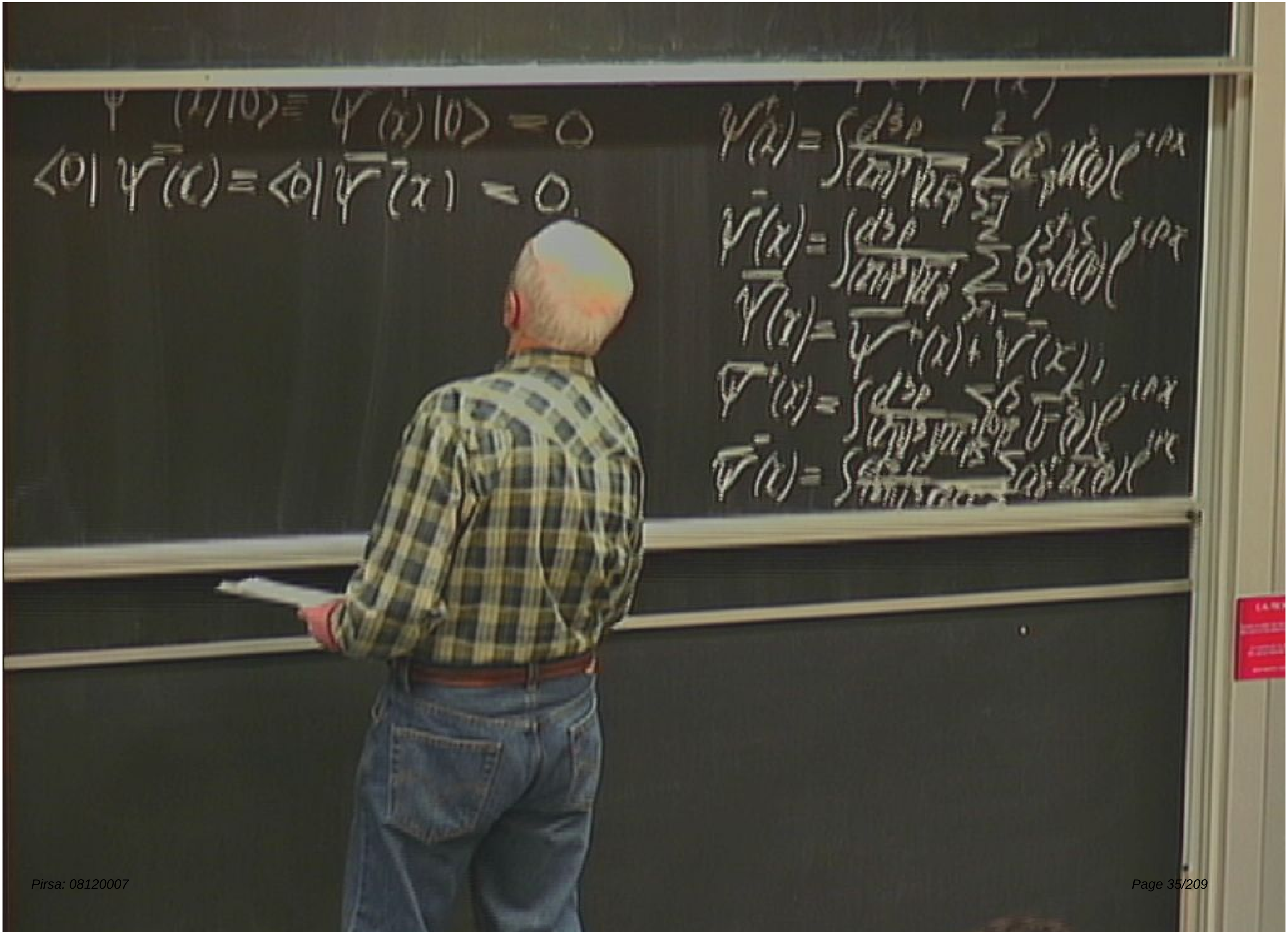


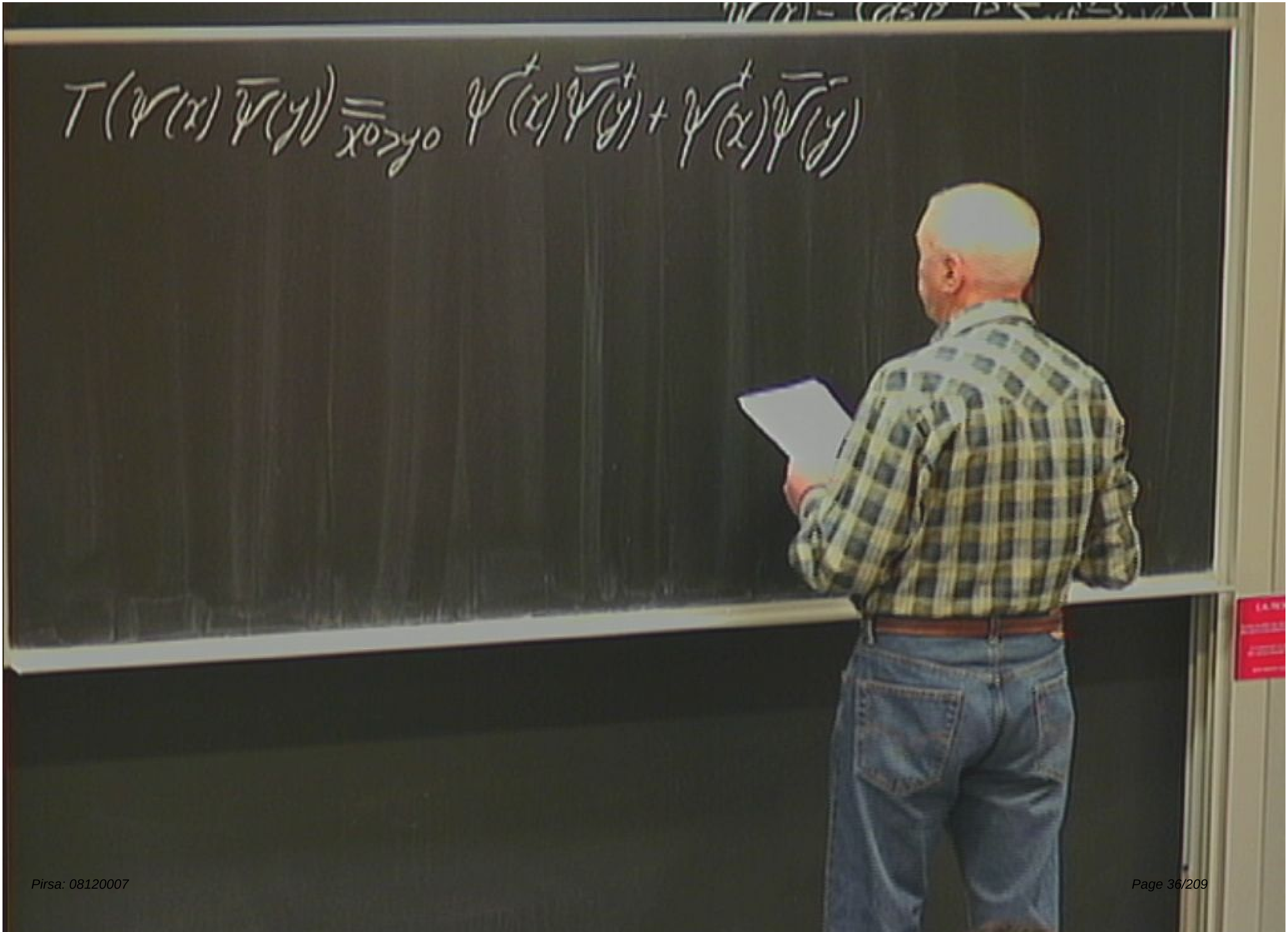

Start from $\Psi(\psi(x)\bar{\psi}(y))$; $\psi(x)=\psi^+(x)+\psi^-(x)$;
$\psi^+(x)=\int\frac{d^3p}{(2\pi)^3\sqrt{2E_p}}\sum_{s=1}^{2} a_p^s u^s(p) e^{-ipx}$
$\psi^-(x)=\int\frac{d^3p}{(2\pi)^3\sqrt{2E_p}}\sum_{s=1}^{2} b_p^{s\dagger} v^s(p) e^{ipx}$
$\bar{\psi}(x)=\bar{\psi}^+(x)+\bar{\psi}^-(x)$;
$\bar{\psi}^+(x)=\int\frac{d^3p}{(2\pi)^3\sqrt{2E_p}}\sum_s b_p^s \bar{v}^s(p) e^{-ipx}$
$\bar{\psi}^-(x)=\int\frac{d^3p}{(2\pi)^3\sqrt{2E_p}}\sum_s a_p^{s\dagger} \bar{u}^s(p) e^{ipx}$

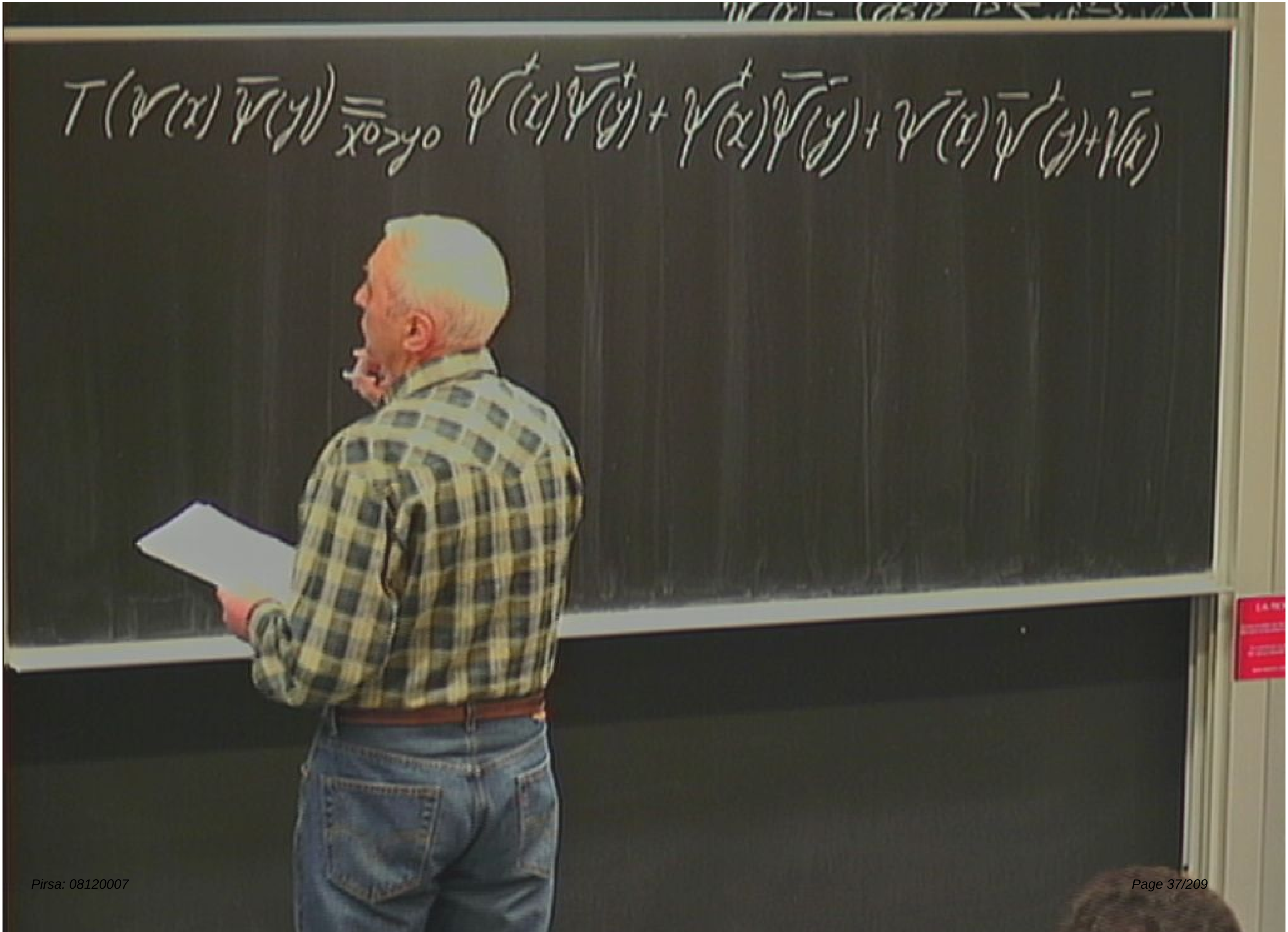


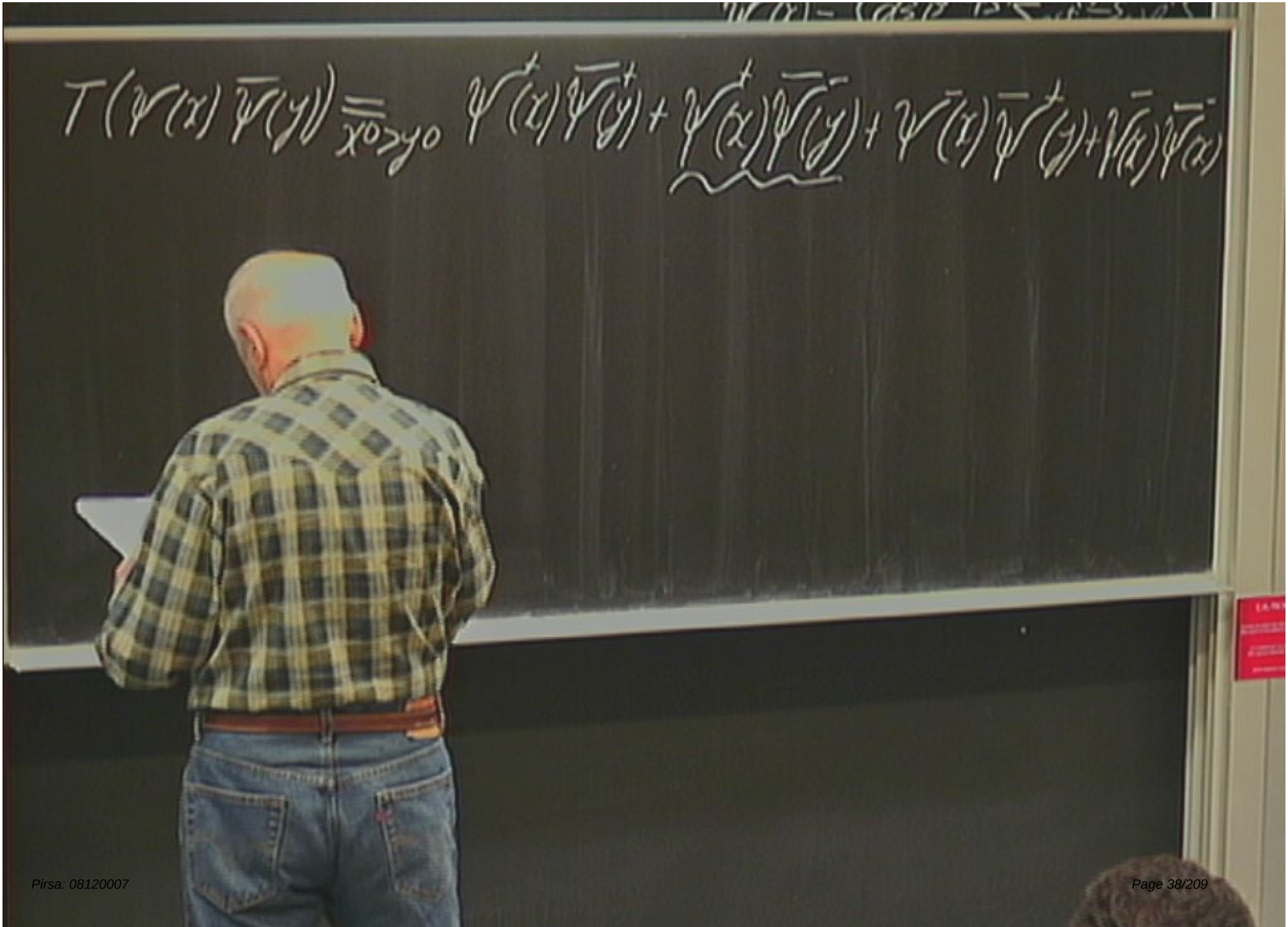
Start from $T(\psi(x)\bar{\psi}(y))$; $\psi(x)=\psi^{+}(x)+\psi^{-}(x)$;
$\psi^{+}(x)|0\rangle$ … $\bar{\psi}^{+}(x)|0\rangle = 0$
$\psi^{+}(x)=\int\frac{d^3p}{(2\pi)^3\sqrt{2E_p}}\sum_{s=1}^{2}a_p^s u^s(p)e^{-ipx}$
$\psi^{-}(x)=\int\frac{d^3p}{(2\pi)^3\sqrt{2E_p}}\sum_{s=1}^{2}b_p^{s\dagger}v^s(p)e^{ipx}$
$\bar{\psi}(x)=\bar{\psi}^{+}(x)+\bar{\psi}^{-}(x)$;
$\bar{\psi}^{+}(x)=\int\frac{d^3p}{(2\pi)^3\sqrt{2E_p}}\sum_s b_p^s\bar{v}^s(p)e^{-ipx}$
$\bar{\psi}^{-}(x)=\int\frac{d^3p}{(2\pi)^3\sqrt{2E_p}}\sum_s a_p^{s\dagger}\bar{u}^s(p)e^{ipx}$

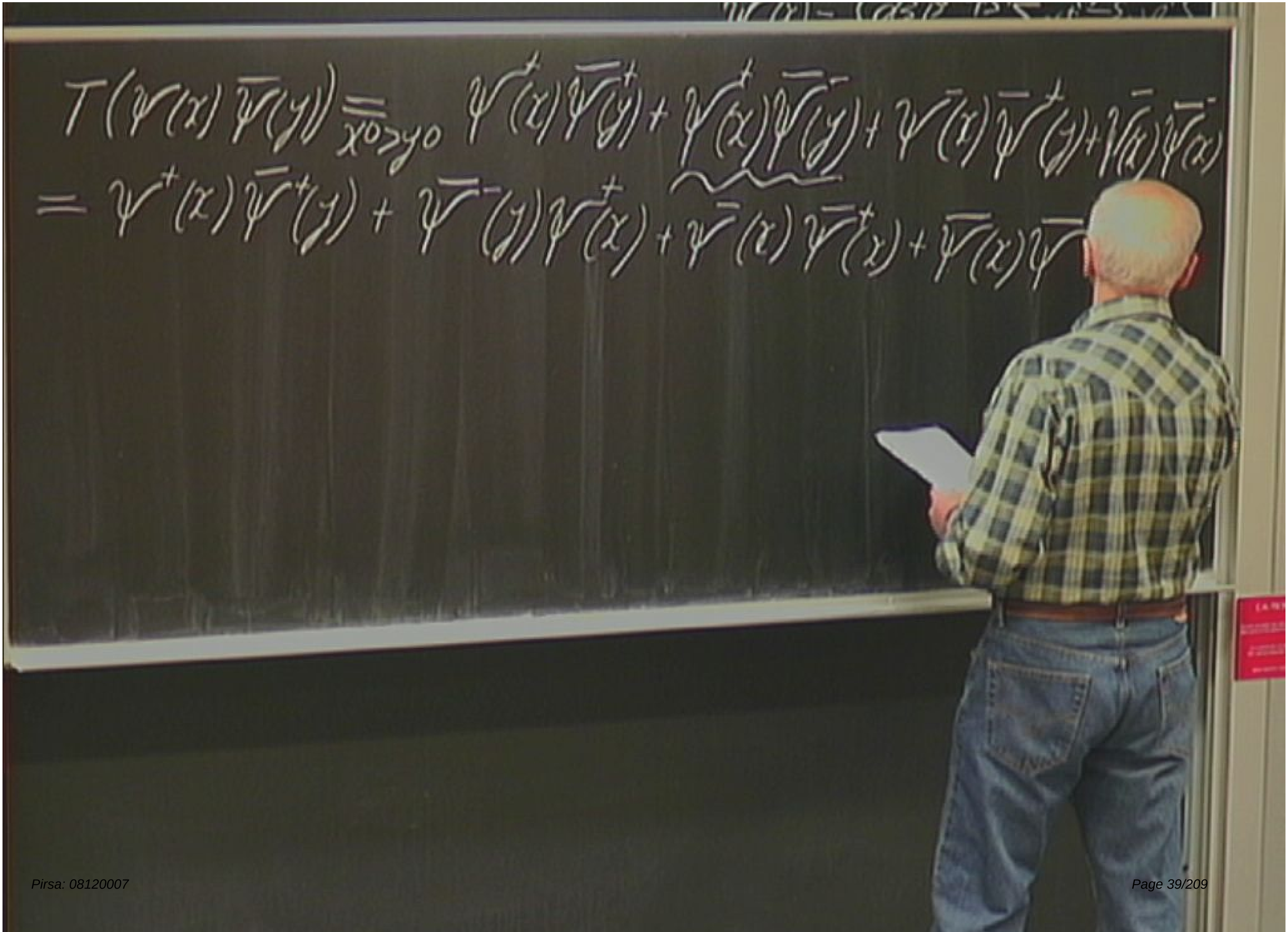

Start from $T(\psi(x)\bar{\psi}(y))$; $\psi(x)=\psi^+(x)+\psi^-(x)$;
$\psi^+(x)|0\rangle = \bar{\psi}^+(x)|0\rangle = 0$
$\langle 0|\psi^-(x) = \langle 0|\bar{\psi}^-(x) = 0$.
$\psi^+(x)=\int \frac{d^3p}{(2\pi)^3\sqrt{2E_p}}\sum_{s=1}^{2} a_p^s u^s(p) e^{-ipx}$
$\psi^-(x)=\int \frac{d^3p}{(2\pi)^3\sqrt{2E_p}}\sum_{s=1}^{2} b_p^{s\dagger} v^s(p) e^{ipx}$
$\bar{\psi}(x)=\bar{\psi}^+(x)+\bar{\psi}^-(x)$;
$\bar{\psi}^+(x)=\int \frac{d^3p}{(2\pi)^3\sqrt{2E_p}}\sum_s b_p^s \bar{v}^s(p) e^{-ipx}$
$\bar{\psi}^-(x)=\int \frac{d^3p}{(2\pi)^3\sqrt{2E_p}}\sum_s a_p^{s\dagger} \bar{u}^s(p) e^{ipx}$

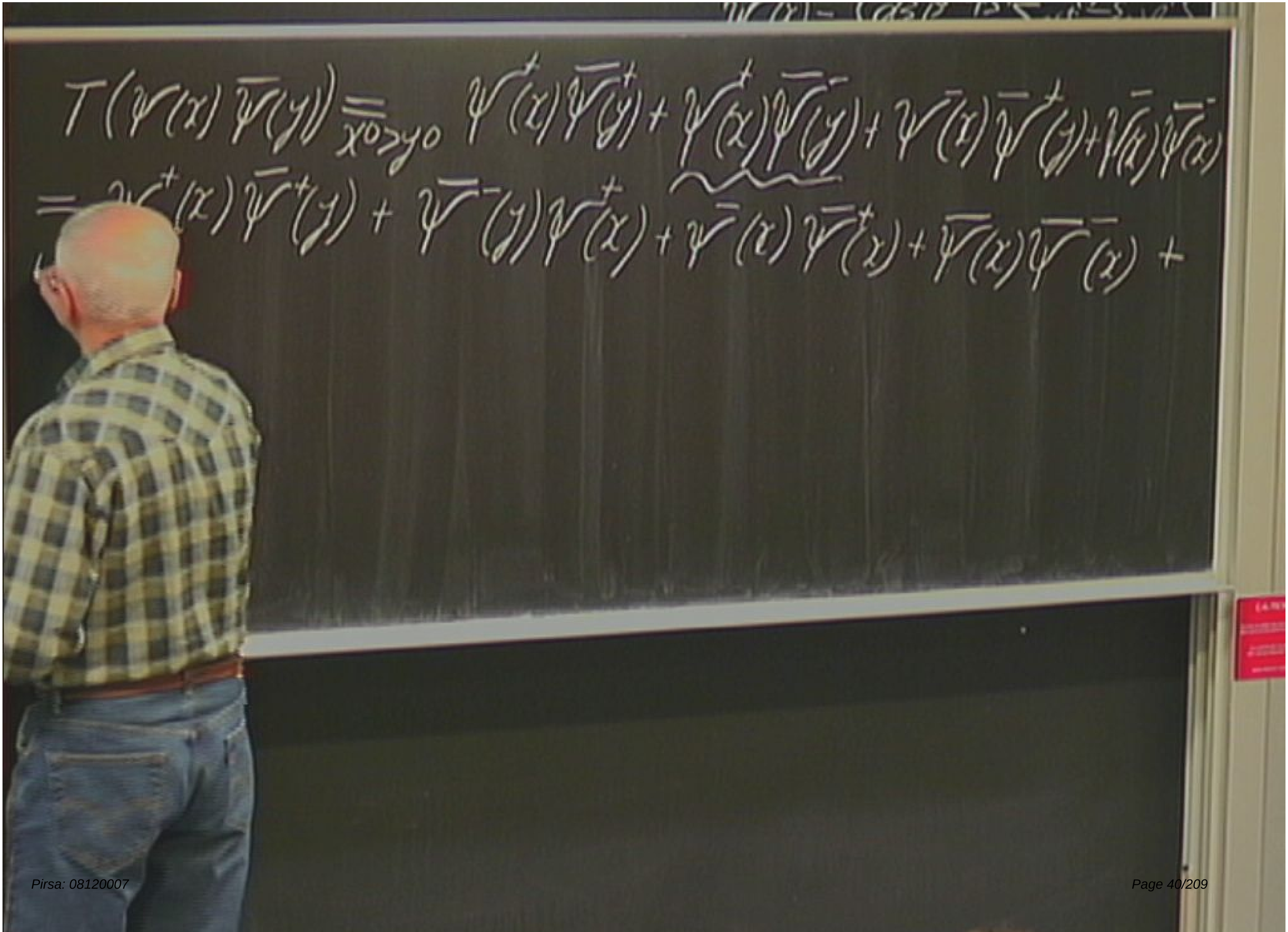



$T(\psi(x)\bar{\psi}(y)) \underset{x^0>y^0}{=} \psi^+(x)\bar{\psi}^+(y) + \psi^+(x)\bar{\psi}^-(y)$

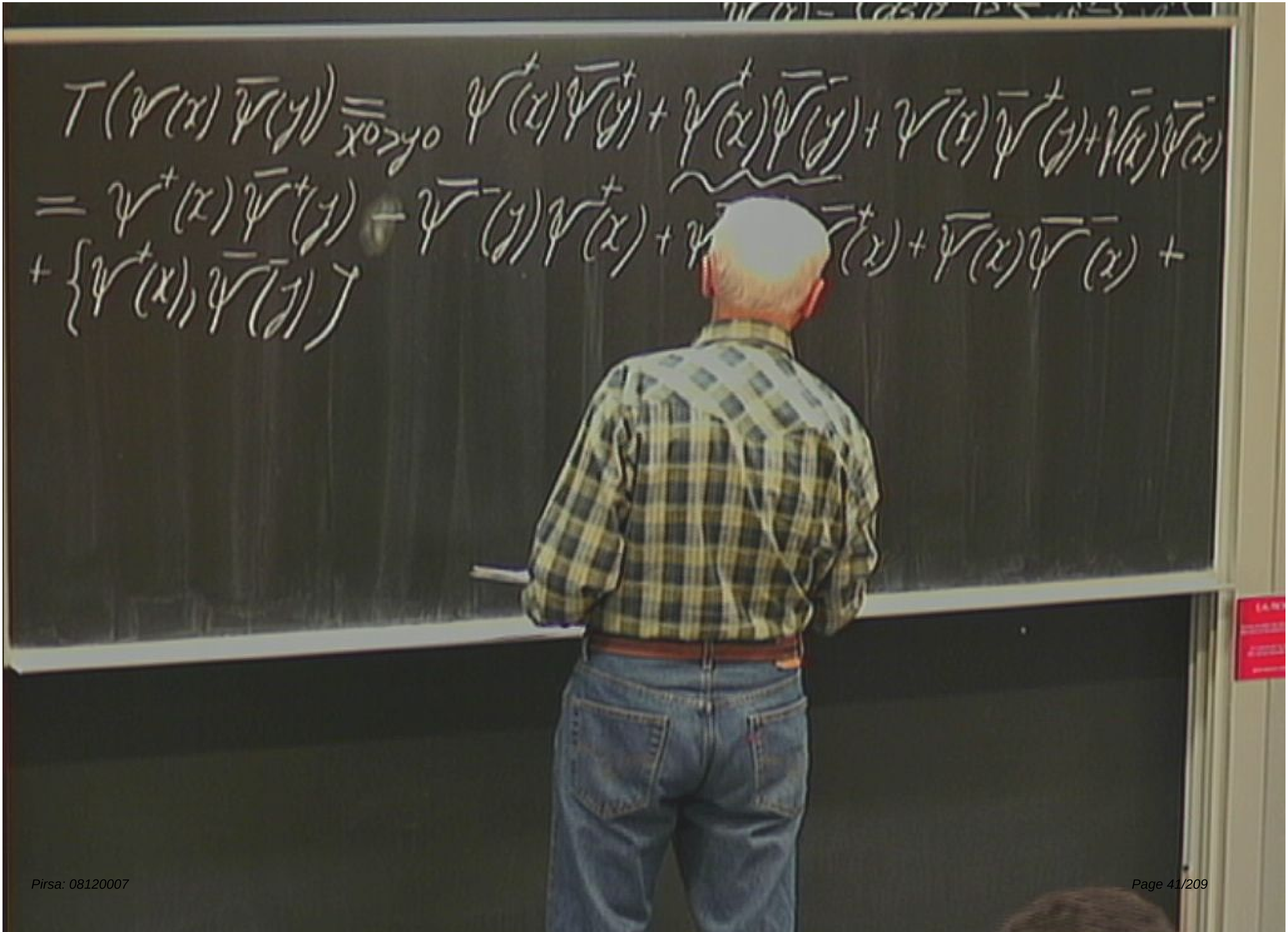

$T(\psi(x)\bar{\psi}(y)) \underset{x^0>y^0}{=} \psi^+(x)\bar{\psi}^+(y) + \psi^+(x)\bar{\psi}^-(y) + \psi^-(x)\bar{\psi}^+(y) + \ldots$

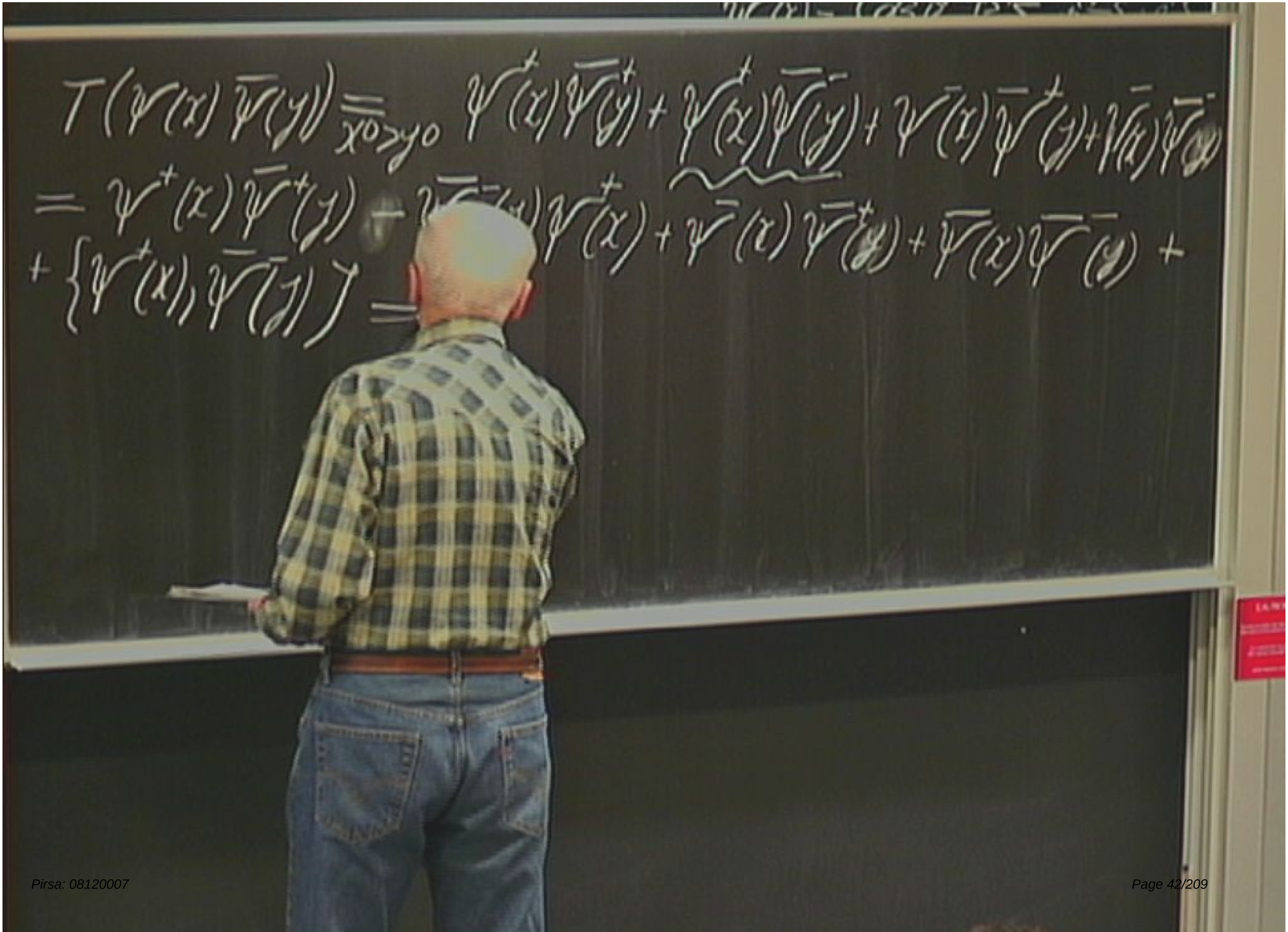

$T(\psi(x)\bar\psi(y)) \underset{x^0>y^0}{=} \psi^+(x)\bar\psi^+(y) + \psi^+(x)\bar\psi^-(y) + \psi^-(x)\bar\psi^+(y) + \psi^-(x)\bar\psi^-(y)$

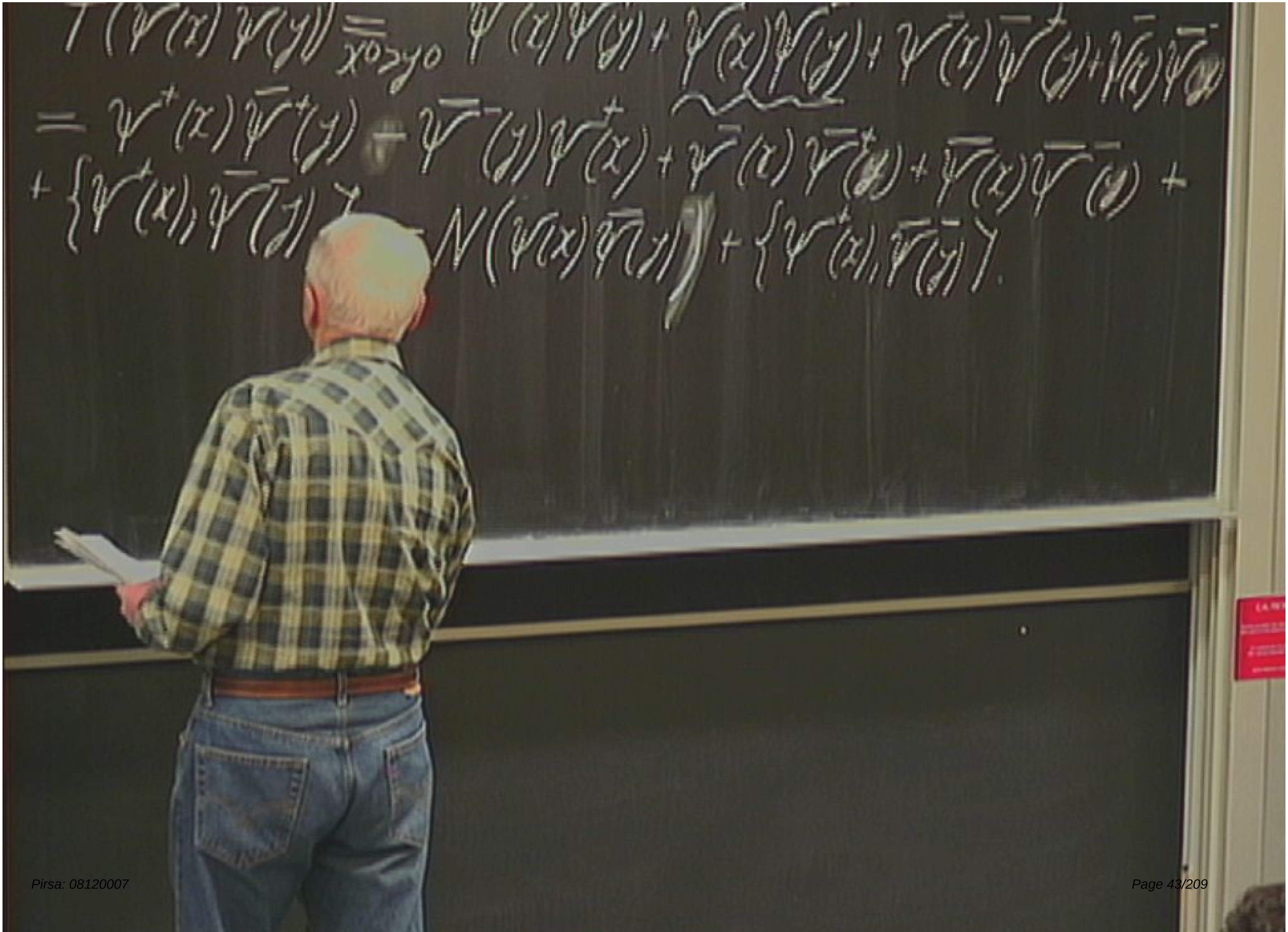

$T(\psi(x)\bar{\psi}(y)) \underset{x^0>y^0}{=} \psi^+(x)\bar{\psi}^+(y) + \psi^+(x)\bar{\psi}^-(y) + \psi^-(x)\bar{\psi}^+(y) + \psi^-(x)\bar{\psi}^-(x)$
$= \psi^+(x)\bar{\psi}^+(y) + \bar{\psi}^-(y)\psi^+(x) + \psi^-(x)\bar{\psi}^+(y) + \bar{\psi}^-(x)\psi^-$

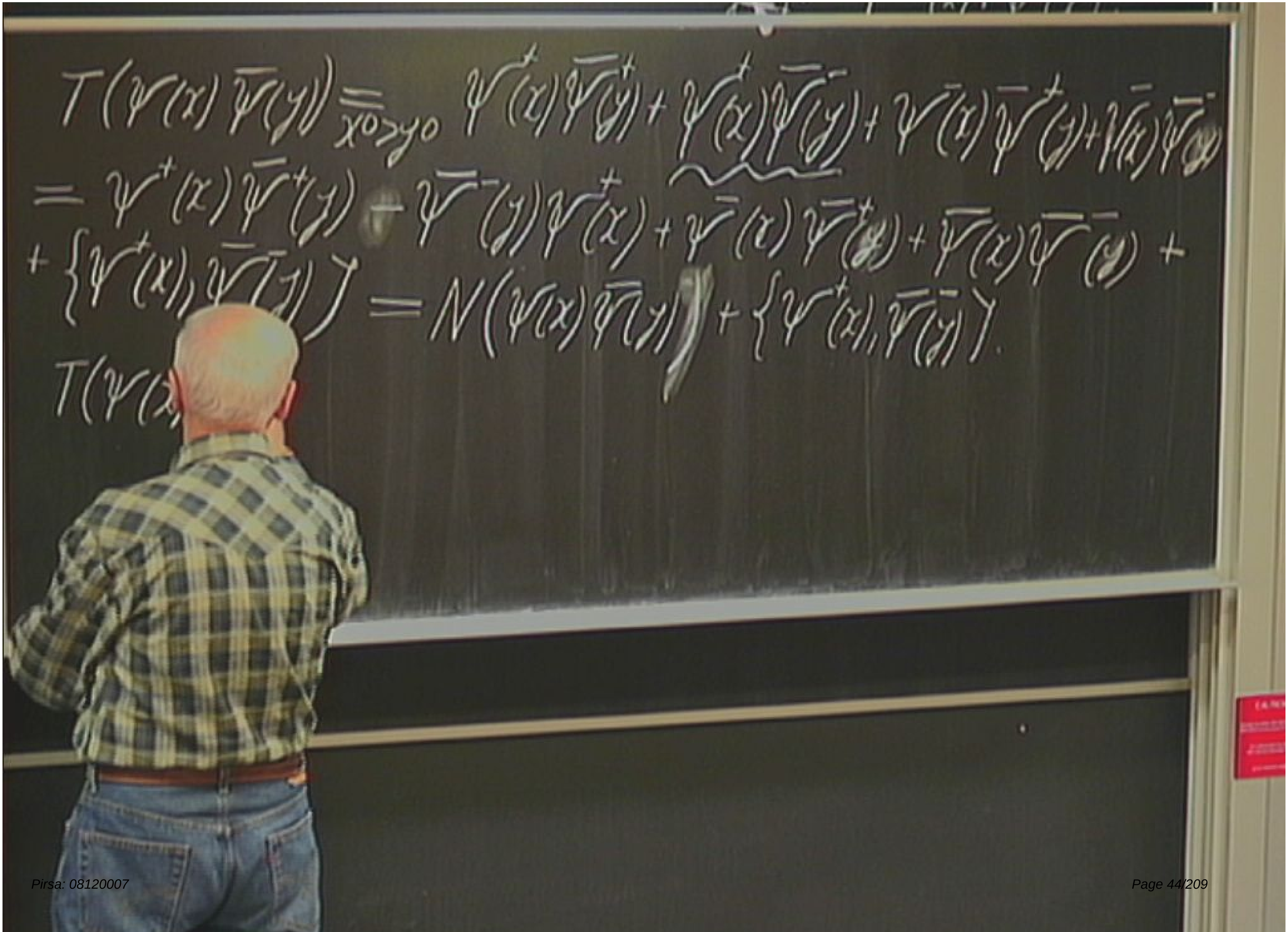


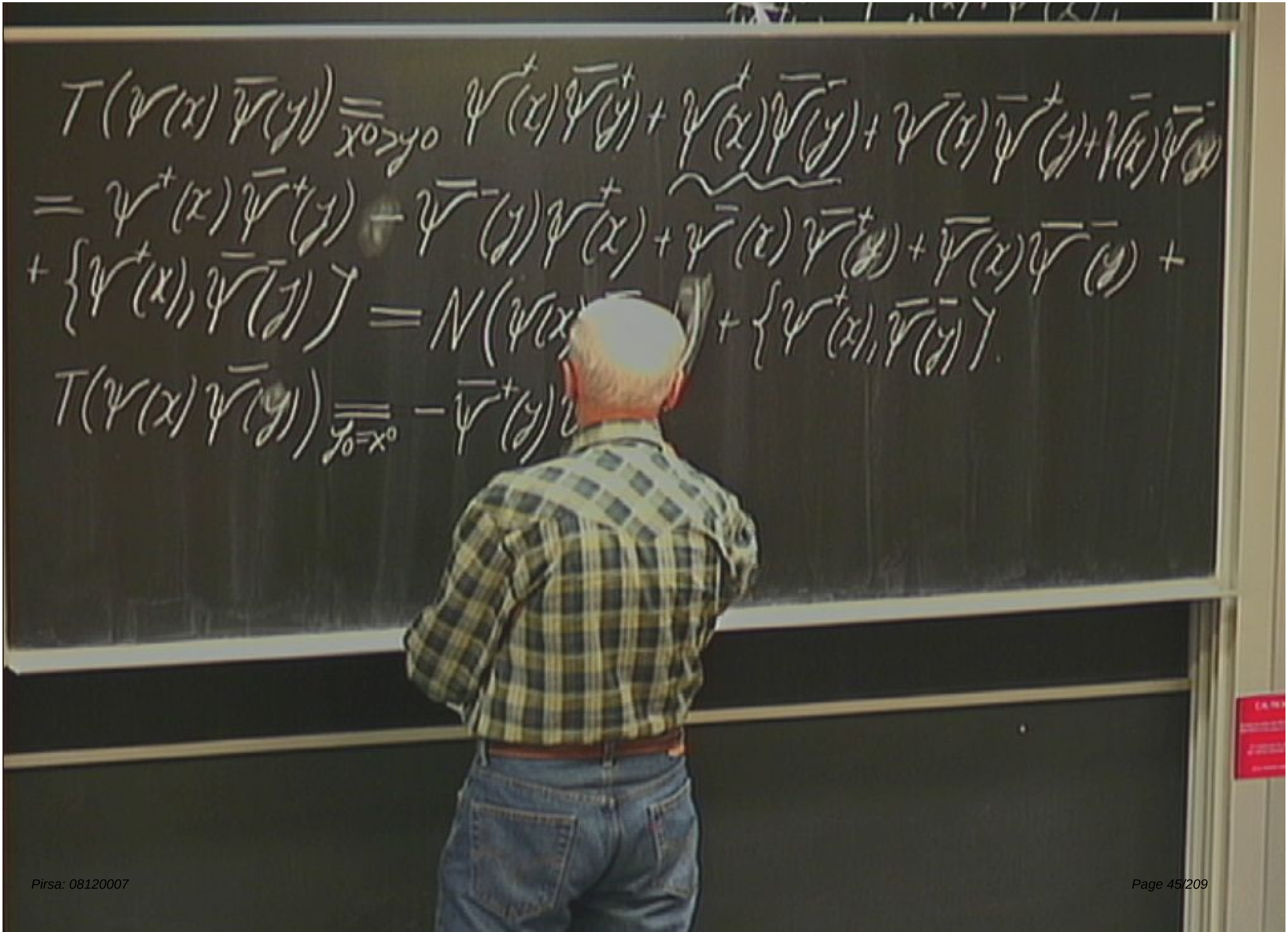
$T(\psi(x)\bar{\psi}(y)) \underset{x^0>y^0}{=} \psi^+(x)\bar{\psi}^+(y) + \psi^+(x)\bar{\psi}^-(y) + \psi^-(x)\bar{\psi}^+(y) + \psi^-(x)\bar{\psi}^-(x)$
$= \psi^+(x)\bar{\psi}^+(y) + \bar{\psi}^-(y)\psi^+(x) + \psi^-(x)\bar{\psi}^+(y) + \bar{\psi}^-(x)\psi^-(x) +$

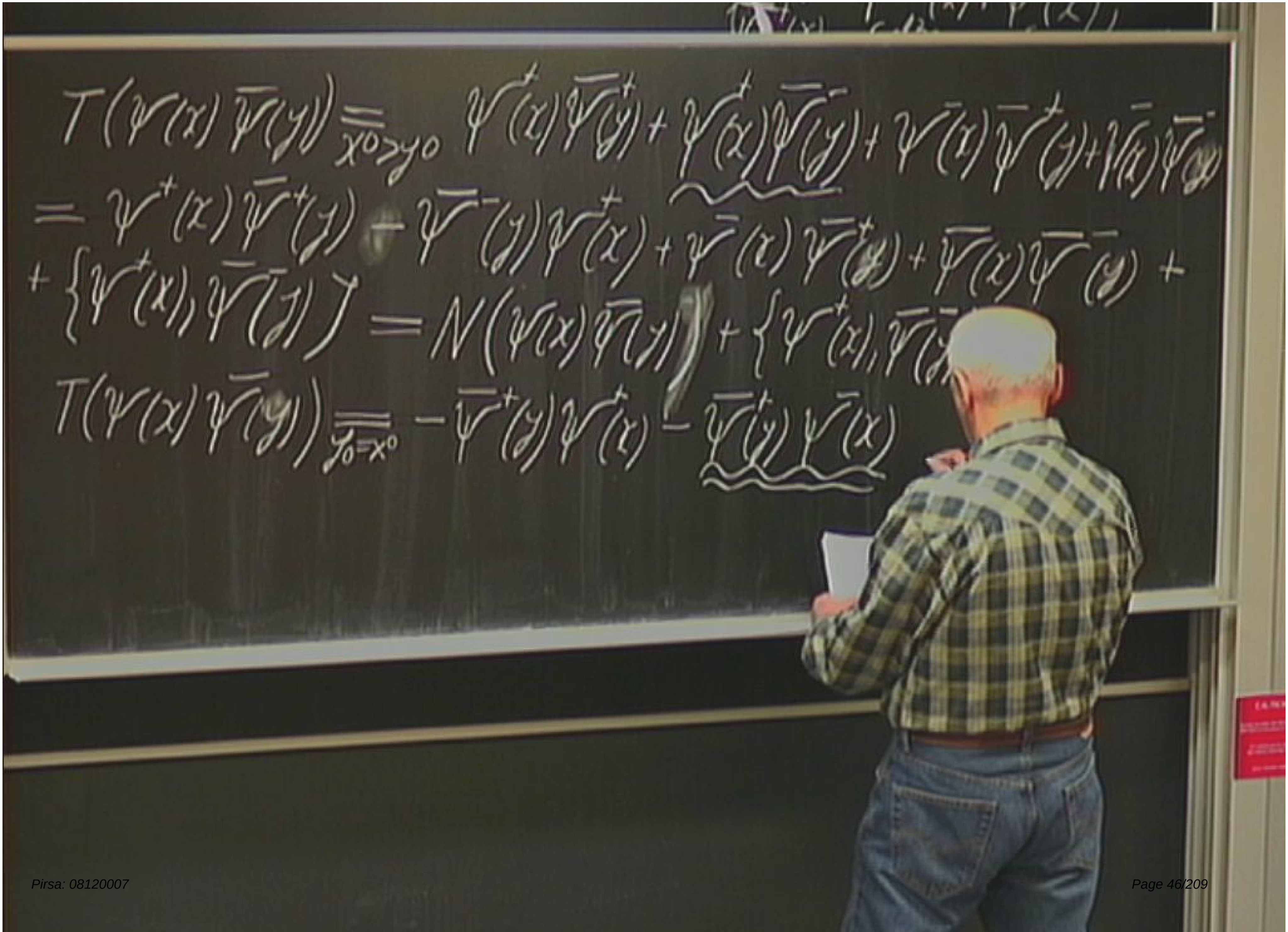

$T(\psi(x)\bar\psi(y)) \underset{x^0>y^0}{=} \psi^+(x)\bar\psi^+(y) + \psi^+(x)\bar\psi^-(y) + \psi^-(x)\bar\psi^+(y) + \psi^-(x)\bar\psi^-(y)$
$= \psi^+(x)\bar\psi^+(y) - \bar\psi^-(y)\psi^+(x) + \ldots + \bar\psi(x)\psi^-(x) +$
$+ \{\psi^+(x), \bar\psi^-(y)\}$

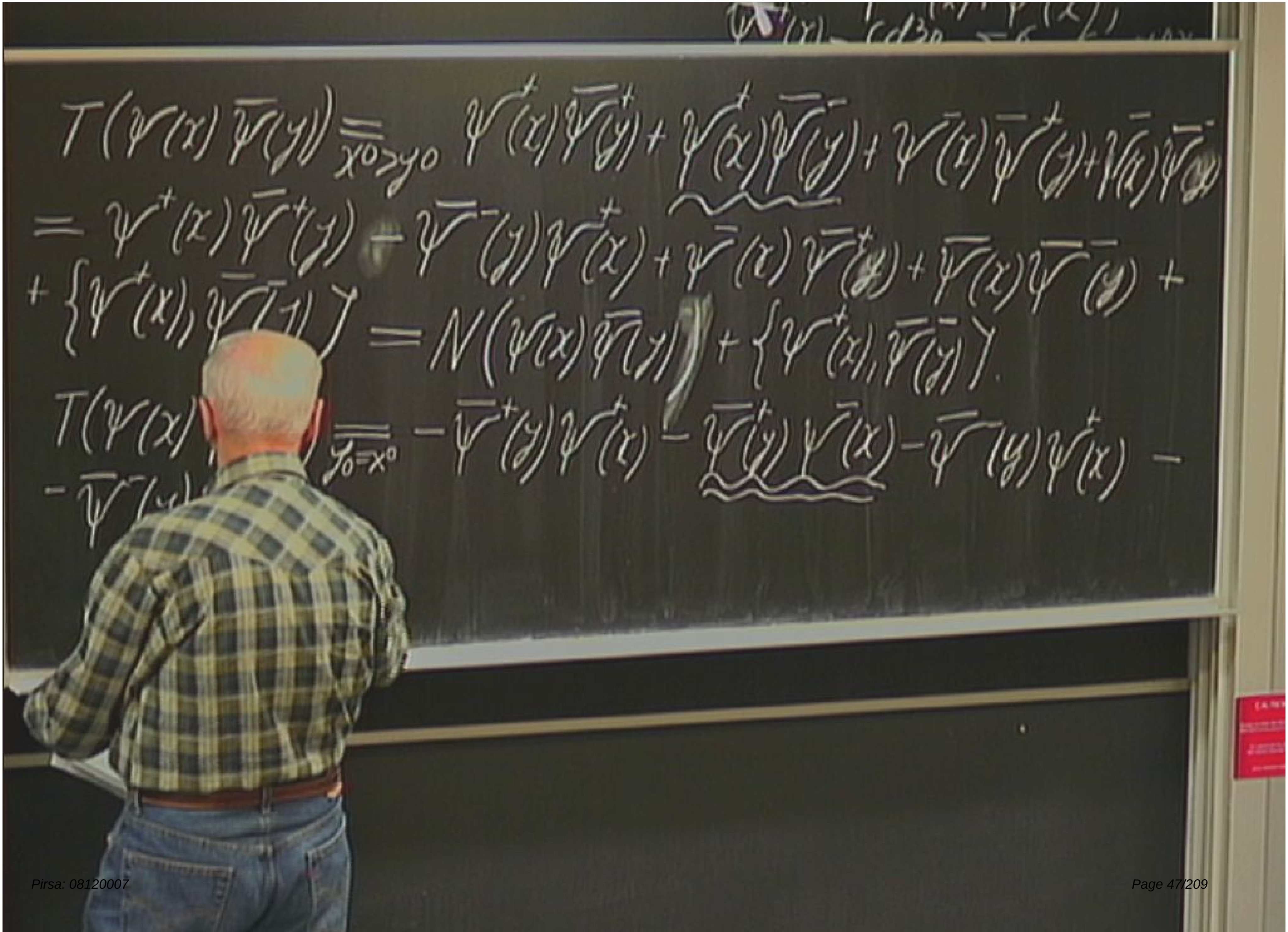

$T(\psi(x)\bar\psi(y)) \underset{x^0>y^0}{=} \psi^+(x)\bar\psi^+(y) + \psi^+(x)\bar\psi^-(y) + \psi^-(x)\bar\psi^+(y) + \psi^-(x)\bar\psi^-(y)$
$= \psi^+(x)\bar\psi^+(y) - \bar\psi^-(y)\psi^+(x) + \psi^-(x)\bar\psi^+(y) + \bar\psi(x)\psi^-(y) +$
$+ \{\psi^+(x), \bar\psi^-(y)\} =$

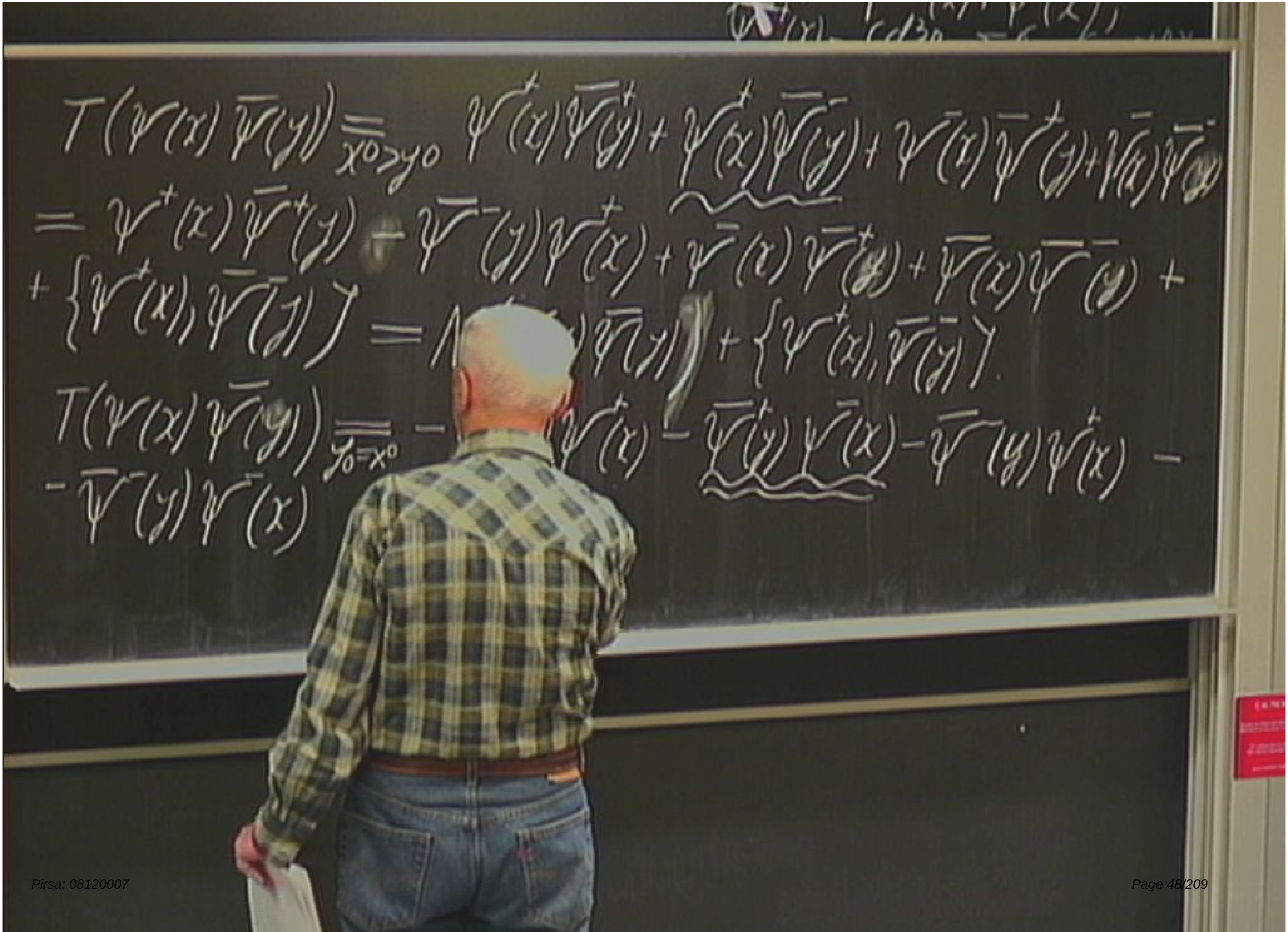



$T(\psi(x)\bar{\psi}(y)) \underset{x^0>y^0}{=} \psi^+(x)\bar{\psi}^+(y) + \psi^+(x)\bar{\psi}^-(y) + \psi^-(x)\bar{\psi}^+(y) + \psi^-(x)\bar{\psi}^-(y)$
$= \psi^+(x)\bar{\psi}^+(y) - \bar{\psi}^-(y)\psi^+(x) + \psi^-(x)\bar{\psi}^+(y) + \psi^-(x)\bar{\psi}^-(y) +$
$+ \{\psi^+(x), \bar{\psi}^-(y)\} = N(\psi(x)\bar{\psi}(y)) + \{\psi^+(x), \bar{\psi}^-(y)\}.$
$T(\psi(x$


$T(\psi(x)\bar\psi(y)) \underset{x^0>y^0}{=} \psi^+(x)\bar\psi^+(y) + \psi^+(x)\bar\psi^-(y) + \psi^-(x)\bar\psi^+(y) + \psi^-(x)\bar\psi^-(y)$
$= \psi^+(x)\bar\psi^+(y) - \bar\psi^-(y)\psi^+(x) + \psi^-(x)\bar\psi^+(y) + \psi^-(x)\bar\psi^-(y) +$
$+ \{\psi^+(x), \bar\psi^-(y)\} = N(\psi(x) \ldots) + \{\psi^+(x), \bar\psi^-(y)\}$
$T(\psi(x)\bar\psi(y)) \underset{y_0>x^0}{=} -\bar\psi^+(y) \ldots$


$T(\psi(x)\bar\psi(y)) \underset{x^0>y^0}{=} \psi^+(x)\bar\psi^+(y) + \psi^+(x)\bar\psi^-(y) + \psi^-(x)\bar\psi^+(y) + \psi^-(x)\bar\psi^-(y)$
$= \psi^+(x)\bar\psi^+(y) - \bar\psi^-(y)\psi^+(x) + \psi^-(x)\bar\psi^+(y) + \bar\psi^-(x)\bar\psi^-(y) +$
$+ \{\psi^+(x), \bar\psi^-(y)\} = N(\psi(x)\bar\psi(y)) + \{\psi^+(x), \bar\psi^-(y)\}$
$T(\psi(x)\bar\psi(y)) \underset{y_0>x^0}{=} -\bar\psi^+(y)\psi^+(x) - \bar\psi^+(y)\psi^-(x)$


$T(\psi(x)\bar\psi(y)) \underset{x^0>y^0}{=} \psi^+(x)\bar\psi^+(y) + \psi^+(x)\bar\psi^-(y) + \psi^-(x)\bar\psi^+(y) + \psi^-(x)\bar\psi^-(y)$
$= \psi^+(x)\bar\psi^+(y) - \bar\psi^-(y)\psi^+(x) + \psi^-(x)\bar\psi^+(y) + \psi^-(x)\bar\psi^-(y) +$
$+ \{\psi^+(x), \bar\psi^-(y)\} = N(\psi(x)\bar\psi(y)) + \{\psi^+(x), \bar\psi^-(y)\}$
$T(\psi(x)$
$\underset{y^0=x^0}{=} -\bar\psi^+(y)\psi^+(x) - \bar\psi^+(y)\psi^-(x) - \bar\psi^-(y)\psi^+(x) -$
$-\bar\psi^-(y)$



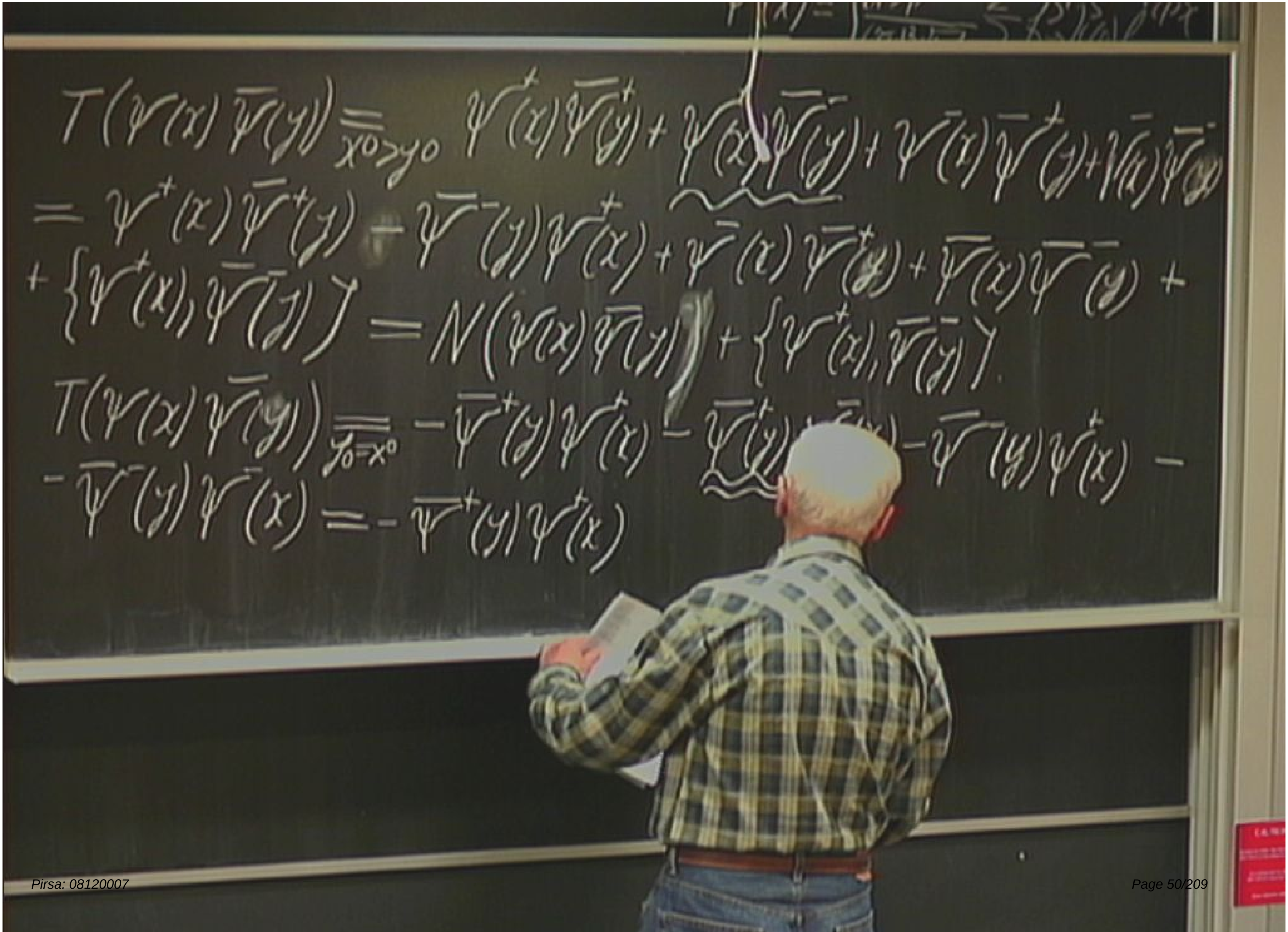
$$T(\psi(x)\bar\psi(y)) \underset{x^0>y^0}{=} \psi^+(x)\bar\psi^+(y) + \psi^+(x)\bar\psi^-(y) + \psi^-(x)\bar\psi^+(y) + \psi^-(x)\bar\psi^-(y)$$

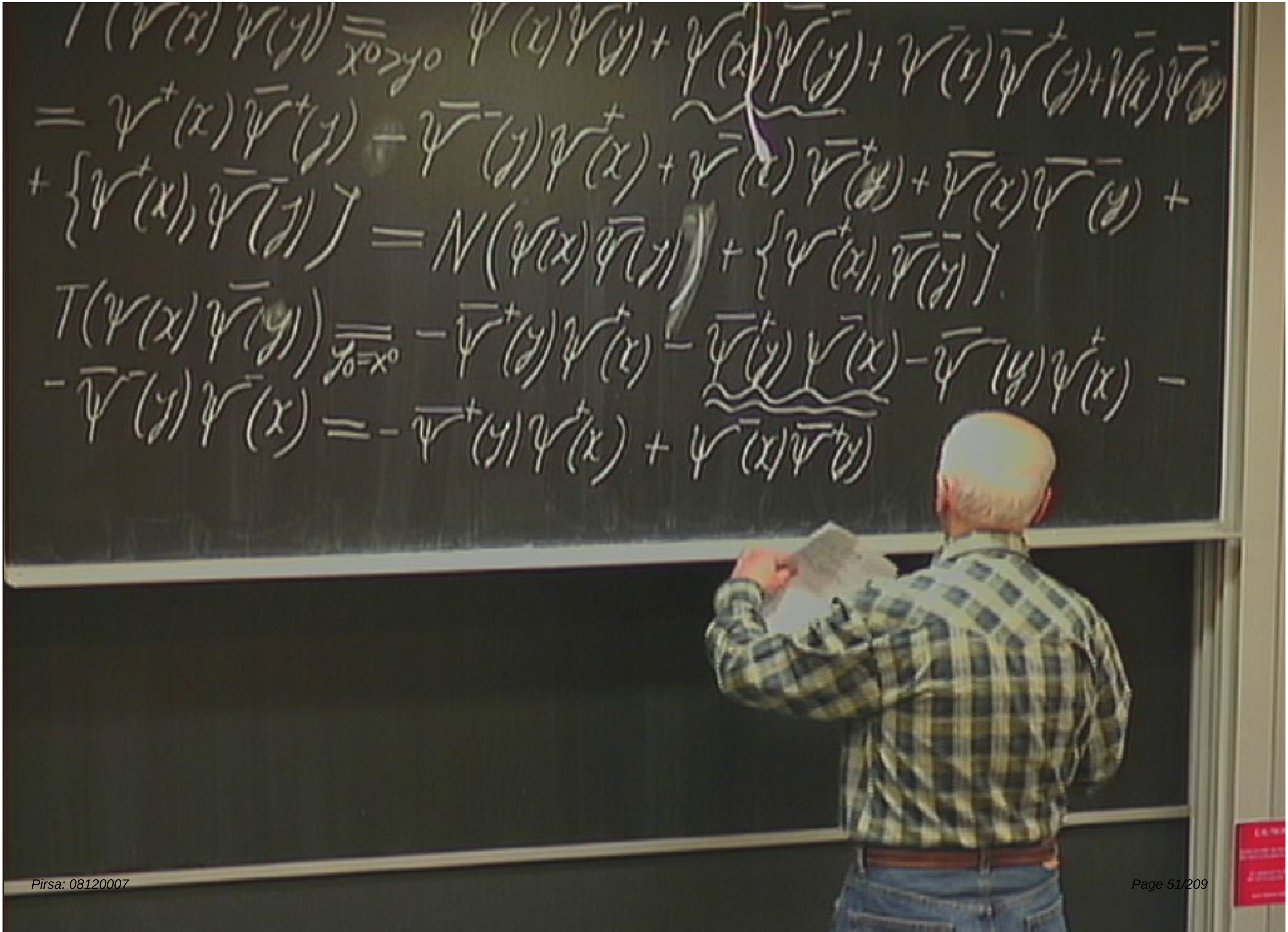
$$= \psi^+(x)\bar\psi^+(y) - \bar\psi^-(y)\psi^+(x) + \psi^-(x)\bar\psi^+(y) + \psi^-(x)\bar\psi^-(y) +$$

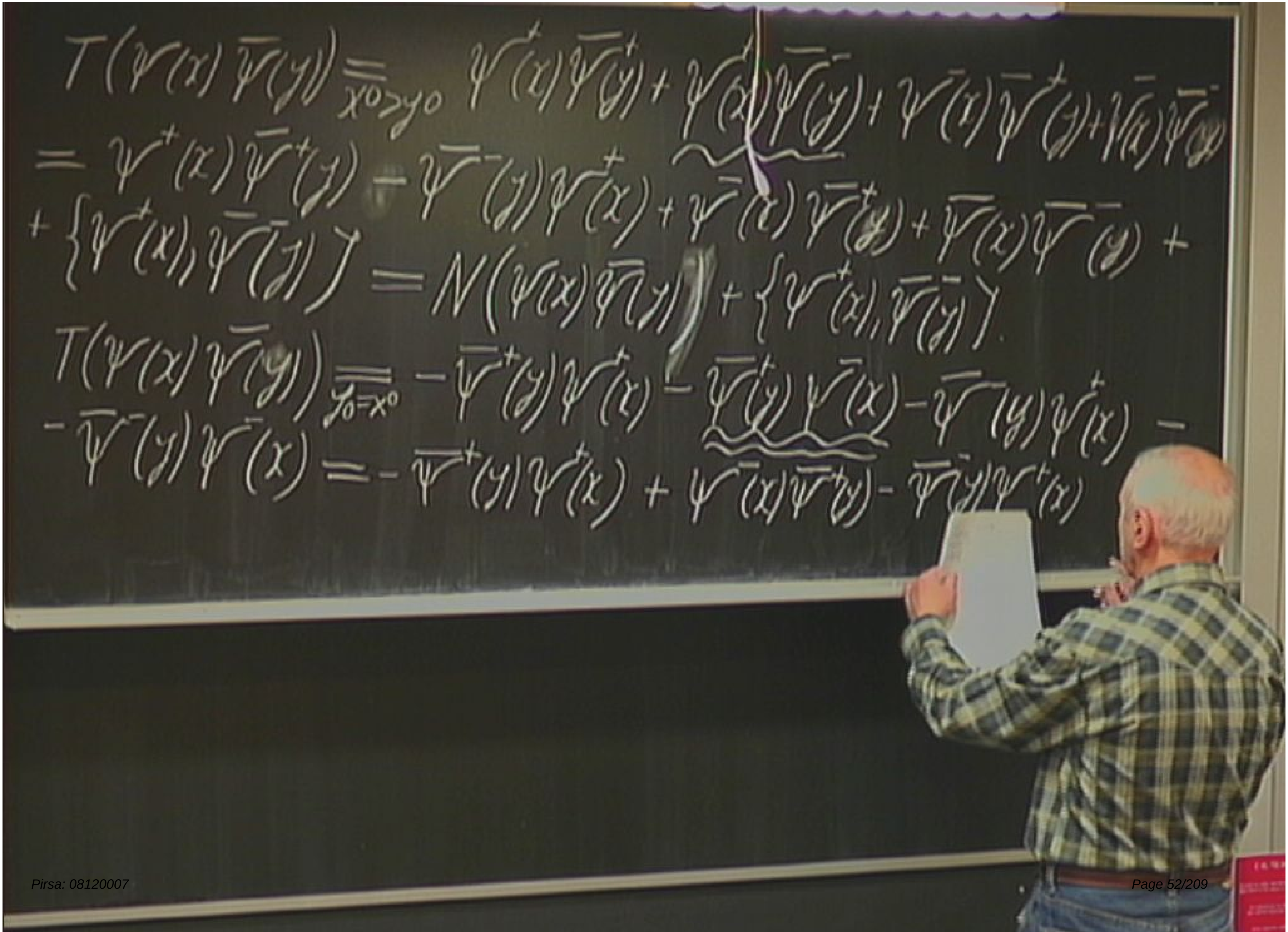
$$+ \{\psi^+(x), \bar\psi^-(y)\} = N(\psi(x)\bar\psi(y)) + \{\psi^+(x), \bar\psi^-(y)\}.$$

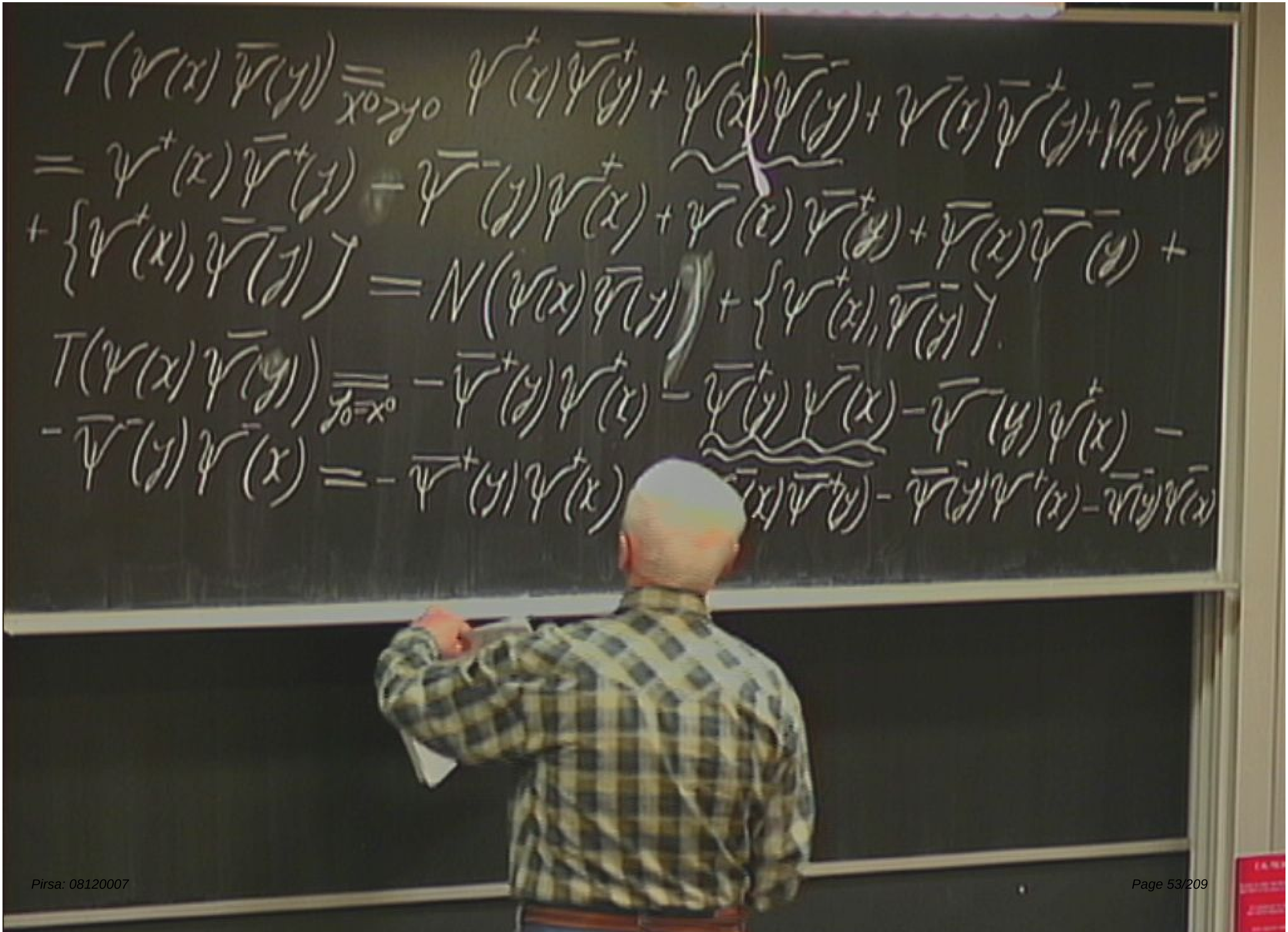
$$T(\psi(x)\bar\psi(y)) \underset{y^0>x^0}{=} -\bar\psi^+(y)\psi^+(x) - \bar\psi^+(y)\psi^-(x) - \bar\psi^- \text{[illegible]} -$$

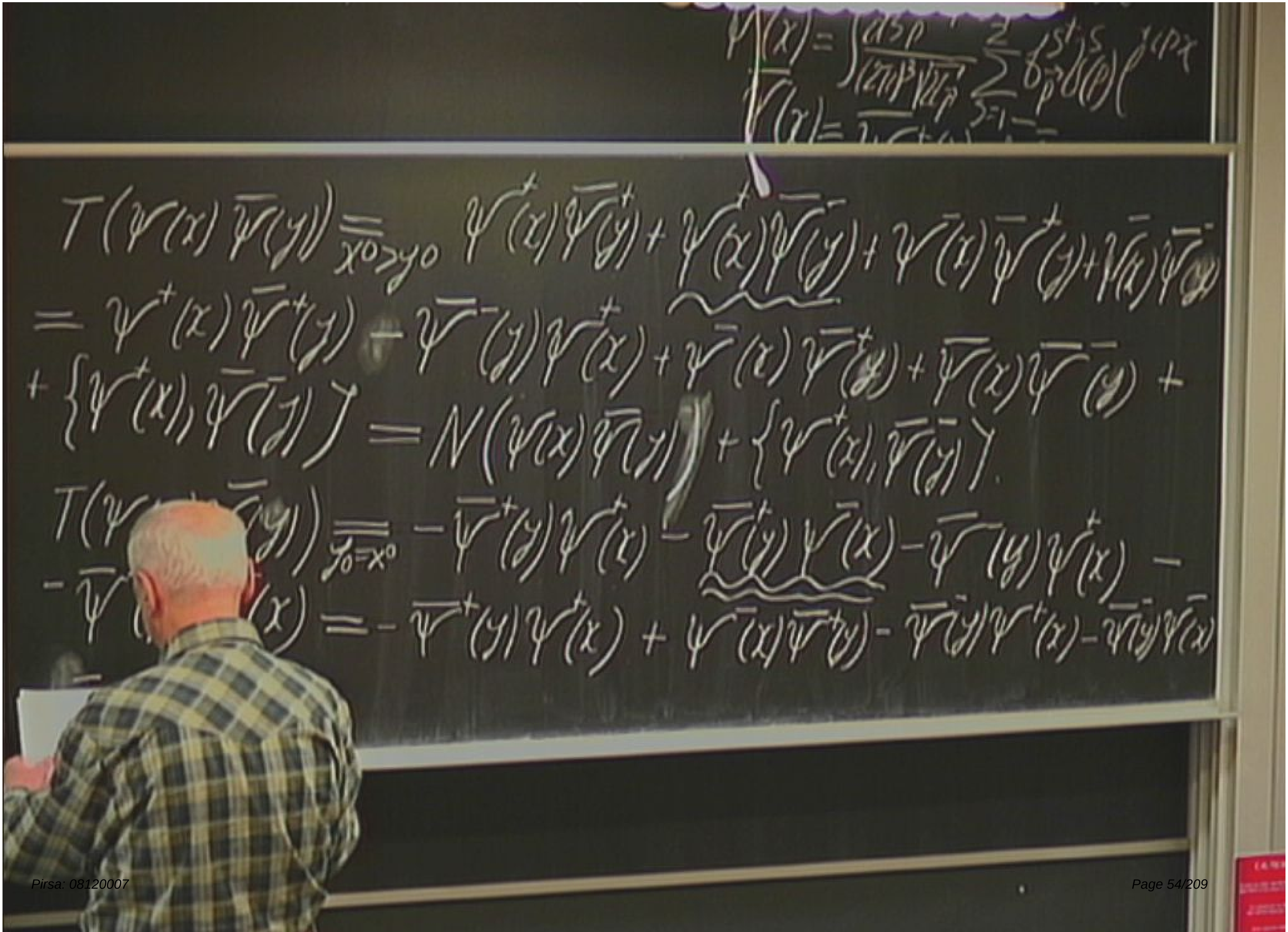
$$-\bar\psi^-(y)\psi^-(x)$$

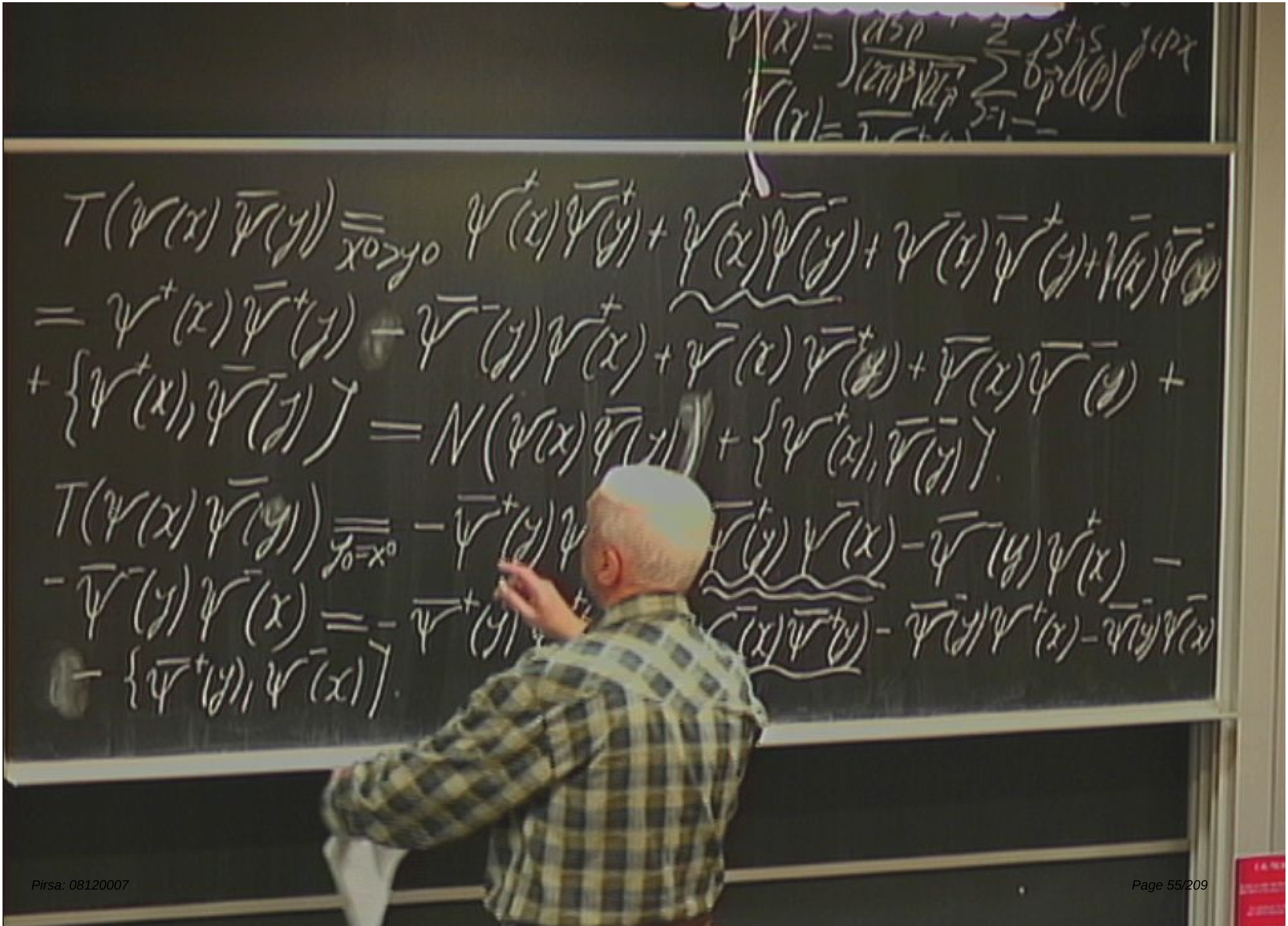

$T(\psi(x)\bar{\psi}(y)) \underset{x^0>y^0}{=} \psi^+(x)\bar{\psi}^+(y) + \psi^+(x)\bar{\psi}^-(y) + \psi^-(x)\bar{\psi}^+(y) + \psi^-(x)\bar{\psi}^-(y)$
$= \psi^+(x)\bar{\psi}^+(y) - \bar{\psi}^-(y)\psi^+(x) + \psi^-(x)\bar{\psi}^+(y) + \bar{\psi}^-(x)\psi^-(y) +$
$+ \{\psi^+(x), \bar{\psi}^-(y)\} = N(\psi(x)\bar{\psi}(y)) + \{\psi^+(x), \bar{\psi}^-(y)\}$
$T(\psi(x)\bar{\psi}(y)) \underset{y^0>x^0}{=} -\bar{\psi}^+(y)\psi^+(x) - \bar{\psi}^+(y)\psi^-(x) - \bar{\psi}^-(y)\psi^+(x) -$
$-\bar{\psi}^-(y)\psi^-(x) = -\bar{\psi}^+(y)\psi^+(x)$

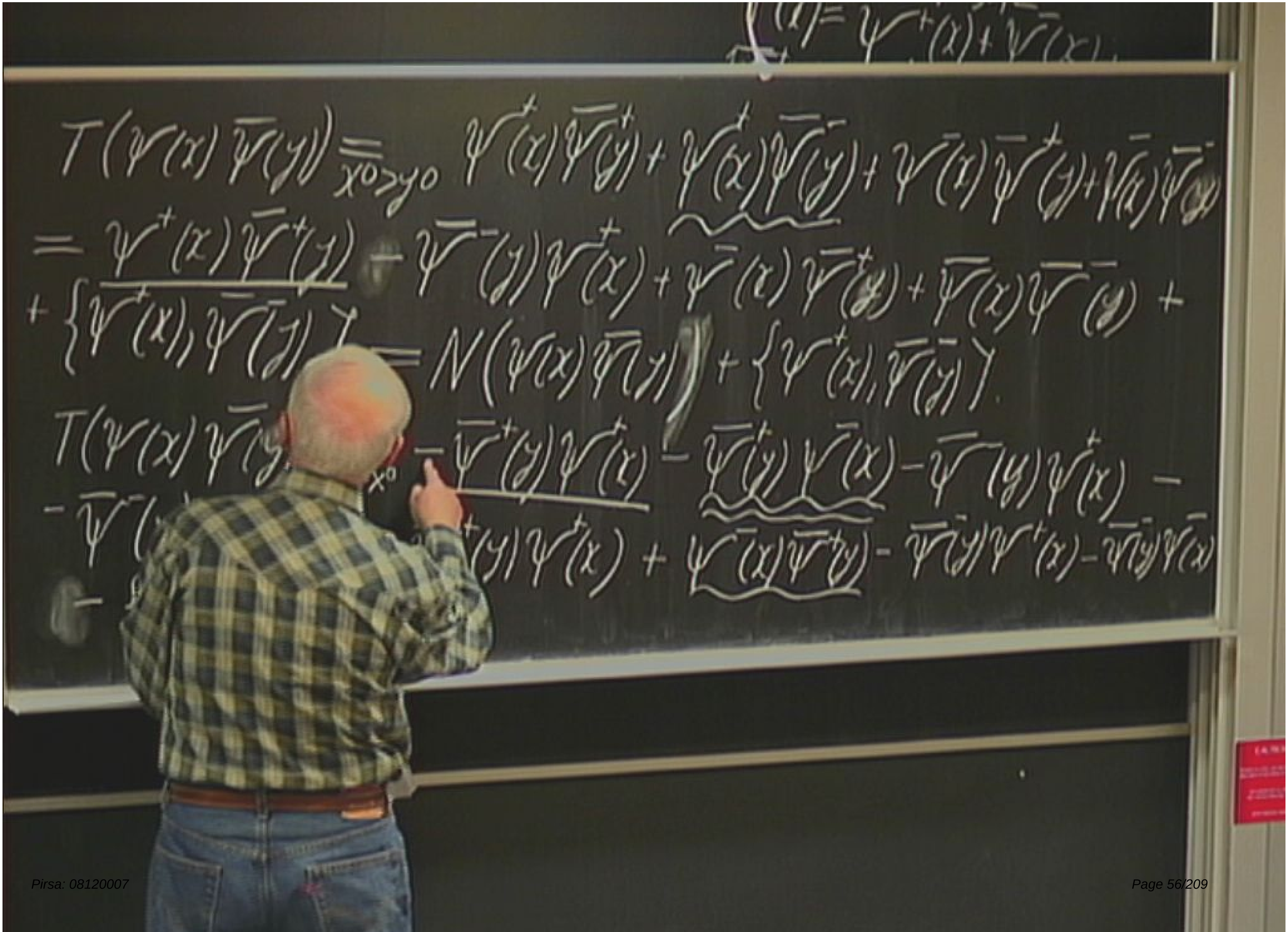

$T(\psi(x)\bar\psi(y)) \underset{x^0>y^0}{=} \psi^+(x)\bar\psi^+(y) + \psi^+(x)\bar\psi^-(y) + \psi^-(x)\bar\psi^+(y) + \psi^-(x)\bar\psi^-(y)$
$= \psi^+(x)\bar\psi^+(y) - \bar\psi^-(y)\psi^+(x) + \psi^-(x)\bar\psi^+(y) + \psi^-(x)\bar\psi^-(y) +$
$+ \{\psi^+(x), \bar\psi^-(y)\} = N(\psi(x)\bar\psi(y)) + \{\psi^+(x), \bar\psi^-(y)\}$
$T(\psi(x)\bar\psi(y)) \underset{y^0>x^0}{=} -\bar\psi^+(y)\psi^+(x) - \bar\psi^+(y)\psi^-(x) - \bar\psi^-(y)\psi^+(x) -$
$-\bar\psi^-(y)\psi^-(x) = -\bar\psi^+(y)\psi^+(x) + \psi^-(x)\bar\psi^+(y)$


$T(\psi(x)\bar\psi(y)) \underset{x^0>y^0}{=} \psi^+(x)\bar\psi^+(y) + \psi^-(x)\bar\psi^-(y) + \psi^-(x)\bar\psi^+(y) + \psi^+(x)\bar\psi^-(y)$
$= \psi^+(x)\bar\psi^+(y) - \bar\psi^-(y)\psi^+(x) + \psi^-(x)\bar\psi^+(y) + \psi^-(x)\bar\psi^-(y) +$
$+\{\psi^+(x),\bar\psi^-(y)\} = N(\psi(x)\bar\psi(y)) + \{\psi^+(x),\bar\psi^-(y)\}$
$T(\psi(x)\bar\psi(y)) \underset{y^0>x^0}{=} -\bar\psi^+(y)\psi^+(x) - \bar\psi^+(y)\psi^-(x) - \bar\psi^-(y)\psi^+(x) -$
$-\bar\psi^-(y)\psi^-(x) = -\bar\psi^+(y)\psi^+(x) + \psi^-(x)\bar\psi^+(y) - \bar\psi^-(y)\psi^+(x)$

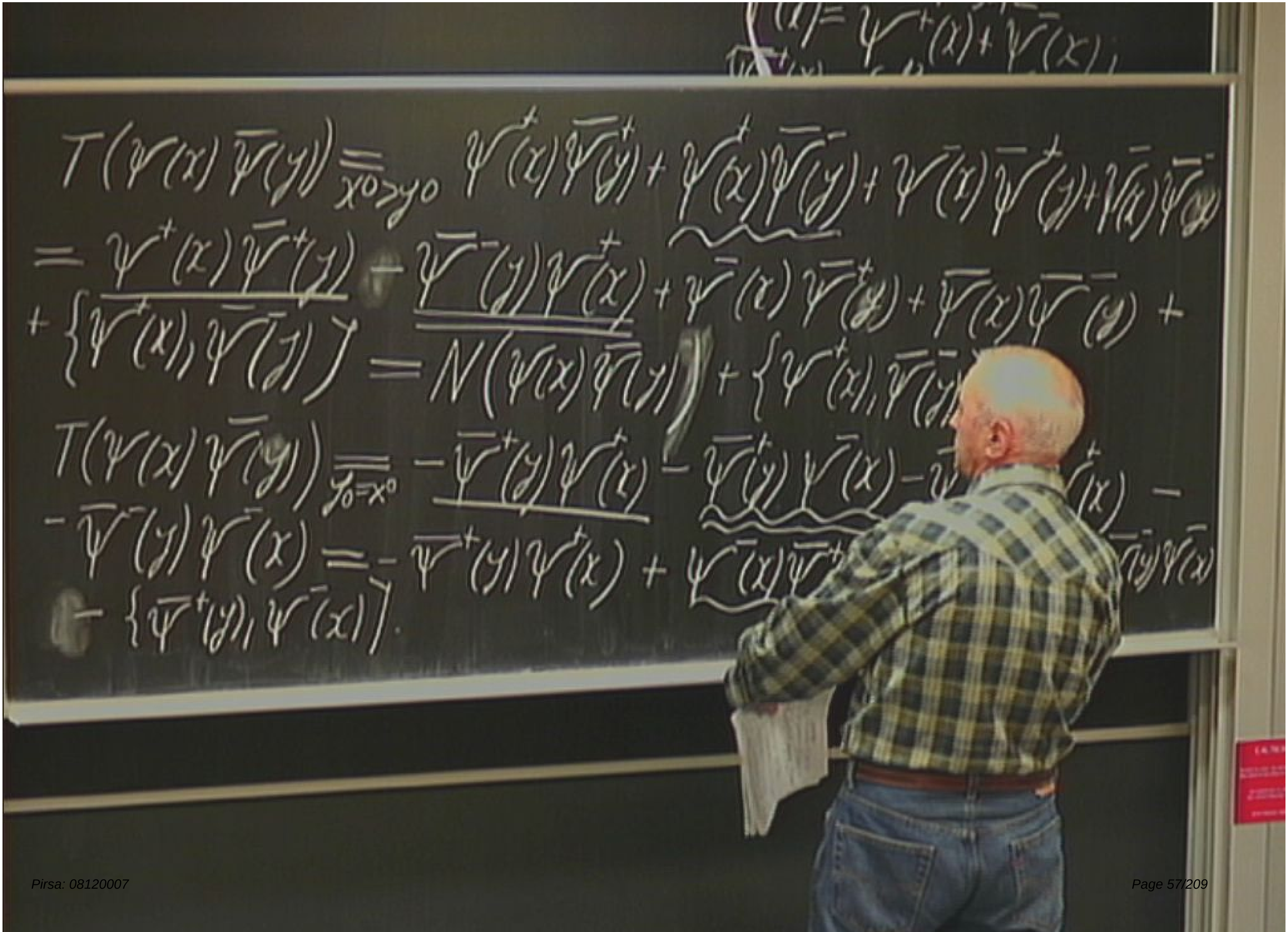

$T(\psi(x)\bar\psi(y)) \underset{x^0>y^0}{=} \psi^+(x)\bar\psi^+(y) + \psi^+(x)\bar\psi^-(y) + \psi^-(x)\bar\psi^+(y) + \psi^-(x)\bar\psi^-(y)$
$= \psi^+(x)\bar\psi^+(y) - \bar\psi^-(y)\psi^+(x) + \psi^-(x)\bar\psi^+(y) + \bar\psi^-(x)\bar\psi^-(y) +$
$+ \{\psi^+(x), \bar\psi^-(y)\} = N(\psi(x)\bar\psi(y)) + \{\psi^+(x), \bar\psi^-(y)\}$
$T(\psi(x)\bar\psi(y)) \underset{y^0=x^0}{=} -\bar\psi^+(y)\psi^+(x) - \bar\psi^+(y)\psi^-(x) - \bar\psi^-(y)\psi^+(x) -$
$-\bar\psi^-(y)\psi^-(x) = -\bar\psi^+(y)\psi^+(x)$


$T(\psi(x)\bar{\psi}(y)) \underset{x^0>y^0}{=} \psi^+(x)\bar{\psi}^+(y) + \psi^+(x)\bar{\psi}^-(y) + \psi^-(x)\bar{\psi}^+(y) + \psi^-(x)\bar{\psi}^-(y)$
$= \psi^+(x)\bar{\psi}^+(y) - \bar{\psi}^-(y)\psi^+(x) + \psi^-(x)\bar{\psi}^+(y) + \psi^-(x)\bar{\psi}^-(y) +$
$+ \{\psi^+(x), \bar{\psi}^-(y)\} = N(\psi(x)\bar{\psi}(y)) + \{\psi^+(x), \bar{\psi}^-(y)\}.$
$T(\psi(x)\bar{\psi}(y)) \underset{y^0>x^0}{=} -\bar{\psi}^+(y)\psi^+(x) - \bar{\psi}^+(y)\psi^-(x) - \bar{\psi}^-(y)\psi^+(x) -$
$-\bar{\psi}^-(y)\psi^-(x) = -\bar{\psi}^+(y)\psi^+(x) + \psi^-(x)\bar{\psi}^+(y) - \bar{\psi}^-(y)\psi^+(x) - \bar{\psi}^-(y)\psi^-(x)$

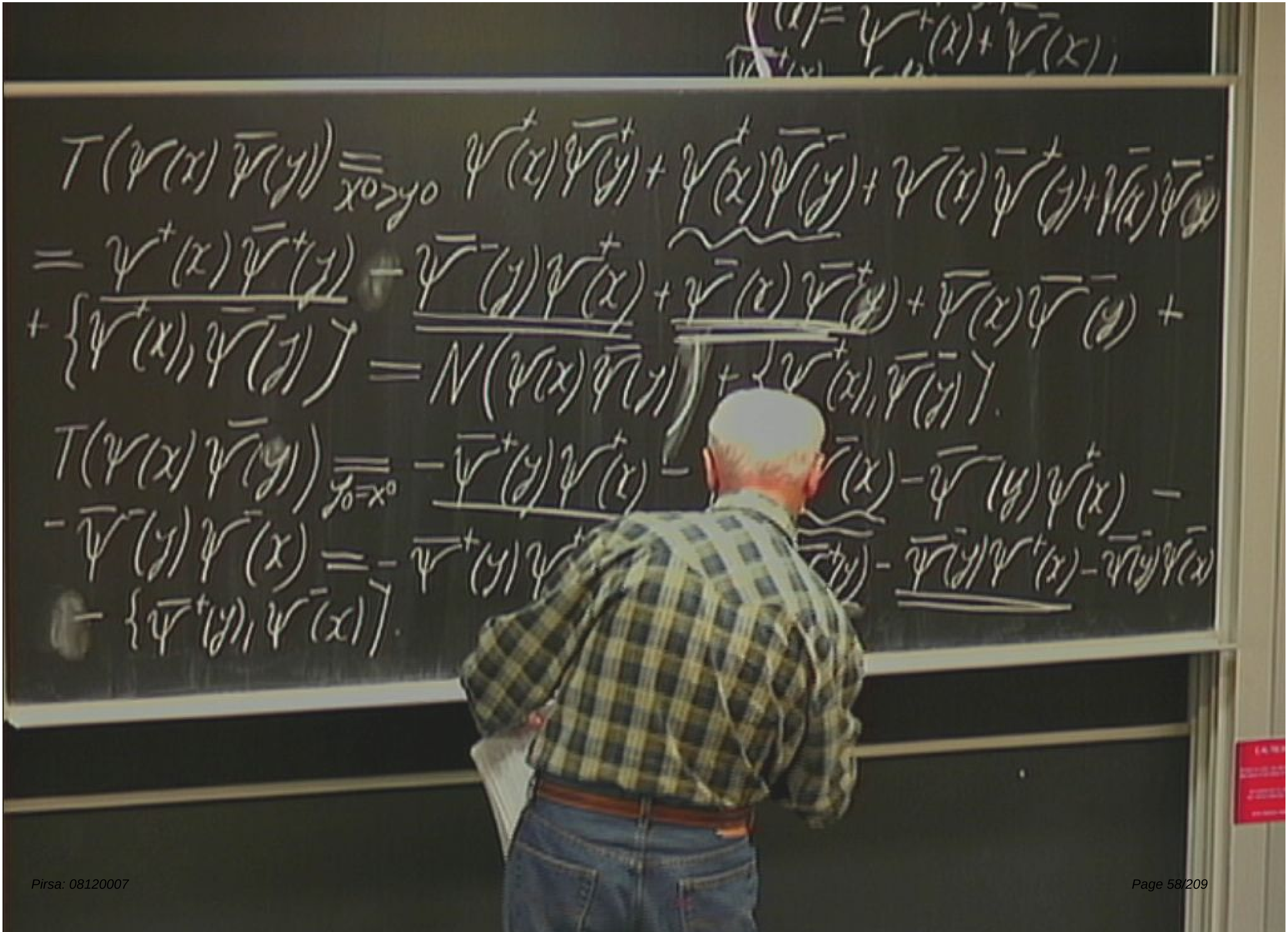

$T(\psi(x)\bar{\psi}(y)) \underset{x^0>y^0}{=} \psi^+(x)\bar{\psi}^+(y) + \psi^+(x)\bar{\psi}^-(y) + \psi^-(x)\bar{\psi}^+(y) + \psi^-(x)\bar{\psi}^-(y)$
$= \psi^+(x)\bar{\psi}^+(y) - \bar{\psi}^-(y)\psi^+(x) + \psi^-(x)\bar{\psi}^+(y) + \psi^-(x)\bar{\psi}^-(y) +$
$+ \{\psi^+(x), \bar{\psi}^-(y)\} = N(\psi(x)\bar{\psi}(y)) + \{\psi^+(x), \bar{\psi}^-(y)\}.$
$T(\psi(x)\bar{\psi}(y)) \underset{y^0>x^0}{=} -\bar{\psi}^+(y)\psi$
$- \{\bar{\psi}^+(y), \psi^-(x)\}.$



$T(\psi(x)\bar\psi(y)) \underset{x^0>y^0}{=} \psi^+(x)\bar\psi^+(y) + \psi^+(x)\bar\psi^-(y) + \psi^-(x)\bar\psi^+(y) + \psi^-(x)\bar\psi^-(y)$
$= \psi^+(x)\bar\psi^+(y) - \bar\psi^-(y)\psi^+(x) + \psi^-(x)\bar\psi^+(y) + \bar\psi^-(x)\psi^-(y) +$
$+ \{\psi^+(x), \bar\psi^-(y)\} = N(\psi(x)\bar\psi(y)) + \{\psi^+(x), \bar\psi^-(y)\}$
$T(\psi(x)\bar\psi(y)) \underset{y^0>x^0}{=} -\bar\psi^+(y)\psi^+(x) - \bar\psi^+(y)\psi^-(x) - \bar\psi^-(y)\psi^+(x) -$
$-\bar\psi^-(y)\psi^-(x)$



T(ψ(x) ψ̄(y)) =_{x⁰>y⁰} ψ⁺(x)ψ̄⁺(y) + ψ⁺(x)ψ̄⁻(y) + ψ⁻(x)ψ̄⁺(y) + ψ⁻(x)ψ̄⁻(y)
= ψ⁺(x)ψ̄⁺(y) − ψ̄⁻(y)ψ⁺(x) + ψ⁻(x)ψ̄⁺(y) + ψ̄⁻(x)ψ⁻(y) +
+ {ψ⁺(x), ψ̄⁻(y)} = N(ψ(x)ψ̄(y)) + {ψ⁺(x), ψ̄⁻(y)}
T(ψ(x)ψ̄(y)) =_{y⁰>x⁰} −ψ̄⁺(y)ψ⁺(x) − ψ̄⁺(y)ψ⁻(x) − ψ̄⁻(y)ψ⁺(x) −
− ψ̄⁻(y)ψ⁻(x) = −ψ̄⁺(y)ψ⁺(x) + ψ⁻(x)ψ̄⁺(y)
− {ψ̄⁺(y), ψ⁻(x)}.


$T(\psi(x)\bar\psi(y)) \underset{x^0>y^0}{=} \psi^+(x)\bar\psi^+(y) + \psi^+(x)\bar\psi^-(y) + \psi^-(x)\bar\psi^+(y) + \psi^-(x)\bar\psi^-(y)$
$= \psi^+(x)\bar\psi^+(y) - \bar\psi^-(y)\psi^+(x) + \psi^-(x)\bar\psi^+(y) + \psi^-(x)\bar\psi^-(y) +$
$+ \{\psi^+(x), \bar\psi^-(y)\} = N(\psi(x)\bar\psi(y)) + \{\psi^+(x), \bar\psi^-(y)\}.$
$- \{\bar\psi^+(y), \psi^-(x)\}.$

$$(x) = \psi^+(x) + \psi^-(x)$$

$$T(\psi(x)\bar\psi(y)) \underset{x^0>y^0}{=} \psi^+(x)\bar\psi^+(y) + \psi^+(x)\bar\psi^-(y) + \psi^-(x)\bar\psi^+(y) + \psi^-(x)\bar\psi^-(y)$$

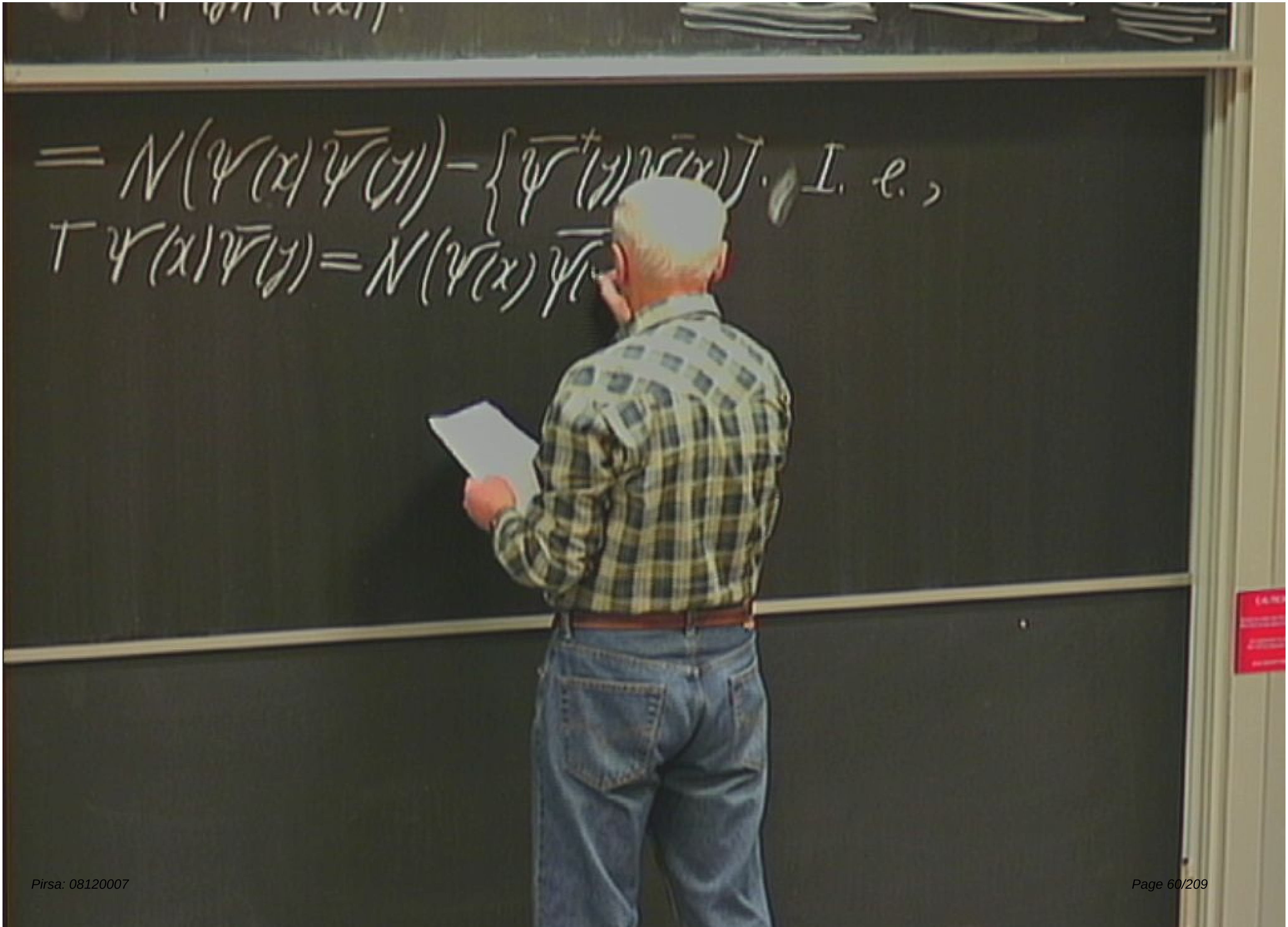
$$= \underline{\psi^+(x)\bar\psi^+(y)} - \underline{\underline{\bar\psi^-(y)\psi^+(x)}} + \underline{\underline{\psi^-(x)\bar\psi^+(y)}} + \bar\psi^-(x)\bar\psi^-(y) +$$

$$+ \{\psi^+(x), \bar\psi^-(y)\} = N(\psi(x)\bar\psi(y)) + \{\psi^+(x), \bar\psi^-(y)\}.$$

$$T(\psi(x)\bar\psi(y)) \underset{y^0>x^0}{=} -\underline{\bar\psi^+(y)\psi^+(x)} - \bar\psi^+(y)\psi^-(x) - \bar\psi^-(y)\psi^+(x) -$$

$$- \bar\psi^-(y)\psi^-(x) = -\bar\psi^+(y)\psi^+(x) + \underline{\underline{\psi^-(x)\bar\psi^+(y)}} - \underline{\underline{\bar\psi^-(y)\psi^+(x)}} - \bar\psi^-(y)\psi^-(x)$$

$$- \{\bar\psi^+(y), \psi^-(x)\}.$$


= N(ψ(x)ψ̄(y)) − {ψ̄ᵀ(y)ψᵀ(x)}. I. e.,
T ψ(x)ψ̄(y) = N(ψ(x)ψ̄(

$$= N(\psi(x)\bar{\psi}(y)) - \{\bar{\psi}^{+}(y)\,\psi^{-}(x)\}. \quad \text{I. e.,}$$

$$\psi(x)\bar{\psi}(y) = N(\psi(x)\,\bar{\psi}(y)) + \psi(x)\,\bar{\psi}(y), \quad \text{where}$$

$$= N(\psi(x)\bar{\psi}(y)) - \{\bar{\psi}^{+}(y), \psi(x)\}. \quad \text{I. e.,}$$

$$T\,\psi(x)\bar{\psi}(y) = N(\psi(x)\bar{\psi}(y)) + \overline{\psi(x)\,\bar{\psi}(y)},$$

$$\overline{\psi(x)\,\bar{\psi}(y)} = \begin{cases} \{\psi^{+}(x), \bar{\psi}^{-}(y)\}, & x^0 > y^0 \\ -\{\bar{\psi}^{+}(y), \psi(x)\}, & y^0 > x^0 \end{cases} = \{$$

$$= N(\psi(x)\bar{\psi}(y)) - \{\bar{\psi}^{+}(y), \psi^{-}(x)\}. \text{ I. e.,}$$

$$T\,\psi(x)\bar{\psi}(y) = N(\psi(x)\bar{\psi}(y)) + \overbrace{\psi(x)\,\bar{\psi}(y)}, \text{ where}$$

$$\overbrace{\psi(x)\,\bar{\psi}(y)} = \begin{cases} \{\psi^{+}(x), \bar{\psi}^{-}(y)\}, & x^0 > y^0 \\ -\{\bar{\psi}^{+}(y), \psi^{-}(x)\}, & y^0 > x^0 \end{cases} = S_F(x-y)$$

$$= N\big(\psi(x)\bar\psi(y)\big) - \{\bar\psi^{+}(y), \psi^{-}(x)\}.$$ I. e.,

$$T\,\psi(x)\bar\psi(y) = N\big(\psi(x)\bar\psi(y)\big) + \overbrace{\psi(x)\bar\psi(y)},$$ where

$$\overbrace{\psi(x)\bar\psi(y)} = \begin{cases} \{\psi^{+}(x), \bar\psi^{-}(y)\}, & x^0 > y^0 \\ -\{\bar\psi^{+}(y), \psi^{-}(x)\}, & y^0 > x^0 \end{cases} = S_F(x-y)$$

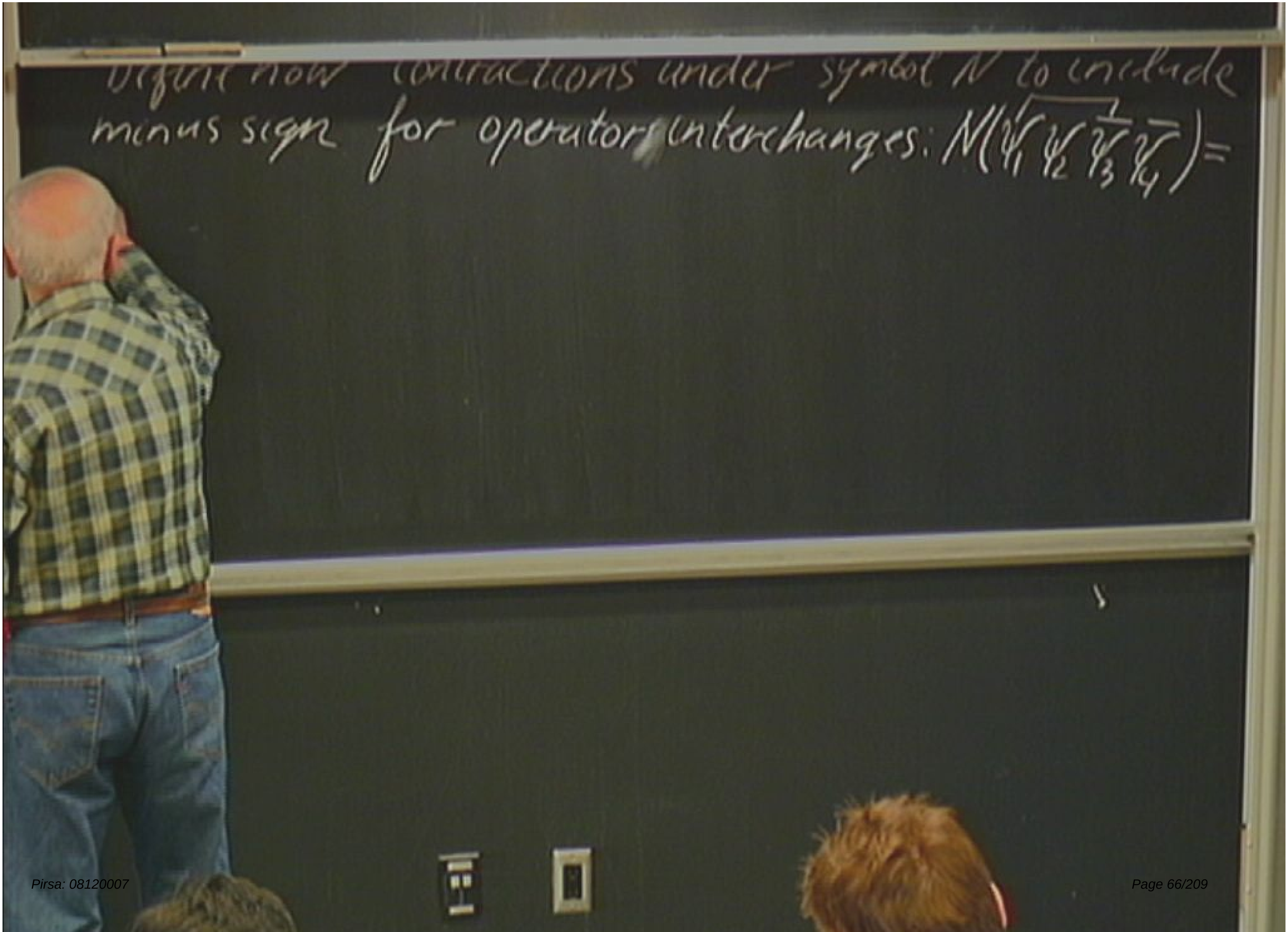
By definition, $\overbrace{\psi(x)\psi(y)} = 0$, $\overbrace{\bar\psi(x)\bar\psi(y)} = 0$.

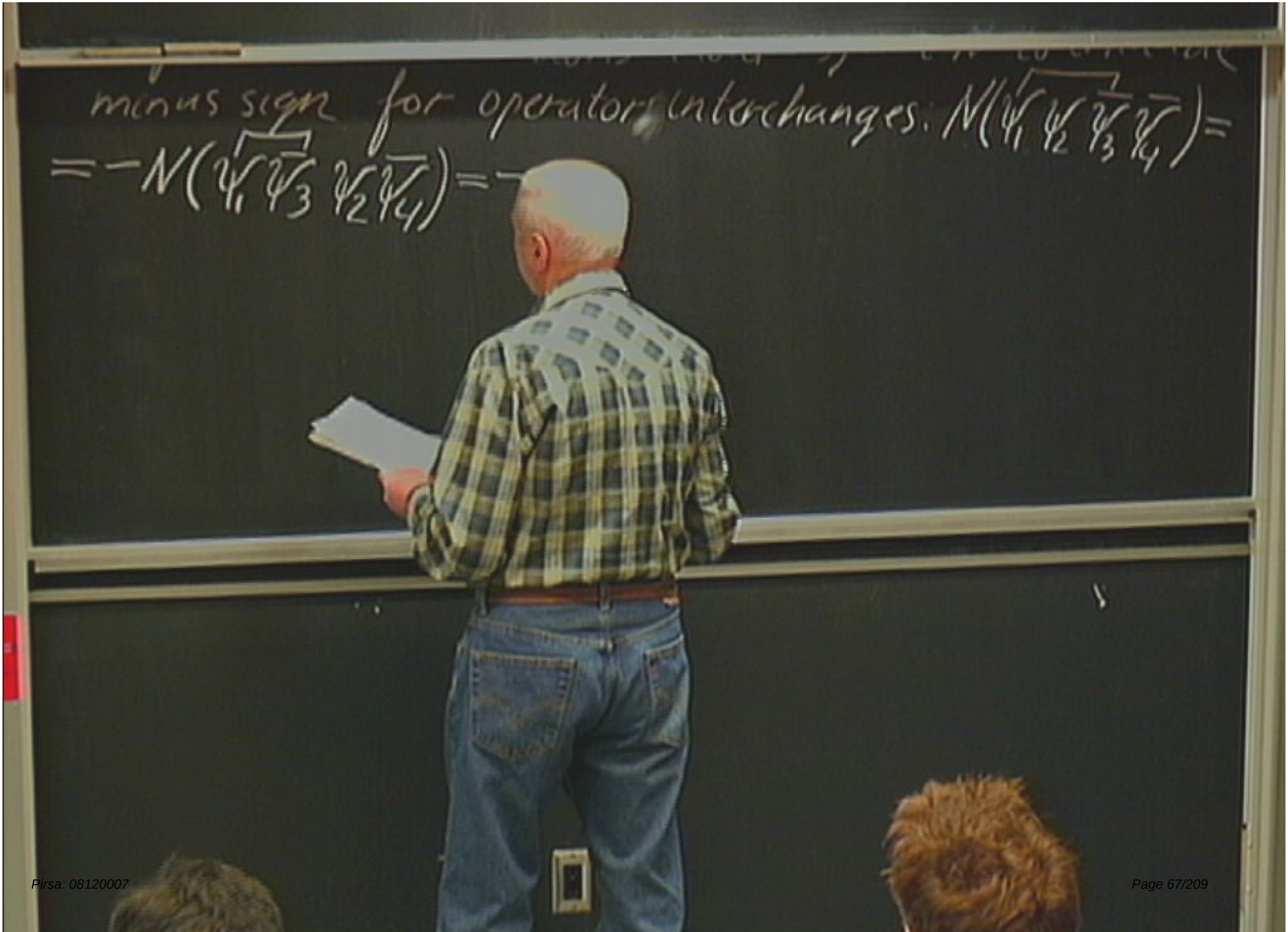
$$= N(\psi(x)\bar{\psi}(y)) - \{\bar{\psi}^{+}(y), \psi^{-}(x)\}.$$ I. e.,

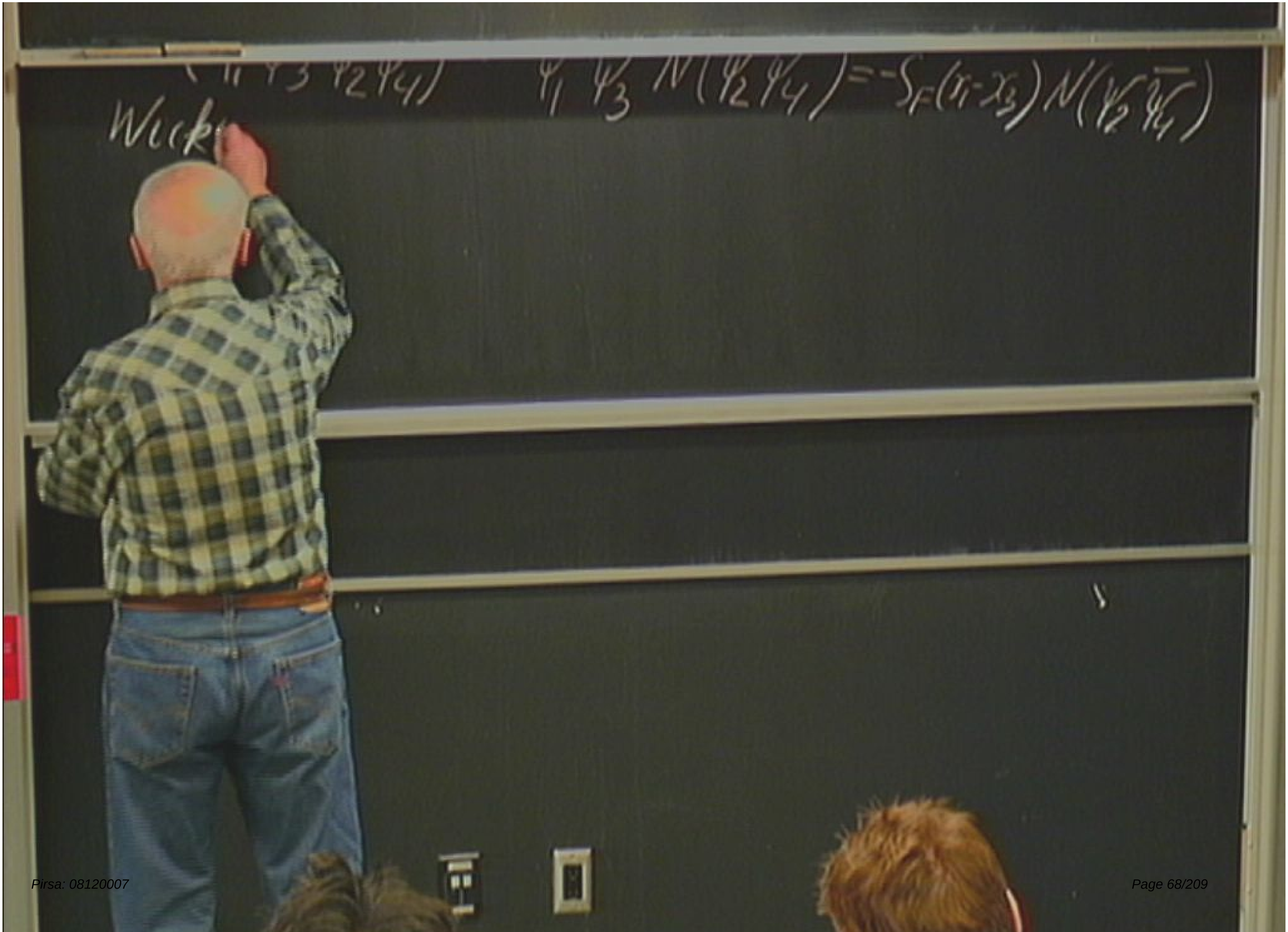
$$T\,\psi(x)\bar{\psi}(y) = N(\psi(x)\bar{\psi}(y)) + \psi(x)\bar{\psi}(y),$$ where

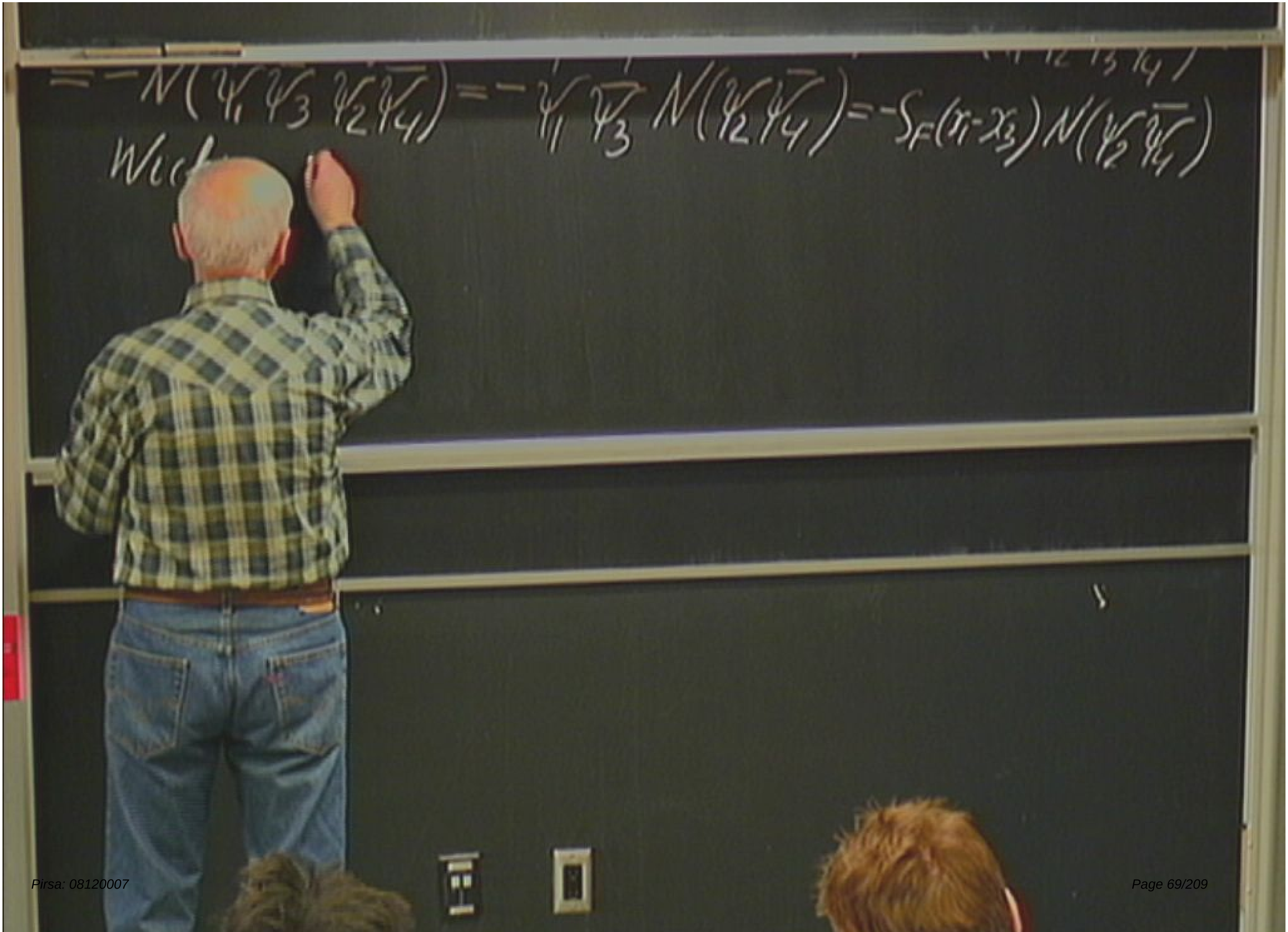
$$\psi(x)\,\bar{\psi}(y) = \begin{cases} \{\psi^{+}(x), \bar{\psi}^{-}(y)\}, & x^0 > y^0 \\ -\{\bar{\psi}^{+}(y), \psi^{-}(x)\}, & y^0 > x^0 \end{cases} = S_F(x-y)$$

By definition, $\psi(x)\psi(y) = 0$, $\bar{\psi}(x)\bar{\psi}(y) = 0$.


Define now contractions under symbol N to include
minus sign for operators interchanges: $N(\psi_1 \psi_2 \bar{\psi}_3 \bar{\psi}_4) =$


minus sign for operators interchanges: $N(\psi_1 \psi_2 \psi_3 \psi_4) =$
$= -N(\psi_1 \psi_3 \psi_2 \psi_4) = -$

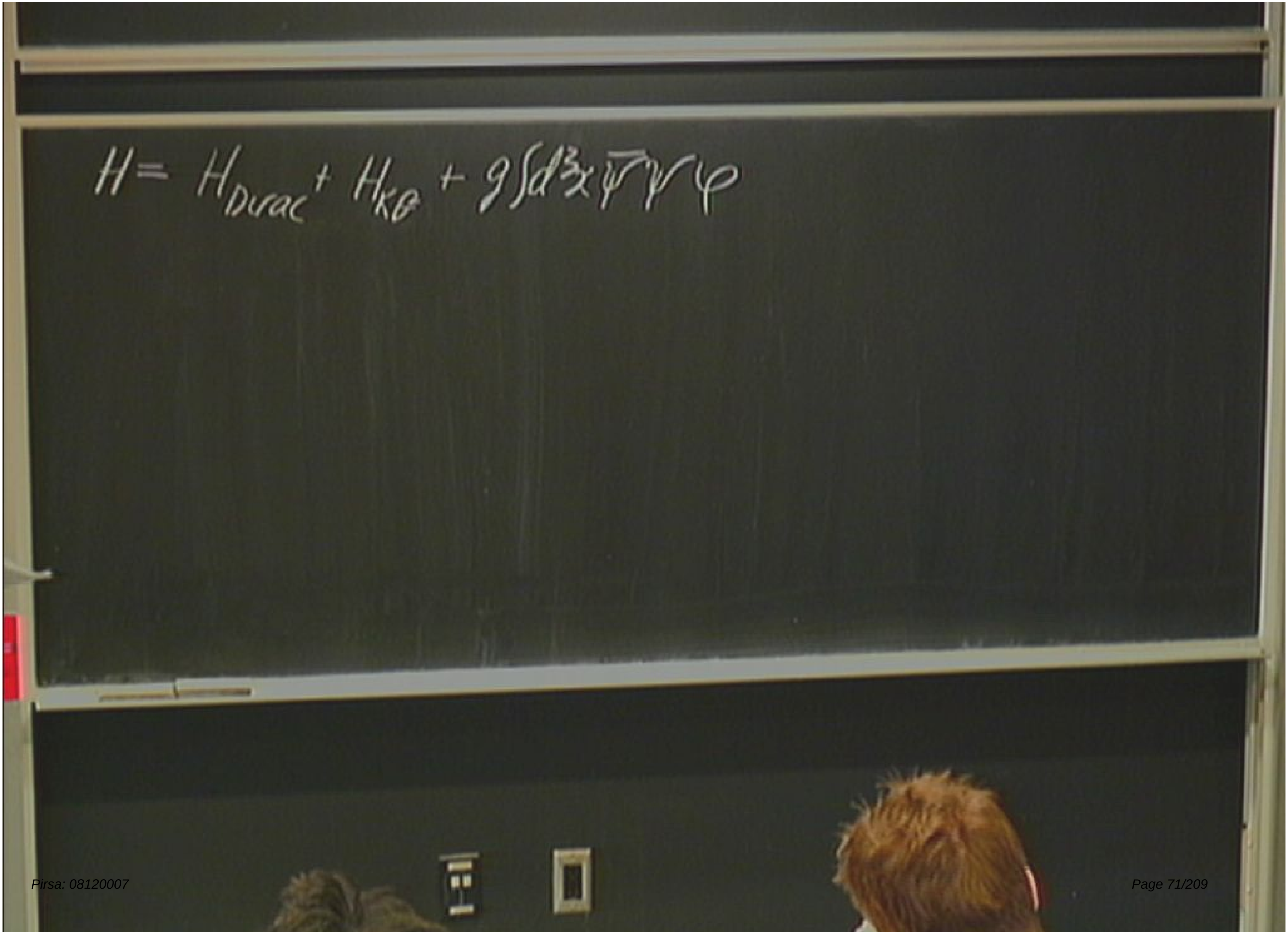

Wick


$= -N(\psi_1 \bar{\psi}_3 \psi_2 \bar{\psi}_4) = -\psi_1 \bar{\psi}_3 N(\psi_2 \bar{\psi}_4) = -S_F(x_1 - x_3) N(\psi_2 \bar{\psi}_4)$
Wic

$(\psi_1 \bar\psi_3 \psi_2 \bar\psi_4) \qquad \psi_1 \bar\psi_3 N(\psi_2 \bar\psi_4) = -S_F(x_1 - x_3) N(\psi_2 \bar\psi_4)$

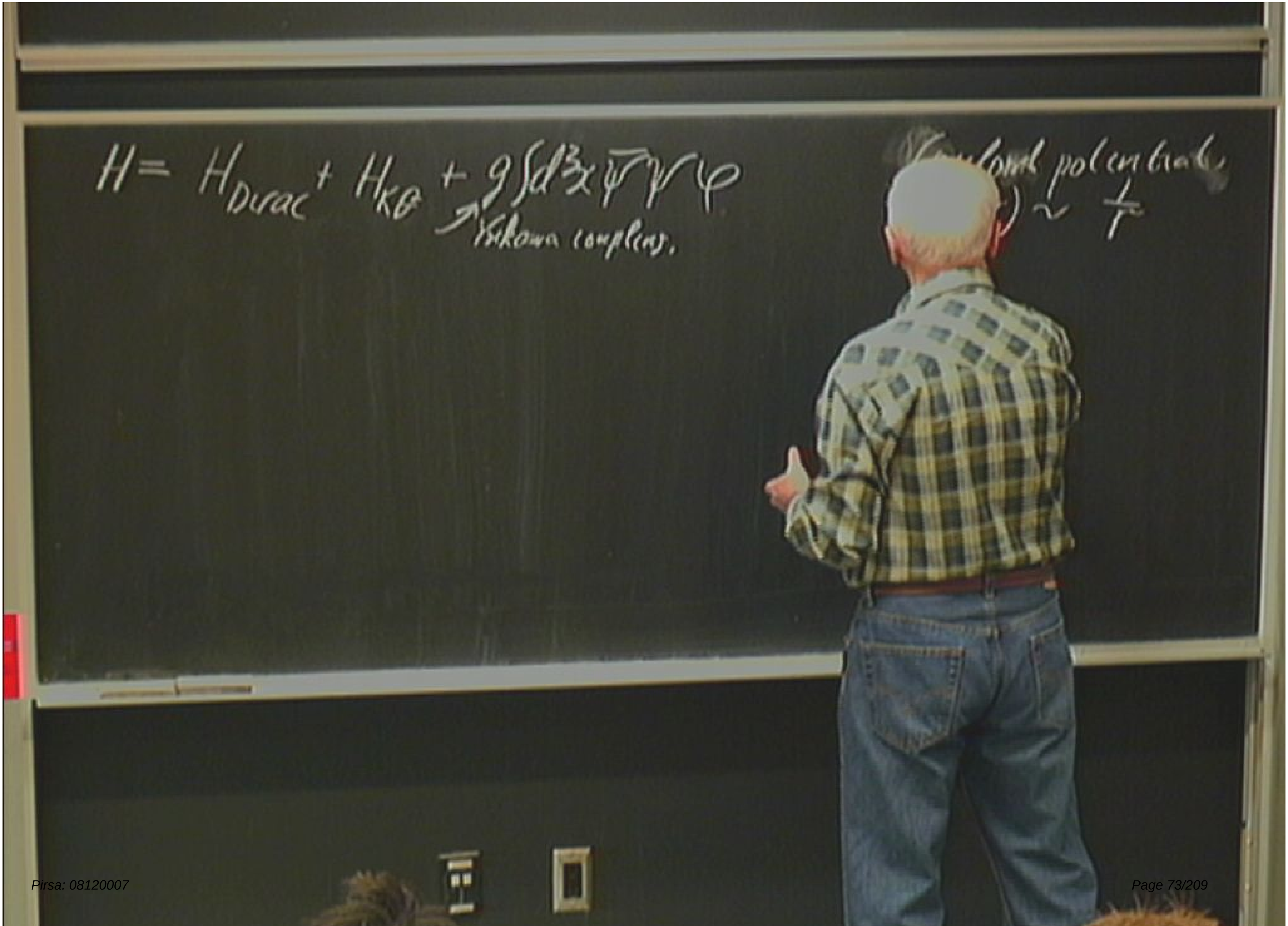
Wick's theorem for fermions:

$T[\psi_1 \bar\psi_2 \psi_3 \ldots] = N[\psi_1 \bar\psi_2 \psi_3 \ldots]$ + all possible contractions


$H = H_{Dirac} + H_{KG} + g\int d^3x\, \bar{\psi}\psi\varphi$

$$H = H_{Dirac} + H_{KG} + g\int d^3x\, \bar{\psi}\psi\varphi$$

Yukawa couplings.


$H = H_{Dirac} + H_{KG} + g\int d^3x\, \bar{\psi}\psi\varphi$
Yukawa couplings.
Coulomb potential

$$H = H_{Dirac} + H_{KG} + g\int d^3x\, \bar{\psi}\psi\varphi$$

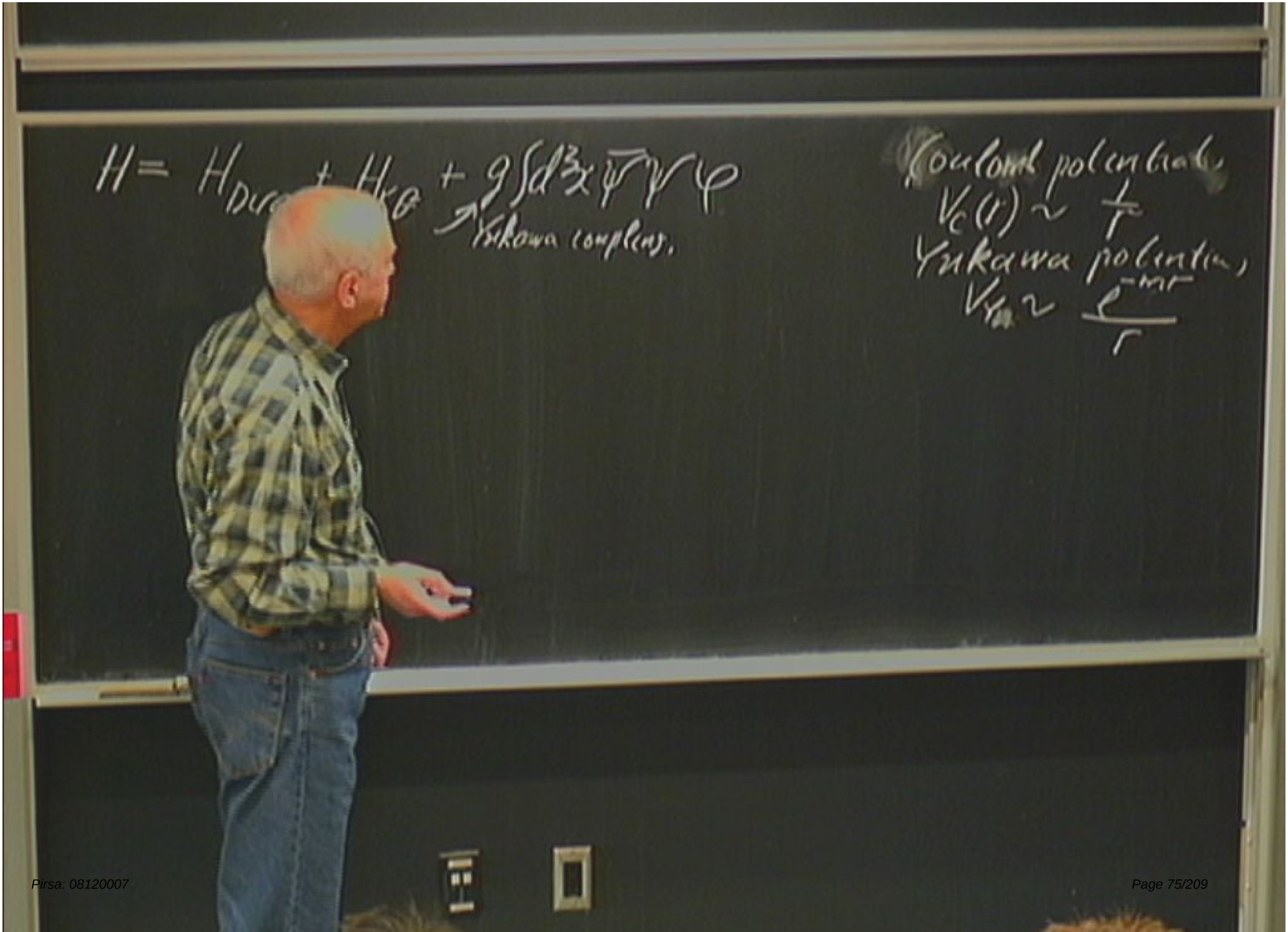
Yukawa coupling.

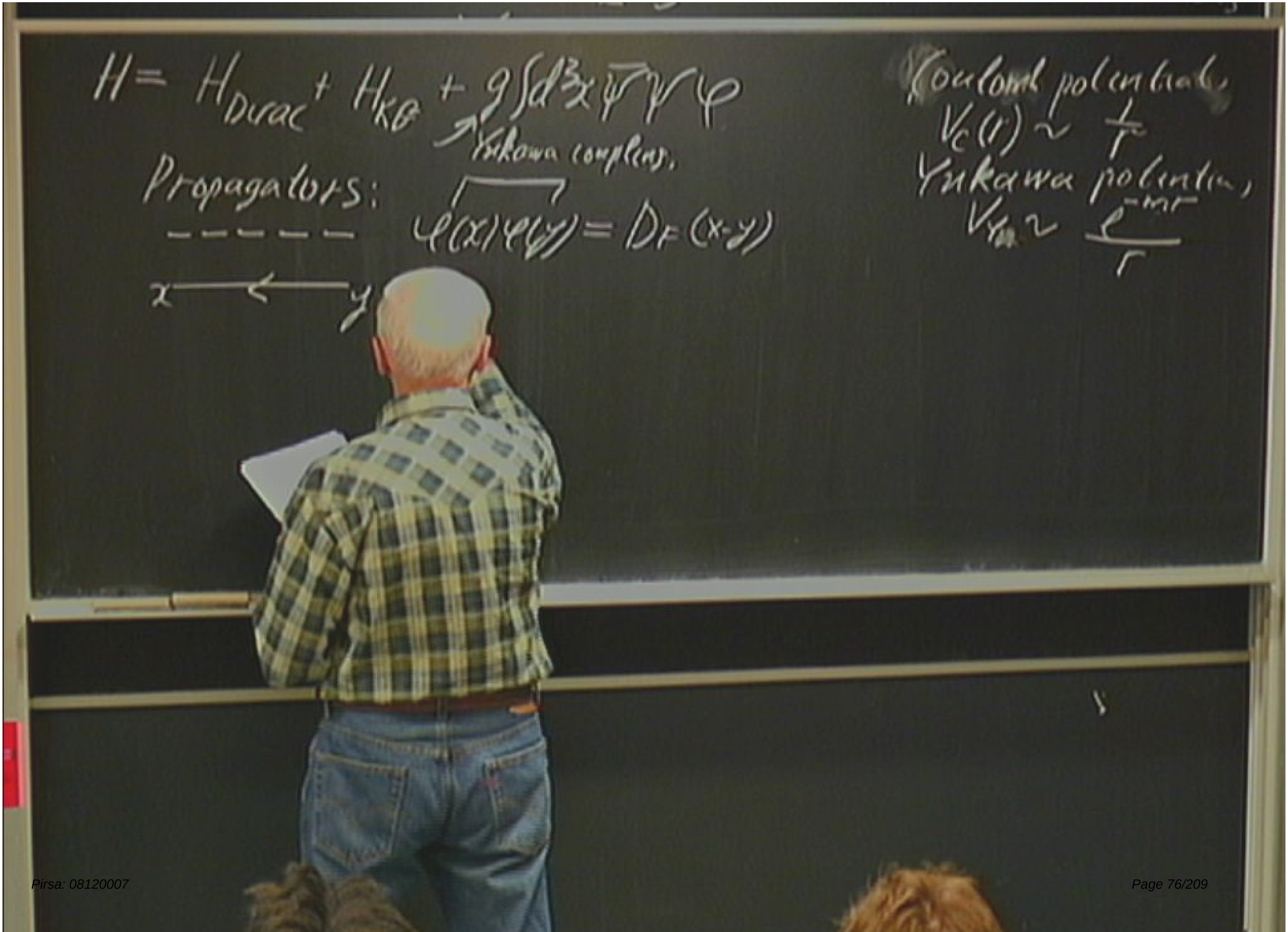
Coulomb potential,

$$V_C(r) \sim \frac{1}{r}$$

Yukawa potential,

$$V_Y \sim \frac{e^{-mr}}{r}$$


$H = H_{Dirac} + H_{KG} + g\int d^3x\, \bar{\psi}\psi\varphi$
Yukawa couplings.
Coulomb potential,
$V_C(r) \sim \frac{1}{r}$
Yukawa potential,
$V_Y \sim \frac{e^{-mr}}{r}$


$H = H_{Dirac} + H_{KG} + g\int d^3x\, \bar\psi \psi \varphi$
Yukawa couplings.
Propagators:
$\varphi(x)\varphi(y) = D_F(x-y)$
x
y
Coulomb potential,
$V_C(r) \sim \frac{1}{r}$
Yukawa potential,
$V_{Yuk} \sim \frac{e^{-mr}}{r}$

$$H = H_{Dirac} + H_{KG} + g\int d^3x\, \bar{\psi}\psi\varphi$$

Yukawa coupling.

Propagators:

$$\varphi(x)\varphi(y) = D_F(x-y)$$

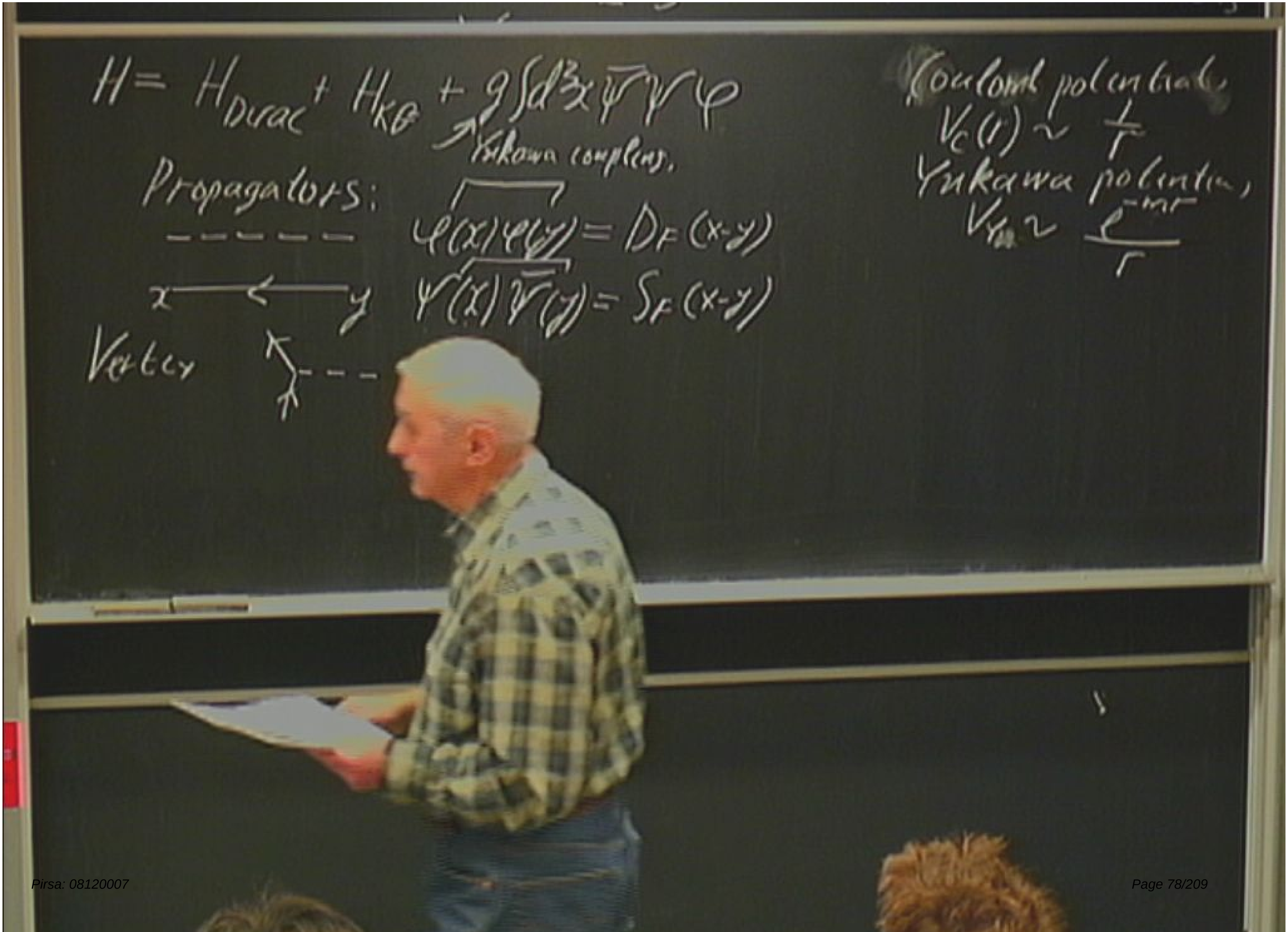
x y $$\psi(x)\bar{\psi}(y) = S_F(x-y)$$

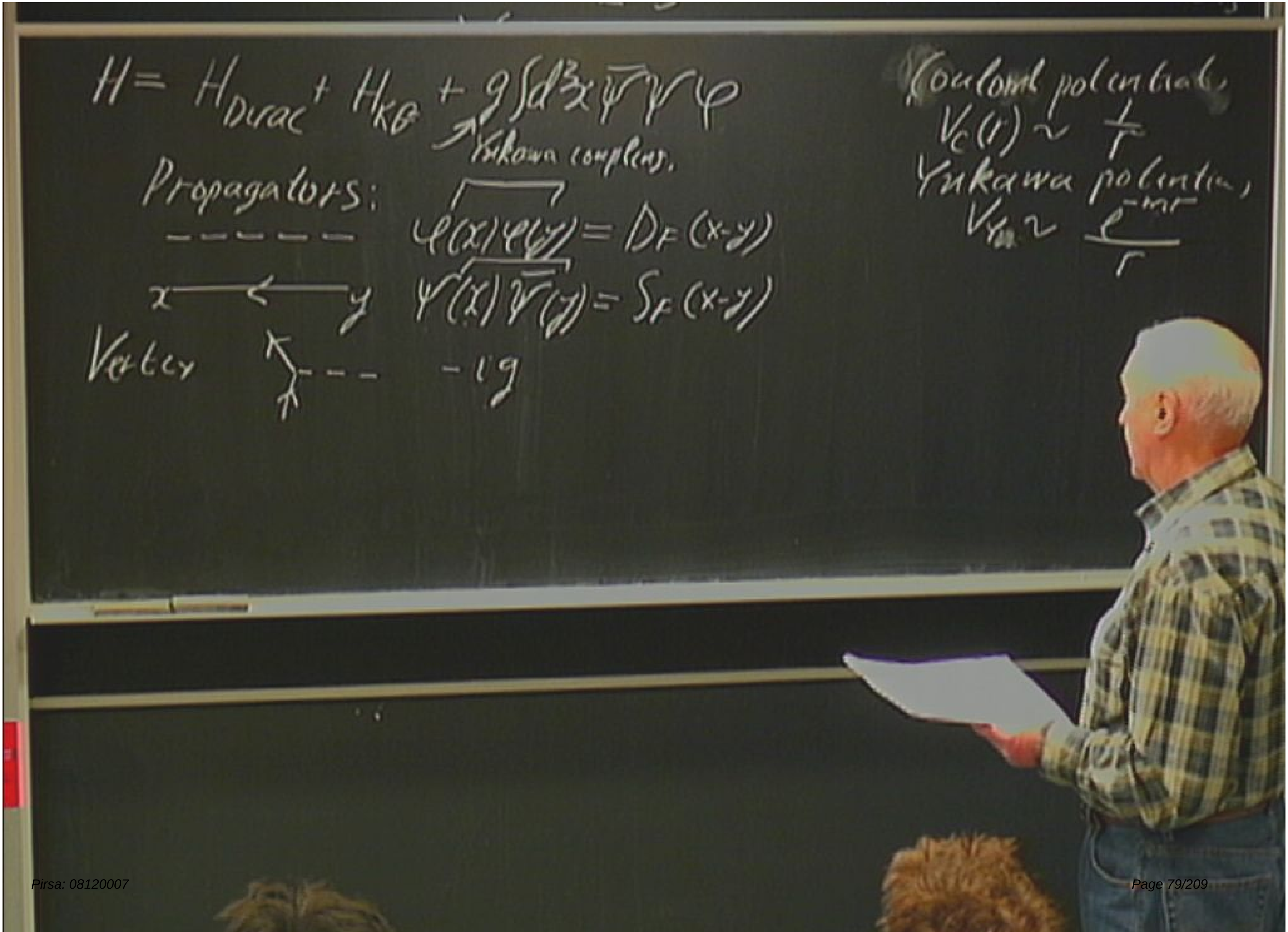
Coulomb potential,

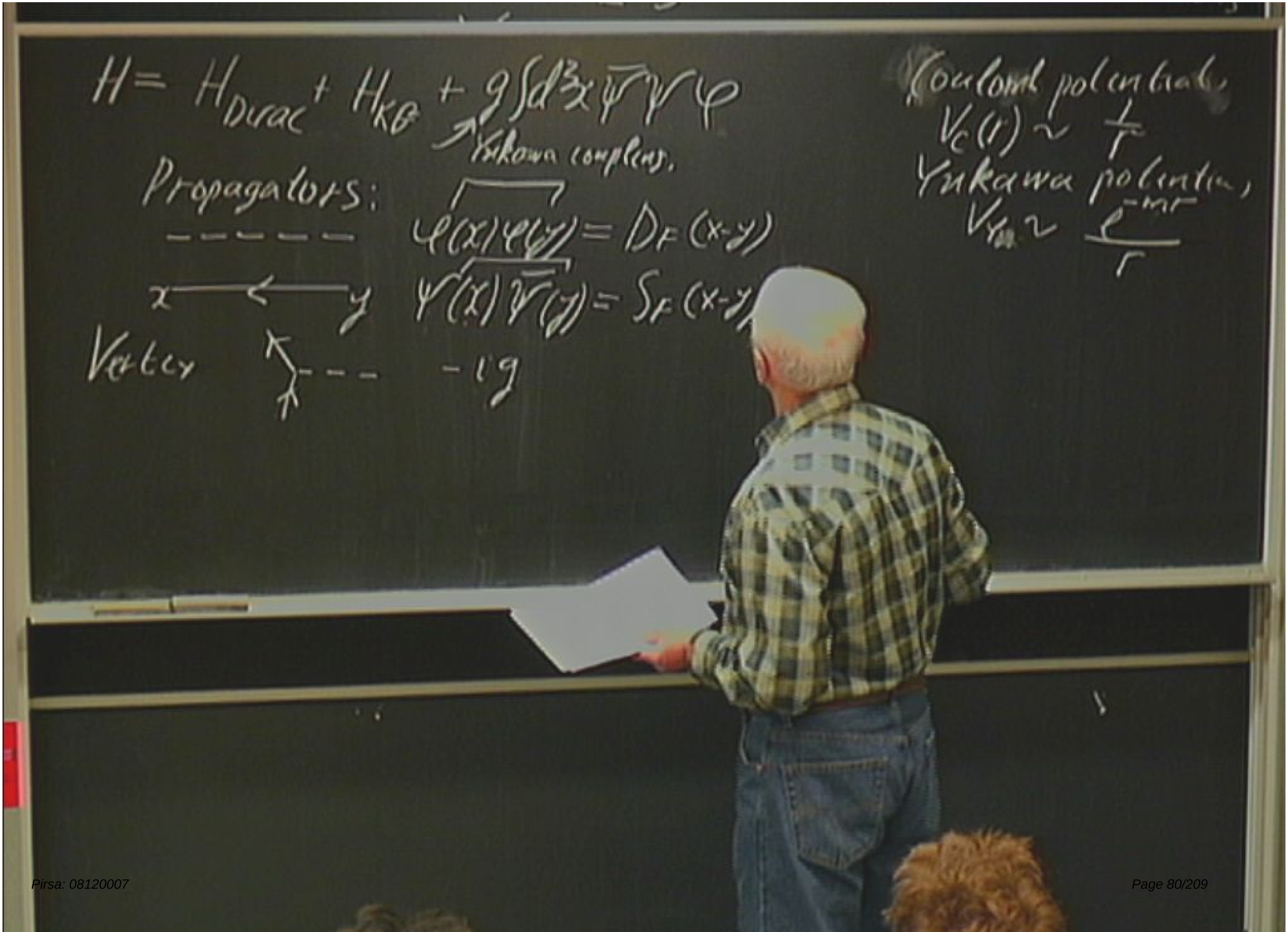
$$V_C(r) \sim \frac{1}{r}$$

Yukawa potential,

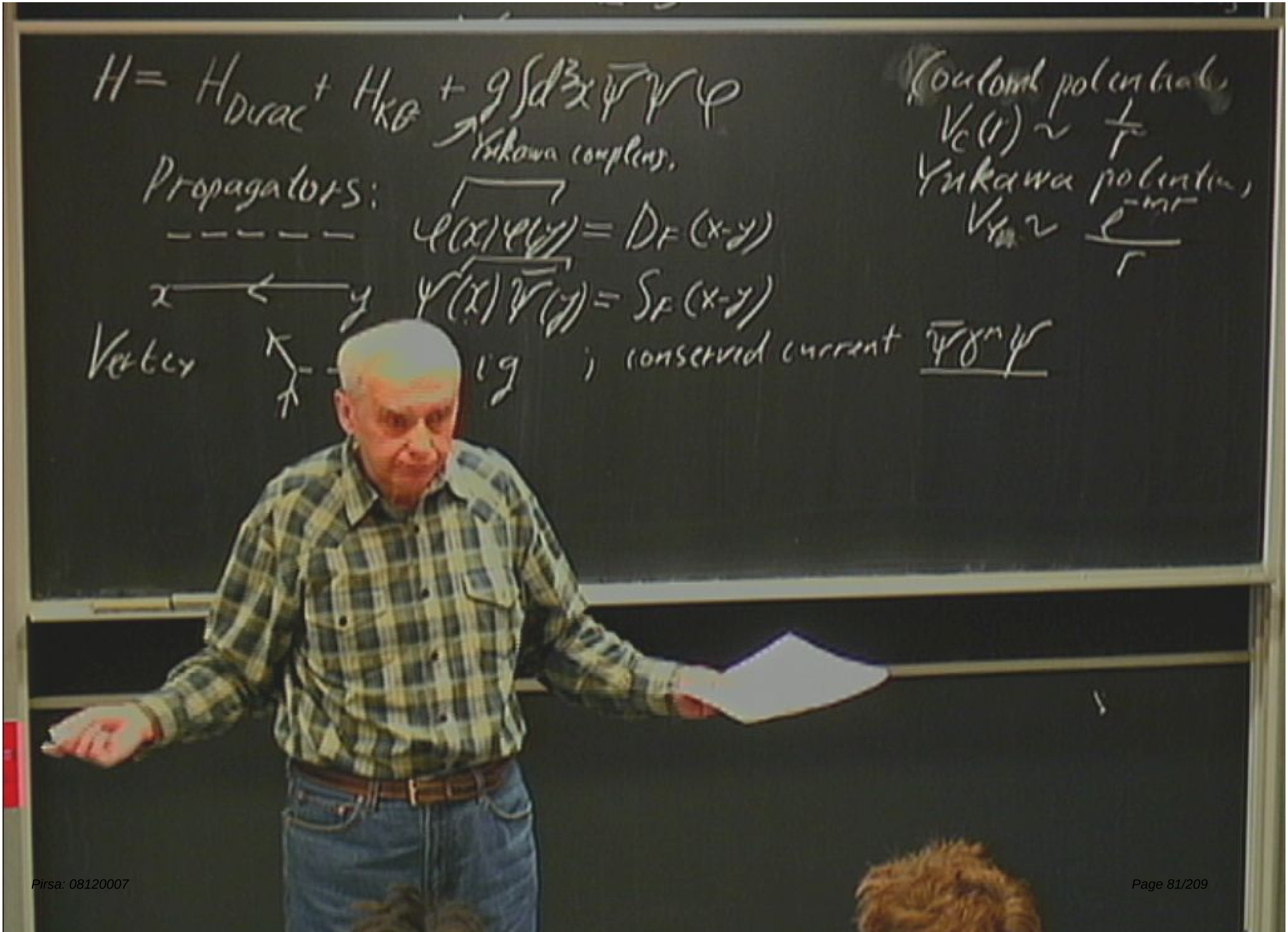
$$V_{Yuk} \sim \frac{e^{-mr}}{r}$$

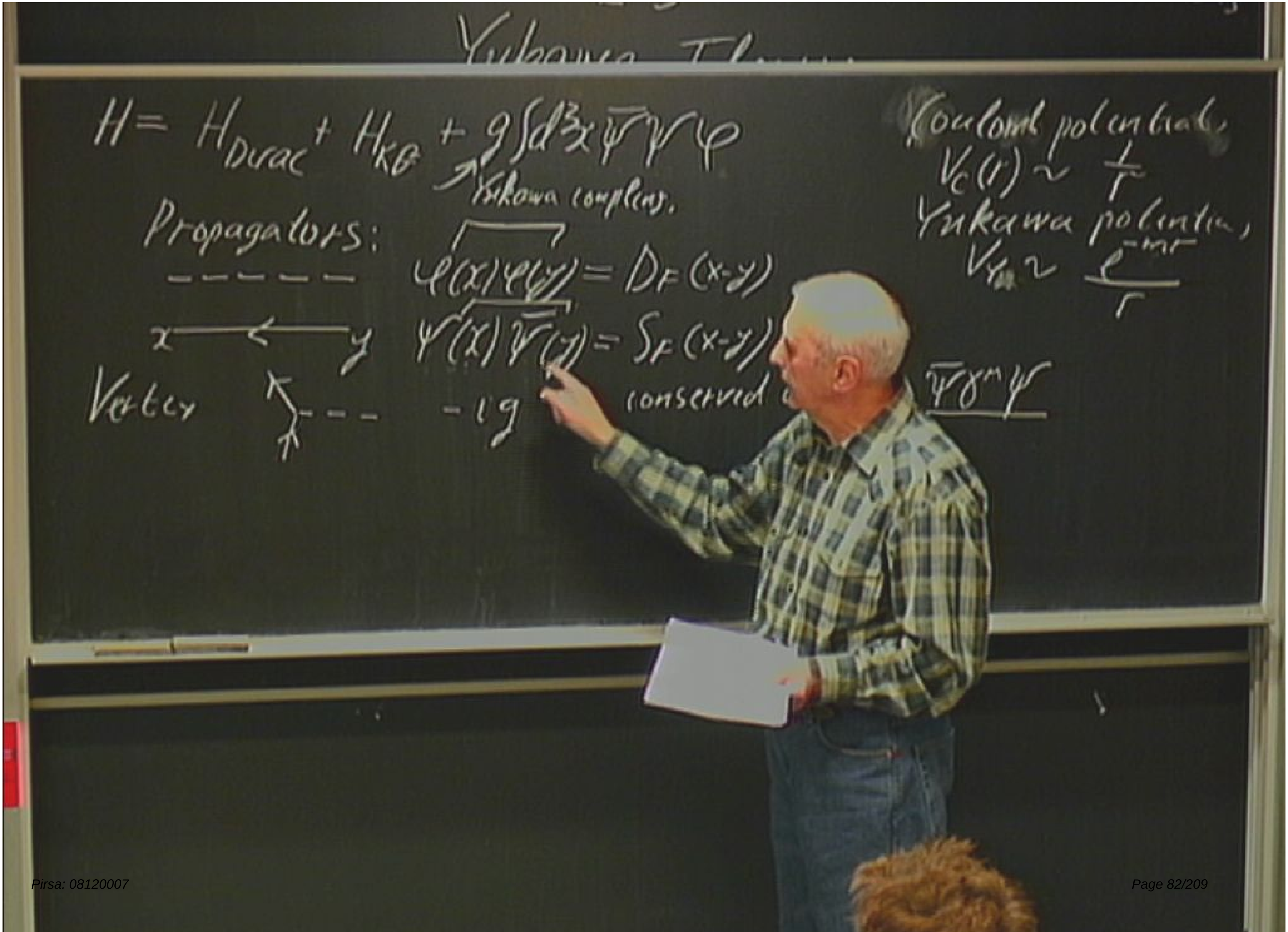

$H = H_{Dirac} + H_{KG} + g\int d^3x\, \bar{\psi}\psi\varphi$
Yukawa coupling.
Propagators:
$\varphi(x)\varphi(y) = D_F(x-y)$
x
y
$\psi(x)\bar{\psi}(y) = S_F(x-y)$
Vertex
Coulomb potential
$V_C(r) \sim \frac{1}{r}$
Yukawa potential,
$V_Y \sim \frac{e^{-mr}}{r}$

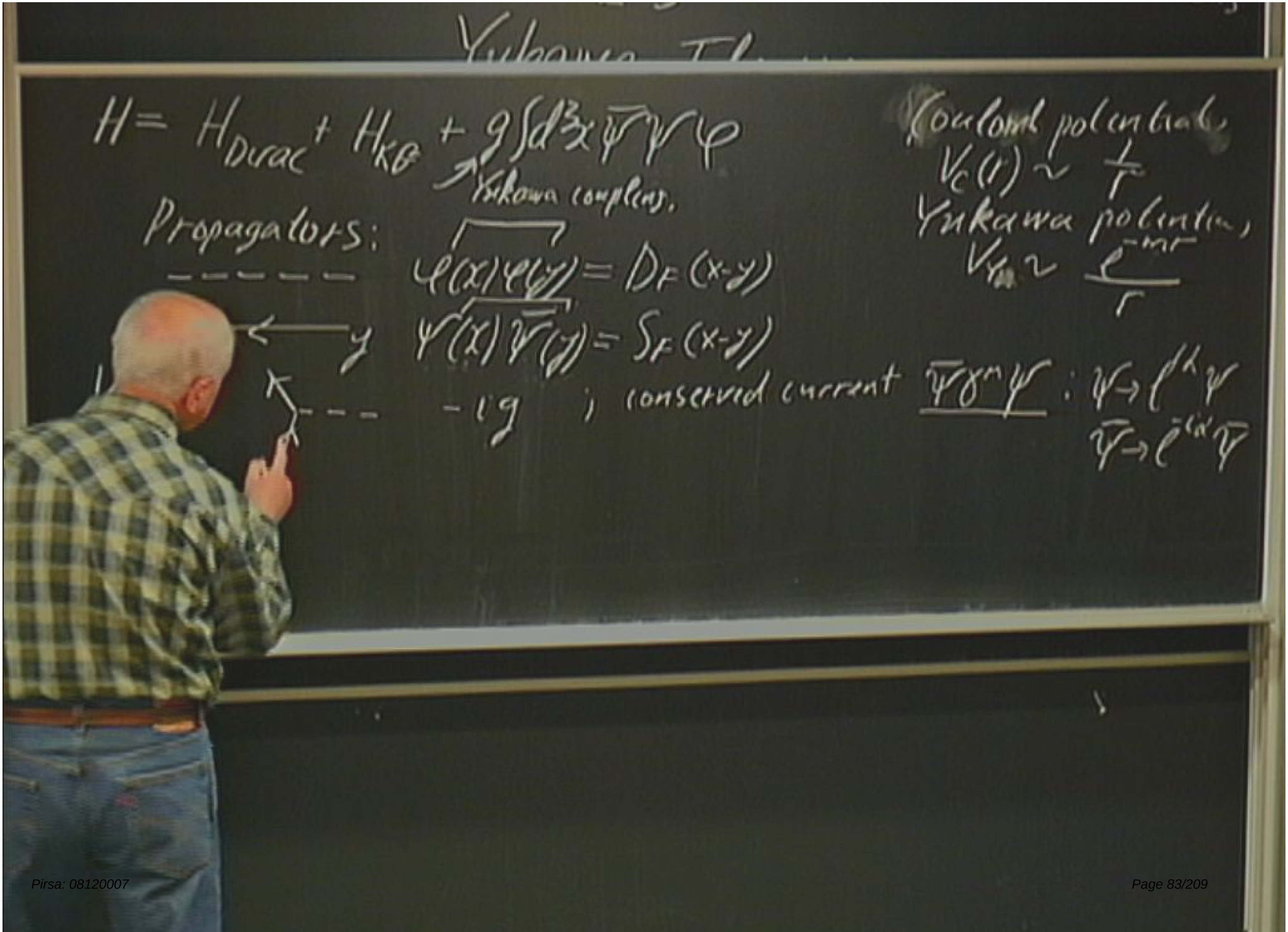

$H = H_{Dirac} + H_{KG} + g\int d^3x\, \bar{\psi}\psi\varphi$
Yukawa couplings.
Propagators:
$\varphi(x)\varphi(y) = D_F(x-y)$
x y $\psi(x)\bar{\psi}(y) = S_F(x-y)$
Vertex $-ig$
Coulomb potential,
$V_C(r) \sim \frac{1}{r}$
Yukawa potential,
$V_{Yu} \sim \frac{e^{-mr}}{r}$



$H = H_{Dirac} + H_{KG} + g\int d^3x\, \bar{\psi}\psi\varphi$
Yukawa coupling.
Propagators:
$\varphi(x)\varphi(y) = D_F(x-y)$
x
y
$\psi(x)\bar{\psi}(y) = S_F(x-y)$
Vertex
$-ig$
Coulomb potential,
$V_C(r) \sim \frac{1}{r}$
Yukawa potential,
$V_{Yu} \sim \frac{e^{-mr}}{r}$


$H = H_{Dirac} + H_{KG} + g\int d^3x\, \bar{\psi}\psi\varphi$
Yukawa coupling
Propagators:
$\varphi(x)\varphi(y) = D_F(x-y)$
x y
$\psi(x)\bar{\psi}(y) = S_F(x-y)$
Vertex
ig ; conserved current $\bar{\psi}\gamma^\mu\psi$
Coulomb potential,
$V_C(r) \sim \frac{1}{r}$
Yukawa potential,
$V_Y \sim \frac{e^{-mr}}{r}$


Yukawa Theory
H = H_Dirac + H_KG + g ∫d³x ψ̄ ψ φ
Yukawa coupling.
Propagators:
φ(x)φ(y) = D_F(x-y)
x y
ψ(x)ψ̄(y) = S_F(x-y)
Vertex
-ig
conserved
ψ̄γ^μψ
Coulomb potential,
V_C(r) ~ 1/r
Yukawa potential,
V_Y ~ e^{-mr}/r


Yukawa Theory
H = H_Dirac + H_KG + g ∫d³x ψ̄ψφ
Yukawa coupling
Propagators:
φ(x)φ(y) = D_F(x-y)
ψ(x)ψ̄(y) = S_F(x-y)
-ig ; conserved current ψ̄γ^μψ : ψ → e^{iα}ψ
ψ̄ → e^{-iα}ψ̄
Coulomb potential,
V_C(r) ~ 1/r
Yukawa potential,
V_Yuk ~ e^{-mr}/r

Yukawa Theory

$$H = H_{Dirac} + H_{KG} + g\int d^3x\, \bar{\psi}\psi\varphi$$

Yukawa couplings.

Propagators:

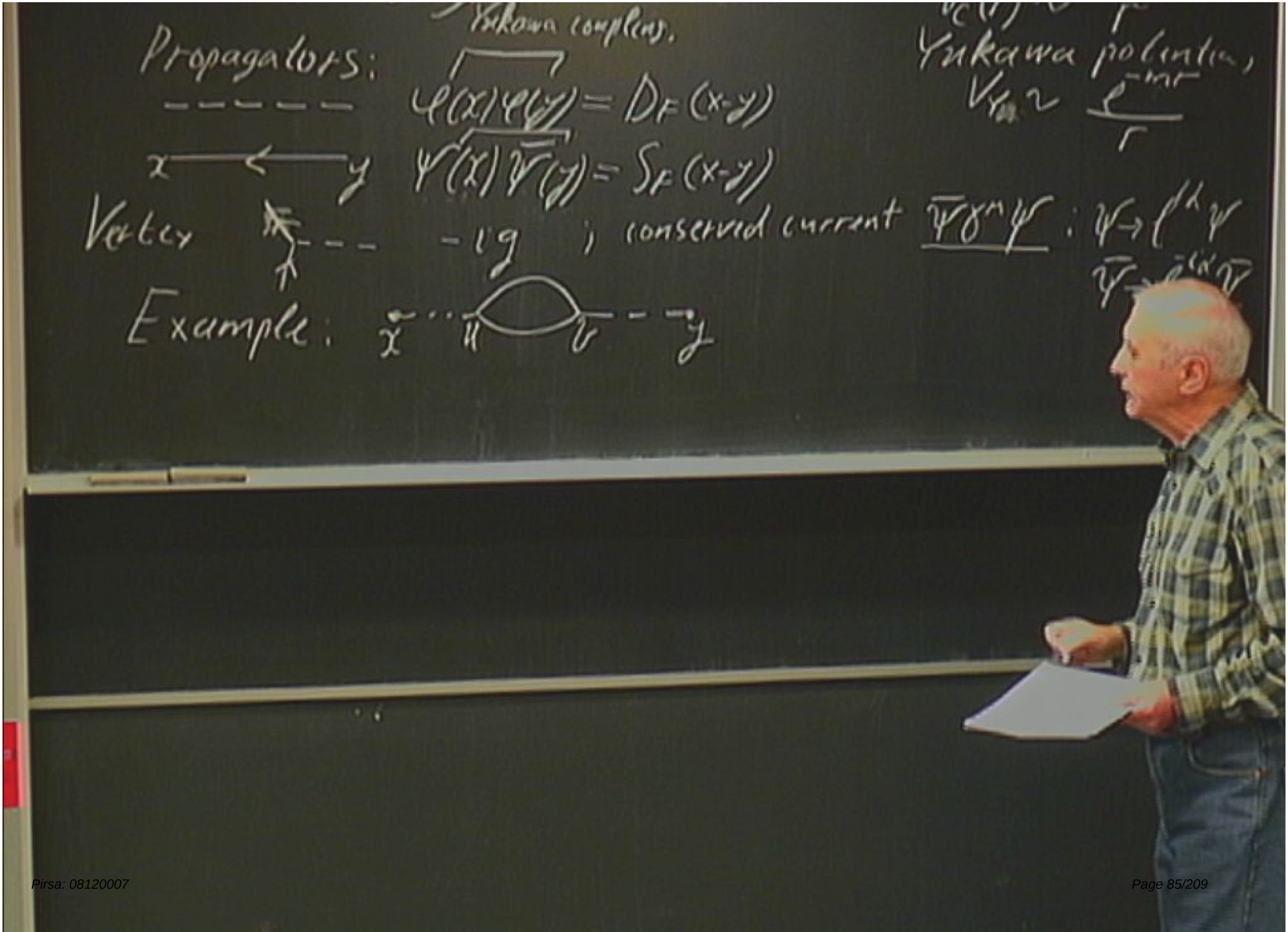
- - - - - - - $\varphi(x)\varphi(y) = D_F(x-y)$

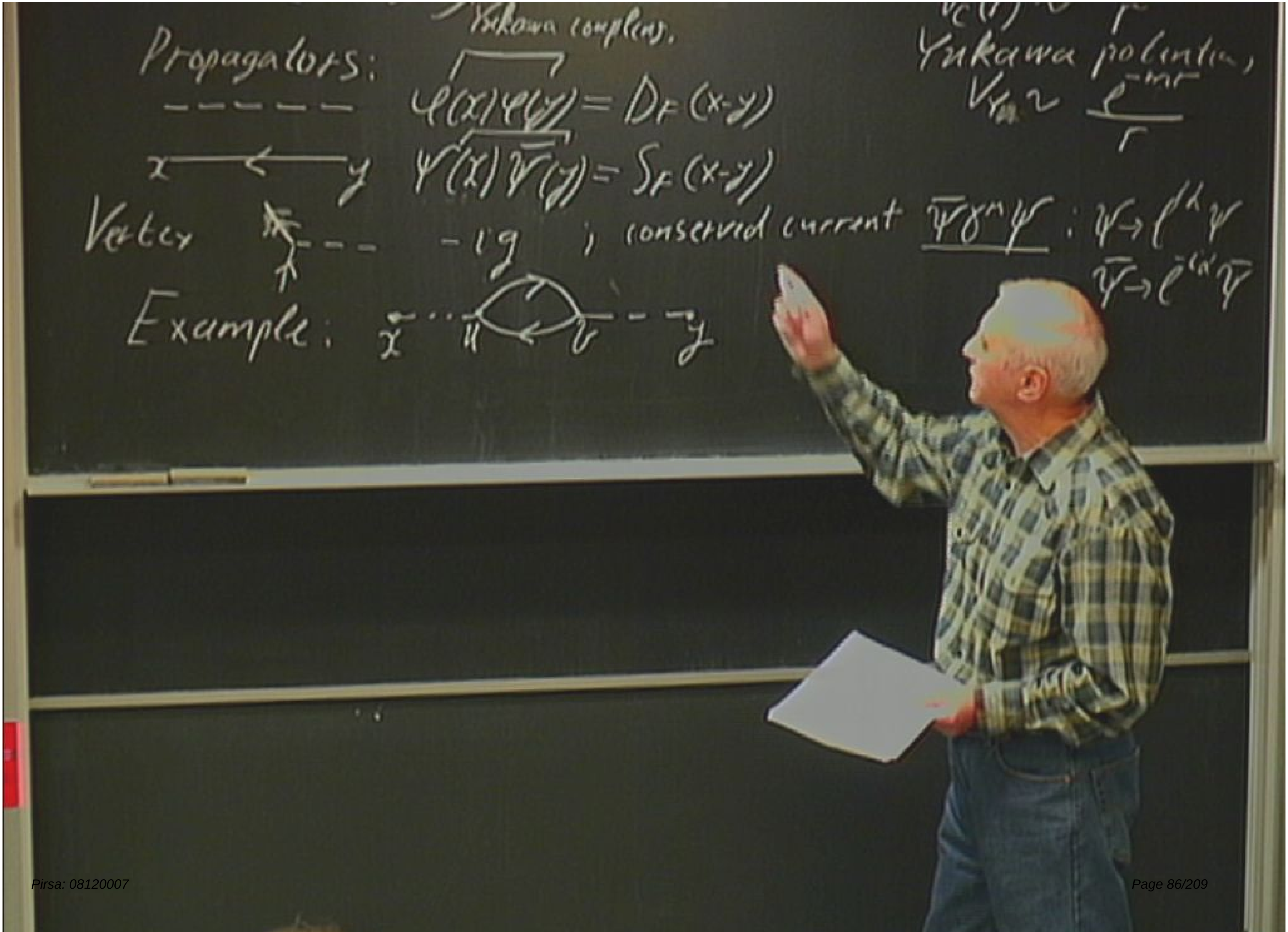
$x \longleftarrow y$ $\psi(x)\bar{\psi}(y) = S_F(x-y)$

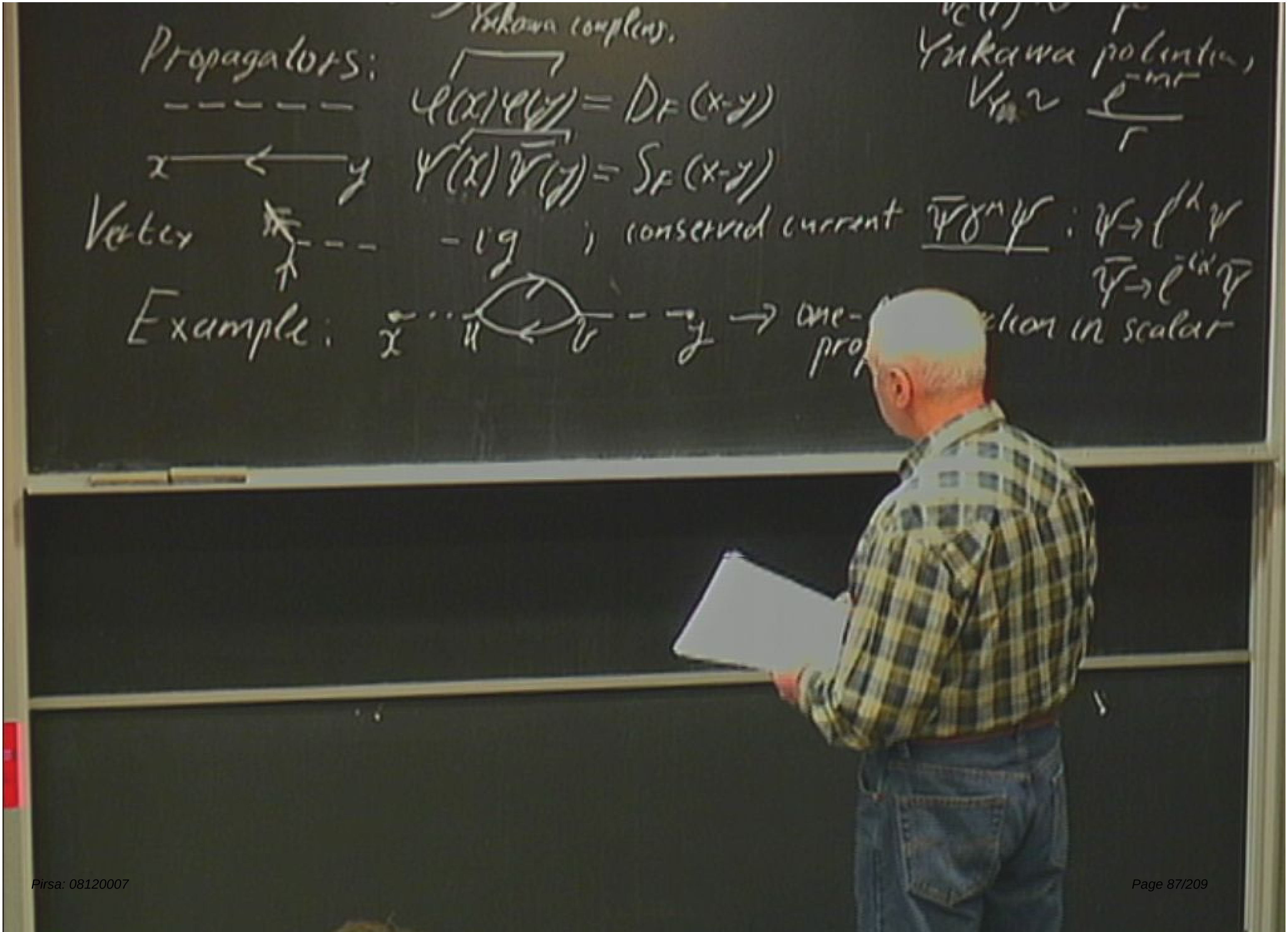
Vertex $-ig$; conserved current $\underline{\bar{\psi}\gamma^\mu\psi}$: $\psi \to e^{i\alpha}\psi$, $\bar{\psi} \to e^{-i\alpha}\bar{\psi}$

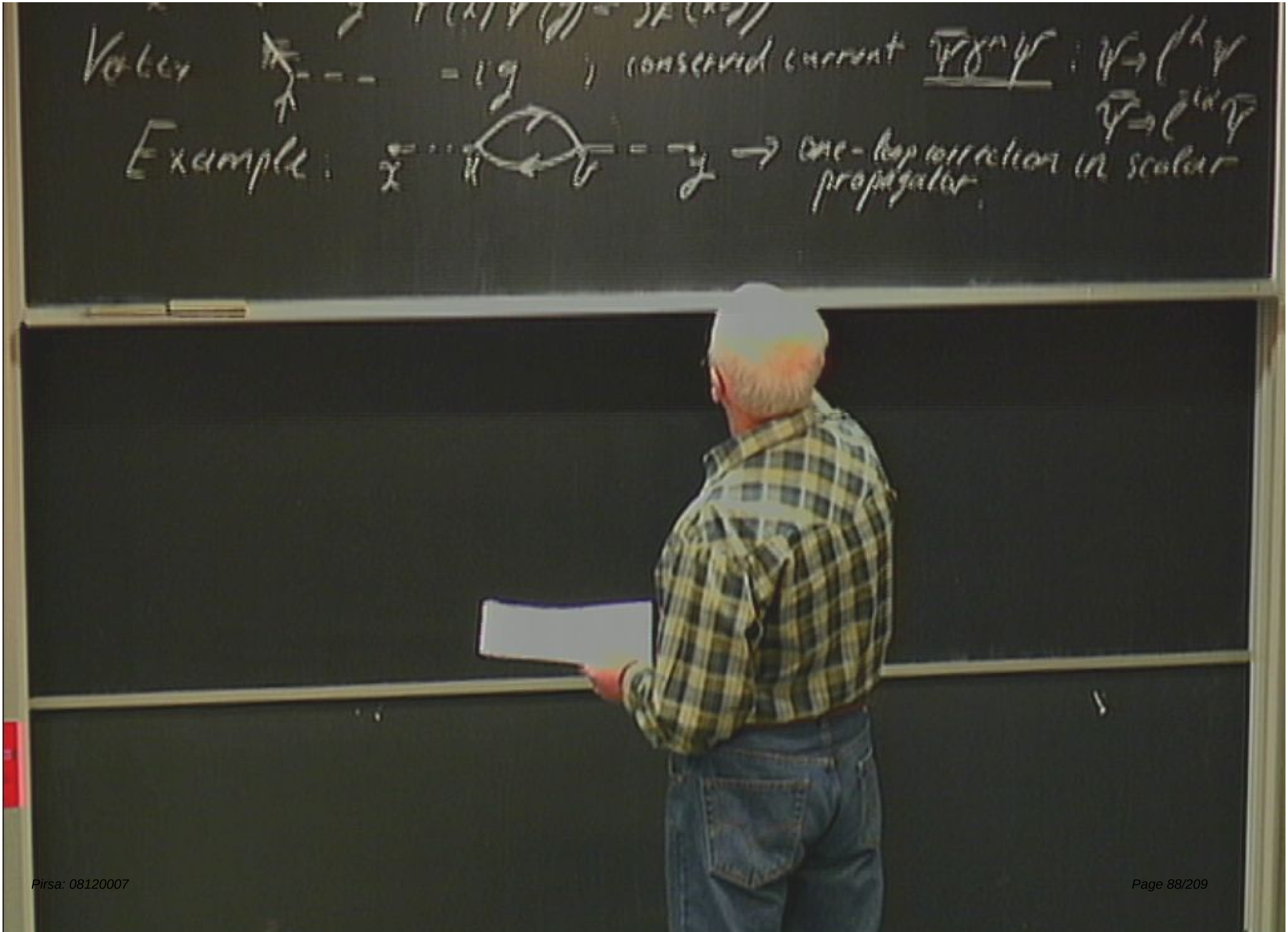
Coulomb potential, $V_C(r) \sim \frac{1}{r}$

Yukawa potential, $V_{Yu} \sim \frac{e^{-mr}}{r}$


Yukawa couplings.
Propagators:
$\varphi(x)\varphi(y) = D_F(x-y)$
$\psi(x)\bar{\psi}(y) = S_F(x-y)$
Yukawa potential,
$V_Y \sim \frac{e^{-mr}}{r}$
Vertex
$-ig$
; conserved current $\bar{\psi}\gamma^\mu\psi$: $\psi \to e^{i\lambda}\psi$
$\bar{\psi} \to e^{-i\lambda}\bar{\psi}$
Example:
x
u
v
y


Yukawa complex.
Propagators:
φ(x)φ(y) = D_F(x-y)
ψ(x)ψ̄(y) = S_F(x-y)
Yukawa potential,
Vertex
-ig
; conserved current
Example:

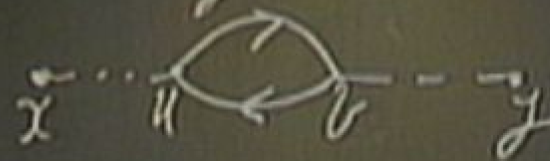

Yukawa couplings.
Propagators:
$\varphi(x)\varphi(y) = D_F(x-y)$
$\psi(x)\bar{\psi}(y) = S_F(x-y)$
Vertex
$-ig$
; conserved current $\bar{\psi}\gamma^\mu\psi$: $\psi \to e^{i\alpha}\psi$
$\bar{\psi} \to e^{-i\alpha}\bar{\psi}$
Example:
in scalar
Yukawa potential,
$V_Y \sim \frac{e^{-mr}}{r}$



Vertex
$-ig$; conserved current $\bar{\psi}\gamma^\mu\psi$: $\psi \to e^{i\lambda}\psi$
$\bar{\psi} \to e^{-i\lambda}\bar{\psi}$
Example:
→ one-loop correction in scalar propagator.

Vertex  $-ig$; conserved current $\underline{\bar{\psi}\gamma^{\mu}\psi}$: $\psi \to e^{i\alpha}\psi$

$\bar{\psi} \to e^{-i\alpha}\bar{\psi}$

Example:  $\to$ one-loop correction in scalar propagator.

$$F = \frac{1}{2!}(-ig)^2 \int d^4u\, d^4v \, \langle 0| \varphi(x)\varphi(y)$$

$$F = \frac{1}{2!}(-ig)^2 \int d^4u\, d^4v \, \langle 0|\, \varphi(x)\varphi(y)\, \bar\psi\psi\varphi(u)\, \bar\psi\psi\varphi(v)\, |0\rangle$$

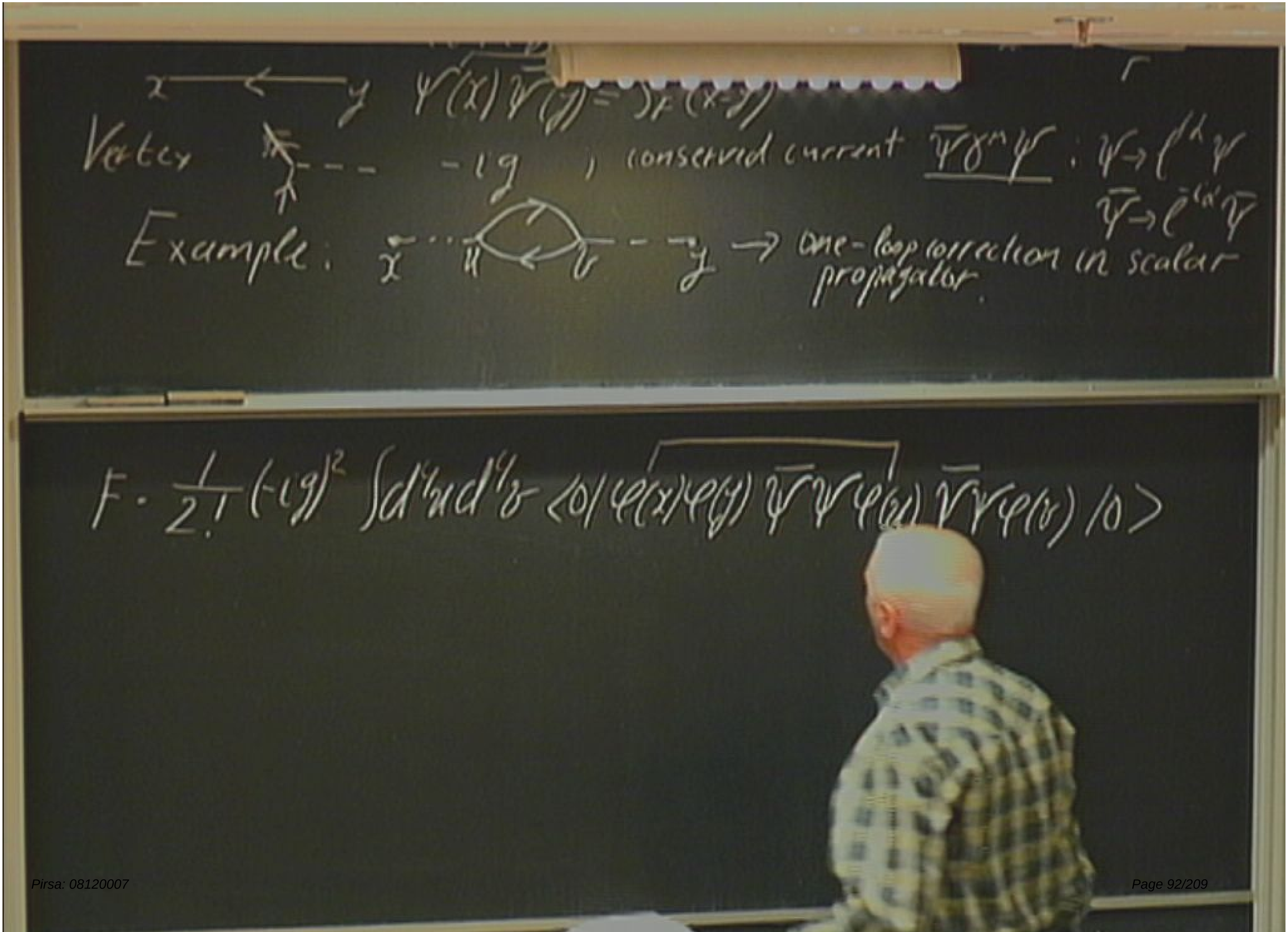
Define now contractions under symbol N to include minus sign for operator interchanges: $N(\psi_1 \psi_2 \bar{\psi}_3 \bar{\psi}_4) =$

$$= -N(\psi_1 \bar{\psi}_3 \psi_2 \bar{\psi}_4) = -\psi_1 \bar{\psi}_3 \, N(\psi_2 \bar{\psi}_4) = -S_F(x_1 - x_3)\, N(\psi_2 \bar{\psi}_4)$$

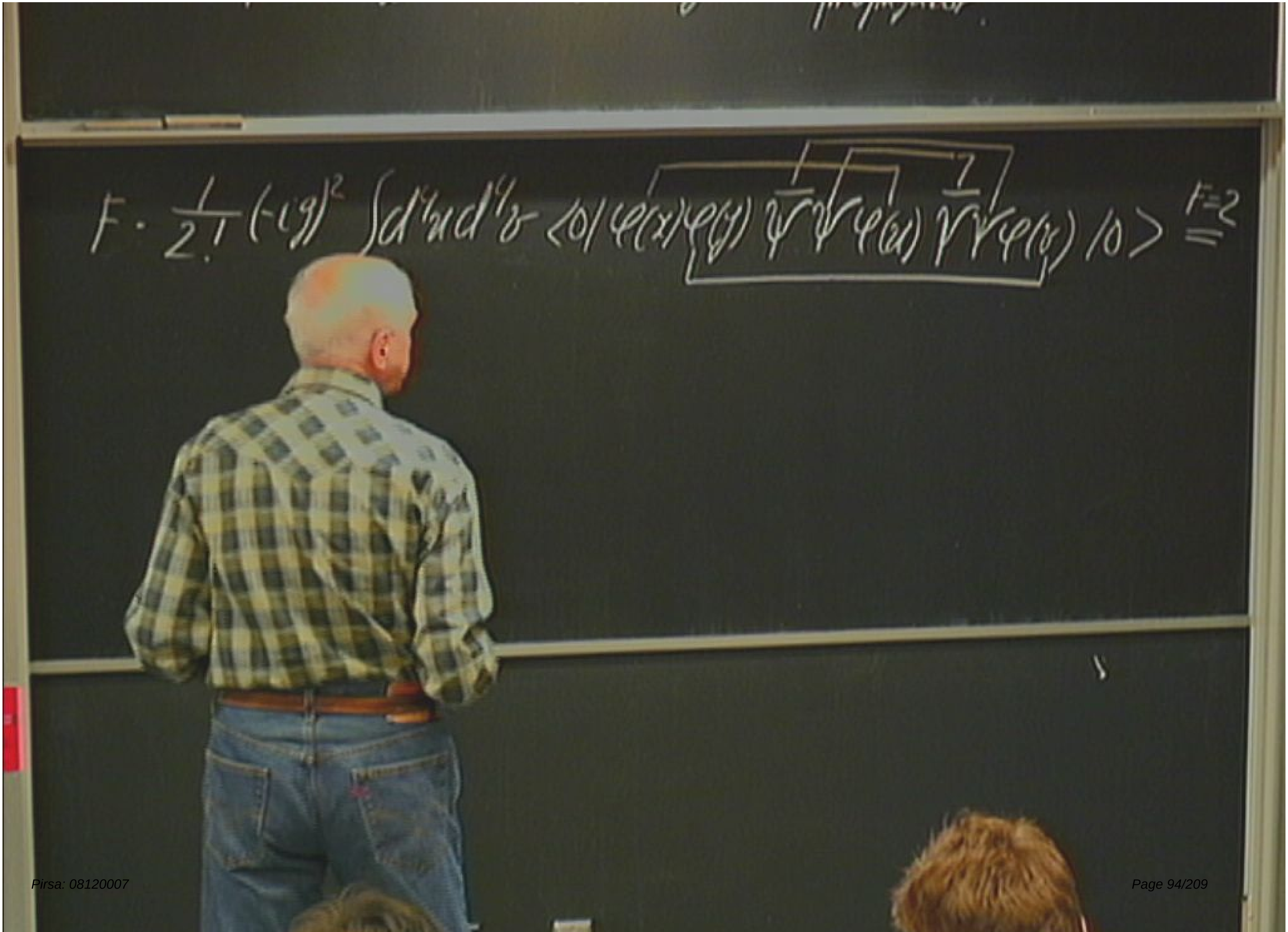
Wick's theorem for fermions:

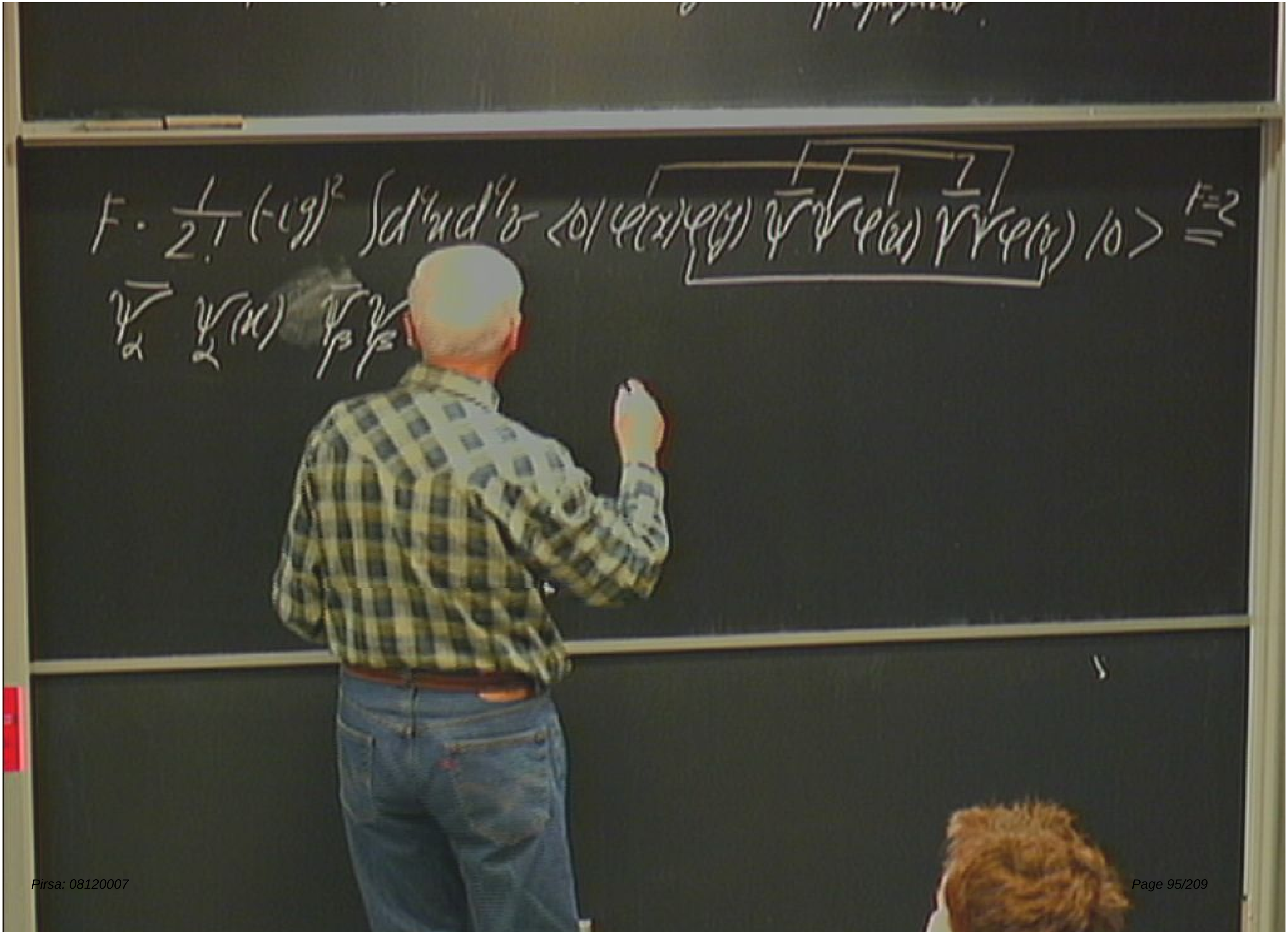
$$T[\psi_1 \bar{\psi}_2 \psi_3 \ldots\ldots] = N[\psi_1 \bar{\psi}_2 \psi_3 \ldots\ldots] + \text{all possible cont}$$

Yukawa Theory


Vertex
-ig
conserved current
Example:
one-loop correction in scalar propagator.

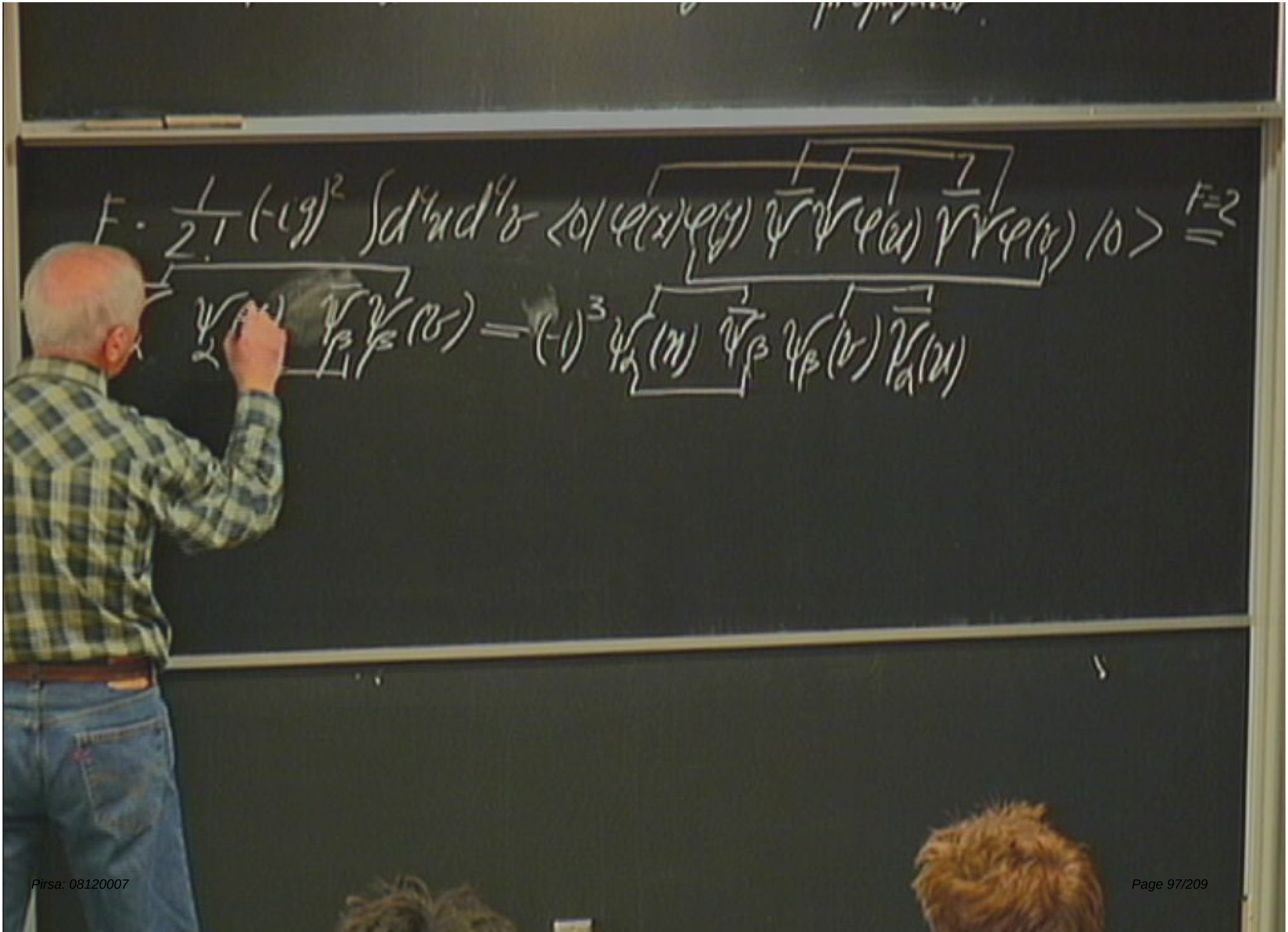
$$F = \frac{1}{2!}(-ig)^2 \int d^4w\, d^4z \langle 0|\varphi(x)\varphi(y)\,\bar\psi\psi\varphi(w)\,\bar\psi\psi\varphi(z)\,|0\rangle$$

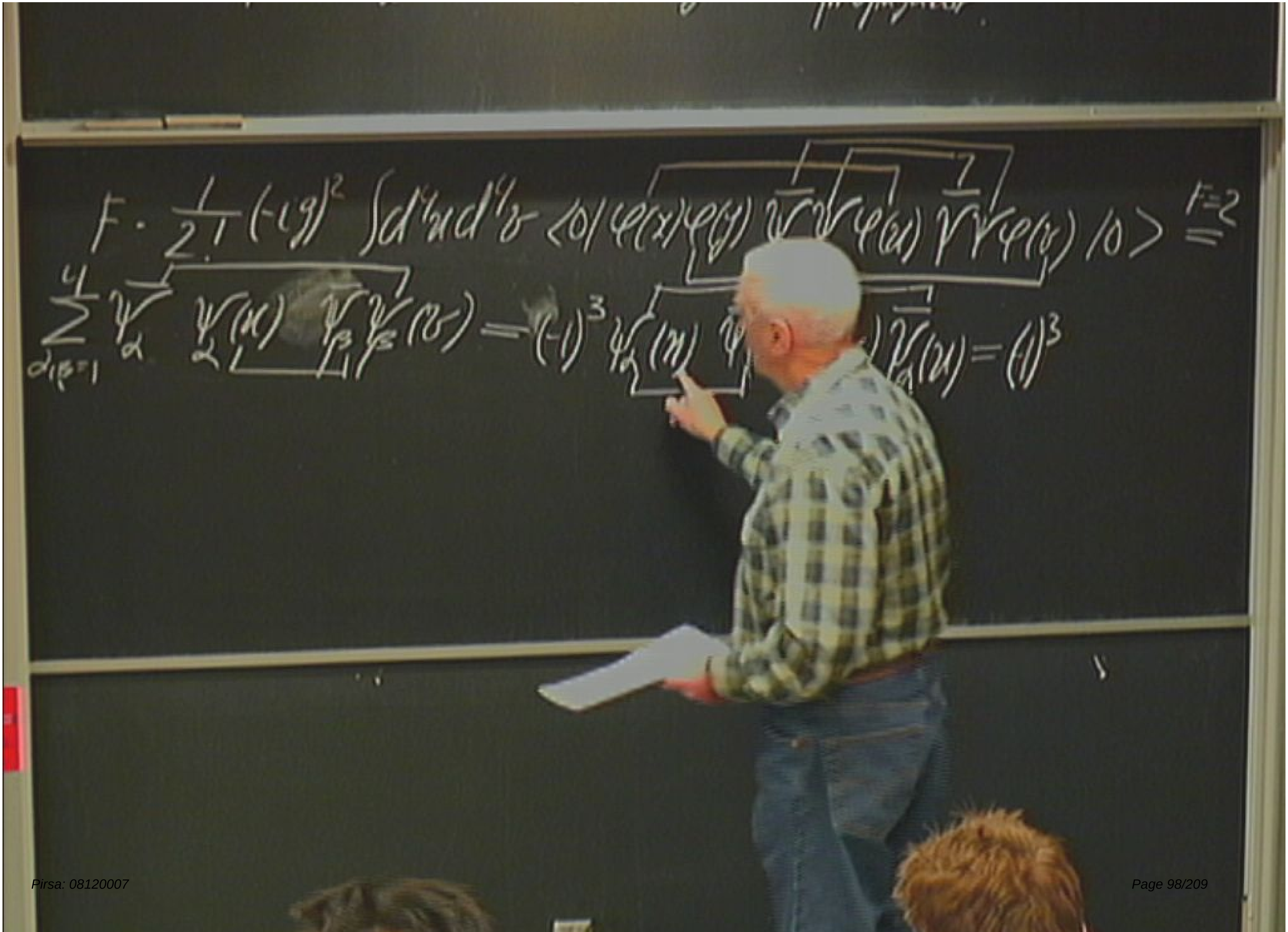

F = 1/2! (-ig)^2 ∫d^4x d^4y <0| φ(x)φ(y) ψ̄ψφ(u) ψ̄ψφ(v) |0> F=2 =


F = 1/2! (-ig)^2 ∫d^4x d^4z ⟨0| φ(x) φ(y) ψ̄ ψ φ(x) ψ̄ ψ φ(z) |0⟩ F=2 =
ψ̄_α ψ_α(x) ψ̄_β ψ_β

$$F \cdot \frac{1}{2!}(-ig)^2 \int d^4x\, d^4y\, \langle 0|\, \varphi(x)\varphi(y)\, \bar{\psi}\psi\varphi(x)\, \bar{\psi}\psi\varphi(y)\, |0\rangle \overset{F=2}{=}$$

$$\sum_{\alpha,\beta=1}^{4} \bar{\psi}_\alpha \;\; \psi_\alpha(x) \;\; \bar{\psi}_\beta \psi_\beta(y) = -(-1)^3 \psi_\alpha(x)$$


F = 1/2! (-ig)^2 ∫d^4x d^4y ⟨0| φ(x) φ(y) ψ̄ψ φ(x) ψ̄ψ φ(y) |0⟩ F=2 =
ψ_α(x) ψ̄_β ψ_β(y) = −(−1)^3 ψ_α(x) ψ̄_β ψ_β(y) ψ̄_α(x)



$$F = \frac{1}{2!}(-ig)^2 \int d^4x\, d^4z \, \langle 0 | \varphi(x)\varphi(y)\, \bar{\psi}\psi\varphi(u)\, \bar{\psi}\psi\varphi(v) | 0 \rangle \overset{F=2}{=}$$

$$\sum_{\alpha,\beta=1}^{4} \bar{\psi}_\alpha \, \psi_\alpha(u) \, \bar{\psi}_\beta \psi_\beta(v) = -(-1)^3 \psi_\alpha(u) \, \bar{\psi}_\beta \, \psi_\beta(v) \, \bar{\psi}_\alpha(u) = (-1)^3$$

$$F = \frac{1}{2!}(-ig)^2 \int d^4x\, d^4y \, \langle 0|\, \varphi(x)\varphi(y)\, \bar{\psi}\psi\varphi(x)\, \bar{\psi}\psi\varphi(y)\, |0\rangle \overset{F=2}{=}$$

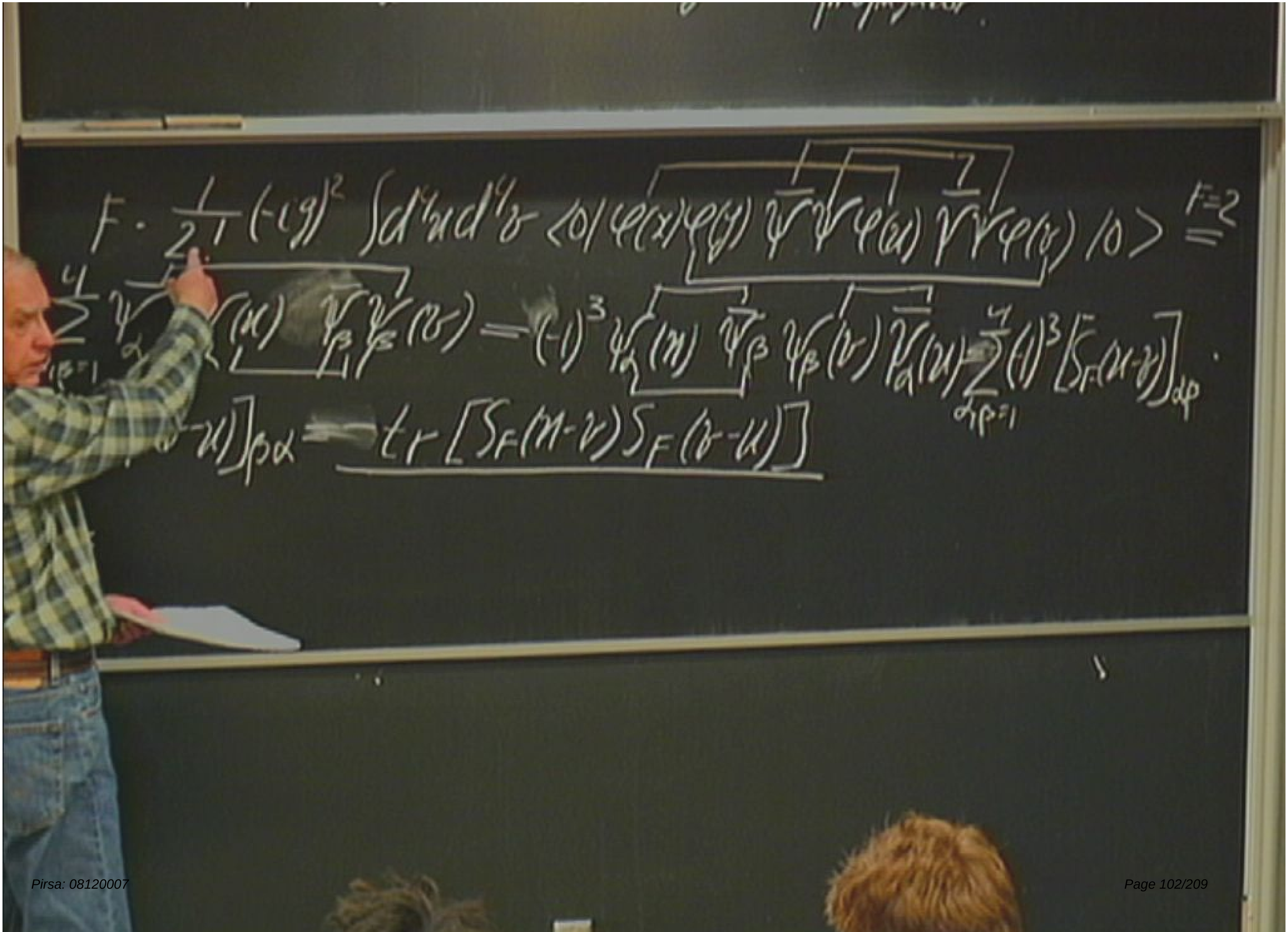
$$\sum_{\alpha,\beta=1}^{4} \bar{\psi}_\alpha \psi_\alpha(x)\, \bar{\psi}_\beta \psi_\beta(y) = -(-1)^3 \psi_\alpha(x)\, \bar{\psi}_\beta\, \psi_\beta(y)\, \bar{\psi}_\alpha(x) = \sum_{\alpha,\beta=1}^{4} (-1)^3 [S_F(x-y)]_{\alpha\beta} \cdot$$

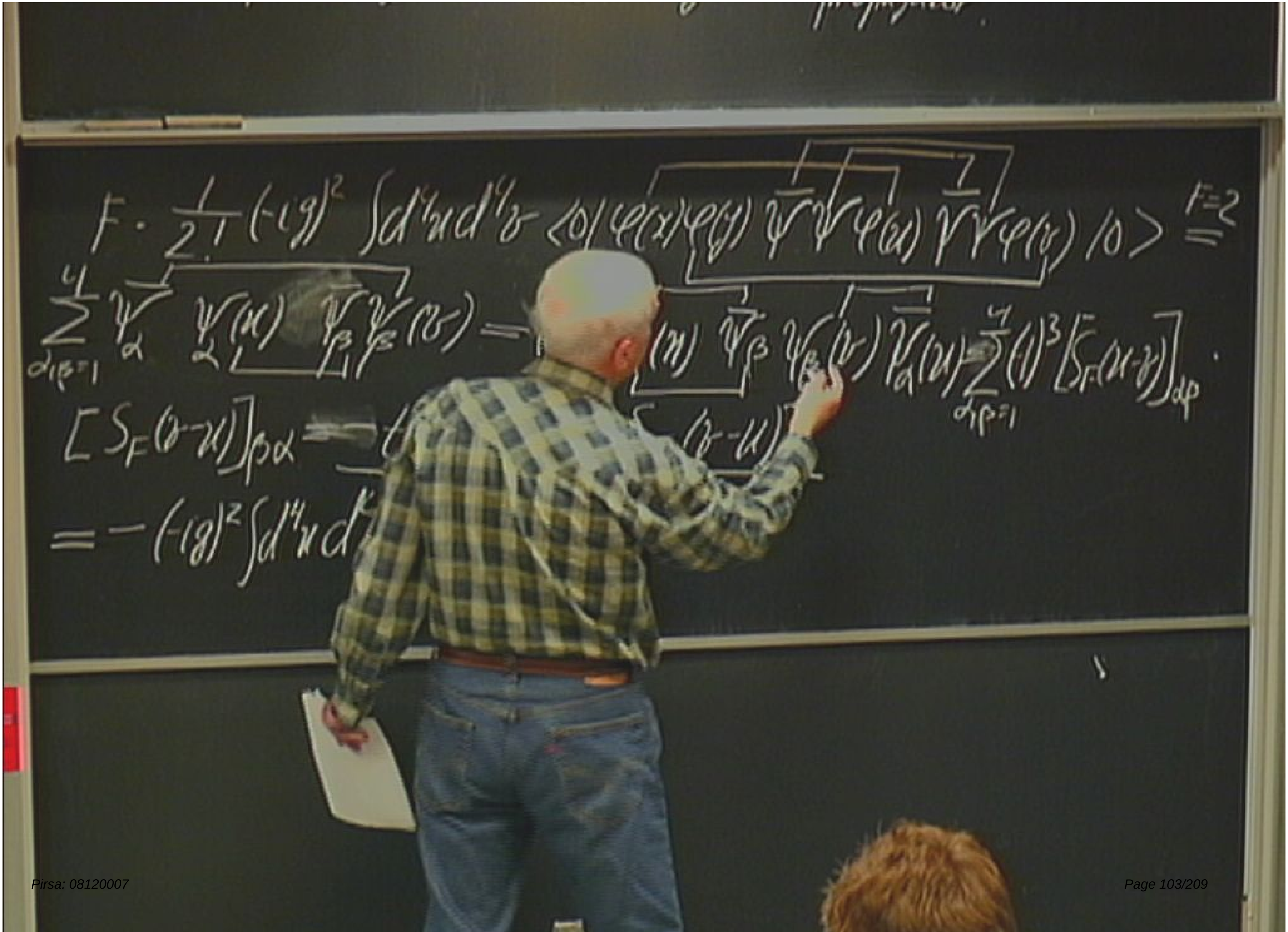
$$[S_F(y-x)]_{\beta\alpha}$$

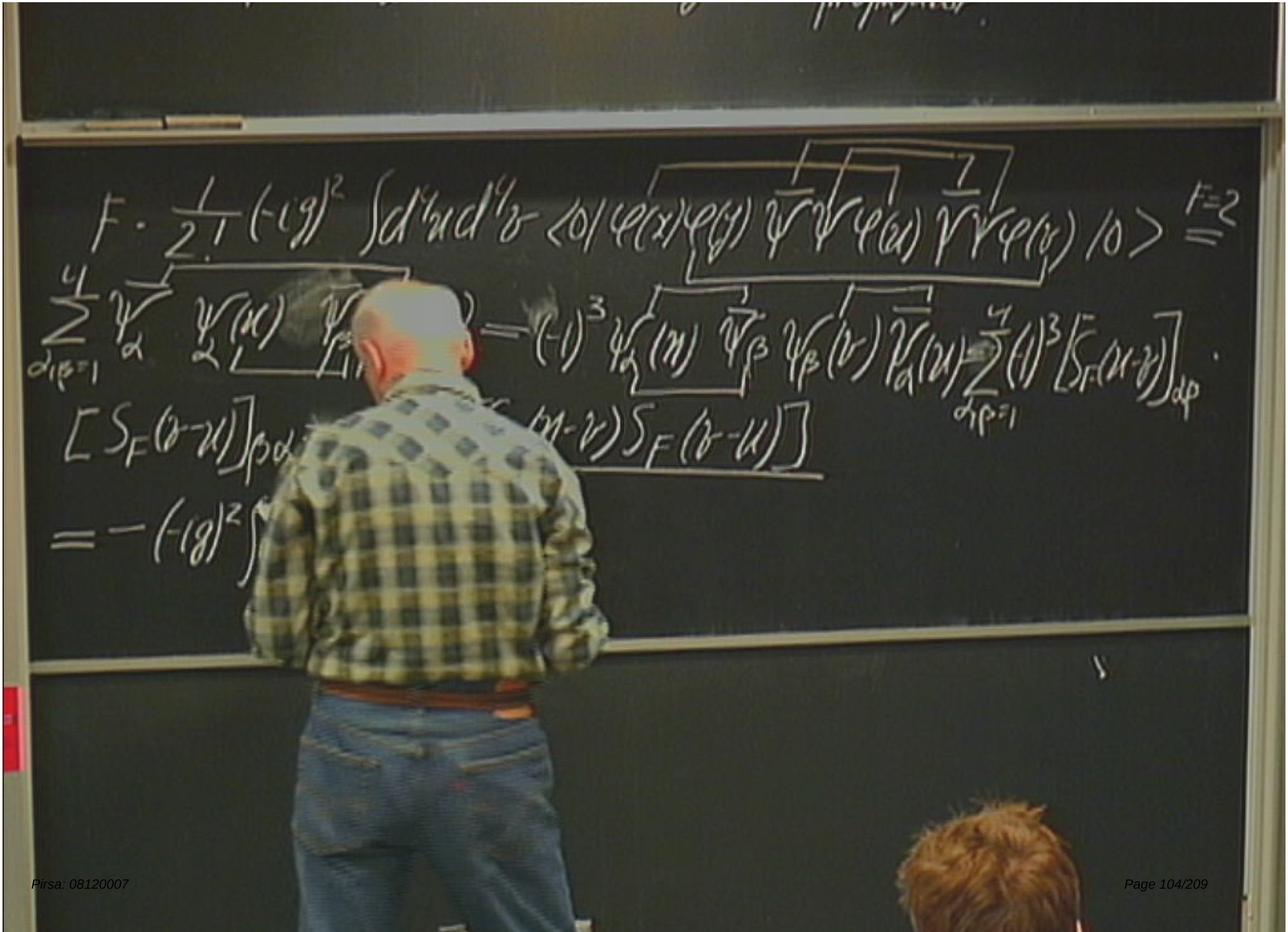
$$F = \frac{1}{2!}(-ig)^2 \int d^4x\, d^4y \,\langle 0|\, \varphi(x)\varphi(y)\, \bar{\psi}\psi\varphi(x)\, \bar{\psi}\psi\varphi(y)\, |0\rangle \overset{F=2}{=}$$

$$\sum_{\alpha,\beta=1}^{4} \bar{\psi}_\alpha \psi_\alpha(x)\, \bar{\psi}_\beta \psi_\beta(y) = -(-1)^3 \psi_\alpha(x)\, \bar{\psi}_\beta\, \psi_\beta(y)\, \bar{\psi}_\alpha(x) = \sum_{\alpha,\beta=1}^{4} (-1)^3 [S_F \ldots$$

$$[S_F(y-x)]_{\beta\alpha} = -\mathrm{tr}\left[S_F(x-y)\, S_F(y-x)\right]$$


$F = \frac{1}{2!}(-ig)^2 \int d^4u\, d^4v \langle 0| \varphi(x)\varphi(y)\, \bar{\psi}\psi\varphi(u)\, \bar{\psi}\psi\varphi(v) |0\rangle \overset{F=2}{=}$
$\sum_{\alpha,\beta=1}^{4} \bar{\psi}_\alpha \psi_\alpha(u)\, \bar{\psi}_\beta \psi_\beta(v) = -(-1)^3 \psi_\alpha(u)\, \bar{\psi}_\beta\, \psi_\beta(v)\, \bar{\psi}_\alpha(u) = \sum_{\alpha,\beta=1}^{4} (-1)^3 [S_F(u-v)]_{\alpha\beta} \cdot$
$[S_F(v-u)]_{\beta\alpha} = -\mathrm{tr}[S_F(u-v) S_F(v-u)]$


F = 1/2! (-ig)^2 ∫d^4x d^4y ⟨0| φ(x) φ(y) ψ̄ ψ φ(x) ψ̄ ψ φ(y) |0⟩ F=2 =
[S_F(y-x)]_{βα}
= −(−ig)^2 ∫d^4x d^4


$F = \frac{1}{2!}(-ig)^2 \int d^4u\, d^4v \langle 0| \varphi(x)\varphi(y)\, \bar\psi\psi\varphi(u)\, \bar\psi\psi\varphi(v) |0\rangle \overset{F=2}{=}$
$\sum_{\alpha,\beta=1}^{4} \bar\psi_\alpha \psi_\alpha(u) \bar\psi$
$= -(-1)^3 \psi_\alpha(u) \bar\psi_\beta \psi_\beta(v) \bar\psi_\alpha(u) = \sum_{\alpha\beta=1}^{4} (-1)^3 [S_F(u-v)]_{\alpha\beta} \cdot$
$[S_F(v-u)]_{\beta\alpha}$
$(u-v) S_F(v-u)]$
$= -(-ig)^2 \int$

$$F \cdot \frac{1}{2!}(-ig)^2 \int d^4u\, d^4v \langle 0| \varphi(x)\varphi(y)\, \bar\psi\psi\varphi(u)\, \bar\psi\psi\varphi(v) |0\rangle \overset{F=2}{=}$$

$$\sum_{\alpha,\beta=1}^{4} \bar\psi_\alpha\; \psi_\alpha(u)\; \bar\psi_\beta\psi_\beta(v) = -(-1)^3 \psi_\alpha(u)\; \bar\psi_\beta\; \psi_\beta(v)\, \bar\psi_\alpha(u) \sum_{\alpha,\beta=1}^{4} (-1)^3 [S_F(u-v)]_{\alpha\beta} \cdot$$

$$[S_F(v-u)]_{\beta\alpha} = -\underline{tr\,[S_F(u-v)S_F(v-u)]}$$

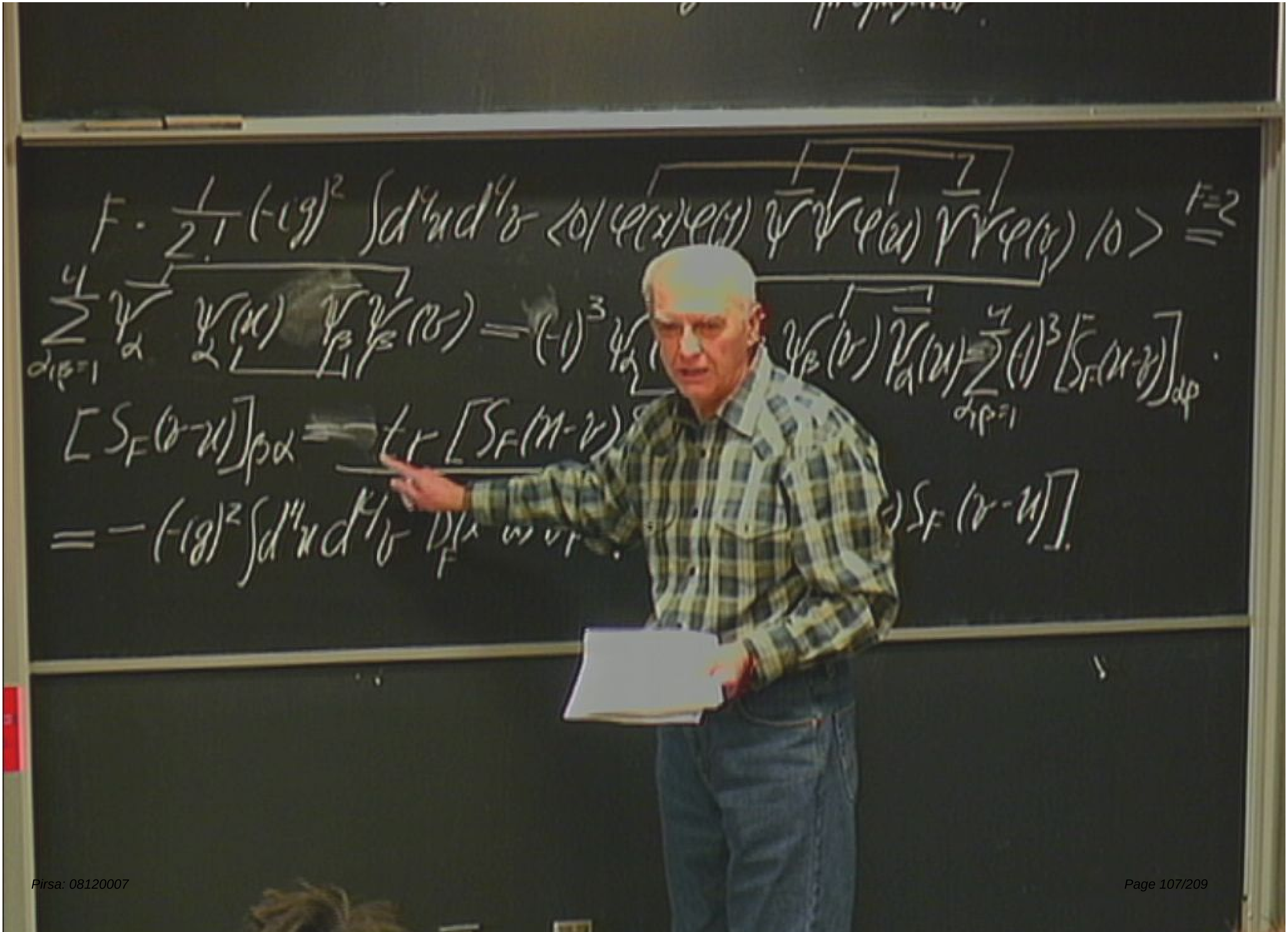
$$= -(-ig)^2 \int d^4u\, d^4v\; D_F(x-u)\, D_F(y-v)\, tr\,[S_F(u-v)\, S_F(v-u)].$$

$$F = \frac{1}{2!}(-ig)^2 \int d^4u\, d^4v \,\langle 0|\varphi(x)\varphi(y)\,\bar\psi\psi\varphi(u)\,\bar\psi\psi\varphi(v)\,|0\rangle \overset{F=2}{=}$$

$$\sum_{\alpha,\beta=1}^{4} \bar\psi_\alpha \psi_\alpha(u)\, \bar\psi_\beta \psi_\beta(v) = -(-1)^3 \psi_\alpha(u)\, \bar\psi_\beta\, \psi_\beta(v)\, \bar\psi_\alpha(u) = \sum_{\alpha,\beta=1}^{4}(-1)^3 [S_F(u-v)]_{\alpha\beta} \cdot$$

$$[S_F(v-u)]_{\beta\alpha} = -\underline{tr[S_F(u-v)S_F(v-u)]}$$

$$= -(-ig)^2 \int d^4u\, d^4v\, D_F(x-u) D_F(y-v)\, tr[S_F(u-v) S_F(v-u)].$$



$$F = \frac{1}{2!}(-ig)^2 \int d^4u\, d^4v \langle 0| \varphi(x)\varphi(y)\, \bar{\psi}\psi\varphi(u)\, \bar{\psi}\psi\varphi(v)|0\rangle \overset{F=2}{=}$$

$$\sum_{\alpha,\beta=1}^{4} \bar{\psi}_\alpha \psi_\alpha(u)\, \bar{\psi}_\beta \psi_\beta(v) = -(-1)^3 \psi_\alpha(u)\, \bar{\psi}_\beta\, \psi_\beta(v)\, \bar{\psi}_\alpha(u) \sum_{\alpha,\beta=1}^{4} (-1)^3 [S_F(u-v)]_{\alpha\beta} \cdot$$

$$[S_F(v-u)]_{\beta\alpha} = -\underline{tr[S_F(u-v) S_F(v-u)]}$$

$$= -(-ig)^2 \int d^4u\, d^4v\, D_F(x-u) D_F(y-v)\, tr[S_F(u-v) S_F(v-u)].$$

In a closed fermion loop, there is always a factor of (-1) and the <u>trace</u> over a product of Dirac matrices

In a closed fermion loop, there is always a factor of (-1) and the trace over a product of Dirac matrices.

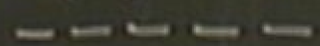
$$H = H_{Dirac} + H_{KG} + g\int d^3x\, \bar{\psi}\psi\varphi$$

↗ Yukawa coupling.

Coulomb potential,
$V_C(r) \sim \frac{1}{r}$

Yukawa potential,
$V_{Y} \sim \frac{e^{-mr}}{r}$

Propagators:

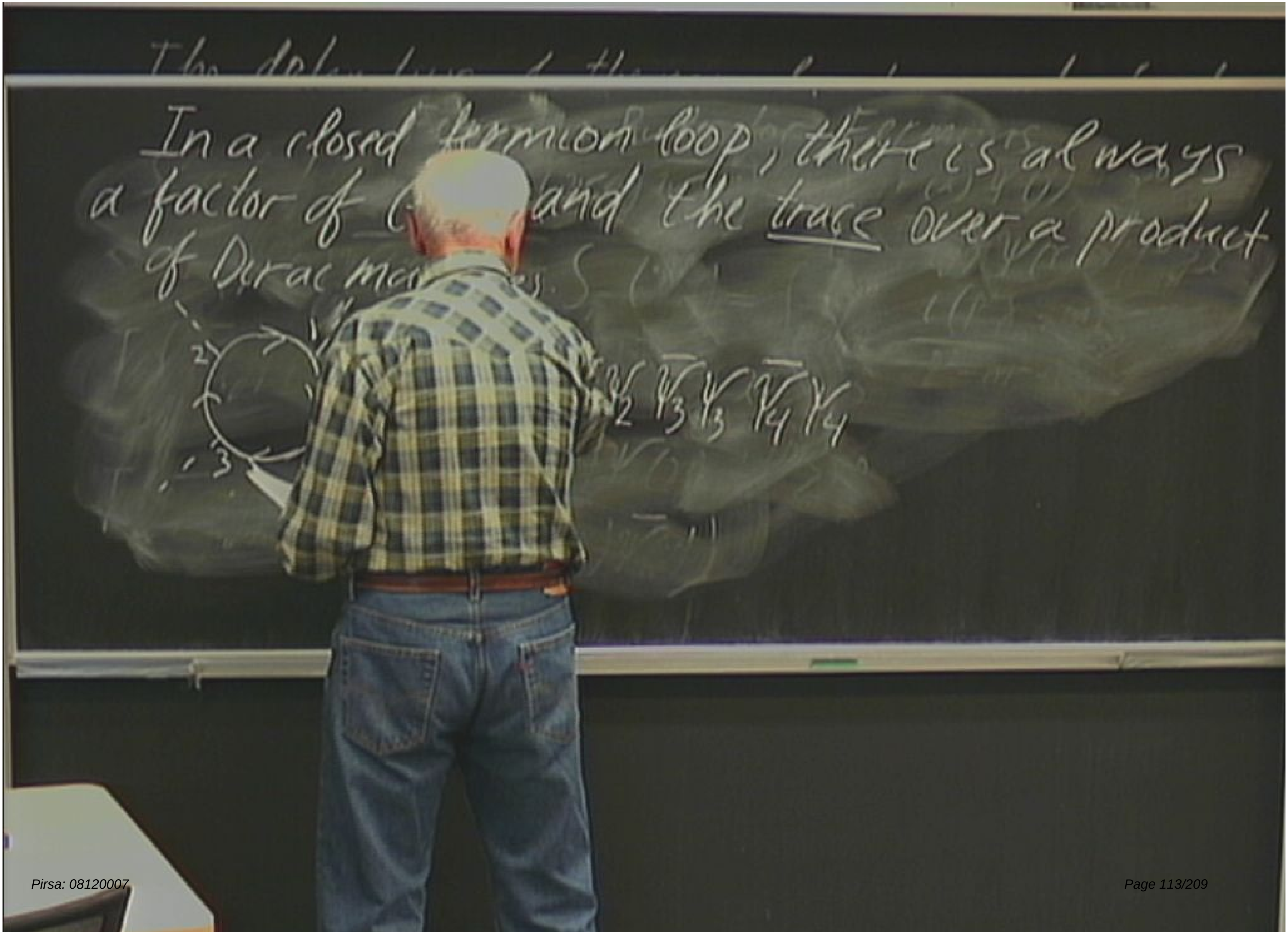
 $\varphi(x)\varphi(y) = D_F(x-y)$

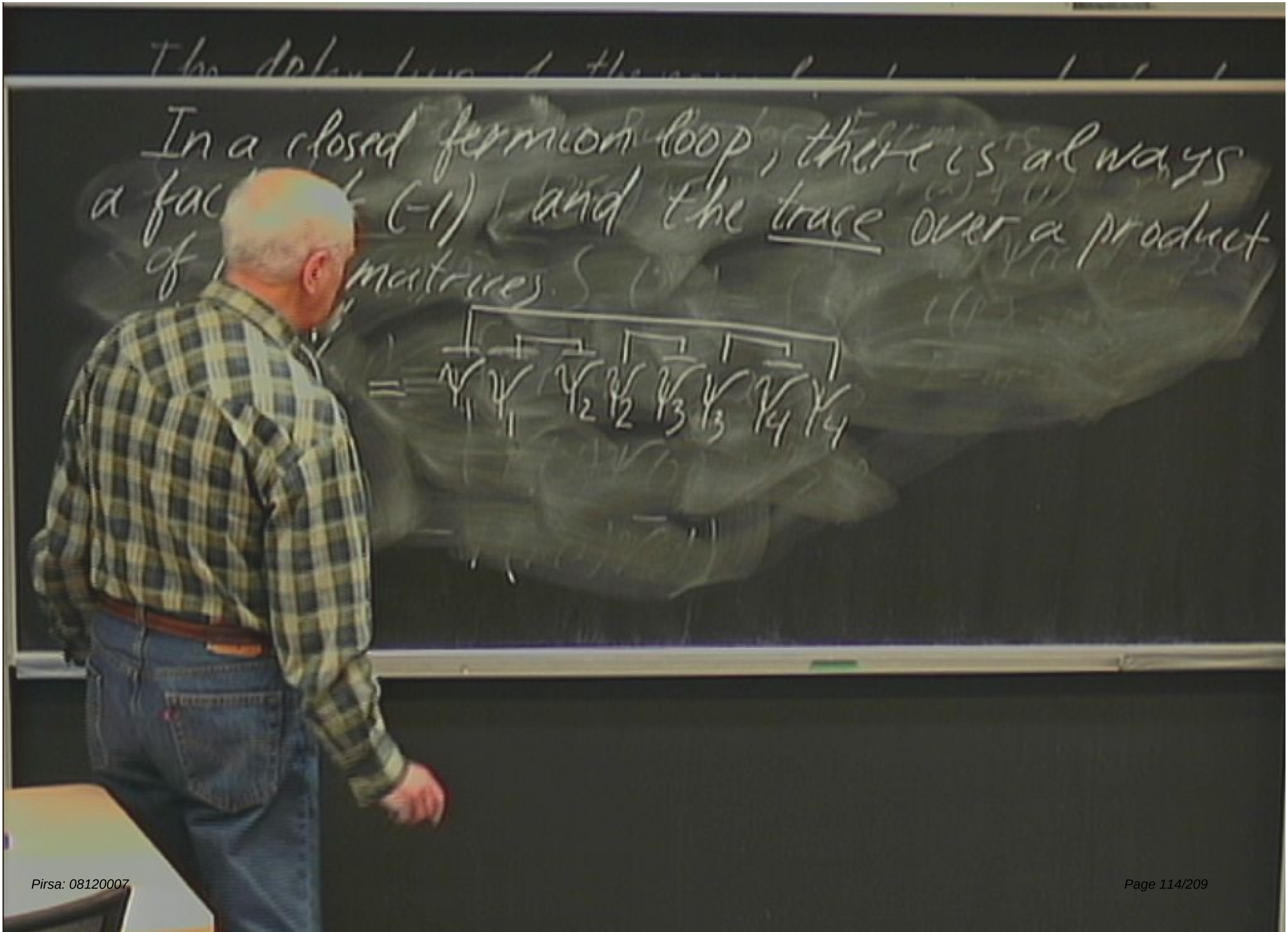
 $\psi(x)\bar{\psi}(y) = S_F(x-y)$

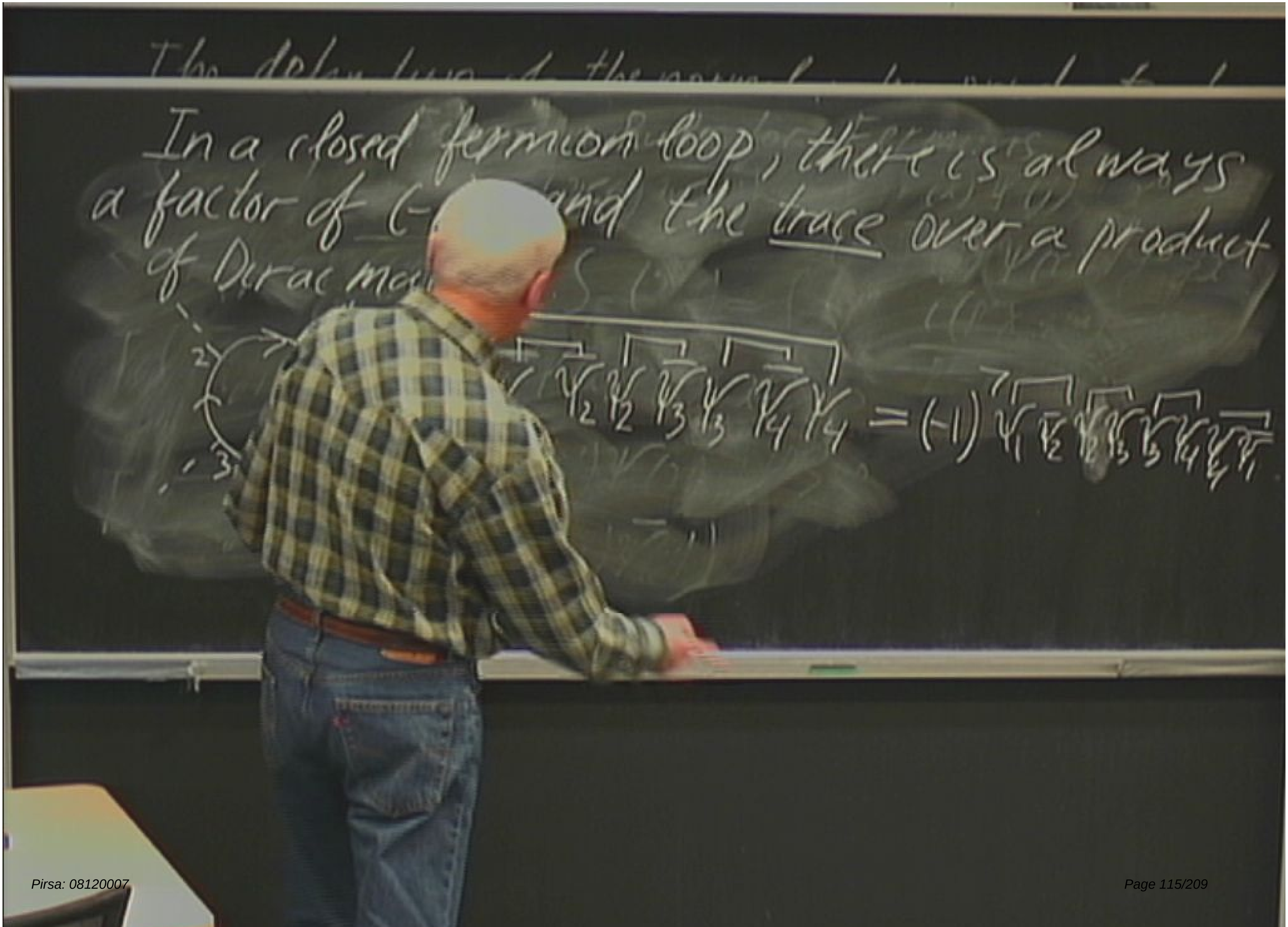
Vertex  $-ig$; conserved current $\bar{\psi}\gamma^\mu\psi$: $\psi \to e^{i\alpha}\psi$, $\bar{\psi} \to e^{-i\alpha}\bar{\psi}$

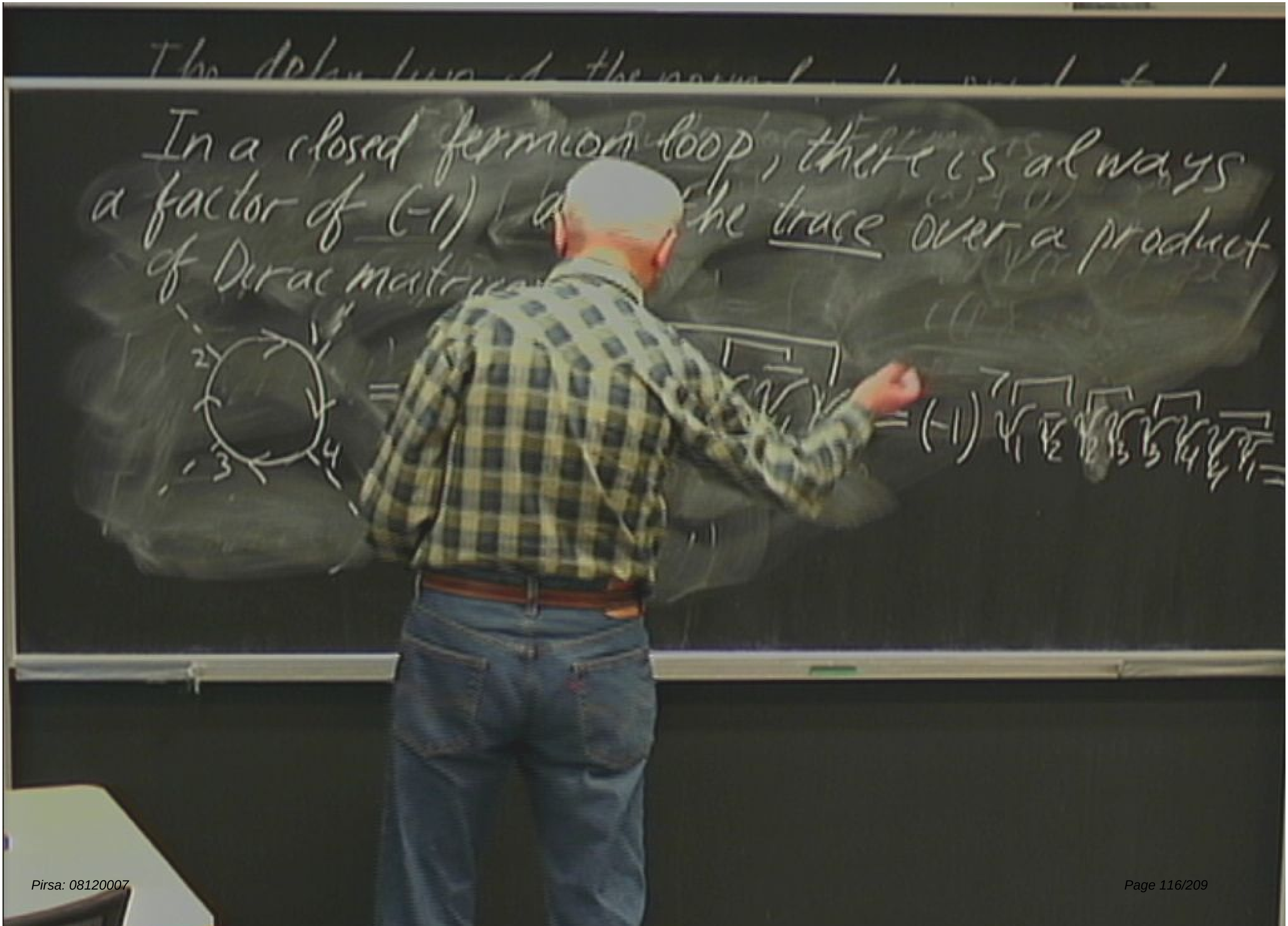
Example:  → one-loop correction in scalar propagator.

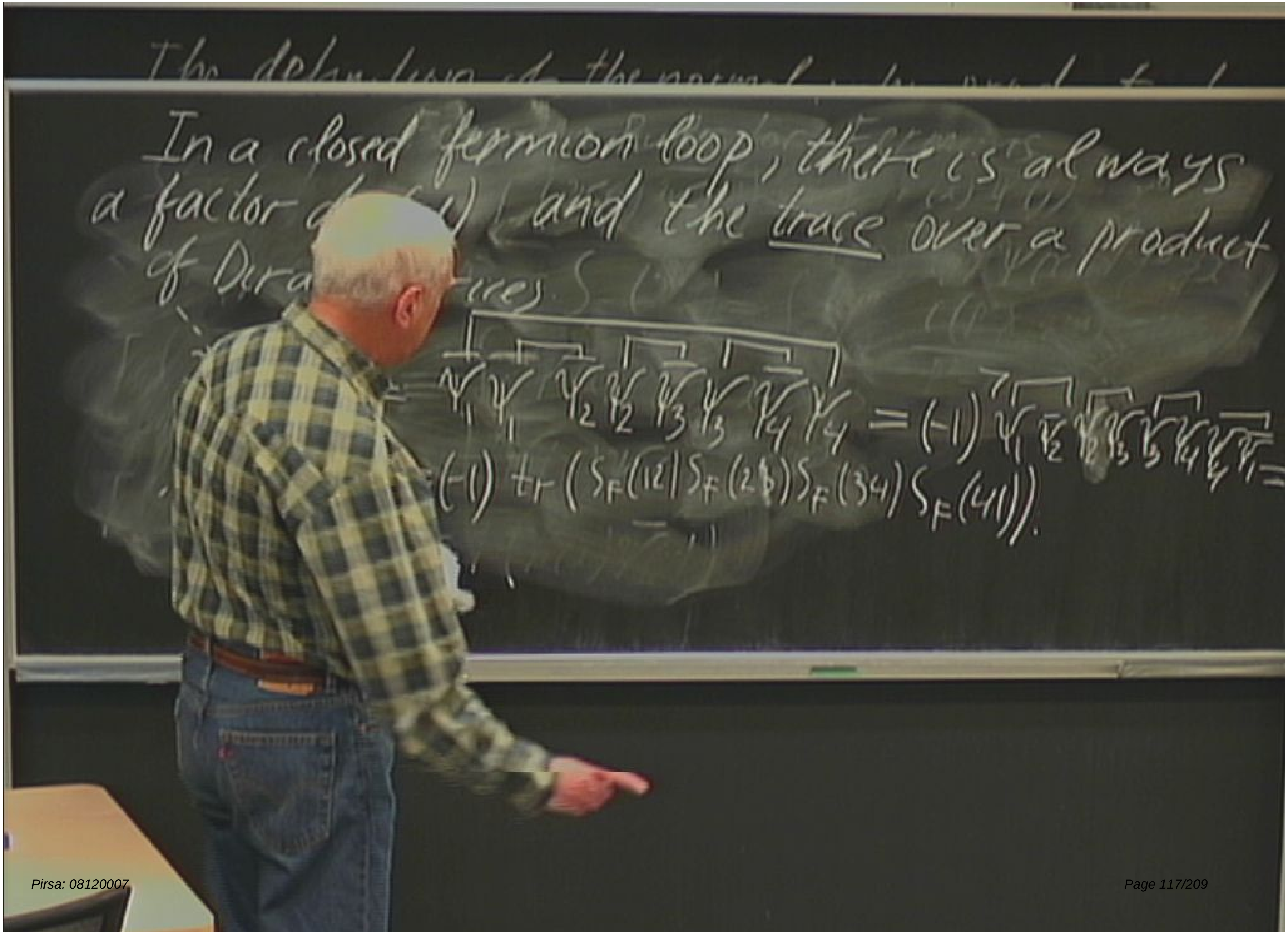
In a closed fermion loop, there is always a factor of (−1) and the trace over a product of Dirac matrices.


In a closed fermion loop, there is always
a factor of (and the trace over a product
of Dirac ma


In a closed fermion loop, there is always
a fac (-1) and the trace over a product
of matrices.
$= \psi \psi_1 \psi_2 \psi_2 \psi_3 \psi_3 \psi_4 \psi_4$


In a closed fermion loop, there is always
a factor of (-
and the trace over a product
of Dirac ma

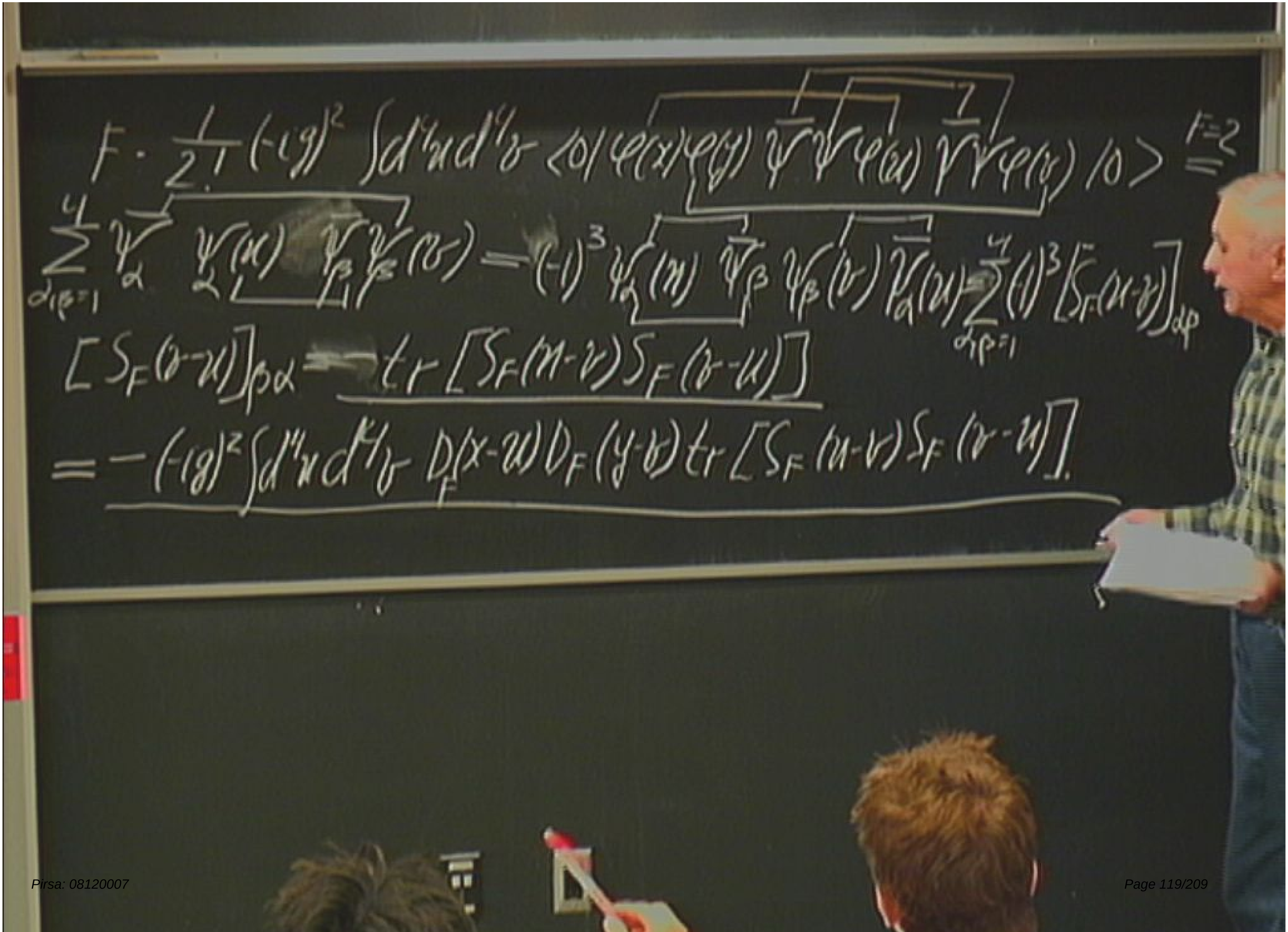

In a closed fermion loop, there is always
a factor of (-1)
the trace over a product
of Dirac
(-1)


In a closed fermion loop, there is always
a factor of (-1) and the trace over a product
of Dirac matrices.
(-1) tr (SF(12) SF(23) SF(34) SF(41)).

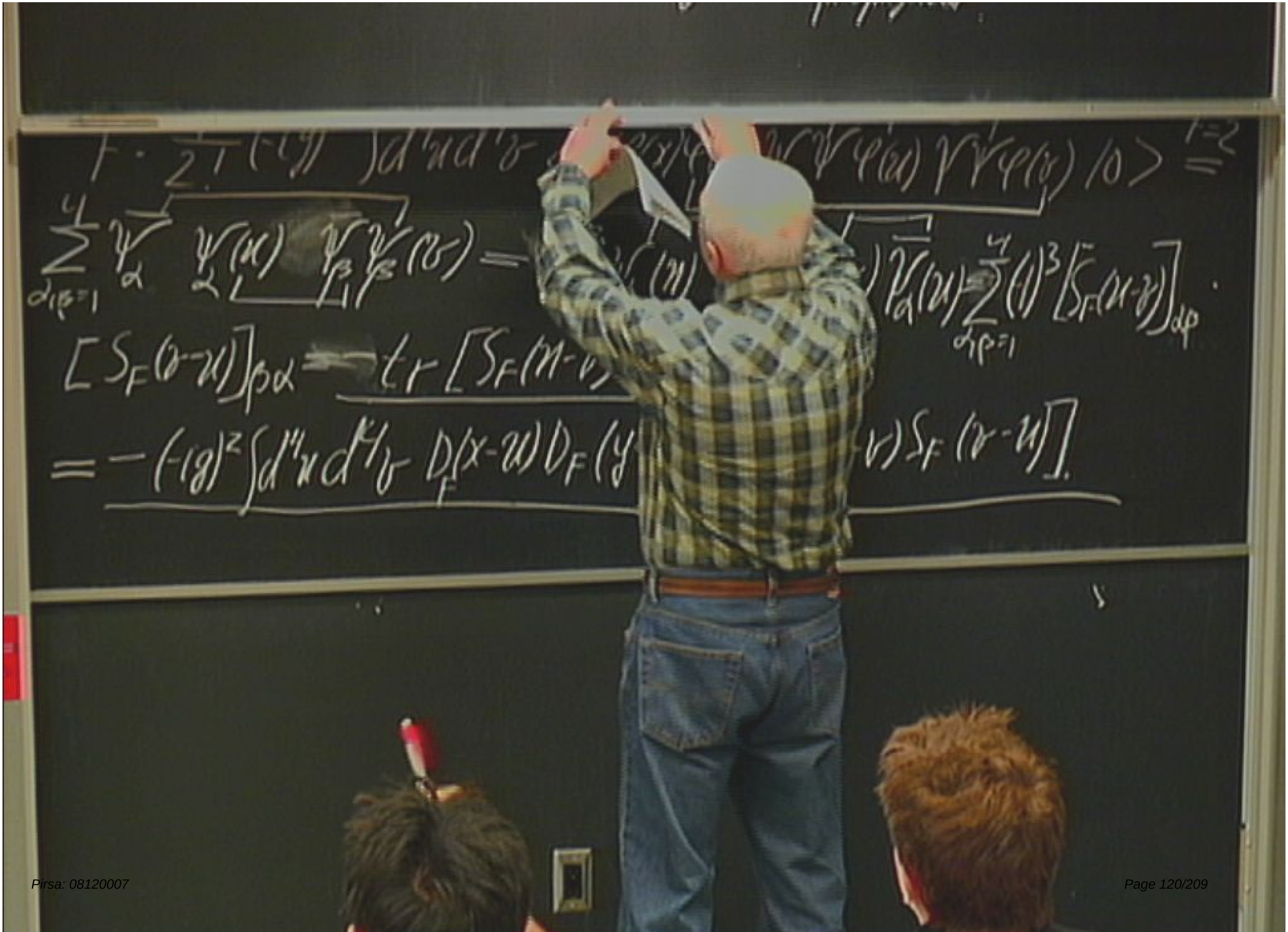
In a closed fermion loop, there is always a factor of (-1) and the <u>trace</u> over a product of Dirac matrices.

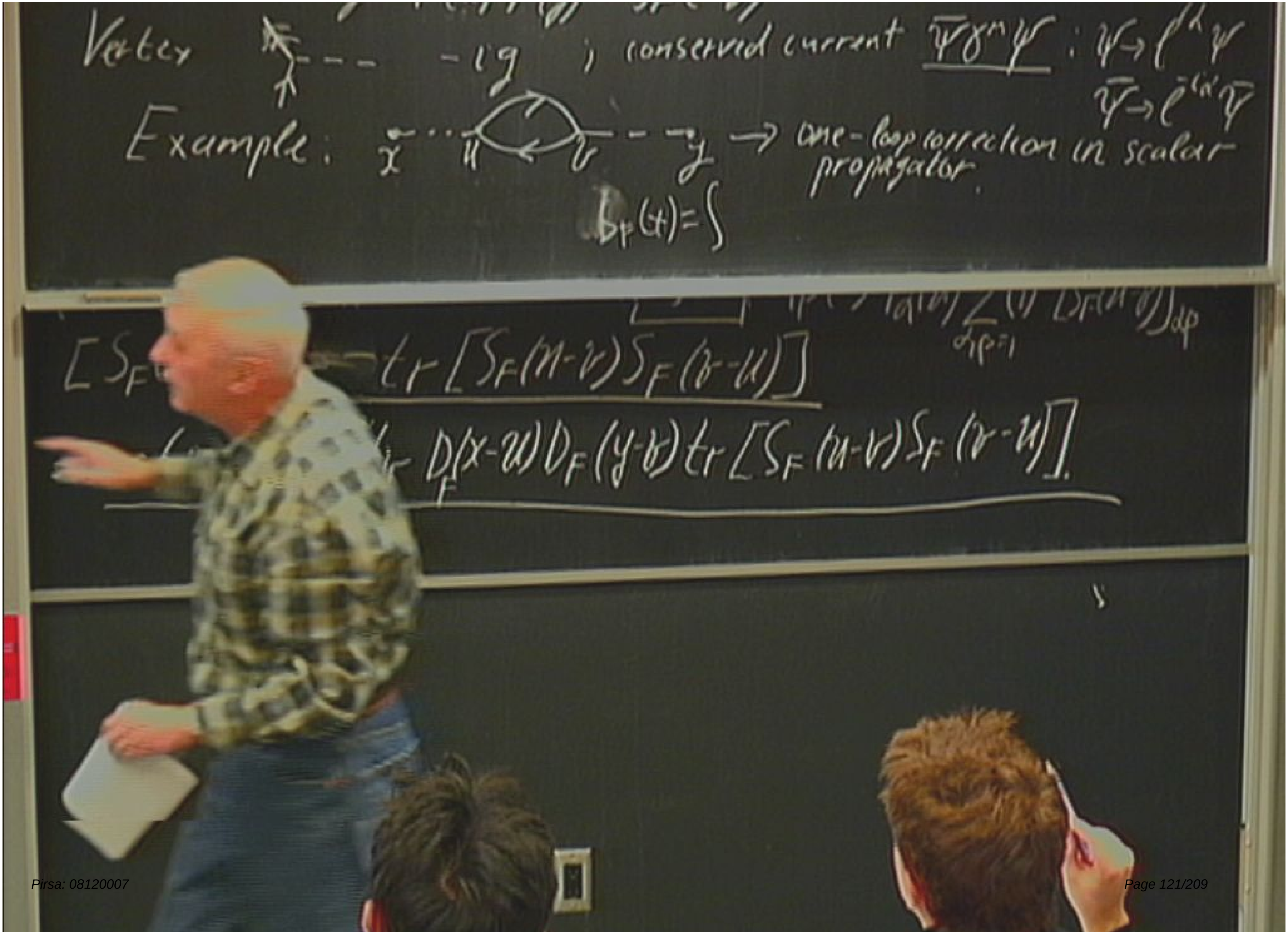
$$= \psi_1 \bar\psi_1 \psi_2 \bar\psi_2 \psi_3 \bar\psi_3 \psi_4 \bar\psi_4 = (-1)\, \psi_1 \bar\psi_2 \psi_2 \bar\psi_3 \psi_3 \bar\psi_4 \psi_4 \bar\psi_1 =$$

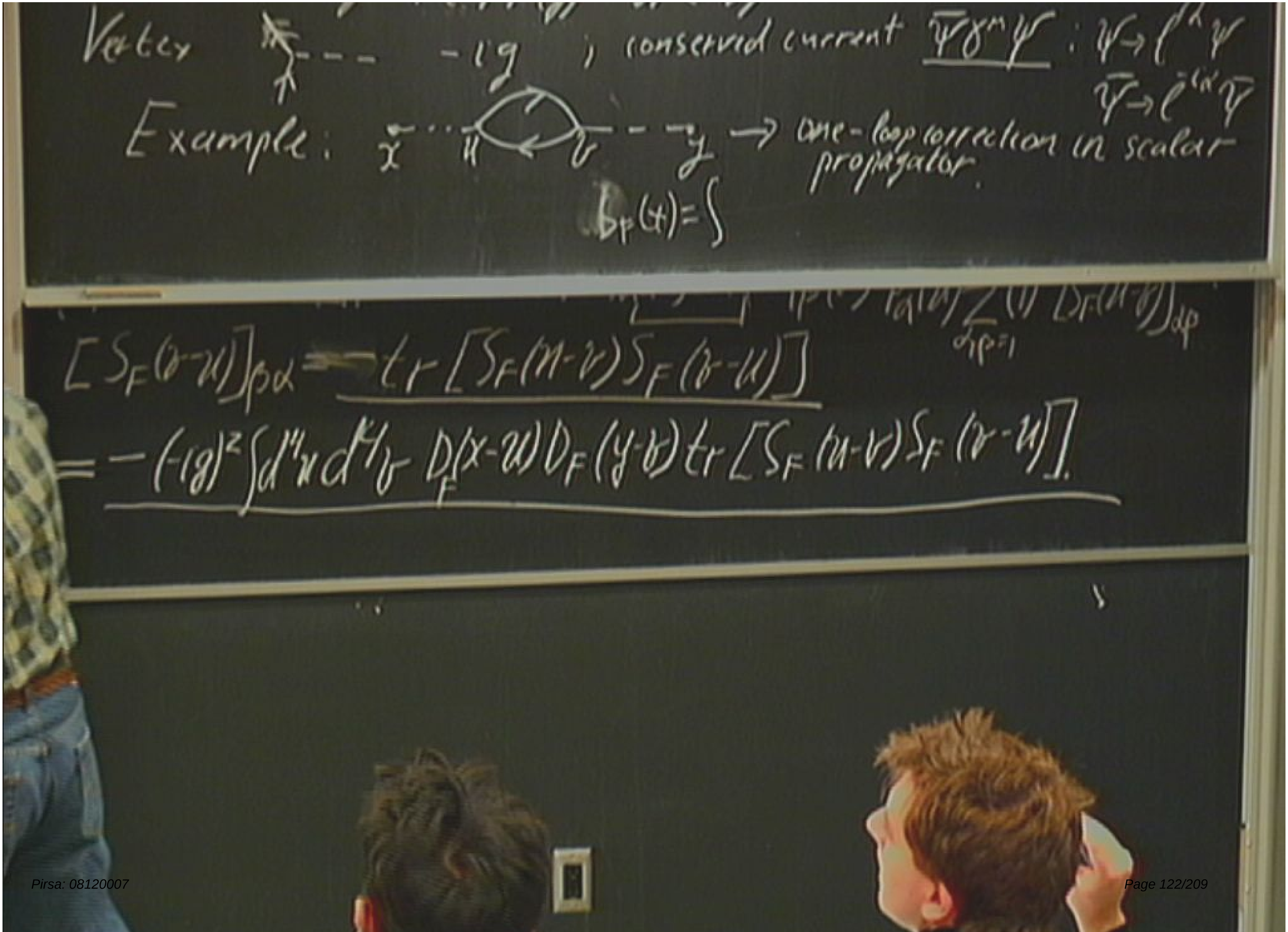
$$= (-1)\, \mathrm{tr}\left(S_F(12)\, S_F(23)\, S_F(34)\, S_F(41) \right).$$

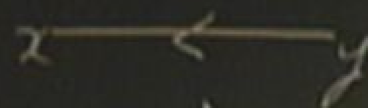


$F = \frac{1}{2!}(-ig)^2 \int d^4x\, d^4y\, \langle 0|\, \varphi(x)\varphi(y)\, \bar\psi\psi\varphi(u)\, \bar\psi\psi\varphi(v)\, |0\rangle \overset{F=2}{=}$
$\sum_{\alpha,\beta=1}^{4} \bar\psi_\alpha \psi_\alpha(u)\, \bar\psi_\beta \psi_\beta(v) = (-1)^3\, \psi_\alpha(u)\, \bar\psi_\beta\, \psi_\beta(v)\, \bar\psi_\alpha(u) = \sum_{\alpha,\beta=1}^{4} (-1)^3 [S_F(u-v)]_{\alpha\beta}$
$[S_F(v-u)]_{\beta\alpha} = -\mathrm{tr}\,[S_F(u-v)\, S_F(v-u)]$
$= -(-ig)^2 \int d^4x\, d^4y\; D_F(x-u)\, D_F(y-v)\, \mathrm{tr}\,[S_F(u-v)\, S_F(v-u)].$




Vertex -ig ; conserved current $\bar\psi\gamma^\mu\psi$: $\psi \to e^{i\alpha}\psi$
$\bar\psi \to e^{-i\alpha}\bar\psi$
Example: x u v y → one-loop correction in scalar propagator.
$D_F(x)=\int$
$tr[S_F(u-v)S_F(v-u)]$
$D_F(x-u)D_F(y-v)\,tr[S_F(u-v)S_F(v-u)]$.


Vertex -ig ; conserved current $\bar\psi\gamma^\mu\psi$: $\psi \to e^{i\alpha}\psi$
$\bar\psi \to e^{-i\alpha}\bar\psi$
Example: x u v y → one-loop correction in scalar propagator.
$D_F(x) = \int$
$[S_F(v-u)]_{\beta\alpha} = -tr[S_F(u-v)S_F(v-u)]$
$= -(-ig)^2 \int d^4u\, d^4v\, D_F(x-u)\, D_F(y-v)\, tr[S_F(u-v)S_F(v-u)]$.



$\psi(x)\bar{\psi}(y) = S_F(x-y)$

Vertex  $-ig$; conserved current $\bar{\psi}\gamma^\mu\psi$: $\psi \to e^{i\alpha}\psi$

$\bar{\psi} \to e^{-i\alpha}\bar{\psi}$

Example:  $\rightarrow$ one-loop correction in scalar propagator

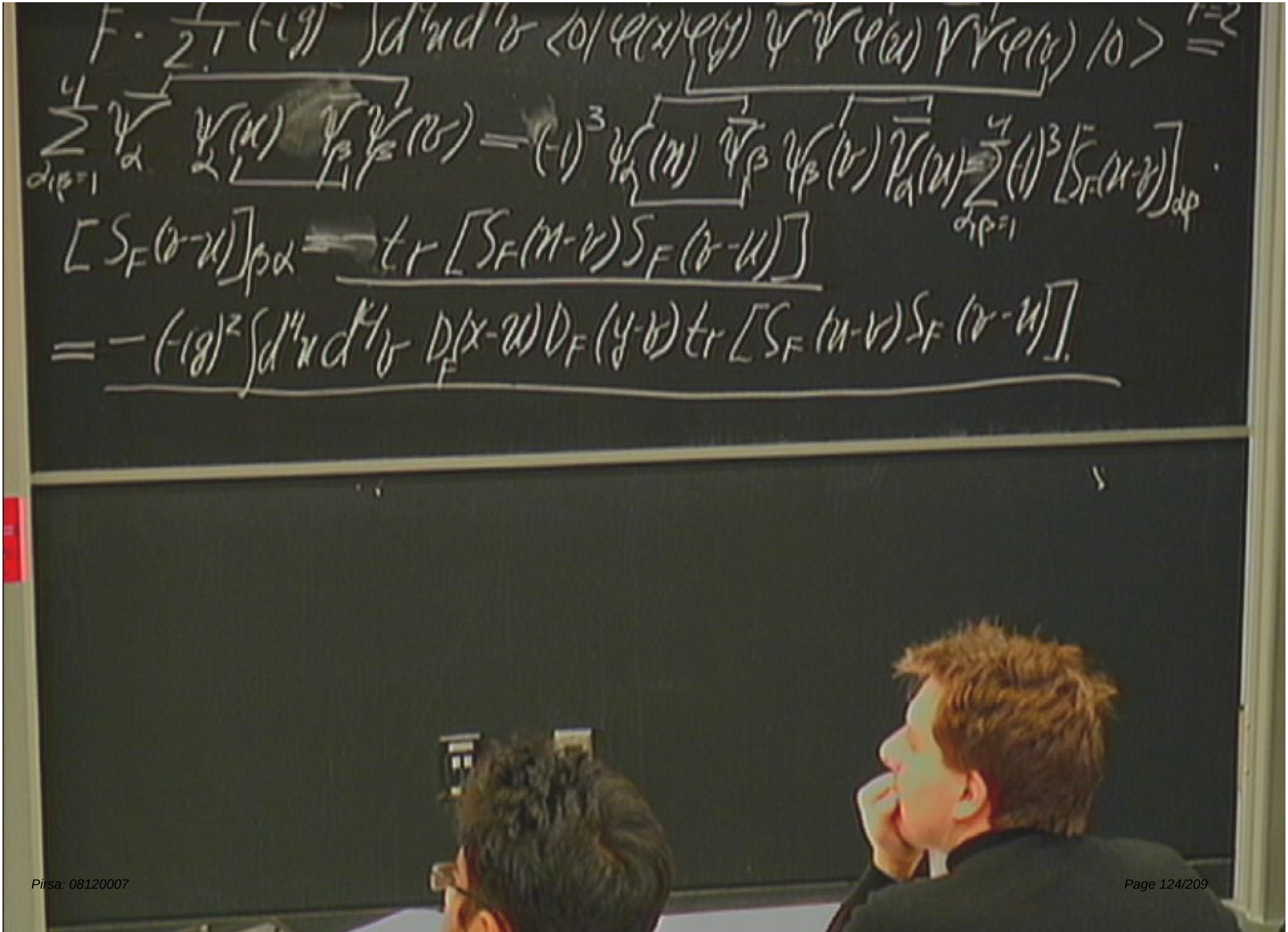
$$S_F(x) = \int d^4p \, e^{-ipx} \frac{p^\mu\gamma_\mu + m}{p^2 - m^2 + i\epsilon}$$

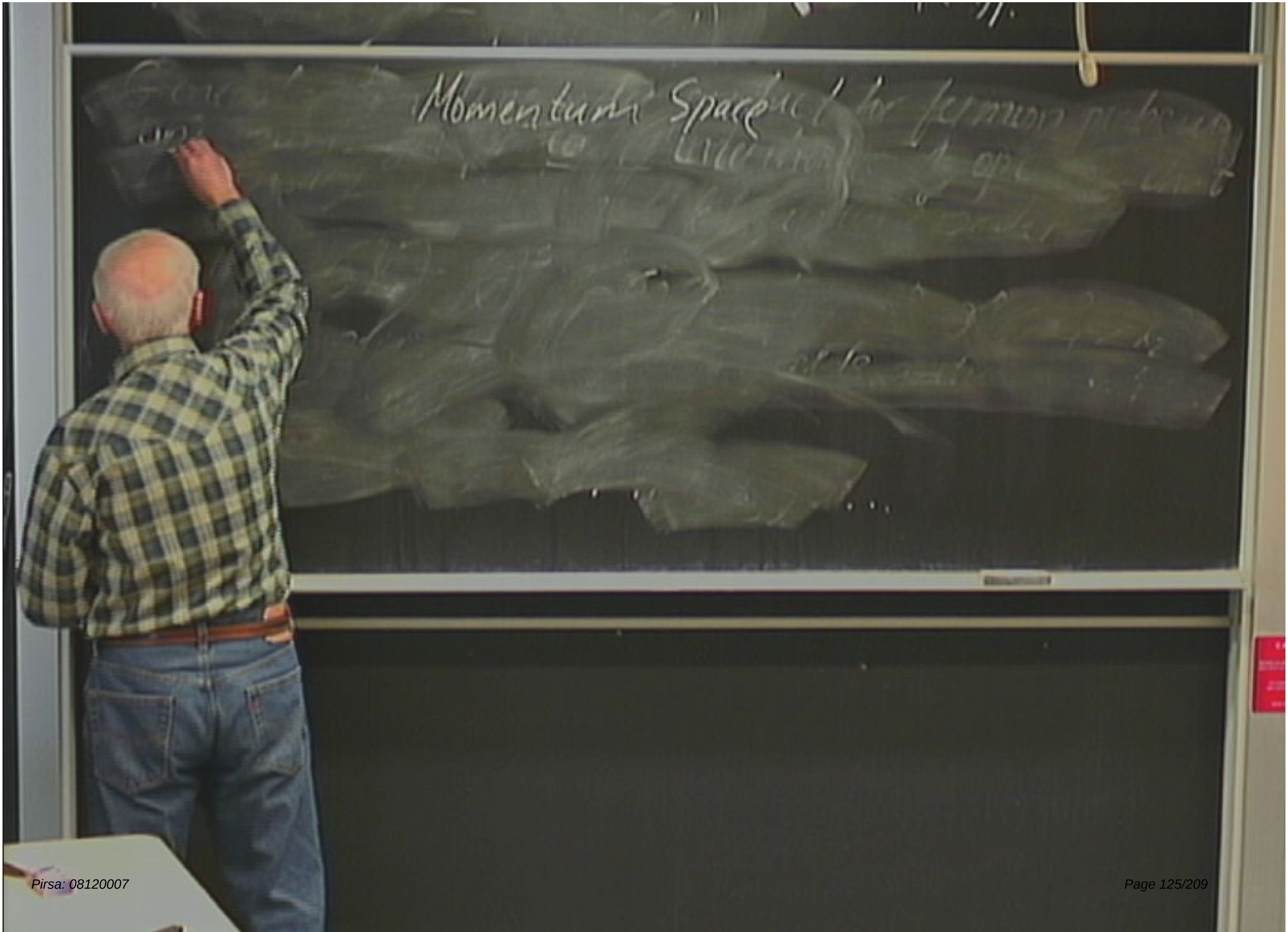
$$F = \frac{1}{2!}(-ig)^2 \int d^4u\, d^4v \, \langle 0| \varphi(x)\varphi(y)\, \bar{\psi}\psi\varphi(u)\, \bar{\psi}\psi\varphi(v) |0\rangle \overset{F=2}{=}$$

$$\sum_{\alpha,\beta=1}^{4} \bar{\psi}_\alpha \psi_\alpha(u)\, \bar{\psi}_\beta \psi_\beta(v) = -(-1)^3 \psi_\alpha(u)\, \bar{\psi}_\beta\, \psi_\beta(v)\, \bar{\psi}_\alpha(u) = \sum_{\alpha,\beta=1}^{4} (-1)^3 [S_F(u-v)]_{\alpha\beta} \cdot$$

$$[S_F(v-u)]_{\beta\alpha} = -\,\mathrm{tr}\left[S_F(u-v) S_F(v-u)\right]$$

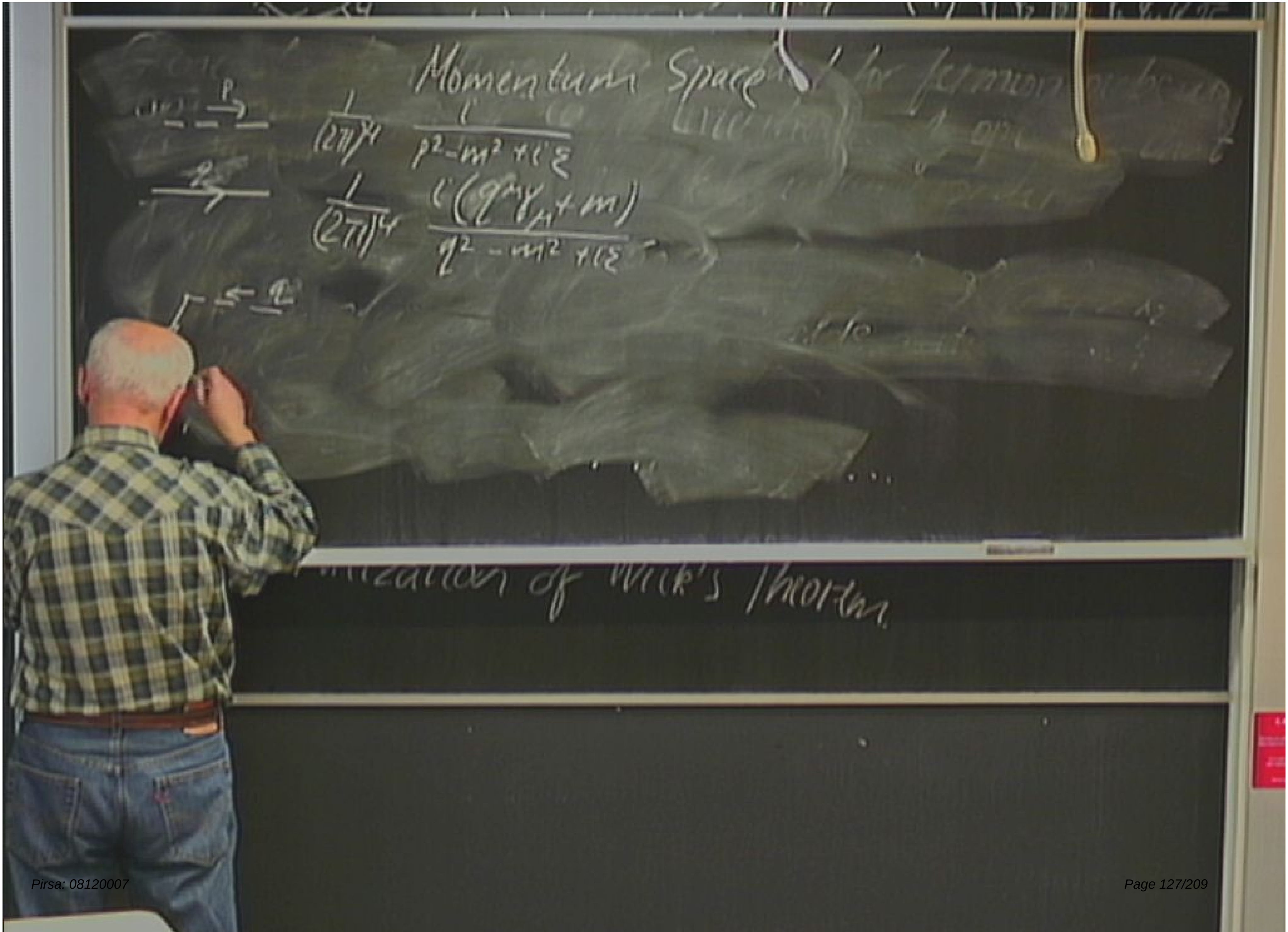
$$= -(-ig)^2 \int d^4u\, d^4v \; D_F(x-u) D_F(y-v)\, \mathrm{tr}\left[S_F(u-v) S_F(v-u)\right].$$

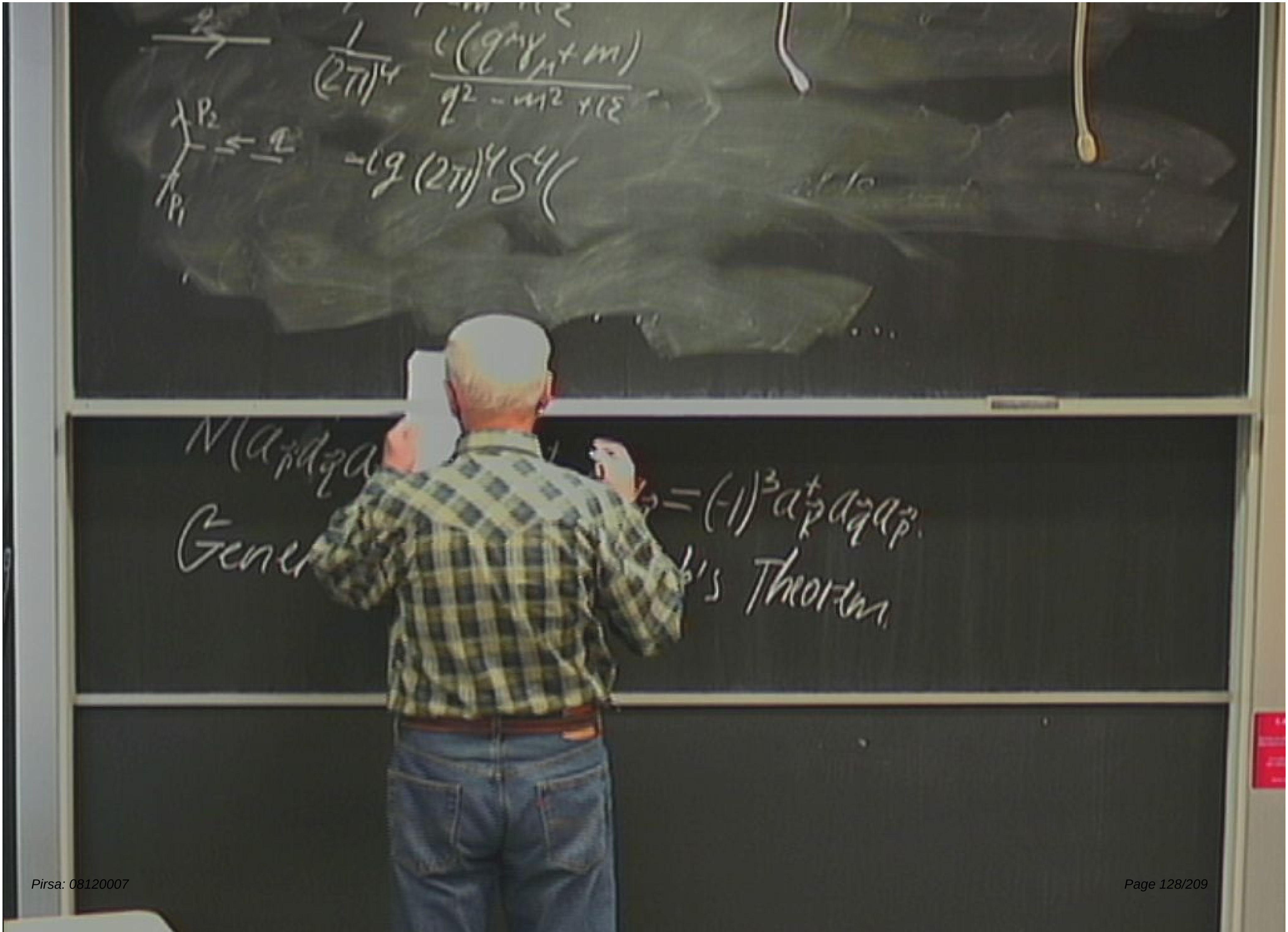

F = 1/2! (-ig)^2 ∫d^4x d^4y ⟨0| φ(x) φ(y) ψ̄ψ φ(u) ψ̄ψ φ(v) |0⟩ =
Σ_{α,β=1}^{4} ψ̄_α ψ_α(u) ψ̄_β ψ_β(v) = -(-1)^3 ψ_α(u) ψ̄_β ψ_β(v) ψ̄_α(u) Σ_{α,β=1}^{4} (-1)^3 [S_F(u-v)]_{αβ} ·
[S_F(v-u)]_{βα} = -tr[S_F(u-v) S_F(v-u)]
= -(-ig)^2 ∫d^4x d^4y D_F(x-u) D_F(y-v) tr[S_F(u-v) S_F(v-u)].

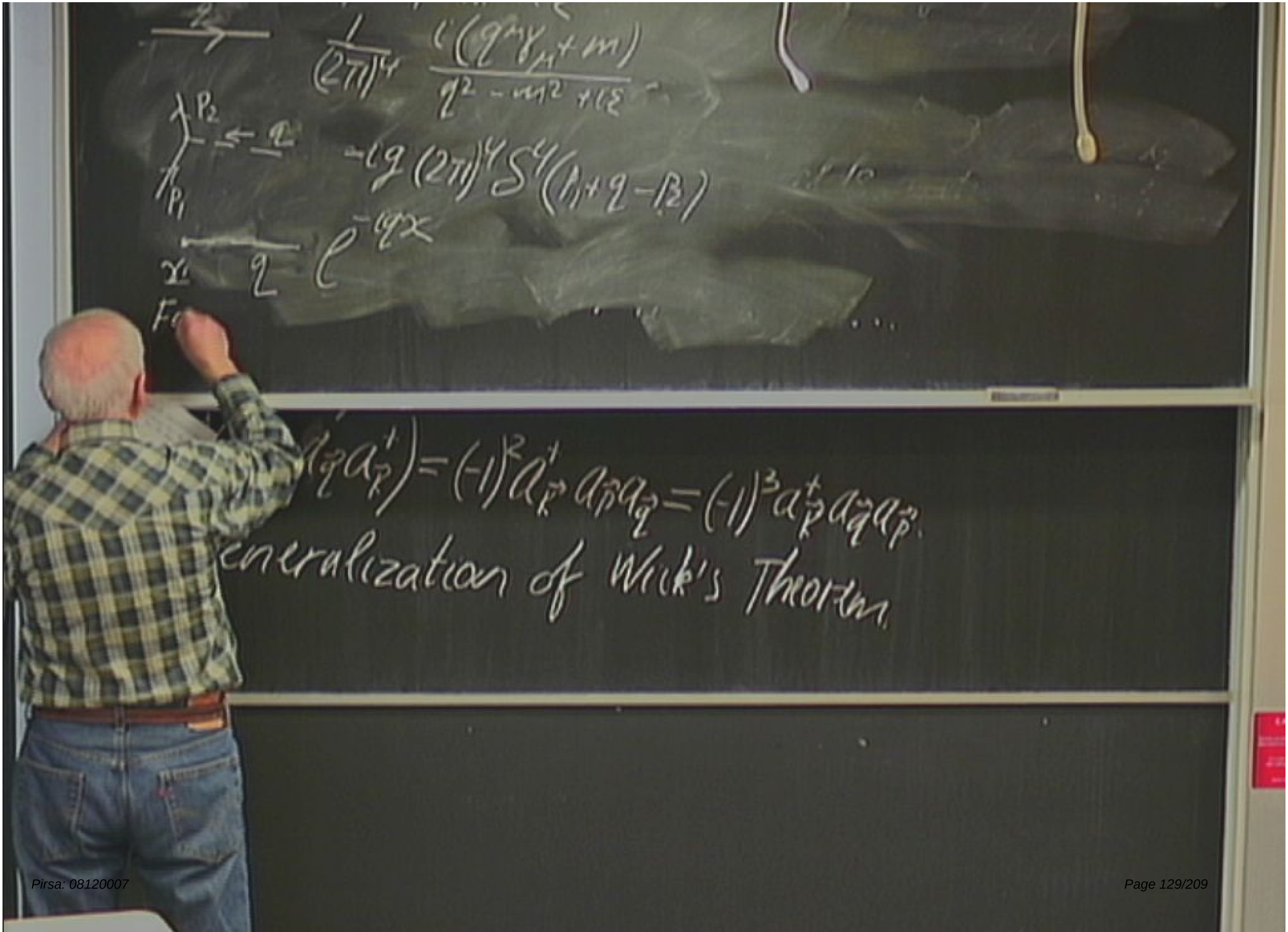

Momentum Space

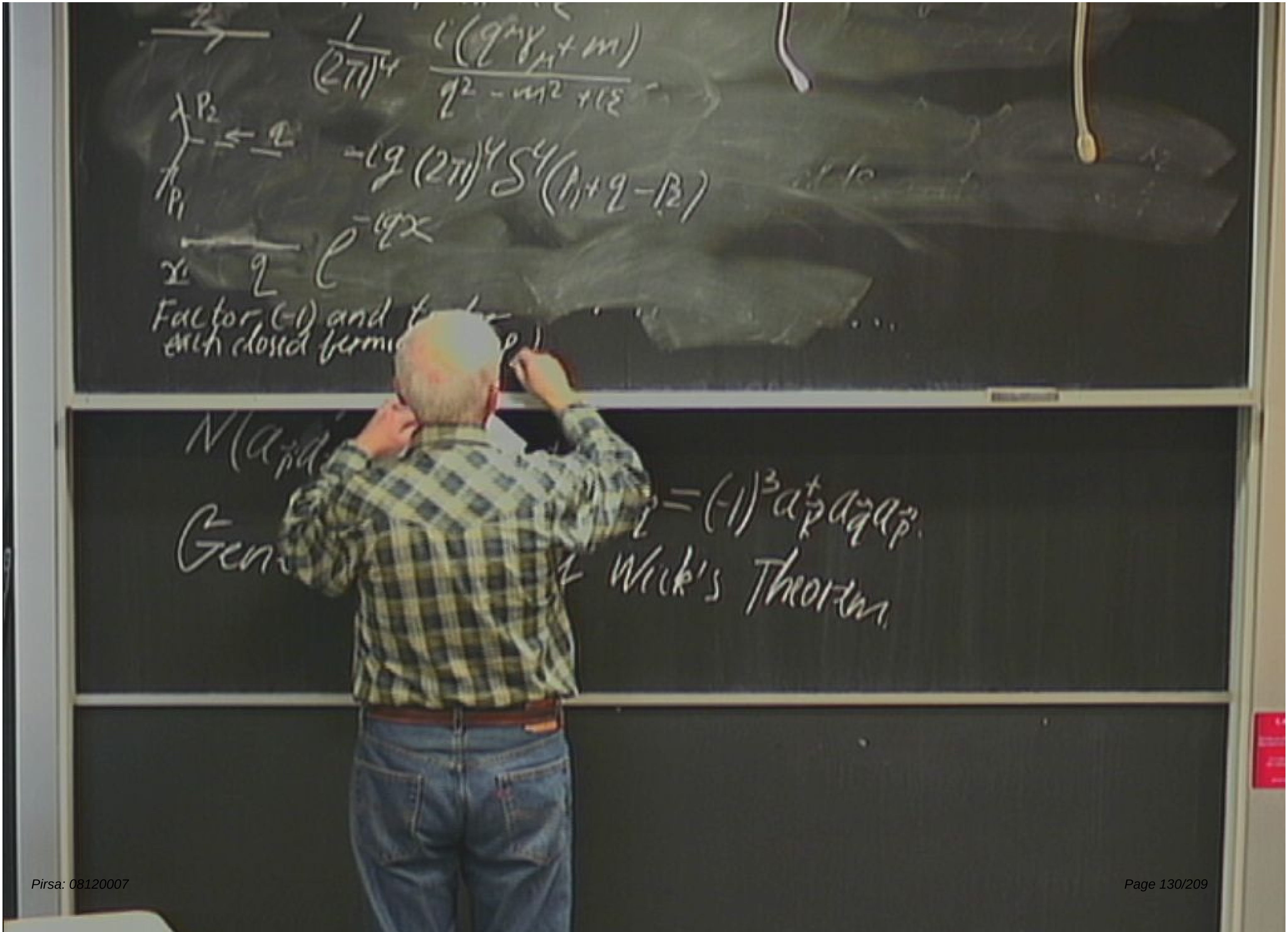
Momentum Space

$\frac{1}{(2\pi)}$

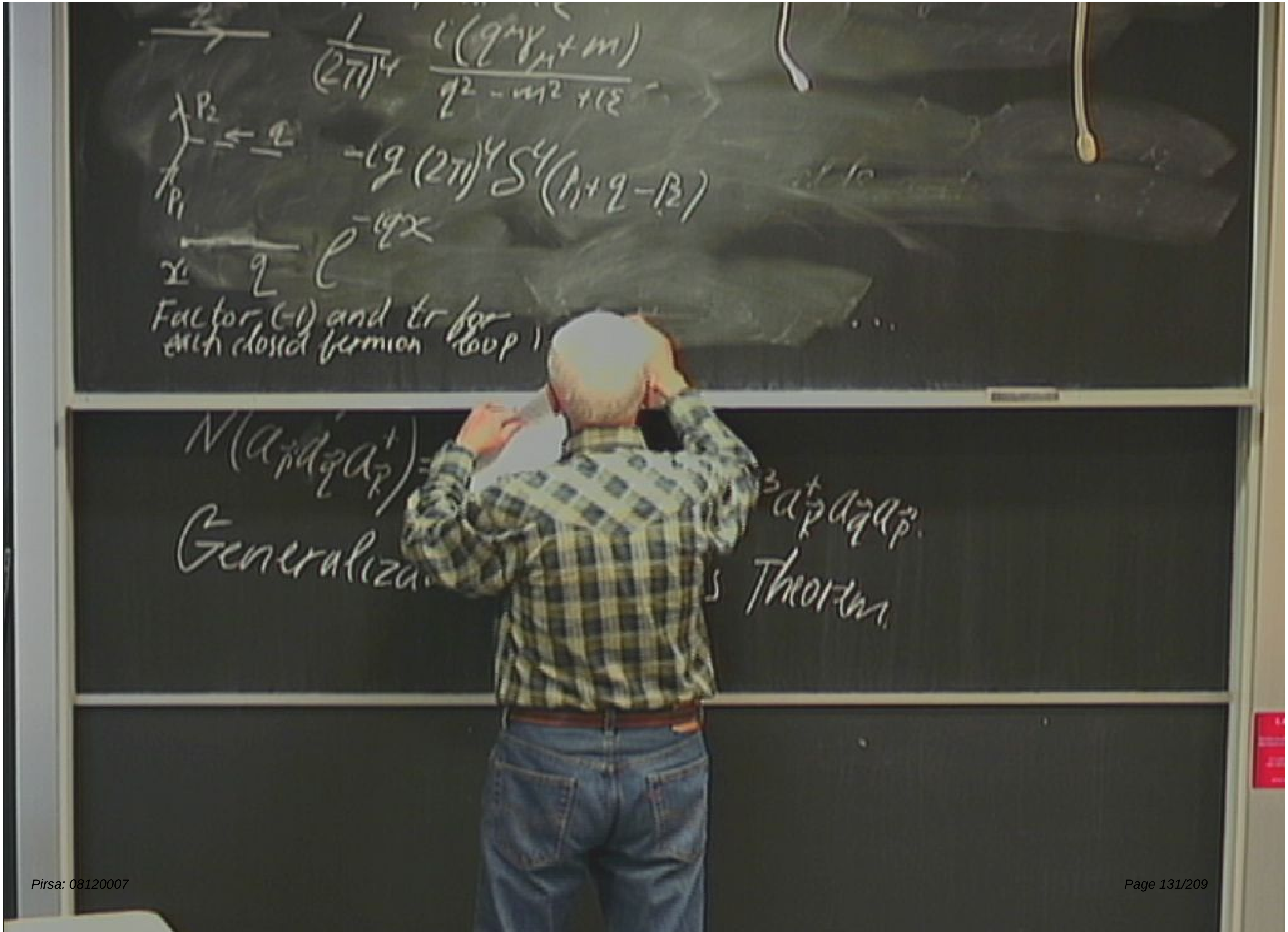

Momentum Space
$\frac{1}{(2\pi)^4} \frac{i}{p^2-m^2+i\epsilon}$
$\frac{1}{(2\pi)^4} \frac{i(q^\mu\gamma_\mu+m)}{q^2-m^2+i\epsilon}$
of Wick's Theorem


Gener
's Theorem


$\frac{1}{(2\pi)^4} \frac{i(q^\mu \gamma_\mu + m)}{q^2 - m^2 + i\epsilon}$
$-ig(2\pi)^4 \delta^4(p_1 + q - p_2)$
e^{-iqx}
eneralization of Wick's Theorem



$\frac{1}{(2\pi)^4} \frac{i(q^\mu \gamma_\mu + m)}{q^2 - m^2 + i\epsilon}$
P_2
q
$-ig(2\pi)^4 \delta^4(p_1 + q - p_2)$
P_1
e^{-iqx}
q
Factor (-1) and
each closed fermi
$= (-1)^3 a_p^\dagger a_q a_{\bar{q}} a_{\bar{p}}$
Wick's Theorem


Factor (-1) and tr for
each closed fermion loop
Theorem

p_1

$(2\pi)^4 \delta^4(p_1+q-p_2)$

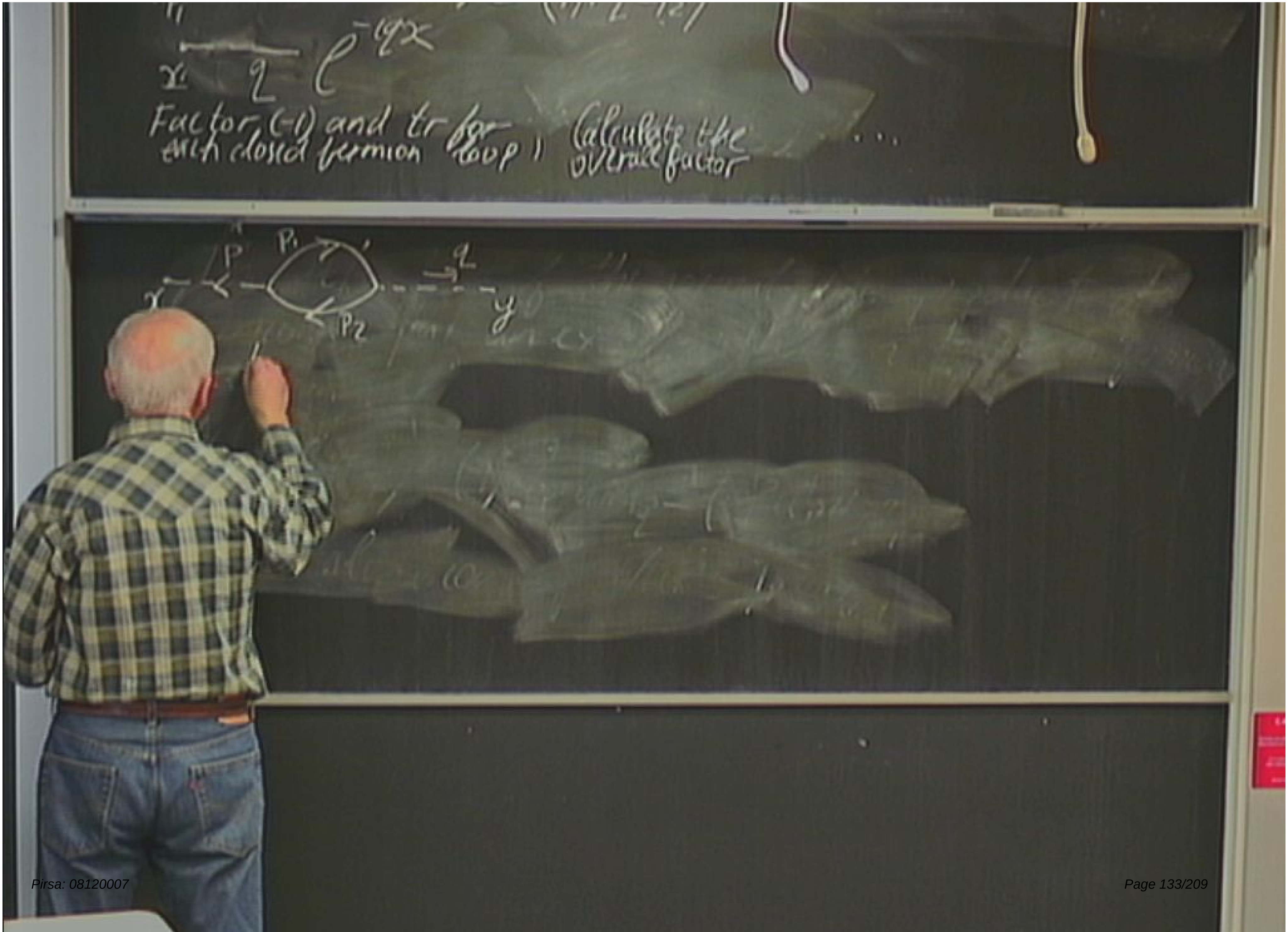
$x_1 \quad q \quad e^{-iqx}$

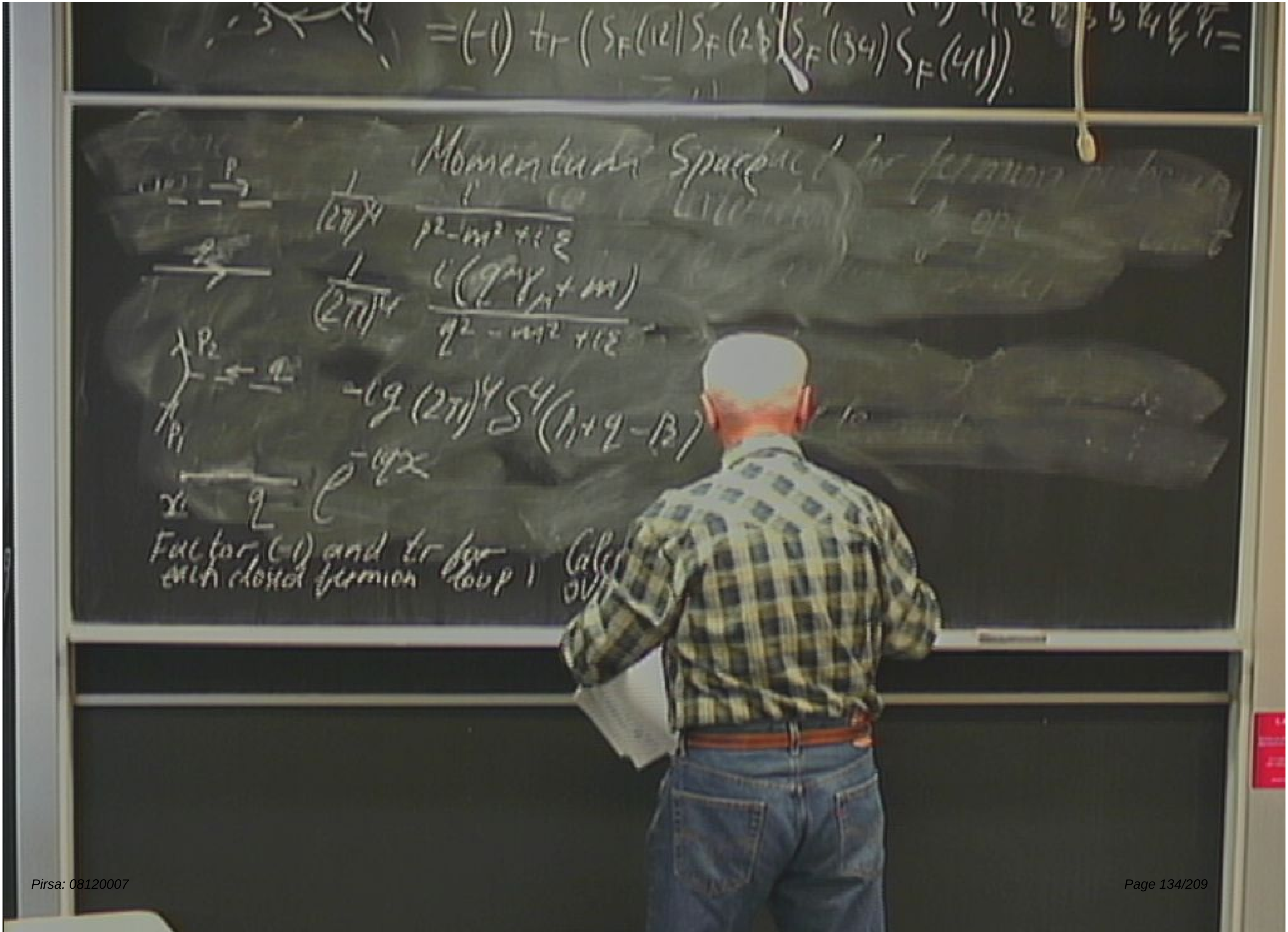
Factor (-1) and tr for each closed fermion loop; Calculate the overall factor ...

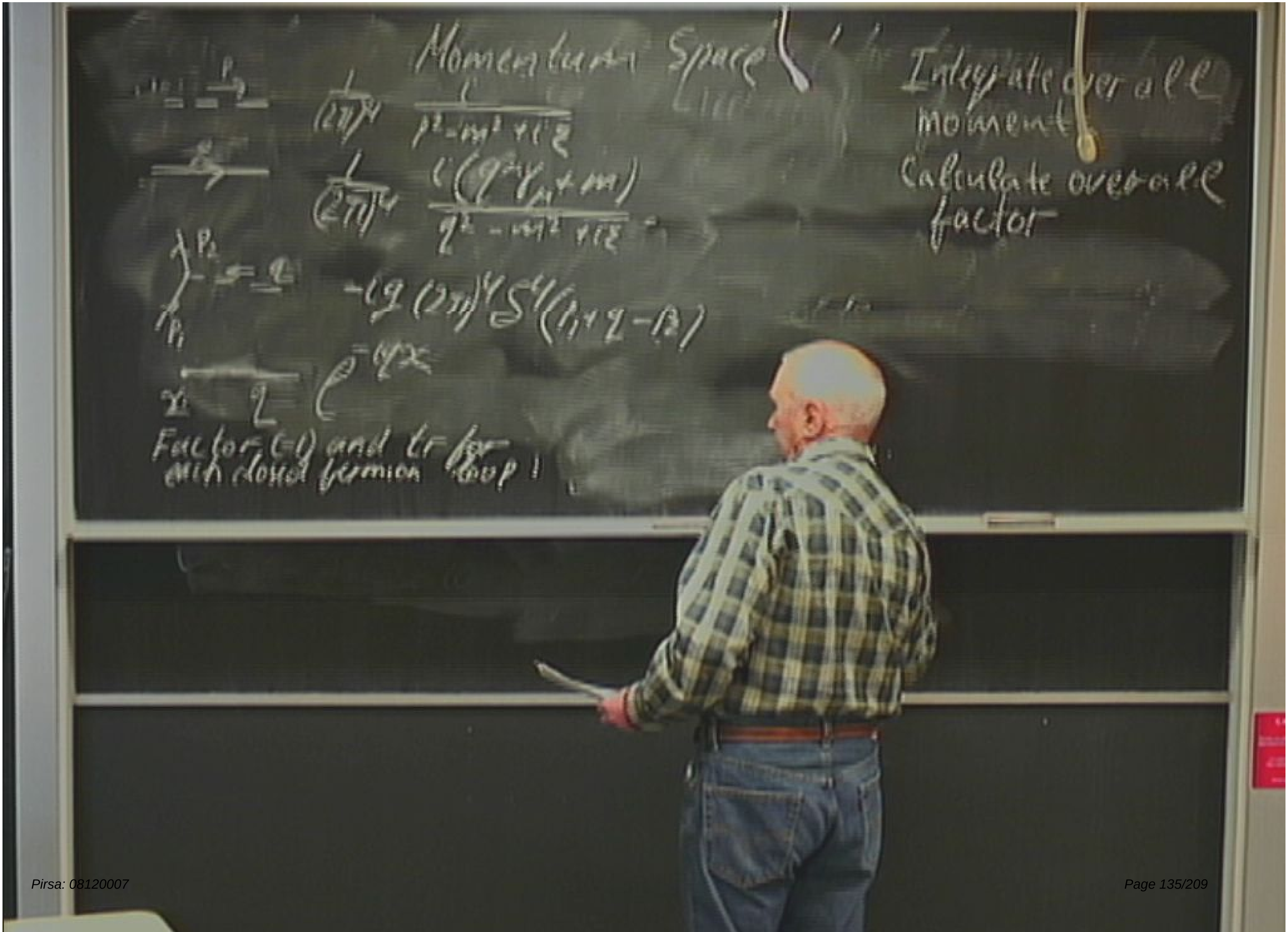
The definition of the normal-order product of fermions: put an extra minus sign for each fermion interchange:

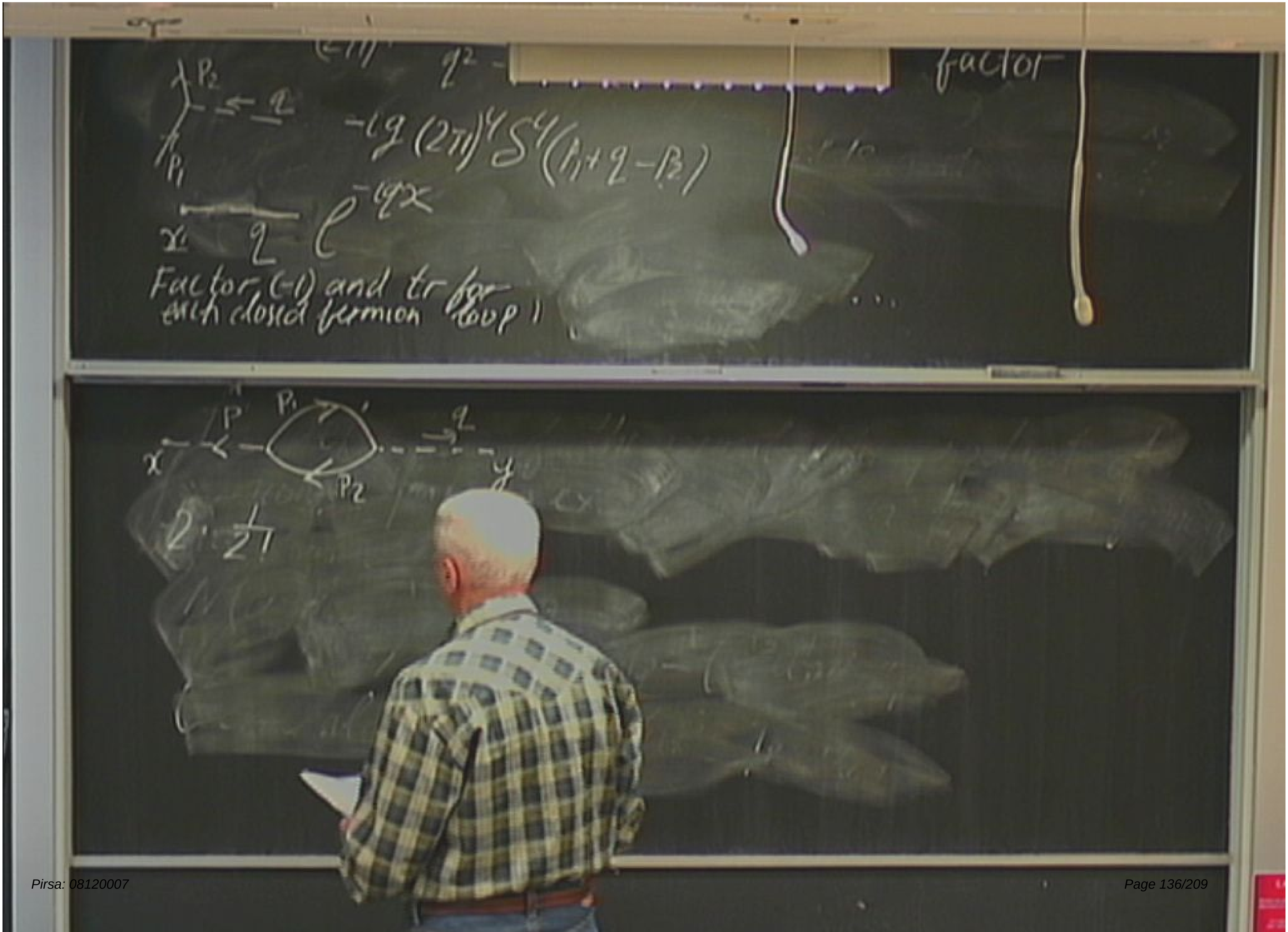
$$N(a_{\vec{p}}\, a_{\vec{q}}\, a^{\dagger}_{\vec{k}}) = (-1)^2 a^{\dagger}_{\vec{k}}\, a_{\vec{p}}\, a_{\vec{q}} = (-1)^3 a^{\dagger}_{\vec{k}}\, a_{\vec{q}}\, a_{\vec{p}}.$$

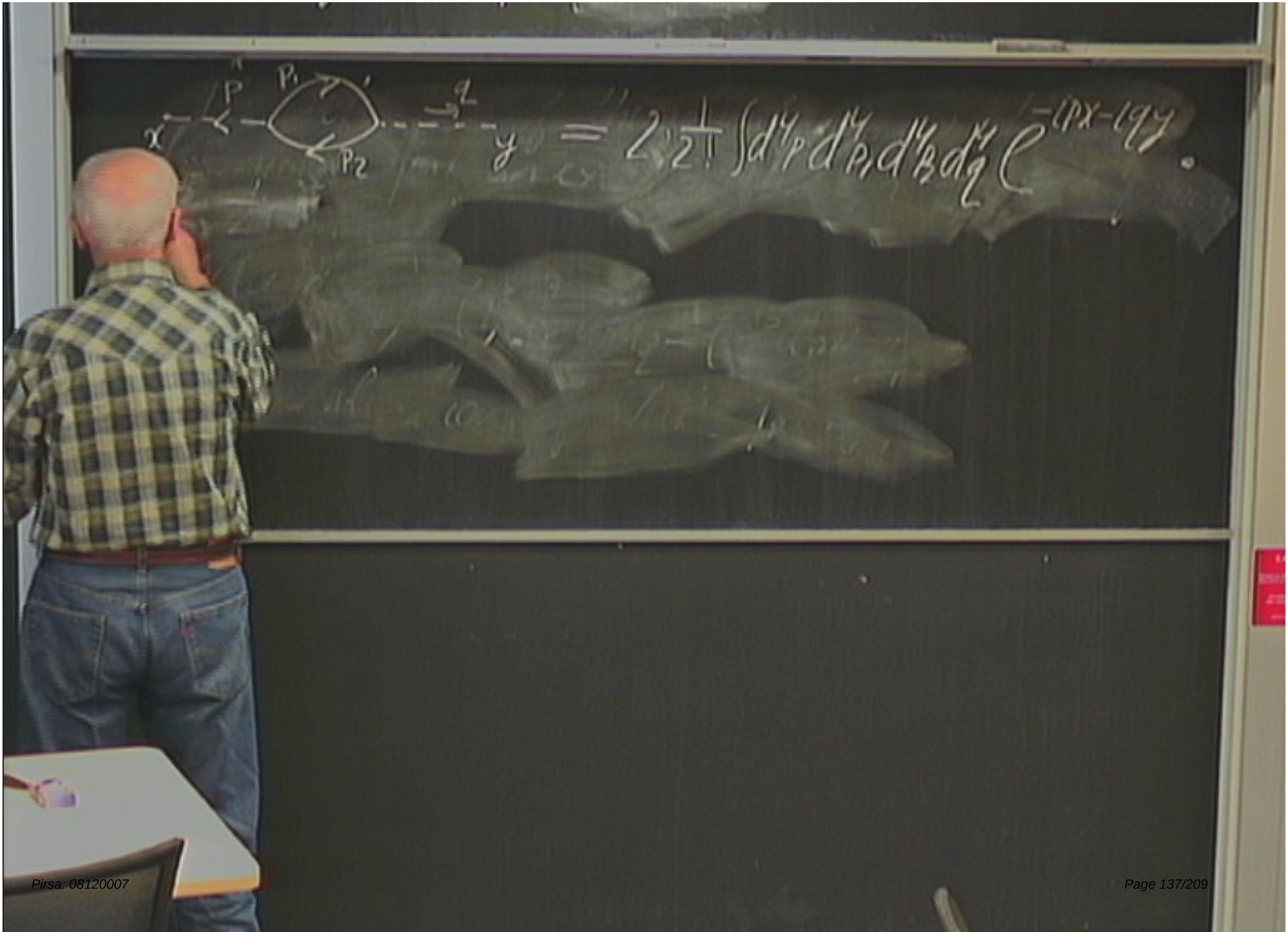
Generalization of Wick's Theorem

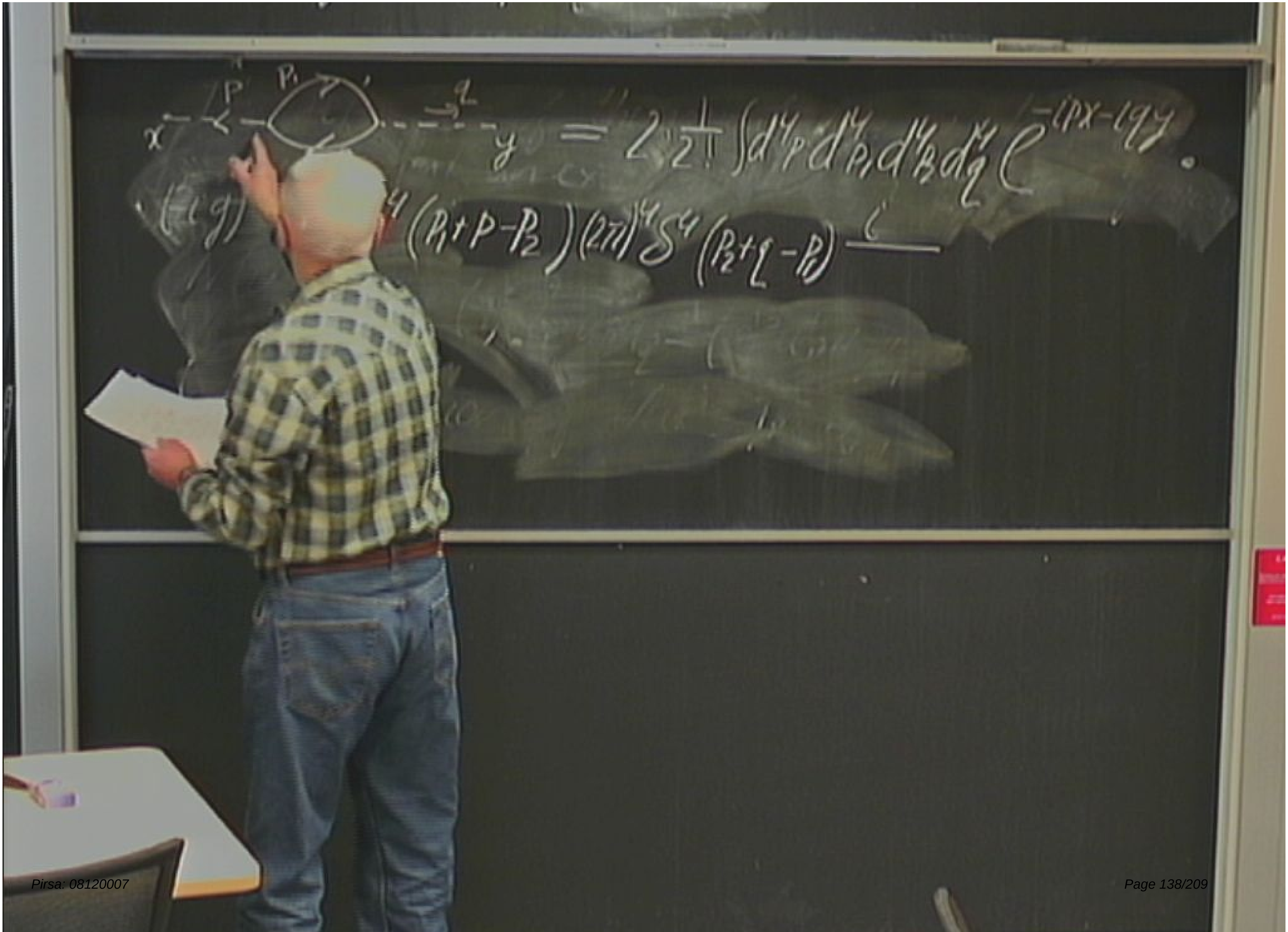

Factor (-1) and tr for each closed fermion loop ;
Calculate the overall factor
...
e^{-iqx}
x
q
p
p_1
p_2
q
y

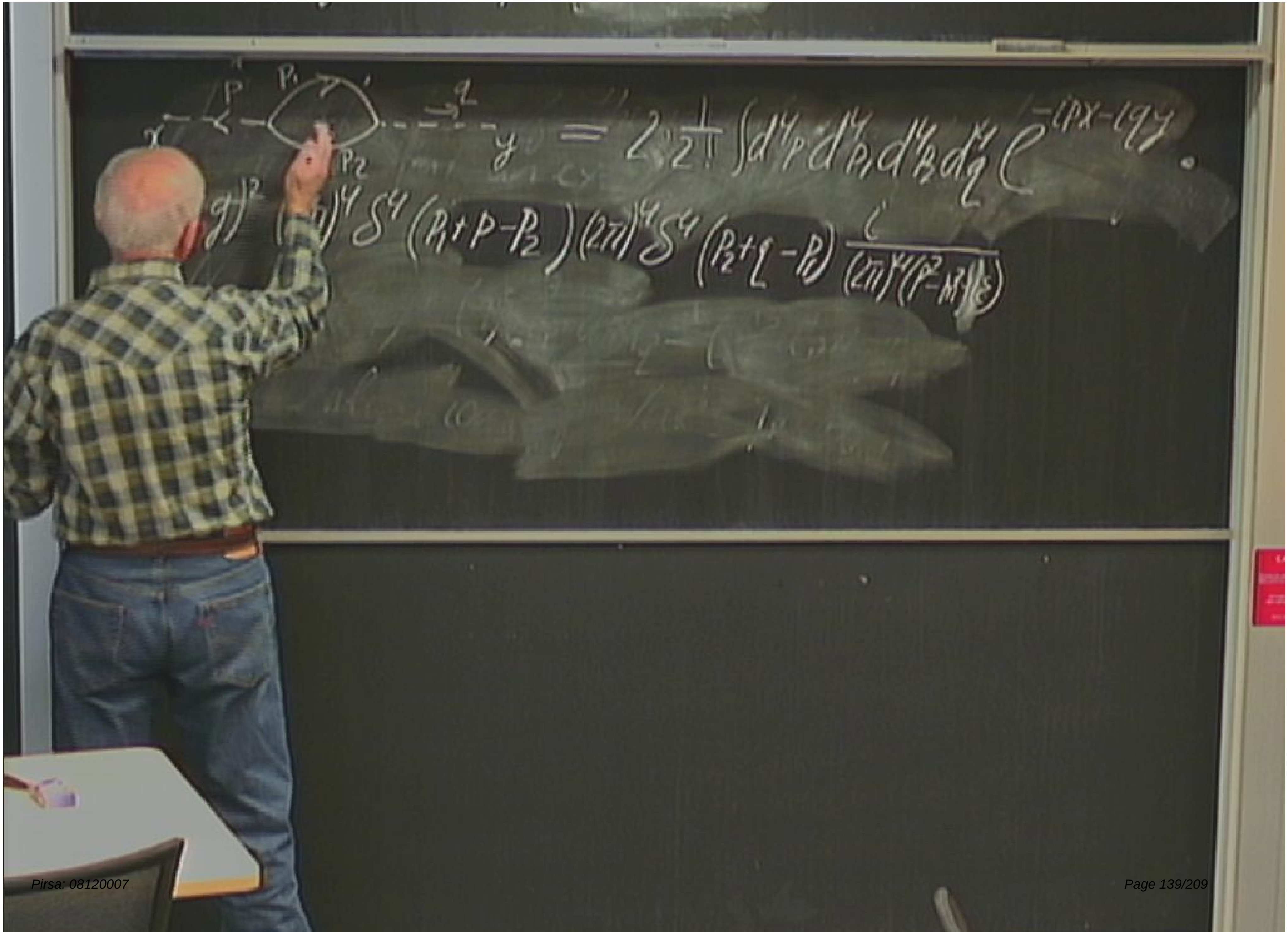

=(-1) tr(S_F(12) S_F(23) S_F(34) S_F(41)).
Momentum Space
1/(2π)^4 i/(p²−m²+iε)
1/(2π)^4 i(q^μ γ_μ + m)/(q² − m² + iε)
−ig (2π)^4 δ^4(p1+q−p2)
e^{−iqx}
Factor (-1) and tr for each closed fermion loop

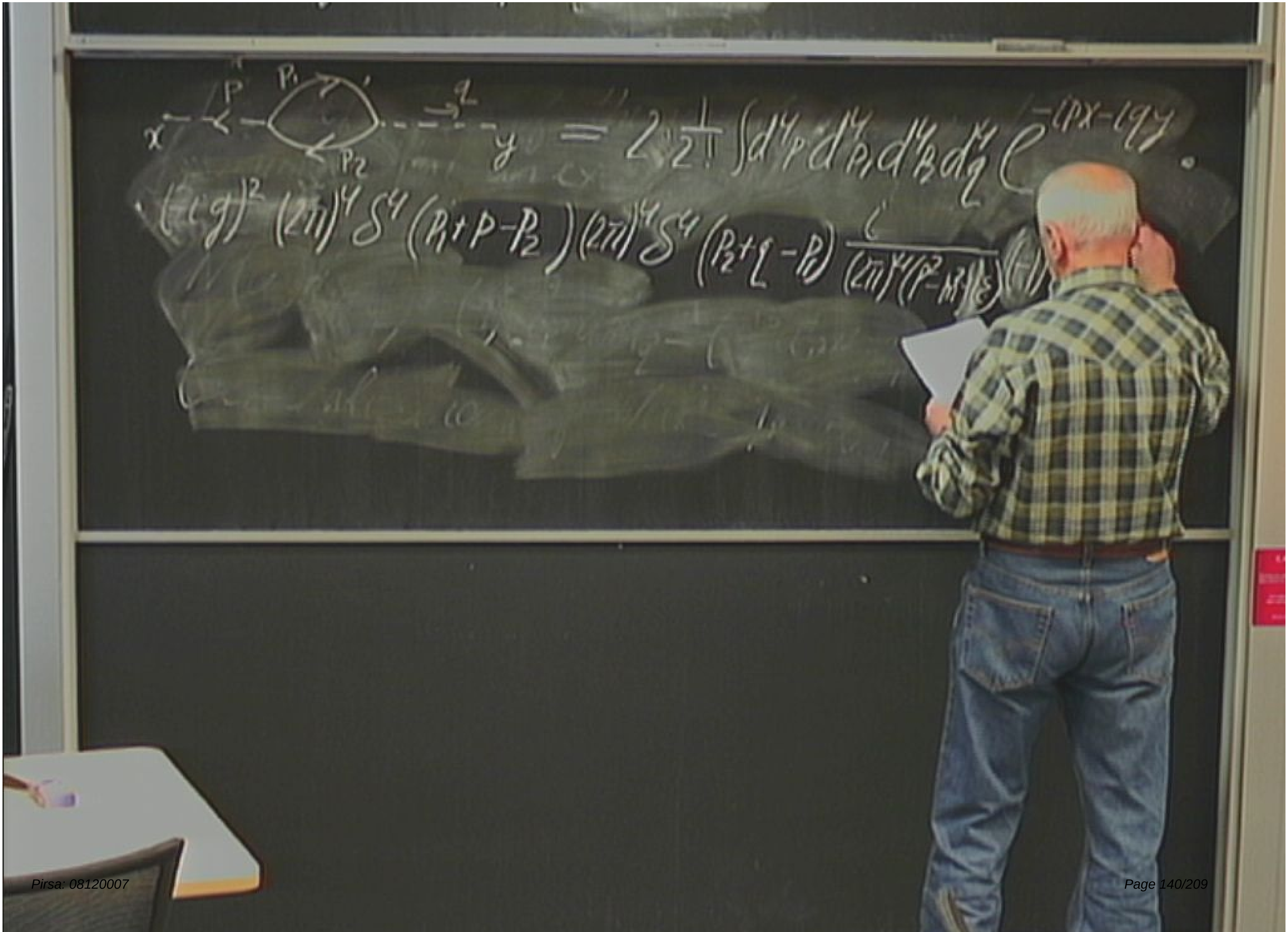

Momentum Space
Integrate over all
moment
Calculate over all
factor
Factor (-1) and tr for
each closed fermion loop!


factor
-ig (2π)^4 δ^4(p_1+q-p_2)
e^{-iqx}
Factor (-1) and tr for each closed fermion loop !
1/2π

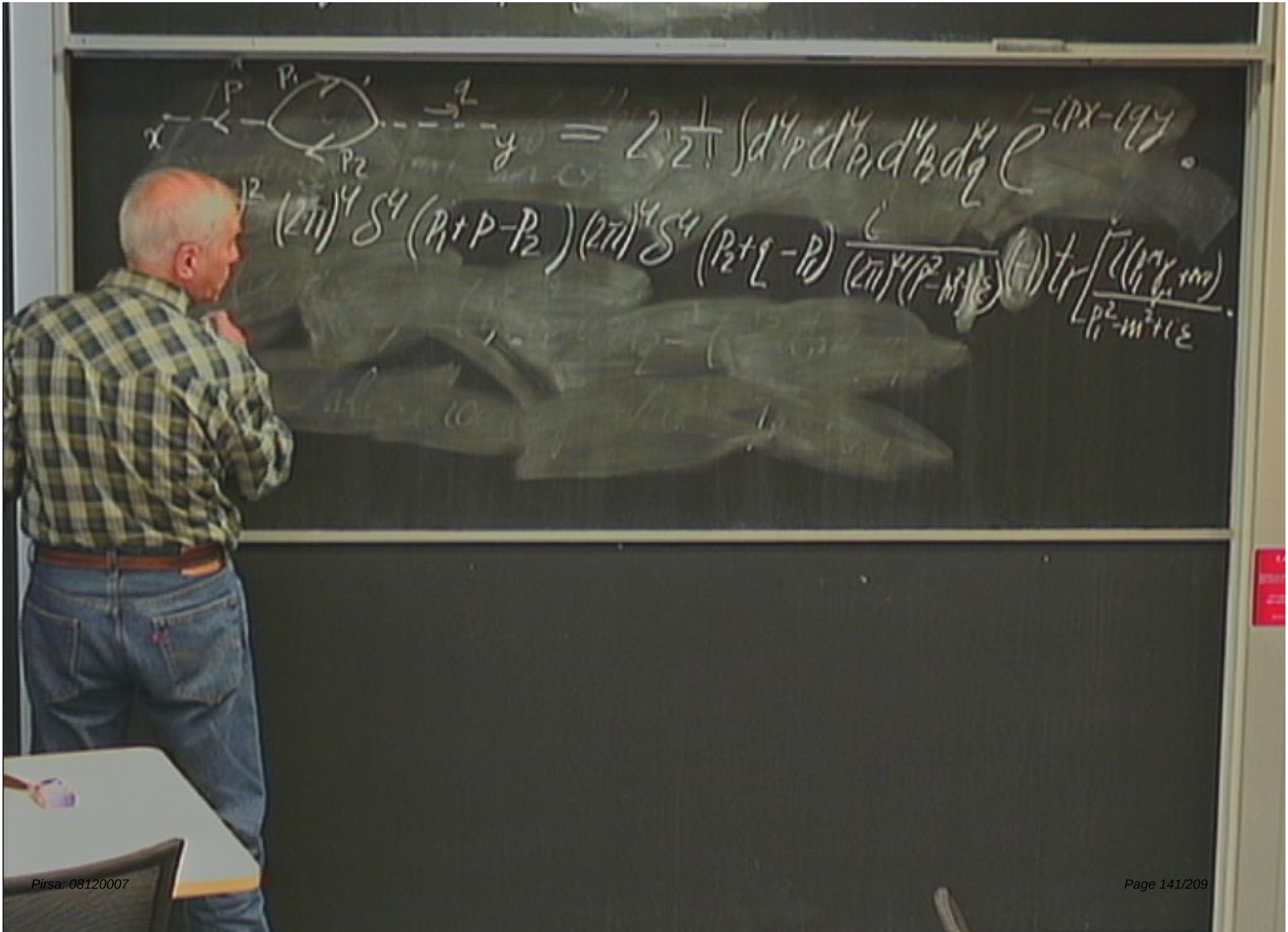

$= \lambda^2 \frac{1}{2!} \int d^4p \, d^4p_1 \, d^4p_2 \, d^4q \, e^{-ipx-iqy}$

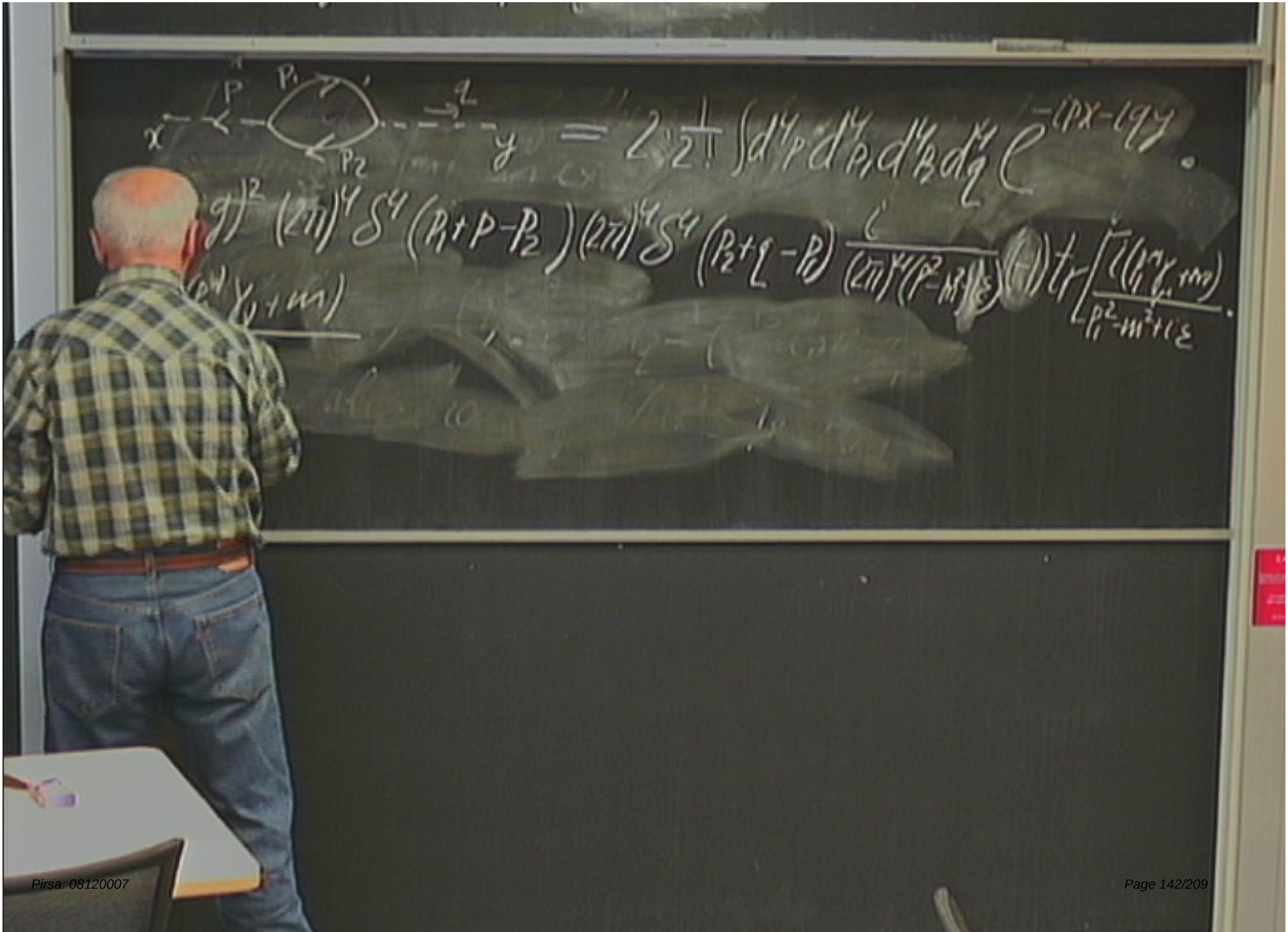

x p p1 q y = i² 1/2! ∫ d⁴p d⁴p1 d⁴p2 d⁴q e^(-ipx-iqy)
(-ig)² ... ⁴(p1+p-p2)(2π)⁴ δ⁴(p2+q-p1) i/

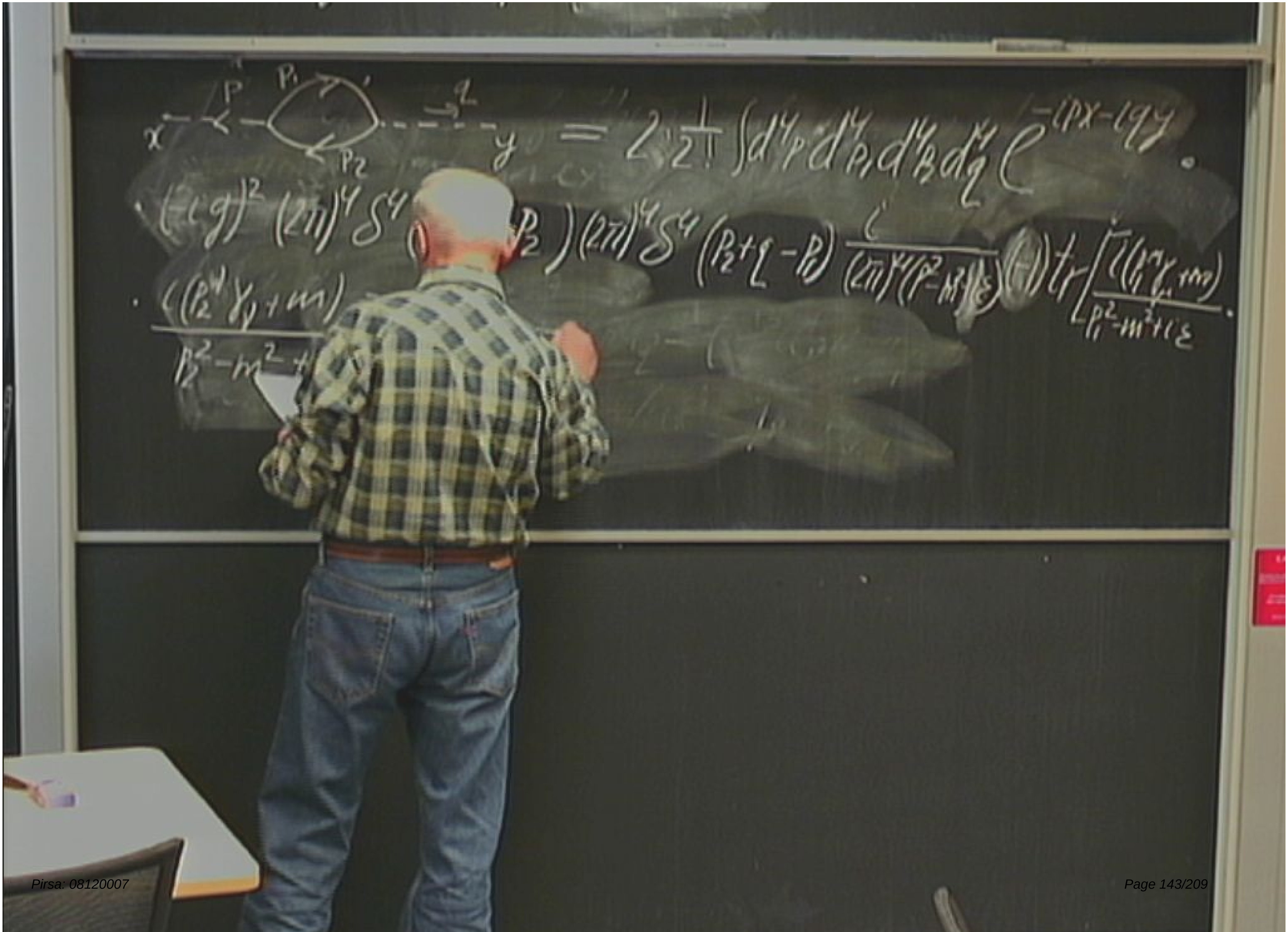

p
p1
p2
q
x
y
= i^2 1/2! ∫ d^4p d^4p1 d^4p2 d^4q e^{-ipx-iqy}
g)^2 (2π)^4 δ^4(p1+p-p2)(2π)^4 δ^4(p2+q-p1) i / (2π)^4(p^2-m^2+iε)

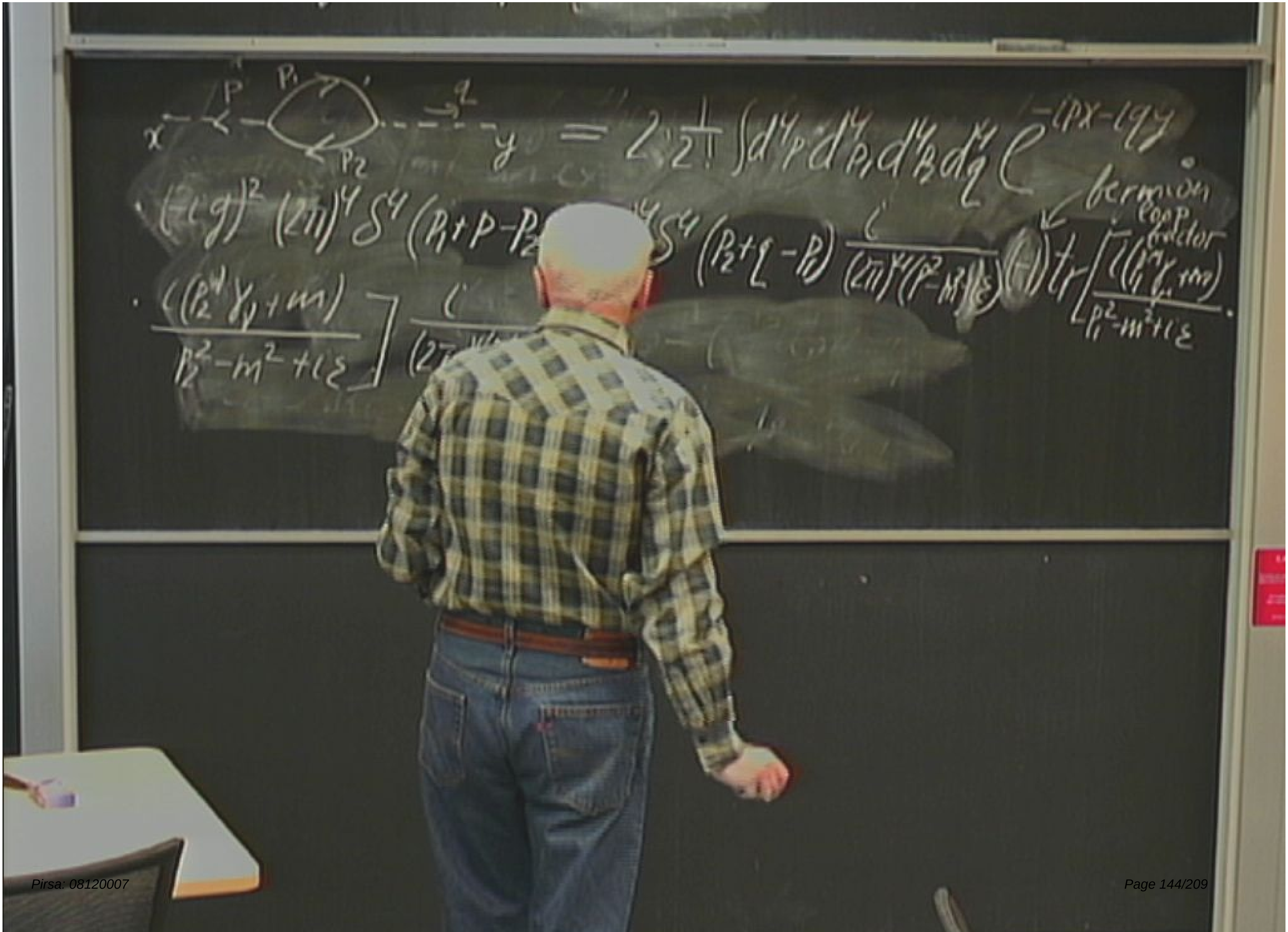


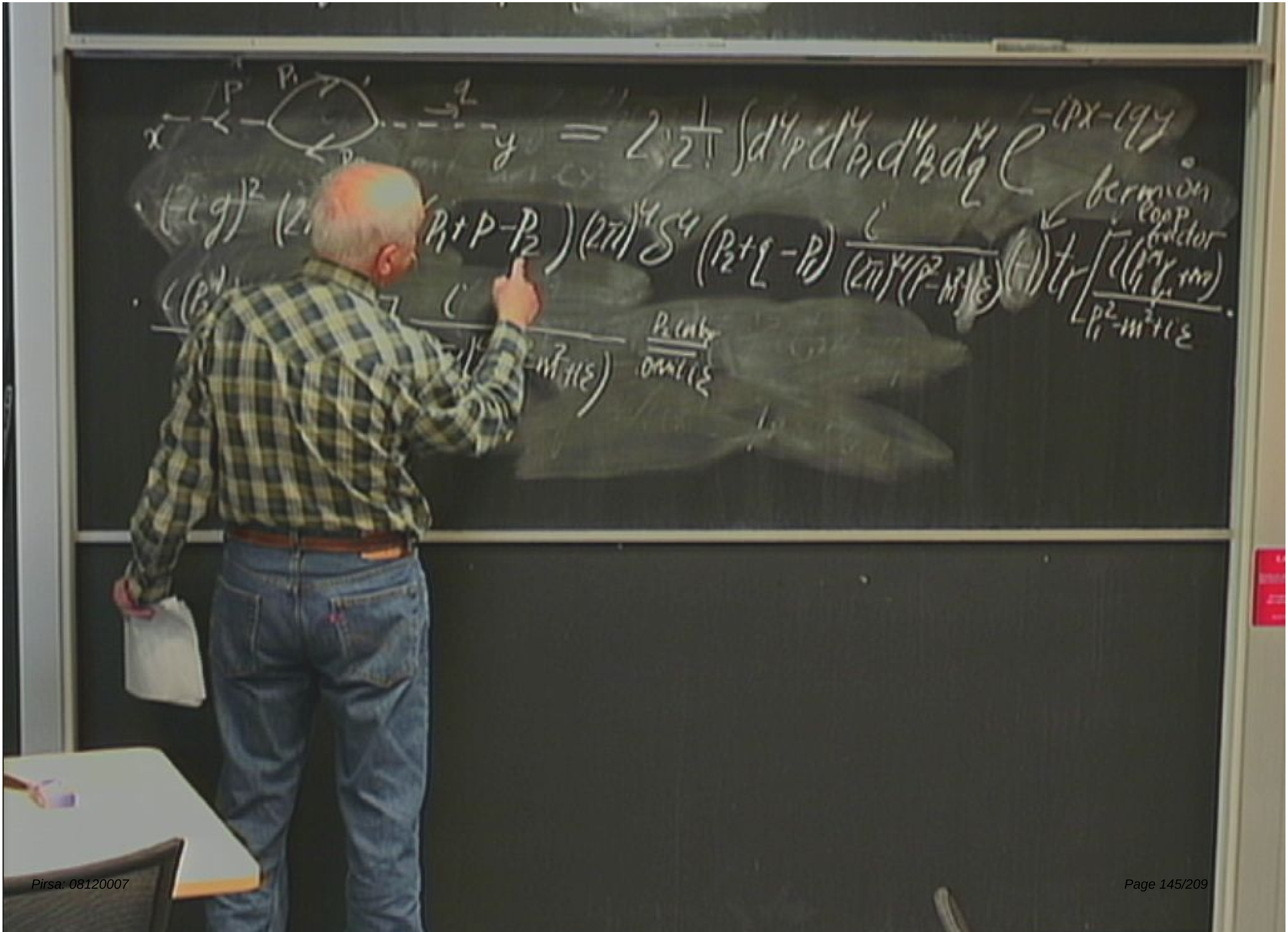
x p p_1 p_2 q y
$= i^2 \frac{1}{2!} \int d^4p\, d^4p_1\, d^4p_2\, d^4q\, e^{-ipx-iqy} \cdot$
$(-ig)^2 (2\pi)^4 \delta^4(p_1+p-p_2)(2\pi)^4 \delta^4(p_2+q-p_1) \frac{i}{(2\pi)^4(p^2-m^2+i\varepsilon)}$

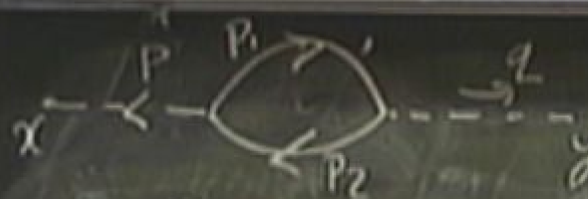

$= i\,\frac{1}{2!}\int d^4p\, d^4p_1\, d^4p_2\, d^4q\, e^{-ipx-iqy}\cdot$
$\lambda^2 (2\pi)^4\delta^4(p_1+p-p_2)(2\pi)^4\delta^4(p_2+q-p_1)\frac{i}{(2\pi)^4(p^2-M^2+i\varepsilon)}(-1)\,tr\left[\frac{i(p_1^\mu\gamma_\mu+m)}{p_1^2-m^2+i\varepsilon}\cdot\right.$


$= i\frac{1}{2!}\int d^4p\, d^4p_1\, d^4p_2\, d^4q\, e^{-ipx-iqy}\,\cdot$
$(2\pi)^4\delta^4(p_1+p-p_2)\,(2\pi)^4\delta^4(p_2+q-p_1)\,\frac{i}{(2\pi)^4(p^2-m^2+i\varepsilon)}\,(-)\,tr\left[\frac{i(p_1^\mu\gamma_\mu+m)}{p_1^2-m^2+i\varepsilon}\right.$


p
p1
p2
q
x
y
= 2 · 1/2! ∫ d^4p d^4p1 d^4p2 d^4q e^{-ipx-iqy} ·
(-ig)^2 (2π)^4 δ^4(p2) (2π)^4 δ^4(p2+q-p1) i/((2π)^4(p^2-m^2+iε)) (-1) tr[i((p1^μ γ_μ + m)/(p1^2-m^2+iε) ·
i((p2^ν γ_ν + m)/(p2^2-m^2+


fermion loop factor


fermion
loop
factor



$$= i^2\frac{1}{2!}\int d^4p\, d^4p_1\, d^4p_2\, d^4q\, e^{-ipx-iqy}\cdot$$

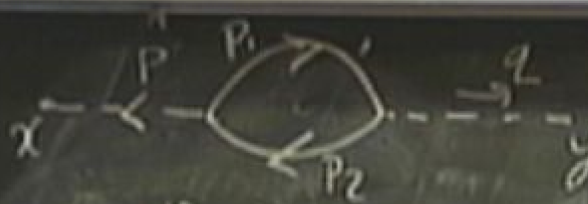
$$(-ig)^2(2\pi)^4\delta^4(p_1+p-p_2)(2\pi)^4\delta^4(p_2+q-p_1)\frac{i}{(2\pi)^4(p^2-m^2+i\epsilon)}(-1)\,tr\left[\frac{i(p_1^\mu\gamma_\mu+m)}{p_1^2-m^2+i\epsilon}\right.\cdot$$

fermion loop factor

$$\cdot\left.\frac{i(p_2^\mu\gamma_\mu+m)}{p_2^2-m^2+i\epsilon}\right]\frac{i}{(2\pi)^4(q^2-m^2+i\epsilon)}\ \underset{\text{omit } i\epsilon}{\overset{p_2\text{ int}}{=}}\ -\frac{(-ig)^2 i^4}{(2\pi)^8}\int d^4p\, d^4p_1\, d^4q\, e^{-ipx-iqy}\,\delta^4(p+q)\cdot$$

$$\frac{1}{(p^2-m^2)(q^2-m^2)}\,tr\left[\frac{p_1^\mu\gamma_\mu+m}{p_1^2-m^2}\,\frac{((p_1-q)^\mu\gamma_\mu+m)}{(p_1-q)^2-m^2}\right]\ \underset{\text{integration}}{\overset{q}{}}$$

Factor (-1) and tr for each closed fermion loop!



$$= i\,\frac{1}{2!}\int d^4p\, d^4p_1\, d^4p_2\, d^4q\; e^{-ipx-iqy}\cdot$$

$$\cdot(-ig)^2(2\pi)^4\delta^4(p_1+p-p_2)(2\pi)^4\delta^4(p_2+q-p_1)\frac{i}{(2\pi)^4(p^2-m^2+i\varepsilon)}(-1)\,\mathrm{tr}\left[\frac{i(p_1^\mu\gamma_\mu+m)}{p_1^2-m^2+i\varepsilon}\right.\cdot$$

fermion loop factor

$$\cdot\left.\frac{i(p_2^\mu\gamma_\mu+m)}{p_2^2-m^2+i\varepsilon}\right]\frac{i}{(2\pi)^4(q^2-m^2+i\varepsilon)}\;\frac{p_1\ \text{only}}{\text{omit}\ i\varepsilon}\;=\frac{(-ig)^2\, i^4}{(2\pi)^8}\int d^4p\, d^4p_1\, d^4q\; e^{-ipx-iqy}\,\delta^4(p-q)\cdot$$

$$\frac{1}{(p^2-m^2)(q^2-m^2)}\,\mathrm{tr}\left[\frac{p_1^\mu\gamma_\mu+m}{p_1^2-m^2}\;\frac{(p_1-q)^\mu\gamma_\mu+m}{(p_1-q)^2-m^2}\right]\;\frac{q}{\text{integration}}$$

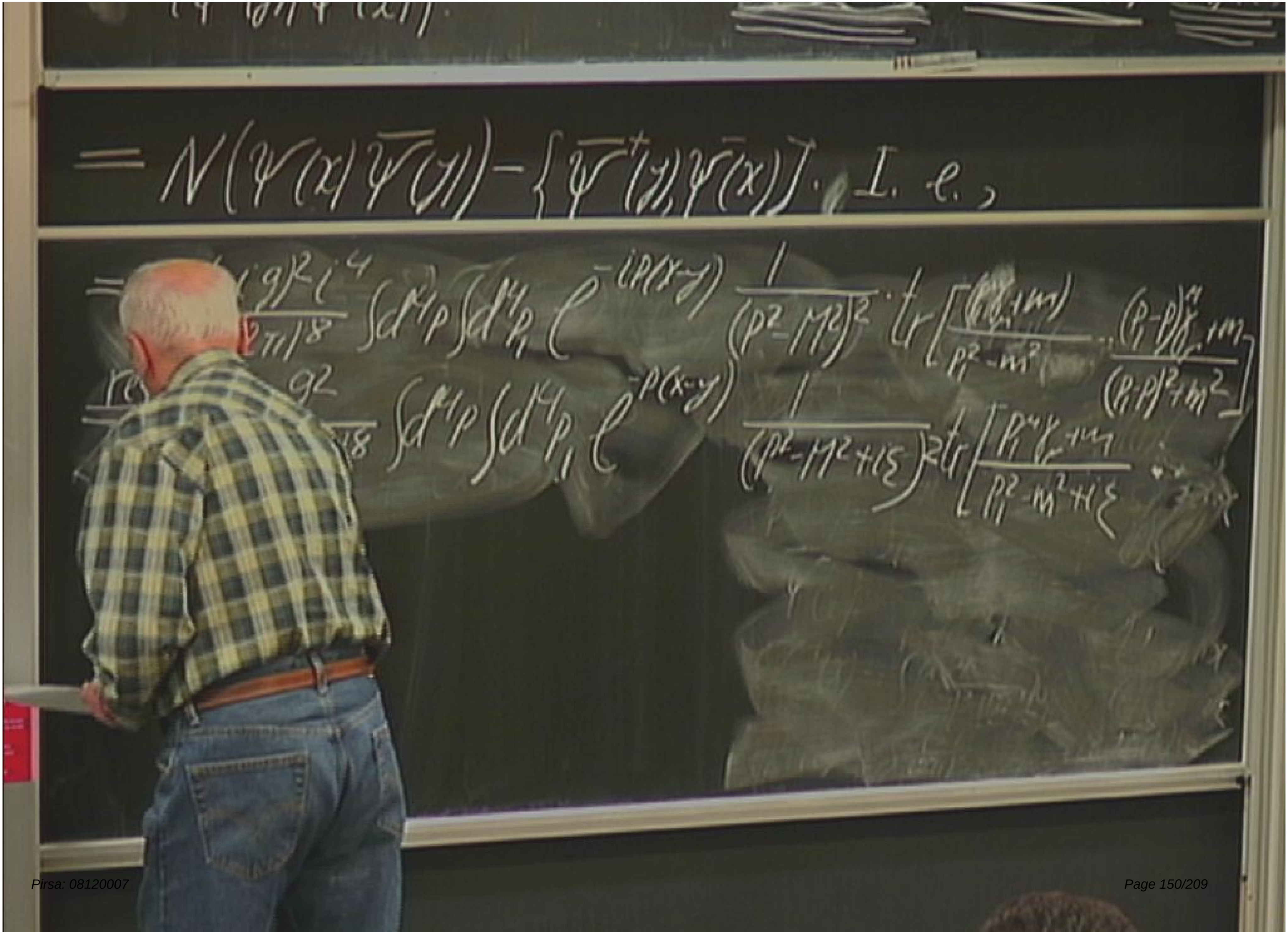
$$= N\left(\psi(x)\bar{\psi}(y)\right) - \int \left[\bar{\psi}^{\dagger}(y)\bar{\psi}(x)\right] \ldots \int e$$

$$= \int - \frac{(-ig)^2 i^4}{(2\pi)^8} \int d^4p \int d^4p_1 \, e^{-ip(x-y)}$$

$$= N(\psi(x)\bar{\psi}(y)) - \{\bar{\psi}^{+}(y), \psi^{-}(x)\}. \text{ I. e.,}$$

$$= \frac{(-ig)^2 i^4}{(2\pi)^8} \int d^4p \int d^4p_1 \, e^{-ip(x-y)} \frac{1}{(p^2-M^2)^2} \cdot tr\left[\frac{(\not p_1 + m)}{p_1^2 - m^2} \cdots\right.$$

$$\xrightarrow[i\xi]{\text{rest.}} \frac{g^2}{(2\pi)^8} \int d^4p \int d^4p_1 \, e^{-ip(x-y)} \frac{1}{(p^2-M^2+i\xi)^2} tr\left[\cdots \right.$$


= N(ψ(x)ψ̄(y)) − {ψ̄^T(y)ψ(x)}. I. e.,

$$= N(\psi(x)\bar\psi(y)) - \{\bar\psi^T(y), \psi(x)\}. \quad \text{I. e.,}$$

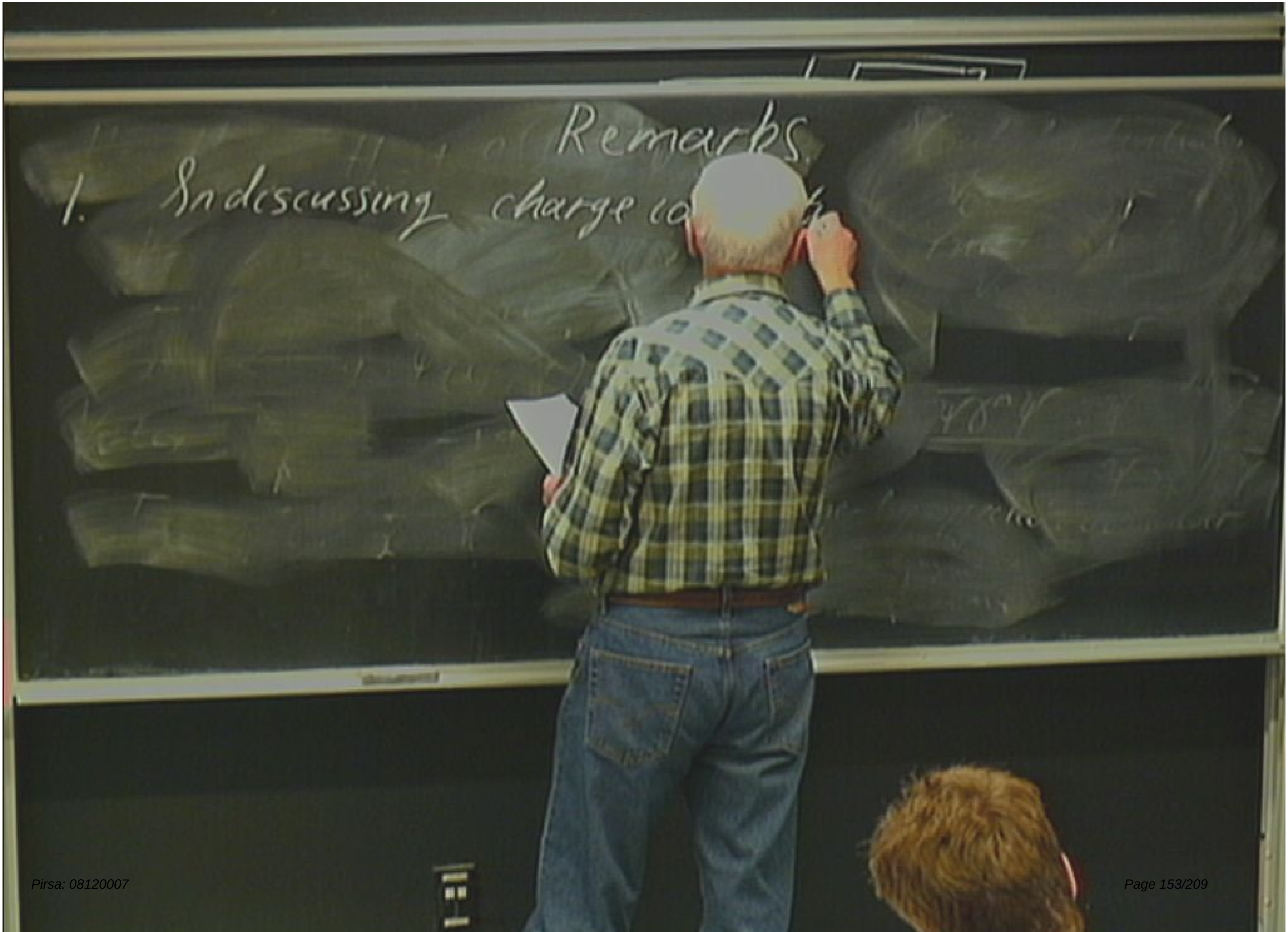
$$= \int -\frac{(-ig)^2 i^4}{(2\pi)^8} \int d^4p \int d^4p_1\, e^{-ip(x-y)} \frac{1}{(p^2-M^2)^2} \cdot tr\left[\frac{(p_1^\mu\gamma_\mu + m)}{p_1^2-m^2} \cdot \frac{(p_1-p)^\mu\gamma_\mu + m}{(p_1-p)^2+m^2}\right]$$

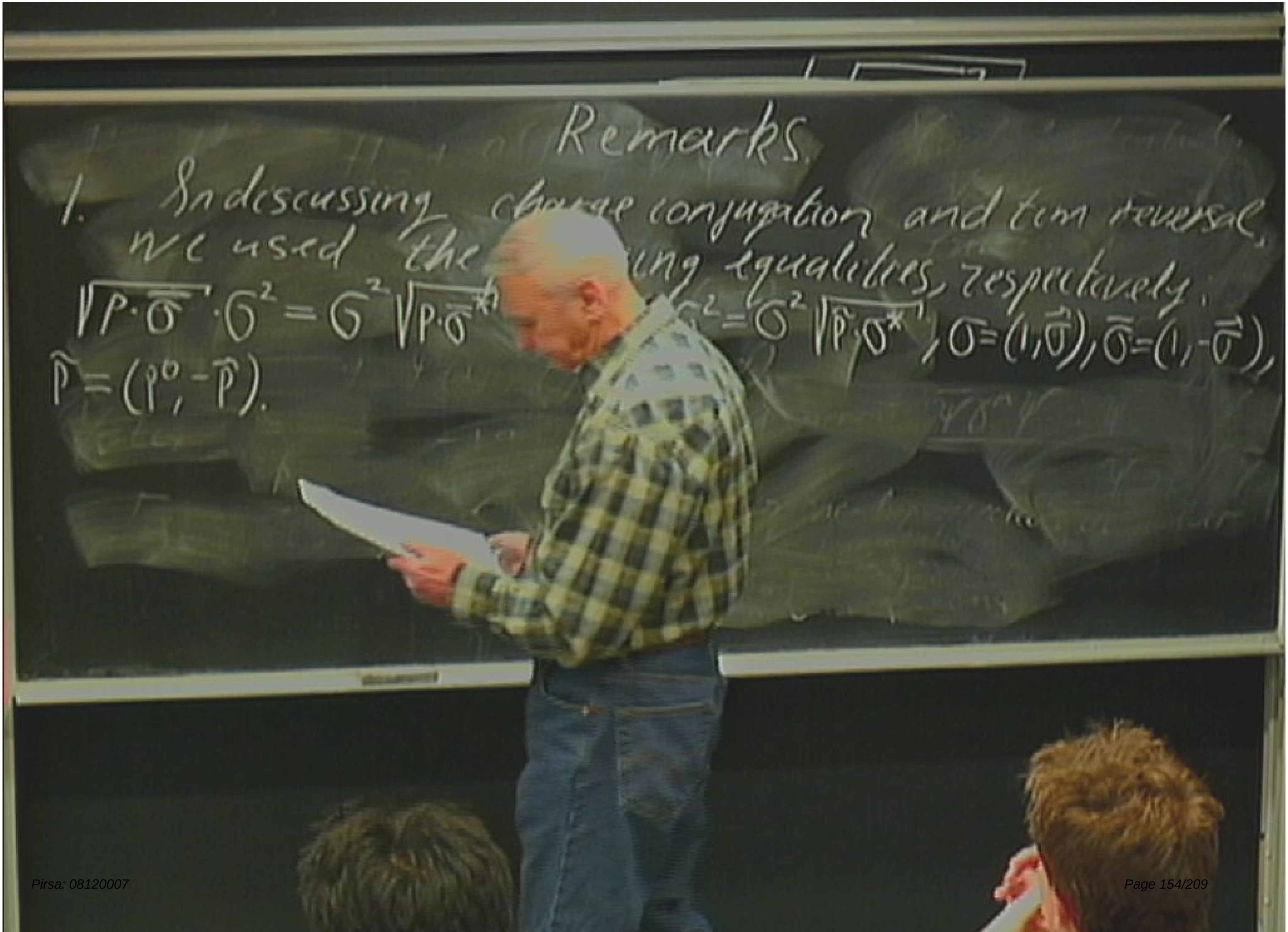
$$\frac{\text{res}(}{i\xi} \quad \frac{g^2}{(2\pi)^8} \int d^4p \int d^4p_1\, e^{-p(x-y)} \frac{1}{(p^2-M^2+i\xi)^2} tr\left[\frac{p_1^\mu\gamma_\mu + m}{p_1^2-m^2+i\xi} \cdot \frac{(p_1-p)^\mu\gamma_\mu + m}{(p_1-p)^2-m^2+i\xi}\right].$$

$$= N(\psi(x)\bar{\psi}(y)) - \{\bar{\psi}^{+}(y), \psi(x)\}. \quad \text{I. e.,}$$

$$= \left\{ -\frac{(-ig)^2 i^4}{(2\pi)^8} \int d^4p \int d^4p_1\, e^{-ip(x-y)} \frac{1}{(p^2-M^2)^2} \cdot \mathrm{tr}\left[\frac{(p_1^\mu\gamma_\mu + m)}{p_1^2 - m^2} \cdot \frac{(p_1-p)^\mu\gamma_\mu + m}{(p_1-p)^2 + m^2} \right] \right.$$

$$\frac{\text{rest}}{i\xi} \quad \frac{g^2}{(2\pi)^8} \int d^4p \int d^4p_1\, e^{-p(x-y)} \frac{1}{(p^2-M^2+i\xi)^2} \mathrm{tr}\left[\frac{p_1^\mu\gamma_\mu + m}{p_1^2 - m^2 + i\xi} \cdot \frac{(p_1-p)^\mu\gamma_\mu + m}{(p_1-p)^2 - m^2 + i\xi} \right].$$


Remarks.
1. In discussing charge co

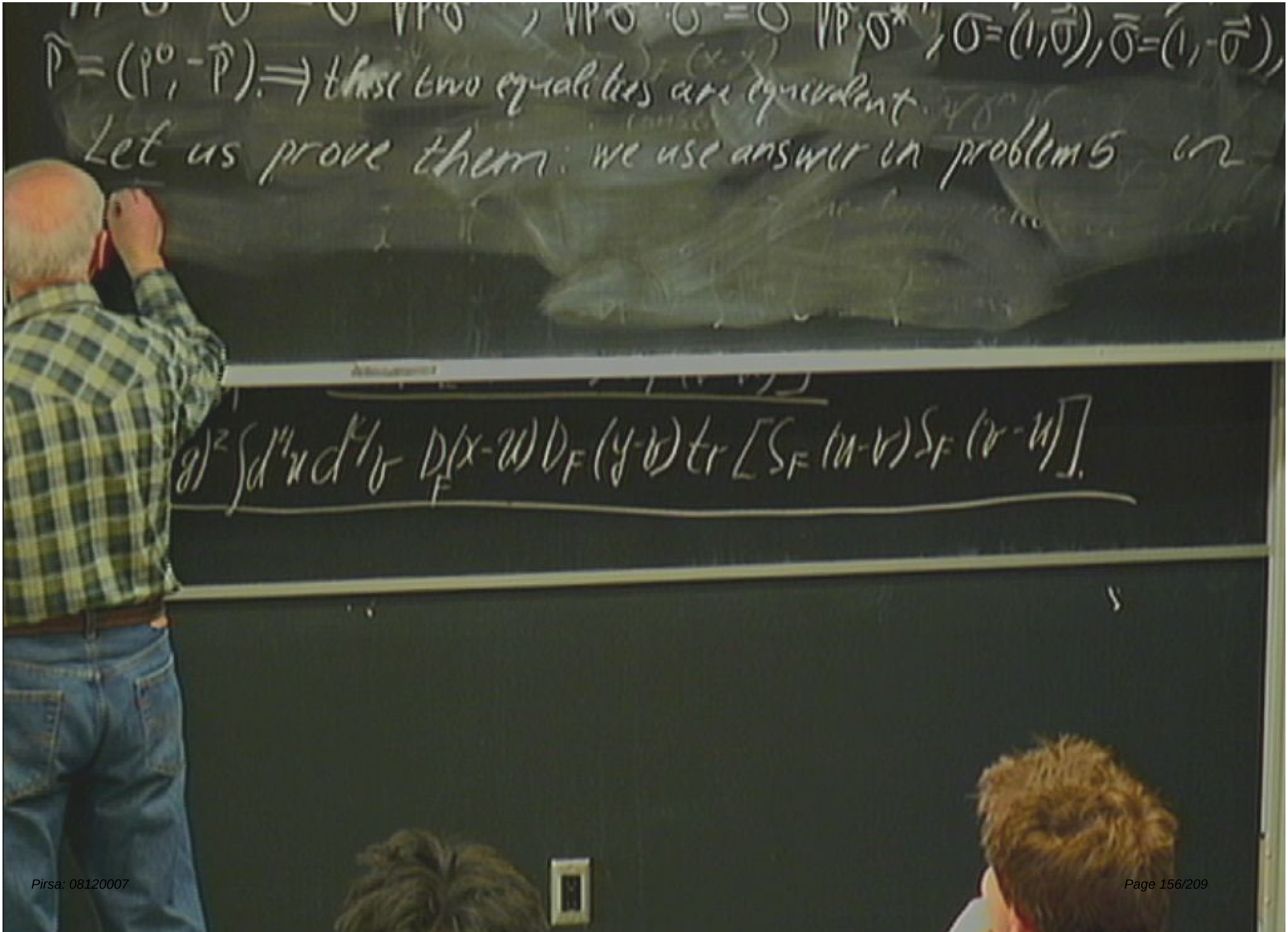

Remarks.
1. In discussing charge conjugation and time reversal,
we used the
ing equalities, respectively:
$\sqrt{p\cdot\bar{\sigma}}\cdot\sigma^2=\sigma^2\sqrt{p\cdot\bar{\sigma}^*}$
$\sigma^2=\sigma^2\sqrt{\tilde{p}\cdot\sigma^*}$, $\sigma=(1,\vec{\sigma})$, $\bar{\sigma}=(1,-\vec{\sigma})$,
$\tilde{p}=(p^0,-\vec{p})$.

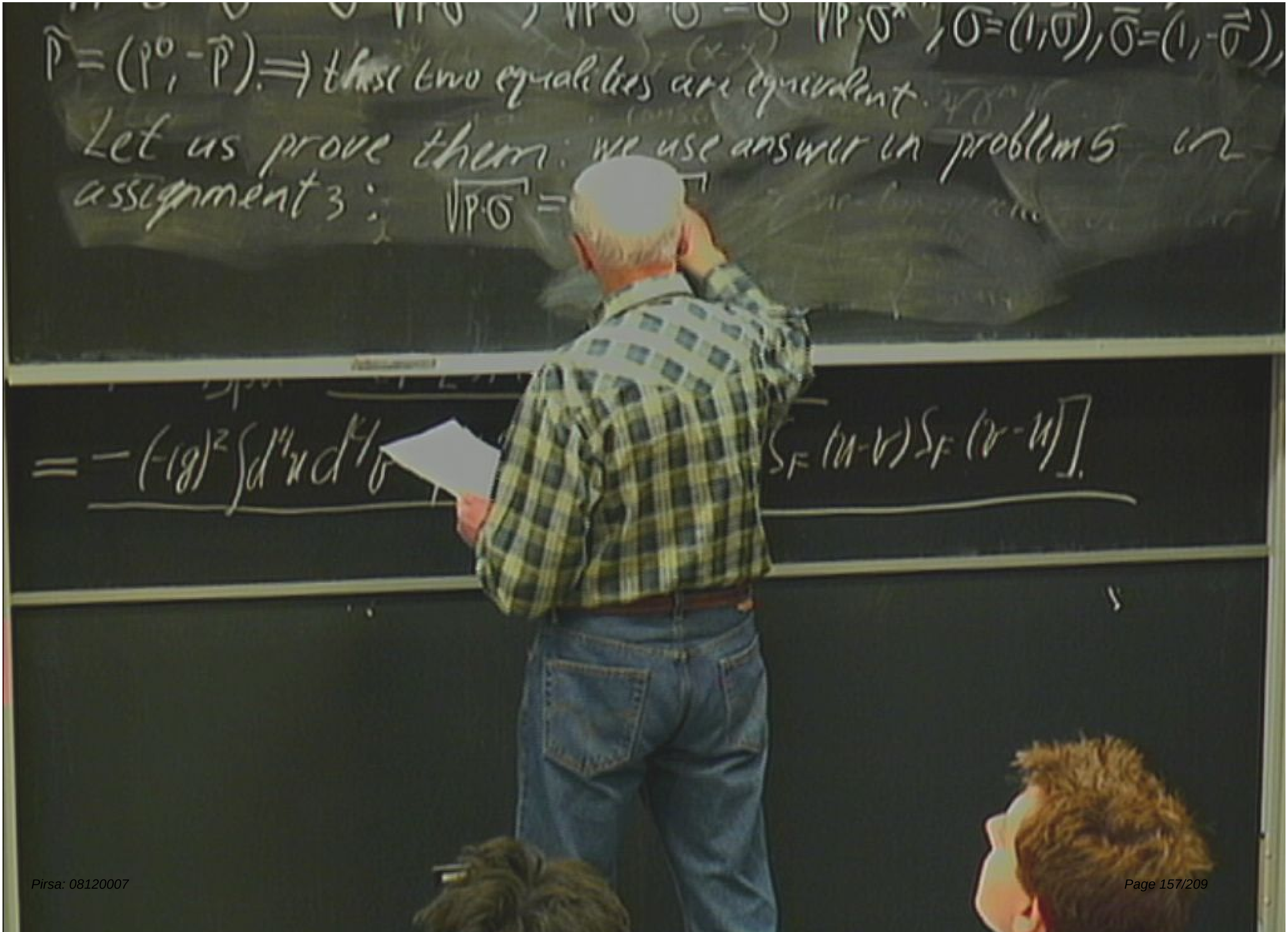
Remarks.

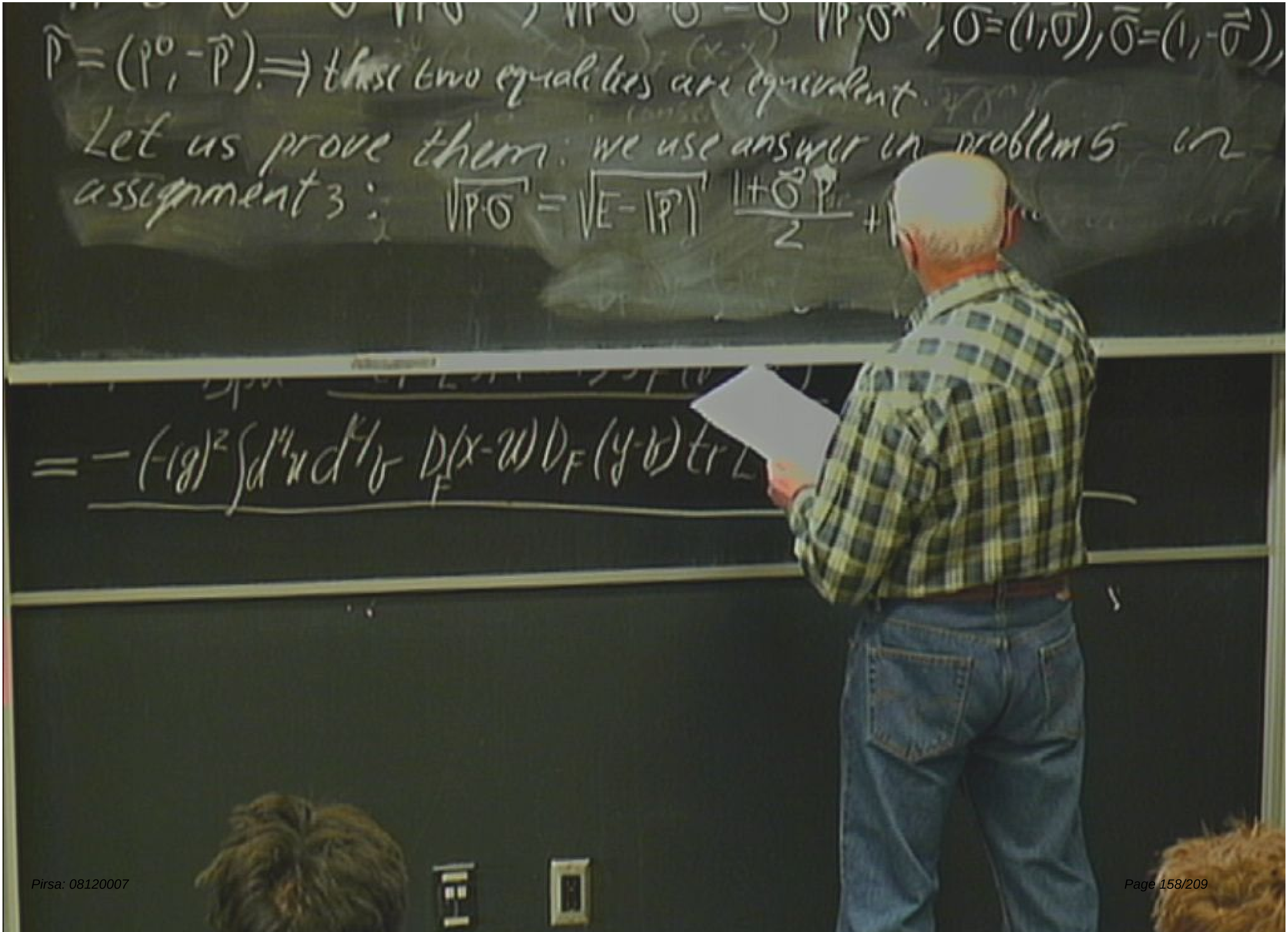
1. In discussing charge conjugation and time reversal, we used the following equalities, respectively:

$$\sqrt{p\cdot\bar{\sigma}}\cdot\sigma^2=\sigma^2\sqrt{p\cdot\bar{\sigma}^*}\,,\quad \sqrt{p\cdot\sigma}\,\sigma^2=\sigma^2\sqrt{\tilde{p}\cdot\sigma^*}\,,\quad \sigma=(1,\vec{\sigma}),\ \bar{\sigma}=(1,-\vec{\sigma}),$$

$\tilde{p}=(p^0,-\vec{p})$. $\Rightarrow$ these two equalities are equivalent


σ=(1,σ⃗), σ̄=(1,-σ⃗),
p̃=(p⁰,-p⃗). ⇒ these two equalities are equivalent.
Let us prove them: we use answer in problems in
(g)² ∫d⁴u d⁴v D_F(x-u) D_F(y-v) tr[S_F(u-v) S_F(v-u)].


σ=(1,σ⃗), σ̄=(1,−σ⃗),
p̃=(p⁰,−p⃗) ⇒ these two equalities are equivalent.
Let us prove them: we use answer in problems in
assignment 3: √(p·σ) =



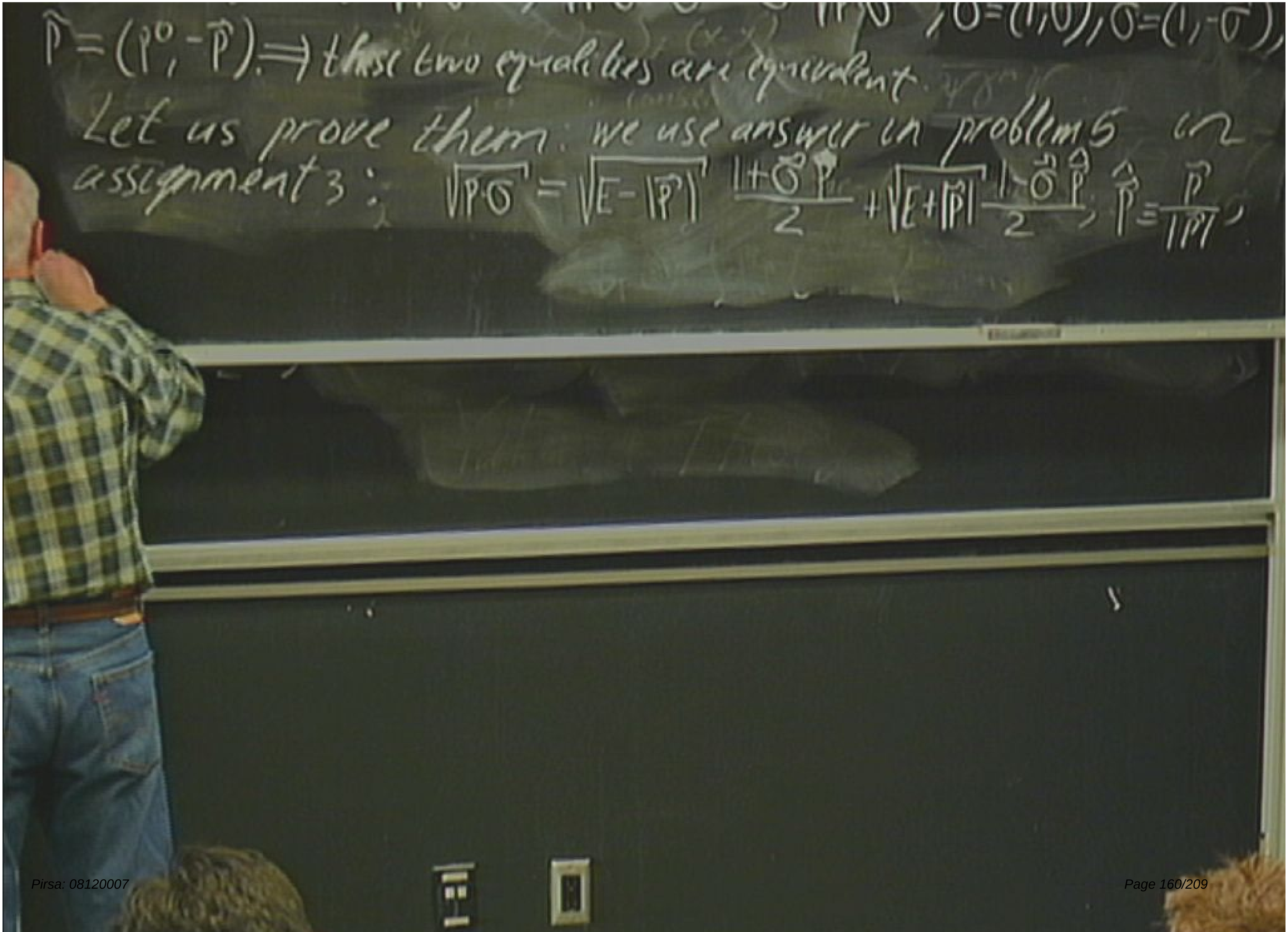
p̃ = (p⁰, −p⃗) ⟹ these two equalities are equivalent.
Let us prove them: we use answer in problem 5 in
assignment 3:

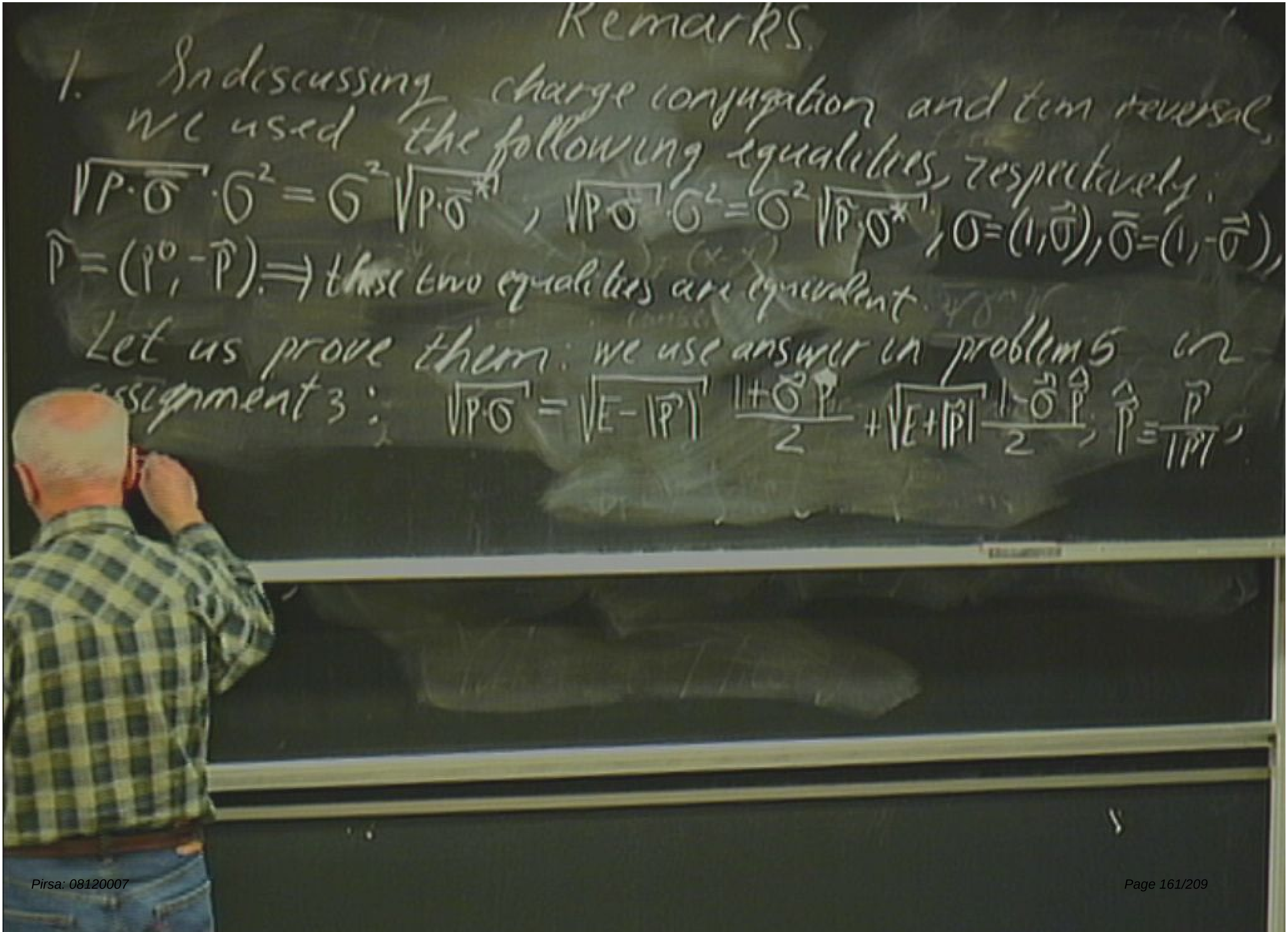
Let us prove them: we use answer in problem 5 in assignment 3:

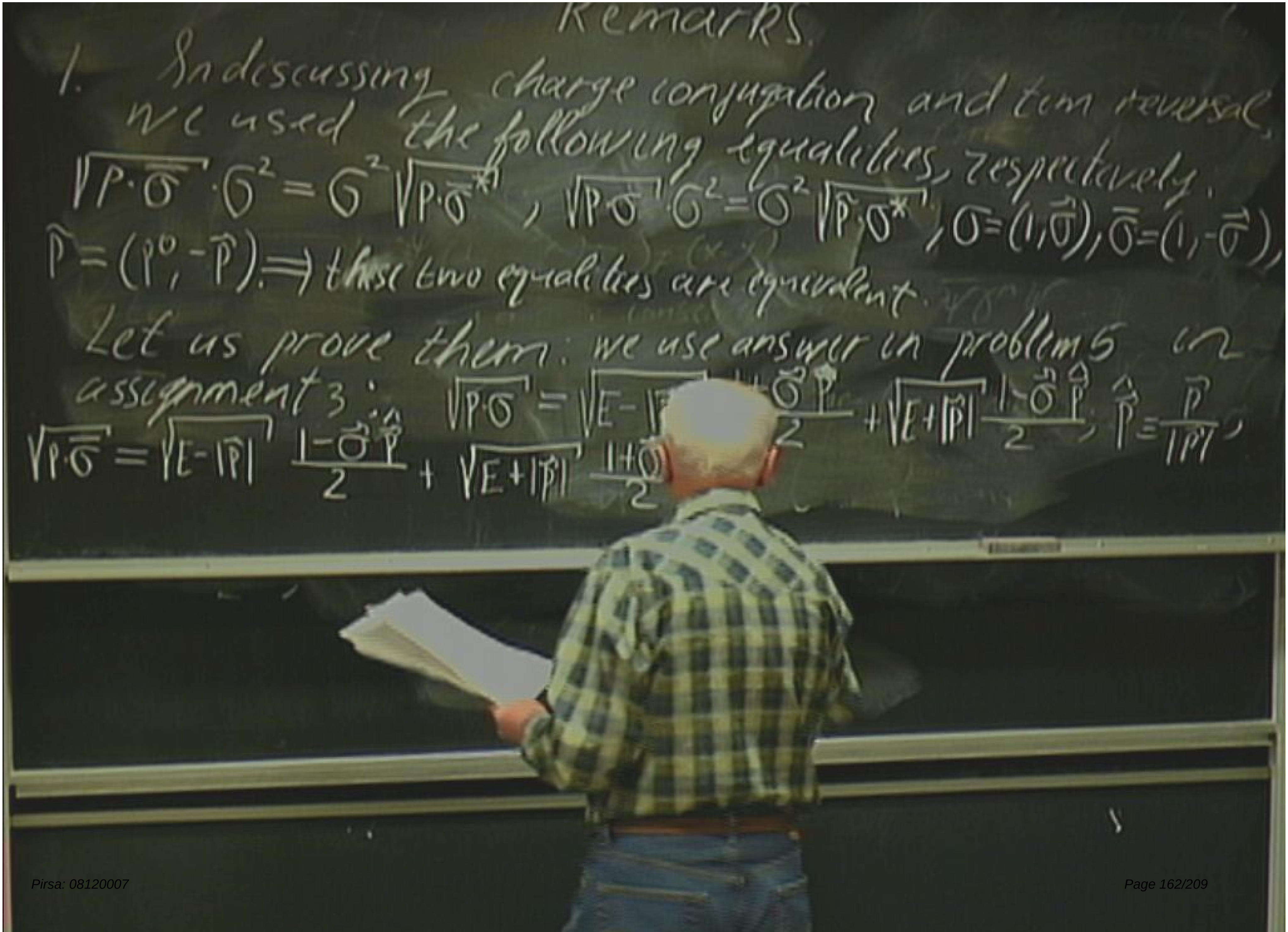
$$\sqrt{p\cdot\sigma} = \sqrt{E-|\vec{p}|}\,\frac{1+\vec{\sigma}\cdot\hat{p}}{2} + \sqrt{E+|\vec{p}|}\,\frac{1-\vec{\sigma}\cdot\hat{p}}{2}, \quad \hat{p} = \frac{\vec{p}}{|\vec{p}|}$$

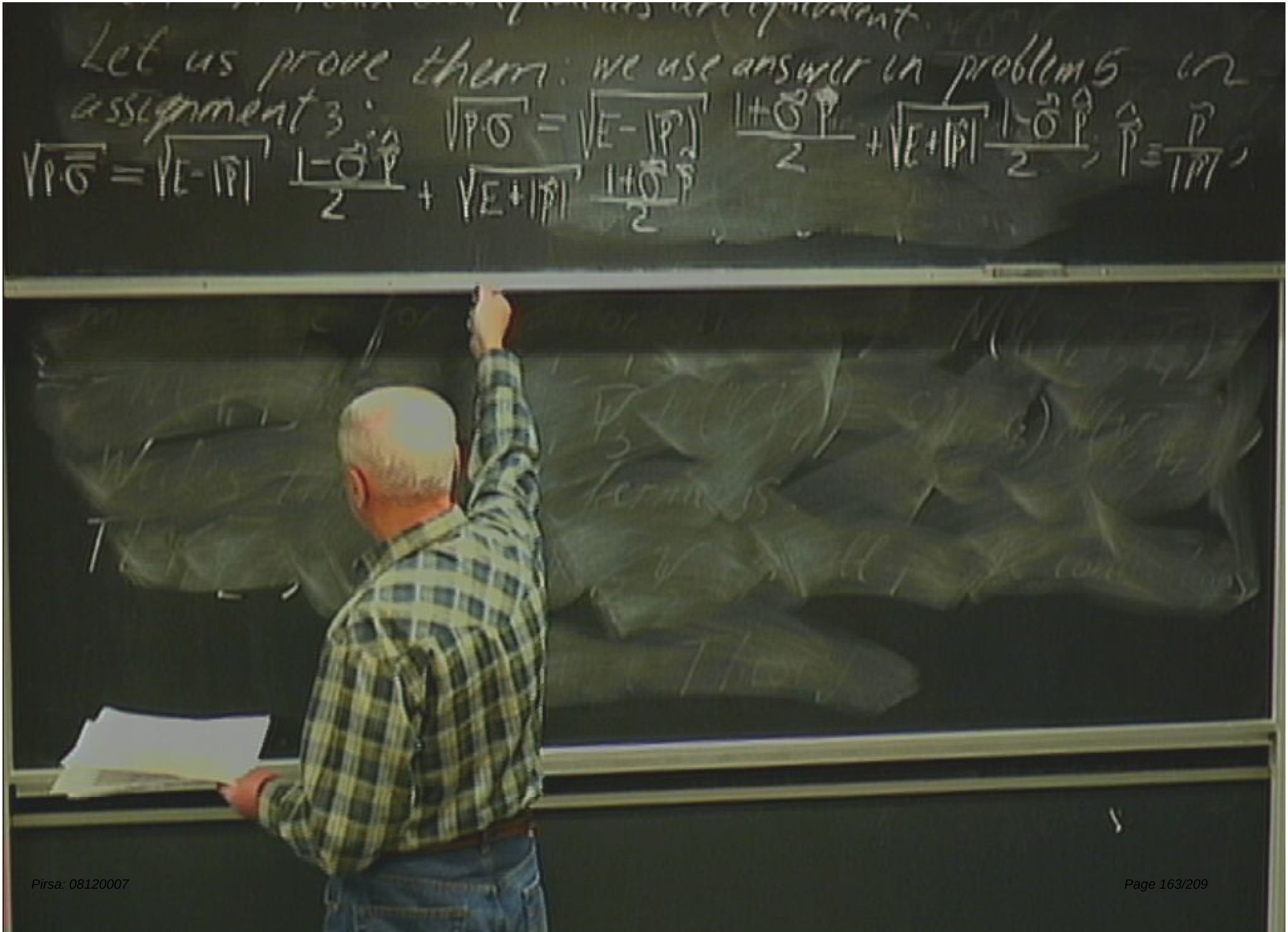
$$[S_F(v-u)]_{\beta\alpha} = \underline{-\mathrm{tr}\,[S_F(u-v)\,S_F(v-u)]}$$

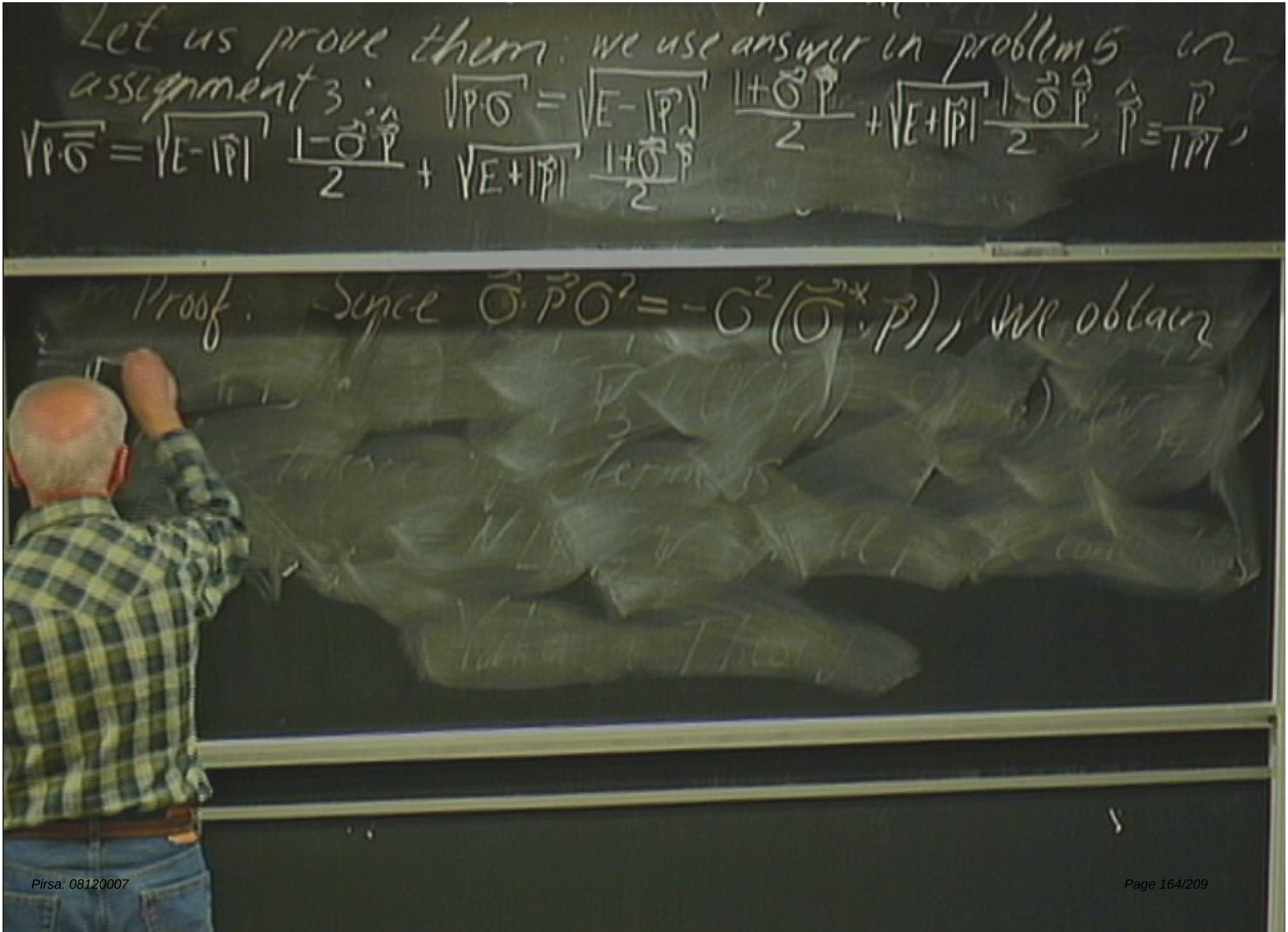
$$\underline{= -(-ig)^2\int d^4u\, d^4v\; D_F(x-u)\, D_F(y-v)\, \mathrm{tr}\,[S_F(u-v)\,S_F(v-u)]}.$$

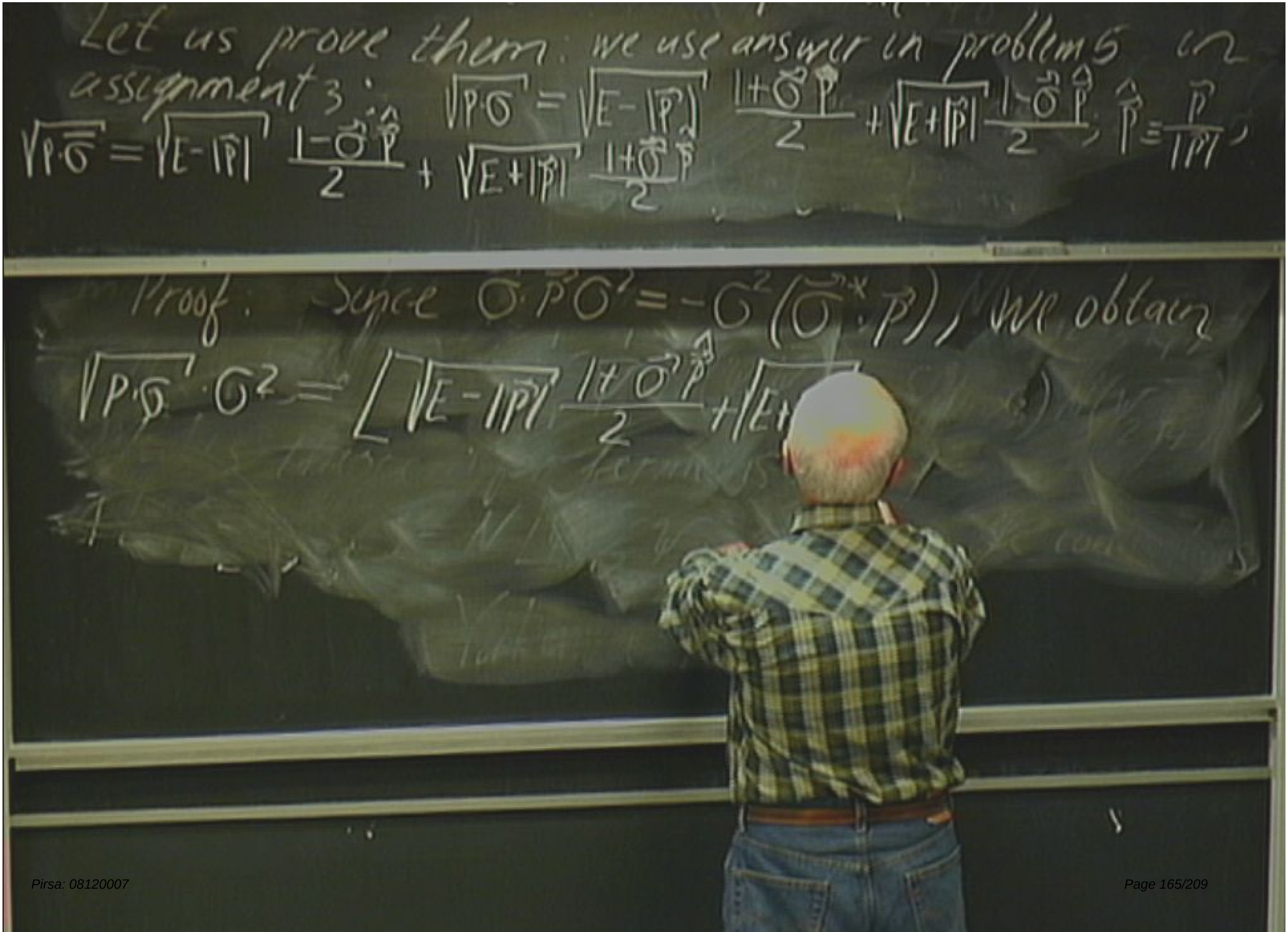

$\sigma=(1,\vec{\sigma}),\ \bar{\sigma}=(1,-\vec{\sigma}),$
$\tilde{p}=(p^0,-\vec{p}) \Rightarrow$ these two equalities are equivalent.
Let us prove them: we use answer in problem 5 in
assignment 3: $\sqrt{p\cdot\sigma}=\sqrt{E-|\vec{p}|}\,\frac{1+\vec{\sigma}\hat{p}}{2}+\sqrt{E+|\vec{p}|}\,\frac{1-\vec{\sigma}\hat{p}}{2},\ \hat{p}=\frac{\vec{p}}{|\vec{p}|},$


Remarks.
1. In discussing charge conjugation and time reversal, we used the following equalities, respectively:
$\sqrt{p\cdot\bar{\sigma}}\cdot\sigma^2=\sigma^2\sqrt{p\cdot\bar{\sigma}^*}$, $\sqrt{p\cdot\sigma}\cdot\sigma^2=\sigma^2\sqrt{\tilde{p}\cdot\sigma^*}$, $\sigma=(1,\vec{\sigma})$, $\bar{\sigma}=(1,-\vec{\sigma})$,
$\tilde{p}=(p^0,-\vec{p})$. $\Rightarrow$ these two equalities are equivalent.
Let us prove them: we use answer in problem 5 in assignment 3:
$\sqrt{p\cdot\sigma}=\sqrt{E-|\vec{p}|}\,\frac{1+\vec{\sigma}\hat{p}}{2}+\sqrt{E+|\vec{p}|}\,\frac{1-\vec{\sigma}\hat{p}}{2}$, $\hat{p}=\frac{\vec{p}}{|\vec{p}|}$,

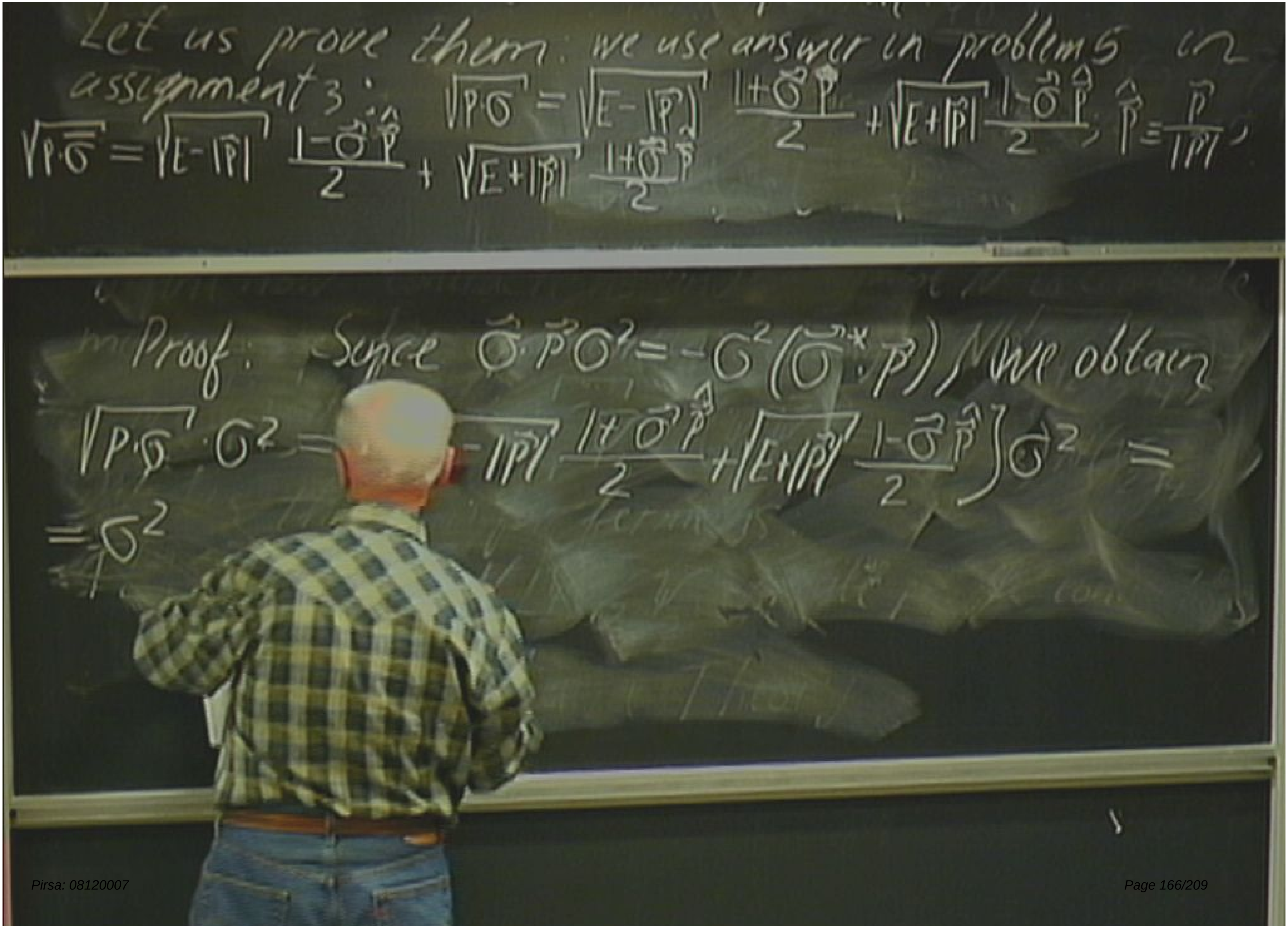

Remarks.
1. In discussing charge conjugation and time reversal, we used the following equalities, respectively:
$\sqrt{p\cdot\bar\sigma}\cdot\sigma^2=\sigma^2\sqrt{p\cdot\sigma^*}$, $\sqrt{p\cdot\sigma}\cdot\sigma^2=\sigma^2\sqrt{\tilde p\cdot\sigma^*}$, $\sigma=(1,\vec\sigma)$, $\bar\sigma=(1,-\vec\sigma)$,
$\tilde p=(p^0,-\vec p)$. $\Longrightarrow$ these two equalities are equivalent.
Let us prove them: we use answer in problems 5 in assignment 3:
$\sqrt{p\cdot\sigma}=\sqrt{E-|\vec p|}\,\frac{1+\vec\sigma\cdot\hat p}{2}+\sqrt{E+|\vec p|}\,\frac{1-\vec\sigma\cdot\hat p}{2}$, $\hat p=\frac{\vec p}{|\vec p|}$,
$\sqrt{p\cdot\bar\sigma}=\sqrt{E-|\vec p|}\,\frac{1-\vec\sigma\cdot\hat p}{2}+\sqrt{E+|\vec p|}\,\frac{1+\vec\sigma\cdot\hat p}{2}$

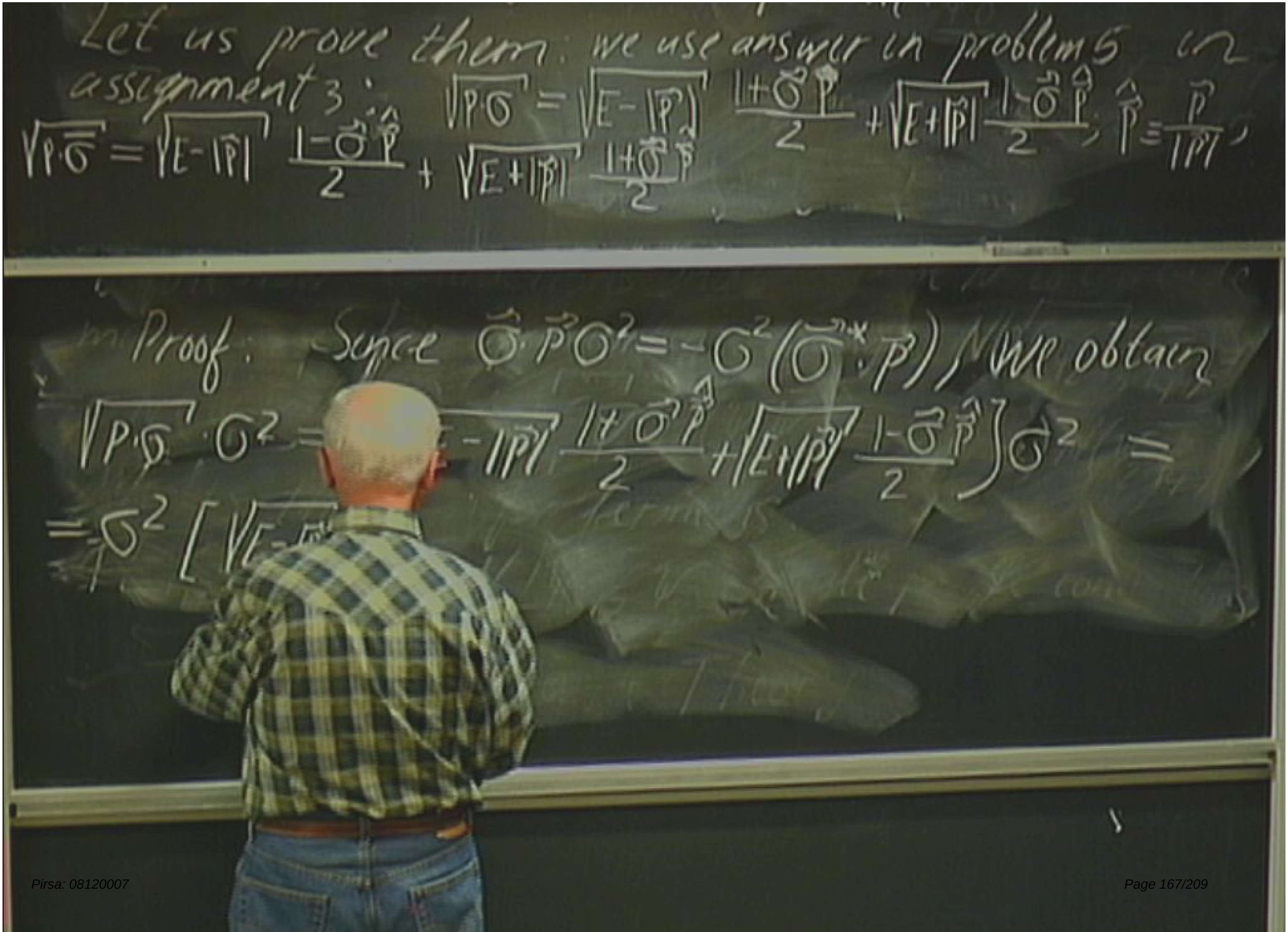

Let us prove them: we use answer in problems in assignment 3:
$\sqrt{p\cdot\sigma} = \sqrt{E-|\vec{p}|}\,\frac{1+\vec{\sigma}\cdot\hat{p}}{2} + \sqrt{E+|\vec{p}|}\,\frac{1-\vec{\sigma}\cdot\hat{p}}{2}$; $\hat{p} = \frac{\vec{p}}{|\vec{p}|}$,
$\sqrt{p\cdot\bar{\sigma}} = \sqrt{E-|\vec{p}|}\,\frac{1-\vec{\sigma}\cdot\hat{p}}{2} + \sqrt{E+|\vec{p}|}\,\frac{1+\vec{\sigma}\cdot\hat{p}}{2}$

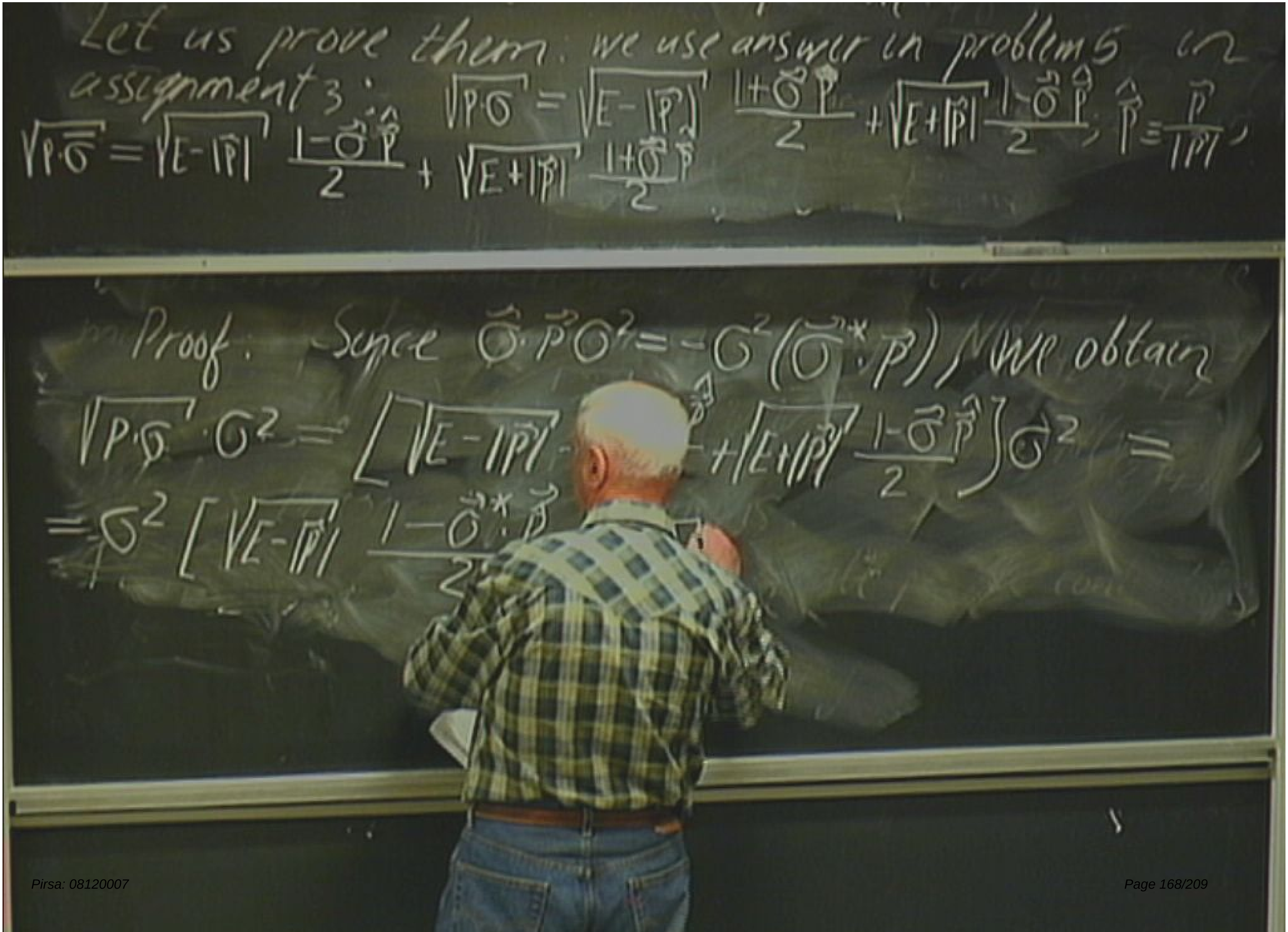

Let us prove them: we use answer in problem 5 in assignment 3:
$\sqrt{p\cdot\sigma} = \sqrt{E-|\vec{p}|}\,\frac{1+\vec{\sigma}\hat{p}}{2} + \sqrt{E+|\vec{p}|}\,\frac{1-\vec{\sigma}\hat{p}}{2}$; $\hat{p} = \frac{\vec{p}}{|\vec{p}|}$,
$\sqrt{p\cdot\bar{\sigma}} = \sqrt{E-|\vec{p}|}\,\frac{1-\vec{\sigma}\hat{p}}{2} + \sqrt{E+|\vec{p}|}\,\frac{1+\vec{\sigma}\hat{p}}{2}$
Proof: Since $\vec{\sigma}\cdot\vec{p}\,\sigma^2 = -\sigma^2(\vec{\sigma}^*\cdot\vec{p})$, we obtain

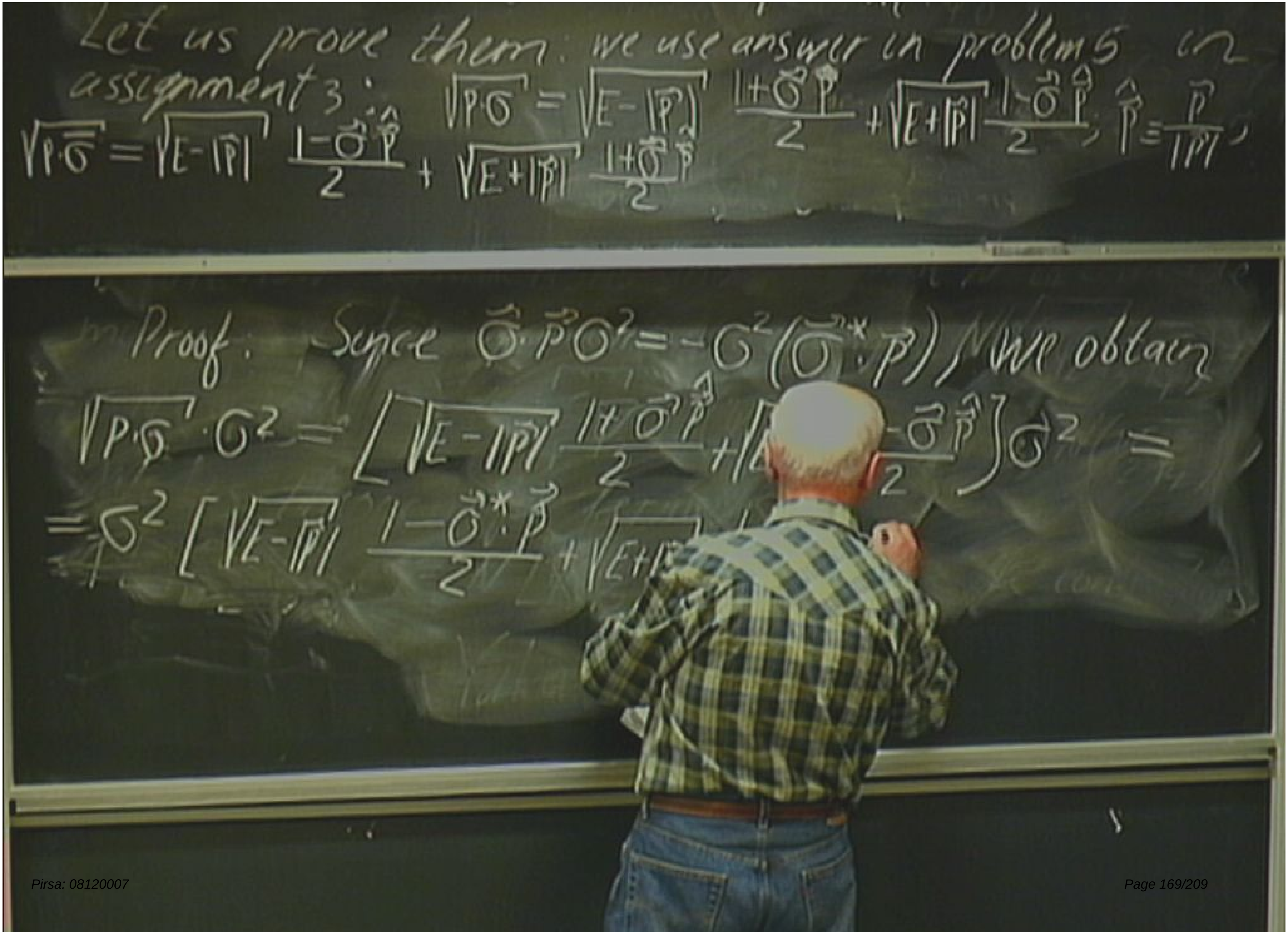


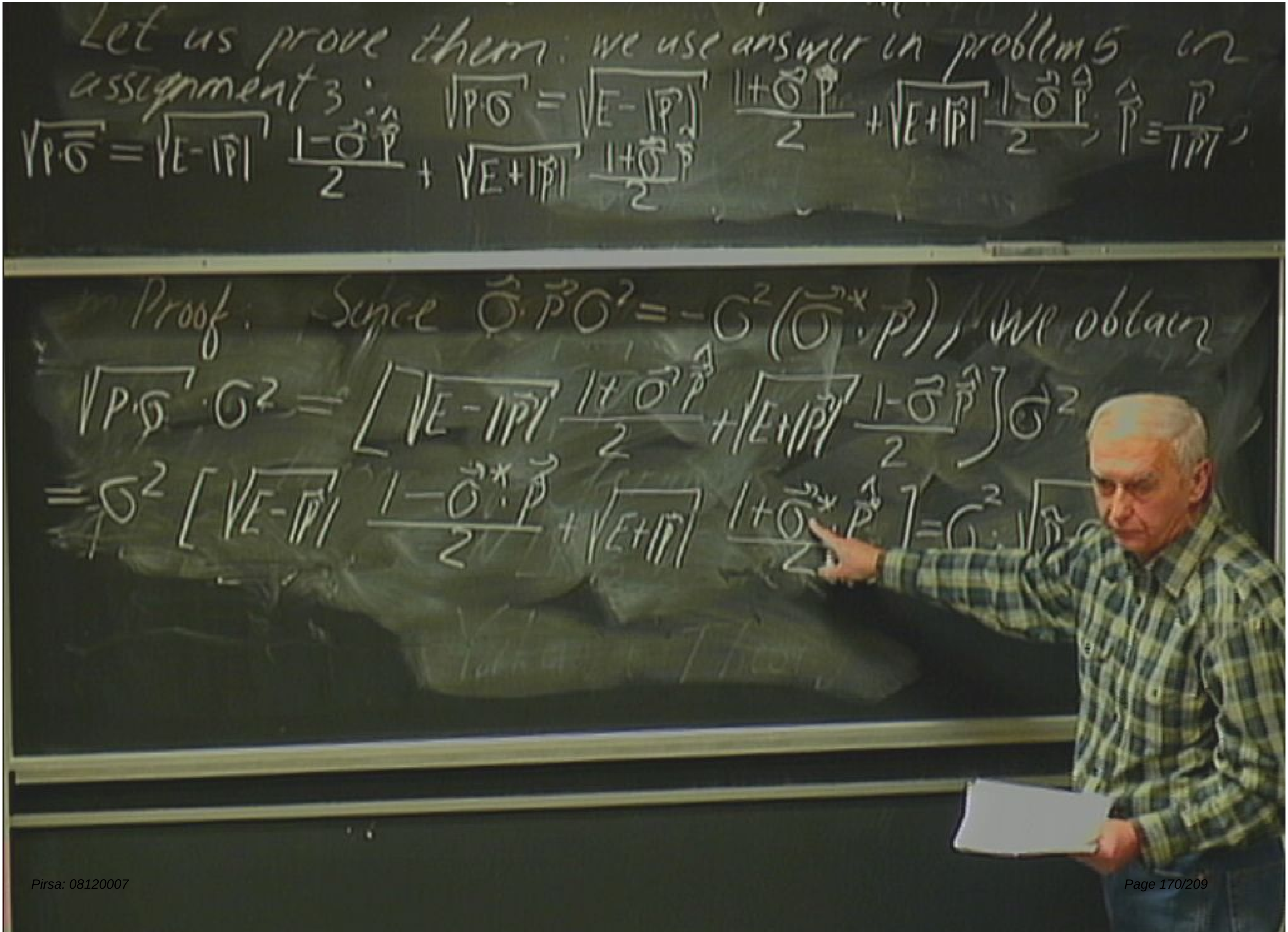
Let us prove them: we use answer in problem 5 in assignment 3: $\sqrt{p\cdot\sigma} = \sqrt{E-|\vec{p}|}\,\frac{1+\vec{\sigma}\cdot\hat{p}}{2} + \sqrt{E+|\vec{p}|}\,\frac{1-\vec{\sigma}\cdot\hat{p}}{2}$; $\hat{p} = \frac{\vec{p}}{|\vec{p}|}$,
$\sqrt{p\cdot\bar{\sigma}} = \sqrt{E-|\vec{p}|}\,\frac{1-\vec{\sigma}\cdot\hat{p}}{2} + \sqrt{E+|\vec{p}|}\,\frac{1+\vec{\sigma}\cdot\hat{p}}{2}$
Proof: Since $\vec{\sigma}\cdot\vec{p}\,\sigma^2 = -\sigma^2(\vec{\sigma}^{\,*}\cdot\vec{p})$, we obtain
$\sqrt{p\cdot\sigma}\cdot\sigma^2 = \Big[\sqrt{E-|\vec{p}|}\,\frac{1+\vec{\sigma}\cdot\hat{p}}{2} + \sqrt{E+}$


Let us prove them: we use answer in problems in assignment 3:
$\sqrt{p\cdot\sigma} = \sqrt{E-|\vec{p}|}\,\frac{1+\vec{\sigma}\cdot\hat{p}}{2} + \sqrt{E+|\vec{p}|}\,\frac{1-\vec{\sigma}\cdot\hat{p}}{2}$; $\hat{p} = \frac{\vec{p}}{|\vec{p}|}$,
$\sqrt{p\cdot\bar{\sigma}} = \sqrt{E-|\vec{p}|}\,\frac{1-\vec{\sigma}\cdot\hat{p}}{2} + \sqrt{E+|\vec{p}|}\,\frac{1+\vec{\sigma}\cdot\hat{p}}{2}$
Proof: Since $\vec{\sigma}\cdot\vec{p}\,\sigma^2 = -\sigma^2(\vec{\sigma}^*\cdot\vec{p})$, we obtain
$\sqrt{p\cdot\sigma}\cdot\sigma^2 =$
$|\vec{p}|}\,\frac{1+\vec{\sigma}\cdot\hat{p}}{2} + \sqrt{E+|\vec{p}|}\,\frac{1-\vec{\sigma}\cdot\hat{p}}{2}]\,\sigma^2 =$
$= \sigma^2$


Let us prove them: we use answer in problems in
assignment 3: $\sqrt{p\cdot\sigma} = \sqrt{E-|\vec{p}|}\,\frac{1+\vec{\sigma}\cdot\hat{p}}{2} + \sqrt{E+|\vec{p}|}\,\frac{1-\vec{\sigma}\cdot\hat{p}}{2}$, $\hat{p} = \frac{\vec{p}}{|\vec{p}|}$,
$\sqrt{p\cdot\bar{\sigma}} = \sqrt{E-|\vec{p}|}\,\frac{1-\vec{\sigma}\cdot\hat{p}}{2} + \sqrt{E+|\vec{p}|}\,\frac{1+\vec{\sigma}\cdot\hat{p}}{2}$
Proof: Since $\vec{\sigma}\cdot\vec{p}\,\sigma^2 = -\sigma^2(\vec{\sigma}^*\cdot\vec{p})$, we obtain
$\sqrt{p\cdot\sigma}\cdot\sigma^2 =$


Let us prove them: we use answer in problem 5 in assignment 3:
$\sqrt{p\cdot\sigma} = \sqrt{E-|\vec p|}\,\frac{1+\vec\sigma\hat p}{2} + \sqrt{E+|\vec p|}\,\frac{1-\vec\sigma\hat p}{2}; \quad \hat p = \frac{\vec p}{|\vec p|},$
$\sqrt{p\cdot\bar\sigma} = \sqrt{E-|\vec p|}\,\frac{1-\vec\sigma\hat p}{2} + \sqrt{E+|\vec p|}\,\frac{1+\vec\sigma\hat p}{2}$
Proof: Since $\vec\sigma\cdot\vec p\,\sigma^2 = -\sigma^2(\vec\sigma^*\cdot\vec p)$, we obtain
$\sqrt{p\cdot\sigma}\cdot\sigma^2 = \Big[\sqrt{E-|\vec p|}$
$+\sqrt{E+|\vec p|}\,\frac{1-\vec\sigma\vec p}{2}\Big]\sigma^2 =$
$= \sigma^2\Big[\sqrt{E-|\vec p|}\,\frac{1-\vec\sigma^*\cdot\vec p}{2}$


Let us prove them: we use answer in problem 5 in
assignment 3: $\sqrt{p\cdot\sigma} = \sqrt{E-|\vec p|}\,\frac{1+\vec\sigma\cdot\hat p}{2} + \sqrt{E+|\vec p|}\,\frac{1-\vec\sigma\cdot\hat p}{2}$; $\hat p = \frac{\vec p}{|\vec p|}$,
$\sqrt{p\cdot\bar\sigma} = \sqrt{E-|\vec p|}\,\frac{1-\vec\sigma\cdot\hat p}{2} + \sqrt{E+|\vec p|}\,\frac{1+\vec\sigma\cdot\hat p}{2}$
Proof: Since $\vec\sigma\cdot\vec p\,\sigma^2 = -\sigma^2(\vec\sigma^*\cdot\vec p)$, we obtain
$\sqrt{p\cdot\sigma}\cdot\sigma^2 = \Big[\sqrt{E-|\vec p|}\,\frac{1+\vec\sigma\cdot\hat p}{2} + \ldots\frac{1-\vec\sigma\cdot\hat p}{2}\Big]\sigma^2 =$
$= \sigma^2\Big[\sqrt{E-|\vec p|}\,\frac{1-\vec\sigma^*\cdot\hat p}{2} + \sqrt{E+|\vec p|}\ldots$


Let us prove them: we use answer in problem 5 in assignment 3: $\sqrt{p\cdot\sigma} = \sqrt{E-|\vec p|}\,\frac{1+\vec\sigma\cdot\hat p}{2} + \sqrt{E+|\vec p|}\,\frac{1-\vec\sigma\cdot\hat p}{2}$, $\hat p = \frac{\vec p}{|\vec p|}$,
$\sqrt{p\cdot\bar\sigma} = \sqrt{E-|\vec p|}\,\frac{1-\vec\sigma\cdot\hat p}{2} + \sqrt{E+|\vec p|}\,\frac{1+\vec\sigma\cdot\hat p}{2}$
Proof: Since $\vec\sigma\cdot\vec p\,\sigma^2 = -\sigma^2(\vec\sigma^*\cdot\vec p)$, we obtain
$\sqrt{p\cdot\sigma}\cdot\sigma^2 = \left[\sqrt{E-|\vec p|}\,\frac{1+\vec\sigma\cdot\hat p}{2} + \sqrt{E+|\vec p|}\,\frac{1-\vec\sigma\cdot\hat p}{2}\right]\sigma^2$
$= \sigma^2\left[\sqrt{E-|\vec p|}\,\frac{1-\vec\sigma^*\cdot\hat p}{2} + \sqrt{E+|\vec p|}\,\frac{1+\vec\sigma^*\cdot\hat p}{2}\right] = \sigma^2\cdot\sqrt{p}$

Let us prove them: we use answer in problem 5 in assignment 3:

$$\sqrt{p\cdot\sigma} = \sqrt{E-|\vec p|}\,\frac{1+\vec\sigma\cdot\hat p}{2} + \sqrt{E+|\vec p|}\,\frac{1-\vec\sigma\cdot\hat p}{2}, \quad \hat p = \frac{\vec p}{|\vec p|},$$

$$\sqrt{p\cdot\bar\sigma} = \sqrt{E-|\vec p|}\,\frac{1-\vec\sigma\cdot\hat p}{2} + \sqrt{E+|\vec p|}\,\frac{1+\vec\sigma\cdot\hat p}{2}$$

Proof: Since $\vec\sigma\cdot\vec p\,\sigma^2 = -\sigma^2(\vec\sigma^*\cdot\vec p)$, we obtain

$$\sqrt{p\cdot\sigma}\cdot\sigma^2 = \left[\sqrt{E-|\vec p|}\,\frac{1+\vec\sigma\cdot\hat p}{2} + \sqrt{E+|\vec p|}\,\frac{1-\vec\sigma\cdot\hat p}{2}\right]\sigma^2 =$$

$$= \sigma^2\left[\sqrt{E-|\vec p|}\,\frac{1-\vec\sigma^*\cdot\hat p}{2} + \sqrt{E+|\vec p|}\,\frac{1+\vec\sigma^*\cdot\hat p}{2}\right] = \sigma^2\cdot\underline{\sqrt{p\cdot\sigma^*}}$$

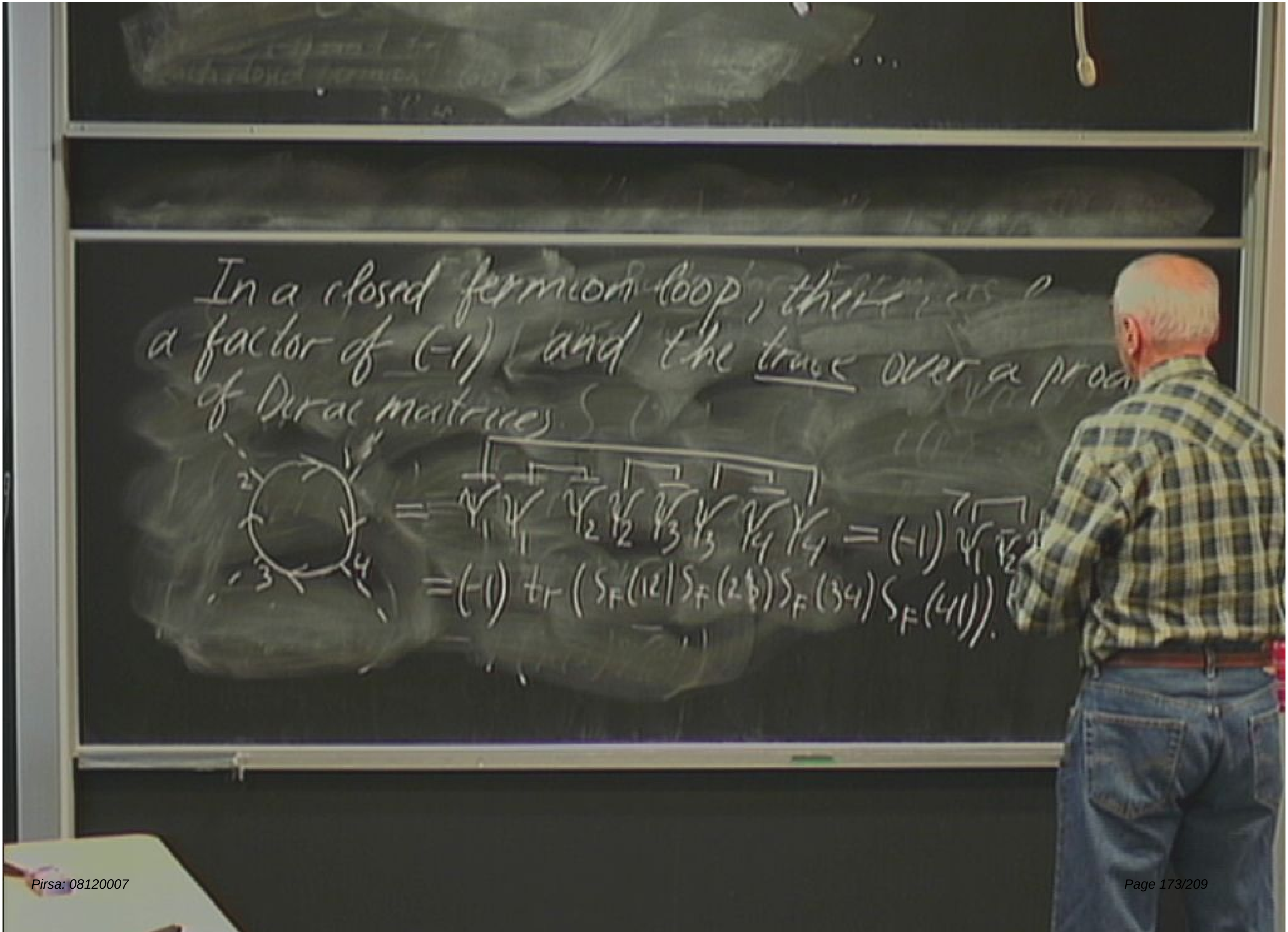
Let us prove them: we use answer in problem 5 in assignment 3: $\sqrt{p\cdot\sigma} = \sqrt{E-|\vec p|}\,\frac{1+\vec\sigma\cdot\hat p}{2} + \sqrt{E+|\vec p|}\,\frac{1-\vec\sigma\cdot\hat p}{2}$, $\hat p = \frac{\vec p}{|\vec p|}$,

$$\sqrt{p\cdot\bar\sigma} = \sqrt{E-|\vec p|}\,\frac{1-\vec\sigma\cdot\hat p}{2} + \sqrt{E+|\vec p|}\,\frac{1+\vec\sigma\cdot\hat p}{2}$$

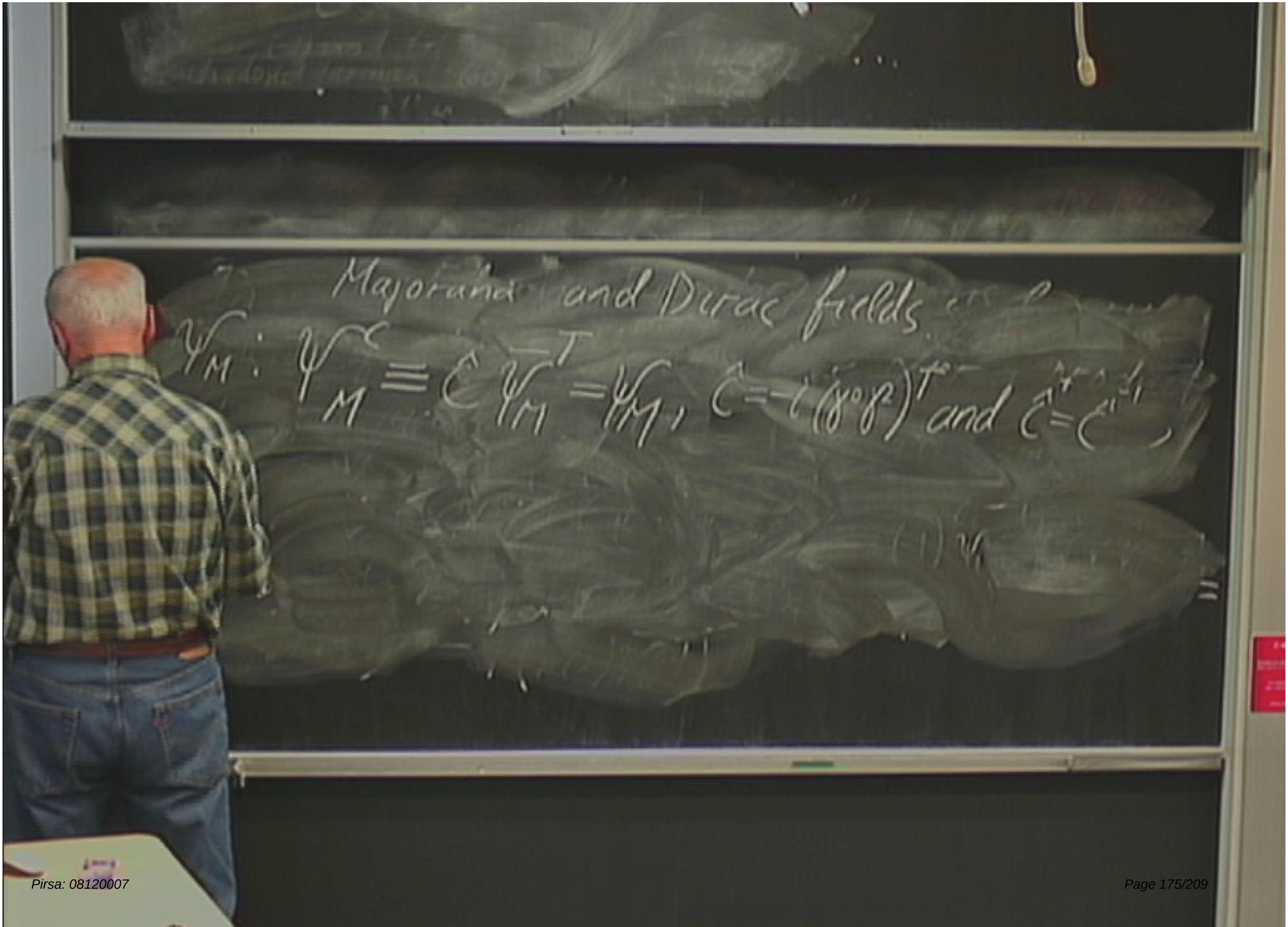
Proof: Since $\vec\sigma\cdot\vec p\,\sigma^2 = -\sigma^2(\vec\sigma^*\cdot\vec p)$, we obtain

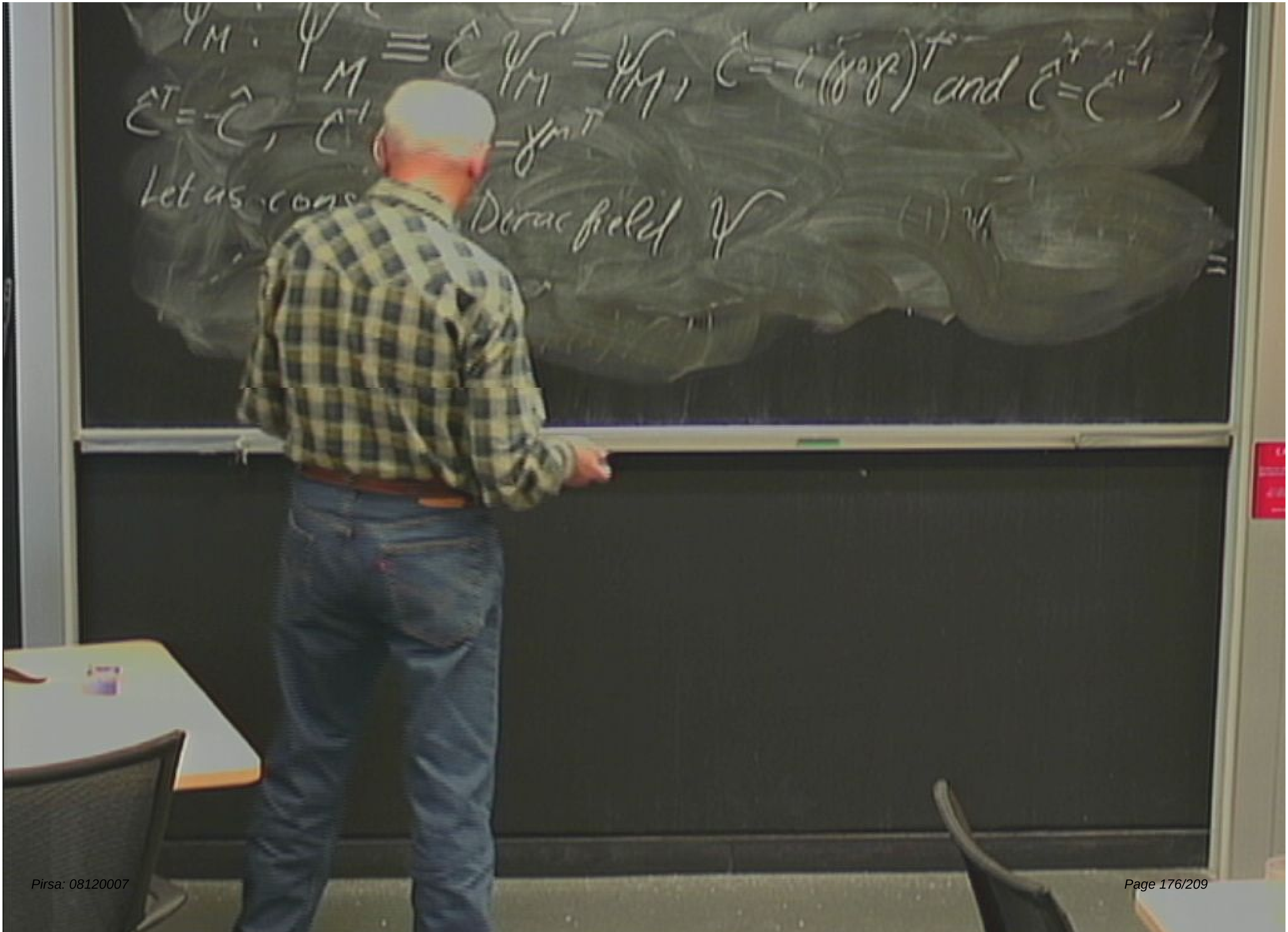
$$\sqrt{p\cdot\sigma}\cdot\sigma^2 = \left[\sqrt{E-|\vec p|}\,\frac{1+\vec\sigma\cdot\hat p}{2} + \sqrt{E+|\vec p|}\,\frac{1-\vec\sigma\cdot\hat p}{2}\right]\sigma^2 =$$

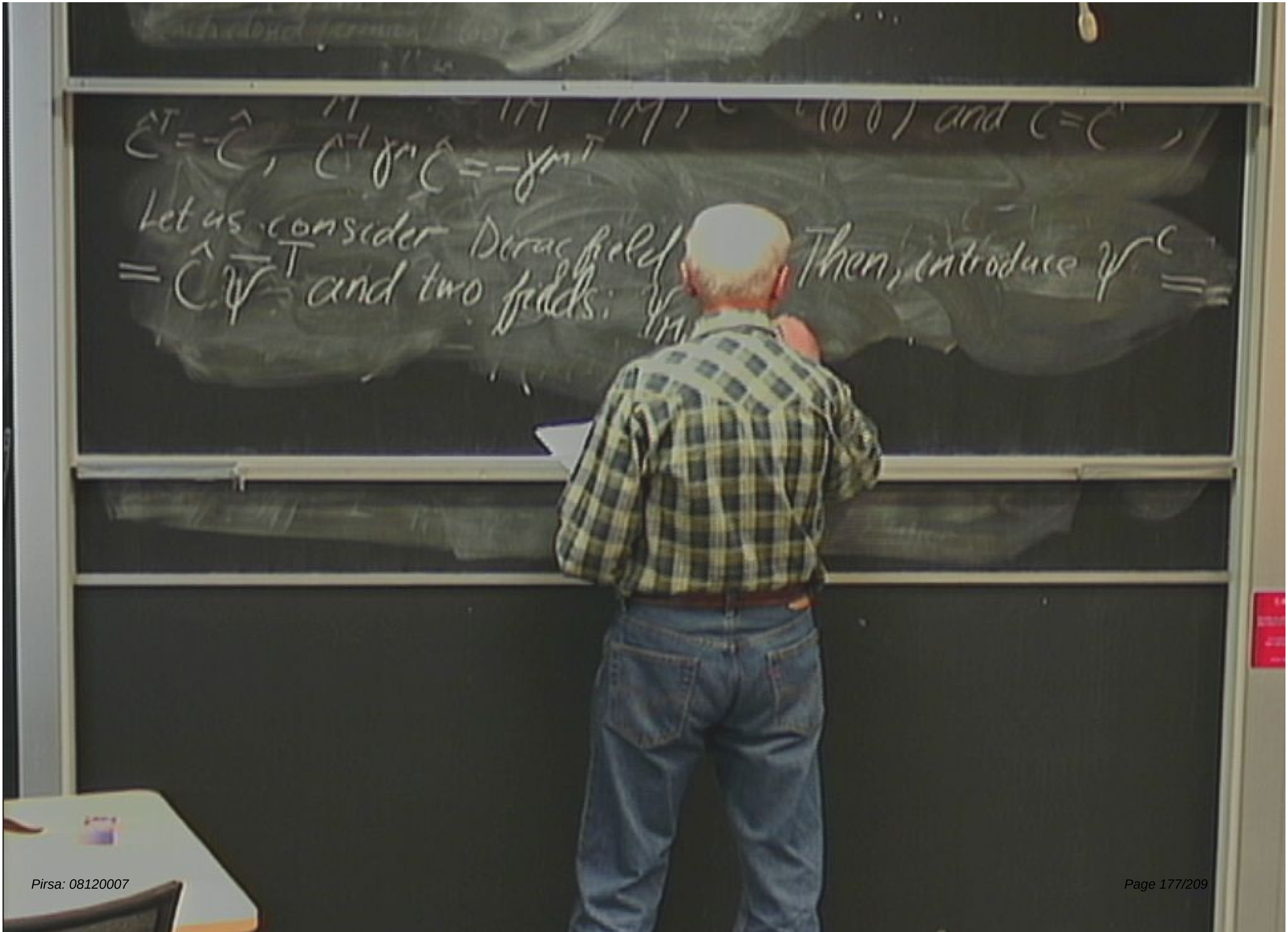
$$= \sigma^2\left[\sqrt{E-|\vec p|}\,\frac{1-\vec\sigma^*\cdot\hat p}{2} + \sqrt{E+|\vec p|}\,\frac{1+\vec\sigma^*\cdot\hat p}{2}\right] = \sigma^2\cdot\sqrt{p\,\sigma^*}$$

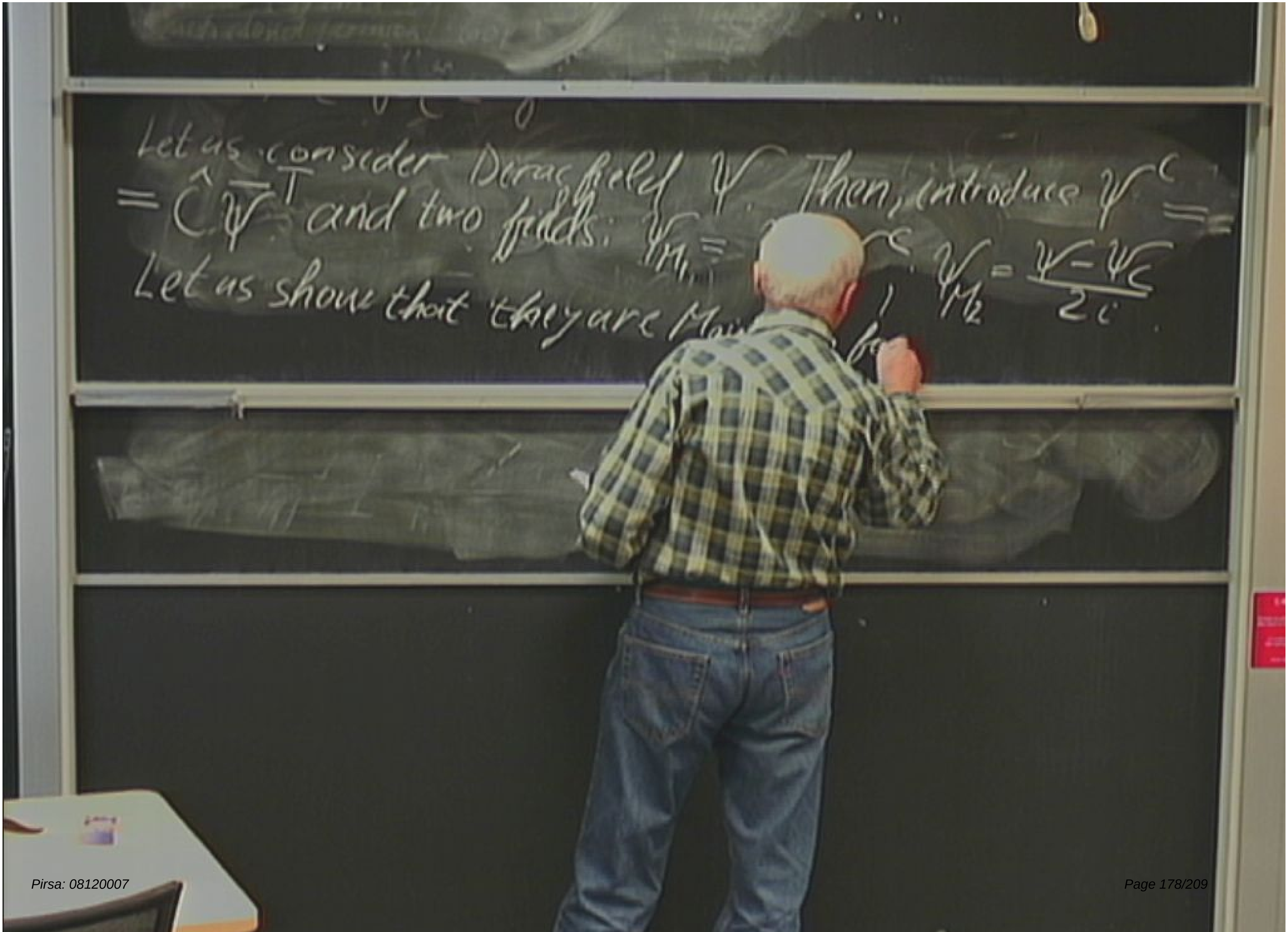

In a closed fermion loop, there is a
a factor of (-1) and the trace over a prod
of Dirac matrices.
2
3
4
=(-1) tr (S_F(12) S_F(23) S_F(34) S_F(41)).

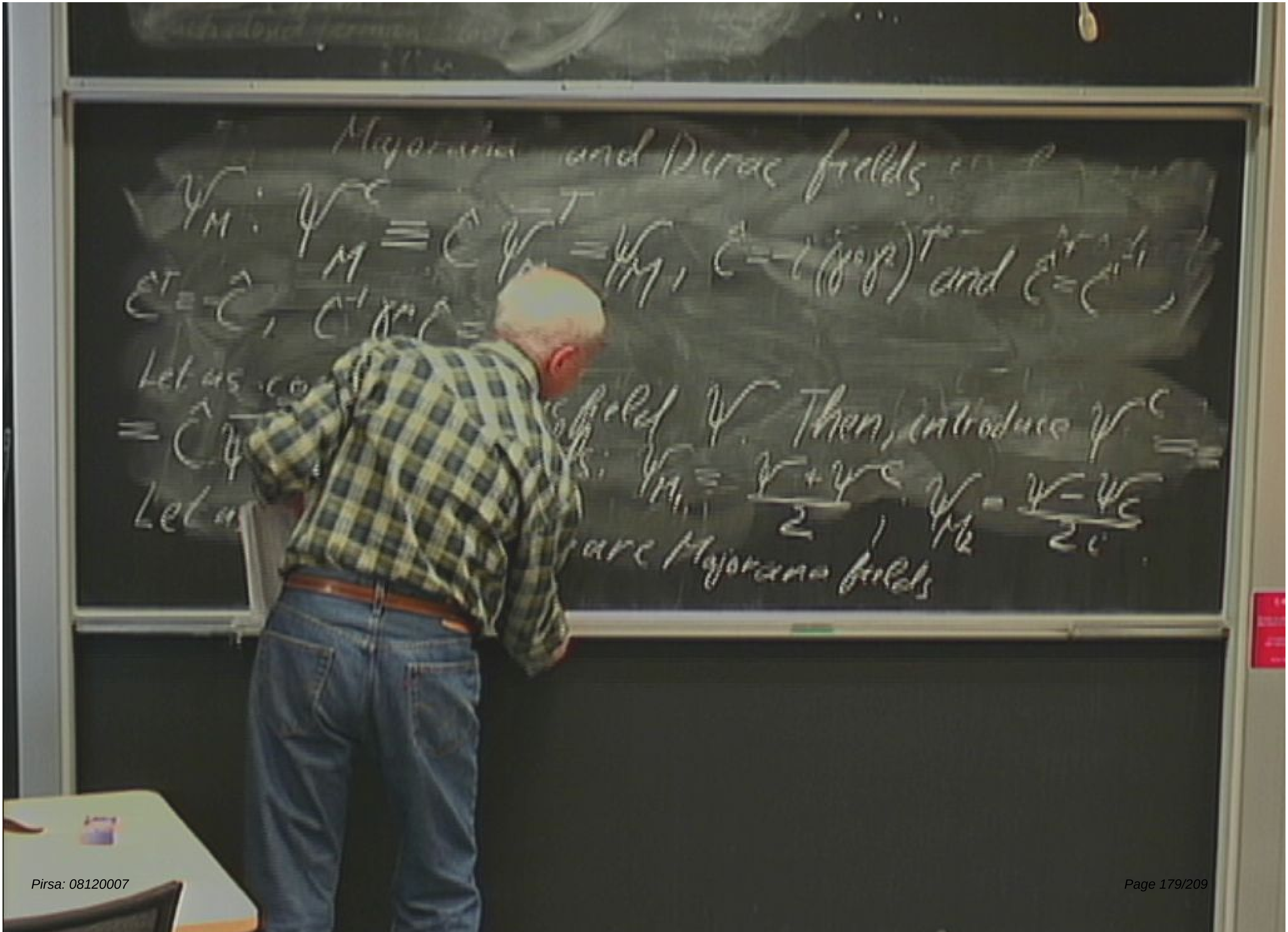
Majorana and Dirac

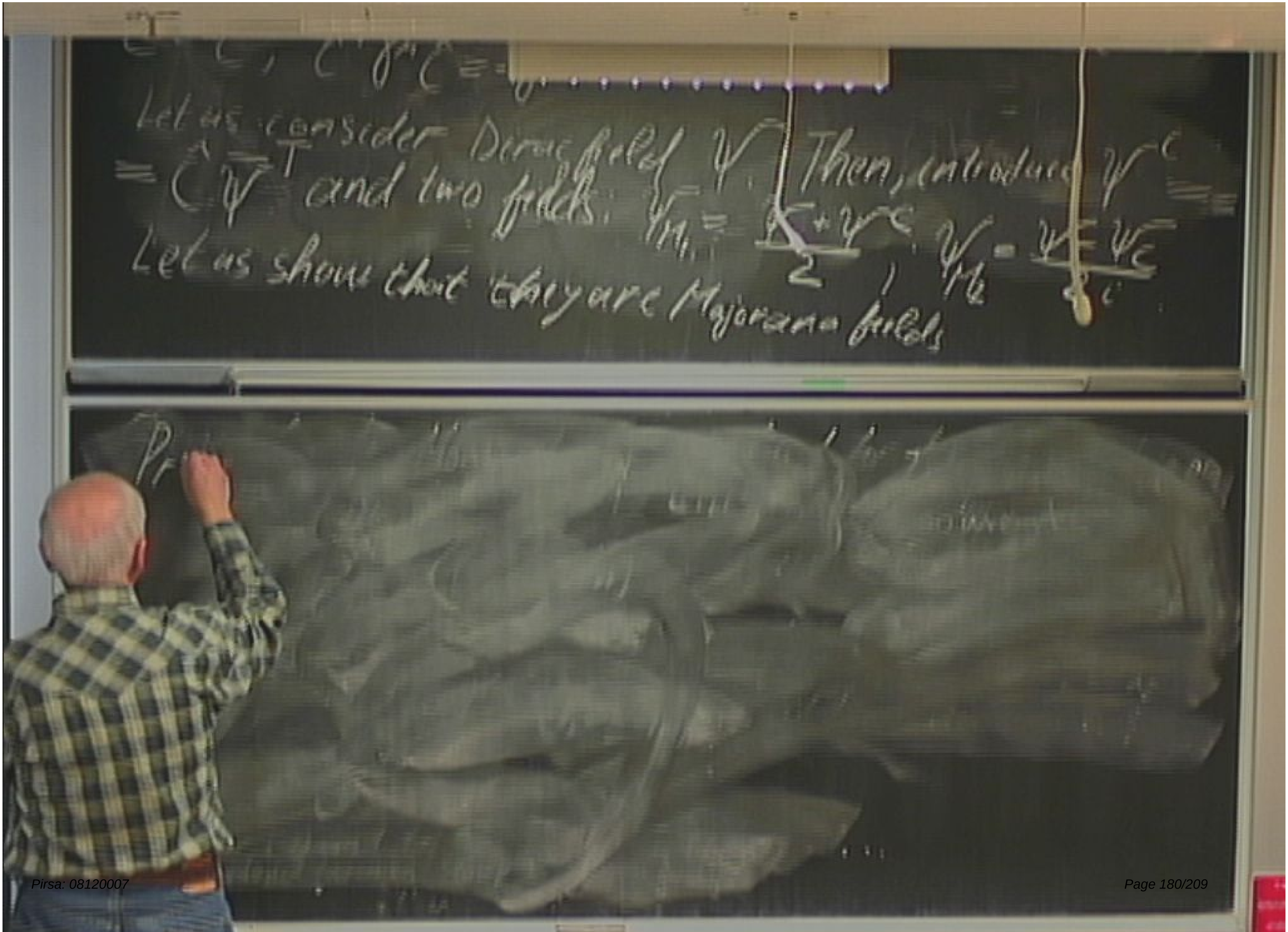

Majorana and Dirac fields
$\psi_M : \psi_M^c \equiv \hat{C}\bar{\psi}_M^T = \psi_M , \hat{C} = -i(\gamma^0\gamma^2)^T$ and $\hat{C}^\dagger = \hat{C}^{-1}$

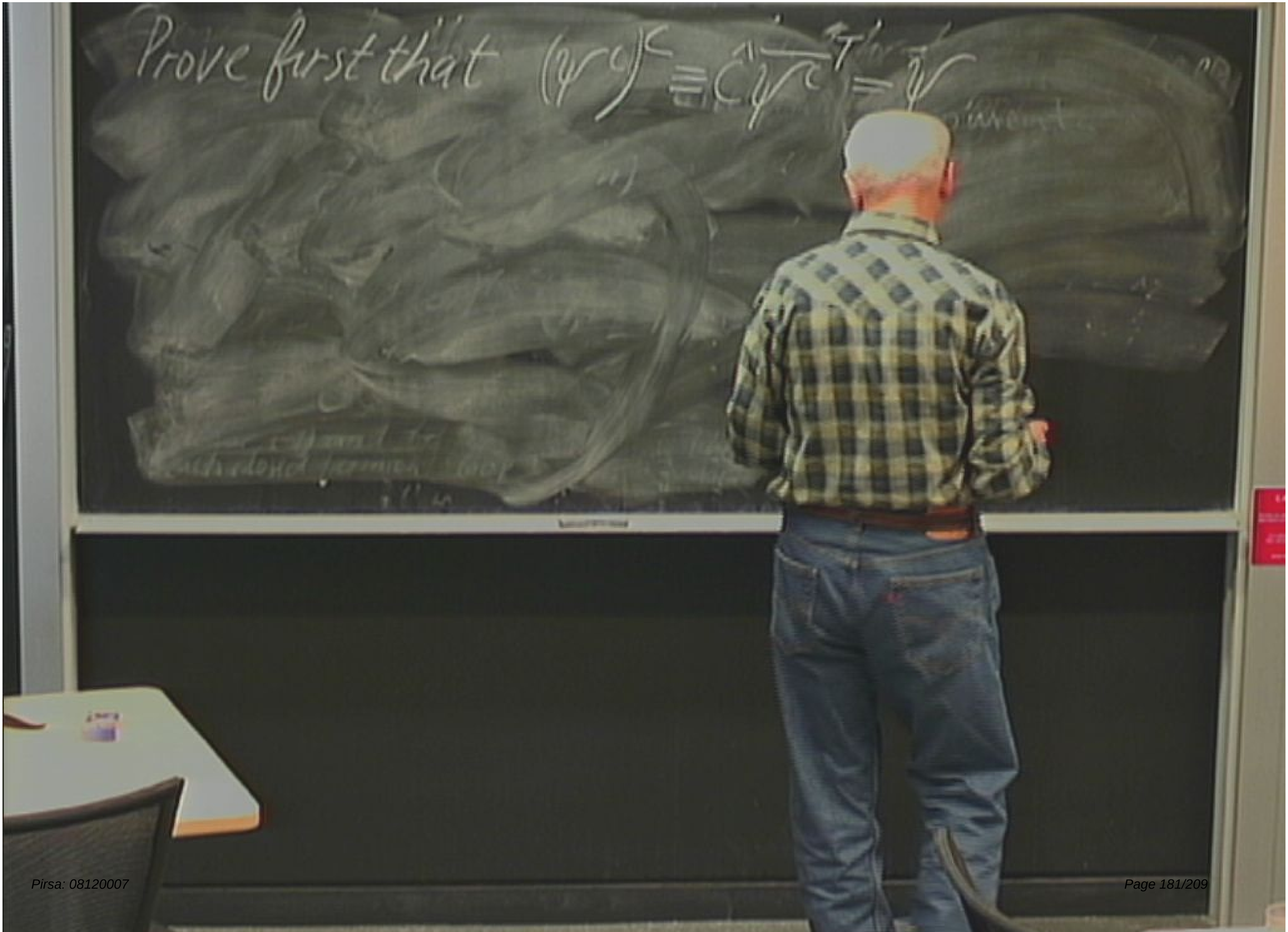

and
Let us cons
Dirac field

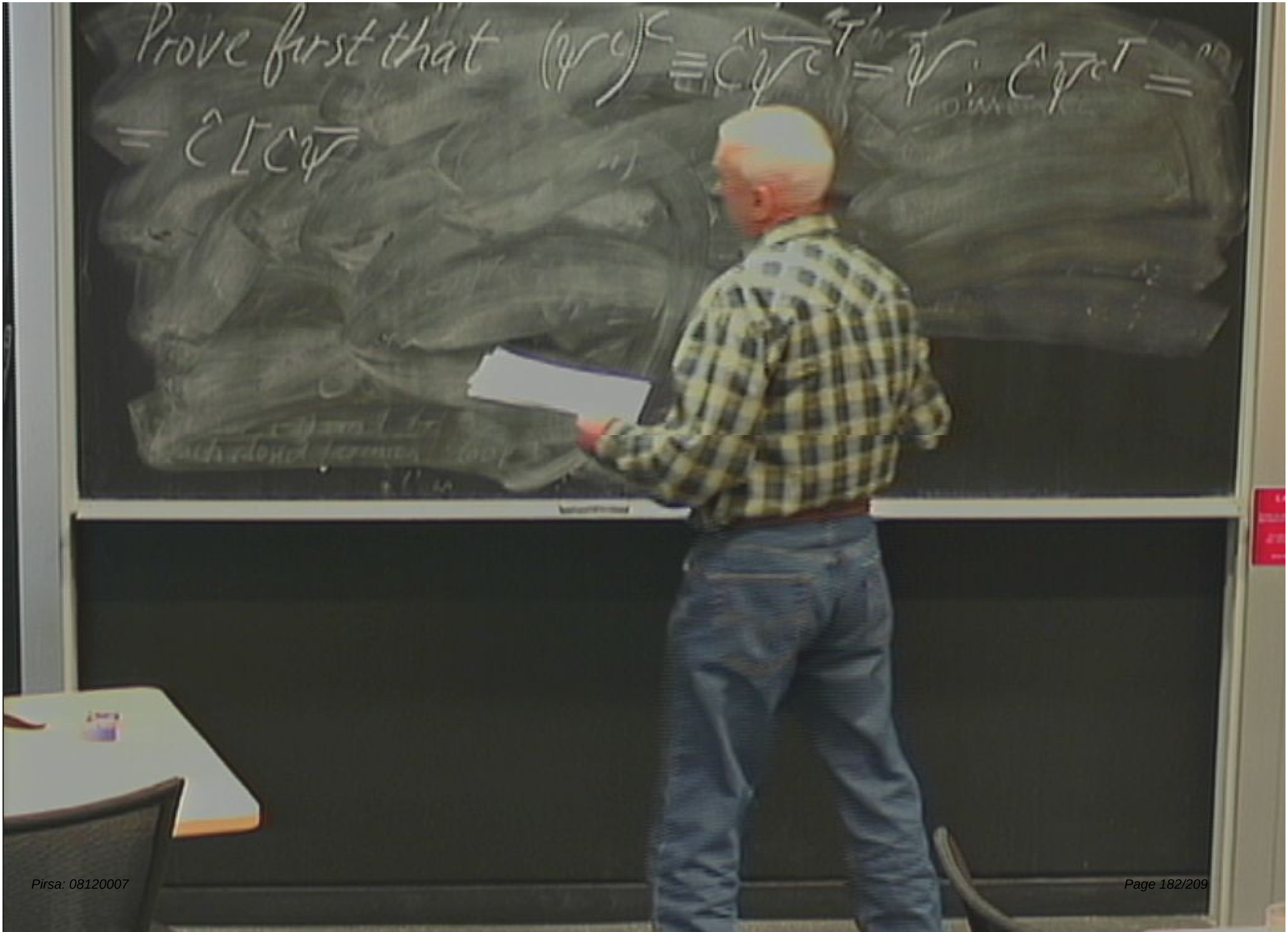

and $C = C$
$\hat{C}^T = -\hat{C}$, $\hat{C}^{-1}\gamma^\mu \hat{C} = -\gamma^{\mu T}$
Let us consider Dirac field
Then, introduce ψ^c =
$= \hat{C}\bar{\psi}^T$ and two fields:

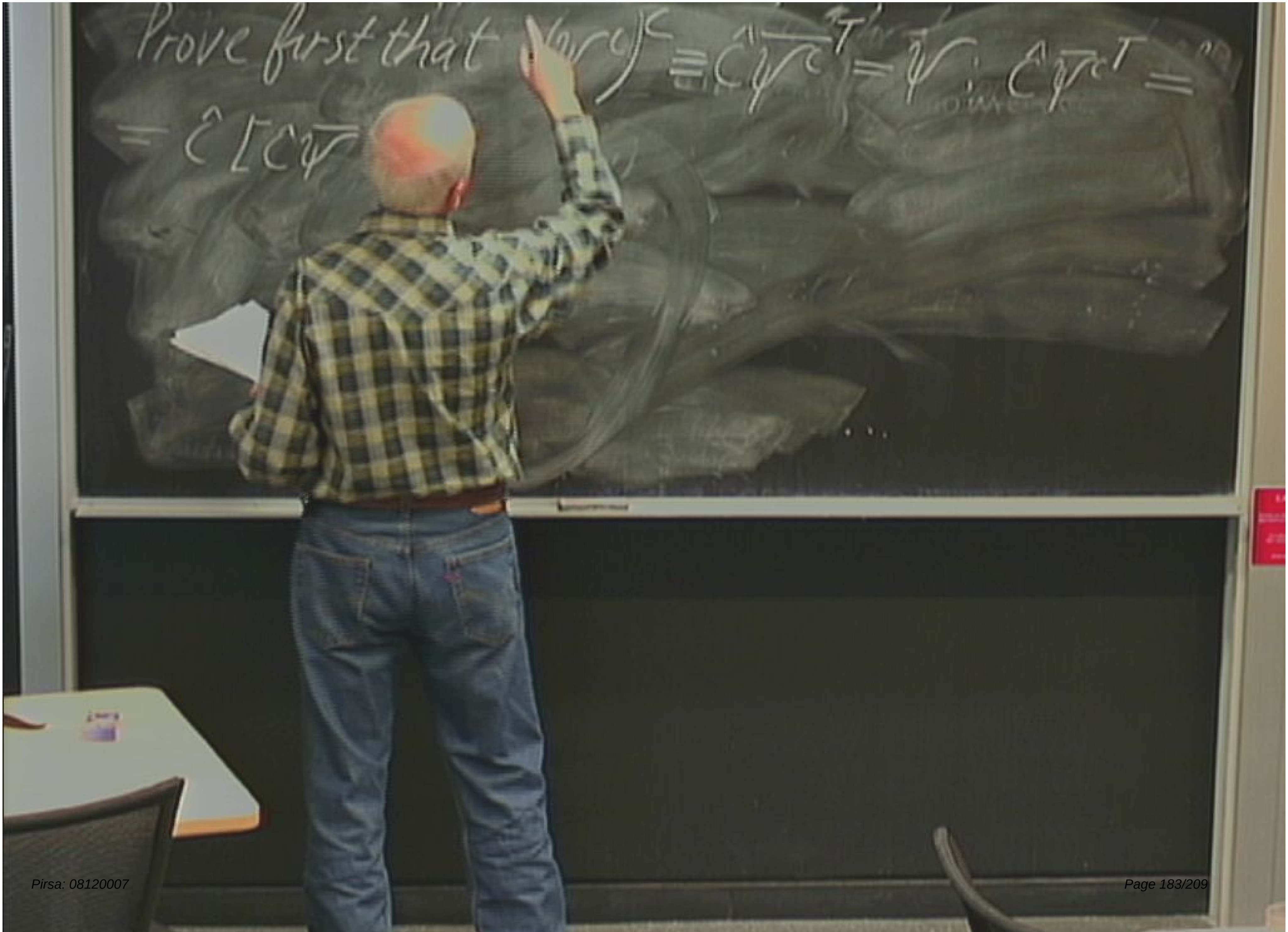

Let us consider Dirac field ψ. Then, introduce ψ^c
= Ĉ ψ̄^T and two fields: ψ_M1 =
ψ_M2 = (ψ − ψ_c)/2i
Let us show that they are

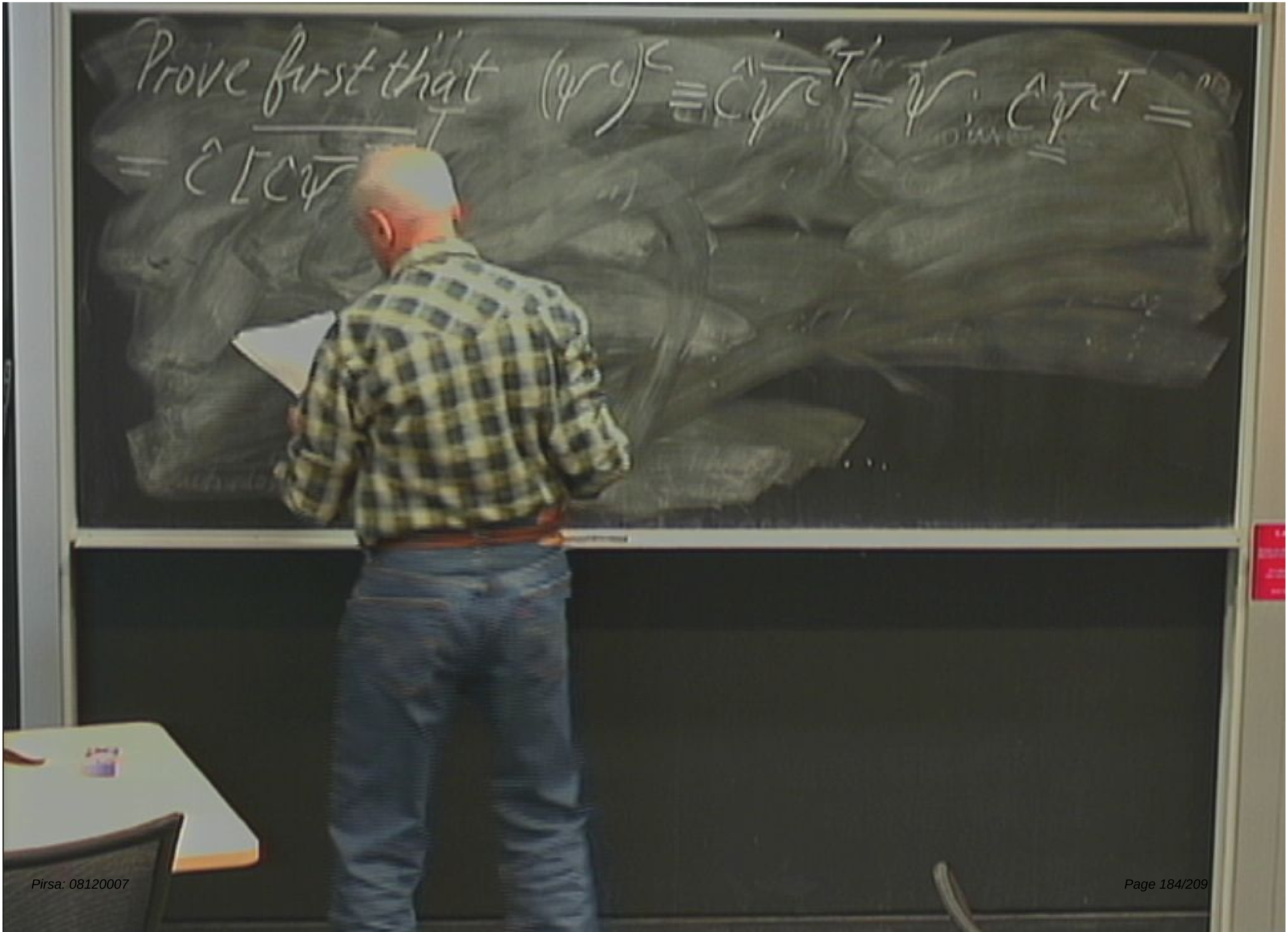

Majorana and Dirac fields
Then, introduce
are Majorana fields

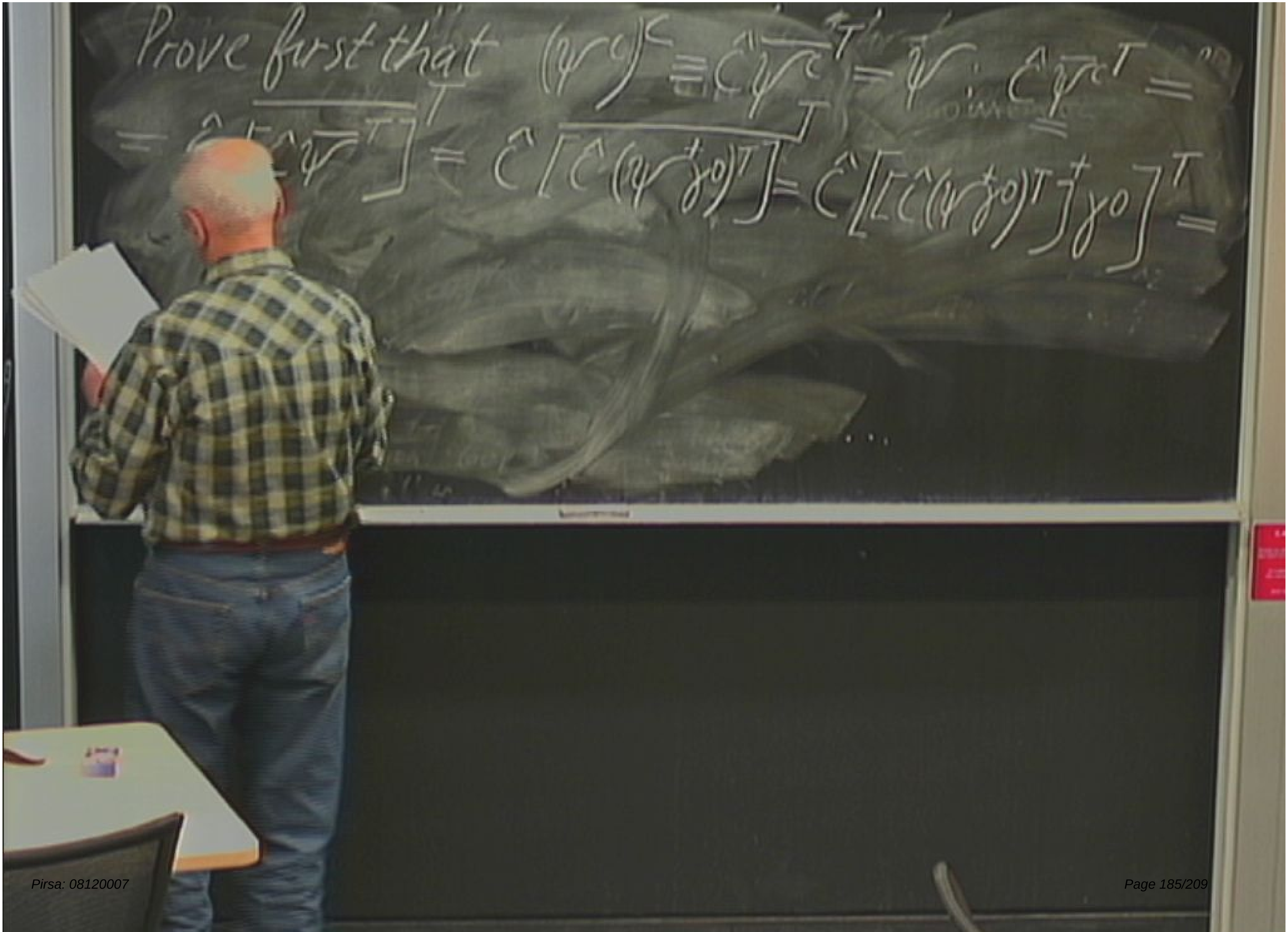

Let us consider Dirac field ψ. Then, introduce $\psi^c =$
$= \hat{C}\bar{\psi}^T$ and two fields: $\psi_{M_1} = \frac{\psi+\psi^c}{2}$, $\psi_{M_2} = \frac{\psi-\psi^c}{2i}$
Let us show that they are Majorana fields
Pr

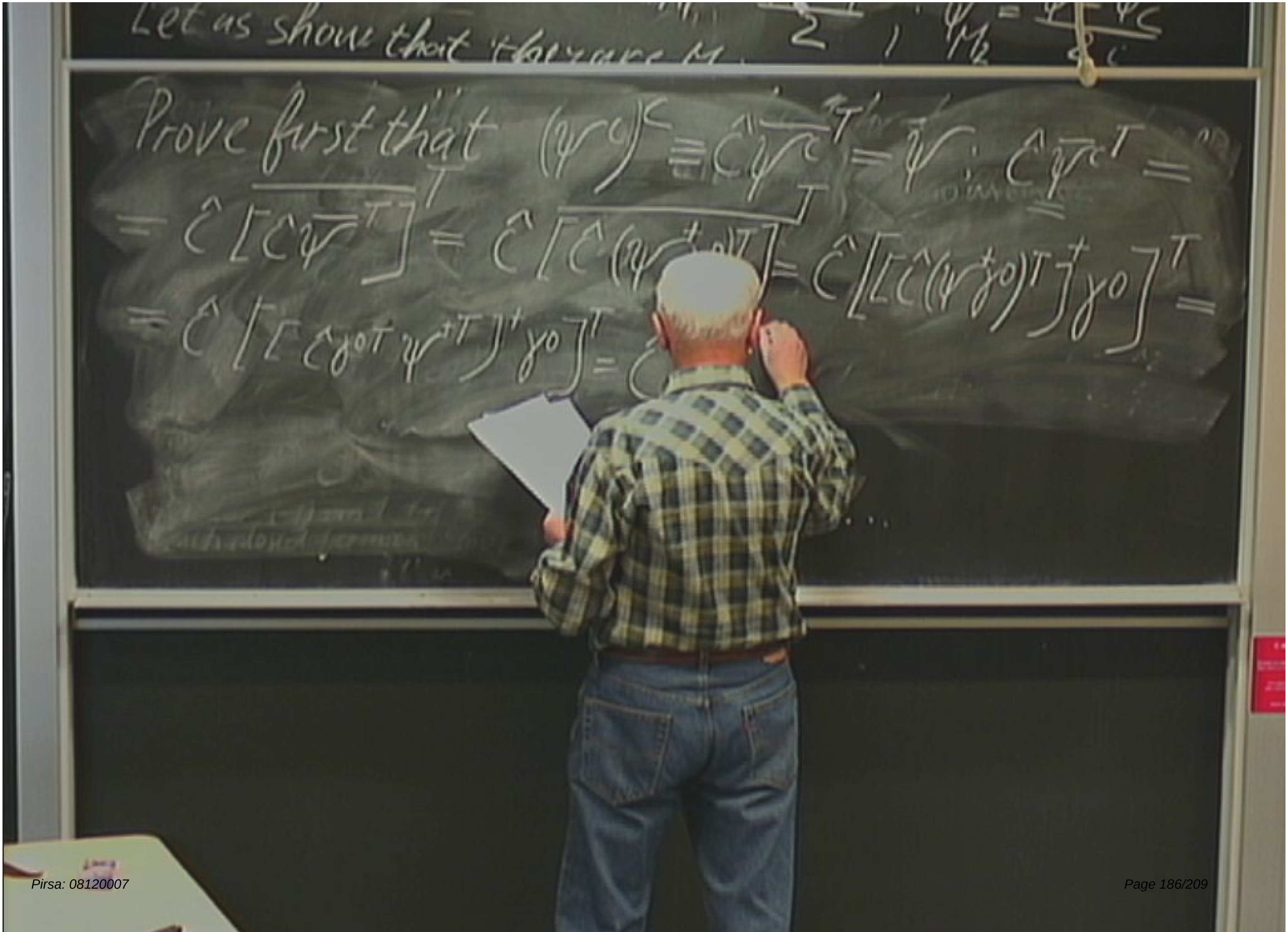

Prove first that

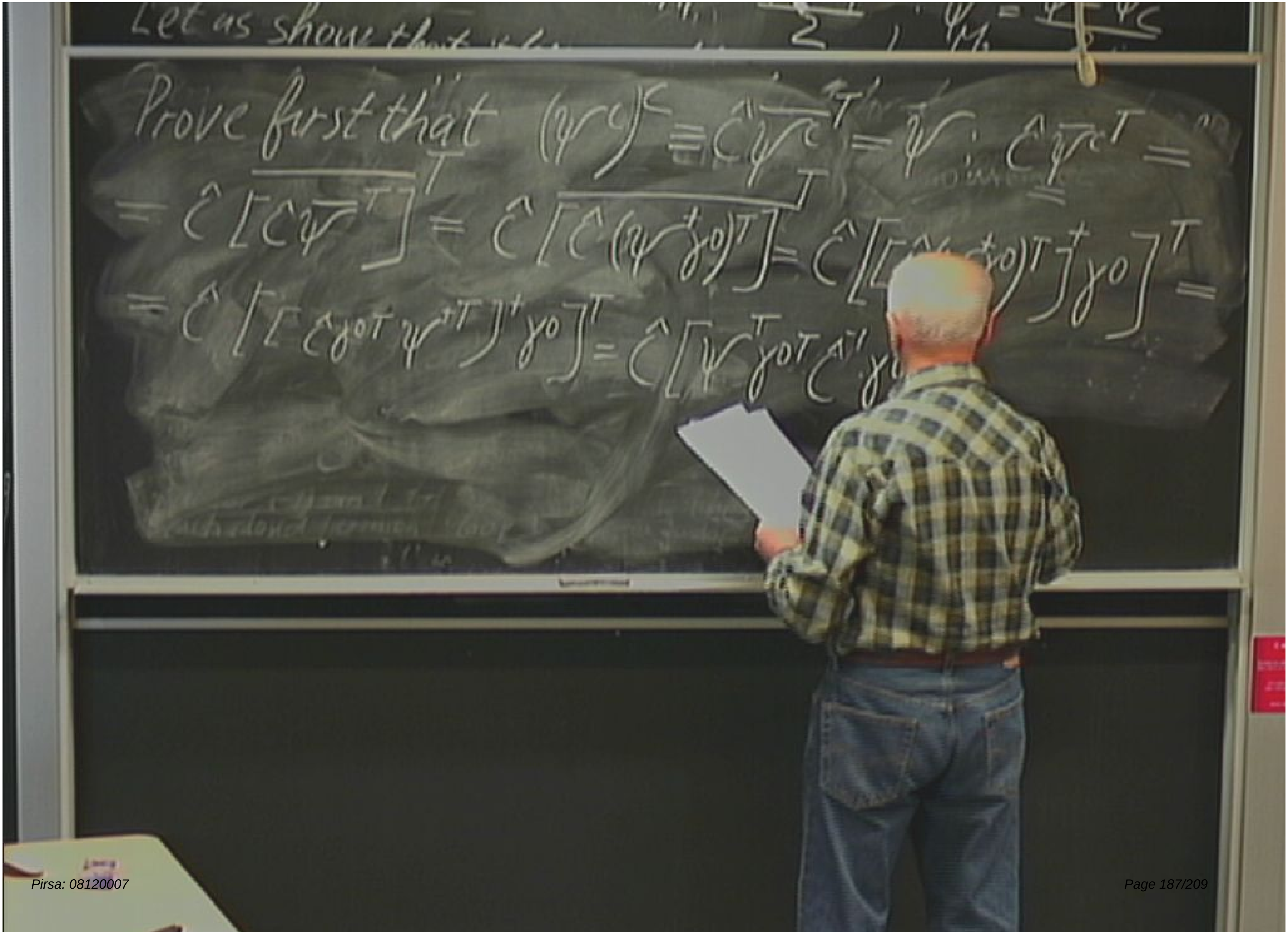

Prove first that

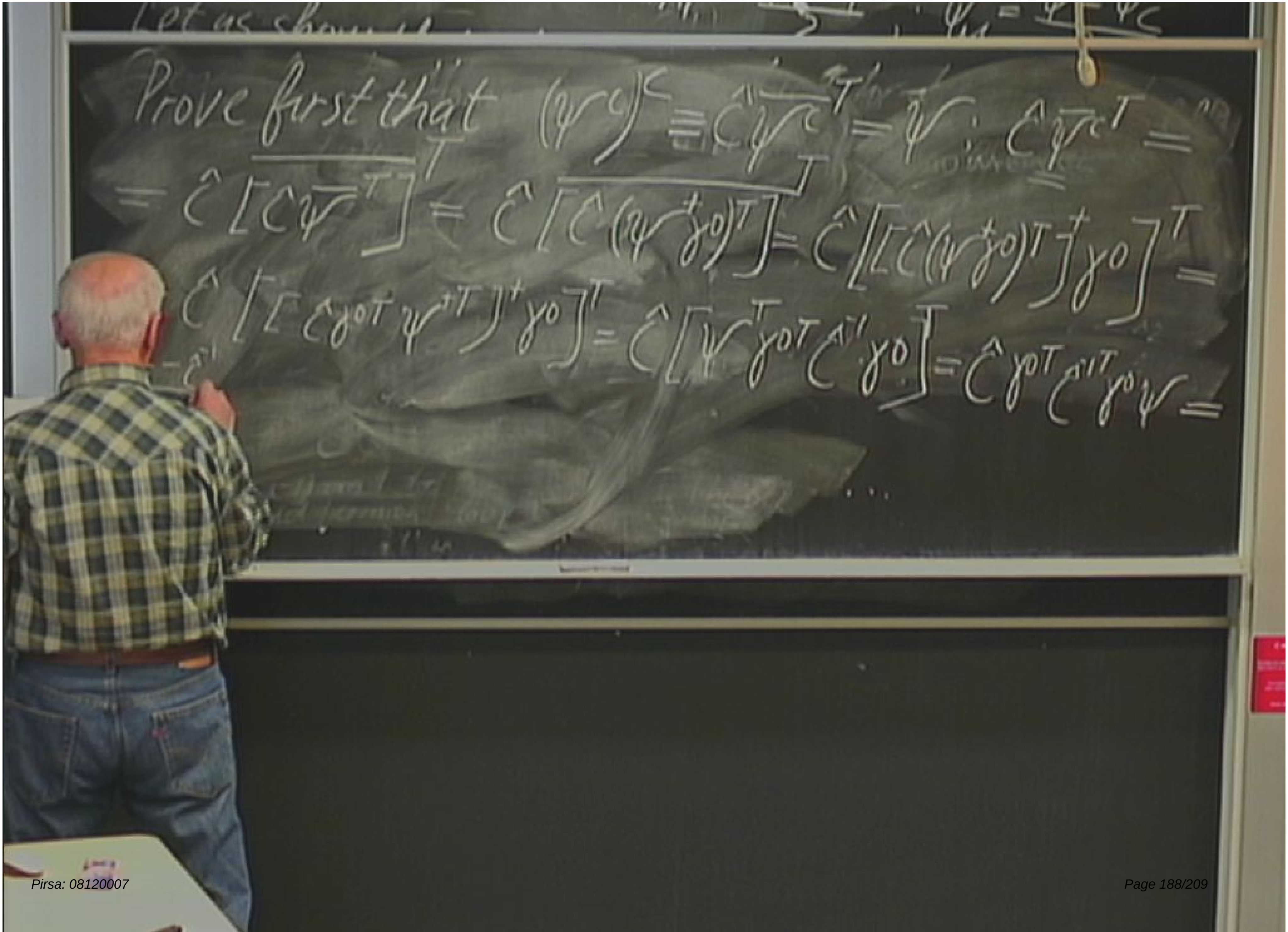

Prove first that

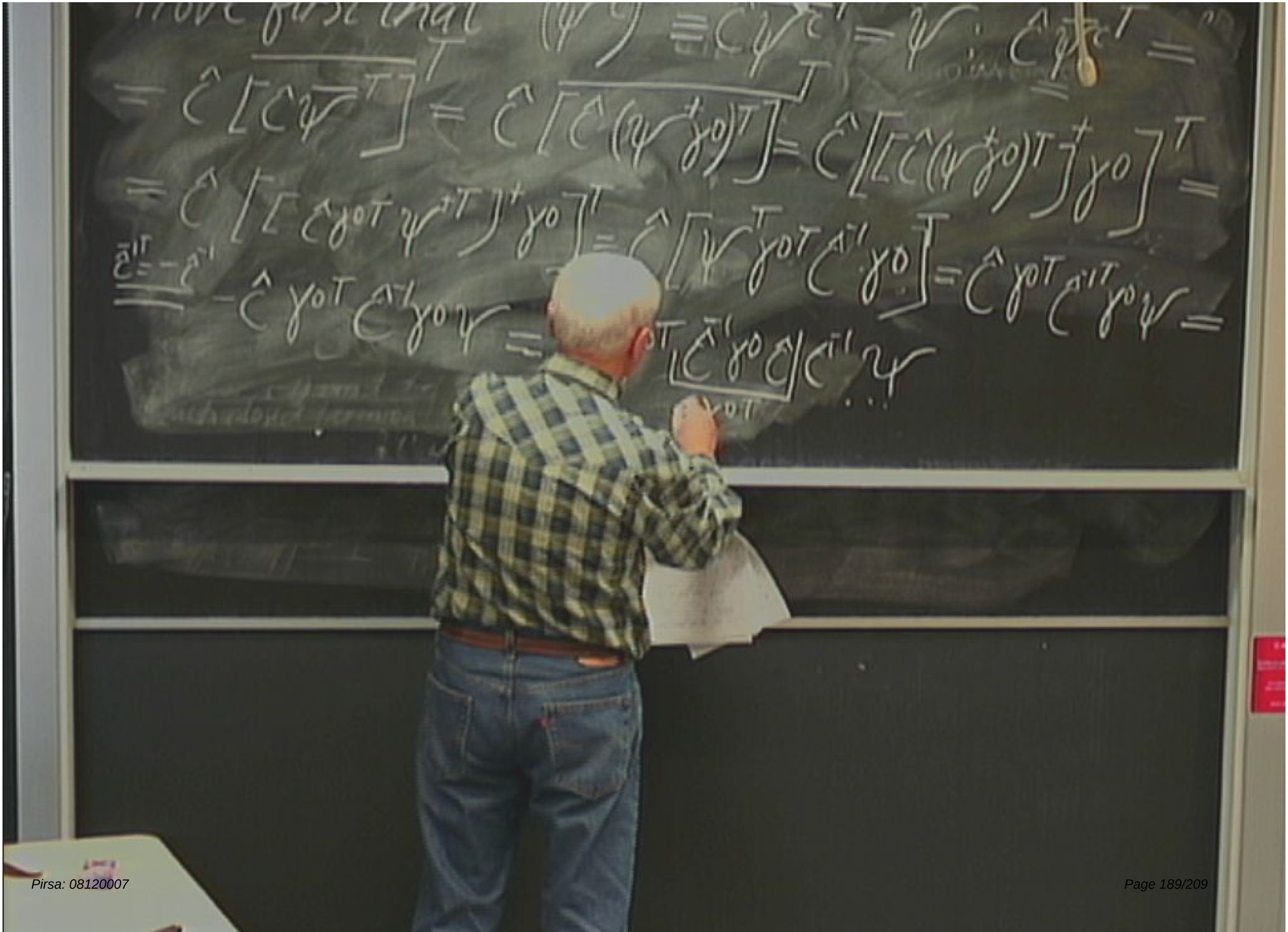

Prove first that

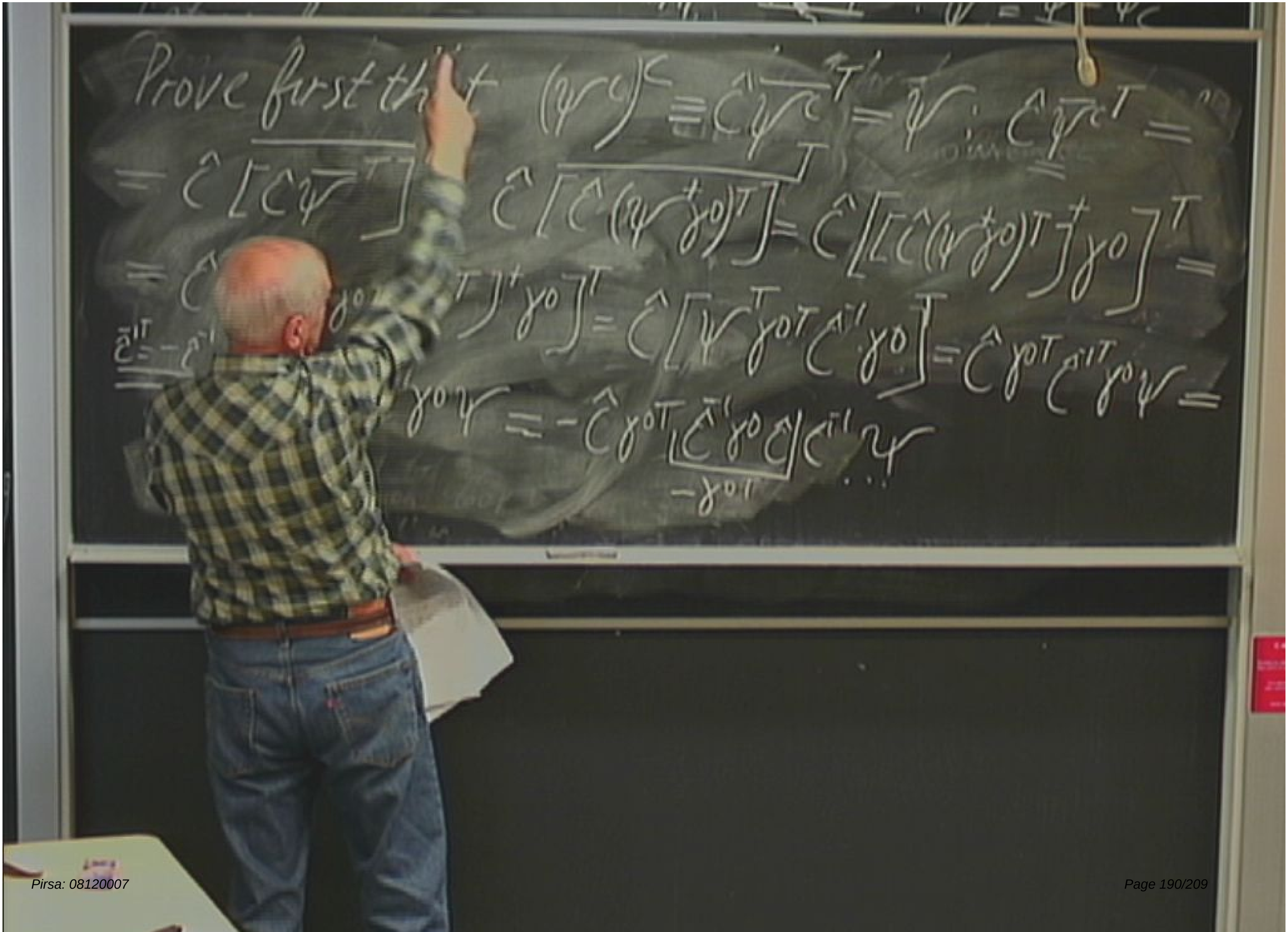

Prove first that

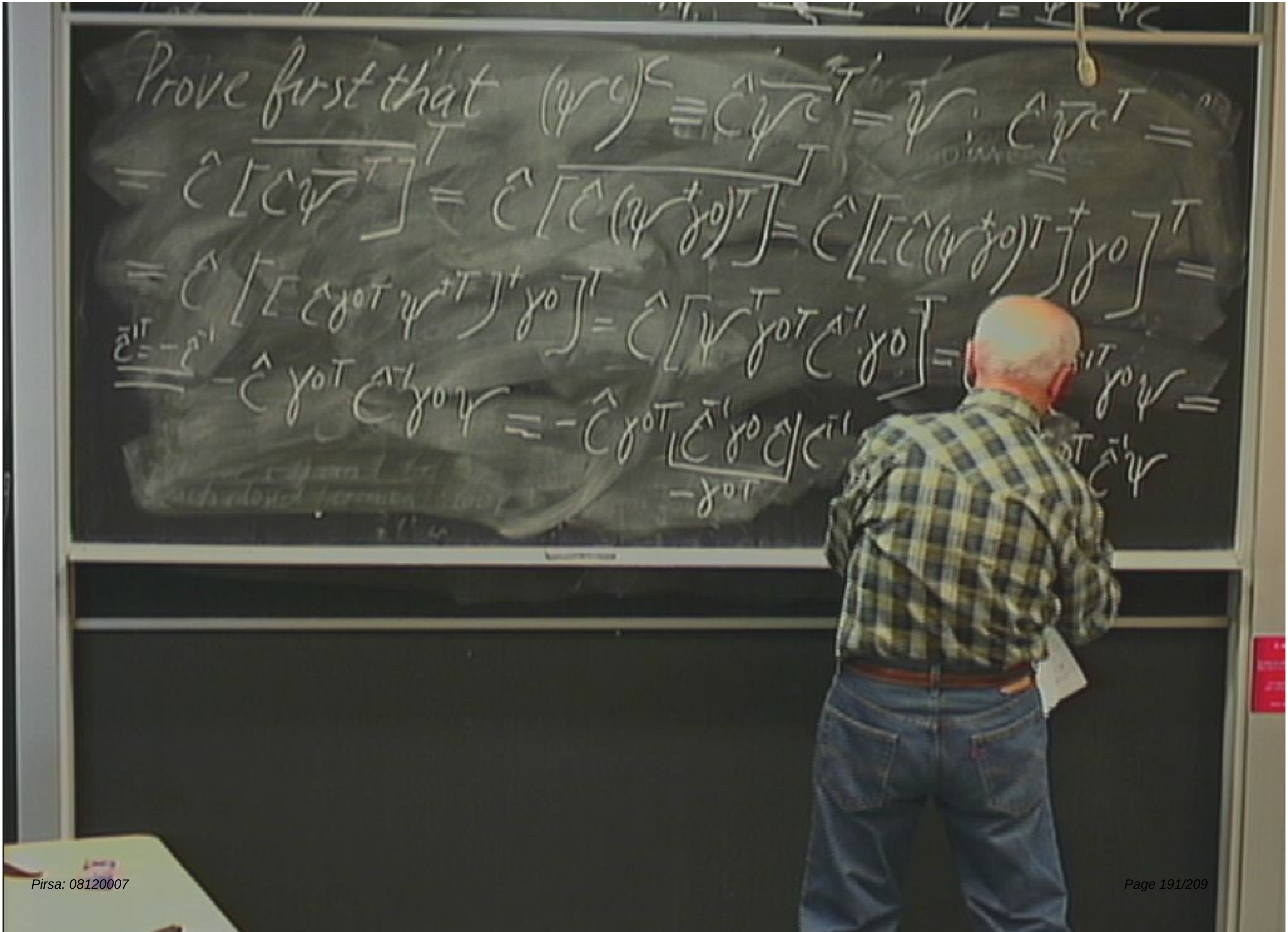

Let us show that
Prove first that

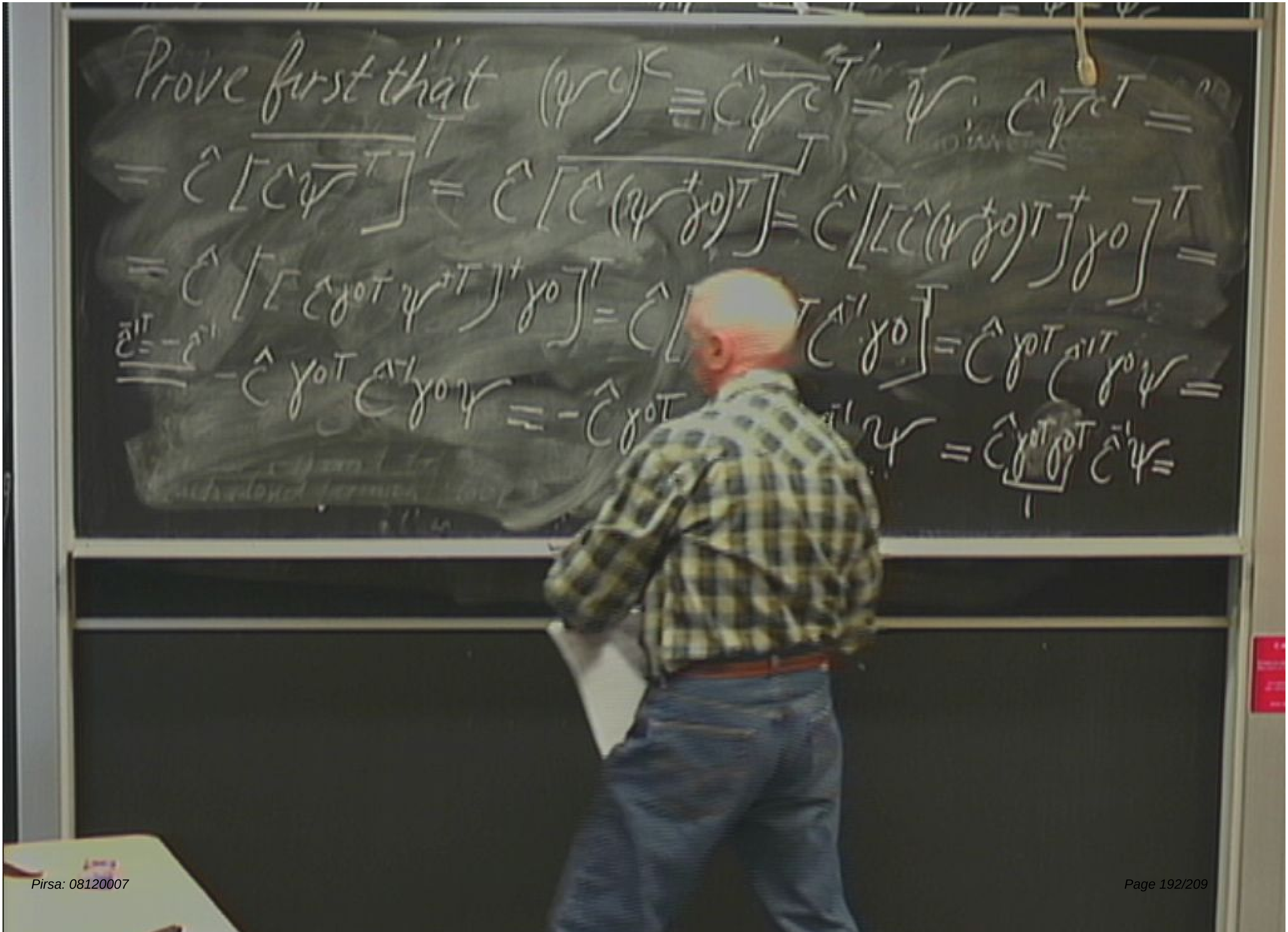

Prove first that

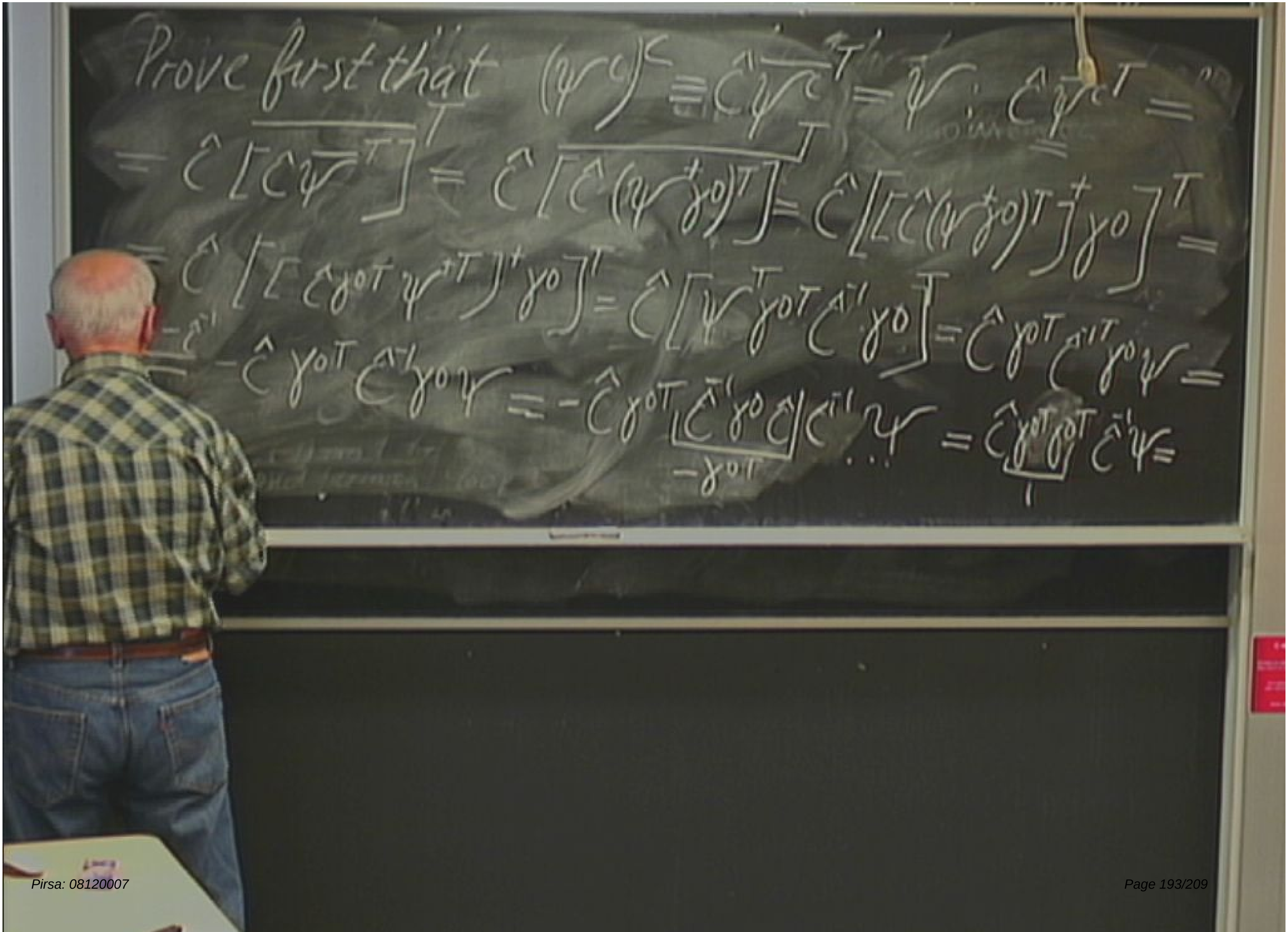

Prove first that

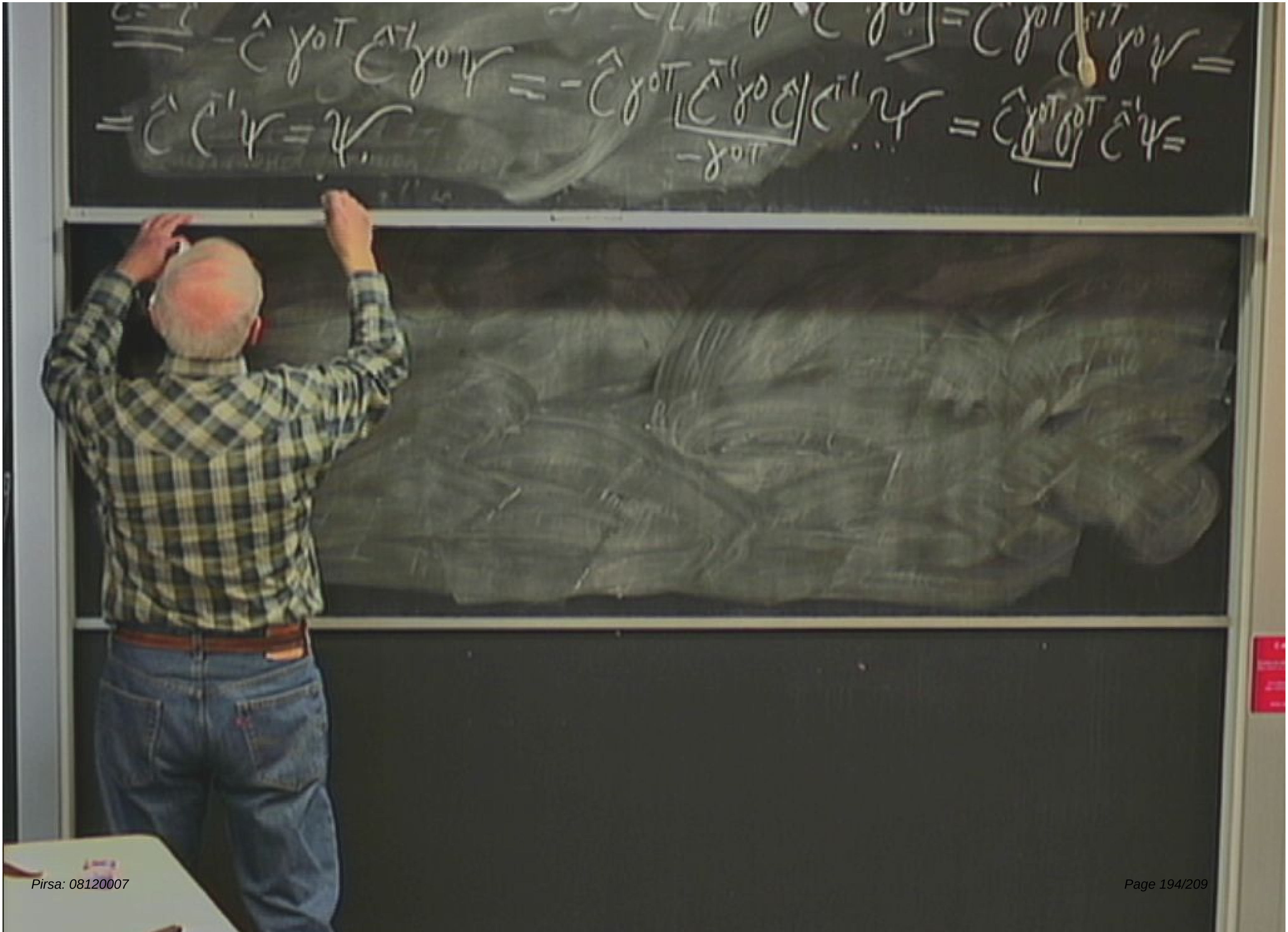


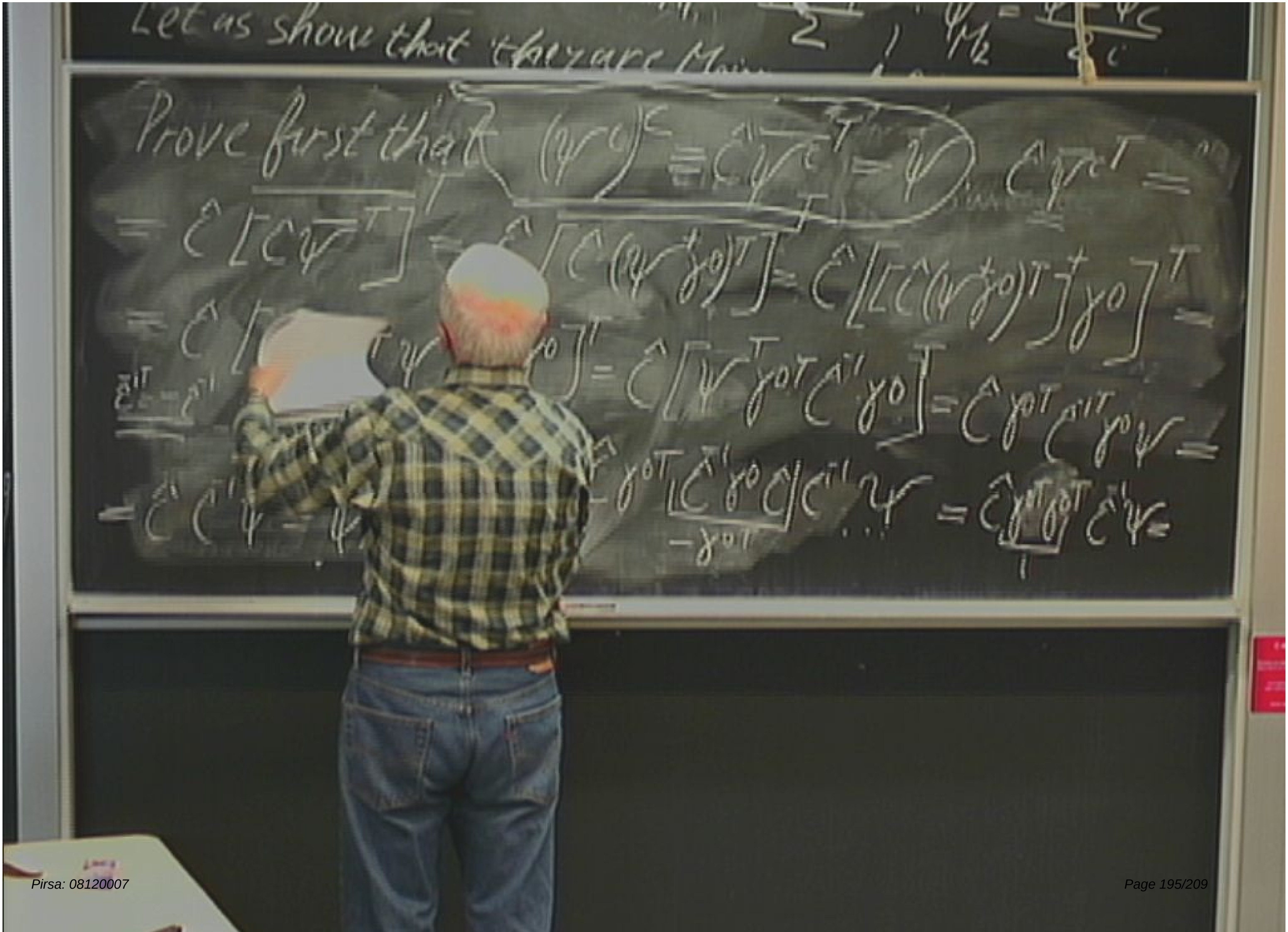

Prove first that

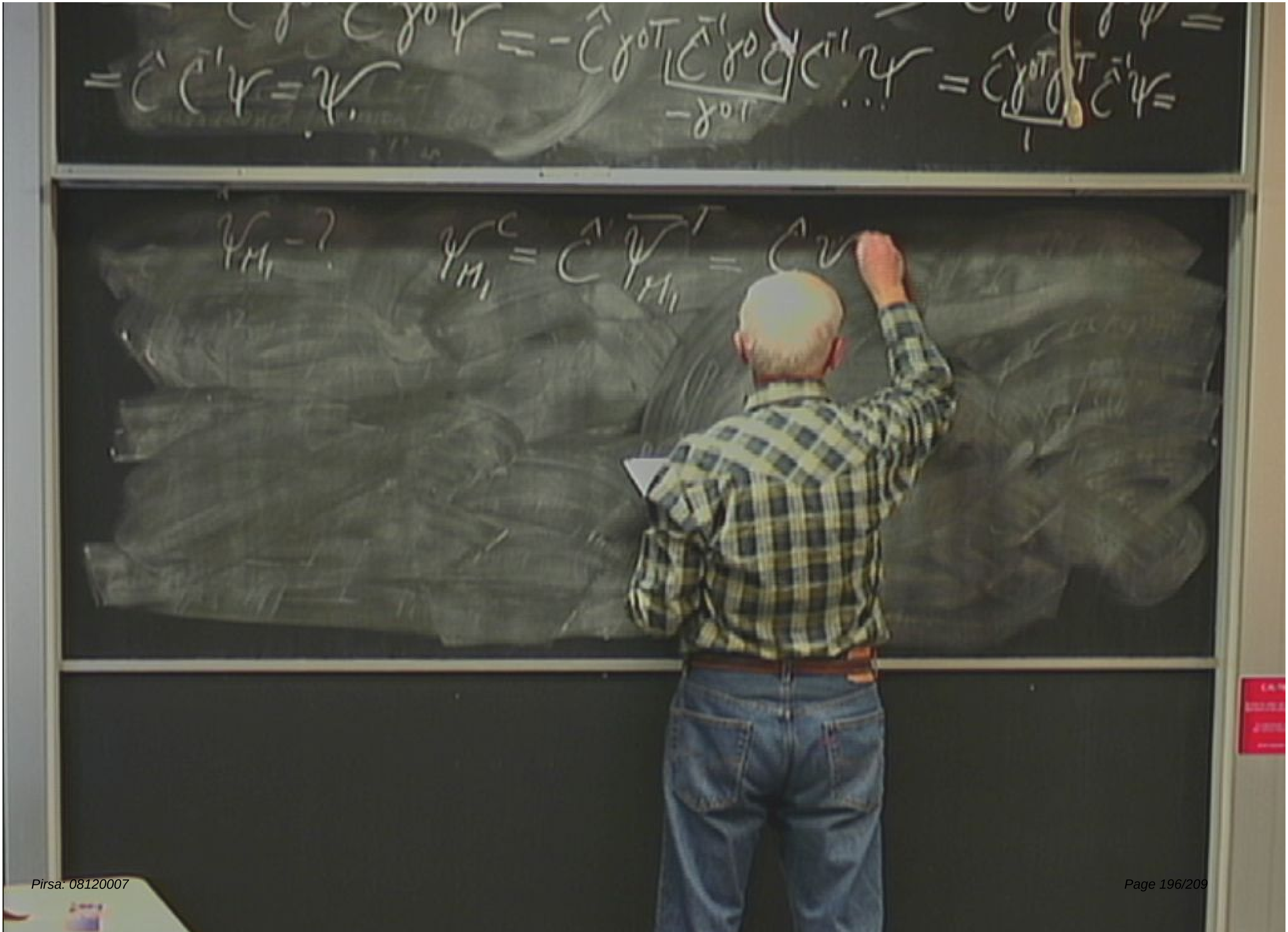

Prove first that (ψ^c)^c ≡ Ĉ ψ̄^{cT} = ψ ; Ĉ ψ̄^{cT} =
= Ĉ [Ĉ ψ̄^T]^T = Ĉ [Ĉ (ψ^† γ0)^T]^T = Ĉ [[Ĉ (ψ^† γ0)^T]^† γ0]^T =
= Ĉ [[Ĉ γ0^T ψ^{†T}]^† γ0]^T = Ĉ [ψ^T γ0^T Ĉ^{-1} γ0]^T =
Ĉ^{-1T} = -Ĉ^{-1}
-Ĉ γ0^T Ĉ^{-1} γ0 ψ = -Ĉ γ0^T Ĉ^{-1} γ0 Ĉ Ĉ^{-1}
-γ0^T

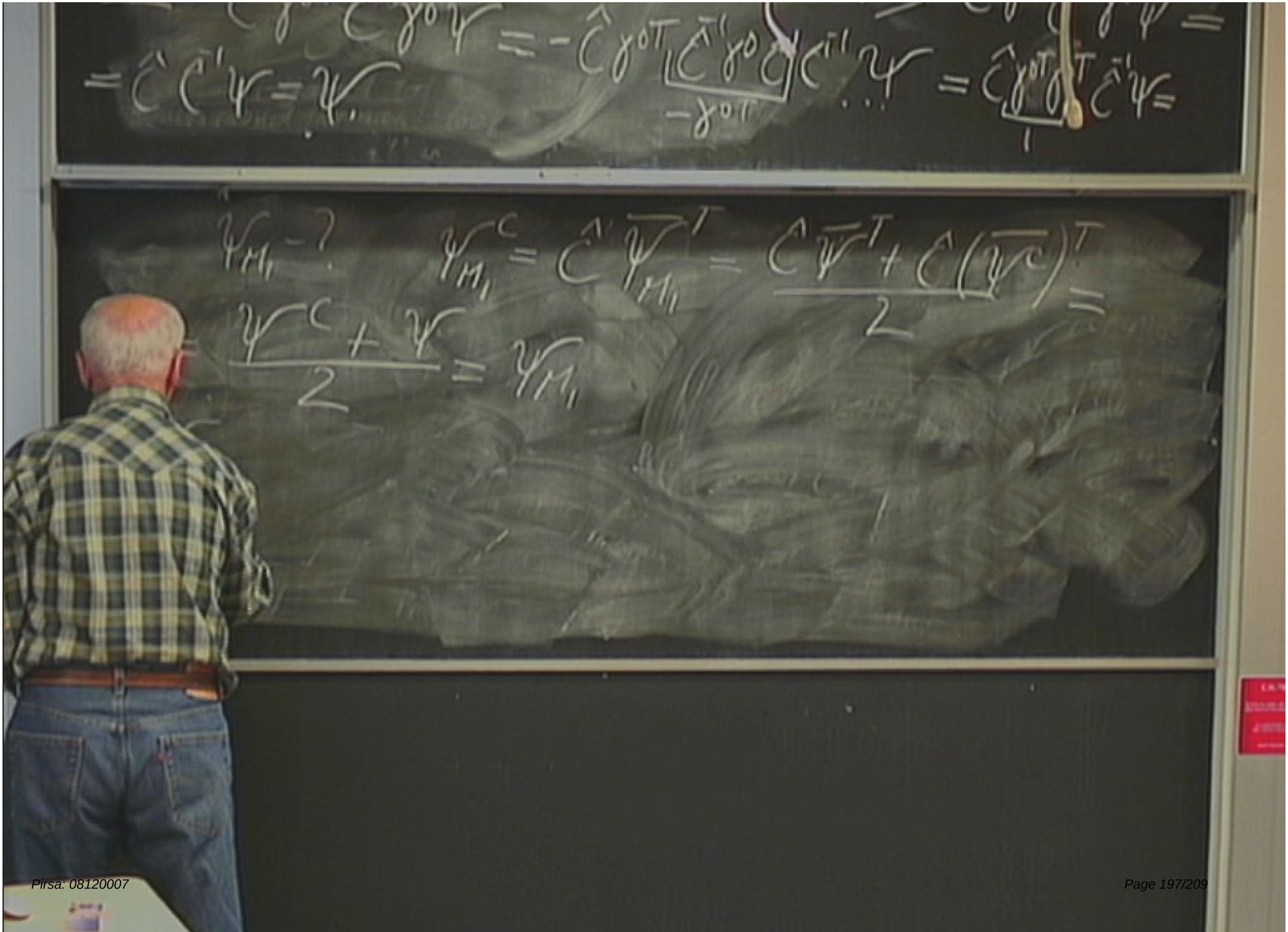

Prove first that

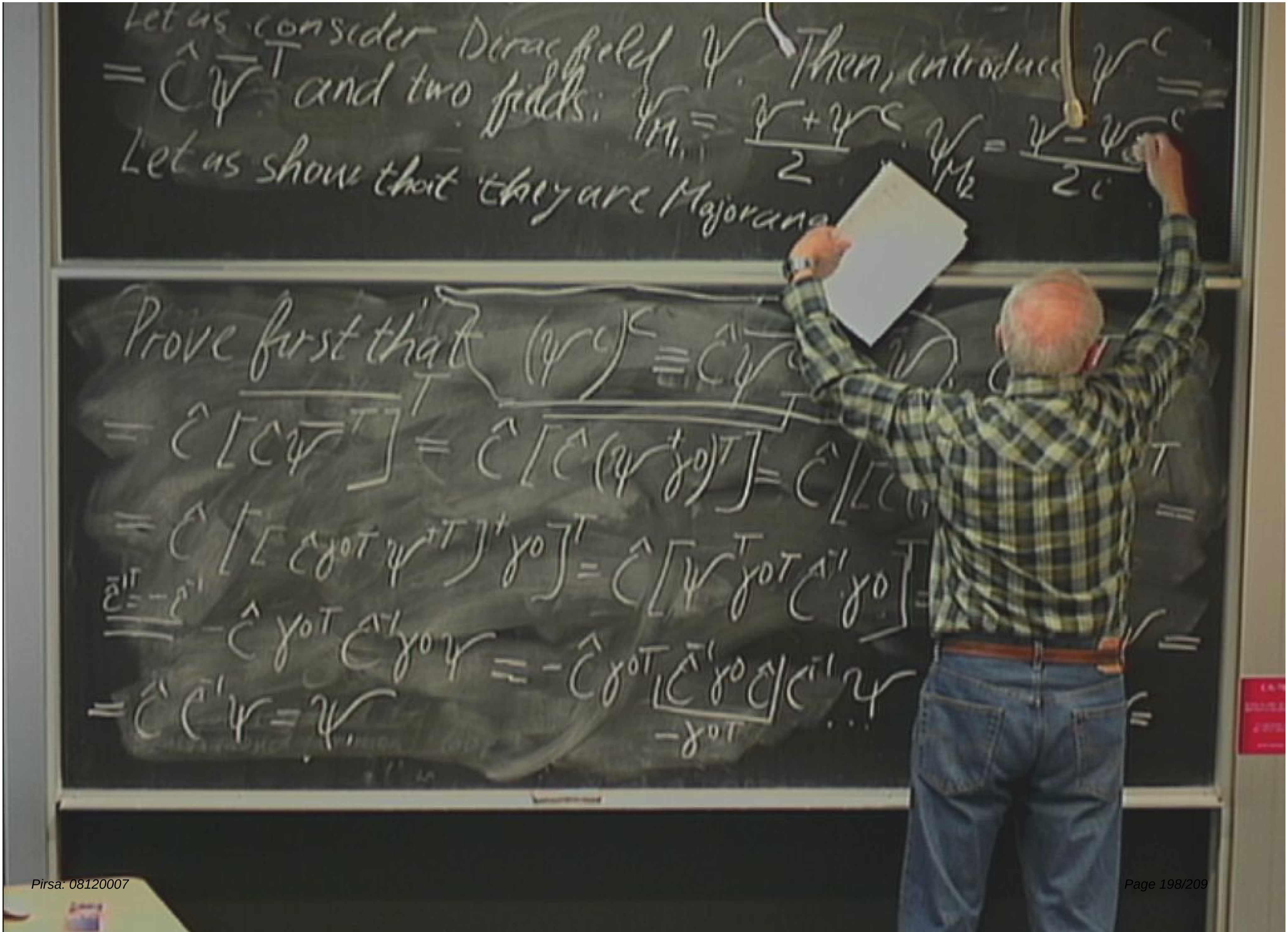

Prove first that $(\psi^c)^c = \hat{C}\,\overline{\psi^c}^T = \psi$; $\hat{C}\,\overline{\psi^c}^T =$
$= \hat{C}\,[\overline{\hat{C}\bar{\psi}^T}]^T = \hat{C}\,[\overline{\hat{C}(\psi^\dagger\gamma^0)^T}] = \hat{C}\,[[\hat{C}(\psi^\dagger\gamma^0)^T]^\dagger\gamma^0]^T =$
$= \hat{C}\,[[\hat{C}\gamma^{0T}\psi^{\dagger T}]^\dagger\gamma^0]^T = \hat{C}\,[\psi^T\gamma^{0T}\hat{C}^{-1}\gamma^0]^T = \hat{C}\gamma^{0T}\hat{C}^{-1T}\gamma^0\psi =$
$-\hat{C}\gamma^{0T}\hat{C}^{-1}\gamma^0\psi = -\hat{C}\gamma^{0T}\hat{C}^{-1}\gamma^0\hat{C}\hat{C}^{-1}\psi = \hat{C}\gamma^{0T}\gamma^{0T}\hat{C}^{-1}\psi =$
$-\gamma^{0T}$
1

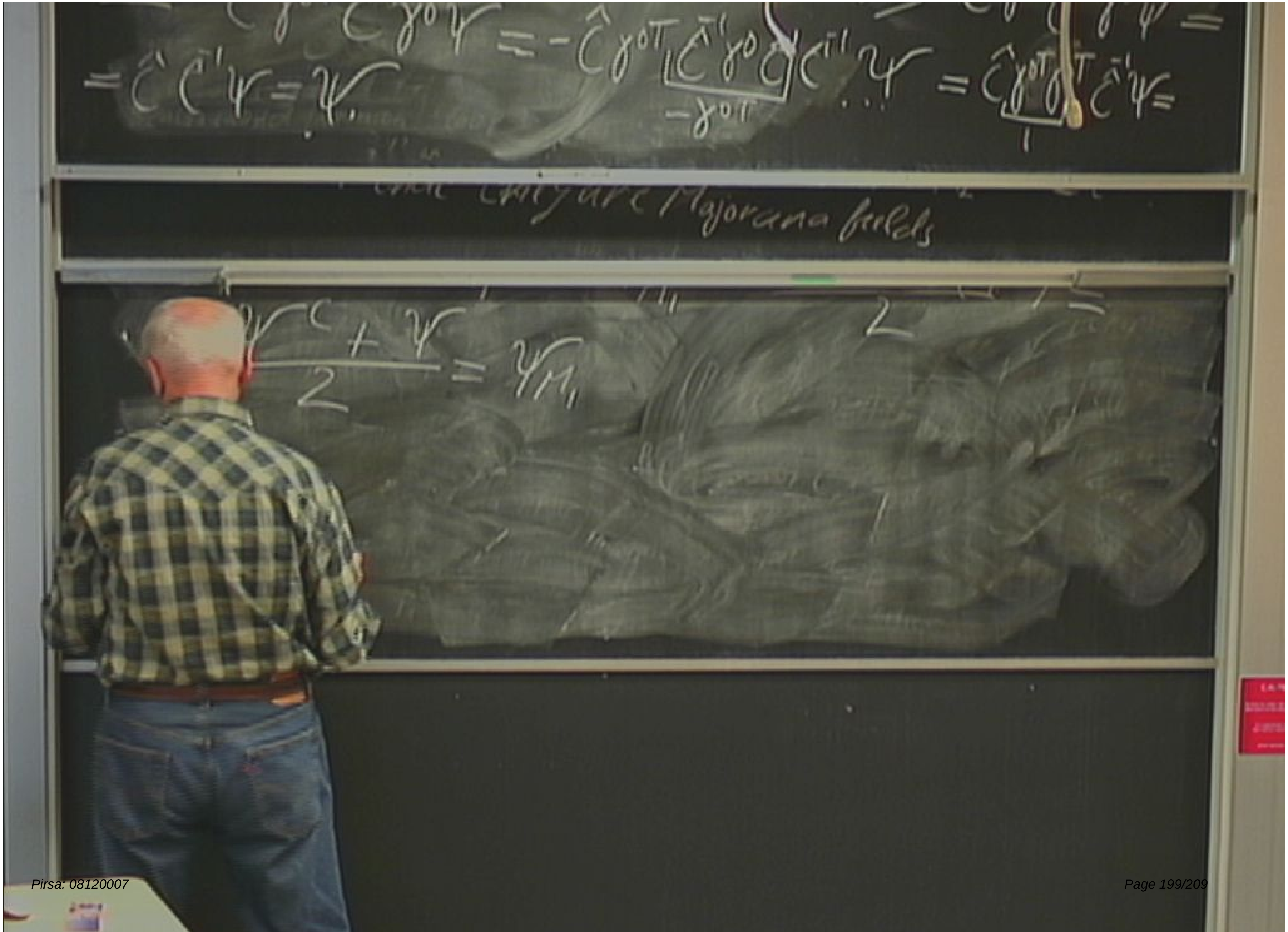

=Ĉ Ĉ⁻¹ψ = ψ.


Let us show that
Prove first that

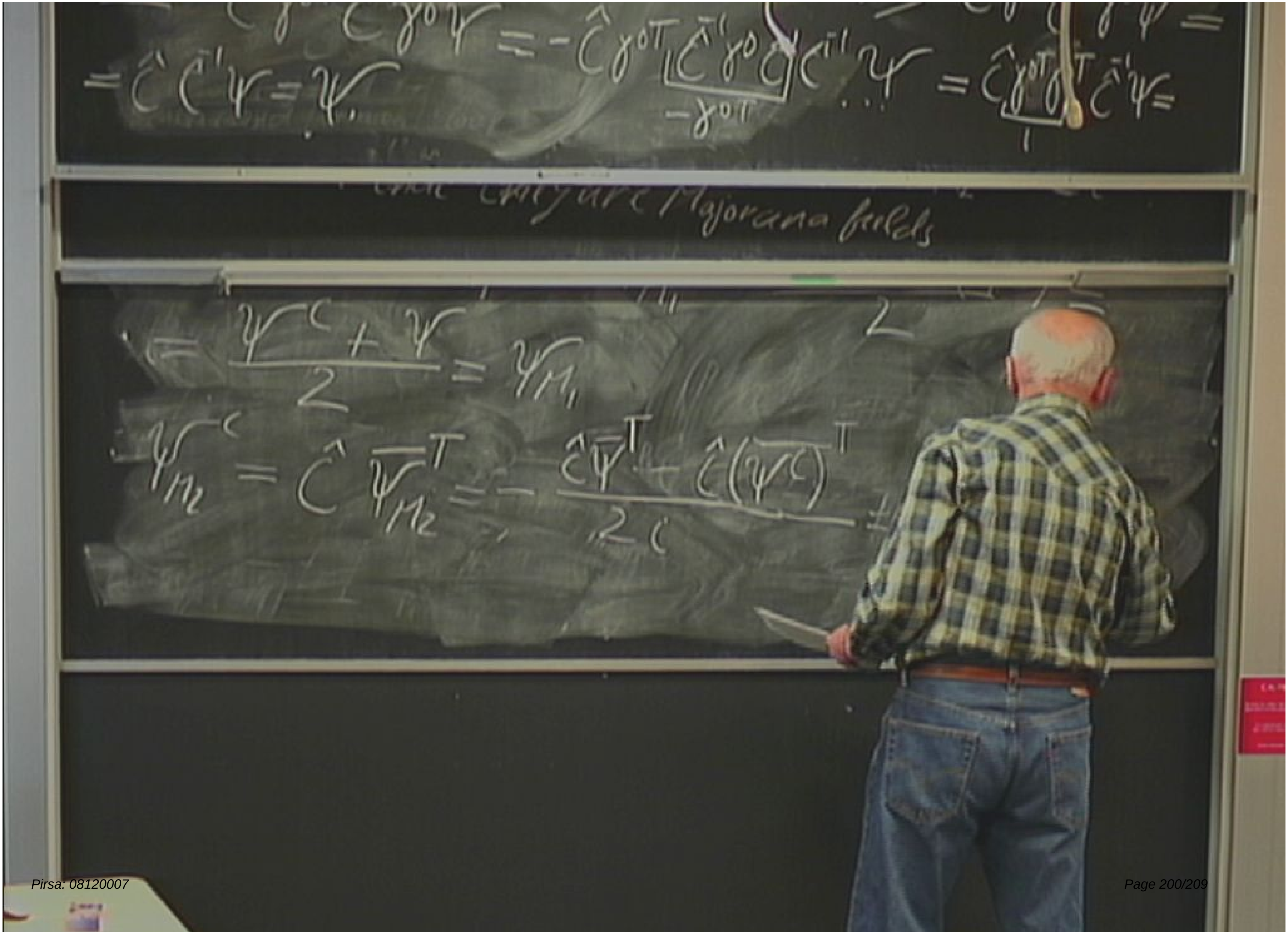

$= \hat{C}\hat{C}^{-1}\psi = \psi$
$\psi_{M_1} - ?$
$\psi_{M_1}^{c} = \hat{C}\,\overline{\psi}_{M_1}^{T} = \hat{C}$


$= \hat{C}\hat{C}^{-1}\psi = \psi$
$= -\hat{C}\gamma^{0T}\hat{C}^{-1}\gamma^{0}C\,C^{-1}\psi = \hat{C}\gamma^{0T}\gamma^{0T}\hat{C}^{-1}\psi =$
ψ_{M_1} - ?
$\psi^{c}_{M_1} = \hat{C}\,\bar{\psi}^{T}_{M_1} = \frac{\hat{C}\bar{\psi}^{T} + \hat{C}(\overline{\psi^{c}})^{T}}{2} =$
$\frac{\psi^{c} + \psi}{2} = \psi_{M_1}$


Let us consider Dirac field ψ. Then, introduce $\psi^c = \hat{C}\bar{\psi}^T$ and two fields: $\psi_{M_1} = \frac{\psi+\psi^c}{2}$, $\psi_{M_2} = \frac{\psi-\psi^c}{2i}$
Let us show that they are Majorana
Prove first that $(\psi^c)^c = \hat{C}\overline{\psi^c}^T$
$= \hat{C}\left[\overline{\hat{C}\bar{\psi}^T}\right]^T = \hat{C}\left[\overline{\hat{C}(\psi^\dagger\gamma^0)^T}\right]^T = \hat{C}\left[\hat{C}\right.$
$= \hat{C}\left[\left[\hat{C}\gamma^{0T}\psi^{\dagger T}\right]^\dagger\gamma^0\right]^T = \hat{C}\left[\psi^T\gamma^{0T}\hat{C}^\dagger\gamma^0\right]^T =$
$\hat{C}^T = -\hat{C}^{-1}$
$-\hat{C}\gamma^{0T}\hat{C}^{-1}\gamma^0\psi = -\hat{C}\gamma^{0T}\underbrace{\hat{C}^{-1}\gamma^0\hat{C}}_{-\gamma^{0T}}\hat{C}^{-1}\psi$
$= \hat{C}\hat{C}^{-1}\psi = \psi.$



$= \hat{C} \hat{C}^{-1} \psi = \psi.$
Majorana fields
$\frac{\psi^c + \psi}{2} = \psi_{M_1}$



Majorana fields
$\frac{\psi^c + \psi}{2} = \psi_{M_1}$
$\psi^c_{M_2} = \hat{C}\,\overline{\psi}_{M_2}^{\,T}$

$$= \hat{C}\hat{C}^{-1}\psi = \psi.$$

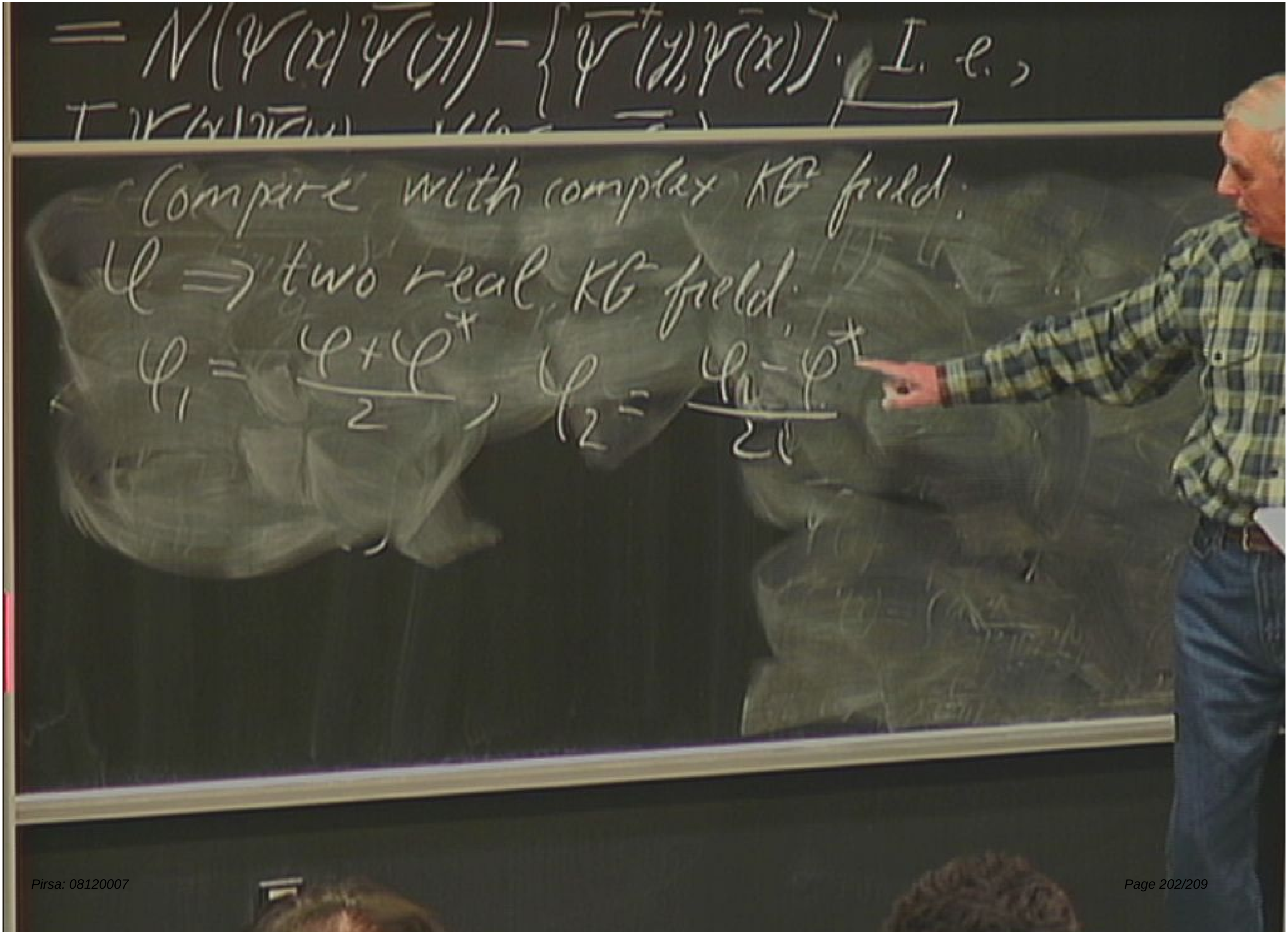
$$= -\hat{C}\gamma^{0T}\underbrace{\hat{C}^{-1}\gamma^{0}\hat{C}}_{-\gamma^{0T}}\hat{C}^{-1}\psi = \hat{C}\underbrace{\gamma^{0T}\gamma^{0T}}_{1}\hat{C}^{-1}\psi =$$

Majorana fields

$$\frac{\psi^{c}+\psi}{2} = \psi_{M_1}$$

$$\psi_{M_2}^{c} = \hat{C}\,\overline{\psi}_{M_2}^{T} = -\frac{\hat{C}\overline{\psi}^{T} - \hat{C}(\overline{\psi^{c}})^{T}}{2i} = -\frac{\psi^{c}-\psi}{2i} =$$

$$= \frac{\psi-\psi^{c}}{2i} = \psi_{M_2}.$$


$= N(\psi(x)\bar{\psi}(y)) - \{\bar{\psi}^{+}(y), \psi(x)\}$. I.e.,
Compare with complex KG field:
$\varphi \Rightarrow$ two real KG field:
$\varphi_1 = \frac{\varphi + \varphi^*}{2}, \ \varphi_2 = \frac{\varphi - \varphi^*}{2i}$

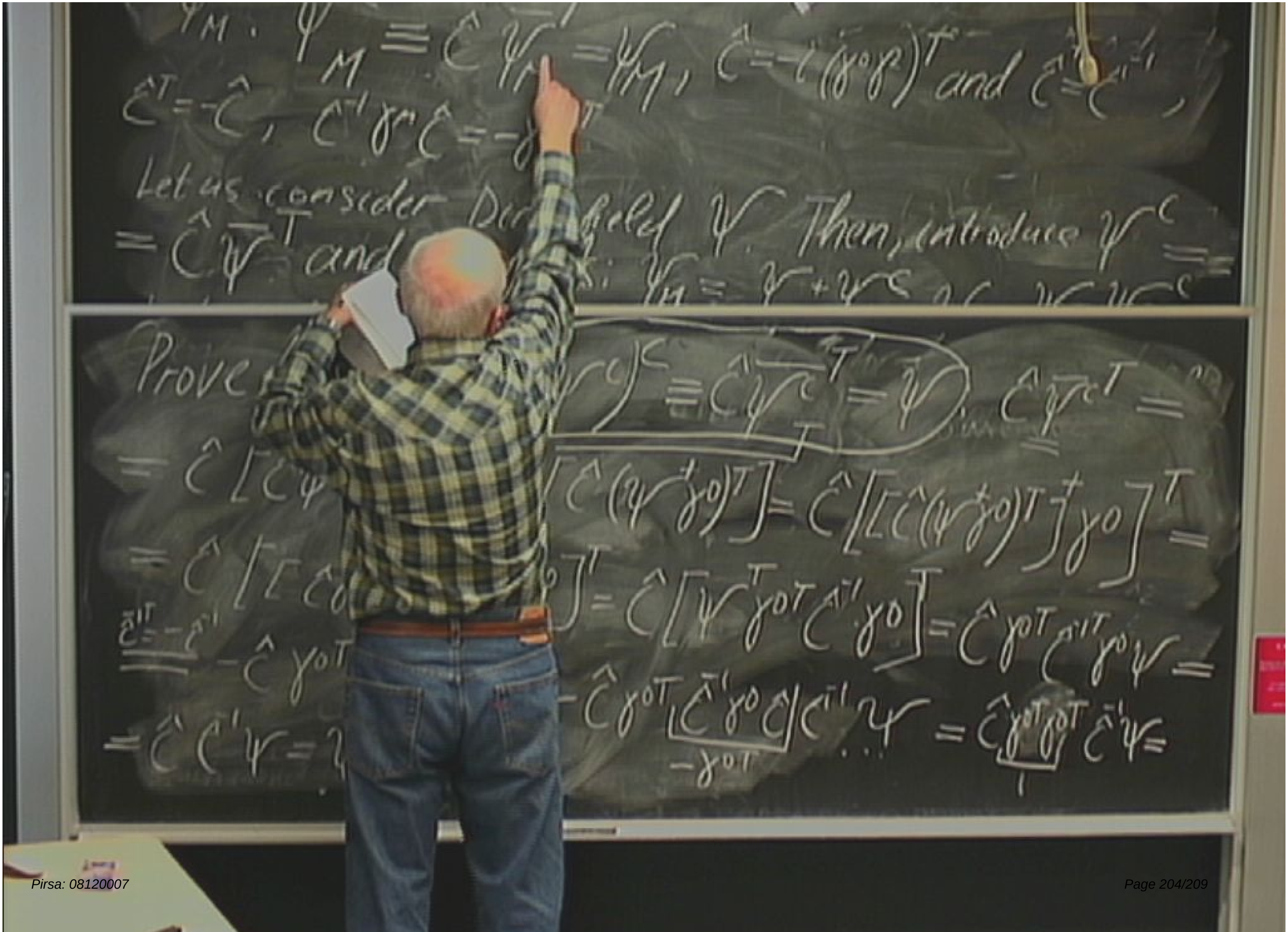
$= N(\psi(x)\bar{\psi}(y)) - \{\bar{\psi}^{+}(y), \psi(x)]$. I.e.,

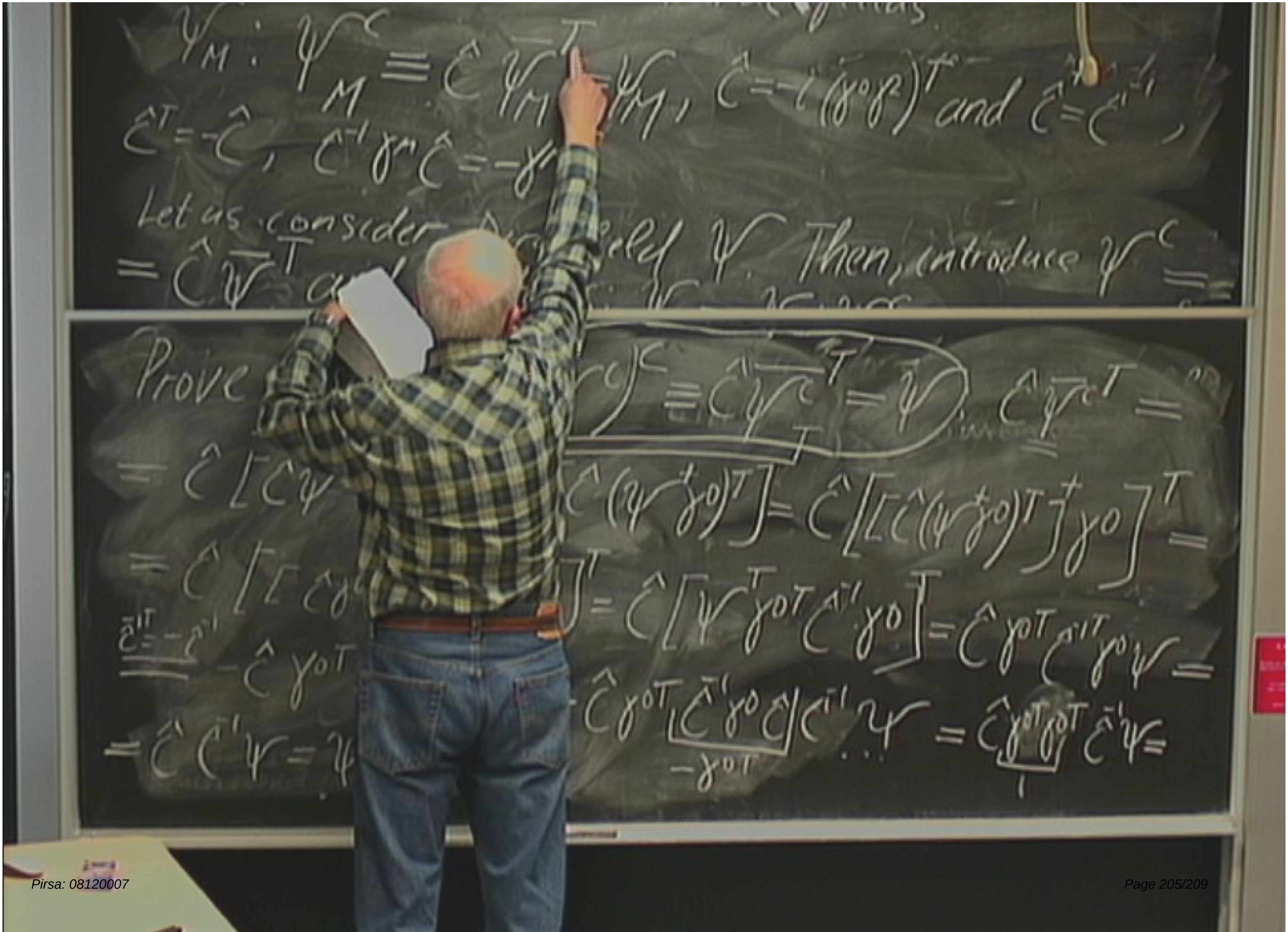
$T[\psi(x)\bar{\psi}(y)]$

Compare with complex KG field:

$\varphi \Rightarrow$ two real KG field:

$$\varphi_1 = \frac{\varphi + \varphi^*}{2}, \quad \varphi_2 = \frac{\varphi - \varphi^*}{2i}$$


Let us consider
field ψ. Then, introduce ψ^c
Prove


Let us consider
Then, introduce
Prove

$$= N(\psi(x)\bar{\psi}(y)) - \{\bar{\psi}^{+}(y), \psi(x)\}.$$ I. e.,

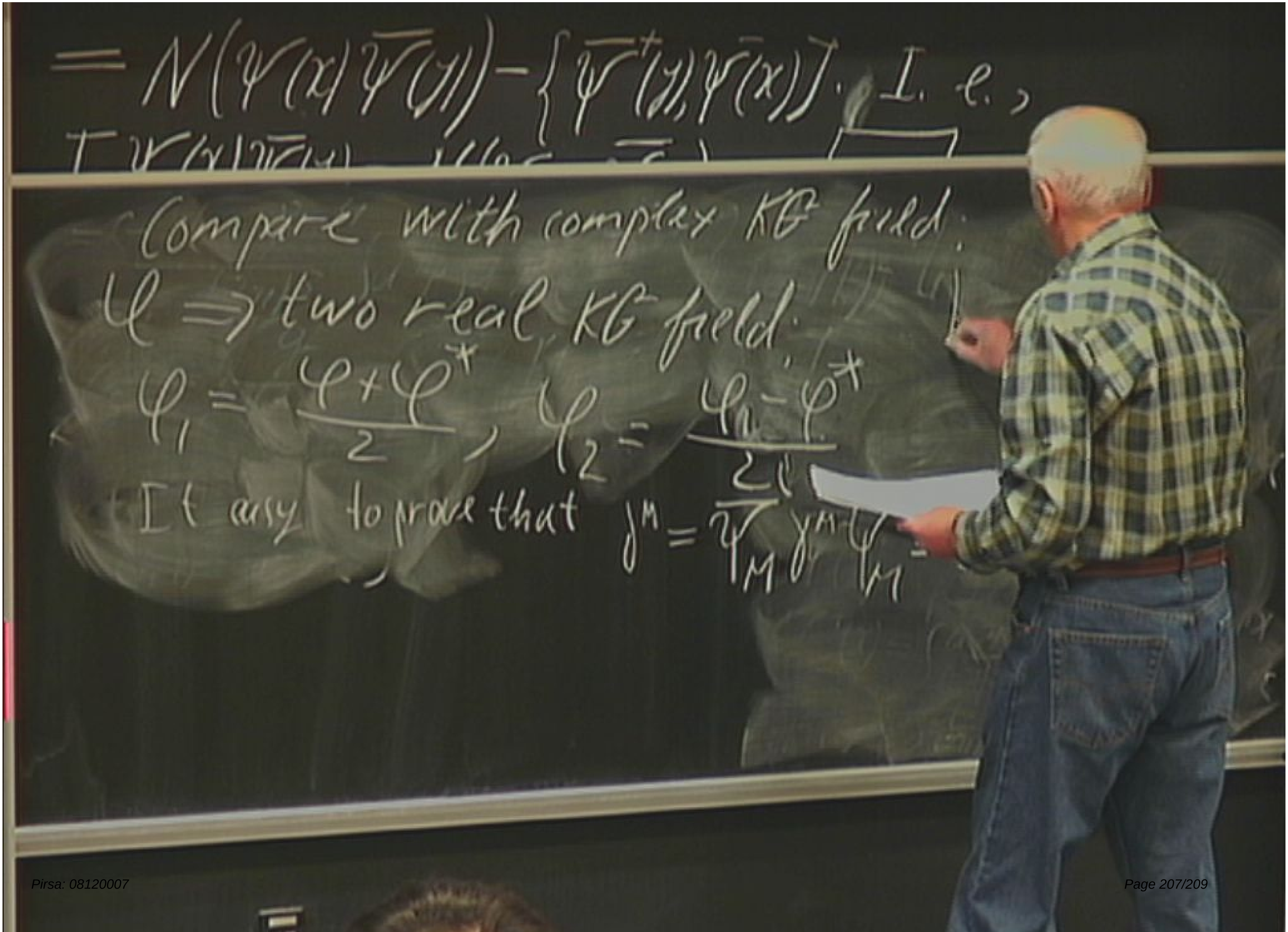
$$T\psi(x)\bar{\psi}(y) = N(\psi\bar{\psi})$$

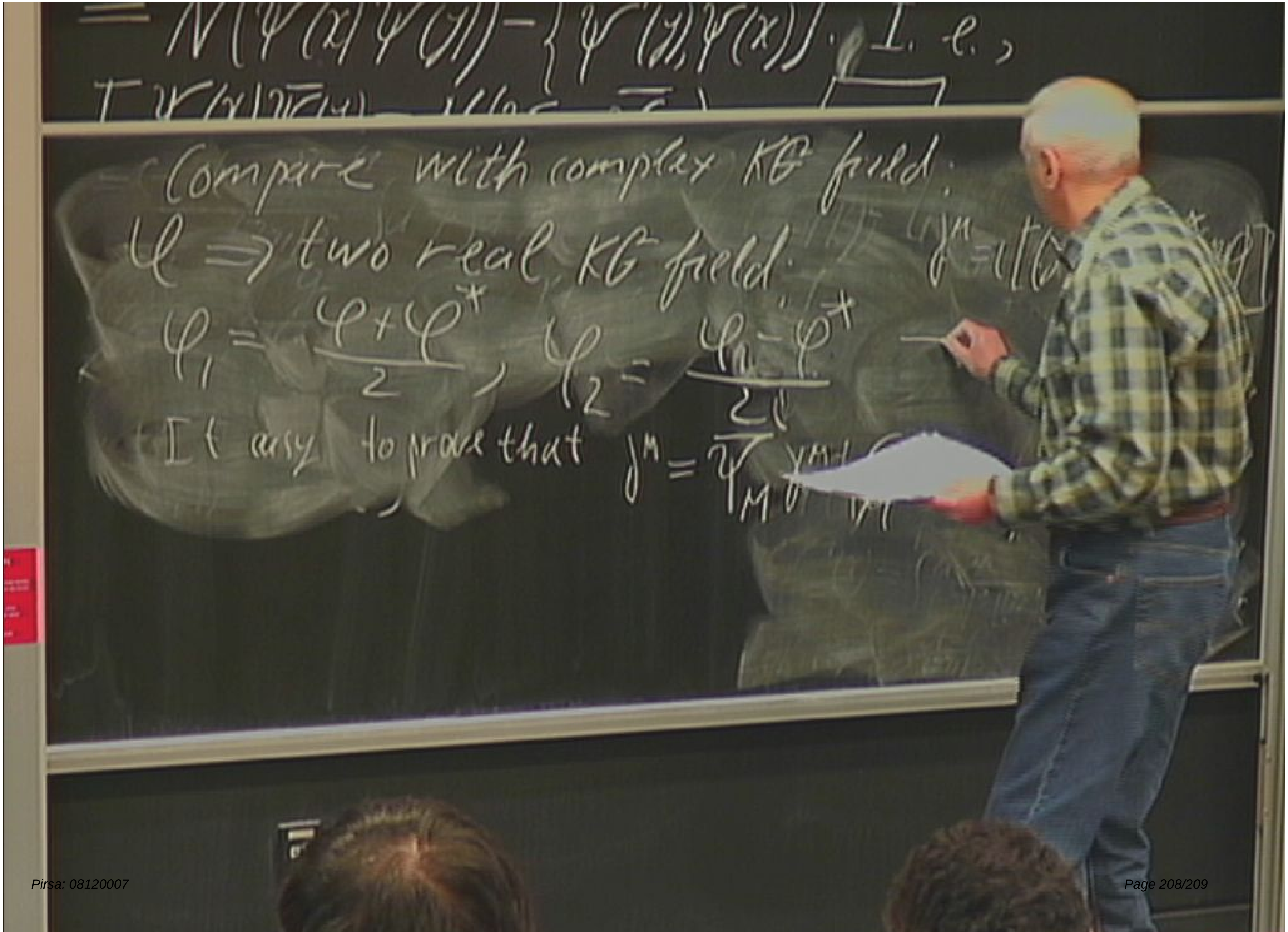
Compare with complex KG field:

$\varphi \Rightarrow$ two real KG field:

$$\varphi_1 = \frac{\varphi + \varphi^*}{2}, \quad \varphi_2 = \frac{\varphi - \varphi^*}{2i}$$

It easy to prove that $j^\mu = \bar{\psi}_M \gamma^\mu \psi_M = 0$


= N(ψ(x)ψ̄(y)) − {ψ̄ᵀ(y), ψ(x)]. I. e.,
Compare with complex KG field:
φ ⇒ two real KG field:
φ1 = (φ + φ*)/2 , φ2 = (φ − φ*)/2i
It easy to prove that j^μ = ψ̄_M γ^μ ψ_M =



$= N(\psi(x)\bar{\psi}(y)) - \{\psi(y), \bar{\psi}(x)\}$. I. e.,
Compare with complex KG field:
$\varphi \Rightarrow$ two real KG field:
$\varphi_1 = \frac{\varphi + \varphi^*}{2}$, $\varphi_2 = \frac{\varphi - \varphi^*}{2i}$
It easy to prove that $j^\mu = \bar{\psi}_M \gamma^\mu \psi_M$
$j^\mu = i[($

$= N(\psi(x)\psi(y)) - \{\psi(y),\psi(x)\}$. I. e.,

$T\psi(x)\bar{\psi}(y) - N(\psi\bar{\psi})$

Compare with complex KG field:

$\varphi \Rightarrow$ two real KG field: $j^\mu = i[(\partial^\mu\varphi^*) - (\varphi^*\partial^\mu\varphi)]$

$\varphi_1 = \frac{\varphi+\varphi^*}{2}$, $\varphi_2 = \frac{\varphi-\varphi^*}{2i} \xrightarrow{\varphi=\varphi^*} 0$.

It easy to prove that $j^\mu = \bar{\psi}_M \gamma^\mu \psi_M = 0$