

Title: Quantum Field Theory 1 - Lecture 15A

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Abstract: Quantum Field Theory I course taught by Volodya Miransky of the University of Western Ontario

Feynman Rules for Fermions

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$\langle 0|T(\psi(x)\bar{\psi}(y))|0\rangle = S_F(x-y) = \int \frac{d^4p}{(2\pi)^4} \frac{i(\not{p}\gamma_4 + m)}{p^2 - m^2 + i\epsilon} e^{-ip(x-y)}$



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$$\begin{aligned}
 \langle 0 | T(\psi(x) \bar{\psi}(y)) | 0 \rangle &= S_F(x-y) = \int \frac{d^4 p}{(2\pi)^4} \bar{\psi}(p) \psi(x), y^0 > x^0 \\
 T(\bar{\psi}(y) \psi(x)) &= -\psi(x) \bar{\psi}(y), x^0 > y^0 \\
 &= \bar{\psi}(y) \psi(x), y^0 > x^0
 \end{aligned}$$

$$\begin{aligned}
 \langle 0 | T (\psi(x) \bar{\psi}(y)) | 0 \rangle &= S_F(x-y) = \int \frac{d^4 p}{(2\pi)^4} \frac{-i(\not{p} \gamma_5 + m)}{p^2 - m^2 + i\epsilon} e^{-i p \cdot (x-y)} \\
 \langle 0 | T (\bar{\psi}(y) \psi(x)) &= \begin{cases} -\psi(x) \bar{\psi}(y), & x^0 > y^0 \\ \bar{\psi}(y) \psi(x), & y^0 > x^0 \end{cases}
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Start from $T(\psi(x)\bar{\psi}(y)) \equiv \begin{cases} \psi(x)\bar{\psi}(y), & x^0 > y^0 \\ -\bar{\psi}(y)\psi(x), & y^0 > x^0 \end{cases}$

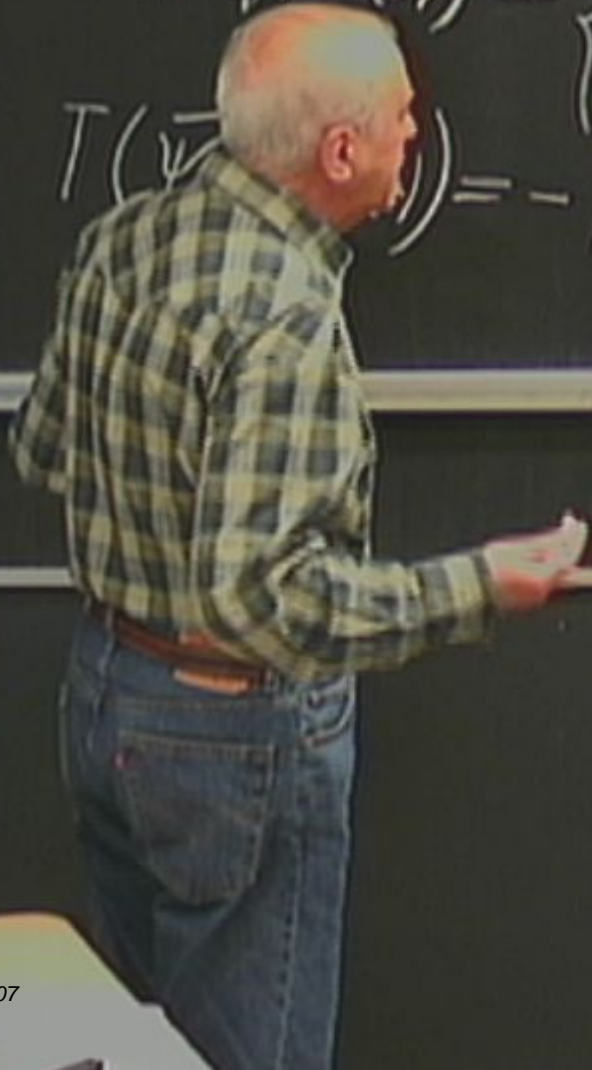
$\langle 0|T(\psi(x)\bar{\psi}(y))|0\rangle = S_F(x-y) = \int \frac{d^4p}{(2\pi)^4} \frac{i(\not{p}\gamma_4 + m)}{p^2 - m^2 + i\epsilon} e^{-ip(x-y)}$

$T(\bar{\psi}(y)\psi(x)) = \begin{cases} -\psi(x)\bar{\psi}(y), & x^0 > y^0 \\ \bar{\psi}(y)\psi(x), & y^0 > x^0 \end{cases}$

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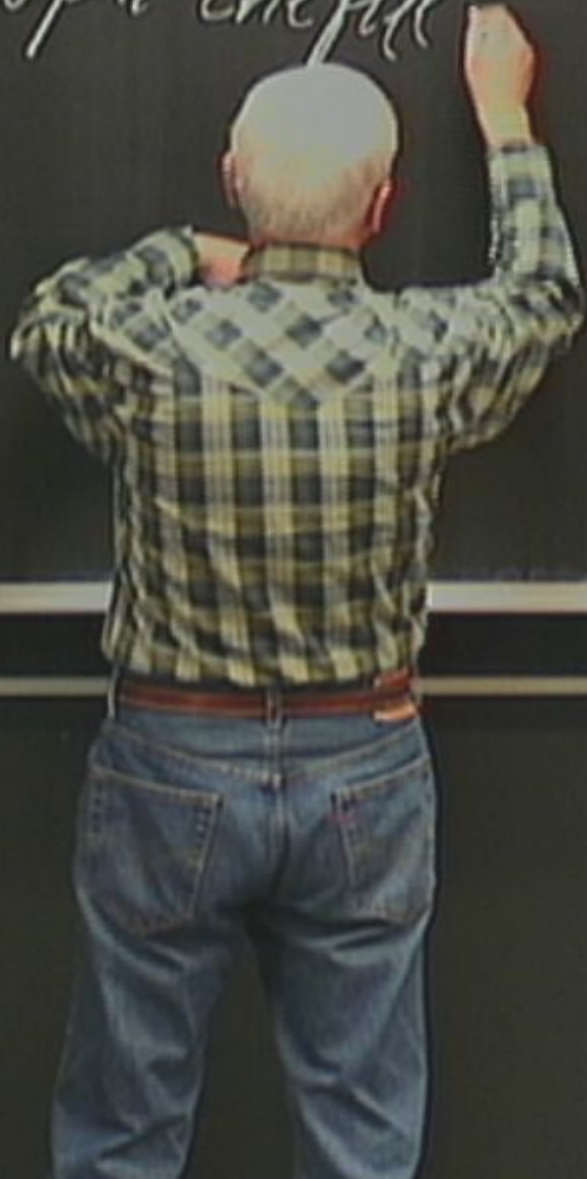
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 T(\bar{\psi}(y)\psi(x)) &= \int \frac{d^4p}{(2\pi)^4} \frac{i(\not{p}\gamma_0 + m)}{p^2 - m^2 + i\epsilon} e^{-ip(x-y)} \\
 T(\bar{\psi}(y)\psi(x)) &= \begin{cases} -\psi(x)\bar{\psi}(y), & x^0 > y^0 \\ \bar{\psi}(y)\psi(x), & y^0 > x^0 \end{cases} \\
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Generalization: Time order product for fermion picks up one minus sign for each interchange of operators that is necessary to put the file



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$$T(\psi_1 \psi_2 \psi_3 \psi_4) = (-1)^3 \psi_3 \psi_1 \psi_4 \psi_2 \quad (x_3^0 > x_1^0)$$

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$$T(\psi_1 \psi_2 \psi_3 \psi_4) = (-1)^3 \psi_3 \psi_1 \psi_4 \psi_2 \text{ for } x_3^0 > x_1^0 > x_4^0 > x_2^0$$



... in any order

$$T(\psi_1, \psi_2, \psi_3, \psi_4) = (-1)^3 \psi_3 \psi_1 \psi_4 \psi_2 \text{ for } x_3^0 > x_1^0 > x_4^0 > x_2^0$$



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to put the fields in time order
 $T(\psi_1 \psi_2 \psi_3 \psi_4) = (-1)^3 \psi_3 \psi_1 \psi_4 \psi_2$ for $x_3^0 > x_1^0 > x_4^0 > x_2^0$
This implies that fermion fields anticommutes
under sign T.

$$T(\psi_1, \psi_2, \psi_3, \psi_4) = -(\psi_4, \psi_1, \psi_2, \psi_3)$$

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$$(\psi_1 \psi_2 \psi_3 \psi_4) = (-1)^{1+1+1} (\psi_4 \psi_3 \psi_2 \psi_1) \quad \text{for } \lambda_3 > \lambda_1 > \lambda_4 > \lambda_2$$

This implies that fermion fields anticommutes under sign T.

$$T(\psi_1 \psi_2 \psi_3 \psi_4) = -(\psi_4 \psi_3 \psi_2 \psi_1) \dots$$



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$$N(a_p^+ a_q) = (-1)^2 a_p^+ a_q = (-1)^3 a_p^+ a_q a_p$$

The definition of the normal-order product of fermions: put an extra minus sign for each fermion interchange:

$$N(a_p a_q a_r^\dagger) = (-1)^2 a_r^\dagger a_p a_q = (-1)^3 a_p^\dagger a_q a_p.$$

Generalization of Wick's Theorem.

Start from $\Psi(\psi(x)\bar{\psi}(y))$; $\Psi(x) = \psi^+(x) + \bar{\psi}(x)$;
 $\psi^+(x) = \int \frac{d^3p}{(2\pi)^3 \sqrt{2E_p}}$

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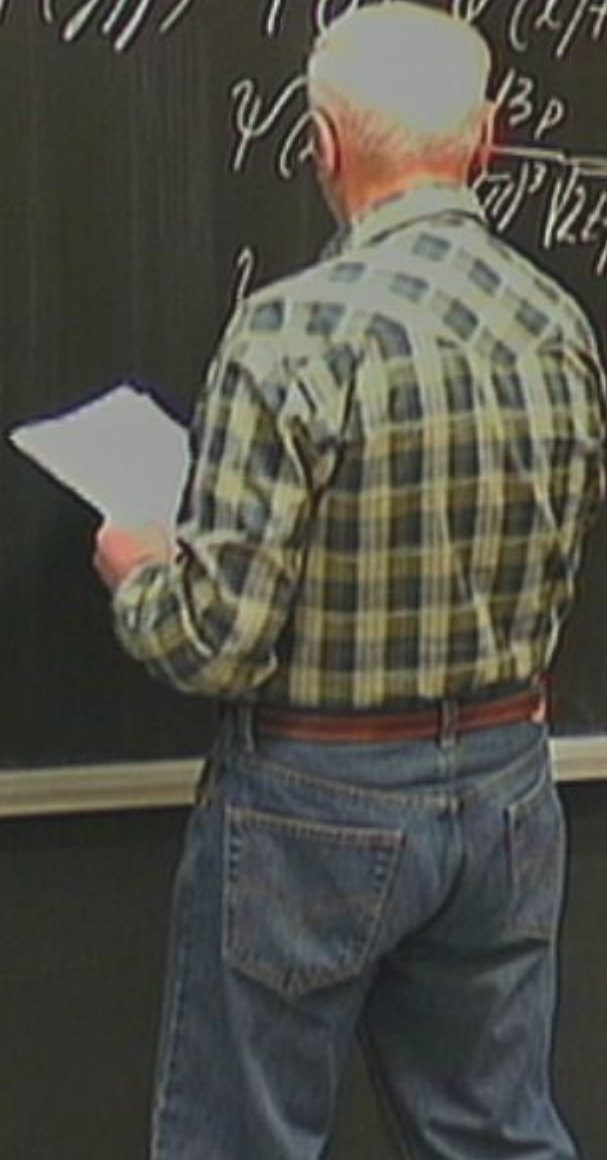
$$T(\psi_1 \psi_2 \psi_3 \psi_4) = -(\psi_4 \psi_3 \psi_2 \psi_1) \dots$$

The definition of the normal-product of fermions: put an extra minus sign for each fermion interchange:

$$N(a_{\vec{p}} a_{\vec{q}} a_{\vec{r}}^{\dagger}) = (-1)^2 a_{\vec{r}}^{\dagger} a_{\vec{p}} a_{\vec{q}} = (-1)^3 a_{\vec{r}}^{\dagger} a_{\vec{q}} a_{\vec{p}}$$

Generalization of Wick's Theorem

Start from $\Psi(\psi(x)\bar{\psi}(y))$; $\Psi(x) = \psi^+(x) + \psi^-(x)$;
 $\Psi(x) = \frac{1}{\sqrt{2\pi}} \sum_{-\infty}^{\infty} a_p^s u(p) e^{-ipx}$



Start from $\Psi(\psi(x)\vec{\psi}(y))$; $\Psi(x) = \psi^+(x) + \psi^-(x)$;

$$\psi^+(x) = \int \frac{d^3p}{(2\pi)^3} \sum_s a_{\vec{p}}^s u^s(p) e^{-ipx}$$

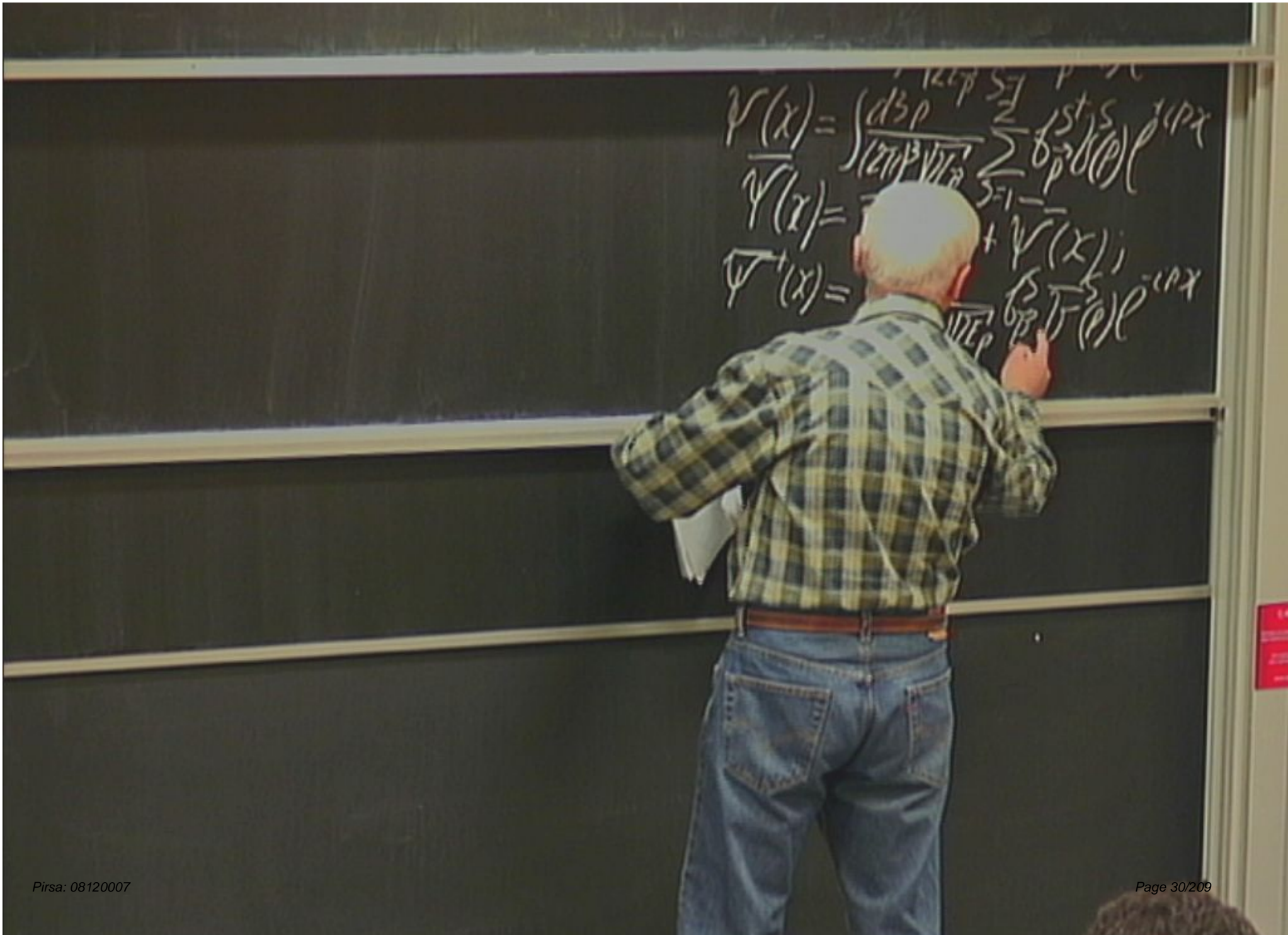
$$\psi^-(x) = \int \frac{d^3p}{(2\pi)^3} \sum_s b_{\vec{p}}^s v^s(p) e^{+ipx}$$

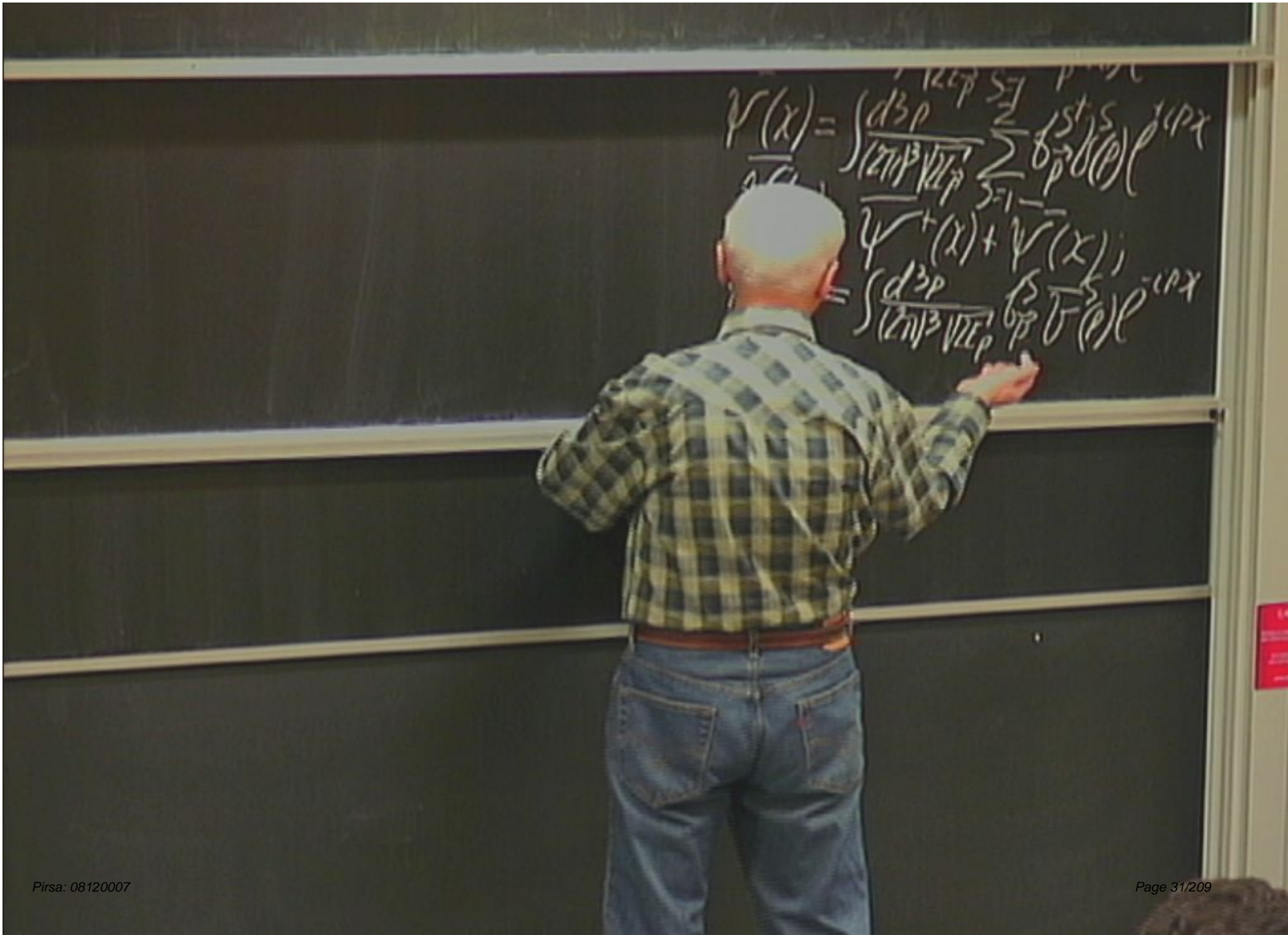
$$\psi(x) = \int \frac{d^3p}{(2\pi)^3 \sqrt{2E_p}} \sum_s \frac{1}{\sqrt{2}} a_{\vec{p}} u^s(p) e^{-i p x}$$

$$\bar{\psi}(x) = \int \frac{d^3p}{(2\pi)^3 \sqrt{2E_p}} \sum_s \frac{1}{\sqrt{2}} b_{\vec{p}}^\dagger v^s(p) e^{+i p x}$$

$$\bar{\psi}(x) \psi(x) + \bar{\psi}(x) \psi(x)$$







Start from $\Psi(\psi(x)\bar{\psi}(y))$, $\Psi(x) = \psi^+(x) + \bar{\psi}(x)$;

$$\psi^+(x) = \int \frac{d^3p}{(2\pi)^3 \sqrt{2E_p}} \sum_{s=1,2} a_{\vec{p}}^s u(\vec{p}) e^{-ipx}$$

$$\bar{\psi}(x) = \int \frac{d^3p}{(2\pi)^3 \sqrt{2E_p}} \sum_{s=1,2} b_{\vec{p}}^{\dagger s} v(\vec{p}) e^{ipx}$$

$$\bar{\Psi}(x) = \bar{\psi}^+(x) + \psi^-(x) ;$$

$$\bar{\psi}^+(x) = \int \frac{d^3p}{(2\pi)^3 \sqrt{2E_p}} \sum_{s=1,2} \bar{v}(\vec{p}) e^{-ipx}$$

$$\psi^-(x) = \int \frac{d^3p}{(2\pi)^3 \sqrt{2E_p}} \sum_{s=1,2} a_{\vec{p}}^{\dagger s} u(\vec{p}) e^{ipx}$$

Start from $\Psi(\psi(x)\bar{\psi}(y))$,

$$\psi^+(x)|0\rangle = \bar{\psi}^+(x)|0\rangle = 0$$

$$\Psi(x) = \psi^+(x) + \bar{\psi}(x)$$

$$\psi^+(x) = \int \frac{d^3p}{(2\pi)^3 \sqrt{2E_p}} \sum_{s=1}^2 a_{\vec{p}}^s u(\vec{p}) e^{-ipx}$$

$$\bar{\psi}(x) = \int \frac{d^3p}{(2\pi)^3 \sqrt{2E_p}} \sum_{s=1}^2 b_{\vec{p}}^{s\dagger} v(\vec{p}) e^{+ipx}$$

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$$\bar{\psi}(x) = \int \frac{d^3p}{(2\pi)^3 \sqrt{2E_p}} \sum_{s=1}^2 b_{\vec{p}}^{st} v(\vec{p}) e^{ipx}$$

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$$\bar{\psi}^+(x) = \int \frac{d^3p}{(2\pi)^3 \sqrt{2E_p}} \sum_{s=1}^2 \bar{b}_{\vec{p}}^s v(\vec{p}) e^{-ipx}$$

$$\psi(x) = \int \frac{d^3p}{(2\pi)^3 \sqrt{2E_p}} \sum_{s=1}^2 a_{\vec{p}}^s u(\vec{p}) e^{ipx}$$

$$\psi(x|t=0) = \psi(x)|_{t=0} = 0$$

$$\langle 0 | \psi(x) = \langle 0 | \psi(x) = 0$$

$$\psi^+(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \sum_{\vec{s}} a_{\vec{p}} u(\vec{p}) e^{-i p x}$$

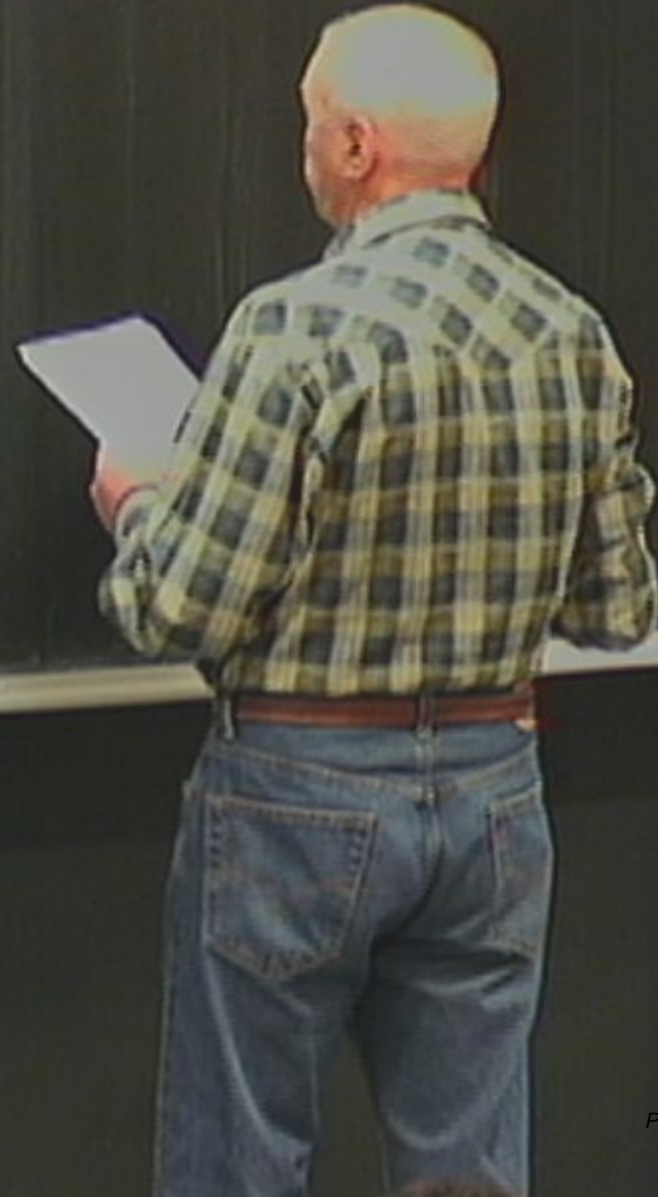
$$\psi^-(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \sum_{\vec{s}} b_{\vec{p}} v(\vec{p}) e^{i p x}$$

$$\psi(x) = \psi^+(x) + \psi^-(x)$$

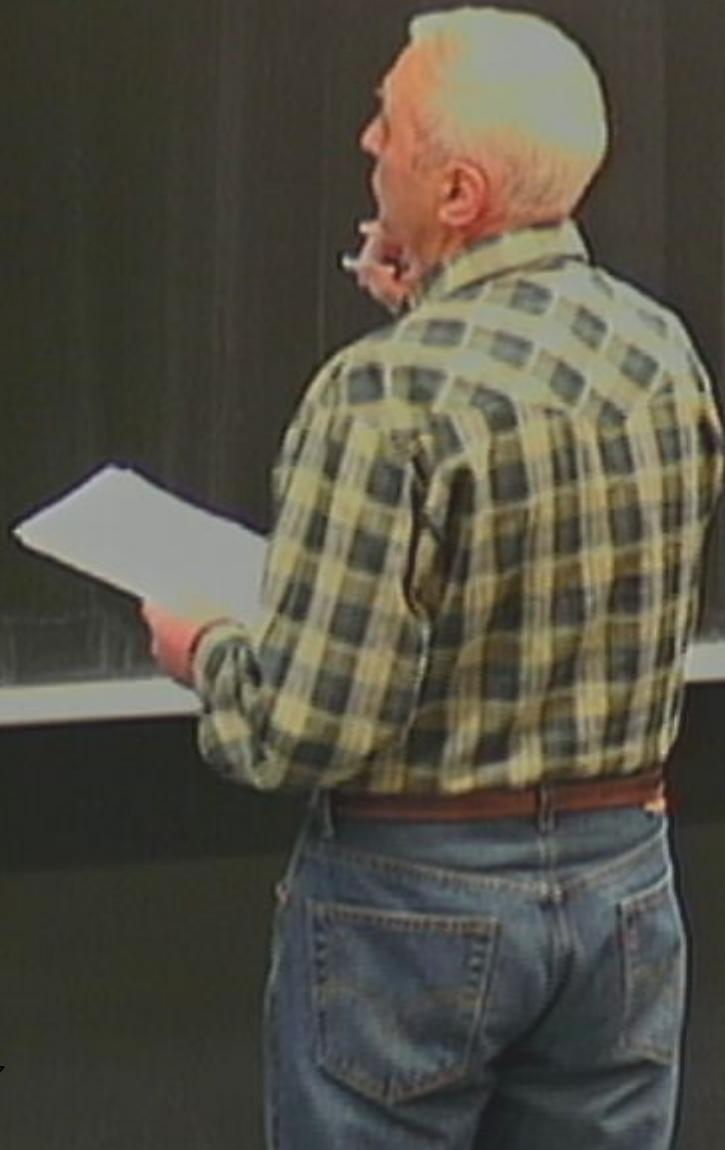
$$\psi^+(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \sum_{\vec{s}} a_{\vec{p}} u(\vec{p}) e^{-i p x}$$

$$\psi^-(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \sum_{\vec{s}} b_{\vec{p}} v(\vec{p}) e^{i p x}$$

$$T(\psi(x)\bar{\psi}(y))_{x_0 > y_0} = \psi^+(x)\bar{\psi}^+(y) + \psi^+(x)\bar{\psi}(y)$$



$$T(\psi(x)\bar{\psi}(y))_{x_0 > y_0} = \psi^+(x)\bar{\psi}^+(y) + \psi^+(x)\bar{\psi}^-(y) + \psi^-(x)\bar{\psi}^+(y) + \psi^-(x)\bar{\psi}^-(y)$$



$$T(\psi(x)\bar{\psi}(y))_{x_0 > y_0} = \psi^+(x)\bar{\psi}^+(y) + \underbrace{\psi^+(x)\bar{\psi}^-(y)} + \psi^-(x)\bar{\psi}^-(y) + \psi^-(x)\bar{\psi}^+(y)$$

$$\begin{aligned}
 T(\psi(x)\bar{\psi}(y))_{x_0 > y_0} &= \psi^+(x)\bar{\psi}^+(y) + \psi^+(x)\bar{\psi}^-(y) + \psi^-(x)\bar{\psi}^+(y) + \psi^-(x)\bar{\psi}^-(y) \\
 &= \psi^+(x)\bar{\psi}^+(y) + \bar{\psi}^-(y)\psi^+(x) + \bar{\psi}^-(x)\psi^-(x) + \bar{\psi}^-(x)\bar{\psi}^-(y)
 \end{aligned}$$

$$\begin{aligned}
 T(\psi(x) \bar{\psi}(y))_{x_0 > y_0} &= \psi^+(x) \bar{\psi}^+(y) + \psi^+(x) \bar{\psi}^-(y) + \psi^-(x) \bar{\psi}^+(y) + \psi^-(x) \bar{\psi}^-(y) \\
 &= \psi^+(x) \bar{\psi}^+(y) + \bar{\psi}^-(y) \psi^+(x) + \psi^-(x) \bar{\psi}^+(x) + \bar{\psi}^-(x) \bar{\psi}^-(x) +
 \end{aligned}$$



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 T(\psi(x)\bar{\psi}(y))_{x_0 > y_0} &= \psi^+(x)\bar{\psi}^+(y) + \psi^+(x)\bar{\psi}^-(y) + \psi^-(x)\bar{\psi}^+(y) + \psi^-(x)\bar{\psi}^-(y) \\
 &= \psi^+(x)\bar{\psi}^+(y) - \bar{\psi}^-(y)\psi^+(x) + \psi^-(x)\bar{\psi}^+(y) + \bar{\psi}^-(y)\psi^-(x) + \\
 &+ \{\psi^+(x), \bar{\psi}^-(y)\}
 \end{aligned}$$

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 &+ \{\psi^+(x), \bar{\psi}^-(y)\} =
 \end{aligned}$$

$$\begin{aligned}
 & \int (\psi(x) \psi(y)) \overline{\psi(x) \psi(y)} = \int (\psi(x) \psi(y) + \psi(x) \psi(y) + \psi(x) \overline{\psi(y)} + \overline{\psi(x)} \psi(y)) \\
 & = \int \psi^+(x) \overline{\psi^+(y)} - \int \overline{\psi^-(y)} \psi^+(x) + \int \overline{\psi^-(x)} \psi^+(y) + \int \overline{\psi^-(x)} \overline{\psi^-(y)} + \\
 & + \int \psi^+(x) \overline{\psi^-(y)} - \int \overline{\psi^-(x)} \psi^+(y) + \int \psi^+(x) \overline{\psi^-(y)}.
 \end{aligned}$$

$$\begin{aligned}
 T(\psi(x) \bar{\psi}(y))_{x_0 > y_0} &= \psi^+(x) \bar{\psi}^+(y) + \psi^+(x) \bar{\psi}^-(y) + \psi^-(x) \bar{\psi}^+(y) + \psi^-(x) \bar{\psi}^-(y) \\
 &= \psi^+(x) \bar{\psi}^+(y) - \bar{\psi}^-(y) \psi^+(x) + \psi^-(x) \bar{\psi}^+(y) + \bar{\psi}^-(y) \psi^-(x) + \psi^-(x) \bar{\psi}^-(y) + \\
 &+ \{\psi^+(x), \bar{\psi}^-(y)\} = N(\psi(x) \bar{\psi}(y)) + \{\psi^+(x), \bar{\psi}^-(y)\}.
 \end{aligned}$$

$T(\psi(x)$

$$\begin{aligned}
 T(\psi(x) \bar{\psi}(y))_{\bar{x}_0 > y_0} &= \psi^+(x) \bar{\psi}^+(y) + \psi^+(x) \bar{\psi}^-(y) + \psi^-(x) \bar{\psi}^+(y) + \psi^-(x) \bar{\psi}^-(y) \\
 &= \psi^+(x) \bar{\psi}^+(y) - \bar{\psi}^-(y) \psi^+(x) + \psi^-(x) \bar{\psi}^+(y) + \bar{\psi}^-(x) \bar{\psi}^-(y) + \\
 &+ \{\psi^+(x), \bar{\psi}^-(y)\} = N(\psi(x), \bar{\psi}(y)) + \{\psi^+(x), \bar{\psi}^-(y)\}.
 \end{aligned}$$

$$T(\psi(x) \bar{\psi}(y))_{\bar{y}_0 = x_0} = -\bar{\psi}^+(y) \psi^-(x)$$



$$\begin{aligned}
T(\psi(x) \bar{\psi}(y))_{\bar{x}_0 > y_0} &= \psi^+(x) \bar{\psi}^+(y) + \psi^+(x) \bar{\psi}^-(y) + \psi^-(x) \bar{\psi}^+(y) + \psi^-(x) \bar{\psi}^-(y) \\
&= \psi^+(x) \bar{\psi}^+(y) - \bar{\psi}^-(y) \psi^+(x) + \psi^-(x) \bar{\psi}^+(y) + \bar{\psi}^-(y) \psi^-(x) + \\
&+ \{\psi^+(x), \bar{\psi}^-(y)\} = N(\psi(x) \bar{\psi}(y)) + \{\psi^+(x), \bar{\psi}^-(y)\} \\
T(\psi(x) \bar{\psi}(y))_{\bar{x}_0 = x_0} &= -\bar{\psi}^+(y) \psi^+(x) - \bar{\psi}^-(y) \psi^-(x)
\end{aligned}$$



$$\begin{aligned}
 T(\psi(x) \bar{\psi}(y)) \Big|_{x_0 > y_0} &= \psi^+(x) \bar{\psi}^+(y) + \psi^+(x) \bar{\psi}^-(y) + \psi^-(x) \bar{\psi}^+(y) + \psi^-(x) \bar{\psi}^-(y) \\
 &= \psi^+(x) \bar{\psi}^+(y) - \bar{\psi}^-(y) \psi^+(x) + \psi^-(x) \bar{\psi}^+(y) + \bar{\psi}^-(x) \bar{\psi}^-(y) + \\
 &+ \{\psi^+(x), \bar{\psi}^-(y)\} = N(\psi(x) \bar{\psi}(y)) + \{\psi^+(x), \bar{\psi}^-(y)\}.
 \end{aligned}$$

$$\begin{aligned}
 T(\psi(x) \bar{\psi}(y)) \Big|_{y_0 = x_0} &= -\bar{\psi}^+(y) \psi^+(x) - \bar{\psi}^-(y) \psi^-(x) - \bar{\psi}^-(y) \psi^+(x) - \\
 &- \bar{\psi}^+(y) \psi^-(x)
 \end{aligned}$$

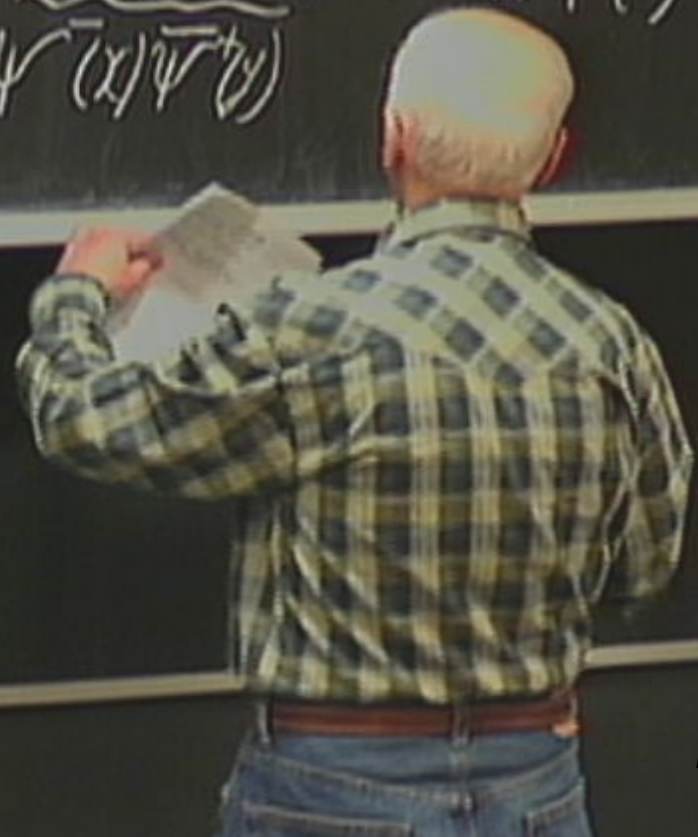
$$\begin{aligned}
T(\psi(x) \bar{\psi}(y))_{x_0 > y_0} &= \psi^+(x) \bar{\psi}^+(y) + \psi^+(x) \bar{\psi}^-(y) + \psi^-(x) \bar{\psi}^+(y) + \psi^-(x) \bar{\psi}^-(y) \\
&= \psi^+(x) \bar{\psi}^+(y) - \bar{\psi}^-(y) \psi^+(x) + \psi^-(x) \bar{\psi}^+(y) + \bar{\psi}^-(x) \bar{\psi}^-(y) + \\
&+ \{\psi^+(x), \bar{\psi}^-(y)\} = 1 + \{\psi^-(x), \bar{\psi}^-(y)\} + \{\psi^+(x), \bar{\psi}^-(y)\} \\
T(\psi(x) \bar{\psi}(y))_{y_0 = x_0} &= \psi^+(x) \bar{\psi}^+(y) - \bar{\psi}^-(y) \psi^+(x) - \bar{\psi}^-(y) \psi^-(x) - \\
&- \bar{\psi}^-(y) \psi^-(x)
\end{aligned}$$

$$\begin{aligned}
 T(\psi(x) \bar{\psi}(y))_{\bar{x}_0 > y_0} &= \psi^+(x) \bar{\psi}^+(y) + \psi^+(x) \bar{\psi}^-(y) + \psi^-(x) \bar{\psi}^+(y) + \psi^-(x) \bar{\psi}^-(y) \\
 &= \psi^+(x) \bar{\psi}^+(y) - \bar{\psi}^-(y) \psi^+(x) + \psi^-(x) \bar{\psi}^+(y) + \bar{\psi}^-(y) \psi^-(x) + \\
 &+ \{\psi^+(x), \bar{\psi}^-(y)\} = N(\psi(x) \bar{\psi}(y)) + \{\psi^+(x), \bar{\psi}^-(y)\} \\
 T(\psi(x) \bar{\psi}(y))_{\bar{y}_0 < x_0} &= -\bar{\psi}^+(y) \psi^+(x) - \bar{\psi}^-(y) \psi^-(x) - \bar{\psi}^-(y) \psi^+(x) \\
 &- \bar{\psi}^+(y) \psi^-(x)
 \end{aligned}$$

$$\begin{aligned}
 T(\psi(x) \bar{\psi}(y))_{x_0 > y_0} &= \psi^+(x) \bar{\psi}^+(y) + \psi^-(x) \bar{\psi}^-(y) + \psi^-(x) \bar{\psi}^+(y) + \psi^+(x) \bar{\psi}^-(y) \\
 &= \psi^+(x) \bar{\psi}^+(y) - \bar{\psi}^-(y) \psi^+(x) + \bar{\psi}^-(x) \bar{\psi}^+(y) + \bar{\psi}^-(x) \bar{\psi}^-(y) + \\
 &+ \{\psi^+(x), \bar{\psi}^-(y)\} = N(\psi(x) \bar{\psi}(y)) + \{\psi^+(x), \bar{\psi}^-(y)\}.
 \end{aligned}$$

$$\begin{aligned}
 T(\psi(x) \bar{\psi}(y))_{x_0 = x_0} &= -\bar{\psi}^+(y) \psi^+(x) - \bar{\psi}^-(y) \psi^-(x) - \bar{\psi}^-(y) \psi^+(x) - \\
 &- \bar{\psi}^+(y) \psi^-(x) = -\bar{\psi}^+(y) \psi^+(x)
 \end{aligned}$$

$$\begin{aligned}
 T(\psi(x)\psi(y))_{x_0 > y_0} &= \psi(x)\psi(y) + \psi(x)\bar{\psi}(y) + \bar{\psi}(x)\psi(y) + \bar{\psi}(x)\bar{\psi}(y) \\
 &= \psi^+(x)\bar{\psi}^+(y) - \bar{\psi}^-(y)\psi^+(x) + \bar{\psi}^-(x)\bar{\psi}^+(y) + \bar{\psi}^-(x)\bar{\psi}^-(y) + \\
 &+ \{\psi^+(x), \bar{\psi}^-(y)\} = N(\psi(x)\bar{\psi}(y)) + \{\psi^+(x), \bar{\psi}^-(y)\}. \\
 T(\psi(x)\bar{\psi}(y))_{y_0 = x_0} &= -\bar{\psi}^+(y)\psi^+(x) - \bar{\psi}^-(y)\psi^-(x) - \bar{\psi}^-(y)\psi^+(x) - \\
 -\bar{\psi}^-(y)\psi^-(x) &= -\bar{\psi}^+(y)\psi^+(x) + \psi^-(x)\bar{\psi}^+(y)
 \end{aligned}$$



$$\begin{aligned}
 T(\psi(x) \bar{\psi}(y))_{\bar{x}_0 > y_0} &= \psi^+(x) \bar{\psi}^+(y) + \psi^-(x) \bar{\psi}^-(y) + \psi^-(x) \bar{\psi}^+(y) + \psi^+(x) \bar{\psi}^-(y) \\
 &= \psi^+(x) \bar{\psi}^+(y) - \bar{\psi}^-(y) \psi^+(x) + \psi^-(x) \bar{\psi}^+(y) + \bar{\psi}^-(y) \psi^-(x) + \\
 &+ \{\psi^+(x), \bar{\psi}^-(y)\} = N(\psi(x) \bar{\psi}(y)) + \{\psi^+(x), \bar{\psi}^-(y)\}.
 \end{aligned}$$

$$\begin{aligned}
 T(\psi(x) \bar{\psi}(y))_{\bar{y}_0 < x_0} &= -\bar{\psi}^+(y) \psi^+(x) - \bar{\psi}^-(y) \psi^-(x) - \bar{\psi}^-(y) \psi^+(x) - \\
 &- \bar{\psi}^+(y) \psi^-(x) = -\bar{\psi}^+(y) \psi^+(x) + \psi^-(x) \bar{\psi}^+(y) - \bar{\psi}^-(y) \psi^+(x)
 \end{aligned}$$

$$\begin{aligned}
 T(\psi(x)\bar{\psi}(y))_{x_0 > y_0} &= \psi^+(x)\bar{\psi}^+(y) + \psi^-(x)\bar{\psi}^-(y) + \psi^-(x)\bar{\psi}^+(y) + \psi^+(x)\bar{\psi}^-(y) \\
 &= \psi^+(x)\bar{\psi}^+(y) - \bar{\psi}^-(y)\psi^+(x) + \psi^-(x)\bar{\psi}^+(y) + \bar{\psi}^-(y)\psi^-(x) + \psi^-(x)\bar{\psi}^-(y) + \\
 &+ \{\psi^+(x), \bar{\psi}^-(y)\} = N(\psi(x)\bar{\psi}(y)) + \{\psi^+(x), \bar{\psi}^-(y)\}.
 \end{aligned}$$

$$\begin{aligned}
 T(\psi(x)\bar{\psi}(y))_{y_0 = x_0} &= -\bar{\psi}^+(y)\psi^+(x) - \bar{\psi}^-(y)\psi^-(x) - \bar{\psi}^-(y)\psi^+(x) - \\
 -\bar{\psi}^+(y)\psi^-(x) &= -\bar{\psi}^+(y)\psi^+(x) - \bar{\psi}^-(y)\psi^-(x) - \bar{\psi}^-(y)\psi^+(x) - \bar{\psi}^+(y)\psi^-(x)
 \end{aligned}$$

$$\psi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \delta(x-p) \psi(p) dp$$

$$T(\psi(x) \bar{\psi}(y))_{x_0 > y_0} = \psi^+(x) \bar{\psi}^+(y) + \psi^+(x) \bar{\psi}^-(y) + \psi^-(x) \bar{\psi}^+(y) + \psi^-(x) \bar{\psi}^-(y)$$

$$= \psi^+(x) \bar{\psi}^+(y) - \bar{\psi}^-(y) \psi^+(x) + \bar{\psi}^-(x) \bar{\psi}^+(y) + \bar{\psi}^-(x) \bar{\psi}^-(y) + \{\psi^+(x), \bar{\psi}^-(y)\} = N(\psi(x) \bar{\psi}(y)) + \{\psi^+(x), \bar{\psi}^-(y)\}$$

$$T(\psi(x) \bar{\psi}(y))_{y_0 > x_0} = -\bar{\psi}^+(y) \psi^+(x) - \bar{\psi}^-(y) \psi^-(x) - \bar{\psi}^-(y) \psi^+(x) - \bar{\psi}^+(y) \psi^-(x)$$

$$= -\bar{\psi}^+(y) \psi^+(x) + \psi^-(x) \bar{\psi}^+(y) - \bar{\psi}^-(y) \psi^+(x) - \bar{\psi}^+(y) \psi^-(x)$$

$$\psi(x) = \frac{1}{(2\pi)^{3/2}} \int_{-\infty}^{\infty} \delta(\vec{p}) \psi(\vec{p}) e^{i\vec{p}\cdot\vec{x}} d^3p$$

$$T(\psi(x) \bar{\psi}(y))_{x_0 > y_0} = \psi^+(x) \bar{\psi}^+(y) + \psi^+(x) \bar{\psi}^-(y) + \psi^-(x) \bar{\psi}^+(y) + \psi^-(x) \bar{\psi}^-(y) \\ = \psi^+(x) \bar{\psi}^+(y) - \bar{\psi}^-(y) \psi^+(x) + \bar{\psi}^-(x) \bar{\psi}^+(y) + \bar{\psi}^-(x) \bar{\psi}^-(y) + \\ + \{\psi^+(x), \bar{\psi}^-(y)\} = N(\psi(x) \bar{\psi}(y)) + \{\psi^+(x), \bar{\psi}^-(y)\}$$

$$T(\psi(x) \bar{\psi}(y))_{y_0 = x_0} = -\bar{\psi}^+(y) \psi^+(x) - \bar{\psi}^-(y) \psi^-(x) - \bar{\psi}^-(y) \psi^+(x) - \\ - \bar{\psi}^+(y) \psi^-(x) = -\bar{\psi}^+(y) \psi^+(x) - \bar{\psi}^-(y) \psi^-(x) - \bar{\psi}^-(y) \psi^+(x) - \\ - \bar{\psi}^+(y) \psi^-(x) - \{\bar{\psi}^+(y), \psi^-(x)\}$$

$$\psi(x) = \psi^+(x) + \psi^-(x)$$

$$T(\psi(x) \bar{\psi}(y))_{x^0 > y^0} = \psi^+(x) \bar{\psi}^+(y) + \psi^+(x) \bar{\psi}^-(y) + \psi^-(x) \bar{\psi}^+(y) + \psi^-(x) \bar{\psi}^-(y)$$

$$= \psi^+(x) \bar{\psi}^+(y) - \psi^-(y) \psi^+(x) + \psi^-(x) \bar{\psi}^+(y) + \psi^-(x) \bar{\psi}^-(y) +$$

$$+ \{ \psi^+(x), \bar{\psi}^-(y) \} = N(\psi(x) \bar{\psi}(y)) + \{ \psi^+(x), \bar{\psi}^-(y) \}$$

$$T(\psi(x) \bar{\psi}(y))_{x^0 < y^0} = -\bar{\psi}^+(y) \psi^+(x) - \bar{\psi}^-(y) \psi^-(x) - \bar{\psi}^-(y) \psi^+(x) -$$

$$-\bar{\psi}^+(y) \psi^-(x) + \psi^-(x) \bar{\psi}^+(y) - \psi^-(y) \psi^+(x) - \bar{\psi}^-(y) \psi^-(x)$$

$$\psi(x) = \psi^+(x) + \psi^-(x)$$

$$T(\psi(x)\bar{\psi}(y))_{x_0 > y_0} = \psi^+(x)\bar{\psi}^+(y) + \psi^+(x)\bar{\psi}^-(y) + \psi^-(x)\bar{\psi}^+(y) + \psi^-(x)\bar{\psi}^-(y)$$

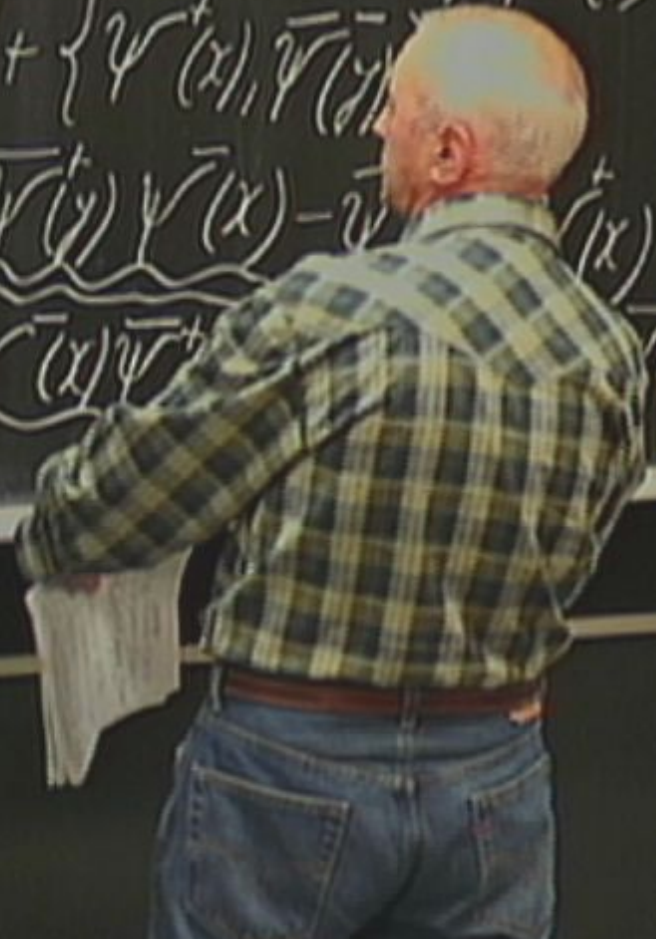
$$= \psi^+(x)\bar{\psi}^+(y) - \psi^-(y)\psi^+(x) + \psi^-(x)\bar{\psi}^+(y) + \psi^-(x)\bar{\psi}^-(y) +$$

$$+ \{\psi^+(x), \bar{\psi}^-(y)\} = N(\psi(x)\bar{\psi}(y)) + \{\psi^+(x), \bar{\psi}^-(y)\}$$

$$T(\psi(x)\bar{\psi}(y))_{y_0 < x_0} = -\bar{\psi}^+(y)\psi^+(x) - \bar{\psi}^-(y)\psi^-(x) - \bar{\psi}^-(y)\psi^+(x) -$$

$$-\bar{\psi}^+(y)\psi^-(x) = -\bar{\psi}^+(y)\psi^+(x) + \psi^-(x)\bar{\psi}^+(y) - \bar{\psi}^-(y)\psi^-(x) -$$

$$- \{\bar{\psi}^+(y), \psi^-(x)\}$$



$$\psi(x) = \psi^+(x) + \psi^-(x)$$

$$T(\psi(x)\bar{\psi}(y))_{x_0 > y_0} = \psi^+(x)\bar{\psi}^+(y) + \psi^+(x)\bar{\psi}^-(y) + \psi^-(x)\bar{\psi}^+(y) + \psi^-(x)\bar{\psi}^-(y)$$

$$= \psi^+(x)\bar{\psi}^+(y) - \bar{\psi}^-(y)\psi^+(x) + \psi^-(x)\bar{\psi}^+(y) + \bar{\psi}^-(y)\psi^-(x) +$$

$$+ \{\psi^+(x), \psi^-(y)\} = N(\psi(x)\bar{\psi}(y)) + \{\psi^+(x), \bar{\psi}^-(y)\}$$

$$T(\psi(x)\bar{\psi}(y))_{y_0 < x_0} = -\bar{\psi}^+(y)\psi^+(x) - \bar{\psi}^-(y)\psi^-(x) -$$

$$-\bar{\psi}^-(y)\psi^+(x) = -\bar{\psi}^+(y)\psi^+(x) - \bar{\psi}^-(y)\psi^-(x) - \bar{\psi}^-(y)\psi^+(x) -$$

$$- \{\bar{\psi}^+(y), \psi^-(x)\}$$

$$\psi(x) = \psi^+(x) + \psi^-(x)$$

$$\begin{aligned}
 T(\psi(x) \bar{\psi}(y))_{x_0 > y_0} &= \psi^+(x) \bar{\psi}^+(y) + \psi^+(x) \bar{\psi}^-(y) + \psi^-(x) \bar{\psi}^+(y) + \psi^-(x) \bar{\psi}^-(y) \\
 &= \psi^+(x) \bar{\psi}^+(y) - \bar{\psi}^-(y) \psi^+(x) + \psi^-(x) \bar{\psi}^+(y) + \bar{\psi}^-(y) \psi^-(x) + \\
 &+ \{\psi^+(x), \bar{\psi}^-(y)\} = N(\psi(x) \bar{\psi}(y)) + \{\psi^+(x), \bar{\psi}^-(y)\}
 \end{aligned}$$

$$\begin{aligned}
 T(\psi(x) \bar{\psi}(y))_{y_0 = x_0} &= -\bar{\psi}^+(y) \psi^+(x) - \bar{\psi}^-(y) \psi^-(x) - \bar{\psi}^-(y) \psi^+(x) - \\
 - \bar{\psi}^+(y) \psi^-(x) &= -\bar{\psi}^+(y) \psi^+(x) + \psi^-(x) \bar{\psi}^-(y) - \bar{\psi}^-(y) \psi^+(x) - \bar{\psi}^+(y) \psi^-(x) \\
 - \{\bar{\psi}^+(y), \psi^-(x)\}
 \end{aligned}$$

$$= N(\psi(x)\bar{\psi}(y)) - \{\bar{\psi}^\dagger(y)\psi(x)\}. \quad \text{I. e.,}$$

$$\int \psi(x)\bar{\psi}(y) = N(\psi(x)\bar{\psi}(y))$$

$$= N(\psi(x)\bar{\psi}(y)) - \{\bar{\psi}^{\dagger}(y)\psi(x)\}. \quad \text{I. e.,}$$

$$\psi(x)\bar{\psi}(y) = N(\psi(x)\bar{\psi}(y)) + \psi(x)\bar{\psi}(y), \quad \text{where}$$

$$= N(\psi(x)\bar{\psi}(y)) - \{\bar{\psi}^+(y)\psi(x)\}. \quad \text{I. e.,}$$

$$T\psi(x)\bar{\psi}(y) = N(\psi(x)\bar{\psi}(y)) + \psi(x)\bar{\psi}(y),$$

$$\overline{\psi(x)\bar{\psi}(y)} = \begin{cases} \{\psi^+(x), \bar{\psi}(y)\}, & x^0 > y^0 \\ -\{\bar{\psi}^+(y), \psi(x)\}, & y^0 > x^0 \end{cases} = \dots$$

$$= N(\psi(x)\bar{\psi}(y)) - \{\bar{\psi}^+(y)\psi(x)\}. \quad \text{I. e.,}$$

$$\overline{\psi(x)\bar{\psi}(y)} = N(\psi(x)\bar{\psi}(y)) + \psi(x)\bar{\psi}(y), \quad \text{where}$$

$$\overline{\psi(x)\bar{\psi}(y)} = \begin{cases} \{\psi(x), \bar{\psi}(y)\}, & x^0 > y^0 \\ -\{\bar{\psi}^+(y), \psi(x)\}, & y^0 > x^0 \end{cases} = S_F(x-y)$$

$$= N(\psi(x)\bar{\psi}(y)) - \{\bar{\psi}^+(y)\psi(x)\}. \quad \text{I. e.,}$$

$$T\psi(x)\bar{\psi}(y) = N(\psi(x)\bar{\psi}(y)) + \overbrace{\psi(x)\bar{\psi}(y)}^{\text{where}},$$

$$\overbrace{\psi(x)\bar{\psi}(y)} = \begin{cases} \{\psi(x), \bar{\psi}(y)\}, & x^0 > y^0 \\ -\{\bar{\psi}^+(y), \psi(x)\}, & y^0 > x^0 \end{cases} = S_F(x-y)$$

By definition, $\psi(x)\psi(y) = 0$, $\bar{\psi}(x)\bar{\psi}(y) = 0$.

$$= N(\psi(x)\bar{\psi}(y)) - \{\bar{\psi}^+(y)\psi(x)\}. \quad \text{I. e.,}$$

$$T \psi(x)\bar{\psi}(y) = N(\psi(x)\bar{\psi}(y)) + \psi(x)\bar{\psi}(y), \quad \text{where}$$

$$\psi(x)\bar{\psi}(y) = \begin{cases} \{\psi(x), \bar{\psi}(y)\}, & x^0 > y^0 \\ -\{\bar{\psi}^+(y), \psi(x)\}, & y^0 > x^0 \end{cases} = S_F(x-y)$$

By definition, $\psi(x)\psi(y) = 0$, $\bar{\psi}(x)\bar{\psi}(y) = 0$.

Define now contractions under symbol N to include
minus sign for operator interchanges: $N(\psi_1 \psi_2 \bar{\psi}_3 \bar{\psi}_4) =$

minus sign for operator interchanges: $N(\psi_1 \psi_2 \psi_3 \psi_4) =$
 $= -N(\psi_1 \psi_3 \psi_2 \psi_4) = -$

$$\psi_1 \psi_3 N(\psi_2 \psi_4) = -S_F(x_1 - x_3) N(\psi_2 \psi_4)$$

Wick's



$$= -N(\psi_1, \psi_3, \psi_2, \psi_4) = -\psi_1 \psi_3 N(\psi_2, \psi_4) = -S_F(x_1 - x_3) N(\psi_2, \psi_4)$$

Wicht



$$(1234) \quad \psi_1 \psi_3 N(\psi_2 \psi_4) = -S_F(x_1 - x_3) N(\psi_2 \bar{\psi}_4)$$

Wick's theorem for fermions:

$$T[\psi_1 \bar{\psi}_2 \psi_3 \dots] = N[\psi_1 \bar{\psi}_2 \psi_3 \dots] + \text{all possible contractions}$$

$$H = H_{\text{Dirac}} + H_{\text{KE}} + g \int d^3x \bar{\psi} \gamma^{\mu} \psi \varphi$$

$$H = H_{\text{Dirac}} + H_{\text{KE}} + g \int d^3x \bar{\psi} \gamma^5 \psi \varphi$$

↑
Yukawa coupling.

$$H = H_{\text{Dirac}} + H_{\text{KE}} + g \int d^3x \bar{\psi} \gamma^5 \psi \varphi$$

↑
Yukawa coupling.

Standard potential, $\sim \frac{1}{r}$



$$H = H_{\text{Dirac}} + H_{\text{KE}} + g \int d^3x \bar{\psi} \gamma^5 \psi \varphi$$

↑
Yukawa coupling.

Coulomb potential,
 $V_C(r) \sim \frac{1}{r}$
Yukawa potential,
 $V_Y \sim \frac{e^{-mr}}{r}$

$$H = H_{Dirac} + H_{EM} + g \int d^3x \bar{\psi} \gamma^5 \psi \varphi$$

↑
Yukawa coupling.

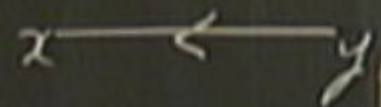
Coulomb potential,
 $V_C(r) \sim \frac{1}{r}$
Yukawa potential,
 $V_Y \sim \frac{e^{-mr}}{r}$

$$H = H_{\text{Dirac}} + H_{\text{KE}} + g \int d^3x \bar{\psi} \gamma \psi \varphi$$

↑ Yukawa coupling.

Propagators:

$$\text{---} \quad \overbrace{\varphi(x)\varphi(y)} = D_F(x-y)$$



(Coulomb potential,
 $V_C(r) \sim \frac{1}{r}$
 Yukawa potential,
 $V_Y \sim \frac{e^{-mr}}{r}$)



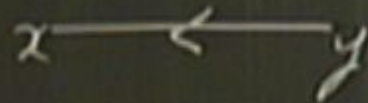
$$H = H_{\text{Dirac}} + H_{\text{KE}} + g \int d^3x \bar{\psi} \psi \varphi$$

↑ Yukawa coupling.

Propagators:

$$\overbrace{\varphi(x) \varphi(y)} = D_F(x-y)$$

$$\overbrace{\psi(x) \bar{\psi}(y)} = S_F(x-y)$$



(Coulomb potential,
 $V_C(r) \sim \frac{1}{r}$
 Yukawa potential,
 $V_Y \sim \frac{e^{-mr}}{r}$)

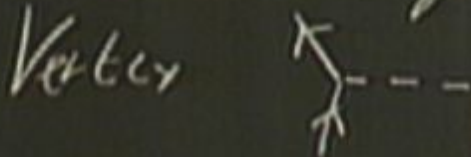
$$H = H_{\text{Dirac}} + H_{\text{KE}} + g \int d^3x \bar{\psi} \psi \phi$$

↑
Yukawa coupling

Propagators:

----- $\overbrace{\quad\quad\quad}^{\quad\quad}$ $\phi(x)\phi(y) = D_F(x-y)$

$x \longleftarrow y$ $\overbrace{\quad\quad\quad}^{\quad\quad}$ $\psi(x)\bar{\psi}(y) = S_F(x-y)$



Coulomb potential,
 $V_C(r) \sim \frac{1}{r}$
 Yukawa potential,
 $V_Y \sim \frac{e^{-mr}}{r}$



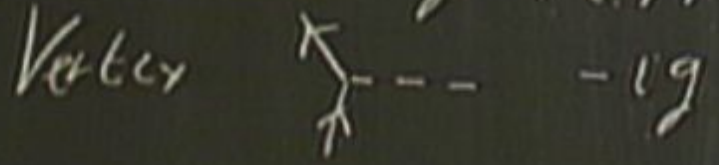
$$H = H_{\text{Dirac}} + H_{\text{KB}} + g \int d^3x \bar{\psi} \psi \phi$$

↑
Yukawa coupling.

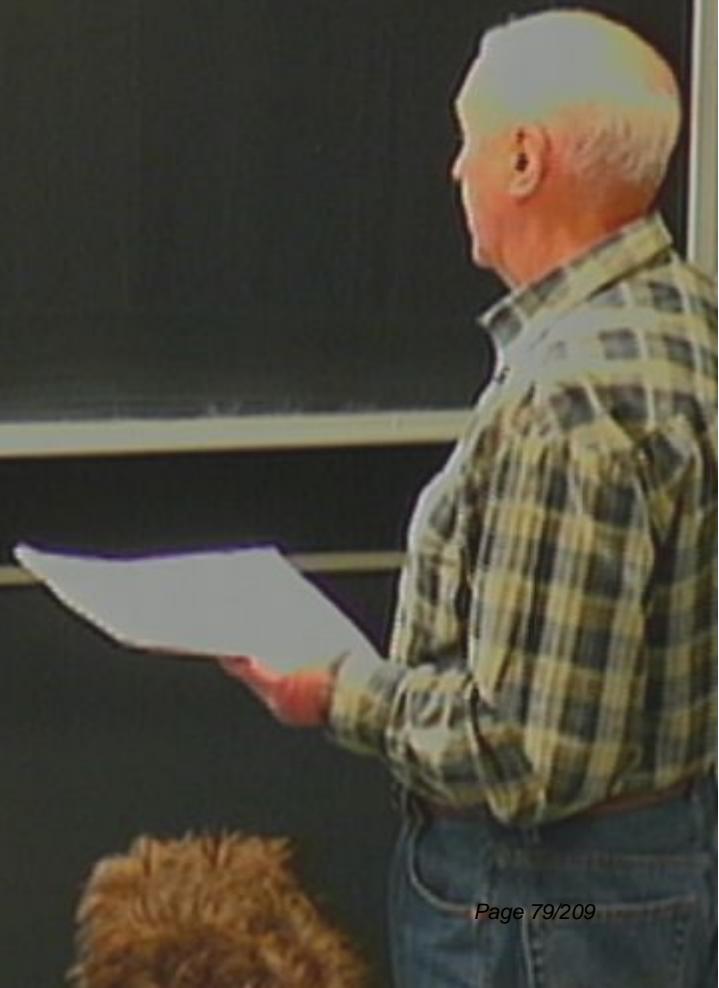
Propagators:

----- $\overbrace{\quad\quad\quad}^{\quad}$ $\phi(x)\phi(y) = D_F(x-y)$

$x \longleftarrow y$ $\overbrace{\quad\quad\quad}^{\quad}$ $\psi(x)\bar{\psi}(y) = S_F(x-y)$



Coulomb potential,
 $V_C(r) \sim \frac{1}{r}$
 Yukawa potential,
 $V_Y \sim \frac{e^{-mr}}{r}$



$$H = H_{\text{Dirac}} + H_{\text{KB}} + g \int d^3x \bar{\psi} \psi \varphi$$

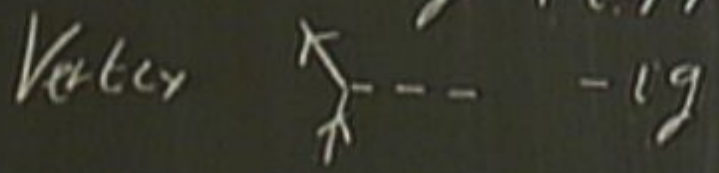
↑ Yukawa coupling.

Coulomb potential,
 $V_C(r) \sim \frac{1}{r}$
 Yukawa potential,
 $V_Y \sim \frac{e^{-mr}}{r}$

Propagators:

----- $\overbrace{\varphi(x)\varphi(y)} = D_F(x-y)$

$x \longleftarrow y$ $\overbrace{\psi(x)\bar{\psi}(y)} = S_F(x-y)$



$$H = H_{\text{Dirac}} + H_{\text{KE}} + g \int d^3x \bar{\psi} \psi \varphi$$

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(Coulomb potential,
 $V_C(r) \sim \frac{1}{r}$
 Yukawa potential,
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Propagators:

----- $\overbrace{\quad\quad\quad}^{\quad\quad\quad}$
 $\varphi(x)\varphi(y) = D_F(x-y)$

$x \longleftarrow y$ $\overbrace{\quad\quad\quad}^{\quad\quad\quad}$
 $\psi(x)\bar{\psi}(y) = S_F(x-y)$

Vertex  ig ; conserved current $\bar{\psi}\gamma^m\psi$

Yukawa Theory

$$H = H_{\text{Dirac}} + H_{\text{KB}} + g \int d^3x \bar{\psi} \gamma^5 \psi \phi$$

↑ Yukawa coupling

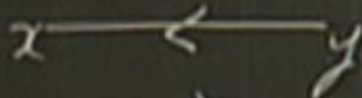
Coulomb potential,
 $V_C(r) \sim \frac{1}{r}$

Yukawa potential,
 $V_Y \sim \frac{e^{-mr}}{r}$

Propagators:

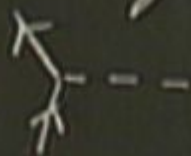


$$\overbrace{\phi(x)\phi(y)} = D_F(x-y)$$



$$\overbrace{\psi(x)\bar{\psi}(y)} = S_F(x-y)$$

Vertex



$-ig$

conserved

$$\bar{\psi} \gamma^5 \psi$$

Yukawa Th.

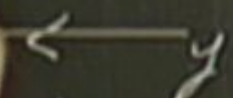
$$H = H_{\text{Dirac}} + H_{\text{KE}} + g \int d^3x \bar{\psi} \psi \phi$$

↑ Yukawa coupling.

Propagators:

$$\overbrace{\quad\quad\quad} \quad \phi(x)\phi(y) = D_F(x-y)$$

$$\overbrace{\quad\quad\quad} \quad \psi(x)\bar{\psi}(y) = S_F(x-y)$$



$-ig$; conserved current $\bar{\psi}\gamma^m\psi : \psi \rightarrow e^{i\alpha}\psi$
 $\bar{\psi} \rightarrow e^{-i\alpha}\bar{\psi}$

Coulomb potential,
 $V_C(r) \sim \frac{1}{r}$
 Yukawa potential,
 $V_Y \sim \frac{e^{-mr}}{r}$



Yukawa Theory

$$H = H_{\text{Dirac}} + H_{\text{KE}} + g \int d^3x \bar{\psi} \psi \phi$$

↑ Yukawa coupling

Coulomb potential,

$$V_C(r) \sim \frac{1}{r}$$

Yukawa potential,

$$V_Y \sim \frac{e^{-mr}}{r}$$

Propagators:

----- $\overbrace{\psi(x)\psi(y)} = D_F(x-y)$

$x \longleftarrow y$ $\overbrace{\psi(x)\bar{\psi}(y)} = S_F(x-y)$

Vertex

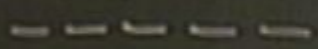


$-ig$

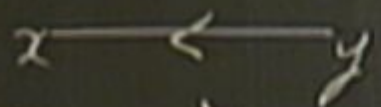
; conserved current

$\bar{\psi} \gamma^\mu \psi : \psi \rightarrow e^{ikx} \psi$
 $\bar{\psi} \rightarrow e^{-ikx} \bar{\psi}$

Propagators:

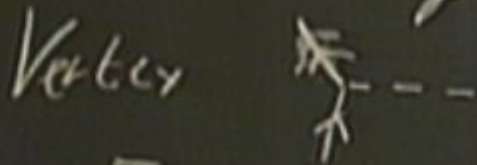


Yukawa complex,
 $\overbrace{\psi(x)\psi(y)} = D_F(x-y)$

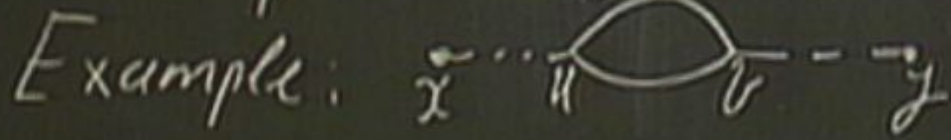


$\overbrace{\psi(x)\bar{\psi}(y)} = S_F(x-y)$

Yukawa potential,
 $V_{\text{int}} \sim \frac{e^{-m|x-y|}}{|x-y|}$



$-ig$; conserved current $\bar{\psi}\gamma^{\mu}\psi : \psi \rightarrow e^{i\alpha}\psi$
 $\bar{\psi} \rightarrow \bar{\psi}e^{-i\alpha}$




Propagators:

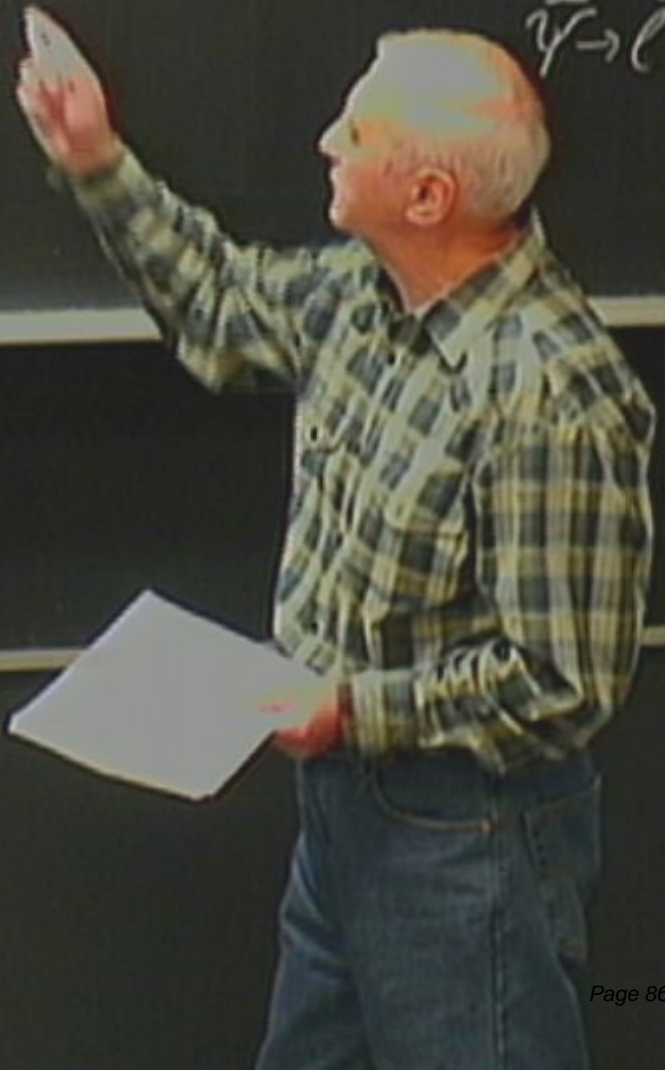
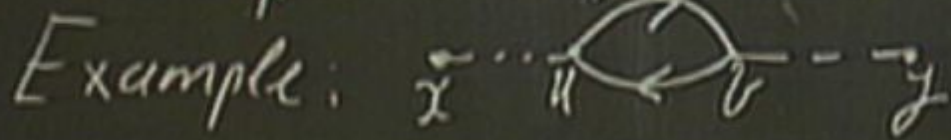
Yukawa complex,
 $\overline{\psi(x)\psi(y)} = D_F(x-y)$

$\psi(x)\overline{\psi(y)} = S_F(x-y)$

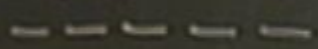
Yukawa potential,
 $V_{int} \sim \frac{e^{-mr}}{r}$

Vertex  $-ig$; conserved current $\overline{\psi}\gamma^m\psi$

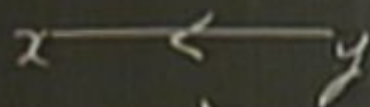
$\psi \rightarrow e^{ikx}\psi$
 $\overline{\psi} \rightarrow e^{-ikx}\overline{\psi}$



Propagators:



Yukawa (complex),
 $\overleftrightarrow{\psi}(x)\overleftrightarrow{\psi}(y) = D_F(x-y)$



$\psi(x)\overline{\psi}(y) = S_F(x-y)$

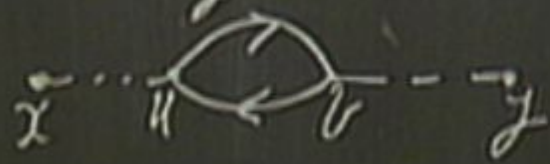
Yukawa potential,
 $V_{int} \sim \frac{e^{-mr}}{r}$

Vertex



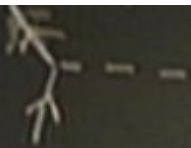
$-ig$; conserved current $\overline{\psi}\gamma^m\psi$; $\psi \rightarrow e^{i\alpha}\psi$
 $\overline{\psi} \rightarrow e^{-i\alpha}\overline{\psi}$

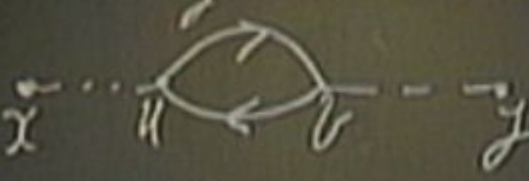
Example:



one-particle exchange in scalar

Vector γ^μ $\int \bar{\psi} \gamma^\mu \psi = \int \bar{\psi} \gamma^\mu \psi$; conserved current $\bar{\psi} \gamma^\mu \psi$: $\psi \rightarrow \psi$
 $\bar{\psi} \rightarrow \bar{\psi}$
 Example: $\bar{x} \dots \text{loop} \dots y \rightarrow$ one-loop correction in scalar propagator

Vertex  $-ig$, conserved current $\bar{\psi}\gamma^\mu\psi$; $\psi \rightarrow e^{i\alpha}\psi$
 $\bar{\psi} \rightarrow \bar{\psi}e^{-i\alpha}$

Example:  \rightarrow one-loop correction in scalar propagator.

$$F = \frac{1}{2!} (-ig)^2 \int d^4x d^4y \langle 0 | \varphi(x) \varphi(y) \rangle$$

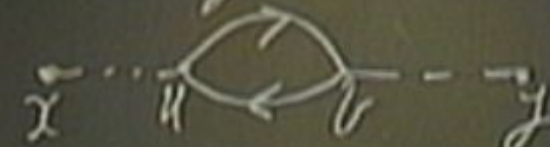
$$F \cdot \frac{1}{2!} (-ig)^2 \int d^4x d^4y \langle 0 | \varphi(x) \varphi(y) \bar{\psi} \psi \varphi(z) \bar{\psi} \psi \varphi(v) | 0 \rangle$$

begin now contractions under symbol N to include minus sign for operator interchanges: $N(\overbrace{\psi_1 \psi_2} \overbrace{\psi_3 \psi_4}) =$
 $= -N(\overbrace{\psi_1 \psi_3} \overbrace{\psi_2 \psi_4}) = -\overbrace{\psi_1 \psi_3} N(\overbrace{\psi_2 \psi_4}) = -S_F(x_1 - x_3) N(\overbrace{\psi_2 \psi_4})$

Wick's theorem for fermions:

$$T[\psi_1 \overline{\psi_2} \psi_3 \dots] = N[\overbrace{\psi_1 \overline{\psi_2}} \psi_3 \dots] + \text{all possible contractions}$$

Yukawa Theory

$\Psi(x) \bar{\Psi}(y) = \langle F(x, y) \rangle$
 Vertex $\begin{array}{c} \leftarrow \\ \text{---} \\ \rightarrow \end{array}$ $-ig$, conserved current $\bar{\Psi} \gamma^\mu \Psi$; $\Psi \rightarrow e^{i\alpha} \Psi$
 $\bar{\Psi} \rightarrow e^{-i\alpha} \bar{\Psi}$
 Example:  \rightarrow one-loop correction in scalar propagator.

$$F = \frac{1}{2!} (-ig)^2 \int d^4x d^4y \langle 0 | \Psi(x) \Psi(y) \bar{\Psi}(y) \bar{\Psi}(x) | 0 \rangle$$

$$F \cdot \frac{1}{2!} (-ig)^2 \int d^4x d^4y \langle 0 | \varphi(x) \varphi(y) \overbrace{\psi \psi \varphi(x) \overbrace{\psi \psi \varphi(y)}^{\gamma}} | 0 \rangle$$

$$F \cdot \frac{1}{2!} (-ig)^2 \int d^4x d^4y \langle 0 | \varphi(x) \varphi(y) \overbrace{\psi \psi \varphi(x) \overbrace{\psi \psi \varphi(y)}^{\gamma}} | 0 \rangle \stackrel{F=2}{=}$$



$$F \cdot \frac{1}{2!} (-ig)^2 \int d^4x d^4y \langle 0 | \psi(x) \psi(y) \overbrace{\psi \psi \psi(x) \psi \psi \psi(y)}^{\text{Feynman diagram}} | 0 \rangle \stackrel{F=2}{=} \dots$$

$$\overline{\psi}_\alpha \psi_\beta \psi_\gamma \psi_\delta$$



$$F \cdot \frac{1}{2!} (-ig)^2 \int d^4x d^4y \langle 0 | \psi(x) \psi(y) \overbrace{\psi \psi \psi \psi}^{\text{Feynman diagrams}} \psi \psi \psi \psi | 0 \rangle \stackrel{F=2}{=} \\ \sum_{\alpha, \beta=1}^4 \overbrace{\psi_{\alpha} \psi_{\beta}}^{\text{Feynman diagrams}} \psi_{\alpha} \psi_{\beta} = (-1)^3 \psi_{\alpha} \psi_{\beta}$$



$$F \cdot \frac{1}{2!} (-ig)^2 \int d^4x d^4y \langle 0 | \psi(x) \psi(y) \overbrace{\overbrace{\overbrace{\overbrace{\psi^\dagger(y) \psi(x)}^{\gamma_1} \psi^\dagger(x) \psi(y)}^{\gamma_2} \psi^\dagger(x) \psi(y)}^{\gamma_3} \psi^\dagger(x) \psi(y)}^{\gamma_4} | 0 \rangle \stackrel{F=2}{=}$$

$$\psi(x) \overbrace{\overbrace{\psi^\dagger(y) \psi(x)}^{\gamma_1} \psi^\dagger(x) \psi(y)}^{\gamma_2} = (-1)^3 \psi(x) \overbrace{\psi^\dagger(y) \psi(x)}^{\gamma_1} \overbrace{\psi^\dagger(x) \psi(y)}^{\gamma_2}$$

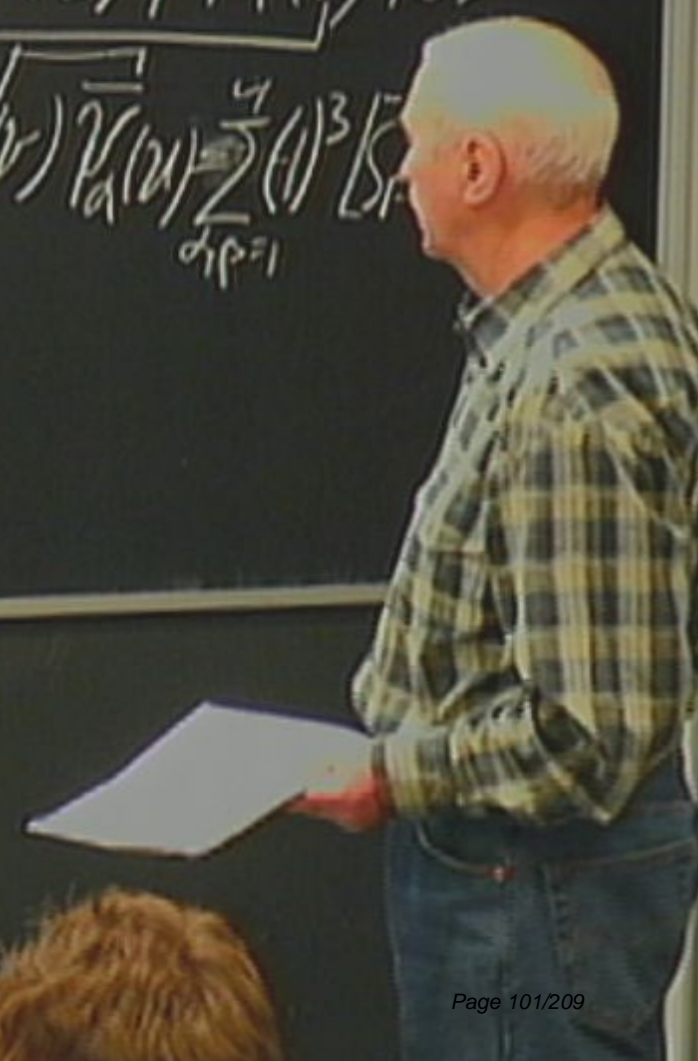


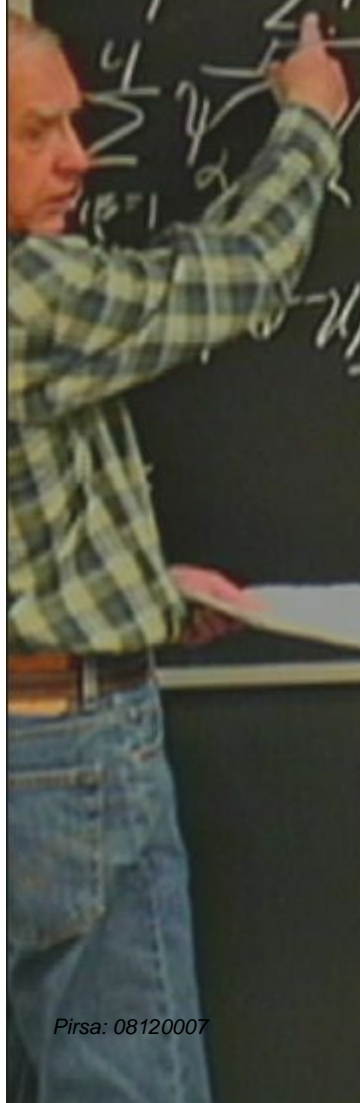
$$F \cdot \frac{1}{2!} (-i)^2 \int d^4x d^4y \langle 0 | \psi(x) \psi(y) \overbrace{\overbrace{\overbrace{\overbrace{\psi(x) \psi(y)}^2}^2}^2}^2}^2 | 0 \rangle \stackrel{F=2}{=} \\ \sum_{\alpha, \beta=1}^4 \overbrace{\overbrace{\overbrace{\psi_\alpha(x) \psi_\beta(y)}^2}^2}^2 = (-1)^3 \overbrace{\overbrace{\overbrace{\psi_\alpha(x) \psi_\beta(y)}^2}^2}^2 = (-1)^3$$

$$F \cdot \frac{1}{2!} (-ig)^2 \int d^4x d^4y \langle 0 | \psi(x) \psi(y) \overbrace{\overbrace{\overbrace{\overbrace{\psi^\dagger(x) \psi^\dagger(y)}^2}^2}^2}^2 \varphi(x) \varphi(y) | 0 \rangle \stackrel{F=2}{=} \\ \sum_{\alpha, \beta=1}^4 \overbrace{\overbrace{\overbrace{\psi_\alpha(x) \psi_\beta(y)}^2}^2}^2 = (-1)^3 \overbrace{\overbrace{\overbrace{\psi_\alpha(x) \psi_\beta(y) \psi_\alpha(x)}^2}^2}^2 = (-1)^3$$

$$\begin{aligned}
 & F \cdot \frac{1}{2!} (-ig)^2 \int d^4x d^4y \langle 0 | \psi(x) \psi(y) \overbrace{\overbrace{\overbrace{\overbrace{\psi^\dagger(y) \psi^\dagger(x)}^{\gamma}}^{\beta}}^{\alpha}}^{\delta} \psi(x) \psi(y) | 0 \rangle \stackrel{F=2}{=} \\
 & \sum_{\alpha, \beta=1}^4 \overbrace{\psi_\alpha(x) \psi_\beta(y)}^{\gamma} \overbrace{\psi_\beta^\dagger(y) \psi_\alpha^\dagger(x)}^{\delta} = (-1)^3 \psi_\alpha(x) \psi_\beta(y) \psi_\beta^\dagger(y) \psi_\alpha^\dagger(x) \sum_{\alpha, \beta=1}^4 (-1)^3 [S_F(x-y)]_{\alpha\beta} \\
 & [S_F(x-y)]_{\beta\alpha}
 \end{aligned}$$

$$F \cdot \frac{1}{2!} (-ig)^2 \int d^4x d^4y \langle 0 | \psi(x) \psi(y) \overbrace{\overbrace{\overbrace{\psi^\dagger(y) \psi(x)}^1}^2}^3 \psi^\dagger(y) \psi(x) | 0 \rangle \stackrel{F=2}{=} \\
\sum_{\alpha, \beta=1}^4 \overbrace{\psi_\alpha(x)}^1 \overbrace{\psi_\beta(y)}^2 = (-1)^3 \overbrace{\psi_\alpha(x)}^1 \overbrace{\psi_\beta(y)}^2 \overbrace{\psi_\alpha(x)}^3 \overbrace{\psi_\beta(y)}^4 = \sum_{\alpha, \beta=1}^4 (-1)^3 [S_F(x-y)]_{\beta\alpha} \\
[S_F(x-y)]_{\beta\alpha} = \text{tr} [S_F(y-x)]$$





$$F \cdot \frac{1}{2!} (-ig)^2 \int d^4x d^4y \langle 0 | \psi(x) \psi(y) \overbrace{\bar{\psi} \psi \psi(x) \bar{\psi} \psi \psi(y)}^{\text{Feynman diagrams}} | 0 \rangle \stackrel{F=2}{=} \\ \sum_{\alpha, \beta} \bar{\psi}_\alpha(x) \overbrace{\psi_\beta \psi_\beta(y)}^{\text{Feynman diagrams}} = (-1)^3 \psi_\alpha(x) \overbrace{\bar{\psi}_\beta \psi_\beta(y)}^{\text{Feynman diagrams}} \bar{\psi}_\alpha(x) \sum_{\alpha, \beta} (-1)^3 [S_F(x-y)]_{\alpha\beta} \\ [S_F(y-x)]_{\beta\alpha} = \underline{\underline{\text{tr} [S_F(x-y) S_F(y-x)]}}$$

$$\begin{aligned}
 & F \cdot \frac{1}{2!} (-ig)^2 \int d^4x d^4y \langle 0 | \psi(x) \psi(y) \overbrace{\overbrace{\overbrace{\psi^\dagger(y) \psi^\dagger(x)}^{\text{Feynman}} \psi(x) \psi(y)}^{\text{Feynman}}}^{\text{Feynman}} | 0 \rangle \stackrel{F=2}{=} \\
 & \sum_{\alpha, \beta=1}^4 \overbrace{\psi_\alpha(x) \psi_\beta(y)}^{\text{Feynman}} = \overbrace{\psi_\beta(y) \psi_\alpha(x)}^{\text{Feynman}} \sum_{\alpha, \beta=1}^4 (-1)^3 [S_F(x-y)]_{\alpha\beta} \\
 & [S_F(x-y)]_{\alpha\beta} = \text{tr} \left[\gamma_5 \gamma_\mu \gamma_\nu \right] \\
 & = -(-ig)^2 \int d^4x d^4y
 \end{aligned}$$

$$F \cdot \frac{1}{2!} (-ig)^2 \int d^4u d^4v \langle 0 | \psi(x) \psi(y) \bar{\psi}(u) \psi(u) \bar{\psi}(v) \psi(v) | 0 \rangle_{F=2}$$

$$= \sum_{\alpha, \beta=1}^4 \bar{\psi}_\alpha(x) \psi_\alpha(u) \bar{\psi}_\beta(v) \psi_\beta(u) = (-1)^3 \psi_\alpha(u) \bar{\psi}_\beta(v) \psi_\beta(u) \bar{\psi}_\alpha(x) = \sum_{\alpha, \beta=1}^4 (-1)^3 [S_F(u-x)]_{\alpha\beta}$$

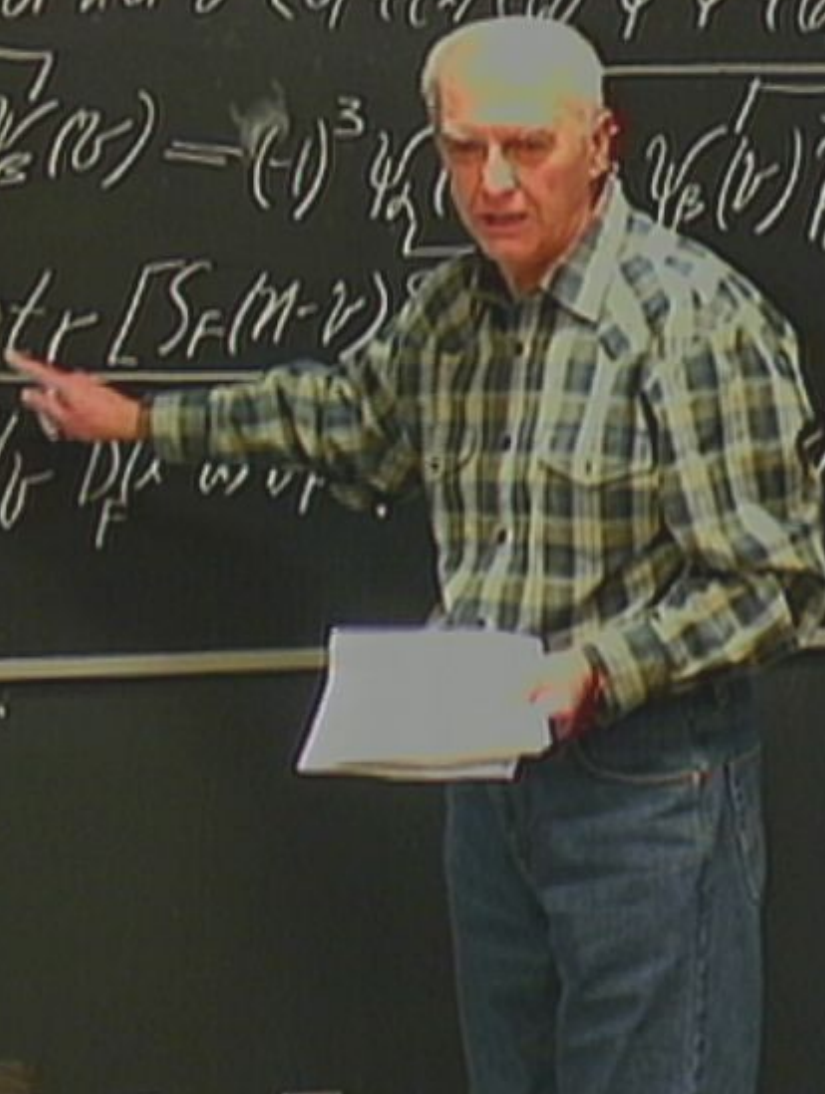
$$[S_F(u-x)]_{\alpha\beta} = [S_F(v-u)]_{\alpha\beta} = [S_F(v-x)]_{\alpha\beta}$$

$$= -(-ig)^2 \int$$

$$\begin{aligned}
 & F \cdot \frac{1}{2!} (-ig)^2 \int d^4x d^4y \langle 0 | \psi(x) \psi(y) \overbrace{\overbrace{\overbrace{\psi^\dagger(y) \psi^\dagger(x)}^2}^2}^2 \psi(x) \psi(y) | 0 \rangle \stackrel{F=2}{=} \\
 & \sum_{\alpha, \beta=1}^4 \overbrace{\psi_\alpha(x) \psi_\beta(y)}^2 = (-1)^3 \psi_\alpha(x) \psi_\beta(y) \overbrace{\psi_\alpha(x) \psi_\beta(y)}^2 \sum_{\alpha, \beta=1}^4 (-1)^3 [S_F(x-y)]_{\alpha\beta} \\
 & [S_F(x-y)]_{\alpha\beta} = \text{tr} [S_F(y-x) S_F(x-y)] \\
 & = -(-ig)^2 \int d^4x d^4y D_F(x-y) D_F(y-x) \text{tr} [S_F(y-x) S_F(x-y)]
 \end{aligned}$$

$$\begin{aligned}
 & F \cdot \frac{1}{2!} (-ig)^2 \int d^4x d^4y \langle 0 | \psi(x) \psi(y) \overbrace{\overbrace{\overbrace{\psi^\dagger(y) \psi^\dagger(x)}^1}^1}^1 \psi(x) \psi(y) | 0 \rangle \stackrel{F=2}{=} \\
 & \sum_{\alpha, \beta=1}^4 \overbrace{\psi_\alpha(x) \psi_\beta(y)}^1 \overbrace{\psi_\beta^\dagger(y) \psi_\alpha^\dagger(x)}^1 = (-1)^3 \psi_\alpha(x) \psi_\beta(y) \overbrace{\psi_\beta^\dagger(y) \psi_\alpha^\dagger(x)}^1 \sum_{\alpha, \beta=1}^4 (-1)^3 [S_F(x-y)]_{\alpha\beta} \\
 & [S_F(x-y)]_{\beta\alpha} = \underline{\text{tr} [S_F(x-y) S_F(y-x)]} \\
 & = -(-ig)^2 \int d^4x d^4y D_F(x-y) D_F(y-x) \text{tr} [S_F(x-y) S_F(y-x)]
 \end{aligned}$$

$$\begin{aligned}
 & F \cdot \frac{1}{2!} (-ig)^2 \int d^4x d^4y \langle 0 | \psi(x) \psi(y) \overbrace{\bar{\psi}(x) \bar{\psi}(y)}^{\text{Feynman diagram}} \psi(u) \bar{\psi}(v) \psi(w) \bar{\psi}(z) | 0 \rangle \stackrel{F=2}{=} \\
 & \sum_{\alpha, \beta=1}^4 \bar{\psi}_\alpha \psi(u) \bar{\psi}_\beta \psi(v) = (-1)^3 \psi_\alpha(u) \psi_\beta(v) \bar{\psi}_\alpha(u) \bar{\psi}_\beta(v) \sum_{\alpha, \beta=1}^4 (-1)^3 [S_F(u-v)]_{\alpha\beta} \\
 & [S_F(u-v)]_{\beta\alpha} = -\text{tr} [S_F(u-v)] \\
 & = -(-ig)^2 \int d^4x d^4y \bar{\psi}_F(x) \psi_F(y) \text{tr} [S_F(u-v)]
 \end{aligned}$$



$$\begin{aligned}
 & F \cdot \frac{1}{2!} (-ig)^2 \int d^4x d^4y \langle 0 | \psi(x) \psi(y) \overbrace{\overbrace{\overbrace{\psi^\dagger(y) \psi^\dagger(x)}^2}^2}^2 \psi^\dagger(y) \psi^\dagger(x) | 0 \rangle \stackrel{F=2}{=} \\
 & \sum_{\alpha, \beta=1}^4 \overbrace{\psi_\alpha(x) \psi_\beta(y)}^2 = (-1)^3 \psi_\alpha(x) \psi_\beta(y) \overbrace{\psi_\alpha(x) \psi_\beta(y)}^2 \sum_{\alpha, \beta=1}^4 (-1)^3 [S_F(x-y)]_{\alpha\beta} \\
 & [S_F(x-y)]_{\alpha\beta} = \text{tr} [S_F(x-y) S_F(y-x)] \\
 & = -(-ig)^2 \int d^4x d^4y D_F(x-y) D_F(y-x) \text{tr} [S_F(x-y) S_F(y-x)]
 \end{aligned}$$

The det of γ_5 is $+1$

In a closed fermion loop, there is always a factor of (-1) and the trace over a product of Dirac matrices

The determinant of the ...

In a closed fermion loop, there is always a factor of (-1) and the trace over a product of Dirac matrices.



$$H = H_{\text{Dirac}} + H_{\text{KE}} + g \int d^3x \bar{\psi} \gamma^0 \psi \phi$$

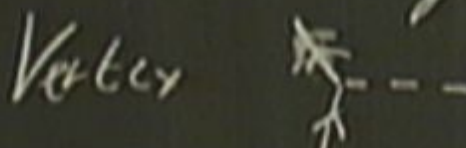
↑ Yukawa coupling.

Coulomb potential,
 $V_C(r) \sim \frac{1}{r}$
 Yukawa potential,
 $V_Y(r) \sim \frac{e^{-mr}}{r}$

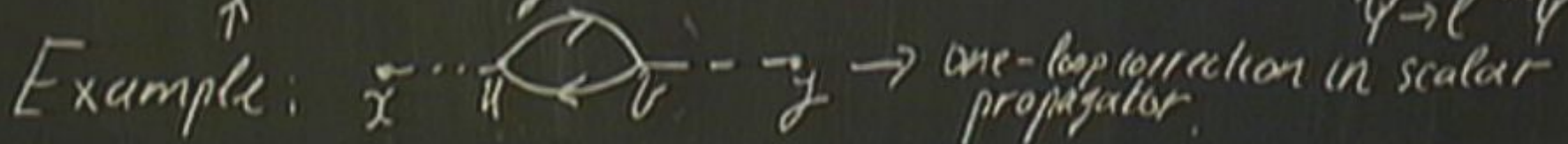
Propagators:

----- $\overbrace{\quad\quad\quad}^{\quad\quad\quad}$ $\phi(x)\phi(y) = D_F(x-y)$

$x \longleftarrow y$ $\overbrace{\quad\quad\quad}^{\quad\quad\quad}$ $\psi(x)\bar{\psi}(y) = S_F(x-y)$



$-ig$; conserved current $\bar{\psi}\gamma^m\psi$: $\psi \rightarrow e^{ik}\psi$
 $\bar{\psi} \rightarrow e^{-ik}\bar{\psi}$



The determinant of the ...

In a closed fermion loop, there is always a factor of (-1) and the trace over a product of Dirac matrices.



The definition of the trace of a product of Dirac matrices

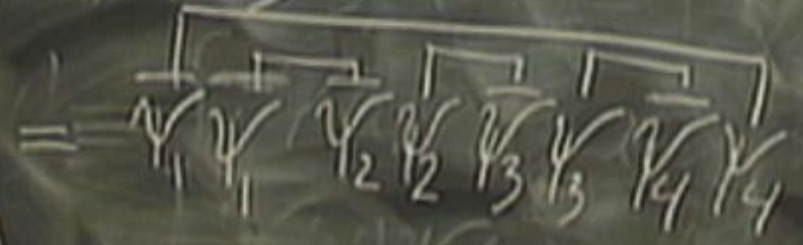
In a closed fermion loop, there is always a factor of -1 and the trace over a product of Dirac matrices



$$\text{Tr}(\gamma_2 \gamma_3 \gamma_3 \gamma_4 \gamma_4)$$

The determinant of the...

In a closed fermion loop, there is always a factor of (-1) and the trace over a product of matrices.



The determinant of the gamma matrices is ± 1

In a closed fermion loop, there is always a factor of (-1) and the trace over a product of Dirac matrices



$$\text{Tr}(\gamma_2 \gamma_2 \gamma_3 \gamma_3 \gamma_4 \gamma_4) = (-1) \text{Tr}(\gamma_1 \gamma_2 \gamma_2 \gamma_3 \gamma_3 \gamma_4 \gamma_4)$$

The determinant of the gamma matrices is ± 1

In a closed fermion loop, there is always a factor of (-1) . The trace over a product of Dirac matrices



$$= (-1) \gamma_1 \gamma_2 \gamma_3 \gamma_4 \gamma_3 \gamma_2 \gamma_1 =$$


The determinant of the normal matrix is ± 1

In a closed fermion loop, there is always a factor of (-1) and the trace over a product of Dirac matrices

$$\begin{aligned}
 & \overline{\psi}_1 \psi_1 \overline{\psi}_2 \psi_2 \overline{\psi}_3 \psi_3 \overline{\psi}_4 \psi_4 = (-1) \overline{\psi}_1 \overline{\psi}_2 \overline{\psi}_3 \overline{\psi}_4 \psi_4 \psi_3 \psi_2 \psi_1 \\
 & = (-1) \text{tr} (S_F(12) S_F(23) S_F(34) S_F(41))
 \end{aligned}$$

The definition of the normal ordering is \dots


In a closed fermion loop, there is always a factor of (-1) and the trace over a product of Dirac matrices.

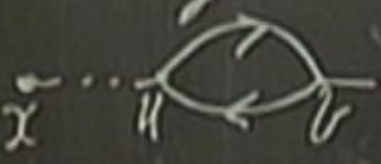


$$\begin{aligned}
 &= \psi_1 \psi_1 \psi_2 \psi_2 \psi_3 \psi_3 \psi_4 \psi_4 = (-1) \psi_1 \psi_2 \psi_3 \psi_4 \psi_4 \psi_3 \psi_2 \psi_1 \\
 &= (-1) \text{tr} (S_F(12) S_F(23) S_F(34) S_F(41))
 \end{aligned}$$

$$\begin{aligned}
& F \cdot \frac{1}{2!} (-ig)^2 \int d^4x d^4y \langle 0 | \psi(x) \psi(y) \overbrace{\bar{\psi}(x) \bar{\psi}(y)}^{\text{Feynman diagram}} \psi(u) \bar{\psi}(v) \psi(w) \bar{\psi}(z) | 0 \rangle \stackrel{F=2}{=} \\
& \sum_{\alpha, \beta=1}^4 \bar{\psi}_\alpha(x) \psi_\alpha(u) \bar{\psi}_\beta(y) \psi_\beta(v) = (-1)^3 \psi_\alpha(u) \bar{\psi}_\beta(v) \bar{\psi}_\alpha(x) \psi_\beta(y) \stackrel{4}{=} \sum_{\alpha, \beta=1}^4 (-1)^3 [S_F(u-v)]_{\alpha\beta} \\
& [S_F(v-u)]_{\beta\alpha} = \text{tr} [S_F(u-v) S_F(v-u)] \\
& = -(-ig)^2 \int d^4x d^4y D_F(x-u) D_F(y-v) \text{tr} [S_F(u-v) S_F(v-u)]
\end{aligned}$$

$$\begin{aligned}
 & \int \frac{1}{2\pi} \int d^4x d^4y \delta(x-w) \delta(y-v) \langle \psi \psi(x) \psi \psi(y) \rangle_{\mathcal{F}} \\
 & \sum_{\alpha, \beta=1}^4 \bar{\psi}_\alpha \psi_\beta(x) \bar{\psi}_\beta \psi_\alpha(y) = \int \bar{\psi}_\alpha(x) \sum_{\beta=1}^4 (\gamma_{\alpha\beta})^3 [S_F(x-y)]_{\alpha\beta} \\
 & [S_F(x-y)]_{\alpha\beta} = \text{tr} [S_F(x-y)] \\
 & = -(-ig)^2 \int d^4x d^4y D_F(x-w) D_F(y-v) S_F(x-y)
 \end{aligned}$$


Vertex  $-ig$; conserved current $\bar{\psi}\gamma^{\mu}\psi$: $\psi \rightarrow e^{ik}\psi$
 $\bar{\psi} \rightarrow e^{-ik}\bar{\psi}$

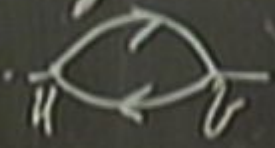
Example:  \rightarrow one-loop correction in scalar propagator.

$b_F(t) = \int$

$$[S_F] \rightarrow \text{tr} [S_F(u-v) S_F(v-u)]$$

$$D_F(x-u) D_F(y-v) \text{tr} [S_F(u-v) S_F(v-u)]$$

Vertex  $-ig$; conserved current $\bar{\psi}\gamma^m\psi$; $\psi \rightarrow e^{ik}\psi$
 $\bar{\psi} \rightarrow e^{-ik}\bar{\psi}$


Example:  \rightarrow one-loop correction in scalar propagator.

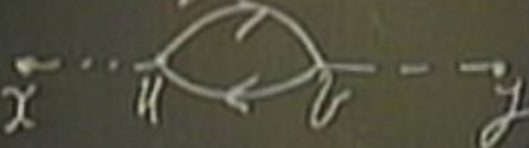
$$G_F(t) = \int$$

$$[S_F(v-u)]_{\alpha\beta} = \text{tr} [S_F(u-v) S_F(v-u)]$$

$$= -(-ig)^2 \int d^4u d^4v D_F(x-u) D_F(y-v) \text{tr} [S_F(u-v) S_F(v-u)]$$

$$x \longleftarrow y \quad \Psi(x) \bar{\Psi}(y) = S_F(x-y)$$

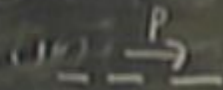
Vertex  $-ig$, conserved current $\bar{\Psi} \gamma^\mu \Psi$: $\Psi \rightarrow e^{i\alpha} \Psi$
 $\bar{\Psi} \rightarrow e^{-i\alpha} \bar{\Psi}$

Example:  \rightarrow one-loop correction in scalar propagator
 $G_F(t) = \int d^4p e^{ipx} \frac{p^2 + m^2}{p^2 - m^2 + i\epsilon}$

$$F \cdot \frac{1}{2!} (-ig)^2 \int d^4u d^4v \langle 0 | \Psi(x) \bar{\Psi}(y) \bar{\Psi}(u) \Psi(v) \Psi(u) \bar{\Psi}(v) | 0 \rangle \stackrel{F=2}{=} \\ \sum_{\alpha, \beta=1}^4 \bar{\Psi}_\alpha \Psi(u) \bar{\Psi}_\beta \Psi(v) = (-1)^3 \Psi_\alpha(u) \bar{\Psi}_\beta \Psi(v) \bar{\Psi}_\alpha(u) \sum_{\alpha, \beta=1}^4 (-1)^3 [S_F(u-v)]_{\alpha\beta} \\ [S_F(v-u)]_{\beta\alpha} = \text{tr} [S_F(u-v) S_F(v-u)] \\ = -(-ig)^2 \int d^4u d^4v D_F(x-u) D_F(y-v) \text{tr} [S_F(u-v) S_F(v-u)]$$

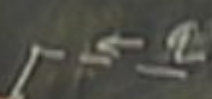
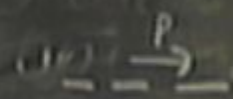
$$\begin{aligned}
 & F \cdot \frac{1}{2!} (-1)^2 \int d^4u d^4v \langle 0 | \psi(x) \psi(y) \psi \psi \psi(u) \psi \psi \psi(v) | 0 \rangle \\
 & \sum_{\alpha, \beta=1}^4 \overbrace{\psi_\alpha(u) \psi_\beta(v)}^{\text{bracket}} = (-1)^3 \overbrace{\psi_\alpha(u) \psi_\beta(v)}^{\text{bracket}} \overbrace{\psi_\alpha(u) \psi_\beta(v)}^{\text{bracket}} \sum_{\alpha, \beta=1}^4 (-1)^3 [S_F(u-v)]_{\alpha\beta} \\
 & [S_F(u-v)]_{\alpha\alpha} = \text{tr} [S_F(u-v) S_F(v-u)] \\
 & = -(-1)^2 \int d^4u d^4v D_F(x-u) D_F(y-v) \text{tr} [S_F(u-v) S_F(v-u)]
 \end{aligned}$$

Momentum Space for fermion problem
at the end



(27)

Momentum Space for fermion problem
... ..
... ..



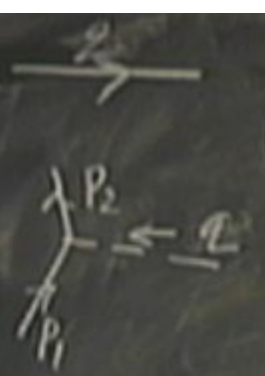
Momentum Space for fermion number

$$\frac{1}{(2\pi)^4} \frac{1}{p^2 - m^2 + i\epsilon}$$

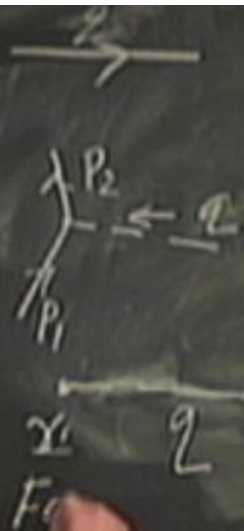
$$\frac{1}{(2\pi)^4} \frac{1}{q^2 - m^2 + i\epsilon} \gamma_\mu$$

Realization of Wick's theorem

$$\frac{1}{(2\pi)^4} \frac{i(q^\mu \gamma_\mu + m)}{q^2 - m^2 + i\epsilon}$$

$$-i g (2\pi)^4 \delta^4(p_2 - p_1)$$


$N(a_p a_q)$
 Generalized Wick's Theorem
 $= (-1)^3 a_p^+ a_q a_p$



$$\frac{1}{(2\pi)^4} \frac{i(q^2 \gamma_\mu + m)}{q^2 - m^2 + i\epsilon}$$

$$-iq (2\pi)^4 \delta^4(p_1 + q - p_2)$$

$$e^{-iqx}$$

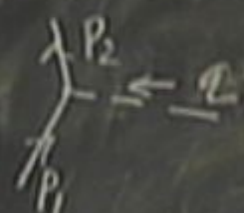
$$(q a_p^\dagger) = (-1)^R a_p^\dagger a_p a_q = (-1)^3 a_p^\dagger a_q a_p$$

generalization of Wick's Theorem

$$\frac{1}{(2\pi)^4} \frac{i(q^2 \gamma_\mu + m)}{q^2 - m^2 + i\epsilon}$$

$$-i q (2\pi)^4 \delta^4(p_1 + q - p_2)$$

$$e^{-iqx}$$



Factor (-1) and to be with closed fermion

$$N(a_p a_p^\dagger) = (-1)^3 a_p^\dagger a_p a_p^\dagger$$

Gen. Wick's Theorem

$$\frac{1}{(2\pi)^4} \frac{i(q^2 \gamma_\mu + m)}{q^2 - m^2 + i\epsilon}$$

$$-i(2\pi)^4 \delta^4(p_1 + q - p_2)$$

$$e^{-iqx}$$

Factor (-1) and tr for each closed fermion loop

$N(a_p^\dagger a_q^\dagger) = a_p^\dagger a_q^\dagger a_p a_q$
 Generalization of Wick's Theorem

$$1/p_1 \quad (1+2-13)$$

$$x \quad 2 \quad e^{-4x}$$

Factor (-1) and tr for each closed fermion loop. Calculate the overall factor...

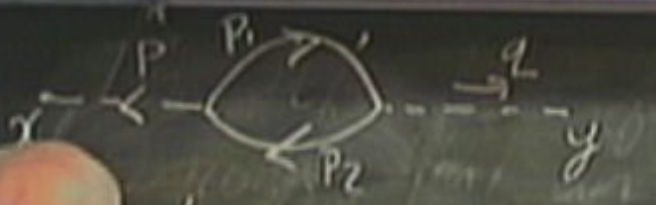
The definition of the normal-order product of fermions: put an extra minus sign for each fermion interchange:

$$N(a_p a_q a_r^+) = (-1)^2 a_r^+ a_p a_q = (-1)^3 a_p^+ a_q a_r$$

Generalization of Wick's Theorem



Factor (-1) and tr for each closed fermion loop. Calculate the overall factor



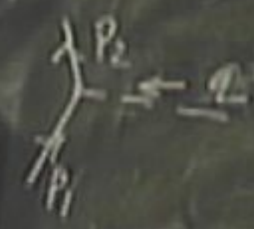
$$= (-1) \text{tr} (S_F(12) S_F(23) S_F(34) S_F(41))$$

Momenta in Spacelike for fermion loop

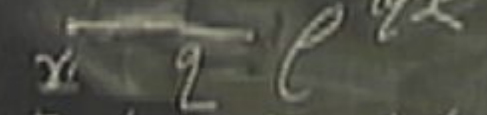


$$\frac{1}{(2\pi)^4} \frac{1}{p^2 - m^2 + i\epsilon}$$

$$\frac{1}{(2\pi)^4} \frac{1}{q^2 - m^2 + i\epsilon}$$



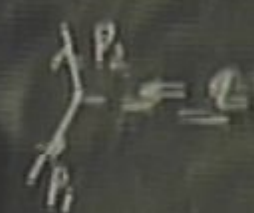
$$-ig (2\pi)^4 \delta^4(p_1 + p_2 - q)$$



Factor (-1) and tr for each closed fermion loop



$$i\epsilon = \frac{p^2}{2}$$



Factor (-1) and tr for
with closed fermion loop!

Momentum Space

$$\frac{1}{(2\pi)^4} \frac{1}{p^2 - m^2 + i\epsilon}$$

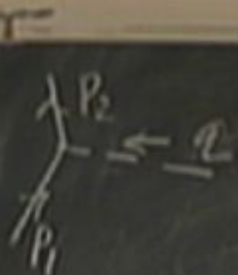
$$\frac{1}{(2\pi)^4} \frac{1}{q^2 - m^2 + i\epsilon} \frac{1}{(q^2 - m^2 + i\epsilon)}$$

$$-(ig)(2\pi)^4 \int \frac{d^4q}{(2\pi)^4} \frac{1}{(q^2 - m^2 + i\epsilon)}$$

$$= -ig^2$$

Integrate over all
momenta
Calculate overall
factor

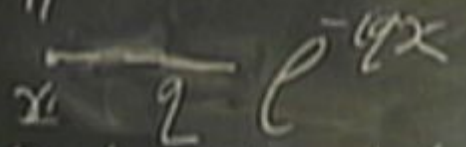




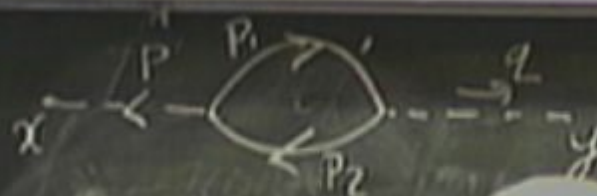
(-1)^q q² -

$$-iq (2\pi)^4 \delta^4(p_1 + q - p_2)$$

factor



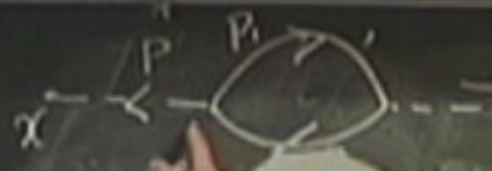
Factor (-1) and tr for each closed fermion loop



$$2 \cdot \frac{1}{2!}$$

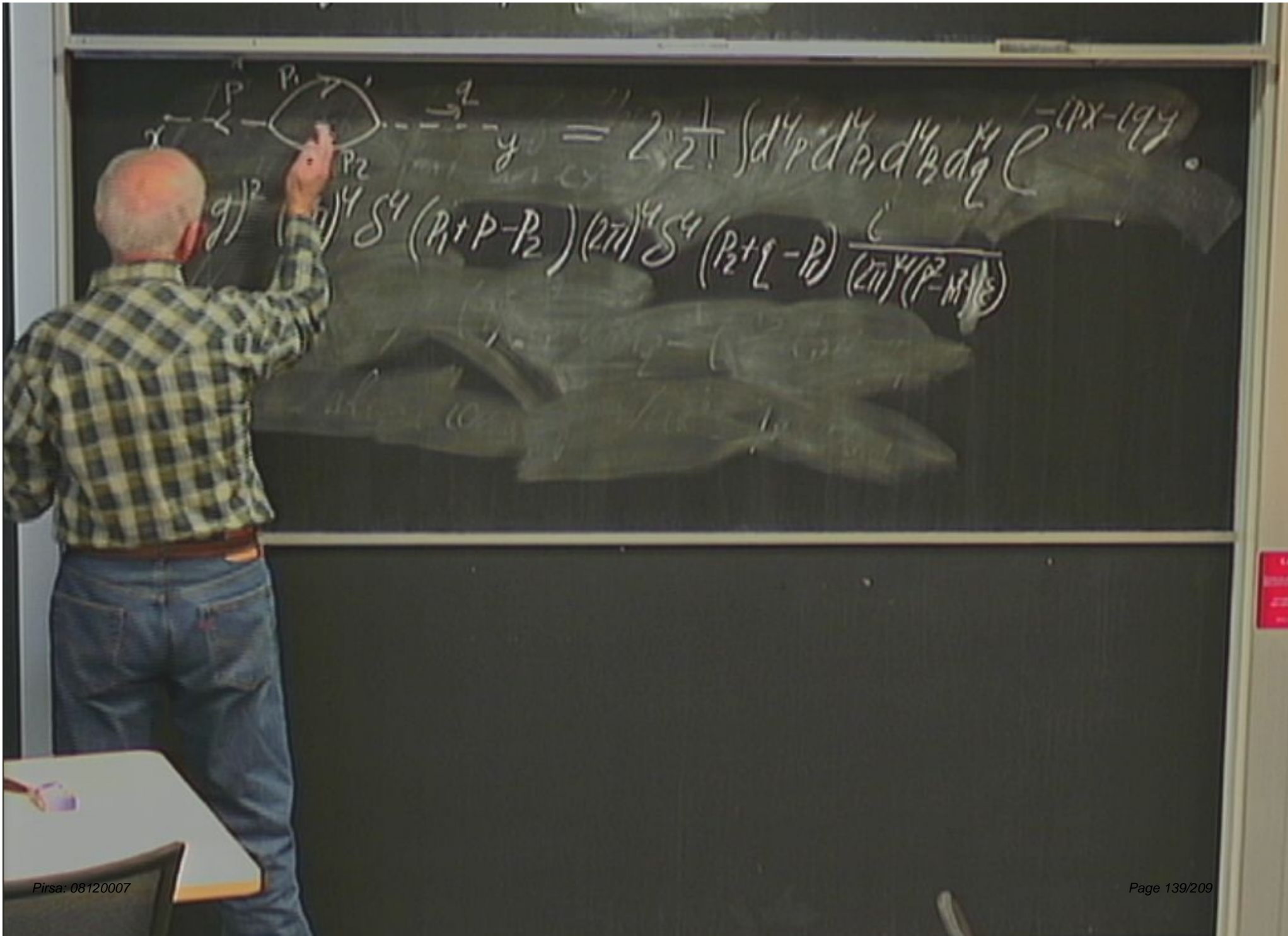


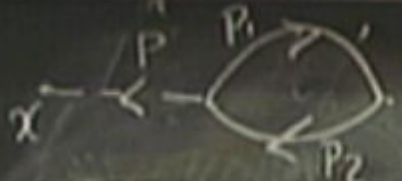
$$y = \frac{1}{2\pi i} \int_{\mathcal{C}} d^4 p d^4 p_1 d^4 p_2 d^4 q \mathcal{L}^{-1(p, q)}$$



$$y = \frac{1}{2!} \int d^4 p d^4 p_1 d^4 q \mathcal{L}^{-1(p, q)}$$

$$(P+P-P_2) (2\pi)^4 \delta^4(P_2+q-P)$$





$$y = 2 \frac{1}{2!} \int d^4 p d^4 p_1 d^4 p_2 d^4 q \mathcal{L}^{-1(p, q)}$$

$$(-ig)^2 (2\pi)^4 \delta^4(p_1 + p - p_2) (2\pi)^4 \delta^4(p_2 + q - p) \frac{1}{(2\pi)^4 (p^2 - m^2 + i\epsilon)}$$



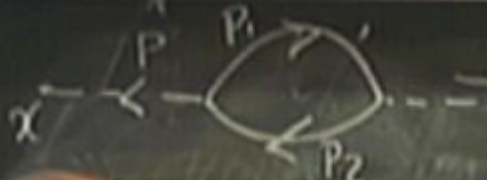
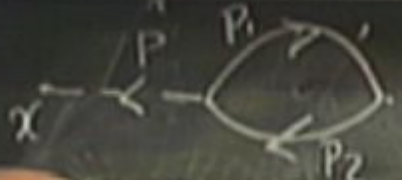


Diagram: A loop with vertices P_1 and P_2 . External lines x and y are attached to the vertices. A dashed line with a '2' above it connects the vertices.

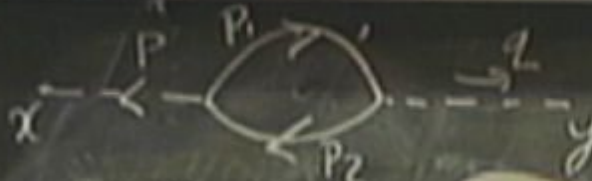
$$y = 2 \cdot 2! \int d^4 p d^4 p_1 d^4 p_2 d^4 q \mathcal{L}^{-1(p_x - l q)}$$

$$(2\pi)^4 \delta^4(p_1 + p - p_2) (2\pi)^4 \delta^4(p_2 + q - p) \frac{1}{(2\pi)^4 (p^2 - m^2 + i\epsilon)} \text{tr} \left[\frac{\gamma_5 \gamma_\mu \gamma_\nu \gamma_5}{p_1^2 - m^2 + i\epsilon} \right]$$

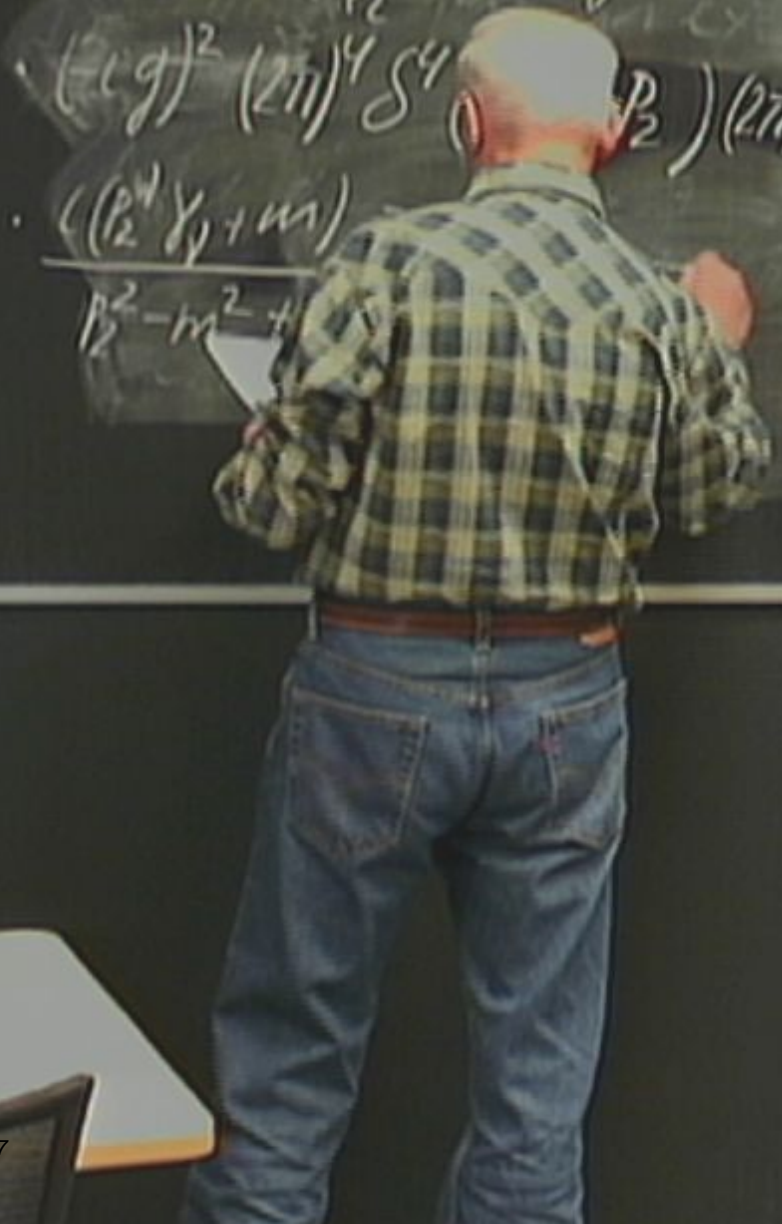


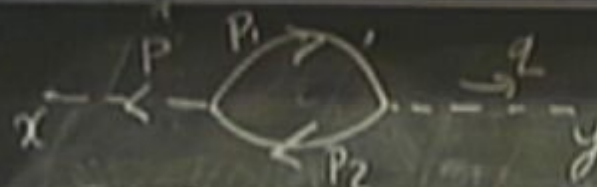
$$\begin{aligned}
 &= 2 \frac{1}{2!} \int d^4 p d^4 p_1 d^4 p_2 d^4 q \mathcal{L}^{-1(p_x - l q)} \\
 &g^2 (2\pi)^4 \delta^4(p_1 + p - p_2) (2\pi)^4 \delta^4(p_2 + q - p) \frac{1}{(2\pi)^4 (p^2 - m^2 + i\epsilon)} \text{tr} \left[\frac{\not{p}_1 \not{p}_2 + m}{p_1^2 - m^2 + i\epsilon} \right]
 \end{aligned}$$





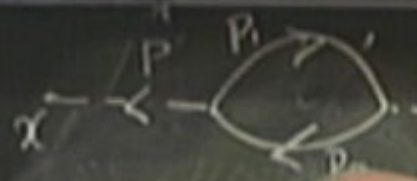
$$\begin{aligned}
 &= 2 \cdot \frac{1}{2!} \int d^4 p \, d^4 p_1 \, d^4 p_2 \, d^4 q \, e^{-i p x - i q y} \\
 &\cdot (-i g)^2 (2\pi)^4 \delta^4(p_2) (2\pi)^4 \delta^4(p_2 + q - p) \frac{i}{(2\pi)^4 (p^2 - m^2 + i\epsilon)} (-i) \text{tr} \left[\frac{i (\not{p}_1 \not{q}_2 + m)}{p_1^2 - m^2 + i\epsilon} \right]
 \end{aligned}$$





$$\begin{aligned}
 &= \int \frac{d^4 p}{(2\pi)^4} \int \frac{d^4 q}{(2\pi)^4} \int \frac{d^4 p'}{(2\pi)^4} \int \frac{d^4 q'}{(2\pi)^4} \mathcal{L}^{-1} p x - l q y \\
 & \cdot (-l g)^2 (2\pi)^4 \delta^4(p_1 + p - p_2) \delta^4(p_2 + q - p) \frac{i}{(2\pi)^4 (p^2 - m^2 + i\epsilon)} \text{tr} \left[\frac{i \gamma_4 \gamma_5 + m}{p_1^2 - m^2 + i\epsilon} \right] \\
 & \cdot \left[\frac{i (p_2^4 \gamma_4 + m)}{p_2^2 - m^2 + i\epsilon} \right] \frac{i}{(2\pi)^4}
 \end{aligned}$$

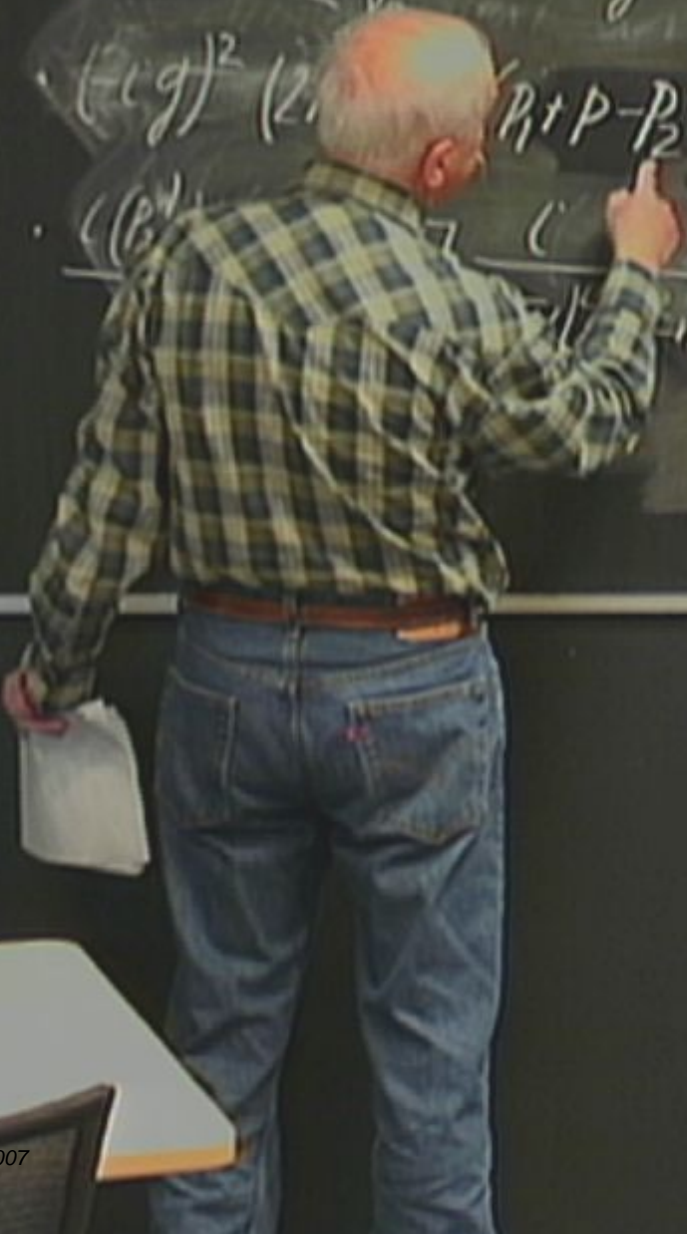
← fermion loop factor
 $\frac{i \gamma_4 \gamma_5 + m}{p_1^2 - m^2 + i\epsilon}$

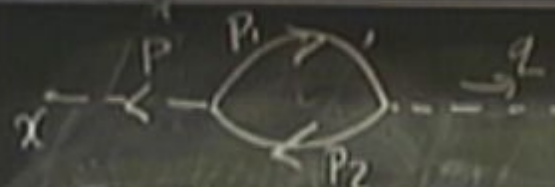


$$y = \int \frac{d^4 p}{(2\pi)^4} \int \frac{d^4 q}{(2\pi)^4} \mathcal{L}^{-1} \mathcal{P} \mathcal{L}^{-1} \mathcal{Q}$$

$$(-ig)^2 (2i) \int \frac{d^4 p}{(2\pi)^4} \int \frac{d^4 q}{(2\pi)^4} \frac{1}{(p^2 - m^2 + i\epsilon)} \frac{1}{(q^2 - m^2 + i\epsilon)} \text{tr} \left[\gamma_5 \frac{\not{p} + \not{q} - \not{p}}{(p+q-p)} \frac{\not{q} - \not{p}}{(q-p)} \right]$$

bermin di loop
factor





$$\begin{aligned}
 &= \frac{2}{2!} \int d^4 p d^4 p_1 d^4 p_2 d^4 q \mathcal{L}^{-i p x - i q y} \\
 & \cdot (-i g)^2 (2\pi)^4 \delta^4(p_1 + p - p_2) (2\pi)^4 \delta^4(p_2 + q - p) \frac{i}{(2\pi)^4 (p^2 - m^2 + i\epsilon)} \text{tr} \left[\frac{\gamma_4 (\not{p}_1 \gamma_4 + m)}{p_1^2 - m^2 + i\epsilon} \right] \\
 & \cdot \frac{i}{(2\pi)^4 (q^2 - m^2 + i\epsilon)} \frac{P_2 \text{ calc} - (-i g)^2 \cdot 4}{0 \text{ on } i\epsilon} \int d^4 p d^4 p_1 d^4 q \mathcal{L}^{-i p x - i q y} \\
 & \cdot \frac{1}{(p^2 - m^2)(q^2 - m^2)} \text{tr} \left[\frac{p_1 \gamma_4 + m}{p_1^2 - m^2} \frac{((p_1 + q) \gamma_4 + m)}{(p_1 + q)^2 - m^2} \right] \frac{2}{\text{integration}} \delta^4(p - q)
 \end{aligned}$$

← term in loop factor
 $\mathcal{L}^{-i p x - i q y}$
 $\mathcal{L}^{-i p x - i q y}$

Factor (-1) and tr for each closed fermion loop

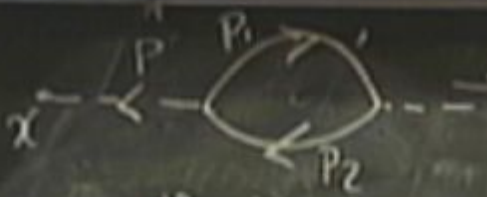


Diagram: A fermion loop with two vertices, x and y. An incoming line with momentum p enters vertex x. A loop with momenta p1 and p2 connects x and y. An outgoing line with momentum q leaves vertex y.

$$\begin{aligned}
 &= 2 \cdot \frac{1}{2!} \int d^4 p d^4 p_1 d^4 p_2 d^4 q \mathcal{L}^{-1(p_x - i q_y)} \\
 &\cdot (-i g)^2 (2\pi)^4 \delta^4(p_1 + p - p_2) (2\pi)^4 \delta^4(p_2 + q - p) \frac{i}{(2\pi)^4 (p^2 - m^2 + i\epsilon)} (-1) \text{tr} \left[\frac{i(\not{p}_1 \gamma + m)}{p_1^2 - m^2 + i\epsilon} \right] \\
 &\cdot \frac{i(\not{p}_2 \gamma + m)}{p_2^2 - m^2 + i\epsilon} \left] \frac{i}{(2\pi)^4 (q^2 - m^2 + i\epsilon)} \frac{p_1 \text{ entry} - (-i g)^2 \cdot 4}{\text{omit } i\epsilon} \frac{1}{(2\pi)^8} \int d^4 p d^4 p_1 d^4 p_2 d^4 q \mathcal{L}^{-1(p_x - i q_y)} \\
 &\frac{1}{(p^2 - m^2)(q^2 - m^2)} \text{tr} \left[\frac{\not{p}_1 \gamma + m}{p_1^2 - m^2} \frac{(\not{p}_2 \gamma + m)}{(p_2 - q)^2 - m^2} \right] \frac{2}{\text{integration}} \delta^4(p, q)
 \end{aligned}$$

Annotations: "fermion loop factor" with an arrow pointing to the trace term; "integration" with an arrow pointing to the final integral.

$$= N(\psi(x)\bar{\psi}(y)) - \int \bar{\psi}^+(y)\psi(x) T \cdot T \cdot e$$

$$= \int \frac{(-ig)^2 i^4}{(2\pi)^8} \int d^4 p \int d^4 q e^{-i p(x-y)}$$

$$= N(\psi(x)\bar{\psi}(y)) - \{\bar{\psi}^+(y)\psi(x)\}. \text{ I. e.,}$$

$$= - \frac{(-ig)^2 i^4}{(2\pi)^8} \int d^4 p \int d^4 p_1 e^{-i p(x-y)} \frac{1}{(p^2 - m^2)^2} \text{tr} \left[\frac{(\not{p} + m)}{p^2 - m^2} \right]$$

rest.

$$\frac{1}{15} \frac{g^2}{(2\pi)^8} \int d^4 p \int d^4 p_1 e^{-i p(x-y)} \frac{1}{(p^2 - m^2 + i\epsilon)^2} \text{tr} \left[\dots \right]$$

$$= N(\Psi(x) \bar{\Psi}(y)) - \{ \bar{\Psi}^{\dagger}(y) \Psi(x) \}. \text{ I. e.,}$$

$$= \frac{ig^2 i^4}{2\pi^{18}} \int d^4 p \int d^4 p_1 e^{-ip(x-y)} \frac{1}{(p^2 - m^2)^2} \text{tr} \left[\frac{(p_1 \not{x} + m)}{p_1^2 - m^2} \frac{(p - p_1) \not{x} + m}{(p - p_1)^2 + m^2} \right]$$

$$\frac{ig^2}{2\pi^{18}} \int d^4 p \int d^4 p_1 e^{-ip(x-y)} \frac{1}{(p^2 - m^2 + i\epsilon)^2} \text{tr} \left[\frac{p_1 \not{x} + m}{p_1^2 - m^2 + i\epsilon} \right]$$

$$= N(\Psi(x) \bar{\Psi}(y)) - \{\bar{\Psi}^{\dagger}(y) \Psi(x)\}. \text{ I. e.,}$$

$$= - \frac{(ig)^2 i^4}{(2\pi)^8} \int d^4 p \int d^4 p_1 e^{-i p(x-y)} \frac{1}{(p^2 - m^2)^2} \text{tr} \left[\frac{(p_1 \not{x} + m)}{p_1^2 - m^2} \frac{(p - p_1) \not{x} + m}{(p - p_1)^2 + m^2} \right]$$

$$\stackrel{\text{rest}}{\sim} \frac{g^2}{(2\pi)^8} \int d^4 p \int d^4 p_1 e^{-i p(x-y)} \frac{1}{(p^2 - m^2 + i\epsilon)^2} \text{tr} \left[\frac{p_1 \not{x} + m}{p_1^2 - m^2 + i\epsilon} \frac{(p - p_1) \not{x} + m}{(p - p_1)^2 + m^2} \right]$$

$$\cdot \left. \frac{(p_1 - p) \not{x} + m}{(p_1 - p)^2 - m^2 + i\epsilon} \right]$$

$$= N(\Psi(x) \bar{\Psi}(y)) - \{ \bar{\Psi}^{\dagger}(y) \Psi(x) \}. \text{ I. e.,}$$

$$= - \frac{(-ig)^2 i^4}{(2\pi)^8} \int d^4 p \int d^4 p_1 e^{-i p(x-y)} \frac{1}{(p^2 - m^2)^2} \text{tr} \left[\frac{(\not{p} + m)}{p^2 - m^2} \cdot \frac{(\not{p}_1 - m)}{(p_1 - p)^2 + m^2} \right]$$

rest

$$\frac{1}{i\epsilon} \frac{g^2}{(2\pi)^8} \int d^4 p \int d^4 p_1 e^{-i p(x-y)} \frac{1}{(p^2 - m^2 + i\epsilon)^2} \text{tr} \left[\frac{\not{p}_1 \not{p} + m}{p^2 - m^2 + i\epsilon} \cdot \frac{(\not{p}_1 - m)}{(p_1 - p)^2 + m^2} \right]$$

$$\cdot \left. \frac{(\not{p}_1 - m)}{(p_1 - p)^2 + m^2} \right]$$

Remarks
1. In discussing charge id

Remarks.

1. In discussing charge conjugation and time reversal, we used the following equalities, respectively.

$$\sqrt{p \cdot \bar{\sigma}} \cdot \sigma^2 = \sigma^2 \sqrt{p \cdot \sigma}^* \quad \sqrt{p \cdot \sigma} \cdot \sigma^2 = \sigma^2 \sqrt{p \cdot \bar{\sigma}}^*, \quad \sigma = (1, \vec{\sigma}), \quad \bar{\sigma} = (1, -\vec{\sigma}),$$
$$\hat{p} = (p^0, -\vec{p}).$$

Remarks

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Let us prove them: we use answer in problems in

$$g)^2 \int d^4x d^4y D_F(x-w) D_F(y-w) \text{tr} [S_F(u-v) S_F(v-w)]$$

$\hat{P} = (P^0, -\vec{P}) \Rightarrow$ these two equalities are equivalent.

Let us prove them: we use answer in problem 5 in assignment 3:

$$\sqrt{P \cdot \sigma} = \dots$$

$$= -(-ig)^2 \left[d^{\mu\nu} d^{\rho\sigma} \dots S_F(u-v) S_F(v-u) \right]$$

$\hat{P} = (P^0, -\vec{P}) \Rightarrow$ these two equalities are equivalent.

Let us prove them: we use answer in problem 5 in assignment 3:

$$\sqrt{P \cdot \sigma} = \sqrt{E - |\vec{P}|} \frac{1 + \vec{\sigma} \cdot \vec{P}}{2} + \dots$$

$$= -(-ig)^2 \int d^4x d^4y D_F(x-w) D_F(y-b) \text{tr} L$$

Let us prove them: we use answer in problem 5 in assignment 3:

$$\sqrt{P \cdot \sigma} = \sqrt{E - |\vec{p}|} \frac{1 + \vec{\sigma} \cdot \hat{p}}{2} + \sqrt{E + |\vec{p}|} \frac{1 - \vec{\sigma} \cdot \hat{p}}{2}, \quad \hat{p} = \frac{\vec{p}}{|\vec{p}|}$$

$$[S_F(v-u)]_{\text{p.d.}} = \text{tr} [S_F(u-v) S_F(v-u)]$$

$$= - (ig)^2 \int d^4x d^4y D_F(x-u) D_F(y-v) \text{tr} [S_F(u-v) S_F(v-u)].$$

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Remarks

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REMARKS

1. In discussing charge conjugation and time reversal, we used the following equalities, respectively.

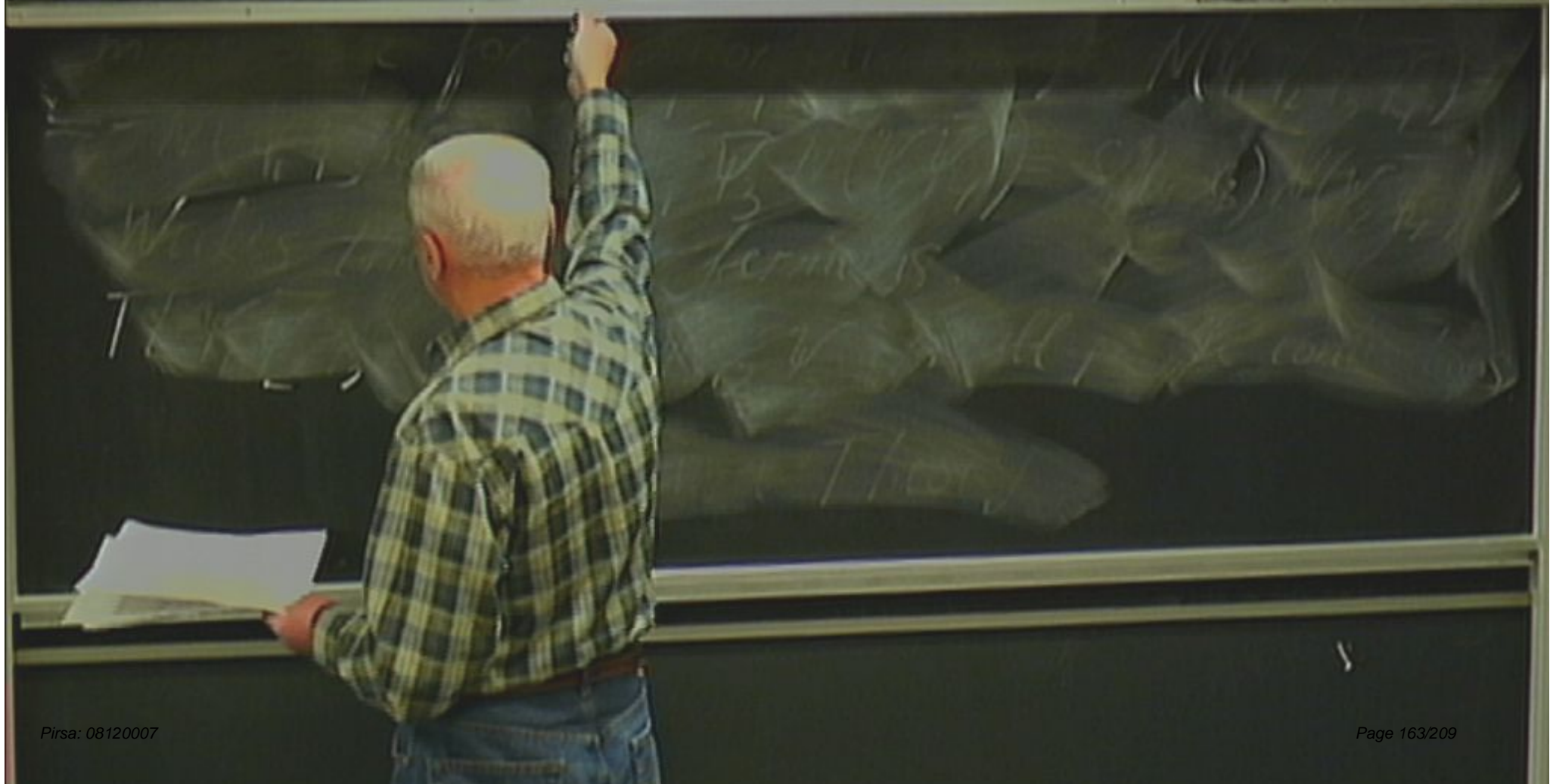
$$\sqrt{P \cdot \vec{\sigma}} \cdot \vec{\sigma}^2 = \vec{\sigma}^2 \sqrt{P \cdot \vec{\sigma}^*}, \quad \sqrt{P \cdot \vec{\sigma}'} \cdot \vec{\sigma}^2 = \vec{\sigma}^2 \sqrt{P \cdot \vec{\sigma}'^*}, \quad \vec{\sigma} = (1, \vec{\sigma}), \quad \vec{\sigma}' = (1, -\vec{\sigma})$$

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Let us prove them: we use answer in problems in assignment 3:

$$\sqrt{p \cdot \sigma} = \sqrt{E - |\vec{p}|} \frac{1 - \vec{\sigma} \cdot \hat{p}}{2} + \sqrt{E + |\vec{p}|} \frac{1 + \vec{\sigma} \cdot \hat{p}}{2}, \quad \hat{p} = \frac{\vec{p}}{|\vec{p}|}$$


Let us prove them. we use answer in problems 5 in assignment 3:

$$\sqrt{p \cdot \sigma} = \sqrt{E - |\vec{p}|} \frac{1 - \vec{\sigma} \cdot \hat{p}}{2} + \sqrt{E + |\vec{p}|} \frac{1 + \vec{\sigma} \cdot \hat{p}}{2}, \quad \hat{p} = \frac{\vec{p}}{|\vec{p}|}$$

$$\sqrt{p \cdot \sigma} = \sqrt{E - |\vec{p}|} \frac{1 - \vec{\sigma} \cdot \hat{p}}{2} + \sqrt{E + |\vec{p}|} \frac{1 + \vec{\sigma} \cdot \hat{p}}{2}$$

Proof. Since $\vec{\sigma} \cdot \vec{p} \sigma^2 = -\sigma^2 (\vec{\sigma} \cdot \vec{p})$, we obtain

Let us prove them. we use answer in problems in assignment 3:

$$\sqrt{P \cdot \vec{\sigma}} = \sqrt{E - |\vec{P}|} \frac{1 + \vec{\sigma} \cdot \hat{P}}{2} + \sqrt{E + |\vec{P}|} \frac{1 - \vec{\sigma} \cdot \hat{P}}{2}, \quad \hat{P} = \frac{\vec{P}}{|\vec{P}|}$$

$$\sqrt{P \cdot \vec{\sigma}} = \sqrt{E - |\vec{P}|} \frac{1 - \vec{\sigma} \cdot \hat{P}}{2} + \sqrt{E + |\vec{P}|} \frac{1 + \vec{\sigma} \cdot \hat{P}}{2}$$

Proof. Since $\vec{\sigma} \cdot \vec{P} \vec{\sigma} = -G^2 (\vec{\sigma} \cdot \vec{P})$, we obtain

$$\sqrt{P \cdot \vec{\sigma}} \cdot G^2 = \left[\sqrt{E - |\vec{P}|} \frac{1 + \vec{\sigma} \cdot \hat{P}}{2} + \sqrt{E + |\vec{P}|} \frac{1 - \vec{\sigma} \cdot \hat{P}}{2} \right] \cdot G^2$$

Let us prove them. we use answer in problems in assignment 3:

$$\sqrt{P \cdot \hat{\sigma}} = \sqrt{E - |\vec{p}|} \frac{1 + \hat{\sigma} \cdot \hat{p}}{2} + \sqrt{E + |\vec{p}|} \frac{1 - \hat{\sigma} \cdot \hat{p}}{2}, \quad \hat{p} = \frac{\vec{p}}{|\vec{p}|}$$

$$\sqrt{P \cdot \vec{\sigma}} = \sqrt{E - |\vec{p}|} \frac{1 - \hat{\sigma} \cdot \hat{p}}{2} + \sqrt{E + |\vec{p}|} \frac{1 + \hat{\sigma} \cdot \hat{p}}{2}$$

in Proof. Since $\vec{\sigma} \cdot \vec{p} \sigma^2 = -\sigma^2 (\vec{\sigma} \cdot \vec{p})$, we obtain

$$\sqrt{P \cdot \vec{\sigma}} \cdot \sigma^2 = \left[-|\vec{p}| \frac{1 + \hat{\sigma} \cdot \hat{p}}{2} + \sqrt{E + |\vec{p}|} \frac{1 - \hat{\sigma} \cdot \hat{p}}{2} \right] \sigma^2 =$$

$$= \sigma^2$$

Let us prove them. we use answer in problems in assignment 3:

$$\sqrt{P \cdot \hat{\sigma}} = \sqrt{E - |\vec{p}|} \frac{1 + \vec{\sigma} \cdot \hat{p}}{2} + \sqrt{E + |\vec{p}|} \frac{1 - \vec{\sigma} \cdot \hat{p}}{2}, \quad \hat{p} = \frac{\vec{p}}{|\vec{p}|}$$

$$\sqrt{P \cdot \hat{\sigma}} = \sqrt{E - |\vec{p}|} \frac{1 - \vec{\sigma} \cdot \hat{p}}{2} + \sqrt{E + |\vec{p}|} \frac{1 + \vec{\sigma} \cdot \hat{p}}{2}$$

In Proof: Since $\vec{\sigma} \cdot \vec{p} \sigma^2 = -\sigma^2 (\vec{\sigma} \cdot \vec{p})$, we obtain

$$\sqrt{P \cdot \hat{\sigma}} \cdot \sigma^2 = \left[\sqrt{E - |\vec{p}|} \frac{1 + \vec{\sigma} \cdot \hat{p}}{2} + \sqrt{E + |\vec{p}|} \frac{1 - \vec{\sigma} \cdot \hat{p}}{2} \right] \sigma^2 =$$

$$= \sigma^2 \left[\sqrt{E - |\vec{p}|} \frac{1 - \vec{\sigma} \cdot \hat{p}}{2} + \sqrt{E + |\vec{p}|} \frac{1 + \vec{\sigma} \cdot \hat{p}}{2} \right]$$

Let us prove them. we use answer in problem 5 in assignment 3:

$$\sqrt{P \cdot \hat{\sigma}} = \sqrt{E - |\vec{p}|} \frac{1 + \vec{\sigma} \cdot \hat{p}}{2} + \sqrt{E + |\vec{p}|} \frac{1 - \vec{\sigma} \cdot \hat{p}}{2}, \quad \hat{p} = \frac{\vec{p}}{|\vec{p}|}$$

$$\sqrt{P \cdot \sigma} = \sqrt{E - |\vec{p}|} \frac{1 - \vec{\sigma} \cdot \hat{p}}{2} + \sqrt{E + |\vec{p}|} \frac{1 + \vec{\sigma} \cdot \hat{p}}{2}$$

Proof. Since $\vec{\sigma} \cdot \vec{p} \sigma^2 = -\sigma^2 (\vec{\sigma}^* \cdot \vec{p})$, we obtain

$$\sqrt{P \cdot \sigma} \cdot \sigma^2 = \left[\sqrt{E - |\vec{p}|} \frac{1 - \vec{\sigma} \cdot \hat{p}}{2} + \sqrt{E + |\vec{p}|} \frac{1 + \vec{\sigma} \cdot \hat{p}}{2} \right] \sigma^2 =$$

$$= \sigma^2 \left[\sqrt{E - |\vec{p}|} \frac{1 - \vec{\sigma}^* \cdot \hat{p}}{2} + \sqrt{E + |\vec{p}|} \frac{1 + \vec{\sigma}^* \cdot \hat{p}}{2} \right]$$

Let us prove them. we use answer in problem 5 in assignment 3:

$$\sqrt{P \cdot \hat{\sigma}} = \sqrt{E - |\vec{p}|} \frac{1 + \vec{\sigma} \cdot \hat{p}}{2} + \sqrt{E + |\vec{p}|} \frac{1 - \vec{\sigma} \cdot \hat{p}}{2}, \quad \hat{p} = \frac{\vec{p}}{|\vec{p}|}$$

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Proof. Since $\vec{\sigma} \cdot \vec{p} \sigma^2 = -\sigma^2 (\vec{\sigma}^* \cdot \vec{p})$, we obtain

$$\sqrt{P \cdot \sigma} \cdot \sigma^2 = \left[\sqrt{E - |\vec{p}|} \frac{1 + \vec{\sigma} \cdot \hat{p}}{2} + \sqrt{E + |\vec{p}|} \frac{1 - \vec{\sigma} \cdot \hat{p}}{2} \right] \sigma^2 =$$

$$= \sigma^2 \left[\sqrt{E - |\vec{p}|} \frac{1 - \vec{\sigma}^* \cdot \hat{p}}{2} + \sqrt{E + |\vec{p}|} \frac{1 + \vec{\sigma}^* \cdot \hat{p}}{2} \right]$$

Let us prove them. we use answer in problems 5 in assignment 3:

$$\sqrt{P \cdot \hat{\sigma}} = \sqrt{E - |\vec{p}|} \frac{1 + \vec{\sigma} \cdot \hat{p}}{2} + \sqrt{E + |\vec{p}|} \frac{1 - \vec{\sigma} \cdot \hat{p}}{2}, \quad \hat{p} = \frac{\vec{p}}{|\vec{p}|}$$

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$$= \sigma^2 \left[\sqrt{E - |\vec{p}|} \frac{1 - \vec{\sigma} \cdot \hat{p}}{2} + \sqrt{E + |\vec{p}|} \frac{1 + \vec{\sigma} \cdot \hat{p}}{2} \right] = \sigma^2 \sqrt{P \cdot \hat{\sigma}}$$

Let us prove them: we use answer in problem 5 in assignment 3:

$$\sqrt{p \cdot \sigma} = \sqrt{E - |\vec{p}|} \frac{1 - \vec{\sigma} \cdot \hat{p}}{2} + \sqrt{E + |\vec{p}|} \frac{1 + \vec{\sigma} \cdot \hat{p}}{2}, \quad \hat{p} = \frac{\vec{p}}{|\vec{p}|}$$

Proof. Since $\vec{\sigma} \cdot \vec{p} \sigma^2 = -\sigma^2 (\vec{\sigma}^* \cdot \vec{p})$, we obtain

$$\begin{aligned} \sqrt{p \cdot \sigma} \cdot \sigma^2 &= \left[\sqrt{E - |\vec{p}|} \frac{1 + \vec{\sigma} \cdot \hat{p}}{2} + \sqrt{E + |\vec{p}|} \frac{1 - \vec{\sigma} \cdot \hat{p}}{2} \right] \sigma^2 = \\ &= \left[\sqrt{E - |\vec{p}|} \frac{1 - \vec{\sigma}^* \cdot \hat{p}}{2} + \sqrt{E + |\vec{p}|} \frac{1 + \vec{\sigma}^* \cdot \hat{p}}{2} \right] = \sigma^2 \sqrt{\vec{p} \cdot \sigma^*} \end{aligned}$$

Let us prove them. we use answer in problems 5 in assignment 3:

$$\sqrt{P \cdot \hat{\sigma}} = \sqrt{E - |\vec{p}|} \frac{1 + \vec{\sigma} \cdot \hat{p}}{2} + \sqrt{E + |\vec{p}|} \frac{1 - \vec{\sigma} \cdot \hat{p}}{2}, \quad \hat{p} = \frac{\vec{p}}{|\vec{p}|}$$

$$\sqrt{P \cdot \vec{\sigma}} = \sqrt{E - |\vec{p}|} \frac{1 - \vec{\sigma} \cdot \hat{p}}{2} + \sqrt{E + |\vec{p}|} \frac{1 + \vec{\sigma} \cdot \hat{p}}{2}$$

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$$= \sigma^2 \left[\sqrt{E - |\vec{p}|} \frac{1 - \vec{\sigma}^* \cdot \hat{p}}{2} + \sqrt{E + |\vec{p}|} \frac{1 + \vec{\sigma}^* \cdot \hat{p}}{2} \right] = \sigma^2 \sqrt{\vec{p} \cdot \sigma^*}$$

In a closed fermion loop, there is a factor of (-1) and the trace over a product of Dirac matrices



$$\begin{aligned}
 &= \psi_1 \psi_1 \psi_2 \psi_2 \psi_3 \psi_3 \psi_4 \psi_4 = (-1) \psi_1 \psi_2 \psi_3 \psi_4 \\
 &= (-1) \text{tr} (S_F(12) S_F(23) S_F(34) S_F(41))
 \end{aligned}$$

~~Handwritten text, possibly a title or definition, mostly obscured by erasing.~~

~~Handwritten text, possibly a definition or theorem, mostly obscured by erasing.~~

Majorana and Dirac

~~Extensive handwritten mathematical notes and equations, including various symbols and integrals, mostly obscured by erasing.~~

Majorana and Dirac fields

$$\Psi_M: \Psi_M^c \equiv C \bar{\Psi}_M^T = \Psi_M, \quad C = -i(\gamma^0 \gamma^2)^T \text{ and } C^{-1} = C^{\dagger-1}$$

$$\Psi_M: \Psi_M \equiv \hat{C} \Psi_M^T = \Psi_M, \quad \hat{C} = -i(\gamma^0 \gamma^2)^T \text{ and } \hat{C} = \hat{C}^{-1},$$

$$\hat{C}^T = -\hat{C}, \quad \hat{C}^{-1} = -\gamma^0 \gamma^2$$

Let us consider Dirac field Ψ

$\hat{C}^T = -\hat{C}$, $\hat{C}^{-1} \gamma^{\mu} \hat{C} = -\gamma^{\mu T}$

Let us consider Dirac field ψ . Then, introduce $\psi^c = \hat{C} \bar{\psi}^T$ and two fields: ψ and ψ^c

Let us consider Dirac field Ψ . Then, introduce $\Psi^c = \hat{C}\bar{\Psi}^T$ and two fields: $\Psi_{M_1} = \Psi$ and $\Psi_{M_2} = \frac{\Psi - \Psi^c}{2i}$.
 Let us show that they are Majorana fermions.

Majorana and Dirac fields

$$\psi_M: \psi^c = \hat{C} \bar{\psi}^T = \psi_M, \quad \hat{C} = -i(\gamma_0 \gamma_2)^T \text{ and } \hat{C}^{-1} = \hat{C}^{-1}$$

$$\hat{C}^T = -\hat{C}, \quad \hat{C}^{-1} \gamma_\mu \hat{C} = \gamma_\mu$$

Let us consider a Dirac field ψ . Then, introduce $\psi^c = \hat{C} \bar{\psi}^T$.

$$\psi_{M1} = \frac{\psi + \psi^c}{2}, \quad \psi_{M2} = \frac{\psi - \psi^c}{2i}$$

These are Majorana fields.

$\psi, \psi^c \equiv \psi^T$
 Let us consider Dirac field ψ . Then, introduce $\psi^c \equiv \bar{\psi}^T$
 and two fields: $\psi_1 = \frac{\psi + \psi^c}{2}, \psi_2 = \frac{\psi - \psi^c}{2i}$
 Let us show that they are Majorana fields

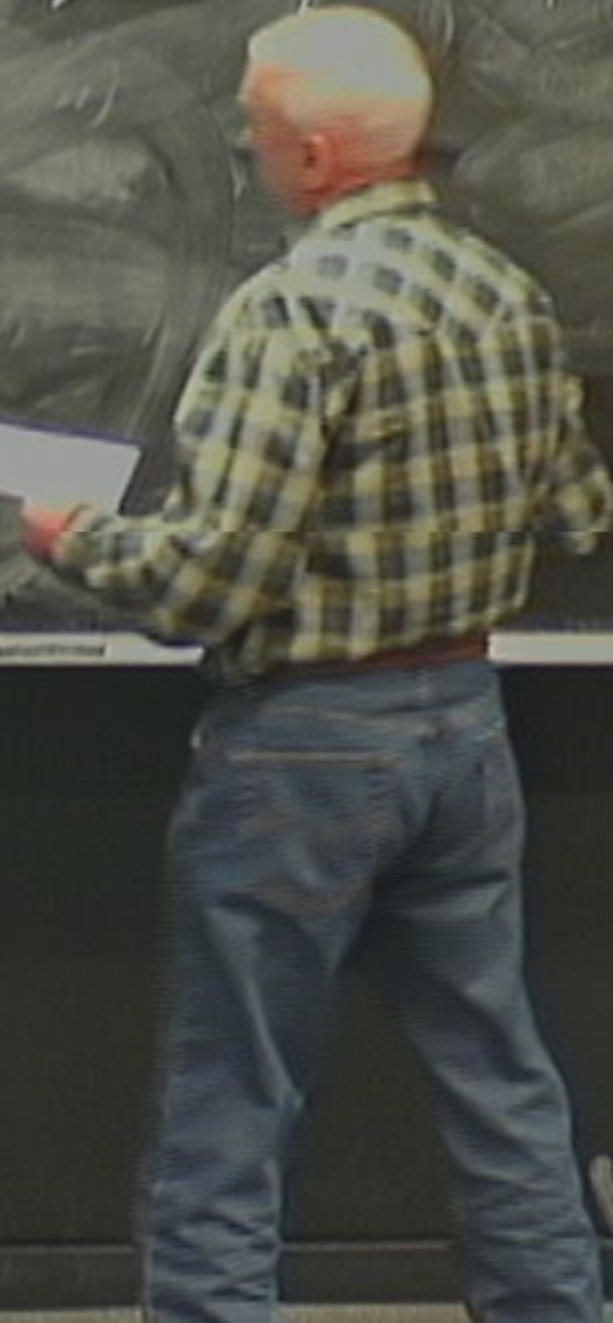


Prove first that $(\psi^c)^c \equiv \psi$

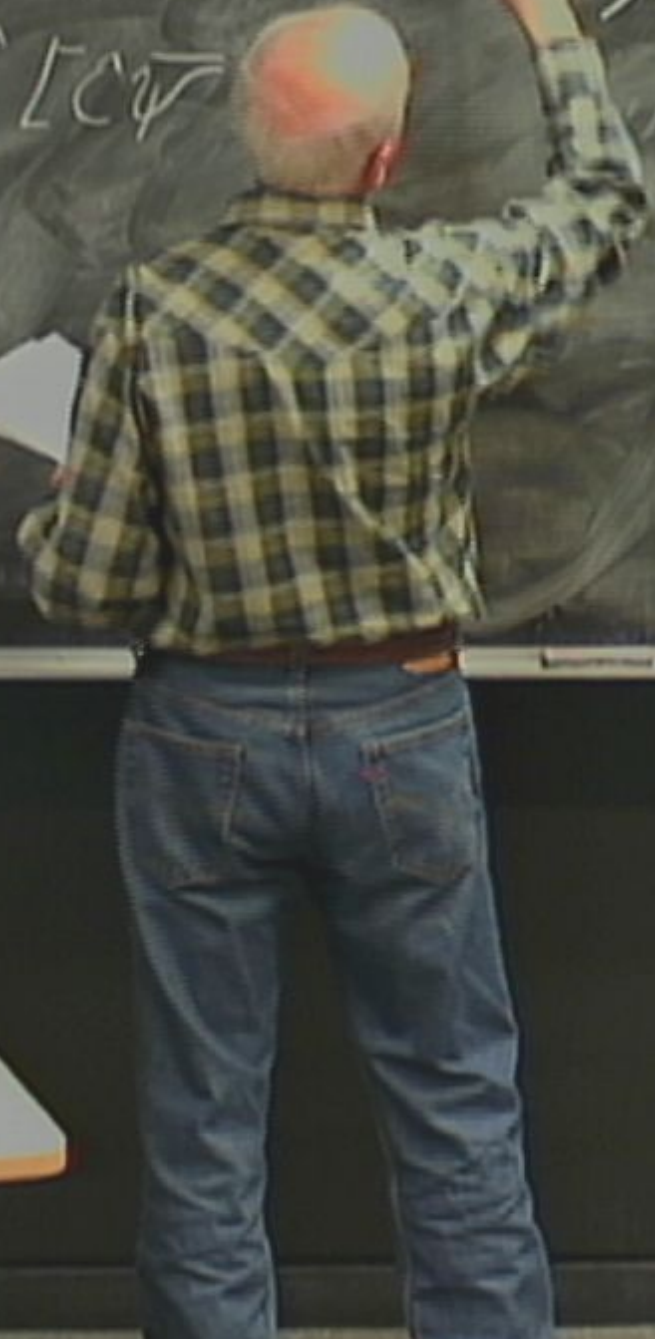


Prove first that $(\psi^\dagger)^\dagger \equiv \hat{C} \psi \hat{C}^T = \psi$; $\hat{C} \psi \hat{C}^T = \psi$
 $= \hat{C} [\hat{C} \psi]$

[Faded handwritten notes on the chalkboard, including the word "operator"]



Prove first that $(\psi^c)^c \equiv \hat{C}\psi^c = \psi$; $\hat{C}\psi^c = \psi$



Prove first that $(\psi)^\dagger \equiv \hat{C} \psi^T = \psi^\dagger; \hat{C} \psi^T = \psi^\dagger$
 $= \hat{C} [\hat{C} \psi]^\dagger$



Prove first that $(\psi^c)^c \equiv \hat{C} \psi^c = \psi$; $\hat{C} \psi^c =$
 $\hat{C} [\hat{C} (\psi^c)^T] = \hat{C} [\hat{C} [\hat{C} (\psi^c)^T] \gamma_0] =$



Let us show that $\hat{C} \hat{C}^T = I$ and $\hat{C}^T \hat{C} = I$

Prove first that $(\psi^c)^c = \hat{C} \hat{C}^T \psi^c = \psi^c$; $\hat{C} \hat{C}^T = I$

$$= \hat{C} [\hat{C} \psi^c]^T = \hat{C} [\hat{C} (\psi^c)^T]^T = \hat{C} [\hat{C} (\psi^c)^T]^T \hat{C}^T =$$

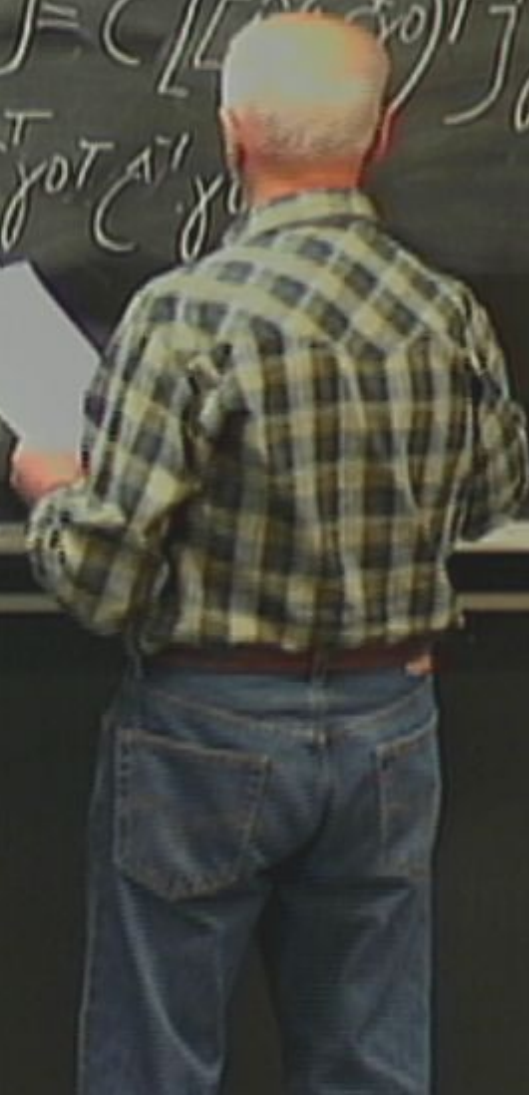
$$= \hat{C} [\hat{C} \hat{C}^T \psi^c]^T = \hat{C} [\psi^c]^T = \psi^c$$



Let us show that

$$\psi_M = \frac{\psi - \psi_C}{\sigma}$$

Prove first that $(\psi)^c \equiv \hat{C} \overline{\psi^c}^T = \psi$; $\hat{C} \overline{\psi^c}^T =$
 $= \hat{C} [\overline{\hat{C} \psi^T}] = \hat{C} [\hat{C} (\psi^T \gamma_0)^T] = \hat{C} [\hat{C} (\gamma_0^T \psi^T)]^T =$
 $= \hat{C} [\hat{C} \gamma_0 \psi^T]^T = \hat{C} [\psi^T \gamma_0^T \hat{C}^T]^T =$



Let us show that

$$\psi_c = \frac{\psi - \psi_c}{\psi_c}$$

Prove first that $(\psi_c)^c \equiv \hat{C} \overline{\psi_c}^T = \psi$; $\hat{C} \overline{\psi_c}^T =$
 $= \hat{C} \overline{[\hat{C} \psi^T]} = \hat{C} [\hat{C} (\psi^+ \delta_0)^T] = \hat{C} [\hat{C} [(\hat{C} (\psi^+ \delta_0)^T)^+ \delta_0]^T] =$
 $\hat{C} [\hat{C} [(\hat{C} \delta_0 \psi^+)^T]^+ \delta_0]^T = \hat{C} [\psi^T \delta_0 \delta_0^T \delta_0^+ \delta_0] = \hat{C} \delta_0 \delta_0^T \delta_0^+ \delta_0 \psi =$

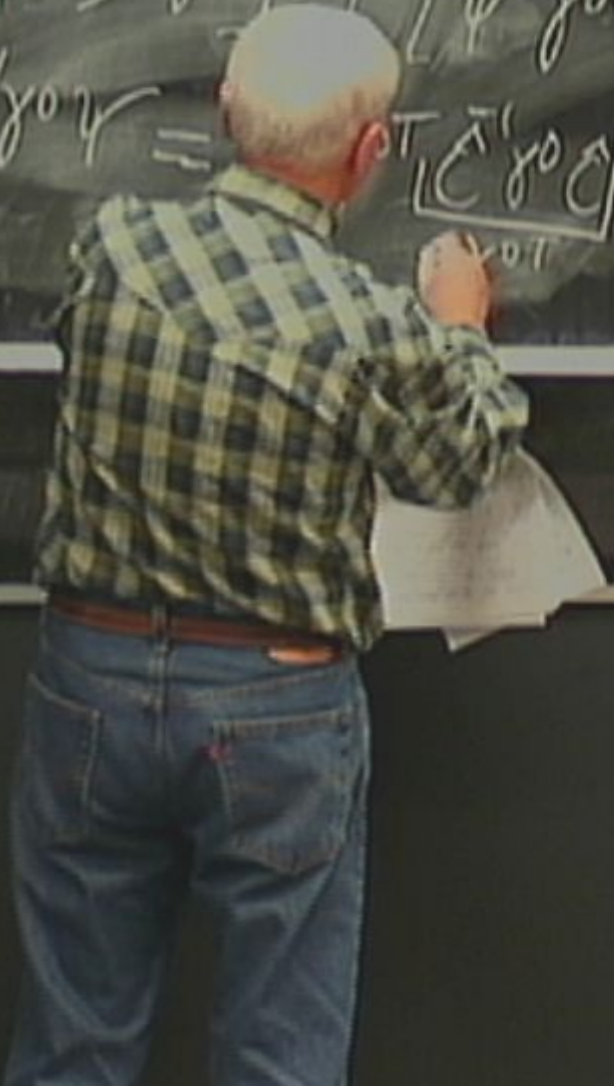


move first into $(\psi) \equiv C\psi^T = \psi$; $C\psi^T = \psi$

$$= \hat{C} [C\psi^T]^T = \hat{C} [C(\psi^T \gamma_0)^T] = \hat{C} [C(\psi^T \gamma_0)^T \gamma_0]^T =$$

$$= \hat{C} [C(\gamma_0^T \psi^T)]^T \gamma_0 = \hat{C} [\psi^T \gamma_0^T C^T \gamma_0] = \hat{C} \gamma_0^T C^T \gamma_0 \psi =$$

$$\hat{C} \gamma_0^T C^T \gamma_0 \psi = \hat{C} \gamma_0^T C^T \gamma_0 \psi$$



Prove first that $(\psi^c)^c \equiv \hat{C} \psi^c = \psi$; $\hat{C} \psi^c =$

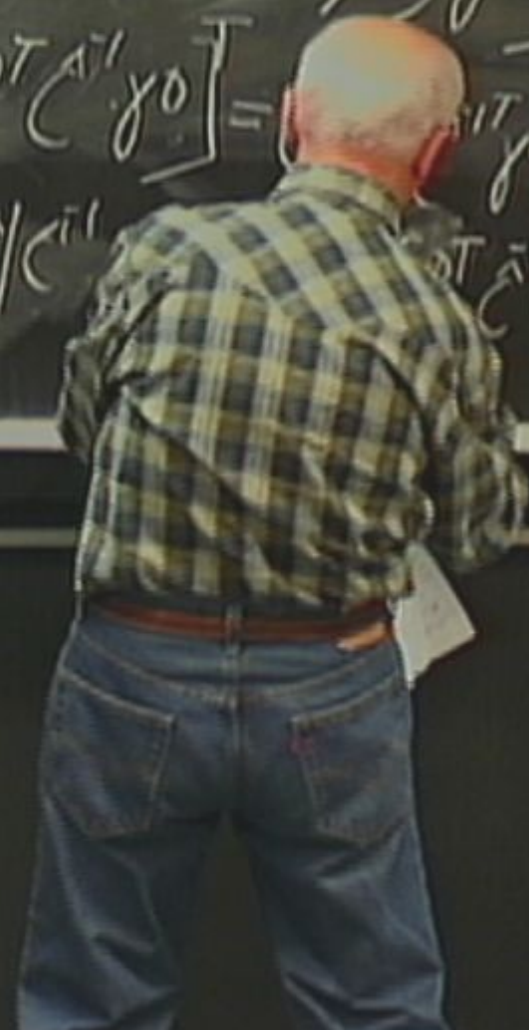
$$= \hat{C} [\hat{C} \psi]^T = \hat{C} [\hat{C} (\psi^+ \gamma_0)^T] = \hat{C} [\hat{C} [\hat{C} (\psi^+ \gamma_0)^T] \gamma_0]^T =$$

$$= \hat{C} [\gamma_0^T \hat{C}^T \hat{C} (\psi^+ \gamma_0)^T] = \hat{C} [\psi^T \gamma_0^T \hat{C}^T \gamma_0] = \hat{C} \gamma_0^T \hat{C}^T \gamma_0 \psi =$$

$$= \psi$$



Prove first that $(\psi^c)^c = \hat{C} \overline{\psi^c}^T = \psi$; $\hat{C} \overline{\psi^c}^T =$
 $= \hat{C} [\overline{\hat{C} \psi^+}] = \hat{C} [\hat{C} (\psi^+ \gamma_0)^T] = \hat{C} [\hat{C} (\psi^+ \gamma_0)^T \gamma_0]^T =$
 $= \hat{C} [\hat{C} \gamma_0^T \psi^+]^T = \hat{C} [\psi^T \gamma_0^T \hat{C}^{-1} \gamma_0] =$
 $\hat{C} \gamma_0^T \hat{C}^{-1} \gamma_0 \psi = -\hat{C} \gamma_0^T \hat{C}^{-1} \gamma_0 \psi$



Prove first that $(\psi^c)^c = \hat{C} \overline{\psi^c}^T = \psi$; $\hat{C} \overline{\psi^c}^T =$
 $= \hat{C} \overline{[\hat{C} \psi^T]} = \hat{C} [\hat{C} (\psi^T \gamma_0)^T] = \hat{C} [\hat{C} (\psi^T \gamma_0)^T \gamma_0]^T =$
 $= \hat{C} [\hat{C} (\gamma_0^T \psi^T)^T \gamma_0]^T = \hat{C} [\hat{C}^T \gamma_0]^T = \hat{C} \gamma_0^T \hat{C}^T \gamma_0 \psi =$
 $\hat{C} \gamma_0^T \hat{C}^T \gamma_0 \psi = -\hat{C} \gamma_0^T \hat{C}^T \gamma_0 \psi = \hat{C} \gamma_0^T \hat{C}^T \gamma_0 \psi = \hat{C} \gamma_0^T \hat{C}^T \gamma_0 \psi =$



Prove first that $(\psi^c)^c \equiv \hat{C} \psi^c = \psi$; $\hat{C} \psi^c =$
 $= \hat{C} [\overline{C \psi^c}]^T = \hat{C} [\overline{C (\psi^+ \gamma_0)^T}] = \hat{C} [\overline{[C (\psi^+ \gamma_0)^T] \gamma_0}]^T =$
 $= \hat{C} [\overline{[C \gamma_0^T \psi^+]} \gamma_0]^T = \hat{C} [\overline{\psi^+ \gamma_0^T C} \gamma_0]^T = \hat{C} \gamma_0^T \overline{C} \gamma_0 \psi =$
 $= \hat{C} \gamma_0^T \overline{C} \gamma_0 \psi = \hat{C} \gamma_0^T \overline{C} \gamma_0 \psi = \hat{C} \gamma_0^T \overline{C} \gamma_0 \psi =$



$$\begin{aligned}
 & \hat{C} \gamma_0 \hat{C}^{-1} \gamma_0 \psi = -\hat{C} \gamma_0 \hat{C}^{-1} \gamma_0 \psi = \hat{C} \gamma_0 \hat{C}^{-1} \gamma_0 \psi = \psi \\
 & \hat{C} \gamma_0 \hat{C}^{-1} \gamma_0 \psi = \psi
 \end{aligned}$$



Let us show that they are Main $\frac{1}{2}$; $\psi_{1/2} = \frac{\psi - \psi_c}{2}$

Prove first that $(\psi^c)^c = C \overline{\psi^c}^T = \psi$ $C \overline{\psi^c}^T =$
 $= \hat{C} [C \overline{\psi}^T] = \hat{C} [C (\psi^* \gamma_0)^T] = \hat{C} [C (\psi^* \gamma_0)^T \gamma_0] =$
 $= \hat{C} [C \overline{\psi}^T \gamma_0] = \hat{C} [\psi^T \gamma_0^T C \gamma_0] = \hat{C} \gamma_0^T C \gamma_0 \psi =$
 $= \hat{C} \psi = \psi$



$$= \hat{C} \hat{C}^{-1} \psi = \psi$$

$$= -\hat{C} \gamma_0^T \left[\hat{C}^{-1} \gamma_0 \right] \psi = \hat{C} \gamma_0^T \gamma_0^{-1} \psi = \psi$$

$$\psi_{M_1} = ?$$

$$\psi_{M_1}^c = \hat{C} \vec{\psi}_{M_1}^T = \hat{C} \psi$$



$$= \hat{C} \hat{C}^{-1} \psi = \psi$$

$$= -\hat{C} \gamma_0^T \left[\hat{C}^{-1} \gamma_0 \right] \psi = \hat{C} \gamma_0^T \hat{C}^{-1} \psi =$$

$$\psi_{M_1}^c = \hat{C} \bar{\psi}_{M_1}^T = \frac{\hat{C} \bar{\psi}^T + \hat{C} (\bar{\psi}^c)^T}{2} =$$

$$\frac{\psi^c + \psi}{2} = \psi_{M_1}$$

$$= \hat{C} \hat{C}^{-1} \psi = \psi$$

$$= -\hat{C} \gamma_0^T \hat{C}^{-1} \gamma_0 \psi = \hat{C} \gamma_0^T \hat{C}^{-1} \psi = \psi$$

Majorana fields

$$\frac{\psi^c + \psi}{2} = \psi_M$$

$$= \hat{C} \hat{C}^{-1} \psi = \psi$$

$$= -\hat{C} \gamma_0^T \hat{C}^{-1} \gamma_0 \psi = \hat{C} \gamma_0^T \hat{C}^{-1} \psi = \psi$$

Majorana fields

$$= \frac{\psi^c + \psi}{2} = \psi_M$$

$$\psi_M^c = \hat{C} \overline{\psi_M}^T = -\frac{\hat{C} \overline{\psi}^T - \hat{C} (\overline{\psi^c})^T}{2}$$

$$= \hat{C} \hat{C}^{-1} \psi = \psi$$

$$= -\hat{C} \gamma_0^T \hat{C}^{-1} \gamma_0 \psi = \hat{C} \gamma_0^T \hat{C}^{-1} \psi = \hat{C} \gamma_0^T \hat{C}^{-1} \psi = \psi$$

These are Majorana fields

$$= \frac{\psi^c + \psi}{2} = \psi_{M1}$$

$$\psi_{M2} = \hat{C} \bar{\psi}_{M2}^T = -\frac{\hat{C} \bar{\psi}^T - \hat{C} (\bar{\psi}^c)^T}{2\hat{C}} = \frac{\psi^c - \psi}{2\hat{C}} = \psi_{M2}$$

$$= N(\psi(x)\psi(y)) - \{\bar{\psi}(y)\psi(x)\}. \text{ I. e.,}$$

Compare with complex KG field:

$\mathcal{U} \Rightarrow$ two real KG field:

$$\varphi_1 = \frac{\varphi + \varphi^*}{2}, \quad \varphi_2 = \frac{\varphi - \varphi^*}{2i}$$

$$= N(\psi(x)\psi(y)) - \{\overline{\psi(y)}\psi(x)\}. \text{ I. e.,}$$

Compare with complex KG field:

$\mathcal{U} \Rightarrow$ two real KG field:

$$\varphi_1 = \frac{\varphi + \varphi^*}{2}, \quad \varphi_2 = \frac{\varphi - \varphi^*}{2i}$$

$$\Psi_M \equiv \hat{C} \bar{\Psi}^T = \Psi_M, \quad \hat{C} = -i(\gamma_0 \gamma)^T \text{ and } \hat{C}^{-1} = \hat{C}^T$$

$$\hat{C}^T = -\hat{C}, \quad \hat{C}^{-1} \gamma^\mu \hat{C} = -\gamma^{\mu T}$$

Let us consider Dirac field Ψ . Then, introduce $\Psi^c = \hat{C} \bar{\Psi}^T$ and $\bar{\Psi}^c = \bar{\Psi} \hat{C}^{-1}$. $\Psi_M = \Psi + \Psi^c$ or $\Psi = \frac{1}{2}(\Psi_M - \Psi^c)$

Prove $\bar{\Psi}^c \gamma^\mu \Psi^c = \bar{\Psi} \gamma^\mu \Psi$

$$\bar{\Psi}^c \gamma^\mu \Psi^c = \bar{\Psi} \hat{C}^{-1} \gamma^\mu \hat{C} \bar{\Psi}^T = \bar{\Psi} \gamma^{\mu T} \bar{\Psi}^T$$

$$= \bar{\Psi} [\hat{C} \gamma^\mu \hat{C}^{-1}] \bar{\Psi}^T = \bar{\Psi} [\hat{C} (\gamma^\mu)^T \hat{C}^{-1}] \bar{\Psi}^T = \bar{\Psi} [\hat{C} \gamma^\mu \hat{C}^{-1}] \bar{\Psi}^T = \bar{\Psi} \gamma^\mu \bar{\Psi}^T = \bar{\Psi} \gamma^\mu \Psi$$

$$\Psi_M: \psi^c \equiv \hat{C} \bar{\psi}^T = \psi_M, \quad \hat{C} = -i(\gamma_0 \gamma^2)^T \text{ and } \hat{C}^{-1} = \hat{C}^T$$

$$\hat{C}^T = -\hat{C}, \quad \hat{C}^{-1} \gamma^\mu \hat{C} = -\gamma^\mu$$

Let us consider Dirac field ψ . Then, introduce $\psi^c \equiv \hat{C} \bar{\psi}^T$ and $\bar{\psi}^c \equiv \bar{\psi} \hat{C}^{-1}$

Prove $\psi^c \equiv \hat{C} \bar{\psi}^T = \psi$, $\bar{\psi}^c \equiv \bar{\psi} \hat{C}^{-1} = \bar{\psi}$

$$= \hat{C} [\hat{C} \psi] = \hat{C} [\hat{C} (\psi \gamma_0)^T] = \hat{C} [\hat{C} (\psi \gamma_0)^T \gamma_0] = \hat{C} [\hat{C} \psi \gamma_0^T \gamma_0] = \hat{C} \psi \gamma_0^T \gamma_0 = \hat{C} \psi = \psi$$

$$= \hat{C} [\hat{C} \bar{\psi}] = \hat{C} [\hat{C} (\bar{\psi} \gamma_0)^T] = \hat{C} [\hat{C} (\bar{\psi} \gamma_0)^T \gamma_0] = \hat{C} [\hat{C} \bar{\psi} \gamma_0^T \gamma_0] = \hat{C} \bar{\psi} \gamma_0^T \gamma_0 = \hat{C} \bar{\psi} = \bar{\psi}$$

$$= N(\psi(x)\bar{\psi}(y)) - \{\bar{\psi}^+(y)\psi(x)\}. \quad \text{I. e.,}$$

Compare with complex KG field:

$\mathcal{U} \Rightarrow$ two real KG field:

$$\varphi_1 = \frac{\varphi + \varphi^*}{2}, \quad \varphi_2 = \frac{\varphi - \varphi^*}{2i}$$

It easy to prove that $\int_M \bar{\psi} \gamma^M \psi = 0$

$$= N(\psi(x)\bar{\psi}(y)) - \{\bar{\psi}^+(y)\psi(x)\}. \text{ I. e.,}$$

Compare with complex KG field:

$\mathcal{U} \Rightarrow$ two real KG field:

$$\varphi_1 = \frac{\varphi + \varphi^*}{2}, \quad \varphi_2 = \frac{\varphi - \varphi^*}{2i}$$

It easy to prove that $j^\mu = \bar{\psi} \gamma^\mu \psi$

$$= N(\Psi(x) \Psi(y)) - \{\Psi(y), \Psi(x)\} \quad \text{I. e.,}$$

Compare with complex KG field:

$\mathcal{L} \Rightarrow$ two real KG field.

$$\varphi_1 = \frac{\varphi + \varphi^*}{2}, \quad \varphi_2 = \frac{\varphi - \varphi^*}{2i}$$

It easy to prove that $j^M = \sqrt{2} \frac{\varphi^M - \varphi^{M*}}{2i}$

$$= N(\Psi(x) \Psi(y)) - \{\Psi(y) \Psi(x)\} \quad \text{I. e.,}$$

$$T \Psi(x) \Psi(y) = \dots$$

Compare with complex KG field:

$\mathcal{L} \Rightarrow$ two real KG field.

$$\varphi_1 = \frac{\varphi + \varphi^*}{2}, \quad \varphi_2 = \frac{\varphi - \varphi^*}{2i}$$

It is easy to prove that $j^M = \sqrt{\frac{2}{M}} \gamma^M \psi = 0$