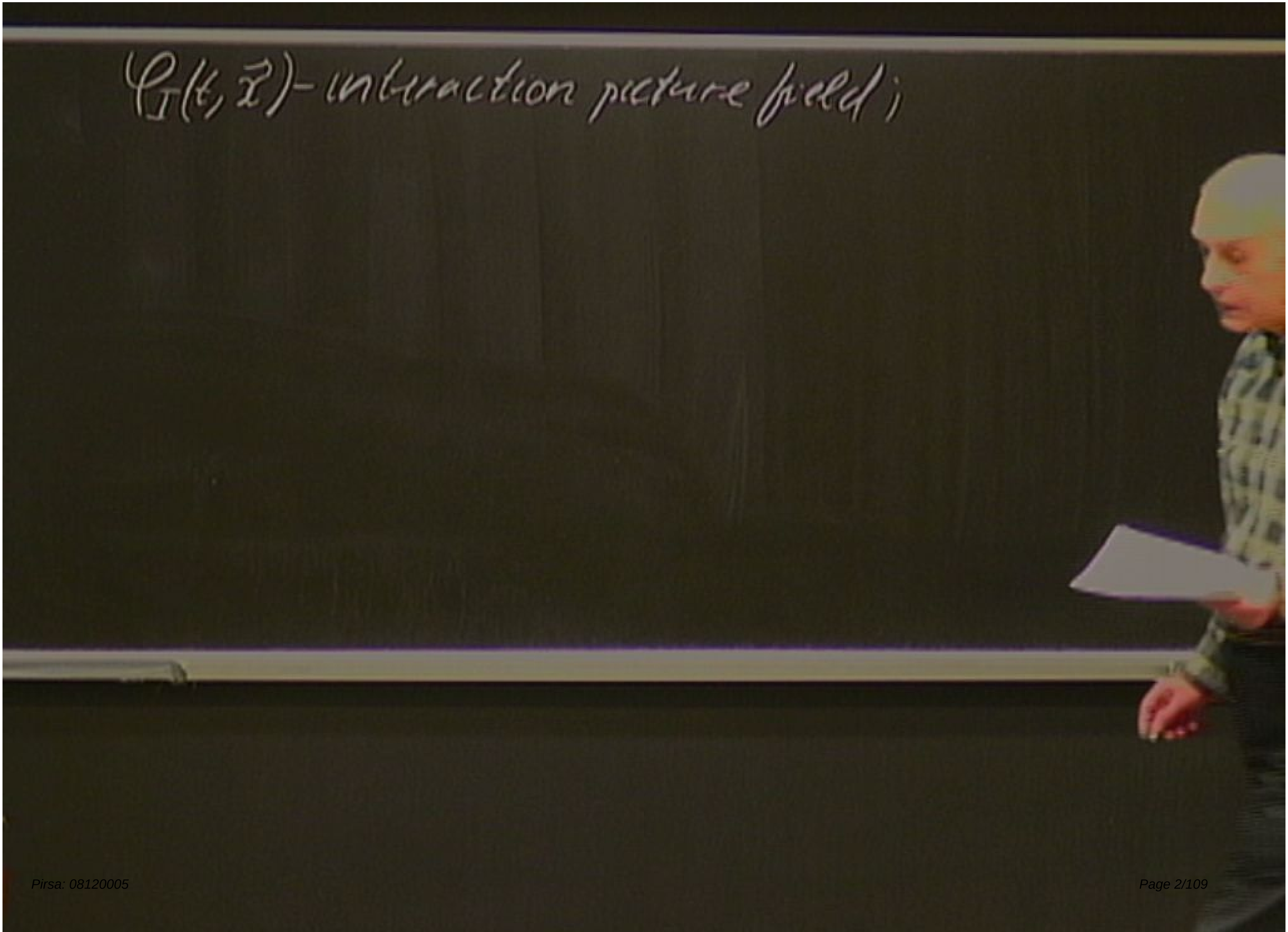
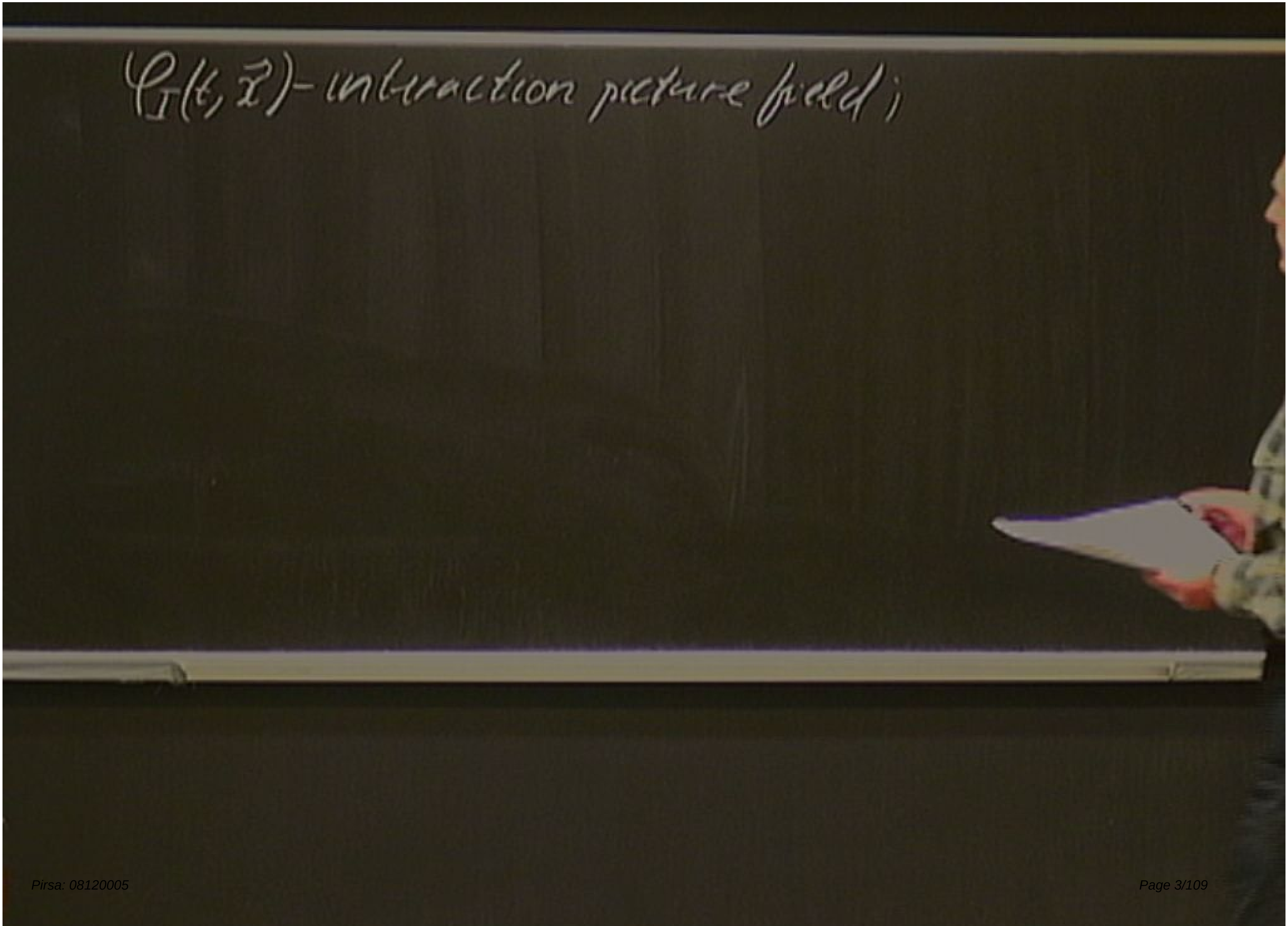
Title: Quantum Field Theory 1 - Lecture 13A

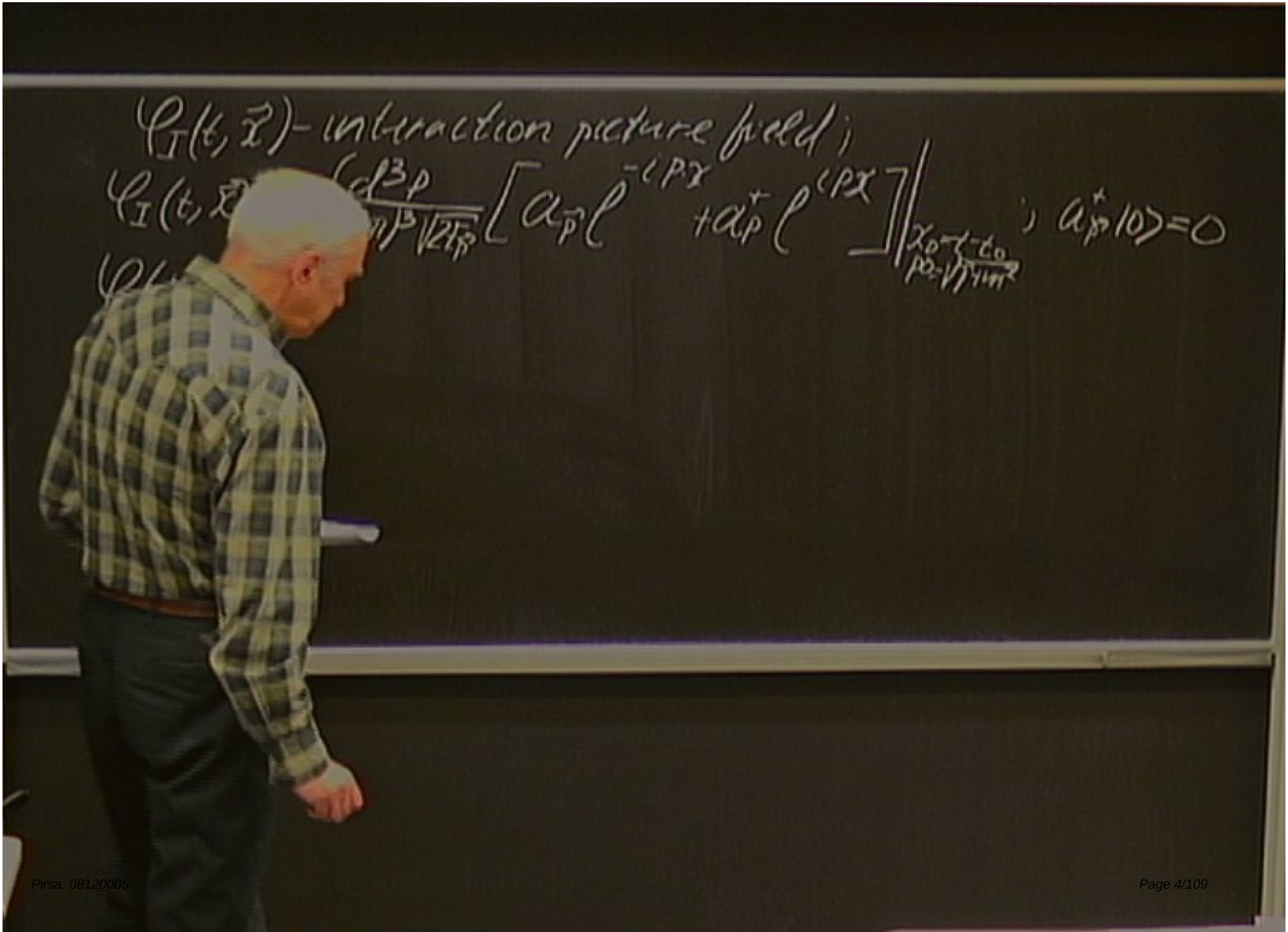
Date: Dec 03, 2008 10:30 AM

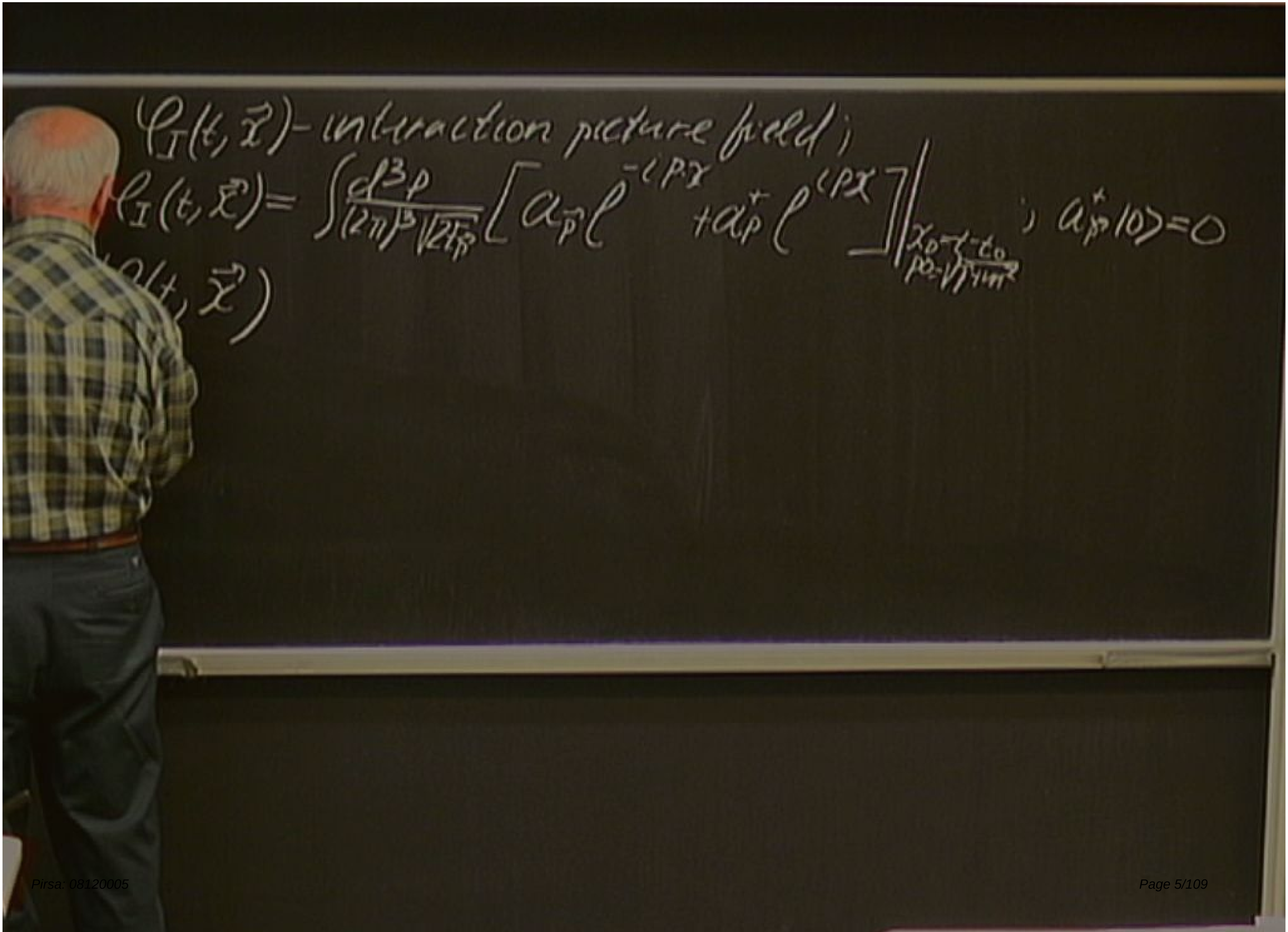
URL: http://pirsa.org/08120005

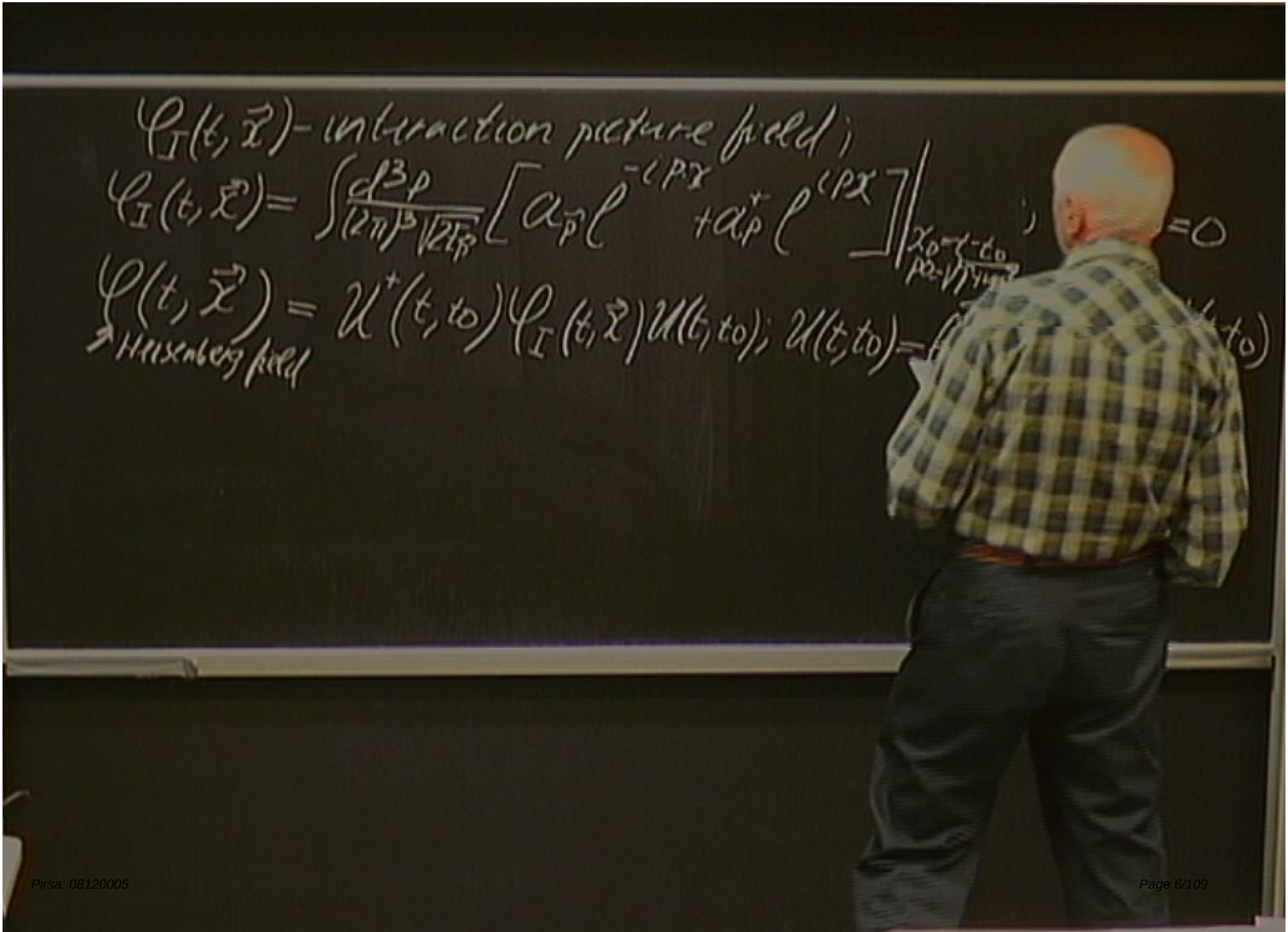
Abstract: Quantum Field Theory I course taught by Volodya Miransky of the University of Western Ontario

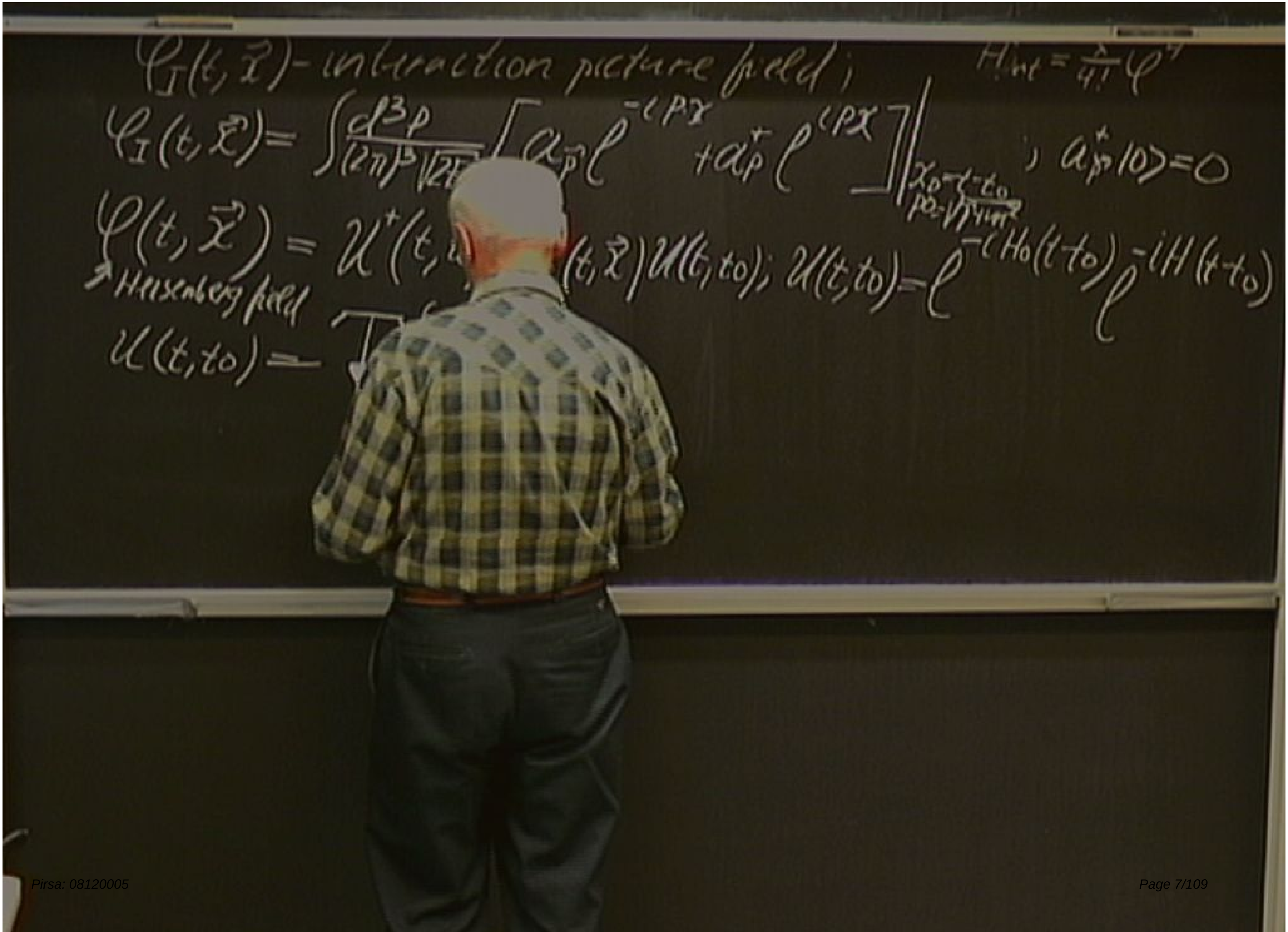

$\varphi_I(t, \vec{x})$ - interaction picture field ;

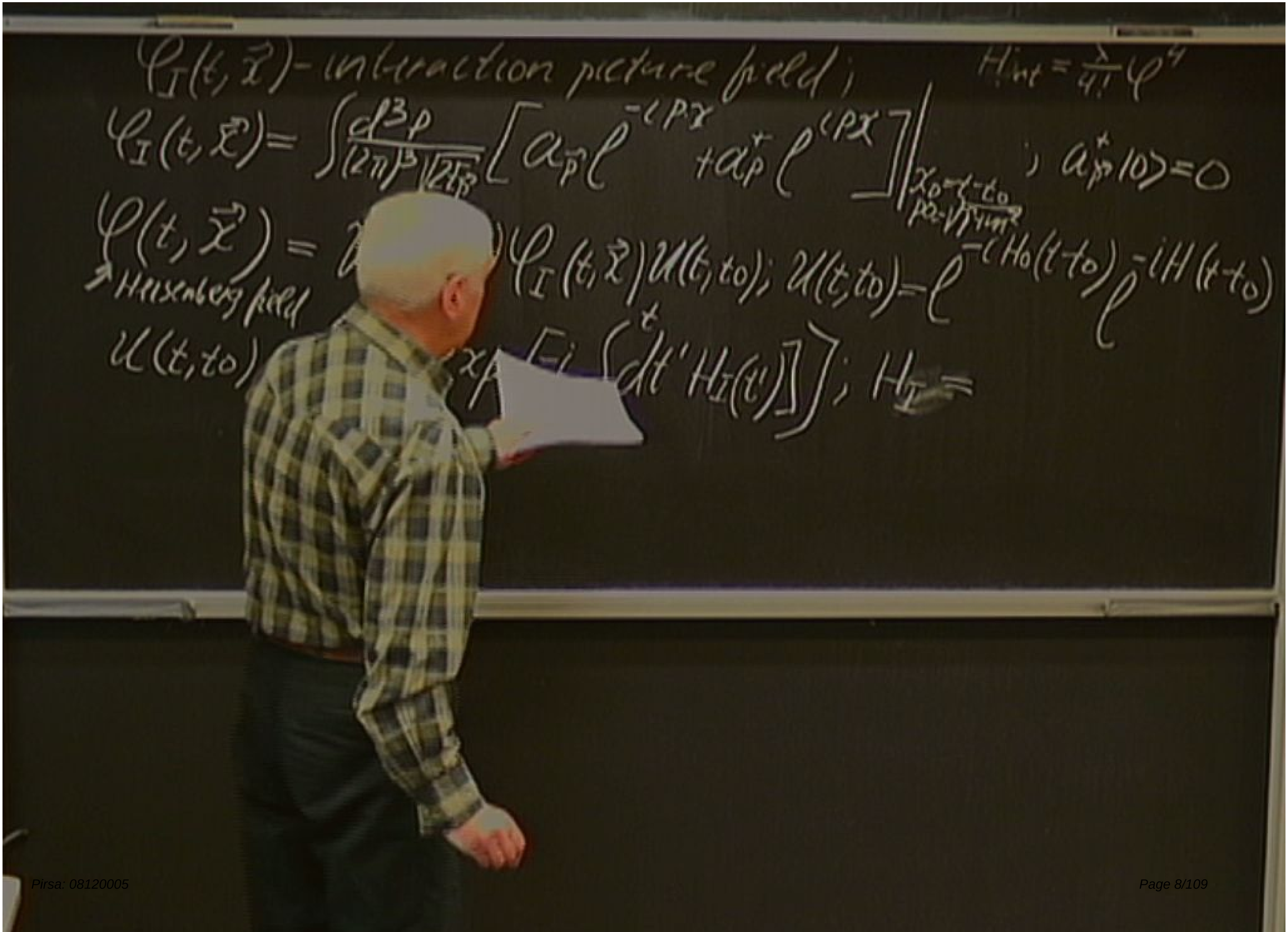

$\varphi_I(t,\vec{x})$ - interaction picture field;

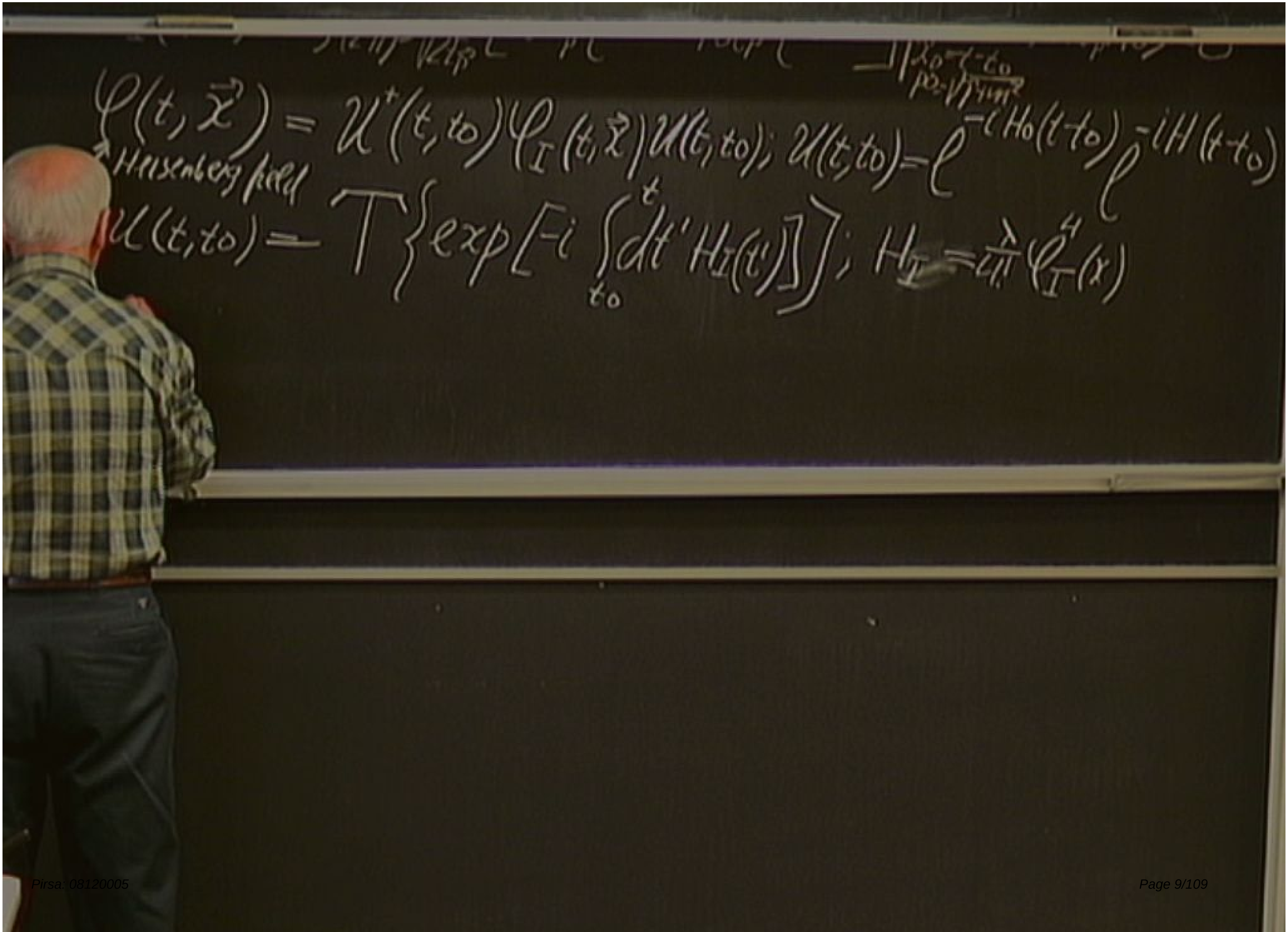

φ_I(t, x⃗) - interaction picture field;
[a_p e^{-ip·x} + a_p^† e^{ipx}] | x_0 = t - t_0, p_0 = √(p² + m²) ; a_p|0⟩ = 0

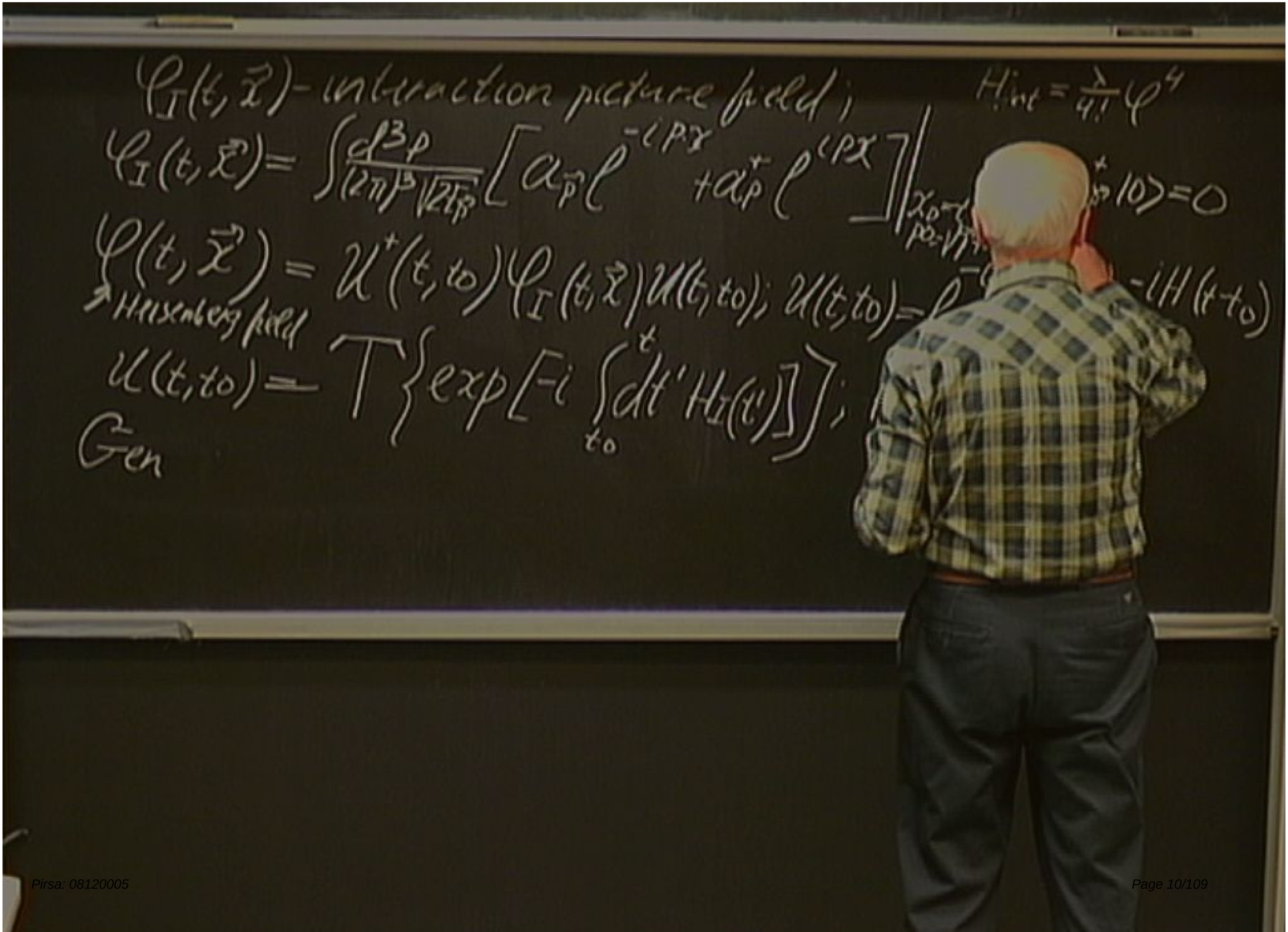

$\varphi_I(t,\vec{x})$ - interaction picture field;
$\varphi_I(t,\vec{x}) = \int \frac{d^3p}{(2\pi)^3\sqrt{2E_p}}\left[a_{\vec{p}}\,e^{-ip\cdot x} + a^+_{\vec{p}}\,e^{ipx}\right]\Big|_{x_0=t-t_0,\ p_0=\sqrt{\vec{p}^2+m^2}}$; $a^+_{\vec{p}}|0\rangle = 0$
$\varphi(t,\vec{x})$

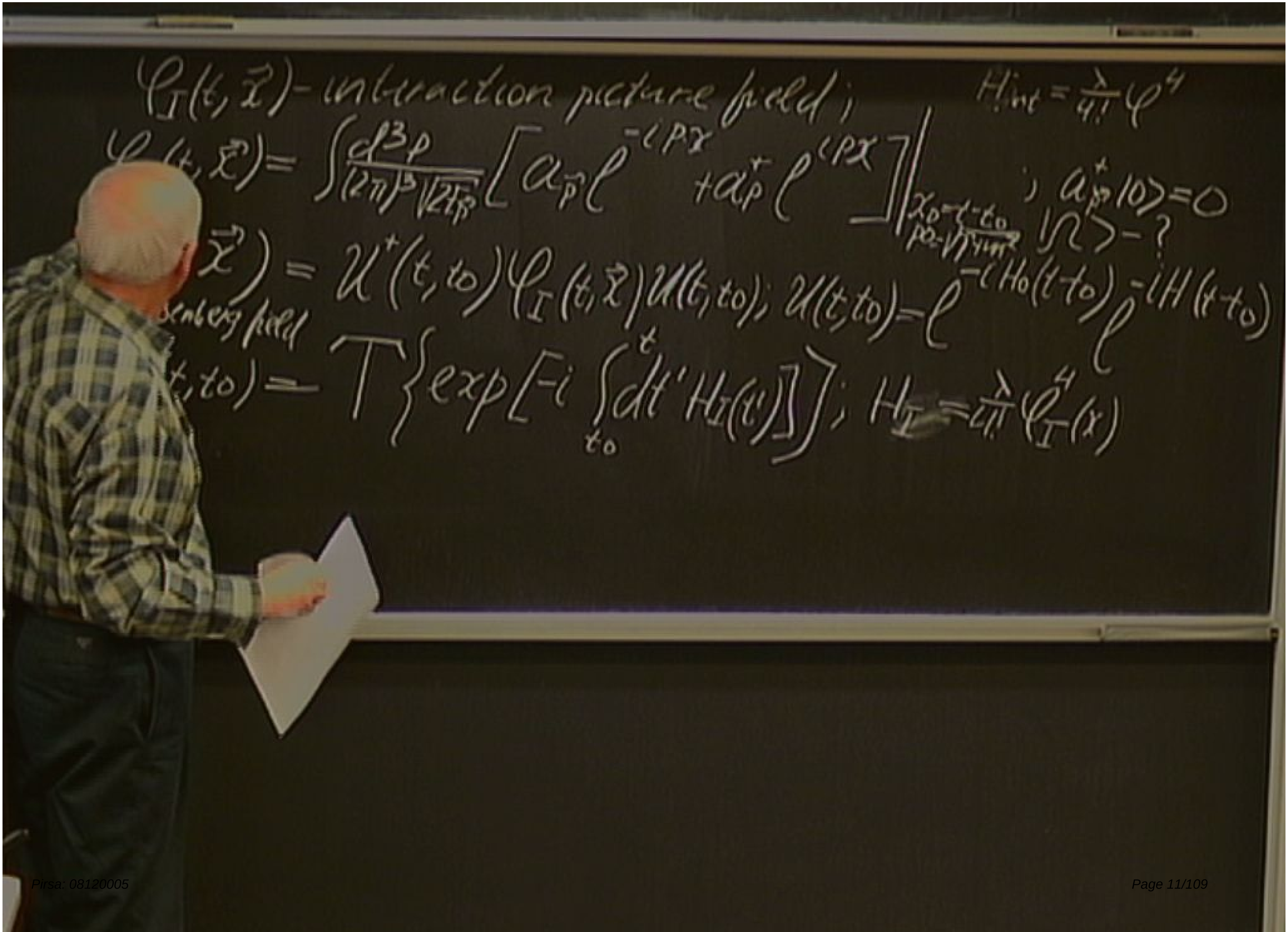

φ_I(t, x⃗) - interaction picture field;
φ_I(t, x⃗) = ∫ d³p / ((2π)³ √(2E_p)) [a_p e^{-ipx} + a_p^+ e^{ipx}] | x₀ = t - t₀
; =0
φ(t, x⃗) = U^+(t, t₀) φ_I(t, x⃗) U(t, t₀); U(t, t₀) =
→ Heisenberg field


φ_I(t, x) - interaction picture field ;
H_int = λ/4! φ^4
; a_p |0> = 0
Heisenberg field
U(t,t0) =


φ_I(t, x) - interaction picture field;
H_int = λ/4! φ^4
φ_I(t, x) = ∫ d^3p/((2π)^3 √(2E_p)) [a_p e^{-ipx} + a_p^+ e^{ipx}] | x_0 = t - t_0 ; a_p |0> = 0
φ(t, x) = U^{-1}(t, t_0) φ_I(t, x) U(t, t_0); U(t, t_0) = e^{-iH_0(t-t_0)} e^{-iH(t-t_0)}
Heisenberg field
U(t, t_0)
dt' H_I(t')]]; H_I =


$\varphi(t,\vec{x}) = U^{+}(t,t_0)\varphi_I(t,\vec{x})U(t,t_0);\ U(t,t_0) = e^{-iH_0(t-t_0)}e^{-iH(t-t_0)}$
Heisenberg field
$U(t,t_0) = T\{\exp[-i\int_{t_0}^{t}dt' H_I(t')]\};\ H_I = \frac{\lambda}{4!}\varphi_I^4(x)$


$\varphi_I(t,\vec{x})$ - interaction picture field;
$H_{int} = \frac{\lambda}{4!}\varphi^4$
$\varphi_I(t,\vec{x}) = \int \frac{d^3p}{(2\pi)^3\sqrt{2E_p}}\left[a_{\vec{p}}\, e^{-ipx} + a^+_{\vec{p}}\, e^{ipx}\right]\Big|_{x_0=t}$
$|0\rangle = 0$
$\varphi(t,\vec{x}) = U^+(t,t_0)\varphi_I(t,\vec{x})U(t,t_0);\ U(t,t_0) =$
$-iH(t-t_0)$
Heisenberg field
$U(t,t_0) = T\{\exp[-i\int_{t_0}^{t} dt' H_I(t')]\};$
Gen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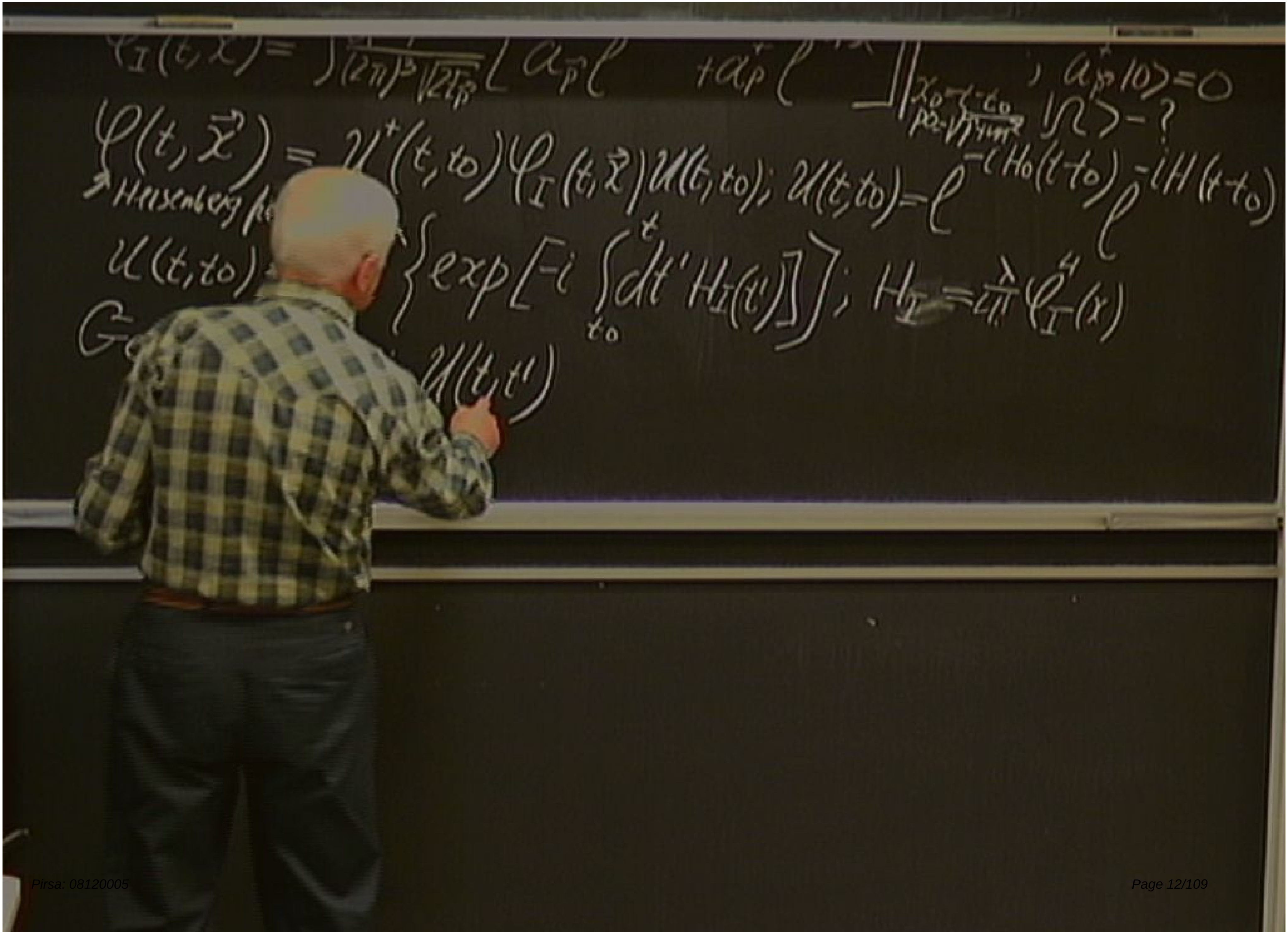
$\varphi_I(t,\vec{x}) = \int \frac{d^3p}{(2\pi)^3\sqrt{2E_p}}\left[a_{\vec p}\, e^{-ipx} + a^+_{\vec p}\, e^{ipx}\right]\Big|_{x_0=t-t_0,\ p_0=\sqrt{p^2+m^2}}$; $a_{\vec p}|0\rangle = 0$
$|\Omega\rangle$ - ?
$\vec{x}) = U^+(t,t_0)\,\varphi_I(t,\vec{x})\,U(t,t_0)$; $U(t,t_0) = e^{-iH_0(t-t_0)}\, e^{-iH(t-t_0)}$
senberg field
$t,t_0) = T\left\{\exp\left[-i\int_{t_0}^{t} dt'\, H_I(t')\right]\right\}$; $H_I = \frac{\lambda}{4!}\varphi_I^4(x)$

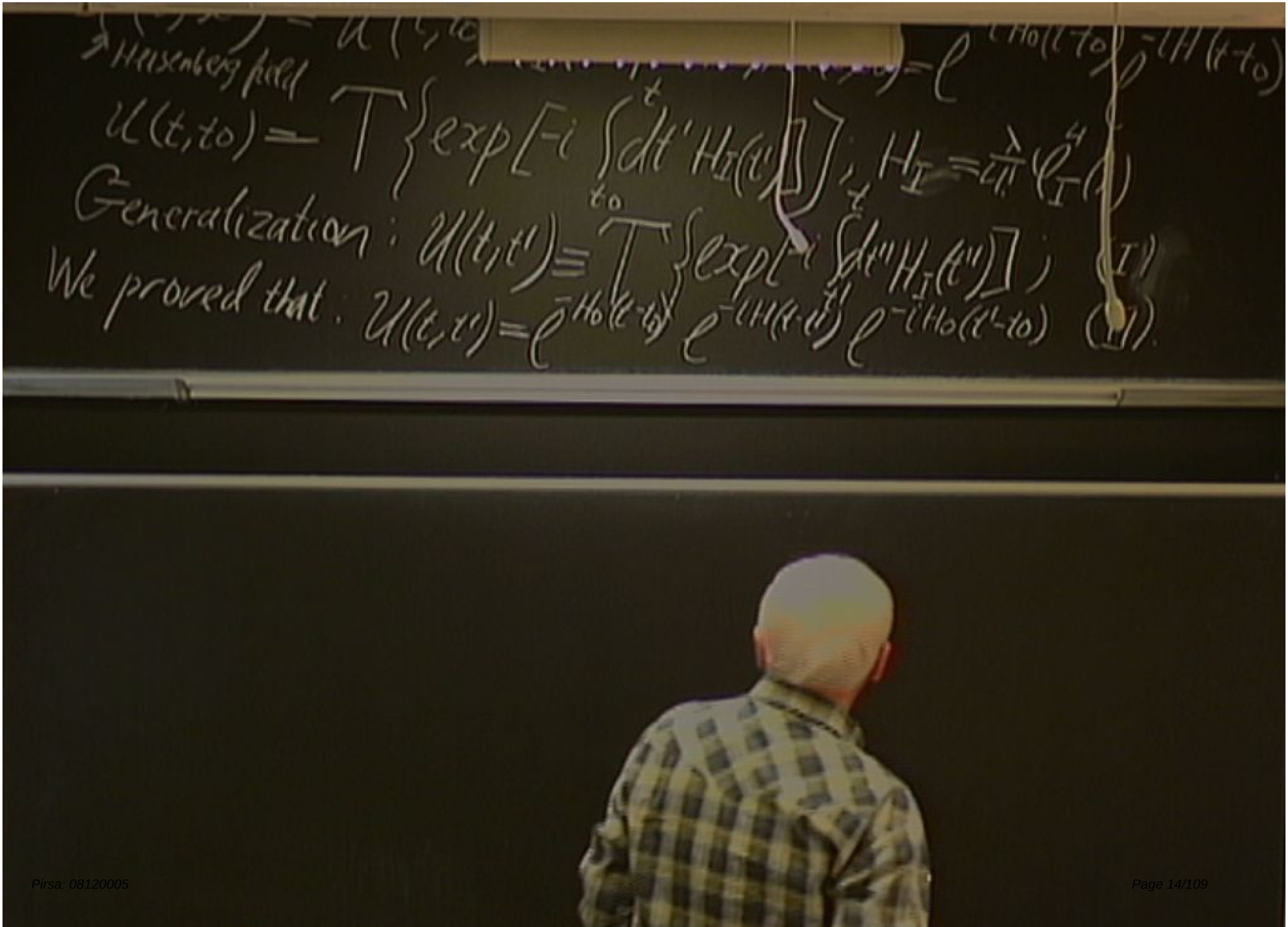

Heisenberg fi

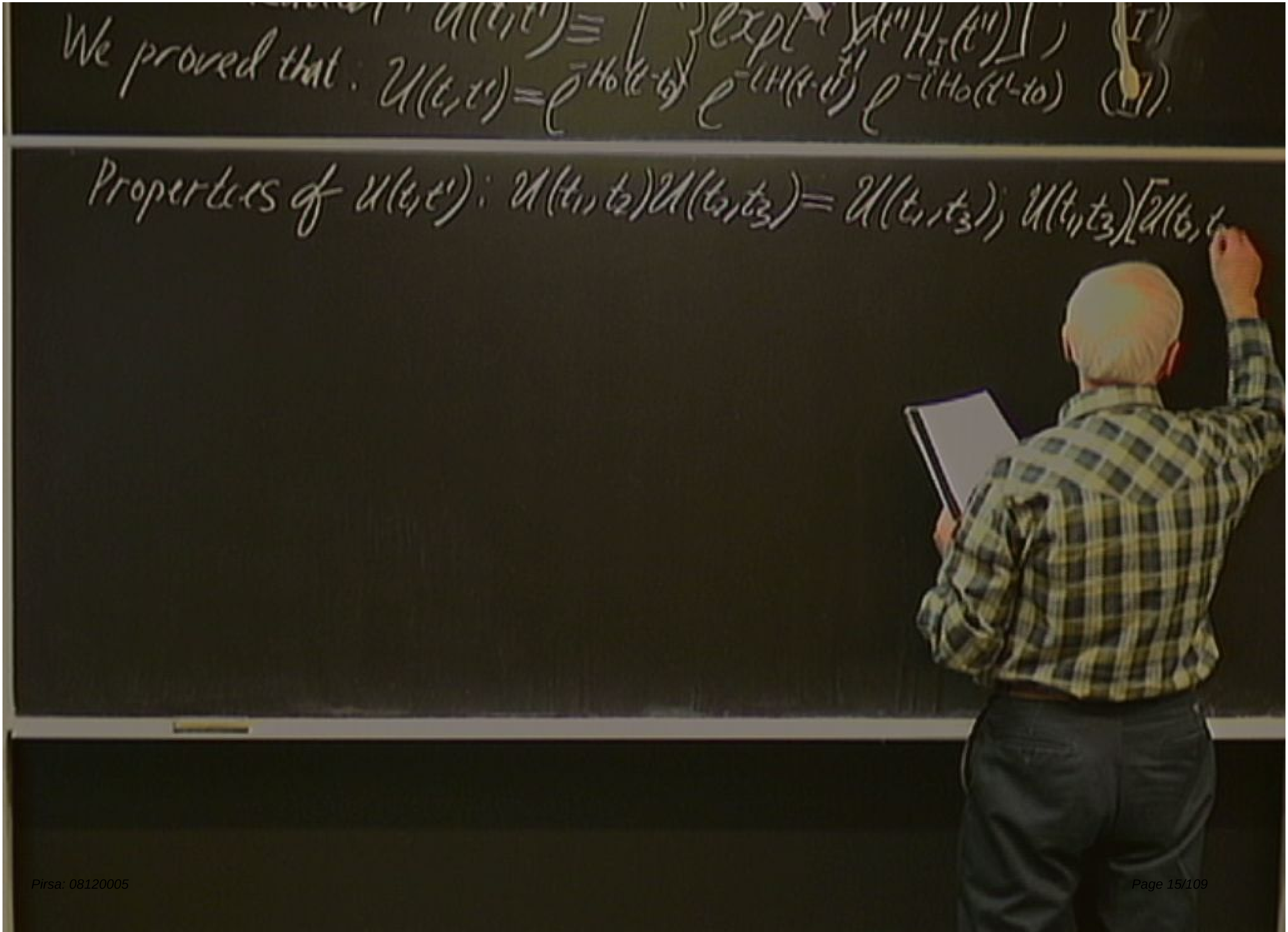
$$\varphi(t,\vec{x}) = \mathcal{U}^{+}(t,t_0)\,\varphi_I(t,\vec{x})\,\mathcal{U}(t,t_0);\quad \mathcal{U}(t,t_0) = e^{-iH_0(t-t_0)}\,e^{-iH(t-t_0)}$$

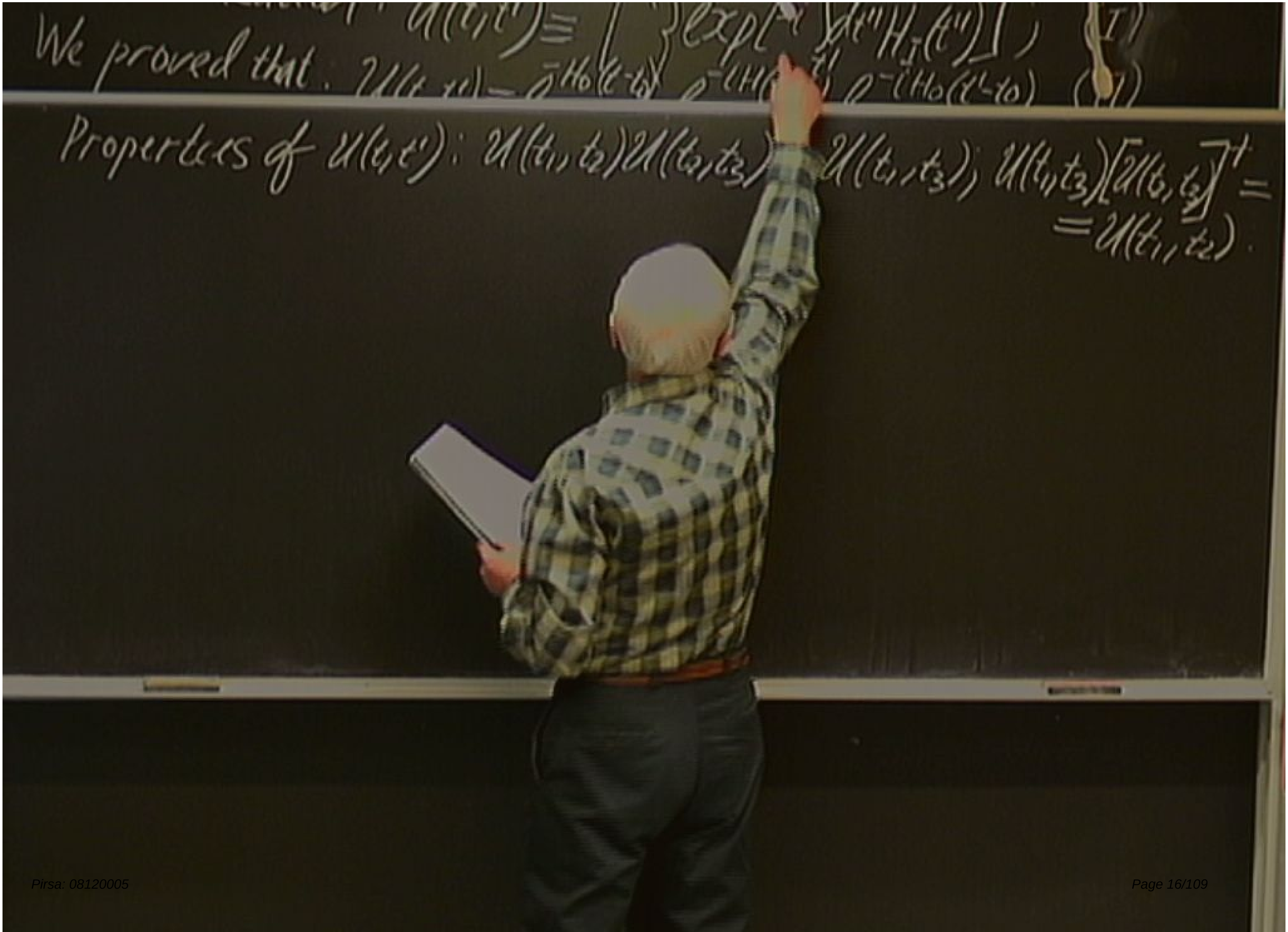
→ Heisenberg field

$$\mathcal{U}(t,t_0) = T\left\{\exp\left[-i\int_{t_0}^{t} dt'\,H_I(t')\right]\right\};\quad H_I = \frac{\lambda}{4!}\varphi_I^4(x)$$

Generalization: $\mathcal{U}(t,t') = T\left\{\exp\left[-i\int_{t'}^{t} dt''\,H_I(t'')\right]\right\};$


→ Heisenberg field
U(t,t0) = T{exp[-i ∫ dt' H_I(t')]}; H_I = λ/4! φ_I^4(t)
Generalization: U(t,t') ≡ T{exp[-i ∫ dt'' H_I(t'')]} ; (I)
We proved that: U(t,t') = e^{iH0(t-t0)} e^{-iH(t-t')} e^{-iH0(t'-t0)} (II).


We proved that: U(t,t')=e^{-iH0(t-t0)} e^{-iH(t-t')} e^{-iH0(t'-t0)} (II).
Properties of U(t,t'): U(t1,t2)U(t2,t3)=U(t1,t3); U(t1,t3)[U(t2,t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Properties of U(t,t'): U(t1,t2)U(t2,t3) = U(t1,t3); U(t1,t3)[U(t2,t3)]† =
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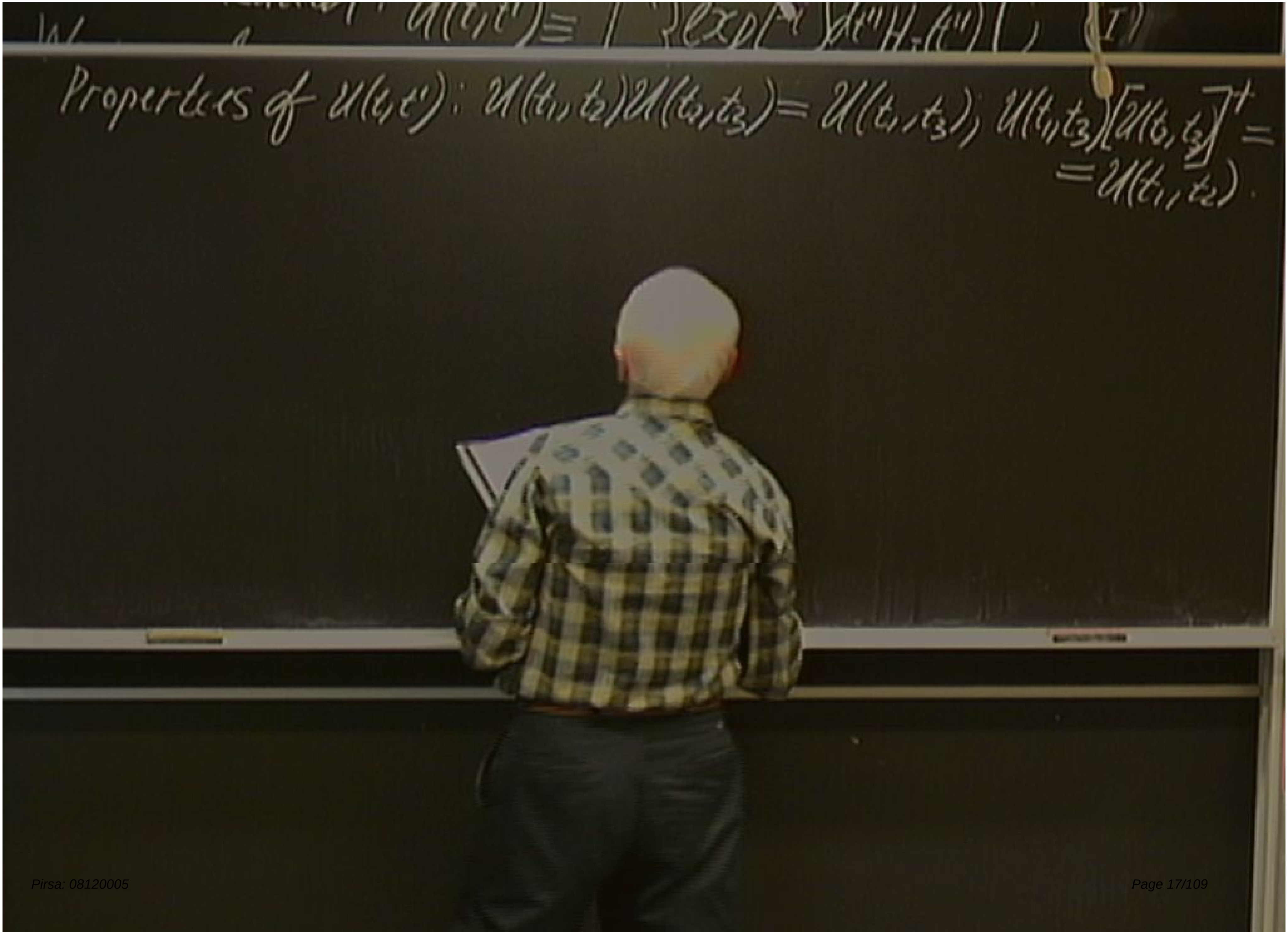

Properties of U(t,t'): U(t1,t2)U(t2,t3) = U(t1,t3); U(t1,t3)[U(t2,t3)]† =
= U(t1,t2).

$$U(t,t') \equiv T\left\{\exp\left[-i\int_{t'}^{t} dt'' H_I(t'')\right]\right\}; \quad (I)$$

We proved that: $U(t,t') = e^{iH_0(t-t_0)}\, e^{-iH(t-t')}\, e^{-iH_0(t'-t_0)}$ (II).

Properties of $U(t,t')$: $U(t_1,t_2)U(t_2,t_3) = U(t_1,t_3)$; $U(t_1,t_3)[U(t_2,t_3)]^{\dagger} =$

Proof: Substitute (II): $e^{iH_0(t_1-t_0)}\, e^{-iH(t_1-t_2)}\, e^{-iH_0(t_2-t_0)} = U(t_1,t_2)$.

$\ldots : U(t,t') \equiv T\{\exp[-i \int dt'' H_I(t'')]\}$; (I)

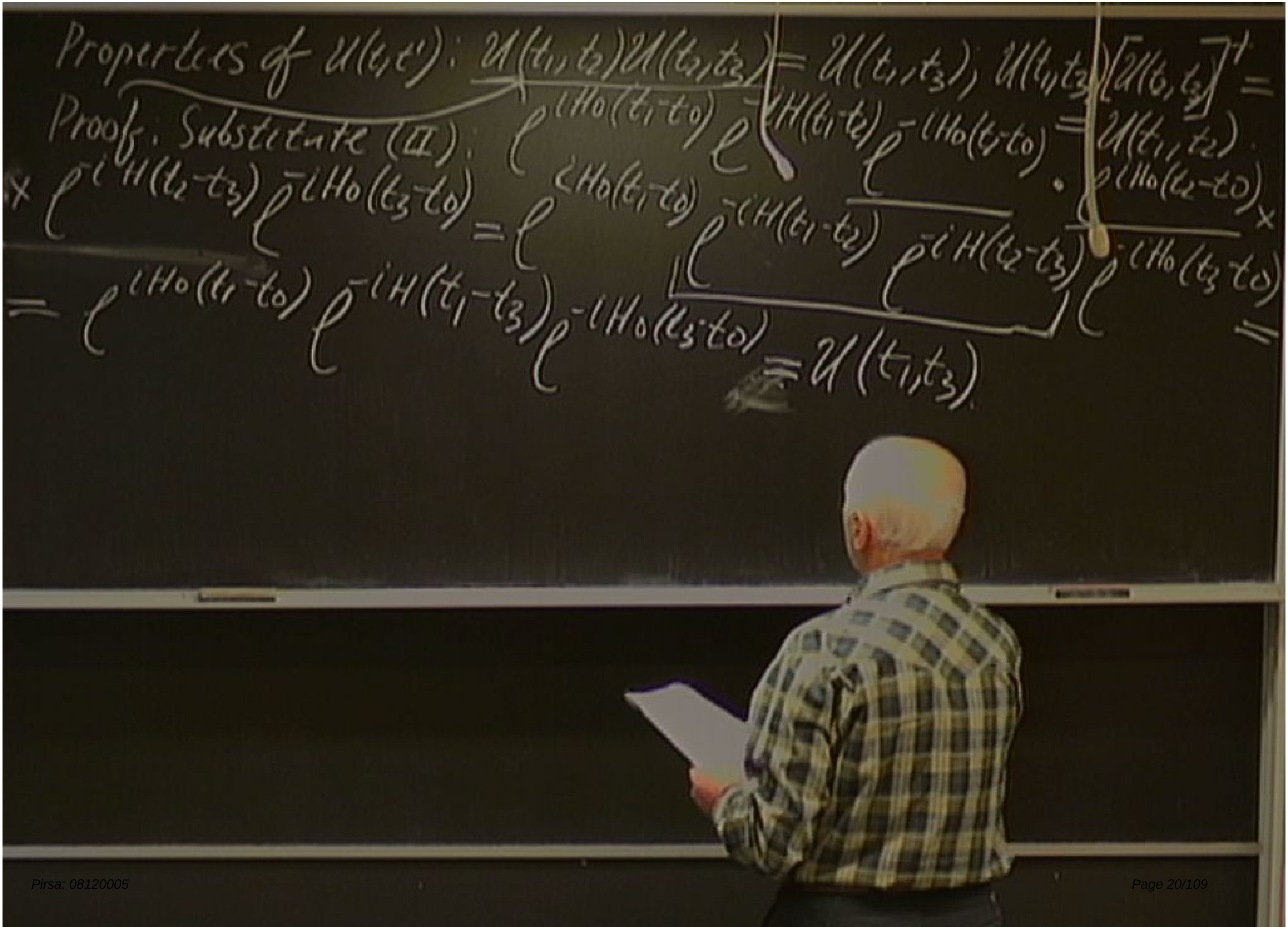
We proved that: $U(t,t') = e^{iH_0(t-t_0)}\, e^{-iH(t-t')}\, e^{-iH_0(t'-t_0)}$ (II).

Properties of $U(t,t')$: $U(t_1,t_2)U(t_2,t_3) = U(t_1,t_3)$; $U(t_1,t_3)[U(t_2,t_3)]^\dagger =$

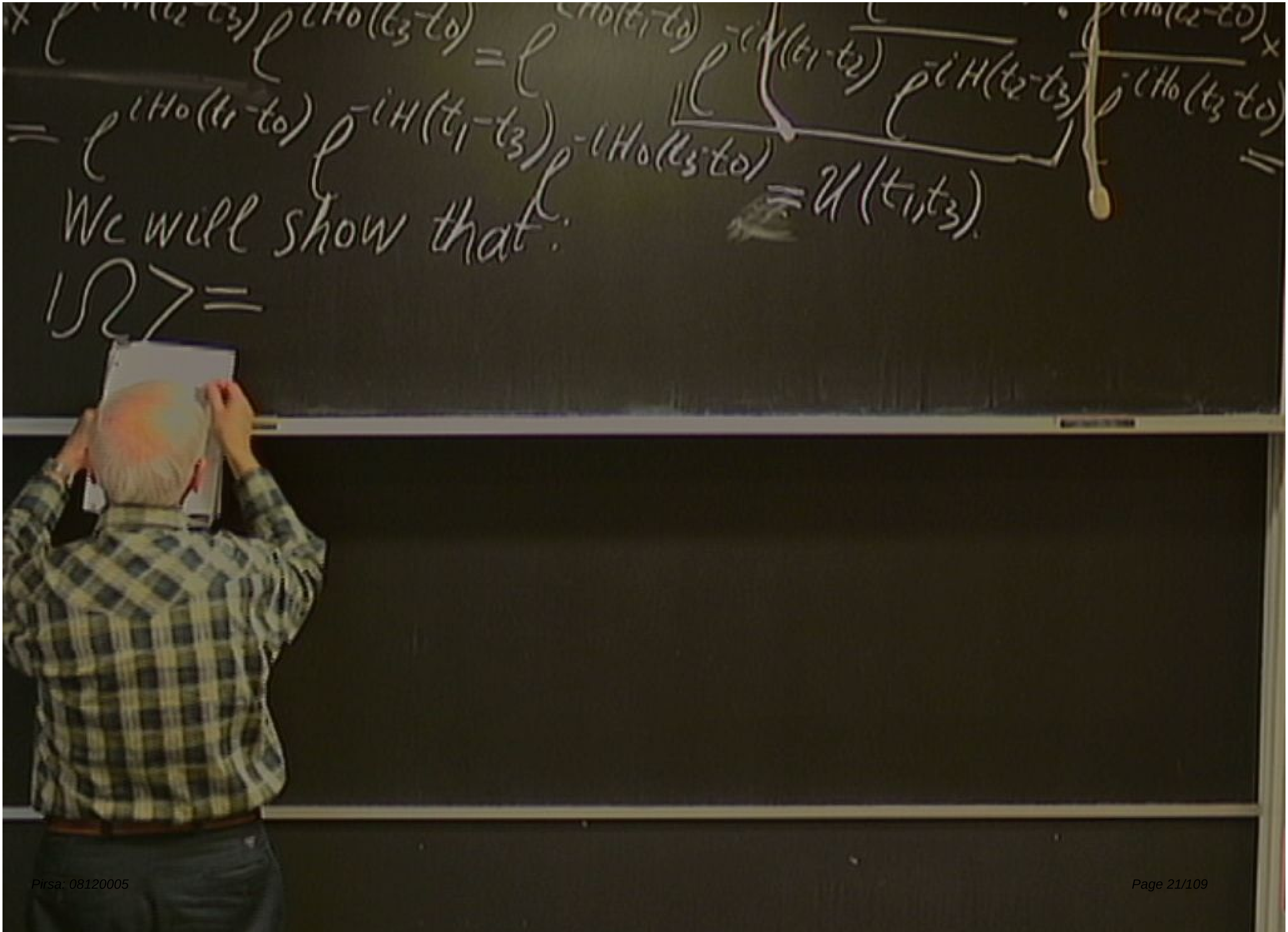
Proof: Substitute (II): $e^{iH_0(t_1-t_0)}\, e^{-iH(t_1-t_2)}\, e^{-iH_0(t_2-t_0)} = U(t_1,t_2)$.

$\cdot\, e^{iH_0(t_2-t_0)} \times$

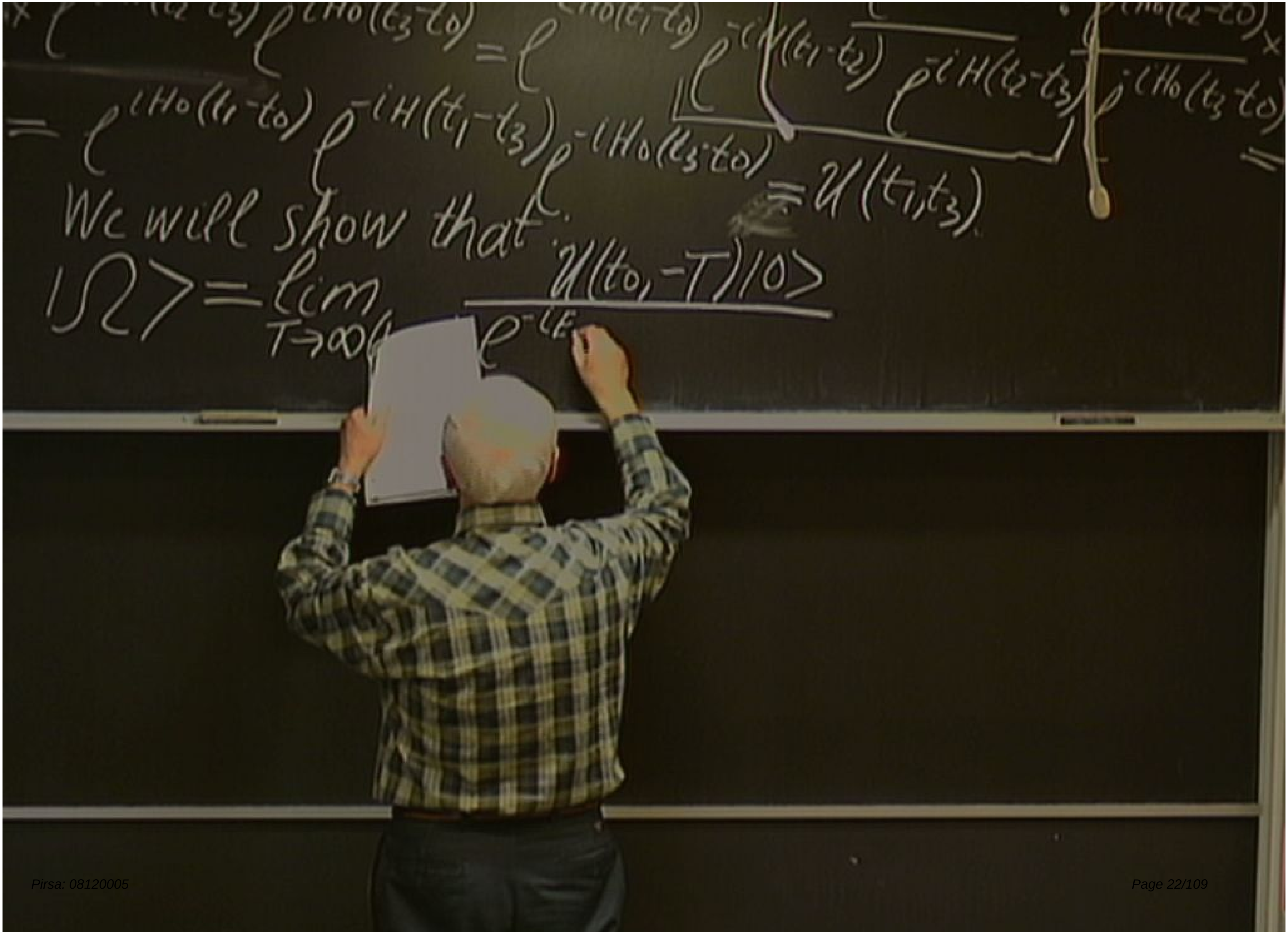
$\times\, e^{-iH(t_2-t_3)}\, e^{-iH_0(t_3-t_0)}$

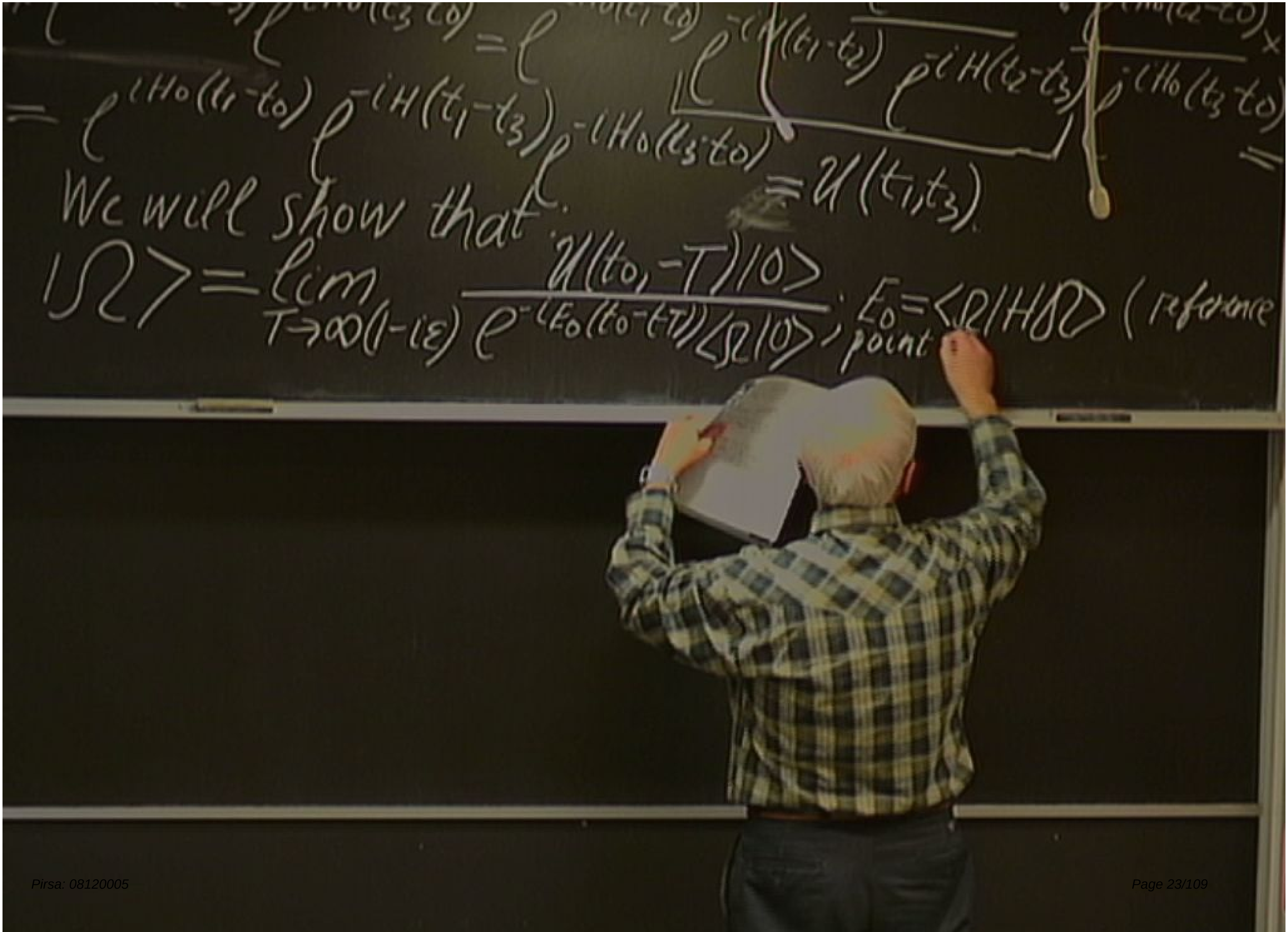


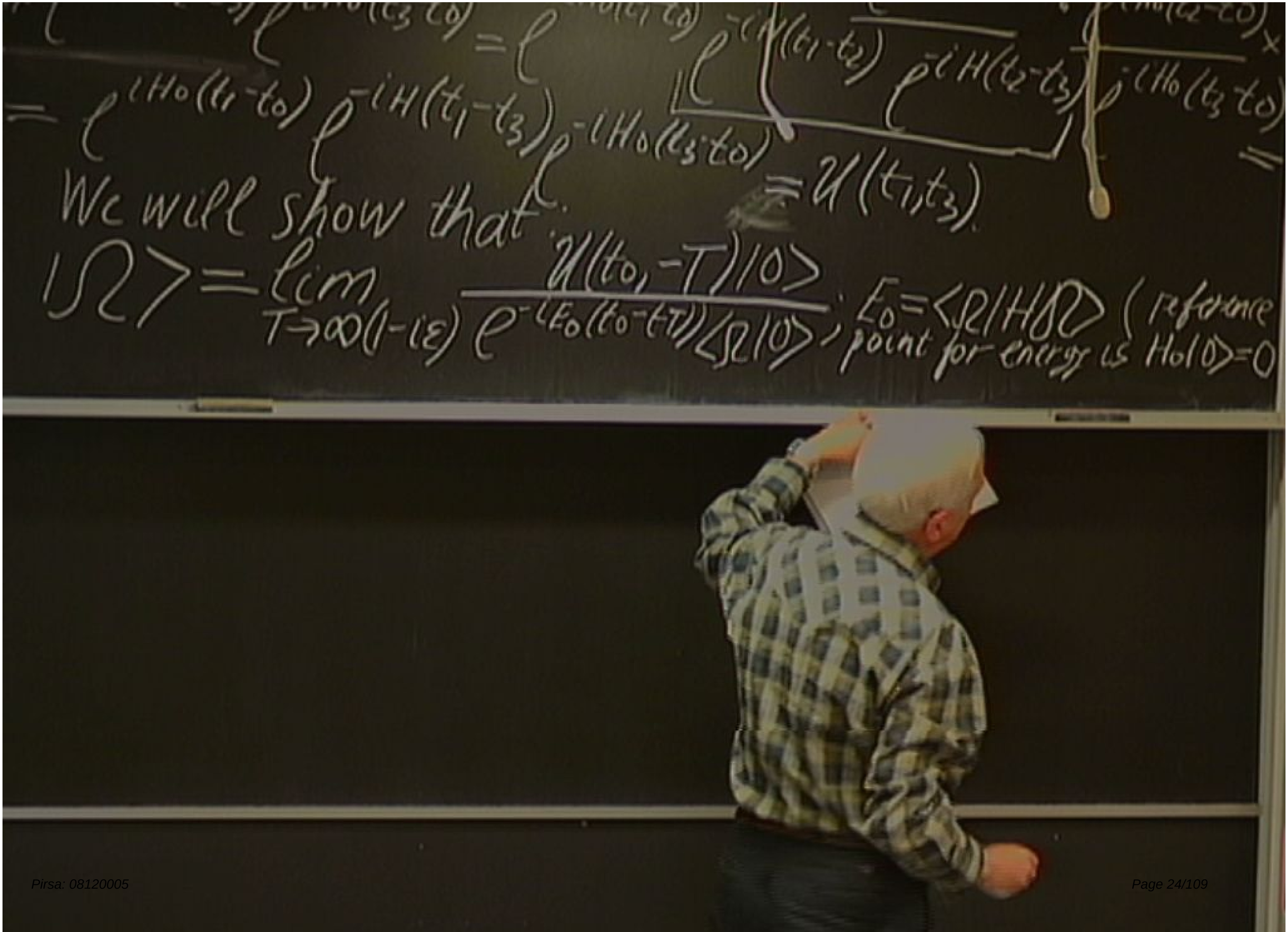
Properties of $U(t,t')$: $U(t_1,t_2)U(t_2,t_3) = U(t_1,t_3)$, $U(t_1,t_3)[U(t_2,t_3)]^{\dagger} =$
Proof: Substitute (II): $e^{iH_0(t_1-t_0)} e^{-iH(t_1-t_2)} e^{-iH_0(t_2-t_0)} = U(t_1,t_2) \cdot e^{iH_0(t_2-t_0)} \times$
$\times e^{-iH(t_2-t_3)} e^{-iH_0(t_3-t_0)} = e^{iH_0(t_1-t_0)} e^{-iH(t_1-t_2)} e^{-iH(t_2-t_3)} e^{-iH_0(t_3-t_0)} =$
$= e^{iH_0(t_1-t_0)} e^{-iH(t_1-t_3)} e^{-iH_0(t_3-t_0)} = U(t_1,t_3)$



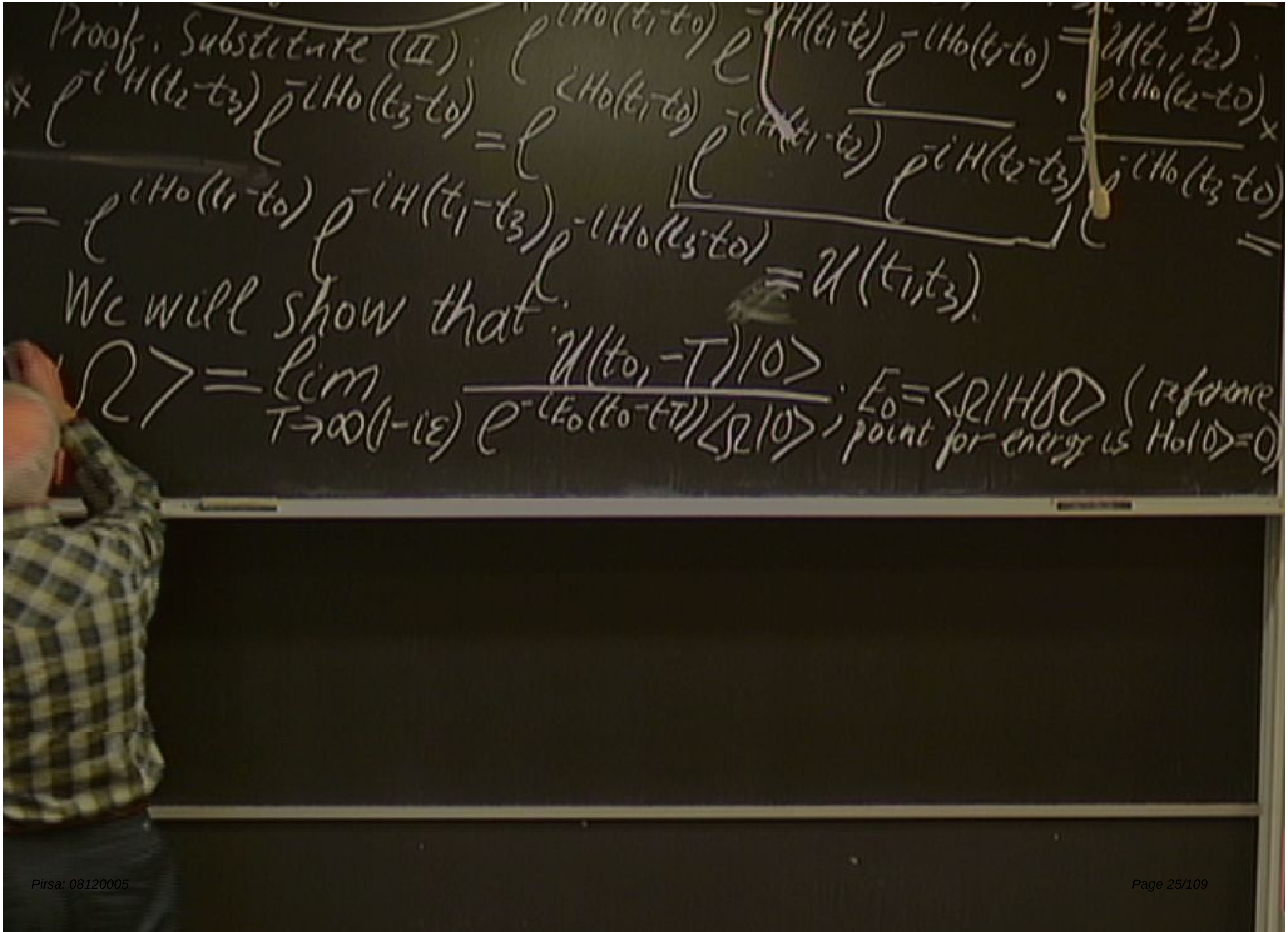
$= e^{iH_0(t_1-t_0)} e^{-iH(t_1-t_3)} e^{-iH_0(t_3-t_0)} = U(t_1,t_3)$
We will show that:
$|\Omega\rangle =$

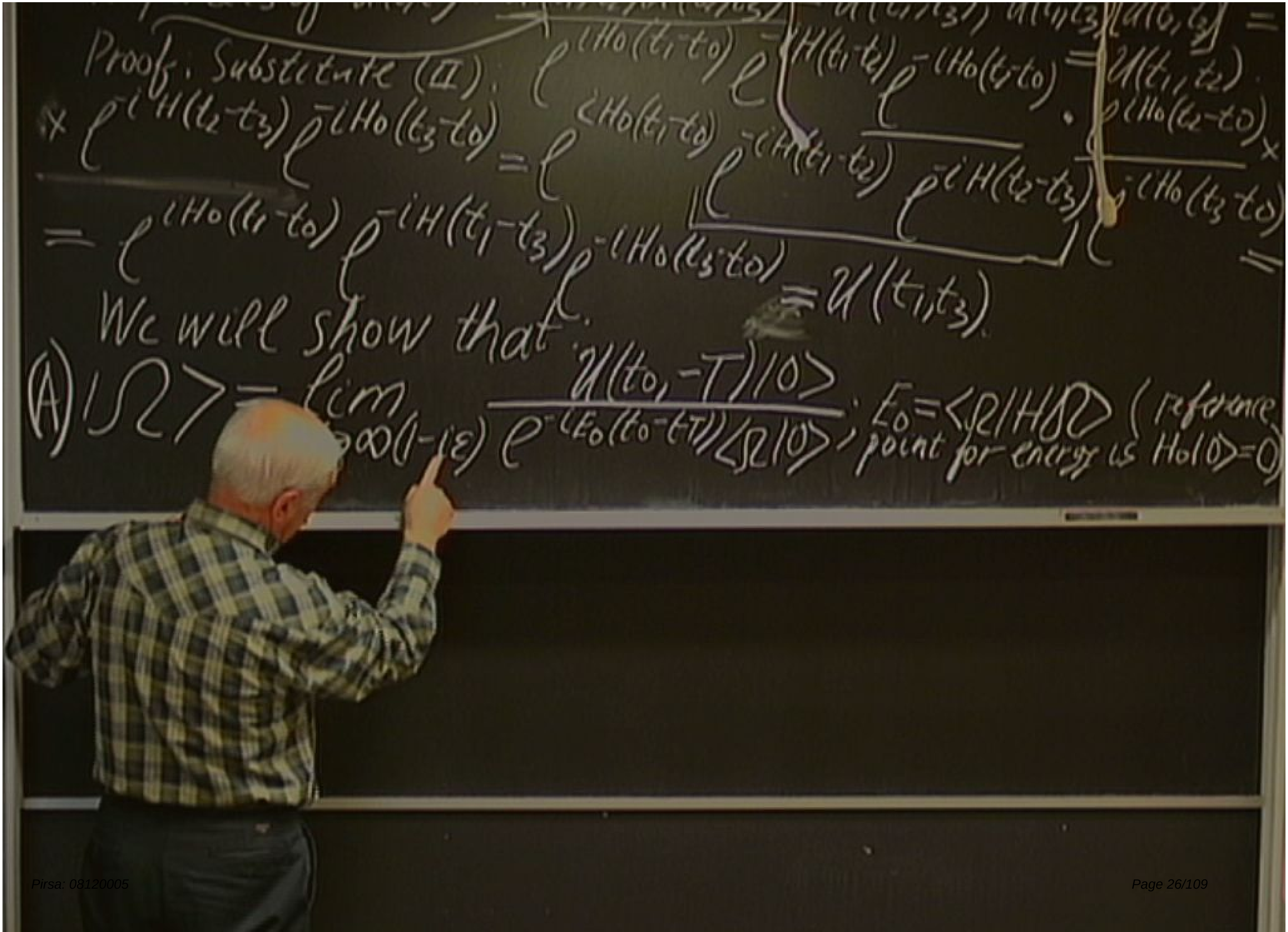

We will show that:
|Ω> = lim U(t0, -T)|0>
T→∞
= U(t1,t3)

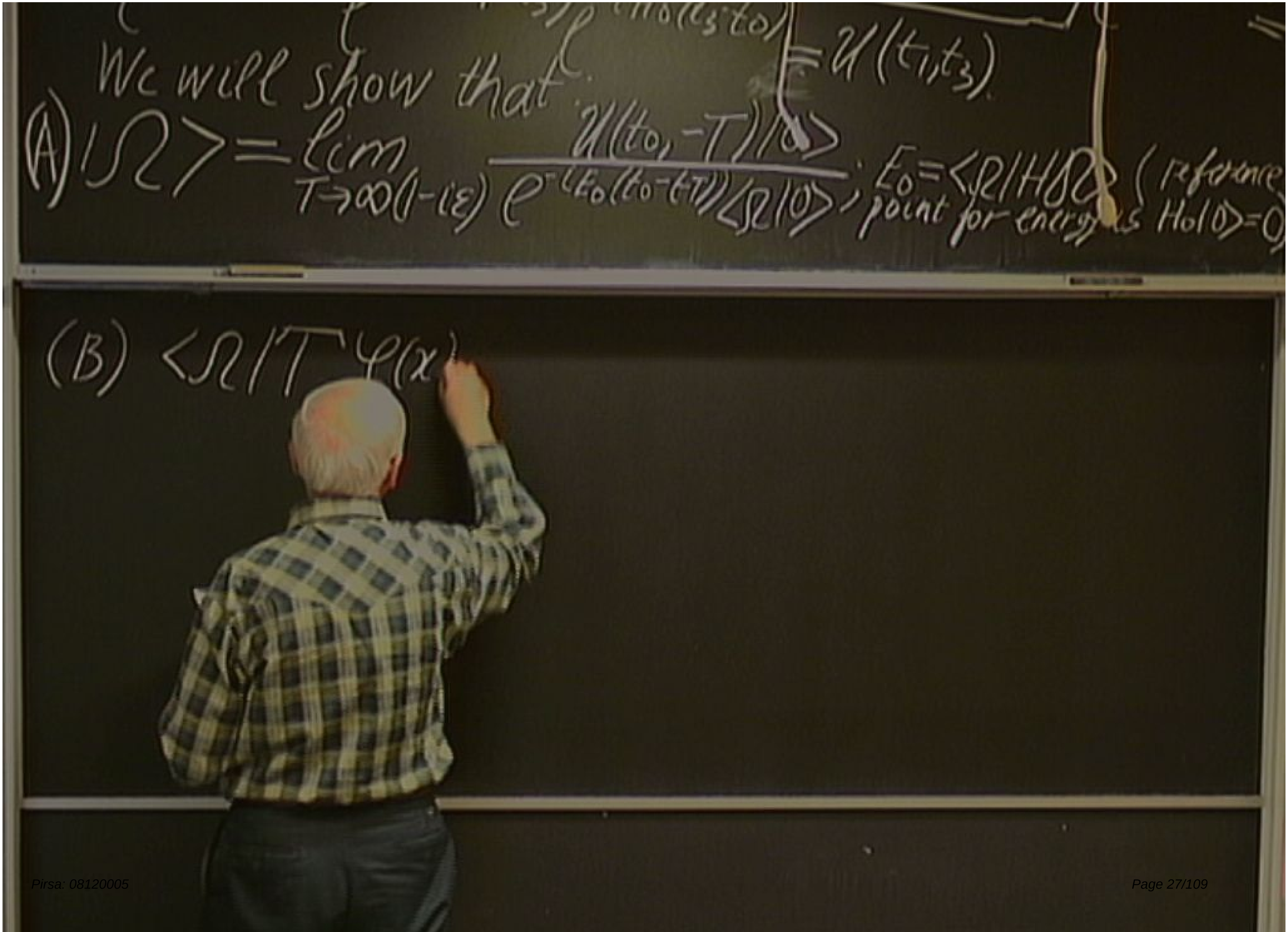

= e^{iH0(t1-t0)} e^{-iH(t1-t3)} e^{-iH0(t3-t0)} = U(t1,t3)
We will show that:
|Ω> = lim_{T→∞(1-iε)} U(t0,-T)|0> / e^{-iE0(t0-(-T))}<Ω|0> ; E0 = <Ω|H|Ω> (reference point



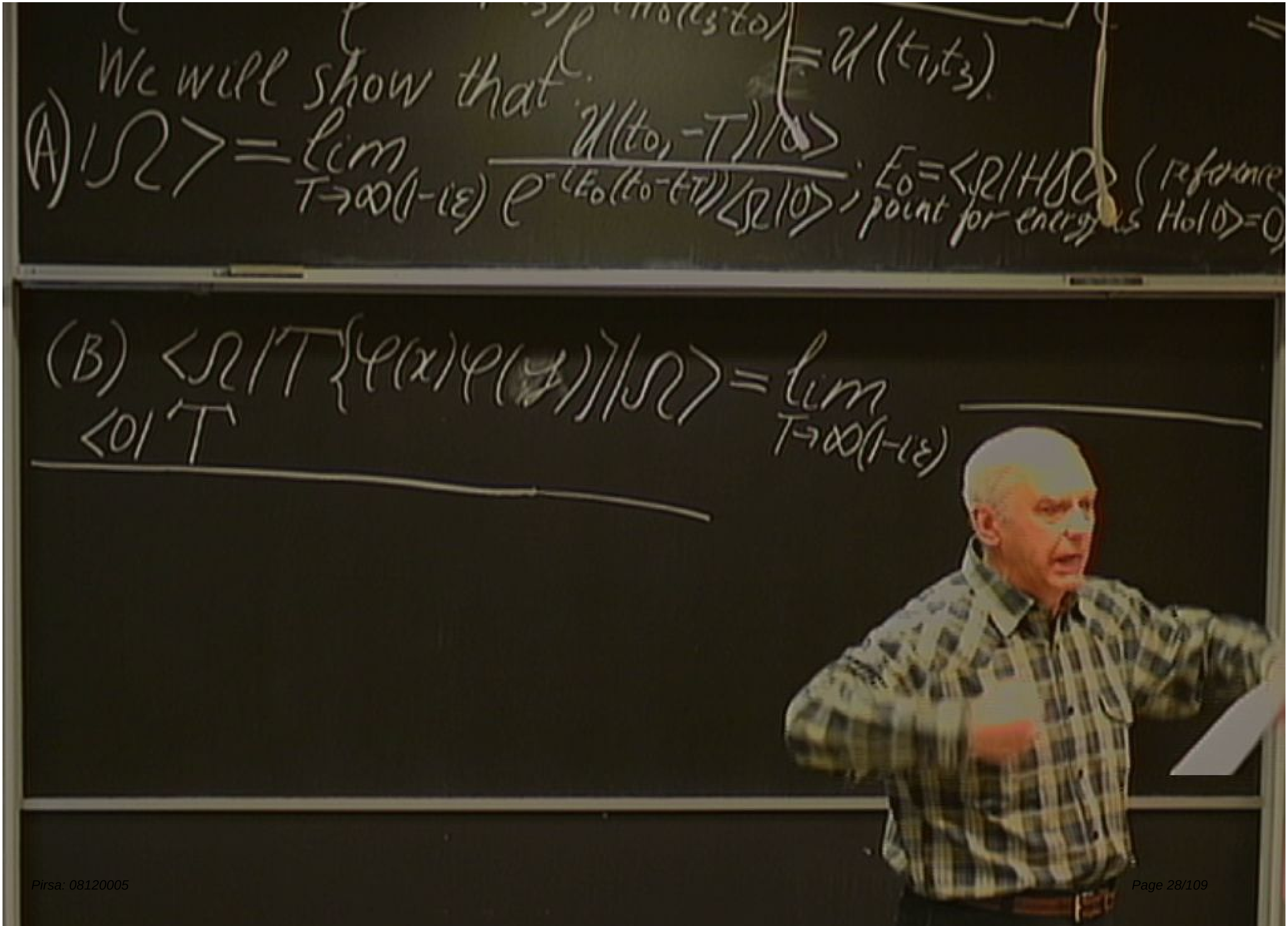
$= e^{iH_0(t_1-t_0)} e^{-iH(t_1-t_3)} e^{-iH_0(t_3-t_0)} = U(t_1,t_3)$.
We will show that:
$|\Omega\rangle = \lim_{T\to\infty(1-i\varepsilon)} \frac{U(t_0,-T)|0\rangle}{e^{-iE_0(t_0-(-T))}\langle\Omega|0\rangle}$; $E_0=\langle\Omega|H|\Omega\rangle$ (reference point for energy is $H_0|0\rangle=0$)

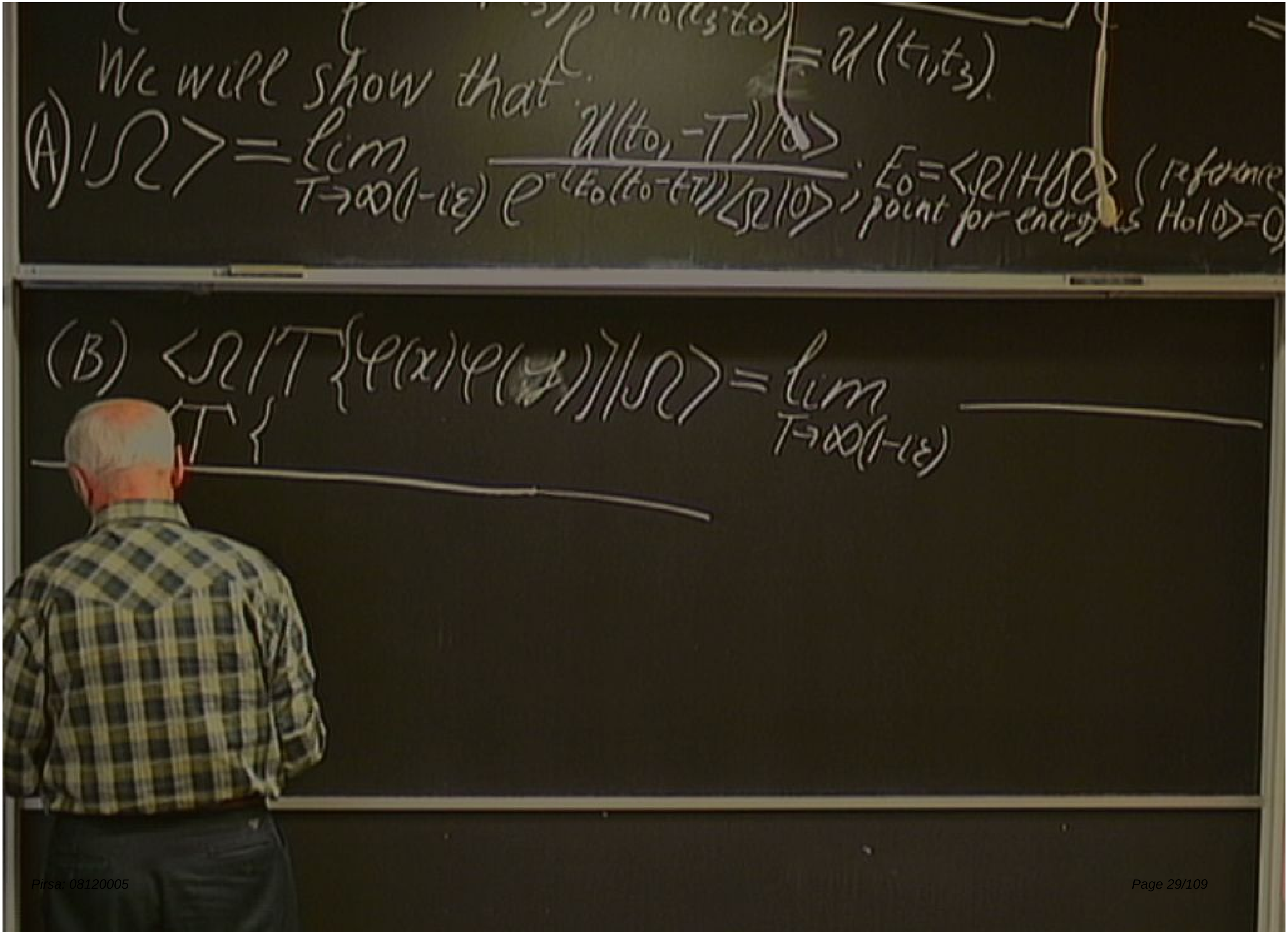

Proof. Substitute (II): e^{iH0(t1-t0)} e^{-iH(t1-t2)} e^{-iH0(t2-t0)} = U(t1,t2) · e^{iH0(t2-t0)} ×
× e^{-iH(t2-t3)} e^{-iH0(t3-t0)} = e^{iH0(t1-t0)} e^{-iH(t1-t2)} e^{-iH(t2-t3)} e^{-iH0(t3-t0)} =
= e^{iH0(t1-t0)} e^{-iH(t1-t3)} e^{-iH0(t3-t0)} = U(t1,t3).
We will show that:
|Ω> = lim_{T→∞(1-iε)} U(t0,-T)|0> / e^{-iE0(t0-(-T))}<Ω|0> ; E0 = <Ω|H|Ω> (reference point for energy is H0|0>=0)


Proof: Substitute (II): e^{iH0(t1-t0)} e^{-iH(t1-t2)} e^{-iH0(t2-t0)} = U(t1,t2) · e^{iH0(t2-t0)} ×
× e^{-iH(t2-t3)} e^{-iH0(t3-t0)} = e^{iH0(t1-t0)} e^{-iH(t1-t2)} e^{-iH(t2-t3)} e^{-iH0(t3-t0)} =
= e^{iH0(t1-t0)} e^{-iH(t1-t3)} e^{-iH0(t3-t0)} = U(t1,t3).
We will show that:
(A) |Ω> = lim U(t0,-T)|0> / e^{-iE0(t0-(-T))} <Ω|0>; E0 = <Ω|H|Ω> (reference point for energy is H0|0> = 0)
T→∞(1-iε)

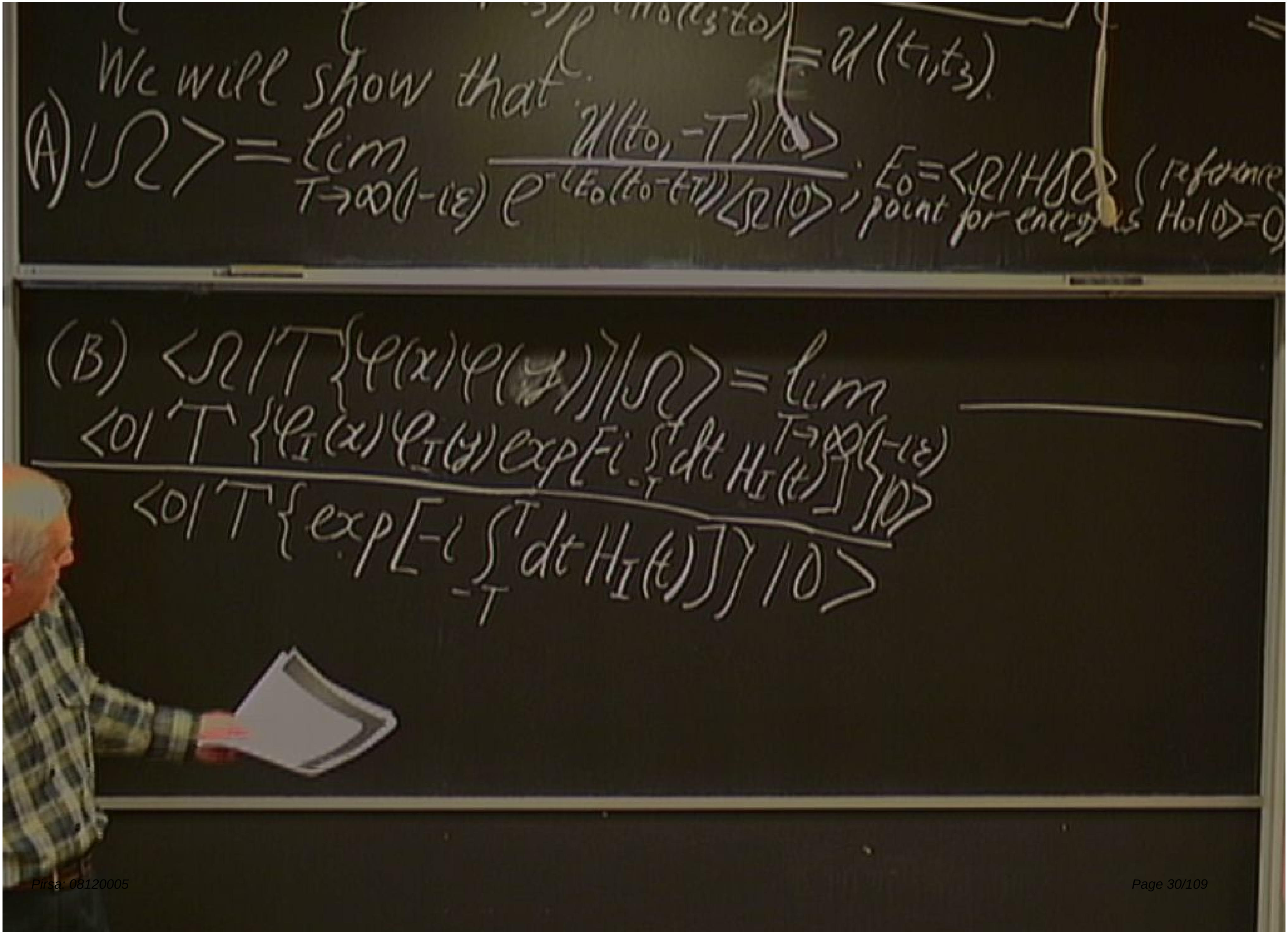


= U(t1,t3).
We will show that:
(A) |Ω> = lim T→∞(1-iε) U(t0,-T)|0> / e^-iE0(t0-(-T))<Ω|0> ; E0=<Ω|H|Ω> (reference point for energy is H0|0>=0)
(B) <Ω|T φ(x)

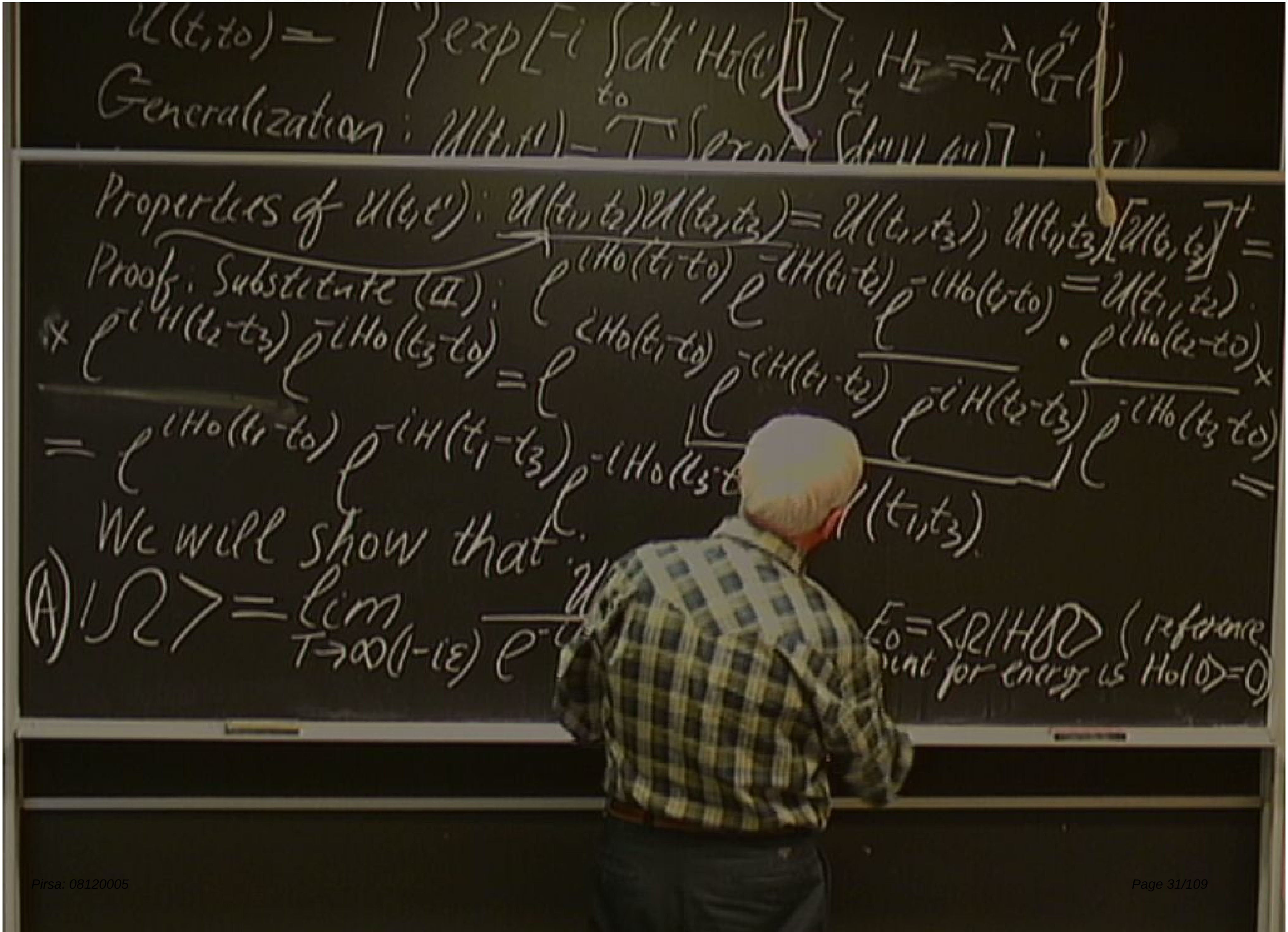

= U(t1,t3)
We will show that:
(A) |Ω> = lim U(t0,−T)|0> / e^(−iE0(t0−(−T)))<Ω|0> ; E0 = <Ω|H|Ω> (reference point for energy is H0|0> = 0)
T→∞(1−iε)
(B) <Ω|T{φ(x)φ(y)}|Ω> = lim
T→∞(1−iε)
<0| T

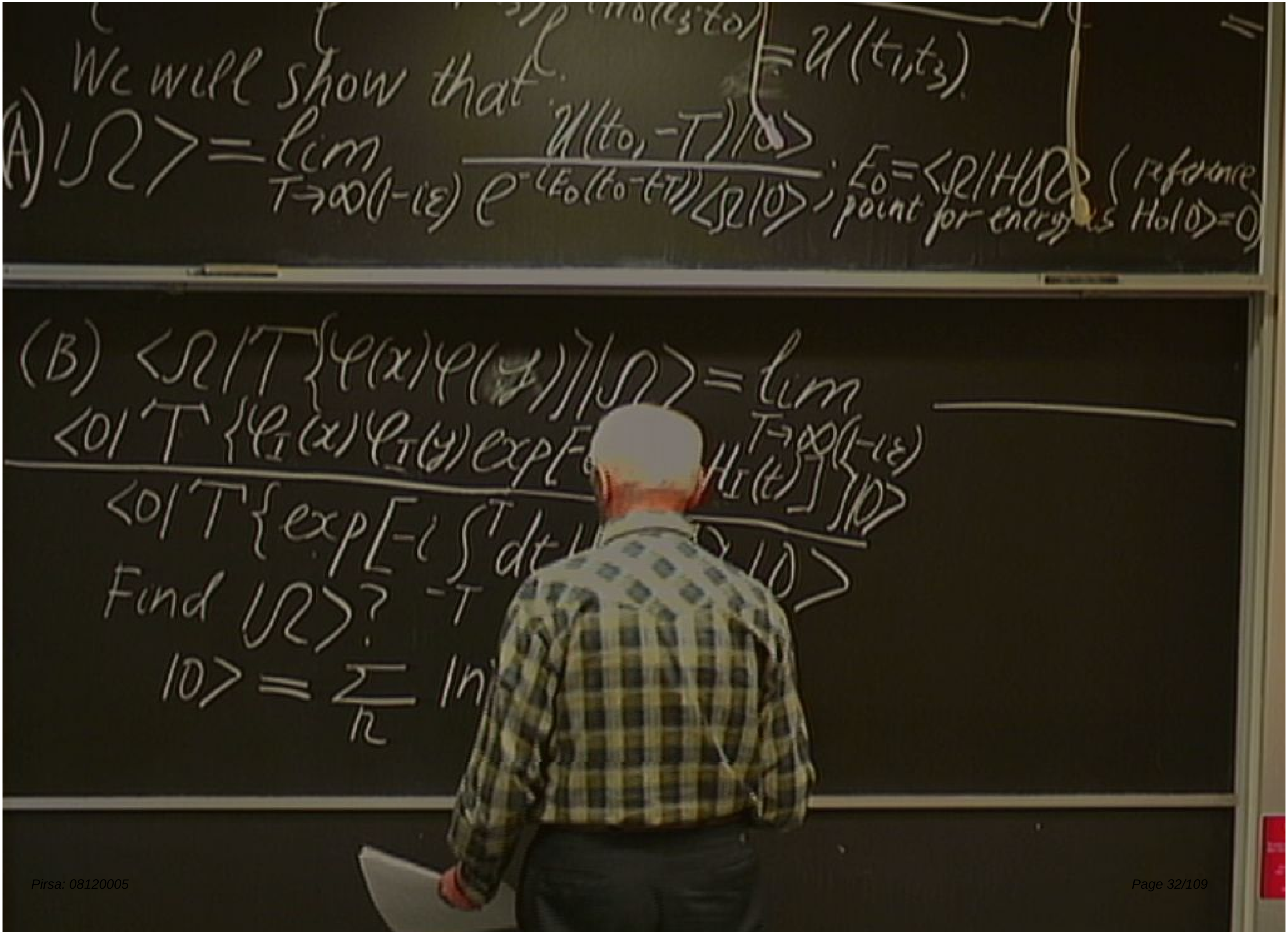


= U(t1,t3)
We will show that:
(A) |Ω> = lim T→∞(1−iε) U(t0,−T)|0> / e^(−iE0(t0−(−T)))<Ω|0> ; E0 = <Ω|H|Ω> (reference point for energy is H0|0> = 0)
(B) <Ω|T{φ(x)φ(y)}|Ω> = lim T→∞(1−iε)
T{

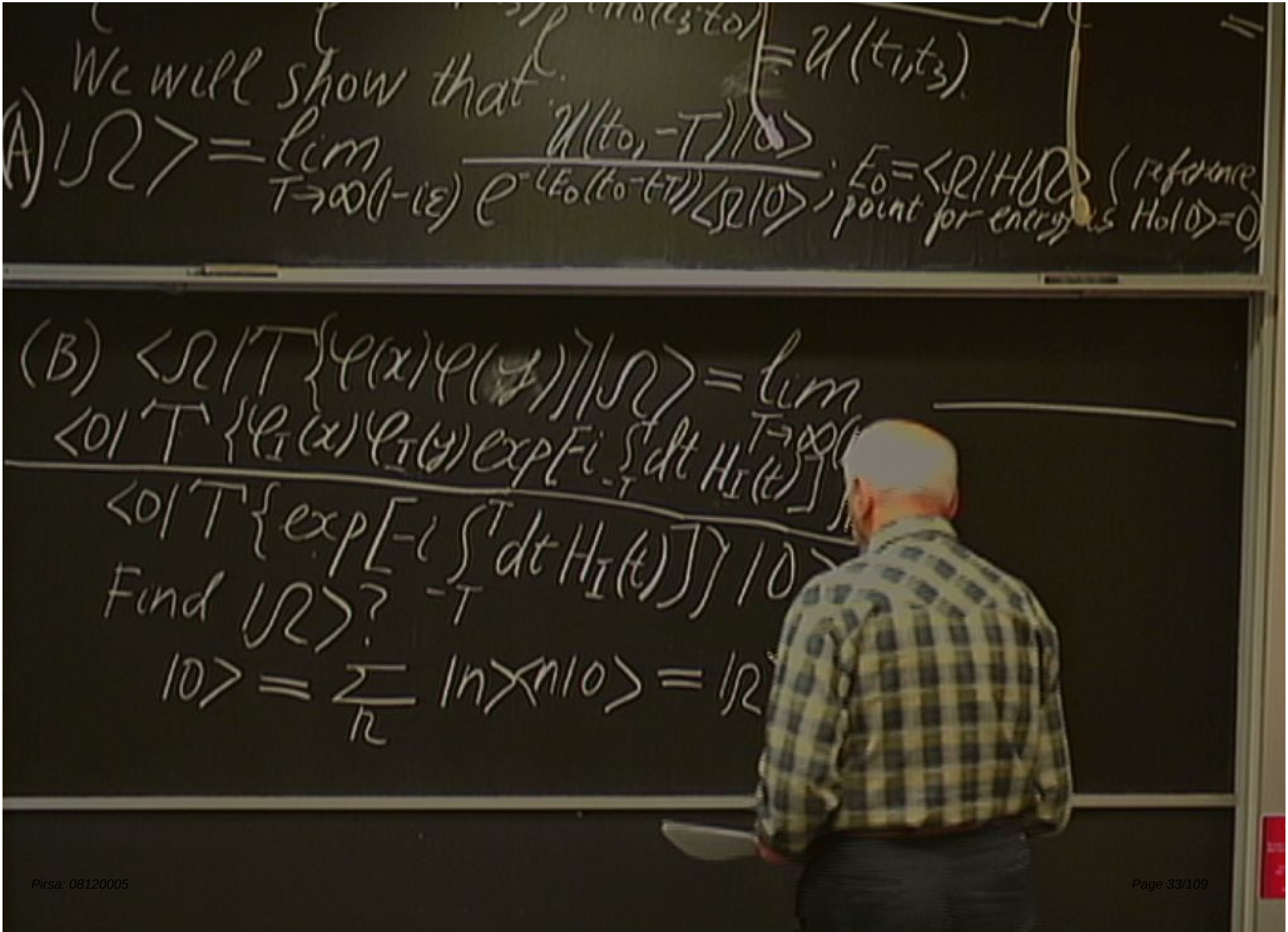


= U(t1,t3)
We will show that:
(A) |Ω> = lim T→∞(1−iε) U(t0,−T)|0> / e^{−iE0(t0−(−T))}<Ω|0> ; E0 = <Ω|H|Ω> (reference point for energy is H0|0> = 0)
(B) <Ω|T{φ(x)φ(y)}|Ω> = lim T→∞(1−iε)
<0|T{φI(x)φI(y) exp[−i ∫_{−T}^{T} dt HI(t)]}|0>
<0|T{exp[−i ∫_{−T}^{T} dt HI(t)]}|0>

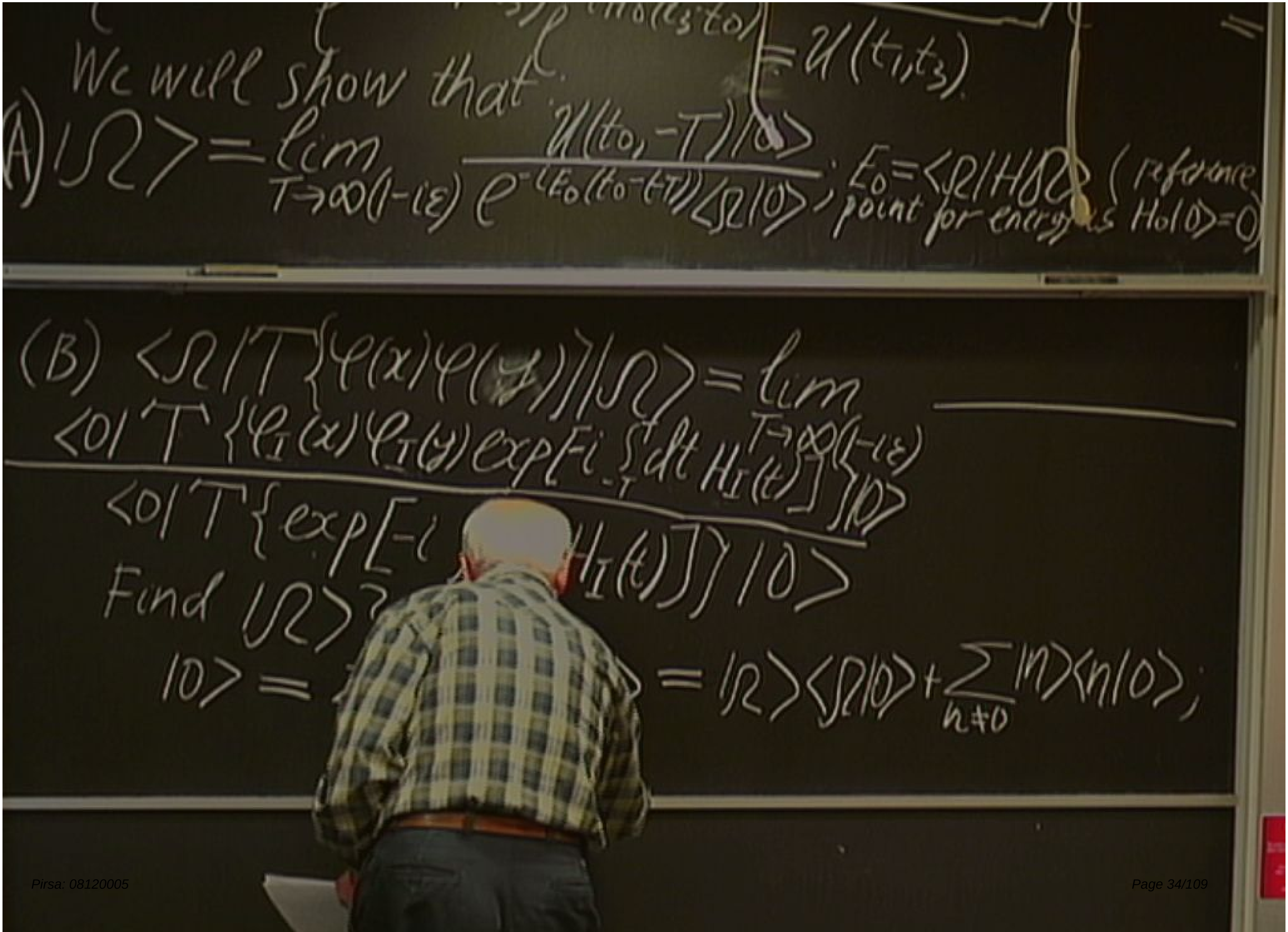

U(t,t0) = T{exp[-i ∫ dt' HI(t')]}; HI = λ/4! (φI(t))^4
Generalization: U(t,t') = T{exp[-i ∫ dt'' HI(t'')]}
Properties of U(t,t'): U(t1,t2)U(t2,t3) = U(t1,t3); U(t1,t3)[U(t2,t3)]† =
Proof: Substitute (II): e^{iH0(t1-t0)} e^{-iH(t1-t2)} e^{-iH0(t2-t0)} = U(t1,t2)
× e^{iH0(t2-t0)} e^{-iH(t2-t3)} e^{-iH0(t3-t0)} = e^{iH0(t1-t0)} e^{-iH(t1-t2)} e^{-iH(t2-t3)} e^{-iH0(t3-t0)}
= e^{iH0(t1-t0)} e^{-iH(t1-t3)} e^{-iH0(t3-t0)} = U(t1,t3)
We will show that:
(A) |Ω> = lim T→∞(1-iε)
E0 = <Ω|H|Ω> (reference point for energy is H0|0> = 0)



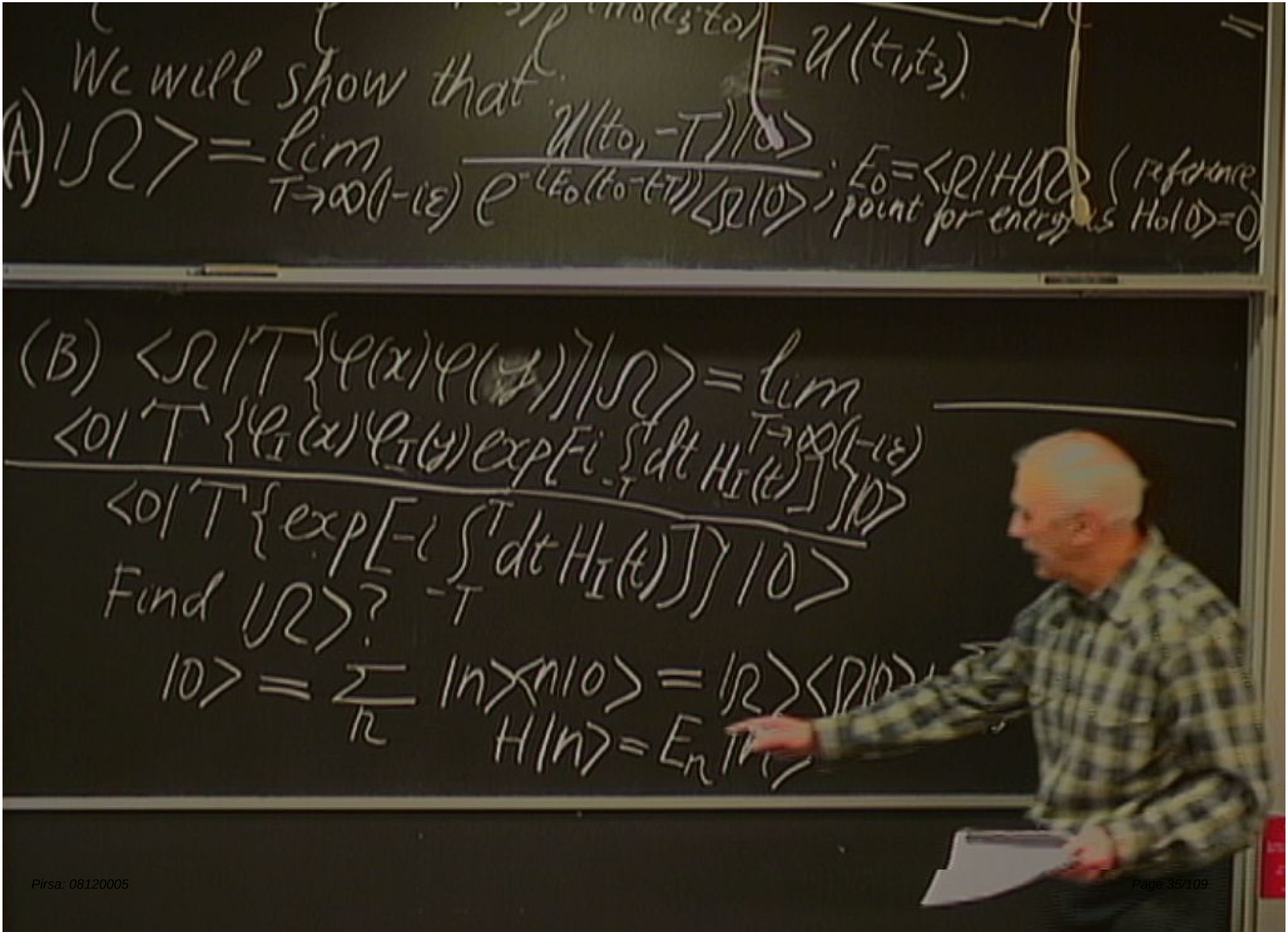
= U(t1,t3)
We will show that:
(A) |Ω> = lim T→∞(1−iε) U(t0,−T)|0> / e^(−iE0(t0−(−T)))<Ω|0> ; E0 = <Ω|H|Ω> (reference point for energy is H0|0> = 0)
(B) <Ω|T{φ(x)φ(y)}|Ω> = lim T→∞(1−iε)
<0|T{φI(x)φI(y) exp[... HI(t)]}|0>
<0|T{exp[−i ∫−T^T dt H ...]}|0>
Find |Ω>?
|0> = Σn |n



$= \mathcal{U}(t_1, t_3)$
We will show that:
(A) $|\Omega\rangle = \lim_{T\to\infty(1-i\varepsilon)} \frac{\mathcal{U}(t_0,-T)|0\rangle}{e^{-iE_0(t_0-(-T))}\langle\Omega|0\rangle}$; $E_0 = \langle\Omega|H|\Omega\rangle$ (reference point for energy is $H_0|0\rangle = 0$)
(B) $\langle\Omega|T\{\varphi(x)\varphi(y)\}|\Omega\rangle = \lim_{T\to\infty(}$
$\langle 0|T\{\varphi_I(x)\varphi_I(y)\exp[-i\int_{-T}^{T} dt\, H_I(t)]\}$
$\langle 0|T\{\exp[-i\int_{-T}^{T} dt\, H_I(t)]\}|0\rangle$
Find $|\Omega\rangle$?
$|0\rangle = \sum_n |n\rangle\langle n|0\rangle = |\Omega\rangle$



= U(t1,t3).
We will show that:
(A) |Ω> = lim T→∞(1−iε) U(t0,−T)|0> / e^{−iE0(t0−(−T))}<Ω|0>; E0 = <Ω|H|Ω> (reference point for energy as H0|0> = 0)
(B) <Ω|T{φ(x)φ(y)}|Ω> = lim T→∞(1−iε)
<0|T{φI(x)φI(y) exp[−i ∫ dt HI(t)]}|0>
<0|T{exp[−i HI(t)]}|0>
Find |Ω>?
|0> = |Ω><Ω|0> + Σ n≠0 |n><n|0>;



We will show that:
(A) |Ω> = lim T→∞(1−iε) U(t₀,−T)|0> / e^{−iE₀(t₀−(−T))}<Ω|0> ; E₀ = <Ω|H|Ω> (reference point for energy is H₀|0> = 0)
(B) <Ω|T{φ(x)φ(y)}|Ω> = lim T→∞(1−iε)
<0|T{φ_I(x)φ_I(y) exp[−i ∫_{−T}^{T} dt H_I(t)]}|0> / <0|T{exp[−i ∫_{−T}^{T} dt H_I(t)]}|0>
Find |Ω>?
|0> = Σ_n |n><n|0> = |Ω><Ω|0> +
H|n> = E_n|n>

We will show that:

(A) $$|\Omega\rangle = \lim_{T\to\infty(1-i\varepsilon)} \frac{U(t_0,-T)|0\rangle}{e^{-iE_0(t_0-(-T))}\langle\Omega|0\rangle};\ E_0=\langle\Omega|H|\Omega\rangle$$ (reference point for energy is $H_0|0\rangle=0$)

$= U(t_1,t_3)$.

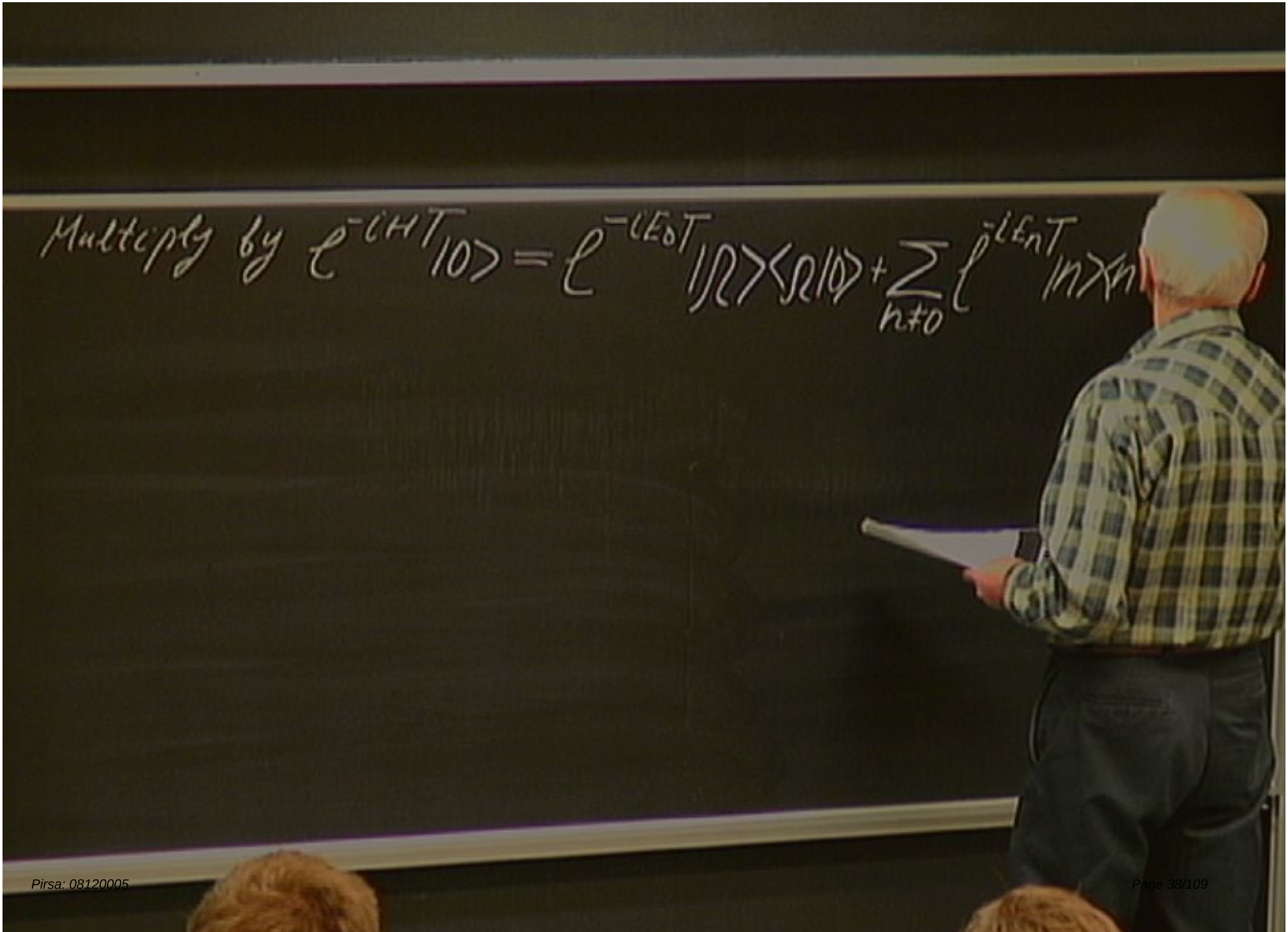
(B) $$\langle\Omega|T\{\varphi(x)\varphi(y)\}|\Omega\rangle = \lim_{T\to\infty(1-i\varepsilon)} \frac{\langle 0|T\{\varphi_I(x)\varphi_I(y)\exp[-i\int_{-T}^{T}dt\,H_I(t)]\}|0\rangle}{\langle 0|T\{\exp[-i\int_{-T}^{T}dt\,H_I(t)]\}|0\rangle}$$

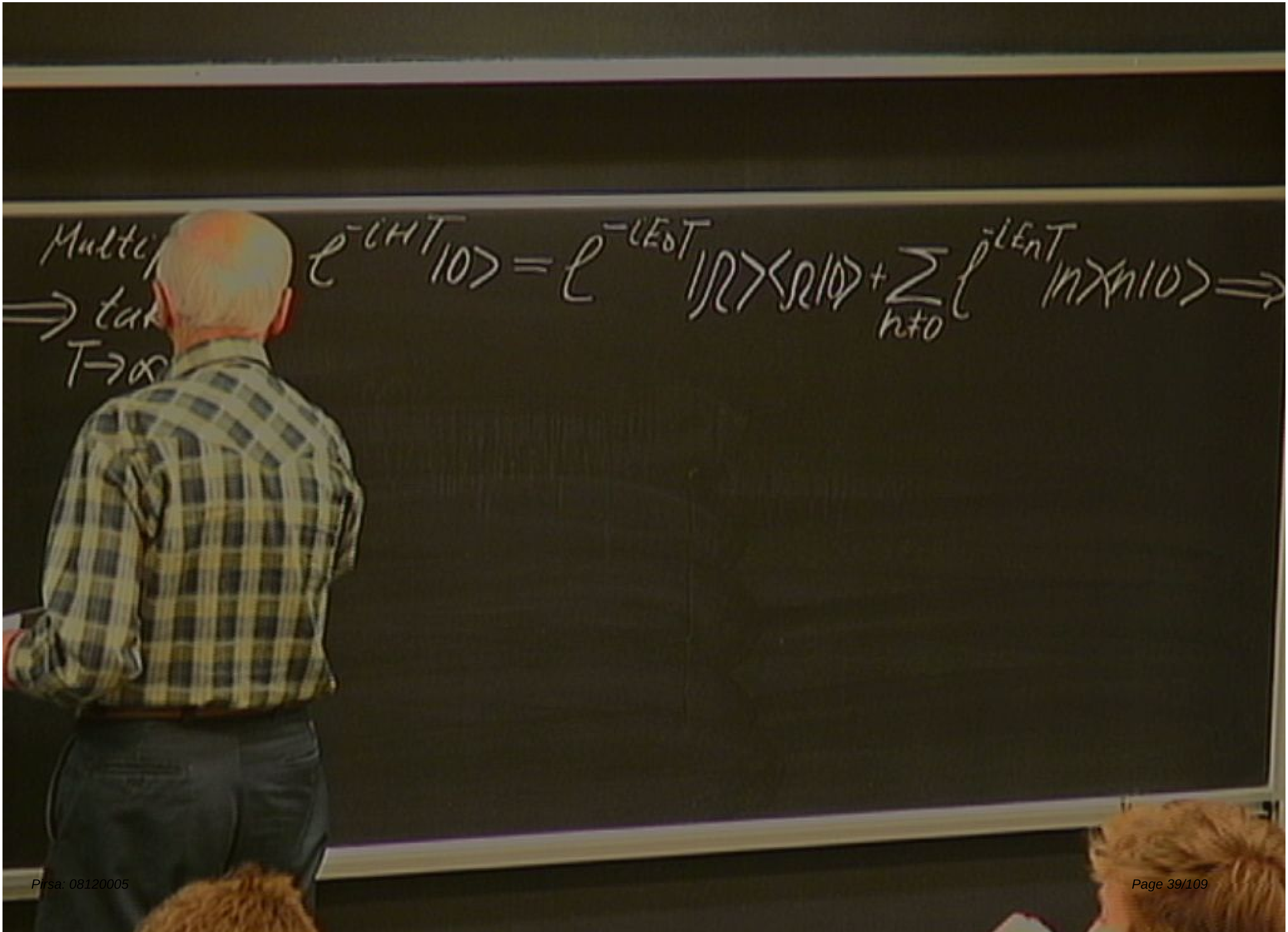
Find $|\Omega\rangle$?

$$|0\rangle = \sum_n |n\rangle\langle n|0\rangle = |\Omega\rangle\langle\Omega|0\rangle + \sum_{n\neq 0}|n\rangle\langle n|0\rangle;$$

$$H|n\rangle = E_n|n\rangle$$

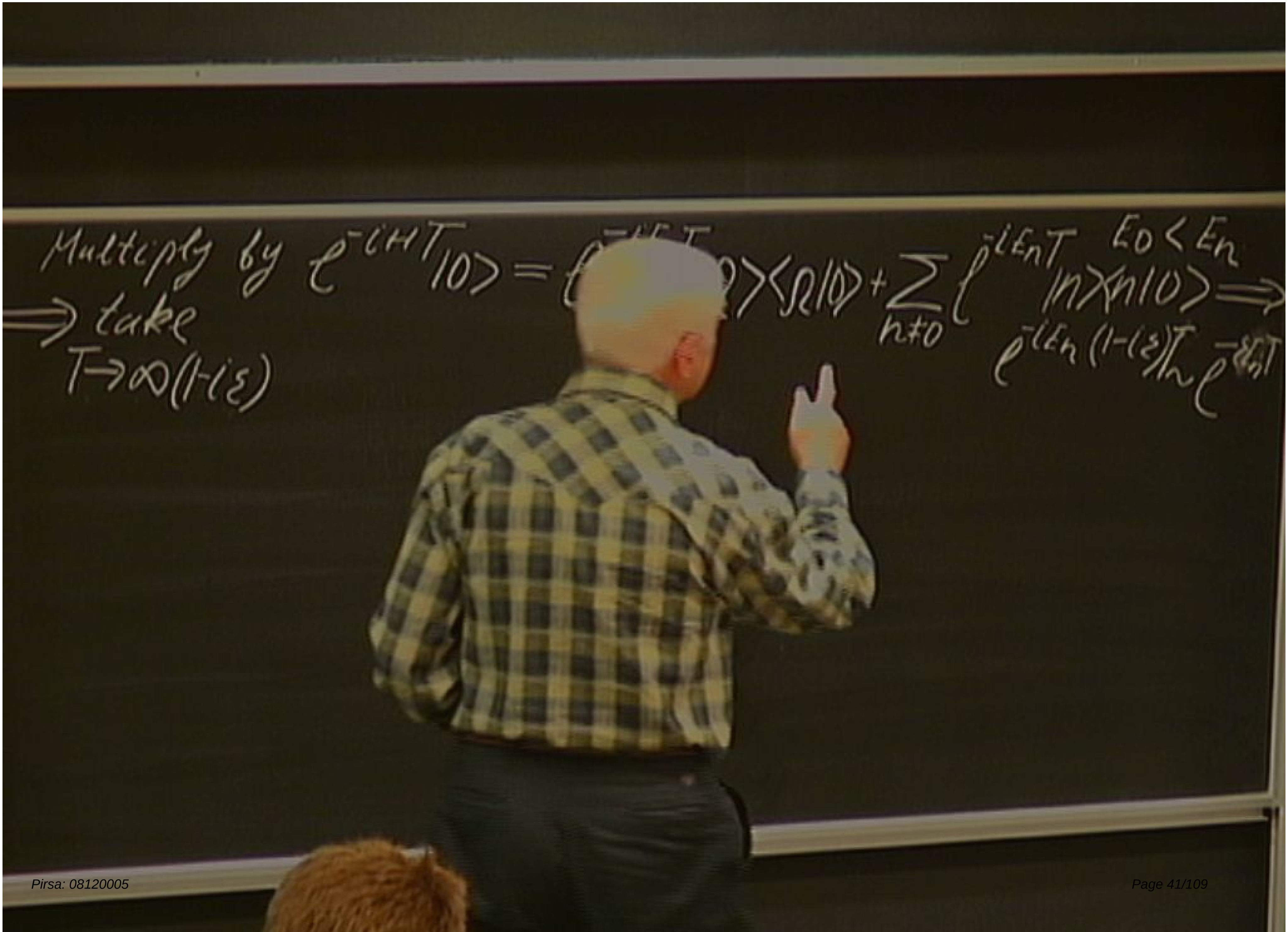
Multiply by $e^{-iHT}|0\rangle = e$


Multiply by $e^{-iHT}|0\rangle = e^{-iE_0T}|\Omega\rangle\langle\Omega|0\rangle + \sum_{n\neq 0} e^{-iE_nT}|n\rangle\langle n|$


Multi
$e^{-iHT}|0\rangle = e^{-iE_0T}|\Omega\rangle\langle\Omega|0\rangle + \sum_{n\neq 0} e^{-iE_nT}|n\rangle\langle n|0\rangle \Rightarrow$
$\Rightarrow$ tak
$T\to\infty$

Multiply by $e^{-iHT}|0\rangle = e^{-iE_0T}|\Omega\rangle\langle\Omega|0\rangle + \sum_{n\neq 0} e^{-iE_nT}|n\rangle\langle n|0\rangle \Longrightarrow$ $E_0 < E_n$

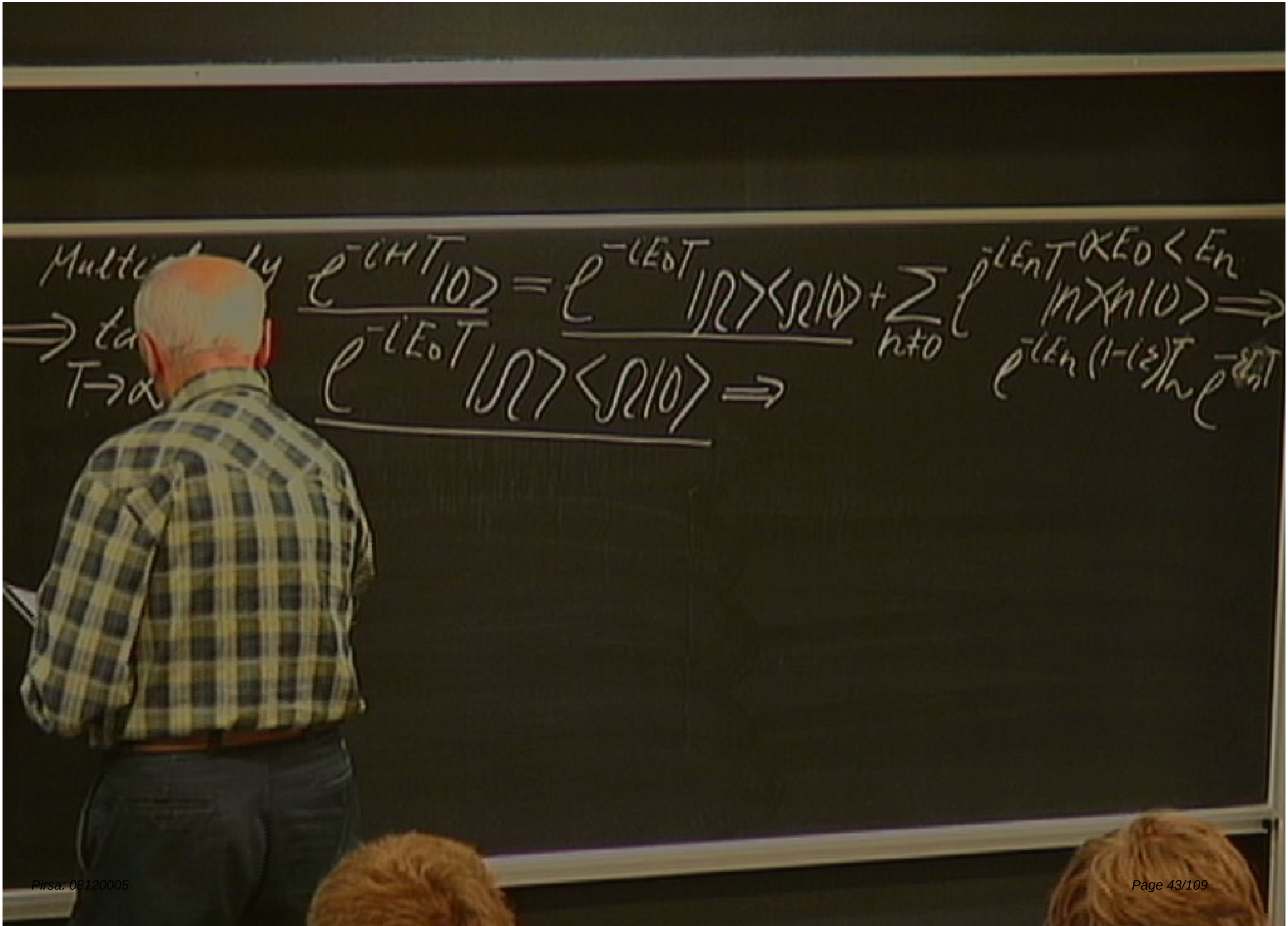
$\Longrightarrow$ take $T \to \infty(1-i\varepsilon)$


Multiply by
take
T→∞(1-iε)
E0 < En

Multiply by $e^{-iHT}|0\rangle = e^{-iE_0T}|\Omega\rangle\langle\Omega|0\rangle + \sum_{n\neq 0} e^{-iE_nT}|n\rangle\langle n|0\rangle \Longrightarrow$ $E_0 < E_n$

$\Longrightarrow$ take $T \to \infty(1-i\varepsilon)$

$e^{-iE_n(1-i\varepsilon)T} \sim e^{-\varepsilon E_n T}$

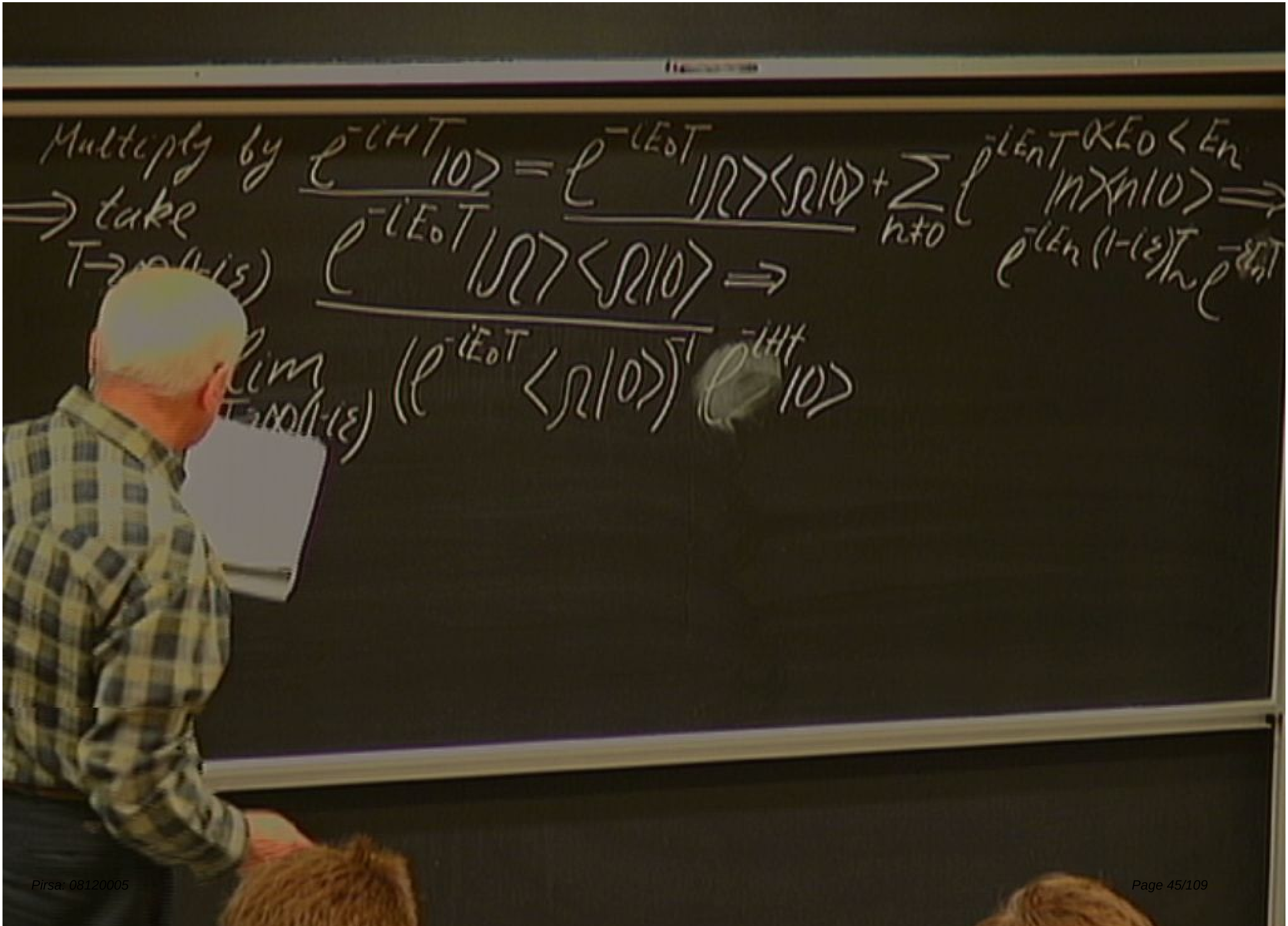

Multi ly
T→∞

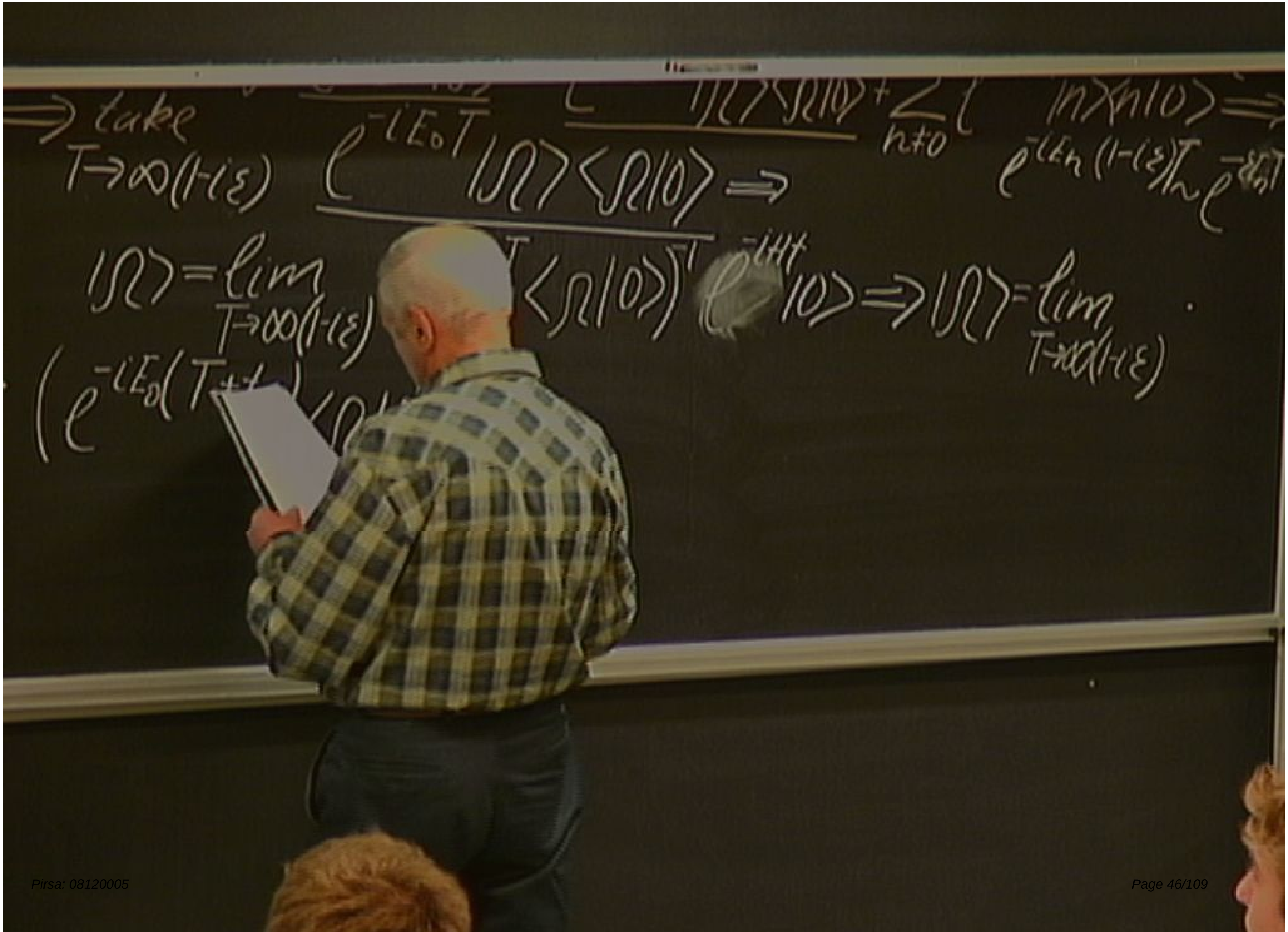
Multiply by $e^{-iHT}|0\rangle = e^{-iE_0T}|\Omega\rangle\langle\Omega|0\rangle + \sum_{n\neq 0} e^{-iE_nT}|n\rangle\langle n|0\rangle \Longrightarrow$ $\quad E_0 < E_n$

$e^{-iE_n(1-i\varepsilon)T} \sim e^{-\varepsilon E_n T}$

$\Longrightarrow$ take $T\to\infty(1-i\varepsilon)$ $\quad e^{-iE_0T}|\Omega\rangle\langle\Omega|0\rangle \Longrightarrow$

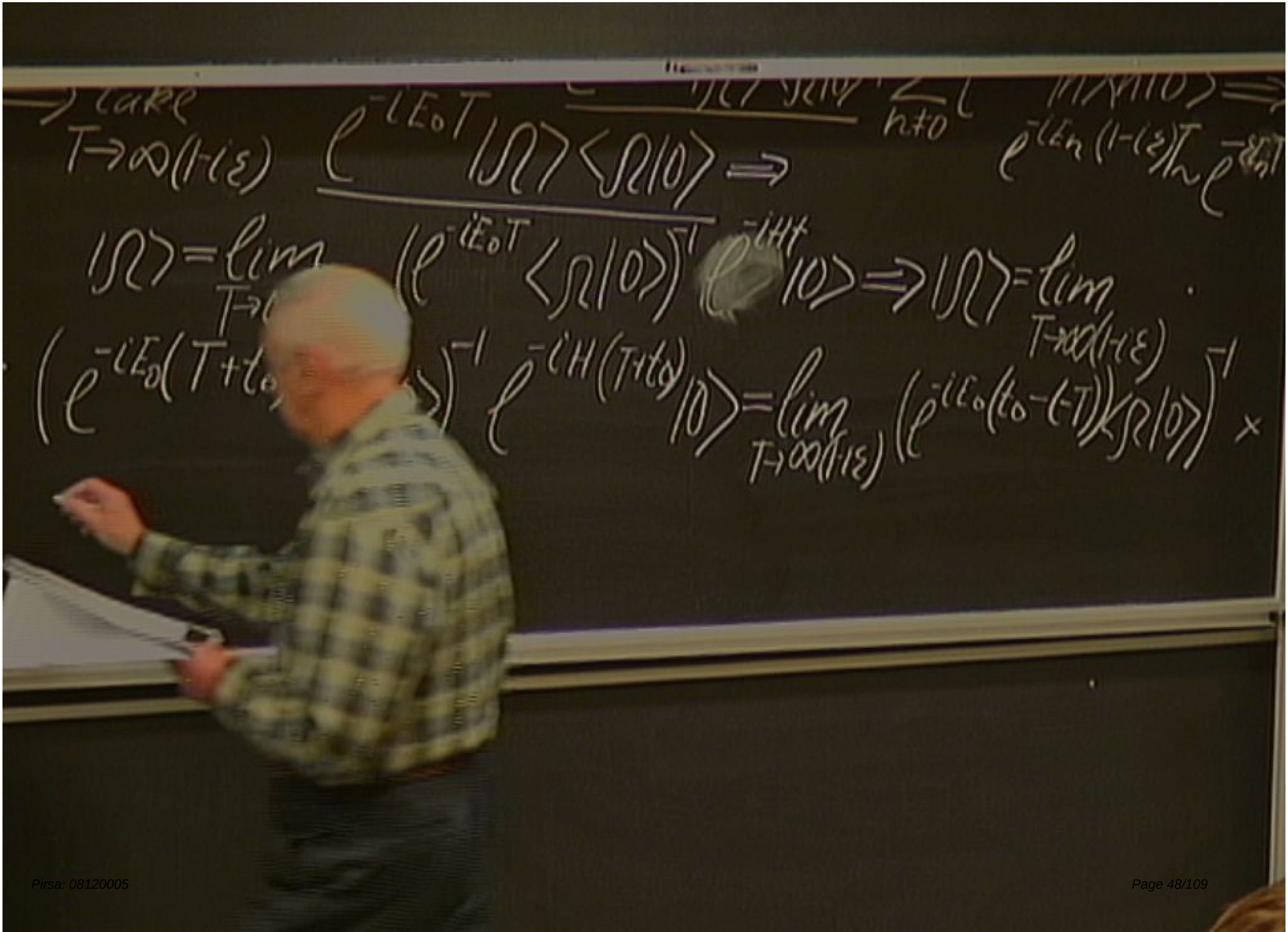
$$|\Omega\rangle = \lim_{T\to\infty(1-i\varepsilon)} \left(e^{-iE_0T}\langle\Omega|0\rangle\right)^{-1}|0\rangle$$


Multiply by e^-iHT |0> = e^-iE0T |Ω><Ω|0> + Σ n≠0 e^-iEnT |n><n|0> ⇒ E0 < En
⇒ take T → ∞(1-iε)
e^-iE0T |Ω><Ω|0> ⇒
e^-iEn(1-iε)T
lim T→∞(1-iε) (e^-iE0T <Ω|0>)^-1 e^-iHT |0>


take

$$\xrightarrow{\text{take } T\to\infty(1-i\varepsilon)} \quad e^{-iE_0T}\,|\Omega\rangle\langle\Omega|0\rangle \Rightarrow \qquad \sum_{n\neq 0} e^{-iE_n(1-i\varepsilon)T} \ldots e^{-\varepsilon E_n T}$$

$$|\Omega\rangle = \lim_{T\to\infty(1-i\varepsilon)} \left(e^{-iE_0T}\langle\Omega|0\rangle\right)^{-1} e^{-iHT}|0\rangle \Rightarrow |\Omega\rangle = \lim_{T\to\infty(1-i\varepsilon)} \left(e^{-iE_0(T+t_0)}\langle\Omega|0\rangle\right)^{-1} e^{-iH(T+t_0)}|0\rangle$$


take
T→∞(1-iε)

$\Longrightarrow$ take $T\to\infty(1-i\varepsilon)$ $\quad e^{-iE_0T}|\Omega\rangle\langle\Omega|0\rangle \Longrightarrow$ $\quad \sum_{n\neq 0} e^{-iE_n(1-i\varepsilon)T}\ldots e^{-E_nT}$

$$|\Omega\rangle = \lim_{T\to\infty(1-i\varepsilon)} \left(e^{-iE_0T}\langle\Omega|0\rangle\right)^{-1} e^{-iHT}|0\rangle \Longrightarrow |\Omega\rangle = \lim_{T\to\infty(1-i\varepsilon)}$$

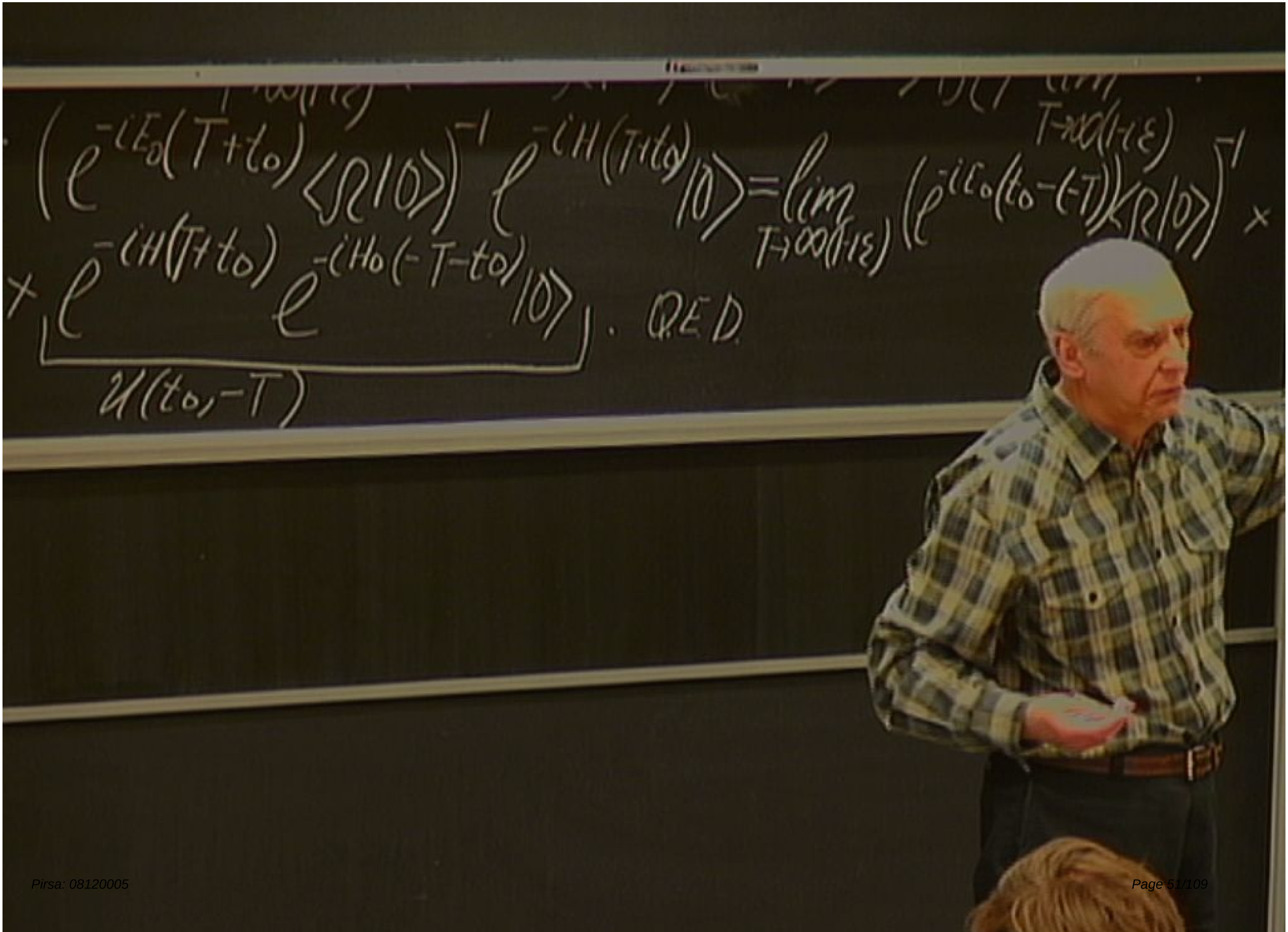
$$\left(e^{-iE_0(T+t_0)}\langle\Omega|0\rangle\right)^{-1} e^{-iH(T+t_0)}|0\rangle = \lim_{T\to\infty(1-i\varepsilon)} \left(e^{-iE_0(t_0-(-T))}\langle\Omega|0\rangle\right)^{-1} \times$$

$$\times\, e^{-iH(T+t_0)}\, e^{-iH_0(-T-t_0)}|0\rangle$$

$$|\Omega\rangle = \lim_{T\to\infty(1-i\varepsilon)} \left(e^{-iE_0 T}\langle\Omega|0\rangle\right)^{-1} e^{-iHT}|0\rangle \Rightarrow |\Omega\rangle = \lim_{T\to\infty(1-i\varepsilon)}$$

$$\left(e^{-iE_0(T+t_0)}\langle\Omega|0\rangle\right)^{-1} e^{-iH(T+t_0)}|0\rangle = \lim_{T\to\infty(1-i\varepsilon)} \left(e^{-iE_0(t_0-(-T))}\langle\Omega|0\rangle\right)^{-1} \times$$

$$\times \underbrace{e^{-iH(T+t_0)} e^{-iH_0(-T-t_0)}|0\rangle}_{U(t_0,-T)}$$



$\left(e^{-iE_0(T+t_0)}\langle\Omega|0\rangle\right)^{-1} e^{-iH(T+t_0)}|0\rangle = \lim_{T\to\infty(1-i\varepsilon)} \left(e^{-iE_0(t_0-(-T))}\langle\Omega|0\rangle\right)^{-1} \times$
$\times\, e^{-iH(T+t_0)} e^{-iH_0(-T-t_0)}|0\rangle$. Q.E.D.
$U(t_0,-T)$

We proved that ...

Properties of $\mathcal{U}(t,t')$: $\mathcal{U}(t_1,t_2)\mathcal{U}(t_2,t_3)=\mathcal{U}(t_1,t_3)$; $\mathcal{U}(t_1,t_3)[\mathcal{U}(t_2,t_3)]^\dagger=$

Proof. Substitute (II): $e^{iH_0(t_1-t_0)}e^{-iH(t_1-t_2)}e^{-iH_0(t_2-t_0)}=\mathcal{U}(t_1,t_2)$.

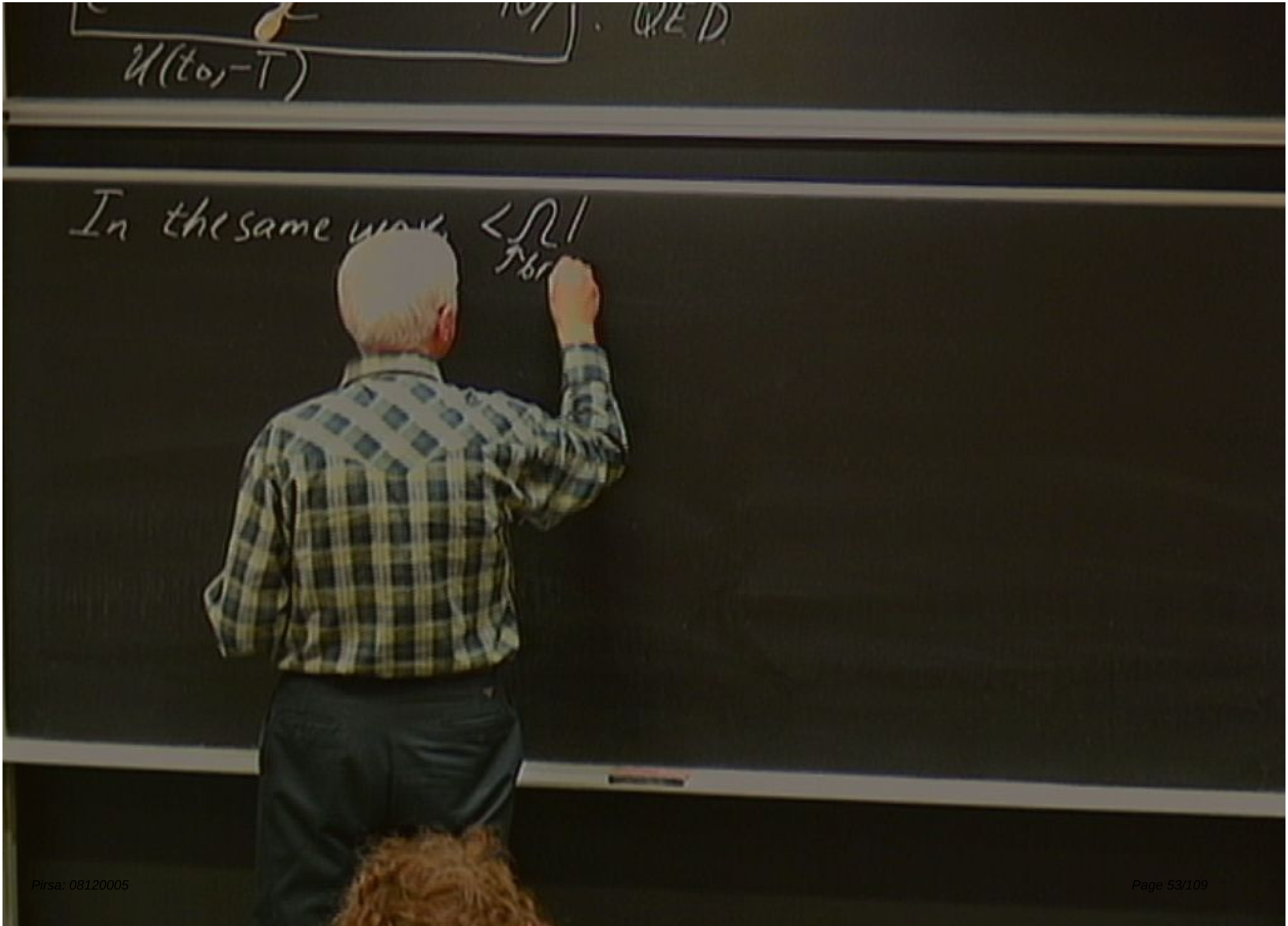
$\times e^{iH_0(t_2-t_0)}\times e^{-iH(t_2-t_3)}e^{-iH_0(t_3-t_0)}=e^{iH_0(t_1-t_0)}e^{-iH(t_1-t_2)}e^{-iH(t_2-t_3)}e^{-iH_0(t_3-t_0)}=$

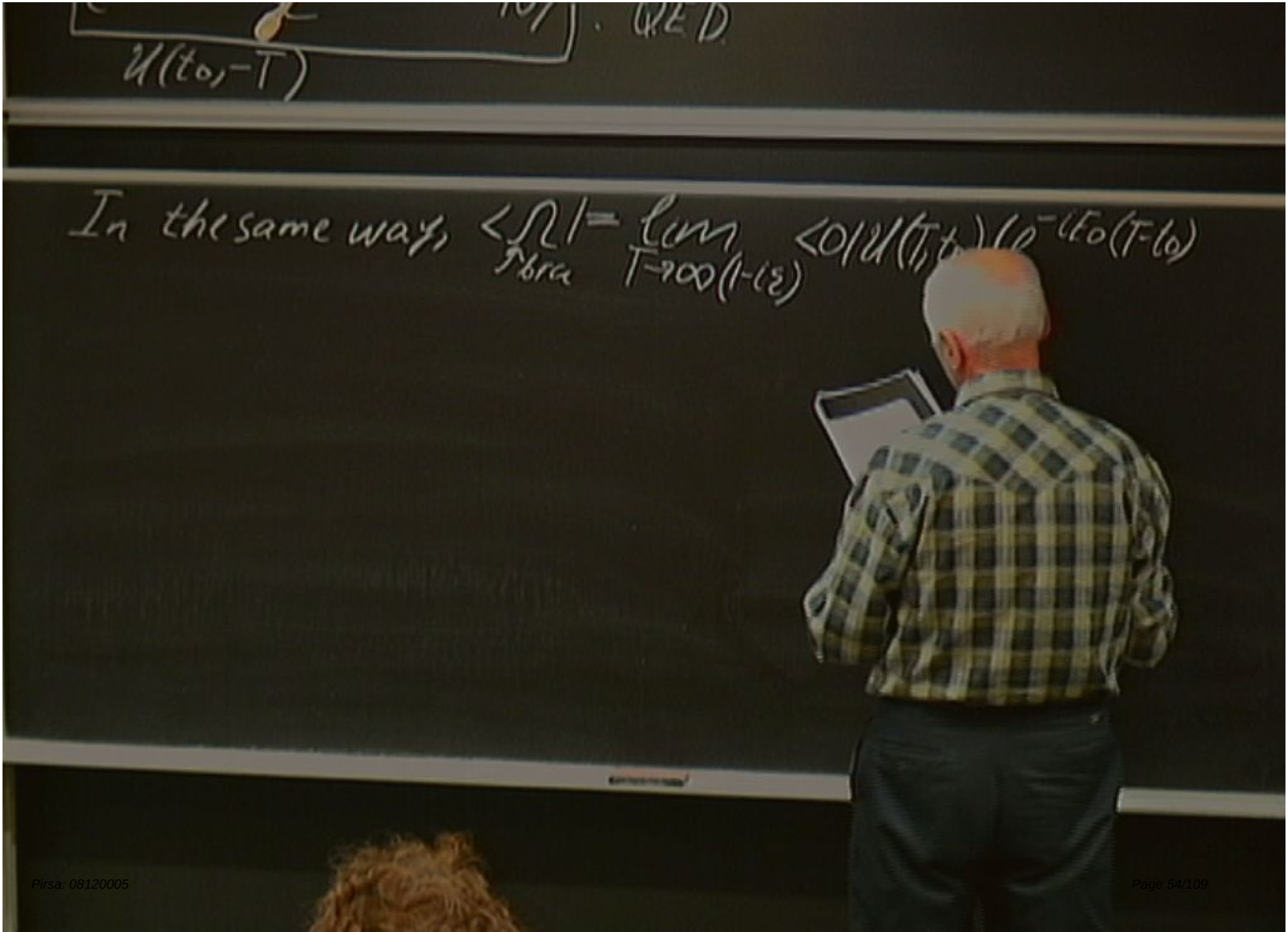
$=e^{iH_0(t_1-t_0)}e^{-iH(t_1-t_3)}e^{-iH_0(t_3-t_0)}=\mathcal{U}(t_1,t_3)$.

We will show that:

(A) $|\Omega\rangle=\lim_{T\to\infty(1-i\varepsilon)}\dfrac{\mathcal{U}(t_0,-T)|0\rangle}{e^{-iE_0(t_0-(-T))}\langle\Omega|0\rangle}$; $E_0=\langle\Omega|H|\Omega\rangle$ (reference point for energy is $H_0|0\rangle=0$)

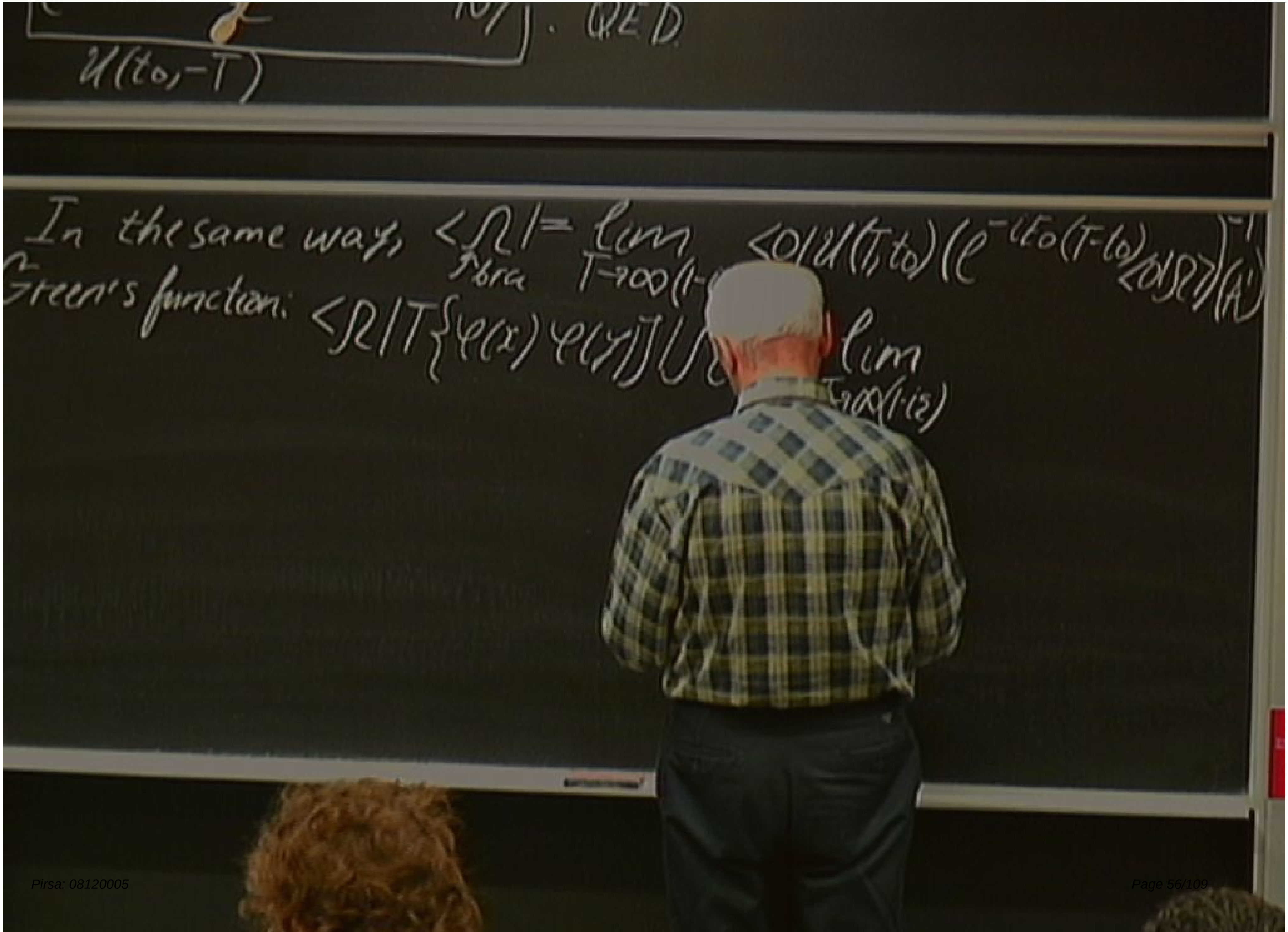
ket

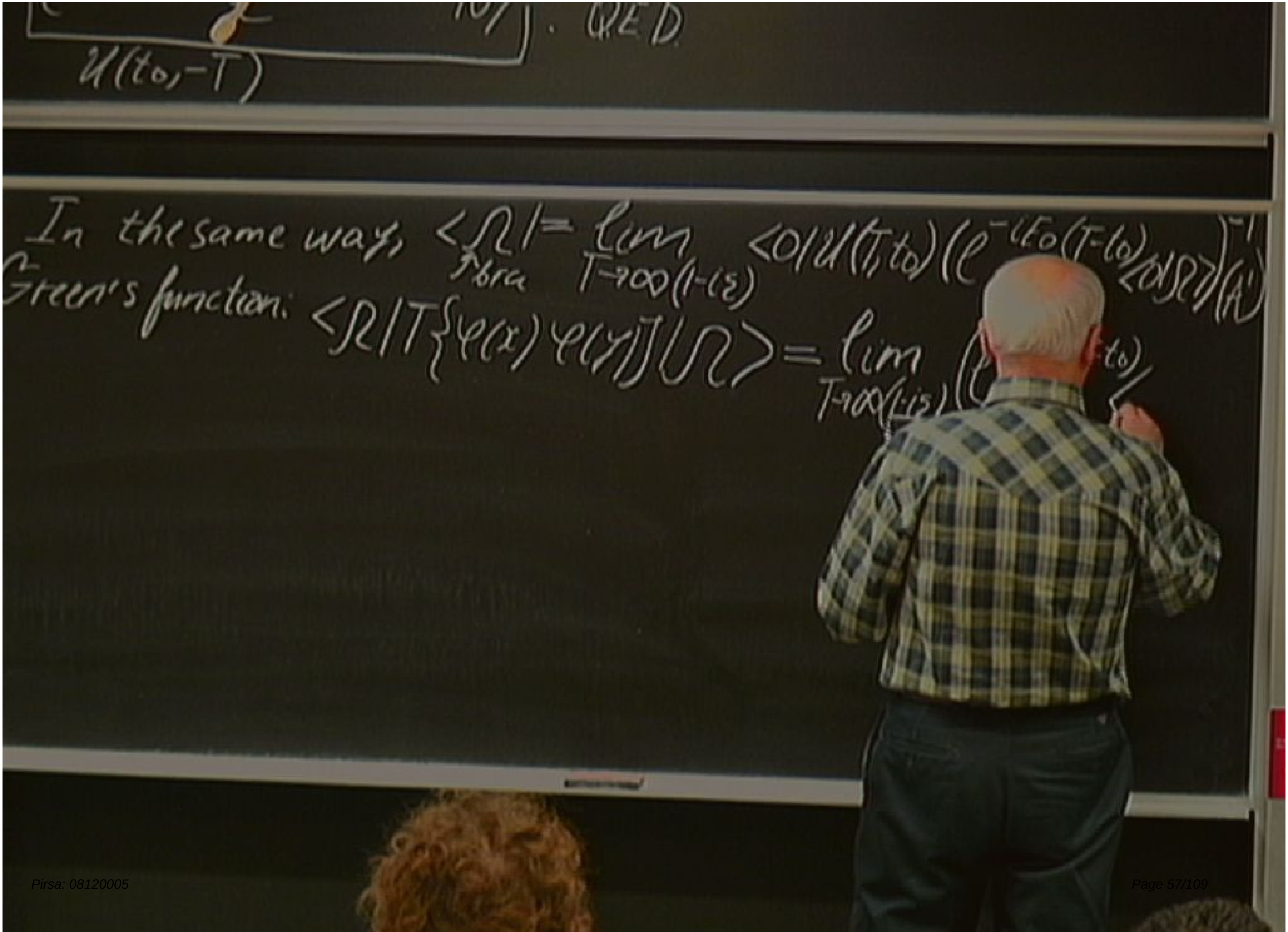

Q.E.D.
$U(t_0,-T)$
In the same


Q.E.D.
U(t0,-T)
In the same way, <Ω| = lim <0|U(T,t0) e^-iE0(T-t0)
for bra
T→∞(1-iε)

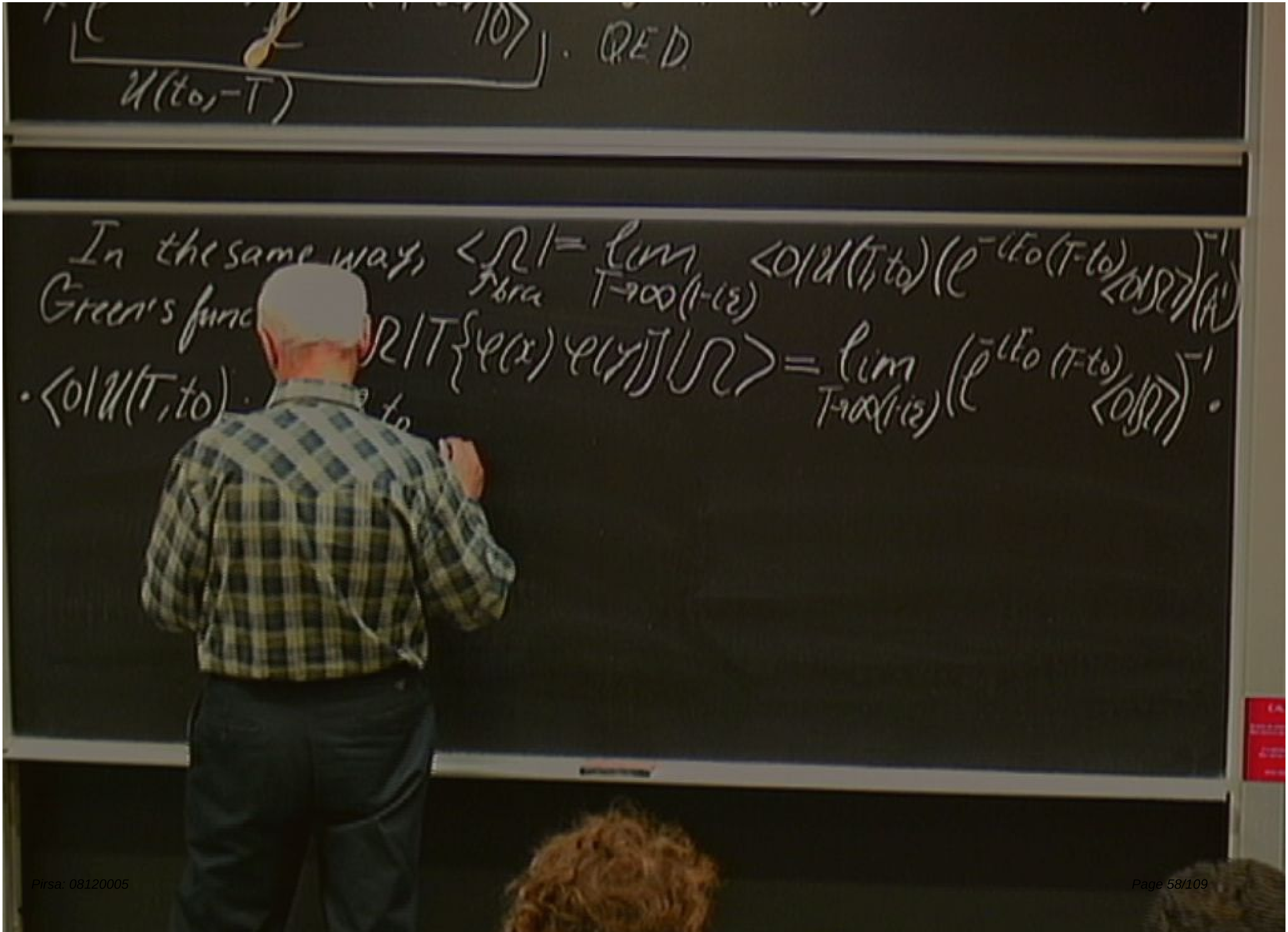
$\mathcal{U}(t_0,-T)$. Q.E.D.

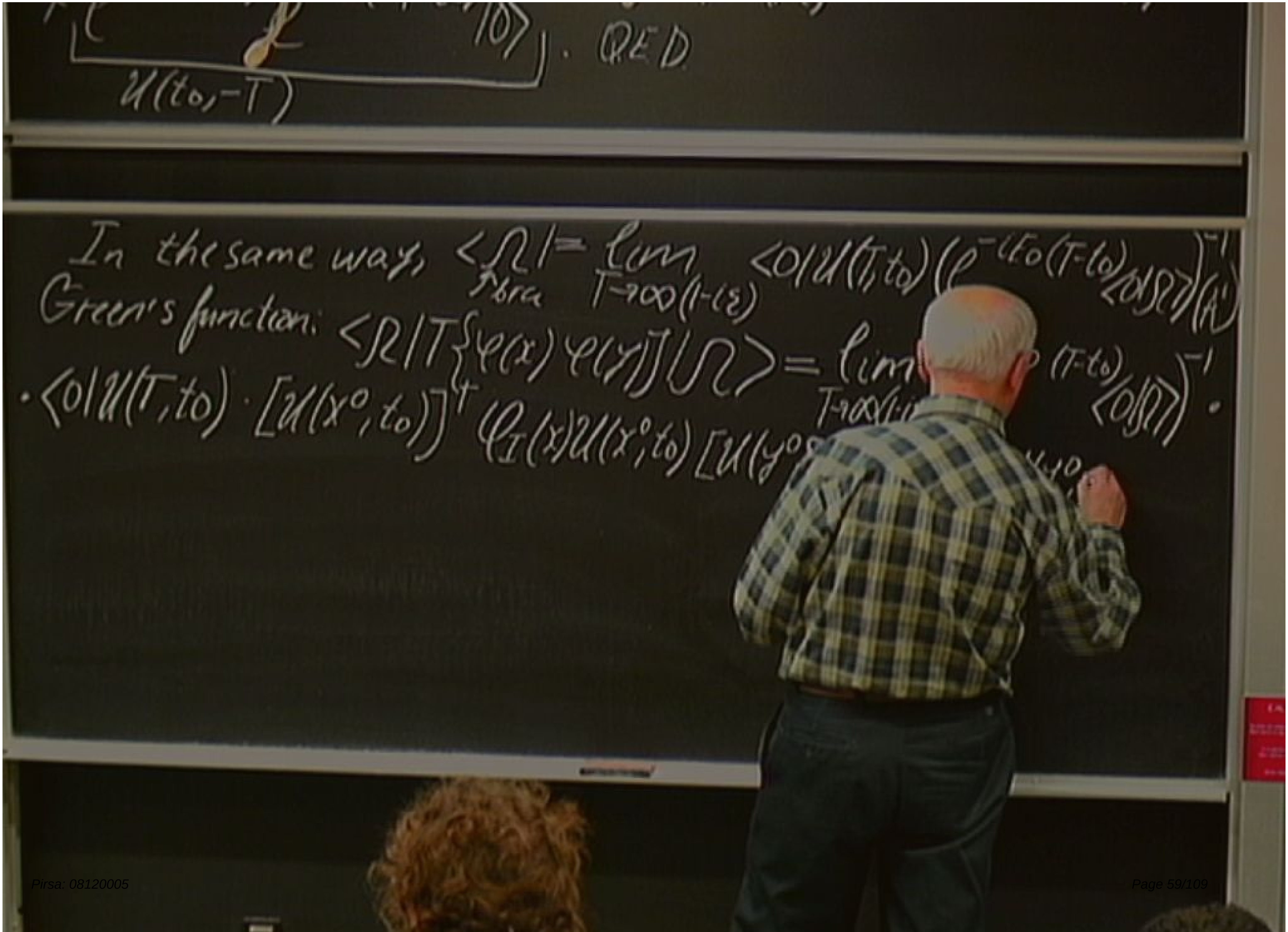
In the same way, $\underset{\text{bra}}{\langle\Omega|} = \lim_{T\to\infty(1-i\varepsilon)} \langle 0|\mathcal{U}(T,t_0)\left(e^{-iE_0(T-t_0)}\langle 0|\Omega\rangle\right)^{-1}$ (A')


Q.E.D.
In the same way, <Ω| = lim
bra
Green's function: <Ω|T{φ(x) φ(y)}
lim



Q.E.D.
In the same way,
Green's function:


|0⟩] . QED
U(t0,−T)
In the same way, ⟨Ω| = lim ⟨0|U(T,t0)(e^−iE0(T−t0)⟨0|Ω⟩)^−1
bra T→∞(1−iε)
Green's func
Ω|T{φ(x) φ(y)}|Ω⟩ = lim (e^−iE0(T−t0)⟨0|Ω⟩)^−1 ·
T→∞(1−iε)
· ⟨0|U(T,t0)



|0>] . QED
U(t0,-T)
In the same way, <Ω| = lim <0|U(T,t0)(e^-iE0(T-t0)<0|Ω>)^-1
bra T→∞(1-iε)
Green's function: <Ω|T{φ(x)φ(y)}|Ω> = lim
(T-t0) <0|Ω>)^-1 ·
·<0|U(T,t0) · [U(x0,t0)]† φI(x)U(x0,t0) [U(y0

$$|0\rangle \Big] \; . \quad \text{QED}$$

$$\mathcal{U}(t_0,-T)$$

In the same way, $\underset{\uparrow \text{bra}}{\langle\Omega|} = \lim\limits_{T\to\infty(1-i\varepsilon)} \langle 0|\mathcal{U}(T,t_0)\left(e^{-iE_0(T-t_0)}\langle 0|\Omega\rangle\right)^{-1}$ (A')

Green's function: $\langle\Omega|T\{\varphi(x)\varphi(y)\}|\Omega\rangle = \lim\limits_{T\to\infty(1-i\varepsilon)} \left(e^{-iE_0(T-t_0)}\langle 0|\Omega\rangle\right)^{-1} \cdot$

$$\cdot\, \langle 0|\mathcal{U}(T,t_0)\cdot[\mathcal{U}(x^0,t_0)]^\dagger \varphi_I(x)\,\mathcal{U}(x^0,t_0)\,[\mathcal{U}(y^0,t_0)]^\dagger \varphi_I(y)\,\mathcal{U}(y^0,t_0) \times$$

$\times$

$|0\rangle \Big)$. QED

$\mathcal{U}(t_0, -T)$

In the same way, $\langle \Omega | = \lim_{T \to \infty(1-i\varepsilon)} \langle 0 | \mathcal{U}(T, t_0) \left(e^{-iE_0(T-t_0)} \langle 0 | \Omega \rangle \right)^{-1}$ (A′)
↑bra

Green's function: $\langle \Omega | T\{\varphi(x)\varphi(y)\} | \Omega \rangle = \lim_{T \to \infty(1-i\varepsilon)} \left(e^{-iE_0(T-t_0)} \langle 0 | \Omega \rangle \right)^{-1} \cdot$

$\cdot \langle 0 | \mathcal{U}(T, t_0) \cdot [\mathcal{U}(x^0, t_0)]^{\dagger} \varphi_I(x) \mathcal{U}(x^0, t_0) [\mathcal{U}(y^0, t_0)]^{\dagger} \varphi_I(y) \mathcal{U}(y^0, t_0) \times$

$\times \mathcal{U}(t_0, -T) | 0 \rangle \left(e^{-iE_0(t_0-(-T))} \langle \Omega | 0 \rangle \right)^{-1}$

In the same way, $\langle\Omega| = \lim_{T\to\infty(1-i\varepsilon)} \langle 0|U(T,t_0)\left(e^{-iE_0(T-t_0)}\langle 0|\Omega\rangle\right)^{-1}$ (bra)

Green's function: $x^0 > y^0$: $\langle\Omega|T\{\varphi(x)\varphi(y)\}|\Omega\rangle = \lim_{T\to\infty(1-i\varepsilon)} \left(e^{-iE_0(T-t_0)}\langle 0|\Omega\rangle\right)^{-1} \cdot$

$\cdot \langle 0|U(T,t_0)\cdot[U(x^0,t_0)]^\dagger \varphi_I(x)U(x^0,t_0)[U(y^0,t_0)]^\dagger \varphi_I(y)U(y^0,t_0) \times$

$\times U(t_0,-T)|0\rangle\left(e^{-iE_0(t_0-(-T))}\langle\Omega|0\rangle\right)^{-1} = \lim_{T\to\infty(1-i\varepsilon)}$

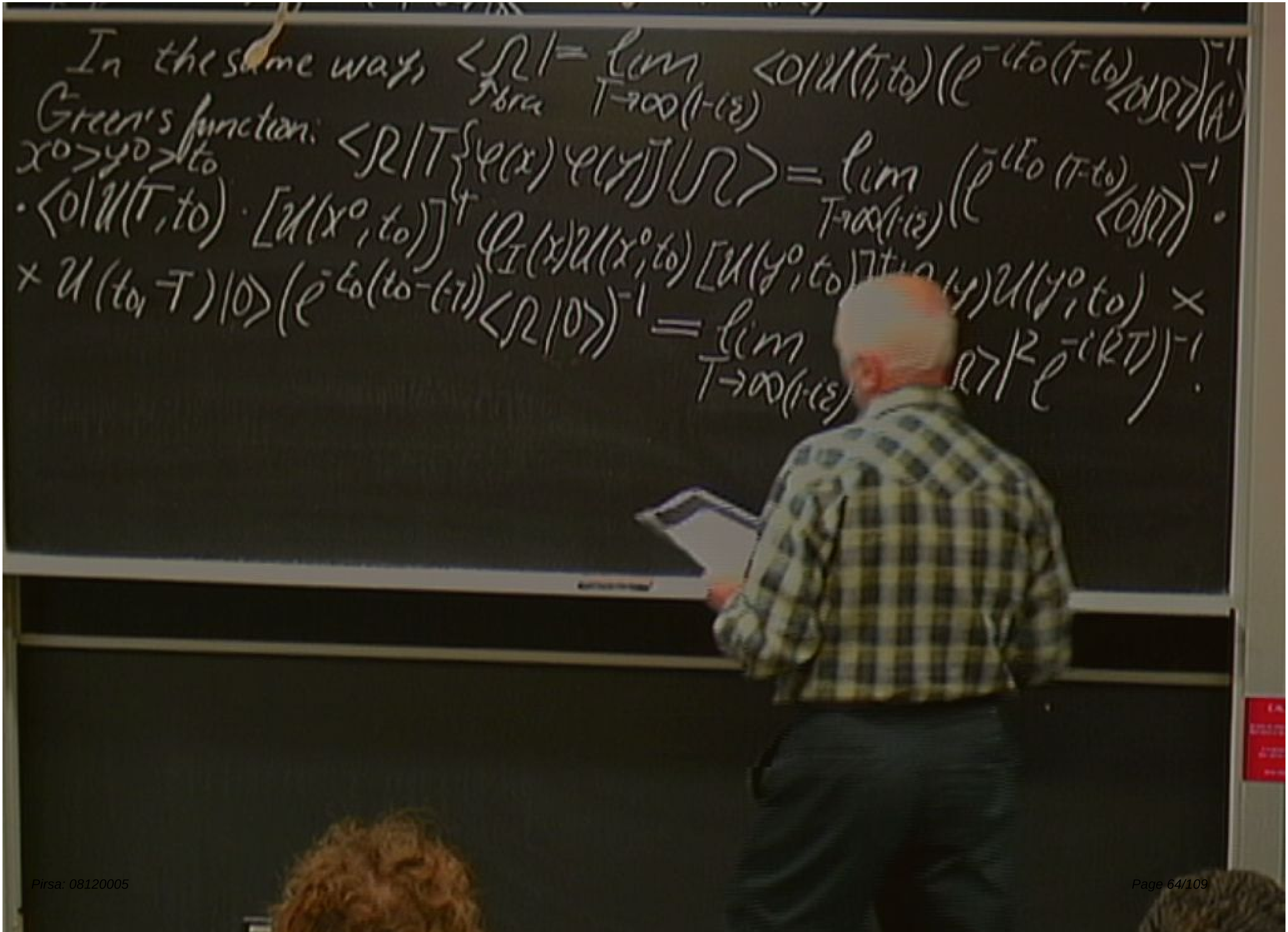
In the same way, $\langle\Omega| = \lim_{T\to\infty(1-i\varepsilon)} \langle 0|U(T,t_0)\left(e^{-iE_0(T-t_0)}\langle 0|\Omega\rangle\right)^{-1}$ (bra)

Green's function: $x^0 > y^0 > t_0$

$$\langle\Omega|T\{\varphi(x)\varphi(y)\}|\Omega\rangle = \lim_{T\to\infty(1-i\varepsilon)} \left(e^{-iE_0(T-t_0)}\langle 0|\Omega\rangle\right)^{-1} \cdot$$

$$\cdot \langle 0|U(T,t_0)\cdot[U(x^0,t_0)]^{+}\varphi_I(x)U(x^0,t_0)[U(y^0,t_0)]^{+}\varphi_I(y)U(y^0,t_0)\times$$

$$\times U(t_0,-T)|0\rangle\left(e^{-iE_0(t_0-(-T))}\langle\Omega|0\rangle\right)^{-1} = \lim_{T\to\infty(1-i\varepsilon)}$$


In the same way, <Ω| = lim <0|U(T,t0)(e^-iE0(T-t0)<0|Ω>)^-1
bra T→∞(1-iε)
Green's function: <Ω|T{φ(x)φ(y)}|Ω> = lim (e^-iE0(T-t0)<0|Ω>)^-1 ·
x0>y0>t0 T→∞(1-iε)
·<0|U(T,t0)·[U(x0,t0)]† φI(x)U(x0,t0)[U(y0,t0)]† φI(y)U(y0,t0) ×
× U(t0,-T)|0>(e^-iE0(t0-(-T))<Ω|0>)^-1 = lim
T→∞(1-iε)

In the same way, $\langle\Omega| = \lim_{T\to\infty(1-i\varepsilon)} \langle 0|U(T,t_0)\left(e^{-iE_0(T-t_0)}\langle 0|\Omega\rangle\right)^{-1}$ (A')

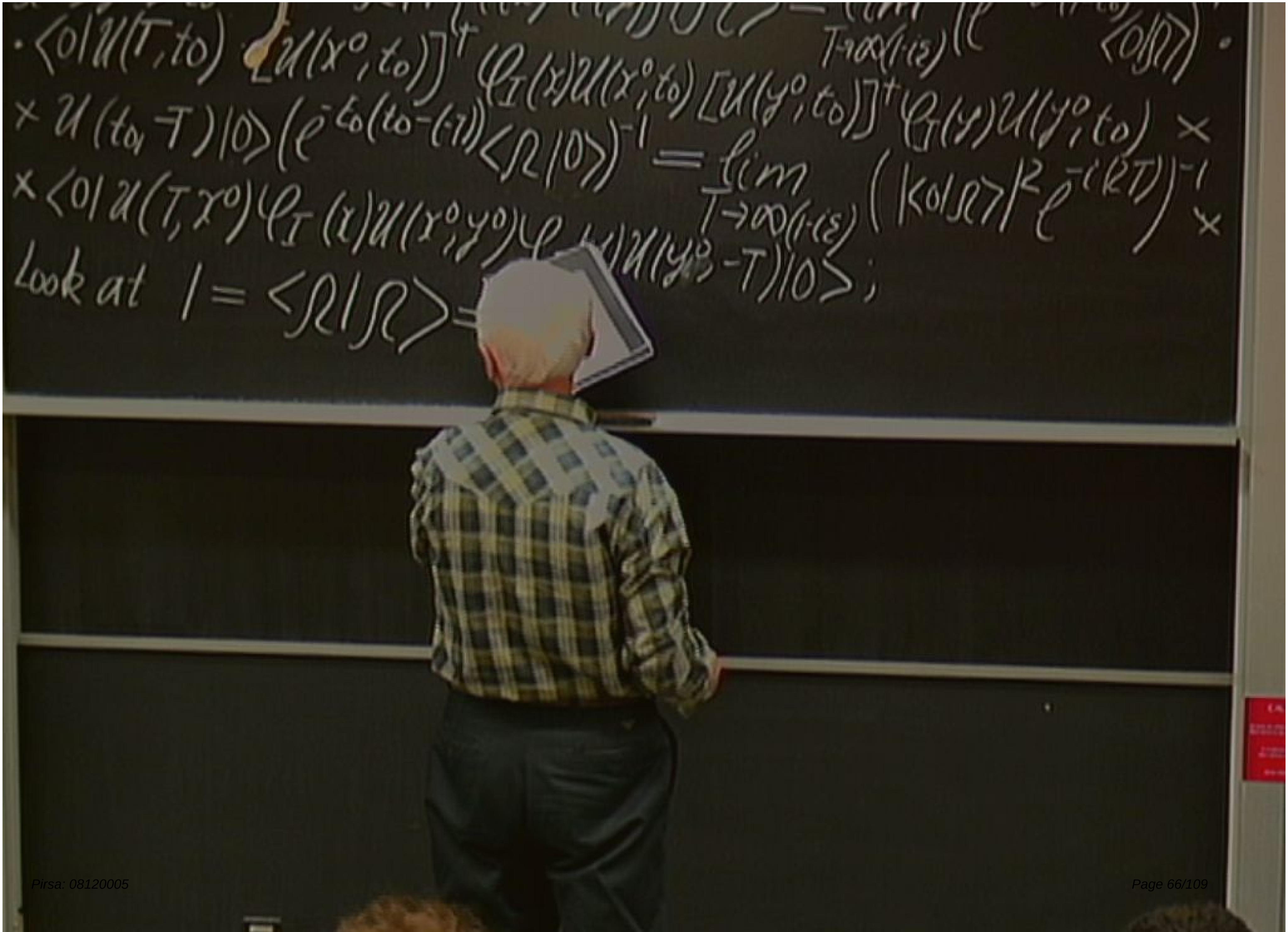
Green's function: $\langle\Omega|T\{\varphi(x)\varphi(y)\}|\Omega\rangle = \lim_{T\to\infty(1-i\varepsilon)}\left(e^{-iE_0(T-t_0)}\langle 0|\Omega\rangle\right)^{-1}\cdot$

$x^0>y^0>t_0$

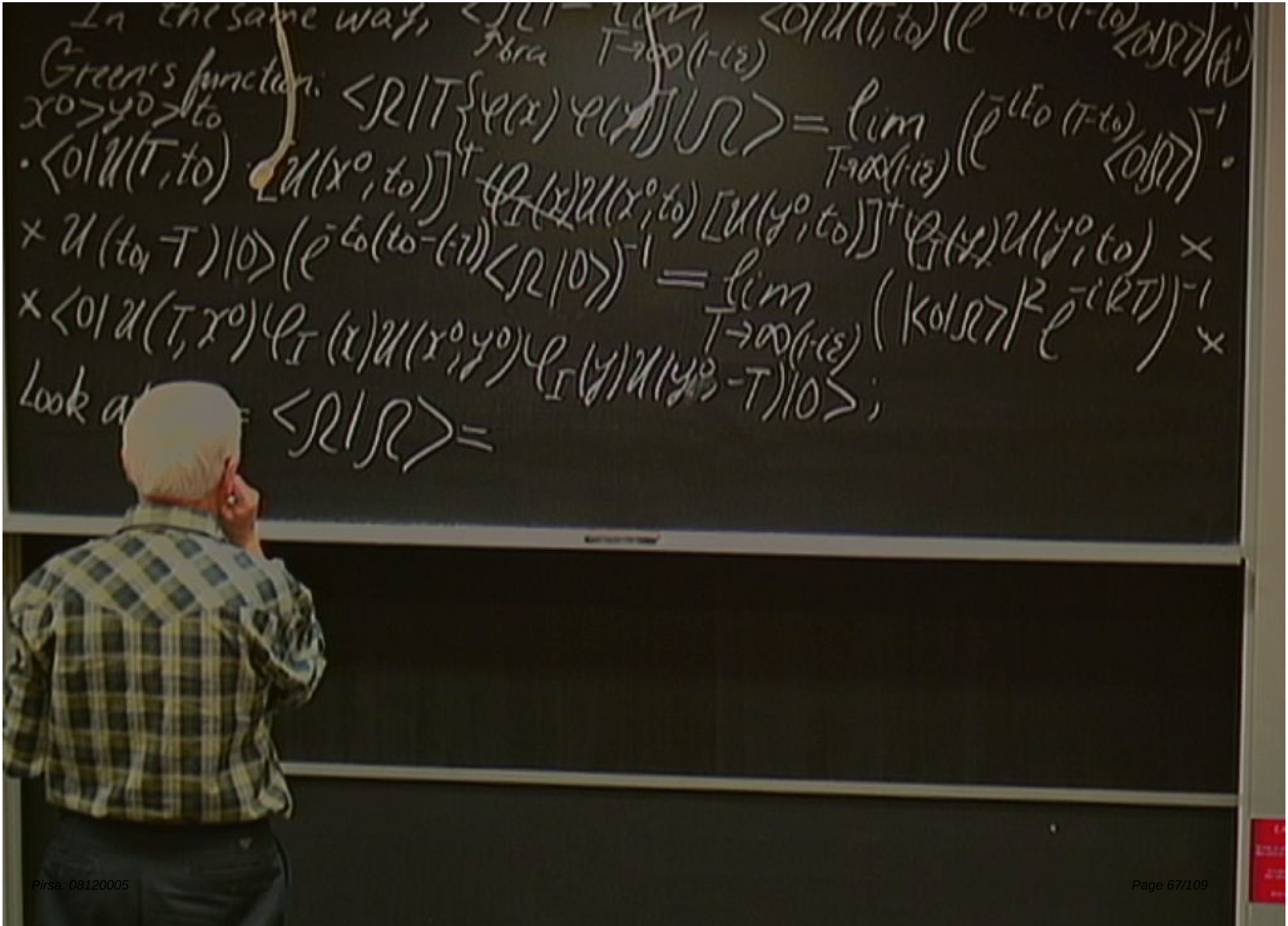
$\cdot\langle 0|U(T,t_0)\cdot[U(x^0,t_0)]^{+}\varphi_I(x)U(x^0,t_0)[U(y^0,t_0)]^{+}\varphi_I(y)U(y^0,t_0)\times$

$\times U(t_0,-T)|0\rangle\left(e^{-iE_0(t_0-(-T))}\langle\Omega|0\rangle\right)^{-1} = \lim_{T\to\infty(1-i\varepsilon)}\left(|\langle 0|\Omega\rangle|^2 e^{-iE(2T)}\right)^{-1}\times$

$\times\langle 0|U(T,x^0)\varphi_I(x)U(x^0,y^0)\varphi_I(y)U(y^0,-T)|0\rangle$



× <0| U(T, x⁰) φ_I (x) U(x⁰, y⁰) φ_I (y) U(y⁰, -T) |0>;
Look at 1 = <Ω|Ω> =



In the same way, <Ω| = lim <0|U(T,t0)(e^{-iE0(T-t0)}<0|Ω>)^{-1}
bra T→∞(1-iε)
Green's function: <Ω|T{φ(x)φ(y)}|Ω> = lim (e^{-iE0(T-t0)}<0|Ω>)^{-1} ·
x0 > y0 > t0 T→∞(1-iε)
· <0|U(T,t0) [U(x0,t0)]^† φI(x)U(x0,t0) [U(y0,t0)]^† φI(y)U(y0,t0) ×
× U(t0,-T)|0>(e^{-iE0(t0-(-T))}<Ω|0>)^{-1} = lim (|<0|Ω>|^2 e^{-iE0(2T)})^{-1} ×
× <0|U(T,x0)φI(x)U(x0,y0)φI(y)U(y0,-T)|0>; T→∞(1-iε)
Look at <Ω|Ω> =

$$\ldots \left(T_I(\ldots\mathcal{U}(x^0,t_0)\right)[\mathcal{U}(y^0;t_0)]^{\dagger}\phi_I(y)\,\mathcal{U}(y^0;t_0)\times$$

$$\times\,\mathcal{U}(t_0,-T)|0\rangle\left(e^{-iE_0(t_0-(-T))}\langle\Omega|0\rangle\right)^{-1} = \lim_{T\to\infty(1-i\epsilon)}\left(|\langle 0|\Omega\rangle|^2 e^{-iE(2T)}\right)^{-1}\times$$

$$\times\langle 0|\mathcal{U}(T,x^0)\,\phi_I(x)\,\mathcal{U}(x^0,y^0)\,\phi_I(y)\,\mathcal{U}(y^0,-T)|0\rangle;$$

Look at $1=\langle\Omega|\Omega\rangle=\left(|\langle 0|\Omega\rangle|^2 e^{-iE(2T)}\right)^{-1}\cdot\langle 0|\mathcal{U}(T,y^0)\,\mathcal{U}(y^0,-T)|0\rangle$

$$=\left(|\langle 0|\Omega\rangle|^2 e^{-iE(2T)}\right)^{-1}\langle 0|\mathcal{U}(T,-T)|0\rangle.$$

Green's function

$x^0 > y^0 > t_0$ $\langle\Omega| T\{\varphi(x)\varphi(y)\}|\Omega\rangle = \lim_{T\to\infty(1-i\varepsilon)} \left(e^{-iE_0(T-t_0)}\langle 0|\Omega\rangle\right)^{-1} \cdot$

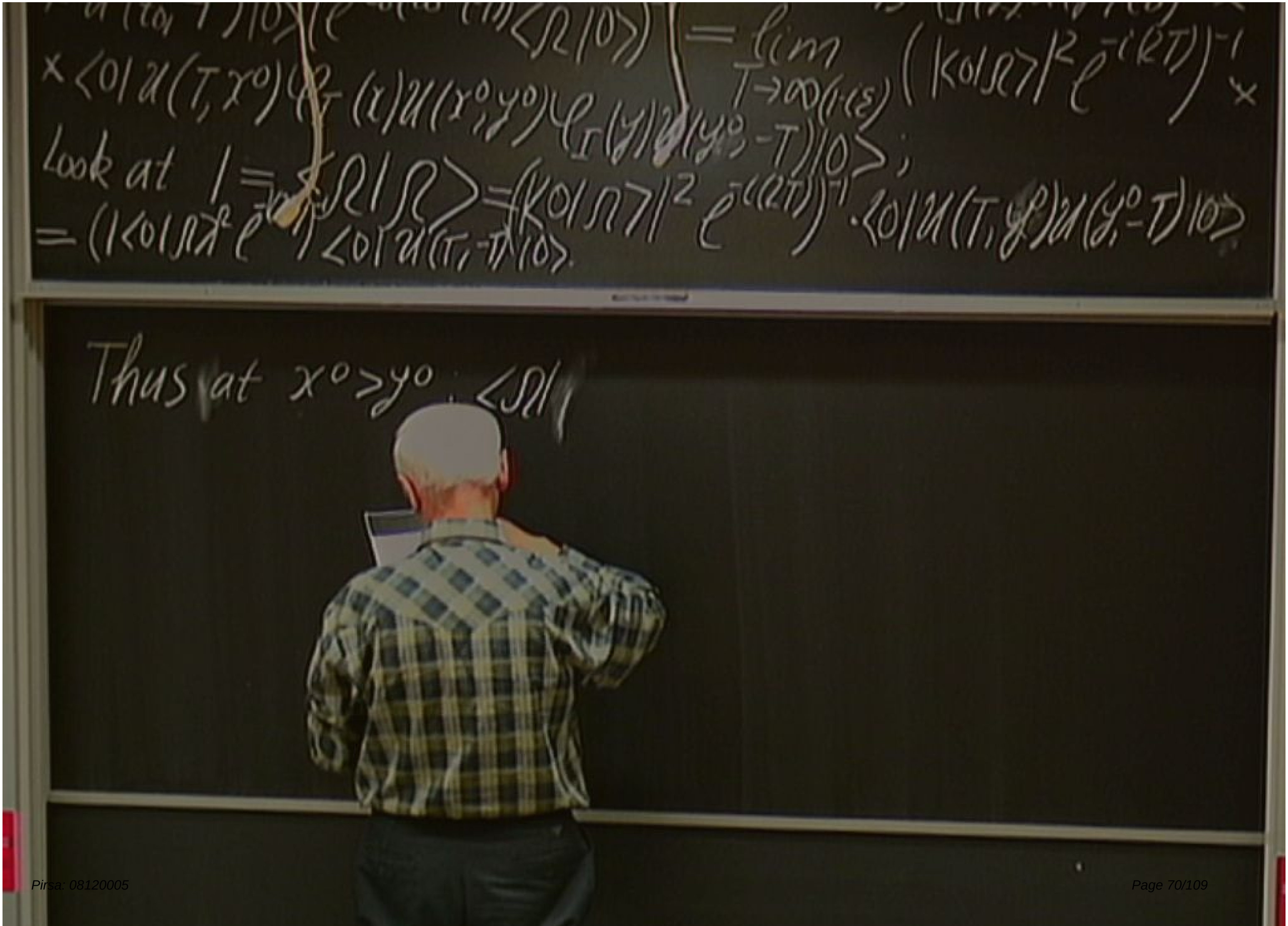
$\cdot \langle 0|U(T,t_0)\cdot[U(x^0,t_0)]^{+}\varphi_I(x)U(x^0,t_0)[U(y^0,t_0)]^{+}\varphi_I(y)U(y^0,t_0)\times$

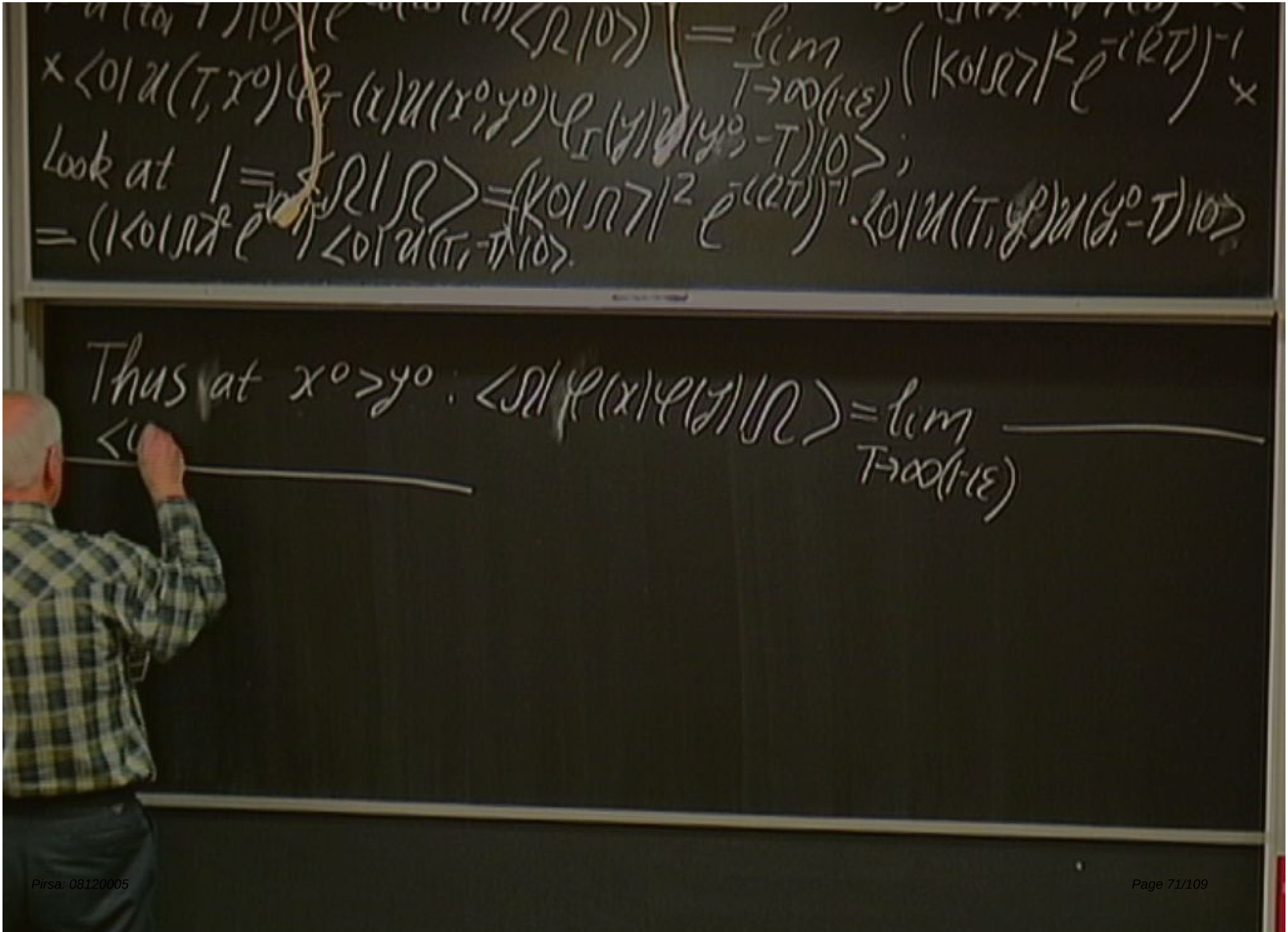
$\times U(t_0,-T)|0\rangle\left(e^{-iE_0(t_0-(-T))}\langle\Omega|0\rangle\right)^{-1} = \lim_{T\to\infty(1-i\varepsilon)}\left(|\langle 0|\Omega\rangle|^2 e^{-i(2T)}\right)^{-1}\times$

$\times\langle 0|U(T,x^0)\varphi_I(x)U(x^0,y^0)\varphi_I(y)U(y^0,-T)|0\rangle;$

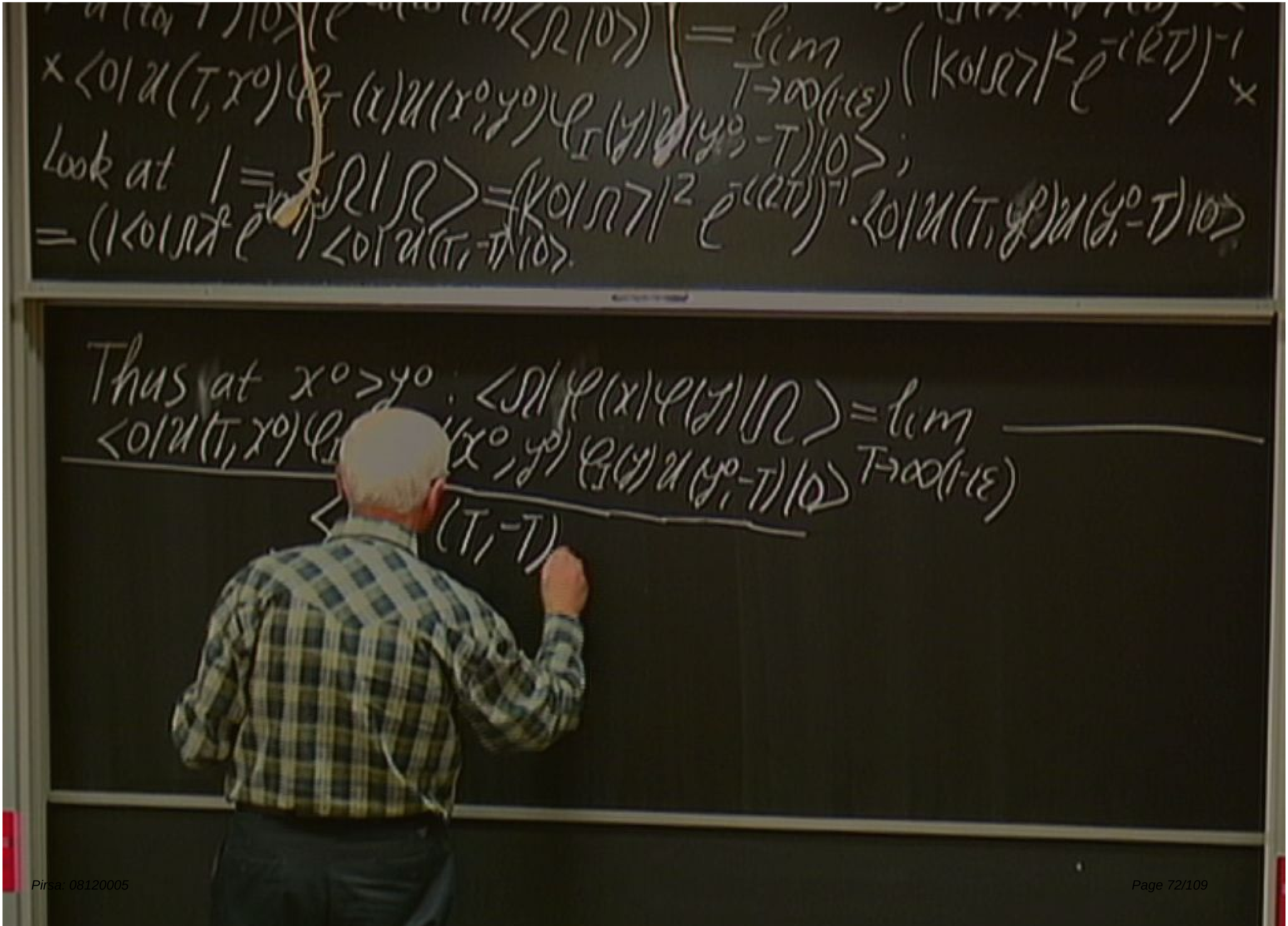
Look at $1 = \langle\Omega|\Omega\rangle = \left(|\langle 0|\Omega\rangle|^2 e^{-i(2T)}\right)^{-1}\cdot\langle 0|U(T,y^0)U(y^0,-T)|0\rangle$

$= \left(|\langle 0|\Omega\rangle|^2 e^{-i(2T)}\right)^{-1}\langle 0|U(T,-T)|0\rangle.$


= lim T→∞(1-iε) (|⟨0|Ω⟩|² e^{-iE(2T)})^{-1} ×
× ⟨0| U(T, x⁰) φ_I(x) U(x⁰, y⁰) φ_I(y) U(y⁰, -T)|0⟩;
Look at 1 = ⟨Ω|Ω⟩ = (|⟨0|Ω⟩|² e^{-iE(2T)})^{-1} · ⟨0|U(T, y⁰) U(y⁰, -T)|0⟩
= (|⟨0|Ω⟩|² e^{-iE(2T)})^{-1} ⟨0|U(T, -T)|0⟩.
Thus at x⁰ > y⁰, ⟨Ω|



Look at
Thus at x⁰ > y⁰ : ⟨Ω| φ(x) φ(y) |Ω⟩ = lim T→∞(1-iε)


Look at
Thus at $x^0 > y^0$: $\langle\Omega|\varphi(x)\varphi(y)|\Omega\rangle = \lim_{T\to\infty(1-i\varepsilon)}$
$(T,-T)$

$$\ldots u(t_0, -T)|0\rangle (e^{\ldots} \langle\Omega|0\rangle) = \lim_{T\to\infty(1-i\varepsilon)} \left(|\langle 0|\Omega\rangle|^2 e^{-i(\Omega T)}\right)^{-1} \times$$

$$\times \langle 0| u(T, x^0) \varphi_I(x) u(x^0, y^0) \varphi_I(y) u(y^0, -T)|0\rangle ;$$

Look at $1 = \langle\Omega|\Omega\rangle = \left(|\langle 0|\Omega\rangle|^2 e^{-i(\Omega T)}\right)^{-1} \cdot \langle 0| u(T, y^0) u(y^0, -T)|0\rangle$

$= \left(|\langle 0|\Omega\rangle|^2 e^{\ldots}\right)^{-1} \langle 0| u(T, -T)|0\rangle .$

Thus at $x^0 > y^0$: $\langle\Omega|\varphi(x)\varphi(y)|\Omega\rangle = \lim_{T\to\infty(1-i\varepsilon)} \dfrac{\langle 0| u(T, x^0) \varphi_I(x) u(x^0, y^0) \varphi_I(y) u(y^0, -T)|0\rangle}{\langle 0| u(T, -T)|0\rangle}$.

$\times \langle 0|U(T,x^0)\varphi_I(x)U(x^0,y^0)\varphi_I(y)U(y^0,-T)|0\rangle$;

$= \lim_{T\to\infty(1-i\varepsilon)} \left(|\langle 0|\Omega\rangle|^2 e^{-iE T}\right)^{-1} \times$

Look at $1 = \langle\Omega|\Omega\rangle = (|\langle 0|\Omega\rangle|^2 e^{-i(ET)})^{-1} \cdot \langle 0|U(T,y^0)U(y^0,-T)|0\rangle$

$= (|\langle 0|\Omega\rangle|^2 e^{-iET})^{-1}\langle 0|U(T,-T)|0\rangle$.

Thus at $x^0 > y^0$: $\langle\Omega|\varphi(x)\varphi(y)|\Omega\rangle = \lim_{T\to\infty(1-i\varepsilon)}$

$$\frac{\langle 0|U(T,x^0)\varphi_I(x)U(x^0,y^0)\varphi_I(y)U(y^0,-T)|0\rangle}{\langle 0|U(T,-T)|0\rangle}$$

Restoring T order, we get answer both for $x^0 > y^0$, $y^0 > x^0$

$\langle 0|\, u(t_0, -T)|0\rangle (e^{\ldots} \langle \Omega|0\rangle) = \lim_{T\to\infty(1-i\varepsilon)} \left(|\langle 0|\Omega\rangle|^2 e^{-i(ET)}\right)^{-1} \times$

$\times \langle 0|\, u(T, x^0)\, \varphi_I(x)\, u(x^0, y^0)\, \varphi_I(y)\, u(y^0, -T)|0\rangle;$

Look at $1 = \langle \Omega|\Omega\rangle = \lim \left(|\langle 0|\Omega\rangle|^2 e^{-i(ET)}\right)^{-1} \cdot \langle 0|\, u(T, y^0)\, u(y^0, -T)|0\rangle$

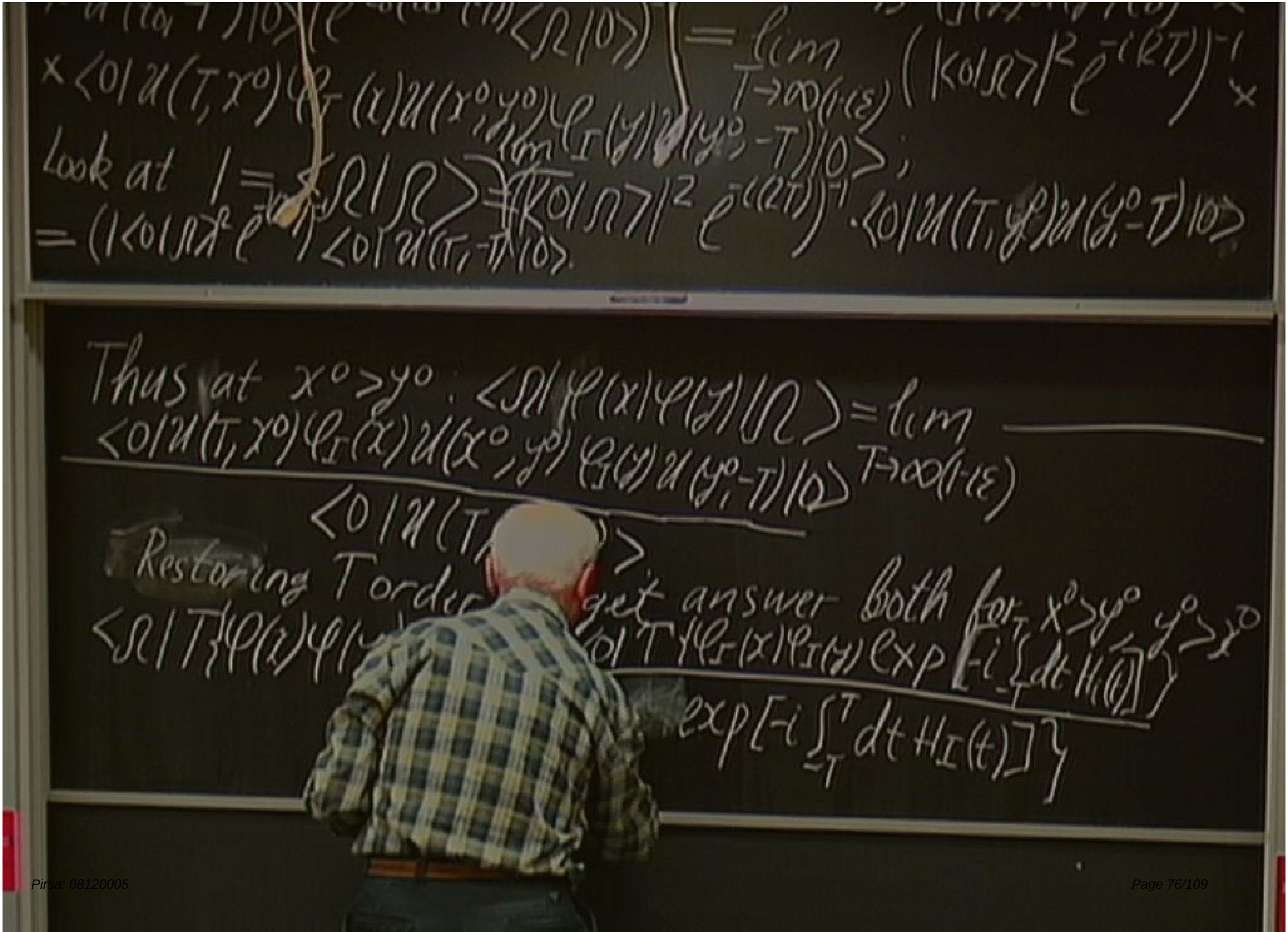
$= \left(|\langle 0|\Omega\rangle|^2 e^{-i(ET)}\right)^{-1} \langle 0|\, u(T, -T)|0\rangle.$

Thus at $x^0 > y^0$: $\langle \Omega|\, \varphi(x)\varphi(y)|\Omega\rangle = \lim_{T\to\infty(1-i\varepsilon)}$

$$\frac{\langle 0|\, u(T, x^0)\, \varphi_I(x)\, u(x^0, y^0)\, \varphi_I(y)\, u(y^0, -T)|0\rangle}{\langle 0|\, u(T, -T)|0\rangle}$$

Restoring T order, we get answer both for $x^0 > y^0$, $y^0 > x^0$

$\langle \Omega|\, T\{\varphi(x)\varphi(y)\}|\Omega\rangle = \dfrac{\langle 0|\, T\{\varphi_I(x)\varphi_I(y)\, \exp}{}$


Look at
Thus at
Restoring T order get answer both for

$$\times \langle 0| U(T,x^0)\varphi_I(x) U(x^0,y^0)\varphi_I(y) U(y^0,-T)|0\rangle ;\quad = \lim_{T\to\infty(1-i\varepsilon)} \left(|\langle 0|\Omega\rangle|^2 e^{-iE_0 T}\right)^{-1} \times$$

Look at $1 = \langle\Omega|\Omega\rangle = \left(|\langle 0|\Omega\rangle|^2 e^{-i(E_0 T)}\right)^{-1} \cdot \langle 0|U(T,y^0)U(y^0,-T)|0\rangle$

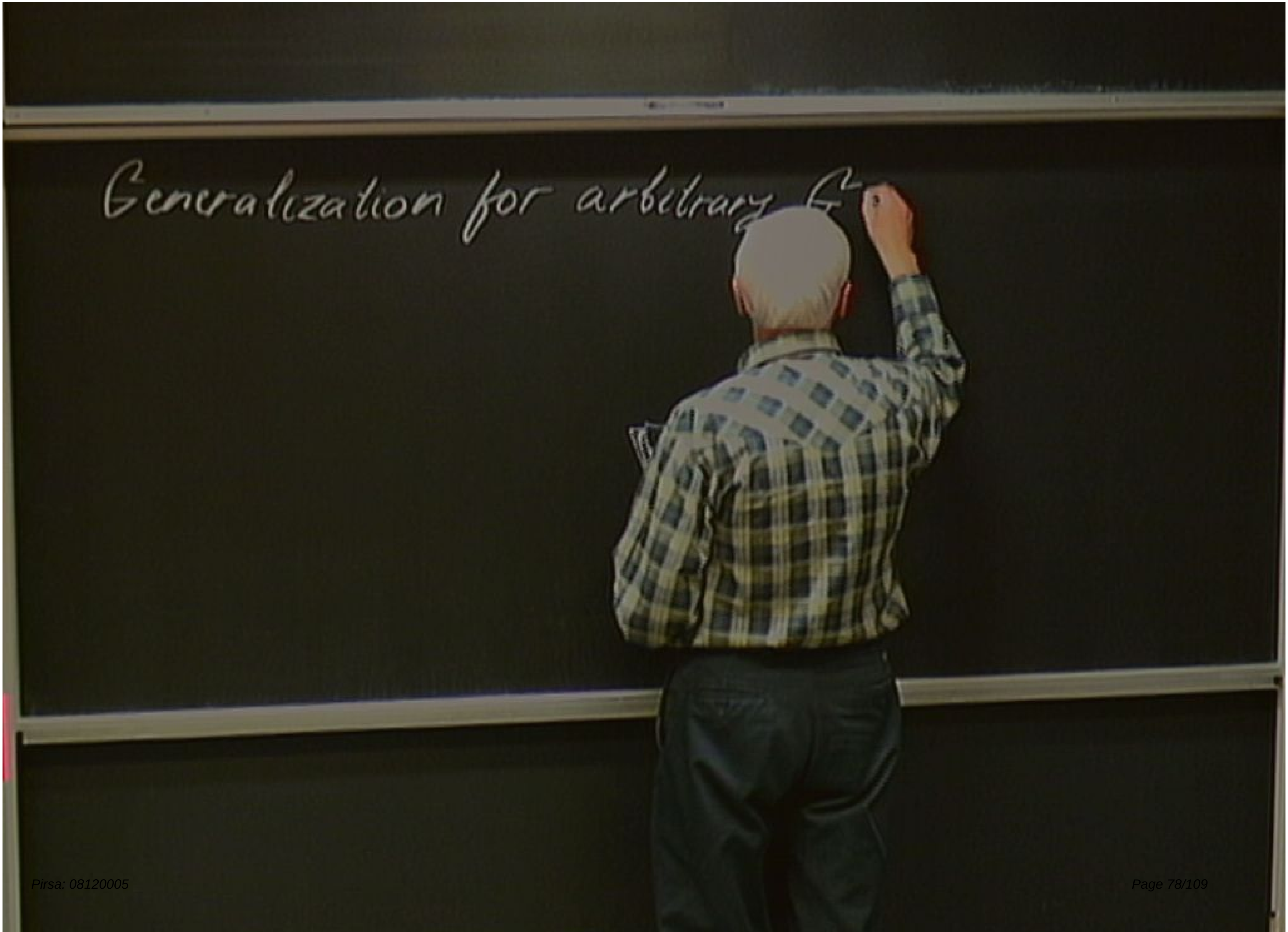
$= \left(|\langle 0|\Omega\rangle|^2 e^{-iE_0 T}\right)^{-1} \langle 0|U(T,-T)|0\rangle.$

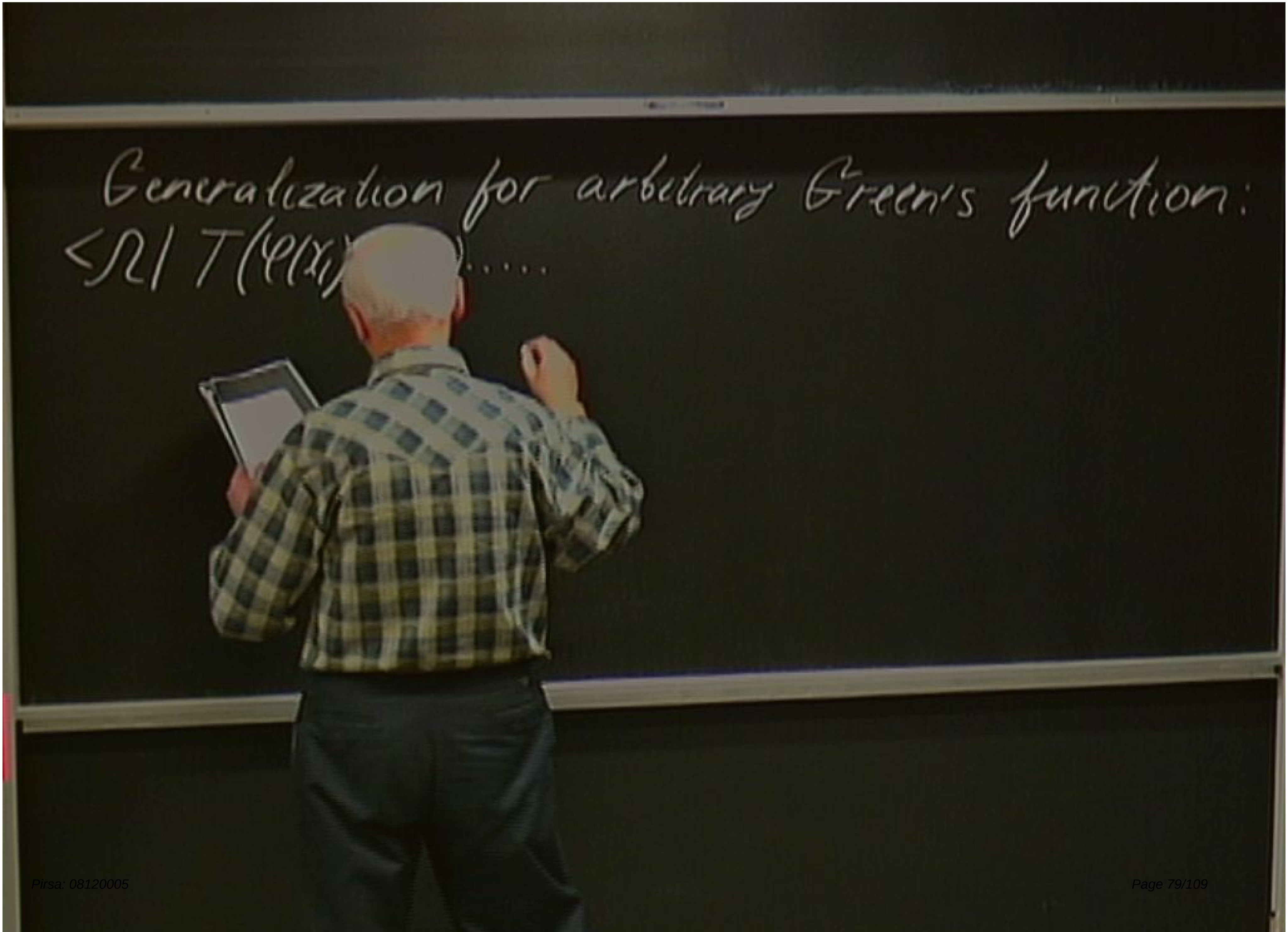
Thus at $x^0 > y^0$:

$$\langle\Omega|\varphi(x)\varphi(y)|\Omega\rangle = \lim_{T\to\infty(1-i\varepsilon)} \frac{\langle 0|U(T,x^0)\varphi_I(x)U(x^0,y^0)\varphi_I(y)U(y^0,-T)|0\rangle}{\langle 0|U(T,-T)|0\rangle}$$

Restoring T order, we get answer both for $x^0 > y^0$, $y^0 > x^0$

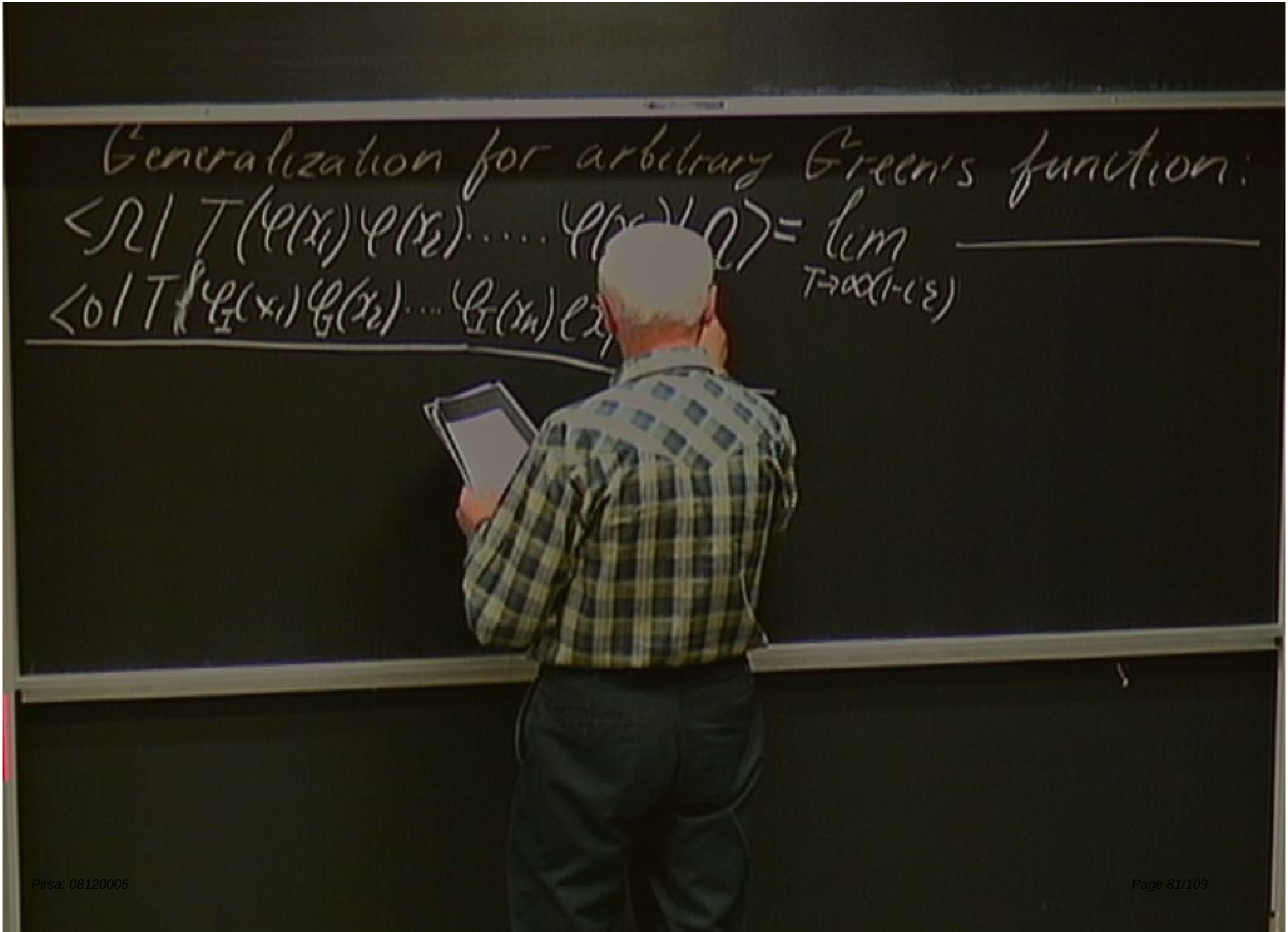
$$\langle\Omega|T\{\varphi(x)\varphi(y)\}|\Omega\rangle = \frac{\langle 0|T\{\varphi_I(x)\varphi_I(y)\exp[-i\int_{-T}^{T} dt\, H_I(t)]\}|0\rangle}{\langle 0|T\{\exp[-i\int_{-T}^{T} dt\, H_I(t)]\}|0\rangle}$$

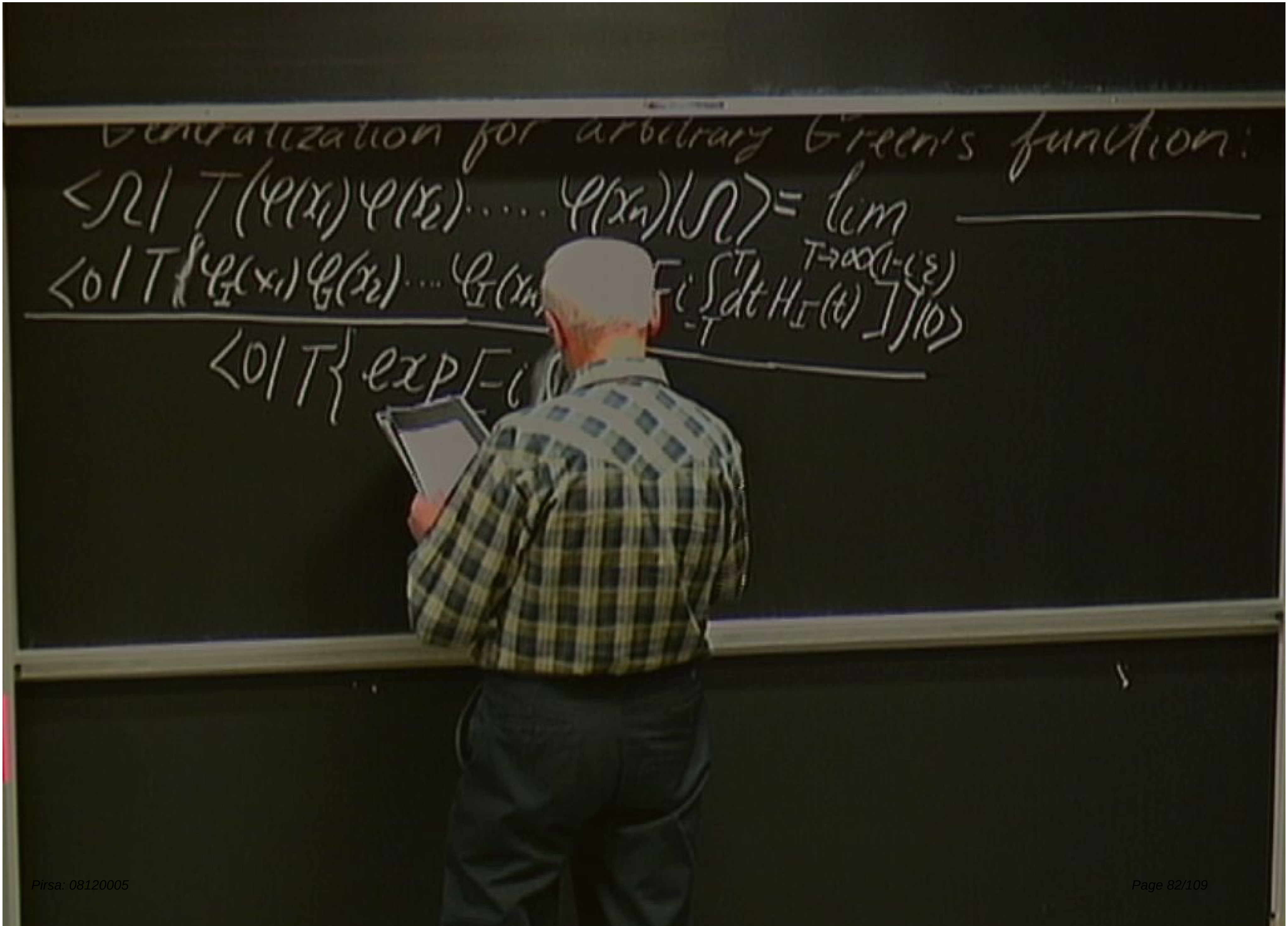

Generalization for arbitrary G

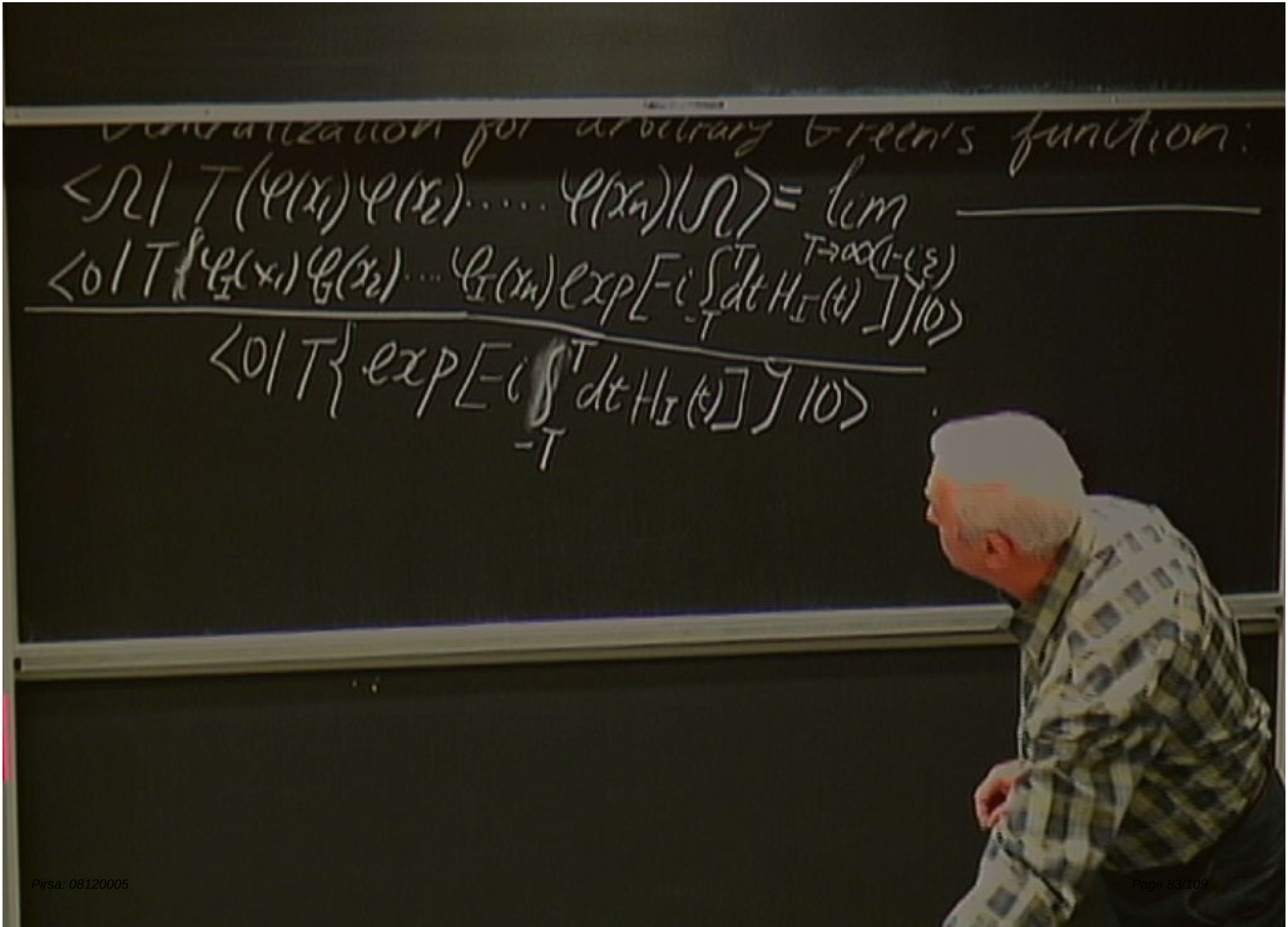

Generalization for arbitrary Green's function:
$\langle \Omega | T(\varphi(x_1)$ ......

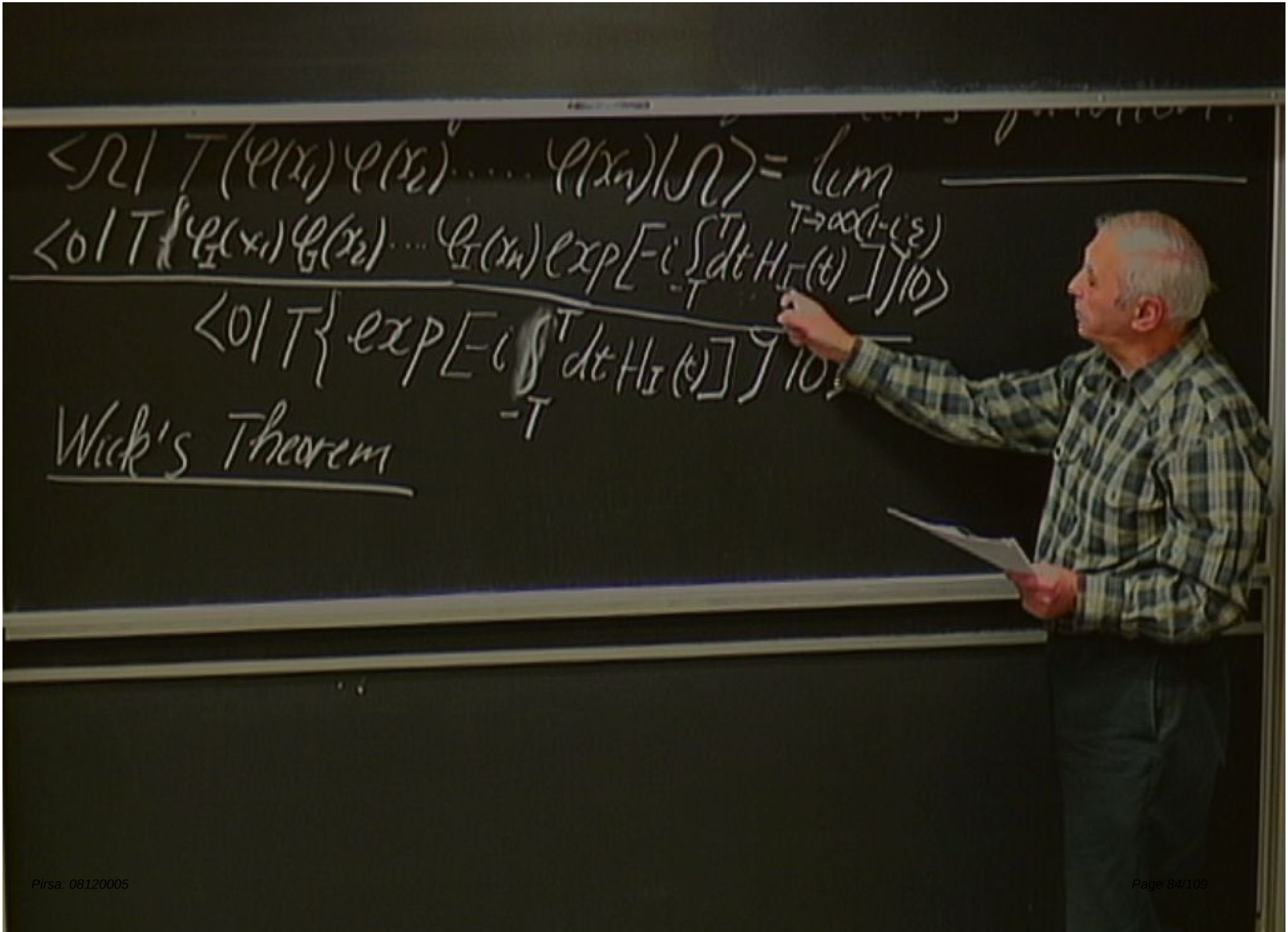
Generalization for arbitrary Green's function:

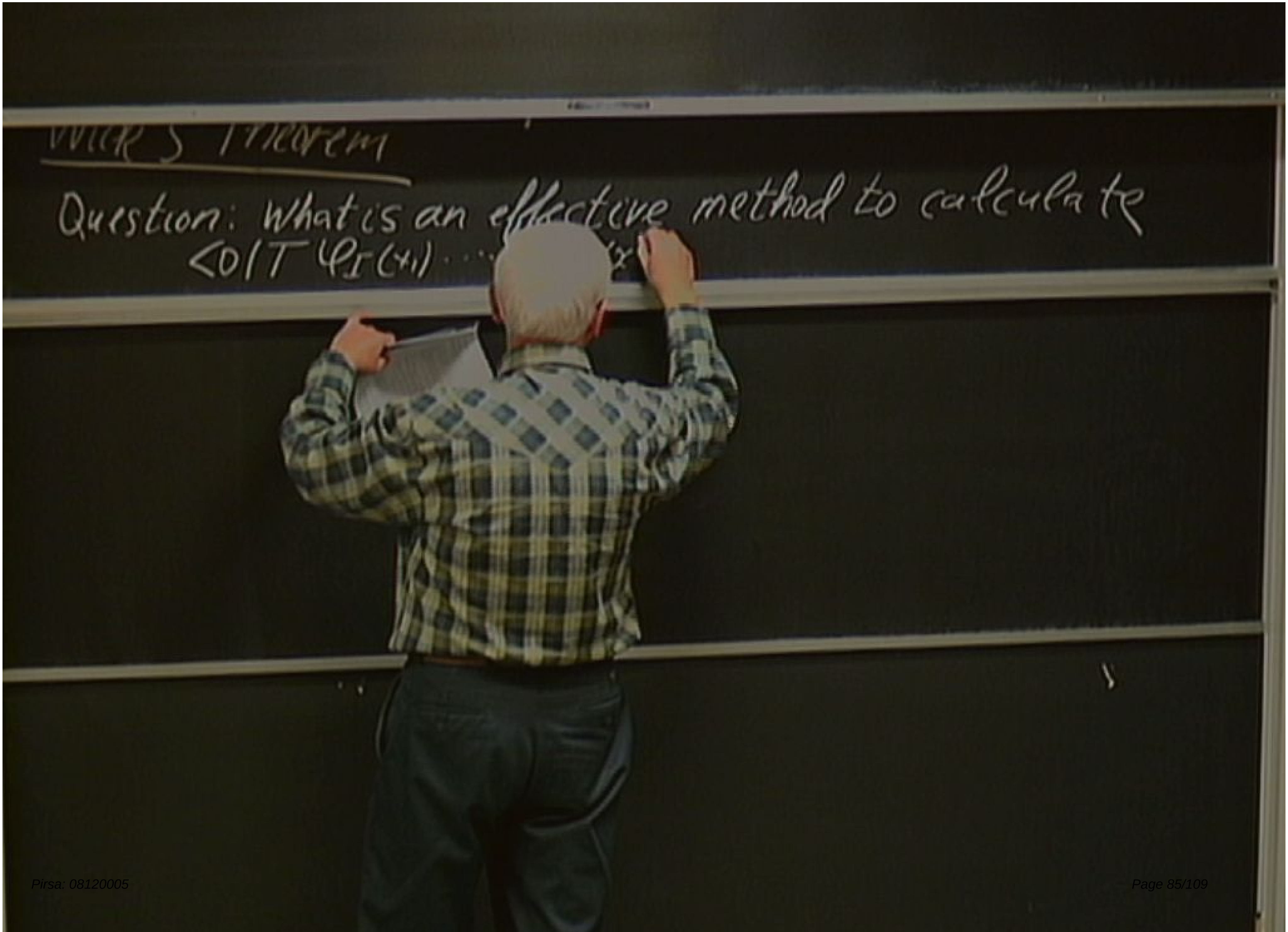
$$\langle \Omega | T(\varphi(x_1)\varphi(x_2)\cdots\cdots\varphi(x_n)) | \Omega \rangle = \lim$$


Generalization for arbitrary Green's function:
lim
T→∞(1−iε)


Generalization for arbitrary Green's function:
<Ω| T(φ(x1)φ(x2).....φ(xn)|Ω> = lim
T→∞(1−iε)
<0| T{φI(x1)φI(x2)....φI(xn) ... −i ∫ dt HI(t)]}|0>
<0| T{ exp[−i

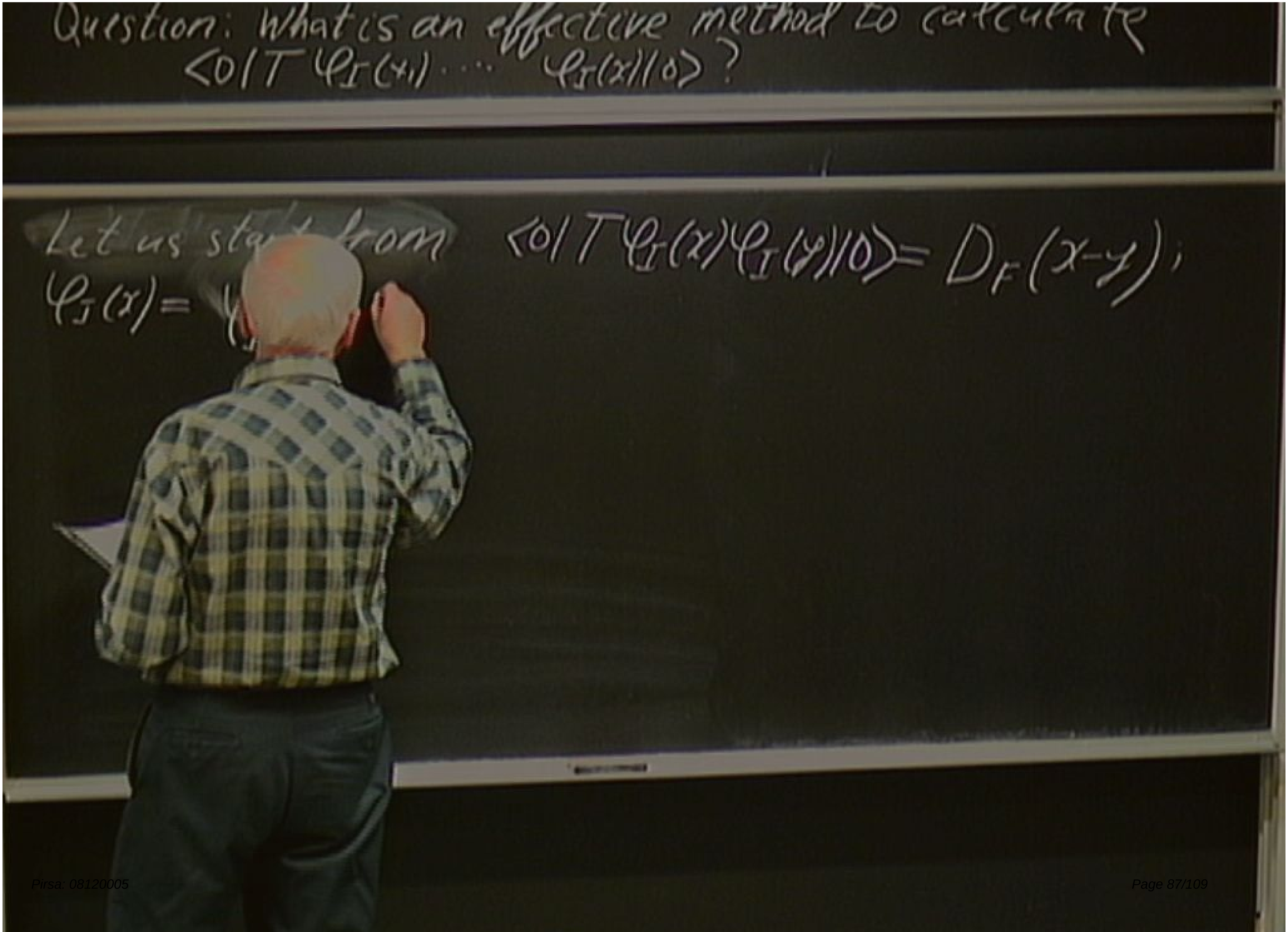

Generalization for arbitrary Green's function:
$\langle\Omega|T(\varphi(x_1)\varphi(x_2)\cdots\varphi(x_n))|\Omega\rangle = \lim_{T\to\infty(1-i\varepsilon)}$
$\langle 0|T\{\varphi_I(x_1)\varphi_I(x_2)\cdots\varphi_I(x_n)\exp[-i\int_{-T}^{T}dt\,H_I(t)]\}|0\rangle$
$\langle 0|T\{\exp[-i\int_{-T}^{T}dt\,H_I(t)]\}|0\rangle$

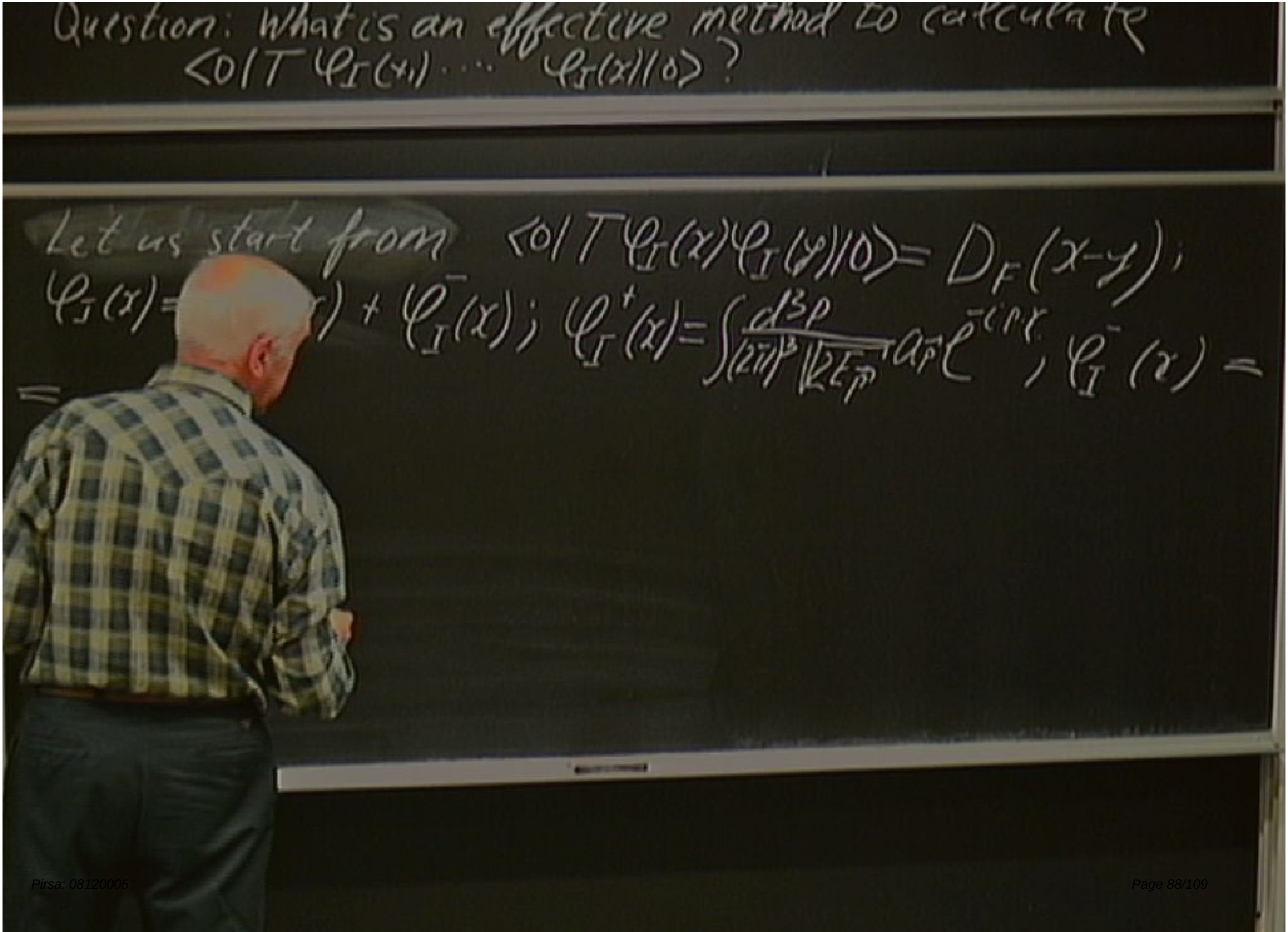

<Ω| T(φ(x1)φ(x2)......φ(xn)|Ω> = lim
T→∞(1-iε)
<0| T{φI(x1)φI(x2).....φI(xn) exp[-i ∫ dt HI(t)]}|0>
-T
<0| T{ exp[-i ∫ dt HI(t)]}|0>
-T
Wick's Theorem

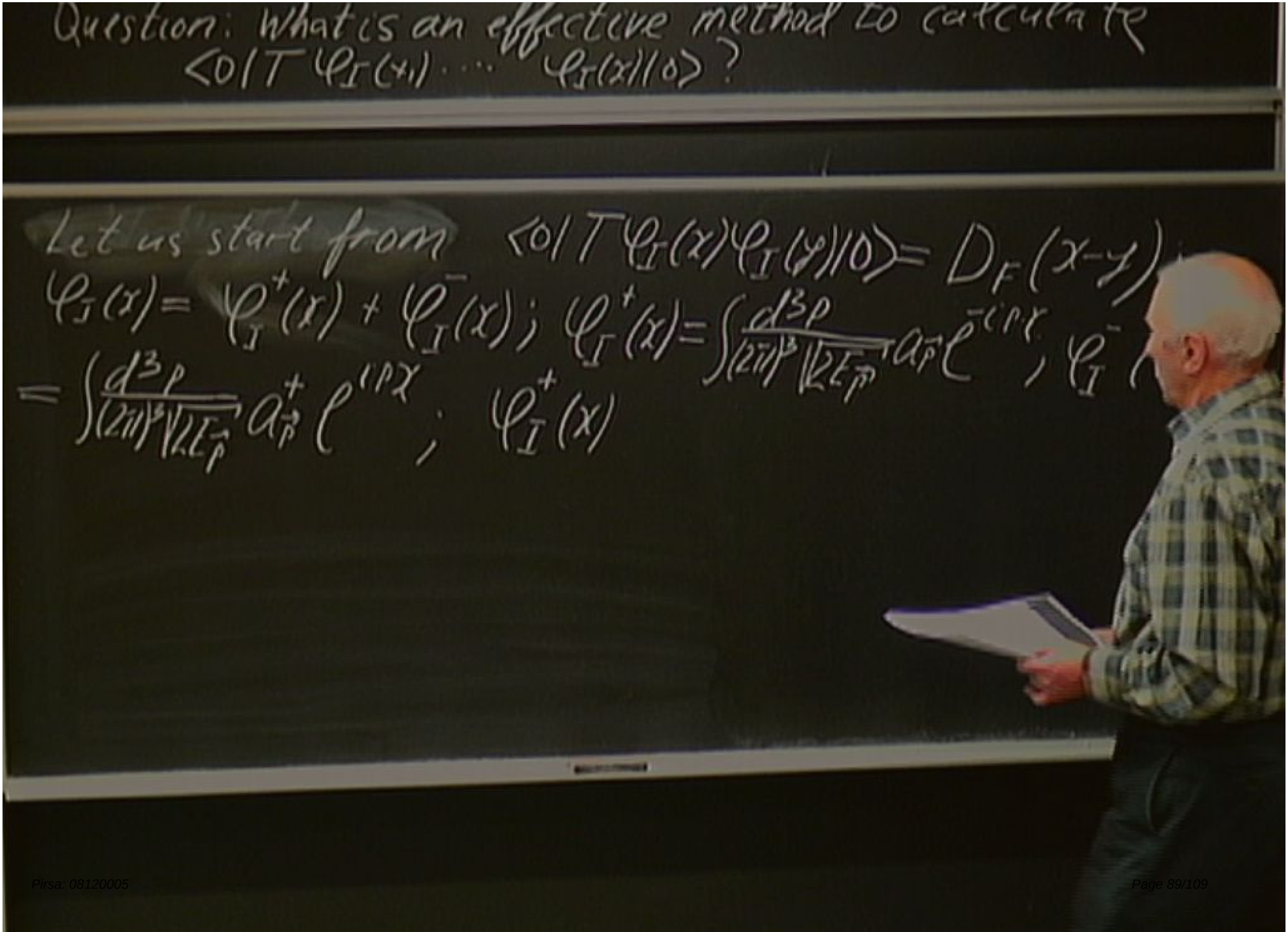

Wick's Theorem
Question: what is an effective method to calculate
<0|T φ_I(x_1) ...

Question: What is an effective method to calculate
$\langle 0|T\varphi_I(x_1)\cdots\varphi_I(x_n)|0\rangle$?

Let us start from $\langle 0|T\varphi_I(x)\varphi_I(y)|0\rangle = D_F(x-y)$


Question: What is an effective method to calculate
$\langle 0|T\varphi_I(x_1)\cdots\varphi_I(x_n)|0\rangle$?
Let us start from $\langle 0|T\varphi_I(x)\varphi_I(y)|0\rangle = D_F(x-y)$;
$\varphi_I(x) =$


Question: What is an effective method to calculate
<0|T φI(x1) ... φI(xl)|0> ?
Let us start from <0|T φI(x)φI(y)|0> = DF(x-y);
φI(x) = ...) + φI⁻(x); φI⁺(x) = ∫ d³p/((2π)³ √(2Ep)) ap e^(-ipx); φI⁻(x) =
=


Question: What is an effective method to calculate
<0|T φI(x1) ··· φI(xn)|0> ?
Let us start from <0|T φI(x)φI(y)|0> = DF(x−y)
φI(x) = φI+(x) + φI−(x); φI+(x) = ∫ d³p/((2π)³ √(2Ep)) ap e^(−ipx); φI−(x)
= ∫ d³p/((2π)³ √(2Ep)) ap+ e^(ipx); φI+(x)

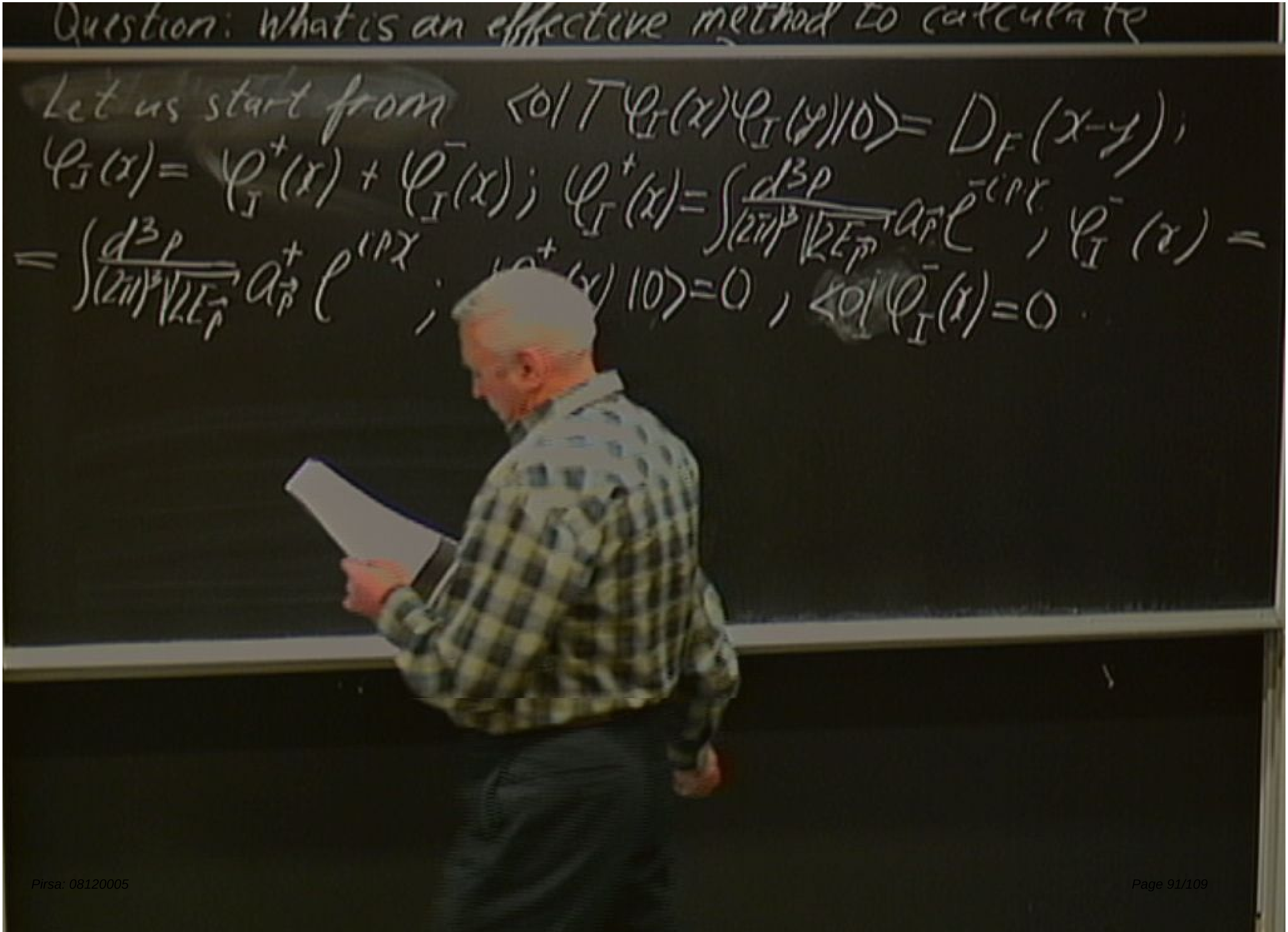
Question: What is an effective method to calculate

$\langle 0|T\varphi_I(x_1)\cdots\varphi_I(x_n)|0\rangle$?

Let us start from $\langle 0|T\varphi_I(x)\varphi_I(y)|0\rangle = D_F(x-y)$

$\varphi_I(x) = \varphi_I^+(x) + \varphi_I^-(x)$; $\varphi_I^+(x) = \int \frac{d^3p}{(2\pi)^3\sqrt{2E_{\vec p}}} a_{\vec p}\, e^{-ipx}$, $\varphi_I^-(x)$

$= \int \frac{d^3p}{(2\pi)^3\sqrt{2E_{\vec p}}} a_{\vec p}^+\, e^{ipx}$; $\varphi_I^+(x)|0\rangle = 0$, $\langle 0|\varphi_I^-(x) = 0$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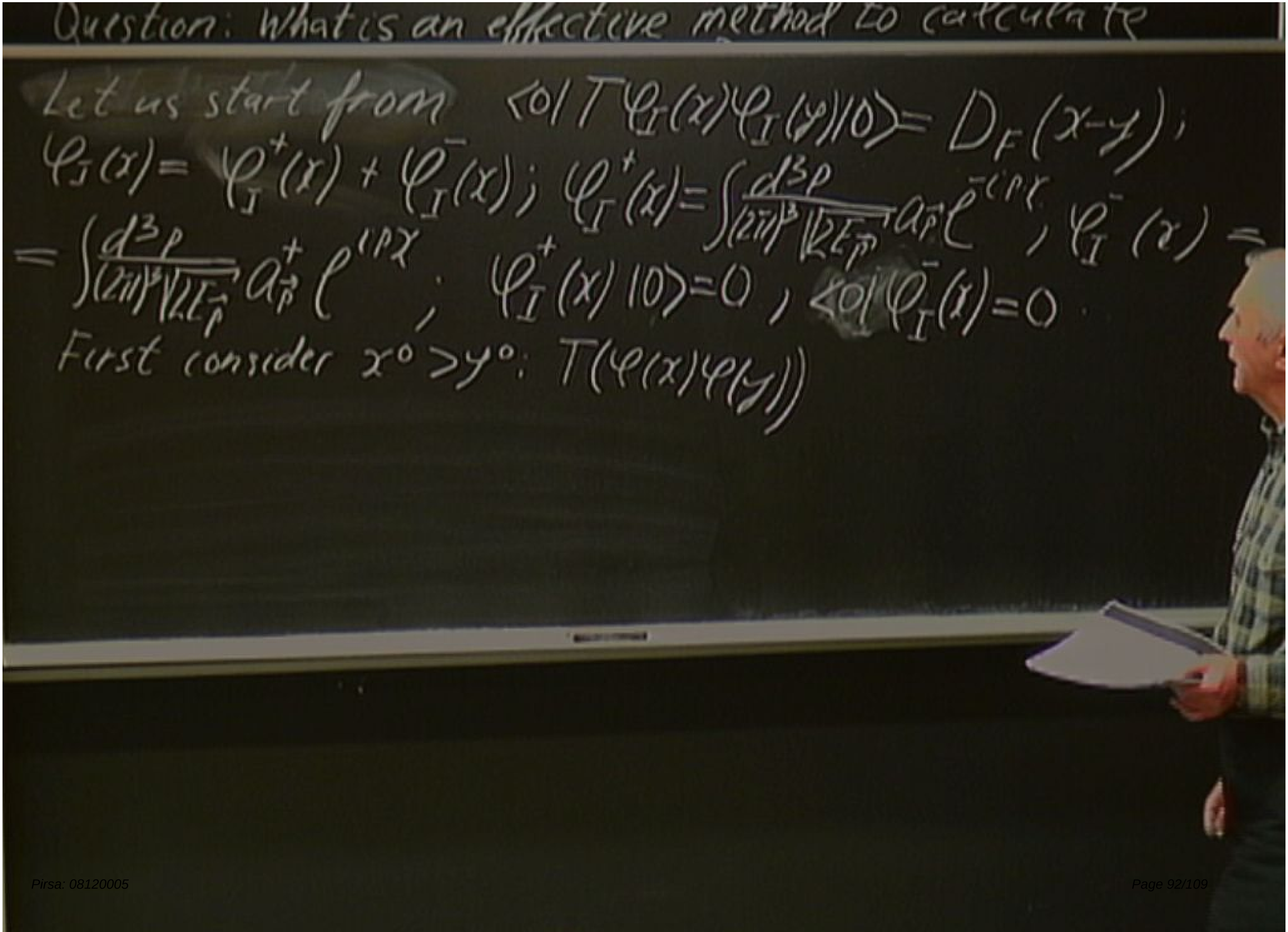
$= \int \frac{d^3p}{(2\pi)^3\sqrt{2E_{\vec{p}}}} a_{\vec{p}}^+ e^{ipx}$; $\varphi_I^+(x)|0\rangle = 0$, $\langle 0|\varphi_I^-(x) = 0$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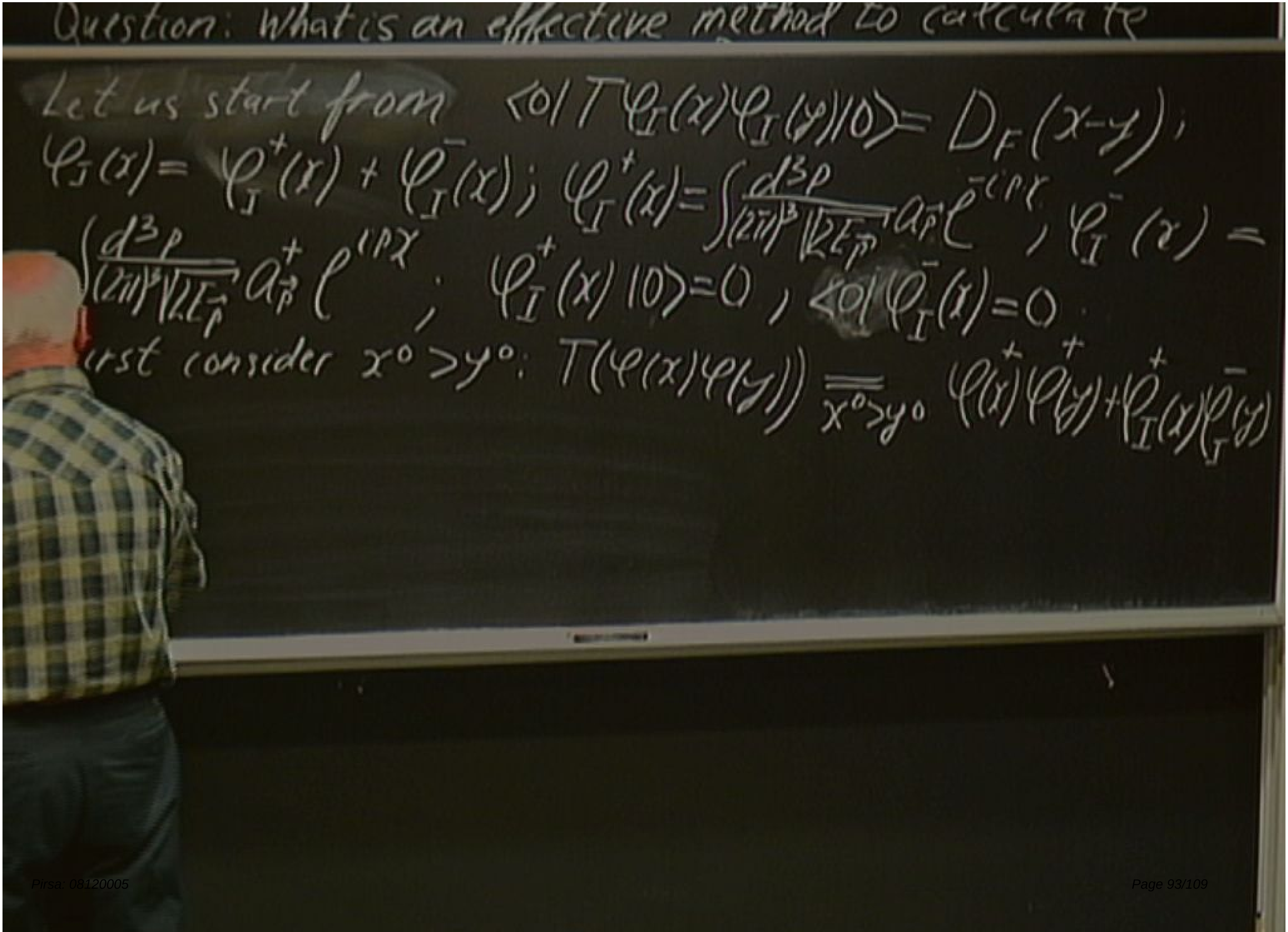
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$\int \frac{d^3p}{(2\pi)^3\sqrt{2E_{\vec p}}} a_{\vec p}^+ e^{ipx}$; $\varphi_I^+(x)|0\rangle = 0$, $\langle 0|\varphi_I^-(x) = 0$
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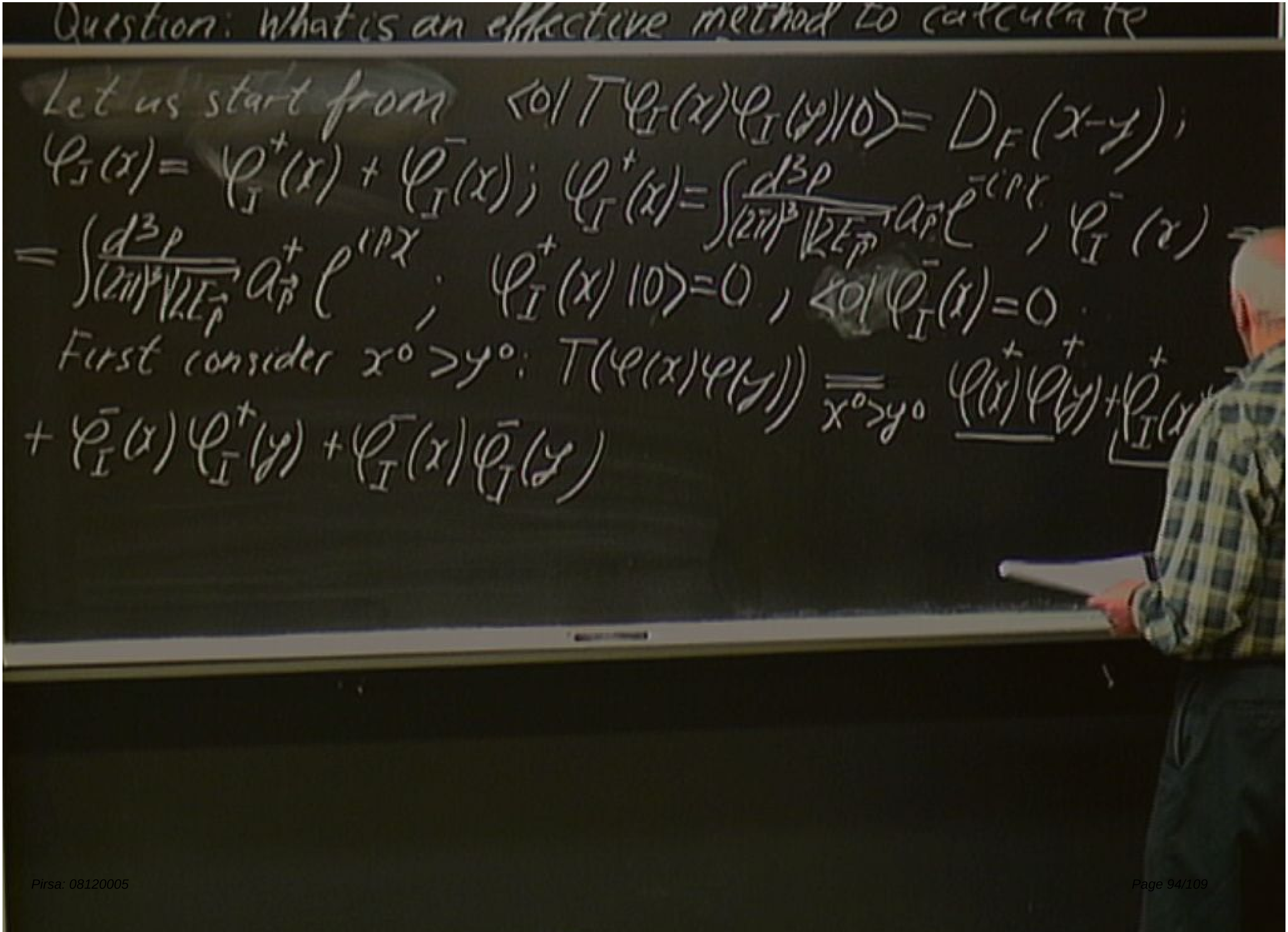
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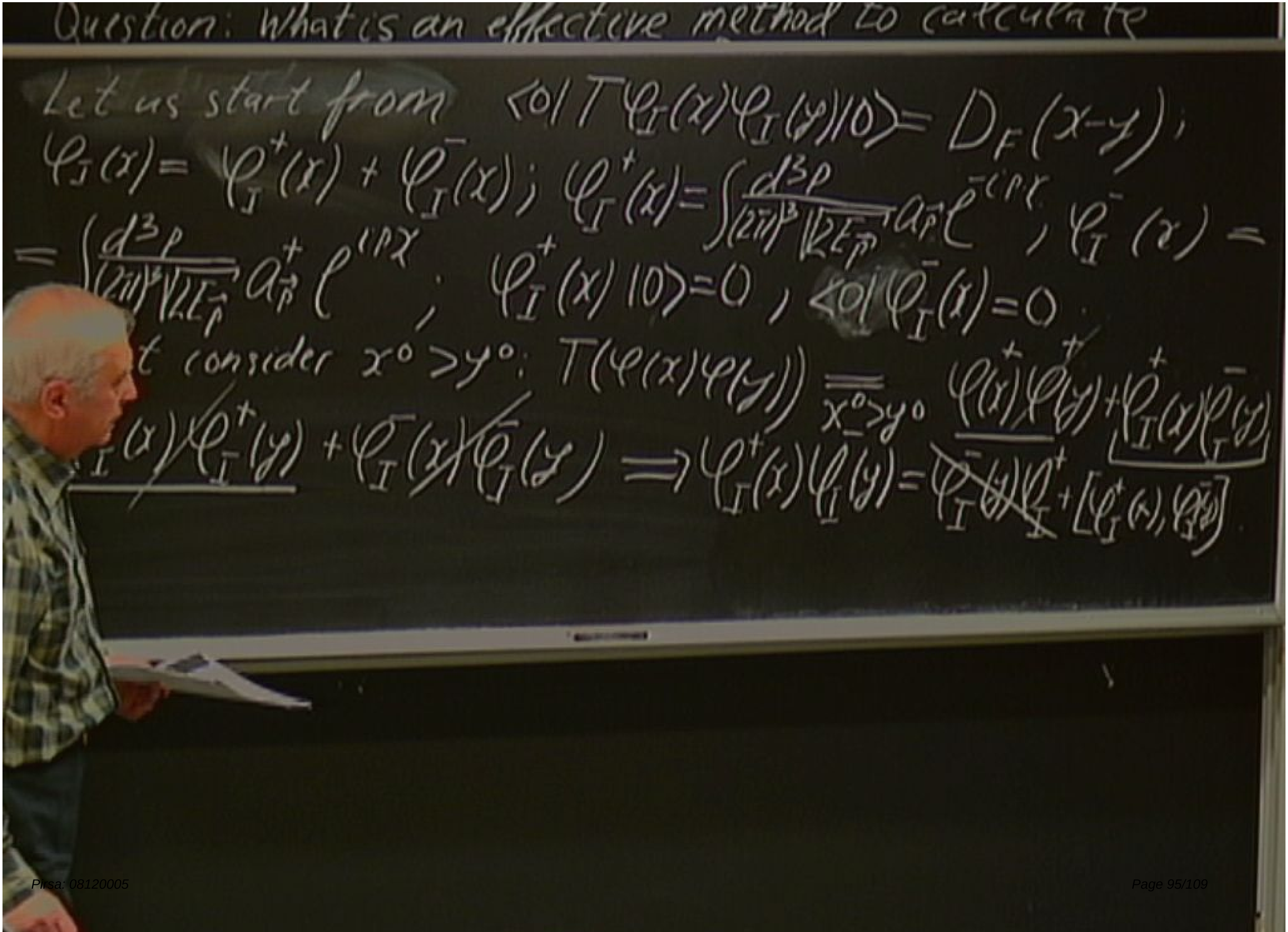
$\varphi_I(x) = \varphi_I^+(x) + \varphi_I^-(x)$; $\varphi_I^+(x) = \int \frac{d^3p}{(2\pi)^3\sqrt{2E_{\vec p}}} a_{\vec p} e^{-ipx}$, $\varphi_I^-(x)$
$= \int \frac{d^3p}{(2\pi)^3\sqrt{2E_{\vec p}}} a_{\vec p}^+ e^{ipx}$; $\varphi_I^+(x)|0\rangle = 0$, $\langle 0|\varphi_I^-(x) = 0$
First consider $x^0 > y^0$: $T(\varphi(x)\varphi(y)) \underset{x^0>y^0}{=} \varphi^+(x)\varphi^+(y) + \varphi_I^+(x)$
$+ \varphi_I^-(x)\varphi_I^+(y) + \varphi_I^-(x)\varphi_I^-(y)$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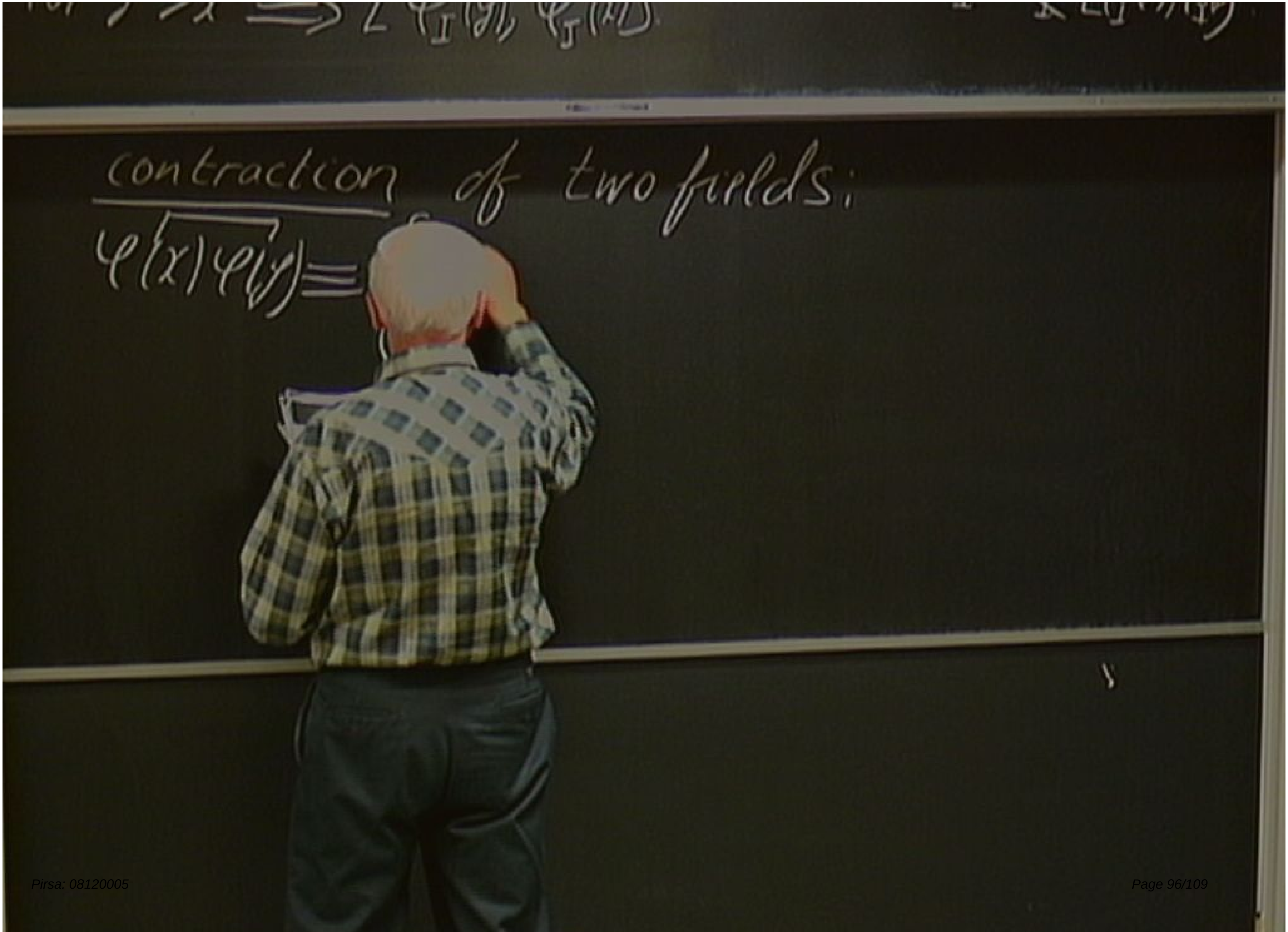
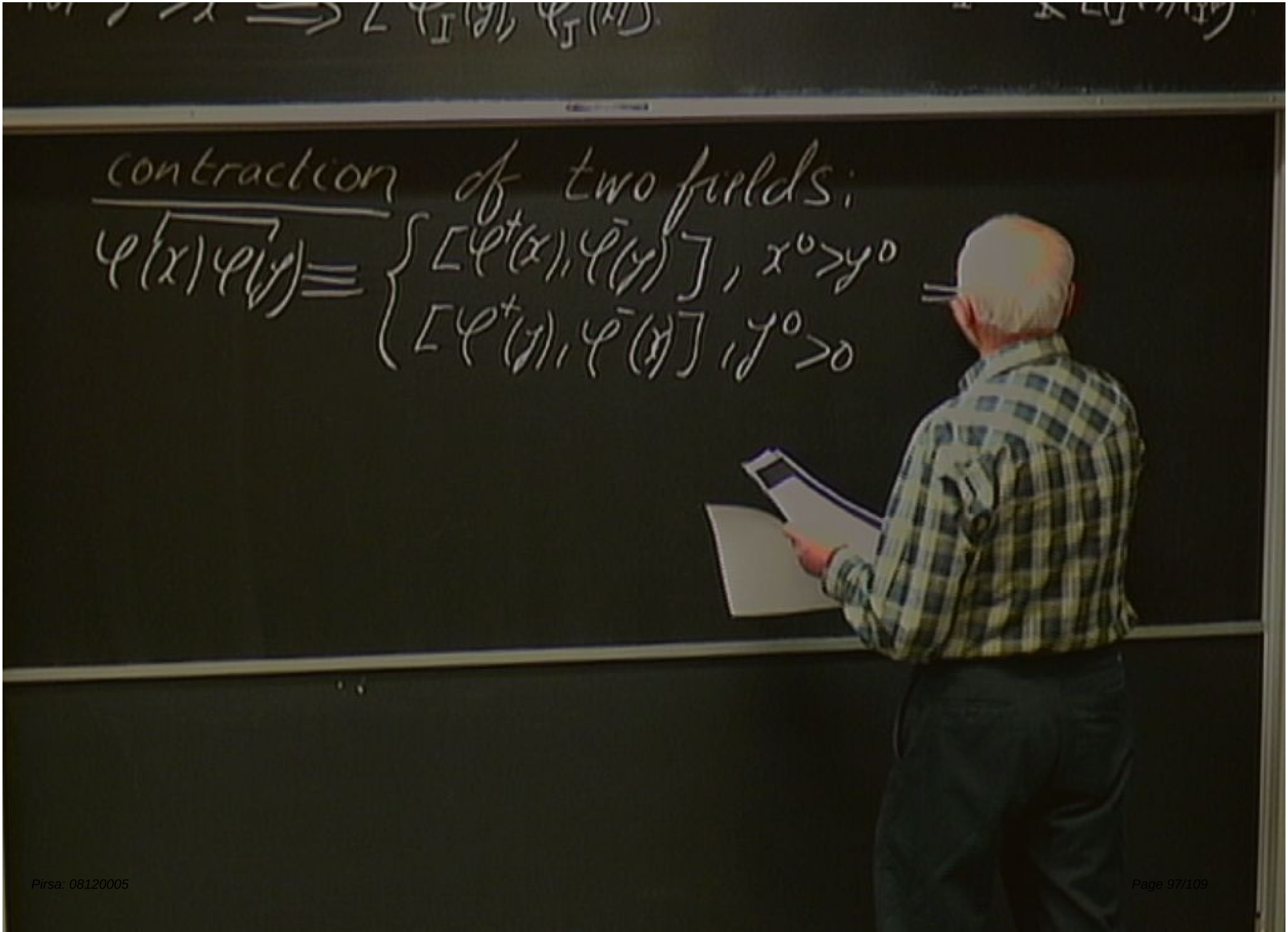
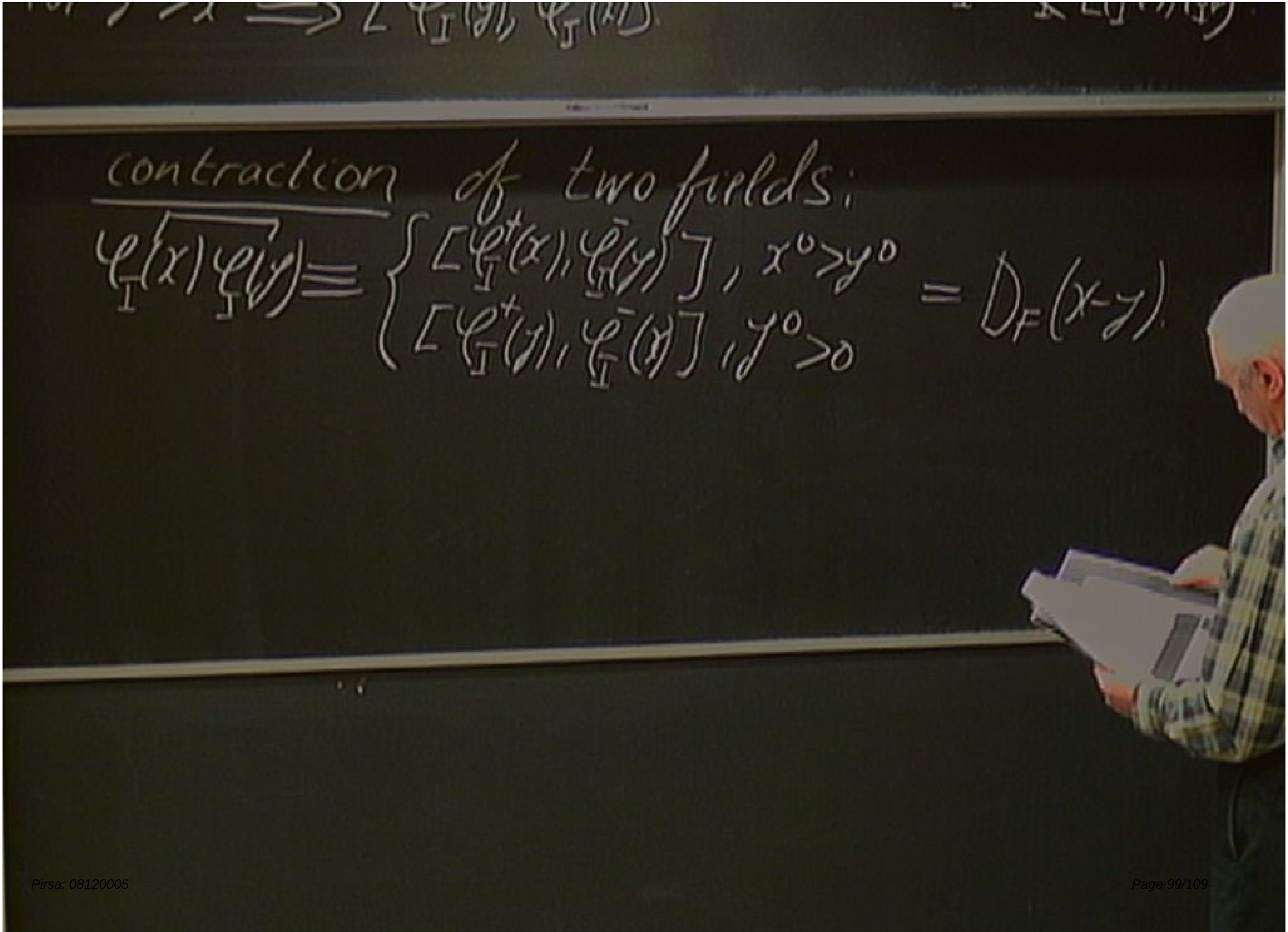
$= \int \frac{d^3p}{(2\pi)^3\sqrt{2E_{\vec p}}} a^+_{\vec p} e^{ipx}$; $\varphi_I^+(x)|0\rangle = 0$, $\langle 0|\varphi_I^-(x) = 0$
t consider $x^0 > y^0$: $T(\varphi(x)\varphi(y)) \underset{x^0>y^0}{=} \varphi^+(x)\varphi^+(y) + \varphi_I^+(x)\varphi_I^-(y)$
$(x)\varphi_I^+(y) + \varphi_I^-(x)\varphi_I^-(y) \Longrightarrow \varphi_I^+(x)\varphi_I^-(y) = \varphi_I^-(y)\varphi_I^+ + [\varphi_I^+(x), \varphi_I^-(y)]$

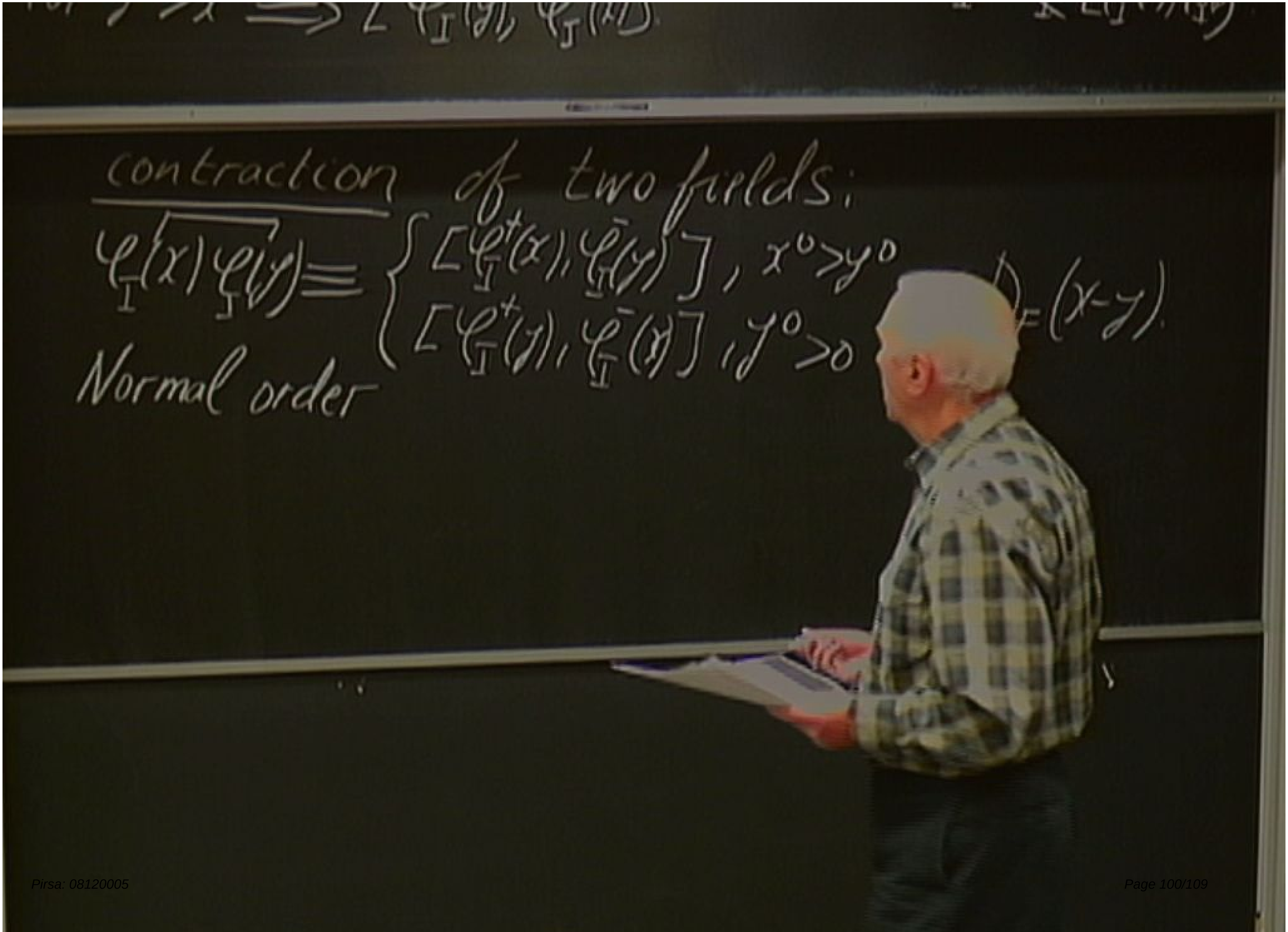

contraction of two fields:
φ(x)φ(y) ≡

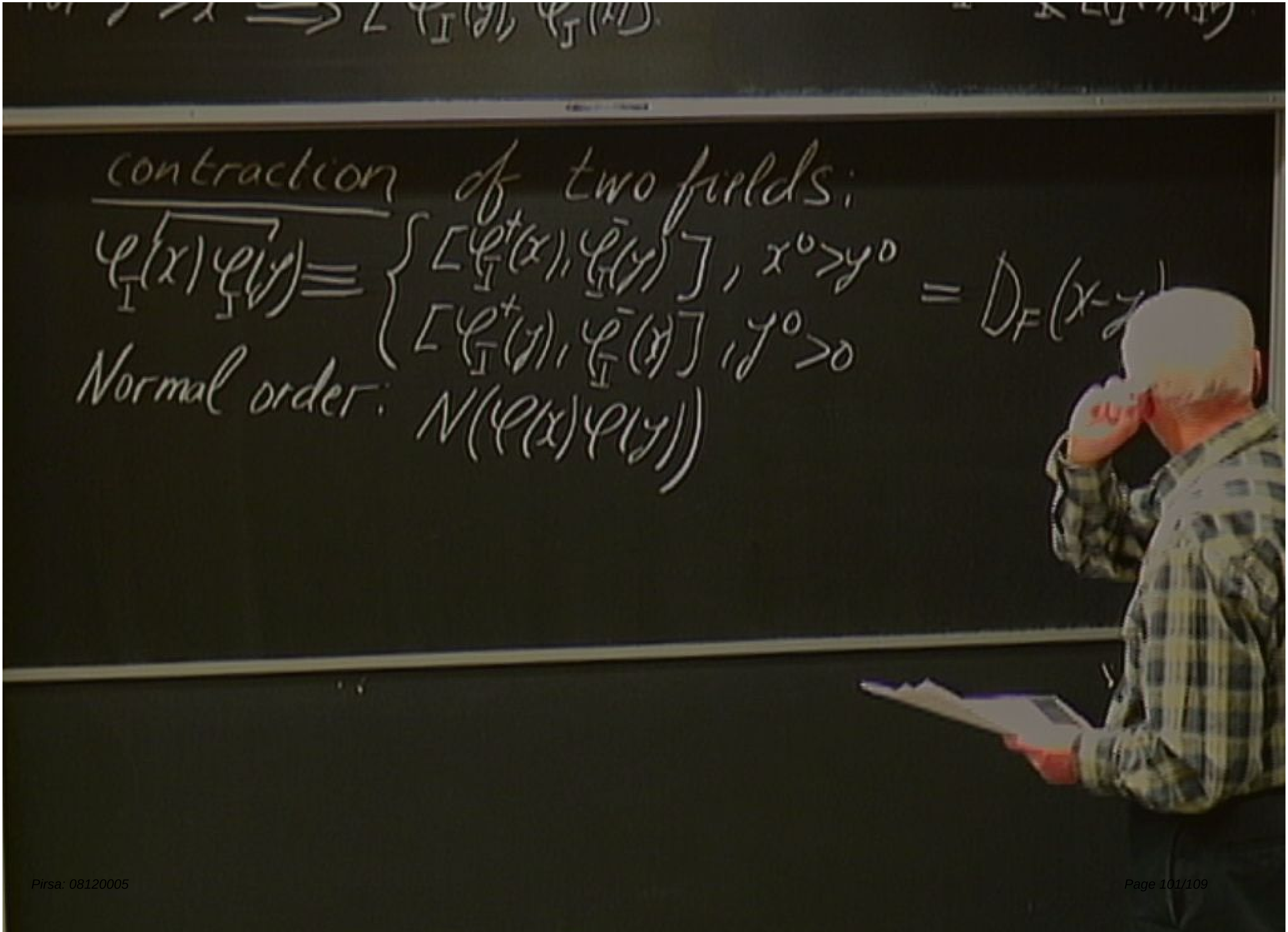

contraction of two fields:
φ(x)φ(y) ≡ { [φ⁺(x), φ⁻(y)], x⁰>y⁰
[φ⁺(y), φ⁻(x)], y⁰>0

contraction of two fields:

$$\varphi(x)\varphi(y) \equiv \begin{cases} [\varphi^+(x), \varphi^-(y)], & x^0>y^0 \\ [\varphi^+(y), \varphi^-(x)], & y^0>0 \end{cases} = D_F(x-y).$$


contraction of two fields:
φ_I(x) φ_J(y) ≡ { [φ_I^+(x), φ_J^-(y)], x^0 > y^0 ; [φ_J^+(y), φ_I^-(x)], y^0 > x^0 } = D_F(x−y).


contraction of two fields:
Normal order


contraction of two fields:
Normal order: N(φ(x)φ(y))

for $y^0 > x^0 \Longrightarrow [\varphi_I(y), \varphi_J(x)]$

<u>contraction</u> of two fields:

$$\varphi_I(x)\,\varphi_J(y) \equiv \begin{cases} [\varphi_I^+(x), \varphi_J^-(y)], & x^0 > y^0 \\ [\varphi_J^+(y), \varphi_I^-(x)], & y^0 > 0 \end{cases} = D_F(x-y).$$

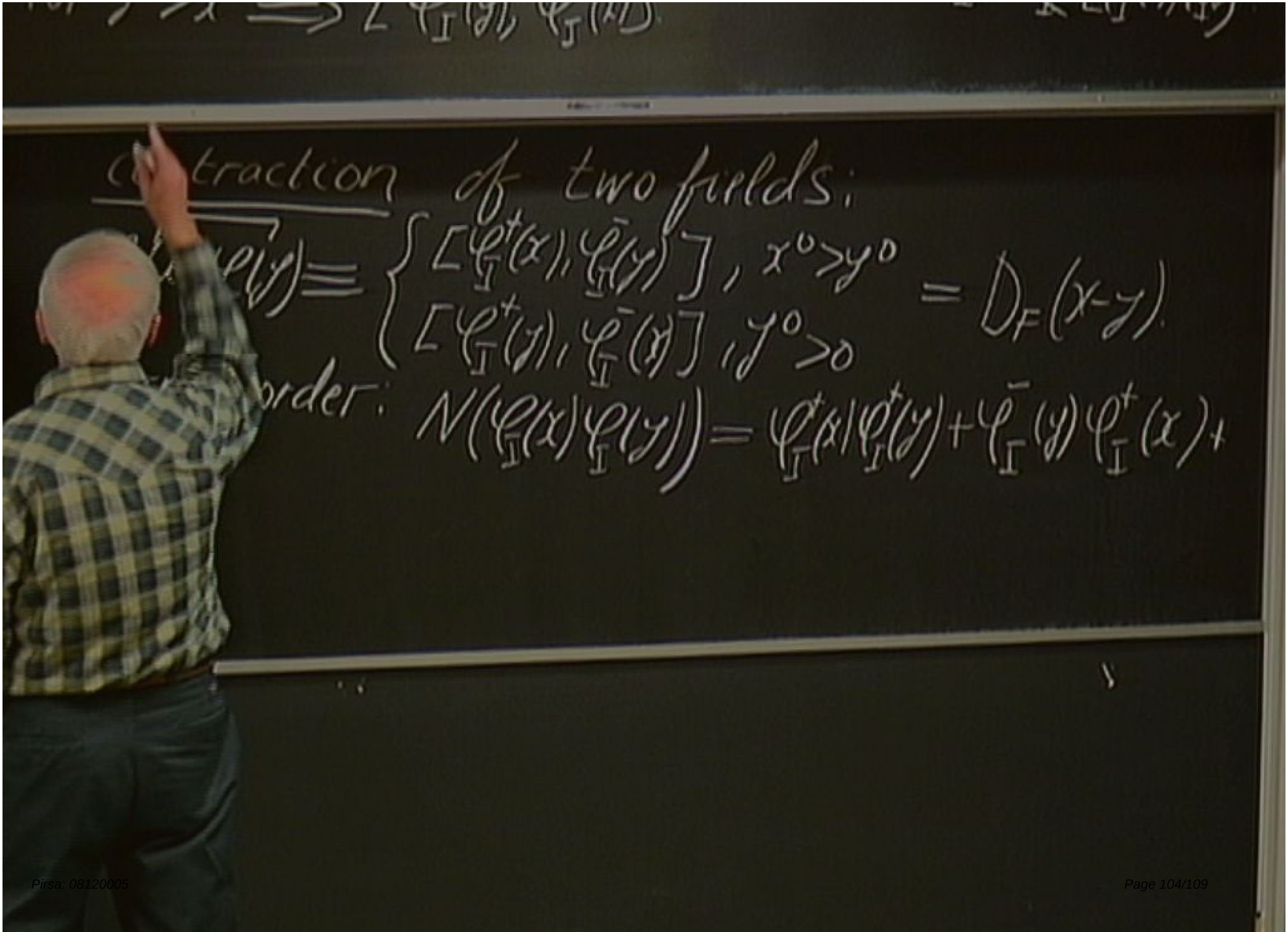
Normal order: $N(\varphi(x)\varphi(y)) =$

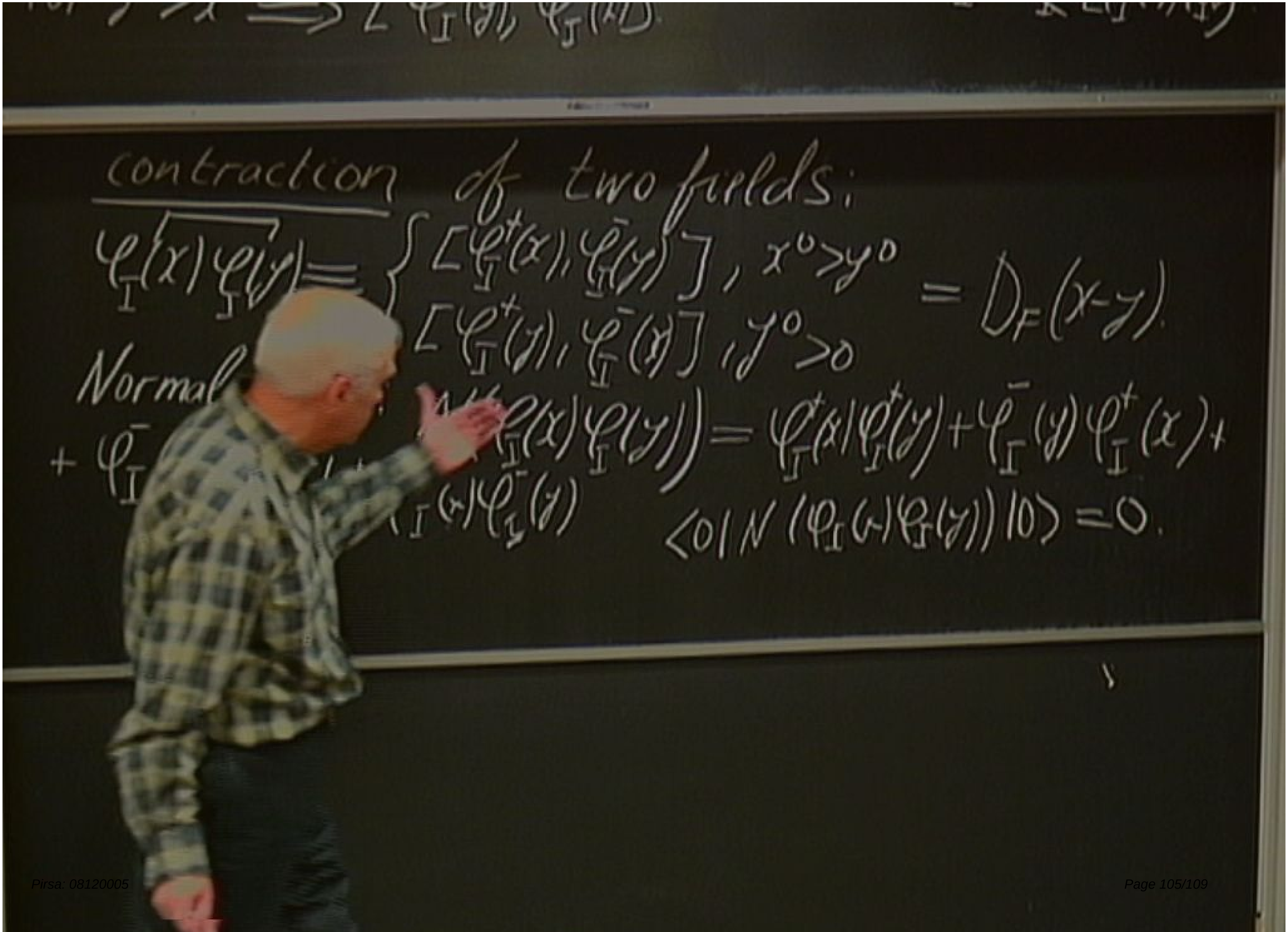
$y > x \Longrightarrow [\varphi_I(y), \varphi_J(x)]$

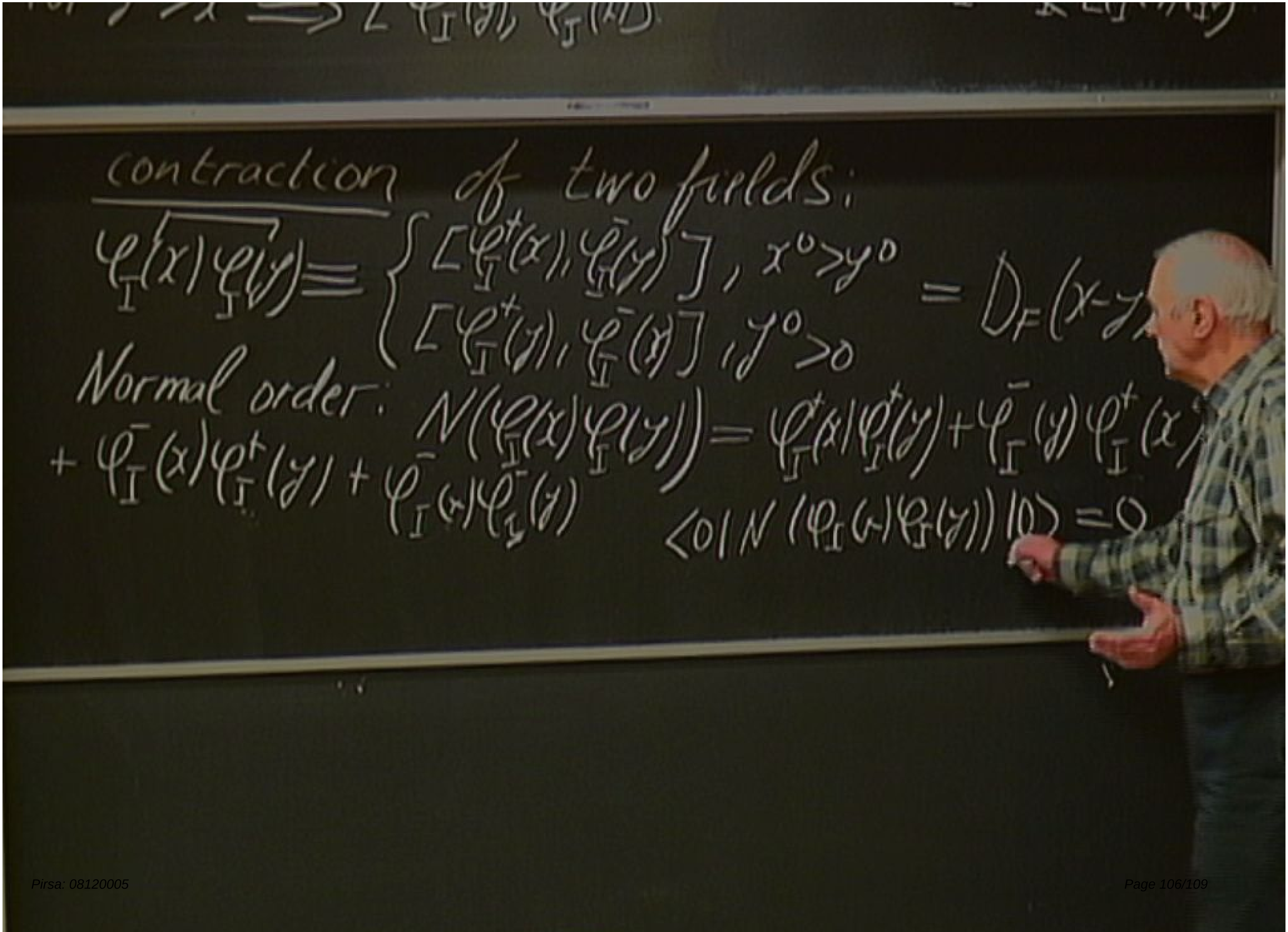
<u>contraction</u> of two fields:

$$\overline{\varphi_I(x)\varphi_I(y)} \equiv \begin{cases} [\varphi_I^+(x), \varphi_I^-(y)], & x^0 > y^0 \\ [\varphi_I^+(y), \varphi_I^-(x)], & y^0 > 0 \end{cases} = D_F(x-y).$$

Normal order: $N(\varphi_I(x)\varphi_I(y)) = \varphi_I^+(x)\varphi_I^+(y)$


traction of two fields:
[φ_I^+(x), φ_I^-(y)], x^0>y^0
[φ_I^+(y), φ_I^-(x)], y^0>0
= D_F(x-y).
order: N(φ_I(x)φ_I(y)) = φ_I^-(x)φ_I^+(y) + φ_I^-(y)φ_I^+(x) +


contraction of two fields:
Normal
$\langle 0| N(\varphi_I(x)\varphi_I(y))|0\rangle = 0.$
$= D_F(x-y).$

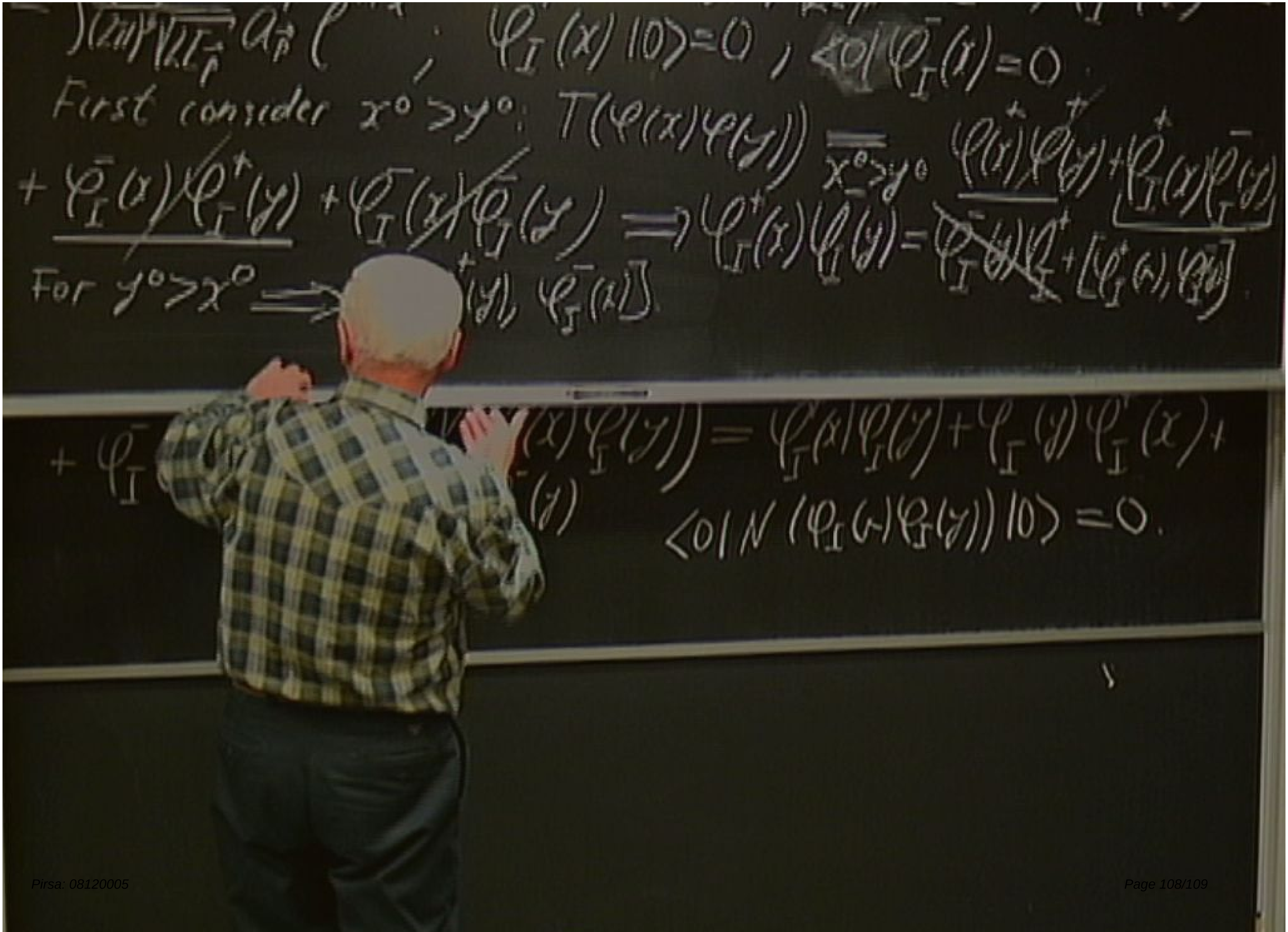

contraction of two fields:
Normal order:
<0| N (φ_I(x) φ_I(y)) |0> = 0

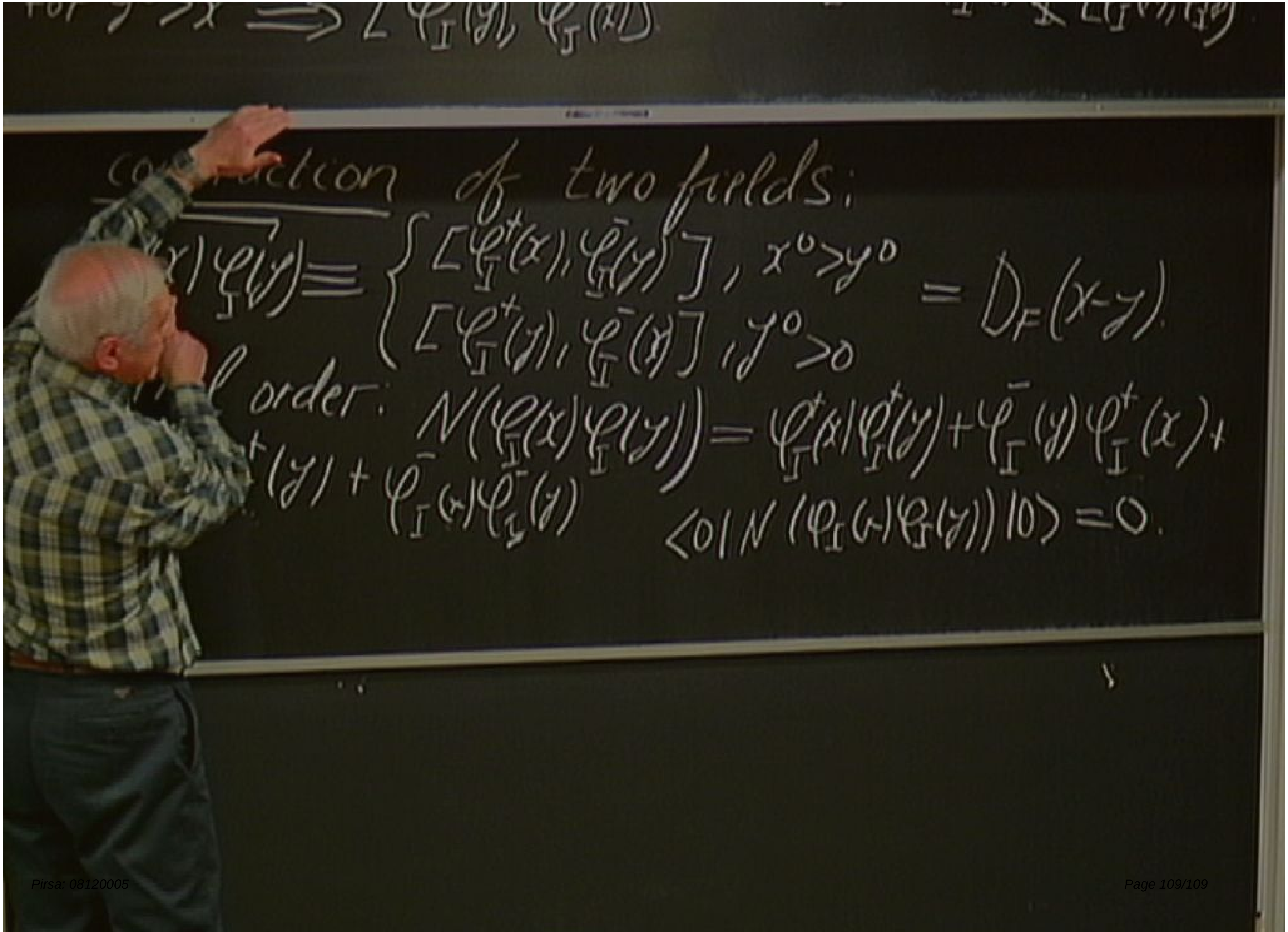
contraction of two fields:

$$\varphi_I(x)\varphi_I(y) \equiv \begin{cases} [\varphi_I^+(x), \varphi_I^-(y)], & x^0 > y^0 \\ [\varphi_I^+(y), \varphi_I^-(x)], & y^0 > 0 \end{cases} = D_F(x-y).$$

Normal order: $N(\varphi_I(x)\varphi_I(y)) = \varphi_I^+(x)\varphi_I^+(y) + \varphi_I^-(y)\varphi_I^+(x) + \varphi_I^-(x)\varphi_I^+(y) + \varphi_I^-(x)\varphi_I^-(y)$

$$\langle 0| N(\varphi_I(x)\varphi_I(y))|0\rangle = 0.$$


First consider
For



of two fields:
order: