

Title: Quantum Field Theory 1 - Lecture 13A

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Abstract: Quantum Field Theory I course taught by Volodya Miransky of the University of Western Ontario

$\Psi_I(t, \vec{x})$ - interaction picture field;

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 $\psi_I(t, \vec{x}) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \left[a_{\vec{p}} e^{-i p \cdot x} + a_{\vec{p}}^\dagger e^{i p \cdot x} \right] \Big|_{x_0=t-t_0, \vec{p}=\vec{p}}$

$a_{\vec{p}} |0\rangle = 0$

$\psi_I(t, \vec{x})$ - interaction picture field;

$$\psi_I(t, \vec{x}) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \left[a_{\vec{p}} e^{-i p \cdot x} + a_{\vec{p}}^\dagger e^{i p \cdot x} \right] \Big|_{x_0 = t - t_0}^{x_0 = t - t_0} \Big|_{p_0 = \sqrt{m^2 + \vec{p}^2}}$$

$a_{\vec{p}} |0\rangle = 0$



$\Psi_I(t, \vec{x})$ - interaction picture field;

$$\Psi_I(t, \vec{x}) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \left[a_{\vec{p}} e^{-i p \cdot x} + a_{\vec{p}}^\dagger e^{i p \cdot x} \right] \Big|_{x_0=t-t_0, \vec{x}=\vec{x}}$$

$\Psi(t, \vec{x}) = U^\dagger(t, t_0) \Psi_I(t, \vec{x}) U(t, t_0); U(t, t_0) = \dots$

$\Psi(t, \vec{x})$ → Heisenberg field

$\varphi_I(t, \vec{x})$ - interaction picture field;

$H_{int} = \frac{\lambda}{4!} \varphi^4$

$$\varphi_I(t, \vec{x}) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E}} \left[a_{\vec{p}} e^{-i p x} + a_{\vec{p}}^\dagger e^{i p x} \right] \Big|_{x_0=t-t_0, \vec{p}=\vec{p}}$$

$a_{\vec{p}} |0\rangle = 0$

$\varphi(t, \vec{x}) = \mathcal{U}^\dagger(t, t_0) \varphi_I(t, \vec{x}) \mathcal{U}(t, t_0); \mathcal{U}(t, t_0) = e^{-i H_0(t-t_0)} e^{-i H(t-t_0)}$

→ Heisenberg field

$\mathcal{U}(t, t_0) = T \dots$

$\psi_I(t, \vec{x})$ - interaction picture field;

Hint = $\frac{1}{4!} \psi^4$

$$\psi_I(t, \vec{x}) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \left[a_{\vec{p}} e^{-i p \cdot x} + a_{\vec{p}}^\dagger e^{i p \cdot x} \right] \Big|_{x_0=t-t_0, \vec{p}=\vec{p}}$$

$a_{\vec{p}} |0\rangle = 0$

$$\psi(t, \vec{x}) = \psi_I(t, \vec{x}) U(t, t_0); \quad U(t, t_0) = e^{-i H_0(t-t_0)} e^{-i H_I(t-t_0)}$$

Heisenberg field

$U(t, t_0)$

$$= \mathcal{T} \exp \left[-i \int_{t_0}^t dt' H_I(t') \right]; \quad H_I =$$

$$\Psi(t, \vec{x}) = U^\dagger(t, t_0) \Psi_I(t, \vec{x}) U(t, t_0); \quad U(t, t_0) = e^{-iH_0(t-t_0)} e^{-iH(t-t_0)}$$

↑ Heisenberg field

$$U(t, t_0) = T \left\{ \exp \left[-i \int_{t_0}^t dt' H_I(t') \right] \right\}; \quad H_I = \frac{\vec{p}^2}{2m} \psi_I^\dagger(x)$$

$\psi_I(t, \vec{x})$ - interaction picture field;

$$\psi_I(t, \vec{x}) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \left[a_{\vec{p}} e^{-i p \cdot x} + a_{\vec{p}}^\dagger e^{i p \cdot x} \right] \Big|_{x_0=t, \vec{p}=\vec{p}}$$

Hint = $\frac{1}{4!} \psi^4$

$|\vec{p}=0\rangle = 0$

$\psi(t, \vec{x}) = U^\dagger(t, t_0) \psi_I(t, \vec{x}) U(t, t_0); U(t, t_0) = \mathcal{T} \exp[-i \int_{t_0}^t H(t') dt']$

→ Heisenberg field

$U(t, t_0) = \mathcal{T} \left\{ \exp \left[-i \int_{t_0}^t H_I(t') dt' \right] \right\};$

Gen



$\psi_I(t, \vec{x})$ - interaction picture field;

$H_{int} = \frac{\lambda}{4!} \psi^4$

$\psi(t, \vec{x}) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \left[a_{\vec{p}} e^{-i p x} + a_{\vec{p}}^\dagger e^{i p x} \right] \Big|_{x_0=t-t_0, \vec{x}=\vec{x}-\vec{v}(t-t_0)}$; $a_{\vec{p}}^\dagger |0\rangle = 0$

$\psi(\vec{x}) = U^\dagger(t, t_0) \psi_I(t, \vec{x}) U(t, t_0)$; $U(t, t_0) = e^{-i H_0(t-t_0)} e^{-i H(t-t_0)}$

Heisenberg field

$U(t, t_0) = T \left\{ \exp \left[-i \int_{t_0}^t dt' H_I(t') \right] \right\}$; $H_I = \frac{\lambda}{4!} \psi_I^4(x)$

$$\psi_I(t, \vec{x}) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \left[a_p e^{-i(E_p t - \vec{p} \cdot \vec{x})} + a_p^\dagger e^{i(E_p t - \vec{p} \cdot \vec{x})} \right], \quad a_p |0\rangle = 0$$

$$\psi(t, \vec{x}) = U^\dagger(t, t_0) \psi_I(t, \vec{x}) U(t, t_0); \quad U(t, t_0) = \mathcal{T} \exp \left[-i \int_{t_0}^t H(t') dt' \right]$$

Heisenberg picture

$$U(t, t_0) = \exp \left[-i \int_{t_0}^t H_I(t') dt' \right]; \quad H_I = \frac{\vec{p}^4}{4m} \psi_I^\dagger \psi_I(x)$$

Go to $U(t, t')$

$$\psi(t, \vec{x}) = U^+(t, t_0) \psi_I(t, \vec{x}) U(t, t_0); \quad U(t, t_0) = e^{-i(H_0(t-t_0) + H(t-t_0))}$$

→ Heisenberg field

$$U(t, t_0) = T \left\{ \exp \left[-i \int_{t_0}^t dt' H_I(t') \right] \right\}; \quad H_I = \frac{\vec{p}^2}{2m} \psi_I^\dagger(x)$$

Generalization: $U(t, t') = T \left\{ \exp \left[-i \int_{t'}^t dt'' H_I(t'') \right] \right\}$

→ Heisenberg field

$$U(t, t_0) = T \left\{ \exp \left[-i \int_{t_0}^t dt' H_I(t') \right] \right\}, \quad H_I = \frac{\hbar}{i} \ln \mathcal{U}_I(t)$$

Generalization: $U(t, t') = T \left\{ \exp \left[-i \int_{t'}^t dt'' H_I(t'') \right] \right\}$ (I)

We proved that: $U(t, t') = e^{-iH_0(t-t_0)} e^{-iH_I(t-t')} e^{-iH_0(t'-t_0)}$ (II)

We proved that: $U(t, t') = \left\{ \exp \left[-i \int_{t'}^t H_I(t'') dt'' \right] \right\} e^{-iH_0(t-t')} \quad (1)$

Properties of $U(t, t')$: $U(t_1, t_2)U(t_2, t_3) = U(t_1, t_3)$; $U(t_1, t_3) = U(t_1, t_2)U(t_2, t_3)$

We proved that $U(t, t_0) = \mathcal{P} \exp \left\{ -\int_{t_0}^t H_I(t') dt' \right\}$ (1)

Properties of $U(t, t')$: $U(t_1, t_2)U(t_2, t_3) = U(t_1, t_3)$; $U(t_1, t_3) [U(t_1, t_2)]^\dagger = U(t_1, t_2)$.

$$U(t, t') = \exp\left[-\int_{t'}^t H(t'') dt''\right] (I)$$

Properties of $U(t, t')$: $U(t_1, t_2)U(t_2, t_3) = U(t_1, t_3)$, $U(t_1, t_3)[U(t_1, t_2)]^\dagger = U(t_1, t_2)$.



We proved that: $U(t, t') = \left\{ \exp \left[\int_{t'}^t H_I(t'') dt'' \right] \right\}^{-1} \quad (I)$
 $U(t, t') = e^{-H_0(t-t')} e^{-U_H(t-t')} e^{-iH_0(t'-t_0)} \quad (II)$

Properties of $U(t, t')$: $U(t_1, t_2)U(t_2, t_3) = U(t_1, t_3)$; $U(t_1, t_3)[U(t_1, t_2)]^\dagger =$
 Proof: Substitute (II): $e^{iH_0(t_1-t_2)} e^{-U_H(t_1-t_2)} e^{-iH_0(t_2-t_1)} = U(t_1, t_2)$.

We proved that $U(t, t') = \left\{ \exp \left[-\int_{t'}^t H_I(t'') dt'' \right] \right\} \cdot \left(\text{I} \right)$
 $U(t, t') = e^{-iH_0(t-t_0)} e^{-iH(t-t')} e^{-iH_0(t'-t_0)} \quad \left(\text{II} \right)$

Properties of $U(t, t')$: $U(t_1, t_2)U(t_2, t_3) = U(t_1, t_3)$, $U(t_1, t_3) [U(t, t_3)]^\dagger = U(t_1, t_2)$.

Proof: Substitute (II):
 $e^{-iH_0(t_1-t_2)} e^{-iH(t_2-t_3)} e^{-iH_0(t_2-t_0)} \cdot e^{-iH_0(t_2-t_0)} e^{-iH(t_2-t_3)} e^{-iH_0(t_1-t_2)} = U(t_1, t_2)$

Properties of $U(t, t')$: $U(t_1, t_2)U(t_2, t_3) = U(t_1, t_3)$, $U(t_1, t_3)[U(t_1, t_2)]^\dagger = U(t_1, t_2)$.

Proof. Substitute (II):

$$\begin{aligned}
 & \left\{ e^{iH(t_2-t_3)} e^{-iH_0(t_3-t_0)} \right\} = \left\{ e^{iH_0(t_1-t_0)} e^{-iH(t_1-t_2)} e^{-iH_0(t_2-t_0)} \right\} \cdot \left\{ e^{iH_0(t_2-t_0)} \right\} \\
 & = \left\{ e^{iH_0(t_1-t_0)} e^{-iH(t_1-t_3)} e^{-iH_0(t_3-t_0)} \right\} = U(t_1, t_3)
 \end{aligned}$$



$$\begin{aligned}
 & \times \left\{ e^{-iH_0(t_2-t_3)} e^{-iH_0(t_3-t_0)} = \left\{ e^{-iH_0(t_1-t_0)} e^{-iH(t_1-t_2)} e^{-iH_0(t_2-t_3)} e^{-iH_0(t_3-t_0)} \right\} \right. \\
 & = \left\{ e^{-iH_0(t_1-t_0)} e^{-iH(t_1-t_3)} e^{-iH_0(t_3-t_0)} \right\} = U(t_1, t_3)
 \end{aligned}$$

We will show that:

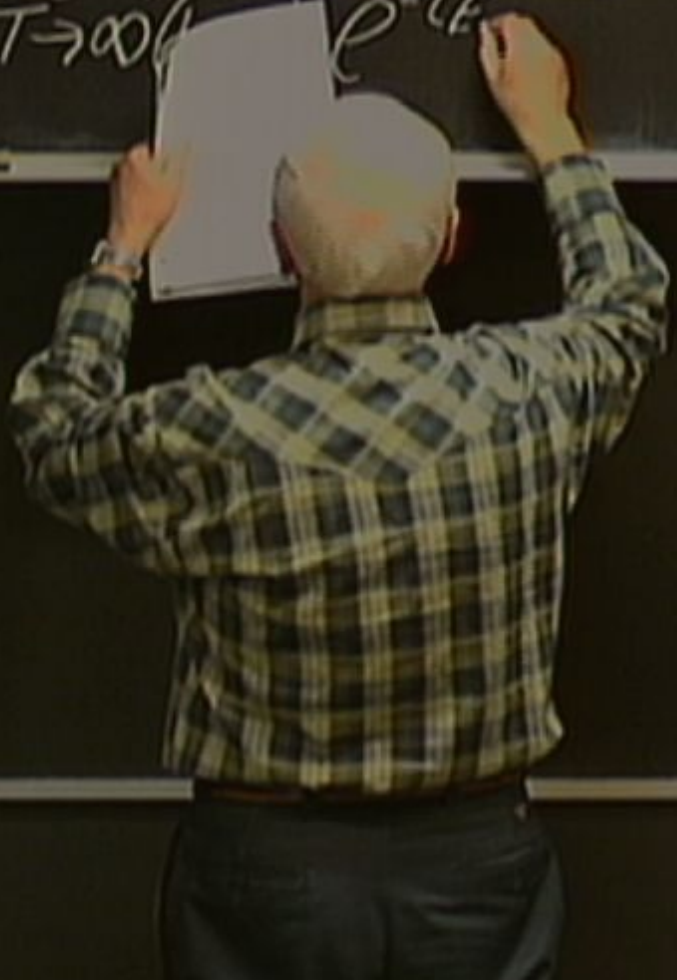
$$|\Omega\rangle =$$



$$\begin{aligned}
 & \times \left\{ e^{-iH_0(t_2-t_3)} e^{-iH_0(t_3-t_0)} = \left\{ e^{-iH_0(t_1-t_0)} e^{-iH(t_1-t_2)} e^{-iH_0(t_2-t_3)} e^{-iH_0(t_3-t_0)} \right\} \right. \\
 & = \left\{ e^{-iH_0(t_1-t_0)} e^{-iH(t_1-t_3)} e^{-iH_0(t_3-t_0)} \right\} = U(t_1, t_3)
 \end{aligned}$$

We will show that:

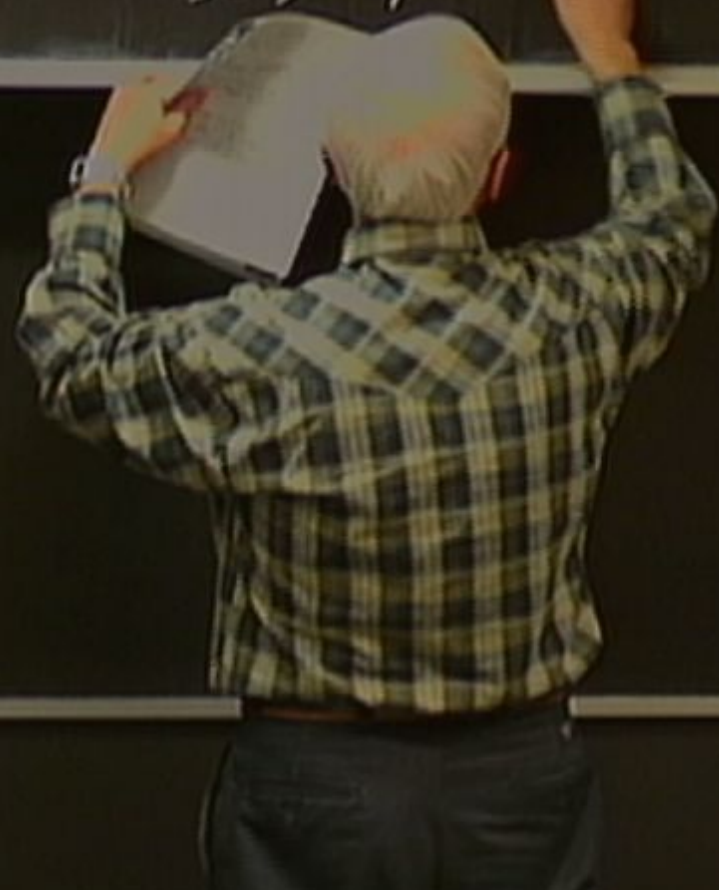
$$|\Omega\rangle = \lim_{T \rightarrow \infty} \frac{U(t_0, -T)|0\rangle}{e^{-LE}}$$



$$\begin{aligned}
 & \dots = \left\{ e^{iH_0(t_3-t_0)} e^{-iH(t_1-t_3)} e^{-iH_0(t_3-t_0)} \right\} = U(t_1, t_3) \\
 & \dots = \left\{ e^{iH_0(t_1-t_0)} e^{-iH(t_1-t_2)} e^{-iH_0(t_2-t_0)} \right\} \left\{ e^{-iH(t_2-t_3)} e^{-iH_0(t_3-t_0)} \right\}
 \end{aligned}$$

We will show that:

$$|\Omega\rangle = \lim_{T \rightarrow \infty} \frac{U(t_0, -T)|0\rangle}{(1-i\epsilon) e^{-iE_0(t_0 - (-T))} \langle \Omega | 0 \rangle}, \quad E_0 = \langle \Omega | H | \Omega \rangle \text{ (reference point)}$$



$$\begin{aligned}
 & \dots = \left\{ e^{iH_0(t_1-t_0)} e^{-iH(t_1-t_3)} e^{-iH_0(t_3-t_0)} \right\} = U(t_1, t_3) \\
 & \dots = \left\{ e^{iH_0(t_1-t_0)} e^{-iH(t_1-t_2)} e^{-iH(t_2-t_3)} e^{-iH_0(t_3-t_0)} \right\}
 \end{aligned}$$

We will show that

$$|\Omega\rangle = \lim_{T \rightarrow \infty} \frac{U(t_0, -T)|0\rangle}{(1-i\epsilon) e^{-iE_0(t_0 - (-T))} \langle \Omega | 0 \rangle} ; E_0 = \langle \Omega | H | \Omega \rangle \text{ (reference point for energy as } H|0\rangle = 0)$$



Proof. Substitute (II).
$$\begin{aligned}
 & \langle \psi | e^{-iH(t_2-t_3)} e^{-iH_0(t_3-t_0)} = \langle \psi | e^{-iH_0(t_1-t_0)} e^{-iH(t_1-t_2)} e^{-iH_0(t_2-t_0)} = \langle \psi | e^{-iH_0(t_1-t_0)} e^{-iH(t_1-t_2)} e^{-iH_0(t_2-t_0)} \\
 & \times e^{-iH(t_2-t_3)} e^{-iH_0(t_3-t_0)} = \langle \psi | e^{-iH_0(t_1-t_0)} e^{-iH(t_1-t_2)} e^{-iH_0(t_2-t_0)} e^{-iH(t_2-t_3)} e^{-iH_0(t_3-t_0)} \\
 & = \langle \psi | e^{-iH_0(t_1-t_0)} e^{-iH(t_1-t_3)} e^{-iH_0(t_3-t_0)} = \langle \psi | U(t_1, t_3)
 \end{aligned}$$

We will show that:

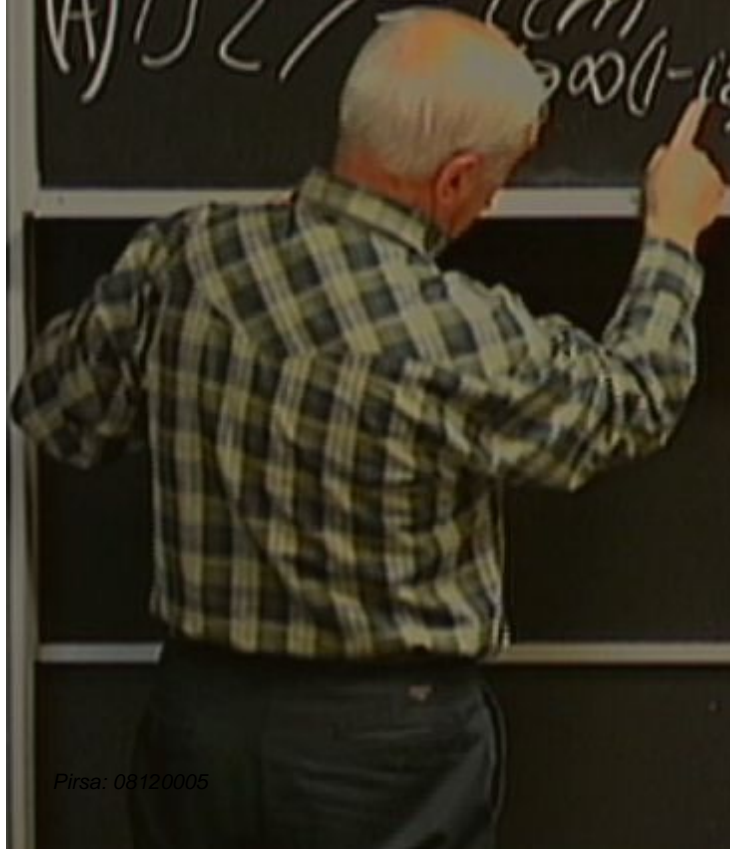
$$|\Omega\rangle = \lim_{T \rightarrow \infty} \frac{U(t_0, -T)|0\rangle}{(1-i\epsilon) e^{-iE_0(t_0 - (-T))} \langle \Omega | 0 \rangle}, \quad E_0 = \langle \Omega | H | \Omega \rangle \quad (\text{reference point for energy is } H|0\rangle = 0)$$

Proof. Substitute (II). $\left\{ \begin{array}{l} U(t_1, t_2) = e^{iH_0(t_1-t_2)} e^{-iH(t_1-t_2)} e^{-iH_0(t_1-t_2)} \\ U(t_2, t_3) = e^{iH_0(t_2-t_3)} e^{-iH(t_2-t_3)} e^{-iH_0(t_2-t_3)} \end{array} \right.$

$$\times e^{-iH_0(t_2-t_3)} e^{-iH_0(t_3-t_0)} = e^{iH_0(t_1-t_0)} e^{-iH(t_1-t_3)} e^{-iH_0(t_3-t_0)} = U(t_1, t_3)$$

We will show that:

$$(A) |\Omega\rangle = \lim_{T \rightarrow \infty} \frac{U(t_0, -T) |0\rangle}{(1-i\epsilon) e^{-iE_0(t_0 - T)} \langle \Omega | 0 \rangle}; \quad E_0 = \langle \Omega | H | \Omega \rangle \text{ (reference point for energy is } H|0\rangle = 0)$$



We will show that $\psi(t_1, t_3) = U(t_1, t_3)$

(A) $|\Omega\rangle = \lim_{T \rightarrow \infty} \frac{U(t_0, -T)|0\rangle}{(1-i\epsilon) e^{-iE_0(t_0 - t_T)} \langle \Omega | 0 \rangle}$; $E_0 = \langle \Omega | H | \Omega \rangle$ (reference point for energy as $H|0\rangle = 0$)

(B) $\langle \Omega | T \psi(x)$

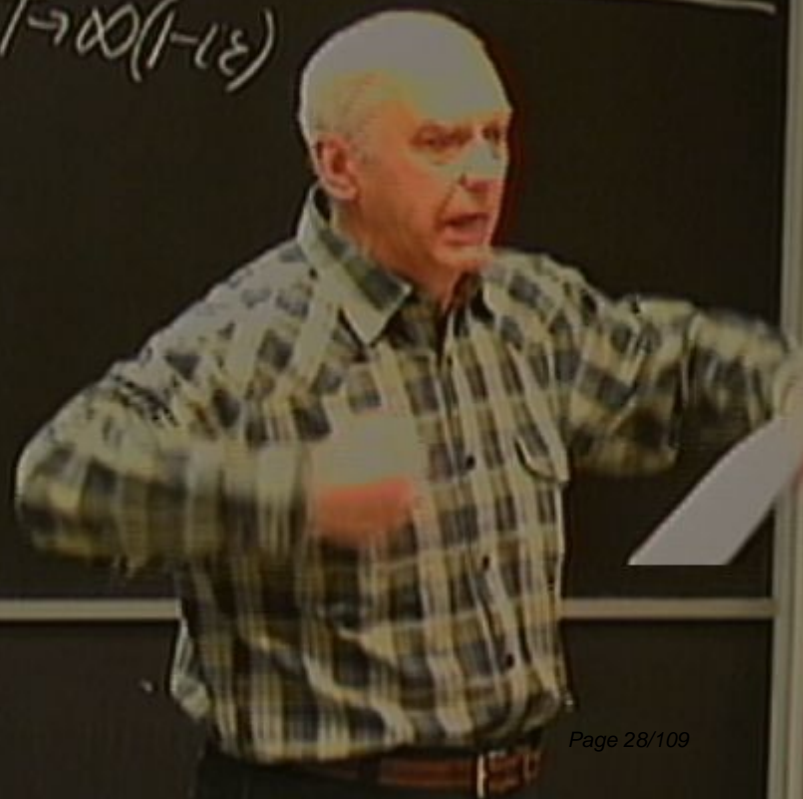


We will show that

$$(A) |\Omega\rangle = \lim_{T \rightarrow \infty} \frac{U(t_0, -T)|0\rangle}{(1-i\epsilon) e^{-iE_0(t_0 - (-T))} \langle \Omega | 0 \rangle}$$

$E_0 = \langle \Omega | H | \Omega \rangle$ (reference point for energy as $H|0\rangle = 0$)

$$(B) \frac{\langle \Omega | T \{ \psi(x) \psi(y) \} | \Omega \rangle}{\langle \Omega | T \{ \psi(x) \psi(y) \} | 0 \rangle} = \lim_{T \rightarrow \infty} \frac{\langle \Omega | T \{ \psi(x) \psi(y) \} | \Omega \rangle}{(1-i\epsilon)}$$



We will show that

$$(A) |\Omega\rangle = \lim_{T \rightarrow \infty} \frac{U(t_0, -T)|0\rangle}{(1-i\epsilon) e^{-iE_0(t_0 - (-T))} \langle \Omega | 0 \rangle}; \quad E_0 = \langle \Omega | H | \Omega \rangle \quad (\text{reference point for energy as } H|0\rangle = 0)$$

$$(B) \langle \Omega | T \{ \psi(x) \psi(y) \} | \Omega \rangle = \lim_{T \rightarrow \infty} \frac{\langle \Omega | T \{ \psi(x) \psi(y) \} | 0 \rangle}{(1-i\epsilon) e^{-iE_0(t_0 - (-T))} \langle \Omega | 0 \rangle}$$

We will show that

$$(A) |\Omega\rangle = \lim_{T \rightarrow \infty} \frac{U(t_0, -T)|0\rangle}{(1-i\epsilon) e^{-iE_0(t_0 - (-T))} \langle \Omega | 0 \rangle}, \quad E_0 = \langle \Omega | H | \Omega \rangle \quad (\text{reference point for energy, } \langle \Omega | 0 \rangle = 0)$$

$$(B) \frac{\langle \Omega | T \{ \psi(x) \psi(y) \} | \Omega \rangle}{\langle \Omega | T \{ \psi_I(x) \psi_I(y) \exp[-i \int_{-T}^T dt H_I(t)] | 0 \rangle}} = \lim_{T \rightarrow \infty} \frac{\langle 0 | T \{ \psi_I(x) \psi_I(y) \exp[-i \int_{-T}^T dt H_I(t)] | 0 \rangle}{\langle 0 | T \{ \exp[-i \int_{-T}^T dt H_I(t)] | 0 \rangle}}$$

$$U(t, t_0) = 1 - i \int_{t_0}^t dt' H_I(t'), \quad H_I = \frac{\lambda}{\hbar} \psi^\dagger \psi$$

Generalization: $U(t, t') = T \left[\exp(-i \int_{t'}^t H_I(t'') dt'') \right]$

Properties of $U(t, t')$: $U(t_1, t_2) U(t_2, t_3) = U(t_1, t_3)$, $U(t_1, t_3) [U(t_2, t_3)]^\dagger = U(t_1, t_2)$

Proof. Substitute (II): $\int_{t_0}^{t_1} dt' H_I(t') - \int_{t_0}^{t_2} dt' H_I(t') - \int_{t_2}^{t_1} dt' H_I(t') = U(t_1, t_2)$

$$\times \int_{t_2}^{t_3} dt' H_I(t') - \int_{t_3}^{t_2} dt' H_I(t') = \int_{t_0}^{t_1} dt' H_I(t') - \int_{t_0}^{t_3} dt' H_I(t') + \int_{t_3}^{t_2} dt' H_I(t')$$

$$= \int_{t_0}^{t_1} dt' H_I(t') - \int_{t_0}^{t_3} dt' H_I(t') + \int_{t_3}^{t_2} dt' H_I(t')$$

We will show that:

(A) $|\Omega\rangle = \lim_{T \rightarrow \infty} \frac{1}{(1-i\epsilon)} e^{-iE_0 T} U(T, -T) |\Omega\rangle$

$E_0 = \langle \Omega | H_0 | \Omega \rangle$ (reference point for energy is $H_0 | \Omega \rangle = 0$)

We will show that

$$(A) |\Omega\rangle = \lim_{T \rightarrow \infty} \frac{U(t_0, -T)|0\rangle}{(1-i\epsilon) e^{-iE_0(t_0 - (-T))} \langle \Omega | 0 \rangle}$$

$E_0 = \langle \Omega | H | \Omega \rangle$ (reference point for energy as $H|0\rangle = 0$)

$$(B) \frac{\langle \Omega | T \{ \psi(x) \psi(y) \} | \Omega \rangle}{\langle 0 | T \{ \psi_I(x) \psi_I(y) \exp[-i \int_{-T}^T dt H_I(t)] | 0 \rangle} = \lim_{T \rightarrow \infty} \frac{\langle \Omega | T \{ \exp[-i \int_{-T}^T dt H_I(t)] \psi(x) \psi(y) \} | 0 \rangle}{\langle 0 | T \{ \exp[-i \int_{-T}^T dt H_I(t)] | 0 \rangle}$$

Find $|\Omega\rangle$?

$$|0\rangle = \sum_n |n\rangle$$

We will show that

$$(A) |\Omega\rangle = \lim_{T \rightarrow \infty} \frac{U(t_0, -T)|0\rangle}{(1-i\epsilon) e^{-iE_0(t_0 - (-T))} \langle \Omega | 0 \rangle}$$

$E_0 = \langle \Omega | H | \Omega \rangle$ (reference point for energies $H|0\rangle = 0$)

$$(B) \langle \Omega | T \{ \psi(x) \psi(y) \} | \Omega \rangle = \lim_{T \rightarrow \infty} \frac{\langle 0 | T \{ \psi_I(x) \psi_I(y) \exp[-i \int_{-T}^T dt H_I(t)] | 0 \rangle}{\langle 0 | T \{ \exp[-i \int_{-T}^T dt H_I(t)] | 0 \rangle}$$

Fund $|\Omega\rangle$?

$$|0\rangle = \sum_n |n\rangle \langle n | 0 \rangle = |\Omega\rangle$$

We will show that

$$(A) |\Omega\rangle = \lim_{T \rightarrow \infty} \frac{U(t_0, -T)|0\rangle}{(1-i\epsilon) e^{-iE_0(t_0 - (-T))} \langle \Omega | 0 \rangle}; E_0 = \langle \Omega | H | \Omega \rangle \text{ (reference point for energies } H|0\rangle = 0)$$

$$(B) \frac{\langle \Omega | T \{ \psi(x) \psi(y) \} | \Omega \rangle}{\langle 0 | T \{ \psi_I(x) \psi_I(y) \exp[-i \int_{-T}^T dt H_I(t)] | 0 \rangle} = \lim_{T \rightarrow \infty} \frac{\langle 0 | T \{ \exp[-i \int_{-T}^T dt H_I(t)] \} | 0 \rangle}{\langle 0 | T \{ \psi_I(x) \psi_I(y) \exp[-i \int_{-T}^T dt H_I(t)] | 0 \rangle}$$

Find $|\Omega\rangle$?

$$|0\rangle = |\Omega\rangle \langle \Omega | 0 \rangle + \sum_{n \neq 0} |n\rangle \langle n | 0 \rangle;$$

We will show that

$$(A) |\Omega\rangle = \lim_{T \rightarrow \infty} \frac{U(t_0, -T)|0\rangle}{(1-i\epsilon) e^{-iE_0(t_0 - (-T))} \langle \Omega | 0 \rangle}; \quad E_0 = \langle \Omega | H | \Omega \rangle \text{ (reference point for energies } H|0\rangle = 0)$$

$$(B) \frac{\langle \Omega | T \{ \psi(x) \psi(y) \} | \Omega \rangle}{\langle 0 | T \{ \psi_I(x) \psi_I(y) \exp[-i \int_{-T}^T dt H_I(t)] | 0 \rangle} = \lim_{T \rightarrow \infty} \frac{\langle 0 | T \{ \exp[-i \int_{-T}^T dt H_I(t)] \} | 0 \rangle}{\langle 0 | T \{ \exp[-i \int_{-T}^T dt H_I(t)] \} | 0 \rangle}$$

Find $|\Omega\rangle$?

$$|0\rangle = \sum_n |n\rangle \langle n | 0 \rangle = |\Omega\rangle \langle \Omega | 0 \rangle$$

$$H |n\rangle = E_n |n\rangle$$



We will show that

$$(A) |\Omega\rangle = \lim_{T \rightarrow \infty} \frac{U(t_0, -T)|0\rangle}{(1-i\epsilon) e^{-iE_0(t_0 - (-T))} \langle \Omega | 0 \rangle}; \quad E_0 = \langle \Omega | H | \Omega \rangle \quad (\text{reference point for energies } H|0\rangle = 0)$$

$$(B) \frac{\langle \Omega | T \{ \psi(x) \psi(y) \} | \Omega \rangle}{\langle 0 | T \{ \psi_I(x) \psi_I(y) \exp[-i \int_{-T}^T dt H_I(t)] | 0 \rangle}} = \lim_{T \rightarrow \infty} \frac{\langle 0 | T \{ \exp[-i \int_{-T}^T dt H_I(t)] \} | 0 \rangle}{\langle 0 | T \{ \psi_I(x) \psi_I(y) \exp[-i \int_{-T}^T dt H_I(t)] | 0 \rangle}}$$

Find $|\Omega\rangle$?

$$|0\rangle = \sum_n |n\rangle \langle n | 0 \rangle = |\Omega\rangle \langle \Omega | 0 \rangle + \sum_{n \neq 0} |n\rangle \langle n | 0 \rangle;$$

$$H |n\rangle = E_n |n\rangle$$

Multiply by $e^{-iHT} |0\rangle = \mathcal{L}$

Multiply by $e^{-i\omega T}|0\rangle = e^{-iE_0 T} |0\rangle \langle 0| + \sum_{n \neq 0} e^{-iE_n T} |n\rangle \langle n|$

Multi $e^{-i\hat{H}T}|0\rangle = e^{-iE_0T}|0\rangle + \sum_{n \neq 0} e^{-iE_nT}|n\rangle \langle n|0\rangle \Rightarrow$
 \Rightarrow take $T \rightarrow \infty$

Multiply by $e^{-iHT}|0\rangle = e^{-iE_0T}|0\rangle + \sum_{n \neq 0} e^{-iE_nT} |n\rangle \langle n|0\rangle \Rightarrow$

\Rightarrow take $T \rightarrow \infty (1-i\epsilon)$

Multiply by $e^{-iHT} |0\rangle = e^{-iHT} |0\rangle + \sum_{n \neq 0} \left\{ \begin{array}{l} e^{-iE_n T} \langle n | \chi | 0 \rangle \\ e^{-iE_n (1-i\epsilon) T} \end{array} \right\} \Rightarrow$

\Rightarrow take $T \rightarrow \infty (1-i\epsilon)$

Multiply by $e^{-i\epsilon T} |0\rangle = e^{-iE_0 T} |0\rangle + \sum_{n \neq 0} \left\{ \frac{e^{-iE_n T} \langle E_0 | E_n \rangle}{e^{-iE_n (1-i\epsilon) T}} \right\} e^{-iE_n T}$

\Rightarrow take $T \rightarrow \infty (1-i\epsilon)$

Multiplying by $e^{-\alpha H T} |0\rangle = e^{-\alpha E_0 T} |0\rangle + \sum_{n \neq 0} e^{-\alpha E_n T} |n\rangle \langle n|0\rangle \Rightarrow$

\Rightarrow take $T \rightarrow \infty$

$$\frac{e^{-\alpha H T} |0\rangle}{e^{-\alpha E_0 T}} \Rightarrow |0\rangle \langle 0|0\rangle$$

$$e^{-\alpha E_0 T} |0\rangle \langle 0|0\rangle \Rightarrow$$

$$e^{-\alpha E_n (1-\epsilon) T} \sim e^{-\alpha E_n T}$$

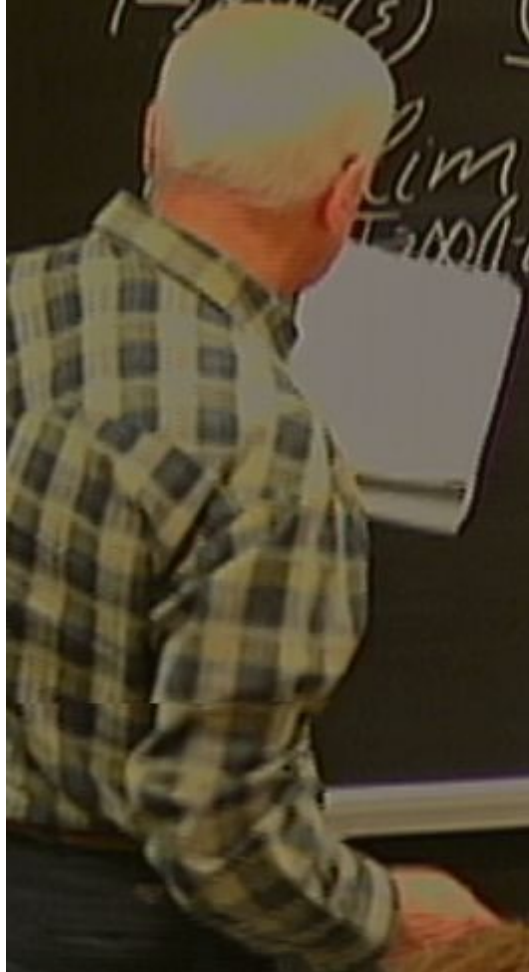
Multiply by $e^{-i\epsilon T} |0\rangle = e^{-iE_0 T} |\Omega\rangle \langle \Omega| + \sum_{n \neq 0} e^{-iE_n T} |\Omega\rangle \langle \Omega|$ \Rightarrow
 \Rightarrow take $T \rightarrow \infty (1-i\epsilon)$ $e^{-iE_0 T} |\Omega\rangle \langle \Omega| \Rightarrow$ $e^{-iE_n (1-i\epsilon) T} \sim e^{-\epsilon T}$

$$|\Omega\rangle = \lim_{T \rightarrow \infty (1-i\epsilon)} (e^{-iE_0 T} \langle \Omega|0\rangle)^{-1} |0\rangle$$

Multiply by $e^{-iHt} |0\rangle = e^{-iE_0 t} |0\rangle + \sum_{n \neq 0} e^{-iE_n t} \langle n|0\rangle |n\rangle$

\Rightarrow take $T \rightarrow \infty(1-i\epsilon)$

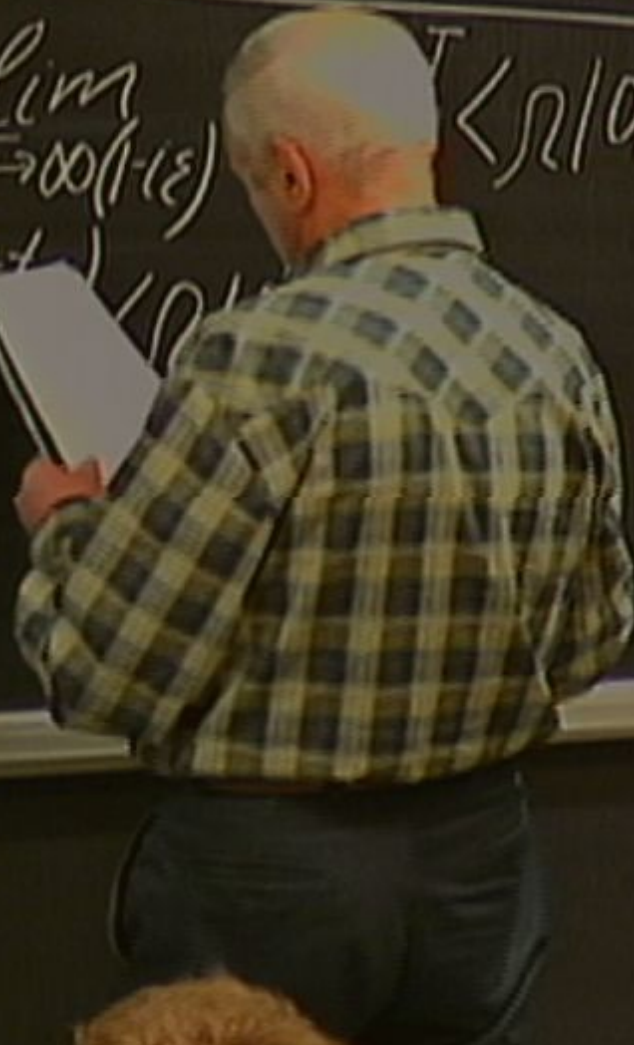
$$\lim_{T \rightarrow \infty(1-i\epsilon)} \frac{e^{-iE_0 T} |0\rangle \langle 0|}{(e^{-iE_0 T} \langle 0|0\rangle)^T} e^{-iHt} |0\rangle$$



\Rightarrow take $T \rightarrow \infty (1-i\varepsilon)$ $\frac{e^{-iE_0 T}}{\langle \Omega | \langle \Omega | 0 \rangle} \Rightarrow$

$\frac{\langle \Omega | \langle \Omega | 0 \rangle + \sum_{n \neq 0} \langle \Omega | \langle n | 0 \rangle}{e^{iE_n (1-i\varepsilon) T} \sim e^{-\varepsilon n T}}$

$\langle \Omega | = \lim_{T \rightarrow \infty (1-i\varepsilon)} \langle \Omega | \langle \Omega | 0 \rangle \langle e^{-iHt} | 0 \rangle \Rightarrow \langle \Omega | = \lim_{T \rightarrow \infty (1-i\varepsilon)} \langle e^{-iE_0 (T+i\varepsilon)} | \Omega | 0 \rangle$



$$T \rightarrow \infty (1 - i\varepsilon) \quad \frac{e^{iE_0 t_0} |\Omega\rangle \langle \Omega|_0\rangle}{h \neq 0} \Rightarrow$$

$$|\Omega\rangle = \lim_{T \rightarrow \infty (1 - i\varepsilon)} \left(e^{-iE_0 T} \langle \Omega|_0 \rangle \right)^{-1} e^{-iH T} |0\rangle \Rightarrow |\Omega\rangle = \lim_{T \rightarrow \infty (1 - i\varepsilon)}$$

$$\left(e^{-iE_0 (T + t_0)} \langle \Omega|_0 \rangle \right)^{-1} e^{-iH (T + t_0)} |0\rangle$$

\rightarrow take $T \rightarrow \infty(1-i\epsilon)$ $\frac{e^{-iE_0 T} |\Omega\rangle \langle \Omega|_0}{h \neq 0} \Rightarrow$

$|\Omega\rangle = \lim_{T \rightarrow \infty} (e^{-iE_0 T} \langle \Omega|_0)^{-1} e^{-iH T} |0\rangle \Rightarrow |\Omega\rangle = \lim_{T \rightarrow \infty(1-i\epsilon)} \cdot$

$(e^{-iE_0(T+t_0)})^{-1} e^{-iH(T+t_0)} |0\rangle = \lim_{T \rightarrow \infty(1-i\epsilon)} (e^{-iE_0(t_0 - (-T))} \langle \Omega|_0)^{-1} \times$

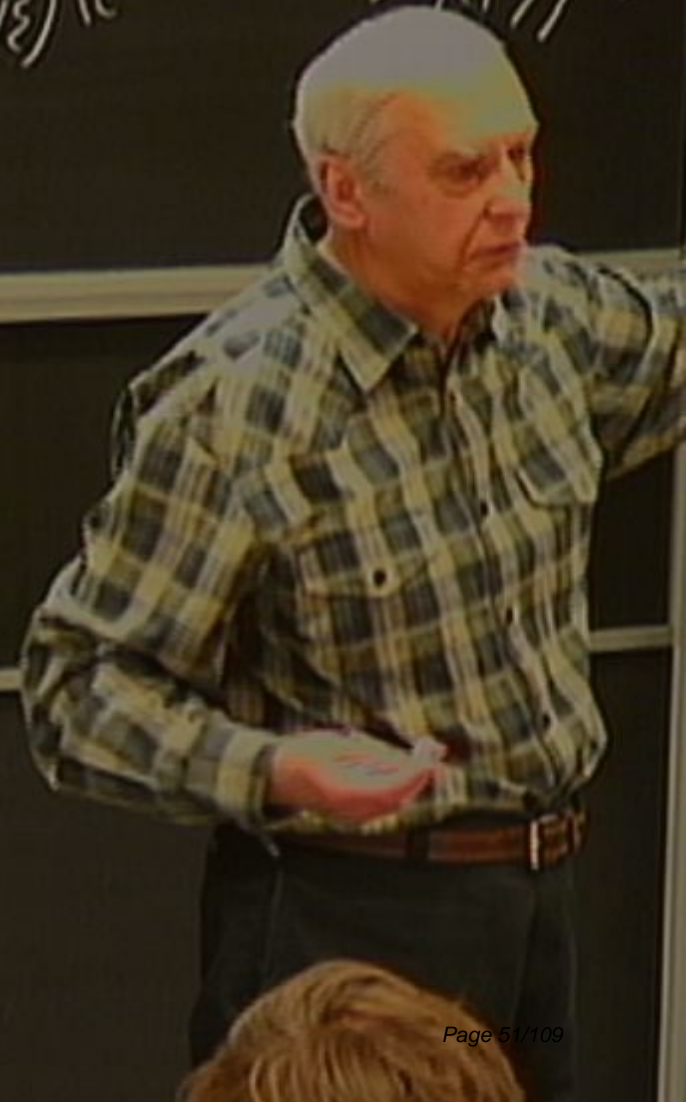
\Rightarrow take $T \rightarrow \infty(1-i\epsilon)$ $\frac{e^{-iE_0 T} \langle \Omega | \Omega \rangle}{\langle \Omega | \Omega \rangle} \Rightarrow$

$\langle \Omega \rangle = \lim_{T \rightarrow \infty(1-i\epsilon)} \left(e^{-iE_0 T} \langle \Omega | 0 \rangle \right)^{-1} e^{-iH T} | 0 \rangle \Rightarrow \langle \Omega \rangle = \lim_{T \rightarrow \infty(1-i\epsilon)}$

$\left(e^{-iE_0(T+t_0)} \langle \Omega | 0 \rangle \right)^{-1} e^{-iH(T+t_0)} | 0 \rangle = \lim_{T \rightarrow \infty(1-i\epsilon)} \left(e^{-iE_0(t_0 - (-T))} \langle \Omega | 0 \rangle \right)^{-1} \times$
 $+ \left[e^{-iH(T+t_0)} e^{-iH_0(-T-t_0)} | 0 \rangle \right]$

$$\begin{aligned}
 |\Omega\rangle &= \lim_{T \rightarrow \infty (1-i\epsilon)} (e^{-iE_0 T} \langle \Omega | 0 \rangle)^{-1} e^{-iH T} |0\rangle \Rightarrow |\Omega\rangle = \lim_{T \rightarrow \infty (1-i\epsilon)} \\
 & (e^{-iE_0(T+t_0)} \langle \Omega | 0 \rangle)^{-1} e^{-iH(T+t_0)} |0\rangle = \lim_{T \rightarrow \infty (1-i\epsilon)} (e^{-iE_0(t_0 - (-T))} \langle \Omega | 0 \rangle)^{-1} \times \\
 & + \underbrace{e^{-iH(T+t_0)} e^{-iH_0(-T-t_0)} |0\rangle}_{U(t_0, -T)}
 \end{aligned}$$

$$\begin{aligned}
 & \left(e^{-iE_0(T+t_0)} \langle R|0 \rangle \right)^{-1} e^{-iH(T+t_0)} |0\rangle = \lim_{T \rightarrow \infty (1-i\epsilon)} \left(e^{-iE_0(t_0 - (-T))} \langle R|0 \rangle \right)^{-1} \times \\
 & + \left[e^{-iH(T+t_0)} e^{-iH_0(-T-t_0)} |0\rangle \right]. \text{ Q.E.D.} \\
 & \mathcal{U}(t_0, -T)
 \end{aligned}$$



We proved that $U(t_1, t_2) = \mathcal{P} \exp(-i \int_{t_2}^{t_1} H_I(t') dt')$

Properties of $U(t, t')$: $U(t_1, t_2)U(t_2, t_3) = U(t_1, t_3)$; $U(t_1, t_3)[U(t_2, t_3)]^\dagger =$

Proof. Substitute (II): $\left\{ \begin{matrix} e^{iH_0(t_1-t_2)} e^{-iH(t_1-t_2)} e^{-iH_0(t_2-t_0)} \\ e^{iH_0(t_2-t_3)} e^{-iH_0(t_3-t_0)} \end{matrix} \right\} = U(t_1, t_2)$

$$\begin{aligned} & \times \left\{ e^{iH(t_2-t_3)} e^{-iH_0(t_3-t_0)} \right\} = \left\{ e^{iH_0(t_1-t_2)} e^{-iH(t_1-t_2)} e^{-iH_0(t_2-t_0)} \right\} \cdot \left\{ e^{iH_0(t_2-t_3)} e^{-iH_0(t_3-t_0)} \right\} \\ & = \left\{ e^{iH_0(t_1-t_2)} e^{-iH(t_1-t_2)} e^{-iH_0(t_2-t_3)} e^{-iH(t_2-t_3)} e^{-iH_0(t_3-t_0)} \right\} = U(t_1, t_3) \end{aligned}$$

We will show that:

$$(A) |\Omega\rangle = \lim_{T \rightarrow \infty} \frac{U(t_0, -T)|0\rangle}{(1-i\epsilon) e^{-iE_0(t_0 - (-T))} \langle \Omega | 0 \rangle}; \quad E_0 = \langle \Omega | H_0 | \Omega \rangle \text{ (reference point for energy is } H_0 | 0 \rangle = 0)$$

$$\left[\frac{\partial}{\partial t} + \mathcal{H} \right] \psi(t_0, -T) = 0 \quad \text{Q.E.D.}$$

In the same way $\langle \Omega |$
 \uparrow
 $\psi(t_0, -T)$

$$U(t_0, -T) \quad \text{WED}$$

In the same way, $\langle \Omega | = \lim_{T \rightarrow \infty} \langle 0 | U(T, t_0) e^{-iE_0(T-t_0)}$
 \uparrow bra

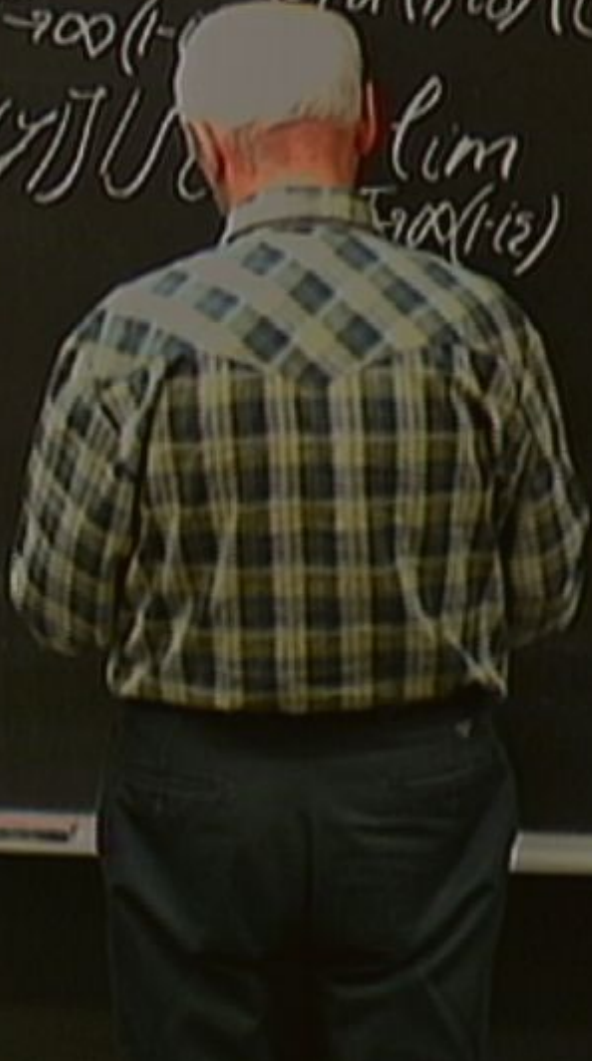


$U(t_0, -T)$ Q.E.D.

In the same way, $\langle \Omega | \uparrow_{\text{bra}} = \lim_{T \rightarrow \infty} \langle 0 | U(T, t_0) (e^{-iE_0(T-t_0)} \langle 0 | \Omega \rangle)^\dagger | A \rangle$

$$U(t_0, -T) \quad \text{Q.E.D.}$$

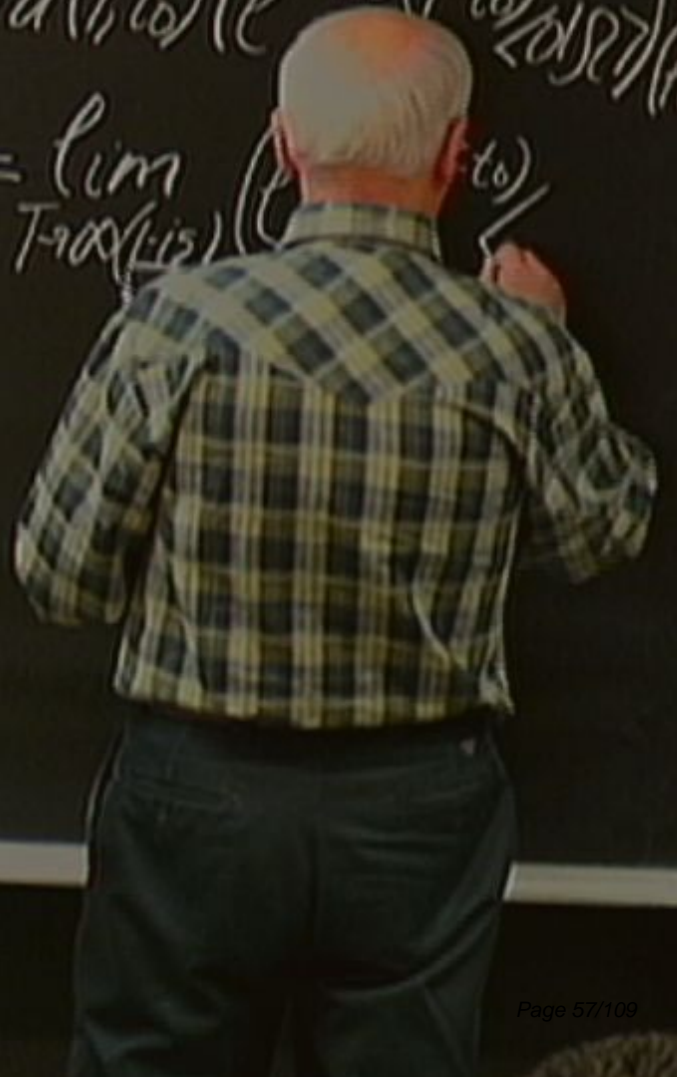
In the same way, $\langle R | = \lim_{T \rightarrow \infty} \langle 0 | U(T, t_0) (e^{-iE_0(T-t_0)} | 0 \rangle \langle 0 | R | A')^{-1}$
 Green's function: $\langle R | T \{ \psi(x) \psi(y) \} | S \rangle = \lim_{T \rightarrow \infty} \langle 0 | U(T, t_0) (e^{-iE_0(T-t_0)} | 0 \rangle \langle 0 | R | A')^{-1}$



$$u(t_0, -T) \quad \text{Q.E.D.}$$

In the same way, $\langle \Omega | \uparrow_{bra} = \lim_{T \rightarrow \infty} \langle 0 | u(T, t_0) (e^{-iE_0(T-t_0)} | \Omega \rangle)^\dagger$

Green's function: $\langle \Omega | T \{ \psi(x) \psi(y) \} | \Omega \rangle = \lim_{T \rightarrow \infty} \langle 0 | u(T, t_0) \psi(x) u(t_0, -T) \psi(y) u(-T, 0) | 0 \rangle$



$$U(t_0, -T) |0\rangle \text{ QED}$$

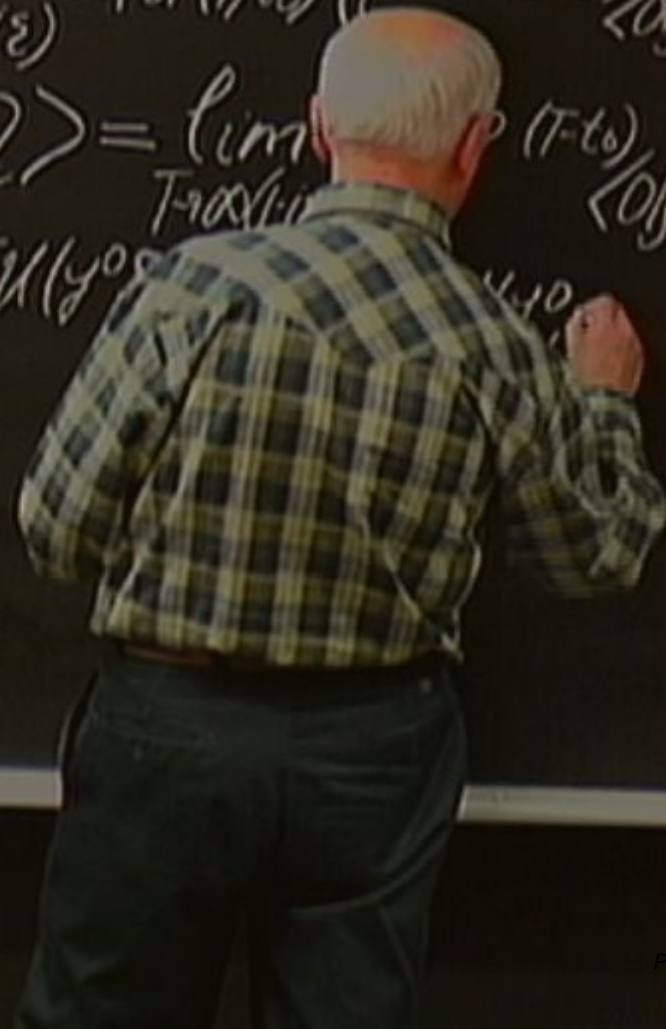
In the same way, $\langle \Omega | = \lim_{T \rightarrow \infty} \langle 0 | U(T, t_0) (e^{-iE_0(T-t_0)})^{-1} | \Omega \rangle$
 Green's func $\langle \Omega | T \{ \psi(x) \psi(y) \} | \Omega \rangle = \lim_{T \rightarrow \infty} (e^{-iE_0(T-t_0)})^{-1} \langle 0 | T \{ \psi(x) \psi(y) \} | 0 \rangle$
 $\cdot \langle 0 | U(T, t_0)$

$$U(t_0, -T) \left[\dots \right] \cdot \text{QED}$$

In the same way, $\langle \Omega | \dots \rangle = \lim_{T \rightarrow \infty} \langle 0 | U(T, t_0) (e^{-iE_0(T-t_0)} | \Omega \rangle)^\dagger$

Green's function: $\langle \Omega | T \{ \psi(x) \psi(y) \} | \Omega \rangle = \lim_{T \rightarrow \infty} \dots$

$\langle 0 | U(T, t_0) \cdot [U(x^0, t_0)]^\dagger \phi_I(x) U(x^0, t_0) [U(y^0, t_0)]^\dagger \dots$



$$U(t_0, -T) \left| 10 \right\rangle \text{ QED}$$

In the same way, $\langle \Omega | = \lim_{T \rightarrow \infty} \langle 0 | U(T, t_0) (e^{-iH_0(T-t_0)} | \Omega \rangle)^{-1}$
 Green's function: $\langle \Omega | T \{ \psi(x) \psi(y) \} | \Omega \rangle = \lim_{T \rightarrow \infty} (e^{-iH_0(T-t_0)} | \Omega \rangle)^{-1} \cdot$
 $\cdot \langle 0 | U(T, t_0) \cdot [U(x^0, t_0)]^\dagger \phi_I(x) U(x^0, t_0) [U(y^0, t_0)]^\dagger \phi_I(y) U(y^0, t_0) \times$
 \times

$$U(t_0, -T) |0\rangle \text{ QED}$$

In the same way, $\langle R| = \lim_{T \rightarrow \infty} \langle 0| U(T, t_0) (e^{-i t_0 (T-t_0)} |0\rangle)^{-1}$

Green's function: $\langle R| T \{ \psi(x) \psi(y) \} |R\rangle = \lim_{T \rightarrow \infty} (e^{-i t_0 (T-t_0)} |0\rangle)^{-1}$

$\cdot \langle 0| U(T, t_0) \cdot [U(x^0, t_0)]^\dagger \psi_I(x) U(x^0, t_0) [U(y^0, t_0)]^\dagger \psi_I(y) U(y^0, t_0) \times$

$\times U(t_0, T) |0\rangle (e^{-i t_0 (t_0 - (-T))} |R\rangle)^{-1}$

In the same way, $\langle R| = \lim_{T \rightarrow \infty} \langle 0|u(T, t_0) (e^{-iE_0(T-t_0)} \langle 0|R\rangle)^{-1}$

Green's function: $\langle R|T\{\psi(x)\psi(y)\}|\Omega\rangle = \lim_{T \rightarrow \infty} (e^{-iE_0(T-t_0)} \langle 0|R\rangle)^{-1}$
 $\cdot \langle 0|u(T, t_0) \cdot [u(x^0, t_0)]^\dagger \phi_I(x)u(x^0, t_0) [u(y^0, t_0)]^\dagger \phi_I(y)u(y^0, t_0) \times$
 $\times u(T, t_0) |0\rangle (e^{-iE_0(t_0-t_0)} \langle R|0\rangle)^{-1} = \lim_{T \rightarrow \infty} (1-i\epsilon)$

In the same way, $\langle R| = \lim_{T \rightarrow \infty} \langle 0|U(T, t_0) (e^{-iE_0(T-t_0)} \langle 0|R\rangle)^{-1}$

Green's function: $\langle R|T\{\psi(x)\psi(y)\}|R\rangle = \lim_{T \rightarrow \infty} (e^{-iE_0(T-t_0)} \langle 0|R\rangle)^{-1}$

$x^0 > y^0 > t_0$

$\cdot \langle 0|U(T, t_0) \cdot [U(x^0, t_0)]^\dagger \phi_I(x)U(x^0, t_0) [U(y^0, t_0)]^\dagger \phi_I(y)U(y^0, t_0) \times$

$\times U(t_0, T)|0\rangle (e^{-iE_0(t_0-t_0)} \langle R|0\rangle)^{-1} = \lim_{T \rightarrow \infty} (1-i\epsilon)$

In the same way, $\langle R| \overset{\text{bra}}{\uparrow} = \lim_{T \rightarrow \infty} \langle 0| u(T, t_0) (e^{-iE_0(T-t_0)} \langle 0| \rho(T) |A\rangle)^{-1}$

Green's function: $\langle R| T \{ \psi(x) \psi(y) \} | \Omega \rangle = \lim_{T \rightarrow \infty} (e^{-iE_0(T-t_0)} \langle 0| \rho(T) \rangle)^{-1}$

$x^0 > y^0 > t_0$

$\cdot \langle 0| u(T, t_0) \cdot [u(x^0, t_0)]^\dagger \phi_I(x) u(x^0, t_0) [u(y^0, t_0)]^\dagger \phi_I(y) u(y^0, t_0) \times$

$\times u(t_0, T) |0\rangle (e^{-iE_0(t_0 - (-T))} \langle R|0\rangle)^{-1} = \lim_{T \rightarrow \infty} \langle R| e^{-i(RT)} \rangle^{-1}$

In the same way, $\langle R| = \lim_{T \rightarrow \infty} \langle 0|u(T, t_0) (e^{-iE_0(T-t_0)} \langle 0|\Omega\rangle)^{-1}$

Green's function: $\langle R|T\{\psi(x)\psi(y)\}|\Omega\rangle = \lim_{T \rightarrow \infty} (e^{-iE_0(T-t_0)} \langle 0|\Omega\rangle)^{-1}$

$x^0 > y^0 > t_0$

$\cdot \langle 0|u(T, t_0) \cdot [u(x^0, t_0)]^\dagger \psi_I(x)u(x^0, t_0) [u(y^0, t_0)]^\dagger \psi_I(y)u(y^0, t_0) \times$

$\times u(t_0, T)|0\rangle (e^{-iE_0(t_0-T)} \langle R|0\rangle)^{-1} = \lim_{T \rightarrow \infty} (| \langle 0|\Omega\rangle |^2 e^{-iE_0 T})^{-1} \times$

$\times \langle 0|u(T, x^0) \psi_I(x)u(x^0, y^0) \psi_I(y)u(y^0, -T)|0\rangle$

$$\begin{aligned}
 & \langle 0 | \psi(T, t_0) \psi(x^0, t_0) \rangle^\dagger \langle \psi(x) \psi(x^0, t_0) | \psi(y^0, t_0) \rangle^\dagger \langle \psi(y) \psi(y^0, t_0) | \psi(x^0, t_0) \rangle \\
 & \times \langle \psi(x^0, t_0) | \psi(x, t_0) \rangle (e^{-t_0(t_0 - (-i))} \langle \mathcal{R} | \mathcal{R} \rangle)^{-1} = \lim_{T \rightarrow \infty (1-i\epsilon)} (|\langle 0 | \mathcal{R} \rangle|^2 e^{-i\epsilon T})^{-1} \\
 & \times \langle 0 | \psi(T, x^0) \psi_I(x) \psi(x^0, y^0) e^{-iH(x^0, y^0, -T)} | 0 \rangle;
 \end{aligned}$$

Look at $1 = \langle \mathcal{R} | \mathcal{R} \rangle =$



In the same way, $\langle R|T \rangle = \lim_{T \rightarrow \infty} \langle 0|U(T, t_0) (e^{-iH_0(t_0 - T)}) \langle 0|S(T) \rangle^{-1}$

Green's function: $\langle R|T \rangle \langle \psi(x) \psi(y) \rangle |\Omega\rangle = \lim_{T \rightarrow \infty} (e^{-iE_0(T-t_0)} \langle 0|S(T) \rangle)^{-1}$

$\cdot \langle 0|U(T, t_0) [U(x^0, t_0)]^\dagger \psi_I(x) U(x^0, t_0) [U(y^0, t_0)]^\dagger \psi_I(y) U(y^0, t_0) \times$
 $\times U(t_0, T) |0\rangle (e^{-iE_0(t_0 - T)}) \langle R|0 \rangle^{-1} = \lim_{T \rightarrow \infty} (|K|S(T)|^2 e^{-iKT})^{-1} \times$
 $\times \langle 0|U(T, x^0) \psi_I(x) U(x^0, y^0) \psi_I(y) U(y^0, -T) |0\rangle;$

Look at $\langle R|R \rangle =$



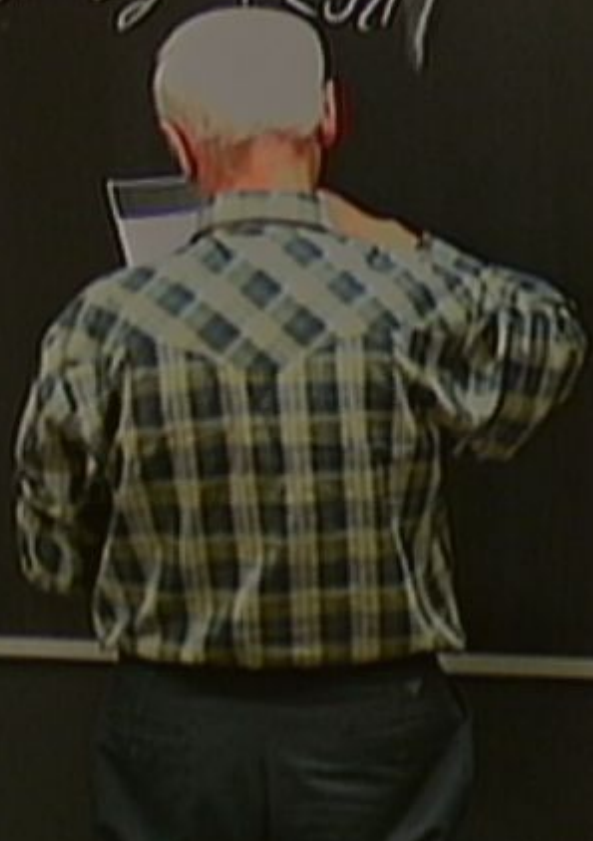
$$\begin{aligned}
 & \times \langle 0 | u(t_0, T) | 0 \rangle \left(e^{-t_0(t_0 - (-T))} \langle \Omega | 0 \rangle \right) = \lim_{T \rightarrow \infty (1-\epsilon)} \left(|K_0| \Omega T \right)^2 e^{-cRT} \times \\
 & \times \langle 0 | u(T, x^0) \rho_I(x) u(x^0, y^0) \rho_I(y) u(y^0, -T) | 0 \rangle; \\
 \text{Look at } 1 &= \langle \Omega | \Omega \rangle = \left(|K_0| \Omega T \right)^2 e^{-c(2T)} \langle 0 | u(T, y^0) u(y^0, -T) | 0 \rangle \\
 & = \left(|K_0| \Omega T e^{-cT} \right)^2 \langle 0 | u(T, -T) | 0 \rangle.
 \end{aligned}$$

Green's function

$$\begin{aligned}
 & \langle 0 | u(T, t_0) \cdot [u(x^0, t_0)]^\dagger \left(\int_I(x) u(x, t_0) [u(y^0, t_0)]^\dagger \int_I(y) u(y, t_0) \times \right. \\
 & \times u(t_0, T) | 0 \rangle \left. e^{-t_0(t_0 - (t))} \langle \Omega | 0 \rangle \right) = \lim_{T \rightarrow \infty (1-\epsilon)} \left(e^{-t_0(T-t_0)} \langle 0 | \Omega \rangle \right)^{-1} \\
 & \times \langle 0 | u(T, x^0) \int_I(x) u(x^0, y^0) \int_I(y) u(y^0, -T) | 0 \rangle = \lim_{T \rightarrow \infty (1-\epsilon)} \left(|\langle 0 | \Omega \rangle|^2 e^{-iKT} \right)^{-1} \\
 & \times \langle 0 | u(T, x^0) u(y^0, -T) | 0 \rangle; \\
 \text{Look at } 1 &= \langle \Omega | \Omega \rangle = \left(|\langle 0 | \Omega \rangle|^2 e^{-i(2T)} \right)^{-1} \cdot \langle 0 | u(T, x^0) u(y^0, -T) | 0 \rangle \\
 & = \left(|\langle 0 | \Omega \rangle|^2 e^{-i(2T)} \right)^{-1} \langle 0 | u(T, -T) | 0 \rangle.
 \end{aligned}$$

$$\begin{aligned}
 & \langle 0 | u(T, x^0) \psi_T(x) u(x^0, y^0) \psi_I(y) u(y^0, -T) | 0 \rangle \\
 & = \lim_{T \rightarrow \infty} \frac{1}{(2\pi)^{1/2}} \left(\frac{K_0 |x^0 - y^0|}{2T} e^{-iK_0 T} \right)^{-1} \times \\
 & \text{Look at } 1 = \int_{\mathbb{R}} \langle \Omega | \Omega \rangle = \left(\frac{K_0 |x^0 - y^0|}{2T} e^{-iK_0 T} \right)^{-1} \langle 0 | u(T, x^0) u(y^0, -T) | 0 \rangle \\
 & = \left(\frac{K_0 |x^0 - y^0|}{2T} e^{-iK_0 T} \right)^{-1} \langle 0 | u(T, x^0) u(y^0, -T) | 0 \rangle
 \end{aligned}$$

Thus at $x^0 > y^0$, $\langle \Omega | \Omega \rangle$



$$\langle \Omega | \psi(x) \psi(y) | \Omega \rangle = \lim_{T \rightarrow \infty} \frac{1}{(2\pi)^{1/2}} \left(\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dk e^{-ikx} \right) \left(\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dk' e^{-iky'} \right) \langle \Omega | \psi(x, T) \psi(y, -T) | \Omega \rangle$$

Look at $1 = \langle \Omega | \Omega \rangle = \left(\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dk e^{-ikx} \right) \left(\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dk' e^{-iky'} \right) \langle \Omega | \psi(x, T) \psi(y, -T) | \Omega \rangle$

$$= \left(\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dk e^{-ikx} \right) \langle \Omega | \psi(x, -T) | \Omega \rangle$$

Thus at $x^0 > y^0$: $\langle \Omega | \psi(x) \psi(y) | \Omega \rangle = \lim_{T \rightarrow \infty} \frac{1}{(2\pi)^{1/2}}$



$$\langle \Omega | \psi(x) \psi(y) | \Omega \rangle = \lim_{T \rightarrow \infty (1-\epsilon)} \left(|K_0| \Omega \right)^{-2} e^{-i(KT)} \times$$

$$\times \langle 0 | \psi(T, x^0) \psi_T(x) \psi(x^0, y^0) \psi_I(y) \psi(y^0, -T) | 0 \rangle,$$

Look at $1 = \langle \Omega | \Omega \rangle = (|K_0| \Omega)^{-2} e^{-i(KT)} \langle 0 | \psi(T, y^0) \psi(y^0, -T) | 0 \rangle$

$$= (|K_0| \Omega)^{-2} e^{-i(KT)} \langle 0 | \psi(T, -T) | 0 \rangle.$$

Thus at $x^0 > y^0$: $\langle \Omega | \psi(x) \psi(y) | \Omega \rangle = \lim_{T \rightarrow \infty (1-\epsilon)}$

$$\frac{\langle 0 | \psi(T, x^0) \psi_T(x) \psi(x^0, y^0) \psi_I(y) \psi(y^0, -T) | 0 \rangle}{\langle 0 | \psi(T, -T) | 0 \rangle}$$

$$\langle 0 | \psi(T, x^0) \psi_T(x) \psi(x^0, y^0) \psi_I(y) \psi(y^0, -T) | 0 \rangle = \lim_{T \rightarrow \infty (1-\epsilon)} \left(\langle 0 | \Omega_T \rangle^2 e^{-\epsilon(2T)} \right)^{-1} \times$$

Look at $1 = \frac{\langle \Omega | \Omega \rangle}{\langle 0 | \Omega \rangle} = \left(\langle 0 | \Omega_T \rangle^2 e^{-\epsilon(2T)} \right)^{-1} \cdot \langle 0 | \psi(T, y^0) \psi(y^0, -T) | 0 \rangle$

$$= \left(\langle 0 | \Omega_T \rangle^2 e^{-\epsilon(2T)} \right)^{-1} \langle 0 | \psi(T, -T) | 0 \rangle$$

Thus at $x^0 > y^0$: $\langle \Omega | \psi(x) \psi(y) | \Omega \rangle = \lim_{T \rightarrow \infty (1-\epsilon)} \frac{\langle 0 | \psi(T, x^0) \psi_I(x) \psi(x^0, y^0) \psi_I(y) \psi(y^0, -T) | 0 \rangle}{\langle 0 | \psi(T, -T) | 0 \rangle}$

$$\langle \Omega | \psi(x) \psi(y) | \Omega \rangle = \lim_{T \rightarrow \infty (1-\epsilon)} \left(\langle \Omega | \psi(T) \psi(x) \psi(x^0, y^0) \psi(y) \psi(y^0, -T) | \Omega \rangle \right)^{-1} \times \langle \Omega | \psi(T, x^0) \psi_T(x) \psi(x^0, y^0) \psi(y) \psi(y^0, -T) | \Omega \rangle$$

Look at $1 = \langle \Omega | \psi(T) \psi(T) | \Omega \rangle = \left(\langle \Omega | \psi(T) \right)^2 e^{-i(2T)} \langle \Omega | \psi(T, y^0) \psi(y^0, -T) | \Omega \rangle$

$$= \left(\langle \Omega | \psi(T) \right)^2 e^{-i(2T)} \langle \Omega | \psi(T, -T) | \Omega \rangle$$

Thus at $x^0 > y^0$: $\langle \Omega | \psi(x) \psi(y) | \Omega \rangle = \lim_{T \rightarrow \infty (1-\epsilon)} \frac{\langle \Omega | \psi(T, x^0) \psi_T(x) \psi(x^0, y^0) \psi(y) \psi(y^0, -T) | \Omega \rangle}{\langle \Omega | \psi(T, -T) | \Omega \rangle}$

Restoring T order, we get answer both for $x^0 > y^0, y^0 > x^0$

$$\langle \Omega | \psi(x) \psi(y) | \Omega \rangle = \lim_{T \rightarrow \infty (1-\epsilon)} \left(\langle \Omega | \psi(T) \right)^{-1} \times \langle 0 | \psi(T, x^0) \psi_T(x) \psi(x^0, y^0) \psi_I(y) \psi(y^0, -T) | 0 \rangle$$

Look at $1 = \frac{\langle \Omega | \Omega \rangle}{\langle \Omega | \Omega \rangle} = \frac{\langle \Omega | \psi(T) \rangle \langle \Omega | \psi(T) \rangle^{-1} \langle 0 | \psi(T, x^0) \psi(y^0, -T) | 0 \rangle}{\langle \Omega | \Omega \rangle e^{-i(E_0 T)}} \langle 0 | \psi(T, x^0) \psi(y^0, -T) | 0 \rangle$

Thus at $x^0 > y^0$: $\langle \Omega | \psi(x) \psi(y) | \Omega \rangle = \lim_{T \rightarrow \infty (1-\epsilon)} \frac{\langle 0 | \psi(T, x^0) \psi_I(x) \psi(x^0, y^0) \psi_I(y) \psi(y^0, -T) | 0 \rangle}{\langle 0 | \psi(T, -T) | 0 \rangle}$

Restoring T order, we get answer both for $x^0 > y^0, y^0 > x^0$

$$\langle \Omega | T \{ \psi(x) \psi(y) \} | \Omega \rangle = \frac{\langle 0 | T \{ \psi_I(x) \psi_I(y) \} \exp$$

$$\langle \Omega | \psi(x) \psi(y) | \Omega \rangle = \lim_{T \rightarrow \infty (1-\epsilon)} \left(|\Omega\rangle \right)^2 e^{-i(2T)} \times \langle 0 | \psi(T, x^0) \psi_I(x) \psi(x^0, y^0) \psi_I(y) \psi(y^0, -T) | 0 \rangle$$

Look at $1 = \langle \Omega | \Omega \rangle = \left(|\Omega\rangle \right)^2 e^{-i(2T)} \langle 0 | \psi(T, y^0) \psi(y^0, -T) | 0 \rangle$

$$= \left(|\Omega\rangle \right)^2 e^{-i(2T)} \langle 0 | \psi(T, -T) | 0 \rangle$$

Thus at $x^0 > y^0$: $\langle \Omega | \psi(x) \psi(y) | \Omega \rangle = \lim_{T \rightarrow \infty (1-\epsilon)} \langle 0 | \psi(T, x^0) \psi_I(x) \psi(x^0, y^0) \psi_I(y) \psi(y^0, -T) | 0 \rangle$



Restoring T order

$$\langle \Omega | T \{ \psi(x) \psi(y) \} | \Omega \rangle = \langle 0 | T \{ \psi_I(x) \psi_I(y) \exp[-i \int_T^t dt H_I(t)] \exp[-i \int_t^T dt H_I(t)] \} | 0 \rangle$$

get answer both for $x^0 > y^0$, $y^0 > x^0$

$$\langle \Omega | \psi(x) \psi(y) | \Omega \rangle = \lim_{T \rightarrow \infty (1-\epsilon)} \left(|\langle \Omega | \Omega \rangle|^2 e^{-i(2T)} \right)^{-1} \times \langle \Omega | \psi(T, x^0) \psi_I(x) \psi(x^0, y^0) \psi_I(y) \psi(y^0, -T) | \Omega \rangle$$

Look at $1 = \frac{\langle \Omega | \Omega \rangle}{\langle \Omega | \Omega \rangle} = \frac{\langle \Omega | \Omega \rangle}{\left(|\langle \Omega | \Omega \rangle|^2 e^{-i(2T)} \right)^{-1} \langle \Omega | \psi(T, x^0) \psi(y^0, -T) | \Omega \rangle}$

$$= \left(|\langle \Omega | \Omega \rangle|^2 e^{-i(2T)} \right)^{-1} \langle \Omega | \psi(T, x^0) \psi(y^0, -T) | \Omega \rangle$$

Thus at $x^0 > y^0$: $\langle \Omega | \psi(x) \psi(y) | \Omega \rangle = \lim_{T \rightarrow \infty (1-\epsilon)} \frac{\langle \Omega | \psi(T, x^0) \psi_I(x) \psi(x^0, y^0) \psi_I(y) \psi(y^0, -T) | \Omega \rangle}{\langle \Omega | \psi(T, -T) | \Omega \rangle}$

Restoring T order, we get answer both for $x^0 > y^0$, $y^0 > x^0$

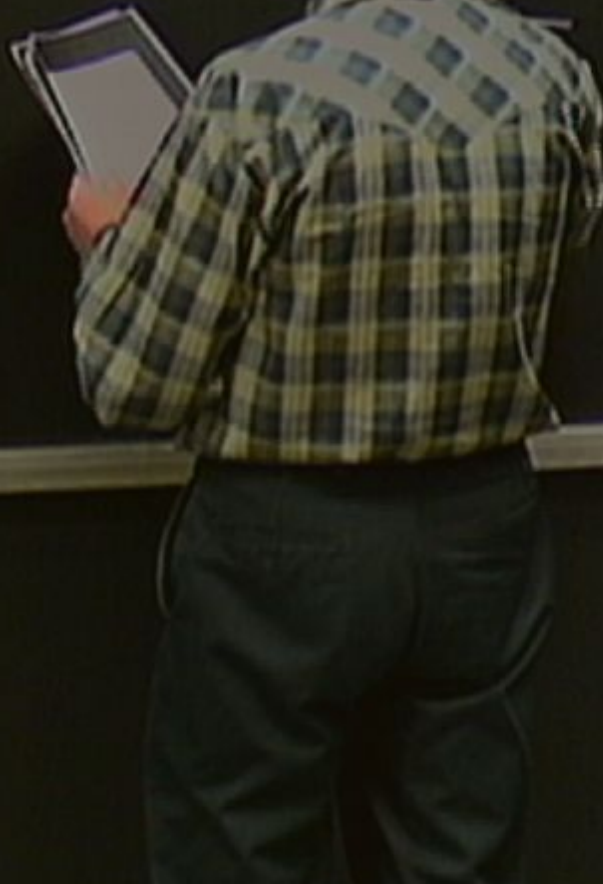
$$\langle \Omega | T \{ \psi(x) \psi(y) | \Omega \rangle = \frac{\langle \Omega | T \{ \psi_I(x) \psi_I(y) \exp[-i \int_T^t dt H_I(t)] \}}{\langle \Omega | T \{ \exp[-i \int_T^T dt H_I(t)] \}}$$

Generalization for arbitrary G

Generalization for arbitrary Green's function:
 $\langle \Omega | T(\psi(x)) \dots$

Generalization for arbitrary Green's function:
 $\langle \Omega | T(\psi(x_1)\psi(x_2)\dots\psi(x_n)) | \Omega \rangle = \text{lim}$

Generalization for arbitrary Green's function:

$$\langle \Omega | T(\psi(x_1)\psi(x_2)\dots\psi(x_n)) | \Omega \rangle = \lim_{T \rightarrow \infty} \frac{\langle 0 | T\{\psi_I(x_1)\psi_I(x_2)\dots\psi_I(x_n)\} | 0 \rangle}{\langle 0 | T\{1\} | 0 \rangle}$$


Generalization for arbitrary Green's function:

$$\langle \Omega | T(\psi(x_1)\psi(x_2)\dots\psi(x_n)) | \Omega \rangle = \lim$$

$$\langle 0 | T \left[\psi_I(x_1)\psi_I(x_2)\dots\psi_I(x_n) \exp \left(i \int_{-T}^T dt H_I(t) \right) \right] | 0 \rangle$$

$$\langle 0 | T \left\{ \exp \left[i \int_{-T}^T dt H_I(t) \right] \right\} | 0 \rangle$$



Generalization for arbitrary Green's function:

$$\langle \Omega | T(\psi(x_1)\psi(x_2)\dots\psi(x_n)) | \Omega \rangle = \lim_{T \rightarrow \infty} \frac{\langle 0 | T \left\{ \psi_I(x_1)\psi_I(x_2)\dots\psi_I(x_n) \exp \left[-i \int_{-T}^T dt H_I(t) \right] \right\} | 0 \rangle}{\langle 0 | T \left\{ \exp \left[-i \int_{-T}^T dt H_I(t) \right] \right\} | 0 \rangle}$$



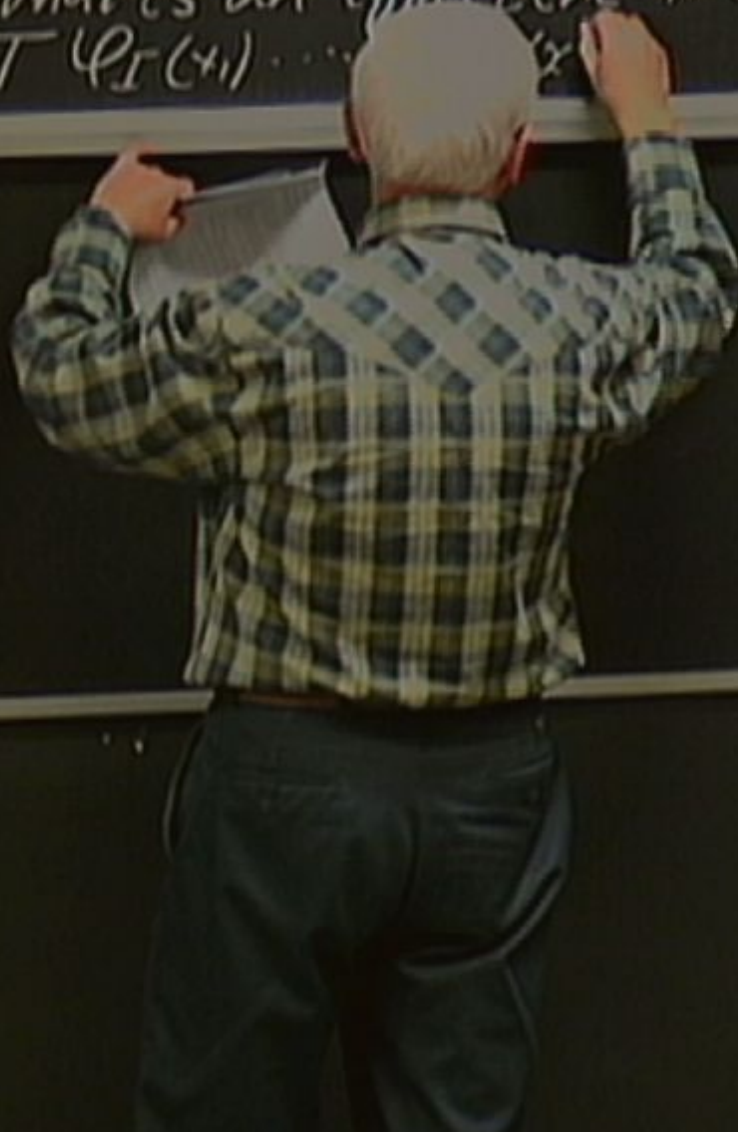
$$\langle \Omega | T(\psi(x_1)\psi(x_2)\dots\psi(x_n)) | \Omega \rangle = \lim_{T \rightarrow \infty(1-i\epsilon)} \langle 0 | T\left\{ \psi_I(x_1)\psi_I(x_2)\dots\psi_I(x_n) \exp\left[-i \int_{-T}^T dt H_I(t)\right] \right\} | 0 \rangle$$

$$\langle 0 | T\left\{ \exp\left[-i \int_{-T}^T dt H_I(t)\right] \right\} | 0 \rangle$$

Wick's Theorem

VIET'S THEOREM

Question: What is an effective method to calculate
 $\langle \text{GIT} \Psi_I(t) \rangle \dots$



Question: What is an effective method to calculate $\langle 0|T \psi_I(x) \dots \psi_I(x)|0\rangle$?

Let us start from $\langle 0|T \psi_I(x) \psi_I(y)|0\rangle = D_F(x-y)$

Question: What is an effective method to calculate $\langle 0|T \psi_I(x) \dots \psi_I(x)|0\rangle$?

Let us start from

$$\psi_I(x) = \psi$$

$$\langle 0|T \psi_I(x) \psi_I(y)|0\rangle = D_F(x-y);$$

Question: What is an effective method to calculate $\langle 0|T \psi_I(x) \dots \psi_I(x)|0\rangle$?

Let us start from $\langle 0|T \psi_I(x) \psi_I(y)|0\rangle = D_F(x-y)$;
 $\psi_I(x) = \psi_I^+(x) + \psi_I^-(x)$; $\psi_I^+(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_{\vec{p}}}} a_{\vec{p}} e^{-ipx}$, $\psi_I^-(x) =$

Question: What is an effective method to calculate $\langle 0|T \psi_I(x) \dots \psi_I(x)|0\rangle$?

Let us start from $\langle 0|T \psi_I(x) \psi_I(y)|0\rangle = D_F(x-y)$
 $\psi_I(x) = \psi_I^+(x) + \psi_I^-(x); \psi_I^+(x) = \int \frac{d^3p}{(2\pi)^3 2E_{\vec{p}}} a_{\vec{p}} e^{-ipx}, \psi_I^-$
 $= \int \frac{d^3p}{(2\pi)^3 2E_{\vec{p}}} a_{\vec{p}}^+ e^{ipx}; \psi_I^+(x)$

Question: What is an effective method to calculate $\langle 0|T \psi_I(x) \dots \psi_I(x)|0\rangle$?

Let us start from $\langle 0|T \psi_I(x) \psi_I(y)|0\rangle = D_F(x-y)$
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 $= \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_{\vec{p}}}} a_{\vec{p}}^+ e^{ipx}; \psi_I^+(x)|0\rangle = 0, \langle 0|\psi_I^-(x) = 0$

Question: What is an effective method to calculate

Let us start from $\langle 0 | T \phi_I(x) \phi_I(y) | 0 \rangle = D_F(x-y)$,
 $\phi_I(x) = \phi_I^+(x) + \phi_I^-(x)$; $\phi_I^+(x) = \int \frac{d^3p}{(2\pi)^3 \sqrt{2E_{\vec{p}}}} a_{\vec{p}} e^{-ipx}$, $\phi_I^-(x) =$
 $= \int \frac{d^3p}{(2\pi)^3 \sqrt{2E_{\vec{p}}}} a_{\vec{p}}^{\dagger} e^{ipx}$; $\langle 0 | \phi_I^+(x) | 0 \rangle = 0$, $\langle 0 | \phi_I^-(x) | 0 \rangle = 0$.

Question: What is an effective method to calculate

Let us start from $\langle 0 | T \psi_I(x) \psi_I(y) | 0 \rangle = D_F(x-y)$;
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 $= \int \frac{d^3p}{(2\pi)^3 \sqrt{2E_{\vec{p}}}} a_{\vec{p}}^{\dagger} e^{ipx}$; $\langle 0 | \psi_I^+(x) | 0 \rangle = 0$, $\langle 0 | \psi_I^-(x) | 0 \rangle = 0$.
First consider $x^0 > y^0$: $T(\psi(x)\psi(y))$

Question: What is an effective method to calculate

Let us start from $\langle 0 | T \phi_I(x) \phi_I(y) | 0 \rangle = D_F(x-y)$,

$$\phi_I(x) = \phi_I^+(x) + \phi_I^-(x); \quad \phi_I^+(x) = \int \frac{d^3p}{(2\pi)^3 2E_{\vec{p}}} a_{\vec{p}} e^{-ipx}, \quad \phi_I^-(x) = \int \frac{d^3p}{(2\pi)^3 2E_{\vec{p}}} a_{\vec{p}}^{\dagger} e^{ipx};$$

$\langle 0 | \phi_I^+(x) | 0 \rangle = 0, \quad \langle 0 | \phi_I^-(x) | 0 \rangle = 0$

First consider $x^0 > y^0$: $T(\phi(x)\phi(y)) \stackrel{x^0 > y^0}{=} \phi^+(x)\phi^+(y) + \phi^+(x)\phi^-(y)$

Question: What is an effective method to calculate

Let us start from $\langle 0 | T \phi_I(x) \phi_I(y) | 0 \rangle = D_F(x-y)$;
 $\phi_I(x) = \phi_I^+(x) + \phi_I^-(x)$; $\phi_I^+(x) = \int \frac{d^3p}{(2\pi)^3 2E_p} a_{\vec{p}} e^{-ipx}$, $\phi_I^-(x) =$
 $= \int \frac{d^3p}{(2\pi)^3 2E_p} a_{\vec{p}}^{\dagger} e^{ipx}$; $\langle 0 | \phi_I^+(x) | 0 \rangle = 0$, $\langle 0 | \phi_I^-(x) | 0 \rangle = 0$.

First consider $x^0 > y^0$: $T(\phi(x)\phi(y)) \stackrel{x^0 > y^0}{=} \phi^+(x)\phi^+(y) + \phi^+(x)\phi^-(y) + \phi^-(x)\phi^+(y) + \phi^-(x)\phi^-(y)$

Question: What is an effective method to calculate

Let us start from $\langle 0 | T \psi_I(x) \psi_I(y) | 0 \rangle = D_F(x-y)$,

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$$\langle 0 | \psi_I^+(x) | 0 \rangle = 0, \quad \langle 0 | \psi_I^-(x) | 0 \rangle = 0.$$

Let us consider $x^0 > y^0$: $T(\psi(x)\psi(y)) \stackrel{x^0 > y^0}{=} \psi(x)\psi(y) + \psi_I^+(x)\psi_I^-(y)$

$$\psi_I^-(x)\psi_I^+(y) + \psi_I^-(x)\psi_I^-(y) \Rightarrow \psi_I^+(x)\psi_I^-(y) = \psi_I^-(y)\psi_I^+(x) + [\psi_I^+(x), \psi_I^-(y)]$$

contraction of two fields:

$$\varphi(x)\varphi(y) \equiv$$



contraction of two fields:
 $\varphi(x)\varphi(y) \equiv \begin{cases} [\varphi^+(x), \varphi^-(y)], & x^0 > y^0 \\ [\varphi^+(y), \varphi^-(x)], & y^0 > x^0 \end{cases}$

contraction of two fields:

$$\overline{\varphi(x)\varphi(y)} \equiv \begin{cases} [\varphi^+(x), \bar{\varphi}(y)], & x^0 > y^0 \\ [\varphi^+(y), \bar{\varphi}(x)], & y^0 > x^0 \end{cases} = D_F(x-y).$$

contraction of two fields:

$$\overline{\psi_I(x)\psi_I(y)} \equiv \begin{cases} [\psi_I^+(x), \bar{\psi}_I(y)], & x^0 > y^0 \\ [\psi_I^+(y), \bar{\psi}_I(x)], & y^0 > x^0 \end{cases} = D_F(x-y)$$

contraction of two fields:

$$\overline{\psi_I(x)\psi_I(y)} \equiv \begin{cases} [\psi_I^+(x), \bar{\psi}_I(y)], & x^0 > y^0 \\ [\psi_I^+(y), \bar{\psi}_I(x)], & y^0 > x^0 \end{cases} \quad \text{if } x^0 - y^0 > |\mathbf{x} - \mathbf{y}|$$

Normal order

contraction of two fields:

$$\overline{\psi_I(x)\psi_I(y)} \equiv \begin{cases} [\psi_I^+(x), \bar{\psi}_I(y)], & x^0 > y^0 \\ [\psi_I^+(y), \bar{\psi}_I(x)], & y^0 > x^0 \end{cases} = D_F(x-y)$$

Normal order: $N(\psi(x)\psi(y))$

contraction of two fields:

$$\overline{\psi_I(x)\psi_I(y)} \equiv \begin{cases} [\psi_I^+(x), \bar{\psi}_I(y)], & x^0 > y^0 \\ [\psi_I^+(y), \bar{\psi}_I(x)], & y^0 > x^0 \end{cases} = D_F(x-y)$$

Normal order: $N(\psi(x)\psi(y)) =$

contraction of two fields:

$$\overline{\psi_I(x)\psi_I(y)} \equiv \begin{cases} [\psi_I^+(x), \bar{\psi}_I(y)], & x^0 > y^0 \\ [\psi_I^+(y), \bar{\psi}_I(x)], & y^0 > x^0 \end{cases} = D_F(x-y).$$

Normal order: $N(\psi_I(x)\psi_I(y)) = \psi_I^+(x)\psi_I^+(y)$

contraction of two fields:

$$D_F(x-y) \equiv \begin{cases} [\psi_{\mathbb{I}}^+(x), \bar{\psi}_{\mathbb{I}}(y)], & x^0 > y^0 \\ [\psi_{\mathbb{I}}^+(y), \bar{\psi}_{\mathbb{I}}(x)], & y^0 > x^0 \end{cases} = D_F(x-y).$$

order: $N(\psi_{\mathbb{I}}(x)\psi_{\mathbb{I}}(y)) = \psi_{\mathbb{I}}^+(x)\psi_{\mathbb{I}}^+(y) + \bar{\psi}_{\mathbb{I}}(y)\psi_{\mathbb{I}}^+(x) +$

contraction of two fields:

$$\overline{\varphi_I(x)\varphi_I(y)} = \begin{cases} [\varphi_I^+(x), \bar{\varphi}_I(y)], & x^0 > y^0 \\ [\varphi_I^+(y), \bar{\varphi}_I(x)], & y^0 > x^0 \end{cases} = D_F(x-y)$$

Normal

$$+ \varphi_I^-(x)\varphi_I^-(y) \quad \langle 0 | N(\varphi_I(x)\varphi_I(y)) | 0 \rangle = 0$$

$$+ \varphi_I^-(x)\varphi_I^-(y) \quad \langle 0 | N(\varphi_I(x)\varphi_I(y)) | 0 \rangle = 0$$

contraction of two fields:

$$\overline{\varphi_I(x)\varphi_I(y)} \equiv \begin{cases} [\varphi_I^+(x), \bar{\varphi}_I(y)], & x^0 > y^0 \\ [\varphi_I^+(y), \bar{\varphi}_I(x)], & y^0 > x^0 \end{cases} = D_F(x-y)$$

Normal order: $N(\varphi_I(x)\varphi_I(y)) = \varphi_I^+(x)\varphi_I^+(y) + \bar{\varphi}_I^-(y)\varphi_I^+(x) + \varphi_I^-(x)\varphi_I^-(y) + \bar{\varphi}_I^-(x)\varphi_I^-(y)$

$$\langle 0 | N(\varphi_I(x)\varphi_I(y)) | 0 \rangle = 0$$

contraction of two fields:

$$\overline{\varphi_I(x)\varphi_I(y)} \equiv \begin{cases} [\varphi_I^+(x), \bar{\varphi}_I(y)], & x^0 > y^0 \\ [\varphi_I^+(y), \bar{\varphi}_I(x)], & y^0 > x^0 \end{cases} = D_F(x-y).$$

Normal order: $N(\varphi_I(x)\varphi_I(y)) = \varphi_I^+(x)\varphi_I^+(y) + \bar{\varphi}_I(y)\varphi_I^+(x) + \varphi_I^-(x)\varphi_I^-(y) + \bar{\varphi}_I(x)\varphi_I^-(y)$

$$\langle 0 | N(\varphi_I(x)\varphi_I(y)) | 0 \rangle = 0.$$

$(211) \sqrt{2} \Gamma_{\vec{p}} a_{\vec{p}} (\quad ; \psi_I(x) | 0 \rangle = 0, \langle 0 | \psi_I(x) = 0$
 First consider $x^0 > y^0$: $T(\psi(x)\psi(y)) \stackrel{x^0 > y^0}{=} \psi(x)\psi(y) + \psi_I^+(x)\psi_I^-(y)$
 $+ \psi_I^-(x)\psi_I^+(y) + \psi_I^-(x)\psi_I^-(y) \Rightarrow \psi_I^+(x)\psi_I^-(y) = \psi_I^-(x)\psi_I^+(y) + [\psi_I^+(x), \psi_I^-(y)]$
 For $y^0 > x^0 \Rightarrow \psi_I^+(y), \psi_I^-(x)$

$(x)\psi_I(y) = \psi_I(x)\psi_I(y) + \psi_I(y)\psi_I(x) +$
 (y)
 $\langle 0 | N(\psi_I(x)\psi_I(y)) | 0 \rangle = 0.$

$\psi_I(x) \Rightarrow \langle \psi_I(y), \psi_I(x) \rangle$

contraction of two fields:

$$\overline{\psi_I(x)\psi_I(y)} \equiv \begin{cases} [\psi_I^+(x), \bar{\psi}_I(y)], & x^0 > y^0 \\ [\psi_I^+(y), \bar{\psi}_I(x)], & y^0 > x^0 \end{cases} = D_F(x-y)$$

order:

$$N(\psi_I(x)\psi_I(y)) = \psi_I^+(x)\psi_I^+(y) + \bar{\psi}_I(y)\psi_I^+(x) + \psi_I^+(y)\psi_I^-(x) + \bar{\psi}_I(x)\psi_I^-(y)$$

$$\langle 0 | N(\psi_I(x)\psi_I(y)) | 0 \rangle = 0$$