Title: Measurement Pattern Interpolation: Theory, and potential applications

Date: Nov 19, 2008 04:00 PM

URL: http://pirsa.org/08110050

Abstract: Quantum computation by single-qubit measurements was proposed by Raussendorf and Briegel [PRL 86, 5188] as a potential scheme for implementing quantum computers. It also offers an unusual means of describing unitary transformations. To better understand which measurement-based procedures perform unitary operations, we may consider the following problem: under what circumstances can a measurement-based procedure for a unitary U be found, provided a similar procedure for U which relies on post-selection? In this talk, I describe the so-called \'Measurement Pattern Interpolation\' problem, the intuition behind the solved special cases, and possible applications of a general solution to this problem.

Measurement Pattern Interpolation

Theory, and potential applications

Niel de Beaudrap

Institute for Quantum Computing, Waterloo

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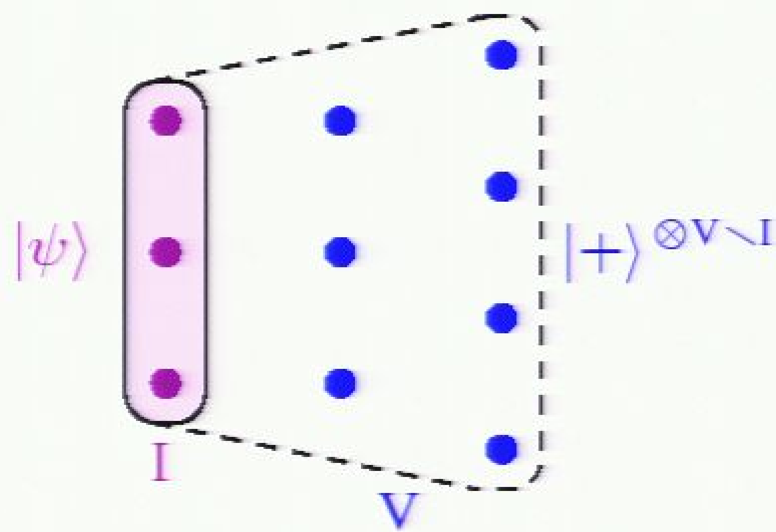

V = set of all qubits

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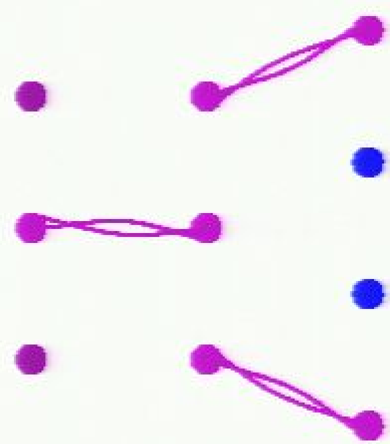

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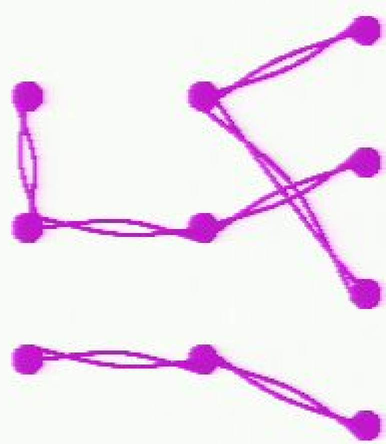

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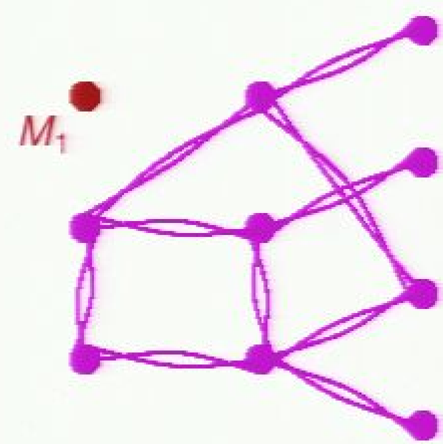

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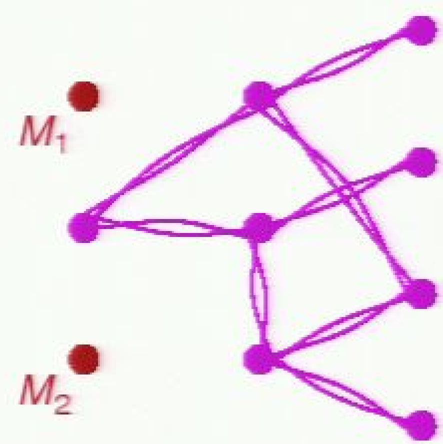

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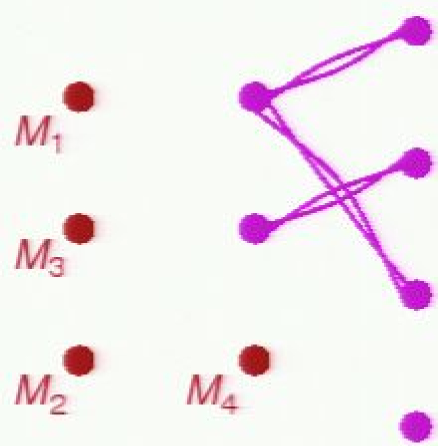

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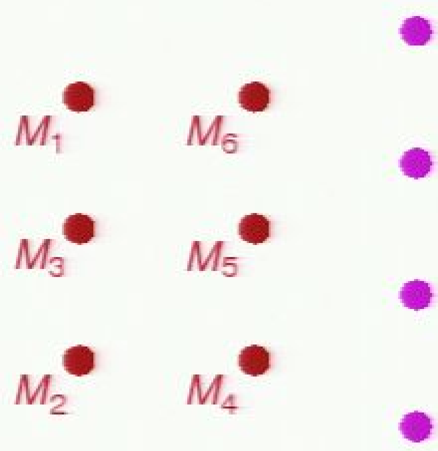

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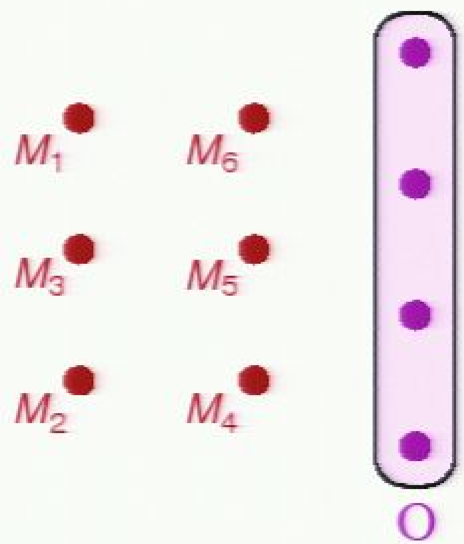

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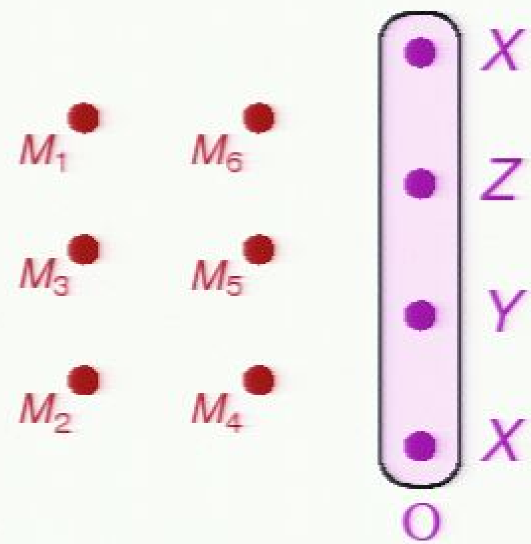
- perform LU "corrections" on any residual quantum state.



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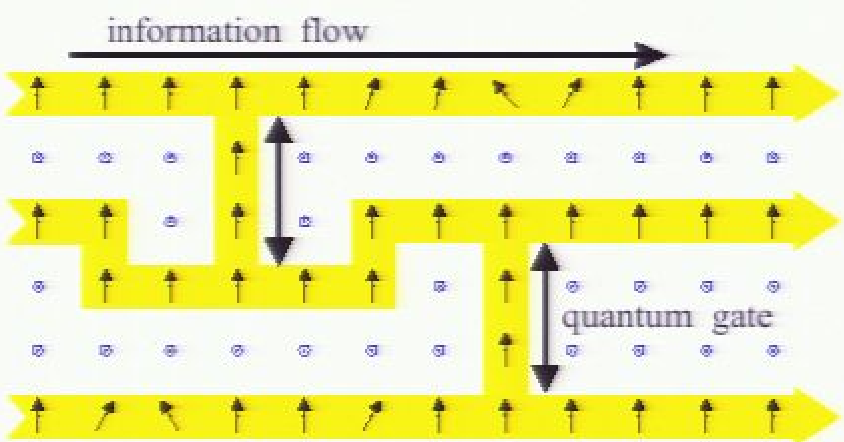
Special cases: teleportation of data + simple transformations (idioms: "quantum wires" & instantaneous interactions between them)



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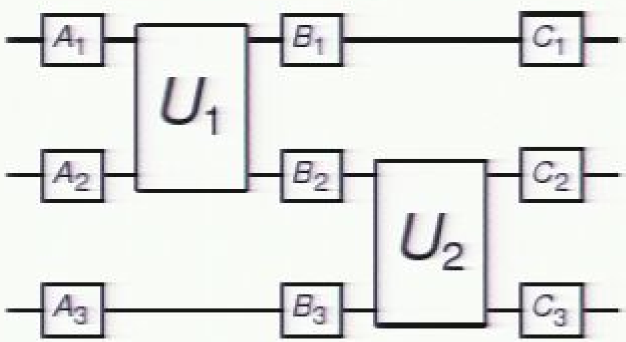

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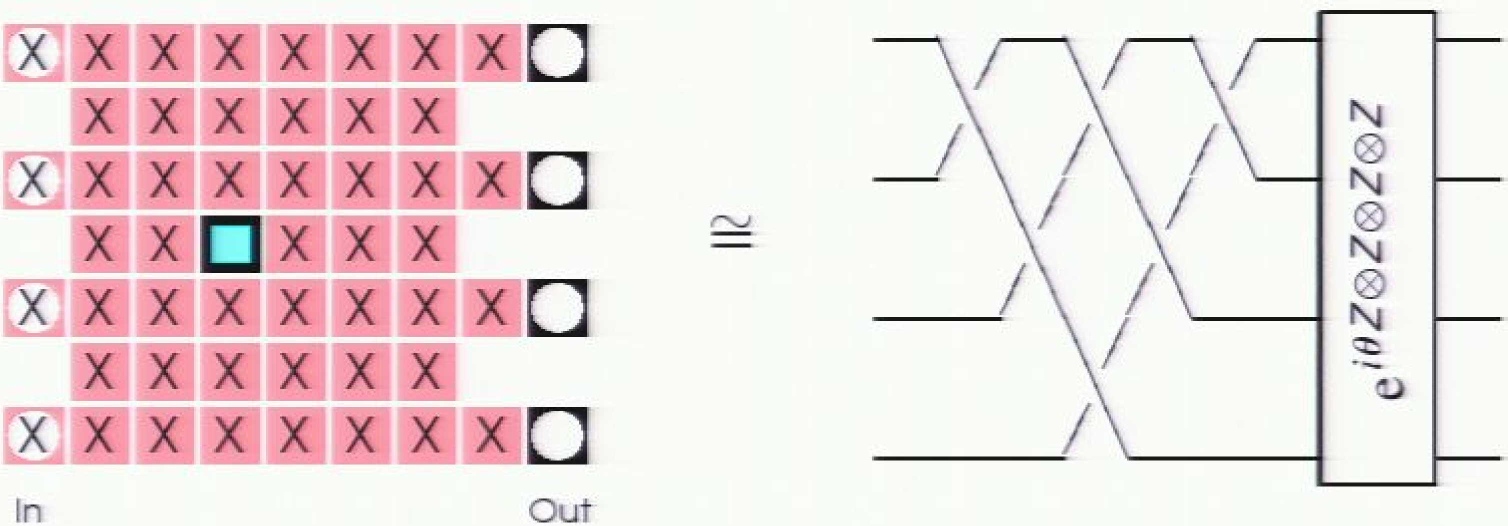

colour diagram from PRA **68** (022312), © 2003 American Physical Society

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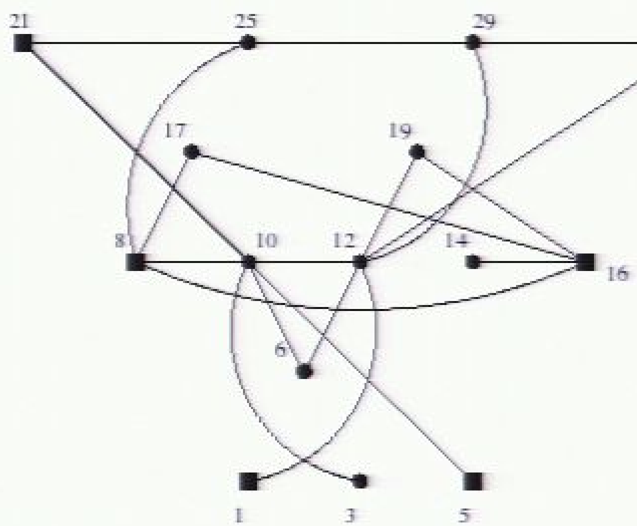

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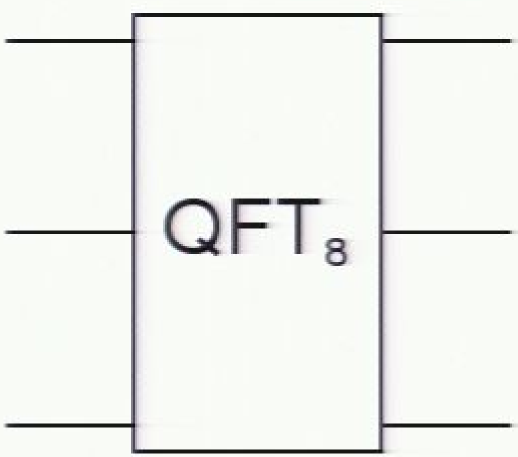

graph diagram from PRA **69** (062311), © 2004 American Physical Society

"Understanding" MBQC

A very broad question.

Given a one-way measurement-based procedure (*measurement pattern*) for a unitary U, what other representations of U can we efficiently obtain?

e.g. unitary circuit, matrix formula, ...

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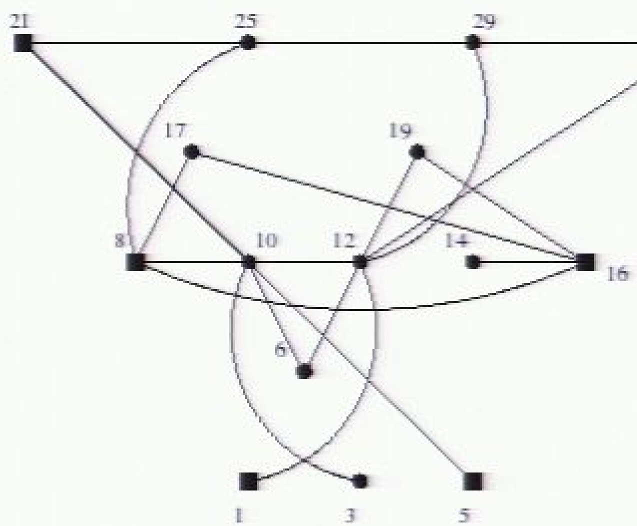

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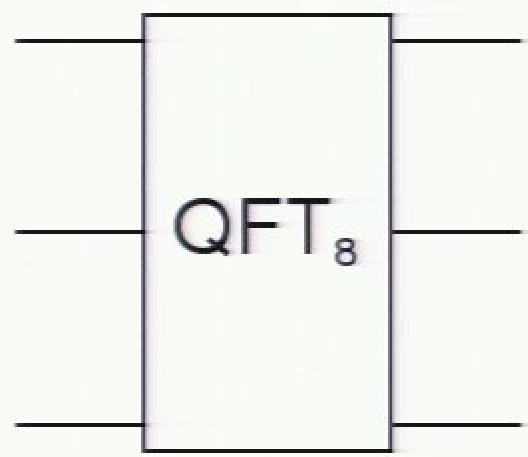

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A more modest question.

Given a measurement pattern, can we identify whether it performs a unitary transformation?

- Does it perform a unitary transformation, conditioned on always obtaining the $+1$ measurement result?
- Does it perform the same CP map, independent of measurement results?

Outline

1. Introducing Measurement Pattern Interpolation (MPI)
   - Potential applications
2. Solved special cases of MPI
3. Outlook for a general solution

Anatomy of a measurement pattern

Non-adaptive aspects of a measurement pattern:

- the entangling unitary embedding
  — given by the set of qubits V, input subsystem I, entanglement graph
- the set of qubits O which are left unmeasured
- the order in which the measurements are performed

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Adaptive aspects of a measurement pattern:

- observables M_v for each qubit to be measured
  — conditioned on prior measurement results
- final local unitary correction operations
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Different measurement results give rise to *by-product* operators
—- effective changes of reference frame for later measurement observables

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- the *default* observables M_v for each qubit to be measured
  — conditioned on all prior measurements yielding the $+1$ result

Adaptive aspects of a measurement pattern:

- byproduct operators arising from measurement results
  — determines observables to actually measure, and final corrections

Different measurement results give rise to *by-product* operators

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Measurement Pattern Interpolation (MPI)

Observation. The non-adaptive aspects of a measurement pattern specify a *postselection* procedure.

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Given a specification of an MBQC-*like* postselection procedure:

- an entanglement graph, with a vertex-subset I (specifying an entangling unitary embedding);
- a vertex-subset O representing the qubits which are to be left unmeasured;
- the default observables M_v for each qubit to be measured (conditioned on all prior measurements yielding the $+1$ result);

Find: a measurement order & suitable byproduct operators, yielding a complete description of a measurement pattern which performs a unitary transformation — if such orders/operators exist.

What solving MPI allows us to do

We would be able to:

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We *might* be able to:

- analyze non-unitary MBQC in terms of unitary MBQC;
- represent measurement patterns in terms of unitary circuits *of similar complexity*; or
- discover interesting new primitives of quantum computation.

Looking through the lens of the MPI

Sketch of a *projection*-based quantum computation:

$$|\psi\rangle = \sum_{\mathbf{z}\in\{0,1\}^{\mathrm{I}}} \alpha_{\mathbf{z}} \, |\mathbf{z}\rangle$$

I = the set of input qubits

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Operator-valued expressions

Quadratic form expansion:
an operator-expression of the form

$$U \;=\; C \sum_{\mathbf{x}\in\{0,1\}^{\mathrm{V}}} e^{i\,Q(\mathbf{x})}\; |\mathbf{x}_{\mathrm{O}}\rangle\langle\mathbf{x}_{\mathrm{I}}|\;, \qquad \text{where} \quad Q(\mathbf{x}) \;=\; \sum_{\{u,v\}\subseteq \mathrm{V}} \theta_{uv}\, x_u x_v$$

Notation: $\mathbf{x}_{\mathrm{I}}$, $\mathbf{x}_{\mathrm{O}}$ are restrictions of $\mathbf{x} \in \{0,1\}^{\mathrm{V}}$ to the index-sets $\mathrm{I}, \mathrm{O} \subseteq \mathrm{V}$

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- Discretization of path integrals?

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Flows: a first approach to MPI

Start with a simple, universal construction for measurement patterns.

Danos, Kashefi, Panangaden. *J. ACM* **54** #2 (p. 8), 2007.

- quantum wires

  — can be constructed from the following primitive pattern:

…where $M(\theta_v) = \cos(\theta_v)X - \sin(\theta_v)Y$

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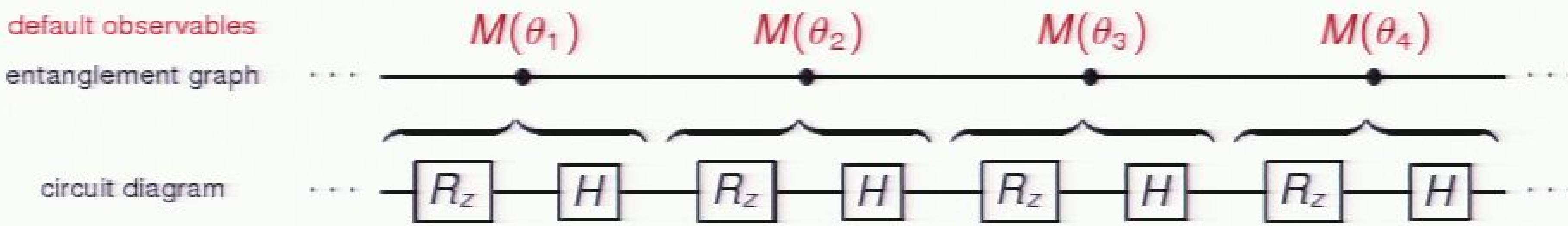

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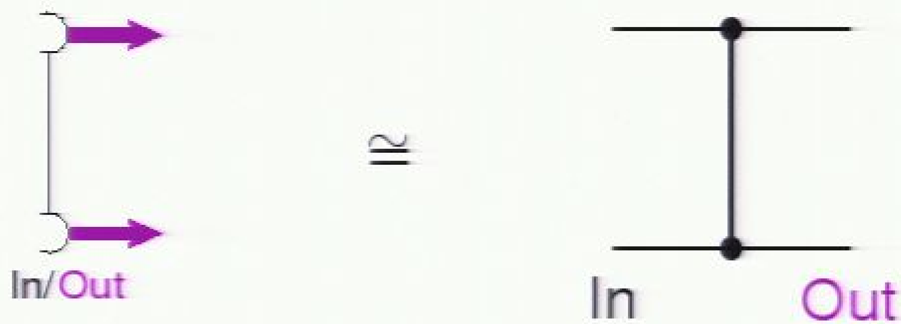

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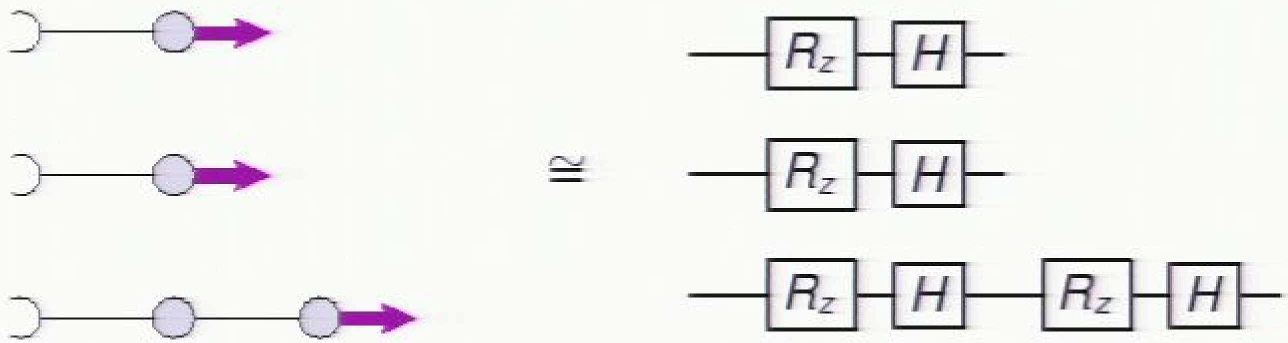

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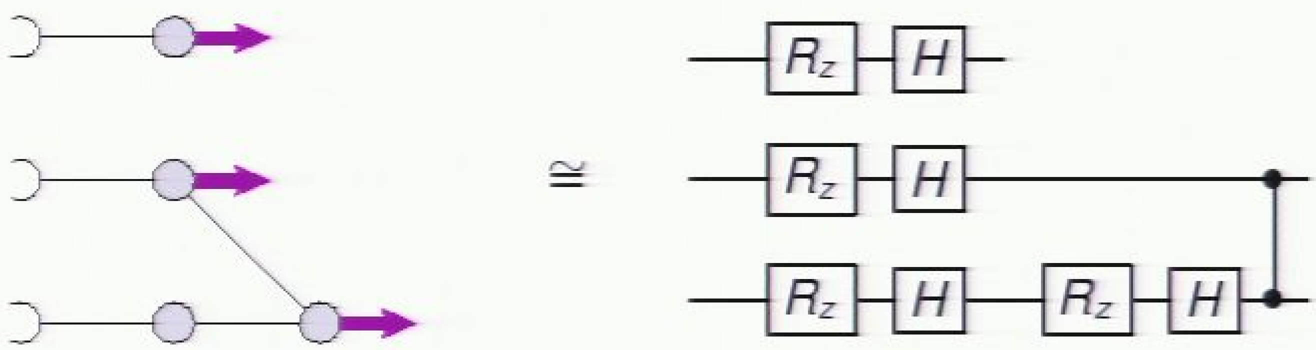

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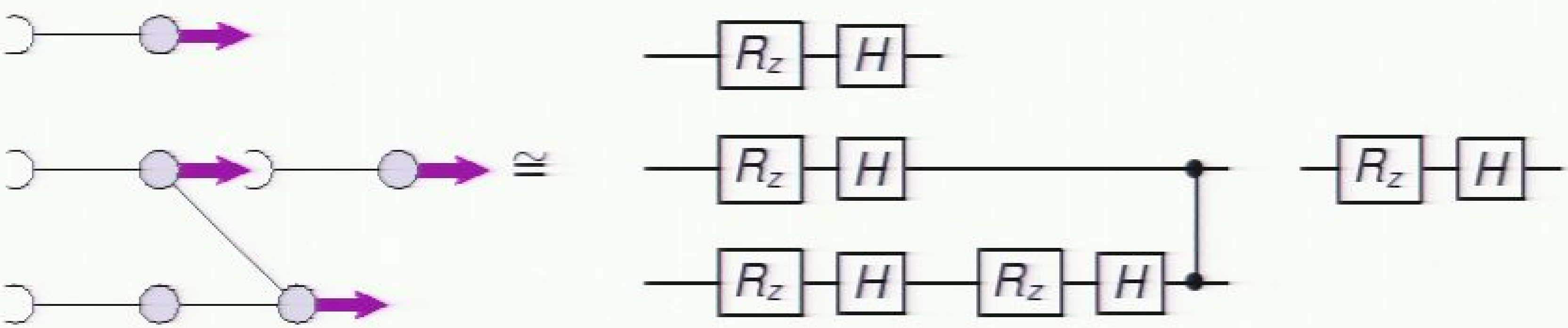

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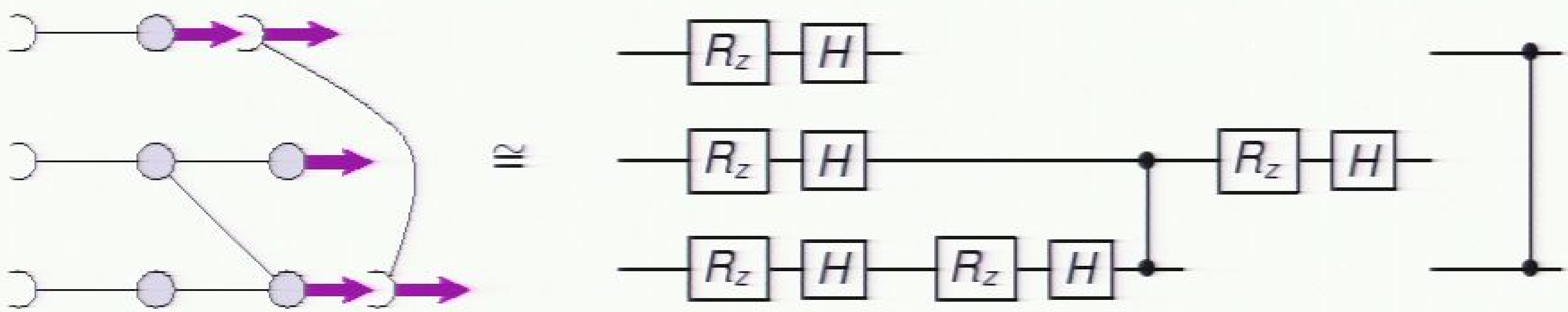

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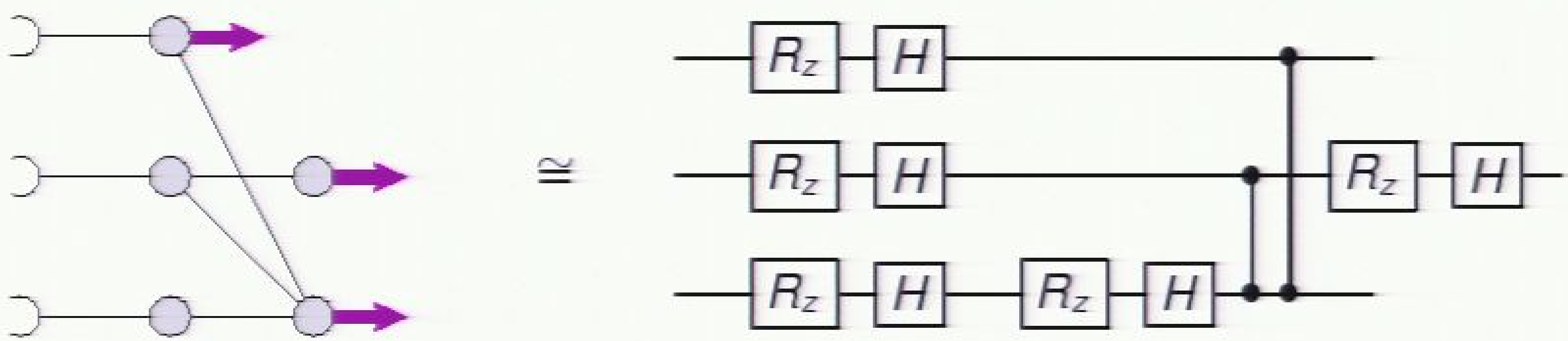

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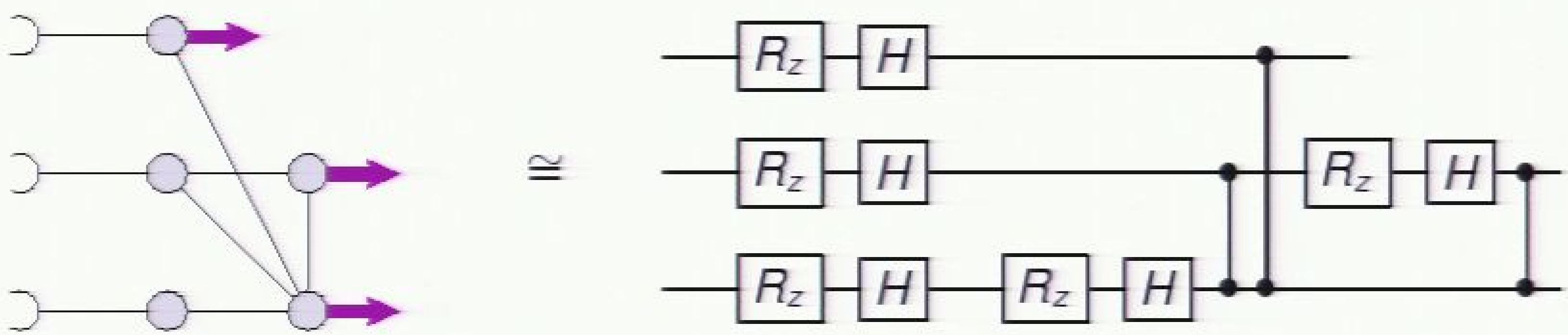

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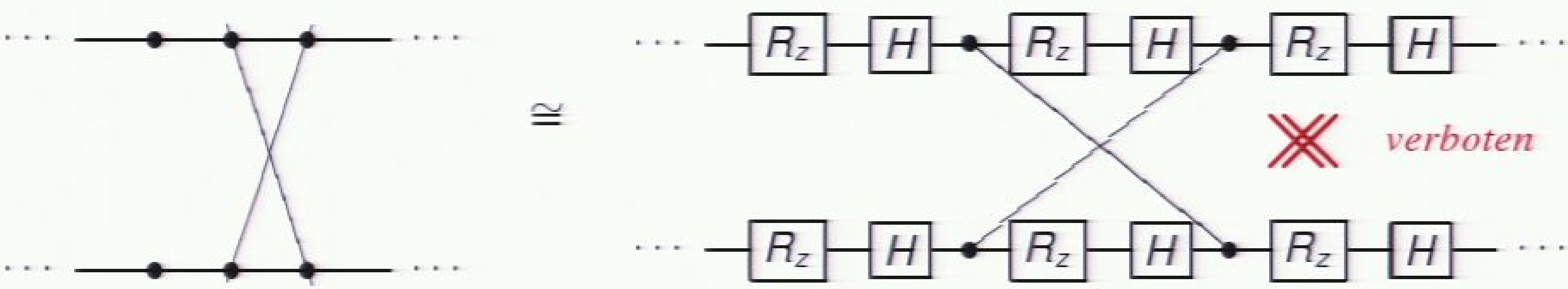
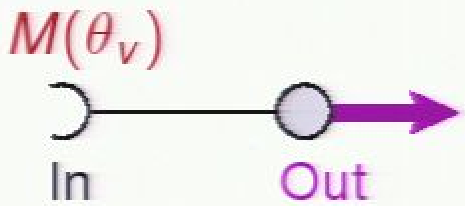
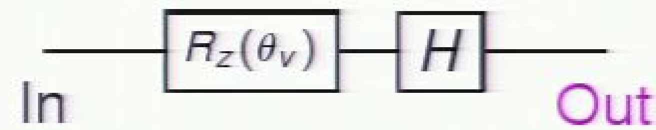


Emergent structures



$\cong$ projection onto $+1$ eigenstate



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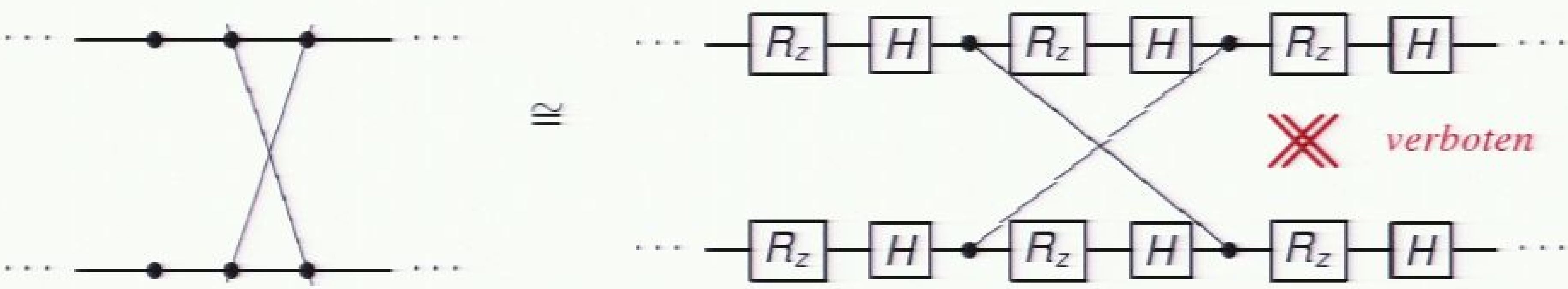

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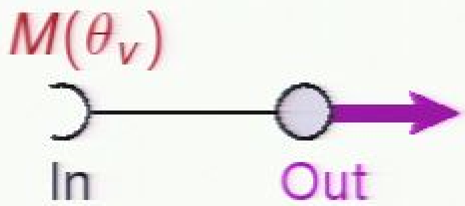

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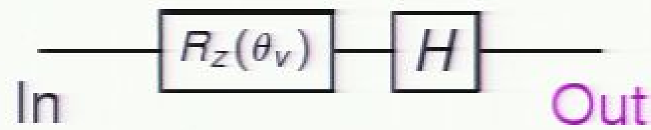

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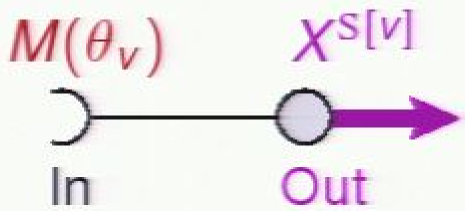

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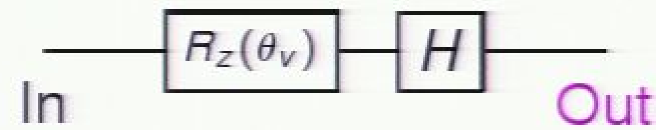

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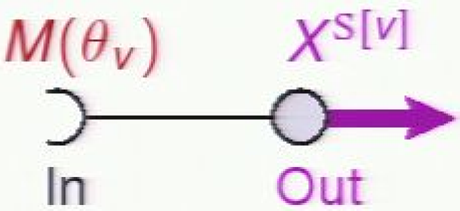

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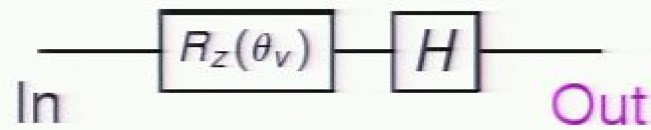

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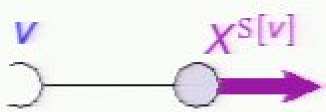

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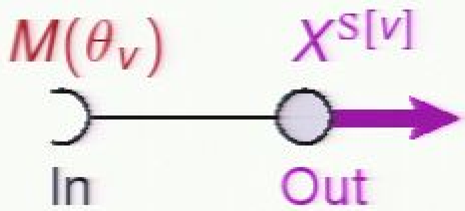

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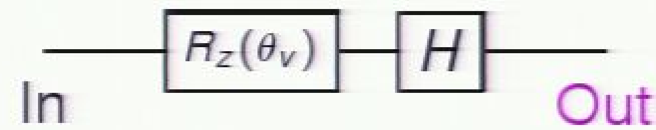

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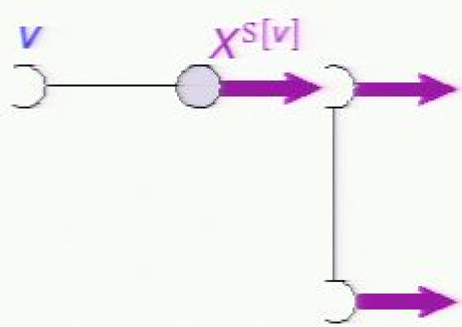

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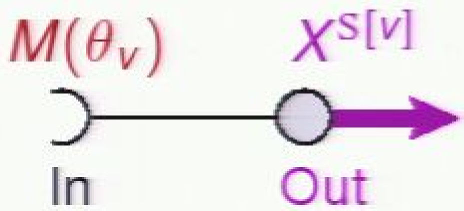

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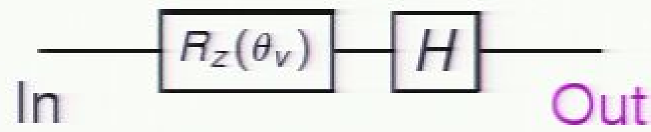

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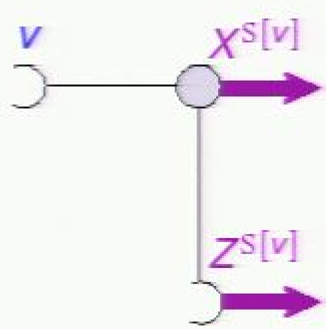

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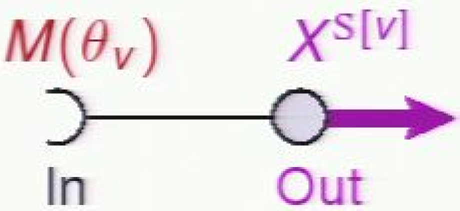

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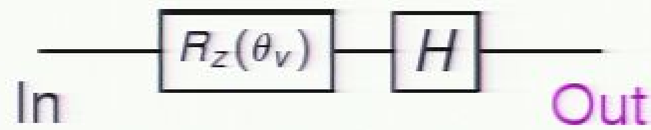

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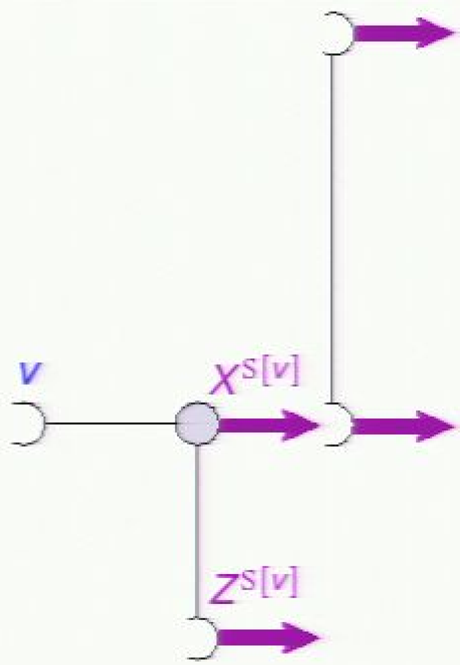

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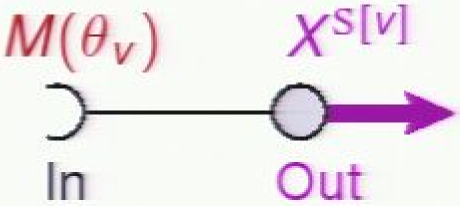

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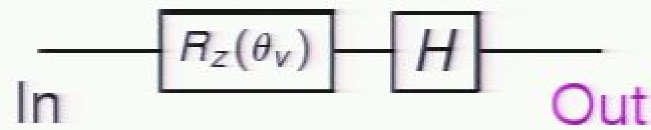

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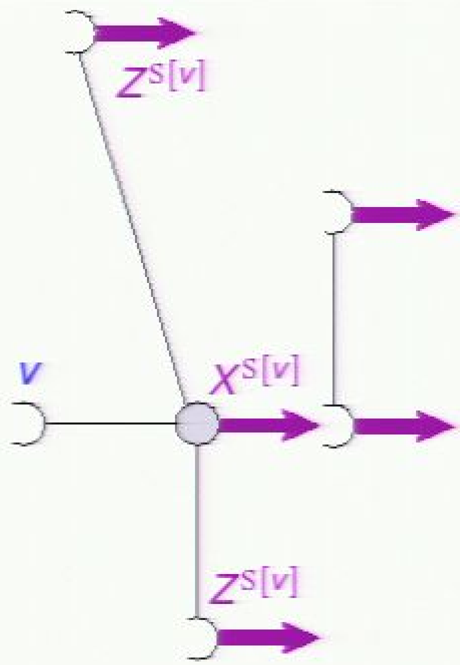

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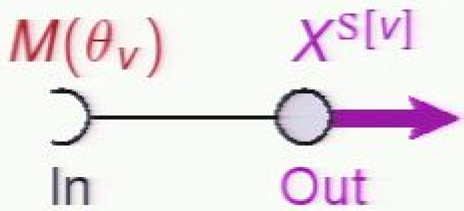

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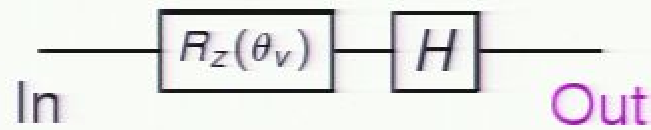

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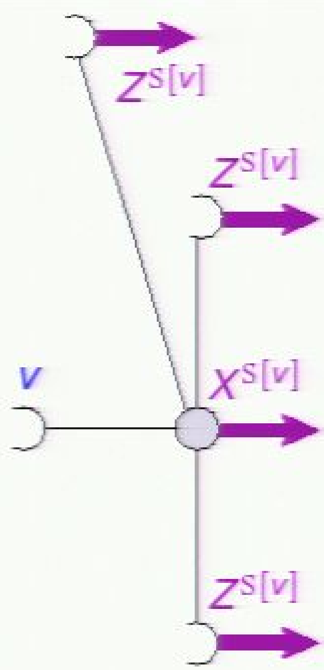

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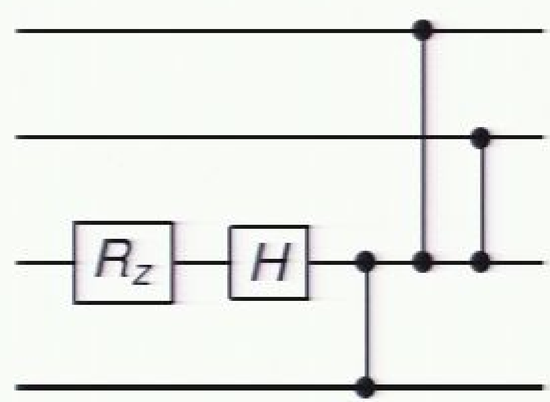

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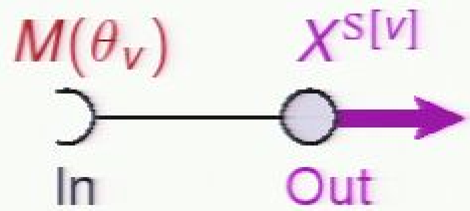

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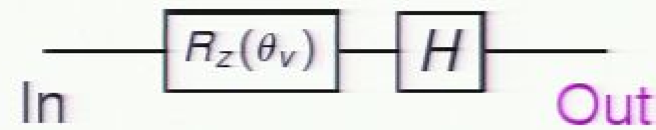

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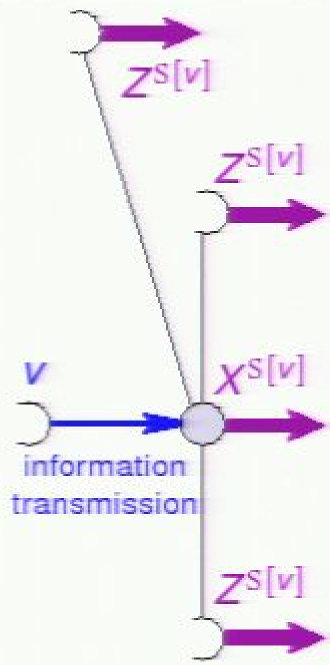

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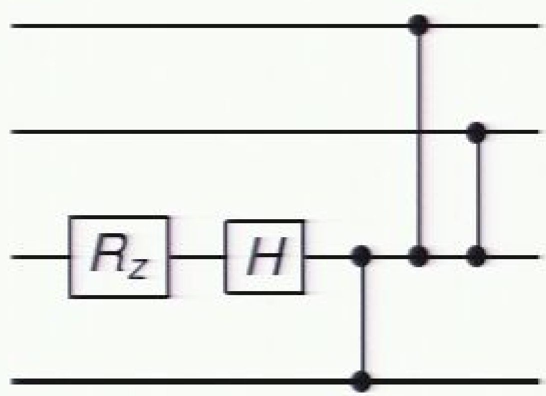

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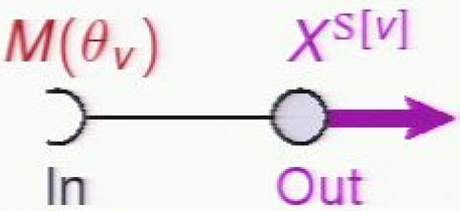

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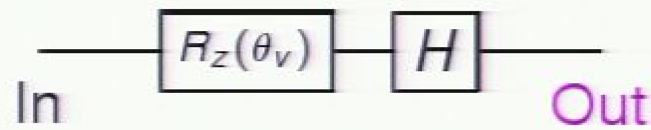

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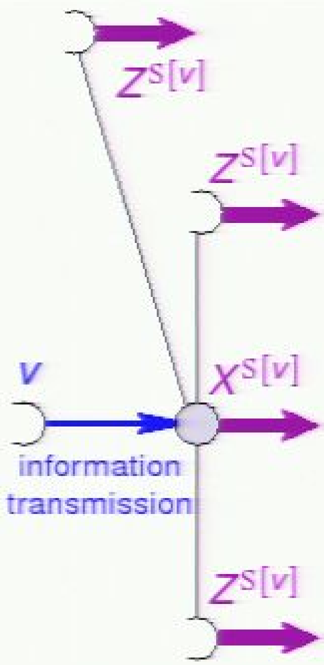

Flow:
define a function f from non-outputs to non-inputs, representing the direction of state transfer:

- v is adjacent to $f(v)$
- measuring v may induce corrections on $f(v)$
- measuring v may induce corrections on the other qubits adjacent to $f(v)$

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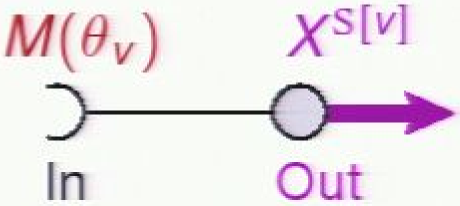

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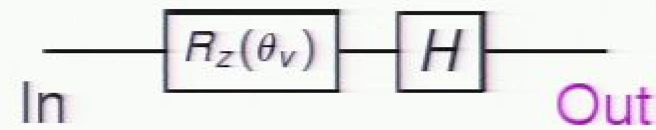

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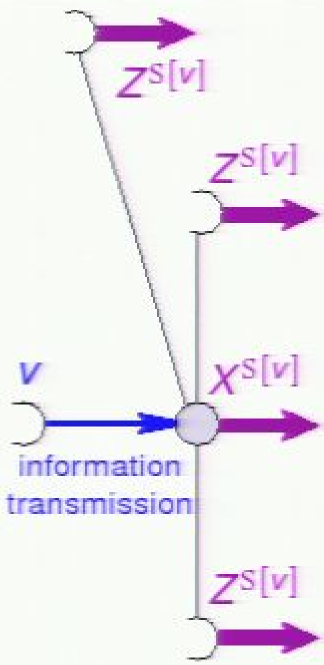

Flow:
define a function f from non-outputs to non-inputs, representing the direction of state transfer:

- v is adjacent to $f(v)$
- v is measured prior to $f(v)$
- v is measured prior to the other qubits neighboring $f(v)$

Flows: a special case of MPI

[Danos, Kashefi. *Phys. Rev. A* **74** (052310), 2006]

Given:

- a triple $(G, \mathrm{I}, \mathrm{O})$ specifying the entangling embedding & qubits to measure;
- that the default observables for the non-output qubits v are all of the form $M_v = \cos(\theta_v)X - \sin(\theta_v)Y$;

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- a function $f : \mathrm{O}^c \longrightarrow \mathrm{I}^c$ and a partial order $\preccurlyeq$ such that:

$$\left.\begin{array}{l} v \text{ is adjacent in } G \text{ to } f(v), \text{ for all } v \in \mathrm{O}^c \\ v \preccurlyeq f(v) \text{ for all } v \in \mathrm{O}^c \\ \text{for any } w \text{ adjacent in } G \text{ to } f(v), \text{ we have } v \preccurlyeq w \end{array}\right\} \text{“\textit{flow conditions}”}$$

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Then: the measurement order $\preccurlyeq$, and byproduct operators given by

$$B_v = X_{f(v)} \prod_{\substack{w \text{ adj. to } f(v) \\ w \neq v}} Z_w$$

yield a measurement pattern performing a unitary embedding.

Flows: a *solved* special case of MPI

Theorem. A flow for $(G, \mathrm{I}, \mathrm{O})$ can be found in time $O(k^2 n)$,
for $|V(G)| = n$ and $|\mathrm{I}| = |\mathrm{O}| = k$.
B. *Phys. Rev. A* **77** (022328), 2008

— takes advantage of a uniqueness result when $|\mathrm{I}| = |\mathrm{O}|$, which may be proven from a graph-theoretic description of flows.

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Mhalla, Perdrix. *ICALP '08*, to appear. [arXiv:0709.2670]

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Dividend: An efficient inverse construction, transforming measurement patterns with flows to a unitary circuit of similar complexity.

A deeper look at flows

Byproduct operator arising from a flow:

$$B_v = X_{f(v)} \prod_{\substack{w \text{ adj. to } f(v) \\ w \neq v}} Z_w$$

compare with:

$$K_{f(v)} = X_{f(v)} \prod_{w \text{ adj. to } f(v)} Z_w$$

- stabilizes the output space of the entangling unitary embedding
- anticommutes with the "default" observable $M_v = \cos(\theta) X_v - \sin(\theta) Y_v$

A deeper look at flows

Byproduct operator arising from a flow:

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$\Longrightarrow$ flow function f corresponds measured qubits, to "suitable" generators of the Pauli stabilizer group
(for the state-space prior to any measurement)

Extending flows

$$K_{f(v)} = X_{f(v)} \prod_{w \text{ adj. to } f(v)} Z_w$$

Does every correspondence f from measured qubits to a "suitable" generator $K_{f(v)}$ yield a flow?

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- For $v \notin S$, we require $f(v)$ adjacent to v, as for flows;
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Extended Flows

correspond to the elementary measurement pattern constructions of:
Raussendorf, Browne, Briegel — *Phys. Rev. A* **68** (022312), 2003
Browne, Briegel — [arXiv:quant-ph/0603226]

Extended flows: a solved special case of MPI

Theorem. For a tuple $(G, \mathrm{I}, \mathrm{O}, S)$ an extended flow can be found in time $O(kn)$, for $|E(G)| = m$, $|V(G)| = n$, and $|\mathrm{I}| \leqslant |\mathrm{O}| = k$.

— adaptation of the algorithm of Mhalla, Perdrix [arXiv:0709.2670] for finding flows, by generalizing the concept of a star geometry

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Dividend: An efficient inverse for the construction using the elementary constructions of *PRA* **68** 022312 and [arXiv:quant-ph/0603226].

Generalized flows

$$K_u \;=\; X_u \prod_{w \text{ adj. to } u} Z_w$$

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Each qubit to be measured has a "Pauli plane" default observable:

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- includes all standard measurement pattern constructions, after reduction techniques of [Hein, Eisert, Briegel — *Phys. Rev. A* **69** (062311), 2004]
- efficient algorithm to find generalized flows when all default observables are of *XY* type, in Mhalla, Perdrix [arXiv:0709.2670] . . . seems generalizable

Outline

1. Introducing Measurement Pattern Interpolation (MPI)
   - Potential applications
2. Solved special cases of MPI
3. Outlook for a general solution

Special cases of MPI

"Generic" Pauli plane observables:

flows $<$ *extended flows* $<$ *generalized flows*

Pauli observables:

stabilizer formalism (arbitrary measurement order)

Solutions for these special cases of MPI:

- find generators for the pre-measurement stabilizer group, which anticommute with corresponding default observables
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Question: can we treat observables which fail to (anti)commute with some stabilizer group operators?

Outline of a general solution to MPI

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   - **candidate**: "uniformly twisted" stabilizer codes

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4. develop theory for ordering an *unordered* set of observables into adaptable, information-preserving sequences of measurements

Twisting the observables to fit the method

$\mathcal{O} \otimes \mathbb{1}^{\otimes n-1}$	observable to measure (general single qubit observable)
$X \otimes S'_1$	stabilizer generator
$Z \otimes S'_2$	"
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Uniformly twisted stabilizer codes

After a few measurements (of observables which commute with preceding rotation operations), the state-space is of the form

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where

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Measurements on uniformly twisted stabilizer codes

. . . equivalent to a "twisted measurement" on the stabilizer code $\mathcal{K}$:

conjugate the observable by the twisting unitaries

$$\text{default observable} \xmapsto[\text{unitaries}]{\text{twisting}} \text{large "superposition" of Pauli operators}$$

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For each equivalence class of non-uniform observables:
do the coefficients cancel out?

Current status

the difficulty of subproblems

Is MPI, at root, a hard problem?

- What relaxations of the problem are "provably" intractable?
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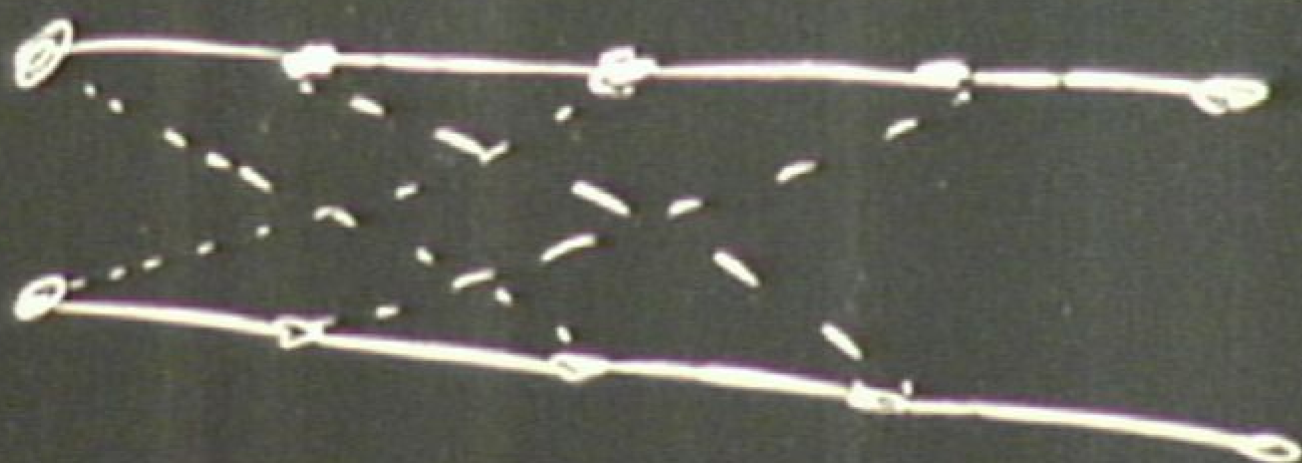
- What constraints can we find from the requirement that the final byproduct maps one last Z observable to a single-qubit observable?



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