Title: Computing Black Hole entropy in LQG from a CFT perspective

Date: Nov 26, 2008 11:00 AM

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Abstract: Motivated by the analogy proposed by Witten between Chern-Simons theories and CFT-Wess-Zumino-Witten models, we explore a new way of computing the entropy of a black hole starting from the isolated horizon framework in Loop Quantum Gravity. The results seem to indicate that this analogy can work in this particular case. This could be a good starting point for the search of a deeper connection between the description of black holes in LQG and a conformal field theory.

PERIMETER INSTITUTE 26-Nov-2008

COMPUTING BLACK HOLE ENTROPY IN LQG FROM A CFT PERSPECTIVE

VNIVERSITAT DE VALENCIA

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Work in collaboration with: **J. Diaz-Polo & E.F. Borja** (U. of Valencia)

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PLAN OF THE TALK

- Black hole entropy in LQG
- Black hole entropy from Number Theory
- New perspective to compute black hole entropy
- Conclusions

BLACK HOLE ENTROPY IN LQG

Ashtekar, Baez, Corichi, Krasnov (2000): Quantum geometry of IH and BH Entropy

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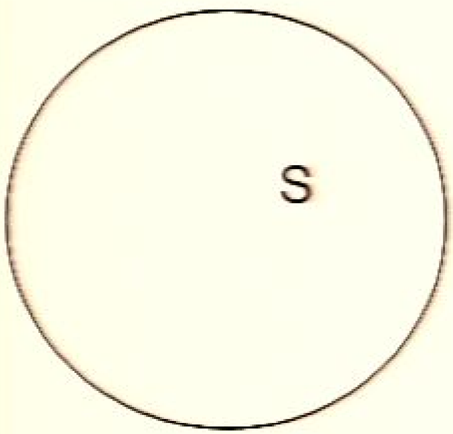
- The black hole is introduced in an effective way: the horizon is introduced a
an inner boundary of the spacetime manifold



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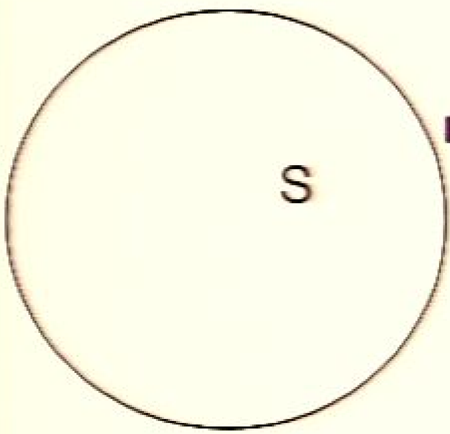

Boundary conditions (at the classical level):

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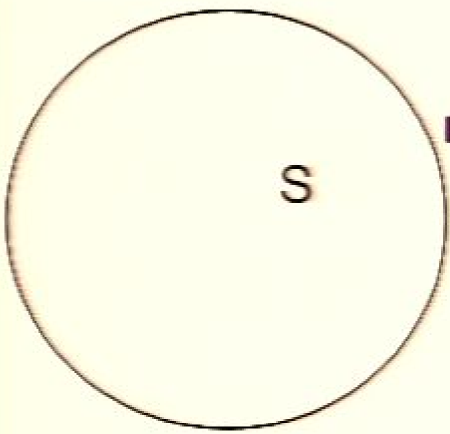


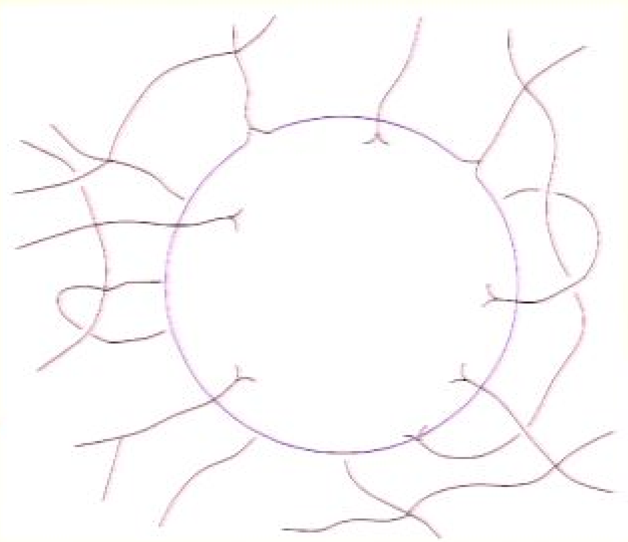
$\Rightarrow$ Boundary conditions (at the classical level):

- SU(2)-connection A_a on the bulk $\longrightarrow$ U(1)-connection W_a on

- U(1)-Chern-Simons Theory on S

BLACK HOLE ENTROPY IN LQG

Quantizing…



BLACK HOLE ENTROPY IN LQG

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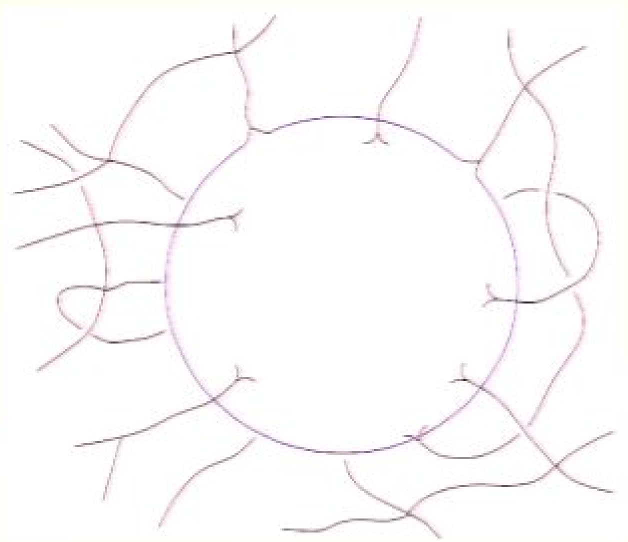
- SU(2) spin networks → j_i label
- finite number of edges piercing the horizon (punctures) → m_i labels



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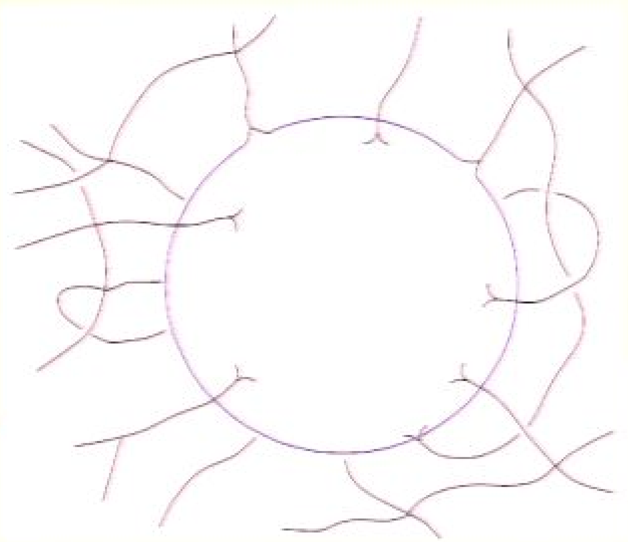

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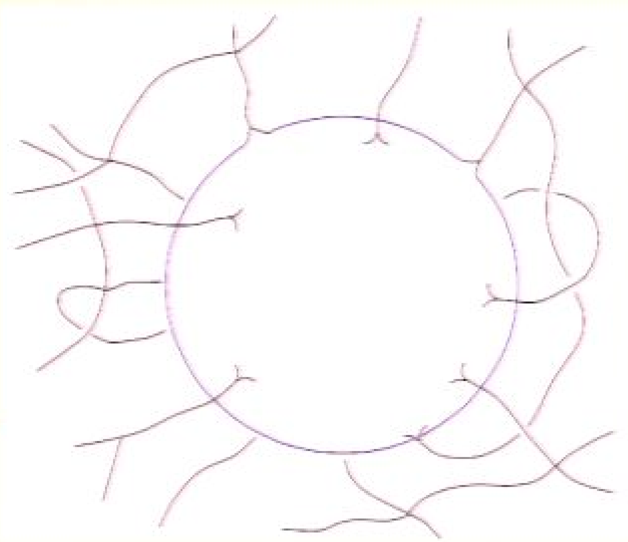

Boundary conditions:

- Area of the horizon $A = 8\pi\gamma\ell_P^2 \sum_i \sqrt{j_i(j_i+1)}$
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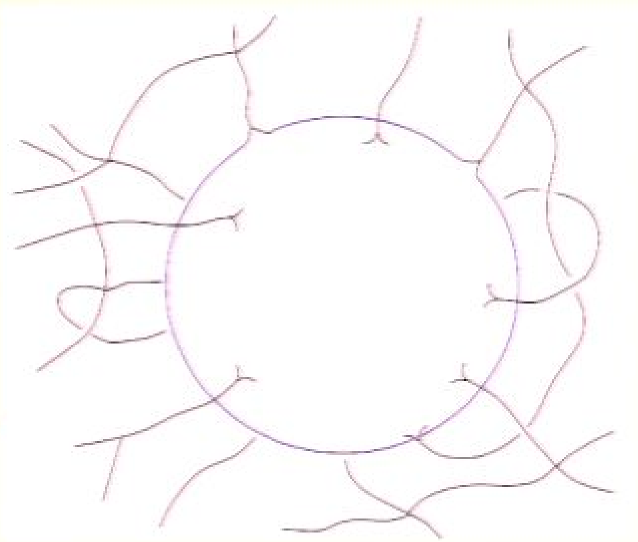

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Entropy: S(A) =log N(A)

N(A)= number of ***ordered*** $\{a_i\}$ sequences compatible with the above conditions for $A - \delta \leq A \geq A + \delta$

BLACK HOLE ENTROPY IN LQG

- Domagala M., Lewandowsky J. Class. & Qunt. Grav. 21, 5233 (2004)

Combinatorial Problem:

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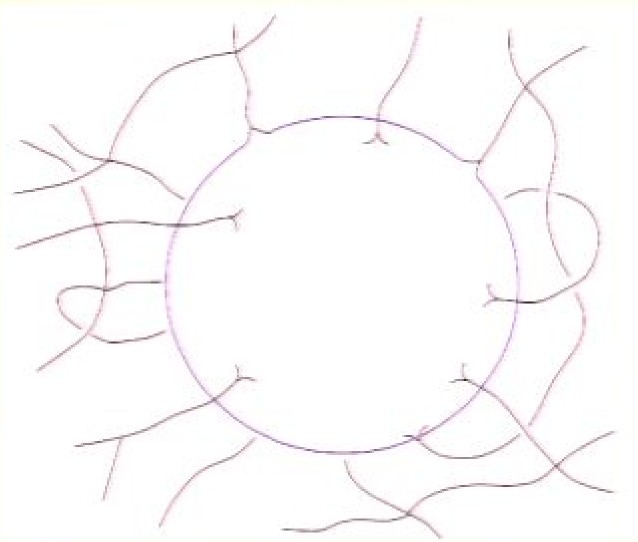

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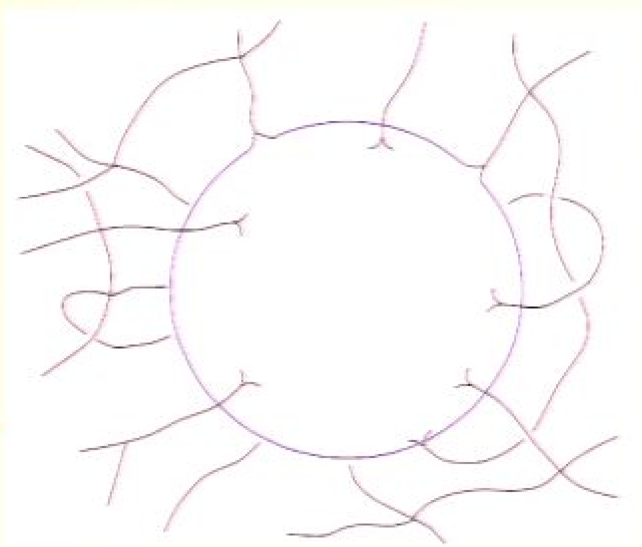

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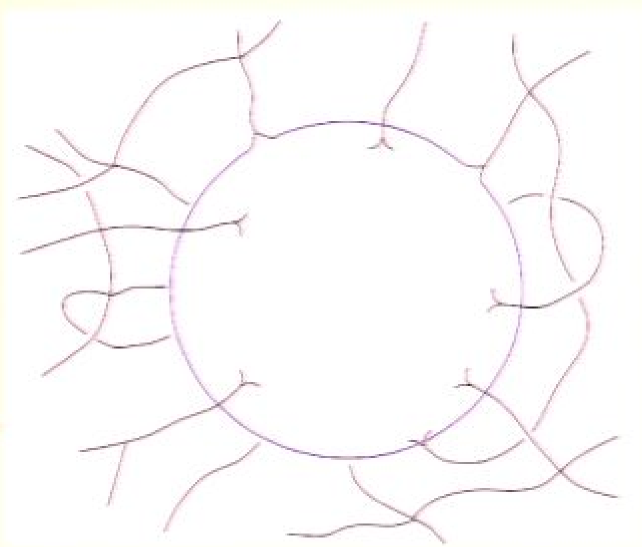

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Using exact methods, (but with some assumptions):

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Using exact methods, (but with some assumptions):

$$N(A) = C_M \frac{e^{\frac{\gamma_M}{4\gamma l_P^2}A}}{\sqrt{A/l_p^2}} \Rightarrow S(A) = \frac{\gamma_M}{4\gamma}\frac{A}{l_P^2} - \frac{1}{2}\ln A/l_P^2 + O(1)$$

$$\gamma_M = 0.23753\ldots$$

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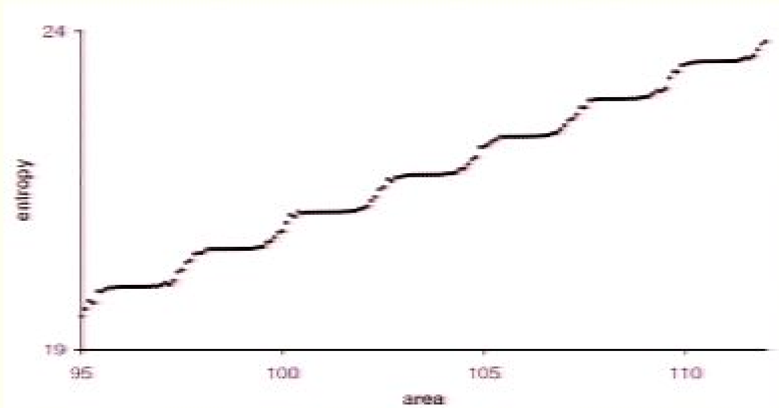

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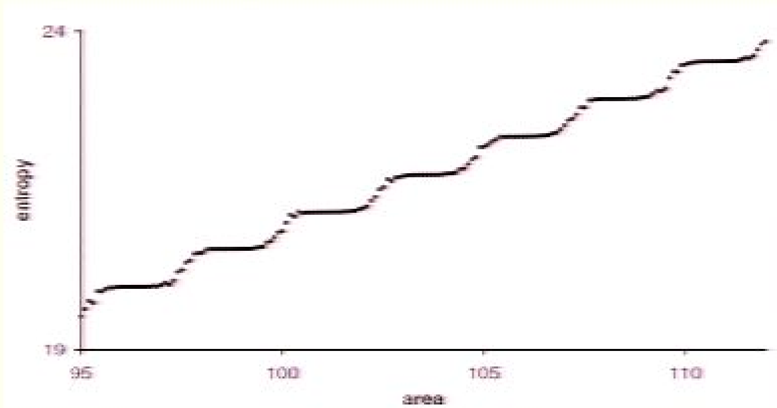

Questions:

- Is this effect an artifact the way of counting?
- Where is this effect coming from?
- Is this effect present for large areas where the Isolate Horizon approximation is justified?

BH ENTROPY FROM NUMBER THEORY

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- Definitions:
  - Use units in which $4\pi\gamma\ell_P^2 = 1$
  - Define k = 2 |m| $\quad A = \sum_i \sqrt{k_i(k_i+2)}$
  - Define n_k as the number of punctures carrying label k

$$A = \sum_k n_k \sqrt{k(k+2)}$$

- A set of $\{n_k\}$ numbers characterize a |m| list (up to reorderings)

BH ENTROPY FROM NUMBER THEORY

d(A) can be expressed in terms of $\{n_k\}$ like:

$$d(A) = \frac{(\sum_k n_k)!}{\prod_k n_k!} P(n_k)$$

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Steps:

1. Find $\{n_k\}$

2. Find $P(n_k)$

BH ENTROPY FROM NUMBER THEORY

Step 1 Finding $\{n_k\}$: Black hole area spectrum in LQG

$$A = 2\sum_i \sqrt{j_i(j_i+1)}$$

BH ENTROPY FROM NUMBER THEORY

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$$A = 2\sum_i \sqrt{j_i(j_i+1)} \Longrightarrow A = \sum_i q_i\sqrt{p_i}$$

q_i: positive integer number

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Then, given an area eigenvalue $A = \sum_i q_i \sqrt{p_i}$ we have

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Set of decoupled diophantine equations $\Longrightarrow$ $\{n_k\}$

BH ENTROPY FROM NUMBER THEORY

Step 1 Finding $\{n_k\}$: Black hole area spectrum in LQG

$$A = 2\sum_i \sqrt{j_i(j_i+1)} \Longrightarrow A = \sum_i q_i \sqrt{p_i}$$

q_i: positive integer number

p_i: squarefree number

$$\sum_k n_k \sqrt{(k+1)^2 - 1} = \sum_i q_i \sqrt{p_i}$$

Each $\sqrt{(k+1)^2-1}$ can be written as an integer times a SRSFN

$$\sqrt{(k+1)^2 - 1} = y\sqrt{p_i} \Longrightarrow$$ **Pell equation**

Given a p_i, the solutions, $\{k_i^m, y_i^m\}$, are known

Then, given an area eigenvalue $A = \sum_i q_i \sqrt{p_i}$ we have

$$\sum_i \sum_m n_{k_i^m} y_i^m \sqrt{p_i} = \sum_i q_i \sqrt{p_i} \Longrightarrow \sum_m n_{k_i^m} y_i^m = q_i \quad i = 1, 2, \ldots$$

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If there are no solutions $\Longrightarrow$ $A = \sum q_i \sqrt{p_i}$ does not belong to area spectrum

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Step 2: Finding $P(\{n_k\})$

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This is a well-known problem in number-theory called Partition Problem

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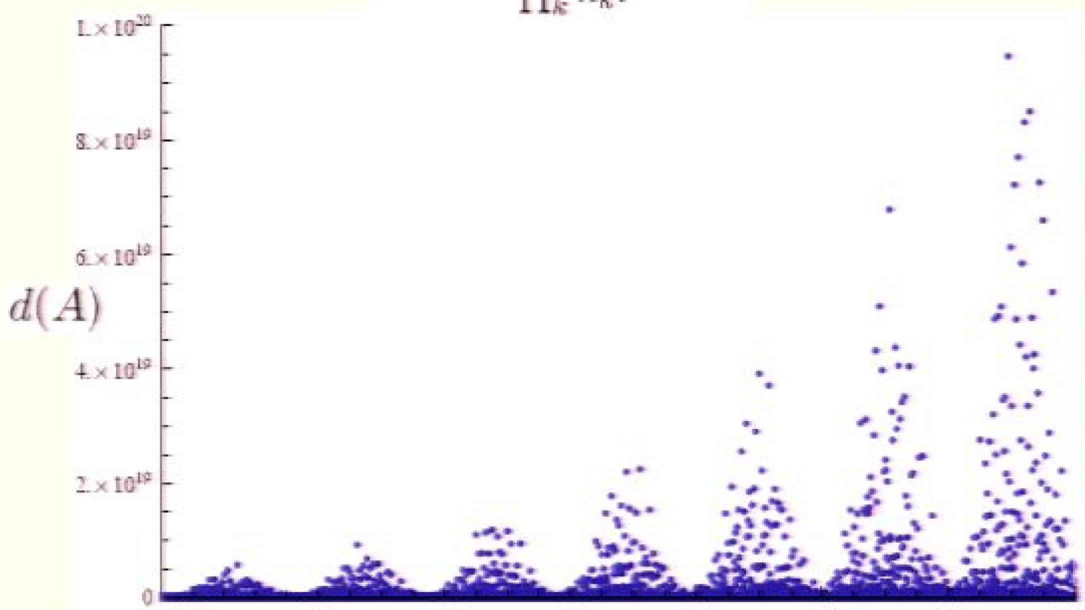

BH ENTROPY FROM NUMBER THEORY

- F. Barbero, E.J. Villaseñor: Phys. Rev. D 77 (2008)

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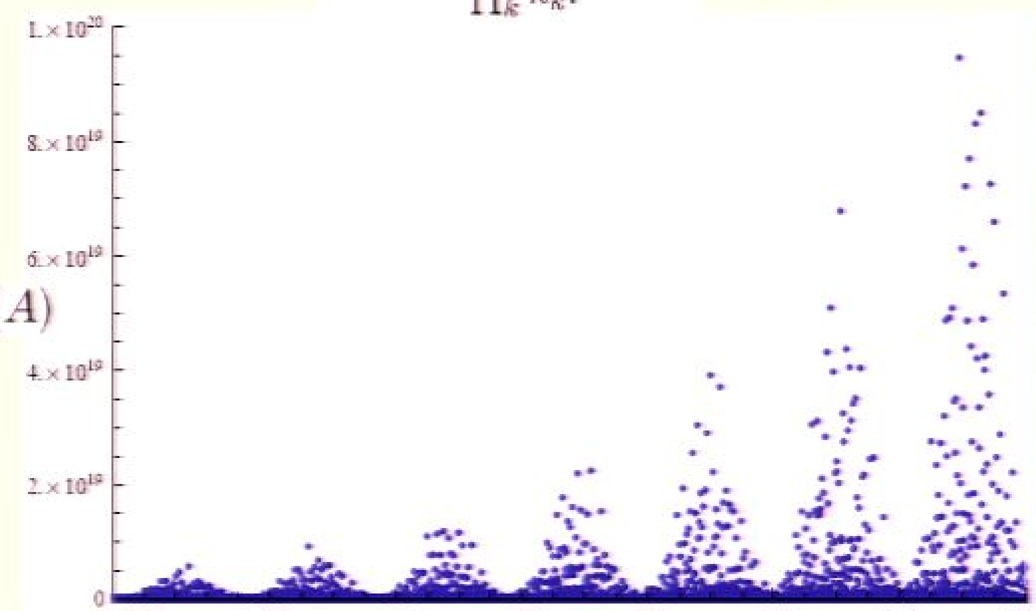

$$N(A) = \sum_{A' \leq A} d(A')$$

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For instance, to obtain $P(\{n_k\})$ the generating functions is:

$$G(x) = \prod_{i=1}^{n} (x^{k_i} + x^{-k_i})$$

Which gives:

$$P(\{n_k\}) = \frac{1}{2\pi} \int_0^{2\pi} d\theta \prod_k n_k 2 \cos k\theta$$

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- **S.Carlip**: an underlying conformal symmetry driving the exponential growing of the number of states of a horizon with area, independently of the particular details of the quantum gravity theory considered.

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PROBLEM: In LQG the horizon d.o.f. are described by a U(1)-CS theory

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How to account for the U(1) degrees of freedom?

GEOMETRIC SYMMETRY REDUCTION

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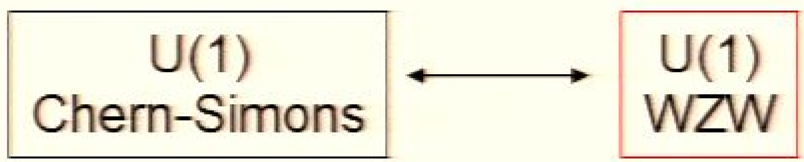

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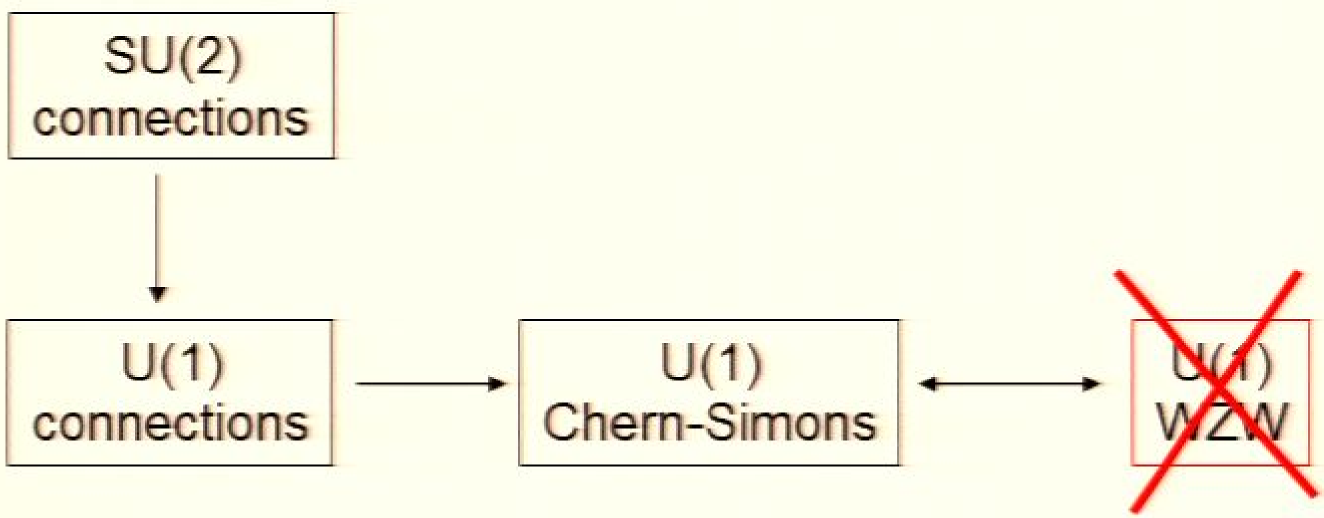

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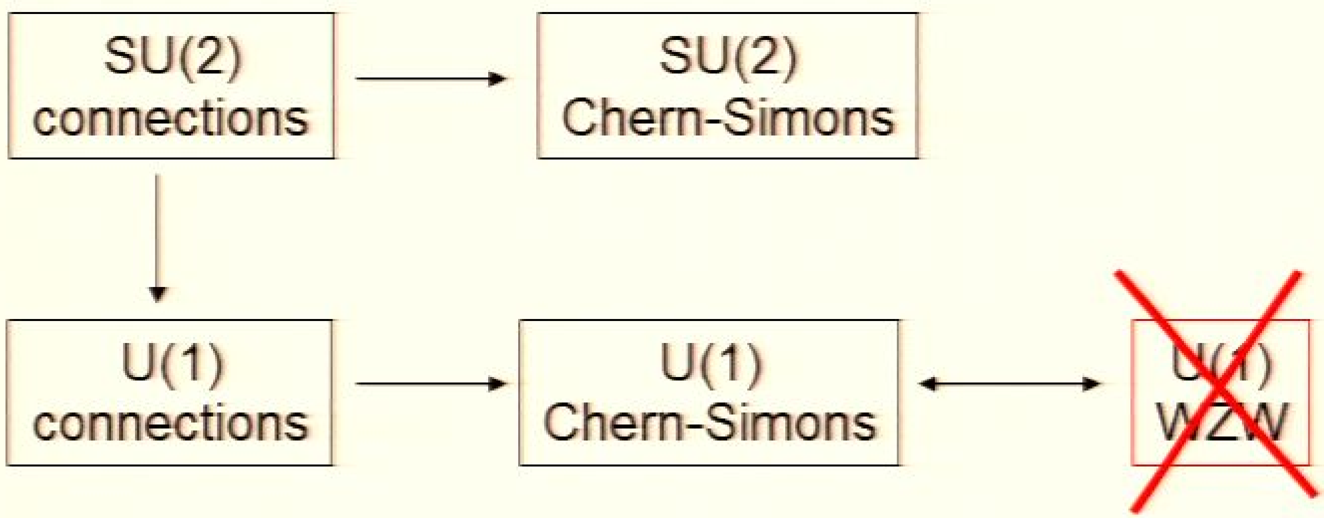

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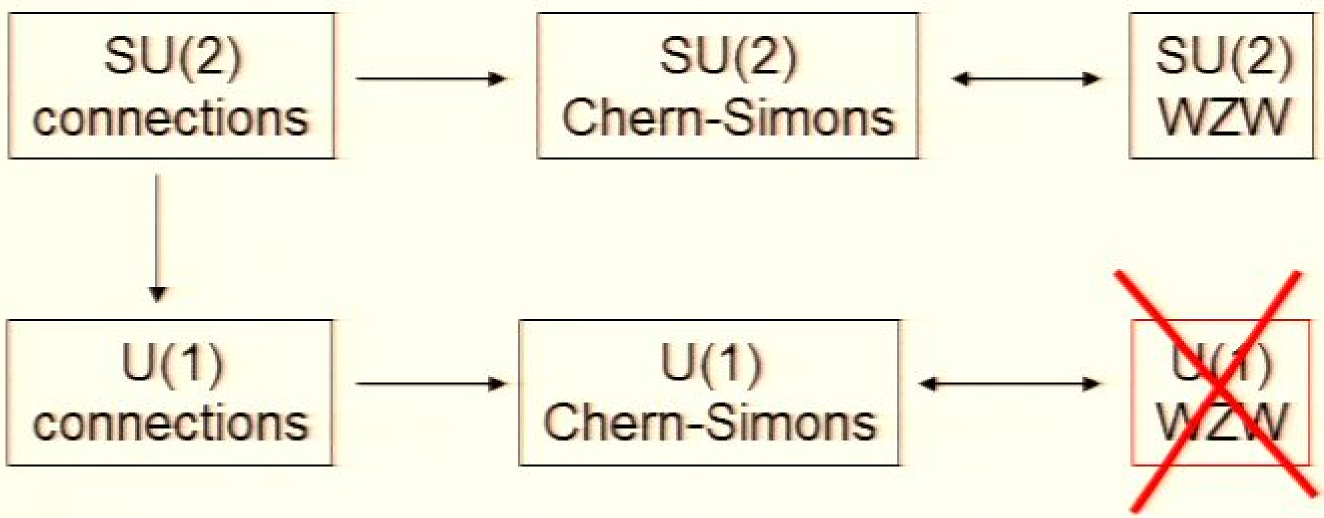

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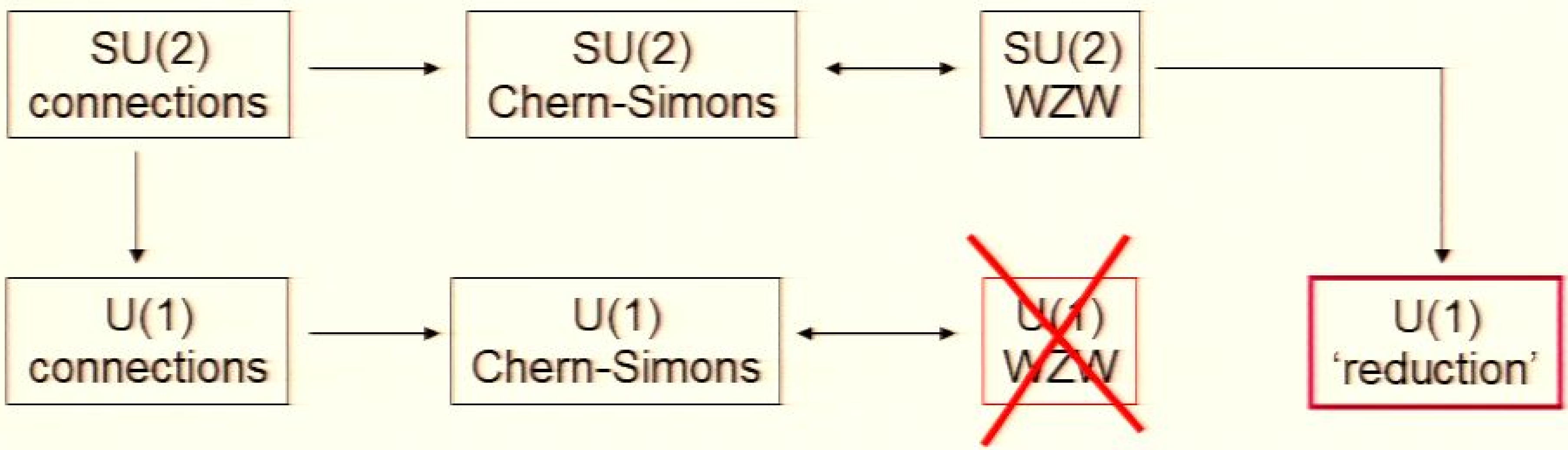

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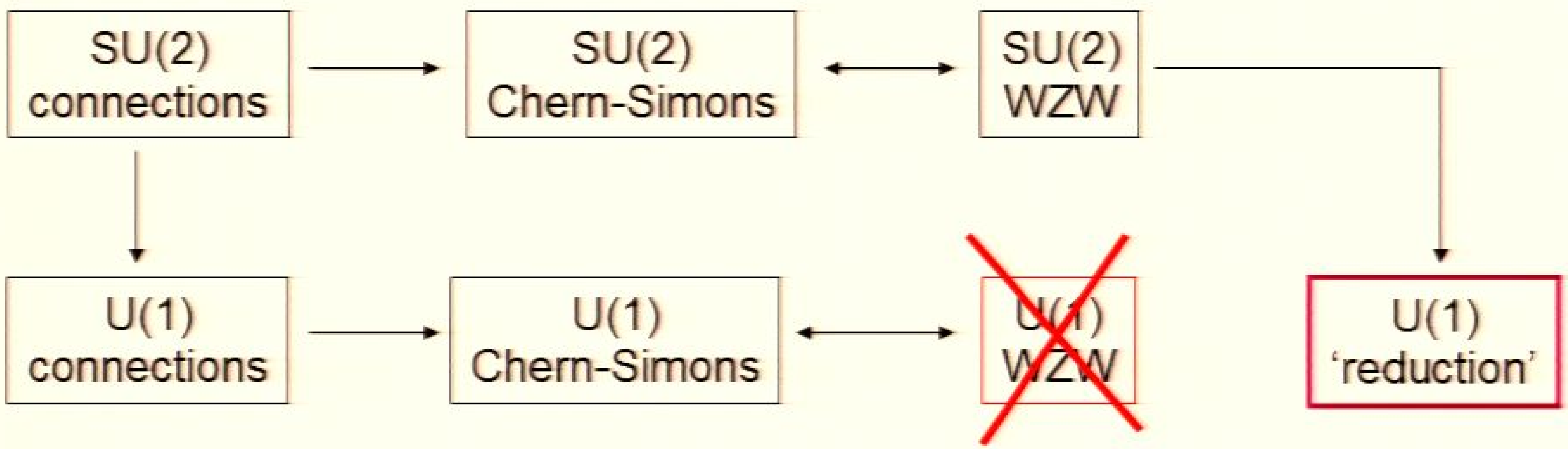

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Given a fiber bundle $P(G,\Sigma)$ with connection w, the homomorfism λ: $H \longrightarrow G$ induce the bundle reduction $P(G,\Sigma) \longrightarrow Q(H,\Sigma)$ with a “reduced” conexión w’

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All homomorphisms in Hom(U(1), T(SU(2))) are given by

$$\lambda_k : z \mapsto diag(z^k, z^{-k})$$

•k and -k conjugated by the Weil group action $\rightarrow k \in N_0$

NEW PERSPECTIVE TO COMPUTE BH ENTROPY

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- This is precisely using the homomorphisms λ_k as U(1) representations

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Given a $\{k_i\}$ list we have to project the product of the characters $\chi_{k_1} \chi_{k_2} \ldots \chi_{k_n}$ over the trivial U(1) irrep to require gage invariance.
(This is the implementation in this context of the projection constrain).

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$$N^{\mathcal{P}} = \sum_{r_i} N^{r_1}_{j_1 j_2} N^{r_2}_{r_1 j_3} \ldots N^{j_n}_{r_{n-2} j_{n-1}} \qquad [\phi_i] \otimes [\phi_j] = \oplus_r N^r_{ij} [\phi_r]$$

We can re-write this expressions using the theory of characters: $\chi_i \chi_j = \sum N^r_{ij} \chi_r$

Using that $\langle \chi_i | \chi_j \rangle = \delta_{ij}$ being $\langle \chi_i | \chi_j \rangle_{SU(2)} = \frac{1}{\pi} \int_0^{2\pi} d\theta \sin^2\theta \chi_i \chi_j$

$$\longrightarrow N^{\mathcal{P}} = \langle \chi_1 \chi_2 \ldots \chi_n | \chi_0 \rangle$$

- For the U(1) case:

The characters we are using are $\chi_k = e^{ik\theta} + e^{-ik\theta} = 2\cos k\theta$

Given a $\{k_i\}$ list we have to project the product of the characters $\chi_{k_1} \chi_{k_2} \ldots \chi_{k_n}$ over the trivial U(1) irrep to require gage invariance.
(This is the implementation in this context of the projection constrain).

Taking into account $\langle \chi_i | \chi_j \rangle_{U(1)} = \frac{1}{2\pi} \int_0^{2\pi} d\theta \chi_i \chi_j$ we obtain

$$\langle \chi_{k_1} \chi_{k_2} \ldots \chi_{k_n} | \chi_0 \rangle = \frac{1}{2\pi} \int_0^{2\pi} d\theta \prod_{\{k_i\}} 2\cos\theta k_i$$

NEW PERSPECTIVE TO COMPUTE BH ENTROPY

- Coming back to the LQG framework

U(1)
Chern-Simons

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$$
\begin{tikzcd}
\boxed{\text{Area } A} \arrow[r, "\text{LQG}"'] \arrow[d] & \boxed{j \text{ lists}} \\
\boxed{\begin{array}{c}\text{U(1)} \\ \text{Chern-Simons}\end{array}} &
\end{tikzcd}
$$

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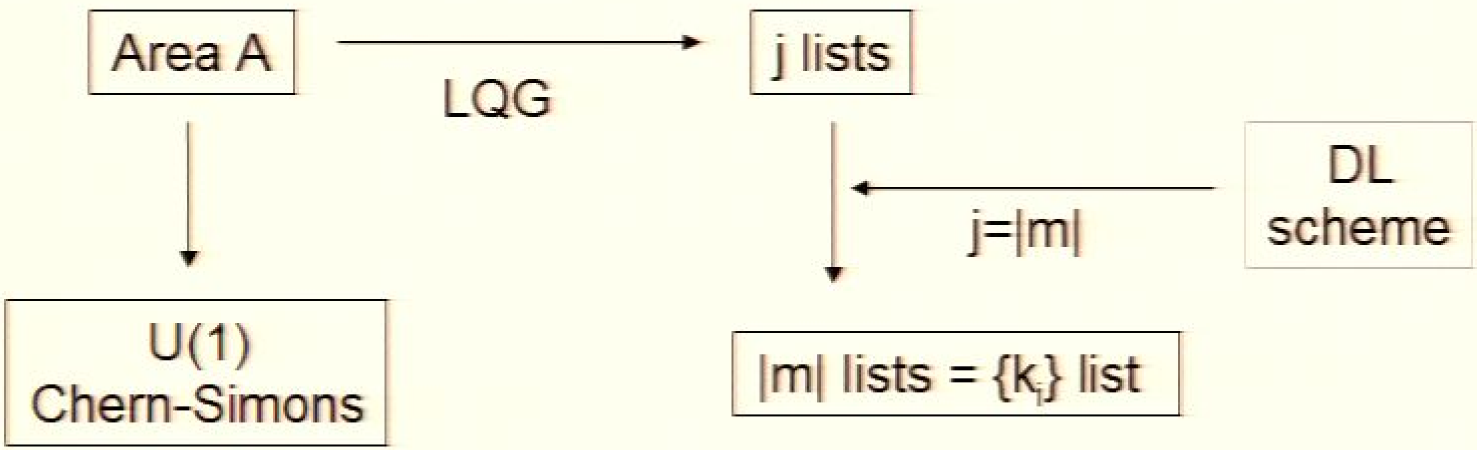

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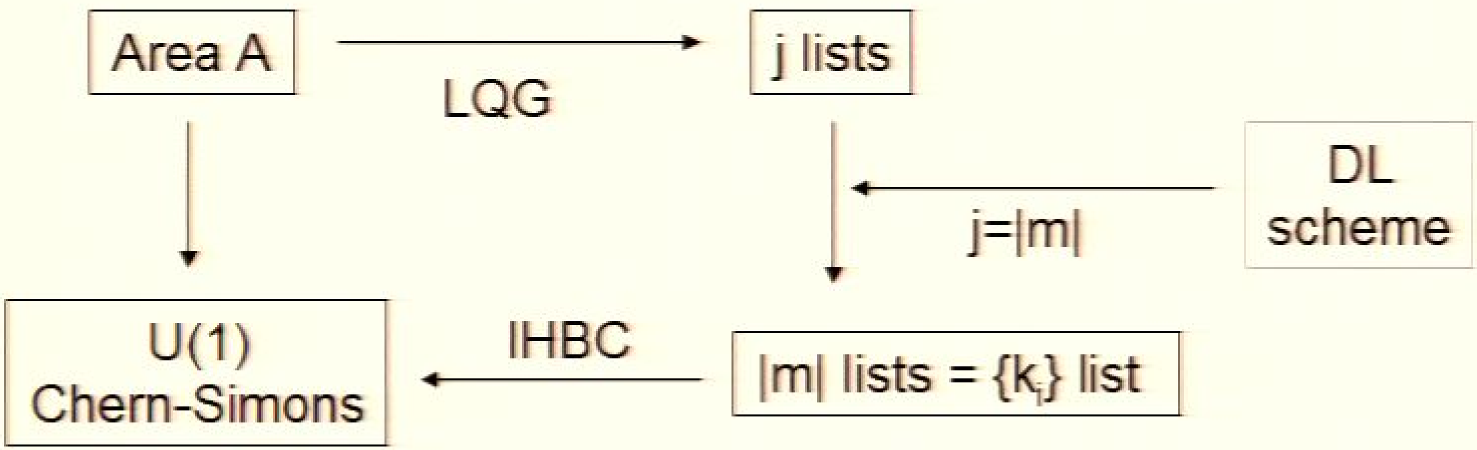

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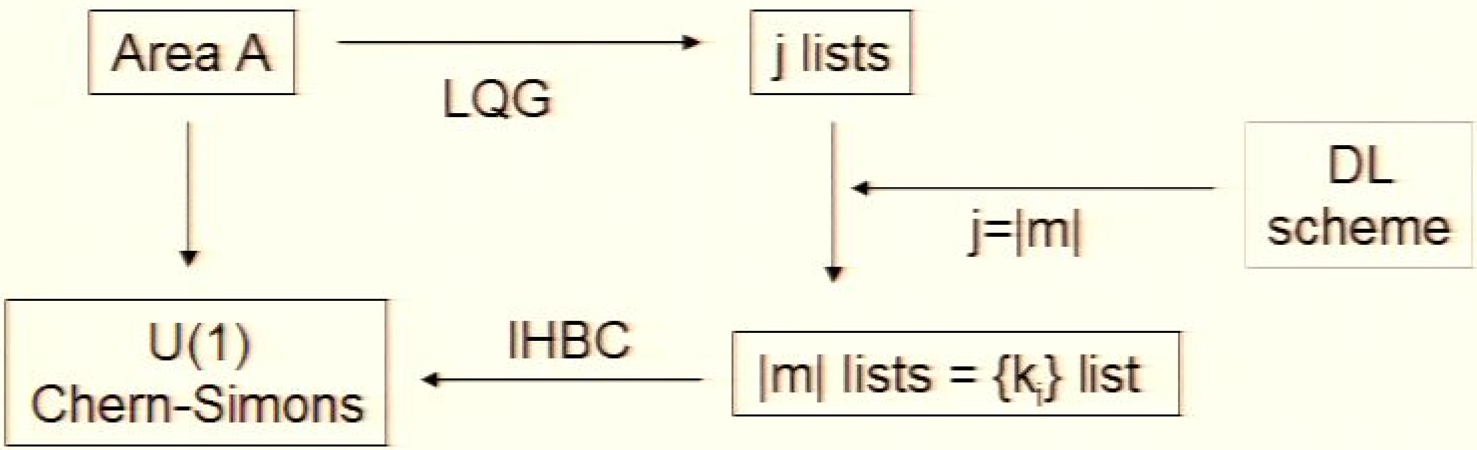

Reorderings

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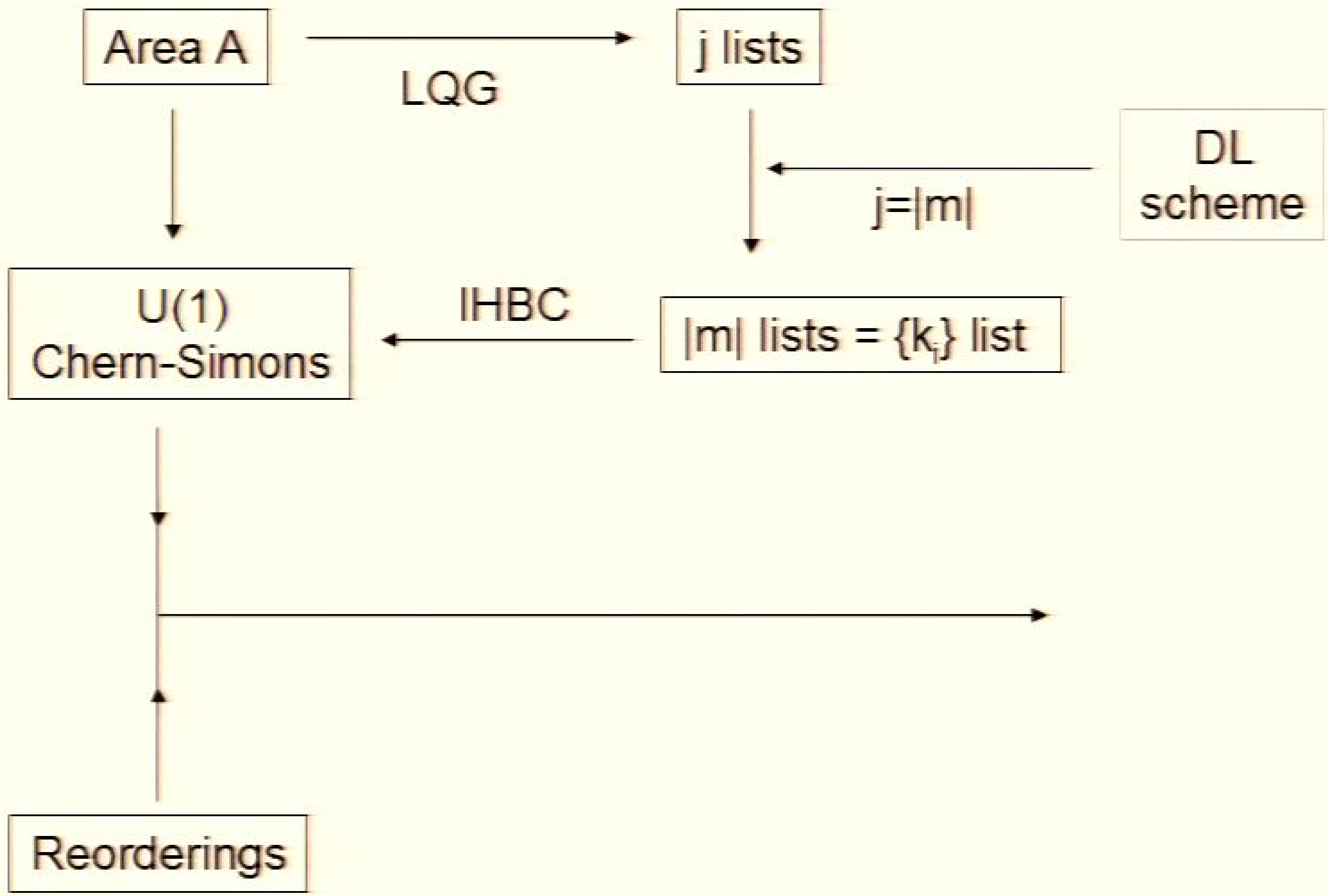

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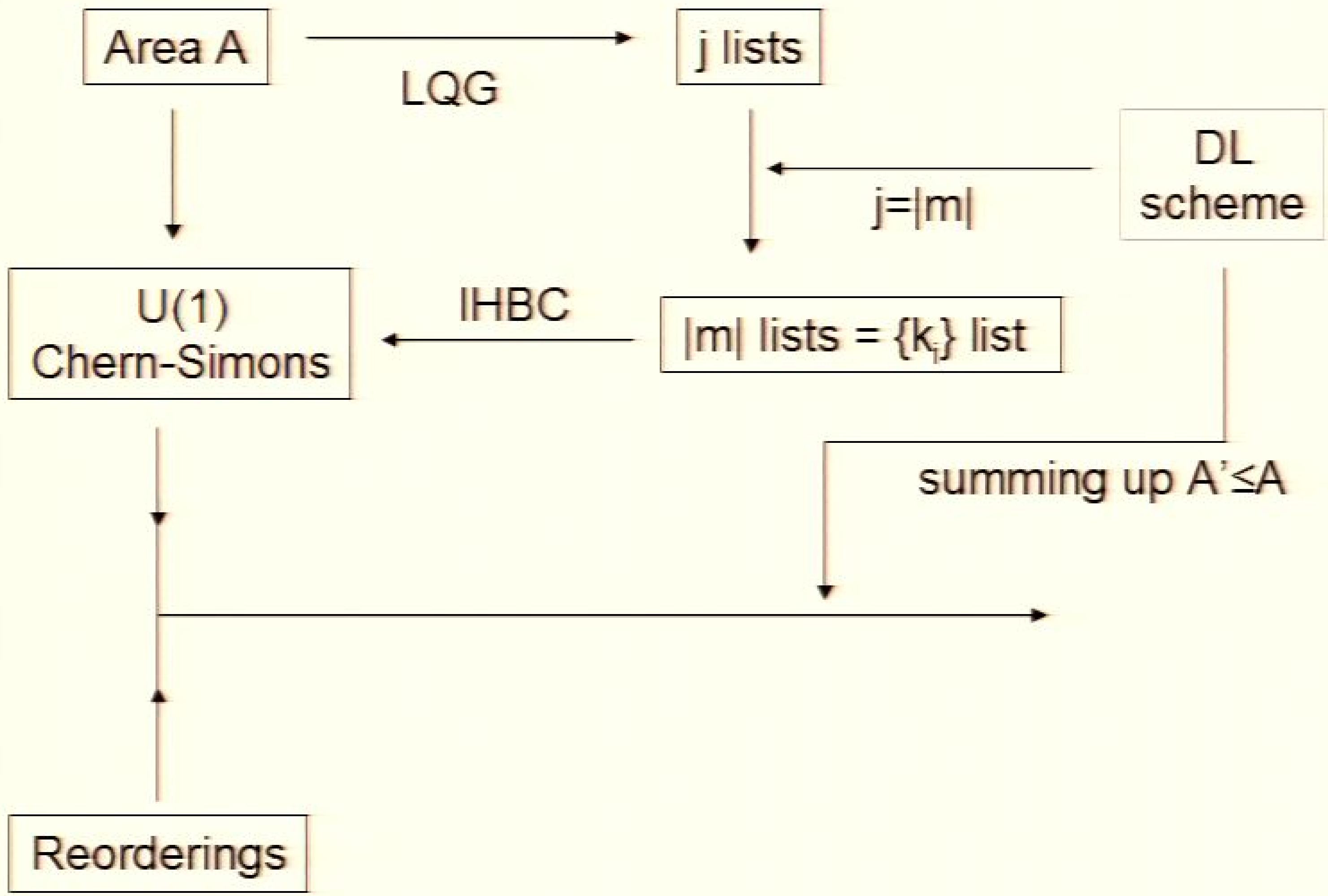

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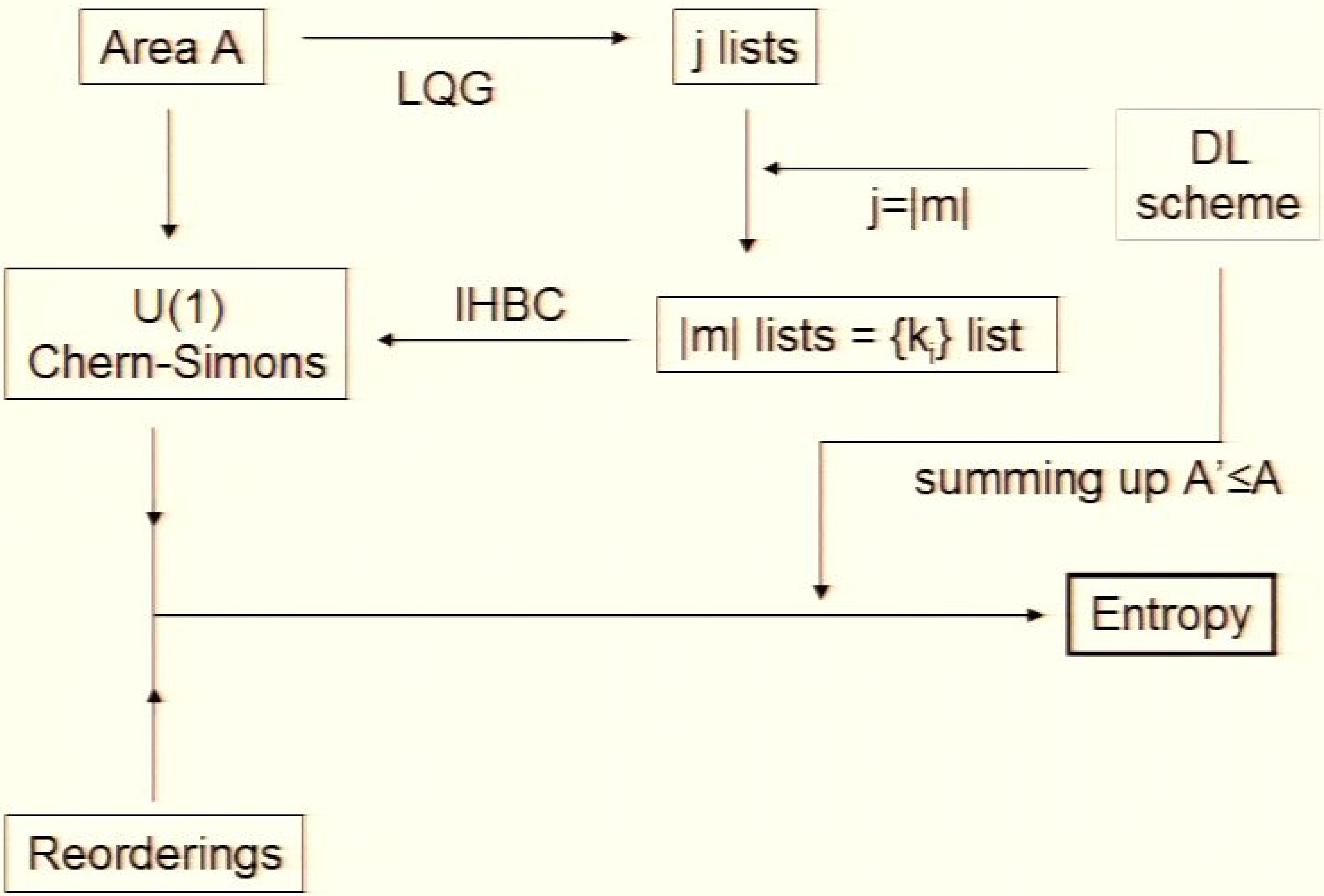

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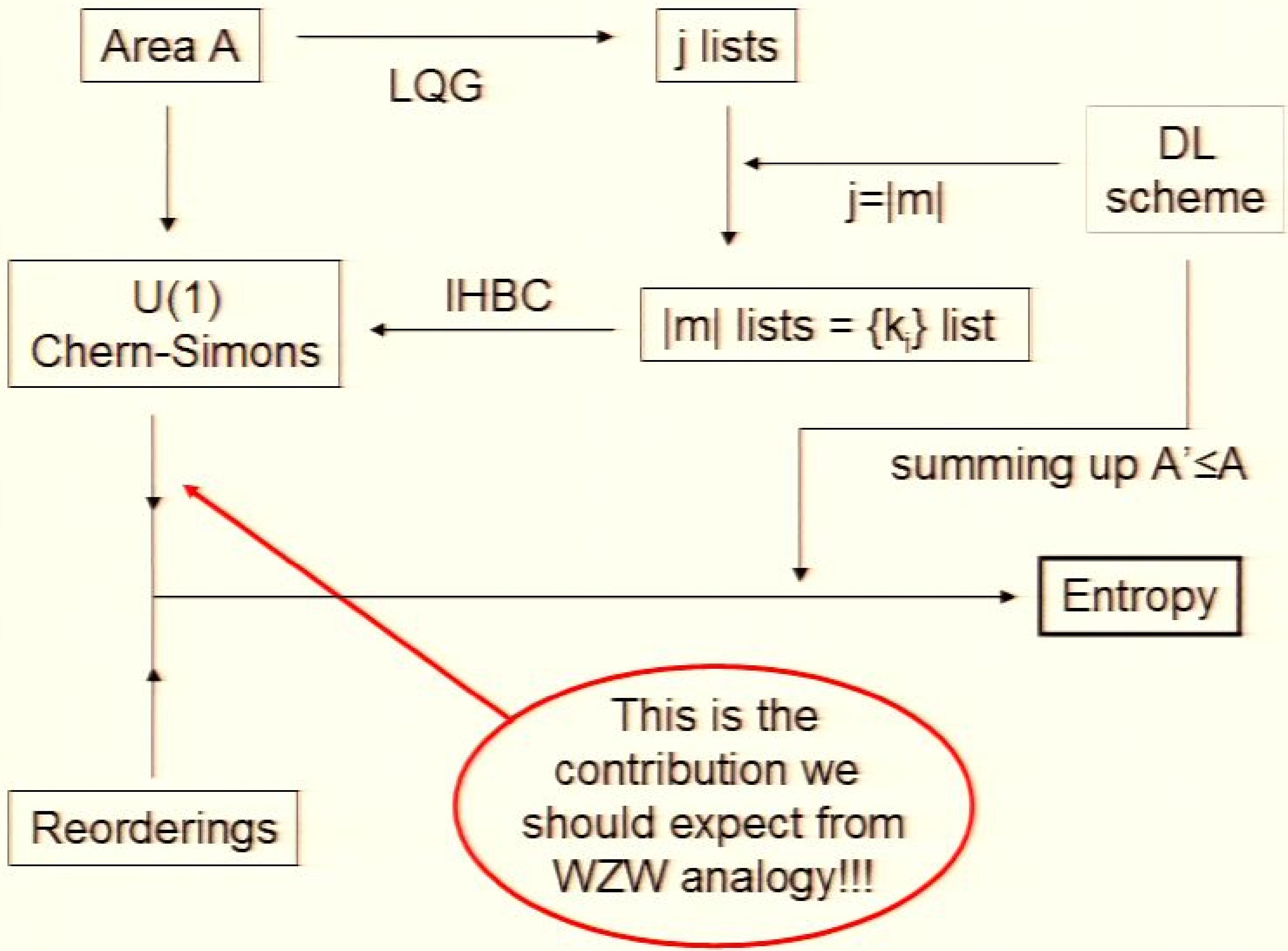

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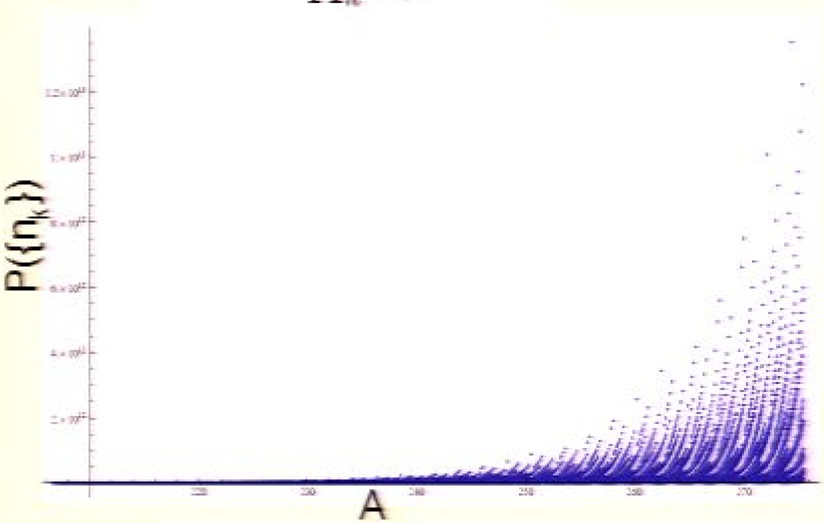

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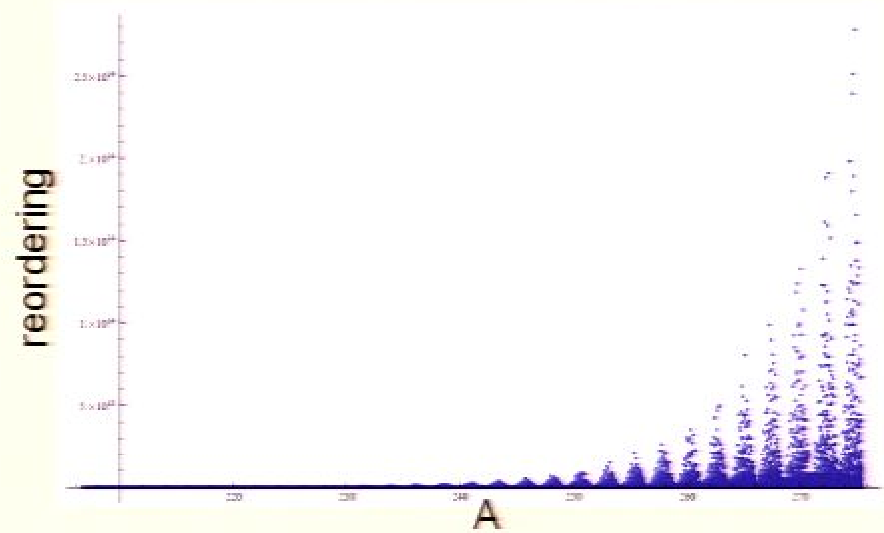

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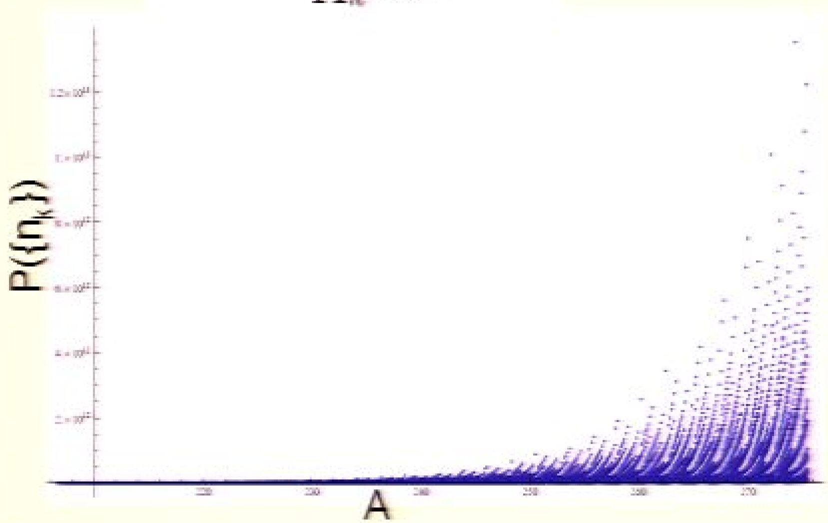

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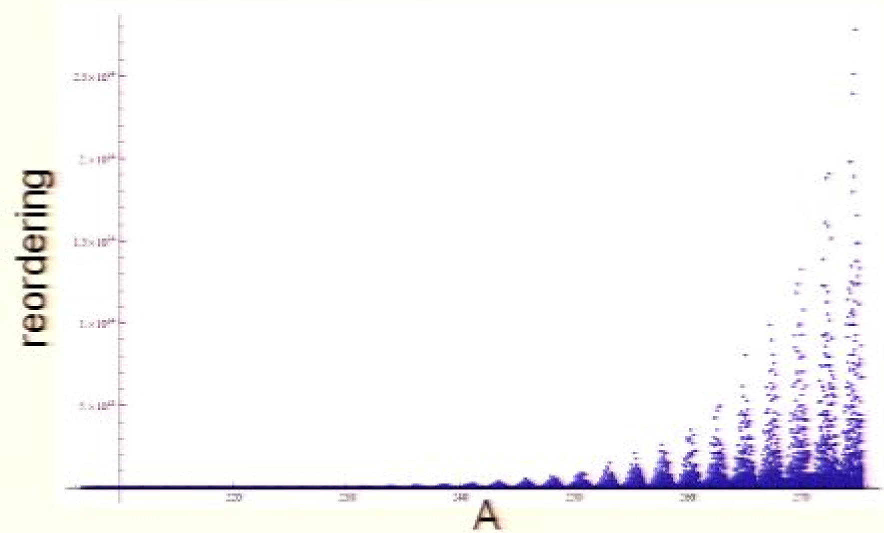

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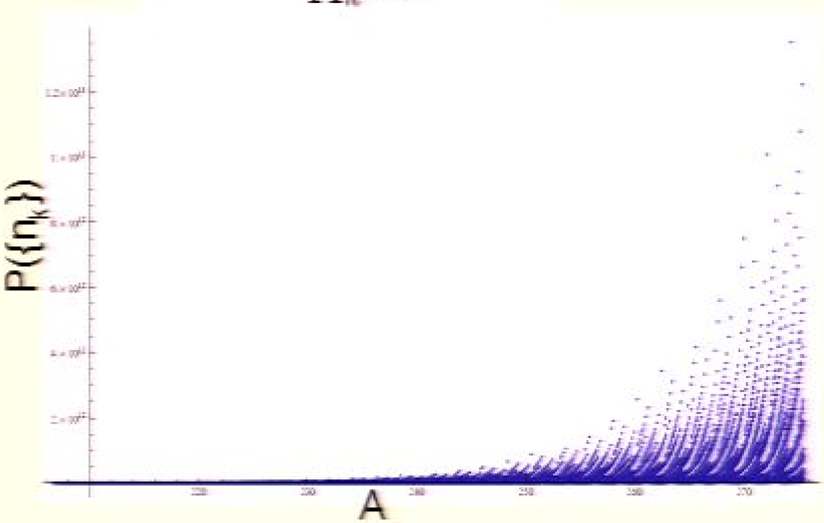

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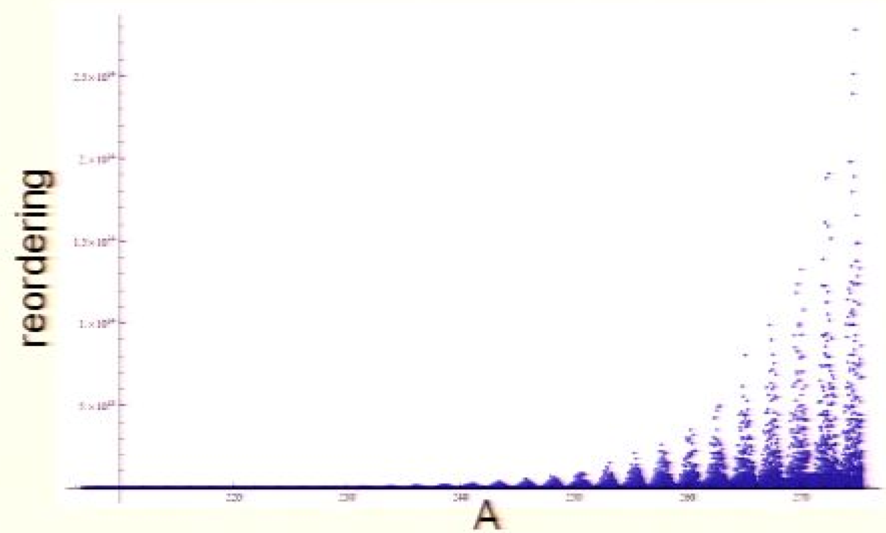

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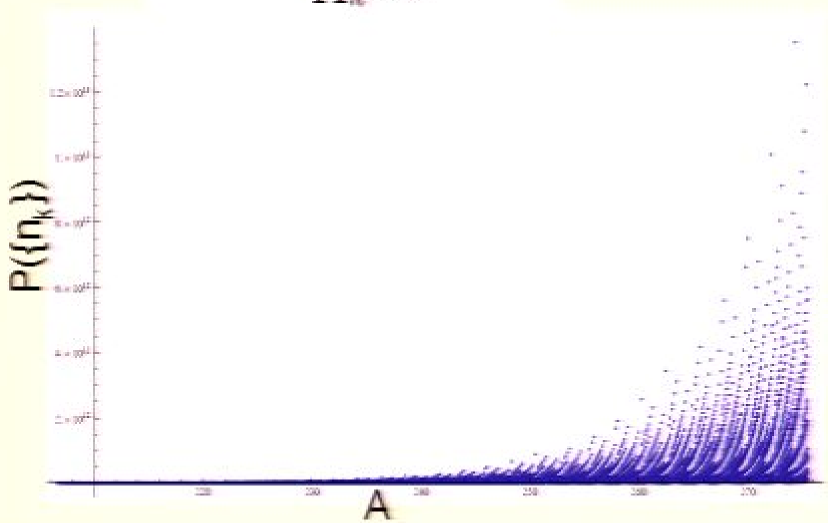

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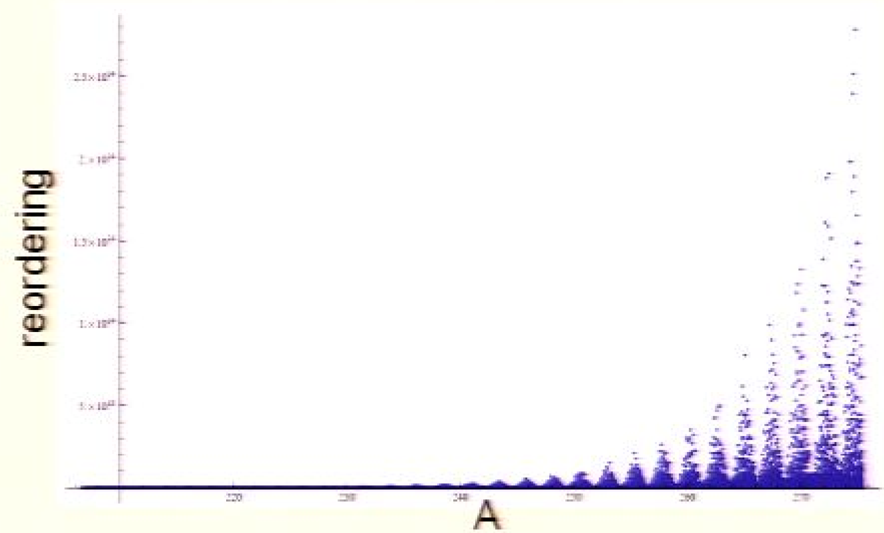

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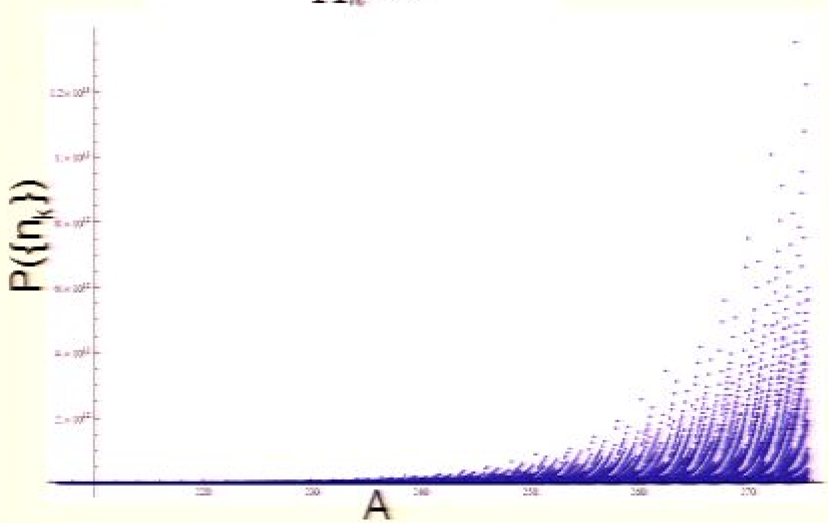

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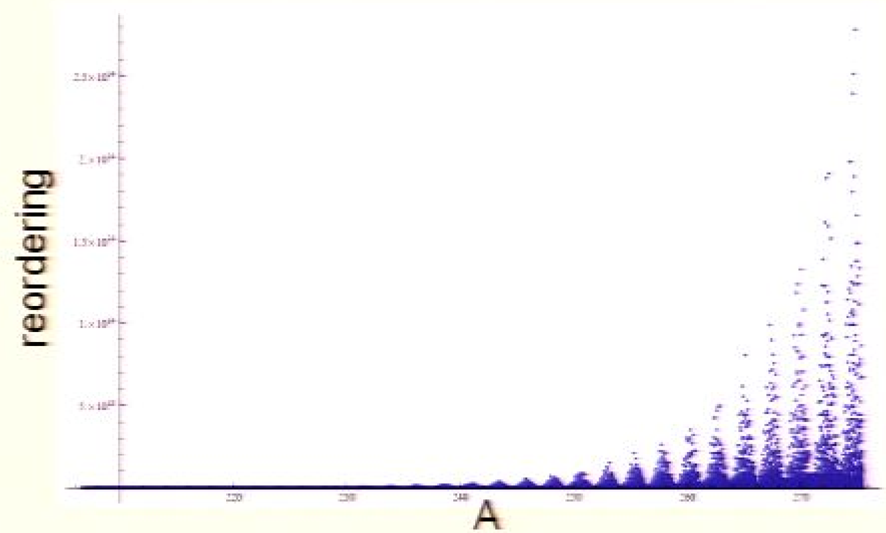

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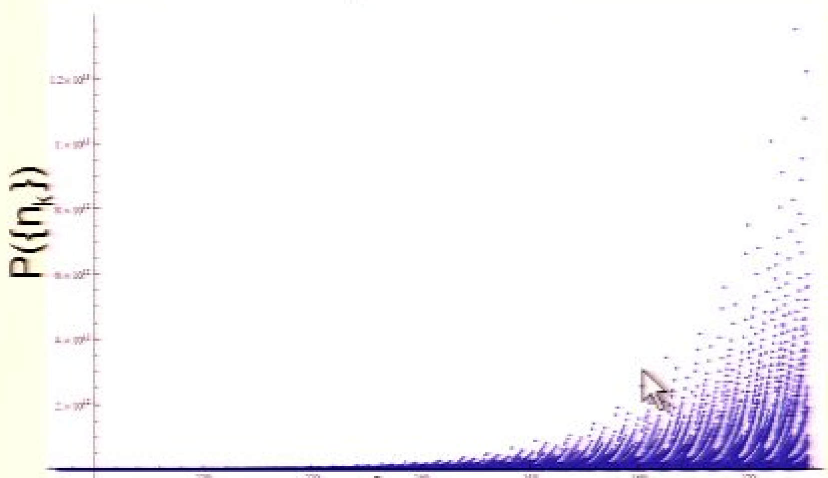

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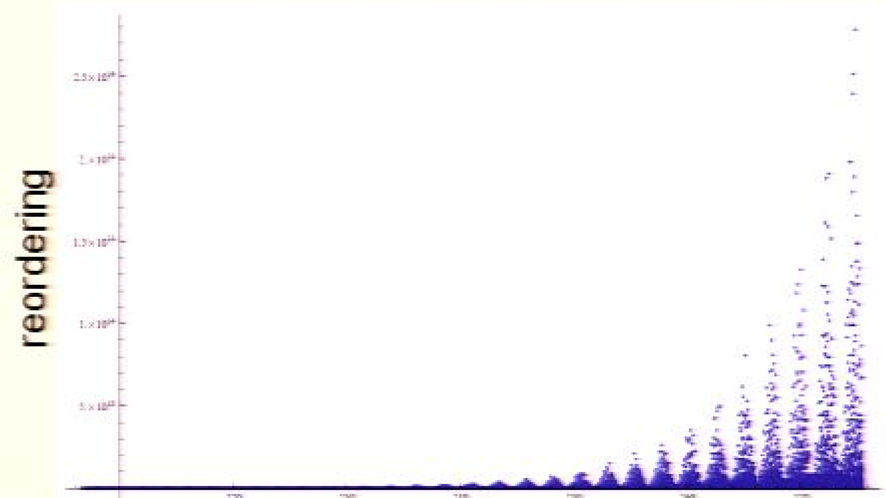

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BLACK HOLE ENTROPY IN LQG

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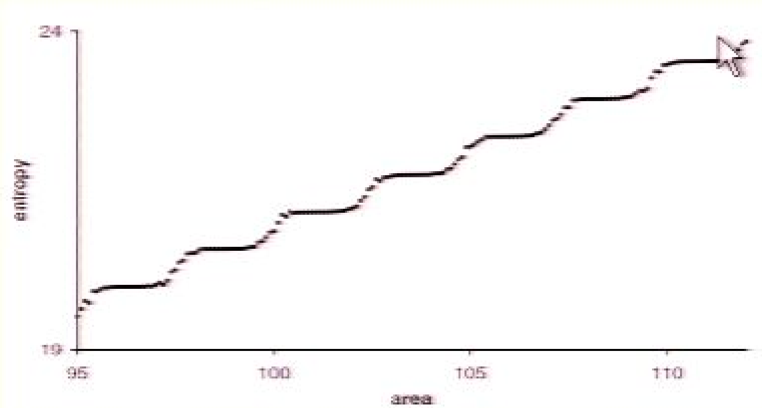

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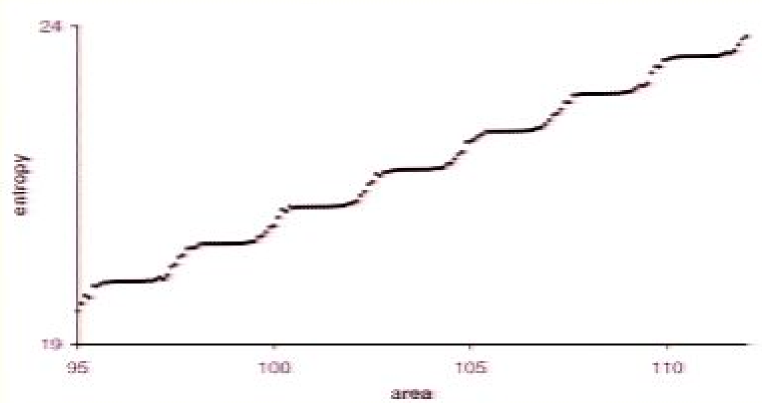

Questions:

- Is this effect an artifact the way of counting?
- Where is this effect coming from?
- Is this effect present for large areas where the Isolate Horizon approximation is justified?