Title: Torsion and the Holography of Parity Breaking

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Abstract: TBA

TORSION AND THE GRAVITY DUAL OF PARITY BREAKING

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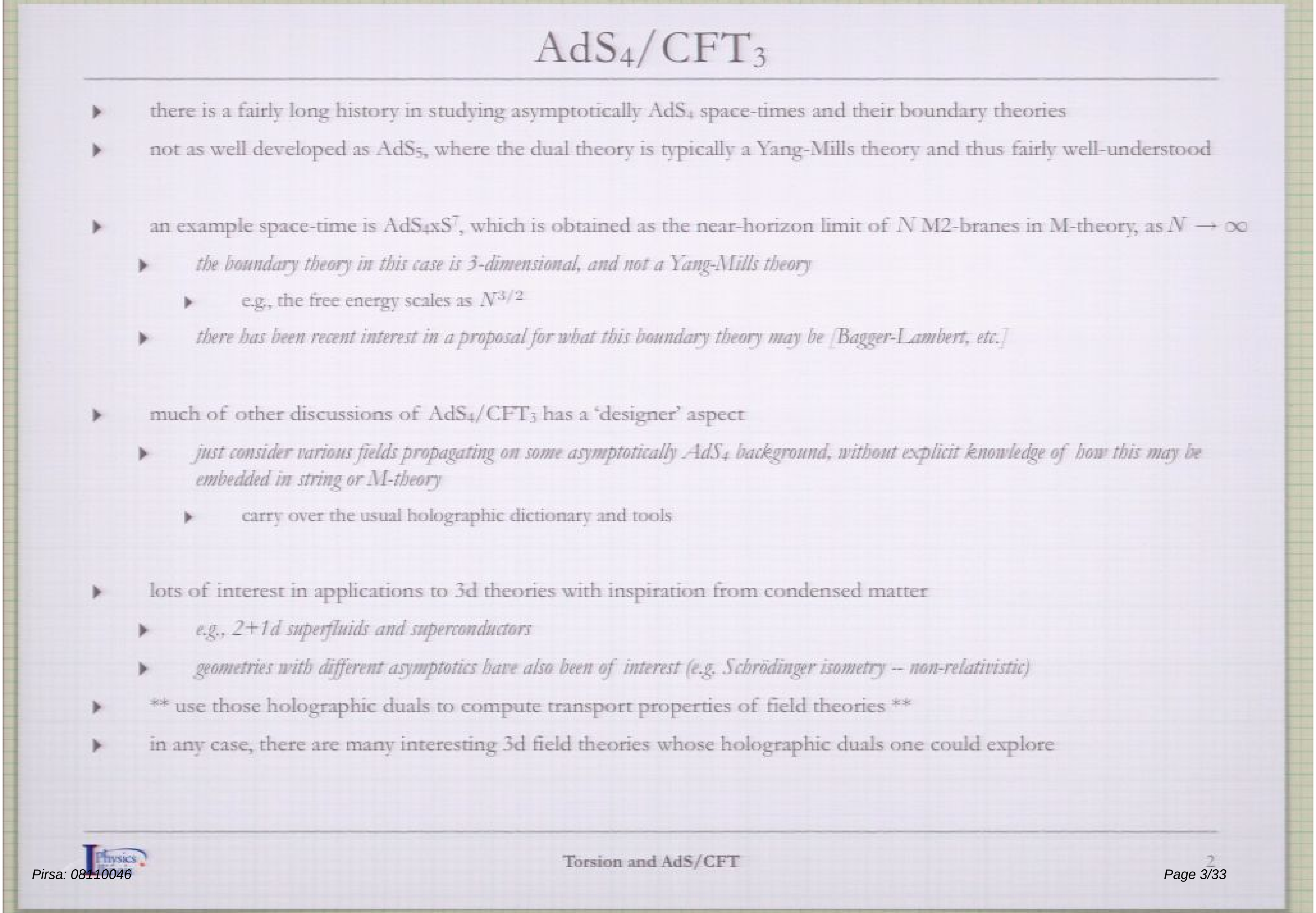


AdS_4/CFT_3

- there is a fairly long history in studying asymptotically AdS_4 space-times and their boundary theories
- not as well developed as AdS_5, where the dual theory is typically a Yang-Mills theory and thus fairly well-understood

- an example space-time is $AdS_4 x S^7$, which is obtained as the near-horizon limit of N M2-branes in M-theory, as $N \to \infty$
    - *the boundary theory in this case is 3-dimensional, and not a Yang-Mills theory*
        - e.g., the free energy scales as $N^{3/2}$
    - *there has been recent interest in a proposal for what this boundary theory may be* [Bagger-Lambert, etc.]

- much of other discussions of AdS_4/CFT_3 has a 'designer' aspect
    - *just consider various fields propagating on some asymptotically AdS_4 background, without explicit knowledge of how this may be embedded in string or M-theory*
        - carry over the usual holographic dictionary and tools

- lots of interest in applications to 3d theories with inspiration from condensed matter
    - *e.g., 2+1d superfluids and superconductors*
    - *geometries with different asymptotics have also been of interest (e.g. Schrödinger isometry -- non-relativistic)*
- ** use those holographic duals to compute transport properties of field theories **
- in any case, there are many interesting 3d field theories whose holographic duals one could explore

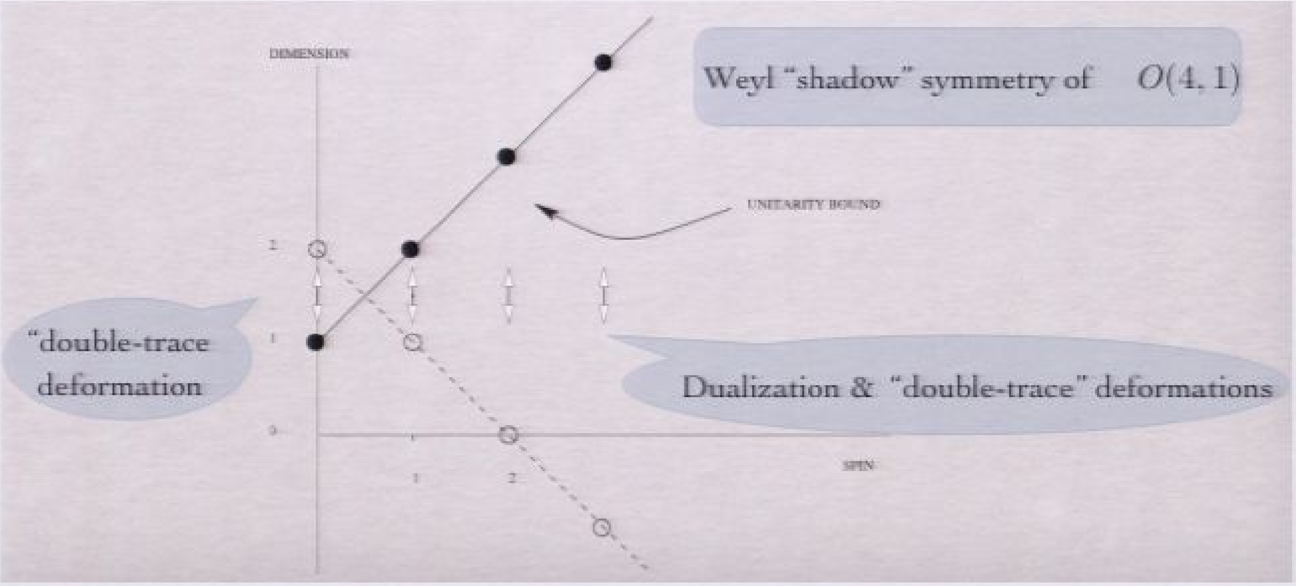
AdS$_4$/CFT$_3$

- One of the earliest examples involved conformally coupled scalars in bulk, dual to $\Delta = 1\ , 2$ operators in bdy
  - *the proposal was that this describes large N fixed points of the O(N) scalar model in 3d*
  - *"double trace" deformation leads to passage from one large N fixed point to the other*

Klebanov & Polyakov '03
RGL & Petkou '03

- in 'mass window' where both asymptotic modes are normalizable
- this structure extends to gauge fields as well
- "duality conjecture" -- linearized higher spin gauge fields on AdS$_4$ possess a generalization of EM duality that is seen holographically in the boundary theory

RGL & Petkou, 0309177



Weyl "shadow" symmetry of $O(4,1)$

"double-trace deformation

Dualization & "double-trace" deformations

EM Duality (Hamiltonian form)

- Maxwell theory (written as '3+1 split'):

$$I = \int_M dt \wedge \left\{ \dot{A} \wedge *_3 E - \frac{1}{2}(E \wedge *_3 E + B \wedge *_3 B) - A_0 \tilde{d} *_3 E \right\}$$

- here, E and B are spatial 1-forms

$$B = *_3 \tilde{d} A$$

- under EM duality

$$E \mapsto -B, \qquad B \mapsto E$$

- we find

$$I \mapsto \int_M dt \wedge \left\{ -\dot{A}_D \wedge *_3 B - \frac{1}{2}(E \wedge *_3 E + B \wedge *_3 B) + A_0 \tilde{d} *_3 B \right\}$$

$$E = *_3 \tilde{d} A_D$$

- or

$$I \mapsto I_D = I - \int_{\partial M} A_D \wedge \tilde{d} A$$

(canonical transformation)

Bianchi identity <---> Gauss' law

- a bulk duality gives a canonical transformation, induced by a boundary term built from the bdy values of bulk fields
- leads to a modification of boundary correlation functions
  - *in fact, extends to an SL(2) action on current-current correlator*

Gravity

- write the Einstein-Hilbert action in the first-order formalism

$$I = \int_M \epsilon_{abcd} \left\{ e^a \wedge e^b \wedge R^{cd} - \frac{\Lambda}{6} e^a \wedge e^b \wedge e^c \wedge e^d \right\}$$

$$\Lambda = -3/L^2 = -3a^2$$

$\sqrt{|g|}R$

$\sqrt{|g|}$

$$R^a{}_b = d\omega^a{}_b + \omega^a{}_c \wedge \omega^c{}_b$$

$$T^a = de^a + \omega^a{}_c \wedge e^c$$

- regard the vielbein and connection as independent
- equations of motion (vacuum):

$$R^{ab} - \frac{\Lambda}{3} e^a \wedge e^b = 0$$

$$T^a = 0$$

first-order formalism: torsion vanishes by equation of motion
metric formalism: torsion vanishes by constraint

- we employ the first-order formalism, because it is straightforward to introduce torsion

Gravity: 3+1 Split

- do radial (t) split

$$\begin{aligned} e^0 &= N dt \\ e^\alpha &= N^\alpha dt + \tilde{e}^\alpha \\ \omega^0{}_\alpha &= q^0{}_\alpha dt + K_\alpha \\ \frac{1}{2}\epsilon_{\alpha\beta}{}^\gamma \omega^\beta{}_\gamma &= Q_\alpha dt + B_\alpha \end{aligned}$$

- the action then takes the form

$$\begin{aligned} I_{EH} + I_{GH} = & \int dt \wedge (\dot{\tilde{e}}^\alpha \wedge (-4\sigma_\perp \epsilon_{\alpha\beta\gamma} \tilde{e}^\beta \wedge K^\gamma) \\ & +2\sigma_\perp N \left\{ 2\tilde{d}\,(B^\alpha \wedge \tilde{e}_\alpha) + 2B^\alpha \wedge \tilde{T}_\alpha + \epsilon_{\alpha\beta\gamma} \left(\sigma B^\alpha \wedge B^\beta - K^\alpha \wedge K^\beta - \frac{\sigma_\perp \Lambda}{3} \tilde{e}^\alpha \wedge \tilde{e}^\beta \right) \wedge \tilde{e}^\gamma \right\} \\ & -4\sigma_\perp N^\alpha \epsilon_{\alpha\beta\gamma} (\tilde{D}K)^\beta \wedge \tilde{e}^\gamma + 4Q^\alpha (K_\beta \wedge \tilde{e}^\beta) \wedge \tilde{e}_\alpha + 4q^0{}_\alpha \left\{ \epsilon^\alpha{}_{\beta\gamma} \tilde{T}^\beta \wedge \tilde{e}^\gamma \right\}) \end{aligned}$$

- as usual, mostly constraints

$\tilde{e}^\alpha$: coordinate
K^α : momentum
B^α : determined

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Extrinsic curvature

Lagrange multipliers

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- as usual, mostly constraints

$\tilde{e}^\alpha$	:	coordinate	'gauge field'
K^α	:	momentum	'electric field'
B^α	:	determined	'magnetic field'

Gravitational EM duality

- linearize around AdS $(\underline{\tilde{e}}^\alpha = e^{at} dx^\alpha, \;\; \underline{K}^\alpha = a\underline{\tilde{e}}^\alpha)$

$$I = \int dt \wedge \left\{ (\dot{E}^\alpha + aE^\alpha) \wedge p_\alpha - 2\epsilon_{\alpha\beta\gamma}(b^\alpha \wedge b^\beta + k^\alpha \wedge k^\beta) \wedge \underline{\tilde{e}}^\gamma + \text{constraints} \right\}$$

- duality here, very similar to EM:

$$\begin{gathered} k^\alpha \mapsto -b^\alpha, \;\; b^\alpha \mapsto k^\alpha \\ E \mapsto E_D, \;\; p \mapsto -p_D \\ \text{constraints} \leftrightarrow \text{Bianchi} \end{gathered}$$

- so again, a bulk duality gives a canonical transformation, induced by a boundary term built from the boundary values of bulk fields
    - *duality acts in boundary on (higher-spin) current-current correlators*

Topological, Boundary terms, etc.

- I_{EH} in the first-order formalism has more degrees of freedom than gravity
  - *can couple to these in interesting ways*
- build SO(3,1) invariants (4-forms of e^a, $R^a{}_b$, T^a)
-

$$I_{top} = \theta \int C_{NY} + \frac{2}{\gamma} \int R_{ab} \wedge e^a \wedge e^b + p \int P_4 + q \int E_4$$

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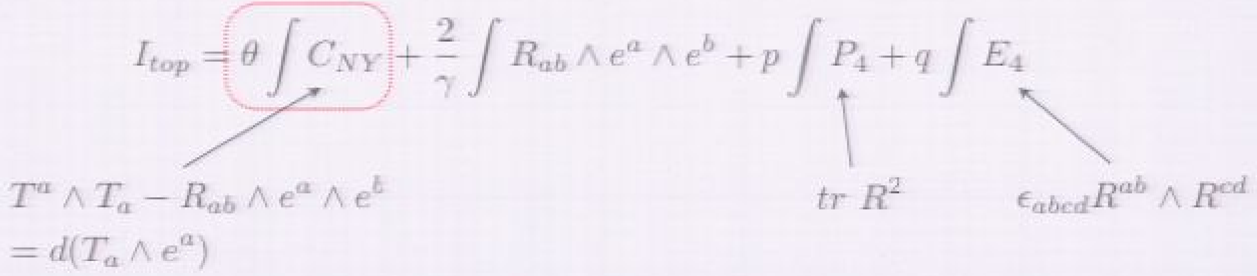


$$I_{top} = \boxed{\theta \int C_{NY}} + \frac{2}{\gamma}\int R_{ab}\wedge e^a \wedge e^b + p\int P_4 + q\int E_4$$

$T^a \wedge T_a - R_{ab}\wedge e^a \wedge e^b$
$= d(T_a \wedge e^a)$

$tr\ R^2$

$\epsilon_{abcd}R^{ab}\wedge R^{cd}$

- will concentrate on the Nieh-Yan term here
- if θ is constant

$$I_{EH} + I_{GH} + \mathcal{I}_{NY} \quad = \quad \int dt \wedge \left(\dot{\tilde{e}}^\alpha \wedge (-4\sigma_\perp \epsilon_{\alpha\beta\gamma}\tilde{e}^\beta \wedge [K^\gamma - \theta B^\gamma]) + 2\sigma_\perp \theta \epsilon_{\alpha\beta\gamma}\dot{B}^\alpha \wedge \tilde{e}^\beta \wedge \tilde{e}^\gamma + \text{ constraints}\right)$$

- equation of motion still gives zero torsion, because NY is a boundary term
- tr B has kinetic term, but e,K still canonical variables
- boundary has been modified: on-shell action now varies as

$$\delta\left(I_{EH} + I_{GH} + \mathcal{I}_{NY}\right)_{on\ shell} = \int_{\partial\mathcal{M}} \delta\tilde{e}^\alpha \wedge \left(-4\sigma_\perp \epsilon_{\alpha\beta\gamma}\tilde{e}^\beta \wedge [K^\gamma - \theta B^\gamma]\right)_{on\ shell}$$

Nieh-Yan Axion

- now promote θ to be a field
- adding a boundary term (gives axion appropriate boundary condition)

$$I = I_{EH}[e,\omega] + I_{GH}[e,\omega] - \frac{2}{3}\int_M d\Theta \wedge T_a \wedge e^a \,.$$

- variation of action now gives

$$T_a \wedge e^a = *_4 d\Theta$$

- various other formulations:
  - *massless pseudoscalar coupled to torsionless gravity*

$$\omega^a{}_b = \mathring{\omega}^a{}_b + \Omega^a{}_b$$

d'Auria & Regge, '82

$$I_{PS} = I_{EH}[e,\mathring{\omega}] + I_{GH}[e,\mathring{\omega}] - \frac{1}{3}\int_M d\Theta \wedge *_4 d\Theta \,.$$

  - *Kalb-Ramond field*

$$H_3 = *_4 d\Theta \sim dC$$

$$I_{KR} = I_{EH}[e,\mathring{\omega}] + I_{GH}[e,\mathring{\omega}] - \frac{1}{2}\int_M dC \wedge *_4 dC + \int_M d(C \wedge *_4 dC) \,.$$

- I'll use the torsional formulation

Torsion ‘Vortex’

- 3+1 split:

$$I \;=\; \int dt \wedge \left(\dot{\tilde{e}}^{\alpha} \wedge (4\epsilon_{\alpha\beta\gamma} K^{\gamma} \wedge \tilde{e}^{\beta} - \frac{2}{3} \tilde{d}\Theta \wedge \tilde{e}_{\alpha}) - \frac{2}{3} \dot{\Theta} (\tilde{e}^{\alpha} \wedge \tilde{T}_{\alpha}) \right) + \text{ modified constraints}$$

$$\Pi_{\Theta} \sim \tilde{e}^{\alpha} \wedge \tilde{T}_{\alpha}$$

- make a simple ansatz -- fields depend only on the radial coordinate

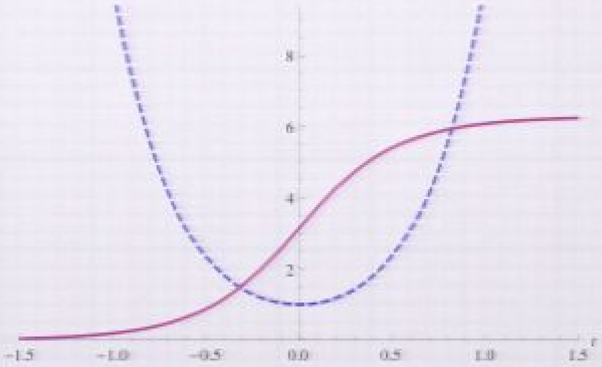
$$\begin{array}{ccc} \tilde{e}^{\alpha} = e^{A(t)} dx^{\alpha}, & N = 1, & N^{\alpha} = 0, \\ K_{\alpha} = k \tilde{e}_{\alpha}, & & B_{\alpha} = b \tilde{e}_{\alpha}, \\ \Pi_{A} = -4k, & & \Pi_{\Theta} = b \end{array}$$

- and equations of motion give

$$\ddot{A} + 3\dot{A}^2 - 3a^2 = 0, \qquad \ddot{A} = \frac{1}{12} \sigma \dot{\Theta}^2, \qquad \ddot{\Theta} + 3\, \dot{\Theta} \dot{A} = 0$$

- in *Euclidean* signature, there is an *exact* solution

$$\begin{aligned} h(t) &\equiv \dot{A}(t) = a \tanh(3a(t - t_0)) \\ \Theta(t) &= \Theta_0 \pm 4 \arctan(e^{3a(t-t_0)}) \\ e^{A(t)} &= \alpha (2 \cosh(3a(t - t_0)))^{1/3} \end{aligned}$$

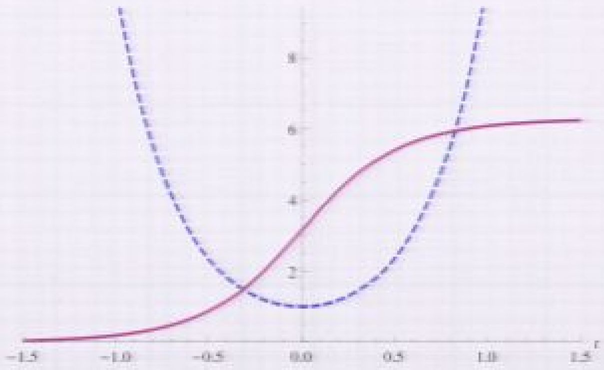


Torsion 'Vortex'

- note that the solution is asymptotically AdS at *both* $t \to \pm\infty$
- the pseudoscalar field is kinked, changing by 2π from one asymptote to the other

$$\int *_4 H = \Delta\Theta = 2\pi.$$

- it is non-singular everywhere
- transverse slices are flat at the core of the vortex
- the energy of the solution diverges, but in precisely the same way as AdS
  - *counterterms on each boundary cancel this divergence*
- in this sense, this is a 'zero energy' Euclidean solution



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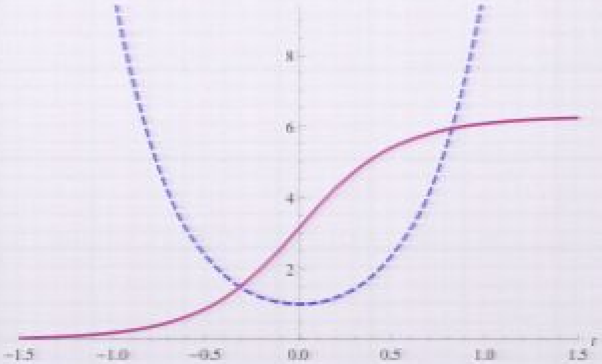

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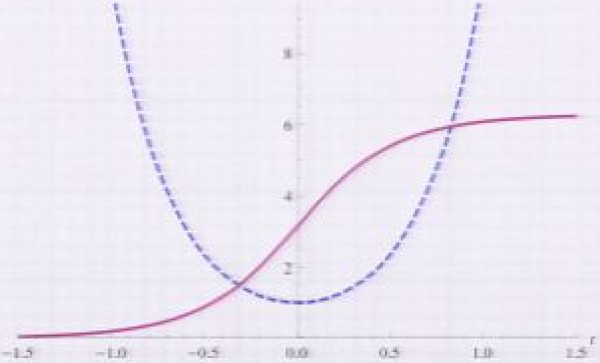

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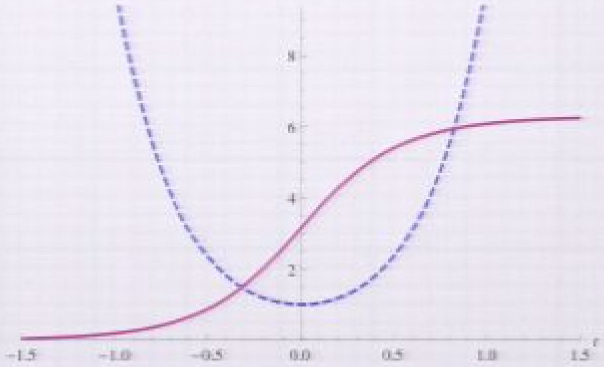

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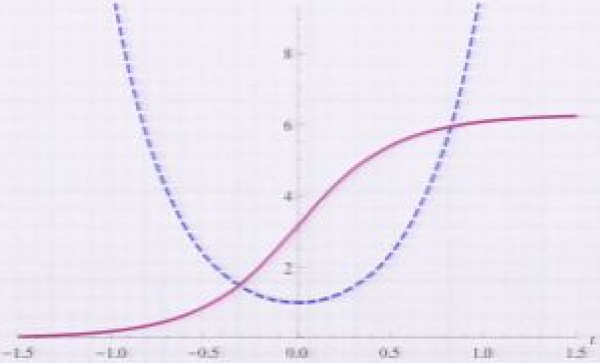


Torsion Vortex -- holographic interpretation

- Holography: expand asymptotically --

$$\tilde{e}^{\alpha} = 2^{-1/3}\alpha e^{\pm a(t-t_0)}\left(1+\frac{1}{3}e^{\mp 6a(t-t_0)}+\cdots\right)dx^{\alpha} \ \text{for} \ t\to\pm\infty$$

- the $e^{\pm 3at}$ term is missing, meaning $\langle T_{\mu\nu}\rangle = 0$
- the pseudoscalar behaves as

$$\begin{aligned}\Theta(t) &\to 4e^{-3a(t-t_0)}-\frac{4}{3}e^{-9a(t-t_0)}+\cdots \ \text{for} \ t\to-\infty\,,\\ \Theta(t) &\to 2\pi-4e^{3a(t-t_0)}+\frac{4}{3}e^{9a(t-t_0)}+\cdots \ \text{for} \ t\to+\infty\,.\end{aligned}$$

- the pseudoscalar is massless, so corresponds to a $\Delta=3$ parity odd operator with

$$\langle\mathcal{O}_3\rangle = \pm 4 \ \text{for} \ t\to\mp\infty$$

- coupling goes from $g: 0\to 2\pi$
- thus each asymptotic regime is in a different parity breaking vacuum state

Holographic Interpretation

- of course, we don't know much about the dual boundary theory, but it does have distinct parity breaking vacua
- a simple toy model fits very well with this structure -- the Gross-Neveu model at large N, coupled to Maxwell

$$I = -\int d^3x \left[\bar{\psi}^a \left(\partial\!\!\!/ - \mathrm{i}eA\!\!\!/\right)\psi^a + \frac{G}{2N}\left(\bar{\psi}^a\psi^a\right)^2 + \frac{1}{4M}F_{\mu\nu}F^{\mu\nu}\right]$$

- do large N analysis by Hubbard-Stratanovic; integrate out fermions and find saddle points with

$$\langle\sigma\rangle = \pm m$$

- since coupled to Abelian gauge field, find effective action in each vacuum of Chern-Simons form

$$S_{CS} = \pm i\frac{ke^2}{4\pi}\int d^3x\epsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho\,, \quad k = \frac{N}{2}$$

- $A \wedge dA$ is a dimension 3 parity odd operator
  - *its coupling shifts as we go from one vacuum to the other*

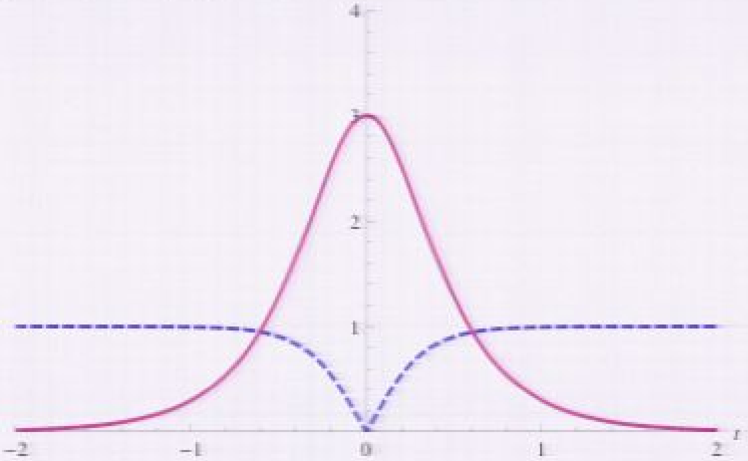
$$I(+) = I(-) + 2\pi\int\mathcal{O}_3$$

  - *because parity is broken, presumably it also gets a vev of opposite sign*

- the topology of the boundary is an open question
  - *e.g., the two asymptotes may correspond to two patches that should be glued along their boundary*
  - *the torsion vortex would then look like a domain wall in the boundary separating parity violating vacua*

Bulk Interpretation

- there is a fun analogue of this solution to a condensed matter system
- to see this make a plot of Π_Θ (red) and $|h|$ (blue)



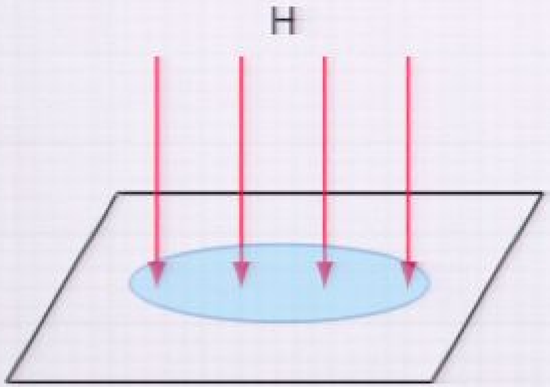
- open any book on superconductivity and you will see this picture, with *condensate* (blue) and *magnetic induction of a vortex* (red)
- look at asymptotics: 'magnetic field' has penetration length $\lambda \sim 1/3a$, order parameter has coherence length $\xi \sim 1/6a$
- furthermore, equation of motion gives

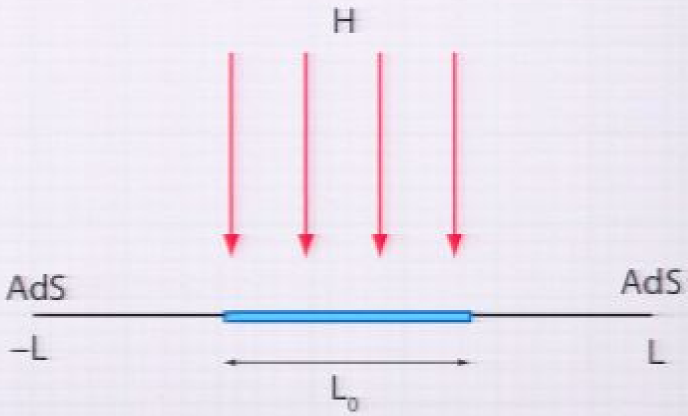
Multiple Vortices

- so the torsion vortex corresponds to the Abrikosov vortex in a superconductor
  - *note though that the codimensions are different*
- let's push the analogy further -- this is in fact a Type I superconductor
  - *vortices attract one another*
- to see this, put two vortices together at distance ℓ and compute the energy and hence a force

$$F = -6a\alpha^3 \cosh(3a\ell/2)$$

- vortices tend to clump together, potentially forming a region of normal phase within the superconductor
- critical magnetic field -- normal phase fills up

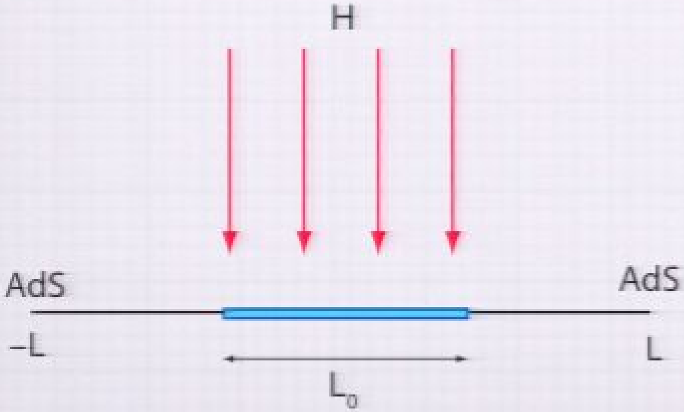




- recall the normal phase (at centre of vortex core) is a flat transverse section

$$\boxed{\text{normal phase} \leftrightarrow \text{flat space}}$$

- the analogue of an external magnetic field is H-flux (torsion)
- if we put on an external field, expect vortices to carry the magnetic field through the system
  - *-- flux quantization -- each vortex carries a flux quantum*
- for a fixed uniform external field, expect a droplet of normal phase to form, with some size
  - *determine by minimizing energy of multi-vortex configuration*
-



$$\Delta\Theta = 2L\hat{H}$$

L = radial cutoff (each end)

$$H = \dot{\Theta}\ Vol_3 = \hat{H}\ \widehat{Vol_3}$$

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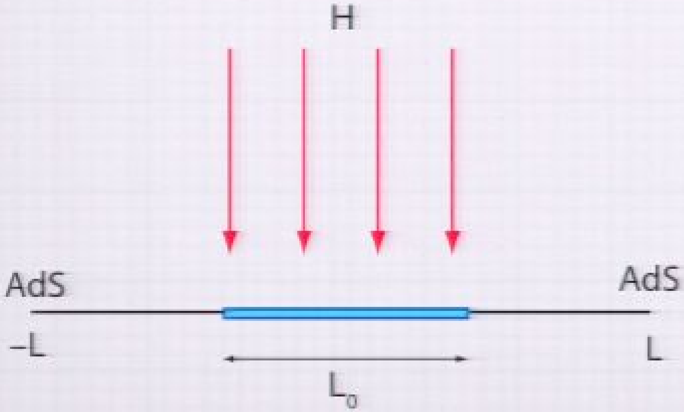

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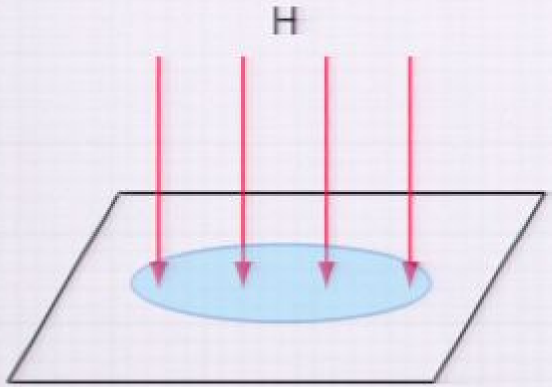
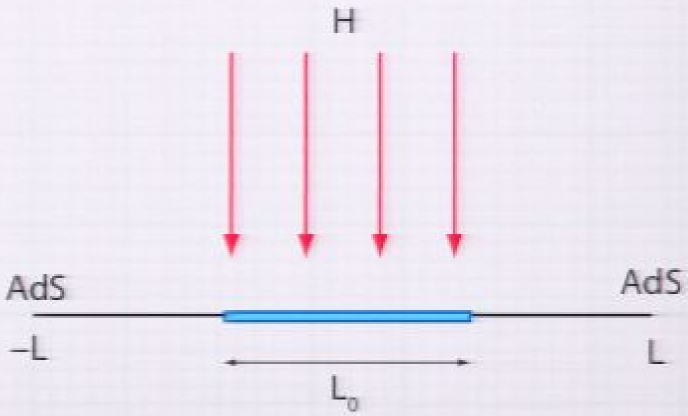
Multiple Vortices

- so the torsion vortex corresponds to the Abrikosov vortex in a superconductor
  - *note though that the codimensions are different*
- let's push the analogy further -- this is in fact a Type I superconductor
  - *vortices attract one another*
- to see this, put two vortices together at distance ℓ and compute the energy and hence a force

$$F = -6a\alpha^3 \cosh(3a\ell/2)$$

- vortices tend to clump together, potentially forming a region of normal phase within the superconductor
- critical magnetic field -- normal phase fills up





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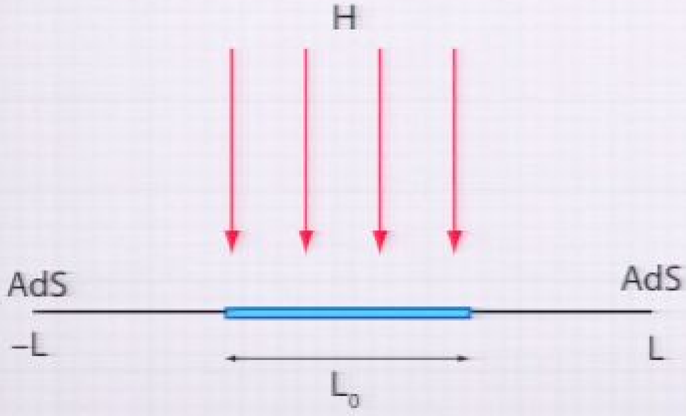

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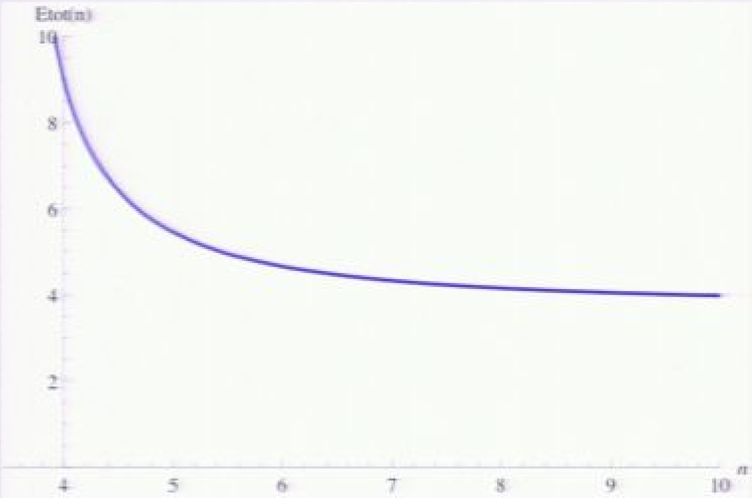
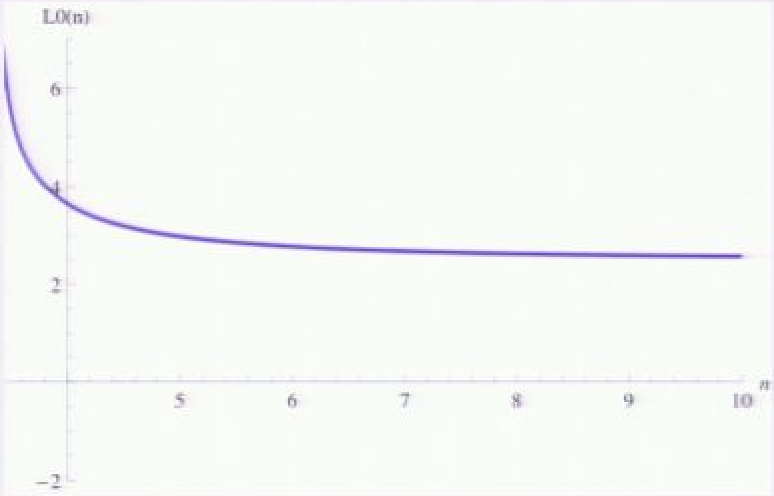
Multiple Vortices

- so in the presence of some external H-flux, expect a corresponding number of vortices
- consider an array of *n* vortices in a box of size L_0





- the energy is minimized for a large number of vortices, and in that case, L_0 is determined

$$L_0 = \frac{\hat{H}}{6a} \cdot 2L = \alpha^3 \cdot 2L$$

- so with external flux, system prefers to have a finite size droplet of normal phase (flat space) inside the superconductor (asymptotically AdS space)
  - *there is a critical H-flux for which* $L_0 \sim 2L$ *-- droplet fills all of (cut off) space*

$$\hat{H}_c \sim (T - T_c)^{1/2}$$

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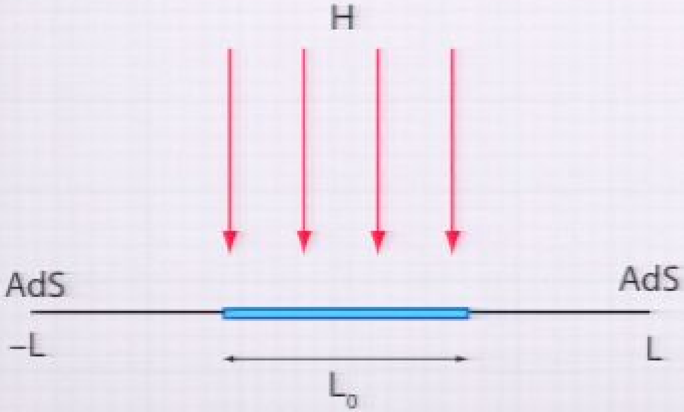

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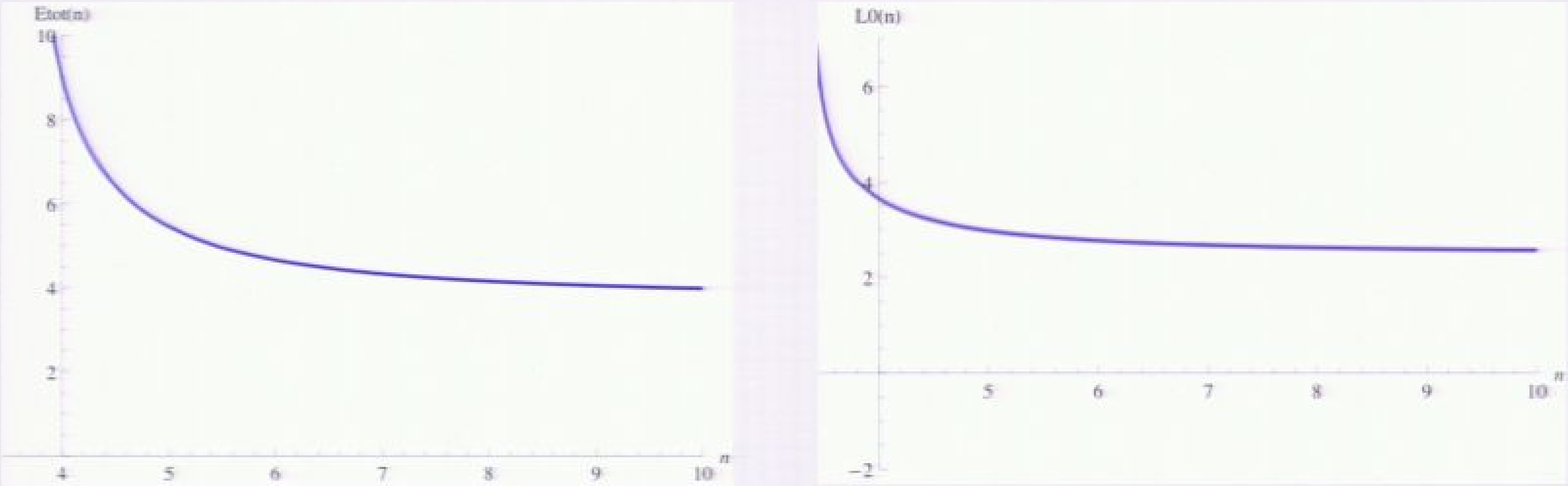

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Conclusion and Outlook

- we have found an interesting exact solution of gravity + torsion with AdS asymptotics
  - *bulk solution is somewhat like a wormhole between two asymptotically AdS regions*
  - *appears to describe a parity violating 'domain wall' holographically*
- an interesting analogy to superconductors appears and suggests that H-flux can support droplets of flat space within asymptotically AdS geometry
  - *cosmological constant plays the role of 'temperature'*

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- an interesting analogy to superconductors appears and suggests that H-flux can support droplets of flat space within asymptotically AdS geometry
    - *cosmological constant plays the role of 'temperature'*
- is there a boundary version of this?
- does this extend to other signatures?
    - *appears to, but only if we keep same overall signature*
    - *so e.g., deSitter with Lorentzian boundary --- corresponds to Type II superconductor!*
- ....