Title: What is a Wavefunction?

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Abstract: Conventional quantum mechanics answers this question by specifying the required mathematical properties of wavefunctions and invoking the Born postulate. The ontological question remains unanswered. There is one exception to this. A variation of the Feynman chessboard model allows a classical stochastic process to assemble a wavefunction, based solely on the geometry of spacetime paths. A direct comparison of how a related process assembles a Probability Density Function reveals both how and why PDFs and wavefunctions differ from the perspective of an underlying kinetic theory. If the fine-scale motion of a particle through spacetime is continuous and position is a single valued function of time, then we are able to describe ensembles of paths directly by PDFs. However, should paths have time reversed portions so that position is not a single-valued function of time, a simple Bernoulli counting of paths fails, breaking the link to PDF\'s! Under certain circumstances, correcting the path-counting to accommodate time-reversed sections results in wavefunctions not PDFs. The result is that a single `switch\' simultaneously turns on both special relativity and quantum propagation. Physically, fine-scale random motion in space alone yields a diffusive process with PDFs governed by the Telegraph equations. If the fine-scale motion includes both directions in time, the result is a wavefunction satisfying the Dirac equation that also provides a detailed answer to the title question.

What is a Wavefunction?

Garnet Ord

Mathematics Department
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- The Classical PDEs
- Entwined paths
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- Quantum mechanics is a *frequency* calculus, yet the object of this calculus is the wavefunction or 'probability amplitude'.
- We know how wavefunctions propagate in time, how to extract them using classical hamiltonians and how we can apply the Born rule to convert them to probability density functions.
- We do not, however, know what they are (or if we do, we disagree with the majority of our colleagues!)

Interpretation: A spectrum

Instramentalist?

"There is no quantum world. There is only an abstract physical description. It is wrong to think that the task of physics is to find out how nature is. Physics concerns what we can say about nature." Niels Bohr

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Realist?

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"[we] do not need deep theories to tell us that parallel universes exist, single-particle interference phenomena tell us that.."
David Deutsch

(on the Many Worlds Interpretation)

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- We do not have a large spectrum of interpretations of diffusion equations.
- Bohr, Einstein and Deutsch would have little problem accepting PDFs (and Weiner integrals) without recourse to 'Many Worlds'.
- In this talk we construct a counting argument for wavefunctions that shows, in a transparent way, exactly what a wavefunction is in a classical context where *we do not have to worry about the ambiguous status of reality in quantum mechanics.*

Classical vs. Quantum

	Classical	**Quantum**
Kinetic 'picture'	Kac (Poisson)	Chessboard
Telegraph/Dirac	$\frac{\partial \mathbf{U}}{\partial t} = \sigma_z \frac{\partial U}{\partial z} + a\sigma_x U$	$\frac{\partial \Psi}{\partial t} = \sigma_z \frac{\partial \Psi}{\partial z} + i\, m \sigma_x \Psi$
Telegraph/KG	$\frac{\partial^2 U}{\partial t^2} = \frac{\partial^2 U}{\partial z^2} + a^2 U$	$\frac{\partial^2 \psi}{\partial t^2} = \frac{\partial^2 \psi}{\partial z^2} + (i\, m)^2 \psi$
Heat/Schrödinger	$\frac{\partial U}{\partial t} = D \frac{\partial^2 U}{\partial x^2}$	$\frac{\partial \psi}{\partial t} = i\, D \frac{\partial^2 \psi}{\partial x^2}$
Characteristic Random Variable	Bernoulli $X \in \{1, 0\}$	

Table: Three sets of partial differential equations are compared. The left column contains phenomenological equations that have a basis in Kinetic theory. The PDF solutions are expected values of sums of the Bernoulli random variable. The right column contains 'quantum' equations obtained from the classical equations through a formal analytic continuation.

Interpretive Differences

	Classical	Quantum
Status	Phenomenology	Fundamental
Ontology	Kinetic Theory	Unknown
Counting Process	Yes	No
Uncertainty Principle	Yes	Yes
Special Relativity	No	Yes
Complex Numbers	No	Yes
Quantum scale physics	No	Yes
Conventional bridge	$1 \longrightarrow i$	$i \longrightarrow 1$

Table: The partial differential equations from the perspective of classical statistical mechanics. The PDF solutions of the classical equations are easy to understand, but do not ultimately illuminate quantum scale or relativistic physics. The Quantum equations are difficult to extend and interpret because there is no known process beneath them. Canonical quantization involves a formal analytic continuation.

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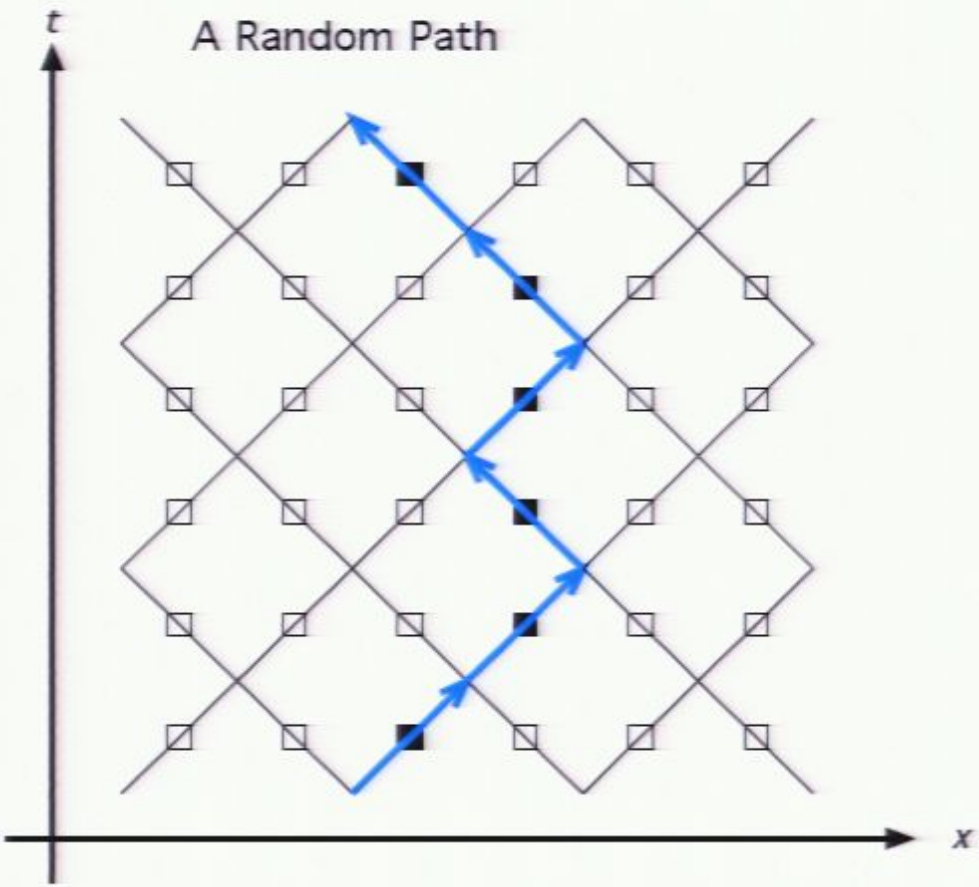
  This is the stochastic version of an indicator function.
- Classically, when we count paths (as in diffusion) it is the expectation value of a normalized sum of Bernoulli random variables that becomes a PDF in the continuum limit. So:

$$(\text{Normalized sum of } X\text{'s}) \xrightarrow{\text{Continuum Limit}} \text{PDF}$$

Bernoulli example

Classical 'counting' stochastic processes are based on a Bernoulli random variable X with $X \in \{0, 1\}$



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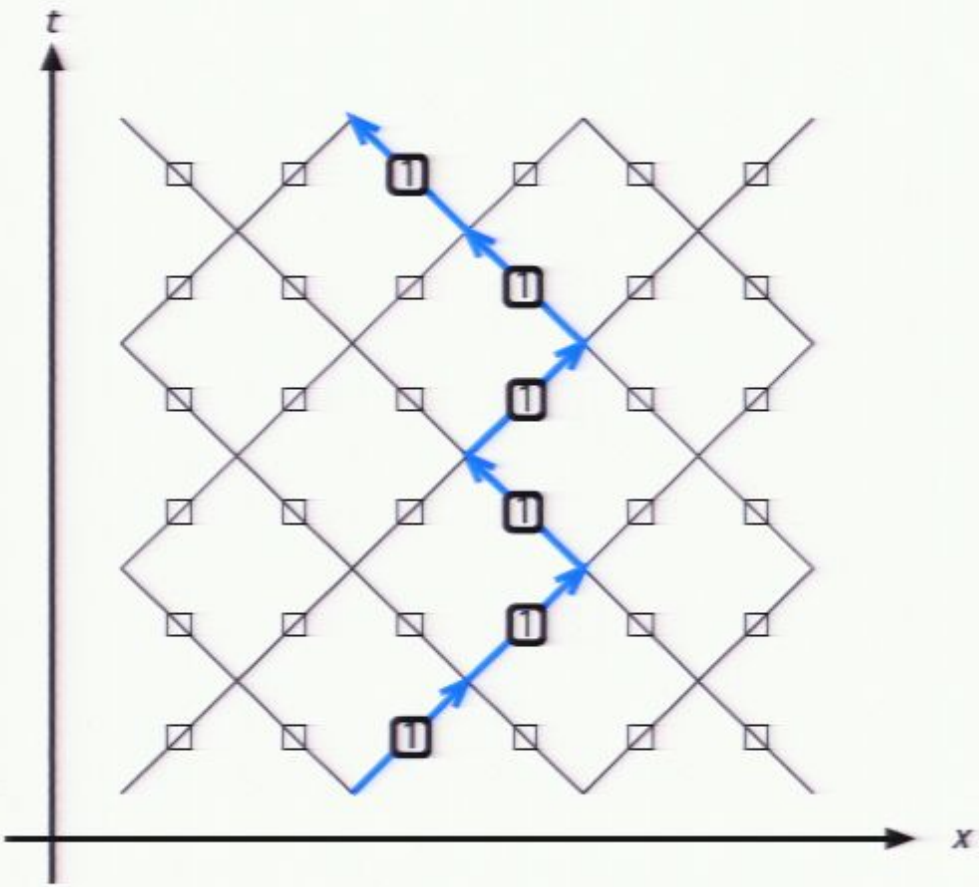

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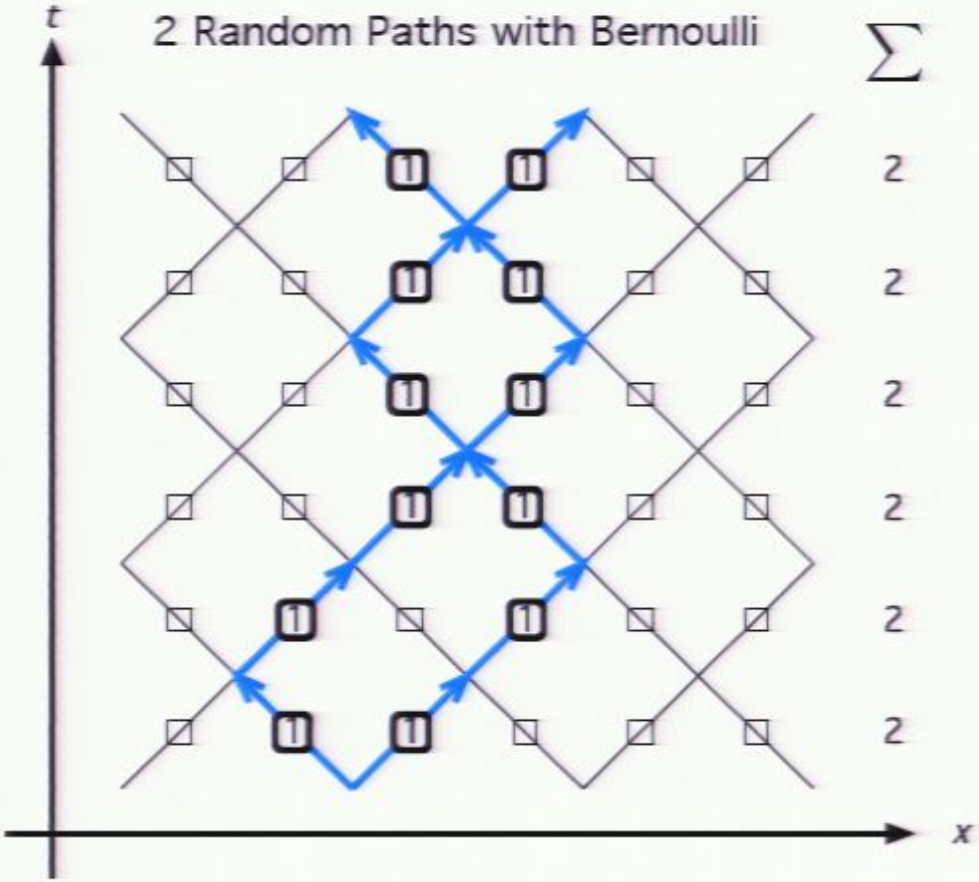

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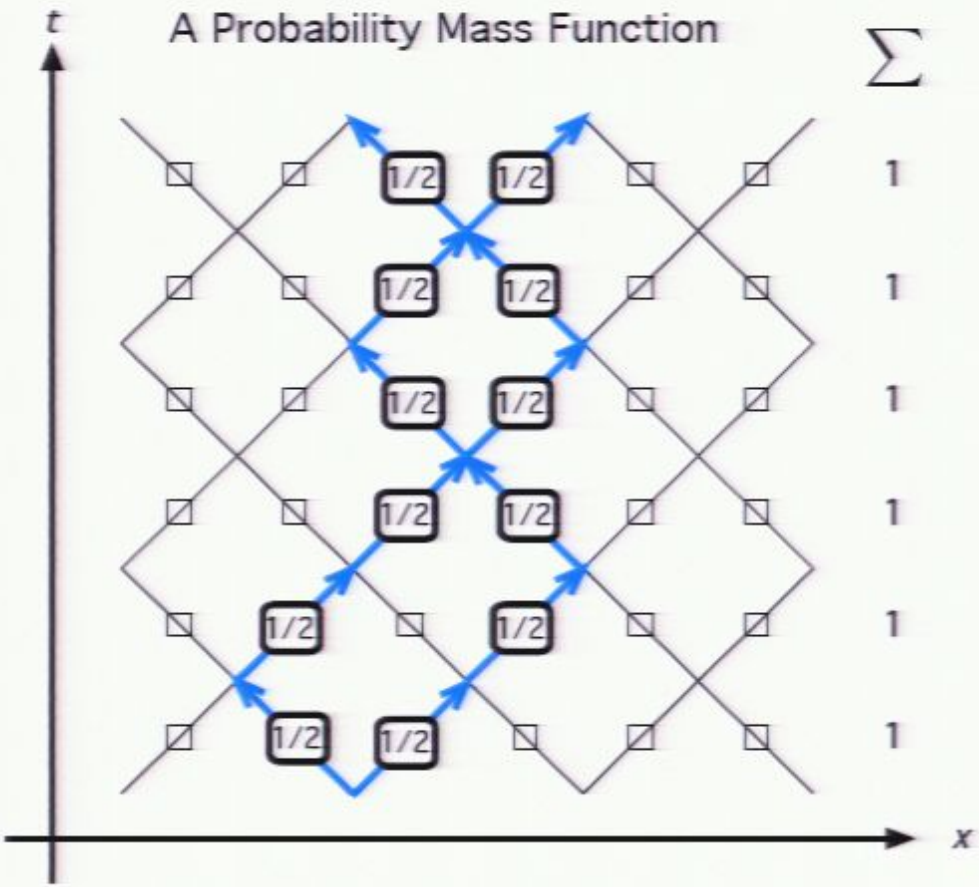

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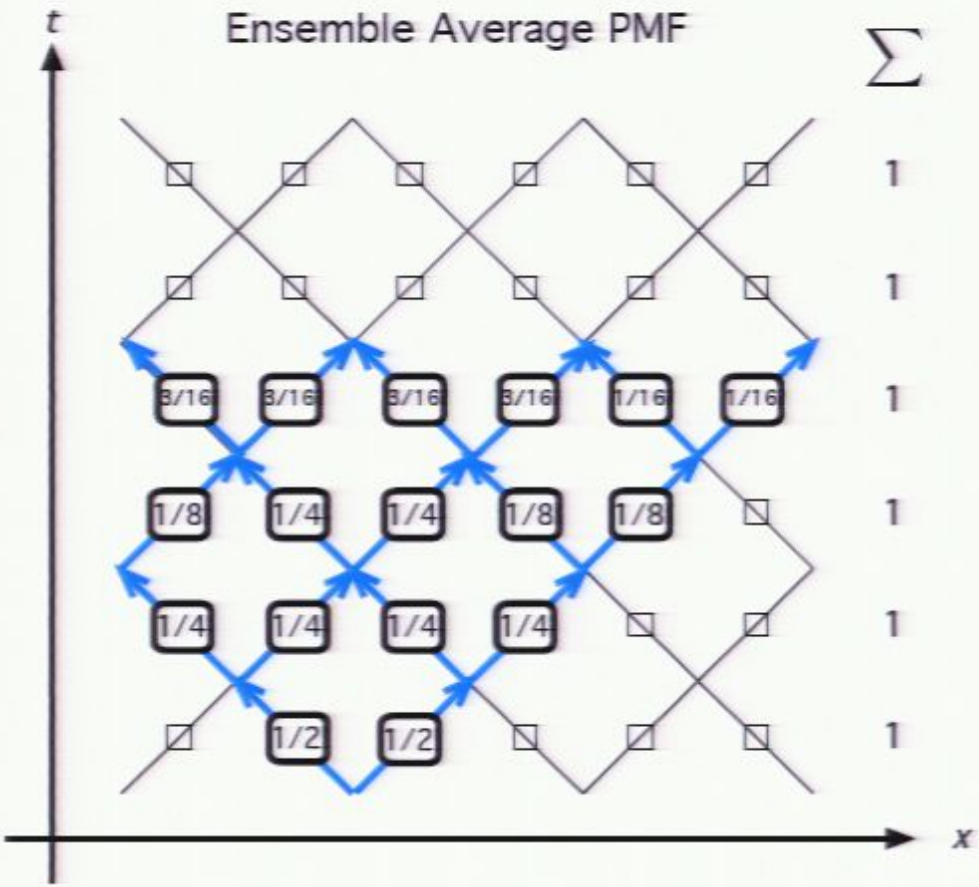

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- Paths on a lattice, lattice spacing ϵ. Direction change probability $\epsilon\, m$.
- Conservation of probability on the lattice gives:

$$
\begin{aligned}
W_+(y+\epsilon) &= (1-\epsilon m)\, W_+(y) + \epsilon m\, W_-(y) \\
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\end{aligned}
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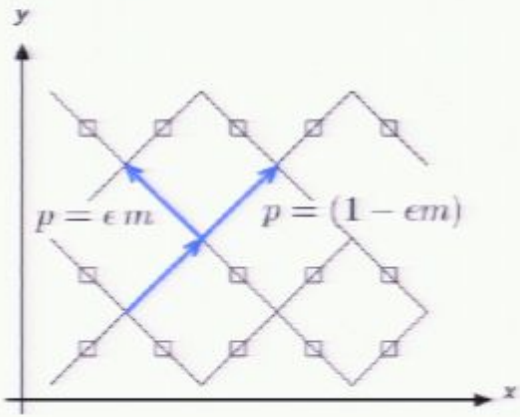

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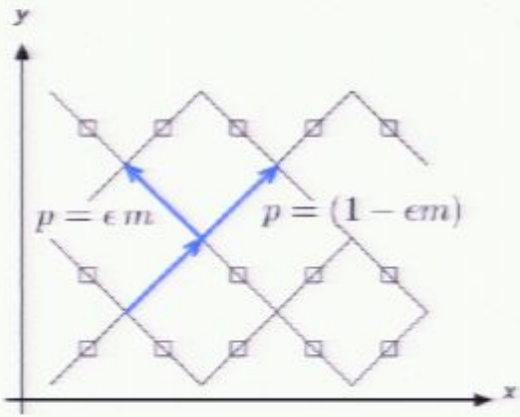

- $W_\pm$ is a probability mass function that is the expected value of the normalized sum of Bernoulli random variables. If we start all paths off at $y = 0$ in the $+$ state we get, in the continuum limit:

$$\mathbf{W}(y) = e^{-my} \begin{pmatrix} \cosh(my) \\ \sinh(my) \end{pmatrix}.$$

Kac Model with x-dependence

- With the x-dependence the difference equations are:

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\begin{aligned}
w_+(x, y+\epsilon) &= (1-\epsilon m)\, w_+(x-\epsilon, y) + \epsilon m\, w_-(x, y) \\
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\frac{\partial w_+}{\partial y} &= \frac{\partial w_+}{\partial x} - m\, w_+ + m\, w_- \\
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- or, using the Pauli matrices and writing $U = e^{-my} W$:

$$\frac{\partial \mathbf{U}}{\partial y} = \sigma_{\mathbf{z}} \frac{\partial \mathbf{U}}{\partial x} + m\, \sigma_{\mathbf{x}} \mathbf{U}$$

- Note that $\mathbf{U}$ *is just the expected value of sums of the Bernoulli random variable*. $\mathbf{U}$ just counts paths.

Counting tentative paths.

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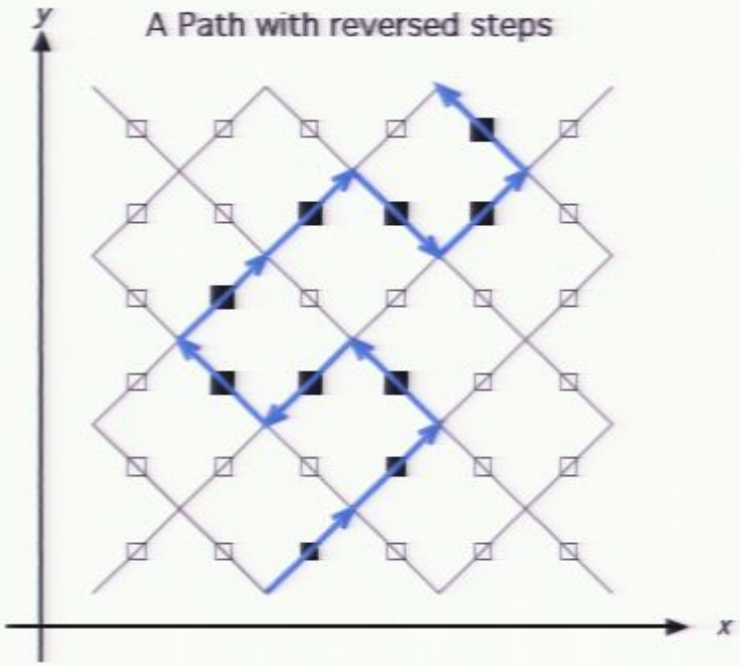

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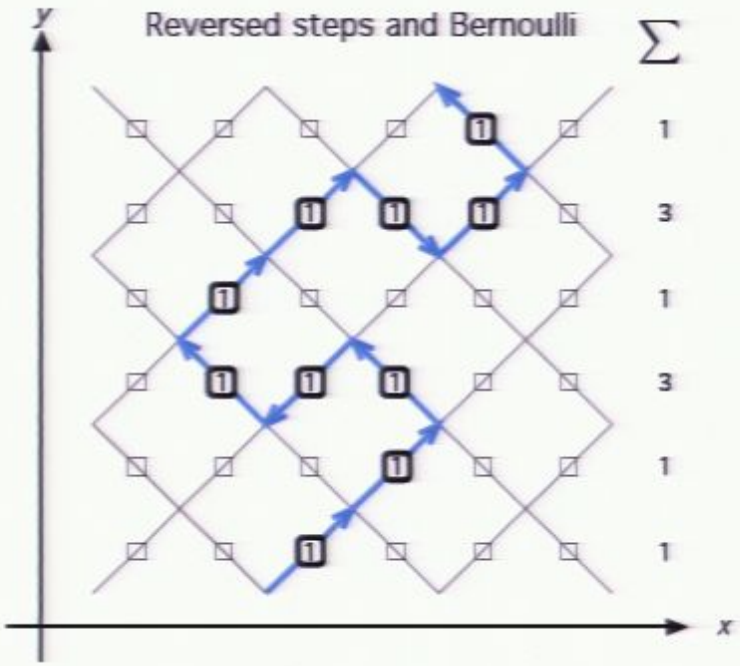

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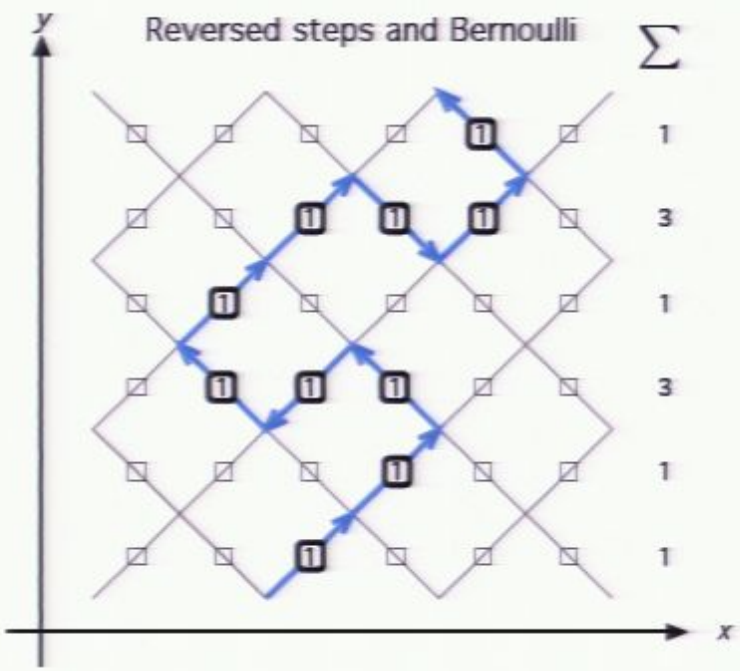

- Consider an (x, y) lattice with a discrete random walk with reversing steps.
- The Bernoulli sum of path links is y-dependent.
- Bernoulli cannot handle paths with reversing steps.

Counting with Anti-Bernoulli

The Bernoulli random variable X with $X \in \{0, 1\}$ cannot be used to count paths that reverse themselves. Instead, consider the random variable Y with $Y \in \{-1, 0, 1\}$. Here:

$$Y = \begin{cases} 1 & \text{link traversed in the } +y \text{ direction.} \\ 0 & \text{link not traversed} \\ -1 & \text{link traversed in the } -y \text{ direction.} \end{cases}$$

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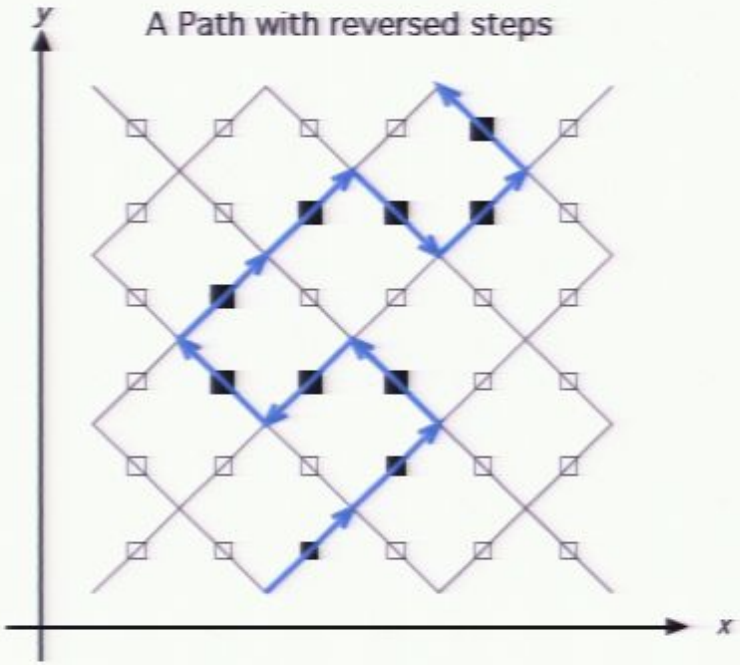
If a discrete path is (discretely) continuous from the minimum y to the maximum y, the Anti-Bernoulli variable takes care of reversed links.

Counting with Anti-Bernoulli

Here is a path with reversed link traversal.



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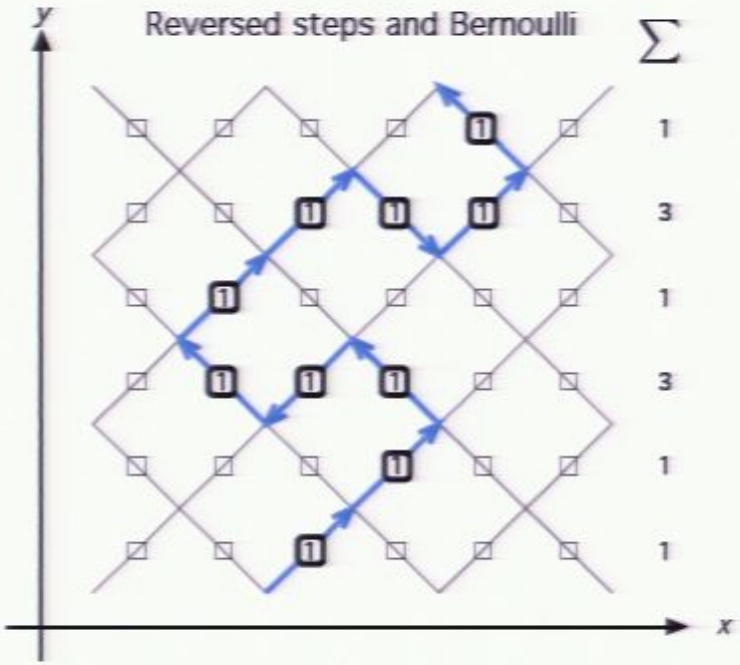

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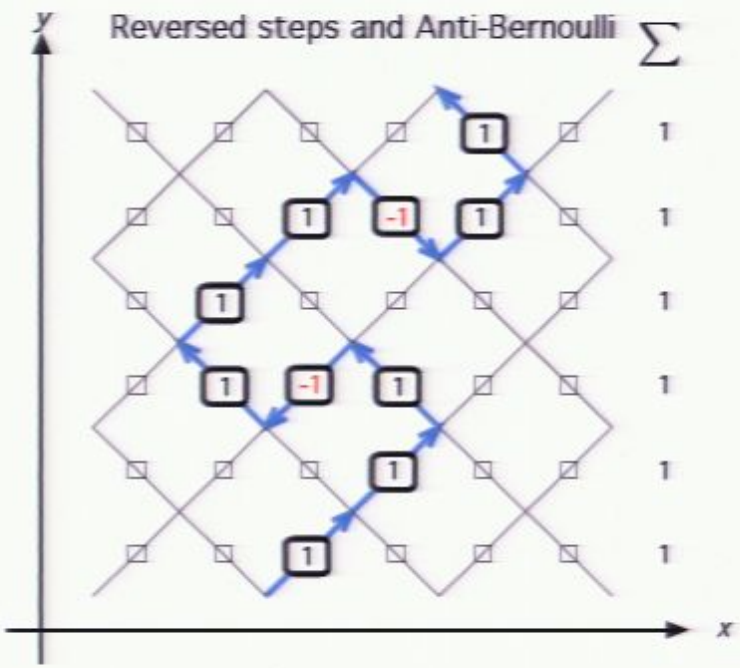

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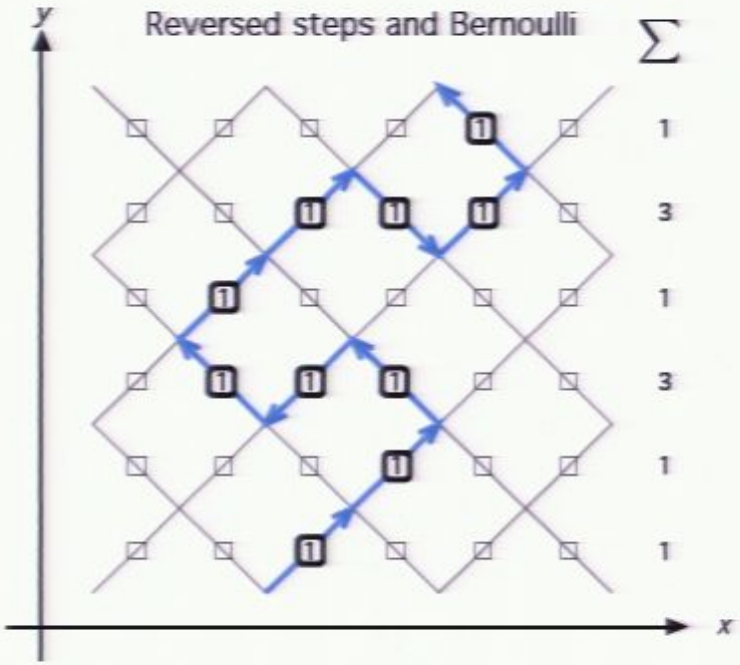

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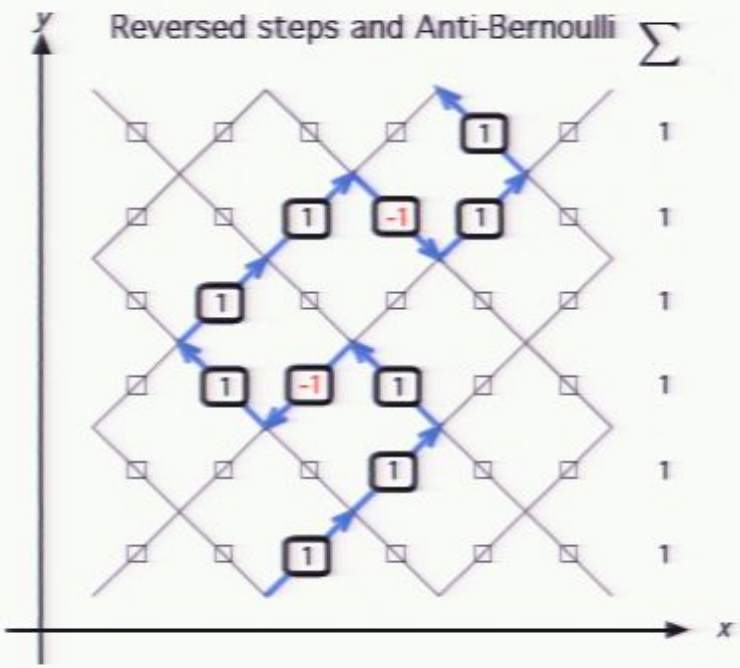

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Continuum Limit

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Continuum Limit

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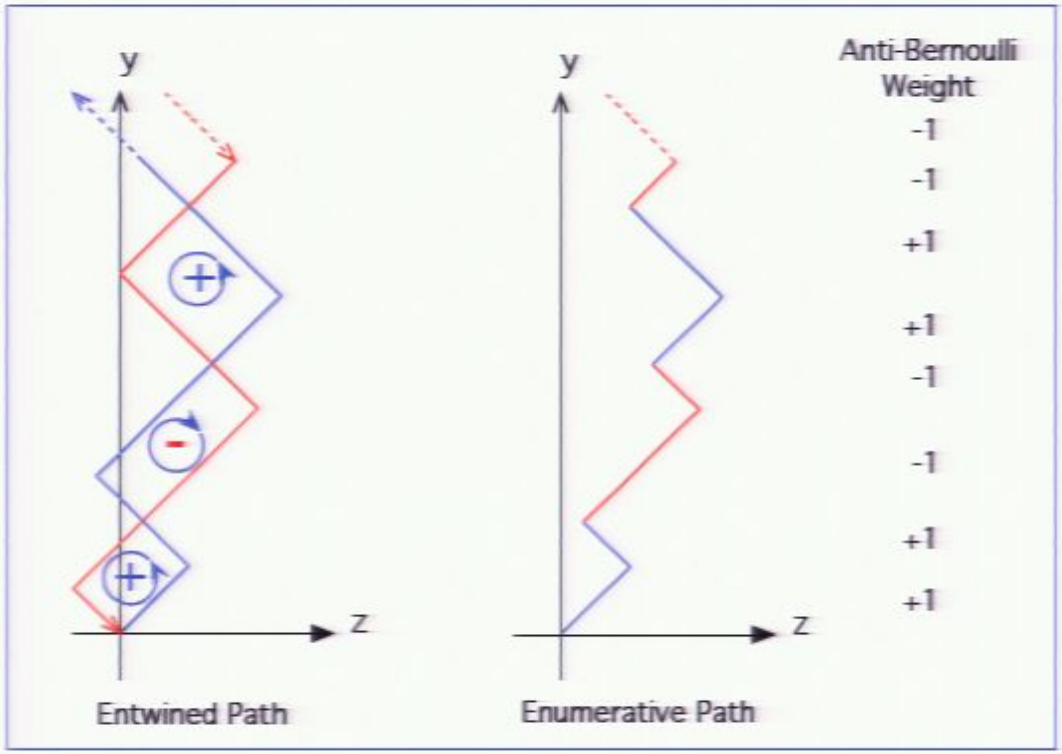
- For walks on a lattice with reversed links the expected value of Bernoulli random variables do not usually yield a continuous distribution with the properties of a PDF.
  However, for some walks with reversed links, the expected value of the Anti-Bernoulli random variable yields an oscillatory density, not a PDF! The following model is an example of this.

Entwined Path Example



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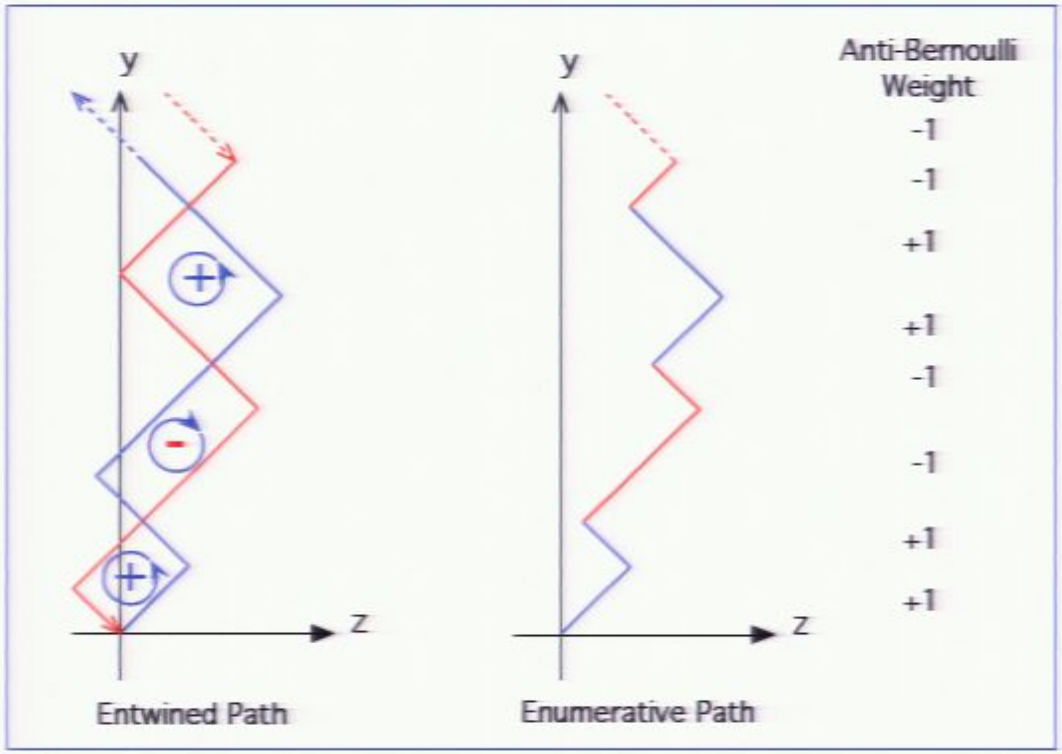

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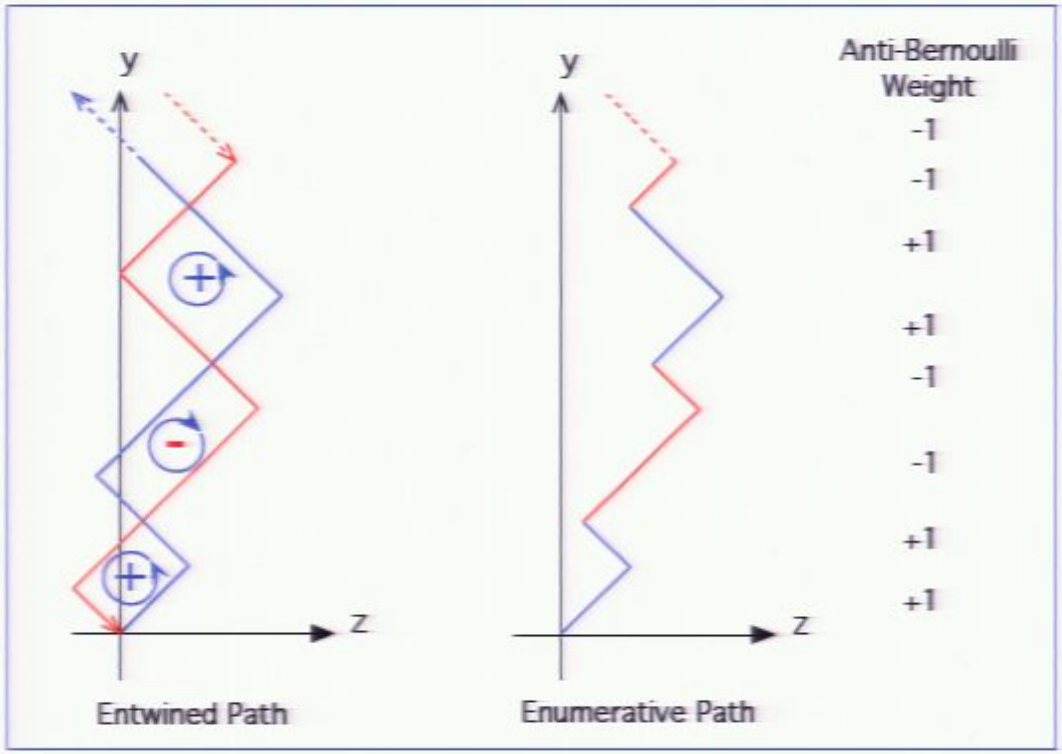

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- Sum over all paths on the right with Anti-Bernoulli RV's yields a 2-component Dirac Eqn. (4-component if other enumerative path kept.)

The Entwined Path Model of the Dirac Equation

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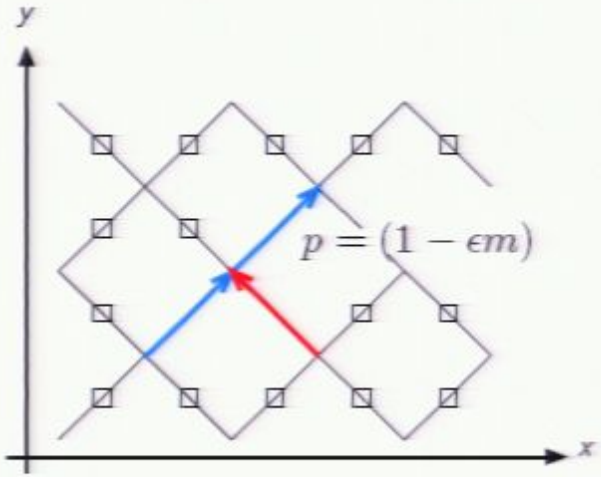
- Paths on a lattice, lattice spacing ϵ. Direction change probability $\epsilon\, m$.
- Continuity of enumerative paths on the lattice gives:

$$\begin{aligned} \Phi_+(y+\epsilon) &= (1-\epsilon m)\,\Phi_+(y) - \epsilon\, m\,\Phi_-(y) \\ \Phi_-(y+\epsilon) &= (1-\epsilon m)\,\Phi_-(y) + \epsilon m\,\Phi_+(y) \end{aligned}$$



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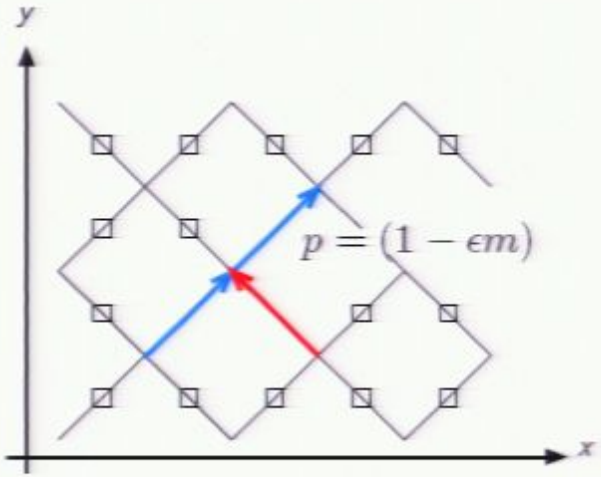

- $\Phi_\pm$ is a mass function that is the expected value of the normalized sum of Anti-Bernoulli random variables. It is *not a probability mass function* since it is not non-negative.

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$$\phi(y) = e^{-my} \begin{pmatrix} \cos(my) \\ \sin(my) \end{pmatrix}$$

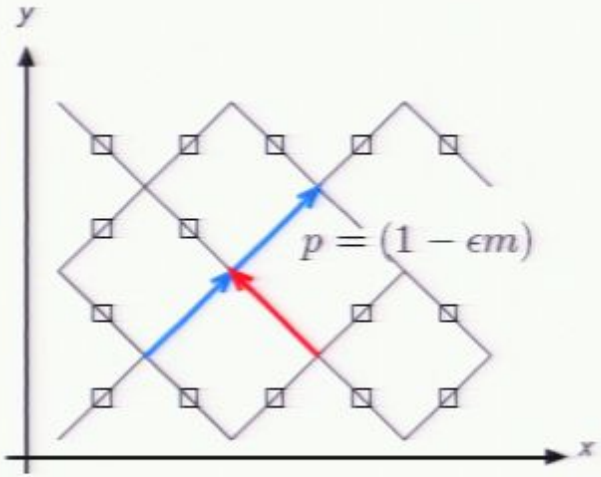
Although we no longer have a PDF, note the two-component density has a rotational feature to its equilibration!

The Entwined Path Model of the Dirac Equation

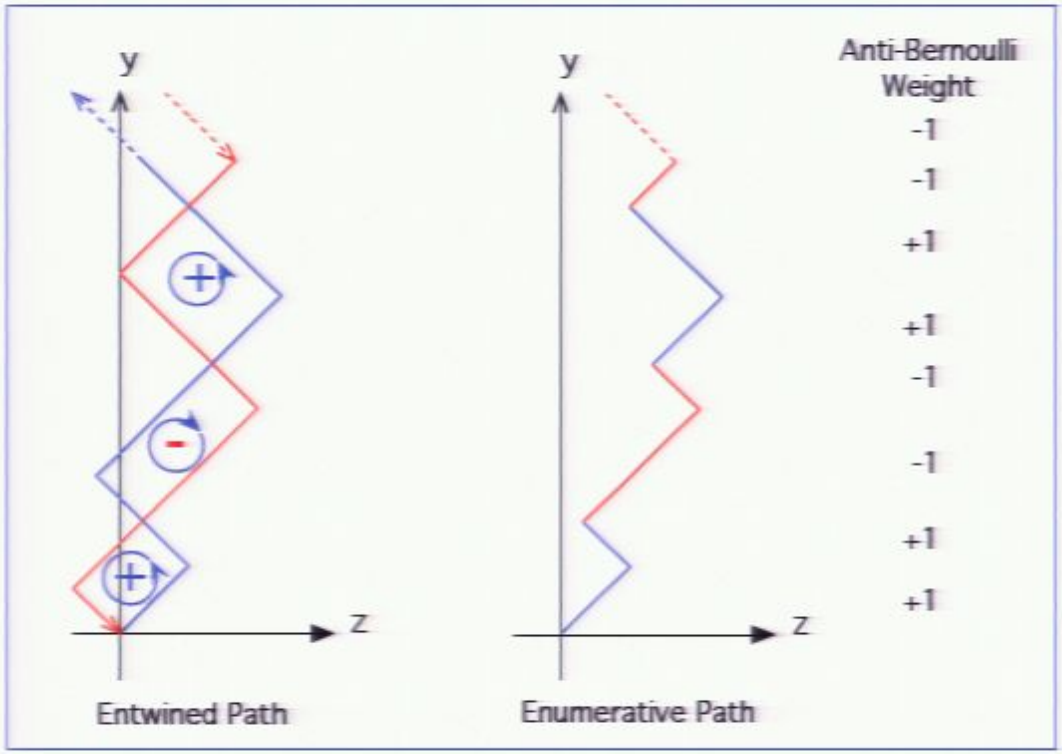
- Using the Anti-Bernoulli random variable $Y \in \{-1, 0, 1\}$ for counting classical paths leads to the formation of density functions under appropriate conditions.

- Paths on a lattice, lattice spacing ϵ. Direction change probability $\epsilon\, m$.
- Continuity of enumerative paths on the lattice gives:

$$\begin{aligned} \Phi_+(y+\epsilon) &= (1-\epsilon m)\,\Phi_+(y) - \epsilon\, m\,\Phi_-(y) \\ \Phi_-(y+\epsilon) &= (1-\epsilon m)\,\Phi_-(y) + \epsilon m\,\Phi_+(y) \end{aligned}$$



Entwined Path Example



- Forward and reversed paths are 'Entwined'.
- Paths cannot be counted by regular Bernoulli.
- Sum over all paths on the right with Anti-Bernoulli RV's yields a 2-component Dirac Eqn. (4-component if other enumerative path kept.)

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$$\begin{aligned} \frac{\partial \phi_+}{\partial y} &= \frac{\partial \phi_+}{\partial x} - m\,\phi_+ - m\,\phi_- \\ \frac{\partial \phi_-}{\partial y} &= -\frac{\partial \phi_-}{\partial x} - m\,\phi_- + m\,\phi_+ \end{aligned}$$

If we write $\phi_\pm = e^{-imt}\psi_\pm$, $\Psi = \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}$, the above becomes:

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Dirac Wavefunctions through Entwined Paths

	Classical	Quantum
Status	Phenomenology	~~Fundamental~~ Phenomenology
Ontology	Kinetic Theory	~~Unknown~~ EPs
Counting Process	Yes	~~No~~ Yes
Uncertainty Principle	Yes	Yes
Special Relativity	No	Yes
Complex Numbers	No	Yes
Quantum scale physics	No	Yes
Bridge	non-reversing paths	reversing paths

Table: The partial differential equations from the perspective of classical statistical mechanics and classical path counting. The Quantum equations are easily incorporated if reversing paths are allowed. The quantum context is invoked with a label change, $\ldots y \to t$.

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Answer

When we think of y as a spatial variable, the reversible paths are easily visualized and the necessity of the Anti-Bernoulli random variable is quite obvious. In the quantum context y is macroscopic time and the implication is that for the Dirac equation to actually appear, a particle has to traverse a spacetime region multiple times!

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The derivation presupposes a 'free boundary' at large values of y. The Dirac equation propagates the 'initial conditions' subject to this supposition. If there is an observation at large y this will change the large-y boundary condition that will, in turn have to 'equilibrate' with the initial conditions via the underlying stochastic process. The initial conditions are always within the past light-cone however.

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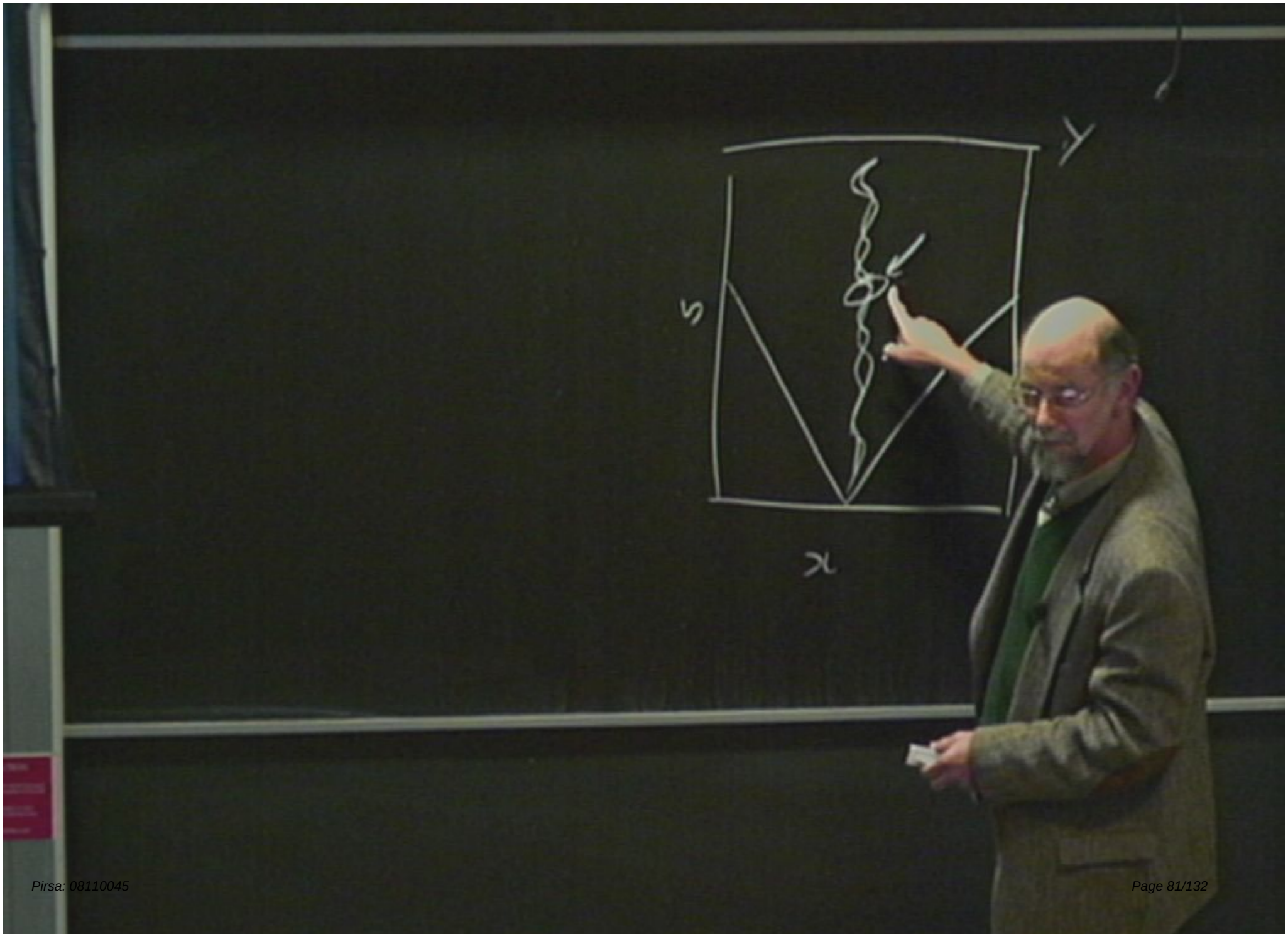
y
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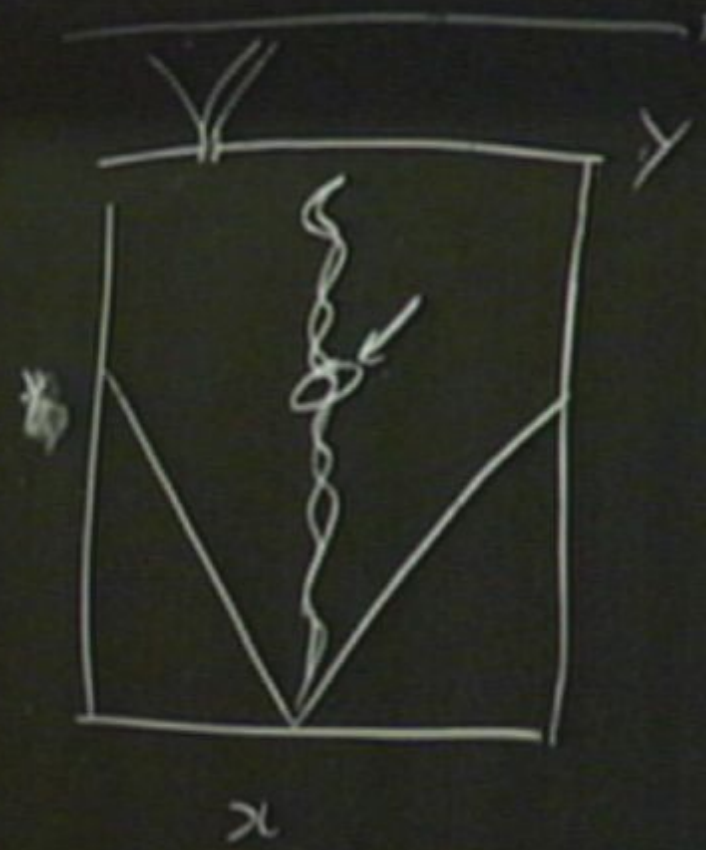
Y
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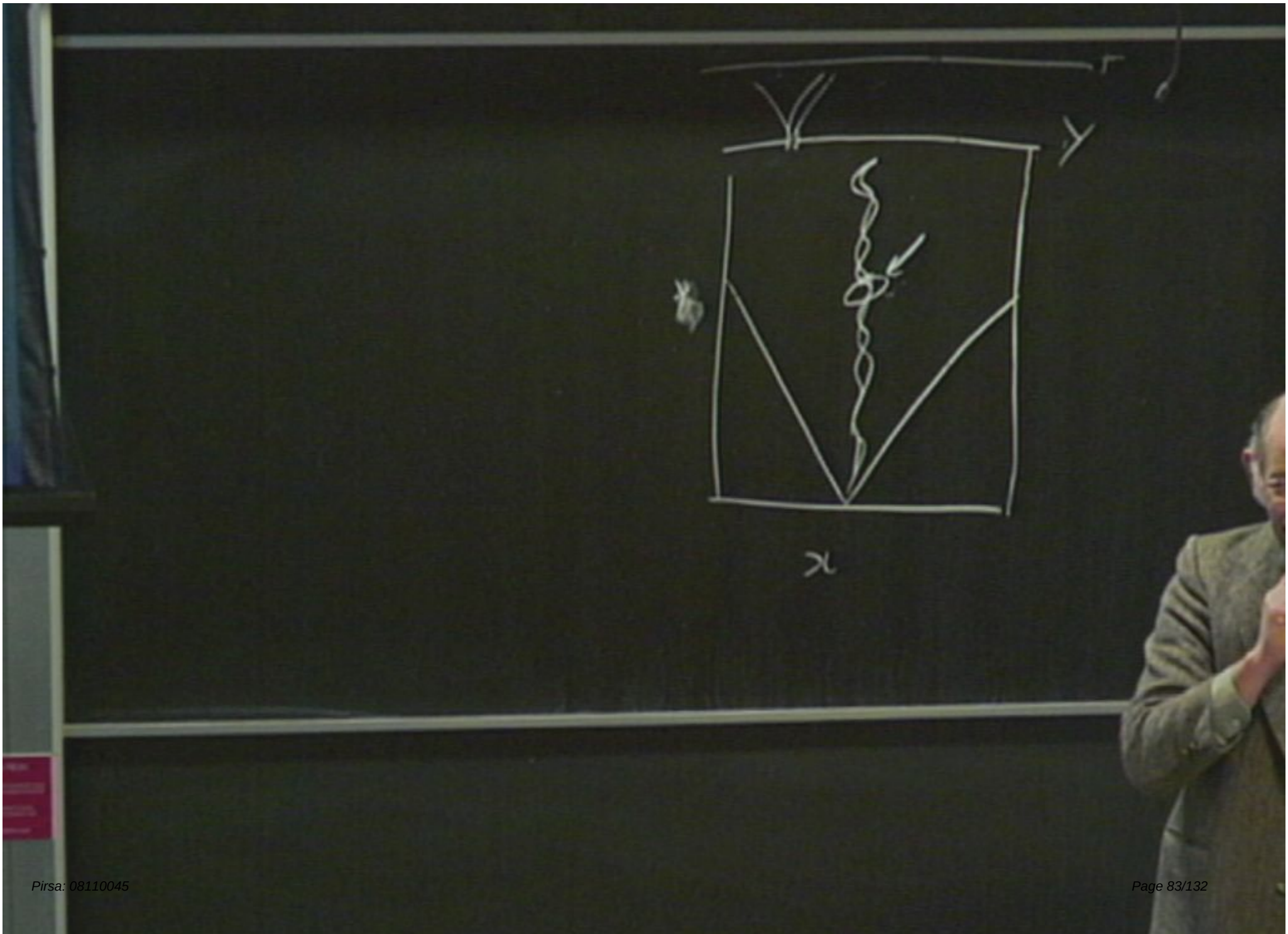
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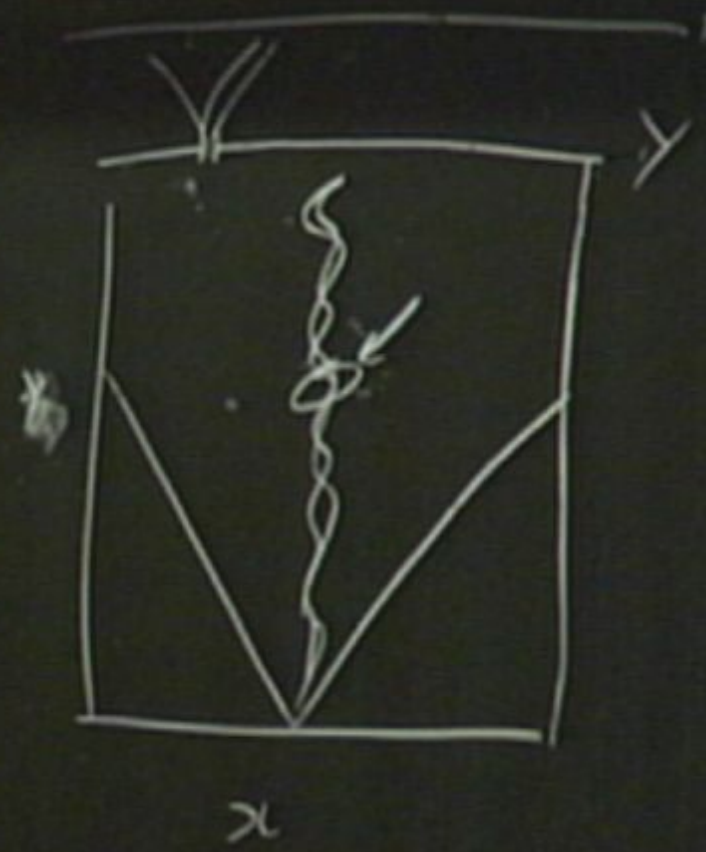
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If two 'particles' are coupled by initial conditions then even if the particles are 'observed' at space-like separations, the measurements communicate through the initial conditions. They are entangled.

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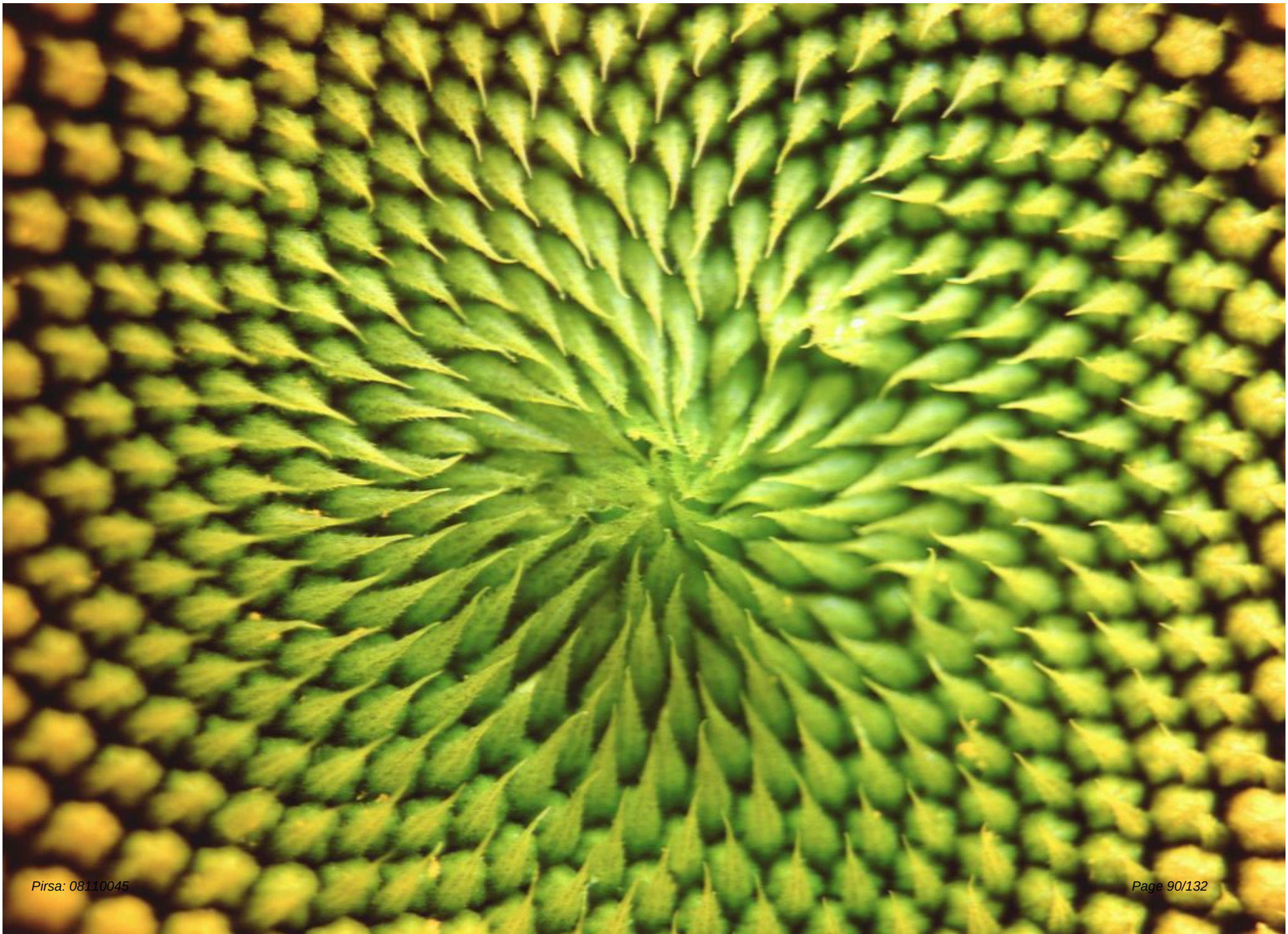

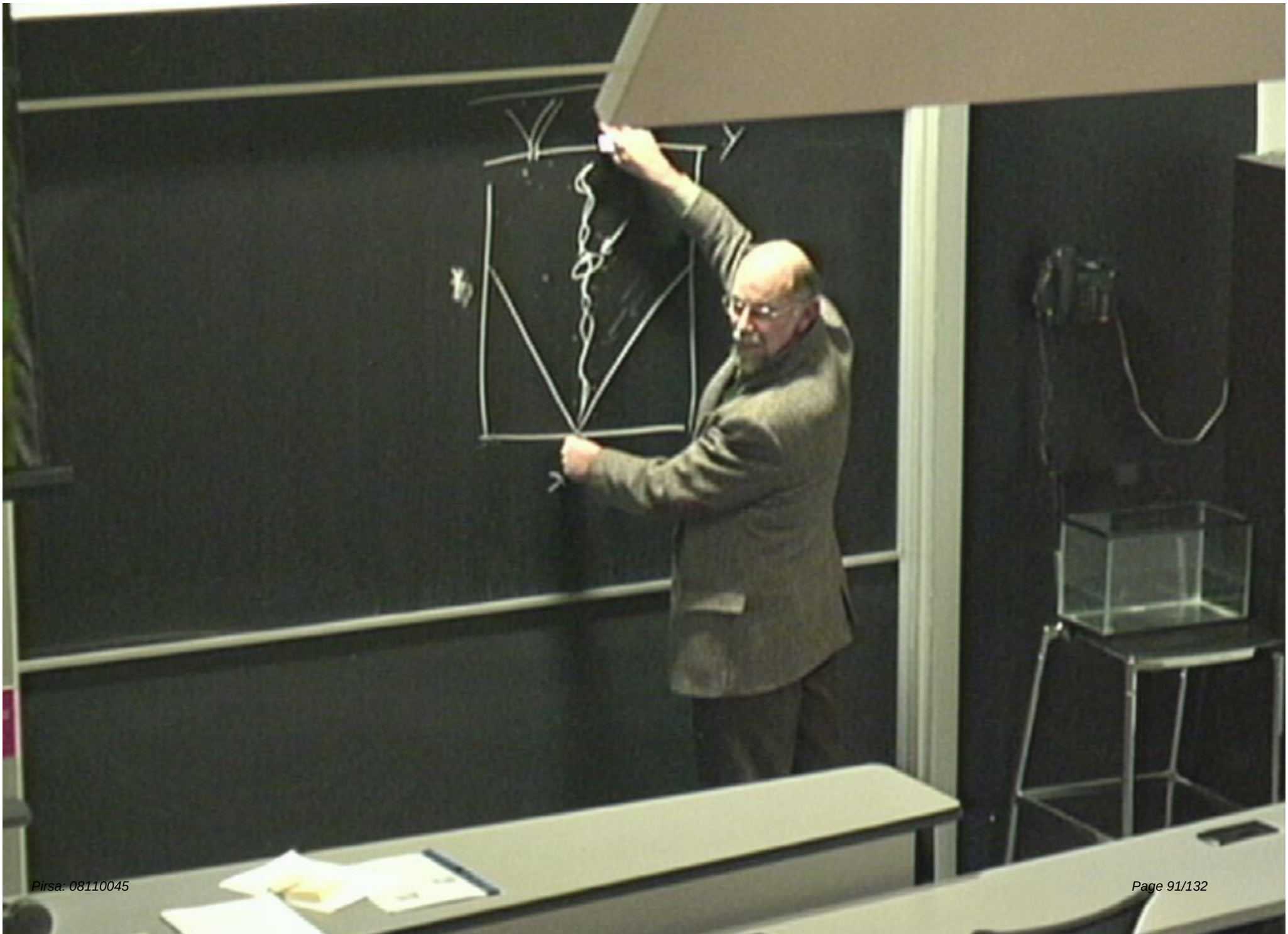

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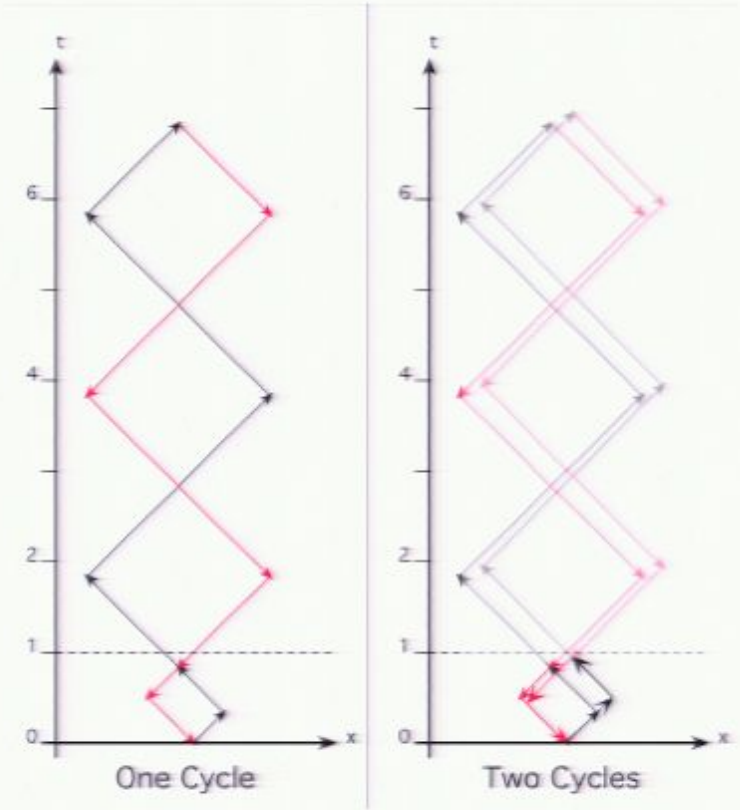
Answer

It turns out that you do not need a stochastic process 'directing traffic' over the whole spacetime area. You only need it in a small region as an initial condition. The rest of the process can be completely deterministic! This removes the exponential decay.

Deterministic Propagation



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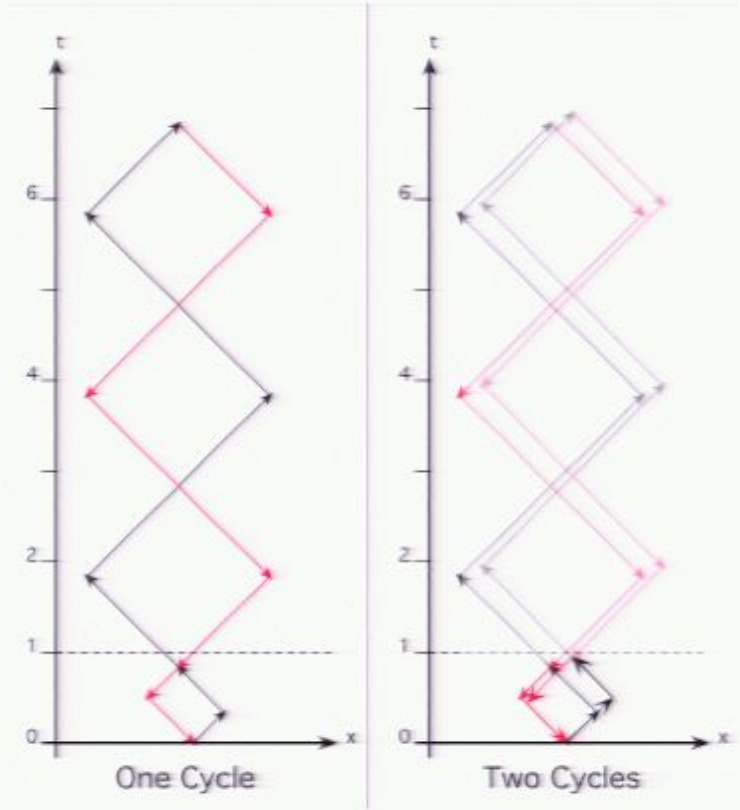
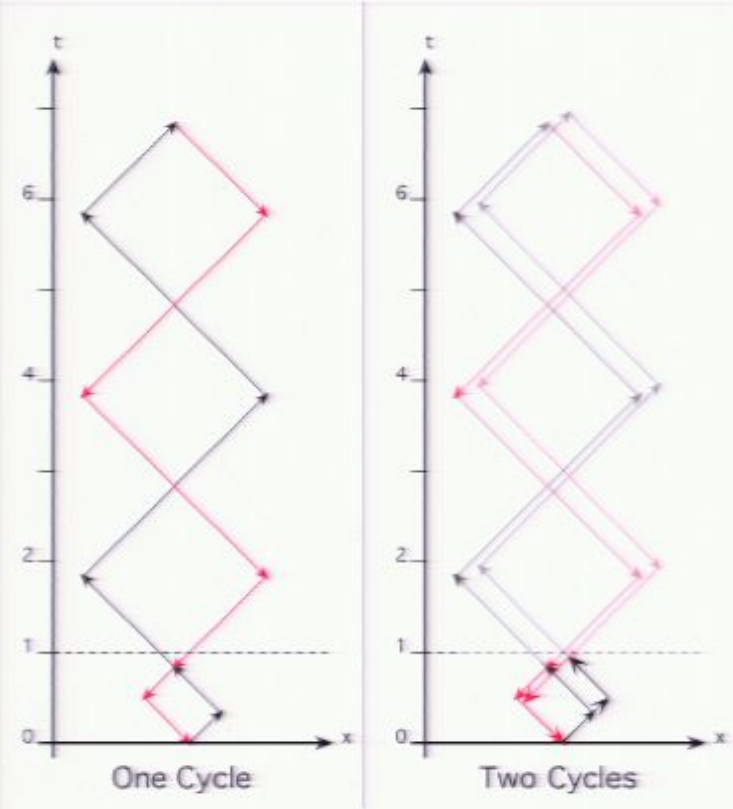
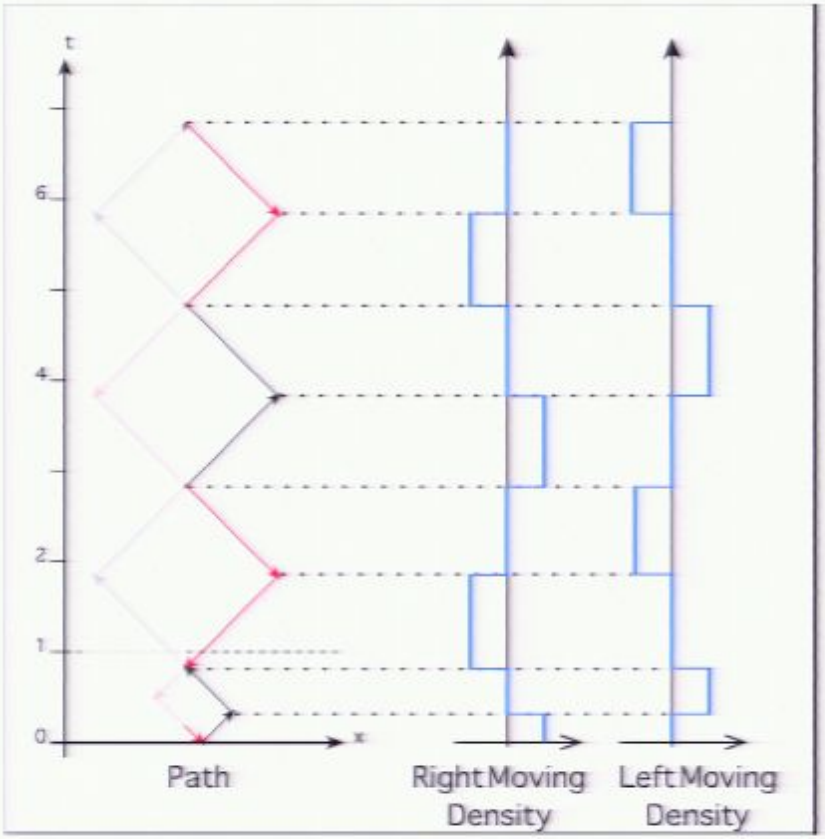


- Each path returns to the origin leaving a trail of Anti-Bernoulli RV's that count paths.
- The stochastic process cycles repeatedly from the origin building up a density of sums of the AB random variable.

Component Simulation





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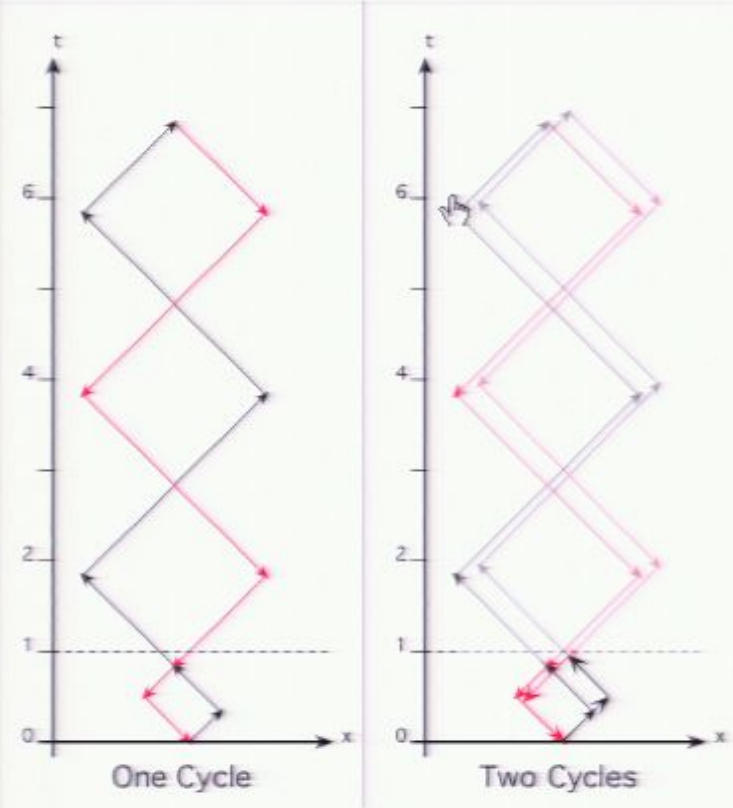

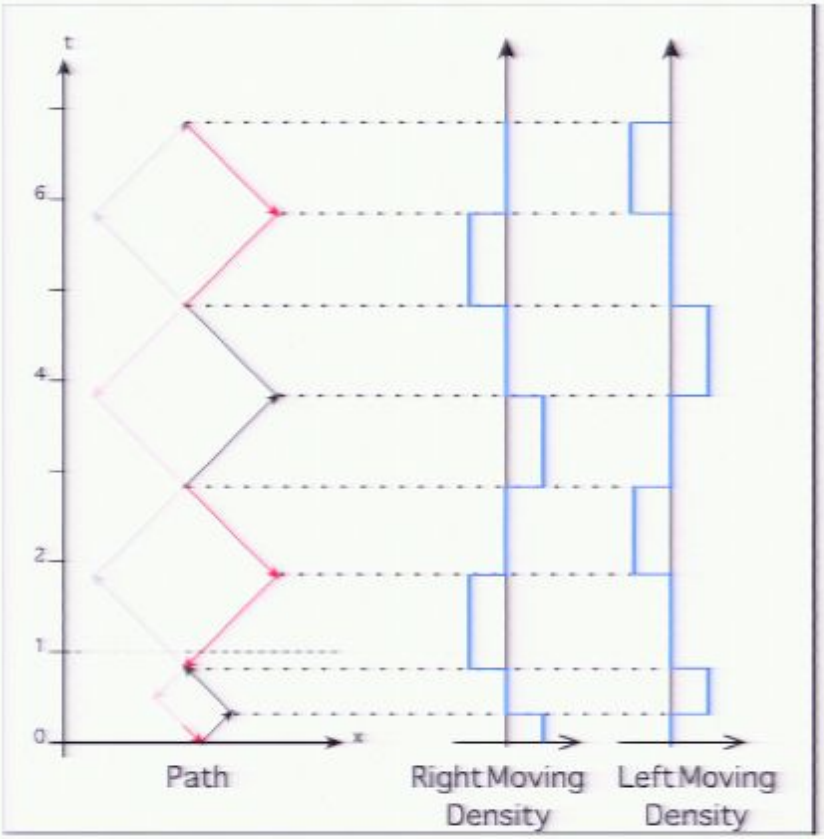

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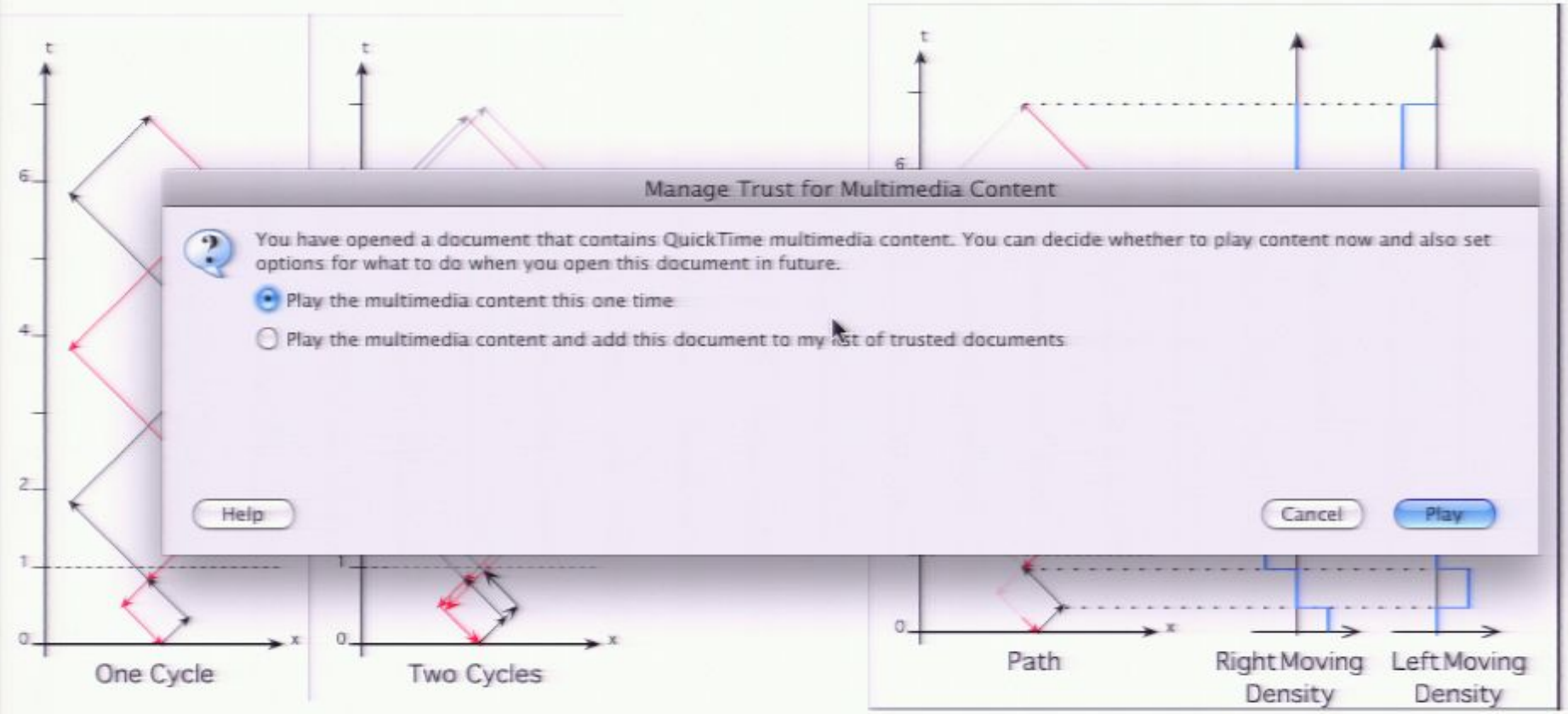

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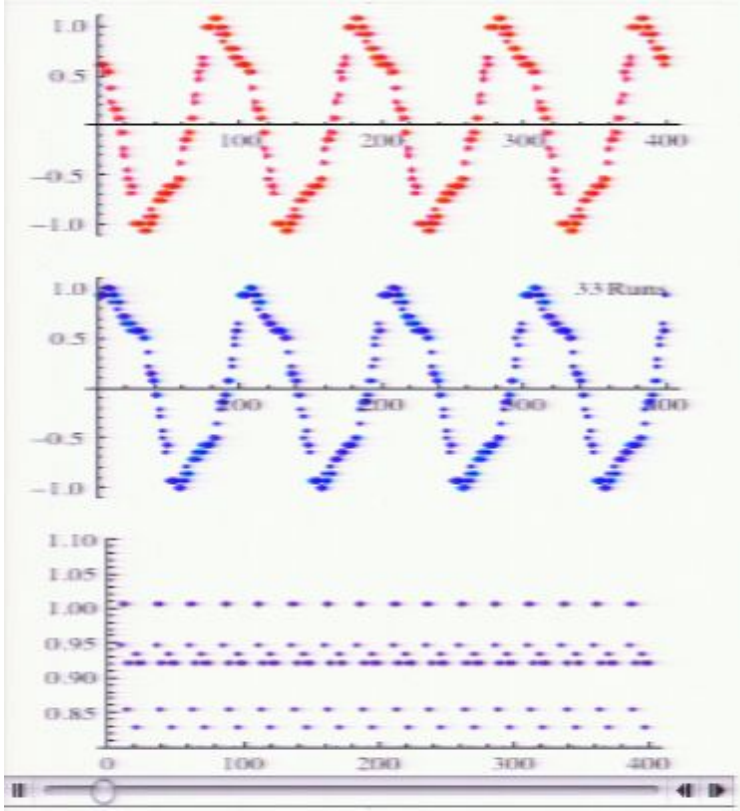

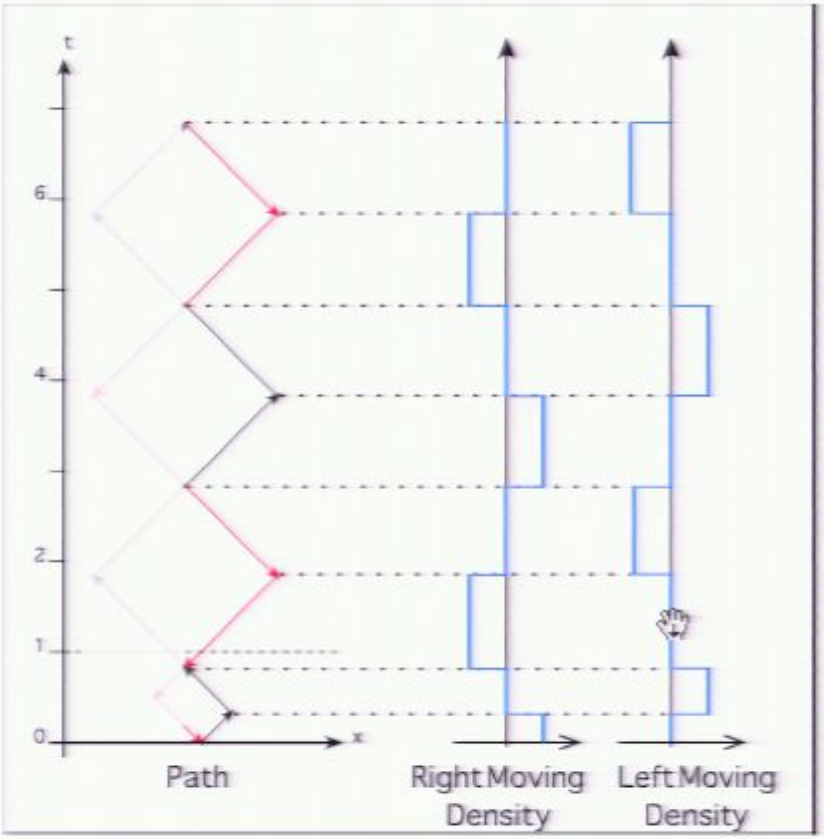

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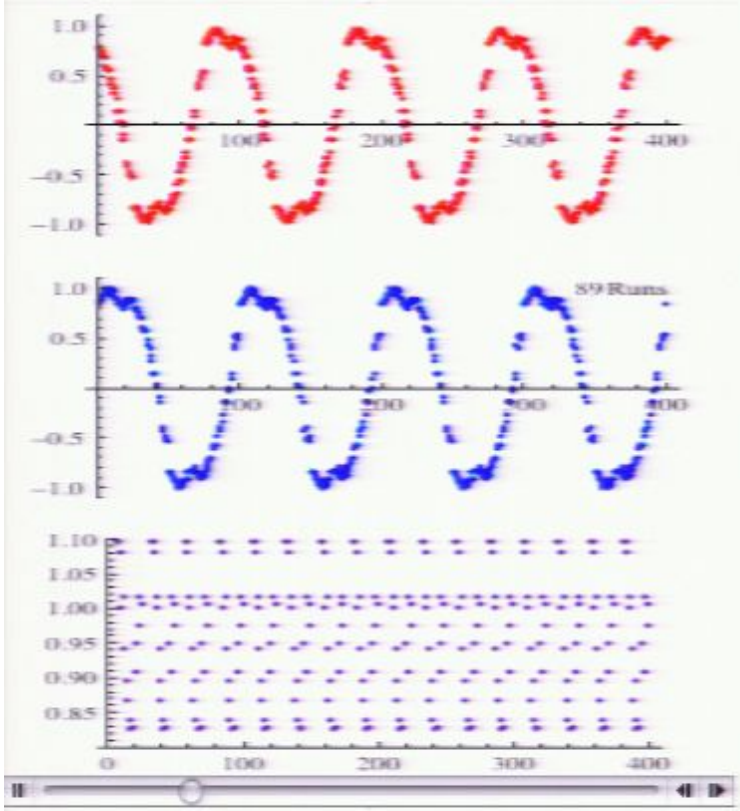

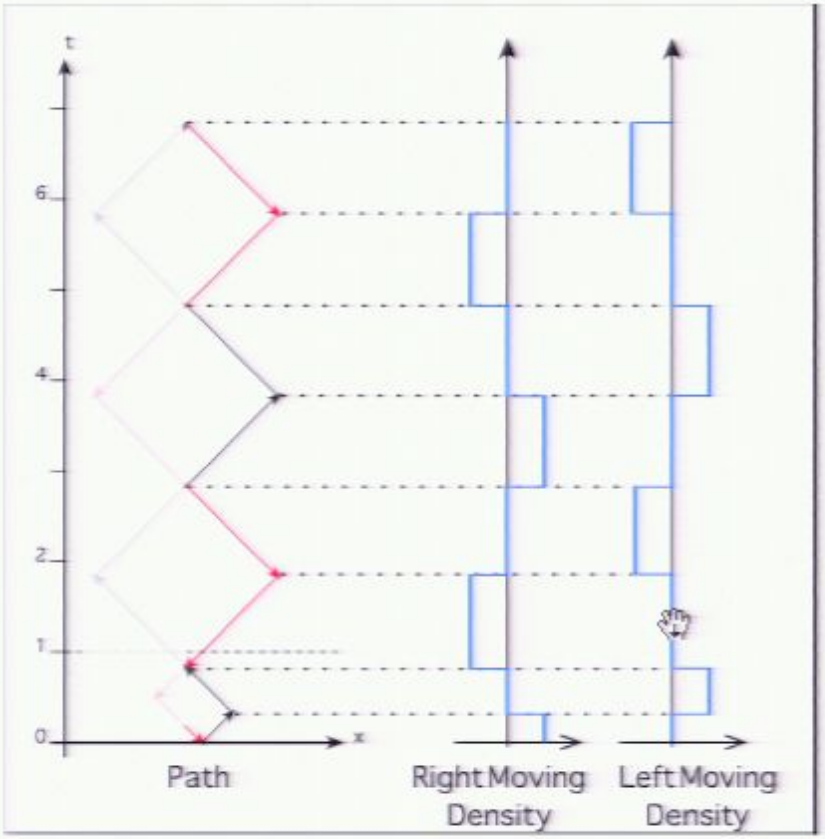

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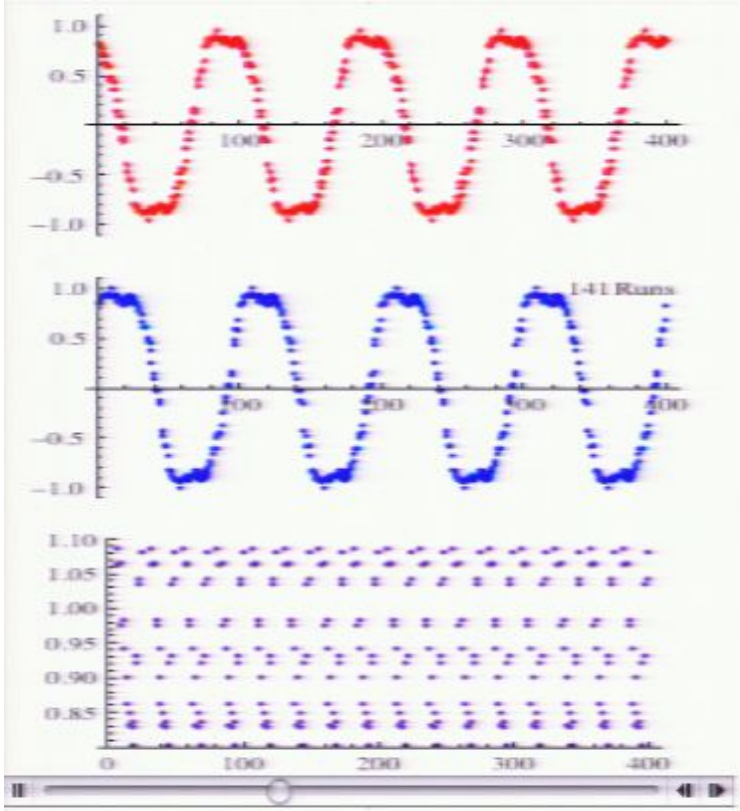

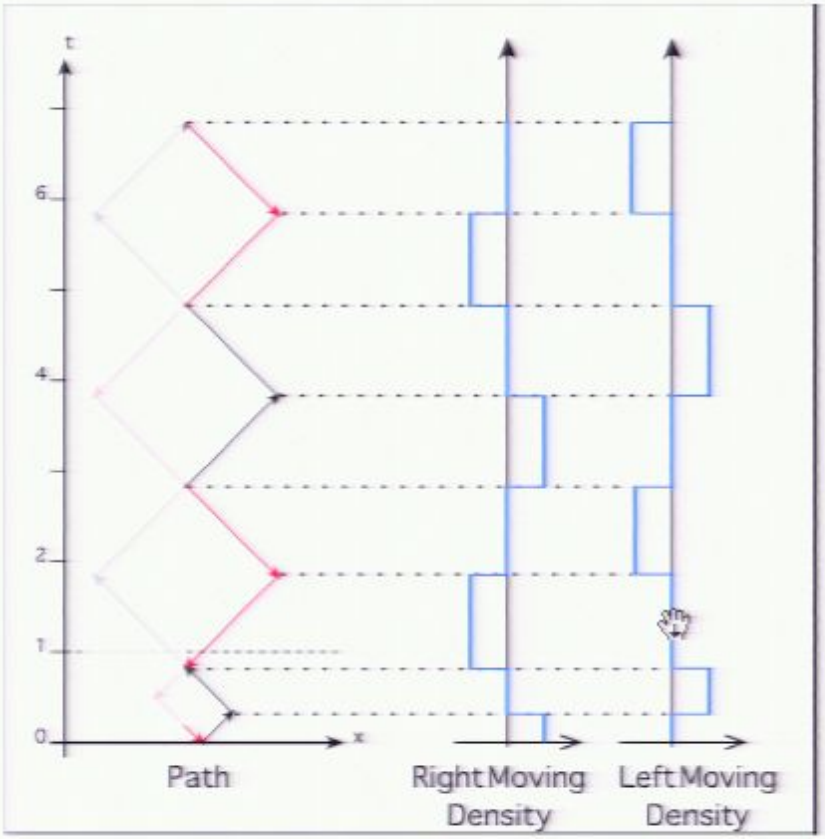

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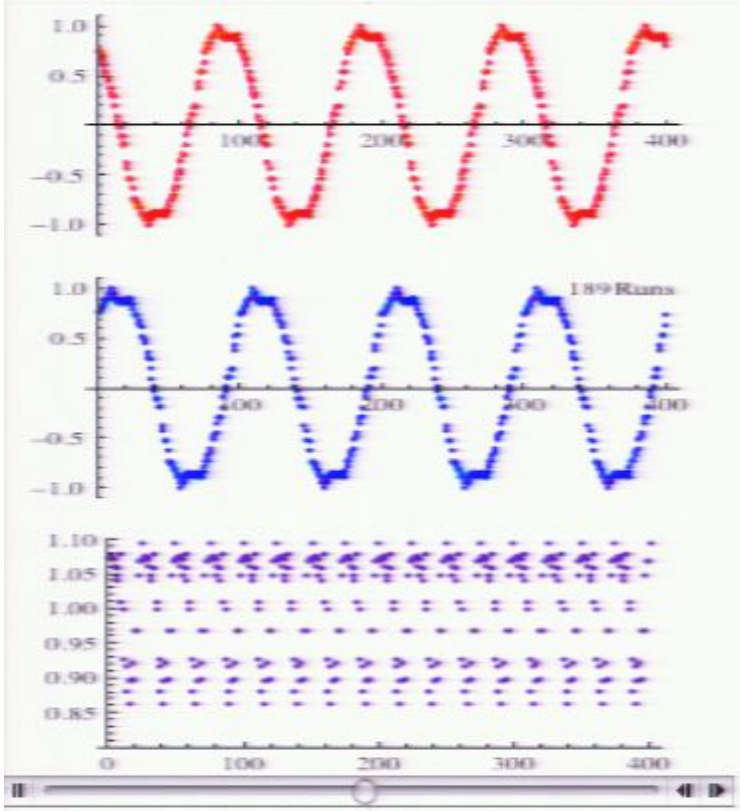

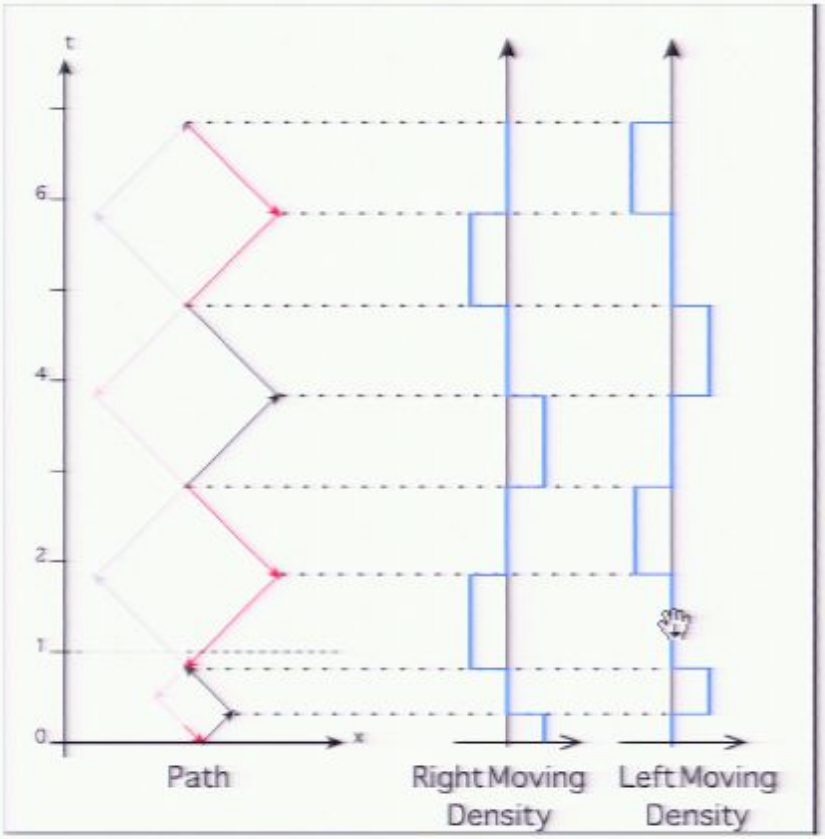

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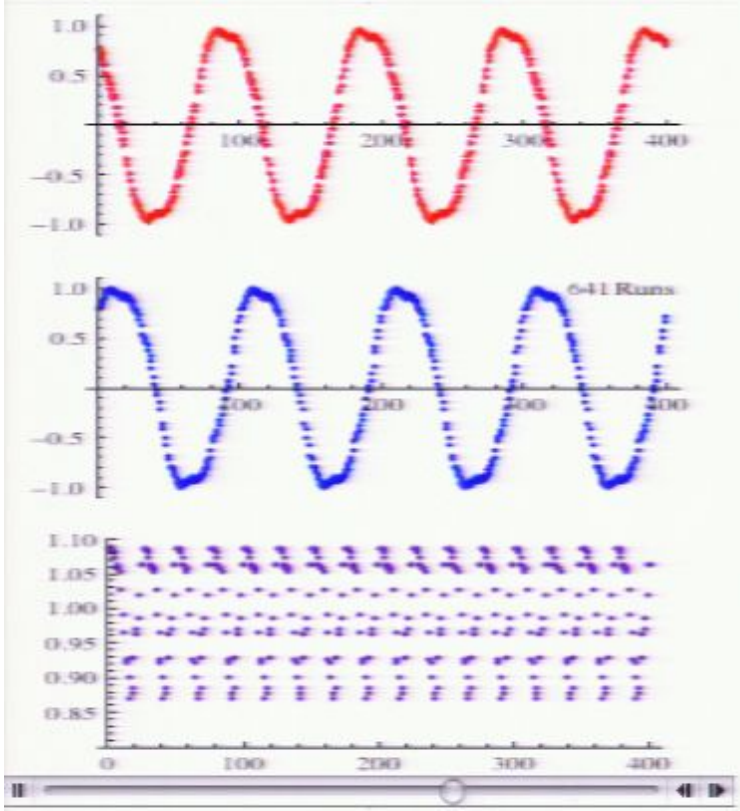

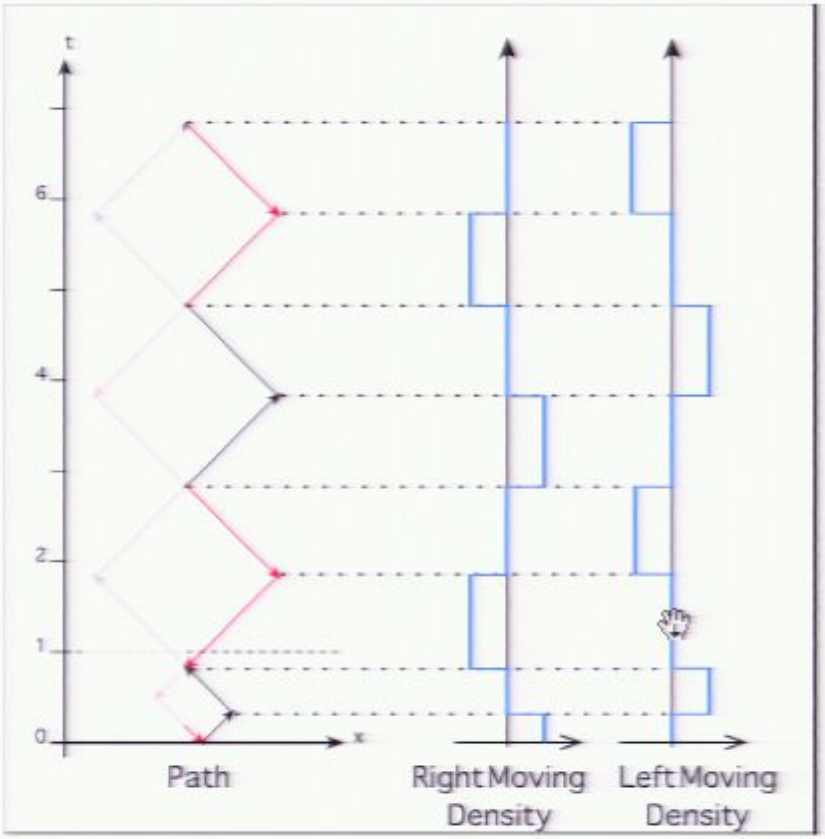

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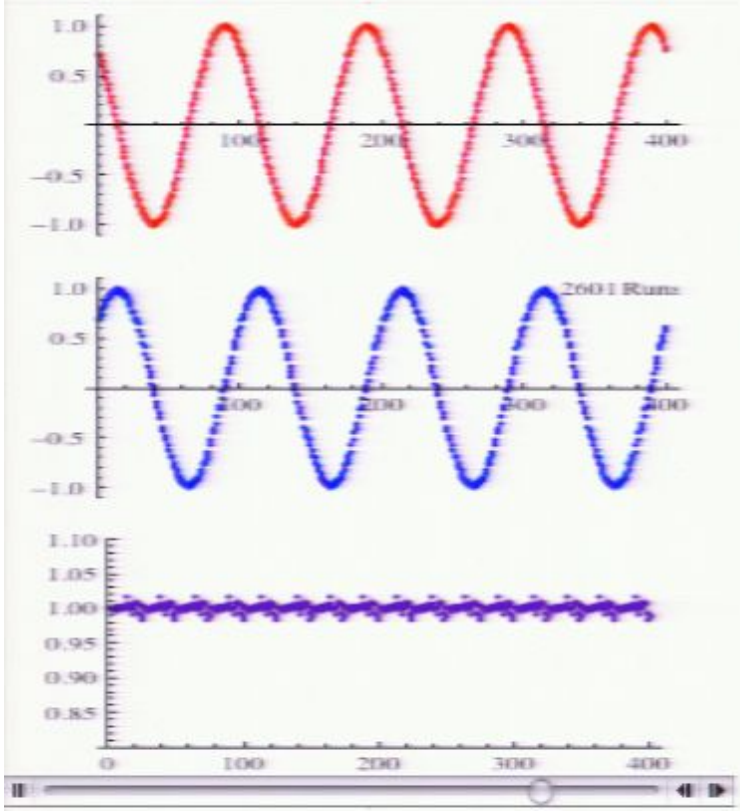

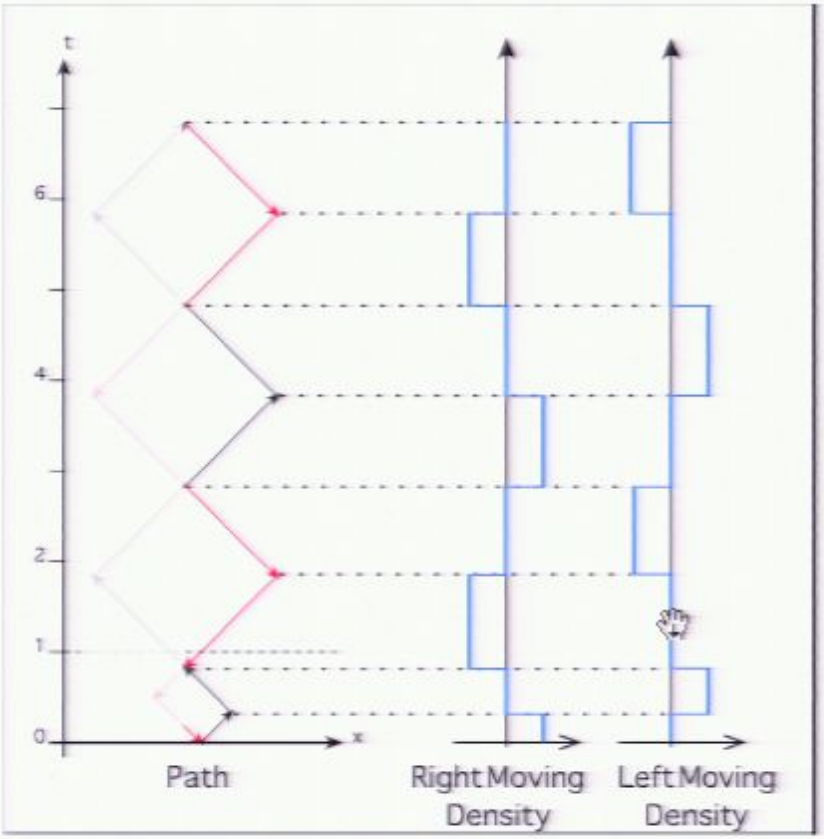

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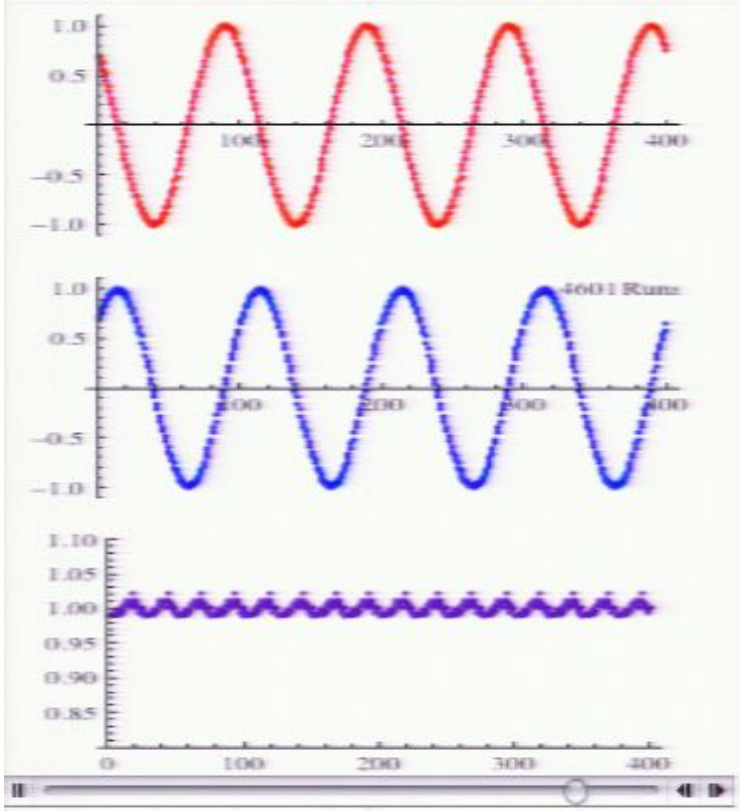

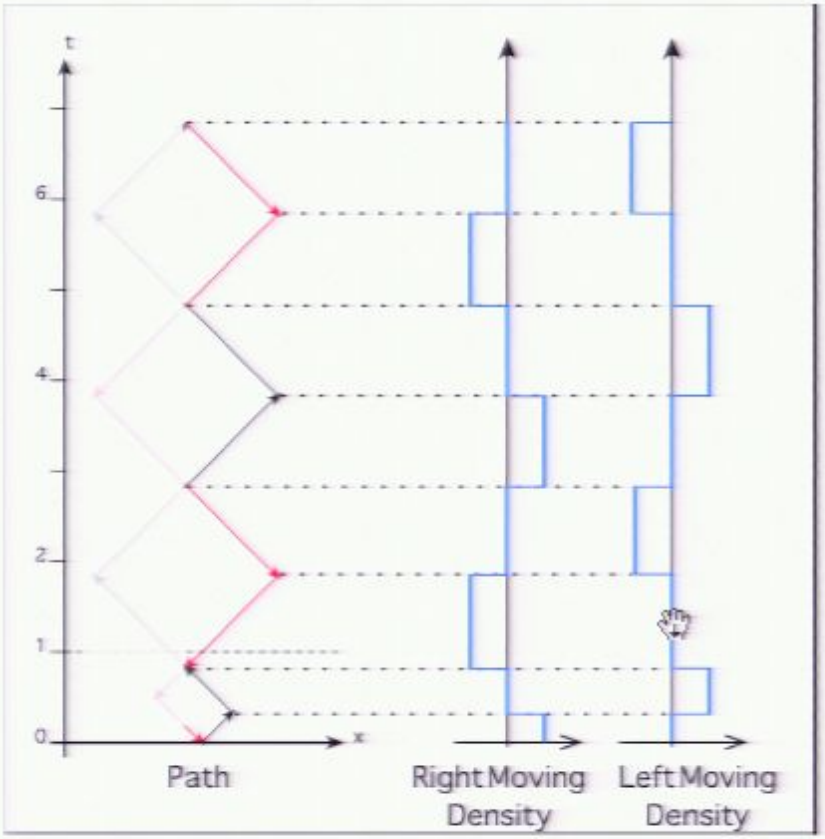

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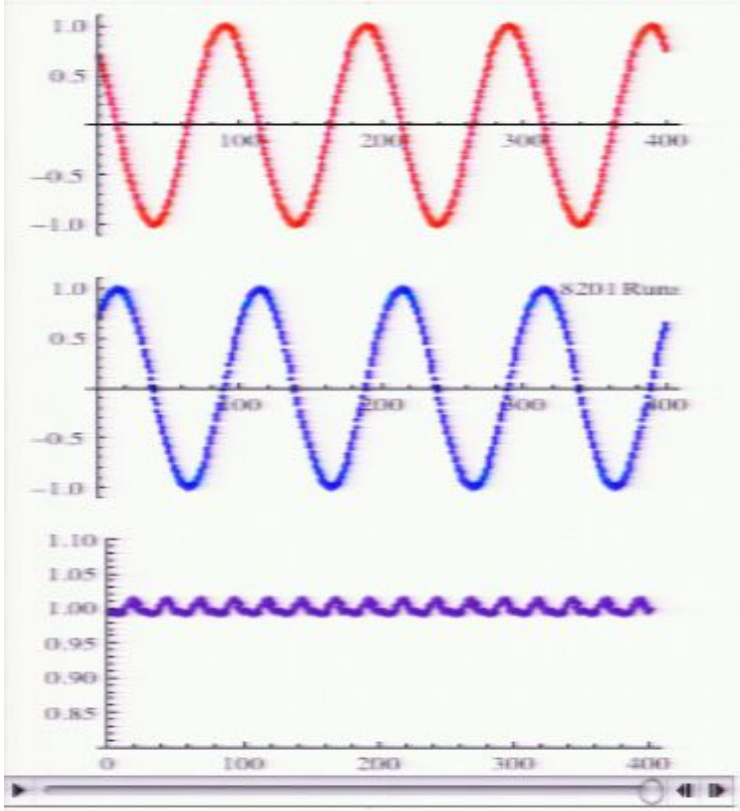
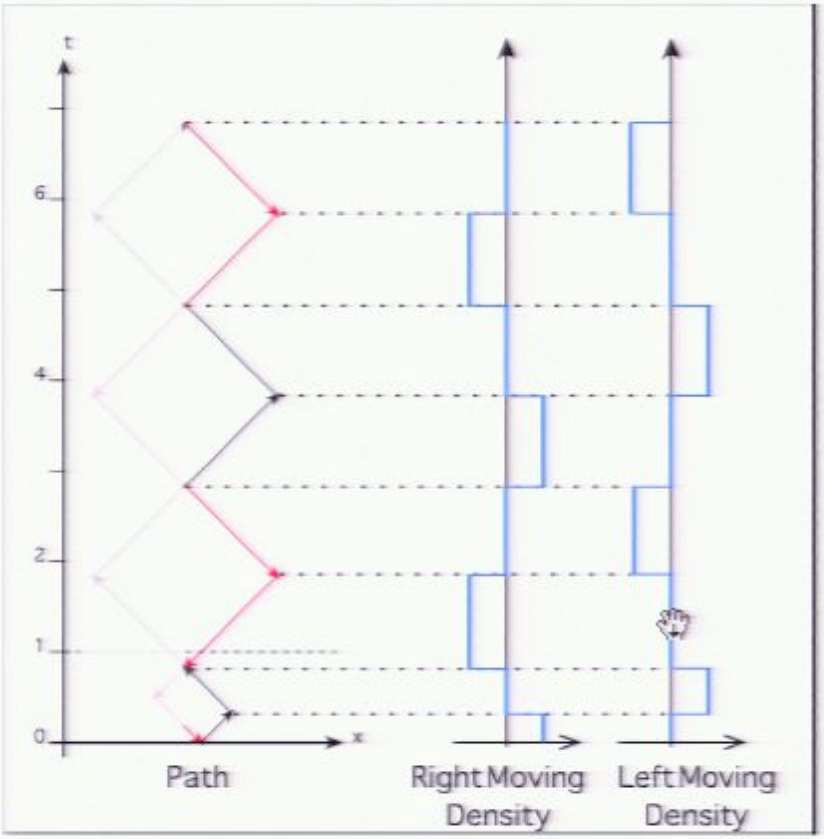
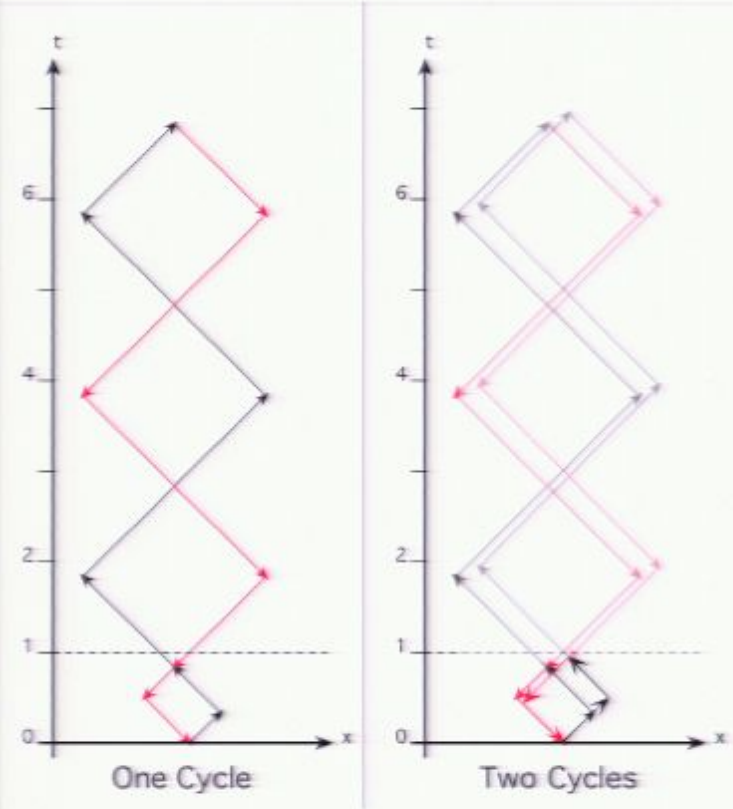
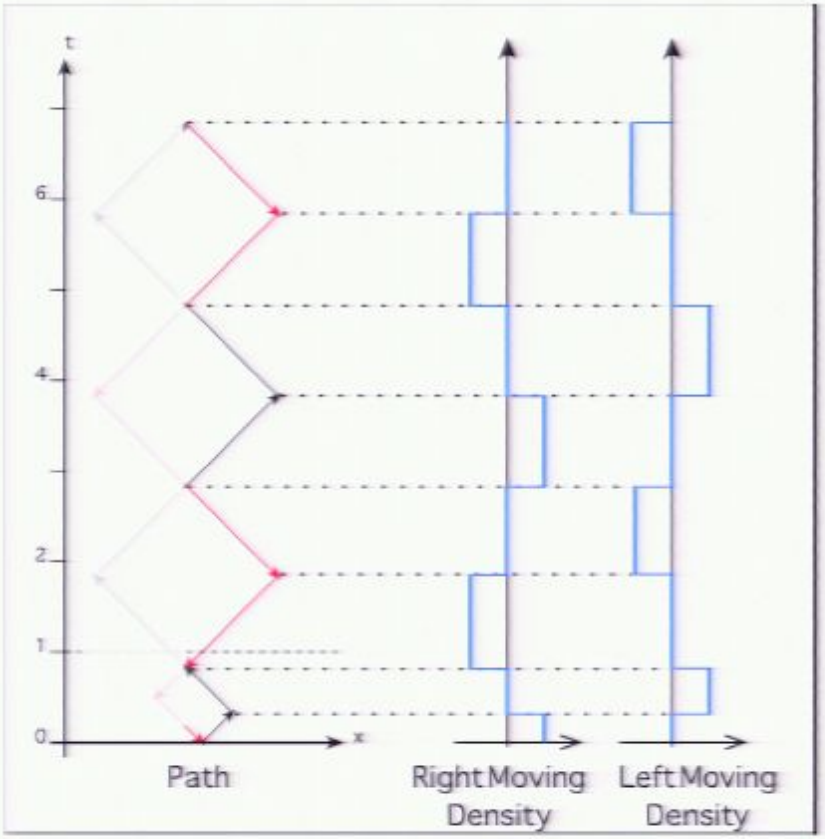




Component Simulation 3D, $e^{\pm i m t}$





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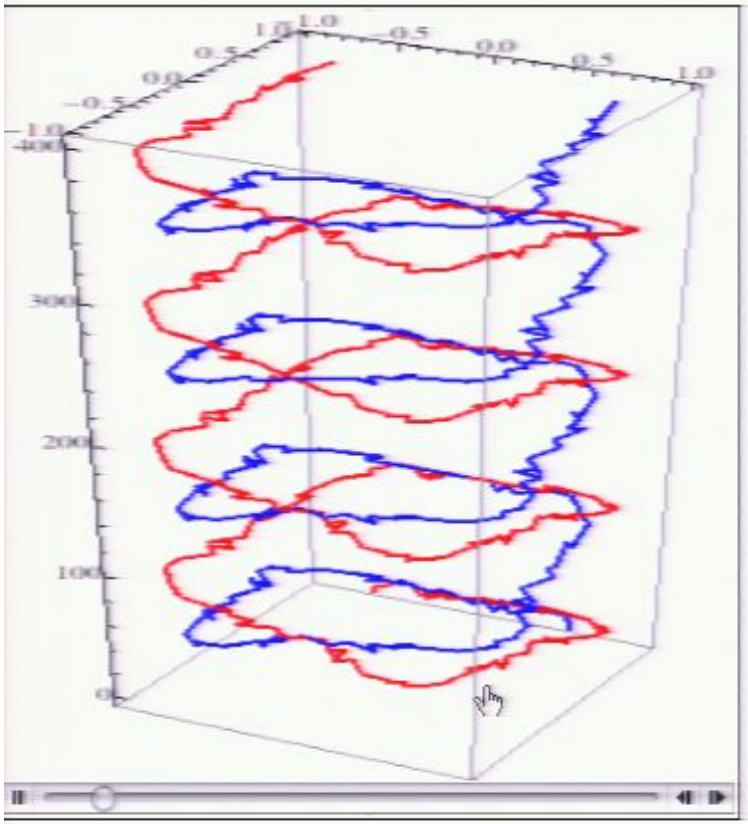

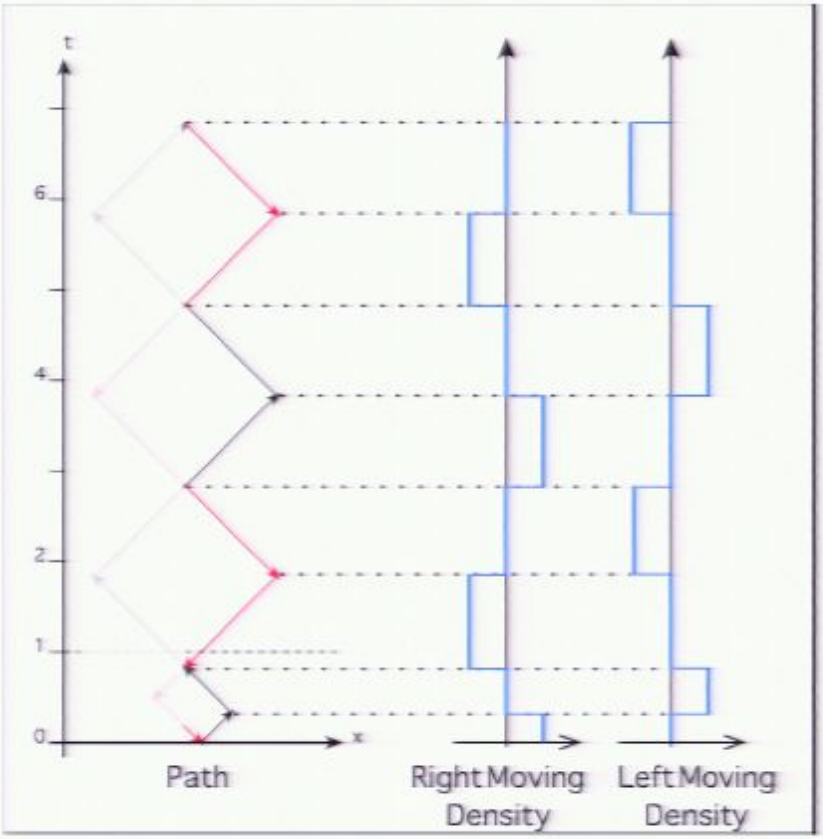

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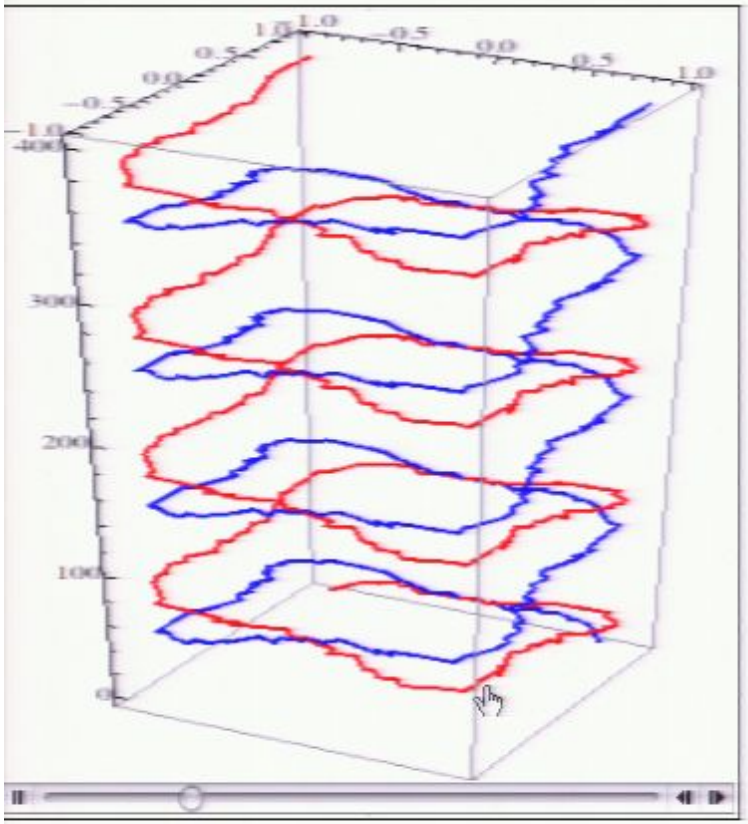

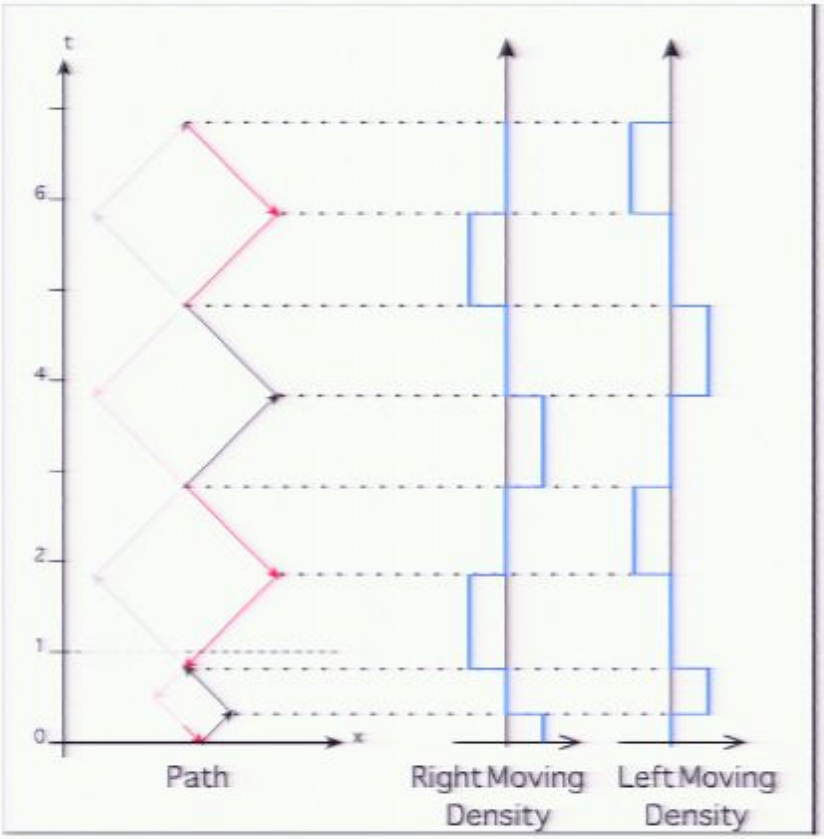

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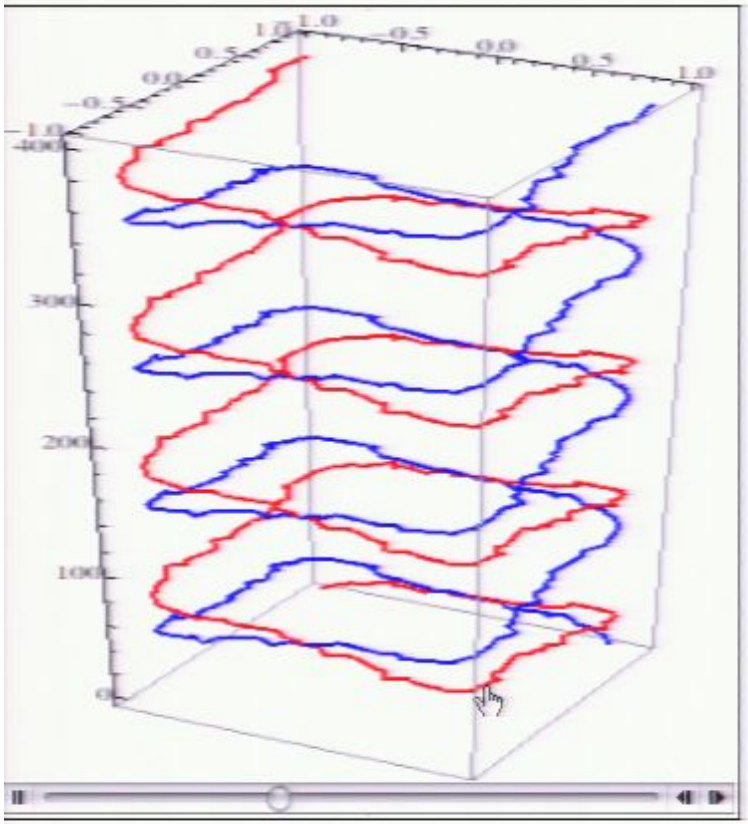

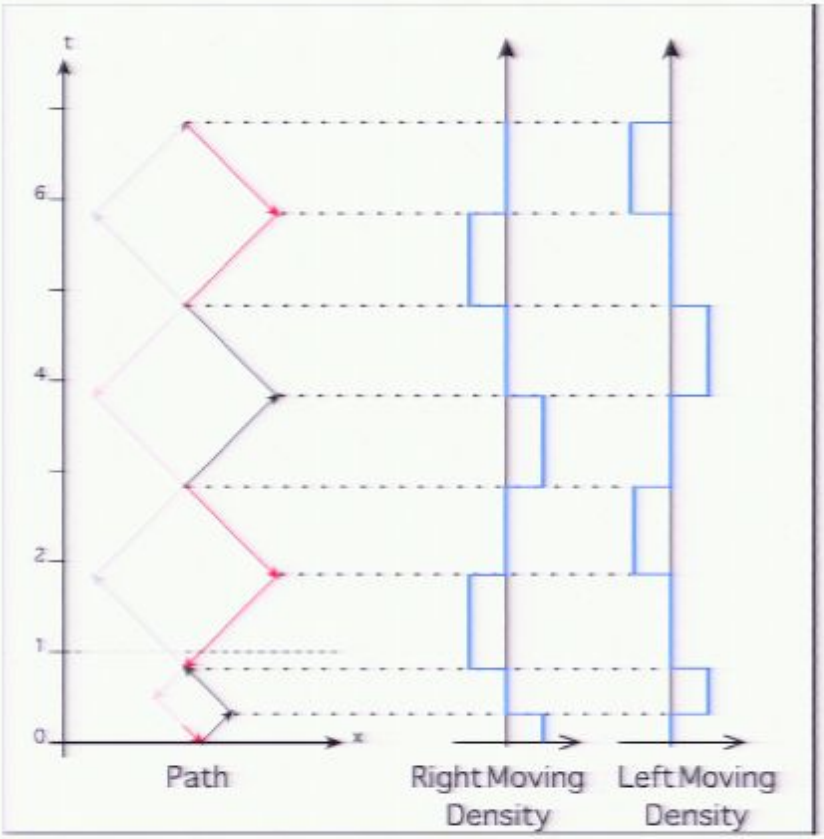

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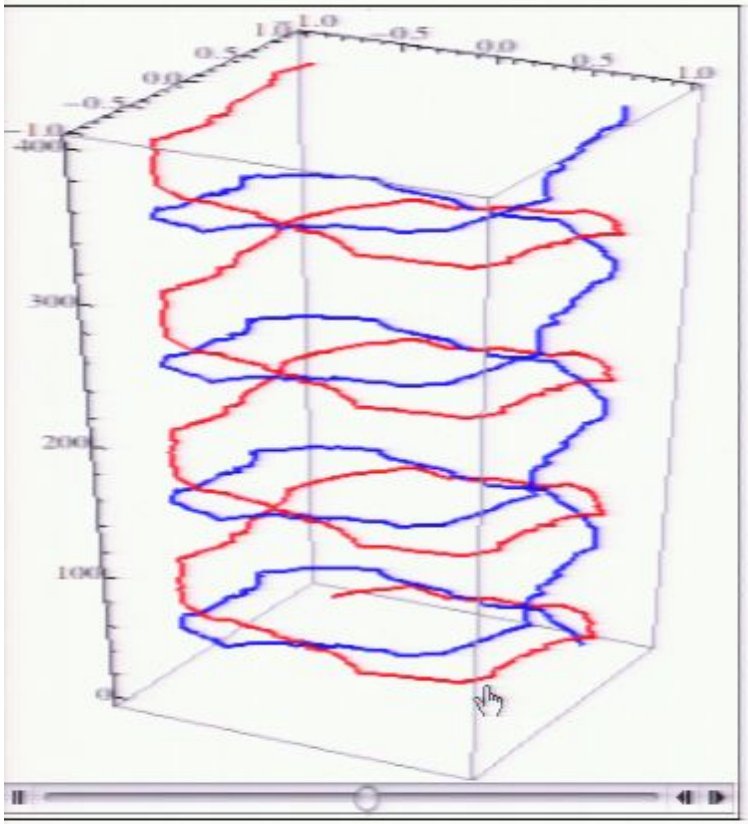

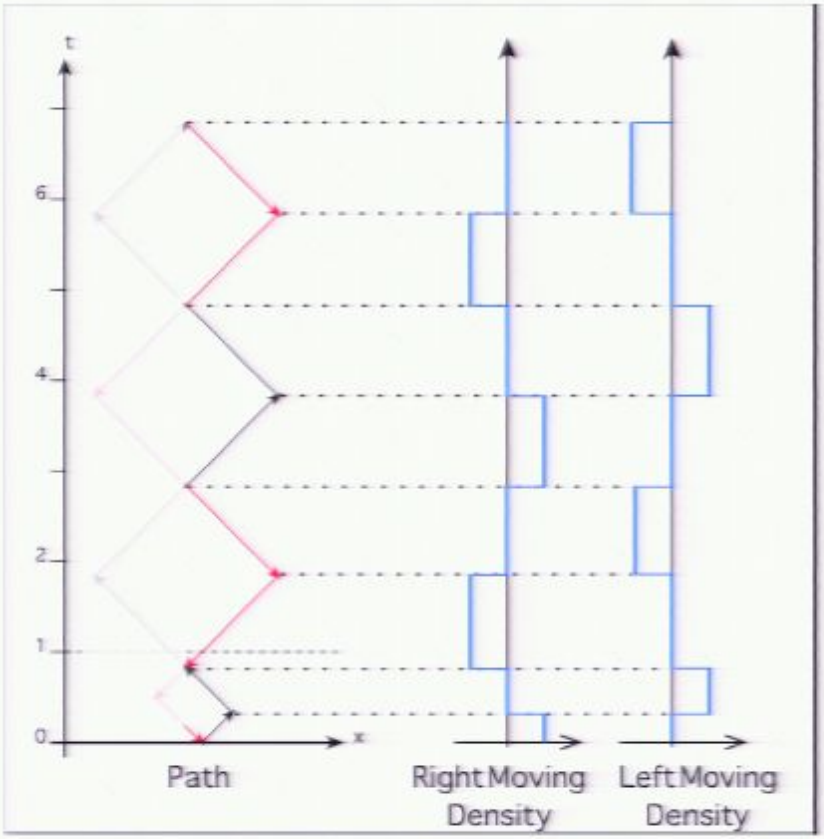

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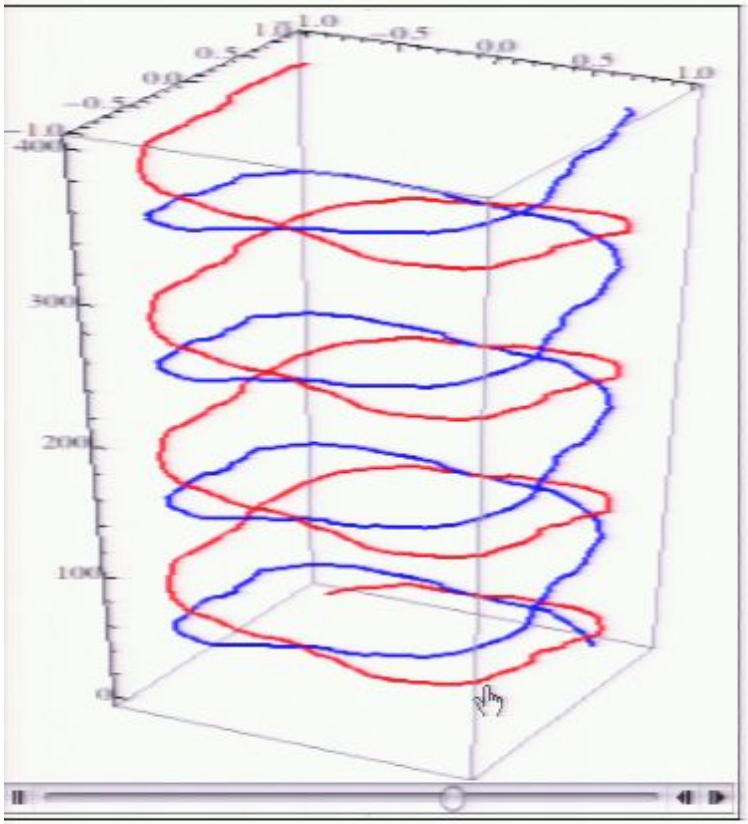

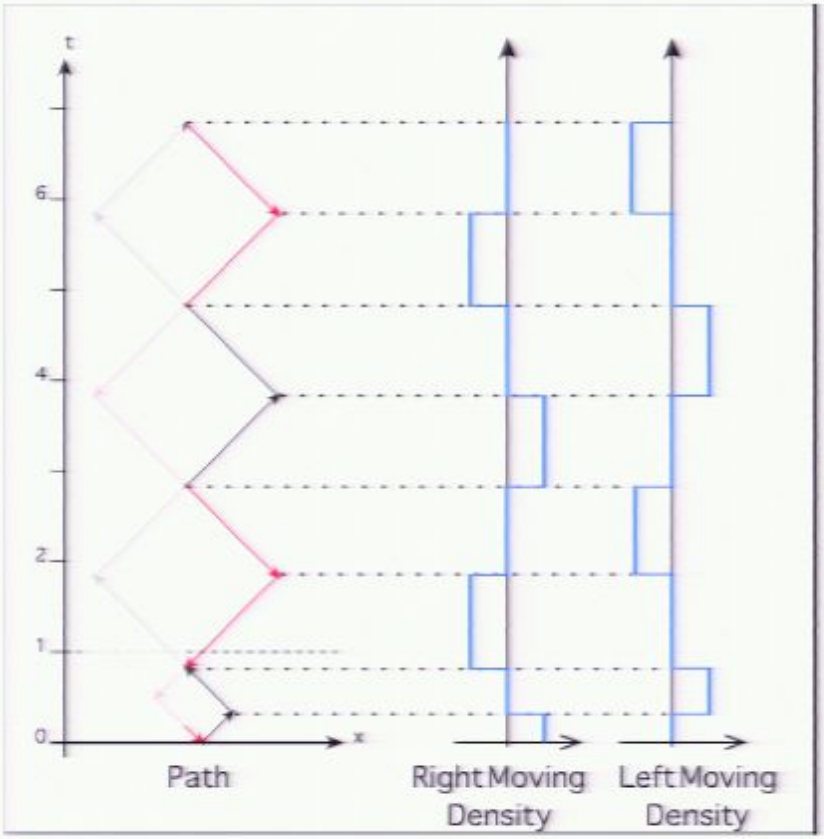

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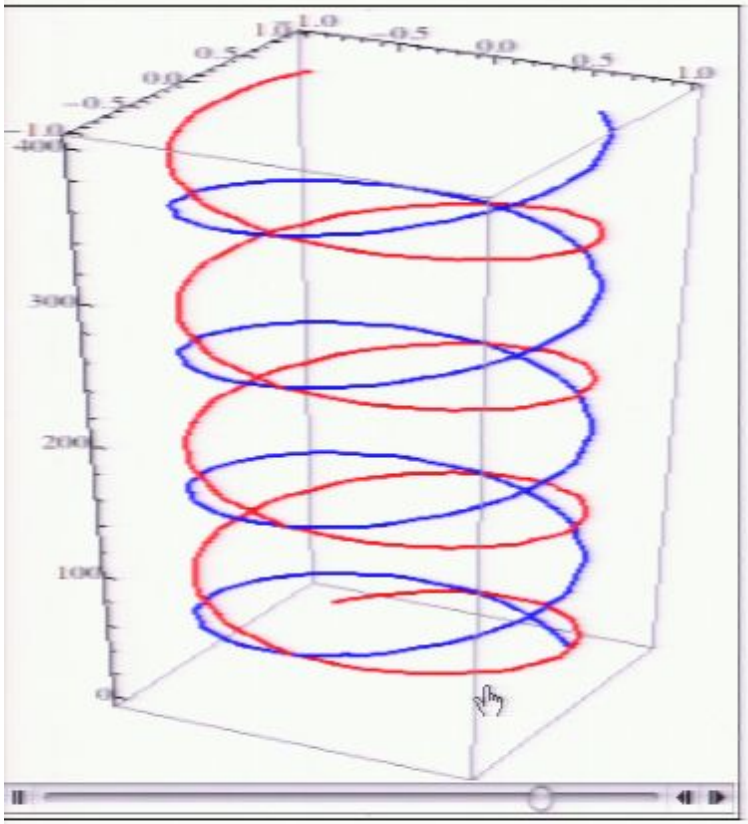

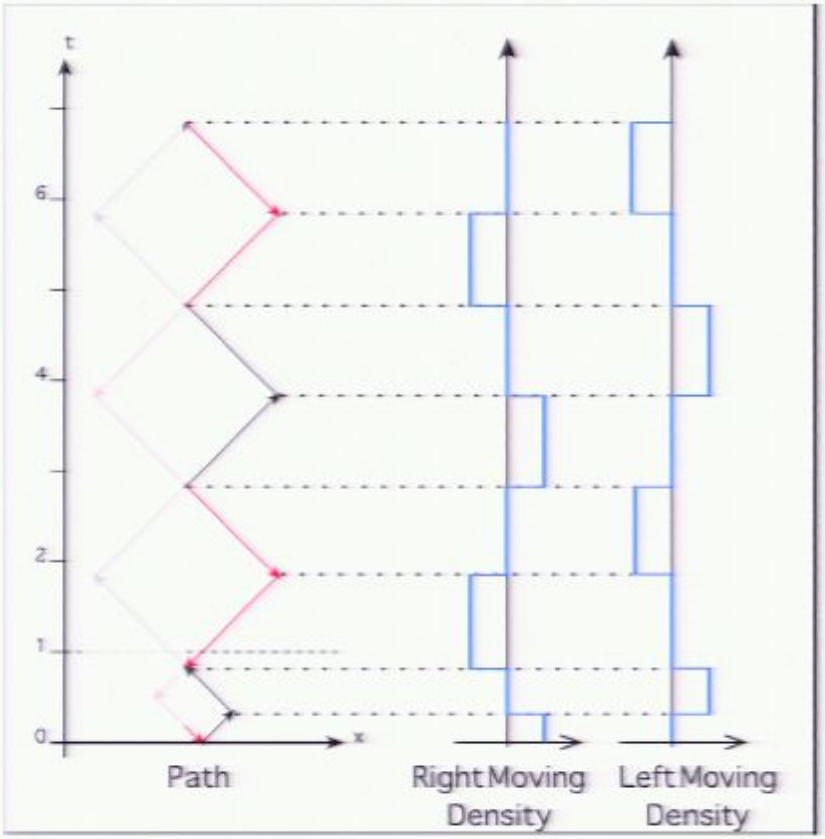

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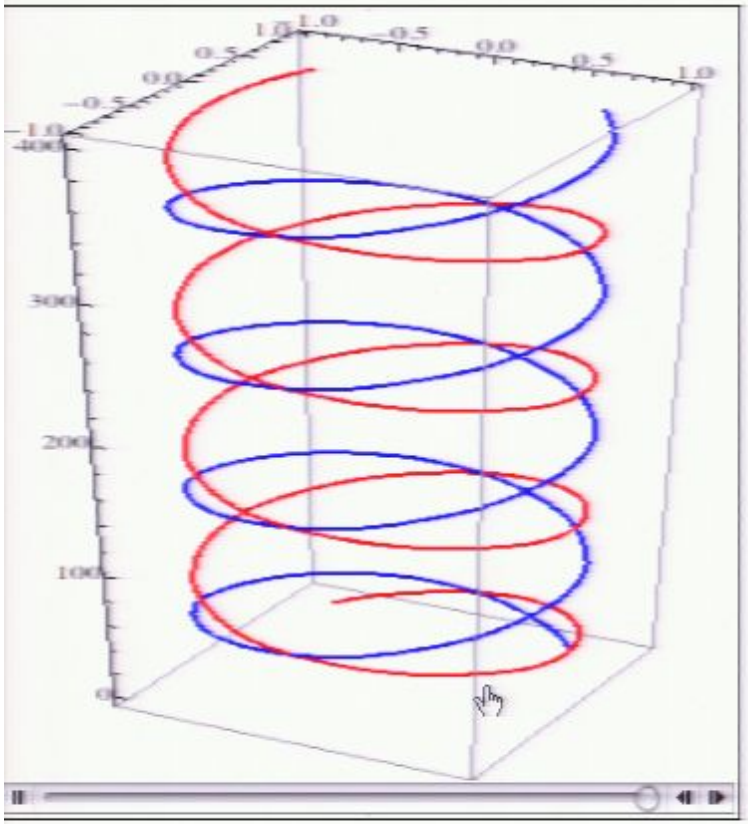
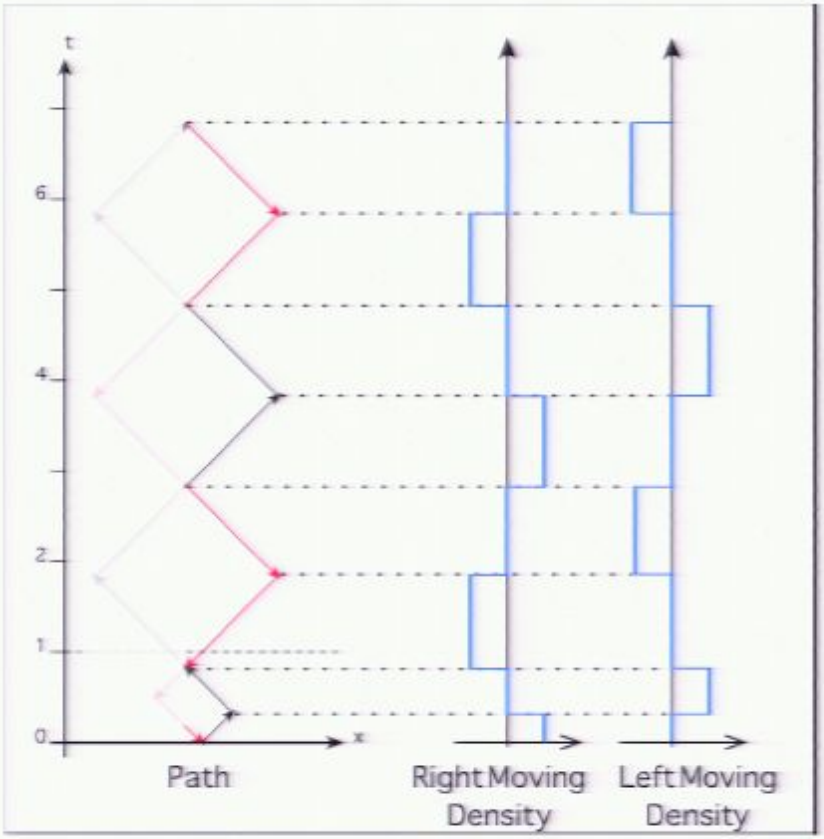
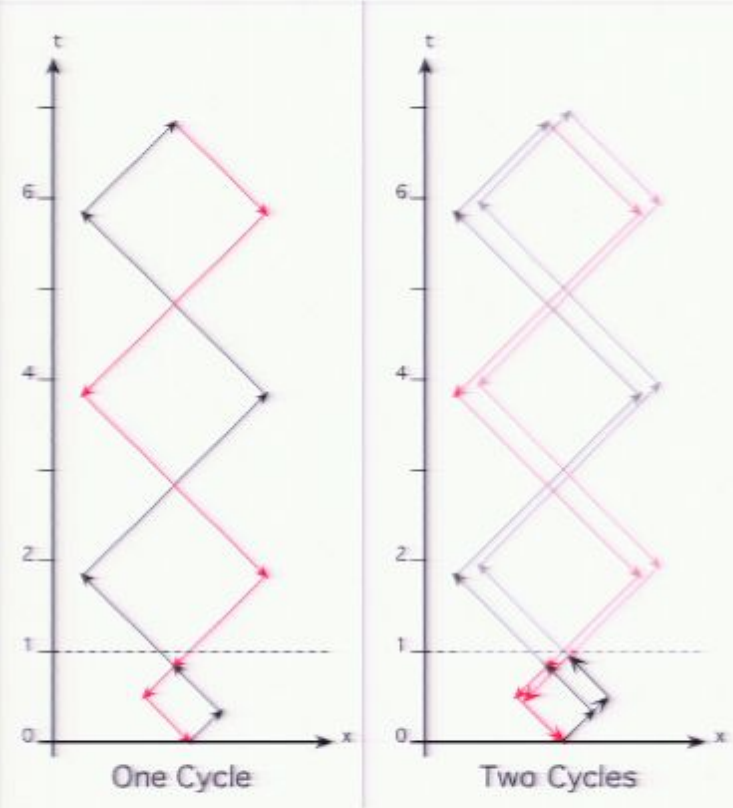
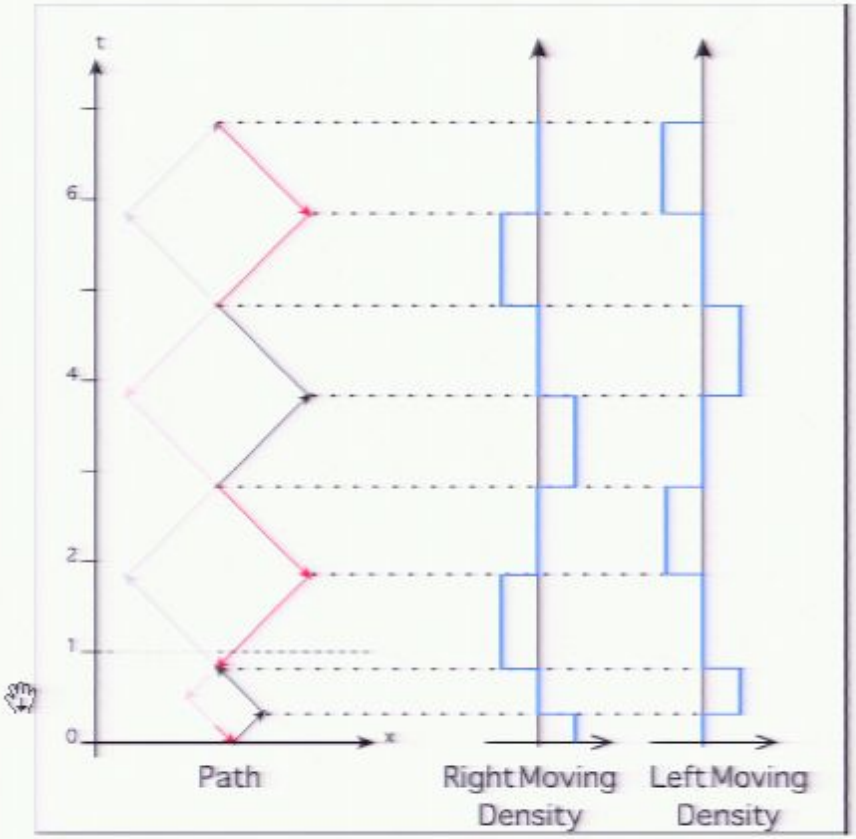




Component Simulation 3D TopView, $e^{\pm i m t}$





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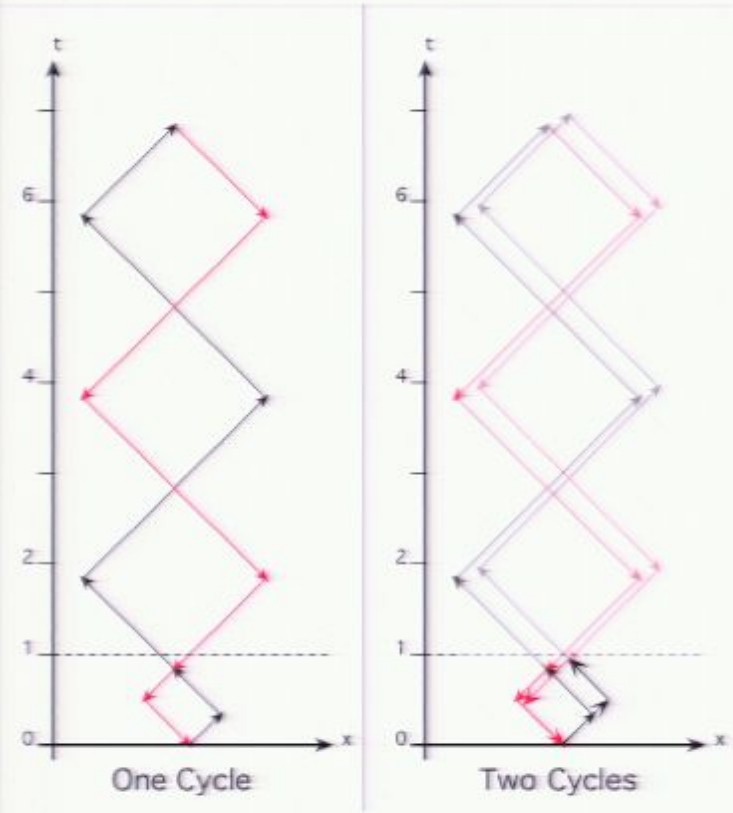

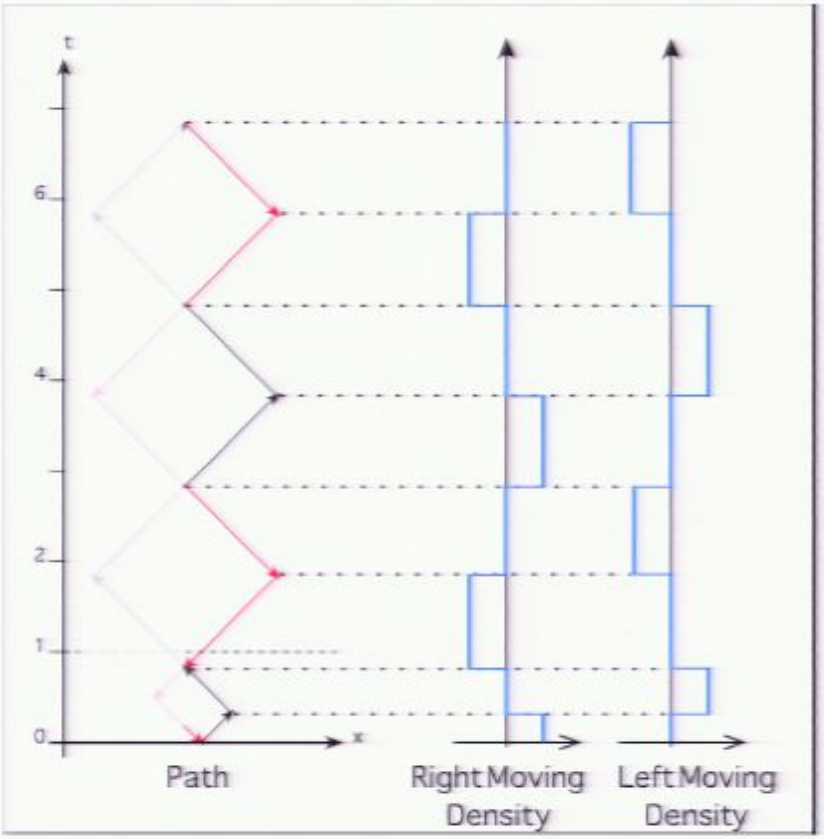

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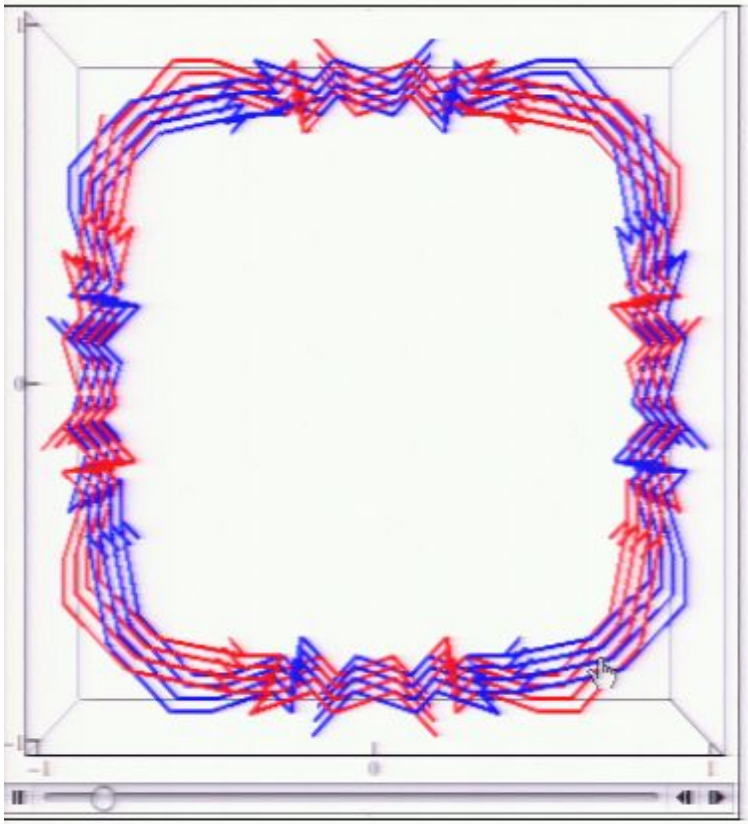

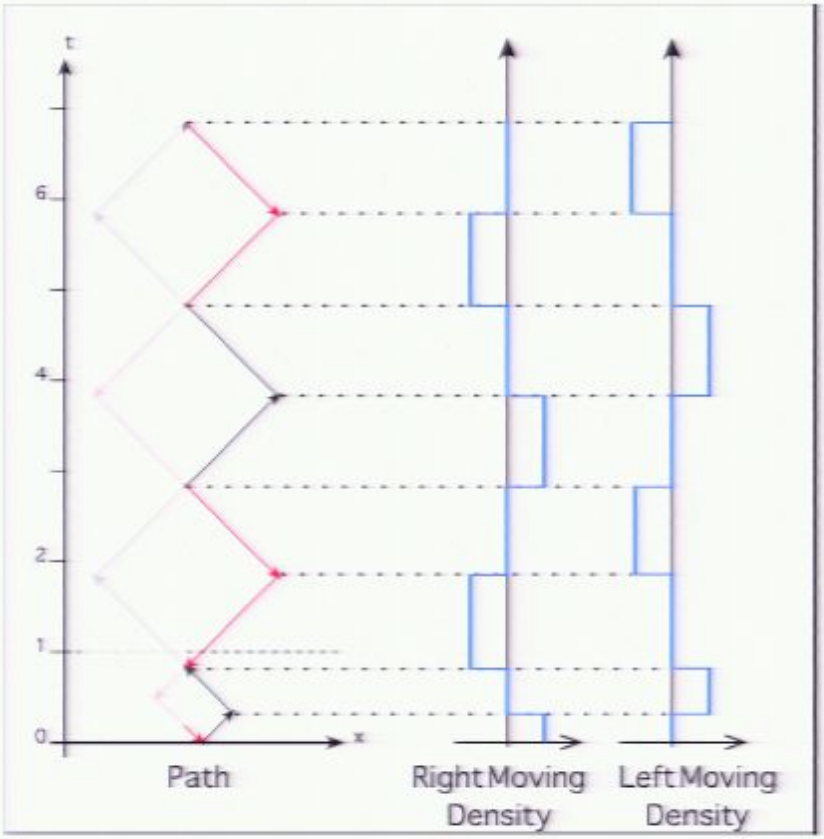

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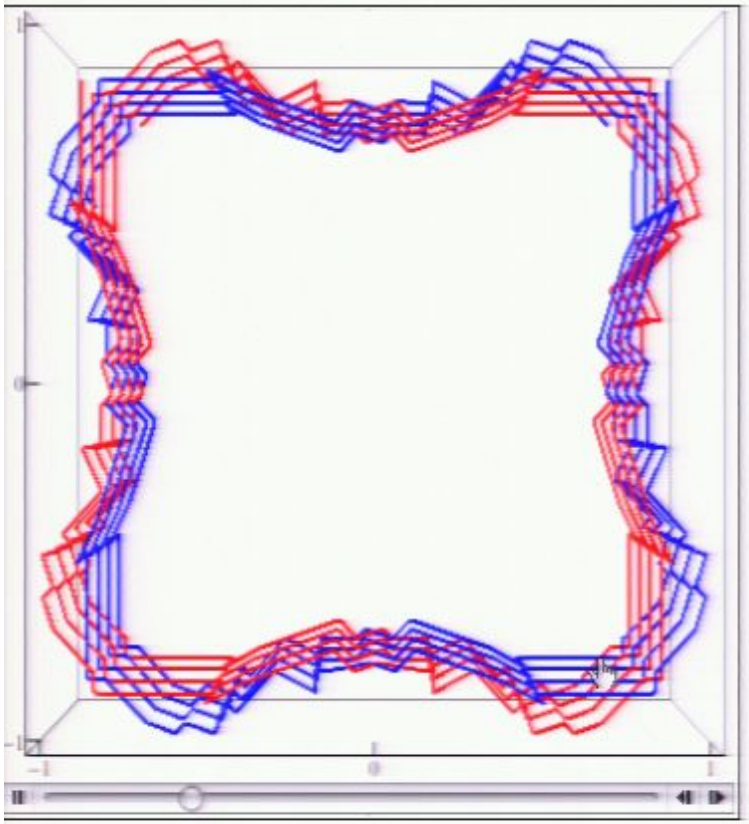

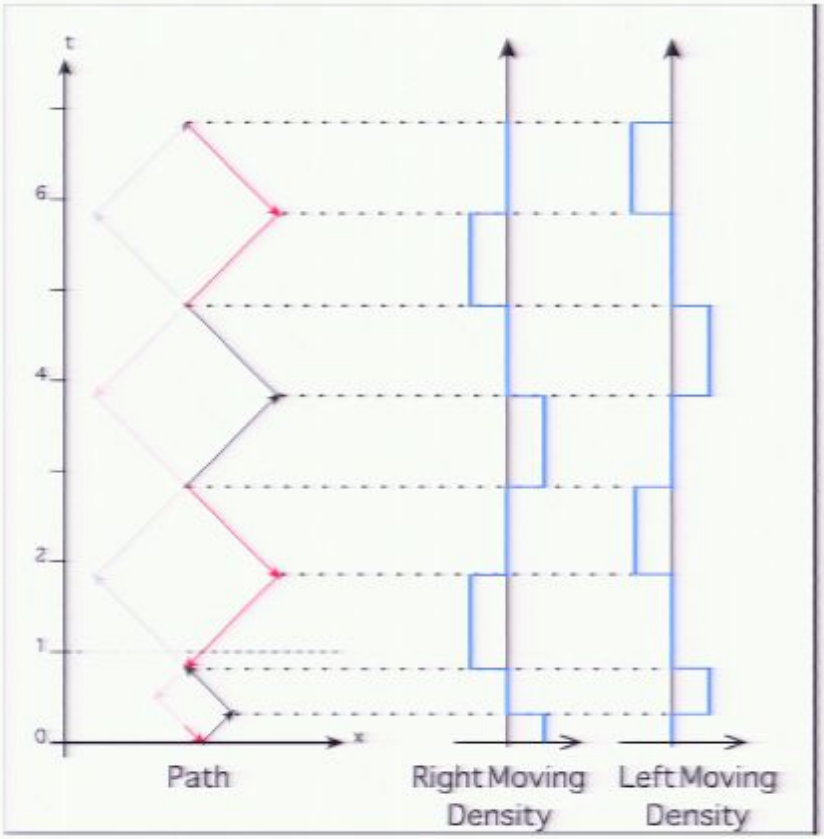

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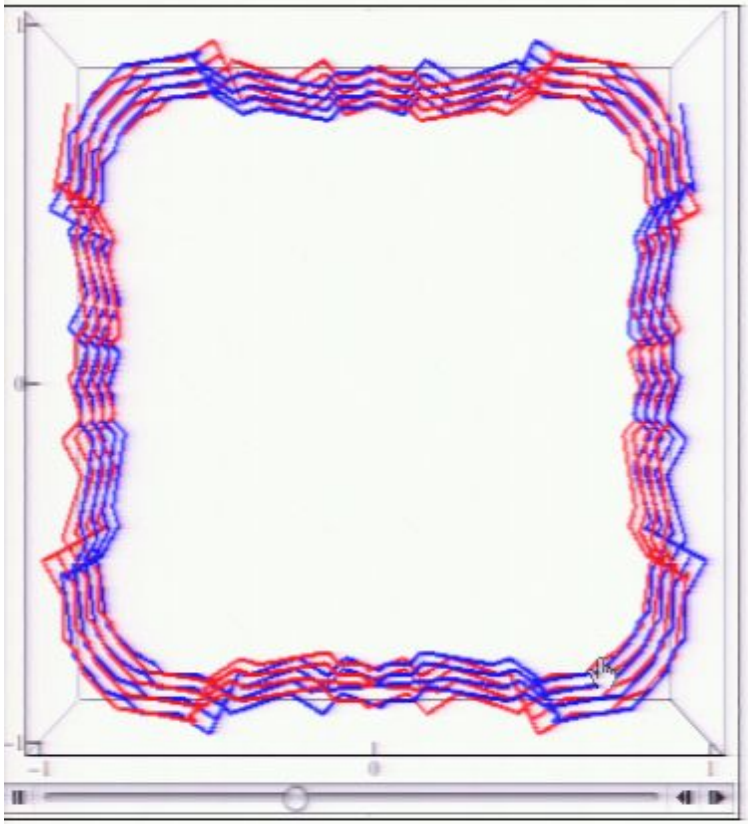

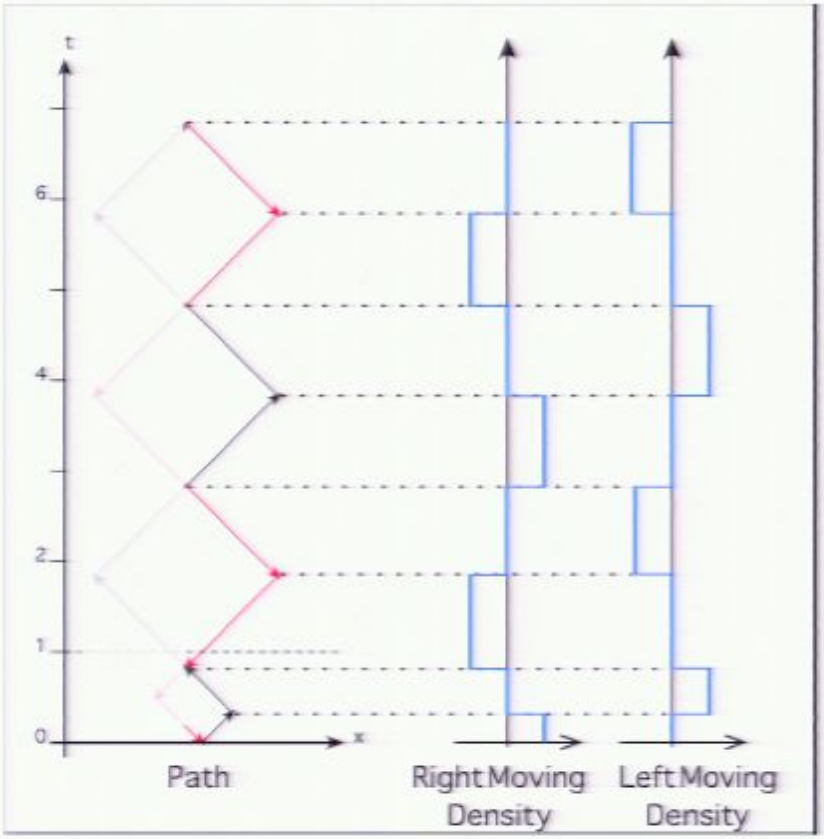

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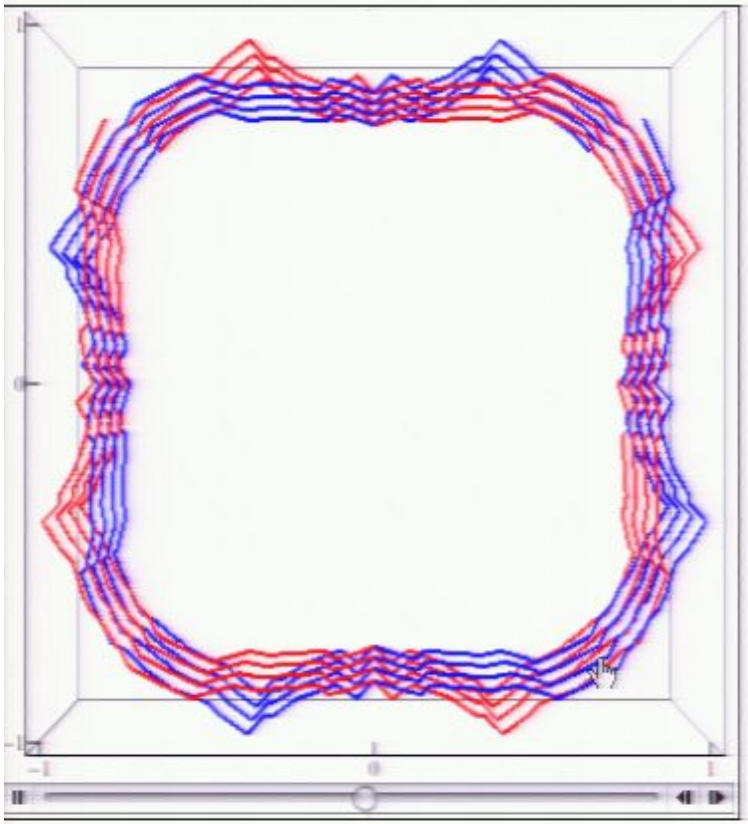

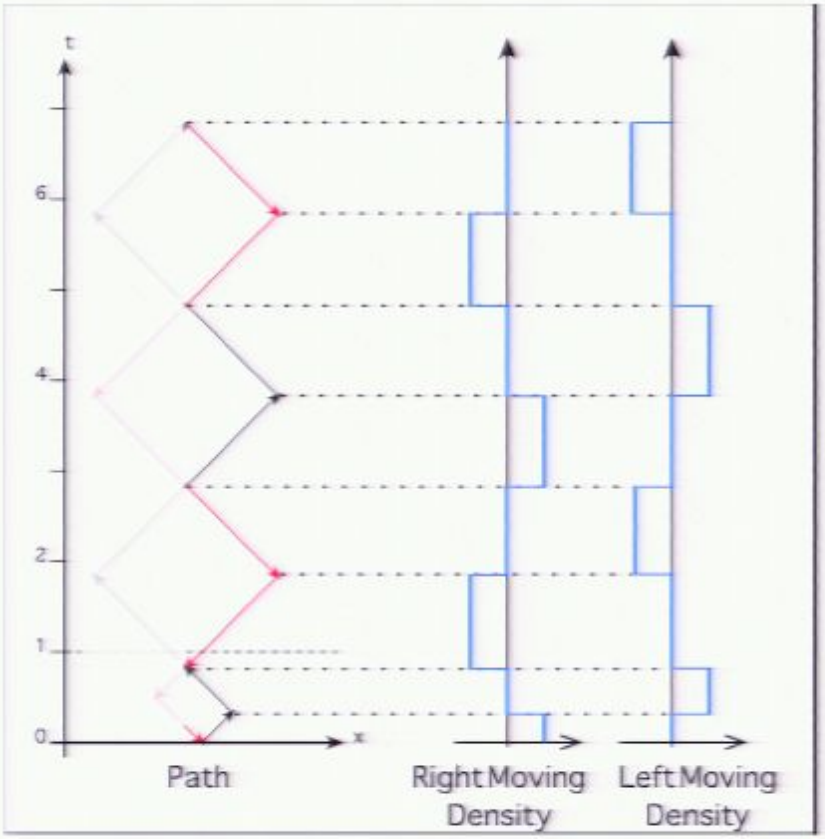

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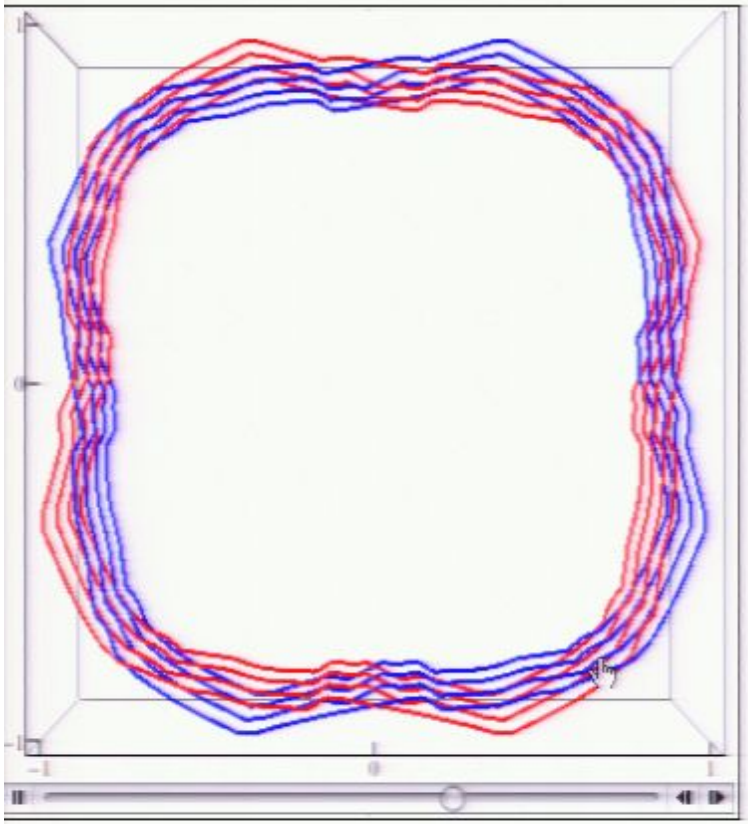

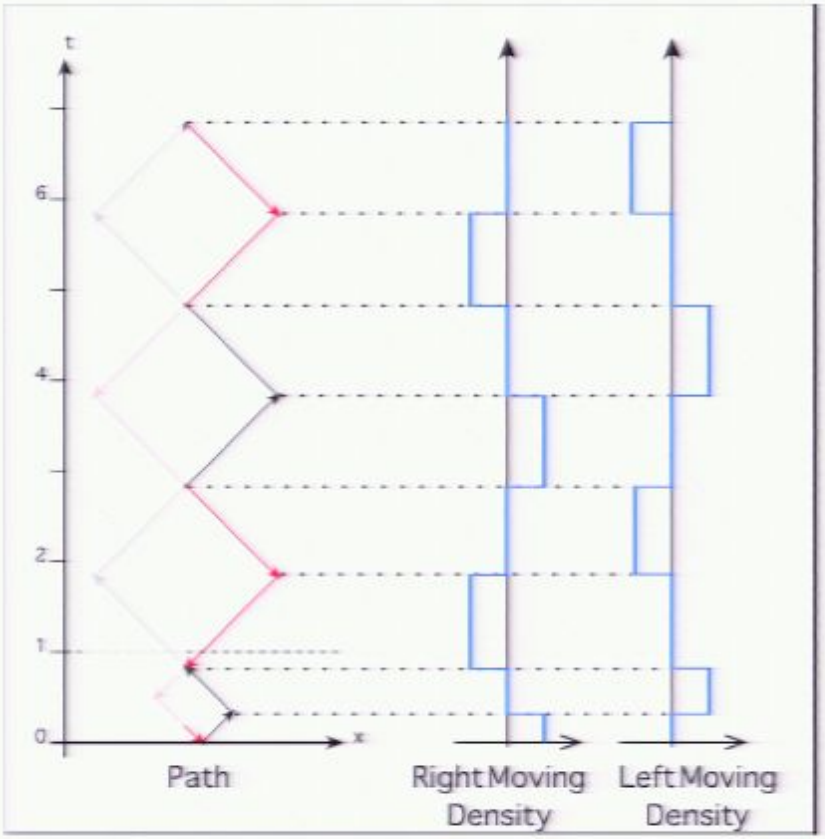

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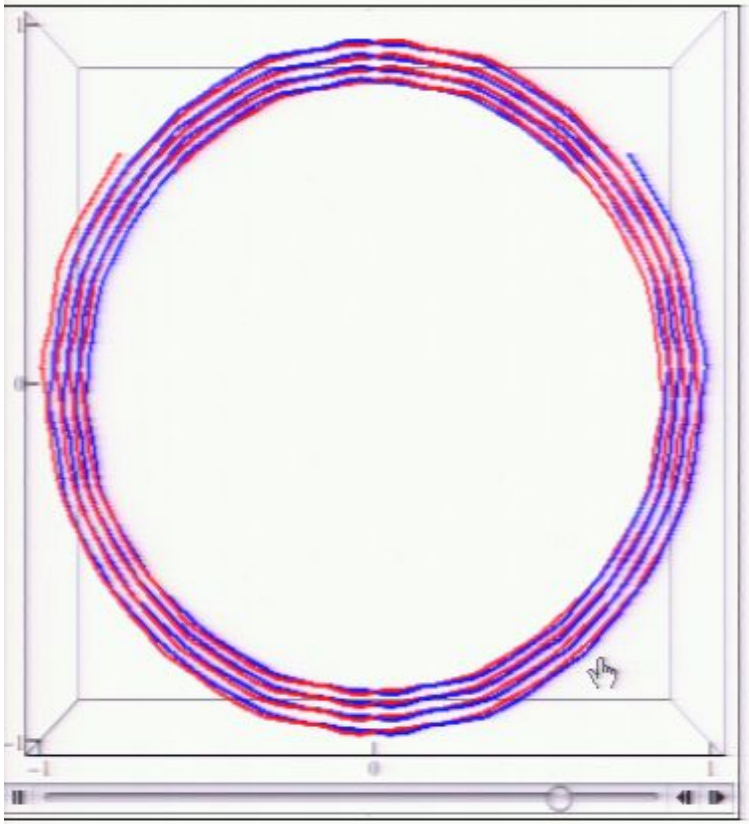

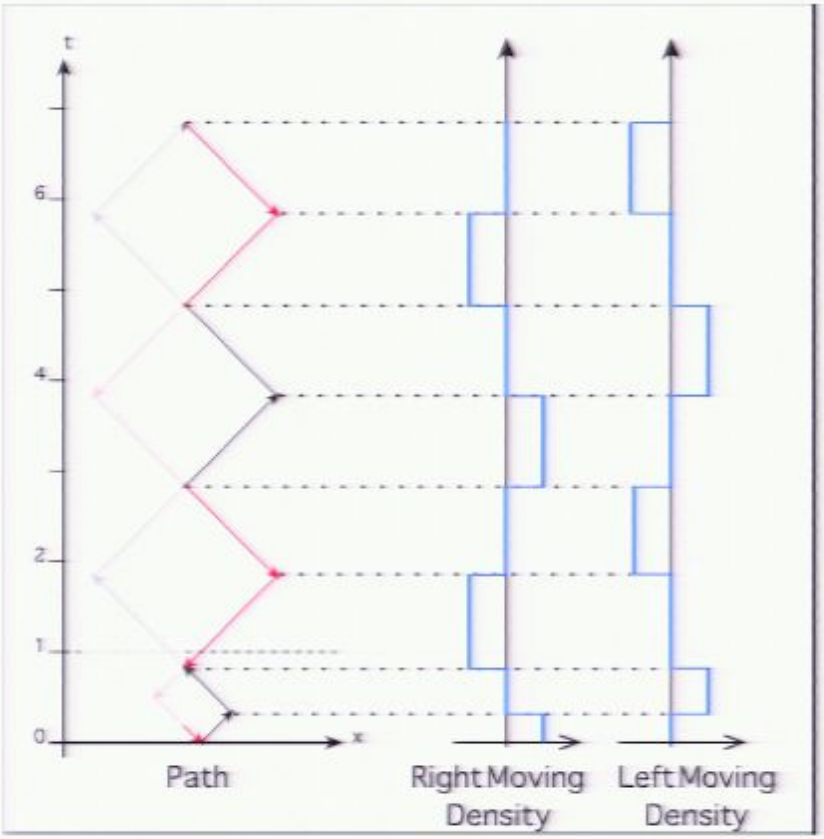

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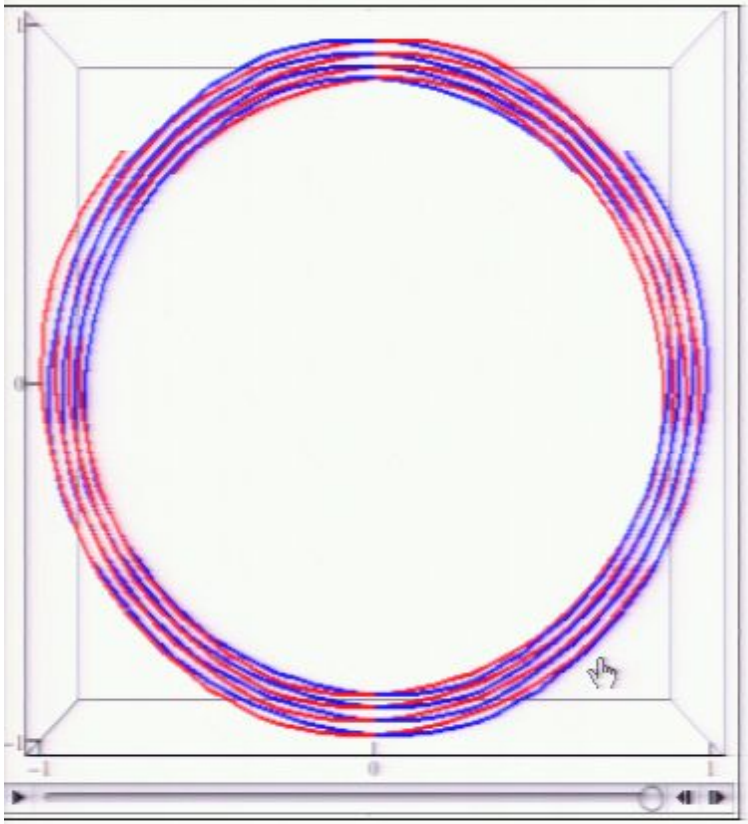

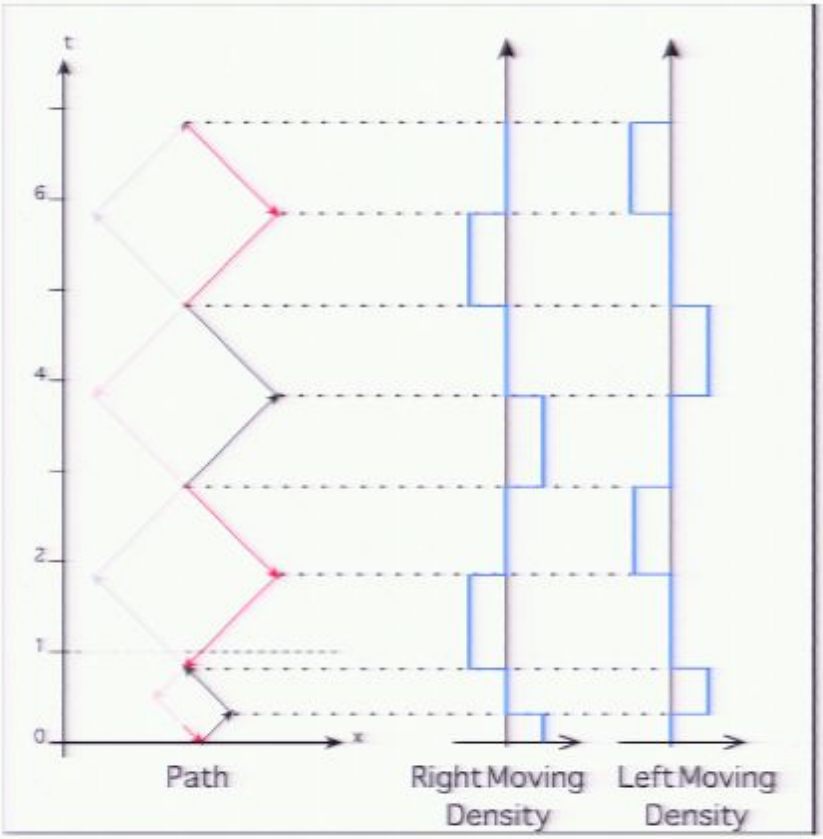

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- Wavefunctions:

$$\text{(Normalized sum of } Y\text{'s)} \xrightarrow{\text{Continuum Limit}} \text{Wavefunction}$$

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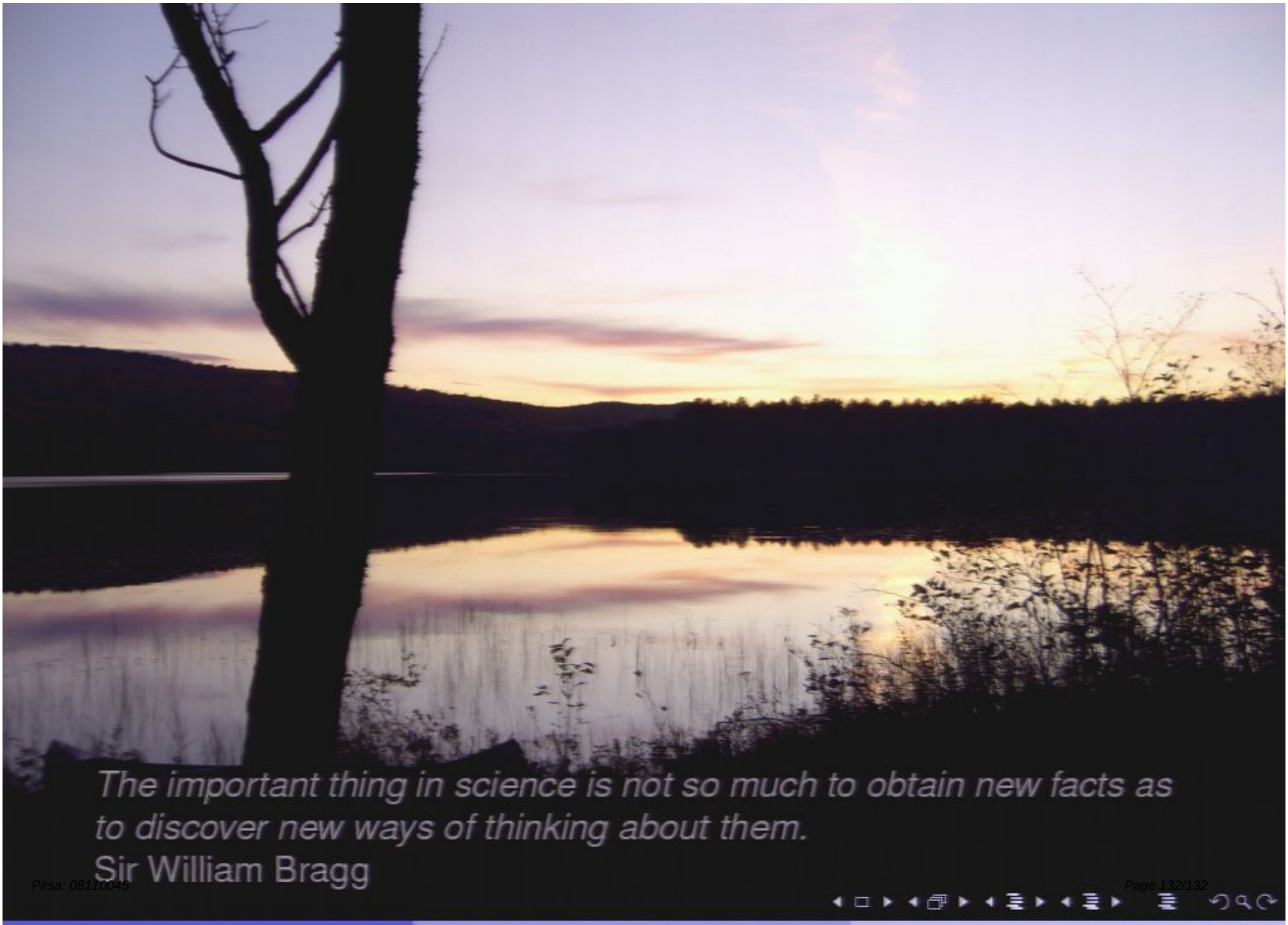
- *This suggests that the Formal Analytic Continuation used in the quantization process may simply be a shortcut for replacing the Bernoulli RV with the Anti-Bernoulli RV, as would be appropriate for the presence of time-reversed paths!*


The important thing in science is not so much to obtain new facts as to discover new ways of thinking about them.
Sir William Bragg