Title: From Information Geometry to Quantum Theory

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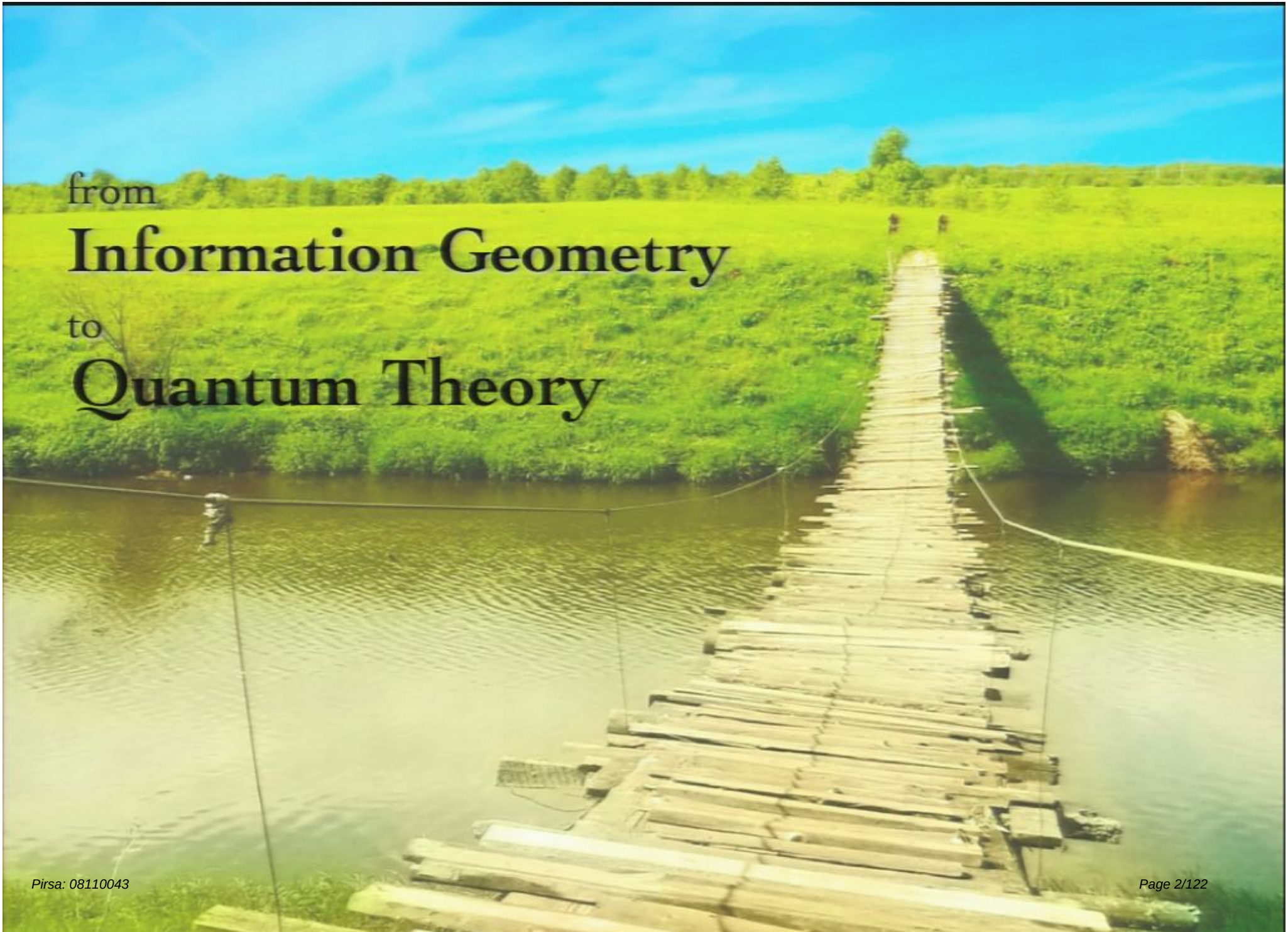
Abstract: The unparalleled empirical success of quantum theory strongly suggests that it accurately captures fundamental aspects of the workings of the physical world. The clear articulation of these aspects is of inestimable value --- not only for the deeper understanding of quantum theory in itself, but for its further development, particularly for the development of a theory of quantum gravity. Recently, there has been growing interest in elucidating these aspects by expressing, in a less abstract mathematical language, what we think quantum theory might be telling us about how nature works, and trying to derive, or reconstruct, quantum theory from these postulates. In this talk, I describe a simple reconstruction of the finite-dimensional quantum formalism. The derivation takes places with a classical probabilistic framework equipped with the information (or Fisher-Rao) metric, and rests upon a small number of elementary ideas (such as complementarity and global gauge invariance). The complex structure of quantum formalism arises very naturally. The derivation provides a number of non-trivial insights into the quantum formalism, such as the extensive nature of the role of information geometry in determining the quantum formalism, the importance of global gauge invariance, and the importance (or lack thereof) of assumptions concerning separated systems.


from
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to
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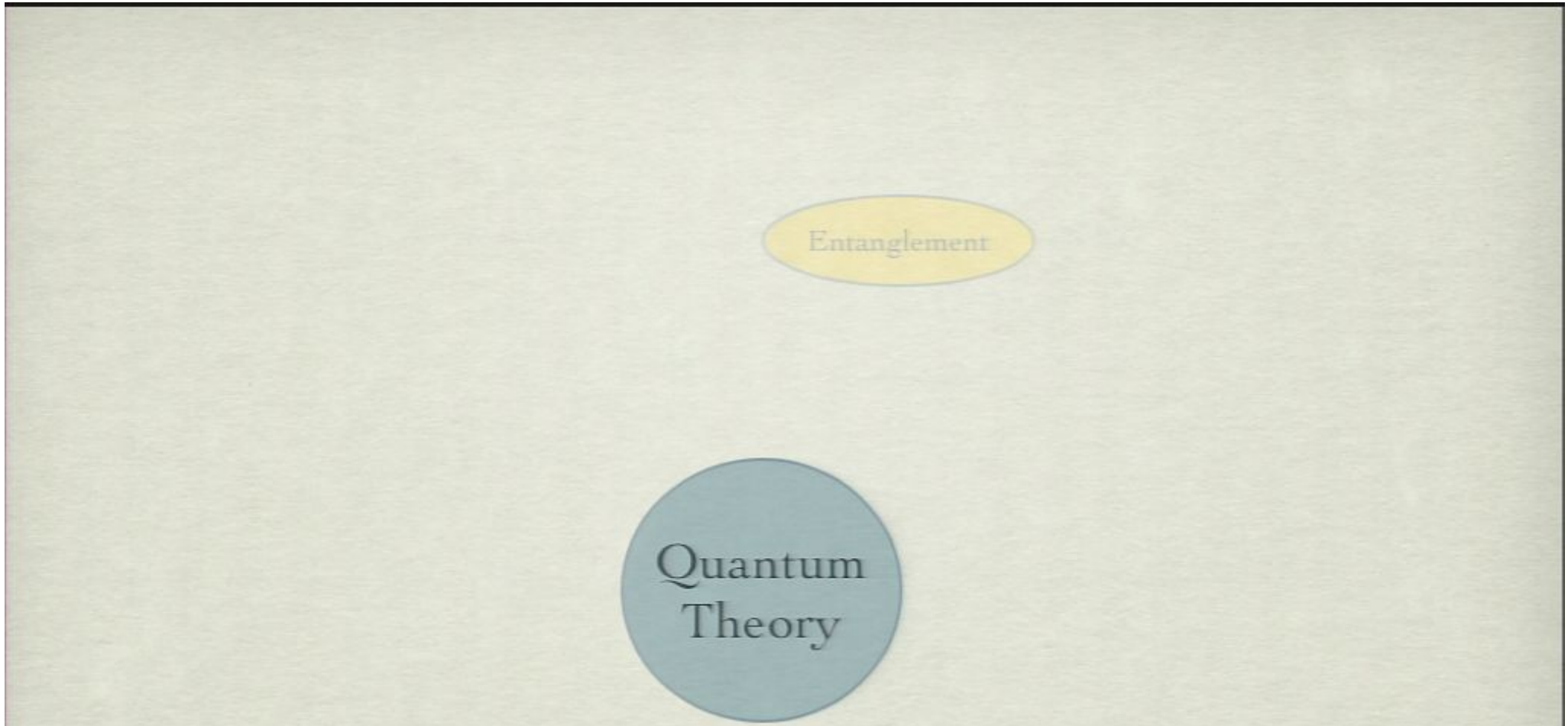
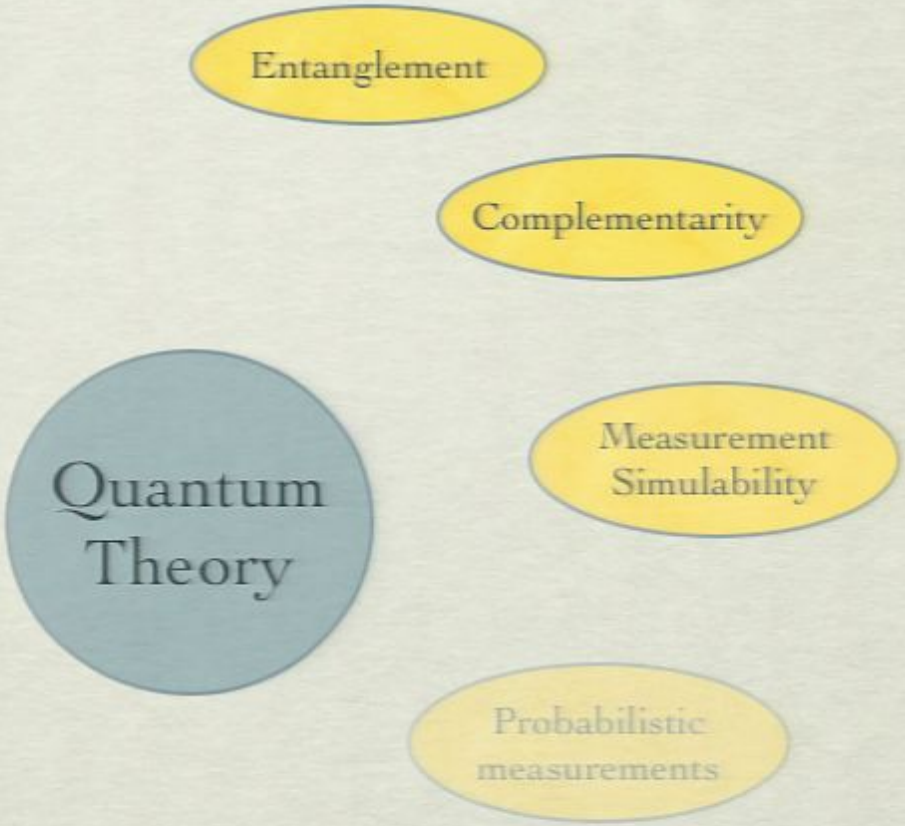
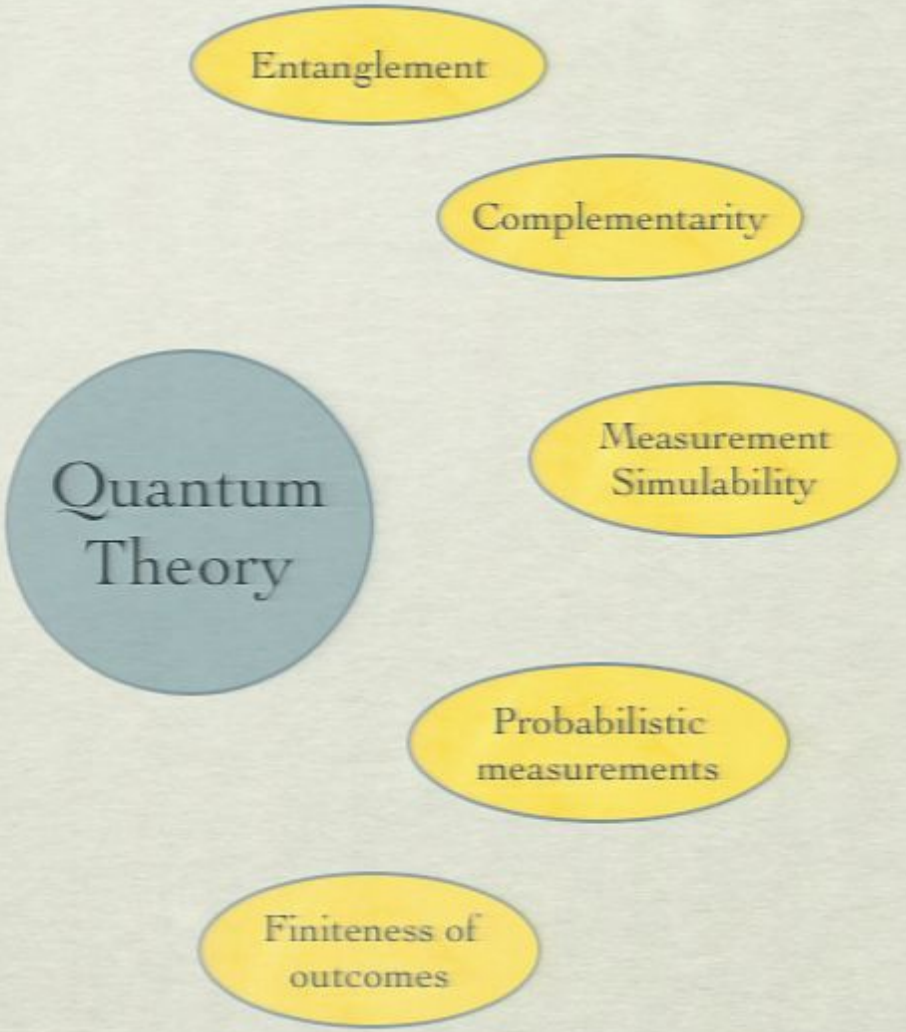

Entanglement
Quantum
Theory


Entanglement
Complementarity
Quantum Theory
Measurement Simulability
Probabilistic measurements


Entanglement
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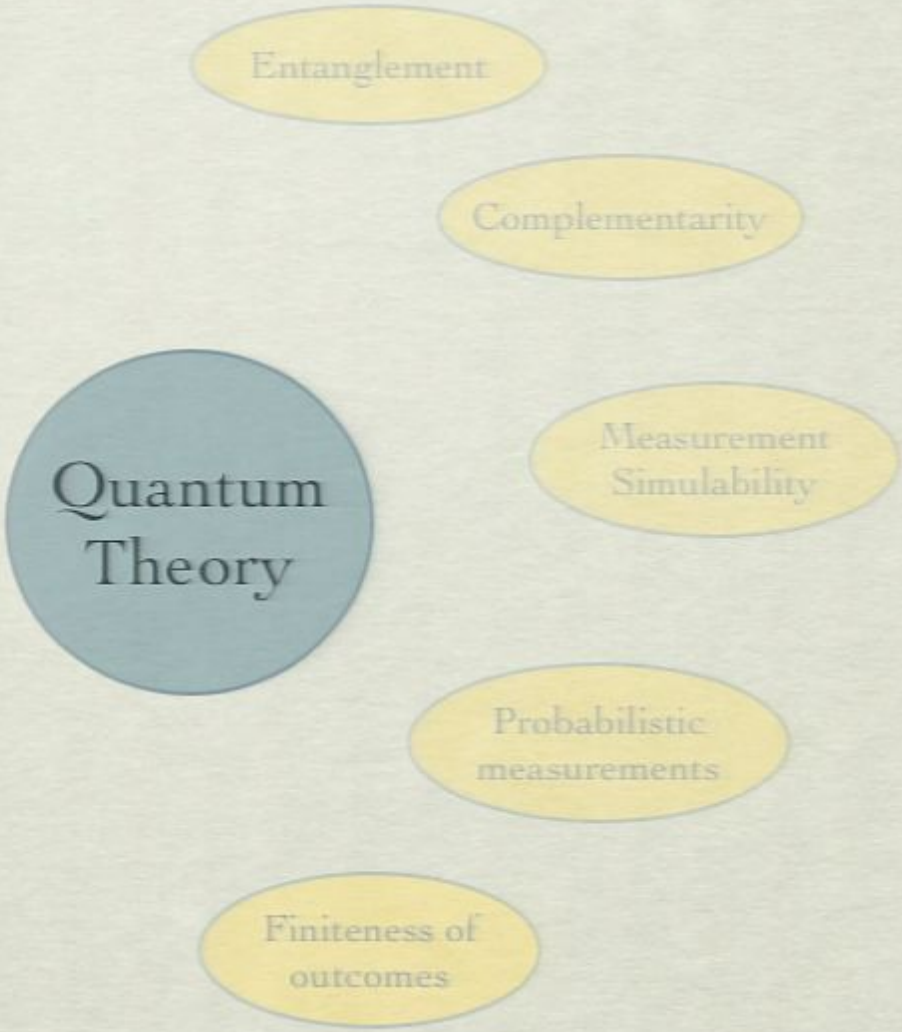
Finiteness of outcomes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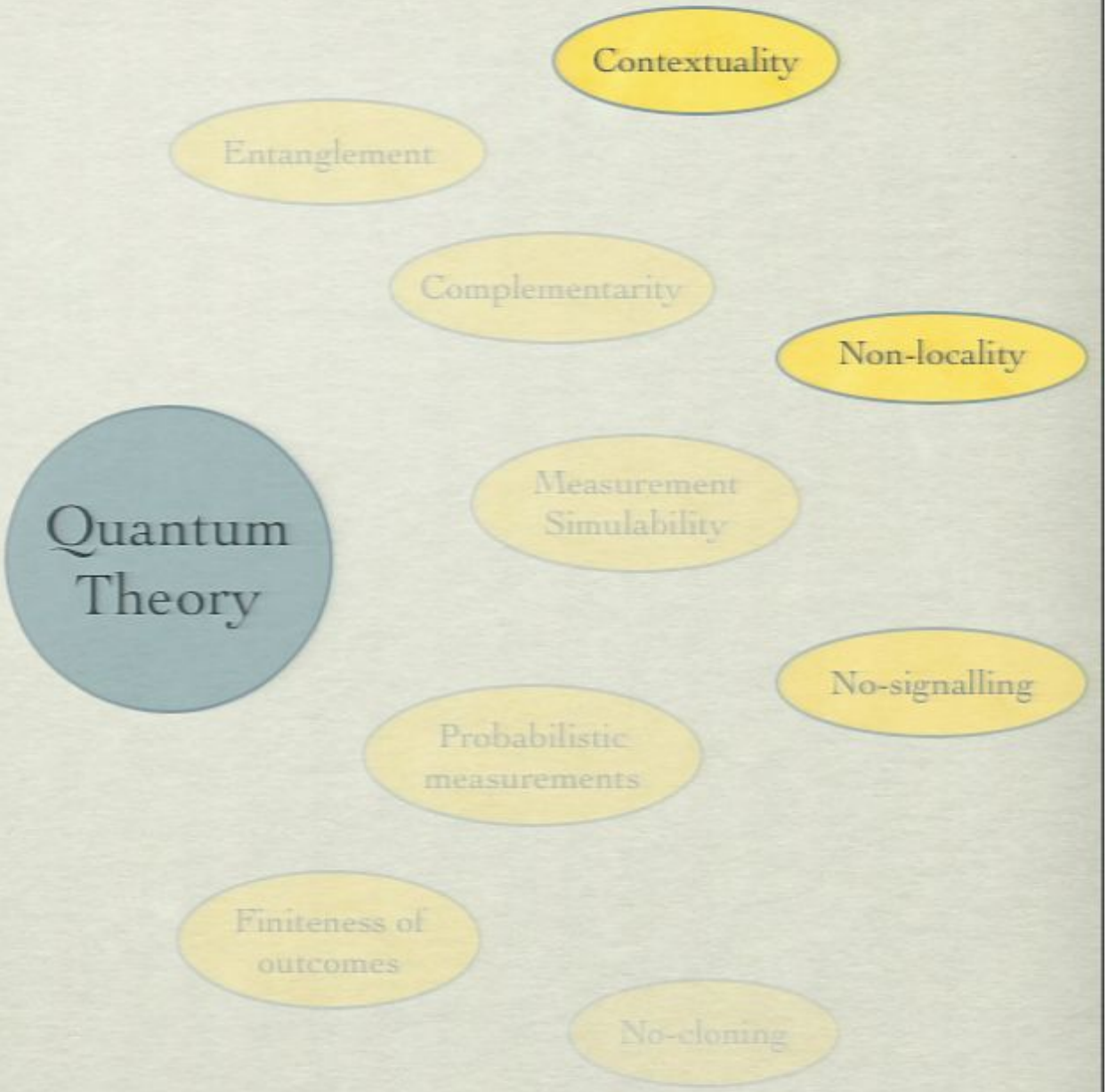
Contextuality
Entanglement
Complementarity
Non-locality
Quantum Theory
Measurement Simulability
No-signalling
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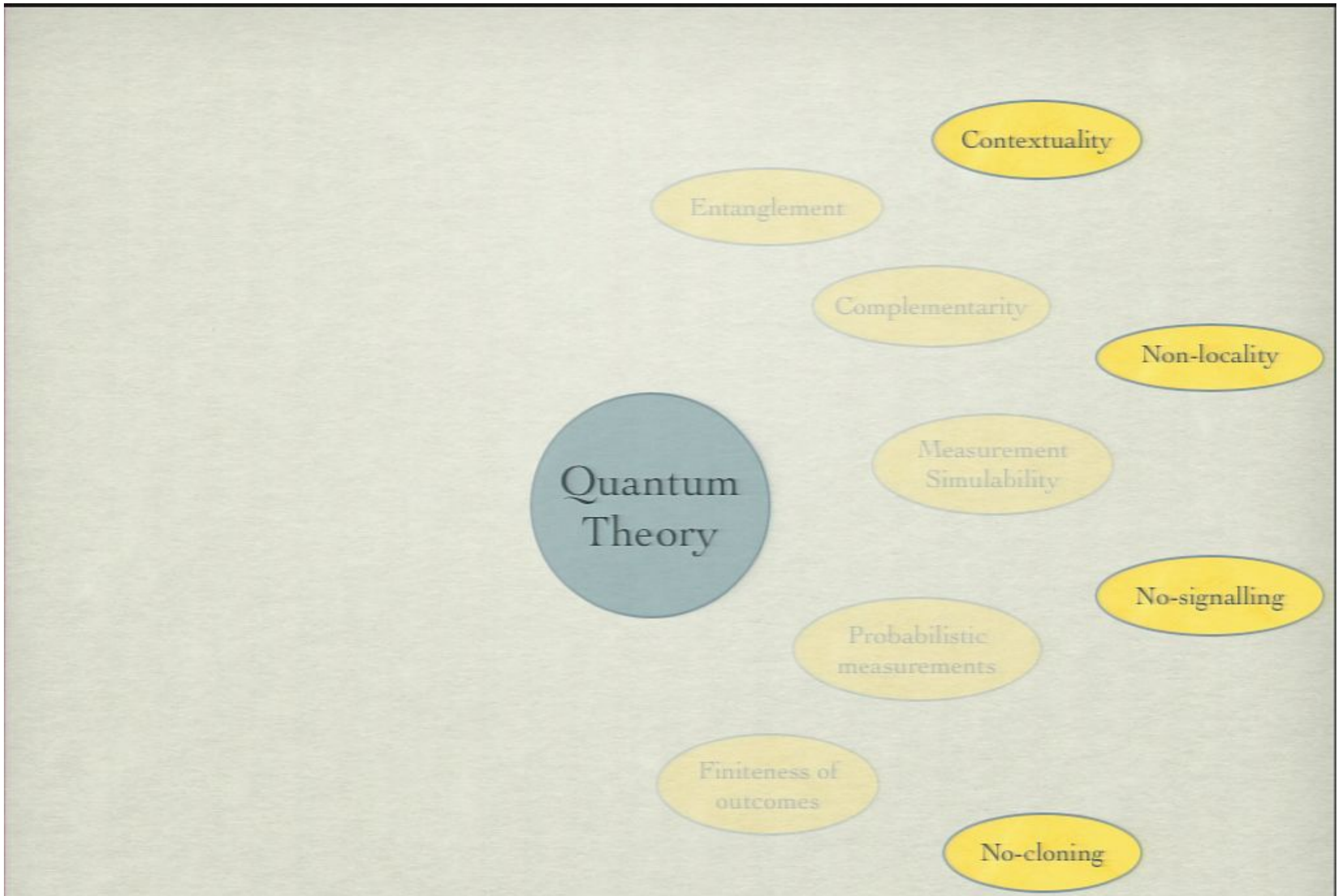
Finiteness of outcomes
No-cloning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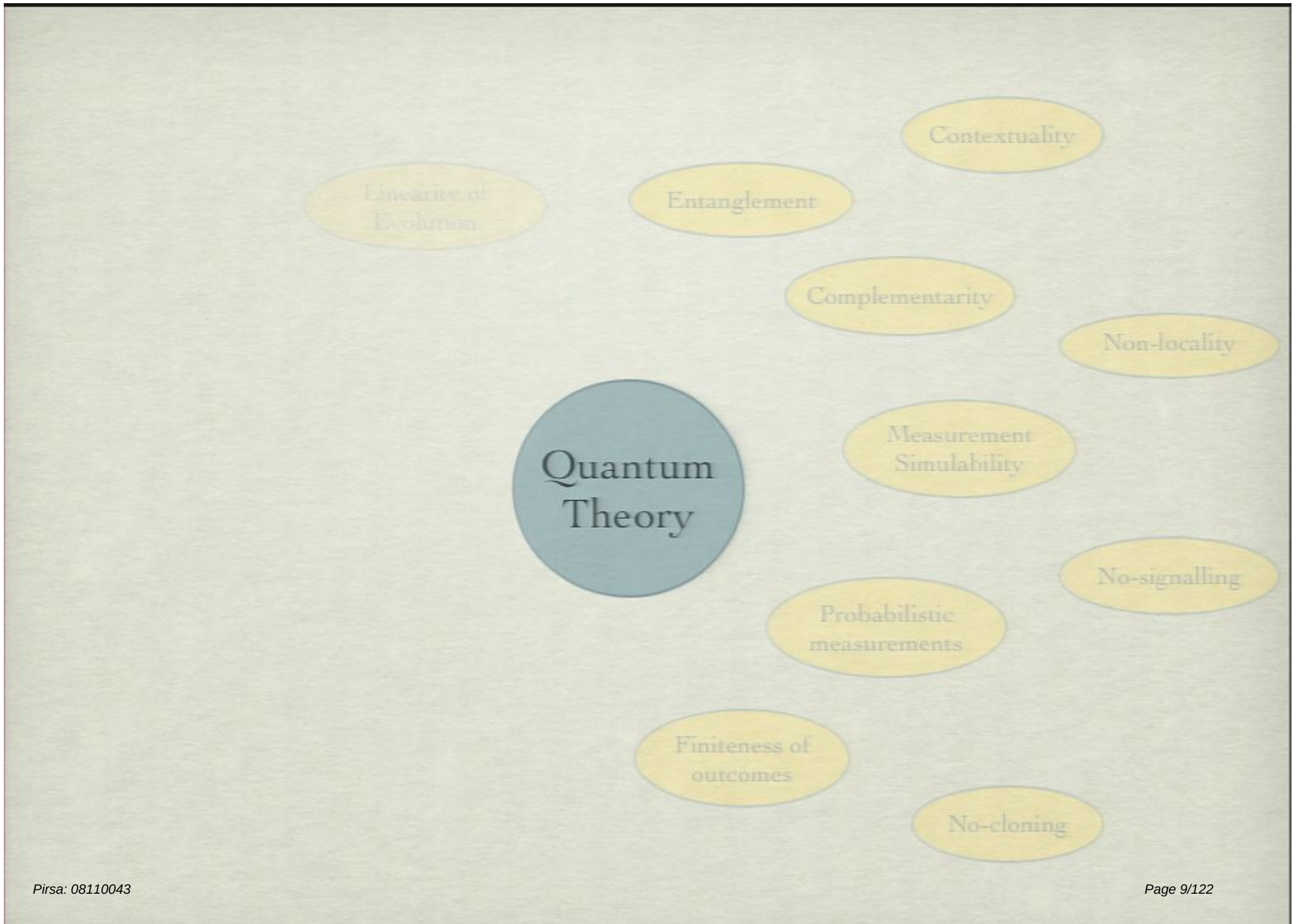
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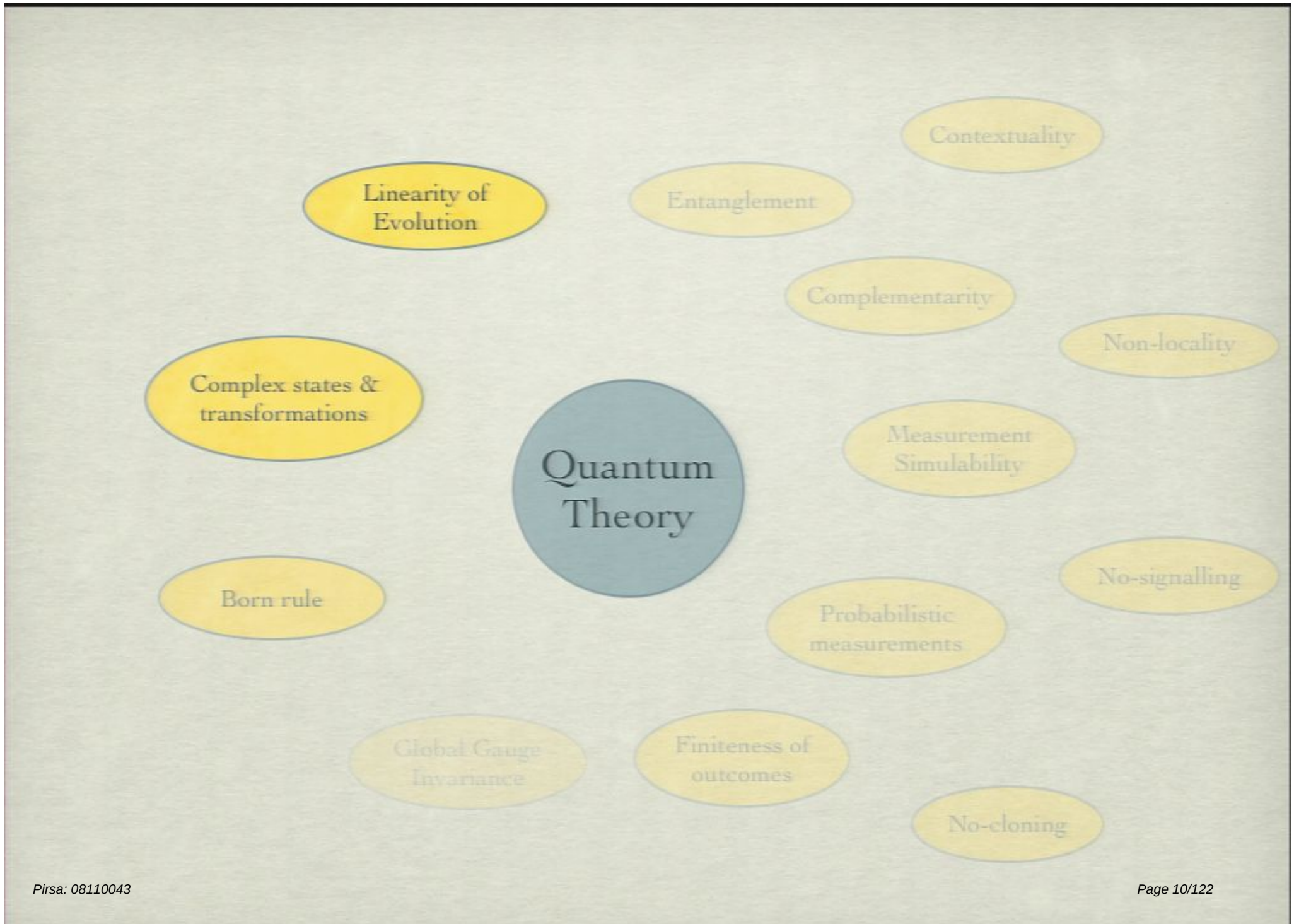
Linearity of Evolution
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Complex states & transformations
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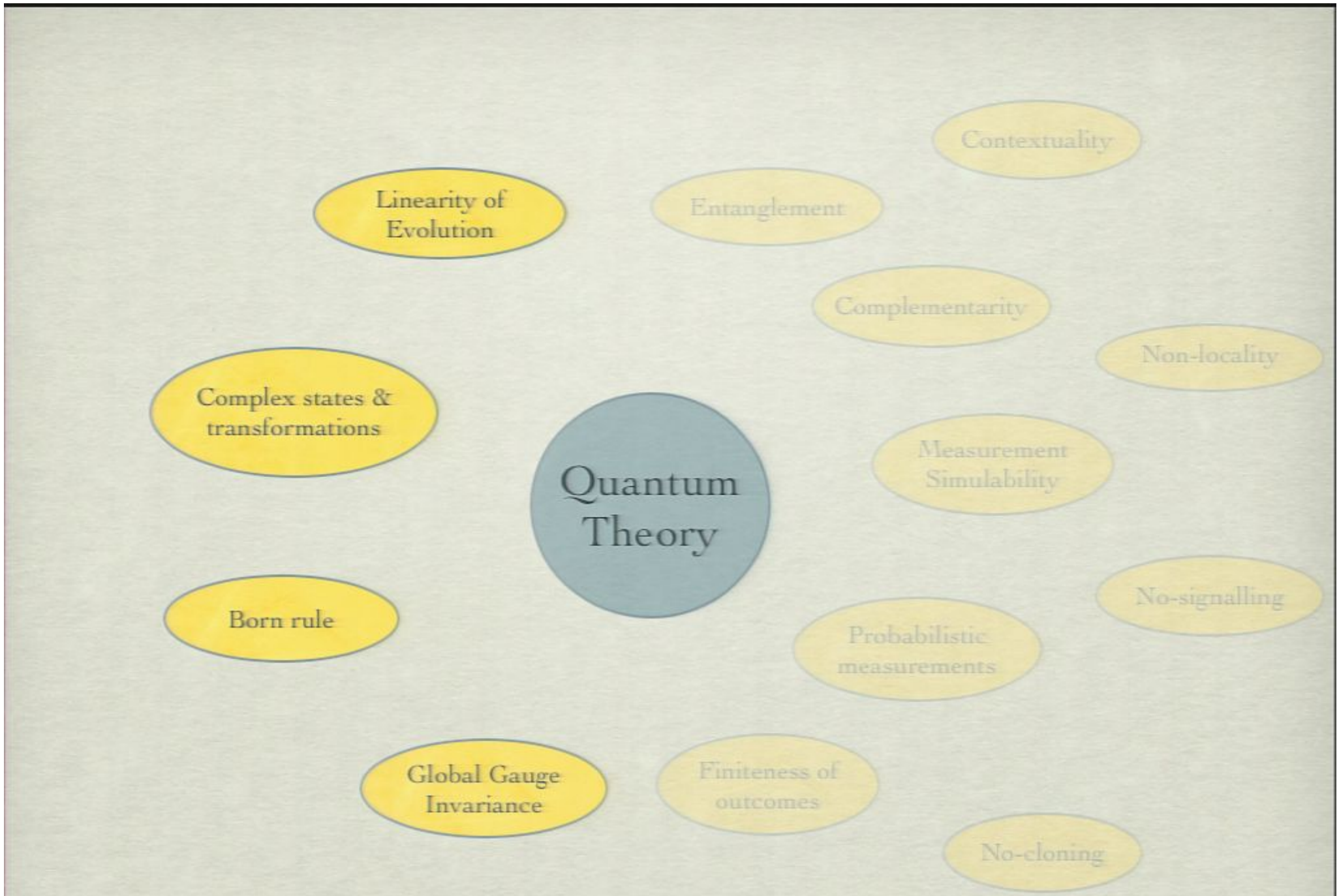
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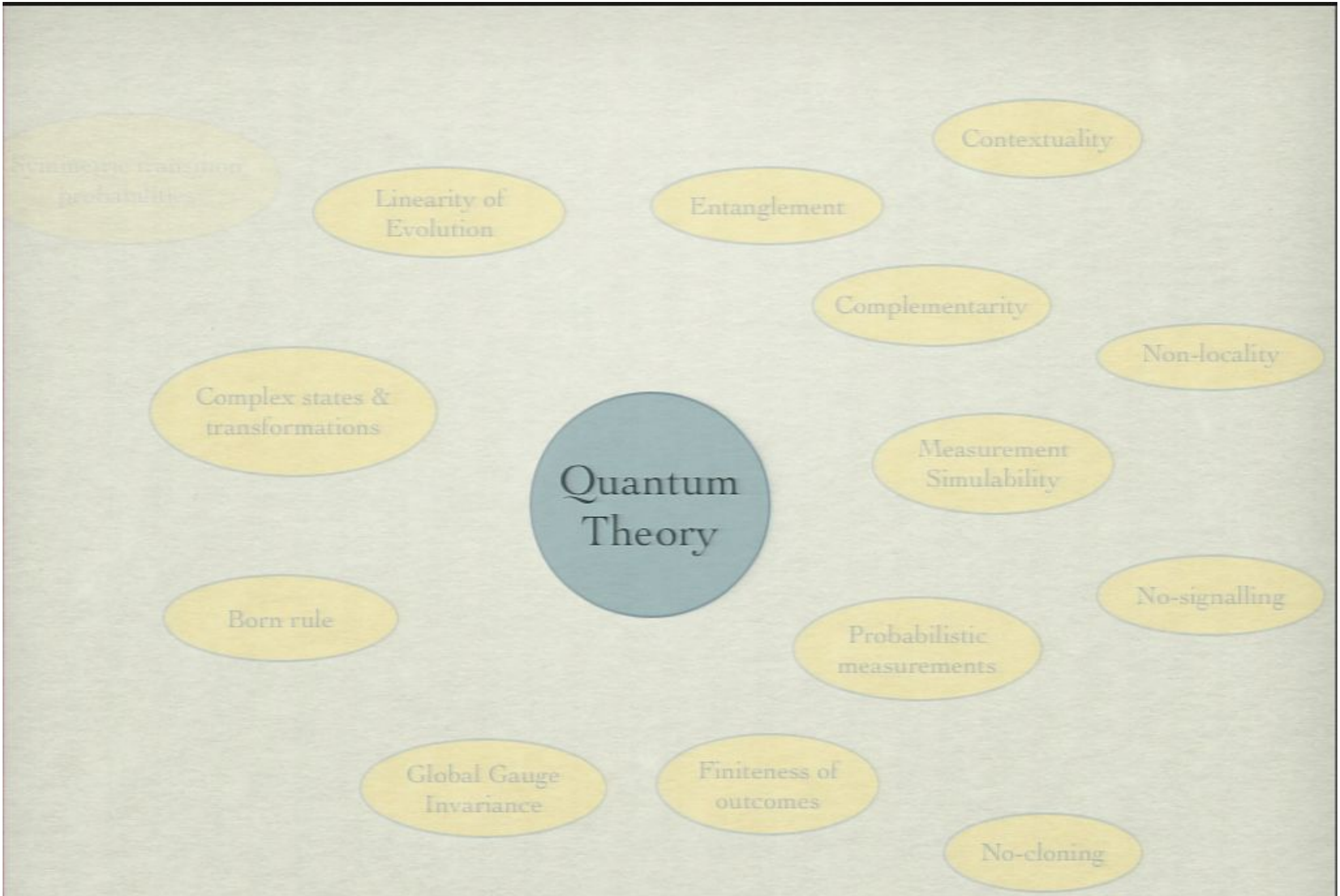
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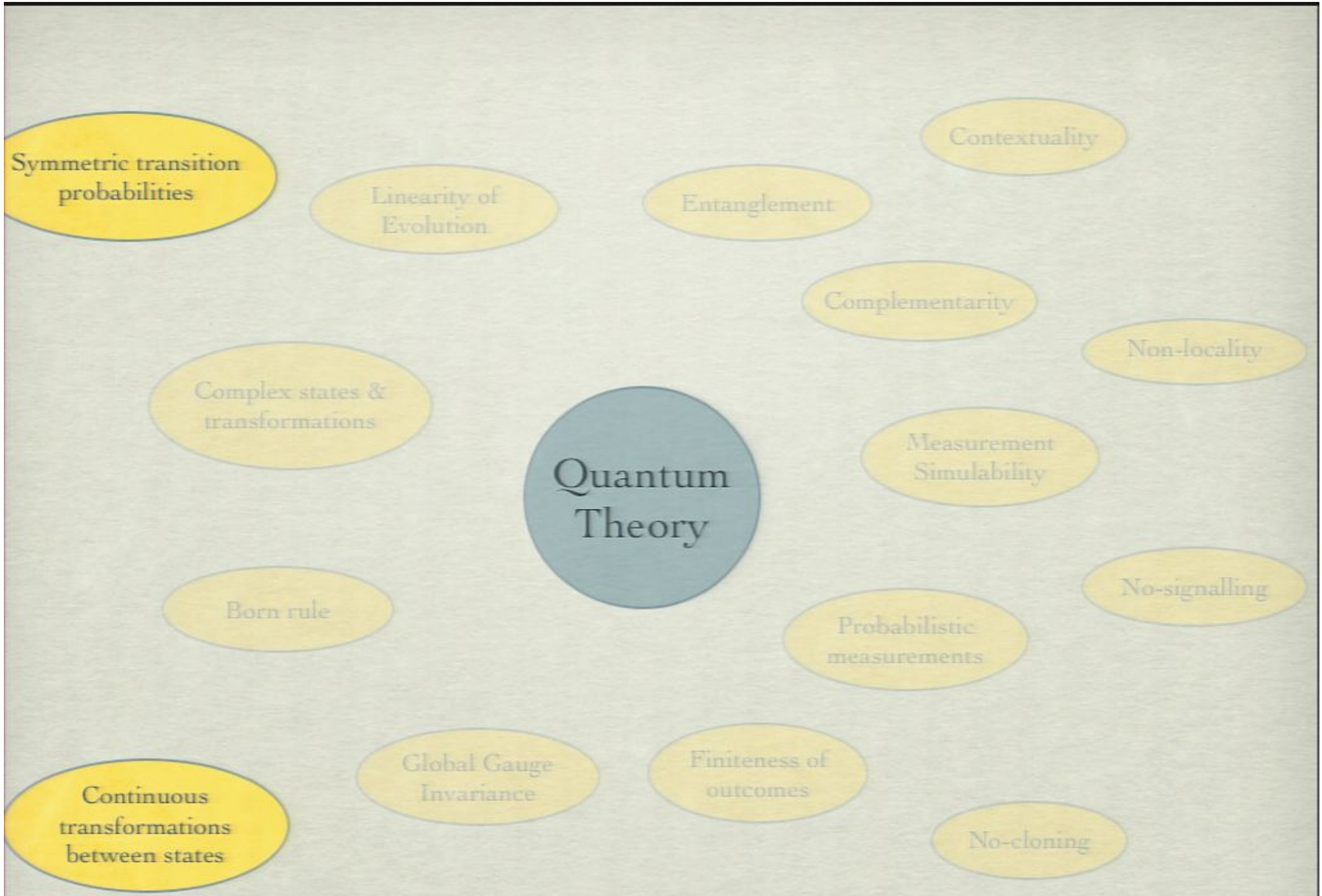
Symmetric transition probabilities
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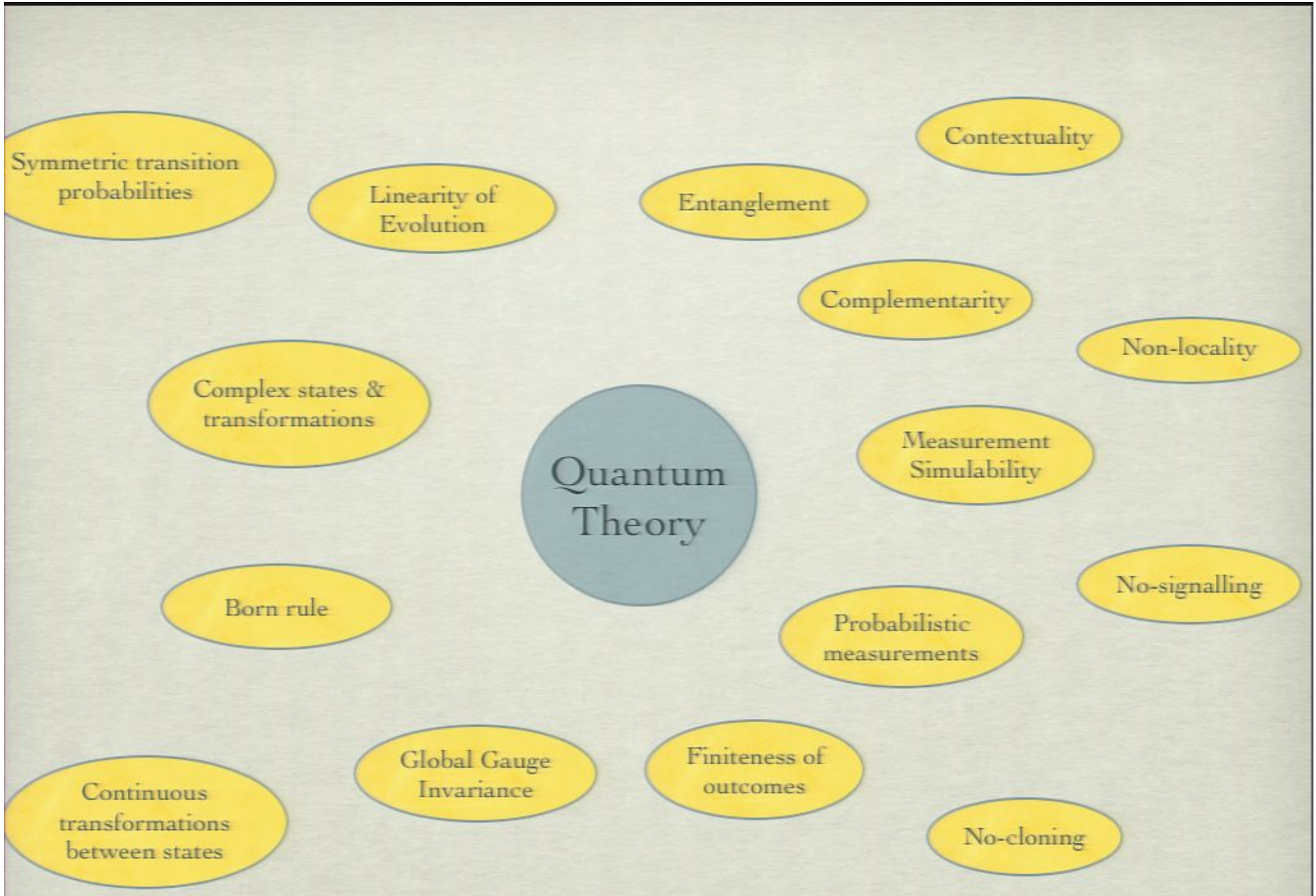
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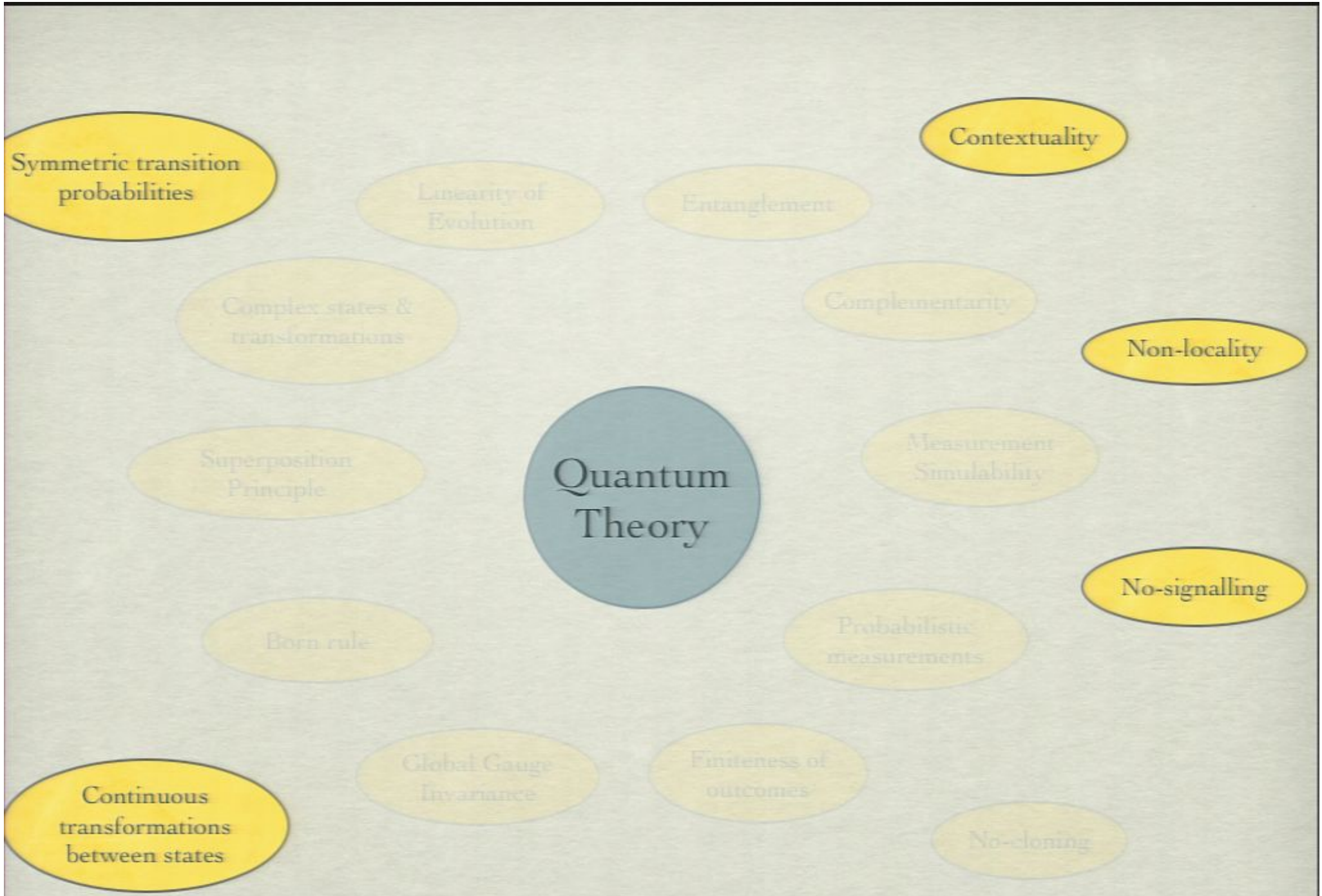
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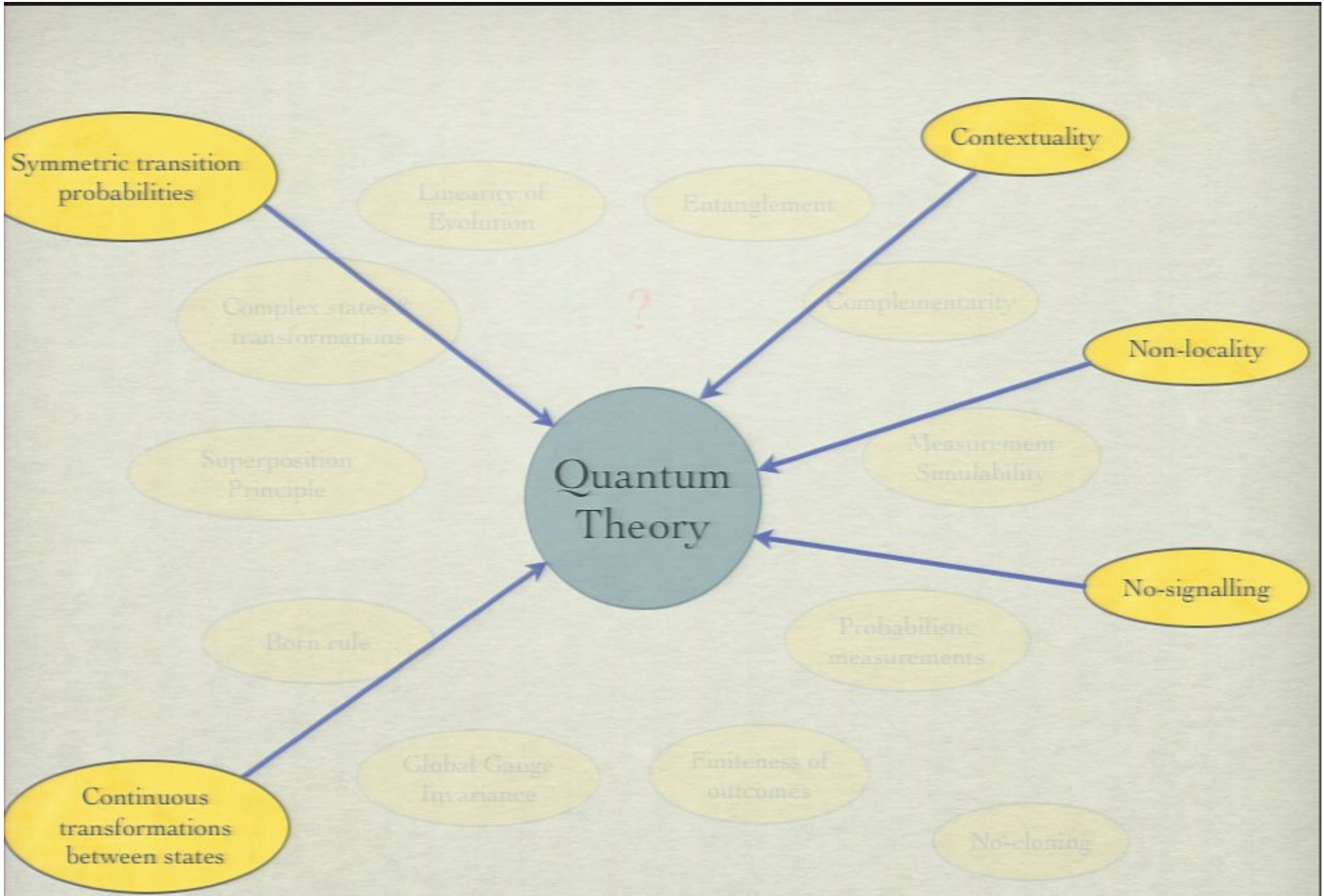
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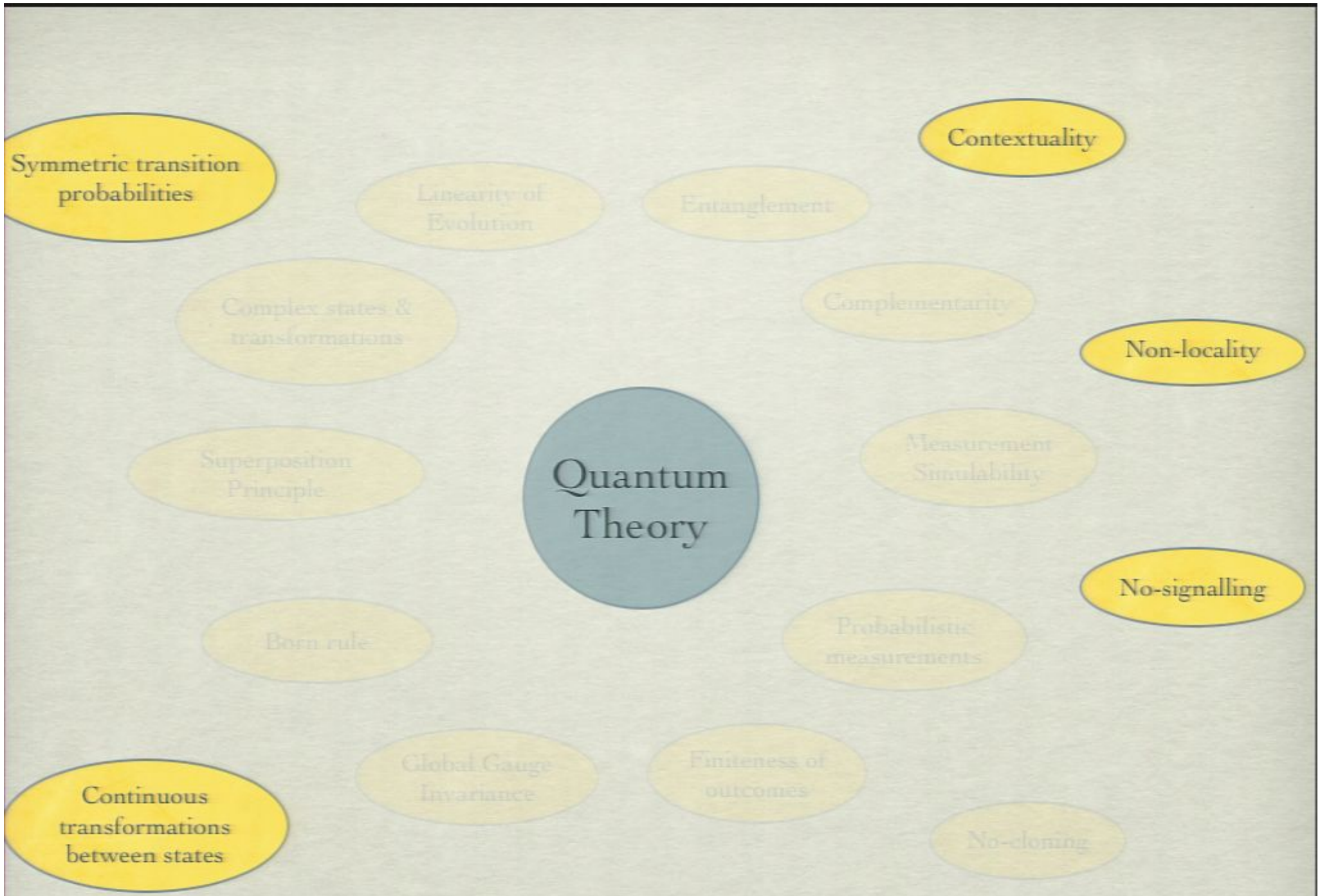
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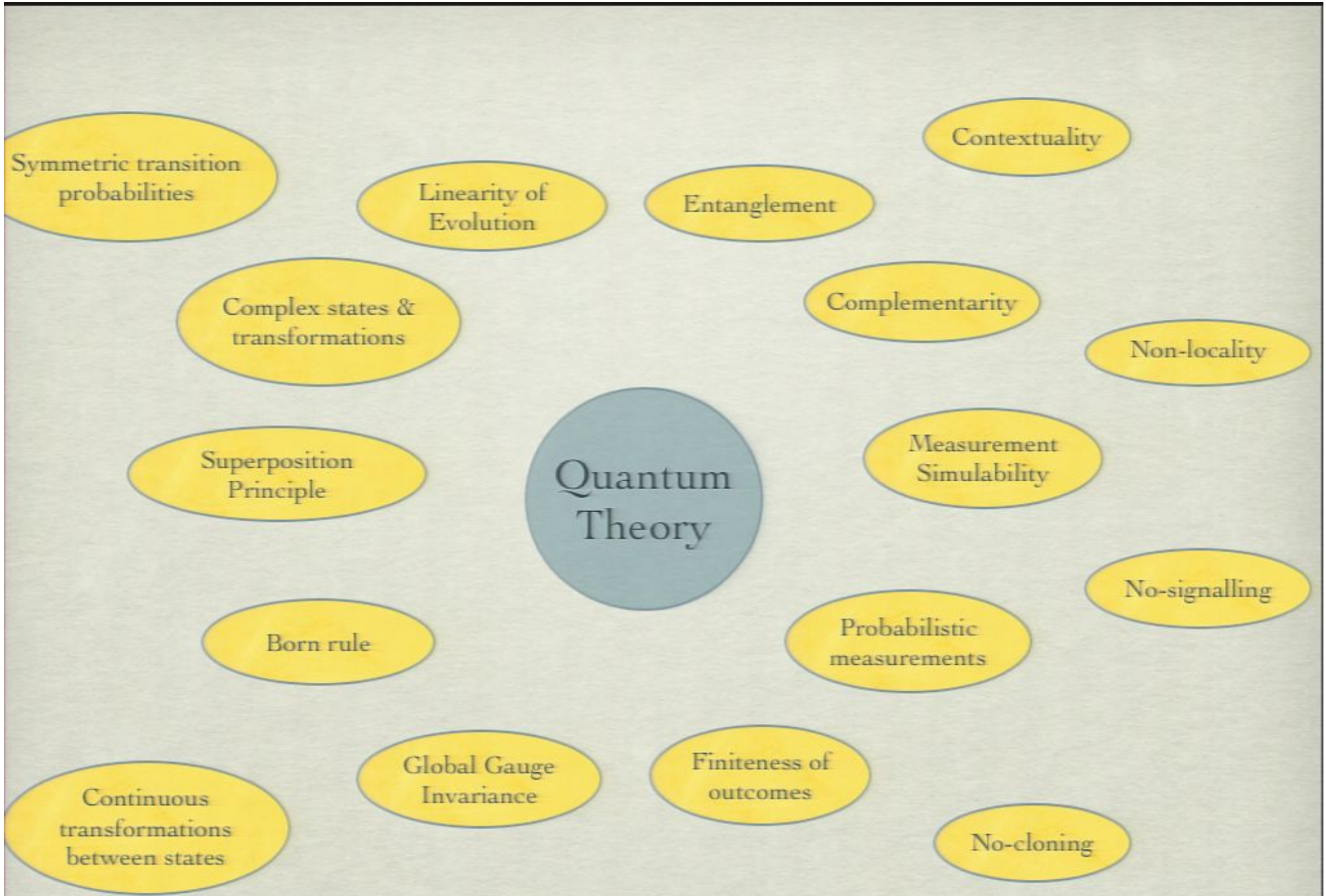
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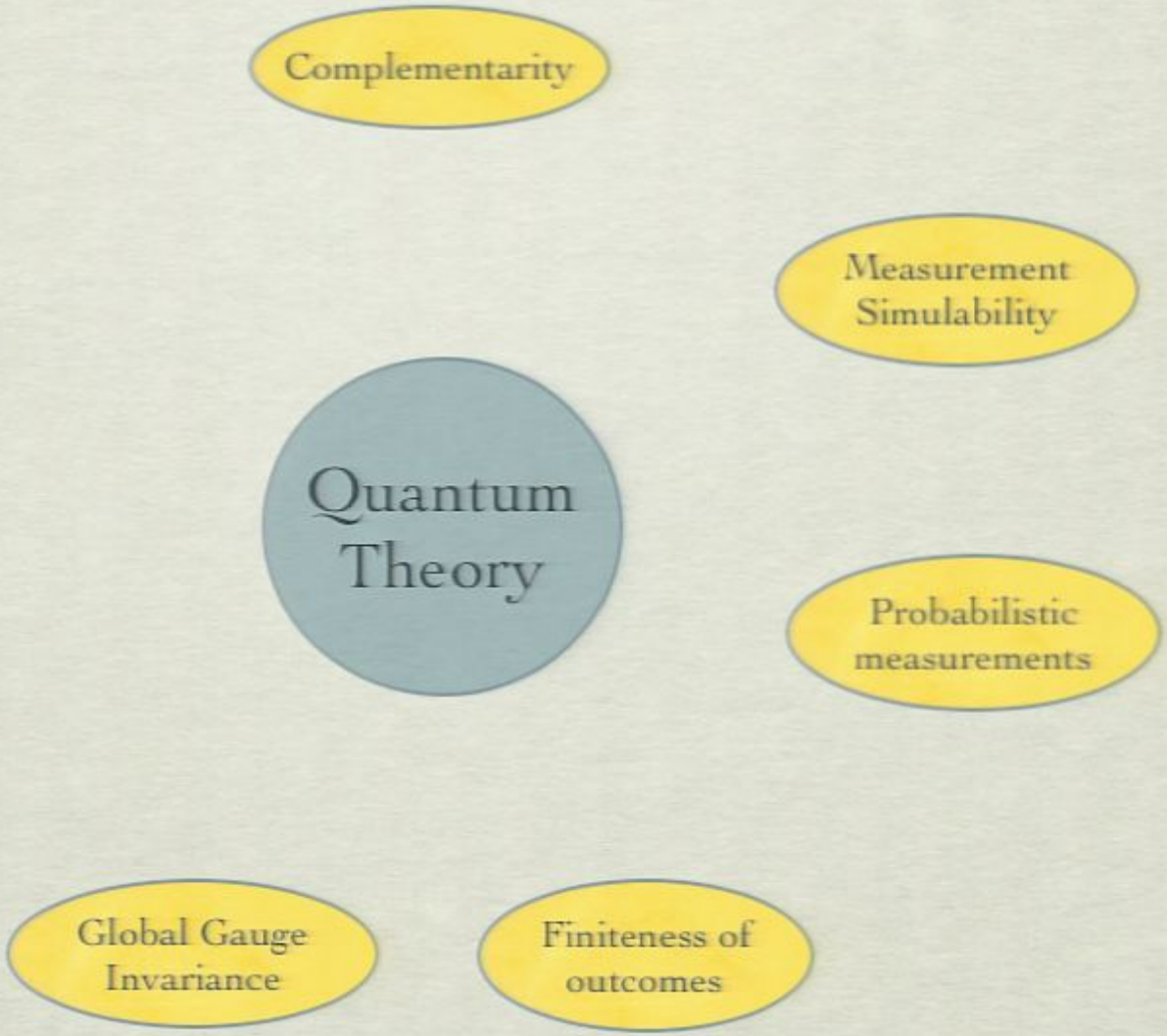
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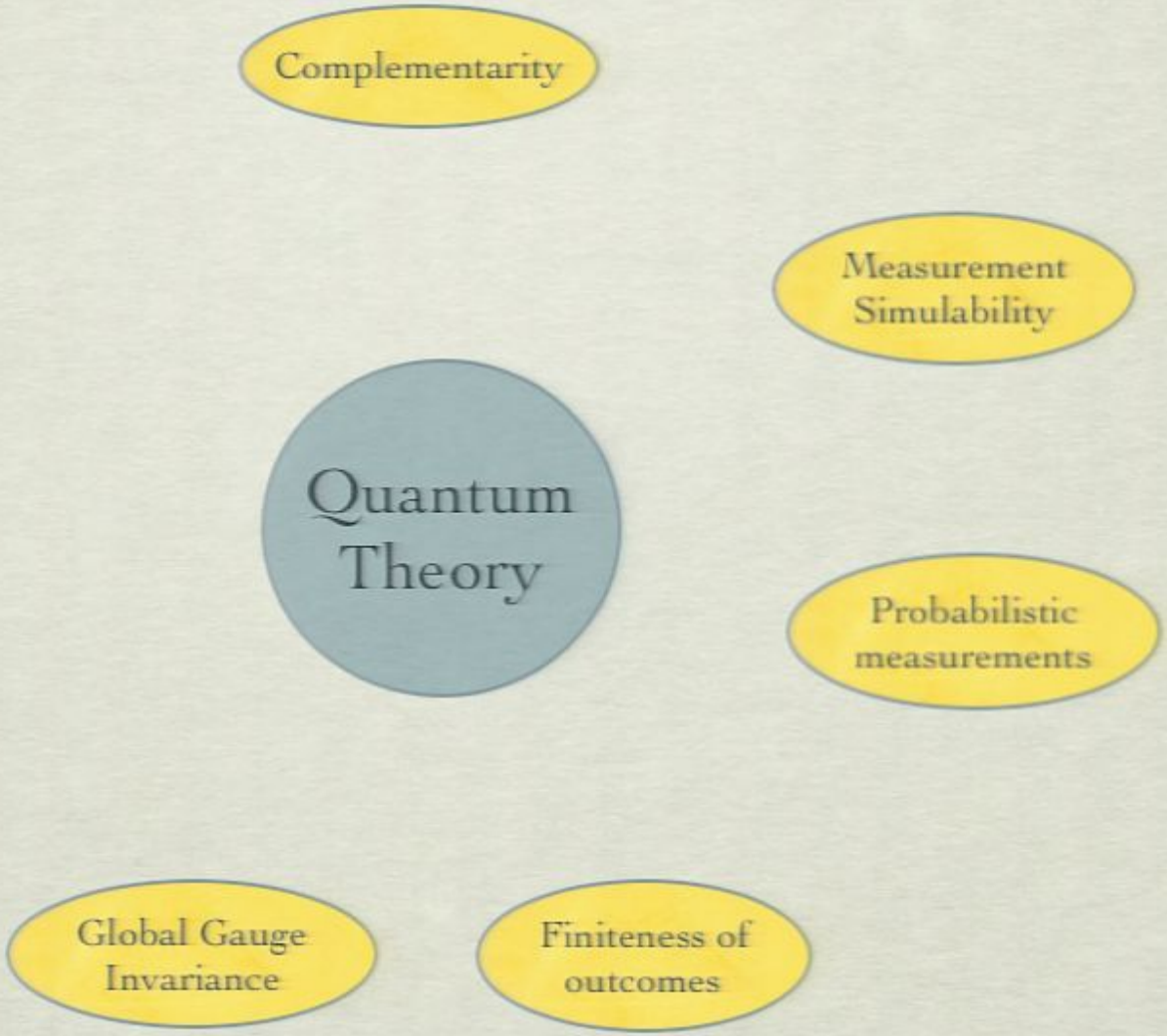
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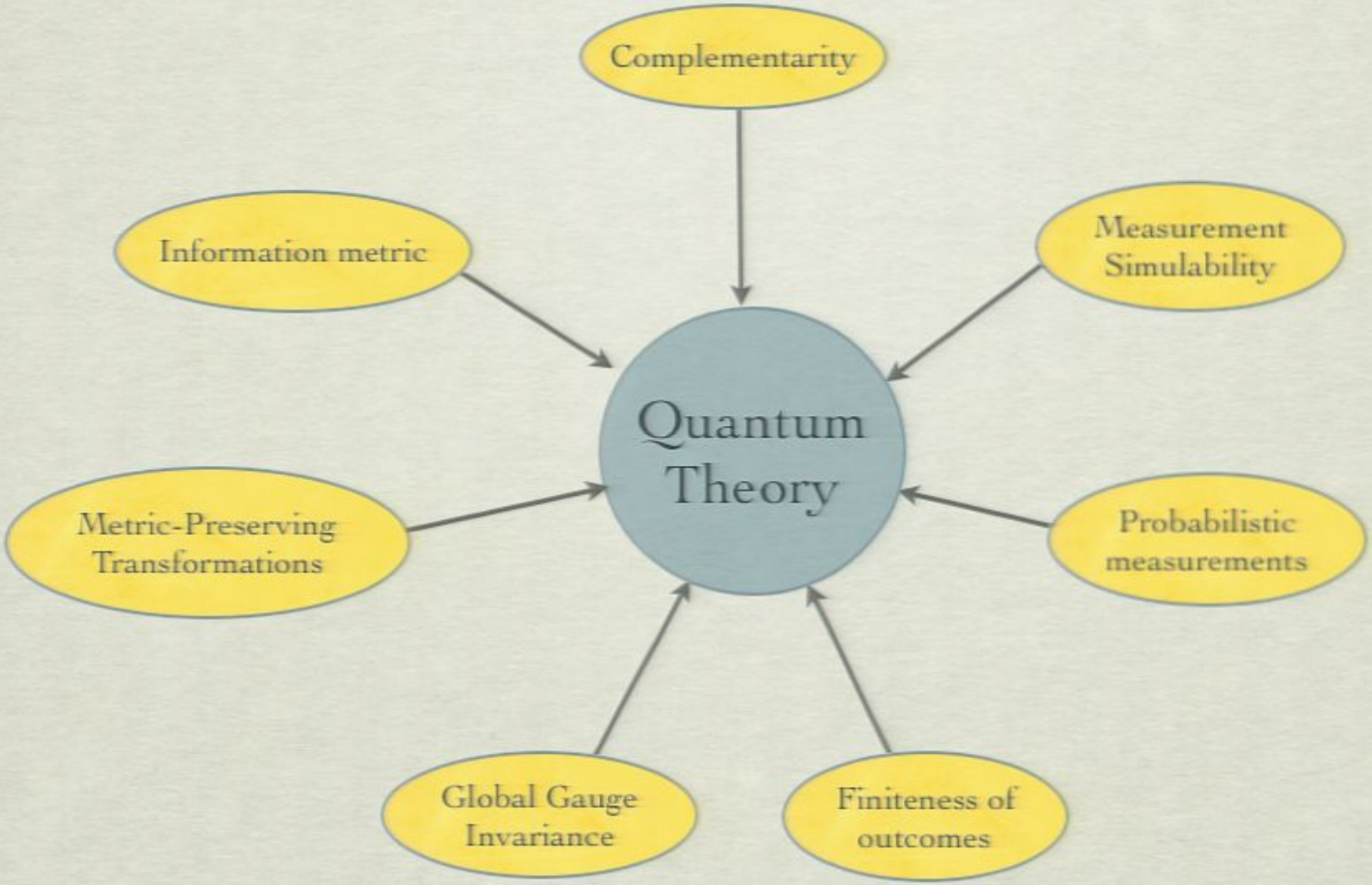
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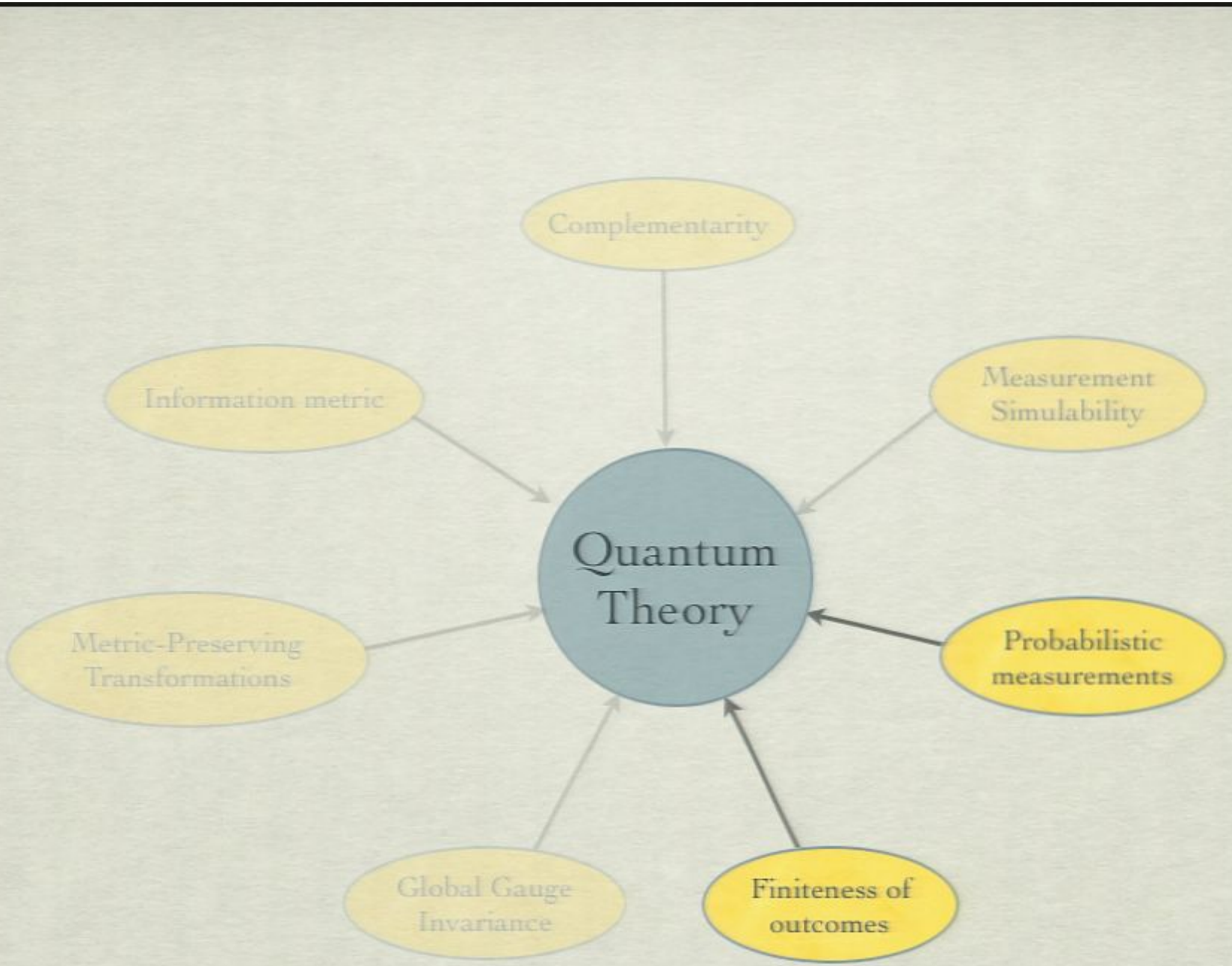
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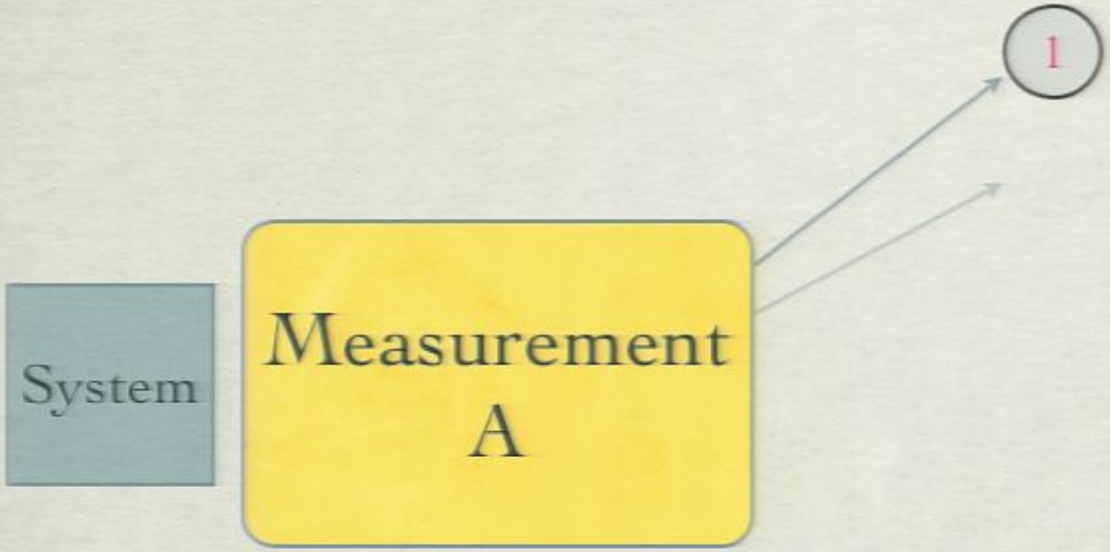
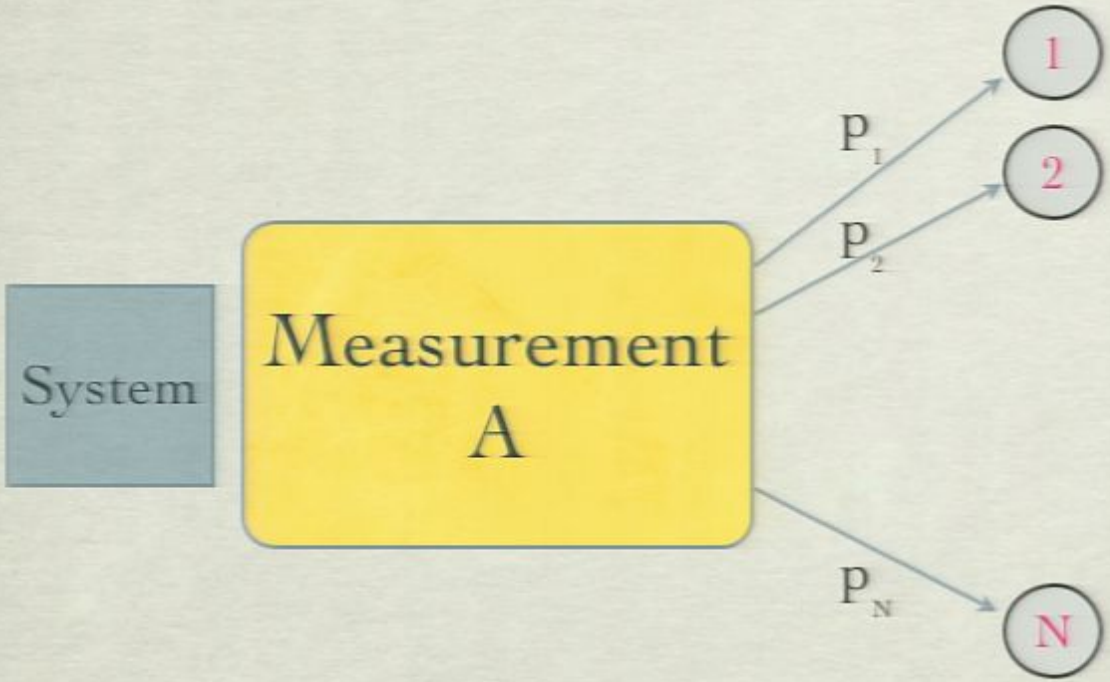
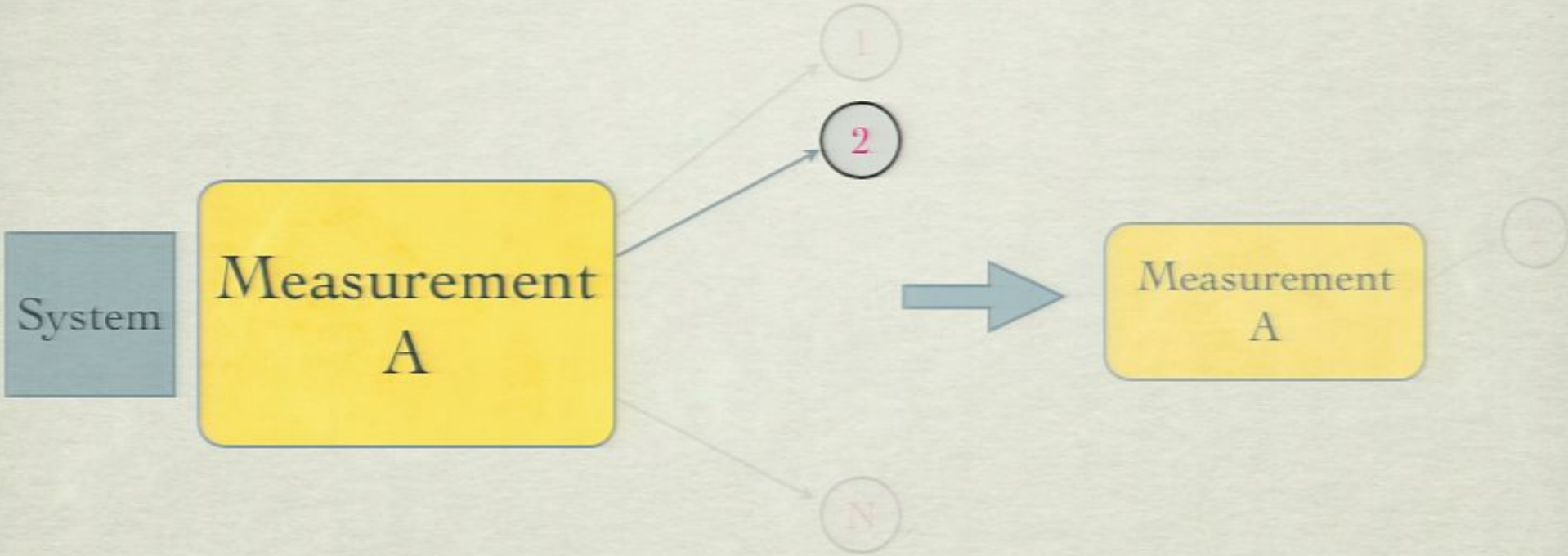
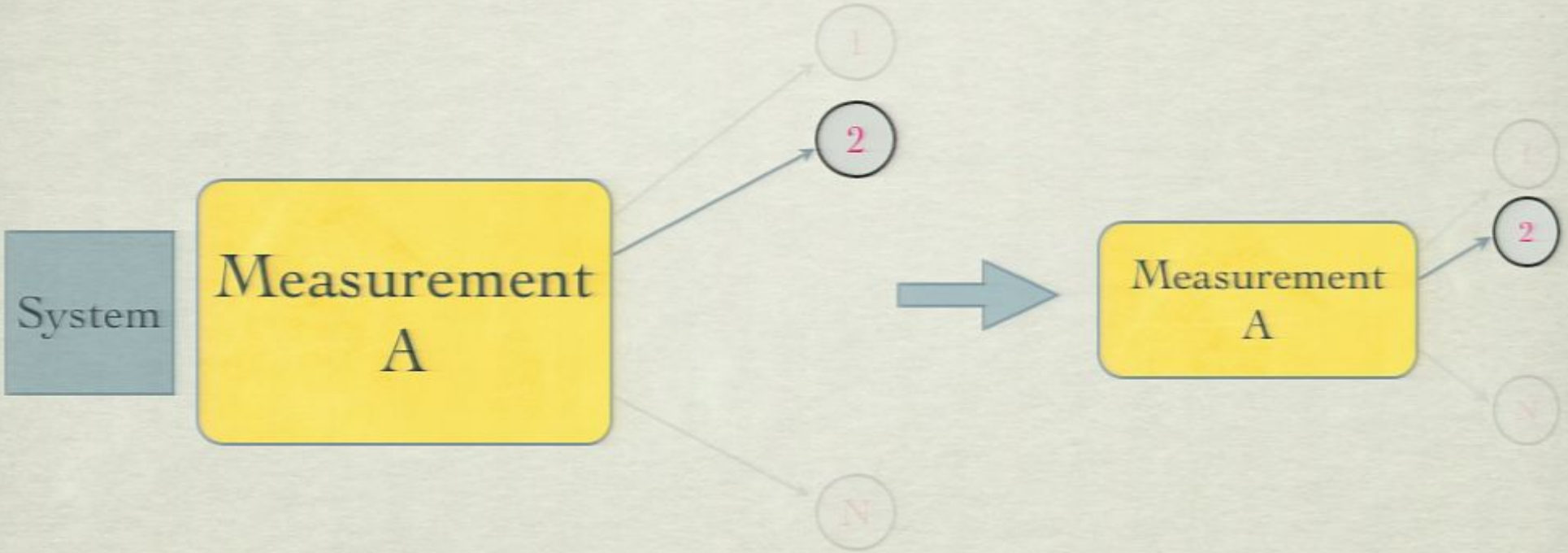
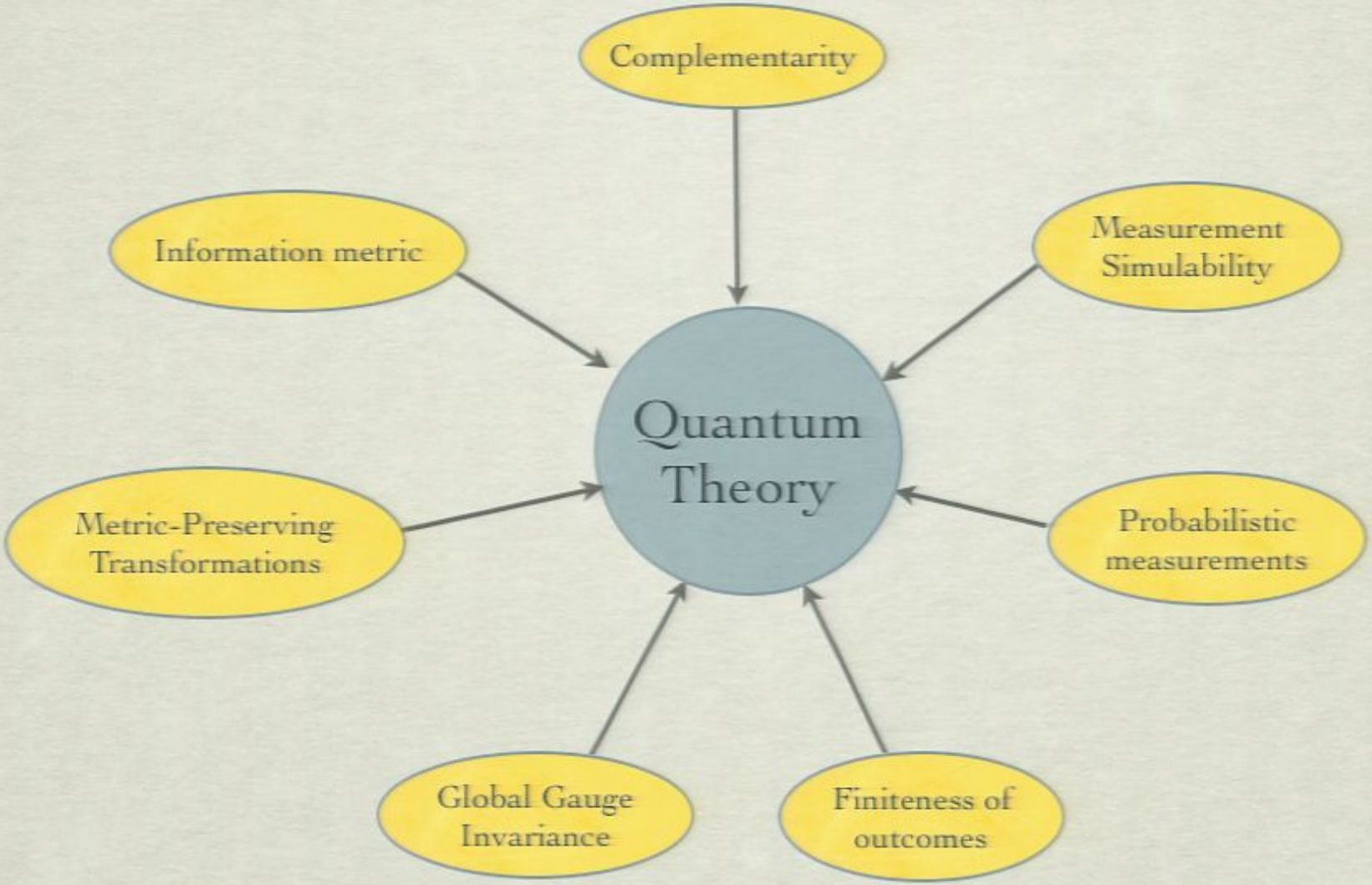
System


1
System
Measurement
A


System
Measurement A
p_1
p_2
p_N
1
2
N


System
Measurement A
1
2
N
Measurement A


System
Measurement A
1
2
N
Measurement A
1
2
N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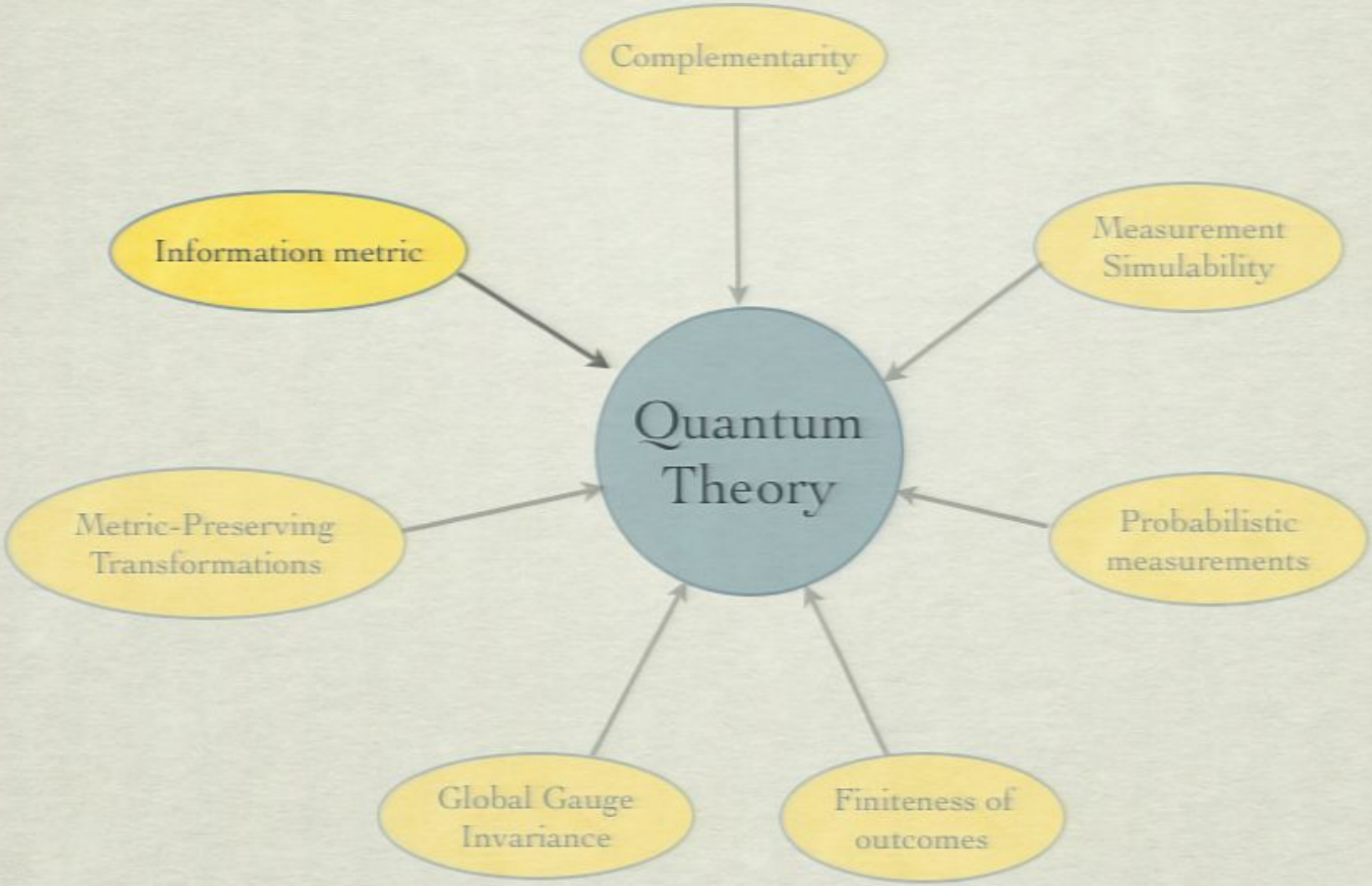
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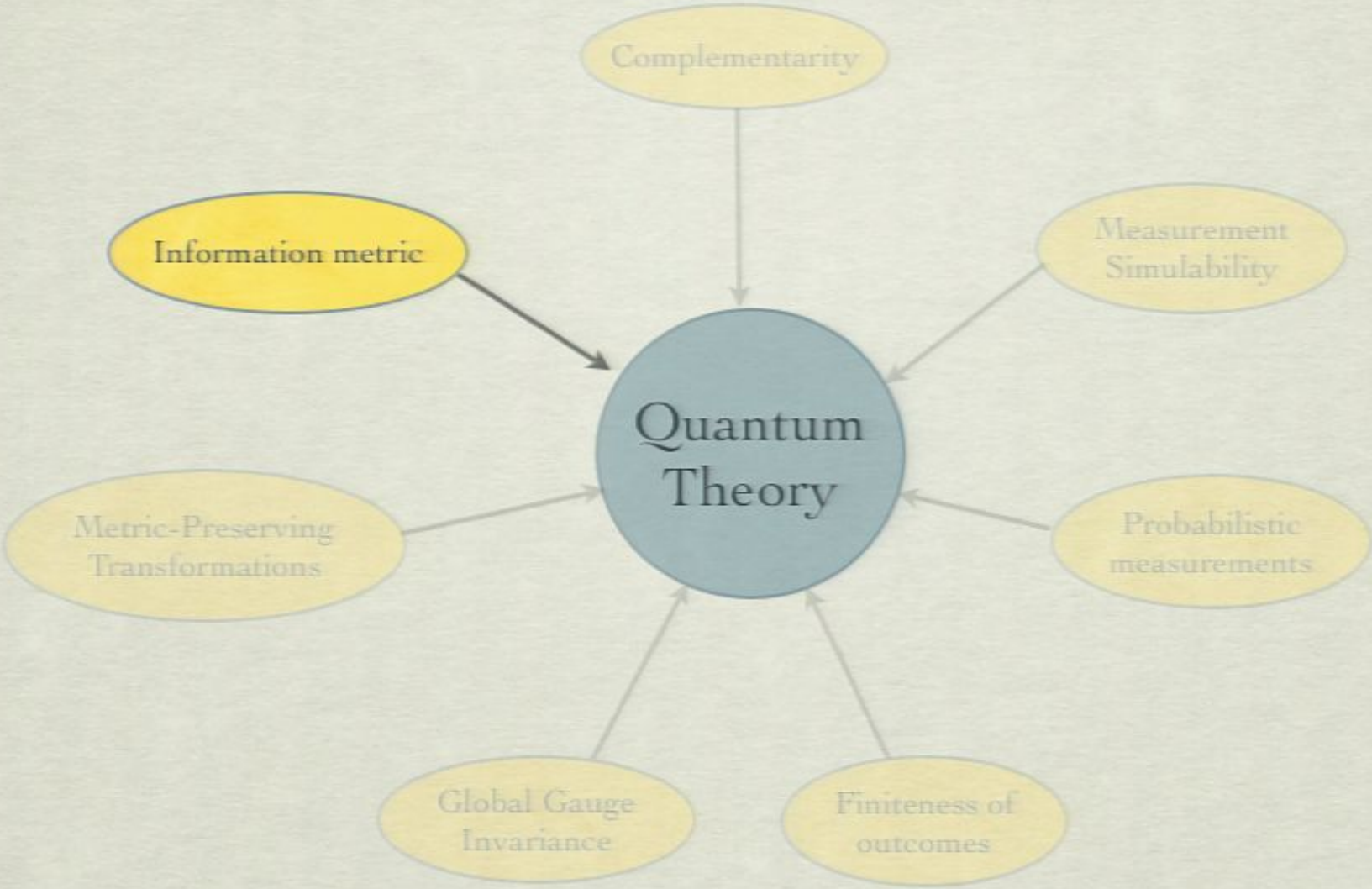
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Coin A: $\mathbf{p} = (p_1, p_2)$

Coin B: $\mathbf{p}' = (p'_1, p'_2)$

Coin A: $\mathbf{p} = (p_1, p_2)$

Coin B: $\mathbf{p}' = (p'_1, p'_2)$

n tosses

"HTHT.....T"

Coin A: $\mathrm{p} = (p_1, p_2)$

Coin B: $\mathrm{p}' = (p_1', p_2')$

n tosses

"HTHT.....T"

Coin A: $\mathbf{p} = (p_1, p_2)$

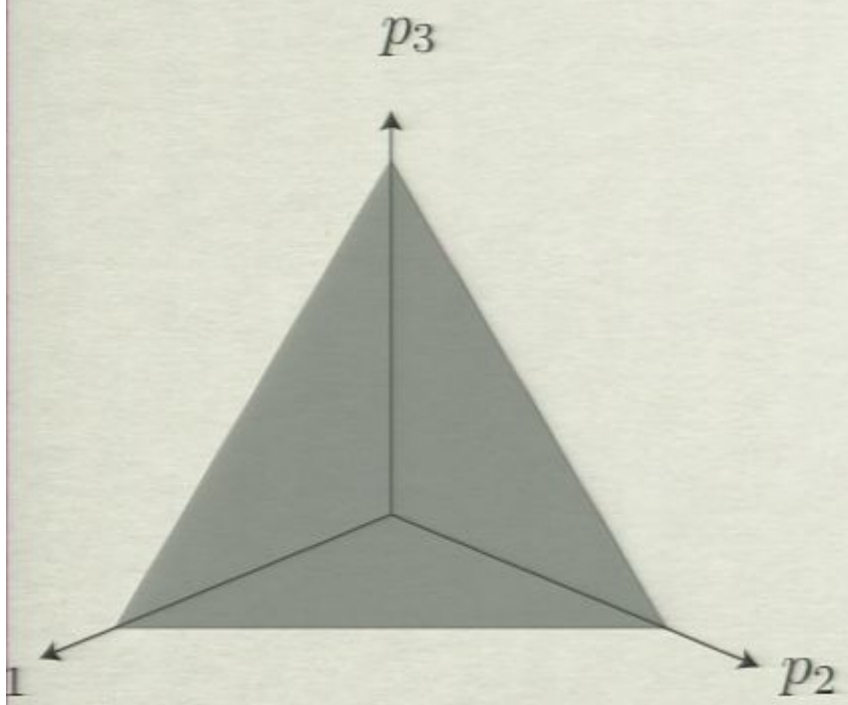
Coin B: $\mathbf{p}' = (p_1', p_2')$

n tosses

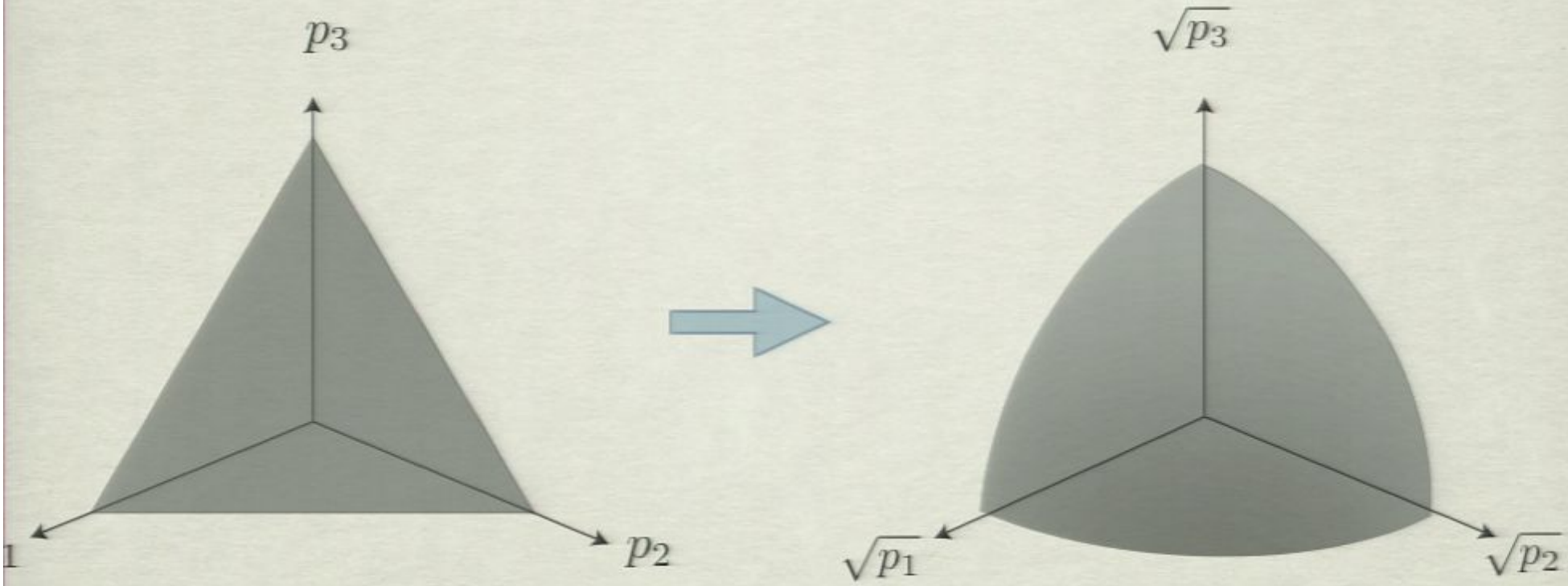
"HTHT.....T"

Information gained $= f(n\,ds^2)$, when $\mathbf{p}' = \mathbf{p} + d\mathbf{p}$

where $ds^2 = \frac{1}{4}\sum_{i=1}^{2} \frac{dp_i^2}{p_i}$



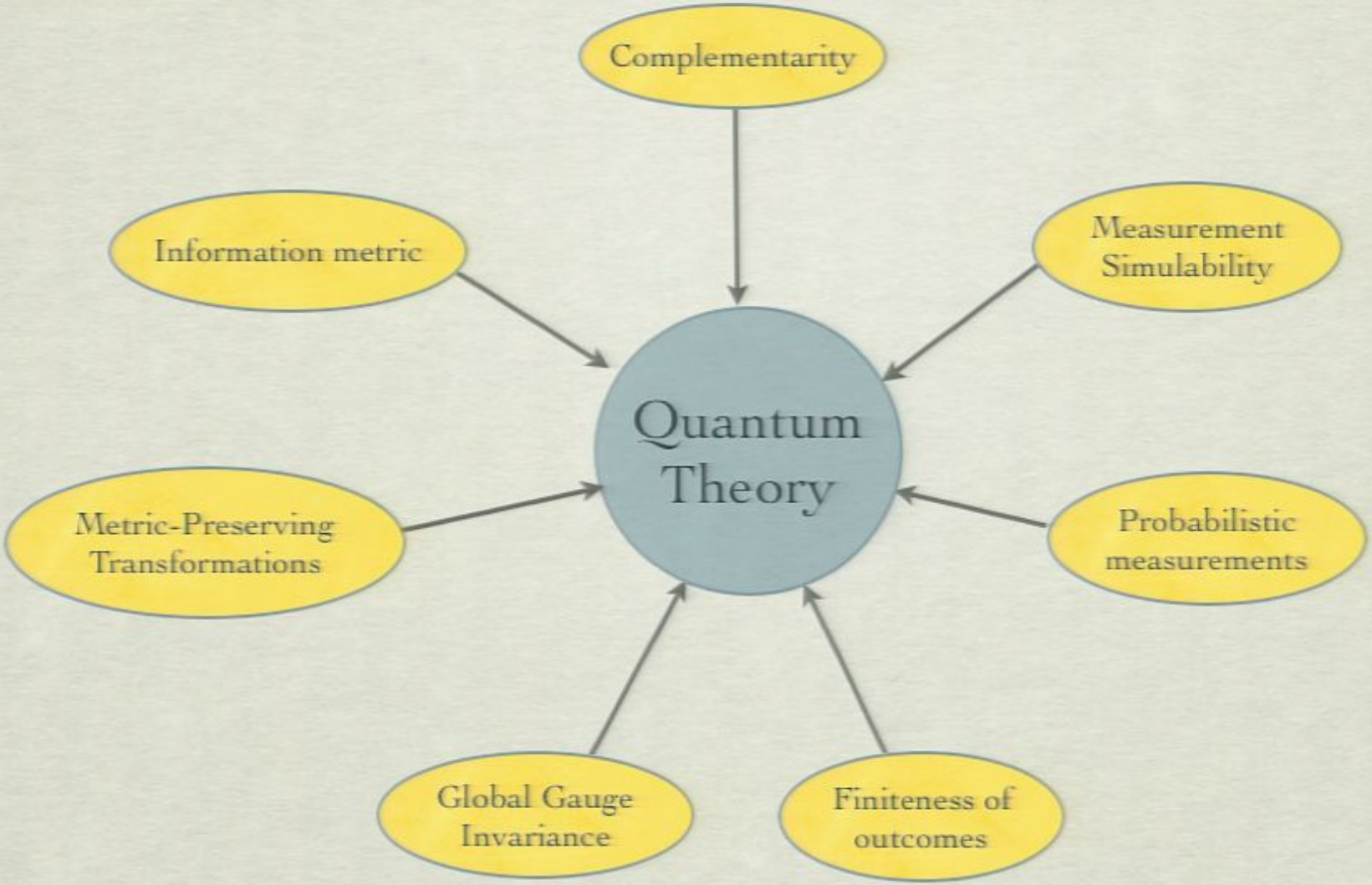
$$ds^2 = \frac{1}{4}\sum_i \frac{dp_i^2}{p_i}$$



$$ds^2 = \frac{1}{4}\sum_i \frac{dp_i^2}{p_i}$$

$$ds^2 = \sum_i dq_i^2$$

where $q_i = \sqrt{p_i}$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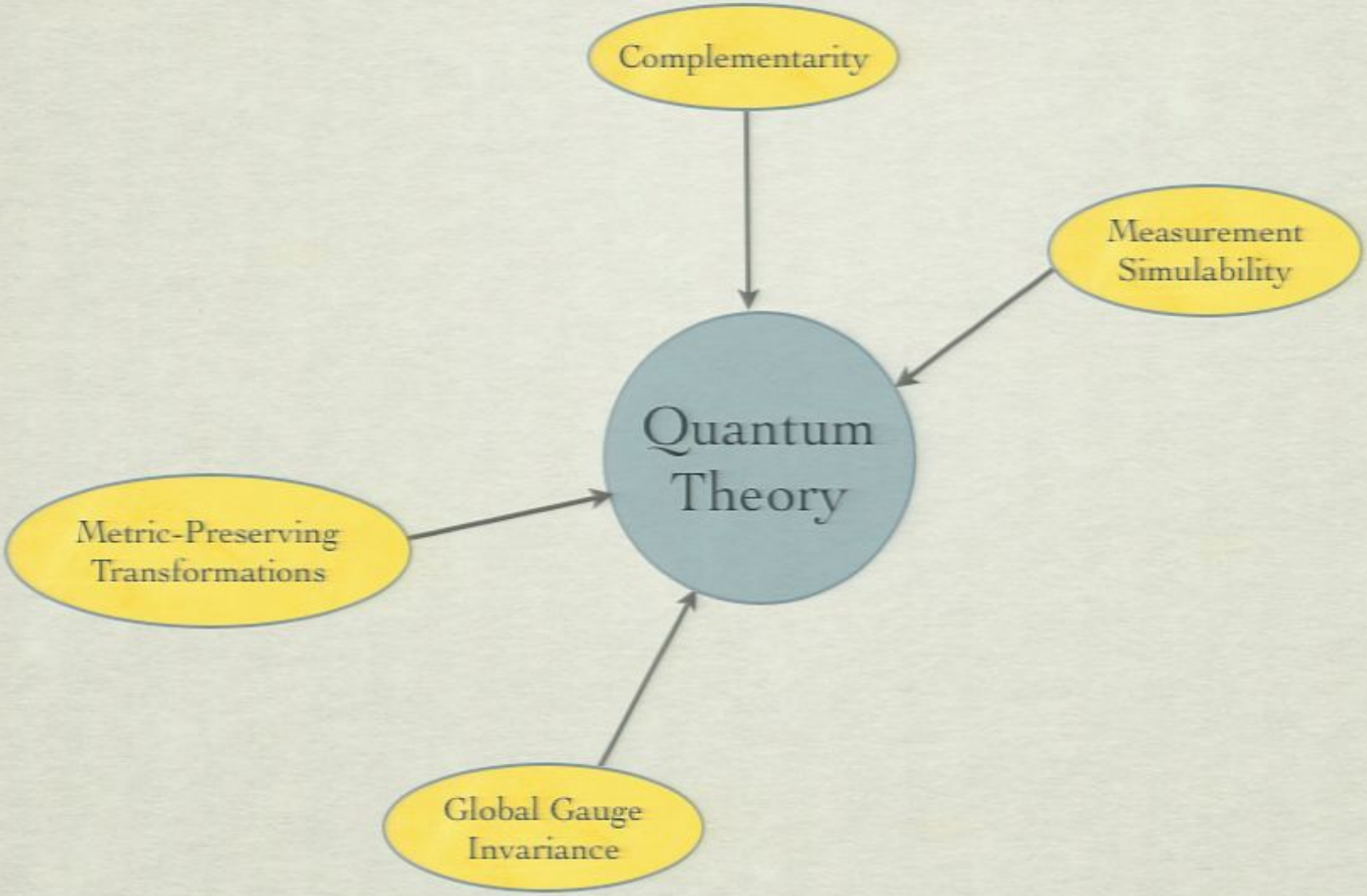
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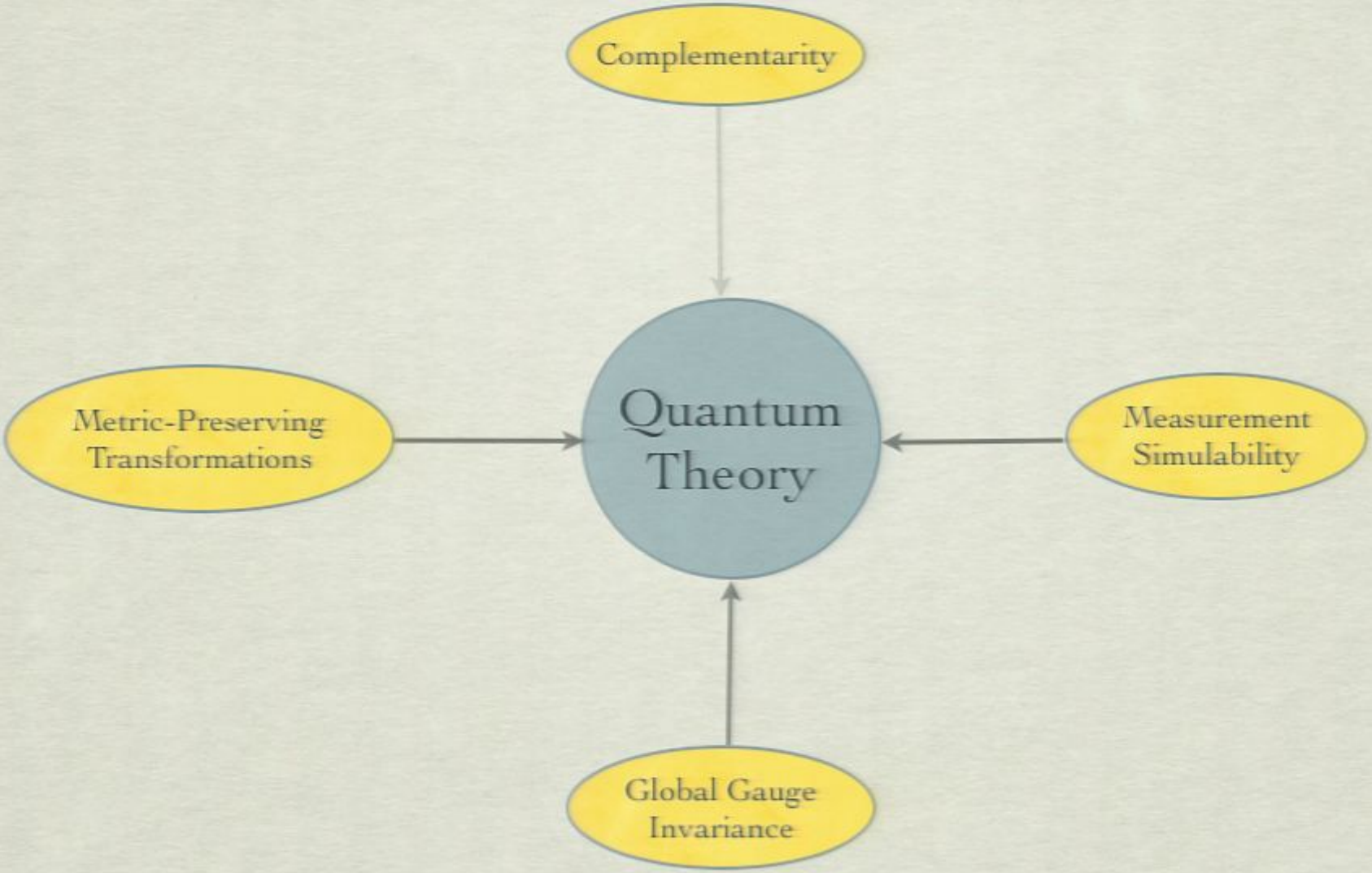
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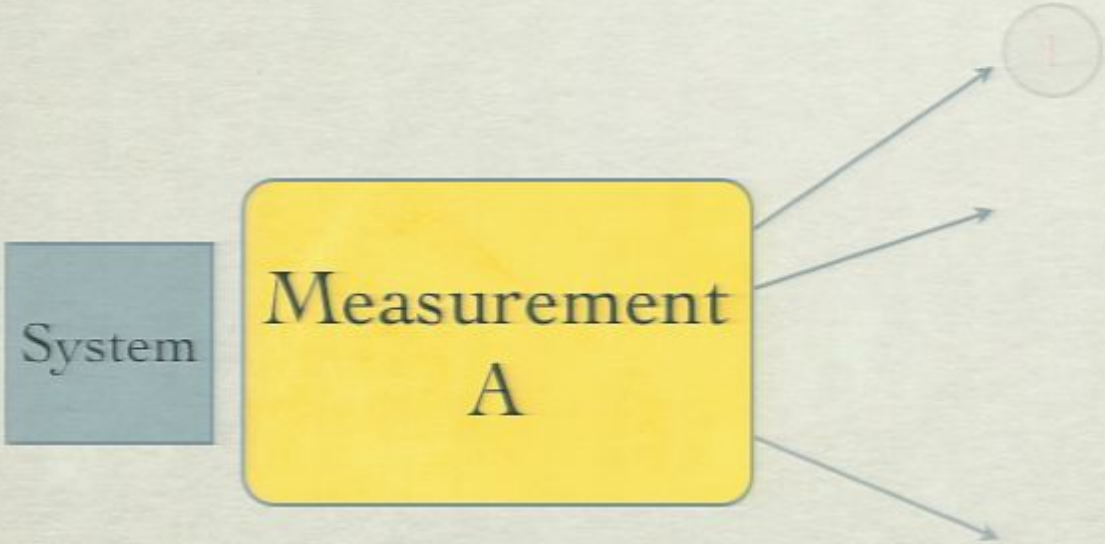
Complementarity

Complementarity



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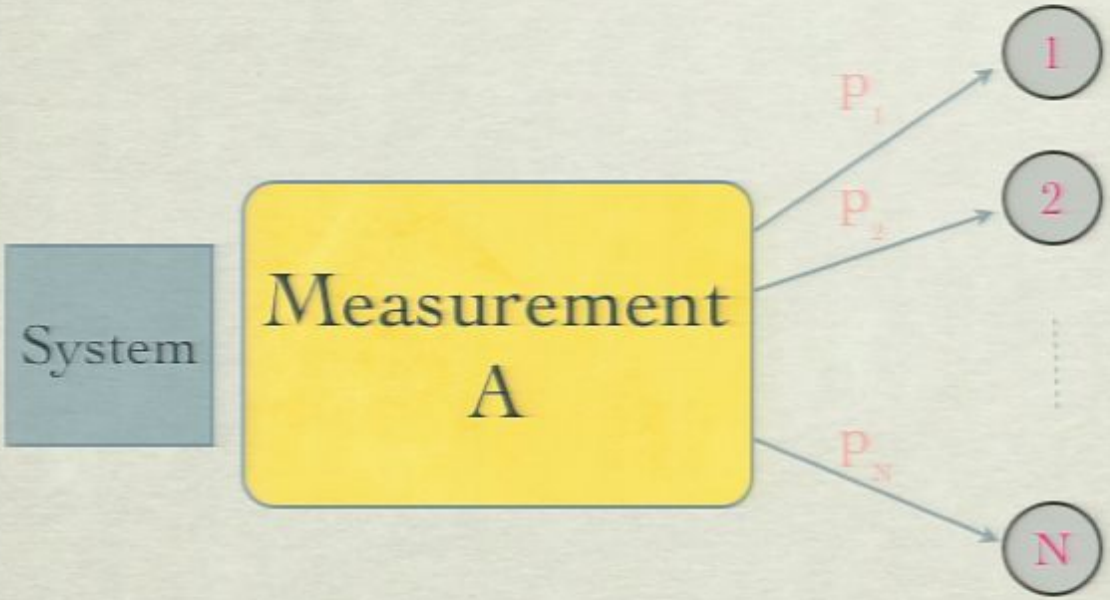

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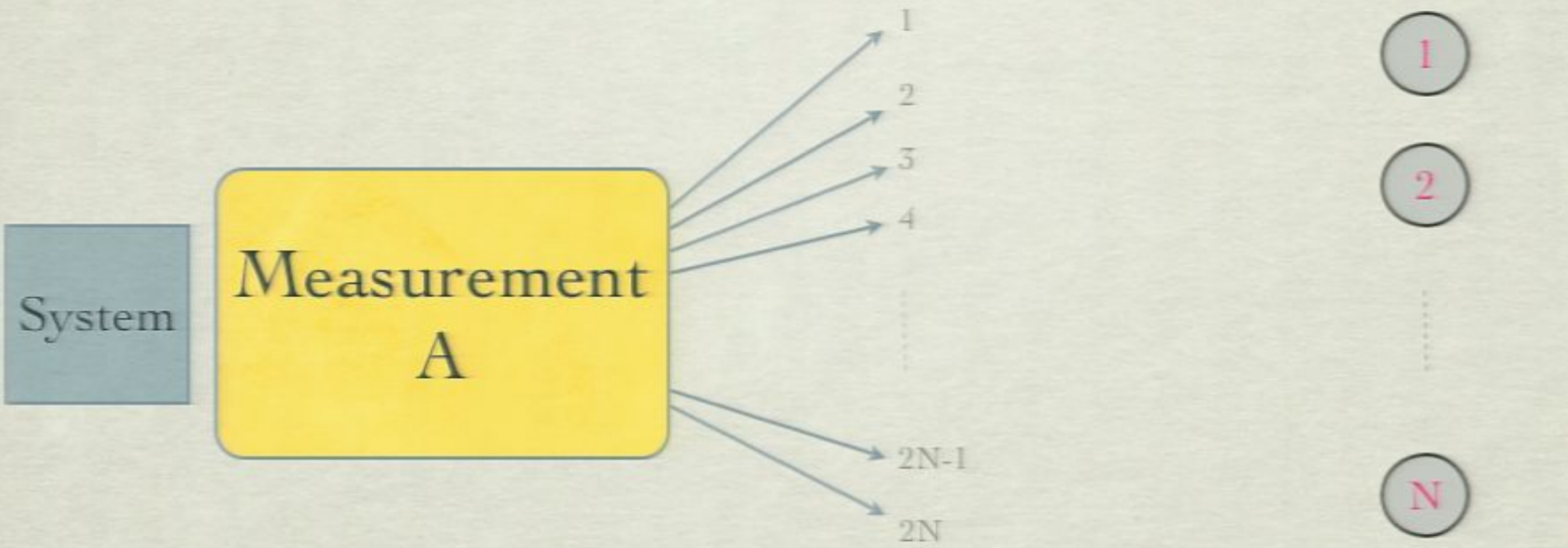

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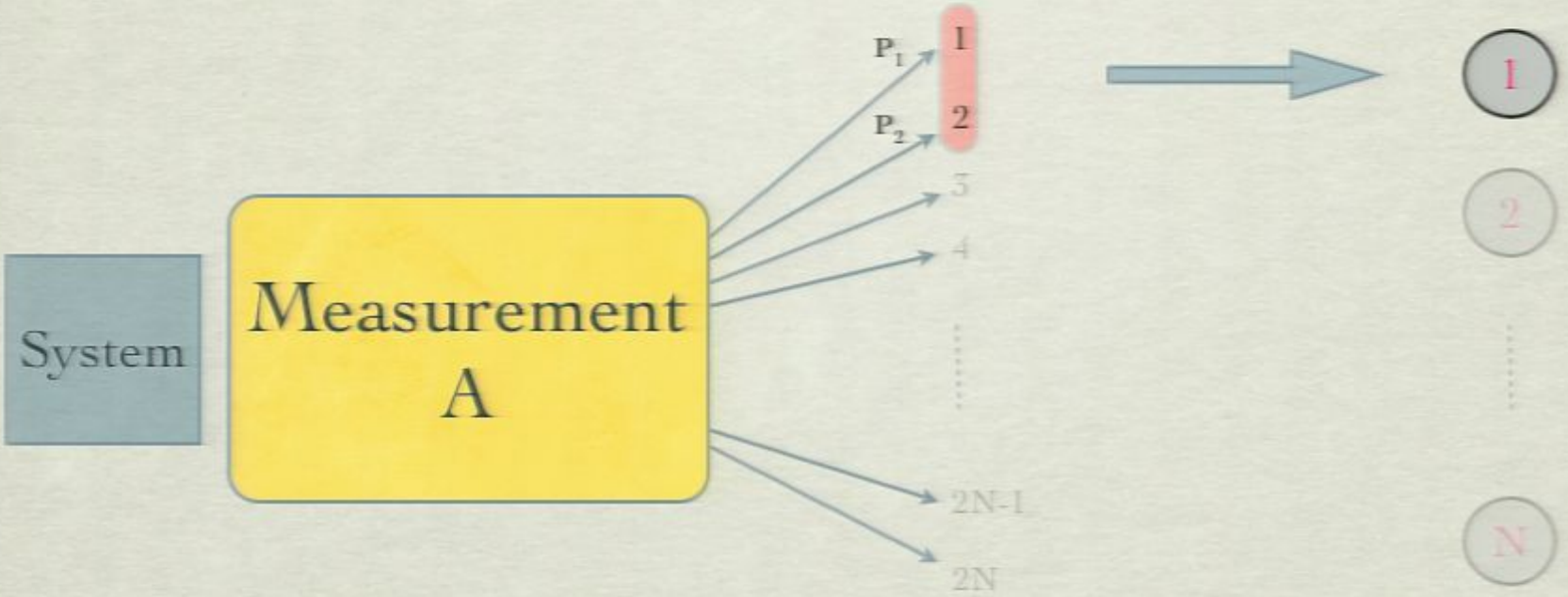

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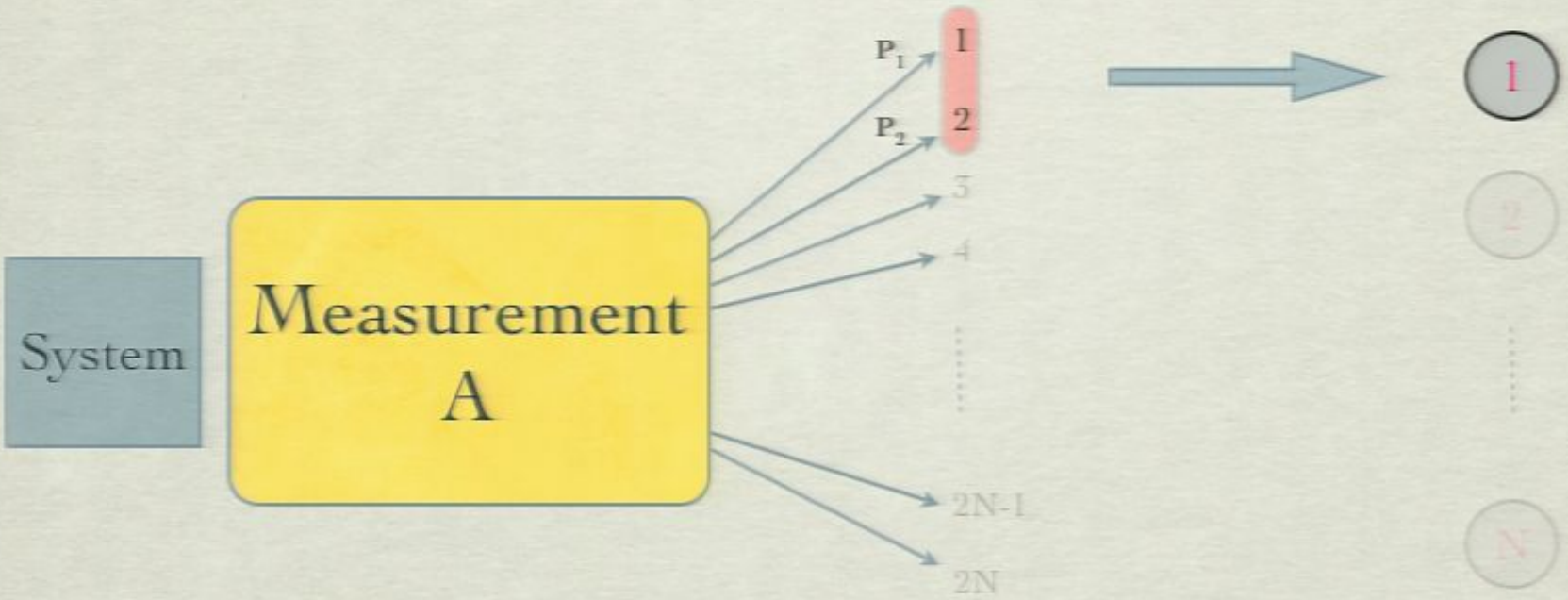
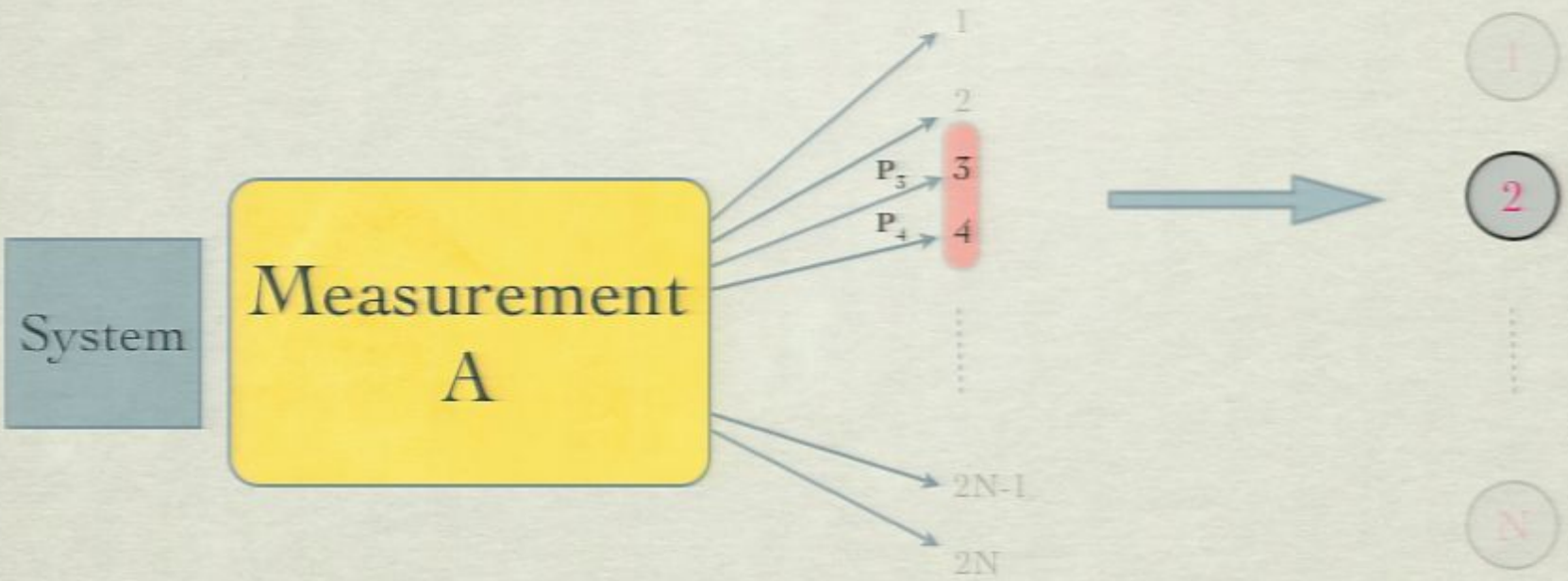
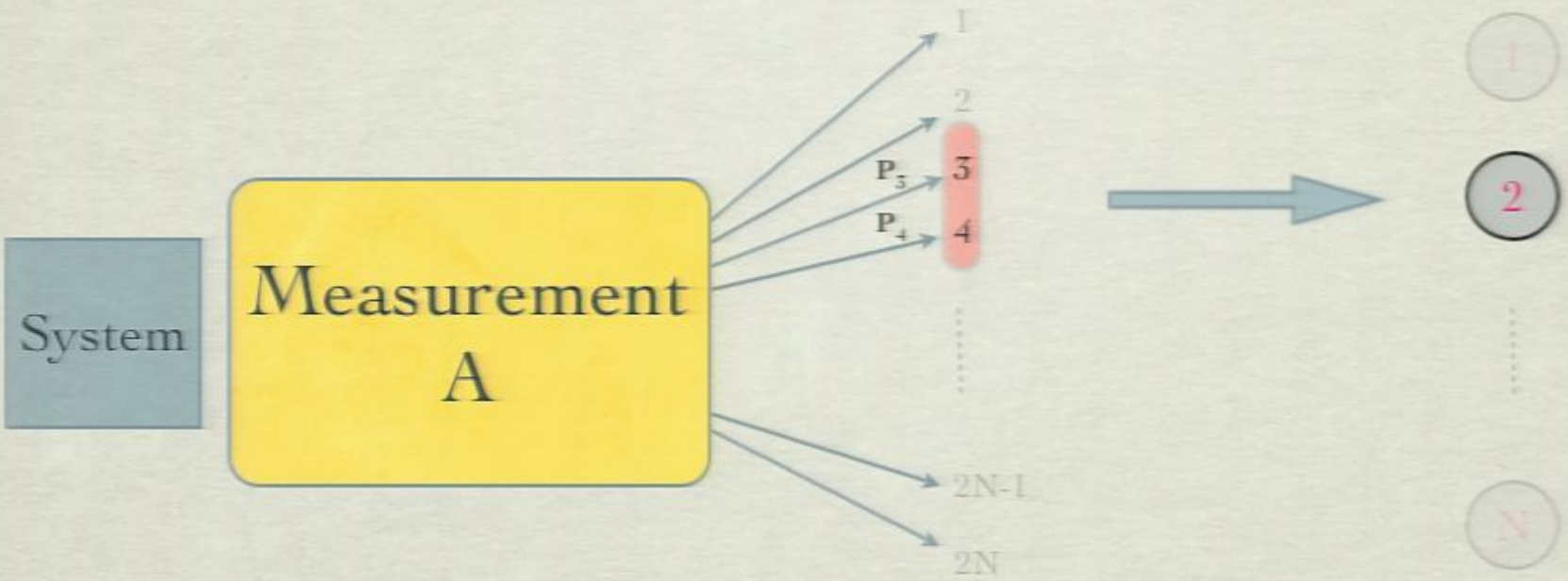


$$P_1 = P_1 + P_2$$

Complementarity



Complementarity



$$P_2 = P_3 + P_4$$

$$\mathbf{P} = \begin{pmatrix} P_1 \\ P_2 \\ \vdots \\ P_{2N-1} \\ P_{2N} \end{pmatrix}$$

$$\mathbf{P} = \begin{pmatrix} P_1 \\ P_2 \\ \vdots \\ P_{2N-1} \\ P_{2N} \end{pmatrix}$$

$$\mathbf{Q} = \begin{pmatrix} \sqrt{P_1} \\ \sqrt{P_2} \\ \vdots \\ \sqrt{P_{2N-1}} \\ \sqrt{P_{2N}} \end{pmatrix}$$

$$\mathbf{P} = \begin{pmatrix} P_1 \\ P_2 \\ \vdots \\ P_{2N-1} \\ P_{2N} \end{pmatrix} \quad \Rightarrow \quad \mathbf{Q} = \begin{pmatrix} \sqrt{P_1} \\ \sqrt{P_2} \\ \vdots \\ \sqrt{P_{2N-1}} \\ \sqrt{P_{2N}} \end{pmatrix}$$

$$p_1 = P_1 + P_2 \quad \Rightarrow \quad p_1 = Q_1^2 + Q_2^2$$

$$\vdots \qquad\qquad\qquad \vdots$$

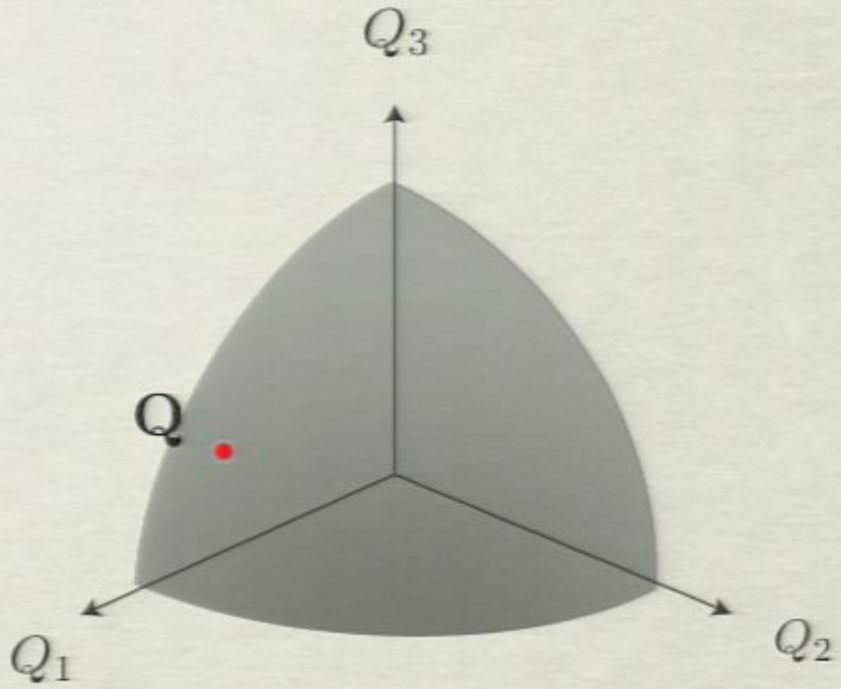
$$p_N = P_{2N-1} + P_{2N} \qquad\qquad p_N = Q_{2N-1}^2 + Q_{2N}^2$$

$$\mathbf{P} = \begin{pmatrix} P_1 \\ P_2 \\ \vdots \\ P_{2N-1} \\ P_{2N} \end{pmatrix} \quad \rightarrow \quad \mathbf{Q} = \begin{pmatrix} \sqrt{P_1} \\ \sqrt{P_2} \\ \vdots \\ \sqrt{P_{2N-1}} \\ \sqrt{P_{2N}} \end{pmatrix}$$

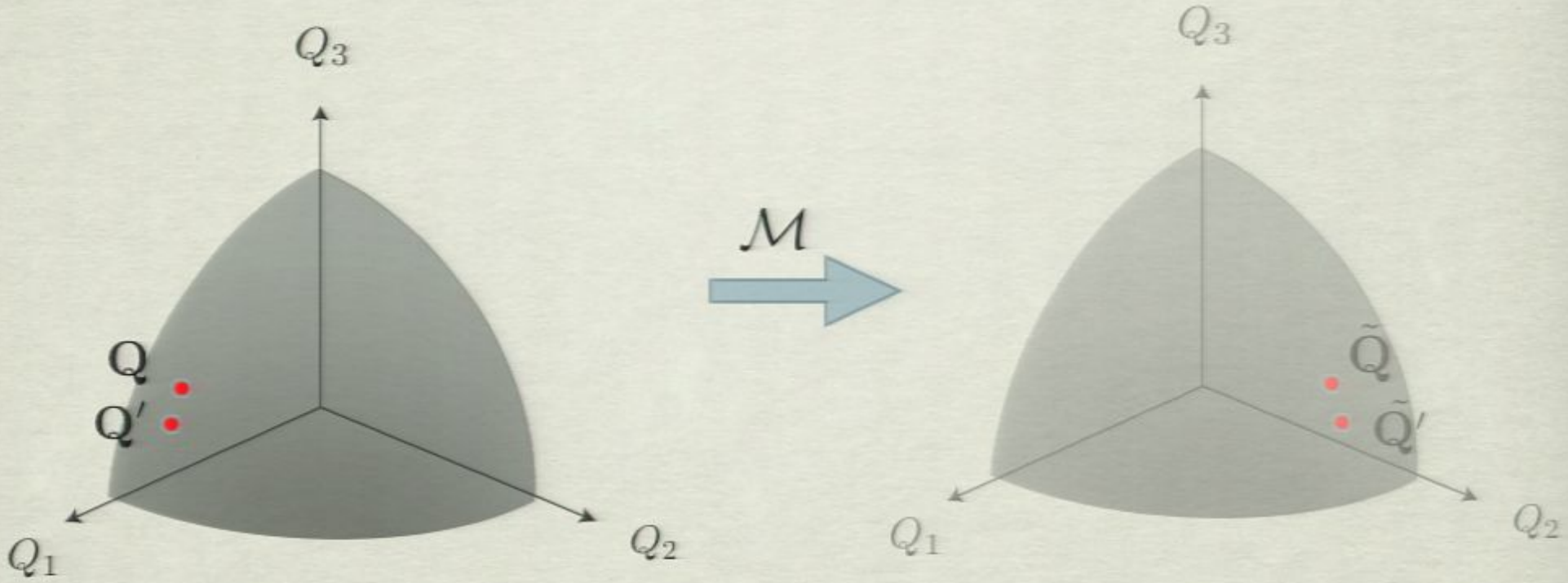
$$ds^2 = \frac{1}{4}\sum_{q=1}^{2N} \frac{dP_q^2}{P_q} \quad \rightarrow \quad ds^2 = \sum_{q=1}^{2N} dQ_q^2$$

$$\mathbf{Q} = \begin{pmatrix} \sqrt{P_1} \\ \sqrt{P_2} \\ \vdots \\ \sqrt{P_{2N-1}} \\ \sqrt{P_{2N}} \end{pmatrix}$$

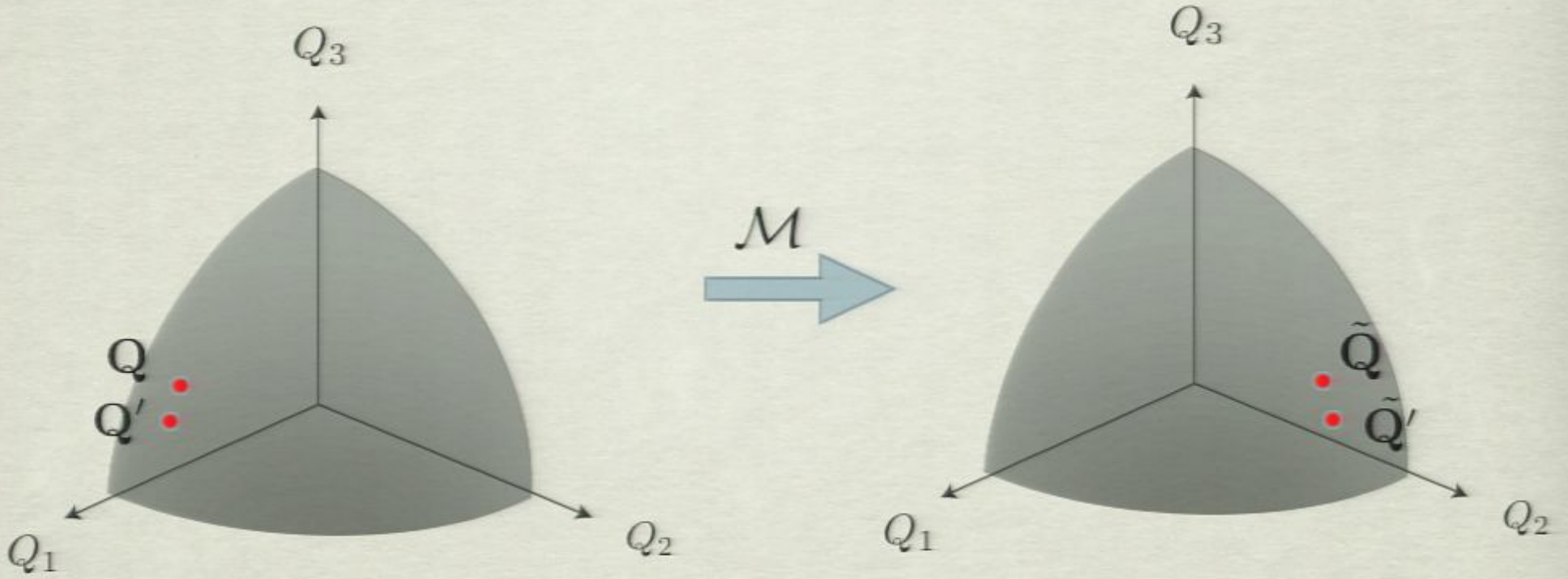
$$ds^2 = \sum_{q=1}^{2N} dQ_q^2$$



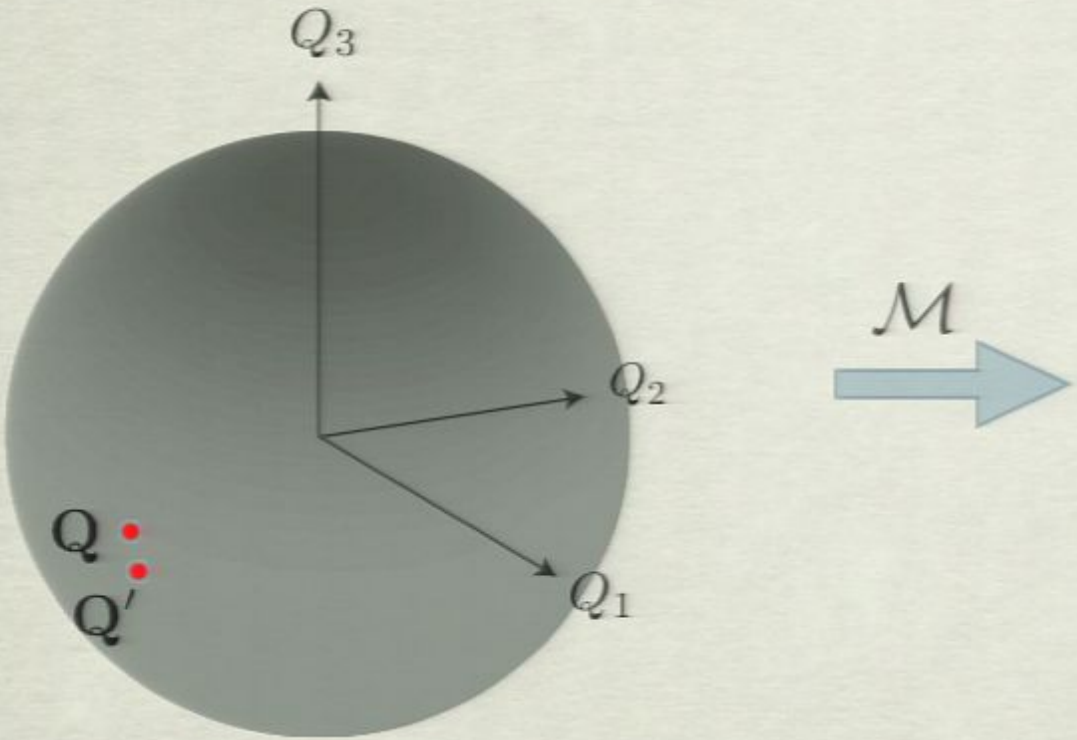
Metric-Preserving Transformations

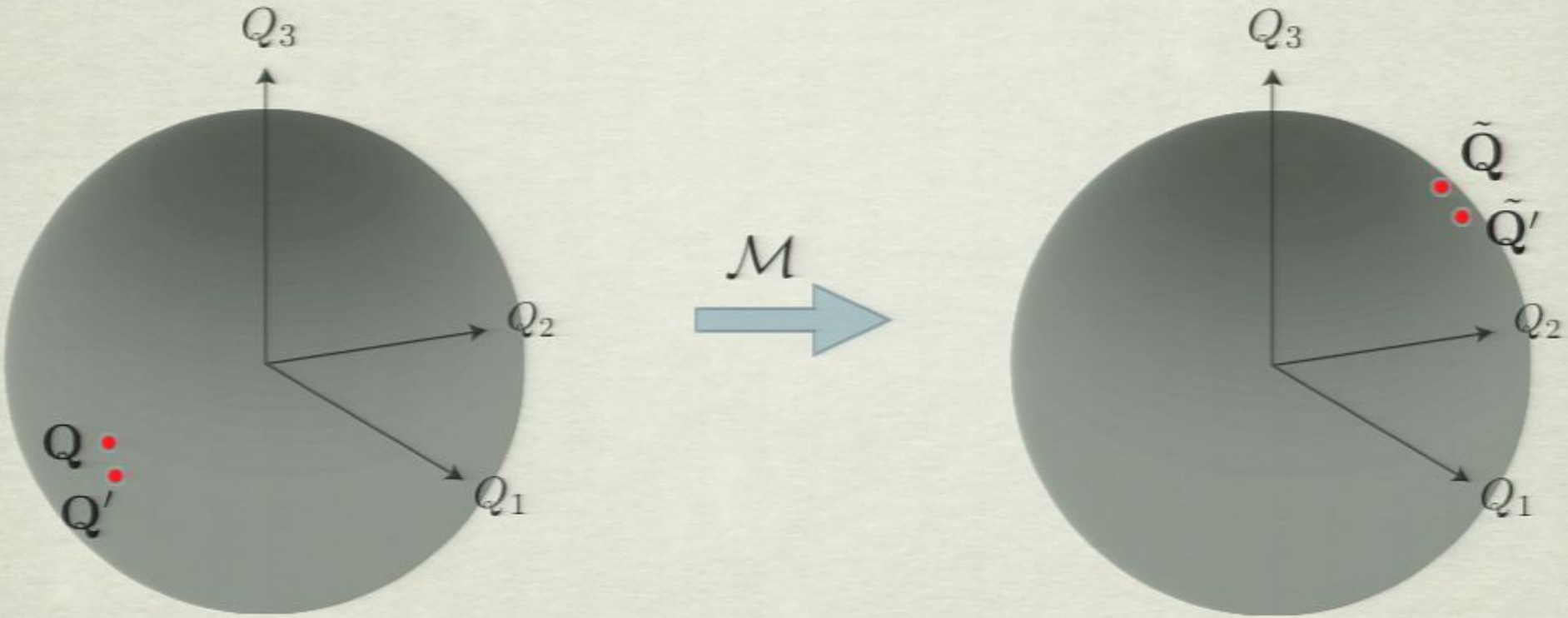


Metric-Preserving Transformations



$$d(\mathbf{Q}, \mathbf{Q}') = d(\mathcal{M}\mathbf{Q}, \mathcal{M}\mathbf{Q}')$$


Q_3
Q_2
Q_1
Q
Q′
$\mathcal{M}$



$$d(\mathbf{Q}, \mathbf{Q}') = d(\mathcal{M}\mathbf{Q}, \mathcal{M}\mathbf{Q}') \quad \Rightarrow \quad \mathcal{M} \text{ is orthogonal}$$

Global Gauge Invariance

$$\mathbf{Q} = \begin{pmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_{2N-1} \\ Q_{2N} \end{pmatrix}$$

$$p_1 = Q_1^2 + Q_2^2$$

$$p_N = Q_{2N-1}^2 + Q_{2N}^2$$

$$Q = \begin{pmatrix} \sqrt{p_1}\cos\theta_1 \\ \sqrt{p_1}\sin\theta_1 \\ \vdots \\ \sqrt{p_N}\cos\theta_N \\ \sqrt{p_N}\sin\theta_N \end{pmatrix}$$

Global Gauge Invariance

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Let $\theta_i = \theta(\chi_i)$

Global Gauge Invariance

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Let $\theta_i = \theta(\chi_i)$

$$\mathbf{Q} = \begin{pmatrix} \sqrt{p_1}\cos\theta(\chi_1) \\ \sqrt{p_1}\sin\theta(\chi_1) \\ \vdots \\ \sqrt{p_N}\cos\theta(\chi_N) \\ \sqrt{p_N}\sin\theta(\chi_N) \end{pmatrix}$$

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Assume $\chi_i \to \chi_i + \chi_0$ is a global gauge transformation

Measure Invariance

Measure Invariance

$$ds^2 = \sum_{q=1}^{2N} dQ_q^2$$

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$$Q_2 = \sqrt{p_1}\sin\theta(\chi_1)$$

$$ds^2 = \frac{1}{4}\sum_{i=1}^{N}\frac{dp_i^2}{p_i} + \sum_{i=1}^{N}\theta'^2(\chi_i)\,d\chi_i^2$$

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$$\sqrt{\det(\cdot)}$$

$$\mu(p_1,\ldots,p_N;\chi_1,\ldots,\chi_N) \propto |\theta'(\chi_1)\ldots\theta'(\chi_N)|$$

$$\mu(p_i; \chi_1, \ldots, \chi_N) = \mu(p_i; \chi_1 + \chi_0, \ldots, \chi_N + \chi_0)$$

$$\mu(p_i; \chi_1, \ldots, \chi_N) = \mu(p_i; \chi_1 + \chi_0, \ldots, \chi_N + \chi_0)$$

$$\theta(\chi) = a\chi + b$$

$$\mu(p_i; \chi_1, \ldots, \chi_N) = \mu(p_i; \chi_1 + \chi_0, \ldots, \chi_N + \chi_0)$$

$$\theta(\chi) = a\chi + b$$

$$\mathbf{Q} = \begin{pmatrix} \sqrt{p_1}\cos\theta(\chi_1) \\ \sqrt{p_1}\sin\theta(\chi_1) \\ \vdots \\ \sqrt{p_N}\cos\theta(\chi_N) \\ \sqrt{p_N}\sin\theta(\chi_N) \end{pmatrix} \xrightarrow{\phi_i = a\chi_i + b} \mathbf{Q} = \begin{pmatrix} \sqrt{p_1}\cos\phi_1 \\ \sqrt{p_1}\sin\phi_1 \\ \vdots \\ \sqrt{p_N}\cos\phi_N \\ \sqrt{p_N}\sin\phi_N \end{pmatrix}$$

Gauge Invariance

$$\mathbf{Q} = \begin{pmatrix} \sqrt{p_1}\cos\phi_1 \\ \sqrt{p_1}\sin\phi_1 \\ \vdots \\ \sqrt{p_N}\cos\phi_N \\ \sqrt{p_N}\sin\phi_N \end{pmatrix} \xrightarrow{M} \mathbf{Q}' = \begin{pmatrix} \sqrt{p'_1}\cos\phi'_1 \\ \sqrt{p'_1}\sin\phi'_1 \\ \vdots \\ \sqrt{p'_N}\cos\phi'_N \\ \sqrt{p'_N}\sin\phi'_N \end{pmatrix}$$

Gauge Invariance

$$\mathbf{Q} = \begin{pmatrix} \sqrt{p_1}\cos\phi_1 \\ \sqrt{p_1}\sin\phi_1 \\ \vdots \\ \sqrt{p_N}\cos\phi_N \\ \sqrt{p_N}\sin\phi_N \end{pmatrix} \xrightarrow{M} \mathbf{Q}' = \begin{pmatrix} \sqrt{p'_1}\cos\phi'_1 \\ \sqrt{p'_1}\sin\phi'_1 \\ \vdots \\ \sqrt{p'_N}\cos\phi'_N \\ \sqrt{p'_N}\sin\phi'_N \end{pmatrix} \longrightarrow p'_1, \ldots, p'_N$$

$\phi_i \to \phi_i + \phi_0$ leaves the p'_i invariant

Orthogonal Transformations

$$\left(\begin{array}{c|c} \alpha_{11}\begin{pmatrix}\cos\varphi_{11} & -\sin\varphi_{11}\\ \sin\varphi_{11} & \cos\varphi_{11}\end{pmatrix} & \alpha_{12}\begin{pmatrix}\cos\varphi_{12} & -\sin\varphi_{12}\\ \sin\varphi_{12} & \cos\varphi_{12}\end{pmatrix} \\ \hline \alpha_{21}\begin{pmatrix}\cos\varphi_{21} & -\sin\varphi_{21}\\ \sin\varphi_{21} & \cos\varphi_{21}\end{pmatrix} & \alpha_{22}\begin{pmatrix}\cos\varphi_{22} & -\sin\varphi_{22}\\ \sin\varphi_{22} & \cos\varphi_{22}\end{pmatrix} \end{array}\right)$$

type 1

Orthogonal Transformations

$$\left(\begin{array}{c|c} \alpha_{11}\begin{pmatrix}\cos\varphi_{11} & -\sin\varphi_{11}\\ \sin\varphi_{11} & \cos\varphi_{11}\end{pmatrix} & \alpha_{12}\begin{pmatrix}\cos\varphi_{12} & -\sin\varphi_{12}\\ \sin\varphi_{12} & \cos\varphi_{12}\end{pmatrix} \\ \hline \alpha_{21}\begin{pmatrix}\cos\varphi_{21} & -\sin\varphi_{21}\\ \sin\varphi_{21} & \cos\varphi_{21}\end{pmatrix} & \alpha_{22}\begin{pmatrix}\cos\varphi_{22} & -\sin\varphi_{22}\\ \sin\varphi_{22} & \cos\varphi_{22}\end{pmatrix} \end{array}\right)$$

type 1

$$\left(\begin{array}{c|c} \alpha_{11}\begin{pmatrix}\cos\varphi_{11} & -\sin\varphi_{11}\\ \sin\varphi_{11} & \cos\varphi_{11}\end{pmatrix}F & \alpha_{12}\begin{pmatrix}\cos\varphi_{12} & -\sin\varphi_{12}\\ \sin\varphi_{12} & \cos\varphi_{12}\end{pmatrix}F \\ \hline \alpha_{21}\begin{pmatrix}\cos\varphi_{21} & -\sin\varphi_{21}\\ \sin\varphi_{21} & \cos\varphi_{21}\end{pmatrix}F & \alpha_{22}\begin{pmatrix}\cos\varphi_{22} & -\sin\varphi_{22}\\ \sin\varphi_{22} & \cos\varphi_{22}\end{pmatrix}F \end{array}\right)$$

type 2

where $F = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$

$$\mathbf{Q} = \begin{pmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_{2N-1} \\ Q_{2N} \end{pmatrix}$$

$$\text{orthogonals} \begin{cases} \text{type 1} \\ \\ \text{type 2} \end{cases}$$

$$\mathbf{Q} = \begin{pmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_{2N-1} \\ Q_{2N} \end{pmatrix} \quad \Longleftrightarrow \quad \mathbf{v} = \begin{pmatrix} Q_1 + iQ_2 \\ \vdots \\ Q_{2N-1} + iQ_{2N} \end{pmatrix}$$

orthogonals $\begin{cases} \text{type 1} \\ \\ \text{type 2} \end{cases}$

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$$\text{orthogonals} \begin{cases} \text{type 1} \\ \\ \text{type 2} \end{cases} \quad \Longleftrightarrow \quad \begin{matrix} \text{unitary} \\ \\ \text{antiunitary} \end{matrix}$$

$$\mathbf{Q} = \begin{pmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_{2N-1} \\ Q_{2N} \end{pmatrix} \quad \Longleftrightarrow \quad \mathbf{v} = \begin{pmatrix} Q_1 + iQ_2 \\ \vdots \\ Q_{2N-1} + iQ_{2N} \end{pmatrix}$$

orthogonals { type 1 / type 2 } $\Longleftrightarrow$ unitary / antiunitary

$$\mathbf{Q} = \begin{pmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_{2N-1} \\ Q_{2N} \end{pmatrix} \quad \Longleftrightarrow \quad \mathbf{v} = \begin{pmatrix} Q_1 + iQ_2 \\ \vdots \\ Q_{2N-1} + iQ_{2N} \end{pmatrix}$$

orthogonals { type 1 ⟺ unitary

type 2 ⟺ antiunitary

$$\mathbf{Q} = \begin{pmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_{2N-1} \\ Q_{2N} \end{pmatrix} \quad \Longleftrightarrow \quad \mathbf{v} = \begin{pmatrix} Q_1 + iQ_2 \\ \vdots \\ Q_{2N-1} + iQ_{2N} \end{pmatrix}$$

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orthogonals { type 1, type 2 } $\longleftrightarrow$ unitary, antiunitary

$$\mathbf{Q} = \begin{pmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_{2N-1} \\ Q_{2N} \end{pmatrix} \quad \Longleftrightarrow \quad \mathbf{v} = \begin{pmatrix} Q_1 + iQ_2 \\ \vdots \\ Q_{2N-1} + iQ_{2N} \end{pmatrix}$$

$$p_i = Q_{2i-1}^2 + Q_{2i}^2$$

$$\mathbf{Q} = \begin{pmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_{2N-1} \\ Q_{2N} \end{pmatrix} \quad \Longleftrightarrow \quad \mathbf{v} = \begin{pmatrix} Q_1 + iQ_2 \\ \vdots \\ Q_{2N-1} + iQ_{2N} \end{pmatrix}$$

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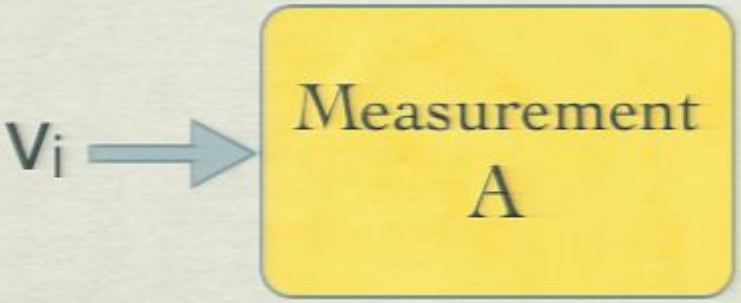
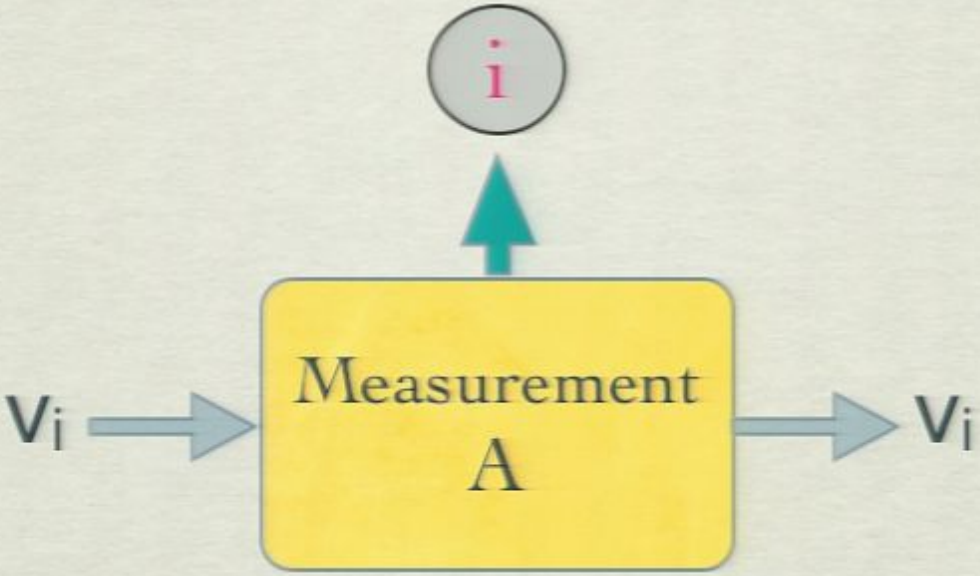
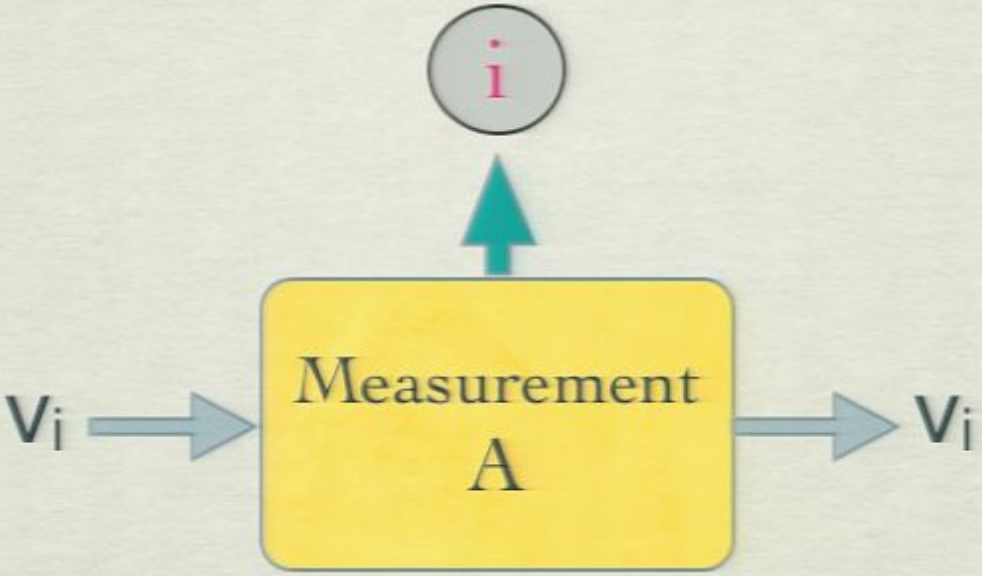
$$p_i = Q_{2i-1}^2 + Q_{2i}^2 \quad \Longleftrightarrow \quad p_i = |v_i|^2$$

Measurement A

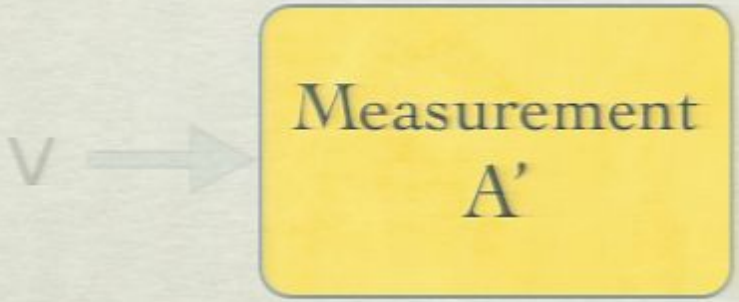

V_i
Measurement
A


i
Measurement
A
V_i
V_i

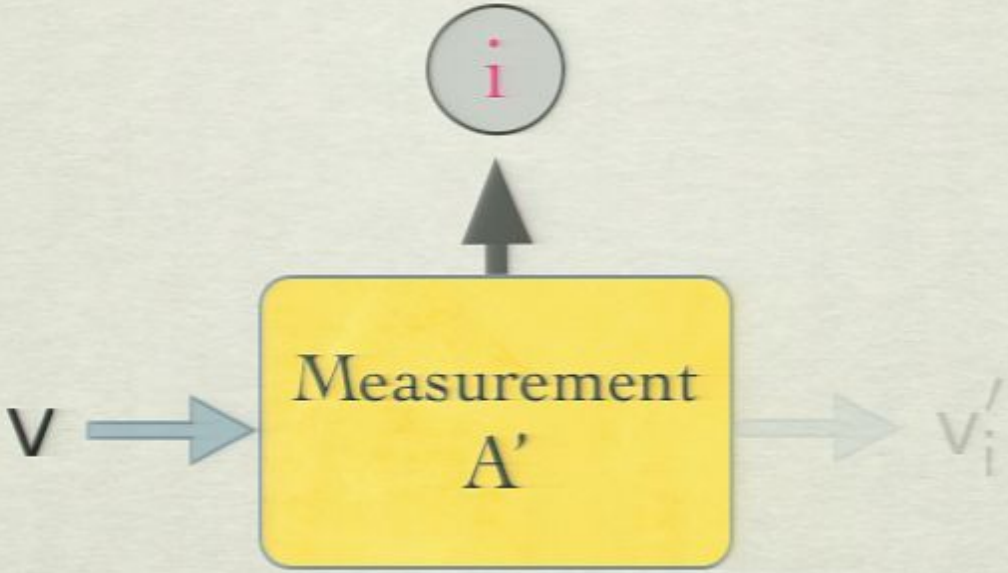


States $\mathbf{v}_1 = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \ldots, \mathbf{v}_N = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ N \end{pmatrix}$ yield outcomes 1,..,N with certainty

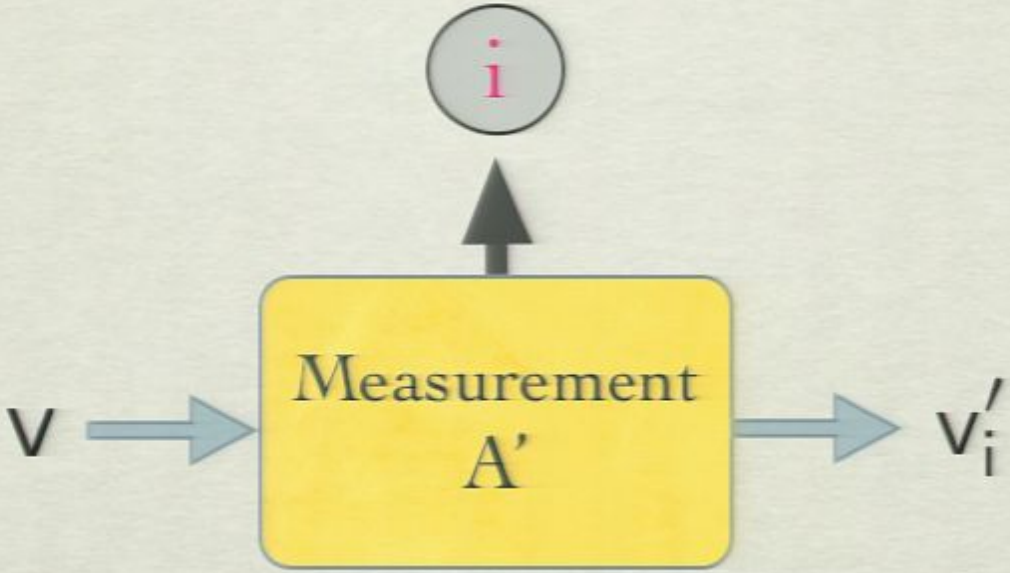
Measurement Simulability



Measurement Simulability

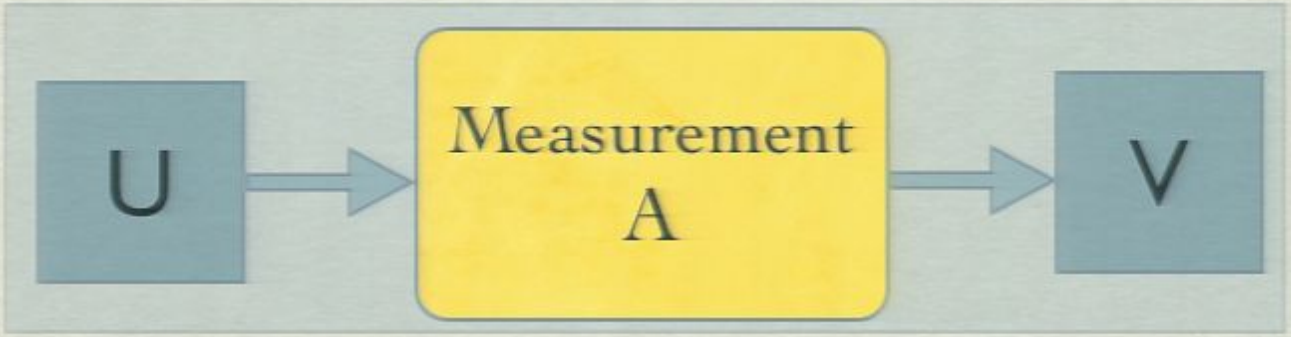


Measurement Simulability



Measurement Simulability

Measurement Simulability



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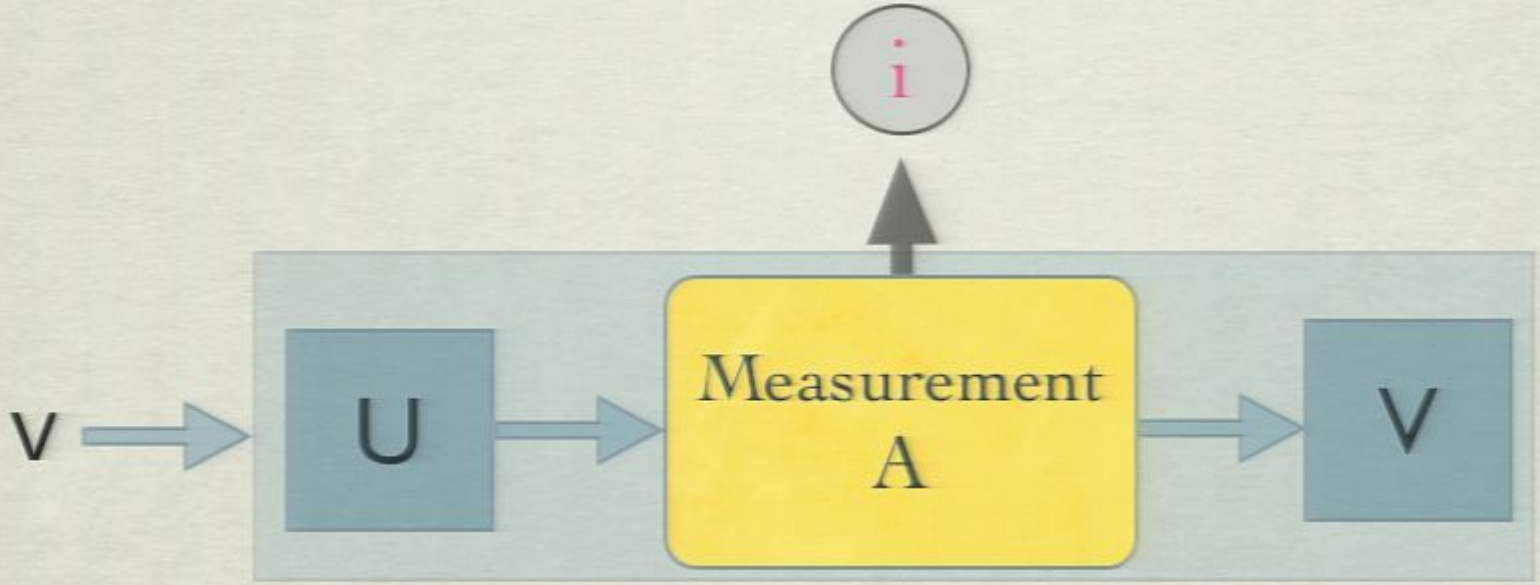

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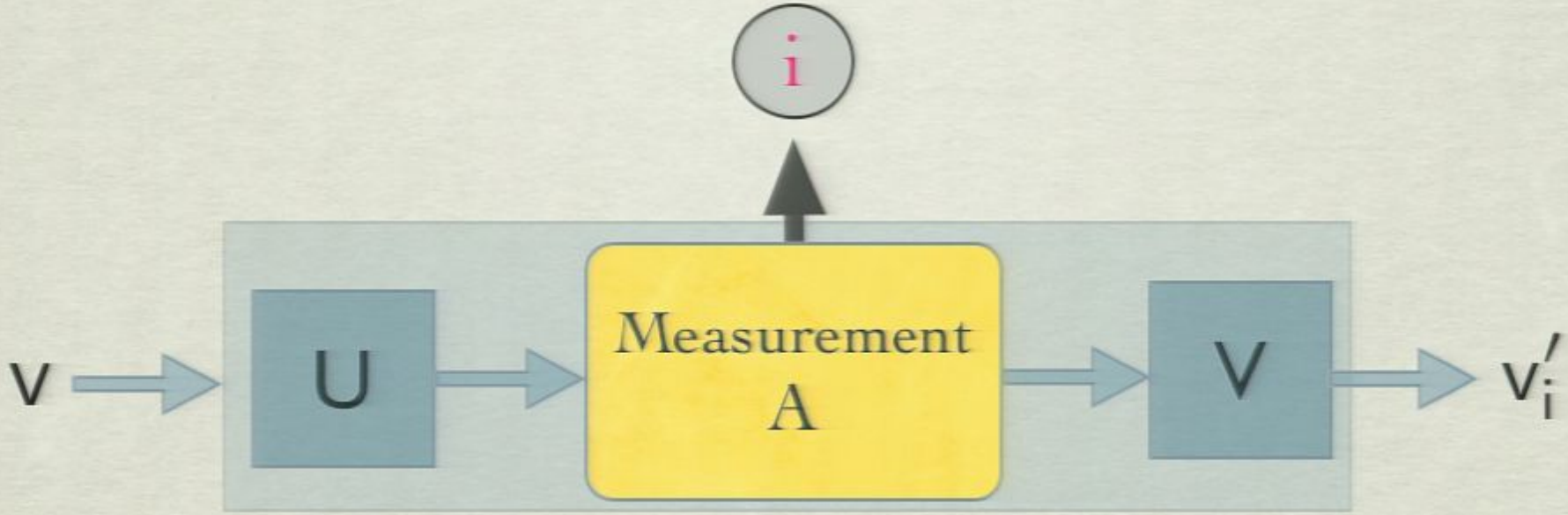
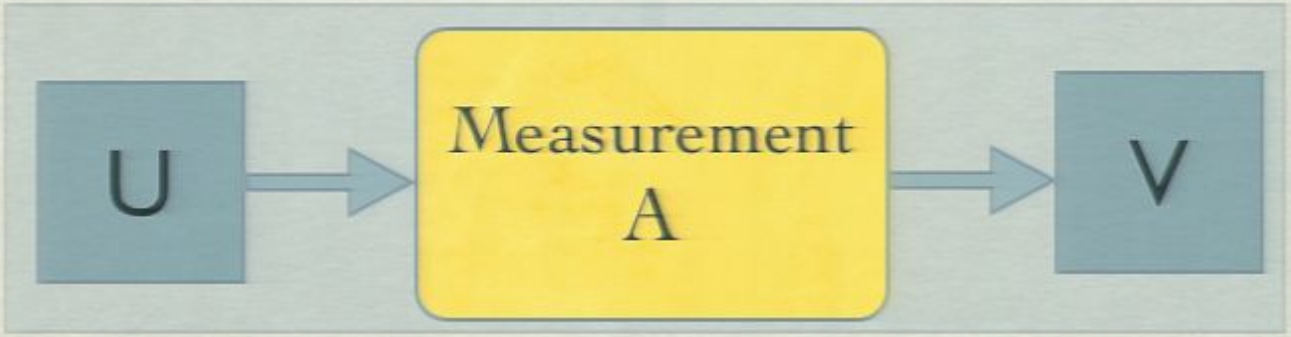


Measurement Simulability



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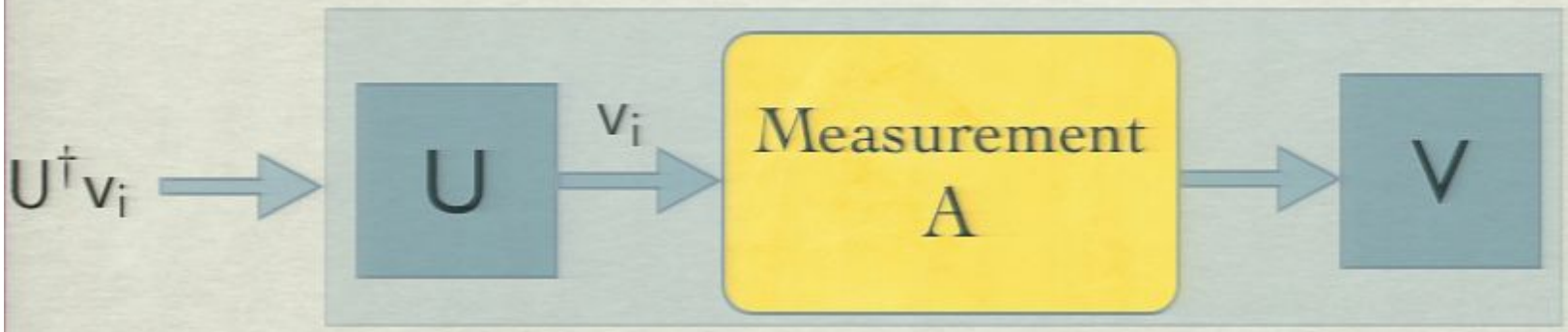
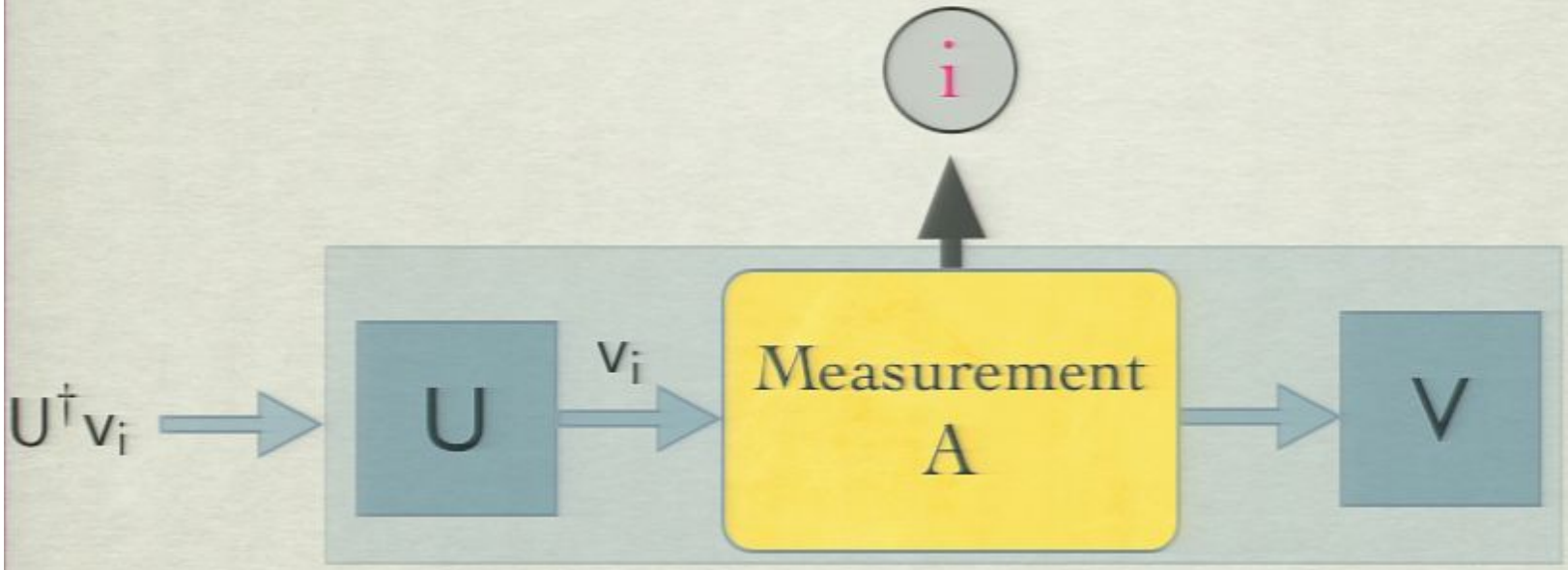
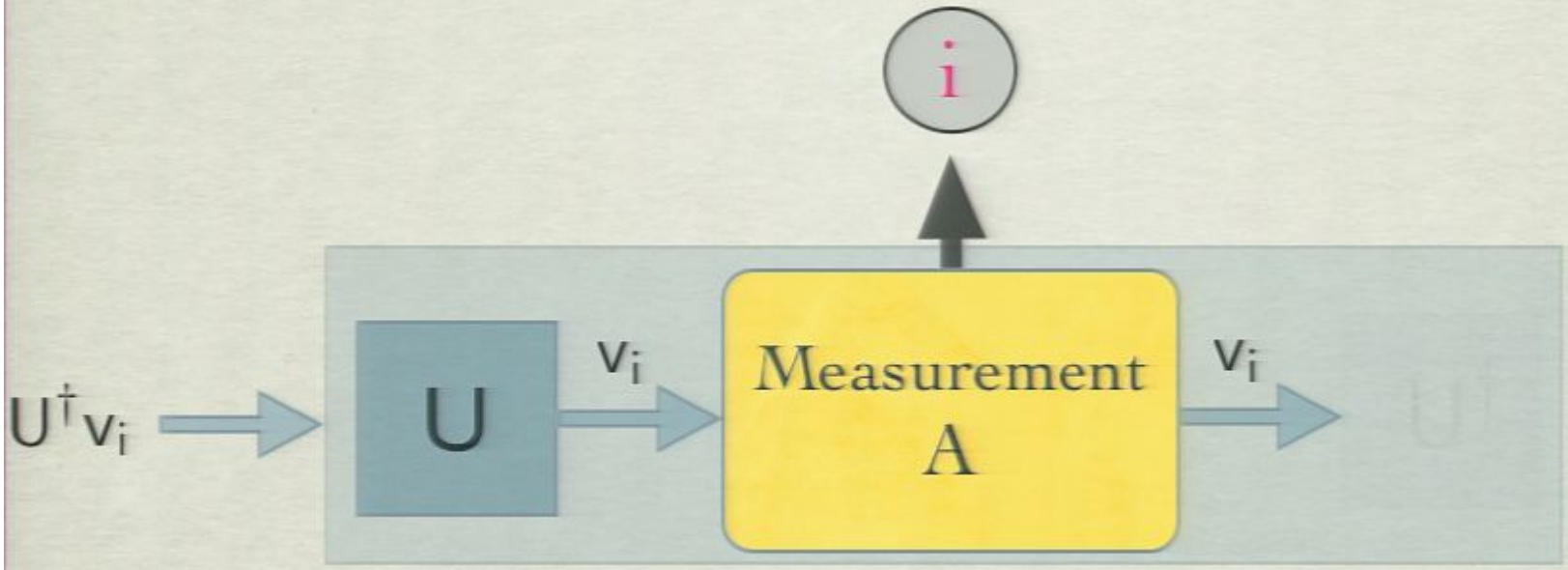


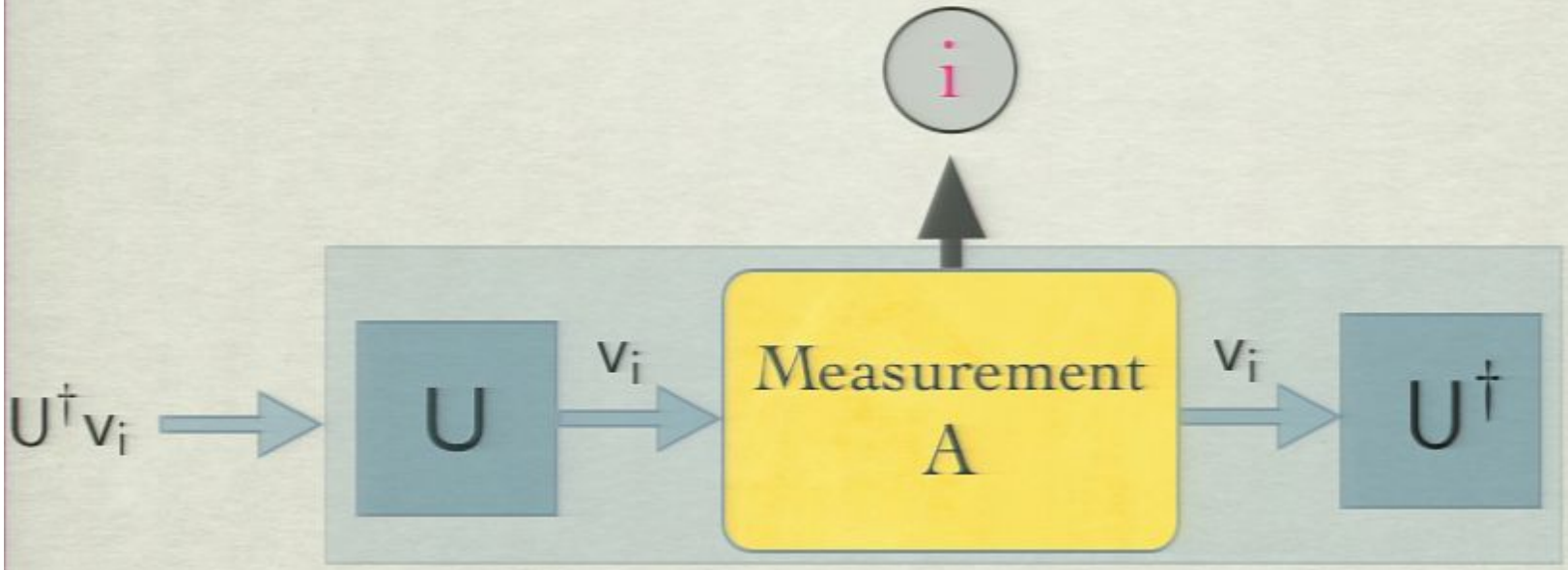
Measurement Simulability



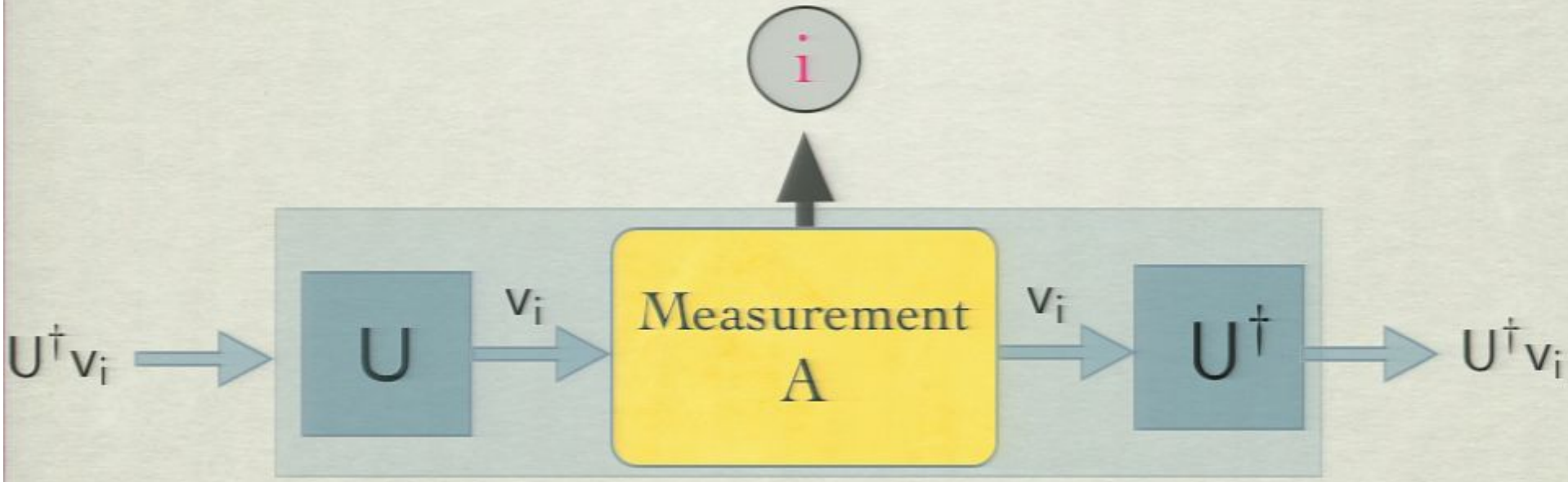
Measurement Simulability



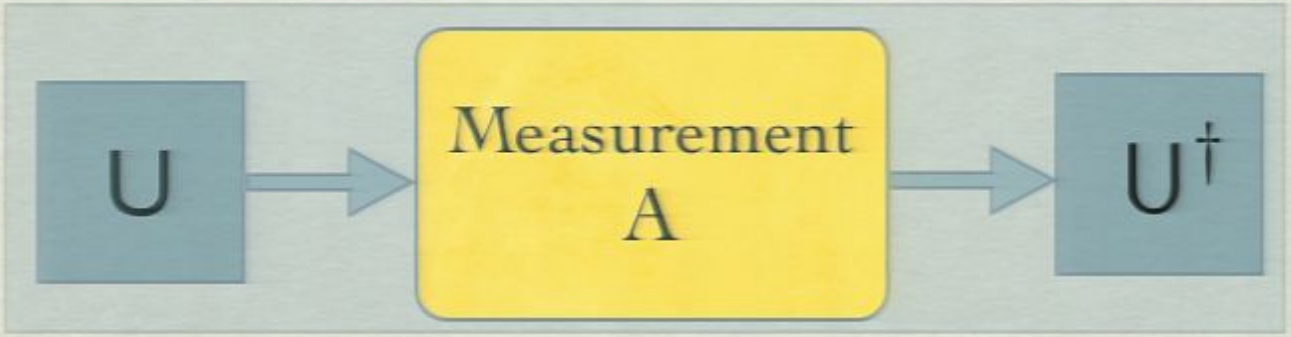
Measurement Simulability



Measurement Simulability

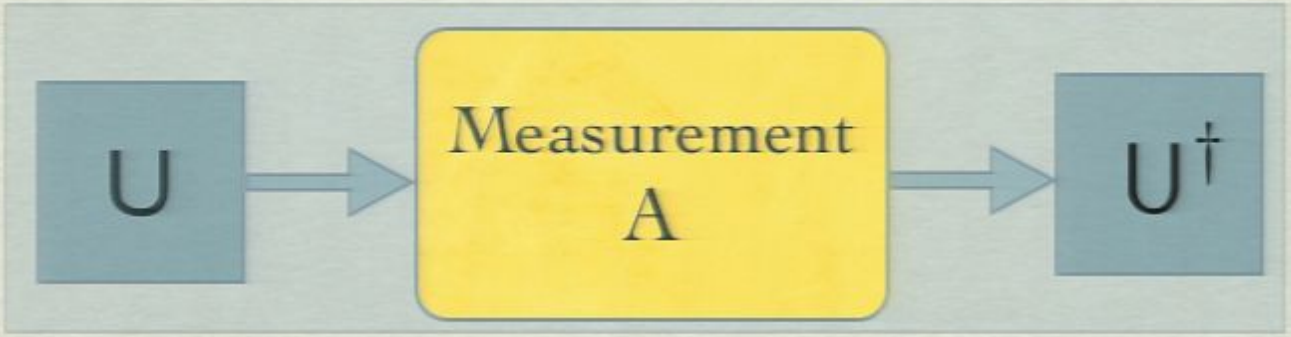


Measurement Simulability



$$\text{Let} \begin{cases} v_i' = U^\dagger v_i \\ v = \sum_i c_i' v_i' \end{cases}$$

Measurement Simulability



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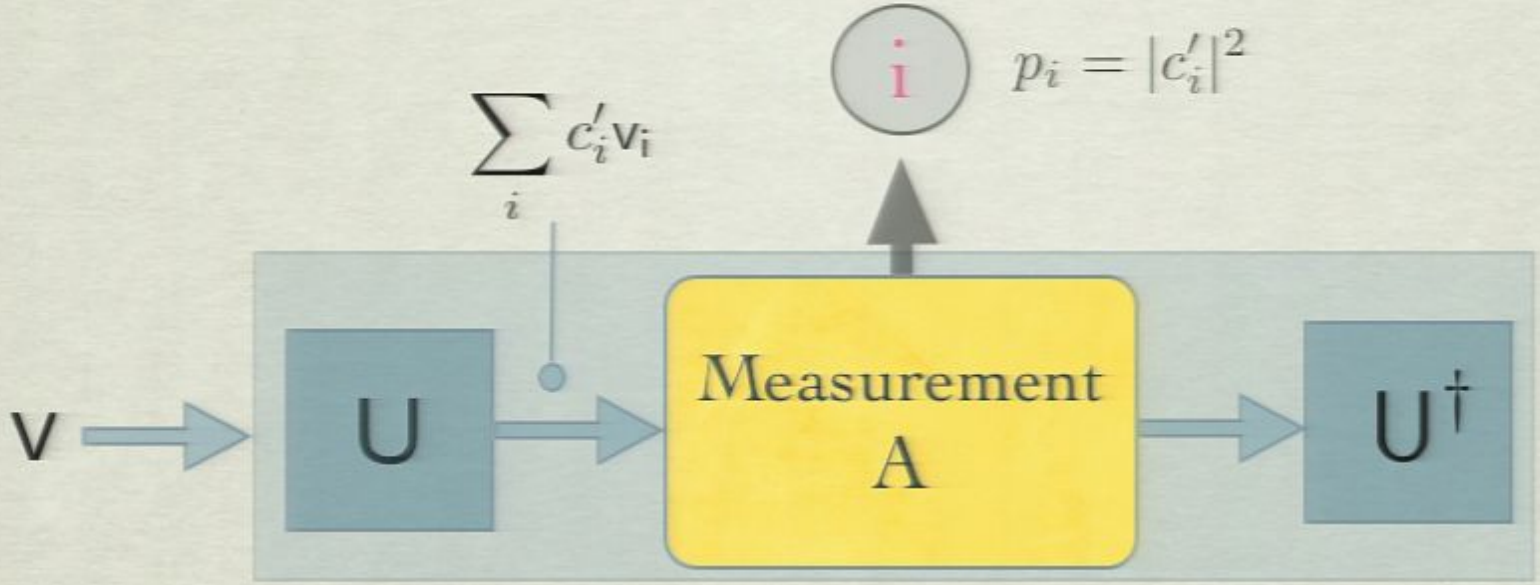
Measurement Simulability



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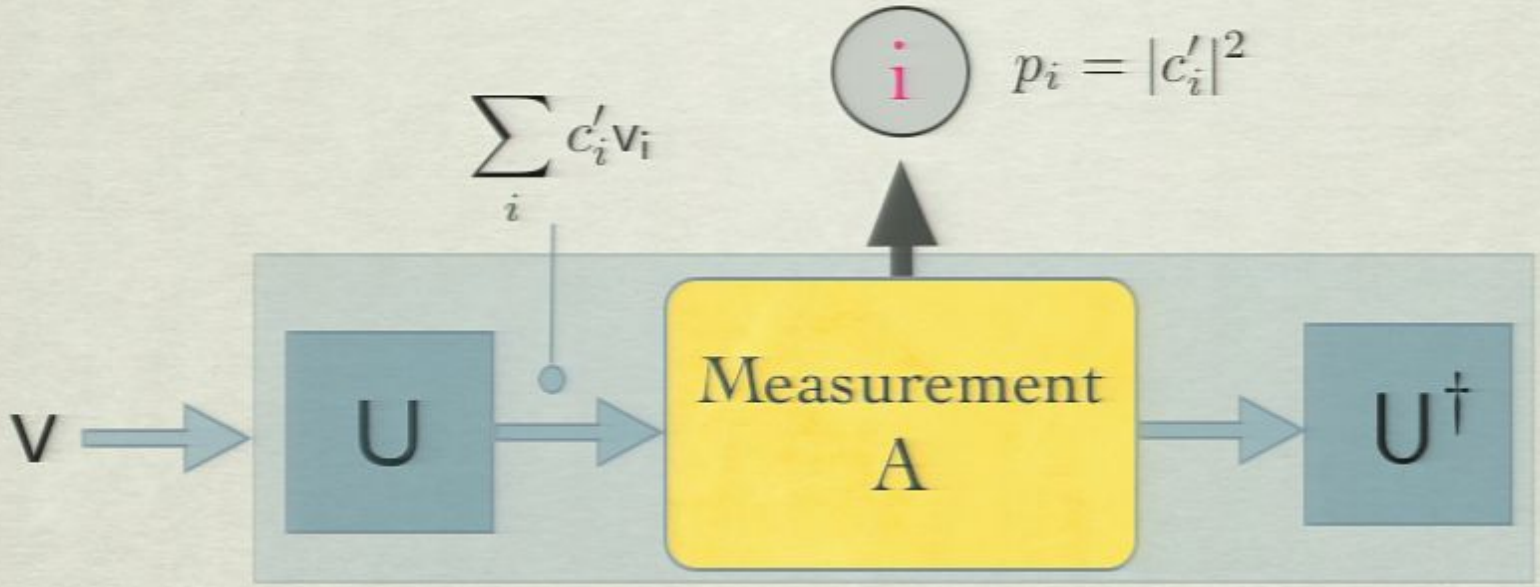
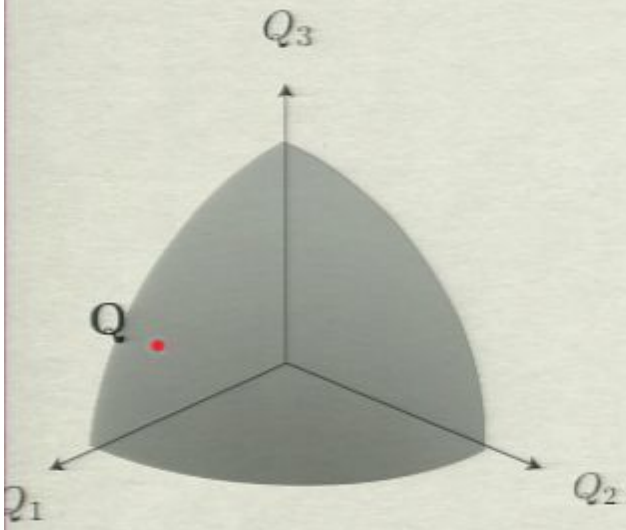
Measurement Simulability



$$\text{Let} \begin{cases} v_i' = U^\dagger v_i \\ v = \sum_i c_i' v_i' \end{cases}$$

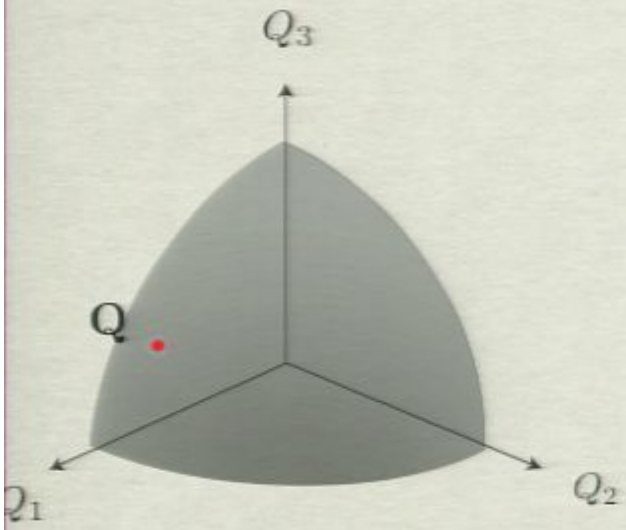
Complementarity

Complementarity



$$ds^2 = \sum_{q=1}^{2N} dQ_q^2$$

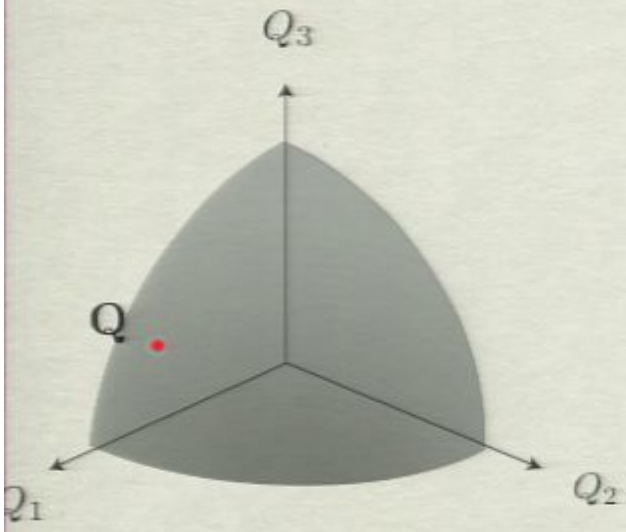
Complementarity



$$ds^2 = \sum_{q=1}^{2N} dQ_q^2$$

Metric-Preservation

Complementarity



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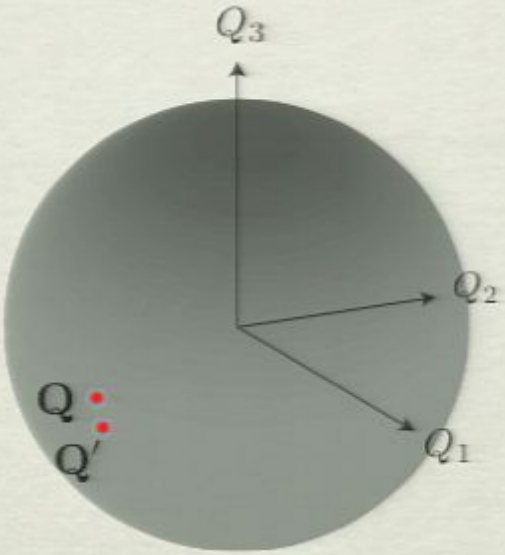
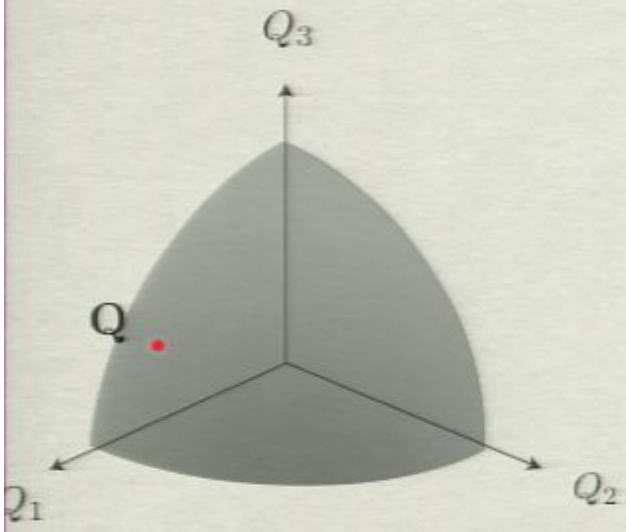
Metric-Preservation



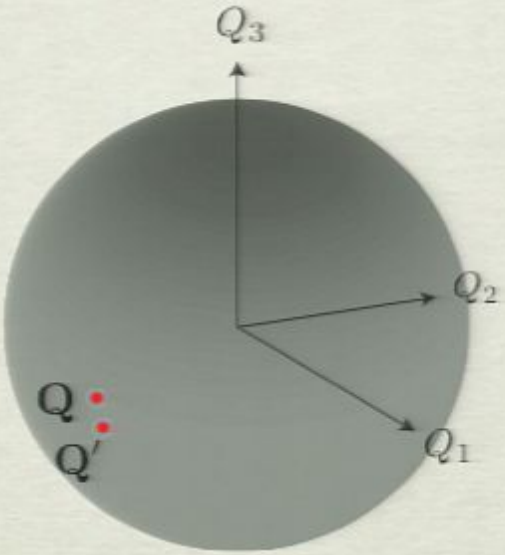
Transformations are orthogonal

Complementarity



$$ds^2 = \sum_{q=1}^{2N} dQ_q^2$$

Metric-Preservation

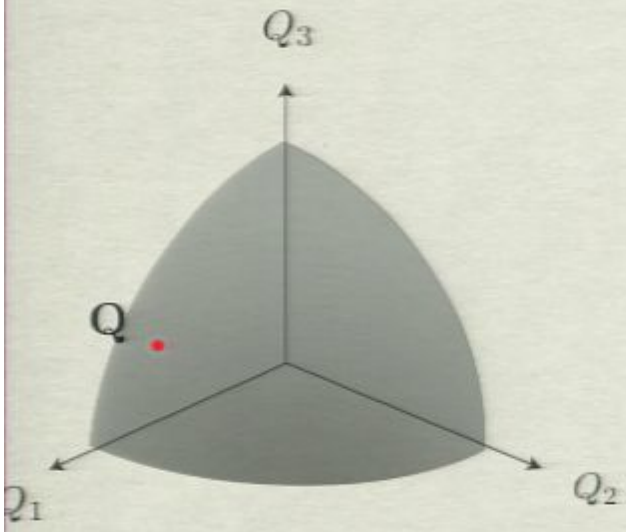


Transformations are orthogonal

Global Gauge Invariance

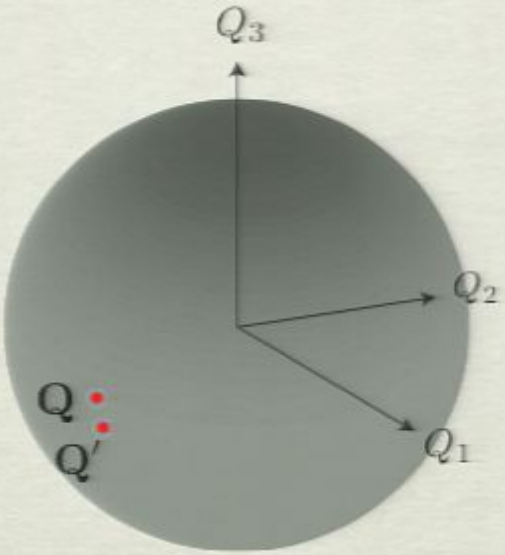
Transformations are orthogonal of *type 1* or *type 2*

Complementarity



$$ds^2 = \sum_{q=1}^{2N} dQ_q^2$$

Metric-Preservation

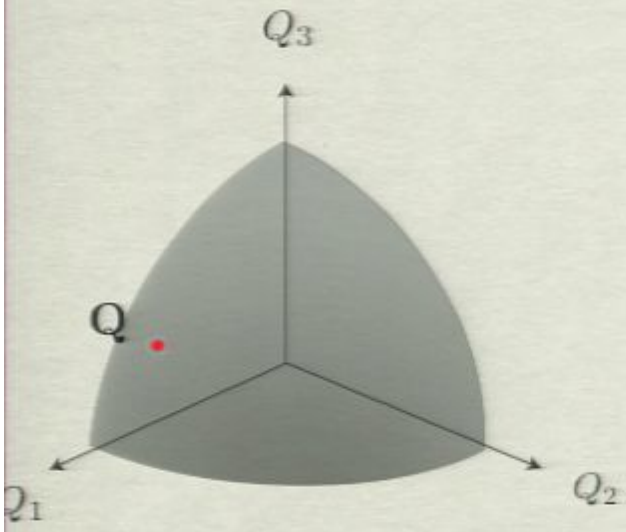


Transformations are orthogonal

Global Gauge Invariance

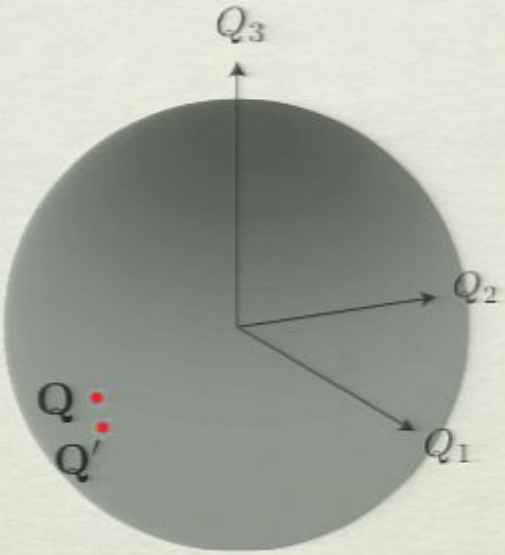
Transformations are orthogonal of *type 1* or *type 2*

Complementarity



$$ds^2 = \sum_{q=1}^{2N} dQ_q^2$$

Metric-Preservation



Transformations are orthogonal

Global Gauge Invariance

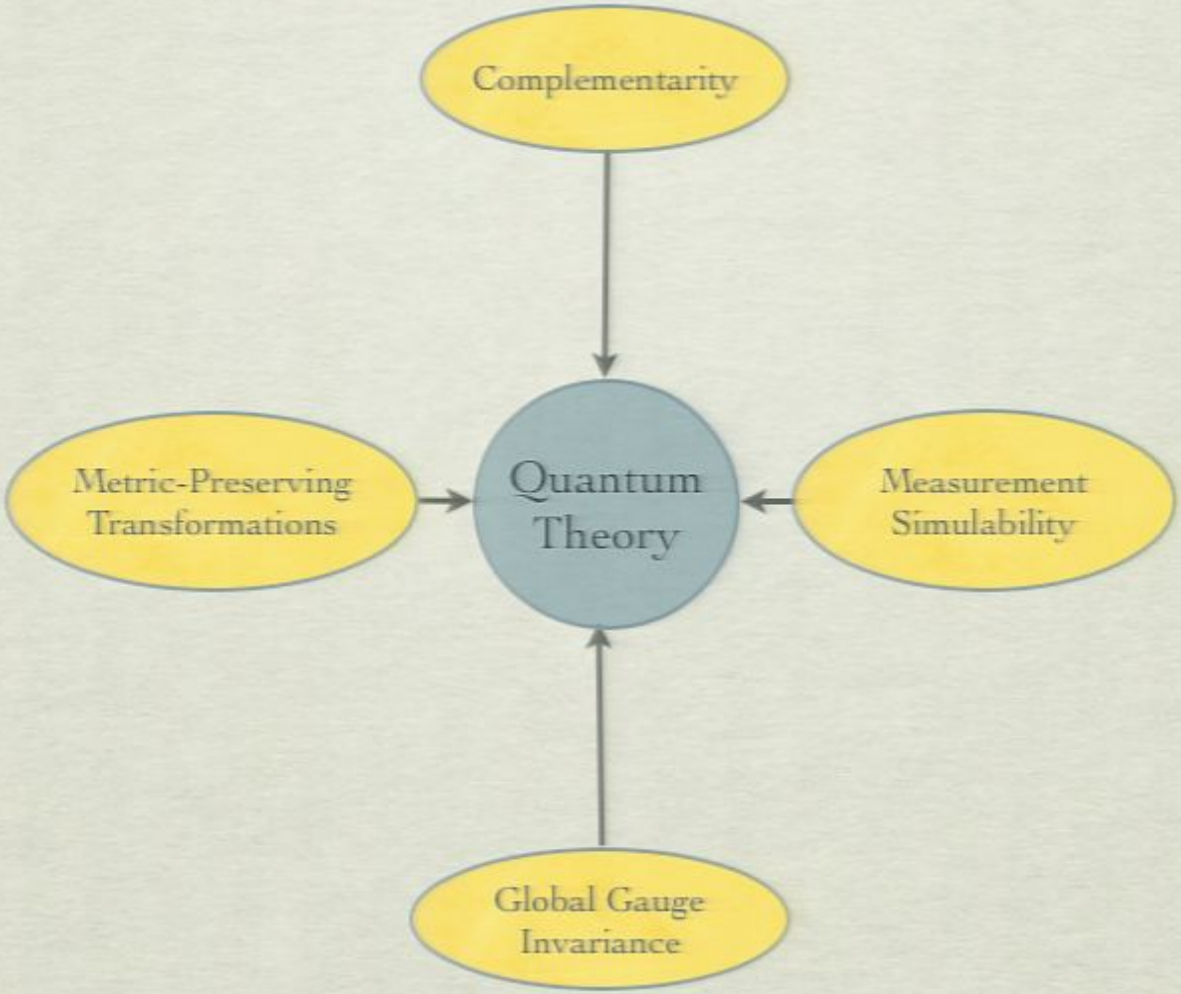
Transformations are orthogonal of *type 1 or type 2*

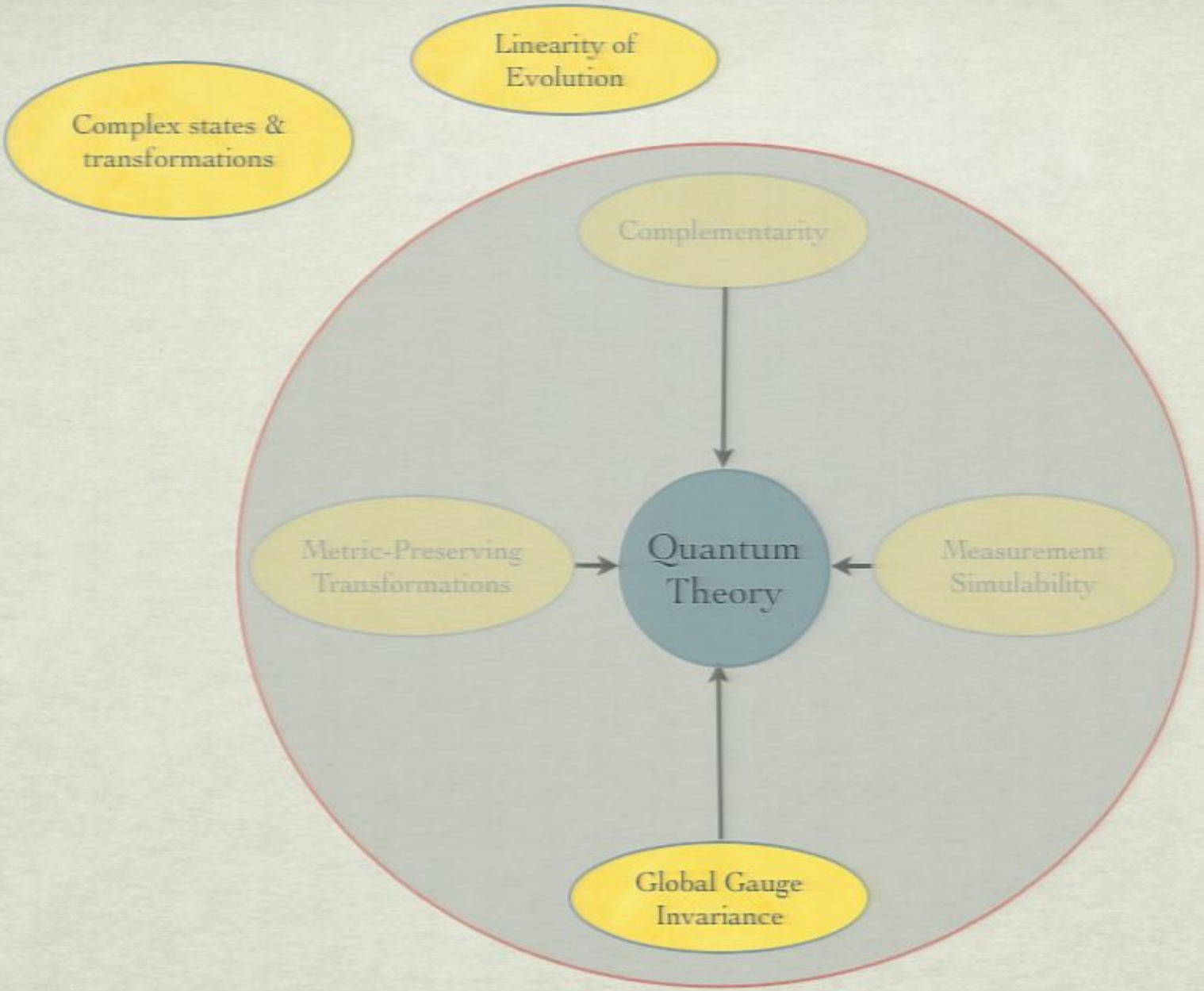
States are complex vectors

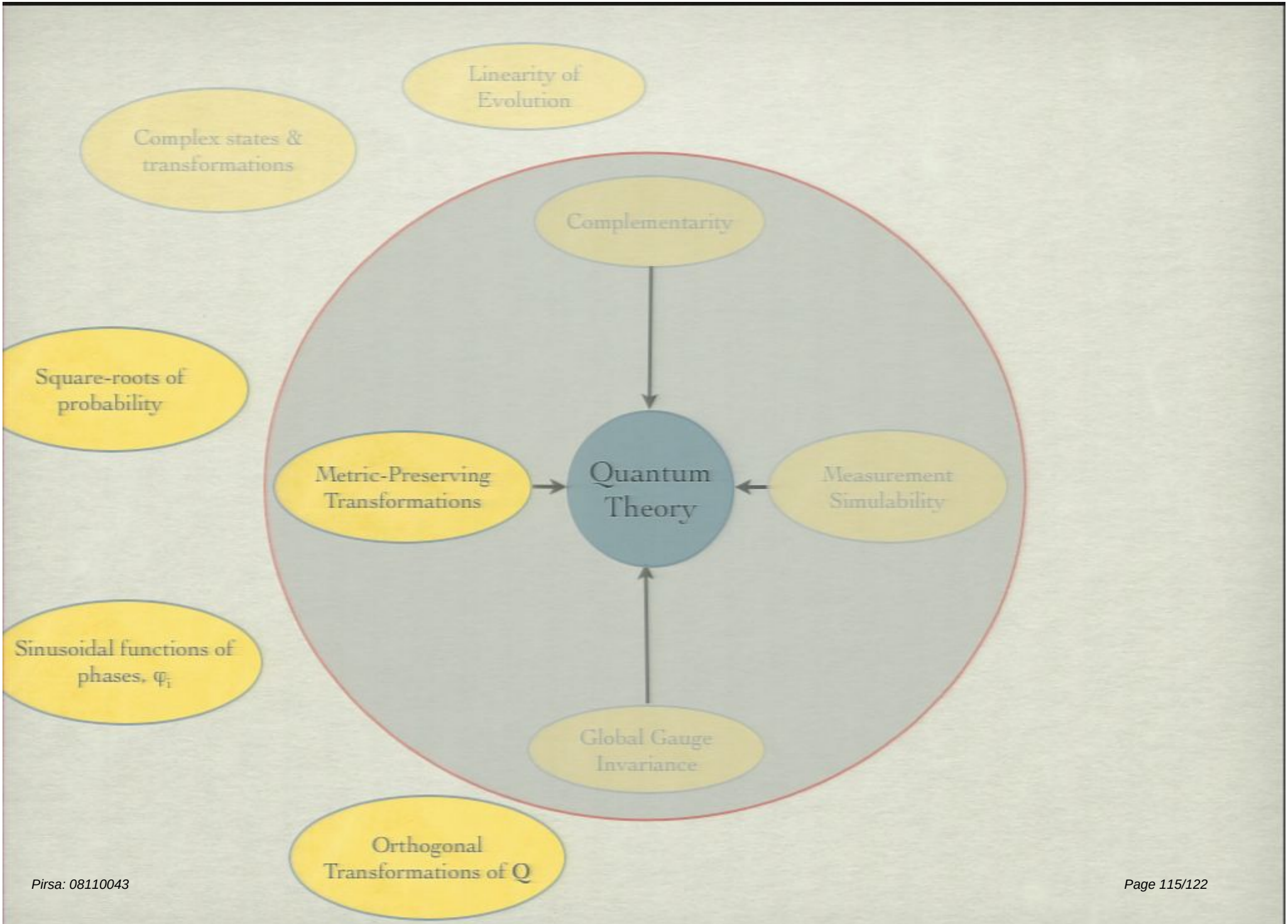
Transformations are unitary or antiunitary

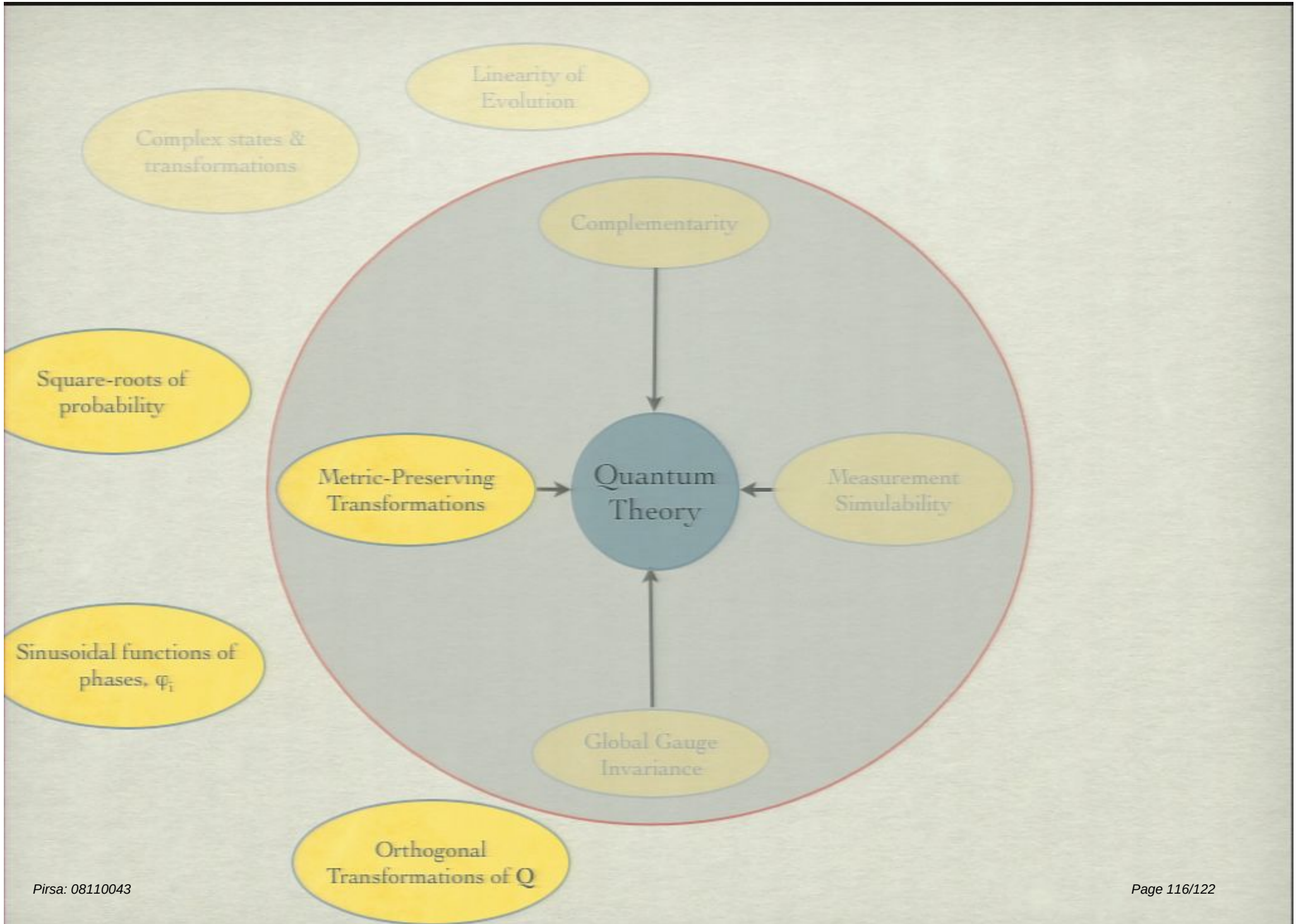
Further Steps

- Tensor Product Rule
  $\mathsf{v} = \mathsf{v}^{(1)} \otimes \mathsf{v}^{(2)}$
- Temporal Evolution Operator
  $\mathsf{U}_t(dt) = \exp(i\mathsf{H}\, dt/\hbar)$
- Correspondence Rules
  $[\mathsf{L}_x, \mathsf{L}_y] = i\hbar\mathsf{L}_z, \text{etc.}$


Complementarity
Metric-Preserving Transformations
Quantum Theory
Measurement Simulability
Global Gauge Invariance


Linearity of Evolution
Complex states & transformations
Complementarity
Metric-Preserving Transformations
Quantum Theory
Measurement Simulability
Global Gauge Invariance


Linearity of Evolution
Complex states & transformations
Complementarity
Square-roots of probability
Metric-Preserving Transformations
Quantum Theory
Measurement Simulability
Sinusoidal functions of phases, φ_i
Global Gauge Invariance
Orthogonal Transformations of Q


Linearity of Evolution
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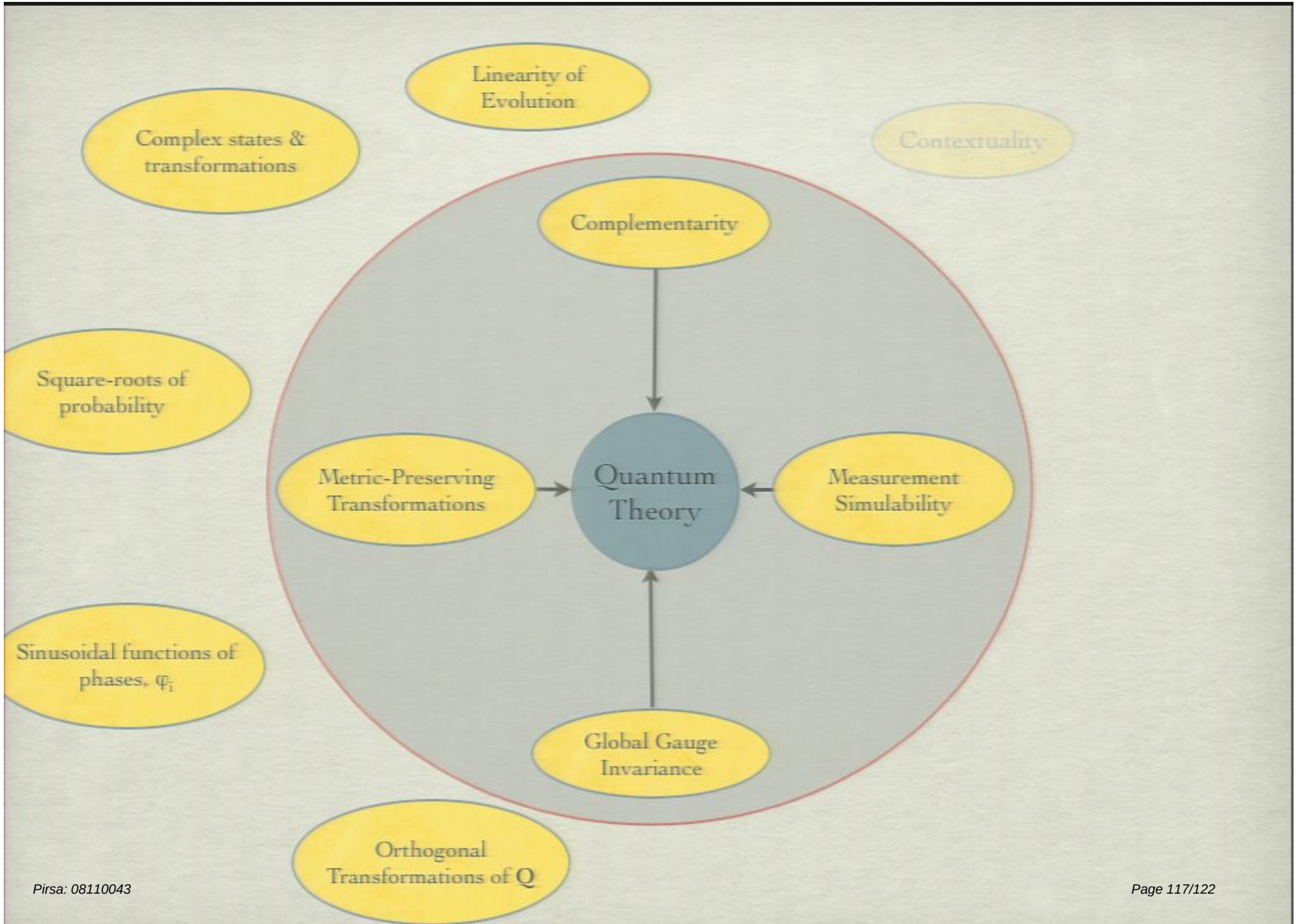
Linearity of Evolution
Complex states & transformations
Contextuality
Complementarity
Square-roots of probability
Metric-Preserving Transformations
Quantum Theory
Measurement Simulability
Sinusoidal functions of phases, φ_i
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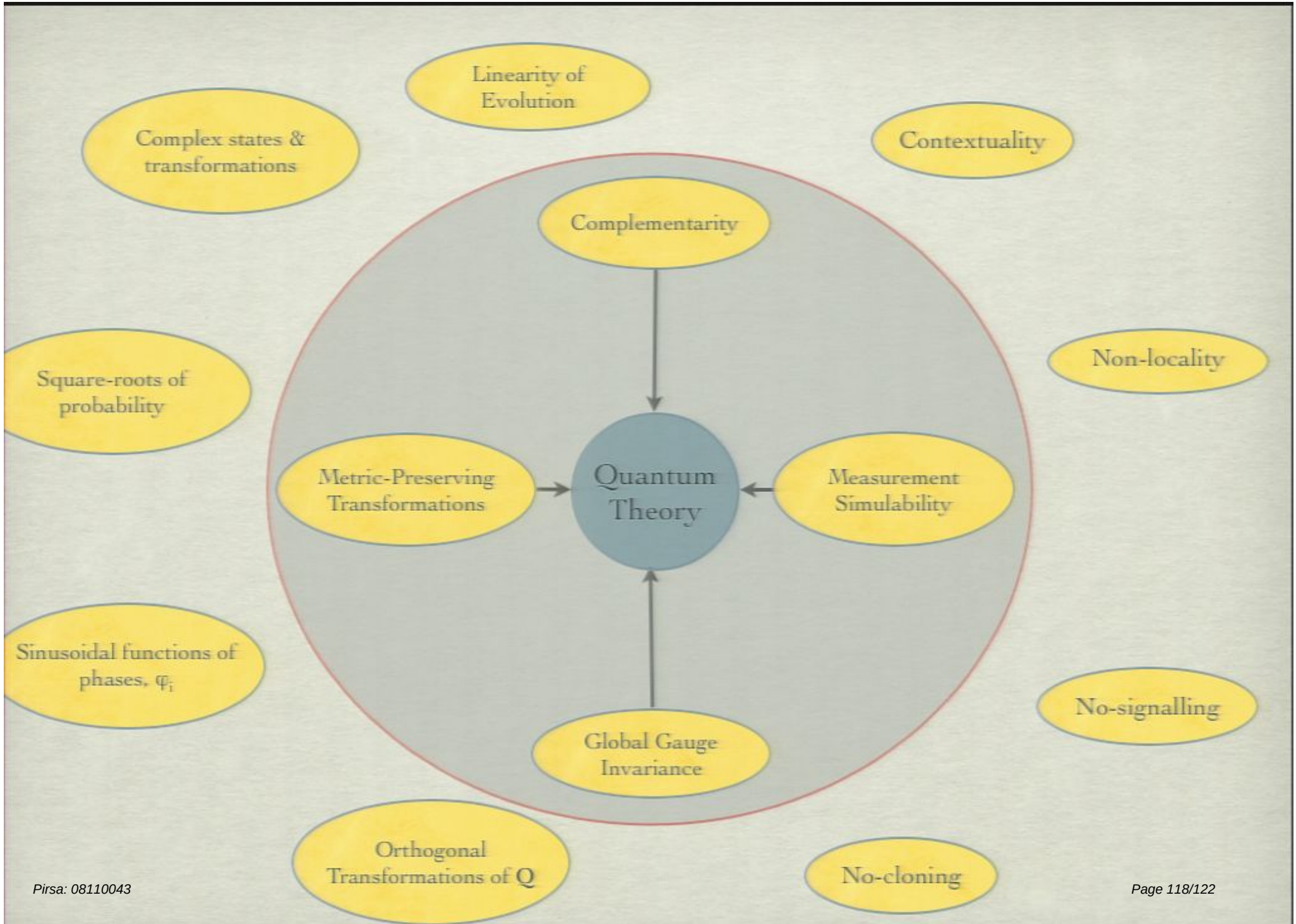
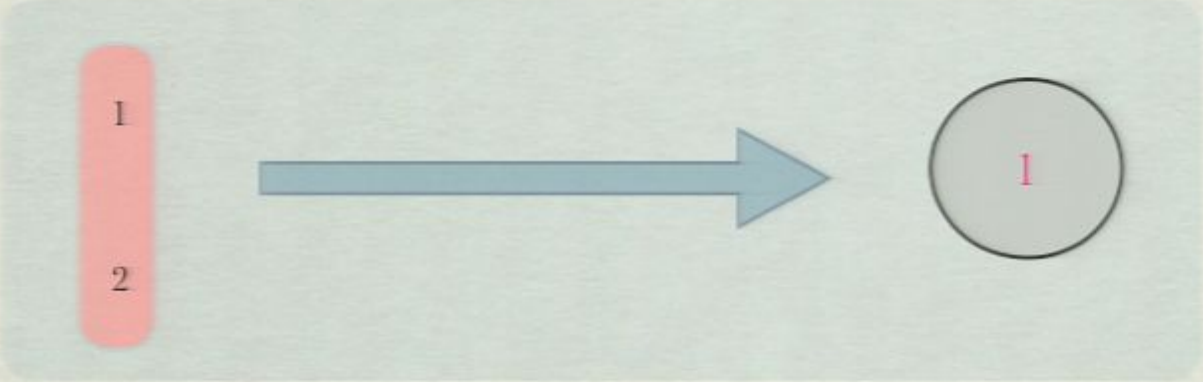
Orthogonal Transformations of Q


Linearity of Evolution
Complex states & transformations
Contextuality
Complementarity
Square-roots of probability
Non-locality
Metric-Preserving Transformations
Quantum Theory
Measurement Simulability
Sinusoidal functions of phases, φ_i
No-signalling
Global Gauge Invariance
Orthogonal Transformations of Q
No-cloning

Questions



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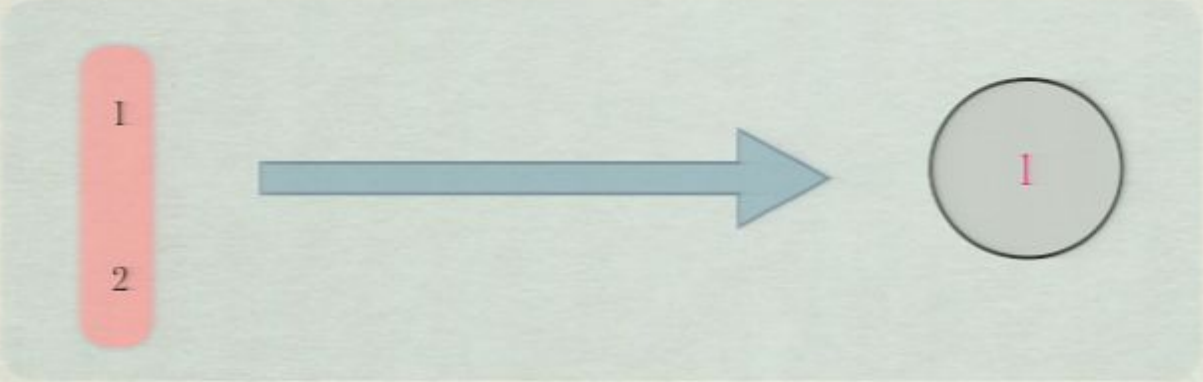
Questions



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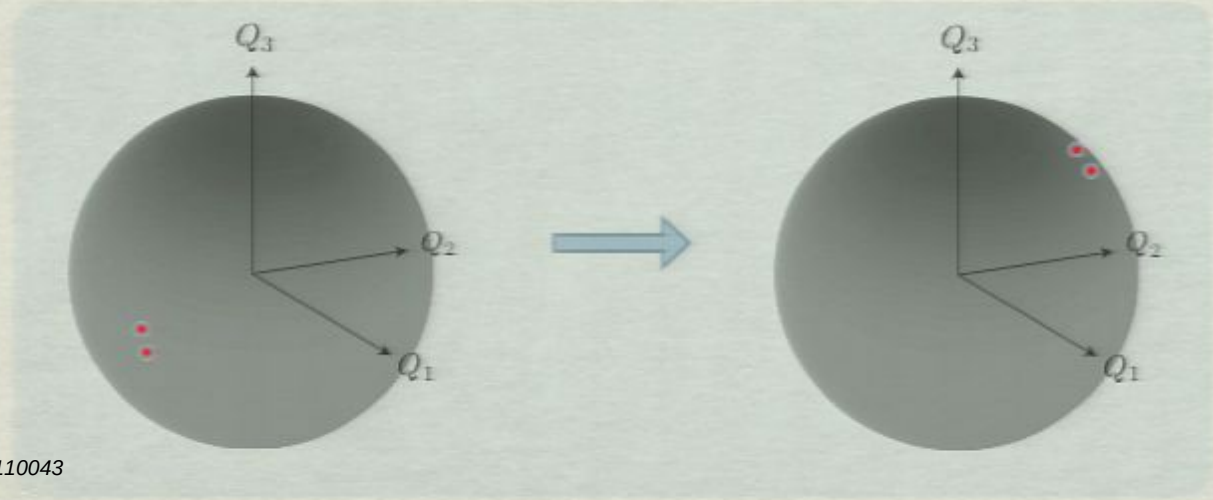

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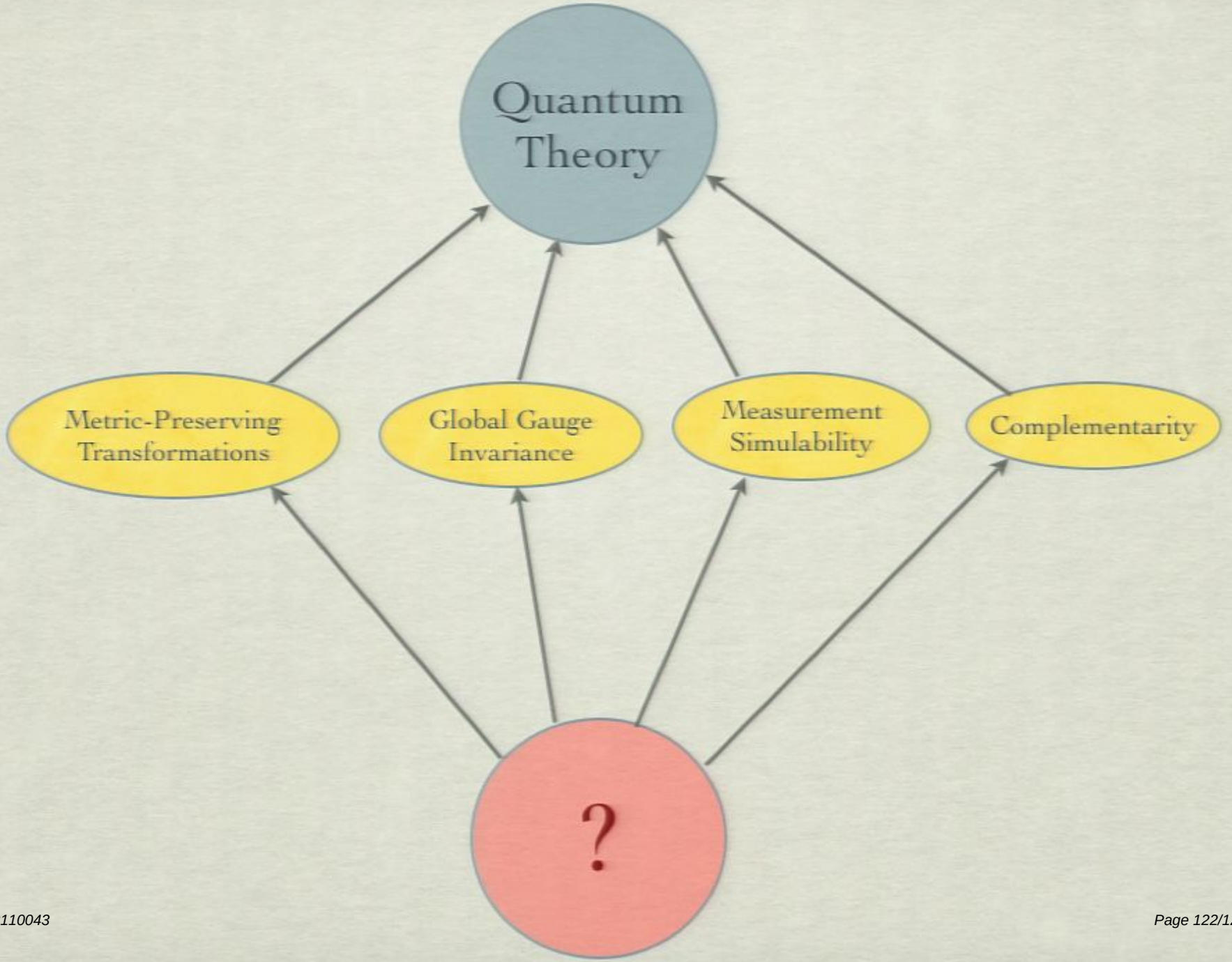
Questions

$$\chi_i \rightarrow \chi_i + \chi_0 \quad ?$$


Quantum Theory
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Measurement Simulability
Complementarity
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