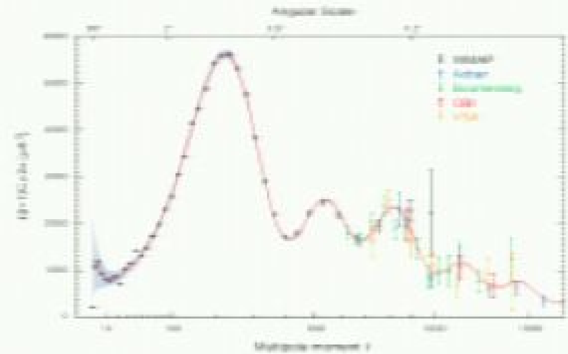
Title: The Volume of the Universe after Inflation and de Sitter Entropy

Date: Dec 05, 2008 11:00 AM

URL: http://pirsa.org/08110034

Abstract: I will show the calculation of the probability distribution for the volume of the Universe after slow-roll inflation both in the eternal and the non-eternal regime. Far from the eternal regime the probability distribution for the number of e-foldings, defined as one third of the logarithm of the volume, is sharply peaked around the number of e-foldings of the classical inflaton trajectory. At the transition to the eternal regime this probability is still peaked (with the width of order one e-foldings) around the average, which however gets twice larger at the transition point. As one enters the eternal regime the probability for the volume to be finite rapidly becomes exponentially small. In addition to developing techniques to study eternal inflation, these results allow us to establish the quantum generalization of the recently proposed bound on the number of e-foldings in non-eternal regime: the probability for slow-roll inflation to produce a finite volume larger than Exp[S_dS/2], where S_dS is the de Sitter entropy at the end of the inflationary stage, is smaller than the uncertainty due to non-perturbative quantum gravity effects. The existence of such a bound provides a consistency check for the idea of de Sitter complementarity.

de Sitter and us



- Quasi dS phase in our present and in our past
- Possibly both are eternal

- Landscape in String Theory
- Weinberg's solution to CC (and to other parameters)

- Eternal Inflation has become extremely important
- Difficult task
- Puzzles with de Sitter Entropy

- This motivates detailed studies

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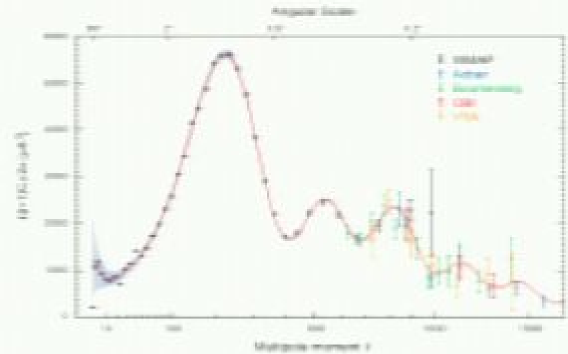

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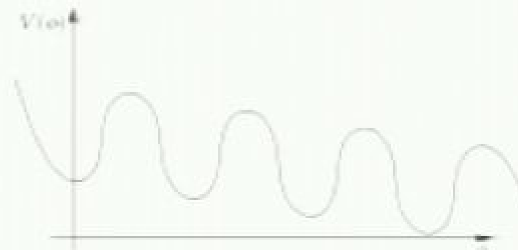

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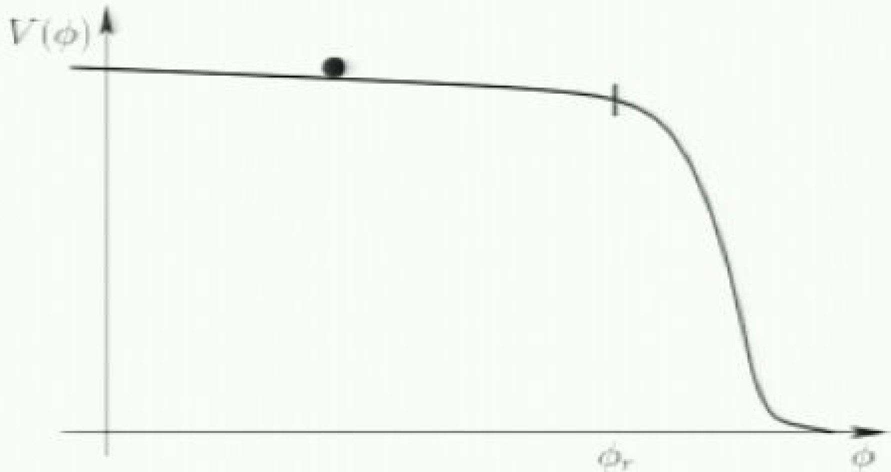
What is Eternal Inflation?

/hat is Inflation?



$$a \sim e^{Ht}$$

/hat is Eternal Inflation?

Classical Motion Vs Quantum Motion

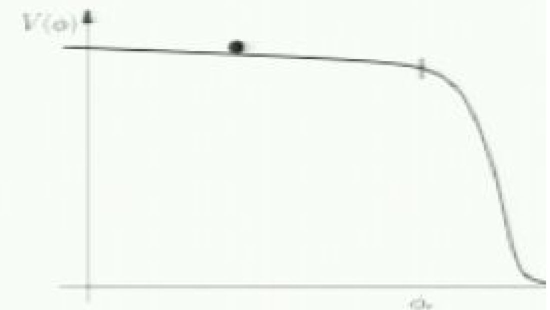
$$\Delta\phi_{Cl} \sim \dot{\phi}\, H^{-1} \quad \text{vs} \quad \Delta\phi_{Q} \sim H$$

Quantum dominates for $\frac{\dot{\phi}}{H^2} \lesssim 1 \Rightarrow$ Eternal Inflation

/lake Eternal Inflation sharp?

- No Semiclassical
- No FRW $\delta\rho/\rho \sim H^2/\dot{\phi} \sim 1$

Perturbativity of the system



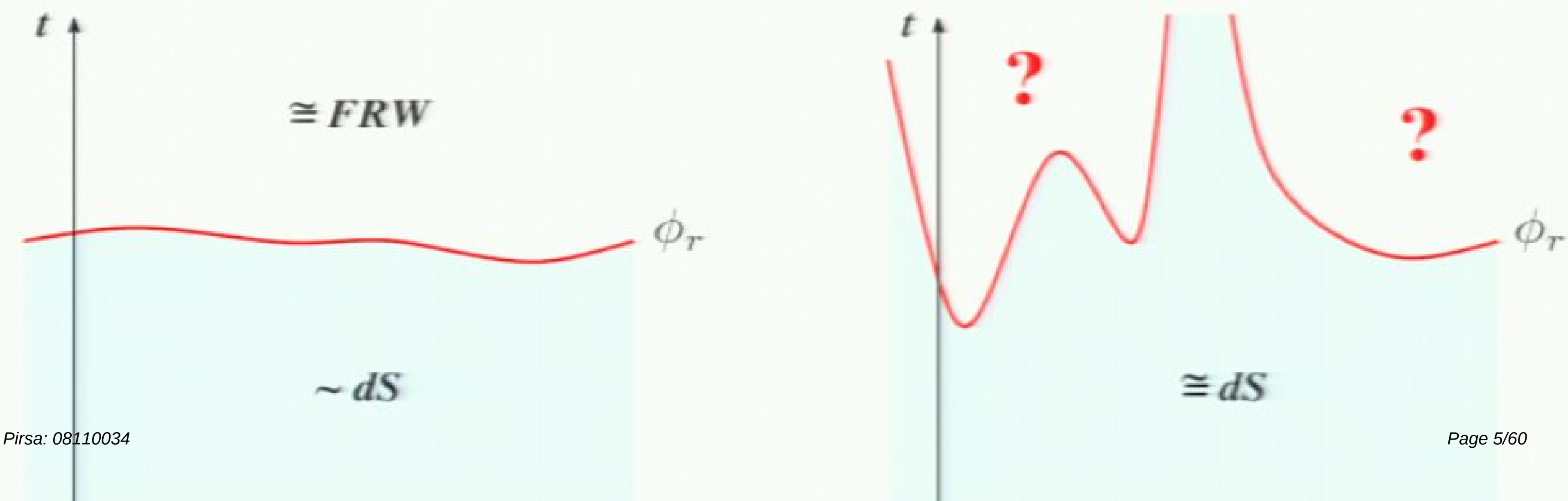
Close to de-Sitter $\epsilon \simeq \frac{\dot{\phi}^2}{V(\phi)} \ll 1$

- Still unperturbed before reheating: $\delta g \sim \sqrt{\epsilon}\frac{H}{M_{\rm Pl}}$

No big interactions: $\frac{S_3}{S_2} \sim \sqrt{\epsilon}\frac{H}{M_{\rm Pl}}$

$\Rightarrow$ Study the volume of the Reheating surface $\phi = \phi_r$

Standard Infl. Eternal Infl.



A random walk

moothing the field: $\Lambda \ll H$:

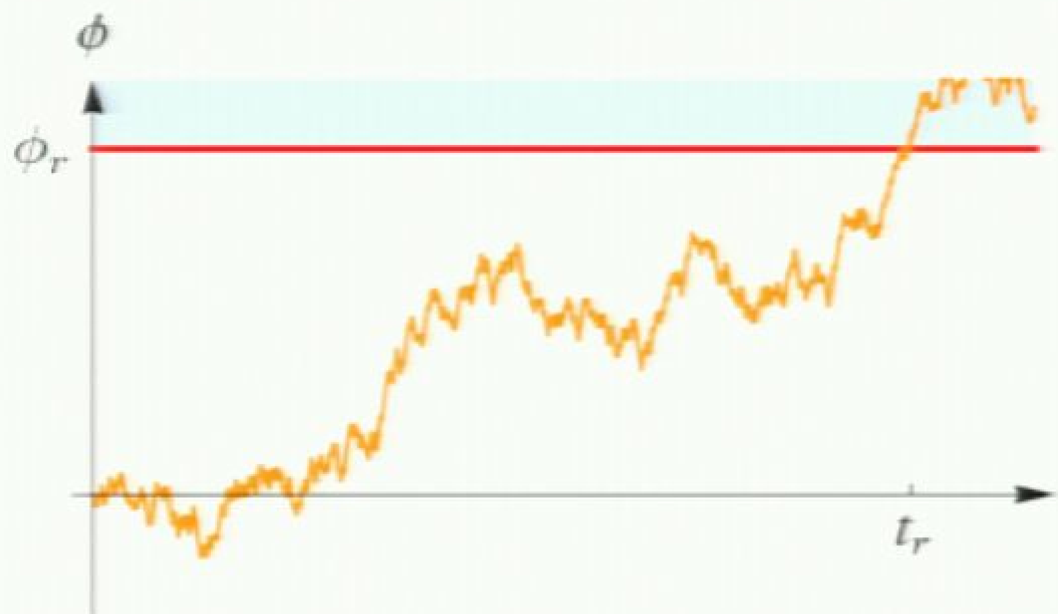
» $\Delta t \gtrsim H^{-1}$ for reheating

» . $[\delta\phi_k, \dot{\delta\phi}_{-k}] \rightarrow 0$

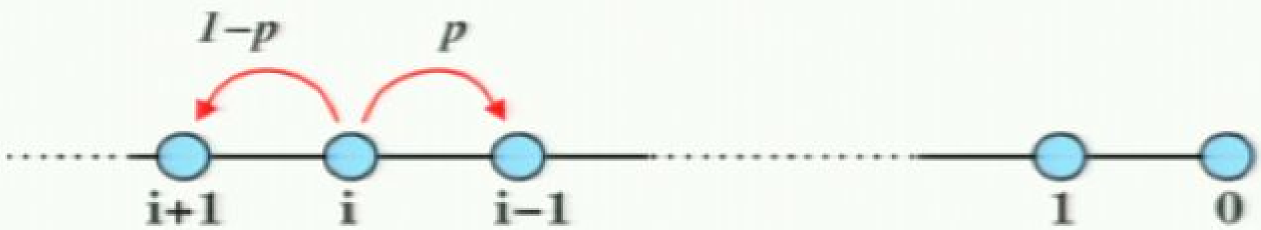
nflaton ~ Classical stochastic system with Gaussian statistics

robability distribution follows a diffusion equation

$$P(\bar{\phi}, \phi, t) \sim e^{-\frac{(\bar{\phi}-\phi-\dot{\phi}t)^2}{H^3 t}}$$



Bacteria Model: a discretization



Reproduction $N \Longrightarrow \sim e^3$ New Hubble volumes

Number of dead bacteria $\Longrightarrow$ Reheated Volume

Classical motion $\dot{\phi} \Longrightarrow p - \frac{1}{2}$,

$$j = -\frac{\phi}{\Delta\phi} \quad , \qquad n = \frac{t}{\Delta t}$$

Continuum Limit: $\bar{P}(j, n+1) = (1-p)\bar{P}(j-1, n) + p\,\bar{P}(j+1, n)$

$$\partial_{\sigma^2} P(\psi, \sigma^2) = \frac{4\pi^2}{H^3}\left((1-2p)\frac{\Delta\phi}{\Delta t} + \dot{\phi}\right)\partial_\psi\, P(\psi, \sigma^2) + \frac{1}{2}\frac{4\pi^2}{H^3}\frac{\Delta\phi^2}{\Delta t}\partial_\psi^2 P(\psi, \sigma^2)$$

Identifications: $-(1-2p)\frac{\Delta\phi}{\Delta t} = \dot{\phi}$ $\qquad \frac{4\pi^2}{H^3}\frac{\Delta\phi^2}{\Delta t} = 1$ $\qquad N = 1 + 3H\Delta t$

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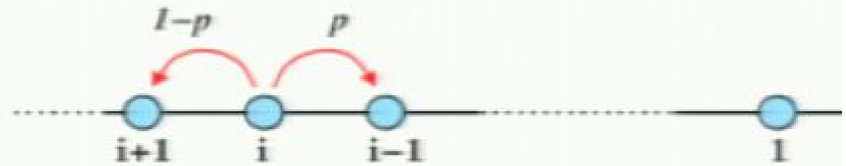

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$$f_i^{(n)}(s_j) = \sum_{k_1 \ldots k_L} p^{(n)}_{i;k_0 \ldots k_L} s_0^{k_0} \ldots s_L^{k_L} \qquad 0 \le s_i \le 1$$

Recursion: $F_{n+1} = F_1(F_n)$, $F_1(F_\infty) = F_\infty$

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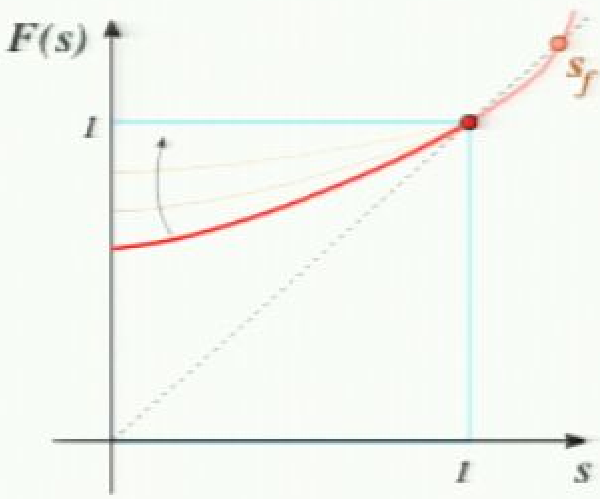

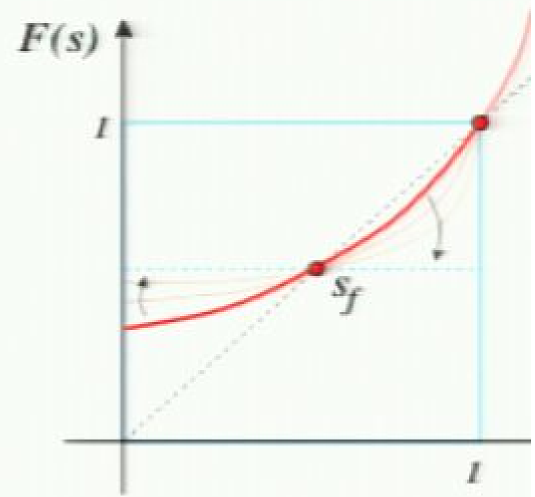

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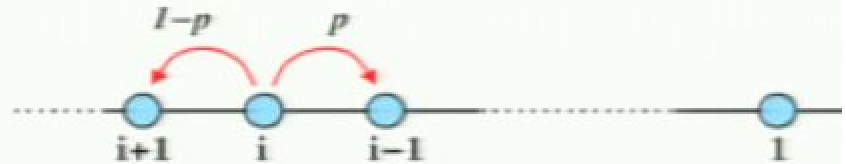

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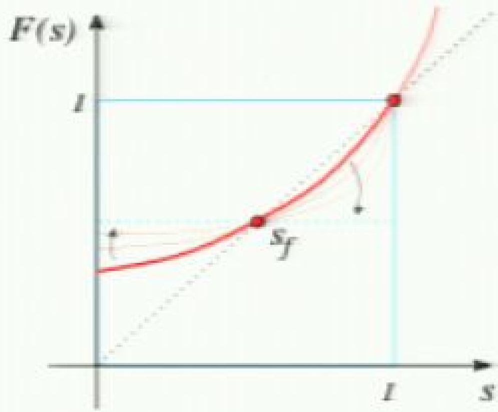

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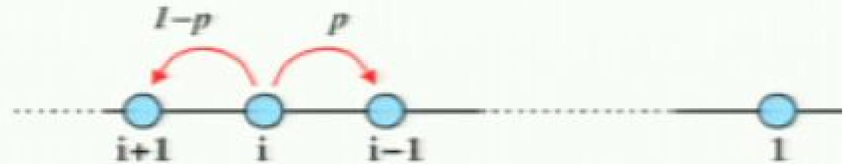

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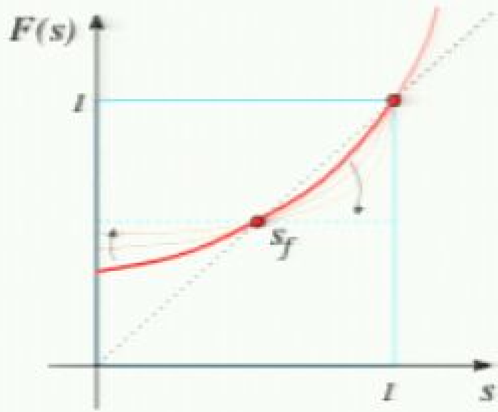

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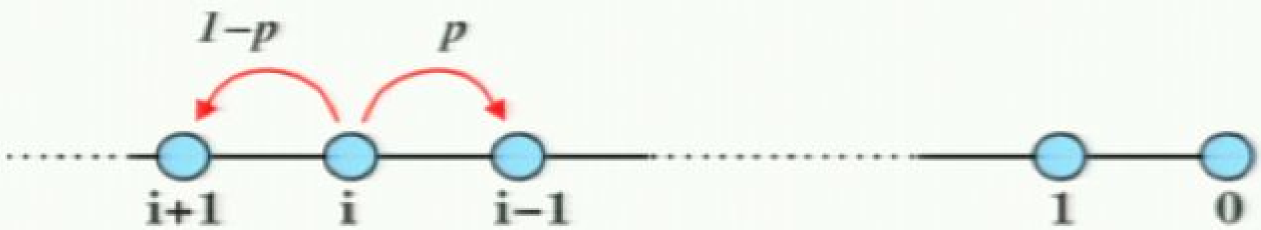

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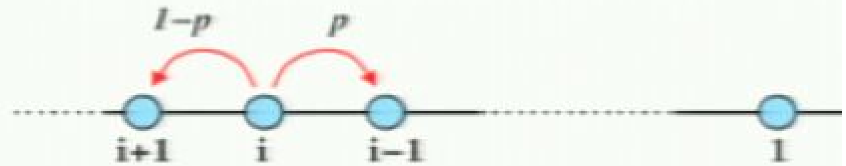

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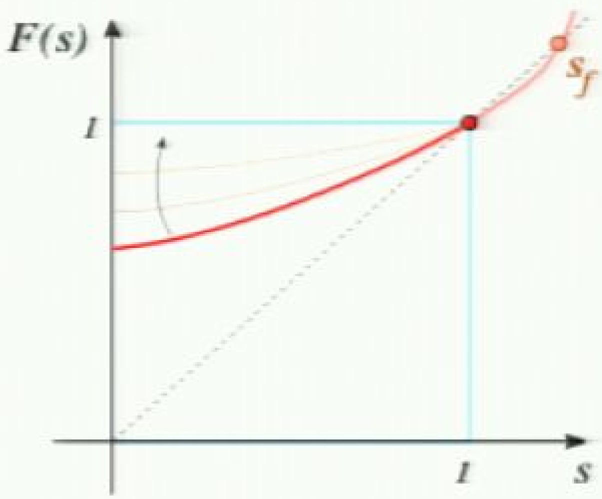

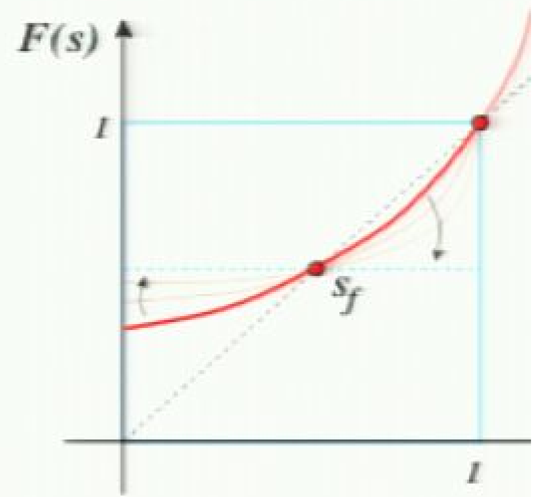

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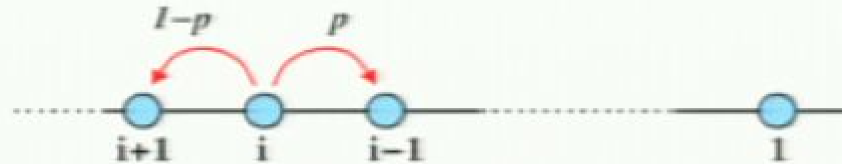

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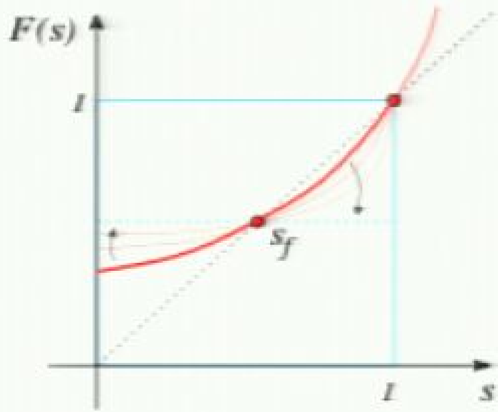

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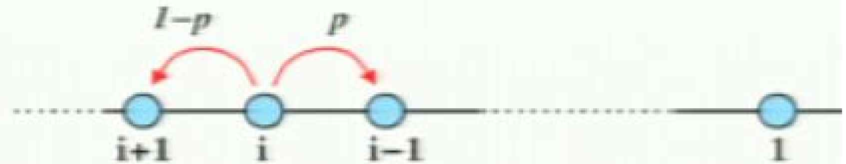

n the sites: $i \Rightarrow \phi$

differential equation:

$$f^{(\infty)}(\phi; s_0) \;=\; \left((1-p) f^{(\infty)}(\phi + \Delta\phi; s_0) + p\, f^{(\infty)}(\phi - \Delta\phi; s_0)\right)^{N_r} \quad \Rightarrow$$

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$$\rho(\phi, V) = \frac{1}{2\pi i}\int_{\gamma - i\infty}^{\gamma + i\infty} d\left(-\log(s_0)\right) f^{(\infty)}(\phi; s_0) e^{-V \log(s_0)}$$

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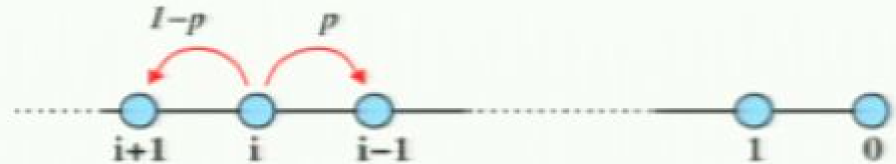

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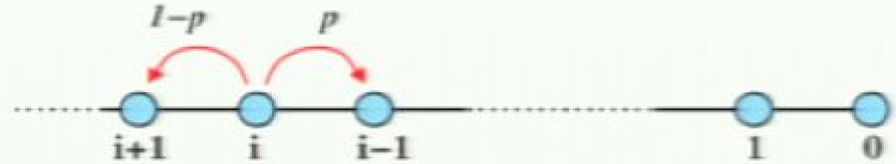

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ere $\tau \propto \phi \propto N_c\,, \qquad z = -\log(s_0)\;, \qquad \Omega \equiv \frac{2\pi^2}{3}\frac{\dot{\phi}^2}{H^4}$

undary conditions:

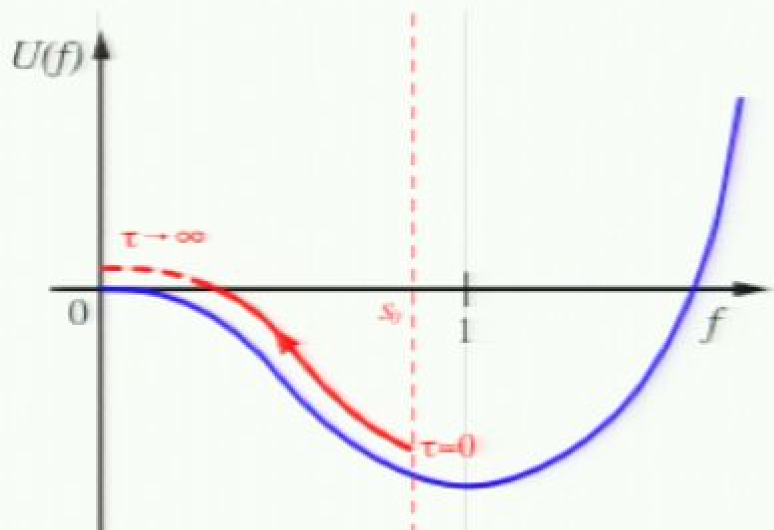
$$f(0;z) = s_0 = e^{-z}\,,$$

$$\dot{f}(\tau_b;z) = 0\,,$$

$$f(\tau;z) \in [0,1]$$

chanical Problem: Anti-friction and potential $\qquad U(f) = \frac{f^2}{4}\left(\log f^2 - 1\right)$

rrier at infinity:

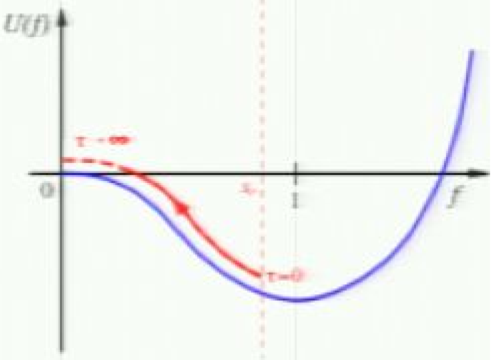


e Phase Transition

$$\ddot{f}(\tau;z) - 2\sqrt{\Omega}\dot{f}(\tau;z) + f(\tau;z)\log\left[f(\tau;z)\right] = 0$$

$$P_{\rm ext} \equiv \int_0^\infty dV\,\rho(V,\tau) = f(\tau;0)$$

$z = -\log(s_0)$



$$s_0 \to 1 \quad \Rightarrow \quad z \to 0$$

$\Rightarrow$ Linearized solution:

$$\ddot{f} - 2\sqrt{\Omega}\dot{f} + f - 1 = 0\,, \qquad \Longrightarrow \qquad f = 1 - e^{\sqrt{\Omega}\tau}\left(Ae^{\sqrt{\Omega-1}\tau} + Be^{-\sqrt{\Omega-1}\tau}\right)$$

ifferent behaviors for $\Omega \gtrless 1$

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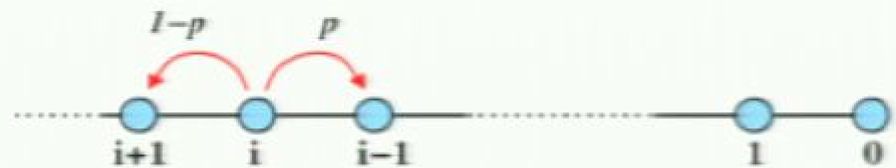

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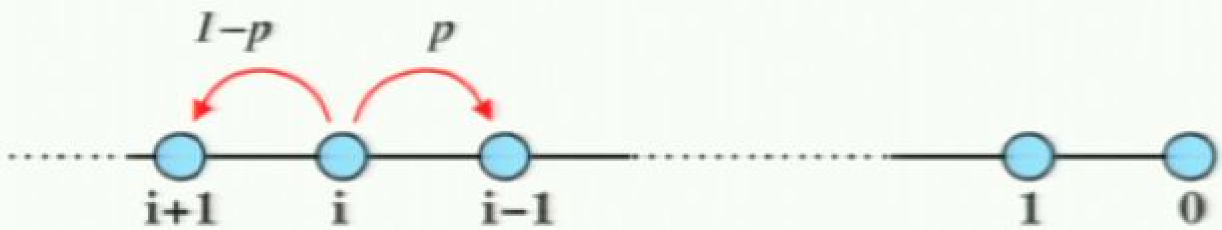

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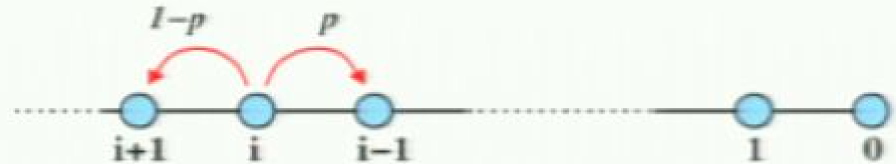
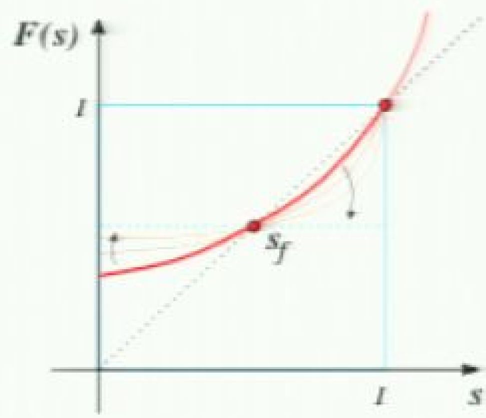


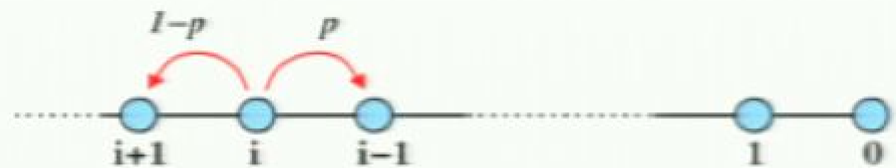
$_\infty$ is only a function of s_0



$_1(F_\infty) = F_\infty$,

$$
\begin{aligned}
s_0,\dots,s_L) &= s_0\,, \\
s_0,\dots,s_L) &= ((1-p)s_2 + p\,s_0)^{N_r}\,, \\
&\vdots \\
s_0,\dots,s_L) &= ((1-p)s_{i+1} + p\,s_{i-1})^{N_r} \\
&\vdots \\
s_0,\dots,s_L) &= ((1-p)s_L + p\,s_{L-1})^{N_r}
\end{aligned}
\Longrightarrow
\begin{aligned}
f_0^{(\infty)}(s_0) &= s_0\,, \\
&\vdots \\
f_i^{(\infty)}(s_0) &= \left((1-p)f_{i+1}^{(\infty)}(s_0) + p\,f_{i-1}^{(\infty)}(s_0)\right)^{N_r} \\
&\vdots \\
f_L^{(\infty)}(s_0) &= \left((1-p)f_L^{(\infty)}(s_0) + p\,f_{L-1}^{(\infty)}(s_0)\right)^{N_r}
\end{aligned}
$$

king the continuum limit:



n the sites: $i \Rightarrow \phi$

differential equation:

$$f^{(\infty)}(\phi; s_0) \;=\; \left((1-p)f^{(\infty)}(\phi+\Delta\phi; s_0) + p\, f^{(\infty)}(\phi-\Delta\phi; s_0)\right)^{N_r} \qquad \Rightarrow$$

$$\frac{\partial^2}{\partial\phi^2} f^{(\infty)}(\phi; s_0) - \frac{2\pi\sqrt{6\Omega}}{H}\frac{\partial}{\partial\phi} f^{(\infty)}(\phi; s_0) + \frac{12\pi^2}{H^2} f^{(\infty)}(\phi; s_0)\log\left[f^{(\infty)}(\phi; s_0)\right] = 0$$

$$\Omega \equiv \frac{2\pi^2}{3}\frac{\dot\phi^2}{H^4}$$

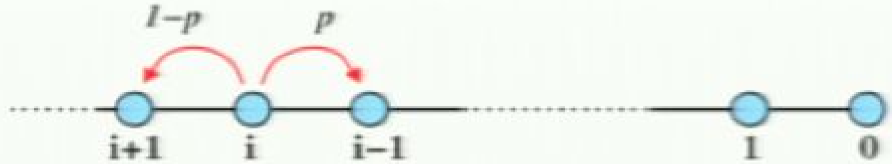
ith boundary conditions: $f^{(\infty)}(\phi=0, s_0) = s_0 \;,\; \frac{\partial}{\partial\phi} f^{(\infty)}(\phi; s_0)\Big|_{\phi_b} \;=\; 0$

or the quantity: $f_j^{(\infty)}(s_0) = \sum_{k=0}^{\infty} p_{j,k}\, s_0^k \;\Rightarrow f^{(\infty)}(\phi; s_0) = \int_0^{\infty} dV\, \rho(\phi, V)\, s_0^V$

e. for the Laplace transform of $\rho(\phi, V)$

$$\rho(\phi, V) = \frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} d\left(-\log(s_0)\right) f^{(\infty)}(\phi; s_0) e^{-V\log(s_0)}$$

ıdying the differential eq.



ff. Eq. :

$$\ddot{f}(\tau;z) - 2\sqrt{\Omega}\dot{f}(\tau;z) + f(\tau;z)\log\left[f(\tau;z)\right] = 0$$

ere $\tau \propto \phi \propto N_c\,, \qquad z = -\log(s_0)\ , \qquad \Omega \equiv \frac{2\pi^2}{3}\frac{\dot{\phi}^2}{H^4}$

ındary conditions:

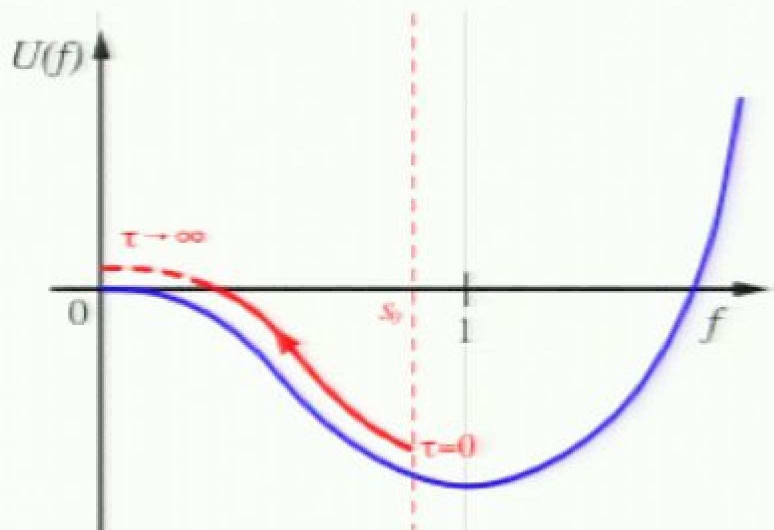
$$f(0;z) = s_0 = e^{-z}\,,$$

$$\dot{f}(\tau_b;z) = 0\,,$$

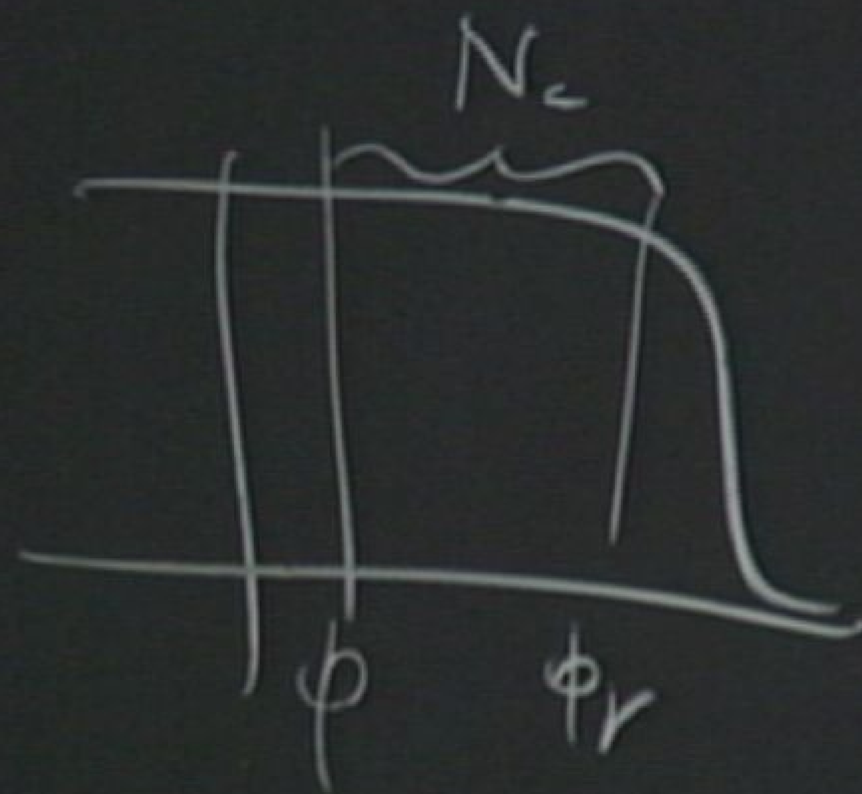
$$f(\tau;z) \in [0,1]$$

:chanical Problem: Anti-friction and potential $\qquad U(f) = \frac{f^2}{4}\left(\log f^2 - 1\right)$

ırrier at infinity:



$$V = e^{3N}$$



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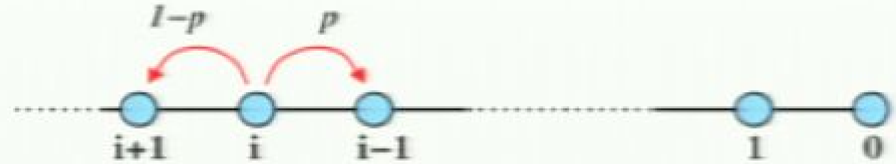

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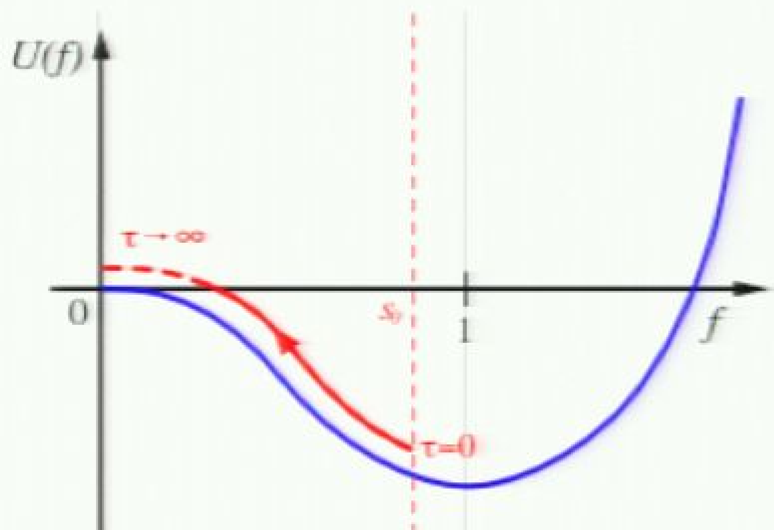

e Phase Transition

$$\ddot{f}(\tau;z) - 2\sqrt{\Omega}\dot{f}(\tau;z) + f(\tau;z)\log\left[f(\tau;z)\right] = 0$$

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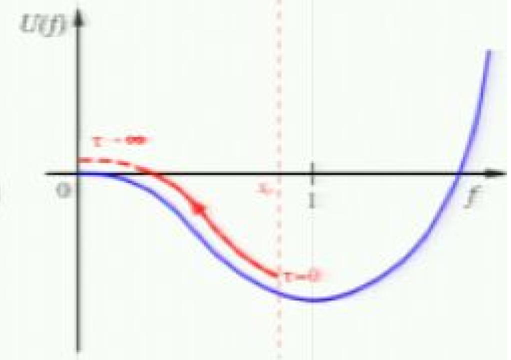
$z = -\log(s_0)$



$$s_0 \to 1 \quad \Rightarrow \quad z \to 0$$

$\Rightarrow$ Linearized solution:

$$\ddot{f} - 2\sqrt{\Omega}\dot{f} + f - 1 = 0\,, \qquad \Longrightarrow \qquad f = 1 - e^{\sqrt{\Omega}\tau}\left(Ae^{\sqrt{\Omega-1}\tau} + Be^{-\sqrt{\Omega-1}\tau}\right)$$

ifferent behaviors for $\Omega \gtrless 1$

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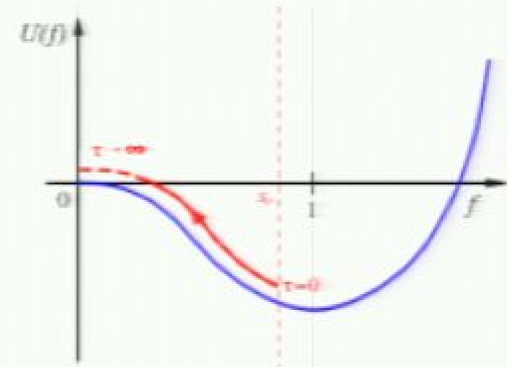

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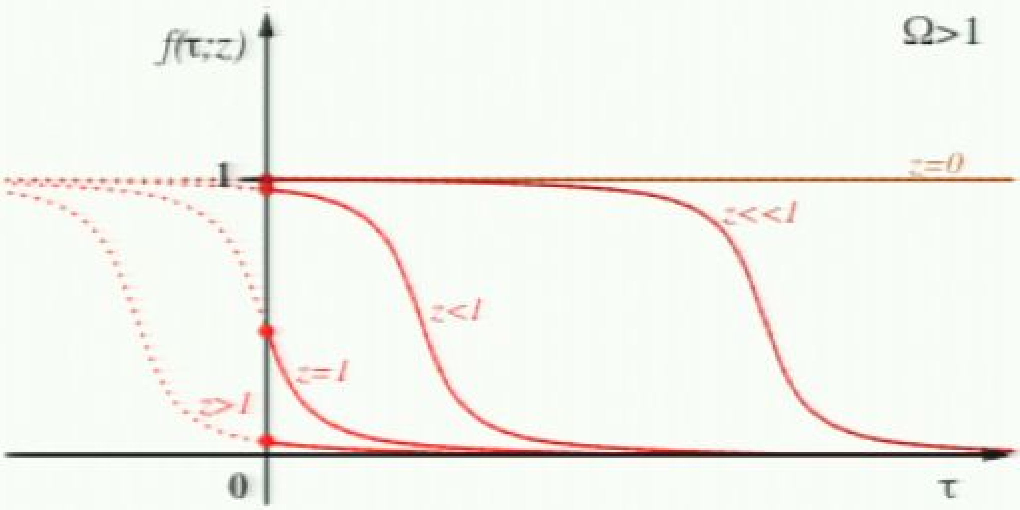

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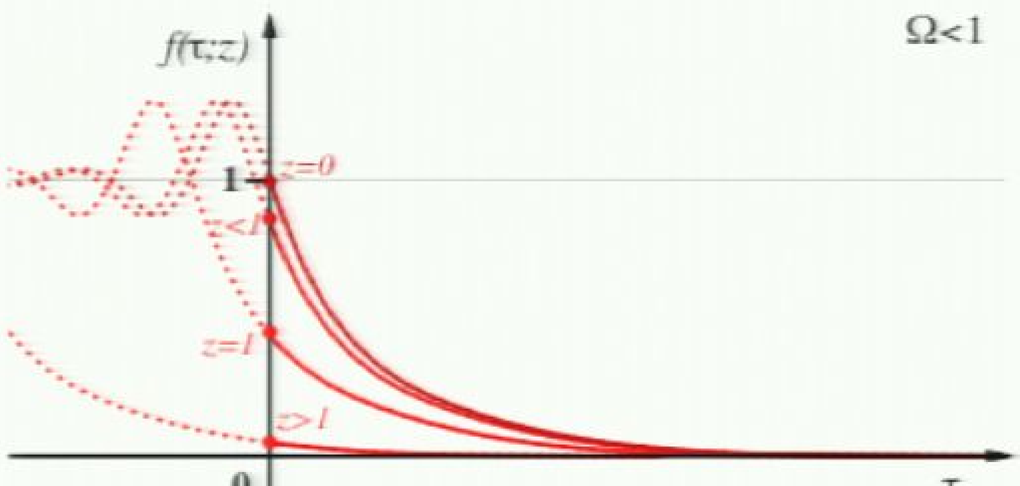

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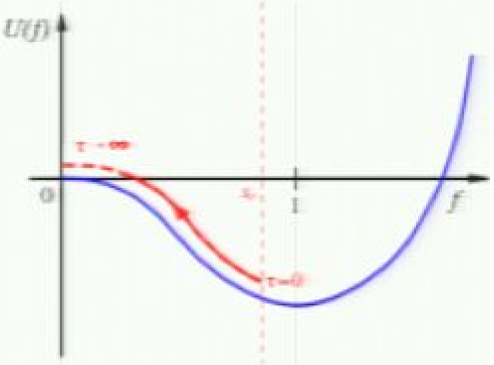

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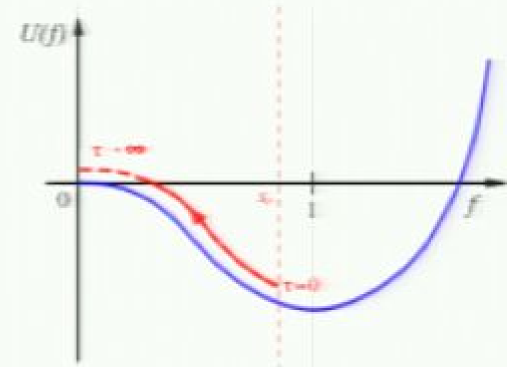

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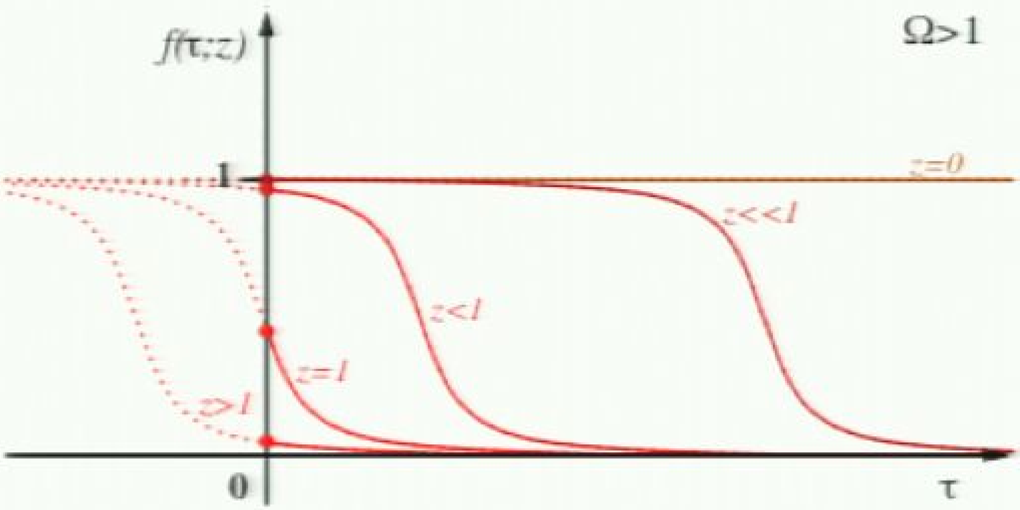

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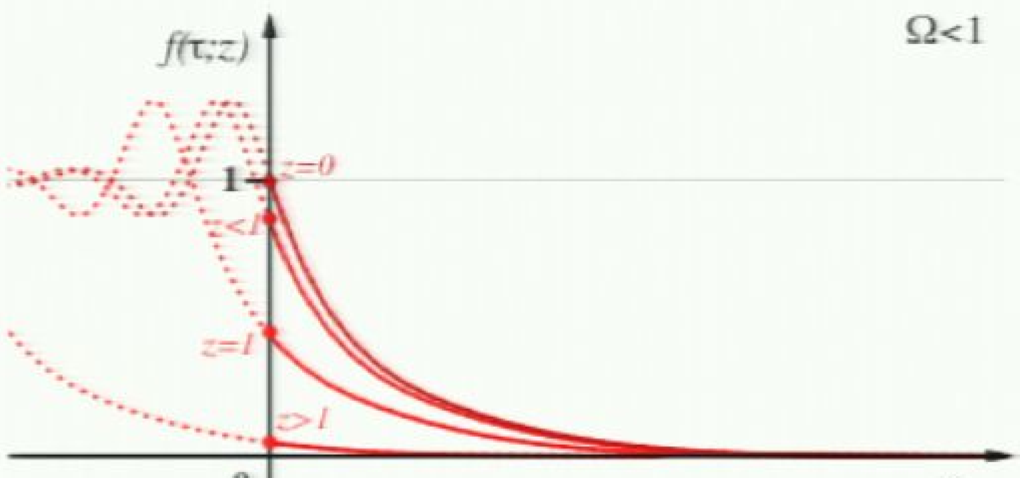

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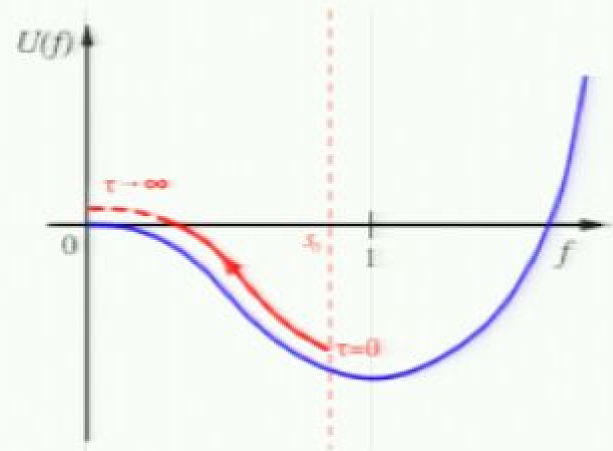

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addle point $$\rho(V,\tau) \approx \frac{1}{\sqrt{2\pi |S''(z_0)|}} e^{-S(z_0)} \equiv \mathcal{N} e^{-S(z_0)},$$

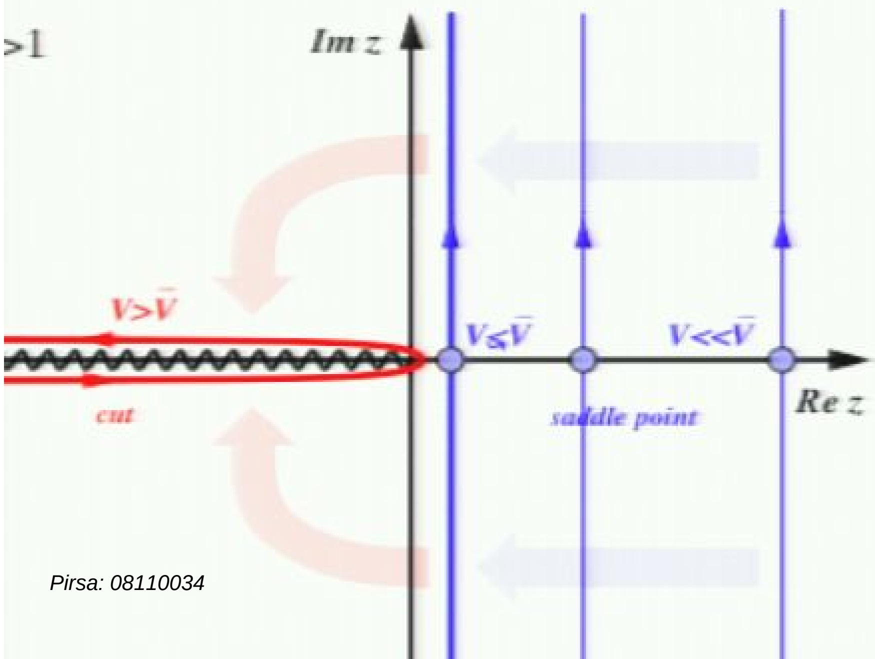
here $$S(z) \simeq (\tau+\tau_0)^2 - zV \simeq \left(\tau - \frac{1}{\omega_-}\log z\right)^2 - Vz$$

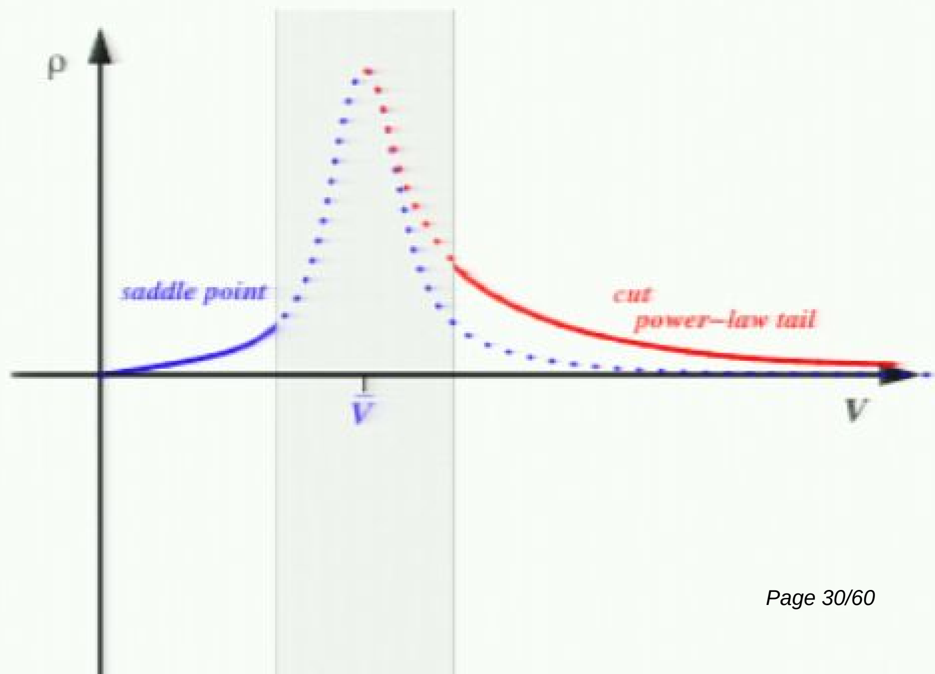
dle point $$z_0 \approx \frac{1}{2\omega_- V}\left[\tau - \frac{1}{\omega_-}\log\left(\frac{2\omega_- V}{\tau}\right)\right]$$

$$, \tau) \approx \mathcal{N} e^{-\frac{1}{4}\Omega\left(1+\sqrt{1-\frac{1}{\Omega}}\right)^2\left[\log\left(\frac{V}{\overline{V}}\right)\right]^2} = \mathcal{N} e^{-\Omega\left[\frac{3N}{2}\left(1+\sqrt{1-\frac{1}{\Omega}}\right)-3N_c\right]^2}, \qquad V \lesssim \overline{V},$$

where $$\overline{V} \equiv e^{\omega_- \tau} = e^{3N_c \frac{2}{1+\sqrt{1-1/\Omega}}} .$$

>1





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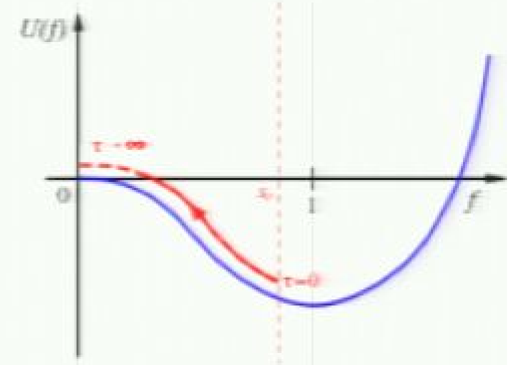

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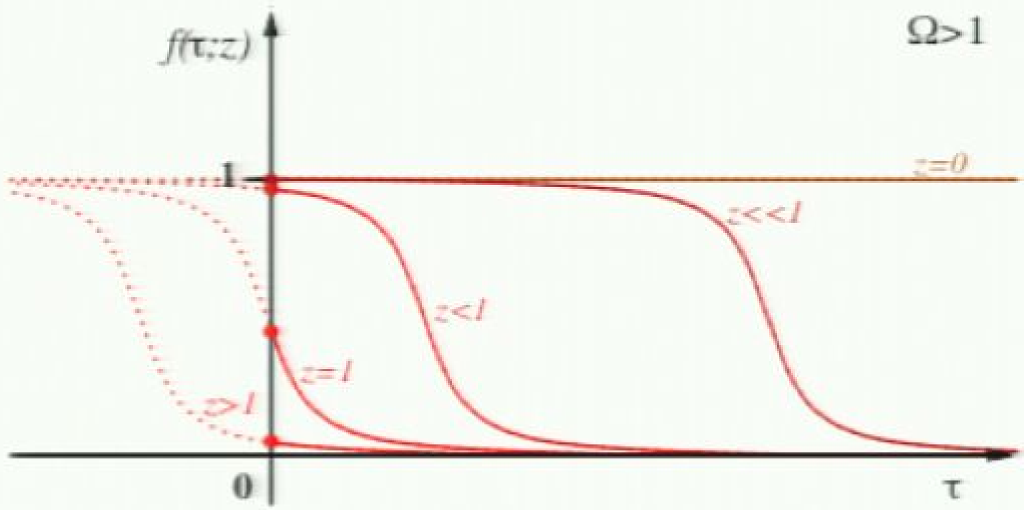

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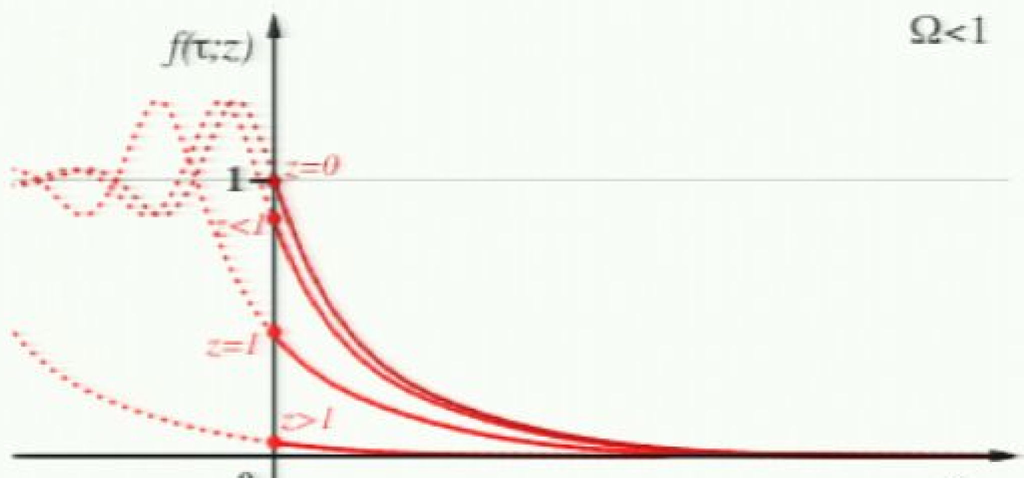

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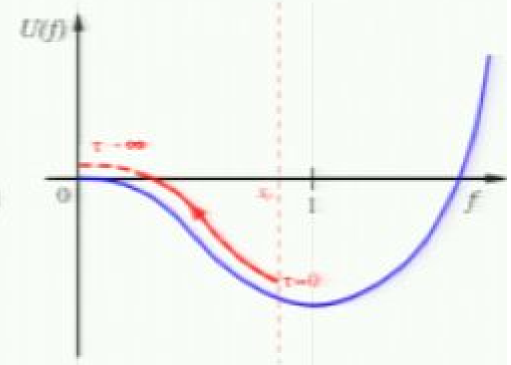

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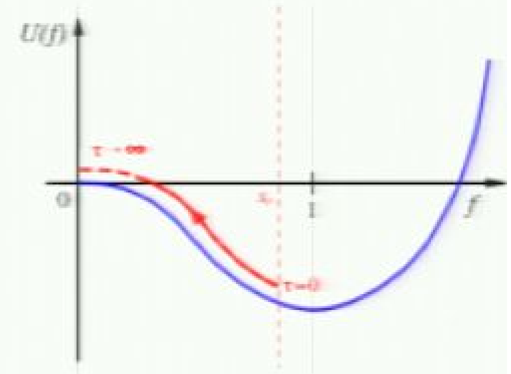

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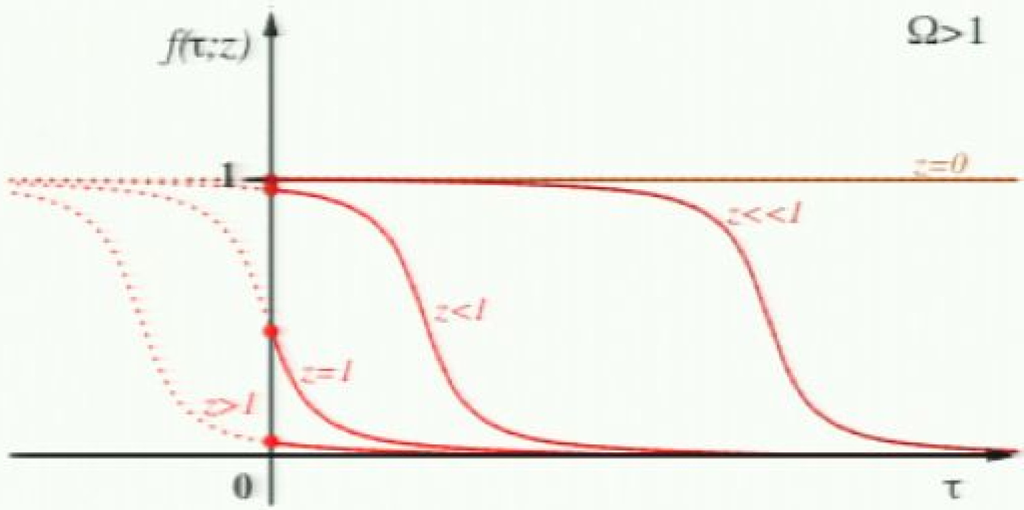

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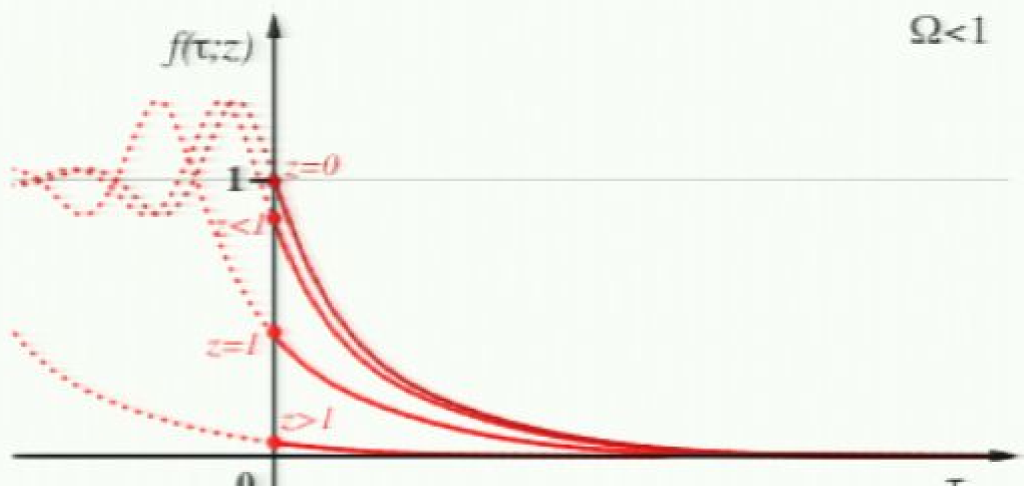

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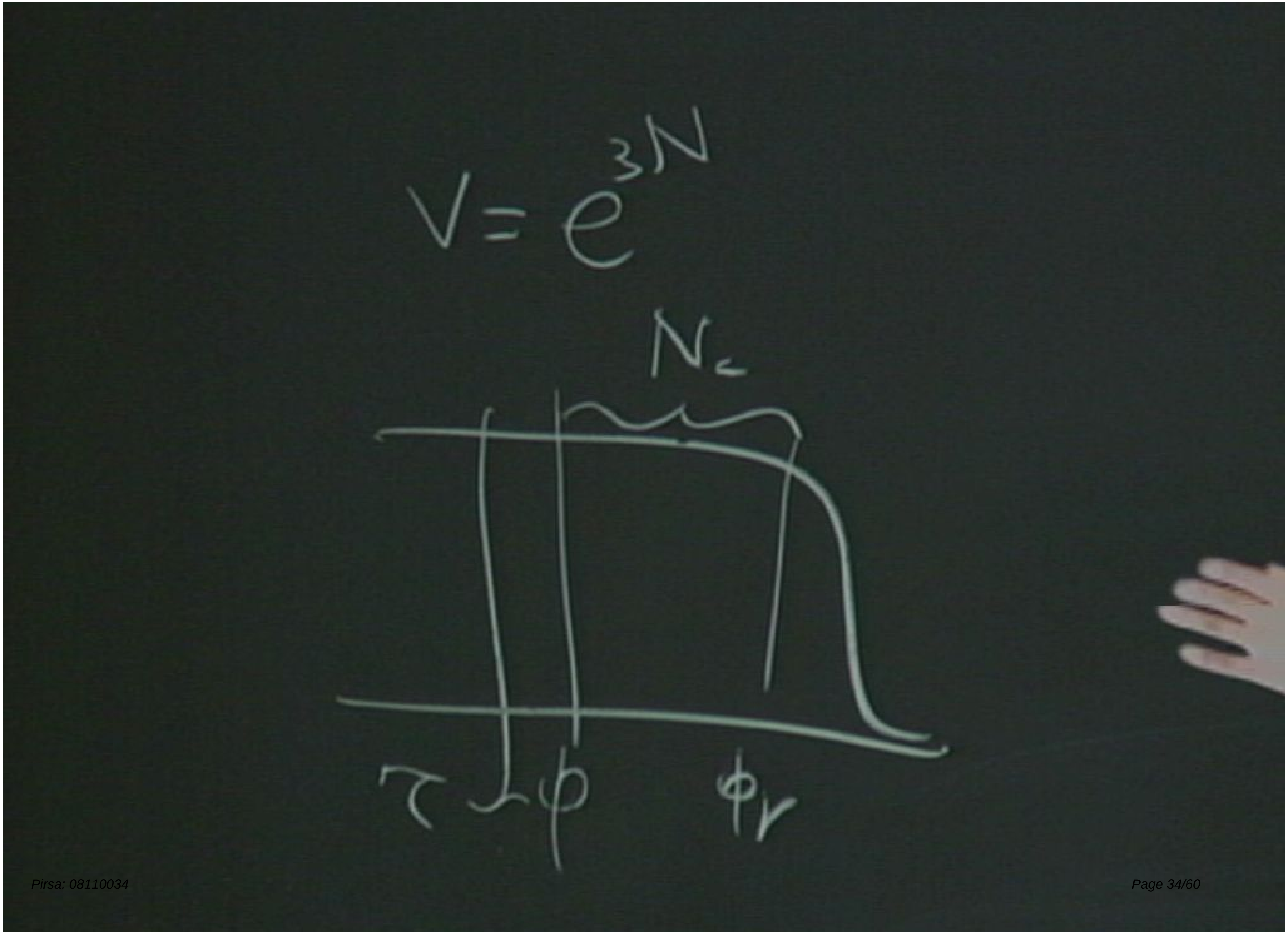
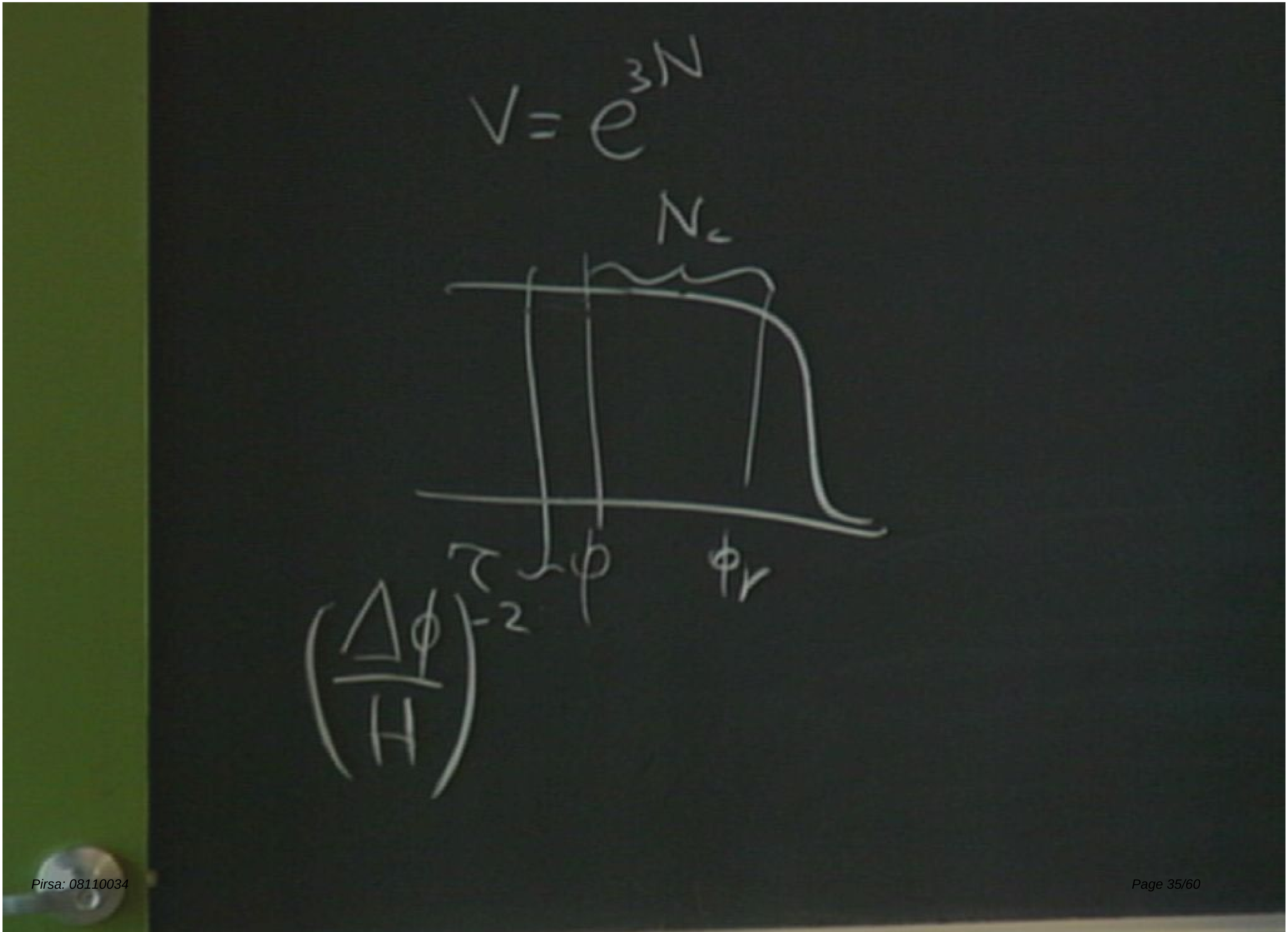

V = e^3N
N_e
τ
φ
φ_r


$V = e^{3N}$
N_c
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ϕ_r
$\left(\frac{\Delta\phi}{H}\right)^{-2}$

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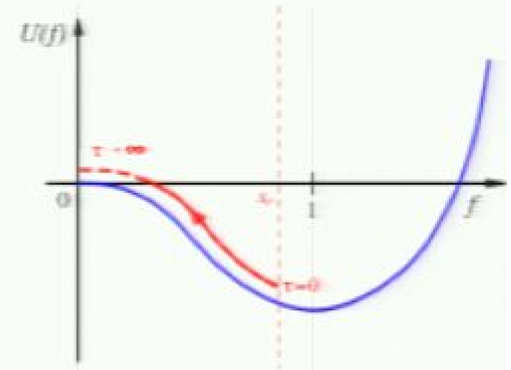

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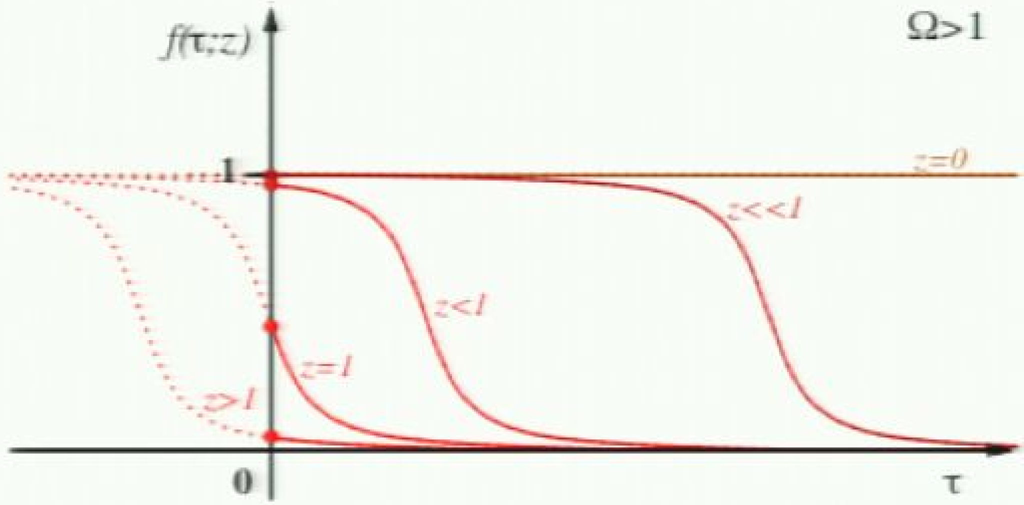

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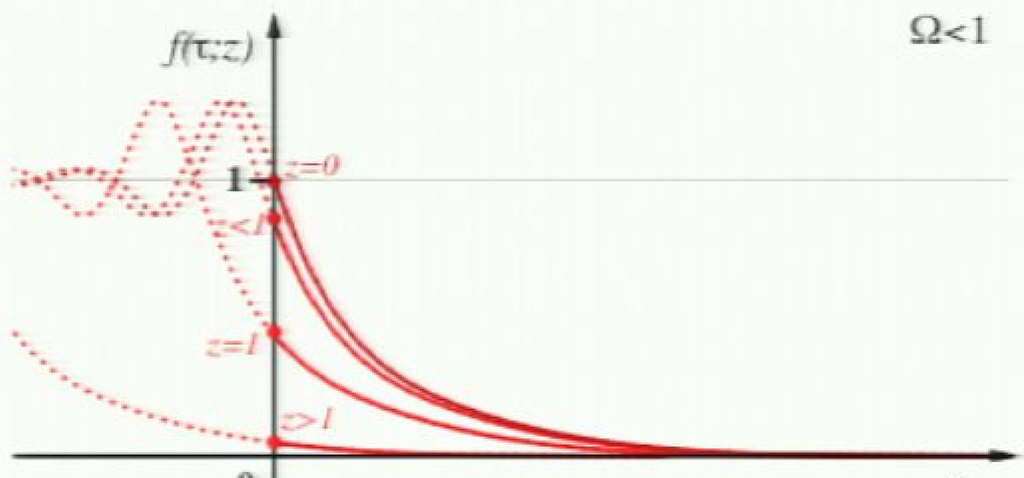

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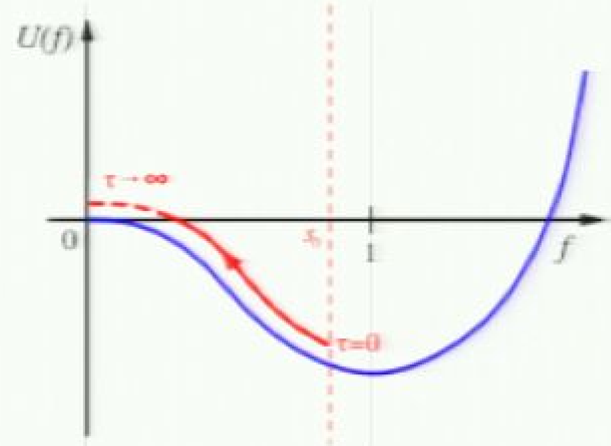

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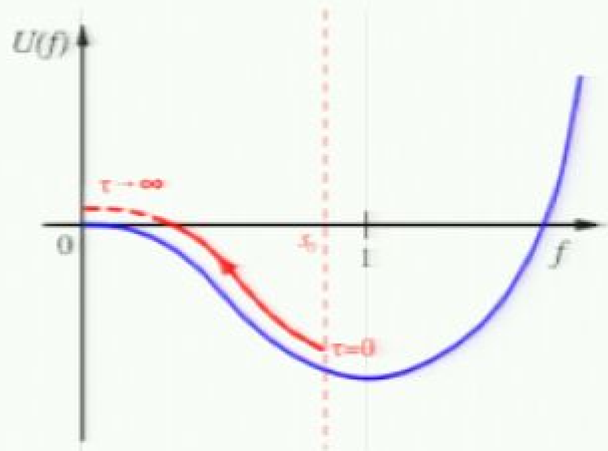

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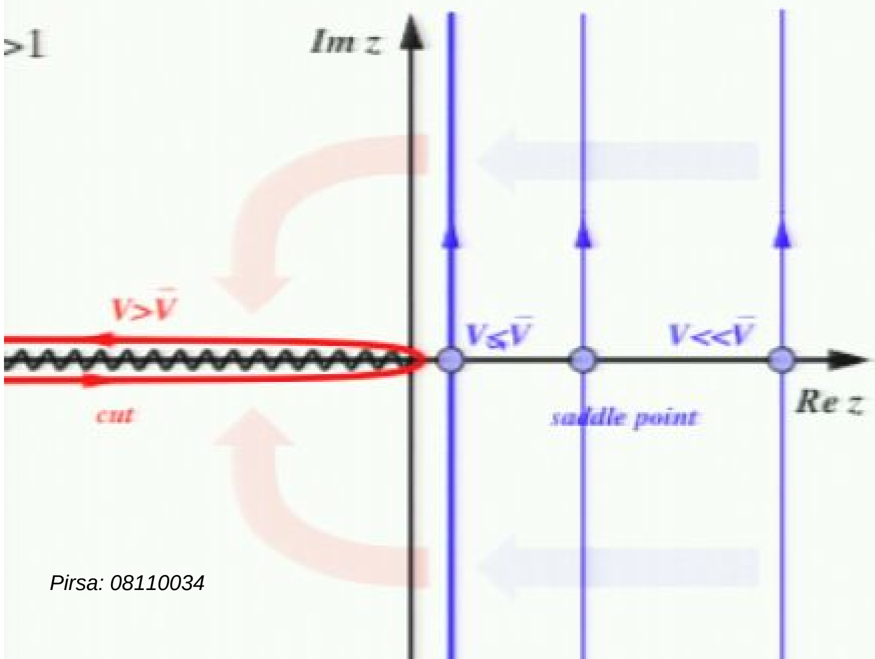
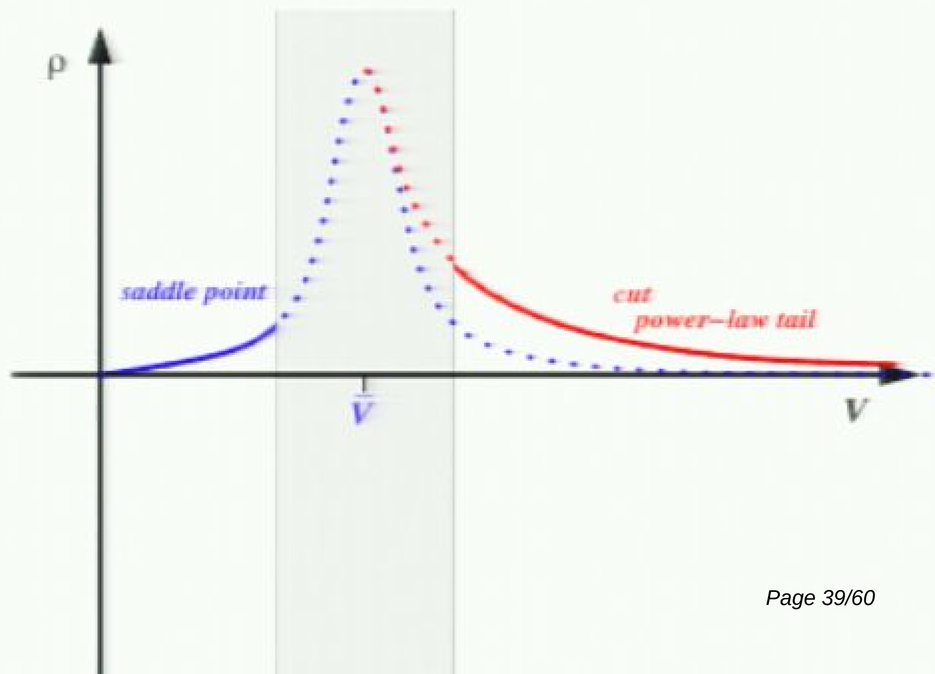
ldle point $\quad z_0 \approx \frac{1}{2\omega_- V}\left[\tau - \frac{1}{\omega_-}\log\left(\frac{2\omega_- V}{\tau}\right)\right]$

$$V,\tau) \approx \mathcal{N} e^{-\frac{1}{4}\Omega\left(1+\sqrt{1-\frac{1}{\Omega}}\right)^2\left[\log\left(\frac{V}{\overline{V}}\right)\right]^2} = \mathcal{N} e^{-\Omega\left[\frac{3N}{2}\left(1+\sqrt{1-\frac{1}{\Omega}}\right)-3N_c\right]^2}\,, \qquad V \lesssim \overline{V}\,,$$

where $\quad \overline{V} \equiv e^{\omega_- \tau} = e^{3N_c \frac{2}{1+\sqrt{1-1/\Omega}}}\,.$

>1

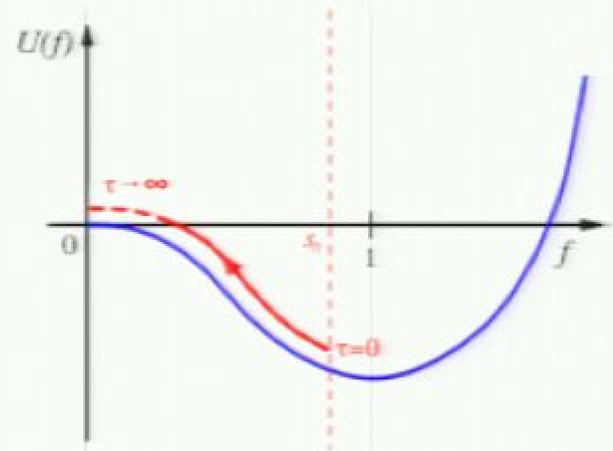




> 1 : Approaching the Transition from above:

$$\ddot{f}(\tau;z) - 2\sqrt{\Omega}\dot{f}(\tau;z) + f(\tau;z)\log\left[f(\tau;z)\right] = 0$$

iolutions in two regimes:



aussian

$f \simeq 0 \quad \Longrightarrow \quad f \approx f_g = e^{-\frac{(\tau+\tau_1)^2}{4}} \qquad |\tau+\tau_1| \gg 1$

inear

$f \simeq 1 \quad \Longrightarrow \quad f_{\rm lin}(\tau;z) = 1 - e^{\omega_-(\tau+\tau_0)} - \sigma e^{\omega_+(\tau+\tau_0)}$

$$\omega_\pm \equiv \sqrt{\Omega} \pm \sqrt{\Omega - 1} \qquad \tau + \tau_0 \ll -1$$

oundary Condition: $\quad f_{\rm lin}(0;z) = 1 - e^{\omega_-\tau_0} - \sigma e^{\omega_+\tau_0} = e^{-z} \quad \Longrightarrow \quad \tau_0 \approx \log(z)/\omega_-$

ut at

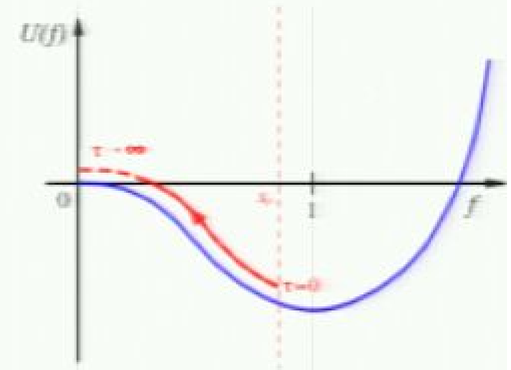
$$z = 0. \quad \Longrightarrow \quad f_{\rm lin}(\tau;z) \ \simeq \ 1 - z e^{\omega_-\tau} - \sigma z^{\omega_+^2} e^{\omega_+\tau}$$

addle point or integral along the cut: $\quad \rho(V,\tau) = \frac{1}{2\pi i}\int_{0^+-i\infty}^{0^++i\infty} dz\, f(\tau;z)e^{zV}$

e Phase Transition

$$P_{\rm ext} \equiv \int_0^\infty dV \rho(V,\tau) = f(\tau;0)$$

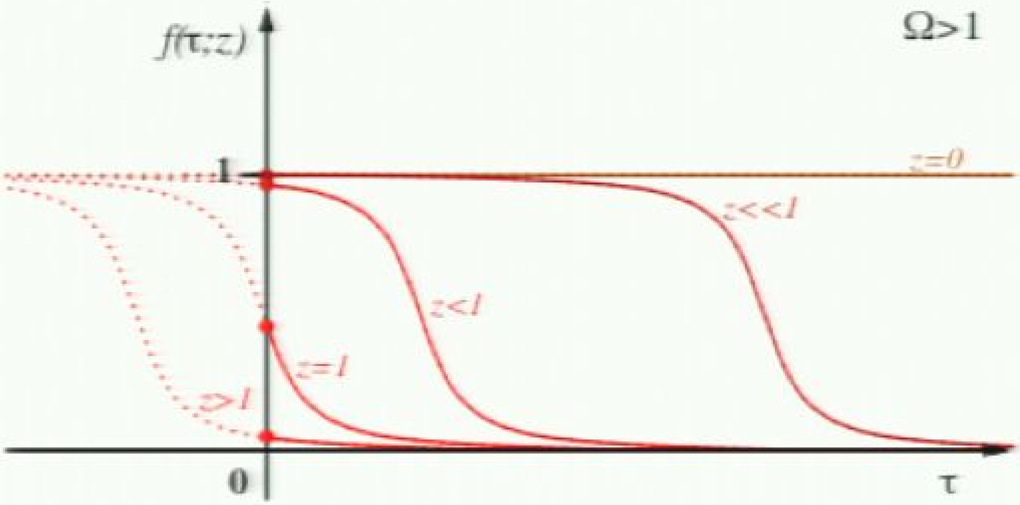
fferent behaviors for $\Omega \gtrless 1$



$$f = 1 - e^{\sqrt{\Omega}\tau}\left(Ae^{\sqrt{\Omega-1}\tau} + Be^{-\sqrt{\Omega-1}\tau}\right)$$

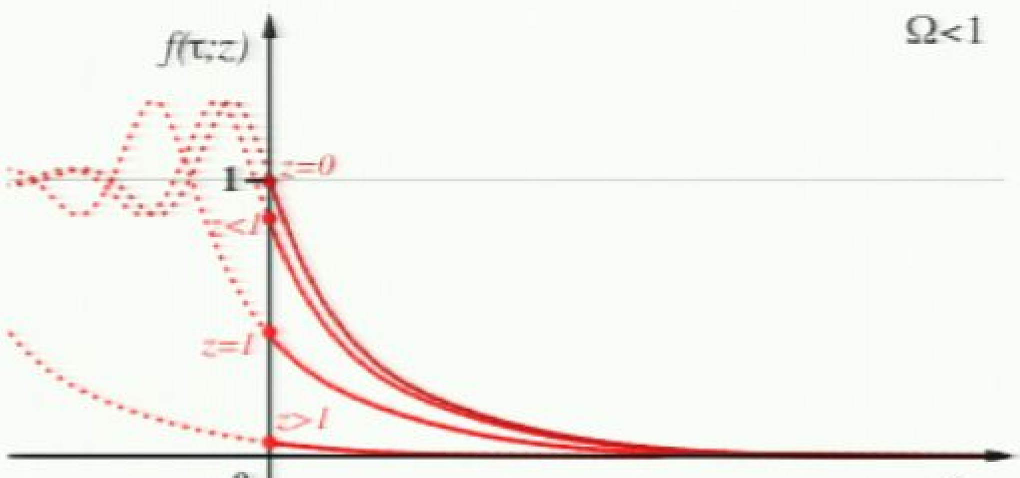
tal Extinction

$\Omega > 1$



› Extinction

$\Omega < 1$



Phase transition:

$$\Omega \equiv \frac{2\pi^2}{3}\frac{\dot{\phi}^2}{H^4} < 1$$

with P. Creminelli, S. Dubovsky
A. Nicolis, M. Zaldarriaga,
JHEP 0809:036 2008

> 1 : Approaching the Transition from above:

$$\ddot{f}(\tau; z) - 2\sqrt{\Omega}\dot{f}(\tau; z) + f(\tau; z)\log[f(\tau; z)] = 0$$

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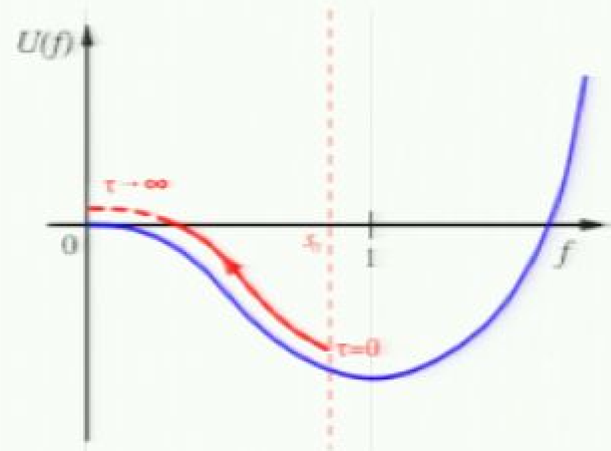

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$$z = 0. \quad \Longrightarrow \quad f_{\text{lin}}(\tau; z) \simeq 1 - z e^{\omega_- \tau} - \sigma z^{\omega_+^2} e^{\omega_+ \tau}$$

addle point or integral along the cut: $\rho(V, \tau) = \frac{1}{2\pi i} \int_{0^+ - i\infty}^{0^+ + i\infty} dz\, f(\tau; z) e^{zV}$

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addle point $\quad \rho(V,\tau) \approx \frac{1}{\sqrt{2\pi|S''(z_0)|}}e^{-S(z_0)} \equiv \mathcal{N}e^{-S(z_0)}\,,$

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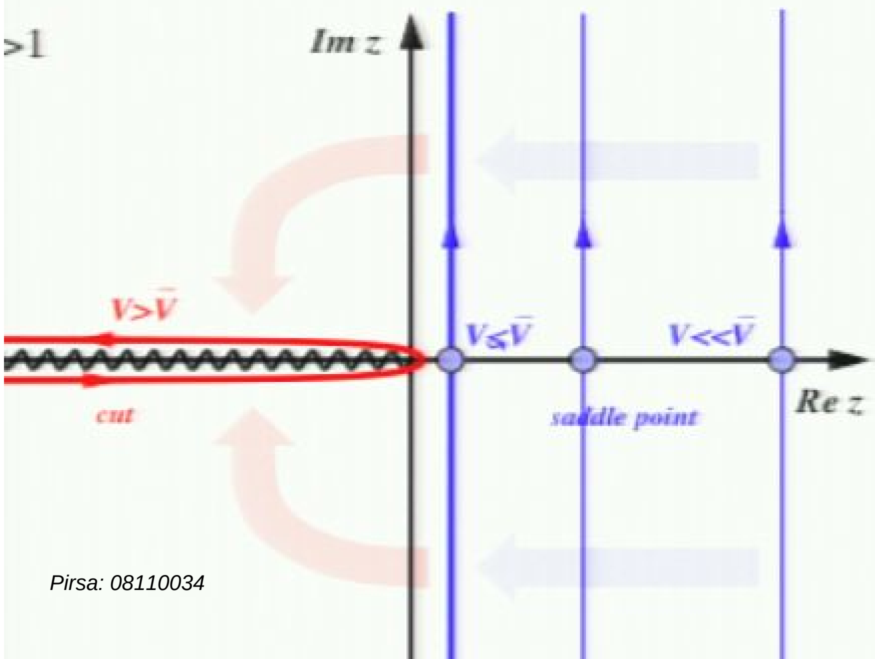

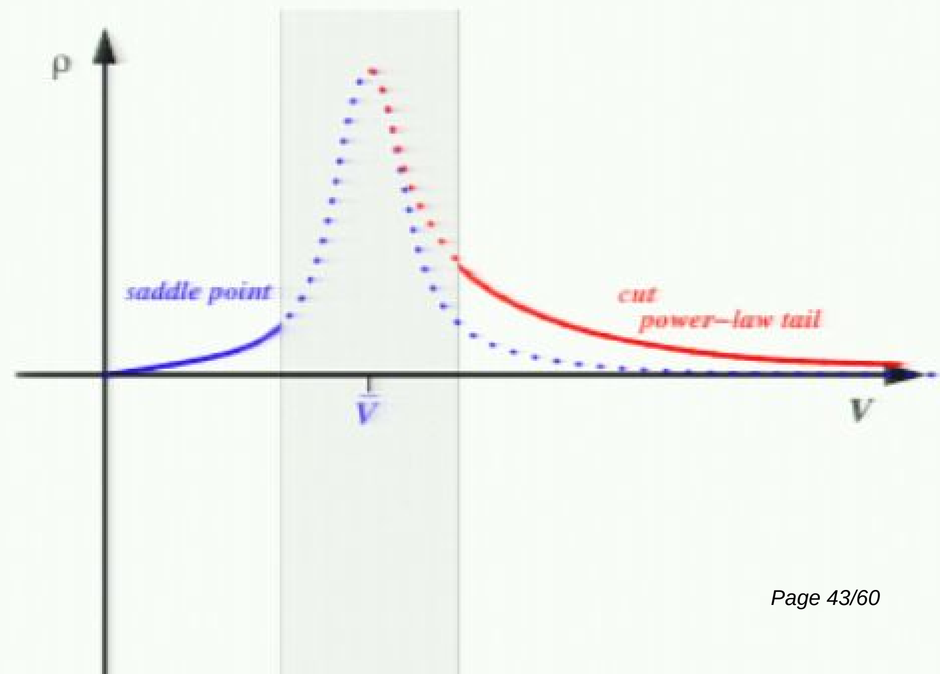

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here $\mathrm{Im}[f_{\mathrm{lin}}(\tau;z)]_{\mathrm{cut}} \sim e^{\omega_+\tau} z^{\frac{\omega_+}{\omega_-}}$

ntegral in linear regime: $\rho(V,\tau) \sim \left(\frac{\overline{V}}{V}\right)^{\frac{\omega_+}{\omega_-}+1}$ for $V \gtrsim \overline{V}\frac{\omega_+}{\omega_-}$

$$\rho(V,\tau) \sim e^{-\Omega\left[\frac{3N}{2}\left(1+\sqrt{1-\frac{1}{\Omega}}\right)-3N_c\right]^2}, \quad \text{for} \quad V \lesssim \overline{V}\,,$$

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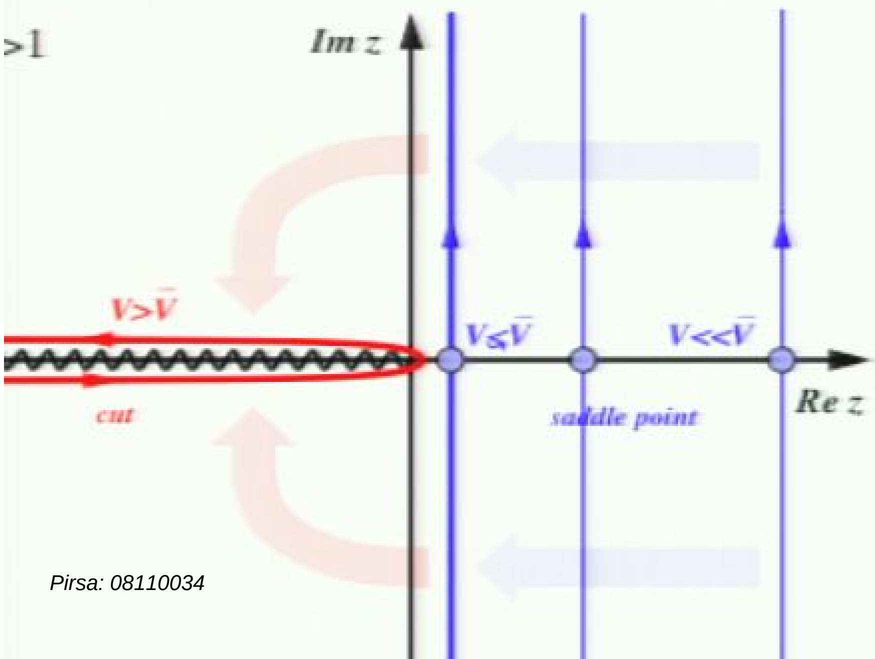

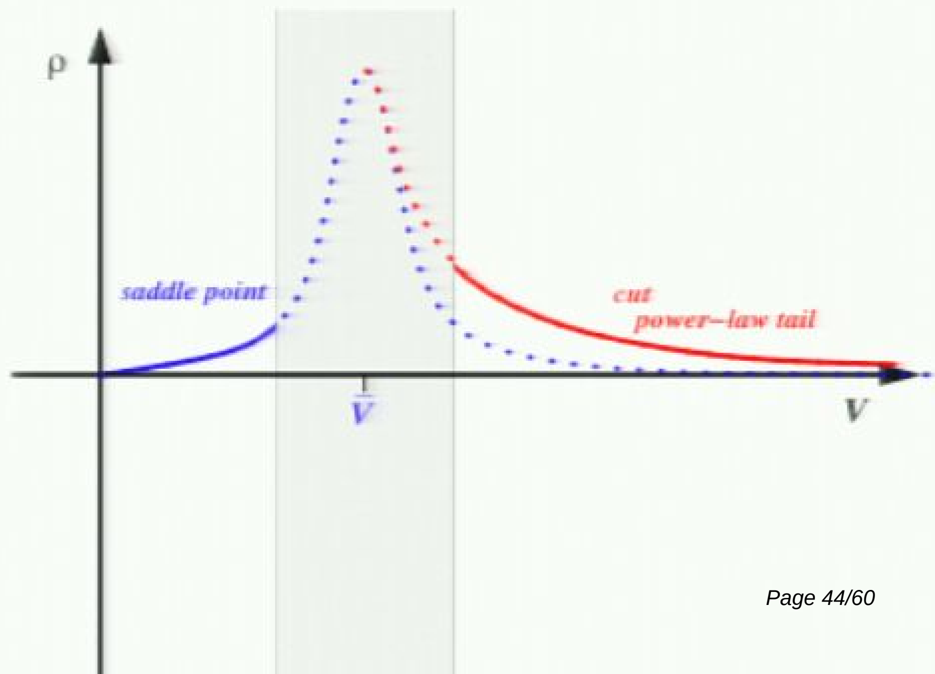

$\lesssim 1$: Inside Eternal Inflation: $\Omega = 1 - \epsilon$

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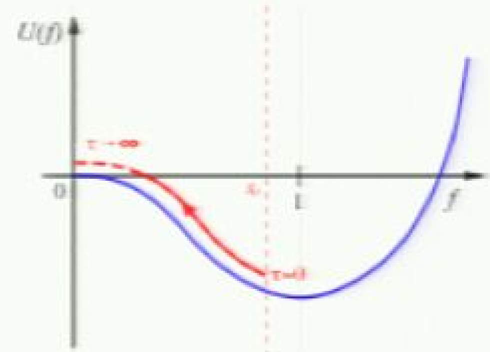

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ut at

$$0 = \frac{dz}{d\tau_0} \quad \Rightarrow \quad \tau_0 \simeq -\frac{\pi}{2\sqrt{\epsilon}} - 1 \quad \Rightarrow \quad z_{\text{cut}} \simeq -\frac{\sigma\sqrt{\epsilon}}{eV_\epsilon}\,, \qquad V_\epsilon \equiv e^{\frac{\pi}{2\sqrt{\epsilon}}}$$

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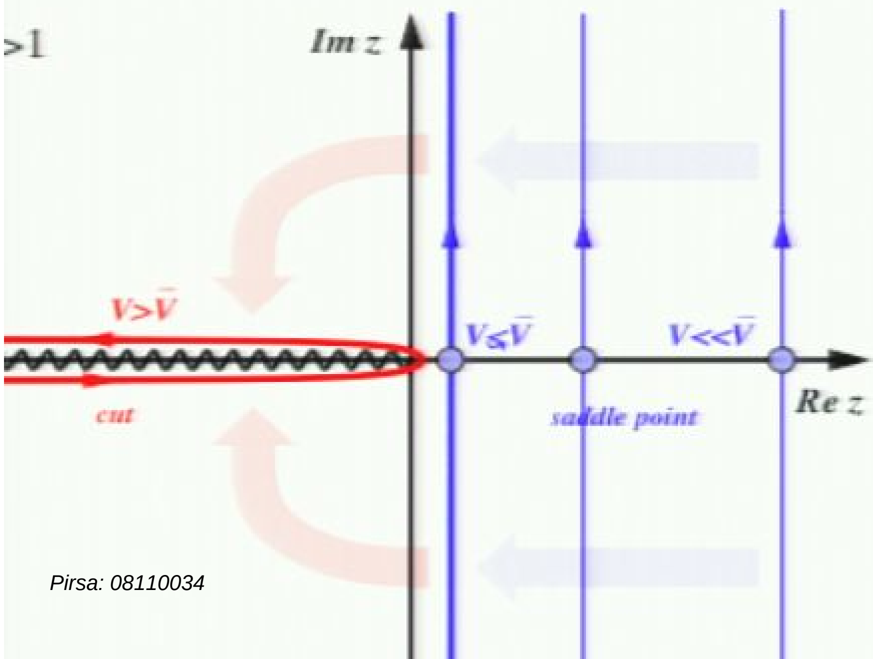

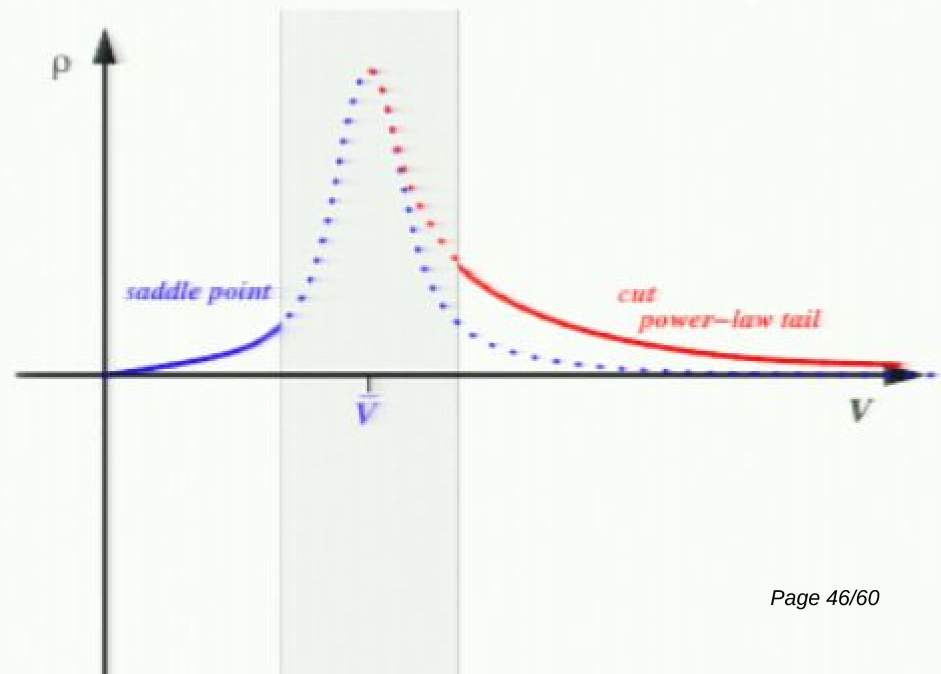

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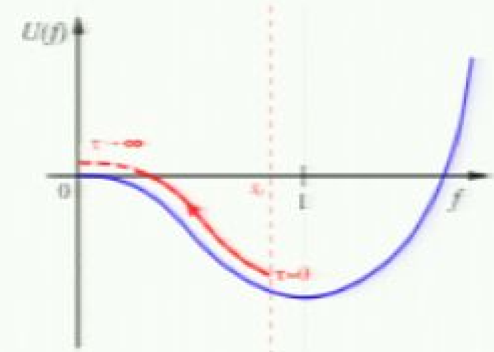

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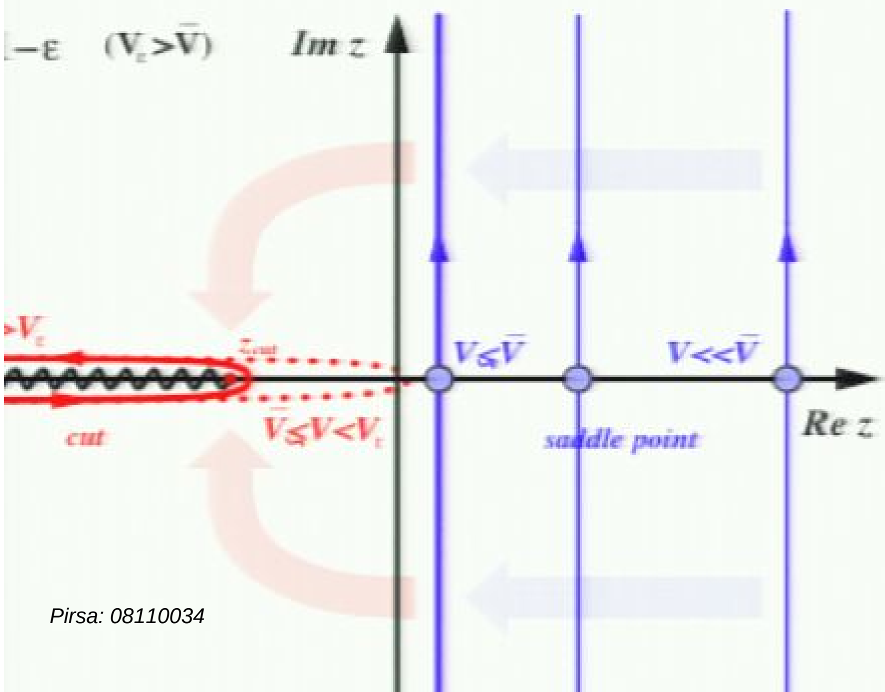
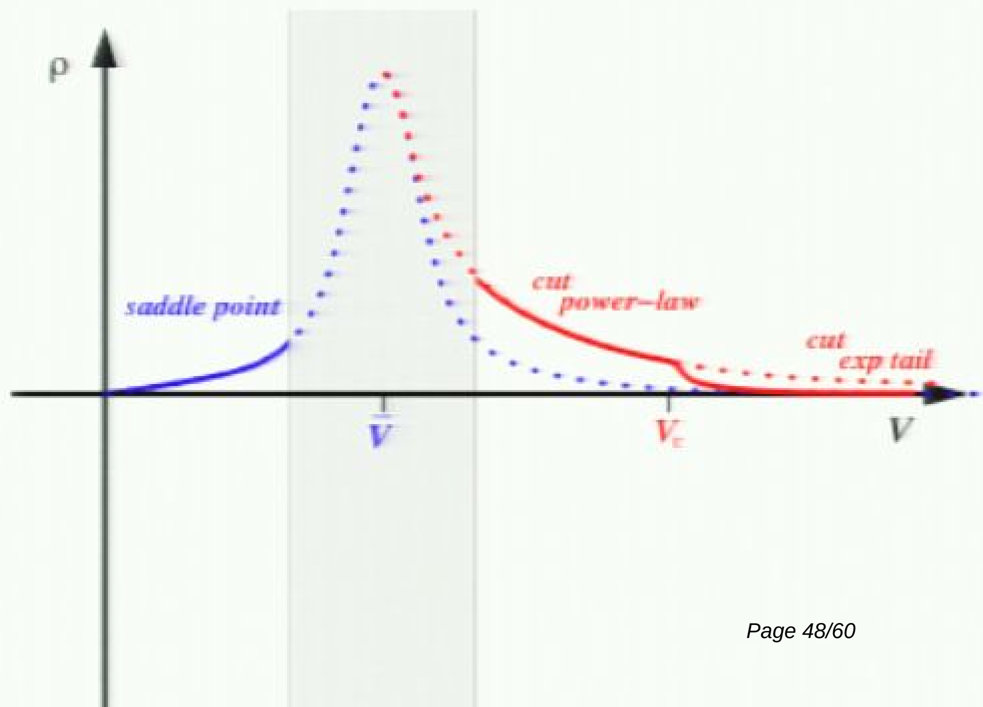
ddle point $\quad V = \frac{(\tau+\tau_0)e^{-\tau_0}}{2\sigma\left[\cos(\sqrt{\epsilon}\tau_0) - \sqrt{\epsilon}\sin(\sqrt{\epsilon}\tau_0)\right]}$ $\quad$ where $\quad z \approx \sigma e^{\tau_0}\cos\left(\sqrt{\epsilon}\tau_0\right)\,,$

s long as $\quad \tau_0 \gtrsim -\log V_\epsilon$, we have $\quad \tau_0 \approx -\log V$

$\rho \approx \mathcal{N} e^{-\frac{1}{4}(\tau - \log V)^2}$ $\quad$ for $\quad 1 \ll V \lesssim \mathrm{Min}\{\overline{V}, V_\epsilon\}$ $\quad$ where $\quad \overline{V} = e^{6N_e}$





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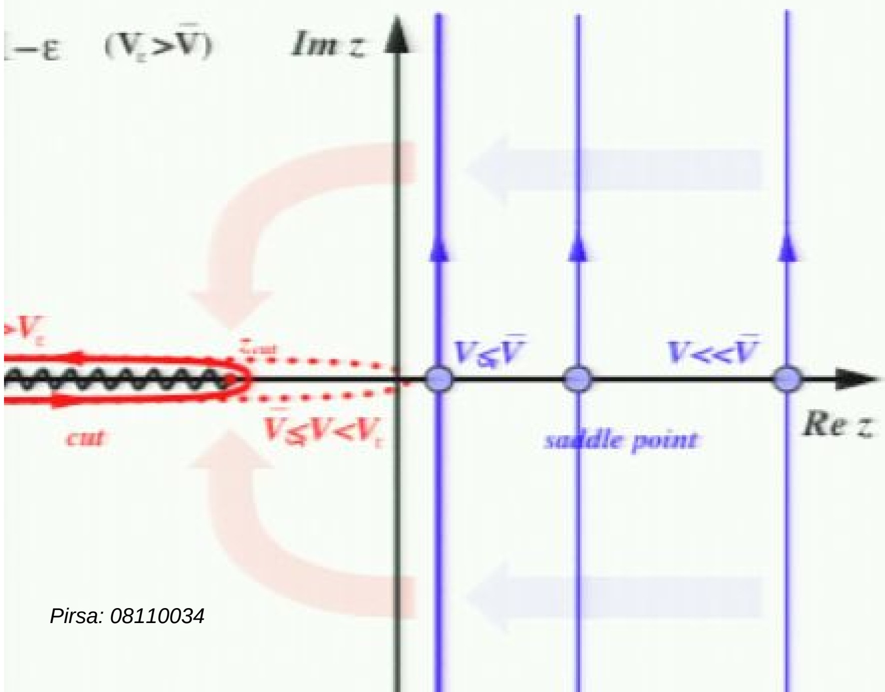
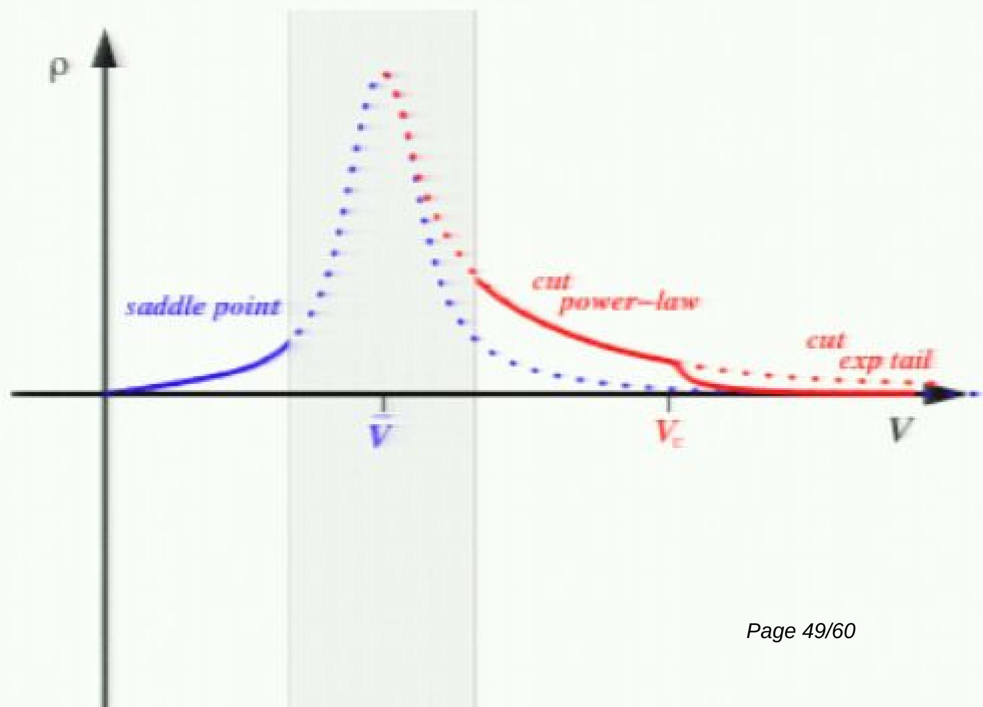




$\lesssim 1$: Inside Eternal Inflation $\quad \overline{V} \lesssim V_\epsilon$

long the cut $\quad \rho(V,\tau) = \frac{1}{2\pi i}\int_{z_{\rm cut}}^{+\infty} d|z|\; 2i\,{\rm Im}[f(\tau;-|z|)]e^{-V|z|} \qquad z_{\rm cut} \simeq -\frac{\sigma\sqrt{\epsilon}}{eV_\epsilon}$.

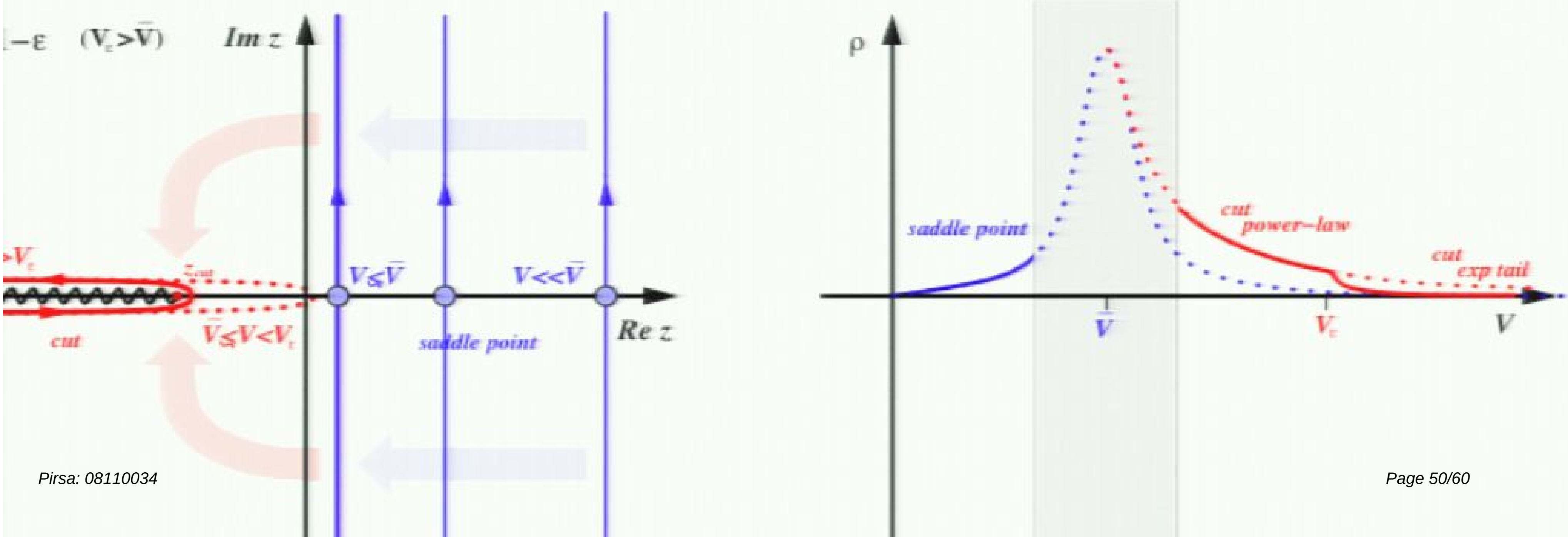
or $\; V < V_\epsilon$, we have $\quad |z| \sim 1/V \quad$, and $\quad f_{\rm lin}(\tau;z) \approx 1 - \sigma e^{\tau+\tau_0}\cos\left(\sqrt{\epsilon}(\tau+\tau_0)\right)$

Power law:

$$\rho(V,\tau) \sim \int_{z_{\rm cut}}^{+\infty} d|z|\; e^{\tau} z\, e^{-V|z|} \sim \frac{\overline{V}}{V^2}\,, \qquad \overline{V} \lesssim V \lesssim V_\epsilon$$

or $\; V > V_\epsilon$, exponential

$$\rho(V,\tau) \sim \int_{z_{\rm cut}}^{+\infty} d|z|\; e^{\tau}\frac{1}{V_\epsilon} e^{-V|z|} \sim \frac{\overline{V}}{V_\epsilon}\frac{1}{V}e^{-\frac{\sigma}{e}\sqrt{\epsilon}V/V_\epsilon}\,, \qquad \overline{V} \lesssim V_\epsilon \lesssim V\,,$$



$\lesssim 1$: Inside Eternal Inflation $\bar{V} \gtrsim V_\epsilon$

addle point $$\rho(V,\tau) \approx \frac{1}{\sqrt{2\pi|S''(z_0)|}} e^{-S(z_0)} \equiv \mathcal{N} e^{-S(z_0)} ,$$

here $$S(z) \;=\; \frac{1}{4}(\tau+\tau_0)^2 - zV$$

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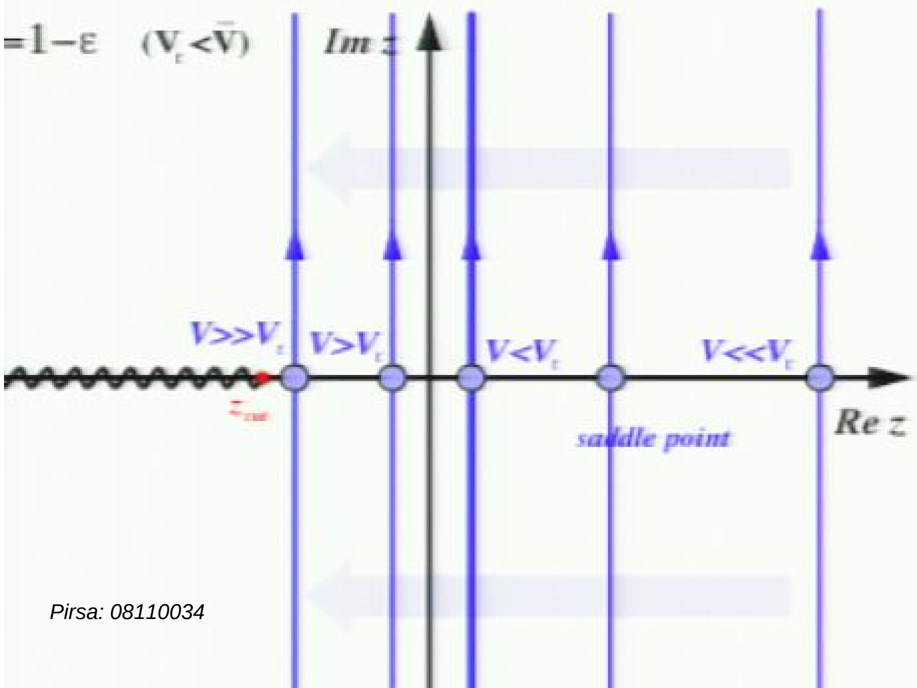
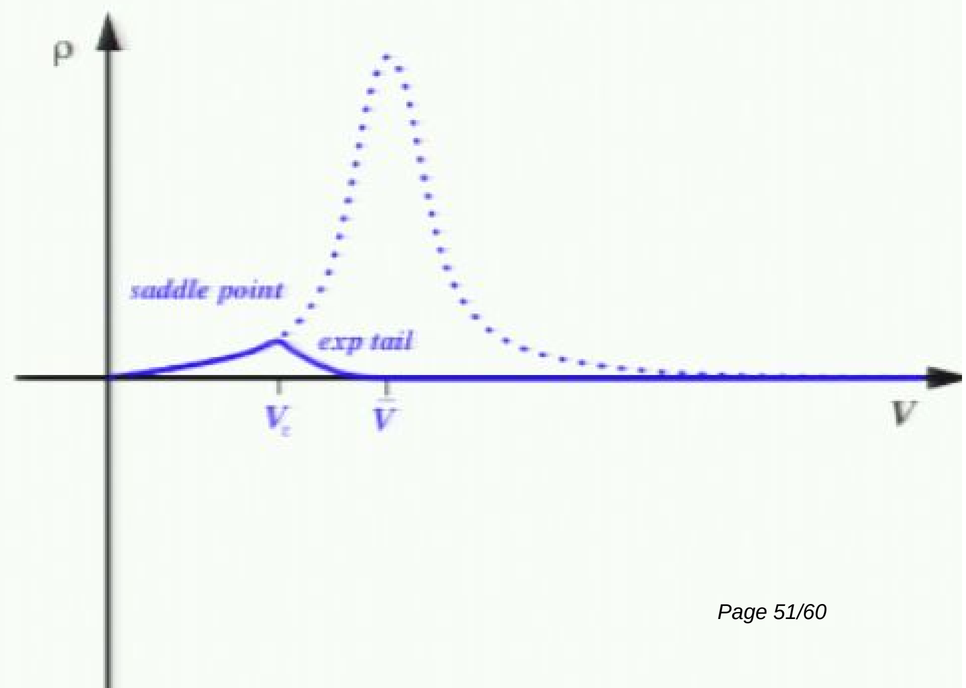
s long as $\tau_0 \gtrsim -\log V_\epsilon$, we have $\tau_0 \approx -\log V$. When $V \gtrsim V_\epsilon$, $\tau_0 \simeq -\frac{\pi}{2\sqrt{\epsilon}} - 1$

$$\rho(V,\tau) \approx \mathcal{N} e^{-\frac{1}{4}\left(\tau - \frac{\pi}{2\sqrt{\epsilon}}\right)^2 - \frac{\sigma}{e}\sqrt{\epsilon}\, V/V_\epsilon} , \qquad V \gtrsim V_\epsilon ,$$

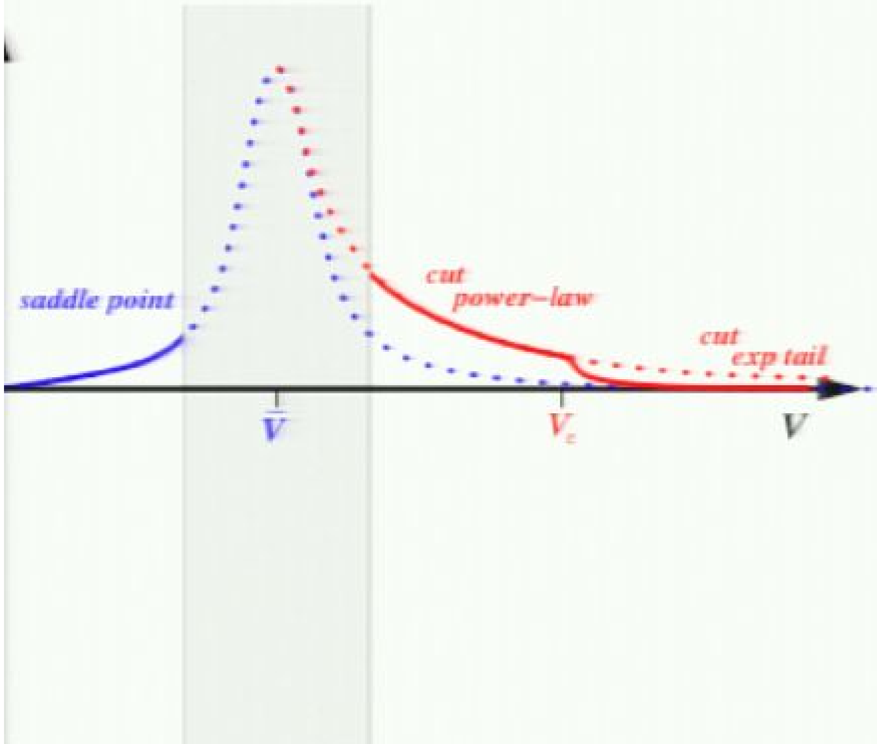
=1−ε (V$_\epsilon$<$\bar{V}$)



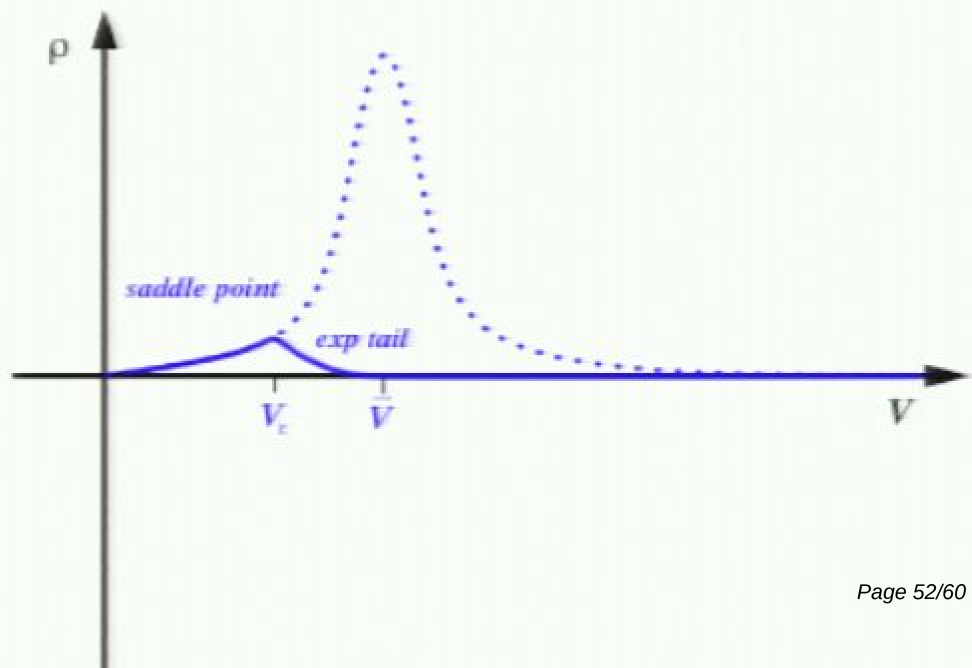


$\lesssim 1$: Inside Eternal Inflation: Summary

Gaussian, power law, and exponential



$\overline{V} \lesssim V_\epsilon$

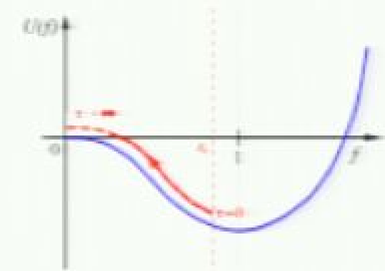


$\overline{V} \gtrsim V_\epsilon$

$\simeq 0$: Deeply inside eternal inflation

gain exact solution

$$\ddot{f}(\tau;z) - 2\sqrt{\Omega}\dot{f}(\tau;z) + f(\tau;z)\log\left[f(\tau;z)\right] = 0$$



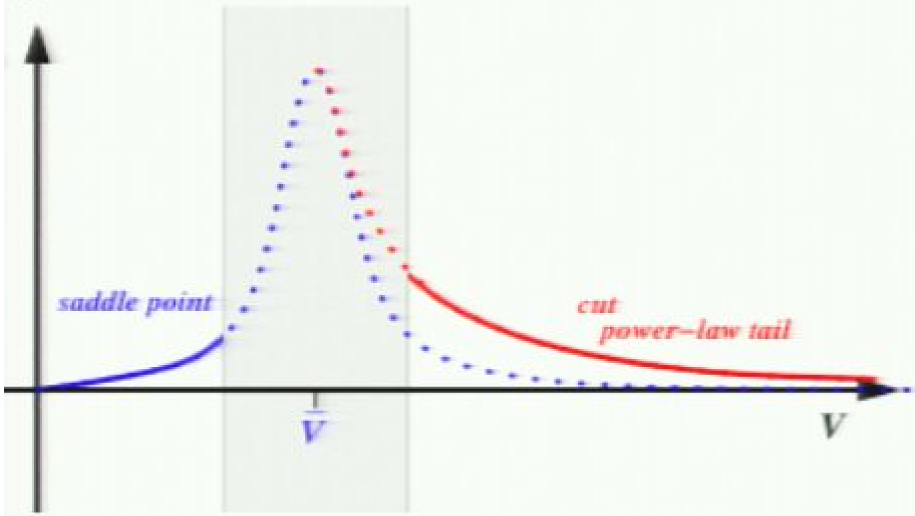
$$z) = e^{\frac{1}{2}-\frac{1}{4}\left(\tau+\sqrt{2+4z}\right)^2} \qquad \Rightarrow$$

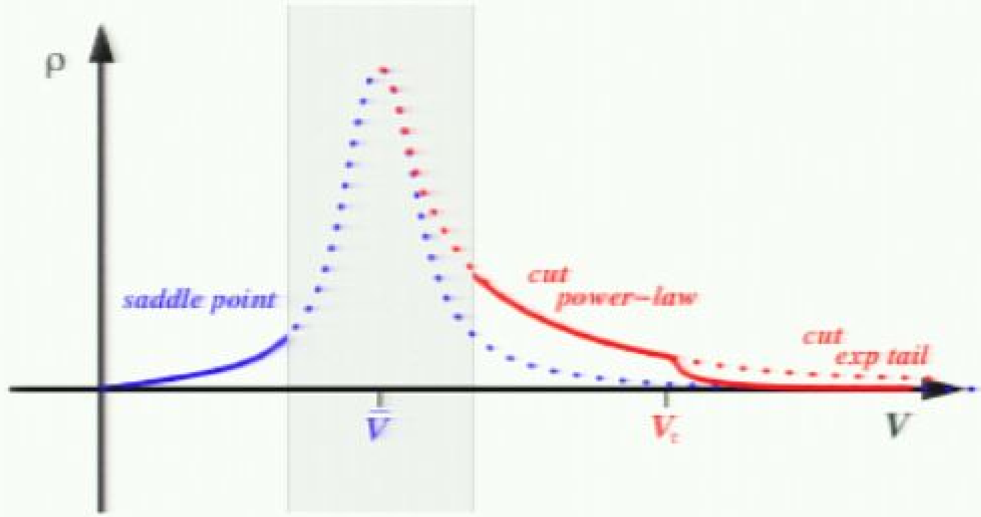
ddle point

$$\rho(V,\tau) \sim e^{-\frac{V-1}{2}-\frac{V}{(V-1)}\frac{\tau^2}{4}}$$

Summary and intuition

$\gtrsim 1$ $\Omega \lesssim 1$



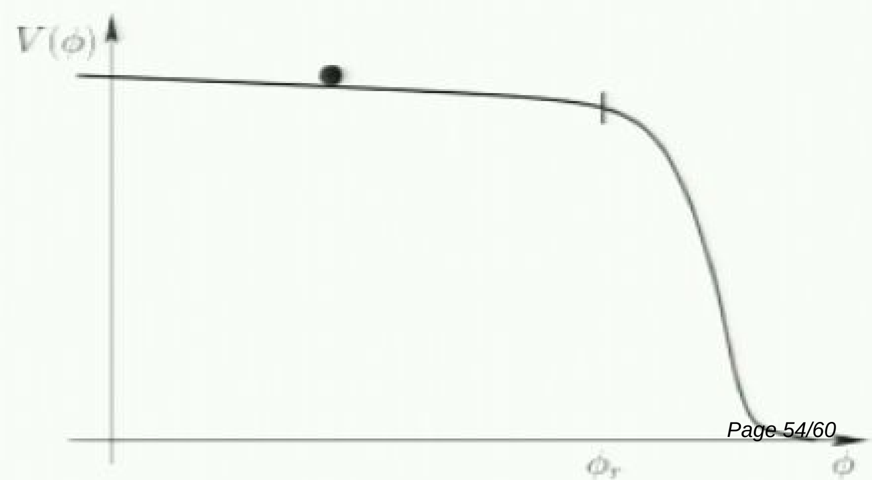


ussian behavior $N \ll N_c$

$$\phi, t) \sim e^{-\frac{(\bar{\phi}-\phi-\dot{\phi}t)^2}{H^3 t}} \sim e^{-\Omega\frac{(N-N_c-Ht)^2}{Ht}} \quad \Rightarrow \quad p \propto e^{-c(N-N_c)^2}\,, \quad \text{for} \quad N \ll N_c\,.$$

ponential $N \gg N_c$

$$p \propto e^{-c_1 \Delta\phi^2/t} \propto e^{-c_2 N}$$

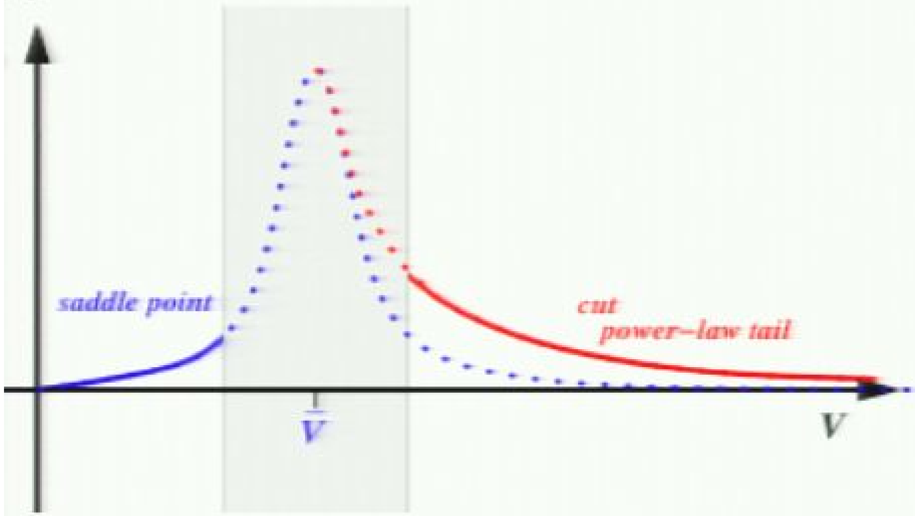


per-Exponential: Eternal Inflation

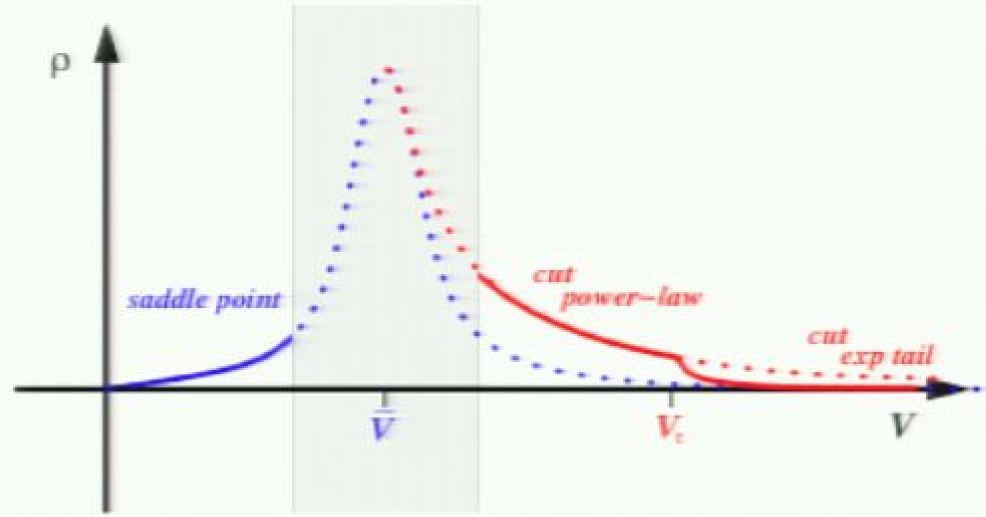
$$p \propto \tilde{p}^{e^{3N}}$$

Summary and intuition

$\gtrsim 1$



$\Omega \lesssim 1$



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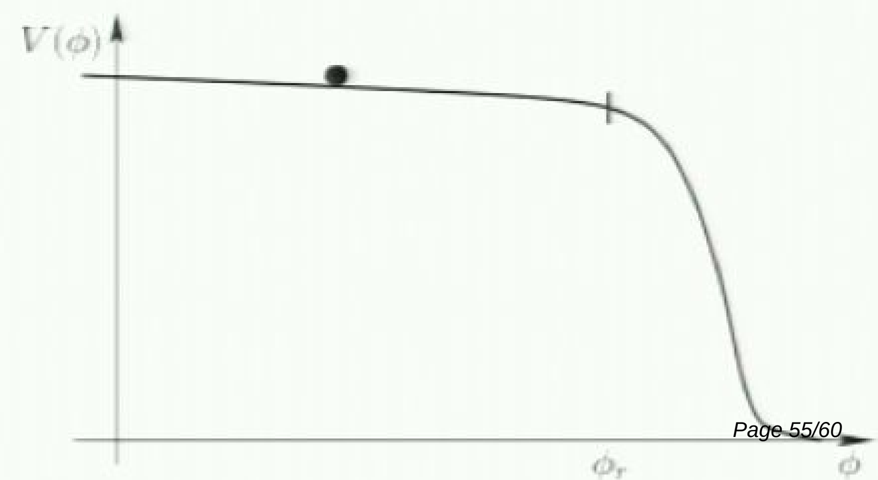

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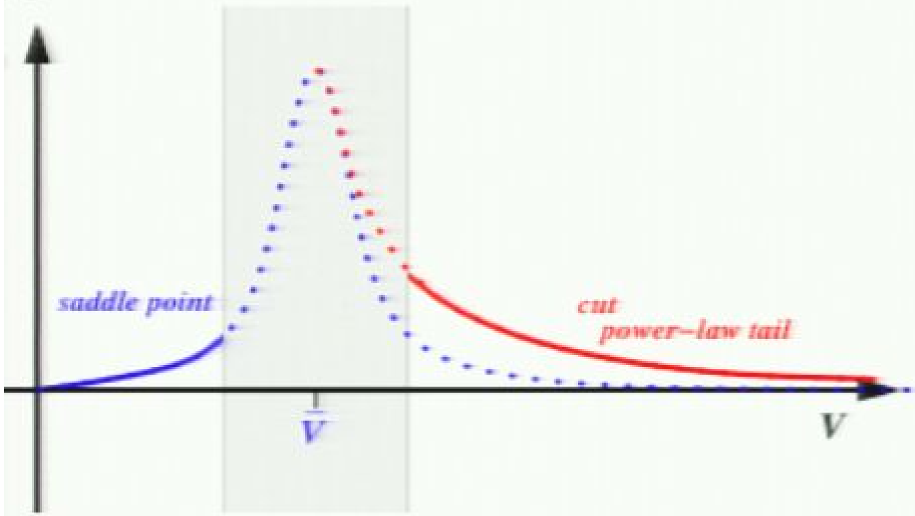
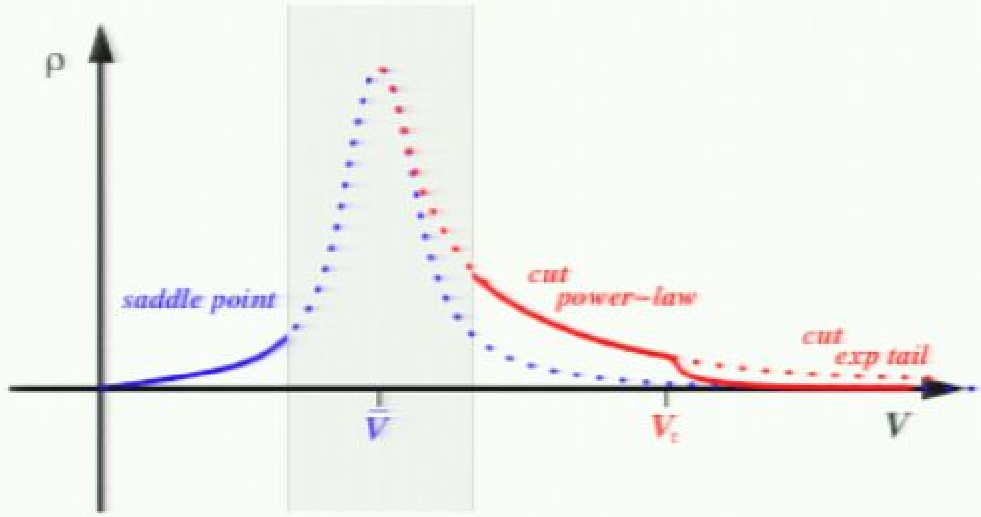
Summary and the de Sitter Entropy

$\gtrsim 1$ $\qquad\qquad$ $\Omega \lesssim 1$





entropy and classical number of e-foldings

$$S_{\rm dS} = \pi \frac{M_{\rm Pl}^2}{H^2}$$

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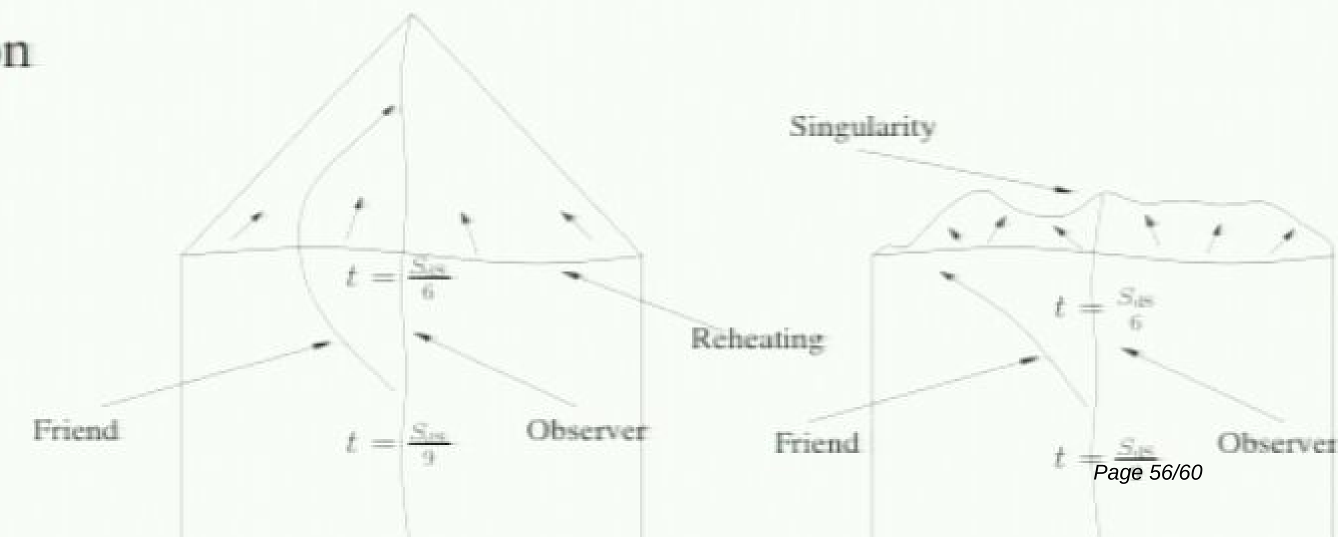
$$\frac{S_{\rm ds}}{N_c} = \pi M_{\rm Pl}^2 \frac{dH^{-2}}{H dt} = 8\pi \frac{\dot\phi^2}{H^4} \quad \Rightarrow \quad N_c \lesssim \frac{S_{ds}}{12}$$

every finite volume realization

$$V > e^{\frac{S_{\rm ds}}{2}}) < e^{-\alpha S_{\rm ds}}$$

$$\text{ite Realization} < e^{\frac{S_{\rm ds}}{2}}$$



eck for de Sitter entropy

void Xerox Paradox)

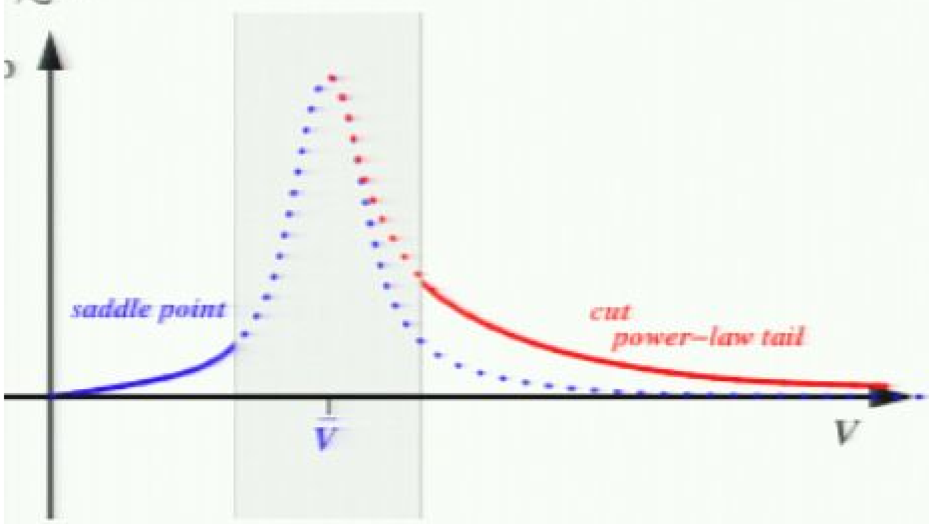
Conclusions

rnal Inflation and the Landscape: detailed calculations

ume of the Universe after Slow Roll Inflation

- Detailed study of the volume in Eternal and non-Eternal Inflation
- A bound of the finite volume produce by Inflation: $V_{\text{Finite Realization}} < e^{\frac{S_{\text{ds}}}{2}}$
  - Educated speculations about de Sitter

$\gtrsim 1$



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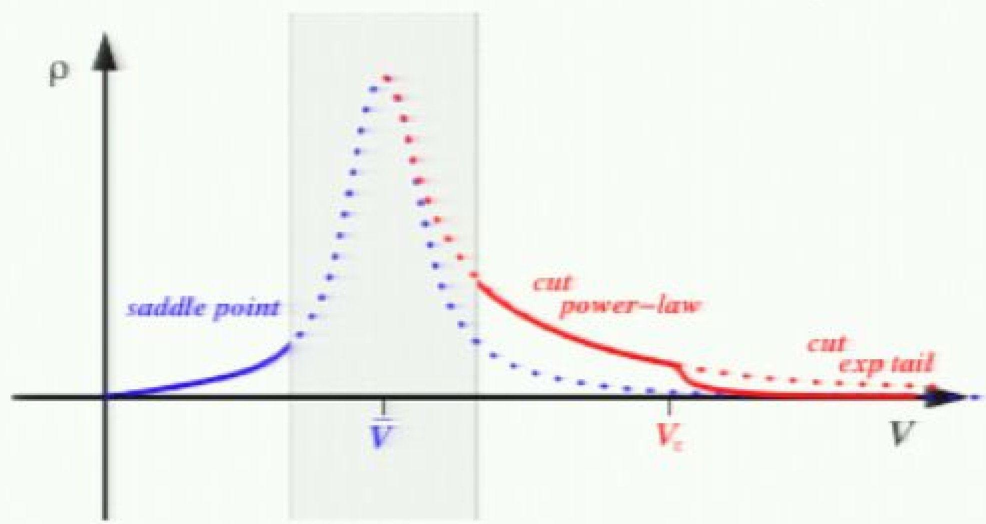

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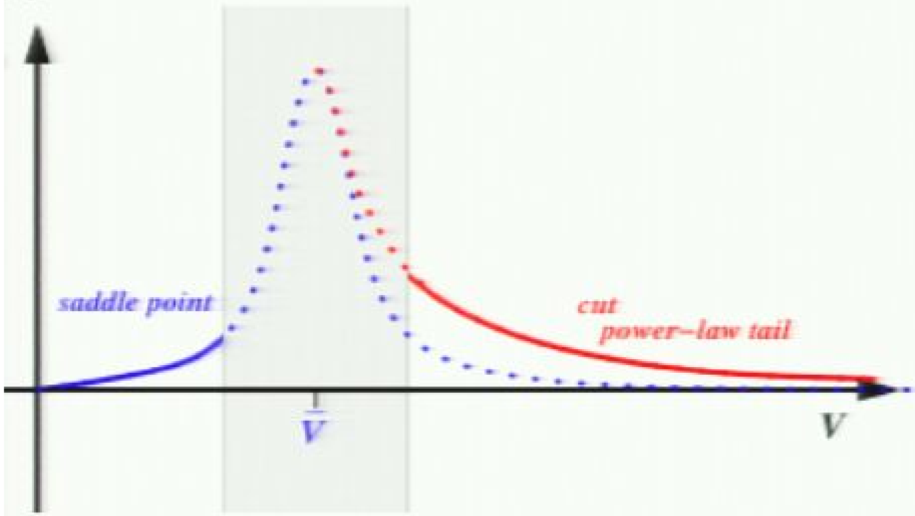

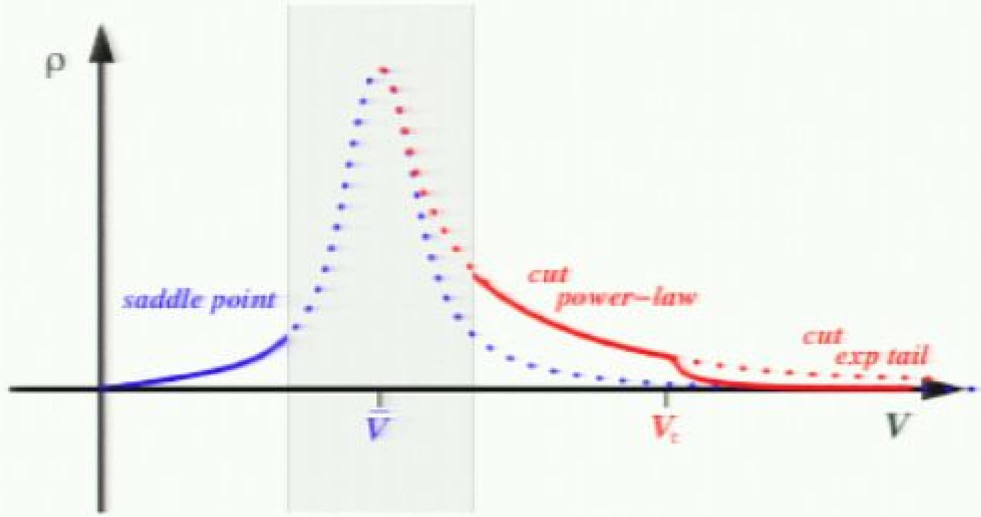

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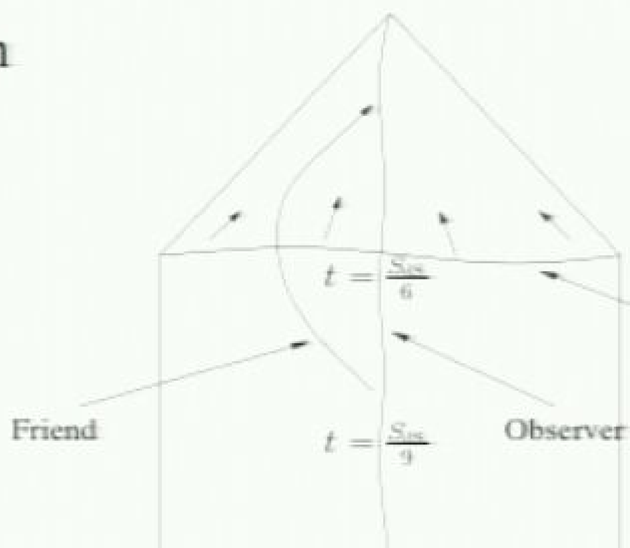

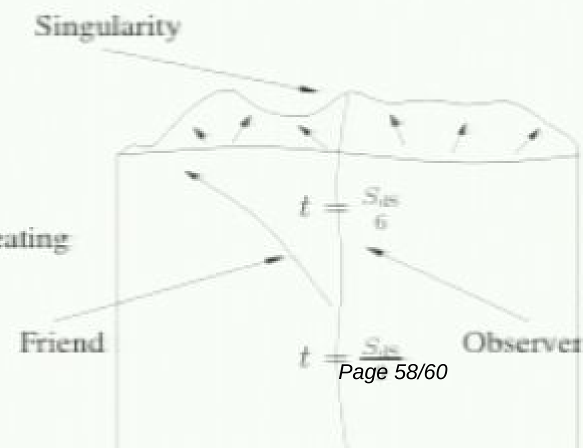

Summary and intuition

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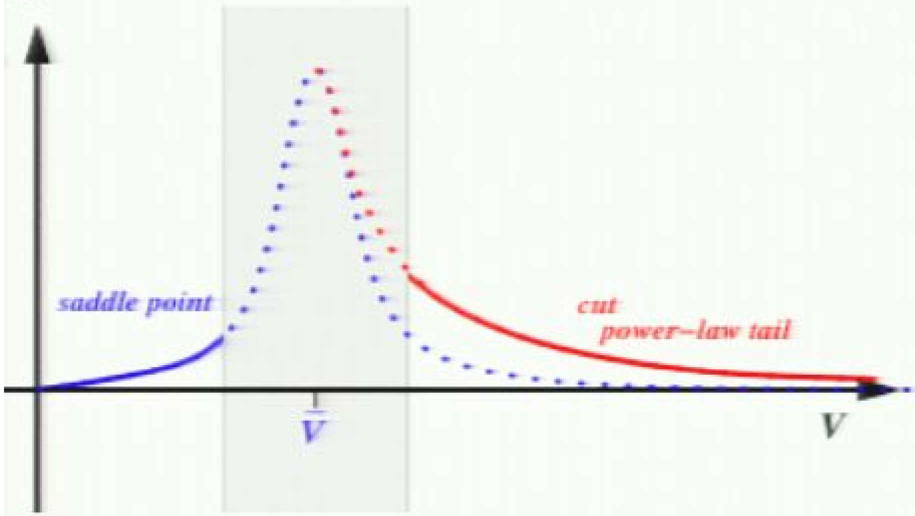

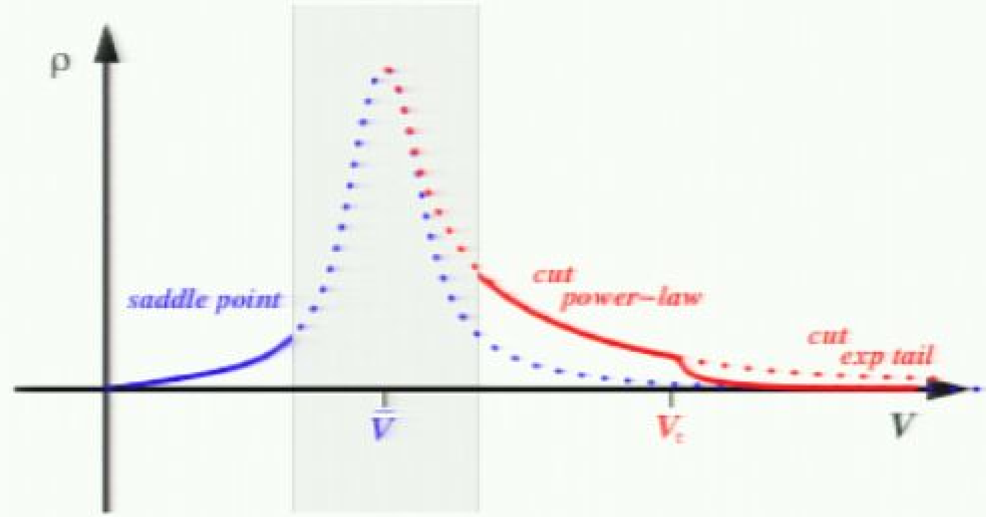

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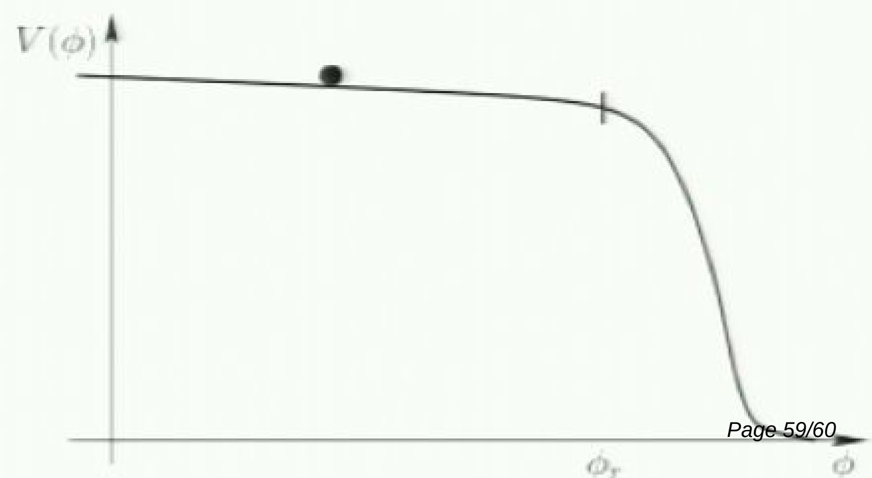

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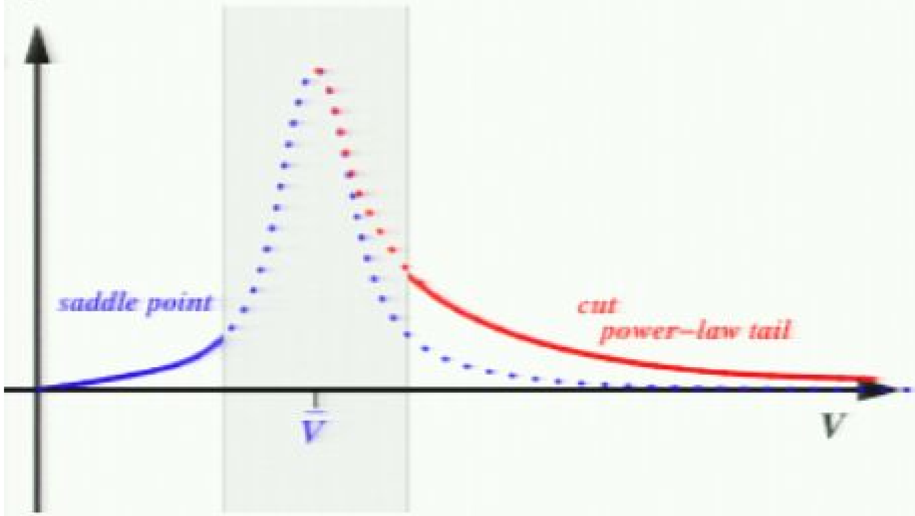

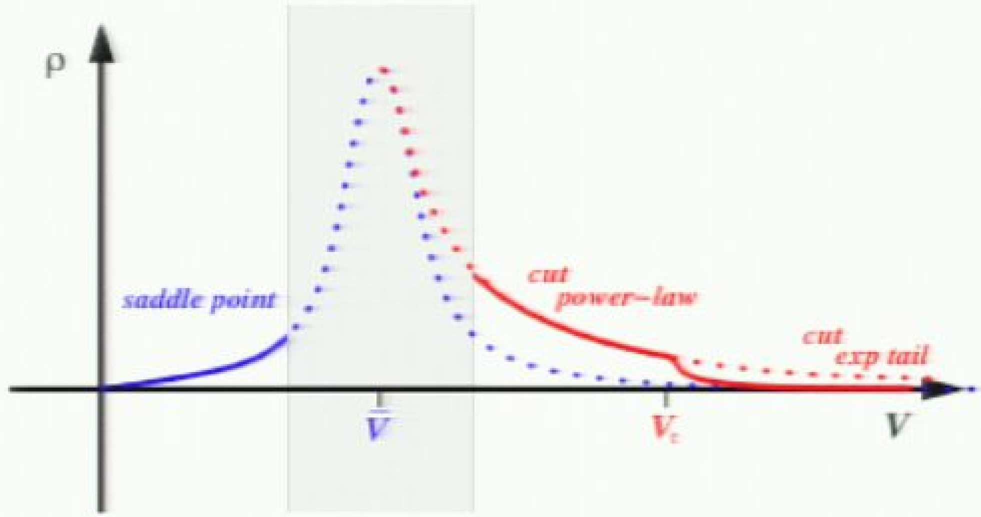

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