Title: Meson widths from string worldsheet instantons.

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Abstract: We discuss various properties of holographic mesons in a deconfined strongly coupled plasma. We show that such mesons obtain a width from a non-perturbative effect. On the string theory side this is due to open string modes on a D-brane tunneling into a black hole through worldsheet instantons. On the field theory side these instantons have the simple interpretation as heavy thermal quarks. We also comment on how this non-perturbative effect has important consequences for the phase structure of the Yang-Mills theory obtained in the classical gravity limit.

Meson widths from string worldsheet instantons

Tom Faulkner, MIT

TF, Hong Liu *arxiv:0807.0063*

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Outline

- Motivation
- Meson screening length from **Semi-classical Strings**
- **Meson dispersion relations** - a subluminal limiting velocity
- **Meson widths** - a nonperturbative calculation
- **Phase diagram** of strongly coupled $\mathcal{N}=2$ plasma revisited
- Conclusions

Motivation: J/Ψ and the QGP

- Charmonium ($J/\Psi, \ldots$) and Bottomonium ($\Upsilon, \ldots$) survive the deconfinement transition $T = T_c$
- Attributed to their small size compared to T_c^{-1}.
- Good probes of the QGP
- Consider *medium* effects:
  1. Color screening - weakens potential between quarks and anti-quarks
     $\rightarrow$ bound state eventually *dissociates* at some $T \rightarrow T_{\text{diss}}$
  2. Collisions with deconfined thermal quarks and gluons can *break apart* bound state
     $\rightarrow$ medium induced width

Dissociation and Suppression

Lattice:

- Screened heavy quark potential
  $\rightarrow$ screening length $L_s(T)$ decreases with T
  Dissociation for $L_s(T_{\rm diss})$ = size of bound state
- Lattice calculation of spectral functions:
  $\rightarrow$ J/Ψ peak becomes broader with increasing T
- For J/Ψ, $T_{\rm diss} = 1.5 - 2.5 T_c$

J/Ψ suppression:

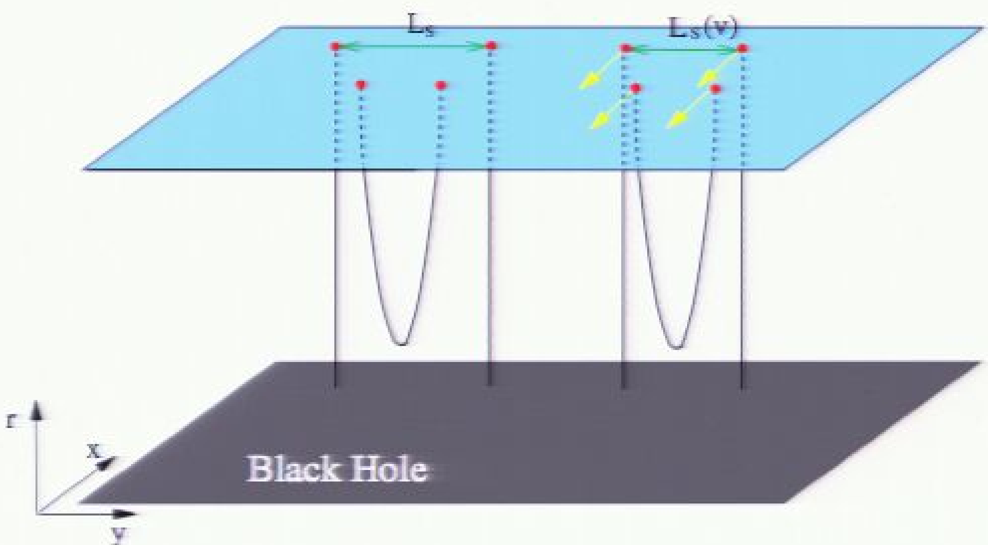
Observed in heavy ion collisions through suppression in production of J/Ψ relative to p-p collisions

Theoretical challenges

- Heavy quark mesons produced in heavy ion collisions can have large momentum relative to the plasma.
- Characterize screening length, dissociation temperature, meson width for a moving meson
- Hard problem for Lattice QCD
- We study a toy model via AdS/CFT. Try to extract such medium effects focusing on their *momentum* dependence.

Semi-classical Strings

$$AdS_5 \times S^5 \leftrightarrow \mathcal{N} = 4\,\mathrm{SYM} \quad (\alpha' \sim 1/\sqrt{\lambda} \quad g_s \sim 1/N_c)$$



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Screening at zero velocity $L_S \sim 1/T$

At non-zero velocity $L_S(v) \sim (1/T)(1-v^2)^{1/4}$

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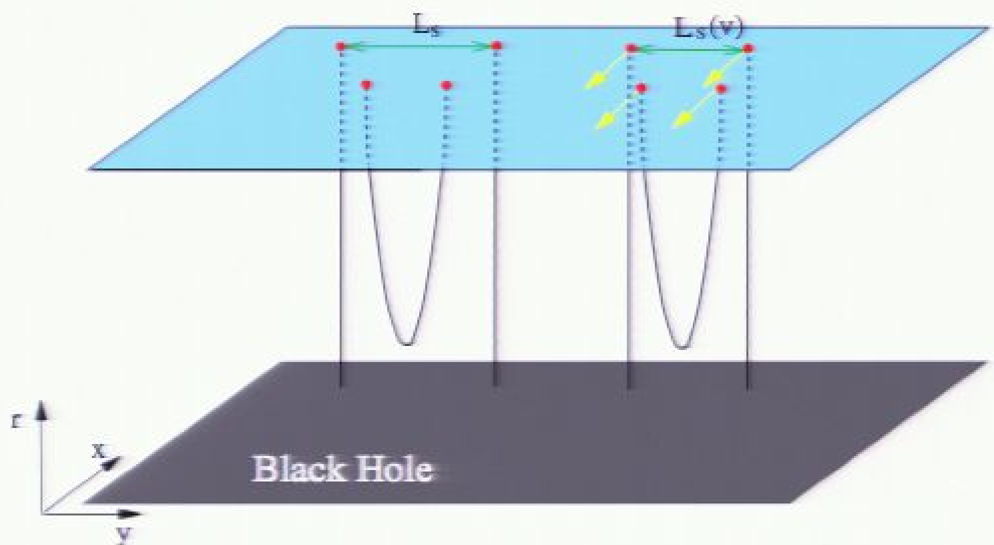

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Semi-classical Strings

Suggests mesons with higher velocity relative to plasma will dissociate at lower T.

In other words: dissociation at $L_S(v) \sim$ *meson size* $\sim M^{-1}$, so at a given T, $L_S(v) \sim (1/T)(1-v^2)^{1/4}$ implies one of two things:

$$v_{\max} \sim 1 - \left(\frac{T}{M}\right)^4 \qquad \frac{q_{\max}}{M} \sim \left(\frac{M}{T}\right)^2$$

where the last relationship has used the relativistic dispersion relation.

We will see evidence for *both*.

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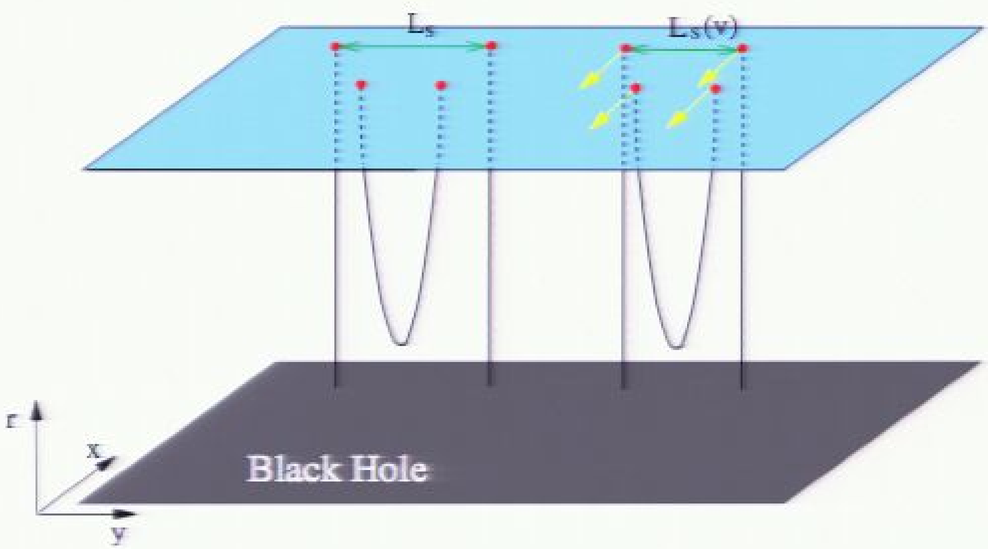

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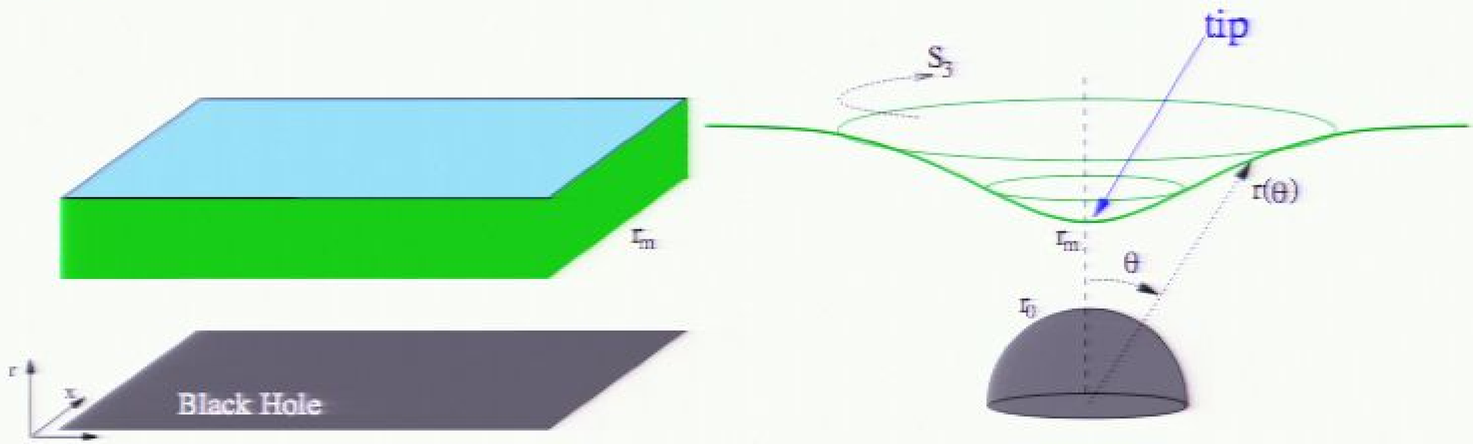
Meson dispersion relations

Dynamical quarks - the D3/D7 system

Adding quarks to $\mathcal{N} = 4$SYM equivalent to adding D branes (open strings) in the string theory. *Karch, Katz*

Specifically: $\mathcal{N} = 4$ SYM $\rightarrow \mathcal{N} = 2$ SYM $+ N_f$ fundamental quarks achieved by embedding $D7$ branes in $AdS_5 \times S^5$. (Minimal area - DBI)



Babington, Erdmenger, Evans, Guralnik, Kirsch (BEEGK);

Kruczenski, Mateos, Myers, Winters.

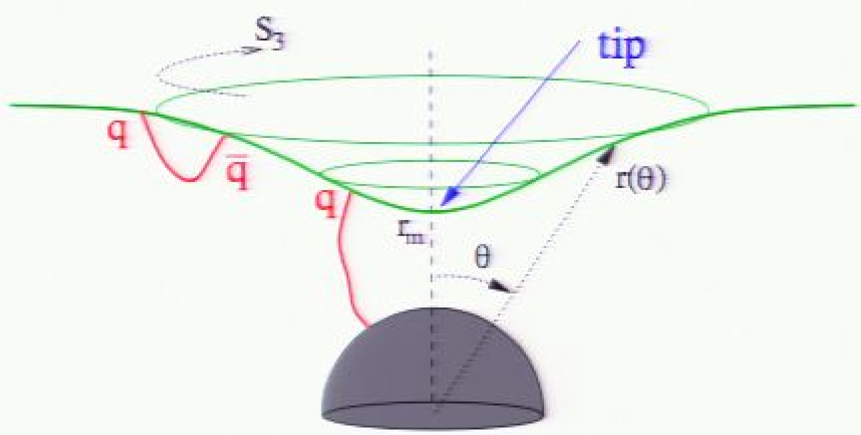
Meson dictionary

Temperature $\equiv$ black hole. $T = r_0/\pi R^2$.

Meson $\equiv$ fluctuations of D7 brane. $M \sim (r_m - r_0)/R^2$

Quark $\equiv$ strings ending on horizon. $m_q^{(T)} = (r_m - r_0)/\alpha'$

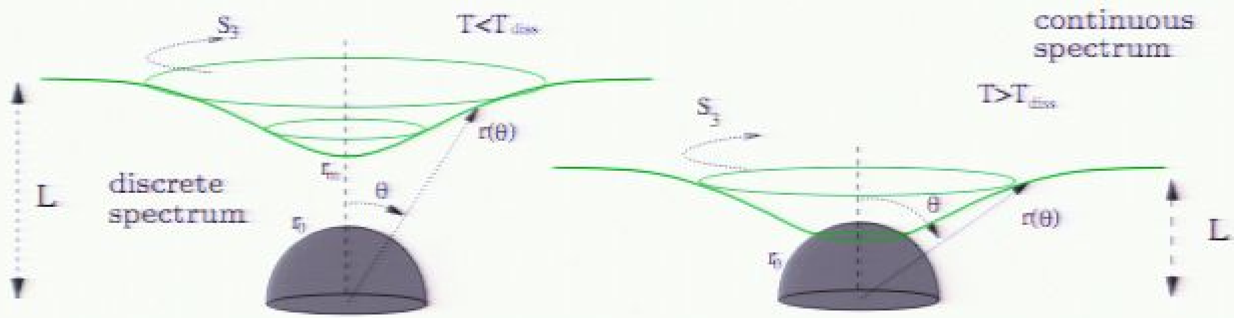
Rough scales: $M \sim m_q/\sqrt{\lambda} \to m_q \gg M \to E_{\mathrm{BE}} \sim 2m_q \gg M$.



$d\Omega_5^2 = d\theta^2 + \sin^2\theta d\Omega_3^2 + \cos^2\theta d\phi^2$ and $\rho = r\sin(\theta)$, $y = r\cos(\theta)$

Dissociation

Increase temperature (equivalent to decreasing the quark mass.)
Eventually D7 brane will fall into horizon.



Mateos, Myers, Thomson; Hoyos, Landsteiner, Montero

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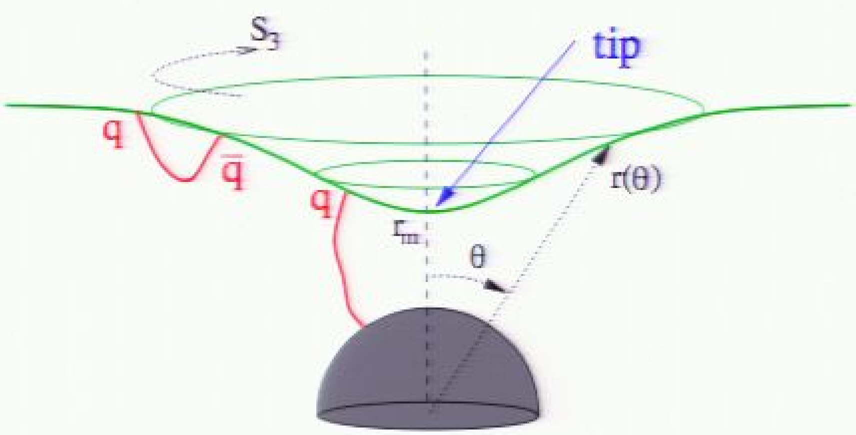

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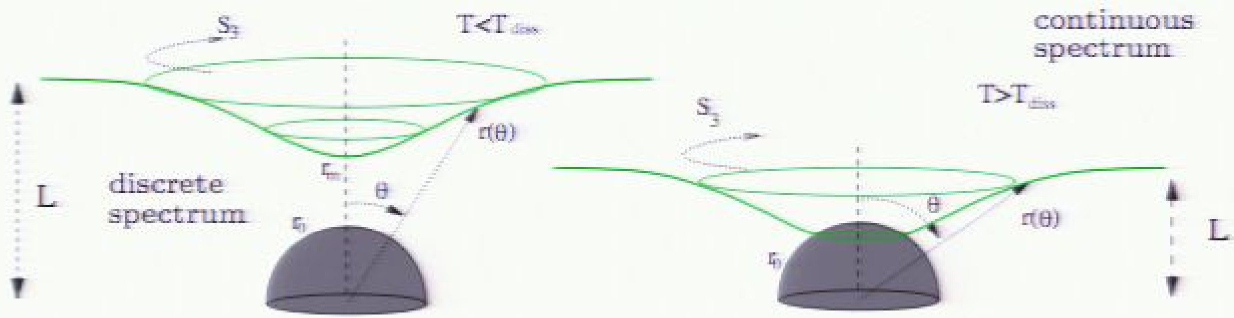

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Meson spectrum

2 transverse directions - radial mode χ_u and angular mode χ_ϕ.
$\rightarrow$ Couple to operators in the field theory $\sim \bar{q}q$.

Choose orthonormal basis of normal space:

$$\delta X^\mu = \chi_u n_u^\mu + \chi_\phi n_\phi^\mu$$

Embedding of D7: $K_u = 0$ and $K_\phi = 0$

Action for the fluctuations

$$S = -\frac{\mu_7}{2} \int d^8\xi \sqrt{h} \left((\partial\chi_u)^2 + (\partial\chi_\phi)^2 + m_u^2 \chi_u^2 + m_\phi^2 \chi_\phi^2 \right)$$

$$m_u^2 = R_{uu} + R_{u\phi\phi u} + 2R_{\phi\phi} + {}^{(8)}R - R$$

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$m_i^2 \rightarrow -3$ which gives $\Delta_{\mathcal{O}} = 3$.

Meson spectrum - dispersion relations

Concentrate on transverse scalar χ_ϕ: $\partial_\alpha(\sqrt{g}\partial^\alpha \chi_\phi) + \sqrt{g} m_\phi^2 \chi_\phi = 0$

$$\chi_\phi = e^{-i\omega t + i\vec{k}\cdot\vec{x}} Y_l(S_3)\psi(\theta) \quad \rightarrow \quad \hat{H}(\vec{k}, l)\psi(\theta) = \omega^2 \psi(\theta)$$

Mateos, Myers, Thomson
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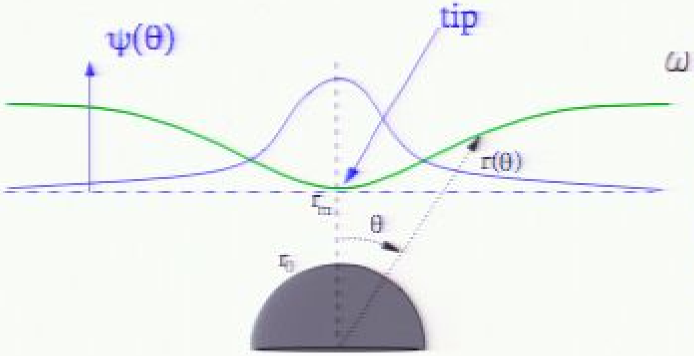

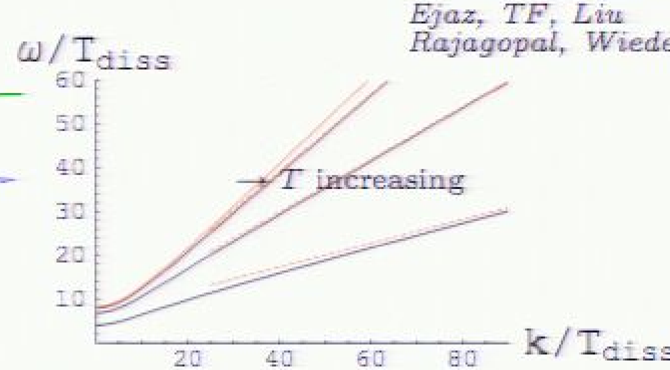

$T < T_{\rm diss}$ - discrete spectrum, $\omega = \omega_n(\vec{k}, l)$.

General argument: Large k - wave function localized at "tip" $\rightarrow$
$\omega = k v_0$ with $v_0 = \sqrt{\frac{-g_{tt}}{g_{xx}}}\Big|_{\rm tip}$

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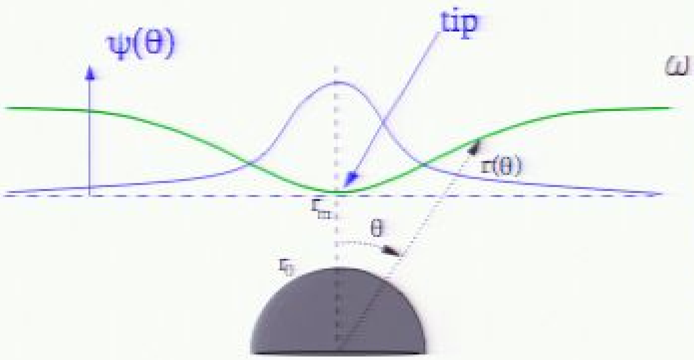

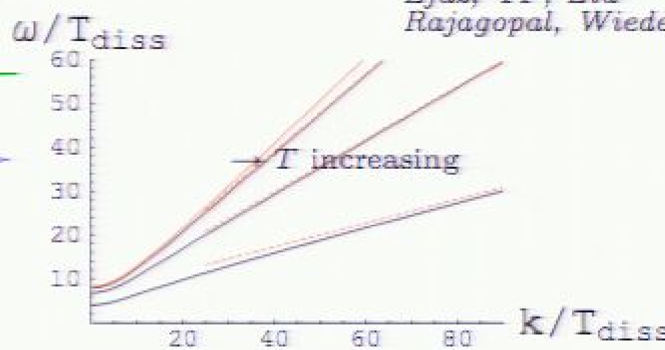

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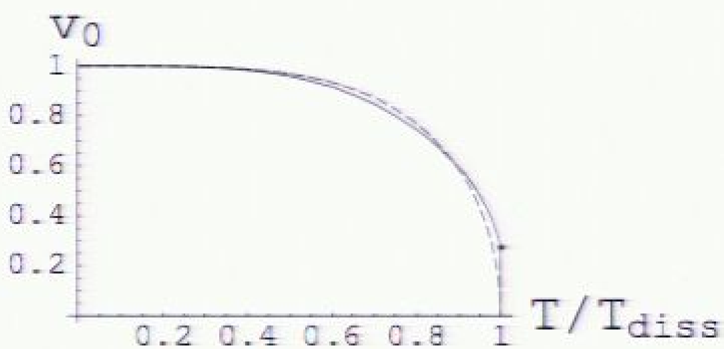
The limiting velocity

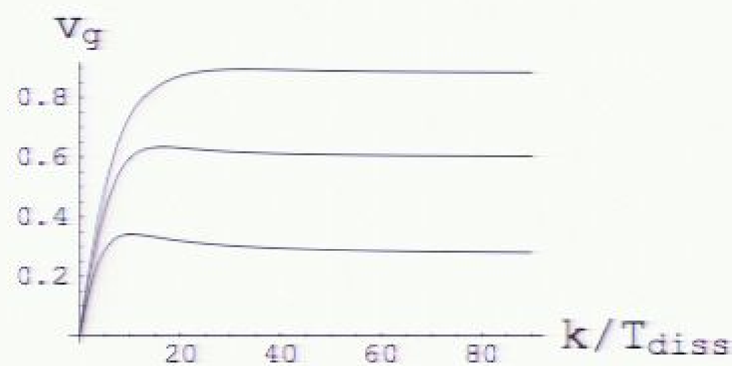
Can analytically solve for the large k dispersion relation:

$$\omega^2 = k^2 v_0^2 + k\Omega(n+2) + \mathcal{O}(1/k)$$

$$v_0 \sim \frac{1-(T/M)^4}{1+(T/M)^4}$$

Consistent with semiclassical strings





Maximum velocity slightly higher than limiting velocity.

Meson widths

Puzzle I - zero width?

- For $T < T_{\text{diss}}$, mesons have zero width. (Recall gluons are deconfined.)
- Due to topology of D7 brane: fluctuations on brane see an induced geometry which is asymptotically AdS and smoothly capped off at some finite radius r_m (in AdS/CFT this is usually associated with confinement.)
- Relax $\alpha' \to 0$ limit : derivative expansion on D brane. Mesons still stable to any order in a perturbative α' expansion.
- But, bound states should always have a non-zero width in a finite temperature medium.
- Implies;
  $\to$ g_s correction
  $\to$ *Nonperturbative* α' correction.

Puzzle II - chemical potential

- Gauge field $A_\mu(r)$ on D7 brane $\leftrightarrow$ conserved baryon/quark current J_μ in field theory. More specifically the behavior of the gauge field at the boundary is,

$$A_t(r) \to \mu + n_q/r^2 + \dots \tag{1}$$

- Turn on constant gauge field $A_t(r) = \mu$: still a solution!. $\to$ So $n_q = 0$ and mesons still stable. This is a puzzle at *non-zero* T.
- Again true in a perturbative α' expansion.

(For $\mu > m_q^{(T)}$ a new phase where D7 falls into horizon is thermodynamically preferred - see later.)

Expectations from field theory

- Thermal contribution:
  1. Recall $\beta m_q^{(T)} \sim \sqrt{\lambda} M/T$, so quark densities are suppressed,

     $$n_\pm \sim e^{-\beta m_q^{(T)} \pm \beta\mu} \to \mathcal{O}(e^{-\sqrt{\lambda}})$$

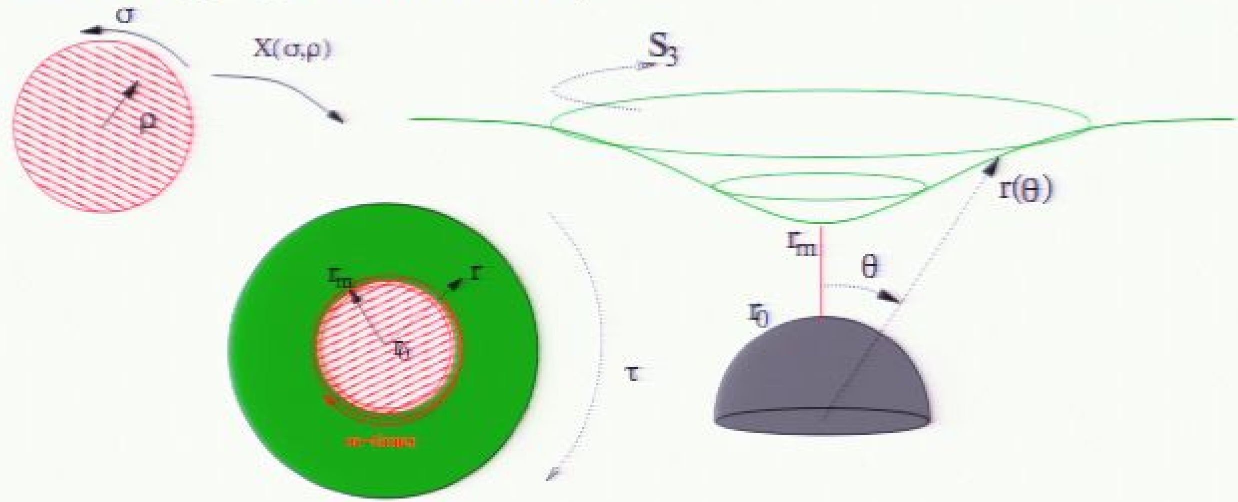
  2. Similarly $E_{\rm BE} \sim 2m_q$. At zero T mesons stable as tightly bound. At finite temperature thermal fluctuations can tear quarks apart, but fluctuations also exponentially suppressed in $\sqrt{\lambda}$.

  Not visible in perturbative $1/\sqrt{\lambda}$ expansion. Can only have non-perturbative origins on the worldsheet.

- $1/N_c$: the annihilation process, meson $\to$ meson + meson etc.
- $1/N_c^2$: there are thermal contributions to the width from processes that do not break apart the meson; see *Dusling, Erdmenger, Kaminski, Rust, Teaney, Young*

World sheet instanton

In *Euclidean time* there exists a non trivial "cycle with boundary" albeit wrapping Euclidean time,



Extremal solution to world sheet action: will contribute to the spacetime effective action.

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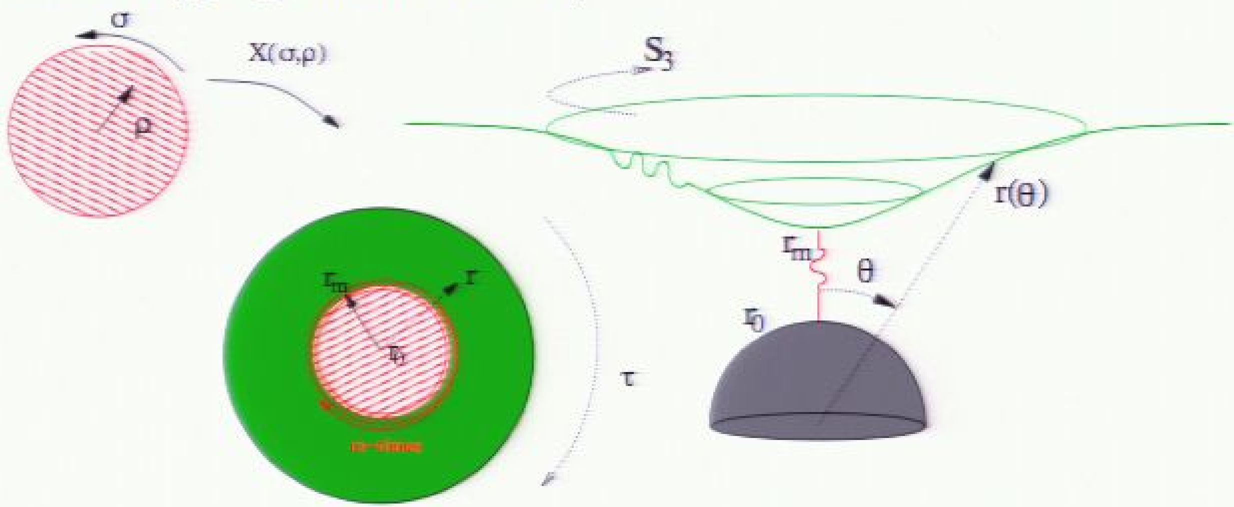

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Effectively changes the topology of the D7 brane.

Spacetime Effective Action

Calculate string partition function

$$S_E[\chi] = \int_{\text{disk}} DX\, e^{-I[X]-I_{\text{bdry}}[\chi,X]}$$

- X denotes the worldsheet fields.
- $I[X]$ is the world sheet action (Nambu-Goto),

$$I[X] = \frac{1}{2\pi\alpha'}\int_C \sqrt{\det P[G]} + \oint_\Sigma P[A] + \dots$$

- $I_B[\chi, X]$ action on boundary of worldsheet, allowing for presence of some background (open string) fields χ.

Saddle point approximation,

$$S_E = S_{m=0} + S_{m=1} + S_{m=-1} + \dots \qquad \text{and} \quad S_{m=0} \to S_{DBI}$$

In $m = \pm 1$ sectors, $I_{m=\pm 1} = \beta m_q^{(T)} \pm \mu\beta \to$ thermally suppressed.

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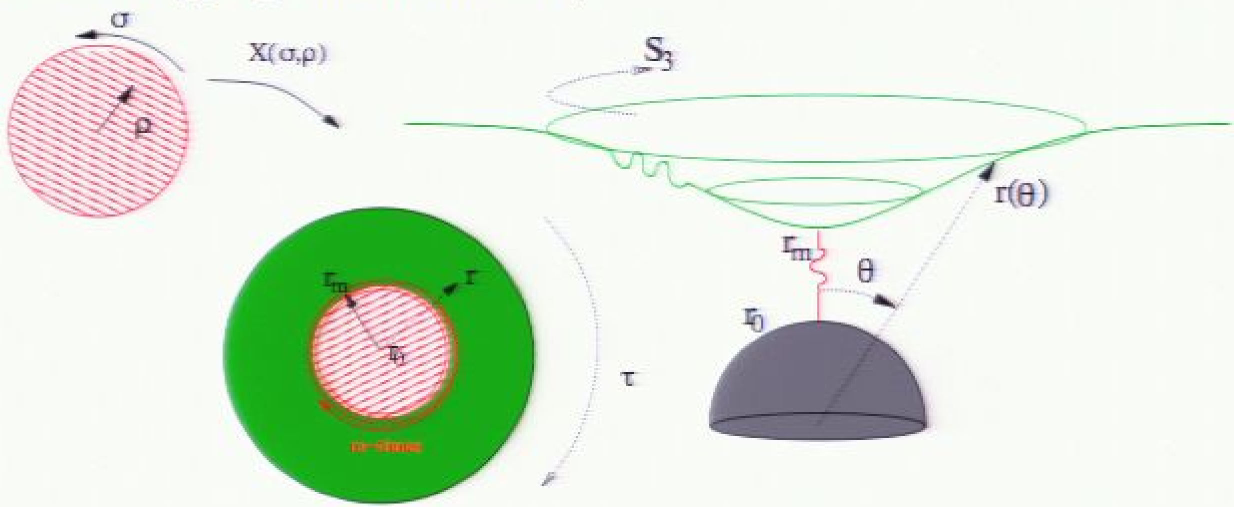

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Free Energy

Firstly, set χ to zero;

$$S_E[0] = \int_{\text{disk}} DX\, e^{-I[X]}$$

Contribution to free energy,

$$\beta F_\pm(\beta,\mu) = S_{m=\pm 1} = e^{\pm\mu\beta} e^{-\beta m_q^{(T)}} \frac{1}{g_s} D V_3 \quad \rightarrow \quad n_\pm = e^{\pm\mu\beta} e^{-\beta m_q^{(T)}} \frac{1}{g_s} D$$

Quark densities! Instantons $\leftrightarrow$ Boltzmann gas of quarks.

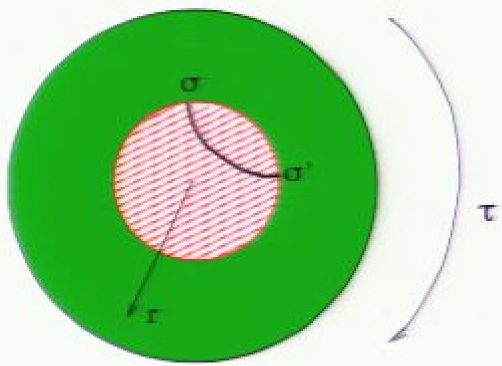
- $\pm$ - quarks, antiquarks
- g_s^{-1} - disk worldsheet! $n_\pm \propto N_c$
- D - functional determinant, includes fermions, hard!
- V_3 - three bosonic zero modes (c.f. DBI: has 8 bosonic zero modes.)

Width Calculation

Integrate out fluctuations on the world sheet.

Quadratic action for χ^ϕ schematically, $S_{DBI}[\chi^\phi]+$

$$S_{m=\pm 1}[\chi_\phi] \sim \int d^3 x_0 d\tau d\tau' \left(\chi^\phi(\tau, \vec{x}_0)\tilde{G}_D(\tau,\tau')\chi^\phi(\tau',\vec{x}_0)\right)_{\theta=0}$$



$\tilde{G}_D(\tau,\tau') \sim$ "boundary" to "boundary" propagator on the worldsheet disk.

Compute meson correlators in Euclidean. After analytic continuation find poles shifted by small imaginary part.

Alternatively, compute directly in real times. Continue worldsheet disk → Rindler spacetime $ds^2 = d\rho^2 - \rho^2 d\eta^2$. Use $\tilde{G}_R(t-t')$!

Width Calculation

$$\partial_\alpha\left(\sqrt{-g}\partial^\alpha\chi^\phi\right) - m_\phi^2\sqrt{-g}\chi^\phi = -\frac{R^2 n_\pm \delta(\theta)}{\mu_7\pi\alpha'\beta^2}\int dt'\,\tilde{G}_R(t-t')\chi^\phi(t')$$

Momentum space ($l=0$):

$$\hat{H}(\vec{k}, l=0)\psi - \frac{i\omega n_\pm}{4\pi^3\alpha'\mu_7 A}\delta(\theta)\psi(\theta) = \omega^2\psi(\theta)$$

Perturbation theory problem with a delta function! Write $\omega = \omega_n - \frac{i}{2}\Gamma_n$ find,

$$\Gamma_n^{\pm 1} = \frac{32\pi^3\sqrt{\lambda}}{N_c m_q^2}|\psi_n(\theta=0)|^2 n_\pm$$

Recall $n_\pm \propto N_c$ so $\Gamma \sim \mathcal{O}(1)$ in N_c. Incredibly simple result.

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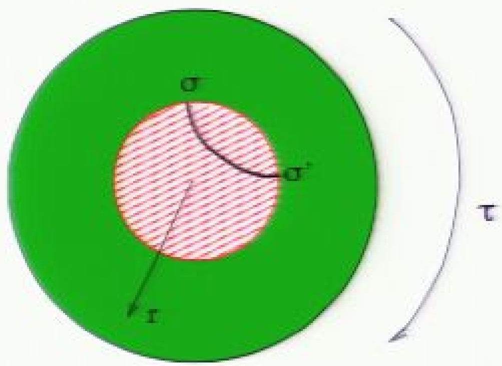

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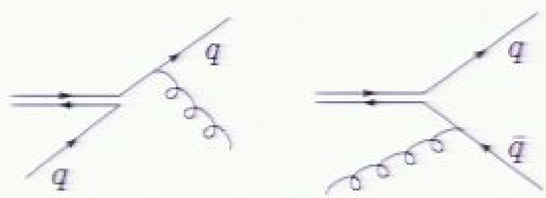
Interpretation

- String theory perspective: open string modes living on a D brane which lies outside a black hole can *tunnel* into the black hole.
- Field theory perspective:

$$\Gamma \sim e^{-\beta m_q^{(T)}}$$



$$\Gamma \sim e^{-\beta E_{\rm BE}} \sim e^{-2\beta m_q^{(T)}}$$

1. Our width from *left* process. Dominant thermal process which breaks apart meson.
2. However *right* process more interesting from QCD perspective. Subdominant in *this* strongly coupled gauge theory.

Momentum dependence of width

Note $n_{\pm}(T,\mu)$ not calculated; depends on functional determinant. Cannot extract width directly; However $n_{\pm}$ is k independent, so we take the ratio:

$$\frac{\Gamma_n(k)}{\Gamma_n(0)} = \frac{|\psi_n(\theta=0;\vec{k})|^2}{|\psi_n(\theta=0;\vec{k}=0)|^2}$$

Use large k analytic results for wave function, grows quadratically;

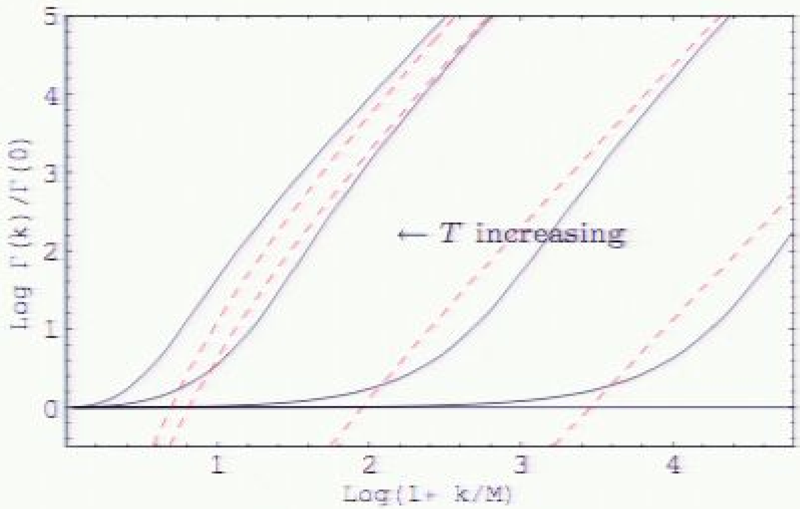
$$\Gamma_n(k)/\Gamma_n(0) = R_n[T/M](k/M)^2 + \mathcal{O}(k)$$

Temperatures $T \ll M$ and for $k \gg \frac{M^3}{T^2}$;

$$\Gamma_n(k)/\Gamma_n(0) \approx \frac{2(4\pi)^4}{(n+2)(n+3/2)}\frac{T^4k^2}{M^6}$$

General argument: wavefunction more peaked at the "tip" for higher momentum.

Momentum dependence of width



- Width turns up at same momentum that the meson group velocity reaches its maximum.
- $\Gamma \sim T^4 k^2/M^6$ consistent with semiclassical string analysis of screening length.

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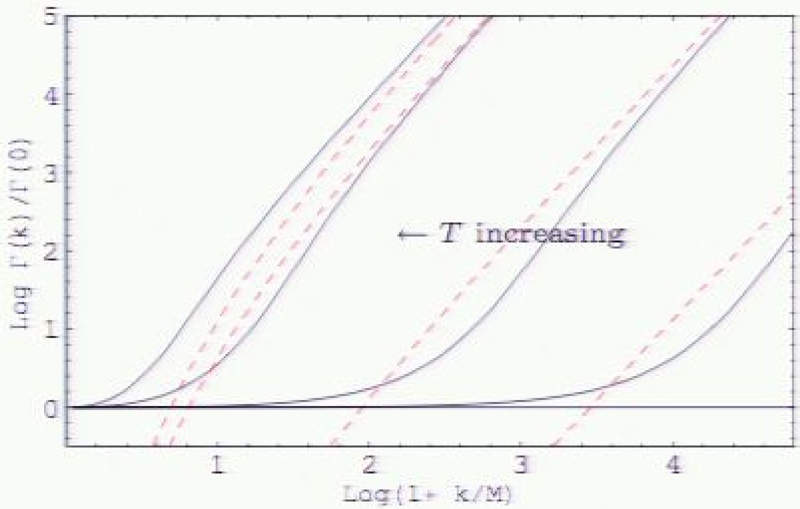

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Width Calculation

$$\partial_\alpha \left(\sqrt{-g}\partial^\alpha \chi^\phi\right) - m_\phi^2 \sqrt{-g}\chi^\phi = -\frac{R^2 n_\pm \delta(\theta)}{\mu_7 \pi \alpha' \beta^2} \int dt'\, \tilde{G}_R(t-t')\chi^\phi(t')$$

Momentum space ($l = 0$):

$$\hat{H}(\vec{k}, l=0)\psi - \frac{i\omega n_\pm}{4\pi^3 \alpha' \mu_7 A}\delta(\theta)\psi(\theta) = \omega^2 \psi(\theta)$$

Perturbation theory problem with a delta function! Write $\omega = \omega_n - \frac{i}{2}\Gamma_n$ find,

$$\Gamma_n^{\pm 1} = \frac{32\pi^3 \sqrt{\lambda}}{N_c m_q^2} |\psi_n(\theta = 0)|^2 n_\pm$$

Recall $n_\pm \propto N_c$ so $\Gamma \sim \mathcal{O}(1)$ in N_c. Incredibly simple result.

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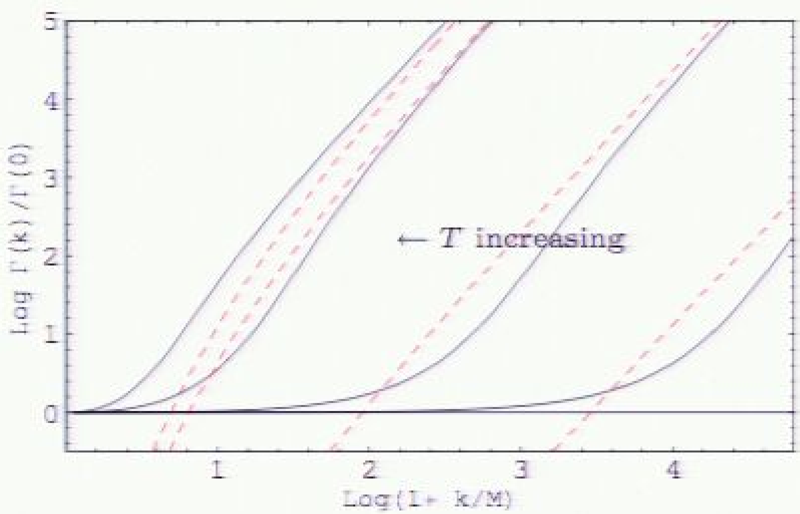

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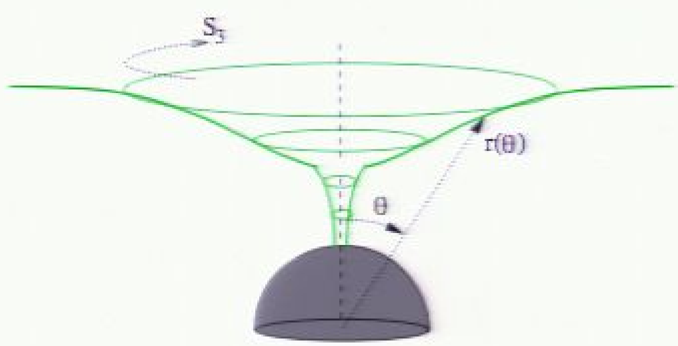
Comparison to phase with $\mu > m_q^{(T)}$ *Kobayashi, Mateos, Matsuura, Myers, Thomson; Erdmenger, Kaminski, Rust; Myers, Sinha*

For $\mu > m_q^{(T)}$ a new phase is thermodynamically favored where D7 branes fall into horizon through a narrow neck.



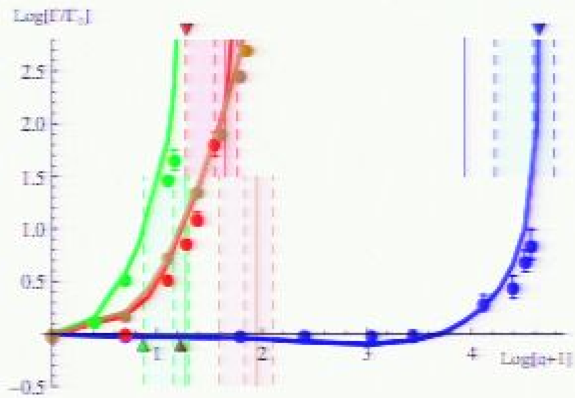
Neck looks like a bundle of strings.

Width of neck $w \propto n_q^{1/2}$. For small quark densities such that $w < \alpha'^{1/2}$, DBI description bad. For $w \ll \alpha'^{1/2}$, instantons (strings) are the better description.

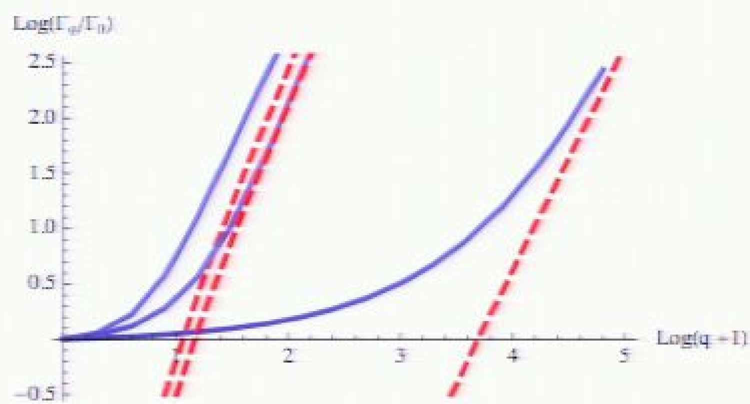
Comparison to phase with $\mu > m_q^{(T)}$

Meson spectrum is now continuous. Width as a function of momentum for this phase has been calculated;

Myers, Sinha



$(\mu > m_q^{(T)})$



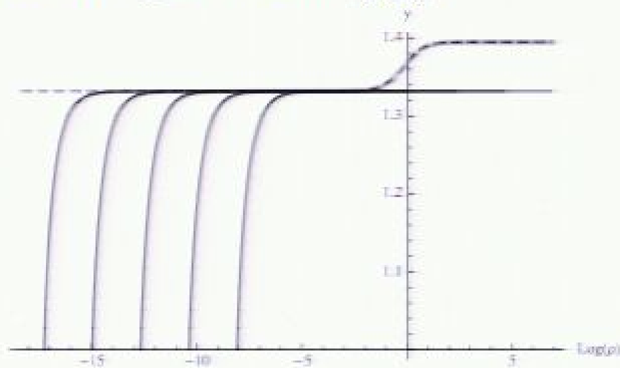
$(\mu < m_q^{(T)})$

Phase Diagram - Redux

(see previous work by: *Mateos, Matsuura, Myers, Thomson; Ghoroku, Ishihara, Nakamurai; Nakamura, Seo, Sin, Yogendran*)

Small density analysis *TF, Hong Liu* to appear.

Embeding becomes singular as $n_q \to 0$. The limit $n_q \to 0$ is not uniform for the embeding functions $y(\rho)$:



Split into regions:

- **Inner** $y(\rho) = Y_0(\sigma) + \mathcal{O}(n_q)$ with $\rho = n_q^{1/2}\sigma$
- **Outer** $y(\rho) = y_0(\rho) + \mathcal{O}(n_q)$

Width: study χ_ϕ fluctuations (to leading order in n_q) in two regions; match; find exact same result as instanton calculation!

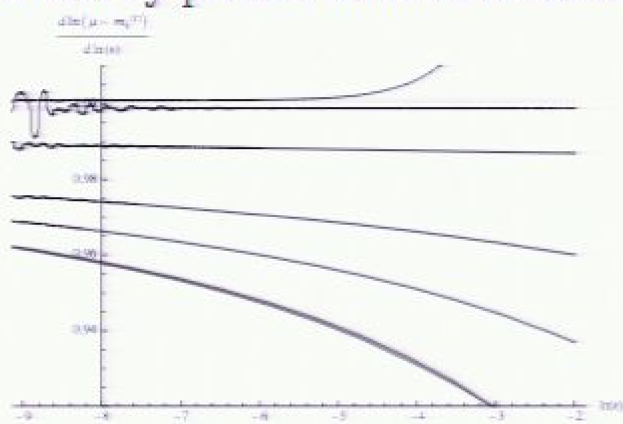
Small density analysis - Phase diagram

Using the two *regions* we compute perturbatively in n_q thermodynamic quantities such as the *chemical potential*:

$$\mu = m_q^{(T)} - B(T) n_q \log n_q + A(T) n_q + O(n_q^2)$$

Curiously: the $n_q \log(n_q)$ comes from canceling of log divergences between the two regions.

This form is confirmed by precise numerical studies:



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Small density analysis - Phase diagram

Using this expansion we can show there is a third order phase transition (in the grand canonical ensemble) as $\mu \to m_q^{(T)}$ and $n_q \to 0$:

- for $\mu < m_q^{(T)}$ we only have Minkowski embedings which are independent of μ:

$$P(T, \mu) = P_0(T)$$

- for $\mu > m_q^{(T)}$ we have:

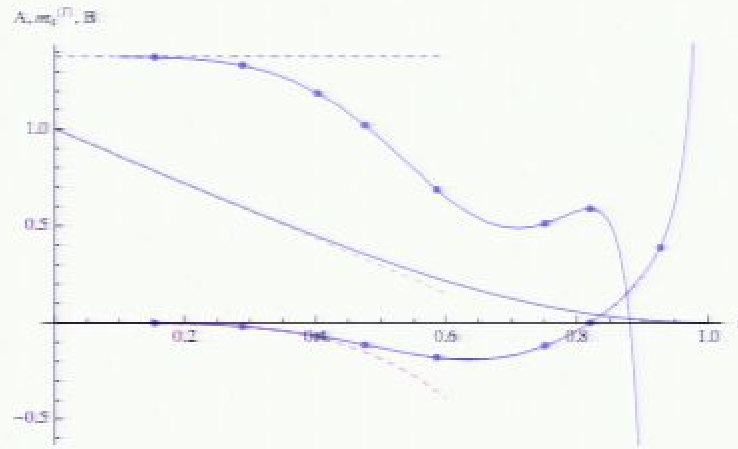
$$\frac{\partial^2 P}{\partial \mu_q^2} \propto \left(\frac{\partial \mu_q}{\partial n_q}\right)^{-1} = \frac{1}{A - B - B \log n_q} \to 0$$

$$\frac{\partial^3 P}{\partial \mu_q^3} \propto -\frac{\partial^2 \mu_q}{\partial n_q^2} \left(\frac{\partial \mu_q}{\partial n_q}\right)^{-2} = \frac{B}{n_q} \frac{1}{(A - B - B \log n_q)^2} \to \infty$$

Small density analysis - Phase diagram

Becomes 2nd order for $B(T) = 0$ and thermodynamically unstable for small ϵ when $B(T) < 0$ (susceptability is negative).

The functions $A(T)$ and $B(T)$ can be evaluated;



Where η is some messy function of T.

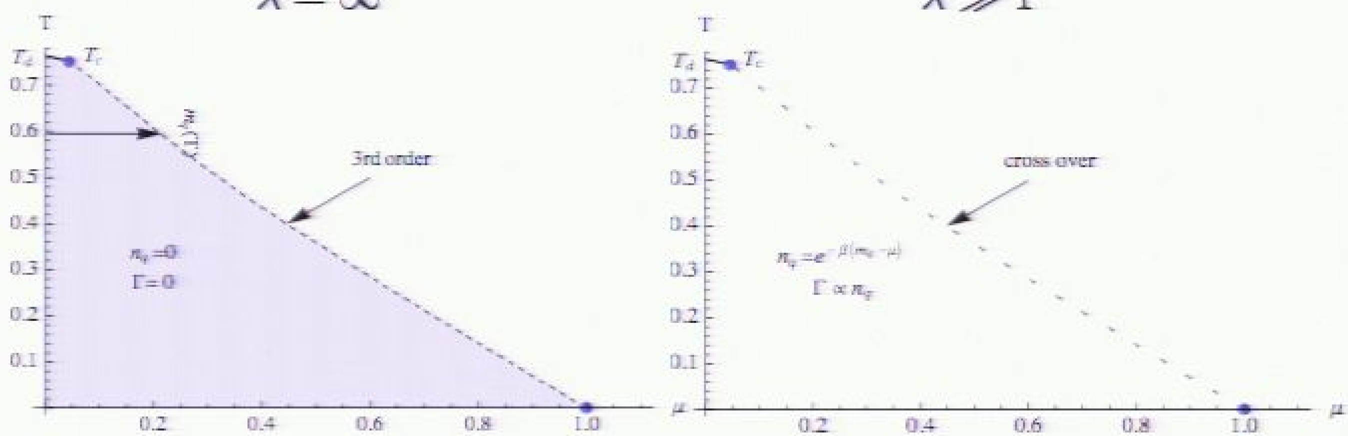
Note: $B(T) = 0$ at two points. $T = 0$ (see *Karch, O'Bannon*) and some $T = T_c$.

Phase diagram of D3/D7 system

TF, Hong Liu in progress.

New picture of the phase structure at finite λ,

$\lambda = \infty$ $\qquad\qquad$ $\lambda \gg 1$



Do these non perturbative effects smooth out the transition between these two phases?

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Conclusions/Open questions

- studied dynamical properties of mesons via AdS/CFT
- characterized the limiting velocity at high momentum
- found a width via a nonperturbative correction, width grows like k^2 for large k.
- do these features apply to QCD?
- gluon contribution to breakup of the meson. Does this also grow with k?
- Nature of the phase transition, $\mu \to m_q^{(T)}$

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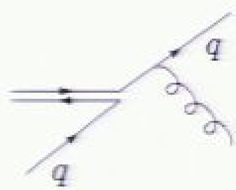
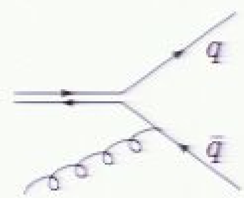
Interpretation

- String theory perspective: open string modes living on a D brane which lies outside a black hole can *tunnel* into the black hole.
- Field theory perspective:

$\Gamma \sim e^{-\beta m_q^{(T)}}$





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